PERFORMANCE ANALYSIS OF A TRANSMIT DIVERSITY SCHEME IN CORRELATED FADING WITH IMPERFECT CHANNEL ESTIMATION

by

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Abstract

Exact closed-form expressions are derived for the bit error rate of Simple Transmit Diversity (STD) with 2 transmit and *M* receive antennas in time-selective, spatially independent Rayleigh fading with imperfect channel estimation and in non time-selective, spatially correlated Rayleigh fading with imperfect channel estimation. The performance analysis is also presented. For spatially independent fading, it is found that for the same values of the channel gain time correlation coefficient ρ_t and the channel gain estimation error correlation coefficient ρ_e, the error performance in non time-selective fading with imperfect channel estimation is worse than in time-selective fading with perfect channel estimation.

The BER floor resulting from channel estimation errors and time-selective fading is determined. For the same values of ρ_t and ρ_e, say ρ, the error floor limits are approached at lower signal to noise ratio (SNR) values for $(\rho_t = 1, \rho_e = \rho)$ than for $(\rho_t = \rho, \rho_e = 1)$.

The effects of channel estimation errors on error performance of STD and Maximum Ratio Combining (MRC) were compared and it was shown that for large values of signal to noise and estimation error to noise ratios, STD suffers a 3 dB loss compared to MRC in non-time selective, spatially independent fading.

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Chapter 1 Introduction

The primary objective of this thesis is to study the effectiveness of space diversity techniques in improving the performance of wireless communication systems such as cellular systems. Currently cellular systems enjoy widespread use around the world since the introduction of the Advanced Mobile Phone Service (AMPS) in the United States in 1983 [1]. Currently the number of cellular phone subscribers worldwide is 1.3 billion [2]. With the introduction of new applications in 3G systems and beyond, it is anticipated that much research and development activity will be needed.

Mobile radio or indoor wireless communication channels commonly suffer from signal fading which can cause severe performance degradation [3], [4]. The adverse effects of fading can be mitigated by employing diversity techniques which exploit the randomness in signal propagation to establish independent (or at least highly uncorrelated) signal paths for communication so as to reduce the probability that all the signal paths will fade simultaneously. There are a number of diversity techniques which can provide significant link improvement. Depending on the propagation mechanisms, these may include [3]: space diversity, frequency diversity and time diversity.

1.1 Space Diversity

Space diversity, also known as antenna diversity, is one of the most popular forms of diversity used in wireless communications. It is conceptually simple and relatively easy to implement. The method is based upon the principle of using two or more antennas at the base station or at the mobile terminal to provide diversity. Conventional cellular communication systems consist of an elevated base station antenna and an antenna at the mobile unit which is close to the ground. The existence of a direct path between the transmitter and the receiver is not guaranteed and the possibility of a number of scatters in the vicinity of the mobile unit suggests Rayleigh fading in the wireless channel. If there are more than one antenna at the base station or the mobile unit, to achieve decorrelation among the received signals, separation between antennas on the order of several tens of wavelengths is required at the base station and at least one half wavelength at the mobile unit [1]. Space diversity techniques include Selection Diversity (SD), Maximal Ratio Combining (MRC) and Equal Gain Combining (EGC) [5].

Among these three diversity techniques, MRC is theoretically the optimum diversity combining method for branch signals [6]. It provides the highest average output signal-to-noise (SNR) and the lowest probability of occurrence of deep fades. All the branch information is used to improve the overall receiver performance by cophasing the signals from different branches, weighting them according to their individual SNR's, and then summing the cophased and weighted signals. It is well-known that the output SNR is equal to the sum of the individual SNRs [3].

1.2 Simple Transmit Diversity

The classical MRC technique uses multiple antennas at the receiver to obtain the optimum performance. However, in cellular communications systems, this approach is not desirable for the mobile handsets because of cost, size and power considerations. Recently, a technique known as simple transmit diversity (STD) was proposed by Alamouti [7]. This technique employs two transmit antennas and one receive antenna to achieve the same diversity order as MRC. Two signals are simultaneously transmitted from the two antennas during a given symbol period and a transformed version of the signal pair is transmitted during the next symbol period. The technique can also be used for space-frequency coding. The proposed scheme was shown to have the same error performance in non time-selective channels as MRC when perfect channel estimation is available at the receiver. STD can be easily generalized to two transmit antennas and M receive antennas to provide a diversity order of $2M$. It does not require any feedback from the receiver to the transmitter and involves small computation complexity. As a result, STD has now been incorporated in third generation cellular communication systems [8], [9]. It is thus important to understand its performance under non-ideal conditions.

With imperfect channel estimation, STD was shown in [10] to have a poorer performance than MRC. Bit error rate (BER) curves for STD in Rayleigh fading with imperfect channel estimation were obtained by computer simulation. In [11], the performance of STD in time-selective Rayleigh fading channels was investigated with perfect channel estimation and an approximate BER expression was obtained.

In this thesis, closed-form expressions are derived for the BER of STD in time-selective, spatially independent Rayleigh fading with imperfect channel estimation and in non time-selective, spatially correlated Rayleigh fading with imperfect channel estimation. This not only obviates the need for time-consuming simulations but also provides greater insight into the effects of channel estimation errors, time-selectivity and spatially correlated fading.

1.3 Thesis Overview

In Chapter 2, some background and previous related studies are reviewed. The MRC combining technique is described. The STD and MRT scheme [12] are also discussed.

In Chapter 3, exact, closed form expressions are derived for the BER of 2 transmit and M receive antennas STD with time-selective spatially independent Rayleigh fading with imperfect channel estimation and with non time-selective spatially correlated Rayleigh fading with imperfect channel estimation. BER expressions for time-selective spatially independent fading channels with perfect channel estimation or non time-selective spatially independent fading channels with imperfect channel estimation are obtained as special cases. Both binary phase shift keying (BPSK) and quadrature phase shift keying (QPSK) modulation methods are considered.

In Chapter 4, numerical results are provided to illustrate the performance of STD in variety of channel model. In particular, the BER performances of STD and MRC are compared in the presence of imperfect channel estimation and non time-selective fading. In Chapter 5, the main contributions and conclusions of this thesis are summarized and some suggestions for future work are given.

Chapter 2 Background and Related Works

The classical maximal ratio combining (MRC) technique has been shown to be optimum if the channel state is know perfectly [13]. This technique gives the best statistical reduction of fading for any known linear diversity combiner [1].

However, mobile station (MS) receiver diversity may not be desirable for wireless systems because of cost, size and power considerations. Recently, a simple but effective technique, the simple transmit diversity (STD) was proposed in [7]. For the same level of radiated power per transmit antenna, it was shown in [7] that STD in non time-selective Rayleigh faded channels has the same error performance as MRC when perfect channel estimation is available at the receiver. The STD technique can be generalized to two transmit antennas and M receive antennas to provide a diversity order of $2M$.

Maximum ratio transmission (MRT) was proposed in [12] to allow a generalization to an arbitrary number of transmit antennas. However, it requires feedback from the receiver to the transmitter so that the latter can estimate the channel.

In a slow fading channel where the fading channel is treated constant over the symbol duration, assume that the phase shift can be estimated from the received signal without error, coherent detection can be used [13]. Coherent combining systems do not suffer degradation from phase transients [14], therefore, coherent detection is considered more desirable when a large number of diversity branches are employed and it is used for the diversity schemes in this study.

In this chapter, we describe the fading channel model to be used and briefly review MRC, STD and MRT schemes.

2.1 Statistical Models for Fading Channels

When there are a large number of scatterers in the channel that contribute to the signal at the receiver, the central limit theorem suggests that the channel gain can be modeled as a complex Gaussian random process. If the process has zero-mean, the envelope of the channel response at any time instant has a Rayleigh distribution and the phase is uniformly distributed in the interval $(0, 2\pi)$ [13].

The complex channel gain corresponding to the kth branch is denoted by $g_k = \alpha_k \exp(j\theta_k), k = 1,2, \ldots M$, where α_k is Rayleigh distributed with variance $2\sigma_K^2$ and θ_k is uniformly distributed in $(0, 2\pi)$. We can express the channel gain as $g_k = x_k + jy_k$, where $x_k = \alpha_k \cos\theta_k$ and $y_k = \alpha_k \sin\theta_k$, are samples of independent zero-mean Gaussian random variables (r.v.'s) with variance $\sigma_X^2 = \sigma_Y^2 = \sigma_K^2$.

2.2 Maximal Ratio Combining (MRC)

Figure 2.1 shows the baseband representation for MRC with diversity order of M. The received signal form each of the M diversity branches are co-phased and weighted to maximized the received SNR. The received signal on the ith branch corresponding to the transmission of signal s_0 can be written as

$$r_{k, MRC} = g_k s_0 + n_k, \; k = 1 \ldots M, \tag{2.1}$$

where g_k is the independent channel gain and n_k is an independent complex Gaussian r.v. representing the noise and interference at the receiver. For clarity, we will use uppercase letters to

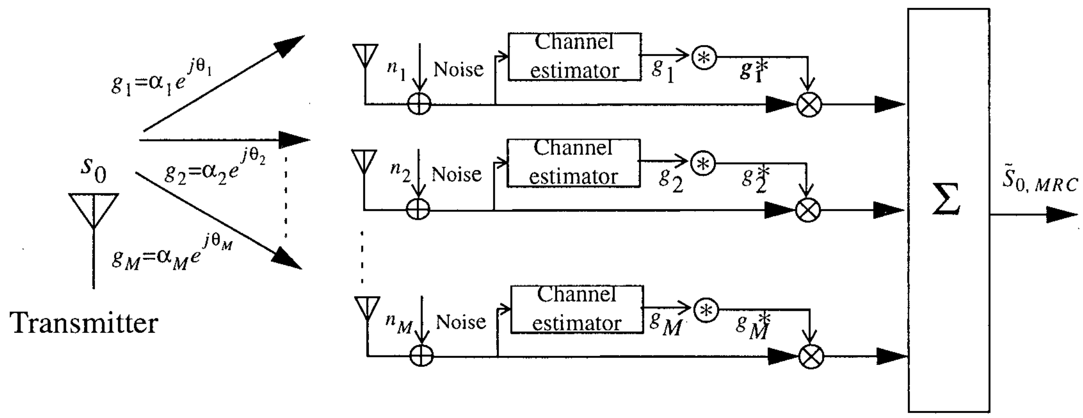

Figure 2.1 MRC with M receive antennas

denote r.v.'s and corresponding lowercase letters to denote their sample values. The receiver combining rule for M branch MRC is as follows:

$$\begin{aligned}
\tilde{s}_{0,\,MRC} &= \sum_{k=1}^{M} g_k^* r_{k,\,MRC} \\
&= \sum_{k=1}^{M} g_k^*(g_k s_0 + n_k) \\
&= \sum_{k=1}^{M} \alpha_k^2 s_0 + \sum_{k=1}^{M} g_k^* n_k \ ,
\end{aligned} \tag{2.2}$$

where $\tilde{s}_{0,\,MRC}$ is the combined signal at the output and * denotes the complex conjugate. The theoretical analysis of the error performance for MRC was discussed in [6] and it was shown that the instantaneous received SNR γ, at the output of the diversity combiner, is the sum of the SNR's on the individual branches, i.e.

$$\begin{aligned}\gamma &= \sum_{k=1}^{M} \gamma_k \\ &= \sum_{k=1}^{M} \frac{\xi \alpha_k^2}{2\sigma_N^2} \\ &= \sum_{k=1}^{M} \frac{\xi}{2\sigma_N^2}(x_k^2 + y_k^2) \ ,\end{aligned} \tag{2.3}$$

where we define the received SNR on the individual branches as $\gamma_k = \frac{\xi \alpha_k^2}{2\sigma_N^2}$. ξ is the energy of s_0 and σ_N^2 is the variance of the real or imaginary component of N_k. It is assumed that the average received energy gain for each diversity channel is equal, i.e. $\sigma_K^2 = \sigma^2$. The average SNR per branch is then

$$\gamma_0 = \frac{\xi}{\sigma_N^2}\sigma^2 \ . \tag{2.4}$$

The probability density function (pdf) of γ is given by [13]

$$p(\gamma) = \frac{\gamma^{M-1}}{\gamma_0^M (M-1)!} e^{-\gamma/\gamma_0}, \gamma \geq 0 , \tag{2.5}$$

The BER is obtained by averaging over the fading channel statistics (2.5), i.e.

$$\begin{aligned}P_{e,MRC} &= \int_0^{\infty} P(\gamma) Q(\sqrt{2\gamma}) d\gamma \\ &= \left[\frac{1}{2}(1-u)\right]^M \sum_{i=0}^{M-1} \left\{ \binom{M-1+i}{i} \left[\frac{1}{2}(1+u)\right]^i \right\},\end{aligned} \tag{2.6}$$

where $u = \sqrt{\frac{\gamma_0}{1+\gamma_0}}$.

2.3 Simple Transmit Diversity (STD)

The STD scheme with one receive antenna is first reviewed, followed by M receive antennas case.

2.3.1 STD with Two Transmit and One Receive Antenna

Figure 2.2 shows the baseband representation of the STD scheme with one receive antenna.

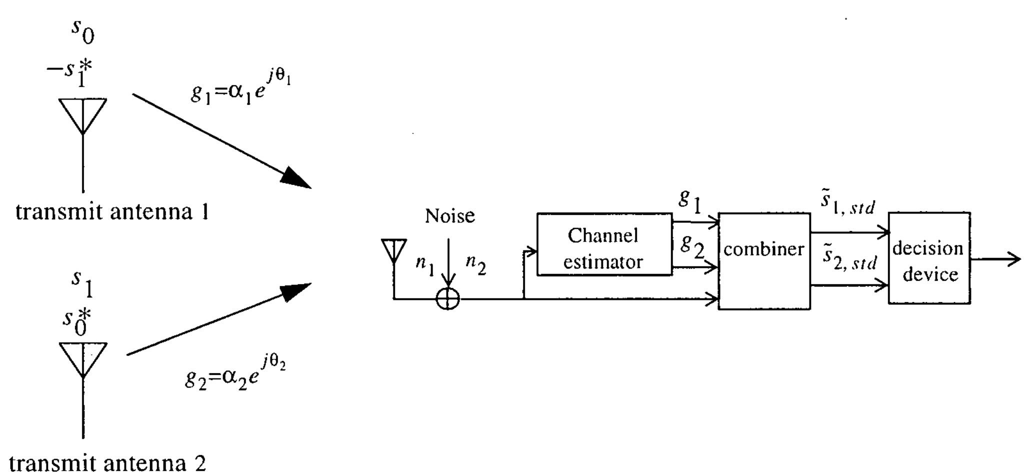

Figure 2.2 STD scheme with one receive antenna

In this scheme, independent and equiprobable data bits are transmitted from each transmit antenna at symbol rate $1/T$. In the first symbol period, s_0 and s_1 are sent from antenna 1 and antenna 2 respectively. In the second symbol period, $-s_1^*$ is transmitted from antenna 1 and s_0^* from antenna 2, where * denotes the complex conjugate operation. The delay spreads are small compared to T and the coherence times are much larger than T, so that the channels are treated as frequency flat and non time-selective, i.e. the received signal in the first and second bit period can be expressed as

$$\begin{aligned} r_1 &= g_1 s_0 + g_2 s_1 + n_1 \\ r_2 &= -g_1 s_1^* + g_2 s_0^* + n_2 \end{aligned} \tag{2.7}$$

where n_1 and n_2 are samples of independent Gaussian r.v.'s representing noise and interference at the receiver at successive intervals.

The decoding of s_0 and s_1 is based on $\tilde{s}_{0,\,std}$ and $\tilde{s}_{1,\,std}$ respectively where

$$\begin{aligned} \tilde{s}_{0,\,std} &= g_1^* r_0 + g_2 r_1^* \\ &= (\alpha_1^2 + \alpha_2^2) s_0 + g_1^* n_0 + g_2 n_1^* \\ \tilde{s}_{1,\,std} &= g_1^* r_0 - g_2 r_1^* \\ &= (\alpha_1^2 + \alpha_2^2) s_1 + g_1^* n_0 - g_2 n_1^* \ . \end{aligned} \tag{2.8}$$

The resulting signals, $\tilde{s}_{0,\,std}$ and $\tilde{s}_{1,\,std}$, are then sent to the maximal likelihood detector. The combined signals in (2.8) are equivalent to those obtained from two-branch MRC except for the phase rotations on the noise component which do not degrade the effective SNR. Thus assuming perfect channel estimation in non time-selective fading, STD scheme provides the same error performance as 2-branch MRC for a fixed value of the radiated power per transmit antenna.

2.3.2 STD with Two Transmit and *M* Receive Antennas

Figure 2.3 shows the baseband representation of the STD scheme with two transmit and M receive antennas.

