ADAPTIVE INVENTORY CONTROL HEURISTICS FOR NON-STATIONARY DEMAND

by

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ABSTRACT

The objective of the current research is to develop an inventory algorithm that determines the ordering periods and the corresponding order quantities for nonstationary demand. The inventory system is a periodic review with lost sales. We propose a heuristic, Wagner Whiting Plus Forecast (WWPF), in which the forecasts are revised and the parameters for inventory control policy parameters are updated periodically. The demand process is non-stationary with a linear trend. The cost function is constituted by a fixed setup cost and a proportional holding cost. In each period, safety stocks are added to the forecast and the dynamic lot sizing is done as per the Wagner-Whitin algorithm. The proposed heuristic is compared with an adaptive (*s, S*) policy proposed by Axsäter (2000). Both WWPF algorithm and Axsäter's heuristic determine inventory parameters for demand data with trend, in a reasonable way. WWPF algorithm exhibits a marginal improvement over Axsäter's heuristic and can be recommended for inventory control in practical settings. WWPF algorithm can address seasonality, by using seasonal forecast models, such as Holt-Winters. Moreover, WWPF algorithm is independent of the forecasting method and it can be modeled with other forecasting methods, too.

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CHAPTER 1: INTRODUCTION & LITERATURE REVIEW

1.1 Introduction:

In January 2001, ***Cisco*** had to write-off inventory worth $ 1.2 billion. It was a result of a harsh slowdown for a company that had been forecasting 30% to 50% annual revenue growth pace. Samsung estimated it will take ***Cisco*** five months to work off the excess DRAM it has built up, said Ygongho Kang, associate director of DRAM at Samsung. ***Cisco*** acknowledged that it overestimated demand. "Our inventory level had gotten higher than we had wanted it and it will be a couple of quarters before it reaches a more acceptable level," a ***Cisco*** spokesman said. During the Christmas season of year 2000, DRAM stockpiles at PC manufacturers had clogged the channels with parts due to overly optimistic forecasts from some of the biggest names in the industry. The tumultuous events following the economic slowdown of the year 2000 present a new area of research for combining sales forecasts and inventory control. Due to the modeling challenges of combining forecasts and inventory control, limited research has been pursued in this area. The Demand Planning and Inventory Control modules available in the standard supply chain solutions offered by companies such as i2 and Manugistics strive to embed algorithms and approximations in their solutions, so as to incorporate a dynamic updating of inventory parameters with respect to demand forecasts. The area of combining forecasts and inventory control has great potential for industry applications and the savings would add to the financial bottomlines.

One major challenge for manufacturing and retail industry practitioners is to obtain effective inventory control policies for non-stationary demand. In practical settings, the demand for an item may follow a trend and / or seasonality patterns. Under such conditions the problem is two fold, first to forecast the demand in future periods and second to decide the optimal inventory control parameters. The research project was motivated by follow-up questions during an Executive MBA module, in which sales forecasting and inventory control techniques were covered. The module participants highlighted the need for a combined decision making framework for optimal forecasting and inventory decisions. The industry practitioners wanted a tool that not only handled the demand uncertainty but also captured the essence of decision-making in practical settings. Later on, we came across a heuristic, proposed by Axsäter, for updating forecasts and inventory control parameters. This further motivated us to investigate the efficacy of the heuristic proposed by Axsäter and if required, propose a better algorithm. Thus the journey towards design of the new algorithm was begun.

Approximate solutions that are easy to understand are usually implemented in practice. The current research proposes a heuristic, we refer to as Wagner Whitin Plus Forecast (hereafter termed as WWPF), which incorporates the above-mentioned attributes for a desired approximate solution. In WWPF, forecasts are revised, and the inventory control parameters are dynamically updated each period. We consider a class of non-stationary demand models exhibiting linear trend. The heuristic integrates sales forecasting and inventory control leading to minimal supply chain costs. In our heuristic, exponential smoothing with trend is used to forecast the demand for future periods. Safety stocks are added to the forecast estimates based on the *MAD* (Mean Absolute Deviation). The safety stock is directly proportional to the forecast inaccuracy.

In every period of the planning horizon, the net requirements for future periods, derived from the forecast estimates, are adjusted for planned order arrivals and stock carried over from previous periods. The net requirements are treated as deterministic demand. Using the Wagner-Whitin algorithm (Wagner and Whitin, 1958), i.e., dynamic lot sizing (called "WW" henceforth), order quantities are computed. The order quantity is determined for each period in the planning horizon, including the current period. Based on the actual demand realized for the previous period, the sales forecast model updates the *MAD* (Mean Absolute Deviation) and safety stocks are added to the order quantities. The order quantity for a period is equal to prescribed WW order quantity plus a safety stock. In the next period, the smoothing parameters are re-optimized and new forecast estimates are computed for the future periods.

Axsäter (2000) proposes an adaptive (s, S) policy and the EOQ (Economic Order Quantity) model is used to compute the optimal order quantity and the re-order level[1]. The

[1] The following transformation is used: s=Reorder point, and S-s=Order quantity

adaptive policy is applicable for periodic review systems and backorder / lost sales environment. To account for the trend component in demand, the average demand rate in each period is updated using exponential smoothing with trend forecast model. Using the updated demand rate in a particular period, the current parameters of order quantity and reorder level are computed using the EOQ model. The reorder level incorporates a safety stock based on the *MAD* values and whenever the inventory position falls below the re-order level, the optimal order quantity is ordered.

In the current research, a numerical study is conducted to compare the performance of the two algorithms. The choice of input parameters for the numerical study is motivated by Porteus (1985). A planning horizon of 24 periods is selected and the horizon is partitioned into three zones; Initialisation, Stabilisation and Comparison. The performance metrics used for comparison are total cost, service level and average stock out level. The algorithms are compared to each other, and to a perfect information solution.

1.2 Scope Of Research Project:

This section outlines the scope of the current research project. The objective is to compute near optimal values of periodic review[2], inventory control parameters for a single-product, independent, nonstationary demand following a linear trend. The demand data are obtained through simulated values, and no effort has been made to collect real demand data.

The research project proposes a heuristic, WWPF, as an inventory control algorithm for nonstationary demand. The WWPF algorithm is compared to an adaptive (*s*, *S*) inventory control policy proposed in Axsäter (2000), and exponential smoothing with trend is used as a forecasting model in both algorithms. Both algorithms are also compared to a perfect information solution, termed as Baseline algorithm. The perfect information solution is computed by assuming that the decision-maker exactly knows the simulated demand data, in advance.

The research excludes an optimal solution to the inventory problem for the nonstationary demand problem using dynamic programming. Moreover, there is no effort towards improving the method for updating forecasts.

The WWPF, Axsäter and Baseline algorithms have been modeled in MS Excel using VBA modules. The MS Excel plus VBA tool incorporates a graphical user interface and it may serve as a classroom example to illustrate the concepts of integrating sales forecasting and inventory control. This tool can be used during executive MBA courses, where managers with operational insight can appreciate the concepts embedded in the tool.

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[2] Using short periods, continuous review aspects can be modeled too.

1.3 Literature Review:

Graves (1999) proposes an inventory control policy for a demand model following the IMA (Integrated Moving Average) process of order (0,1,1) in which, the demand model behaves like a "random-walk". A first order exponential weighted moving average, which results in the minimum MSE (Mean Square Error), is used as the forecast updating method. The inventory control follows an adaptive base stock control policy and any unsatisfied demand is backordered. In each period *t,* a demand, d_t, is observed and fulfilled from the stock-on-hand. An order quantity, q_t, is placed using an adaptive base stock policy and follows the relation:

$$q_t = d_t + L(F_{t+1} - F_t).$$

This is the myopic policy (Veniott, 1965) for a *L*-period lead-time, assuming stationary parameters and it minimizes the expected one-period cost a lead-time into the future. The order quantity has two components. The first part replenishes demand for the immediate period and the second part adjusts the base stock level to accommodate any changes in the forecast. The demand over the lead-time is considered to be $L*F_{t+1}$ rather than $L*F_t$, as F_t is the forecast from period *t* as seen from period *t-1*. Using the critical fractile policy, the order quantity can be set to a level so that specified stock out probability is satisfied. The inventory control policy is not optimal but a reasonable extension of base stock policy to the case of non-stationary demand.

Bollapragada and Morton (1999) present a myopic heuristic to compute (*s, S*) parameters for nonstationary demand. The heuristic is based on stationary approximation to the portions of the nonstationary problem. It involves precomputing (*s, S*) policy values for several stationary problems with different values of mean demand and tabulating the results. The non-stationary problem is approximated to a stationary problem by averaging the demand parameters over an estimate of time between two orders. The corresponding (*s, S*) values are then read from the pre-computed stationary table. The length of time over which demand parameters are averaged is equal to the "optimal expected replenishment time period for the equivalent stationary problem". A numerical study is conducted to evaluate the effectiveness of the heuristic under geometric trend and seasonal demand models. The heuristic is compared to Askin's heuristic (Askin, 1981). Askin's heuristic is an extension of the Silver-Meal (1973) heuristic to the case of stochastic demand and the order quantities are computed for minimising the average total cost of inventory per period. The heuristic is also compared to an optimal solution, obtained using dynamic programming, which needs the demand data for the entire time horizon.

Porteus (1985) presents a number of shortcuts and approximations for computing periodic review (*s, S*) parameters for stationary demand. Some of the shortcuts are based on the EOQ approximations and consider a penalty cost for backorders. The goal is to obtain approximately optimal policies with little computational effort. The shortcuts and approximations are compared to the optimal solution in terms of the error percentage and the computation time associated with each method. A numerical study is conducted for 1200 combinations of input parameters[3]. For every unique combination of input parameters, total costs and computation times associated with each shortcut method are compiled. The methods are compared on the averages for error percentages in cost and computing times. Based on error percentages and saving in computing times, the efficacy of shortcuts is highlighted. The numerical study in Porteus (1985) motivates the numerical study discussed later in the thesis.

As compared to the above-mentioned literature, WWPF deals with a single product, nonstationary demand, under a finite horizon, periodic review and a "lost-sales" inventory control model. The nonstationary demand model follows a linear trend, and exponential smoothing with trend is used as the method for updating forecasts. In each period *t,* net requirements for the future periods are computed from the forecast values adjusted for inventory position, and the order quantity is computed using the WW algorithm. The order quantities are augmented by adding safety stocks to account for forecast inaccuracies. The order quantities and the safety stocks are dynamically updated every period. A numerical study is designed to test the performance of WWPF and it is compared to Axsäter's adaptive (*s, S*) policy and a perfect information solution.

[3] Mean demand, variance, lead-time, holding cost, setup cost, and backorder penalty.

CHAPTER 2: ALGORITHMS FOR NON-STATIONARY DEMAND

This chapter describes the demand model, the modeling assumptions and the algorithms for nonstationary demand. In section 2.1, the nonstationary demand model is explained. The same demand model is used for all three algorithms. In section 2.2, the use of exponential smoothing with trend forecasting is described. In section 2.3, the assumptions associated with modeling are explained. The rationale of the modeling assumptions would be dealt later in the thesis. In remaining sections 2.4, 2.5, and 2.6, WWPF, Axsäter and Baseline algorithms are described, respectively. Finally, in section 2.7, the design of an optimal solution using dynamic programming is discussed.

2.1 Demand model:

A single-product, independent, stochastic demand with nonstationary parameters is treated as the demand model for the inventory control algorithms. The stochastic demand follows a normal distribution and the mean of the random variable follows a linear trend.

The demand, D_t, can be mathematically expressed as the following.

$D_t \sim N(\mu_t, \sigma^2)$, and

$\mu_t \equiv mt + \mu_0$,

where, D_t is the demand at time period t, μ_t is the mean of the normal demand generating process at time t and σ is the standard deviation of the demand. The mean of the demand process, μ_t, follows a linear trend and its definition is pictorially represented in Figure 1. By choosing different values for the slope and the intercept, a wide range of values for the mean can be obtained.

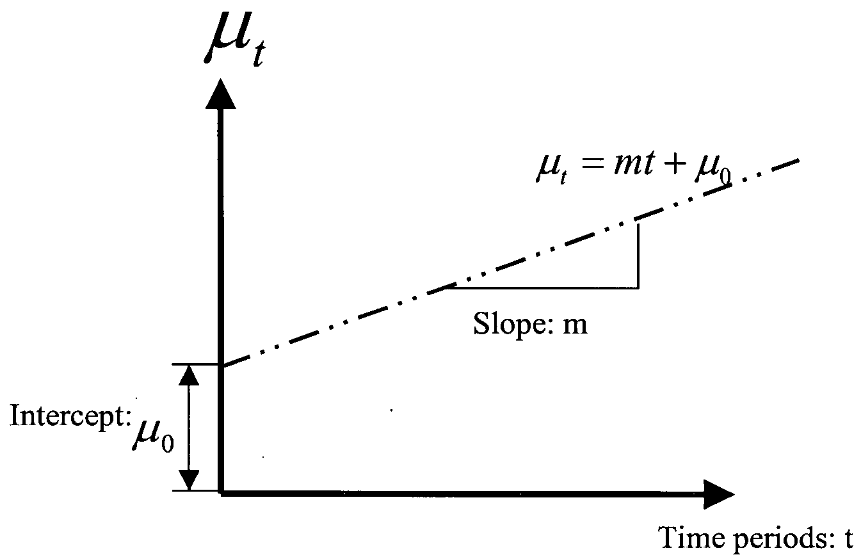

Figure 1: Mean of the stochastic demand with linear trend

2.2 Modeling assumptions:

In order to make the inventory system model mathematically manageable, the following simplifying assumptions are made.

