STAFF SCHEDULING AND WORKSTATION ALLOCATION
AT UBC LIBRARIES

by
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FACULTY OF COMMERCE AND BUSINESS ADMINISTRATION

We accept this thesis as conforming to
the required standard

THE UNIVERSITY OF BRITISH COLUMBIA
Mar, 2003

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Date Apr 22, 2003
ABSTRACT

Two projects that have been carried out for the UBC Libraries are the UBC Library Human Resource Project and the UBC Library Workstation Project. The UBC Libraries provide multiple services such as reference desk, circulation desk, computer and photocopiers to satisfy needs of UBC students and faculties. It was noted that utilization of the reference desks in some of the libraries was extremely variable. There was the belief that the current staffing rules were inadequate for the variation in demand that the branches experience. The UBC Library Human Resource Project was conducted at the Koerner Library, the Woodward Biomedical Library, and the David Lam Library. This project was undertaken to determine a set of rules to help the libraries to schedule the staff at the reference desks in these three libraries. A regression model, a queuing model, and a simulation model were built to analyze the demand for reference desks and derive corresponding staffing levels to achieve certain service level.

In recent years, investments in computers and new technologies have been increasing at the UBC libraries. The UBC Library Workstation Project was conducted at eleven libraries of UBC to analyze the usage of the workstations in the computer labs and different areas. Utilization analysis was carried out to determine the minimum number of workstations needed in each library to achieve certain utilization performance level. A queuing model was developed to derive the minimum number of workstations required in busy computer labs to satisfy certain waiting time service level. These rules will be used to support libraries' decision-making in workstations allocation and updating.
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Finally, I appreciate all of the encouragement from my family and friends throughout the study of the M.Sc. program.
1. **Background**

1.1.1 The UBC Library Reference Service Introduction

The UBC Library is the second largest research library in Canada and includes 21 branches and divisions at UBC and at other locations. The Library's collections are large and diverse, attracting researchers from around the world and contributing significantly to establishing UBC as a leading academic institution.

The libraries provide circulation and reference services. Circulation desks deal with charges, discharges, renewal, fines and charges. Most transactions are handled by the library staff, while a portion of the transactions such as renewals and checking requests can be done on the web by the users directly. Library staff also provides reference service, helping clients with reference questions. There are three types of questions asked at reference desk: directional questions, reference questions and research questions. Directional questions are the questions regarding locations of materials and services provided in the library. Reference questions and research questions are questions concerning looking for information in the library collections and on the web sites. In practice, reference questions are defined as questions that take less than 15 minutes to handle while research questions take more than 15 minutes to handle.

The project focused on the reference desks in the David Lam Library, the Woodward Biomedical Library and the Koerner Library. There is one reference desk in the David Lam Library and one reference desk in the Woodward Biomedical Library. In the Koerner Library there are two types of reference desks; one called the Journals and Microform Desk and the other called the Reference Desk. The Journals and Microform Desk in the Koerner Library provides reference service regarding journals, newspaper and microforms while the Reference Desk in the Koerner Library answers general reference questions.

The desks in different libraries have different open time based on different demand pattern. The David Lam Library reference desk is open from Monday to Friday with open hours from 10:00
am to 16:00 pm. The Koerner Library reference desks are open from Monday to Sunday from 9:00 to 22:00. For the Woodward Library, the open time is from 10:00 to 16:00 each weekday.

1.1.2 Problem Definition

It was noted that utilization of the reference desks in the Koerner Library, the Woodward Biomedical Library, and the David Lam Library was highly variable. The library management believed that the current staffing rules may be sub-optimum for the variation in demand that the branches experience. The library no longer wanted to use subjective methods when it comes to determine the number of staff needed in each time period, but rather needed a methodology to assist them in making the decision to achieve certain customer service levels. They wanted to know how the demand varied and how many people are needed with varying demand for service. This project was conducted to determine demand for reference desk services in the David Lam Library, the Woodward Biomedical Library and the Koerner Library and derive a set of staffing rules that satisfy a given service level for each of the reference desks in the three libraries. In the study for this project, we consider scheduling performance criteria as service level, that is, the probability that a person waits less than a certain number of minutes is greater than 85%. The library selected the service criteria since they believe a probability of 85% is appropriate to assess the performance of the reference desk.

1.1.3 Project Scope and Assumptions

The development of the models involved the following phases: analysis of demand, construction of efficient scheduling rules and verification of the scheduling rules. The staff scheduling problem can be divided into two sub-problems. The first problem is to determine the number of staff required to meet minimum service levels. Computing agent requirements is a problem well covered by queuing theory. Queuing models were established in this study. The second problem is the task of constructing efficient daily work schedules that satisfy agent requirements. In this project, constructing daily schedules was not covered because the number of people needed at each reference desk is small and because cost data and constraint data are not available for effective daily scheduling.

We assume each reference area is an M/M/s system in developing queuing models. It is assumed that historical data of demand last year applies to the next year.
1.2 Literature Review

1.2.1 Library Studies

For many service organizations, to schedule employees to match customer demand for service at different time while keeping cost under control and satisfying all applicable regulations is a challenge. There are many examples of scheduling problems although few of the examples have focused on the libraries.

Ashley (1995) explored the library staff scheduling problem. In this study, the author investigated the weekly scheduling for staffing at a reference and circulation desk at a university library. The scheduling requirement problem is formulated as a binary integer linear program. The model solves a tour scheduling problem for a work pattern of several single periods, each requiring a recovery time of at least one period. The objective is to minimize the number of unfilled slots. Requirements are expressed as ordinary constraints. Although the paper investigated the scheduling problem for the reference desk in the library, it is quite different from the UBC Library staffing scheduling problem in that the UBC Library scheduling model only determines minimum number of staff in each period, while Ashley’s model focused on assigning specific staff numbers to time slots.

1.2.2 Scheduling Problem Applications

Buffa et al. (1976) proposed a stepwise approach to employee scheduling, which is the approach employed by many researchers. Buffa et al.’s approach is as follows:

- Forecast period-by-period demand rates.
- Convert demand forecasts to period-by-period minimum employee requirements.
- Determine the set of permissible shifts and select a set of shifts that minimize labor cost, while providing at least the minimum number of employees in each time period.

1.2.3 Demand Forecasting

Most forecasting uses time series for industry applications. The COE study (Tse, 2000) in determining the optimal staffing levels at the Whistler Blackcomb Ski and Snowboard School
explores the topic of demand forecasting for the ski industry. In the study, regression models and ARIMA models was developed to predict demand for ski lessons. The models are economically compared by calculating the cost of using the forecasting model with certain service levels.

