The Role of Capital Flows and Savings in the Growth Process

by

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Abstract

The goal of this dissertation is to address the following questions: Why do growing economies borrow? Are the neoclassical forces that drive accumulation the main impulse behind long-term cross-country movements in capital? What class of models best describes observed capital flows? A standard view of capital flows is that they are driven by scarcity, and act as a substitute to domestic savings. Countries borrow to accumulate capital, and by using international capital markets, can increase investment with no cost in current consumption: foreign financing replaces domestic savings. An alternative view is that factors other than the domestic scarcity of capital may drive inflows from abroad. For example, in the presence of collateral constraints, countries that are willing to cut current consumption to accumulate domestic capital may be rewarded by additional inflows, i.e. capital inflows may complement domestic savings.

In the first essay, I examine whether the qualitative implications of a simple neoclassical model with collateral constraints fit the long-term movements in external debt. The most surprising prediction of this class of models is that, contrary to a pure neoclassical model, domestic savings should act as a complement rather than a substitute to capital inflows. Nevertheless, this class of models still predicts that, ceteris paribus, capital should flow to the countries where it is most scarce. Using data on debt between 1970 and 1997 for high-income as well as middle-income developing economies, I find evidence that supports the prediction that domestic savings play a complementary role. These result suggest that policies that affect national savings may potentially be important for capital accumulation and growth, even in open economies.

The second essay of the thesis evaluates the quantitative performance of this class of models, by focusing on the model developed by Barro, Mankiw and Sala-i-Martin [1995]. Specifically, I ask what factor shares are implied by the cross-country variations in debt accumulation examined in the first essay. The model, which features decreasing returns and a complementary role for savings, is solved and simulated using non-linear techniques. Factor shares are then estimated using a method of indirect inference by comparing simulated and actual debt data. The model implies fairly high, though not implausible, shares of composite capital, without completely matching the effect of the savings rate. It does however, consistently predict unrealistically high debt-to-GDP ratios.

In the third essay, I consider and evaluate an alternative model of debt. A neoclassical model with capital adjustment costs and a debt-elastic interest rate is examined to determine whether an alternative model, more standard in the literature, can match the observed convergence equation on debt. I find that although qualitatively the model allows for the possibility that capital inflows can be both complements and substitutes to domestic savings, quantitatively, the model is odds with the data.

These results suggest that two mechanisms — decreasing returns and the complementarity between domestic savings and foreign financing — are quantitatively important. They cannot however, fully account for the observed variations in debt and income. Modeling the dynamics of the debt-to-output ratio is a promising avenue for future research.


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Dedication

Cette thèse est dédiée à ma mère et mon père, Evelyne et Antoine, et ma soeur, Dominique, qui ne m'ont jamais rien refusé, ni leur attention, ni leur affection, ni leur indulgence; et à mes oncles, Marc et Henri, qui m'ont inspirée à aller jusqu'au bout de mes ambitions.
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Chapter 1

Introduction

In recent years, dramatic events in world asset and commodity markets have placed the role played by capital flows and international debt markets squarely in the public eye. The so-called 'globalisation' phenomenon has elicited varying degrees of support from the general public. Economists however, tend to view capital markets as an important component of growth and development. Two arguments in particular, relevant to this thesis and favourable to the liberalisation of capital flows, often appear in the growth and international macroeconomic literature. The first concerns the direction of capital flows: poor countries with high capital productivity attract capital flows from abroad which permit increased investment and output; rich countries with low capital productivity can earn higher returns by investing their savings abroad. This is simply the principle of decreasing returns at work. The ability to tap into world markets for capital is seen as an accelerator of convergence.

The second argument concerns the role of capital flows: they fill the gap between savings and investment, by reducing the need for saving. A country that has access to international capital markets can borrow from abroad, increase investment and consumption. Capital flows substitute for domestic savings: they reduce the burden of development on consumers, which can smooth their consumption over the transition to the steady state. Much of this thesis will focus on these concepts — decreasing returns and the role of capital flows as substitutes to savings — both empirically and theoretically.

The objective of this dissertation is to address the following questions: What role do
capital flows play in the growth process? How important are savings? And what class of models can capture the economic mechanisms that best describe long-run movements in capital flows? This thesis will examine the role of capital flows and challenge the notion that capital flows are substitutes for domestic savings. In particular, I will assess the possibility that high-saving countries attract capital flows. Chapter 2 will present empirical evidence that capital flows are **complements** to domestic savings. In addition, it will show that this prediction is consistent with a class of models that features collateral constraints and decreasing returns. Chapter 3 will build on this evidence and evaluate the quantitative relevance of this class of models. Chapter 4 will examine whether more standard class of models are also consistent with the evidence on the complementarity of savings.

Throughout the thesis, the analysis will be undertaken under the assumption of decreasing returns and convergence, i.e. we will focus on neoclassical models of growth. This maintained hypothesis is made as a starting point to understanding the role of capital flows from the point of view of a class of models that are well understood. In fact, one contribution of this work is to demonstrate how much one can learn from very simple models. Yet, evidence of convergence and decreasing returns seems limited, as noted in Chapter 2. Studies of cross-sectional variation in output growth suggest that the convergence rate is approximately 2 per cent per year, which implies that it would take several generations to eliminate an initial gap from the steady state (see Mankiw, Romer and Weil [1992], Barro and Sala-i-Martin [1995]). It is that type of evidence which has prompted Lucas [1990] to ask why capital does not flow from rich to poor countries, and to suggest more attention be paid to imperfect capital mobility as a possible culprit. This thesis follows this suggestion and examines the role of capital flows under two types of imperfections: quantity and price constraints.

In Chapter 2, the analysis is undertaken within the framework of a model developed by Barro, Mankiw and Sala-i-Martin [1995] in which countries face a collateral constraint. Production takes place with two types of complementary capital, foreign and domestic. Countries can borrow to finance investment in foreign capital but must save in order to
accumulate domestic capital. Because the two types of capital are complements in production, higher savings in domestic capital increases the productivity of foreign capital, and consequently, the flow of capital from abroad. In this model, a convergence equation for debt, in which debt accumulation is a function of initial condition and the savings rate, can be derived and estimated. Using a new source of data on net external debt, Chapter 2 presents evidence that capital flows in the long term are complements to domestic savings. In addition, the results are consistent with the convergence hypothesis, i.e. scarcity can play a role in driving long-term capital flows. These results appear robust to varying assumptions about variables that affect the steady state and the flow of technology.

Chapter 3 builds on these empirical results. Since convergence and the complementarity of savings appear to be relevant qualitatively, this chapter attempts to provide a quantitative evaluation of the model developed by Barro, Mankiw and Sala-i-Martin [1995]. More specifically, we try to find the share of capital for which simulated data from the model comes close to reproducing the convergence equation for debt estimated from the data. In order to undertake this quantitative exercise, we must obtain a solution for the decision rules for consumption and capital in the model. Since most countries considered in our data samples are likely to be far away from their steady state, linear solution methods are inappropriate. We propose a non-linear solution and describe an algorithm which leads to the estimation of the capital shares in the model. In the version of the model with exogenous savings, we find that the model can match the convergence equation for debt for reasonable values of the capital shares if we believe them to apply to both physical and human capital. Even so, the model does tend to predict implausibly high debt-to-output ratios. The surprising conclusion from Chapters 2 and 3 is that although the quantitative performance of the model is imperfect, a very simple model can help us understand the workings of international capital markets.

Although Chapters 2 and 3 suggest that models with collateral constraints are a promising avenue of research, most of the international macroeconomics literature has not focused on this class of models. In Chapter 4, we turn to the evaluation of an alternative model in
the class of neoclassical models with imperfect capital markets. We choose to examine a
model with adjustment costs to capital and a debt-elastic interest rate premium, a standard
example of models used in the open-economy macroeconomic literature. The objective of
this chapter is first, to present the prediction of this model for the role of capital flows,
and second, to determine whether it can match the evidence presented in Chapter 2. We
find that this class of models predicts that capital flows will be driven by two forces. More
patient countries with a higher propensity to save will tend to rely less on borrowing, i.e.
capital flows are substitutes to domestic savings. Imperfections in capital markets however,
will tend to cause a positive relationship between savings and capital flows, i.e. savings
play a complementary role. In order to determine which effect dominates, we repeat the
quantitative exercise undertaken in Chapter 3. We find that this model cannot match the
convergence equation for debt for reasonable values of the parameters. In fact, quanti­
tatively, the substitution channel seems to dominate the complementarity channel in this
class of models.
Chapter 2


2.1 Introduction

Why do growing economies borrow? Are the neoclassical forces that drive accumulation the main impulse behind long-term cross-country movements in capital? A standard view of capital flows is that they are driven by scarcity, and act as a substitute to domestic savings. Countries borrow to accumulate capital, and by using international capital markets, can increase investment with no cost in current consumption: foreign financing replaces domestic savings. An alternative view is that factors other than the domestic scarcity of capital may also drive inflows from abroad. In particular, countries that are willing to cut current consumption to accumulate domestic capital may be rewarded by additional inflows, i.e. capital inflows may complement domestic savings. The objective of this chapter is to illustrate how well this latter view fits the long-term movements in external debt. More specifically, the evidence presented here suggests that the cross-country variations in net foreign liabilities conform rather well to the qualitative predictions of a simple neoclassical model with collateral constraints in which countries with high savings rates tend to accumulate debt faster.
Surprisingly, little is known about the role of savings, convergence or imperfect capital markets in driving cross-country variations in external debt \(^1\). The limited research on long-term cross-sectional variation in external positions stems in part from the paucity of consistent data on net foreign debt. A newly constructed database by Lane and Milesi-Ferretti [1999] now allows the investigation of these questions. This chapter exploits these new data to estimate a reduced form for debt accumulation derived from a neoclassical model with credit constraints.

The predictions of neoclassical growth models for movements in capital are well known. In the benchmark neoclassical model with labour-augmenting technological progress, growth is driven by accumulation, the returns to which decline with development. With perfect capital markets, convergence accelerates: less developed economies can use international markets to finance capital accumulation, while richer countries lend them the necessary funds thereby earning higher returns on their savings. Poor countries borrow and rich countries lend to equalise the marginal products of capital. As a starting point to address questions about long-term debt accumulation, the thesis focuses on this class of models with decreasing returns, which predicts convergence — a standard and well-understood phenomenon.

These models however, tend to predict infinite rates of convergence when capital is allowed to flow freely. Observed rates of convergence are not only finite, but fairly low, around 2 per cent a year (e.g. Mankiw, Romer and Weil [1992], MRW hereafter). In addition, there is evidence that the ability to borrow is somewhat limited by capital market imperfections (see Obstfeld and Rogoff [2000]). This suggests that for a neoclassical model to help us understand variations in borrowing behaviour, some type of imperfection in capital markets is needed. What form should this imperfection take? This chapter suggests that a modelled imperfection that allows for the possible complementary role of domestic savings is an appropriate choice. The analysis is undertaken with a two-sector neoclassi-

\(^1\)Throughout this thesis, the terms 'net foreign liabilities', 'net external debt' and 'debt' are used interchangeably.
cal model developed by Barro, Mankiw and Sala-i-Martin [1995] (BMS hereafter). In the framework developed by BMS, there are two types of capital, foreign and domestic. Constrained countries can borrow freely on world markets for their foreign capital needs, but must save in order to accumulate domestic capital. Nevertheless, this framework retains the feature that, everything else being equal, capital flows to where it is most scarce. Since foreign and domestic capital are complementary in production, the collateral constraint slows down the rate of income convergence compared to the standard model. Perhaps more importantly, this complementarity in production leads to a complementarity between savings in domestic capital and foreign financing of capital. This model offers a natural way to examine cross-country long-run debt accumulation and is the starting point of the empirical work presented here.

Previous empirical work in the field of international finance has focused on short run determinants of capital flows with emphasis on current account movements (e.g. Obstfeld and Rogoff [1996], Glick and Rogoff [1995]). Other studies have focused on the long-term flows of savings (Edwards [1996], Masson, Bayoumi and Samiei [1998]) and the current account (Chinn and Prasad [2000]) while another branch of the literature has shed some light on the long term determinants of debt within countries and across time (Masson, Kremers and Horne [1994], Calderon, Loayza and Serven [1999]). Yet these studies, do not explicitly allow for the forces of convergence to explain long-run dynamics either because they do not specify a model in particular or because they do not attempt to explain cross-sectional variation.

There exists some evidence of quantity constraints in international capital markets directly from measures of debt. One early example is Eaton and Gersovitz [1981] who develop a model of sovereign debt in which countries that default are excluded from world markets. The possibility of default leads lenders to establish a credit ceiling which is a function of the cost of exclusion from capital markets for the borrower. Using this model and data on gross public debt, they find that a number of low-income countries are constrained. Similarly, Adda and Eaton [1998] develop a methodology for estimating the...
level of expenditure of a credit-constrained country and infer the level of the credit ceiling. They find that GDP, openness and investment are significant determinants of constraints. More recent papers also include Lane [2000] and Lane and Milesi-Ferretti [2001] who report a positive relationship between debt and income. Lane [2001] also develops a version of the BMS model. In his version of the model, a small open economy can only use capital from the tradable sector as collateral for borrowing on international capital markets. His study however, focuses on the determinants of the steady state debt-output ratio, not on the cross-country dynamics of debt.

This chapter will exploit the net foreign assets data constructed by Lane and Milesi-Ferretti [1999] in order to offer an empirical assessment of the role of savings as well as convergence within the context of a growth model. The vast literature on economic growth has mainly focused on the determinants of income across countries in a closed-economy framework. When the assumption of capital immobility is inappropriate, studies have relied instead on partial openness. For example, the application of the Solow model to US states \(^2\) relies on the results of the BMS model which suggests that a neoclassical model with partial capital mobility will mimic the behaviour of a closed economy Ramsey model. This chapter makes use of this feature of the model. Specifically, convergence equations for debt, similar to equations from the convergence literature, are derived from the model, and estimated using the Lane and Milesi-Ferretti database. Results give some qualitative support to the idea of convergence. Countries with low levels of initial debt did increase their borrowing over the sample period, at a rate of about 2 per cent per year, an estimate surprisingly close to those of income convergence found in the literature. In addition, the model predicts a positive correlation between domestic savings and debt accumulation which results from the combination of the technological complementarity between the two types of capital and the collateral constraint. This is supported by the data and is robust to the econometric specification, the sample of countries considered, as well as assumptions about technology and the measurement of domestic savings. However, the tight relationship between debt

\(^2\)See Sala-i-Martin [1996].
and income implied by the model only seems to be partially borne out by the data. In particular, in the samples considered here, income exhibits low conditional convergence, implying that the convergence observed in debt is not entirely a consequence of decreasing returns. These results suggest that to successfully explain cross-country variations in levels of debt, a model should exhibit the convergence property, as well as a complementarity between domestic savings and foreign financing. In addition, it should allow for a more complex and flexible relationship between debt and output.

The chapter is organized as follows. Section 2.2 describes the framework based on the BMS growth model and derives empirically-testable convergence equations. Section 2.3 presents the results of estimation. Section 2.4 concludes.

### 2.2 The Model

In order to study the long-term determinants of debt accumulation, this chapter makes use of a neoclassical open-economy growth model with credit constraints developed by BMS. This choice deserves some justification. First, the use of a model with decreasing returns allows us to determine how far we can go in explaining cross-country variations in debt accumulation using the standard and well-understood mechanism of convergence. Second, to derive an empirically implementable reduced form for external debt, we must consider a model with some form of imperfection in international capital markets since the predictions of the open-economy version of the Ramsey model with perfect capital mobility for net foreign assets are problematic. In particular, for constant consumption, the value of net foreign assets is indeterminate in the steady state (it depends on initial conditions), or is excessively large for declining consumptions paths (a small open economy ends up mortgaging all of its wealth). In addition, a neoclassical open-economy model with perfect capital markets exhibits infinite speeds of convergence for capital and output. Assuming a debt-elastic interest rate, an endogenous discount factor or portfolio adjustment costs are
ways to solve for the indeterminacy of debt in these models \(^3\) (see Schmitt-Grohé and Uribe [2001]). An alternative is to have a quantity constraint as in the BMS model. This is the avenue pursued here. Furthermore, as will become apparent, the presence of the collateral constraint — as opposed to these other imperfections — leads to a role for domestic savings that is rarely emphasised. We discuss this point further below.

The BMS model is one in which credit-constrained small-open economies can use foreign financing to accumulate part of their capital and must save in order to finance the remaining fraction. Technology takes the form of a Cobb-Douglas production function over three inputs — two types of capital, \(K\) and \(Z\) which are defined below, and raw labour \(L\), so that

\[
Y_t = K_t^\alpha Z_t^\eta (\theta_t L_t)^{1-\alpha-\eta}
\]  

(2.1)

where \(\alpha, \eta > 0, \alpha + \eta < 1\). \(\theta_t\) is the exogenous source of technology and grows at rate \(g\) while raw labour grows at rate \(n\). The production function can be expressed in units of effective labour (where \(x_t = \frac{L_t}{\theta_t L_t}\)):

\[
y_t = k_t^\alpha z_t^\eta
\]  

(2.2)

Profit maximization then requires that factor prices equal the marginal productivities of inputs so that

\[
R_{kt} = \alpha k_t^{\alpha-1} z_t^\eta = \frac{y_t}{k_t}
\]

\[
R_{zt} = \eta k_t^\alpha z_t^{\eta-1} = \frac{y_t}{z_t}
\]

\[
w_t = k_t^\alpha z_t^\eta - R_{kt} k_t - R_{zt} z_t = (1 - \alpha - \eta) y_t
\]

(2.3)

where \(R_{kt}\) is the rental rate of \(k\), \(R_{zt}\) is the rental rate of \(z\) and \(w_t\) is the wage rate.

Households collect income from their labour input and from ownership of the two types of capital. This income is used to consume and accumulate capital (of both types) and debt, \(d\), on which they pay the constant world interest rate, \(r\). The budget constraint faced

\(^3\)In Chapter 4, we explore the implications for the role of savings of a model that features a debt-elastic interest rate.
by the infinitely-lived representative consumer is

$$(1+g)(1+n)(k_{t+1} + z_{t+1} - d_{t+1}) = (1 + R_k t - 6)k_t + (1 + z_{t+1} - 6)z_t - (1 + r)d_t + w_t - c_t$$  \text{(2.4)}

where we have assumed that foreign and domestic capital depreciate at the same rate, $\delta$.

Under the small-open-economy assumption, $r = R_k - \delta$.

What distinguishes the two types of capital? In this model, the credit constraint takes an extreme form. Debt cannot exceed the amount of $k$ — what we will call foreign capital — i.e. $k$ can be used as collateral whereas $z$ — the domestic capital stock — cannot. In the original BMS model, $k$ corresponds to physical capital and $z$ to human capital. Yet as noted by the authors, one need not be that specific about the type of capital that can be used for collateral. All the credit constraint assumption requires is that some types of capital are easy to borrow against internationally whereas some others are not. It is reasonable to think that foreign investors would be reluctant to accept human capital as collateral, but one can think of other types of capital that are more difficult to acquire abroad. In fact, in this thesis, we want to think of domestic capital as a broader concept than just human capital. Domestic firms might find it difficult to convince foreign investors to channel capital to their projects because of moral hazard problems, or risk of debt repudiation.

For example, foreign investors may finance machinery and equipment, which are easier to repossess, whereas domestic savers invest in building structures. Differences between the two types of capital may also be sectoral. In that case, foreign capital is used in the formal sector of the economy, while domestic capital operates in the informal sector: firms in the informal sector may not easily collateralise their assets. In all these examples — human vs physical capital, structures vs equipment or formal vs informal capital sectors — $k$ and $z$ can be complements in production.

---

4Cohen and Sachs [1986] develop a model of borrowing with one capital good in which borrowers can choose to default on their debt each period. Default however, results in loss of output (through loss of efficiency because of lost trade) and exclusion from capital markets in the future. To insure repayment, lenders ration credit and debt cannot exceed a lending limit which is a function of productive capital in the economy, i.e. in the notation used here $d_t \leq v k_t$, where $v \in [0, 1]$ — a constraint similar to equation (2.5). When the constraint binds, debt and output grow at the same rate. Gertler and Rogoff [1990] construct a model with moral hazard where borrowers actions after the loan has been made are not observable. The optimal lending contract also involves a perfect correlation between debt and output net of investment.
Whether a country is constrained or not is determined by initial asset holdings relative to steady state domestic capital $z^*$. More specifically, if $k_0 + z_0 - d_0 \geq z^*$, the constraint does not bind and the model behaves as the open-economy Ramsey model with infinite speed of convergence. If $k_0 + z_0 - d_0 < z^*$, the constraint binds ($k_t = d_t$). In that case the combination of the credit constraint, the small-open-economy assumption and profit maximization (equation (2.3)) imply that

$$d_t = k_t = \frac{\alpha}{r + \delta} y_t$$

so that $\frac{\delta}{r}$ has a constant path to the steady state. The production function can therefore be written as

$$y_t = B z_t^\varepsilon$$

where $B = \left(\frac{\alpha}{r + \delta}\right)^{1-\alpha}$ and $\varepsilon = \frac{\eta}{1-\alpha}$. Given profit maximisation (equation (2.3)) and the collapsed production function (equation (2.6)), market clearing implies

$$(1 + n)(1 + g)z_{t+1} = (1 - \alpha)y_t - c_t + (1 - \delta)z_t$$

(2.7)

Note that $(1 - \alpha)y_t$ is the gross national product, and $-\alpha y$ is net factor income from abroad.

In the original BMS paper, households chose their consumption paths optimally and the savings rate can rise or fall during the transition to the steady state. Here we choose to make the Solow-growth-model assumption of a constant savings rate for two reasons. First, we are interested in long-term phenomena for periods of time over which the savings rate does not vary much. In addition, the use of a constant exogenous savings rate will be useful for estimation.⁶

Suppose that domestic consumers save a fixed fraction of income. Let $s_y$ denote the rate at which consumers save out of gross domestic product $y$ to accumulate domestic capital, i.e. $s_yy_t = y_t - c_t$. Domestic savings must equal investment in domestic capital minus net factor payments on debt ($i_t^f = (s_y - \alpha)B z_t^\varepsilon$), so that

$$(1 + n)(1 + g)z_{t+1} = s B z_t^\varepsilon + (1 - \delta)z_t$$

(2.8)

⁶See Chapter 3 for a derivation of the estimated equation in a version of the model with endogenous savings.
where \( s = s_y - \alpha \). In the steady state, \( z_{t+1} = z_t = z^* \) so that

\[
z^* = \left[ \frac{sB}{(1 + n(1 + g) - (1 - \delta))} \right]^{\frac{1}{1-\gamma}}
\]  \hspace{1cm} \text{(2.9)}

Since \( k_t = d_t = \frac{\alpha}{r + \delta} y_t \),

\[
d^* = \frac{\alpha B}{r + \delta} \left[ \frac{sB}{(1 + n(1 + g) - (1 - \delta))} \right]^{\frac{\epsilon}{1-\gamma}}
\]  \hspace{1cm} \text{(2.10)}

Note that equation (2.8) will behave just like the dynamic equation in capital of a closed-economy Solow growth model with a broad capital share less than \( \alpha + \eta \). Consequently, the convergence rate is higher than in a closed economy but lower than with perfect capital markets. \( \frac{k}{z} \) falls during the transition: the possibility of tapping into world markets means that \( k \) is relatively high initially. \( k \) does not jump immediately to its steady state since domestic capital accumulation is constrained and \( k \) and \( z \) are complementary in production.

We can easily solve equation (2.8). Log-linearizing to approximate around the steady state, we have:

\[
\log z_t = \lambda^* \log z_0 + (1 - \lambda^*) \log z^*
\]  \hspace{1cm} \text{(2.11)}

where \( 1 - \lambda \) is the convergence rate and \( \lambda = \frac{\alpha}{r + \delta} \frac{(1 - \epsilon)(1 - \delta)}{(1 + n)(1 + g)} \).

In this model the rate of convergence is a function of \( \epsilon = \frac{\alpha}{1-\alpha} \) which governs how fast decreasing returns set in. Individual shares however, govern the degree of capital mobility. The relative importance of the two types of capital determine the degree to which countries are constrained. As \( \alpha \) — the income share of capital that can be used as collateral — rises, the degree of capital mobility increases. Foreign capital is more important in production and the economy behaves more like an open economy. On the other hand, a higher \( \eta \) means that the importance of domestic savings has increased as the relative importance of domestic capital in production rises. Thus when \( \alpha = 0 \), the economy behaves like a closed economy, and when \( \eta = 0 \), it exhibits an infinite rate of convergence. By raising \( \frac{\alpha}{\eta} \) for a given \( \alpha + \eta \), one can increase the degree of capital mobility and therefore the rate of convergence.
Equation (2.11) implies that the change in net foreign debt takes the form

\[ \log d_t - \log d_0 = -(1 - \lambda^t) \log d_0 + (1 - \lambda^t) \log d^* \]  

(2.12)

Manipulating this equation yields a convergence equation for debt of the form

\[
\log d_t - \log d_0 = (1 - \lambda^t) \frac{e}{1 - e} \log B + (1 - \lambda^t) \log \frac{\alpha}{\delta + \tau} B - (1 - \lambda^t) \log d_0 \\
+ (1 - \lambda^t) \frac{e}{1 - e} \log s - (1 - \lambda^t) \frac{e}{1 - e} \log ((1 + n)(1 + g) - (1 - \delta))
\]  

(2.13)

This specification with exogenous savings sheds light on one of the predictions of the model not emphasised in the BMS original paper: a positive relationship between domestic savings and debt accumulation. Typically, we think of domestic and foreign sources of finance as substitutes: one of the advantages of open economies is that they need not save in order to accumulate capital — they can borrow. In this model, domestic savings and foreign investment are complements. This is a direct consequence of the assumption about the production function, in which domestic and foreign capital are complements in production. Even though the economy has access to foreign sources of financing, domestic agents must still accumulate domestic capital. As it rises, their ability to attract foreign funds is enhanced and debt increases. Lucas [1990] has noted that the (unmodified) neoclassical model implies marginal product differentials so large that, no investment should take place in rich countries, as all capital would flow to low-income economies. Here, differences in marginal products are lowered by the combination of the credit constraint and the complementarity of savings and capital inflows.

In this thesis, we emphasise the complementary role of savings as a source of differences in growth rates within a neoclassical setting from a technological point of view. It is also possible to consider a preference-based explanation to the role of savings. Rebelo [1992] focuses on the role of savings, but in endogenous growth models. He argues that endogenous growth models cannot explain differences in income growth in a world with perfect capital markets. In these models, liberalising capital markets leads capital to flow from poor to rich countries — where the rate of return is high — and to the equalisation of growth rates across countries. He suggests that assuming Stone-Geary preferences with subsistence-level consumption instead of the standard isoelastic preferences partly addresses this issue. Under this assumption, poor countries in which consumption is close to subsistence level, would have low savings rates, and perfect capital markets would not lead to 'capital flight'. This alternative model would also predict that savings and capital inflows are positively correlated.
It is this feature of the model that makes it an appropriate choice for the exercise in this chapter. As noted above, there are alternative ways of introducing capital market imperfections in a neoclassical model. Kremer and Thompson [1998], and Duczynski [2000] and [2002] have argued in favour of models with adjustment costs. They note that this model relies on binding constraints — and is thus only relevant for a limited set of economies. Nevertheless, if we interpret the domestic capital as human capital, this may be relevant for numerous countries. In addition, although a model with adjustment costs would also predict some form of convergence in debt, it would not predict a possible complementarity between domestic savings and foreign financing. As shown in the results below, this prediction is supported by the data.

2.3 Estimation

2.3.1 Empirical Approach

The empirical exercise has a qualitative flavour. That is, we ask whether we can understand the cross-country variation in net external debt observed in the data within the framework set up in the previous section. Although the exercise is empirical in nature, it is qualitative in the sense that we want to determine whether the data exhibit the correlations predicted by the model, without defining a metric of how far they are from it. More specifically, we will estimate (2.13) in its average form:

$$\frac{1}{t} \left( \log \frac{D_{it}}{\theta_{it} L_{it}} - \log \frac{D_{i0}}{\theta_{i0} L_{i0}} \right) = -\frac{(1 - \lambda^t)}{t} \log \frac{D_{i0}}{\theta_{i0} L_{i0}} + \frac{(1 - \lambda^t)}{t} \log d^*$$  \hspace{1cm} (2.14)

for each country $i$, where $d^*$ takes on the value defined in (2.10). In order to estimate this equation, we must address three issues. First, since we only observe debt per worker ($\frac{D}{L}$) and not debt per efficiency units of labour ($\frac{D}{AL}$), we must make some assumptions about how technology flows across countries. Second, we must correctly control for cross-country variations in the level of steady state net external debt, or equivalently of output.

