

TWO FACES OF PRIVATIZATION

by

Ivan Alves

S.B., Massachusetts Institute of Technology, 1991

M.A., University of British Columbia, 1992

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

in

THE FACULTY OF GRADUATE STUDIES

(Department of Economics)

We accept this thesis as conforming
to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

August 2001

©Ivan Alves, 2001

In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the head of my department or by his or her representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of Economics

The University of British Columbia
Vancouver, Canada

Date 27 July 2001

Abstract

Few countries in Latin America, and the developing world in general, have not implemented some type of economic reform, with profound economic consequences. Potential income effects for large social groups have been deemed insignificant when set against the expected overall efficiency improvements. This dissertation presents two models addressing important potential shortcomings of recently implemented reforms: they question standard assumptions about the positive underlying economic processes and characterize some conditions where reform may have perverse effects. In particular, the first paper argues that capitalization may serve as a vehicle for increasing the consumption of a self-serving political class that improves the lot of some income groups at the expense of aggregate welfare, while the second suggests that the sale of large public utilities may result, depending on the resulting ownership distribution, in inefficient production plans.

The first chapter presents a variation of the infinite horizon model where we introduce the *weight* of private and public capital in aggregate capital, an input for the production of the consumption good. Capitalization, by decreasing the state's weight in aggregate capital, serves as an important vehicle altering the income of various groups, most importantly that of the political class. In particular, it is shown that the capitalization package may be Pareto inferior at the steady state, despite creating benefits for both the capitalist and the political classes. We illustrate the possibility of policy being influenced by the evolution of other variables in the model and show that the experience with reform typical of developing countries can be well explained in a context of a rational political class, shifting between coalitions in alternative steady-states.

The second chapter develops a model where the price setting behavior of a privatized

monopoly is determined endogenously by shareholders. The population of owners, who are also consumers, and perhaps workers, is heterogenous with respect to their autonomous wealth, their participation as workers in the production of the monopoly good and their share of the monopoly's rent. The model reveals the sensitivity of the post-privatization pricing behavior of the firm to the distribution of ownership that results from privatization. In particular, policy makers, by determining the concentration and distribution of ownership, gain some freedom in choosing a given policy objective, such as efficient pricing. In this context, we look at the impact of alternative policy objectives of the central planner on the post-divestiture pricing behavior of the monopoly and find out that well-intended objectives may have perverse outcomes. We also contrast the freedom available to the central planner under alternative divestiture programs, and suggest that whereas auction places a considerably lower information demand on the central planner, it reduces significantly the planner's policy freedom. However, there remains the caveat that freeing the trade of shares is likely to result in renewed concentration and more inefficient production plans.

Contents

Abstract	ii
List of Figures	ix
Acknowledgements	x
Dedication	xi
1 Introduction	1
1.1 Historical Background	1
1.1.1 The Bolivian Experience	4
1.2 Theory and Key Issues in Reform	6
1.2.1 Why reform?	8
1.2.2 How much to reform?	9
1.2.3 The Impact of Reform	11
1.3 The Two Issues Covered	13
1.3.1 Privatization and Policy Equilibria	15
1.3.2 Divesting of a Monopoly	17
2 Privatization, its Long-term Effect on Income and Policy Equilibria	20
2.1 Introduction	20

2.1.1	General Background	20
2.1.2	Our Model of Privatization	23
2.1.3	The paper's sections	25
2.2	The Basic Model	27
2.2.1	The Agents	28
	The politician	29
	The entrepreneur	31
	The worker	31
2.2.2	Production	32
2.3	Solving the Model	35
2.3.1	Optimization Solutions	36
	Representative Politician	36
	Representative Entrepreneur	36
	Aggregate Firm	36
2.3.2	Market Clearing	37
2.3.3	Equilibrium	38
2.3.4	Steady State Equilibrium	38
2.4	Comparative Statics on the Steady State	41
2.4.1	Privatization	42
	Capital Stocks	43
	Output	43
	Consumption	44
	Worker's Consumption	45

	Entrepreneur's Consumption	46
	Politician's Consumption	47
2.4.2	Fiscal Policy	50
	Capital Stocks	50
	Output	51
	Consumption	51
	The Worker	51
	The Entrepreneur	51
	The Politician	51
2.5	Policy Implications	53
2.5.1	Coalitions	54
2.5.2	Transfer mechanisms	57
	A transfer from the entrepreneur to the politician	57
	A transfer between the politician and the worker	59
2.5.3	Output-decreasing Coalitions	61
2.5.4	Choosing a Tax	63
2.5.5	A Historical Interpretation	66
2.6	Concluding Remarks	69
3	Privatizing a State Monopoly	71
3.1	Introduction	71
3.1.1	Conceptual issues	72
3.1.2	Why share privatization is important	74
3.1.3	How to divest?	76

3.1.4	Outline	77
3.2	The Model	77
3.2.1	The Agents	78
3.2.2	The Labour Market	80
3.2.3	The Monopoly	81
3.3	Production Plans	84
3.3.1	The Pure Monopoly Plan	85
3.3.2	The Consumer as Manager Plan	85
	Existence and Uniqueness of the Private Plan	87
	Properties of the Private Plan	90
3.4	Voting for a Production Plan	96
3.4.1	A concept of equilibrium: the median share	98
3.4.2	On the importance of who decides	100
3.5	Privatization Schemes	102
3.5.1	Distribution of Shares	104
	Maximization of aggregate utility	105
	Maximization of the Minimum Utility	108
3.5.2	Auction of Shares	112
	Risk Neutrality and a Proportional Allocation Auction	113
	Risk Aversion and a Random Allocation Auction	117
3.6	Trading of Shares	121
3.7	Concluding Remarks	126

A	Appendix to Chapter 3	128
----------	------------------------------	------------

A.1	Proofs	128
A.1.1	Monopoly Production	128
A.1.2	Properties of the Private Plan	130
A.1.3	Voting	131
A.2	CES Preferences	132
A.3	The Aggregated Space \mathcal{J}_k and the Translation of Measures	136
A.3.1	The Aggregated Space	137
A.3.2	Measures in the Quotient Space	139
A.4	Auxiliary Results	141
A.4.1	Product of Concave Functions	141
A.4.2	Concave and a Convex Functions	142

Bibliography	143
---------------------	------------

List of Figures

2.1	Output as a function of market orientation and fiscal policy $\{\theta, \tau\}$	40
2.2	Worker's consumption as a function of the policy pair $\{\theta, \tau\}$	46
2.3	Entrepreneur's consumption response for policy profiles $\{\theta, \tau\}$	47
2.4	Politician's consumption for policy profiles $\{\theta, \tau\}$	49
2.5	The politician's preferred policy directions	52
2.6	Worker and politician policy support profiles $\{\theta, \tau\}$	55
2.7	Capitalization (Υ_φ) more likely than privatization (Υ_0	58
2.8	Effect of transfer between the politician and worker on the privatization- support sets	60
2.9	Politician-entrepreneur output-decreasing coalition is possible ($\Upsilon \cap \Gamma \cap \Psi$) .	63
2.10	Capital tax maximizing the politician's consumption for two different depre- ciation rates	66
2.11	Existence of the set $\Upsilon \cap \Gamma \cap \Psi$ with tax being set by the politician	67
3.1	Mapping between the preferred price $p(\cdot, w)$ and the characteristic function $k(\cdot, w)$ for Cobb-Douglas-type preferences, with parameters $\alpha = .48$, $\lambda = 1.5$, $w = 0.5$, and normally distributed (with different means) shares.	96

Acknowledgements

I much appreciate the feedback and useful discussions with Hugh Neary and Francisco Gonzales, as well as the rigour and patience of Barbara and Crtomir Zupančič. The content of the paper gained much from their support and guidance. Any remaining errors are the responsibility of the author.

For my family

Chapter 1

Introduction

1.1 Historical Background

Four decades of state intervention characterize the post-war period of many countries in Latin America. One of the key elements of the political program during these years was the nationalization of important economic sectors, such as oil production, telecommunications or air transport.¹ This strong presence of the state in a variety of economic activities was motivated by a number of factors including significant developments in the global political economy and/or equity considerations.

However, a marked process of *policy reversal* can be evidenced in the last two decades. The number of countries selling state-owned enterprises (SOEs) increased in the 1980s, as divestiture spread from industrial countries to developing countries throughout the world. By the beginning of the nineties the reversal was dramatic: the majority of Latin American countries privatized a large number of their state-owned enterprises (SOEs), establishing privatization as the dominant policy instrument. According to World Bank (1995), there

¹As late as the early eighties, the banking sector in Mexico, for instance, was nationalized.

were almost five times as many transactions in Latin America during 1988-1993 than in 1980-87, with a value of divestiture transactions representing 57% of the worldwide value.²

Such profound restructuring of economic relationships brought about by divestiture was to deliver a marked improvement in economic performance. The privatization process in emerging economies could in this context embody some revelation mechanism acting on policy makers who, disappointed with previous *dirigiste* policies of state-driven industrialization that had failed to deliver industrial development, turned to the private sector for the better management of public enterprises. Assisted by a period of re-structuring following the debt-crisis of the early 1980s, and with the goal of bringing economic benefits to the under privileged, politicians presumably found the appropriate conditions and objective for a change in policy direction.

However, this picture is barely, if at all, supported by the facts. First, the intervention of the state had long before failed to deliver on its initial promises (for many of the smaller countries in Latin America) or had succeeded admirably well in doing so (Brazil and Mexico stand out): reform did not result from a sudden realization of failure or from policy evolution. Second, the argument that a period of crisis could facilitate the overcoming of some form of political threshold does not hold in the case of many reforming countries. These could not implement privatization programs immediately following the crisis (Bolivia, Brazil) and others did not undergo an economic crisis before privatizing large sectors of their economies (Chile). Finally, reform packages, faithfully implemented by a series of administrations of diverse political orientation in a number of countries, failed to address *social* issues, a core component that would underlie the thesis of enlightenment, and imposed a significant cost

²The original source is Sader (1993).

on various income and social groups. Although reform has in many cases consolidated and achieved significant microeconomic improvements,³ it is not clear whether the transformation it brought about has served to improve the living standards of the population as a whole. More than a decade later, mediocre growth, increasing income disparities and persistent overall economic instability call into question the virtues of the change in policy.

Therefore, the jury is still out regarding the mechanism, economic or not, that brought about such radical reversal in policy orientation.

At the same time, these countries' experience with privatization has not permitted the formation of a consensus regarding the best method of divestiture once reform has been agreed upon. Whereas auctions have been preferred in more developed economies, smaller economies or economies with massive privatization programs have explored alternative methods, such as the issue of stocks among targeted groups in the population. Lack of developed financial markets and instruments to assess the worth of the public enterprises being divested, insufficient resources to undertake this work independently, and difficulties with the process' transparency forced the search for alternatives. Equally important in this search, however, was the motivation of shielding some classes from the rigor of the market or the objective of achieving greater efficiency of the post-privatization utilities.

One characteristic of the divestiture process stands out in this regard: by and large the type of SOEs being privatized are large utilities that are typically associated with natural monopolies. As state-driven industrialization meant that the public sector would take a pre-

³Recent empirical studies show evidence of better profitability, operating efficiency and capital investment spending; higher output and employment levels; and larger dividends following privatization. See for instance Bourbakri and Cosset (1998) for evidence from developing countries and Megginson, Nash, and van Randenborgh (1994) for a more general coverage.

dominant role in the provision of infrastructure, the ensuing process of divestiture involved the sale of large firms in non-competitive markets. Furthermore, as the state had traditionally played a central role in these economies, supervisory powers were only poorly developed, suggesting that a significant effort (and cost) would be invested in the design of appropriate incentive structures within which the post-privatization enterprises would operate. Accordingly, supervision bodies were created to address the future monitoring of divested utilities. Little thought was dedicated, however, to the study of the incentive structures that would follow from the particular privatization method chosen.

1.1.1 The Bolivian Experience

The author's close experience with Bolivia, one of the early reformer countries, is both an inspiration and a benchmark for the material presented below. We believe that Bolivia's experience with reform serves an interesting example to illustrate the issues that are also present, in another scale, in larger economies in the region.

Following widespread macroeconomic reform during 1985 and in the aftermath of the most serious economic crisis experienced by that country in this century, a privatization programme failed to materialize in what Cariaga (1996) describes as a failure to gain the necessary support from the various economic players (labour unions and some economic sectors weary of the management of the privatization process). Ten years later, however, six important public utilities had been divested under a program coined *capitalization*, an offspring from traditional privatization, introduced by a political party with a solid historical record favoring nationalist policies.⁴ In the decade that had passed, and through its mutation

⁴Capitalization is a particular instance of privatization where the state does not receive any payment for the public enterprise. That is, there is no purchase by the private entrepreneur. The government grants the

into a capitalization initiative, divestiture was supported and opposed by parties from all over the political spectrum, despite the initial opposition to it of many of them.⁵ In spite of the widespread support of the program and of it being characterized as a model for other privatization programs in the region, Bolivia's growth record has been poor, and the result on its income distribution have been calamitous.

The relatively long duration of the privatization process in Bolivia and its evolution points at a few issues. First, gathering sufficient support to implement microeconomic reform after macroeconomic cleansing takes a substantial amount of time. Second, privatization is not necessarily a policy supported by parties inclined to favor market-oriented policies. Third, the purported benefits in terms of high economic growth economic recovery and stability are not assured, and important additional elements may be missing. Finally, there is evidence suggesting that it was clear, not only to the policy makers involved in the process, but also to other interest groups, that the mechanism to be chosen for divesting the large state utilities would have an important role in gathering support for the program, and also for the consecutive performance of the divested utilities.

In light of the capitalization program in Bolivia, we investigate two aspects of reform that appear of central importance. Our *first thesis* is that divestiture was largely an outcome of

control of the firm and keeps ownership of a fraction thereof, thus establishing the capitalization of the firm.

⁵In 1985, the Paz Estenssoro party MNR -traditionally a revolutionary party- implemented the New Economic Plan (NEP). The main opposition parties, ADN -a right-wing party- and the MIR -a left wing party- strongly criticized the policy, presumably seeking the support of a median voter (See Schultz (1996) and references therein for more on median voter platforms). Despite winning the majority of votes in the 1989 election, the MNR lost power due to an alliance between the ADN and the MIR, who then *continued to pursue the NEP*. The MNR won the 1993 election and partly accomplished the capitalization program, yet again under strong opposition of the ADN-MIR alliance. Losing the 1997 election, the job was later partly finished by the ADN-MIR alliance.

a political, rather than economic, transition. Many of the elements of the capitalization program, of the events that surrounded its implementation and the dynamics of support and opposition, indicate that this form of privatization was the outcome of an important political transition. While political, the elements that underlie the convergence of interest of the different political actors have a core economic nature and a transition mechanism that can be illustrated with standard economic models. Our *second thesis*, based on the fact that capitalization involved primarily the divestiture of large national monopolies, is that the distribution of ownership (both in terms of the form of divestiture and in terms of the post-privatization distribution) may have far-reaching consequences to the welfare of groups in the population these programs were meant to target.

1.2 Theory and Key Issues in Reform

Parallel to the policy experience in the region and partly motivated by it, ideas in the field of reform evolved from being centered on the economics of the “how to” undertake reform, towards greater focus on the political economic aspects of the process. The work of Williamson (1994) stands out in bringing together a comprehensive empirical evaluation, and in its attempt to characterize the main elements comprising a successful reform program. He finds a lack of support for the public choice theory depicting an “homo economicus” public servant concerned with some sort of utility-maximization which may involve interest in re-election. Evidence from over 12 countries covered in the study suggests that technopols are, by and large, the main promoters of reform often sacrificing political support for the

sake of economic efficiency.⁶

It is difficult not to wonder, however, about the origin and underlying elements of this, in Williamson's terms, *stampede* towards reform. Why have these technopols emerged, and how have they abandoned their grip on power? If indeed the contributors to the Williamson volume quite accurately and with much detail describe the conditions that make a reform programme successful, not much is said about how the consensus was reached to undertake reform. In this context, as mentioned above and important to keep in mind, is that, at least as far as Latin America is concerned, the previous three and a half decades were characterized by exactly the opposite process of nationalization. Therefore, the stampede is even more dramatic, as it involved the *reversal* of policies consistently implemented during a long period of time.

These issues are also present in Rodrik (1996, p. 10), who gives an economist's perspective on the political economy of reform where he discusses central puzzles in the growing field of policy reform. Two issues in his presentation are relevant in our context and are discussed in turn below. First, he enquires about *the origin* of reform or, in his own words, "... why are so many governments reforming now, after decades of adherence to policies of an opposite kind?". He then questions stylized facts about the *extent* of the implemented reforms: if reform in Latin America has been inspired by the success of the South East Asian economies, why has it by far exceeded the latter in the implementation of liberal economic policies as well as in the speed of adjustment?

⁶See Williamson (1994, p. 11) for the precise definition of "technopol". A technopol is a technocrat who assume positions of political responsibility.

1.2.1 Why reform?

On the first issue, Rodrik refutes the view that a preceding crisis is necessary for reform to be implemented.⁷ Instead he suggests that one of the principal mechanisms of reform is an underlying war of attrition between different classes, much along the lines of Alesina and Drazen (1991). In particular, the more uneven is the wealth distribution and the poorer the distributive track record of the government, the more likely is reform to be postponed. However, this argument leaves unexplained the degree of *simultaneity* in the implementation of reform by countries where income distribution patterns were likely to differ when privatization was decided upon. Even if wealth distribution were equal in all countries that implemented reform, it is unlikely that the war of attrition came to an end at the very same time in all of them. Furthermore, if income distribution were the true mechanism driving policy makers to reform, the ordering of implementation should reflect increasing income disparities: Bolivia and Mexico, the pioneers in the wave of reforms that swept through the region, should have a more homogeneous distribution of income, whereas Argentina and Brazil should have a more unequal one. This is not consistent with the facts, as made evident by distributive indicators published by Rodrik (1994).⁸ Instead, he suggests that the simultaneity is a consequence of the type of advice that governments in the region were given by international organizations, such as the World Bank and International Monetary Fund.

The official perspective of these organizations, however, as reflected by Kikeri, Nellis, and Shirley (1992, p. 32) suggest the opposite causation.⁹ According to their view, international

⁷Among the authors that implicitly or explicitly support the view that as state of crisis is a precondition for reform one finds Krueger (1993), Bresser-Pereira, Maravall, and Przeworski (1993) and de la Dehesa (1994).

⁸If they were, we would have expected reform to first occur in Argentina, followed by Brazil and Mexico.

⁹"The World Bank has long assisted its borrowers' efforts to improve the performance of SOEs [state

organizations adapted their position to a policy realignment of the incumbent governments, rather than the other way around. Admittedly, it is hard to believe that sovereign states would flock to the directives of a new international order dictated by the World Bank. Instead, a new order placing emphasis on the virtues of a market economy would have replaced by some mechanism the order of strong state intervention prevailing until then. Policy makers were driven to simultaneously implement new policies and were convinced that new order would be better than the freshly abandoned alternative.

One is lead to wonder about the nature of this mechanism driving the different governments to implement a previously demonized policy, *all at the same time*. It is even more difficult to understand the *degree* of support for policies with clear distribution effects in the context of clear class differences and obvious beneficiaries and losers.

1.2.2 How much to reform?

The *extent* of reform implementation is Rodrik's second puzzle, as he finds no concrete mechanism to explain the rapid and extreme process of microeconomic reform. He suggests that influential interest groups were compelled to go along with the implementation of simultaneous macroeconomic and microeconomic reform because a consensus on macroeconomic reform had been reached and the two very different types of reform were offered as a package.¹⁰ But why would influential interest groups agree to microeconomic policy that had obvious and significant consequences to the distribution of income once they already owned enterprises] without changing ownership, ... But changing perceptions and attitudes in member governments, combined with difficulties in sustaining SOE reforms and the growing body of evidence on the benefits of privatization, have created opportunities for the World Bank Group to support privatization."

¹⁰Hence reform entails costs to agents at the microeconomic level which ought to be compensated by the gains derived from macroeconomic stability.

tained the benefits of macroeconomic reform? Indeed, in Bolivia and Brazil microeconomic reform was proposed and implemented *well after* the macroeconomic stabilization package had already consolidated. In general, considering other reform experiences in the region, one finds that microeconomic adjustment generally follows macroeconomic reform by a significant margin of time. As Rodrik remarks, there is insufficient evidence suggesting that the implemented microeconomic adjustment was at all necessary for the success of the overall reform package. He finds it surprising that, given that reform was largely motivated by the success of the Asian tigers, so many countries, specially in the region of Latin America, have implemented the totality of the Washington consensus policy recommendations within such a short period of time.¹¹ In the context prior to the 1997 Asian crisis, Rodrik (1996, p.18) states that

Mexico, Bolivia, and Argentina, to cite some of the more distinguished examples, have undertaken more trade and financial liberalization and capitalization within five years than the East Asian countries have managed in three decades.

An interesting and often striking characteristic of reform is the widespread political support for it, even by political parties opposed to its implementation in the electoral campaign. Cuckierman and Tommasi (1998) characterize the electoral mechanism underlying the observed comparative advantage of politicians from the left for implementing market reforms.¹²

¹¹A key element in Rodrik's assessment is the so called *Washington Consensus*, a policy desiderata first introduced by Williamson (1994). Rodrik's conclusion on whether the different elements of the Washington consensus played an important role in the success of the development policies of South Korea and Taiwan is unambiguous: only policies aimed at macroeconomic stability were implemented in both, while keeping -contrary to orthodox policy recommendation- microeconomic intervention to a minimum.

¹²In the case of Bolivia, this characterizes well the experience of the initially left-wing party which, immediately after the electoral victory, first implemented the reform program of which capitalization was one of

There appears to be something intrinsically attractive to insiders in continuing with reform independently of the electoral platform of their electoral victory. This is in stark contradiction of the reputation literature¹³ where politicians nurture reputation with re-election in mind. A policy reversal establishes a poor record of credibility indeed, suggesting the existence of other rents in being part of the *incumbent* government when implementing microeconomic reform.

1.2.3 The Impact of Reform

Closely related to the conditions that foster the implementation of reform is the extent of its impact, and underlying this relationship is the concept of economic efficiency. However, in establishing the impact of reform it is also important not to forget the simple fact that reform induces winners and losers. Indeed, this fact is present beyond the aggregate level alone, where presumably entrenched interest groups oppose reform even though it may be optimal from a social point of view. It is surprising that relatively little effort has been devoted to the study of the micro-economic effects of the privatization of large state utilities in economies where markets are at their infancy and there exists a large scope for market imperfections.¹⁴

the latter stages. It is curious that other left-wing parties part of the opposition, protesting the perceived betrayal, did nothing to steer away from the reform package once they had the subsequent electoral victory. Indeed a great deal of the privatization process was also pursued during their administrations.

¹³See for example Bernhardt and Ingberman (1985) or Drazen and Masson (1994) and the references therein.

¹⁴Spulber (1989) and Laffont and Tirole (1993, Chapter 7) look at different issues in the regulation and divestiture of natural monopolies. However, they work with the standard model of the firm, where profit maximization by an outside manager is the norm. We believe this model is only partially applicable to small developing countries where large utilities are controlled by relatively few individuals.

In the context of a small economy where public utilities play a central role, it is evident that ownership of a natural monopoly is important for the post-privatization pricing rule. This point is made, for instance, by Vickers and Yarrow (1988, page 11), who model explicitly the interaction between owners and managers with the aid of principal-agent techniques. They postulate that “shareholders seek to maximize their expected financial return (profit) from the company”, but that, in the case of the privatization of utility companies, shareholders are also consumers, thus having an interest in a production plan going beyond financial returns alone. Two further specific characteristics of the privatization experience in small emerging economies seem relevant. First, several privatization programs in these regions incorporated some form of distribution of shares among economic agents with little previous experience with capital markets. Thus, it is unlikely that recipients would have had sufficiently diversified portfolios ensuring indifference to the monopoly’s pricing rule. Second, as the output of state utilities is likely to represent a large share of the consumers’ budgets, agents deciding on the pricing rule would have an intrinsic interest in pricing the monopoly good lower.

Looking at the decision making process in monopolies in these type of economies would suggest considering general equilibrium models of monopolistic competition. This literature dates back to at least the seminal paper by Negishi (1961), establishing the existence of a *Monopolistically Competitive Equilibrium* under the strong assumption of a subjective demand.¹⁵

It is somewhat un-natural to think of a monopoly’s production plan as a collection of the shareholders’ interests. The economist’s typical objective of a monopoly is profit max-

¹⁵A good overview of the issues in this literature can be found in Hart (1985).

imization. At best, monopolies are modelled in terms of a principal-agent model where *an* owner may have an objective that differs from that of a manager in charge of running the firm. Very little is written about the role shareholders play and about how their possibly different objectives converge to a decision on the production plan of the firm.

This issue has gained importance throughout the developing world in the context of the privatization of state utilities, where the concentration of a firm's control is seen as a desirable outcome of privatization schemes.¹⁶ Here again, not much is said about the possibly positive effects of widespread share ownership in lowering the price of the utility's output. Generally preserving a profit objective is considered central in achieving economic efficiency, and the consumption effect of the higher price that is likely to follow privatization is disregarded. Thus one has generally imposed a presumed profit objective common to all shareholders.

However, the benefits of widespread distribution underlie the Bolivian capitalization program, for instance. By giving 50% of the monopoly's control to the prospective investor, the programme aimed at attaining both effective management and the positive effect of distributing half of the profit among the population. Requena (1996) describes some of the benefits to the investor from this scheme when he states that shareholders profited from an immediate increase in the value of the shares.

1.3 The Two Issues Covered

The two chapters below cover in turn two issues we understand to be of keen interest in the reform literature. We first provide an alternative interpretation of the origins of reform

¹⁶Gupta, Schiller, and Ma (1999) look at the fiscal implications of the adjustment costs of privatization in the context of various privatization methods. They do not consider the incentive aspects that we find important for state utilities being privatized.

in general, and privatization in particular. We suggest that conventional wisdom about privatization may, under some circumstances, be wrong: what could appear to be a transition to more efficient market mechanisms may in fact be a short-lived coalition shift resulting in an overall worse situation. The converse is also true: nationalization may bring about an increase in welfare. We look at the conditions that drive the optimality of a privatization or nationalization program in a context where policy changes require the support of large social groups.

In the second chapter we turn to the pricing behavior of large utilities that have been privatized. We argue that the eventual behavior will depend in a large measure on the procedure employed in transferring state monopolies to private owners. The distribution of shares of the state-owned utility can be a mechanism for altering the decision making mechanism within the then-private enterprise. By granting a different weight in a share-holding decision making process to agents with distinct characteristics and objectives, different pricing equilibria can be achieved, thus altering the welfare of different segments of the economy. We suggest that market-oriented or egalitarian objectives may not always be attained, as the outcome may turn out to be as desired or predicted by conventional wisdom. In light of the nature and large size of firms being privatized throughout Eastern Europe, the issue we address pertains more than just the Latin America region, and the insight applies to the issue of divestiture of a natural monopoly in general.

We conclude that careful consideration of the origin, goals and outcomes of reform policy may benefit from a greater understanding of the surrounding social, political and economic context where reform takes place. Whereas economic efficiency is a worthy objective, the social planner would be wise not to ignore imperfections characterizing developing economies

and the unique dynamics of the local social and political institutions.

1.3.1 Privatization and Policy Equilibria

In chapter 2 we rationalize privatization as a possible outcome of a successful political coalition transformation. We argue that a simple framework of coalitions between income groups can explain much of the stylized facts about the political economy of Bolivia in particular, and Latin America in general. In doing so, we identify conditions which may have been pivotal in the shift to a new market-oriented paradigm.

Underlying our argument is a mechanism for reallocating, at a given policy status quo, wealth to members of different income groups. We exploit the fact that privatization fundamentally alters the private sector's responsibilities vis-à-vis, and benefits from, the state and the private and public sectors joint role vis-à-vis the overall economy.

Where such synergies between a private and a public sector exist and is significantly affected by a policy of divestiture, we ask whether the divestiture or acquisition of public enterprises can create benefits among different social groups. Departing from this well defined change in the *function* of the state in the economy, we go on to construct a simple parametrization capturing essential elements of this transformation. For instance, we look at possible coalitions supporting different policy positions (privatization or nationalization). We discover that the policy status quo matters significantly in establishing the magnitude of the gains and losses that result from a change in the function of the state, as well as on who gains or loses from alternative policies. In other words, the existing institutional and economic framework serves as a catalyst for, or inhibits, a given policy relative to the alternative: the current state of the policies matters in making one type of policy more attractive

from the point of view of the different economic groups and thus in cementing alternative coalitions.

Motivating these issues is the case of the Bolivian capitalization program whose objectives Antelo (1995) describes as follows :¹⁷

... the capitalization process has as objective the generation of greater levels of investment and technology transfer from abroad, thereby increasing the export potential of the country and permitting the substitution of state property for private participation. The distinctive characteristic is the distribution of the state shares (50%), through pension funds, among the adult citizens of Bolivia. This process will permit freeing state resources for social and infrastructure investment, necessary for assuring long-term growth.

However, capitalization could conceivably also reduce synergies at the aggregate level, where the new dynamic of the capitalized utility can create phase-outs with other sectors of the economy which may or may not remain under the control of the state. Supply chains, contractual agreements and interest-group dynamics, can be subject to strains resulting from the changed priorities of the privatized enterprises which are generally large and concentrated in the infrastructure sector.

Against a backdrop of a regime transition favoring privatization, we suggest that historically or technologically-driven changes in the structure of the economy may have been

¹⁷ ... la capitalización tiene como objeto generar una mayor inversión y transferencia de tecnología del exterior, incrementando el potencial exportador del país y permitiendo la sustitución de la propiedad estatal por la participación privada, con la característica singular de la distribución de las acciones estatales (50%) a través de los fondos de pensiones, a los ciudadanos bolivianos mayores de edad. Este proceso permitirá liberar recursos estatales, para la inversión social e infraestructura, necesarios para asegurar un crecimiento económico de largo plazo

important in eroding the ruling status quo of support for nationalization, thus favoring a policy orientation over to the alternative. In effect we maintain that a *distinctive* policy change took place in Bolivia -in the form of capitalization- portraying a political and economic realignment between different income groups. Reform has become “fashionable” because it has proved to be a *workable* political transition from a coalition between a working and a governing class to one between the latter and a capitalist class. By capturing some of the given policy’s benefits, policy makers are in a position to implement a menu of policies with appeal to specific social groups. Support can therefore be obtained for a variety of programs, only one of which is liberalization. The policy selection is irrespective of its socially desirable characteristics: privatization may result in an output fall under some conditions, and policy makers may nevertheless go ahead with its implementation.

1.3.2 Divesting of a Monopoly

In the last chapter we look at the impact of privatization on the resulting pricing behavior of a privatized utility. In this context, we look at equity issues and how consumers of the utility’s output will eventually be affected by the particular privatization mechanism chosen.

We propose a formal setting of a small economy to address this issue, aiming to arrive at a sensible framework clarifying the constraints a reformer central planner faces when contemplating the privatization of public enterprises. The model is therefore presented as generally as possible and its relevance is illustrated with common functional forms. Nevertheless, some non-trivial assumptions are taken to arrive at concrete results. In particular, ours is a very simple economy, closely resembling a partial equilibrium framework, but with important elements of general equilibrium that are relevant to the specific analysis we seek.

