INTELLIGENT FUSION OF SENSOR DATA FOR PRODUCT QUALITY
ASSESSMENT IN AN INDUSTRIAL MACHINE

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Abstract

Sensor fusion (or data fusion) refers to the combined and synergistic use of information from multiple sensors in order to increase the reliability, accuracy, and overall effectiveness of the sensor-based operation. This thesis investigates knowledge-based fuzzy sensor fusion methods that reliably fuse both redundant and complementary data. Fuzzy sensor fusion methods use fuzzy logic in arriving at a “fusion decision”, and they can fuse data that are imprecise, incomplete, or even conflicting, to provide meaningful information required for decision-making related to a process. Such decisions are useful, for example, in assigning quality grades to products, assessing the performance of a process, and in feedback control, both direct and supervisory. Fuzzy techniques are particularly useful when the errors and uncertainties associated with the sensors are subjectively assessed rather than based on precise statistics. These methods are applicable when the plant/process is complex, incompletely known, and difficult to model either analytically or experimentally.

In this thesis, the uncertainty and error in sensor data are considered to be both subjective and qualitative and hence related to the concept of fuzziness. The thesis specifically considers the implementation of fuzzy sensor fusion methods for the quality assessment in industrial production machines. Primary attention is given in this regard to an industrial fishcutting machine developed in the Industrial Automation Laboratory, University of British Columbia. Also some attention is given to CNC router machines used in material cutting.

Three appropriate methods of multiple-sensor fusion using fuzzy logic are adopted, implemented, and evaluated in the present work. The first sensor fusion method is based on Mamdani’s max-prod (or, sup-prod) composition, and it places equal weights on all the data sources, without considering their merit or importance. The second sensor fusion method is based on degree of certainty. It assigns weights proportional to the degree of certainty of sensor data, and in addition to the fused output, it provides information about the certainty of the output. The third sensor fusion method is based on the concept of compatibility of data. It provides a fused output and additional knowledge
about the degree of confidence in that output. This method is specifically effective, when sensors provide conflicting information. Thus, fusion is expected to improve the reliability of the sensor information and hence the knowledge of the system. The three knowledge-based sensor fusion techniques are implemented in a prototype fishcutting machine. The results are critically examined to determine the effectiveness and relative merits of the techniques.
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Chapter 1

1 Introduction

The primary objective of the research reported in this thesis is to develop, implement and evaluate appropriate methods for knowledge-based fuzzy fusion of sensor data for product quality assessment in an industrial cutting machine. Humans and animals instinctively use (or integrate) information from different senses to perform complex daily chores, with adroitness. Incorporating this ability in the present-day industrial machines and systems that use multiple sensors can increase their capabilities and performance. In this introductory chapter, first the objectives of the research are highlighted. Then the motivation for the present study is outlined. Next, the approach taken in achieving the research goals is presented. Also the rationale for taking the specific approach is illustrated. Further, possible application areas for the developed methods are indicated. The chapter concludes with a brief overview of the thesis.

1.1 Research Objectives

The primary objective of the research given in this thesis is to develop, implement and evaluate appropriate methods for knowledge-based fusion of sensor data for product quality assessment in an industrial cutting machine. In particular, a fish cutting machine and a router table are considered as suitable candidates for applying the developed technology. The implemented methods will be especially applicable to systems that use fuzzy logic for information representation and fusion. Hence, the research aims at developing methods, which are based on fuzzy logic, for sensor fusion. The techniques
are expected to increase the reliability and accuracy, and reduce the uncertainties of the fused information. It should be noted that the emphasis of this research is on quality assessment and inspection of cut products and operations; however, it is expected to be applicable to a variety of tasks of industrial automation, which may use multiple sensor systems. This work does not deal with the issues of using fused data to control the actuators, however, this topic has been addressed in related work in the Industrial Automation Laboratory [1].

1.2 Motivation

This research was motivated primarily by the requirements of a novel and automated fishcutting machine, which has been developed in the Industrial Automation Laboratory, University of British Columbia, Canada. This machine was developed to address the shortcomings of the machine known as the “Iron Butcher,” which is commonly used in the fish processing industry for head removal of fish. This machine has been designed and built at the turn of the century and has seen little modification or improvement since then. It is a purely mechanical device, which uses friction to sense the position of the gill of a fish and a tactile engagement mechanism to align the collarbone of the fish with the cutter blade. Due to the lack of active sensing and feedback control, this mechanism is known to cause considerable wastage by frequently overfeeding the fish into the cutter. For example, in the salmon processing industry of British Columbia, Canada, on the average 5% of the useful fish meat is wasted during processing, due to poor cutting accuracy, which represents a loss of 25 million dollars per year [2]. Also, underfeed of fish causes a part of the head to remain behind with the processed meat, thereby hampering the product quality. Such cases need reprocessing of the fish manually, which results in a reduction of the throughput rate and a wastage of labor. Another drawback of the “Iron Butcher” is the possible poor quality of the fish-cut (due to undesirable movements of fish during cutting and problems with the cutting blades,
resulting in an irregular and rough cut) even with accurate positioning; this degrades the aesthetic appeal of the product.

In order to address these shortcomings of the “Iron Butcher”, Industrial Automation Laboratory has developed a fish-processing machine, which uses modern sensing, actuation, and control technologies to detect and position the fish accurately at the cutter so as to minimize the wastage of useful meat and maximize the cut quality. The research prototype employs a low-level CCD camera (primary), which grabs the image of each fish on the conveyor, and determines the position of the collarbone, which is the desired position of cut [2]. Also a high-level CCD camera (secondary) grabs the image of each processed fish, and provides information about the visual quality of the fish-cut. The main source of product quality information is this high-level vision system. However, the information in the images from the high-level CCD camera is not enough to accurately and completely determine the product quality, as it is possible that the quality of cut may be good in terms of appearance, but at the same time the cut may not be accurate.

Single sensor systems have met with limited success in cases where there are multiple factors in assessing (inspection of) the quality of the product. Although vision system (color camera) produces a wealth of information, still it can only provide a two dimensional array of intensities (grey-scale) from a single direction of view. Also a considerable amount of time and computational effort is required in processing the large amount of image data. Another drawback of a vision system is that the images lack a tactile “feel” of an object under inspection. These issues can be remedied by using other sensors in conjunction with vision sensors. Besides providing complementary information as mentioned above, multiple sensors also provide redundant information to improve the accuracy and robustness of the system. Specifically, sensor fusion techniques [3] can be employed to overcome the drawbacks of single sensor systems used in the areas of inspection and quality control.
1.3 Proposed Approach

The approach proposed in the present research for the fusion of multiple sensor data is knowledge-based fuzzy fusion. Complexity of regular industrial tasks such as inspection, assembly, and manipulation (of robots/tools), increases when performed in a dynamic and unstructured environment. In such situations the use of multiple sensors provide a certain degree of intelligence, which is required for effective (error-free) performance of desired task, autonomously or with minimal human assistance. The possible benefits of fusing and/or integrating information from multiple sensors, include:

- Fusion of redundant data from multiple sensors reduces the uncertainty, thus improving the accuracy; also redundant information increases reliability and robustness, in case of sensor failure.
- Complementary information from multiple sensors provides information that cannot be otherwise obtained with individual sensors operating separately.
- Multiple sensors, because of the processing parallelism provide more timely information that improves the speed and quality/accuracy of decisions.
- Replacing one expensive and complex (sophisticated) sensor by many inexpensive sensors with fusion capability, can reduce the cost of the overall system, while providing information that is of the same or superior quality.

In summary, sensor (or data) fusion is the synergistic use or combination of information from independent, similar or disparate multiple sensors of limited reliability and accuracy to give information (decisions) of higher accuracy and increased reliability [4].

Sensor fusion can occur at different levels. A common differentiation is among the three levels: high-level fusion, which fuses locally made decisions; mid-level fusion, which fuses parameters concerning features locally; and lower-level fusion, which fuses the raw data from disparate sensors [5]. The techniques of data fusion can be classified into probabilistic: Bayesian theory, Dempster-Shafer theory, and evidence theory;
statistical: Kalman filtering, weighted mean, uncertainty ellipsoids; and soft computing: fuzzy logic, neural networks, and genetic algorithms. In the present research, the emphasis is on applications in which the process/plant is complex and ill-defined, and sensors are disparate. The sensory information is incomplete, imprecise, inconsistent, and sometimes contradictory. For such cases fusion methods that utilize the humanoid capability to acquire qualitative information and knowledge, perform reasoning, and integrate sensor data and a priori information, are suitable. Thus the proposed sensor fusion approach falls within a framework of fuzzy logic and uses a knowledge-based method. The knowledge in the form of a rule-base is extracted from human-experts or by running experiments.

1.4 Application Areas

The application areas involving multiple sensors include industrial, defense, space, medical, and remote sensing. Most techniques of multi-sensor integration and fusion are particularly suitable for industrial applications as industrial environments are relatively well structured (unvarying). The industrial applications [5] can be categorized into four general areas: material handling, manufacturing, inspection (quality assurance), and assembly. In the tasks of material handling industrial robots can be used for in-process workpiece handling, and the loading and unloading of industrial trucks (automated guided vehicles or AGVs) and conveyors. In the area of manufacturing, robots using vision, laser range, and ultrasound sensors are used for welding applications, which include both spot and arc welding of automotive parts. Tasks of inspection and quality assurance include gauging, verification, flaw detection, sorting, grading and recognition. In the context of this research, the emphasis is on the applications of inspection and product quality assurance related to food industry, such as sorting and grading of fruits, fish, and poultry. Assembly tasks are amongst the most complex industrial tasks, as they need complex operations such as part recognition and insertion. Industrial robots that employ vision, ultrasonic, tactile and force/torque sensors are used
Chapter 1. Introduction

in various automotive assemblies. Military applications have provided the major boost to the research and development of the technologies of multi-sensor fusion. Applications in the military areas include: surveillance, threat assessment, target detection, and tracking. In the area of remote sensing, satellites and airplanes use combinations of microwave, millimeter-wave, and infrared sensors for providing weather forecasting, and earth resource (crops, oil, minerals) detection and measurement. In the last decade medical applications have seen widespread introduction of computer tomography, fiber-optic probes for examining intestines, and devices that use sound waves to display the images of human organs. These technologies can greatly benefit from multi-sensor fusion.

1.5 Thesis Outline

The structure of thesis is summarized below:

Chapter 1  Introduction: The present introductory chapter.

Chapter 2  Literature Review: Presents review of literature related to this thesis, which focuses on multi-sensor fusion and integration; general techniques of sensor fusion; fuzzy logic and methods of knowledge-based fuzzy sensor fusion; and industrial applications.

Chapter 3  Experimental System: Describes the industrial prototype of automated fish processing machine, which is used for the implementation of sensor fusion techniques, detailed in chapters 4 and 5. Also this chapter describes the three-axis CNC router table machine, manufactured produced by Precix Advanced Cutting Technologies, which is also a possible application platform for the developed technology.

Chapter 4  Fuzzy Sensor Fusion Methods: Presents the formulation and development of three fuzzy sensor fusion techniques. The methods are applied to an illustrative example. Also presented are the simulation results to demonstrate the efficacy and usefulness of the applied sensor fusion techniques.
Chapter 5  Implementation in a Fishcutting Machine: Presents the implementation of the three sensor fusion methods for product quality (quality of fish-cut) assessment for three different sets of sensor data. Comparison and analysis of the results obtained by the three methods are presented.

Chapter 6  Conclusion and Recommendations: Concludes the thesis with a summary of contributions of the thesis, and providing directions for future research.
Chapter 2

2 Literature Review

This chapter presents some selected literature relevant to the concepts, techniques, algorithms and applications of multisensor fusion. Most of the pioneering work in the area of multisensor fusion has been done in the defense applications such as, target detection, classification, tracking and threat assessment. This chapter highlights the existing fuzzy sensor fusion techniques along with their applications. The literature in the area of industrial applications is mostly related to mobile robot navigation, part identification, manufacturing, assembly, and quality control in automated manufacturing systems. This chapter also outlines the key literature related to the concepts of fuzzy logic, to provide the necessary background for the developments undertaken in the thesis. The work done in these fields provide useful information that can be applied for automated tasks of industrial inspection and quality assessment, which is the focus of this thesis.

2.1 Multisensor Fusion

The processes of multisensor integration and fusion are closely related; however, Luo and Kay define them while making a distinction between integration and fusion. Multisensor integration refers to the synergistic use of information provided by multiple sensory devices for the accomplishment of a task by the system. The somewhat more restricted notion of multi-sensor fusion refers to any stage in the integration process where there is actual combination (or aggregation) of different sources of sensory
information into one representational format. These definitions serve to distinguish the system-level issues related to the integration of multiple sensory devices at the architecture and control from the more specific mathematical and statistical issues presented by the actual technique of fusion of the sensory information. The present work in this thesis is focused on developing algorithms for fusion of data and testing them in a specific industrial application.

Sensor fusion can occur at all levels in a three-level hierarchy [6]; namely, low-level dealing with signals or pixels, mid-level dealing with features, and high-level dealing with decisions or symbols. The type of information they process and generate differentiates these levels. In the low-level fusion raw data from multiple sensors are fused, mid-level fusion involves fusion of parameters related to features that are sensed locally, and in high-level fusion locally-made decisions are fused. De Silva [6] has indicated that higher-level decision-making generally requires lesser degree of detail (low information resolution), but a higher degree of intelligence, and uses smaller bandwidth (speed). Moving down the hierarchy the information resolution and the bandwidth increase. In the present research emphasis is on sensor fusion at a high level, for making decisions of product quality assessment.

Dasarathy [7] has grouped sensor fusion models in a number of distinctly different ways: by the level at which fusion takes place (low, medium, high); by the objective or the purpose of fusion (detection, identification, classification); by the type of sensors used (active, passive or combination); by the application area (defense, robotics, medical, space); and by the sensor suite configuration employed (parallel, serial or tandem).

According to the U.S. Department of Defense, data fusion refers to a multilevel, multifaceted process dealing with the automatic detection, association, correlation, and combination of data and information from multiple sources. Stover et al. [8] have discussed a four level hierarchy of sensor fusion based on the stage of processing, which
is specifically well suited for defense applications. But, a three-level hierarchy is adequate for most applications.

2.2 Multisensor Fusion Techniques

The multisensor fusion techniques provide mathematical tools for fusion (or aggregation) of data from multiple sensors to provide a fused output. The fusion techniques can be basically classified in to three categories; namely, the probabilistic models, least square methods, and soft computing approaches. Figure 2.1 illustrates this classification.

Figure 2.1 Techniques of multisensor fusion.

Bayesian Estimation is a probabilistic sensor fusion algorithm based on the well-known Bayes’ rule of conditional probability:

\[
P(H_i | E) = \frac{P(E | H_i)P(H_i)}{\sum_j P(E | H_j)P(H_j)}
\]  
(2.1)

with \(\sum_j P(H_j) = 1\)  
(2.2)
where, $P(H_i | E)$ is a posteriori probability that hypothesis $H_i$ is true given evidence $E$. $P(E | H_i)$ is probability of observing evidence $E$ given that $H_i$ is true, $P(H_i)$ is a priori probability that hypothesis $H_i$ is true.

Note that $\sum_j P(E | H_j)P(H_j) = P(E)$ = probability of the evidence $E$ (for all possible hypotheses $H_j$).

Bayesian estimation belongs to the class of algorithms which uses a priori knowledge about the system, and it is more suitable for fusion in high-level. Brusmark, et al. [10] have successfully applied Bayesian estimation for the detection of mines, by fusing the data from a metal detector and a ground detecting radar. However, Bayesian methods are not effective in the following cases: when multiple hypotheses can hold at a given time and multiple condition dependent events are present; when the competing hypotheses are not mutually exclusive; and when a hypotheses cannot support an uncertainty class.

Dempster-Shafer Evidence Theory [11] is another probabilistic sensor fusion algorithm, which operates on belief or mass functions as the Bayes’ rule does on probability functions. This theory is particularly useful when the sensors contributing the information cannot associate a 100 percent probability of certainty to their output decisions. Probability mass may be used to define an uncertainty interval which represents support and plausibility for a proposition. Support or belief is the sum of direct sensor evidence for proposition. Plausibility is the sum of all probability masses not directly assigned by the sensor to the negation of proposition. According to Figure 2.2, the uncertainty interval is represented as the interval between the higher bounds of belief and plausibility or mathematically defined as $[bel(A), pls(A)]$.

![Figure 2.2 Uncertainty interval for a proposition.](image-url)
According to Figure 2.2 belief, doubt, plausibility, and uncertainty interval can be computed using the following relations:

Belief or support function for an event or proposition $A$,

$$bel_i(A) = \sum_{A_i \subseteq A} m_i(A_i)$$

(2.3)

Doubt can be represented as $dbt_i(A) = beU(A_c)$

Plausibility: $pls_i(A) = 1 - dbt_i(A)$

Uncertainty interval: $u_i(A) = pls_i(A) - bel_i(A)$ or $[bel_i(A), pls_i(A)]$.

Note that $A_c$ is the complement of $A$.