\footnote{See Chapter 3 for a quantitative evaluation of the model.}

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Finally, we must determine whether and how to discriminate between constrained and unconstrained countries.

One possibility to account for the fact that we do not observe the level of technology in each country $i$ is to follow MRW and assume that the level of technology is common across countries, i.e.

$$\log \theta_{i0} = c$$

with

$$\frac{\theta_{it+1}}{\theta_t} = (1 + g)$$

This suggest a regression of the form

$$\frac{1}{t} \left( \log \frac{D_{it}}{L_{it}} - \log \frac{D_{i0}}{L_{i0}} \right) = k - \frac{(1 - \lambda^t)}{t} \log \frac{D_{i0}}{L_{i0}} + X \beta + u_t \quad (2.15)$$

where $k$ is a constant, $X$ is a matrix of variables that capture differences in steady states across countries and $u_t$ the error term assumed to be uncorrelated with $X$. This assumption about technology is relaxed later.

In the model, steady state differences can arise from differences in savings behaviour and technology parameters, depreciation and interest rates as well as labour force growth. What variables should we include in $X$? We have already assumed that countries share the same production function, and implicitly, that differences in output stem from disparities in factor inputs so the common initial technology level will be included in the constant. Contrary to the previous growth literature, we cannot use average investment rates as controls for steady state output since it is endogenous to the domestic savings decisions. Equation (2.13) however, suggests variables that may be included in $X$ which I choose to focus on: first, the model predicts that the domestic savings rate $s$ positively affects debt accumulation; second, labour force growth which is a major force behind accumulation in all neoclassical models of growth. Low income countries, more likely to be constrained, generally have a numerous and growing population that needs to be equipped with new capital in order to produce output. Additional controls are considered in a later section.
The basic convergence regression will therefore take the form

\[
\frac{1}{t} \left( \log \frac{D_{it}}{L_{it}} - \log \frac{D_{i0}}{L_{i0}} \right) = k + \beta_0 \frac{1}{t} \log \frac{D_{i0}}{L_{i0}} + \beta_L \frac{1}{t} \log ((1 + n_t)(1 + g) - (1 - \delta)) + \beta_s \log s_t + u_t
\]

(2.16)

A final issue concerns the determination of the countries which are credit constrained. The convergence equation for debt will be estimated on two types of samples. One possible test of the model is to consider whether the estimated reduced form holds for all countries in the sample, whether or not we consider them to be constrained. A second possibility is to discriminate between constrained and unconstrained countries. I divide the sample between countries likely to be constrained and those likely to be unconstrained. In the model, constrained countries are determined by their initial wealth relative to the steady state domestic capital stock. The data on assets are limited but income can be used as a proxy. The approach taken here is to choose income as a criterion as in Lane [2000] and [2001] where the sample consists of low and middle-income countries. First I will estimate equation (2.16) on the entire sample including both high and low-income countries. I will also use a sample of countries with income below the median.

2.3.2 Results

The Data

The data on net external debt are taken from Lane and Milesi-Ferretti [1999]. The authors construct data for 66 countries between 1970 and 1997 using data on current account balances supplemented by available stock data on foreign direct investment, portfolio equity and debt assets and liabilities. They justify their use of flow data by noting that changes in net foreign assets are equal to current account balances net of capital transfers (transactions that do not give rise to an asset or liabilities such as debt forgiveness) and capital gains. They can therefore construct measures of net foreign assets using an initial value and cumulating current account balances. When possible, they also use direct measures of stocks. These measures are adjusted for valuation effects such as exchange rate changes, variations in the price of capital goods and changes in the values of stock market indices.
In all regressions, debt per worker is computed as

\[ \frac{D}{L} = \frac{-B}{pL} \]

where \( B \) is a measure of net foreign assets in US dollars, \( p \) is the US GDP deflator, and \( L \) is working-age population from the Penn World Tables.

I consider two measures for \( B \). The first, CUMCA, is based on cumulative current accounts. It is available for both industrial and developing countries. The second, NFA, is based on direct stock measures of the various assets included in debt and is available for developing countries. The main difference between these two measures is the treatment of unrecorded capital flows. As unrecorded capital flows are large — the world had a current account deficit on the order of $US70 billion in 1998 — this is of some importance. CUMCA implicitly assumes that unrecorded capital flows reflect accumulation of foreign debt assets by domestic residents. The second measure, NFA, only includes unrecorded capital flows to the extent that they are recorded in net errors and omissions. If capital flight is important and often goes unreported, NFA will tend to overstate external debt levels. Finally, despite the great care with which these data were constructed, they have the same measurement drawbacks as all balance-of-payments data.

Figure 2.1 illustrates the movements in external positions over the past 30 years. The figure shows density estimates of demeaned net debt per worker in 1970 and 1997. The last few decades have seen large increases in both levels and perhaps more strikingly, in the variance of external positions. This must reflect in part, the much publicised globalisation: countries seem to have found it much easier to access world capital markets.

The BMS model is only relevant for small-open economies with negative assets, and that may be constrained in borrowing. Some restrictions are therefore in order. First, estimation will be restricted to countries with positive net external debt in both beginning and end of sample. This implies a sample which excludes countries that have switched from being net lenders to net borrowers, and vice versa. Countries less likely to be small-open

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8 More details on the debt measures, as well as all other data used in the thesis are available in Appendix D.
economies such as the U.S. and Japan are also excluded. The remaining group of countries also excludes important members of the G7 countries, such as France and the U.K. These restrictions reduce the sample size to 42 observations.

These data will be used in three subgroups. Since there are two available debt measures, the data will be organised along those lines. Sample I contains all 42 observations and measures net foreign debt with the CUMCA variable. Sample II contains the countries for which the NFA measure is available and has 29 observations. Finally, since the model implies that countries are credit constrained, it is more likely to be relevant for low-income countries. Consequently, Sample III corresponds to the poorest half of Sample I in terms of income in 1970. Sample compositions are described in Appendix D. Note that among developing nations, the samples are dominated by middle-income countries. Possibly then, the results presented here cannot be assumed to extend to much poorer economies, such as those of sub-saharan Africa. In fact, Lane and Milesi-Ferretti [2001] find that the relationship between external debt and output is non-linear, suggesting that the simple mechanisms considered here cannot explain all cross-country variation. Data limitations however, prevent the further exploration of this issue.

Figures 2.2 to 2.4 illustrate the two mechanisms emphasised in this chapter: convergence and the complementarity between domestic savings and debt accumulation. On the left panel, these figures show raw correlations between the average annual growth in debt between 1970 and 1997 and initial conditions, measured by the log of debt per worker or the log of GDP per worker at the beginning of the sample period. In all three samples, the negative correlation between initial debt and subsequent accumulation is consistent with decreasing returns. The model also predicts a positive correlation between domestic savings and debt accumulation. Still, it does not specify the type of domestic savings that might exhibit this type of complementarity. In this chapter, I will focus on the fraction of gross domestic product not consumed \(^9\). The right panel of these figures show the correlation

\(^9\)The model suggests that a better measure of savings would be the fraction of gross national product not consumed (see equation (2.8)). Yet, the use of GNP presents certain problems — other than the obvious measurement issues. First, GNP is not available in PWT 6.0. It is available in PWT 5.6 but only between
between average annual debt growth and this savings rate. It is measured as $1 - \frac{\xi}{\eta}$ where $\frac{\xi}{\eta}$ is the average consumption-to-output ratio between 1970 and 1997 from the Penn World Tables version 6.0. Again, in all three samples, the positive raw correlations are consistent with the prediction from the model. The model relies heavily on the assumption that output and debt are tightly linked. Figures 2.5 to 2.7 show that output growth and debt accumulation do tend to move together. The correlation is higher in the samples dominated by low-income countries. The relationship between debt and output is investigated further below.

The presentation of the empirical results will proceed as follows. I will first present the basic convergence results. Second, the robustness of the results to assumptions about technology, the measurement of savings and other steady state controls will be examined. Finally, the relationship between debt and output will be studied.

**Basic Convergence Results**

Tables 2.1 to 2.2 present the basic reference convergence equations for debt. In all regressions, the dependent variable is the average annual log change in real net debt per worker between 1970 and 1997 for most countries. Since there are missing data for some countries, the sample period for each country varies between fifteen and twenty eight years. The coefficient on initial debt is thus an estimate of the average annual convergence rate.

Data on population and national accounts are from the Penn World Table 6.0. $n$ is the average annual growth rate of the working-age population between 1970 and 1997. As in MRW, $g = 0.02$ and $\delta = 0.03$. In the model, $s$ corresponds to the fraction of income that goes into domestic capital investment. This domestic capital can represent human capital, as well as any other type of capital that cannot be funded by foreign sources. This type of

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1970 and 1992. This means that a measure of savings from this source can only span part of the period considered. In addition, in the remainder of the chapter, savings out of GDP are often instrumented using the value of the savings rate before 1970 — an option clearly unavailable for GNP. The measure of savings based on GDP however, may not be inappropriate. First, GNP and GDP are highly correlated. Second, the results are fairly robust to the use of the GNP measure. The results are more fragile in Sample II, but this is driven by one country, Jordan. Once it is excluded, the results are similar to those found using the GDP measure.
savings is first measured by the private domestic savings rate as measured by 

\[ s = 1 - \frac{\xi}{y}, \]

where \( \xi \) is the average consumption to output ratio between 1970 and 1997. In the tables below, all variables are in logs. In addition, both asymptotic and bootstrap p-values are shown. These bootstrap statistics are robust to the presence of heteroscedasticity.

Table 2.1 presents the results for ordinary least squares. In all three samples, the estimated convergence rate is about 2 percent per year and is significant. This is surprisingly close to the estimates of income convergence in the literature. In this model, labour force growth lowers output-per-worker through the usual neoclassical channel: new entrants in the labour force must be equipped with capital. Consequently, because of the credit constraint, countries with high labour force growth should have low output, and thus low net external debt. The labour force variable however, fails to significantly affect debt accumulation, except in Sample I which includes high-income countries. Savings seems to increase debt accumulation, particularly in samples I and II. This positive association may be surprising to one used to thinking about foreign sources of finance as substitutes for domestic ones. In fact, we often think of the benefits of open borders as resulting from the consumption gain capital flows create by reducing the need for saving. In the model, capital flows do not eliminate domestic savings. Countries with a high savings rate also have higher domestic capital. Since domestic and foreign capital are complementary, this leads to higher debt accumulation. Savings rates however, are also likely to be partly endogenous. A positive shock on income — through terms of trade for example — could lead consumers to increase both consumption and savings, possibly their savings rate. Table 2.2 shows the results of estimation by instrumental variables. The instruments are

10 See Appendix E for details on bootstrap estimation.
11 Labour force growth could be endogenous. In the model, faster debt accumulation is synonymous with higher output growth. The model abstracts from the effect of higher output growth on fertility choices and hence on labour force growth. If fertility choices are endogenous, labour force growth and debt accumulation could be simultaneously determined. However, instrumenting with the average labour force between 1960 and 1969 does not significantly change the results.
12 The model implies that domestic savings should be substitutes to capital inflows for countries that are net creditors. When the convergence equation for debt is estimated for creditors countries at the beginning and end of sample, we find that the coefficient on the savings rate is negative, but insignificant. The results on creditor countries is thus consistent with the model, though the sample of countries with positive assets is small.
average labour force growth and savings rate between 1960 and 1969. The savings rate is now significant in all samples.

**Controlling for Productivity**

So far, we have assumed that $\log \theta_{i0} = c$ so that initial technology levels are identical across countries. This is not a prediction of the model. If initial technology levels vary across countries and are excluded from the matrix of explanatory variables, their resulting inclusion in the error term would bias the estimate of the convergence rate and of the effect of savings. Klenow and Rodriguez-Clare [1997] have criticised the income growth literature for failing to recognise the importance of productivity differences. If feasible then, one would like to account for the possibility that $\theta_{i0}$ varies across countries. In the literature on output convergence, this seems difficult: any measure of initial TFP — obtained by growth accounting for example — is likely to be highly correlated with initial output, a variable already included in the regression to capture convergence. This is less of a concern when dealing with debt. Suppose we allow $\theta_0$ to vary across countries so that

$$\log \theta_{i0} = c + \log A_{i0}$$

with

$$A_{it} = (1 + g)^t A_{i0}$$

Our estimating equation now takes the form

$$\frac{1}{t} \left( \log \frac{D_{it}}{L_{it}} - \log \frac{D_{i0}}{L_{i0}} \right) = k - \frac{(1 - \lambda^t)}{t} \log \frac{D_{i0}}{L_{i0}} + \frac{(1 - \lambda^t)}{t} \log A_{i0} + \frac{(1 - \lambda^t)}{t} \frac{\varepsilon}{1 - \varepsilon} \log s_i$$

$$- \frac{(1 - \lambda^t)}{t} \frac{\varepsilon}{1 - \varepsilon} \log ((1 + n_i)(1 + g) - (1 - \delta)) + u_i \quad (2.17)$$

Our measure of $A_{i0}$ is obtained by standard growth accounting methods. Initial physical capital stocks are computed from Penn World Tables investment data using the perpetual inventory method. A TFP measure is computed under the assumption that $\alpha = 0.3$ \textsuperscript{13}.

\textsuperscript{13}See Appendix D for details.
The results, shown in the top panel of Table 2.3, are robust to alternative assumptions about capital shares\textsuperscript{14}. The addition of $A_{i0}$ does not alter the results. Both the estimated coefficient on initial debt and savings are still individually significant. This suggests that the savings variable was not controlling for differences in technology levels in Tables 2.1 and 2.2. As before the labour force variable does not seem to matter for debt accumulation.

When using samples of these sizes, it is useful to adopt a more parsimonious specification. Recall from equation (2.17) that the model offers two restrictions on the parameters of the model. First, the coefficients on the labour force variable and the savings rate should be equal in magnitude but with opposite signs. In the empirical income growth literature, a similar prediction from the Solow growth model is often imposed on the estimated coefficients. From equation (2.8), we know that this corresponds to the domestic capital-to-output ratio in the steady state,

$$\frac{z}{(1-\alpha)y} = \frac{s}{(1+n)(1+g)-(1-\delta)}$$

This measures each country's long-run domestic capital intensity. In addition, the coefficient on initial debt and technology level should be equal in absolute value. Imposing these restrictions allows us to obtain more precise estimates, and the potential inclusion of other additional controls.

The bottom panel of Table 2.3 shows the result of estimation when these restrictions are imposed. First note that the fit of the regression is greatly improved by imposing these restrictions. The estimate of the convergence rate remains higher than when initial technology levels are assumed to be the same across countries. The complementary effect of savings however, is more precisely estimated. Somewhat surprisingly, the estimated least squares coefficients are very close in magnitude across samples. In addition, the null hypothesis that the restrictions are true cannot be rejected as shown by the $F$-statistic at the bottom of the table.

Are these results driven by the average behaviour of debt accumulation across coun-

\textsuperscript{14}The results are robust to assuming $\alpha = 0.5$. 

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tries, or are they the result of a few outliers? One way to answer this question graphically is by looking at the correlation between debt accumulation and our accumulation variable once the effect of convergence has been removed. This is easily achieved by regressing both \( \log d_t - \log d_0 \) and \( \frac{s^t}{1+n(1+g)-(1-\delta)} \) on a constant and \( \frac{d_0}{n_0} \) and graphing the resulting residuals. The residuals from these regressions correspond to \( \log d_t - \log d_0 \) and \( \frac{s^t}{1+n(1+g)-(1-\delta)} \) once the effect of convergence has been removed. These residuals are plotted in Figures 2.8 to 2.10. The right panel of these figures shows the residual correlation once the outliers have been removed. In all three samples, the correlation is positive, though this seems to be partly driven by Egypt and Syria, which both have very low savings and have accumulated little debt on average compared to the rest of the sample. Nonetheless, once these countries are removed, the correlation still appears strong and positive in all three samples. These results do not seem to be driven by outliers.

2.3.3 Robustness

The basic convergence results suggest that debt does converge at a rate similar to that observed in output. In addition, savings seem to play a role that is not explained by endogenous movements. These results however, are obtained under several assumptions: first, that investment in domestic capital is appropriately measured by savings out of gross domestic product; second, that other steady state controls cannot improve the fit of the regression. In addition, although the results seem to support the model, we have implicitly assumed that movements in debt were mirrored in output. We turn to each of these issues in turn.

The measurement of savings: human capital

So far we have ignored the potential role of human capital. Yet, education could be important in several ways within the framework of the BMS model. First, as noted in the original paper by BMS, human capital is likely to be the type of capital that cannot be borrowed against. Consequently, a measure of human capital investment may be an
alternative to \( s \), the fraction of income not consumed. If \( s \) measures savings in domestic physical capital, adding human capital investment to the estimation of the convergence equation for debt may add another dimension to the results. In that case, a convergence equation for debt would be

\[
\frac{1}{t} \left( \log \frac{D_{it}}{L_{it}} - \log \frac{D_{i0}}{L_{i0}} \right) = k - \frac{(1 - \lambda_t)}{t} \log \frac{D_{i0}}{L_{i0}} + \frac{(1 - \lambda_t)}{t} \log A_{i0} \\
+ \frac{(1 - \lambda_t)}{t} \frac{\varepsilon}{1 - \varepsilon} \log s^*_t + \frac{(1 - \lambda_t)}{t} \frac{\varepsilon}{1 - \varepsilon} \log s^h_t \\
- \frac{(1 - \lambda_t)}{t} \frac{\varepsilon}{1 - \varepsilon} \log ((1 + n_t)(1 + g) - (1 - \delta)) + u_i
\]  

(2.18)

where \( s^* \) is savings in domestic physical capital and \( s^h \) is savings in human capital.

The measurement of human capital investment is difficult. MRW construct a measure based on secondary enrollment rates. The focus on secondary schooling however, has been criticised by Klenow and Rodriguez-Clare [1997]. They note that cross-sectional variation in non-secondary school enrollment is much less than that of secondary school. This would tend to lower the effect of human capital investment on output. I use a measure that is more in line with what these authors propose. It is a weighted average of primary, secondary and higher schooling enrolment rates, \( e = \log \frac{6xP + 6xS + 4xH}{16} \) where \( P, S \) and \( H \)

\[\text{with} \]

\[A_{it} = (1 + g)^t A_{i0}\]

Technology levels are still partly exogenous through \( \Lambda \); the rate of change of \( \Theta \) however, is now partially driven by human capital accumulation. In this case the convergence equation is modified as follows

\[
\frac{1}{t} \left( \log \frac{D_{it}}{L_{it}} - \log \frac{D_{i0}}{L_{i0}} \right) = k - \frac{(1 - \lambda_t)}{t} \log \frac{D_{i0}}{L_{i0}} + \frac{(1 - \lambda_t)}{t} \log A_{i0} + \frac{(1 - \lambda_t)}{t} \frac{\varepsilon}{1 - \varepsilon} \log s^*_t \\
+ \log h_{it} - \frac{\lambda_t}{t} \log h_{i0} - \frac{(1 - \lambda_t)}{t} \frac{\varepsilon}{1 - \varepsilon} \log ((1 + n_t)(1 + g) - (1 - \delta)) + u_i
\]

Note that this view of human capital as a determinant of technology, and the alternative, human capital investment as a complement to foreign financing, are observationally equivalent. Both involve including a measure of the change in the stock of human capital in the convergence equation for debt.
denote primary, secondary and higher schooling enrolment rates. The data are from Barro and Lee [1993].

The results are shown in Tables 2.4 and 2.5. In both tables, we have used the accumulation formulation for human capital, \( \frac{e}{(1+n)(1+g)-(1-\delta)} \) (in logs), a restriction that is not rejected. The estimate of the convergence rate is robust to the inclusion of measures of human capital investment, but the results on savings vary. Table 2.4 shows the results of estimation by ordinary least squares. The measure of human capital seems to capture part of the complementarity of domestic savings when included on its own: it is precisely estimated in Sample I, and very marginally significant in Samples II and III in the top half of Table 2.4. Figure 2.11 shows the residual correlation between the education variable and the average debt accumulation once the effect of convergence has been removed as in Figures 2.8 to 2.10. It is a positive correlation, though much lower than when \( s \) is used as a measure of savings. When both measures of savings are included, private savings seem to be robust to the addition of human capital investment, whereas the reverse is not true.

These measures of investment in education however, have notorious measurement problems. In order to address this measurement issue, we instrument \( e \) with the average years of schooling in the population over 15 between 1960 and 1969. Table 2.5 shows the results of estimation by two-stage least squares, where the private savings rate is also instrumented. Private savings remain significant in Sample I only. It seems difficult to discriminate between the types of savings that have a complementary role.

This may be due to measurement problems in \( e \), though we have addressed this issue by using instrumental variable estimation. But note that private savings and human capital investment measure similar things. Presumably, part of the income not consumed is used to finance education, i.e. \( e \) may contain no additional information than that already contained in \( s \). On the other hand, part of human capital investment is often attributed to consumption in the national accounts (books, tuition, uniforms, part of government spending on schools, etc). In this latter case, \( s \) would underststate savings, and a measure of human capital would contain additional information. Finally, if a large fraction of
human capital investment is financed by governments through taxes $s$ may or may not capture human capital investment depending on whether these education expenditures are financed by consumption or income taxes. As a result, even if investment in knowledge were properly measured by $e$, private savings and human capital investment may not be independent sources of information.

Nevertheless, the results from these estimations are consistent with the predictions of the model. The results from Table 2.4 are strongest in the largest Sample I, which also includes higher income countries. We do not tend to think of wealthier countries as constrained, but it is reasonable to think they may not be able to borrow human capital from abroad. Measurement problems however, make it difficult to go further in investigating the role of human capital.

Other Steady State Controls

The growth literature has explored a plethora of variables that could control for the steady state beyond the savings rate and labour force growth. Sala-i-Martin [1997] provides a list of robust regressors. Among them are variables that capture market distortions and market performance, openness and institutions. As suggested by this author, I include three variables along these lines: (i) a measure of capital controls taken from Calderón et al [2000] that accounts for the presence of current and capital account restrictions, multiple exchange rate practices and mandatory surrender of export proceeds; (ii) an index of openness constructed by Sachs and Warner [1995] that measures the fraction of years between 1950 and 1994 that a country has been open to trade; (iii) a measure of institutional performance from Hall and Jones [1999] that captures the extent of corruption, bureaucracy, risk of expropriation, and the ability of government to maintain law and order. A higher value of the index indicates institutions that support growth. In addition, a measure of government spending — the ratio of spending to GDP from the Penn World Tables — is added to the list of controls. Results are shown in Tables 2.6 and 2.7. In general, these variables fail to improve the fit of the data appreciably. Capital controls seem to affect
debt accumulation, but mostly in Sample II. Other variables, such as the Sachs and Warner measure of openness and the measure of institutional performance, do not add to the fit of the regression. In fact, what is surprising is how robust both the estimates of convergence and the savings effect are robust to the addition of other steady state controls 16.

The marginal significance of government spending is intriguing. There are ways to explain the negative correlation between $\bar{y}$ and the change in debt within the context of the model. If government spending is wasteful and distorts domestic savings and capital accumulation, it may reduce the marginal product of foreign capital, discouraging flows of capital. On the other hand, $\bar{y}$ may be highly correlated with government borrowing abroad. A negative correlation between $\bar{y}$ and debt accumulation could then arise because foreign funds serve as a substitute for government savings. Both possibilities are consistent with this result.

**Debt and Output**

So far, we have found that predictions of the model about the complementary role of domestic savings as well as the presence of decreasing returns are consistent with the data. The model also predicts a tight link between net liabilities and output. The convergence rate for debt, estimated to be around 2 per cent, is robust to specifications with different assumptions about technology and savings, and consistent with the results from the income convergence literature. This seems to indicate a strong relationship between debt and output. There is however, evidence that this relationship is not as close as the one predicted by the model. Debt accumulation and output growth are correlated across countries as shown in Figures 2.5 to 2.7, but this correlation is much lower than the one-for-one relation predicted by the model (recall that in the model $d_t = \frac{\Delta \bar{y}}{\gamma + \delta} y_t$). In addition, labour force growth seems to have no individual effect on debt accumulation. This is surprising since previous empirical work suggests that labour force growth has a negative effect on output 16.

16 Other variables such as the terms of trade also fail to improve the fit of the regression.
growth. Since debt is proportional to output in this model, we should expect the same qualitative impact on debt as on output. It is easy to determine whether debt and output follow the same dynamics. For output, the BMS model predicts a convergence equation of the form

\[
\frac{1}{t} \left( \log y_t - \log y_0 \right) = \frac{(1 - \lambda t^t)}{1 - \varepsilon} \log B + \frac{1 - \lambda t^t}{1 - \varepsilon} \log y_0
\]

\[
+ \frac{1 - \lambda t^t}{1 - \varepsilon} \log s - \frac{1 - \lambda t^t}{1 - \varepsilon} \log ((1 + n)(1 + g) - (1 - \delta))
\]

when savings are exogenous, assuming technology is identical across countries.

Table 2.8 shows the convergence estimates for output per worker in the samples used here. A few results are worth noting. First, the estimated convergence rate appears much lower than that from the debt data, closer to 1 per cent. Second, the savings rate has a smaller estimated effect on output growth in this sample, and it is not completely robust to IV estimation.

Output and debt however, may be subject to similar shocks. It may be useful to take advantage of this empirically in order to gain efficiency by using systems methods. The debt and output convergence equations are estimated by SUR and three-stage least squares in Tables 2.9 and 2.10. The coefficient on initial debt is slightly lower than before, though if anything the effect of the savings rate on debt is a bit higher in the debt equation. On the output side, the estimated convergence rate is still fairly low. As in Table 2.8, the effect of the savings rate in Table 2.10 on output is much lower than that on debt.

What can we conclude from these results? The dynamics and predictions of the model depend crucially on the assumed perfect correlation between debt and output. As a result, convergence in debt is a direct consequence of convergence in output in the model. We know that debt accumulation is associated with output growth (Figures 2.5, 2.6 and 2.7). But...
these output regressions indicate that the convergence observed in debt can only partially be attributed to movements in output. This also means that the debt-to-GDP ratio cannot be constant as predicted by the model 18.

2.4 Conclusion

This chapter has emphasized an important mechanism for the long-term dynamics of net external debt across countries: the complementary role of savings. Theoretically, we can allow for two roles for foreign capital flows. In many models, foreign funds are substitutes for domestic savings: borrowing from abroad allows an economy to increase investment with no cost in consumption. Alternatively, foreign financing may act as a complement to domestic savings: countries with higher savings are rewarded with higher flows of capital. The framework developed by BMS allows for this possibility by specifying two types of capital, one that is accumulated domestically and one that comes from abroad. These capital inputs are complementary in production. When an economy increases its savings, and thereby its domestic capital stock, this increases the marginal product of the foreign capital. This will tend to raise output, and the closely-linked stock of net external debt. This model also allows for the standard convergence story: low-income countries borrow in order to accumulate capital. The incentive to borrow diminishes with development as the economy hits diminishing returns.

The original BMS paper mostly focused on the ability of the model to reproduce estimates of convergence found in the data. This simple model however, also allows us to determine how domestic savings and capital flows are linked. To address this question, a simple convergence equation for debt is derived from the model. It predicts that debt accumulation should exhibit convergence, and should be positively correlated to measures of domestic savings. The results are consistent with both of these mechanisms: Debt per worker converged at a rate between 2 and 2.5 per cent per year, and savings consistently increased debt accumulation. These results were robust to the econometric specification,

18In fact it converges at a rate of about 2 percent per year.
the measure of debt, and the sample of countries. It was more difficult to discriminate between the types of domestic savings that could act as complements to foreign financing.

These results suggest that this model is clearly useful in explaining cross-country variation in debt accumulation. Even so, it may not be quantitatively sufficient. First, in the samples considered here, the convergence found in debt was not mirrored in output. In fact, convergence in output was consistently lower than that of debt. This suggests that the debt-to-GDP ratio exhibits some convergence, and is far from constant as assumed in the model.

Does this evidence support a model with decreasing returns and limited access to credit markets quantitatively? To match long-term observed movements in debt, it appears a model must exhibit some decreasing returns, but also allow for a mechanism that permits foreign financing to complement domestic savings. Nevertheless, this is not enough to explain all cross-country variations in borrowing behaviour. The model partly fails because the assumption about capital flow restrictions is too rigid. The tight relationship between income, human and physical capital stemming from the assumption about capital markets leaves little room for a more complex and independent relationship between debt accumulation and other assets. In Chapter 3, we attempt to address the quantitative relevance of this model.