The non-trivial exogenous restrictions we impose reflect some aspects we believe are very relevant for the economies studied, and help us illustrate real choices that a central planner privatizing a public utility may be confronted with.

The specific mechanism we consider to model an individuals' choice looks at the relative importance to the share-holder of a profit increase compared to the cost she bears, as a consumer of the monopoly's output, embodied in a higher price. In particular, we ask whether selecting a board, representing the interests of the public, to select the price of the private utility's output would improve the well being of consumers (who are also owners) of the utility's output compared to a situation of leaving the pricing decision to the shareholders. Clearly, not all agents are better off with the socially optimum pricing behavior of the firm. Agents with a higher share of profit and with lesser alternative sources of income would prefer at the margin a higher profit income.

The diversity of preferences that characterizes the shareholders of the privatized monopoly calls for a mechanism mapping their preferences into a unique and coherent production plan. In this regard, we construct the concept of *shareholders' majority voting* from first principles of voting equilibria, thus obtaining an extended form of Black's median voter theorem that is suitable to our economic context.

The notion of an equilibrium is complicated by the trading of shares that may follow privatization. Justified by the trading restrictions of many privatization programs, we do not address this important issue, or provide support for observed patterns, until the end, where we explore some aspects of share trading that illustrate the issues involved.

Our model gives us a transparent and plausible mechanism to study the constraints a central planner is confronted with when deciding on privatization. We derive simple rules

to assess the efficiency of different, alternative, privatization programs that may depend on both the mechanism of divestiture and the objectives of the central planner regarding, for example, promoting equality or reaching production efficiency.

Our conclusions are discouraging. First, large demands are placed on central planners if they are to arrive at socially optimum equilibria, something unlikely in industrialized economies, let alone developing countries. Second, auctions, while placing less demands on the central planner, also require generating post-privatization rents and thus are socially costly, in particular when agents are well informed. Third, objectives of income redistribution require abandoning the objective of overall efficiency, as economic rents need to be generated by the monopoly in order to improve the welfare of the worst-off. Finally, there is a natural tendency towards concentration of ownership when the trading of shares is allowed, that may result in a further departure from the social optimum.

Chapter 2

Privatization, its Long-term Effect on Income and Policy Equilibria

2.1 Introduction

2.1.1 General Background

The post-war period in Latin America was characterized by four decades of state intervention. One of the key elements of the political programme during these years was the nationalization of important economic sectors, such as oil production, telecommunications or air transport. The strong presence of the state was motivated by a variety of economic factors such as developments in the global political economy, or equity considerations.

However, at the beginning of the nineties, the majority of Latin American countries privatized a large number of their state-owned enterprises (SOEs), establishing privatization as the dominant policy instrument. According to the World Bank (2000), Brazil, Argentina, Mexico and Chile are the four largest privatizers in the world during the period 1990-1998, as measured by the proceeds from privatization. Between 1988 and 1993, the value of divestiture

transactions in Latin America represented 57% of the worldwide value.¹

This vast transition from state intervention to freer market economies fundamentally transformed the production structure and the organization of markets, and established a key and long-lasting transformation in the relationship between the state and the markets. Not only did the private sector expand, but this transition also led to a substantial rethinking of the role of the state. It was also believed that this restructuring would lead to an improvement in economic performance, despite the general awareness of evident difficulties involved in the future oversight of the natural monopolies that were divested.² It would appear that whatever prerogatives prevailed to go ahead with privatization, they offered significant compensation for the efficiency losses foreseen in the aftermath of privatization.

Almost a decade later, mediocre growth, increasing income disparities and persistent overall economic instability call into question the virtues of the change in policy.³

The fact that so many Latin American countries experienced a similar policy reversal almost simultaneously and to such degree, motivates the search for a mechanism that may underlie such a trend.⁴

¹Sader (1993) as cited in World Bank (1995).

²The recognition of limited capacity in developing countries of providing effective regulation was recognized early on in the case of the Bolivian capitalization programme by Sinn and Sinn (1993, p. 10). Although provisions were foreseen to regulate monopolies, limited resources and technical expertise remain a formidable challenge for effective supervision.

³Furthermore, not all the state enterprises to be privatized were inefficiently run, and those that were were less likely to attract investors.

⁴This relates to what Rodrik (1996) poses as the remaining central puzzles in the field of reform: on the one hand *the origin* of reform or why so many governments decided to reform at the same time, after decades of adherence to policies of an opposite kind; on the other the *extent* to which the policies were applied in these countries, by far exceeding those of South East Asian economies which were supposedly the source of inspiration.

While it has been argued that the policy reversal followed the policy-makers' realization of the advantages of private management, it seems unlikely that *the vast majority* of countries in the region embraced such a discovery at the same time and with the same conviction. The unique transition instead suggests a consistent and reproducible mechanism creating incentives for policy-makers to divest. In particular, the mechanism must have a political component providing for sustained support (so as to be consistent), presumably from groups in society poised to gain from the new framework. This mechanism should also depend to some extent on conditions that are common to the countries implementing reform so that the policy is reproducible and attractive to a large number of policy makers. In this regard, Baer and Birch (1994, p. xii) and Baer and Birch (1992, p. 5) identify the impact on the distribution of income and on government finances as two key elements characterizing the changing role of the state and its relation to markets. Therefore, an explanation of the privatization phenomenon must, at the very least, address these two issues.

In this context, the capitalization programme of Bolivia serves as a good example. We shall concentrate in this paper on issues regarding privatization of state-owned enterprises (SOEs), in general, and their capitalization in the context of the Bolivian reform, in particular. In doing so we shall consider some specific institutional characteristics and topics that are at the center of that country's experience, such as strong class divisions, an imperfect income distribution mechanism, a self-serving political class, and the partial failure of growth to materialize in the aftermath of reform.

2.1.2 Our Model of Privatization

The aspect of privatization that we are most concerned to emphasize here has to do with changes in the institutional rules under which private and public sectors interact. Whereas we agree with the empirical literature suggesting that there are significant efficiency gains in the privatization of public enterprises, we do not dwell on this issue. Therefore we assume that private capital is more efficient than public capital.⁵ We also attempt to go beyond the common conception of privatization, for example in the U.K. case, as being a mere capital transfer, ΔK , such that the pre-privatization sectorial stocks (K_p, K_e) (p stands for public and e for private) become $(K_p - \Delta K, K_e + \Delta K)$. In this regard, we wish to focus attention on the more fundamental change in the economy's aggregate technology brought about by changes in the institutional rules of the game, whereby private and public capital interact in technology. The changed roles of the public and private sectors is summarized by an aggregate parameter in the production function of the economy.

Accordingly, our approach focuses on the impact of this important policy instrument on the interaction between the state and "markets" which is denoted by Yarrow (1999) as *the market for regulation*.

These factors include interest group pressures, transparency and scrutiny of decision, the influence of public opinion, the degree of regulatory discretion, bureaucratic goals, the balance of political forces, and the extent of judicial oversight of regulation. As a first approximation, privatization can be viewed as one means of reducing the impact of political factors on economic incentives, behavior and

⁵The greater efficiency may for instance result from better allocation of resources, such as lesser over-staffing and better management. A significant body of literature has focused on this issue, which is surveyed by Megginson and Netter (2001).

performance.⁶

This type of interaction between the two sectors is natural in the context of developing economies. Nationalization and state intervention in these economies involves much more than state ownership and economic control of specific enterprises; it involves a particular ideology and set of attitudes of the political class, and an associated system of legislation on economic behavior and policy that extends to all economic activity and that heavily favors the beneficiaries of state capital relative to private capital. Likewise, while a privatization policy must include a transfer of assets from the state to private sector, these transfers presuppose that the previous ideology and its associated economic attitudes and institutions have been overthrown, and that the replacement ideology of market-orientation and decentralization to the private sector have been enshrined in new institutional forms and behaviors.

It is the impact of this change in the rules of the game under which public and private capital interact in the technology that we wish to analyze in what follows. If the key specific characteristic of a privatization policy is the surrender by the political class of asset ownership, then its key global impact is the surrender by the political class of some of its power to intervene in the economy, in a way that preserves some rules and institutions that favor the political class' public capital or result in alternative rents for the political class.⁷

In what follows we ignore the capital-transfer aspects of privatization entirely, and focus only on the effects it implies for the parameters of the production technology, which describes

⁶Yarrow (1999, p. 158) in reference to Vickers and Yarrow (1991).

⁷See Shleifer and Vishny (1994) for a related discussion that emphasizes the difference between political ownership and political control of firms, and that emphasizes the idea that privatization of ownership may have negative economic consequences if it is not accompanied by surrender of political control. The kind of technological inefficiency that they discuss in relation to political control of an individual firm will be mirrored here at the level of aggregate technology in the relationship between capital stocks.

the manner in which the public and private capital stocks are combined in the aggregate technology. We achieve this by studying the *steady-state effect* of a policy change from an established status-quo, instead of the instantaneous impact of a transfer of capital stock. In doing so we see departures from the given status-quo as politically difficult, thus requiring the support of a majority of the population. The technology-parameter changes will of course induce a steady-state equilibrium change in the capital stocks of the two sectors/capital-owning classes, but this is generated here as an outcome of the privatization policy rather than as a direct component of this policy. It is the surrender of regulatory power by the political class rather than the surrender of capital stocks that are of interest here. Yarrow (1999, p. 162) emphasizes this point as follows:

[It is] the overall shape of the package of regulatory reforms that is likely to be of most economic significance, not privatization per se.

2.1.3 The paper's sections

Section 2.2 sets up the basic model by laying out the elements of the infinite-horizon model and introducing the notion of aggregate capital used to model the market for regulation.

In section 2.3 we characterize the steady-state equilibrium of the dynamic model in order to focus on the long-run aspects of policy changes.

In section 2.4 we identify the comparative static effects on the steady-state capital stocks of the policy instruments (privatization and fiscal policy), given some initial *policy status-quo*. We then characterize the impact of the two policy instruments on output and on the steady-state consumption of the capitalist, the politician, and the worker.

In section 2.5, we look at policy implications of the model. We first postulate that the

coalitions can be formed by agents whose consumption increases when the given policy is implemented. The politician is unique in that he is the only one who can propose a policy, while the other two social classes either support or reject the proposed policy. Having defined the criteria for political support, we identify the policy status-quo from which different policies are supported by different income groups. We then turn to answering key policy issues that appear important in the context of privatization programs, in general, and the Bolivian capitalization, in particular. We first establish that lump-sum transfer between the entrepreneur and the politician, and the worker and the politician, make some policy outcomes more likely. More specifically, the former transfer increases the set of status-quo policies from which a coalition between the political and the entrepreneurial classes would support privatization, while the latter alter the status-quo that result in a worker-politician coalition. We see whether privatization may cause output to fall. In this context we also show that nationalization, supported by the politician and the worker, increases output. We also look at the model's prediction regarding the tax policy when, as has been observed in periods of reform, the politician is free to set it to maximize his own welfare. The model predicts that under some conditions the tax rate maximizing the politician's welfare is positively correlated with the degree of market orientation: a self-serving politician will, in this situation, tax higher to compensate for lower income derived from public capital. Finally we ask whether our simple model can provide an explanation for the change in the economic paradigm from state intervention to divestiture in Bolivia in particular and Latin America in general. We see that we can think of this fundamental change as resulting from a shift in a political alliance from one between the politician and the worker supporting nationalization to one between the politician and the capitalist supporting privatization.

Section 2.6 concludes.

2.2 The Basic Model

In subsection 2.2.1 the consumption-saving decision of the agents is specified. A class assumption is introduced separating investment opportunities for public and private capital owners, as well as for workers. Within each class, agents are numerous, so that they do not affect each others decisions and there are no game-theoretic considerations. In particular, there is no bargaining between the different classes about their actions, thus obviating the need for a formal bargaining solution. While no explicit reference is made to a specific class *size*, we view the process as one where the consensus of two classes is needed to implement policy initiatives. The formulation of classes we use portrays an important characteristic we believe exists in developing economies, where the consumption decision of a self-serving politician is different from that of a resident private investor or that of a worker. Also characteristic of these economies is the lack of markets for securities, thus barring the political class from borrowing in capital markets. As a result there is no public debt in our formulation.

In a similar fashion to Abel and Blanchard (1983) or Brock and Turnovsky (1981), we assume that access to capital markets is restricted to members of each class, so that the politician can only hold assets in the form of public capital and the entrepreneur in the form of private capital. This interpretation reflects an imperfection of capital markets and is natural in the context of developing economies, given that public enterprises normally have no tradeable stocks, nor are governments in a typical developing country involved in the purchase and sale of traded stocks, as capital markets are underdeveloped.

To make the distinction between the two capital owning classes that are observed in

a typical developing country, on the one hand, and their workers, on the other, we also specify that workers have no access to savings, thus consuming the residual proceeds of production. Analytically, the separation of the investment decisions allows representing the decision making process as one taken by a representative agent for each class, as in Chamley (1981, 1986) and Judd (1985a). Therefore there are a *politician*, an *entrepreneur* and a *worker*. Our economy therefore has three agents bound by a regulatory framework and policy instruments, changes to which can only be proposed by one of the agents: the politician.

The two policy instruments available to the political class are a tax policy τ accrued on gross private capital returns and a privatization or nationalization policy that affects the relative weight of private and public capital stocks in the economy θ . The two policies can be changed, subject to some coalition support (explained in section 2.5), by the politician.

In subsection 2.2.2 the production technology is specified, under the assumption that an exogenous manager of an aggregate firm maximizes profit. Special care is taken in describing a complementarity between the capital inputs we have formulated, which stems from the institutional structure linking the private and the public sectors.

2.2.1 The Agents

Three economic “classes” populate the economy and are indexed by $j \in \{w, e, p\}$: workers, entrepreneurs (holders of private capital) and politicians (manage and receive the proceeds from public capital). The latter two have possession of class-specific capital stocks which they rent to an aggregate firm producing the consumption good. Workers supply their labour inelastically, get paid their marginal product, and do not save.

There are N_e entrepreneurs and N_w workers. Politicians represent the remaining share of the total population N , $N_p = N - N_w - N_e$. In each class agents are sufficiently small, as there is a sufficient number of them, so that their decisions do not affect the decision making process of other agents.

Agents are infinitely lived, have a common discount rate ρ on the consumption of a unique good, and have identical and homothetic Bernoulli utility function of the form

$$u(c_j(t)) = \frac{c_j(t)^{1-\sigma}}{1-\sigma},$$

where $c_j(t)$ stands for instantaneous consumption of a member of class j at time t . Since there is only one good, utility is tantamount to consumption as far as the agent's welfare is concerned.

The two capital-owning agents are confronted with an investment decision: they can save by purchasing assets. As agents are small, capital markets are competitive. Therefore the politician and entrepreneur receive a *net* rate of return $r_j(t)$, $j \in \{e, p\}$, which they *take as given*. While this assumption is natural when there are a large number of private investors, strategic considerations may affect public investment decisions. We overcome this possibility by specifying a significantly large number of political appointees charged with the control of a number of public enterprises who are unable to affect the rate of return of their investment.

The politician

The consumption/investment decision of the politician is independent of the other decision she meets as a policy maker, i.e. the taxation of the raw returns to private capital at a rate τ . We consider an exogenous tax whose rate τ initially cannot be chosen by the politician.⁸

⁸In section 2.5.4, we look at the tax which maximizes the politician's steady-state consumption.

The tax is used for illustrating the policy space of the politician and the effect of changes in τ .⁹ As an example of a tax which is not optimally set, consider a situation where the politician does not have sufficient information to levy a tax which maximizes his welfare (or the welfare of a target class). Under imperfect information of this type, the politician would look for changes from the given status quo that make him better off.

Given the net rate of return r_p on public assets to the politician and the gross rental rate R_e of the private capital stock K_e , the politician increases her holdings of public assets a_p at time t according to

$$\dot{a}_p = r_p(t) a_p(t) + \frac{\tau R_e(t) K_e(t)}{N_p} - c_p(t),$$

from where it is clear that a politician receives a fraction $1/N_p$ of the total tax receipts $\tau R_e(t) K_e$.¹⁰ The tax collected is distributed equally among the members of the political class.¹¹

At time s the politician's maximization of the discounted stream of future utility is formally given by :

$$\max_{\{c_p(t)\}} V_p(s) = \int_s^\infty u(c_p(t)) e^{-\rho(t-s)} dt, \quad (2.1)$$

⁹Fiscal reform is often implemented in the prelude of privatization in developing economies, as tax collection is virtually non-existent prior to it. Rodrik (1996) interprets this phenomenon as a *precondition* of reform.

¹⁰The notation \dot{y} denotes the time derivative of variable $y(t)$, $dy(t)/dt$.

¹¹We somewhat relax this assumption in section 2.5.2, where we look at inter-class transfer mechanisms.

subject to the resource constraint and transversality and feasibility conditions

$$\begin{aligned}\dot{a}_p &= r_p(t) a_p(t) + \frac{\tau R_e(t) K_e(t)}{N_p} - c_p(t) \\ \lim_{t \rightarrow \infty} a_p(t) u'(c_p(t)) e^{-\rho t} &= 0; \\ a_p(0) &> 0, \quad c_p(t), a_p(t) > 0.\end{aligned}$$

The entrepreneur

The entrepreneur's problem is simpler, as he only earns a net return r_e on his holdings of the private asset, a_e . The private entrepreneur's formal problem is given at time s by

$$\max_{\{c_e(t)\}} V_e(s) = \int_s^\infty u(c_e(t)) e^{-\rho(t-s)} dt, \quad (2.2)$$

subject to the capital accumulation, transversality and feasibility conditions

$$\begin{aligned}\dot{a}_e &= r_e(t) a_e(t) - c_e(t); \\ \lim_{t \rightarrow \infty} a_e(t) u'(c_e(t)) e^{-\rho t} &= 0; \\ a_e(0) &> 0, \quad c_e(t), a_e(t) > 0.\end{aligned}$$

The worker

The worker, having no capital of her own, receives only the product of her unit labour which she supplies inelastically. She has no savings decision and consumes the given wage rate $w(t)$.

$$c_w(t) = w(t). \quad (2.3)$$

2.2.2 Production

We noted in section 2.1.2 that we are concerned with a policy reform which affects the rules of the game defining what is, at large, an economy's technology. When specifying a production function we are used to thinking of the technology's parameters in terms of engineering rules and data. However, social proscription can be important in ruling out the possibility of achieving some subsets of technically feasible input-output combinations. A parameterization of technology failing to take account of such factual social restrictions would give a false picture of what was possible for the production sector to achieve. The aggregate production function for the economy in this paper, therefore, is bound by a technology which includes in its parameterization the various restrictions and biases with respect to the interaction of the capital stocks that have been imposed by the political class.

A number of possible arrangements can be specified for the interaction, at the production level, between private capital K_e and public capital K_p . Consider for instance a constant returns to scale capital aggregation process using the two types of capital, represented by the CES function below

$$K = \left(\theta (A K_e)^\nu + (1 - \theta) K_p^\nu \right)^{1/\nu}; \quad (2.4)$$

$$\theta \in (0, 1), \quad \nu \in (-\infty, 1], \quad A \geq 1,$$

where $\sigma = 1/(\nu - 1)$ is the elasticity of substitution between private and public capital, θ is a factor representing the relative weight of the two types of capital in the production technology, and A is a factor measuring the relative efficiency of one unit of each capital type. Thus $\theta > 1/2$ would indicate that private capital is a larger component of the technology than public capital, and $A > 1$ portrays private capital to be more efficient than public

capital. Likewise, $\nu = -\infty$ and $\nu = 1$ would portray an economy where the two types of capital are perfect complements and substitutes respectively.

The two issues that stand out in this functional form are, first, the relative weight and efficiency of the capital stocks in the aggregation process and, second, how substitutable the two stocks of capital are. On the issue of substitutability there is a tendency away from a view that deemed both capital stocks to be compliments (the role of the state is distinctly different from that of the private sector) to one emphasizing a more effective role of the private sector in functions traditionally reserved for the state. Instead of adhering to the extreme view of this tendency, which foresees a role for the private sector in *all* matters run by the state, it is our view that the two types of capital are imperfect substitutes. Underlying this view is the belief that, whereas the private sector may deliver the same type of service possibly more efficiently, the public sector's service cannot be replicated by a privately run firm.¹² In particular, we make the simplifying assumption of a unitary elasticity of substitution. Thus aggregate capital will take a Cobb-Douglas functional form.

$$K = (A K_e)^\theta (K_p)^{1-\theta}, \quad \theta \in (0, 1). \quad (2.5)$$

We find this aggregation specification acceptable because it summarizes a myriad of markets and relationships that exist in fact, while simplifying the relationship between an "aggregate private sector" and an "aggregate public sector" that we view as both contesting and co-operative at the same time. Implicit in our model of capital aggregation, we postulate a symbiotic yet *contrasting* relationship between public (which can be thought of as providing

¹²This notion is very evident in the health sector, for instance, where privately-provided services are distinctly different from publicly-provided ones. Leaving aside the relative virtues of one or the other, it is difficult to argue that both services are perfect substitutes. This is also true, to a less evident extent, for other infrastructure sectors traditionally run by the state.

infrastructure) and private capital; the interaction taking place between the private and public sectors we observe in fact. This is particularly evident in developing countries, where the lack of an overall financial infrastructure and the absence of a well developed private sector is evident in many sectors, making the private sector dependent on the public one in a variety of ways.¹³

Our specification of an intermediate interaction between public and private capital is close in spirit to Yarrow's idea of *the market for regulation*. The private and public sectors (capital stocks) interact in a market where regulation plays an important role and where there are distinct rules of the game, captured by the parameter θ . Therefore, θ measures the *weight* or *bargaining power* of the private relative to the public sector in the market for regulation. It measures the "importance" of private sector decisions within the process whereby the aggregate capital of our economy is assembled.¹⁴ In this regard, θ is part of the technology of the aggregate economy, even though it is affected in the long run by the impact of some policy decisions, such as privatization, and to historical developments, such as changing perceptions on the roles of the state and the private sector. The reader can think of the parameters $\theta = \theta(p, \cdot)$ and $\tau = \tau(p, \cdot)$ as functions of a policy decision p (and many other relevant factors), and of the presentation below simplifying the explicit dependence of the

¹³In this regard we note that of the six enterprises capitalized in Bolivia, at least four would clearly qualify as providing an intermediate good (telecommunications, electricity, rail transport and oil production), thus emphasizing the complementary relationship we have in mind. The sceptical reader is reminded that the most basic complementary production relationship used by economists is that between capital and labour, even though a great degree of substitutability between the two is often natural.

¹⁴In the case of an economy with a small or no private sector, such as the former Soviet Union, investment decisions by the central planner influence all notions of capital stock (factories, land, infrastructure, etc.). At the other extreme, in an economy we call Hong Kong, the government plays a purely legislative role and it does not have a direct decision making on the capital stock and investment decisions associated therewith.

parameters on the policy decision p . However, exogenously generated changes to θ are not anticipated by agents in advance, as they only imperfectly observe the impact of deviations from its current state. We present in section 2.5.5 below a possible process of change in θ that is not anticipated by agents.

The two capital inputs, $K_p = N_p k_p$ and $K_e = N_e k_e$ aggregate to a capital stock input given in condition (2.5). This process can be interpreted as the first step of an *instantaneous* two-step procedure. In the first step the politician and the entrepreneur interact in the market for regulation to bring about the aggregate capital used. In the second stage the economy's aggregate capital is combined with labour in the production of Y , the sole output in our economy.

$$Y = \left[(A N_e k_e)^\theta (N_p k_p)^{1-\theta} \right]^\alpha N_w^{1-\alpha}, \quad \alpha \in (0, 1). \quad (2.6)$$

An independent manager is charged with the production of Y . She maximizes profit under price-taking conditions.

2.3 Solving the Model

We begin by looking at the optimization solutions in section 2.3.1, in order to turn to market clearing conditions in section 2.3.2. In subsection 2.3.3, the rental rates of the capital inputs equate, in equilibrium, the demand and supply of capital inputs. The equilibrium determines the stocks and flows in the capital stocks. We finish this section with the steady-state characterization of equilibrium in subsection 2.3.4.

2.3.1 Optimization Solutions

Only the two representative capital owners strictly optimize. Workers consume all their wage income, as specified by (2.3).

Representative Politician

The first order conditions to the representative politician's maximization problem (2.1) are the Ramsey-Euler equation, the public capital accumulation constraint, and the transversality or no Ponzi-game conditions given by

$$\sigma \frac{\dot{c}_p(t)}{c_p(t)} = r_p(t) - \rho; \quad (2.7a)$$

$$\dot{a}_p = r_p(t) a_p(t) + \tau R_e(t) k_e(t) \left(\frac{N_e}{N_p} \right) - c_p(t); \quad (2.7b)$$

$$\lim_{t \rightarrow \infty} a_p(t) v u'(c_p(t)) e^{-\rho t} = 0. \quad (2.7c)$$

Representative Entrepreneur

First order conditions for the representative entrepreneur's maximization problem (2.2) are analogous to equations (2.7), and are given by

$$\sigma \frac{\dot{c}_e(t)}{c_e(t)} = r_e(t) - \rho; \quad (2.8a)$$

$$\dot{a}_e = r_e(t) a_e(t) - c_e(t); \quad (2.8b)$$

$$\lim_{t \rightarrow \infty} a_e(t) u'(c_e(t)) e^{-\rho t} = 0. \quad (2.8c)$$

Aggregate Firm

In light of the production function (2.6), and given the *gross* rental rates of private capital $R_e(t)$, public capital $R_p(t)$, and labour $w(t)$, the manager will demand levels of private and

public capital satisfying the following optimality conditions

$$\frac{\partial Y}{\partial(N_p k_p)} = R_p(t); \quad (2.9a)$$

$$\frac{\partial Y}{\partial(N_e k_e)} = R_e(t); \quad (2.9b)$$

$$\frac{\partial Y}{\partial N_w} = w(t). \quad (2.9c)$$

2.3.2 Market Clearing

In order for the asset market to clear, assets held by politicians and entrepreneurs must be matched by public and private capital stocks respectively. Thus,

$$N_p a_p(t) = N_p k_p(t); \quad (2.10a)$$

$$N_e a_e(t) = N_e k_e(t). \quad (2.10b)$$

Capital markets also clear, that is, the net return to asset holders must equal the net rental rate of capital

$$r_p(t) = R_p(t) - \delta; \quad (2.11a)$$

$$r_e(t) = (1 - \tau) R_e(t) - \delta. \quad (2.11b)$$

It is clear from equations (2.11) that depreciation is accounted in both markets. Equation (2.11b) shows that we have specified the capital tax on the *gross rental rate* of private capital. That is, the tax is imposed before depreciation. This allows some simplification of the calculations without affecting the main results.

The assumption of constant returns to scale ensures that all output accrues to the factors of production

$$Y = R_p N_p k_p + R_e N_e k_e + w N_w. \quad (2.12)$$

2.3.3 Equilibrium

The adjustment path is characterized by a system of four differential equations for the consumption and capital accumulation of capital owners (given by the Euler-Ramsey equations (2.8a) and (2.7a), and the capital investment conditions given by (2.8b) and (2.7b)), and the static consumption equation for the worker.

$$\frac{\dot{c}_p(t)}{c_p(t)} = \frac{1}{\sigma} \left(\frac{\partial Y}{\partial (N_p k_p)} - (\rho + \delta) \right); \quad (2.13a)$$

$$\frac{\dot{c}_e(t)}{c_e(t)} = \frac{1}{\sigma} \left((1 - \tau) \frac{\partial Y}{\partial (N_e k_e)} - (\rho + \delta) \right); \quad (2.13b)$$

$$\dot{k}_p = \left(\frac{\partial Y}{\partial (N_p k_p)} - \delta \right) k_p(t) + \left(\frac{N_e}{N_p} \right) \left(\tau \frac{\partial Y}{\partial (N_e k_e)} \right) k_e(t) - c_p(t); \quad (2.13c)$$

$$\dot{k}_e = \left((1 - \tau) \frac{\partial Y}{\partial (N_e k_e)} - \delta \right) k_e(t) - c_e(t); \quad (2.13d)$$

$$c_w(t) = \frac{\partial Y(t)}{\partial N_w}. \quad (2.13e)$$

The overall dynamic system is constrained by the two transversality conditions (2.8c) and (2.7c), and the feasibility conditions in (2.1) and (2.2) ensure that a zero level of activity is not feasible.

The properties of the production function guarantee the unique equilibrium given in equations (2.13).

2.3.4 Steady State Equilibrium

Values for the steady state returns on public and private capital stocks follow from conditions (2.13a) and (2.13b) above. Given these rates, capital stocks are also established. Steady-state consumption streams are in turn derived from conditions (2.13c), (2.13d) and (2.13e).

The following five equations thus characterize the steady state

$$\frac{\partial Y}{\partial (N_p k_p^*)} = \rho + \delta; \quad (2.14a)$$

$$\frac{\partial Y}{\partial (N_e k_e^*)} = \frac{\rho + \delta}{(1 - \tau)}; \quad (2.14b)$$

$$c_p^* = \rho k_p^* + \left(\frac{\tau(\rho + \delta)}{1 - \tau} \right) \left(\frac{N_e}{N_p} \right) k_e^*; \quad (2.14c)$$

$$c_e^* = \rho k_e^*; \quad (2.14d)$$

$$c_w = \frac{Y^*}{N_w} - (\rho + \delta) \left(\frac{1}{1 - \tau} \left(\frac{N_e}{N_w} \right) k_e^* + \left(\frac{N_p}{N_w} \right) k_p^* \right). \quad (2.14e)$$

Explicit solutions for the capital stocks and output are easily derived from equations (2.14a), (2.14b) and (2.14e). After some simplifications we obtain

$$k_p^* = \left(\frac{N_w}{N_p} \right) \left[\frac{\alpha(1 - \theta)}{\rho + \delta} \left(\frac{A\theta(1 - \tau)}{1 - \theta} \right)^{\alpha\theta} \right]^{\frac{1}{1 - \alpha}} \quad (2.15a)$$

$$k_e^* = \left(\frac{N_w}{N_e} \right) \left(\frac{\theta(1 - \tau)}{1 - \theta} \right) \left[\frac{\alpha(1 - \theta)}{\rho + \delta} \left(\frac{A\theta(1 - \tau)}{1 - \theta} \right)^{\alpha\theta} \right]^{\frac{1}{1 - \alpha}} \quad (2.15b)$$

whereas steady-state output is given by

$$Y^* = \left[\frac{\alpha(1 - \theta)}{\rho + \delta} \left(\frac{A\theta(1 - \tau)}{1 - \theta} \right)^{\alpha\theta} \right]^{\frac{\alpha}{1 - \alpha}} N_w \quad (2.16)$$

Output per-capita as a function of the policy parameter pair $\{\theta, \tau\}$ is illustrated in figure 2.1 below.¹⁵

In the absence of market imperfections or economies of scale the highest level of output is achieved when all capital is managed privately (or when θ is one, as private capital is more efficient by assumption) and there are no taxes (τ creates distortions in production). This result, embedded in the construction of the model, would not hold when, as is usually

¹⁵Throughout the presentation of figures we use parameter values $N_w = N_e = N_p = 1$, $A = 7/3$, $\alpha = 1/3$, $\delta = 1/5$ and $\rho = 1/5$.