Dempster-Shafer method provides the combination operators to fuse the evidence from several sensor sources. The following equation indicates the fusion of propositions $X$ and $Y$ from two sensors $S_i$ and $S_j$, and whenever $A \neq \phi$, and where $m_{i,j}$ is the orthogonal sum $m_i \oplus m_j$ and $X, Y \in A$:

$$m_{i,j}(A) = \frac{\sum_{X \cap Y = A} m_i(X)m_j(Y)}{1 - \sum_{X \cap Y = \phi} m_i(X)m_j(Y)}$$

(2.4)

where $X \cap Y = A$ indicates degree of agreement, while $X \cap Y = \phi$ indicates complete disagreement between the propositions $X$ and $Y$. The denominator term in Equation (2.4) is a normalization factor and it represents the degree of the conflict between the two sensor sources.

Dempster-Shafer method has been extensively used in applications such as obstacle avoidance, navigation of autonomous vehicles in dynamic environments, map making, and target tracking. This method is also suitable for fusion at high level.

Robust statistics [12] is a part of statistical decision theory, in which fusion takes place in two stages. In the first stage the multi-sensor data are tested for consistency, i.e., verifies within the uncertainty of the system that the data are in fact measurements of the same parameter. In the second stage the data is fused optimally using a class of robust minimax decision rules.
Chapter 2. Literature Review.

The Kalman filter [13] is a set of mathematical equations that provides an optimal estimation for a dynamic state, which uses a priori knowledge of the system. In essence it is an efficient computational (recursive) solution of the least-squares method. It is used in multisensor data fusion for fusing low-level dynamic redundant data in real-time. When the system can be described by a linear model and both the system and the sensor noise can be modeled as white Gaussian noise, the Kalman filter provides a statistically optimum (i.e., minimum variance) estimate of fused data. The filter uses statistical characteristics of the measurement model to determine the estimates recursively, for the fused data. In case of nonlinear models an extended Kalman filter is used as it linearizes the system about the current mean and covariance. Examples of use of the Kalman filter in multisensor fusion include: robot navigation, multi-target tracking, inertial navigation and remote sensing.

Weighted Average is one of the simplest and most intuitive methods for the fusion of redundant multi-sensory data in real-time. Essentially every multi-sensor fusion technique involves weighted average of sensory data, and different techniques use different discriminating parameters to determine the weights. The weights can be used to account for the differences in parameters like accuracy, reliability, and fuzziness measure. This method works well with real-time data; however, it does not work well with very noisy data.

Uncertainty ellipsoid fusion [14] is a statistical fusion method based on the geometry of uncertainties of the data measured by multiple sensors. Uncertainty ellipsoid is associated with the covariance matrix of the error of the sensory data. In this method optimal fusion is defined as the one that minimizes the geometric volume of the ellipsoid among all the linear combinations of sensory data.
2.3 Fuzzy Logic

Lotfi A. Zadeh [15] introduced the concept of fuzzy logic. Fuzzy logic is based on the idea that humans do not think in terms of numbers, but rather linguistic labels or classes of objects in which transition from belonging to non belonging is gradual rather than abrupt. The theory of fuzzy logic is based on human knowledge and non-crisp logic, in which a proposition, unlike in crisp binary logic, can belong to a hypothesis to a varying degree. This definition of fuzzy logic provides a framework to couple human judgment with standard mathematical tools to provide approximate solutions, in a simple and robust way, to complex engineering problems. This approach is suitable for problems that are too complex or ill-defined to yield analytical solutions, whose boundaries are not crisp, and where data are incomplete, imprecise and inconsistent. Fuzzy logic is a multi-valued logic technique that has proven to be useful in areas such as control, image processing, object recognition, and data fusion in intelligent systems.

2.3.1 Fuzzy Sets and Membership Functions

Fuzzy logic is an extension or generalization of the conventional (binary) logic, and analogously, fuzzy set theory is an extension of the conventional set theory. A fuzzy set is a collection of elements, which has no crisp boundary, and is defined or characterized by a membership function. Corresponding to the characteristic function of a crisp set, the membership function of a fuzzy set assigns a grade of membership, which gives the set membership. It can assume any real value between 0 and 1. In particular a value greater than 0 and less than 1 represents a partial membership where the element has a degree of a presence inside the set and a complementary presence outside the set. Figure 2.3 (a) shows the Venn diagram of a crisp set (with crisp boundary) and Figure 2.3 (b) shows a fuzzy set (with fuzzy boundary).

A fuzzy set \( A \) in the universe \( X \) is defined by a membership function \( \mu_A(x) \), which assigns a degree of membership in the set, to any element \( x \) of the universe of discourse. The
membership function $\mu_A$ maps the universe of discourse $X$ to the interval $[0,1]$, and can be mathematically expressed as

$$\mu_A(x): X \rightarrow [0,1] \quad (2.5)$$

A membership value of 1 ($\mu_A(x) = 1$) implies that the corresponding element definitely belongs to the fuzzy set $A$. If the membership value is zero ($\mu_A(x) = 0$), the corresponding element definitely does not belong to the fuzzy set $A$. The graphical representation of the characteristic function of a crisp set and the membership function of a fuzzy set are illustrated in Figure 2.4 (a) and (b), respectively. The membership functions commonly take triangular, trapezoidal, or Gaussian shapes.

According to de Silva [16], a fuzzy set can be completely defined as a universe of discourse and membership function spanning the universe. The support set of a fuzzy set is the crisp set obtained by omitting the elements of zero grade of membership.
Chapter 2. Literature Review.

Consequently, a fuzzy set is also a subset of its support set. A fuzzy set $A$ with the elements $x$ can be expressed as:

$$ A = \{(x, \mu_A(x)) \mid x \in X\} \quad (2.7) $$

Fuzzy set may be specified using a convenient form of notation, in which each element is paired with its grade of membership in the following forms:

For discrete universe:

$$ A = \sum_{x_j \in X} \frac{\mu_A(x_j)}{x_j} \quad (2.8) $$

For continuous universe:

$$ A = \int_{x \in X} \frac{\mu_A(x)}{x} \quad (2.9) $$

Equations (2.7) and (2.8) are symbolic forms of notion only.

Fuzzy sets may also be used for representing fuzzy variables. For example, in Figure 2.5, the fuzzy variables representing a product quality index is defined to have five fuzzy states; namely, Very Poor (VP), Poor (PR), Medium (MD), Good (GD), and Excellent (EX). Each of these fuzzy states is a fuzzy set with a uniquely defined membership function (Figure 2.6). The fuzzy resolution represents the number of fuzzy states that a fuzzy variable may assume. The fuzzy resolution is five in this particular example of product quality index ($PQI$). The fuzzy resolution $r_f$ can also be represented quantitatively as:

$$ r_f = \frac{w_s}{w_m} \quad (2.10) $$

where

- $w_s =$ width of the support set,
- $w_m =$ intermodal spacing.
2.3.2 Fuzzy Operations and Reasoning

In the boolean logic “union”, “intersection”, “complement”, and “implication” of sets correspond to the logical operations OR, AND, NOT, and IF-THEN respectively. These logical operations of fuzzy sets are used in knowledge representation and processing (reasoning). In fuzzy logic these connectives are expressed in terms of the membership functions of the sets, which are operated on.

**Union (OR, ∨)** \( A \cup B \)

The union of two fuzzy sets \( A \) and \( B \) in the same universe \( X \) is a fuzzy set in that universe, with membership function given by

\[
\mu_{A \cup B}(x) = \max \{\mu_A(x), \mu_B(x)\} \quad \forall x \in X
\]  

(2.11)
**Intersection (AND, \( \land \))** \( A \cap B \)

The intersection of two fuzzy sets \( A \) and \( B \) in the same universe \( X \) is a fuzzy set in that universe, with membership function given by

\[
\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\} \quad \forall x \in X
\]  

(2.12)

**Complement (NOT)** \( A' \)

The complement \( A' \) of a fuzzy set \( A \) is a fuzzy set in the same universe whose membership is given by

\[
\mu_{A'}(x) = 1 - \mu_A(x) \quad \forall x \in X
\]  

(2.13)

**Cartesian Product**

Consider fuzzy sets \( A_1, A_2, \ldots, A_n \) defined independently in the one-dimensional universes \( X_1, X_2, \ldots, X_n \), respectively. The Cartesian product \( A_1 \times A_2 \times \ldots \times A_n \) is a fuzzy subset of Cartesian space \( X_1 \times X_2 \times \ldots \times X_n \), whose membership function is given by

\[
\mu_{A_1 \times A_2 \times \ldots \times A_n}(x_1, x_2, \ldots, x_n) = \min\{\mu_{A_1}(x_1), \mu_{A_2}(x_2), \ldots, \mu_{A_n}(x_n)\}
\]  

(2.14)

\[ \forall x_1 \in X_1, \forall x_2 \in X_2, \ldots, \forall x_n \in X_n \]

Note that Cartesian product corresponds to and “AND” operation in a multi-dimensional universe (product space).

**Fuzzy Relation**

Fuzzy relation is given by a fuzzy set in multidimensional space, and is an extension of the concept of relation in conventional function (crisp set) theory. A fuzzy relation \( R \), in \( n \)-dimensional Cartesian product space \( X_1 \times X_2 \times \ldots \times X_n \), is a fuzzy set denoted by

\[
R_{X_1 \times X_2 \times \ldots \times X_n} = \{(x_1, \ldots, x_n), \mu_R(x_1, \ldots, x_n) | (x_1, \ldots, x_n) \in X_1 \times X_2 \times \ldots \times X_n\}
\]  

(2.15)

where \( \mu_R \) is the membership function of \( R \) given as

\[
\mu_R : X_1 \times X_2 \times \ldots \times X_n \rightarrow [0,1]
\]
In summary, a fuzzy relation is simply given by a membership function expressed in terms of the variables in the particular relation \((x_1, x_2, \ldots, x_n)\).

**Implication (IF-THEN) \(A \rightarrow B\)**

Consider two fuzzy sets \(A\) and \(B\) in two different universes \(X\) and \(Y\), respectively. The fuzzy implication of \(A\) and \(B\) (denoted by \(A \rightarrow B\)) is a fuzzy relation in the Cartesian product space \(X \times Y\). The most commonly used relation for obtaining the membership function of the fuzzy implication, also known as Mamdani implication, is given as

\[
\mu_{A \rightarrow B}(x, y) = \min\{\mu_A(x), \mu_B(y)\} \quad \forall x \in X, \forall y \in Y
\]  

(2.16)

### 2.3.3 RULE-BASE

In fuzzy logic a rule-base comprises a set of linguistic conditional statements in the form of IF-THEN rules. These rules are generally generated on the basis of expert knowledge of humans and take the following form:

If \(X\) is \(A\) and \(Y\) is \(B\) then \(Z\) is \(C\)

The “if” parts of the rule “\(X\) is \(A\)” and “\(Y\) is \(B\)” are called the rule antecedent or premise, while the “then” part “\(Z\) is \(C\)” of the rule is called the consequent or conclusion. In the fuzzy rulebase, \(X\) and \(Y\) are the inputs and \(Z\) is the output, to the fuzzy system. In the context of inspection and grading, the expert knowledge available from experienced (seasoned) human inspectors, is used by knowledge engineers to develop a fuzzy rule-base. For example, in a bakery one of the fuzzy rules for estimating the process endpoint can be

If Color is Brown and Aroma is Sweet then Baking is Complete.

The fuzzy linguistic variables such as, Brown and Sweet are associated with membership functions, which describe a fuzzy subset of the universe of discourse.
2.3.4 Compositional Rules of Inference (CRI)

Inference:

In fuzzy logic, the decision-making procedures are done by an inference mechanism or "inference engine", using the compositional rules of inference. Decisions are made according to fuzzy IF-THEN rules (fuzzy implications), which are embedded in the knowledge-base using fuzzy approximate reasoning. Composition is used for combining fuzzy relations defined in different Cartesian product spaces. Consider two fuzzy relations \( R_1(x,y) \) and \( R_2(y,z) \) defined over the Cartesian products \( X \times Y \) and \( Y \times Z \), respectively. The popular compositions are the sup-min and sup-product compositions. The sup-min composition of \( R_1 \) and \( R_2 \) is a new relation \( R_1 \circ R_2 \), defined on \( X \times Z \) as

\[
R_1 \circ R_2 \equiv \mu_{R_1 \circ R_2}(x,z) = \sup_{y} \{ \min_{y} \{ \mu_{R_1}(x,y), \mu_{R_2}(y,z) \} \} \quad (2.17)
\]

where the symbol "\( \circ \)" is the composition operator. Similarly the sup-product composition is represented as

\[
R_1 \circ R_2 \equiv \mu_{R_1 \circ R_2}(x,z) = \sup_{y} \{ \prod_{y} \{ \mu_{R_1}(x,y), \mu_{R_2}(y,z) \} \} \quad (2.18)
\]

Forming the Rule-Base:

There exist several methods for forming the membership function of a rule-base, depending on the choice of the combination operators, implication, and the aggregation method. If we consider the rule:

\[
\text{IF } x \text{ is A AND } y \text{ is B THEN } z \text{ is C}
\]

Two commonly used combination operators are AND operator and OR operator. The AND operator may be represented by operations such as \( \min \) and \( \text{algebraic product} \). The OR operator may use logical operations such as \( \max \) and \( \text{algebraic sum} \). There are several methods that have been employed for the implication operation "IF-THEN" [18];
for example, min, algebraic product, bounded product, drastic product, Goedel logic, etc.
The *min* implication clips (truncates) the output fuzzy set, and the *product* implication scales the output fuzzy sets. Two types of operations are used for rule aggregation: *max* and *sum*.

**Combined Inference:**
Instead of forming the rule-base first and then making inferences by applying the CRI to it in conjunction with observed data, one may conveniently carry out an individual rule-based inference. Here inferences are made by using the individual rules and then aggregated. In Mamdani’s inference method *min* is chosen for AND combination operator; *min* or *product* is chosen for implication “IF-THEN”; and *max* is chosen for rule aggregation. All the fuzzy rules that are fired for a particular input result in output fuzzy sets, which are aggregated into a single output fuzzy set. This aggregate is defuzzified to obtain a crisp numerical value, which can be used for providing a crisp control action. Defuzzification is a process that converts a fuzzy inference into a crisp numerical output. Defuzzification is commonly performed by using the *centroid* method.

### 2.3.5 Uncertainty and Fuzziness Measure

Both uncertainty and fuzziness are concepts of approximation, but the underlying principles are not same. Uncertainty is caused by probability. Fuzziness is caused by imprecise perception (non-crisp nature) of membership set. The former is related to the lack of complete information that precludes the certainty that an element either belongs to or does not belong to some crisp set [19]. The latter relates to the possibility that the set does not have a crisp boundary. If the membership grade of an element is close to unity, the element is almost definitely a member of the set. Conversely, if the membership grade is close to zero, the set is nearly out of the set. This observation makes it clear that the membership function of an element *x* in *A* is most fuzzy when the membership grade \( \mu_A(x) = 0.5 \). Accordingly, the fuzziness of a set may be measured by the closeness of its
elements to the membership grade of 0.5. Thus, fuzziness measure can be computed by calculating the distance between the membership grade of the element and the membership grade of maximum fuzzy point, i.e., 0.5. The fuzziness measure can be represented as the inverse of

\[
c_A = 2 \int_{x \in X} |\mu_A(x) - 0.5| \, dx = \int_{x \in X} |2\mu_A(x) - 1| \, dx \tag{2.19}
\]

Fuzziness measure is also related to lack of distinction between a set \( A \) and its complement \( \overline{A} \), the closer \( A \) and \( \overline{A} \), the fuzzier \( A \). The fuzzy measure can be computed by calculating the distance between \( A \) and \( \overline{A} \), and can be expressed as

\[
c_A = \int_{x \in X} |\mu_A(x) - \mu_{\overline{A}}(x)| \, dx \tag{2.20}
\]

**\( \alpha \)-Cut of a fuzzy set**

The \( \alpha \)-Cut of a fuzzy set \( A \) is the crisp set formed by elements \( A \) whose membership grade is greater than or equal to a given value \( \alpha \). It is denoted by \( A_\alpha \)

\[
A_\alpha = \{ x \in X \mid \mu_A(x) \geq \alpha \} \tag{2.21}
\]

in which \( A \) belongs to the \( X \) universe. In other words \( \alpha \)-Cut of \( A \) is a crisp set that contains all the elements of \( A \), which have a membership grade greater than or equal to \( \alpha \). Index of fuzziness may be computed by calculating the distance between a set \( A \) and its \( \frac{1}{2} \) cut. It is expressed as

\[
f_A = \int_{x \in X} |\mu_A(x) - \mu_{A_{\frac{1}{2}}}(x)| \, dx \tag{2.22}
\]

where \( \mu_{A_{\frac{1}{2}}}(x) \) is the \( \frac{1}{2} \) cut of \( \mu_A(x) \).
2.4 Fuzzy Sensor Fusion Techniques

Fuzzy sensor fusion techniques are particularly suitable in systems where the uncertainty is subjectively assessed (i.e., using fuzzy logic) rather than based on precise statistics (i.e., using probability theory). For systems that are ill-defined, complex and employ dissimilar sensors, fuzzy fusion techniques have proved useful. The information available can be in the form of numerical data received from sensor measurements, and linguistic data obtained from human operators and domain experts. The numerical data may be noisy, incomplete, and unreliable, and the linguistic information is qualitative, imprecise and subjective. Fuzzy fusion techniques provide a systematic framework to deal with these two types of data simultaneously. This section describes the fuzzy fusion methods available in the surveyed literature.