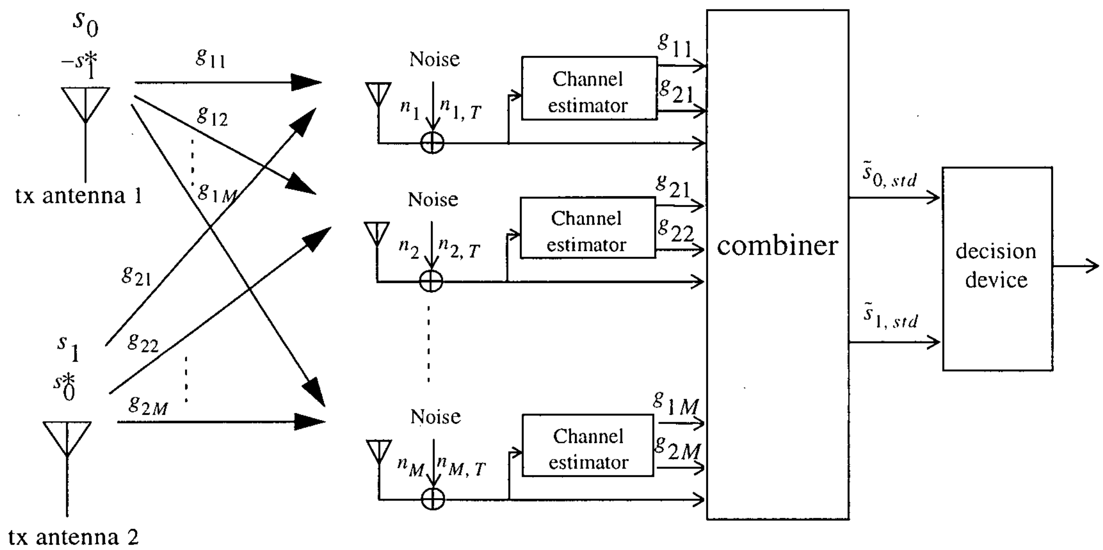

Figure 2.3 The STD scheme with M receivers

The encoding and transmission sequence of the information symbols is identical to the case of a single receiver. The channel gains from transmit antenna j to receive antenna k is denoted by $g_{jk}, j = 1, 2, k = 1, 2, \ldots M$.

The received signals at the ith receive antenna are:

$$\begin{aligned} r_k &= g_{1k}s_0 + g_{2k}s_1 + n_k \\ r_{k,T} &= -g_{1k}s_1^* + g_{2k}s_0^* + n_{k,T} \qquad k = 1, 2, \ldots M \end{aligned} \tag{2.9}$$

where r_k is the signal received in the first symbol period and $r_{k,T}$ is the signal received in the second symbol period. The output signals are expressed as

$$\begin{aligned}
\tilde{s}_{0,std} &= \sum_{k=1}^{M} g_{1k}^{*} r_k + \sum_{k=1}^{M} g_{2k} r_{k,T}^{*} \\
&= \sum_{k=1}^{M} (\alpha_{1k}^2 + \alpha_{2k}^2) s_0 + \sum_{k=1}^{M} g_{1k}^{*} n_k + \sum_{k=1}^{M} g_{2k} n_{k,T}^{*} \\
\tilde{s}_{1,std} &= \sum_{k=1}^{M} g_{2k}^{*} r_k - \sum_{k=1}^{M} g_{1k} r_{k,T}^{*} \\
&= \sum_{k=1}^{M} (\alpha_{1k}^2 + \alpha_{2k}^2) s_1 + \sum_{k=1}^{M} g^{*}_{2k} n_k - \sum_{k=1}^{M} g_{1k} n_{k,T}^{*} .
\end{aligned} \tag{2.10}$$

Equation (2.10) shows that the error performance of STD with two transmit and M receive antennas is equal to that of $2M$ branch MRC.

2.4 Maximum Ratio Transmission (MRT)

The MRT scheme [12] was proposed to make use of an arbitrary number of transmit antennas. The channel gain matrix can be represented by

$$G = \begin{bmatrix} g_{11} & \cdots & g_{1M} \\ \vdots & \ddots & \vdots \\ g_{N1} & \cdots & g_{NM} \end{bmatrix} \tag{2.11}$$

where $g_{jk}, j = 1, 2 \ldots, N, k = 1, 2 \ldots, M$ represents the channel gain from transmit antenna j to receive antenna k.

As shown in Figure 2.4, the symbol to be transmitted, s, is weighted by a transmit weighting vector $\underline{V} = [v_1, v_2 \ldots v_N]$ with

$$\underline{V} = \frac{1}{a}(G\underline{W})^H \tag{2.12}$$

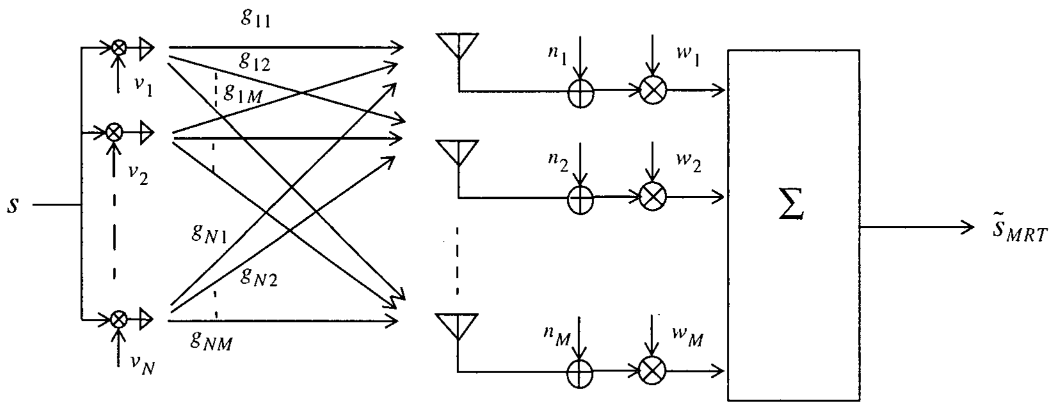

Figure 2.4 MRT with N transmit and M receive antennas

where $\underline{W}$ is a $M \times 1$ receive weighting vector and $a = |G\underline{W}|$. $[\,.\,]^H$ denotes the Hermitian operation, i.e. complex conjugate transpose.

The received signal vector is given by

$$\underline{r} = \frac{s}{a}(G\underline{W})^H G + \underline{n} \tag{2.13}$$

where $\underline{n} = [n_1 \ldots n_M]^T$ and $[\,.\,]^T$ denotes the transpose operation. n_i is an independent Gaussian r.v representing noise and interference. The estimate of the signal is given by

$$\begin{aligned} \tilde{s}_{MRT} &= \frac{s}{a}(G\underline{W})^H G\underline{W} + \underline{n}^T \underline{W} \\ &= as + \underline{n}^T \underline{W} \end{aligned} \tag{2.14}$$

The overall output SNR can be written as

$$\gamma_{MRT} = \frac{a^2\xi}{2\underline{W}\underline{W}^H\sigma_N^2} = \frac{a^2\xi}{2\sum_{i=1}^{M}|w_i|^2\sigma_N^2} \tag{2.15}$$

In [12], it is assumed that $|w_1| = |w_2| = \ldots = |w_M|$, and

$$(w_p w_q^*)^* = \frac{\sum_{j=1}^{N} g_{pi}g_{qi}^*}{\left|\sum_{j=1}^{N} g_{pi}g_{qi}^*\right|}, \tag{2.16}$$

where p and q take on values in the set $\{1, 2, \ldots, N\}$.

For $(N \times 1)$ MRT, i.e. N transmit and one receive antennas, the weighting function at the receiver, w_1, is set to unity for convenience. From (2.15), the resulting output SNR can be written as

$$\gamma_{N\times 1, MRT} = \frac{a_{Nx1}^2\xi}{2\sigma_N^2} \tag{2.17}$$

where $a_{N\times 1} = |G|^{\frac{1}{2}} = \left(\sum_{j=1}^{N}\alpha_{j1}^2\right)^{\frac{1}{2}}$. Equation (2.17) is the same as (2.3), the output from MRC combiner. Since the pdf of the output SNR for both $N \times 1$ MRT and $1 \times N$ MRC are the same, they have the same error performance.

For $(1 \times M)$ MRT with one transmit and M receive antennas, the output SNR can be written as

$$\gamma_{1 \times M, MRT} = \frac{\xi}{2M\sigma_N^2}\left(\sum_{k=1}^{M} |g_{1k}|\right)^2 \tag{2.18}$$

which is the same as the output SNR from equal gain combiner (EGC) [3].

However, the constraint that $|w_1| = |w_2| = \ldots = |w_M|$ in [12] results in degraded performance and a new scheme, maximum ratio transmission and combining (MRTC) was proposed recently in [15]. It is shown that MRTC can offer significant gains over MRT by using optimum transmit and receive weights. $(N \times 1)$ MRTC has the same error performance as MRC with same diversity order.

Chapter 3 Performance Analysis of STD with Correlated Fading and Channel Estimation

STD was shown to have the same error performance in non time-selective Rayleigh fading as MRC when perfect channel knowledge is available at the receiver. However, with imperfect channel estimation, STD has a poorer performance [10]. BER curves for STD in Rayleigh fading with imperfect channel estimation were obtained using computer simulation in [10]. The performance of STD in time-selective Rayleigh fading was investigated in [11] assuming perfect channel knowledge. An approximate expression for the BER was obtained.

In this chapter, exact closed-form expressions are derived for the BER of STD with two transmit and M receive antennas in time-selective, spatially independent Rayleigh fading with imperfect channel estimation and in non time-selective, spatially correlated Rayleigh fading with imperfect channel estimation. BER expressions for time-selective spatially independent Rayleigh fading with perfect channel estimation or non time-selective spatially independent Rayleigh fading with imperfect channel estimation are obtained as special cases. BPSK and quadrature phase shift keying (QPSK) modulation methods are considered.

3.1 2*M*-Branch STD in Time-selective, Spatially Independent Fading with Imperfect Channel Estimation

In this section, we present the performance analysis of STD with two transmit and M receive antennas in time-selective, spatially independent Rayleigh fading with imperfect channel estimation using BPSK modulation.

3.1.1 System Model

Figure 3.1 shows the baseband representation of the STD scheme with M receive antennas in

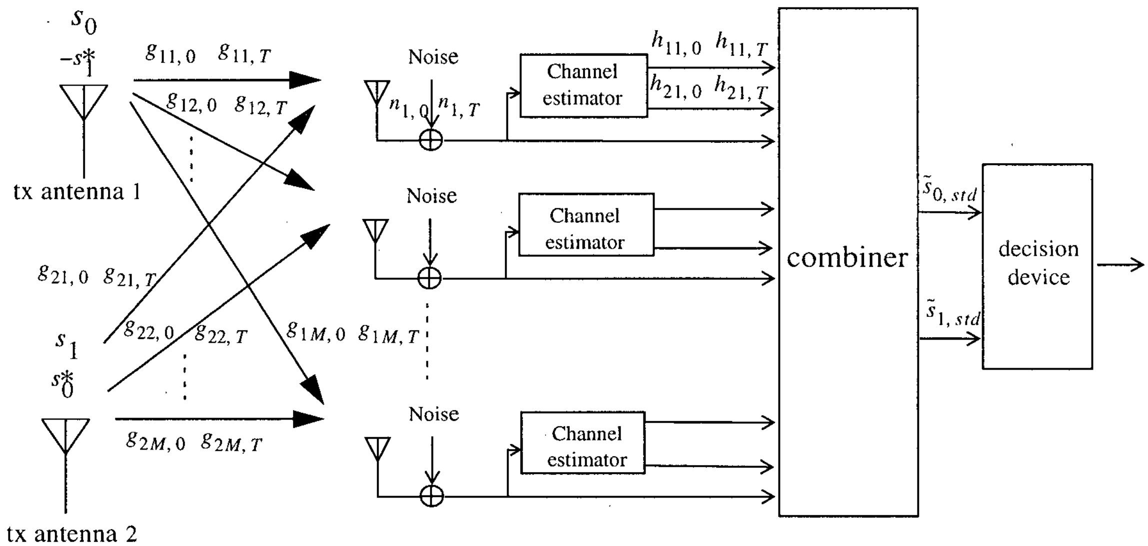

Figure 3.1 The STD scheme with M receivers in time-selective, spatially independent fading with imperfect channel estimation.

time-selective, spatially independent fading with imperfect channel estimation. Independent and equiprobable data bits, each of duration T, are transmitted. With BPSK modulation, the transmitted signals s_0 and s_1 from the two transmit antennas are either $+1$ or -1. It is assumed that the bandwidth of the signal is narrow compared to the channel coherence bandwidth so that the channel can be considered as non frequency-selective [1]. We use the time-selective fading model in [11] in which the channel gain is constant over a symbol duration but can change in successive symbol periods. The channel gains from transmit antennas 1 and 2 to receive antennas i, $i = 1, \ldots, M$ at time 0 and time T are denoted by r.v.'s $G_{1i,0}, G_{2i,0}, G_{1i,T}, G_{2i,T}$. In STD, the received signals at time 0 and time T at receive antenna i can be written as:

$$
\begin{aligned}
r_i &= g_{1i,0}s_0 + g_{2i,0}s_1 + n_{i,0} \\
r_{i,T} &= -g_{1i,T}s_1 + g_{2i,T}s_0 + n_{i,T} \qquad i = 1, \ldots, M
\end{aligned}
\tag{3.1}
$$

where g_{1i,t_1}, g_{2i,t_2}, $n_{i,t}$, $(t_1, t_2, t) \in \{0, T\}$ are outcomes of independent complex Gaussian distributed r.v.'s with zero means, i.e. the channel gains are spatially independent. The variances of the corresponding r.v.'s $G_{ji,t}$ and $N_{i,t}$ are denoted by σ_G^2 and σ_N^2 respectively. In this thesis, we define the variance of a complex r.v. as the variance of either its real or imaginary component. $G_{ji,0}$ and $G_{ji,T}$, $j = 1, 2$, are correlated with correlation coefficient ρ_t which is defined by [3] as

$$
\begin{aligned}
\rho_t &= \frac{E[G_{ji,0}G_{ji,T}^*]}{\sqrt{E[|G_{ji,0}|^2]E[|G_{ji,T}|^2]}} \\
&= \frac{E[Re(G_{ji,0})Re(G_{ji,T})] + E[Im(G_{ji,0})Im(G_{ji,T})]}{2\sigma_G^2} ,
\end{aligned}
\tag{3.2}
$$

where $E[\,.\,]$ denotes the expected value, $Re(G_{ji,t})$ and $Im(G_{ji,t})$ are the real and imaginary components of $G_{ji,t}$. In STD, the decoding of s_0 and s_1 is based on

$$
\begin{aligned}
\tilde{s}_{0,std} &= \sum_{i=1}^{M} h_{1i,0}^* \, r_i + \sum_{i=1}^{M} h_{2i,T} r_{i,T}^* \\
\tilde{s}_{1,std} &= \sum_{i=1}^{M} h_{2i,0}^* \, r_i - \sum_{i=1}^{M} h_{1i,T} r_{i,T}^*
\end{aligned}
\tag{3.3}
$$

where $h_{ji,t}$ is the estimate for $g_{ji,t}$. If the real part, $Re(\tilde{s}_{k,STD})$, of $\tilde{s}_{k,STD}$, $k = 0, 1$, is greater than 0, s_k =+1 is chosen; otherwise s_k =−1 is chosen. Following [10], we write

$h_{ji,t} = g_{ji,t} + z_{ji,t}$ where the channel estimation error, $z_{ji,t}$, is a sample of a zero mean, variance σ_Z^2 complex Gaussian r.v which is independent of $G_{ji,t}$. $H_{ji,t}$ is thus a zero mean complex Gaussian r.v. with variance $\sigma_H^2 = \sigma_G^2 + \sigma_Z^2$. It is shown in Appendix A that the correlation coefficient of $G_{ji,t}$ and $H_{ji,t}$ is $\rho_e = \sigma_G/\sigma_H$.

3.1.2 BER Analysis

Due to the symmetry in the STD scheme, the BER for the two transmitted signals are equal and we need only consider one of the signals, say s_0. We will make use of the following result for joint Gaussian r.v.'s [16].

For the two jointly Gaussian r.v.'s X and Y with zero means, i.e. $E(X) = E(Y) = 0$, assuming $X = x'$, then

$$\begin{aligned} E\{Y|X = x'\} &= \frac{\rho\sigma_y}{\sigma_x}x' \\ \sigma_{Y|X = x'} &= \sigma_Y\sqrt{1-\rho^2} \end{aligned}, \tag{3.4}$$

where σ_X^2, σ_Y^2 are the variances of X and Y respectively and ρ is the correlation coefficient of X and Y.