Despite its obvious flaws, the model does predict the importance of convergence and the complementary role of savings in explaining cross-country debt data. The policy implications are potentially important. If capital inflows are substitutes for domestic savings, policies that affect savings will be of little importance for capital accumulation and growth in open economies. On the other hand, if savings raises the marginal product of capital from abroad as suggested by the results presented here, domestic consumption and accumulation decisions are no longer segmented. Within the framework developed here, the positive correlation between domestic savings and investment first observed by Feldstein and Horioka [1980] is interpreted naturally, in a context in which at least a fraction of
capital flows freely. Feldstein [1994] argues that the capital mobility is not inconsistent with this observed correlation. Countries in which investors are risk averse or uninformed will also show high savings retention. The alternative suggested in this chapter, is that some types of capital are difficult to obtain from abroad and must be accumulated domestically through domestic savings even with a world with (some) capital mobility. The consequences for policy however, are similar to those advanced by Feldstein. Policies that affect national savings may potentially be important for capital accumulation and growth.

\[1^9\text{Coakley, Kulasi and Smith [1998] for a review of the literature on the Feldstein-Horioka puzzle.}\]
Chapter 3

External Debt in Neoclassical Models with Collateral Constraints: A Quantitative Investigation

3.1 Introduction

The standard neoclassical growth model predicts that capital flows are driven by decreasing returns. In economies where capital is scarce, its high return attracts foreign savings. The performance of this model however, has often been criticised. In a well-known paper, Lucas [1990] has argued that the model does poorly in explaining capital flows without some form of capital market imperfection. One class of models has focused on imperfections in the form of collateral constraints. In Chapter 2, we noted that the most intriguing prediction of this class of models is that domestic savings act as complements to capital inflows, which ceteris paribus, should still be driven by decreasing returns and domestic scarcity: countries that save more attract more capital inflows. Using newly constructed data on net foreign liabilities (Lane and Milesi-Ferretti [1999]), we find qualitative support for both these mechanisms — the complementary role of savings and the neoclassical force of decreasing returns. The objective of this chapter is to evaluate the quantitative relevance for explaining long-term capital flows of a class of models that features these two forces.
More specifically, this chapter estimates the capital shares implied by the data on net external debt.

In the framework developed by BMS, debt is proportional to output. Therefore, the model predicts that debt should exhibit the same type of convergence-like dynamics as income. Capital should flow to locations where it is scarce in efficiency units, i.e. for constant technology levels. When convergence equations for debt derived from the model are estimated in Chapter 2, I find evidence consistent with both decreasing returns and a complementarity between domestic savings and debt accumulation. These results are robust to the econometric specification, the sample of countries considered as well as assumptions about technology and the measurement of domestic savings. The model also predicts that the debt-to-GDP ratio should be constant, i.e. that debt and output should have the same dynamics. This prediction is not supported by the data.

These results prompt some additional questions. They suggest that a simple model of growth that features decreasing returns and domestic saving complementarity is useful in thinking about long-term movements in capital. This conclusion however, is qualitative. How relevant is this framework quantitatively? The objective of this chapter is to answer this question. More specifically, we take the model seriously and estimate the factor shares implied by the convergence equations estimated in Chapter 2. This is achieved by a method of indirect inference. That is, we estimate the factor shares by matching the regression coefficients from the data to their simulated counterparts in the model. To obtain these simulated data, the model must be solved. Non-linear model solution methods are often appropriate when countries to which the model applies are far from their steady state. Since this is likely to be the case for many of the countries in our sample, the model is solved non-linearly and simulated for all countries in the sample. These simulated data are then used to determine whether reasonable technology parameters can be found to match the convergence equations.

We consider two versions of the model. In the first version, savings behaviour is endogenous. Under the assumption of identical preferences, the potential complementary role
of domestic savings is obscured. Consequently, the focus is on matching the convergence rate. When the model with endogenous savings behaviour is used, I find that the model implies unrealistic shares of capital, that are either too low or too high. In a second Solow-type version of the model, I assume that the savings rate is exogenous. We can therefore allow each country to have a different savings rate in simulation. In this framework, we can estimate the factor shares by matching both the effect of decreasing returns and of domestic savings. This estimation yields more plausible factor shares. Nevertheless, two potential problems remain after this modification. First, the model cannot both match the convergence rate and the effect of the savings rate on debt accumulation. In particular, the simulated coefficient on the savings rate is low compared to the data. In addition, the estimated share of foreign capital implies unrealistically high debt-to-GDP ratios, unless one allows for very high institutional disincentives to capital accumulation. These results highlight the weakness of the model: the tight relationship between debt and output. The form of the credit constraint implies that debt and output are proportional, and that their ratio is constant and uniform across countries. A successful model of debt dynamics must allow for a more complex relationship between debt and output while retaining the prediction of savings complementarity and convergence.

The chapter is organized as follows. Section 3.2 provides a brief description of the BMS growth model with endogenous savings. Section 3.3 presents the method and results of estimation. Section 3.4 concludes.

3.2 The Model

The objective of this chapter is to evaluate the quantitative performance of neoclassical models with quantity constraints. What form should this constraint take? In order to achieve our objective — the estimation of capital shares — a simple modelling device is needed. The credit constraint should not be completely arbitrary, static or unrelated to the economic fundamentals of the model. For these reasons, the framework developed by BMS is appropriate. In the BMS model, the debt ceiling is a direct function of the capital
stock, and thus, changes over time as the economy develops. We start by briefly describing our version of the original BMS model with endogenous savings which focuses on the role of convergence. The initial objective of the model was to emphasise how a simple collateral constraint could help a neoclassical model match the observed income convergence. A Solow-type version of the model which allows for a possible role for savings is introduced in a later section.

As in Chapter 2, the production function can be expressed in units of effective labour (where \( x_t = \frac{X_t}{5_t} \)):

\[
y_t = k_t^\alpha z_t^\eta
\]  
(3.1)

where \( \alpha, \eta > 0 \) and \( \alpha + \eta < 1 \) and profit maximisation requires

\[
R_{kt} = \alpha k_t^{\alpha-1} z_t^\eta = \alpha \frac{y_t}{k_t}
\]

\[
R_{zt} = \eta k_t^\alpha z_t^{\eta-1} = \eta \frac{y_t}{z_t}
\]

\[
w_t = k_t^\alpha z_t^\eta - R_{kt}k_t - R_{zt}z_t = (1 - \alpha - \eta)y_t
\]

where \( R_{kt} \) is the rental rate of \( k \), \( R_{zt} \) is the rental rate of \( z \) and \( w_t \) is the wage rate. We maintain the small open-economy assumption, so that \( r = R_k - \delta \).

Recall that in this model a credit-constrained economy can use capital \( k \) as collateral, but cannot use \( z \). For a country that cannot finance the transition to its steady state from its initial wealth, the constraint will bind and debt will be proportional to output:

\[
d_t = k_t = \frac{\alpha}{r + \delta} y_t
\]  
(3.3)

The production function can therefore be written as

\[
y_t = B z_t^\varepsilon
\]  
(3.4)

where \( B = \left( \frac{\alpha}{r+\delta} \right)^{\alpha} \) and \( \varepsilon = \frac{\eta}{1-\alpha} \).

We now endogenise the savings decision. Under these assumptions, the household
problem takes the form:

\[
\text{Max} \quad \{c_t, z_{t+1}\}_{t=0}^{\infty} \quad \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \\
\text{s.t.} \quad (1+n)(1+g)z_{t+1} = (1-\alpha)Bz_t + (1-\delta)z_t - c_t
\]

(3.5)

The dynamics of the system are governed by the Euler equation

\[
\left(\frac{c_{t+1}}{c_t}\right)^\sigma = \beta^* \left((1-\alpha)eBz_t^{\sigma-1} + (1-\delta)\right)
\]

(3.6)

where \(\beta^* = \frac{\beta}{(1+n)(1+g)}\), and market clearing

\[
(1+n)(1+g)z_{t+1} = (1-\alpha)Bz_t + (1-\delta)z_t - c_t
\]

(3.7)

Note that with the exception of the gross national product term (here \((1-\alpha)Bz_t\) is GNP and \(-\alpha Bz_t\) is net factor payments), these two equations in consumption \(c\) and domestic capital \(z\) are nearly identical to those of a closed-economy Ramsey growth model. The model will therefore behave like one.

We can easily solve this system. Log-linearizing the system and approximating around the steady state, we have:

\[
\log z_t = \lambda^t \log z_0 + (1-\lambda^t) \log z^*
\]

(3.8)

where \(1-\lambda\) is the convergence rate. This implies that the change in net foreign debt takes the form

\[
\log d_t - \log d_0 = -(1-\lambda^t) \log d_0 + (1-\lambda^t) \log d^*
\]

(3.9)

where

\[
d^* = \frac{\alpha}{\tau + \delta} B \left[\frac{(1+n)(1+g) - \beta(1-\delta)}{B(1-\alpha)e\beta}\right]^{\frac{1}{1-\sigma}}
\]

(3.10)

The model thus predicts that countries that have low debt initially — which in this model is equivalent to low initial income — will accumulate debt faster. This prediction is a direct result of the assumption of decreasing returns. Countries that have low income
and debt are constrained and have high return to capital. We first focus on this standard neoclassical prediction and later allow for the possibility that domestic savings play an important role.

Manipulating this equation yields a convergence equation for debt of the form

\[
\log d_t - \log d_0 = (1 - \lambda t) \log \frac{\alpha B}{r + \delta} + (1 - \lambda t) \frac{\epsilon}{1 - \epsilon} \log B(1 - \alpha) \beta - (1 - \lambda t) \log d_0 \\
- (1 - \lambda t) \frac{\epsilon}{1 - \epsilon} \log ((1 + n)(1 + g) - \beta(1 - \delta))
\]

(3.11)

This is the reduced form we will use to estimate the capital shares implied by the model. To understand the results, it will be useful to write the constant as a function of the original capital shares as follows

\[
\log d_t - \log d_0 = (1 - \lambda t) \log \beta + (1 - \lambda t) \log \eta \\
+ (1 - \lambda t) \frac{1 - \eta}{1 - \alpha - \eta} \log \frac{\alpha}{r + \delta} - (1 - \lambda t) \log d_0 \\
- (1 - \lambda t) \frac{\epsilon}{1 - \epsilon} \log ((1 + n)(1 + g) - \beta(1 - \delta))
\]

(3.12)

3.3 A Quantitative Assessment

3.3.1 Methodology

How far off is this model quantitatively? Can we find reasonable values of the technology parameters that can reproduce the data? One way to determine whether the model is quantitatively relevant is to use moments of the data and see how closely we can reproduce them using the model. A first step then is to find appropriate moments. Second, we must decide on an estimation method. I address each of these questions below.

The moments

In Chapter 2, I estimate convergence equations of a form similar to equation (3.12) and find qualitative support for the predictions of the model. In particular, I find that countries with low levels of initial debt did accumulate more debt subsequently — which suggests convergence. More specifically, an equation of the form
\[
\frac{1}{T} \left( \log \frac{D_{iT}}{\theta_{iT} L_{iT}} - \log \frac{D_{i0}}{\theta_{i0} L_{i0}} \right) = -\frac{(1 - \lambda^T)}{T} \log \frac{D_{i0}}{\theta_{i0} L_{i0}} + \frac{(1 - \lambda^T)}{T} \log d^* \quad (3.13)
\]

is estimated using least squares. I choose these least squares coefficient as the moments to reproduce.

In order to estimate this equation, some simplifying assumptions must be made. First, since we observe debt-per capita \( \frac{D}{L} \) and not debt per efficiency units \( \frac{D}{\overline{L}} \), we must make some assumption about technology. Second, we must find variables to control for the steady state level \( d^* \). Finally, since the model applies to credit-constrained economies, we must find a way to identify them in the sample countries we use. I address these issues here briefly, but a more elaborate discussion can be found in Chapter 2.

Estimation is first undertaken under the assumption that the initial level of technology is identical across countries as in Mankiw, Romer and Weil [1992] (MRW hereafter), i.e.

\[\log \theta_{i0} = c\]

with

\[\frac{\theta_{it}}{\theta_{i0}} = (1 + g)^t\]

This assumption is relaxed later.

In order to control for the steady state, we will restrict ourselves to the variables suggested by the model. In the version of the model developed in the previous section, this corresponds to the labour force growth rate. In a subsequent section, we will consider a version of the model with a fixed savings rate, and add it as a control variable \(^1\). Finally, we must find a way to discriminate between constrained and unconstrained countries. First, I focus on countries that have positive net liabilities, and that are small open economies as in Chapter 2. These however, may include high-income countries. In the model, countries are constrained if they have low levels of initial wealth. As a second step, samples will be divided according to the level of initial income. A more thorough discussion of sample composition is provided in the data section below.

\(^1\)In Chapter 2 I have found that additional controls do not improve the fit of the regression.
The estimation method

The quantitative assessment offered here will attempt to match the coefficients from the convergence equation. Specifically, we can simulate the model for all countries in the sample and estimate the convergence equation data for debt using the simulated data. We can then choose the values of the capital shares that minimise the distance (by some metric) between actual and simulated moments. This is the method of indirect inference.

Under what assumptions should we simulate the model? In the model, countries can be different along many dimensions. The convergence equation for debt suggests two dimensions in which we may want to allow differences in the simulated data: first, initial conditions; second, steady states. We turn to each in turn.

In order to simulate the data, we must provide a set of initial conditions for $x$. We do not have direct measures of the initial distribution of capital that was saved for domestically. Since GDP data is the most reliable, we use the distribution of initial output in the data to generate a distribution of initial domestic capital stocks across countries. Countries may also have different initial levels of technology, $\theta_0$. We will first assume initial technology levels are identical across countries, and relax this assumption below.

Countries also have different steady states. First, we only allow the steady state to be defined by the labour force growth rate taken from the data, an assumption which we relax later.

Since we have a limited set of moments, we cannot estimate all the model parameters. I choose to estimate the technology parameter $\eta$ and $\alpha$, the shares of domestic and foreign capital in income. This quantitative exercise can therefore allow us to determine the magnitude of decreasing returns necessary to reproduce the variation in debt accumulation observed in the data.

The complete algorithm for estimating $\Gamma = \begin{pmatrix} \alpha \\ \eta \end{pmatrix}$ is thus as follows:

1. Estimate a vector of moments $\bar{m}$ from the data. Here these moments are the least

\(^2\)See section A.2 in Appendix A for details on the choice of initial conditions.
squares coefficients in the convergence equations on debt, i.e. the least squares coefficients obtained by regressing \( \frac{1}{t} (\log d_t - \log d_0) \) on

\[
X = \begin{bmatrix} 1 & \log d_0 & \log (1 + n)(1 + g) - \beta(1 - \delta) \end{bmatrix}
\]

\( \hat{m} \) is the vector of least squares coefficients from this regression.

2. Given values for \( n, z_0 \) taken from the data, and an initial value for \( \Gamma \), simulate the non-linear model for each country, i.e. obtain the transition path for consumption, domestic capital, output and debt:

- Find decision rules for consumption \( c = f_i(z) \) in each country \( i \). Obtain the simulated series by computing

\[
c_{it}^s = f_i(z_{it}^s)
\]

\[
y_{it}^s = B(z_{it}^s)^e
\]

\[
d_{it}^s = \frac{\alpha}{r + \delta} y_{it}^s
\]

\[
z_{it+1}^s = \frac{1}{(1 + n)(1 + g)} ((1 - \alpha)y_{it}^s + (1 - \delta)z_{it}^s - c_{it}^s)
\]

\[
\frac{D_{it}^s}{L_{it}^s} = A_{it}^s d_{it}^s
\]

(3.14)

for \( t = 1, 2, ..., T \) for each country \( i \) given \( z_0 \), and where the superscript \( s \) denotes the simulated series.

3. Use the simulated data to estimate \( \hat{m}(\Gamma) \), the same moments as in the data, that is regress \( \frac{1}{t} (\log d_t^s - \log d_0^s) \) on

\[
X^s = \begin{bmatrix} 1 & \log d_0^s & \log (1 + n)(1 + g) - \beta(1 - \delta) \end{bmatrix}
\]

\( \hat{m}(\Gamma) \) is the vector of least squares coefficients from this regression.
4. Choose $\Gamma$ to minimise

$$J(\Gamma) = (\hat{m} - \hat{m}(\Gamma))^T \mathcal{W} (\hat{m} - \hat{m}(\Gamma))$$

$\mathcal{W}$ is a positive semi-definite weighing matrix that can take any value. In this exercise, we will choose to put more weight on the more precisely estimated coefficient, i.e., $\mathcal{W}$ is a diagonal matrix with the inverse of the variance of the least square coefficients on the diagonal.

Since the BMS model is non-linear, Step 2 requires a numerical solution method to approximate the decision rule for consumption. A non-linear approach is used for two reasons. First, linear approximation are notoriously inaccurate when used to approximate decision rules for economies far away from their steady state. Since we have no reason to believe these countries are near their steady state, a non-linear method seems appropriate. Second, since we want to determine how much variation in debt can be explained by concavity in the production function, it seems important to capture the curvature in the decision rule correctly. Details on the solution method are available in Appendix A. 

Since we cannot estimate all the parameters of the model, we must calibrate some preference and technology parameters. As in MRW, I choose a value of 3 per cent for the depreciation rate and of 2 per cent for the exogenous rate of technological progress. Several authors estimate that the world interest rate has been between 0 and 5 per cent over the past thirty years (see IMF [1995], Allsopp and Glyn [1999] and Chadha and Dimsdale [1999]). I choose $r = 0.05$ a value at the high range of these estimates. Since the samples considered here will be dominated by low-income and potentially credit-constrained economies, it seems appropriate to choose a higher value for $r$. Finally, the data are simulated under the assumptions that the intertemporal elasticity of substitution $\sigma$ is 1.5 and the discount factor $\beta$ is 0.95. These are standard values in the business cycle literature and are also in line with other studies where models are calibrated for developing countries (see Chari, Kehoe and McGrattan [1997]).

\[^{3}\text{Judd [1999] provides a good overview of solution methods for non-linear models.}\]
Finally, note that during the estimation of the capital shares, both $\alpha$ and $\eta$ are constrained to be within the interval $[0.05, 0.95]$.

3.3.2 The Data

As in Chapter 2, the data on net external debt are taken from Lane and Milesi-Ferretti [1999]. As noted above, the BMS model is only relevant for indebted credit-constrained small-open economies. We consider the same three samples as in Chapter 2 for small open economies with positive net external debt. These restrictions reduce the sample size to 42 observations for the CUMCA measure (Sample I), 29 observations for the NFA measure (Sample II). A third sample corresponds to the poorest half of the CUMCA sample in terms of income in 1970 (Sample III). Sample compositions are described in Appendix D. Note that among developing nations, the samples are dominated by middle-income countries from Latin America and Asia.

The data on output and the labour force are taken from the Penn World Tables 6.0.

3.3.3 The Basic Model

Common Technology

What should we expect from the model for 'reasonable' capital shares? For illustrative purposes, Figures 3.1 and 3.2 show the approximated consumption decision rules and the transition paths to the steady state for each country in Sample I when $\alpha = \eta = 0.35$. The decision rule function has the expected features: it is concave and increasing in domestic capital $z$. In Figure 3.2, each variable is expressed as a ratio to its steady state value. The transition paths are similar to those obtained from a closed-economy Ramsey model, as expected. Table 3.1 shows the magnitudes of the least square coefficient for our basic model. For $\alpha = \eta = 0.35$, the fraction of capital that can be used as collateral $\left(\frac{\alpha}{\alpha + \eta}\right)$ is one half. The convergence rate is approximately 3 per cent, a bit higher than estimated in the income growth literature. As noted in BMS, the model comes close to reproducing the convergence rates observed in the data on output with reasonable parameter values.

43
Table 3.2 shows the result of the estimation of $\alpha$ and $\eta$. The capital shares are chosen so as to match the reference regressions estimated by ordinary least squares. How does matching occur? Recall that what matters for matching the convergence rate is $\varepsilon = \frac{\eta}{1-\alpha}$, the coefficient on domestic capital in the collapsed production function, or alternatively the total share of capital $\alpha + \eta$. The higher the share of broad capital, the lower the speed of convergence since a high share of capital means that diminishing returns set in more slowly. Individual shares however, can serve as a measure of how open the economy is. If $\alpha = 0$, none of the accumulated capital can serve as collateral, and it is the equivalent of a closed economy. If $\eta = 0$, all capital serves as collateral, and the model exhibits infinite convergence. Raising $\frac{\alpha}{\eta}$ for a given $\alpha + \eta$ raises the degree of capital mobility because it raises $\frac{\alpha}{\alpha+\eta}$, the fraction of capital that serves as collateral.

Consider first the results shown in Table 3.2. The standard errors are shown in the Data column in parenthesis below the least square estimates. These, squared, serve as weights in estimation: more weight is put on matching the estimates that are more precisely estimated. In all estimations, the convergence rate has the lowest variance, and hence receives more weight in estimating the capital shares. Compared to Table 3.1, the estimated convergence rate is low in all samples. To match it, we must choose a higher value of $\varepsilon$ and increase the share of broad capital. For Sample I, this tendency to increase $\varepsilon$ is reinforced by the size of the coefficient on labour force growth, which is large in absolute value relative to Table 3.1. Increasing the share of broad capital in the model will increase the relative importance of the domestic sector, and raise the negative effect of labour force growth: as $\varepsilon$ increases, each new generation must be equipped with more capital. An increase in $\varepsilon$ is better achieved by an increase in $\eta$. The estimate of the constant in Sample I however, is low compared to the standard case. Recall from equation (3.12), that the constant is partly a function of the debt-to-GDP ratio $\frac{d}{y} = \frac{\alpha}{r+\delta}$. The model partly interprets the constant as this ratio, and lowers $\alpha$ to match it. In Sample I, $\alpha$ hits the lower constraint imposed on estimation of the capital shares. In Sample II, the constant is fairly high and this results in a high value of $\alpha$ and a labour share that is less than 0.1. Sample III shows more plausible
shares.

**Varying Technology**

How seriously should we take these estimates? Although the convergence rate is precisely estimated, the other least square coefficients have high variance. So far, we have considered matching a regression under very strict assumptions about preferences and technology. We have assumed there are no differences in saving behaviour — an assumption we relax later — and that initial technology levels are identical — an issue we address now. The regression in Table 3.2 is not a very good fit, in part perhaps because we have imposed the assumption of common technology levels across countries. This is not a prediction of the model. Indeed, the model predicts that capital should flow where it is most scarce *in efficiency units*. If there are uncontrolled differences in $\theta_0$, i.e. if efficiency units differ across countries, the least square coefficients will be biased. Suppose technology takes the following form:

$$\log \theta_{i0} = c + \log A_{i0}$$

with

$$A_{it} = (1 + g)^t A_{i0}$$

We can control for these differences by using traditional growth accounting methods. An estimate of total factor productivity $A_{i0}$ is computed under the assumption that $\alpha = 0.3^4$.

Debt accumulation has dynamics of the form:

$$\frac{1}{T} (\log d_T - \log d_0) = \frac{(1 - \lambda T)}{T} \log \beta + \frac{(1 - \lambda T)}{T} \log \eta + \frac{(1 - \lambda T)}{T} \frac{1 - \eta}{1 - \alpha - \eta} \log \frac{\alpha}{r + \delta}$$

$$- \frac{(1 - \lambda T)}{T} \log d_0 + \frac{(1 - \lambda T)}{T} \log A_0$$

$$- \frac{(1 - \lambda T)}{T} \frac{\varepsilon}{1 - \varepsilon} \log ((1 + n)(1 + g) - \beta(1 - \delta))$$

(3.15)

Table 3.3 shows the least square coefficient predicted by the model for $\alpha = \eta = 0.35$.

Note from equation (3.15) that the coefficients on $d_0$ and $A_0$ should be equal in absolute

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4See Appendix D for details.
value. Since the convergence equation is a property of the linearised model, there are some discrepancies between the two coefficients on these variables. Other than the coefficient on technology, the coefficients here are similar to those in the case of common technology levels.

The estimates of the capital shares shown in Table 3.4 are similar to those in obtained under the assumption of common technology, with the exception of Sample III. For all three samples, the broad share of capital is between 0.7 and 0.8. The estimate of \( \alpha \) however, hits the lower constraint imposed on estimation.

How reasonable are these shares? A common calculation from the National Income and Product Accounts usually puts the labour share of income anywhere between 0.05 to 0.8 — a share that applies to a composite of raw labour and human capital. Low-income countries often have low estimates of the labour share. These estimates typically measure labour shares by the ratio of employee compensation to national income. Recent work by Gollin [2002] and Bernanke and Gürkaynak [2001] suggests that employee compensation understates total labour compensation, more so in low-income countries since these economies devote a large portion of their labour force to self-employment or employment outside of corporate businesses. Taking this into account, these authors find that the labour share has much less variation across countries \(^5\) and ranges between 0.65 and 0.8. This implies a share for physical capital between 0.2 and 0.35. If we interpret \( \alpha \) and \( \eta \) narrowly as shares of physical and human capital as in the original BMS paper, our estimate for \( \alpha \) tends to fall outside the range suggested by these authors. By this measure, these estimates are not plausible.

Even if \( \alpha \) corresponds only to physical capital it is likely that \( \eta \) applies to a composite of physical and human capital. Alternatively, we can judge the plausibility of these estimates by looking at the share of total capital \( \alpha + \eta \). In the growth literature, estimates of the broad capital share vary between 0.6 and 0.8 (see MRW and Barro and Sala-i-Martin

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\(^5\) Hence, justifying the use of a constant and unique-factor-shares production function like Cobb-Douglas technology.
The results shown here are consistent with these estimates.

Should we conclude that the model is consistent with the data on debt? One prediction of the data provides another way of judging whether these estimated factor shares are plausible. The model predicts that the debt-to-GDP ratio is a constant function of the foreign share $\alpha$, i.e. $\frac{d}{y} = \frac{\alpha}{r+\delta}$. Can we reproduce the debt-to-GDP ratios observed in our sample? As shown in Table 3.4, for Sample II, the estimated shares imply a debt-to-GDP ratios of 7.1. As an illustration of the external positions of the countries considered, Figure 3.3 shows histograms of debt-to-GDP ratios in all three samples in 1997. The highest ratio observed is less than 2 and the average ratio is approximately 0.4. For Samples I and III, the estimated $\alpha = 0.05$ implies a debt-to-GDP ratio of 0.63 which is close to the average observed ratio. $\alpha$ is constrained to be no lower than 0.05 in estimation, which implies that had it not been constrained, the estimation would have chosen a negative value for $\alpha$. The resulting debt-to-GDP ratio is thus not believable.

These results seem to imply that while the model seems to find plausible values for the broad share of capital, they are inconsistent with observed debt-to-output ratios. The moments we have considered however, are poorly estimated, often with large variances. With the exception of the convergence rate, the model does not appear to have great relevance for the debt data considered here. We have assumed however, that saving behaviour was identical across countries and could be captured in a variable encompassing both the labour force and discounting effects $(1+n)(1+g) - \beta(1-\delta)$. In Chapter 2, I presented evidence that a model of the kind considered here with exogenous savings was a useful framework in which to examine debt data. We turn to this next.

3.3.4 Exogenous Savings

We can constrain the behaviour of savings by assuming that domestic consumers save a fixed fraction of income. Let $s_y$ denote the domestic savings rate, or the rate at which

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A similar point is made by Duczynski [2000]. He does not however, provide estimates of the capital shares. His criticism of the BMS model does not take into account the prediction for savings, which I address below.
consumers save out of gross domestic product $y$ to accumulate domestic capital so that $s_y y_t = y_t - c_t$. Since domestic savings must equal investment in human capital ($i_t^h = (s_y - \alpha)B z_t^e$), we have

$$(1 + n)(1 + g)z_{t+1} = sB z_t^e + (1 - \delta)z_t$$

(3.16)

where $s = s_y - \alpha$. In the steady state, $z_{t+1} = z_t = z^*$ so that

$$z^* = \left[ \frac{sB}{(1 + n(1 + g) - (1 - \delta))} \right]^{\frac{1}{1-\delta}}$$

(3.17)

Since $k_t = d_t = \frac{\alpha}{r + \delta} y_t$,

$$d^* = \frac{\alpha B}{r + \delta} \left( \frac{sB}{(1 + n(1 + g) - (1 - \delta))} \right)^{\frac{\varepsilon}{1-\varepsilon}}$$

(3.18)

and the convergence equation for debt is

$$\log d_t - \log d_0 = (1 - \lambda^t) \frac{1 - \eta}{1 - \alpha - \eta} \log \frac{\alpha}{r + \delta} - (1 - \lambda^t) \log d_0$$

$$+ (1 - \lambda^t) \frac{\varepsilon}{1 - \varepsilon} \log s - (1 - \lambda^t) \frac{\varepsilon}{1 - \varepsilon} \log ((1 + n)(1 + g) - (1 - \delta))$$

(3.19)

where $\lambda = \varepsilon + \frac{(1 - \varepsilon)(1 - \delta)}{(1 + n)(1 + g)}$. This equation corresponds to the one estimated in Chapter 2. As before, the model predicts that countries that initially have low debt levels will accumulate it faster, which in this model, is equivalent to convergence. In addition, a second mechanism comes into play in this version of the model: the role of domestic savings. A standard view of capital inflows is that they act as substitutes to domestic savings. Open economies can increase investment with no cost in current consumption. In this model, domestic savings act as complements to capital inflows. This is the result of the combination of two features of the model: first, the complementarity in production of the two types of capital; and second, the collateral constraint. Production cannot take place without both factors, and these factors have different sources, one foreign and the other domestic. An increase in domestic savings raises the marginal product of foreign capital, which increases an economy’s ability to attract financing from abroad.

