Purchasing Power Parity Puzzle as a Trade Phenomenon

by

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A THESIS SUBMITTED IN PARTIAL FULFILMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

in

The Faculty of Graduate Studies

(Economics)

THE UNIVERSITY OF BRITISH COLUMBIA

May 2005

Abstract

The purchasing power parity puzzle is among the central issues of international macroeconomics. In my thesis, I document and explain the persistence and volatility of its empirical counterpart - the real exchange rate - as a trade phenomenon arising from heterogeneous physical characteristics of products and geography. In the first chapter, a general equilibrium model with shipping costs that depend on physical characteristics of goods and distance leads to endogenous tradability of goods. Deviations of prices from parity are sustained as long as they do not exceed the heterogeneous trade frictions. The real exchange rate exhibits deviations whose persistence matches the data and, when quadratic adjustment costs in change of trade volume are added to the model, also the volatility of the real exchange rate deviations matches the data as well. The second chapter studies monthly deviations from the law of one price for a group of 63 goods and services in Canada and USA between 1970 and 2000 and relates them to a separate dataset of price-to-weight and price-to-volume ratios. Threshold estimates are significantly negatively related to the estimates of the price-to-weight ratios and price-to-volume ratios, respectively. Physical characteristics of goods are important empirical determinants of heterogeneous non-linear behavior of deviations of their prices from parity. The third chapter studies the implications for monetary policy in a general equilibrium model where credit crunches occur due to shifts in the distribution of assets among heterogeneous households.

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Acknowledgements

I am greatly indebted to my supervisory committee, Professor Michael B. Devereux and Professor John Helliwell for their excellent guidance and support. I would like to thank Professor Christian Zimmermann, our common work is the foundation of my last essay. I would also like to thank Professor James Nason, Professor Paul Beaudry, Professor Brian Copeland, Professor Patrick Francois, Professor Francisco González, and Professor Henry Siu for helpful suggestions and discussions at various stages of my dissertation. I am also grateful to my classmates, colleagues and friends, Lilia Karnizova, Kristin Løseth, Christoph Schleicher, Geneviève Verdier, Jenny Xu, Jacob Wong and many others for helpful suggestions, discussions and support. I am solely responsible for any errors and misinterpretations.

Summary

The first chapter of my thesis shows how modelling of physical characteristics of goods and geography can explain the puzzling persistence and volatility in the deviations of the prices from parity. Consequently, it can also explain the purchasing power parity puzzle. In a two-country, three-good general equilibrium model, arbitrage firms trade goods across borders. Shipping costs depend on distance and physical characteristics of the product (weight or volume). Tradability of goods is endogenous - goods are traded only when the difference in their price from parity exceeds the shipping cost. The adjustment of prices across borders is non-linear, with heterogeneous thresholds that depend positively on the physical characteristic of a product and the distance an empirical regularity. Aggregation of the law of one price deviations implies a smooth threshold non-linearity in the real exchange rate, justifying a reoccurring finding in the recent empirical literature. When the stochastic endowments process is calibrated to match the quarterly HP-filtered US and EU GDPs, and trade costs equal on average less than 1% of good's value, the half-life of deviation in the real exchange rate matches the persistence found in the data. A model with quadratic adjustment costs in change of trade volume also matches real exchange rate volatility. The PPP puzzle can thus be explained entirely as a trade phenomenon.

The second paper analyzes the law of one price deviations across 63 groups of goods and services between the US and Canada and combines them with a separate dataset of physical characteristics of products (weight and volume). The estimation and testing of linear- and threshold-autoregressive models on monthly data (1970-2000) shows that the adjustment of prices of goods and services is often non-linear: there are heterogeneous no-trade bands within which deviations of prices from parity do not revert towards the mean. The estimates of the band size vary from 0.5% for footwear to 30% for tobacco. First, in terms of the speed of convergence, services have an average half-life of 200 months while goods have an average half-life of 47 months. Second, one half of the persistence in the real exchange rate deviations can be accounted for by a proper modeling of the prices of tradable goods, in that it is contained within the aforementioned thresholds.

Third, there is a large heterogeneity across goods and services in the dynamics of the law of one price deviations. Threshold estimates of tradable goods are significantly negatively related their price-to-weight ratios, as well as to price-to-volume ratios. Less bulky goods (relative to price) goods have narrower no-trade bounds which is why their price adjusts faster to a shock to the law of one price deviation. A similar relationship is found between the heterogeneous half lives and price-to-weight ratios. The paper establishes stylized facts about the heterogeneity in the price adjustment across goods and finds a significant empirical support for the model from the first chapter of my dissertation.

The third paper studies the loan activity in a context where banks have to follow Basle Accord type rules and need to find financing from households. Loan activity typically decreases when investment returns of entrepreneurs decline, and we study policies that may address this event in a general equilibrium model with heterogeneous agents. Active capital requirement policy can be effective as well if it implies tightening of regulation in bad times. Basle Accord rules are more powerful in this respect. We also show that an identification of a credit crunch may be difficult even when banks are clearly tightening credit, due to somewhat counterintuitive behaviour of some indicators. The paper also develops new solution methods for transition paths in dynamic heterogeneous agent models with aggregate shocks.

Chapter 1

General Equilibrium Model of Arbitrage Trade and Real Exchange Rate Persistence

1.1 Introduction

Why do prices of tradable goods between countries diverge in such a persistent manner? How do physical characteristics of products and the geographical distances influence the deviations of good prices from the law of one price? Can this explain the persistent purchasing power parity puzzle? This chapter aims to answer these questions.

The concept of purchasing power parity (PPP) maintains that national price levels should be equal when expressed in the units of a common currency (Cassel (1918)). Translated into observables, it states that the real exchange rate (a ratio of price indexes in two countries expressed in terms of a single currency) should be constant. The central puzzle in the international business cycle literature is that fluctuations in the real exchange rate are very large and very persistent. Traditionally, attempts to address this puzzle were based on the Harrod-Balassa-Samuelson objection to PPP, utilizing the distinction between traded and non-traded goods (Balassa (1961)). The real exchange rate then equals the relative price of non-traded goods to traded goods. However, these models were shown to be empirically unwarranted for developed economies[1]. Most notably, Engel (1999) shows that in the U.S. data, no more than 2% of the variation in the real exchange rate can be attributed to the fluctuations in the relative price of non-traded to traded goods. Dozens of empirical studies document large, volatile and persistent deviations in the prices of *traded goods* across countries.

Therefore, deviations in the prices of traded goods are the empirically relevant cornerstone of

[1] Harrod-Balassa-Samuelson proposition holds holds better for emerging and developing economies, and at lower frequencies. See, i.a., Choudhri & Khan (2004)

the current theoretical approaches[2]. Several avenues have been explored to motivate the deviations of prices of traded goods from the law of one price. Pricing to market combined with nominal rigidities has been used widely in creating volatile deviations in the real exchange rate (Betts & Devereux (2000), Bergin & Feenstra (2001)). In particular, a year-long price stickiness combined with a low degree of intertemporal elasticity of substitution and consumption - leisure separable preferences generates sufficient volatility but not sufficient persistence in the real exchange rate (Devereux (1997), Chari, Kehoe & McGrattan (2002)). A distribution costs approach (e.g., Corsetti & Dedola (2002), Burstein, Neves & Rebello (2001)) justifies wedges between the prices of tradable goods but has to rely on very large costs to product distribution to match the volatility of the real exchange rate. Differences in preferences across countries have also been used to rationalize deviations from the law of one price (e.g., Lapham & Vigneault (2001)) but must resort to volatile and highly persistent shocks to the preference substitution parameters in order to match the observed fluctuations in the prices of traded goods. Finally, models of the costs of arbitrage trade were relatively unsuccessful in generating deviations from the law of one price (e.g., Obstfeld & Rogoff (2000), Dumas (1992), Ohanian & Stockman (1997), Prakash & Taylor (1997), Sercu, Uppal & van Hulle (1995)). This paper fits into the last strand of literature.

Recent empirical literature stresses the finding that the law of one price deviations behave in a non-linear and heterogeneous way (e.g., O'Connel & Wei (2002), Crucini, Telmer & Zachariadis (2001)). The causal relationship of this heterogeneity has not been understood so far. In the second paper of my dissertation I use post-Bretton Woods monthly price series for 63 traded goods in the U.S. and Canada and a separate dataset of product weights and volumes to show that the thresholds and half-lives in the deviations of the law of one price are significantly positively related to product weights and volumes and are also negatively related to the price-to-weight and price-to-volume ratios. Understanding of the thresholds is important for the understanding of the real exchange rate because they alone account for about 50% of the persistence in the real exchange rate.

The two general equilibrium models presented in this chapter show how geography and physical characteristics of goods explain the persistence in the deviations of good prices – and the real exchange rate – from parity. In a world inhabited by identical consumers, three consumption

[2] Real exchange rate differences can also result from differences in consumption expenditure weights. Given that the RER is very volatile and highly persistent, this modeling strategy alone may not be ideal.

goods are traded across borders for arbitrage purposes. The goods only differ by their weight (a physical mass). The trade is carried out by arbitrage trading firms who decide on the timing and magnitude of trade in order to maximize their profits. Arbitrage revenue depends on law of one price deviations, and the volume of trade. Arbitrage costs in the first model depend on the distance between the countries and some physical characteristic of the shipment (e.g., weight of the product). When a profitable arbitrage opportunity arises, firms engage in trade until prices adjust, so that no viable arbitrage opportunities remain. All trade takes place within the period. Arbitrage costs in the second model depend on all of the above characteristics and also include quadratic adjustment costs in the volume of trade. Arbitrage trading firms find large changes in the volume of trade costly due to needed adjustment to legal contracts, infrastructure, etc. Consequently, firms choose to adjust the amount of goods more gradually following changes in their environment, which leads to more volatile behaviour of price differences.

Prices of goods at Home and Abroad can deviate from parity as long as their difference does not exceed marginal costs of trade. In this sense, arbitrage costs create symmetric thresholds: for law of one price deviations below the thresholds, no trade occurs and prices deviate freely. Prices differentials in excess of the thresholds are immediately arbitraged away. The first implication of this is that the tradability of goods is determined endogenously. The second implication is that prices of tradable goods behave in a non-linear manner. Third, the non-linearity depends on product weights and distance. Non-linearity is a result of the thresholds, which delimit a maximal deviation in the law of one price for every traded good. Thresholds are heterogeneous across goods and increase in the weight of a good and the distance of shipment. Heavier and more distant goods require a larger deviation in the law of one price in order to become traded. In a stochastic environment, there is a concave relationship between the volatility of the law of one price deviations and the volatility of the endowment shock process. Large endowment shocks push the law of one price deviations above the thresholds, triggering immediate arbitrage.

Physical characteristics of goods are not the only candidate source of heterogeneity in estimates of no-arbitrage thresholds. As has been highlighted by other studies (Obstfeld & Taylor (1997), Zussman (2002), etc.), such thresholds tend to be significantly related to import taxes, distance between countries and nominal exchange rate volatility. In the spirit of the border effect literature (Helliwell (1998)), other potential candidates for heterogeneity in threshold estimates may include home bias in preferences, some degree of domestic networking for certain types of goods (primarily non-homogeneous), as well as distribution costs.

The logarithm of a real exchange rate in this model is approximately a weighted average of the logs of the law of one price deviations. It therefore inherits the volatility and persistence of its components and some of their non-linear behaviour. The non-linearity in the real exchange rate is not of an on-off switch type present in its components. Instead, it is akin to a string: for large deviations of RER from parity, thresholds of many goods are crossed, creating a relatively strong mean-reverting tendency. This leads to a smooth threshold non-linearity in the real exchange rate, justifying a reoccurring finding in the recent empirical literature (e.g., Taylor, Peel & Sarno (2001), Kilian & Taylor (2003)). When the stochastic endowments follow an AR(1) process, a simple arbitrage trade model without nominal variables and real rigidities can generate arbitrarily large half lives of the real exchange rate deviations. Real exchange rate persistence depends positively on the size of the trade friction, negatively on the volatility of the endowment shocks and positively on the persistence of the endowment process. Volatility of the real exchange rate depends positively on all three of the above factors.

When both foreign and home countries are subject to endowment shocks, the correlation of the two shock processes influences the stochastic properties of the real exchange rate. Estimates of the half lives of deviations in the real exchange rate increase in the shock correlation parameter[3] while the standard deviation of the real exchange rate decreases monotonically. When the shocks are sector-specific, their negative correlation has a minimal impact on volatility and persistence of the real exchange rate. For positive values of the shock correlation, the half life of deviations in the real exchange rate is strongly positively related to the correlation coefficient.

The stochastic endowment process at Home is calibrated to match the autoregressive root in the post-Bretton Woods quarterly U.S. GDP series and the endowment process Abroad to match the correlation between U.S. and EU GDP data over that period[4]. The physical weights of the goods are calibrated using the dataset from the second chapter of my dissertation. The parameters of the symmetric preferences are calibrated according to literature standards. The model simulation matches exactly the persistence of the real exchange rate found in the data. It

[3] This relationship is not monotonic, it reverses for correlation values near 1. See section 1.4.2.

[4] The calibration is taken from Chari, Kehoe & McGrattan (2002) who use France, Italy, UK and Germany in their EU data.

also generates a quantitatively close match to the persistence and co-movements of various price- and quantity- constructs. However, the model fails to create sufficiently volatile prices.

The difficulty in generating high real exchange rate volatility in the model with linear trade costs is that, when the model is realistically calibrated, arbitrage is triggered for relatively small movements in the real exchange rate. In response to this, the model is extended to incorporate quadratic adjustment costs in the volume of trade. The second model differs from the first only in the structure of the costs faced by the arbitrage trading firms. In addition to the shipping costs which depend on the distance and physical characteristics of goods, firms also have to face quadratic adjustment costs in the volume of trade. Sudden changes in the volume of trade are costly because they require additional legal and infrastructure costs, such as costs of establishing new (or changing existing) business relationships and distribution networks.

Introducing a quadratic adjustment cost into the decision of the firm leads to a dynamic and highly non-linear model but improves the results. An additional friction preserves the core feature of the linear shipping cost model – the heterogeneous trade frictions – while creating a dynamic environment from the point of view of the firm. Quadratic adjustment costs lead to larger and more long-lived deviations of law of one price for traded goods in equilibrium, because firms are more careful in reacting to changes in endowments by large adjustments in the volume of trade. Consequently, the arbitrage trade between countries and co-movement between their consumptions is limited. A simulation of the model matches both the persistence and volatility of the real exchange rate while giving qualitatively meaningful results along other dimensions. Although a more extensive quantitative investigation should be done in the future, it appears that both the persistence and volatility sides of the purchasing power parity puzzle can be explained by careful modeling of trade costs.

We know that prices, like trade volumes, exhibit an apparent "border effect" anomaly when comparing equidistant locations. When border exists between locations, price volatility and persistence appears to be much higher than for equidistant locations within the same country (see, i.a., Engel & Rogers (1996) and Jenkins & Rogers (1995)). Although the model in this chapter does not distinguish transport costs between locations within the same country and across countries, it would be easy to create such environment by including more than two locations. A tariff could be introduced between the countries that would apply to shipments of all 3 goods. As well, adjustment costs could differ between countries due to additional language, legal, culture and other barriers. Such addition could potentially explain the Engel & Rogers (1996) findings. The empirical framework of the second chapter (threshold non-linear AR estimations) could be used on city data to establish whether (by how much) the border affects the threshold estimates.

More generally, an extension of the model into a multi location (country) framework could improve our understanding of trade flows, bilateral exchange rate persistence and volatility and perhaps other multilateral economic phenomena. It is in particular possible to envision an extension of the model in which trade technology exhibits increasing returns to scale which could, in multi-location framework, help explain the "missing trade" puzzle. It may also be feasible to link the trade costs to development: should country A be producing goods that are more costly to transport than goods produced by country B, subset of prices in country A will be more isolated from some "world shocks" leading to lower integration into the world economy. If we assume that price response is desirable and that country A is poorer than country B (both of which are not unlikely), such a version of the model could be understood as a trade-related source of a development trap. On the empirical side, a multi country estimation framework could be used to analyze the persistence and volatility of price deviations for bilateral pairs of goods across many countries (in the same way Engel and Rogers (1996) do for city data using linear models).

The paper is structured as follows. Section 1.2 exposes the general equilibrium model of arbitrage trade with linear heterogeneous shipping costs and analyzes its equilibrium properties. Section 1.3 analyzes the properties of the arbitrage trade model that also has quadratic adjustment costs in trade volume. Section 1.4 constructs and analyzes stochastic properties of the real exchange rate in a variety of environments. Section 1.5 discusses parameter calibration. Section 1.6 analyzes persistence, comovement and volatility of the real exchange rate and other variables. Section 1.7 concludes.

1.2 General equilibrium model of arbitrage trade

The two-country world consists of households and arbitrage trading firms. Each country is endowed with positive amounts of three tradable goods. Goods differ in their physical characteristics, proxied here by their weight. The trade is costly because the arbitrage firm has to use resources to ship goods across borders. As shown in the transportation literature, physical characteristics of goods are an important determinant of the shipment costs (e.g., Hummels (1999)). One of the goods is used by the firms as an input in the transportation technology (in addition to being consumed by households). This numeraire good has a zero trade friction[5]. I assume that the cost of shipment is linear in the weight of the product: shipment X tons of any good between any two equidistant locations will cost X times the shipping of one ton of a good[6].

1.2.1 Households

A representative household at Home chooses its consumption path to maximize an instantaneous CES utility function subject to a resource budget constraint:

$$\max_{C_{1t},C_{2t},C_{3t}} \sum_{t=1}^{\infty} \beta^t \left\{ \frac{1}{1-\theta} \left[\gamma_1^{\frac{1}{\theta}} C_{1t}^{1-\frac{1}{\theta}} + \gamma_2^{\frac{1}{\theta}} C_{2t}^{1-\frac{1}{\theta}} + \gamma_3^{\frac{1}{\theta}} C_{3t}^{1-\frac{1}{\theta}} \right]^{1-\theta} \right\}$$

$$\text{s.t.} \qquad p_{1t}C_{1t} + p_{2t}C_{2t} + C_{3t} = p_{1t}Y_{1t} + p_{2t}Y_{2t} + Y_{3t} + \frac{1}{2}AP_t \tag{1.1}$$

given AP_t and Y_{it}, $i = \{1, 2\}$, where Y_{it} is an endowment of good i at time t, $\sum_{i=1}^{3} \gamma_i = 1$, $\theta > 1$, prices $p_{it}, i = 1, 2$ are relative prices denoted in the units of the local currency and AP_t is the amount of current-period arbitrage profits transferred to the household from a firm, assuming an equal splitting rule between households at home and abroad. The first order conditions for this problem imply the usual demand functions:

$$C_{1t} = \gamma_1 p_{1t}^{-\theta} \frac{Y_t}{P_t^{1-\theta}} \tag{1.2}$$

$$C_{2t} = \gamma_2 p_{2t}^{-\theta} \frac{Y_t}{P_t^{1-\theta}} \tag{1.3}$$

$$C_{3t} = \gamma_3 \frac{Y_t}{P_t^{1-\theta}} \tag{1.4}$$

where Y_t is home country's real GDP measured in the units of good 3 ($Y_t = p_{1t}Y_{1t} + p_{2t}Y_{2t} + Y_{3t} + \frac{1}{2}AP_t$) and P_t is a composite price index $P_t = (\gamma_1 p_{1t}^{1-\theta} + \gamma_2 p_{2t}^{1-\theta} + \gamma_3)^{\frac{1}{1-\theta}}$. The problem for the representative household abroad is identical, with prices and quantities denoted with an asterix. Preferences of households at Home and Abroad are identical.

[5] The zero trade friction is an innocuous assumption. A positive friction for good 3 would make the computation more complicated but would not change the nature of the results.

[6] Insurance costs, costs of setting up distribution networks, and other costs are ignored in this specification of the transportation technology. This simplification is for the purpose of expositional clarity and mathematical simplicity.

1.2.2 Arbitrage trading firms

There is a representative arbitrage trading firm in each country. It chooses the time and the amount traded in each good, taking into account the transportation costs. Transportation costs are introduced in a way of a transportation technology, which uses good 3 as an input.

$$\begin{aligned}\max_{N_1,N_2,N_3} A\Pi_t &= \max_{N_1,N_2,N_3} \sum_{t=1}^{\infty} \beta^t AP_t \\ &= \max_{N_1,N_2,N_3} \sum_{t=1}^{\infty} \beta^t \left[\sum_{i=1}^{3} (p_{it}^* - p_{it}) N_{it} - T(N_{1t}, N_{2t}) \right] \end{aligned} \tag{1.5}$$

where N_{it} is the amount of trade in good i ($N > 0$ implies exports from Home to Abroad) and $T(N_{1t}, N_{2t})$ is the cost function of the arbitrage trading firm. An arbitrage firm has to purchase $T(N_1, N_2)$ units of good 3 to trade $\{N_1, N_2\}$. Because the trade in good 3 is costless, N_3 does not enter T(.)[7]. For a start, the cost function takes a simple linear form, accounting only for the shipment cost of N_i units of a good. A more realistic technology would also include fixed costs of arbitrage: legal costs, insurance costs, labor costs, etc. A second model in section 1.3 proxies for these costs of trade by introducing a quadratic adjustment cost in the volume of trade. For now,

$$T(N_{1t}, N_{2t}) = (aw_1|N_{1t}| + aw_2|N_{2t}|) = a(w_1|N_{1t}| + w_2|N_{2t}|)$$

where w_i is the weight of a good i and a is a constant[8]. A ratio of the shipment cost of two goods between the same locations then equals the ratio of their weights. The first order conditions for the arbitrage trading firm yield:

$$\mathrm{I(N)}(p_1^* - p_1) = aw_1 \quad \text{iff } |N_1| > 0 \tag{1.6}$$
$$\mathrm{I(N)}(p_1^* - p_1) < aw_1 \quad \text{iff } N_1 = 0$$
$$\mathrm{I(N)}(p_2^* - p_2) = aw_2 \quad \text{iff } |N_2| > 0 \tag{1.7}$$
$$\mathrm{I(N)}(p_2^* - p_2) < aw_2 \quad \text{iff } N_2 = 0$$
$$p_3^* - p_3 = 0$$

[7]This is an innocuous assumption, one could calculate a more complicated version of the model with a positive trade friction in all three goods. The parameters t_1 and t_2 can be thought of as trade frictions of goods 1 and 2 *relative* to the trade friction of good 3.

[8]Intuitively, a is a constant, per-kilogram fraction of good 3 which disappears when a good is transported between these two particular locations.

where I(N) is an indicator function, such that I(N)= 1 when $N \geq 0$, I(N)= -1 otherwise. The left hand side of the first order conditions is the marginal revenue of arbitrage trade and the right hand side the marginal cost. Trade occurs when the marginal revenue of arbitrage exceeds the marginal cost. The trade brings about price convergence, and stops when all profit opportunities are eliminated. Hence, following the trade, law of one price deviation equals marginal trade friction and the FOC holds with equality. The first order conditions for goods 1 and 2 hold with inequality in an autarky solution when the law of one price deviation does not exceed the marginal costs of trade. The first two pricing equations can be rewritten to show the first-order effect of good heterogeneity:

$$\mathrm{I(N)}\frac{p_i^* - p_i}{w_i} \leq p_3 a, \quad i = 1,2 \tag{1.8}$$

or, more intuitively,

$$-p_3 a \leq \overbrace{\frac{p_i^* - p_i}{w_i}}^{\text{MRA per kg}} \leq \overbrace{p_3 a}^{\text{MCA per kg}} \qquad i = 1,2$$

The left-hand side of equation (1.8) captures the marginal arbitrage revenue per kilogram of good i (MR_i) and the right-hand side the marginal arbitrage cost per kilogram of good i (MC_i). While the right-hand side is identical across goods, the left-hand side is not. Goods that are relatively heavy will need a larger deviation in the law of one price in order for MRA to exceed MCA and trigger arbitrage trade. Denoting $t_1 \equiv aw_1$ and $t_2 \equiv aw_2$, the first order conditions for the firm can be expressed as:

$$\overbrace{|p_1^* - p_1|}^{\text{LOPD}} \leq t_1 \tag{1.9}$$

$$|p_2^* - p_2| \leq t_2 \tag{1.10}$$

$$p_3 - p_3^* = 0 \tag{1.11}$$

The law of one price ensues for good 3 because the trade in it is costless. Equations (1.9) and (1.10) imply there exists a maximum law of one price deviation for goods 1 and 2: $\max(LOPD_i) = t_i$ for $i = 1,2$. Maximum LOPD is proportional to the weight of a product.

This leads to *heterogeneous filtering.* Consider a shock x that affects the law of one price deviations for goods 1 and 2 identically (for example, an identical increase in home endowments of both goods). It follows that the value of such shock can be divided into three subsets. First, for a positive law of one price deviation, $x \in [0, x_1^*)$ leads to autarky because the law of one price deviations for goods 1 and 2 are in a no-trade region ($|LOPD_i| < t_i \iff MR_i < MC_i \;\; i = 1, 2$). Second, for $x \in [x_1^*, x_2^*)$, only the lighter good (thereafter good 1) is traded because while the law of one price deviation for good 1 is initially outside the no-trade region, this is not true for good 2: $|LOPD_1| > t_1 \iff MR_1 > MC_1$, $|LOPD_2| < t_2 \iff MR_2 < MC_2$. Third, for $x \in [x_2*, \infty)$, all goods are traded as their initial law of one price deviations are outside the no-trade region ($|LOPD_i| > t_i \iff MR_i > MC_i \;\; i = 1, 2$). Identical shocks to the law of one price deviation can have differential effects on the price deviations[9]. Some goods will become traded while others will not. Consequently, prices will adjust in a heterogeneous fashion.

1.2.3 Market clearing

There are no asset markets in this model. All three goods markets clear at home as well as abroad. The direction of trade in goods 1 and 2 depends on the size and sign of the initial deviation from a law of one price, as determined by the endowments of these goods. Because there are only two countries in this model, $N_i \equiv EXP_i = IMP_i^* = -EXP_i^* \equiv -N_i^*$. The market clearing conditions can then be written as follows.

$$C_1 + N_1 = Y_1, \qquad C_1^* - N_1 = Y_1^* \tag{1.12}$$

$$C_2 + N_2 = Y_2, \qquad C_2^* - N_2 = Y_2^* \tag{1.13}$$

$$C_3 + N_3 + \tfrac{1}{2}T(N_1, N_2) = Y_3, \quad C_3^* - N_3 + \frac{1}{2}T(N_1, N_2) = Y_3^* \tag{1.14}$$

1.2.4 Equilibrium

The equilibrium is a set of prices and quantities $\{p_1, p_1^*, p_2, p_2^*, C_1, C_1^*, C_2, C_2^*, C_3, C_3^*, N_1, N_2, N_3\}$ such that the households maximize their utility (equations (1.1)-(1.4)), arbitrage trading firms maximize their profits (equations (1.9) to (1.11)) and markets clear (equations (1.12) - (1.14)).

Frictionless trade

Here I analyze the model with zero transportation costs ($a = 0$), which implies $T(N_1, N_2) = 0 \;\forall\; N_1, N_2$. The profit maximization problem faced by the arbitrage trading firm (eq. (1.5))

[9]In the case 2, p_2 of the heavier good will adjust due to substitution and welfare effects of a change in p_1.

yields a law of one price equality for all goods:

$$p_i^* = p_i \qquad i \in \{1,2\}. \tag{1.15}$$

The equilibrium relative prices then depend on the world endowments and the preference parameters:

$$\frac{p_i}{p_j} = \frac{p_i^*}{p_j^*} = \left[\frac{Y_i^W}{Y_j^W}\frac{\gamma_j}{\gamma_i}\right]^{-\frac{1}{\theta}} \quad \forall\, i \tag{1.16}$$

where $Y_i^W \equiv Y_i + Y_i^*$. The equilibrium consumption levels are

$$C_1 = Y_1 \frac{\left(\frac{\gamma_1}{\gamma_2}\right)^{\frac{1}{\theta}}\left(\frac{Y_2^W}{Y_1^W}\right)^{\frac{1}{\theta}} + \frac{Y_2}{Y_1}}{\left(\frac{\gamma_1}{\gamma_2}\right)^{\frac{1}{\theta}}\left(\frac{Y_2^W}{Y_1^W}\right)^{\frac{1}{\theta}} + \frac{Y_2^W}{Y_1^W}}$$

$$C_2 = Y_1 \frac{Y_2^W}{Y_1^W} \frac{\left(\frac{\gamma_1}{\gamma_2}\right)^{\frac{1}{\theta}}\left(\frac{Y_2^W}{Y_1^W}\right)^{\frac{1}{\theta}} + \frac{Y_2}{Y_1}}{\left(\frac{\gamma_1}{\gamma_2}\right)^{\frac{1}{\theta}}\left(\frac{Y_2^W}{Y_1^W}\right)^{\frac{1}{\theta}} + \frac{Y_2^W}{Y_1^W}}$$

similarly for C_1^* and C_2^* when all Y_i in equations (1.17) and (1.17) are changed for Y_i^*. $C_i = Y_i$, $C_i^* = Y_i^*$ iff $\frac{Y_1}{Y_2} = \frac{Y_1^*}{Y_2^*}$. The usual comparative advantage argument ensues: the country that is endowed with a *relatively* larger amount of good i will export good i and import good j.

Equilibrium with positive trade frictions

When borders open to trade, three cases can arise. First, if endowments are such that the autarky prices do not bring the law of one price deviation above the marginal costs (i.e., (1.9) and (1.10) hold with inequality), neither of the goods 1 and 2 is traded. Second, if the endowments lead to autarky prices which exceed the marginal cost of arbitrage for one good but not the other, trade occurs in the good with a relatively smaller friction ((1.9) holds with equality) while the good with a relatively larger friction is not traded ((1.10) holds with inequality). Finally, for endowments which lead to autarky prices such that the law of one price exceeds MC_i $\forall i \in \{1,2\}$, both goods are traded. In this subsection I study the properties of the equilibrium in all three of these cases.

Case 1: No trade in goods 1 & 2 When the autarky prices $\{p_{1a}, p_{1a}^*, p_{2a}, p_{2a}^*\}$ are such that the law of one price deviation stays inside of the no-trade region for both good 1 and 2, pricing expressions (1.9) and (1.10) hold with inequalities. $N_1 = N_2 = 0$ in this equilibrium. Using the law of one price equation (1.11) the system can be written as:

$$\gamma_1 p_1^{-\theta} \frac{Y}{P^{1-\theta}} = Y_1, \qquad \gamma_1 p_1^{*-\theta} \frac{Y^*}{P^{*1-\theta}} = Y_1^*$$
$$\gamma_2 p_2^{-\theta} \frac{Y}{P^{1-\theta}} = Y_2, \qquad \gamma_2 p_2^{*-\theta} \frac{Y^*}{P^{*1-\theta}} = Y_2^*$$
$$\gamma_3 \left(\frac{Y}{P^{1-\theta}} + \frac{Y^*}{P^{*1-\theta}} \right) = Y_3 + Y_3^*$$

where $Y = p_1 Y_1 + p_2 Y_2 + Y_3$, $Y^* = p_1^* Y_1^* + p_2 Y_2^* + Y_3^*$, $P = (\gamma_1 p_1^{1-\theta} + \gamma_2 p_2^{1-\theta} + \gamma_3)^{1/(1-\theta)}$ and $P^* = (\gamma_1 p_1^{*1-\theta} + \gamma_2 p_2^{*1-\theta} + \gamma_3)^{1/(1-\theta)}$. Due to Walras' law, this system can be uniquely solved for *relative* prices $\{p_1, p_2, p_1^*, p_2^*\}$, which recursively define all other equilibrium values.

Case 2: No trade in good 2 When $\{p_{1a}, p_{1a}^*, p_{2a}, p_{2a}^*\}$ are such that the law of one price deviation stays inside of the no-trade region for good 2 but exceeds the marginal costs of arbitrage for good 1, pricing expression (1.9) holds with an equality while (1.10) hold with an inequality. The relative price of good 2 is determined in the local markets while prices for goods 1 and 3 are determined internationally. The equilibrium is then constructed as defined above with $N_2 = 0$:

$$\begin{aligned}
\gamma_1 \left(p_1^* - I(N_1)t_1\right)^{-\theta} \frac{Y}{P^{1-\theta}} + \gamma_1 p_1^{*-\theta} \frac{Y^*}{P^{*1-\theta}} &= Y_1 + Y_1^* \\
\gamma_2 p_2^{*-\theta} \frac{Y}{P^{1-\theta}} &= Y_2 \\
\gamma_2 p_2^{*-\theta} \frac{Y^*}{P^{*1-\theta}} &= Y_2^* \\
\gamma_3 \frac{Y}{P^{1-\theta}} + \gamma_3 \frac{Y^*}{P^{*1-\theta}} + t_1 \left[Y_1 - \gamma_1 \left(p_1^* - I(N_1)t_1\right)^{-\theta} \frac{Y}{P^{1-\theta}} \right] &= Y_3 + Y_3^*
\end{aligned}$$

where $Y = (p_1^* - I(N_1)t_1)Y_1 + p_2 Y_2 + Y_3)$ and $P = (\gamma_1(p_1^* - I(N_1)t_1)^{1-\theta} + \gamma_2(p_2)^{1-\theta} + \gamma_3)^{1/(1-\theta)}$. Due to Walras' law, this system uniquely determines $\{p_1^*, p_2^*, p_2\}$ and consequently all other equilibrium values as functions of preferences, endowments, and the trade friction t_1.

Case 3: All goods traded Finally, when the law of one price deviations for both goods under autarky prices exceed their respective marginal costs of arbitrage, both goods are traded in equilibrium. The equilibrium prices then solve a reduced system

$$\begin{aligned}
\left(p_1^* - I(N)t_1\right)^{-\theta} \frac{Y}{P^{1-\theta}} + p_1^{*-\theta} \frac{Y^*}{P^{*1-\theta}} &= \frac{1}{\gamma_1} \left(Y_1 + Y_1^*\right) \\
\left(p_2^* - I(N)t_2\right)^{-\theta} \frac{Y}{P^{1-\theta}} + p_2^{*-\theta} \frac{Y^*}{P^{*1-\theta}} &= \frac{1}{\gamma_2} \left(Y_2 + Y_2^*\right)
\end{aligned}$$

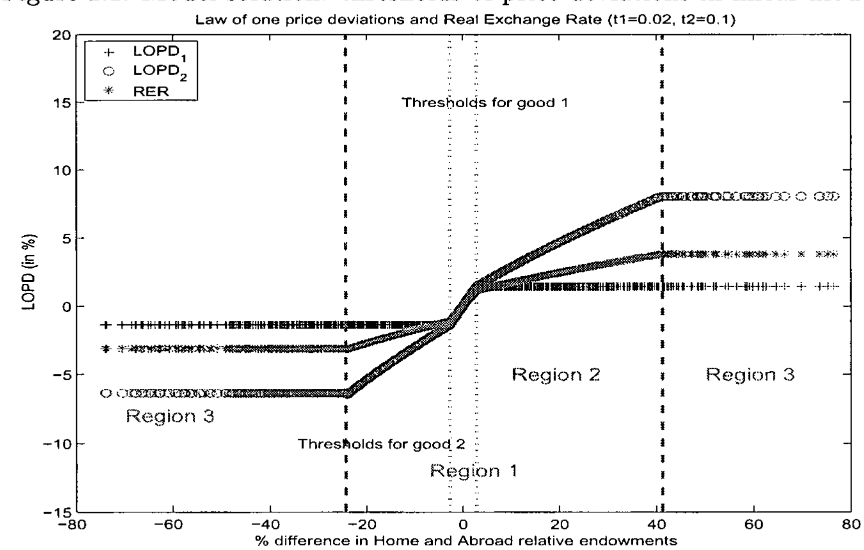
$$\gamma_3 \frac{Y}{P^{1-\theta}} + \gamma_3 \frac{Y^*}{P^{*1-\theta}} + t_1 \left[Y_1 - \gamma_1 \left(p_1^* - I(N_1)t_1\right)^{-\theta} \frac{Y}{P^{1-\theta}} \right] \quad = \quad Y_3 + Y_3^*$$

where $Y = (p_1^* - I(N_1)t_1)Y_1 + (p_2^* - I(N_2)t_2)Y_2 + Y_3)$ and $P = (\gamma_1(p_1^* - I(N_1)t_1)^{1-\theta} + \gamma_2(p_2 - I(N_2)t_2)^{1-\theta} + \gamma_3)^{1/(1-\theta)}$. Walras' law reduces the above system into two equations that solve uniquely for $\{p_1^*, p_2^*\}$ and implicitly all other variables as functions of endowments, preferences, and the trade frictions t_1, t_2.

Properties of the equilibrium

The trade frictions affect the equilibrium prices and allocations in all three cases described above. The equilibrium values in case 3 depend directly on t_1 and t_2, and all equilibrium values in case 2 depend directly on t_1. However, t_1 and t_2 also define the set of endowments for which the autarky (case 1 applies). Likewise, t_2 defines the set of endowments for which only good 1 is traded (case 2).

Figure 1.1: Model solution: thresholds of price deviations in linear model



To understand the properties of the equilibrium, I solve the model numerically and analyze its properties across a range of values for relative endowments. Keeping the endowments Abroad fixed, I vary the size of the Home endowments of goods 1 and 2 by the same amount. This is the simplest way to model an environmental change in this model and it is chosen to clearly illustrate its properties. Figure 1.1 plots the law of one price deviations from the solution of the model and Figure A.3 plots the volume of trade. Depending on the level of the relative endowment (shown as a percentage change from the initial endowment), deviations from the law of one price fall into one of the three regions. In region 1 with deviations in the law of one price below the trade friction, no trade takes place. This corresponds to the case when both expressions (1.9) and (1.10) hold with inequalities. Marginal costs of trade are higher than marginal revenues for any volume of trade, therefore no trade takes place.

When endowments exceed the first set of thresholds, trade occurs in good 1 but good 2 remains non-traded (region 2). Trade brings about reversion in the price differential: in the case of exports, price at Home rises while price Abroad falls and vice versa in the case of imports. After all arbitrage trade opportunities are extinguished, the law of one price deviation for good 1 reverses back to the threshold. Good 2 remains non-traded and its LOPD increases. Notice that the increase in the deviation from the law of one price for good 2 is smaller in region 2 than in region 1, although good 2 is non-traded in both cases. There are two reasons why the slope of the LOPD_2 is less in region 2 than in region 1. First, as good 3 vanishes in transportation, its price increases (hence a small relative decrease in p_2). Second, households substitute from the more expensive into a cheaper good[10].

Finally, after the shocks exceed the second set of thresholds, both goods become traded, and the law of one price deviation for good 2 reverses towards its threshold. It is clear that the model generates heterogeneity in the thresholds of the law of one price deviations. The effects of thresholds on the persistence and volatility of the law of price deviations will be explored in the subsection 1.4 below.

Next, I run a Monte Carlo simulation to illustrate stochastic properties of the model. Starting with identical endowments of all 3 goods in both countries, I introduce a country-specific shock which alters the balance of endowments of good 1 and 2 at home relative to abroad. The shock affects goods 1 and 2 symmetrically. The relative endowment shock is normally distributed ($\epsilon \sim \mathrm{N}(0, \sigma^2)$) and alters the endowments in the following way: $Y_1 = Y_2 = Y + \epsilon$, $Y_1^* = Y_2^* = Y_3^* = Y_3 = Y$, and changes the autarky relative prices of goods 1 and 2 proportionally at home while keeping them unchanged abroad. Depending on the size of the shock, one of cases 1, 2 or 3

[10] If good 1 is exported, Home buyers substitute from it into good 2 which is now relatively cheaper. Buyers abroad substitute from good 2 into 1. Both imply a lower LOPD_2 relative to its autarky value. If good 1 is imported an inverse of the argument applies.

ensues. I simulate the model 1000 times and record equilibrium allocations after shocks. I repeat this process for a range of values of the standard deviation of the endowment shock σ.

Figure A.4 shows standard deviations and mean price deviations as a function of the variability of the endowment shocks. Price deviations of heavier goods are more larger and more volatile than those of lighter good. Volatility of price deviations is concave in shock volatility. For a range of values of $\sigma \in [1, \hat{\sigma}_i]$ where $\hat{\sigma}_2 > \hat{\sigma}_1$, price volatility rises with shock volatility. For $\sigma > \hat{\sigma}_i$, volatility remains constant as excessive price differences get arbitraged away.

Figure A.5 plots the average thresholds of price deviations for different values of standard deviation of the shock volatility. The thresholds are symmetric, and depend positively on the trade friction (here $t_2 = 5t_1$).