Since $G_{ji,t}$ and $H_{ji,t}$ are jointly Gaussian, hence the channel fading gain $G_{ji,t}$ conditioned on $H_{ji,t} = h_{ji,t}$ is a complex Gaussian r.v. with mean $\rho_{e,n}h_{ji,t}$ where $\rho_{e,n} = \frac{\sigma_G^2}{\sigma_H^2} = \rho_e^2$ and variance $(1-\rho_e^2)\sigma_G^2$. Thus, we can write $g_{ji,t}$ as

$$\begin{aligned} g_{1i,0} &= \rho_{e,n} h_{1i,0} + d_{1i} \\ g_{2i,T} &= \rho_{e,n} h_{2i,T} + d_{2i} \end{aligned} \tag{3.5}$$

where d_{1i} and d_{2i} are samples of zero mean, variance $\sigma_D^2 = (1-\rho_e^2)\sigma_G^2$, complex Gaussian r.v.'s, D_{1i} is independent of $H_{1i,0}$ and D_{2i} is independent of $H_{2i,T}$. Similarly, given $g_{1i,0}$ and $g_{2i,T}$, we can express $g_{1i,T}$ and $g_{2i,0}$ as

$$\begin{aligned} g_{1i,T} &= \rho_t g_{1i,0} + v_{1i} \\ g_{2i,0} &= \rho_t g_{2i,T} + v_{2i} \end{aligned} \tag{3.6}$$

where v_{1i} and v_{2i} are samples of zero mean, variance $\sigma_V^2 = (1-\rho_t^2)\sigma_G^2$, complex Gaussian r.v.'s, V_{1i} is independent of $G_{1i,0}$ and V_{2i} is independent of $G_{2i,T}$. Using (3.1) - (3.6), $\tilde{s}_{0,std}$ can be written as

$$\begin{aligned} \tilde{s}_{0,std} = & \sum_{i=1}^{M} \rho_{e,n}(|h_{1i,0}|^2 + |h_{2i,T}|^2)s_0 + \sum_{i=1}^{M} [h_{1i,0}^* \, d_{1i} + h_{2i,T} \, d_{2i}^*]s_0 \\ & + \sum_{i=1}^{M} [h_{1i,0}^* \, v_{2i} - h_{2i,T} \, v_{1i}^* + h_{1i,0}^* \, \rho_t d_{2i} - h_{2i,T} \, \rho_t \, d_{1i}^*]s_1 \\ & + \sum_{i=1}^{M} h_{1i,0}^* \, n_{i,0} + \sum_{i=1}^{M} h_{2i,T} n_{i,T}^* \quad . \end{aligned} \tag{3.7}$$

Since $s_1 = s_0$ and $s_1 = -s_0$, each with probability $\frac{1}{2}$, we can calculate the BER for STD as

$p_{e,std} = \frac{1}{2}(p_{e,s_1=s_0} + p_{e,s_1=-s_0})$.

For the case $s_1 = s_0$, from (3.7) we can write the decision variable for $\tilde{s}_{0,STD}$ as

$$\begin{aligned} Re(\tilde{s}_{0,\,std}) = {} & \sum_{i=1}^{M} \rho_{e,\,n}(|h_{1i,\,0}|^2 + |h_{2i,\,T}|^2)s_0 + Re\left\{\sum_{i=1}^{M}[h^*_{1i,\,0}\ (m_{1i}+v_{2i})]\right\}s_0 \\ & + Re\left\{\sum_{i=1}^{M}[h_{2i,\,T}(m_{2i}-v_{1i})^*]\right\}s_0 + Re\left[\sum_{i=1}^{M} h^*_{1i,\,0}\ n_{i,\,0}\right] \\ & + Re\left[\sum_{i=1}^{M} h_{2i,\,T}n^*_{i,\,T}\right], \end{aligned} \tag{3.8}$$

where $m_{1i} = d_{1i} + \rho_t d_{2i}$ and $m_{2i} = (d_{2i} - \rho_t d_{1i})$. It is shown in Appendix B that $Re(h^*_{1i,\,0}\ M_{1i})$, $Re(h_{2i,\,T}\,M^*_{2i})$, $Re(h^*_{1i,\,0}\ V_{2i})$, $Re(h_{2i,\,T}\ V^*_{1i})$, $Re(h^*_{1i,\,0}\ N_{i,\,0})$ and $Re(h_{2i,\,T}\,N^*_{i,\,T})$ are independent, zero mean Gaussian r.v.'s with variances $(1+\rho_t^2)\sigma_D^2|h_{1i,\,0}|^2$, $(1+\rho_t^2)\sigma_D^2|h_{2i,\,T}|^2$, $\sigma_V^2|h_{1i,\,0}|^2$, $\sigma_V^2|h_{2i,\,T}|^2$, $\sigma_N^2|h_{1i,\,0}|^2$ and $\sigma_N^2|h_{2i,\,T}|^2$ respectively. Thus, $Re(\tilde{s}_{0,\,std})$ is the sum of $\sum_{i=1}^{M}\rho_{e,\,0}[|h_{1i,\,0}|^2 + |h_{2i,\,T}|^2]s_0$ and an independent, zero mean Gaussian r.v. with variance $\sum_{i=1}^{M}[(1+\rho_t^2)\sigma_D^2 + \sigma_V^2 + \sigma_N^2](|h_{1i,\,0}|^2 + |h_{2i,\,T}|^2)$. The BER is given by

$$\begin{aligned} p_{e,\,s_1 = s_0} &= Q\left(\sqrt{\frac{\left[\sum_{i=1}^{M}\rho_{e,\,n}(|h_{1i,\,0}|^2 + |h_{2i,\,T}|^2)\right]^2}{\sum_{i=1}^{M}[(1+\rho_t^2)\sigma_D^2 + \sigma_V^2 + \sigma_N^2](|h_{1i,\,0}|^2 + |h_{2i,\,T}|^2)}}\right), \\ &= Q(\sqrt{2a}) \end{aligned} \tag{3.9}$$

where

$$a = \sum_{i=1}^{M} K(|h_{1i,\,0}|^2 + |h_{2i,\,T}|^2), \tag{3.10}$$

with

$$K = \rho_{e,n}^2/\{2[(1+\rho_t^2)\sigma_D^2 + \sigma_V^2 + \sigma_N^2]\}. \tag{3.11}$$

Similarly, it can been shown that the BER for $s_1 = -s_0$ is also given by (3.9). For non time-selective fading with imperfect channel estimation, (3.10) reduces to

$$a_{nts} = \frac{\rho_{e,n}^2}{2[2\sigma_D^2 + \sigma_N^2]} \sum_{i=1}^{M} (|h_{1i,0}|^2 + |h_{2i,T}|^2) \tag{3.12}$$

Since $H_{1i,0}$ and $H_{2i,T}$ are independent and identically distributed zero-mean complex Gaussian r.v.'s, $A = \sum_{i=1}^{M} K(|H_{1i,0}|^2 + |H_{2i,T}|^2)$ has a chi-square distribution with $4M$ degrees of freedom and its pdf is given by [13]

$$f_A(a) = \frac{(2M)^{2M} a^{2M-1}}{\mu_A^{2M}(2M-1)!} e^{-(2Ma)/\mu_A}, a \geq 0, \tag{3.13}$$

where

$$\begin{aligned}\mu_A &= 4MK\sigma_H^2 = 2M\rho_e^2\sigma_G^2/[(1+\rho_t^2)(1-\rho_e^2)\sigma_G^2 + (1-\rho_t^2)\sigma_G^2 + \sigma_N^2] \\ &= 2M\sigma_G^4/[(1+\rho_t^2)\sigma_Z^2\sigma_G^2 + \sigma_G^2(\sigma_V^2 + \sigma_N^2) + \sigma_Z^2(\sigma_V^2 + \sigma_N^2)]\ .\end{aligned} \tag{3.14}$$

The overall BER for STD with BPSK modulation on a Rayleigh fading channel can then be obtained by averaging over the fading channel statistics (3.13), i.e.

$$p_{f,STD} = \int_0^{\infty} f_A(a) Q(\sqrt{2a}) da . \tag{3.15}$$

The integral of (3.15) can be simplified as [13]

$$p_{f,STD} = \left[\frac{1}{2}(1-u)\right]^{2M} \sum_{i=0}^{2M-1} \left\{ \binom{2M-1+i}{i} \left[\frac{1}{2}(1+u)\right]^i \right\} . \tag{3.16}$$

where $u = \sqrt{\frac{\mu_A}{2M+\mu_A}}$. For the special case of STD with two transmit and one receive antenna, the overall BER is given by (3.16) with $M = 1$, i.e.

$$p_{f,STD} = \frac{1}{4}\left(1 - \sqrt{\frac{u_A}{2+u_A}}\right)^2 \left(2 + \sqrt{\frac{u_A}{2+u_A}}\right). \tag{3.17}$$

For given values of M, ρ_e, ρ_t and σ_N^2, μ_A increases monotonically with σ_G^2. The limiting value of μ_A as $\sigma_G^2 \to \infty$ is $\mu_{A,max} = \frac{2M\rho_e^2}{2 - \rho_e^2 - \rho_e^2\rho_t^2}$. The BER thus has an error floor expression given by replacing μ_A by $\mu_{A,max}$ in (3.16), i.e. $u = \sqrt{\frac{\rho_e^2}{2 - \rho_e^2\rho_t^2}}$.

For a non time-selective Rayleigh fading channel, $\rho_t = 1, \sigma_V^2 = 0$ and (3.14) reduces to

$$\mu_{A,nts} = 2M\sigma_G^4/[2\sigma_Z^2\sigma_G^2 + \sigma_G^2\sigma_N^2 + \sigma_Z^2\sigma_N^2] . \tag{3.18}$$

With perfect channel estimation, $\rho_e = 1, \sigma_D^2 = 0, \sigma_H^2 = \sigma_G^2$ and (3.14) reduces to

$$\mu_{A,\,pce} = 2M\sigma_G^2/(\sigma_V^2 + \sigma_N^2)\ . \tag{3.19}$$

With both non time-selective Rayleigh fading and perfect channel estimation, (3.14) reduces to the result in [7], i.e.

$$\mu_{A,\,nts/pcs} = 2M\sigma_G^2/\sigma_N^2\ . \tag{3.20}$$

The corresponding BER is given by replacing μ_A by $\mu_{A,\,nts}$, $\mu_{A,\,pcs}$ and $\mu_{A,\,nts/pcs}$ in (3.16).

For comparison purposes, we note that the BER of MRC with M receive antennas and imperfect channel estimation is given by [17]

$$p_{e,\,MRC} = Q\left(\sqrt{\frac{\rho_{e,\,n}^2\left(\sum_{i=1}^{M}|h_i|^2\right)}{\sigma_D^2 + \sigma_N^2}}\right). \tag{3.21}$$

where h_i is the estimated channel gain for the i th receive antenna. A comparison of (3.12) for the non time-selective fading case and (3.21) shows that, for the same diversity order, MRC has a smaller BER and for $\sigma_D^2 \gg \sigma_N^2$, STD is 3 dB worse than MRC.

3.2 2*M*-Branch STD with QPSK Modulation

The performance of 2*M*-branch STD with QPSK modulation is now considered. To calculate the symbol error rate (SER), we note that coherent demodulation ideally results in the two demodulated signals being separated at the outputs of the quadrature mixers at the receiver [18].

Thus a coherent QPSK system can be considered equivalent to two parallel independent coherent BPSK systems [13].

Hence using (3.16), the SER is given by [13]

$$\begin{aligned} p_{e,\,QPSK} &= 1-(1-p_{f,\,STD})^2 \\ &= 2p_{f,\,STD}-p_{f,\,STD}^2 \;. \end{aligned} \tag{3.22}$$

3.3 2*M*-Branch STD in Non Time-selective, Spatially Correlated Fading with Imperfect Channel Estimation

In this section, we investigate the performance of 2*M*-branch STD with BPSK modulation in non time-selective, spatially correlated Rayleigh fading with imperfect channel estimation.

3.3.1 System Model

Figure 3.2 shows the complex baseband representation of the STD scheme with M receive antennas in non time-selective, spatially correlated fading with imperfect channel estimation. Independent and equiprobable data bits, each of duration T, are transmitted. With BPSK modulation, the transmitted signals s_0 and s_1 from the two transmit antennas are either $+1$ or -1. It is assumed that the bandwidth of the signal is narrow compared to the channel coherence bandwidth and the channel coherence time is much larger than T so that the channel can be considered as non frequency-selective and non time selective [1]. The gains of the 2*M* diversity paths, denoted by $G_{11}, G_{12}, \ldots, G_{1M}, G_{21}, \ldots, G_{2M}$ are zero mean, variance σ_G^2 correlated complex Gaussian r.v.'s. The $2M \times 2M$ covariance matrix, C_g, of these r.v.'s is assumed to be of the form [19]:

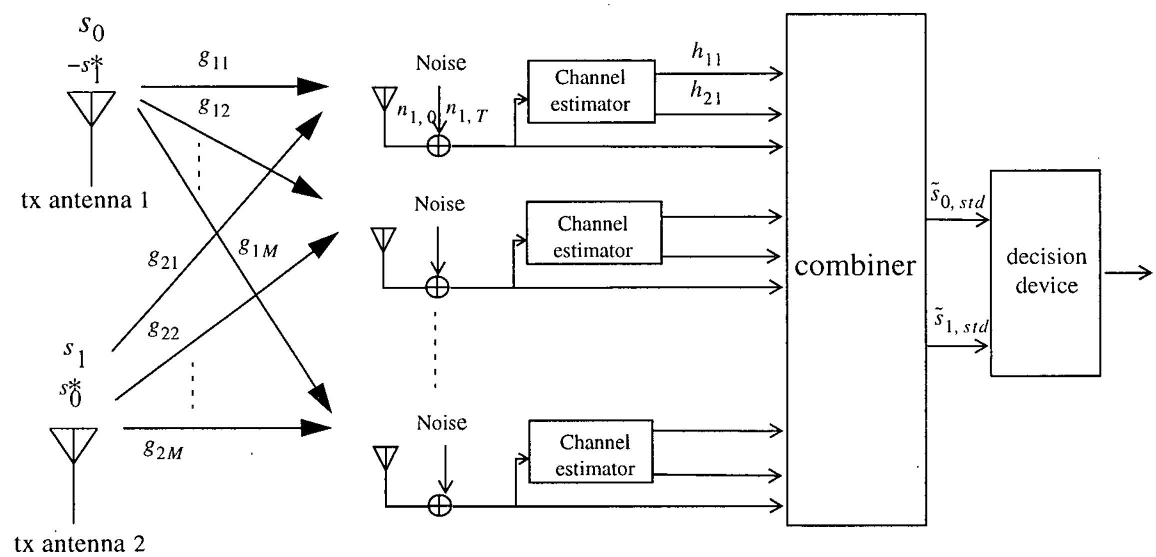

Figure 3.2 The STD scheme with M receivers in non time-selective, spatially correlated fading with imperfect channel estimation.

$$C_g = \begin{bmatrix} \sigma_G^2 & \sigma_G^2\rho_s & 0 & \dots & 0 \\ \sigma_G^2\rho_s^* & \sigma_G^2 & \sigma_G^2\rho_s & \ddots & \vdots \\ 0 & \sigma_G^2\rho_s^* & \sigma_G^2 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \sigma_G^2\rho_s \\ 0 & \dots & 0 & \sigma_G^2\rho_s^* & \sigma_G^2 \end{bmatrix}. \tag{3.23}$$