- The demand values in successive periods are independent random variables.
- Whenever the actual demand exceeds stock-on-hand it results in loss of sales, and there is no penalty associated with unsatisfied demand. A stock out condition may lead to the loss of goodwill and it can affect the demand in the future periods but due to the difficulties associated with modeling these losses, the simplifying assumption has been made.
- A review period is the length of time between successive moments at which the inventory position is reviewed and the ordering decisions are taken. The duration of the review period is fixed and integer valued. For the purpose of this research the review period has been fixed as unity, i.e., inventory position is reviewed every period. A frequent review of inventory would be beneficial towards the dynamic updating of the inventory parameters. Though a frequent review leads to

cost increases, the cost savings from the dynamic updating would more than compensate the cost increases.

- The inventory is reviewed at the start of the period and the opening stock is equal to the closing stock of the previous period.

- The holding cost incurred during the current period is proportional to the excess inventory[4] carried from the immediate previous period. The units held during the current period are not charged for holding cost.

- A setup cost of $\$K$ is incurred for every order release.

- The lead times are deterministic and integer valued.

- All shipments scheduled from the previous orders arrive at the beginning of a review period. These shipments are available for use in the current period. When the lead-time is zero, an order is placed and received at the same instant. The order arrival adds to the opening stock.

- The capacity of the source of supply is considered as infinite. This ensures that any order placed by the retailer or the manufacturer can be fulfilled.

2.3 Forecasting model:

Exponential smoothing with trend is used to forecast the demand for future periods. This forecasting model is also known as Holt's linear method. Exponential smoothing methods behave like control processes, where the gap between the forecast value and the actual value is corrected with the use of smoothing parameters. The method allocates exponentially decreasing weights, as the observations get older. There are two smoothing parameters, one for level smoothing and the other for trend smoothing. The forecast at any point in time is made of a level and a trend component. The forecasting method updates the level and the trend using two smoothing parameters.

[4]Positive opening stock at the start of current period

The following notation is used to describe the forecasting model.

a_t = Level component of time series at time t

b_t = Trend component of time series at time t

α = Level smoothing parameter

β = Trend smoothing parameter

n = time periods in integers

F_{t+n} = Forecast for demand at time $t+n$, i.e., n periods in future, computed at time t

The exponential smoothing with trend model consists of the following equations.

$$a_t = \alpha D_t + (1-\alpha)(a_{t-1} + b_{t-1})$$

$$b_t = \beta(a_t - a_{t-1}) + (1-\beta)b_{t-1}$$

$$F_{t+1} = a_t + b_t$$

$$F_{t+n} = a_t + b_t n \qquad (1)$$

Table 1 illustrates the forecasting model as setup in MS Excel.

Table 1:Forecasting model setup in MS Excel using exponential smoothing with trend.

Period	Demand	Level	Trend	forecast	AbsError	Alpha	Beta	
t	D_t	a_t	b_t	F_t	e_t	$\boxed{\alpha}$	$\boxed{\beta}$	*MAD (t)*
1	61.00	61.00	6.00					
2	67.00	67.00	6.00	67.00	0.00	0.85	0.50	0.00
3	64.00	65.31	2.17	73.00	9.00	0.85	0.50	4.50
4	54.00	55.97	-3.56	68.00	14.00	0.85	0.50	7.67
5	68.00	65.73	3.07	53.00	15.00	0.85	0.50	9.50
6	60.00	61.28	-0.67	69.00	9.00	0.85	0.50	9.40
7	70.00	68.63	3.32	61.00	9.00	0.85	0.50	9.33
8	64.00	65.16	-0.06	72.00	8.00	0.85	0.50	9.14
9	71.00	70.14	2.45	66.00	5.00	0.85	0.50	8.63
10	76.00	75.50	3.90	73.00	3.00	0.85	0.50	8.00
11	88.00	86.75	7.56	80.00	8.00	0.85	0.50	8.00
12	69.00	72.69	-3.20	95.00	26.00	0.85	0.50	9.64

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The absolute error in time period t, e_t, is defined as

$$e_t = |a_t - D_t|$$

The mean absolute deviation in time period t, MAD (t), is defined as

$$MAD\,(t) = Average\ \{e_1 + e_2 + ...e_{t-1}\}$$

The mean squared error in time period t, MSE (t), is defined as

$$MSE(t) = Average\left(e_1^{\ 2} + e_2^{\ 2} + ...e_{t-1}^{\ 2}\right)$$

The smoothing parameters for the level and the trend, α and β respectively, are computed using a non-linear optimization for minimizing the MSE. The selection of optimal smoothing parameters and the selection criteria is addressed in section 3.3.1.

In order to set the forecasting model, the following initializing conditions are applied. The initialization has been quoted from Makridakis (1998, pp-159).

- The level in the first period, a_1, is set equal to the demand in the same period, i.e., $a_1 = D_1$
- The trend in the first period, b_1, is set equal to the difference in the demand in the first and second periods, i.e., $b_1 = D_2 - D_1$

Gardner (1985) remarks the use of the above initialization approach in practice, and the popularity is attributable to its simplicity. An alternative, proposed by Makridakis (1998, pp-161), is to use least squares regression on the first few values of the series for finding a_1 and b_1. The details for using least squares regression have not been discussed. Another alternative, with a limited number of data, is to use Bayesian methods to combine a prior estimate of the level with an average of the available data-see Cohen (1966), Johnson and Montgomery (1974) and Taylor (1981). In case historical data is available, Brown (1959) recommends using the mean of the data for an initializing value of the level.

Using the above-mentioned forecasting model, it is possible to have negative values for demand forecasts, due to the negative trend values. As a modification to the exponential smoothing with trend model, a negative value of demand forecast is substituted with zero for modeling convenience.

2.4 Wagner Whitin Plus Forecasting (WWPF):

In this section the Wagner Whitin Plus Forecasting algorithm is described. A short outline of the algorithm along with a pictorial representation is presented first, followed by the details.

2.4.1 *Inventory Control algorithm*:

The inventory control algorithm of WWPF has been pictorially summarised in Figure 2. The following notation is used for describing the WWPF algorithm.

$T=$	planning horizon, i.e., if $T = 24$, there are 24 periods in the horizon for which the inventory control policy is being formulated
$t=$	current period
$R(t) =$	Net requirements for period t (explained in the next sub-section)
$O(t) =$	Order released in period t, to be received in period $t + L$
$OS(t)=$	Opening Stock for period t
W-W:	Wagner-Whitin algorithm, also known as, dynamic lot-sizing
L	Lead-time in periods
D_t	Actual demand realised in period t

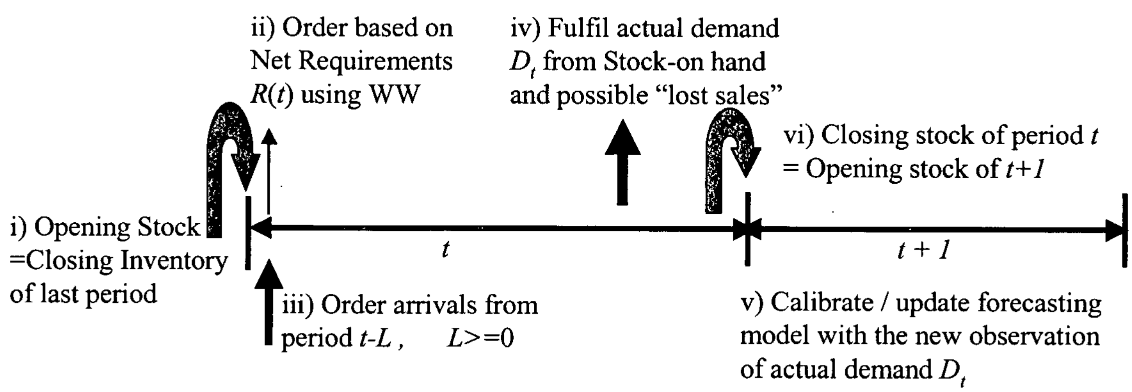

Figure 2: Pictorial presentation of WWPF algorithm

2.4.2 *Outline of WWPF algorithm*:

Based on the sequential steps described in Figure 2, an outline of WWPF algorithm is presented. At any time t, WWPF algorithm can be summarised by the following sequential steps.

i. **Carry forward the closing inventory of the previous period**:

The closing inventory of the last period, $t-1$, is carried over to the current period, t, and treated as opening stock. In case the demand in $t-1$ exceeds the stock-on-hand, the opening stock for the period t is zero.

ii. **Compute net requirements and order quantities using WW algorithm**:

The net requirements, $R(t)$, are computed for periods t, $t+1$, $t+2$, ...,T. The net requirements for the future periods are computed by subtracting planned order arrivals[5] and opening stock from the forecast values of the future periods. Mathematically, this can be expressed as:

$R(t) = F_t - O(t-L) - OS(t)$

The net requirements R (t) are computed for all values up to T including the current period t, and then W-W is used to determine the ordering quantities for all the periods up to T, including the current period. The WW order quantity for the current period is considered for order release and the order quantities for the other periods are ignored. The order quantity in the current period is augmented by adding a safety stock to the order quantity. The augmented amount is ordered as order release $O(t)$ in the current period t.

iii. **Receive $O(t-L)$**:

The order quantity placed in $t-L$, $O(t-L)$, is received as planned order arrival in period t and this increases the stock-on-hand.

[5] Due to orders in earlier periods i.e. O ($t - L$)

iv. **Fulfill D_t with Stock-on-hand**:

The actual demand, D_t, is satisfied from stock-on-hand. The demand is fulfilled as and when the demand[6] arises and there are possible "lost-sales" when the actual demand exceeds the stock-on-hand.

v. **Update forecasting model**:

At the end of the current period, t, the actual demand for the period is known and the smoothing parameters for the forecast model are updated based on a measure of forecast accuracy.

vi. **Closing stock of current period:**

The closing stock equals the stock-on-hand, after receipt of the planned orders, minus the actual demand. For practical purposes, the closing stock is zero when the actual demand exceeds the stock-on-hand. However, during the numerical study, explained in Chapter 3, the number of units that were not satisfied from stock-on-hand is recorded.

[6] We do not go into the details of how the demand appears during the period. The demand can be realised at an instant or with a uniform distribution.

2.4.3 *Details of WWPF algorithm*:

Now we discuss some of the steps in details. Several steps have been sufficiently dealt in the outlines and are skipped in the detailed discussion.

2.4.3.1 *Net Requirements*:

The net requirements $R(t)$ are computed for periods t, $t+1$, $t+2$, ……,T by adjusting the forecast values with the planned order arrivals and the opening stock. Table 2 illustrates an example where we have the actual demand data points for periods 1 to 11, and the net requirements have been computed for the periods 12,13….,24.

The notations and computations referred in Table 2 are explained as follows. At

Table 2:Sample computations for Net Requirements in Excel sheet

Notation:

D1= F_t - $O(t\text{-}L)$

D2= **D1** - $OS(t)$

D3= Max{**D2**, 0} = $\boldsymbol{R(t)}$

L= 3 periods

$OS(12)$= 54 units

Period t		Demand	Order Release $O(t)$	D1	D2	D3
1	D_t	61				
2	D_t	67				
3	D_t	64				
4	D_t	54				
5	D_t	68				
6	D_t	60				
7	D_t	70	185			
8	D_t	64	0			
9	D_t	71	126			
10	D_t	76	0			
11	D_t	88	134			
12	F_t	70		-56	-110	0
13	F_t	67		67	-43	0
14	F_t	64		-70	-113	0
15	F_t	60		60	-53	0
16	F_t	57		57	4	4
17	F_t	54		54	54	54
18	F_t	51		51	51	51
19	F_t	48		48	48	48
20	F_t	44		44	44	44
21	F_t	41		41	41	41
22	F_t	38		38	38	38
23	F_t	35		35	35	35
24	F_t	32		32	32	32

the start of period 12, the actual demand figures of periods 1 to 11 have been observed. Now based on the forecast model as described in the section 2.3, the level a_t and trend b_t at t = 12, are computed. Using exponential smoothing with trend forecasting model, we forecast the demand figures for all future periods. Using the equation (1) discussed in section 2.3, the equivalent equation for forecasts in periods 12,13...24 becomes the following.

$$F_{12+n} = a_{12} + (n+1) * b_{12}$$

The demand forecast value, adjusted with planned order arrivals, referred as **D1** in Table 2, is computed using the expression

F_t - $O(t\text{-}L)$.

The demand forecast value is further adjusted for the opening stock and is referred as D2 in Table 2. The value of **D2** is computed using the expression:

D1- $OS(t)$.