The algorithm for estimation is essentially the same as before:
1. Estimate a vector of moments \( \hat{m} \) from the data. Here these moments are the least squares coefficients in the convergence equations on debt, i.e. the least squares coefficients obtained by regressing \( \frac{1}{T} \) (log \( d_t \) - log \( d_0 \)) on

\[
X = \begin{bmatrix}
1 & \log d_0 & \log (1 + n)(1 + g) - (1 - \delta) & \log s
\end{bmatrix}
\]

with the possible addition of log \( A_0 \). \( \hat{m} \) is the vector of least squares coefficients from this regression.

2. Given values for \( n, s, z_0 \) and \( A_0 \) taken from the data, and an initial value for \( \Gamma \) simulate the non-linear model for each country, i.e. obtain the transition path for consumption, domestic capital, output and debt, i.e.

- Obtain the simulated series by computing

\[
\begin{align*}
\dot{y}_t & = B(z_t) \dot{r} \\
\dot{d}_t & = \frac{\alpha}{r + \delta} \dot{y}_t \\
\dot{z}_{it+1} & = \frac{1}{(1 + n)(1 + g)} (s y_t + (1 - \delta) z_t) \\
\frac{D_t}{L_t} & = A_t d_t
\end{align*}
\]

for \( t = 1, 2, ..., T \) for each country \( i \) given \( z_{i0} \) and \( A_{i0} \).

3. Use the simulated data to estimate \( \hat{m}(\Gamma) \), the same moments as in the data, that is regress log \( d_t^e \) - log \( d_0^e \) on

\[
X^e = \begin{bmatrix}
1 & \log d_0^e & \log (1 + n)(1 + g) - (1 - \delta) & \log s
\end{bmatrix}
\]

with the possible addition of log \( A_0 \). \( \hat{m}(\Gamma) \) is the vector of least squares coefficients from this regression.

4. Choose \( \Gamma \) to minimise

\[
J(\Gamma) = (\hat{m} - \hat{m}(\Gamma))^\top \hat{W} (\hat{m} - \hat{m}(\Gamma))
\]
As noted above, the data are simulated using the average values of \( n \) and \( s \) in the data. \( s \) is measured as \( 1 - \frac{\xi}{g} \) which is taken from the Penn World Tables 6.0 \(^7\). If however, the least squares coefficients from the data, are estimated using instruments, we can no longer use the raw data for savings and labour force growth in simulation. We therefore use the first-stage fitted values of these variables, i.e. the values of \( n \) and \( s \) used in simulation are the fitted values obtained by regressing \( \log (1 + n)(1 + g) - (1 - \delta) \) and \( \log s \) on the matrix of instruments \( Z \) where

\[
Z = \begin{bmatrix}
1 & \log d_0 & \log (1 + n_{1960-1969})(1 + g) - (1 - \delta) & \log s_{1960-1969}
\end{bmatrix}
\]

Results from exogenous savings

Table 3.5 shows the results for the model with exogenous savings for \( \alpha = \eta = 0.35 \). The convergence rate is now lower than in the model with endogenous savings, and the coefficient on savings is around 0.03. How does this version of the model perform?

The column entitled Data in Table 3.6 is a reproduction of the results in Chapter 2. The first result to note is that the fit of the regression is much better than before. Both \( A_0 \) and the savings rate consistently enter the equation significantly. As noted in Chapter 2, these results are robust to the samples considered, the assumptions about technology and to additional controls. From equation (3.19), recall that the coefficients on \( (1 + n)(1 + g) - (1 - \delta) \) and \( s \) are equal in absolute value. Both of these coefficients are larger then in the reference Table 3.5, causing the model to choose a higher value of \( \varepsilon \). In Samples I and III, when debt is measured by CUMCA, the constant is fairly low, causing the model to choose a low value of \( \alpha \). In fact, in Sample III, the level of the constant causes the estimation to hit the lower constraint for the capital shares. In Sample II however, the constant is high: the estimate of \( \alpha \) is over 0.5. Are these shares reasonable? One potential problem is that they vary according to the measure of debt used. In Samples II and III,

\(^7\)The model suggests that a better measure of savings would be the rate at which consumers save out of gross national product, not gross domestic product, but availability of GNP data from the Penn World Tables is more limited than for GDP. In addition, the regression results are fairly robust to the use of GNP. See Chapter 2.
two groups of relatively low-income countries, the fraction of foreign capital, or the degree of openness varies between 7 and over 60 per cent.

As noted before, equation (3.19) implies two restrictions on the least square coefficients: the coefficient on \( d_0 \) and \( A_0 \) as well as those on \( s \) and \( (1 + n)(1 + g) - (1 - \delta) \) are equal in absolute value respectively. We can therefore use a more parsimonious specification, where we impose these restrictions \(^8\). There are two positive results from the results of estimation under these restrictions as shown in Table 3.7. First, the factor shares are similar across samples. Second, the estimated broad capital share is between 0.6 and 0.8. Many authors have argued that a broad share of capital (that includes both physical and human capital) of this magnitude is reasonable (e.g. MRW, Barro and Sala-i-Martin [1995]). By this criterion, the broad share of capital \( \alpha + \eta \) estimated here is reasonable. Yet this may be overly optimistic. In Chapter 2, we noted that adding measures of human capital investment did not improve the fit of the convergence equation for debt. Although education data are often of low quality, these results suggest that our estimated equation is perhaps only relevant for physical capital, i.e. the broad share of capital refers exclusively to physical capital, such as structures and equipment. In that case, our estimates of the broad share of capital are implausibly high.

It is difficult to judge whether these estimates are individually plausible since they cannot strictly be interpreted as human and physical capital shares. Even if we could interpret them as such, and found them plausible on that basis, some problems remain, namely the simulated coefficient on the savings rate, and the level of the implied debt-to-GDP ratio. We turn to each of these issues in turn.

**The effect of savings** The estimated coefficient on the savings rate appears high relative to what was predicted in Table 3.5 for standard values of the capital shares. The simulated value for this coefficient is consistently below what is estimated in the data. Why is the simulated effect of the savings rate so low? To increase the coefficient on savings in the

\(^8\)An \( F \)-test shows that these restrictions are not rejected by the data.
simulated data, a higher value of $\frac{\varepsilon}{1-\varepsilon}$ is needed, a goal better achieved by lowering $\frac{\alpha}{\eta}$. This however, would tend to reduce the convergence rate: recall that for a given $\alpha + \eta$, reducing $\alpha$ reduces the convergence rate because the fraction of capital from abroad has decreased. Thus, the model cannot fully match both the effect of savings and the convergence rate. This is illustrated in Table 3.8 where each panel shows a pair of two coefficients being matched together. Compare the first and second panels. To match the effect of the savings rate, $\varepsilon$ must be close to or above 0.8. To match the convergence rate, $\varepsilon$ is close or below 0.6. As shown in Table 3.9, matching both is a compromise that requires higher values of both $\alpha$ and $\eta$.

**The level of the debt-to-GDP ratio** The implied debt-to-GDP ratio is still outside the observed range in the data with a value well over 2. In the original BMS model, the authors interpret $\alpha$ as the share of physical capital. For a plausible value of 0.3, they noted that the model implied high levels of the current account, levels not often observed in developing economies. The paper offers two potential explanation for this. First, the authors note that certain low-income countries may be insufficiently productive to be credit-constrained. The samples considered here are dominated by middle income developing countries, which suggests that this explanation is unlikely to apply. Second, the authors observe that the fraction of capital that can be used as collateral is likely to be less than the share of physical capital. Our estimate of $\alpha$ however, corresponds to the share of capital that comes from abroad, not necessarily the share of physical capital.

A third explanation proposed in the BMS paper is that firms pay a proportional tax $\tau$ on output, so that profits are

$$(1 - \tau)k_t^p z_t^p - \omega_t - R_{kt}k_t - R_{zt}z_t$$

This tax is meant to include any disincentive to invest in capital such as low levels of property rights protection. In this case the market-clearing condition becomes

$$(1 + n)(1 + g)z_{t+1} = s(1 - \tau)Bz_t^p + (1 - \delta)z_t$$

(3.20)
and the constraint takes the form \( k_t = d_t = (1 - \tau)\alpha y_t \), so that in the steady state

\[
z^* = \left[ \frac{(1 - \tau)sB}{(1 + n(1 + g) - (1 - \delta))} \right]^{1/\tau}
\]

(3.21)

\[
d^* = \frac{\alpha B}{\tau + \delta} \left( \frac{(1 - \tau)sB}{(1 + n)(1 + g) - (1 - \delta)} \right)^{1/\tau - \epsilon}
\]

(3.22)

and the convergence equation for debt is

\[
\log d_t - \log d_0 = (1 - \lambda^t) \frac{1 - \eta}{1 - \alpha - \eta} \log \left( \frac{1 - \tau}{\tau + \delta} \right) - (1 - \lambda^t) \log d_0
\]

(3.23)

\[
+ (1 - \lambda^t) \frac{\epsilon}{1 - \epsilon} \log s - (1 - \lambda^t) \frac{\epsilon}{1 - \epsilon} \log \left( (1 + n)(1 + g) - (1 - \delta) \right)
\]

Can such a modification lower the value of \( \alpha \) predicted by the model? Table 3.10 shows the results of estimation. The top panel shows the results under the assumption that \( \tau = 0.3 \), the value suggested in BMS. To lower the resulting debt-to-GDP ratio, the value of its denominator in the model has also been increased. As suggested in BMS, the world interest rate is assumed to be 0.06. The value of the the depreciation rate \( \delta \) has also been raised to 0.06 from 0.03, to be more in line with other calibrations in the literature (see Chari, Kehoe and McGrattan [1997]). The three factor shares have plausible values for all three samples, between 0.2 and 0.5. Although the value of \( \epsilon \) is still high, the fraction of foreign capital has a believable value, below 0.5. As could be expected it is lowest in the low-income Sample III. The value of the the debt-to-GDP ratio however, is still high at close to 2.

It is possible that the value of \( \tau \) that we have chosen is too low. Many of the countries in the sample are known for severe controls on capital flows. But this parameter is difficult to calibrate. In recent work, Chari, Kehoe and McGrattan [1997] attempt to explain cross-country income differences by variations in disincentives to invest in capital. They measure these disincentives with the price of investment relative to consumption. The average value of this relative price is 0.78 in the samples they use. The bottom panel of Table 3.10 uses this value for \( \tau \). Note that this is a very high value as it implies producers appropriate less than thirty per cent of their output. The resulting estimate of the share of broad
capital is now above 0.8, while $\frac{d}{y}$ is close to one, at the high end of observed values of the debt-to-GDP ratio.

What can we conclude from these results? This model can match the convergence rate implied by debt data with plausible values of factor shares if we believe that they apply to both physical and human capital. Even so, these values are inconsistent with at least two data regularities. First, to match the effect of savings on debt accumulation, the model needs very high values for the capital shares. Second, it predicts implausibly high values of the debt-to-GDP ratio.

### 3.4 Conclusion

This chapter has attempted to offer a quantitative assessment of neoclassical models with collateral constraints. Previous work has shown that the predictions of these models for debt accumulation are supported qualitatively. First, the accumulation of debt does seem to be partly driven by decreasing returns and the domestic scarcity of capital. Second, domestic savings do seem to positively affect debt and act as complements to capital inflows. Nevertheless, these results are not sufficient to determine how relevant such a framework might be.

In order to offer a quantitative evaluation, this chapter takes the BMS model seriously and estimates the factor shares implied by the convergence equation on debt. As a first step, we emphasise the convergence rate in a model with endogenous saving decisions. In general, this version of the model tends to produce factor estimates that are either too high or too low. When we consider a model with exogenous savings behaviour that varies across countries, the estimates of the capital and labour shares fall within a range that seems reasonable at first pass. The share of foreign capital is around 0.2 and the share of

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9One potential way of addressing this problem is to relax the assumption of a Cobb-Douglas production function. A CES production function for example, would allow dynamics in $\frac{d}{y}$ — as noted in the original BMS paper — and consequently in $\frac{d}{y}$. This will be explored in future research.
domestic capital is approximately 0.5.

The estimated broad share of capital however, is only plausible if we believe it applies to both physical and human capital, an assumption that may be unreasonable. In addition, the estimated share of foreign capital implies a debt-to-GDP of over 600 per cent, a level rarely observed in the data. In fact, in the sample considered here, the debt-to-GDP ratio rarely exceeds 100 per cent. To reduce the debt-to-GDP implied by the model, we must assume that an implausibly large portion of output cannot be appropriated by producers due to corruption, taxes and disincentives to capital accumulation.

What can we conclude from these results? We may have some reservations about the quantitative performance of the BMS model. But the surprising conclusion from this work is that such a simple model, with only two mechanisms — decreasing returns and a complementary role for savings — can be this useful in understanding international capital markets. The results suggest that these two mechanisms may play an important role in the long-term dynamics of external debt. Both the strength and the weakness of the model are encompassed in the same feature. It is the collateral constraint that leads to the complementarity of savings. At the same time, this rigid constraint implies that debt and output are proportional, that their ratio is constant and identical across countries and is a function of the share of foreign output — three predictions we know to be false. The modelling challenge is to retain the predictions of savings complementarity and convergence while allowing for a more flexible relationship between debt and output.
Chapter 4

The Role of Capital Flows in Neoclassical Open-Economy Models with Imperfect Capital Markets

4.1 Introduction

What have we learned so far? In Chapter 2, I present evidence that capital flows are complements to domestic savings: it is high-saving economies that have attracted the most flows from abroad over the last thirty years. Such a view of the world is consistent with a class of neoclassical models that feature a collateral constraint. In Chapter 3, I find that such models can be quantitatively relevant, although with some reservations. Nevertheless, a question remains: can this class of models explain the role of capital flows to the exclusion of more traditional open-economy models of growth? The objective of this chapter is to address this question.

In Chapters 2 and 3, the analysis is undertaken with a model developed by Barro, Mankiw and Sala-i-Martin (1995) in which a credit-constrained country can borrow to acquire foreign capital but must save in order to accumulate domestic capital. The open-economy macroeconomic literature however, has relied on more traditional models of capital flows to investigate a variety of issues, namely international business cycles (Mendoza
[1991], Mendoza and Tesar [1998]), the correlation between savings and investment (Baxter and Crucini [1993]) or the mechanics of the current account (Nason and Rodgers [2001]). The results in Chapters 2 and 3 seem to suggest that many of these questions would be better addressed in models with collateral constraints. Can alternative open-economy neoclassical models predict complementarity between savings and capital flows? And could they be consistent with the observed correlations between foreign and domestic sources of financing? The objective of the chapter is twofold. First, we want to determine whether such models predict that capital flows are complements to domestic saving. If so, we want to know whether this alternative class of models matches the long-term dynamics of debt quantitatively.

In order to address these questions, I first restrict the class of models considered to those with capital market imperfections. A model with perfect markets is inconsistent with the evidence of output convergence, and cannot predict that savings and capital flows are complements. We consider a model with adjustment costs in capital and a premium on international borrowing. This is a fairly standard model in the international macroeconomic literature (see for example Nason and Rodgers [2001] or Schmitt-Grohé and Uribe [2003]). We find that generally this class of models allows for two forces to drive capital flows. The first operates through a substitution channel: the propensity to save tends to reduce the reliance on foreign borrowing relative to income — savings and capital flows are substitutes. The second operates through a complementary channel: because capital markets are imperfect, the propensity to save will increase foreign debt — savings and capital flows are complements.

This second effect however, does not seem to be large enough quantitatively to offset the substitution effect. We reproduce the quantitative exercise from Chapter 3 in which the distance between the model and the data is minimised by choice of the capital share. We find that it is not possible to find a share of capital that can match the positive conditional correlation between debt and accumulation and savings. We conclude from this exercise that although qualitatively this class of models could be consistent with the evidence on
the role of savings, quantitatively, it cannot match this evidence. This suggests that models with collateral constraints may be a more promising avenue of research.

The chapter is organised as follows. Section 4.2 discusses the role of savings in the BMS model. Section 4.3 presents a standard neoclassical model with imperfect capital markets and describes the role of savings in that context. Section 4.4 offers a quantitative assessment of the standard model. Section 4.5 concludes.

4.2 The Role of Capital Flows in Models with Collateral Constraints: A Review of the BMS Model

In the BMS model, capital flows play the role of complements to domestic savings. The objective of this chapter is partly to show that such a prediction may be at odds with standard open-economy neoclassical models of growth quantitatively. First, it is useful to review how the prediction of complementarity occurs in the BMS model. In this chapter, we will focus on the role of the discount factor $\beta$ in the endogenous savings model, as the parameter that governs the preference for saving. The reason for this emphasis will become apparent in the next section.

Recall from Chapter 3 that in the dynamics of the BMS model are governed by the Euler equation

$$\left(\frac{c_{t+1}}{c_t}\right) = \beta^* \left( (1 - \alpha)\epsilon B z_{t+1}^{-1} + 1 - \delta \right)$$

(4.1)

where $\beta^* = \frac{\beta}{(1+n)(1+g)}$, and market clearing

$$ (1 + n)(1 + g)z_{t+1} = (1 - \alpha)Bz_t + (1 - \delta)z_t - c_t $$

(4.2)

Solving this system linearly, we have:

$$ \log z_t = \lambda^t \log z_0 + (1 - \lambda^t) \log z^* $$

(4.3)

where $1 - \lambda$ is the convergence rate. This implies that the change in net foreign debt takes the form

$$ \log d_t - \log d_0 = -(1 - \lambda^t) \log d_0 + (1 - \lambda^t) \log d^* $$

(4.4)
where

\[
d^* = \frac{\alpha}{r + \delta} B \left[ \frac{\beta^*_r - (1 - \delta)}{(1 - \alpha)\epsilon B} \right]^{\frac{\alpha}{\gamma}}
\]  

(4.5)

Consider the role of the effective discount factor \( \beta^* \). As \( \beta^* \) increases, consumers put a higher weight on future consumption and more importance on current saving. It can be characterised as the 'propensity' to save. In this model, steady state debt is a positive function of the discount factor, i.e. \( \frac{\partial d^*}{\partial \beta^*} > 0 \). Countries that have a higher propensity to save tend to have higher debt levels. In the context of a convergence equation, this also means that countries that save more during the transition will attract more capital flows. In the BMS model, domestic savings and foreign investment are complements. This prediction is the result of the combination of two assumptions: the credit constraint and the complementarity of foreign and domestic capital in production. Countries can borrow foreign capital freely. It remains true however, that capital flows where it is most scarce, i.e. the model predicts convergence. Because of the complementarity, higher savings in the domestic capital increases the marginal product of foreign capital, thereby attracting more capital flows from abroad. In Chapter 2, I estimate the empirical counterpart to equation (4.4) to test this prediction. The estimated equation takes the following form

\[
\frac{1}{t} \left( \log \frac{D_{it}}{L_{it}} - \log \frac{D_{i0}}{L_{i0}} \right) = k - \frac{(1 - \lambda^i)}{t} \log \frac{D_{i0}}{L_{i0}}
+ \frac{(1 - \lambda^i)}{t} \frac{\epsilon}{1 - \epsilon} \log \frac{s_i}{(1 + n_i)(1 + g) - (1 - \delta)} + u_i
\]

(4.6)

where \( n_i \) is the labour force growth rate and \( s_i \) is the average savings rate for country \( i \). It is estimated under the assumption that all countries share the same depreciation rate and share of capital, and borrow at the same fixed world interest rate. The variable \( \frac{s}{(1+n)(1+g)-(1-\delta)} \) controls for the steady state \(^1\). The rate of savings out of income \( s \) is meant to capture savings in domestic capital \( z \). In fact, it should capture as well preference parameters that affect savings behaviour — in particular the discount factor. I find a positive coefficient on the savings rate, and interpret this result as evidence of the role of

\(^1\)See Chapter 2 for a discussion of additional steady-state controls
savings as complements to capital flows.

This same convergence equation is used in Chapter 3 to conduct a quantitative exercise. The BMS model is solved and simulated for all countries in the sample used to estimate equation (4.6). The distance between the observed convergence regression and the simulated convergence regression is minimised by choice of the capital shares $\alpha$ and $\eta$. I find that the model can match the convergence equation for debt $^2$.

A number of questions arise. First, can the complementarity prediction be reproduced in a more standard open-economy model, i.e. do these results exclude alternative models of capital flows? Second, can a positive conditional relationship between debt accumulation and savings be taken as evidence of the complementary role of savings? To address these questions, we turn to the description of a model more representative of the standard open-economy framework.

4.3 The Role of Capital Flows in a Debt-Elastic Interest-Rate-Premium Model of Growth

4.3.1 The Model

What role do capital flows play in more standard neoclassical models of growth? In this section, we show that these models, even those with imperfect capital markets, allow for the possibility that capital flows are substitutes to domestic savings.

We present a neoclassical model of capital accumulation for a small open economy. Each country's actions can be summarised by the decisions of a representative agent which consumes, accumulates capital and trades an international bond with the rest of the world. We assume that capital accumulation is hindered by adjustment costs. In addition, agents must pay a debt-elastic interest premium on borrowing from abroad. This type of model has been widely used in the international real business cycles and open-economy macroeconomic literature (see for example Nason and Rogers [2001], Schmitt-Grohé and Uribe [2003] and $^2$The model does imply however, implausibly high debt-to-GDP ratios in the range of 100 to 600 per cent.

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Senhadji [1997]; Eichengreen and Mody [1998] also present evidence of the existence of emerging-market spreads. The objective is to present a fairly standard neoclassical growth model in the open economy.

Output is produced from a Cobb-Douglas technology, with capital and labour as inputs such that

\[ Y_t = K_t^\alpha (\theta_t L_t)^{1-\alpha} \]  

(4.7)

where \( 0 < \alpha < 1 \). \( \alpha \) is meant to capture the share of broad capital, encompassing both human and physical capital. \( \theta_t \) is the exogenous source of technological progress which grows at rate \( g \). Population \( L_t \) grows at rate \( n \). The law of motion of capital is, as usual

\[ K_{t+1} = I_t + (1 - \delta)K_t \]  

(4.8)

where \( I_t \) is investment and \( \delta \) is the depreciation rate. Investment in capital is subject to convex adjustment costs à la Abel and Blanchard [1983] so that for each additional unit of investment, an additional cost of \( \frac{1}{2} \left( \frac{I_t}{K_t} - \frac{I^*}{K^*} \right)^2 K_t \) must be paid over and above \( I_t \). This assumption is necessary to avoid excess volatility of investment outside the steady state. As the economy approaches its steady state, adjustment costs fall to zero.

The small open economy has access to one international bond, \( D_t \). Each period, the change in net foreign liabilities is the difference between output \( Y_t \) and the sum of consumption \( C_t \), gross investment \( I_t \) and factor payments

\[ D_{t+1} - D_t = R_tD_t - Y_t + C_t + I_t + \mu \left( \frac{I_t}{K_t} - \frac{I^*}{K^*} \right)^2 K_t \]  

(4.9)

where \( R_t \) is the time-varying interest rate to be paid on the stock of debt \( D_t \). \( R_t \) does partly reflect domestic conditions. Each country faces a premium over the world interest rate that is a function of that country’s international net foreign asset position relative to output. The debt-elastic interest rate \( R_t \) takes the following form

\[ R_t = r + \varphi \frac{D_t}{Y_t} \]  

(4.10)

where \( r \) is the constant world interest rate. The parameter \( \varphi \) reflects the degree of capital mobility. With \( \varphi > 0 \), countries that increase their debt holdings relative to income must
pay a higher interest on their liabilities. Countries that increase their asset position relative to output will see their interest premium fall. If \( \phi = 0 \), international capital markets are perfect, and countries can borrow at the world rate \( r \), a price that is independent of their own actions. In this case however, external debt becomes non-stationary and indeterminate in the steady state. This indeterminacy problem is well-known in the international macroeconomics literature \(^3\).

We now turn to consumption decisions. In this model, a constant savings rate out of income is inconsistent with balanced growth. Consequently, we cannot make the simplifying assumption of constant savings made in Chapters 2 and 3 and must consider endogenous household behaviour. We will emphasise the role of the discount factor \( \beta \) as a parameter that captures the 'propensity' to save. When attempting to match the model to the data, this choice will require a way to link the observed savings rate to the discount factor. We address this issue later.

The representative agent in each country has preferences over consumption in each period \( t \) and supplies labour inelastically. The social planner's problem (equivalent here to the decentralized equilibrium) takes the following form

\[
\max_{\{c_t, x_t, k_{t+1}, d_{t+1}\}} \sum_{t=0}^{\infty} \frac{\beta^t c_t^{1-\sigma}}{1-\sigma} \\
\text{s.t.} \\
(1+n)(1+g)k_{t+1} = x_t + (1-\delta)k_t \\
(1+n)(1+g)d_{t+1} = \left(1 + r + \phi \frac{d_t}{y_t}\right) d_t - y_t + x_t + x_t + \frac{\mu}{2} \left(\frac{x_t}{k_t} - \frac{x^*}{k^*}\right)^2 k_t
\]

where all variables are expressed in efficiency units (i.e. \( v_t \) refers to \( v_t \equiv \frac{V_t}{\delta_t L_t} \)). \( x_t \) is investment \( I_t \) in efficiency units of labour.

The optimal set of allocation satisfies the two resource constraints

\[
(1+n)(1+g)k_{t+1} = x_t + (1-\delta)k_t \tag{4.12}
\]

\(^3\)Schmitt-Grohé and Uribe [2003] discuss alternative ways of addressing this problem.
\[(1 + n)(1 + g)\Delta t = \left(1 + r + \varphi \frac{d_t}{y_t}\right) dt - y_t + c_t + x_t + \frac{\mu}{2} \left(\frac{x_t}{k_t} - \frac{x^*}{k^*}\right)^2 k_t \quad (4.13)\]

as well as two Euler equations

\[
\left(\frac{c_{t+1}}{c_t}\right)^\sigma = \beta^* \left(1 + r + 2\varphi \frac{d_{t+1}}{y_{t+1}}\right) \quad (4.14)
\]

\[
\left(\frac{c_{t+1}}{c_t}\right)^\sigma = \frac{\beta^*}{\left(1 + \mu \left(\frac{y_t}{k_t} - \frac{x^*}{k^*}\right)\right)} \left[\frac{y_{t+1}}{k_{t+1}} \left(1 + \varphi \left(\frac{d_{t+1}}{y_{t+1}}\right)^2\right) + \mu \left(\frac{x_{t+1}}{k_{t+1}} - \frac{x^*}{k^*}\right) \left(\frac{x_{t+1}}{k_{t+1}} - \frac{x^*}{k^*}\right)\right] + \left(1 + \mu \left(\frac{x_{t+1}}{k_{t+1}} - \frac{x^*}{k^*}\right)\right) (1 - \delta) \quad (4.15)
\]

where \(\beta^* = \frac{\beta}{(1+n)(1+\rho)}\).

Equation (4.14) reflects equilibrium in the market for international bonds. It implies that the net cost from borrowing — which includes both the world interest rate as well as the interest premium — must be equated to the net benefit in the form of increased consumption. In equation (4.15), optimality requires that the marginal cost of increasing investment be equated to the marginal benefit of an extra unit of capital at time \(t + 1\). Additional capital at \(t + 1\) contributes to additional output through its marginal product in the following period as well as its marginal contribution to lower adjustment costs net of interest costs through the interest premium and depreciation.

\[^4\text{Equation (4.15) can also be decomposed into two equations that reflect the evolution of the shadow price of capital, commonly referred to as Tobin’s } q. \text{ The relevant equations are}\]

\[
q_t = c_t^{-\sigma} \left(1 + \mu \left(\frac{x_t}{k_t} - \frac{x^*}{k^*}\right)\right)
\]

\[
q_t = \beta^* \left[c_{t+1}^{-\sigma} \left(\frac{y_{t+1}}{k_{t+1}} \left(1 + \varphi \left(\frac{d_{t+1}}{y_{t+1}}\right)^2\right) + \mu \left(\frac{x_{t+1}}{k_{t+1}} - \frac{x^*}{k^*}\right) \left(\frac{x_{t+1}}{k_{t+1}} - \frac{x^*}{k^*}\right)\right) + q_t \left(1 - \delta\right)\right]
\]

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In the steady state,

\[ k^* = \left[ \frac{1}{\alpha} \left( \frac{1 - \delta}{\beta^* - (1 + \delta)} \right) \right]^{\frac{1}{\alpha-1}} \]  

(4.16)

and

\[ d^* = \frac{1}{2\varphi} \left( \frac{1}{\beta^* - (1 + \delta)} \right) \left[ \frac{1}{\alpha} \left( \frac{1 - \delta}{\beta^* - (1 + \delta)} \right) \right]^{\frac{\varphi}{\alpha-1}} \]  

(4.17)

which is a non-linear function of the discount factor \( \beta \). The debt-to-output ratio is

\[ \frac{d^*}{y^*} = \frac{1}{2\varphi} \left( \frac{1}{\beta^* - (1 + \delta)} \right) \]  

(4.18)

Log-linearising in the neighbourhood of the steady state, we can solve the system defined by equations (4.12) to (4.15) \(^5\). The solution for debt takes the form

\[ \log dt - \log do = a_0 + a_1 \log d_0 + a_2 \log k_0 + a_3 \log d^* + a_4 \log k^* \]  

(4.19)

where the \( a_i \) parameters are functions of the eigenvalues of the system defined by equations (4.12) to (4.15). Contrary to the BMS model, this model has two state variables, and its 'convergence' equation reflects that fact.