In STD, the received signals at time 0 and time T at receive antenna i can be written as [7] [10]:

$$\begin{aligned} r_i &= g_{1i}s_0 + g_{2i}s_1 + n_{i,0} \\ r_{i,T} &= -g_{1i}s_1 + g_{2i}s_0 + n_{i,T} \qquad i = 1, \dots, M \end{aligned} \tag{3.24}$$

where g_{1i} and g_{2i} are path gain samples and $n_{i,t}$, $t \in \{0, T\}$ is a sample of a zero mean, variance σ_N^2 independent complex Gaussian r.v. which represents the channel noise. In STD, the decoding of s_0 and s_1 is based on

$$
\begin{aligned}
\tilde{s}_{0,std} &= \sum_{i=1}^{M} h_{1i}^{*} r_i + \sum_{i=1}^{M} h_{2i} r_{i,T}^{*} \\
\tilde{s}_{1,std} &= \sum_{i=1}^{M} h_{2i}^{*} r_i - \sum_{i=1}^{M} h_{1i} r_{i,T}^{*}
\end{aligned}
\tag{3.25}
$$

where h_{ji} is the estimate for g_{ji}. If the real part, $Re(\tilde{s}_{k,STD})$, of $\tilde{s}_{k,STD}$, $k = 0, 1$, is greater than 0, $s_k = +1$ is chosen; otherwise $s_k = -1$ is chosen. Following [10], we write $h_{ji} = g_{ji} + z_{ji}$ where the channel estimation error, z_{ji}, is a sample of a zero mean, variance σ_Z^2 complex Gaussian r.v. which is independent of G_{ji}. H_{ji} is thus a zero mean complex Gaussian r.v. with variance $\sigma_H^2 = \sigma_G^2 + \sigma_Z^2$. It is shown in Appendix C that the correlation coefficient of H_{1i} and H_{2i} is $\rho_h = \dfrac{\sigma_G^2}{\sigma_H^2}\rho_s$. The $2M \times 2M$ covariance matrix, C_h, of $H_{11}, H_{12}, \ldots, H_{1M}, H_{21}, \ldots, H_{2M}$ can be expressed as

$$
C_h = \begin{bmatrix}
\sigma_H^2 & \sigma_H^2\rho_h & 0 & \cdots & 0 \\
\sigma_H^2\rho_h^* & \sigma_H^2 & \sigma_H^2\rho_h & \ddots & \vdots \\
0 & \sigma_G^2\rho_h^* & \sigma_H^2 & \ddots & 0 \\
\vdots & \ddots & \ddots & \ddots & \sigma_H^2\rho_h \\
0 & \cdots & 0 & \sigma_G^2\rho_h^* & \sigma_H^2
\end{bmatrix}
\tag{3.26}
$$

It is shown in Appendix A that G_{ji} and H_{ji} are correlated with correlation coefficient $\rho_e = \sigma_G/\sigma_H$. Using (3.4), we can write g_{ji} as

$$\begin{aligned} g_{1i} &= \rho_{e,n} h_{1i} + d_{1i} \\ g_{2i} &= \rho_{e,n} h_{2i} + d_{2i} \end{aligned} \tag{3.27}$$

where $\rho_{e,n} = \rho_e^2 = \sigma_{G_j}^2 / \sigma_{H_j}^2$, d_{1i} and d_{2i} are samples of zero mean, variance $\sigma_D^2 = (1 - \rho_e^2)\sigma_G^2$, complex Gaussian r.v.'s, D_{1i} is independent of H_{1i} and D_{2i} is independent of H_{2i}. It is shown in Appendix D that D_{1i} and D_{2i} are correlated with correlation coefficient $\rho_d = (1 - \rho_e^2)\rho_s$. Using (3.24), (3.25) and (3.27), $\tilde{s}_{0,\,std}$ can be written as

$$\begin{aligned} \tilde{s}_{0,\,std} = {} & \sum_{i=1}^{M} \rho_{e,n}(|h_{1i}|^2 + |h_{2i}|^2)s_0 + \sum_{i=1}^{M} (h_{1i}^* \; d_{1i} + h_{2i}\, d_{2i}^*)s_0 \\ & + \sum_{i=1}^{M} (h_{1i}^* \; d_{2i} - h_{2i} \; d_{1i}^*)s_1 + \sum_{i=1}^{M} h_{1i}^* \; n_{i,0} + \sum_{i=1}^{M} h_{2i} n_{i,T}^* \quad . \end{aligned} \tag{3.28}$$

Since $s_1 = s_0$ or $s_1 = -s_0$, each with probability $\frac{1}{2}$, we can calculate the BER for STD as

$p_{e,\,std} = \frac{1}{2}(p_{e,\,s_1 = s_0} + p_{e,\,s_1 = -s_0})$.

For the case $s_1 = s_0$, we can write the decision variable for $\tilde{s}_{0,\,std}$ as

$$\begin{aligned} Re(\tilde{s}_{0,\,std}) = {} & \sum_{i=1}^{M} \rho_{e,n}(|h_{1i}|^2 + |h_{2i}|^2)s_0 + Re\left\{ \sum_{i=1}^{M} [h_{1i}^* \; (d_{1i} + d_{2i})] \right\} s_0 \\ & + Re\left\{ \sum_{i=1}^{M} [h_{2i}(d_{2i} - d_{1i})^*] \right\} s_0 + Re\left[\sum_{i=1}^{M} h_{1i}^* \; n_{i,0} \right] \\ & + Re\left[\sum_{i=1}^{M} h_{2i} n_{i,T}^* \right] \quad . \end{aligned} \tag{3.29}$$

It is shown in Appendix D that $Re(h_{1i}^* \; (D_{1i} + D_{2i}))$, $Re(h_{2i}(D_{2i} - D_{1i})^*)$, $Re(h_{1i}^* \; N_{i,0})$ and

$Re(h_{2i}N^*_{i,T})$ are independent, zero mean Gaussian r.v.'s with variances $[2(1+\rho_d)\sigma_D^2]|h_{1i}|^2$, $[[2(1+\rho_d)\sigma_D^2]]|h_{2i}|^2$, $\sigma_N^2|h_{1i}|^2$ and $\sigma_N^2|h_{2i}|^2$ respectively. Thus, $Re(\tilde{s}_{0,std})$ is the sum of $\sum_{i=1}^{M}\rho_{e,n}[|h_{1i}|^2+|h_{2i}|^2]s_0$ and an independent, zero mean Gaussian r.v. with variance $\sum_{i=1}^{M}[[2(1+\rho_d)\sigma_D^2]+\sigma_N^2](|h_{1i}|^2+|h_{2i}|^2)$. The BER is given by

$$p_{e,s_1=s_0} = Q\left(\sqrt{\frac{\left[\sum_{i=1}^{M}\rho_{e,n}(|h_{1i}|^2+|h_{2i}|^2)\right]^2}{\sum_{i=1}^{M}[[2(1+\rho_d)\sigma_D^2]+\sigma_N^2](|h_{1i}|^2+|h_{2i}|^2)}}\right) = Q(\sqrt{2a}) \ , \tag{3.30}$$

where

$$a = \frac{\sum_{i=1}^{M}\rho_{e,n}^2(|h_{1i}|^2+|h_{2i}|^2)}{2\{[2(1+\rho_d)\sigma_D^2]+\sigma_N^2\}} = \sum_{i=1}^{M}K(|h_{1i}|^2+|h_{2i}|^2) \ , \tag{3.31}$$

with

$$K = \frac{\rho_{e,n}^2}{2\{[2(1+\rho_d)\sigma_D^2]+\sigma_N^2\}} . \tag{3.32}$$

Similarly, it can be shown that the BER for $s_1 = -s_0$ is also given by (3.30).

Since H_{1i} and H_{2i} are correlated and identically distributed zero-mean complex Gaussian r.v.'s, if the pdf of $A = \sum_{i=1}^{M} K(|H_{1i}|^2 + |H_{2i}|^2)$ is $f_A(a)$, then its Laplace transform, $P(s)$, can be written as [3]

$$P(s) = \prod_{k=1}^{2M} \frac{1}{1 + s\Gamma_l} \tag{3.33}$$

where $\Gamma_l = 2K\lambda_l$ and λ_l are the eigenvalues of (3.26) and are given by [19]

$$\lambda_l = \sigma_H^2\left[1 - 2\rho_h \cos\left(\frac{l\pi}{2M+1}\right)\right], \ l = 1, 2, \ldots, 2M \tag{3.34}$$

Then $f_A(a)$ is given by [19]

$$f_A(a) = \sum_{p=1}^{2M} d_p \exp(s_p a), \quad a \geq 0 \tag{3.35}$$

where s_p are the poles of $P(s)$ and d_p are the corresponding residues of $P(s)$. The overall BER is given by

$$p_{f,STD} = \int_0^{\infty} f_A(a) Q\sqrt{2a} da \tag{3.36}$$

By using [20], [21], (3.36) reduces to

$$p_{f,STD} = \sum_{p=1}^{2M} \frac{d_p}{2s_p}\left(\sqrt{\frac{1}{1-s_p}} - 1\right) \tag{3.37}$$

For the special case of STD with two transmit and one receive antennas, the overall BER is given by (3.36) with $M = 1$, i.e.

$$p_{f,STD} = \frac{1}{2(\Gamma_1 - \Gamma_2)}\left[\Gamma_1\left(1 - \sqrt{\frac{\Gamma_1}{1+\Gamma_1}}\right) - \Gamma_2\left(1 - \sqrt{\frac{\Gamma_2}{1+\Gamma_2}}\right)\right] \tag{3.38}$$

where

$$\begin{aligned} \Gamma_1 &= 2K\sigma_H^2(1+\rho_h) = \frac{\rho_e^2\sigma_G^2(1+\rho_h)}{[2(1+\rho_d)\sigma_D^2] + \sigma_N^2} \\ \Gamma_2 &= 2K\sigma_H^2(1-\rho_h) = \frac{\rho_e^2\sigma_G^2(1-\rho_h)}{[2(1+\rho_d)\sigma_D^2] + \sigma_N^2} \end{aligned} \tag{3.39}$$

Chapter 4 Numerical Results

Numerical results based on the analytic results derived in Chapter 3 are presented in this chapter. For convenience, we define the signal-to-noise ratio (SNR) as the ratio of the variance, σ_G^2, of the channel gain, to the variance, σ_N^2, of the additive Gaussian noise, i.e. σ_G^2/σ_N^2 and the estimation error-to-signal ratio (ESR) as σ_Z^2/σ_G^2, where σ_Z^2 is the variance of the channel estimation error. A fixed value of ESR corresponds to a fixed value of ρ_e since $\rho_e^2 = 1/(1+ESR)$.

The theoretical BER of two branch STD in non time-selective Rayleigh fading with imperfect channel estimation is given by substituting (3.18) into (3.17). The corresponding curves are plotted in Figure 4.1 as a function of SNR for different correlation coefficient values, ρ_e, between the estimated channel gain and actual channel gain. It can be seen that the error performance degrades rapidly as ρ_e decreases from 1. The performance difference with perfect channel estimation increases with SNR. For a target BER of 10^{-3}, there is about 3.5 dB degradation for $\rho_e = 0.99$ relative to perfect channel estimation, i.e. $\rho_e = 1$. For $\rho_e < 1$, the BER curve exhibits an error floor with a value obtained by substituting $u_{A,max} = \frac{\rho_e^2}{1-\rho_e^2}$ in (3.17). It can be seen that the error floor limit is approached for lower SNR values as ρ_e decreases. When $\rho_e = 0$, the estimated channel phase is completely random and hence the BER is 0.5.

The BER of two branch STD in time-selective Rayleigh fading with perfect channel estima-

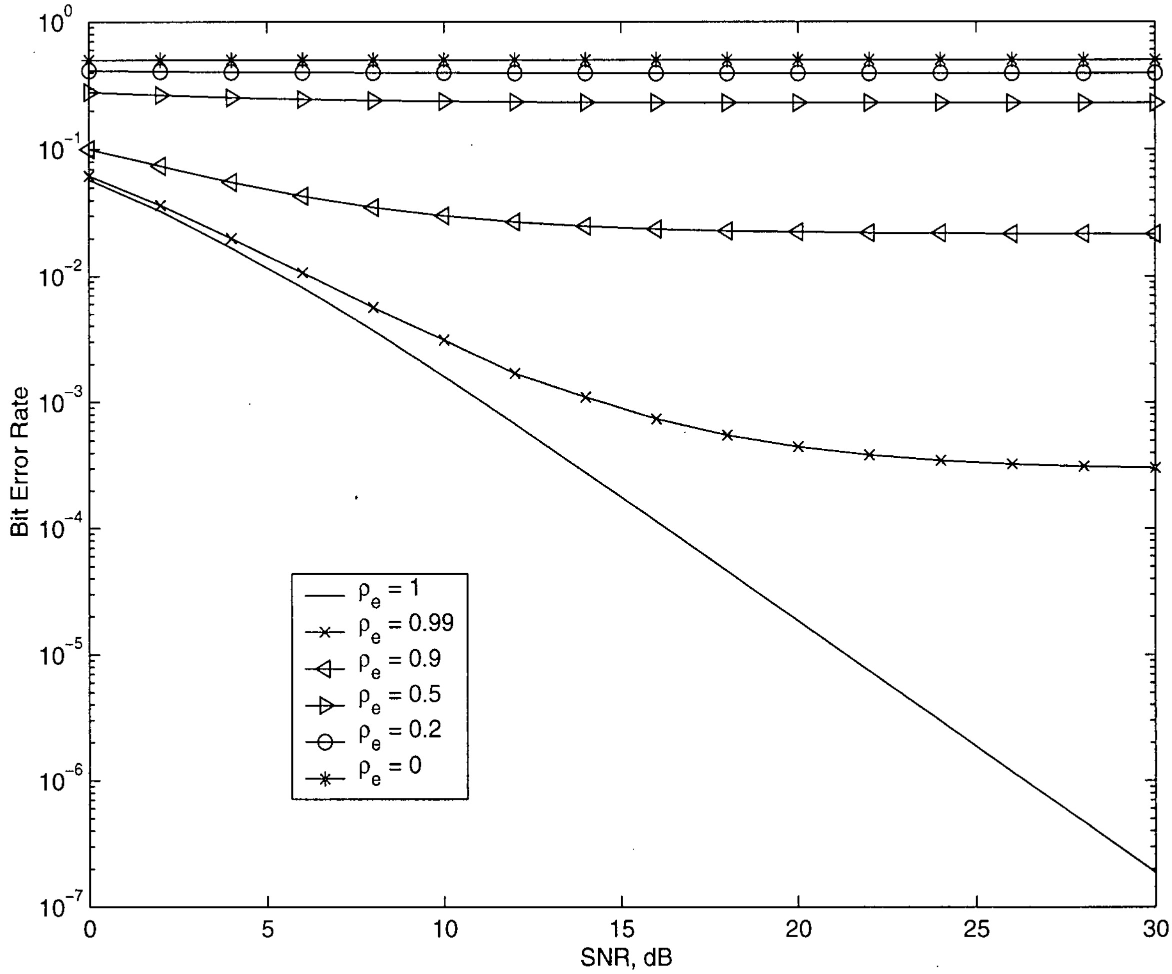

Figure 4.1 BER of two-branch STD as a function of SNR for different values of ρ_e when $\rho_t = 1$.

tion as given by substituting (3.19) into (3.17) is plotted in Figure 4.2 as a function of SNR for different correlation coefficient, ρ_t, between the channel gains at time 0 and time T. The error performance degrades rapidly as ρ_t decreases from 1. The performance difference with non time-selective fading increases with SNR. For a target BER of 10^{-2}, there is about 4dB degradation for $\rho_t = 0.9$ relative to $\rho_t = 0.99$ and there is only about 0.4 dB degradation for $\rho_t = 0.99$ relative

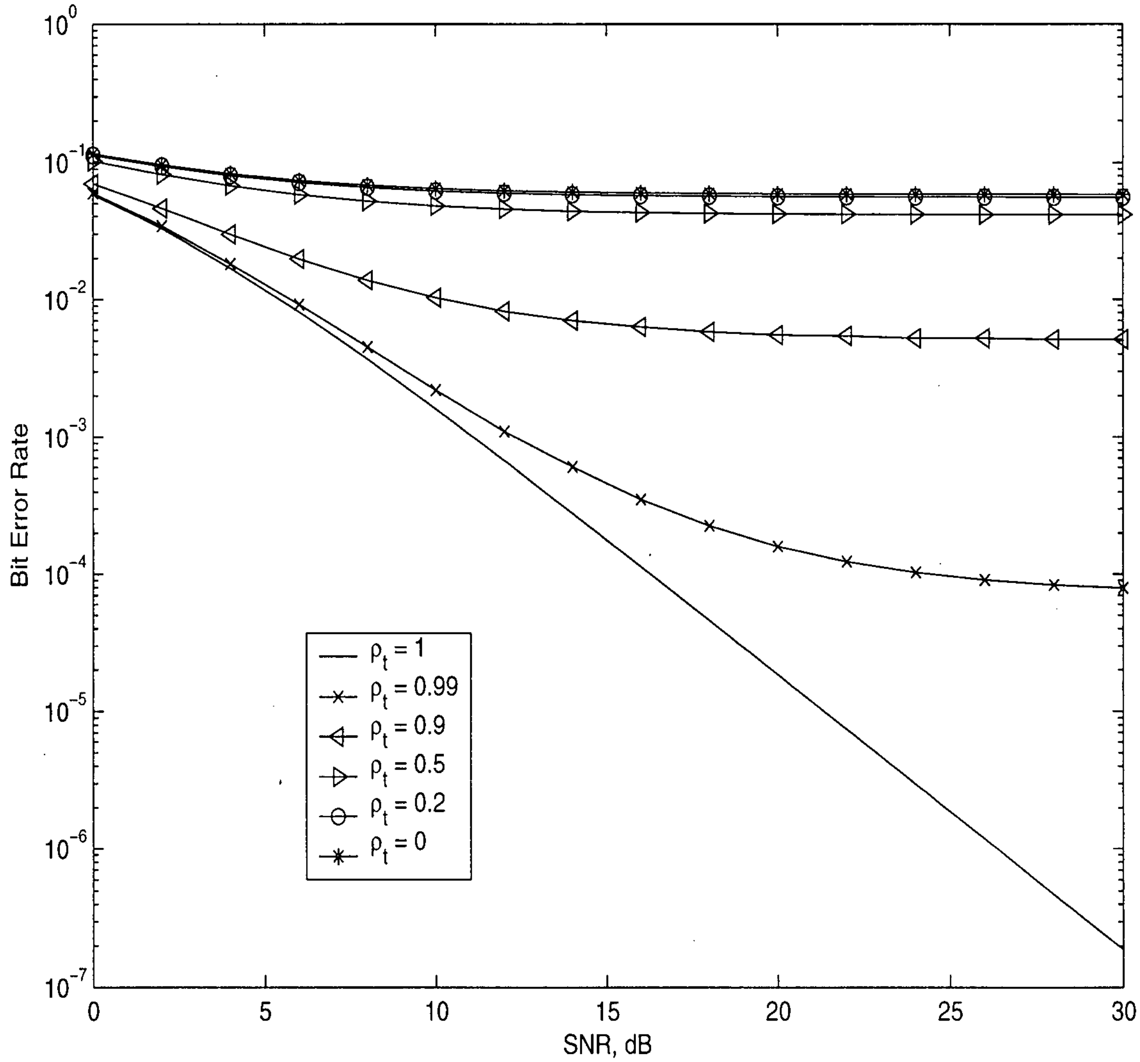

Figure 4.2 BER of two-branch STD as a function of SNR for different values of ρ_t when $\rho_e = 1$.

to non time-selective fading case, i.e. $\rho_t = 1$. For small values of ρ_t, $0 \le \rho_t \le 0.2$, the performance is almost identical.

