Methodology for Scheduling Pre-Board Screening Staff at the Vancouver International Airport

by

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A THESIS SUBMITTED IN PARTIAL FULFILMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER of SCIENCE

in

THE FACULTY OF GRADUATE STUDIES

(Department of Business Administration)

We accept this thesis as conforming

to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

February 2003

Abstract

A scheduling methodology has been developed for the staffing of pre-board screening at the Vancouver International Airport. Pre-board screening is the process of x-raying hand baggage and checking passengers to ensure no prohibited items are carried onto an aircraft. At the Vancouver International Airport there are four screening locations and the demand fluctuates differently at each location.

Given a day's departing flight schedule, for each 10-minute interval over the day, the methodology forecasts the passenger demand at each of the four pre-board screening locations, determines optimal staffing level to meet the service criteria of less than 10 percent of passengers spending more than 10 minutes in the system, then determines a staff shift schedule that will ensure there are sufficient staff in the airport to meet the minimum numbers. The methodology has been implemented using Excel spreadsheets and solves quickly.

In the forecasting methodology, a distribution is applied to each departing flight to distribute the departing passengers over the time period preceding the departure time. The results agree well with available data. The scheduling period is divided into 10-minute intervals and for the passenger demand in each interval at each screening location, a staffing level is determined using queuing theory. Finally, a shift schedule is developed that will cover the aggregated demand of the four screening locations using linear programming. It is possible for screening agents to move from one screening location to another during the shift making satisfying the aggregate demand feasible.

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Acknowledgements

The successful completion of this project would not have been possible without the hard work of my project team and the support of the COE staff and the Operations Group at YVRAA. I am especially appreciative of the support of my advisor, Dr. Derek Atkins. Despite a very full schedule, he was actively involved in all aspects of the project and always impressed me by his support.

I would also like to express my thanks to my fellow COE students. It was a tough haul and a rough year marked by lots of changes, but we pulled together and had fun too. It is reassuring to know that at a time when I am venturing into a new career path I already have a strong peer network to rely on. Thanks especially to Kelly and Dave for suggestions and listening.

Finally, thanks to all my friends and family who let me disappear into a world of homework and were still there when I emerged in June. I am always grateful for the support of my sister, but she came through again, following me into a world of new vocabulary and ideas even wanting to see my big presentation while on vacation. Thank you.

1 Introduction

1.1 Background

The Vancouver International Airport, known by the call letters YVR, is operated and managed by the Vancouver International Airport Authority (YVRAA). This is a community-based, not for-profit organization operating under a long term lease from the Government of Canada. Their focus on safety, security and customer service has contributed to YVR's ranking among the 10 top airports in the world. In addition to managing the physical assets of the airport, they are responsible for perimeter security and safety.

In response to the events of September 11th, the Government of Canada formed a new entity, the Canadian Airline Transport Security Authority (CATSA), responsible for the provision of key air security services. Included in these services is pre-board screening which the Government of Canada has committed to enhancing and funding. Pre-board screening (PBS) is the process of checking that passengers are not carrying any prohibited items when they board the aircraft. At pre-board screening, the passenger walks through a metal detector while hand luggage is simultaneously checked using an x-ray machine. Explosive detection equipment has recently been added.

Prior to the formation of CATSA, pre-board screening was the responsibility of the airlines, under the guidelines of Transport Canada. At most airports in Canada, Air Canada has managed the pre-board screening process on behalf of the other airlines who contribute financially. At the Vancouver International Airport, Air Canada has contracted out the day-to-day operation of pre-board screening to Aeroguard Company. Aeroguard has been in operation since 1986 and its core business is passenger and baggage pre-board screening. Current operations consist of twenty five airports across Canada and the cruise ship terminals in Vancouver.

With the formation of CATSA, YVRAA saw an opportunity to assume management of the pre-board screening process and incorporate it into the other security roles they manage. In order to prepare themselves for this opportunity, the Operations group asked the COE to work with them to understand the process and to efficiently schedule staff.

1.2 Pre-Board Screening

Pre-board screening takes place at four locations within the main terminal at YVR. The two domestic locations, Domestic North and Domestic South, screen passengers departing on flights to Canadian destinations, the Transborder location screens passengers departing to destinations in the United States of America, and the International location screens passengers traveling to locations other than the United States of America or Canada. While most of us have likely passed through pre-board screening at some point, it is unlikely that we have observed the full process so a more detailed description follows.

Pre-board screening consists of a number of steps, carried out by different people, some of which occur simultaneously. When a passenger approaches the pre-board screening location they see a number of equipment lines consisting of a walk through metal detector, an x-ray machine and a baggage search table. After selecting an equipment line, the passenger places any hand luggage and metal items on the belt feeding into the x-ray machine, then waits until the metal detector attendant indicates that they should walk through the metal detector. If metal is detected, an audible beep is sounded and the metal detector attendant must conduct a manual search. If the audible beep is not heard, there is no need for a manual search. The manual search is conducted by the attendant using a hand held metal detector that is passed over the body. The hand held metal detector is known as a wand, the manual search as wanding and the attendant as a **wander**. The wanding continues until all metal items have been identified. At this point the passenger moves on to claim their hand luggage and the next passenger can pass through the metal detector.

Simultaneously, the hand baggage has passed through the x-ray machine and the x-ray attendant has determined if a hand search is required. If no search is required, the bag remains on the belt until the passenger claims it and exits the system. If a hand search is required one of two things will happen. Either the x-ray attendant will leave the x-ray post and move to conduct the search or the x-ray attendant will remain at the x-ray post and the search will be conducted by another attendant. In the first case, the next piece of baggage will not pass through the x-ray machine until there is a free attendant to operate it, and in the second, there may be a delay until an attendant becomes available to conduct the bag search. Once the appropriate attendant takes the bag, permission must be received from the passenger before a search can be conducted. This may involve a delay if the passenger has not completed the metal detection step. When permission has been received, the attendant, known as the **searcher**, conducts the search removing any restricted items and returns the bag to the passenger who exits the system. Typically the system is run with one wander and between 2 and 6 searchers who also operate the x-ray machine.

During the course of this study, new equipment was added to the pre-board screening operation. Explosive trace detection equipment is used to detect explosive materials on items carried by passengers onboard the aircraft. Electronic equipment and other items are wiped with a pad that is analyzed for explosives. At the time of the project, each equipment line had one explosive detection operator who randomly tested items during the course of a bag search.

1.3 Previous Work

This thesis describes the second phase of the work done by COE for the Operations Group at YVRAA. In the first phase, a computer simulation was created and used to better understand the process and investigate ways to reduce the queues that were forming. Process maps were developed to document the steps and work rules involved in pre-board screening. Next the animated simulation was developed in ARENA 6 and data was collected for the simulation and for validation. When the simulation was complete, it was used to evaluate configuration ideas, identify bottlenecks in the current operation and determine an achievable service standard criterion.

1.4 Objectives

Pre-board screening is conducted at four locations within the Vancouver International Airport. Each location experiences fluctuations in demand over the day in response to the daily flight schedule. While a minimum staff level is required by Transport Canada at each location, staffing levels fluctuate with passenger demand. The peak times for the four locations do not always coincide, providing the opportunity to gain efficiency by moving staff between the locations. For the most efficient operation, exactly the right amount of staff should be working at any given time, at any given location. The real world presents three significant obstacles to reaching this state. The first obstacle is passenger demand forecasting. Staff cannot instantly appear as the passengers appear or as the queue grows. A forecast is needed to predict the demand so that staff can be available to screen passengers. The second obstacle is determining the minimum number of staff required to meet the demand and satisfy the service criteria. Finally, the third obstacle is that staff can work 4, 6, 8 or 10 hour shifts while the peak demand at a given location fluctuates every 10 minutes. Thus, staffing up for a short peak period results in excess staff after the peak for the remainder of the shift.

Scheduling attempts to mitigate these factors and overcome these obstacles. The intention of the staff scheduling project is to forecast demand, translate demand into staffing levels that will meet the service criteria, then determine the shifts to fill to ensure the staffing levels are most efficiently met. Specifically, the forecast and staffing are done for each of the four screening locations before the shift determination is done as an aggregate of the four piers. Scheduling at the airport level implies that staff can move between screening locations.

The Operations group at YVRAA, under the guidance of Paul Levy, Director of Security and Emergency Planning, asked the COE to develop a methodology for scheduling pre-board screening staff to meet the desired service standard.

It was understood that the methodology should be easy to understand, relatively quick to execute and should be accessible to members of the Operations group. Based on this, the aim was to create a methodology that could be run in Excel and that would minimize the running of simulations in ARENA 6.0. In addition, the input data for the methodology should be readily available.

The following assumptions were made about the pre-board screening operations:

- Each staff member can work in all roles (searcher, wander, explosive detection trace)
- There can be one or two wanders at each equipment line
- There can be one to six searchers at each equipment line
- There will be one explosive detection trace operator in each equipment line
- Searcher refers to x-ray attendants and bag searchers. The searchers move to the x-ray machine when they are free and remain there until a bag requires searching. At that point they move with the bag to the search table and conduct the search.
- All staff hours are equally weighted
- Service criteria is that 90% of passengers spend less than 10 minutes in the system (waiting in line and undergoing pre-board screening)
- Boarding pass checkers and staff controlling the line are not included.

1.5 Literature Review

Previous work on staffing and scheduling in an airport setting includes Mason, Ryan and Panton (1998) who undertook a scheduling project at the Aukland International Airport. They combined a heuristic search with their simulation model to determine staffing levels for customs agents. This was followed by an integer programming model to optimally allocate full and part-time staff to each period of the working day. The customs agents were required to screen both arriving and departing passengers and customs agents could move between the two locations throughout their shift. The simulation model included a "Departing Passenger Simulator" and an "Arrivals Passenger Simulator" which transformed one day's expected flight times and passenger levels into a deterministically calculated profile of expected passengers at each customs location. These simulators were developed by the Aukland International Airport Limited and included distributions for time prior to a flight that a passenger arrives at the airport, the rate of airline check-in and the time spent shopping at the airport. Further details of these simulators are not given. Once the demand profiles were established, the simulation evaluated proposed staffing levels for each 15-minute interval over the day. The staffing schedule was tested for feasibility, and government-specified service requirements. For departing passengers, all passengers must be processed at least 10 minutes prior to the scheduled departure of the flight. The staffing schedule must also cover the required minimum staffing requirements as well as re-queuing restrictions that state that if a queue behind a custom booth is longer than some threshold, that booth could not be closed.

Starting from a feasible schedule, the heuristic systematically reduced the staffing level, one 15-minute period at a time, until any further reductions would result in an infeasible schedule. The authors solved for schedules for the departing and arriving passenger locations independently. They then solved for the two schedules that resulted in the lowest combined staff schedule.

Other literature discusses the use of queuing theory as an alternative to the use of simulation to set staffing levels. Most commonly this is done in a call center where multiple operators are available to answer calls in parallel. For example, Agnihothri and Taylor (1991) used the Erlang formula to staff a call center for booking hospital appointments to meet a service criteria of less than 10% of calls waiting for service. In their model, they assumed that the system followed an M/M/c queuing model with inter-arrival rate between calls and the service time each having an exponential distribution. Using the Erlang formula, the service and arrival rates, and the service criteria, the optimal number of operators (c) could be determined for each 15-minute interval in the working day. A heuristic procedure was used to determine the work-shift schedule that would minimize the total differences between the optimal and actual staffing levels. By this method, staffing shortages did occur in some time periods.

Data was collected to validate the assumption of an M/M/c model which assumes exponential inter-arrival times and service times. The inter-arrival time was shown to be exponential, but the service time was fitted by a mixture of exponential and Erlang distributions. It was shown that the results were insensitive to the service time distribution, demonstrating the robustness of the M/M/c queuing model.

2 Passenger Demand Forecasting

2.1 Methodology

The scheduling project was broken into three tasks and each was approached as an independent step. The three tasks are forecasting, staffing level determination and scheduling.

The objective of the forecasting task was to predict the pattern of passenger demand at a pre-board screening pier over the scheduling day. In the current operation, Aeroguard, who have the contract to staff the pre-board locations at YVR, estimates staffing requirements from a gate schedule listing the departure times and seating capacity of the flights from each screening location. In addition, the airlines are required to provide estimates of daily load factors for each flight 48 hours in advance. A load factor is the ratio of bought tickets to number of seats on the aircraft, and can be multiplied by the number of seats on a flight to estimate the number of passengers that will be on the flight. The proposed forecasting task uses the same information as is currently used, namely the list of flights for the screening location. The flight list includes the departure time, number of seats on the plane and a load factor.

The main assumption behind the forecasting method is that passengers appear at pre-board screening (PBS) in a predictable pattern based on the departure time. That is, the passenger will change the time they will appear at PBS if the departure time of the flight changes. Also considered was an alternative model in which passengers arrive in a pattern based on time of day and day of the week that can be predicted from past PBS demand. This methodology had previously been used to predict demand for US Customs agents at the Transborder departure point. Basing the forecast on the flight schedule should be more accurate than basing it solely on historical data.

The pattern used to describe passenger demand in this methodology is the triangular distribution, as shown in Figure 1. The horizontal axis is time and the vertical axis is the probability density function for passenger demand at any given time relative to flight departure. The area under the probability density function is equal to one, so the area of any slice of the graph from time t to time t + Δt represents the probability of passenger demand in that time interval. The probability can be equated to the percentage of passengers on the flight that will appear at PBS in the given time interval. Multiplying the percentage by the total number of passengers expected on the flight provides an estimate of the number of passengers expected to appear at PBS in the given time interval.

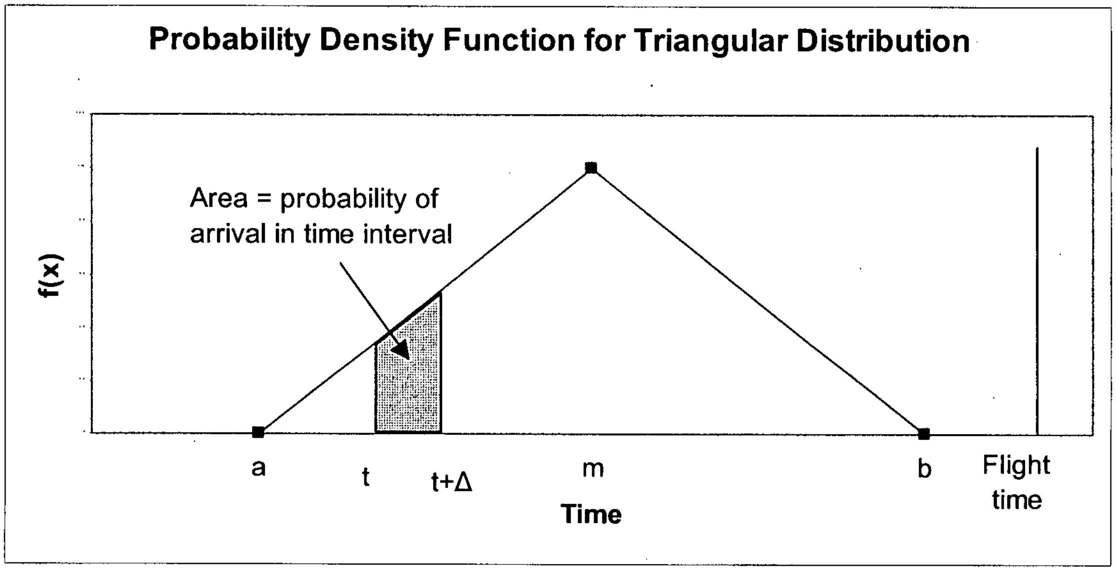

Figure 1: Probability Density Function for Triangular Distribution

Specifying a triangular distribution requires three points: "a" is the time the first passenger appears at PBS, "b" is the time the last passenger appears at PBS and "m" is the time a passenger is most likely to appear at PBS. Prior to time "a" and after time "b" no passengers will appear. Starting at time "a" the probability of a passenger appearing increases until time "m" when a passenger is most likely appear, then decreases again until time "b". When this is applied to a scheduled flight and converted to number of passengers, the result is a profile of how many passengers are expected at any time over the day.