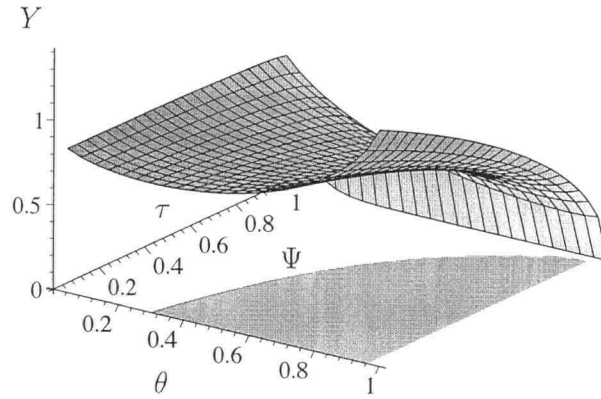


Figure 2.1: Output as a function of market orientation and fiscal policy $\{\theta, \tau\}$

the case, there are benefits for the public management of some industries, such as natural monopolies or sectors characterized by market imperfections.

Note that, as illustrated in figure 2.1, the output surface will have two local maxima, at points $\{\theta = 0, \tau = 0\}$ and $\{\theta = 1, \tau = 0\}$. This is clear, as taxation distorts production and lowers output, thus making the output-maximizing tax rate equal to 0. Likewise, the local maxima of high and low θ result from the Cobb-Douglas specification of aggregate capital, as there are greater benefits for more concentrated use of either type of capital (either high or low θ). If $A > 1$, it is socially optimal to use only private capital in the production of the consumption good.

2.4 Comparative Statics on the Steady State

Political decisions are taken in the context of an existing regulatory framework which is in itself the product of past policy decisions. Whereas it is conceivable that in the very long-run the regulatory framework and the relationship between the various sectors in the economy are endogenous, this is not the case for any given central planner, who is confronted with a set of existing constraints. Our politician takes decisions on changes in the policy parameters θ and τ from an *already existing* value of these parameters, and is possibly unable to reach, or even know about, the overall value of these parameters that are optimal for herself or other groups.

With this in mind, we look below at the impact of policy changes from a starting, or status-quo, value of the policy parameters on the consumption streams of the different groups and on output. In section 2.5 we motivate how this impact will establish whether agreement about a given policy can be reached or not, thus allowing us to assess when group coalitions can be created. Also, by looking at a policy' impact on output we can assess its overall virtues and determine, for example, when privatization results in an overall welfare decrease.

Given the nature of the consumption streams, we first look at the response of the capital stocks in order to then focus on output and the consumption streams. As illustrated by conditions (2.15), the capital stocks in the steady-state are determined, among others, by the two policies of interest: the tax rate on private capital returns τ and the weight of private capital in the market for regulation, θ .

2.4.1 Privatization

Whereas the concession to the entrepreneur of control over a fraction ΔK_t of public capital would affect the immediate dynamics of the economy, it would not on its own alter the steady state capital stocks, as capital stocks would slowly revert to the steady states consistent with the fundamentals of the economy. The steady-state levels of capital stocks reflect deeply rooted characteristics of the interaction between the private and the public sectors. In light of our discussion in sections 2.1.2 and 2.2.2, it is clear that we understand θ as embodying the ensemble of historical and institutional factors establishing the degree of *private sector participation* or *market orientation* in the productive process. In the long-run, this component also reflects past policy decisions, such as privatization or nationalization, which change the nature of the interaction of the two sectors. Public policy affects the productive process by altering the interaction between the two sectors, thus changing the long-run composition of aggregate capital. This is nicely put by Yarrow (1999, p. 161):

... state ownership can be a very flexible policy instrument, or, putting it more accurately, that state ownership is a means of increasing the flexibility of policy by increasing the range of immediately available policy instruments. For, under state ownership, all the decision variables of the firm (prices, investment, employment, location, and so on) potentially become instruments of public policy.

Clearly, various other factors also affect θ , including the credibility and enforceability of rules and regulations, or external constraints or incentives binding the policy spectrum. We will consider some of these in section 2.5.5 below, where we motivate the history of nationalization and privatization.

Capital Stocks

It is evident from conditions (2.14) that an increase in market orientation, i.e. privatization ($d\theta > 0$), affects the steady-state capital stocks. From equations (2.15), after simplifying, the effect of divestiture on the steady-state capital stocks can be expressed as

$$\frac{dk_p^*}{d\theta} = \left[\frac{\alpha}{1-\alpha} \ln \left(\frac{A\theta(1-\tau)}{1-\theta} \right) - \frac{1}{1-\theta} \right] k_p^* \geq 0; \quad (2.17a)$$

$$\frac{dk_e^*}{d\theta} = \left[\frac{\alpha}{1-\alpha} \ln \left(\frac{A\theta(1-\tau)}{1-\theta} \right) + \frac{1}{\theta} \right] k_e^* \geq 0. \quad (2.17b)$$

Depending on the parameters of the economy, the per-class-member capital stocks may increase or decrease in response to a divestiture policy in the form of an increase in θ . The sign of this impact will depend, in particular, on the initial level of the policy parameters θ and τ .

Output

The effect of a change in θ on the steady-state level of output can be derived directly from (2.16) or indirectly from (2.6) by using conditions (2.17). Privatization affects output directly through the parameter θ in the functional form, and indirectly through its impact on capital stocks. After some simplification, it can be shown that

$$\begin{aligned} \frac{dY^*}{d\theta} &= \frac{\partial Y}{\partial \theta} + N_p \left(\frac{\partial Y}{\partial (N_p k_p)} \right) \left(\frac{dk_p^*}{d\theta} \right) + N_e \left(\frac{\partial Y}{\partial (N_e k_e)} \right) \left(\frac{dk_e^*}{d\theta} \right) \\ &= \frac{\partial Y}{\partial \theta} + (\rho + \delta) \left(N_p \frac{dk_p^*}{d\theta} + \frac{N_e}{1-\tau} \frac{dk_e^*}{d\theta} \right) \\ &= \alpha \ln \left(\frac{A\theta(1-\tau)}{1-\theta} \right) \left[Y^* + \frac{N_p k_p^* (\rho + \delta)}{(1-\alpha)(1-\theta)} \right] \geq 0. \end{aligned} \quad (2.18)$$

The impact of privatization on output is therefore undetermined, as the logarithm may be negative. The sign of the effect will depend on the magnitude of the term θ , the relative efficiency of private over public capital A , and on the tax rate τ . If market orientation is high

and/or private capital is very efficient, increasing the level of market orientation will have a positive direct effect on output. Clearly, this impact depends on the distortion introduced by the way in which gross capital returns are taxed, as private capital with greater weight (high θ) results in a larger distortion (therefore the negative relationship between the tax rate and the sign of the logarithm). A *necessary and sufficient* condition for output to decrease following privatization is that

$$\ln \left(\frac{A\theta(1-\tau)}{1-\theta} \right) < 0, \quad (2.19)$$

that is when the argument of the logarithm is smaller than one.¹⁶

The status-quo policy parameter pairs $\{\theta, \tau\}$ consistent with an output fall after privatization are illustrated in figure 2.1 by the non-shaded area in the θ - τ plane at the bottom, which we denoted by Ψ . If the status-quo policy parameters are in Ψ , privatization decreases output and therefore cannot be optimal from the average consumer's perspective, irrespective of the benefit-sharing arrangement the entrepreneur and politician may have (and which we see in greater detail in section 2.5 below). Clearly, a policy of nationalization (a decrease in θ) will lower the level of output if the status-quo of the policies $\{\theta, \tau\}$ is in the complement of Ψ , Ψ^c . Policy parameters that lead to this outcome are shown by the shaded area on the θ - τ plane in figure 2.1.

Consumption

The net impact on consumption of an increase in the degree of market orientation θ also depends on the initial parameters. In section 2.5 we look at how the impact of a policy change on the steady-state consumption of the different actors determines the support of

¹⁶This condition will be used in the discussion of section 2.5 below.

these for a given policy (privatization vs. nationalization).

The effect of a change in θ on the steady-state consumption streams of the three agents can be derived from conditions (2.14) and (2.17). The capitalist's and bureaucrat's consumption streams are positively correlated with their respective steady-state levels of capital. The bureaucrat's consumption will furthermore be positively related to the capital stock of the private entrepreneur. That of the worker, although less clear, is simpler, and we therefore begin with it.

Worker's Consumption As is evident from expressions (2.14e) and (2.16), the worker's consumption is affected in the same way as output, which is given in (2.18). Indeed, it is not difficult to show after some simplification, that the response of a worker's consumption is given by

$$\begin{aligned}\frac{dc_w^*}{d\theta} &= \frac{1}{N_w} \left[\frac{dY^*}{d\theta} - (\rho + \delta) \left(\frac{N_e}{1 - \tau} \frac{dk_e^*}{d\theta} + N_p \frac{dk_p^*}{d\theta} \right) \right]; \\ &= \frac{1}{N_w} \frac{\partial Y^*}{\partial \theta}; \\ &= \frac{\alpha Y^*}{N_w} \ln \left(\frac{A \theta (1 - \tau)}{1 - \theta} \right) \geq 0.\end{aligned}\tag{2.20}$$

Privatization only affects the worker through the *partial effect on technology*. As the worker has no capital income, and does not profit from higher rates of return on the capital stocks, she will only benefit from the technology impact. If privatization increases output directly, workers will have higher earnings, reflecting the higher productivity of capital.

As we saw above in section 2.4.1 for steady-state output, the sign of this impact depends on the relative efficiency of private capital vis-à-vis public capital A , the level of capital income taxes τ , and the starting value of θ , our measure of market orientation. We denote by Ω the status quo policy parameters where the worker's steady-state consumption increases

with privatization, i.e. such that condition (2.20) is positive.

Figure 2.2 shows the worker's consumption surface in the policy space, where we have marked the sets Ω and Ψ .

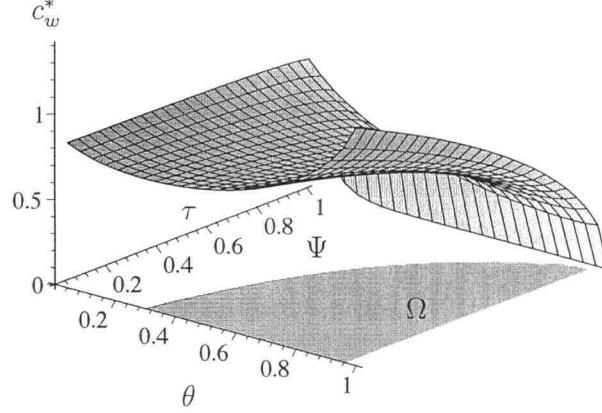


Figure 2.2: Worker's consumption as a function of the policy pair $\{\theta, \tau\}$

Entrepreneur's Consumption It is clear from (2.14d) that the entrepreneur's steady-state consumption is proportional to the steady-state stock of private capital. Using condition (2.17b) it can be shown that the response of the entrepreneur's steady-state consumption is given by

$$\begin{aligned} \frac{dc_e^*}{d\theta} &= \rho \frac{dk_e^*}{d\theta}; \\ &= \left[\frac{\alpha}{1-\alpha} \ln \left(\frac{A\theta(1-\tau)}{1-\theta} \right) + \frac{1}{\theta} \right] c_e^* \geq 0. \end{aligned} \tag{2.21}$$

Two results are evident from (2.21). First, a high tax rate is a necessary condition for the entrepreneur's consumption to decrease with higher values of θ . Second, even for high

tax rates, low values of θ ensure that the entrepreneur's consumption will increase. This is illustrated in figure 2.3, where the response of the entrepreneur's steady-state consumption to privatization, given in equation (2.21), is shown.

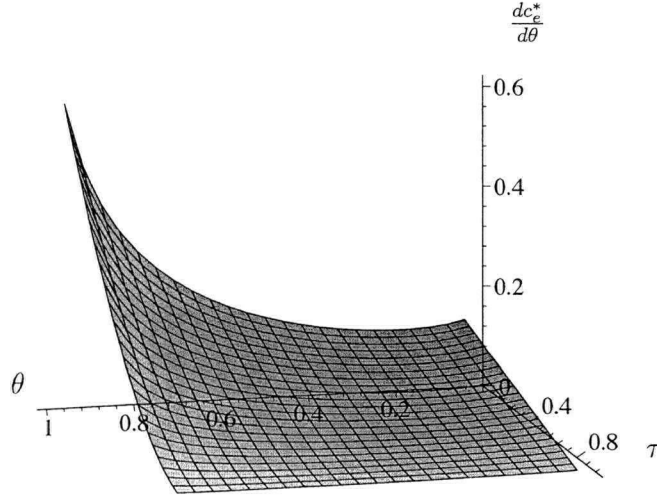


Figure 2.3: Entrepreneur's consumption response for policy profiles $\{\theta, \tau\}$

Almost the whole plane θ - τ represent status-quo points from which an increase in θ increases the entrepreneur's consumption. We denote by Γ the set of status-quo policy parameters $\{\theta, \tau\}$ resulting in the entrepreneur's consumption increasing with privatization (positive values of $dc_e^*/d\theta$ in figure 2.3).

In light of condition (2.21), if the entrepreneur's consumption falls with θ , it must be that the ratio $\frac{A\theta(1-\tau)}{1-\theta}$ is smaller than one. We saw in (2.20) that when this condition holds the worker's consumption has fallen.

Politician's Consumption A glance at expression (2.14c) suggests that the politician's consumption substantially differs from that of the entrepreneur or the worker. The politi-

cian's fiscal income responds positively to an increase in θ . However, her income also responds to changes in steady-state public capital, this effect dominating when the capital tax rate is lower (thus the lower the politician's share of the positive impact on private capital). Therefore the net impact on the politician's consumption depends on the joint response of the two steady-state capitals,

$$\frac{dc_p^*}{d\theta} = \rho \frac{dk_p^*}{d\theta} + \frac{\tau(\rho + \delta)}{1 - \tau} \left(\frac{N_e}{N_p} \right) \frac{dk_e^*}{d\theta} \geq 0, \quad (2.22)$$

which makes evident that the politician will face a trade-off only when public capital shrinks with privatization.

Using conditions (2.17), condition (2.22) can be written as

$$\frac{dc_p^*}{d\theta} = \left[\frac{\alpha}{1 - \alpha} \ln \left(\frac{A\theta(1 - \tau)}{1 - \theta} \right) + \frac{\tau(\rho + \delta) - \rho}{\rho - \theta(\rho - \tau(\rho + \delta))} \right] c_p^*. \quad (2.23)$$

It is evident from (2.23) that the sign will depend on the term in square brackets.

In order to visualize it consider figure 2.4, which shows the response of the politician's consumption as a function of both the tax rate τ and the level of private presence, θ . In this figure we show by the shaded area on the θ - τ plane, the policy parameter pairs $\{\theta, \tau\}$ starting from which the politician's consumption increases with higher θ , thus satisfying condition (2.23). We refer to this set of status-quo parameter pairs by Υ and we note that for the technology parameters used for the figures such policy parameters exist.

The saddle shape of the consumption surface on the plane θ - τ is the result of a shift in the relative importance of the elements in the privatization trade-off, along the θ axis, and the typical laffer-curve effect of taxation along the τ axis.

Consider the consumption profile of the policy maker given in figure 2.4. For sufficiently high values of θ and moderate tax rates, it is in the interest of the politician to privatize.

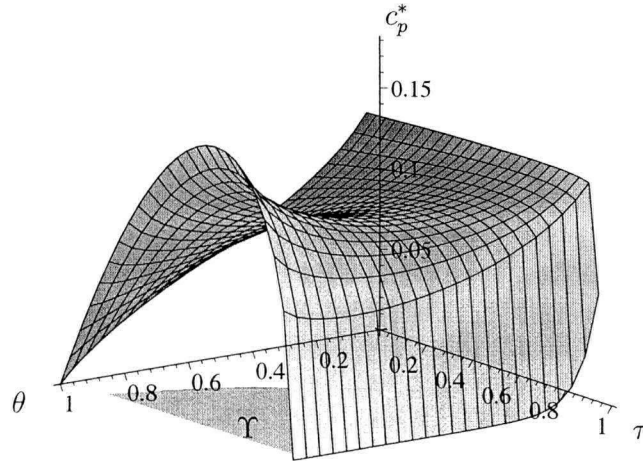


Figure 2.4: Politician's consumption for policy profiles $\{\theta, \tau\}$

The set of values of policy parameters $\{\theta, \tau\}$ consistent with this situation, Υ is given by the shaded area in figure 2.4 (and also in figure 2.5 below). For sufficiently low values of θ , on the other hand, it is in their interest to pursue the opposite policy, that is to nationalize. A number of policy possibilities result from the shape of the consumption profile of the politician as shown below.

The following proposition summarizes how steady-state consumption responds to privatization.

Proposition 2.1. *The worker's steady-state consumption responds to privatization for a given status-quo in the same way as output ($\Omega = \Psi^c$). The entrepreneur's steady-state consumption falls as a result of privatization only when the capital tax τ is sufficiently large. When the entrepreneur's steady-state consumption falls, it must be the case that the worker's steady-state consumption has also fallen, but the converse is not necessarily true ($\Gamma^c \subset \Omega^c$).*

2.4.2 Fiscal Policy

We now consider the impact of changes in the capital tax on the steady state capital stocks and consumption levels. We begin with the capital stocks given in (2.15) and then use these results to look at the impact of such policy change on the steady-state consumption of the three classes. These results are used in the political economy analysis of section 2.5 below.

Capital Stocks

In a similar fashion to the derivation of the response of the capital stocks to a change in θ given above in (2.17), we derive, after some simplification, the comparative static response of the capital stocks to a higher tax rate:

$$\frac{dk_p^*}{d\tau} = - \left[\frac{\alpha\theta}{(1-\alpha)(1-\tau)} \right] k_p^* < 0; \quad (2.24a)$$

$$\frac{dk_e^*}{d\tau} = - \left[\frac{1-\alpha(1-\theta)}{(1-\alpha)(1-\tau)} \right] k_e^* < 0, \quad (2.24b)$$

from where it is evident that the steady-state level of private and public capital holdings are decreasing in τ .

A higher value of the tax rate increases the steady-state marginal productivity of k_e , lowering k_e^* : by decreasing the incentives for saving, a higher tax results in lower levels of private capital. In addition to this channel, the impact on public capital also depends on the indirect effect of a lower private capital stock. Because of the complementarity between the two capital stocks in the production function (i.e. a fall in k_e^* decreases the marginal product of k_p^*), the tax lowers the marginal product of public capital. But, as shown in (2.14a), given that the steady-state marginal product of public capital is unaffected by the higher tax, the public capital stock must decrease so as to leave its marginal product at the original value.

Output

It is clear that, by decreasing both steady-state capital stocks, a tax on capital τ will also lower per-capita output. This is illustrated in figure 2.1 by the fact that the surface declines at any given value of θ for higher values of the tax rate τ . This can be calculated directly from (2.16).

$$\frac{dY^*}{d\tau} = - \left[\frac{\alpha\theta}{(1-\alpha)(1-\tau)} \right] Y^* < 0 \quad (2.25)$$

Consumption

The Worker Taking the derivative of (2.14e) with respect to τ and simplifying, the worker's consumption's response to tax τ can be shown to satisfy

$$\frac{dc_w^*}{d\tau} = - \left[\frac{\alpha\theta}{(1-\alpha)(1-\tau)} \right] c_w^* < 0 \quad (2.26)$$

where we have used (2.24) to simplify. The behavior of the worker's consumption can also be derived from figure 2.2, where it is clear that the worker benefits from lower taxes.

The Entrepreneur The entrepreneur's steady-state consumption will unambiguously decrease as a consequence of a lower capital stock following a tax increase, as the private capital stock decrease is shown in (2.24b). More specifically,

$$\frac{dc_e^*}{d\tau} = - \left[\frac{1-\alpha(1-\theta)}{(1-\alpha)(1-\tau)} \right] c_e^* < 0. \quad (2.27)$$

The Politician As illustrated by the saddle-shape of the politician's steady-state consumption surface in figure 2.4, a change in τ creates a trade-off between higher revenue and a lower return for public capital. This trade-off will depend on the prevailing fiscal policy and the degree of market orientation θ . It is clear from this figure, that the impact of capital

taxes on the politician's consumption is less significant, the smaller is the value of θ . The exact form of the response of the politician's consumption is given by

$$\frac{dc_p^*}{d\tau} = \left[\frac{\theta \left((\rho + \delta) \left[(1 - \alpha)(1 - \tau) - \alpha\theta\tau \right] - \alpha\rho(1 - \theta) \right)}{(1 - \alpha)(1 - \tau) \left(\rho - \theta(\rho - \tau(\rho + \delta)) \right)} \right] c_p^* \geq 0. \quad (2.28)$$

We denote by Σ the policy status quo where the politician's steady-state consumption would increase if the tax rate τ would be raised. The policy preference of the politician as a function of the status-quo policy parameter pair $\{\theta, \tau\}$ is shown in figure 2.5 below, where we distinguish parameter pairs in Σ (lower segment) and Υ (shaded area).

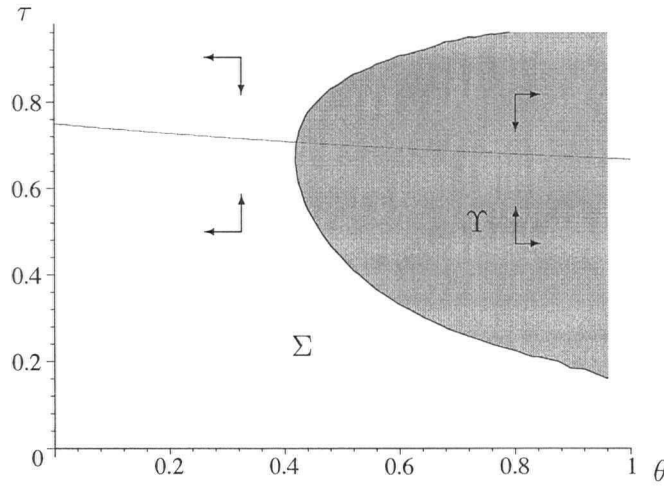


Figure 2.5: The politician's preferred policy directions

The impact of an increase of the tax rate on the different steady-state consumption streams is summarized in the following proposition.

Proposition 2.2. *Whereas output, and the entrepreneur's and worker's steady-state consumption decrease as a consequence of a capital tax increase $d\tau > 0$, the impact of the tax on*

the politician's consumption depends on the given tax rate and degree of market orientation of the economy.

2.5 Policy Implications

We characterized in section 2.4 the impact of policy changes on steady-state consumption of the different agents for a given status-quo of these policies. It would be natural to associate the support of a given group towards a policy change with the impact of the policy on the steady-state consumption stream of the group's representative agent. The status quo policy parameters that generate support from the different classes is summarized in table 2.1 below.

<i>Class</i>	<i>Policy</i>			
	$d\theta > 0$	$d\theta < 0$	$d\tau > 0$	$d\tau < 0$
workers	Ω	Ω^c	no support	support always
capitalist	Γ	Γ^c	no support	support always
politician	Υ	Υ^c	Σ	Σ^c

Table 2.1: Status quo parameters that determine support for policy change

Having characterized the support, or lack thereof, for a given policy, we illustrate how to use this formal framework to address five questions that are motivated by the Bolivian Reform Program:

1. Can the model broadly explain different policies towards state ownership during distinctive periods in the region as resulting from coalitions unified by differing policy objectives? Which policies and under what coalitions are feasible?
2. Does a lump-sum transfer mechanism φ increase the set of status-quo policy parameters where capitalization is supported by a coalition between the politician and the

entrepreneur (does $\Upsilon \cap \Gamma$ increase with φ)? What is the impact of a lump-sum transfer arrangement between the worker and the politician?

3. Can we establish the possibility of a coalition between the politician and the capitalist in support of privatization that results in lower output (is $\Gamma \cap \Upsilon \cap \Psi$ non-empty)? Could nationalization result in an increase in output? Who would support such a policy?
4. If the politician is free to set the capital tax rate τ to maximize his consumption, will the rate be positively correlated with θ ?
5. Can the historical reversal from nationalization (statization) to privatization (capitalization) be explained by a underlying change in status quo policy parameters?

2.5.1 Coalitions

In order to be implemented a policy has to be *proposed* by the political class and *supported by a majority* of agents. We assume that support of two classes (one of which is necessarily the political class) is necessary and sufficient for the policy to be implemented.

There is no feasible taxation policy, as the political class cannot form a coalition to support it and the coalition of workers and capitalists cannot propose a decrease in the rate. Similarly, privatization will be implemented if the policy status quo is in $\Upsilon \cap (\Gamma \cup \Omega)$. When the policy status-quo is in $\Upsilon \cap \Gamma$, privatization will be supported by a coalition of the political and the entrepreneurial classes, whereas in $\Upsilon \cap \Omega$ it will be supported by one between the political and the working classes. On the contrary a coalition between workers and politicians supporting *nationalization* (a decrease in θ) is possible for status quo policy parameters in $\Upsilon^c \cap \Omega^c$. The sets of policy parameters consistent with the different possible

coalitions is given in table 2.2 below.

<i>Coalition</i>	<i>Policy</i>			
	<i>Privatization $d\theta > 0$</i>		<i>Nationalization $d\theta > 0$</i>	
	Y^* up	Y^* down	Y^* up	Y^* down
worker/politician	$\Upsilon \cap \Omega$	empty	$\Upsilon^c \cap \Omega^c$	empty
capitalist/politician	$\Upsilon \cap \Gamma \cap \Psi^c$	$\Upsilon \cap \Gamma \cap \Psi$	$\Upsilon^c \cap \Gamma^c \cap \Psi$	$\Upsilon^c \cap \Gamma^c \cap \Psi^c$

Table 2.2: Policy coalitions

Consider, for example, figure 2.6 where the intersection of preferred sets of politician and worker are shown. For status-quo policies within Υ (resp. Υ^c), the politician's policy proposal to privatize (resp. nationalize) will find support of workers if the status-quo is also in Ω (resp. Ω^c).

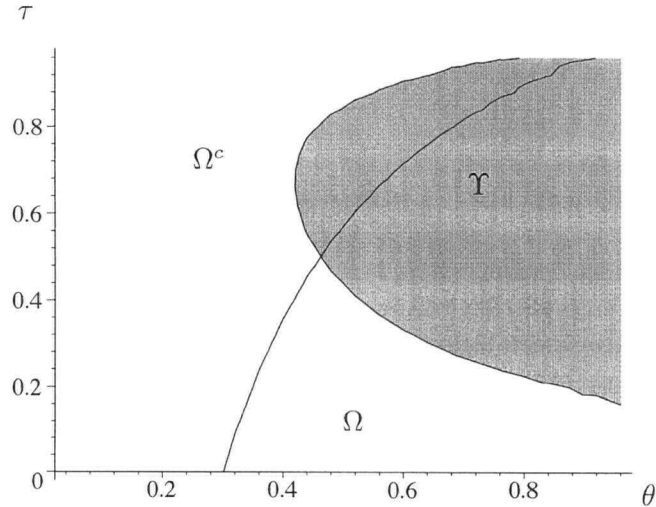


Figure 2.6: Worker and politician policy support profiles $\{\theta, \tau\}$

For the parameter values chosen, nationalization is favored by a coalition between the politician and the worker in much of the policy space (the intersection of the sets Ω^c and Υ^c).

With this policy, the entrepreneur's consumption decreases, as her capital stock falls. This situation arises in the upper-left segment of the space θ - τ . Nationalization in our model, although possibly globally inferior to privatization, does not lower output when implemented by a worker-politician coalition.

Privatization can be the preferred policy in the context of either growing or falling output. Output will increase for policy status-quo in the set $\Psi^c = \Omega$, possible under a coalition of all three agents in set $\Upsilon \cap \Omega \cap \Gamma$. However, a possible coalition in support of privatization is that between capitalists and bureaucrats at the expense of workers and output (in $\Upsilon \cap \Gamma \cap \Omega^c$, covered in section 2.5.3 below).

Some status quo could result in a political stalemate: even though both capitalists and workers would prefer privatization when taxes are low and the degree of market orientation high (in $\Gamma \cap \Omega \cap \Upsilon^c$), the politician will be reluctant to implement such policy, as it would lower her consumption.

In terms of the options *globally* preferred by the politician, we can distinguish two local maxima in figure 2.4: one of full state ownership and control which would be consistent with a coalition between the worker and the politician at the expense of the capitalist; and another product of a politician-capitalist coalition. This important characteristic is stated in the following proposition.

Proposition 2.3. *For the non-policy parameters chosen in the illustrations, two coalitions, one formed by the politician, the entrepreneur and the worker, and another conformed by the politician and the worker, lead to local maxima for the politician's consumption. Whereas privatization leads to the former, nationalization does the job for the latter.*

2.5.2 Transfer mechanisms

A transfer from the entrepreneur to the politician

The Bolivian capitalization programme was conceived in the aftermath of a standard privatization plan which failed to gather the necessary support. Cariaga (1996, p. 220) describes the situation following the early reform in the late 1980s as one where substantial opposition to the privatization programme delayed its implementation until it was *converted* into a capitalization programme.¹⁷ By creating a rent to be shared by the politician (partial ownership of the capitalized SOEs), private entrepreneurs could have prompted a policy of capitalization that would increase their consumption at a cost.

In order to illustrate this possibility suppose through some mechanism, each capitalist transfers a *lump-sum* amount φ to politicians.¹⁸ This transfer may expand the set of status-quo policies consistent with the politician wanting to capitalize. Indeed, it is not necessary that capitalization increases the politician's consumption (before the transfer), since she can now be compensated for a loss by φ . The maximum amount that the politician is capable of extracting from the entrepreneur (if the transfer mechanism is perfect), $\bar{\varphi}$, and still be able to receive support of the capitalist is the increase in the capitalist's consumption that results from the policy; that is

$$\bar{\varphi} = \Delta c_e^*.$$

In this extreme situation, because the politician captures all the increase, it is only necessary that *the joint consumption per class member* be positively affected by the policy

¹⁷Despite a broader support for the capitalization of six enterprises, a substantial part of the labour sector remained in opposition. The programme's legacy of ineffective implementation, remains until today and raises questions about the optimality of the original programme.

¹⁸We can think of this as "stock bribe" which may precede or follow capitalization.

change. Considering the steady-state consumption equations (2.14), the politician's effective consumption is given by

$$c_p^* + \left(\frac{N_e}{N_p}\right) c_e^* = \rho k_p^* + \left[\rho + \frac{\tau(\rho + \delta)}{1 - \tau}\right] \left(\frac{N_e}{N_p}\right) k_e^*. \quad (2.29)$$

Taking the derivative of (2.29) with respect to θ , the politician will favor capitalization provided that

$$\rho \frac{dk_p^*}{d\theta} + \left[\rho + \frac{\tau(\rho + \delta)}{1 - \tau}\right] \left(\frac{N_e}{N_p}\right) \frac{dk_e^*}{d\theta} > 0. \quad (2.30)$$

Comparing (2.30) with the condition for the politician's output to increase following privatization (2.22), it is clear that, because of the transfer, the set of status-quo policies where capitalization is proposed by the politician is larger than that where privatization is supported by both. We denote this expansion of the status-quo set where divestiture is agreed upon by a subscript φ on the set Υ , that is Υ_φ , and is illustrated in figure 2.7 below.