Odeberg [20] did some original work in developing a fusion method based on the fuzzy measure of sensor data for product classification and discrimination. Sensory opinion is presented by a fuzzy measure, a number in the range [0,1], describing the degree of support for a hypothesis. A fuzzy measure value $\mu = 1$ indicates the sensor is in complete agreement with the hypothesis, and $\mu = 0$ indicates complete disagreement. Also $\mu = 0.5$ means the sensor data is of no relevance to the hypothesis, and represents the state of highest fuzziness. The fusion formula is based on the distance between the sensory opinions and is expressed as:

$$f(\mu_1, \mu_2) = \frac{k(\mu_1 + \mu_2) + (k - 1)^2 \mu_1 \mu_2}{1 + k^2 - (k - 2)^2(\mu_1 + \mu_2 - 2\mu_1 \mu_2)}$$

(2.23)

where $k \in [1, \infty]$. The parameter $k$ controls the degree to which strong opinions are weighted in comparison to the weaker. However, there are two drawbacks in this method. First, only two sensor opinions could be fused, second the fusion method is not associative i.e., the fused result depends on the order in which measurements are fused.
Karlsson [21] addressed these issues and extended the method to multisensor fusion. He modified the fusion formula as:

\[
f(\mu_1, \ldots, \mu_n) = \frac{1}{k^n - 1} \left( \frac{-1 + k^n G_{\mu_1 \cdots \mu_n}}{1 + G_{\mu_1 \cdots \mu_n}} \right), \quad G_{\mu_1 \cdots \mu_n} = \prod_{i=1}^{n} \frac{1 + (k - 1)\mu_i}{k - (k - 1)\mu_i}
\]

where \( k \in [1, \infty] \), and \( \mu_i \), \( i = 1, 2, \ldots, n \) is the opinion of the \( i^{th} \) sensor. It was observed that a higher value of parameter \( k \) gave better results when the number of sensors was few and sensory opinions were close to the position of maximum fuzziness, i.e., 0.5. Karlsson selected a value for \( k \) equal to two for classifying electrical motors in a disassembly operation. As well, his paper did not provide a rule for selecting the value of \( k \), as it varies from case to case depending on whether the sensory opinions are close to 0.5 or not, how many sensors are used, and the level of uncertainty. Tong et al. [22] further improved on this fusion method by incorporating the effect of the sensor reliabilities, and used a genetic algorithm to find the optimal sensor reliabilities and fuzzy rules for determining optimal value of \( k \).

Abdulghafour et al., [23] developed a fuzzy fusion formula based on the fuzziness measure of the sensor measurements, which applies weights proportional to the degree of certainty associated with the data. They implemented and tested the fusion formula for fusing the segmented real range and intensity images, which were obtained using an Odetics Laser Scanner. The goal was to achieve a better scene description.

Singh and Bailey [24] applied a fuzzy logic technique to resolve data association problems in the two dimensional multi-sensor multi-target tracking system. They used the possibility/probability consistency principle as proposed by Zadeh [25], which states that the degree of possibility of an event is greater than or equal to its degree of probability. For a given value of confidence level of statistical data and its probability density function, the possibility probability consistency is applied to obtain an optimal membership function. Laboratory simulation results reflect better data correlation between position error and velocity error.
Mahajan et al. [26] have applied a zeroth order Sugeno fuzzy inference system for weighted average redundant fusion of measurement data from dissimilar sensors with different resolutions. It is one of the few papers in which different sensors, such as strain gauge, accelerometer, and piezoceramic transducer, have been used for similar information. The desired information about strain is directly measured with a strain gauge and a piezoceramic transducer, and using preprocessed information from an accelerometer. Another important point about this method is the use of additional information on sensory performance, such as operating temperature, frequency range, life cycle, and signal to noise ratio. This information along with sensor measurements are used as inputs to the fuzzy inference system, which in turn provides the weights that are assigned to the different sensor measurement data, which reflect the confidence in the sensor’s behavior and performance. The proposed fusion formula assigns weights on the basis of the sensors’ reliability and confidence levels, which are obtained either from the manufacturer or by experiments. However, it is a limitation that such information on sensor performance is not always easily available or obtainable.

Yager [27] introduced the concept of compatibility function and the ordered weighting averaging (OWA) operator for the fuzzy fusion of redundant data. Depending on the problem a compatibility function based on distance between data is set up, and the data to be fused is checked for the degree of compatibility or conflict. The basic idea is to put higher emphasis on the data that is close to each other, and to avoid or put less emphasis on those that are not close. The method is based on the $\alpha$-level fuzzy sets. The fused output not only indicates the location or aggregated value, but also a measure of conflict in the fused data, and it can be used to provide the confidence in the fused result.

In the fusion method based on the OWA operator the data is arranged according to their values, and weights are assigned on the basis of their relative ordered position, rather than on their values. This method provides a way of applying different aggregation operators; specifically by selecting a value for the parameter $\alpha \in [0,1]$, known as the degree of
optimism, differential weights can be assigned to lower or higher valued data. The closer \( \alpha \) is to one, the more preference is given to higher valued data; the closer to zero the more preference is given to lower valued data; and a value close to 0.5 indicates an equal preference to all the data. The paper does not clearly show the method of determining the compatibility function, and the method considers the fusion of data from similar sensors only.

Luo et al. [28] developed a method for the fusion of redundant data within a statistical framework, using a confidence distance measure and a relational matrix. This method eliminates the sensor data that is likely to be in error. The remaining sensors, which are in consensus, then determine the fused output. This method becomes less effective as the number of sensors decreases, which is generally the case with industrial workcells.

Wide and Driankov [29] applied a fusion approach based on fuzzy measure for the fusion of data from olfactory sensors (electronic nose), which are used for air quality classification in and outside a car in various traffic conditions. Different sensors point to different quality categories, to a certain degree, and these diverse sensory opinions are merged to obtain one that points to a predominant quality category.

Duflos et al. [30] employed the Mamdani method of inference to fuse the indicators extracted from the images provided by a ground penetrating radar, a metal detector and an infrared camera, for the detection of underground mines.

Choi and Dickerson [31] presented a fuzzy fusion method for the redundant fusion of data from similar (homogeneous) sensors. This method is based on the concepts of fusability measure and expected output membership function (EOMF). EOMF is the expected fuzzy output, based on the inputs and their variance values. The fusability measure is calculated by taking the average of the degrees of intersection of the possible fuzzy output set with the fuzzified inputs. The EOMF method creates a fuzzy confidence distance measurement by assessing the fusability of the data. The fused output is obtained by defuzzifying the EOMF, and the position of EOMF is calculated by calculating the
fusability measure at all possible positions. The point with the maximum fusability measure corresponds to the most likely position of the EOMF. The support length of the EOMF can be obtained by the weighted average of the input variances. The method has been applied to a simple case of ten similar sensors. It does not provide a systematic method for calculating the proper support length of the EOMF. The fused results are more accurate than those obtained by the methods such as the weighted average, Yager's compatibility function method [27] and Luo's method [28]; however, the authors have arbitrarily chosen certain constants to calculate the support length of input and output (EOMF) fuzzy sets. Another drawback is its inability to deal with dissimilar sensors with different membership functions of different shapes.

2.5 Industrial Applications

Most of the industrial applications of multi-sensor data fusion systems have hampered their commercial success due to various constraints such as sensor technology, cost-effectiveness, availability of cheap labor, and limited throughput rate (speed). Industrial applications are witnessed in the areas of manufacturing, assembly, robotics, and automated quality assurance.

In 1998, Toyota became the first to introduce the Adaptive Cruise Controller (ACC) system on a production vehicle, the Luxury Sedan [32]. ACC employs a fusion processor developed by Fujitsu Ten Ltd. to fuse the data from a stereo camera and a millimeter wave radar or laser sensor to move the car automatically at varying speeds. The camera and radar units report on the width, distance, and speed of the objects ahead. The fused information is used to automatically control the brake and throttle to accelerate or decelerate the vehicle. It keeps the vehicle speed below a set value in order to maintain a constant distance between the vehicles. When the vehicle in front suddenly slows down or stops, the brake is automatically applied. The vehicle returns to its set speed when the
vehicle in front moves out of the lane or increases its speed. The system status is shown on the meter display, and a vehicle distance warning is provided.

Thien and Hill [33] used force, tactile, proximity and vision sensors to control an industrial robot for the assembly of electric motors. The assembly task requires the acquisition and placement of four different parts – the stator, the rotor, and the two bearing plates. For assembly of a motor the robot picks the workpieces up in a specified order and places them on top of one another. A force sensor is used to ensure proper mating of parts. In case of an excessive force, an error recovery routine moves the robot arm to a safe zone, and then attempts the mating stage again, until the allowable forces are recorded by the force sensor.

Insertion of shafts into holes is an important operation in assembling products using industrial robots. Masayuki et al. [34] at Toyota Motor Corporation, have developed an automatic shaft inserting system. It uses a position and inclination sensor to determine the center of gravity and inclination of the hole. The experiments indicate that shafts can be successfully inserted into holes that have an eccentricity of 50 microns and move at 3 mm/s, and this operation takes 35 seconds.

Ghosh et al. [35] have employed fusion of sensors such as a CCD camera and encoders for the purpose of visual robotic manipulation. The experimental workcell comprises a turntable having an encoder, a vision system, and a robot manipulator with encoders. The task consists of manipulating a part that is randomly placed on the turntable, while the camera and the robot end effector are uncalibrated.

Grading of herring roe skeins [36] has been done in the Industrial Automation Laboratory at the University of British Columbia. Images obtained with a CCD camera are processed to extract color, contour, size, and texture information. Skein weight is calculated from the two-dimensional area using a multiple-regression estimator. Firmness is estimated from the brightness of ultrasonic echo images. All of this information is combined to determine a classification for each roe. Grading accuracy ranges from 72%
to 95%. Classification accuracy between Grade1 and Grade 2 roe is about 95%; however, the system is less successful at subclassifying the Grade2 roe into various sub-grades. Additional sensors are required to improve the overall performance of the system.

Other applications that make use of sensory information for grading and classification include potato grading [37], shrimp inspection [38], visual inspection of canned salmon [39], material surface inspection, and printed circuit board inspection.

A number of commercial industrial systems exist for product inspection and classification. These include the QualiVision system from Dipix Technology Inc. for the quality assessment of bakery and snack food products. The system uses three-dimensional imaging to assess the product consistency to 10 microns [40]. Lumetech A/S has developed the Fisheye Waterjet Portion Cutter for trimming and portioning fish fillets [41]. Key Technologies Inc. provide Tegra system for grading agricultural products according to size and color [42]. Some level of industrial success has been achieved by all these applications. Yet more work is needed to realize robust and reliable techniques that are fast and cost effective. The work reported in the present thesis is an effort towards this end.
Chapter 3

3 Experimental System

To test the fuzzy sensor fusion methods investigated and developed in this research they have to be implemented in practical systems. To this end, we have considered two industrial systems. The developed techniques are implemented for the product quality assessment in the industrial prototype of an automated fishcutting machine developed in the industrial Automation Laboratory. First, this chapter explains the “Intelligent Iron Butcher” which is used as the test bed for the present research. Next the sensors and actuators employed in the machine are described. Further, the control system of the machine is illustrated. Then the three axes CNC router table, and its sensors and actuators are presented, as another possible system for implementing the developed techniques. The control system of the router table is presented in the end.

3.1 The Intelligent Iron Butcher

A prototype fishcutting machine, also known as the “Intelligent Iron Butcher,” has been developed at the Industrial Automation Laboratory of the University of British Columbia, Vancouver, Canada. The machine is intended for the head removal of fish, which is the first phase of processing in an automated production line of fish canning. The objective of the Intelligent Iron Butcher is to minimize the wastage of useful meat and maximize the cut quality. The original production machine, which is currently used in the fish processing industry, does not perform well in this regard due to frequent over-
feeding or under-feeding of the fish into the cutter. The improvements of the new machine are realized through an innovative mechanical design and by incorporating modern and intelligent sensing and actuation technology to detect and position fish accurately at the cutter.

A view of the prototype machine is shown in Figure 3.1. It has two vision sensory systems, one (primary) for determining the fish dimensions in order to position a fish prior to cutting, and the other (secondary) for determining the cutting quality after the head removal process. The evaluation of the product quality by fusing the multi-sensory
information is the main focus in the present work. A speed-controlled conveyor is used for feeding the fish into the cutter. The cutter system consists of two servo-controlled axes. One axes drives a platform for positioning a fish to be processed vertically (platform axis) so that it is delivered symmetrically into the cutter. The other axis (blade axis) is for horizontal positioning of the cutter blades, aligning them with the collarbone of the fish. The cutter system and the positioning platform are shown in Figure 3.2, and illustrated schematically in Figure 3.3.

A pair of rotary blades driven by two independent high-speed induction motors through flexible shafts, and arranged in a “V” configuration is used as the cutter. This particular configuration is employed to maximize the recovery of useful meat during fish processing. During a typical cycle of head removal, an image of a fish on the conveyor is captured by the primary vision system, after being triggered by means of an object-
detection ultrasonic displacement sensor mounted on the conveyor. The captured image is analyzed to detect and establish the cutting locations, which are later used in the lateral positioning of the cutter in alignment with the collarbone of a fish. As each fish enters the primary imaging station, an ultrasonic displacement sensor measures the thickness of the fish at or near the general area of the collarbone. This measurement is required to align the fish in the vertical direction so that each fish is delivered symmetrically into the cutter blades.

Figure 3.3 A schematic diagram of cutter assembly.

These locations are then communicated to the host computer, which subsequently commands the cutter and platform system to move to the correct locations. Two DC permanent magnet servomotors with lead screws and nuts are used to actuate the two axes (cutter and platform). Both motors are driven by PWM type servo-amplifiers (Galil PC-Mate-10). The feedback of position and speed is achieved using sensor signals from the two optical encoders of the positioning motors. The motions of the cutter unit and the platform are controlled by a two-axis programmable servo-controller (Galil DMC-620), which
communicates directly with the PC-compatible host computer. The machine also has a grasping mechanism to avoid the possibility of any unwanted lateral movement of the fish across the conveyor while they are being transported from the imaging station up to the cutting platform. The typical throughput of the machine is two cutting actions per second, leaving only 500 ms for each sequence of operation. The cutter positioning mechanism is required to have a response time on the order of 100 ms since image capturing and processing takes approximately 300-400 ms.

3.1.1 Sensors and Actuators

The main focus of the practical implementation of the present work is the Intelligent Iron Butcher, which is used as the test bed for experimental investigation of the fuzzy sensor fusion techniques for quality assessment. Figure 3.4 shows a schematic diagram of the overall system. The functional components of the iron butcher are presented here and those related to the cutter module are shown in Figure 3.3. Further details and inner working of the overall iron butcher can be found in [43].

Conveyor:

The conveyor carries fish continuously from the feeding end to the cutter. The fish are separated by means of push lugs, which are attached to a set of chains around the conveyor. A variable-speed DC motor through a belt drive drives the conveyor shaft. For a throughput rate of 2 fish/s, the linear speed of the conveyor is 46 cm/s.

Primary Vision Sensor:

The primary vision system has a low-level CCD camera, which grabs the image of a fish on the conveyor, when triggered by an object-detection sensor on the conveyor table. The low-level vision system then analyzes the image and determines the gill position of the fish. This system comprises

a) Primary CCD camera for image grabbing.
b) Ultrasonic sensor for height measurement.
Figure 3.4 Schematic diagram of the overall system.
c) Trigger switch for detecting a fish on the conveyor.
d) SHARP GPB-1 image processing board for image analysis.
e) PCL-I/O board for data communication.

**Cutter and Platform Module:**

The cutter and platform module has two key functions. It is required to perform the horizontal placement of the cutter blades upon receiving a position command. This horizontal placement makes sure that the cutter blades are aligned accurately with the collarbone of the fish to be processed. Simultaneously with the horizontal placement, the platform should also perform its vertical placement of the platform so that each fish is delivered symmetrically into the cutter. The positioning accuracy in the horizontal (Blade-axis) and vertical (Platform-axis) directions must be ±0.1 mm and ±0.5 mm, respectively. The components of the cutter module are carefully chosen so that the aforementioned requirements are met. The relevant specifications are as follows:

a) DC permanent magnet servomotor for blade-axis: Model 500/1000B Galil,
   Voltage = 64 VDC, Peak Torque = 5.1 Nm, Continuous torque = 1.11 Nm,
   Moment of Inertia = $1.9 \times 10^{-4}$ kgm$^2$, and Maximum Speed = 3750 rpm.
b) DC permanent magnet servomotor for platform-axis: Model 50/1000 Galil,
   Voltage = 32 VDC, Peak Torque = 1.45 Nm, Continuous torque = 0.21 Nm,
   Moment of Inertia = $2.6 \times 10^{-5}$ kgm$^2$, and Maximum Speed = 3750 rpm.
c) A backlash-free lead-screw arrangement for the blade-axis with ½ inch pitch.
d) A backlash-free lead-screw arrangement for the platform-axis with ¼ inch pitch.
e) Two rotary blades, each 6 inch in diameter, arranged in "V" configuration and mounted on the blade-axis of the cutter module.
f) Two high-speed induction motors: 3F, 230 V, 3600 rpm, 1.5 hp with two flexible shafts for individually coupling the induction motors to the cutter blades.
g) Optical encoders for position and speed feedback of the servomotors and for speed (cutter load) sensing of the induction motors.