Figure 2 below compares the expected distribution of passenger demand at PBS from two flights of 100 passengers. A different triangle has been applied to each flight.

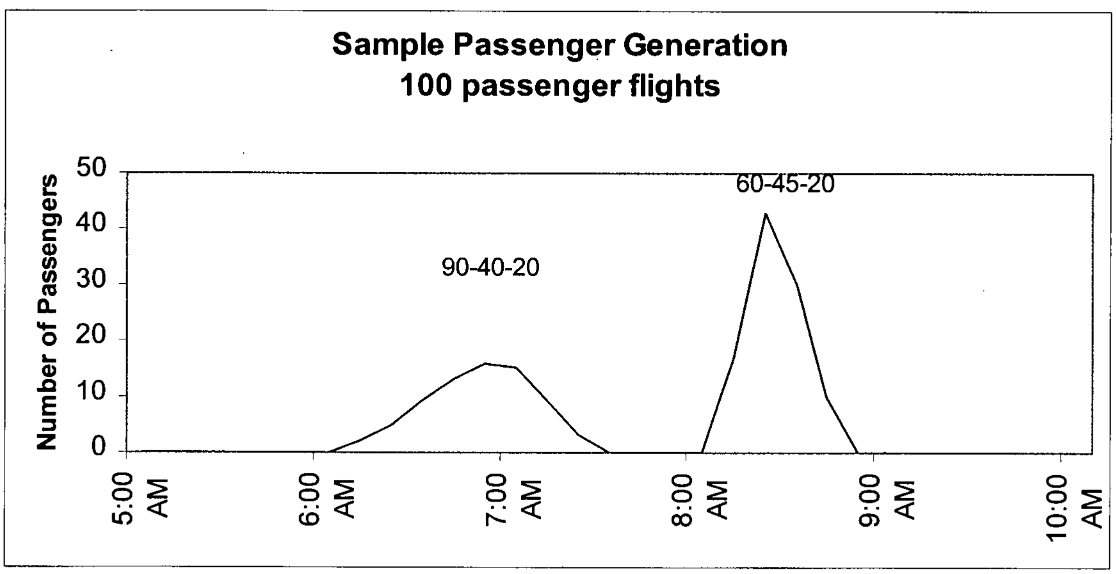

Figure 2: Sample Passenger Generator

The first flight is assumed to have a 90-40-20 distribution (first passenger appears 90 minutes before flight, last passenger 20 minutes before flight and a passenger is most likely to appear 40 minutes before the flight) and the second a 60-45-20 distribution.

It can be seen that the two different triangles distribute passengers in different ways and would result in different passenger profiles when applied to an entire flight schedule.

In developing the methodology, first the applicability of using the triangular distribution to predict passenger demand patterns at PBS was evaluated. Once it was found to be applicable, historical flight schedules and load factors would be used to determine the parameters of the distribution, "a", "b" and "m" that best match actual patterns.

2.2 Data

The passenger generation step was the most data-intensive component of the project. Past flight schedules, load factors and demand data were used.

2.2.1 Flight Schedules

Past flight schedules were used to test the passenger generation methodology. The schedules for all of 2001 and January to July 2002 were provided by YVRAA. Some seating capacity numbers were missing, so in these instances, values were estimated from aircraft manufacturers data.

2.2.2 Load Factors

Estimates of load factors for each flight were required for developing the forecasting model. The load factor is the ratio of tickets purchased to available seats and is used to estimate the number of passengers on a flight from the seating capacity of the aircraft. In the absence of historical individual flight load factor data, estimates of monthly load factors were supplied by YVRAA for January, July, August and December 2001. These were calculated from monthly sums of passengers and seating capacity for domestic, transborder and international destinations. For the remainder of 2001, monthly ticket sales were provided and seat capacity was estimated from the flight schedules. Table 1 shows the monthly load factors used as well as the percentage of passengers that connect to other flights through YVR .

Table 1: Monthly Load Factors

	January 2001	February 2001	March 2001	April 2001	May 2001	June 2001	July 2001	August 2001	December 2001
Load Factors									
Domestic	64%	73%	76%	74%	71%	70%	70%	73%	73%
Transborder	68%	75%	76%	67%	70%	74%	76%	81%	58%
International	70%	63%	65%	61%	59%	62%	71%	82%	80%
Connecting Passengers	36%	32%	38%	37%	42%	46%	33%	31%	36%

2.2.3 AIF

The Airport Improvement Fee (AIF) is a charge paid by every passenger originating in Vancouver on the day of the flight. Proof of payment is a computer-generated coupon that is surrendered immediately before entering PBS. Since the coupons are computer generated, and sales are logged, there is a wealth of data available. In the airport, the AIF can be purchased at an Automated Teller Machine (ATM) or from an AIF sales person seated at a booth. In general, the AIF sales points are located in close proximity to the entrance to the PBS locations and it is assumed that the majority of passengers purchase their AIF tickets as they prepare to enter the PBS. Approximately 6% of AIF tickets are bought offsite, not at the airport.

It was assumed that the AIF data, segmented by proximity to PBS location, was a good representation of the passenger demand at the PBS location. This data is available in 10-minute intervals and is in a format that is easy to use. Plotting the forecasted PBS demand generated from applying the triangles to past flight schedules with the AIF data on the same graph provides a means of validating the forecasting method.

As of November 2001, zero value AIF tickets or stickers were issued to passengers not originating in Vancouver who required PBS. The zero value tickets were computer generated and recorded in system but the stickers, only distributed during busy times, were pre-printed and were not recorded in the computer system. At busy times, an AIF agent could print a large number of zero value tickets and walk the PBS queue distributing them to connecting passengers. This could result in a spike in the AIF data at the time the tickets were printed. With the issuance of zero value AIF tickets, the AIF ticket data now captures a high percentage of those connecting passengers who require PBS.

It should be noted that the approach to the Transborder PBS location is different from the other locations. At this location, there are a number of AIF purchase points immediately preceding PBS, as there are at other locations. In addition, there are two AIF ATM's located immediately following the check-in desks. Once a passenger has checked in, they pass through the duty free store, US immigration and US customs before appearing at PBS. Thus there is a delay between the time an AIF ticket is purchased at an ATM located immediately following check-in and the time the passenger appears at the PBS.

2.3 Results

The work was done in two stages. In the first stage, the validity of the method was tested, and the triangle parameters were developed. This work was done using the available data from January, July, August and December 2001. Once the forecast was determined to be a good representation of the AIF data for these months, the forecast methodology was applied to January through June of 2002 using the monthly load factors from 2001.

In the first stage, an Excel sheet was created to demonstrate the methodology. Once the methodology was established, it was tested on a day in August 2001 using the monthly load factors for August 2001, and plotted against the AIF data. The plot for Domestic North is shown in Figure 3. For each plot, the horizontal axis is the time of day and the vertical axis the number of passengers every 10 minutes. The shaded area is the AIF data and the solid line is the generated passenger demand.

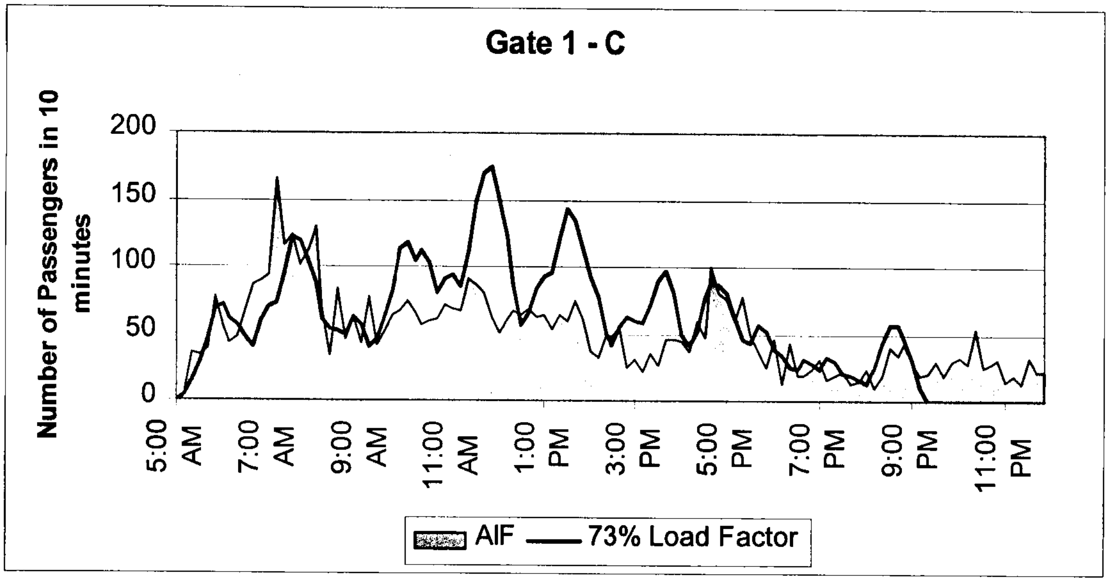

Figure 3: Comparison of Generated Passengers to AIF data for Domestic PBS Location

It seemed that the forecast correctly predicted the peaks and valleys, but overestimated after approximately 10:00 am. In general, the locations of the peaks and valleys are a result of the chosen distribution, and the magnitude is influenced by the load factor. Figure 3 suggests that the correct load factor was selected for the beginning and end of the day, but a different load factor should be used in the middle of the day. To test this theory, the day was divided into three time periods, morning, midday and evening, and different load factors were applied to each time period. The result is plotted in Figure 4 below with the vertical lines separating the time periods.

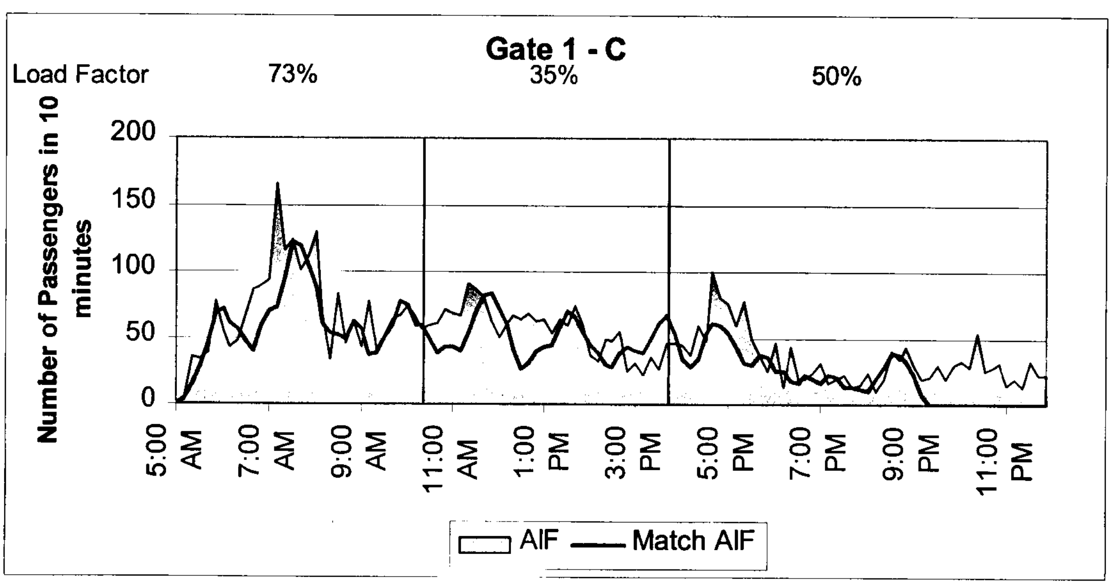

Figure 4: Passengers Generated with Variable Load Factor Over the Day

With the changes in load factor, it was easier to see that triangular distribution based forecast successfully created the peaks and valleys, but load factor was important. This revealed that the AIF data did not truly represent the PBS demand. For the domestic and international gates, the actual PBS demand is the number of passengers originating in Vancouver *plus* the number of connecting passengers that leave the secure area *plus* any connecting passengers that arrive from outside of Canada and pass through Canadian customs and immigration. At the time of our study, passengers passing through Canadian customs and immigration left the secure area and were required to go through PBS before boarding a connecting flight. For the Transborder gates, **all** passengers are required to pass through PBS. Any connecting passengers, denoted "In Transit Pre-Clearance" (ITPCF), use a separate queue for US immigration and customs, but join a common PBS queue. The available AIF data for flights prior to November 2001 only accounts for the passengers originating in Vancouver as connecting passengers were not required to pay the AIF and no zero value tickets were issued. Based on this, it was concluded that the AIF ticket sales data represented a lower bound on PBS demand.

The AIF data was a very useful data resource for validating the forecast for PBS demand. Prior to November 2001, it provided a lower bound on PBS demand with the difference as the number of connecting passengers requiring PBS. Following November 2001, AIF data captured the majority of the connecting passengers and became a much better estimate of PBS demand. Approximately 6% of AIF sales are conducted offsite and are not captured in the AIF data. It should be noted that this work was conducted in the first half of 2002 and most of the available data was from prior to November 2001.

Testing of different triangles revealed that while the pattern did change, it was not as significant as the load factor, and did not improve the fit at different points during the day. The graph, shown in Figure 5, is an example of two different triangles applied to a Domestic North flight schedule. Both triangles result in very similar patterns with similar peaks and valleys. Following similar tests on other days in August 2001, it was decided that a constant triangular distribution could be used over the day.