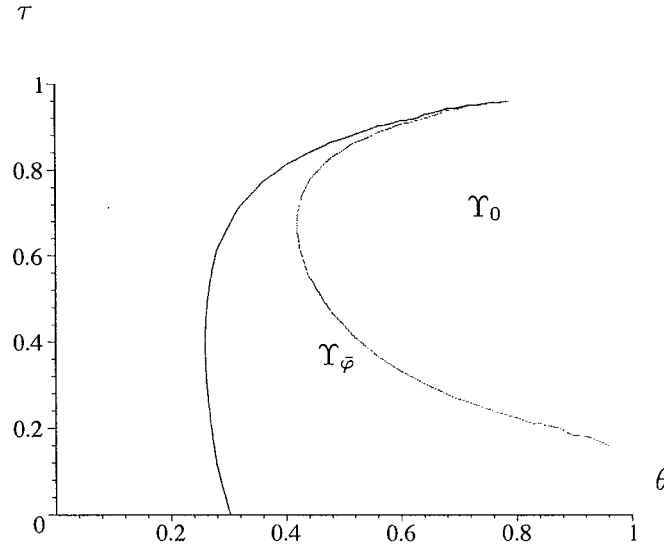


Figure 2.7: Capitalization (Υ_φ) more likely than privatization (Υ_0)

A transfer between the politician and the worker

A lump-sum transfer mechanism is also conceivable between the politician worker. Unlike transfer φ which the entrepreneur offers to the politician in order to have reform implemented, however, the transfer between the worker and the politician can go in both directions. The worker may attempt to bribe the politician to support nationalization when the politician would prefer to privatize (the policy status-quo is in the set $\Omega^c \cap \Upsilon$, thus τ is relatively high). The worker may also want to bribe the politician to privatize when this policy is not preferred by the politician (the policy status-quo is in the set $\Omega^c \cap \Upsilon$, τ is relatively low).

As with the transfer from the entrepreneur, the maximum amount that the worker can transfer to the politician is the total increase in her consumption. Therefore, as in the case of the transfer from the entrepreneur to the politician with a full transfer, the decision of whether to support privatization or not, depends on whether the *joint consumption of the worker and the politician* increases. Noting that $N_e k_e^* = N_p k_p^* \frac{\theta(1-\tau)}{1-\theta}$, condition (2.14c), and the joint consumption of the worker and the politician can be written as

$$c_p^* = \left(\rho + \theta\tau \left(\frac{\rho + \delta}{1 - \theta} \right) \right) k_p^*; \quad (2.14c')$$

$$c_p^* + \left(\frac{N_w}{N_p} \right) c_w^* = \left(\rho + \left(\frac{1 - \alpha}{\alpha} + \theta\tau \right) \left(\frac{\rho + \delta}{1 - \theta} \right) \right) k_p^*, \quad (2.31)$$

from where it is clear that the term $\alpha/(1 - \alpha)$ alters the relationship between the two policy parameters τ and θ underlying the politician's decision to privatize or nationalize.

The effect of this type of transfer is shown shown below in figure 2.8. The two sets Υ and Ω , denoting the status-quo where privatization and capitalization are supported by the politician and worker respectively, are merged into the set in between where they *jointly* support privatization.

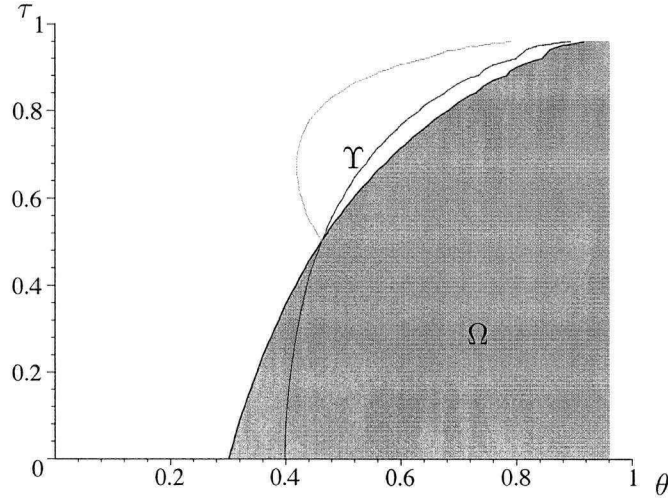


Figure 2.8: Effect of transfer between the politician and worker on the privatization-support sets

The transfer may also go from the politician to the worker, for instance when the tax collected from the entrepreneur is shared by a lump-sum arrangement by the two groups.¹⁹

The result of the lump-sum transfer mechanisms between the politician and the entrepreneur, on the one hand, and the worker, on the other, is stated in the following proposition.

Proposition 2.4. *A lump-sum transfer from the entrepreneur to the politician, makes the latter willing to implement a capitalization programme for status-quo policy parameters where he would not have done so in the absence of the transfer. A similar transfer from the worker may influence the politician either way depending on the policy status-quo.*

¹⁹If the arrangement is proportional instead of lump-sum, a higher share of taxes transferred will gradually reduce the set of status quo policies (Υ) where the politician supports privatization, while increasing that where the workers support this policy (Ω). The shift in the decision boundaries on the plane θ - τ does not alter the results, however.

2.5.3 Output-decreasing Coalitions

It is evident that some status-quo policy pairs $\{\theta, \tau\}$ are supportive of a politician-workers coalition supporting nationalization ($d\theta < 0$). It also follows from proposition 2.1 that in this situation output will not decrease.

The possibility of an output-decreasing coalition is interesting, as it would defy the common perception of privatization being consistent with higher levels of output. In fact, there is some evidence that this is a likely outcome. For instance, in the aftermath of the Bolivian capitalization program, growth failed to take-off as expected, registering mediocre performance accompanied by a worsening of the economic situation of workers. Is a politician-capitalist coalition supporting privatization possible which lowers output (is $\Gamma \cap \Upsilon \cap \Psi$ non-empty)? If this is the case, a welfare-decreasing capitalization program is possible, raising questions about the purported invariably positive nature of reform, in general, and capitalization, in particular.

As stated in proposition 2.1, if output per capita falls as a consequence of privatization (thus making the outcome Pareto-inefficient) then the worker's consumption has also fallen. Therefore, such an outcome is only possible within the framework of a coalition between the capitalist and the politician, who would have jointly benefited therefrom. Analytically, this question is reduced to establishing whether the sets Υ , Γ and $\Psi = \Omega^c$ intersect. Finding an output-decreasing coalition in favor of capitalization thus amounts to establishing whether a pair $\{\theta, \tau\}$ can be found such that:

$$\frac{dc_p^*}{d\theta}, \frac{dc_e^*}{d\theta} > 0 > \frac{dc_w^*}{d\theta} \quad (2.32)$$

We provide a partial analytical answer for this possibility and illustrate the existence of such status-quo policy parameters by way of an example.

Suppose output falls following privatization, so that the status-quo is in the set Ψ . We have seen in (2.19) that a necessary and sufficient condition for this to happen is that the ratio $\frac{A\theta(1-\tau)}{1-\theta}$ be less than one, thus resulting in a negative value of its logarithm. In order for the politician to generate such a proposal, it must be that the term in square brackets in equation (2.23) is positive. Since the logarithm within the square brackets is negative, a necessary and sufficient condition for the expression in square brackets in (2.23) to be positive is

$$\frac{1-\alpha}{\alpha} \left[\frac{\tau(\rho+\delta)-\rho}{\rho-\theta(\rho-\tau(\rho+\delta))} \right] > -\ln \left(\frac{A\theta(1-\tau)}{1-\theta} \right) > 0. \quad (2.33)$$

Therefore θ and τ sufficiently large and δ and ρ sufficiently small will make this possible. Note that this is not possible when there are no taxes or when there is no depreciation, as then either the logarithm cannot be negative or the first inequality in (2.33) cannot be satisfied (θ cannot be larger than 1).

When condition (2.33) is satisfied, the consumption of the politician increases with privatization, while the worker's consumption decreases (i.e. the intersection $\Omega^c \cap \Upsilon$ is not empty). Is the value $\ln \left(\frac{A\theta(1-\tau)}{1-\theta} \right)$ not so negative so that the entrepreneur opposes the policy (we are in Γ^c)? It turns out that this is not the case for some status quo, and the possibility of the existence of the set $\Upsilon \cap \Gamma \cap \Psi$ is illustrated for the parameter values in our previous examples in figure 2.9 below.

This motivates the following result.

Proposition 2.5. *Privatization could be implemented by a coalition between the politician and the entrepreneur at the expense of the worker ($\Upsilon \cap \Gamma \cap \Omega^c$ is not empty). Furthermore, some status-quo will result in per-capita output decreasing ($\Upsilon \cap \Gamma \cap \Psi$ is not empty).*

The set of status-quo policy parameters $\{\theta, \tau\}$ that are compatible with a output-decreasing

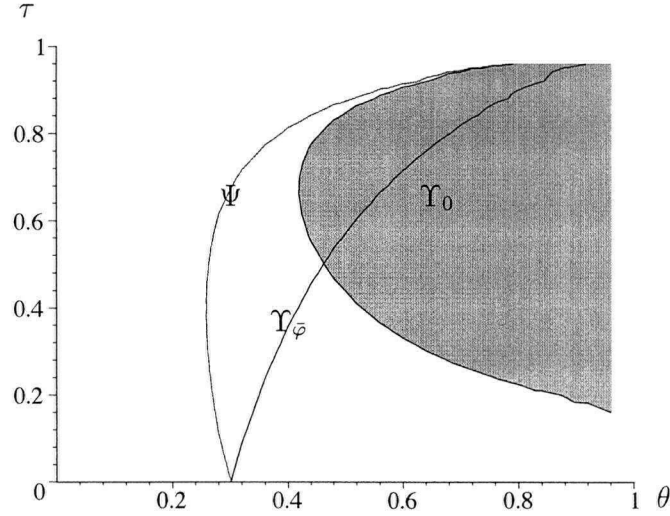


Figure 2.9: Politician-entrepreneur output-decreasing coalition is possible ($\Upsilon \cap \Gamma \cap \Psi$)

coalition between the capitalist and politician is made larger by the transfer from the entrepreneur covered in section 2.5.2. The set of status-quo supporting an increase in the politician's consumption, Υ_φ , progressively expands with higher values of φ . Just to how much was illustrated in figure 2.7. By giving the politician access to the private benefits of a lump-sum transfer, the entrepreneur may prompt privatization, thus benefiting both the capitalist and the politician. At the same time, it makes the possibility of a output-decreasing policy more likely, as the intersection $\Upsilon_\varphi \cap \Psi$ grows.

2.5.4 Choosing a Tax

A distinctive characteristic of privatization programs in general, and the Bolivian capitalization in particular, is that there is usually an associated tax reform. A stylized fact is that tax collections, and often rates themselves, increase following privatization. Can our model predict the increase in the tax rate, provided that it is set by the politician so as to maximize

her consumption?

In the context of our model, we would expect that a decrease in one source of income of our politician will be partly compensated with an increase in the other source. Consider figure 2.4, suggestive of how the politician would, when no support from other group were needed, choose the tax rate $\hat{\tau}$ in the steady-state so as to maximize her consumption.²⁰

We had excluded this possibility on the grounds that the tax rate would not receive support from a coalition involving the politician, and hence was not in the choice set of the latter. We now consider a situation where tax policy is not subject to vote.

The tax rate $\hat{\tau}$ that would maximize the politician's consumption would normally depend on the degree of market orientation θ , and would be represented in figure 2.4 as the projection of the peak in the consumption saddle onto the θ - τ plane. In figure 2.5, $\hat{\tau}$ is the horizontal line crossing the plane. Recognizing that a capital tax policy that benefits the politician could be implemented both simplifies and complicates the issue. On the one hand, it reduces the dimensionality of the problem: the consumption surface of the politician now becomes a consumption line, a function only of the level of market orientation. On the other hand, however, this introduces the analytical difficulty of tracking the wider equilibrium effect of privatization on our variables of interest, *including* the effect on the now endogenous tax rate.

The tax rate $\hat{\tau}$ is set to reach, for a given value of θ , the highest point of the politician's

²⁰Since we focus on the steady-state formulation, we will address the capital tax in that state. A study of the tax dynamics, similar to that of Judd (1985a) increases the level of complexity and goes beyond the objective of the paper.

consumption surface. That is, it must satisfy

$$\left. \frac{dc_p^*}{d\tau} \right|_{\hat{\tau}} = 0.$$

Using equations (2.24), $\hat{\tau}$ is given as a function of θ

$$\hat{\tau} = \frac{(\rho + \delta)(1 - \alpha) - \alpha\rho(1 - \theta)}{(\rho + \delta)(1 - \alpha(1 - \theta))}, \quad (2.34)$$

and it is evident that the tax rate does not depend on the relative efficiency of private capital A , nor the size of the classes.

The question of whether the tax rate is positively correlated with θ is whether the projection of the top of the saddle line onto the θ - τ plane in figure 2.4 is upward or downward sloping in the 2 dimensional plane θ - τ . Figure 2.10 shows the response of the tax to different levels of market orientation θ , under different assumptions of the remaining non-policy parameters.²¹

Under some parameter specifications, the relation proves to be positive: higher levels of private ownership result in a higher levels of capital income tax. This results in the following lemma.

Proposition 2.6. *There are feasible production parameters such that the capital tax that maximizes the consumption of the politician is positively correlated with the degree of private orientation in the market for regulation. In these cases, privatization will be associated with a higher taxation of capital earnings.*

How about the possibility of output-decreasing privatization? The possibility of having an output-decreasing coalition may disappear because the set $\Upsilon \cap \Psi$ may be out of the range where we would see the “tax consumption” path. However, both the original policy support

²¹The values assumed for the two schedules are $\delta = 1/5$ (decreasing) and $\delta = 1/20$ (increasing). Higher depreciation rates make the relationship decreasing.

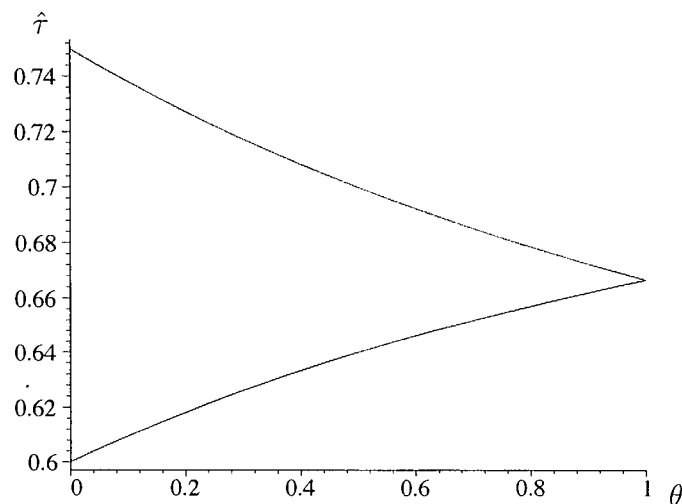


Figure 2.10: Capital tax maximizing the politician's consumption for two different depreciation rates

set of the politician Υ_0 and the “expansion” of the set Υ_φ that the transfer mechanism of section 2.5.2 brings about, touch on the path. The results of section 2.5.3 still hold under a tax rate maximizing the politician's consumption. This is illustrated in figure 2.11 below.

2.5.5 A Historical Interpretation

It is clear that the market orientation of the economy embodied in θ has a wider number of arguments in addition to privatization. For instance, entrepreneurs' decisions and their interaction with policy and policy-making affect the equilibrium of the market for regulation, and thus aggregate output. Furthermore, changes in technology, or the evolution of new economic activities subsidiary to those of the private sector, or a more efficient management of resources exogenous to our model, as well as the emergence of new markets complementary to the existing ones, all affect the value of θ at any point in time. In the context of the

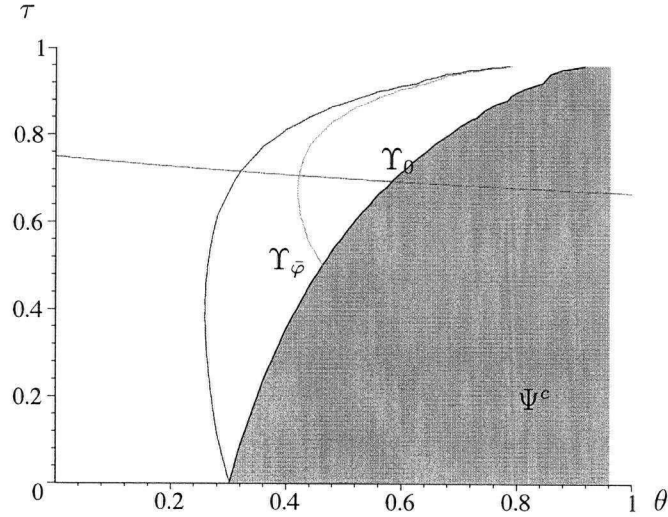


Figure 2.11: Existence of the set $\Upsilon \cap \Gamma \cap \Psi$ with tax being set by the politician

discussion so far, all of these factors were assumed constant. We now informally refer to their effect on the key parameter θ , and how they motivate an interpretation of different historical policy scenarios observed in the second half of this century in many of the countries in Latin America.

Latin America's post-war period's economic infrastructure was characterized by strong state intervention, an important public sector and a close relationship between the political class and labour movements. In terms of the parameter-plane θ - τ of figure 2.5, this would put us on the left (low θ), and possibly on the upper (high τ) corner, of the non shaded area. Within this set of status-quo policy values, it is in the interest of the politician to continue to pursue a policy of state intervention and nationalization, for this will increase her steady-state consumption. Also note that the policy of nationalization, although detrimental to the entrepreneur, may be output-increasing (in $\Psi \cap \Upsilon^c$), or not (in $\Psi^c \cap \Upsilon^c$).

Now consider a positive drift factor in the value of $\theta(\cdot)$ beyond the control of policy

makers, driving θ naturally to higher values. One can think, for instance, about the pressure put by international organizations to open the market, create transparent rules making the state redundant, innovation in the private sector's ability to interact in the market for regulation, etc. As the level of influence of the private sector on the economy grows, the economy's policy status-quo may well approach the set Υ , whose boundary we denote by $\hat{\theta}(\tau)$. Once this set is reached and $\hat{\theta}(\tau)$ is crossed, there is a reversal in the politician's preferred policy. A new coalition between the political and the entrepreneurial classes, benefiting from the new policy of divestiture, is ripe to be formed. The subsequent *policy-driven* increase in θ pushes the status-quo towards the other local maximum in the consumption profile of the politician (in figure 2.4) where θ is high. The new paradigm is dictated by the inability of the politician to effectively control the parameter θ and her subsequent recognition that an alternative local maximum is attainable within its regulatory powers. Paradoxically, for workers and the average consumer, it is at least possible that the parameters at the time of policy reversal were in the set $\Upsilon \cap \Psi$, thus making the new policy package pareto suboptimal.

An exchange mechanism, a distinctive characteristic of capitalization, generates an earlier policy reversal, as it provides rent-sharing between the capitalist and the politician. Indeed, it expands the set Υ and its boundary $\hat{\theta}(\tau)$ in figure 2.9 towards the left side. It also reduces the set Γ setting it away from high taxes. If we understand the historical evolution of the parameter θ as moving from the far left, capitalization in fact accelerates the arrival of the policy reversal.

Much in the common timing of privatization in the Latin American region suggests that there was a process of *revelation* in the late 80s and early 90s. The vigor and simultaneity of the implementation of the privatization programs hints at the realization of an alternative

policy equilibrium with the private sector playing a more central role. Furthermore, the strong opposition of labour groups, the penetration of technocrats in the policy-making infrastructure, and the income redistribution that followed, all suggest a radical shift in paradigm. A considerable amount of this can be interpreted from this paper's model.

2.6 Concluding Remarks

Possible extensions to the model include the analysis of the transition dynamics of the policy implementation. This could help in clarifying the standing question on the short-run costs and effects of capitalization as well as the immediate effect on consumption. An empirical estimation of the the transition could then be used to establish the validity of the results.

A second extension could study the impact of conditional aid, thus shedding light on the controversial issue of whether foreign aid accelerates the adoption of reform. It seems natural that a conditional grant to the political class following capitalization would increase the payoff from divestiture, thus moving the frontier $\hat{\theta}(\tau)$ closer to the status-quo tax rate and θ and making capitalization a preferred policy while prompting policy reform. The aid could also be used to gain the support of marginalized groups. This seems to have been the norm regarding the support programs associated with reform financed by the IMF and the World Bank in the region. This venue also opens empirical possibilities. For instance, one could establish the degree of correlation between aid supplied by international agencies and the degree and nature of implementation of reform packages.

Finally, from a modelling perspective, it may be of interest pursuing in detail the dynamic optimal tax and its impact on welfare. This issue is covered in the context of the more standard model by Judd (1987, 1985b, 1985a), Chamley (1986, 1981). More recent work

in this area can be found in Correia (1996b) and (1996a), and Jones, Manuelli, and Rossi (1993) and (1997).

Chapter 3

Privatizing a State Monopoly

3.1 Introduction

This paper looks at the role of the distribution of ownership in the production plan of a monopoly in the context of privatization. It is in the interest of a central planner deciding on privatization to understand the subsequent monopoly's pricing rules, as they have a direct impact on the market and the income of different economic groups. We consider three central elements in the pricing rule of the monopoly which we deem important: profit, labour and autonomous income. Not only does the eventual pricing rule clearly affect the former two elements, but all three are likely going to feed back into the pricing rule by influencing the decision of shareholders voting for the monopoly's pricing rule. We therefore look at a plausible mechanism of interaction between these elements.

In order to look at this issue, and contrary to traditional models of the monopoly that focus on decision-making based on profit maximization, we consider a production plan equilibrium where shareholders are also consumers of the firm's output and/or possibly employees of the firm. Where economic agents are simultaneously monopoly owners, consumers of the

monopoly's product, and employees of the firm, there are tensions in their preferences over price: *qua* owners they prefer a higher, profit-maximizing, price whereas *qua* consumers they prefer a lower, utility-maximizing, price, while *qua* workers they prefer a lower price and hence a higher employment income.

Simultaneous profit-maximizing and utility-maximizing behavior is likely to exist in the case of privatized public utilities, for instance. The person or group in charge of the production plan may be a corporate or industrial group interested in both the profitability of the firm, as well as a low price to be charged as consumers of the utility's product.¹

3.1.1 Conceptual issues

In our view, two hurdles have preserved the standard outside-manager approach, even in the case of a privatized public utility, where there are important issues neglected by the standard approach. First, the assumption of an outside manager obviates the need to formulate the shareholder's information and objectives: the sole objective is profit maximization. Second, having a group of decision makers would require a selection mechanism bringing together their possibly disparate interests into *one* consistent production plan for the firm. We look at one way to address these two issues in a formal setting, with the goal of developing a sensible framework to model the constraints a reformist central planner faces when contemplating the privatization of public enterprises.

There are a number of assumptions that can be made with respect to agents' information.

¹In the context of privatization, Vickers and Yarrow (1988), model explicitly the interaction between owners and managers with the aid of principal-agent techniques. They postulate that "shareholders seek to maximize their expected financial return (profit) from the company" (page 11). However, in the case of the privatization of utility companies, shareholders are also consumers, thus having an interest in a production plan going beyond solely financial returns (page 12).

Here we look at the simplest scenario, where agents are aware of all facets of aggregate demand, wage income and profit returns. That is, agents correctly conjecture the composition of aggregate demand facing the monopoly and are thus capable of assessing the impact of a price change on the economy as a whole.² Despite this simplification, the model as a whole captures important aspects of the real economy and the challenges faced by reformers.

Another element that is added to explore some of the policy issues related to the problem is the specific formulation of the labour market. We impose a particular disequilibrium or dual-market condition that permits us to have “insider” and “outsider” workers in a fashion similar to that of the employed vs. unemployed formulation of Harris and Todaro (1960). Thus insiders are the workers that, because of some rigidity, remain employed by the firm, whereas outsiders are those unemployed (for the firm provides the only source of employment in the economy).

As we consider agents behaving as consumers and producers in a shareholder economy with only one firm with market power, we need to address the issue of aggregating their various preferences.³ With respect to the selection mechanism mapping shareholders’ diverse preferences into a unique and coherent production plan, we use the concept of *sharehold-*

²Our approach shares much of the motivation of the study of general equilibrium models of monopolistic competition, dating back to at least the seminal paper by Negishi (1961), establishing the existence of a *Monopolistically Competitive Equilibrium* under the strong assumption of a subjective demand. Problems associated with the assumption of subjectivity were made evident by Nikaido (1975). Gabszewicz and Vial (1972) look at the endogeneity of pricing decisions in a general equilibrium context, imposing the behavioral restriction of profit maximization on the part of the consumer-owners. A good overview of the issues in this literature can be found in Hart (1985).

³This is similar in spirit to Cornwall (1977). Unlike him, however, we enrich the equilibrium concept to include majority voting among all shareholders instead of imposing one agent to decide on the production plan of the firm.

ers' majority voting derived from first principles of voting equilibria. For this purpose, we construct a generalization of Black's median voter theorem.

These elements together give a plausible mechanism for the analysis of some of the constraints a privatization program might face. Furthermore, we derive simple rules to estimate an efficiency measure of different, alternative, privatization programs.

3.1.2 Why share privatization is important

Greater exposure to competitive forces and a more competent management have been main motivating factors behind the drive towards privatization of state monopolies.⁴ Greater exposure to competitive forces is often not the outcome following privatization in the case of natural monopolies, as they still may require some form of regulation. One possible form of regulation is that provided by shareholders interested in low prices. As pointed out by Vickers and Yarrow (1988), consumption considerations of decision makers of the firm play an important role following the privatization of state utilities. This is particularly so in emerging market economies, where public enterprises played an integral component in the pre-privatization industrialization strategies. Two specific characteristics of the privatization experience in these economies underlie the present paper. On the one hand, several privatization programs in these regions incorporated some form of distribution of shares among

⁴This idea is explicitly mentioned in the World Bank's support of privatization in the context of its wider goals of economic development and poverty reduction, as presented by Kikeri, Nellis, and Shirley (1992). For instance, on page 6, they write "The economic benefits of privatization are maximized when governments make improved efficiency the number one goal-by using privatization to enhance competition and by ensuring a competitive market that reinforces the benefits of privatization." Furthermore they go on to state that "short-run distributional considerations, although they cannot be ignored, should not be pursued at the cost of managerial competence."

economic agents with no previous experience with capital markets. Thus, it is unlikely that the share recipients would have had sufficiently diversified portfolios ensuring a sufficient level of risk diversification rendering them indifferent to the consumption effect of changes in price. On the other hand, the output of state utilities is likely to have represented a large share of the consumers' budget. Agents on whom the production decision falls, would have in this situation an intrinsic interest in pricing the monopoly good lower.

The radical change from a centrally planned management to one based on shareholder boards would suggest an equally radical change of production objectives. Clearly, the procedure for arriving at a production plan in a shared enterprise entails a bargaining or electoral process with an outcome reflecting the diversity of preferences of the shareholders. Likewise, bringing on board labor's concerns in production decisions is likely to change effective marginal cost and revenue assessments. Also, small shareholders with keen interest on the firm's output could, when large as a group, put some downward pressure on price, thus serving as a channel for more efficient production. Therefore, besides the goal of empirical accuracy there is another important motive in formally addressing share-privatization in Eastern Europe and the developing world, namely the necessity to provide an assessment of the *efficiency* of such programs. As privatization is motivated precisely by the promise of sizable improvements in this area, one needs to be reminded that the *means* of privatization are an important aspect of the post-divestiture organization and behavior of the privatized monopoly.

One of the goals of reformers has been, in this respect, the achievement of a widespread ownership structure rather than a concentrated one. Kornai (1995), among others, has strongly made this point. His suggestion (page 81) of ascribing to an *embourgeoisement*

of society as a precondition to the success of a privatization program in Eastern Europe, for instance, is reminiscent of the idea of a uniform ownership presented below. He is also adamant about the central role ownership concentration plays for the economic structure that will evolve, when stating that "rather than the vast majority of the property being concentrated in the hands of a small group, there should also be a broad middle class that includes the masses of owners of small and medium enterprises". Furthermore, as argued by Weitzman (1985), profit sharing may also, through a regularizing role, have important efficiency implications for the production behavior of the firm. A central objective of emerging markets' privatization is the fostering of entrepreneurial skills, and thus requires a not insignificant degree of distribution of shares among workers and the population at large.

3.1.3 How to divest?

The scheme by which governments divest from public enterprises impinges upon the ultimate behavior of the privatized monopoly, and is therefore pivotal in establishing the effectiveness of the privatization program. Whether the combination of an auction and the redistribution of its proceeds is better than the outright distribution of shares among the population is an important question for policy makers in the newly reforming governments throughout the developing world.

Auction of public enterprises has tended to be favored in previous privatization programs. Besides offering clear advantages in managing the divestment, it is widely believed to provide well defined post-divestiture management structures. However, this option may be less attractive for planners in emerging markets, as it requires a considerable amount of resources and planning, especially considering the scale of some of the privatization efforts. Also, as

bidders incorporate future monopoly rents in their offers, auctions may perpetuate monopoly behavior and the associated production inefficiency.

3.1.4 Outline

In section 3.2 the structure of the model is specified. Section 3.3 presents the production problem and the choice of output that the different agents would prefer. In section 3.4 the voting mechanism that selects from among the diverse preferred plans of the owners/consumers/workers is presented.

Once the notion of equilibrium in the economy has been characterized, section 3.5 looks at some alternative objectives and forms of privatization program that central planners might consider, and evaluates their relative performance in terms of efficiency and distribution objectives.

In section 3.6 we explore the evolution of the ownership distribution once agents are allowed to trade shares. This section provides one rationale for understanding the post-privatization economy. Section 3.7 concludes.

3.2 The Model

We consider a simple economy that can be thought of as a small island. Two goods are consumed and people supply labor elastically up to an upper bound, which for the economy as a whole we denote by \bar{L} . Only one of the two goods consumed is produced in the island and we denote this good by C . We can think of the other, non-produced, good Z , as an import, or a natural resource that consumers have to pay for.⁵

⁵The non-produced good serves an important role in this kind of model, as it assures that part of the consumer's budget is spent on a good other than the good produced by the monopoly. See for example Hart

The monopoly charges a price p for good C and pays the ongoing wage rate w for the labor it hires. Because we only have three goods, we normalize the price of the non-produced good to unity.⁶ There is a population \mathcal{J} where agents are indexed by i .

There are two periods. Policy decisions, such as the production objectives of the monopoly or the distribution of shares for the monopoly, are taken in the first period. In the second period exchanges take place.

3.2.1 The Agents

Agents $i \in \mathcal{J}$ have identical, twice continuously differentiable, strictly quasi-concave and homogeneous of degree one utility functions $U = U(C, Z)$. This homotheticity formulation allows us to abstract from issues related to the effect on aggregate demand and welfare of changes in the distribution of income. The marginal utility of the non-produced good is well defined and equal to infinity for the first unit of consumption, thus specifying that Z is always consumed when income is positive (not all income is spent on the monopoly good).

The agent derives no disutility from labor, supplying it completely elastically at the going wage rate and up to a maximum level \bar{l} .