**Secondary Vision System:**

The secondary vision system has a high-level CCD camera at the exit end of the cutting zone. It captures the image of a processed fish and the visual information is processed via GPB board. In this manner information is extracted related to the quality of the processed fish and in turn the performance of machine. This system is comprised of:

a) A CCD camera for image grabbing.

b) SHARP GPB-1 image processing board for visual data analysis.

c) A 486/33MHz PC as the host machine.

### 3.1.2 Control System

Mesarovic, Mystel, and Takahara [45] studied the hierarchical control for complex control situations such as robotic manipulators and industrial process control. Saridis [46] introduced the concept of “Intelligent Hierarchical Control” as a unified approach of cognitive and control system methodologies. Inspired by Saridis’ principle of increasing precision with decreasing intelligence and the work done by de Silva and Macfarlane [47] on hierarchical control. Wickramarachchi [43] developed and implemented a three-level knowledge-based hierarchical fuzzy control system on the prototype fishcutting machine. The hierarchy assigns tasks into different levels of hierarchical structure according to the level of abstraction, level of complexity of decision-making, and the level of priority of action. This hierarchy combines the integration of expert knowledge into a knowledge-based soft control scheme, combined with conventional crisp control algorithms. A fuzzy controller, which consists of fuzzy IF-THEN rules as its knowledge-base, is used for servo tuning. The main advantage of this control architecture is its ability to incorporate human knowledge and flexibility in multi-layered decision making for control. Figure 3.5 shows the hierarchical control
structure of the prototype fishcutting machine. The hierarchical structure has been designed in such a way that the lowest level of the hierarchy is occupied by various controllers, and the sensors that provide feedback information for process control. The information generated at this level is mainly visual data from the primary CCD camera, position and speed data from the encoders, and cutter load data from the slip as determined by the encoders of the induction motors. In this level all the information is present in large numerical quantities and at high resolution.

Figure 3.5 Hierarchical supervisory control system of the prototype fishcutting machine.

The second level of the hierarchy is a data processing level where high-resolution crisp data from the sensors are filtered. This filtered (pre-processed) data provides an abstract representation of the current state of the machine. The filter operations include averaging the signal, peak detection, pattern recognition and computation of performance.
attributes such as rise time, damping ratio, and settling time. These parameters are interpreted according to some subjective criterion of performance evaluation and then transformed into fuzzy sets.

The top level of the hierarchy comprises knowledge-base and inference engine, which makes decisions based on current fuzzy information about the process. The knowledge-base can be considered as a collection of expert instructions on how to improve the yield while maintaining the product quality requirements and keeping the requirements at their optimum performance. The information at this level as well as the inferences are exclusively represented by fuzzy linguistic variables.

3.2 CNC Router Table

A CNC router table is like a CNC milling machine and it is used for machining materials such as foam, wood, plastics, aluminum and bronze. Precix Advanced Cutting Technologies, in Vancouver designs and manufactures three axes CNC router tables for dispensing and laser cutting applications. These machines are primarily used by sign makers, custom cabinet makers, and pattern makers. Typical applications include: wooden furniture design, jewellery design, electrical control panels, wooden, plastic and neon signs, and injection molding prototypes. Typical machining operations include: milling, drilling, contouring, routing inlays, cutouts and engraving.

The CNC router system basically consists of a motorized XYZ router table, a control unit, and programming software. Figure 3.6 shows the three axes CNC signmaker [48] series router table manufactured by Precix Advanced Cutting Technologies. The mechanical structure of the router consists of aluminum T-Slot table-top supported on the gantries, and the drive system. For each axis the drive system uses zero-backlash ground ball screws, as they have excellent power transmission capability due to the rolling ball contact between the nut and the screws.
The part to be fabricated is designed using standard 3D CAD software such as AutoCAD and Rhino3D, or by scanning an existing master part/template using a probe type scanner or a hand-held pendant type scanner. CAM software like MasterCAM and ArtCAM, use the drawing files to generate the toolpath instructions in the format of G codes, or HPGL codes to produce the part [49]. The CNC computer processes the data and generates discrete numerical position command for each feed drive and velocity command for the spindle drive.

### 3.2.3 Sensors and Actuators

The three axes CNC router table uses one brushless DC servomotor per axis. Each servomotor has an integral optical encoder and a tachometer for position and velocity feedback, respectively. Also there is one limit switch per axis for home reference and one at the far end. These limit switches have positioning accuracy of 0.1 mm and repeatability of 0.01 mm. The spindle motors provided by Colombo, Italy are used and they have following specifications:
• Power – 4.5, 7.5, and 10 hp
• RPM –300 to 21,000 r.p.m.

A CCD camera can be used along with other sensors in the CNC router. The vision sensing can be employed for automated part inspection and identification. The vision system allows for quick setup of jobs. This is especially useful for jobs that have multiple jigs and when it is desired to switch between jigs without having to reconfigure the setup every time. The vision system may include following components:

• JVC color video CCD camera.
• Matrox Meteor MC4 frame grabber.
• A Pentium III 500 MHz. as host PC.

3.2.4 Control System

The control system of the CNC router is basically similar to that of a three-axis milling machine. The CNC consists of an IBM compatible PC with QNX real-time operating system, and uses a three-axis PMDI MFI03A motion control interface card. Encoders provide the position and velocity feedback. First the NC program is entered for the desired toolpath. Then the CNC computer processes the data and generates discrete numerical position commands for each feed drive and the velocity command for the spindle drive. The numerical commands are sent to the servo amplifiers, which process and amplify them to the high voltage levels required by the drive motors.

![Diagram of the control system of the CNC router table](image-url)

Figure 3.7 Block diagram of the control system of the CNC router table.
Presently a proportional-integral-derivative (PID) control scheme is used in the Precix CNC router machine, and the controller gains can only be changed off-line. Figure 3.7 indicates the control system of the three axes CNC router table machine. The CNC router tables or milling machines can employ a range of sensors such as force/torque, vibration, optical, acoustic emission, profile laser, and vision. In these machines multi-sensor fusion can be effective, particularly for monitoring the tool wear, in-process machining, and inspection. The fusion technology developed in the present work may be conveniently implemented in CNC machines of this type.
Chapter 4

4 Fuzzy Sensor Fusion Methods

This chapter presents the three fuzzy sensor fusion methods that have been applied in the evaluation of the quality of the processed fish. First, the application area and the applicable sensor fusion methodology are presented. Next, the data fusion employing Mamdani's max-prod (or, sup-prod) composition is highlighted. Further the data fusion method based on the concept of the degree of certainty is described. The subsequent section explains the development of the method based on the idea of compatibility of the data to be fused. Finally, the three fusion methods are implemented on an example case to compare and evaluate their performance.

4.1 Application Area

In this research the application area is fish processing, and the application platform is the fishcutting machine developed in the University of British Columbia, Vancouver, Canada. The fishcutting machine employs various disparate sensors (see section 3.1.1) that provide information necessary for the control of the actuators. To understand about the type of sensor fusion useful in the fishcutting machine first we discuss the two main categories of sensor fusion.

Redundant Data Fusion – Redundant data is provided by similar or dissimilar sensors while measuring the same feature, possibly with a different resolution or fidelity. The mechanism used to generate the redundant measurements can be entirely dissimilar. For
example an ultrasound sensor, a laser range finder and a digital camera can redundantly measure the position of an object in two dimensions.

**Complementary Data Fusion** - Complementary data is provided by similar or dissimilar sensors when they measure different features. Here, the sensors do not depend on each other directly, but can be combined to give more complete information about the measured features. The mechanism of sensing can be similar or dissimilar. For example, three similar CCD cameras or an ultrasound sensor, a laser range finder and a CCD camera may be monitoring different views of an object. No single sensor alone can give the complete picture about the position of the object in three dimensions; however, by fusing the complementary information from two or more sensors, the three-dimensional information can be acquired.

In the case of the assessment of the quality of processed fish, the sensor fusion is complementary. To evaluate the quality of processed fish in terms of accuracy and aesthetics, different sensors that measure different features are employed to acquire complementary data. Optical encoders of the servomotors provide position offset data on the cutter and platform, and encoders of the induction motors provide cutter load information. The offset data and the cutter load data provide information for the assessment of the accuracy of a cut. The secondary CCD camera at the exit side captures images of the processed fish, and this provides information for assessment of the visual quality or the aesthetic appearance of the processed fish. The acquired data is processed through the quality assessment knowledge-base to provide meaningful information about the overall quality of the product.

Following are the factors that warrant the need of knowledge-based fuzzy sensor fusion techniques in the case of fishcutting:

1. The mechanics of fishcutting is not clearly and completely understood, and the variation in the product, i.e., the fish, is immense.
2. The criteria of product quality assessment employed by the quality inspectors are subjective. The uncertainty and approximation associated with the data guide the selection of a suitable data fusion method for a particular application. In the case of fishcutting the "quality" is subjectively assessed rather than exclusively based on precise statistics.

4.2 Mamdani Max-Prod Composition

Mamdani [18] proposed the compositional rule of inference for decision making in fuzzy logic control. The same method can also be used for fusing (or combining) fuzzified data from different sensors. Amongst various types of composition methods, max-min and max-prod are commonly used in engineering applications. If $A_1$ and $A_2$ are fuzzy sets with membership functions $\mu_{A_1} (x_1,y)$ and $\mu_{A_2} (x_2,y)$ then max-prod (or sup-prod) composition of $A_1$ and $A_2$ is given by following equation:

$$A_1 \circ A_2 = \mu_{A_1 \circ A_2} (y) = \sup_{x_1, x_2} \prod \{ \mu_{A_1} (x_1, y), \mu_{A_2} (x_2, y) \}$$

(4.1a)

where the symbol " $\circ$ " denotes the sup-prod composition. In particular, for a rule-base $R(x,y)$ and a fuzzy input $A(x)$ the corresponding inference $P(y)$ is obtained using

$$\mu_{R \circ A} (y) = \sup_{x} \prod \{ \mu_{R} (x, y), \mu_{A} (x) \}$$

(4.1b)

The decision making process comprises the following five steps, in the case of individual rule-based composition:

a) fuzzification of inputs using the membership functions, for each rule.

b) aggregation of fuzzified input variables using AND/OR (min/max) operator depending on the connectives in the rule.

c) implication by min or prod operator for each rule.

d) aggregation of output fuzzy sets from all the rules by the OR (max) operator.

e) defuzzification of the aggregated output by centroid method.
In each rule that is fired there are multiple number of fuzzified input values, which are aggregated by applying fuzzy operators such as AND (min) or the OR (max), depending on the connectives that are present in the rule. An implication operator such as min or prod is applied to establish the output fuzzy set from each rule. Note that the output fuzzy set is clipped by the min operator and scaled by the prod. The outputs generated by all the rules in the rule-base are then aggregated using the max operator. The last step is the defuzzification of the aggregated output, which produces a crisp numerical value from the aggregated output fuzzy sets. Note that, instead of this individual rule approach, the combined rule-base approach may be applied according to Equation (4.1) so as to get the fuzzy output in one step. Figure 4.1 indicates a schematic diagram for knowledge-based fuzzy sensor fusion. Mamdani’s max-prod composition is used in the inference mechanism.

![Diagram of knowledge-based fuzzy sensor fusion](image)

Figure 4.1 Knowledge-based fuzzy sensor fusion in a process application.

Defuzzification using the centroid method produces the fused output as given by:

\[
y^* = \frac{\sum_{i=1}^{n} y_i \cdot \mu_{out}(y_i)}{\sum_{i=1}^{n} \mu_{out}(y_i)} \quad \text{for the discrete case} \tag{4.2}
\]
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or \[ y^* = \left[ \frac{\int y \cdot \mu_{out}(y) dy}{\int \mu_{out}(y) dy} \right] \] for the continuous case \[ (4.3) \]

where \( y^* \) is the crisp output value.

An example is shown in Figure 4.2 to illustrate the application of Mamdani’s max-prod composition for sensor fusion. Temperature and pressure sensors are used to control the opening of the throttle valve in an internal combustion engine. The fuzzy sets for temperature, pressure, and throttle opening are represented by \( T, P, \) and \( TH \) respectively. \( T \) and \( P \) each assume two fuzzy states High and Low, and \( TH \) takes three fuzzy states viz., Small, Medium, and Large. Figure 4.3 indicates the fuzzy rule-base for controlling the opening of the throttle valve.

1. If \( T \) is Low and \( P \) is Low then \( TH \) is Large
2. If \( T \) is Low and \( P \) is High then \( TH \) is Medium
3. If \( T \) is High and \( P \) is Low then \( TH \) is Medium
4. If \( T \) is High and \( P \) is High then \( TH \) is Small

Figure 4.2 A rule-base for control of a throttle valve.

For a given input the temperature measurements belong to fuzzy sets Low and High, while the pressure measurement belongs to only High, and as a result rules number two and four would fire. Figure 4.3 indicates the two rules that fire, and the defuzzified fused value for the throttle valve opening.

In each rule the lower value of two fuzzy states (Low or High) is used to scale (implication–prod operator) the corresponding membership function of \( TH \). The resulting two scaled membership functions of \( TH \) are aggregated (max operator). This aggregated fuzzy set is defuzzified via centroid method to obtain a crisp numerical value for the fused output \( TH \); i.e., throttle opening. Fusion using Mamdani’s max-prod composition places equal weights on all the sources, without considering their merit or importance,
though a weighted combination via assigning differential weights to different rules is possible. However, this method does not provide a strong basis required to assign the weights. Furthermore, the use of Mamdani inference amounts to using conventional fuzzy decision making through a rule-base, to arrive at an inference, which takes the meaning of "fused output" in the present context.

<table>
<thead>
<tr>
<th>Rules</th>
<th>IF</th>
<th>Temperature</th>
<th>AND</th>
<th>Pressure</th>
<th>Implication THEN (Prod)</th>
<th>Throttle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Low</td>
<td>High</td>
<td></td>
<td>Small Medium Large</td>
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<td>Rule 2:</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td>Low</td>
<td>High</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rule 4:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Small Medium Large</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.3 Data fusion using Mamdani's max-product composition.

4.3 Degree of Certainty Method

The data fusion method as described in [23] is used now, to present an alternative approach, which is based on the degree of certainty associated with each given assessment (or measurement). Suppose that \( \mu_1, \mu_2, \ldots, \mu_n \) are the membership values of a set of evidences elated to a proposition \( \theta \subset \Theta \), where \( 0 \leq \mu_i \leq 1 \), for \( i = 1, 2, \ldots, n \).
and \( n \) is the number of evidences to be fused. The membership values have a range \([0,1]\), and \( \mu = e = 0.5 \) corresponds to the membership grade of maximum fuzziness, meaning that at this value the data is of no relevance to the hypothesis and thus its degree of certainty is zero. The degree of certainty of a measurement depends on how far the committed support is from the maximally fuzzy location.

Figure 4.4 Relationship of fuzziness measure and membership values.

Figure 4.4 indicates that greatest weight is associated with an evidence at \( \mu = 0 \) and \( \mu = 1 \) as at these points the degree of fuzziness is zero, thus providing maximum certainty. Evidence \( \mu_i \) is more certain than \( \mu_j \) if and only if for \( i \neq j \) and \( i, j = 1, 2, \ldots, n \)

\[
|\mu_i - 0.5| > |\mu_j - 0.5|
\]

Note, however that \( \mu = 0 \) corresponds to certain “non-supportive” evidence and \( \mu = 1 \) corresponds to certain “supportive” evidence. In practice, these two aspects have to be incorporated separately. The combined deviation of \( \mu_1, \mu_2, \ldots, \mu_n \) from the location of maximum fuzziness is obtained by calculating the average Minkowski Distance given by the following equation:

\[
d = \left[ \frac{1}{n} \sum_{i=1}^{n} (\mu_i - e)^a \right]^{\frac{1}{a}},
\]

(4.4)
where $\alpha$ is odd; i.e., $\alpha = 1, 3, 5, \ldots, \infty$, and represents a normalization factor that preserves the degree of certainty associated with each $\mu_i$ and discriminates between supportive and non-supportive evidence. The fused value can be calculated as the sum of the combined deviations, $d$, offset by the location of maximum fuzziness ($\mu = e = 0.5$), and is given by following equation:

$$\mu = e + \left[ \frac{1}{n} \sum_{i=1}^{n} (\mu_i - e)^{\alpha} \right]^{\frac{1}{\alpha}}, \tag{4.5}$$

As $\alpha$ is an odd integer, i.e., $\alpha = 1, 3, 5, \ldots, \infty$ the value of $(\mu_i - e)^{\alpha}$ is negative if $\mu_i$ is less than $e = 0.5$; i.e., non-supportive evidences, and is positive if $\mu_i > 0.5$; i.e., supportive evidences. The parameter $\alpha$ assigns a weight to each evidence, which in turn expresses the relative importance of each evidence in the combined evidence (fused value). The fusion function as defined in Equation (4.4) adjusts the weights related to each evidence in such a manner that a support that is closer to the maximum fuzziness location, has a smaller effect in the fused output value, while those nearer to the crisp values have a quite larger effect. The characteristics of the fusion function change with the parameter $\alpha$.