Beaumont (1997) calculated average demand in each of 20-min periods of the week across a year when they investigated staffing a workforce to meet demand that varies markedly with the time of day and moderately with the day of week. This demand forecasting is only an average of historical data.

These methods cannot be directly applied to the UBC library reference desk scheduling since very limited historical data was available for reference desks. In the UBC library study, a regression model was developed to derive the correlation between demand for circulation desk and demand for reference desk. Then demand of circulation desks was averaged for each period.

1.2.4 Queuing Model

The minimum staffing requirements are often generated using a queuing model. Scheduling problems usually fall into the category of Service Process Control in Dynamic (control) model in which the arrival rates or service rates may vary over time and there is more than one server. Servers can be added or taken away and the waiting time of customers depends on the number of servers available at different times. The typical method is to apply steady-state formulas for a M/M/s queuing system.

Agnihotri and Taylor (1991) investigated the use of M/M/s model to find the optimal staffing levels to handle the variation in call arrivals within a day. In this study, they grouped together the intervals with similar arrival rates and analyzing each time group using an M/M/s model with stationary arrival rate. First they found the system behavior within each time group should have been modeled with Poisson arrivals and mixed Erlang service time distribution. As exact results are not available for an M/G/s queue, they used M/M/s approximation in this case. They argued that that the delay probability for the M/Ek/s system is bounded by the delay probability for the M/M/s system. Whenever the difference between the required service level and the upper bound is small, M/M/s approximation provides very good results.
Some of the scheduling problems generate staff schedule shifts based on minimum staffing requirements as well as schedule constraints and staff preference. A set of shifts are selected to minimize labor cost or achieving certain aggregated service level while providing at least the minimum number of employees in each period. Integer programming or network modeling may provide an optimal solution. In this UBC library study, however, daily assignments or work shifts were not within the project scope and thus not formulated.
1.3 Methodology

1.3.1 Data Requirement

Arrival rates and service rates for the reference desks were required to develop queuing models. The option to collect data on arrival times and on service times was not available. The only data available for the reference analysis is the daily gate counts of each library, hourly circulation charge, and the monthly total of questions asked at each reference desk.

The gate count data is electronic data that counts the number of people going through the turnstile gates of each library each day. This data was available for the period of September 1, 2001 to Apr 30, 2002.

The circulation charge data was the electronic data that is collected by the computers for the circulation desk. This data consists of the total number of circulation charges at each library for each hour of the day. This data was available for the period September 1, 2001 to May 31, 2002. The data on the questions asked at each reference desk was available for the month of September 2001 through to the month of March 2002.

Because of the limitation of the data, average service time for each reference desk was assumed from a small historical sample data. Since circulation charge data were available, a regression model was developed to find if there is a relationship between demand for circulation desks and reference desks.

1.3.2 Regression Model

Since the option to collect the relevant data for the analysis was not available, an alternate approach is needed. Recall that the only data available for the reference analysis is the daily gate counts of each library, hourly number of circulation charges, and monthly totals of the questions asked at each reference desk. Because the data for reference desks were very limited, the correlation between circulation data and reference data was explored in order to determine the arrival rates at the reference desks.
Regression models were developed to determine if there is a relationship between circulation data and reference data. The data on questions asked at the reference desk includes questions asked in person and queries made by phone for each month at each desk. Staff at the reference desk has the option of putting a phone call on hold and dealing with phone queries at a more appropriate time, so questions asked by phone were assumed to not influence the service time and queue length at the desks. As a result, the number of questions asked in person was the only data of this type that was considered in the regression model. The data on the tallies of questions asked at each of the circulation desks is grouped per month. Hence, gate counts and the circulation charges were each summed for all months and these totals were used in the regression analysis.

Two types of models were considered. The first model is comprised of the number of questions asked at each reference desk and the number of circulation charges. The former is the dependent variable and the latter is the independent variable. The linear regression model is:

\[ y = \beta_0 + \beta_1 x_1 + \epsilon \]

where \( \epsilon \) is unobserved random variation.

The second model has the number of questions asked at each reference desk as the dependent variable with both the gate counts and circulation charge as the independent variable. The multiple regression model is:

\[ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon \]

where \( \epsilon \) is unobserved random variation.

See Section 1.4.1 for the relationship between reference demand and circulation demand.

1.3.3 Periods, Arrival Rates and Service Time

The next step in the analysis was to determine which periods have similar demand. It was found from the analysis of circulation desks that demand is variable, not only throughout the year, but also throughout the week and throughout the day, with some timeframes experiencing similar demand. In the project on the analysis of circulation desks, the similar time periods were grouped
together to be used for the staffing rules. There were three timeframes that were considered for the groupings of periods. The first grouping consisted of similar months, the second grouping consisted of similar days of the week, and the third grouping consisted of similar hours of the day, for each of the libraries independently. Due to the correlation between circulation desk and reference desk, it is assumed that the reference desk has the same time periods with the corresponding circulation desk.

The 18 time periods for each library were provided by the circulation desk project. Each time period is comprised by three variables: month (low, moderate, peak), day (low, peak) and hour (low, moderate, peak). Each element of a period has similar demand, corresponding to a specific arrival rate at the desk. A summary of the period combinations can be found in Appendix A.

The next step was to determine the arrival rate for each reference desk for each period combination. For each period combination, the average number of charges for each month was calculated. The average number of questions asked hourly at each reference desk in each period was then calculated using the relationship between circulation charges and questions asked at the reference desk that was found in the regression analysis. Let $y =$ Monthly number of questions asked at reference desk, $x =$ Monthly number of circulation charges, the fitted model for the Woodward Biomedical Library is:

$$
\hat{y} = 0.378x
$$

The relationship derived from the regression model is the number of questions asked at the reference desk is equal to 0.378 times the number of circulation charges at the Woodward Biomedical Library. The average arrival rate for each reference desk in each period corresponds to the number of questions asked at the reference desk in each period, so we have:

Arrival Rate for each period = $0.378 \times$ Circulation Charges for each period

For each reference desk, a small sample of data of around 100 to 200 arrivals was collected by observation over a span of three days. Average service time for each was assumed to be static.