The negative values in **D2** imply excess stock and so the net requirements are zero, until this excess stock is consumed. Therefore, the column of **D3** follows the transformation Max (**D2,** 0) and these values are used as the net requirements for the future periods.

2.4.3.2 *Wagner-Whitin algorithm*:

The Wagner-Whitin (WW)[7] algorithm was proposed in Wagner and Whitin (1958) as dynamic version of the EOQ model. The WW algorithm computes the periods in which orders are placed and the corresponding order quantities, so as to incur the least total cost. WW assumes a deterministic demand and the demand is realised at the start of the corresponding period. It is also assumed that replenishments always cover the demand for an integer number of consecutive periods. The cost function for the algorithm is a fixed cost of \$$K$ per order, and a proportional holding cost of \$$h$ per unit per period of holding[8].

[7] Wagner-Whitin algorithm is also known as dynamic lot-sizing

[8] The demand in various periods is assumed to occur at the beginning of the period, so that the holding costs are charged only for inventory held for future periods and not for the current period i.e. no holding charge for D_t

To apply WW in WWPF algorithm, the net requirements, shown as **D3** in Table 2, are called adjusted demand. Now for the current period, we consider these adjusted demand figures as deterministic and the ordering policy is computed using a backward induction method. The algorithm is explained in the following paragraphs.

The WW algorithm is explained by means of an example. The details of the algorithm can be found in Wagner and Whitin (1958). Suppose we have a certain demand to be fulfilled in the various periods as shown in Table 3, and using WW algorithm we need to compute how much to order and when to order. A setup cost, *K,* is $5000 per order, and a holding cost, h , of $ 1 per unit per period of holding constitute the cost function. The objective is to incur the least total cost and satisfy all demand.

Table 3: Demands for periods 1 to 10

Period t:	1	2	3	4	5	6	7	8	9	10
Demand: D_t	600	698	726	770	820	874	866	916	930	981

The following definitions are used to derive the WW solution to the demand shown in Table 3. The notation and equations for WW algorithm have been cited from Axsäter (2000, pp 43).

Let,

D_t = adjusted demand in period t

f_k = Minimum cost over periods 1,2,…, k ,i.e., when periods $k+1,k+2,...,T$ are disregarded.

$f_{k,t}$ = Minimum cost of satisfying demand in periods $t, t+1, ..., k$, given that the last delivery was in period t ($1 \le t \le k$)

Now $f_k = \min\limits_{1 \le t \le k} f_{k,t}$ (2)

The boundary condition values are:

$$f_0 = 0$$

$$f_1 = f_{1,1} = \begin{cases} K, D_1 > 0 \\ 0, D_1 = 0 \end{cases}$$

Let us assume that we know f_{t-1} for some $t > 0$; then we can obtain $f_{k,t}$ for $k > t$ by the following expression.

$$f_{k,t} = f_{t-1} + K + h(D_{t+1} + 2D_{t+2} + \dots\dots\dots\dots + (k-1)D_k) \qquad (3)$$

The demands are assumed to take place at the beginning of the periods. This implies that the demand in any period (D_t) will not cause any holding costs, and the holding costs would be charged only when an inventory is stocked to meet the demand in future periods.

The term $f_{k,t}$ in equation (3) represents the total cost of ordering in t to satisfy the demand in periods t, $t+1$, $t+2 \ldots k$. In that case the order quantity covering demand in $t+1$ is held for one period, order quantity for covering demand in $t+2$ is held for two periods, and so on. Therefore the demand D_{t+1} incurs a holding cost of $h\ D_{t+1}$, the demand D_{t+2} incurs $2hD_{t+2}$, and the equation (3) holds.
For $k = t$, equation (3) becomes

$$f_{t,t} = f_{t-1} + K .$$

At any given time t, assume that f_{t-1} is known and then the values for $f_{k,t}$ are computed for all the values of k until the following condition is achieved.

$$h(k-t)D_k \geq K . \qquad (4)$$

The idea is that if we order for the demand in a future period and the cost of holding for that future period equals or exceeds the setup cost, then we would order for that period later on.

A cost matrix is prepared from these considerations and is shown in Table 4. In this matrix, we have the periods, the corresponding demands, and the values for $f_{k,\,t}$.

Table 4: Cost matrix and backward induction using WW

Period t:	1	2	3	4	5	6	7	8	9	10
Demand: D_t	600	698	726	770	820	874	866	916	930	981
COST MATRIX										
$k=t$	5000	10000	10698	12150	14460	17740	19718	22066	24814	27494
$k=t+1$	5698	10726	11468	12970	15334	18606	20634	22996	25795	0
$k=t+2$	7150	12266	13108	14718	17066	20438	22494	24958	0	0
$k=t+3$	9460	14726	15730	17316	19814	23228	25437	0	0	0
$k=t+4$	12740	18222	19194	20980	23534	27152	0	0	0	0
$k=t+5$	17110	22552	23774	25630	28439	0	0	0	0	0
BACKWARD INDUCTION & OPTIMAL SOLUTION										
Min Values	9460				17066			24958		
Order Quantity	**2794**				**2560**			**2827**		

In Table 4, the cost matrix begins from the column for period 1(i.e., $t=1$) and $f_{k,t}$ values are computed as per equations (2) , (3) and (4). In period 1, the $f_{1,1}$ value of 5000 represents the cost when an order is placed in period 1 to cover the demand of period 1 only. The second value in the first column, 5698 ($f_{2,1}$), represents the cost of ordering in period 1 when the order quantity covers the demand for periods 1 and 2. Similarly, the third value in column 1, 7150 ($f_{3,1}$), represents the cost of ordering in period 1 when the order quantity covers the demand in periods 1, 2, and 3. These $f_{k,t}$ values are computed in column t until the holding costs for future periods demand is less than the setup cost (See equation (4)). The values of $f_{k,t}$ in cost matrix, where the condition expressed in equation (4) is attained, are replaced by zero.

After generating $f_{k,t}$ values for period 1, we go to period 2 (i.e. $t=2$) column. The first entry in this column, 10000 ($f_{2,2}$) represents the cost of ordering in period 2 to cover demand in period 2, given that the minimum cost of ordering for period 1 is known. Using equation (7), the value for $f_{2,2}$ can be computed as:

$$f_{2,2} = f_1 + K$$

where f_1 is known from the boundary condition values. The second entry in this column, 10726 ($f_{3,2}$), represents the cost of ordering in period 2, given that the optimal order was placed in period 1, and the order quantity covers the demand in periods 2 and 3. Similarly, the rest of the entries in column 2 are computed till the condition expressed in equation (4) is reached.

In the column for period 3, the first entry 10698 ($f_{3,3}$) represents the cost of ordering in period 2, given that the minimum cost of ordering for periods 1and 2 is known. Using equation (3), the value for $f_{3,3}$ can be computed as:

$$f_{3,3} = f_2 + K$$

The minimum cost for periods upto 2 is given by corresponding substitution in equation (2) as

$$f_2 = \min(f_{2,1}, f_{2,2}),$$

i.e., the minimum of values 10000 ($f_{2,2}$) and 5698 ($f_{2,1}$). Similarly the cost matrix is completed for all the periods upto 10.

Once the cost matrix is ready, the optimal solution is computed using backward induction. The backward induction begins from the last period of the planning horizon. For $k=T$ and $t=T$, find the minimum of $f_{k,t}$, $f_{k,t-1}$, $f_{k,t-2}$, ... $f_{k,t-r-1}$. The $(r+1)^{th}$ term has a value that achieves the condition expressed in equation (4) and r can take any value as 1,2.....t.

In our example, for $k=10$, we compare the following values:

- 27494 ($f_{10,10}$= cost of satisfying the demand in period 10 by ordering in period10)
- 25795 ($f_{10,9}$= cost of satisfying the demands of periods 9 and 10 by ordering in 9)
- 24958 ($f_{10,8}$=cost of satisfying the demand of periods 8,9,10 by ordering in 8)
- 25437 ($f_{10,7}$ =cost of satisfying the demand of periods 7,8,9,10 by ordering in 7)
- 27152 ($f_{10,6}$. =cost of satisfying the demand of periods 6,7,8,9,10 by ordering in 6)

- 28439 ($f_{10,5}$=cost of satisfying the demand of periods 5,6,7,8,9,10 by ordering in period 5)

The iteration stops after the value 28439 as the next value is zero, corresponding to the condition expressed in equation (4), and therefore not shown in the table. Out of the above values, the minimum value is 24958, corresponding to an order in period 8 covering the demands in periods 8,9 and 10. The ordering quantity is the sum of demand in periods 8,9 and 10, i.e., 2827 units.

As we have a solution for periods 8, 9, and 10 so we next consider $k=7$ and $t=7$, and compare values of $f_{7,7}$, $f_{7,6}$, $f_{7,5}$ until the condition in equation (4) is reached. We compare the values 19718, 18606, 17066, 17316, 19194, and 22552. In this case the minimum value is 17066, corresponding to an order of 2560 units in period 5, covering the demand in periods 5, 6 and 7. Similarly, we continue our backward induction process till we reach the current period. Thus the optimal ordering periods and the corresponding order quantities for the entire planning horizon are computed.

The solution for the dynamic lot-sizing problem using the Wagner-Whitin algorithm is shown in Table 5.

Table 5: Least total cost solution using backward induction as applied in WW algorithm

Period t:	1	2	3	4	5	6	7	8	9	10
Demand: D_t	600	698	726	770	820	874	866	916	930	981
BACKWARD INDUCTION & OPTIMAL SOLUTION										
Order Quantity	**2794**				**2560**			**2827**		

The WW algorithm assumes that there is no lead-time, but in our model we do consider lead times. In order to model lead times in WWPF, the ordering decisions are computed for the future periods beyond the lead-time. Say, in our 10-period example the lead-time is 2 periods, then the ordering decisions, starting at the current period, would be computed for periods 3,4….,10. The first two periods, 1 and 2, corresponding to the lead-time are just ignored. Due to the 2-period lead-time constraint, we cannot fulfill the demand of periods 1 and 2 from ordering at the start of period 1. On the other hand, for a lead-time of 2 periods, we can order in period 1 and satisfy the demand for periods 3 and onwards.

In our modified version of Wagner-Whitin algorithm, termed as WWPF, the demand in future periods are computed from the net requirements as discussed in the previous section. Once the order quantities are determined using the Wagner-Whitin algorithm, a safety stock is added to account for the forecast inaccuracy. In order to determine a suitable safety stock, the decision maker needs to know how uncertain the forecast is, i.e., how large forecast errors tend to be. The variations are usually described using standard deviation. For forecast errors, it is traditional to estimate the MAD instead of directly estimating the standard deviation of forecast errors. Using the common assumption that the forecast errors are normally distributed, the standard deviation of demand during one period, σ_t, can be estimated from MAD using the following relation.

$$\sigma_t = \sqrt{\frac{\pi}{2}} \times MAD_t \approx 1.25 MAD_t$$

where, MAD_t is the mean absolute deviation for the forecast model at time t. MAD computations have been discussed in section 2.3.

Usually, safety stocks are added to account for variations in the "vulnerable periods", i.e., periods in which no contingent shipment can arrive. This period of risk is defined as the time interval between inventory position reaching the re-order level and the arrival of the resulting replenishment order. The demand during this period of risk has two components – the dropping of inventory position below the re-order level and the demand during the lead-time. The difficulty with modeling the first component in a periodic review, lost sales environment would be addressed in the section 5.2.3. However, safety stocks should be added to cover the demand in the lead-time, i.e., the second component of demand in the period of risk. On the contrary, WWPF has been modeled in a way that ordering decisions are based on order receipts in future periods. Consequently, in case of any unanticipated variation in demand during the lead-time, a contingent replenishment can be ordered during the next review. In WWPF, safety stocks are added to account for variations during the number of periods covered by the order quantity. Say, at any time t, the order quantity obtained using WW algorithm covers the demand for n periods, inclusive of the current period. In that case, the standard deviation of demand during the periods covered by the order quantity is given by the following expression.

$$\sigma_n = \sigma_t \sqrt{n} = \sqrt{\frac{\pi}{2}} \times MAD_t \sqrt{n} \approx 1.25 MAD_t \sqrt{n}$$

where, σ_n is the standard deviation of the total demand during the n periods that are covered by the order quantity in t. The forecast errors in the n periods are assumed to be independent.

The safety stocks can then be computed as per the following formula:

$$SS = k\sqrt{\frac{\pi}{2}} MAD\sqrt{n}\,, \tag{5}$$

where,

k = Safety Factor corresponding to service level constraints

SS= Safety stock

MAD = Mean absolute deviation of forecast model

n = number of periods covered by the order quantity.

MAD value in period t is computed at the start of each period, is the average of the absolute deviation for the previous t-1 periods (Refer forecasting in section 2.3).

If we use lead time instead of n for computing the safety stock values, then we might incur increased ordering and holding costs. In case the lead time is greater than n, excess safety stock would be added to the order quantities leading to increased holding costs. On the contrary, for lead time values less than n, the safety stocks would be insufficient and a premature order would be warranted leading to greater ordering costs. Therefore, it is optimal to use n for computing safety stocks in WWPF algorithm.