Given price p of good C and the income function $\omega : \mathcal{J} \mapsto \mathbb{R}^+$, a consumer i has two multiplicably separable twice continuously differentiable demand functions denoted by $c(p)\omega(i)$ and $z(p)\omega(i)$ for the consumption and non-produced goods respectively. The term $c(p)$ (resp. $z(p)$) represents the share of income spent on good C (resp. Z). Due to the multiplicative (1982). We could alternatively assume that a large (relative to the monopoly) constant returns technology with an additional assumption that changes in employment are small enough so as to be ignored

⁶Note that the price normalization in this non-competitive general equilibrium model is permissible given that there is no optimization made for the non-produced good's price. In the context of the island, the price of the non-produced good is given exogenously.

separability of the demand functions $c(p)\omega(i)$ and $z(p)\omega(i)$, the ratio of their expenditures, and thus of aggregate expenditure on C and Z , is decreasing in the price ratio p , that is

$$\frac{pc(p)\omega(i)}{z(p)\omega(i)} = \frac{pc(p)}{1 - pc(p)},$$

is non-increasing in p .

The associated indirect utility of individual i is given by

$$V(i, p, \omega(i)) = b(p)\omega(i), \quad (3.1)$$

where $b(\cdot)$ reflects the price-dependent proportionality of the indirect utility function to income characteristic of homothetic preferences. This term is decreasing and quasi-convex in p and continuous for strictly positive values of p .⁷ Integrating the demand functions over the space of agents \mathcal{J} , the corresponding *aggregate* functions are $c(p)\Omega$ and $z(p)\Omega$, where Ω is aggregate income.

An employed agent will receive a wage rate w for which she will inelastically supply labour up to a maximum given by \bar{l} . Therefore aggregate supply of labour is bounded above by $\bar{L} := \int_{i \in Q} \bar{l} di$.

In addition, consumer i has a share $a(i)$ of aggregate autonomous income A . Autonomous income held by a group of individuals $Q \subset \mathcal{J}$ is given by $\int_{i \in Q} a(i) A di$. Finally, an agent i receives a share $s(i)$ of monopoly profit Π . The group of agents $Q \subset \mathcal{J}$ has a fraction $\int_{i \in Q} s(i) di$ of firm's profit. A different ownership density s is, effectively, a different structure of ownership of the firm and thus of the overall distribution of income in the economy. The

⁷For example, the Cobb-Douglas utility function $U(C, Z) = C^\alpha Z^{1-\alpha}$, $\alpha \in (0, 1)$, has the associated indirect utility function $V(i, p, \omega(i)) = (1 - \alpha)^{1-\alpha} \left(\frac{\alpha}{p}\right)^\alpha \omega(i) := b(p)\omega(i)$ and demand functions $C(p, \omega(i)) = \frac{\alpha}{p} \omega(i) := c(p)\omega(i)$ and $Z(p, \omega(i)) = (1 - \alpha)\omega(i) := z(p)\omega(i)$.

share function is useful in modelling comparative distribution schemes for the initially public monopoly.

3.2.2 The Labour Market

We could easily model an equilibrating labour market: the wage rate w would guarantee that the labour demanded by the public sector monopoly be equal to the aggregate supply of labour \bar{L} . There would be full employment and all agents would have an identical labour income given by $w\bar{l}$.

However, we introduce an important rigidity in the labour market. We do so in order to address some central issues relevant in the context of reform or transition economies, where a dual-market is generally present: employment of less than the work-force in the public and monopolistic sector at an essentially given (or unionized) wage rate, and employment in the private sector at a lower wage rate.

We furthermore assume that the wage rate outside the monopoly sector is zero, an extreme but simplifying assumption implying that there is no employment outside the public sector.⁸ Thus, the first distinction we make is that between employed and unemployed agents, thereby partitioning the population in two classes: the employed E and the unemployed $J \setminus E$. There is systematic unemployment given by $\bar{L} - \int_{i \in E} \bar{l} := \bar{L} - \bar{L}_E > 0$. The assumption of a constant labour force E is reasonable for low fluctuations in the demand for labor.⁹ Transition economies often exhibit this kind of labour market rigidity, where current employees are important formal and informal stakeholders and the firm adjust employment

⁸The key aspect is that there be a difference between the wage rate paid in the monopoly sector and that there be no ready mobility between the sectors.

⁹That is, an increase (resp. decrease) in labor demand will result in rising (resp. falling) levels of employment for all the employed.

within this pool only.

A second rigidity we assume is that the labour demanded by the monopoly does not exceed the maximum labour supply available to it, \bar{L}_E . Since there is less than full employment of labour, the excess leisure could for example be spread equally over all the employed. However, here we also suppose that the burden of labour falls unevenly between the employed: an employed agent $i \in E$ is assigned a given share $l(i)$ of the labor demanded by the monopoly. The set of employed agents constitute the support of a labor share function $l(i)$, $E = \{i : l(i) > 0\}$.

The rigidities imposed on the labour market will help clarify the discussion in the context of the economy of interest about the role of a differential labour income for some privatization programs.

3.2.3 The Monopoly

Consider the problem facing a hypothetical manager of the firm when formulating a production plan. First, the specification of the information set of the manager should be consistent with the eventual outcome. Then, the production plan has to satisfy a specified objective. A production plan satisfying these two conditions is an *equilibrium* to the economy. Since the agent may or may not know the functioning of the economy as a whole, the representation of such agent's problem requires a specification of beliefs regarding market demand and its response to changes in p . Different beliefs can result in different conjectural equilibria.

A common specification which we use is presented in D'Aspremont, Ferreira, and Gérard-Varet (1990), Hart (1982) or Marschak and Selten (1974), where it is assumed that the agent has full knowledge of both the profit and labor effect of a change in p .¹⁰ It is natural to

¹⁰Alternatively we could have also specified that only the impact of p on profit, or the labor alone be

assume that the impact of a price change on profit will be known to the manager when she is also the sole owner. Similarly, it is also natural that agents employed are fully aware of the impact of a price change on labor income. We assume that whoever decides about the production plan of the firm is fully informed about both labor and profit earnings and can thus objectively specify aggregate income.

Consider the agent assessing aggregate income. Given the firm's labour demand $L(w, p)$, let $W(p)$ denote the conjectured aggregate income function and let \hat{A} denote the conjectured autonomous aggregate income, so that

$$\begin{aligned} W(p) &= \hat{A} + \underbrace{w L(p, w)}_{\text{conjectured labor earnings}} + \underbrace{p c(p) W(p) - w L(p, w)}_{\text{conjectured profit earnings}} \\ &= p c(p) W(p) + \hat{A} = \frac{\hat{A}}{1 - p c(p)}. \end{aligned}$$

Indeed, if autonomous income is correctly conjectured, conjectured income will be equal to actual income: defining the firm's profit by $\Pi(p, w) := p c(p) \Omega - w L(w, p)$, and integrating income over all agents gives aggregate income as

$$\begin{aligned} \Omega(p) &:= \int_{i \in \mathbf{I}} (a(i) A + l(i) w L(p, w) + s(i) \Pi(p, w)) di \\ &= A + p c(p) \Omega(p) = \frac{A}{1 - p c(p)}. \end{aligned}$$

Since some non-produced good must be consumed by all agents, as noted in section 3.2.1, it follows that $1 > p c(p) > 0$ for all feasible prices. Accordingly, the aggregate demand facing the monopoly, $C_d(p)$, is given by

$$C_d(p) = c(p) \Omega(p) = \frac{c(p)}{1 - p c(p)} A := H(p) A,$$

considered by the agent.

where $H(p)$ is twice continuously differentiable in p , and serves the role in aggregate demand that the function $c(p)$ plays in the individual's demand for the monopoly good.¹¹ $H(p)$ is strictly decreasing in p , as implied by the fact that $pH(p) = \frac{pc(p)}{1-pc(p)}$ is non-increasing in p , and thus gives the demand curve its downward sloping form. A smooth and downward sloping aggregate demand curve is guaranteed by the restrictions we have imposed on the agents' preferences.

For simplicity we assume that the monopoly's technology is linear, with output being proportional to labour

$$y = \left(\frac{1}{\lambda}\right) L, \quad \lambda > 0.$$

The linear technology significantly simplifies the structure of the firm's profit function. In market equilibrium, output of the firm will equal aggregate demand so that the profit function can be written as

$$\Pi(p, w) = (p - w\lambda) H(p) A, \quad (3.2)$$

where costs enter profit in a linear fashion. The profit function inherits the smoothness properties of the aggregate demand function $C_d(p)$ and is thus twice continuously differentiable in p .

We assume that the monopoly cannot price C below average cost, as the privatization takes place in a context of hard budget constraints. Therefore, given the wage rate w , the price is bounded below by $w\lambda$ and $p \geq w\lambda$. Letting \bar{p} denote the maximum price any agent is willing to pay for the monopoly-produced good and still derive benefit from its consumption, the set of prices feasible to the monopoly's manager is therefore $[w\lambda, \bar{p}]$.

¹¹If the monopoly good is the only good, $H(p) = c(p) = 1/p$.

Production possibilities are also bounded since $\frac{\bar{L}}{\lambda}$ is the maximum level of output the firm can produce.

We furthermore require the aggregate demand function not to be ‘too’ convex in price, so that the profit function is convex. That is, the second derivative $H''(p)$ satisfies for all prices $p \in [w\lambda, \bar{p}]$, the following constraint¹²

$$H''(p) < -\frac{2H'(p)}{p - w\lambda}.$$

3.3 Production Plans

We first consider the benchmark specification of profit maximization in the firm. We refer to this production plan by *pure monopoly plan*. In reference to this benchmark we then illustrate the effect of introducing a consumption decision in the maximization exercise of the firm’s manager.

This arrangement is referred to as *the consumer as a manager plan*. It describes the situation when a given consumer i is selected from the population of consumers to decide on the production of the monopoly. In particular, we look in particular at the trade-off between i ’s interests as consumer, as possible employee, and as owner of a share $s(i)$ of the profit of the firm. Agent i ’s optimal decision will set the marginal benefit of a price increase on profit equal to the marginal consumption loss and, when she is also a worker, its marginal impact on labour income.

This simple decision mechanism will underlie the remaining sections of the paper.

¹²The conjectured marginal propensity to consume in the case of the Cobb Douglas preferences is given by $H(p) = \frac{\alpha}{(1-\alpha)p}$, which is decreasing in p for all $\alpha \in (0, 1)$. It can be shown this requirement in the Cobb-Douglas case is $\frac{1}{p} < \frac{1}{p-w\lambda}$, which clearly holds since $p \geq w\lambda > 0$.

3.3.1 The Pure Monopoly Plan

In the pure monopoly benchmark case, we calculate the price that results from pure profit maximization. The decision-maker in this case does not consume the output of the monopoly, and so ignores the effect of price on consumption. The decision-maker chooses the price $p^{**}(w)$ at a given wage rate $w \geq 0$ so that

$$p^{**}(w) \in \arg \max_{p \in [w\lambda, \bar{p}]} \Pi(p, w);$$

$$W(p^{**}(w)) = \Omega(p^{**}(w))$$

are jointly satisfied.

Proposition 3.1. *A pure monopoly plan exists and is unique.*

Proof. See Appendix A.1.1.

The pure monopoly plan will serve as a reference for the results of section 3.3.2 below. In particular, we note that given the concavity of the profit function (3.2), profit is increasing in prices for $p \in [w\lambda, p^{**}(w)]$.¹³

3.3.2 The Consumer as Manager Plan

Now consider the decision problem facing a consumer i when she is chosen to decide the pricing strategy of the monopoly. Her decision is enriched by the fact that she has a share $s(i)$ of the firm's profit, while being a consumer of the monopoly-produced good. As consumer,

¹³In the Cobb Douglas case the monopoly has constant returns to scale. Therefore, whenever $p \geq w\lambda$ the pure monopoly plan is the price defined by full capacity, i.e.

$$p^{**}(w) = \{p : L(p, w) = \bar{L}\}.$$

she benefits from lower prices, while on the other hand, higher prices represent higher profit earnings. The choice of output depends on the the agent's employment and consumption levels and her autonomous income, as well as on her share of the monopoly's profit. This trade-off is the first element central to this paper.

We begin by considering agent i 's income, consisting of its autonomous income, its labor income (when employed) and its profit income. Rewriting income within the indirect utility function (3.1) and factoring out her profit share $s(i) A$, results in the following reduced form indirect utility function¹⁴

$$\begin{aligned} V(i, p, w) &= b(p) \left(a(i) A + l(i) w L(p) + s(i) \Pi(p, w) \right), \\ &= s(i) A b(p) \left[\frac{a(i)}{s(i)} + \left(p - w \lambda \left(1 - \frac{l(i)}{s(i)} \right) \right) H(p) \right]. \end{aligned} \quad (3.3)$$

An important implication for the monopoly's pricing exercise is evident from expression (3.3). Consider a consumer i_o , the sole owner of the profit of the firm, with $s(i_o) = 1$. Consider also that she does not work as an employee of the firm, and has no income. Thus $l(i_o) = a(i_o) = 0$. The price preferred by her is the upper bound for all other agents, for no other agent will have a greater incentive to raise the price. It is interesting, however, that this agent's preferred price will be lower than the pure profit maximization one whenever $b'(p) < 0$.¹⁵

¹⁴The dependence of the reduced form indirect utility function on the profit share distribution $s(\cdot)$ and other density functions is implicit.

¹⁵For prices higher than $p^{**}(w)$, the marginal increase in profit will not fully compensate the consumer-owner since at $p^{**}(w)$, the first derivative of effective profit is zero and the derivative of the objective (3.3) with respect to prices is

$$D_p V(i_o, p, w) = b'(p^{**}(w)) \left[a(i_o) A + \Pi(i_o, p^{**}(w), w) \right] < 0.$$

We use the operator D_x denoting the partial derivative with respect to argument x , $\frac{\partial}{\partial x}$. Accordingly D_{xx}

The selection of a manager from among consumers effectively reduces the price the monopoly charges for the consumption good. If the chosen consumer is also a worker, the selected price is further reduced, for the optimal price reflects the wage income loss resulting from lower production levels.¹⁶ This serves as proof for the following lemma.¹⁷

Lemma 3.2. *No agent will select a price higher than that of the pure monopoly plan.*

Existence and Uniqueness of the Private Plan

A *private plan* is the solution to the maximization by a consumer-owner of her objective function (3.3), given that her demand conjecture is satisfied. It represents the hypothetical situation where agent i is called upon to decide the production plan of the firm and her conjectures are satisfied.

Definition 3.1. Given an agent i (the manager), a given density $s(\cdot)$ (the ownership distribution) denotes $D_x D_x = \frac{\partial^2}{\partial x^2}$ and D_{xy} denotes $D_x D_y = \frac{\partial^2}{\partial x \partial y}$.

¹⁶Labor income lowers the perceived cost component of profit. This is evident from (3.3), where we observe an *effective profit function* to agent i , with marginal cost equal to

$$\lambda(i) := \left(1 - \frac{l(i)}{s(i)}\right) \lambda < \lambda.$$

Defining

$$\Pi(i, p, w) := (p - w\lambda(i))H(p)A$$

we can write the objective function of the individual in terms of effective profit:

$$V(i, p, w) = b(p) [a(i) + s(i) \Pi(i, p, w)].$$

Noting that $\lambda(i) \leq \lambda$ we can show that the price that maximizes the objective of a worker-owner is lower than that of the pure manager. Moreover, this price increases with $s(\cdot)$ for it is clear that $\lambda(i)$ is larger the larger is $s(\cdot)$.

¹⁷Lemma 3.2 is useful in the proof of the existence of solution to the maximization problem of the selected decision maker. We use the fact that the preferred price of all agents is bound above by the pure monopoly price so as to make the range of prices available to the agents compact.

bution) and a wage rate w , we define a *private plan* price $p^*(i, w)$ (the price preferred by the manage) in agent i 's private plan correspondence $P^*(i, w)$ (which is subject to a feasibility restriction on the price domain) when both

$$p^*(i, w) \in P^*(i, w) = \arg \max_{p \in [w\lambda, p^{**}(w)]} V(i, p, w);$$

$$\Omega(p^*(i, w)) = \frac{\hat{A}}{1 - p^*(i, w)c(p^*(i, w))},$$

are satisfied.

The first property specifies the private plan as the manager's optimal pricing strategy given a domain constraint on prices, the second one requires her conjectures to be fulfilled.¹⁸ An additional desirable property to impose on the population parameter distributions $a(\cdot)$, $l(\cdot)$ and $s(\cdot)$ is that of continuity across our indexed consumers, for then the maximization problem of the consumer-owner is simplified.¹⁹

Proposition 3.3. *For each agent $i \in \mathcal{I}$ there exists a private plan where the maximized objective and maximizing price, given by*

$$M(i, w) = V(i, p^*(i, w), w);$$

$$p^*(i, w) \in P^*(i, w),$$

are both continuous functions of i and $P^(i, w)$ is an upper-continuous correspondence.*

Proof. Straight application of the Theorem of the Maximum in the version provided by Varian (1992, p.506). □

¹⁸Note that an agent may well prefer a price inferior to $w\lambda$ in the maximization of the first condition, but this price is not feasible, thus resulting in a corner solution.

¹⁹The continuity requirement of the autonomous income and the share densities can be achieved even for a finite population by endowing the space of agents \mathcal{I} with an appropriate topology. We present the paper so as to allow for both a continuum of agents and for a finite number of them. In the case of a continuum of agents, even if there was a finite number of discontinuities, the problem could be arbitrarily approximated with a continuous function.

The issue remains as to whether the optimal price for an agent i is *unique* over $P(i, w)$, i.e. whether the correspondence $P^*(i, w)$ is single-valued.²⁰ We assume a sufficient condition for the uniqueness of the solution to the private plan, thus guaranteeing the uniqueness of function $p^*(i, w)$ and we show it holds for CES preferences in appendix A.2.

Assumption 3.1. *Given the terms*

$$X(p) := p H(p) + \frac{b(p)}{b'(p)} [H(p) + p H'(p)] > 0;$$

$$Y(p) := H(p) + \frac{b(p)}{b'(p)} H'(p) > 0,$$

$-X(p)$ and $\frac{X(p)}{Y(p)}$ are both either jointly increasing or jointly decreasing in p with at least one being strictly monotone.

Assumption 3.1 guarantees that the objective function over $[w\lambda, p^{**}(w)]$ is *single-peaked* and thus makes the consumer-owner's preferred price unique. The use of this assumption is clarified in the proof of the following proposition.

Proposition 3.4. *Under assumption 3.1, correspondence $P^*(i, w)$ of proposition 3.3 is single valued and thus $p^*(i, w)$ is a continuous function of i .*

Proof. See Appendix A.1.1.

So far we have constructed our model so that each agent, if selected to manage the monopoly, prefers a unique price $p^*(i, w)$. We now turn to the properties of this preferred price.

²⁰This would of course result if the objective was concave in prices over this range. From appendix A.4.1 and the observation that $b(p)$ and $D_p \Pi(i, p, w)$ have opposite slopes over $[w\lambda, p^{**}(w)]$, it would suffice if $b(p)$ was concave. However, it can be shown that for Cobb-Douglas and CES preferences $b(p)$ is in fact convex. Nevertheless, a not too convex function $b(p)$ would suffice, as noted in appendix A.4.2. This is the case for the Cobb-Douglas specification of preferences. However, even if this is not the case and the objective function $V(i, p, w)$ is convex for $p \in [w\lambda, p^{**}(w)]$, further restrictions preventing multiple turning points in the objective function would result in a unique private equilibrium for all agents, albeit with the possibility of being at the lower edge of $[w\lambda, p^{**}(w)]$.

Properties of the Private Plan

Consumer i 's preferred price depends on both her profit and autonomous income shares. The larger the profit share, the more willing she will be to forego the benefits as a consumer at lower prices. If the pricing decision falls upon a consumer with no ownership, she will choose a price as low as possible, for she has no incentive to increase prices. The agent's share of labor $l(\cdot)$, her autonomous income share, $a(\cdot)$, and her share of profits, $s(\cdot)$, fully characterize her pricing behavior. Two agents with an equal triple of values for these three values $(a(i), l(i), s(i))$, will be identical in terms of their desired price $p^*(i, w)$.

We now characterize the sensitivity of the preferred price function $p^*(i, w)$ of an agent i to changes in her share of autonomous income, labor and profit.²¹

Proposition 3.5. *A density $a(\cdot)$ assigning agent i a larger autonomous income or a density $l(\cdot)$ assigning agent i a larger labor share do not increase her preferred price. Similarly, a density $s(\cdot)$ assigning agent i a larger profit share does not decrease her preferred price. That is, the preferred price $p^*(i, w)$ of any agent $i \in \mathcal{I}$ is*

- (i) *decreasing in her labor share,*
- (ii) *decreasing in her autonomous income share,*
- (iii) *increasing in her profit share.*²²

Proof. See Appendix A.1.2.

Since the focus of the paper is the decision-making process of shareholders, we concentrate on the segment of the population in the support of the ownership distribution, that is $\{i \in \mathcal{I} : s(i) > 0\}$. It is legitimate for this subset of the population to divide the objective function

²¹For a given agent i , a change in her parameters $a(i)$, $l(i)$ and $s(i)$ results from a change in the parameter densities $a(\cdot)$, $l(\cdot)$ and $s(\cdot)$ over the population as a whole.

²²When either $b(p)$ or $\Pi(p, w)$ is strictly monotone in p , the relationships are all strict.

by $s(i)A$ a positive value, thus giving

$$\frac{V(i, p, w)}{s(i)A} = b(p) \left[r_a(i) + (p - w(1 - r_l(i)) - \lambda) H(p) \right], \quad (3.4)$$

where

$$r_a(i) := \frac{a(i)}{s(i)} \quad r_l(i) := \frac{l(i)}{s(i)},$$

and it is clear that we can reduce the dimensionality of the agent's objective function from three "exogenous" densities to two ratios of such densities. Note that this transformation of an agent's indirect utility function leaves the first order condition unaffected, and thus leaves unaltered the agent's preferred price.

At the same time as serving as a 'modified income parameter', the ratio $r_a(\cdot)$ measures the "trade-off" between $a(\cdot)$ and $s(\cdot)$. That is, the effect of the level of autonomous income on the agent's optimal price will be weighted by its share of profit. The higher the profit share, the lower will be the impact of more autonomous wealth on the optimal price $p^*(i, w)$. Similarly, the ratio $r_l(i)$ captures the relative importance of labor income to profit earnings. Both terms "weighted" by the ownership share, reflect an inherent trade-off that exists between both non-profit income shares and $s(\cdot)$ within the indirect utility function. Particularly interesting is the ratio $r_l(\cdot)$, as it illustrates that the trade-off the agents faces as a worker and as a claimant to a share of profit is special in the formulation of the agent's preferred price. In order to see this, consider an equal (may be egalitarian or not) distribution of shares and labour, such that $r_l(i) = 1$ for all i . Condition (3.4) becomes

$$\frac{V(i, p, w)}{s(i)A} = b(p) \left[r_a(i) + pH(p) \right], \quad (3.5)$$

from where it is clear, since both $b(p)$ and $pH(p)$ are decreasing functions of prices, that *all* agents will prefer the lowest possible price and the only possible equilibrium is the pro-

ductively efficient average cost pricing, *regardless of the distribution of autonomous income*. Clearly, an egalitarian society (where all agents are identical) will be characterized by the same outcome.

In the more general case of arbitrary autonomous, labour and profit income distributions, proposition 3.5 characterizes the unambiguous response of agent i 's preferred price $p^*(i, w)$ to changes in her ratios $r_a(i)$ and $r_l(i)$: an increase in either of the ratios will not raise her preferred price. Agents with lower autonomous or labor income ratios will prefer a higher optimal price. Furthermore, because both ratios have the same effect on an agent's optimal price, proposition 3.5 suggests that it should be possible to synthesize both ratios within a unique indicator function of the agents *characteristics*. It is clear from (3.3) that a well-chosen weighting of the two ratios within the objective function results in

$$\frac{V(i, p, w)}{s(i) A} = b(p) \left[k(i, p, w) + (p - w\lambda)H(p) \right],$$

where

$$k(i, p, w) := r_a(i) + w\lambda H(p) r_l(i),$$

is a function that synthesizes the two ratios $r_a(\cdot)$ and $r_l(\cdot)$ encompassing the agent's characteristics. It is now evident how to define an index of the consumer's characteristics. We call this index *the characteristic function* of an agent i , and we denote it by $k(i, w)$. We substitute the agent's optimal price in the expression for $k(i, p, w)$ above, resulting in

$$k(i, w) := k(i, p^*(i, w), w), \quad (3.6)$$

Two agents with identical values of $k(i, w)$ are, when selecting a production plan, identical. Hence they will both choose the same price for the monopoly if asked to do so. Note

that, being a continuous function of continuous functions, the characteristic function is also continuous in its arguments. For example, in the case of Cobb Douglas-type preferences, it can be shown that the preferred price $p^*(i, w)$ is given by

$$p^*(i, w) = \begin{cases} \frac{(1+\alpha)(1-r_l(i))}{(1-\alpha)r_a(i)+\alpha} w\lambda & \text{for } 1 > (1-\alpha)r_a(i) + (1+\alpha)r_l(i), \\ w\lambda & \text{otherwise,} \end{cases}$$

where the second case follows from the non-negativity requirement we have imposed on profit. Thus, agents with $1 > (1-\alpha)r_a(i) + (1+\alpha)r_l(i)$ prefer prices above average cost, whereas other agents have to be content with prices equal to average cost.²³

The dependence of the characteristic function on the ownership distribution of the monopoly $s(\cdot)$ is implicit in the ratios $r_a(\cdot)$ and $r_l(\cdot)$, and the agent's optimal price $p^*(\cdot)$. We are interested in modelling how the choice of such distribution may affect agents' preferences. As an illustration of what prices agents with different characteristics may prefer, consider a finite number N of agents with ownership and work distributions assigning each a share $s(\cdot) = \frac{1}{N}$ of profits and labor, an *egalitarian distribution in labour and profits*. The term $r_l(i)$ is equal to unity for all agents in this economy: all agents are employed and have an equal share of profit and labor. Thus, from the discussion of page 91 it is clear that in this economy average cost is all stock owners get.

As another example, consider an economy where the labor force consists of E identical employed agents, with $E < N$, and where profit is, as before, distributed equally among the

²³Although it is not the characteristic function, the function $(1-\alpha)r_a(i) + (1+\alpha)r_l(i)$ could also serve the same purpose.

N agents. Thus,

$$r_l(i) = \begin{cases} N/E & \text{if employed} \\ 0 & \text{otherwise,} \end{cases}$$

and because $N > E$ ($r_l(i) > 1$ for the employed), she would prefer a lower price than in a situation where all agents are employed ($N = E$, $r_l(i) = 1$, and the characteristic is lower). An unemployed agent, on the other hand, would prefer a higher price than in the previous case, since now she won't take into account the labor component of effective profit. In the Cobb Douglas case of page 93, for instance, her preferred price will equal $\frac{1+\alpha}{(1-\alpha)r_a(i)+\alpha}w\lambda$, which is higher than average cost provided that $r_a(i) < 1/(1-\alpha)$. A majority of unemployed agents with a share of autonomous income that is sufficiently low will, in this case, vote for a price higher than average cost.

As a final example, consider a population with three types of agents: the employed E who now have no autonomous income, the rich R who do not work and who own a positive and equal share of the totality of autonomous income, and the remaining destitute who neither work nor possess autonomous income. The values of the characteristic ratios are accordingly,

$$r_l(i) = \begin{cases} N/E > 1 & \text{if a worker} \\ 0 & \text{otherwise,} \end{cases} \quad r_a(i) = \begin{cases} N/R > 1 & \text{if rich} \\ 0 & \text{otherwise.} \end{cases}$$

The destitute have a characteristic value of zero, whereas workers will behave as in the previous example. However, the further decomposition of the non-working population into the destitute and the rich classes adds a twist to the story. For Cobb-Douglas-type preferences, the rich, with $r_l(\cdot) = 0$ and $r_a(\cdot) > 1$, prefer a price equal to $\frac{1+\alpha}{(1-\alpha)N/R+\alpha}w\lambda$ and therefore higher than average cost if $R > (1-\alpha)N$. A rich agent's preferred price,

however, is lower than that preferred by a destitute agent, as the latter prefers a price equal to $\frac{1+\alpha}{\alpha}w\lambda$. The agent demanding the highest price and consequently the most inefficient production plan, is the destitute one, as her profit share is her sole source of income.

Although other illustrative distributions of labor, income and shares are possible, it is evident from these examples, that the allocation of employment, autonomous income and ownership, as well as the identity of the decision maker are essential in establishing the production plan of the firm.

In essence, proposition 3.5 substantiates the statement that the agent's preferred price is also characterized by her characteristic function. To arrive at this result, we chose a convenient definition of the characteristic function and used the fact that $H'(p) < 0$. This close relationship between $p^*(i, w)$ and $k(i, w)$ is captured in the following proposition:

Proposition 3.6. *The price $p^*(i, w)$ a consumer $i \in \mathcal{I}$ would select when deciding on the price of the consumption good is uniquely for $p^* > w\lambda$ and inversely related to her characteristic function $k(i, w)$.*

Proof. Follows immediately from the definitions of $k(i, p, w)$ and the characteristic function (3.6), noting that $H'(p) < 0$. □

According to proposition 3.6, an ordering of representatives in \mathcal{I} according to their characteristic value is possible. This ordering is monotone with respect to the agent's preferred price. Thus, we have defined a continuous mapping between the set of preferred prices, $\mathbf{P}^* = \{p^*(i, w) \mid i \in \mathcal{I}\}$ and the characteristic values of the agents. An example of such a mapping is given in figure 3.1.

This mapping will prove useful in the next section, where a characterization of a shareholders' equilibrium is introduced. We thereby propose a mechanism for choosing the identity of the decision maker j_m based on majority shareholders' vote.

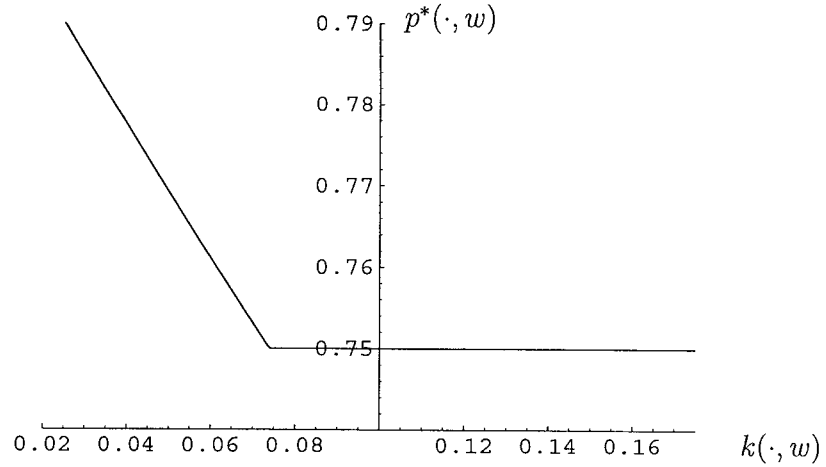


Figure 3.1: Mapping between the preferred price $p(\cdot, w)$ and the characteristic function $k(\cdot, w)$ for Cobb-Douglas-type preferences, with parameters $\alpha = .48$, $\lambda = 1.5$, $w = 0.5$, and normally distributed (with different means) shares.