1.3.4 The Queuing Model

Queuing theory was used to determine the minimum number of staff needed at each reference desk for each period with similar demand to meet a particular service level.
Both service time and inter-arrival time in the queuing model were assumed to be exponentially distributed. This assumption allows the queuing system to be modeled as an M/M/s queue. The service rate for each desk was assumed to be static while arrival rates vary with different time periods for each desk. For each desk across all period combinations, analysis was done to determine the mean time in queue and the mean time in system, with a certain number of staff at a desk. Also, probabilities that a client will wait less than a certain number of minutes with a certain number of staff at the desk were calculated. The Queuing Tool-Pak software (Armann Ingolfsson, Fraser Gallop, 2002) was used to do the queuing analysis.

At UBC Libraries, several libraries may occupy a single reference desk. Assume each desk is an M/M/M/s system with inter-arrival time and service time exponentially distributed and with s staff, representing s servers. The problem can be formulated as follows:

Let:
- $\lambda$ represent the arrival rate
- $\mu$ represent the service rate
- $k$ represent number of clients in system
- $s$ represent number of staff in system
- $\pi_k$ represent the probability that $k$ people are in the system in steady state

Then:

$$
\pi_0 = \left\{ \sum_{j=0}^{s-1} \frac{1}{j!} (\frac{\lambda}{\mu})^j + (\frac{\lambda}{\mu})^s \right\}^{-1}
$$

$$
\pi_k = \frac{1}{k!} (\frac{\lambda}{\mu})^k \pi_0 \quad \text{for } k = 0,1, \ldots, s,
$$

$$
= \frac{1}{s! \mu} (\frac{\lambda}{\mu s})^{k-s} \pi_0 \quad \text{for } k \geq s
$$

$L_0$ is the mean number of customers in the system waiting for service, then

$$
L_0 = \frac{\pi_0}{s!} (\frac{\lambda}{\mu})^s \frac{\frac{\lambda}{s\mu}}{(1 - \frac{\lambda}{s\mu})^2}
$$

$L$ is the mean number of customers in the system, then

$$
L = L_0 + \frac{\lambda}{\mu}
$$
Average waiting time in queue $W_0$ is given by

$$W_0 = \frac{L_0}{\lambda}$$

Let $T_q$ denote the random variable "time spent waiting in the queue" and $W_q(t)$ denote its cumulative probability distribution, that is, the probability that time spent waiting in the queue is less than $t$ minutes. $W_q(0)$ is the probability that people don’t wait for service.

$$W_q(t) = 1 - \frac{s(\lambda/\mu)^s}{s!(s - \lambda/\mu)} \pi_0$$

$$= \frac{(\lambda/\mu)^s (1 - e^{-(\mu - \lambda)})}{(s - 1)! (s - \lambda/\mu)} \pi_0 + W_q(0) \quad (t > 0)$$

Thus, given the arrival rates, service rates, and the number of staff in the system, the average number in the queue, the average waiting time, and the probability that a person waits less than a certain number of minutes can be determined from the M/M/s model.

In this project, the service level is defined as the probability that a client waits less than 2 or 5 minutes is greater than 80%. Given the desired service level, the minimum number of staff needed at each reference desk was determined for each period. Then a set of staffing rules for each time period was set up for each reference desk. See Section 1.4.2 for an example of a staffing rule.

1.3.5 Simulation Model

To determine a schedule to achieve a specific service level for each defined period, queuing theory was used. Since arrival rates are non-stationary, it is more useful to assess aggregated service levels. For this assessment, a simulation model was built to simulate each reference desk as a non-stationary queuing system throughout a day, using the staffing schedule that was determined with queuing theory. Each day is comprised of certain period combinations. Arrival rates corresponding to these period combinations were input into the models. Again, inter-arrival time and service time were assumed to be exponentially distributed. The distribution of average
waiting time of a client for each day-structure was found and analyzed. See Section 1.4.3 for results of simulation.

In terms of reference desk traffic, each desk experiences 6 different types of days. These days are dependent of the period classification of the month (low, moderate, peak) and the day (low, peak). For each reference desk, the hour structure is fixed for each day. Table 1 is an example of the six day-structure.

<table>
<thead>
<tr>
<th>Structure</th>
<th>Month</th>
<th>Day</th>
<th>Hour</th>
<th>Time-frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Low</td>
<td>Low</td>
<td>Moderate</td>
<td>10:00</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>Low</td>
<td>Peak</td>
<td>11:00 – 15:00</td>
</tr>
<tr>
<td>2</td>
<td>Low</td>
<td>Peak</td>
<td>Moderate</td>
<td>10:00</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>Peak</td>
<td>Peak</td>
<td>11:00 – 15:00</td>
</tr>
<tr>
<td>3</td>
<td>Moderate</td>
<td>Low</td>
<td>Moderate</td>
<td>10:00</td>
</tr>
<tr>
<td></td>
<td>Moderate</td>
<td>Low</td>
<td>Peak</td>
<td>11:00 – 15:00</td>
</tr>
<tr>
<td>4</td>
<td>Moderate</td>
<td>Peak</td>
<td>Moderate</td>
<td>10:00</td>
</tr>
<tr>
<td></td>
<td>Moderate</td>
<td>Peak</td>
<td>Peak</td>
<td>11:00 – 15:00</td>
</tr>
<tr>
<td>5</td>
<td>Peak</td>
<td>Low</td>
<td>Moderate</td>
<td>10:00</td>
</tr>
<tr>
<td></td>
<td>Peak</td>
<td>Low</td>
<td>Peak</td>
<td>11:00 – 15:00</td>
</tr>
<tr>
<td>6</td>
<td>Peak</td>
<td>Peak</td>
<td>Moderate</td>
<td>10:00</td>
</tr>
<tr>
<td></td>
<td>Peak</td>
<td>Peak</td>
<td>Peak</td>
<td>11:00 – 15:00</td>
</tr>
</tbody>
</table>

Table 1: Periods day-structure for David Lam Reference Desk

In the simulation model, six day-structures were developed for each desk, each structure with corresponding arrival rates and staff number for that period combination. All day-structures were replicated for 2 years. For each reference desk, the average waiting time of clients for each day-structure was collected in the simulation. The 95% lower limits and 95% upper limits of the average waiting time for each day-scenario were also calculated.
1.4 Results

1.4.1 Regression Relationships

It was noted from the regression analysis that gate count data and circulation charge data should not be used simultaneously as independent variable in the regression models, as there is high correlation between the two variables. Considering that gate count data is daily data and would not be practical in deriving the hourly arrival rates, the circulation charge data was selected as the only independent variable in the regression models.