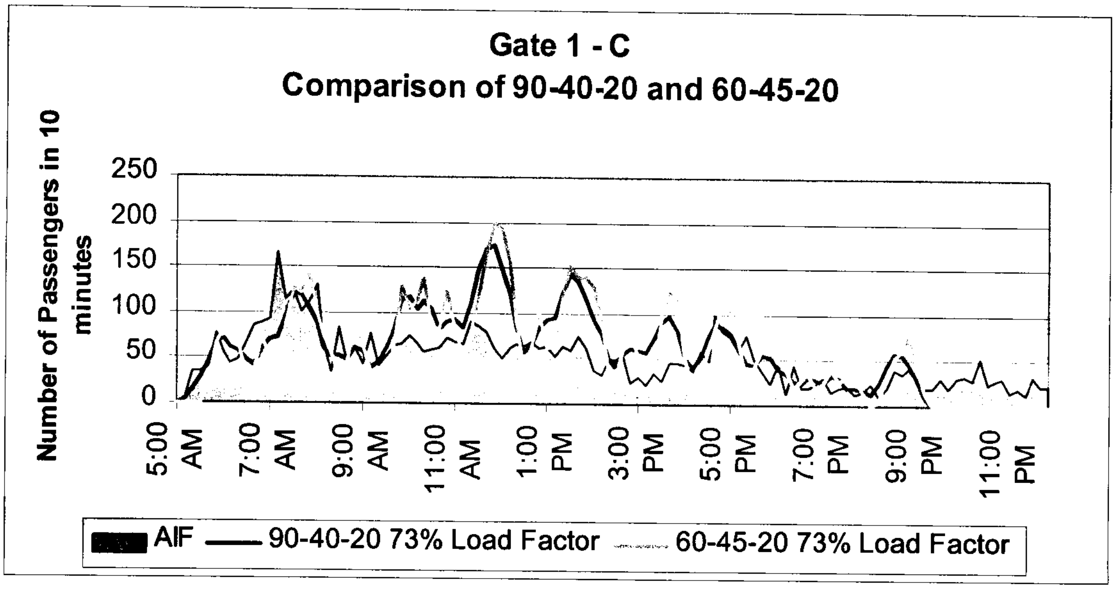

Figure 5: Comparison of Passenger Generation using Different Triangular Distributions

2.3.1 Domestic PBS Locations

Trials with different triangles were run on flight schedules for Domestic North and Domestic South PBS locations for January, July, August and December 2001. Following the trials, a triangle pattern of 90-40-20 was selected for the domestic PBS locations.

2.3.2 International PBS Location

A similar process was followed for the international flights. A 150-80-20 triangular distribution was determined to be the appropriate distribution. Not only was this a good fit, matching the peaks and the perimeter of the AIF data, but it also followed passenger behaviour patterns as related by YVRAA staff. In general, the first passengers check in 2½ to 3 hours before departure. They then spend some time in the airport shopping, etc then proceed to PBS 150 minutes before departure. The majority of passengers check in 1½ to 2 hours before departure, approximately an hour later. This translates to appearance at PBS 80 minutes before departure.

The plot in Figure 6 shows the results of the forecast for the international PBS using a 150-80-20 triangular distribution.

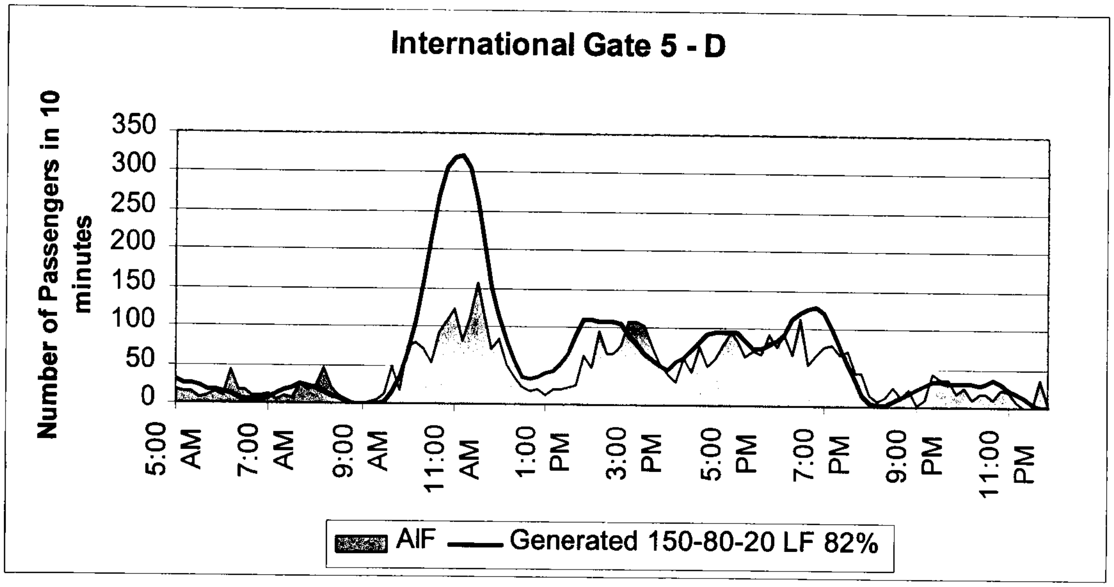

Figure 6: Comparison of Generated Passengers to AIF Data for International PBS Location

As can be seen in the plot, there is a good matching of the peaks and valleys of the AIF data with the exception of the period 9:00 to 12:00. During this time period, the forecast value is far greater than the AIF tickets sold. This pattern is consistently repeated across the four months of data, January, July, August and December 2001.

Further thought led to the theory that there is a higher than normal connecting passenger ratio for international flights departing in the beginning of the day. These passengers would not be required to purchase AIF tickets and only would only require PBS if they left the secure area during their stop over. Inspection of the corresponding flight schedule, shown in Figure 7, showed that between 11:30 and 15:00 the majority of the flights were to Asia, while the evening was dominated by flights to Europe. We were able to obtain connecting passenger estimates for one day that showed that approximately 50% of the passengers on the Asia bound flights were connecting passengers. That YVR serves as an Asian gateway and that a large number of Asia-bound passengers originate in cities other than Vancouver support the idea of a high connecting passenger ratio.

Carrier		Destination	VIA	Scheduled Departure	Estimated Departure	Actual Departure	Aircraft Type	Passenger Capacity
BR	TPE	Taipei		**2:15**	2:15	2:30	744	**392**
CX	HKG	Hong Kong		**2:50**	4:00	4:17	744	**392**
AC	TPE	Taipei		**11:50**	12:05	12:05	343	**284**
AC	HKG	Hong Kong		**12:00**	12:45	12:45	343	**284**
AC	PVG	Shanghai		**12:00**	14:45	14:45	763	**240**
SSV	JFK	JFK New York		**12:15**	12:55	13:00	320	**150**
CI	TPE	Taipei		**12:20**	12:20	12:20	744	**392**
AC	PEK	Beijing		**12:30**	12:30	12:30	763	**240**
JL	NRT	Tokyo		**12:30**	12:30	12:30	747	**426**
AC	NGO	Nagoya		**12:35**	18:30	18:30	763	**240**
AC	ICN	Incheon		**12:40**	12:55	12:55	343	**284**
AC	KIX	Kanasai Osaka		**12:45**	12:45	12:45	744	**392**
AC	NRT	Tokyo		**13:30**	16:05	16:05	744	**392**
PR	SFO	San Francisco		**14:00**	14:30	14:30	343	**284**
KE	ICN	Incheon		**14:20**	14:45	14:45	744	**392**
JL	NRT	Tokyo		**15:05**	16:00	16:00	747	**426**
TS	GLA	Glasgow	MAN	**15:30**	16:00	16:00	332	**284**
LH	FRA	Frankfurt		**15:55**	16:20	16:20	742	**374**
CX	HKG	Hong Kong		**16:00**	16:20	16:20	744	**392**
2T	DUS	Dusseldorf	YYC	**17:45**	18:15	18:15	332	**284**
AC	HKG	Hong Kong		**17:45**	18:30	18:30	343	**284**
AC	LHR	London		**18:25**	18:45	18:45	744	**392**
KL	AMS	Amsterdam		**19:00**	19:00	19:00	763	**240**
AC	SYD	Sydney	HNL	**20:00**	20:35	20:35	763	**240**
AC	LHR	London		**20:00**	20:00	20:00	763	**240**
BA	LHR	London		**20:15**	20:15	20:15	744	**392**
PR	MNL	Manila		**23:10**	23:10	23:10	343	**284**

Figure 7: Sample International Flight Schedule

Figure 8 shows the forecast for a constant load factor as well as the forecast if the load factor for flights between 11:30 and 15:00 is reduced to 50%, the value suggested by the connecting passenger data. This adjustment resulted in a much better fit of the forecast to the AIF data throughout all the months tested, namely January through June 2002.

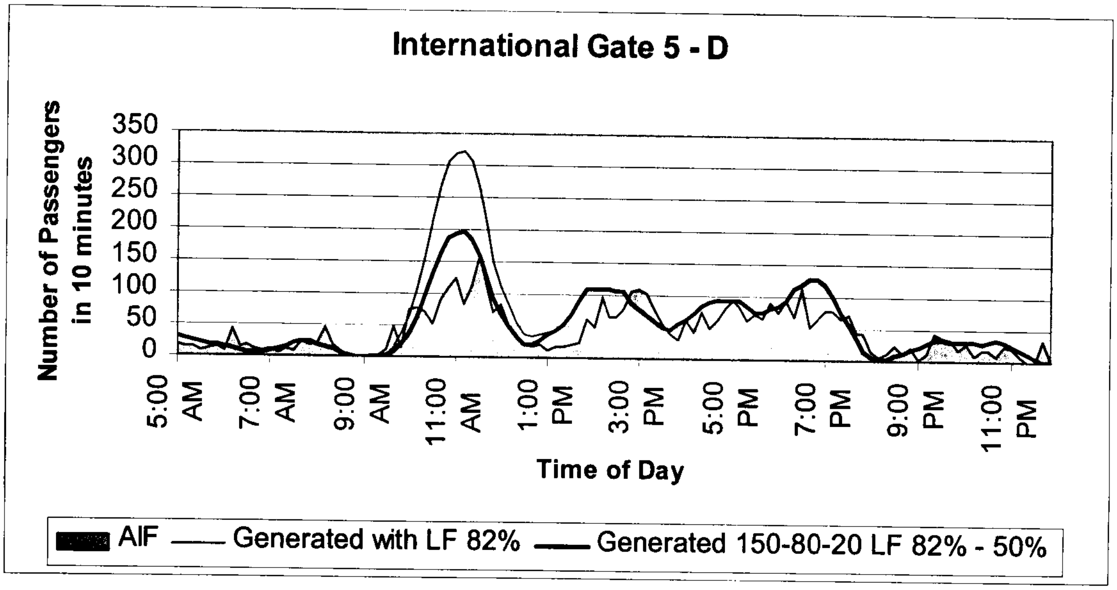

Figure 8: Passengers Generated for International PBS Location

2.3.3 Transborder PBS Location

The Transborder data is also well estimated with a 150-80-20 triangle, as can be seen from Figure 9 and Figure 10.

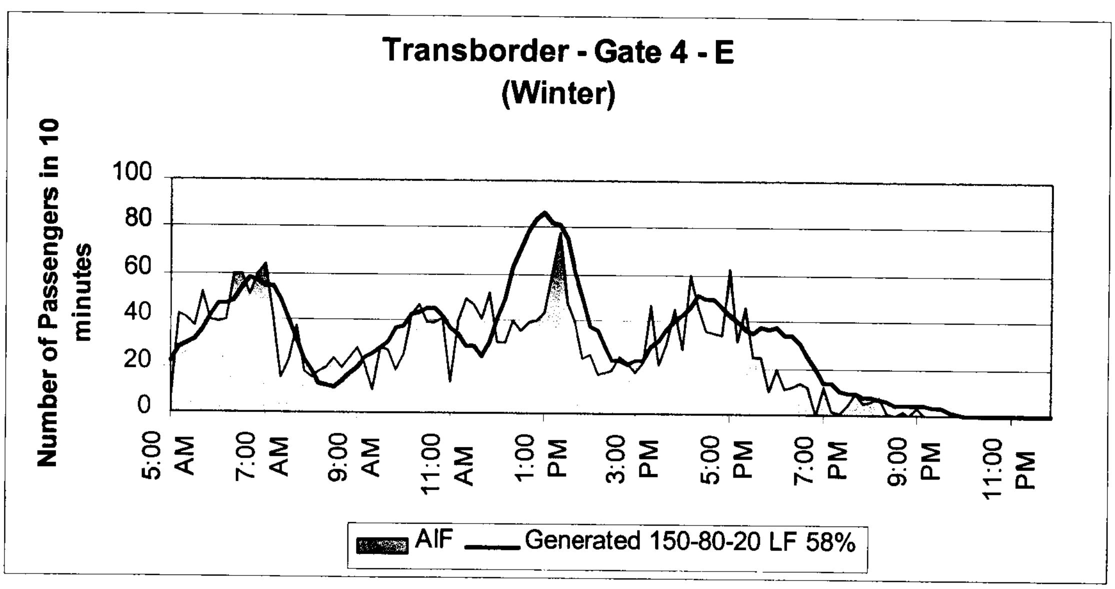

Figure 9: Comparison of Winter Generated Passengers to AIF Data for Transborder PBS Location

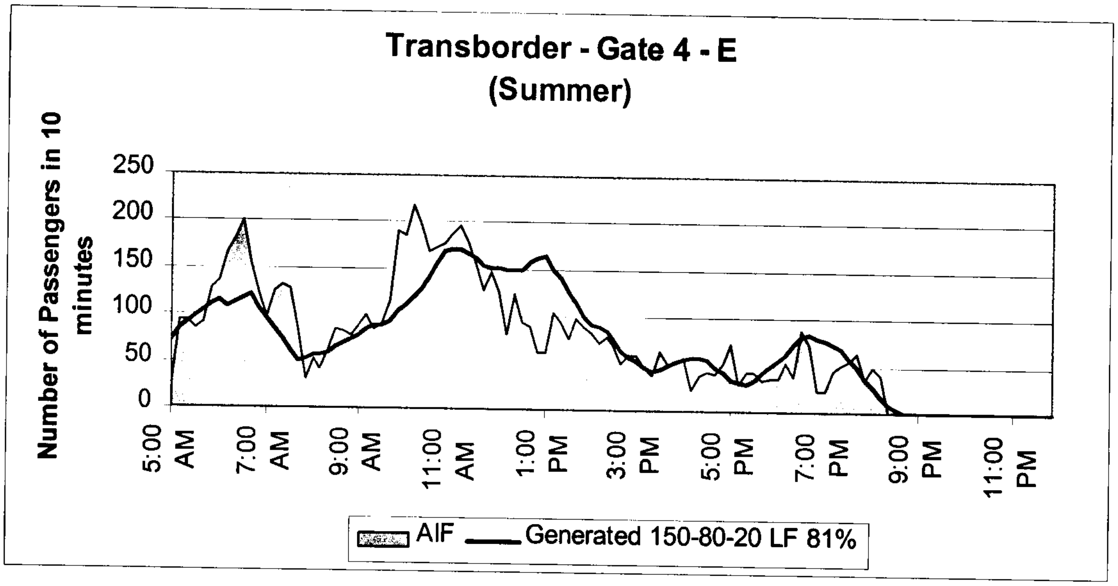

Figure 10: Comparison of Summer Generated Passengers to AIF Data for Transborder PBS Location

Figures 9 and 10 show a comparison of generated passenger demand for a summer day and a winter day for the Transborder PBS location. In the summer, there is a change in the pattern as can be seen above. There is a large block of passengers between 10:00 and 12:00 that is not seen in the winter profile. It is known that the cruise ships have an impact on the Transborder operation. In general, the cruise ships arrive in Vancouver early in the day and the passengers leave the boat throughout the morning. There are busses to transport the passengers between the cruise ship terminal and the airport. These busses arrive at the airport between 9:00 and 12:00. It is reported that with so many busses arriving within a short period of time, the Transborder area becomes congested. Newly arriving passengers see the congestion and join the line in order to ensure they are checked-in in time. Many of these passengers could have waited until closer to their flight time but check-in early due to the congestion. This activity explains why the passenger forecast does not match the AIF data as well in the summer as in the winter.