As a robustness check, figures A.6 - A.8 show the behavior of the means, standard deviations and thresholds of price deviations across a range of values of the trade frictions. Because the shipping costs are linear, the properties of the model described above remain qualitatively unchanged for any t_2/t_1[11]. As t_2/t_1 rises so does the maximum of $LOPD_2/LOPD_1$ (figure A.6). This is independent of the value of standard deviation of the endowment shock, as long as it sufficiently large to allow good 2 to cross its threshold. Secondly, $STD_{LOPD_2}/STD_{LOPD_1}$ is increasing in t_2/t_1. It appears that this relationship is concave: LOPD variability increases less than the trade friction (panel 2 in figure A.7). Third, the thresholds are proportional to the levels of t_2/t_1 (figure A.8).

1.3 Arbitrage trade model with adjustment costs to trade

The endowment and preference setting of the second model is identical to the first. However, trade costs consist of shipping costs as well as quadratic adjustment costs in the change of trade volume. Changes in the volume of trade from previous to current time period require hiring of labour resources, adjustment in the distribution system and possibly investment in (or a changes in the existing) trade infrastructure. Adjustment of trade to endowment shocks also requires time. Quadratic adjustment costs get at this idea in a smoother way than assuming pre-determined

[11]Please refer to the earlier version of the paper with a partial equilibrium model in which a friction also consists of a fixed cost and a value-related cost (e.g., an insurance cost). It is shown how fixed costs of arbitrage brings about non-linearities to the relationships discussed here.

shipment volume. Consequently, large swings in trade volume are costly. The arbitrage firms' problem can be written as follows:

$$\begin{aligned}
\max_{N_{1t},N_{2t}} A\Pi_t &= \max_{N_{1t},N_{2t}} \sum_{t=1}^{\infty} \beta^{(t-1)} AP_t \\
&= \max_{N_{1t},N_{2t}} \sum_{t=1}^{\infty} \beta^{(t-1)} \left[\sum_{i=1}^{2} (p_{it}^{*} - p_{it}) N_{it} - T(N_{1t}, N_{2t}) \right] \quad (1.17) \\
\text{s.t. } T(N_{1t}, N_{2t}) &= t_1 |N_{1t}| + t_2 |N_{2t}| + c_1 \Delta N_{1t}^2 + c_2 \Delta N_{2t}^2 \quad (1.18)
\end{aligned}$$

where N_{it} is the amount of trade in good i at time t ($N > 0$ implies exports from Home to Abroad) and $T(N_{1t}, N_{2t})$ represents the cost function of the arbitrage trading firm, and p_{it} is the price of good i relative to good 3. An arbitrage firm has to purchase $T(N_{1t}, N_{2t})$ units of good 3 to trade $\{N_{1t}, N_{2t}\}$. Because the trade in good 3 is assumed to be costless, N_{3t} does not enter T(.) [12] and its trade volume is determined according to the market clearing conditions and the preferences of households. The arbitrage trade cost function consists of a shipment cost and adjustment cost component. The shipping cost between two locations is again assumed linear in weight of the good, leading to identical trade frictions $t_i = aw_i,\ i = 1, 2$ where w_i is the weight of good i and a is a constant. The adjustment cost component is quadratic in the change of volume of trade from the previous period to the current period. The quadratic adjustment cost parameters c_1 and c_2 are not related to the physical characteristics of goods.

Summarizing the behaviour of the firm by the first order conditions of its profit maximization decision is challenging because of the non-differentiability of the absolute value function at 0. I use a smooth approximation $G(.)$ to the absolute value function to allow a continuous mapping between the first order conditions and the objective function. Function $g(.) \equiv \mathrm{d}G(.)$ denotes the first order derivative of the "smooth" absolute value function, and is itself a smooth approximation to a step function. See the appendix for details. The first order optimality conditions with respect to goods $i = 1, 2$ yield:

$$\begin{aligned}
0 &= \left\{ (p_{i,t}^{*} - p_{i,t}) - \frac{\partial T(._t)}{\partial N_{i,t}} - \beta \mathrm{E}_t \frac{\partial T(._{t+1})}{\partial N_{i,t+1}} \right\} \\
0 &= p_{i,t}^{*} - p_{i,t} - [t_i \mathrm{I}(N_{i,t}) + 2c_i(N_{i,t} - N_{i,t-1})] - \beta \mathrm{E}_t[-2c_i(N_{i,t+1} - N_{i,t})]
\end{aligned}$$

[12]This is an innocuous assumption, one could calculate a more complicated version of the model with a positive trade friction for trade in each of the three goods. The parameters t_1 and t_2 can be thought of as trade frictions of goods 1 and 2 *relative* to the trade friction of good 3.

Rearranging, we get

$$\begin{aligned}\frac{1}{2c_i}\left[p^*_{i,t}-p_{i,t}-t_i\mathrm{I}(N_{i,t})\right] &= -\beta\mathrm{E}_tN_{i,t+1}+(1+\beta)N_{i,t}-N_{i,t-1}\\ &= (-\beta\mathrm{B}^{-2}+(1+\beta)\mathrm{B}^{-1}-1)\mathrm{E}_tN_{i,t-1}\\ &= [-\beta+(1+\beta)\mathrm{B}-\mathrm{B}^2]\mathrm{E}_tN_{i,t+1}\ \forall i\end{aligned}$$

where B is a backshift operator and $\mathrm{I}(N_{i,t})$ is an approximation to the indicator function, such that $\mathrm{I}(N_{i,t})=1$ when $N_{i,t}>0$, $\mathrm{I}(N_{i,t})=-1$ when $N_{i,t}<0$ and $\mathrm{I}(N_{i,t})=0$ when $N_{i,t}=0$. The quadratic form on the right hand side of the last equality has one stationary and one non-stationary root and can be re-written as $-(\mathrm{B}-1)(\mathrm{B}-\beta)$. The first order conditions for the firm can then be written as

$$-\frac{1}{2c_i}\left[p^*_{i,t}-p_{i,t}-t_i\mathrm{I}(N_{i,t})\right]=(1-\mathrm{B}^{-1})(1-\beta\mathrm{B}^{-1})\mathrm{E}_tN_{i,t-1}\ \forall i$$

After expanding the stable eigenvalue forward and the unstable one backward, the first order condition for the arbitrage trading firm can be re-written as

$$N_{i,t}=N_{i,t-1}+\frac{1}{2c_i}\mathrm{E}_t\sum_{j=0}^{\infty}\beta^j\left(p^*_{i,t+j}-p_{i,t+j}-t_i\mathrm{I}(N_{i,t+j})\right)\ \forall i \tag{1.19}$$

The optimal amount of trade in good i in period t depends positively on the volume of trade in the past period and on the expected future path of deviations of prices from parity in excess of the trade friction. Firms care about the future path of price differentials so that their trade patterns are smooth over time. An expectation of a deviation from parity in excess of the trade friction in the future period $t+j$ increases trade in all future periods from t onwards.

Note that the expectation of the *direction* of the trade flow in the future periods $\mathrm{E}_t\mathrm{I}(N_{i,t+j})$ enters a firm's decision rule. If a firm expects a future price process in which a set of time periods with expected export regime is followed by a set of time periods with an expected import regime[13], it optimally lowers the trade volume relative to a scenario in which the firm expects only one regime to prevail.

1.3.1 Equilibrium

The equilibrium is a set of prices and quantities $\{p_{1,t},p^*_{1,t},p_{2,t},p^*_{2,t},\ C_{1,t},C^*_{1,t},C_{2,t},\ C^*_{2,t},C_{3,t},$ $C^*_{3,t},N_{1,t},N_{2,t},\ N_{3,t}\}_{t=0}^{\infty}$ such that the representative household maximizes its utility (equations

[13]A process in which $p^*_{i,t+j+l}>p_{i,t+j+l}+t_i\Rightarrow\mathrm{I}(N_{i,t+j})=1$ is followed by $p^*_{i,t+j+k}+t_i<p_{i,t+j+k}\Rightarrow\mathrm{I}(N_{i,t+j+k})=-1$ for $k>l$.

(1.1)-(1.4)), arbitrage trading firms maximize their profits (equation (1.19) for both goods) and all markets clear (equations (1.12) - (1.14)).

Household optimality conditions at home and abroad (equations (1.2), (1.3), (1.4), (1.2*), (1.3*), (1.4*)), firm's two optimality conditions described by (1.19), and 6 market clearing conditions ((1.12), (1.13), (1.14), (1.12*), (1.13*), (1.14*)) can be simplified into a 4-by-4 system in $\{p_{1,t}, p_{2,t}, p^*_{1,t}, p^*_{2,t}\}$:

$$\Delta Y_{1,t} - \gamma_1 \left[p_{1,t}^{-\theta} \frac{Y_t}{P_t^{1-\theta}} - p_{1,t-1}^{-\theta} \frac{Y_{t-1}}{P_{t-1}^{1-\theta}} \right] = \\ = \frac{1}{2c_1} \mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left[p^*_{1,t+j} - p_{1,t+j} - t_1 \mathrm{I}(N_{1,t+j}) \right] \tag{1.20}$$

$$\Delta Y_{2,t} - \gamma_2 \left[p_{2,t}^{-\theta} \frac{Y_t}{P_t^{1-\theta}} - p_{2,t-1}^{-\theta} \frac{Y_{t-1}}{P_{t-1}^{1-\theta}} \right] = \\ = \frac{1}{2c_2} \mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left[p^*_{2,t+j} - p_{2,t+j} - t_2 \mathrm{I}(N_{2,t+j}) \right] \tag{1.21}$$

$$\gamma_1 p_{1,t}^{-\theta} \frac{Y_t}{P_t^{1-\theta}} + \gamma_1 p_{1,t}^{*-\theta} \frac{Y_t^*}{P_t^{*1-\theta}} = Y_{1,t} + Y^*_{1,t} \tag{1.22}$$

$$\gamma_2 p_{2,t}^{-\theta} \frac{Y_t}{P_t^{1-\theta}} + \gamma_2 p_{2,t}^{*-\theta} \frac{Y_t^*}{P_t^{*1-\theta}} = Y_{2,t} + Y^*_{2,t} \tag{1.23}$$

where $Y_t = p_{1,t}Y_{1,t} + p_{2,t}Y_{2,t} + Y_{3,t} + \frac{1}{2}AP_t$, P_t is a composite price index ($P_t = (\gamma_1 p_{1,t}^{1-\theta} + \gamma_2 p_{2,t}^{1-\theta} + \gamma_3)^{\frac{1}{1-\theta}}$), $\mathrm{I}(N)$ is a smooth approximation to the indicator function such that $\mathrm{I}(N_{i,t}) = 1 \iff N_{i,t} > 0$ and $\mathrm{I}(N_{i,t}) = -1 \iff N_{i,t} < 0$ and AP_t are the current-period arbitrage profits. Equations (1.20) and (1.21) are the two intertemporal equilibrium conditions. Equations (1.22) and (1.23) are the two intratemporal equilibrium conditions.

Intuition

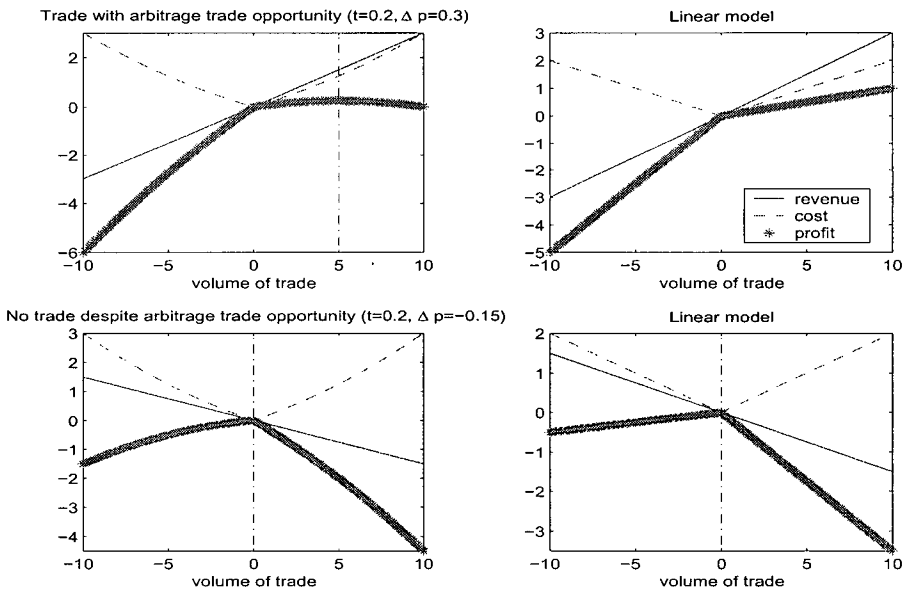
Before solving the infinite version of the model, I illustrate some of its main features using a simpler one-period version. The first obvious improvement is the lack of inteterminacy of trade volume in partial equilibrium setting. If a price deviation exceeds marginal cost of shipment in a model *without* quadratic adjustment costs, firms prefer to ship an infinite amount of goods in a partial equilibrium. Market clearing imposed in the general equilibrium forces prices to adjust until price difference reaches the threshold and trading stops (therefore the volume traded is finite). Trade volume is effectively determined outside the firm. On the other hand, in a model *with* quadratic adjustment costs, the optimal volume of trade chosen by the firm is finite in a partial equilibrium.

The quadratic adjustment cost model encompasses the linear shipping cost model. If we multiply the inter-temporal Euler equations (1.20) and (1.21) by c_1 and c_2, respectively, so as to avoid having adjustment cost parameters in the denominator of a ratio, we can study the properties of a system as c_i approaches 0. Smaller c limits the linkage between today's and yesterday's volumes of trade $N_{i,t}$ and $N_{i,t-1}$, respectively (note that the expressions on the left-hand side of equations (1.20) and (1.21) are just $\Delta N_{i,t}$ $i = 1, 2$). When $c = 0$, the two inter-temporal equations become identical to equations (1.6) and (1.7) from the linear shipping cost model. The inter-temporal linkage ceases to exist, and the system collapses onto system described in cases 1, 2, or 3 of the linear shipping cost model equilibrium (section 1.2.4).

Consider a one-period version of the model, with only one good with a positive trade friction. Then, the firm's first order condition implies: $p^* - p - \mathrm{I}(N)t = 2c(N - N_{-1})$ where N_{-1} is the last period's trade volume. Without the quadratic adjustment cost function ($c = 0$), as long as the law of one price deviation exceeds the trade friction t, the volume of trade is indeterminate in a partial equilibrium. From the point of view of the firm, an infinite volume of trade is optimal because that maximizes its profits. With $c > 0$ and $|p^* - p| > t$, the firm chooses a finite volume of trade that depends positively on the law of one price deviation $(p^* - p)$ and last period's trade volume N_{-1}, and negatively on the shipping cost t and quadratic adjustment cost parameter c. The general equilibrium conditions imposed by market clearing and price adjustment further limit the volume of trade because adjustment of prices narrows the profit margin of the firm. Note that with $|p^* - p| < t$, and $N_{-1} = 0$, the firm optimally chooses not to trade. When $|p^* - p| < t$ and $N_{-1} \neq 0$, balance has to be struck between the loss-making trade and the costs of trade deceleration (note that profits are negative in this situation).

Figure (1.2) plots the revenue, cost and profit functions for linear and quadratic models in two situations, assuming $N_{-1} = 0$. In the upper segment, price abroad is 30% higher than price at home. In the lower segment, price abroad is 15% lower than the price at home. Trade friction $t = 0.2$ in both cases, i.e., 20% of good 3 is consumed by shipment. Note that the revenue functions are identical in both models, while the cost function in the quadratic adjustment cost model lies always above the cost function in the linear model (due to the additional cost). It

Figure 1.2: Simplified profit functions in a quadratic and linear model when $N_{-1} = 0$



follows that the profit function in a quadratic adjustment cost model is a parabola with a kink and lies below the profit function of the linear cost model in all cases other than $N = 0$, when the two profit functions are identical. Consequently, trade will occur in the QAC model for law of one price deviations only when it also occurs in the linear model. We know that trade occurs in a linear model if and only if $|p^* - p| > t$. *Therefore* $|LOPD| > t$ *is a necessary condition for trade to occur in the QAC model.*

Also note that the no-trade bands in a quadratic adjustment cost model are identical to the no-trade bands in a linear cost model. As has been shown above, if $|p^* - p| \leq t$ while $N_{-1} = 0$, firms will not trade. If $|p^* - p| > t$, trade does occur, but in a smaller (finite) volume than it would in a linear cost model. In particular, when $c > 0$ and $N_{-1} = 0$,

$$\frac{\partial N_i}{\partial Y_i} = \frac{\pi}{2c\theta} \left[\frac{Y_i^{-\frac{1}{\theta}-1}}{Y_3^{-\frac{1}{\theta}}} - \theta t \right] < 1 \tag{1.24}$$

where $\pi \equiv prob(|p^* - p| > t)$. The volume traded will be smaller than the increase in endowment (see the Appendix for proof).

Because the arbitrage trade is the only source of price adjustment, a smaller trade volume leads to smaller price adjustment. Consumption closer to the autarky level requires that the equilibrium prices are closer to the autarky prices than they were in a linear model. Therefore, equilibrium law of one price deviations in excess of the shipment thresholds t can be sustained. *While the no-trade region in a model with quadratic adjustment costs is identical to the one in a model with linear shipment costs, law of one price deviation can exceed the shipping threshold in equilibrium.* In particular, it can be shown that, in the simple one-period model, for any N_{-1},

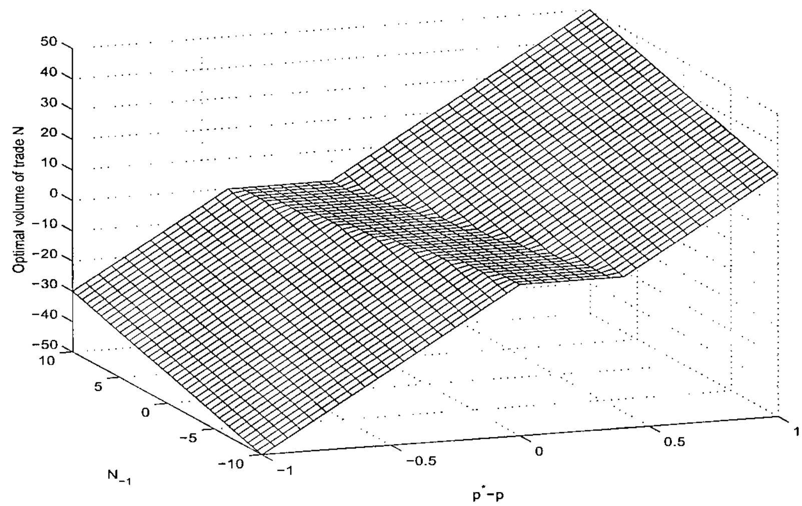
$$\frac{\partial p_i}{\partial Y_i} = \mathrm{D}\left[-2c - t\frac{\partial \mathrm{I}(N)}{\partial Y_i}\left(1 + \frac{1}{A}\right)\right] \tag{1.25}$$

where D>0 and $A < -1$ (see the Appendix for derivation). Therefore, an increase in endowment will decrease the home price, i.e., increase the law of one price deviation $p^* - p$ when $c > 0$. Moreover, the deviation from the law of one price is increasing in c. Intuitively, larger quadratic adjustment costs lead to smaller adjustment in volumes, lessening the pressure on prices to converge.

Figures (A.13)-(A.16) show initial responses of deviations from the law of one price for a range of $\mathrm{d}Y_{1,i}, i = 1, 2$, as well as for a range of cost adjustment parameters c.. As in the linear shipping cost model, three regions exist for any level of c: region 1 in which none of the goods are traded, region 2 in which only the lighter good is traded and region 3 with both goods traded. Price of good 2 is affected by the fact that good 1 is traded even when good 2 is not (region 2): as home price of good 1 rises, home consumers substitute to good 2. This expenditure switching effect from traded to non-traded goods at home increases home demand for the non-traded good 2 and increases its prices above what it would have been otherwise. Consequently, law of one price deviation for good 2 is smaller than it would have been if good 1 was not traded. The figures also show how total volume of trade changes with the endowment. Trade volume depends negatively on the quadratic adjustment cost c as well as on the trade friction t. Also, cumulative trade volume expressed as a percentage of the change in endowment is non-monotonic. Finally, costly adjustment leads firms to spread out trade in more steps of smaller magnitude: time period of adjustment depends positively on c and $\mathrm{d}Y_i$.

The intuition presented can change slightly when $N_{-1} \neq 0$. Figures (A.13) - (A.16) showed that when $N_{-1} = 0$, volume of trade N increases in $p^* - p$ when $|p^* - p| > t$ and is zero otherwise. $N = N_{-1} + \frac{1}{2c}(p^* - p - \mathrm{I}(N)t)$ and so as long as the good remains traded ($\mathrm{I}(N)$ does not change), qualitative relationship between the optimal trade volume and price deviation stays the same. But the range of autarkic values of deviations of prices from parity decreases in N_{-1} (see figure 1.3). If $N_{-1} > 0$, positive trade volume may be optimal even when $p^* - p < t$ due to costs of trade deceleration. The reverse can also hold. Therefore, optimality does not guarantee positive profits when $c > 0$ (see figure A.17). Large values of $|N_{-1}|$ require larger $|p^* - p|$ for arbitrage profits to be positive.

Figure 1.3: Optimal trade volume in partial equilibrium in QAC model for various price deviations and N_{-1}. c=0.01, t=0.2



1.3.2 Solution method

Due to a high degree of non-linearity, I solve the model numerically. A differentiable approximation to the absolute value function $G(.)$ allows us to summarize the system by its first order conditions. The derivative of $G(.)$ is a smooth approximation to a step function $I(N_{i,t}) = I(\{p_{1,t+i}, p^*_{1,t+i}, p_{2,t+i}, p^*_{2,t+i}\}_{i=0}^{\infty})$, resulting in a highly non-linear, although differentiable, first order system.

First, the time span of the model needs to be limited. I assume the existence of a steady state equilibrium to which countries converge following a shock. A maximum number of time periods T available for an adjustment is necessary to limit the size of the system. Conditional on T, the model can in principle be solved recursively. However, efforts to apply recursive solution methods have failed to find the equilibrium paths of the variables, primarily due to the non-linearity in the indicator function. Therefore, a method of relaxation (see Boucekkine (1995)) is applied in which a finite-period approximation $\widehat{f(.)}_{t=1:T} = 0$ to the system $f(.)_{t=1:\infty} = 0$ is solved by stacking all equations for all time periods into one large system $F(.) \equiv [\widehat{f(.)}_{t=1} \dots \widehat{f(.)}_{t=T}]' = 0$ which is then solved numerically. The four unknown price variables in the equilibrium system $f(.)$ above imply that the size of the Jacobian is $4T \times 4T$.

Second, in order to compute the Jacobian of the stacked system $F(.)$ in one step, a smooth $I(\widehat{N_{i,t}})$ approximation to the indicator function $I(N_{i,t})$ is necessary. Some inventive work leads to

$$I(\widehat{N_{i,t}}) = \frac{2}{\pi} \arctan(\lambda N_{i,t})$$

where λ is a choice parameter which governs the approximation error. An inverse of a trigonometric function $\tan(x)$, $\arctan(x)$ has a range of $[-\pi/2, \pi/2]$ for $x \in \mathbf{R}$ and is monotonically increasing, continuously differentiable, and has a convenient property that $\arctan(x) < 0$ when $x < 0$, $\arctan(x) > 0$ when $x > 0$ and $\arctan(x) = 0$ when $x = 0$. Because of the bounded range, and because $\arctan(\lambda x)_{x=0} = 0 \ \forall \lambda$, $\arctan(\lambda x)$ can reach the bounds very quickly. Finally, premultiplying the function by $2/\pi$, changes the range to [-1,1] – very convenient for a step function. High λ lowers the approximation error, as illustrated in figure (A.12). A choice of $\lambda = 10^{40}$ makes the approximation error indistinguishable from zero for any feasible stopping criterion of the numerical solver. $I(\widehat{N_{i,t}})$ is a derivative of $G(N_{i,t})$.

Third, to facilitate the numerical solver in finding an equilibrium, the inter-temporal Euler equations (1.20) and (1.21) are replaced with their simpler forms which do not include an infinite forward-looking sum. An error in p_{it} by the numerical solver then affects only the $4(t-1) : 4(t+1)$ partition of the Jacobian ($= 12^2 = 144$ values), whereas before it would have influenced all $4T^2$ ($= 1600$ if $T = 10$, 6400 if $T = 20$) values of the Jacobian, making convergence exceedingly difficult for relatively small error levels.

$$\begin{aligned}
\frac{1}{2c_1}(p^*_{1,t} - p_{1,t} - t_1 I(\widehat{N_{1,t}})) &= (1+\beta)Y_{1,t} - \beta Y_{1,t+1} - Y_{1,t-1} \\
&\quad -(1+\beta)\gamma_1 p_{1,t}^{-\theta} \frac{Y_t}{P_t^{1-\theta}} + \beta\gamma_1 p_{1,t+1}^{-\theta} \frac{Y_{t+1}}{P_{t+1}^{1-\theta}} \\
&\quad +\gamma_1 p_{1,t-1}^{-\theta} \frac{Y_{t-1}}{P_{t-1}^{1-\theta}} \qquad (1.26)
\end{aligned}$$

$$\begin{aligned}
\frac{1}{2c_2}(p^*_{2,t} - p_{2,t} - t_2 I(\widehat{N_{2,t}})) &= (1+\beta)Y_{2,t} - \beta Y_{2,t+1} - Y_{2,t-1} \\
&\quad -(1+\beta)\gamma_2 p_{2,t}^{-\theta} \frac{Y_t}{P_t^{1-\theta}} + \gamma_2 \beta p_{2,t+1}^{-\theta} \frac{Y_{t+1}}{P_{t+1}^{1-\theta}} \\
&\quad +\gamma_2 p_{2,t-1}^{-\theta} \frac{Y_{t-1}}{P_{t-1}^{1-\theta}} \qquad (1.27)
\end{aligned}$$

A system $\widehat{f(.)}_{t=i}$, part of the large stacked system $F(.)$, consists of equations (1.26), (1.27), (1.22) and (1.23). Period $T+1$ values which are found in the inter-temporal Euler equations of $\widehat{f(.)}_T$ are set to steady-state equilibrium values associated with a full adjustment to the shock.

Finally, a two-step method is used in the calculation to ensure that the layers of approximation do not lead us to a wrong solution. First, a system $F(.)$ above is solved, and time paths of all variables are computed. Because this is merely an approximation to the true system (for smaller levels of λ, $I(\widehat{N_{i,t}})$ can be distinguished from $1, -1$, or 0), a second step involves replacing the $I(\widehat{N_{i,t}}))$ with $1, -1$, or 0 using the first step estimates, and solving the system again.

1.4 Real exchange rate

This section explains the behaviour of the real exchange rate in the model. Because the model is solved numerically, properties of the real exchange rate are explained using simulations across a range of ad-hoc parameter values. Section 1.5 calibrates most realistic parameter values for a two-country simulation of the model which is then compared to the moments of the data.

A model-specific logarithm of the real exchange rate is a weighted average of the three law of one price deviations: $\log(RER) = \gamma_1 \log(LOPD_1) + \gamma_2 \log(LOPD_2) + \gamma_3 \log(LOPD_3)$. To understand the model's implications for the persistence, an endowment process at Home is introduced to the model: $Y_{i,t} = \alpha Y_{i,t-1} + (1-\alpha)\bar{Y} + u_t \; i = 1, 2$ where $u_t \sim \mathrm{N}(0, \sigma^2)$.

Lack of price reversion to parity for tradable goods creates *persistence* in their deviations from the law of one price and consequently in the real exchange rate. As long as the deviation from parity stays within the thresholds, reversion does not occur. A sufficiently small endowment shock can generate an infinite half-life of deviation in the real exchange rate from PPP. Conditional on the trade friction, the degree of persistence in the real exchange rate is positively related to the persistence of the endowment shock process (see Figure 1.4 and Table 1.1). For $\alpha < 0.9$, half lives of the real exchange rate deviations do not exceed 7 time periods. Half life increases sharply in α for values near 1, to about 12 time periods when $\alpha = 0.95$, 30 time periods for $\alpha = 0.975$ up to 525 time periods when the endowments have a unit root ($\alpha = 1$). Variability of the shocks inversely affects the half life in this region: for $\sigma = 6.8\%$ of GDP, the maximum half does not exceed 19 periods. For given thresholds, high variability of the endowment shocks negatively affects the real exchange rate persistence because it increases the likelihood that the price deviation will exceed the threshold, thus leading to price equalization.

Figure 1.4: RER persistence and volatility and the shock process in linear model

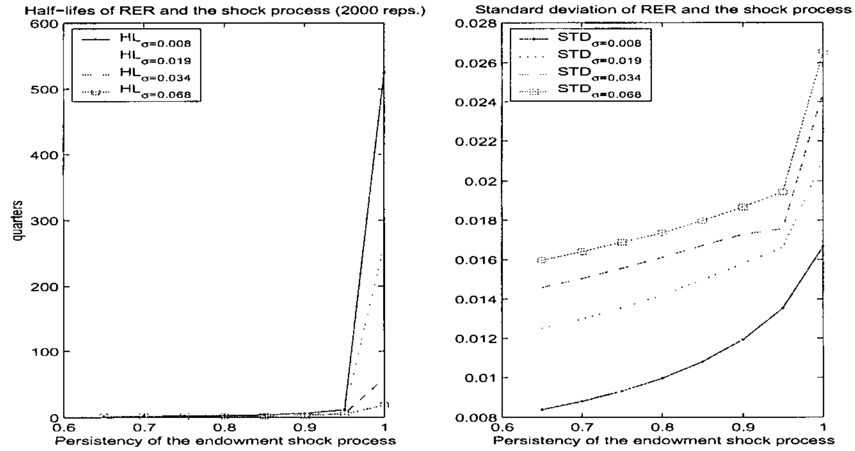


Table 1.1: RER half lives (in quarters) and the shock process

	$\alpha = 0.65$	$\alpha = 0.7$	$\alpha = 0.75$	$\alpha = 0.8$	$\alpha = 0.85$	$\alpha = 0.9$	$\alpha = 0.95$	$\alpha = 1$
$\sigma = 0.008$	1.66	1.99	2.44	3.11	4.16	6.13	11.52	525.12
$\sigma = 0.019$	1.42	1.67	2.00	2.48	3.18	4.55	7.93	263.46
$\sigma = 0.034$	1.21	1.41	1.65	1.98	2.50	3.50	5.86	59.39
$\sigma = 0.068$	1.02	1.16	1.34	1.59	2.03	2.82	4.99	18.56

Each result is based on 2000 Monte Carlo simulations of the linear shipping cost model ($c_i = 0$ $i = 1, 2$) when t_1=0.02 and t_2=0.04. α is the AR(1) coefficient of the shock process, σ is the standard deviation as a proportion of the mean GDP.

The standard deviation of the real exchange rate is increasing in both persistence α and volatility σ of the shock process (see figure 1.4 and table 1.2). High volatility of the endowment shocks leads to high volatility in the law of one price deviations, as long as the shocks do not exceed the thresholds. Once a shock exceeds the threshold, additional volatility is neutral. More persistent shocks lead to longer-lived deviations from the law of one price (the shocks effectively last longer) and so increases their volatility. This effect is particularly visible when σ is small enough so that most shocks leave LOPD below the thresholds. In such cases, std(RER) can exceed σ (see table 1.2). As σ rises, price deviations are more frequently outside of their thresholds, and get arbitraged away. Therefore, higher shock persistence α has minimal influence on real exchange rate volatility when σ is relatively high.

Size of the trade (shipping) friction is a major determinant of the persistence of the real exchange rate. For any given α, a larger shipping friction t requires a larger endowment shock

Figure 1.5: Properties of the real exchange rate and the trade friction in linear model

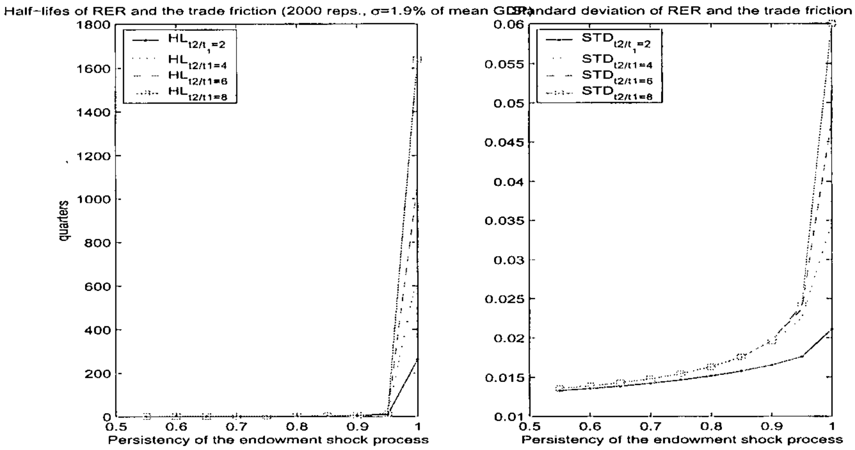


Table 1.2: Standard deviation of the RER and the shock process

	$\alpha=0.65$	$\alpha=0.7$	$\alpha=0.75$	$\alpha=0.8$	$\alpha=0.85$	$\alpha=0.9$	$\alpha=0.95$	$\alpha=1$
$\sigma=0.008$	0.008	0.009	0.009	0.010	0.011	0.012	0.014	0.017
$\sigma=0.019$	0.013	0.013	0.014	0.014	0.015	0.016	0.017	0.021
$\sigma=0.034$	0.015	0.015	0.016	0.016	0.017	0.017	0.018	0.024
$\sigma=0.068$	0.016	0.016	0.017	0.017	0.018	0.019	0.019	0.027

Each result is based on 2000 Monte Carlo simulations of the linear shipping cost model ($c_i = 0$ $i = 1,2$) when t_1=0.02 and t_2=0.04. α is the AR(1) coefficient of the shock process, σ is the standard deviation as a proportion of the mean GDP.

in order for arbitrage trade to occur (see figure 1.5). For any given shock process, a larger trade friction t will increase RER volatility by making a larger subset of the endowment shocks non-neutral.

Shipping costs generate persistence in the real exchange rate by creating a wedge between the prices of goods in two locations. The size of the wedge varies: it increases for shocks that are small relatively to the weight of a product, and stays constant for relatively large shocks above the threshold. The half life then depends *negatively on the volatility* of the endowment shocks because small (and persistent) shocks will need more time to cross a threshold. On the other hand, the half life of deviation in the RER depends *positively on the weights* of the products, as these determine the location of a threshold.

Table 1.3: RER half-lives and the relative trade friction

	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$
$t_2/t_1 = 2$	1.4	1.9	2.9	5.0	263.5
$t_2/t_1 = 4$	1.4	2.0	3.1	6.1	653.4
$t_2/t_1 = 6$	1.4	2.0	3.1	6.2	1060.2
$t_2/t_1 = 8$	1.4	2.0	3.1	6.2	1641.0

Each result is based on 2000 Monte Carlo simulations of the linear shipping cost model ($c_i = 0$ $i = 1, 2$) when t_1=0.02 and t_2=0.04. α is the AR(1) coefficient of the shock process, σ is the standard deviation as a proportion of the mean GDP.

1.4.1 Real exchange rate in a model with quadratic adjustment costs

The real exchange rate in a quadratic adjustment cost model is *more volatile and more persistent* than in the linear shipping cost model. The additional friction of quadratic adjustment costs in changes in trade volume narrows the profitability of the arbitrage opportunity to the firm after an endowment shock. As we have seen in section (1.3.1), the optimal choice set is narrowed in a non-linear fashion. The no-trade zones in the QAC model are identical to those in a linear shipping cost model (these are determined only by the cost-of-shipping parameters t_1 and t_2). Thresholds of the law of one price deviations are now upward sloping in the size of the endowment shock (price deviations for good i can exceed friction t_i). Additional trading costs lower the trade volume because consumption sharing is now more costly. Equilibrium consumption sets are closer to the endowments, and prices are closer to autarky prices. This implies that the law of one price deviations are larger and reverse more slowly than in the case without additional costs[14].

[14]Recalling that the $\text{LOPD}_i = t_i$ in the linear model, it follows that $\text{LOPD}_i > t_i$ in the QAC model.

Firms choose to adjust in more than one time period to a profitable shock in order to minimize the losses from trade deceleration. Negative contemporaneous arbitrage profits in the post-shock periods (there is no arbitrage opportunity that would warrant trade in post-shock periods) are smaller in discounted net present value than the costs of shutting down the trade channel. Both of the above differences lower the responses of firms to changes in endowments and make the real exchange rate more volatile and sticky in a model with quadratic adjustment costs.

A word of caution needs to be said about the comparability of results between the two models. The linear shipping cost model is static, and the dynamics are introduced by an autoregressive endowment process. The computation of the equilibrium is not restricted by the time horizon and long time spans can be used to increase the precision persistence and volatility estimates. On the other hand, the quadratic adjustment cost model is inherently dynamic. The numerical solution method has to limit the maximum number of time periods allowed for adjustment (in this case, $T = 20$ - compare to $T = 2000$ in the linear shipping cost model) which makes the AR estimations less precise. Moreover, a shock to the autoregressive endowment process only occurs in the first period, and then gradually dies off. This condition is necessary for the convergence of the numerical solver[15].

A simulation of a length T is repeated M times. Half-life and volatility of the real exchange rate deviations are computed at the end of each simulation, and a mean of that distribution of statistics is reported in the tables below. This contrasts with the result tables for the linear shipping cost model which report one statistic based on a simulation of a considerably longer length. The endowment process is $Y_{i,t} = \alpha Y_{i,t-1} + (1-\alpha)\bar{Y} + u_t \; i = 1, 2$ where $u_t \sim \mathrm{N}(0, \sigma^2)$. Moreover, I assume that $u_t = 0$ for $t > 1$. I then simulate the model for a variety of adjustment cost parameters c.

Table 1.4 shows that the half life of deviation of the real exchange rate behaves in a qualitatively similar manner when quadratic adjustment costs are present – it is decreasing in the shock volatility and increasing in the shock persistence. A more volatile endowment process lowers the half-life of convergence because profitable arbitrage opportunities arise more frequently. A more persistent endowment process makes an arbitrage opportunity last longer and so increases the

[15] If the AR(1) endowment process is shocked in every period, numerical solver may not converge because the assumption of full adjustment to the equilibrium by $t = T$ may require very fast adjustment, especially in the latter periods. Shocking the system in later periods leaves less time for adjustment. Moreover, shocking the system in each period makes the adjustment more complicated due to the changes in the state variable ($N_{i,t-1}$).

Table 1.4: Mean half-life of convergence estimate of log(RER) when c=0.1

	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 0.99$
$\sigma = 0.8\%$	1.9444	3.12	7	–
$\sigma = 1.9\%$	1.9438	3.11	6.8	–
$\sigma = 3.4\%$	1.9436	3.11	6.7	–
$\sigma = 6.8\%$	1.9433	3.1	6.6	–

Each result is based on 1000 simulations of the model when $T = 20$, $t_1 = 0.0054$ and $t_2 = 0.0174$ are the shipping trade frictions (see Calibration section 1.5). α is the AR(1) coefficient of the shock process.

adjustment time. Compared with Table 1.1, half lives of deviation are longer with quadratic adjustment costs for any combination of σ and α. Larger adjustment costs magnify these differences (tables 1.4), 1.5 and 1.1). The intuition from the one-period model carries through: quadratic adjustment costs lower the speed of adjustment of trade volume to shocks, leading to more sluggish adjustment in prices and consequently the real exchange rate.

Table 1.5: Mean half-life of convergence estimate of log(RER) when c=0.01

	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 0.99$
$\sigma = 0.8\%$	1.943	3.105	6.567	68
$\sigma = 1.9\%$	1.9429	3.1048	6.567	66
$\sigma = 3.4\%$	1.9428	3.1047	6.515	65
$\sigma = 6.8\%$	1.9428	3.09	6.522	63

Each result is based on 100 simulations of the model when $T = 20$, $t_1 = 0.0054$ and $t_2 = 0.0174$ are the shipping trade frictions (see Calibration section 1.5). α is the AR(1) coefficient of the shock process.

Law of one price deviations are more volatile than in the linear shipping cost model (see Table 1.6). Unlike in the linear shipping cost model, estimates of real exchange rate volatility relative to that of the GDP are not sensitive to endowment volatility, as more volatile endowments lead to higher volatility in prices (therefore their ratio stays unchanged). The standard deviation estimates decline in α because of a smoother adjustment in prices imposed by the quadratic adjustment costs.