The regression models indicated that there was indeed a linear relationship between the number of questions at a reference desk and the corresponding number of circulation charges. The relationship between the two variables was derived for each of the reference desks. T-tests show that the coefficient of the regression model is significantly different from zero for each reference desk.

The regression results are summarized in Table 2 below. The t-statistics for the coefficients are shown.

<table>
<thead>
<tr>
<th>Desk</th>
<th>Unstandardized Coefficient</th>
<th>Standard Error</th>
<th>t-statistic</th>
<th>P-value</th>
<th>R Squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>Koerner Journal &amp; Microform</td>
<td>0.06227</td>
<td>.004</td>
<td>15.013</td>
<td>&lt;.001</td>
<td>0.974</td>
</tr>
<tr>
<td>Koerner Reference</td>
<td>0.119</td>
<td>.009</td>
<td>13.861</td>
<td>&lt;.001</td>
<td>0.970</td>
</tr>
<tr>
<td>Woodward Reference</td>
<td>0.378</td>
<td>.020</td>
<td>18.824</td>
<td>&lt;.001</td>
<td>0.983</td>
</tr>
<tr>
<td>David Lam Reference</td>
<td>2.363</td>
<td>.090</td>
<td>26.162</td>
<td>&lt;.001</td>
<td>0.991</td>
</tr>
</tbody>
</table>

Table 2: Linear relationship between reference question and circulation charge

For each desk, the number of reference questions is equal to the coefficient times the number of circulation charges. The coefficients show that these desks differ in the relationship between
number of reference questions and circulation charges. The coefficient for the Koerner Journal &
Microform desk is the smallest, indicating relative small demand in Journal and Microform. The
coefficient of David Lam Library is the biggest. This suggested that average user in Commerce
might have more research related questions than users in other libraries. It is assumed that the
relationship between the monthly number of circulation charges and the monthly number of
questions asked at reference desk can be applied to any hour and any time period.

1.4.2 Staffing Rules and Sensitivity Analysis

The queuing models indicated the probability that a person waits less than certain number of
minutes with certain number of staff in the system. For each desk, two sets of staffing standards
were determined for each period. The first set is the minimum number of staff needed to ensure
that the probability that a client waits less than 2 minutes is greater than 85%, while the second
set is the minimum number of staff needed to ensure that the probability that a client waits less
than 5 minutes is greater than 85%. See Table 3 for scheduling rules derived. Compared to a full
schedule, this schedule can help the UBC libraries save costs by having less staff in service in less
busier time and better accommodate clients’ needs in peak time.
Table 3: Number of staff required at each reference desk as a function of type of period for queue service criteria
For the David Lam Library, the results seemed overestimated. It is speculated that the reason may be that the assumed average service time for the analysis is not an adequate representative of the queuing system. It is also suspected that some questions might be answered by circulation desk, which is very close to reference desk in the library. Since real data was not accessible in the project, sensitivity analysis was done to determine how changes in arrival rates and service time would affect the staffing rules. The service level defined in the sensitivity analysis is the probability that a client waits less than 2 minute is greater than 85%. The queuing tool package was again used to derive the minimum number of staff required to achieve this service level. The analysis indicates the minimum number of staff needed at a desk for known arrival rate and service time. If there is change in the arrival rate or the service time, one can find the resulting minimum number of staff needed at the desk to meet this service level simply by looking at the intersection of corresponding arrival rate and service time. The sensitivity analysis table is applicable to all the desks in the three libraries. See Appendix B.

Compared to what was used previously in practice, this set of schedule rules will result in potential cost reduction by reducing the number of staff needed in some periods. More importantly, it better accommodates the clients’ needs in peak periods by matching demand with the number of staff at the desks.

1.4.3 Simulation Verification

The scheduling rules from the queuing models are used to determine the minimum number of staff needed at a desk to achieve certain service level at each period. The simulation models were developed to assess the average waiting time of clients for each day. The minimum number of staff needed to ensure that 80% of the clients wait less than 5 minutes in each period that were found in the queuing analysis were used as the staff level in the simulation model.

It was found from the simulation that for the Woodward Biomedical Library and the Koerner Library Journals & Microforms Desk, the average waiting times in all 6 types of days are all less than 1 minute or very close to 1 minute. For David Lam Library, the average waiting times are below 2 minutes. For Koerner Library Reference Desk, the average waiting times for the scenarios are between 1 minute and 2 minutes. The results verified that the staffing schedule established using queuing theory meet the specified service level.
For each desk, the distribution of average waiting times of a client for each of the 6 types of days was found and plotted. It was noted that for almost all 6 types of days for all desks, the relative frequency that the average waiting time of one typical day is less than 2 minutes is between 80% and 90%. The simulation results suggest that the staffing schedule derived from the queuing model can achieve the defined service level.
1.5 Summary and Areas for Further Research

In this project, we have established a regression model to derive the arrival rates of reference desks using the relationship between circulation desk and reference desk, a queuing model to set up the minimum number of staff needed at each reference desk for each period while achieving certain service level. Finally, a simulation model was built to assess the average waiting time in each typical day given the scheduling rules found by the queuing theory. The project has provided the library with a useful tool to assist their decision-making in staff scheduling.

The queuing model was set up as a M/M/s model. Inter-arrival time and service time were assumed to be exponentially distributed. However, the exponential assumptions may be rather limiting, especially the assumption concerning service times being distributed exponentially. There is lost demand due to balking. We could expand the model to more general model and do more research on applying more complicated queuing theories.

A possible expansion of the project is to generate staff schedule shifts based on minimum staffing requirements. There are schedule constraints and staff preference for shifts. We could take these factors into consideration and formulate an integer programming model. A set of shifts could be derived to minimize cost or achieve certain aggregated service level while meeting operational constraints.
II. UBC LIBRARY WORKSTATION ALLOCATION

2.1 Background

2.1.1 Introduction

A study of workstation allocation was also conducted for the UBC Libraries. In this study, workstation is defined as a personal computer for people stay to work on. The library uses this term to distinguish this type of personal computer with the more powerful central servers. With the growth of UBC library technology, the number of workstations in the libraries increased from about 800 in 1997 to over 1000 in 2002. Between 1997 and 1999, there is significant increase of the number of workstations due primarily to the implementation of several instructional and public computer labs in Koerner, Main and Woodward Libraries. UBC Library owns over 1000 workstations located in, 4 computer machine rooms. 600 of these workstations are available for public use with the remaining 400 are installed for staff use.