AIF sales points are distributed differently in the Transborder area than in the other terminals. In addition to the sales points immediately preceding the pre-board screening location, there are two AIF ATM's located in the check-in area, as well as one between check-in and the Duty Free shop. As a result, the AIF sales data may be skewed earlier than actual demand at the PBS location.

3 Required Staffing Levels

The second step in the methodology is translating demand into required staffing levels. The objective was to develop a lookup table or formula so that the required number of staff for that 10 minute period could be determined given the number of passengers that were expected to appear in each 10 minute interval in the scheduling day. The staffing level would be sufficient so that the service criteria, 90% of the passengers spend 10 minutes or less in the system, would be met.

An alternate method of using the ARENA simulation was considered. For each demand profile, proposed staffing levels could be evaluated by running them on the simulation. A proposed staffing level which met the service criteria and used the minimum number of staff could be selected using a heuristic similar to the one used by Mason, Ryan and Panton (1998). This methodology was rejected as each iteration of the simulation required approximately 10 minutes of computation time and a full solution required many iterations. In addition, a methodology on ARENA would require extensive staff training on ARENA if it were to be implemented and maintained.

Based on observations of the PBS process, it was determined that up to 6 searchers and 2 wanders could staff one equipment line. The staffing configurations shown in Table 2 were considered for each line in each pier.

Table 2: Possible Staffing Configurations

Number of Searchers	Number of Wanders	Number of Explosive Trace Workers
2	1	1
3	1	1
4	1	1
5	1	1
6	1	1
2	2	1
3	2	1
4	2	1
5	2	1
6	2	1

The explosive detection trace was not fully functional at the time of the study, so it was assumed that one staff member per equipment line would be dedicated to conducting traces. No change was made to the simulation model since it was not clear the impact that explosive detection trace would have on service time.

3.1 Methodology

The methodology is based on queuing theory and is borrowed from call centre scheduling. In a call centre, the operators work in parallel, answering one call at a time. Scheduling for a call centre requires the determination of the number of operators required over a day using the Erlang formula. Based on queuing theory, the Erlang formula relates the service criteria, the average service time and the number of calls expected, to the number of operators required. Thus for each time period in which demand is known, a number of operators can be determined. Since the number of operators in each period is enough to meet the service criteria, the service criteria will be met over the entire day.

For the PBS system, the challenge was to find a form of the Erlang equation that could be applied to the more complicated configuration. As a precaution against not finding a useable formula, an alternate method was developed. This involved using the ARENA simulation to create a lookup table that related staffing requirements to demand. This table could also be a validation of the queuing theory model.

The question that guided this task was "For a given number of passengers arriving in a 10 minute period, what is the minimum number of staff required to ensure that 90% of the passengers spend 10 minutes or less in the system?" The second question, required both for implementation and to use the simulation, is "How should the staff be deployed?" Specifically, once the number has been set, how many equipment lines should be open and how many searchers and wanders on each? For example, with 10 staff should one line be open with all the staff working there, or should two lines be open, each with five staff, or should five lines be open, each with two staff?

3.1.1 Determining Optimal Staff Allocation

The second question, how to deploy a given number of staff, had to be answered before optimal staffing levels could be determined. In order to run the simulation at a given staffing level, the staff had to be divided between the available equipment lines and assigned roles as searchers or wanders. It was realized that different staffing configurations would result in different number of passengers screened and that there must be an optimal way to deploy the staff. *In this project optimal staff allocation is defined as the configuration of staff that maximizes the number of passenger screened in a given unit of time.*

The optimal staff allocation tables are based on a series of simulation runs that estimated the maximum number of passengers screened per hour that could be achieved for a given staffing configuration. That is, the average number of passengers screened per hour if the staff were working non-stop. For each of the runs, a staffing configuration was specified for a single screening line, and was not changed over the duration of the simulation. The arrivals were set much higher than normal to ensure there was always a queue and the screening officers were always busy. The simulation was run over 20 hours and the number of passengers screened was counted. The hourly throughput is the total number of passengers screened divided by the number of hours of the simulation. The throughput simulation was done for each of the 10 staff configurations on a single equipment line, listed in Table 2 above. With this data, and the assumption that each line operates independently and follows the same patterns as one equipment line, it is now possible to calculate throughputs for any staff configuration across any or all of the equipment lines.

An Excel macro was created that enumerated all the possible staff configurations, then found the configuration with the maximum throughput for each possible number of staff.

For example, 8 staff could be allocated in any of the three configurations shown in Table 3 below.

Table 3: Alternate Staffing Configurations for 8 Staff

Configuration	Equipment Line 1			Equipment Line 2			Total Staff
	Searchers	Wanders	Trace	Searchers	Wanders	Trace	
1	6	1	1				8
2	5	2	1				8
3	2	1	1	2	1	1	8

The throughput for each configuration is either taken directly from the throughput table, or, in the case of multiple equipment lines, the sum of the appropriate entries in the table.

3.1.2 Determining Required Staff Levels

3.1.2.1 Simulation

With the optimum staff allocation determined for each PBS location, the simulation was used to determine the minimum staffing level required to meet the service standard for each number of passengers that can arrive in 10 minutes. For the simulation, the passenger arrival rate was set and an optimal staffing allocation chosen. The simulation was run for 10 repetitions and the ratio of passengers spending less than 10 minutes in the system was determined too see if the service criteria had been met. If more than 90% of passengers spent longer than 10 minutes in the system, then the optimum configuration for one less total staff was tested. If less than 90% of passengers spent longer than 10 minutes in the system, then the optimum configuration for one more total staff was tested. In this manner, the minimum staffing level that meets the service criteria was determined. This procedure can be repeated for a range of demands at each PBS.

3.1.2.2 Queuing Theory

Queuing theory helps explain and predict waiting time, queue length and time in system for queuing systems. The simplest queuing system is a single resource that services clients in the order that they arrive. The service is similar for all clients and the resource provides the service to one client at a time. Under certain distributions for the arrival pattern of clients and the service times, the behaviour of the queue can be predicted. The simplest of these is the M/M/1 queuing model in which the inter-arrival time and service time are assumed to be exponentially distributed. The model can then be expanded to include a number of resources, each working in parallel. If the same assumptions are made about the arrival pattern of the clients and the service time distribution, the Erlang formula can be used to model the queue behaviour.

The Erlang formula is used most often in call centres to determine the number of telephone operators required to process a given number of calls in a given time period. Since there is some variability in the times calls arrive and the service time, the model calculates the probability that a given service level can be met. The service level can be expressed in a number of ways for example, the long run percentage of time that there is no queue or that there is one person in the queue. In this project, the service criterion is average time in system. Gross and Harris derived the cumulative distribution function of the waiting time in queue.

$W_q(t,c)$ = Probability(time in queue $\le$ t) when c staff are working.

$$W_q(t,c)=\begin{cases} 1-\dfrac{c\left(\lambda/\mu\right)^c}{c!\left(c-\lambda/\mu\right)}p_0 & (t=0) \\ \dfrac{\left(\lambda/\mu\right)^c\left(1-e^{-(\mu c-\lambda)t}\right)}{(c-1)!\left(c-\lambda/\mu\right)}p_0+W_q(0,c) & (t>0) \end{cases}$$

where

c = number of resources
λ = number of clients to arrive in a unit of time
μ = number of clients one resource can serve in a unit of time
p_0 = steady state percentage of time there are no clients in the queue

$$p_0=\left[\sum_{n=0}^{c-1}\frac{1}{n!}\left(\lambda/\mu\right)^n+\frac{1}{c!}\left(\lambda/\mu\right)^c\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-1} \qquad \lambda/c\mu<1$$

There is an underlying assumption that $c>\lambda/\mu$ or that the average passenger demand is greater than the number of servers times the average service rate. This sets a theoretical minimum number of servers for each passenger demand.

Gross and Harris also provide the probability density function for the time in system w(t,c).

$$w(t,c)=\frac{\mu e^{-\mu t}\left(\lambda-c\mu+\mu W_q(0,c)\right)-\left(1-W_q(0,c)\right)(\lambda-c\mu)\mu e^{-(\mu c-\lambda)t}}{\lambda-(c-1)\mu} \quad (t>0)$$

This can be integrated to obtain the cumulative distribution function of time in system W(t,c).

$$W(t,c)=\frac{\left(1-\mu e^{-\mu t}\right)\left(\lambda-c\mu+\mu W_q(0,c)\right)-\left(1-W_q(0,c)\right)\mu\left(1-e^{-(\mu c-\lambda)t}\right)}{\lambda-(c-1)\mu} \quad (t>0)$$

where

W(t,c) = Probability(time in system $\le$ t) with c staff working
$W_q(0,c)$ = Probability(time in queue $\le$ 0) with c staff working

In order to determine the number of staff required to meet the service criterion, we need to solve

$$\min_{all\ c} \quad W(10,c)$$
$$subject\ to \quad W(10,c) \geq 0.9$$

It is clear if that the formulae are used to determine number of staff required to ensure the probability of time in queue or time in system is less than 10 minutes, then the result from the time in system formula should never be less than the result from the time in queue formula. That is the number of staff required to ensure 10 minutes or less in system should never be less than the number of staff required to ensure 10 minutes or less in queue.

The formulae given above have been derived for an M/M/c system in which c servers work in parallel, each with exponentially distributed service times serving customers who arrive with exponentially distributed inter-arrival times. Exponentially distributed inter-arrival times is often used to describe random customer arrival. Random inter arrival times is a good assumption for PBS, though there are occasionally large groups travelling together who will appear at PBS at the same time.

As described earlier, the PBS process is not as simple as c servers working in parallel. Instead of servers working in parallel, each processing one customer, the PBS system is made up of equipment lines working in parallel. Each equipment line consists of two procedures, wanding and searching, with wanding conducted by one or two people working in parallel and searching conducted by up to 6 people working in parallel. At a macro scale, however, each equipment line works in parallel and could be seen as an equivalent to a server. At this scale, the service time is a compilation of the wanding time and the search time, both of which may include waiting for an available agent. The collected data showed each of these to be approximately exponentially distributed. Based on this, it seemed reasonable that the overall service time was exponentially distributed.

In attempting to model the PBS location with a queuing model it becomes apparent that while each equipment line works in parallel, the service time for one equipment line is a function of the number of staff working, and how they are deployed between the wanding and searching positions. Thus there may be three equipment lines operating, but unless they are staffed in exactly the same manner, they do not have the same service time distribution. It is not even enough to say that the same number of staff on each line will result in the same service time distribution, as there should be an optimal or highest throughput configuration. It was not intuitive that the average service time for one equipment line, staffed at optimal configuration, was a linear function of the number of staff. However, plotting the optimal configuration results as throughput vs. total number of staff produced a remarkably straight line, with a slope having units passengers screened per hour per staff member. This slope had the correct units for service rate and represented the improvement in passenger throughput that could be realized by one more staff, optimally deployed. While it is clear that PBS does not operate as c servers in parallel, the effect of adding one more staff to the PBS system has the same effect on the overall service rate as adding another server to a M/M/c system and the model is applicable.

To apply queuing theory to a system, three inputs are required. The first is the arrival rate of the clients, or the number of clients per time unit. The PBS demand data, the number of passengers in a 10-minute interval, can be used directly for this value. The second is the service rate or number of clients served by one resource per time unit and can be estimated by the slope of the optimal configuration results as throughput vs. total number of staff plot. The third is the service criteria in terms of percentage of clients and how long they wait, spend in the system or spend receiving service. The service criteria of 90% of passengers spending less than 10 minutes in the system can be directly used. Hence, all data should be available to apply the queuing model.

3.2 Results

3.2.1 Determining Optimal Staff Allocation

The simulation was used to create maximum throughput tables for each of the PBS locations for each of the staffing configurations in Table 1. The throughputs are considered maximums as they are the results of simulations in which there were so many passengers that the staff were always busy. The results are shown in Table 4. The differences in throughputs result from differences across the PBS locations in search and wand ratios as well as number of bags each passenger carried. The search ratio is the percentage of passengers whose bags require searching and similarly the wand ratio is the fraction of passengers who require wanding after passing through the metal detector.

Table 4: Maximum Throughput Values

Searchers	Wanders	Maximum Throughput (passengers/hour) Domestic	Transborder	International
2	1	91	87	109
3	1	129	129	157
4	1	163	165	199
5	1	191	194	234
6	1	211	221	260
2	2	95	91	114
3	2	138	134	165
4	2	179	173	211
5	2	213	211	251
6	2	239	243	278

	Domestic	Transborder	International
Search Ratio	50%	54%	42%
Wand Ratio	42%	32%	25%
Passengers with 0 bags	2%	1%	2%
Passengers with 1 bags	35%	46%	51%
Passengers with 2 bags	46%	44%	40%
Passengers with 3 or more bags	17%	9%	7%

Using these three sets of throughput values, optimum staff configuration tables were produced for each pier by enumerating all possible configurations for a given number of staff, and selecting the configuration resulting in the highest throughput. All PBS locations were configured for 5 working equipment lines, except for Domestic South, which had 3 working equipment lines. The maximum number of staff per equipment line is 6 searchers, 2 wanders and 1 EDT for a total of 9 staff.

The optimal allocation guidelines can be summarized as follows:

Distribute staff evenly over all operating equipment lines

Open additional equipment lines according to Table 5

Table 5: Guidelines for Opening Additional Equipment Lines

	1st line Number of staff	2nd line Number of staff	3rd line Number of staff	4th line Number of staff	5th line Number of staff
Domestic South	1 to 9	10 to 17	17 to 27		
Domestic North	1 to 9	10 to 17	17 to 24	24 to 30	30 to 45
International	1 to 9	10 to 17	17 to 24	24 to 30	30 to 45
Transborder	1 to 9	10 to 18	18 to 25	25 to 33	33 to 45

Tabular and graphical representations for the Transborder location are included in the following pages.