Table 1.6: [Mean std(log(RER))]/[Mean std(log(GDP))] when c=0.1

	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 0.99$
$\sigma = 0.8\%$	27.62	14.39	12.59	8.49
$\sigma = 1.9\%$	17.83	14.88	12.35	8.42
$\sigma = 3.4\%$	17.68	14.52	12.24	8.54
$\sigma = 6.8\%$	17.07	14.25	12.14	8.75

Each result is based on 1000 simulations of the model when $T = 20$, $t_1 = 0.0054$ and $t_2 = 0.0174$ are the shipping trade frictions (see Calibration section 1.5). α is the AR(1) coefficient of the shock process.

Table 1.7: [Median std(log(RER))]/[Median std(log(GDP))] when c=0.1

	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 0.99$
$\sigma = 0.8\%$	1.951	1.952	1.953	2.03
$\sigma = 1.9\%$	1.950	1.951	1.952	2.08
$\sigma = 3.4\%$	1.950	1.950	1.951	2.36
$\sigma = 6.8\%$	1.949	1.949	1.950	2.71

Each result is based on 1000 simulations of the model when $T = 20$, $t_1 = 0.0054$ and $t_2 = 0.0174$ are the shipping trade frictions (see Calibration section 1.5). α is the AR(1) coefficient of the shock process.

1.4.2 Country-specific shocks

This section studies the addition to the linear shipping cost model of endowment shocks Abroad. To isolate the effect of a second shock process, the persistence of shocks at Home and Abroad is constrained to be the same across sectors as well as across countries:

$$\hat{Y}_{i,t} = \alpha\hat{Y}_{i,t-1} + (1-\alpha)\bar{Y} + \hat{u}_t \text{ for } i = 1,2$$

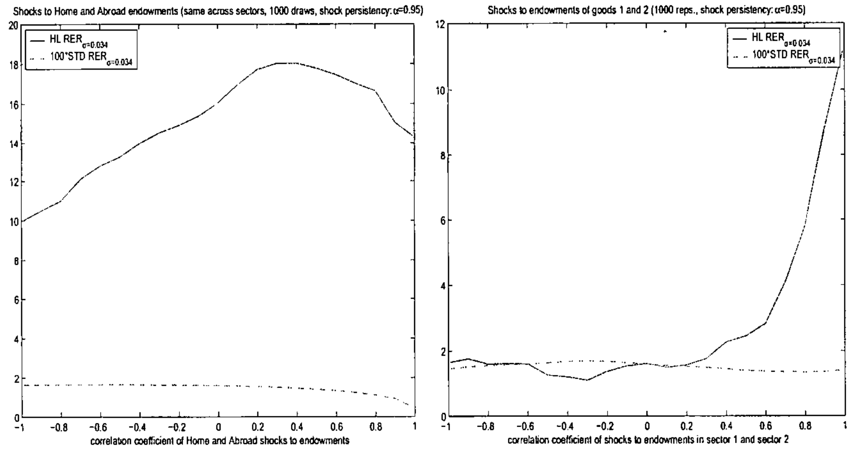
where $\hat{Y}_{i,t} = \begin{bmatrix} Y_{i,t} \\ Y^*_{i,t} \end{bmatrix}$, $\hat{u}_t = \begin{bmatrix} u_t \\ u^*_t \end{bmatrix}$ and $\hat{u}_t \sim \mathrm{N}(0, \hat{\Omega})$ where $\hat{\Omega} = \begin{pmatrix} \sigma^2 & \gamma \\ \gamma & \sigma^2 \end{pmatrix}$. The correlation parameter $\eta = \frac{\gamma}{\sigma^2}$ of the shocks at Home and Abroad affects the properties of the real exchange rate. Persistence of the real exchange rate deviations is lowest when the correlation of the shocks is close to zero (figure 1.6). Positively correlated shocks tend to produce relatively small average price deviations and lengthen the time needed to cross a threshold. Highly negatively correlated shocks lead to large price deviations which lower the persistence of the real exchange rate. Therefore, the real exchange rate *persistence* increases in η. On the other hand, *volatility* of the real exchange rate is a monotonically decreasing function of η. Negatively correlated shocks increase the real exchange rate volatility because they virtually act like a single unilateral shock of a larger magnitude. Positively correlated shocks lead to a minimal average deviation in the law of one price. Standard deviation of the real exchange rate approaches 0 as η approaches 1.

1.4.3 Sector-specific shocks

Here, endowment shocks differ across goods, possibly to proxy for sector-specific productivity shocks. Sectoral endowment shocks at Home are correlated, with no endowment shocks Abroad:

$$\tilde{Y}_t = \alpha\tilde{Y}_{t-1} + (1-\alpha)\bar{Y} + \tilde{u}_t \text{ for } i = 1,2$$

Figure 1.6: RER properties with country – and sector – specific endowment shocks in linear model



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[1] In both simulations, shock volatility as a proportion of GDP is set to $\sigma = 0.034$.

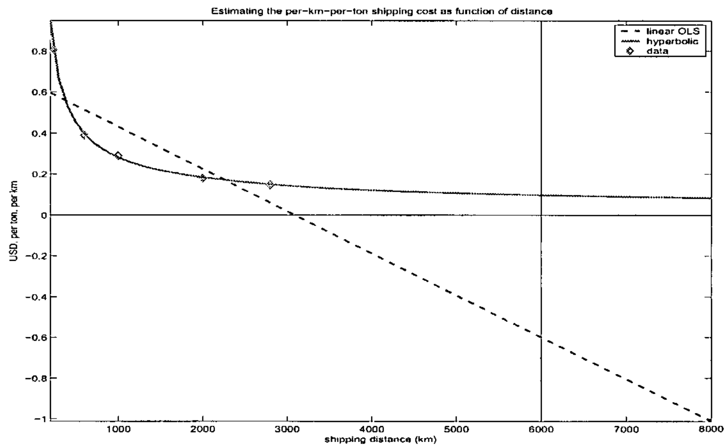
where $\tilde{Y}_t = \begin{bmatrix} Y_{1,t} \\ \cdot Y_{2,t} \end{bmatrix}$, $\tilde{u}_t = \begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix}$ and $\tilde{u}_t \sim \mathrm{N}(0, \tilde{\Omega})$ where $\tilde{\Omega} = \begin{pmatrix} \sigma^2 & \gamma \\ \gamma & \sigma^2 \end{pmatrix}$. Both persistence and standard deviation of the real exchange rate are increasing in $\tilde{\eta}$ (right panel of figure 1.6). When $\tilde{\eta} < 0$ (shocks to goods 1 and 2 are negatively correlated), a positive law of one price deviation to good one tends to be offset by a negative law of one price deviation for another good, resulting in a small deviation in the real exchange rate. Even small real exchange rate deviations will be arbitraged away because they originate in two law of one price deviations of opposite signs but larger magnitudes. This lowers the half life of persistence in the real exchange rate (such offsetting effect disappears as η approaches zero). For $\tilde{\eta} > 0$, shocks to goods 1 and 2 grow more similar and, as $\tilde{\eta} = 1$, persistence of the real exchange rate approaches the value when good endowments are subject to a single shock (see table 1.1 above).

The standard deviation of the real exchange rate is an increasing function of $\tilde{\eta}$. Negatively correlated shocks tend to offset each other and lower the volatility of the real exchange rate. Positive correlation makes the two shocks resemble a single country-specific shock which moves law of one price deviations of both goods in unison, leading to a larger real exchange rate deviation. Note again that as $\tilde{\eta}$ approaches 1, the real exchange rate volatility approaches the value for a single country-specific shock.

1.5 Calibration

In this section I describe the parameter choices for the calibration of the model. I choose the utility function weights symmetrically: $\gamma_i = \frac{1}{3}\ \forall i$. The inverse of the elasticity of substitution θ assumes the standard value in the literature 1.5 (see Chari, Christiano & Kehoe (1994) and McGrattan (1994)).

Figure 1.7: Calibration of per-kg-per-km shipping cost for cooled sea transport



Many authors calibrate the transportation costs using indirect estimates (e.g., Ravn & Mazzegna (2002)). I calibrate the shipping costs directly as a tax (or heterogeneous iceberg cost) that disappears in the course of good's shipment. I assume that, for any particular route, this tax depends multiplicatively on the distance and the weight of a good. Although unit shipping costs tend to decline with distance (possibly due to increasing returns to scale in transportation), this is not a problem here because the model has only two locations (one distance). Weight-dependance of shipping costs has been established by already (see Table 7 in Hummels (1999)). US and EU are chosen as locations because of their similar size. The distance is set equal to the New York - Hamburg shipping distance of 6000km. Most goods are shipped by sea between Europe and the US. I find two sources for shipping cost estimates.

In a large survey of all modes of transportation, Runhaar et. al (2001) quote an average price in 2001 a standard 40' container on a route Rotterdam – Singapore (9650km) of NLG 3060 including a NLG 360 fuel surcharge (equal to USD 1220 in 2001). They estimate the average load of a 40' container is 16.25 ton. This yields an average rate of USD 0.0077 per ton per km in 2001.

Perishable items such as most of foodstuffs are shipped in chilled containers. In their survey of shipping costs for fish containers Brox et. al. (1984) survey costs across a range of distances. The implied per ton per km shipping costs can be well approximated by a hyperbolic function (see figure 1.7). At the 6000km distance these imply a unit cost of USD 0.11 per ton per km between US and Europe.

In order to compute the share of the good that is used up, I use the the dataset of physical weights and average prices in 2001 collected for chapter two. Approximately 24% of the goods in the dataset require refrigerating for transport. Using this as a weight, the average shipping cost per ton per km is USD 0.033. An average weight of a good in the dataset is 43kg, and the average price USD 745 (2001 prices). Because I need to calibrate two fricitions, I arbitrarily pick two weights (20kg and 66kg) which equal the average weight of a good in my dataset. Consequently, the per-kg-per-km fractions t_i of good $i = 1, 2$ equal 0.0054 and 0.0174, respectively. That is, about 0.54% of good 1 and 1.74% of good 2 get used in transportation. These cost estimates are conservative compared to those found in the literature: Harrigan (1993) finds transportation barriers of 20%. Hummels (1999) uses 2-digit SITC data to estimate a transportation costs of 9%. Using 4-digit SITC data, Ravn & Mazzegna (2002) find that the weighted average of transportation costs declined from 6.31% in 1974 to 3.49% in 1994. IMF frequently uses 11% as a rule of thumb for transportation costs. All these are greater than the 1.14% average in my calibration.

The stochastic endowment process at Home is calibrated to match the logged and H-P-filtered quarterly U.S. GDP series from 1973:1 to 1994:4 (see Chari, Kehoe & McGrattan (2002)). That implies the AR(1) coefficient $\alpha = 0.88$ and the standard deviation of the residuals of this process is 0.8% of the mean GDP. Because the two countries in the model are approximately of the same size, the output process Abroad is calibrated to the total GDP of the main EU members: France, Italy, UK and Germany. From Chari, Kehoe & McGrattan (2002), this implies a correlation between the two output processes $corr(Y, Y^*) \equiv \hat{\eta} = \frac{\gamma}{\sigma^2} = 0.6$. The mean endowment of each good is normalized to 100.

Calibration of the quadratic adjustment cost model is identical to that of a linear shipping cost model. No values for the quadratic adjustment cost parameter c were found in the literature. I therefore use an ad-hoc value $c = 0.1$. We saw in the discussion of the equilibrium properties of the real exchange rate (section 1.4.1) that persistence will increase in c, as will the standard deviation of RER relative to GDP. At the same time, at $c = 0.1$, the thresholds of law of one price deviations are still distinctly different between the two goods.

1.6 Simulation results

1.6.1 Simulation results in a linear shipping cost model

A bivariate vector of 10,000 normally distributed shocks u_t and u_t^* are used to generate the stochastic endowment vectors at Home and Abroad: $\hat{Y}_{i,t} = \alpha\hat{Y}_{i,t-1} + (1-\alpha)\bar{Y} + \hat{u}_t$ for $i = 1,2$ where $\hat{Y}_{i,t} = \begin{bmatrix} Y_{i,t} \\ Y_{i,t}^* \end{bmatrix}$, $\hat{u}_t = \begin{bmatrix} u_t \\ u_t^* \end{bmatrix}$ and $\hat{u}_t \sim \mathrm{N}(0, \hat{\Omega})$ where $\hat{\Omega} = \begin{pmatrix} \sigma^2 & \gamma \\ \gamma & \sigma^2 \end{pmatrix}$. The model is then solved for equilibrium price levels and allocations. The solution can be described qualitatively in the same way as in section 1.2.4. Due to trade frictions, tradability in goods is determined endogenously. Good 1 is traded more frequently (86% of the time periods) than good 2 (32%). Histogram in Figure A.9 illustrates the differential trade volume in the two goods. Consequently, prices of goods 1 and 2 deviate from parity in a persistent but heterogeneous manner. Figure A.10 shows the bimodal distributions of the law of one price deviations. This bimodal nature is a direct result of the thresholds. Shocks that bring the law of one price deviations in autarky above a threshold also trigger arbitrage, equilibrium price adjustment and reversion of the the deviation from the law of one price back to the threshold. In a linear shipping cost model, this happens immediately which is why we observe bunching of equilibrium price deviations around the thresholds – lighter goods have narrower thresholds than heavier goods, and their law of one price deviations reverse more frequently. I analyze the persistence, comovements and volatility of the simulated series. Table 1.8 shows selected statistics.

Persistence

The model-generated logarithm of the real exchange rate matches the persistence in the data, as in Chari, Kehoe & McGrattan (2002) (see section 1.5 above). Model's AR(1) estimate in $RER_t = \hat{\alpha}\ RER_{t-1}$ is $\hat{\alpha} = 0.8286$ with a standard error 0.0056[16]. The persistence of the real exchange rate found in the data is 0.83. Deviations from parity for good 1 are more persistent than for good 2 (AR(1) estimates of 0.7379 vs. 0.847, respectively).

[16] This implies a half-life of convergence of about 3.7 quarters.

Table 1.8: Properties of the US-EU model simulation

	data	linear model[1]	QAC $c = 0.05$	model[2] $c = 0.1$	CKMcG[3]
Autocorrelations					
Ex. rates & prices					
RER	0.83	0.8286	0.868	0.92	0.62
Business cycle stat					
GDP	0.88	0.88*	0.88*	0.88*	0.62
Consumption	0.89	0.88	0.854	0.69	0.61
Net Exports	0.82	0.87	0.700	0.49	0.72
STD rel. to GDP					
Ex. rates & prices					
RER	4.36	0.002	6.41 (1.65)	7.66 (2.17)	4.27
Business cycle stat					
Consumption	0.83	0.75	1	1	0.83
Net Exports	0.11	0.19	0.001	0.001 (0.49)	0.09
Cross-Correlat.					
GDPs	0.6	0.6*	0.6*	0.6*	0.49
Consumptions	0.38	0.62	0.283	0.27	0.49
NX & GDP	-0.41	-0.03	0.050	0.04 (0.87)	0.04
RER & GDP	0.08	0.69	-0.02	0.13	0.51
RER & NX	0.14	0.88 (-0.02)	0.027	0.024	-0.04
RER & Relat. C	-0.35	0.96	0.956	0.95	1.00

[1] based on 10,000 simulations of the linear shipping model with parameter calibration as described in section 1.5.
[2] based on 5,000 simulations (each 20 periods long) of the quadratic adjustment cost model with $c = 0.1$.
[3] results of the model simulation in Chari, Kehoe & McGrattan (2002).
* denotes a calibrated value.

The persistence of consumption and net exports in the model is driven by the persistence of the stochastic endowment process. Because of the risk-sharing role played by the international trade, consumption is less volatile than the endowments in the two countries ($\text{STD}_C/\text{STD}_Y$=0.72 in model and 0.83 in the data.). On the other hand, zero trade volume in the linear shipping cost model lowers the volatility of exports relative to that of output ($\text{STD}_N/\text{STD}_Y$=0.19 in model and 0.11 in the data.). The existence of thresholds increases the volatility of consumption and decrease the volatility of net exports.

Comovements

Consumption levels at Home and Abroad have the same cross-country correlation as the GDPs. This is a result of the calibration of Y and Y* and of the fact that consumption dynamics is qualitatively similar to the shock dynamics, although smaller in magnitude.

Because this is a real model, the good with the lowest friction sometimes flows in the opposite direction than goods 1 and 2 for the aggregate constraint to be satisfied[17]. While the exports of goods 1 and 2 are positively correlated with the domestic GDP, $corr(N_3, Y)$ is negative. Consequently total net exports are slightly negatively correlated with the GDP (-0.03 in the model and -0.41 in the data).

The thresholds provide only a partial disconnect of the real exchange rate in from the real economy. A sufficiently large endowment change lowers the price at Home relative to Abroad – a depreciation (increase) in the real exchange rate that leads to a positive $corr(RER, Y)$ [18]. Backhus-Smith puzzle remains due to the expenditure-switching effect of the changes in the real exchange rate. Partial expenditure switching brings model's correlation of RER and relative consumptions away from unity and closer to the data ($corr(RER, C/C^*)$=0.91 in the model and -0.35 in the data). Therefore, consumption correlation across countries is negatively related to the trade friction of a particular product: it changes from -1 (good 3) to 0.834 (good 1) to 0.686 (good 2).

Volatility

The linear shipping cost model fails to generate sufficient volatility in the real exchange rate, for three reasons. First, all adjustment takes place within the time period when the shock arrives. Second, maximal deviation is a weighted average of the the trade friction of the real exchange rate's components – all of them goods. A more realistic model would see the real exchange rate adjust over a longer time horizon (during which RER could exceed the thresholds), and would also model the real exchange rate components more realistically by including services with potentially large thresholds[19]. Third, in the absence of nominal and real rigidities, volatility of the price aggregates is limited by the volatility of the driving force of the model (see Figure A.11). The standard deviation of shocks is calibrated to 0.8% of GDP. Therefore, irrespective of other parameters, it must be that $\mathrm{STD}_{RER}^{model} < \sigma = 0.8\%$ of mean GDP. In this calibration, $\mathrm{STD}_{RER}^{model} = 0.2\%$ of mean GDP. This is inconsistent with the observation of $\mathrm{STD}_{RER} \approx 436\%$ of GDP in the data.

[17] When goods 1 and 2 do not flow in the same directions, net flow of good 3 depends on the prices of goods 1 and 2 and the volume of trade.

[18] $corr(RER, Y)$=0.69 in the model but only 0.08 in the data

[19] Nominal rigidities could also serve to increase the volatility of price aggregates over that of the endowment shocks. Unlike quadratic adjustment costs, these nominal rigidities would be orthogonal to the current workings of the model.

The volatility of consumption and net exports is roughly consistent with the data (STD_C^{model} =0.72, STD_C^{data}=0.83). Trade creates a quantitatively relevant consumption insurance. While the ratio of marginal utilities at home and abroad is not constant, it is less volatile than in the data. This is a necessary implication of a marginally higher volatility of the arbitrage trade in the model than in data ($\text{STD}_N^{model}/\text{STD}_{GDP}^{model}$=0.19, $\text{STD}_N^{data}/\text{STD}_{GDP}^{data}$=0.11). In turn, high volatility in trade comes from the the fact that the arbitrage trade is the only adjustment mechanism in this model. Trade frequently changes from autarky (N=0) to full trade equilibria. A model that would also include non-arbitrage trade motive would generate a smoother $\{N\}$ process and so increase the volatility of consumption.

1.6.2 Simulation results in a quadratic adjustment cost model

The addition of the quadratic adjustment costs (QAC) in change of volume of trade improves the qualitative results of the model. The additional trade friction brings countries closer to their autarkic consumption sets[20] and leaves the equilibrium prices further away from parity for a longer time. Therefore, both persistence and volatility of the real exchange rate increase compared to the results of a linear shipping cost model. The trade volume between countries declines and becomes smoother due to the quadratic nature of the additional friction. Consumption behaviour therefore mimics the endowment dynamics.

Persistence

On average, the real exchange rate is more persistent in the QAC model than in the data, with an AR(1) coefficient estimate of 0.92. A result of the upward-sloping thresholds in the law of one price deviations, the real exchange rate is more persistent than in the linear shipping cost model. Consumptions and trade are persistent to a lesser degree than the endowments, due to a limited pass-through of endowment shocks to prices and trade. Although the volatility of consumption aggregate is as high as that of the endowment shocks, the relatively small trade volume can not match the volatility of net exports from the data. As has been mentioned above, the adjustment costs further disconnect the two economies. Therefore, consumption co-moves less between countries than it did in the shipping cost model, although its value of 0.27 is closer to

[20]This happens in a more complicated way than by "increasing the distance" as would be the case in a linear shipping model (see section 1.3.1).

that of 0.38 found in the data. Similar results hold qualitatively for co-movement of RER and GDP, RER and NX, and RER and the relative consumption. The results are further improved when $c = 0.05$.

Volatility

The calibration of the quadratic adjustment cost model is significantly more successful in creating volatility of prices relative to GDP (see table 1.8). The average standard deviation estimate of 7.66 is well above the 4.35 found in the data (when $c = 0.05$, volatility is 6.4 times higher than that of GDP). Histogram of all standard deviation estimates can be found in Figure A.18. Law of one price deviations for good 1 are more volatile than that of the real exchange rate. As can be seen in Figure A.15, volatility of the law of one price deviation for good 2 is only marginally higher than for good 1 at $c = 0.1$ (see Figure A.19). The distribution of mean law of one price deviations appears to be bimodal with higher mass near the thresholds (figure A.19). The bimodality is not as pronounced as in the linear model due to the existence of quadratic adjustment costs.

1.7 Conclusion

Geography and physics delimit our lives but are somewhat marginalized in economic analysis. In a vast majority of cases, this is a justifiable omission due to a *static* nature of these constraints and a *uniform* way in which they are thought to affect economic variables. However, there are cases when geography and physics affect the marginal decisions in an economically relevant way. This chapter analyzes a particular example – the persistence and volatility dimensions of the purchasing power parity puzzle.

The persistence in deviations of the real exchange rate - and of prices of traded goods - from parity is not puzzling. It can be explained as a byproduct of heterogeneous trade costs. This idea is intuitive and, as the following chapter shows, supported by the data. Nominal rigidities may not be necessary in explaining persistence and volatility in price differences across countries[21]. The price stickiness is a byproduct of the shipment costs which depend on distance and physical characteristics of goods.

Because weight and volume of products matter in shipment (in a way which is unrelated to their price) differences in physical characteristics leads to a heterogeneity in the unit shipping

[21] Alternatively, the model can be thought of as justifying an optimal level of nominal price stickyness

costs. Only price deviations in excess of the shipping costs are arbitraged away in the model. Consequently, tradability is endogenous and adjustment of prices over distance is non-linear. Goods with a larger trade friction need a larger deviation from parity to become traded - they are traded relatively less frequently, and can sustain larger deviations in prices from parity.

This chapter studies two real models: a simple linear shipping cost model and a model with both linear shipping costs and quadratic adjustment costs in the changes of trade volume. In both models, the real exchange rate exhibits persistent deviations from parity due to trade frictions. Calibration exercise shows that half life in real exchange rate deviations can match the estimates observed in the data. The linear model also generates a quantitatively close match to the persistence and co-movements of various price- and quantity- constructs. However, volatility of price deviations is insufficient.

Arbitrage firms in the second model also pay quadratic adjustment cost in the change of trade volume, due to additional legal and infrastructure expenses and to capture the time dimension of shipping. The core feature of the linear shipping cost model stay unchanged. However, a dynamic and highly non-linear environment emerges. Firms' aversion to react strongly to endowment shocks by large trade volume adjustments creates more pronounced and long-lived real exchange rate deviations. This further limits the arbitrage trade between countries and the co-movement between their consumptions. Simulation results of the second model can easily match both persistence and volatility of the real exchange rate while giving qualitatively meaningful results along other dimensions. Persistence and volatility of the real exchange rate need not be puzzling, they can be explained as a result of trade costs.

Chapter 2

Non-linear Adjustment in International Prices and Physical Characteristics of Goods

This chapter analyzes the heterogeneous adjustment of international relative prices across a range of goods. It examines the relationship between the physical characteristics of goods and the heterogeneity in the adjustment of their prices to parity. The main result of this chapter is the significant negative relationship between price-to-weight and price-to-volume ratios and non-linear threshold estimates across goods in US and Canada[1].

Post-Bretton Woods CPI data on 64 groups of products and services in Canada and the US are used to study the dynamics of the law of one price deviations. It is found that changes in the LOPD behave in a non-linear way which can be described well as a threshold-type non-linearity. The setup of this estimating model is rooted in the theoretical model presented in the previous chapter: deviations of prices from parity do not adjust inside of thresholds (observationally equivalent to the no-trade bounds in the linear shipping cost model), but exhibit relatively high speeds of adjustment outside of thresholds. It is found that the no-adjustment zones account for approximately one half of the persistence in the deviations of the real exchange rate. Furthermore, these zones of no adjustment and half-lives of deviation are heterogeneous across goods. The most interesting result emerges when the no-adjustment zones are related to a separately collected dataset of product weights and prices. The threshold width is significantly negatively related to the price-to-weight ratios, and also positively to prices and negatively to weights when these are considered individually.

The purchasing power parity puzzle is a special case of a broader phenomenon labelled "exchange rate disconnect" for unexplainably low short-term links between real macroeconomic variables and the nominal exchange rate. A fully encompassing explanation of high volatility and persistence in nominal exchange rates, it is sometimes argued, requires a model that would gen-

[1] A similarly significant relationship is present when prices and weights are considered individually.

erate similar behavior in all asset markets (see, i.a., Obstfeld & Rogoff 1996, chapter 9) [2]. One way to approach this problem is from "the bottom up". If the frictions in the goods markets are modelled explicitly, what is there to be learned about the adjustment of prices across borders? Do physical characteristics of goods matter for the behaviour of law of one price deviations? This chapter attempts to contribute to answering these questions.

The general equilibrium model introduced in the previous chapter lays out the framework for the empirical investigation carried out here. In that model, heterogeneous goods in two countries are traded by profit-maximizing arbitrage firms who take prices as given and decide on the volume of trade for each good. The goods only differ in their physical characteristics which in part determine the shipping costs[3]. These differences lead to different marginal costs of arbitrage trade across goods. Consequently, an identical increase in an endowment for all goods may lead to profitable trade opportunities (hence mean reversion in prices) for some goods while other goods remain non-traded (and their prices in the two countries will continue to deviate). Because arbitrage trade is the only source of price adjustment in the model, endogenous tradability of goods leads to heterogeneous but symmetric thresholds: for law of one price deviations below the thresholds, no trade occurs and prices deviate freely. Prices differentials in excess of the thresholds are eventually arbitraged away. The linear shipping cost model has two predictions relevant for my empirical exercise: that *prices of tradable goods behave in a non-linear manner* as a consequence of the thresholds, and that the *non-linearity thresholds depend on product weights and distance*[4].

[2]It is not difficult to generate arguments for why the nominal exchange rate - and asset prices in general - is more volatile than the prices of other goods. First, the utility of holding assets, as opposed to consumption goods, is derived from a stream of future dividends which have much longer time horizon (a few days up to maximum of near 20 years for durables, versus potentially hundreds of years for assets). Second, once a good is purchased, utility derived from it's consumption is, up to the first-order approximation, independent of any future (non-preference) shocks while the utility derived from holding an asset varies with the income stream the asset is expected to generate. Expectations of future shocks can then magnify the variance in subjective asset valuations relative to consumption products leading to larger asset price volatility (which is consistent with large observed trading volume). A third reason for high price variance specific to the foreign exchange market is that a high volatility in the foreign exchange market can additionally be caused by the role of nominal exchange rate as a medium of exchange.

[3]Distance is the other determinant. Although physical weight is an important determinant of the shipping costs for air freight, other physical characteristics such as volume are essential in container transport.

[4]This also has the potential to explain why smooth threshold autoregressive models (STARs) are a very good fit for real exchange rates: the model predicts that the real exchange rate is more persistent around its equilibrium value when none of the components is far enough from the parity to trigger price reversion. The persistence decreases in the distance from the equilibrium as more of the components' thresholds are crossed. Consequently, the non-linearity of such construct is not an "on-off switch" non-linearity but a smooth nonlinearity akin to a string - just the idea behind a STAR econometric model

The remainder of this section reviews the empirical and theoretical literature to motivate the topic. Section 2.2 discusses the data, section 2.3 presents an empirical analysis of the two datasets and section 2.4 concludes.

2.1 Motivation and the literature

Purchasing power parity is a very old topic in economics. While the term was coined by Cassel (1918), a question of whether national price levels are equal when expressed in terms of a common currency had been analyzed for a very long time before that[5]. No doubt, the appeal lies in part in the topic's accessability[6], especially when much of the "puzzle" remains to be explained. The puzzle also reflects the strength of the priors most economists have about one variant of PPP or another as an anchor for the long-run real exchange rate (Rogoff (1996)). This idea is imbedded in most of the "old" (e.g., Dornbush (1980)) and many of the "new" (e.g., Obstfeld & Rogoff (1995, 1996)) open macroeconomic models. Typical general equilibrium models of exchange rates (e.g., Chari, Kehoe & McGrattan (2002)) contain a first order condition which relates the real exchange rate to the marginal rate of substitution between two countries' relative consumption baskets. In these models, properties of the real exchange rate are limited by the dynamics of relative consumptions. The theory of PPP implies that the real exchange rate (as measured by the price indexes) should be constant. Yet in the data, real exchange rates (RER) exhibit large degrees of volatility, and their deviations from the mean are highly persistent, particularly in the post Bretton-Woods era of flexible nominal exchange rates. This is why the Euler equation described above, combined with sticky prices, fares poorly.

Empirical literature

The empirical literature has largely followed two strands: studies of the law of one price deviations for the individual commodities and studies of aggregate indexes such as the real exchange rate. Using 7-digit ISTC categories, Isard (1977) finds large and persistent deviations from the law of one price. This is a recurring finding in many later studies. Giovannini (1988) reaches similar conclusion for extremely homogeneous products such as screws. Parsley & Wei (2001) find large

[5]The issue has first been articulated in the sixteenth century by scholars of Salamanca school. For a survey see, e.g., Officer (1976a).

[6]"A decade ago, PPP seemed like a fairly dull research topic. [...] Fortunately, the past decade has witnessed a tremendous degree of progress in the area [...] and several important results have emerged." Froot & Rogoff (1995).

deviations in the price levels of mainly food product across cities in US and Japan and lend further weight to the presence of a "border effect" (see below). Similarly, Crucini, Telmer & Zachariadis (2000, 2001) find that prices of consumer products vary greatly across the EU member states, but that overall they tend to cancel each other out so that some variant of PPP holds. Froot, Kim & Rogoff (2000) construct data of grains and dairy products over the past 700 years to find that although law of one price holds on average, there are low frequency trends of about 200 years that tend to reverse themselves.

Studies of aggregated indexes (either the real exchange rates or some of its aggregated components) tend to conclude that the Purchasing Power Parity holds over long horizons. While the earlier work (Lee (1976), Officer (1982)) found strong evidence in favor of PPP, it later became clear that real exchange rates do not revert or revert very slowly in post-Bretton Woods data[7]. In an important paper in terms of its theoretical implications, Engel (1999) found that nearly all of the deviations of bilateral US real exchange rates from mean (at all time horizons from one month to 30 years, and for five available definition of non-traded goods) can be attributed to the movements in the international relative price of the traded goods, rather than non-traded goods. Similarly, Chari, Kehoe & McGrattan (2000) showed that less than 2% of the variance of RER between the US and the EU is due to fluctuations in the relative price of nontradables. These findings imply that, in short and medium run, deviations of prices of traded goods from parity are at the heart of the nonstationarity of the real exchange rate.

The quest for more precise estimates of the real exchange rates persistence has often been addressed either by longer time series or by panel datasets[8]. It was found that purchasing power parity holds fairly well over longer horizons (e.g., Cheung & Lai (1994) and Lothian & Taylor (1996)). Due to low power of augmented Dickey-Fuller univariate unit root tests, the use of panel datasets has increased. These reject H_0 of nonstationarity for groups of real exchange rates (e.g., Frankel & Rose (1996), Flood & Taylor (1996), Papell (1997), Taylor & Sarno (1998)). Imbs et al. (2004) caution about interpreting the RER persistence estimates due to sectoral aggregation

[7]A popular estimate range for half-lifes of convergence of *detrended* series is 3-5 years. This measure varies significantly across base countries. Chari, Kehoe & McGrattan (2002) find the post-Bretton Woods half life for US-EU real exchange rate to be approximately 4 quarters. However, Murray & Papell (2002) show that the half-life estimates are highly misspecified and contain virtually no information regarding the true half-lifes. Yet, the correctly estimated half-lifes still range in the 3-5 year range.

[8]Distinguishing long memory processes from various alternatives is difficult. Diebold & Inoue (2001) show that even small amounts of a stochastic regime switching is very easily confused (even asymptotically) with long memory process.

bias[9]. RER half-life estimates do not imply that the half-lifes of RER components are as persistent - they are not.

It remains unclear how much weight should one place on the results of the early panel unit root tests. Taylor & Sarno (1998) show that such tests tend to reject non-stationarity if just one of the series in the panel does not have a unit root. Lyhagen (2000), Bornhorst (2003) and Banerjee, Marcellino and Osbat (2001) show that common stochastic trends - largely unaccounted for and introduced by the base currency's nominal exchange rate - yield large sizes of most of the panel tests.

Nonlinear econometric models also became popular in analyzing the dynamics of the real exchange rate. Dumas (1992), Ohanian & Stockman (1997) are a few examples of theoretical models that generate bands where the real exchange rate does not revert due existence of transportation costs. Empirical studies of this mold fit some type of a self-exciting threshold autoregressive (SETAR) model on log-deviations of real exchange rate or its major components[10]. It is known that an evidence of non-linear mean reversion invalidates the standard long-term tests of RER based on linear models (see, e.g., Obstfeld & Taylor (1997), Kilian & Taylor (2003)). Obstfeld & Taylor (1997) estimate a TAR model for CPI and its four major components across cities in Europe and North America and show that AR estimations of half life in the literature have to be biased upward if a data-generating process is nonlinear. Zussman (2002) estimates TAR(2,1,1) models on annual, quarterly and monthly bilateral real exchange rates for 108 countries and finds that the threshold estimates are significantly positively related to distance, import taxes, and nominal exchange rate volatility between countries[11]. Imbs et. al. (2003) find non-linear adjustment at sectoral level and show that the heterogeneity relates to distance and nominal exchange rate volatility. Studies that use smooth threshold autoregressive models (STAR - see Granger & Teräsvirta (1993)) with smooth transition between stationary and non-stationary states find quicker estimated speeds of convergence than the linear AR models (e.g., Taylor, Peel & Sarno (2001), Kilian & Taylor (2003)). Theoretical justification of the STAR models remains the stumbling block of this strand of research. Taylor (2000) uses multivariate as well as univariate tests

[9]More volatile components of RER receive a larger weight in the estimation, leading to upward bias.

[10]A SETAR model implies that the process is more stationary the bigger the deviation from the mean. A most basic model is the Threshold autoregressive (TAR) model with two symmetric bounds within which the process is a unit root while it reverts outside of the bounds.

[11]The use of White heteroskedasticity-consistent standard errors as well as the ad-hoc choice - and lack of testing - of TAR(2,1,1) model is questionable

of higher power over long time horizons to show that while volatility of the RER varies across different exchange-rate regimes, persistence of the RER deviations *is not significantly different* from one exchange rate regime to another.

A related "border effect" literature shows how the strength of dependance of prices and trade volume on distance changes dramatically for within- and cross- border comparisons. The cross-border price differences are much larger (and trade densities much lower) than what can be accounted for by standard gravity models. Engel (1993) finds that prices of similar goods across borders are a lot more volatile than relative prices of dissimilar goods within the US. Engel & Rogers (1996) show such effect in terms of price volatility and Jenkins & Rogers (1995) in terms of persistence. Engel & Rogers (2001) use monthly CPI data for 55 European cities and find that at monthly frequency, variance of changes in nominal exchange rates is the most important explanatory variable of the variance of changes in prices across locations. Yet at 48-month horizon, impact of the border dummy on the variance of prices across borders is 40-times larger (yet equally significant) than the impact of changes in nominal exchange rates. These results suggest that the sticky-prices and pricing-to-market hypothesis may well be present at frequencies below 48 months. O'Connel & Wei (2002) show how various bands of no-trade may arise from trade frictions in an optimal control framework of Dixit (1993). The authors analyze quarterly cost-of-living price level survey data covering 48 goods (mainly foods) and services in 24 US cities[12] and find stationarity in either an AR, TAR, or ESTAR specifications.

Theoretical literature

Although the real exchange rates are aggregate objects consisting of nearly all goods and services sold in the economy, their theoretical analysis has traditionally been approached in a binary framework of tradable versus non-tradable goods. Such framework suggests two sources of real exchange rate deviations: from the deviations of the law of one price for tradable goods or from the movements in the relative prices of non-tradable to tradable goods. In the evidence of the importance of deviations of prices from parity for traded goods, the latter approach has been addressed by modeling endogeneous tradability.

[12]This is an identical dataset to Parsley & Wei (1996).

Pricing to market Models of pricing to market (a term coined by Krugman (1997)) assume *market segmentation* for a subset of the tradable goods, and have a positive share of goods prices quoted in a different currency (*local currency pricing*). When preferences do not differ across countries market segmentation is non-vacuous only in the presence of real frictions (frequently *imperfect competition*). The former allows firms to choose their prices as a markup over their marginal costs and warrants a demand-determined and inefficiently low level of output. *If prices are sticky*, an excessive response of the exchange rate to a shock is necessary to clear the market for real balances, increasing the volatility.

If prices are quoted in the currency of a producer and preferences are identical, an exchange rate change will leave unaffected home and foreign prices of home goods, and change proportionately home and foreign prices of foreign goods. Thus, nominal exchange rate movements will *not* induce deviations from the law of one price for traded goods. With different yet constant preference parameters, the ratio of prices in two countries is different from 1 but constant (relative PPP holds). Only in the presence of preference shocks is producer pricing capable of generating LOP deviations (e.g., in Lapham & Vigneault (2001)).

Betts & Devereux (1996) build a model of pricing to market and show how it limits the exchange rate pass-through to foreign prices. In a dynamic model, Betts & Devereux (2000) show that pricing to market leads to overshooting in the exchange rate and, because of sticky prices, increases the volatility and persistence of the RER relative to a model with producer pricing. In models of this kind (Chari, Kehoe & McGrattan (2002), Bergin & Feenstra (2001), etc.), behavior of the real exchange rate around a stationary equilibrium $\hat{q}_t$ is a function of marginal utilities of consumption and a risk-aversion parameter[13]:

$$\hat{q} = A(\hat{c} - \hat{c}^*) + B(\hat{m} - \hat{m}^*) + D(\hat{l} - \hat{l}^*)$$

where c, m, l are logarithms of consumption, money and labor, "*" denotes a value of a variable abroad, " ˆ " is a deviation from a steady state and A, B, D are constants of which only A is significantly different from zero. The variance of the real exchange rate is then determined by the variance of the consumption growth differential between countries, while the persistence of real

[13]Chari, Kehoe & McGrattan (2000) show that preferences need to be additively separable in leisure and a consumption-money aggregate to generate this result.

exchange rate deviations comes from autocorrelation in such consumption growth differentials:

$$\begin{aligned} std(\hat{q}) &\cong \sigma std(\hat{c} - \hat{c}^*) \\ corr(\hat{q}_t, \hat{q}_{t-1}) &\cong corr(\hat{c}_t - \hat{c}_t^*, \hat{c}_{t-1} - \hat{c}_{t-1}^*) \end{aligned}$$

Models of this type tend to generate sufficient variance but insufficient persistence in the RER movements, even with 12-month price rigidity. In Ravn (2001) endogenous (optimal) pricing-to-market leads to persistent and volatile RER fluctuations. Ghironi & Melitz (2004) model changes in the consumption basket composition due to monopolistically competitive firms facing heterogeneous productivity shocks and trade barriers.

Nontraded goods Even when the law of one price holds for traded goods, price of non-traded goods relative to the price of traded goods depends on the local conditions, which can create a wedge in the real exchange rate. Therefore, purchasing power parity need not hold in the models with non-traded goods. Models of this sort tend to be nested in the *Redux* model of Obstfeld & Rogoff (1995) (see Obstfeld & Rogoff (1996, 2000a), Hau (2000)).

Specifically, non-traded goods change the transmission of a positive monetary shock in three ways. First, the "relief effect" of a international price adjustment of tradables is curbed, which in equilibrium requires a larger nominal exchange rate adjustment (an *exchange rate magnification effect*). Second, domestic demand expands following a depreciation in home currency because the non-tradables become cheaper relative to the tradables. This *home consumption bias* effect limits the demand transmission of the shock and lowers the international comovements of C, Y. Third, a price differential drives a wedge between the real returns on foreign assets to home and foreign consumers which affects the *intertemporal consumption smoothing* effect.