Given that the demand is normally distributed and the standard deviation of the demand during the n periods is known, a standard z-value, corresponding to the probability of no stock out in the vulnerable period, can be used as the safety factor k. For example, if the decision maker wants to achieve 95 % rate of no stock-out and the MAD is exactly known, then using the standard normal distribution, 95% of all demand observations, during the n periods, would be under 1.645 times of the standard deviation. This explains the use of the safety factor k in equation (5).

For a given *MAD*, say 100, and a safety factor of 1.645, the actual order quantities for the same example (see Table 3), are computed in Table 6. The safety factor of 1.645 approximately corresponds to 95% probability of no stock out in the vulnerable period, as the MAD is an estimate. The values of D_t represent the deterministic demand for a planning horizon of ten periods.

Table 6: Ordering quantities using WWPF with safety stocks

Period t:	1	2	3	4	5	6	7	8	9	10
Demand: D_t	600	698	726	770	820	874	866	916	930	981
BACKWARD INDUCTION & OPTIMAL SOLUTION										
Order Quantity	**2794**				**2560**			**2827**		
Safety Stocks	$SS_1 =$ 412				$SS_5 =$ 357			$SS_8 =$ 357		
Actual Order Quantity	**3206**				**2917**			**3184**		

Using equation (5), we have the following safety stock quantities.

$SS_1 = 1.645 * 1.25 * 10 * \text{sqrt}(4) = 411.25$

$SS_5 = SS_8 = 1.645 * 1.25 * 10 * \text{sqrt}(3) = 356.14$

Therefore, the safety stock added to the order quantity in period 1 (SS_1) is 412 units, and 357 units[9] in period 8. Similarly, the safety stock values are computed for all ordering periods depending on the number of periods covered by the order quantity.

The *MAD* values for determining safety stock values in various future periods are the same, i.e., 100 as assumed in the example.

The dynamic lot-sizing algorithm described in Wagner and Whitin (1958) does not allow any backordering. In case backorders are allowed, a backorder penalty is charged. Silver and Peterson (1985, pp-263) present three variants for backorder penalty costs depending on the way the penalty is charged, i.e., per stock out occasion, per unit short, and per unit short per unit time. The choice of backorder cost definition should reflect the real costs incurred by backorders. However, there are difficulties associated with assessing backorder costs. Axsäter (2000, pp-58) highlights the problems associated with shortage costs as:

"One problem with shortage costs is that practitioners usually find it difficult to determine how high they should be."

Therefore, backorder penalty has not been considered in the current research.

WW algorithm provides 100% service level and it cannot be modeled for a target service level other than 100%. However, a network flow integer program for modeling a deterministic lot sizing with service level constraints is described in section 2.6.3.

[9] Safety stock values are rounded up to the next integer values to obtain integral order quantities.

2.4.3.3 *Ordering for the current period*:

After order quantity computations, only the order for the current period is placed. The order quantities for other periods are ignored. Referring to Table 6, we order **3206** units in the current period and ignore the actual orders in periods 5 and 8. This is done, as the objective is to dynamically update the order quantities in line with the changing demand.

2.4.3.4 *Update the forecasting model*:

At the end of the current period t, the actual demand for the period is realised. It is at this point that we realise the deviation of the forecast estimate from the actual demand. The forecasting model is then updated in the following steps.

i. Compute a new value of MAD_{t+1} (mean absolute deviation) including the D_t value, the most recent demand. This value of MAD is used to compute the safety stock for determining the order quantities at the start of the next period $t+1$.
ii. The smoothing parameters for the exponential smoothing with trend model can be recalculated as we have a new data point from the recent demand[10]. The new forecasting model would be used in the next period for computing the forecast values for the future periods.

[10] Refer to the forecasting model section

2.5 *Axsäter's Heuristic:*

Axsäter (2000) proposed a heuristic that updates the inventory control parameters with changes in forecast model. The heuristic is proposed for demand data with trends and utilises a modified EOQ lot-sizing algorithm. The forecast model is exponential smoothing with trend and the forecast components of the demand are used to dynamically update the average demand rate of the EOQ model. The proposed inventory model is applicable in both continuous and periodic review inventory control models. The demand and forecasting model for Axsäter's heuristic is the same as that for WWPF algorithm. The modeling assumptions discussed in the section 2.2 hold good for Axsäter's heuristic, too.

2.5.1 *Inventory control for Axsäter's heuristic:*

In this section, we present the inventory control steps for Axsäter's heuristic. The following notation and terms are used for describing the algorithm.

t =	current time period
R =	Re-order level
Q =	Order placed in any period and received L periods later.
I.P. =	Inventory Position (Stock-on-hand + Outstanding orders[11])
EOQ =	Economic order quantity
D_t =	Actual demand for period t
F_t =	Demand forecast for period t

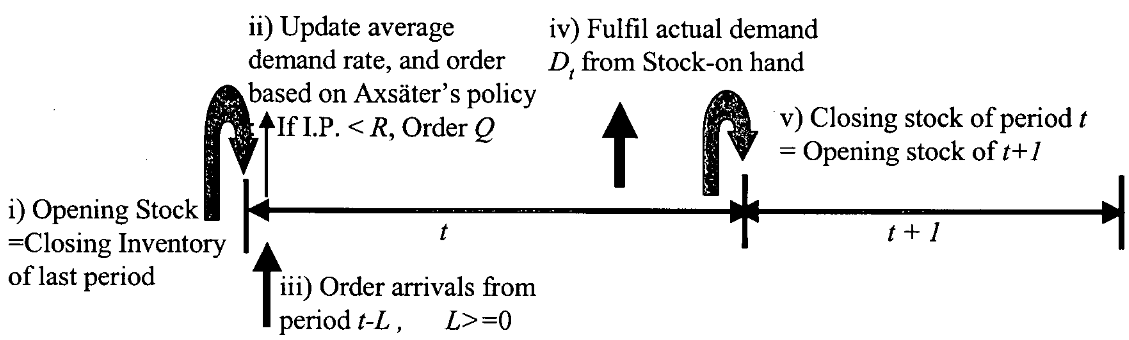

Figure 3:Inventory Control Policy using Axsäter's Heuristic

[11] Orders released but yet to be received.

Figure 3 summarises a pictorial representation of the inventory control policy followed by Axsäter's heuristic. Using the sequential steps shown in Figure 3, an outline of Axsäter's heuristic is presented in this section. Some of the steps would be dealt in details in the subsequent section.

At any current time period t, the following steps are performed in sequence.

i) **Carry forward the Closing Inventory of the previous period**: The closing stock of last period t-1 is received as the opening stock for the current period t.

ii) **Update the average demand rate and order for *t***: The average demand rate is updated and this updated demand rate is used in the Economic Order Quantity (EOQ) formula for determining the order quantity, Q. Whenever the inventory position falls below the re-order level, a shipment of Q units is ordered in period t for receipt in period $t+L$.

According to the formula for EOQ, the order quantity, Q, is computed using the expression:

$$Q = \sqrt{\frac{2K\mu}{h}}$$ where,

K=Setup cost ($ per order),

h=per unit holding cost ($ per unit per period), and

μ=average demand rate.

The updating of the demand rate and computations of the inventory position is explained in the next section.

iii) **Planned order arrival**: The order quantity, Q, placed in t-L, is received as planned order arrival in period t.

iv) **Fulfill D_t with the Stock-on-hand**: The actual demand D_t in period t, is satisfied from the stock-on-hand, as and when the demand arises[12].

v) **Closing stock of current period:** The closing stock of the current period t is computed, and this closing stock appears as the opening stock for the next period $t+1$. The closing stock in t is equal to the Stock-on-hand (t) minus the actual demand D_t. If the actual demand exceeds the stock-on-hand, the opening stock for the next period is zero.

2.5.2 *Details of Axsäter's heuristic:*

This section describes the details of Axsäter's heuristic. The details of the heuristic can be found in Axsäter (2000,pp-82). The computations of inventory parameters, i.e., order quantity and re-order level, and the inventory control policy are addressed in the subsections. Both continuous and periodic review versions of Axsäter's heuristic are discussed. Finally, the inventory control policy is explained.

2.5.2.1 *Continuous Review Inventory model:*

The following notation is used for describing Axsäter's adaptive inventory model and the sections are have been quoted from Axsäter (2000).

t_f = forecast period (periodicity of updating forecasts)

$\hat{a}_t$ = average forecast demand at the end of forecast period t (Level at time period t)

b_t = average forecast trend at the forecast period t

$\mu_t = \dfrac{\hat{a}_t}{t_f}$ is the average demand rate per unit time at time t

[12] We do not go into the details of how the demand appears during the period. The demand can be realised at an instant or with uniform distribution. No holding costs charged for D_t.

The average demand rate at time t, just after forecast update, is $\mu_t = \frac{\hat{a}_t}{t_f}$, and τ time units later, the average demand rate is modeled as:

$$\mu_t = \frac{\hat{a}_t}{t_f} + \tau \frac{b_t}{t_f}.$$

The standard deviation of the demand per unit time is given by:

$$\sigma_t = \frac{1}{(t_f)^c} \sqrt{\frac{\pi}{2}} MAD_t,$$

where the parameter c equals 0.5, if we assume that forecast errors in different time periods are independent.

The average demand during time interval $(t, t+\tau)$ can be estimated as

$$E\{D(t,t+\tau)\} = \int_0^\tau \frac{1}{t_f}(\hat{a}_t + \hat{b}_t u)du = \frac{1}{t_f}(\hat{a}_t \tau + \hat{b}_t \frac{\tau^2}{2}) \quad (6)$$

and by setting equation (6) equal to a certain quantity d, the expected time $\tau(d)$ in which this amount d is generated can be computed by solving a second-order differential equation. Upon solving the differential equation, we get the following result for $\tau(d)$.

$$\tau(d) = -\frac{\hat{a}_t}{\hat{b}_t} + \sqrt{\left[\frac{\hat{a}_t}{\hat{b}_t}\right]^2 + \frac{2dt_f}{\hat{b}_t}}. \quad (7)$$

Assume that reorder point and order quantity before average demand rate update are R' and Q', respectively. Axsäter's heuristic requires an initialisation for the values of R' and Q'. If we order when the inventory position reaches R', we will start to consume the batch around time $\tau(R')$, and the whole batch will be consumed around time $\tau(R'+Q')$. Considering equation (7), a reasonable estimate of the average demand rate during the time when the batch is consumed can be expressed as

$$\mu = \frac{\hat{a}_t}{t_f} + \frac{\tau(R') + \tau(R'+Q')}{2} \frac{\hat{b}_t}{t_f}, \quad (8)$$

and this value of the average demand per unit time is used in the EOQ formula for computation of the order quantity Q. The EOQ formula performs a lot sizing to minimise the sum of ordering and holding costs. The EOQ formula is expressed as:

$$Q = \sqrt{\frac{2K\mu}{h}} \tag{9}$$

where,

K = fixed ordering cost ($ per order),

h = per unit holding cost ($ per unit per period).

2.5.2.2 *Distribution of lead-time demand and safety-stock:*

In section 2.5.2.1, the average demand rate is updated but we still need to determine the lead-time demand parameters to account for safety stock. Axsäter (2000) defines the following mean and standard deviation for characterizing the lead-time demand distribution.

$$\mu' = \left[\frac{\hat{a}_t}{t_f} + \frac{\hat{b}_t}{t_f}\frac{L}{2}\right]L$$

$$\sigma' = \sigma L^c = \sqrt{\frac{\pi}{2}} MAD_t \left[\frac{L}{t_f}\right]^c$$

where, μ' is the mean demand during lead-time, and σ' is the standard deviation of the lead-time demand.

Following our modeling assumptions, we update the forecasts every period (i.e., t_f=1). The forecast errors in different periods are assumed to be independent, therefore the value of c is 0.5. The expressions for lead time parameters can then be written as:

$$\mu' = (\hat{a}_t + \hat{b}_t \frac{L}{2})(L) \tag{10}$$

$$\sigma' = \sqrt{\frac{\pi}{2}} MAD_t \sqrt{L} \tag{11}$$

The reorder level equals the average demand and safety stock during lead-time. The average demand during lead-time is given by equation (10) and the standard deviation of lead-time demand is given by equation (11). Assuming that the lead time demand is normally distributed, the re-order point can be expressed as,

$$R = \mu' + k\sigma' \qquad (12)$$

where, k is the safety factor[13].

2.5.2.3 *Periodic review transformation:*

Axsäter (2000) proposed the dynamic inventory updating heuristic in a continuous review setting, but the heuristic is also applicable using a periodic review (*s, S*) inventory control policy. The periodic review (*s, S*) policy is derived from a continuous review (*R, Q*) model by the following transformation:

$$s = R \text{, and}$$

$$S - s = Q$$

The safety stock computations for periodic review setting use different levels as compared to that of continuous review. In a periodic review setting, the safety stocks are planned for $L+1$ periods instead of L periods, as in a continuous review. This can be explained by the fact that orders can be placed only once every period. Consider a situation, where an order is placed in current period t and received in period $t+L$. If the demand during the lead time is more than expected then a new order to account for this unanticipated demand can be placed only in the next period. This contingent shipment would be received L periods later. Therefore, at any given time safety stock should be enough to cover uncertainties in $L+1$ vulnerable periods, i.e., the shortest possible time period for the receipt of a contingent shipment.