As in the BMS model, the role of savings hinges on the relationship between savings and the steady state stock of debt.

### 4.3.2 The Discount Factor and the Role of Savings

What role does the discount factor play in this model? In the steady state

\[ \frac{\partial d^*}{\partial \beta^*} = \frac{\partial d^*}{\partial y^*} \frac{\partial y^*}{\partial \beta^*} + \frac{d^*}{y^*} \frac{\partial y^*}{\partial \beta^*} \]  

(4.20)

The sign of \( \frac{\partial d^*}{\partial \beta^*} \) has two components. First, the debt-to-output ratio unambiguously falls when the discount factor rises from equation (4.18). This is the sense in which capital

\(^5\)See Appendix C for the linearised set of equations.
flows are substitutes to domestic savings in this model. The level of debt however, also
depends on output because capital markets are imperfect. The sign of $\frac{\partial c^*}{\partial y}$ is unknown
because the sign of $\frac{\partial c^*}{\partial y} \frac{\partial y^*}{\partial y}$ is ambiguous. For a net debtor country, $\frac{\partial c^*}{\partial y} \frac{\partial y^*}{\partial y}$ is positive (see
Appendix B). The correlation between debt and $\beta$ could be positive to the extent that $\beta$, or savings, will tend to increase output by much more than it decreases the debt-to-output
ratio. For a net creditor country, $\frac{\partial c^*}{\partial y} \frac{\partial y^*}{\partial y}$ will generally be negative for reasonable parameter
values, so that $\frac{\partial c^*}{\partial y}$ will generally be negative (see Appendix B).

In the convergence equation for debt, the sign of the coefficient on savings hinges on
the relationship between the propensity to save and steady state debt. Contrary to the
BMS model, this model allows for two possibilities for net debtors: savings are substitutes
— through $\frac{\partial c^*}{\partial y} \frac{\partial y^*}{\partial y}$ — or complements to capital flows — $\frac{\partial c^*}{\partial y} \frac{\partial y^*}{\partial y}$. Which effect dominates is
an empirical question.

### 4.4 A Quantitative Assessment

We have seen that in a standard neoclassical model with imperfect capital markets, capital
flows can be substitutes to domestic savings, more specifically, that countries that tend
to save more will attract less capital flows. In Chapter 2, I found evidence that capital
flows respond positively to an increase in savings. In addition, in Chapter 3, I show that
the BMS model is quantitatively relevant. That is, it can find reasonable values of capital
share parameters so as to reproduce the convergence equation shown in (4.6).

Although the relationship between the propensity to save and debt is generally nega­tive in a standard neoclassical model with imperfect capital markets, a positive relation­ship cannot be ruled out. A quantitative exploration can determine whether reasonable
parameters exists that could explain the observed positive relationship between savings
and capital flows. The objective of this section is to reproduce the exercise undertaken
in Chapter 3 to determine whether the standard neoclassical model described above can
match the convergence equation for debt.
4.4.1 Methodology

The Moments

In Chapter 3, I estimate convergence equations of a form similar to equation (4.6) and find that the BMS model can reproduce the estimated coefficients for reasonable capital shares\(^6\). Can the model presented here also match the convergence equation on debt? The estimated equation is as follows

\[
\frac{1}{t} \left( \log \frac{D_t}{L_t} - \log \frac{D_0}{L_0} \right) = a_0 + a_1 \log \frac{L_{t0}}{A_{t0}} + a_2 \log \frac{K_{t0}}{A_{t0}} + a_3 \log \frac{s_i}{(1+n_i)(1+g) - (1-\delta) + u_i} 
\]

In the data, \(a_1 < 0\): this is consistent with the prediction of convergence of both models. However, \(a_3 > 0\), which may not be consistent with the model presented here.

The equation above is estimated under a number of assumptions regarding the types of countries to which it applies, the steady state controls and the role of technology. All these issues are discussed at length in Chapter 2. I address these issues here briefly.

First, equation (4.21) was estimated for countries believed to be credit-constrained since the equation is a reduced form of the BMS model. In order to determine whether the neoclassical model can also reproduce this equation, this assumption is maintained.

For estimation purposes, we use \(\log \frac{s_i}{(1+n_i)(1+g) - (1-\delta) + u_i}\) to control for the steady state. We assume, as we did in Chapters 2 and 3, that differences between countries in the steady state arise because of variations in labour force growth and the savings rate.

Finally, we control for the possibility that countries start with different levels of technology, \(\theta_0\) but share a common growth rate of technological progress. More specifically, initial technology for some country \(i\) has a component shared by all countries, \(c\), and captured by the constant, as well as a country-specific component \(A_{t0}\):

\[
\log \theta_{t0} = c
\]

\(^6\)The BMS model does fail in one dimension. It implies implausibly high debt-to-GDP ratios in the range of 100 to 600 percent.
with
\[ \frac{A_{it}}{A_{i0}} = (1 + g)^t \]

The Estimation Method

How does the matching between the model and the data occur? In order to reproduce the convergence regression observed in the data with the model, we must solve the model numerically. Once we have obtained decision rules for consumption and investment, we can simulate the model, and obtain time series for debt. These series can then be used to reproduce the convergence equation. Once that is accomplished, we can choose the capital share so as to minimise the distance between the simulated convergence equation and the actual one.

This is summarised in the algorithm that follows:

1. Estimate a vector of moments \( \hat{m} \) from the data. Here these moments are the least squares coefficients in the convergence equations on debt, i.e. the least squares coefficients obtained by regressing \( \frac{1}{2} (\log d_t - \log d_0) \) on

\[ X = \left[ 1 \quad \log \frac{d_0}{A_0} \quad \log \frac{k_0}{A_0} \quad \log \frac{d_0}{(1+n)(1+g)-(1-\beta)} \right] \]

\( \hat{m} \) is the vector of least squares coefficients from this regression.

2. Given values for \( n, d_0 \) and \( k_0 \) taken from the data, and an initial value for \( \alpha \), solve and simulate the non-linear model for each country, i.e. obtain the transition path for consumption, domestic capital, output and debt:

- For each country \( i \), solve the problem outlined in (4.28) to obtain a value for the discount factor \( \beta \)
- Find decision rules for consumption \( c = f_i(k, d) \) and investment \( x = g_i(k, d) \) in each country \( i \). Obtain the simulated series by computing
\[ c_{it}^s = f_i(k_{it}^s, d_{it}^s) \]
\[ x_{it}^s = g_i(k_{it}^s, d_{it}^s) \]
\[ y_{it}^s = (k_{it}^s)\alpha \]
\[ k_{it+1}^s = \left( \frac{1}{(1+n)(1+g)} \right) (x_{it}^s + (1-\delta)k_{it}^s) \]
\[ d_{it+1}^s = \left( \frac{1}{(1+n)(1+g)} \right) \left( (1+r + \varphi \frac{d_{it}^s}{y_{it}^s}) d_{it}^s - y_{it}^s + c_{it}^s + x_{it}^s \right. \]
\[ + \frac{\mu}{2} \left( \frac{x_{it}^s}{k_{it}^s} - \frac{x_{it}^s}{k_{it}^s} \right)^2 k_{it}^s \right) \]
\[ \frac{D_{it}^s}{L_{it}^s} = A_{it}^s d_{it}^s \]

for \( t = 1, 2, ..., T \) for each country \( i \) given \( k_{i0} \) and \( d_{i0} \), and where the superscript \( s \) denotes the simulated series.

3. Use the simulated data to estimate \( \hat{m}(\alpha) \), the same moments as in the data, that is regress \( \frac{1}{t} (\log d_{t}^s - \log d_{0}^s) \) on

\[ X^s = \begin{bmatrix} 1 & \log \frac{D_{t}^s}{A_{0}} & \log \frac{k_{t}^s}{A_{0}} & \log \frac{s}{(1+n)(1+g)-(1-\delta)} \end{bmatrix} \]

\( \hat{m}(\alpha) \) is the vector of least squares coefficients from this regression.

4. Choose \( \alpha \) to minimise

\[ J(\Gamma) = (\hat{m} - \hat{m}(\alpha))^T W (\hat{m} - \hat{m}(\alpha)) \]

\( W \) is a positive semi-definite weighing matrix that can take any value. In this exercise, we will choose to put more weight on the more precisely estimated coefficient, i.e., \( W \) is a diagonal matrix with the inverse of the variance of the least square coefficients on the diagonal.
The Steady State: The savings rate as reflection of the discount factor  Con­trary to the BMS model, this model does not lend itself easily to a constant-savings-rate Solow version. Consequently, we must find a way to link observed savings behaviour to a parameter of the model. I choose to focus on the discount factor. Since we do not observe the discount factor directly, I assume it is reflected in the observed long-run savings rate.

We will choose \( \beta \) so that the observed average savings rate \( \bar{s} \) corresponds to the steady state savings rate. In the steady state

\[
\frac{d^*}{y^*} = \frac{1}{2\varphi} \left( \frac{1}{\beta^*} - (1 + r) \right)
\]

(4.22)

\[
\frac{x^*}{k^*} = (1 + n)(1 + g) - (1 - \delta)
\]

(4.23)

\[
y^* = \frac{1}{\alpha} \left( \frac{1}{\beta^*} - (1 - \delta) \right)
\]

(4.24)

\[
k^* = \left( \frac{y^*}{k^*} \right)^{1/\alpha - 1}
\]

(4.25)

\[
\frac{c^*}{d^*} = (1 + n)(1 + g) - \left( 1 + \varphi \frac{d^*}{y^*} \right) + \frac{y^*}{d^*} - \frac{x^*}{d^*}
\]

(4.26)

We can therefore choose \( \beta \) by implementing the following algorithm:

1. Given \( \varphi, n, g, r, \delta, \alpha \), and an initial guess for the discount factor, \( \beta_0 \), compute the steady state for \( \frac{d^*}{y^*}, \frac{x^*}{k^*}, \frac{y^*}{k^*}, \frac{c^*}{d^*} \) as defined by equations (4.22) to (4.26).

2. Let \( 1 - s = \frac{c^*}{y^*} \). Then choose \( \beta \) as follows

\[
\min_{\beta} \left| \frac{d^*}{y^*} - (1 - s) \frac{d^*}{c^*} \right|
\]

s.t.

\[
\Omega_{\min} \leq \frac{d^*}{y^*} \leq \Omega_{\max}
\]

\[
0 < \beta \leq 0.999
\]

\[
\beta^* = \frac{\beta}{(1 + n)(1 + g)} \geq 0.9
\]

(4.27)

(4.28)

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What range should the steady state debt-to-GDP ratio be constrained to? In Chapters 2 and 3, estimation was restricted to countries with positive debt holdings, countries that could be believed to be constrained. In the adjustment cost model, the complementarity of savings will only generally arise for net debtors as well. The matching exercise to be undertaken here will also be done under these same assumptions. $\beta$ is therefore chosen so as to match the observed savings rate subject to a positive debt-to-output ratio\(^7\). Basically we are choosing $\beta$ so that the model steady state debt-to-GDP ratio is consistent with the observed average savings rate as the steady state savings rate given the observed average population growth rate. We also restrict the effective discount factor $\beta^*$ to be less than 0.9 so as to avoid unrealistic steady-state interest rates.

The Transition Path: Solving the model  Step 2 of the Algorithm requires a solution method. This model is more difficult to solve than the BMS model because it features two state variables, capital and debt\(^8\). In addition, many of the countries included in estimation are far away from their steady state at the beginning of the sample period, by as much as 90 per cent. In fact, we assume that the difference in output per worker with the richest country in the sample is a measure of this distance. Because we want to focus on situations where countries start far away from their steady state, a linear solution is not appropriate. For this reason, we choose to approximate the decision rules for consumption and investment with Chebychev polynomials as functions of debt and capital. Appendix C contains a detailed description of the solution method.

4.4.2 The Data

As in Chapters 2 and 3, the data on net external debt are taken from Lane and Milesi-Ferretti [1999]\(^9\). We restrict ourselves to the same samples of net debtors from Chapters

\(^7\)We choose $\Omega_{\text{min}} = 10^{-5}$ and $\Omega_{\text{max}} = 80$. We impose the lower bound to avoid exploding consumption-debt ratios. We impose an upper bound, although a high one, in order to avoid exploding debt levels.

\(^8\)Although in principle the BMS model has three state variables – foreign capital, domestic capital and debt – the model elegantly collapses into a one-state variable model: debt and foreign capital are equal and proportional to output, itself a function of domestic capital.

\(^9\)See Chapters 2 and 3 for a more detailed description of the data.
2 and 3. The samples used — of which there will be three — will include countries with positive net external debt in both beginning and end of sample. These restrictions reduce the sample size to 42 observations (Sample I) for the measure of debt based on cumulative current accounts (CUMCA), 29 observations (Sample II) for the variable based on direct stock measures (NFA). We also use a third sample that corresponds to the poorest half of the CUMCA sample in terms of income in 1970 (Sample III). Sample compositions are described in Appendix D. Note that among developing nations, the samples are dominated by middle-income countries from Latin America and Asia.

The data on output and the labour force are taken from the Penn World Tables 6.0. The capital stock data is taken from the Penn World Tables 5.6.

4.4.3 Results

The Discount Factor

To illustrate the choice of $\beta$, Figure 4.1 presents some steady state correlations in Sample I. These unconditional correlations should not be confused with the conditional correlations from the convergence equations. Nevertheless, these are informative. The steady state relationships between various variables is shown as a function of the capital share $a$ and for descending values for $\varphi$, the parameter that governs the degree of capital mobility, the size of the interest premium. The correlations are computed under following assumptions: $\sigma = 1.5$, $\delta = 0.03$, $g = 0.02$, $r = 0.05$. The savings rate $s$ and the labour force growth rate $n$ are taken from the data. For each observation of $s$ and $n$, a value for the discount factor $\beta$ is chosen according to the algorithm described in (4.28).

In the debt-elastic-interest-rate model, the correlation between the effective discount rate ($\beta^*$) and steady state debt is generally negative: savings are substitutes to capital inflows as shown in the first line of the panel. The correlation between steady state savings and steady state debt depends on the size of the interest premium, or more specifically on the parameter $\varphi$, in a non-linear way. Although it is always negative for $\varphi = 0.001$, the model then produces debt-to-GDP ratios well above 100 per cent as shown in the second
line of figures. Note that these figures show the average debt-to-output ratio, which as we will see later is not a good representation of the distribution of values for that ratio across countries. For $\varphi = 0.1$ or $\varphi = 0.01$, the average debt-to-GDP ratio stays within a reasonable range, between 0 and 1, so that we will concentrate on these two cases.

The last lines of the panel of figures shows a potential problem. Our contention is that the observed savings rate is a reflection of underlying preference parameters, namely the propensity to save. In fact, the average savings rate on the transition path should capture which countries have tended to save more. However, in the model, we are using the average savings rate as the steady state savings rate. The steady state savings rate does not necessarily capture whether a country has saved more on the path to the steady state. In fact, when a country has a high steady state savings rate, it may be a reflection of the fact that it did not save a lot in the transition and has a high debt burden to carry as a result. This is reflected in the varying sign of the correlation between $\beta$ and the savings rate. This suggests that it would be possible to find a positive conditional correlation between debt accumulation and savings even if capital flows were in fact substitutes to savings. Choosing the discount factor this way should actually make it easier for the model to match the data. The impact of the choice of $\beta$ on our estimates is addressed later.

Solving the Model

We start by showing an example of the model solution in Figures 4.2 and 4.3. This solution is obtained under the assumption that $\sigma = 1.5$, $r = 0.05$, $n = g = 0.02$, $\varphi = 0.01$ and $\beta = 0.95$. The graphs show the transition path when capital $k$ and debt $d$ start 90 per cent below their steady state respectively. All variables are shown as a ratio of their steady state with the exception of debt – which is expressed in level change – and the risk premium – expressed in level.

Figure 4.2 presents the dynamics of the model when capital is initially below the steady state. Debt tends to be overly volatile despite the risk premium. This feature of the model makes finding a solution difficult, as $d$ tends to vary greatly outside of the steady state.
The movements in debt and capital are consistent with the optimality conditions expressed in the Euler equations (4.14) and (4.15). A large increase in borrowing is used to finance both consumption and investment. Adjustment costs ensure that capital is smooth while a preference for an even consumption profile explains the smooth transition of consumption.

Figure 4.3 shows the transition path of key variables when debt starts below the steady state. Since debt is below its steady state, consumers can enjoy a period of high consumption because they can borrow to increase capital temporarily. The savings rate has to increase to accommodate a higher debt burden in the steady state.

Note that in both cases the time series correlation between debt and savings is positive. But this positive correlation does not give insight into role of capital market; this is only revealed by cross-sectional correlation. In order to gain some insight into this issue, we solve the model for all countries for varying values for the capital share $\alpha$, the interest premium parameter $\varphi$ and and the adjustment cost parameter $\mu$. We then use the solution to simulate the model and run the convergence equation for debt.

The Choice of $\beta$ We have noted that our choice of $\beta$ is somewhat arbitrary. To what extent will the results be attributed to our choice of the discount factor? Recall that we have chosen the discount factor $\beta$ so that the observed average savings rate corresponds to the steady state savings rate in the model given the calibration of the parameter and the solution for the share of capital $\alpha$. Ideally, we would choose the discount factor so that it corresponds to the average savings rate observed over the transition to the steady state in the simulation. As we have seen, this means that in the simulated data a high savings rate in the steady state will not necessarily translate into a high propensity to save. In this section we consider whether the matching exercise to be undertaken in the next section might be affected by our choice of $\beta$.

Computationally, it is difficult to choose the discount factor that corresponds to the savings rate over the transition while estimating the capital share $\alpha$. We can however, parametrise $\alpha$, solve the model for that calibrated value, and estimate the convergence
equations with the simulated savings rate. The results from this exercise are shown in Table 4.3. Each convergence equation is estimated entirely from the simulated data (with the exception of the labour force growth rate $n$ and the initial value of total factor productivity $A_0$ taken from Sample III) under various assumptions for the parameters of the model. The share of capital $\alpha$ varies between 5 and 40 per cent and the adjustment cost parameter $\mu$ between 0.05 and 12. The parameter that governs the degree of capital mobility $\varphi$ takes on two values 0.01 and 0.1. We choose a high value for the discount factor $\beta = 0.98$, to ensure that countries have a high propensity to save. The table shows both the simulated regression results for debt and output.

The first thing to note from this table is that the coefficient on savings is generally negative with one exception. For constant values of the adjustment cost parameter $\mu$, the capital-mobility parameter $\varphi$, and the discount factor $\beta$, decreasing the capital share $\alpha$ increases the coefficient on savings, which may be a problem for our estimation of $\alpha$ as we will see later. The choices of $\mu$ and $\varphi$ however, also affect the results. When capital market imperfections are low ($\varphi = 0.01$), savings are stronger substitutes to capital inflows. For low adjustment costs, both output and debt converge more quickly. When capital market imperfections are high ($\varphi = 0.1$), savings are stronger complements to capital inflows. For low adjustment costs, debt converges more slowly. A low $\mu$ means that the economy can accumulate capital faster, and thus will require more capital inflows over the transition. But debt is slower to return to its steady state because as the economy receives more capital inflows, it requires a higher degree of domestic savings.

For adjustment costs that are low enough, savings can become complements to capital inflows. Consider columns 1 and 3 of Table 4.3. For $\mu = 0.05$, the effect of savings becomes positive. This is a direct result of the dynamics of debt. This is illustrated in Figures 4.7 and 4.8 which show the transition paths of the variables of interest for $\mu = 12$ and $\mu = 0.05$. All variables are expressed as a ratio to their steady state, with the exception of debt, which is expressed in level change, and the interest premium, expressed in level. When adjustment costs in capital are low, agents borrow more over the transition which
requires higher savings.

A lower capital share $\alpha$ also tends to increase the coefficient on the savings rate by reducing the importance of capital, and therefore the need to borrow for investment purposes. We investigate how the choice of $\alpha$ affects the coefficient on savings in this model in the next section.

What should we conclude from this exercise? It seems to indicate that any potential failure of the model to match the effect of savings cannot be blamed on our somewhat arbitrary choice of the discount factor. The use of a simulated savings rate will not generally change the way in which capital inflows and savings interact in this model for reasonable values of the adjustment cost parameter. It also suggests that the model is unlikely to produce a reasonable value of the capital share that can match the evidence from the debt data presented in Chapter 2.

**Finding $\alpha$**

We now turn to the matching exercise. We want to choose the share of capital $\alpha$ so as to minimise the distance between the debt convergence equation in the actual and simulated data. With the exception of $\beta$ — which is chosen to match the observed savings rate — the other parameters are chosen to be in line with the literature. In fact, the calibration follows Chapter 3. The adjustment cost parameter is set to $\mu = 12$ following work in the growth literature by Barro and Sala-i-Martin [1995] and in the international RBC literature by many authors (e.g. Mendoza and Tesar [1998]). We consider two values for the interest premium parameter, $\varphi = 0.01$ and $\varphi = 0.1$. As noted previously, when $\varphi = 0.001$ or less, the debt-to-output ratio explodes. I follow Mankiw, Romer and Weil [1992] and set the depreciation rate to $\delta = 0.03$ and the growth rate of technological progress to $g = 0.02$. I choose the world interest rate $r$ to be 5 per cent $^{10}$, and the elasticity of substitution $\sigma$ to 1.5 $^{11}$.


$^{11}$Chari, Kehoe and McGrattan [1997] make a similar calibration for developing countries.
Table 4.1 presents the results of matching when the risk premium parameter $\phi$ is 0.01. For each sample, the table shows two columns: the ‘Data’ column presents the results of estimation with actual data; the ‘Simulation’ column refers to the regression results from the simulated data. For illustration purposes, Figure 5 shows the time series used in the Simulation column for Sample I. Table 4.1 also presents the estimated share of capital $\alpha$, the average discount factor $\beta$, the average debt-to-output ratio $\frac{d}{y}$, the average risk premium $\frac{\phi}{y}$ and the value of the objective function $J$ defined in Step 4 of the Algorithm.

What share of capital should we expect? Gollin [2002] and Bernanke and Gürkaynak [2001] argue that the share of labour in output is between 65 and 80 per cent — a value that includes both educated and uneducated labour. This would imply a share of physical capital between 20 and 35 per cent. On the other hand, in our model, $\alpha$ could represent a share of broad capital that includes both human and physical capital. Many authors in the growth literature have argued that such a broad share should be between 60 and 80 per cent (see Mankiw, Romer and Weil [1992] as well as Barro and Sala-i-Martin [1995]). This leaves a large range of 20 to 80 per cent for $\alpha$.

The striking result from Table 4.1 is that the share of capital $\alpha$ has to be 5 or 6 per cent to match the estimated coefficients from the convergence regression. Note that for computational reasons, the parameter $\alpha$ is restricted to be between 5 and 95 per cent. Estimation therefore hits the lower bound of that range. Nonetheless, the model cannot reproduce the positive conditional correlation between the savings rate and debt accumulation.

Why does the model choose such low values for the capital share $\alpha$? Recall that the coefficient on savings in this model comes from the sign of the partial

$$\frac{\partial d^*}{\partial \beta^*} = \frac{\partial d^*}{\partial y^*} y^* + \frac{d^*}{y^*} \frac{\partial y^*}{\partial \beta^*}$$

To make this partial positive, the model can act on its two components: first, it can make the term $\frac{\partial d^*}{\partial y^*} y^*$ — which is negative — as small as possible, and second make the term $\frac{d^*}{y^*} \frac{\partial y^*}{\partial \beta^*}$ — which is positive when $\frac{d^*}{y^*} > 0$ — as large as possible.

Recall from equation (4.22) that the debt-output ratio does not depend on the share
of capital in the steady state. In order to reduce \( \frac{\partial y^*}{\partial y} \), the model must make output \( y^* \), and hence capital per efficient worker small, i.e. choose a small \( \alpha \).

How \( \frac{\partial^2 y^*}{\partial y^2} \) changes with the share of capital is more complicated. As shown in Appendix B, \( \frac{\partial y^*}{\partial y^2} \) is a fairly complicated function of the capital share \( \alpha \). But this second derivative term is actually fairly small in the estimation of \( \alpha \). Table 4.1 shows the average debt-to-output ratio, which is higher than 50 per cent in all samples. This however, is misleading. As shown in Figure 4.5, for most countries in the sample the debt-to-output ratio implied by the model is less than 10 per cent. That means that this second derivative term is fairly small.

It should be possible to improve matters by raising the degree of capital market imperfection by increasing the risk premium parameter \( \varphi \). Because capital markets are imperfect, debt levels depend on output. It is through this channel that a positive correlation between debt and savings can be engineered. Estimation is undertaken under the assumption that \( \varphi = 0.1 \) and the results shown in Table 4.2. This does not seem to increase the share of capital \( \alpha \). By raising the risk premium parameter, we have made it difficult to sustain a high debt-to-output ratio in the steady state. When \( \varphi = 0.1 \), the debt-to-output ratio falls below 10 per cent for all countries as shown in Figure 4.6.

Together, these results suggest that although in principle this model could be qualitatively consistent with the complementarity of savings, quantitatively, it fails to match the evidence provided by the long-term dynamics of debt.

4.5 Conclusion

In Chapters 2 and 3 we have seen that long-term debt accumulation is partly driven by scarcity and decreasing returns, as well as savings complementarity. These results are consistent with a class of neoclassical models with collateral constraints that predict convergence in output and debt. These predictions however, could possibly be consistent with any neoclassical model with imperfect capital markets. This chapter has attempted
to show that the prediction of savings complementarity will generally not be consistent with models that do not feature a collateral constraint in the way the BMS model does.

First, we examine an open-economy model with adjustment cost and a debt-elastic interest rate. Such models are widely used in the open-economy macroeconomics literature to investigate a variety of questions. For net debtor countries, these types of models allow for two possibilities for capital flows: they can be complements or substitutes to domestic savings. They are substitutes to the extent that the rate of borrowing relative to income falls as the propensity to save increases. They are complements to the extent that debt is positively linked to output because of capital market imperfections. For net creditor countries, capital flows will generally be substitutes to domestic savings, a result also predicted by the BMS model.

In order to determine whether the interest premium model is quantitatively relevant, we attempt to reproduce the exercise undertaken in Chapter 3 and estimate the capital share implied by the convergence equation on debt. The interest premium model produces an implausibly low share of capital of 5 per cent. In order to match the observed positive coefficient on savings, the model must minimise the substitution channel by making the economy's reliance on capital small. But this also means that the debt-to-output ratio is small and makes the complementary channel less relevant.

The size of the debt-to-output ratio in this model is also a problem. For most countries in the sample considered, the debt-to-output ratio is arbitrarily close to zero. In fact, if the choice of the discount factor were left unconstrained, the model would make net creditors of most of the countries considered. To increase this ratio, we must choose to increase the degree of capital mobility but such a choice will cause the level of debt to explode relative to output.

Matching the actual level of debt appears difficult for most neoclassical models. The BMS model also predicted implausible values for the debt-to-GDP ratio (although to a lesser extent than the interest premium model). It did so however, while matching the evidence for the complementary role of savings. This chapter suggests that it maybe more
fruitful to investigate this problem within the framework of the BMS model.
Chapter 5

Concluding Remarks

This dissertation examines the role of savings and capital flows in the long-run. In particular, this thesis has put forward the notion that we should think of capital flows as complements rather than substitutes to capital flows.