Table 6: Optimal Staff Allocation Table for Transborder PBS

Total Staff	Throughput (Passengers /hour)	Equipment Line 1		Equipment Line 2		Equipment Line 3		Equipment Line 4		Equipment Line 5	
		Number of Searchers	Number of Wanders	Number of Searchers	Number of Wanders	Number of Searchers	Number of Wanders	Number of Searchers	Number of Wanders	Number of Searchers	Number of Wanders
4	87	2	1								
5	129	3	1								
6	165	4	1								
7	194	5	1								
8	221	6	1								
9	243	6	2								
10	258	3	1	3	1						
11	294	4	1	3	1						
12	330	4	1	4	1						
13	359	5	1	4	1						
14	388	5	1	5	1						
15	415	6	1	5	1						
16	442	6	1	6	1						
17	464	6	2	6	1						
18	495	4	1	4	1	4	1				
19	524	5	1	4	1	4	1				
20	553	5	1	5	1	4	1				
21	582	5	1	5	1	5	1				
22	609	6	1	5	1	5	1				
23	636	6	1	6	1	5	1				
24	663	6	1	6	1	6	1				
25	689	5	1	4	1	4	1	4	1		
26	718	5	1	5	1	4	1	4	1		
27	747	5	1	5	1	5	1	4	1		
28	776	5	1	5	1	5	1	5	1		
29	803	6	1	5	1	5	1	5	1		
30	830	6	1	6	1	5	1	5	1		
31	857	6	1	6	1	6	1	5	1		
32	884	6	1	6	1	6	1	6	1		
33	912	5	1	5	1	5	1	4	1	4	1
34	941	5	1	5	1	5	1	5	1	4	1
35	970	5	1	5	1	5	1	5	1	5	1
36	997	6	1	5	1	5	1	5	1	5	1
37	1024	6	1	6	1	5	1	5	1	5	1
38	1051	6	1	6	1	6	1	5	1	5	1
39	1078	6	1	6	1	6	1	6	1	5	1
40	1105	6	1	6	1	6	1	6	1	6	1
41	1127	6	2	6	1	6	1	6	1	6	1
42	1149	6	2	6	2	6	1	6	1	6	1
43	1171	6	2	6	2	6	2	6	1	6	1
44	1193	6	2	6	2	6	2	6	2	6	1
45	1215	6	2	6	2	6	2	6	2	6	2

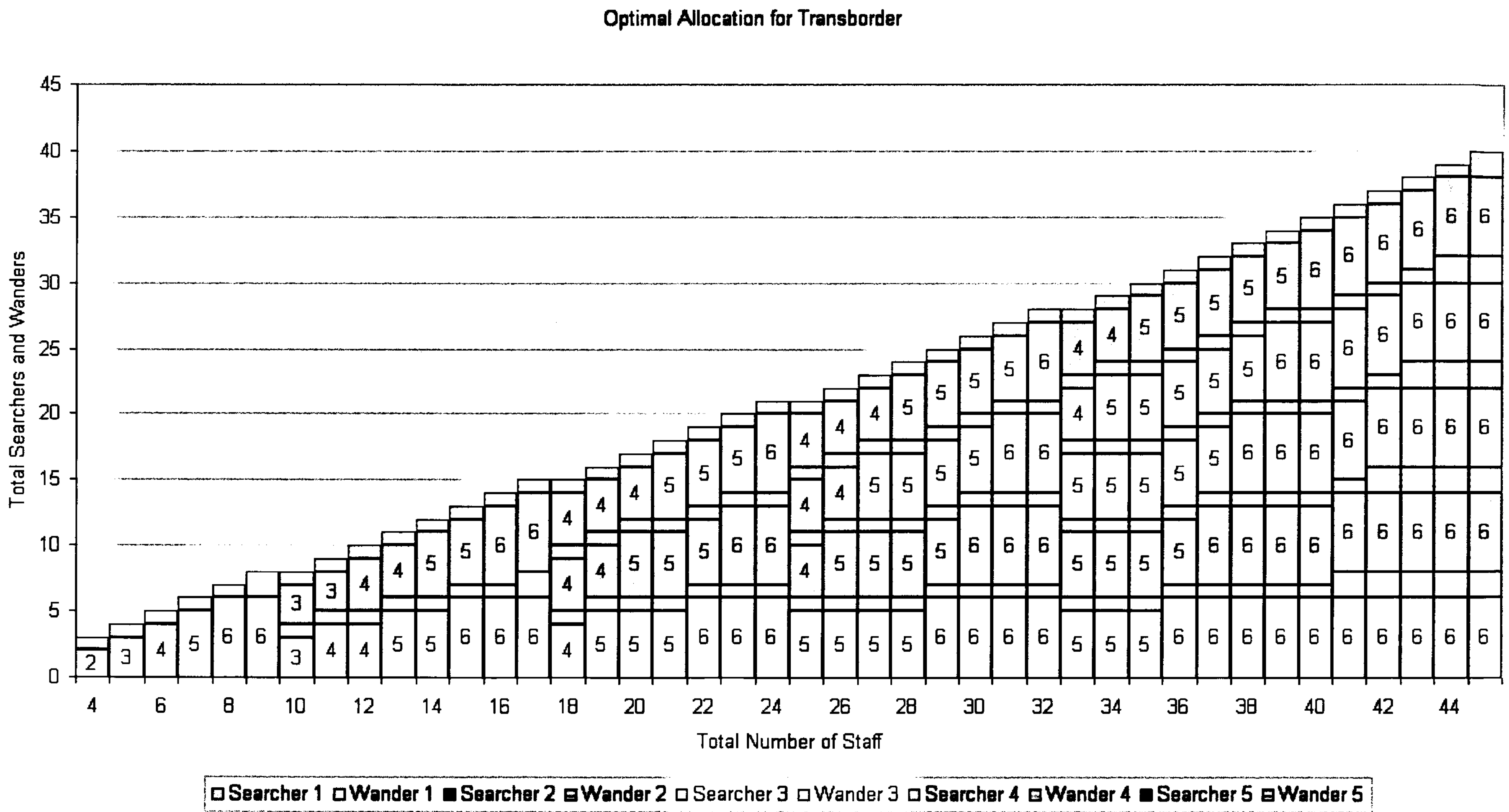

Figure 11: Optimal Staff Allocation for Transborder PBS

With the throughput values and the optimal staffing allocation tables, the effect of alternate staffing configurations can be evaluated. The following example demonstrates the benefits of using the optimal staff allocations at the Transborder pier.

In this example, 16 staff members are available to work at the Transborder pier. From the Optimal Staff Allocation Table for Transborder, the optimal configuration is two equipment lines each with 6 searchers and 1 wander. The estimated maximum throughput is 442 passengers per hour. As a comparison, the staff could be allocated to four equipment lines, each with 2 searchers and 1 wander. One explosive trace worker is assigned to each equipment line. For this configuration, the maximum throughput is estimated using the throughput values from Table 7.

Table 7: Maximum Throughput Values for Transborder PBS

Searchers	Wanders	Maximum Throughput at Transborder PBS (passengers/hour)
2	1	87
3	1	129
4	1	165
5	1	194
6	1	221
2	2	91
3	2	134
4	2	173
5	2	211
6	2	243

The maximum throughput for this alternate configuration is estimated as 4 * 87 = 348 passengers per hour. This is approximately 25% lower than the optimal configuration. Similarly, other configurations can be compared and are shown in Table 8.

Table 8: Comparison of Alternate Staffing Configurations

	Equipment Line 1		Equipment Line 2		Equipment Line 3		Equipment Line 4		
	Number of Searchers	Number of Wanders	Number of Searchers	Number of Wanders	Number of Searchers	Number of Wanders	Number of Searchers	Number of Wanders	Maximum Throughput (Passengers /hour)
Optimal	6	1	6	1					442
Alternate 1	2	1	2	1	2	1	2	1	348
Alternate 2	5	1	3	1	2	1			410

Note: One person per screening line must work the explosives detection machine.

3.2.2 Determining Required Staff Levels

3.2.2.1 Simulation

With the optimal staff allocation determined, the simulation was used to determine minimum required staffing levels to meet the service criteria over a range of passenger arrival rates. Table 9 summarizes the results.

Table 9: Minimum Staff Levels from Simulation

	Minimum Number of Staff Required			
Number of Passengers in 10 minutes	**Domestic South**	**Domestic North**	**Transborder**	**International**
20	5	5	5	5
40	10	10	10	8
60	14	14	14	12
80	18	18	18	15
100	23	23	22	19
120		27	26	22
140		31	31	26
160		36	35	30
180		41	39	33
200			45	36

From Table 9 it can be seen that the staffing requirements at Domestic and Transborder PBS locations are similar and slightly higher than at the International PBS location. The variations are mostly due to the bag search ratios that are similar for the Domestic and Transborder PBS locations and slightly lower at the International PBS location.

Plotting the results in Figure 12 shows the similarities between the PBS locations.

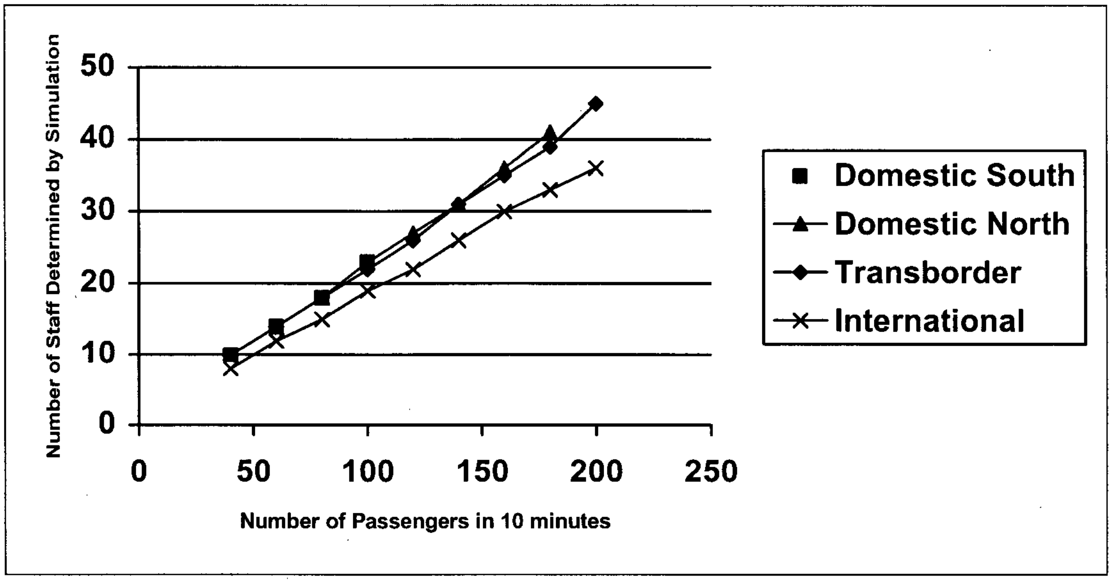

Figure 12: Simulation Results

There is a linear relationship between the number of staff and the number of passengers requiring screening. This supported the use of a lookup table or linear relationship to predict required staffing levels given the forecasted demand.

In determining the minimum staffing levels using the simulation, there was often a clear divide in the percentage of passengers spending less than 10 minutes in the system between the staffing levels that were below 90% and the first that exceeded 90%. That is, for one more staff, the percentage of passengers spending less than 10 minutes in the system jumped from 50% on average to over 90%.

3.2.2.2 Queuing Theory

In parallel to the staffing level determination by simulation, the use of queuing theory to determine the required staffing levels was pursued through application of the Erlang formula. The first step was to determine a service rate or number of passengers screened per hour per staff member. Once the allocation tables were determined, the results were plotted to see if a linear relationship existed. Figures 13, 14, 15, and 16 are the plots for each of the four PBS locations. It can be seen that for all four PBS locations, there is a strong linear relationship between throughput and number of staff working. The slope of the line has units "passengers per hour per staff" and was used to estimate the service rate for one staff member.