[13] k value of 1.645 corresponds to 95% of service level (probability of no stock out in the vulnerable period)

Accordingly, the lead-time demand parameters for periodic review control policy can be expressed as:

$$\mu' = (\hat{a}_t + \hat{b}_t \frac{L+1}{2})(L+1) \text{ , and}$$

$$\sigma' = \sqrt{\frac{\pi}{2}} MAD_t \sqrt{L+1} \text{ .}$$

2.5.2.4 *Inventory Control policy:*

Using a periodic review inventory control model, the ordering decision for Axsäter's heuristic is based on the inventory position. The inventory position at any given time is defined as the sum of stock-on-hand and all outstanding orders. Whenever the inventory position falls below re-order level, computed using equation (12), a batch quantity of Q units, computed using equation (9), is ordered and received L time periods later. The ordering decision can be expressed as:

If $IP < R$, then order Q.

The reorder level computation is different for continuous and periodic review control systems as discussed in earlier sections.

2.5.2.5 *Approximations in Axsäter's heuristic for current research:*

The frequency of updating the forecast and the inventory control parameters updating is the same, so t_f can be taken as 1. On substituting the values for $\tau(R')$ and $\tau(R'+Q')$, derived from using equation (7), in equation (8), the average demand per unit time at time t is updated as per the following expression:

$$\mu = \frac{1}{2}\sqrt{(\hat{a}_t)^2 + 2R'\hat{b}_t} + \frac{1}{2}\sqrt{(\hat{a}_t)^2 + 2(R' + Q')\hat{b}_t} \qquad (13)$$

where, R' and Q' are the re-order level and the order quantity for the previous period.

Using equation (13), it is possible that some mathematical complexities arise. This is possible in the following situation, where $b_t < 0$ and

$$(\hat{a}_t)^2 + 2R'\hat{b}_t < 0, \text{ or } (\hat{a}_t)^2 + 2(R' + Q')\hat{b}_t < 0. \tag{14}$$

This would make equation (14) a complex expression due to negative terms under the square root sign and thus it would be computationally infeasible to use equation (11). Therefore, under circumstances expressed in equation (14) no demand rate updating is performed, and the average demand per unit time is expressed as:

$\mu = a_t$.

By doing so, we pretend that the Axsäter's heuristic is ordering for a deterministic demand. Another alternative would be to modify the heuristic for positive and negative values of trend, which was intentionally avoided in the current research.

Similarly it is possible to get negative values for the mean demand during the lead-time as expressed in equation (10). A negative value for the lead-time mean demand can result in negative reorder levels, and to avoid such situations zero replaces the negative mean demand during the lead-time. However, we believe that these approximations do not affect the efficacy of the heuristic modeling.

2.5.3 *Limitations of Safety Stock in periodic review systems with lost sales*:

Hadley and Whitin (1963) highlight the difficulty of calculating the average annual cost expressions for lost sales case vis-à-vis backorders. For continuous review systems, Hadley and Whitin (1963) develop exact equations to calculate the average annual cost expressions for lost sales environment, provided that there is a single order outstanding (pp.197), but in case of periodic review it is stated that an exact formulation is not possible even with a single order outstanding. The difficulty in exact modeling of lost sales is described in Hadley and Whitin (1963, pp.197) as:

"In a lost sales environment, when the system is out of stock, the amount on hand plus on order does not change when a demand occurs. Unlike the inventory position in backorders case, it is not possible to treat the changes in the amount on hand plus on order independent of the amount on hand. It is necessary to take explicit account of the number of orders outstanding and the times at which they were placed."

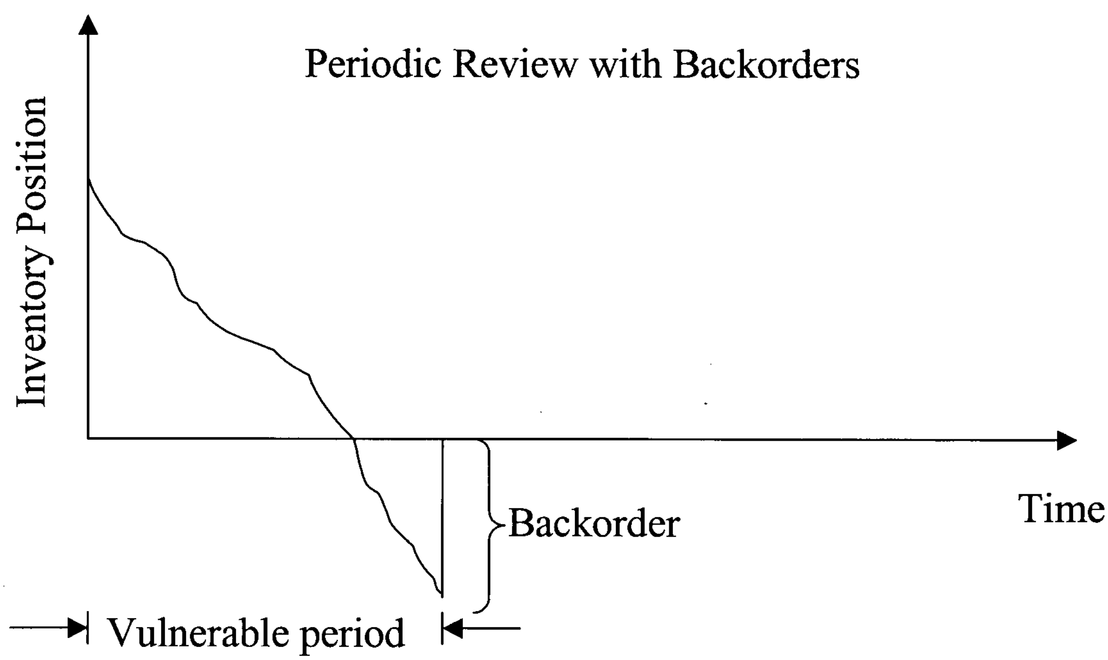

Figure 4: Periodic review system with Backorders

Figure 4 shows the inventory position for a periodic review system where backorders have been allowed. The vulnerable period for a periodic review system is the sum of lead-time and review interval. The inventory position is defined as:

Inventory Position = Stock-on-hand + Outstanding orders – Backorders.

For a backorder environment, all backorders during these two phases are accumulated and covered in the next replenishment decision. Therefore, in a backorder environment, a safety stock can be specified using equation (14), i.e., safety factor times the standard deviation of the lead-time demand. But equation (14) is not exact for the lost sales model.

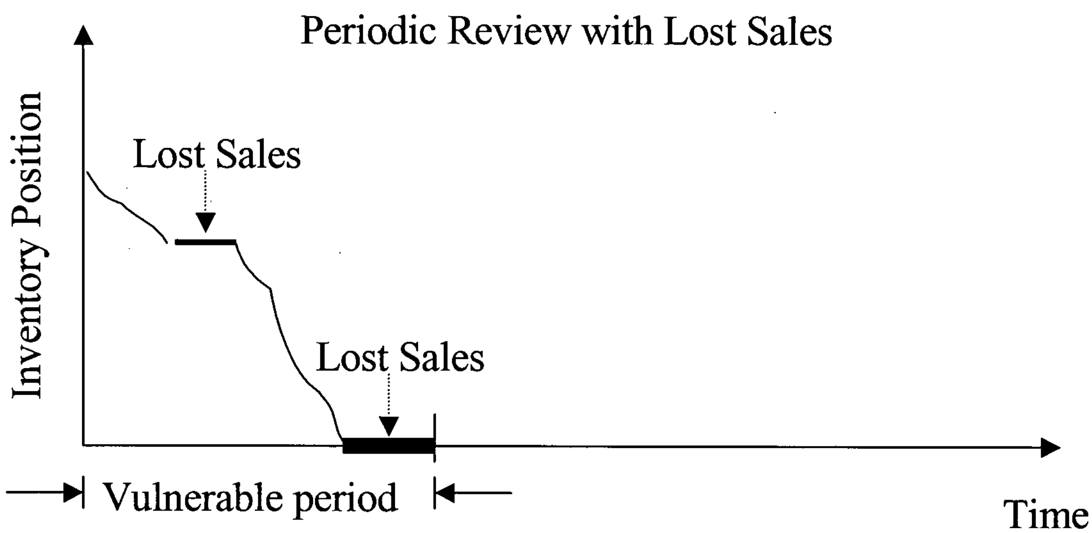

Figure 5: Periodic review with Lost Sales

Figure 5 shows the inventory position with respect to time it can be observed that the inventory position remains unchanged when a demand occurs during stock-out. Therefore, as highlighted by Hadley and Whitin (1963), the decision maker needs to know explicitly the number of outstanding orders and the timings of the stock-out. Johansen and Hill (2000) explore (*r, Q*) policies for periodic review systems with lost sales and fixed lead times. The results of an asymptotic renewal theory are used to estimate the "undershoot" of the re-order level, *r*. The way safety stock is accounted in Axsäter's heuristic does not correspond to optimality under lost sales conditions, but it is a good approximation for minimising any stock out during the vulnerable period. This is a limitation of the current research that the exact safety stock equations have not been developed for the lost sales model; however, the adopted methodology for safety stock derives its merit from computational simplicity.

In the current research, we have assumed lead times to be deterministic but it is possible to encounter variability in lead times. Nowadays, there is an increased awareness towards reducing the lead-time variability, and Availability-To-Promise (ATP) is one measure that is an integral part of service level agreements in supply contracts. Silver and Peterson (1985) recommend the use of an approximate mathematical model to ascertain the standard deviation of total demand in a lead time. This model assumes that the lead time (L) and the demand rate (D) are independent random variables, which is a reasonable approximation to reality, and it is shown that

$E(x) = E(L)E(D)$, and

$$\sigma_x = \sqrt{E(L)\text{var}(D) + [E(D)]^2 \text{var}(L)}$$

where x, with mean $E(x)$ and standard deviation σ_x is the total demand in a replenishment lead time, in units; L, with mean $E(L)$ and variance $\text{var}(L)$, is the length of a lead time, in unit time periods; and D, with mean $E(D)$ and variance $\text{var}(D)$, is the demand rate, in units per unit time period. Therefore the parameters μ' and σ' in equation (14) are substituted with $E(x)$ and σ_x, for incorporating variability in lead time.

2.6 Baseline:

The Baseline (B/L) is the perfect information solution for the inventory problem. In order to compute this solution we assume that the exact demand information for the future periods is known in advance, and this solution will be used for simulation. The ordering policy[14] is then decided using the Wagner-Whitin algorithm. The Service level for B/L is 100%, i.e., no stock-out as this method has the perfect demand information. The B/L solution is the least cost solution and given the demand information and cost parameters, the B/L achieves the least cost and 100% service level.

As the perfect information solution is computed using the actual demand information, there is no need for forecasting future periods demand. The backward induction routine and the lead-time incorporation is the same for WWPF and the B/L algorithms (Refer to the section 2.4). The demand model and inventory control policy for B/L are the same as that of WWPF algorithm.

The differences between B/L and WWPF can be summarised as:

- The demand is perfectly known for the B/L solution
- The ordering quantities are never revised during the planning horizon and no safety stock is needed.

2.6.1 *Steps of B/L*:

The following steps outline the algorithm for computing the B/L solution.

i. Take the actual demand for all periods in the planning horizon
ii. Compute the ordering policy at the start of the planning horizon using WW (Refer section 2.4) for the entire planning horizon and calculate the total inventory cost.
iii. For incorporating a lead-time L, we disregard the first L periods and then compute the ordering policy using the WW method.

[14] Determines the ordering periods and the respective order quantities for the entire horizon

2.6.2 *Example for illustrating B/L*:

Consider an eighteen period problem, where the setup cost K is \$1000 per order, and the proportional holding cost h is \$1 per unit per period. The lead-time L is 3 periods and the review period is unity. The planning horizon is of 18 periods and the demand for each period is accurately known in advance. Based on these considerations, the problem is setup using the Wagner-Whitin algorithm and the backward induction principle yields the least total cost for 100% service level.

The B/L solution is shown in Table 7.

Table 7: Baseline solution for an eighteen period inventory problem

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
D_t	153	87	157	240	178	242	182	214	297	245	255	322	299	294	309	320	320	387
k=t				1000	2000	2178	2662	3208	3788	4470	4995	5543	6317	6842	7430	8151	8750	9390
k=t+1				1178	2242	2360	2876	3505	4033	4725	5317	5842	6611	7151	7750	8471	9137	0
k=t+2				1662	2726	2788	3470	3995	4543	5369	5915	6430	7229	7791	8390	9245	0	0
k=t+3				2208	3272	3679	4205	4760	5509	6266	6797	7357	8189	8751	0	0	0	0
k=t+4				3064	4128	4659	0	0	0	0	0	0	0	0	0	0	0	0
k=t+5				0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D_t				240	178	242	182	214	297	245	255	322	299	294	309	320	320	387
Min Value				1178		2788			4543			6430			7750		9137	
Order Periods				4		6			9			12			15		17	
Order Receipt				418		638			797			915			629		707	
Order Release	**418**		**638**			**797**			**915**			**629**		**707**				

The demand figures in the Table 7 represent the actual figures and this information is assumed to be known, in advance. Based on the actual demand figures, WW algorithm is run and the ordering quantities and the corresponding periods are determined. The cost matrix preparation, backward induction and lead-time incorporation are the same as that of WWPF and the details have been discussed earlier in section 2.4.