The BER of two branch STD in time-selective Rayleigh fading with imperfect channel

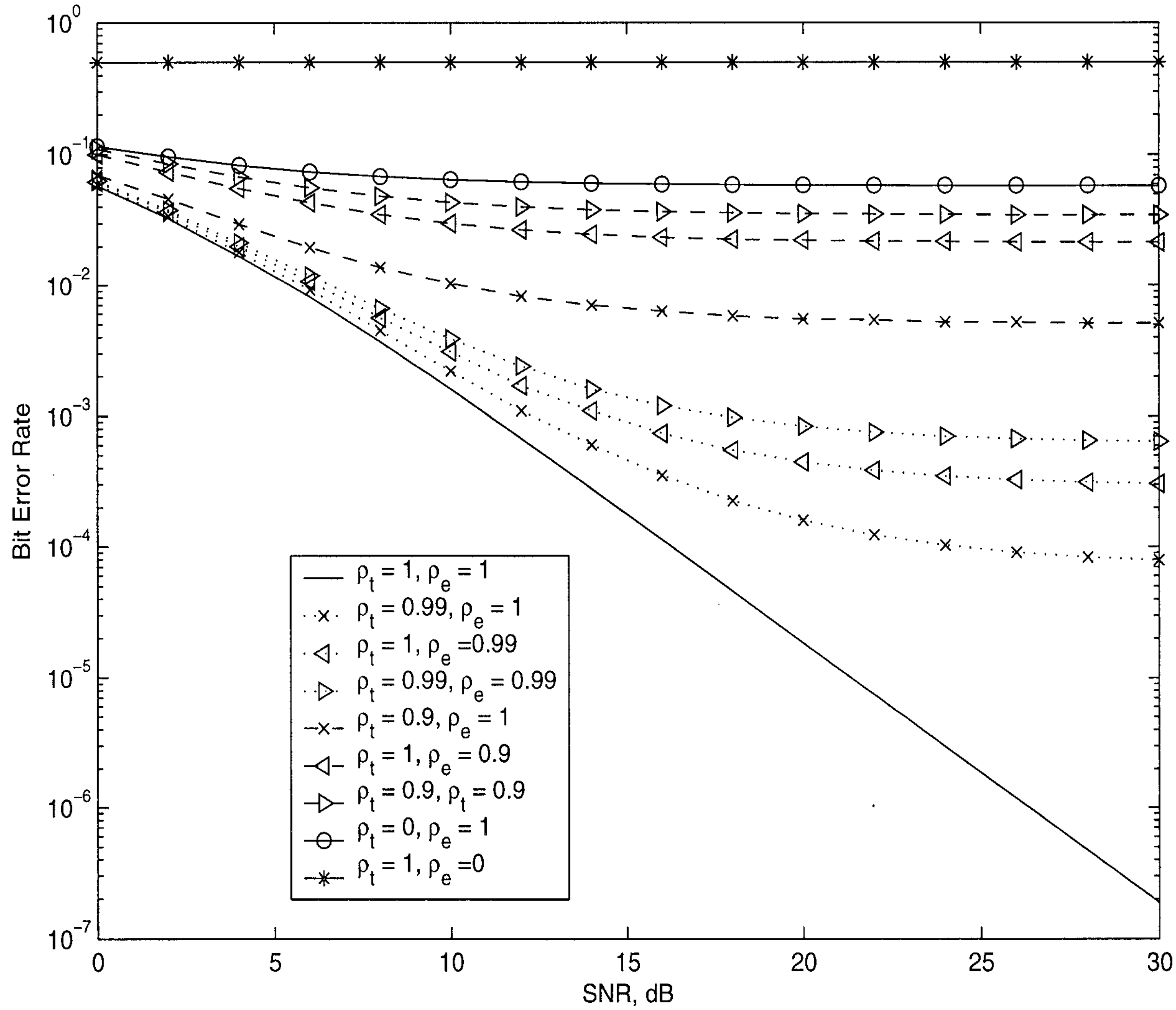

Figure 4.3 BER of two-branch STD as a function of SNR for different values of ρ_t and ρ_e.

estimation as given by substituting (3.14) into (3.17) is plotted as a function of SNR for several ρ_t and ρ_e values in Figure 4.3. For the same ρ_t and ρ_e values, the performance in non time-selective fading with imperfect channel estimation ($\rho_t = 1, 0 \le \rho_e \le 1$) is worse than in time-selective fading with perfect channel estimation ($0 \le \rho_t \le 1, \rho_e = 1$). For a target BER of 10^{-3}, there is about 1 dB degradation for $(\rho_t = 0.99, \rho_e = 1)$, about 3 dB degradation for $(\rho_t = 1, \rho_e = 0.99)$ and about 7 dB degradation for $(\rho_t = 0.99, \rho_e = 0.99)$ relative to non time-selective fading with perfect channel estimation i.e. $(\rho_t = 1, \rho_e = 1)$. For $\rho_t < 1$ or $\rho_e < 1$, the BER curve exhibits an error floor with a value obtained by substituting $\mu_{A,\,max} = \dfrac{2\rho_e^2}{2 - \rho_e^2 - \rho_e^2\rho_t^2}$ in (3.17).

Figure 4.4 shows the BER curves for STD and MRC in non time-selective Rayleigh fading with diversity order of two as a function of SNR for four different values of ESR. It can be seen that the BER for STD or MRC degrades quiet rapidly with increase in ESR. STD is more sensitive to channel estimation error than MRC. For a target BER of 10^{-4}, an ESR of -20 dB results in an SNR loss of about 2 dB for MRC and about 6 dB for STD relative to the perfect channel estimation case. For ESR > 0, the BER curve exhibits an error floor with a value given by substituting $u_{A,\,MAX} = \dfrac{1}{ESR}$ in (3.17).

Figure 4.5 shows the BER curves for STD and MRC in non time-selective Rayleigh fading with a diversity order of two as a function of ESR for three different SNR values. The BER difference between STD and MRC increases with SNR and ESR. For $\sigma_D^2 \gg \sigma_N^2$, or equivalently $SNR \gg 1 + 1/ESR$, STD is 3 dB worse than MRC as expected from (3.12) and (3.21).

The theoretical BER performance of STD and MRC with diversity order of 4 in time-selective Rayleigh fading with imperfect channel estimation, as given by (3.16) and (3.21), is plotted in Figure 4.6 as a function of SNR for different values of ρ_e and ρ_t. As in the case of a

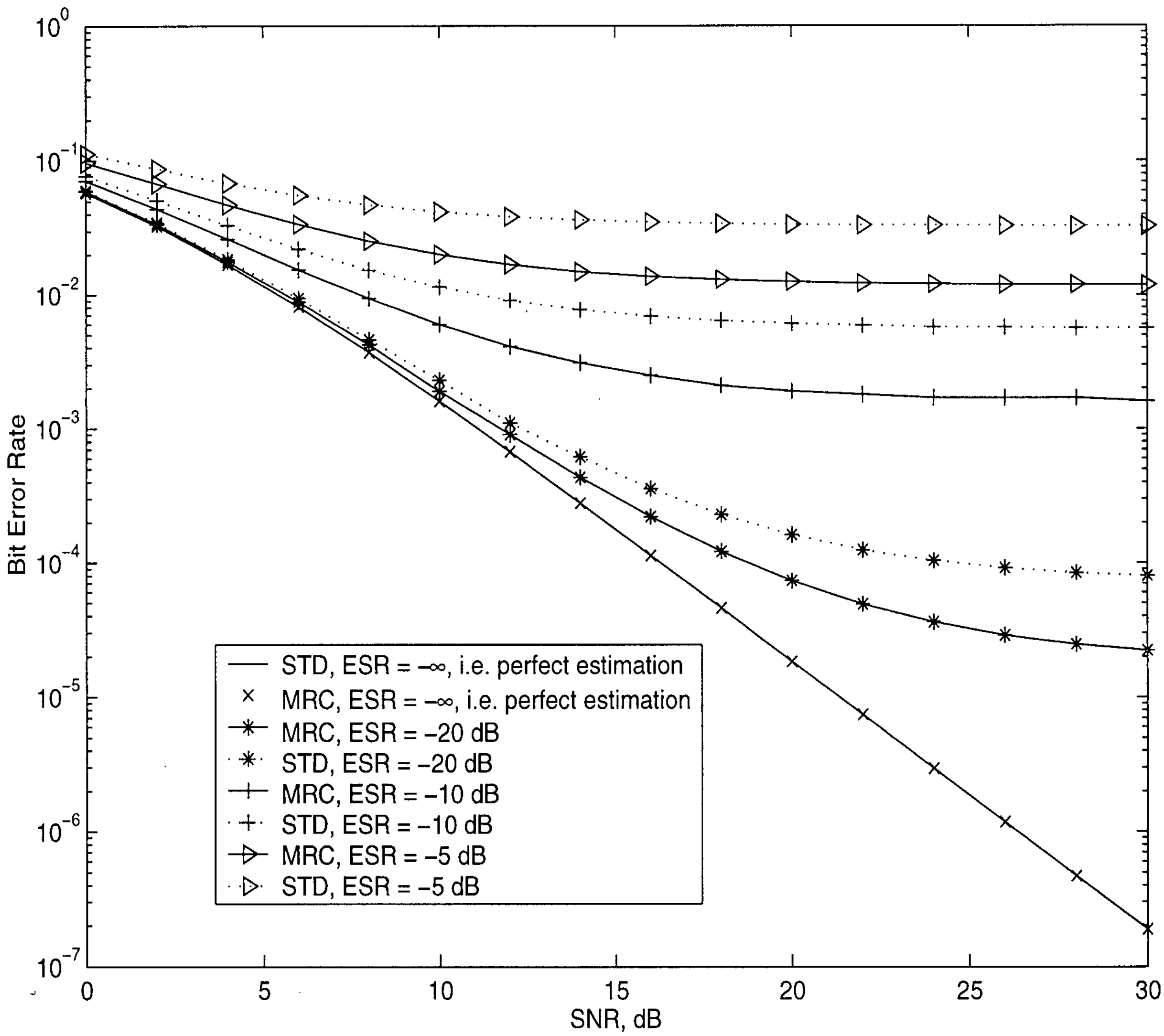

Figure 4.4 BER for MRC and STD in non time-selective Rayleigh fading with diversity order of two as a function of SNR for ESR = −5 dB, −10 dB, −20 dB and $-\infty$ dB.

diversity order of two, the error performance of STD degrades rapidly as ρ_t and ρ_e decrease from 1 and the error performance of MRC degrades only as ρ_e decrease from 1. For values of ρ_t or ρ_e less than 1, each curve exhibits an error floor. As expected, STD is more sensitive to channel estimation error than MRC. For a target BER of 10^{-4}, the error performance of MRC with

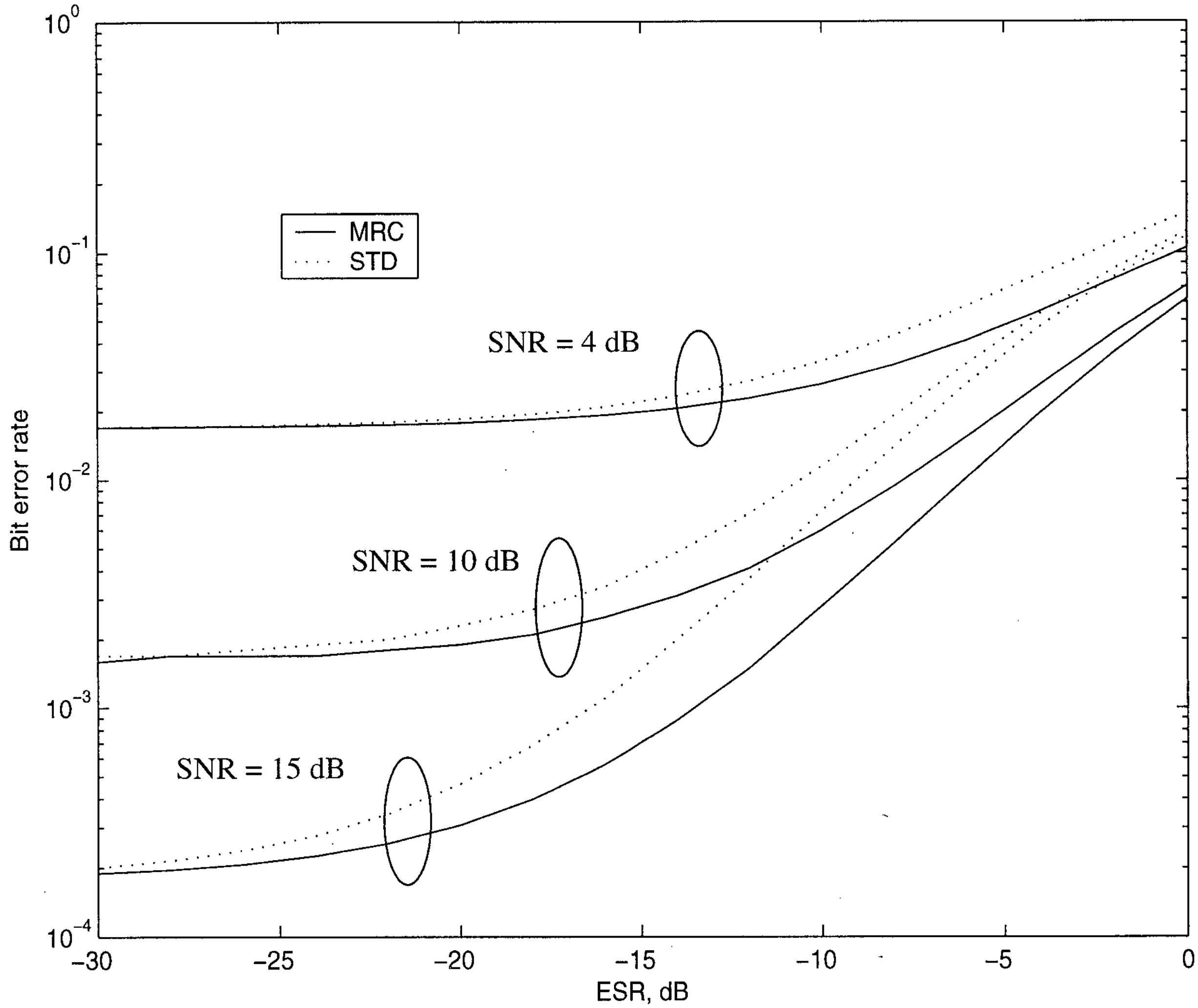

Figure 4.5 BER for MRC and STD in non time-selective, spatially independent Rayleigh fading with diversity order of two as a function of ESR for SNR = 4 dB, 10 dB and 15 dB.