If $\alpha = 1$, then fusion function acts as an arithmetic mean.

$$\mu = \frac{\mu_1 + \mu_2 + \ldots + \mu_n}{n} \tag{4.6}$$

For $\alpha \geq 3$, the evidences with higher fuzziness have less effect on the fused output, and for $\alpha = \infty$ the fusion function acts as a max operator. A value of $\alpha = 3$ provides a practical tradeoff between these two extremes, and is used for obtaining the fused output in the present work.
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This fusion formula has the following desirable properties:

**Commutativity:** A combination function $f$ is commutative if

$$f(\mu_i, \mu_j) = f(\mu_j, \mu_i), \forall i,j = 1,2,\ldots,n$$

(4.7)

The commutative property guarantees that the order in which the pieces of evidence are received, does not affect the final outcome of fusion.

**Convexity:** $\forall \theta \in \Theta$, a combination of fuzzy memberships $\mu_i(\theta), \mu_j(\theta), \ldots, \mu_n(\theta)$ that is given by $\mu(\theta) = f(\mu_i, \mu_j, \ldots, \mu_n)$ is said to be convex if and only if

$$\min(\mu_1, \mu_2, \ldots, \mu_n) \leq \mu \leq \max(\mu_1, \mu_2, \ldots, \mu_n)$$

(4.8)

The convexity of a combination allows for a compromise among pieces of evidence.

**Indempotence:** A combination function is said to be indempotent if

when $\mu = \mu_1 = \mu_2 = \ldots = \mu_n$, one has $f(\mu_1, \mu_2, \ldots, \mu_n) = \mu$

(4.9)

The indempotence property guarantees that if all the sources indicate the same support to an event, the outcome of the data fusion must also have the same support to that event.

**Monotonicity:** The property of monotonicity guarantees the existence of a unique point (identity value) within the belief region, which divides the belief region into two segments. Values greater than the identity value are supportive of a given hypothesis and those less than identity value are non-supportive of the hypothesis. Also, it guarantees that a stronger piece of evidence makes a stronger support (or contribution) to the fusion. It can be represented as

$$f(\mu_i, \mu_j) > f(\mu_i, \mu_k), \forall \mu_j > \mu_k, \forall i,j = 1,2,\ldots,n$$

(4.10)

Data fusion formula based on the degree of certainty, in contrast to other fusion methods, tests for the certainty and accounts for a relative importance of aggregated bodies of evidence. This method is typically useful when the sensor sources disagree and modeling the evidence on the basis of its degree of certainty results in a reliable fusion.
4.4 Compatibility Function Fusion Method

Yager [27] introduced the concept of the compatibility of the measured data in the fusion of fuzzy numbers. In many cases the sources of information, whether sensors or experts, are pervaded with uncertainty, imprecision and contradiction. The aggregation operator should to some extent be based on the compatibility of the measured data. The aggregation operator may be chosen among min, mean, median, max, etc., depending on the concept of compatibility of the measured data. The importance of the concept of compatibility of data can be explained using a simple example. Consider the fusion of two temperature measurements $T_1 = 60^\circ C$, and $T_2 = 20^\circ C$ obtained by two similar thermocouples. The mean operator provides the fused result as $T_{\text{mean}} = 40^\circ C$, which is not compatible with either of the temperature measurements. The reason for this lack of compatibility is the attempt to aggregate conflicting values. In developing intelligent fusion systems the very dissimilar data should not be directly aggregated, and should be assigned low weights, or, at the very least, should be associated with some indication of the conflict or level of confidence in the aggregated result.

4.4.1 Extension Principle

In order to fuse the data Zadeh’s extension principle [25] is used to extend to the case of fuzzy inputs (fuzzy numbers) the usual arithmetic fusion functions (min, mean, max), which are applicable to ordinary numbers. Let $F$ be an ordinary (crisp) fusion function defined as

$$F : X \rightarrow Y \quad (4.11)$$

that is, $F$ takes points in the $\mathbb{R}$ space $X$ and maps a value in the space $Y$. The extension principle for this case can be expressed as

$$B(y) = \{A(x)/F(x) : y = F(x), x \in \mathbb{R}\} \quad (4.12)$$
where $A(x)$ and $B(y)$ are the values of the membership functions of the fuzzy sets $A$ and $B$

at the elements $x$ and $y$, respectively.

The extension principle allows extending any crisp function to deal with fuzzy sets. In the

generalized case of mapping a function from the $\mathbb{R}^n$ space (Cartesian space $X_1 \times X_2 \times \ldots \times X_n$) into the space of $Y$, according to the extension principle [16] the

min of the membership values should be taken of all the points $(x_1 \times x_2 \times \ldots \times x_n)$

which map to a point $y$.

If there are two or more such combinations of $(x_1 \times x_2 \times \ldots \times x_n)$ that map to the

same $y$, then max of the result should be taken. If $F$ is a crisp fusion function, a mapping

of $\mathbb{R}^n$ into $\mathbb{R}$, and $A_1, A_2, \ldots, A_n$ are the fuzzy data that is to be aggregated, the fuzzy

fusion is defined by

$$F(A_1, A_2, \ldots, A_n) = B$$

Then according to the general extension principle, the membership function of the fused

output is obtained according to

$$B(y) = \left\{ \min \frac{A_i(x_i)}{F(x_1, x_2, \ldots, x_n)} \right\}_{x_i \in X_i} = \left\{ \min \frac{A_i(x_i)}{F(x)} \right\}_{x \in \mathbb{R}^n} (4.13)$$

Note that the max operation is used when there are two or more points in $\mathbb{R}^n$ space

mapping to the same point $B$ in space. Any fusion function should satisfy the constraints

of commutativity, monotonicity, and idempotence. The mean fusion operator $F$ meets all

these requirements and is defined as

$$F(A_1, A_2, \ldots, A_n) = \frac{1}{n} \sum_{i=1}^{n} A_i \quad (4.14)$$

$$F(A_1, A_2, \ldots, A_n) = \frac{A_1 \oplus A_2 \oplus \ldots \oplus A_n}{n} = B \quad (4.15)$$
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and thus $B = \left\{ \min_i A_i / \frac{1}{n} \sum_{i=1}^{n} A_i \right\}$

(4.16)

Note that max operation is needed here since the "mean" function is a one-to-one transformation.

4.4.2 Inclusion of Compatibility

Consider the mean as the fusion function $F$. Let $X$ be the space from which the observations are drawn. A compatibility relationship $R$ is a mapping

$R : X \times X \rightarrow [0, 1],$

such that for two values $a$ and $b$ drawn from $X$, $R(a,b)$ indicates the degree to which it is acceptable to fuse the values $a$ and $b$ under $F$. Alternatively, $R(a,b)$ can be viewed as indicating the level of confidence in fusing $a$ and $b$. The actual form of this relationship is problem dependent to a great extent and as such can be seen as synthesizing a knowledge-base that contains meta information about the application in the fusion operation is being used. For example, if two incompatible temperatures $20^\circ C$ and $60^\circ C$ are fused, then one may use the compatibility relationship $R(20,60) = 0$. On the other hand, fusion of the temperatures $60^\circ C$ and $80^\circ C$ may be considered fully compatible resulting in $R(60,80) = 1$.

While the form of $R$ is problem dependent for the most part, some problem independent properties can be associated with $R$. It is natural to require that $R(a,a) = 1$ for all $a \in X$. Furthermore, as the distance between $a$ and $b$ on the metric associated with $X$ increases then $R(a,b)$ should not at least increase; that is, if $\text{Distance}(a,b) \geq \text{Distance}(a,b')$ then $R(a,b) \leq R(a,b')$. On the real line $\mathbb{R}$ it can be assumed that compatibility function $R$ depends on the absolute distance, $\text{Distance}(a,b) = |a-b|$ only, and is a decreasing function of $|a-b|$.

$R$ can be extended to act on $n$-tuples from $X$
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\[ R(a_1, a_2, \ldots, a_n) = \min_{a_i, a_j} R(a_i, a_j) \quad (4.17) \]

It is clear from this relation that extension of \( R \) taken as the minimum of the compatibilities of all pairs data in the considered set. To fuse the data \((a_1, a_2, \ldots, a_n)\), then one may calculate the fused value \( F(a_1, a_2, \ldots, a_n) \), and the value of \( R(a_1, a_2, \ldots, a_n) \) provides the degree of confidence in this fused value. In spirit, this compatibility value is related to the inverse of the uncertainty or variance.

Assume that \( A_1, A_2, \ldots, A_n \) are a collection of fuzzy subsets, over the space \( X \), which are to be fused. Then fused value \( B \) for mean fusion function \( F \) is given by

\[ F(A_1, A_2, \ldots, A_n) = \{ A(x_1, x_2, \ldots, x_n) / F(x_1, x_2, \ldots, x_n) \} = B \quad (4.18) \]

Where \( A(x_1, x_2, \ldots, x_n) = \min_i A_i(x_i) \)

Thus \( B \), a fuzzy subset of \( X \) is defined as:

\[ B(x) = \max_i \{ A(x_1, x_2, \ldots, x_n); x_i \in X \} \]

Now to include the compatibility function \( R \), it is required that the resulting function be one that only allows the aggregation of data that are combinable. This additional requirement leads to a fused value that must satisfy

\[ F(A_1, A_2, \ldots, A_n) \text{ and } R(A_1, A_2, \ldots, A_n) \]

Thus the mean fusion function is modified to take compatibility function \( R \) into consideration as:

\[ F_R(A_1, A_2, \ldots, A_n) = \{ R(x_1, x_2, \ldots, x_n) \wedge A(x_1, x_2, \ldots, x_n) / F(x_1, x_2, \ldots, x_n) \} = B \quad (4.19) \]

where \( \wedge \) denotes the min operator and \( B \) denotes the fuzzy set of \( X \) that corresponds to the fused value. In the above relation the effect of compatibility \( R \) is to essentially reduce the membership grade associated with the tuples that are incompatible. Also \( \max \_ \_ B(x) \) provides an indication of the level of confidence (or, lack of conflict) in the fused value.

The compatibility function \( R \) can range from complete compatibility to complete conflict.
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For the complete compatibility case, i.e., when it is acceptable to fuse any values from $X$, one has:

$$\forall (x, y) \in X^2, \quad R(x, y) = 1$$

$$F_R(A_1, A_2, \ldots, A_n) = \left\{ \frac{R(x_1, x_2, \ldots, x_n) \land A(x_1, x_2, \ldots, x_n)}{F(x_1, x_2, \ldots, x_n)} \right\}$$  \hspace{1cm} (4.20)

Since $R(x_1, x_2, \ldots, x_n) = 1$ for all tuples, we get

$$F_R(A_1, A_2, \ldots, A_n) = F(A_1, A_2, \ldots, A_n)$$

Thus the ordinary fusion function is a special case of the constrained fusion when all fusions are completely compatible.

Considering the extreme case where the fusion of only the identical elements is allowed, one has:

$$\forall (x, y) \in X^2, \text{ if } x \neq y \text{ then } R(x, y) = 0$$

With the mean fusion operator the fused value is $B$ where

$$F_M(A_1, A_2, \ldots, A_n) = \left\{ \frac{R(x_1, x_2, \ldots, x) \land A(x_1, x_2, \ldots, x)}{F(x_1, x_2, \ldots, x)} \right\} = B$$  \hspace{1cm} (4.21)

With $R(x_1, x_2, \ldots, x_n) = 1$ if all $x_i$ are identical else $R(x_1, x_2, \ldots, x_n) = 0$.

Since $F(x_1, x_2, \ldots, x) = \frac{1}{n} \sum_{i=1}^{n} x = x$, we have

$$F_M(A_1, A_2, \ldots, A_n) = \left\{ \min_i A_i(x) / x \right\} = B$$  \hspace{1cm} (4.22)

Note that $B = \min_i A_i(x)$ corresponds to $B = A_1 \cap A_2 \cap \ldots \cap A_n$, which is simply the conjunction of fuzzy numbers. Thus with this choice of compatibility relationship, fusion based on the mean operator reduces to the intersection of the fuzzy values to be fused.
4.4.3 Fusion of Fuzzy Numbers using Compatibility

The data fusion method developed in this section is based on the concept of compatibility proposed by Yager [27]. This section describes the approach using $\alpha$-level fuzzy sets to fuse triangular shaped fuzzy numbers $A_1, A_2, \ldots, A_n$. The mean is used as the fusion operator $F$.

$$F(A_1, A_2, \ldots, A_n) = \frac{A_1 \oplus A_2 \oplus \cdots \oplus A_n}{n} = B \quad (4.23)$$

For a given $\alpha \in [0,1]$, let the $\alpha$-level set of the fuzzy set $A_i$ be denoted by $A_{i\alpha}$.

Then, $A_{i\alpha} = [a_{i\alpha}, b_{i\alpha}] \quad (4.24)$

Where $[a_{i\alpha}, b_{i\alpha}]$ is the interval in the support set of $A_i$ in which the membership grade of $A_i$ is $\geq \alpha$.

Let $a_\alpha^* = \max_i[a_{i\alpha}]$, and $b_\alpha^* = \min_i[b_{i\alpha}]$

where $a_\alpha^*$ is the largest lower bound of the $\alpha$-level intervals of fuzzy sets and $b_\alpha^*$ is the smallest upper bound on the $\alpha$-level interval of the fuzzy sets.

Let $U_\alpha^*$ be the minimum $x$ such that $R(a_\alpha^*, x) \geq \alpha$. Thus $U_\alpha^*$ is the smallest data value that is compatible with $a_\alpha^*$ at a compatibility level of at least $\alpha$. We also let $V_\alpha^*$ be the maximum $x$ such that $R(b_\alpha^*, x) \geq \alpha$. Thus $V_\alpha^*$ is the largest data value that is compatible with $b_\alpha^*$ at a compatibility level of at least $\alpha$. That is,

$$U_\alpha^* = \min\left\{x \mid R(a_\alpha^*, x) \geq \alpha\right\}, \quad V_\alpha^* = \max\left\{x \mid R(b_\alpha^*, x) \geq \alpha\right\} \quad (4.25)$$

Then, $\alpha$-level intervals of the aggregated set value $B$ can be calculated as

Then, $B = [d_\alpha, e_\alpha] \quad (4.26)$

Where $d_\alpha = f(g_1, g_2, \ldots, g_n)$ and $e_\alpha = f(h_1, h_2, \ldots, h_n)$
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Where \(g_{ia}\) are the components of \(d_a\), which is the left side of the fused set \(B\), and similarly \(h_{ia}\) are the components of \(e_a\), which is the right side of the fused set \(B\).

In particular, using the mean operator for \(F\) one has

\[
d_a = \frac{1}{n} \sum_{i=1}^{n} g_{ia}
\]

(4.27)

where

\[
g_{ia} = a_{ia} \quad \text{if} \quad a_{ia} \geq U^{*}_{a}
\]

\[
g_{ia} = U_{ia} \quad \text{if} \quad a_{ia} < U^{*}_{a}
\]

and

\[
e_a = \frac{1}{n} \sum_{i=1}^{n} h_{ia}
\]

(4.28)

where

\[
h_{ia} = b_{ia} \quad \text{if} \quad b_{ia} \leq V^{*}_{a}
\]

\[
h_{ia} = V_{ia} \quad \text{if} \quad b_{ia} > V^{*}_{a}
\]

An example is given now to illustrate this method. Suppose that it is required to fuse two fuzzy sets related to the temperature measurements obtained from two thermocouples. The first temperature measurement is 9°C and the second measurement is 30°C. Suppose that the variance of these measurements is 10°C, as given by their support sets. Figure 4.5 shows the temperatures to be fused as the fuzzy sets \(A_1\) and \(A_2\) and the fused set as \(B\).

We select the compatibility function as

\[
R(x, y) = \begin{cases} 
0 & \text{if } |x - y| > 15, \\
1 - \frac{1}{15} |x - y| & \text{if } |x - y| \leq 15.
\end{cases}
\]

(4.29)

This compatibility function indicates that any two temperature measurements that are more than 15°C apart are not compatible, and if they are within 15°C then compatibility
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A level of conflict is assigned as a membership grade that is inversely proportional to the difference in the temperatures.

<table>
<thead>
<tr>
<th>Level of Conflict</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.16</td>
</tr>
<tr>
<td>0.4</td>
<td>9</td>
</tr>
<tr>
<td>0.3</td>
<td>14</td>
</tr>
<tr>
<td>0.2</td>
<td>17.5</td>
</tr>
<tr>
<td>0.1</td>
<td>21.5</td>
</tr>
<tr>
<td>0</td>
<td>25</td>
</tr>
<tr>
<td>0.2</td>
<td>30</td>
</tr>
<tr>
<td>0.4</td>
<td>35</td>
</tr>
</tbody>
</table>

Figure 4.5 Fusion of two disparate temperatures using compatibility function.

The \( \alpha \)-level set associated with the fuzzy sets \( A_1 \) and \( A_2 \) are given by

\[
A_{1\alpha} = [4 + 5\alpha, 14 - 5\alpha] \quad \text{and} \quad A_{2\alpha} = [25 + 5\alpha, 35 - 5\alpha].
\]

It follows that

\[
a_{1\alpha} = [4 + 5\alpha, 25 + 5\alpha] \quad \text{and} \quad b_{1\alpha} = [14 - 5\alpha, 35 - 5\alpha]
\]

Here, \( a^*_{1\alpha} = \max_i[a_{1\alpha}] = 25 + 5\alpha \) and \( b^*_{1\alpha} = \min_i[b_{1\alpha}] = 14 - 5\alpha \)

where \( a^*_{1\alpha} \) is the largest lower bound of the \( \alpha \)-level sets and \( b^*_{1\alpha} \) is the smallest upper bound of the \( \alpha \)-level sets.