2.6.3 *Limitations of the Baseline*:

The Baseline has certain limitations and it may not be a perfect benchmark for comparing the WWPF algorithm and Axsäter's heuristic. The Baseline computes an ordering policy for satisfying the entire demand and thereby provides 100% service level. On the contrary, the WWPF algorithm and Axsäter's heuristic cannot achieve 100% service level, due to the extended tail encountered in a normal distribution. This tail comprises values with very low probabilities, and the tail extends to infinity in an asymptotic manner. Therefore, it would be unrealistic to expect both these algorithms to yield 100% service level for a normally distributed demand. A target service level of 95% no stock out is incorporated in WWPF and Axsäter's heuristic, so that a majority of the normally distributed demand values are captured.

To ensure a fair comparison, the Baseline would have to be modeled in a way that a target service level can be specified. But such a solution, where the least total cost is incurred subject to service level constraints, cannot be modeled using the WW algorithm. An alternate Baseline, subject to service level constraints, can be formulated using integer programming.

Figure 3 gives the pictorial representation of the integer programming formulation for an alternate Baseline.

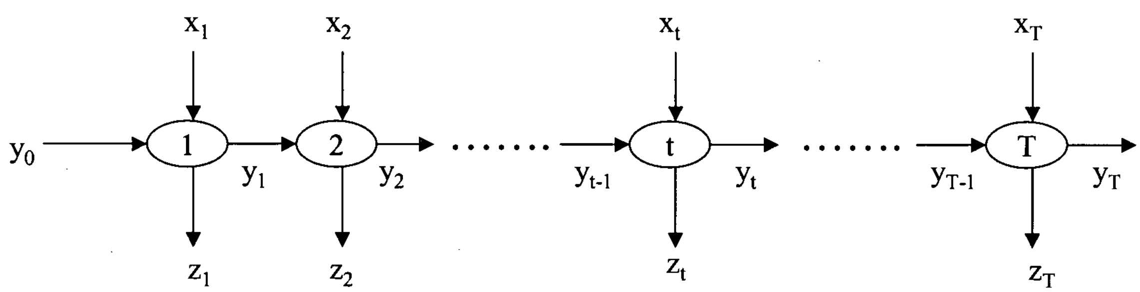

Figure 6: Alternate Baseline solution for target service level using Integer Programming

The following notation is used for the integer programming formulation:

y_t = Opening stock in period *t*.

x_t = Order receipt in period *t*

z_t = Actual units delivered in period *t*

d_t = Actual demand in period t

K = Ordering cost ($ per order)

h = holding cost ($ per unit per period)

F = Fill-rate target for service level, i.e., proportion of demand satisfied from stock-on-hand.

The integer formulation can be expressed as:

$$\text{Min.} \sum_{t=1}^{t=T} KI_{x_t>0} + hy_{t-1}$$

$$\text{where, } I_{x_t>0} = \begin{cases} 0, x_t = 0 \\ 1, x_t > 0 \end{cases}$$

s.t. $y_t + x_t = y_{t+1} + z_t$ (Inventory balance equation)

$z_t \le d_t$ (Delivery is always less or equal to actual demand)

$\sum_{t=1}^{t=T} z_t \ge F\left(\sum_{t=1}^{t=T} d_t\right)$ (Actual delivery is fill rate times the actual demand)

$y_T = 0$ (No stock left at the end of the planning horizon)

$x, y, z \ge 0$

The above integer program would give a solution for the values of x_t, i.e., order receipt. In order to compute the order releases for the corresponding periods, the order receipt values would be accounted for the lead-time. The above integer program can be reduced to 0-1 Knapsack problem.

Thus the integer programming formulation computes the Baseline with service level constraints. However, we chose a Baseline to satisfy 100% demand by using WW algorithm to take advantage of the modeling ease. Moreover, one would expect the Baseline solution with a 95% service level constraint to cost less than the Baseline for satisfying 100% demand. In this way, we are penalizing the Baseline and incurring more cost by constraining it to 100% service level. However, we expect the inflated Baseline to serve as a good benchmark for comparing the two algorithms.

Chapter 3: Numerical Study

This chapter presents the design of a numerical study to compare WWPF algorithm, Axsäter's heuristic, and the B/L solution. The performance metrics, selection of the factor levels for input parameters, and the partitioning of planning horizon are discussed in the following sections.

3.1 *Performance metrics for comparison of algorithms*:

The following performance metrics are used to judge the performance of the three algorithms.

i. Total Cost: Sum of all setup costs and holding costs in the planning horizon.
ii. Service Level: Proportion of periods in the planning horizon, where the entire actual demand is fulfilled from stock-on-hand.
iii. Stock out level as compared to the average demand: Sum of stock out quantities divided by the average demand in the test periods. This can be considered equivalent to

    1-Fill Rate

    where, "fill rate"[15] is the fraction of demand that can be satisfied immediately from stock-on-hand.

The measures of Total Cost and Service level are the traditional measures used to evaluate performance of inventory control algorithms and policies. The measure of stock out level is inspired from Song (2001) where the author claims that service level (i.e. fill rate or no stock out probability) is the not a sufficient measure to evaluate service performance. The paper establishes the magnitude of backorder as an alternate measure to be used in conjunction with fill rate for monitoring vendor performance. In our lost-sales inventory model, there is no backordering but the magnitude of stock-out is

[15] Denoted as S_2 in service level terminology.

measured. The average stock out measure would be explained in details later in the chapter.

3.2 *Factor levels for input parameters*:

The factor levels for input parameters have been motivated by a similar numerical study conducted in Porteus (1985). The relevant factor levels in our numerical study have been chosen to be the same as in Porteus (1985).

The following factor levels are chosen for the input parameters in our numerical study.

- Holding cost h (1) \$ per unit per period
- Setup cost per period K (1,10,100,1000,10000) \$ per order
- Lead time L (0,1,3,5) periods
- Non-stationary demand[16] process: $D_t \sim N(\mu_t, \sigma^2)$ where $\mu_t \equiv mt + \mu_0$
    - Intercept of non-stationary mean μ_0 (2,6,20,60)
    - Slope of non-stationary mean normalized by μ_0, $\frac{m}{\mu_0}$, (0,0.02,0.05,0.1,0.25),
    - Variance / mean, σ^2 / μ_0, (0.3,0.75,1.5,10)

The higher values of setup cost, in the range of 100, 1000 and 10000 times the holding cost, capture typical cost functions in a manufacturing setting. For example, a die shop has a high setup cost compared to the holding cost. The high setup cost can be attributed to opportunity cost attributed to production loss[17], cost of changeover, material handling, and initial quality failures after a die changeover. The holding costs are low as compared to setup cost and comprise opportunity cost on blocked capital and interest payments, material handling, labour, and pilferage costs.

[16] Demand data by using Norminv function in MS Excel, and the probabilities for Norminv are modeled using Rnd function of VBA.

[17] Die changeover times may vary from 8 to 36 hours in some automotive applications such as forgings

The low values of setup cost, i.e., 1 and 10 times the holding cost, might be applicable to the retail settings. With recent advances in communication technologies especially EDI (Electronic Data Interchange), B2B (Business To Business) exchanges and applications, XML (Extensible Markup Language), some retail companies have invested in such modern tools. The management of retail corporations consider it as a "sunk cost" that needs to be borne in order to do business and survive in the competitive world. Consequently, a large setup cost is no longer valid for such retail situations.
During discussions with executives at Canadian Tire and Future Shop, managers expressed their inclination for more frequent orders. An executive at Future Shop said that he encouraged the buyers / planners to place 52 purchase orders instead of one. On the other hand, the lot sizing approach is applicable to the shipment coming from Asia. There are two aspects regarding the shipment from the Far East that introduces high setup costs. Firstly, more frequent shipment means processing time and processing cost for customs and related freight clearances. So it would be beneficial for the retail firms to order items in bulk. Secondly, the shipments from Asia are primarily transported in full container loads, which may contain different items in containers. The holding costs are low as compared to setup cost and comprise opportunity cost on blocked capital and interest payments, markdowns and obsolescence costs, material handling, labour, and pilferage costs.

The slope of the mean with linear trend, *m*, is varied as 0, 0.02, 0.05, 0.1, and 0.25 corresponding to 0%, 2%, 5%, 10%, and 25% growth w.r.t. the values of μ_0. The value of zero in terms of slope captures the case of stationary demand. The lower values of 2% and 5% might be more realistic in terms of growth. The value of 10% growth is a significant increase in 24 periods, and can be used to model successful electronic goods like MP3 players, where the demand increases rapidly upon introduction. In real life these parameters of non-stationary demand would not be known and the objective would be to estimate these parameters from the actual demand pattern. However, by choosing a wide range of slopes we hope to cover a broad spectrum of scenarios in "real-world". The slope of 25% is extremely high and the demand for the item could be as high as eight times in 24 periods. The selection of this slope is to explore the robustness of the algorithms at high values of non-stationarity. In practice, greater revenue potential exists in the growth phase of a product introduction. Therefore, inventory control has greater relevance for positive values of *m* and the objective of inventory managers is to minimise stock outs during the growth phase. However, we believe that in order to conduct an exhaustive numerical study, negative values of *m* should be included.

The values for mean-intercept, μ_0, and variance/mean, σ^2 / μ_0, for the random normal variate, are motivated by the values chosen in Porteus (1985). The intercept varies as 2, 6, 20, and 60. These intercept values generate demand values in low and intermediate ranges. The demand could be as low as 0 and as high as 500 in a single period. If we need to address even higher values of demand, a proportionate[18] change in the setup cost values would be needed.

By choosing 24 periods as the planning horizon, we can plan for 2 years, 1 year, 6 months, 1 month; with corresponding review periods as 1 month, 2 weeks, 1 week, and 1day. Both WWPF and Axsäter plan for safety stocks and the safety factor value corresponds to the probability of no stock out in the vulnerable period. For the sake of numerical study, we add safety stock to achieve 95% probability of no stock out during the vulnerable period. This corresponds to a safety factor value *k* value of 1.645.

[18] Reasonable setup and holding cost values for the demand level

3.3 *Partitioning of Planning Horizon*:

The planning horizon of 24 periods has been partitioned into three zones viz. Initialisation, Stabilization, and Comparison zones.

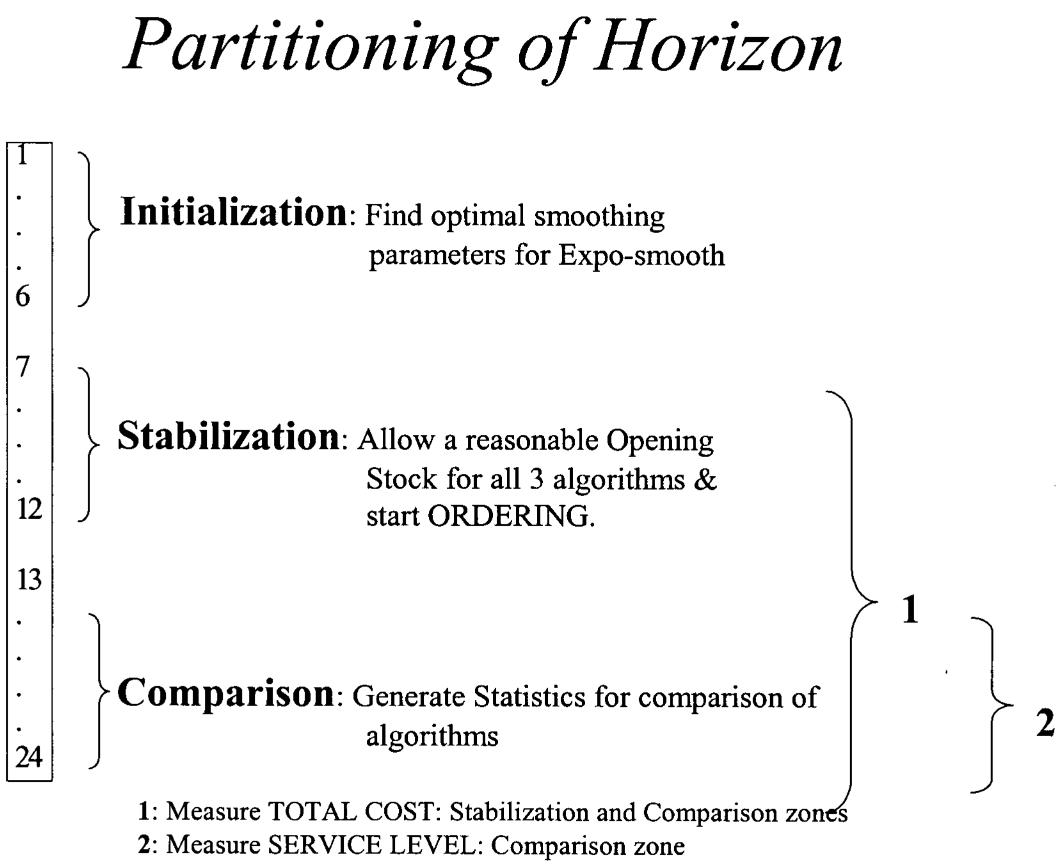

Figure 7:Partitioning of planning horizon periods during numerical study to compare algorithms

A pictorial presentation of the planning horizon is presented in Figure 7. The planning horizon of 24 periods is partitioned into initialisation, Stabilization and comparison zones.