3.4 Voting for a Production Plan

The price the monopoly eventually charges for the consumption good is determined by the preferences of the given consumer taking the pricing decision. Who this consumer is, in turn depends on the *selection procedure* selecting this consumer from the population. The situation the paper considers is one where all consumer-owners are active in the decision making process of the monopoly, and their opinion counts according to their *weight* $s(\cdot)$: the decision about production is a process undertaken jointly by shareholders. We look at shareholders' majority voting since we consider it a possible, and indeed frequently practiced, process for deciding on a firm's production plan. Instead of the one-person-one-vote type of voting, however, in our context we deal with a decision making process involving one-share one-vote. Accordingly, we define the notion of a shareholder voting equilibrium establishing the median-share holder.

Since agents with the same characteristic value prefer an identical price for the consumption good, we first “bundle” them in groups. Having so aggregated agents with identical characteristic, we assign a *representative* for each group of bundled agents. A representative has the price mandate and the share weight given by those of the group she represents. The representative of agents with characteristic $k(i, w) = j$ will be denoted by j . The population of all representatives is denoted by \mathcal{J}_k . We derive the space \mathcal{J}_k of representatives from the original population \mathcal{J} in Appendix A.3. Important in this exercise is that the notion of “shareholding” be preserved: the representative is assigned a weight in relation to the shares of the members of the group it represents.²⁴ A representative votes for the price that maximizes the indirect utility of the agents in the group she represents, and has the “weight” of the total amount of shares of the group.

Since the characteristic across the population of representatives is uniquely and inversely related to the preferred price, by ordering them according to the characteristic value, we “line up” representatives according to the price they have mandate for.²⁵

With this strictly monotone ordering of representatives according to their characteristics k in \mathbf{K} , and the associated monotonic preferred price schedule (in the range \mathbf{P}^*), we use a transformation of Black’s median voter theorem to select the representative whose preferred price is charged by the monopoly.²⁶ Accordingly, we denote this result *the median-share*

²⁴If $S(Q) = \int_{i \in Q} (s \circ \varphi) di$ is the share of profits held by agents of a group Q in the original population, we formulate the equivalent “size” function $\varsigma(Q_k) = \int_{j \in Q_k} s dj$ for the share of profits held by their representatives (conforming group Q_k), properly defined in their population \mathcal{J}_k .

²⁵Indeed, a formal equivalence between the original population of agents and the range of the characteristic function permits the formulation of the results in terms of the latter alone. We note that the range \mathbf{K} of the characteristic function of the initial agents and that of their representatives is, by construction, the same.

²⁶Whereas in the context of the median voter theorem each agent has one vote, in our context the representative has voting weight proportional to the number of shares of the agents represented.

holder theorem.

3.4.1 A concept of equilibrium: the median share

We begin with the definition of the median shareholder and then state the main result.²⁷

Given a price $p \in \mathbf{P}^*$ desired by one of the representatives, consider the following groups of representatives

$$B(p) = \{j \in \mathcal{J}_k \mid p^*(j) \leq p\};$$

$$U(p) = \{j \in \mathcal{J}_k \mid p^*(j) \geq p\},$$

denoting representatives with lower and higher preferred prices. The respective shares held by each group is given by $\varsigma(B(p))$ and $\varsigma(U(p))$.²⁸ The *lower and upper median shareholders* j_m^l and j_m^u are those representatives satisfying

$$\begin{aligned}\varsigma[B(p^*(j_m^l))] &\geq \frac{1}{2}; \\ \varsigma[U(p^*(j_m^u))] &\geq \frac{1}{2}.\end{aligned}$$

In a population with a continuum of representatives, the lower and the upper median-share holders are, by the continuity of $k(\cdot)$, the same and thus the median-share holder is unique. With a finite number of representatives, the lower and higher median-share holders may be different, reflecting a situation similar to the even number of agents in Black's medium voter theorem. In the present context, because the representative's weight is given by her ownership share and not by her vote, we need to modify the standard odd-number requirement of medium voter schemes. In order to do this, we note that a problem exists

²⁷In order to simplify notation we drop the wage rate from the list of arguments.

²⁸The profit-share ownership density $\varsigma(\cdot)$ for representatives is derived from that of the original agents, $s(\cdot)$ in Appendix A.3.

only when

$$\varsigma [B (p^*(j_m^l))] = \varsigma [U (p^*(j_m^u))] \quad \text{and} \quad j_m^u \neq j_m^l.$$

The requirement of an odd number of shareholders does not prevent two separate groups, each with fifty percent ownership and different supported price, to exist, as the problem arises in relation to the ownership density of the groups and not with the number of group members. If the lower and the higher median shareholder are different, a situation where the shareholders cannot reach a pricing agreement, and thus with no equilibrium, may exist.²⁹ We need to specify a condition, equivalent to the odd-number requirement in Black's theorem, that guarantees an agreement will be reached. We therefore *assume that the median-share holder j_m is unique*, such that

$$\begin{aligned} \varsigma [B (p^*(j_m))] &> \frac{1}{2}; \\ \varsigma [U (p^*(j_m))] &> \frac{1}{2}. \end{aligned}$$

The median-share (representative) holder is pivotal for the majority voting equilibrium and uniquely establishes the pricing behavior of the privatized monopoly.

Theorem 3.7 (The median-share holder theorem). *Under a shareholder majority voting rule, the price the privatized monopoly charges for the consumption good C is that preferred by the median-share holder, i.e. $p^*(j_m)$.*

Proof. See Appendix A.1.3

While this theorem enables to compute the equilibrium, it does not provide a ready method of doing comparative statics on the equilibrium price with respect to changes in the

²⁹For instance, three representatives (an odd number) with shares 1/4, 1/4 and 1/2 would have the second one as the lower median-share holder, and the third one as the upper median-share holder.

joint distribution $\{a(\cdot), l(\cdot), s(\cdot)\}$. In particular, the indexing scheme j must be recomputed for each selection of a joint distribution $\{a(\cdot), l(\cdot), s(\cdot)\}$ from the set of such distributions. This re-indexing makes it difficult to associate a change in the joint distribution with the consequent equilibrium change in the identity of the median-share's owner and that individual's value of $k(\cdot)$. It is clear that the mapping between the underlying joint distribution $\{a(\cdot), l(\cdot), s(\cdot)\}$ to the equilibrium value of $k(\cdot)$ is not one-to-one.

A particular value of $k(\cdot)$ for the owner of the median share, and hence the equilibrium value of p , can be supported by an open subset of the set of joint distributions $\{a(\cdot), l(\cdot), s(\cdot)\}$. It is not required that the identity of the median-share's owner be preserved for the value of $k(\cdot)$ to recur. On the other hand, changes in the joint distribution may also be envisaged for which the identity of the median-share's owner is maintained in the new equilibrium, though her value of $k(\cdot)$, and hence the equilibrium price, changes.

In view of this general indeterminacy of comparative static analysis, we turn below to providing analysis of very specific examples, where the relation between changes in the joint distribution $\{a(\cdot), l(\cdot), s(\cdot)\}$ and the equilibrium value of $k(\cdot)$ can be tracked precisely.

3.4.2 On the importance of who decides

Any redistribution that affects the characteristic value of the resulting ex-post median-share holder will affect the equilibrium price. Furthermore, the median-share holder's shares of autonomous and labor income, by defining her characteristic value, are central to the price that she chooses. For instance, it may matter whether the median-share holder belongs to a poor (destitute of autonomous income) or unemployed (destitute of labour income) segment of the population, as was illustrated in the example of page 94. The lower the median-share

representative's ratios $r_a(j_m)$ and $r_l(j_m)$, the lower is her characteristic, and the higher the price she will choose. *Location*, defined in this sense, is important for the selection of an equilibrium. It is not, however, the only important aspect.³⁰

A second, and equally important, element is the role of the median-share holder's share of profits, i.e. the *concentration* of the share distribution $\varsigma(\cdot)$: the higher her share of profit, the lower is her characteristic value, and thus the higher her preferred price. Given her autonomous and labour income, a representative with full ownership (100%) will have her lowest possible characteristic value, and will accordingly prefer her highest possible personal preferred price. This is so because a higher concentration decreases the median-share holder's $r_a(j_m)$ and $r_l(j_m)$ ratios, thus lowering her characteristic value and increasing her preferred price, while at the same time concentrating the decision making among fewer individuals "near" the median-share holder.³¹

In order to characterize this effect of $\varsigma(\cdot)$ on the equilibrium price, we define a clear, though not unique, notion of *concentration*. A distribution of shares $\varsigma_1(\cdot)$ having a median-share representative j_m is said to be *more concentrated* than an alternative distribution $\varsigma_2(\cdot)$, if, while preserving the representative j_m , it assigns her a larger share of the firm's profit. Using this definition of concentration that preserves the median-share holders we formalize the comments on location and concentration.

Proposition 3.8. *Under majority voting, ceteris paribus, share ownership schedules $\varsigma(\cdot)$*

³⁰One would otherwise be tempted to concentrate ownership among the groups of the population with an incentive to charge low prices, such as the wealthy or employees of the monopoly.

³¹A policy makers' legitimate equity considerations in choosing the ownership schedule may be associated with a perverse outcome: a well intended policy directed to the poor or unemployed, thus concentrating ownership among individuals with low autonomous and labor incomes, may have a perverse outcome since ownership is concentrated on a high-pricing group.

that

- (i) are more concentrated;
 - (ii) allocate a larger proportion of the shares among the poor;
 - (iii) allocate a smaller proportion of shares among employees;
- result in a higher price for the monopoly-produced good C .

3.5 Privatization Schemes

Having defined a notion of equilibrium subject to a distribution $s(\cdot)$ of shares, we look into the question of distributing the shares to attain a pre-determined objective. The mechanism establishing the endogenously-decided-upon price works through the selection of a median-share holder representative and the price preferred by her. We now turn to the question of how the planner, knowing the median-share pricing decision mechanism, might decide on the initial shareholding distribution $s(\cdot)$ in a privatization scheme. In the framework we have developed, we look at a static allocation of the shares first (where the shares are fixed by law or where trading is not allowed after the initial divestiture), and briefly discuss the implication of posterior trading in the pricing evolution.

As a benchmark with which to contrast the distribution mechanisms below, consider first the *pricing* decision taken by a benevolent central planner ‘adding up’ with equal weighting the indirect utility of all agents. The central planner’s price selection problem is

$$\max_p \left\{ V(p) = \int_{j \in J_k} V(j, p, w) dj = b(p) [1 + p H(p)] A \right\}, \quad (3.7)$$

such that $p \geq w\lambda$.

Since both $b(p)$ and $p H(p)$ are decreasing in the price of the consumption good, it is clear that the central planner should set a price as low as possible. The restriction of non-negative

profits results in a corner solution with price equal to average cost $w\lambda$. Thus, selecting the “optimal” price results in average cost pricing.

The planner’s price-selection optimum can occur as an equilibrium of the system. For example, if we have a homogenous distribution, such that $s(j) = l(j) = a(j) = 1/N$ for all j , then, in particular, $r_l = 1$ for all j . It follows from equation (3.4) on page 91 that the optimal p in this case satisfies $p = w\lambda$. More generally, for any joint distribution $\{a(\cdot), l(\cdot), s(\cdot)\}$ such that the owner of the median-share, j_m , has $s(j_m) = l(j_m)$, then $r_l = 1$ and price must equal average cost. That is, whenever the median share holder has equal stakes in the firm as a worker and as a profit claimant, average cost pricing will result, as the contradicting incentives that the median-share owner cancel out and only the incentive as a consumer prevails.

On the other hand, and as figure 3.1 suggests, if the median share is held by a pure consumer ($l(j_m) = 0$), then the combination of high $a(j_m)$ and low $s(j_m)$ will raise the value of $k(j_m)$ such that average cost pricing results.

We illustrate this issue in the context of a simple example: that of an egalitarian distribution of shares among agents (a flat share distribution, $s(\cdot) = 1/N$), thus following the examples given on page 94. The outcome is decided in this case by the population proportions of the different income groups. As there are N agents with Cobb-Douglas type preferences who belong to three “classes”, there are W workers, R rich and $N - W - R$ destitute. Since within classes agents have an identical characteristic, aggregation results in at most three representatives, say j^w , j^r , and j^d . The derived ownership share of the three representatives is, accordingly, $\varsigma(j^w) = W/N$, $\varsigma(j^r) = R/N$ and $\varsigma(j^d) = (N - R - W)/N$. Given j^w ’s, j^r ’s and j^d ’s shares of other income sources, we know (see the final example of

page 94) that

$$p^*(j^d) > p^*(j^r) \geq p^*(j^w) = w\lambda.$$

As agents have distinct characteristic values, $p^*(j^r) = p^*(j^w)$ can only hold when both representatives' preferred price is equal or lower to the average cost $w\lambda$. Recalling the results of page 94, this depends on whether $(1 - \alpha)N/R$ equals or is less than one. Suppose $\alpha \leq \frac{1}{2}$ and that the classes' sizes R and N are such that $R > (1 - \alpha)N$. Then $p^*(j^r) > p^*(j^w)$ and the rich provide the median-share holder, as $\varsigma(j^r) = R/N > 1/2$. The price selected by their representative is $p^*(j^r) = w\lambda(1 + \alpha)/((1 - \alpha)N/R + \alpha)$. With analogous reasoning we can show that a plurality of workers will choose average cost pricing, and that the destitute would set the price equal to $\frac{1+\alpha}{\alpha}w\lambda$. If there is no single group with share holding dominance, the rich hold the median-share and decide on the price.

We look at two alternative mechanisms for the divesting of the monopoly. In the first case the central planner decides on the distribution $s(\cdot)$. For instance, she may decide to give one share to each agent (as in the examples previously touched upon), or concentrate the distribution among a favored group in the population, such as the the destitute, the workers or the rich. Some of the core issues in the choice of a distribution mechanism are illustrated in the first subsection where a maximization of the minimum utility is carried out. We then turn our discussion to the use of auctions, noting this method's close relationship with the distribution of autonomous income.

3.5.1 Distribution of Shares

The central planner may have legitimate equity concerns in the choice of the share distribution. For instance, and as mentioned by Kornai (1995), one goal may be to distribute the

shares among the members of a specific social class, such as the workers or those agents with no income. The choice of $s(\cdot)$ allows policy makers to influence the desired market price of good C . As just noted and as illustrated in the example on page 94, a uniform distribution of shares only among workers, ensures average cost pricing as the voted outcome.

The monopolistic behavior of the firm is endogenous to the rule selected to allocate shares. More specifically, policy makers need only consider the ownership mapping, if one exists, that results in a pivotal median-share holder j_m with a preferred price $p^*(j_m)$ as close to $w\lambda$ as possible.

Maximization of aggregate utility

Consider, therefore, the objective of maximizing average utility subject to keeping the price above average cost. Denoting the lowest-characteristic-agent preferring a price equal to $w\lambda$ (the target agent as a median-share holder) by j^0 and the set of agents with higher characteristic $\{j \in \mathcal{J}_k \mid k(j) \geq k(j^0)\}$ by $\bar{K}(j^0)$, the problem the central planner faces when maximizing an equally weighted aggregate utility can be expressed as

$$\begin{aligned} \max_{s(\cdot)} \left\{ V(p^*(j_m)) = \int_{j \in \mathcal{J}_k} V(j, p^*(j_m), w) dj \right\}; \\ \text{subject to} \quad \int_{\bar{K}(j^0)} s(j) dj \geq 1/2. \end{aligned} \quad (3.8)$$

The constraint specifies that the majority of agents have a preferred price equal or lower than average cost (by specifying a characteristic higher or equal to $k(j^0)$ for the median-share owner). As the median-share owner j_m and her preferred price depend on the chosen distribution $s(\cdot)$, different allocation rules will have median-share holders with different characteristics. This is the mechanism the central planner has at its disposition in order to influence the process establishing the eventual price of good C . The central planner must first

foresee the minimum characteristic $k(j^0)$ that gives average cost pricing while manipulating the distribution of shares $s(\cdot)$ so as to get a median shareholder with this characteristic.

It is not always possible to find a distribution schedule $s(\cdot)$ solving problem (3.8) which reproduces the solution to problem (3.7) above. If there is no autonomous wealth and labour equally divided, for instance, a solution is less likely to exist. We have seen an example of Cobb-Douglas-type preferences on page 93 that helps illustrate the issues. We saw there the pseudo-characteristic function $(1 - \alpha)r_a(j) + (1 + \alpha)r_l(j)$. The critical value $k(j^0)$ for this function is given by 1. Individuals with values bigger than 1 would prefer to forego profit and would accordingly price the good at average cost; those with lower values prefer profit and thus higher prices. More generally, an agent j would prefer average cost pricing in that context whenever

$$s(j) \leq (1 - \alpha)a(j) + (1 + \alpha)l(j).$$

Therefore, if wealth is concentrated in the minority shareholding group, i.e. $a(\cdot) = 0$ for all members in the policy target group, and if their preferences lean to the non-produced good, α is low, only a very egalitarian distribution of 51% of the shares so that $s(j) < (1 + \alpha)l(j)$, will result in average cost pricing. However, if workers are not given the majority of shares, there is a majority of destitute preferring a price higher than average cost. Or if the distribution of shares among workers is too concentrated, average cost pricing is unlikely for high levels of α , since it may not be possible to find a sufficient mass of owners such that $s(j) \leq (1 + \alpha)l(j)$. Similar arguments can be formulated when there is autonomous wealth.

It is clear from this example and, more generally from the nature of the problem, that more equally distributed autonomous and labor income will allow a flatter distribution of shares to reach average cost pricing, since for low values of $a(\cdot)$ and $l(\cdot)$, only low values of

the share of profits $s(\cdot)$ are sufficient to reach sufficiently high characteristic values. These comments are summarized in the following proposition

Proposition 3.9. *The maximization of an egalitarian welfare function having the distribution $s(\cdot)$ as the policy instrument (privatization) may not result in the same level of welfare that results from having the price as an instrument. Furthermore, the incentives that result from the use of this instrument work only through the assigning of profit income to groups strategic in the ex-post equilibrium, and may cause a further distancing from the price-selection equilibrium.*

Privatization in this framework has the appeal of possibly serving as an income balancing policy instrument. By selecting an appropriate share distribution, the policy maker can use privatization to “neutralize” the role autonomous income plays in the price selection mechanism. This is evident from the fact that the median-share holder’s preferred price depends on the ratios r_a and r_l . By assigning few shares among the destitute, a higher ratio $r_a(\cdot)$ is achieved for this group, thus shifting their preferred price lower.³² Of course, this idea also applies to labor allocation among the population, through the ratio $r_l(\cdot)$. Distribution schemes centered on workers, thus lowering values of r_l , will be characterized by the undesirable effect of neutralizing their tendency to otherwise prefer low prices (and higher production levels). We illustrate the consequences of following a welfare-balancing objective in the next section.

³²The possibility of rendering less important the initial distribution of autonomous income is in sharp contrast to Weitzman (1977), where preference heterogeneity and the absence of homothetic preferences gives income distribution a central role in the optimality of a given distribution program.

Maximization of the Minimum Utility

We consider as an example the maximization of the minimum utility as elaborated, for instance, by Rawls (1971). If pricing were the only instrument the central planner has to maximize the utility of the worst off, the solution would be simply to find the agent with the lowest utility (henceforth referred to as the “worst-off agent” and denoted by j^w) and then chose this agent’s preferred price.³³ When distributing shares, however, the central planner cannot directly decide on the price. The price is chosen ex-post by the median-share holder resulting from the chosen share distribution scheme $s(\cdot)$. In this framework, increasing a worst-off agent’s income will only be possible by giving him some profit. Thus the central planner’s decision is complicated by having to resort to an overall inefficient production plan in order to positively affect the income of the worst-off.

Furthermore, when considering the welfare of the worst-off, the choice of a given scheme involves a trade-off between lower prices favoring the consumer, and a higher ownership concentration (on the worst-off agent) and a consequently higher equilibrium price. Caring for the welfare of the worst off, the central planner will chose a distribution $s(\cdot)$ generating benefits for this group that compensate their welfare loss of a higher price. We refer to the endogenous median-share holder representative under this distribution scheme by j_m^w , and note that it is her preferred price $p^*(j_m^w)$ that will be effective. The planer’s problem is formally,

$$\max_{s(\cdot)} \left\{ \min_{j \in J_k} \{V(j, p^*(j_m^w), w)\} = V(j^w, p^*(j_m^w), w) \right\}; \quad (3.9)$$

such that $\int_{\bar{K}(j_m^w)} s(j) dj < 1/2.$

³³The simplest case is when a dispossessed agent exists, for then the minimum utility is clearly this agent’s, and the solution is the lowest permissible price, as the characteristic of this agent is equal to zero.

However, inducing a positive profit may be harmful for this agent if the marginal benefit of the first unit of profit cannot compensate for the marginal loss of a consequent higher price. In fact, it must at least be possible to make this agent better off by granting him all of the profit (the resulting price thus being the one chosen by him) than by a policy with zero profit and the minimum price (maximizing his utility as a consumer). That is,

$$V(j^w, p^*(j^w), w) \Big|_{s(j^w)=1} > \underbrace{V(j^w, w\lambda, w)}_{\text{utility when } p=w\lambda} = b(w\lambda) a(j^w) A,$$

which is clearly possible for the dispossessed, as in this case $a(j^w) = 0$.

Given an ex-post profit of $\bar{\Pi}$, the policy maker will gradually give a fraction of $\bar{\Pi}$ to the worst-off agent (the destitute and/or unemployed) raising his utility to that of the next worse-off agent. This will be done, while at the same time considering the effect of the changing ownership distribution (and its changing median-share holder) on the voted-price (and ex-post profit). Then she will equally distribute more profit between the two of them, bringing them to the level of the third worse-off agent; and so forth, until there is no more $\bar{\Pi}$ to distribute. The difficulty stems from doing this while ensuring that the price preferred by the median-share holder is consistent with the level of profits $\bar{\Pi}$. This requires the chosen distribution to endogenously generate median voter j_m^w , consistent with profit $\bar{\Pi}$. This may not be possible.

In general the central planner will reach an equilibrium distribution in the following manner:

- The central planner starts by observing the utilities resulting from giving each agent the median share. From all of them, the central planner chooses the worst-off, j^w , and grants him the first share.

- Given the price that j^w would chose holding the median (so far only) share, the central planner verifies that there is nobody else worse-off. If this is the case the problem is finished, for no instrument exists to raise the welfare of agent j^w , nor that of any other agent. Profit is given by $\Pi(p^*(j^w), w)$, all of which accrues to j^w .
- If there is someone worse-off, say n agents j^{w_n} , the central planner distributes additional shares among them making sure that a unique median-share holder among them exists at each stage. If the n agents have the same level of welfare, once again the central planner sees whether there is anybody else worse than them using the same procedure. Gradually the central planer distributes shares among the worst-off while observing the preferred price of the median-share holder among them. At any given point, there exists an interim median-share holder for the distributed shares.
- This process may or may not end with an equal distribution of income for the population as a whole, but clearly so for the (ex-ante worst-off) shareholders (if a continuum of agents exists).

It is clear that this procedure concentrates ownership among the poorest agents, that is, agents with lower values of $a(\cdot)$ and $l(\cdot)$. The worse-off are shareholders and their distribution of characteristics is the relevant one for the ex-post price of the consumption good. Furthermore, utility of the worst-off is raised *homogeneously*, bringing among the shareholders an increasing number of worse-off agents.

We can say the following about the characteristic of the median-share holder. First, it is lowest (and thus the highest ex-post price results) if there exist dispossessed in the economy and profit is only shared among them, as $a(j^w)$ and $l(j^w)$ are equal to zero and

the median-share holder's characteristic is nil. Second, even if there are no dispossessed, the price is higher at earlier stages of the distribution process, as the characteristic value of the median share holder is lower. For instance, were the process to end with agent j^w (the single worst-off), the agent with the single lowest characteristic is the median-share holder ($s(j^w) = 1$, and $a(j^w)$ and $l(j^w)$ are jointly the lowest in the population). At further stages of the distribution of shares, the median-share holder is among agents with increasingly higher characteristic: as $s(\cdot)$ falls for any one agent holding shares, agents with increasingly higher values of $a(\cdot)$ and $l(\cdot)$ are brought on board the distribution scheme, and those with smaller characteristics among them hold increasingly a smaller proportion of the shares. Third, the extent of initial inequality determines how early the redistribution ends and therefore who the ex-post median-share holder is. If autonomous and labour income are more evenly distributed, shares are distributed among a wider population, $s(\cdot)$ is small for anyone shareholder, and the median-share holder is more likely to have a high characteristic.

The maximization of the minimum strongly illustrates how the location and concentration of the distribution indirectly, through the selection of the median voter, affect the overall ex-post efficiency in the economy. These results are summarized in the following proposition.

Proposition 3.10. *Maximization of the minimum utility, by forcing a positive profit level, necessarily results in a lower level of aggregate welfare than that resulting from having the price as an instrument. The ex-ante inequality determines the extent of the deviation of the post-distribution price from the overall optimal price, as more evenly distributed autonomous and labour income levels result in a lower post-distribution price.*

A formidable amount of information is required from the central planner for the distribution of shares. In order to correctly assess the ex-post equilibrium price resulting from a distribution scheme $s(\cdot)$, she needs to know the distribution of autonomous and labour

income over the whole population. A less demanding mechanism for the distribution of shares is provided by an auction: the central planner decides only the share's price while guaranteeing a minimum level of profits for the post-auction firm.

3.5.2 Auction of Shares

We study the elements that comprise an equilibrium of the auction and illustrate some characteristics that stand in contrast with the distribution case. In working through the auction cases below, we attempt to depict restrictions that exist ex-ante on the central planner and ex-post on the price of the consumption good. In particular, we discuss how a rationality requirement on agents linking the ex-ante price of a stock with its ex-post yield, restricts the policy options of the central planner.

The timing of the events is similar to the distribution case: in the first period the central planner auctions shares of the firm (claims on the firm's next period profit) at a given price, say q . Faced with a demand of shares amounting to $T(q)$, and a distribution of this demand among agents, $n(\cdot)$, the central planner distributes shares according to $s(\cdot) = n(\cdot)$ and transfers the receipts of the auction to the population according to a distribution $t(\cdot)$. Given the ownership distribution $n(\cdot)$ generated by the demand for shares in the auction, the transformed autonomous and profit income distributions, and the post-auction median-share holder j_m^a choosing the monopoly good's price, profit $\Pi(p^*(j_m^a))$ is realized in the second period according to the equilibrium mechanism described earlier.

We assume that the value associated with a share by an agent is independent of whether she participates in the auction or not, i.e the population of buyers is sufficiently large. Furthermore, it is reasonable to assume that agents consider themselves small enough ex-

post, therefore they ignore ex-ante their ex-post role in the firm's production plan.

Suppose there are no capital markets, and thus borrowing is not allowed, so that agents can acquire stocks only by foregoing autonomous income. Agent i , having acquired a fraction $n(i)$ of the total number of shares issued, has autonomous income amounting to $a(i)A - n(i)T(q)$. Furthermore, she receives a fraction $t(\cdot)T(q)$ of the auction proceeds. When entering the second period, agent i 's autonomous net income is therefore given by $a(i)A + (t(i) - n(i))T(q)$.

The central planner, aware of the decision-making mechanism of the firm and concerned about the (unweighted) average welfare, therefore has formally the following problem:

$$\max_{t(\cdot), q} \left\{ V(p^*(j_m^a)) = \int_{j \in J_k} V(j, p^*(j_m^a), w) dj \right\}. \quad (3.10)$$

For the population as a whole, a total A is available for the purchase of shares being auctioned. Agent i can purchase shares amounting to a maximum of $a(i)A$ (labour income cannot be invested before it is realized). The decision on whether to buy shares depends on i 's expectation of the value of the shares in terms of profit earnings and the agent's preference for risk. We consider two scenarios, one with risk neutral agents, and the other with risk averse agents.

Risk Neutrality and a Proportional Allocation Auction

Risk neutrality and the absence of capital markets considerably simplify an agent's decision. If a share's price q is larger than expected future dividends, a rational agent prefers not to invest her autonomous income, as the purchase results in an expected loss. Therefore, the share price chosen by the central planner must provide non-negative expected profit, that is, a stock's profit share must be at least equal to its value in terms of autonomous income.

Only then are risk-neutral agents inclined to purchase shares at all. Indeed, if the price of the stock is lower, thus granting more profits than the autonomous income foregone, they would want to invest the totality of their autonomous income in the purchase of shares, as they expect more profit than the stock's price.

Formally this can be formulated as follows. Suppose that N_s shares are offered by the central planner. Then agent i is interested in a share offered at price q if

$$q(i) := E_i \left[\frac{1}{N_s} \Pi(p, w) \right] > q, \quad (3.11)$$

i.e. when the price q of the share is lower than agent i 's expected profits derived from it, $q(i)$.³⁴ When this condition is satisfied (strict inequality), agents use *all* of their autonomous income in the purchase, as they exchange a given amount of autonomous income for a larger expected profit income. Therefore, proceeds collected from the auction at price q , $T(q)$, are equal to A , and the distribution of bids $n(\cdot)$, to the ex-ante distribution $a(\cdot)$ of autonomous income. The ex-post share distribution $s(\cdot) = n(\cdot)$ should thus also replicate the initial distribution of autonomous income $a(\cdot)$. That is, we have $s(i) = a(i)$ for all $i \in J$. Agent i 's autonomous net income is therefore given by

$$a(i)A + (t(i) - n(i))T(q) = t(i)A.$$

Let us first illustrate the importance of the ex-post redistribution of autonomous income with an example. Recall that on page 94, the ratio $r_a(i)$ under the equal share distribution scheme is given by $N a(i)$ ($s(i)$ was $1/N$). Suppose that the central planner distributes autonomous income either *equally* to everyone, $t(i) = 1/N$, or *proportionally* according to what was paid, $t(i) = n(i) = s(i)$. The latter situation is one where the government pursues

³⁴ E_i denotes agent i 's expectation of the share's value in terms of profit to be realized in the following period.

no active redistribution policy and the former one where the government takes an egalitarian redistribution policy. After the distribution of the auction's receipts, the ratio r_a is $1/Na(i)$ in the first case. Note that r_a is the exact inverse of the one of the share distribution: agents that would have a high r_a in the context of the share distribution, have a low ratio with the auction, and vice-versa. If a desirable objective is to have a flat characteristic function across agents (inversely correlated $r_a(i)$ and $r_l(i)$ ratios across the population), the choice of divestiture mechanism matters.³⁵ An auction may thus harmonize agents' characteristics and is characterized by shifting decision making weight to agents with "middle" characteristics.

The proportional distribution offers a different outcome, as the ratio r_a becomes unity and therefore homogeneous across all agents with the auction. The characteristic function only varies across agents because of differences in the ratio $r_l(\cdot)$. If the ex-ante autonomous income share is correlated with the labour share, the ratio $r_l(\cdot)$ is also "flattened" across agents and the proportional-give-back policy has a neutralizing effect as it homogenizes the autonomous income component of agents' characteristic. Note that, while this option has a harmonizing effect on agents' characteristics, it also has important consequences for the ex-post distribution of overall income.³⁶

Coming back to the more general situation, it is important that the price of stocks q and the transfer distribution $t(\cdot)$, through their effect on the *ex-post* profit level decided by the eventual median-share holder, be consistent with the *ex-ante* expectation $q(\cdot)$ agents have

³⁵If under share distribution the characteristic profile was to be flat, with this type of auction it will show accentuated disparity. Naturally, the converse is also true, with this scenario neutralizing disparities between $r_a(\cdot)$ and $r_l(\cdot)$ that may have arisen in the distribution case.