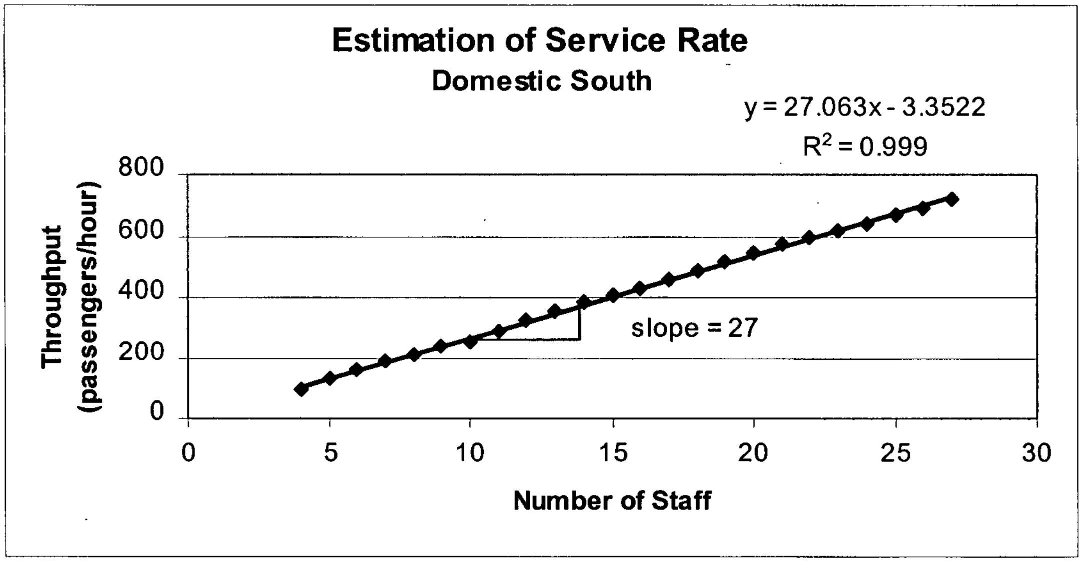

Figure 13: Estimation of Service Rate for Domestic South PBS Location

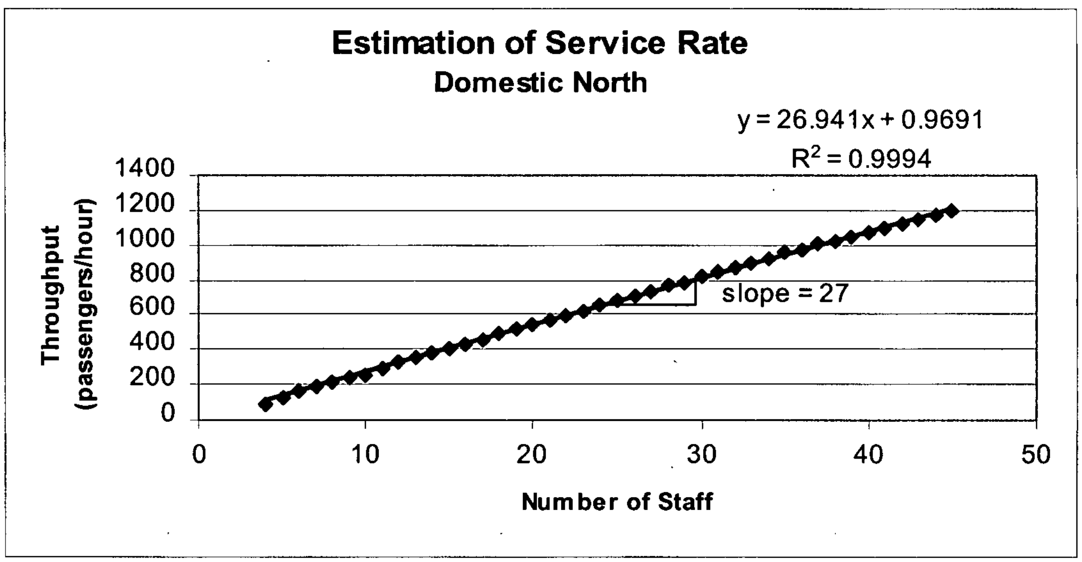

Figure 14: Estimation of Service Rate for Domestic North PBS Location

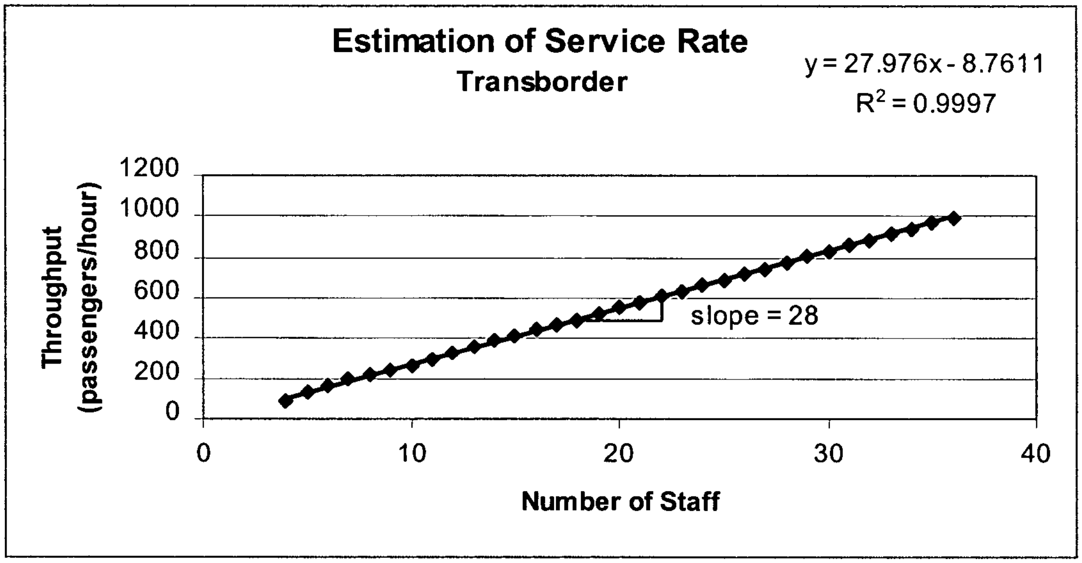

Figure 15: Estimation of Service Rate for Transborder PBS Location

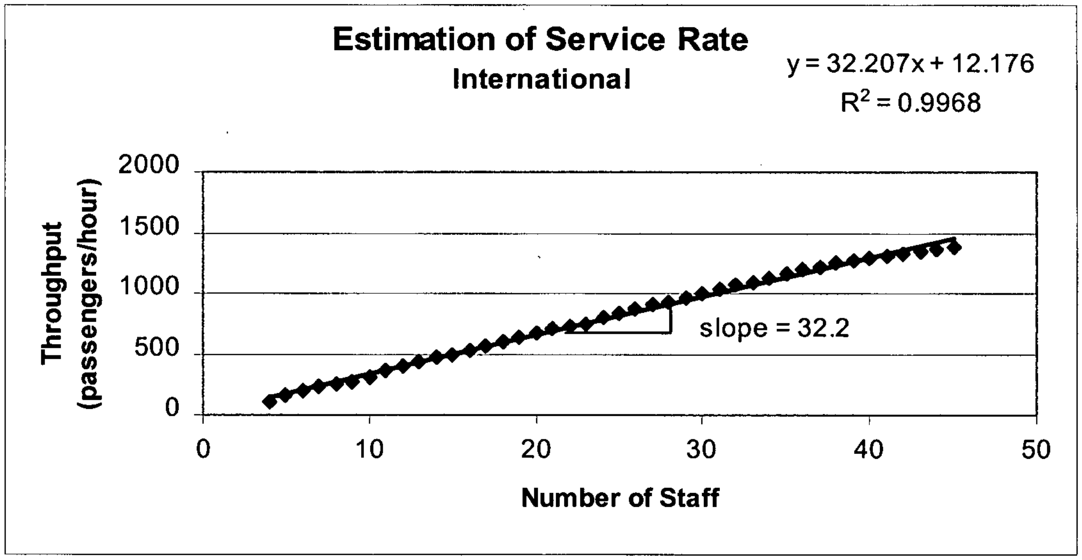

Figure 16: Estimation of Service Rate for International PBS Location

The service rate estimations are summarized in Table 10.

Table 10: Estimation of Service Rates

	Domestic North	Domestic South	Transborder	International
Service Rate (Customers/hour/staff)	27	27	28	32.2

With the service rate calculated, the service criterion formula was used to calculate the required number of staff to meet the service criterion of less than 10% of passengers spending more than 10 minutes at PBS. Originally, this was done with the aid of Queuing ToolPak, an Excel add-in developed by Armann Ingolfsson and Fraser Gallop. While the service criterion was based on total time in system, the toolpack only included a function based on the time in queue. Since determining staffing levels based on time in queue would err on the overstaffing or conservative side, it was considered an acceptable approach. Due to difficulties in accessing the toolpak from all network computers, an Excel macro was later written to replace the function used from the toolpak. The code for this function is included in the appendix.

The results of the queuing theory calculations are compared to the simulation results in Table 11. For each of the PBS locations, the minimum number of staff required as calculated from the queuing model and the simulation are shown for a range of passenger demand.

Table 11: Comparison of Simulation and Queuing Theory Results

	Minimum Number of Staff Required to Meet Service Criterion							
	Domestic South		Domestic North		Transborder		International	
Arrivals (10 minutes)	Queuing Prediction	Simulation Value	Queuing Prediction	Simulation Value	Queuing Prediction	Simulation Value	Queuing Prediction	Simulation Value
20	5	5	5	5	5	5	5	5
40	10	10	10	10	10	10	8	8
60	14	14	14	14	14	14	12	12
80	19	18	19	18	18	18	16	15
100	23	23	23	23	22	22	20	19
120			28	27	27	26	23	22
140			32	31	31	31	27	26
160			37	36	35	35	31	30
180			41	41	40	39	34	33
200					44	45	38	36

There is very good agreement between the simulation results and those determined using queuing theory. For every case except one, the simulation and queuing theory agree exactly or are within one staff member. In all cases where there is a discrepancy between the two results, the queuing theory calculates a higher value and is therefore a conservative solution.

Since there is such good agreement between the simulation and queuing theory, queuing theory is used to estimate the number of staff required for each 10-minute period, given the predicted number of passengers that will appear in that 10-minute period.

In applying the queuing theory, the upper limit on number of staff, 9 per equipment line, must be enforced as well as the minimum number of 3 staff set by Transport Canada.

4 Determination of Shifts

The third task is to provide a set of shifts that will ensure that there are enough staff working in the airport to meet the required staffing levels in each 10 minute interval at all four PBS locations. The schedule is at an aggregate level and covers the combined staffing requirements of the four locations.

4.1 Methodology

Linear programming is used to solve the shift determination problem. Linear programming is a mathematical technique used to find the maximum or minimum of an objective function, subject to a list of constraints. Small linear programming problems can be solved within Microsoft Excel using the Solver. Larger problems must use linear programming software packages.

For this scheduling application, the linear program can be phrased as "Determine which shifts must be worked to minimize the number of hours worked subject to having enough staff working that the required staff in each 10 minute period at each PBS location is met".

Currently, Aeroguard staff are scheduled for 4, 6, 8 or 10 hour shifts. The staff members are paid for their breaks, which are taken at the discretion of the manager. A 4-hour shift does not include a break, but a 10-hour shift includes a 1 hour 20 minute break – usually taken over two periods, an 8-hour shift includes a 1 hour break– usually taken over two periods and a 6 hour shift includes a 30 minute break. At YVRAA's request, 12-hour shift were added to the list of possible shifts. It was decided that breaks would be accounted for by scheduling "surplus" staff. By this method, there will be enough staff scheduled to cover all the breaks required over the day. A constraint is added that the surplus number of staff in each 10 minutes be greater than or equal to the surplus percentage of the required number of staff. Surplus number of staff is the difference between the scheduled number of staff and the minimum required as specified in Task 2. The default value of the surplus percent is 15% and represents the 1-hour break in an 8-hour shift.

An additional constraint was added that allows for a desired mix of shifts of different duration. For example "at least 50% of the shifts be 8 hour shifts".

The methodology assumes that the required staff levels for each 10 minute time interval is known as well as the list of all possible shifts.

The model is formulated as follows:

Model for Linear Program to Determine Number of Each Shift to be Worked in the Airport

Given a set of **s** shifts and a day covering **d** 10 minute intervals

Data:

A_{ij} = 1 if shift j covers time interval i   j = 1 to s, i = 1 to d

0 else

l_i = duration or length of shift i   i = 1 to s

r_i = sum of required staff across the 4 piers for 10 minute time interval i, i = 1 to d

lb_{4hour} = lower bound on percentage of shifts that are 4 hours long

lb_{6hour} = lower bound on percentage of shifts that are 6 hours long

lb_{8hour} = lower bound on percentage of shifts that are 8 hours long

lb_{10hour} = lower bound on percentage of shifts that are 10 hours long

lb_{12hour} = lower bound on percentage of shifts that are 12 hours long

ub_{4hour} = upper bound on percentage of shifts that are 4 hours long

ub_{6hour} = upper bound on percentage of shifts that are 6 hours long

ub_{8hour} = upper bound on percentage of shifts that are 8 hours long

ub_{10hour} = upper bound on percentage of shifts that are 10 hours long

ub_{12hour} = upper bound on percentage of shifts that are 12 hours long

surplus percent = additional hours to be staffed to cover breaks, as percentage of required

Decision variables:

x_i = number of shift i selected, i = 1 to s

Objective function:

Minimize the number of hours worked.

Minimize $\sum x_i l_i$ where l_i = length of shift i

Constraints:

For each of the d 10-minute intervals, the number of staff working must be greater than or equal to the minimum required across all piers.

$Ax \geq r$

For each of the d 10-minute intervals, the surplus staff scheduled must be greater than or equal to the specified surplus percentage multiplied by the minimum required across all piers.

$$Ax - r \geq \text{surplus percent} * r$$

For each of the shift durations, the percentage of the total number of shifts must be greater than or equal to the lower bound.

$$\frac{\sum_{\text{all 4 hour shifts}} x_i}{\sum_{i=1}^{s} x_i} \geq lb_{4\,hour}, \quad \frac{\sum_{\text{all 6 hour shifts}} x_i}{\sum_{i=1}^{s} x_i} \geq lb_{6\,hour}, \quad \frac{\sum_{\text{all 8 hour shifts}} x_i}{\sum_{i=1}^{s} x_i} \geq lb_{8\,hour},$$

$$\frac{\sum_{\text{all 10 hour shifts}} x_i}{\sum_{i=1}^{s} x_i} \geq lb_{10\,hour}, \quad \frac{\sum_{\text{all 12 hour shifts}} x_i}{\sum_{i=1}^{s} x_i} \geq lb_{12\,hour}$$

For each of the shift durations, the percentage of the total number of shifts must be less than or equal to the upper bound

$$\frac{\sum_{\text{all 4 hour shifts}} x_i}{\sum_{i=1}^{s} x_i} \leq ub_{4\,hour}, \quad \frac{\sum_{\text{all 6 hour shifts}} x_i}{\sum_{i=1}^{s} x_i} \leq ub_{6\,hour}, \quad \frac{\sum_{\text{all 8 hour shifts}} x_i}{\sum_{i=1}^{s} x_i} \leq ub_{8\,hour},$$

$$\frac{\sum_{\text{all 10 hour shifts}} x_i}{\sum_{i=1}^{s} x_i} \leq ub_{10\,hour}, \quad \frac{\sum_{\text{all 12 hour shifts}} x_i}{\sum_{i=1}^{s} x_i} \leq ub_{12\,hour}$$

Non-negativity

$x \geq 0$

Integrality

$x = \text{integer}$ Note: this constraint only required if "surplus percent" > 0

The result is the number of each of the s shifts required to cover the total demand

Number of variables s

Number of constraints d + d + 10

4.2 Data

The inputs to the model are the required staffing levels for each PBS location, in the format produced in the previous step, as well as a list of shifts that can be worked. The shifts are specified by the start time and the duration.

Information is also required for the additional constraints regarding surplus staff and shift mix. The default value for the surplus percentage is 15% as this represents the 1-hour break assigned to an 8-hour shift. The user must also specify if there are constraints to the percentage of number of each length of shift. For example, at least 50% of shifts must be 8 hours.

4.3 Results

The output of the linear program is in three forms. The first is a list of all shifts and how many of each shift should be worked.

Aggregate Schedule

	Start Time	End Time	Shift Length	Number Required
1	5:00:00 AM	1:00:00 PM	8 hours	48
2	6:00:00 AM	2:00:00 PM	8 hours	8
3	7:00:00 AM	3:00:00 PM	8 hours	0
4	8:00:00 AM	4:00:00 PM	8 hours	0
5	9:00:00 AM	5:00:00 PM	8 hours	26
6	10:00:00 AM	6:00:00 PM	8 hours	11
7	11:00:00 AM	7:00:00 PM	8 hours	1
8	12:00:00 PM	8:00:00 PM	8 hours	6
9	1:00:00 PM	9:00:00 PM	8 hours	0
10	2:00:00 PM	10:00:00 PM	8 hours	5
11	3:00:00 PM	11:00:00 PM	8 hours	0
12	4:00:00 PM	12:00:00 AM	8 hours	33
13	5:00:00 AM	9:00:00 AM	4 hours	0
14	10:00:00 AM	2:00:00 PM	4 hours	0
15	7:00:00 PM	11:00:00 PM	4 hours	0
16	8:00:00 PM	12:00:00 AM	4 hours	21
17	5:00:00 AM	3:00:00 PM	10 hours	18
18	8:00:00 AM	6:00:00 PM	10 hours	4
19	9:00:00 AM	7:00:00 PM	10 hours	0
20	1:00:00 PM	11:00:00 PM	10 hours	0
21	10:00:00 AM	10:00:00 PM	12 hours	0
22	11:00:00 AM	11:00:00 PM	12 hours	11
			Total	192

Figure 17: Sample Shift Schedule

The second is a graphical representation of the required number of staff and the number of staff scheduled.