$\rho_e = 0.99$ and STD with $(\rho_t = 0.99, \rho_e = 1)$ are almost identical with about 0.6 dB degradation relative to the curve for non time-selective fading with perfect channel estimation i.e. $(\rho_t = 1, \rho_e = 1)$. However, there is about 0.6 dB degradation for STD with $(\rho_t = 1, \rho_e = 0.99)$ and about 1.2 dB degradation for STD $(\rho_t = 0.99, \rho_e = 0.99)$ relative to MRC with $\rho_e = 0.99$.

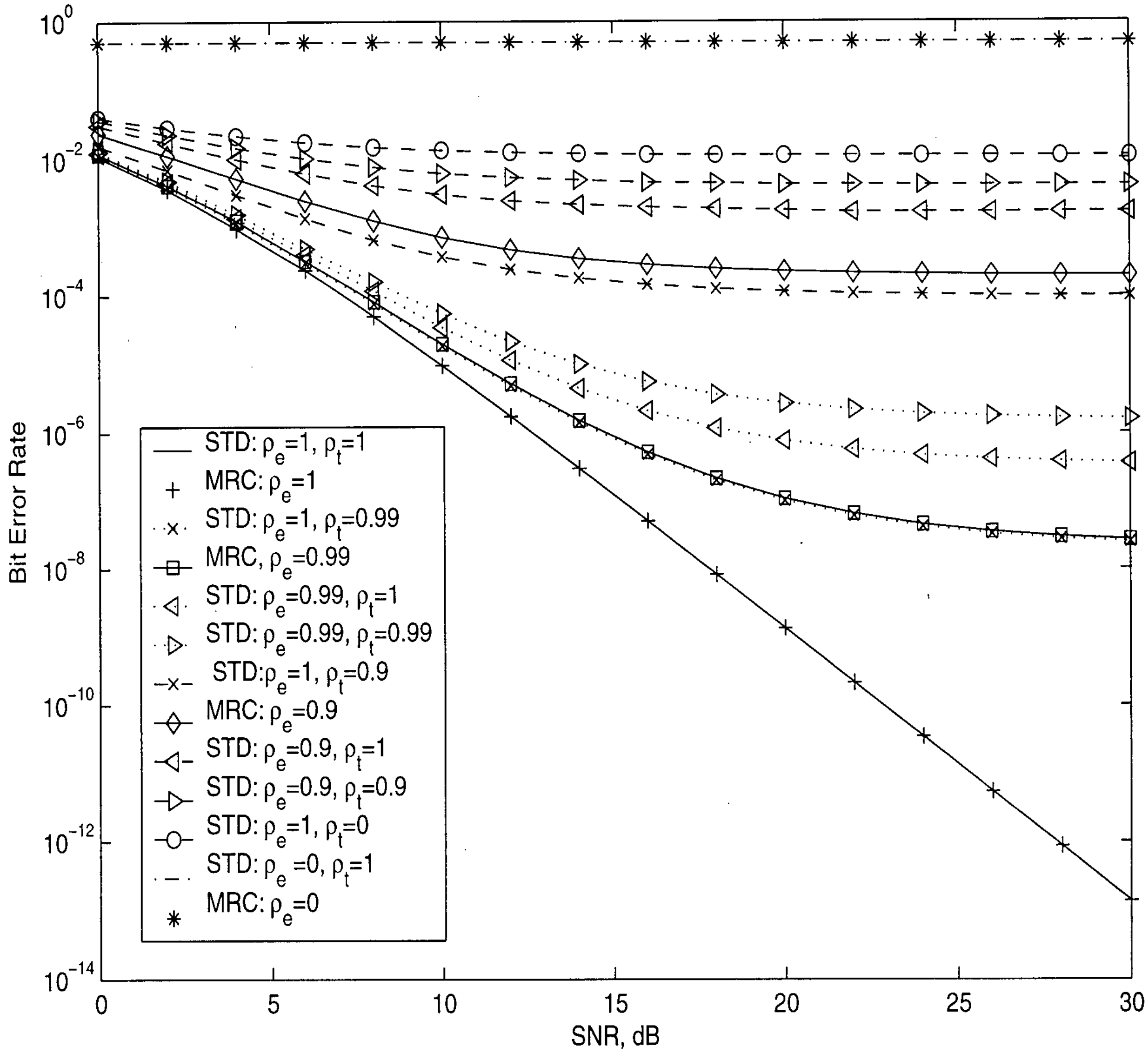

Figure 4.6 BER for STD and MRC with diversity order of four as a function of SNR for different values of ρ_t and ρ_e.

The theoretical BER performance of two branch STD in spatially correlated Rayleigh fading with imperfect channel estimation, as given by (3.38), is plotted in Figure 4.7 as a function of SNR for different values of ρ_s and ρ_e. It can be seen that the error performance degrades as ρ_s increases from 0 and as ρ_e decreases from 1. For $\rho_e = 1$ and a target BER of 10^{-3}, there is

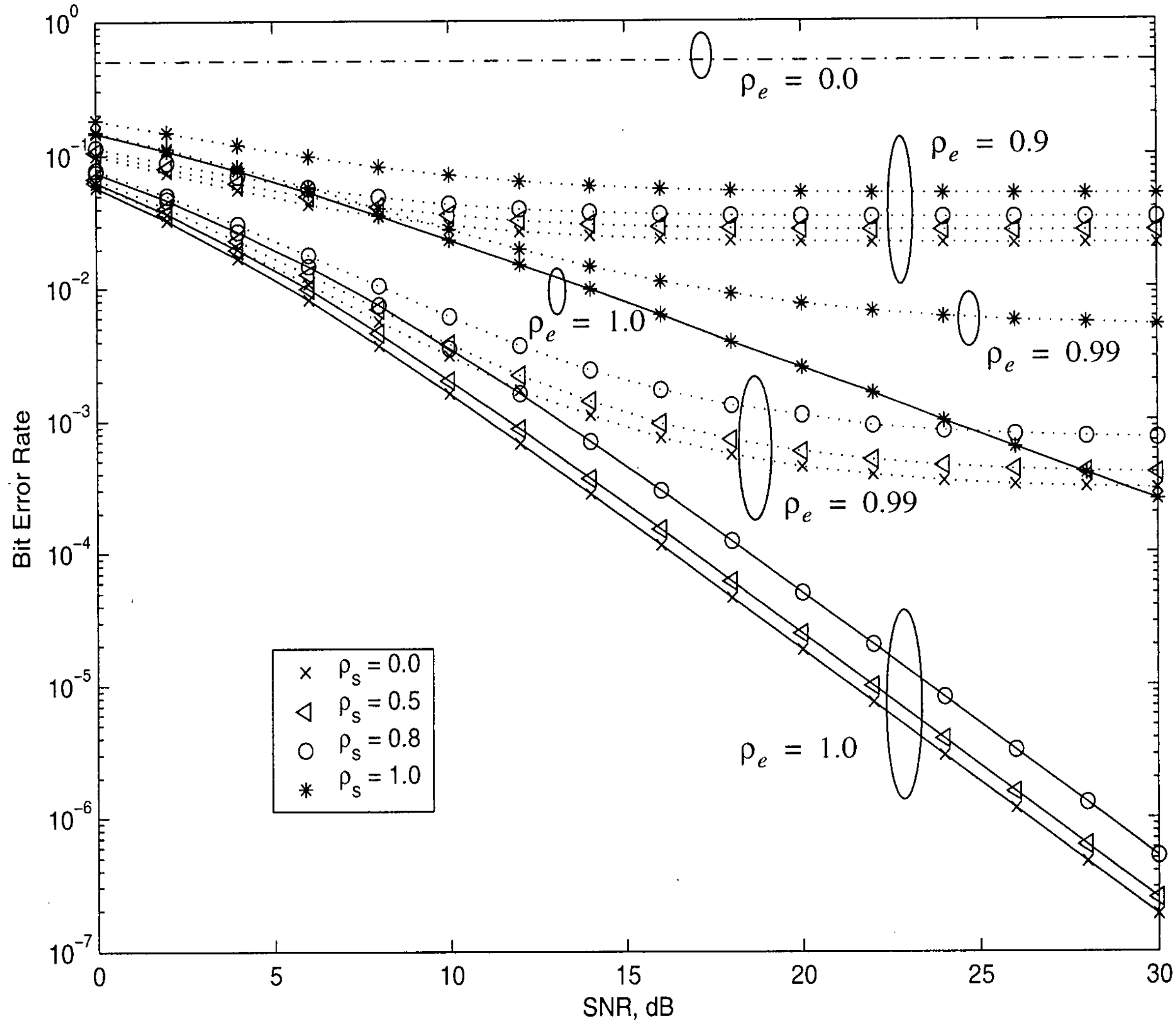

Figure 4.7 BER of two-branch STD as a function of SNR for different values of ρ_s and ρ_e.

about 1 dB degradation for $\rho_s = 0.5$ and about 2 dB degradation for $\rho_s = 0.8$ relative to the spatially independent case, i.e. $\rho_s = 0$. For $\rho_e = 0.99$, there is about 4.6 dB degradation for $\rho_s = 0.5$ and about 10 dB degradation for $\rho_s = 0.8$ relative to spatially independent fading with perfect channel estimation. It can also be seen that, in spatially correlated fading with imperfect channel estimation, the error floor limit is approached because of the estimation error while there is no error floor for STD in spatially correlated fading with perfect channel estimation.

Figure 4.8 shows the BER curves for STD and MRC in spatially correlated Rayleigh fading with diversity order of two as a function of SNR for three different values of ρ_e. It can be seen that the BER for STD or MRC degrades quiet rapidly with decrease in ρ_e and increase in ρ_s.

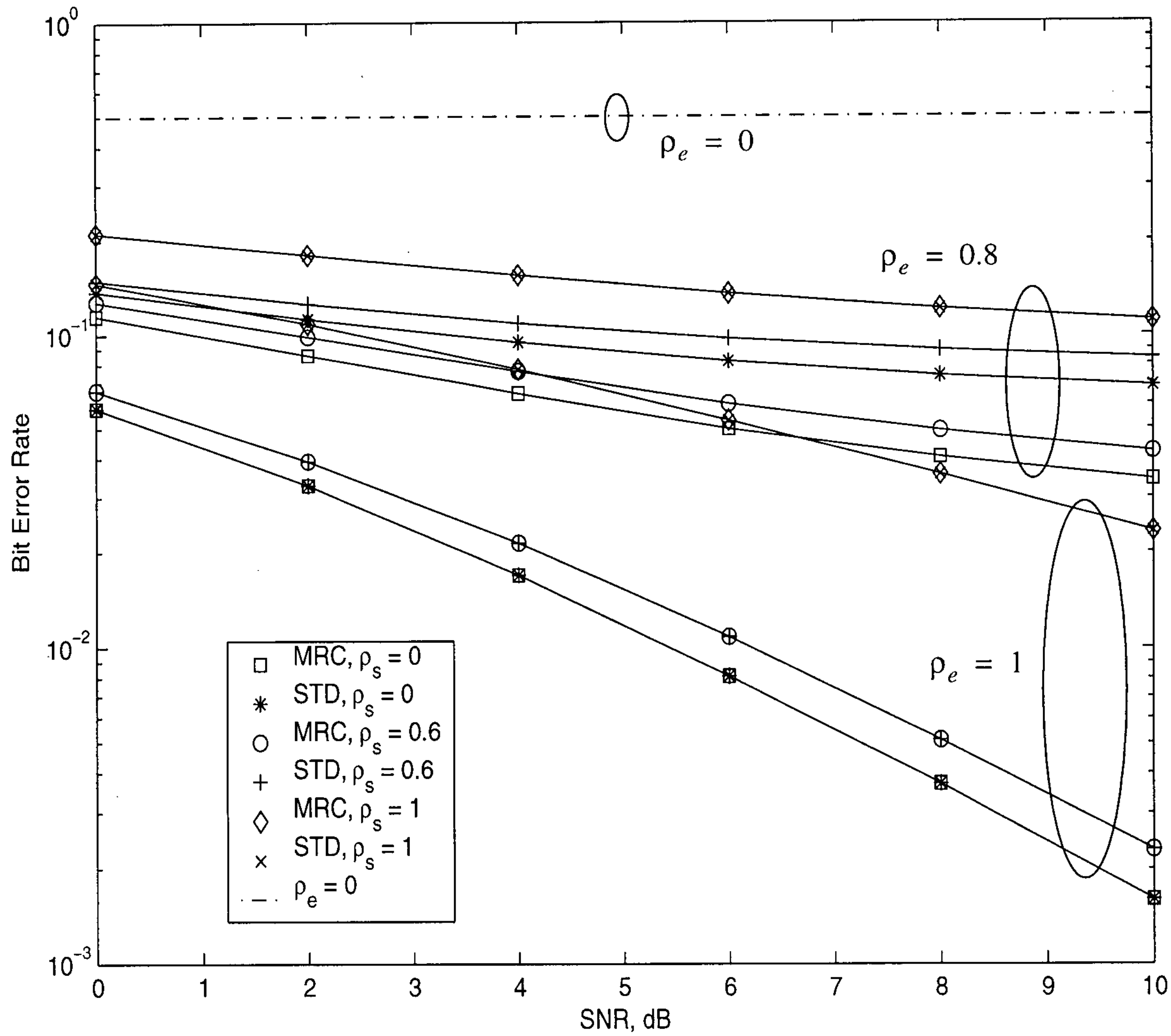

Figure 4.8 BER of two-branch STD and MRC as a function of SNR for different values of ρ_s and ρ_e.

With perfect channel estimation, i.e. $\rho_e = 1$, the error performance of STD and MRC in spatially correlated fading is identical; with $\rho_e = 0$, the estimated channel phase is completely random and the BER is 0.5 regardless of the value of ρ_s. With channel estimation error, the error performance of STD is worse than that of MRC in both spatially independent and correlated fading. For $\rho_e = 0.8$ and a target BER of 10^{-1}, there is 2.3 dB degradation in independent fading $(\rho_s = 0)$ and 3.2 dB degradation in correlated fading with $\rho_s = 0.6$ for STD relative to MRC.

Figure 4.9 shows the BER curves for STD and MRC in spatially correlated Rayleigh fading with a diversity order of two as a function of ESR for two different SNR values. The BER difference between STD and MRC increases with SNR and ESR.

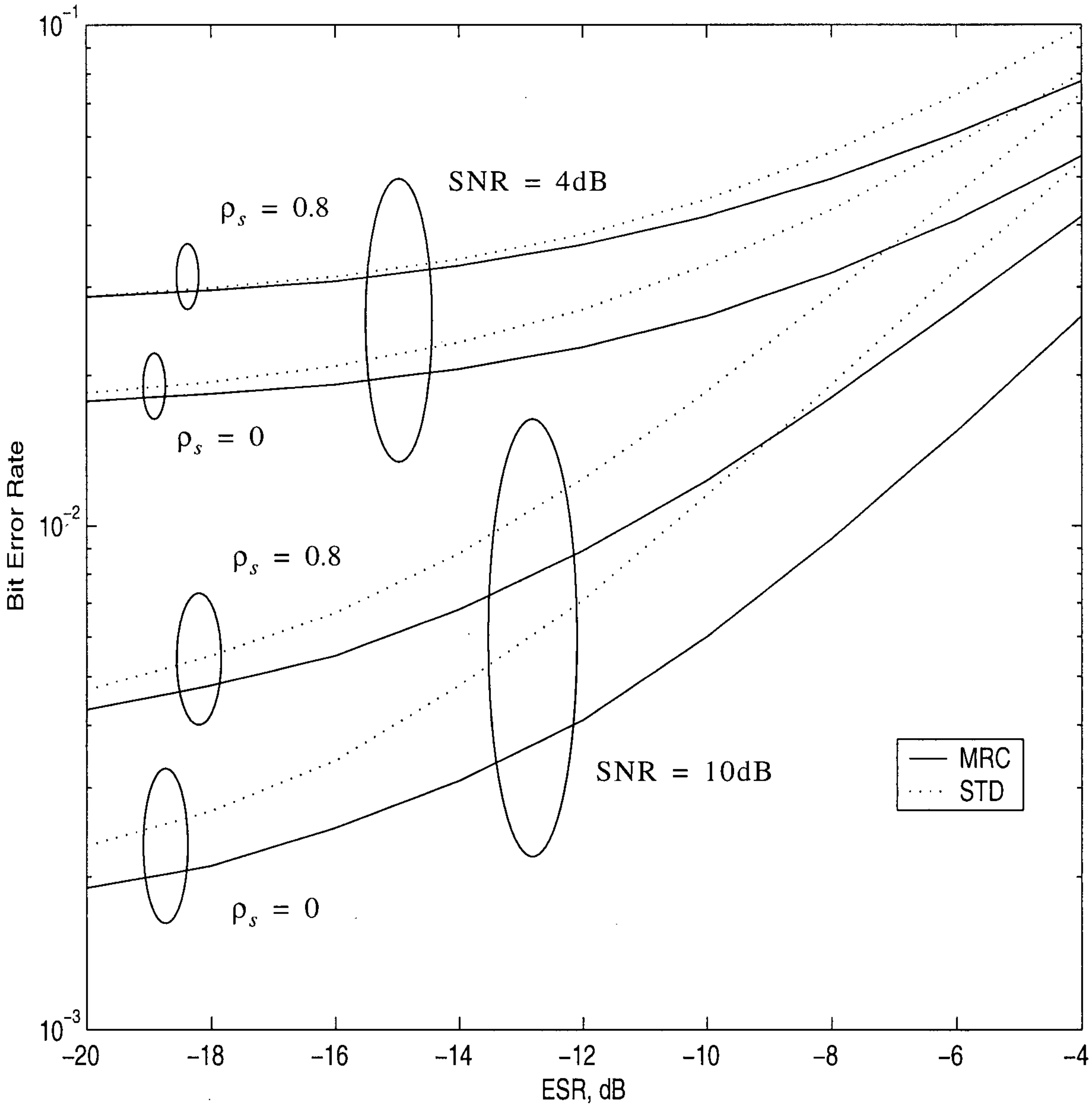

Figure 4.9 BER of two-branch STD as a function of ESR for two different values of ρ_s and two different SNR values

Chapter 5 Conclusion

5.1 Main Thesis Contributions

- In this thesis, exact closed-form expressions for the bit error rate of the simple transmit diversity scheme (STD) [7] in time-selective, spatially independent Rayleigh fading with imperfect channel estimation and in non time-selective, spatially correlated Rayleigh fading with imperfect channel estimation are derived. For spatially independent fading, it is found that for the same values of the channel gain time correlation coefficient ρ_t and the channel gain estimation error correlation coefficient ρ_e, the error performance in non time-selective fading with imperfect channel estimation is worse than in time-selective fading with perfect channel estimation. For a target BER of 10^{-3}, there is about 1 dB degradation for $(\rho_t = 0.99, \rho_e = 1)$ and about 3 dB degradation for $(\rho_t = 1, \rho_e = 0.99)$ relative to non time-selective fading with perfect channel estimation i.e. $(\rho_t = 1, \rho_e = 1)$.