Bergin & Glick (2004) introduce endogeneous non-tradability by the way of heterogeneous iceberg costs. Good on the margin provides the linkage between prices of traded and non-traded goods. Ravn & Mazzenga (2002) generate persistence and volatility in RER deviations in a model where foreign imports and domestic factors of production are complements.

The remainder of this chapter is structured as follows: section 2.2 describes the data, section 2.3 studies the dynamics of the deviations from parity for 63 goods and services, by estimating appropriate non-linear models and analyzes potential sources of heterogeneity in threshold estimates. Section 2.4 concludes.

2.2 Data

This section aims to assess the evidence in support (or against) of the model presented in the first chapter. It first describes why data limitations do not constrain analysis of the dynamics of the law of one price deviations, and then describes the datasets.

2.2.1 Limitations of studying index-based RER

Study of disaggregated real exchange rates based on price indexes can only answer a subset of question related to the PPP puzzle. A level of an index does not have an economic meaning on its own, only when compared to other observations of the same index. The same applies to any construct that uses at least one index, for example the index-based real exchange rate. An observation of log(RER) = 3 does not tell us anything about the deviation from PPP: one goods basket could be overvalued, undervalued or at parity with the other one[14]. Therefore, questions related to the *magnitudes* of the law of one price deviations and PPP deviations can only be addressed directly with price *level* data. Consequently, one can not test for speed of adjustment to parity using index data because such data does not specify where the parity is. Index data can only be used for the analysis of deviations from a mean, interpretation of which is open to discussion[15]. If there are bands where prices do not adjust and if the deviations from parity within these bands are tied to nominal exchange rates, then the width of such bounds may be negatively related to the likelihood that the mean of the real exchange rate coincides with parity. In models which generate no-trade bounds, the exact location of the mean of the real exchange rate within no-arbitrage bounds is pinned down by the mean of the nominal exchange rate.

Although most theories predict that the mean of $\log(RER)$ is zero, this is not true in the data. Demeaning of the series is then essential for two reasons. First, a constant has no meaning in terms of an index, and second, there is no theoretical justification for a mean log-deviation to have a non-zero value[16]. Detrending of the index-based series may be harmful because it makes the data more stationary and so increases the likelihood of rejecting nonstationarity (see Tables

[14]If one knew the size of the deviation from parity at any point of time, one could construct the series on actual deviations from law of one price deviation.

[15]See Crucini, Telmer & Zachariadis (2001) for a price *level* analysis that documents widespread law of one price violations (hence mean does not equal parity) across the EU.

[16]A less convenient alternative to demeaning is to remove the starting value of the deviation from a law of one price (or a real exchange rate) for all series and study the dynamics of the relative prices starting from this assumed parity

B.7, B.8 and B.9). Although some argue that detrending removes a long-run trend and allows to capture better the short-run dynamics (Obstfeld & Taylor (1997)), this does not withstand closer scrutiny. The dynamics of an index series carries the same meaning as the dynamics of level series. Consequently, if there is a trend in the index series, that same trend is also present in the price level series. Such trend needs to be explained, not removed. I do not detrend the data.

2.2.2 Dataset description

Tables B.7, B.8 and B.9 illustrate basic properties of the data and the issues raised above. First, the non-stationarity of the demeaned real exchange rate differs across the sample periods (see Table B.7). Augmented Dickey Fuller tests show that in the 1947-1970 period, levels of nominal as well as real exchange rates were more stationary than in the 1970-2000 period. This holds irrespective of whether and how the data is detrended (either linearly in Table B.8 or by Hodrick-Prescott filter in Table B.9). Second, detrending makes data more stationary and so lowers their half lives of convergence. Between 1947 and 1970, estimates of the half life of convergence of the real exchange rate decline from 27 to 15 to 4.3 months as we move from raw to linearly detrended and then to HP-detrended series, respectively. If we consider the full sample period (1970-2000), this measure drops from infinity to 228 to 4.6 months, respectively. Properties of the nominal exchange rates are similar.

Price index dataset

The price index dataset contains disaggregated price series of 63 groups of goods and services in the United States and Canada, as well as the aggregate consumer price index and the nominal exchange rate. The maximal time coverage is from 1970:1 to 2000:8 (some series start after 1970). The series were obtained from Bureau of Labor Statistics and Statistics Canada, respectively, and are all demeaned. Of the 63 groups included in the dataset, 49 cover goods and 14 services. The goods in the dataset cover 24.1% of the CPI while the services and nontradable goods cover 46.7% of the CPI[17]. That is, my dataset covers approximately 73.5% of the CPI. Using the taxonomy of Lebow & Rudd (2001), 77% of durable goods, 70% of nondurable goods and 39% of services are included in the data (see tables tables B.1 and B.2 in Appendix B.1).

[17]Source: CPI all urban consumers, Bureau of Labor Statistics, December 2001. Some of the groups are a subset of other groups - all such double accounts are accounted for in this reported measure.

Physical weights dataset

I construct the dataset of physical weights and individual prices for each good (or group). These data can not be obtained from one source, leading me to the following data-collection algorithm. First, when available, weights are obtained from statistical agencies or government bodies. When unavailable, documents of manufacturers' associations are searched for average weights of particular goods or product groups. In a minority of cases when neither of the the first two techniques is available, the weights are estimated as an average of the market's large manufacturer's product range (e.g., for watches, an average weight is set equal to a current average weight of a Timex watch). Average prices are obtained in a similar manner, US data sources being searched first. When a US price is unavailable, similar search is performed across the Canadian data sources. The price corresponds to an average price in year 2000 in US dollars.

When the describing a group of products rather an individual product (e.g., womens' apparel), weight and price are computed as weighted averages of weights and prices of components using the expenditure shares from US urban average CPI in December 2001 as weights. When the price is not available for year 2000, the last available price is inflated by the CPI inflation rate of the relevant country. Composition of all groups, data sources, as well as price and weight estimates are documented in tables B.3 and B.4 in the Appendix B.2.

Volume dataset

The dataset of physical volumes of is calculated indirectly using data on stowage factors from the German Transportation Information Service database[18]. A stowage factor of a cargo is the ratio of weight to stowage space (the unit is ton/m^3) required under normal conditions, including all required packaging. Consequently, the volume of a unit of the good can be calculated using the stowage factor and weight of the good. Because stowage factors for goods can vary depending on packaging, water contents, and compression, I use the average of all quoted stowage factors in calculating the volume of a good. I find stowage ratios for products that are not included in the German database from other sources (see the Appendix B.2 for a detailed list).

[18]A website run by the German Insurance Association `http://www.tis-gdv.de/tis e/inhalt.html`

2.3 Empirical analysis and results

This section studies the dynamics of the law of one price deviations across goods and services, and attempts to account for its sources. It has two goals: first, to establish stylized facts about the heterogeneity in international price adjustment across goods and second, to account for relevant features of the observed behaviour using the lens of the theoretical model from the first chapter. The second task tests the theory from chapter one.

The variable of interest z_t^g is a logarithm of law of one price deviation: $z_t^g = p_t^g - p_t^{g*} + s_t$, where t is a time index and g is a good (service) index, p and p^* denote logarithm price indexes in US and Canada, respectively, and s_t is the logarithm of the nominal exchange rate.

2.3.1 Model selection

A general discrete form of a threshold autoregressive model is chosen for analyzing the time series. The theoretical model from chapter one implies good-specific thresholds of no arbitrage which can be well captured by a group of self-exciting threshold autoregressive models[19]. A choice has to be between two kinds of models: "smooth" and not "smooth". Frequently used smooth transition autoregressive models (STAR) are, for the purposes of RER analysis, a fluid combination of a non-stationary and a stationary regime akin to a string[20]. However, good-specific no-arbitrage bounds from the linear shipping cost model imply discrete thresholds determined by the physical characteristics, distance, unit shipping costs, insurance costs, trade barriers, etc. With two locations, and assuming that the unit shipping costs do not change over time, such a model implies a 'sharp switch' non-linearity with two separate autoregressive regimes: one within and another outside the thresholds.

Other choices need to be made about the exact form of a TAR model before estimation: selection of the number of thresholds, selection of a number of autoregressive lags p and of an optimal delay parameter d. Because the theoretical model predicts two thresholds for each good

[19]Self-exciting threshold autoregressive (SETAR) models can be thought of as a combination of several (typically two) regimes which differ in the degree of stationarity they impose on the series. The decision on which regime shall the variable observe depends on a position of a control variable - in "self-exciting" models this is just a lagged value of the examined series.

[20]A larger deviation of the RER rises with the weight placed on the stationary regime relative to the non-stationary one. Therefore, reversion occurs for *any* deviation and its strength rises in the size of the deviation (see Tong (1990), Granger & Teräsvirta (1993) for reference on non-linear time series analysis)

a TAR(2,p,d) is selected[21]. Moreover, I assume that deviations from the law of one price behave symmetrically and therefore the two thresholds γ_1 and γ_2 are symmetric around zero, i.e., $\gamma_1^g = -\gamma_2^g \equiv \gamma^g$ for each good g. Letting the optimal delay lag parameter be d_p, the vector of the appropriate lagged values z_t be $\bar{z}_t$, the symmetric threshold be γ, and g again index the goods, an "equilibrium threshold autoregressive model" (EQ-TAR) can be written as:

$$\Delta z_t^g = \begin{cases} \bar{\beta}^{g,out}\bar{z}_t^g + e_t^{out} & \text{if } z_{t-d_p}^g > \gamma^g \\ \bar{\beta}^{g,in}\bar{z}_t^g + e_t^{in} & \text{if } \gamma^g \geq z_{t-d_p}^g \geq -\gamma^g \\ \bar{\beta}^{g,out}\bar{z}_t^g + e_t^{out} & \text{if } -\gamma^g > z_{t-d_p}^g \end{cases}$$

where $e_t^{out} \sim N(0, \sigma_E^{out2})$ and $e_t^{in} \sim N(0, \sigma_E^{in2})$. As an alternative econometric model, band-threshold autoregressive model (BAND-TAR) is estimated as well. BAND-TAR forces data to converge towards the bound, rather than towards its mean as in EQ-TAR:

$$\Delta z_t^g = \begin{cases} \bar{\beta}^{g,out}(\bar{z}_t^g - \gamma^g) + e_t^{out} & \text{if } z_{t-d_p}^g > \gamma^g \\ \bar{\beta}^{g,in}\bar{z}_t^g + e_t^{in} & \text{if } \gamma^g \geq z_{t-d_p}^g \geq -\gamma^g \\ \bar{\beta}^{g,out}(\bar{z}_t^g + \gamma^g) + e_t^{out} & \text{if } -\gamma^g > z_{t-d_p}^g \end{cases}$$

where $e_t^{out} \sim N(0, \sigma_B^{out2})$ and $e_t^{in} \sim N(0, \sigma_B^{in2})$. Because large deviations from the mean in a BAND-TAR model gravitate more strongly towards the band than the smaller ones, this model necessarily produces faster conditional speeds of convergence. Vectors of estimates $\bar{\beta}^{g,in}$ and $\bar{\beta}^{g,out}$ determine the conditional convergence speed of the law of one price deviation Δz_t^g outside the no-arbitrage bound γ^g. It is assumed that a law of one price deviation that lies within these bounds does not exhibit mean reversion due to transportation-related costs, good characteristics, trade barriers and other impediments preventing arbitrage: a restriction of $\bar{\beta}^{g,in} = 0$.

2.3.2 Methodology

Specification and estimation of a nonlinear model (in this case TAR(2,p,d)) proceeds in several steps[22] that are repeated for all 63 product groups and for the CPI-based real exchange rate. First, an appropriate lag-structure p of the linear model is determined. Secondly, the delay parameter d is chosen. Finally, the non-linear model is estimated and tested against its linear alternative. Specifically:

[21]One threshold following sufficient appreciation, another one after depreciation.

[22]See Granger and Teräsvirta (1993), Teräsvirta(1994), Tsay (1986), Tsay(1989).

1. With monthly data, up to 12 lags are considered. Examination of the partial autocorrelogram (Granger and Teräsvirta (1993)) narrows the potential candidates for lags. Out of the potential candidates, I choose a combination with the lowest Akaike information criterion (or Schwarz Bayesian information criterion) as long as the residuals are not serially correlated and are normally distributed[23]. An appropriate model specification is important at this stage because omitted autocorrelation may lead to a rejection of the null hypothesis of a linear model and in general make if difficult to interpret test results (Kilian and Taylor (2003)).

2. For a given set S of feasible values of the delay parameter d (here, $S = \{1, 2, ..., 12\}$) and for a given lag structure of the AR model determined in the previous step, the optimal d_p is selected by a procedure suggested in Tsay (1989):

$$\hat{F}(p, d_p) = \max_{\nu \in S} \hat{F}(p, \nu)$$

where $\hat{F}$ is the F-statistic described in the Appendix B.1. This procedure selects the value of the delay parameter which gives the most significant result in testing for a non-linearity. P-values of the optimal d_p can then also be used as a general nonparametric test of non-linearity.

3. For a given lag structure and a delay parameter d_p, parametric estimation uses maximum likelihood estimation in a procedure described in Obstfeld and Taylor (1997) (who in turn follow Fanizza (1990), Balke and Fomby (1997) and Prakash (1996)). The procedure is a best-fit grid search for a threshold parameter γ that maximizes the log-likelihood ratio $LLR = 2(L_a - L_0)$ where

$$\begin{aligned}
L_a(\beta_{in}, \beta_{out}, \sigma_{in}, \sigma_{out}, \gamma) &= -\frac{1}{2} n_{in} \Big[\log(2\pi) + \log\Big[\frac{\sum_{I_{in}} e_{t,in}^2}{n_{in} - 2}\Big] + \frac{n_{in} - 2}{n_{in}} \Big] \\
&\quad -\frac{1}{2} n_{out} \Big[\log(2\pi) + \log\Big[\frac{\sum_{I_{out}} e_{t,out}^2}{n_{out} - 2}\Big] + \frac{n_{out} - 2}{n_{out}} \Big], \\
L_0(\beta, \sigma) &= -\frac{1}{2} n \Big[\log(2\pi) + \log\Big[\frac{\sum e_t^2}{n - 2}\Big] + \frac{n - 2}{n} \Big] \\
\text{where } e_{t,in} &= \Delta z_{t,in} - \beta_{in} \bar{z}_{t,in}
\end{aligned}$$

[23]Selected combination is tested for residual serial correlation using Breusch-Goodfrey LM test and by examining the Q-statistic, and for the residual normality using Lomnicki-Jarque-Berra statistic. Residual normality is frequently rejected which can be result of the sample size. Most of the time, these criteria select the same model.

$$e_{t,out} = \Delta z_{t,out} - \beta_{out}\bar{z}_{t,out}$$
$$\text{and } e_t = \Delta z_t - \beta \bar{z}_t$$

Choices of γ with less than 10 observation above the threshold are not considered in order to lower the sample bias. The half-life of adjustment is often used as a way of assessing the speed with which deviations from LOP return to their respective bands. When the regression includes more than one lag, the half-life of convergence to a shock is computed numerically[24].

4. Two tests are used to assess the non-linear TAR against the linear alternative: likelihood ratio test and Tsay's general nonparametric F-test. First, the likelihood ratio test uses a statistic obtained during the grid-search. The likelihood ratio statistic does not follow the asymptotic χ^2 distribution in a non-linear model. Because the threshold parameter γ is not identified under H_0 of linearity[25], Monte Carlo simulation is used to obtain p-values of the LLR statistic for this case Second, Tsay's general nonparametric F-test uses the minimal p-value of the two F-statistic described in Appendix B.1: one from an arranged regression using ascending ordering of the case data, another with descending orderings of the case data (see Obstfeld and Taylor (1997)). So far, I have only run this test on a TAR(2,1,1), not the general TAR(2,p,d) model.

2.3.3 Non-linear estimation results

This subsection documents the findings of the time series estimations. In summary, the non-linear models perform significantly better than linear autoregressive models in characterizing the law of one price deviations. Most of the time series are non-stationary and significantly non-linear, well suited for analysis by threshold autoregressive models[26]. Application of augmented Dickey Fuller tests in this paper takes into account the appropriate lag structure chosen by analyzing the

[24]For AR(1), half-life $= -\frac{\log(2)}{\log(1+\hat{\beta}^{*g})}$ where $\hat{\beta}^{*g}$ is Kendall's bias-adjusted slope coefficient: $\hat{\beta}^{*g} = \frac{n\hat{\beta}+1}{n-3}$.

[25]This is another way of saying that there exist other sets of restrictions that make the TAR model linear.

[26]The precision with which we can conclude non-linearity or non-stationarity depends on the length and breadth of the sample as well as on whether the test statistic controls for the serial correlation of the error terms. O'Connel (1998) shows how failure to account for serial correlation leads to serious size distortions. Papell (1997) shows that various panel datasets provide stronger rejection of the unit root hypothesis than a similar time-series analyses. While panels improve the power of unit root tests, they suffer from series of other problems (see, e.g., Lyhagen (2000), Bornhorst (2003), Banerjee, Marcellino & Osbat (2001)). It should also be noted that, in addition to all problems mentioned above, the power of the unit root tests further drops when the underlying DGP is not linear.

partial autocorrelation function. Table B.10 reports the results of the Augmented Dickey Fuller tests for all series. In the majority of cases, ADF can not reject H_0 of a unit root. Unit roots appear to be rejected for the more valuable series with the notable exception of foods. Tables B.11 and B.12 summarize the results for two versions of the threshold autoregressive model.

AR(1)

Although the slope coefficients on all stationary AR(1) regressions are significant, the majority of the regressions are misspecified because the optimal lag structure is rarely AR(1) (misspecification results are not reported). Unit root hypothesis can be rejected for just 16 of the initial 47 goods (see Table B.10). The half-life estimates show a large degree of heterogeneity in mean reversion across goods: they are 8 years on average with a standard deviation of 17 years. Pooling products into groups (see Tables B.11 and B.12), we see that vehicles and car parts, clothing and footwear have the lowest half-life of 20 months. Following them are toys (31 months), foods (46), fuels (51), furniture (60), vice goods and jewelry (72), laundry appliances (93), educational books and supplies (119), technological goods and equipment (156) and medical products and house chemicals (235 months). Two sources of model misspecification complicate interpretation of half-lives: a potential nonlinearity of the underlying series[27], and an inappropriate lag structure even when the series is linear. These problems are to some extent eliminated by threshold autoregressive models.

TAR(2,1,1)

A TAR(2,1,1) eliminates one of the two sources of misspecification. However, it does so in an imprecise way by assuming that the optimal delay parameter $d = 1$ which need not be the case. Nevertheless, the threshold heterogeneity is clearly visible in the results (table B.15). The conditional half-lifes decline relative to the AR(1) estimates - a necessity with any SETAR model. The average estimate of a half-life declines from 96 months in AR(1) to 49 months in EQ-TAR (2,1,1) and 70 months in Band-TAR(2,1,1). Half-lives in BAND-TAR range from 2 months for watches to 630 months for medical care products, and in EQ-TAR from 2 months for potatoes to 346 months for car maintenance. The average size of a threshold is 11.7%, and varies from 0.8% for watches to 40% (31% in BAND-TAR) for potatoes.

[27] Obstfeld & Taylor (1997) and Kilian & Taylor (2003) show how nonlinearity leads to an upward bias of the half-life estimates.

I then estimate an appropriate lag-structure p (see table B.10) and the optimal delay parameter d for each of the 63 series. The appropriate lag structure is AR(1) in less than half of the situations. The optimal delay parameter equals one in only 10 cases and equals on average 6.8 months. This suggests that prices react to shocks with an average delay of more than half a year. The non-linearity of the series is also clearer when the autocovariance structure of the series is taken into account. Tests reject linearity for 66% of all stationary series.

TAR(2,p,d)

Due to treatment of miss-specification, the number of non-stationary series drops from 7 in AR(p) to one in a BAND-TAR and 5 in EQ-TAR.

Thresholds Threshold estimates γ^g denote the width of a symmetric no-arbitrage band: nominal exchange rate deviation has to cross the threshold before, d months later, reversion towards a mean (towards threshold in case of a BAND-TAR) occurs. The average threshold of price deviation is $\pm 7.9\%$ (a threshold of 0.0785) from the mean. The thresholds vary from $\pm 0.6\%$ for footwear to $\pm 30\%$ for tobacco[28]. A weighted average of thresholds where weights follow the RER weights equals $\pm 4\%$, which is in between the TAR(2,1,1) and TAR(2,p,d) thresholds of $\pm 7.1\%$ and $\pm 1.2\%$.

Because the thresholds are symmetric, their estimates are obviously bounded from above by the degree of variability of the series. The correlation of $std(z_t)$ and γ is 0.55. The relationship between the thresholds and physical characteristics of goods that has been suggested by the model of chapter one is examined in subsection 2.3.5 below.

Persistence The half lives of convergence reported in the tables are *conditional* on a shock that brings z_t outside the no-arbitrage bounds. Moreover, the unconditional half lives vary greatly across goods as they combine the effects of heterogenous threshold with that of a conditional half-life. A simple average of the half-life of deviation of a CPI component contained in the dataset is 94 months according to a BAND-TAR model. This amounts to 48% of the persistence of the CPI-based RER as measured by a half-life of convergence in a BAND-TAR model. A weighted average of the components in the dataset is 188 months, 97% of the persistence of a CPI-based

[28]The standard deviation of thresholds is 0.074.

RER according to a BAND-TAR model[29]. This excercise illustrates that the persistence of the real exchange rate can be decomposed into persistence of its components[30]. Consequently, our understanding of the adjustment of individual goods' prices across borders is essential.

2.3.4 Testing

The estimation results are tested in two ways. First, I test the appropriateness of the threshold non-linearity assumption in all threshold-autoregressive models using Tsay's F-test for non-linearity. Second, I test the likelihood that the non-linear model describes the data better than the linear autoregressive model using a log-likelihood ratio test. Both tests are meant to assess the robustness of the findings. As another robustness check, note that Equilibrium TAR and Band TAR give identical estimates of threshold in 67% of cases, while their estimates of speeds conditional on the thresholds coincide in only 15% of all cases.

Tsay's F-test for non-linearity

Tsay's test for non-linearity is described in detail in Appendix B.1. When applied under the assumption that AR(1) is the data generating process, Tsay's test rejects the H_0 of linearity for 16 out of the 42 stationary series. The non-linearity of the series is clearer when the autocovariance structure of the series is taken into account. Under AR(p), linearity is rejected for 66% of the stationary series.

Log-likelihood ratio test

Log likelihood ratio (LLR) test statistic has been conveniently computed during the TAR(2,p,d) estimation. It measures the difference between log-likelihoods of an optimal TAR(2,p,d) and a corresponding AR(p) model. However, LLR statistic of a non-linear model does not follow the usual χ^2 distribution because the parameters of the nonlinear alternative are not identified under H_0 of linearity (Obstfeld & Taylor (1997), Granger & Teräsvirta (1993)), i.e., there exists more than one set of restrictions which makes a TAR(2,p,d) model linear. I use Monte Carlo simulation to obtain an empirical distribution of LLR for all goods and from it compute the empirical p-values of LLR statistics Tables B.16 and B.18 provide empirical p-values of the hypothesis that

[29]Conditional TAR half-life is used when LLR test shows that TAR(2,p,d) model is preferred to AR(p). Otherwise, AR(p) half-life is used. Also, categories of goods that overlap are only used once.

[30]Imbs et. al (2004) show how inferences can not be made in opposite direction: RER persistence measure does not imply much for the persistence of the component series.

a TAR(2,p,d) is better than AR(p) for EQ-TAR and BAND-TAR models, respectively. At 10% significance level, LLR test results imply that TAR(2,p,d) is a better model than the linear AR(p) for 40% of series (30% in Equilibrium TAR).

2.3.5 Determinants of no-arbitrage bounds

I analyze the determinants of thresholds through a lens of the linear shipping cost model of chapter one. Estimates of the no-adjustment bands are related to the estimates of weights (and price-to-weight ratios) as well as to estimates of volume (and price-to-volume ratios) across goods. This amounts to an indirect test of the theory.

Product weight and no-arbitrage bounds

When a full dataset of goods is used, price-to-weight ratios are negatively but insignificantly correlated with the thresholds estimates (see the first estimation in table B.20). After removing tobacco from the dataset (tobacco data exhibits a structural break due to a differential tax change in 1984 - see figure B.2), the estimation results are significant at a 10% level (estimation 2 in table B.20). An increase in the price-to-weight ratio of a good by one \$/kg narrows its no-adjustment threshold by 2.4 percentage points on average. This is a major finding. It shows that certain characteristic of a time-series estimation across a range of goods is significantly related to a completely different dataset of physical characteristics of those goods. Moreover, this is also a prediction of the model in chapter one.

There are groups of products for which shipping costs present only a small fraction of arbitrage trade costs. Trade in natural gas or gasoline is controlled, and significant distribution costs are necessary for trade in these goods. Licensing requirements present a barrier to trade in alcoholic drinks. For these groups of products, shipping costs constitute a small fraction of the arbitrage costs in these products. Indeed, estimations that exclude these goods are more significant. Table B.23 presents findings of such regressions. If both alcohol and energy are excluded from the dataset, price-to-weight ratios are significant at a level of less than 1%.

Figures B.2 and B.3 plot the relationship between thresholds and price-to-weight estimates. A non-linear least squares estimation which assumes hyperbolic relationship between thresholds and price-to-weight ratios delivers a better fit: 23% of variability in thresholds can be accounted for by the variability in price-to-weight ratios (see table B.21)[31].

Relaxing the ad-hoc restriction about identical magnitude and opposite directions in which prices and weights affect the threshold improves the results[32]. The R^2 of the regression doubles and the significance of the regressors increases (see table B.22). In the absolute value, price elasticity of the threshold is 20 times lower than the weight elasticity. The effect of product weight is now 8-times larger than when price-to-weight ratio was considered. Standard deviation of the law of one price deviation is significantly negatively correlated with the weights of the products and their prices. All else constant, an increase in the weight of a product by 10kg is predicted to increase its no-trade bounds by 0.93 percentage points (on each side). Similarly, an increase in the price of a product by USD 100 will lower the no-arbitrage bound by 0.5 percentage points, ceteris paribus. The constant term (7.1%) absorbs all effects homogeneous across goods: unit shipping costs, identical trade barriers, etc.

Product volume and no-arbitrage bound

For many transportation modes, volume of the product is more important than weight for determining the cost of shipping. This is in particular true of sea shipping. Shipping costs are quoted per container (with various sizes available). Therefore goods that take less space (relative to their price) will have a relatively smaller per-dollar shipping cost. At times, Leontief-type combination of weight and volume is used to determine shipping costs. I estimate regressions identical to those above with price-to-volume data. Table B.20 shows that price-to-volume ratios are negatively correlated to thresholds at 5% level of significance: more voluminous products have wider thresholds in price deviations. The coefficient is smaller than for price-to-weight ratios. This finding is present more significantly when goods with high non-shipping cost component of arbitrage trade costs are excluded (see table B.23).

2.4 Conclusion

Physical characteristics of goods are important in explaining the dynamics of deviations of international prices of goods from parity. In particular, they are important in explaining the threshold

[31] A source of such nonlinearity could lie in an additional effect of fixed costs, different degrees of tradability or in any other factor I do not control for.

[32] Such assumption is only supported in a model without fixed costs to arbitrage (which in this model can be thought of as wage bill, legal, or other trade-related expenses homogeneous across goods).

nonlinearity across goods. In this chapter, an econometric model with a threshold nonlinearity shows that about one half of the persistence of the RER is *accounted for* by the existence of good-specific thresholds. Moreover, good-specific thresholds are in turn significantly related to the price-to-weight ratios as well as price-to-volume ratios obtained from a different dataset in a direction predicted by the arbitrage trade models from chapter one.

This chapter analyzes the law of one price deviations across 63 groups of goods and services between the US and Canada and combines them with a separate dataset on product weights. The estimation and testing of linear- and threshold-autoregressive models on monthly data (1970-2000) shows that the adjustment of prices of goods and services is often non-linear: there are bands of no adjustment where deviations of prices from parity do not revert towards the mean. First, in terms of the speed of convergence, services have an average half-life of 200 months while goods have an average half-life of 47 months. Second, one half of the persistence in the real exchange rate deviations can be accounted for by a proper econometric modeling of the prices of tradable goods, in that it is contained within the aforementioned thresholds that are left out as part of misspecification error in the AR models. Third, there is a large heterogeneity across goods and services in the dynamics of the law of one price deviations. Heterogeneity across thresholds is significantly negatively related to the estimates of the price-to-weight ratios of the tradable goods (the size of the no-adjustment bands varies from 0.5% for footwear to 30% for tobacco). More "valuable" goods have narrower no-trade bounds which is why their price adjusts faster to a shock to the law of one price deviation. A similar relationship is found between the heterogeneous half lives and price-to-volume ratios. These findings explain the heterogeneity of behaviour of price deviations from parity as a result of heterogeneity in physical characteristics of goods. They also yield support to the theoretical models outlined in chapter one which explain PPP puzzle as a trade-based phenomenon.

Chapter 3

Basle Accord and Financial Intermediation: The Impact of Policy

3.1 Introduction

[1] Traditionally, the literature on financial intermediation and credit channels, especially credit crunches, emphasized the relationship between banks and entrepreneurs requiring credit and neglected the funding of banks. With this paper, we want to be more precise in this respect and study the impact of funding on credit. Indeed, regulation that has become world wide with the Basel Accord limits the amount of bank loans with minimal levels of bank equity. How much equity the banks can issue depends in particular on the demand for equity by the households.

In our model economy, households have heterogenous asset holdings because they have different labor histories and because only some of them get credit as entrepreneurs (among those, the return on investment is stochastic). Non-entrepreneur households invest in bank deposits and bank equity, and banks maximize profits while following regulations. A central bank conducts the monetary policy and regulates the banks.

When banks need to reduce their loan portfolio, the displaced entrepreneurs become new equity holders, thereby acting as "automatic stabilizers". However, banks typically cut loans as a consequence of their loan portfolio becoming too risky, and households may then want to hold less equity in banks that are now more risky. Whether and how much banks have to tighten credit depends primarily on the distribution of assets across households and their equity decisions.

We solve the model using numerical methods, in particular for the transitional dynamics that may lead an economy into a possible credit crunch. We then analyze policies that may help the economy out of a trough. We find that the endogenous distribution of assets has strong implications that should not be neglected in future research. Also, monetary policy can only have positive real effects if the central bank is able to commit.

We find some evidence in our model that a credit crunch can arise in the presence of capital

[1] This chapter is based on work with Christian Zimmermann.

requirements. Numerical simulations show that the crunch is not very large. It is natural to look at whether flexible capital requirements can change the loan volume dynamics. Although one may think that loosening those requirements in a trough will expand the loan mass, the contrary is the case. As tighter capital requirements increase the demand for equity, they facilitate the financing of banks sufficiently to offset the reduction of allowable loans for a given level of equity. Again, this highlights the importance of household savings decisions. This result is particularly important as the new Basel Accord with its more flexible requirements would essentially tighten requirements when the economy undergoes a recession.

We are not the first to highlight the real impact of monetary policy through bank lending. Bernanke and Gertler (1995) highlight two channels. In the balance sheet channel, central bank's policy affects the financial position of borrowers and hence there ability to post collateral or self-finance. In the bank lending channel, central bank's policy shifts the supply of bank credit, in particular loans. They argue the importance of the latter channel has declined with deregulation, as this channel relies on reserves. Van de Heuvel (2001) identifies another channel stemming specifically from Basel Accord like rules. The "bank capital channel" arises from maturity transformation through banks: higher short term interest rates depress profits, thus equity and capital adequacy. Their model has a very detailed banking structure, but neglects the problems of households and firms. Our model has a simpler banking structure but emphasizes the source of financing (households) and the demand for loans (entrepreneurs) by modeling occupational choice, savings and bankruptcy.[2]

Chami and Cosimano (2001) identify a similar channel, called "bank-balance sheet channel", using the concept of increasing marginal cost of external financing. As Van den Heuvel, they need market power in the banking industry to obtain the result. Our model has fully competitive banks. Furthermore, they summarize the demands for loans with a reduced form while we try to come closer to a general equilibrium framework. Bolton and Freixas (2001) find that capital requirements can be the origin of a credit crunch. Their model is very detailed on the lending market and asymmetric information. Our model puts more emphasis on the financing side and does not require asymmetric information.

The structure of this chapter is as follows: section 3.2.2 analyzes the heterogenous behavior

[2]The heterogeneity of firms we obtain is then endogenous. Bernanke, Gertler and Gilchrist (1998) also have heterogeous firms, but they exogenously fix a share of firms to have easy access to credit.

of households, sections 3.2.3 and 3.2.4 analyze the (homogeneous) financial sector and the central bank, section 3.2.6 defines and analyzes the equilibrium and section 3.3 presents the calibration of the model. Section 3.4 analyses bank lending and optimal monetary policy behavior following negative shocks. Section 3.5 concludes. The appendices give additional details about various aspects of the model and the solution strategy.

3.2 Model

3.2.1 Overview

There are three types of agents in the economy: households, banks, and a central bank. Households in a productive stage of their lives want to become entrepreneurs, but a shortage of internal financing forces them to apply for external funds. Successful applicants become entrepreneurs and the others remain workers. Each worker faces an idiosyncratic shock of becoming unemployed while entrepreneurs have risky returns on investment. All households in a productive stage of life (entrepreneurs, employed and unemployed workers) face a risk of becoming permanently retired, and all retirees face a risk of dying. New households are born to replace the deceased ones.

When the households make their consumption–savings decision, savings are invested in bank deposits and bank equity. Banks collect deposits and equity, provide loans to entrepreneurs and purchase risk-free government bonds in order to maximize their profits. Banks screen loan applications and accept them according to the level of household's net worth. Banks have to purchase deposit insurance and are subject to a capital adequacy requirement imposed by the central bank. The central bank also controls the government bond rate.

We now go through the model in more detail. The economy is subject to aggregate shocks and thus can be represented by an aggregate state vector including the current shock and the current distribution of assets and occupations that we ignore in the following to simplify notation.

3.2.2 Households

In the model economy, there is a continuum of measure one of households, each maximizing their expected discounted lifetime utility by choosing an optimal consumption–savings path. A household can either be productive or retired, and the probability of a productive household retiring τ is exogenous[3].

[3] Once retired, household cannot become productive again.

A productive household i is endowed with one investment project of size x^i, which is always greater than the household's net worth m^i. We assume that the total investment is a fixed multiple of household's net worth: $x^i = \phi m^i$ where $\phi > 1$. The project is indivisible, and so $(\phi - 1)m^i$ has to be funded by the bank in order for a project to be undertaken[4]. If a household receives a loan it becomes an entrepreneur and invests into a project, receiving a return r^i drawn from a trinomial distribution. In equilibrium, distribution of returns is such that households always prefer investing into projects and becoming entrepreneurs to becoming workers[5]. The returns are drawn independently across households (i.e. projects) and time. The lowest of the returns is sufficiently negative with a positive probability to lead to bankruptcy, in which case a household is guaranteed a minimal amount of consumption c_{min} and starts next period with no assets.

When the bank rejects a loan application, the household enters the work force and faces exogenous probabilities $1 - u$ of becoming employed and u of becoming unemployed. Workers inelastically supply their labor and receive an after tax wage income y. Unemployed workers receive unemployment benefits θy where θ is the replacement ratio.

Labor supply is inelastic at an individual level. At the aggregate level, labor supply is determined by moves between the pools of workers, entrepreneurs, unemployed and retirees. This assumption increases the role asset accumulation plays in the economy. We use aggregate labor input data on the average hours per worker to calibrate the labor demand.

After retirement, the household earns income from its savings and pension (which equals unemployment benefit payments). Retirees face a probability δ of dying. They are then replaced by agents with no assets and any remaining assets are lost (no bequests).

The households make their consumption–savings decision to maximize their expected lifetime utility. The contemporaneous utility function is a CRRA type:

$$U(c)_j = \frac{(l_j^{\sigma} c^{1-\sigma})^{1-\rho} - 1}{1 - \rho}$$

where $j \in \{W, U, E, R\}$, l denotes leisure, c consumption and ρ is a risk-aversion parameter. As mentioned above, the labor supply is inelastic and the values l_j represent market-clearing values for leisure.

[4]Therefore at a household level, demand for loans is uniquely determined by the net worth and so by the history of consumption–savings decisions and luck.

[5]For entrepreneurs that receive loans, the participation constraint is always satisfied in our simulations.

Let V_j denote the value functions and m^* be the minimum net worth necessary for external financing. A worker with a net worth m ($< m^*$) faces probability $(1-u)$ of being employed, following which he receives labor income $y = (1-l_W)w$ and interest income $R^d m$, pays a banking fee ξ[6], consumes a desired level and invests his remaining net worth m'[7] in a bank. If unemployed, he receives unemployment benefit payment θy and makes a similar consumption–savings decision. In the next period, depending on the level of m', a worker may either become an entrepreneur (borrower) or remain a worker (depositor).

For an employed worker, the Bellman equation is:

$$\begin{aligned} V_W(m^i) = \max_{c^i, m^{i'}} \{ & U_W(l_W, c^i) + \beta[(1-\tau)[(1-u)V_W(m^{i'}) + \\ & uV_U(m^{i'}) + E_{r'}V_E(m^{i'}, r^{i'})] + \tau V_R(m^{i'})]\} \end{aligned} \tag{3.1}$$

s.t.

$$c^i + m^{i'} = (1 + r^d)m^i + y - \xi$$

For an unemployed worker:

$$\begin{aligned} V_U(m^i) = \max_{c^i, m^{i'}} \{ & U_U(l_U, c^i) + \beta[(1-\tau)[(1-u)V_W(m^{i'}) + \\ & uV_U(m^{i'}) + E_{r'}V_E(m^{i'}, r^{i'})] + \tau V_R(m^{i'})]\} \end{aligned} \tag{3.2}$$

s.t.

$$c^i + m^{i'} = (1 + r^d)m^i + \theta y - \xi$$

An entrepreneur i invests in a project of size x^i, earns a stochastic net return r^i and labor income $y = (1 - l_E)w$ and pays the borrowing cost $r^l(x^i - m^i)$, while making a consumption–savings decision to maximize her expected utility. Because the net wealth is constrained to be non-negative, significant project losses may drive the entrepreneur into bankruptcy. When bankrupt, an entrepreneur defaults on the portion of the debt he can not repay less a minimal consumption allowance c_{min} which has to be granted by the bank. Upon default, entrepreneur starts the next period as a household with no assets and no liabilities. The returns on project r_i are drawn independently across time and individuals and follow a trinomial distribution. The lowest

[6]We will justify in the calibration the use of ξ.

[7]A prime $'$ denotes variable values in the next period.

of the returns is sufficiently negative to lead the entrepreneur to bankruptcy. For an entrepreneur:

$$V_E(m^i, r^i) = \max_{c^i, m^{i'}} \{U_E(l_E, c^i) + \beta[(1-\tau)[(1-u)V_W(m^{i'}) + uV_U(m^{i'}) + E_{r'}V_E(m^{i'}, r^{i'})] + \tau V_R(m^{i'})]\} \quad (3.3)$$

s.t.

$$c^i = \max\{c_{min}, m^i + y + (1+r^i)x^i - r^l(x^i - m^i) - \xi - m^{i'}\}$$

$$x^i = \phi m^i$$

Note that we assumed the size of the project is proportional to the entrepreneur's asset holdings. This can be justified by collateral requirements typically observed in the credit markets. In addition, ϕ can easily be quantified in the data. To stress the effects of the supply of credit, we assume that households *ex ante* always prefer to apply for a loan. This implies a participation constraint for households in a productive stage of their lives that needs to be satisfied for all households that effectively obtain a loan:

$$E_r V_E(m, r) \geq (1-u)V_W(m) + uV_U(m), \quad \forall m \geq m^* \quad (3.4)$$

Every household faces an exogenous probability of retirement τ. Once retired, the household collects retirement income $y_R = \theta w$ and manages its assets subject to the risk of death δ.

$$V_R(m) = \max_{c^i, m^{i'}} \{U_R(1, c^i) + \beta[(1-\delta)V_R(m^{i'})]\} \quad (3.5)$$

s.t.

$$c^i + m^{i'} = (1 + r^d)m + y_R - \xi.$$

Because of their risk aversion, the agents smooth their consumption over time. The presence of heterogeneous risks of unemployment and retirement as well as the heterogeneity in project returns lead to a non-degenerate distribution of assets in the economy. Intuitively, the individual risks along these dimensions substitute for the uncertainty of income which is modeled as fixed. Without these risks, there would be no reason to save other than to invest in a project, and the asset distribution would unrealistically collapse along $m = 0$ and $m = m^*$. This would not allow for financial intermediation because of lack of funds (no depositors). Any equilibrium in this bimodal distribution is very unstable because all entrepreneurs can drift to zero assets following a shock. The distribution of assets plays a crucial role in determining the dynamics of the aggregate variables.