Workstation software has continued to evolve and increase at the same time. Workstation operating systems have been regularly upgraded to remain reasonably current with industry trends. For the application software, most public workstations have installed email access and internet explorer. In 2002, the MS software suite was added to the public workstations in the Chapman Learning Commons in Main Library and the E-space Lab in Koerner Library. In the coming year it is likely MS Office will be installed on more public workstations.

Since MS office software cannot be installed on relatively old machines, the libraries' current policy is to invest on new machines in heavily used areas and shift older machines to less busy areas. On some low traffic floors, only a few machines are kept to facilitate usage.

2.1.2 Problem Definition

The increasing investments in workstations and workstation software have prompted an initiative at UBC Library management to review resource allocation. They recognized the need to understand the utilization of workstations in various libraries. The objective of the project is to
analyze the usage of the workstations and determine the number of machines needed in each library to support UBC Library management decision-making in workstations allocation and updating.

2.1.3 System Performance Measure

The project was first undertaken in three of UBC's libraries: the Koerner Library, the Law Library, and the Main Library. Then some analysis was extended to other libraries: Asian Library, Biomedical Branch Library, David Lam Library, Education Library, Hamber Library, MacMillan Library, Robson Square, St. Paul's Hospital Library and Woodward Biomedical Library.

In low traffic libraries, people seldom need to wait to use the computers. The percentage of time there are no people waiting in the system was set up as the criteria to assess the service level for these low traffic libraries. The minimum number of computers was determined for each library each month by analyzing the utilization of workstations. For heavy traffic libraries or busy labs in certain libraries, the probability that a person waits less than certain minutes was used as system performance measure. A queuing model was set up to determine the minimum number of machines for each system. Since arrival rates were unknown, some research was conducted to derive the unknown parameter in queuing model using utilization data.
2.2 Methodology

2.2.1 Data Requirement

The data available is the log-in and log-off time of each user at each workstation in all the libraries from Dec 2001 to Dec 2002. For the Main Library Learning Commons Lab, the data is from Feb 2002 to Dec 2002. There are 100,000 to 650,000 records for each library. The data was provided in text files, showing IP address and workstation name for each record. From the IP address and workstation name, the location of the workstation can be determined. Guest stations were excluded from the analysis since they don’t have log-in and log-off data.

2.2.2 Service Time

The data was input into Microsoft Access for analysis. Each log in and log-off time was paired and the service time was derived for each user by calculating the difference between log-off time and login time. Service time that exceeded 420 minutes was removed from the analysis to improve the accuracy since some users don’t log off when they leave the computer. 420 minutes is the cut-off criteria used because it is believed that is the longest time people would work at one time. Visual Basic code was developed to recognize each pair of log in and log-off time by giving each pair an identification number. Data without corresponding log-in or log-off time was recognized and removed. See Appendix C for the Visual Basic Code.

The workstations were classified according to libraries and labs in the libraries. Average service times were grouped according to different workstations, different months, weekdays and hours. The analysis indicated which areas, which time periods and which machines have higher utilization. See Section 2.3.1 for the results.

2.2.3 Utilization Analysis

The service time analysis showed that some libraries are much busier than others. The percentage of time that a specific number of people was logged in in each system was computed. This was done by using queries to derive the number of log-ins for each minute in the whole year for each workstation group and then computing the percentage. In Microsoft Access, a table for each
minute in one year was created and used in the queries. See Appendix D for Visual Basic code used to create this table. See Section 2.3.2 Table 5 for an example of a utilization table.

It was found that for many libraries the number of people who are logged in the system was always less than the number of machines in the systems, indicating that there are never people waiting for computers in these libraries. For these less busy libraries, the service criteria is defined to ensure 85% or 90% of time there is no queue in the system.

The cumulative percentage was also calculated which indicates the percentage of time there is no greater than a certain number of people logged in the system. In other words, this number of machine in the system can ensure that there are no people waiting for the percentage of time. Given the service requirement that 85% or 90% of time there is no queue in the system, the minimum number of machines for each system was determined. See Section 2.3.2 for an example of a cumulative table and the recommended number of machines.

2.2.4 Queuing Model

For a busy system, service time would be a better criterion to assess the system performance than the percentage of time there is no queue. In this study, the service level for busy system was defined to ensure that the probability that a person waits less than 10 minutes is greater than 90%. A queuing model was developed to derive the optimal number of machines for the busy systems. We assume a M/M/s system for the queuing model. Both service time and inter-arrival time in the queuing model were assumed to be exponentially distributed.

Service times were grouped according to month, weekday, and starting hour. High and low demand for different month, weekday, and starting hour was identified. Periods with similar demand were defined as a combination of month (low, peak), weekday (low, peak), and hour (low, middle, peak).

Average service time was calculated for each period in each system. In the periods without queues in the system, arrival rates are equal to log-in rates. In the periods with people waiting in the system, arrival rates are unknown. When both arrival rates and service rates are known, the queuing model is the same as that used in the Human Resource studies. (See Section 1.3.4)
Assuming that arrival rates are proportional to log-in rates, the minimum number of machines was determined to ensure that the probability that a person wait less than 10 minutes is greater than 90% for each busy area. See Section 2.3.3 for the results.

2.2.5 Estimating Unknown Parameter in a Queuing Model

When arrival rates are unknown, the methodology below was developed to investigate if the unknown parameter in the queuing model can be estimated using the observed utilization data. Estimating arrival rate is the same as estimating the ratio of arrival rate to service since an estimate of the service rate is available from the existing service time data.

Assume the system is an M/M/s system

Let

- $\lambda$ represent arrival rate
- $\mu$ represent service rate
- $k$ represent number of people in system
- $s$ represent number of servers in system
- $\pi_k$ represent the probability that $k$ number of people are in the system
- $\rho = \lambda / \mu$

The probability that $k$ number of people are in the system in the long run is:

$$
\pi_0 = \sum_{j=0}^{s-1} \frac{1}{j!} (\rho)^j + \frac{\rho^s}{s!(1 - \rho / s)}
$$

$$
\pi_k = \frac{1}{k!} (\rho)^k \pi_0 \quad \text{for } k = 0, 1, ..., s
$$

$$
= \frac{1}{s!} \rho^s \left( \frac{\rho}{s} \right)^{k-s} \pi_0 \quad \text{for } k \geq s
$$

When the number of people in service is less than the number of machines, the number of people who are logged-in equals the number of people in the system. Thus, we have observed $\pi_k$ by computing the probability that $k$ people are logged in the system for $k = 0, 1, ..., s - 1$.  