- An expression for the BER floor resulting from channel estimation errors and time-selective fading is determined. For the same values of ρ_t and ρ_e, say ρ, the error floor limits are approached at lower SNR values for $(\rho_t = 1, \rho_e = \rho)$ than for $(\rho_t = \rho, \rho_e = 1)$.

- The effects of channel estimation errors on error performance of STD and MRC were compared and it was shown that for large values of signal to noise and estimation error to noise ratios, STD suffers a 3 dB loss compared to MRC in non-time selective, spatially independent fading.

5.2 Topics for Future Study

- This thesis investigated STD in a frequency flat Rayleigh fading channel. It would be useful to analyze the error performance of STD in frequency selective channels with Rican fading.

- The BER result for time selective fading is based on the assumption that the channel gains are spatially independent. It would be interesting to study the effects of spatial correlation on performance in time selective fading.

Glossary

Acronyms

AMPS	Advanced Mobile Phone Service
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
dB	decibel
EGC	Equal Gain Combining
ESR	Estimation Error to Signal Ratio
MRC	Maximal Ratio Combining
MRT	Maximum Ratio Transmission
MRTC	Maximum ratio transmission and combining
MS	Mobile station
pdf	Probability of density function
QPSK	Quadrature Phase Shift Keying
SD	Selection Diversity
SER	Symbol Error Rate
SNR	Signal to Noise Ratio
STD	Simple Transmit Diversity

Symbols

N	Number of transmit antennas
M	Number of receive antennas
$G_{ji,\,t}$	The channel gain from transmit antenna j to receive antenna i at time t
$g_{ji,\,t}$	The sample of $G_{ji,\,t}$
g_k	The sample of the channel gain corresponding to kth branch
α_k	Amplitude of g_k
θ_k	Phase of g_k
x_k	The real part of g_k
y_k	The imaginary part of g_k
σ_X^2	Variance of X_k
σ_Y^2	Variance of Y_k
σ_G^2	Variance of $G_{ji,\,t}$
ζ	Signal energy
$r_{k,\,MRC}$	Received signal on branch k in MRC scheme
s_0, s_1	Transmitted signal
n_k	noise and interference on branch K
σ_N^2	Variance of the real (or imaginary) component of N_i
$\tilde{s}_{0,\,MRC}$	Signal output from MRC combiner
γ	Output SNR from combiner
γ_k	SNR on ith branch
γ_0	Average SNR per branch
$p(\gamma)$	pdf of γ
$p_{e,\,MRC}$	BER of MRC
T	Symbol duration
$\tilde{s}_{0,\,STD}$, $\tilde{s}_{1,\,STD}$	Signal output from STD combiner
r_i	Received signal on branch i in the first symbol period
$r_{i,\,T}$	Received signal on branch i in the second symbol period

G	Channel coefficient matrix for MRT
$\underline{V}$	Transmit weighting vector for MRT
$\underline{W}$	Receive weighting vector for MRT
^{H}	Hermitian operation
^{T}	Transpose operation
$\underline{n}$	Noise vector for MRT
$\tilde{s}_{MRT}$	Signal output from MRT combiner
a	A normalization factor
$H_{ji,t}$	Estimated channel gain for $G_{ji,t}$
σ_H^2	Variance of $H_{ji,t}$
$Z_{ji,t}$	Channel estimation error for $G_{ji,t}$
σ_Z^2	Variance of $Z_i(t)$
ρ_t	Correlation coefficient of $G_{ji,0}$ and $G_{ji,T}$
ρ_e	Correlation coefficient of $G_{ji,0}$ and $H_{ji,t}$
$\rho_{e,n}$	Normalized correlation coefficient equal to ρ_e^2
Re $(\tilde{s}_{0,STD})$	The real component of $(\tilde{s}_{0,STD})$
$p_{e,std}$	BER for STD
$p_{f,STD}$	The overall BER for STD
$p_{e,QPSK}$	SER for QPSK
C	The covariance matrix of the channel gains
ρ_s	Spatially correlation coefficient of the channel gains
ρ_h	Spatially correlation coefficient of the estimated channel gains
λ_l	Eigenvalues of C

Appendix A. Derivation of the Correlation Coefficient Between the Estimated Channel Gain and the Actual Channel Gain

Here we derive the correlation coefficient, ρ_e, between the channel gain, $G_{ji,\,t}$ and the estimated channel gain $H_{ji,\,t}$. In the following, we will leave out the index t and i for brevity. Following [10], we can express the estimated channel gain H_1 and H_2 as

$$\begin{aligned} h_1 &= g_1 + z_1 \\ h_2 &= g_2 + z_2 \end{aligned} \tag{A.1}$$

where g_j and z_j are samples of independent zero mean complex Gaussian r.v.'s. We express G_j as

$$\begin{aligned} g_1 &= x_1 + jy_1 \\ g_2 &= x_2 + jy_2 \end{aligned} \tag{A.2}$$

where x_1, y_1, x_2, y_2 are samples of independent zero mean Gaussian r.v.'s. The variances of X_1, Y_1, X_2, Y_2 can be expressed as

$$\begin{aligned} E(X_1^2) &= E(Y_1^2) = \sigma_G^2 \\ E(X_2^2) &= E(Y_2^2) = \sigma_G^2 \end{aligned} \tag{A.3}$$

We express Z_j as

$$\begin{aligned} z_1 &= u_1 + jw_1 \\ z_2 &= u_2 + jw_2 \end{aligned} \tag{A.4}$$

where u_1, w_1, u_2, w_2 are samples of independent zero mean Gaussian r.v.'s. The variances of

U_1, W_1, U_2, W_2 can be expressed as

$$\begin{aligned} E(U_1^2) &= E(W_1^2) = \sigma_Z^2 \\ E(U_2^2) &= E(W_2^2) = \sigma_Z^2 \end{aligned} \tag{A.5}$$

Thus

$$\begin{aligned} h_1 &= x_1 + u_1 + j(y_1 + w_1) \\ h_2 &= x_2 + u_2 + j(y_2 + w_2) \end{aligned} \tag{A.6}$$

The correlation coefficient ρ_e is given by [3]

$$\begin{aligned} \rho_e &= \frac{E[G_j H_j^*]}{\sqrt{E[|G_j|^2]E[|H_j|^2]}} \\ &= \frac{E\{(X_j + jY_j)[(X_j + U_j) - j(Y_j + W_j)]\}}{\sqrt{E(X_j^2 + Y_j^2)E[(X_j + U_j)^2 + (Y_j + W_j)^2]}} \\ &= \frac{E[X_j^2 + X_j U_j + Y_j^2 + Y_j W_j + j(U_j Y_j - W_j X_j)]}{\sqrt{E(X_j^2 + Y_j^2)E[X_j^2 + U_j^2 + 2X_j U_j + Y_j^2 + W_j^2 + 2Y_j W_j]}} \end{aligned} \tag{A.7}$$

Since X_j, Y_j, U_j, W_j are independent zero mean Gaussian r.v.'s, $E(X_j Y_j) = E(X_j W_j) = E(X_j U_j) = E(Y_j W_j) = E(U_j Y_j) = E(W_j X_j) = 0$. From 3.1.1, we have

$$\sigma_{H_j}^2 = \sigma_{G_j}^2 + \sigma_{Z_j}^2 \tag{A.8}$$

Using (A.5), (A.7) and (A.8), ρ_e can be expressed as

$$\rho_e = \sigma_{G_j} / \sigma_{H_j}. \tag{A.9}$$

Appendix B. Derivation of the Means and Variances of the Random Variables in Equation (3.8)

Here we derive the means and variances of $Re(h^{*}_{1i,0} M_{1i})$, $Re(h_{2i,T} M^{*}_{2i})$, $Re(h^{*}_{1i,0} V_{2i})$, $Re(h_{2i,T} V^{*}_{1i})$, $Re(h^{*}_{1i,0} N_{i,0})$ and $Re(h_{2i,T} N^{*}_{i,T})$ as given in (3.8). $H_{ji,t}$, $j \in (1,2)$, $i = 1 \ldots M$, $t \in (0, T)$ can be denoted by $h_{ji,t} = \alpha_{ji,t} e^{j\theta_{ji,t}} = u_{ji,t} + jw_{ji,t}$ where $u_{ji,t} = \alpha_{ji,t} \cos\theta_{ji,t}$ and $w_{ji,t} = \alpha_{ji,t} \sin\theta_{ji,t}$. First of all, we prove that D_{1i}, D_{2i}, V_{1i} and V_{2i} are independent with each other.

From (3.1) and (3.2), we have

$$\begin{aligned} g_{1i,0} &= \rho_{e,n} h_{1i,0} + d_{1i} \\ g_{2i,T} &= \rho_{e,n} h_{2i,T} + d_{2i} \end{aligned} \tag{B.1}$$

$$\begin{aligned} g_{1i,T} &= \rho_t g_{1i,0} + v_{1i} \\ g_{2i,0} &= \rho_t g_{2i,T} + v_{2i} \end{aligned} \tag{B.2}$$

where g_{1i,t_1} and g_{2i,t_2}, $t_1, t_2 \in (0, T)$, each with variance σ_G^2, are samples of zero mean complex Gaussian r.v.'s which are independent with each other; d_{1i} and d_{2i}, each with variance $\sigma_D^2 = (1-\rho_e^2)\sigma_G^2$, are samples of zero mean complex Gaussian r.v.'s which are independent of $h_{1i,0}$ and $h_{2i,T}$, v_{1i} and v_{2i}, each with variances $\sigma_V^2 = (1-\rho_t^2)\sigma_G^2$, are samples of zero mean complex Gaussian r.v.'s which are independent of $g_{1i,0}$ and $g_{2i,T}$. $h_{ji,t} = g_{ji,t} + z_{ji,t}$ where z_{ji,t_1} is a sample of a zero mean, variance σ_Z^2, complex Gaussian r.v which is independent of

G_{ji,t_2}. $z_{ji,0}$ is independent of $z_{ji,T}$. Using (B.1), the covariance of $G_{1i,0}$ and $H_{2i,T}$ can be expressed as [3]

$$\begin{aligned} E(G_{1i,0}H^*_{2i,T}) &= E(G_{1i,0}G^*_{2i,T} + G_{1i,0}Z^*_{2i,T}) \\ &= E(\rho_{e,n}H_{1i,0}H^*_{2i,T} + D_{1i}H^*_{2i,T}) \end{aligned} \tag{B.3}$$

Since G_{1i,t_1} is independent of G_{1i,t_2}; z_{ji,t_1} is independent of G_{ji,t_2} and $z_{ji,0}$ is independent of $z_{ji,T}$, we have

$$E(G_{1i,0}G^*_{2i,T}) = E(G_{1i,0}Z^*_{2i,T}) = E(H_{1i,0}H^*_{2i,T}) = 0, \tag{B.4}$$

thus using (B.3) and (3.3), we have

$$E(D_{1i}H^*_{2i,T}) = 0. \tag{B.5}$$

Similarly, it can been shown that

$$E(D^*_{2i}H_{1i,0}) = 0. \tag{B.6}$$

Using (B.1) and (B.2), the covariance of $G_{1i,0}$ and $G_{2i,T}$ can be expressed as

$$E(G_{1i,0}G^*_{2i,T}) = E(\rho^2_{e,n}H_{1i,0}H^*_{2i,T} + \rho_{e,n}H^*_{2i,T}D_{1i} + \rho_{e,n}D^*_{2i}H_{1i,0} + D_{1i}D^*_{2i}) \tag{B.7}$$

Using (B.4), (B.5) and (B.6), thus we have

$$E(D_{1i}D^*_{2i}) = 0 \tag{B.8}$$

Since D_{1i} and D_{2i} are zero mean complex Gaussian r.v.'s, they are uncorrelated and statistically independent [23]. Similarly we can show D_{li} and V_{ki}, V_{li} and V_{ki}, $l, k \in (1, 2), l \neq k$, are independent with each other.

In the following, we will show that D_{ji} is independent of V_{ji}. Using (B.2), the covariance of $H_{1i,0}$ and $G_{1i,T}$ can be expressed as

$$E(H_{1i,0}G^*_{1i,T}) = E(\rho_t H_{1i,0}G^*_{1i,0} + H_{1i,0}V^*_{1i}) \tag{B.9}$$

Since

$$\begin{aligned} E(H_{1i,0}G^*_{1i,T}) &= E(G_{1i,0}G^*_{1i,T} + Z_{1i,0}G^*_{1i,T}) \\ &= E(G_{1i,0}G^*_{1i,T}) \\ &= 2\rho_t\sigma_G^2 \end{aligned} \tag{B.10}$$

$$E(H_{1i,0}G^*_{1i,0}) = 2\sigma_G^2 \tag{B.11}$$

thus using (B.9) and (B.10), we have

$$E(H_{1i,0}V^*_{1i}) = 0 \tag{B.12}$$

Since D_{1i} is independent of $H_{1i,0}$, using (B.1), the covariance of $G_{1i,0}$ and D_{1i} can be expressed as

$$\begin{aligned} E(G_{1i,0}D^*_{1i}) &= E(\rho_{e,n}H_{1i,0}D^*_{1i} + D_{1i}D^*_{1i}) \\ &= 2\sigma_D^2 \end{aligned} \tag{B.13}$$

Using (B.1) and (B.2), the covariance of $G_{1i,0}$ and $G_{1i,T}$ can be expressed as

$$E(G_{1i,0}G^*_{1i,T}) = E(\rho_{e,n}\rho_t H_{1i,0}G^*_{1i,0} + \rho_{e,n}H_{1i,0}V^*_{1i} + \rho_t D_{1i}G^*_{1i,0} + D_{1i}V^*_{1i}) \quad \text{(B.14)}$$

Using (B.10) - (B.13), we have

$$E(D_{1i}V^*_{1i}) = 0 \quad \text{(B.15)}$$

D_{1i} and V_{1i} are uncorrelated and statistically independent. Similarly we can show D_{2i} and V_{2i} are also independent with each other. Thus D_{1i}, D_{2i}, V_{1i} and V_{2i} are independent with each other.

In the following, we derive the means and variances for $Re(h^*_{1i,0}\ M_{1i})$, $Re(h_{2i,T}M^*_{2i})$, $Re(h^*_{1i,0}\ V_{2i})$, $Re(h_{2i,T}\ V^*_{1i})$, $Re(h^*_{1i,0}\ N_{i,0})$ and $Re(h_{2i,T}N^*_{i,T})$ in (3.8).