³⁶On the other hand, if there was a negative ex-ante correlation between $a(\cdot)$ and $l(\cdot)$, differences of characteristic values across the population are accentuated, as high values of $l(\cdot)$ are divided in $r_l(\cdot)$ by low values of $a(\cdot)$ and viceversa.

about the firm's profit (and thus are willing to pay for a share thereof). That the central planner cannot fool rational agents, is a reasonable requirement for the equilibrium, but it imposes a non-trivial restriction on both the price q and the transfer schedule $t(\cdot)$.

The restriction on the choice of q is evident in light of condition (3.11): the central planner must support rational agents' expectations of a profit level. If each agent is to obtain profit at least as large as autonomous income, then in the aggregate profits must be at least as large as A . If N_s shares are issued, then their price must at least satisfy

$$q = \frac{A}{N_s},$$

where equality denotes the implicit assumption that the central planner has no interest in larger profits than strictly necessary.³⁷

The choice of the ex-post allocation $t(\cdot)$ is more complicated, as it affects the ex-post profit directly. Since the ex-post share ownership distribution replicates the ex-ante distribution of autonomous wealth, the choice of $t(\cdot)$ must be such that the median-share owner in the following period, j_m^a , fulfills the specific requirement of providing profits at least as large as A .³⁸ This restricts objective (3.10) on the one hand by requiring that the transfer $t(\cdot)$ result in

$$\int_{\bar{K}(j_m^a)} s(j) dj < \frac{1}{2},$$

and on the other, that agent j_m^a 's characteristic be such that his preferred price $p^*(j_m^a)$

³⁷Note that we are also implicitly assuming a partition of shares N_s sufficiently fine to allow replicating the initial autonomous income distribution $a(\cdot)$.

³⁸Note that, unlike the case of the simple distribution of shares we saw before, the central planner is now better informed about the ex-post distribution of autonomous and profit income.

satisfies

$$\Pi(p^*(j_m^a)) \geq A.$$

Because only the transfer $t(\cdot)$ can be used (and thus only affect the r_a ratio), only one of the two components of the characteristic function is subject to fine-tuning. Furthermore, the “policy range” on r_a changes from $[a(\cdot)/1, a(\cdot)/\epsilon]$ that was possible under the distribution of shares, to $[0, 1/a(\cdot)]$, which is possible with the auction. The more significant negative side of the auction, however, is the fact that it forces, in the presence of rational agents, an ex-post equilibrium with positive profits, and thus productively inefficient.

Proposition 3.11. *An auction of shares among risk neutral agents, while proving a more effective mechanism to inform the central planner about the characteristics of the population, reduces the range of effect on the agents’ characteristics available to achieve a given objective. Furthermore, if agents are rational, it compels an ex-post level of profit equal to ex-ante autonomous wealth, and thus may force a significant and negative deviation from the unconstrained optimal equilibrium (having price as a policy instrument).*

Risk Aversion and a Random Allocation Auction

In the previous section agents know the distributions of labour and autonomous income so as to rationally foresee the future distribution of shares for the population as a whole. This section relaxes the knowledge assumptions on the agents thereby introducing risk averse preferences at the expense of the homotheticity assumption of section 3.2. We assume preferences to be a particular instance of the generalized Gorman polar form.³⁹ The setting for this section is the *firm commitment offerings* type of new issues market discussed, for instance, in Rock (1996). The government agrees on the price of the firm’s first issuance of

³⁹See Deaton and Muellbauer (1980, p.130-1) for the definition and typical uses of this functional form.

equity. If there is excess demand, the underwriter rations the shares and if there is excess supply, the offer ends with unsold shares. The condition of excess demand or supply is not observed until after the offering date.

Agents have a state contingent indirect utility function (or henceforth Bernoulli utility function) of the form

$$V(p, \omega) = b(p) \psi(\omega),$$

where $\psi(\omega) = \omega^\gamma$, with $0 < \gamma < 1$.⁴⁰

Under the requirement that agents are small relative to the population as a whole, the subjective probability that profit will have a given value $\Pi(\cdot) = \pi$, is independent of the agents' actions, and it is the same across agents. We denote the joint and independent cumulative probability function that profit will have a value π by $F(\pi)$.

The central planner sets a price q for the share, whereas investor i decides on a fraction $\delta \leq a(i)A$ of her autonomous income to invest on shares. The investor exchanges one unit of autonomous income for $1/q$ units of shares. She chooses the fraction δ optimally to maximize expected utility

$$U(\delta, i, q) = \int_0^{\pi_m} \psi \left(a(i) - \delta + \frac{\delta}{q} \pi \right) dF(\pi), \quad (3.12)$$

where π_m is the maximum possible profit. Using Leibnitz's rule and noting that $q > 0$ the first order condition is given by

$$\int_0^{\pi_m} \psi'(\omega) (\pi - q) dF(\pi) \leq 0.$$

First note that agents demand shares provided that $E[\pi] = \int_0^{\pi_m} \pi dF(\pi) > q$, that is, when the share's price is set below its expected worth. Second, it is well known that for a

⁴⁰This specifies (constant) risk averse preferences allowing a less trivial representation of a given agent's expected utility maximization.

constant relative risk aversion Bernoulli utility function the fraction δ of autonomous income $a(\cdot)$ spent on the risky stock will be constant across agents.⁴¹ That is, the rich buy more than the poor, but will do so proportionally. Finally, it is evident from (3.12) that agents will demand more stock whenever its price is lower. The central planner is therefore free to regulate by the choice of q the aggregate demand for stocks, thus being able to select a price where demand equals supply and there is no over-subscription for the shares.

The demand for stocks that the central planner faces has aspects that are reminiscent of the previous section: agents will demand shares as a proportion of their autonomous income. In contrast with the discussion on page 114, an agent would only give up a fraction δ of her share of autonomous income $a(i)$ for a share of profit, while keeping only $(1 - \delta)a(i)$ autonomous income. In the aggregate, the central planner receives bids amounting to δA . It is clear that the central planner only has to generate a fraction δ of the ex-post profits he had to previously under perfect information, while still distributing, however, shares according to $n(i) = a(i)$.

The ratio r_a now equals $\frac{(1-\delta)a(i)+\delta t(i)}{\delta a(i)}$. In the case of a proportional distribution $t(i) = a(i)$, the ratio equals $1/\delta$, which is larger than 1 (the case without risk aversion), and thus resulting in agents with a higher characteristic value and lower preferred price. The central planner extract an additional benefit, as she may now either retain some non-voting shares, in addition from profiting from lower profits to convince agents to purchase shares.⁴²

Let us illustrate how this additional mechanism works. Suppose a fraction $1 - \beta$ of claims

⁴¹See for instance Andreu Mas-Colell and Green (1995, Section 6C).

⁴²This is only possible if the holding of shares does not affect the ex-ante probability distribution $F(\pi)$. Whereas this is clearly not true ex-post (as the government reduces the profits available to shareholders), it may be possible from the perspective of the investor, as the shares retained by the government may not affect the *perceived* profitability of the shares.

to profit are held by the central planner (who may then distribute them to the dispossessed, for instance, or not). The effective share of profits accruing to shareholder i is $\beta s(i)$. If we now use this effective ownership distribution to evaluate the ratios r_a and r_l , we observe that, because the denominator in both r_a and r_l is lower, the characteristic of any agent is higher and her preferred price thus lower. Furthermore, because the ownership distribution of the population of voting shareholders exactly replicates the autonomous income distribution, the median share holder is the same. The central planner is therefore in a position to achieve an equilibrium with less profits (and thus more efficient).⁴³ These results are summarized in the following proposition.

Proposition 3.12. *Risk aversion in an environment of imperfect information, by raising the shareholders' characteristics and by permitting the central planner to retain non-voting shares, may serve to reduce ex-post profits. In doing so, it allows the central planner a mechanism to induce an equilibrium where the consumption good's price more closely resembles the un-constrained optimal equilibrium (having price as a policy instrument).*

The qualitative nature of the auction with risk averse agents is the same as that with perfect foresight, and emphasizes the important constraints that this form of divestiture entails for the policy maker and the nature of the resulting distribution and equilibrium. It nevertheless grants the central planner an extra degree of freedom in achieving a more efficient post-privatization activity level of the monopoly.

⁴³This result is even greater when the profits held by the central planner are distributed among the shareholders, as then the ratios become larger still.

3.6 Trading of Shares

No matter how well orchestrated is the divestiture of the monopoly, the post-divestiture equilibrium is likely to evolve as a result of trading in a new market for the shares of the monopoly. Although some privatization programs require a transition period where agents are not allowed to trade their shares, eventually agents decide whether to hold on to the issued shares or sell them in capital markets that may be created for such purpose. In the present context, we can think of share trading taking place in a sequence of periods following the initial policy of divestiture. It is clear that once secondary trading is allowed, the pricing behavior of the utility is determined by the ownership structure that results from trading. In particular, the ownership distribution in each period of transaction has an endogenous median shareholder deciding on the price of the consumption good C . What can we say about a given period's equilibrium?

We saw that, *ceteris paribus*, shareholders with higher ownership share have a *lower* $r_a(\cdot)$ ratio and agents with lower ownership share have a *higher* one. To illustrate the nature of the outcome that results from trading, first consider share trading resulting in the *same* median-share holder. If the median-share holder turned out to be a buyer, her preferred price will be larger following trade, and conversely if she happened to be a seller. How much of this simple logic is applicable in a trading situation where the median-share holder is likely to be a different agent?

Clearly, the post-trading ownership distribution depends on the *extent* of the trading and on who is *buying* and who is *selling*. It is possible to depict some outcomes that are more likely.

Among traders we find buyers B and sellers S . We know that $\varsigma[B]$, the proportion

of shares held by the buyers, increases during trade, whereas that held by sellers, $\varsigma[\mathbf{S}]$, decreases. The *extent* of the transfer determines whether $\varsigma[\mathbf{B}] > \frac{1}{2}$, that is whether the new median-share owner is among the buyers. If this is so, we know that she has a higher preferred price than before trading, but not whether that price is higher or lower than the previous median-share holder.⁴⁴ What information regarding traders can be useful in characterizing whether the price charged for the consumption good increases following trade?

As trade involves the accumulation of shares, looking at the population in terms of the characteristic of the buyers provides some guidance in answering this question. In particular, ordering the population according to their characteristic (thus separating two groups to the “right” and to the “left” of the median-share holder) and then measuring whether those with higher characteristic are all buyers or sellers, can provide an answer.

For instance, when agents with ‘low’ characteristic value relative to the original median shareholder j_m buy more from agents with a ‘high’ one. Given the reference (pre-trade) median voter j_m denoted by

$$\begin{aligned}\underline{K}(j_m) &:= \{j \in \mathcal{J}_k \mid k(j) \leq k(j_m)\}; \\ \overline{K}(j_m) &:= \{j \in \mathcal{J}_k \mid k(j) \geq k(j_m)\},\end{aligned}$$

the sets of agents with lower characteristic (higher preferred price) and higher characteristic (lower preferred price). When $\underline{K}(j_m)$ are all buyers of shares (and $\overline{K}(j_m)$ are therefore all sellers), the price following trade is certain to have increased. This result is summarized by the following corollary from proposition 3.8.

⁴⁴To illustrate this point consider a pre-trade equilibrium with a unique share holder who then sells 51% of the shares during trading to a new and otherwise identical agent. The price falls following this trade, as the new median-share holder has a higher characteristic (her $s(\cdot)$ is 51% rather than 100%) than the previous sole shareholder.

Corollary 3.13. *Once trading is allowed, given the reference median voter j_m^0 , and when all agents in $\underline{K}(j_m^0)$ are buyers, the new monopoly price voted under majority of the new shareholders will be higher than before, i.e. $p(j_m^1) \geq p(j_m^0)$, with j_m^1 being the new median-share owner.*

We can no longer say anything about the resulting equilibrium if not *all* agents with lower characteristic than the reference median-share holder are buyers, as we can show that even if they are *net* buyers, we can find an equilibrium with lower price.⁴⁵

If monitoring of the traders' characteristics were possible, the outcome of the post-trade equilibrium would be known. Such information, however, is not known to the central planner. The incentives facing agents sheds some insight as to the likelihood of the conditions of corollary 3.13 being satisfied: agents with low characteristic prefer a higher price, because they marginally benefit more from higher profits than from the consumption gains of a lower price. The association of ownership for these agents is, therefore, direct: increasing ownership allows a greater say in the pricing-decision of the monopoly and automatically substantiates the desired outcome of a higher price. On the other hand, agents with a high characteristic marginally prefer the benefit as a consumer associated with a lower price, than the benefit, as an owner, of larger profit earnings. The association of ownership for these agents is, therefore, indirect: increasing ownership is associated with more saying in the pricing decision of the monopoly that indirectly leads to a lower price. This simple mechanism of incentives therefore suggests that ownership gets concentrated and thus that

⁴⁵For instance consider three agents with increasing characteristic and $\{4, 5, 4\}$ shares each. If the median-share holder and the agent with highest characteristic sell each a share to the agent with lowest characteristic, the new distribution is $\{6, 4, 3\}$: the median share holder has not changed, agents in $\underline{K}(j_m^0)$ as a whole are net buyers (bought 1 share as a whole), but the chosen price decreases, as now the median-share holder has a higher characteristic (sold one share).

the conditions for corollary 3.13 are likely to be satisfied.

The distribution of the beliefs $q(i)$ is clearly also an important factor for the post-trade equilibrium and the eventual pricing behavior of the monopoly. In fact, the sole requirement for a pair $\{q, q(i)\}$ (a given trading price for the shares and a distribution of ex-ante beliefs on the value of the share) to be a trading equilibrium is that the amount of shares offered at the price q be equal to those demanded given the expectations $q(i)$. It is clear that an agent, unless the price offered is identical to that she expects, will choose to either sell all of her shares (if $q > q(i)$) or buy until her autonomous income is exhausted (if $q < q(i)$). For the population as a whole, a trading equilibrium requires that the shares of the two groups

$$\underline{Q}(q) := \{j : q(j) < q\}$$

$$\overline{Q}(q) := \{j : q(j) > q\},$$

of agents with lower and higher expected price respectively, have equal number of shares. That is $\varsigma[\underline{Q}(q)] = \varsigma[\overline{Q}(q)]$, and the price of a share q would adjust to satisfy this requirement.

Empirical evidence is not favorable to advocates of achieving distribution and pricing efficiency by means of privatization, for concentration of ownership usually follows privatization. The effect of post-privatization trading on the distribution of shares is described by Kikeri, Nellis, and Shirley (1992, p. 47) as follows: "Although over time, share ownership tends to reconcentrate, many governments . . . see wide distribution of shares as an important benefit of privatization."⁴⁶ If concentration is an inevitable outcome of secondary trading, it would appear that agents with a higher preferred price than the pre-trading median-share holder

⁴⁶The footnote accompanying the last statement which is found on page 81, elaborates on the point as follows: "Experience shows that share ownership tends to concentrate over time, despite the mechanisms used to attract and retain small shareholders (such as bonuses or matching shares, pricing discounts, and

are the likely buyers indeed.

Are there other shareholder characteristics which might contribute to explaining the concentration following the extent of secondary trading described by Kikeri, Nellis, and Shirley (1992)?

One important aspect is access to information: managers would likely be better informed about the firm's technology and would therefore have a more accurate perception of the impact of their purchase on profits. This is of course also true for agents who in the first round got a significant amount of shares. Higher ownership shares grant more decision making power and the knowledge of this creates incentives on big shareholders to bid a high price for a share that would grant them control of the price. This aspect is embodied in the high reservation price (a large value of $p^*(\cdot)$) of an agent and also works in an opposite sense: a limited participation in the decision making of a small shareholder creates the sense of isolation from influencing the price, making the agent more prone to accept selling her shares.

In general, the extent of the concentration will be important for the post-privatization pricing strategy of the firm, increasing levels of concentration are likely to be detrimental to efficiency, and privatization may likely bypass the objective of widespread ownership and its reduced taxes on dividends). In the United Kingdom, although concerted efforts were made to spread shares widely in the privatization flotations, reconcentration of ownership occurred quickly. Similarly, at the time of the public issue of the Malaysian Shipping Corporation, there were approximately 60,000 shareholders; the number fell to fewer than 5,000 after a brief round of secondary trading. Similar patterns emerge in other countries divesting through their stock markets (see Adam and Cavendish (1990)). This appears to be true even in the former socialist countries. A 1992 case study on Russia notes that a worker-management buyout in 1991 resulted in an initial capital structure of 50 percent workers, 30 percent management, and 20 percent others; one year later management controlled more than 50 percent as some workers took the opportunity for a quick cash profit."

desirable implications for the pricing strategy of the firm.

3.7 Concluding Remarks

Greater exposure to competitive forces is often not the outcome following privatization in the case of natural monopolies and may require some form of regulation. We considered a form of self-regulation of the monopoly pricing through the strategic distribution of ownership shares among groups or agents that exert pressure on the enterprise to price its output as close as possible to a social optimum.⁴⁷ Under the restrictions imposed to reach clear conclusions, the model has underlined important points about the share privatization of large state-owned enterprises. Through the mechanism of ownership distribution in the initial phase of the privatization program, governments have a powerful influence on the ex-post pricing behavior of the private monopoly.

With the aid of this framework, we have considered desirable welfare properties of the ex-post equilibrium. The distribution of shares implies that the goals of production efficiency and equality may run hand-in-hand. That is, a widespread distribution of shares, by lowering the characteristic of the agent deciding on the production of the monopoly, may be consistent with average cost pricing. Such a scheme is not void of negative side effects, as absence of governability may result from a wide distribution of shares and substantial information is required from the central planer. However, it is not true that an objective of wealth *redistribution* is always consistent with production efficiency. In order to have any effect on the distribution of wealth in the economy, the central planer will have to induce ex-post

⁴⁷In doing so, we have ignored the issue of managerial competence which may result from concentrated forms of ownership. To the extent that management boards prove effective in exerting managerial control, however, the question remains open as to the necessity of concentration.

positive profits and thus introduce an important inefficiency.

We have also compared direct distribution of shares with a framework of an auction of the public utility, and have provided one way of assessing which may be better than the alternative. We have seen that the control of the central planner over the equilibrium production plan is larger with distribution of shares than with an auction. The auction restricts redistribution objectives in a very concrete manner: it grants ownership only to agents with some form of income (the autonomously wealthy). An auction, especially when agents are well informed, also makes the ex-post production plan of the firm inefficient by requiring positive profits in the post-privatization firm that attract bidders in the auction phase.

Finally, we have also seen that the opening of secondary trading of shares will likely dilute the initial gains of an initial strategic distribution of ownership in a very natural way: buyers demand higher prices on the firm (more inefficient production plans) while becoming more dominant in the selection of the production plan of the firm. Furthermore expectations on the resulting behavior of the firm and the resulting agents' behavior in secondary markets are argued to be consistent with empirical evidence on the concentration of ownership. This evidence suggests that ownership tends to concentrate and therefore, in the context of our model, provides a bleak outlook of the longer-term outcome of well-intended privatization efforts.

Appendix A

Appendix to Chapter 3

A.1 Proofs

A.1.1 Monopoly Production

Proof of Proposition 3.1. The profit function is twice continuously differentiable in p and the constraint set is a non-empty and compact-valued correspondence. It follows from the maximum theorem as presented in Varian (1992, p. 505) that a solution exists to the profit maximization problem. Furthermore, the profit function is strictly concave by the requirements of section 3.2.3, since

$$D_{pp}\Pi(p, w) = A[(p - w\lambda) H''(p) + 2H'(p)] < 0,$$

for $p \in [w\lambda, \bar{p}]$. Therefore by the maximum theorem and the constraint set being convex, the solution to the profit maximization problem is unique. \square

Proof of Proposition 3.4. If there is no interior solution to the maximization problem, we know that the objective function is monotone over the compact range $p \in [w\lambda, p^{**}(w)]$. However, because $D_p V(p^{**}, i) < 0$, the maximizing price is equal to $w\lambda$, as the objective function is decreasing at the upper edge of the interval $[w\lambda, p^{**}(w)]$.

An interior solution to the agent's maximization problem, on the other hand, requires

that the first derivative of agent i 's objective function be equal to zero, i.e. that

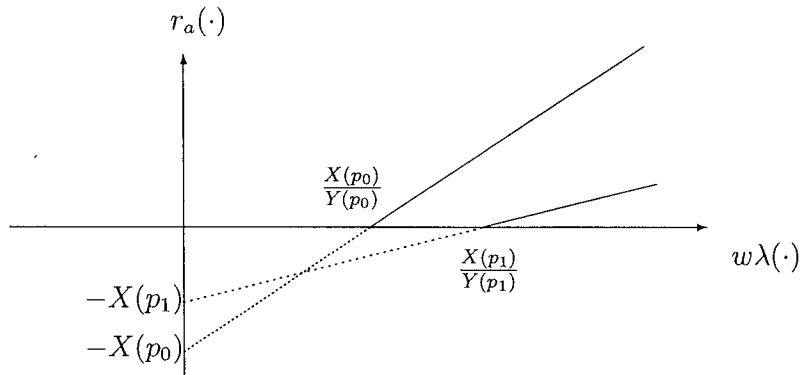
$$D_p V(i, p, w) = b'(p) (a(i) + s(i) \Pi(i, p, w)) + s(i) b(p) D_p \Pi(i, p, w) = 0, \quad (\mathfrak{b})$$

at $p = p^*(i, w)$. Dividing (b) by $s(i)$, A and $b'(p)$, simplifying notation by defining $r_a(i) := \frac{a(i)/A}{s(i)}$, and $\lambda(i) := \left(1 - \frac{l(i)}{s(i)}\right)$; and rearranging we obtain

$$r_a(i) = -X(p) + Y(p)w\lambda(i), \quad (\mathfrak{J})$$

where $X(p)$ and $Y(p)$ are defined in assumption 3.1, and by definition $r_a(i) \geq 0$.

(J) defines the solution manifold for $(r_a(\cdot), w\lambda(\cdot), p)$ in $\mathbb{R}^+ \times \mathbb{R} \times [w\lambda, p^{**}(w)]$. A sufficient condition for the solution to be unique, is that this manifold should not “bend” in $[w\lambda, p^{**}(w)]$, i.e. there are no two prices p_0 and p_1 such that the lines $r_a(\cdot) = -X(p_0) + Y(p_0)w\lambda(\cdot)$ and $r_a(\cdot) = -X(p_1) + Y(p_1)w\lambda(\cdot)$ in $\mathbb{R}^+ \times \mathbb{R}$ cross. For $r_a(i)$ and $w\lambda(i)$ given, two prices p_0 and p_1 are a solution if their implicit functions of the form (J) in $\mathbb{R}^+ \times \mathbb{R}$ cross at the point $(w\lambda(i), r_a(i))$ in $\mathbb{R}^+ \times \mathbb{R}$.



If no such crossing point exists for positive values of $r_a(\cdot)$, then the solution price to the agent's maximization problem is unique. Under assumption 3.1, the terms $-X(p)$ and $\frac{X(p)}{Y(p)}$ either jointly decrease or increase, so that the lines do not cross for $(w\lambda(i), r_a(i))$ in the positive quadrant $\mathbb{R}^+ \times \mathbb{R}$. Therefore the solution is unique. \square

A.1.2 Properties of the Private Plan

Proof of Lemma 3.5. We begin with the autonomous wealth density and divide the proof between the case when $p^*(i, w)$ lies in the interior of $P(i, w)$,¹ and the lower edge. We begin with the proof for interior suprema and then prove for the corner solution by contradiction.

Taking the total derivative of (b), rearranging terms, evaluating at the point where (b) and $ds(\cdot)$ equal zero we get

$$\left. \frac{dp^*}{da(\cdot)} \right|_{p^*} = \frac{-b'(p^*)}{D_{pp}V(p^*, i)},$$

where we know that the denominator of the right hand side is negative at the point p^* , since $D_{pp}V(p^*, i) < 0$ at the interior solution. Thus the claim is established for interior suprema.

If $p^*(i, w) = w\lambda$ and (b) is not equal to zero, we know that the objective function is decreasing at $p = w\lambda$. We also know that the objective function is increasing in $a(\cdot)$. For $p^*(i, w)$ to increase with an arbitrarily small increase in $a(\cdot)$ the objective function would have to change from being decreasing to being increasing in prices, since otherwise $p^*(i, w)$ would not have increased. This would violate the smoothness of the objective function.

The proof for the corner solution of the labor density is similar to the above argument and we thus omit the details. For the interior solution we take the derivative of the first order solution to get

$$\left. \frac{\partial p^*}{\partial l(\cdot)} \right|_{p^*} = -\frac{[b(p^*)\Pi'(p^*) + b'(p^*)\Pi(p^*)]w\lambda}{D_{pp}V(p^*, i)},$$

which, by the fact that the denominator is negative, and the observation that both $b(p)$ and $H(p)$ are decreasing in p , is non-positive.

As for the profit share the case of the corner solution is clearly satisfied, for $p^*(i, w) = w\lambda$ cannot decrease. For the interior solution we need to show that

$$\left. \frac{\partial p^*}{\partial s(\cdot)} \right|_{p^*} = -\frac{b(p^*)\Pi'(p^*) + b'(p^*)\Pi(p^*)}{D_{pp}V(p^*, i)} \geq 0,$$

¹Or lies at the edge but where the derivative of the objective function with respect to price is equal to zero.

which, by the fact that for the interior solution $D_{pp}V(p^*, i) < 0$, requires that we establish that $b(p^*) \Pi'(p^*) + b'(p^*) \Pi(p^*)$ is non-negative.

Consider agent i 's objective function (3.3). Taking its derivative with respect to price and evaluating at price p^* we obtain

$$\begin{aligned} D_p V(p^*, i) = \lambda [b'(p^*) H(p^*) + b(p^*) H'(p^*)] + b'(p^*) a(i) \\ + s(i) [b(p^*) \Pi'(p^*) + b'(p^*) \Pi(p^*)] = 0, \end{aligned}$$

where the first two elements on the right hand side are non-positive, thus forcing $b(p^*) \Pi'(p^*) + b'(p^*) \Pi(p^*)$ to be non-negative and establishing the result. \square

A.1.3 Voting

Proof of Theorem 3.7. Given an reference representative j and a status quo price $p \in \mathbf{P}^*$, consider the sets

$$\begin{aligned} \mathbf{B}(j, p) &:= \{ \tilde{j} \in \mathcal{J}_k \mid V(p^*(\tilde{j}), j) \geq V(p, j) \}, \\ \underline{K}(j) &:= \{ \tilde{j} \in \mathcal{J}_k \mid k(\tilde{j}) \leq k(j) \}, \\ \overline{K}(j) &:= \{ \tilde{j} \in \mathcal{J}_k \mid k(\tilde{j}) \geq k(j) \} \end{aligned}$$

of representatives \tilde{j} with preferred price giving the reference representative j a higher utility than that she would get at price p ; of representatives with characteristic values lower than the reference representative j ; and higher than j .

From the monotonicity of the characteristic function we have

$$\begin{aligned} \underline{K}(j) &\subset U(p^*(j)); \\ \overline{K}(j) &\subset B(p^*(j)), \end{aligned}$$

where the sets $U(\cdot)$ and $B(\cdot)$ respectively denote agents with lower and higher preferred prices.

Since the objective function $V(\cdot)$ is single peaked, representatives with a characteristic lower than that of j (higher preferred price) would prefer a higher price than $p^*(j)$ (j 's

favorite). The converse is true for representatives with a higher characteristic. Therefore, it follows that

$$\begin{aligned}\forall \bar{j} \in \underline{K}(j) \quad & \mathbf{B}(\bar{j}, p^*(j)) \subset U(p^*(j)); \\ \forall \bar{j} \in \overline{K}(j) \quad & \mathbf{B}(\bar{j}, p^*(j)) \subset B(p^*(j)).\end{aligned}$$

From the fact that we have a homomorphism we know that, given agent j , agents belong either to $\underline{K}(j)$ or to $\overline{K}(j)$. In the context of simple majority voting, agents in $\underline{K}(j)$ (respectively $\overline{K}(j)$) do not want to deviate from price p if and only if $\varsigma(B(p)) > \frac{1}{2}$ (respectively $\varsigma(U(p)) > \frac{1}{2}$) since a price giving each a higher utility cannot obtain majority. However, the preferred price of the median agent is the only one satisfying both inequalities. Therefore, for all agents either

$$\mathbf{B}(\bar{j}, p^*(j_m)) \subset U(p^*(j_m)),$$

or

$$\mathbf{B}(\bar{j}, p^*(j_m)) \subset B(p^*(j_m)).$$

□

A.2 CES Preferences

Consider the CES utility function

$$U(C, Z) = (C^\rho + Z^\rho)^{\frac{1}{\rho}},$$

with $\rho \in (0, 1)$ and where $\frac{1}{1-\rho}$ is the constant elasticity of substitution between consumption and the non-produced good. The indirect utility function associated with the CES utility function for agent i with income $\omega(i)$ is given by

$$V(p, \omega(i)) = (p^{\frac{\rho}{\rho-1}} + 1)^{\frac{1-\rho}{\rho}} \omega(i) := b(p) \omega(i)$$

with associated demand functions

$$C(p, \omega(i)) = \frac{p^{\frac{1}{\rho-1}}}{p^{\frac{\rho}{\rho-1}} + 1} \omega(i) := c(p) \omega(i),$$

$$Z(p, \omega(i)) = \frac{1}{p^{\frac{\rho}{\rho-1}} + 1} \omega(i) := z(p) \omega(i).$$

Clearly,

$$\frac{p c(p) \omega(i)}{z(p) \omega(i)} = p^{\frac{\rho}{\rho-1}},$$

is decreasing in p for all $\rho < 1$.

The smoothness and continuity assumption requires the elasticity of substitution to be strictly larger than one and bounded away from infinity, i.e. it requires $\rho \in (0, 1)$. It can be shown after some manipulation, that preferences of the CES type satisfy

$$p < (2 - \rho) \frac{w\lambda}{\rho},$$

requiring the preferred price of agents to be in the set $[w\lambda, \frac{2-\rho}{\rho}w\lambda)$. Note that for $\rho \in (0, 1)$, the interval is well defined, and that the solution is unique over $[\lambda, \infty]$. The latter follows from the convexity of the profit function for prices higher than $\frac{2-\rho}{\rho}\lambda$ being weak enough, thus preventing the profit function from increasing at higher prices. Therefore, local concavity of the profit function is sufficient to show the uniqueness property over \mathbb{R}^+ in the case of CES preferences.

The pure monopoly equilibrium for these preferences can be shown to be given by

$$p^{**}(w) = \frac{w\lambda}{\rho},$$

whereas the price which maximizes agent i 's *effective profit* is given by

$$p^{**}(i, w) = \begin{cases} \frac{1-r_l(i)}{\rho} w \lambda & \text{for } 1 + \rho > r_l(i), \\ w \lambda & \text{otherwise} \end{cases}$$

from where it is evident that, as in the case of Cobb-Douglas preferences, a larger labor to profit income share will drive an agent's preferred price towards average cost pricing.