Then,

\[
U^*_\alpha = 1 - \frac{1}{15}|a^*_\alpha - x| = \alpha \quad \text{and} \quad V^*_\alpha = 1 - \frac{1}{15}|x - b^*_\alpha| = \alpha
\]

\[
\Rightarrow U^*_\alpha = 10 + 20\alpha \quad \text{and} \quad \Rightarrow V^*_\alpha = 29 - 20\alpha
\]

where \( U^*_\alpha \) is the smallest temperature value that is within \( \alpha \) the compatibility level of \( a^*_\alpha \), and \( V^*_\alpha \) is the largest temperature value that is within the compatibility level of \( b^*_\alpha \).

Now for an aggregate \( \alpha \)-level fuzzy set of \( B \) to exist one must have

\[
U^*_\alpha \leq b^*_\alpha \Rightarrow \alpha \leq 0.16 \quad \text{and} \quad V^*_\alpha \geq a^*_\alpha \Rightarrow \alpha \leq 0.16
\]

Now for \( \alpha \leq 0.16 \)
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\[ a_{1a} < U^*_a \quad \therefore \quad g_{1a} = U^*_a = 10 + 20\alpha \]
\[ a_{2a} > U^*_a \quad \therefore \quad g_{2a} = a_{2a} = 25 + 5\alpha \]

\[ d_a = \frac{1}{2} \sum_{i=1}^{2} g_{ia} = 17.5 + 12.5\alpha \]

\[ b_{1a} < V^*_a \quad \therefore \quad h_{1a} = b_{1a} = 14 - 5\alpha \]
\[ b_{2a} > V^*_a \quad \therefore \quad h_{2a} = V^*_a = 29 - 20\alpha \]

\[ e_a = \frac{1}{2} \sum_{i=1}^{2} h_{ia} = 21.5 - 12.5\alpha \]

Thus, aggregated \( \alpha \)-level set of \( B = [d_a, e_a] = [17.5 + 12.5\alpha, 21.5 - 12.5\alpha] \).

The centroid defuzzification of fuzzy set \( B \) provides fused temperature \( T_{\text{mean}} = 19.5^\circ \text{C} \), and the height of \( B \) is 0.16, which indicates level of confidence in the fused result. The two temperature measurements are clearly conflicting and as expected the level of confidence clearly indicates the fact.

Figure 4.6 indicates the fused temperature with a softer compatibility function \( R \) as given by

\[
R(x, y) = \begin{cases} 
0 & \text{if } |x - y| > 20 \\
1 - \frac{1}{20}|x - y| & \text{if } |x - y| \leq 20
\end{cases}
\]  
(4.30)

Figure 4.6 Fusion of two disparate temperatures using softer compatibility function.
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As shown in Figure 4.6 the centroid defuzzification of the fuzzy set \( B' \) provides the fused temperature \( T_{\text{mea}} = 19.2^\circ \text{C} \). The height of \( B' \) is 0.3, which indicates level of confidence in the fused result. The two temperature measurements as per the new compatibility function are less conflicting and as this fact is reflected by the increase in the level of confidence. However, the numerical value of fused temperature is still close to the one obtained with the harder (or stricter) compatibility function.

Thus this method provides a useful tool when all sources of data are considered equally reliable and all of them are to be satisfied at the same time, which means that a conflict between the sources leads to a low confidence value. When utilizing the fused information for taking a control action the confidence value should be considered along with the fused result.

Depending on the problem, compatibility function \( R \) can be generated by an appropriate knowledge-base. It provides a means for incorporating expert meta-knowledge about the observation space, and helps in making a more intelligent fusion decision.

4.5 Illustrative Example

An illustrative example is provided here to compare the three methods of fusion. Cars with automatic cruise control (ACC) maintain a constant preset distance between the cars. Suppose that these cars are equipped with three sensors viz., millimeter-wave radar (MMWR), laser range finder (LRF), and stereo camera (SC). Each measures the distance between the two cars in the same lane. The distance measurements acquired by the three sensors are fused and, on the basis of this fused distance, the car is accelerated or decelerated via throttle and/or brakes to maintain a desired safe distance. In this example let the preset safe distance be 10m. A fuzzy system is developed for the fusion of measured distances by the three sensors. At an instance when the correct distance between two cars is 10m, suppose that the distances measured by the three sensors are:
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MMWR–10m, SC–7m, LRF–9.5m. According to Figure 4.7, all the inputs and the output take the five states: Very Small (VSM), Small (SM), Correct (CR), Large (LG), and Very Large (VLG).

For this system 125 rules are possible, however Figure 4.8 indicates the only the four rules that fire for the given input, i.e, MMWR–10m, SM–7m, and LFR–9.5m.

1. If $MMWR$ is CR and $SC$ is SM and $LRF$ is CR then $FD$ is CR
2. If $MMWR$ is CR and $SC$ is SM and $LRF$ is SM then $FD$ is SM
3. If $MMWR$ is CR and $SC$ is VS and $LRF$ is CR then $FD$ is CR
4. If $MMWR$ is CR and $SC$ is VS and $LRF$ is CR then $FD$ is SM

The fuzzy system ACC as shown in Figure 4.9 has three inputs $MMWR$, $LRF$, $SC$ and one output variable, which is the Fused Distance ($FD$).
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Fuzzy system for adaptive cruise control of a car.

The fused distance ($FD$) obtained by Mamdani’s max-prod composition based fusion method, as shown in Figure 4.10, is 9.4m.

**Figure 4.9** Fuzzy system for adaptive cruise control of a car.

**Figure 4.10** Fused distance obtained by the data fusion method based on Mamdani’s max-prod composition.

<table>
<thead>
<tr>
<th>MMWR</th>
<th>SC</th>
<th>LRF</th>
<th>$FD$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>7</td>
<td>9.50</td>
<td>9.40</td>
</tr>
</tbody>
</table>
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Table 4.1 shows the input variables, the fuzzy sets they belong to, and their membership grades. Applying the data fusion based on degree of certainty for each rule that fires, one obtains the degree of support of the fused distance \( (FD) \), as given by

\[
\mu_i(FD) = 0.5 + \left[ \frac{\left( \mu_i(MMWR) - 0.5 \right)^3 + \left( \mu_i(SC) - 0.5 \right)^3 + \left( \mu_i(SC) - 0.5 \right)^3}{3} \right]^{1/3}, \forall i = 1, 2, \ldots, 4
\]

and the centroid of the output represented by \( \mu_c(FD) \).

Table 4.1 Crisp inputs, their fuzzy sets, and Membership values from the three sensors.

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Crisp Values</th>
<th>Fuzzy Sets</th>
<th>Membership Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>MMWR</td>
<td>10.0</td>
<td>CR</td>
<td>1.0</td>
</tr>
<tr>
<td>SC</td>
<td>7.0</td>
<td>SM, VSM</td>
<td>0.5, 0.5</td>
</tr>
<tr>
<td>LRF</td>
<td>9.5</td>
<td>CR, SM</td>
<td>0.75, 0.25</td>
</tr>
</tbody>
</table>

\[
\mu_i(FD) = 0.5 + \left[ \frac{(1 - 0.5)^3 + (0.5 - 0.5)^3 + (0.75 - 0.5)^3}{3} \right]^{1/3} = 0.86, \text{ and } \mu_c(FD_{c1}) = 10
\]

\[
\mu_2(FD) = 0.5 + \left[ \frac{(1 - 0.5)^3 + (0.5 - 0.5)^3 + (0.25 - 0.5)^3}{3} \right]^{1/3} = 0.832, \text{ and } \mu_c(FD_{c2}) = 9
\]

\[
\mu_3(FD) = 0.5 + \left[ \frac{(1 - 0.5)^3 + (0.5 - 0.5)^3 + (0.75 - 0.5)^3}{3} \right]^{1/3} = 0.86, \text{ and } \mu_c(FD_{c3}) = 10
\]

\[
\mu_4(FD) = 0.5 + \left[ \frac{(1 - 0.5)^3 + (0.5 - 0.5)^3 + (0.25 - 0.5)^3}{3} \right]^{1/3} = 0.832, \text{ and } \mu_c(FD_{c4}) = 9
\]

By using the fusion based on degree of certainty, the final value for fused distance \( (FD) \) is calculated as:

\[
FD = \frac{\sum_{i=1}^{4} \mu_i(FD) \mu_c(FD_{ci})}{\sum_{i=1}^{4} \mu_i(FD)} = 9.6 \text{ m.} \quad (4.31)
\]
Fuzzy sets $A_1, A_2, A_3$, as shown in Figure 4.11, represent the distance measurements by SC, LFR, MMWR, and $B$ indicates the fuzzy set for the fused distance ($FD$).

Fuzzy set $A_x$, $A_2$, and $A_3$, as shown in Figure 4.11, represent the distance measurements by SC, LFR, MMWR, and $B$ indicates the fuzzy set for the fused distance ($FD$).

The following compatibility function is selected for the fusion:

$$R(x, y) = \begin{cases} 0 & \text{if } |x - y| > 3 \\ 1 - \frac{1}{3}|x - y| & \text{if } |x - y| \leq 3 \end{cases}$$

(4.32)

For any level $\alpha$, the $\alpha$-level set associated with fuzzy set $A_i$ is given by

$$A_{i\alpha} = \left[a_{i\alpha}, b_{i\alpha}\right]$$

We have,

$$A_{1\alpha} = [6 + \alpha, 8 - \alpha], \quad A_{2\alpha} = [8.5 + \alpha, 10.5 - \alpha], \quad A_{3\alpha} = [9 + \alpha, 11 - \alpha]$$

Now, $\{a_{i\alpha}\} = \{6 + \alpha, 8.5 + \alpha, 9 + \alpha\}$ and $\{b_{i\alpha}\} = \{8 - \alpha, 10.5 - \alpha, 11 - \alpha\}$

Hence, $a^*_{i\alpha} = \max_i a_{i\alpha} = 9 + \alpha$, and $b^*_{i\alpha} = \min_i b_{i\alpha} = 8 - \alpha$

where $a^*_{\alpha}$ is the largest lower bound of the $\alpha$-level sets and $b^*_{\alpha}$ is the smallest upper bound of the $\alpha$-level sets.

$$U^*_{\alpha} = 1 - \frac{1}{3}|a^*_{\alpha} - x| = \alpha$$

and

$$V^*_{\alpha} = 1 - \frac{1}{3}|x - b^*_{\alpha}| = \alpha$$
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\[ U^*_a = 6 + 4\alpha \quad \text{and} \quad V^*_a = 11 - 4\alpha \]

where \( U^*_a \) is the smallest value that is within \( \alpha \) the compatibility level of \( a^*_a \), and \( V^*_a \) is the largest value that is within the compatibility level of \( b^*_a \).

Now for aggregate \( \alpha \)-level fuzzy set \( B \) to exist

\[ U^*_a \leq b^*_a \Rightarrow \alpha \leq 0.4 \quad \text{and} \quad V^*_a \geq a^*_a \Rightarrow \alpha \leq 0.4 \]

Now for \( \alpha \leq 0.4 \)

\[ a_{1a} < U^*_a \quad \therefore \quad g_{1a} = U^*_a = 6 + 4\alpha \]
\[ a_{2a} > U^*_a \quad \therefore \quad g_{2a} = a_{2a} = 8.5 + \alpha \]
\[ a_{3a} > U^*_a \quad \therefore \quad g_{3a} = a_{3a} = 9 + \alpha \]

Then,

\[ d_a = \frac{1}{3} \sum_{i=1}^{3} g_{ia} = 7.8 + 2\alpha \]

Also,

\[ b_{1a} < V^*_a \quad \therefore \quad h_{1a} = b_{1a} = 8 - \alpha \]
\[ b_{2a} > V^*_a \quad \therefore \quad h_{2a} = V^*_a = 11 - 4\alpha \]
\[ b_{3a} > V^*_a \quad \therefore \quad h_{3a} = V^*_a = 11 - 4\alpha \]

\[ e_a = \frac{1}{3} \sum_{i=1}^{3} h_{ia} = 10 - 2\alpha \]

Thus, aggregated set \( B \), as shown in Figure 4.11 has the \( \alpha \)-level set \([d_a, e_a] = [7.8 + 2\alpha, 10 - 2\alpha]\), and the centroid of \( B \) gives the fused distance (FD) as 8.9m. The level of confidence value 0.4 indicates that there is significant conflict in the distances measured by the three sensors.
Figure 4.12 Fused distance results obtained by the three data fusion methods.

Table 4.2 gives the fused distances and percentage error values obtained by the three methods of fusion. Percentage error results indicate that the fused distance achieved by the data fusion method based on degree of certainty is the most accurate in this case.

Table 4.2 Fusion distance and percentage error results obtained by the three data fusion methods.

<table>
<thead>
<tr>
<th>Fusion Method based on</th>
<th>Fused Distance</th>
<th>Percentage Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max – Prod Composition</td>
<td>9.4</td>
<td>6</td>
</tr>
<tr>
<td>Degree of Certainty</td>
<td>9.6</td>
<td>4</td>
</tr>
<tr>
<td>Compatibility Function</td>
<td>8.9</td>
<td>11</td>
</tr>
</tbody>
</table>

Also Figure 4.12 shows that the results obtained by the degree of certainty method are most crisp, i.e., least fuzzy thus most reliable. However, like in any application of fuzzy logic, the fused results depend heavily on the vagaries of membership functions for the input/output variables and the rule-base that is used. It cannot be easily concluded that the fusion based on the degree of certainty method always provides the most accurate result, as the results heavily depend on the nature (or context) of the application.
Chapter 5

5 Implementation in a Fishcutting Machine

This chapter outlines the utilization of the fuzzy sensor fusion methods discussed in chapter 4, for the product quality assessment in a fishcutting machine. First, the chapter presents the fuzzy system for the fishcutting machine, describing the input and output parameters, their generation, and relation to product quality. Then the fuzzy rule-base and defuzzification methods are given. Next, the experimental results obtained by the implementation of the three fuzzy sensor fusion methods on three sets of fish-cut data are presented. The chapter concludes with an analysis and comparative evaluation of the results.

5.1 Fuzzy System of the Fishcutting Machine

The implementation of data fusion consists of the following four steps:

1) Fuzzification of raw data
2) Rule-base generation
3) Fusion
4) Defuzzification.

For the implementation of data fusion a fuzzy system for the fishcutting machine is developed. This section presents the development of the fuzzy system for the fishcutting machine and it involves steps 1, 2, and 4, as listed above and is presented now.
5.1.1 Fuzzification

This section describes the preprocessing carried out on the input sensor data. The sensors involved in the fish cutting process are the CCD cameras (primary and secondary), the optical encoders, and the ultrasonic displacement sensor. At the lowest level of the control system (see section 3.1.2) raw (crisp) signals from these sensors are preprocessed through specific filters to extract context based fuzzy sets (linguistic variables). The membership functions chosen have symmetrical triangular and trapezoidal shapes, as they can be represented by parametric functional descriptions. Also these shapes provide computational effectiveness, efficient use of memory, and relative ease of performance analysis. The support lengths of the selected fuzzy sets are based on the sensor variances.

a) Fuzzification of Vision Data

The primary purpose of the image processor is to extract context information from the raw images that indicates high-level information such as product quality. The quality of processed fish, unlike the quality of a manufactured product, is not amenable to simple quantification based on few dimensions, shape, or surface defects. This should be clear from the raw image captured by the secondary camera, for a good cut and a bad cut, as shown in Figure 5.1, and Figure 5.2, respectively. These images are filtered to obtain the arc-length profiles as shown in Figure 5.3 and Figure 5.4 for the good cut and the bad cut, respectively. Now quality indicative parameters in the form of linguistic variables are extracted from these arc-length profiles. The quality indicative parameters are Cut_Depth (\(D_c\)), Cut_Contour (\(S_c\)), and Cut_Surface (\(S_r\)).
Figure 5.1 Raw Image of a processed fish where processing quality is good.

Figure 5.2 Raw image of a processed fish where processed quality is bad.

Figure 5.3 Arc-Length profile for the good cut.

Figure 5.4 Arc-Length profile for the bad cut.

The arc-length profiles have certain points that are related to the product quality. These are: Point \( a \) which signifies the beginning of the cut contour; Point \( b \) – the depth of solid cut; and Point \( c \) – the end of the cutting contour. These performance indicative parameters can be defined as follows:

\[
D_c = \frac{y_{bc}}{y_{af}} \tag{5.1}
\]

where, \( y_{bc} = \) depth of solid cut from \( b \) to \( c \) in pixel co-ordinates.
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\[ y_{af} = \text{height of fish from } a \text{ to } f \text{ in pixel co-ordinates.} \]

Smoothness of cut contour \( S_c = \frac{y_{bc}}{l_{bc}} \) \hspace{1cm} (5.2)

where, \( l_{bc} = \text{arc-length from } b \text{ to } c \).

Smoothness of cutting region \( S_r = N_i \) \hspace{1cm} (5.3)

where, \( N_i = \text{Number of labeled image objects within the region of interest.} \)

A larger value of \( D_c \) indicates a better cut, a larger value of \( S_c \) indicates a smoother cut profile, and a larger value of \( S_r \) indicates a smoother cut over the cutting region. As shown in Figure 5.5 these parameters may take the three fuzzy states: Poor (PR), Medium (MD), and Good (GD).