The decision to allocate savings between bank equity and bank deposits is obtained by maximizing a risk-adjusted return on protfolio (r^{port}):

$$\max_{\omega_r} r^{port} - \frac{1}{2}\lambda\sigma^2_{port}$$

where $r^{port} = r^e \frac{E}{M} + r^d\left(\frac{D}{M}\right) = \omega_r r^e + (1-\omega_r)r^d$, $\omega_r \equiv E/M$ is a weight on the risky (equity) investment, λ is a risk-aversion parameter and σ^2_{port} is a variance of the portfolio return. Because bank deposits carry no risk ($\sigma^2_d = 0$), the household maximizes:

$$\max_{\omega_r} \omega_r r^e + (1-\omega_r)r^d - \frac{1}{2}\lambda\omega_r^2\sigma_e^2$$

which yields the optimal share of equity $\omega_r^* = \frac{r^e - r^d}{\lambda\sigma_e^2}$. This in turn defines the demand for equity (and implicitly for deposits) given savings M:

$$\frac{E}{M} = \frac{r^e - r^d}{\lambda\sigma_e^2} \tag{3.6}$$

Note that we have separated this portfolio problem from the intertemporal utility maximization of the household. This is for computational reasons: given that with aggregate shocks we need to include the entire asset distribution in the state space, we need to avoid having to track for each household two separate assets to keep the dimension of the state space within reasonable bounds. This means also that the share of equity is independent of the asset level. Such assumption is not necessarily innocuous. As the Appendix C.2 shows, as long as the households have the same labor income, their optimal splitting rule between equity and deposits is constant and identical for all households due to CRRA preferences. With labor income varying between workers and unemployed/retired, the optimal splitting rule may change.

3.2.3 Financial Sector

Bank

The representative bank maximizes its expected profits, taking the asset distribution in the economy as given. Profits equal asset returns less the funding costs, deposit insurance payments and the expected loan losses and liquidation costs. The bank's choice variables are loans L, bonds B, equity E and deposits D. Because the bank takes the distribution of assets as well as all returns as given, the choice of loan volume is identical to choice of a threshold level of net worth m^*. Formally, the problem can be stated as:

$$\max_{L,B,D,E} r^l L + r^b B - r^d D - r^e E - \delta\Big(\frac{D}{E}\Big)^{\gamma} D - (1 + lc)\epsilon L + \xi \tag{3.7}$$

subject to

$$\begin{aligned} B + L &= D + E = M && (3.8) \\ \frac{E}{L} &\geq \alpha && (3.9) \\ D + E &\geq L && (3.10) \end{aligned}$$

where M is the total amount of loanable funds that are exogenous from the point of view of the bank[8], δ is a per-unit deposit insurance cost parameter, ϵ is an expected share of loan losses, ϵ determines the loans facing bankruptcy losses, and lc is a liquidation cost parameter. Equation (3.8) is the usual balance sheet constraint, (3.9) is the regulatory requirement on capital adequacy and (3.10) is a non-negativity constraint on bond holdings. The profit function (3.7) is non-linear due to the inclusion of deposit insurance costs which are an increasing function of the deposit/equity ratio. Because profits increase in loans for any given asset distribution, one and only one of the constraints (3.9) and (3.10) will bind at any time[9].

The solution of the profit maximization is described in the appendix.

3.2.4 Central bank

The central bank in the model sets the bond interest rate r^b and elastically supplies (government) bonds at this rate. It also determines the capital asset ratio parameter α. α and r^b are the only monetary policy instruments. In the simulation section 3.4 we show how different monetary policy actions, as represented by mean preserving changes in r^b across the aggregate states, influence the behavior of the different types of households and of the representative bank.

[8]The total amount of assets flowing through the financial sector is determined by households' decisions. Half of the total "financial" assets (note that the self-financed part of entrepreneur's project does not enter financial sector) has to equal total bank liabilities=assets (see equation 3.14).

[9]The chances that both of them bind at the same time can be dismissed as arbitrarily low.

3.2.5 Market clearing

On the financial side, markets for loans, bonds, equity and deposits must clear. The bond market clears automatically because of an infinitely elastic supply of bonds[10]. The remaining market clearing conditions are:

$$D^S = D^D = \sum_{m^i < m^*} m^i(1-\omega_r) \tag{3.11}$$

$$E^S = E^D = \sum_{m^i < m^*} m^i\omega_r \tag{3.12}$$

$$L = \sum_{m^i \geq m^*} (\phi - 1)m^i \tag{3.13}$$

$$M = \sum_{m^i < m^*} m^i = D + E = B + L = \sum_{m^i \geq m^*} (\phi - 1)m^i \tag{3.14}$$

Moreover, expected losses of the bank must in equilibrium equal the realized loan losses:

$$\epsilon = \sum_{m^i \geq m^*} \max\Big\{0, (1+\mu)\Big[r^l(\phi-1)m^i - \phi m^i(1+r^i)\Big] + c_{min}\Big\}$$

where μ are auditing costs. The market clearing equations (3.11) – (3.14) connect the new homogeneous part with the heterogenous part of the model. The sum of individual demands for deposits, equity and loans on the right-hand sides must equal the supply levels decided on an aggregate level.

Equity market clearing implicitly defines the return on equity r^e as a function of all other returns. In the case of an *interior* solution, equations (C.2) and (3.6) imply:

$$\frac{1}{\delta}(r^e - r^d)^3 - \Big[\frac{1}{\alpha\delta}\Big(r^l - r^d - (1+l_c)\epsilon\Big) + 1\Big](r^e - r^d)^2 + 2\lambda\sigma_e^2(r^e - r^d) - \lambda^2\sigma_e^4 = 0 \tag{3.15}$$

In the case of a *corner* solution, equations (C.5) and (3.6) imply:

$$\begin{aligned} r^{e3} - r^{e2}\Big[2r^d + r^l - (1+l_c)\epsilon + 1\Big] - r^e\Big[r^{d2} + 2r^d(r^l - (1+l_c)\epsilon + 1) + 2\lambda\sigma_e^2\Big] \\ -\Big[r^{d2}(r^l - (1+l_c)\epsilon + 1) + 2\lambda\sigma_e^2 r^d + \delta\lambda^2\sigma_e^4\Big] = 0 \end{aligned} \tag{3.16}$$

To illustrate the functioning of the equity market, it is useful to undergo a following thought experiment. Consider a case of an increase in the lending interest rate r^l, possibly because of an increase in the demand for loans. As long as the ratio of expected losses as a proportion of loans ϵ rises less than r^l, the bank's profit margin on each new loan goes up, which prompts the

[10]One can think of banks depositing their non-loanable investments at the central bank which also sets the deposit rate in this model.

bank to lend more. To do so, bank has to raise more equity (it starts without any excess of it: $E = \alpha L$), which is why the equity supply equation (C.2) is increasing in the loan profit margin. The demand for equity (3.6) is unaffected by the return on loans, and so to raise more of equity, the bank's offered r^e has to increase. Note that because the government bond rate is exogenous and it determines the deposit rate in an interior solution (which is the norm), and because the bank can not choose the size of its balance sheet M, r^e plays an important role in the bank's liability management. Its increase will lead to a rise in the total amount of equity raised and to a more-than-proportional increase in the E/D ratio for any size of the balance sheet M[11].

It is therefore easy to see that when the bank increases the share of loans in its portfolio, it has to fund the higher equity holdings at an ever-increasing price. Eventually, the original profit margin disappears and a new optimal loan level is achieved. Two cases can occur. First, the total amount of new loans is less than the new balance sheet level, loan market clearing conditions are satisfied and constitute a potential equilibrium. Secondly, the total amount of new loans may exceed the new balance sheet volume M, which is what we defined earlier as a *corner solution.* In the latter case, the loan market does not clear and the banks ration some of the eligible loan applicants. Because there is no asymmetric information problem in this model (hence no adverse selection), an increase in the price of loans does not affect their quality and a higher r^l is needed to clear the market. Therefore we have a choice of focusing on market-clearing equilibria which rule out corner solutions and equity "hoarding", or allowing credit rationing when multiple equilibria may arise and excess equity is kept as a backup in case the total amount of loanable funds M increases. For simplicity, we only focus on the market-clearing equilibria, and only equation (3.15) becomes relevant. One of the implications of this is that we will never observe banks hold excess equity in equilibrium, and so regulatory changes in capital adequacy ratio ρ will have a direct effect on the loan volume.

The above market clearing condition (3.15) defines a return on equity as function of all other returns and some parameters: $r^e = r^e(r^l, r^d, \sigma_e^2, \lambda, \alpha)$. The above cubic equation can be solved analytically but does not determine the r^e uniquely. Depending on the parameter values, two of the three roots are complex and disregarded in the following calculations.

We now have a recursive system. Conditional on M, equation (C.2) determines the optimal

[11] This follows from the fact that $\frac{E}{D} = \frac{\omega_r}{1-\omega_r}$ and ω_r increases in r^e.

level of equity E, equation (C.4) determines the optimal level of deposits D, equation (3.14) determines the optimal level of bonds B and equation (C.3) determines the optimal level of loans L. We therefore have $\{r^e, r^d, E, D, L, B\}$ as a function of $\{r^l, M\}$ and exogenous variables.

3.2.6 Equilibrium

A recursive equilibrium in this model economy are four value functions $V_j(m,s)$, where s represents the aggregate state (current shock, distribution of m), for $j \in \{E, W, U, R\}$, decision rules $\{g_j^m(m,s), g_M^d(s), g_M^e(s), g_B^{m^*}(s), g_B^{r^l}(s)\}$, government policies $\{\alpha(s), r^b(s)\}$, prices $\{r^d(s), r^{port}(s), r^e(s)\}$, aggregate asset levels $\{L, D, B, E\}$, and a function $\Psi(\mu)$ such that:

1. decision rules $g_j^m(m,s)$ solve each household's problem with the associated value functions $V_j(m,s)$.

2. decision rules $g_M^d(s)$ and $g_M^e(s)$ solve portfolio problem of the household.

3. decision rules $g_B^{m^*}(s)$ and $g_B^{r^l}(s)$ solve the banks' problems.

4. loan, equity and deposit markets clear:

$$L(s) = \sum_{m \geq m^*} (\phi - 1) m \mu(m,s) \tag{3.17}$$

$$E(s) = \frac{r^e - r^d}{\gamma \sigma_e^2} \sum_{m < m^*} m \mu(m,s) \tag{3.18}$$

$$D(s) = \left(1 - \frac{r^e - r^d}{\gamma \sigma_e^2}\right) \sum_{m < m^*} m \mu(m,s) \tag{3.19}$$

5. the distribution of households is the fixed point of the law of motion Φ:

$$\mu'(m,s) = \Psi(m,s)$$

3.3 Parametrization

To simulate the economy and obtain numerical results, we parametrize the model to the Canadian economy in the years of 1988 to 1992, in accordance with the available data on project return distributions. Indeed, these are the only years for which Statistics Canada published such data.

First we calibrate the household sector. Several parameters are set in accordance with the literature: $\rho = 2.5$, $\beta = 0.96$ and $\sigma = 0.67$. In accordance with the models that include explicit leisure specification, $l_E = l_W = l_U = 0.55$ while l_R=1, as a result of which the labor input of entrepreneurs and workers, and the search effort of unemployed are set to 0.45. Wages are exogenous and while they completely characterize the labor income of entrepreneurs and workers, the incomes of unemployed and retired are determined by the ratio of unemployment insurance benefits to wages $\theta = 0.2929$[12].

The probability of unemployment is set equal to the average Canadian unemployment rate for the considered period: $u = 0.0924$. The probability of retirement τ and the mortality rate δ are set at 0.05 and 0.1, so that the number of expected periods while worker and retiree are 20 and 10, respectively. Longer expected lifetime horizon allows us to utilize the effect of savings over time more fully than in the usual 2-period models (e.g. Williamson (1987) and Bernanke and Gertler (1989).

Now we turn to the financial side. Following the calibration in Yuan and Zimmermann (1999), we set the real bond rate r^b at 1%, such that the deposit rate r^d is about 0.9%, which corresponds to an average of savings rates and guaranteed investment certificate rates. The parameter α of the capital adequacy constraint is taken to represent the tier-1 capital requirements imposed by the Basle Accord (1988) and set to $\alpha = 0.08$. The deposit insurance parameter δ is calibrated using the premium rates of the Canadian Deposit Insurance Corporation for banks in 2000/2001 (0.0417% of insured deposits). This per-unit rate corresponds to $\delta = 0.0000417$ for an average D/E ratio of 10. The loan administration cost l_c is assumed to equal 0. The account flat fee ξ is set at 0.0003 by trial and error in order to get the banks to break even. The parameters of the equity market that need to be calibrated are λ and σ_E^2. The variance of returns on equity of the banks is calculated from the TSE monthly series on financial enterprises' returns on equity from September 1978 until December 2000, which are deflated by the CPI. Therefore, $\sigma_E^2 = 0.24$. The risk-aversion parameter of the portfolio optimization problem λ is calibrated from the market clearing condition (3.15) using the observed average real deposit, lending and ROE rates. This implies $\lambda = 16$.

The distribution of returns follows a two-state Markov process calibrated such that the high state occurs 75% of the time. Specifically, a high state has a 75% chance of reoccuring the next period, while a low state can repeat itself with a 25% chance.

[12]This measure is based on the replacement rate of Hornstein and Yuan (1999).

The distributions of project returns in both aggregate states are calibrated from firms' return on equity data. Statistics Canada (1994) reports the distribution of return on equity by non-financial enterprises from the fourth quarter of 1988 until the fourth quarter of 1992. Average returns in each quarter are reported for the top, middle and bottom tertile. Assuming the underlying distribution is normal, we find the returns and associated probabilities for trinomial distributions such that a) average returns are replicated, b) we have have two extreme returns, one implying bankruptcy. We compute two such distributions, one for the high aggregate state, corresponding to the average of the 75% best quarters in the sample period, and the other for the low state. The returns and the associated distributions are the following:

$$\text{High:} \begin{pmatrix} -50\% & 5.2\% & 60\% \\ 0.71\% & 98.48\% & 0.81\% \end{pmatrix} \text{Low:} \begin{pmatrix} -50\% & 2.57\% & 60\% \\ 1.79\% & 97.42\% & 0.79\% \end{pmatrix}$$

The ratio of investment to net worth $(\phi - 1)$ is calibrated to equal the average debt-equity ratio during the reference period, and so $\phi = 2.2$. With a minimum return on investment of -50%, we have occasional bankruptcies. The auditing costs ν are assumed to equal 0.03.

3.4 Capital requirements, bank lending and monetary policy

We now want to understand the behavior of the model economy. This task is made more difficult by the complexity of the model and richness of the aggregate state space. Many histories can be imagined in this environment, and discretion needs to be used in selecting those we want to focus on. In this section we consider a history which is empirically relevant from the business cycle perspective. Starting from a steady state, the experiment begins with a succession of five Low shocks followed by five High shocks. Thus, the model economy goes through the whole cycle, bottoming out in the middle. Note that this a particular history of shocks among many others, and that this history is not anticipated. In Figure C.1, we show the behavior of various indicators in a benchmark economy, that is with no policy intervention from the central bank on bond rates or capital requirements.

When the initial bad shock hits the economy, the lending rate jumps up, essentially to cover higher than expected loan losses. As more bad news accumulate, the lending rate decreases as m^* reacts and the households adapt their asset levels. Indeed, banks ration more and more as bad shocks accumulate, but revert to "normal" behavior as soon as good news come in. From peak to trough, the amount of loans decreases by 3.0%, and 3.6% of all entrepreneurs are driven out. The consequence is that the size of an average loan increases by 0.6%, corresponding to the empirically documented phenomenon that small businesses are hurt more when credit conditions worsen.

Do we have evidence of a credit crunch in this benchmark economy? Despite the fact that banks can increase the loan rate to compensate for higher rates, they have to decrease the total loan mass. The reason is the following. Facing increased risk, more entrepreneurs are forced to become workers, as the bankruptcy rate is higher. With more agents that save, the volume of assets increases. However, a smaller share of those assets are channeled to bank equity because its return is too low given its risk. The banks are then squeezed by the capital requirement and have to ration credit and invest more into "unproductive" government bonds. Without the capital requirement, banks could give more loans, in principle, by charging even higher loan rates, and entrepreneurs would still be ready to pay these rates. Although all agents behave optimally, we have a situation that can be described as a credit crunch, where marginal return and marginal costs of loans are not equal.

Capital requirements imply that changes in the compostition of banks' liabilities affect the amount of credit in the economy. An adverse productivity shock increases the number of depositors and lowers the number of borrowers. Yet risk averse depositors shy away from the highly risky bank equity which leads to a further credit decline (due to the capital requirements). However, the movements described above are relatively small.

3.4.1 Countercyclical monetary policy

The following experiments will help us understand what are the consequences of various policy actions. The first policy experiment, described in Figure C.2, involves a 25 basis point reduction of the bond rate in the worst aggregate state (current shock Low, long history of Low shocks).[13] Thus, the central bank reacts only after a prolonged decline in the economy. Note that the decisions of the banks are changed only in this specific state: m^* and the lending rate are unaffected when the central bank does not move, but when it does banks reduce the lending rate by the same margin and, more importantly, significantly relax their loan threshold m^*. Thus the situation for entrepreneurs should improve noticeably: easier access to credit at better conditions. Loan ac-

[13]Note that all experiments are designed such that the average r^b or m^* stay at the same level.

tivity is negatively affected, however, and equity is reduced compared to the benchmark. This is because workers' savings decline (interest rates are lower) and with them the total amount of equity (lower return). Note that household decisions are affected even when the central bank has left the bond rate untouched, in anticipation of possible changes. Ultimately, the same number of entrepreneurs gets loans and the average loan is now smaller.

A one-time drop in the interest rate therefore does not appear to be an effective policy. What now if the interest rate is gradually reduced by 5 points after each bad shock, and goes back to normal whenever a good shock comes by? This policy takes better into account the anticipations the households formulate. On Figure C.3, we see that the outcome is quite different. Banks become much more generous to entrepreneurs in bad times, both in terms of lower lending rates in bad times (but higher in good ones) and quite significantly in terms of m^*. In all states, there are more entrepreneurs, loans, deposits and equity. While the average loan is larger in normal times compared to the benchmark, it is smaller in almost any other. This means that asset accumulation has increased for households: entrepreneurship is more interesting as monetary policy counterbalances the increased risk in bad times. Indeed, while firms face lower average returns and higher bankruptcy rates, monetary policy forces banks to offer better conditions. This has an impact on asset accumulation even in good times. We conclude that an active countercyclical monetary policy can have a significant positive impact. Note, however, that it cannot remove the cyclical nature of loans.

3.4.2 Procyclical monetary policy

If some policy of interest rate reduction may have negative consequences, one may naturally ask whether an interest rate increase can do some good. Indeed, higher bond rates mean higher returns on savings, and potentially more equity to satisfy the loan needs in the presence of capital requirements.

In Figure C.4, we find that the model economy does not behave in a symmetric way, as compared to Figure C.2. While the lending rate increases as expected, m^* stays essentially put rather then shoot up. Consequently, loan activity does not change much as households barely change their decisons compared to the benchmark. The sum of all tiny changes results, however, in a noticeably decrease in the average loan size, but not as strong as in the opposite policy.

Comparing Figures C.3 and C.5, it appears that the same kind of asymmetry exists for a gradual policy. A gradual increase of the bond rate has a negative, but much smaller impact on the various assets.

An explanation of this asymmetry is as follows. Procyclical monetary policy induces a drop in m^*, leading to an increase in the loan volume as more smaller agents can become entrepreneurs. Moreover, a lower m^* induces workers to save more (consumption drops) at any given deposit rate because the entrepreneurship is more likely to be attained (this move is slightly offset by the distributional movements as there are fewer workers and more entrepreneurs). Because of such boom in banks' liabilities, asset sides of banks' balance sheets expand which reinforces the initial loan volume increse.

On the other hand, a countercyclical monetary policy induces a small rise in m^*. This is a strong disincentive for saving for workers who want to become entrepreneurs, and leads to a drop in the volume of deposits and equity. Such drop is partly offset by an increase of the pool of depositors and a rise in the deposit interest rate. These offsetting moves are behind the relatively small changes in the volume and the composition of banks' balance sheets.

Banks' decision to change m^* in an asymmetric way is just a reflection of the equilibrium nature of the problem. With procyclical monetary policy, banks' desire to give more loans requires a rise in their equity funding (capital requirements bind). Yet equity is more risky in bad states and households channel their savings away from equity and into deposits. Therefore, in order to expand their loans, banks must make the vision of entrepreneurship (a motivation for saving) highly desirable to get sufficient equity - hence a sharp drop in m^*. On the contrary, a countercyclical monetary policy motivates a loan volume drop which is achieved by an increase in m^*. Such increase can be small because for any amount of savings, risk-averse households prefer deposits in bad times anyway.

The heterogenous agent setup of this model highlights the effects of the changes in distribution of assets and bank financing on loan activity. In particular, it shows the asymmetric propagation of the monetary policy.

3.4.3 Countercyclical capital requirements

The interest rate is one of two instruments the central bank can use. The other is to modify the capital requirements, which in the benchmark economy are set at a 8% equity/loan ratio, as in the Basle Accord. As it appears capital requirements have an impact on the model economy, one may want to establish whether it can be used for cyclical purposes as well. In the first experiment, Figure C.6, the equity/loan ratio is allowed to be reduced to 7% in the worst aggregate state only. While the banks can now offer more generous conditions, in this state only, households observe higher bank risk and shift from equity to deposits sufficiently to counterbalance and decrease the loan mass. As for a bond rate reduction, the average loan size decreases as the number of entrepreurs barely changes compared to the benchmark economy.

The next experiment involves a gradual decline of the capital requirements during the bad shocks, Figure C.7. One would expect that the regulator allowing the banks to take more risks during a downturn may generate more loans. To the contrary, equity declines even more, resulting in a smaller loan mass. Interestingly, loans are lower even when the regulator does not intervene and has in fact slightly more stringent capital requirements to maintain the same average as in the benchmark. The reasons are the same as previously: households shy away from banks when they take on more risk.

3.4.4 Procyclical capital requirements

If countercyclical capital requirements have adverse effects, maybe procyclical ones have a positive impact on lending ability. Figure C.8 looks at the punctual policy, Figure C.9 at the gradual one. Both policies have positive effects, locally and small for the first one, globally and massively for the second one. Thus it appears that tightening capital requirements is good for loan activity because it improves the financing of the banks. In this case the arguments are symmetric to the countercyclical policies.

Note that we have no informational problem in the model economy that would actually require the imposition of capital requirements. One can easily imagine that if the model would include this it would only reinforce the result: the presence of more entrepreneurial risk leads to a higher impact of asymmetric information and risk, thus furthering the need for regulation.

3.4.5 Credit crunch? What exactly happens in the model?

A negative aggregate shock lowers the expected project returns and increases their volatility. This affects the loan volume and the lending rate in four ways. *First*, both these effects decrease the expected value of risk-averse entrepreneurs ($E_r V_E$) while the value functions of non-entrepreneurial households do not change.[14] Therefore the incentive to accumulate assets in order to be eligible for a loan declines. This lowers the demand for credit because fewer agents save enough to pass the m^* cutoff. *Second,* the risk-neutral banks only care about the expected return of projects. The relative net payoff of bonds versus loans rises and induces a substitution from loans to bonds. The loan supply drops and the lending rate r^L increases to compensate for higher loan losses. This is the credit supply effect (i.e. the "crunch"). *Third,* an increase in r^L further discourages loan applicants because their net return on investment declines, and the equilibrium credit level drops further. Therefore the post-shock equilibrium exhibits a higher lending rate and a lower level of loans which further propagates the shock. Note that the decline in the market-clearing volume of credit is partly demand-driven, and cannot be only attributed to the the credit crunch behavior of the banks. *Fourth,* the household perceives more risk in the bank when entrepreneurial risk increases. It then shifts, on behalf of households, from equity to insured deposits, thus making it harder for banks to meet the capital requirements.

3.4.6 Does the equity market worsen or soften the credit decline?

The existence of equity market can either amplify or reduce the impact of a negative shock on a volume of credit. Only the second and fourth of the above mentioned four effects is directly affected by the existence of an equity market. The equilibrium condition (3.15) shows that only changes in r^L and ϵ affect credit behavior through the equity channel, and they do so in an offsetting manner. An increase in ϵ (higher loan losses) increases the return on equity r^E, while an increase in r^L lowers it. We therefore distinguish two cases. (A) If $\mathrm{d}\left(r^L - (1 + l_c\right)\epsilon) < 0$, then a rise in r^E increases the cost of funds to the bank which squeezes the profit margin further and leads to an additional substitution from loans to bonds (L drops) as well as an increase in r^L. At the same time r^{PORT} increases, making borrowing relatively less attractive (demand for credit drops). In this case, the presence of the equity market worsens the credit decline: a higher r^E is only compatible with a lower amount of equity E on the market[15], which in turn requires an additional drop in loans due to a binding capital adequacy constraint (see equation (C.3)). Case (B) when $\mathrm{d}(r^L - (1 + l_c)\epsilon) > 0$ has the opposite implication – it softens the effects of financial accelerator. According to the simulations (comparing peak and trough states), $\mathrm{d}(r^L - (1 + l_c)\epsilon) = 0.0002$ and

[14]There is only a second order effect coming from expectations to be an entrepeneur in the future.
[15]This is because households are risk averse while banks are risk neutral.

we can conclude that the presence of the equity market softens the credit crunch.

3.5 Conclusion

We study the interaction of household saving decisions, project returns, Basel Accord type banking regulation and credit activity. We find that the Basel Accord has a noticeable impact on loans when project returns decline through the cycle. Active monetary policy through interest rate reductions in bad times increases the loan activity but does not remove its cyclical nature.

A relaxation of the Basel Accord capital requirements in bad times obtains negative results, as households shy away from the equity banks need to make loans. As in models with informational problems, of which there are none here, a monetary policy tightening is needed. This calls for an active regulatory policy through the business cycle, not fixed policy rules currently in place.

Our results also emphasize the importance of taking into account the financing of banks. Given capital requirements, banks are limited in their lending by the amount of bank equity that households are willing to hold. As this decision is influenced by interest rates, another channel of monetary policy arises. This channel has also been identified by Chami and Cosimano (2001) and van der Heuvel (2001). Unlike these papers, we do not require asymmetric information, market power in the banking industry or increasing marginal cost of financing.

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Appendix A

A.1 Approximating the absolute value function

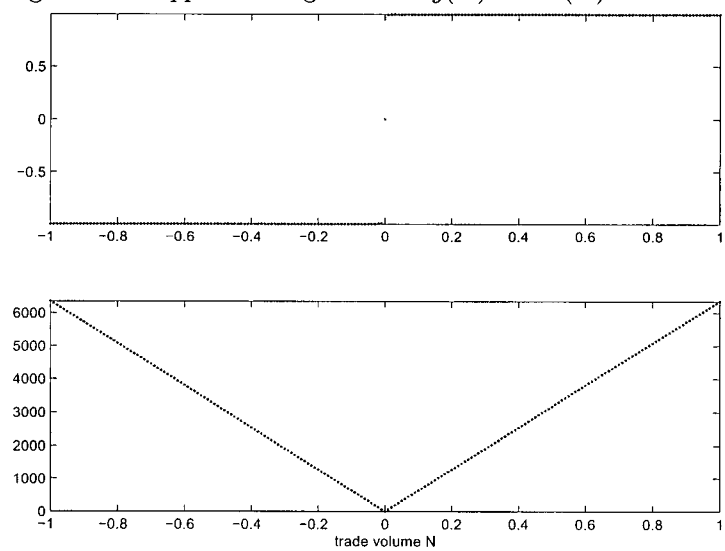
As mentioned in section (1.3.2), function $g(N_{i,t}) \equiv I(\widehat{N_{i,t}}) = \frac{2}{\pi}\arctan(\lambda N_{i,t})$ approximates a step function in the volume of trade $N_{i,t}$. An inverse of a trigonometric function $\tan(x)$, $\arctan(x)$ has a range of $[-\pi/2, \pi/2]$ for $x \in \mathbf{R}$ and is monotonically increasing, continuously differentiable, and has a convenient property that $\arctan(x) < 0$ when $x < 0$, $\arctan(x) > 0$ when $x > 0$ and $\arctan(x) = 0$ when $x = 0$. Because of the bounded range, and because $\arctan(\lambda x)_{x=0} = 0\ \forall\lambda$, $\arctan(\lambda x)$ can reach the bounds very quickly. Finally, if we premultiply the function by $2/\pi$, the range becomes [-1,1] – very convenient for a step function.

The choice parameter λ is inversely related to the approximation error, a relationship that can be seen in figure (A.12). However, it is misleading to use this approximation to describe the first order conditions of a system with $|N_{i,t}|$ because the absolute value function is not differentiable at 0. Therefore, a smooth approximation $G(N_{i,t})$ to $|N_{i,t}|$ needs to be constructed first, and then differentiated. Conveniently, function

$$G(N_{i,t}) \equiv \int g(N_{it}) = \frac{2}{\pi}\left[\lambda N_{i,t}\left(\frac{2}{\pi}\arctan(\lambda N_{i,t}) - 0.5\log(1+(\lambda N_{i,t})^2)\right)\right]$$

can be used to arbitrarily closely approximate $|N_{i,t}|$ by a choice of λ (see figure (A.1)).

Figure A.1: Approximating functions $g(N)$ and $G(N)$ for $\lambda = 10^5$.



Derivation of equation (1.24)

$$
\begin{aligned}
N &= N_{-1} + \frac{1}{2c}(p^* - p - \mathrm{I}(N)t) \\
\text{therefore} \partial N_i/\partial Y_i &= \frac{1}{2c}(\partial p^*/\partial Y_i - \partial p/\partial Y_i - t)\pi + 0(1-\pi) \;\; \text{where } \pi \equiv prob(|p^* - p| > t) \\
\text{Noting that } p^{aut} &= \left[\frac{Y_i}{Y_3}\frac{\gamma_3}{\gamma_i}\right]^{-\frac{1}{\theta}} \Rightarrow \partial p/\partial Y_i = -\frac{1}{\theta}\left[\frac{Y_i}{Y_3}\frac{\gamma_3}{\gamma_i}\right]^{-\frac{1}{\theta}-1}\frac{1}{Y_3}\frac{\gamma_3}{\gamma_i} \\
\text{and that} & \quad \partial p^*/\partial Y_i = 0 \\
\partial N/\partial Y_i &= \frac{\pi}{2c\theta}\left(\left[\frac{Y_i}{Y_3}\frac{\gamma_3}{\gamma_i}\right]^{-\frac{1}{\theta}-1}\frac{1}{Y_3}\frac{\gamma_3}{\gamma_i} - t\theta\right), \text{ identical to equation (1.24)}
\end{aligned}
$$

Assuming that the shares of 3 goods in the utility function are identical ($\gamma_i = \gamma_j \; \forall i, j$), $\partial N_i/\partial Y_i < 1$ when c is sufficiently large.

$$
c > \bar{c} \equiv \max\left\{0, \frac{\pi}{2\theta}\left(\frac{Y_i^{-\frac{1}{\theta}-1}}{Y_3^{-\frac{1}{\theta}}} - t\theta\right)\right\}
$$

which is automatically satisfied ($\bar{c} = 0$) if

$$
Y_i > \bar{Y}_i \equiv \left[\frac{Y_3}{(t\theta)^{\theta}}\right]^{\frac{1}{1+\theta}} \quad \text{when exporting}
$$

When $Y_3 = 100$, $\theta = 1.5$ and $t = 0.0174$ (as in the model's calibration), $\bar{Y}_i = 51.7$. Therefore, this condition always holds when exporting. An analogous derivation can be made for the case of imports.

Figures

Figure A.2: Linear model: Autarky solution

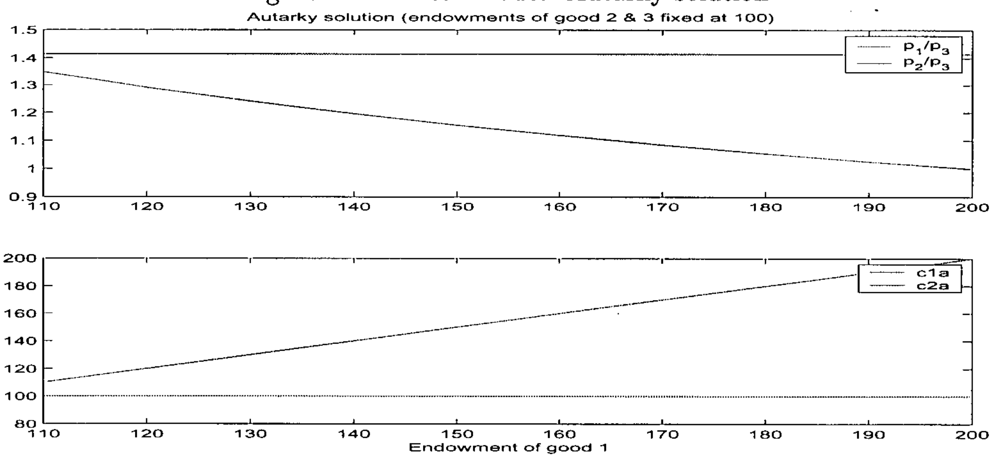


Figure A.3: Linear model: International trade

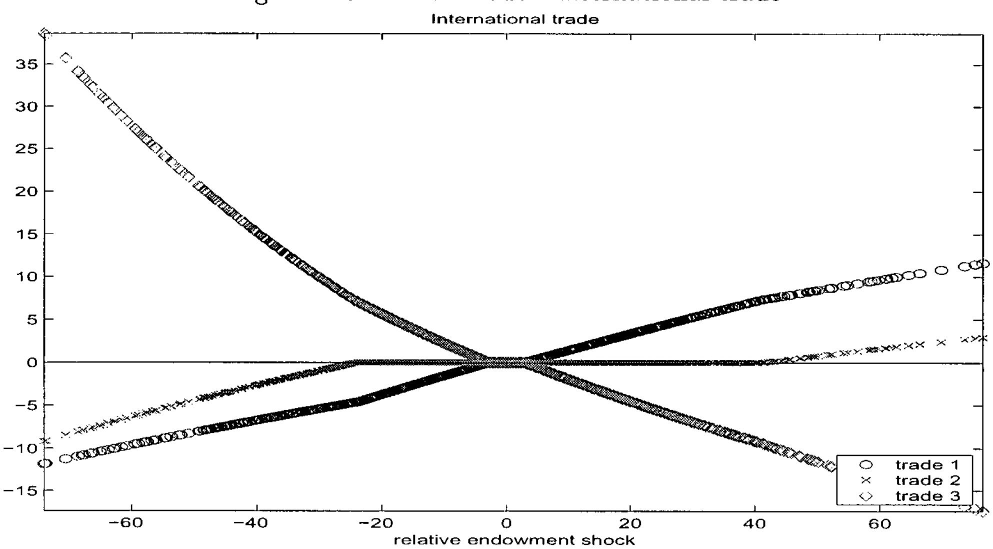


Figure A.4: Linear model: means and standard deviations of the price deviations as functions of shock volatility.