From (3.8), we have

$$\begin{aligned} m_{1i} &= d_{1i} + \rho_t d_{2i} \\ m_{2i} &= d_{2i} - \rho_t d_{1i} \end{aligned} \quad \text{(B.16)}$$

Since D_{1i} and D_{2i} are independent, zero mean complex Gaussian r.v.'s, we assume

$$\begin{aligned} d_{1i} &= x_{1i} + jy_{1i} \\ d_{2i} &= x_{2i} + jy_{2i} \end{aligned} \quad \text{(B.17)}$$

where $x_{1i}, y_{1i}, x_{2i}, y_{2i}$ are samples of independent, zero mean Gaussian r.v.'s. Thus

$$
\begin{aligned}
m_{1i} &= x_{1i} + \rho_t x_{2i} + j(y_{1i} + \rho_t y_{2i}) \\
m_{2i} &= x_{2i} - \rho_t x_{1i} + j(y_{2i} - \rho_t y_{1i})
\end{aligned}
\tag{B.18}
$$

and

$$
\begin{aligned}
Re(h_{1i,0}^{*}\ M_{1i}) &= u_{1i,0}(x_{1i} + \rho_t x_{2i}) + w_{1i,0}(y_{1i} + \rho_t y_{2i}) \\
Re(h_{2i,T}\, M_{2i}^{*}) &= u_{2i,T}(x_{2i} - \rho_t x_{1i}) + w_{2i,T}(y_{2i} - \rho_t y_{1i})
\end{aligned}
\tag{B.19}
$$

Since the means of $X_{1i}, Y_{1i}, X_{2i}, Y_{2i}$, $E(X_{1i}) = E(X_{2i}) = E(Y_{1i}) = E(Y_{2i}) = 0$, the means of M_{1i}, M_{2i}, $Re(h_{1i,0}^{*}\ M_{1i})$ and $Re(h_{2i,T}\, M_{2i}^{*})$ are also equal to zero.

From (3.5), The variances of $X_{1i}, Y_{1i}, X_{2i}, Y_{2i}$ can be expressed as

$$
\begin{aligned}
E(X_{1i}^2) &= E(Y_{1i}^2) = \sigma_D^2 = (1-\rho_e^2)\sigma_G^2 \\
E(X_{2}^2) &= E(Y_{2i}^2) = \sigma_D^2 = (1-\rho_e^2)\sigma_G^2
\end{aligned}
\tag{B.20}
$$

and the variances of $Re(h_{1i,0}^{*}\ M_{1i})$ and $Re(h_{2i,T}\, M_{2i}^{*})$ can be expressed as

$$
\begin{aligned}
E\{\mathrm{Re}[(h_{1i,0}^{*}\ M_{1i})]^2\} &= E[u_{1i,0}^2(x_{1i}^2 + \rho_t^2 x_{2i}^2 + 2\rho_t x_{1i} x_{2i}) + w_{1i,0}^2(y_{1i}^2 + \rho_t^2 y_{2i}^2 + 2\rho_t y_{1i} y_{2i}) \\
&\quad + 2u_{1i,0} w_{1i,0}(x_{1i} + \rho_t x_{2i})(y_{1i} + \rho_t y_{2i})] \\
E\{\mathrm{Re}[(h_{2i,T}\, M_{2i}^{*})]^2\} &= E[u_{2i,T}^2(x_{2i}^2 + \rho_t^2 x_{1i}^2 - 2\rho_t x_{1i} x_{2i}) + w_{2i,T}^2(y_{2i}^2 + \rho_t^2 y_{1i}^2 - 2\rho_t y_{1i} y_{2i}) \\
&\quad + 2u_{2i,T} w_{2i,T}(x_{2i} - \rho_t x_{1i})(y_{2i} - \rho_t y_{1i})]
\end{aligned}
\tag{B.21}
$$

Since $X_{1i}, Y_{1i}, X_{2i}, Y_{2i}$ are independent zero mean Gaussian r.v.'s, $E(X_{ji} Y_{li}) = 0$, $l \in \{1, 2\}, l \neq j$. using (B.20), the variances of $\mathrm{Re}(h_{1i,0}^{*}\ M_{1i})$ and $\mathrm{Re}(h_{2i,T}\, M_{2i}^{*})$ can be expressed as

$$\begin{aligned} E\{Re[(h^*_{1i,0}\ M_{1i})]^2\} &= \alpha^2_{1i,0}(\sigma^2_D + \rho^2_t \sigma^2_D) \\ E\{Re[(h_{2i,T}\, M^*_{2i})]^2\} &= \alpha^2_{2i,T}(\sigma^2_D + \rho^2_t \sigma^2_D) \end{aligned} \tag{B.22}$$

and the covariance of $Re(h^*_{1i,0}\ M_{1i})$ and $Re(h_{2i,T}\, M^*_{2i})$ can be expressed as

$$E[(\mathrm{Re}h^*_{1i,0} M_{1i})\mathrm{Re}(h_{2i,T}\, M^*_{2i})] = 0 \tag{B.23}$$

It indicates that $\mathrm{Re}(h^*_{1i,0} M_{1i})$ and $\mathrm{Re}(h_{2i,T}\, M^*_{2i})$ are uncorrelated and statistically independent.The variance of M, σ^2_M, where $M = \mathrm{Re}(h^*_{1i,0}\ M_{1i}) + \mathrm{Re}(h_{2i,T}\, M^*_{2i})$, can be derived as

$$\begin{aligned} \sigma^2_M &= E\{\{\mathrm{Re}[h_0{}^*(0) M_0]\}^2\} + E\{\{\mathrm{Re}[h_1(T)\, M^*_1]\}^2\} \\ &= [\alpha^2_{1i,0} + \alpha^2_{2i,T}](1 + \rho^2_t)\sigma^2_D \end{aligned} \tag{B.24}$$

Since D_{1i}, D_{2i}, V_{1i}, V_{2i}, $N_{i,0}$ and $N_{i,T}$ are zero mean independent Gaussian r.v.'s with variances σ^2_D, σ^2_V and σ^2_N, M, $\mathrm{Re}(h^*_{1i,0}\ V_{2i})$, $\mathrm{Re}(h_{2i,T}\, V^*_{1i})$, $\mathrm{Re}(h^*_{1i,0}\ N_{i,0})$ and $\mathrm{Re}(h_{2i,T}\, N^*_{i,T})$ are also zero mean independent Gaussian r.v.'s. The corresponding variances are $[\alpha^2_{1i,0} + \alpha^2_{2i,T}](1 + \rho^2_t)\sigma^2_D$, $\sigma^2_V\, a^2_{1i,0}$, $\sigma^2_V\, \alpha^2_{2i,T}$, $\sigma^2_N\, a^2_{1i,0}$ and $\sigma^2_N\, \alpha^2_{2i,T}$.

Appendix C. Derivation of the Correlation Coefficient between the Estimated Channel Gains

From 3.3.1, we assume that G_{1i} and G_{2i} are correlated with correlation coefficient ρ_s. The estimated channel gain, H_{1i} and H_{2i} are zero mean complex Gaussian r.v.'s with variances $\sigma_H^2 = \sigma_G^2 + \sigma_Z^2$ where the channel estimation error, Z_{1i} and Z_{2i}, are independent zero means complex Gaussian r.v's with variances σ_Z^2.

The correlation coefficient, ρ_h, of H_{1i} and H_{2i}, is given by [3]

$$\begin{aligned}\rho_h &= \frac{E[H_{1i}H^*{}_{2i}]}{\sqrt{E[|H_{1i}|^2]E[|H_{2i}|^2]}} \\ &= \frac{E[(G_{1i}+Z_{1i})(G_{2i}+Z_{2i})^*]}{2\sigma_H^2}\end{aligned} \tag{C.1}$$

Since $E[G_{1i}\, G_{2i}^*] = \rho_s\sigma_G^2$, also Z_{1i} and Z_{2i} are independent Gaussian r.v.'s, $E[G_{1i}Z_{2i}{}^*] = E[G_{2i}Z_{2i}{}^*] = E[G_{2i}{}^*Z_{1i}] = E[G_{1i}{}^*Z_{1i}] = E[Z_{1i}Z_{2i}{}^*] = 0$, and hence the correlation coefficient ρ_h can be expressed as

$$\rho_h = \frac{\sigma_G^2}{\sigma_H^2}\rho_s \tag{C.2}$$

Appendix D. Derivation of Means and Variances of Random Variables in Equation (3.29)

Here we derive the variances of $Re(h^*{}_{1i}(D_{1i}+D_{2i}))$, $Re(h_{2i}(D_{2i}-D_{1i})^*)$, $Re(h^*{}_{1i}N_{i,0})$ and $Re(h_{2i}N^*{}_{i,T})$ as given in (3.29). $h_{ji}, j = 1, 2, i = 1, \ldots, M$ can be denoted by $h_{ji} = \alpha_{ji}e^{j\theta_{ji}} = u_{ji} + jw_{ji}$ where $u_{ji} = \alpha_{ji}\cos\theta_{ji}$ and $w_{ji} = \alpha_{ji}\sin\theta_{ji}$. First of all, we prove that D_{1i}, D_{2i} are correlated with correlation coefficient $\rho_d = (1-\rho_e^2)\rho_s$.

From (3.27), we have

$$\begin{aligned} g_{1i} &= \rho_e^2 h_{1i} + d_{1i} \\ g_{2i} &= \rho_e^2 h_{2i} + d_{2i} \end{aligned} \tag{D.1}$$

where $\rho_e^2 = \sigma_{G_j}^2/\sigma_{H_j}^2$, d_{1i} and d_{2i} are samples of zero mean, variance $\sigma_D^2 = (1-\rho_e^2)\sigma_G^2$ complex Gaussian r.v.'s which are independent of H_{1i} and H_{2i}. The complex Gaussian r.v.'s G_{1i} and G_{2i} are correlated with the correlation coefficient ρ_s, i.e.

$$\rho_s = \frac{E(G_{1i}G_{2i}^*)}{2\sigma_G^2}. \tag{D.2}$$

Using (D.1), the covariance of G_{1i} and H_{2i} can be expressed as

$$\begin{aligned} E(G_{1i}H_{2i}^*) &= E(G_{1i}G_{2i}^* + G_{1i}Z_{2i}^*) \\ &= E(\rho_e^2 H_{1i}H_{2i}^* + D_{1i}H_{2i}^*) \end{aligned} \tag{D.3}$$

since

$$E(G_{1i}Z^*_{2i}) = 0, \tag{D.4}$$

$$E(G_{1i}H^*_{2i}) = E(H_{1i}H^*_{2i}) = 2\rho_s\sigma_G^2, \tag{D.5}$$

Thus using (D.2) - (D.5), we have

$$E(D_{1i}H^*_{2i}) = 2(1-\rho_e^2)\rho_s\sigma_G^2. \tag{D.6}$$

Similarly, the covariance of H_{1i} and D_{2i} can be expressed as

$$E(H_{1i}D^*_{2i}) = 2(1-\rho_e^2)\rho_s\sigma_G^2. \tag{D.7}$$

Using (D.1), the covariance of G_{1i} and G_{2i} can be expressed as

$$E(G_{1i}G^*_{2i}) = E(\rho_e^4 H_{1i}H^*_{2i} + \rho_e^2 D_{1i}H^*_{2i} + \rho_e^2 H_{1i}D^*_{2i} + D_{1i}D^*_{2i}) \tag{D.8}$$

Using (D.2), (D.5) - (D.7), we have

$$E(D_{1i}D^*_{2i}) = 2(1-\rho_e^2)^2\rho_s\sigma_G^2 \tag{D.9}$$

The correlation coefficient of D_{1i} and D_{2i}, ρ_d, can be expressed as

$$\begin{aligned}\rho_d &= \frac{E[D_{1i}D_{2i}^*]}{\sqrt{E[|D_{1i}|^2]E[|D_{2i}|^2]}} \\ &= \frac{2(1-\rho_e^2)^2\rho_s\sigma_G^2}{2(1-\rho_e^2)\sigma_G^2} \\ &= (1-\rho_e^2)\rho_s \ .\end{aligned} \tag{D.10}$$

In the following, we derive the variances of $Re(h_{1i}^{*}\ (D_{1i}+D_{2i}))$, $Re(h_{2i}(D_{2i}-D_{1i})^{*})$ and prove that $Re(h_{1i}^{*}\ (D_{1i}+D_{2i}))$ and $Re(h_{2i}(D_{2i}-D_{1i})^{*})$ are independent. Since D_{1i} and D_{2i} are correlated, zero mean complex Gaussian r.v.'s, we can express the samples of D_{1i} and D_{2i}, d_{1i} and d_{2i} as

$$\begin{aligned} d_{1i} &= x_{1i} + jy_{1i} \\ d_{2i} &= x_{2i} + jy_{2i} \end{aligned} \tag{D.11}$$

where $x_{1i}, y_{1i}, x_{2i}, y_{2i}$ are samples of zero mean correlated Gaussian r.v.'s with variances σ_D^2, i.e.

$$E[X_{ji}^2] = E[Y_{ji}^2] = \sigma_D^2 \tag{D.12}$$

$$E[X_{li}Y_{ki}] = 0 \qquad l = 1,2;\ k = 1,2 \tag{D.13}$$

$$E[X_{1i}X_{2i}] = E[Y_{1i}Y_{2i}] = \rho_d\sigma_D^2 \tag{D.14}$$

Then we have

$$\begin{aligned} d_{1i} + d_{2i} &= x_{1i} + x_{2i} + j(y_{1i} + y_{2i}) \\ d_{2i} - d_{1i} &= x_{2i} - x_{1i} + j(y_{2i} - y_{1i}) \end{aligned} \tag{D.15}$$

and

$$\begin{aligned} Re(h_{1i}^{*}\ (d_{1i} + d_{2i})) &= u_{1i}(x_{1i} + x_{2i}) + w_{1i}(y_{1i} + y_{2i}) \\ Re(h_{2i}(d_{2i} - d_{1i})^{*}) &= u_{2i}(x_{2i} - x_{1i}) + w_{2i}(y_{2i} - y_{1i}) \end{aligned} \tag{D.16}$$

The covariance of $Re(h_{1i}^{*}\ (D_{1i}+D_{2i}))$, $Re(h_{2i}(D_{2i}-D_{1i})^{*})$ can be expressed as

$$\begin{aligned} &E[Re(h^*_{1i}\ (D_{1i}+D_{2i}))\mathrm{Re}(h_{2i}(D_{2i}-D_{1i})^*)] = \\ &E[u_{1i}(X_{1i}+X_{2i})+w_{1i}(Y_{1i}+Y_{2i})][u_{2i}(X_{2i}-X_{1i})+w_{2i}(Y_{2i}-Y_{1i})]\ . \end{aligned} \tag{D.17}$$

Using (D.12), (D.13) and (D.14), (D.17) can be reduced to

$$E[Re(h^*_{1i}\ (D_{1i}+D_{2i}))\mathrm{Re}(h_{2i}(D_{2i}-D_{1i})^*)] = 0 \tag{D.18}$$

This shows that $Re(h^*_{1i}\ (D_{1i}+D_{2i}))$ and $Re(h_{2i}(D_{2i}-D_{1i})^*)$ are uncorrelated and statistically independent with each other. The variances of $Re(h^*_{1i}\ (D_{1i}+D_{2i}))$, $Re(h_{2i}(D_{2i}-D_{1i})^*)$ can be written as

$$\begin{aligned} E\{Re[h^*_{1i}\ (D_{1i}+D_{2i})]^2\} &= E[u^2_{1i}(X^2_{1i}+X^2_{2i}+2X_{1i}X_{2i})+w^2_{1i}(Y^2_{1i}+Y^2_{2i}+2Y_{1i}Y_{2i}) \\ &\quad +2u_{1i}w_{1i}(X_{1i}+X_{2i})(Y_{1i}+Y_{2i})] \\ E\{Re[h_{2i}(D_{2i}-D_{1i})^*]^2\} &= E[u^2_{2i}(X^2_{2i}+X^2_{1i}-2X_{1i}X_{2i})+w^2_{2i}(Y^2_{2i}+Y^2_{1i}-2Y_{1i}Y_{2i}) \\ &\quad +2u_{2i}w_{2i}(X_{2i}-X_{1i})(Y_{2i}-Y_{1i})] \end{aligned} \tag{D.19}$$

Using (D.12), (D.13) and (D.14), (D.19) can be reduced to

$$\begin{aligned} E\{Re[h^*_{1i}\ (D_{1i}+D_{2i})]^2\} &= 2(1+\rho_d)\alpha^2_{1i}\sigma^2_D \\ E\{Re[h_{2i}(D_{2i}-D_{1i})^*]^2\} &= 2(1+\rho_d)\alpha^2_{2i}\sigma^2_D \end{aligned} \tag{D.20}$$

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