In Chapter 2, I have presented evidence that long-term capital flows are consistent with this view of the world. The predictions of a class of simple neoclassical models that feature decreasing returns and collateral constraints fit surprisingly well with data on net external debt. The theoretical and empirical analysis is based on the BMS model which predicts first, that scarcity drives capital flows, and second, that countries that save more attract more foreign financing. Using this framework, we derive a convergence equation for debt in which long-term cross-sectional variation in debt was driven by initial conditions and savings. We find evidence of both decreasing returns and the complementary role of savings.

In Chapter 3, the thesis turns to a quantitative evaluation of the model. Having shown that the predictions of neoclassical models with collateral constraints are relevant qualitatively, we consider whether the effects of decreasing returns and savings were relevant quantitatively. This is achieved by estimating the shares of capital that minimise the distance between the convergence equation estimated with observed data on debt and savings and the convergence equation estimated with simulated data on debt. We find that the model can find plausible values for a broad share of capital, but implies implausibly large
debt-to-output ratio. This seems to result from the rigid and constant relationship between debt and output in the model, a direct consequence of the form of the quantity constraint. This suggests that this class of models is promising, both qualitatively and quantitatively, though future research should focus on modelling a more flexible collateral constraint.

Finally, in Chapter 4 we look at alternative open-economy neoclassical growth models. The international macroeconomic literature has focused on a class of models that features capital adjustment costs as well as some type of market imperfection. We consider a model with a debt-elastic interest rate: the interest paid on debt is an increasing function of the debt-to-output ratio. We find that qualitatively, this class of models is consistent with savings as both complements and substitutes to capital inflows. Nevertheless, this model cannot quantitatively reproduce the convergence equation for debt observed in the data.

In sum, the results presented in this thesis suggest a class of models that features decreasing returns and a complementary role for savings may be useful in understanding the size and direction of capital flows. The BMS model, though elegant, has some flaws. In particular, it cannot reproduce a plausible level for the debt-to-output ratio, a ratio also assumed to be constant over time. A promising avenue for future research is to find a way to match the behaviour of the debt-to-output ratio. To see savings as a necessary part of development however, has important implications for policy. It suggests that developing countries should not only work on increasing access to world markets but improve domestic capital markets in order to foster national savings and capital accumulation.
Figures and Tables
Figure 2.1: Distribution of net external debt

![Distribution of net external debt](image-url)
Figure 2.2: Debt accumulation, initial conditions and savings - Sample I

Figure 2.3: Debt accumulation, initial conditions and savings - Sample II
Figure 2.4: Debt accumulation, initial conditions and savings- Sample III

correlation: -0.74805

correlation: 0.52607
Figure 2.5: Debt accumulation and output growth - Sample I

correlation: 0.19969

Average annual output growth 1970-97

Average annual debt growth 1970-97
Figure 2.6: Debt accumulation and output growth - Sample II

correlation: 0.3561

Average annual debt growth 1970-97

Average annual output growth 1970-97
Figure 2.7: Debt accumulation and output growth - Sample III

correlation: 0.38066
Table 2.1: Basic Convergence - OLS estimation

<table>
<thead>
<tr>
<th>Dependent variable: $\log d_t - \log d_0$</th>
<th>Sample I</th>
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<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
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<td>0.297</td>
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<td></td>
<td>(0.738)</td>
<td>(0.019)</td>
<td>(0.571)</td>
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<td>[0.736]</td>
<td>[0.009]</td>
<td>[0.400]</td>
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<tr>
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<td>-0.021</td>
<td>-0.024</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
<tr>
<td>$(1 + n)(1 + g) - (1 - \delta)$</td>
<td>-0.083</td>
<td>0.015</td>
<td>-0.069</td>
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<tr>
<td></td>
<td>(0.058)</td>
<td>(0.737)</td>
<td>(0.319)</td>
</tr>
<tr>
<td></td>
<td>[0.052]</td>
<td>[0.604]</td>
<td>[0.237]</td>
</tr>
<tr>
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<td>0.573</td>
<td>0.538</td>
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<tr>
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<td>30.000</td>
<td>21.000</td>
</tr>
</tbody>
</table>

<table>
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<tr>
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<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
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<td>[0.011]</td>
<td>[0.000]</td>
<td>[0.075]</td>
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<td>-0.019</td>
<td>-0.021</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
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<tr>
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<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
<tr>
<td>$(1 + n)(1 + g) - (1 - \delta)$</td>
<td>-0.050</td>
<td>0.005</td>
<td>-0.064</td>
</tr>
<tr>
<td></td>
<td>(0.173)</td>
<td>(0.887)</td>
<td>(0.293)</td>
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<tr>
<td></td>
<td>[0.068]</td>
<td>[0.006]</td>
<td>[0.133]</td>
</tr>
<tr>
<td>$s$</td>
<td>0.071</td>
<td>0.051</td>
<td>0.055</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.001)</td>
<td>(0.025)</td>
</tr>
<tr>
<td></td>
<td>[0.005]</td>
<td>[0.001]</td>
<td>[0.140]</td>
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</table>

Note: Asymptotic p-values are in parenthesis. Bootstrap p-values are in brackets.
Table 2.2: Basic Convergence - IV estimation

<table>
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<th>Dependent variable: $\log d_t - \log d_0$</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
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<td>(0.063)</td>
<td>(0.003)</td>
<td>(0.340)</td>
</tr>
<tr>
<td></td>
<td>[0.005]</td>
<td>[0.006]</td>
<td>[0.313]</td>
</tr>
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<td>-0.019</td>
<td>-0.019</td>
<td>-0.020</td>
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<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
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<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.004]</td>
</tr>
<tr>
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<td>-0.049</td>
<td>0.004</td>
<td>-0.064</td>
</tr>
<tr>
<td></td>
<td>(0.192)</td>
<td>(0.903)</td>
<td>(0.294)</td>
</tr>
<tr>
<td></td>
<td>[0.175]</td>
<td>[0.898]</td>
<td>[0.291]</td>
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<tr>
<td>$s$</td>
<td>0.074</td>
<td>0.055</td>
<td>0.058</td>
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<tr>
<td></td>
<td>(0.001)</td>
<td>(0.003)</td>
<td>(0.030)</td>
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<td>[0.002]</td>
<td>[0.047]</td>
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<td>0.675</td>
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<td>number of obs.</td>
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<td>30,000</td>
<td>21,000</td>
</tr>
</tbody>
</table>

Note: Asymptotic p-values are in parenthesis. Bootstrap p-values are in brackets. The savings rate is instrumented using the average savings rate between 1960-69.
Table 2.3: Controlling for $A_0$

<table>
<thead>
<tr>
<th></th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent variable:</strong></td>
<td>log $d_t - \log d_0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>constant</strong></td>
<td>0.194</td>
<td>0.185</td>
<td>-0.072</td>
</tr>
<tr>
<td></td>
<td>(0.262)</td>
<td>(0.757)</td>
<td>(0.734)</td>
</tr>
<tr>
<td>$d_0$</td>
<td>-0.023</td>
<td>-0.022</td>
<td>-0.023</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>$(1 + n)(1 + g) - (1 - \delta)$</td>
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<td>-0.074</td>
</tr>
<tr>
<td></td>
<td>(0.092)</td>
<td>(0.958)</td>
<td>(0.212)</td>
</tr>
<tr>
<td>$A_0$</td>
<td>0.036</td>
<td>0.018</td>
<td>0.038</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.404)</td>
<td>(0.143)</td>
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<td>$s$</td>
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<td>0.052</td>
<td>0.066</td>
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<tr>
<td></td>
<td>(0.003)</td>
<td>(0.009)</td>
<td>(0.018)</td>
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<tr>
<td>$\bar{R}^2$</td>
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<td>0.674</td>
<td>0.633</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent variable:</strong></td>
<td>log $d_t - \log d_0$</td>
<td></td>
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</tr>
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<td><strong>constant</strong></td>
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<td>0.050</td>
<td>0.044</td>
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<tr>
<td></td>
<td>(0.046)</td>
<td>(0.064)</td>
<td>(0.190)</td>
</tr>
<tr>
<td>$d_0$</td>
<td>-0.022</td>
<td>-0.022</td>
<td>-0.023</td>
</tr>
<tr>
<td>$A_0$</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>$(1 + n)(1 + g) - (1 - \delta)$</td>
<td>0.055</td>
<td>0.054</td>
<td>0.063</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.004)</td>
<td>(0.008)</td>
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<td>$\bar{R}^2$</td>
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<td>0.655</td>
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<td>21,000</td>
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</table>

Note: $(1 + n)(1 + g) - (1 - \delta)$ is instrumented with average labour force and the average savings rate between 1960 and 1969. $A_0$ is measured as $\log y_0 - 0.3 \log k_0$. The line denoted $F$-test shows the $F$ statistic for testing the null that the restricted model is true. The last line shows the p-value for this test.
Figure 2.8: Savings and Debt Accumulation, Residual Correlation (convergence removed):
Sample I

Figure 2.9: Savings and Debt Accumulation, Residual Correlation (convergence removed):
Sample II
Figure 2.10: Savings and Debt Accumulation, Residual Correlation (convergence removed):
Sample III
Table 2.4: Controlling for human capital I

<table>
<thead>
<tr>
<th>Dependent variable: log $d_t$ - log $d_0$</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
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<td>0.064</td>
<td>0.092</td>
<td>0.071</td>
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<td>(0.210)</td>
</tr>
<tr>
<td></td>
<td>(0.077)</td>
<td>[0.003]</td>
<td>[0.151]</td>
</tr>
<tr>
<td>$d_0/A_0$</td>
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<td>-0.025</td>
<td>-0.027</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td></td>
<td>[0.001]</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
<tr>
<td>$e$</td>
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<td>0.042</td>
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<tr>
<td></td>
<td>(0.063)</td>
<td>(0.294)</td>
<td>(0.235)</td>
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<td></td>
<td>[0.027]</td>
<td>[0.131]</td>
<td>[0.162]</td>
</tr>
<tr>
<td>$(1+n)(1+g)-(1-\delta)$</td>
<td>0.070</td>
<td>0.053</td>
<td>0.067</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.469</td>
<td>0.590</td>
<td>0.519</td>
</tr>
<tr>
<td>number of obs.</td>
<td>41.000</td>
<td>29.000</td>
<td>20.000</td>
</tr>
<tr>
<td>$F$-test</td>
<td>0.250</td>
<td>0.159</td>
<td>0.503</td>
</tr>
<tr>
<td></td>
<td>(0.620)</td>
<td>(0.693)</td>
<td>(0.488)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dependent variable: log $d_t$ - log $d_0$</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.049</td>
<td>0.060</td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td>(0.114)</td>
<td>(0.065)</td>
<td>(0.500)</td>
</tr>
<tr>
<td></td>
<td>[0.142]</td>
<td>[0.038]</td>
<td>[0.527]</td>
</tr>
<tr>
<td>$d_0/A_0$</td>
<td>-0.022</td>
<td>-0.023</td>
<td>-0.024</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
<tr>
<td>$s$</td>
<td>0.070</td>
<td>0.053</td>
<td>0.067</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.001)</td>
<td>(0.007)</td>
</tr>
<tr>
<td></td>
<td>[0.023]</td>
<td>[0.001]</td>
<td>[0.121]</td>
</tr>
<tr>
<td>$e$</td>
<td>-0.010</td>
<td>-0.005</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.638)</td>
<td>(0.796)</td>
<td>(0.915)</td>
</tr>
<tr>
<td></td>
<td>[0.639]</td>
<td>[0.732]</td>
<td>[0.886]</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.627</td>
<td>0.736</td>
<td>0.683</td>
</tr>
<tr>
<td>number of obs.</td>
<td>41.000</td>
<td>29.000</td>
<td>20.000</td>
</tr>
<tr>
<td>$F$-test</td>
<td>0.262</td>
<td>1.636</td>
<td>0.043</td>
</tr>
<tr>
<td></td>
<td>(0.612)</td>
<td>(0.213)</td>
<td>(0.839)</td>
</tr>
</tbody>
</table>

Note: OLS estimation. $e$ is measured as $\log \frac{6xP + 6xS + 4xH}{10}$. The line denoted $F$-test shows the $F$ statistic for testing the null that the restricted model is true. The last line shows the p-value for this test.
Figure 2.11: Education and Debt Accumulation, Residual Correlation (convergence removed)
### Table 2.5: Controlling for human capital II

<table>
<thead>
<tr>
<th>Dependent variable: ( \log d_t - \log d_0 )</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.066</td>
<td>-0.085</td>
<td>-0.327</td>
</tr>
<tr>
<td>( d_0 )</td>
<td>(0.173)</td>
<td>(0.461)</td>
<td>(0.431)</td>
</tr>
<tr>
<td>( \Delta_0 )</td>
<td>[0.133]</td>
<td>[0.444]</td>
<td>[0.552]</td>
</tr>
<tr>
<td>( \frac{e}{(1+n)(1+g)-(1-\delta)} )</td>
<td>0.026</td>
<td>-0.024</td>
<td>-0.027</td>
</tr>
<tr>
<td>( (1+n)(1+g)-(1-\delta) )</td>
<td>(0.000)</td>
<td>(0.001)</td>
<td>(0.103)</td>
</tr>
<tr>
<td>( \Delta_0 )</td>
<td>[0.000]</td>
<td>[0.004]</td>
<td>[0.070]</td>
</tr>
<tr>
<td>( \bar{R}^2 )</td>
<td>0.046</td>
<td>0.132</td>
<td>0.292</td>
</tr>
<tr>
<td>number of obs.</td>
<td>36.000</td>
<td>24.000</td>
<td>17.000</td>
</tr>
<tr>
<td>F-test</td>
<td>2.374</td>
<td>0.388</td>
<td>0.001</td>
</tr>
<tr>
<td>( (1+n)(1+g)-(1-\delta) )</td>
<td>(0.133)</td>
<td>(0.540)</td>
<td>(0.981)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dependent variable: ( \log d_t - \log d_0 )</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.070</td>
<td>-0.046</td>
<td>-1.046</td>
</tr>
<tr>
<td>( d_0 )</td>
<td>(0.122)</td>
<td>(0.687)</td>
<td>(0.849)</td>
</tr>
<tr>
<td>( \Delta_0 )</td>
<td>[0.109]</td>
<td>[0.602]</td>
<td>[0.964]</td>
</tr>
<tr>
<td>( \frac{s}{(1+n)(1+g)-(1-\delta)} )</td>
<td>0.025</td>
<td>-0.024</td>
<td>-0.034</td>
</tr>
<tr>
<td>( (1+n)(1+g)-(1-\delta) )</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.599)</td>
</tr>
<tr>
<td>( \Delta_0 )</td>
<td>[0.000]</td>
<td>[0.001]</td>
<td>[0.625]</td>
</tr>
<tr>
<td>( \bar{R}^2 )</td>
<td>0.096</td>
<td>0.027</td>
<td>-0.245</td>
</tr>
<tr>
<td>number of obs.</td>
<td>36.000</td>
<td>24.000</td>
<td>17.000</td>
</tr>
<tr>
<td>F-test</td>
<td>2.079</td>
<td>0.249</td>
<td>0.000</td>
</tr>
<tr>
<td>( (1+n)(1+g)-(1-\delta) )</td>
<td>(0.159)</td>
<td>(0.624)</td>
<td>(0.994)</td>
</tr>
</tbody>
</table>

Note: IV estimation. \( e \) is measured as \( \log \frac{6 \times P + 6 \times X + 6 \times N}{4} \) and is instrumented using the average years of schooling in the population over 15 at the beginning of the period and average labour force growth between 1960 and 1969. \( \frac{s}{(1+n)(1+g)-(1-\delta)} \) is instrumented with the average savings rate and labour force growth between 1960 and 1969. The line denoted F-test shows the F statistic for testing the null that the restricted model is true. The last line shows the p-value for this test.
Table 2.6: Controlling for the steady state I

<table>
<thead>
<tr>
<th>Dependent variable: ( \log d_t - \log d_0 )</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.088</td>
<td>0.079</td>
<td>0.075</td>
</tr>
<tr>
<td></td>
<td>(0.038)</td>
<td>(0.025)</td>
<td>(0.113)</td>
</tr>
<tr>
<td></td>
<td>[0.042]</td>
<td>[0.029]</td>
<td>[0.131]</td>
</tr>
<tr>
<td>( \frac{d_0}{A_0} )</td>
<td>-0.022</td>
<td>-0.022</td>
<td>-0.022</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.001]</td>
</tr>
<tr>
<td>( \frac{s}{(1+n)(1+g)-(1-\delta)} )</td>
<td>0.047</td>
<td>0.052</td>
<td>0.060</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.004)</td>
<td>(0.012)</td>
</tr>
<tr>
<td></td>
<td>[0.020]</td>
<td>[0.002]</td>
<td>[0.041]</td>
</tr>
<tr>
<td>capital controls</td>
<td>-0.009</td>
<td>-0.010</td>
<td>-0.010</td>
</tr>
<tr>
<td></td>
<td>(0.150)</td>
<td>(0.071)</td>
<td>(0.221)</td>
</tr>
<tr>
<td></td>
<td>[0.151]</td>
<td>[0.081]</td>
<td>[0.267]</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.597</td>
<td>0.779</td>
<td>0.714</td>
</tr>
<tr>
<td>number of obs.</td>
<td>41.000</td>
<td>29.000</td>
<td>21.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dependent variable: ( \log d_t - \log d_0 )</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.062</td>
<td>0.052</td>
<td>0.053</td>
</tr>
<tr>
<td></td>
<td>(0.043)</td>
<td>(0.007)</td>
<td>(0.132)</td>
</tr>
<tr>
<td></td>
<td>[0.044]</td>
<td>[0.004]</td>
<td>[0.142]</td>
</tr>
<tr>
<td>( \frac{d_0}{A_0} )</td>
<td>-0.023</td>
<td>-0.023</td>
<td>-0.024</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
<tr>
<td>( \frac{s}{(1+n)(1+g)-(1-\delta)} )</td>
<td>0.044</td>
<td>0.050</td>
<td>0.049</td>
</tr>
<tr>
<td></td>
<td>(0.049)</td>
<td>(0.016)</td>
<td>(0.069)</td>
</tr>
<tr>
<td></td>
<td>[0.047]</td>
<td>[0.011]</td>
<td>[0.085]</td>
</tr>
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<td>openness</td>
<td>0.023</td>
<td>0.009</td>
<td>0.031</td>
</tr>
<tr>
<td></td>
<td>(0.277)</td>
<td>(0.591)</td>
<td>(0.216)</td>
</tr>
<tr>
<td></td>
<td>[0.255]</td>
<td>[0.577]</td>
<td>[0.250]</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.561</td>
<td>0.700</td>
<td>0.709</td>
</tr>
<tr>
<td>number of obs.</td>
<td>41.000</td>
<td>29.000</td>
<td>21.000</td>
</tr>
</tbody>
</table>

Note: \( \frac{s}{(1+n)(1+g)-(1-\delta)} \) is instrumented with the average savings rate and the average labour force growth rate between 1960 and 1969.
Table 2.7: Controlling for the steady state II

<table>
<thead>
<tr>
<th>Dependent variable: log $d_t - \log d_0$</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.051</td>
<td>0.047</td>
<td>0.044</td>
</tr>
<tr>
<td></td>
<td>(0.055)</td>
<td>(0.057)</td>
<td>(0.067)</td>
</tr>
<tr>
<td></td>
<td>[0.047]</td>
<td>[0.067]</td>
<td>[0.086]</td>
</tr>
<tr>
<td>$\frac{d_0}{A_0}$</td>
<td>-0.023</td>
<td>-0.023</td>
<td>-0.023</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
<tr>
<td>$s(1+n)(1+g)-(1-\delta)$</td>
<td>0.055</td>
<td>0.051</td>
<td>0.060</td>
</tr>
<tr>
<td></td>
<td>(0.065)</td>
<td>(0.038)</td>
<td>(0.043)</td>
</tr>
<tr>
<td></td>
<td>[0.056]</td>
<td>[0.025]</td>
<td>[0.057]</td>
</tr>
<tr>
<td>pol. inst.</td>
<td>0.007</td>
<td>0.014</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>(0.886)</td>
<td>(0.826)</td>
<td>(0.928)</td>
</tr>
<tr>
<td></td>
<td>[0.885]</td>
<td>[0.819]</td>
<td>[0.925]</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.543</td>
<td>0.702</td>
<td>0.634</td>
</tr>
<tr>
<td>number of obs.</td>
<td>41.000</td>
<td>29.000</td>
<td>21.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dependent variable: log $d_t - \log d_0$</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>-0.023</td>
<td>0.012</td>
<td>-0.069</td>
</tr>
<tr>
<td></td>
<td>(0.567)</td>
<td>(0.799)</td>
<td>(0.403)</td>
</tr>
<tr>
<td></td>
<td>[0.575]</td>
<td>[0.782]</td>
<td>[0.394]</td>
</tr>
<tr>
<td>$\frac{d_0}{A_0}$</td>
<td>-0.022</td>
<td>-0.021</td>
<td>-0.023</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
<tr>
<td>$s(1+n)(1+g)-(1-\delta)$</td>
<td>0.070</td>
<td>0.065</td>
<td>0.086</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.005]</td>
<td>[0.008]</td>
</tr>
<tr>
<td>$\bar{G} \bar{Y}$</td>
<td>-0.033</td>
<td>-0.013</td>
<td>-0.046</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.394)</td>
<td>(0.150)</td>
</tr>
<tr>
<td></td>
<td>[0.035]</td>
<td>[0.356]</td>
<td>[0.169]</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.579</td>
<td>0.695</td>
<td>0.680</td>
</tr>
<tr>
<td>number of obs.</td>
<td>41.000</td>
<td>29.000</td>
<td>21.000</td>
</tr>
</tbody>
</table>
Table 2.8: Output convergence

<table>
<thead>
<tr>
<th>Dependent variable: $\log y_t - \log y_0$</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.099</td>
<td>0.096</td>
<td>0.060</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.005)</td>
<td>(0.068)</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.003]</td>
<td>[0.003]</td>
</tr>
<tr>
<td>$y_0$</td>
<td>-0.013</td>
<td>-0.013</td>
<td>-0.009</td>
</tr>
<tr>
<td></td>
<td>(0.060)</td>
<td>(0.018)</td>
<td>(0.307)</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.004]</td>
<td>[0.267]</td>
</tr>
<tr>
<td>$s(1+n)(1+g)-(1-\delta)$</td>
<td>0.025</td>
<td>0.024</td>
<td>0.027</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.009)</td>
<td>(0.027)</td>
</tr>
<tr>
<td></td>
<td>[0.003]</td>
<td>[0.027]</td>
<td>[0.059]</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.341</td>
<td>0.309</td>
<td>0.246</td>
</tr>
<tr>
<td>number of obs.</td>
<td>42,000</td>
<td>30,000</td>
<td>21,000</td>
</tr>
</tbody>
</table>

Dependent variable: $\log y_t - \log y_0$

| constant                               | 0.098    | 0.113     | 0.091      |
|                                        | (0.003)  | (0.037)   | (0.200)    |
|                                        | [0.007]  | [0.029]   | [0.260]    |
| $y_0$                                  | -0.011   | -0.013    | -0.011     |
|                                        | (0.003)  | (0.019)   | (0.234)    |
|                                        | [0.005]  | [0.016]   | [0.217]    |
| $s(1+n)(1+g)-(1-\delta)$ (IV)         | 0.016    | 0.015     | 0.016      |
|                                        | (0.074)  | (0.011)   | (0.216)    |
|                                        | [0.065]  | [0.017]   | [0.227]    |
| $R^2$                                  | 0.129    | 0.149     | 0.075      |
| number of obs.                          | 42,000   | 30,000    | 21,000     |

Note: The accumulation variable is instrumented using average labour force growth and the average savings rate between 1960 and 1969.
Table 2.9: Convergence for debt and output — SUR Estimation

<table>
<thead>
<tr>
<th>Dependent variable: $\log d_t - \log d_0$</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.153</td>
<td>0.178</td>
<td>0.174</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.001)</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.015]</td>
</tr>
<tr>
<td>$d_0$</td>
<td>-0.019</td>
<td>-0.019</td>
<td>-0.020</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.001]</td>
<td>[0.003]</td>
</tr>
<tr>
<td>$\frac{s}{(1+n)(1+g)-(1-\delta)}$</td>
<td>0.066</td>
<td>0.045</td>
<td>0.056</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.001)</td>
<td>(0.008)</td>
</tr>
<tr>
<td></td>
<td>[0.001]</td>
<td>[0.002]</td>
<td>[0.034]</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.592</td>
<td>0.702</td>
<td>0.657</td>
</tr>
<tr>
<td>number of obs.</td>
<td>42.000</td>
<td>30.000</td>
<td>21.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dependent variable: $\log y_t - \log y_0$</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.092</td>
<td>0.092</td>
<td>0.067</td>
</tr>
<tr>
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<td>(0.004)</td>
<td>(0.378)</td>
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<td>[0.009]</td>
<td>[0.441]</td>
</tr>
<tr>
<td>$y_0$</td>
<td>-0.012</td>
<td>-0.012</td>
<td>-0.010</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.017)</td>
<td>(0.230)</td>
</tr>
<tr>
<td></td>
<td>[0.004]</td>
<td>[0.047]</td>
<td>[0.335]</td>
</tr>
<tr>
<td>$\frac{s}{(1+n)(1+g)-(1-\delta)}$</td>
<td>0.024</td>
<td>0.024</td>
<td>0.027</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.005)</td>
<td>(0.010)</td>
</tr>
<tr>
<td></td>
<td>[0.001]</td>
<td>[0.015]</td>
<td>[0.037]</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.340</td>
<td>0.309</td>
<td>0.245</td>
</tr>
<tr>
<td>number of obs.</td>
<td>42.000</td>
<td>30.000</td>
<td>21.000</td>
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</table>

100
### Table 2.10: Convergence for debt and output — 3SLS Estimation

<table>
<thead>
<tr>
<th>Dependent variable: $\log d_t - \log d_0$</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.165</td>
<td>0.173</td>
<td>0.180</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$d_0$</td>
<td>-0.020</td>
<td>-0.020</td>
<td>-0.021</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.001]</td>
</tr>
<tr>
<td>$\frac{s}{(1+n)(1+g)-(1-\delta)}$</td>
<td>0.062</td>
<td>0.050</td>
<td>0.053</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.004)</td>
<td>(0.019)</td>
</tr>
<tr>
<td></td>
<td>[0.002]</td>
<td>[0.003]</td>
<td>[0.042]</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.593</td>
<td>0.705</td>
<td>0.656</td>
</tr>
<tr>
<td>number of obs.</td>
<td>41.000</td>
<td>29.000</td>
<td>21.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dependent variable: $\log y_t - \log y_0$</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.091</td>
<td>0.106</td>
<td>0.099</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.040)</td>
<td>(0.212)</td>
</tr>
<tr>
<td></td>
<td>[0.005]</td>
<td>[0.038]</td>
<td>[0.300]</td>
</tr>
<tr>
<td>$y_0$</td>
<td>-0.011</td>
<td>-0.012</td>
<td>-0.012</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.019)</td>
<td>(0.161)</td>
</tr>
<tr>
<td></td>
<td>[0.007]</td>
<td>[0.023]</td>
<td>[0.242]</td>
</tr>
<tr>
<td>$\frac{s}{(1+n)(1+g)-(1-\delta)}$</td>
<td>0.016</td>
<td>0.016</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>(0.053)</td>
<td>(0.149)</td>
<td>(0.172)</td>
</tr>
<tr>
<td></td>
<td>[0.062]</td>
<td>[0.094]</td>
<td>[0.202]</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.301</td>
<td>0.279</td>
<td>0.209</td>
</tr>
<tr>
<td>number of obs.</td>
<td>41.000</td>
<td>29.000</td>
<td>21.000</td>
</tr>
</tbody>
</table>

Note: $\frac{s}{(1+n)(1+g)-(1-\delta)}$ is instrumented with the average savings rate and the average labour force growth rate between 1960 and 1969.
Figure 3.2: Transition Paths - Sample I

Consumption

Domestic capital

Output

Debt

Domestic capital investment rate

Foreign capital investment rate
Table 3.1: Endogenous Savings: Common Technology - Parametrised capital shares

<table>
<thead>
<tr>
<th>OLS estimation</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.068</td>
<td>0.046</td>
<td>0.052</td>
</tr>
<tr>
<td>$d_0$</td>
<td>-0.032</td>
<td>-0.030</td>
<td>-0.030</td>
</tr>
<tr>
<td>$(1+n)(1+g)-(1-\delta)$</td>
<td>-0.034</td>
<td>-0.042</td>
<td>-0.037</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.350</td>
<td>0.350</td>
<td>0.350</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
</tr>
<tr>
<td>$\frac{\alpha}{\alpha+\eta}$</td>
<td>0.538</td>
<td>0.538</td>
<td>0.538</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.350</td>
<td>0.350</td>
<td>0.350</td>
</tr>
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</table>