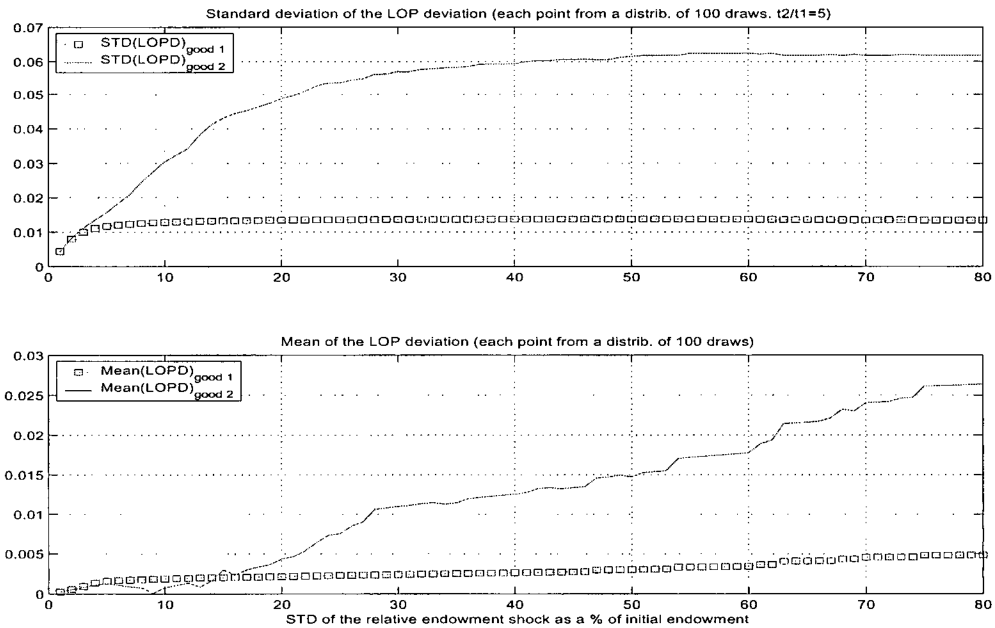


Figure A.5: Linear model: average thresholds of the law of one price deviation as function of shock volatility

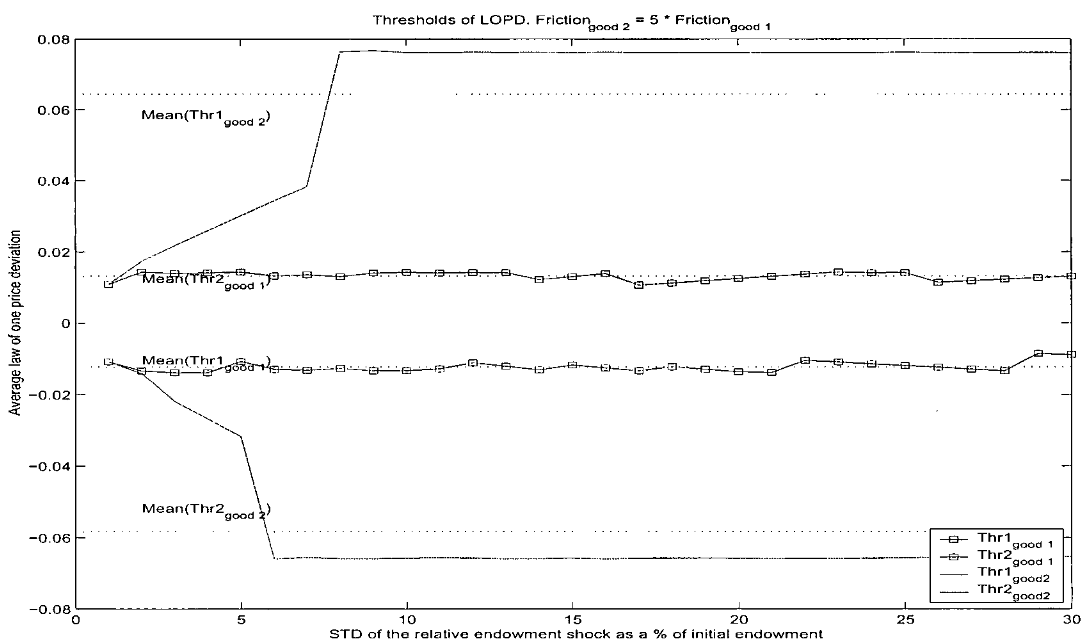


Figure A.6: Linear model robustness: mean price deviations as a function of shock volatility and relative friction

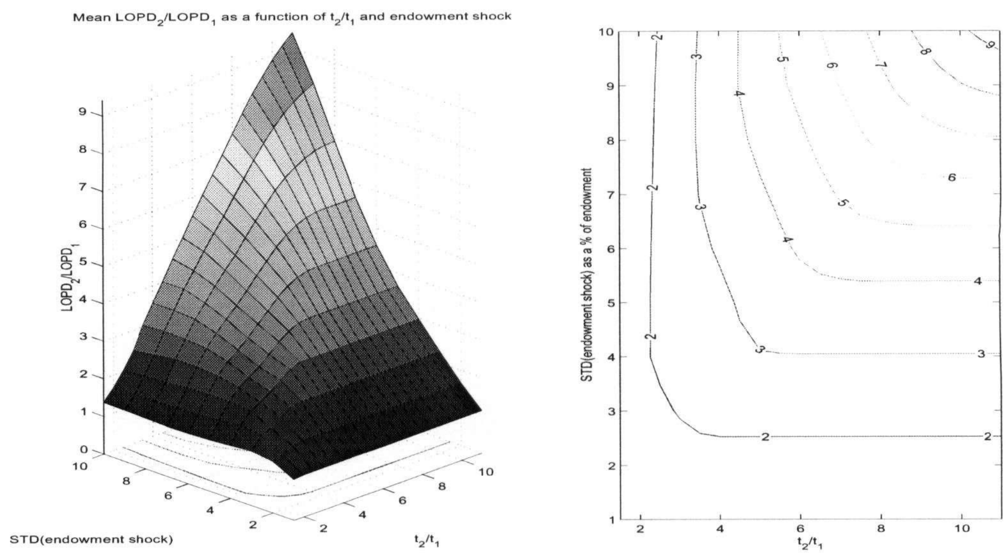


Figure A.7: Linear model robustness: volatility of price deviations as a function of shock volatility and relative friction

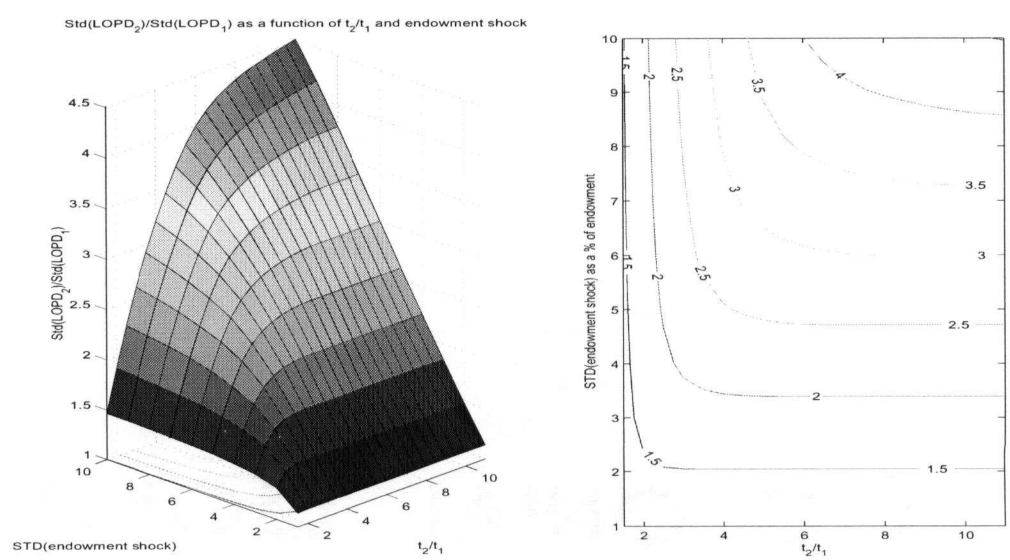


Figure A.8: Linear model robustness: price deviations thresholds as a function of shock volatility and relative friction

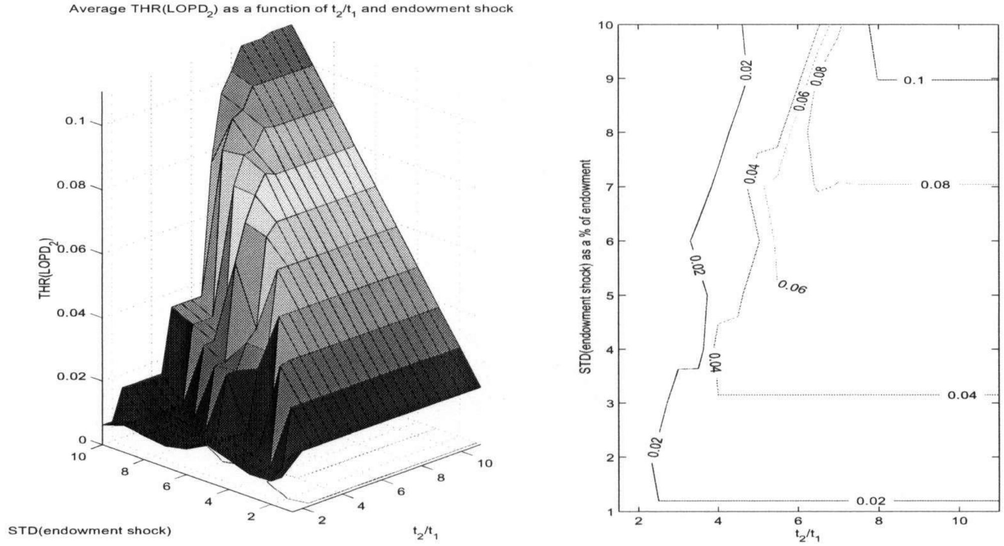


Figure A.9: Distribution of trade. US-EU simulation of the linear model

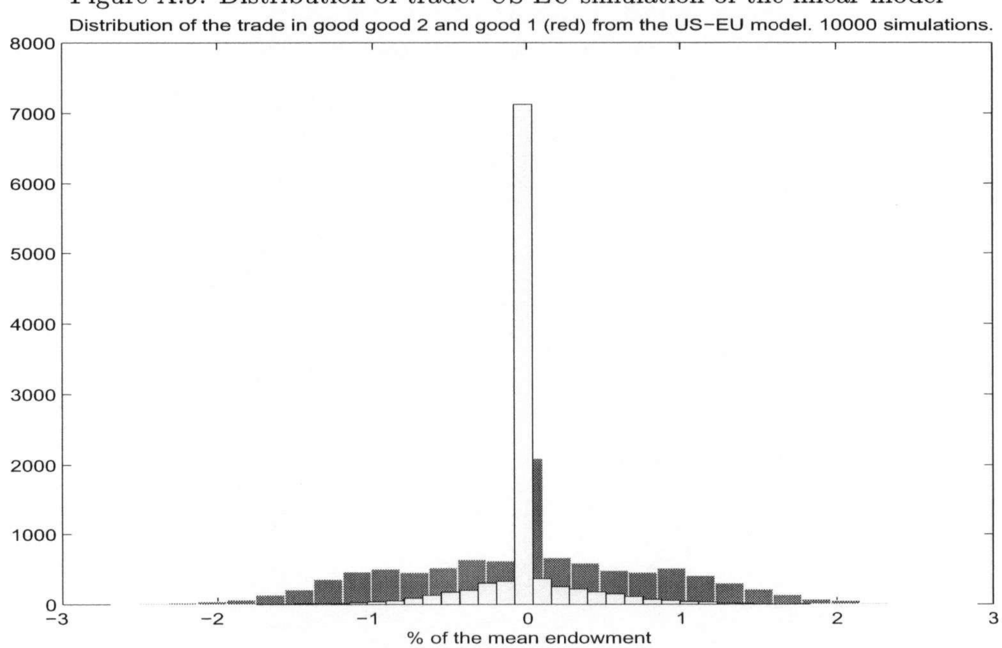


Figure A.10: Distribution of international prices. US-EU simulation of the linear model

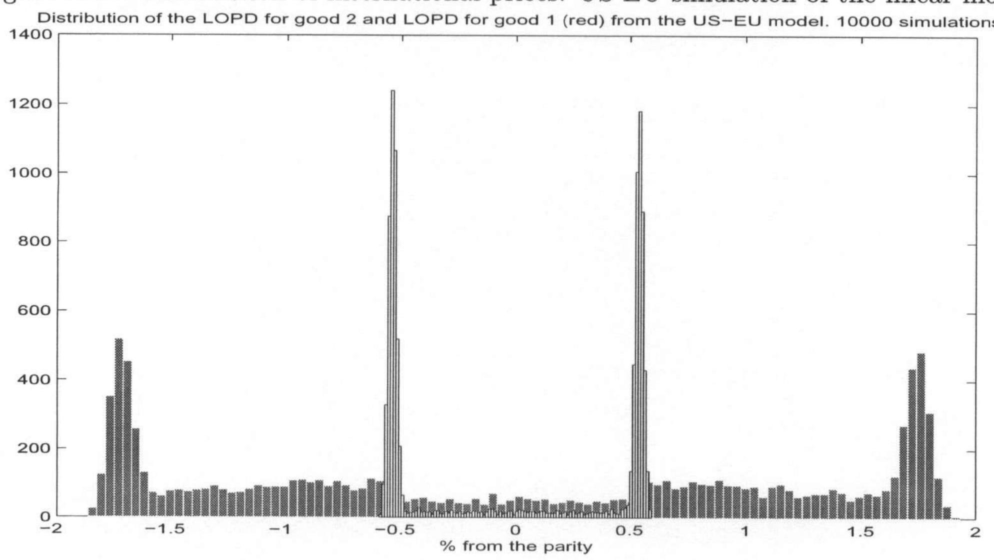


Figure A.11: Distribution of the real exchange rate and the shock process. US-EU simulation of the linear model

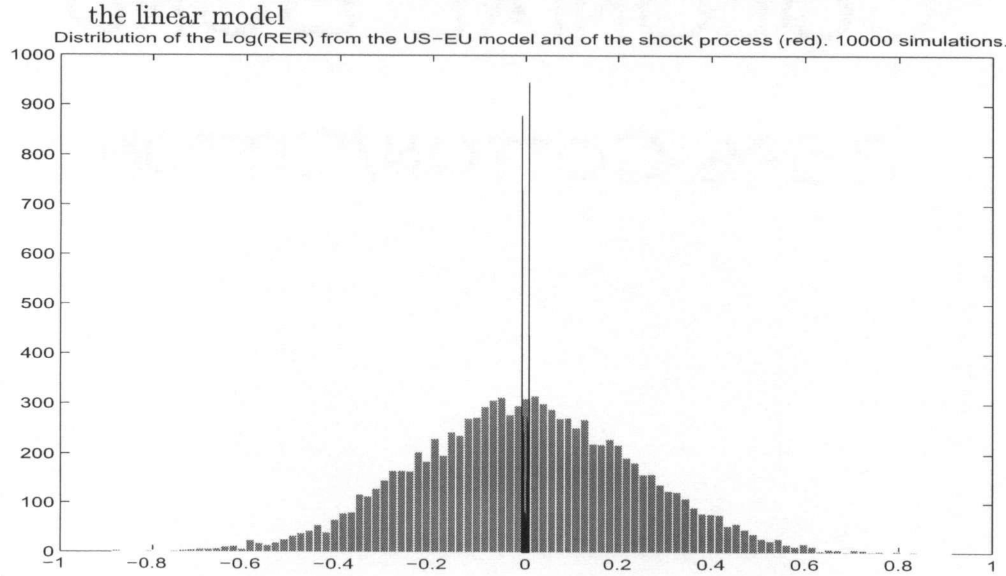


Figure A.12: Approximating the indicator function in QAC model

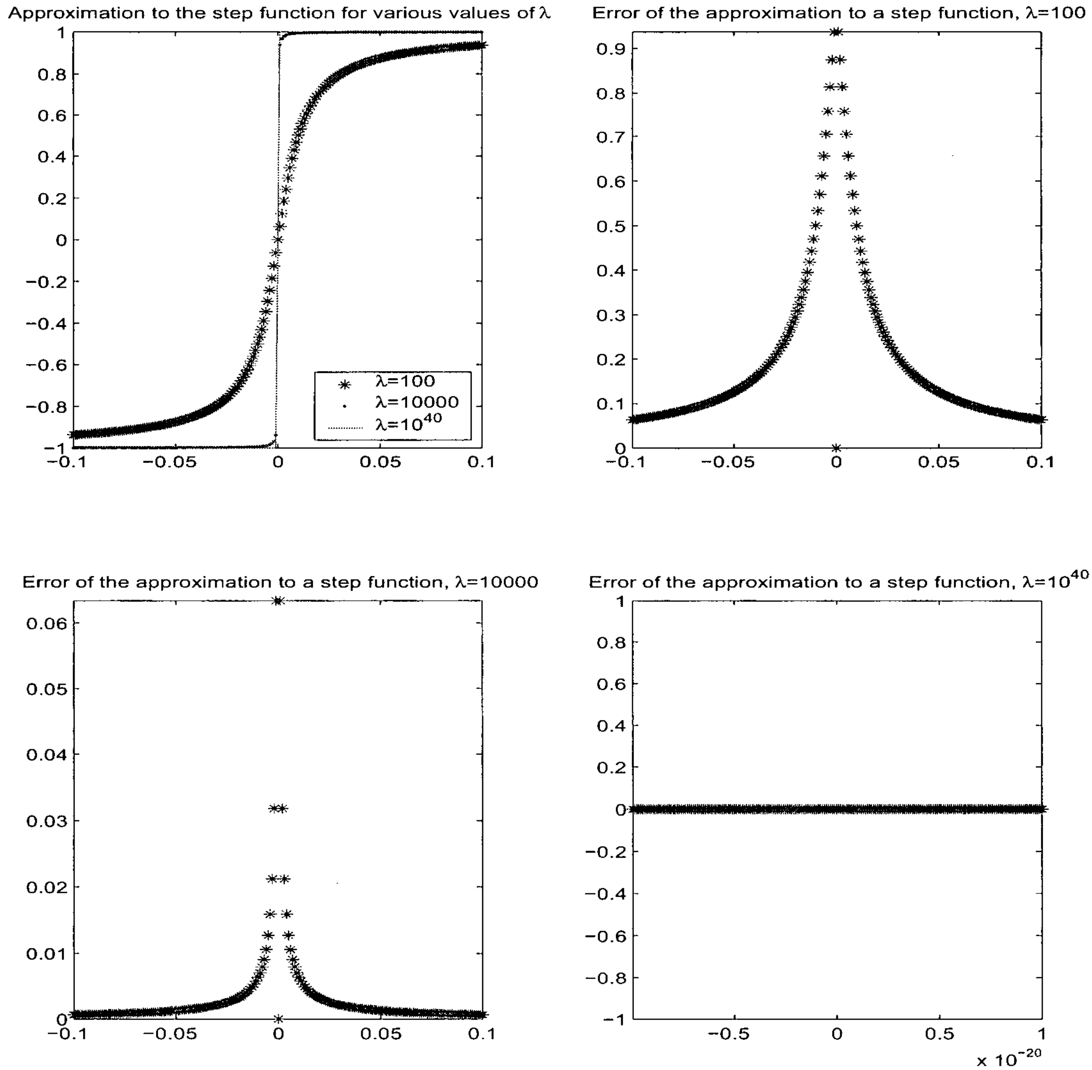


Figure A.13: Thresholds in QAC model when c=0.000001

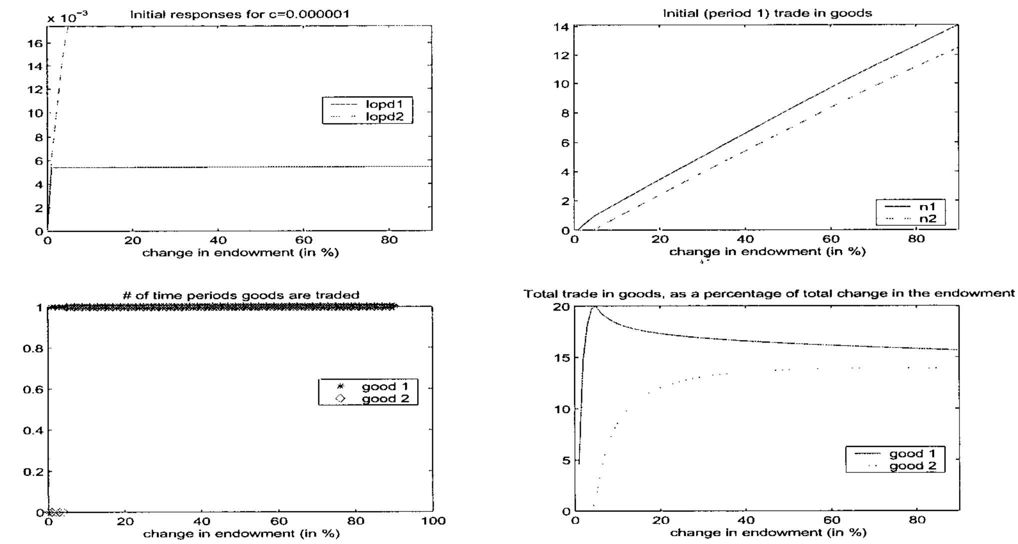


Figure A.14: Thresholds in QAC model when c=0.001

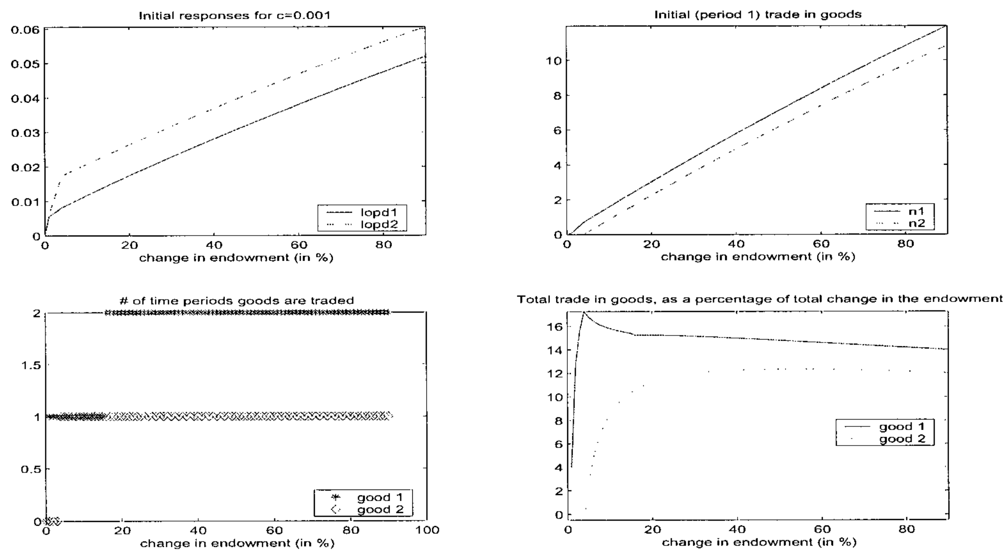


Figure A.15: Thresholds in QAC model when c=0.1

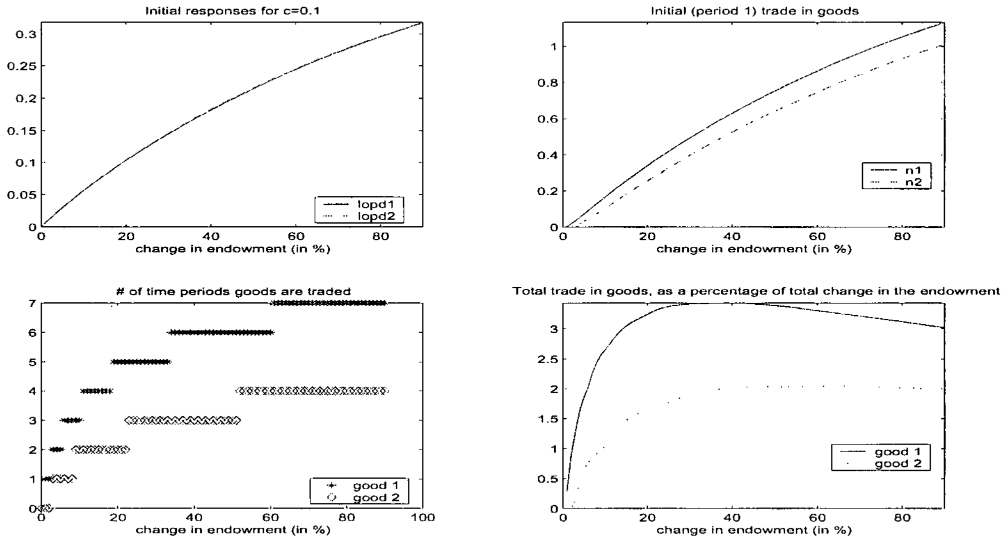


Figure A.16: Thresholds in QAC model when c=1

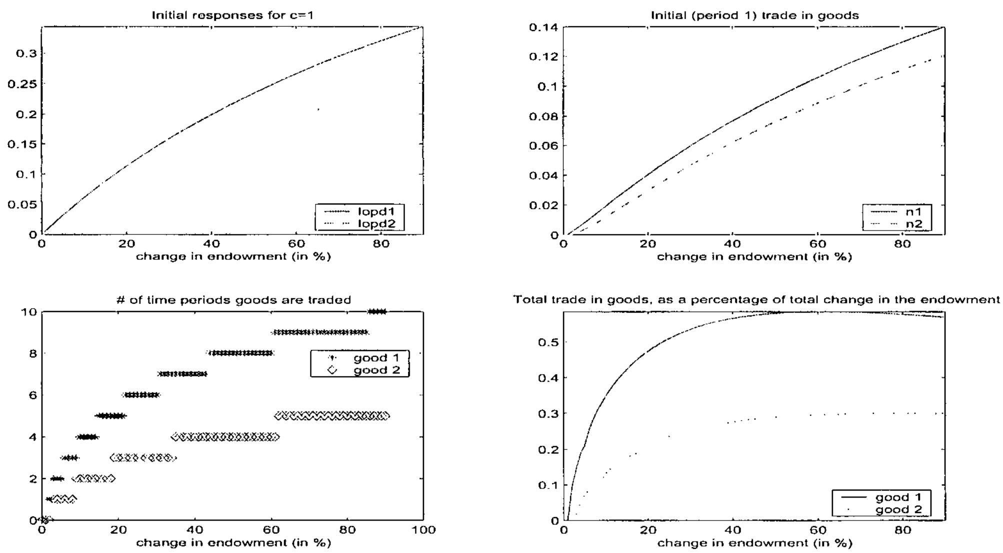


Figure A.17: Profits in partial equilibrium, for various price deviations and N_{-1}. c=0.01, t=0.2

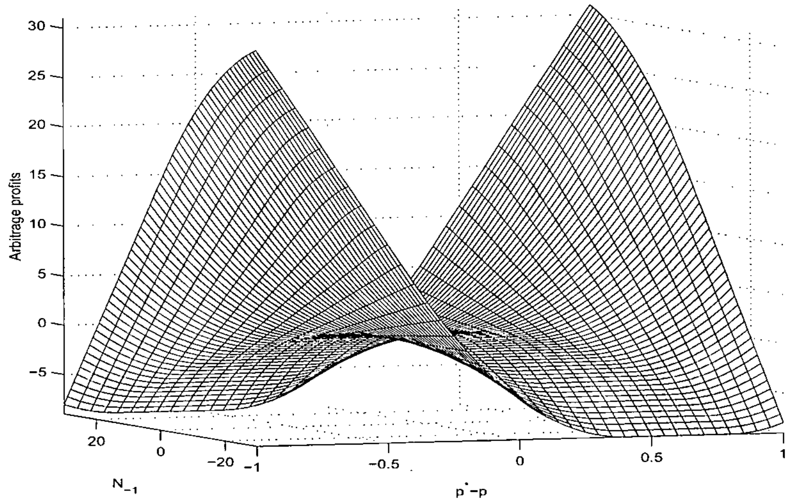


Figure A.18: Distribution of standard deviation estimates in a QAC model, c=0.1

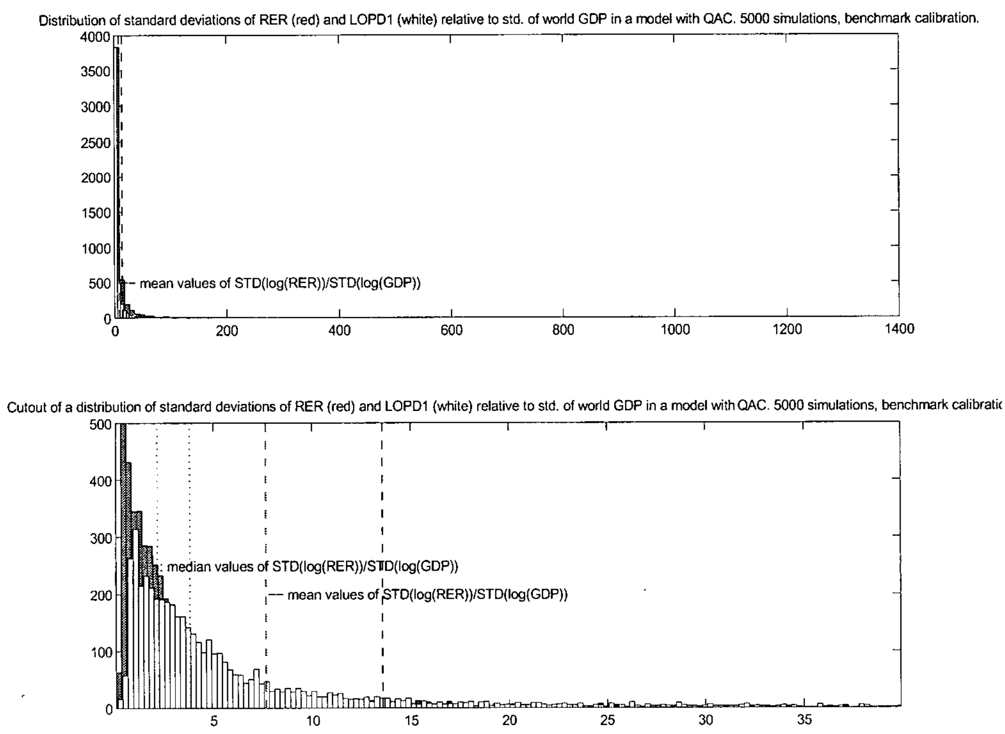


Figure A.19: Price deviations in a QAC model, c=0.1

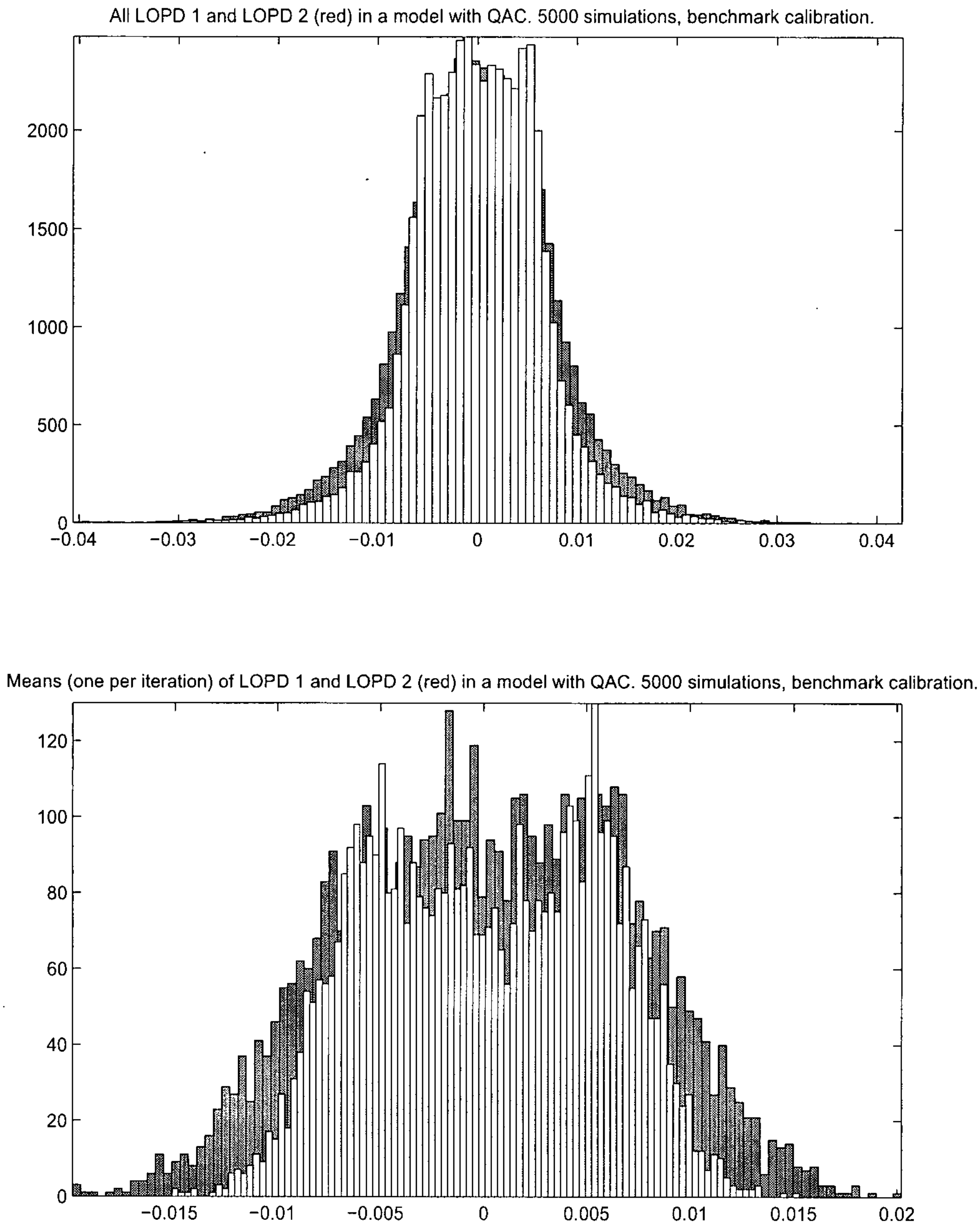


Figure A.20: Endowment shocks and RER in a QAC model, c=0.1

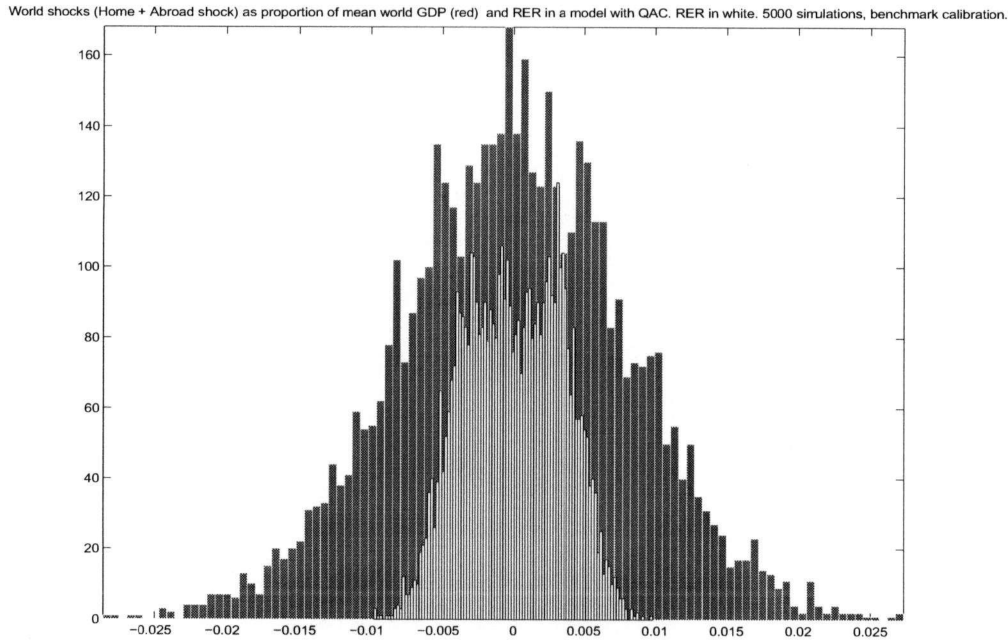


Figure A.21: Trade in a QAC model, c=0.1

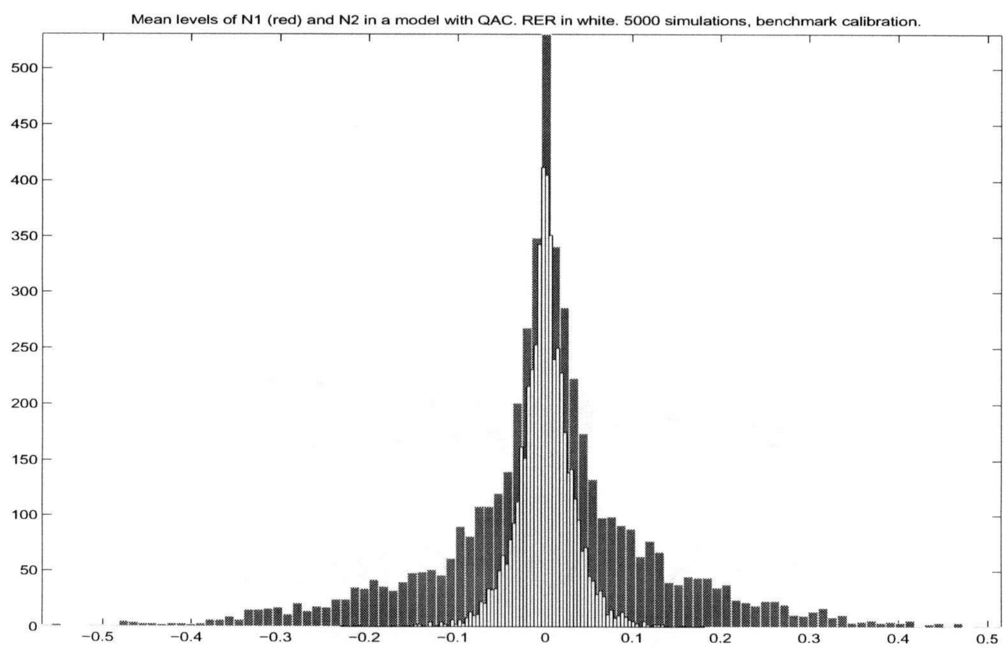
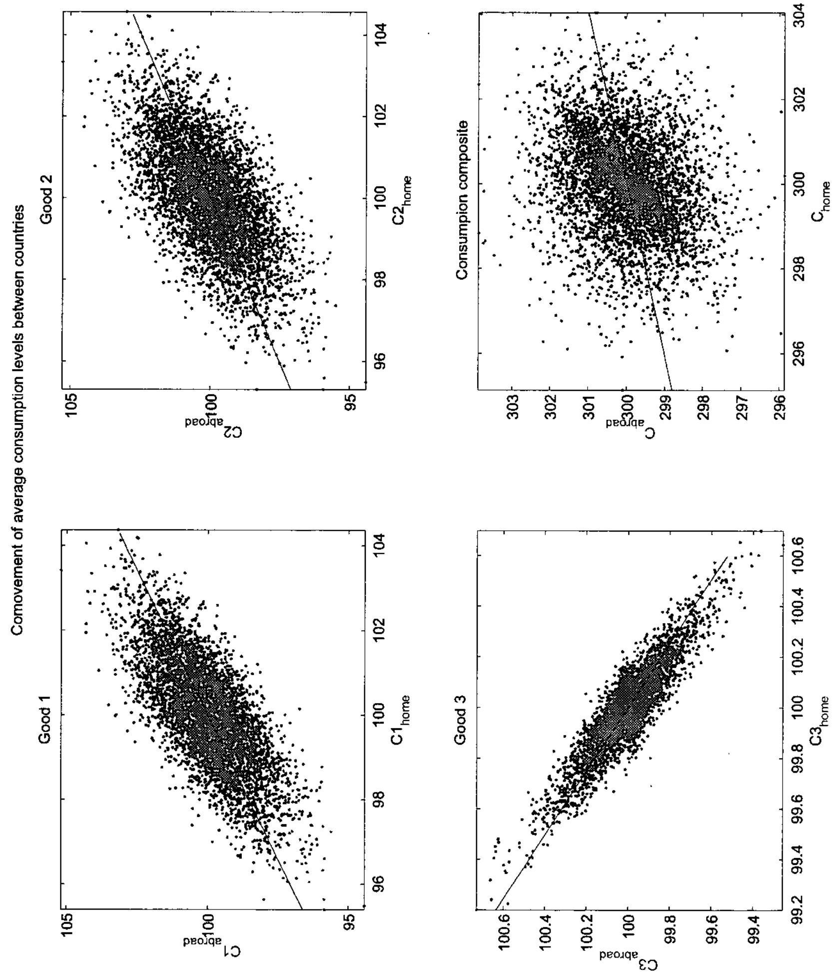


Figure A.22: Consumption co-movement in a QAC model, c=0.1



Appendix B

B.1 Tsay's F-test for non-linearity

The F-test for non-linearity consists of several steps. First, data is arranged into *cases* of $(\Delta z_t, 1, z_{t-1}, ..., z_{t-k})$ such that $k \in M$ where M is a set of all relevant lags as determined in stage 1 of the model-specification (see section(2.3.2) above). Second, these cases are arranged in ascending order according to z_{t-d} where d is the threshold delay parameter[1] (see Tsay (1989), Obstfeld and Taylor (1997)). Third, an arranged autoregression is run on the ordered case data using recursive least squares:

$$\Delta z_t = \sum_{k \in M}^{k^{max}} \alpha_k z_{t-k} + u_t \tag{B.1}$$

Recursive least square technique provides us with predictive residuals which are then used in Tsay's nonlinearity test. The recursive estimates are updated as follows (see Tsay (1989), Tong (1990), Barlet(????), Ertel and Fowlkes (1976)):

$$\begin{aligned}
\hat{\beta}_{m+1} &= \hat{\beta}_m + K_{m+1}[\Delta z_{m+1} - \beta_m \bar{z}_m] \\
K_{m+1} &= P_m \bar{z}_{m+1} / D_{m+1} \\
D_{m+1} &= 1 + \bar{z}'_{m+1} P_{m+1} \bar{z}_{m+1} \\
P_{m+1} &= (I - P_m \frac{\bar{z}_{m+1} \bar{z}'_{m+1}}{D_{m+1}}) P_m
\end{aligned}$$

where m denotes a case, $\bar{z}_m$ is a vector of all RHS variables in equation (B.1) (hence $\beta = (\alpha_1 \ ... \ \alpha_{k^{max}})'$) and I is an identity matrix. The recursion is initiated by a regular OLS on the first b cases where $b = n/10 + p$, n is the total number of observations and p is the number of elements in M^2. The first b cases are then scrapped. The predictive $(\hat{a}_m)$ and standardized predictive $(\hat{e}_m)$ residuals are obtained as follows:

$$\begin{aligned}
\hat{a}_{m+1} &= \Delta z_{m+1} - \beta_m \bar{z}_m \\
\hat{e}_{m+1} &= \hat{a}_{m+1} / \sqrt{D_{m+1}}
\end{aligned}$$

Fourth, standardized predictive residuals are regressed on the RHS variables:

$$\begin{aligned}
\hat{e}_m &= \sum_{k \in M}^{k^{max}} \gamma_k z_{m-k} + \epsilon_m \\
F &= \frac{(\sum \hat{e}_t^2 - \sum \epsilon_t^2)/(p+1)}{\sum \epsilon_t^2/(n-d-b-p-h)}
\end{aligned} \tag{B.2}$$

[1] Cases are analyzed in an ordered fashion because of the lack of knowledge of the position of a threshold ex-ante.

[2] Therefore $P_b = (\bar{z}_b' \bar{z}_b^{-1})$.

The associated F-statistic follows an F distribution with $p+1$ and $n-d-b-p-h$ degrees of freedom where $h \equiv \max\{1, p+1-d\}$.

Intuitively, a threshold implies a parameter change in the arranged autoregression (B.1) at the threshold level. Therefore, while predictive residuals will be orthogonal to the regressors for the cases that fall below the threshold, they will become biased above the threshold, destroying the orthogonality with the regressors. Such regime change then leads to a rejection of orthogonality and can be tested by the F-statistic (B.2). Rejection of orthogonality implies a rejection of a linear AR model for a non-linear TAR alternative.

B.2 Constructing the price and weight, and volume datasets

Table B.1: Coverage of CPI by the data.

Relative importance of components in the Consumer Price Indexes (1999-2000 weights): U.S. city average, December 2001. **Bold** series are **not included** in the dataset.