![Figure 5.5 Membership functions for visual quality indicative parameters.](image)

b) Fuzzification of Cutter Load Data

The purpose of preprocessing the cutter load data is to extract certain high-level information related to the performance of the cutter blades during the period of cutting
action of the fishcutting machine. The speeds of both top and bottom blades, driven by induction motors, are obtained by optical encoders. The speeds of the induction motors are first converted into “slip” parameters [50] and then to percentage average loads by means of the following equations:

$$\text{Measured Slip} = \frac{(\text{Synchronous speed}) - (\text{Measured shaft speed})}{\text{Synchronous speed}}$$  \hspace{1cm} (5.4)

where, synchronous speed for the given induction motors is 3600 rpm.

$$\text{Cutter Load} = \frac{\text{Measured slip}}{\text{Slip at full load}} \times 100\%$$  \hspace{1cm} (5.5)

A typical load profile indicating the load on two cutter blades is shown in Figure 5.6, where load on each of the two cutter blades during a typical cutting operation is plotted against time. Now, the following parameters can be measured from the two profiles: peak load, average load, and magnitude of secondary peaks if present and their times of occurrence with respect to the first peak.

The average load of each curve over a single cutting cycle signifies the average load on the induction motors; and the difference of magnitudes between the two peak loads of the top and bottom blades is typically an indication of the degree of asymmetry in feeding the fish in to the cutter.

Figure 5.6 Typical load profile of the two cutter blades.
The quality indicative parameters such as Average_Load ($AL$) and Assymetry_Index ($AI$), are calculated, which are defined as:

$$\text{Average_Load (} AL \text{)} = \frac{1}{2} (L_{mt} + L_{mb})$$  \hspace{1cm} (5.6)$$

$$\text{Asymmetry_Index (} AI \text{)} = \frac{|L_{mt} - L_{mb}|}{(L_{mt} + L_{mb})/2}$$  \hspace{1cm} (5.7)$$

where the subscripts $t$ and $b$ represent the top and the bottom blade, respectively. As shown in Figure 5.7 and Figure 5.8, these parameters are given the three fuzzy states: Small (SM), Medium (MD) and Large (LG). The parameter Average_Load ($AL$) indicates the load on the induction motors over a single cutting cycle, and Assymetry_Index ($AI$), indicates the level of asymmetry in feeding a fish into the cutter. Asymmetry is represented by the difference of loads between the top and the bottom blade, and also by the offset of the platform position. Small values of these parameters indicate a better quality of cut.

![Figure 5.7 Membership function for average load.](image-url)
c) **Fuzzification of Servomotor Data**

The fishcutting machine has two DC servomotors, viz., the cutter motor and the platform motor, which are used for the horizontal positioning of the cutter unit and the vertical positioning of the platform, respectively. Preprocessing of the position servo responses from the two servomotors provides performance parameters such as: rise time; damping ratio; damped natural frequency; overshoot, if underdamped; and offset at steady state. Most important of these parameters that is related to product quality is the offset. Quality of cut can then be deduced from the quality indicative parameters, Cutter_Offset (CO) and Platform_Offset (PO). As shown in Figure 5.9 and Figure 5.10 these parameters are given three fuzzy states: Small (SM), Medium (MD) and Large (LG). A larger value of offset will represent a higher wastage of meat, and a smaller value will indicate a higher accuracy of cut; i.e., the distance between the desired and the actual cut positions is small, thus the smaller the offset the better the quality of cut.
Chapter 5. Implementation in a fishcutting machine.

Figure 5.9 Membership function for Cutter Offset.

Figure 5.10 Membership function for platform offset.
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d) Fuzzification of Product Quality Index

The product quality assessment which is the focus of this thesis is determined on the basis of sensory information such as: a) Depth of the solid cut region ($D_c$), b) Smoothness of the cutting contour ($S_c$), c) Smoothness of the surface over the cutting region ($S_r$), d) Average_Load ($AL$), e) Asymmetry_Index ($AI$), f) Cutter_Offset ($CO$), and g) Platform_Offset ($PO$). The single output of the fusion process is the parameter, Product_Quality_Index ($PQI$). It indicates the grade or level of product quality, calibrated on a scale of one to ten. As represented in Figure 5.11 it is assigned to take the five fuzzy states: Excellent (EX), Good (GD), Medium (MD), Poor (PR), and Very Poor (VP). Generally in the fish processing industry, seasoned quality inspectors sort fish into two grads, viz., acceptable and unacceptable; however in our case there are five levels of classification of product quality, indicating better discrimination and choice of inspection.

![Figure 5.11 Membership function for product quality index.](image-url)
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The complete multiple-input and single-output MISO fuzzy model of the sensor-fusion system for the fishcutting machine, with seven inputs, one output, and 45 rules, is shown Figure 5.12.

**Fuzzified Quality Parameters**

- Dc (3)
- Sc (3)
- Sr (3)
- AL (3)
- Al (3)
- CO (3)
- PO (3)

**Fusion System**

- PQI
- (Mamdani)
- 45 rules

**Fused Quality Decision**

- PQI (5)

Figure 5.12 Fuzzy sensor-fusion system for the fishcutting machine.

### 5.1.2 Rule-Base Generation

A fuzzy rule-base is characterized by a collection of linguistic statements expressed in the form of IF-Then rules, which contain fuzzy descriptors. The rules for sensor fusion in the fishcutting machine have been generated by making use of the experience and the knowledge gained by operators in the fish processing industry and extensive experiments performed in the laboratory [43]. In the present work, the fusion system has 45 rules, which relate the preprocessed inputs from the sensors to the fused output, i.e., the product quality index ($PQI$) [44]. Table 5.1 indicates the 27 rules in the
form of linguistic matrices (a), (b) and (c). These rules relate the input parameters from the CCD cameras, such as Cut_Depth ($D_c$), Cut_Contour ($S_c$) and Cut_Surface ($S_r$), to the product quality index. Matrices (a), (b) and (c) in Table 5.1 represent the rules for the condition that Cut_Depth ($D_c$) is Good, Medium, and Poor, respectively. Table 5.2, matrix (a) represents the 9 rules, which relate the input parameters from the induction motors, such as Average_Load ($AL$) and Assymetry_Index ($AI$), to the fused output, i.e., the PQI. Similarly, Table 5.2, matrix (b) represents the 9 rules, which relate the input parameters from the cutter servo and the platform servo, such as the Cutter_Offset ($CO$) and the Platform_Offset ($PO$), to the fused output, i.e., the PQI.

Table 5.1 Rule-bases (a) $D_c = GD$  (b) $D_c = MD$  (c) $D_c = PR$

<table>
<thead>
<tr>
<th>$S_r$</th>
<th>$S_c$</th>
<th>GD</th>
<th>MD</th>
<th>PR</th>
</tr>
</thead>
<tbody>
<tr>
<td>GD</td>
<td>EX</td>
<td>GD</td>
<td>GD</td>
<td>GD</td>
</tr>
<tr>
<td>MD</td>
<td>GD</td>
<td>MD</td>
<td>MD</td>
<td>MD</td>
</tr>
<tr>
<td>PR</td>
<td>MD</td>
<td>PR</td>
<td>PR</td>
<td>PR</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$S_r$</th>
<th>$S_c$</th>
<th>GD</th>
<th>MD</th>
<th>PR</th>
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<td>GD</td>
<td>GD</td>
<td>MD</td>
<td>MD</td>
<td>MD</td>
</tr>
<tr>
<td>MD</td>
<td>MD</td>
<td>MD</td>
<td>MD</td>
<td>MD</td>
</tr>
<tr>
<td>PR</td>
<td>MD</td>
<td>PR</td>
<td>PR</td>
<td>PR</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$S_r$</th>
<th>$S_c$</th>
<th>GD</th>
<th>MD</th>
<th>PR</th>
</tr>
</thead>
<tbody>
<tr>
<td>GD</td>
<td>GD</td>
<td>MD</td>
<td>MD</td>
<td>MD</td>
</tr>
<tr>
<td>MD</td>
<td>MD</td>
<td>PR</td>
<td>PR</td>
<td>PR</td>
</tr>
<tr>
<td>PR</td>
<td>MD</td>
<td>PR</td>
<td>VP</td>
<td>VP</td>
</tr>
</tbody>
</table>

Table 5.2 Rule-bases (a) $AL$ and $AI$

<table>
<thead>
<tr>
<th>$AL$</th>
<th>$SM$</th>
<th>MD</th>
<th>LG</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM</td>
<td>EX</td>
<td>GD</td>
<td>GD</td>
</tr>
<tr>
<td>MD</td>
<td>GD</td>
<td>MD</td>
<td>PR</td>
</tr>
<tr>
<td>LG</td>
<td>MD</td>
<td>PR</td>
<td>VP</td>
</tr>
</tbody>
</table>

(b) $CO$ and $PO$

<table>
<thead>
<tr>
<th>$PO$</th>
<th>$SM$</th>
<th>MD</th>
<th>LG</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM</td>
<td>EX</td>
<td>GD</td>
<td>MD</td>
</tr>
<tr>
<td>MD</td>
<td>GD</td>
<td>MD</td>
<td>PR</td>
</tr>
<tr>
<td>LG</td>
<td>MD</td>
<td>PR</td>
<td>VP</td>
</tr>
</tbody>
</table>

The complete rule-base can be considered to comprise the three sets of rules related to vision data, cutter-load data, and servomotor offset data.

### 5.1.3 Defuzzification

Defuzzification is used to transform the fused fuzzy output (product quality index) into a crisp numerical value. The defuzzification method employed is based on the centroid method. For each fuzzy output decision, the crisp value is calculated using the formula:

\[ \text{Crisp Value} = \frac{\sum_{i} \text{Fuzzy Value}_i \times \text{Membership}_i}{\sum_{i} \text{Membership}_i} \]
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\[
PQI = \left[ \frac{\sum_{i=1}^{n} \mu_i(PQI)(PQI)_{ci}}{\sum_{i=1}^{n} \mu_i(PQI)} \right]
\]

(5.8)

where \( n \) is the number of rules that are fired for a given input, \( \mu_i(PQI) \) is the output membership function from the \( i \)th rule fired, and \( (PQI)_{ci} \) is the centroid of the fired \( i \)th output membership function.

5.2 Case Studies

Three cases of fish-cut are tested now to investigate the performance of the implemented fusion methods. Table 5.3 gives the crisp input parameters and the fuzzy sets which they belong to, for the three representative cases considered; namely, good cut, bad cut, and conflicting-data cut. In the case of the good and the bad cuts most of the sensors agree. However, a case with conflicting-data has been selected to show the efficacy of the methods in dealing with disagreeing sensors.

Table 5.3 Input parameter values for the example cases.

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Good Cut</th>
<th>Bad Cut</th>
<th>Conflicting-Data Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Crisp Values</td>
<td>Fuzzy Sets</td>
<td>Crisp Values</td>
</tr>
<tr>
<td>( D_c )</td>
<td>0.638</td>
<td>GD</td>
<td>0.384</td>
</tr>
<tr>
<td>( S_c )</td>
<td>0.697</td>
<td>GD</td>
<td>0.550</td>
</tr>
<tr>
<td>( S_r )</td>
<td>0.730</td>
<td>GD</td>
<td>0.440</td>
</tr>
<tr>
<td>( AL )</td>
<td>99</td>
<td>MD, LG</td>
<td>149</td>
</tr>
<tr>
<td>( AI )</td>
<td>17.63</td>
<td>SM, MD</td>
<td>21.90</td>
</tr>
<tr>
<td>( CO )</td>
<td>0.1</td>
<td>SM</td>
<td>1.1</td>
</tr>
<tr>
<td>( PO )</td>
<td>0.1</td>
<td>SM</td>
<td>2.0</td>
</tr>
</tbody>
</table>

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5.3 Fusion by Mamdani Max -Prod Composition

The fuzzy data fusion method based on Mamdani prod-max composition method has been applied to the three cases of fish cut. For this MATLAB's Fuzzy Logic Toolbox [51] with prod implication, max aggregation, and centroid defuzzification is employed. For any given input comprising of seven numerical values, depending on the fuzzy sets, which they belong to, different rules will fire. Depending on the membership grades of the input fuzzy sets the output fuzzy set is scaled, and all such output fuzzy sets are aggregated, which on defuzzification provide a crisp numerical output i.e., product quality index. Figure 5.13, Figure 5.14, and Figure 5.15 represent the product quality indices for the good, the bad, and the conflicting-data cut cases, respectively. These figures show all the 45 rules and the contribution of those that fire; the aggregation of output sets; and finally the defuzzified result which is the product quality index.
$D_c = 0.638 \quad S_c = 0.697 \quad S_r = 0.73 \quad AL = 99 \quad AI = 17.63 \quad CO = 0.1 \quad PO = 0.1 \quad PQI = 6.72$

Figure 5.13 Product quality index result by max-product composition, for the case of good cut.
Figure 5.14 Product quality index result by max-product composition, for the case of bad cut.
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Table 5.4 shows the product quality indices for the three cases of fish cut obtained by the data fusion method that uses max-prod composition.

Table 5.4 Product quality index results by max-product composition.

<table>
<thead>
<tr>
<th>Fusion Method</th>
<th>PQI Good Cut</th>
<th>PQI Bad Cut</th>
<th>PQI Conflicting-Data Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max – Prod Composition</td>
<td>6.72</td>
<td>2.51</td>
<td>3.99</td>
</tr>
</tbody>
</table>

$D_c = 0.653 \ S_c = 0.697 \ S_r = 0.73 \ \ AL = 38.4 \ AI = 42.4 \ CO = 0.75 \ PO = 1.9 \ PQI = 5.99$

Figure 5.15 Product quality index result by max-product composition, for the case of conflicting-data cut.
5.4 Fusion by Degree of Certainty Method

In the present application, there are three sets of rules within the rule-base. The first set comprises 27 rules, which relate the three input parameters from the CCD cameras, specifically, Cut_Depth ($D_c$), Cut_Contour ($S_c$) and Cut_Surface ($S_r$), to the Product_Quality_Index ($PQI$). Each rule that is fired, for a given input, provides an output ($PQI$), with a support level $\mu(PQI)$. The following fusion formula combines the degrees of support $\mu(PQI)$ of the outputs corresponding to all the fuzzy input values ($D_c$, $S_c$ and $S_r$) that make up the fired rules:

$$
\mu_i(PQI) = e + \frac{1}{3} \left[ (\mu_i(D_c) - e)^3 + (\mu_i(S_c) - e)^3 + (\mu_i(S_r) - e)^3 \right]^{\frac{1}{3}}
$$

(5.9)

where $i = 1, 2, \ldots, 27$.

For the second set of 9 rules corresponding to the cutter load data provided by the induction motors, there are two fuzzy inputs: Average_Load ($AL$) and Assymetry_Index ($AI$). The following formula combines the degrees of support $\mu(PQI)$ of the outputs of the fired rules:

$$
\mu_i(PQI) = e + \frac{1}{2} \left[ (\mu_i(AL) - e)^3 + (\mu_i(AI) - e)^3 \right]^{\frac{1}{3}}
$$

(5.10)

where $i = 1, 2, \ldots, 9$. Similarly, for the third set of 9 rules corresponding to the cutter and platform servo, there are two fuzzy inputs: Cutter_Offset ($CO$) and Platform_Offset ($PO$). The following formula combines the degrees of support $\mu(PQI)$ of outputs of the fired rules:

$$
\mu_i(PQI) = e + \frac{1}{2} \left[ (\mu_i(CO) - e)^3 + (\mu_i(PO) - e)^3 \right]^{\frac{1}{3}}
$$

(5.11)
where $i = 1, 2, \ldots, 9$. By adding the Equations (5.9), (5.10), and (5.11) the cumulative degree of support of the fused output $\mu(PQI)$ is obtained. The corresponding crisp output is determined by the following defuzzification method:

$$PQI = \left[ \frac{\sum_{i=1}^{n} \mu_i(PQI)(PQI)_{ci}}{\sum_{i=1}^{n} \mu_i(PQI)} \right]$$

(5.12)

where $n$ is the number of rules that are fired for a given input, $\mu_i(PQI)$ is the membership function of the $i^{th}$ fired output, and $(PQI)_{ci}$ is the centroid of the fuzzy output from the $i^{th}$ rule that is fired.

Table 5.5 shows for the good cut the input parameters, their membership values and the fuzzy sets which they belong to.

Table 5.5 Input parameters and their membership values for the case of good cut.

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Crisp Values</th>
<th>Membership Values</th>
<th>Fuzzy Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_c$</td>
<td>0.638</td>
<td>1.0</td>
<td>GD</td>
</tr>
<tr>
<td>$S_c$</td>
<td>0.697</td>
<td>1.0</td>
<td>GD</td>
</tr>
<tr>
<td>$S_r$</td>
<td>0.730</td>
<td>1.0</td>
<td>GD</td>
</tr>
<tr>
<td>$AL$</td>
<td>99</td>
<td>0.5, 0.5</td>
<td>MD, LG</td>
</tr>
<tr>
<td>$AI$</td>
<td>17.63</td>
<td>0.5, 0.5</td>
<td>SM, MD</td>
</tr>
<tr>
<td>$CO$</td>
<td>0.1</td>
<td>1.0</td>
<td>SM</td>
</tr>
<tr>
<td>$PO$</td>
<td>0.1</td>
<td>1.0</td>
<td>SM</td>
</tr>
</tbody>
</table>
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The sample calculations for the product quality index in the case of good cut are shown below.