Table 3.2: Endogenous Savings: Common Technology - Estimated Parameters

<table>
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<tr>
<th>Dependent variable: $\log d_t - \log d_0$</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS estimation</td>
<td>Data</td>
<td>Simulation</td>
<td>Data</td>
</tr>
<tr>
<td>constant</td>
<td>-0.046</td>
<td>-0.020</td>
<td>0.309</td>
</tr>
<tr>
<td>$d_0$</td>
<td>-0.019</td>
<td>-0.019</td>
<td>-0.021</td>
</tr>
<tr>
<td>$(1+n)(1+g)-\beta(1-\delta)$</td>
<td>-0.144</td>
<td>-0.052</td>
<td>0.024</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-</td>
<td>0.050</td>
<td>-</td>
</tr>
<tr>
<td>$\eta$</td>
<td>-</td>
<td>0.719</td>
<td>-</td>
</tr>
<tr>
<td>$1-\alpha-\eta$</td>
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<td>0.231</td>
<td>-</td>
</tr>
<tr>
<td>$\frac{\alpha}{\alpha+\eta}$</td>
<td>-</td>
<td>0.065</td>
<td>-</td>
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<tr>
<td>$\varepsilon$</td>
<td>-</td>
<td>0.757</td>
<td>-</td>
</tr>
<tr>
<td>$d$</td>
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<td>0.625</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma$</td>
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<td>1.585</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: Standard errors — which act as weight in the estimation of the parameters — are in parenthesis.
Table 3.3: Endogenous Savings: Varying Technology - Parametrised capital shares

<table>
<thead>
<tr>
<th></th>
<th>OLS estimation</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.093</td>
<td>0.079</td>
<td>0.072</td>
<td></td>
</tr>
<tr>
<td>( d_0 )</td>
<td>-0.030</td>
<td>-0.027</td>
<td>-0.031</td>
<td></td>
</tr>
<tr>
<td>((1 + n)(1 + g) - (1 - \delta))</td>
<td>-0.032</td>
<td>-0.038</td>
<td>-0.037</td>
<td></td>
</tr>
<tr>
<td>( A_0 )</td>
<td>0.028</td>
<td>0.024</td>
<td>0.032</td>
<td></td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.350</td>
<td>0.350</td>
<td>0.350</td>
<td></td>
</tr>
<tr>
<td>( \eta )</td>
<td>0.350</td>
<td>0.350</td>
<td>0.350</td>
<td></td>
</tr>
<tr>
<td>( \alpha + \eta )</td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
<td></td>
</tr>
<tr>
<td>( \xi )</td>
<td>0.538</td>
<td>0.538</td>
<td>0.538</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.4: Endogenous Savings: Varying Technology - Estimated Parameters

<table>
<thead>
<tr>
<th>Dependent variable: ( \log d_t - \log d_0 )</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS estimation</td>
<td>Data</td>
<td>Simulation</td>
<td>Data</td>
</tr>
<tr>
<td>constant</td>
<td>-0.199</td>
<td>-0.006</td>
<td>0.201</td>
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<tr>
<td></td>
<td>(0.162)</td>
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<td></td>
</tr>
<tr>
<td>( d_0 )</td>
<td>-0.023</td>
<td>-0.024</td>
<td>-0.023</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>((1 + n)(1 + g) - \beta(1 - \delta))</td>
<td>-0.116</td>
<td>-0.046</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>(0.079)</td>
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<td>( A_0 )</td>
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<td>(0.015)</td>
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<td></td>
</tr>
<tr>
<td>( \alpha )</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.017)</td>
</tr>
<tr>
<td>( \eta )</td>
<td>-</td>
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<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>( 1 - \alpha - \eta )</td>
<td>-</td>
<td>0.304</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha + \eta )</td>
<td>-</td>
<td>0.072</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \xi )</td>
<td>-</td>
<td>0.680</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( d )</td>
<td>-</td>
<td>0.625</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y )</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
</tr>
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</table>

Note: Standard errors — which act as weight in the estimation of the parameters — are in parenthesis.
Figure 3.3: Distribution of debt-to-GDP ratios, 1997
Table 3.5: Exogenous Savings: Varying Technology - Parametrised capital shares

<table>
<thead>
<tr>
<th>OLS estimation</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.113</td>
<td>0.106</td>
<td>0.100</td>
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<tr>
<td>$d_0$</td>
<td>-0.024</td>
<td>-0.025</td>
<td>-0.025</td>
</tr>
<tr>
<td>$(1 + n)(1 + g) - (1 - \delta)$</td>
<td>-0.024</td>
<td>-0.026</td>
<td>-0.027</td>
</tr>
<tr>
<td>$s$</td>
<td>0.029</td>
<td>0.029</td>
<td>0.029</td>
</tr>
<tr>
<td>$A_0$</td>
<td>0.023</td>
<td>0.025</td>
<td>0.026</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.350</td>
<td>0.350</td>
<td>0.350</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.350</td>
<td>0.350</td>
<td>0.350</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
</tr>
<tr>
<td>$\sigma + \eta$</td>
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<td>0.538</td>
<td>0.538</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.538</td>
<td>0.538</td>
<td>0.538</td>
</tr>
</tbody>
</table>
Table 3.6: Exogenous Savings: Varying Technology

<table>
<thead>
<tr>
<th>Dependent variable: log (d_t) – log (d_0)</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OLS estimation</strong></td>
<td>Data</td>
<td>Simulation</td>
<td>Data</td>
</tr>
<tr>
<td>constant</td>
<td>0.052</td>
<td>0.052</td>
<td>0.200</td>
</tr>
<tr>
<td>(d_0)</td>
<td>-0.022</td>
<td>-0.020</td>
<td>-0.022</td>
</tr>
<tr>
<td>((1+n)(1+g)-(1-\delta))</td>
<td>-0.040</td>
<td>-0.036</td>
<td>-0.005</td>
</tr>
<tr>
<td>(s)</td>
<td>0.066</td>
<td>0.041</td>
<td>0.052</td>
</tr>
<tr>
<td>(A_0)</td>
<td>0.031</td>
<td>0.021</td>
<td>0.018</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.164</td>
<td>0.563</td>
<td>0.149</td>
</tr>
<tr>
<td>(\eta)</td>
<td>0.274</td>
<td>0.225</td>
<td>0.679</td>
</tr>
<tr>
<td>(1-\alpha-\eta)</td>
<td>0.673</td>
<td>2.046</td>
<td>6.710</td>
</tr>
<tr>
<td>(\alpha+\eta)</td>
<td>3.290</td>
<td>1.641</td>
<td>2.778</td>
</tr>
<tr>
<td>(\varepsilon)</td>
<td>0.033</td>
<td>0.037</td>
<td>0.053</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>0.059</td>
<td>0.018</td>
<td>0.022</td>
</tr>
<tr>
<td>(\delta)</td>
<td>1.01</td>
<td>0.576</td>
<td>0.323</td>
</tr>
<tr>
<td>(\delta+\gamma)</td>
<td>0.640</td>
<td>1.259</td>
<td>6.612</td>
</tr>
<tr>
<td>(\varepsilon)</td>
<td>1.915</td>
<td>1.390</td>
<td>2.331</td>
</tr>
</tbody>
</table>

| **IV estimation**                           | Data    | Simulation| Data       | Simulation |
| constant                                    | 0.033   | 0.200     | 0.200      | -0.072     |
| \(d_0\)                                     | -0.022  | -0.021    | -0.022     | -0.023     |
| \((1+n)(1+g)-(1-\delta)\)                  | -0.042  | -0.033    | -0.005     | -0.036     |
| \(s\)                                       | 0.059   | 0.037     | 0.053      | 0.041      |
| \(A_0\)                                     | 0.031   | 0.022     | 0.018      | 0.022      |
| \(\alpha\)                                  | 0.101   | 0.529     | 0.323      | 0.149      |
| \(\eta\)                                    | 0.576   | 0.313     | 0.158      | 0.628      |
| \(1-\alpha-\eta\)                          | 0.323   | 0.632     | 0.665      | 0.661      |
| \(\alpha+\eta\)                             | 0.640   | 1.259     | 6.612      | 6.225      |
| \(\delta\)                                  | 1.915   | 1.390     | 2.331      |            |

Note: Standard errors — which act as weight in the estimation of the parameters — are in parenthesis. \(s\) and \((1+n)(1+g)-(1-\delta)\) are instrumented using the average savings rate and average labour force growth between 1960 and 1969.
Table 3.7: Exogenous Savings: Varying Technology

<table>
<thead>
<tr>
<th>Dependent variable: log $d_t - log d_0$</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS estimation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>constant</td>
<td>0.044</td>
<td>0.044</td>
<td>0.059</td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.020)</td>
<td>(0.029)</td>
</tr>
<tr>
<td>$\frac{d_0}{A_0}$</td>
<td>-0.021</td>
<td>-0.019</td>
<td>-0.022</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>$s$</td>
<td>0.062</td>
<td>0.041</td>
<td>0.048</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.012)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>$(1+n)(1+g)-(1-\delta)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-0.165</td>
<td>-0.220</td>
<td>-0.264</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.006)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>-0.579</td>
<td>-0.516</td>
<td>-0.631</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.015)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>$1-\alpha - \eta$</td>
<td>-0.256</td>
<td>-0.264</td>
<td>-0.207</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.012)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>$\frac{1}{\alpha+\eta}$</td>
<td>-0.222</td>
<td>-0.299</td>
<td>-0.204</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.010)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.694</td>
<td>0.662</td>
<td>0.753</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.009)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>$\bar{d}$</td>
<td>2.067</td>
<td>2.751</td>
<td>2.024</td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(0.031)</td>
<td>(0.029)</td>
</tr>
<tr>
<td>$J$</td>
<td>3.336</td>
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<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
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<tr>
<td>IV estimation</td>
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<td>0.055</td>
<td>0.050</td>
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<tr>
<td></td>
<td>(0.022)</td>
<td>(0.021)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>$\frac{d_0}{A_0}$</td>
<td>-0.022</td>
<td>-0.021</td>
<td>-0.022</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>$s$</td>
<td>0.055</td>
<td>0.034</td>
<td>0.054</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.017)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>$(1+n)(1+g)-(1-\delta)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-0.209</td>
<td>-0.191</td>
<td>-0.161</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.022)</td>
<td>(0.021)</td>
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<td>-0.583</td>
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<tr>
<td></td>
<td>(0.016)</td>
<td>(0.015)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>$1-\alpha - \eta$</td>
<td>-0.294</td>
<td>-0.299</td>
<td>-0.256</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.016)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>$\frac{1}{\alpha+\eta}$</td>
<td>-0.296</td>
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<tr>
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<td>(0.021)</td>
<td>(0.020)</td>
</tr>
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<td>$\epsilon$</td>
<td>0.628</td>
<td>0.630</td>
<td>0.695</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.009)</td>
<td>(0.008)</td>
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<tr>
<td>$\bar{d}$</td>
<td>2.611</td>
<td>2.384</td>
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<td></td>
<td>(0.100)</td>
<td>(0.099)</td>
<td>(0.098)</td>
</tr>
<tr>
<td>$J$</td>
<td>1.934</td>
<td>1.413</td>
<td>1.479</td>
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<td>(0.100)</td>
<td>(0.099)</td>
<td>(0.098)</td>
</tr>
</tbody>
</table>

Note: Standard errors — which act as weight in the estimation of the parameters — are in parenthesis. $s$ and $(1+n)(1+g)-(1-\delta)$ are instrumented using the average savings rate and average labour force growth between 1960 and 1969.
Table 3.8: Exogenous Savings: Limited matching I

<table>
<thead>
<tr>
<th>Dependent variable: $\log d_t - \log d_0$</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matching constant and $\frac{d_0}{A_0}$</td>
<td>Data</td>
<td>Simulation</td>
<td>Data</td>
</tr>
<tr>
<td>constant</td>
<td>0.055</td>
<td>0.055</td>
<td>0.050</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>-</td>
<td>(0.026)</td>
</tr>
<tr>
<td>$\frac{d_0}{A_0}$</td>
<td>-0.022</td>
<td>-0.022</td>
<td>-0.022</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>-</td>
<td>(0.003)</td>
</tr>
<tr>
<td>$s$</td>
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<td>0.032</td>
<td>0.054</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>-</td>
<td>(0.017)</td>
</tr>
<tr>
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<td>0.205</td>
<td>-</td>
</tr>
<tr>
<td>$\eta$</td>
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<td>-</td>
</tr>
<tr>
<td>$1 - \alpha - \eta$</td>
<td>-</td>
<td>0.324</td>
<td>-</td>
</tr>
<tr>
<td>$\frac{\alpha}{\alpha + \eta}$</td>
<td>-</td>
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<td>-</td>
</tr>
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<td>$\beta$</td>
<td>-</td>
<td>0.593</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-</td>
<td>2.567</td>
<td>-</td>
</tr>
<tr>
<td>$\phi$</td>
<td>-</td>
<td>0.000</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dependent variable: $\log d_t - \log d_0$</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matching constant and $\frac{d_0}{A_0}$</td>
<td>Data</td>
<td>Simulation</td>
<td>Data</td>
</tr>
<tr>
<td>constant</td>
<td>0.055</td>
<td>0.055</td>
<td>0.050</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>-</td>
<td>(0.026)</td>
</tr>
<tr>
<td>$\frac{d_0}{A_0}$</td>
<td>-0.022</td>
<td>-0.014</td>
<td>-0.022</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>-</td>
<td>(0.003)</td>
</tr>
<tr>
<td>$s$</td>
<td>0.055</td>
<td>0.055</td>
<td>0.054</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>-</td>
<td>(0.017)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-</td>
<td>0.212</td>
<td>-</td>
</tr>
<tr>
<td>$\eta$</td>
<td>-</td>
<td>0.631</td>
<td>-</td>
</tr>
<tr>
<td>$1 - \alpha - \eta$</td>
<td>-</td>
<td>0.157</td>
<td>-</td>
</tr>
<tr>
<td>$\frac{\alpha}{\alpha + \eta}$</td>
<td>-</td>
<td>0.251</td>
<td>-</td>
</tr>
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<td>$\beta$</td>
<td>-</td>
<td>0.801</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-</td>
<td>2.647</td>
<td>-</td>
</tr>
<tr>
<td>$\phi$</td>
<td>-</td>
<td>0.000</td>
<td>-</td>
</tr>
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</table>
Table 3.9: Exogenous Savings: Limited matching II

<table>
<thead>
<tr>
<th>Matching $\frac{s}{(1+n)(1+g)-(1-\delta)}$</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.055</td>
<td>0.116</td>
<td>0.044</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>(0.026)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>$\frac{d_0}{A_0}$</td>
<td>-0.022</td>
<td>-0.021</td>
<td>-0.023</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>$\frac{s}{(1+n)(1+g)-(1-\delta)}$</td>
<td>0.055</td>
<td>0.034</td>
<td>0.063</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.017)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-</td>
<td>0.404</td>
<td>0.446</td>
</tr>
<tr>
<td>$\eta$</td>
<td>-</td>
<td>0.374</td>
<td>0.349</td>
</tr>
<tr>
<td>$1 - \alpha - \eta$</td>
<td>-</td>
<td>0.221</td>
<td>0.206</td>
</tr>
<tr>
<td>$\frac{\alpha}{\alpha + \eta}$</td>
<td>-</td>
<td>0.519</td>
<td>0.561</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>-</td>
<td>0.628</td>
<td>0.629</td>
</tr>
<tr>
<td>$d$</td>
<td>-</td>
<td>5.052</td>
<td>5.571</td>
</tr>
<tr>
<td>$J$</td>
<td>-</td>
<td>1.934</td>
<td>1.413</td>
</tr>
</tbody>
</table>

Note: Standard errors — which act as weight in the estimation of the parameters — are in parenthesis. $s$ and $(1+n)(1+g)-(1-\delta)$ are instrumented using the average savings rate and average labour force growth between 1960 and 1969.
### Table 3.10: Exogenous Savings: Varying Technology - Institutional Tax

<table>
<thead>
<tr>
<th>Dependent variable: $\log d_t - \log d_0$</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau = 0.3$</td>
<td>Data</td>
<td>Simulation</td>
<td>Data</td>
</tr>
<tr>
<td>constant</td>
<td>0.055</td>
<td>0.055</td>
<td>0.050</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td></td>
<td>(0.026)</td>
</tr>
<tr>
<td>$d_0$</td>
<td>-0.022</td>
<td>-0.021</td>
<td>-0.022</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td></td>
<td>(0.003)</td>
</tr>
<tr>
<td>$s$</td>
<td>0.055</td>
<td>0.035</td>
<td>0.054</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td></td>
<td>(0.017)</td>
</tr>
<tr>
<td>$\frac{(1+n)(1+g)-(1-\delta)}{\alpha \eta}$</td>
<td>-0.347</td>
<td>-0.330</td>
<td>-0.306</td>
</tr>
<tr>
<td>$\eta$</td>
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<td></td>
<td>-0.427</td>
</tr>
<tr>
<td>$1 - \alpha - \eta$</td>
<td>-0.238</td>
<td></td>
<td>-0.242</td>
</tr>
<tr>
<td>$\frac{1 - \alpha - \eta}{\alpha \eta}$</td>
<td>-0.455</td>
<td></td>
<td>-0.436</td>
</tr>
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<td>$\varepsilon$</td>
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<td></td>
<td>-0.638</td>
</tr>
<tr>
<td>$d$</td>
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<td></td>
<td>1.928</td>
</tr>
<tr>
<td>$y$</td>
<td>1.736</td>
<td></td>
<td>1.237</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dependent variable: $\log d_t - \log d_0$</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
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<td>Data</td>
<td>Simulation</td>
<td>Data</td>
</tr>
<tr>
<td>constant</td>
<td>0.055</td>
<td>0.055</td>
<td>0.050</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td></td>
<td>(0.026)</td>
</tr>
<tr>
<td>$d_0$</td>
<td>-0.022</td>
<td>-0.021</td>
<td>-0.022</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td></td>
<td>(0.003)</td>
</tr>
<tr>
<td>$s$</td>
<td>0.055</td>
<td>0.035</td>
<td>0.054</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td></td>
<td>(0.017)</td>
</tr>
<tr>
<td>$\frac{(1+n)(1+g)-(1-\delta)}{\alpha \eta}$</td>
<td>-0.518</td>
<td></td>
<td>-0.509</td>
</tr>
<tr>
<td>$\eta$</td>
<td>-0.306</td>
<td></td>
<td>-0.313</td>
</tr>
<tr>
<td>$1 - \alpha - \eta$</td>
<td>-0.176</td>
<td></td>
<td>-0.177</td>
</tr>
<tr>
<td>$\frac{1 - \alpha - \eta}{\alpha \eta}$</td>
<td>-0.629</td>
<td></td>
<td>-0.619</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>-0.635</td>
<td></td>
<td>-0.638</td>
</tr>
<tr>
<td>$d$</td>
<td>0.950</td>
<td></td>
<td>0.934</td>
</tr>
<tr>
<td>$y$</td>
<td>1.736</td>
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<td>1.237</td>
</tr>
</tbody>
</table>

Note: Standard errors — which act as weight in the estimation of the parameters — are in parenthesis. $s$ and $(1+n)(1+g)-(1-\delta)$ are instrumented using the average savings rate and average labour force growth between 1960 and 1969. The data are simulated under the assumption that $\tau + \delta = 0.12$. 

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Figure 4.1: Steady-State Correlations

\( \phi = 0.1 \)

\( \phi = 0.01 \)

\( \phi = 0.001 \)
Figure 4.2: Transition path: $k_0 = 0.1k^*$

Figure 4.3: Transition path: $d_0 = 0.1d^*$
Figure 4.4: Transition path - Whole sample

- Debt
- Capital
- Consumption
- Investment
- Premium
- Saving rate
Table 4.1: Estimated $\alpha$ ($\varphi = 0.01$)

<table>
<thead>
<tr>
<th>OLS estimation</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Simulation</td>
<td>Data</td>
</tr>
<tr>
<td>constant</td>
<td>0.072</td>
<td>0.379</td>
<td>0.086</td>
</tr>
<tr>
<td></td>
<td>(0.052)</td>
<td></td>
<td>(0.053)</td>
</tr>
<tr>
<td>$\Delta t / \Delta_0$</td>
<td>-0.023</td>
<td>-0.021</td>
<td>-0.025</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td></td>
<td>(0.004)</td>
</tr>
<tr>
<td>$s_{(1+\nu)(1+\varphi)-(1-\delta)}$</td>
<td>0.054</td>
<td>1.380</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td></td>
<td>(0.014)</td>
</tr>
<tr>
<td>$\Delta_0 / \Delta_0$</td>
<td>-0.000</td>
<td>-0.043</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td></td>
<td>(0.006)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Standard errors — which act as weight in the estimation of the parameters — are in parenthesis.
Figure 4.5: Distribution of debt-to-GDP ratios ($\phi = 0.1$)

Sample I

Sample II

Sample III

debt-to-GDP ratio
Table 4.2: Estimated $\alpha$ ($\varphi = 0.1$)

<table>
<thead>
<tr>
<th>OLS estimation</th>
<th>Sample I</th>
<th>Sample II</th>
<th>Sample III</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>Data</td>
<td>Simulation</td>
<td>Data</td>
</tr>
<tr>
<td></td>
<td>0.072</td>
<td>-0.056</td>
<td>0.086</td>
</tr>
<tr>
<td></td>
<td>(0.052)</td>
<td></td>
<td>(0.053)</td>
</tr>
<tr>
<td>$\frac{d_d}{d_0}$</td>
<td>-0.023</td>
<td>-0.049</td>
<td>-0.025</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td></td>
<td>(0.004)</td>
</tr>
<tr>
<td>$\frac{\delta}{(1+n)(1+\gamma)-(1-\delta)}$</td>
<td>0.054</td>
<td>-0.183</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td></td>
<td>(0.014)</td>
</tr>
<tr>
<td>$\frac{k_d}{k_0}$</td>
<td>-0.000</td>
<td>-0.073</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td></td>
<td>(0.006)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-</td>
<td>0.060</td>
<td>-</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-</td>
<td>0.989</td>
<td>-</td>
</tr>
<tr>
<td>$\frac{\gamma}{\nu}$</td>
<td>-</td>
<td>0.004</td>
<td>-</td>
</tr>
<tr>
<td>$J$</td>
<td>-</td>
<td></td>
<td>-</td>
</tr>
</tbody>
</table>

Note: Standard errors — which act as weight in the estimation of the parameters — are in parenthesis.
Figure 4.6: Distribution of debt-to-GDP ratios ($\varphi = 0.1$)
Table 4.3: Simulated Regressions with Simulated Savings Rate

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\mu$</th>
<th>$\varphi$</th>
<th>$\beta$</th>
<th>$\frac{x}{\varphi}$</th>
<th>$\frac{\Delta x}{\Delta n}$</th>
<th>$\frac{1}{\gamma} \left(1+n\right)\left(1+\delta\right)$</th>
<th>$\frac{y}{x}$</th>
<th>$\frac{\Delta y}{\Delta n}$</th>
<th>$\frac{1}{\gamma} \left(1+n\right)\left(1+\delta\right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>0.33333</td>
<td>0.33333</td>
<td>0.33333</td>
<td>0.05000</td>
<td>0.050000</td>
<td>0.33333</td>
<td>0.33333</td>
<td>0.008300</td>
<td>0.010789</td>
<td>0.018993</td>
</tr>
</tbody>
</table>

Note: The table presents the simulated regressions with the simulated savings rate. The values are used to analyze the relationship between the variables under study.
Figure 4.7: Simulated Transition Path, $\alpha = \frac{1}{3}$, $\mu = 12$, $\varphi = 0.01$
Figure 4.8: Simulated Transition Path, $\alpha = \frac{1}{3}$, $\mu = 0.05$, $\varphi = 0.01$
Bibliography


Appendices
Appendix A

Solution Method for the BMS Model

A.1 A Non-Linear Solution Method for the BMS Model with Endogenous Savings

Non-linear methods for approximating decision rules are useful when one wants to ask about the behaviour of an economy away from the steady state. Indeed, Taylor approximation methods remain local in nature. A popular approach to approximate non-linear decision rules is to use least squares methods. In its simplest form, this method involves choosing a non-linear function of the state variables (e.g. a polynomial of physical capital for consumption). This however, is often problematic since powers of the state variables are co-linear. An alternative is least squares orthogonal polynomial approximation. This involves choosing functions of the state variables that are orthogonal to each other so as to avoid co-linearity. Chebychev polynomial are an example of such functions.

The reduced form of the model takes the following form:

\[
\begin{align*}
  z_{t+1} &= \frac{1}{(1+a)(1+b)} \left( (1-a)Bz_t^e + (1-b)n_t - c_t \right) \\
  c_{t+1} &= \beta c_t^e \left( (1-a)Bz_t^{e-1} + 1 - \delta \right)
\end{align*}
\]  

(A.1) (A.2)
We want to find a decision rule for consumption as a function of current domestic capital which satisfies the Euler equation and the market-clearing condition. Here we assume that over the domain \([\bar{z}, \tilde{z}]\) the consumption decision rule takes the form

\[c_t = \phi(z_t, \theta) = \sum_{i=0}^{n_z} \theta_i T_i(\varphi(z_t))\]  

(A.3)

where \(T_i\) is the Chebychev polynomial of order \(i = 0, \ldots, n_h\) and \(\varphi(z)\) is a linear function mapping the domain of \(z\) into \([-1, 1]\). Chebychev polynomials are computed as

\[T_n(x) = \cos(n \cos^{-1}(x))\]  

(A.4)

Let the Euler equation residual be

\[R(z_t; \phi, \theta) \equiv \phi(z_{t+1}, z_{t}, \theta)^{\sigma} - \phi(z_t, \theta)^{\sigma} \beta^* ((1 - \alpha)\epsilon B z_{t+1}(z_t, \theta)^{\rho - 1} + 1 - \delta)\]  

(A.5)

where \(z_j \in [\bar{z}, \tilde{z}]\). Then the problem is to choose \(\theta\) such that

\[\sum_{j=1}^{n_{\text{nodes}}} R(z_j; \phi, \theta) T_i(\varphi(z_j)) = 0\]  

(A.6)

for \(i = 0, \ldots, n_h\). This can be written as

\[T(\varphi(z)) R(z; \phi, \theta) = 0\]  

(A.7)

where

\[T(z) = \begin{pmatrix} T_0(\varphi(z_1)) & \cdots & T_0(\varphi(z_{n_{\text{nodes}}})) \\ \vdots & \ddots & \vdots \\ T_{n_z}(\varphi(z_1)) & \cdots & T_{n_z}(\varphi(z_{n_{\text{nodes}}})) \end{pmatrix}\]  

(A.8)

The algorithm is as follows

1. Choose the order of approximation \(n_z\). We choose \(n_z = 5\)

2. Choose an interval \([\bar{z}, \tilde{z}]\) over which to estimate the decision rule \(^1\).

\(^1\)See Section A.2 for a discussion of the choice of the interval
3. Set up a grid of \( \text{node}_z \) (\( \text{node}_z = 20 \)) data points \( \{z_i\}_{i=1}^{\text{node}_z} \) over \([\bar{z}, \hat{z}]\).

4. Compute the \( \text{node}_z > n_z \) roots of the Chebychev polynomial of order \( \text{node}_z > n_z \) as

\[
x_i = \cos \left( \frac{(2i - 1)\pi}{2\text{node}_z} \right)
\]

for \( i = 1, \ldots, \text{node}_z \).

5. Compute the matrix

\[
aT(z) = \begin{pmatrix}
T_0(\varphi(x_1)) & \ldots & T_0(\varphi(x_{\text{node}_z})) \\
\vdots & \ddots & \vdots \\
T_{n_z}(\varphi(x_1)) & \ldots & T_{n_z}(\varphi(x_{\text{node}_z}))
\end{pmatrix}
\]  

(A.9)

6. Choose an initial value for \( \theta \).

7. Compute \( z_i \) as

\[
z_i = \bar{z} + (x_i + 1) \frac{(\bar{z} - \hat{z})}{2}
\]

for \( i = 1, \ldots, \text{node}_z \) to map \([-1, 1]\) into \([\bar{z}, \hat{z}]\).

8. Compute \( R (z_j; \phi, \theta) \) for \( j = 1, \ldots, \text{node}_z \) and evaluate

\[
T (\varphi(z)) R (z; \phi, \theta)
\]

9. If it is close enough to zero, i.e. if

\[
T (\varphi(z)) R (z; \phi, \theta) < \text{tol}
\]

where \( \text{tol} \) is the tolerance level, then stop and form

\[
c_t = \phi(z_t, \theta) = \sum_{i=0}^{n_z} \theta_i T_i (\varphi(z_t))
\]

else update \( \theta \) and go back to Step 7.