Item	CPI-U	CPI-W
All items	100.000	100.000
Food and beverages	15.719	17.229
Food	14.688	16.228
Food at home	8.468	9.798
Cereals and bakery products	1.298	1.468
Cereals and cereal products	.444	.525
Flour and prepared flour mixes	.058	.070
Breakfast cereal	**.249**	**.278**
Rice, pasta, cornmeal	**.137**	**.177**
Bakery products	**.854**	**.944**
Meats, poultry, fish, and eggs	2.271	2.831
Meats, poultry, and fish	2.178	2.712
Meats	1.450	1.832
Beef and veal	.693	.868
Uncooked ground beef	.255	.334
Uncooked beef roasts	.115	.132
Uncooked beef steaks	.278	.351
Uncooked other beef and veal	.045	.051
Pork	.468	.610
Bacon, breakfast sausage, & rel. products	.148	.190
Ham	**.104**	**.132**
Pork chops	.112	.156
Other pork including roasts and picnics	.105	.132
Other meats	**.289**	**.355**
Poultry	.414	.518
Chicken	.329	.423
Other poultry including turkey	.085	.095
Fish and seafood	.314	.362
Fresh fish and seafood	.187	.219
Processed fish and seafood	.126	.143
Eggs	.093	.119
Dairy and related products	**.916**	**1.021**
Fruits and vegetables	1.204	1.307
Fresh fruits and vegetables	.928	.995
Fresh fruits	.466	.490
Apples	.084	.095
Bananas	.088	.100
Citrus fruits	.079	.085
Other fresh fruits	.215	.210
Fresh vegetables	.462	.505
Potatoes	.080	.092
Lettuce	**.059**	**.066**
Tomatoes	.094	.109
Other fresh vegetables	.230	.238
Processed fruits and vegetables	**.276**	**.312**
Nonalc. beverages and bev. materials	**.967**	**1.132**
Juices and nonalcoholic drinks	.710	.853
Carbonated drinks	.364	.468
Frozen noncarbonated juices and drinks	.036	.039
Nonfrozen noncarbonated juices and drinks	.310	.346
Beverage materials including coffee and tea	.257	.280
Coffee	.100	.105
Other beverage materials including tea	**.157**	**.174**
Other food at home	1.811	2.038
Sugar and sweets	.315	.339
Sugar and artificial sweeteners	.056	.069
Candy and chewing gum	**.200**	**.207**
Other sweets	**.059**	**.063**
Fats and oils	.265	.316
Butter and margarine	.090	.103
Salad dressing	.076	.089
Other fats and oils including peanut butter	.098	.125
Other foods	1.232	1.383
Food away from home	6.220	6.430
Full service meals and snacks	2.649	2.198
Limited service meals and snacks	**2.741**	**3.354**
Food at employee sites and schools	**.296**	**.375**
Food - vending mach. & mobile vendors	**.151**	**.229**
Other food away from home	**.383**	**.275**
Alcoholic beverages	1.031	1.001
Alcoholic beverages at home	.682	.709
Beer, ale, and other malt beverages at home	.352	.469
Distilled spirits at home	.109	.102
Wine at home	.221	.139
Alcoholic beverages away from home	.348	.292
Housing	40.873	38.141
Shelter	31.522	29.212
Rent of primary residence	6.421	8.395
Lodging away from home	2.702	1.523
Housing at school, excluding board	.241	.176
Other lodg. away from home incl. hotels	2.461	1.347
Owners' equivalent rent of primary residence	22.046	18.980
Tenants' and household insurance	.353	.314
Fuels and utilities	4.511	4.829
Fuels	3.654	3.955
Fuel oil and other fuels	.188	.177
Fuel oil	.121	.105
Other household fuels	.068	.072
Gas (piped) and electricity	3.466	3.778
Electricity	2.521	2.762
Utility natural gas service	.945	1.017
Water and sewer and trash collection services	.857	.873
Water and sewerage maintenance	.633	.660
Garbage and trash collection	**.224**	**.213**
Household furnishings and operations	4.840	4.101
Window & floor coverings & other linens	**.289**	**.254**
Furniture and bedding	1.051	.955
Bedroom furniture	.306	.284
Living room, kitchen, & dining room furniture	.555	.495
Other furniture	.181	.154
Unsampled furniture	.010	.021
Appliances	.364	.416
Major appliances	.199	.226
Other appliances	**.151**	**.176**
Unsampled appliances	**.013**	**.015**
Other household equip. & furnishings	**.806**	**.565**
Tools, hardware, outdoor eq. & supplies	**.649**	**.595**
Housekeeping supplies	.862	.959
Household cleaning products	.392	.459
Household paper products	**.200**	**.221**
Miscellaneous household products	**.270**	**.279**
Household operations	**.820**	**.357**
Apparel	4.399	4.831
Men's and boys' apparel	1.122	1.243
Men's apparel	.880	.927
Men's suits, sport coats, and outerwear	.201	.189
Men's furnishings	.191	.196
Men's shirts and sweaters	.263	.279
Men's pants and shorts	.203	.241
Unsampled men's apparel	.021	.023
Boys' apparel	**.242**	**.316**
Women's and girls' apparel	1.807	1.864
Women's apparel	1.515	1.504
Women's outerwear	.108	.111
Women's dresses	.214	.247
Women's suits and separates	.762	.712
Women's underwear, nightwear, sportswear	.400	.399
Unsampled women's apparel	.032	.036
Girls' apparel	**.292**	**.360**
Footwear	.874	1.165
Men's footwear	.290	.416
Boys' and girls' footwear	**.177**	**.269**
Women's footwear	.407	.480
Infants' and toddlers' apparel	**.203**	**.256**
Jewelry and watches	.394	.303
Watches	.058	.057
Jewelry	.336	.245
Transportation	17.055	19.393
Private transportation	15.845	18.452
New and used motor vehicles	8.614	10.145
New vehicles	5.083	4.897
Used cars and trucks	**2.195**	**4.099**
Leased cars and trucks	**1.061**	**.925**
Car and truck rental	**.120**	**.085**
Unsampled new & used motor veh.	**.155**	**.140**
Motor fuel	2.564	3.153
Gasoline (all types)	2.536	3.120
Other motor fuels	.028	.033
Motor vehicle parts and equipment	.421	.530
Tires	.234	.262
Vehicle accessories other than tires	**.187**	**.268**
Motor vehicle maintenance and repair	1.400	1.438
Motor vehicle body work	.082	.077
Motor vehicle maintenance and servicing	.478	.475
Motor vehicle repair	.821	.868
Unsampled service policies	.020	.019
Motor vehicle insurance	2.288	2.679
Motor vehicle fees	**.558**	**.506**
Public transportation	1.211	.941
Airline fare	.761	.508
Other intercity transportation	**.187**	**.124**
Intracity transportation	.256	.300
Unsampled public transportation	**.006**	**.008**
Medical care	5.810	4.620
Medical care commodities	1.377	1.006
Prescription drugs and medical supplies	.959	.680
Nonprescription drugs and medical supplies	.418	.326
Internal & respiratory over-the-counter drugs	.304	.250
Nonprescription medical equip. & supplies	.114	.076
Medical care services	4.434	3.614
Professional services	2.784	2.245
Physicians' services	1.503	1.280
Dental services	.747	.584
Eyeglasses and eye care	.288	.240
Services by other medical professionals	.247	.142
Hospital and related services	1.353	1.092

Table B.2: Part 2 of the CPI coverage table. **Bold** series are **not included** in the dataset.

Item	CPI-U	CPI-W	Item	CPI-U	CPI-W
Hospital services	1.271	1.075	Cigarettes	.864	1.360
Nursing homes and adult daycare	.082	.017	Tobacco products other than cigarettes	.057	.073
Health Insurance	.297	.276	Unsampled tobacco and smoking prods	.007	.008
Recreation	6.019	5.649	Personal care	3.384	3.059
Video and audio	1.645	1.803	Personal care products	.706	.815
Televisions	.150	.157	Hair, dental, shaving, & pers. care	.374	.434
Cable television	.928	1.034	Cosmetics, perfume, bath, nail preps	.327	.374
Other video equipment	.055	.064	Unsampled personal care products	.005	.006
Video casset., discs, & other media incl. rental	.148	.182	**Personal care services**	**.901**	**.900**
Audio equipment	.117	.131	**Miscellaneous personal services**	**1.562**	**1.161**
Audio discs, tapes and other media	**.147**	**.159**	Miscellaneous personal goods	.215	.183
Unsampled video and audio	**.099**	**.076**	**Special aggregate indexes**		
Pets, pet products and services	**.711**	**.703**	All items	100.000	100.000
Sporting goods	.628	.728	Commodities	41.300	45.559
Sports vehicles including bicycles	.286	.413	Commodities less food and beverages	25.582	28.330
Sports equipment	.333	.309	Nondurables less food and beverages	13.493	14.685
Unsampled sporting goods	**.009**	**.006**	Nondurables less food, bev. & apparel	9.094	9.854
Photography	.241	.215	Durables	12.089	13.645
Photographic equipment and supplies	.110	.092	Services	58.700	54.441
Photographers and film processing	**.129**	**.122**	Rent of shelter	31.169	28.898
Unsampled photography	**.001**	**.001**	Transportation services	6.638	6.573
Other recreational goods	.497	.512	Other services	10.963	10.033
Toys	.360	.399	All items less food	85.312	83.772
Sewing machines, fabric and supplies	**.058**	**.052**	All items less shelter	68.478	70.788
Music instruments and accessories	**.062**	**.049**	All items less medical care	94.190	95.380
Unsampled recreation commodities	**.016**	**.012**	Commodities less food	26.612	29.331
Recreation services	**1.861**	**1.364**	Nondurables less food	14.524	15.687
Recreational reading materials	.436	.324	Nondurables less food and apparel	10.125	10.855
Newspapers and magazines	.265	.210	Nondurables	29.212	31.915
Recreational books	.170	.114	Nondurables less food	14.524	15.687
Unsampled recreational reading materials	.001	.000	Nondurables less food and apparel	10.125	10.855
Education and communication	5.813	5.637	Nondurables	29.212	31.915
Education	2.726	2.382	Apparel less footwear	3.525	3.666
Educational books and supplies	.220	.203	Services less rent of shelter	27.531	25.543
Tuition, other school fees, and childcare	2.506	2.178	Services less medical care services	54.266	50.827
College tuition and fees	1.162	.877	Energy	6.218	7.109
Elementary & high school tuition & fees	**.338**	**.258**	All items less energy	93.782	92.891
Child care and nursery school	**.840**	**.895**	All items less food and energy	79.094	76.663
Technical & business sch. tuition & fees	**.084**	**.077**	Commodities less food & energy	23.860	26.001
Unsampled tuition, fees, & childcare	**.083**	**.071**	Energy commodities	2.752	3.330
Communication	**3.087**	**3.255**	Services less energy services	55.234	50.663
Other goods and services	4.312	4.499	Domestically produced farm food	7.099	8.204
Tobacco and smoking products	.928	1.441	Utilities and public transportation	8.785	9.217

Table B.3: Data sources on weights and prices - part 1

item	unit	price	curr.	weight (kg)	p/w (USD/kg)	note
Total RER-CPI						
Apples	kg	2.57	CND	1	1.7	05/00-05/01 average, Statcan Table 326-0012
Audio equipment	stereo unit	150	USD	6	25	www.jandr.com (the largest retailer in US), includes packaging
Beef	ground, 1kg	4.63	CND	1	3.06	05/00-05/01 average, Statcan Table 326-0012
Beer	six pack	5.40	USD	2.30	2.35	See Grossmann & Markowitz (1999)
Car purchase	car	24,923	USD	1326.13	18.79	1996 avg. extrapolated to 2000, American Automobile Manufacturers' Association 1996
Car parts	tire	100	USD	10	10	
Cheese	kg	8.69	USD	1	8.69	Avg. of American processed cheese (Series APU0000710211) and Cheddar cheese (Series APU0000710212) BLS, 2001 average monthly
Clothes						
Clothes (men)	basket#	USD			50.52	U.S. Department of Commerce, 2000
Clothes (women)	basket#	USD			52.93	U.S. Department of Commerce, 2000
Coffee	roast, 300g	3.27	CND	0.3	7.20	05/00-05/01 average, Statcan Table 326-0012
Educ. books & supplies						
Eggs	dozen	1.91	CND	0.73	1.74	05/00-05/01 average, Statcan Table 326-0012 weight: a 30-dozen egg container weighs 47lb.
Electricity	500 kWh	48.55	USD	–	–	BLS, average 2001 price (Series APU000072621)
Fats and oils	basket*	1.81	USD	0.598	3.68	StatCan, Avg price in Calgary in Nov 2001 for Salad dressing, avg. price in NYC, Feb 2001
Fish and seafood	basket+	2.85	USD	1	2.85	Fish processing industry data, wholesale prices.
Flour	2.5kg	3.37	CND	2.50	0.89	05/00-05/01 average, Statcan Table 326-0012
Footwear						
Footwear (men)	pair, avg of casual and athletic	46.50	USD	0.73	63.70	
Footwear (women)	pair, athletic	43.88	USD	0.56	81.00	
Fuel oil	liter	0.34	USD	0.86	0.39	Avg price, BLS 2001, Series APU000072511
Furniture	bed	200	CND	46.7	4.3	IKEA
Gas	1000 ft^3	7.45	USD	18.16	0.41	Avg price for year 2000, Energy Information Administration, Natural Gas Monthly, Jan 2002
Gasoline	liter	0.38	USD	0.70	0.54	Avg. price, BLS, 2001, Series APU000074714

#Men's basket contains: coats, blazers, trousers, suits, women's basket contains: coats, dresses, blazers, trousers, suits, and skirts.

*In accordance with the CPI definition of the category, the basket contains: Margarine (Canola, 1.36kg), Butter (Parchment, 454g), Shortening (454g), Oil (Canola, 1l), Lard (454g), Peanut butter (500g), and Salad dressing (8oz). Weights are equal to the CPI weights.

+Canned fish composition matches the composition of the fish processing industry data. **Canned**: Tuna (48%), Salmon (12%), Clams (8%), Sardines, Shrimp,

Fillets: Cod (4.7%), Flounder (1.7%), Haddock, Rockfish, Pollock (11%) and Other (11%), **Fresh fish** approximated by 50% tuna and 50% salmon.

Table B.4: Data sources on weights and prices - part 2

item	unit	price	curr.	weight (kg)	p/w (USD/kg)	note
House chemicals	75oz pack of laundry deterg.	2.30	USD	2.13	1.16	1997 NYC price extrapolated into 2001
Jewelry	–					
Laundry appliances	washer	887	USD	158.9	5.58	2002 avg. price for Maytag
Liquor	750ml whiskey	11.74	USD	0.75	15.65	BLS avg. price for 1986, adjusted by CPI inflation (series APU0000720211)
Medical care products						
Non-prescription med.						
Pants	pair, jeans, avg.	50.18	USD	1.36	36.86	Parsley & Wei (2001) and US Department of Commerce, avg. price 01/00-07/00
PC	unit	1000	USD	20	50	Dell.com average price in 2002.
Personal care products	a basket[1]	12.58	CND	8.31	2.77	05/00-05/01 average, Statcan Table 326-0012
Photo equipment	–					
Pork	kg, chops	9.29	CND	1	6.14	05/00-05/01 average, Statcan Table 326-0012
Potatoes	4.54kg	3.83	CND	4.54	0.56	05/00-05/01 average, Statcan Table 326-0012
Poultry	kg	4.45	CND	1	2.94	05/00-05/01 average, Statcan Table 326-0012
Prescription medicine	–					
Sport equipment	basket#	99.67	USD	2.10	65.00	http://www.usolympicteam.com/sports2/ih/az equip.html
Sport vehicles	bicycle	225	USD	15	15.00	
Sugar	1lb	0.43	USD	0.45	0.95	BLS avg. price for 2001 (Series APU0000715212)
Tobacco	200 cigs	37.78	CND	0.25	99.80	05/00-05/01 average, Statcan Table 326-0012
Toys	basket	31.33	USD	2.55	13.19	average of 5 age-group categories from Toys'R'Us 2001.
Video equipment	basket*	226.67	USD	8.73	25.96	from J&R website, the largest US retailer, includes packaging.
Watches	piece	50	USD	0.2	250	Timex website avg. price, weight approximated
Wine	liter	5.96	USD	1.3	4.58	BLS avg. price, 2001 (series APU0000720311)
Fresh fruits	basket	19.36	USD	8	2.42	BLS avg. price, 2001
Reading materials	book	30	USD	0.5	60	
Tomatoes	kg	2.90	USD	1	2.9	BLS avg. price, 2001 (series APU0000712311)

#Sports basket contains ski boots, skis and bindings, tennis racquet, basketball, golf set (11pc), dozen golf balls, hockey stick, hockey skates, inline skates and hockey helmet

*Average of a TV set, a VCR, and a camcorder.

Figure B.1: Some prices in NYC, February 2001

Dear Consumer:

To ensure Passover shoppers are protected from stores that try to take advantage by raising prices during the holiday season, NYC Department of Consumer Affairs is once again releasing a price survey of kosher for Passover items

For the past 17 years, this handy price guide has helped consumers get the best value when purchasing Passover foods. It lists the average prices of the lowest priced Passover food items available for sale **before** the Passover season, arming shoppers with fair pricing information when they undertake their holiday shopping.

To compile this survey, DCA inspectors visited supermarkets, grocery stores, fish and meat markets all over New York City between February 27 - March 1st.

I hope consumers will once again use the results of our annual survey to comparison shop and save this holiday season.

Best wishes for a happy and healthy Passover.

Rudolph W. Giuliani
Mayor

Commissioner Jane S. Hoffman

Pre-Passover Prices

Average for lowest-priced brand, February, 2001

Independent Grocers	Kosher Products	Size	Supermarkets
$3.74	Gefilta Fish	24 oz	$4.58
$1.87	Applesauce	34/35 oz	$1.69
$1.92	Grape Juice	22 oz	$1.81
$2.02	Apple Juice	64 oz	$2.15
$1.12	Salad Dressing	8 oz	$1.09
$2.36	Mayonnaise	16 oz	$2.10
$1.55	Borscht	32/33 oz	$1.39
$1.77	Tuna Fish	6 oz	$1.37
$1.09	Extra Large Eggs	1 dz	$1.86
$3.31	Cooking Oil	48 oz	$2.53
$2.80	Sugar	5 lb	$2.78
$1.49	Cider Vinegar	16 oz	$1.32
$1.59	Horseradish	6/8 oz	$1.83
$1.49	Potato Starch	12/16 oz	$2.34
$2.27	Butter	8 oz	$2.42
$3.70	Am. Hard Cheese	12 oz	$3.89
$1.94	Cottage Cheese	16 oz	$2.16

Meat and Fish Stores

Meats	Price Per Pound
Beef Roast, Shoulder	$6.72
Beef Roast, Chuck	$5.87
First Cut Brisket (medium size)	$5.10
Breast Flanken	$7.12
Veal, Ground	$4.86
Veal Roast, Boneless Shoulder	$9.96
Veal Chop, First Cut	$11.40
Turkey, Fresh Whole	$1.79
Chicken, Fresh Whole Roaster	$2.23
Chicken, Boneless Cutlets	$6.29
Fresh Fish	
Michigan Whitefish	$4.82
Michigan Carp	$1.99
Michigan Carp, Sliced	$4.12
Michigan Pike	$6.16

Price survey conducted by the NYC Department of Consumer Affairs.

Table B.5: Data sources on volume - part 1

item	unit	price	stowage factor	volume (m^3)	p/v (USD/m^3)	note
Total RER-CPI						
Apples	kg	2.57	2.622	0.003	647.4	boxes, http://www.tis-gdv.de/tis e/ware/obst/apfel/apfel.htm
Audio equipment	stereo unit	150	5.495	0.055	2730	http://www.jr.com/JRProductPage.process?Product=3967701
Beef	ground, 1kg	4.63	1	0.001	3057.8	http://www.tis-gdv.de/tis e/ware/fleisch/gekuehlt/gekuehlt.htm
Beer	six pack	5.40	1.556	0.004	1508.9	http://www.tis-gdv.de/tis e/ware/lebensmi/bier/bier.htm
Car purchase	car	24,923	8.399	11.138	2237.7	http://www.fordvehicles.com/Cars/focus/features/specdimensions/
Car parts	tire	100	4.041	0.04	2474.6	http://amchouston.home.att.net/stowage factors.htm
Cheese	kg	8.69	1.397	0.001	6222	http://www.tis-gdv.de/tis e/ware/milchpro/kaese/kaese.htm
Clothes						
Clothes (men)	basket#		4.728		10686.4	http://www.tis-gdv.de/tis e/ware/textil/konfektion/konfektion.htm
Clothes (women)	basket#		4.728		11208.1	http://www.tis-gdv.de/tis e/ware/textil/konfektion/konfektion.htm
Coffee	roast, 300g	3.27	1.961	0.001	3671.3	Rodrigues et. al. (2003)
Educ. books & supplies						
Eggs	dozen	1.91	2.755	0.002	630.7	measure
Electricity	500 kWh	48.55		–	–	
Fats and oils	basket*	1.81	1.25	–	2944	German transportation database source for each component
Fish and seafood	basket+	2.85	1.85	–	1537.8	German transportation database source for most components
Flour	2.5kg	3.37	1.33	0.003	669.4	http://amchouston.home.att.net/stowage factors.htm
Footwear						
Footwear (men)	pair, avg of casual and athletic	46.50	21.918	0.016	2906.3	Mens shoe box 14-3/4" x 10-1/8" x 5-5/8"
Footwear (women)	pair, athletic	43.88	28.351	0.014	2857.1	
Fuel oil	liter	0.34	1.163	0.001	338	
Furniture	bed	200	4.73	0.22	909.1	http://www.ikea-usa.com/webapp/wcs/stores/servlet/... ...ProductDisplay?catalogId=10101&storeId=12&productId=32145&... ...langId=-1&parentCats=10103*10144
Gas	1000 ft^3	7.45	1559.298	28.317	0.3	
Gasoline	liter	0.38	1.434	0.001	337	

#Men's basket contains: coats, blazers, trousers, suits, women's basket contains: coats, dresses, blazers, trousers, suits, and skirts.

*In accordance with the CPI definition of the category, the basket contains: Margarine (Canola, 1.36kg), Butter (Parchment, 454g), Shortening (454g), Oil (Canola, 1l), Lard (454g), Peanut butter (500g), and Salad dressing (8oz). Weights are equal to the CPI weights.

+Canned fish composition matches the composition of the fish processing industry data. **Canned**: Tuna (48%), Salmon (12%), Clams (8%), Sardines, Shrimp,

Fillets: Cod (4.7%), Flounder (1.7%), Haddock, Rockfish, Pollock (11%) and Other (11%), **Fresh fish** approximated by 50% tuna and 50% salmon.

Table B.6: Data sources on volume - part 2

item	unit	price	stowage factor	volume (m^3)	p/v (USD/m^3)	note
House chemicals	75oz pack of laundry deterg.	2.30	10.591	0.021	109.5	measurement
Jewelry	–					
Laundry appliances	washer	887	4.506	0.716	1238.8	http://www.maytag.com/products/images/products/dmsearcywash.pdf
Liquor	750ml whiskey	11.74	1.75	0.001	8944.8	http://www.tis-gdv.de/tis e/ware/genuss/rum/rum.htm
Medical care products						
Non-prescription med.						
Pants	pair, jeans, avg.	50.18	3.57	0005	10328	http://www.tis-gdv.de/tis e/ware/textil/konfektion/konfektion.htm
PC	unit	1000	25	0.5	2000	http://www.shipit.co.uk/Overseas Removals Companies Volumes.htm
Personal care products	a basket[1]	12.58	8.664	0.024	346.2	measurement of basket items
Photo equipment	–					
Pork	kg, chops	9.29		1	6.14	
Potatoes	4.54kg	3.83	1.7	0.002	3609.1	http://www.tis-gdv.de/tis e/ware/gemuese/kartoffe/kartoffe.htm
Poultry	kg	4.45	1	0.005	557.1	assume same volume as beef
Prescription medicine	–					
Sport equipment	basket#	99.67	23.61	0.036	2753.3	various sources for items#
Sport vehicles	bicycle	225	17.864	0.268	839.7	http://www.crateworks.com/frameset.html?page=features
Sugar	1lb	0.43	1.354	0.001	699.5	http://www.tis-gdv.de/tis e/ware/zucker/weiszuck/weiszuck.htm
Tobacco	200 cigs	37.78	0.002	6	13861	http://www.discount-cigarettes-online.biz/templates/faq.php
Toys	basket	31.33	–	0.2	156.7	guess
Video equipment	basket*	226.67	0.044	5	5191.4	http://www.tis-gdv.de/tis e/ware/maschinen/unterhaltung/unterhaltung.htm
Watches	piece	50	–	0.0012	41667	dims: 20x10x5cm, volume direct
Wine	liter	5.96	1.175	0.0015	3973.3	same stowage factor as liquor
Fresh fruits	basket	19.36	2.95	0.024	820.3	German transportation database source for each component
Reading materials	book	30	1.78	0.001	33707.9	http://www.tis-gdv.de/tis e/ware/papier/zeitung/zeitung.htm
Tomatoes	kg	2.90	2.373	0.002	1221.9	http://www.tis-gdv.de/tis e/ware/gemuese/tomaten/tomaten.htm

#Sports basket contains ski boots (http://www.snowshack.com/head-boot-bag.html),
skis and bindings (http://www.snowshack.com/salomon-equipe-2pr-skibag.html),
tennis racquet, basketball (http://experts.about.com/q/2551/1184149.htm),
golf set (11pc, length 44in = 111cm), dozen golf balls (http://www.overstock.com/cgi-bin/d2.cgi?PAGE=PROFRAME&PROD ID=676397),
hockey stick (http://www.unleash.com/picks/sportinggoods/topsportinggoodshockeysticks.asp),
hockey skates (15-in x 9-in x 15-in bag), and
inline skates and hockey helmet http://secure1.esportspartners.com/store-redskins/main detail.cfm?nCategoryID=4&nObjGroupID=134&nProductID=56453
*Average of a TV set, a VCR, and a camcorder.

B.3 Tables and Figures

Table B.7: Long run properties of exchange rates

	Sample: 1947:1–2000:8			Sample: 1947:1–1969:12			Sample: 1970:1–2000:8		
	NER	RER	Rel. CPI	NER	RER	Rel. CPI	NER	RER	Rel. CPI
Lags[0]	1	11	13	6	12	13	1	11	13
ADF stat.[1]	0.057	-0.923	-1.923*	-0.667	-1.195	-1.444	0.471	-0.614	-1.495
ADF stat.[2]	0.067	-0.884	-1.931	-1.545	-2.083	-3.19**	-0.225	-0.881	-1.882
ADF stat.[3]	-16.6***	-5.4***	-5.5***	-9.9***	-3.35**	-4.2***	-12.6***	-4.2***	-4.0***
Half life[4]	– (1.001)	– (1.0014)	146.4	74.2	27.3	10.5	14145	– (1.0015)	140

Table B.8: Long run properties of linearly detrended exchange rates

	Sample: 1947:1–2000:8			Sample: 1947:1–1969:12			Sample: 1970:1–2000:8		
	NER	RER	Rel. CPI	NER	RER	Rel. CPI	NER	RER	Rel. CPI
Lags[0]	11	1	13	13	12	12	11	13	13
ADF stat.[1]	-2.93***	-2.09**	-2.26***	-2.57***	-3.22***	-2.11**	-1.87*	-1.74*	-1.98**
ADF stat.[2]	-2.93**	-2.09	-2.25	-2.74*	3.37**	-2.1	-1.86	-1.72	-1.99
ADF stat.[3]	-6.01***	-5.3***	-5.35***	-4.39***	-3.35**	-3.95***	-4.48***	-4.16***	-4.3***
Half life[4]	98.7	64.1	128	66	15.4	13	154.3	228.3	321.9

Table B.9: Long run properties of HP-detrended exchange rates

	Sample: 1947:1–2000:8			Sample: 1947:1–1969:12			Sample: 1970:1–2000:8		
	NER	RER	Rel. CPI	NER	RER	Rel. CPI	NER	RER	Rel. CPI
Lags[0]	11	11	13	15	13	13	12	11	17
ADF stat.[1]	-8.2***	-7.4***	-7.8***	-4***	-4.6***	-5.3***	-5.4***	-5.9***	-5.6***
ADF stat.[2]	-8.2***	-7.4***	-7.9***	-4***	-4.6***	-5.3***	-5.4***	-5.9***	-5.6***
ADF stat.[3]	-9.2***	-8.6***	7.5***	-6***	-5.7***	-5.1***	-5.9***	-5.8***	-5.8***
Half life[4]	5	4.5	4.9	5	4.3	3.2	4.9	4.6	5.9

All series are demeaned and in logarithms.

[0]Lags may vary for ADF tests in differences, depending on the PACF criterions.

[1]Augmented Dickey-Fuller test in levels, no intercept, no trend.

[2]Augmented Dickey-Fuller test in levels, with intercept, no trend.

[3]Augmented Dickey-Fuller test in differences, with intercept, no trend.

[4]AR(1) half-life is calculated without a constant.

Table B.10: Long run properties:ADF and half-life convergence

Good type	obs	τ_c	# lags[1]	half-life^2_A	half-life^3_B
Apples	334	-1.49	13	5.6	6.2
Audio equipment	189	-0.005	4	187	–
Beef	334	-1.197	4	20	31
Beer	334	-1.359	2	120	–
Car maintenance	265	-0.917	9	1391	–
Car parts	117	-2.233^{**}	8	22	–
Car purchase	334	-2.58^{***}	0	20	30
Cheese	189	$-.151$	3	–	–
Clothes	223	-1.563	6	26	84
Clothes (men)	223	-1.51	6	34	274
Clothes (women)	223	-1.581	4	15	26
Coffee	334	-0.842	5	78	–
Educational books and supplies	70	-0.095	2	140	–
Eggs	334	-4.716^{***}	2	5.2	6
Electricity	334	-1.182	1	74	–
Fats and oils	141	-1.333	0	35	–
Fish and seafood	189	-0.315	2	180	–
Flour	273	-2.098^{**}	1	40	262
Footwear	334	-2.437^{**}	7	22	35
Footwear (men)	189	-1.826^{*}	4	15	27
Footwear (women)	273	-2.46^{**}	8	19	32
Fuel oil	333	-1.418	5	39	118
Furniture	334	-1.319	0	60	–
Gas	334	-2.69^{***}	2	16	22
Gasoline	334	-1.554	2	45	202
House chemicals	334	-0.146	4	859	–
Jewelry	117	-2.704^{***}	2	21	–
Laundry appliances	189	-0.589	4	94	–
Liquor	273	-1.271	1	103	–
Medical care products	265	-1.027	11	–	–
Non-prescription medicine	166	-0.616	1	133	–
Pants	273	-2.147^{**}	3	11	14
PC	34	-0.485	8	6	–
Personal care products	334	-0.624	2	151	–
Photo equipment	189	-0.137	0	639	–
Pork	265	0.917	10	23	47
Potatoes	344	-2.374^{**}	13	3.6	3.8
Poultry	334	-2.589^{***}	5	11	14
Prescribtion medicine	189	0.6	3	–	–
Sport equipment	265	-1.519	1	37	203
Sport vehicles	265	-1.589	2	48	–
Sugar	134	-1.904^{*}	7	39	–
Tobacco	177	0.053	0	–	–
Toys	189	-2.09^{**}	12	31	707
Video equipment	57	-1.459	2	10	–
Watches	93	-2.710^{***}	5	2.4	3
Wine	330	-1.758^{*}	2	69	–

[1] The number of lags L in Δy_{t-L} where Δy_t is the LHS variable of ADF test is one which minimizes Akaike Information Criterion. [2] Half-life is calculated using sample-adjusted AR(1) coefficient from Kendall's formula. In cases denoted "+", adjustment for bias leads to non-stationary AR(1) process in which case there is no half-life. There, I compute half-life without adjustment for sample-size bias.

Table B.11: EQ-TAR Summary

	STD	AR(1) half life	TAR(2,1,1) threshold	TAR(2,1,1) half life	AR(p) half life	TAR(2,p,d) threshold	TAR(2,p,d) half life
Foods	0.147	45	0.144	34	41	0.083	58
Vice goods	0.188	72	0.115	79	91	0.134	105
Clothing and footwear	0.075	20	0.035	21	26	0.041	19
Tech stuff	0.085	156	0.077	45	540	0.063	38
Fuels	0.149	51	0.109	45	50	0.070	48
Medical and chemical	0.146	235	0.194	100	244	0.131	90
Cars and car parts	0.074	20	0.039	19	27	0.046	23
Laundry appliances	0.099	94	0.134	26	98	0.154	25
Furniture	0.092	59	0.125	30	67	0.145	34
Services	0.133				224	0.054	241
CPI-RER	0.111		0.071		162	0.012	213

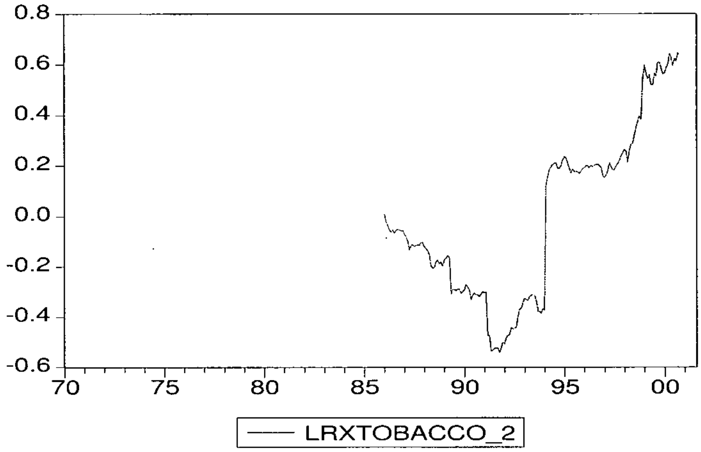
Table B.12: BAND-TAR Summary

	STD	AR(1) half life	TAR(2,1,1) threshold	TAR(2,1,1) half life	AR(p) half life	TAR(2,p,d) threshold	TAR(2,p,d) half life
Foods	0.147	45	0.146	22	41	0.083	29
Vice goods	0.188	72	0.149	70	55	0.144	149
Clothing and footwear	0.075	20	0.027	23	26	0.022	31
Tech stuff	0.085	156	0.079	33	540	0.063	27
Fuels	0.149	51	0.097	43	50	0.069	50
Medical and chemical	0.146	235	0.193	332	244	0.105	527
Cars and car parts	0.074	20	0.039	22	27	0.035	26
Laundry appliances	0.099	94	0.074	111	98	0.154	45
Furniture	0.092	59	0.127	43	67	0.145	60
Services	0.133				224	0.065	160
CPI-RER	0.111		0.071	1733	162	0.012	193

Table B.13: AR(1) Results - Part I

	AR(1) lambda	p-value	AR(1) half life	Tsay's nonlinearity test
Total RER-CPI				0.354
Apples	-0.120	0.025	5	0.000
Audio equipment	-0.004	0.011	182	0.019
Beef	-0.035	0.015	20	0.169
Beer	-0.006	0.005	114	0.087
Car purchase	-0.036	0.014	19	0.218
Car maintenance	-0.001	0.008	1216	0.104
Car parts	-0.031	0.013	22	0.490
Cheese	0.001	0.010		0.106
Clothes	-0.026	0.015	26	0.504
Clothes (men)	-0.020	0.014	34	0.779
Clothes (women)	-0.044	0.019	15	0.634
Coffee	-0.005	0.007	130	0.022
Educational books and supplies	-0.006	0.022	119	0.011
Eggs	-0.127	0.025	5	0.003
Electricity	-0.006	0.006	107	0.243
Fats and oils	-0.020	0.015	35	0.316
Fish and seafood	-0.004	0.011	175	0.203
Flour	-0.017	0.010	40	0.572
Footwear	-0.033	0.013	21	0.076
Footwear (men)	-0.046	0.021	15	0.457
Footwear (women)	-0.036	0.015	19	0.838
Fuel oil	-0.018	0.010	39	0.704
Furniture	-0.012	0.009	59	0.563
Gas	-0.043	0.017	16	0.020
Gasoline	-0.016	0.009	43	0.671
House chemicals	0.001	0.005		0.805
Jewelry	-0.033	0.018	21	0.008
Laundry appliances	-0.007	0.011	94	0.246
Liquor	-0.007	0.008	103	0.327
Medical care products	0.003	0.006		0.030
Non-prescription medicine	-0.005	0.009	132	0.159
Pants	-0.062	0.021	11	0.118
PC	-0.116	0.065	6	0.644
Personal care products	-0.002	0.004	338	0.299
Photo equipment	-0.001	0.009	579	0.056
Pork	-0.030	0.017	23	0.489
Potatoes	-0.171	0.029	4	0.029
Poultry	-0.050	0.015	14	0.030
Prescription medicine	0.004	0.005		0.138
Sport equipment	-0.018	0.011	37	0.419
Sport vehicles	-0.015	0.009	48	0.000
Sugar	-0.018	0.015	39	0.041
Tobacco	0.001	0.011		0.019
Toys	-0.022	0.015	31	0.407
Video equipment	-0.066	0.049	10	0.135
Watches	-0.249	0.061	2	0.566
Wine	-0.010	0.006	69	0.384

Figure B.2: Nonlinear fit of price-weight ratios and thresholds



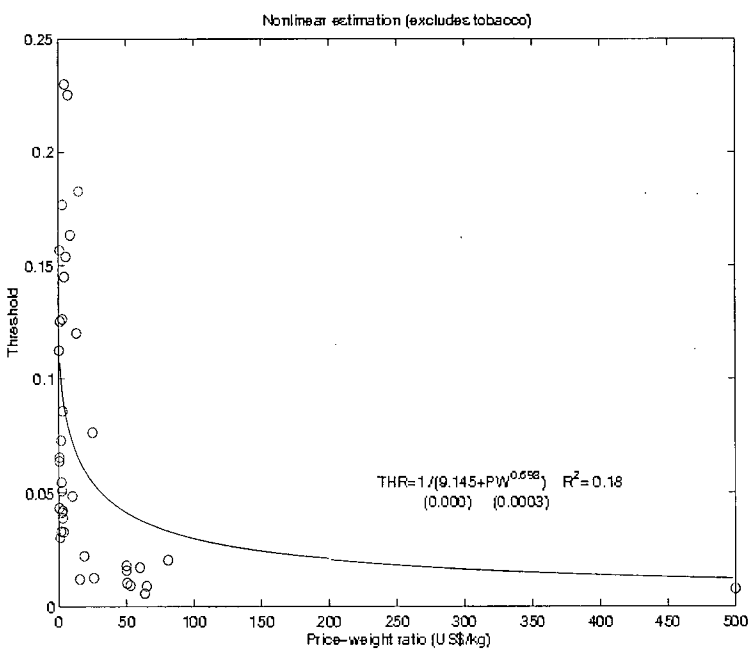


Table B.14: AR(1) Results - part II

	AR(1) lambda	p-value	AR(1) half life	Tsay's nonlinearity test
Airfare			11	
Cable TV			60	
Car insurance			51	
Child care				
Dental services				
Fresh fruits			11	
Intra-city transport			43	
Margarine			31	
Medical services				
Reading materials			48	
Rent			1175	
Restaurant meals			65	
Shelter			150	
Tomatoes			6	
Tuition			17	
Water and sewerage			69	