For vision data, all three parameters belong to the fuzzy set Good (GD) with the membership values 1.0. Therefore only one rule fires whose output is the fuzzy set Excellent (EX):

1. If (Dc is GD) and (Sc is GD) and (Sr is GD) then (PQI is EX)

Accordingly,

\[ \mu_1(PQI) = 0.5 \left[ \frac{(1 - 0.5)^3 + (1 - 0.5)^3 + (1 - 0.5)^3}{3} \right] = 1.0, \text{ and centroid } PQI_{c1} = 8.9 \]

For cutter load data, the input parameters Average_Load and Asymmetry_Index belong to the fuzzy sets (Medium, Large) and (Small, Large) respectively, with the membership values 0.5. Hence, the following four rules fire for the cutter load data:

2. If (AL is MD) and (AI is SM) then (PQI is GD)
3. If (AL is MD) and (AI is MD) then (PQI is MD)
4. If (AL is LG) and (AI is SM) then (PQI is MD)
5. If (AL is LG) and (AI is MD) then (PQI is PR)

For these rules PQI is obtained as follows:

\[ \mu_2(PQI) = 0.5 \left[ \frac{(0.5 - 0.5)^3 + (0.5 - 0.5)^3}{2} \right] = 0.5, \text{ and centroid } PQI_{c2} = 7 \]
\[ \mu_3(PQI) = 0.5 \left[ \frac{(0.5 - 0.5)^3 + (0.5 - 0.5)^3}{2} \right] = 0.5, \text{ and centroid } PQI_{c3} = 5 \]
\[ \mu_4(PQI) = 0.5 \left[ \frac{(0.5 - 0.5)^3 + (0.5 - 0.5)^3}{2} \right] = 0.5, \text{ and centroid } PQI_{c4} = 5 \]
\[ \mu_5(PQI) = 0.5 \left[ \frac{(0.5 - 0.5)^3 + (0.5 - 0.5)^3}{2} \right] = 0.5, \text{ and centroid } PQI_{c5} = 3 \]
Chapter 5. Implementation in a fishcutting machine.

For Cutter and Platform offsets, the following rule fires:

If (Cutter_Offset is GD) and (Platform_offset is GD) then (PQI is EX)

Hence, we have

$$\mu_s(PQI) = 0.5 + \left[ \frac{(1 - 0.5)^3 + (1 - 0.5)^3}{2} \right] = 1.0, \text{ and centroid } PQI_{c6} = 8.9$$

The overall product quality index for the good cut is given by

$$PQI_{\text{Good Cut}} = \frac{\sum_{i=1}^{6} \mu_i(PQI)(PQI)_{ci}}{\sum_{i=1}^{6} \mu_i(PQI)} = 7.0$$

Table 5.6 shows the product quality indices for the three cases of fish cut, obtained by the data fusion method based on degree of certainty:

<table>
<thead>
<tr>
<th>Fusion Method</th>
<th>PQI Good Cut</th>
<th>PQI Bad Cut</th>
<th>PQI Conflicting Data Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degree of Certainty</td>
<td>7.0</td>
<td>3.52</td>
<td>6.40</td>
</tr>
</tbody>
</table>

5.5 Fusion based on Compatibility Function Method

By applying this method, for any given input we get three crisp numerical values of output—the product quality index, using the three set of rules corresponding to vision, cutter-load, and offset data. These outputs are obtained using the Mamdani’s max-prod composition. The resulting three product quality indices are fused based on the compatibility function. The variance depends on the uncertainties in the measured values, which arise from the sensor errors, and the data processing approximations. The variance also depends on the level of confidence in the accuracy, precision, and repeatability of the particular sensor.
Following steps are followed when applying the data fusion method based on compatibility function:

- Fuzzify the inputs and decide their support length based on variance of the input parameters
- Develop the compatibility function
- Fuse the fuzzy sets using the fusion method based on α-level sets (see section 4.4)

This method has been applied to the three cases of fish cut. Sample calculations for the conflicting-data cut are presented now.

Figure 5.16 shows the $PQI$'s corresponding to vision, cutter-load and servomotor offset data.

<table>
<thead>
<tr>
<th>Compatibility Function</th>
<th>Servomotor Offsets</th>
<th>Cutter Load</th>
<th>Vision</th>
</tr>
</thead>
<tbody>
<tr>
<td>$PQI$</td>
<td>3.1</td>
<td>7.0</td>
<td>8.9</td>
</tr>
<tr>
<td>Support Length</td>
<td>2.0</td>
<td>4.0</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Figure 5.16 Product quality index results from the vision, cutter-load, and offset data.

Table 5.7 indicates the product quality index obtained from the offset, vision and cutter-load data. Also the variances, which represent the support length, are indicated. The variances for the offset, cutter-load, and vision data have been chosen as 2, 4, and 3 units, respectively.

Table 5.7 Product quality index results from offset, cutter-loaded, and vision data, and their variances.
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In Figure 5.17 $A_1$, $A_2$, and $A_3$ indicate the triangular fuzzy sets $PQI$, corresponding to the servomotor offset, cutter-load, and the vision data, respectively.

![Figure 5.17 Product quality index result for the case of conflicting-data cut.](image)

The PQI values can exist in the range of [1.1 to 8.9]; from very poor to excellent, and for this range the compatibility function developed for the fusion of three PQIs is:

$$
R(x,y) = \begin{cases} 
0 & \text{if } |x-y| > 5, \\
\frac{1}{5}|x-y| & \text{if } |x-y| \leq 5.
\end{cases}
$$

(5.13)

This compatibility function indicates that any two observations that are more than 5 units apart are assigned zero membership grades, and those less than five units apart are assigned a membership grade that is inversely proportional to the distance.

The average fusion function is applied to obtain the fused fuzzy set $B$:

$$F(A_1 + A_2 + A_3) = \frac{A_1 \oplus A_2 \oplus A_3}{3} = B$$

(5.14)

For any level $\alpha$, the $\alpha$-level set associated with the fuzzy set $A_i$ is denoted by

$$A_{i\alpha} = [a_{i\alpha}, b_{i\alpha}]$$

(5.15)

We have,

$$A_{1\alpha} = [2.1 + \alpha, 4.1 - \alpha], \quad A_{2\alpha} = [5 + 2\alpha, 9 - 2\alpha], \quad A_{3\alpha} = [7.4 + 1.5\alpha, 10.4 - 1.5\alpha]$$
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\[ \{a_{ia}\} = \{2.1+\alpha,5+2\alpha,7.4+1.5\alpha\} \quad \text{and} \quad \{b_{ia}\} = \{4.1-\alpha,9-2\alpha,10.4-1.5\alpha\} \]

\[ a^*_a = \max_i\{a_{ia}\} = 7.4 + 1.5\alpha, \quad \text{and} \quad b^*_a = \min_i\{b_{ia}\} = 4.1 - \alpha \]

where \( a^*_a \) is the largest lower bound of any \( \alpha \)-level set interval and \( b^*_a \) is the smallest upper bound of any \( \alpha \)-level set interval.

\[ U^*_a = 1 - \frac{1}{5}|a^*_a - x| = \alpha \quad \text{and} \quad V^*_a = 1 - \frac{1}{5}|x - b^*_a| = \alpha \]

\[ \Rightarrow U^*_a = 6.5\alpha + 2.4 \quad \text{and} \quad \Rightarrow V^*_a = 9.1 - 6\alpha \]

where \( U^*_a \) is the smallest value that is within \( \alpha \) the compatibility level of \( a^*_a \), and \( V^*_a \) is the largest value that is within the compatibility level of \( b^*_a \).

Now for an aggregate \( \alpha \)-level set \( B \) to exist, we must have

\[ U^*_a \leq b^*_a \Rightarrow \alpha \leq 0.2 \quad \text{and} \quad V^*_a \geq a^*_a \Rightarrow \alpha \leq 0.2 \]

Now for \( \alpha \leq 0.2 \)

\[ a_{ia} < U^*_a \Rightarrow g_{ia} = U^*_a = 2.4 + 6.5\alpha \]

\[ a_{2a} > U^*_a \Rightarrow g_{2a} = a_{2a} = 5 + 2\alpha \]

\[ a_{3a} > U^*_a \Rightarrow g_{3a} = a_{3a} = 7.4 + 1.5\alpha \]

Then,

\[ d_\alpha = \frac{1}{3} \sum_{i=1}^{3} g_{ia} = 4.9 + 3.3\alpha \quad (5.16) \]

Also,

\[ b_{ia} < V^*_a \Rightarrow h_{ia} = b_{ia} = 4 - \alpha \]

\[ b_{2a} > V^*_a \Rightarrow h_{2a} = V^*_a = 9.1 - 6\alpha \]

\[ b_{3a} > V^*_a \Rightarrow h_{3a} = V^*_a = 9.1 - 6\alpha \]
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Hence,

\[ e_\alpha = \frac{1}{3} \sum_{i=1}^{3} h_{i\alpha} = 7.4 + 4.3\alpha \]  

(5.17)

Thus, aggregated set \( B = [d_\alpha, e_\alpha] = [4.9 + 3.3\alpha, 7.4 + 4.3\alpha] \).

Figure 5.17 (shown again) indicates the fuzzy output set \( B \), which represents final \( PQI \).

This fuzzy set is truncated by the level of conflict line \( \alpha = 0.2 \). The centroid defuzzification of fuzzy set \( B \) provides the final \( PQI \) as 5.4.

Table 5.8 shows the product quality indices for the three cases of fish cut, obtained by the fusion method based on compatibility function.

Table 5.8 Product quality index results by the compatibility function method.

<table>
<thead>
<tr>
<th>Fusion Method</th>
<th>PQI Good Cut</th>
<th>PQI Bad Cut</th>
<th>PQI Conflicting-Data Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compatibility Function</td>
<td>7.6</td>
<td>3.01</td>
<td>5.40</td>
</tr>
</tbody>
</table>
5.6 Comparative Analysis of Results

Table 5.9 summarizes the results for the product quality indices their membership values as obtained by the three fuzzy data fusion methods for three example cases of fish cut.

Table 5.9 PQI and level of confidence results for three cases by the three fusion methods.

<table>
<thead>
<tr>
<th>Fusion Method Based on</th>
<th>PQI/μ for Good Cut</th>
<th>PQI/μ for Bad Cut</th>
<th>PQI/μ for Conflicting-Data Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max – Prod Composition</td>
<td>6.72/0.86</td>
<td>2.51/0.75</td>
<td>5.99/0.5</td>
</tr>
<tr>
<td>Degree of Certainty</td>
<td>7.0/1.0</td>
<td>3.52/0.85</td>
<td>6.40/0.7</td>
</tr>
<tr>
<td>Compatibility Function</td>
<td>7.6/0.8</td>
<td>3.0/1.0</td>
<td>5.40/0.4</td>
</tr>
<tr>
<td>Level of Confidence</td>
<td>68%</td>
<td>54%</td>
<td>21%</td>
</tr>
</tbody>
</table>

Figure 5.18 Legend for line types used for data fusion methods.

Figure 5.19 Product quality index results obtained by the three fusion methods for the case of good cut.

The results for the case good cut as shown in Figure 5.19 do not indicate much difference (i.e., less than 10%) in the defuzzified value of product quality index, for the three methods. The results in terms of certainty; specifically crispness, indicate that the best results, in the descending order, are provided by the degree of certainty method, the
compatibility function method, and the max-product method. The level of conflict in the fused output by the compatibility function method is 68%, and it indicates the contradiction that is present in the data sources.

Figure 5.20 Product quality index results obtained by the three fusion methods for the case of bad cut.

The results for the case of bad cut as shown in Figure 5.20 do not indicate much difference (i.e., less than 10%) in the defuzzified values of product quality index, for the three methods. The results in terms of certainty; specifically, crispness, indicate that the best results, in the descending order, are provided by the compatibility function method, the degree of certainty method, and the max-product method. The level of confidence in the fused output determined by the compatibility function method is 54%, and it points to the level compatibility of the data sources.

Figure 5.21 Product quality index results obtained by the three fusion methods for the case of conflicting-data cut.

The results for the case of conflicting data as shown in Figure 5.21 do not indicate much difference (i.e., less than 10%) in the defuzzified values of product quality index,
for the three methods. The results in terms of certainty; specifically, crispness, indicate that the best results, in the descending order are provided by the degree of certainty method, the compatibility function method, and the max-product method. The level of conflict in the fused output determined by the compatibility function method is 21%, and it clearly points to the contradiction that exists in the data sources.

The three fuzzy sensor fusion methods in terms of the computational load (required computational effort) appear to follow the descending order: max-product method, degree of certainty method, and compatibility function method. Note that no direct performance evaluation has been performed, and the results are based on the number of operations required by each method in computing the output, which is the product quality index. In the present case and on current processors, these differences do not make a substantial difference; however, for a complex industrial plant with a much larger knowledge-base, these observations have important performance implications.

Analysis of the membership values clearly indicates that the results obtained by the degree of certainty method are more "certain" for the good and the conflicting-data cut cases. Whether these results are than accurate than those from max-prod method can only be determined if the quality data are available a priori for a set of test measurements, using for example, the opinion of fish cutting experts.

The results in terms of the product quality index reflect the accuracy of the system. In the present application of fish cutting, however, since no two fish are similar, the fish cutting process is not easily repeatable, and the quality assessment parameters are subjective. Hence, it is not easy to a claim high degree of confidence in the achieved accuracy. The sensor data has been achieved on the basis of a single cut at a time, rather than processing batches of fish. Another challenge is to identify the sensor that should be trusted most in the case of conflicting data, and this issue becomes even more challenging, as in the present example, where the fusion sensor data are not redundant, but rather complementary.
Chapter 6

6 Conclusion

6.1 Overview of the Thesis

This thesis investigated knowledge-based fuzzy sensor fusion methods that reliably fuse redundant as well as complementary data. The fuzzy sensor fusion methods are useful when the errors and uncertainties associated with the sensors originate and are assessed subjectively rather than based on precise statistics. These methods are particularly suitable when the plant/process is complex, incompletely known, and difficult to model either analytically or experimentally. In this thesis, the uncertainty and error in sensor data are considered to be both subjective and qualitative and hence related to the concept of fuzziness. The thesis specifically considered the implementation of fuzzy sensor fusion methods for the quality assessment of fish as processed by an industrial fishcutting machine developed in the Industrial Automation Laboratory, University of British Columbia. Three appropriate methods of multiple-sensor fusion using fuzzy logic were adopted, implemented, and evaluated in the present work. The first sensor fusion method is based on Mamdani's max-prod composition. In this method equal weights are placed on all the sources, without considering their merit or importance. The second sensor fusion method is based on degree of certainty. It assigns weights proportional to the degree of certainty of sensor data, and in addition to the fused output, it provides the information about the certainty of the output. The third sensor fusion method is based on the concept of compatibility of data. It provides a fused output and
additional knowledge about the degree of confidence in the fused output. Thus fusion is expected to improve the reliability of the sensor information and hence the knowledge of the system. In particular, when sensors are in agreement, the uncertainty of the fused result is reduced, and the resulting knowledge is augmented. However, if the sensors are in disagreement, then there exists the possibility of sensor faults, uncertainties, incompleteness, and other errors, which increase the uncertainty of fused output. The sensor fusion methods can fuse data that are imprecise, incomplete, or even conflicting, to provide meaningful information required for decision-making related to the process. Such decisions are useful, for example, in assigning quality grades to products, assessing the performance of a process, and in feedback control, both direct and supervisory.

6.2 Major Contributions

The major contributions, which this thesis has made, are the following.

- Investigation of various sensor fusion methods for quality assessment in performance of an industrial process.
- Adoption of three methods of knowledge-based sensor fusion, and development of a method based on the compatibility of measured data, with the capability to fuse the conflicting complementary data.
- Implementation of the three sensor fusion methods for assessment of the quality of fish processed by an industrial fishcutting machine.
- Investigation of the possible application to other industrial processes.

6.3 Directions for Future Work

The work in the present thesis has provided a reliable fusion methodology for practical applications. However, the larger and more diverse area of complementary fusion requires further attention. Much of the work in this thesis has been specific to the application of product quality assessment. A sufficiently general framework for
complementary data fusion would benefit both research and industrial application. The implementation of the fusion methods can be conveniently extended to tasks of inspection and quality assessment in CNC router machines and similar industrial machines. That extension to the present work is left as a future activity. The representation of uncertainty in domains other than fuzzy logic can be useful in the cases where crisp sensor measurements are directly employed in decision-making. A framework needs to be developed under which the additional knowledge generated by fusion, in terms of degree of certainty and level of confidence, can be exploited in decision-making. Fused output by any fuzzy sensor fusion method relies heavily on issues such as: number of variables considered and the associated number of fuzzy states used, shape of the membership functions, generation of the rule-base, and the defuzzification method. All these issues need further analytical study and practical evaluation to determine their effect on the fusion process and the fused output. Other aspects that require further exploration include validation of sensor data prior to fusion, incorporation of the system diagnosis and fault-detection within a robust fusion system.
7 References


References


References