\(^2\text{See Section A.3 for a discussion of the choice of initial values}\)
A.2 Choice of the grid

The estimation of $\alpha$ and $\eta$ requires a solution for consumption for each country. In the simulated data, countries are assumed to be different in two dimensions. First, they have different labour force growth rates, the values of which are taken from the data. Second, they start at different initial values for human capital. We cannot directly use data values for human capital since the model has no scale. Data values must therefore be scaled down in some fashion. Here we will use the distribution of income as a scale. The country with the highest level of per worker GDP at the beginning of the sample is chosen as the standard country. Specifically, for each country $j$ we construct the distribution of income relative to the standard country $s$ at the beginning of the sample

$$\nu_{j0} = \frac{y_{j0}}{y_{s0}}$$

where $y_{j0}$ is income at the beginning of the sample for country $j$ and $y_{s0}$ is income at the beginning of the sample for the standard country. We also construct the distribution of income implied by labour force growth rates in the steady state

$$\nu^* = \frac{y_j^*}{y_s^*}$$

where the '*' denotes the steady state value. If we assume that the standard country is $1 - \varphi$ away from its steady state at the beginning of the sample period, we can compute the deviation of country $j$ from its steady state as

$$\Delta_j = \varphi \frac{\nu_{j0}}{\nu^*}$$

For each country $j$ the approximation grid is then taken to be in the interval $[\bar{z}, \bar{z}]$ where

$$\bar{z} = (1 - \Delta_j)z^*$$

and

$$\bar{z} = (1 + \Delta_j)z^*$$

We choose node $z$ equally spaced points within this interval.

We choose $\phi = 0$
A.3 Choice of initial value

One fairly straightforward way of obtaining an initial value for $\theta$ is to log-linearise the model around the steady state:

1. Find a solution to the model by using standard first-order Taylor approximation methods. That is, solve the following log-linear system

$$
\frac{\tilde{y}_t}{\tilde{i} + c^*} = \frac{\tilde{z}_t}{\tilde{i}^* + c^*}
$$

where $\tilde{q}$ denotes the percentage deviation of $q$ from its steady state. The steady state is defined by

$$
y^* = \frac{1}{(1-\alpha)e} \left( \frac{1}{\beta^*} - (1-\delta) \right)
$$

$$
z^* = \left( \frac{1}{B^*} \right)^{\frac{1}{z-1}}
$$

$$
y^* = Bz^{*e}
$$

$$
i^* = [(1+n)(1+g) - (1-\delta)] z^*
$$

$$
c^* = (1-\alpha)y^* - i^*
$$

2. For each value of $z_t \in [\underline{z}, \bar{z}]$, find the corresponding value of $c_t$ using the linear solution.

3. Regress log $c$ on $T_i(\varphi(z))$ and use the OLS coefficients as initial values for $\theta$.

A.4 Solving the BMS Model with Exogenous Savings

Let $s$ denote the domestic savings rate, or the rate at which consumers save out of output to accumulate domestic capital. Since domestic savings must equal investment in domestic
capital \( (i_t^n = sBh_t) \):
\[
(1 + n)(1 + g)z_{t+1} = sBz_t + (1 - \delta)z_t
\]

(A.12)

Solving this equation yields
\[
z^* = \left( \frac{sB}{(1 + n)(1 + g) - (1 - \delta)} \right)^{-1}
\]

(A.13)

and the steady state is defined by
\[
y^* = Bz^*
\]
\[
k^* = \frac{\alpha}{\tau + \delta} y^*
\]

(A.14)
\[
d^* = k^*
\]

(A.15)

### A.5 Convergence rate

#### A.5.1 Endogenous Savings

Reducing the linearised system defined by (A.11), we have
\[
\begin{align*}
\hat{c}_{t+1} + \frac{1-\varepsilon}{\sigma} (1 - \beta^*(1 - \delta)) \hat{z}_{t+1} &= \hat{c}_t \\
\hat{z}_{t+1} &= \frac{1}{\beta} \hat{z}_t - \frac{1}{(1 + n)(1 + g) z^*} c^* \hat{c}_t
\end{align*}
\]

(A.16)

or
\[
\begin{bmatrix}
\hat{z}_{t+1} \\
\hat{c}_{t+1}
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{\beta} & \frac{1}{(1 + n)(1 + g) z^*} c^* \\
\frac{\varepsilon - 1}{\beta \sigma} (1 - \beta^*(1 - \delta)) & \frac{1 - \varepsilon}{\sigma} (1 - \beta^*(1 - \delta)) \frac{c^*}{z^*} + 1
\end{bmatrix}
\begin{bmatrix}
\hat{z}_t \\
\hat{c}_t
\end{bmatrix}
\]

(A.17)

The convergence rate is governed by the eigenvalue of the transition matrix that solves
\[
2\lambda = \frac{1}{\beta} + 1 + \left( \frac{1-\varepsilon}{\sigma} \right) \frac{(1 - \beta^*(1 - \delta)) c^*}{(1 + n)(1 + g) z^*}
\]

\[
- \left[ \frac{1}{\beta} + \left( \frac{1-\varepsilon}{\sigma} \right) \frac{(1 - \beta^*(1 - \delta)) c^*}{(1 + n)(1 + g) z^*} \right] - 4 \left( \frac{\varepsilon - 1}{\sigma \beta} \right) \frac{(1 - \beta^*(1 - \delta)) c^*}{(1 + n)(1 + g) z^*} \]

(A.18)

As \( \varepsilon \to 1 \), \( \lambda \to 1 \) and the model does not exhibit convergence.
A.5.2 Exogenous Savings

Linearising equation (A.12) yields

\[ \hat{z}_{t+1} = \frac{\varepsilon B z^{\varepsilon - 1} + 1 - \delta}{(1 + g)(1 + n)} \hat{z}_t \]  \hspace{1cm} (A.19)

so that

\[ \hat{z}_{t+1} = \left[ \varepsilon + \frac{(1 - \varepsilon)(1 - \delta)}{(1 + n)(1 + g)} \right] \hat{z}_t \]  \hspace{1cm} (A.20)

with solution

\[ \hat{z}_t = \hat{z}_0 \left[ \varepsilon + \frac{(1 - \varepsilon)(1 - \delta)}{(1 + n)(1 + g)} \right]^t \]  \hspace{1cm} (A.21)

Let \( \lambda = \varepsilon + \frac{(1 - \varepsilon)(1 - \delta)}{(1 + n)(1 + g)} \). Again, as \( \varepsilon \to 1 \), \( \lambda \to 1 \) and the model does not exhibit convergence.
Appendix B

The Role of the Discount Factor

B.1 The relationship between debt and the discount factor

How are $\beta^*$ and the level of debt related in the steady state? We know that

$$
\frac{\partial d^*}{\partial \beta^*} = \frac{\partial d^*}{\partial y^*} + \frac{\partial y^*}{\partial y^*} \frac{\partial y^*}{\partial \beta^*}
$$

(B.1)

Since the first term, $\frac{\partial y^*}{\partial y^*}$, is always negative, we have to determine the sign of the second term.

$$
\frac{\partial y^*}{\partial \beta^*} = \alpha k^{\alpha-1} \frac{\partial k^*}{\partial \beta^*}
$$

(B.2)

In the steady state,

$$
k^* = \left[ \frac{1}{\alpha} \left( \frac{1}{\beta^*} - (1 - \delta) \right) \right]^{\frac{1}{\alpha-1}}
$$

(B.3)

Taking derivatives, we find

$$
\frac{\partial k^*}{\partial \beta^*} = \frac{\alpha^{1-\alpha}}{(1 + \varphi \left( \frac{d^*}{y^*} \right)^2)^{\alpha-1}} \left[ \frac{1}{\alpha-1} \left( \frac{-1}{\beta^*} - (1 - \delta) \right)^{\frac{1}{\alpha-1}-1} \left( 1 + \varphi \left( \frac{d^*}{y^*} \right)^2 \right)^{\frac{1}{\alpha-1}}
\right.
\left. + \left( \frac{1}{\beta^*} - (1 - \delta) \right)^{\frac{1}{\alpha-1}} \left( - \frac{1}{\alpha-1} \left( \frac{-1}{\beta^*} \right) \frac{d^*}{y^*} \right) \left( 1 + \varphi \left( \frac{d^*}{y^*} \right)^2 \right)^{\frac{1}{\alpha-1}-1} \right]
$$

(B.4)
so that the sign of $\frac{\partial k^*}{\partial z}$ will depend on the sign of

$$
\Psi = \left( \frac{1}{\alpha - 1} \right) \left( \frac{-1}{\beta^*} \right) \left( \frac{1}{\beta^*} - (1 - \delta) \right) \frac{a^{1-1}}{a^{1-1}} \left( 1 + \varphi \left( \frac{d^*}{y^*} \right) \right) \frac{a^{1-1}}{a^{1-1}}

\times \left[ 1 + \varphi \left( \frac{d^*}{y^*} \right)^2 + \frac{d^*}{y^*} \left( \frac{1}{\beta^*} - (1 - \delta) \right) \right]

(B.5)

If $\frac{d^*}{y^*} > 0$, $\Psi$ is unambiguously positive. If $\frac{d^*}{y^*} < 0$, $\Psi$ is also likely to be positive for reasonable values of $\beta$ and $\delta$ if $\frac{d^*}{y^*} \leq 1$. Consequently, we should expect that $\frac{\partial k^*}{\partial z} \geq 0$.

For net creditor countries in the steady state, this implies a negative correlation between the discount factor and steady state debt. For debtor countries, the relationship between debt and the discount factor is ambiguous.
Appendix C

Solution Method for the Debt-Elastic Interest Premium Model

C.1 Solution Method

The system of equations to be solved is

\[ y_t = k_t^\sigma \]  \hspace{1cm} (C.1)

\[ (1 + n)(1 + g)k_{t+1} = x_t + (1 - \delta)k_t \]  \hspace{1cm} (C.2)

\[ (1 + n)(1 + g)dt = \left(1 + r + \phi \frac{dt}{yt} \right) dt - y_t + c_t + x_t + \frac{\mu}{2} \left( \frac{x_t}{k_t} - \frac{x^*}{k^*} \right)^2 k_t \]  \hspace{1cm} (C.3)

\[ \left( \frac{c_{t+1}}{c_t} \right)^\sigma = \beta^* \left( 1 + r + 2\phi \frac{dt_{t+1}}{yt_{t+1}} \right) \]  \hspace{1cm} (C.4)

\[ \left( \frac{c_{t+1}}{c_t} \right)^\sigma = \frac{\beta^*}{\left( 1 + \mu \left( \frac{x_t}{k_t} - \frac{x^*}{k^*} \right) \left( \frac{x_{t+1}}{k_{t+1}} - \frac{x^*}{k^*} \right) \left( \frac{x_{t+1}}{k_{t+1}} - \frac{1}{2} \left( \frac{x_{t+1}}{k_{t+1}} - \frac{x^*}{k^*} \right) \right) + \left( 1 + \mu \left( \frac{x_{t+1}}{k_{t+1}} - \frac{x^*}{k^*} \right) \right) (1 - \delta) \right) \]  \hspace{1cm} (C.5)
where $\beta^* = \frac{\beta}{(1+n)(1+\beta)}$.

We want the decision rules for consumption $c_t$ and investment $x_t$ as functions of the two states, capital $k_t$ and debt $d_t$. Here we assume that over the domain $K \times D$ (where $K = [\bar{k}, \bar{k}]$ and $D = [\bar{d}, \bar{d}]$) the consumption and investment decision rules take the form

\[
c_t = \phi(k_t, d_t, \theta_c) = T^k (\varphi(k)) \theta_c T^d (\varphi(d))
\]

\[
x_t = \psi(k_t, d_t, \theta_x) = T^k (\varphi(k)) \theta_x T^d (\varphi(d))
\]

where $T^k$ and $T^d$ are the Chebychev polynomials of order $n_k$ and $n_d$, and $\varphi(h)$ is a linear function mapping the domains $K$ and $D$ into $[-1, 1]$. Chebychev polynomials are computed as

\[
T_n(x) = \cos(n \cos^{-1}(x)) \quad (C.6)
\]

Let the Euler equation residuals corresponding to equations (C.4) and (C.5) be denoted

\[
R_1 (k_{jt}, d_{jt}; \phi, \psi, \theta)
\]

\[
R_2 (k_{jt}, d_{jt}; \phi, \psi, \theta)
\]

where $\theta = \begin{pmatrix} \theta_c \\ \theta_x \end{pmatrix}$. We want to find $\theta$ such that

\[
T (\varphi(k), \varphi(d)) R_1 (k_{jt}, d_{jt}; \phi, \psi, \theta) = 0
\]

and

\[
T (\varphi(k), \varphi(d)) R_2 (k_{jt}, d_{jt}; \phi, \psi, \theta) = 0
\]

where

\[
T (\varphi(k), \varphi(d)) = T^k (\varphi(k)) \otimes T^d (\varphi(d))
\]

and

\[
T^k (\varphi(k)) = \begin{pmatrix} T^k_0 (\varphi(k_1)) & \cdots & T^k_0 (\varphi(k_{n_{de,b}})) \\
\vdots & \ddots & \vdots \\
T^k_{n_k} (\varphi(k_1)) & \cdots & T^k_{n_k} (\varphi(k_{n_{de,b}})) \end{pmatrix}
\]
The algorithm to find $\theta$ is as follows

1. Choose orders of approximation $n_k$ and $n_d$.

2. Choose intervals $\mathcal{K}$ and $\mathcal{D}$ over which to estimate the decision rules.

3. Set up grids of $node_k$ ($\{k_i\}_{i=1}^{node_k}$ over $\mathcal{K}$) and $node_d$ ($\{d_i\}_{i=1}^{node_d}$ over $\mathcal{B}$) data points.

4. Compute the $node_k > n_k$ and $node_d > n_d$ roots of the Chebychev polynomials as

$$z_i^k = \cos \frac{(2i-1)\pi}{2node_k}$$

for $i = 1, \ldots, node_k$

$$z_i^b = \cos \frac{(2i-1)\pi}{2node_d}$$

for $i = 1, \ldots, node_d$.

5. Compute the matrix

$$T(d, \mathcal{V}) = T^k \left( z^k \right) \otimes T^d \left( z^d \right)$$

where

$$T^m \left( z^m \right) = \begin{pmatrix}
T^m_0 \left( z_1^m \right) & \cdots & T^m_0 \left( z_{node_k}^m \right) \\
\vdots & \ddots & \vdots \\
T^m_{node_k} \left( z_1^m \right) & \cdots & T^m_{node_k} \left( z_{node_k}^m \right)
\end{pmatrix}$$

for $m = k, d$.

6. Choose an initial value for $\theta$.

7. Compute $k_i$ and $d_i$ as

$$k_i = k + (z_i^k + 1) \frac{(k - k)}{2}$$
for $i = 1, \ldots, node_k$ and
\[
d_i = d + (z_i^d + 1) \frac{(d - d)}{2}
\]
for $i = 1, \ldots, node_k$ to map $[-1,1]$ into $\mathcal{K}$ and $\mathcal{D}$.

8. Compute
\[
R_1(k_{it}, d_{jt}; \phi, \psi, \theta)
\]
\[
R_2(k_{it}, d_{jt}; \phi, \psi, \theta)
\]
for $i = 1, \ldots, node_k$ and $j = 1, \ldots, node_d$ and evaluate
\[
T(\varphi(k), \varphi(d)) R_1(k_{jt}, d_{jt}; \phi, \psi, \theta)
\]
and
\[
T(\varphi(k), \varphi(d)) R_2(k_{jt}, d_{jt}; \phi, \psi, \theta)
\]

9. If they are close enough to zero then stop and form
\[
c_t = \phi(k_t, d_t, \theta_c) = T^k(\varphi(k)) \theta_c T^d(\varphi(d))'
\]
\[
x_t = \psi(k_t, d_t, \theta_x) = T^k(\varphi(k)) \theta_x T^d(\varphi(d))'
\]
else update $\theta$ and go back to Step 7.

C.2 Choice of the grid

The estimation of $\alpha$ requires a solution for consumption and investment for each country. To solve the model and simulate the data, we need initial values for capital and debt. We cannot directly use data values for capital and debt since the model has no scale. Data values must therefore be scaled down in some fashion. Here we will use the distribution of income as a scale. The country with the highest level of per worker GDP at the beginning of the sample is chosen as the standard country. Specifically, for each country $j$ we construct the distribution of income relative to the standard country $s$ at the beginning of the sample
\[
\nu_{j0} = \frac{y_{j0}}{y_{s0}}
\]
where \( y_{j0} \) is income at the beginning of the sample for country \( j \) and \( y_{s0} \) is income at the beginning of the sample for the standard country. We also construct the distribution of income implied by labour force growth rates in the steady state

\[
\nu^* = \frac{y_j^*}{y_s^*}
\]

where the \('*\)' denotes the steady state value. If we assume that the standard country is \( 1 - \varphi \) away from its steady state at the beginning of the sample period \(^1\), we can compute the deviation of country \( j \) from its steady state as

\[
\Delta_j = \varphi \frac{y_{j0}}{\nu^*}
\]

For each country \( j \) the approximation grid is then taken to be in the interval \([z, \bar{z}]\) where

\[
z = (1 - \Delta_j)z^*
\]

and

\[
\bar{z} = (1 + \Delta_j)z^*
\]

for \( z = k, d \) We choose node \( z \) equally spaced points within this interval.

### C.3 Choice of initial value

To obtain an initial value for \( \theta \) we linearise the model around the steady state:

1. Find a solution to the model by using standard first-order Taylor approximation methods. That is, solve the following log-linear system

\[
\dot{y}_t = \alpha \dot{k}_t
\]

\[
(1 + n)(1 + g)\dot{k}_{t+1} - (1 - \delta)\dot{k}_t - \frac{x^*}{\dot{k}^*}\ddot{x}_t = 0
\]

\(^1\)We choose \( \varphi = 0 \)
\[(1 + n)(1 + g)\hat{d}_{t+1} - \frac{1}{\beta^*} \hat{d}_t - \frac{c^*}{d^*} \hat{c}_t - \frac{x^*}{d^*} \hat{x}_t = \left(\varphi \frac{d^*}{y^*} + \frac{y^*}{d^*}\right) \hat{y}_t\]

\[(\beta^*(1 + q) - 1) \hat{d}_{t+1} + \sigma \hat{c}_{t+1} - \sigma \hat{c}_t = (\beta^*(1 + q) - 1) \hat{y}_{t+1}\]

\[
\left(1 - \beta^*(1 - \delta) + \beta^* \frac{x^*}{k^*} u \left(1 - \delta + \frac{x^*}{k^*}\right)\right) \hat{k}_{t+1} = 2\alpha \frac{y^*}{k^*} \beta^* \varphi \left(\frac{d^*}{y^*}\right)^2 \hat{d}_{t+1} + \sigma \hat{c}_{t+1}
\]

\[-\beta^* \frac{x^*}{k^*} u \left(1 - \delta + \frac{x^*}{k^*}\right) - \mu \frac{x^*}{k^*} \hat{k}_t - \sigma \hat{c}_t + \mu \frac{x^*}{k^*} \hat{x}_t = \alpha \frac{y^*}{k^*} \beta^* \left(1 - \varphi \left(\frac{d^*}{y^*}\right)^2\right)\]

where \(\hat{z}\) denotes the deviation of \(z\) from its steady state. The steady state is defined by

\[
\frac{d^*}{y^*} = \frac{1}{2\varphi} \left(\frac{1}{\beta^*} - (1 + q)\right)
\]

\[
\frac{x^*}{k^*} = (1 + n)(1 + g) - (1 - \delta)
\]

\[
\frac{y^*}{k^*} = \frac{1}{\alpha} \left(\frac{1}{\beta^*} - (1 - \delta)\right)
\]

\[
k^* = \left(\frac{y^*}{k^*}\right)^{\alpha^{-1}}
\]

\[
\frac{c^*}{d^*} = (1 + n)(1 + g) - \left(1 + q + \varphi \frac{d^*}{y^*}\right) + \frac{y^*}{d^*} - \frac{x^*}{d^*}
\]

2. For each combination of \(k_i \in [\underline{k}, \overline{k}]\) and \(d_i \in [\underline{d}, \overline{d}]\), find the corresponding value of \(c_i\) and \(x_i\) using the linear solution.

3. Form the initial values for \(\theta^c\) and \(\theta^x\)

\[
\theta^0_{\theta} = \frac{T_k^c c_i T_d}{X_k X_d^l}
\]

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\[
\theta_6^* = \frac{T_k x_d T_d}{X_k X_d'}
\]

where \( X_k = \text{diag}(T_k' T_k) \) and \( X_d = \text{diag}(T_d' T_d) \).
Appendix D

Data

D.1 Debt

The debt data are from Lane and Milesi-Ferretti (1999). They construct net foreign asset positions for 66 countries between 1970 and 1997. Their approach essentially consists in using available stock data and supplementing it with flows from balance-of-payments data. More specifically, they note that the balance-of-payments identity implies that the sum of the current account (CA), financial flows – which include foreign direct investment, portfolio equity, debt flows and capital transfers (e.g. debt forgiveness) – and the change in reserves equals zero plus net errors and omissions. The change in the value of net foreign assets thus corresponds to the sum of the current account, capital transfers and capital gains or losses on the stock of assets. The first measure used in this paper, CUMCA, corresponds to the cumulative sum of current account balances. It is available for industrial and developing countries between 1970 and 1997. The second measure NFA corresponds to the sum of stock measures of the various assets and liabilities. These measures are either cumulative flows or direct stock measures. NFA is available for developing countries between 1970 and 1997. Both measures are adjusted for debt reductions and forgiveness. In addition, these measures take into account valuations changes, such as exchange rate changes, and variations in the price of capital goods, as well as changes in stock market values.
The main difference between the two measures is the treatment of unrecorded capital flows. By cumulating current accounts, the CUMCA measure implies that unrecorded capital flows— including but over and above net errors and omissions— correspond to assets held by domestic investors abroad. On the other hand, NFA only reflects unrecorded capital outflows to the extent that they are recorded in net errors and omissions. In countries with periods of unrecorded capital flight, debt measured by NFA will tend to be larger than debt measured by CUMCA since the latter records a larger portion of unrecorded capital holdings.

The debt per worker measure used in this paper corresponds to

\[ \frac{D}{L} = \frac{D^m}{p_{US}L} \]

where \( D^m = -\text{CUMCA} \) or \( D^m = -\text{NFA} \). The debt data are measured in US dollars. To obtain a real value, they are divided by \( p_{US} \), the US GDP deflator obtained from the IMF’s *International Financial Statistics.* \( L \) corresponds to the labour force. It is measured by the population between 15 and 64 computed from output and population data from the Penn World Tables, version 6.0 as

\[ L = \frac{\text{RGDPL}}{\text{RGDPW}} \times \text{POP} \]

where RGDPL is real per capita chain GDP, RGDPW is real per worker chain GDP, and POP is total population. The dependent variable is the average annual growth rate of debt

\[ \Delta d = \frac{\log d_T - \log d_t}{T - t} \]

where \([t; T]\) is the sample period.

D.2 Output, Savings, Capital and Other Controls

The output measure is real per worker GDP from the Penn World Tables 6.0 (RGDPW). The labour force growth rate corresponds to the average annual growth rate of \( L \) computed as

\[ 1 + n = \left( \frac{L_T}{L_t} \right)^{\frac{1}{T-t}} \]
The labour force growth rate variable used in the regression is the log of \((1 + n)(1 + g) - (1 - \delta)\). I follow Mankiw, Romer and Weil (1992) and assume a growth rate of technological progress of \(g = 0.02\) and a depreciation rate \(\delta = 0.03\).

The savings rate is measured as \(1 - \frac{\xi}{\hat{y}}\) where \(\frac{\xi}{\hat{y}}\) corresponds to the average value of \(\kappa c\) between 1970 and 1997 in the Penn World Tables 6.0.

In Chapter 4, initial capital is taken from the Penn World Tables 5.6. The variable used is non-residential capital stock per worker (KAPW).

The government variable is also taken from PWT. It is the ratio of government expenditures to GDP (KG). In the regressions, \(\frac{G}{Y}\) corresponds to the log of the average government expenditures to GDP ratio between \(t\) and \(T\) divided by 100.

\(\frac{PM}{PE}\) is the average ratio of import and export prices between \(t\) and \(T\). It is taken from the IMF's International Financial Statistics.

The capital controls variable is taken from Calderón et al (2000). The index constitutes of dummies that account for the presence of current and capital account restrictions, multiple exchange rate practices and mandatory surrender of export proceeds. The variable used in the regression corresponds to the average sum of these dummies between \(t\) and \(T\). Higher values indicate more restrictive controls.

The variable on openness corresponds to the index of trade openness from Sachs and Warner (1995). It measures the fraction of years between 1950 and 1994 that the economy has been open. An open country is defined by the following criteria: (i) non-tariff barriers cover less than 40 percent of trade, (ii) average tariff rates are less then 40 percent, (iii) any black market premium was less than 20 percent during the 1970s and 1980s, (iv) the country does not operate under a communist regime and (v) the government does not monopolise major export.

The variable on political institutions is taken from Hall and Jones (1999). The original source of the data is the International Country Risk Guide which ranks 130 countries according to 24 categories. The authors construct an index on a scale of zero to one between 1985 and 1995 from 5 of these categories: (i) law and order, (ii) bureaucratic quality, (iii)
corruption, (iv) risk of expropriation and (v) government repudiation of contracts. A higher value of the index indicates institutions that support growth.

D.3 Education

The education variables are taken from Barro and Lee (1993). The measure $e$ follows Klenow and Rodriguez-Clare (1997) and assumes that primary and secondary school has an average duration of 6 years whereas higher schooling has an average of 4 years:

$$\frac{I_H}{Y} = \log \left( \frac{6 \times P + 6 \times S + 4 \times H}{16} \right)$$

where $P$, $S$ and $H$ denote the average gross enrollment rates for primary, secondary and higher schooling between 1960 and 1970.

$e$ is instrumented by the initial stock of human capital

$$\log h_0$$

where $h_0$ is the years of education per person in the population over 15 averaged between 1960 and 1965.

D.4 TFP

PWT 6.0 does not provide estimates of the stock of physical capital. To compute total factor productivity at the beginning of sample, capital per worker in 1970 is estimated using the permanent inventory scheme

$$(1 + \bar{n}) \frac{K_{t+1}}{L_{t+1}} = \frac{L_t}{L_t} + (1 - \delta) \frac{K_t}{L_t}$$

Under the assumption that capital and output per worker grow at the same rate $g$ — as they do in the model —, the initial physical capital stock is

$$\frac{K_0}{L_0} = \frac{L_0}{(1 + \bar{n})(1 + g) - (1 - \delta)}$$
where we estimate initial investment per worker as

\[
\frac{I_0}{L_0} = \left( \frac{1}{10} \sum_{t=1970}^{1980} Kt \right) RGDPW_{1970}
\]

and labour force growth as

\[
\bar{n} = \frac{\log(L_{1980}) - \log(L_{1970})}{10}
\]

The TFP measure is computed as

\[
A_0 = \log(RGDPW_0) - \alpha \log \frac{K_0}{L_0}
\]

with \(\alpha = 0.3\).

**D.5 Missing Values**

Many variables are not available for all the years and countries in the full Lane and Milesi-Ferretti database. In order to retain the largest number of countries for estimation, the sample period differs across countries from a maximum of 28 years to a minimum of 13 years. This justifies the use of annual averages for both levels and growth rates.

**D.6 Distribution**

The distributions shown in Figure 2.1 is estimated using a Gaussian kernel with bandwidth

\[
h = 1.08 \sigma n^{-\frac{1}{5}}
\]

where \(\sigma\) is the standard deviation of the sample and \(n\) is the number of observations.
D.7 Sample Composition

<table>
<thead>
<tr>
<th>Sample I (CUMCA)</th>
<th>Sample II (NFA)</th>
<th>Sample III (CUMCA) a</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina (ARG)</td>
<td>Argentina (ARG)</td>
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<td>Australia (AUS)</td>
<td>Bolivia (BOL)</td>
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<tr>
<td>Uruguay (URY)</td>
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</tr>
<tr>
<td>42 countries</td>
<td>30 countries</td>
<td>24 countries</td>
</tr>
</tbody>
</table>

aThe low-income Sample III corresponds to countries whose 1970 GDP per worker is lower than the median for that variable in that year in the whole CUMCA sample.
Appendix E

Bootstrap

The unrestricted model is

\[ \Delta d_t = \log \frac{D_t}{L_t} - \log \frac{D_{t0}}{L_{t0}} = \gamma + \beta X \]  

(E.1)

999 bootstrap samples are generated under the null hypothesis that the unrestricted model is true. Bootstrap samples are generated by re-sampling both regressand and regressors with replacement. The \( p \)-value is constructed as the proportion of test statistics (in this case, the \( t \) ratio) that are more extreme than the empirical test statistics \(^1\).