22
The objective is to find the $\rho$ which minimizes the difference between the observed $\pi_k$ and the $\hat{\pi}_k$ derived from the queuing formula.

Three criteria were tested and the results were compared.

1) Sum of squares method

$\rho$ was selected to make the sum of the squared differences as small as possible.

$$\min \sum_{i=0}^{k-1} (\pi_i - \hat{\pi}_i)^2$$

Since this measurement is sensitive to extreme value, the other two measurements were also tested.

2) Sum of relative squares method

The objective is to make the sum of the relative squared error as small as possible. The measure is less influenced by extreme values than the squared error measurement.

$$\min \sum_{i=0}^{k-1} ((\pi_i - \hat{\pi}_i)/\pi_i)^2$$

3) Sum of $\chi^2$ square method

The objective is to make the sum of $\chi^2$ statistic as small as possible. The basis of the measure is Chi-square goodness of fit.

$$\min \sum_{i=0}^{k-1} (\pi_i - \hat{\pi}_i)^2 / \pi_i$$

See Section 2.3.5 for the results. Arrival rates can be derived by multiplying service rates by $\rho$. 


2.3 RESULTS

2.3.1 Demand Pattern

The analysis shows that the average monthly service time per workstation in Main Learning Commons Lab and Koerner E-space Lab are the biggest among all the libraries and library labs. The two labs have workstations with Microsoft office software installed, indicating a high demand for office software. See Table 4. The libraries have varied hours of operation. Most libraries are open from 8am to 10pm Monday to Thursday, 8am to 6pm on Friday, 10am to 6pm on Saturday, and 12 noon to 10pm on Sunday.

The analysis also shows that workstations have lower usage on Monday and Sunday than on other days of the week. From a monthly perspective, May, June, July and August have very low demand; February, March, September, October, November generally have high demand while April and December have intermediate demand. From an hourly perspective, 10:00 am to 3:00 pm have the highest usage. For Koerner Library, Main Library and Law Library, average service time for each workstation was computed to understand the usage of each workstation.
2.3.2 Recommended Number of Workstations for Less Busy Libraries

The service time analysis shows except Main Learning Commons Lab and Koerner E-space Lab, most libraries and areas are not busy and have similar average service time. The utilization analysis shows that for most libraries the number of people logged on is less than the number of machines at almost each point of time. See Table 5 for an example of a utilization table, which shows the percentage of time each number of people is logged on into the system.
Table 5: Utilization of workstations in Koerner BC-Hydro Lab

The cumulative utilization tables were produced by computing cumulative percentage, indicating the percentage of time there are no people waiting in the system with certain number of machines in the system. See Table 6 for the cumulative utilization table for the Koerner Library BC-Hydro Lab.
workstations derived from the utilization analysis are believed to accommodate users' needs for
in each library to satisfy users' needs for Internet search and email and invest in new machines
below. There are 21 workstations
the Koerner BC-Hydro Lab are summarized in
minimum number of machines for each month was recommended for each library. The results for
90%
85%
Table 6: Cumulative utilization table for Koerner BC-Hydro Lab
Given the service level that 85% and 90% of time there are no people waiting in the system, the
minimum number of machines for each month was recommended for each library. The results for
the Koerner BC-Hydro Lab are summarized in Table 7 below. There are 21 workstations
currently in this area.

Table 7: Minimum number of workstations to achieve utilization criteria

Since MS office software was not installed at these less busy libraries, the minimum numbers of
workstations derived from the utilization analysis are believed to accommodate users' needs for
Internet search and email. The UBC libraries would like to keep a minimum number of machines
in each library to satisfy users’ needs for Internet search and email and invest in new machines
with MS office software.

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<td>13</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>11</td>
<td>4</td>
<td>8</td>
<td>5</td>
</tr>
</tbody>
</table>
Since it might not be practical to vary the number of machines every month, it is suggested the libraries use the average of the minimum numbers of workstations needed for peak, middle, and low demand months separately. See Section 2.3.1 for general demand pattern of different months.

It was observed that queues tend to accrue in big lab rooms rather than other areas in the library probably because of the former's comfortable spaces and study tables. Thus it is recommended that relatively new surplus machines be put together in lab rooms with sitting space provided to users. MS office software or specific application software targeting specific user groups may be installed in these machines.

In low demand months, especially in summer, there are much more surplus machines. The libraries could consider renting some computers to students to improve machine utilization.

2.3.3 Recommended Number of Workstations for Busy Libraries

For the Main Learning Commons and the Koerner E-space Lab, a queuing model was established to derive the minimum numbers of workstations needed to ensure that probability that a person waits less than 10 minutes is greater than 90%.

Periods were defined with different month (low, peak), day (low, peak) and hour (low, middle, peak) according to service time analysis. The Table 8 below gives periods of Main Learning Commons Lab for an example.
Table 8: Classification of periods of Main Learning Commons Lab

Since it is assumed that the libraries allocate the workstations on a monthly base, periods of “Month (low) Day (peak) Hour (peak)” and “Month (peak) Day (peak) Hour (peak)” were picked to derive the number of machine required to achieve the defined service level each month. Figure 1 and Figure 2 show the growth trend of workstations in the two labs if arrival rate of each period was assumed to be proportional to log-in rate. A 20% increase of log-in rate will result in a required increase of 1 or 2 workstation for Koerner E-Space lab and 4 or 5 workstations for the Main Learning Commons Lab. Although the two labs may not have the capacity to hold many more machines, the numbers are believed to be indicative of the future investment in computers with office software in other areas of the two libraries. There are 28 workstations currently in the Main Learning Commons Lab and 9 workstations in the Koerner E-Space Lab.
2.3.4 Estimating $\rho$

The queuing model to estimate $\rho$ was developed using the Main Library Learning Commons Lab data. The optimal $\rho$ was found by minimizing the sum of difference between observed $\pi_k$ and predicted $\pi_k$. Three measurements for estimating the total difference of $\pi_k$ were used and the results were compared.
Figure 3: Estimated $\rho$ from project data

The figure shows estimated $\rho$ using three different criteria compared with observed Login Rate/Service Rate in each of 11 periods. The periods were grouped in three different traffic groups by an ascending order of observed Login Rate/Service Rate. Table 9 below depicts the 11 periods with corresponding Login Rate/Service Rate in an ascending order. We can see that there are apparent differences between the Login Rate/Service Rate in period 3 and period 4, and the rate in period 7 and period 8. Thus, periods 1 to 3 were defined to be Light Traffic, periods 4 to 7 Middle Traffic, and periods 8 to 11 Heavy Traffic.