3.3.1 *Initialisation:*

Using the demand data for periods 1 to 6, the exponential smoothing with trend forecasting model is formulated. The forecasting starts at period 2, as at least two data points are required for formulating the exponential smoothing with trend model. The level and trend values are initialized as discussed in the forecasting model in section 2.3.

The forecast formulation for periods 1 to 6 is shown in Table 8.

Initialisation: Optimal smoothing parameters from periods 1-6

Period	Demand	Level	Trend	Forecast	AbsError	SqError
t	D_t	a_t	b_t	F_t	e_t	e_t^2
1	18.00	18.00	4.00			
2	22.00	22.00	4.00	26.00	4.00	16.00
3	28.00	27.46	4.97	32.42	4.42	19.56
4	19.00	22.65	-1.52	21.13	2.13	4.55
5	33.00	29.77	4.21	33.99	0.99	0.97
6	37.00	36.18	5.67	41.85	4.85	23.53
Smoothing Parameters:						
	α	0.727986			MSE=	14.07
	β	0.663565				

Table 8: Setup of forecasting model in Initialisation zone of planning horizon

α and β represent the smoothing parameter for level and trend components of the forecast, respectively. The optimal smoothing parameters for the forecasting model can be chosen by testing different combination of α and β to obtain the combination with the least error for some criterion. The criterion can be chosen as MAD, MSE, or MAPE (Mean Absolute Percent Error). Makridakis (1998) mentions that it is customary to use MSE as a criterion, as MSE is a smooth function of the smoothing parameters. MAD used a criterion would be less susceptible to outliers vis-à-vis MSE, as the errors are squared in the latter criterion. To obtain the optimal values of smoothing parameters, the MSE or the error value using some other criterion can be evaluated over a grid of values of α and β (e.g., each combination of $\alpha = 0.1, 0.2, \ldots\ldots, 0.9$, and $\beta = 0.1, 0.2, \ldots\ldots, 0.9$), and then the optimal combination that yield the minimum value can be selected. Gardner (1985) mentions that smoothing parameters are usually chosen by a grid search to minimize the *ex post* MSE .

Alternatively, a non-linear optimization could be used for computing the optimal smoothing parameters. Segura and Vercher (2001) describe the spreadsheet modeling of the Holt-Winters optimal forecasting using MS Excel Solver. The Holt-Winters method introduces demand data with trend and seasonality, and the spreadsheet modeling can be used for the current research by ignoring the seasonal factors and the corresponding modeling constructs. The non-linear problem, modified from original problem discussed in Segura and Vercher (2001), can be expressed as:

$$\text{Min. } \frac{1}{5}\sum_{t=2}^{t=6} \left[D_t - \{a_t + b_t\}\right]^2$$

$$\text{s.t. } (\alpha, \beta) \in (0,1)$$

where, D_t is the actual demand, a_t and b_t are level and trend components (Refer Forecasting model, section 2.3), α and β are the smoothing parameters. Segura and Vercher (2001) highlight the limitation of using a non-linear optimization that it is possible to get a local minimum. However, a multi-start strategy, which resolves the non-linear problem, could give a collection of local minima for the objective function. Moreover, since functional managers when using exponential smoothing can interpret the selection of parameters, the solutions computed by the non-linear optimization can be used with discretion. Segura and Vercher (2001) claim that Microsoft® Excel is capable of solving non-linear optimization problems and has generalized reduced gradient algorithm GRG2 implemented in the SOLVER module.

In the initialisation zone, optimal smoothing parameters are computed by minimising the MSE (Mean Square Error) for the periods of 1-6 and the non-linear optimization is solved using MS Excel Solver. The solver parameters are setup as shown in Figure 8. The solver has a target cell equal to the MSE, the objective function is set to minimise the MSE. The smoothing parameters for level and trend, modeled as changing cells in the solver dialog, are constrained for:

- Non-negative values for smoothing parameters

- Smoothing parameter values less or equal to 1.

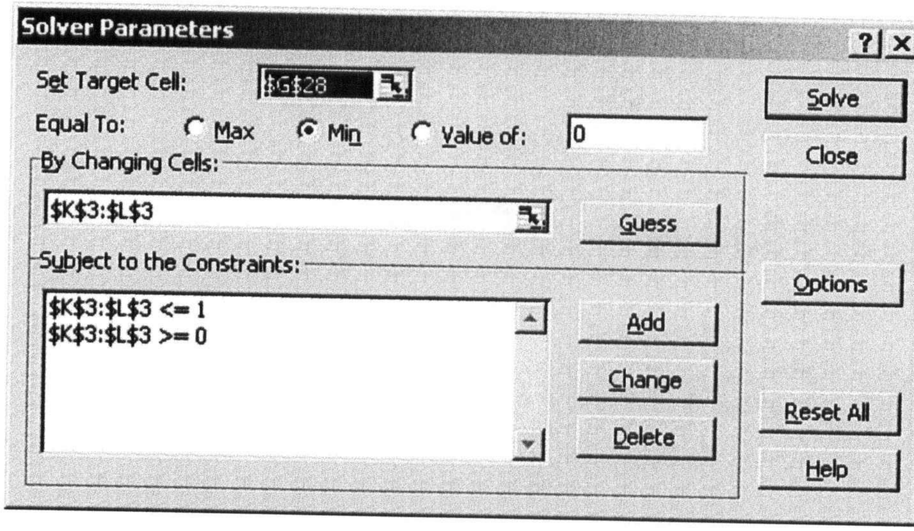

Figure 8:Solver parameters for computing optimal smoothing parameters in forecasting model

Other options like maximum time, maximum # of iterations, tolerance are set in the solver options dialog, as shown in Figure 9.

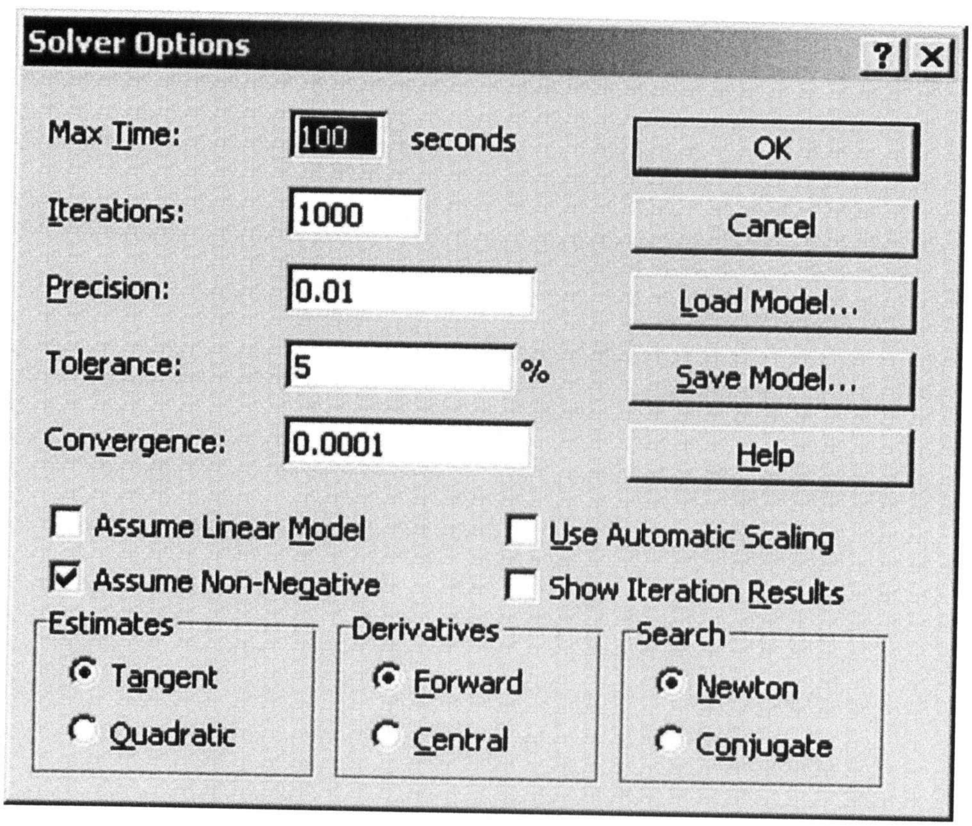

Figure 9: Solver options for computing optimal smoothing parameters

The optimal values for smoothing parameters from data points of the Initialisation zone are used in the entire forecasting model. Ideally, new optimal values for the level and trend smoothing parameters should be computed at the start of every period; but such a step would have increased the computing time significantly, as we run 48 000 iterations. Moreover, the optimization for smoothing parameters at the start of every new period, for the given planning horizon, did not show significant improvements to the algorithms. This was manually verified by running different input parameters on the MS Excel plus VBA code.

3.3.2 *Stabilization:*

The periods 7-12 of the planning horizon constitute the Stabilization zone. From period 7, the algorithms, i.e., WWPF, Axsäter and B/ L start ordering and incur setup and / or holding costs. When ordering is allowed for both WWPF and Axsäter, the algorithms tend to place high order quantities in the initial periods. The six period Stabilization zone mitigates this initial ordering spree by allowing both algorithms to stabilize. The selection of a six period Stabilization zone is based on the fact that lead-time values for the numerical study vary from zero to five periods, and by the end of the Stabilization zone, both the algorithms have received at least one order and accumulated some stock-on-hand. The ordering pattern for the algorithms, especially Axsäter's heuristic, stabilizes once an appreciable stock-on-hand has accumulated.

Apart from allowing six periods of ordering, without measuring the service level, a reasonable amount of opening stock is allocated to all algorithms. An equal amount of opening stock is allocated to all three algorithms at the start of the Stabilization zone, i.e., 7^{th} period of the planning horizon.

The opening stock, *OS*, is expressed as:

$$OS = (\frac{F_7 + F_{7+L}}{2})L + 1.645\sqrt{\frac{\pi}{2}}(MAD)_{t=6}\sqrt{L}$$

where,

F_7 = Demand forecast for the 7^{th} period, and

F_{7+L} = Demand forecast for the $(L+7)^{th}$ period

The value of 1.645 is the safety factor, k, corresponding to 95% probability of no stock out during the lead time.

The opening stock *OS* is the sum of average demand and safety stock during lead-time, where,

$$(\frac{F_7 + F_{7+L}}{2})L$$

is an estimate of demand during the time interval $(7, 7+L)$, and

$$1.645\sqrt{\frac{\pi}{2}}(MAD)_{t=6}\sqrt{L}$$

is an estimate of safety stock during the time interval (7, 7+L).

OS is added to the algorithms because even if the algorithms place an order at the start of the 7th period, it is not going to be available till the lapse of subsequent L periods. Moreover, we assume that the inventory decisions are being taken on a rolling horizon basis and at the start of the 7th period, there is enough stock-on-hand to fulfil demand till the arrival of the planned orders.

The setup costs and / or holding costs are measured from the start of the Stabilization zone, i.e., 7th period. We do not start measuring the service level in this zone. The rationale is that the service level of the first L periods could be poor even though *OS* is added at start of 7th period. The maximum value of L considered in our study is 5 periods, so we do not measure service level in periods 7-12, i.e., Stabilization zone.

3.3.3 *Comparison*:

The periods 13 to 24 of planning horizon are termed as Comparison zone, where we start collecting the service level measures along with total cost figures. The following cost and service level measures are collected in the Comparison zone.

i. *Service Level*: Service level is defined in terms of probability of no stock out and any period in which the actual demand exceeds the stock-on-hand, the service level is considered zero. The comparison zone consists of 12 periods and the service level is collected as a performance metric only in this zone. After the running the algorithm upto 24 periods, service level is computed as:
   - Service Level = 100*{(# of periods where $D_t \leq$ Stock-on-hand) / 12}

ii. *Average Stock out level*: Average stock out is the measure of the magnitude of stock out, i.e., what demand went unfilled. Moreover, this measure would make sense when the stock out level is defined w.r.t. the demand figures. To illustrate the point: stocking out by 50 units in 12 periods when the average demand was 100 per period is significant compared to a stock out of 50 units in 12 periods when the average demand was 500 units per period. The average SO per period is defined as:

$$\frac{\dfrac{\sum_{12 \leq t \leq 24} |S_t - D_t|}{\sum_{12 \leq t \leq 24} D_t}}{12}$$

s.t. $D_t > S_t$

where, S_t is the stock-on-hand in period t, and D_t is the actual demand in t.