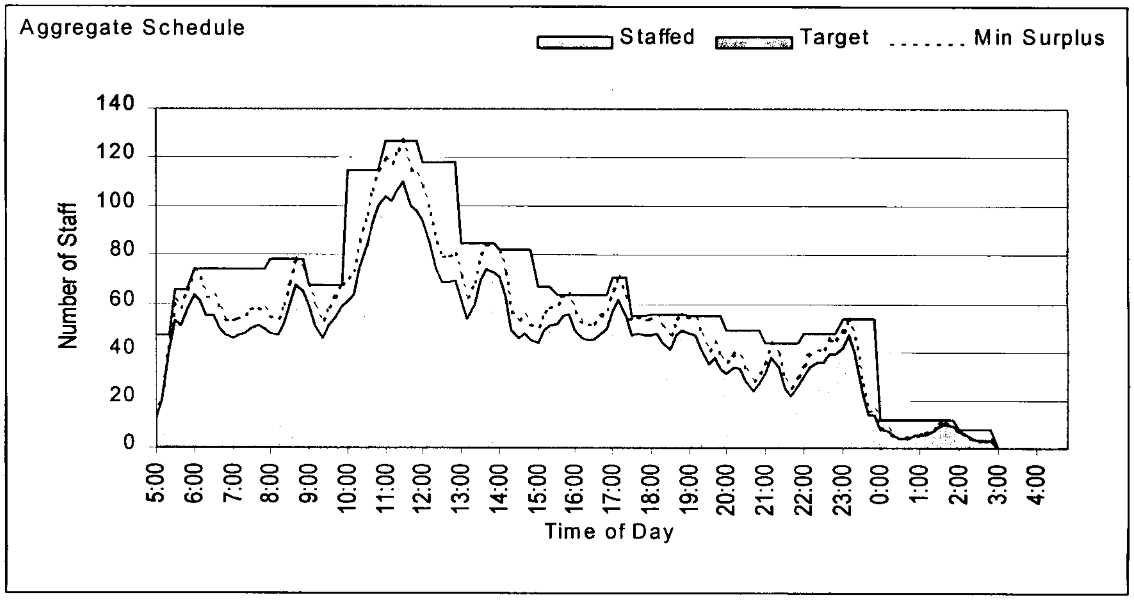

Figure 18: Graphical Representation of Shift Schedule

On the horizontal axis is the time of day and on the vertical axis is the number of staff. The number of staff includes bag searchers, wanders and one explosive detection trace worker per equipment line. The dark shaded area is the sum of the required staff at each of the four PBS locations for each 10-minute interval. The perimeter of the light shaded area is the total number of staff scheduled to be working in the airport and by deduction, the light shaded area is the surplus staff. The surplus is defined as the difference between the scheduled staff and the sum of the required staff at each of the four PBS locations. The dashed line represents the required surplus as specified by the user using the surplus percent variable; the default value is 15%.

The light grey or surplus area indicates times when it may be appropriate for breaks to be taken.

The third output is an hourly staffing allocation table shown in Figure 19. This staffing allocation table provides managerial information to assist in assigning staff to PBS locations and moving staff between PBS locations.

Hourly Staffing Pier Assignment	Total Scheduled		Required DTB North		Required DTB South		Required DTB Transborder		Required Pier DTB International		Excess
		Change		Change		Change		Change		Change	
5:00 - 5:50	**66**		**3**		**17**		**34**		**17**		-5
6:00 - 6:50	**74**	8	**13**	10	**20**	3	**36**	2	**3**	-14	2
7:00 - 7:50	**74**	0	**27**	14	**9**	-11	**22**	-14	**3**	0	13
8:00 - 8:50	**78**	4	**44**	17	**12**	3	**16**	-6	**3**	0	3
9:00 - 9:50	**68**	-10	**31**	-13	**7**	-5	**30**	14	**6**	3	-6
10:00 - 10:50	**115**	47	**34**	3	**9**	2	**34**	4	**29**	23	9
11:00 - 11:50	**127**	12	**45**	11	**11**	2	**31**	-3	**30**	1	10
12:00 - 12:50	**118**	-9	**36**	-9	**14**	3	**38**	7	**16**	-14	14
13:00 - 13:50	**85**	-33	**25**	-11	**19**	5	**36**	-2	**7**	-9	-2
14:00 - 14:50	**82**	-3	**19**	-6	**24**	5	**21**	-15	**10**	3	8
15:00 - 15:50	**67**	-15	**25**	6	**19**	-5	**13**	-8	**9**	-1	1
16:00 - 16:50	**64**	-3	**17**	-8	**16**	-3	**13**	0	**16**	7	2
17:00 - 17:50	**71**	7	**21**	4	**11**	-5	**13**	0	**18**	2	8
18:00 - 18:50	**56**	-15	**18**	-3	**12**	1	**10**	-3	**16**	-2	0
19:00 - 19:50	**55**	-1	**16**	-2	**21**	9	**7**	-3	**16**	0	-5
20:00 - 20:50	**49**	-6	**10**	-6	**24**	3	**3**	-4	**3**	-13	9
21:00 - 21:50	**44**	-5	**15**	5	**20**	-4	**0**	-3	**3**	0	6
22:00 - 22:50	**48**	4	**19**	4	**15**	-5	**0**	0	**9**	6	5
23:00 - 23:50	**54**	6	**17**	-2	**21**	6	**0**	0	**9**	0	7
0:00 - 0:50	**12**	-42	**0**	-17	**0**	-21	**0**	0	**8**	-1	4
1:00 - 1:50	**12**	0	**0**	0	**0**	0	**0**	0	**10**	2	2
2:00 - 2:50	**8**	-4	**0**	0	**0**	0	**0**	0	**7**	-3	1
3:00 - 3:50	**0**	-8	**0**	0	**0**	0	**0**	0	**0**	-7	0

Figure 19: Sample Hourly Staffing Pier Assignment Table

5 Further Discussion of Required Staffing Level

Throughout the project, there was limited access to Aeroguard. With notification, we were able to do data collection, but we were not given information on their current staffing procedures. As a result, validation of the methodologies against current practice was not feasible. Instead, the ARENA simulation was used to conduct a study to assess how well the Required Staffing Level methodology discussed in Chapter 3 performed.

In the study, actual flight schedules were translated into passenger demand using the Passenger Demand Methodology discussed in Chapter 2. Staffing levels were then determined using the Required Staffing Level methodology discussed in Chapter 3. The ARENA simulation would then be run using the demand and staffing levels as input and statistics collected to determine if the service criteria was met. From the results of the simulation runs to determine staffing levels, it was expected that the methodology would create staffing levels that would meet the service criteria. The purpose of this study was to confirm that the service criteria was met, and quantify the realized service level.

The flight schedule for March 2002 was used as input to the Passenger Demand Forecasting methodology to produce daily demand profiles for each of the four PBS locations. This generated 31 * 4 = 124 daily profiles. For each demand profile, the Required Staffing Level methodology was used to determine staffing levels for each 10 minute period in the day. Finally, the optimal staffing allocation tables were used to determine staffing configurations for each of the staffing levels.

10 replications of the ARENA simulation were run with the demand profile and required staffing levels, and the time in system for each of the passengers was recorded. The percentage of passengers spending 10 minutes or less in the system was calculated for each day, then these numbers were averaged over the 31 days. The following tables of results show that the staffing rules meet or exceed the desired service criteria.

Table 12 shows the overall percentage of passengers who spent less than 10 minutes in the system. These numbers can be compared to the service criteria of 90% of passengers spending less than 10 minutes in the system to see that the service criteria is met for all PBS Locations.

Table 12: Service Performance for Staffing at Methodology Levels

	Percentage of Passengers Spending 10 minutes or less in the System			
	Domestic South	**Domestic North**	**International**	**Transborder**
Average	99.9%	100.0%	99.7%	99.8%
Maximum	100.0%	100.0%	99.9%	100.0%
Minimum	99.7%	99.9%	99.0%	99.5%

Since the 10-minute level was so easily met, further analysis was done to determine the time that 90% of passengers spent in system. Over all simulation days, 90% of passengers spent less than the times in Table 13, in the system. On average, 90% of the passengers spent less than 4 minutes in the system when staffed according to the methodologies. These numbers represent the averages over the 31 days of March and the small standard deviation shows that there was little variation over the 31 days.

Table 13: 90th Percentile Service Values for Staffing at Methodology Levels

	90th Percentile of Passenger Time in System			
	Domestic South (minutes)	**Domestic North** (minutes)	**International** (minutes)	**Transborder** (minutes)
Average	3.36	3.16	3.36	3.56
Standard Deviation	0.09	0.02	0.12	0.05
Maximum	3.55	3.20	3.79	3.64
Minimum	3.19	3.12	3.21	3.46

It is interesting to note that there is little variation between PBS locations, and across the 31 days. March 2002 included the peak travel week corresponding to Spring Break for school children. This would suggest that the staffing methodology is adaptable to both peak and low periods and performs consistently over many time periods.

These results of this study show that the proposed staffing methodology succeeds in meeting the service criteria across all four PBS locations, different days of the week and over peak and low periods. These results also open the question, "Is the staffing methodology too generous?"

As was discussed in Chapter 3, two methods were investigated for setting the minimum staffing levels. In the first, the simulation was run at constant passenger demand rate and the staffing level was increased until the service criterion was met. The table below is a sample result of those results for the Transborder PBS location.

Table 14: Simulation Results for Setting Staffing Levels at Transborder PBS Location

Number of Passengers in 10 minutes	Number of Staff	Percentage of Passengers Spending less than 10 minutes in the System
20	4	3.07%
20	5	**93.57%**
40	8	9.20%
40	9	87.25%
40	10	**98.55%**
60	12	9.50%
60	13	78.17%
60	14	**99.91%**
80	16	8.94%
80	17	24.66%
80	18	**99.43%**
100	20	13.11%
100	21	53.07%
100	22	**99.78%**
120	25	14.68%
120	26	**95.65%**
140	29	23.56%
140	30	75.21%
140	31	**99.93%**
160	33	14.18%
160	34	56.69%
160	35	**99.68%**
180	38	32.83%
180	39	**96.90%**
200	42	19.76%
200	43	24.99%
200	44	66.26%
200	45	**99.89%**

A few things can be said about the results in Table 14. First, when a staffing level was reached that exceeded the service criteria, a very high proportion of the passengers spent less than 10 minutes in the system. For the Transborder location, for all but two of the passenger demand rates, over 95% of passengers spent less than 10 minutes in the system. With this information, it is not surprising that the staffing methodology resulted in such a high percentage of passengers meeting the service criteria. The second point of interest in the tables is the considerable jump in service level between the staffing level meeting the service criteria and one staff member below this level. In only one case is the service level at one fewer staff member even close to 90%. This would suggest that all the staff demanded by the staffing methodology are required to meet the service criteria.

The second method investigated was queuing theory, and this was selected for the methodology as the results matched the simulation results so well. One of the fundamental assumptions in the queuing theory is that the average service rate must exceed the average demand rate. This translates into a minimum staffing level for each passenger demand. In the case of the multiple server model, the product of the number of servers and the average service rate must exceed the average passenger demand rate. It was noted during the development of the staffing level methodology that for many of the passenger demand rates, the staffing level was set by this minimum staffing level. A comparison was done of the minimum staffing level and staffing levels from the methodologies to determine how the two varied. The following table and graph shows that for all values of passengers in 10 minutes, the methodology is at most one staff member above the minimum level.

Table 15: Differences in Staffing Levels

Passengers in 10 minutes	Minimum Staffing Level	Staffing Level from Methodology	Difference	Passengers in 10 minutes	Minimum Staffing Level	Staffing Level from Methodology	Difference
0	0	0	-	115	25	26	1
1	1	1	-	120	26	27	1
2	1	1	-	125	27	28	1
3	1	2	1	130	28	29	1
4	1	2	1	135	29	30	1
5	2	2	-	140	31	31	-
10	3	3	-	145	32	32	-
15	4	4	-	150	33	33	-
20	5	5	-	155	34	34	-
25	6	6	-	160	35	35	-
30	7	7	-	165	36	36	-
35	8	8	-	170	37	37	-
40	9	10	1	175	38	39	1
45	10	11	1	180	39	40	1
50	11	12	1	185	40	41	1
55	12	13	1	190	41	42	1
60	13	14	1	195	42	43	1
65	14	15	1	200	43	44	1
70	16	16	-	205	44	45	1
75	17	17	-	210	46	46	-
80	18	18	-	215	47	47	-
85	19	19	-	220	48	48	-
90	20	20	-	225	49	49	-
95	21	21	-	230	50	50	-
100	22	22	-	235	51	51	-
105	23	23	-	240	52	52	-
110	24	25	1	245	53	54	1

Indeed there are many passenger demands for which the methodology sets the staffing level above the minimum required. Had the staffing for each demand been set by the minimum level, it may have been interesting to see if different time intervals, other than 10-minutes, produced different staffing levels.

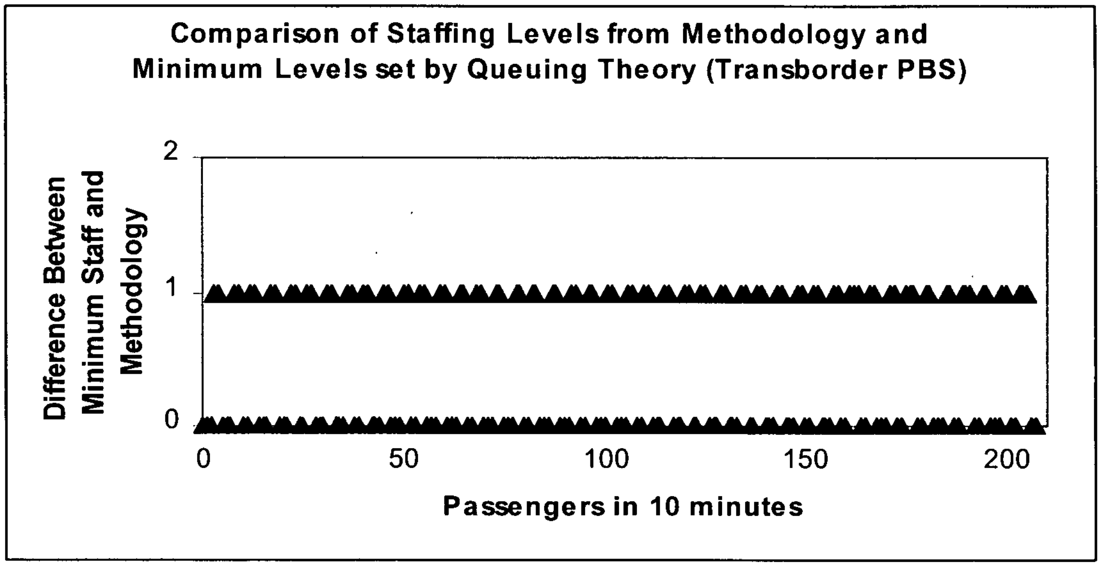

Figure 20: Comparison of Methodology to Minimum Staffing Levels

These results give some guidelines in reducing staffing levels from those generated by the methodology. The results from the simulation suggests a large drop in service if one too few staff members are scheduled. The results from the queuing theory remind us that there is theoretical minimum number of staff for each passenger demand, and that this is at most, one staff member below the level suggested by the methodology.

The first step in reducing staffing levels from those generated by the methodology was to repeat the evaluation study using the theoretical minimum staffing levels. It should be noted that Transport Canada requires a minimum staffing level of three (3) for each PBS location and this is used in this study as well. The results are summarized in the tables below.