Figure B.3: Nonlinear fit of price-weight ratios and thresholds

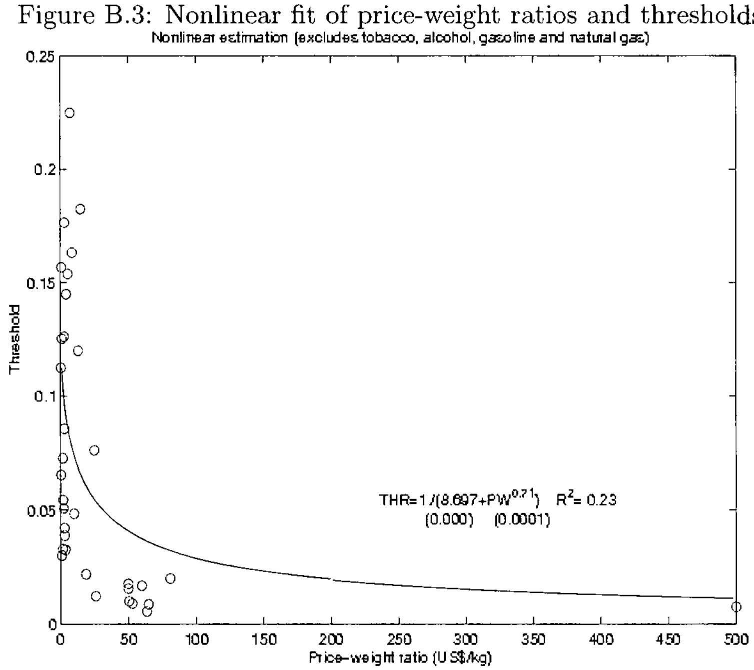


Table B.15: EQ-TAR(2,1,1) results

	LLR	TAR(2,1,1) threshold	TAR(2,1,1) lambda	TAR (2,1,1) half life	p-value
Total RER-CPI	-0.8	0.071	0.003		0.129
Apples	34.7	0.109	-0.124	5.2	0.200
Audio equipment	7.3	0.098	-0.004	160.9	0.134
Beef	5.8	0.024	-0.036	19.1	0.295
Beer	6.7	0.045	-0.006	115.2	0.130
Car purchase	-1.9	0.034	-0.037	18.5	0.266
Car maintenance	1.6	0.041	-0.002	346.2	0.136
Car parts	-0.8	0.043	-0.034	20.3	0.344
Cheese	-3.5	0.042	0.002		0.129
Clothes	-3.4	0.019	-0.026	26.3	0.205
Clothes (men)	-0.7	0.010	-0.020	34.5	0.176
Clothes (women)	-2.9	0.062	-0.037	18.6	0.313
Coffee	24.2	0.223	-0.005	144.1	0.113
Educational books and supplies	3.6	0.126	-0.102	6.4	0.134
Eggs	19.8	0.138	-0.174	3.6	0.335
Electricity	1.9	0.183	-0.009	81.2	0.151
Fats and oils	1.8	0.074	-0.029	23.4	0.213
Fish and seafood	-4	0.141	-0.025	27.6	0.141
Flour	17.6	0.176	-0.033	20.9	0.152
Footwear	-1.1	0.009	-0.033	20.6	0.249
Footwear (men)	-5.2	0.026	-0.042	16	0.301
Footwear (women)	12.4	0.059	-0.029	23.7	0.237
Fuel oil	11.2	0.101	-0.016	43.8	0.198
Furniture	5.6	0.125	-0.023	29.9	0.163
Gas	24.8	0.085	-0.072	9.2	0.211
Gasoline	26.6	0.066	-0.016	43.8	0.129
House chemicals	1.3	0.164	0.004		0.145
Jewelry	-0.9	0.214	-0.023	29.5	0.244
Laundry appliances	-3.6	0.134	-0.026	25.8	0.153
Liquor	-1	0.012	-0.007	103.1	0.153
Medical care products	4	0.072	0.003		0.115
Non-prescription medicine	-2.1	0.156	-0.019	35.4	0.114
Pants	0.3	0.058	-0.076	8.8	0.314
PC	-2	0.059	-0.163	3.9	0.390
Personal care products	3.6	0.288	-0.005	144.1	0.207
Photo equipment	1	0.194	-0.014	48.1	0.144
Pork	-2.4	0.102	-0.009	80.3	0.245
Potatoes	15.2	0.399	-0.298	2	0.414
Poultry	47.1	0.180	-0.063	10.6	0.083
Prescription medicine	-4.8	0.288	-0.006	121.3	0.162
Sport equipment	-1.8	0.139	-0.021	32	0.199
Sport vehicles	30.3	0.189	-0.046	14.8	0.079
Sugar	-3	0.048	-0.023	30.2	0.193
Tobacco	100.2	0.299	-0.005	150.3	0.000
Toys	8.9	0.148	-0.059	11.3	0.186
Video equipment	0.8	0.028	-0.078	8.6	0.203
Watches	7.9	0.008	-0.249	2.4	0.361
Wine	38.2	0.252	-0.028	24.8	0.055

Table B.16: EQ-TAR(2,p,d) results - part I

	LLR	TAR(2,p,d) threshold	TAR(2,p,d) halflife	p-value (old, 600)
Total RER-CPI	-9.1	0.012	213	0.056
Apples	20.9	0.073	6	0.126
Audio equipment	12.6	0.078	115	0.103
Beef	13.4	0.039	23	0.235
Beer	1.8	0.041	113	0.260
Car purchase	2.9	0.022	25	0.278
Car maintenance	-1.9	0.086	470	0.114
Car parts	1	0.070	20	0.425
Cheese	-2.7	0.081	563	0.113
Clothes	1	0.084	21	0.201
Clothes (men)	-11.8	0.132	8	0.189
Clothes (women)	-7.1	0.009	23	0.270
Coffee	1.3	0.223		0.070
Educational books and supplies	1.4	0.116	9	0.131
Eggs	-1.9	0.034	7	0.414
Electricity	1.4	0.059	79	0.218
Fats and oils	2.7	0.033	53	0.211
Fish and seafood	-2.1	0.152	27	0.107
Flour	20.5	0.157	27	0.124
Footwear	-6.3	0.006	34	0.192
Footwear (men)	-2.4	0.019	22	0.367
Footwear (women)	-13.4	0.020	13	0.214
Fuel oil	4.1	0.112	45	0.204
Furniture	10.4	0.145	34	0.194
Gas	44.5	0.043	18	0.029
Gasoline	28.3	0.064	48	0.190
House chemicals	-1.3	0.166		0.152
Jewelry	-2.1	0.080	18	0.386
Laundry appliances	-2.2	0.154	25	0.185
Liquor	-4.1	0.012	86	0.075
Medical care products	-4.9	0.027		0.091
Non-prescription medicine	2	0.150	42	0.143
Pants	-5.1	0.016	11	0.327
PC	-1.4	0.018	18	0.000
Personal care products	-3.2	0.263	138	0.211
Photo equipment	-1.1	0.202	37	0.114
Pork	-3.6	0.013	15	0.288
Potatoes	1.3	0.065	14	0.356
Poultry	20.5	0.086	15	0.103
Prescription medicine	-3.7	0.047		0.077
Sport equipment	-4.3	0.055	49	0.183
Sport vehicles	43.1	0.183	11	0.066
Sugar	1.6	0.030	23	0.410
Tobacco	106.8	0.301	172	0
Toys	10.8	0.145	18	0.201
Video equipment	5.9	0.012	18	0.177
Watches	-5.2	0.007	3	0.332
Wine	30.1	0.230	47	0.029

Table B.17: EQ-TAR(2,p,d) results - part II

	LLR	TAR(2,p,d) threshold	TAR(2,p,d) halflife
Airfare	-2.3	0.015	11
Cable TV	22.1	0.042	51
Car insurance	23.2	0.025	48
Child care	10.1	0.018	
Dental services	9.4	0.046	
Fresh fruits	-12.5	0.051	11
Intra-city transport	4.9	0.036	51
Margarine	0.9	0.117	15
Medical services	0.3	0.161	
Reading materials	-2.2	0.017	48
Rent	-3.3	0.016	1066
Restaurant meals	5	0.040	63
Shelter	-2.2	0.033	146
Tomatoes	-28.5	0.042	6
Tuition	-0.05	0.017	17
Water and sewerage	-2.7	0.114	75

Figure B.4: Nonlinear fit of price-weight ratios and thresholds

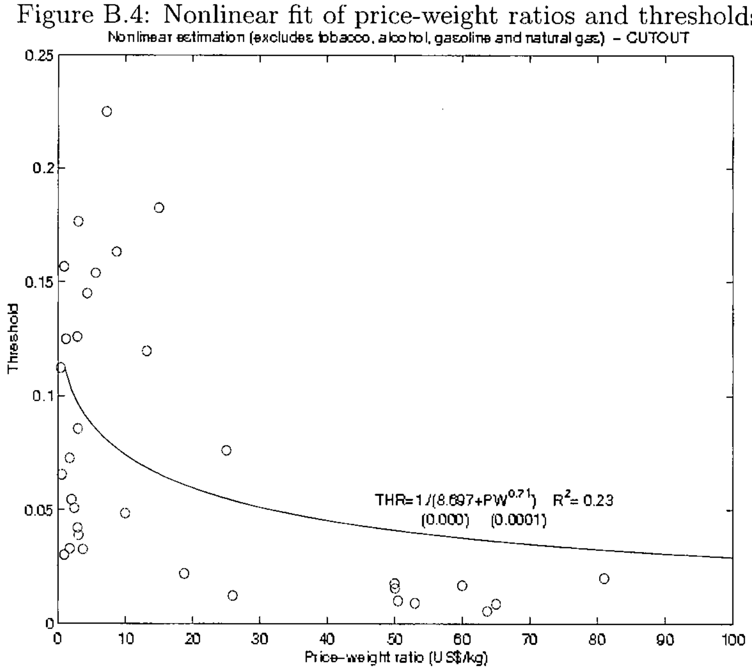


Table B.18: BAND-TAR(2,p,d) results - part I

	LLR	TAR(2,p,d) threshold	TAR(2,p,d) halflife	p-value
Total RER-CPI	-9	0.012	193	0.057
Apples	23	0.073	6	0.086
Audio equipment	13	0.076	86	0.096
Beef	15.1	0.039	23	0.253
Beer	1.3	0.041	144	0.172
Car purchase	1.2	0.022	30	0.273
Car maintenance	-1.9	0.167	58	0.106
Car parts	6.5	0.048	21	0.396
Cheese	-1	0.163	24	0.099
Clothes	-2.2	0.087	36	0.209
Clothes (men)	-11.8	0.010	46	0.204
Clothes (women)	-7.3	0.009	24	0.238
Coffee	1.4	0.225	107	0.067
Educational books and supplies	-1.7	0.116	22	0.086
Eggs	-4	0.033	8	0.412
Electricity	2.5	0.055	71	0.188
Fats and oils	3.1	0.033	49	0.191
Fish and seafood	-2.5	0.126	50	0.116
Flour	17.3	0.157	48	0.182
Footwear	-6.6	0.006	35	0.175
Footwear (men)	-3	0.006	22	0.355
Footwear (women)	-14.1	0.020	39	0.221
Fuel oil	5.9	0.112	48	0.219
Furniture	10.4	0.145	60	0.161
Gas	41.9	0.043	28	0.075
Gasoline	28.7	0.064	51	0.126
House chemicals	-1.3	0.125	188	0.180
Jewelry	-1.3	0.029	17	0.407
Laundry appliances	-2.3	0.154	45	0.188
Liquor	-4.1	0.012	86	0.095
Medical care products	-4.9	0.027	9634	0.084
Non-prescription medicine	0.9	0.150	101	0.166
Pants	-5.6	0.016	13	0.272
PC	-1.6	0.018	22	0.000
Personal care products	-2.2	0.177	165	0.242
Photo equipment	1.2	0.202	5	0.099
Pork	-3.6	0.013	15	0.271
Potatoes	-0.6	0.065	14	0.209
Poultry	18.6	0.086	34	0.291
Prescription medicine	-4	0.047	1655	0.053
Sport equipment	-5.6	0.009	50	0.241
Sport vehicles	40.5	0.183	13	0.075
Sugar	2.7	0.030	21	0.327
Tobacco	107.5	0.303	106	0.000
Toys	8.7	0.120	161	0.247
Video equipment	5.6	0.012	20	0.179
Watches	-3.4	0.008	3	0.282
Wine	27	0.230	409	0.068

Table B.19: BAND-TAR(2,p,d) results - part II

	LLR	TAR(2,p,d) threshold	TAR(2,p,d) halflife	p-value
Airfare	-2.4	0.015	11	0.266
Cable TV	22.4	0.042	50	0.068
Car insurance	23.2	0.025	51	0.157
Child care	10	0.019	461	0.159
Dental services	8.4	0.046		0.161
Fresh fruits	-13	0.051	11	0.311
Intra-city transport	5.2	0.061	60	0.218
Margarine	1	0.054	32	0.245
Medical services	0.9	0.166	165	0.178
Reading materials	-2.4	0.017	51	0.106
Rent	-3.1	0.016	542	0.145
Restaurant meals	5.1	0.040	68	0.061
Shelter	-1.7	0.033	119	0.036
Tomatoes	-28.5	0.042	6	0.460
Tuition	1.3	0.028	18	0.463
Water and sewerage	-1.6	0.114	76	0.205

Table B.20: Price-to-weight ratios, Price-to-volume ratios, and thresholds

	Cnst	P/W	P/W_{NT}	P/V_{NT}	R^2	p-value (F)	LogL	DW
Thr_B	8.4^{**}	-0.017						
	(0.000)	(0.245)			0.03	0.245	-135	2.31
$\mathbf{Thr}_{B,NT}$	$\mathbf{8.68}^{***}$		$\mathbf{-0.059}^{***}$					
	(0.000)		(0.012)		0.16	0.067	-126	2.36
$\mathbf{Thr}_{B,NT}$	$\mathbf{8.42}^{***}$			$\mathbf{-0.0002}^{**}$				
	(0.000)			(0.052)	0.099	0.059	-118	2.48
$Thr_{B,NT}$	8.68^{***}		-0.06	0.00001				
	(0.000)		(0.123)	(0.96)	0.157		-135	2.37

p-values in parentheses. A * denotes a significance at 10% level, ** at 5% level and *** at 1% level.

1) "$_{NT}$" is a regression excluding tobacco due to a large discrete jump in its relative price in 1984/85 following a tax change.

Table B.21: Nonlinear relationship between price-to-weights and thresholds

Hyperbola: $\text{Thr}_{B,NT} = 1/(C(1) + (P/W_{NT})^{C(2)})$

	C(1)	C(2)	R^2	LogL	DW
$\text{Thr}_{B,NT}$	9.144**	0.697**			
	(0.000)	(0.000)	0.18	55.6	2.57
$\text{Thr}_{B,NTAE}$	8.697**	0.71**			
	(0.000)	(0.000)	0.23	50.9	2.59

1) "$_{NT}$" is a regression excluding tobacco due to a large discrete jump in its relative price in 1984/85 following a tax change.
2) "$_{NTAE}$" is a regression excluding tobacco, alcohol and energies (gasoline, natural gas) due to poor tradability.

Table B.22: Prices, weights and thresholds

	Cnst	P	W	R^2	p-value (F)	LogL	DW
Thr_B	7.7**	-0.0046	0.082				
	(0.000)	(0.161)	(0.181)	0.06	0.3	-135	2.4
$\text{Thr}_{B,NT}$	**7.12****	**-0.0049***	**0.088***				
	(0.000)	(0.090)	(0.10)	0.08	0.19	-126	2.6
$\text{Thr}_{B,NTE}$	**7.256****	**-0.0051***	**0.092***				
	(0.000)	(0.086)	(0.098)	0.095	0.18	-120	2.5
100*STD_{NT}	12.2	-0.005					
	(0.000)		(0.221)	0.041	0.23	-113	1.72
100*STD_{NT}	12.1	-0.0003					
	(0.000)	(0.232)		0.04	0.23	-113	1.73

p-values in parentheses. A * denotes a significance at 10% level, ** at 5% level and *** at 1% level.
1) $R_{SW,NT}$ is a residual from estimation of STD on W and a constant due to endogeneity of STD (not shown).
2) "$_{NT}$" is a regression excluding tobacco due to a large discrete jump in its relative price in 1984/85 following a tax change. "$_{NTE}$" also excludes energy (gasoline, natural gas) due to poor tradability.

Table B.23: Price-to-weight ratios, Price-to-volume ratios, and thresholds - robustness

	Cnst	P/W	P/V	R^2	DW
$\mathbf{Thr}_{NTE}$	**8.6*** **	**-0.06*** **			
	(0.000)	(0.012)		0.17	2.29
$\mathbf{Thr}_{NTE}$	**8.6*** **		**-0.0003** **		
	(0.000)		(0.049)	0.11	2.42
Thr_{NTE}	8.9***	-0.06	0.0001		
	(0.000)	(0.123)	(0.99)	0.17	2.30
$\mathbf{Thr}_{NTW}$	**9.8*** **	**-0.14*** **			
	(0.000)	(0.016)		0.24	2.53
Thr_{NTW}	**8.6*** **		**-0.0003* **		
	(0.000)		(0.082)	0.08	2.66
Thr_{NTW}	9.9***	-0.13*	-0.0001		
	(0.000)	(0.095)	(0.80)	0.24	2.56
$\mathbf{Thr}_{NTAE}$	**8.8*** **	**-0.06*** **			
	(0.000)	(0.009)		0.19	2.57
$\mathbf{Thr}_{NTAE}$	**8.4*** **		**-0.0002** **		
	(0.000)		(0.043)	0.12	2.57
Thr_{NTAE}	8.78***	-0.06	0.0001		
	(0.000)	(0.11)	(0.96)	0.19	2.57

p-values in parentheses. A * denotes a significance at 10% level, ** at 5% level and *** at 1% level.
"$_{NTE}$" excludes tobacco and energies (gasoline, natural gas)
"$_{NTAE}$" excludes tobacco, alcohol (liquor, beer, wine) and energies (gasoline, natural gas)
"$_{NTW}$" excludes tobacco and watches

Appendix C

C.1 Solving the banks' problem

Due to the inequality constraints, we have to use a Kuhn-Tucker approach and be careful about the corner solutions. The Lagrangean for this problem is:

$$\begin{aligned}\mathcal{L} &= r^l L + r^b B - r^r D - r^e E - \delta\Big(\frac{D}{E}\Big)^{\gamma} D - (1+l_c)\epsilon L \\ &\quad + \lambda_1(D+E-B-L) + \lambda_2(E/L-\alpha) + \lambda_3(D+E-L)\end{aligned}$$

Then the first order conditions are:

$$\begin{aligned} r^l - \lambda_1 - \lambda_2 E/L^2 - \lambda_3 - \epsilon(1+l_c) &= 0 \\ r^b - \lambda_1 &= 0 \\ -r^d - \delta(\gamma-1)\Big(\frac{D}{E}\Big)^{\gamma} + \lambda_1 + \lambda_3 &= 0 \\ -r^e + \delta\gamma\Big(\frac{D}{E}\Big)^{\gamma+1} + \lambda_1 + \lambda_2/L + \lambda_3 &= 0 \end{aligned}$$

As noted above, there are two possibilities: either constraint (3.9) or constraint (3.10) bind. In terms of the Lagrangean we therefore need to consider two cases. The one where $\lambda_2 > 0$ and $\lambda_3 = 0$ (i.e. (3.9) binds while (3.10) does not) will be referred to as an "interior solution" because not all loanable funds are invested into loans. The opposite case where $\lambda_3 > 0$ and $\lambda_2 = 0$ will be referred to as a "corner solution". For simplicity, in what follows we assume $\gamma = 1$.

Interior solution

This is the case when bank holds just enough equity to satisfy the capital adequacy requirement ($E/L = \alpha$ and therefore $D + E > L$). The above first order conditions can be combined into:

$$r^d = r^b - 2\delta\frac{D}{E} \tag{C.1}$$

$$\frac{M}{E} = 1 + \Big[\frac{1}{\delta}(r^e - r^d) - \frac{1}{\alpha\delta}(r^l - r^d - (1+l_c)\epsilon\Big]^{\frac{1}{2}} \tag{C.2}$$

$$L = \frac{1}{\alpha}E \tag{C.3}$$

$$D = M - E \tag{C.4}$$

where (C.2) is an equity (or implicitly deposit) supply equation. Conditional on particular values of M and all levels of prices, equations (C.1) to (C.4) form a recursive system which uniquely determines all quantities.

Corner solution

In a corner solution, bank holds more equity than required by the capital adequacy requirement ($D+E=L$ and therefore $E/L>\alpha$). Now, $r^b > r^d$ [1], and the above first order conditions can be combined into:

$$\frac{M}{E} = 1+\left[\frac{r^e - r^l + (1+l_c)\epsilon}{\delta}\right]^{\frac{1}{2}} \tag{C.5}$$

$$L = M \tag{C.6}$$

$$D = M - E \tag{C.7}$$

$$r^l - r^b - (1+l_c)\epsilon = r^b - r^d \tag{C.8}$$

where (C.5) is again an equity supply equation. Note that now loans and equity supply decisions are disconnected. Equation (C.8) shows a wedge between the bond and deposit rates. The bond "premium" on the right hand side equals the profit differential between net returns on loans and bonds that would equal zero in an interior solution.

C.2 On the assumption of a single portfolio optimization

It may seem problematic to assume that the asset portfolio is allocated in an identical manner for all households. Here we show that as long as the labor income remains the same across all depositors, the optimal splitting rules derived from their preferences will be identical across all of the households.

To prove this point, we use a simplified version of the problem. Households maximize $\max_{\{c_{t,i},m^i,t+1,d_{t,i},e_{t,i}\}} \mathrm{E}_0\left[\sum_{t=0}^{\infty}\beta^t U(c_{t,i})\right]$ s.t. $c_{t,i}+d_{t,i}+e_{t,i}=m_{t,i}+y_i$, where $U(c_{t,i})=\frac{(l_{oc}^{\sigma}c_{t,i}^{1-\sigma})^{1-\rho}-1}{1-\rho}$, $m_{i,t+1}=d_{t,i}(1+r_t^d)+e_{t,i}(1+r_t^e)$ and $e_{t,i}, d_{t,i}$ denote individual equity and deposit holdings, respectively. The Euler equations for this problem are:

$$c_{t,i}^{\chi} = \beta \mathrm{E_t}\Big[c_{t+1,i}^{\chi}(1+r_{t+1}^d)\Big] = \beta\Big[\mathrm{E_t}(1+r_{t+1}^d)\mathrm{E_t}[c_{t+1,i}^{\chi}]\Big]$$

$$c_{t,i}^{\chi} = \beta \mathrm{E_t}\Big[c_{t+1,i}^{\chi}(1+r_{t+1}^e)\Big] = \beta\Big[\mathrm{E_t}(1+r_{t+1}^e)\mathrm{E_t}[c_{t+1,i}^{\chi}] + cov[(1+r_{t+1}^e), c_{t+1,i}^{\chi}]\Big]$$

where $\chi=-(\sigma+\rho(1-\sigma))$. Solving with a method of undetermined coefficients, we make an educated guess that $e_{t,i}=\gamma_e m_{t,i}$ and $d_{t,i}=\gamma_d m_{t,i}$ and rewrite the Euler equations as:

$$\Big[(1-\gamma_{e,i}-\gamma_{d,i})m_{t,i}+y_i\Big]^{\chi} = \beta(1+r^d)\mathrm{E_t}\Big[(1-\gamma_{e,i}-\gamma_{d,i})m_{t+1,i}+y_i\Big]^{\chi}$$

$$\Big[(1-\gamma_{e,i}-\gamma_{d,i})m_{t,i}+y_i\Big]^{\chi} = \beta\mathrm{E_t}\Big[(1+r_{t+1}^e)^{1/\chi}[(1-\gamma_{e,i}-\gamma_{d,i})m_{t+1,i}+y_i]\Big]^{\chi}$$

The above two equations give determine the shares of equity $\gamma_{e,i}$ and deposits $\gamma_{d,i}$ in $m_{t+1,i}$ as functions of individual as well as aggregate variables. The important point for our argument is that if all agents have the same labor income $y_i = y\ \forall\ i$, then we can harmlessly assume that $y=0$ and these two equations collapse into:

$$1 = \beta(1+r^d)\mathrm{E_t}\Big[[\gamma_d(1+r^d)+\gamma_e(1+r_{t+1}^e)]\Big]^{\chi} \tag{C.9}$$

$$1 = \beta\mathrm{E_t}\Big[(1+r^e)^{1/\chi}[\gamma_d(1+r^d)+\gamma_e(1+r_{t+1}^e)]\Big]^{\chi} \tag{C.10}$$

[1] A lower demand for bank's financing by deposits (relative to equity) depresses their price.

Note that in equations (C.9) and (C.10), γ_e and γ_d are *independent* of any individual variables. They are only functions of the rates of return and the parameters of the utility function. This way we have shown that as long as the agents have an identical labor income (and as long as their deposit and equity demands are linear in their asset holdings which can be proved for the case of $y = 0$), the portfolio-splitting decisions can be assumed to be made uniformly.

Because in this model we work with two types of depositors (workers and retirees / unemployed), the assumption of an identical labor income is only justified *within* these two groups. In the future research, we should therefore model the number of distinct portfolio optimizations equal to 2. The optimality conditions will then present additional identification restrictions on the parameter values of the household's risk-aversion parameter λ. We ignore this in the current version of the paper due to the computational difficulty.

C.3 The solution procedure

Heterogeneous agents models with aggregate shocks are difficult to solve because the distribution of agents is not invariant and becomes a highly dimensional state variable. The two main strategies to solve this problem is to either find a good way to summarize the distribution with very few variables, as Krusell and Smith (1998) demonstrate, or to work with linearization, as Cooley and Quadrini (1999) do. Unfortunately, neither is possible here due to some highly non-linear phenomena that are crucial in our model. For example, decision rules change abruptly in the vicinity of m^*. Finally, second degree effects appear to be quite important, and they are likely to vanish with linearization.

Our strategy uses the realization that aggregate shocks in a two-state Markov process lead to transitional states somewhere between two steady-states corresponding to repeated identical shocks. We therefore choose a sufficient number of aggregate states to represent a large proportion of actual aggregate states.

The aggregate state space is assumed two dimensional: one dimension is the current shock, High or Low, the other is a counter of how far from the the High steady-state the economy is. Specifically, this counter is incremented by one each time a Low shocks occurred in the previous period, or decreased by one if a High shock occurred. The minimum counter value is one, the maximum is chosen such that this state occurs infrequently. We choose a maximum of 5, implying with the transition probabilities of the Markov process that the economy will in any of the aggregate states $S_{sc}\%$ of the time, where

$$S = \begin{pmatrix} 50.2 & 16.7 & 5.6 & 1.9 & 0.6 \\ 16.7 & 5.6 & 1.9 & 0.6 & 0.2 \end{pmatrix}.$$

We then solve this model economy with the standard tools for heterogeneous agent economies, that is value function iterations followed by iterations on the invariant distribution (defined over the aggregate states as well). The equilibrium is reached by finding the set of lending rates r^l and loan eligibility rules m^* that balance all markets and satisfy all constraints.

C.4 Figures

Figure C.1: Benchmark economy as it cycles through all possible aggregate states

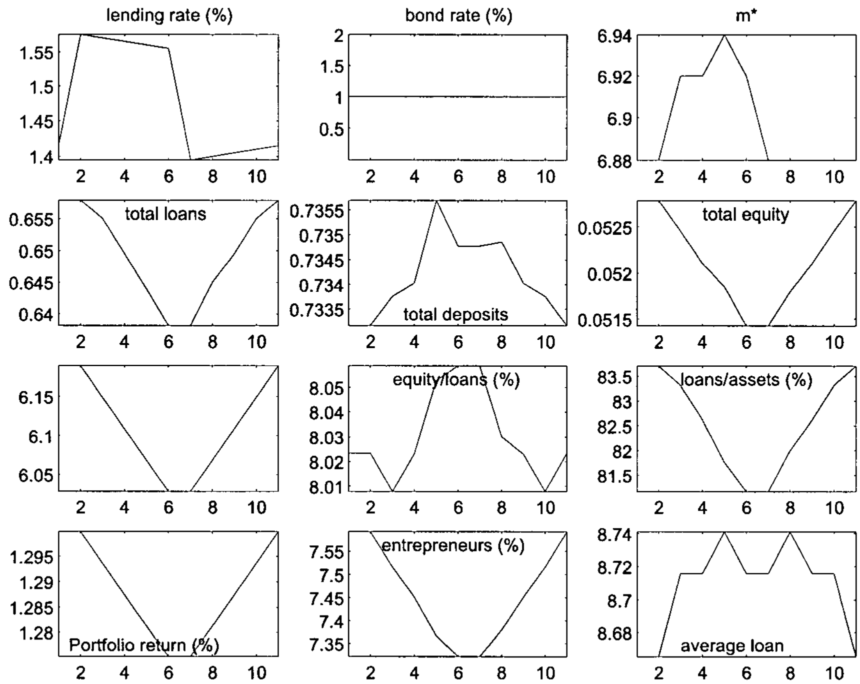


Figure C.2: Benchmark and policy with interest rate reduction in worst case only

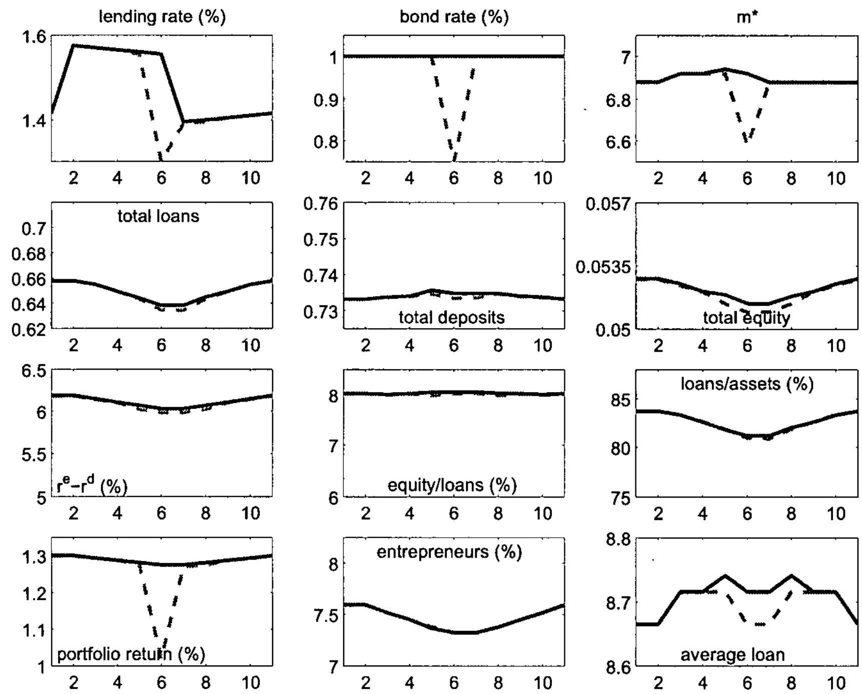


Figure C.3: Benchmark and policy with gradual interest rate reduction in bad return situations

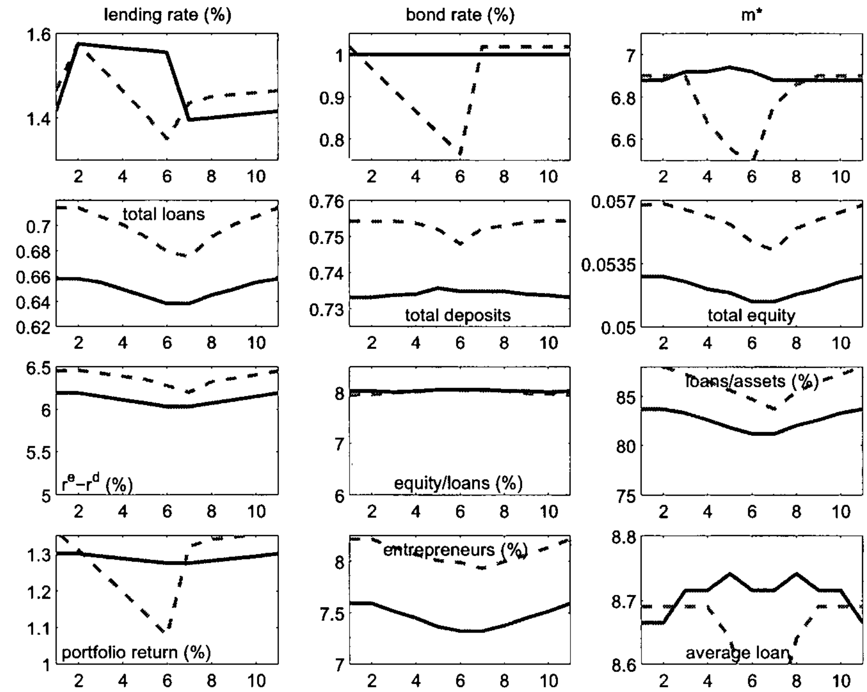


Figure C.4: Benchmark and policy with interest rate increase in worst case only

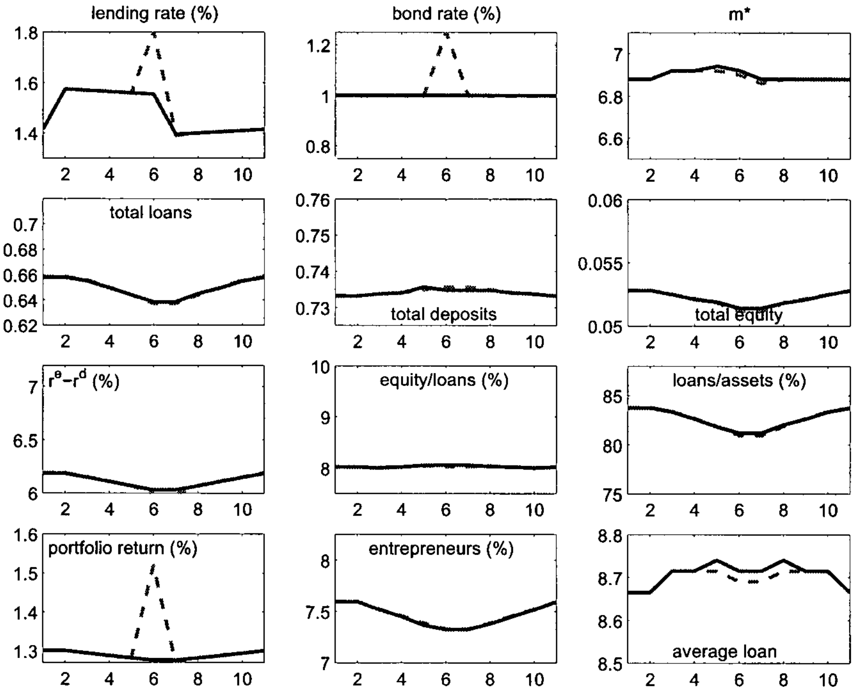


Figure C.5: Benchmark and policy with gradual interest rate increase as aggregate states worsen

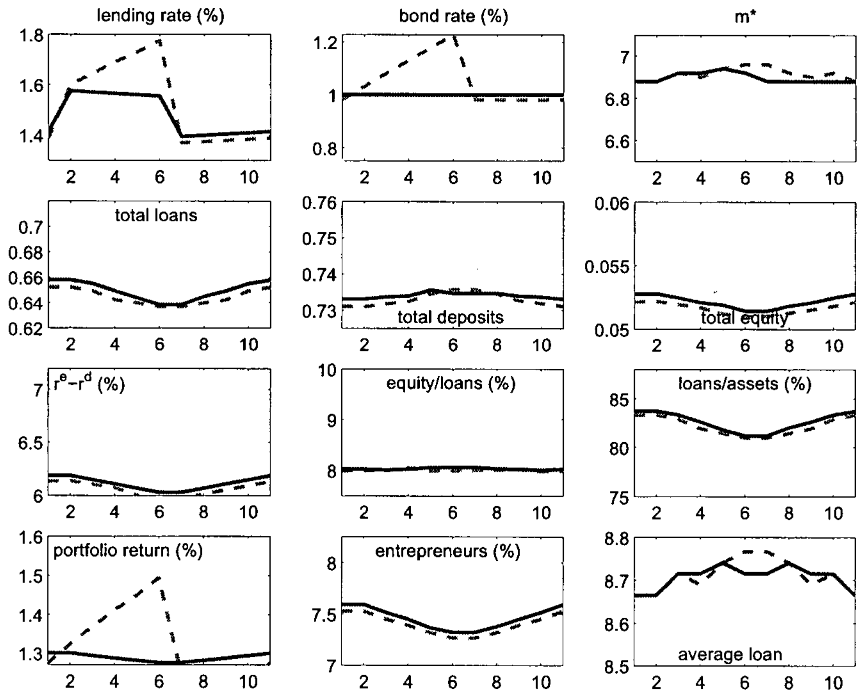


Figure C.6: Benchmark and policy with relaxing of capital requirements in worst case only

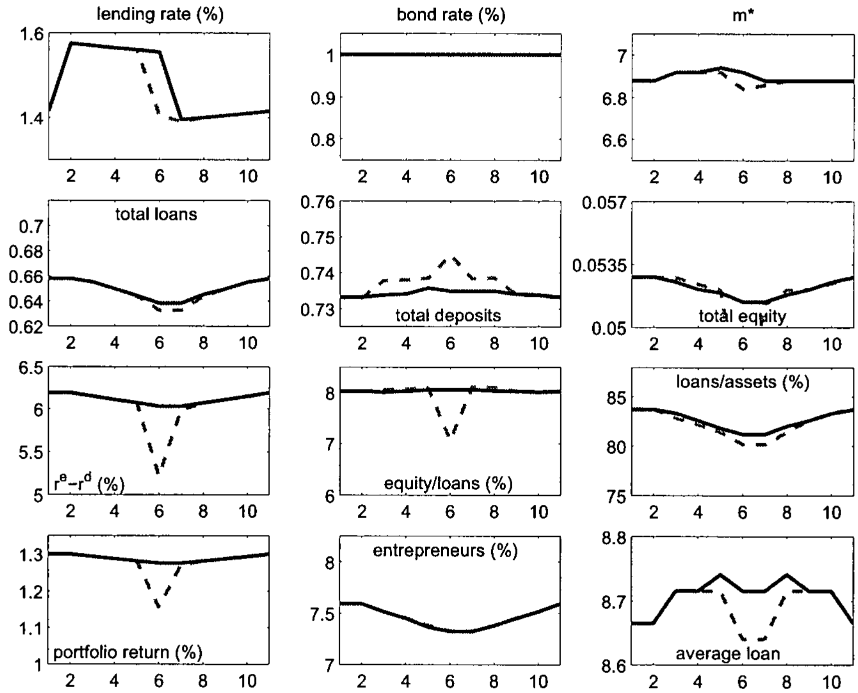


Figure C.7: Benchmark and policy with gradual relaxing of capital requirements as aggregate states worsen

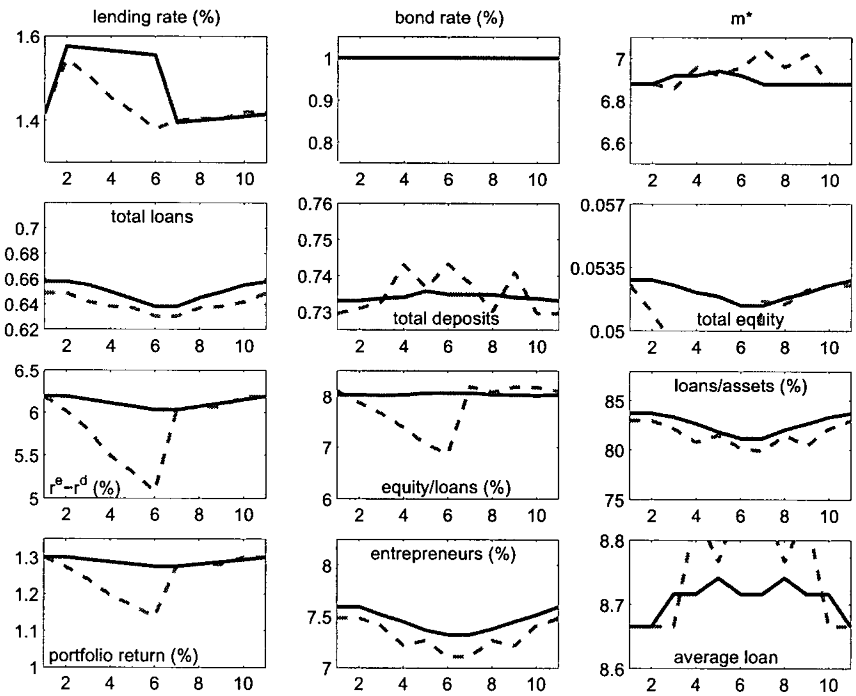


Figure C.8: Benchmark and policy with tightening of capital requirements in worst case only

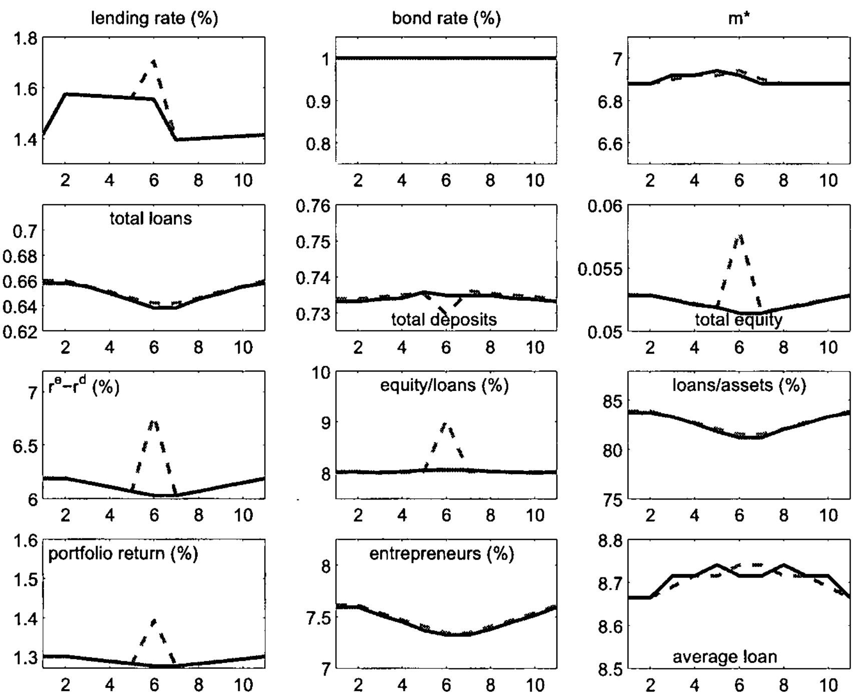


Figure C.9: Benchmark and policy with gradual tightening of capital requirements as aggregate states worsen