Finally, the uniqueness of the private equilibrium is established by showing that the conditions of assumption 3.1 are satisfied for CES preferences. The corresponding terms of assumption 3.1 are

$$\begin{aligned} X(p) &= p H(p) \left[1 + \frac{b(p)}{b'(p)} \left(\frac{1}{p} + \frac{H'(p)}{H(p)} \right) \right] = \frac{1}{1-\rho} (1 + p^{\frac{\rho}{\rho-1}}) - 1; \\ \frac{X(p)}{Y(p)} &= p + \frac{\frac{b(p)}{b'(p)} H(p)}{H(p) + \frac{b(p)}{b'(p)} H'(p)} = p \left[\frac{1 - T(p)}{1 + p^{\frac{\rho}{\rho-1}} T(p)} \right]; \end{aligned}$$

where

$$T(p) = \frac{1 - \rho}{1 + p^{\frac{\rho}{\rho-1}}}.$$

Clearly $X(p)$ is strictly decreasing when $\rho < 1$ thus $-X(p)$ is strictly increasing in p . It is less evident to show that the term $\frac{X(p)}{Y(p)}$ is also strictly increasing. Recall that

$$\frac{X(p)}{Y(p)} = \frac{\frac{(1+p^{\frac{\rho}{\rho-1}})}{1-\rho} - 1}{\frac{1}{p} \left[\frac{(1+p^{\frac{\rho}{\rho-1}})}{1-\rho} + p^{\frac{\rho}{\rho-1}} \right]} = p \left[\frac{1 - T(p)}{1 + p^{\frac{\rho}{\rho-1}} T(p)} \right];$$

where

$$\begin{aligned} T(p) &= \frac{1 - \rho}{1 + p^{\frac{\rho}{\rho-1}}}; \\ Z'(p) &= \rho \frac{p^{\frac{1}{\rho-1}}}{(1 + p^{\frac{\rho}{\rho-1}})^2} = - \frac{d \left[p^{\frac{\rho}{\rho-1}} T(p) \right]}{d p} > 0. \end{aligned}$$

The derivative of $\frac{X(p)}{Y(p)}$ can therefore be expanded and simplified as follows

$$\begin{aligned}\frac{d \left[\frac{X(p)}{Y(p)} \right]}{d p} &= \frac{X(p)}{p Y(p)} - \frac{p}{1 + p^{\frac{\rho}{\rho-1}} T(p)} \left[Z'(p) + \frac{X(p)}{p Y(p)} \frac{d \left[p^{\frac{\rho}{\rho-1}} T(p) \right]}{d p} \right] \\ &= \frac{X(p)}{p Y(p)} - \frac{p Z'(p)}{1 + p^{\frac{\rho}{\rho-1}} T(p)} \left[1 - \frac{X(p)}{p Y(p)} \right] \\ &= \frac{X(p)}{p Y(p)} - \frac{p Z'(p)}{1 + p^{\frac{\rho}{\rho-1}} T(p)} \left[\frac{p Y(p) - X(p)}{p Y(p)} \right].\end{aligned}$$

But from the definitions of $X(p)$ and $Y(p)$ we know that $p Y(p) - X(p) = -\frac{b(p)}{b'(p)} H(p) = (1 + p^{\frac{\rho}{\rho-1}})$. Moreover, by construction $p Z'(p) = \rho \frac{p^{\frac{\rho}{\rho-1}}}{(1 + p^{\frac{\rho}{\rho-1}})^2}$. Thus the expression above becomes

$$\begin{aligned}\frac{d \left[\frac{X(p)}{Y(p)} \right]}{d p} &= \frac{1}{p Y(p)} \left[X(p) - \frac{p Z'(p)}{1 + p^{\frac{\rho}{\rho-1}} T(p)} \left(-\frac{b(p)}{b'(p)} H(p) \right) \right] \\ &= \frac{1}{p Y(p)} \left[X(p) - \frac{\rho \frac{p^{\frac{\rho}{\rho-1}}}{(1 + p^{\frac{\rho}{\rho-1}})}}{1 + p^{\frac{\rho}{\rho-1}} T(p)} \right] \\ &= \frac{1}{p Y(p)} \left[X(p) - \rho \frac{p^{\frac{\rho}{\rho-1}}}{(1 - \rho) p^{\frac{\rho}{\rho-1}} + (1 + p^{\frac{\rho}{\rho-1}})} \right],\end{aligned}$$

where the first term in the right hand side of the last inequality is positive. Therefore the ratio $\frac{X(p)}{Y(p)}$ will be non-decreasing provided that

$$X(p) = \frac{(1 + p^{\frac{\rho}{\rho-1}})}{1 - \rho} - 1 \geq \rho \frac{p^{\frac{\rho}{\rho-1}}}{(1 - \rho) p^{\frac{\rho}{\rho-1}} + (1 + p^{\frac{\rho}{\rho-1}})},$$

or, after rearranging,

$$\begin{aligned}((1 + p^{\frac{\rho}{\rho-1}}) - (1 - \rho)) &\left[(1 - \rho) p^{\frac{\rho}{\rho-1}} + (1 + p^{\frac{\rho}{\rho-1}}) \right] \\ &= (\rho + p^{\frac{\rho}{\rho-1}}) \left[(1 - \rho) p^{\frac{\rho}{\rho-1}} + (1 + p^{\frac{\rho}{\rho-1}}) \right] \\ &\geq \rho (1 - \rho) p^{\frac{\rho}{\rho-1}}\end{aligned}$$

Subtracting $\rho (1 - \rho) p^{\frac{\rho}{\rho-1}}$ from both sides we obtain the following requirement

$$p^{\frac{\rho}{\rho-1}} \left[(1 - \rho) p^{\frac{\rho}{\rho-1}} + (1 + p^{\frac{\rho}{\rho-1}}) \right] + \rho (1 + p^{\frac{\rho}{\rho-1}}) \geq 0,$$

which is clearly satisfied.

A.3 The Aggregated Space \mathcal{J}_k and the Translation of Measures

In order to construct the median voter/shareholder voting equilibrium we require an ordering of agents. The characteristic function $k(\cdot)$ is a natural candidate, were it not for the possibility of having two agents with the same characteristic value. This possibility, however, does not pose a problem, for both agents would also share a common preferred price $p^*(\cdot)$. This fact allows aggregating agents with equal values, that is, ‘joining’ agents with identical characteristic. We take some care in first defining the space of the original agents \mathcal{J} more carefully so as to use techniques for *grouping and re-labelling* agents.²

Definition A.1. $(\mathcal{J}, \mathcal{A}, \mu)$ is a compact measure space, \mathcal{A} is a Borel σ -algebra generated by subsets of \mathcal{J} , and μ is a measure defined on \mathcal{A} .

Whenever we talk about a function defined on \mathcal{J} we will assume below that it is μ -integrable.³ We consider the class of μ -integrable functions f on \mathcal{J} , satisfying $\int_{i \in \mathcal{I}} f(i) di = 1$, where all three $a(\cdot)$, $l(\cdot)$ and $s(\cdot)$ are examples of. Note that a density on \mathcal{J} implicitly defines a measure or set function normalized to unity. That is, the “size” of \mathcal{J} under the density-generated measure is unity. In fact, and as an example of such a “size” function, the profit share function $s(\cdot)$ serves to define the group ownership set function $S : \mathcal{A} \mapsto [0, 1]$ describing the joint ownership of groups of agents, where $Q \in \mathcal{A}$ is a group of individuals. Therefore, Q ’s share of the monopoly profit $S(Q)$ is given by $S(Q) = \int_{i \in Q} s(i) di$. As we change the cardinality of the space of agents when aggregating, we also need to transform measures

²The following section profits from sections 1.7 and 15.1 of Royden (1988) as well as section 8.39 of Halmos (1974a).

³One can always define an appropriate topology on \mathcal{J} depending on whether we are dealing with a continuum of agents or a finite number of them so that the notion of continuity is not violated.

defined on the original space \mathcal{I} (such as $S(\cdot)$) into measures on the transformed space \mathcal{J}_k . This means we need to formalize what intuitively would simply be the addition of shares. If in the original population a group of agents have a joint share of profits, in the population of representatives (the aggregates of agents with identical characteristics), the representative would have the share of the group of agents she represents.

We begin with the construction of the transformed space and then turn to the translation of measures.

A.3.1 The Aggregated Space

The aggregation is achieved by defining *an equivalence relation* or *equivalence classes* among the agents. The choice of equivalence relation is obvious in light of the desired objective of achieving a strictly monotone ordering of equivalent groups.

Definition A.2. We define the characteristic equivalence relation \equiv on \mathcal{I} by

$$i_1 \equiv i_2 \iff k(i_1) = k(i_2).$$

For a given $i \in \mathcal{I}$, let j be the set of agents equivalent to i , that is, $j = \{x : x \equiv i\}$. j is, therefore, a group of agents with identical characteristic value. The collection of equivalence sets or classes of \mathcal{I} under \equiv , $\{j : i \in \mathcal{I}\}$, is called the *quotient* of \mathcal{I} with respect to \equiv and we denote it by \mathcal{J}_k .⁴ This is simply the modified index space and indexes a population of groups where the agents with identical characteristics have been joined within a representative group.⁵

⁴Note that different ownership distributions $s(\cdot)$ will generate a different quotient space \mathcal{J}_k . Given that we are considering a static distribution of shares, the quotient space is given once the share distribution $s(\cdot)$ is decided upon.

⁵For example, consider a finite population of N agents $\mathcal{I} = \{1, \dots, N\}$. Suppose that agents i

The mapping $\varphi : i \rightarrow j$ is called *the natural mapping* of \mathcal{J} onto \mathcal{J}_k and it has an associated *set mapping* Φ defined by $\Phi(E) = \varphi^{-1}[E]$ mapping subsets of \mathcal{J}_k into subsets of \mathcal{J} . We denote by \mathcal{A}_k the collection of sets $E \subset \mathcal{J}_k$ induced on \mathcal{J}_k by the condition that $\Phi(E) \in \mathcal{A}$, i.e. $\mathcal{A}_k = \{E \subset \mathcal{J}_k : \Phi(E) \in \mathcal{A}\}$.

Lemma A.1. *Given $(\mathcal{J}, \mathcal{A}, \mu)$ and $(\mathcal{J}_k, \mathcal{A}_k)$, the mapping φ and its inverse φ^{-1} are both continuous.*

Proof. It is clear from the construction of \mathcal{A}_k , that φ^{-1} is continuous, since for $\Phi[O]$ open in \mathcal{J} , O is open in \mathcal{J}_k . It remains to show that Φ maps open sets in $(\mathcal{J}_k, \mathcal{A}_k)$ into open sets in $(\mathcal{J}, \mathcal{A})$. We will establish this by contradiction. Suppose that for $O \in \mathcal{A}_k$ an open set, its map under φ^{-1} , $\Phi[O] \in \mathcal{A}$, is closed in \mathcal{J} . Then $\sim \Phi[O]$ is open and thus in \mathcal{A} . Therefore $\varphi[\sim \Phi[O]]$ is open in \mathcal{J}_k . But $\varphi[\sim \Phi[O]] = \sim O$ a closed set. \square

Clearly, the natural mapping is also measurable, since for $E \in \mathcal{A}_k$, it follows that $\varphi^{-1}[E] \in \mathcal{A}$. By construction, its inverse φ^{-1} restores the characteristic and, thereby, the preferred price functions, i.e. for all $i \in \mathcal{J}$ and $j \in \mathcal{J}_k$, i.e.

$$(k \circ \varphi^{-1})(j) = k(i)$$

$$(p^* \circ \varphi^{-1})(j) = p^*(i)$$

Thus we refer to the characteristic and preferred price functions on \mathcal{J}_k simply as k : $\mathcal{J}_k \mapsto \mathbb{R}^+$ and $p^* : \mathcal{J}_k \mapsto \mathbf{P}^*$, when it is understood that we are really talking about $(k \circ \varphi^{-1}) : \mathcal{J}_k \mapsto \mathbb{R}^+$ and $(p^* \circ \varphi^{-1}) : \mathcal{J}_k \mapsto \mathbf{P}^*$.

The most important contribution of the reformulation of the space of agents is that the space of representatives \mathcal{J}_k and the range of k , $\mathbf{K} = \{k(j) : j \in \mathcal{J}_k\}$, are now *homeomorphic* and j have the same characteristic value, that is $k(i) = k(j)$. Then the quotient of \mathcal{J} is given by $\mathcal{J}_k = \{1, \dots, \{u, v\}, \dots, N\}$ and has cardinality $N - 1$. The group $\{u, v\}$ inherits the *joint* densities of the individual agents and becomes a single element of \mathcal{J}_k .

to one another. That is, we are able to ‘label’ representatives in \mathcal{J}_k by their characteristic and thus achieve a useful ordering of the population where the preferred price function $p^*(\cdot)$ is monotone. We state this central result in the following proposition.

Proposition A.2. *$k(\cdot)$ is a homeomorphism between \mathcal{J}_k and \mathbf{K} and thus it serves as an ordering in \mathcal{J}_k such that $p^* : \mathcal{J}_k \mapsto \mathbf{P}^*$ is monotone.*

Proof. Clearly k is a mapping from \mathcal{J}_k onto \mathbf{K} . Let u and v be two distinct elements of \mathbf{K} , then $k(u)$ and $k(v)$ are two distinct elements of \mathcal{J}_k , thus k is one-to-one. By lemma A.1, φ^{-1} is continuous and $k = (k \circ \varphi^{-1})$, a composition of two continuous functions, is also continuous (See Rudin (1987, Theorem 1.7)). It remains to be established that k^{-1} is continuous or that k maps open sets into open sets or equivalently closed sets into closed sets. Let C be a closed set in \mathcal{J}_k , then

$$k[C] = (k \circ \varphi^{-1})[C] = k[\varphi^{-1}[C]].$$

By the continuity of φ , $\varphi^{-1}[C]$ is closed and, by the compactness of \mathcal{J} , compact (Royden (1988, Proposition 9.2)). Moreover, the continuous image of a compact set is also compact (Royden (1988, Proposition 9.4)). $\mathbf{K} \cup \{\infty\}$ being a compact space, $k[\varphi^{-1}[C]]$ is closed. \square

A representative $j \in \mathcal{J}_k$ can, therefore, be uniquely identified by its characteristic $k(j) \in \mathbf{K}$ and, accordingly, by their preferred price $p^*(j)$.

A.3.2 Measures in the Quotient Space

In algebraic terms the spaces \mathcal{J} and \mathcal{J}_k are equivalent and therefore the simple change-of-variable formula we defined preserves the measure-theoretic properties of \mathcal{J} in its quotient \mathcal{J}_k . This is important in “translating” the measure of ownership in the original population to the new population of “representatives”.

Lemma A.3. *The collection \mathcal{A}_k of sets of \mathcal{J}_k induced by the map φ is an σ -algebra and Φ is a σ -homomorphism.*

Proof. Follows from the fact that set operations are preserved by inverse functions, thus if we consider a countable collection of sets E_n belonging to \mathcal{A}_k , we know from Kolmogorov and Fomin (1970, sect. 1.3) or Halmos (1974b, section 10) that

$$\begin{aligned}\bigcup_n \Phi[E]_n &= \bigcup_n \varphi^{-1}[E_n] = \varphi^{-1} \left[\bigcup_n E_n \right] = \Phi \left[\bigcup_n E_n \right]; \\ \sim \Phi[E] &= \sim \varphi^{-1}[E] = \varphi^{-1}[\tilde{E}] = \Phi[\tilde{E}];\end{aligned}$$

thus establishing that \mathcal{A}_k is a σ -algebra. In order to establish that it is a σ -homomorphism we simply note that $\Phi[\mathcal{J}_k] = \mathcal{J}$ holds thus establishing the result. \square

Since Φ is a σ -homomorphism, it induces a mapping Φ^* of measures on $(\mathcal{J}, \mathcal{A})$ into measures on $(\mathcal{J}_k, \mathcal{A}_k)$, by defining $\Phi^* \mu$ as $(\Phi^* \mu)(E) = \mu(\Phi^*(E))$. The new measures simply take into account the fact that we have now grouped individuals with identical characteristic functions into a representative group $j \in \mathcal{J}_k$ with the accumulated autonomous wealth, labor and ownership share of all the members $\{x : x \equiv i\}$ of the class, thus formalizing the intuitive notion of ‘summing’ within groups.

Proposition A.4. *For φ , a measurable point mapping of the measure space $(\mathcal{J}, \mathcal{A}, \mu)$ into the measurable space $(\mathcal{J}_k, \mathcal{A}_k)$, let Φ be the induced set mapping of \mathcal{A}_k into \mathcal{A} and $\nu := (\Phi^* \mu)$ its induced measure. Then for each non-negative measurable function f on \mathcal{J}_k we have*

$$\int_{\mathcal{J}_k} f d\nu = \int_{\mathcal{I}} (f \circ \varphi) d\mu.$$

Proof. See Royden (1988, Proposition 15.1).

We denote the μ -induced measure by $\nu := (\Phi^* \mu)$, and also define the S -induced ownership share $\varsigma := (\Phi^* S)$. Note that proposition A.4 holds for ς . It also holds for the non-negative

characteristic function $k : \mathcal{J}_k \mapsto \mathbb{R}^+$ and non-negative preferred price function $p^* : \mathcal{J}_k \mapsto \mathbf{P}^*$ where

$$(k \circ \varphi) = (k \circ \varphi^{-1} \circ \varphi) = k$$

$$(p^* \circ \varphi) = (p^* \circ \varphi^{-1} \circ \varphi) = p^*$$

A.4 Auxiliary Results

A.4.1 Product of Concave Functions

Consider two concave functions $h(x)$ and $g(x)$ defined on X . By the definition of concavity it follows that for $x_{1,2} \in X$, $\lambda \in (0, 1)$ and $\bar{x} = \lambda x_1 + (1 - \lambda)x_2$,

$$g(\bar{x}) \geq \lambda g(x_1) + (1 - \lambda)g(x_2) = \overline{g(x)};$$

$$h(\bar{x}) \geq \lambda h(x_1) + (1 - \lambda)h(x_2) = \overline{h(x)},$$

where we have defined the *corresponding* weighted (by λ) average of function $i(x)$, $i \in \{g, h\}$ evaluated at points x_1 and x_2 as $\overline{i(x)}$.

Therefore the product $g(\cdot)h(\cdot)$ satisfies

$$\begin{aligned} g(\bar{x})h(\bar{x}) &\geq \overline{g(x)} \overline{h(x)} \\ &= \overline{g(x)h(x)} + \lambda(1 - \lambda)(g(x_1) - g(x_2))(h(x_2) - h(x_1)). \end{aligned}$$

Hence where the functions have opposite gradients between points x_1 and x_2 , the product of the two functions will also be concave. Note that when this is so, even if the two functions are linear and not constant, the product will be strictly concave.

A.4.2 Concave and a Convex Functions

Consider a convex function $g(x)$ and a concave function $h(x)$. By the definition of concavity and convexity it follows that for $x_{1,2} \in X$, $\lambda \in (0, 1)$ and $\bar{x} = \lambda x_1 + (1 - \lambda)x_2$,

$$g(\bar{x}) \leq \lambda g(x_1) + (1 - \lambda)g(x_2) = \overline{g(x)};$$

$$h(\bar{x}) \geq \lambda h(x_1) + (1 - \lambda)h(x_2) = \overline{h(x)},$$

where we use the definition in the previous section of corresponding weighted (by λ) average function.

It can be shown that their product $g(\cdot)h(\cdot)$ satisfies

$$g(\bar{x})h(\bar{x}) \geq \overline{g(x)h(x)} + \left[\frac{g(\bar{x})}{\overline{g(x)}} - 1 \right] \overline{g(x)h(x)} + \frac{g(\bar{x})}{\overline{g(x)}} f(x),$$

where we simplified the expression $\lambda(1 - \lambda)(g(x_1) - g(x_2))(h(x_2) - h(x_1))$ as $f(x)$, and the ratio $\frac{g(\bar{x})}{\overline{g(x)}}$ is less than unity.

Since for functions with opposite gradients between points x_1 and x_2 , $f(x)$ is positive, the closer the ratio $\frac{g(\bar{x})}{\overline{g(x)}}$ is to unity, the possibility of concavity of the product of $g(x)$ and $h(x)$ increases. This possibility also increases the larger the gradient of either $g(x)$ or $h(x)$, thus increasing $f(x)$.

Bibliography

ABEL, A. B., AND O. J. BLANCHARD (1983): "An Intertemporal Model of Saving and Investment," *Econometrica*, 51(3), 675–692.

ADAM, C., AND W. CAVENDISH (1990): *Can Privatization Succeed? Economic Structure and Programme Design in Eight Commonwealth Countries*. Oxford University Press.

ALESINA, A., AND A. DRAZEN (1991): "Why Are Stabilizations Delayed?," *American Economic Review*, 81(5), 1170–88.

ANDREU MAS-COLELL, M. D. W., AND J. R. GREEN (1995): *Microeconomic Theory*. Oxford University Press, Oxford, UK.

ANTELO, E. (1995): "La Capitalización en Bolivia," *Análisis Económico*, 11.

BAER, W., AND M. H. BIRCH (1992): "Privatization and the Changing Role of the State in Latin America," *NYU Journal of International Law and Politics*, 25(1), 1–25.

——— (eds.) (1994): *Privatization in Latin America: New Roles for the Public and Private Sectors*. Praeger Publishers, Westport, CT.

- BANQUE PARIBAS, PROUDFOOT USA AND F. GONZALEZ & PARTNERS (1995): "Capitalization of Lloyd Aereo Boliviano, Information Memorandum," Discussion paper, Ministry of Capitalization of the Republic of Bolivia.
- BERNHARDT, M. D., AND D. E. INGBERMAN (1985): "Candidate Reputations and the Incumbency Effect," *Journal of Public Economics*, 27(1), 47-67.
- BOURBAKRI, N., AND J.-C. COSSET (1998): "The Financial and Operating Performance of Newly Privatized Firms: Evidence from Developing Countries," *Journal of Finance*, 53(3), 1081-1110.
- BRESSER-PEREIRA, L. C., J. M. MARAVALL, AND A. PRZEWORSKI (1993): *Economic Reforms in New Democracies: A Social Democratic Approach*. Cambridge University Press, Cambridge.
- BROCK, W. A., AND S. J. TURNOVSKY (1981): "The Analysis of Macroeconomic Policies in Perfect Foresight Equilibrium," *International Economic Review*, 22(1), 179-209.
- CARIAGA, J. I. (1996): *Estabilización y Desarrollo: Importantes lecciones del Programa Económico de Bolivia*. Los Amigos del Libro, La Paz, Bolivia.
- CHAMLEY, C. (1981): "The Welfare Cost of Capital Income Taxation in a Growing Economy," *Journal of Political Economy*, 89(3), 468-496.
- (1986): "Optimal Taxation of Capital Income in General Equilibrium with Finite Lives," *Econometrica*, 54(3), 607-622.
- J. HENRY SCHRODER WAGG & CO (1995): "Briefing Memorandum in respect of the Capitalization of Empresa Corani S.A.M. Empresa Guaracachi S.A.M. Empresa Valle

- Hermoso S.A.M.," Discussion paper, The Ministry of Capitalization and Investment of the Republic of Bolivia.
- CORNWALL, R. R. (1977): "The Concept of a General Equilibrium in a Market Economy with Imperfectly Competitive Producers," *Metroeconomica*, 29, 55-72.
- CORPORACIÓN MINERA DE BOLIVIA (1995): "Information Memorandum," Discussion paper, Ministry of Capitalization of the Republic of Bolivia.
- CORREIA, I. H. (1996a): "Dynamic Optimal Taxation in Small Open Economies," *Journal of Economic Dynamics and Control*, 20(4), 691-708.
- (1996b): "Should capital income be taxed in the steady state?," *Journal of Public Economics*, 60(1), 147-151.
- CUCKIERMAN, A., AND M. TOMMASI (1998): "When Does It Take a Nixon to Go to China?," *American Economic Review*, 88(1), 180-97.
- D'ASPREMONT, C., R. D. S. FERREIRA, AND L.-A. GÉRARD-VARET (1990): "On Monopolistic Competition and Involuntary Unemployment," *Quarterly Journal of Economics*, 105(4), 895-919.
- DE LA DEHESA, G. (1994): "Spain," in *The Political Economy of Reform*, ed. by J. Williamson. Institute for International Economics, Washington, DC.
- DEATON, A., AND J. MUELLBAUER (1980): *Economics and Consumer Behaviour*. Cambridge University Press.
- DRAZEN, A., AND P. R. MASSON (1994): "Credibility of Policies Versus Credibility of Policymakers," *Quarterly Journal of Economics*, 109(3), 735-754.

- ENFE (1995): "Memorandum de Informacion," Discussion paper, Empresa Nacional de Ferrocarriles.
- GABSZEWICZ, J. J., AND J.-P. VIAL (1972): "Oligopoly: 'A la Cournot' in a General Equilibrium Analysis," *Journal of Economic Theory*, 4(3), 381–400.
- GUPTA, S., C. SCHILLER, AND H. MA (1999): "Privatization, Social Impact, and Social Safety Nets," Discussion paper, International Monetary Fund.
- HALMOS, P. R. (1974a): *Measure Theory*, vol. 18 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, NY.
- (1974b): *Naive Set Theory*, Undergraduate Texts in Mathematics. Springer-Verlag, New York, NY.
- HARRIS, J. R., AND M. P. TODARO (1960): "Migration, Unemployment and Development: A Two Sector Analysis," *American Economic Review*, 60(1), 126–142.
- HART, O. (1982): "A Model of Imperfect Competition with Keynesian Features," *Quarterly Journal of Economics*, 97(1), 109–138.
- (1985): "Imperfect Competition in General Equilibrium: An Overview of Recent Work," in *Frontiers of Economics*, ed. by K. J. Arrow, and S. Honkapohja, pp. 100–149. Basil Blackwell.
- JONES, L. E., R. E. MANUELLI, AND P. E. ROSSI (1993): "Optimal Taxation in Models of Endogenous Growth," *Journal of Political Economy*, 101(3), 485–517.
- (1997): "On the Optimal Taxation of Capital Income," *Journal of Economic Theory*, 73, 93–117.

- JUDD, K. (1985a): "Redistributive Taxation in a Simple Perfect Foresight Model," *Journal of Public Economics*, 28, 59–83.
- (1985b): "Short-Run Analysis of Fiscal Policy in a Simple Perfect Foresight Model," *Journal of Political Economy*, 93(2), 298–319.
- (1987): "The Welfare Cost of Factor Taxation in a Perfect-Foresight Model," *Journal of Political Economy*, 95(4), 675–709.
- KIKERI, S., J. NELLIS, AND M. SHIRLEY (1992): *Privatization: The Lessons of Experience*. The World Bank.
- KOLMOGOROV, A. N., AND S. V. FOMIN (1970): *Introductory Real Analysis*, Dover Books on Advanced Mathematics. Dover Publications, Inc., Mineola, NY, Revised English Edition Translated and Edited by *Richard A. Silverman*.
- KORNAI, J. (1995): *Highways and Byways*. The MIT Press, Cambridge, MA.
- KRUEGER, A. O. (1993): *Political Economy of Policy Reform in Developing Countries*. The MIT Press, Cambridge, MA.
- LAFFONT, J. J., AND J. TIROLE (1993): *A theory of incentives in procurement and regulation*. MIT Press.
- MARSCHAK, T., AND R. SELTEN (1974): *General Equilibrium with Price-Making Firms*, vol. 91 of *Lecture Notes in Economics and Mathematical Systems*. Springer-Verlag, Heidelberg, Germany.

- MEGGINSON, W. L., R. C. NASH, AND M. VAN RANDENBORGH (1994): "The Financial and Operating Performance of Newly Privatized Firms: An International Empirical Analysis," *Journal of Finance*, 49(2), 403-452.
- MEGGINSON, W. L. AND JEFFRY M. NETTER (2001): "From State to Market: A Survey of Empirical Studies on Privatization" *Journal of Economic Literature*, 49, 321-389.
- MORGAN GREENFELL & CO. LIMITED (1995): "Information Memorandum on Empresa Nacional de Telecomunicaciones S.A.M. Entel - Bolivia, Part I," Discussion paper, Ministry of Capitalisation of Bolivia.
- NEGISHI, T. (1961): "Monopolistic Competition and General Equilibrium," *The Review of Economic Studies*, 28(77), 196-201.
- NIKAIDO, H. (1975): *Monopolistic Competition and Effective Demand*. Princeton University Press.
- RAWLS, J. (1971): *A Theory of Justice*. The Belknap Press of Harvard University Press, Cambridge, MA.
- REQUENA, B. (1996): "Estrategias y Perspectivas de la Capitalización en Bolivia," *Análisis Económico*, 16.
- ROCK, K. (1996): "Why New Issues are Underpriced," *Journal of Financial Economics*, 15, 187-212.
- RODRIK, D. (1994): "King Kong meets Godzilla: The World Bank and the East Asian Miracle," in *Miracle or Design? Lessons from East Asia*. Overseas Development Council, Washington, DC.

- (1996): "Understanding Economic Policy Reform," *Journal of Economic Literature*, 34(1), 9–41.
- ROYDEN, H. L. (1988): *Real Analysis*. Macmillan Publishing Company, New York, NY, third edn.
- RUDIN, W. (1987): *Real and Complex Analysis*. McGraw-Hill Book Company, New York, NY, third edn.
- SADER, F. (1993): "Privatization and Foreign Direct Investment in the Developing World, 1988-92," Discussion paper, The World Bank, Washington, D.C., Policy Research Working Paper 1202.
- SALOMON BROTHERS (1996): "Yacimientos Petrolíferos Fiscales Bolivianos, Information Memorandum," Discussion paper, Government of Bolivia's Ministry of Capitalization.
- SCHULTZ, C. (1996): "Polarization and Inefficient Policies," *Review of Economic Studies*, 63(2), 331–344.
- SHLEIFER, A., AND R. W. VISHNY (1994): "Politicians and Firms," *Quarterly Journal of Economics*, pp. 995–1025.
- SINN, G. AND H. W. SINN (1993): "Participation, Capitalization and Privatization, Report on Bolivia's Current Political Privatization Debate" Working paper No. 10, University of Munich.
- SPULBER, D. F. (1989): *Regulation and Markets*. MIT Press, Cambridge, Mass.
- VARIAN, H. (1992): *Microeconomic Analysis*. W.W. Norton & Company, New York, NY.

- VICKERS, J., AND G. YARROW (1988): *Privatization: An Economic Analysis*, vol. 18 of *MIT Press Series on the Regulation of Economic Activity*. MIT Press, Cambridge, Mass.
- (1991): "Economic Perspectives on Privatization," *Journal of Economic Perspectives*, 5(2), 111–132.
- WEITZMAN, M. L. (1977): "Is the Price System or Rationing More Effective in Getting a Commodity to Those who Need it Most?," *The Bell Journal of Economics*, 8(2), 517–524.
- (1985): "The Simple Macroeconomics of Profit Sharing," *American Economic Review*, 75(5), 937–953.
- WILLIAMSON, J. (ed.) (1994): *The Political Economy of Reform*. Institute for International Economics, Washington, DC.
- WORLD BANK (1995): *Bureaucrats in Business: The Economics and Politics of Government Ownership*. Oxford University Press, Washington, D.C., World Bank Policy Research Report.
- (2000): *World Development Indicators 2000*. The World Bank.
- YARROW, G. (1999): "A Theory of Privatization, or Why Bureaucrats are Still in Business," *World Development*, 27(1), 157–168.