Table 16: Service Performance for Staffing at Minimum Levels

	Percentage of Passengers Spending 10 minutes or less in the System			
	Domestic South	**Domestic North**	**International**	**Transborder**
Average	99.7%	99.8%	99.3%	99.6%
Maximum	99.9%	100.0%	99.7%	99.9%
Minimum	99.2%	99.5%	98.7%	99.1%

Table 17: 90th Percentile Service Values for Staffing at Minimum Levels

	90th Percentile of Passenger Time in System			
	Domestic South (minutes)	Domestic North (minutes)	International (minutes)	Transborder (minutes)
Average	3.71	3.61	3.67	3.72
Standard Deviation	0.14	0.12	0.18	0.06
Maximum	4.24	3.95	4.09	3.82
Minimum	3.53	3.38	3.44	3.57

From Tables 16 and 17 we can see that the service criterion is still met if the staffing levels are set by the minimum levels required by the queuing theory and Transport Canada. This may not be surprising as the staffing levels are equal to the theoretical minimum for many passenger demand rates, as was shown in Chapter 3.

Finally, the evaluation exercise was repeated at the minimum staffing level less one. In theory these levels should not meet the service level as the demand exceeds the available service except in the periods when minimum staffing is set by the Transport Canada regulations.

Table 18: Service Performance for Staffing at Minimum Levels Less One

	Percentage of Passengers Spending 10 minutes or less in the System			
	Domestic South	Domestic North	International	Transborder
Average	91.6%	96.6%	63.8%	96.6%
Maximum	96.3%	99.2%	78.0%	98.6%
Minimum	76.6%	94.4%	43.5%	94.4%

Table 19: 90th Percentile Service Values for Staffing at Minimum Levels Less One

	90th Percentile of Passenger Time in System			
	Domestic South (minutes)	Domestic North (minutes)	International (minutes)	Transborder (minutes)
Average	8.86	5.15	40.6	5.33
Standard Deviation	2.86	0.68	13.3	0.43
Maximum	16.18	6.64	79.98	6.29
Minimum	6.11	4.78	21.62	5.13

The results in Tables 18 and 19 show a decrease in the service levels when the staffing level is allowed to drop below the theoretical minimum. The results are not entirely consistent, as it appears that that the Domestic North and Transborder PBS locations still meet the service criterion while the Domestic South and International PBS locations do not. When averaged over the 31 days of the March, the Domestic South results are above the 90% level, but individual days can drop as low as 76.6%. The results for the International PBS location are far worse without ever achieving even 80% of passengers spending less than 10 minutes in the system over the 31 days. We can see from Table 19 that the low staffing can result in some passengers spending over 80 minutes in the system.

These results suggest that the methodology could be modified to set the staffing levels at the theoretical minimum without failing the service criterion.

6 Conclusions

6.1 Summary

In this work we have utilized many techniques from the Operations Research toolbox to develop a methodology for scheduling of Pre-Board Screening staff at the YVR Airport. Forecasting passenger demand was done through application of a triangular distribution to a flight schedule. This simple distribution allows for both an intuitive understanding and implementation in Excel which is accessible to many people. Determining staffing levels to meet the forecasted demand required both the use of simulation and queuing theory and again demonstrated the robustness of the simple queuing model to approximate more complicated systems. Finally, linear programming was used to assign shifts to cover the calculated staffing requirement. The entire methodology is possible using Excel spreadsheets and each of the steps can be solved quickly.

6.2 Future Research

The focus of this project was to find a methodology to aid in scheduling that could be easily executed. The result is a daily forecast of demand and a daily shift schedule. Obviously, shift scheduling is more involved than the generation of daily requirements and an obvious extension would be to create weekly schedules and assign staff members to the shifts. This work is not as simple as it first sounds and would be a project on its own, well beyond the scope of this work.

The forecasting methodology is both simple and hopefully somewhat intuitive. While we have shown that one triangular distribution for each PBS location works well, it would be interesting to add the flexibility of different triangles for different time of day. For example, it is known that the current triangles are not appropriate for early morning flights, though an adjustment has been made in the methodology to account for this. In order to compare different distributions, a quantatative measure of goodness of fit is required. Some thought is required to develop the criteria of a good approximation. It is our feeling that capturing the peaks and valleys at a comparable magnitude is more important than a sum of errors.

Also with the Passenger Generator, it would be useful to add the effects of external influences that might alter the triangular distribution. For example, the cruise ship busses that deliver Transborder passengers to the airport on their schedule rather than when the passengers may have chosen based on their departure time. This effect has been seen in the AIF data and it would be interesting to include it in the methodology.

As discussed in Chapter 5, the Required Staffing Level methodology appears to over staff the PBS locations. Further studies could be done to investigate why the general queuing model appears to be applicable to this more complicated system. It would also be interesting to determine if the methodology is sensitive to the length of time interval. 10 minutes was chosen as the original AIF data was available in this time increment. It seemed that there was sufficient fluctuation at this level that aggregation to higher increments would lose some detail. Also, observation showed that staffing configurations on the PBS locations could vary as often as every 10 minutes during peak times.

7 References

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8 Appendix

8.1 Appendix: Passenger Distribution Calculation

The triangular distribution was used to allocate passengers from each flight in the daily flight schedule, to 10-minute intervals over the day.

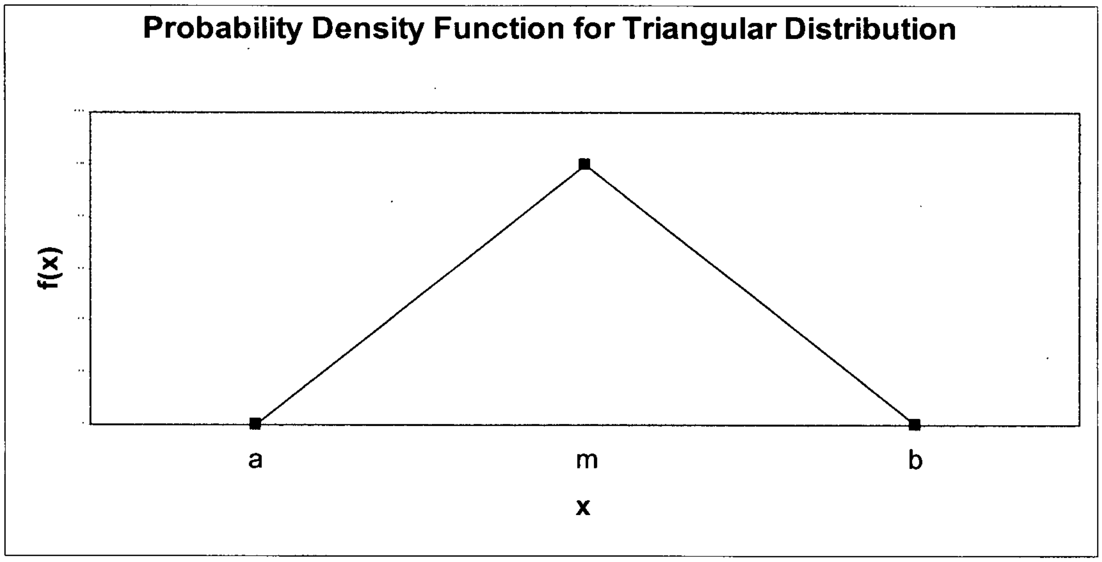

Figure 21: Triangular Distribution

The triangle is broken into 10-minute intervals, and the number of passengers expected to appear in each 10-minute interval is calculated.

As shown in Figure 21, combining the triangles for a day's flights results in the forecast for demand at a pre-board screening pier.

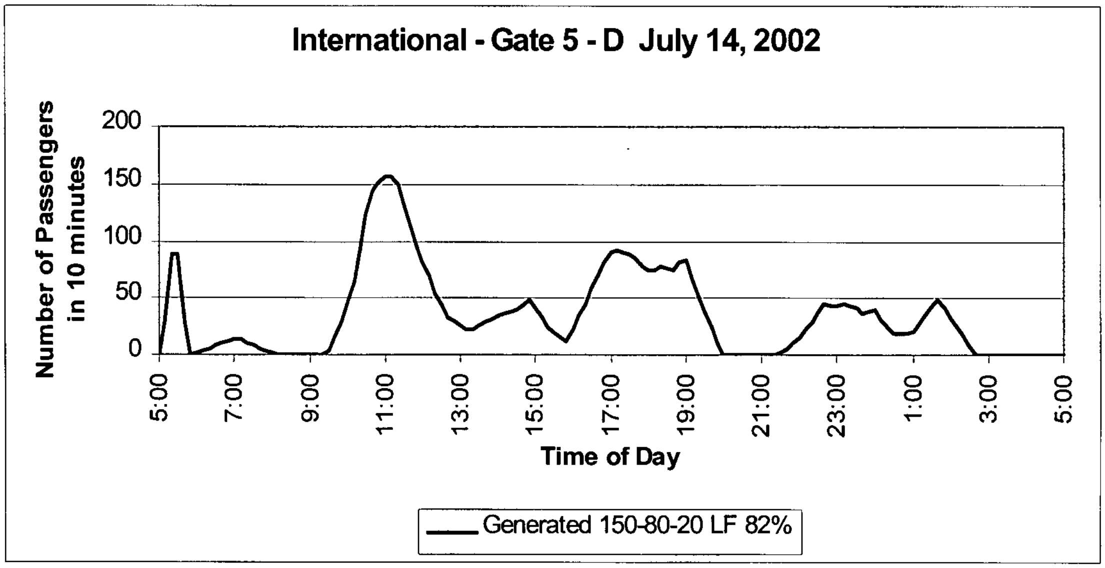

Figure 22: Sample of Daily Passenger Demand

The only exceptions to the triangle are at the beginning of the day. Some flights depart within an hour of the airport opening. For these flights, the first passenger cannot arrive earlier than 5:00, when the airport opens. For these flights, the time of the first passenger is set as the opening time of the airport, and the most likely time is set as either the default most likely time or half way between the first and last passenger, which ever is closer to the last passenger time.

An Excel function has been written to calculate the number of passengers expected to appear for screening in the next 10-minute interval, given the flight departure time, the expected number of passengers on the flight, and parameters on the distribution.

To find the number of passengers arriving in a 10-minute time interval, determine the probability of being in that 10-minute time interval, then multiply by the number of passengers. The probability of being in time interval t, t +10 is F(t+10) – F(t).

Number of passengers expected in time interval t, t+10 is number of seats* load factor *[F(t+10) – F(t)]

F(t) is calculated from the probability distribution function for a triangular distribution which is given as

$$f(x)=\begin{cases}\dfrac{2(x-a)}{(m-a)(b-a)} & for\ a\le x\le m\\ \dfrac{2(b-x)}{(b-m)(b-a)} & for\ m\le x\le b\\ 0 & otherwise\end{cases}$$

$$F(x)=Probability(X\le x)$$

$$F(x)=\int_0^x f(y)dy$$

$$F(x)=0 \qquad for\ x\le a$$

$$F(x)=\int_a^x \frac{2(y-a)}{(m-a)(b-a)}dy$$

$$=\frac{(x-a)^2}{(m-a)(b-a)} \qquad for\ a\le x\le m$$

$$F(x)=\int_a^m \frac{2(y-a)}{(m-a)(b-a)}dy+\int_m^x \frac{2(b-y)}{(b-m)(b-a)}dy$$

$$=\frac{m-a}{b-a}-\frac{(b-m)^2-(b-x)^2}{(b-m)(b-a)}$$

$$=1-\frac{(b-x)^2}{(b-m)(b-a)} \qquad for\ m\le x\le b$$

$$F(x)=1 \qquad for\ x>b$$

8.2 Appendix: Queuing Theory Calculations

Queuing theory applies to processes in which a client arrives and waits for service from a resource. In the simplest queues, service consists of time spent with a single resource. If there are multiple resources then multiple clients can receive service in parallel. In an M/M/c queue, there are c resources working in parallel. Each of the c resources works the same way and delivers service in a service time that is exponentially distributed. The clients arrive in such a way that the time between arrivals is also exponentially distributed. For this case, there are formulae to estimate c, the number of staff required, to serve all clients in a given time interval, a given percentage of the time. These calculations are commonly used in "Call Centre Calculators" which estimate staffing levels for call centers.

The Queuing ToolPak (QTP) a Microsoft Excel add-in developed by Armann Ingolfsson and Fraser Gallop at the University of Alberta contains functions for these calculations. http://www.bus.ualberta.ca/aingolfsson/QTP/

The formula can be found in Fundamentals of Queuing Theory 2nd Edition by Donald Gross and Carl M. Harris.

Probability distributions of the waiting times $W_q(t)$ (From page 90)

$$W_q(t) = \begin{cases} 1 - \dfrac{c\left(\lambda/\mu\right)^c}{c!\left(c - \lambda/\mu\right)} p_0 & (t = 0) \\[2ex] \dfrac{\left(\lambda/\mu\right)^c \left(1 - e^{-(\mu c - \lambda)t}\right)}{(c-1)!\left(c - \lambda/\mu\right)} p_0 + W_q(0) & (t > 0) \end{cases}$$

where

c = number of resources
λ = number of clients to arrive in a unit of time
μ = number of clients one resource can serve in a unit of time
p_0 = steady state percentage of time there are no clients in the system (waiting or being served)

$$p_0 = \left[\sum_{n=0}^{c-1} \frac{1}{n!}\left(\lambda/\mu\right)^n + \frac{1}{c!}\left(\lambda/\mu\right)^c \left(\frac{c\mu}{c\mu - \lambda}\right)\right]^{-1} \qquad \lambda/c\mu < 1$$

The solution method is to solve for $W_q(t)$ for t = service criteria time and c incrementing from integer (λ/μ) until $W_q(t) \geq$ service percentage. In this case t = 10 minutes and the service percentage is 90%.

The Excel function is listed below:

Function CalcServers(lambda, mu, t, Service_percent)

'***Uses formulae from Fundamentals of Queuing Theory 2nd Edition
'Donald Gross and Carl M Harris
'Based on function in Queuing Toolpack from U of A website
'lambda - arrival rate of passengers passengers/unit time
'mu - service rate of one staff passengers served/ unit time

```
't = service time criteria            in unit time
'Service_percent = percentage of passengers that must be screened in t time   fraction
Dim factorial_matrix(100)
c_factorial = 1
For i = 1 To 100
   c_factorial = c_factorial * I
   factorial_matrix(i) = c_factorial
Next I
'if no passengers arrive, then no staff is required
'minimum staff levels set on the spreadsheet
If lambda = 0 Then
   c = 0
   GoTo done
End If
r = lambda / mu
'theory only valid if c*mu< lambda
min_allowed = Int(r)
'Perform the calculation for each c until the service criteria (90%) is met
For c = min_allowed + 1 To 100


   n_factorial = 1
   p_zero_sum = 1
   r_to_power_of_n = 1
   For n = 1 To c – 1
      n_factorial = n_factorial * n
      r_to_power_of_n = r_to_power_of_n * r
      p_zero_sum = p_zero_sum + r_to_power_of_n / n_factorial
   Next n
   c_factorial = factorial_matrix(c)
      p_zero = p_zero_sum + ((r ^ c) / c_factorial) * (c * mu / (c * mu - lambda))
   p_zero = 1 / p_zero
   W_queue_zero = 1 - (c * (r ^ c)) / (c_factorial * (c - r)) * p_zero
   w_queue = (r ^ c) * (1 - Exp(-(mu * c - lambda) * t)) / ((c_factorial / c) * (c - r)) * p_zero
+ W_queue_zero
   If w_queue > Service_percent Then GoTo done
Next c
done:
   CalcServers = c
End Function
```