<table>
<thead>
<tr>
<th>Periods</th>
<th>Month</th>
<th>Day</th>
<th>Hour</th>
<th>Login Rate/Service Rate</th>
<th>Difference Between Periods</th>
<th>Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Low</td>
<td>Low</td>
<td>Moderate</td>
<td>1.69</td>
<td>0</td>
<td>Light Traffic</td>
</tr>
<tr>
<td>2</td>
<td>Low</td>
<td>Peak</td>
<td>Low</td>
<td>4.27</td>
<td>2.58</td>
<td>Light Traffic</td>
</tr>
<tr>
<td>3</td>
<td>Peak</td>
<td>Low</td>
<td>Low</td>
<td>5.60</td>
<td>1.34</td>
<td>Middle Traffic</td>
</tr>
<tr>
<td>4</td>
<td>Peak</td>
<td>Low</td>
<td>Moderate</td>
<td>9.23</td>
<td>3.62</td>
<td>Middle Traffic</td>
</tr>
<tr>
<td>5</td>
<td>Peak</td>
<td>Low</td>
<td>Moderate</td>
<td>9.45</td>
<td>0.22</td>
<td>Middle Traffic</td>
</tr>
<tr>
<td>6</td>
<td>Low</td>
<td>Peak</td>
<td>Moderate</td>
<td>10.89</td>
<td>1.44</td>
<td>Middle Traffic</td>
</tr>
<tr>
<td>7</td>
<td>Low</td>
<td>Low</td>
<td>Peak</td>
<td>12.91</td>
<td>2.02</td>
<td>Heavy Traffic</td>
</tr>
<tr>
<td>8</td>
<td>Peak</td>
<td>Peak</td>
<td>Moderate</td>
<td>17.23</td>
<td>4.32</td>
<td>Heavy Traffic</td>
</tr>
<tr>
<td>9</td>
<td>Peak</td>
<td>Low</td>
<td>Peak</td>
<td>18.62</td>
<td>1.39</td>
<td>Heavy Traffic</td>
</tr>
<tr>
<td>10</td>
<td>Low</td>
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<td>Peak</td>
<td>18.75</td>
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<td>Heavy Traffic</td>
</tr>
<tr>
<td>11</td>
<td>Peak</td>
<td>Peak</td>
<td>Peak</td>
<td>22.03</td>
<td>3.28</td>
<td>Heavy Traffic</td>
</tr>
</tbody>
</table>

Table 9: Traffic Groups Definition
Comparing the results of the three criteria, we have the following findings:

1. In estimating the optimal $\rho$, the method of minimizing Sum of Relative Squared Error and the method of minimizing Sum of Chi Squared Error get similar results while minimizing squared error appears different.

2. When the system is in light traffic, Login rate/Service rate should be close to real $\rho$. The Sum of Squared Error tends to significantly underestimate $\rho$ while the other two methods tend to significantly overestimate $\rho$. The reason might be that in light traffic the data set is too small to obtain percentage of time a certain number of people in the system in the long run.

3. When real Login Rate/Service Rate equals 12.91, the estimate of $\rho$ by the minimizing Squared Error method is extraordinary small. It was suspected this method is not stable in estimating $\rho$ when the assumption of M/M/s does not strictly hold.

4. When traffic is in light, there might still be an effect of insufficient data. When traffic gets heavier, the estimated values get closer to Login Rate/Service Rates. Since real $\rho$ is larger than Login Rate/Service Rate when system is busy and we don’t know how much more arrival rates are larger than login rates, it is unclear if these methods work well in estimating $\rho$.

There are some obstacles that make it difficult to assess these methods in estimating $\rho$ in this project.

1. The assumption that the workstation system is M/M/s might not hold. The inter-arrival times and service times may not be exponentially distributed.

2. When system is idle, observed $\pi_k$ might not be accurate because the data set might be too small.

3. When arrival rates are too high and there are long queues, balking occurred in the libraries, which is not captured by the simple M/M/s model. The lab has lots of informal sitting space with students studying or waiting to use computers, making it difficult to use M/M/s/k model because of the uncertainty of k. This also makes the estimation of $\rho$ inaccurate.

4. The unavailability of observed $\pi_k$ when $k \geq s$ (the number of machines in the system) tends to underestimate $\rho$. 

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2.3.5 Simulation Results

To overcome the obstacles, a simulation model was set up using Arena Software to produce all \( \pi_k \) by simulating an M/M/s system with s equal to five. In the simulation, the input of \( \rho \) is less than 5 to avoid infinite queues. The average inter-arrival time was defined as 5 minutes. To represent different traffic levels, the average service times were defined as 2, 15, 16.5, 20, 22 minutes with \( \rho \) equals 0.4, 3, 3.5, 4, 4.5 respectively \( (\rho < 5) \). The simulation was run for one year with around 105,000 arrivals for each \( \rho \). It is assumed that there is no balking.

The three measurements were used to estimate \( \rho \) in two experiments. One experiment used all \( \pi_k \) while the other experiments only used \( \pi_k \) when k is less than s.

When all \( \pi_k \) were used to estimate \( \rho \), see Figure 4 as follows:

![Figure 4](image)

**Figure 4**: Estimated \( \rho \) from simulated data using all \( \pi_k \)

The three methods work well in estimating \( \rho \) in almost all cases when all \( \pi_k \) are used. Performance of Min \( \chi^2 \) method appears more stable than the other two methods. When only \( \pi_k \) \( (k<s) \) were used to estimate \( \rho \), see Figure 5.
The three methods work well in estimating \( \rho \) when traffic is very light. When traffic gets heavier, the value of \( \rho \) appear to be underestimated due to the unavailability of \( \pi_k \) when \( k \) is larger than \( s \). It seems that estimated values are close to the value in heavy traffic.

In conclusion, if the system is a strict M/M/s and there are enough data points, the methodology to minimize sum of difference between observed and estimated \( \pi_k \) is acceptable to estimate \( \rho \) if all \( \pi_k \) are known. If only \( \pi_k \) (k<s) are known, the methodology tends to underestimate \( \rho \). It seems in case of heavy traffic, the performance of the methodology is better than middle traffic.

It is not obvious which measure of minimizing sum of errors is better as model objective. It seems the performance of Min \( \chi^2 \) method is more stable than other two. We may consider combining these measurements to produce an estimate for \( \rho \). One type of combination that is frequently employed is the weighted average method. The optimum weights for the three measurements could be investigated in future research. However, the methodology could not estimate \( \rho \) when arrival rate is very high and there is lost demand due to balking.
2.4 Summary and Areas for Further Investigation

In this project, workstation usage in different libraries and labs was analyzed. Libraries were differentiated based on how busy they were. The minimum number of machines required in each library was determined to satisfy different service performance levels. Workstation utilization was analyzed for all libraries while a queuing model was developed for heavy traffic library labs. The UBC Library Management believe the optimal numbers of machines indicated by the study will be very helpful in assisting them to allocate workstations and invest new workstations. They extended the study from initial three libraries to almost all the UBC libraries with public workstation services.

There are several areas that worth further investigation. The queuing model research indicates that the unavailability of $\pi_k$ when the number of people in the system is larger than the number of machines would make the methodology inaccurate in estimating $\rho$. Further investigation to quantify the effect is necessary.

Further, the present queuing model compared three objectives of minimizing estimation errors. More measurements may be introduced in further studies. The weighted average of different methods could be explored to obtain more reasonable values.

In this study, queries were made in Microsoft Access to derive the number of log-in in each minute. Since there are hundreds of thousands records in busy library, it takes long time to run the queries. It is possible to further reduce the running time by improving the structure of queries in future studies.
III. CONCLUSION

The focus of the study of the UBC Library Human Resource is to develop a set of scheduling rules that can be used at the reference desks in three of the UBC libraries. The goal is to determine the minimum number of staff needed at each of the reference desks in different periods to achieve the service level that the probability that a person waits less than a certain number of minutes is greater than 85%. In order to solve this problem efficiently, a queuing model was developed to derive the schedule rules and a simulation model was developed to assess the effects of these rules.

The focus of the study of the UBC Library Workstation Allocation is to analyze the usage of the workstations in each library to support UBC Library management decision-making in workstation allocation and updating. For less busy libraries, the goal is to determine the minimum number of workstations needed at each library to satisfy the service criteria that the percentage of time that no people wait for service is greater than 85% or 90%. For busy labs, the goal is to determine the minimum number of workstations needed to achieve the service level that a person waits less than 10 minutes is greater than 90%. Utilization analysis and a queuing model were set up to derive the number of machines. A methodology to derive the unknown parameter \( \rho \) using existing data was investigated in a queuing model and a simulation model.

The main similarity of the two studies is that an M/M/s queuing model was used in both studies to determine the minimum number of people or machines to achieve a certain waiting time criteria. The main differences of the two studies are that the workstation study also has utilization analysis to derive the number of machines to satisfy utilization criteria and that a methodology was developed to investigate how to estimate an unknown parameter in queuing theory.

These studies will help the UBC libraries in their decision-making in scheduling and allocating resources. The UBC Libraries believe that the workstation study is the first time some meaningful data was generated on a library-wide basis. This study has a potential to be extended to future analysis after the libraries make some new investment and reengineering.
REFERENCES


Arena 6.0 (2001-2002, Rockwell Software Inc.).


A. Summary Of Period Combinations

Depicted is a summary of the period breakdowns for each branch that was found in the Circulation Desk Project. Down the left hand side is the timeframe and across the top is the level of activity. The values in the chart correspond to the grouping of like time periods with respect to activity level. Consider a month with low activity for Koerner Library. December and May would be the timeframes that are grouped in that period combination.

### Koerner Library

<table>
<thead>
<tr>
<th>Month</th>
<th>Low</th>
<th>Moderate</th>
<th>High</th>
</tr>
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<tbody>
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<td>September</td>
<td>November</td>
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<tr>
<td>May</td>
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<td>February</td>
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### Woodward Library

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<tr>
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### David Lam Library

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</tbody>
</table>

### B. Sensitivity Analysis

The numbers in the tables show the minimum number of staff needed to ensure that the probability that a user waits less than the waiting time is greater than or equal to the service level. Each number has a corresponding arrival rate shown in the first row and a corresponding service time in the first column.

The service level shown in the sensitivity analysis is to ensure that the probability that a user waits less than 2 minutes is greater than or equal to 85%. Note that D corresponds to David Lam Reference Desk, W corresponds to Woodward Reference Desk, KM corresponds to Koerner Journal & Microform Desk, and KR corresponds to Koerner Reference Desk. KC Corresponds to Koerner Combined Reference Desk.
C. Visual Basic Code to Smooth Data

Sub Identify_Session_Id()

Dim rsttable_smoothdata As New ADODB.Recordset
Dim SessionCounter
Dim LogoutCounter, PreviousLogoutCounter As Integer

rsttable_smoothdata.Open "table_library", CurrentProject.Connection, adOpenDynamicStatic, adLockOptimistic
rsttable_smoothdata.MoveFirst

SessionCounter = 0
PreviousLogoutCounter = 0

Do Until rsttable_smoothdata.EOF

LogoutCounter = rsttable_smoothdata.Fields("logout")

If LogoutCounter = 1 Then
    If PreviousLogoutCounter = 0 Then
        rsttable_smoothdata.Fields("session_id") = SessionCounter
    ElseIf PreviousLogoutCounter = 1 Then
        rsttable_smoothdata.MovePrevious
        rsttable_smoothdata.Fields("session_id") = 
        rsttable_smoothdata.MoveNext
        rsttable_smoothdata.Fields("session_id") = SessionCounter
    End If
ElseIf LogoutCounter = 0 Then
    If PreviousLogoutCounter = 1 Then
        rsttable_smoothdata.Fields("session_id") = SessionCounter
        SessionCounter = SessionCounter + 1
    End If
End If

End If
PreviousLogoutCounter = LogoutCounter

rsttable_smoothdata.MoveNext
Loop

rsttable_smoothdata.Close

End Sub
D. Visual Basic Code to Create Minute Reference for One Year

Sub Add_New_Records()

Dim rsttable_minuteref As New ADODB.Recordset
Dim MinuteCounter As Double
Dim Minute_X As Date

rsttable_minuteref.Open "Minute_Reference", CurrentProject.Connection, adOpenDynamicStatic, adLockOptimistic
rsttable_minuteref.MoveNext

MinuteCounter = 1
Minute_X = #12/1/2002#

For MinuteCounter = 1 To 525600
    Minute_X = DateAdd("s", 60, Minute_X)
    rsttable_minuteref.AddNew
    rsttable_minuteref.Fields("minute") = Minute_X
    rsttable_minuteref.MoveNext
Next MinuteCounter

rsttable_minuteref.Close

End Sub