The performance measure of total cost is collected in the Stabilization and the Comparison zones. The total cost is the sum of ordering and holding costs. The total cost can be expressed as:

$$\sum_{t=7}^{t=24} K \times o_t + \sum_{t=7}^{t=24} h \times OS(t)$$

where, $o_t = 1$ for $OR(t)>0$, and $o_t = 0$ for $OR(t)=0$, and

$OR(t)$ = Order quantity received in period t (due to the order placed L periods earlier)

$OS(t)$ = Opening Stock in period t

h = Holding cost per unit per period ($ per unit per period)

K = Ordering cost per order ($ per order).

The ordering cost is usually incurred when an order is placed, but we charge ordering cost of $*K* per order, at the time of receipt. This is done to ensure a fair comparison between WWPF and Axsäter's heuristic. It is possible that Axsäter's heuristic would place a certain order during the last periods of the Comparison zone and due to the lead times, these shipments would not be received by the end of the 24th period. Such a problem would not be encountered in WWPF, as the algorithm determines the ordering policy, i.e., the ordering periods and the corresponding order quantities, for a given number of periods. Therefore, by charging ordering cost at the receipt of an order instead of an order release, we avoid any disadvantages for Axsäter's heuristic.

3.4 *Computational Complexity*:

The WWPF algorithm, Axsäter's heuristic and the Baseline solution have been coded in MS Excel and VBA. For a planning horizon of 24 periods, there are 18 periods in which the three algorithms are run and the performance metrics are collected. For conducting an exhaustive numerical study, the planning horizon needs to be long enough to overcome the effects of initialisation, stabilization, and end of horizon. But the computation times of the algorithms would increase rapidly with increase in the length of the planning horizon. Let us analyse the increase in computation times of each algorithm with increase in periods in the planning horizons.

The basic operations in WWPF algorithm are cost matrix generation, backward induction, and order quantity determination. For an 18-period problem, as required in the planning horizon of 24 periods, there are:

$18\times 18+17\times 17+16\times 16+............+1\times 1$ operations for determining the cost matrix in all the 18 periods. Similarly there are,

$18\times 18+17\times 17+16\times 16+............+1\times 1$

operations each for the backward induction and order quantity determination during the 18 periods. By substituting 18, the original number of periods, with n periods in the planning horizon, the operations performed for each step in WWPF algorithm would equal,

$$n\times n+(n-1)\times(n-1)+(n-2)\times(n-2)+........+1\times 1 \qquad (15)$$

By increasing the number of periods in the planning horizon ten folds, i.e., $10n$, the number of operations performed by in each step of WWPF algorithm would equal,

$$10n\times 10n+(10n-1)\times(10n-1)+(10n-2)\times(10n-2)+........+1\times 1. \qquad (16)$$

By increasing the number of periods ten fold, the increase in number of operations for each step of WWPF equals the difference between equation (16) and equation (15). For a planning horizon of n periods, the total number of operations for WWPF algorithm is on the order of n^2. Similarly, for a planning horizon of $10n$, the total number of operations is on the order of $(10n)^2$.

If a simplifying assumption is made that each operation takes unit time for execution, then it can be estimated that the computation times would increase in a quadratic manner with increase in the number of periods in the planning horizon. This was one of the reasons why a longer planning horizon was not chosen for conducting the numerical study.

Axsäter's heuristic involves computation for μ, R, Q, and with increase in the number of periods, a linear increase in computation times is expected. The Baseline computes the cost matrix, backward induction, and order quantity determination, only at the start of the planning horizon. Therefore, the increase in computational operations is expected to be exponential. For an n-period planning horizon, Baseline involves,

$3 \times (n \times n)$ computations.

If the planning horizon were increased ten fold, the Baseline would involve,

$3 \times (10n \times 10n)$ computations.

It can be observed that the increase is quadratic.

CHAPTER 4: RESULTS

This chapter presents the results of the numerical study described in Chapter 3. The performance of the three algorithms is summarized over all factors and the performance of algorithms is analyzed for each individual factor.

There are 1600 possible combinations of the input parameters and 30 replications are run for each combination. Each algorithm yields output values for total cost, service level and average stock out level for a unique combination of the input parameters. So we generate 1600*30 data points for each algorithm. These data points were recorded in a MS Excel database and then by means of database query functions in Excel, averages were computed.

4.1 Grand Summary:

Table 9 lists the average for total cost, service level and average stock out level computed for the WWPF, Axsäter and B/L algorithms, over all replications and all factor levels.

Table 9: Grand Summary of performance measures for algorithms over all factors and replications

	Total Cost	Service Level	Avg. Stock out Level
WWPF	6213.512	94.2401	0.230217
AXSÄTER	5973.037	73.16059	1.994076
Baseline (B/L)	4382.691	100	0

WWPF achieves a service level of 94% at a cost of $ 6213. Axsäter's heuristic achieves a service level of 74% at a cost of $ 5974. The Baseline solution represents the perfect information solution. The B/L costs $ 4383 and gives an absolute 100% service level. It can be read from Table 9 that upon averaging over all replications and all factor levels, WWPF gives a cost of 42% higher than the B/L and Axsäter's heuristic gives a cost of 36%, higher than the B/L cost. From the cost perspective, Axsäter's heuristic seems to perform better than WWPF, but we also need to look at the service level figures for a fair comparison. WWPF achieves a 94% service level whereas Axsäter gives a 74% service level. The B/L is always 100%. Going back to the cost figures, we can say that WWPF performs better than Axsäter's heuristic. With an increase of 6% in the total cost, WWPF yields an increase of 21% in service level. The stock out figures for WWPF and Axsäter algorithms are 0.23 and 1.99, respectively.

The figure of 0.23 implies that in a test period of 12 periods, regardless of the demand level and other input parameters, by using WWPF a quantity equal to 23% of the average demand would not be satisfied from stock-on-hand. By this measure, the stock-out level of 1.99 implies that Axsäter's heuristic would not fulfill the demand for approximately 2 periods out of 12 periods test horizon. The stock-out level is significant in case of Axsäter. In practice, the effort of inventory managers is aimed towards minimising stock-out levels and in order to achieve this a reasonable increase in cost is not alarming. This is so because in most of the industries the loss of a prospective customer is significant in terms of monetary as well as other aspects of business.

4.2 Effect of Setup cost on performance of algorithms:

Figure 10 graphs the dollar value costs of the WWPF, Axsäter and B/L algorithms for different setup cost values.

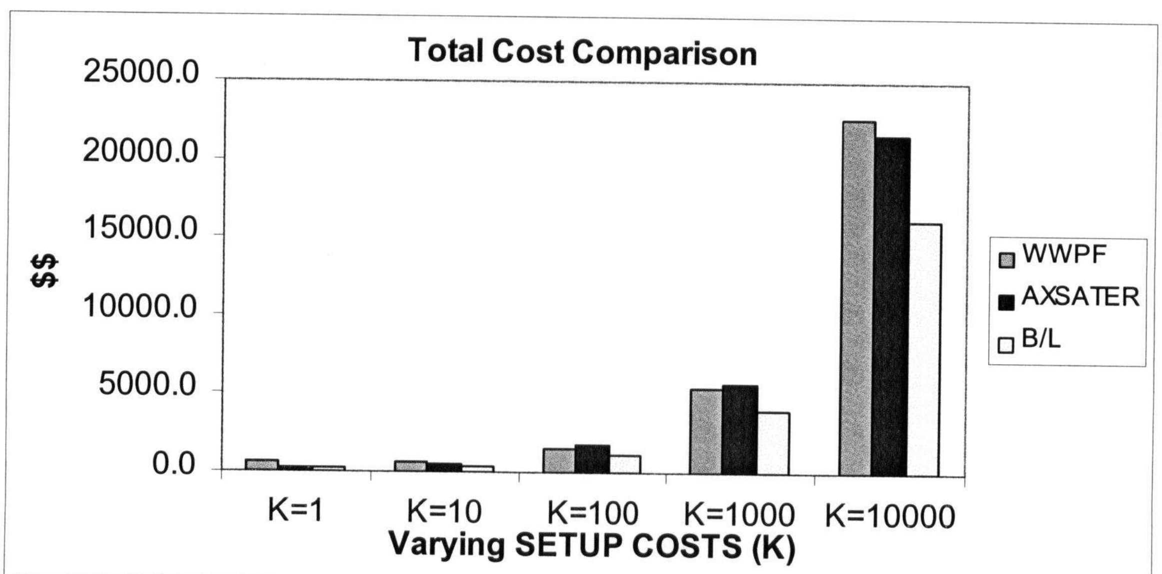

Figure 10: Total cost comparisons for WWPF, Axsäter, and B/L for varying setup costs

Figure 11 gives the total cost figures for WWPF and Axsäter's heuristic w.r.t. B/L cost.

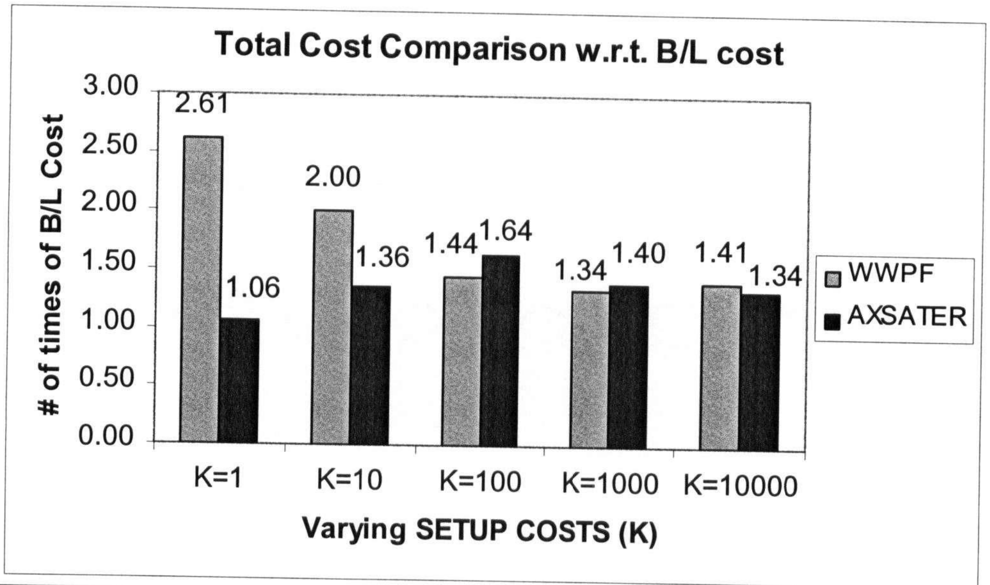

Figure 11: Total Cost comparison of WWPF and Axsäter w.r.t to Baseline

The graphs showing total cost for varying setup costs need to be viewed along with the service levels.

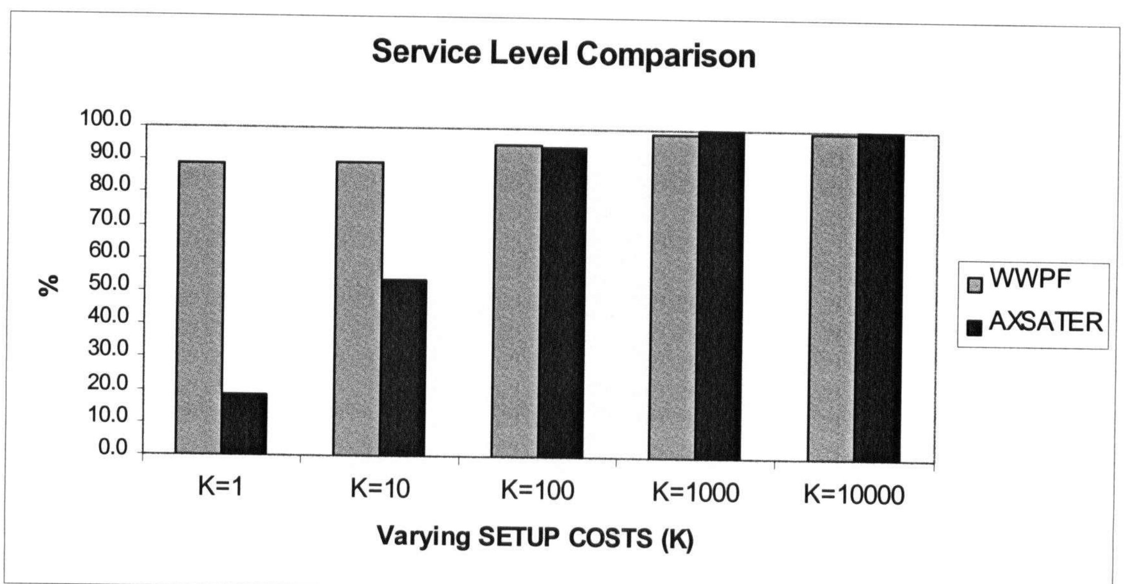

Figure 12: Service levels for WWPF and Axsäter algorithms for varying setup costs

Figure 12 presents the service level figures for WWPF and Axsäter algorithms for varying setup costs. The service level for B/L is always 100%. The total cost figures of WWPF for K= 1and 10 are higher than that of Axsäter. But the service level of Axsäter's heuristic is significantly lagging as compared to that of WWPF. For K value equal to 1, the service level achieved by Axsäter is as low as 20%, and for K equal to 10, the service level improves upto 55%.

Apart from poor service level, the stock-out levels of Axsäter's heuristic need to be explored.

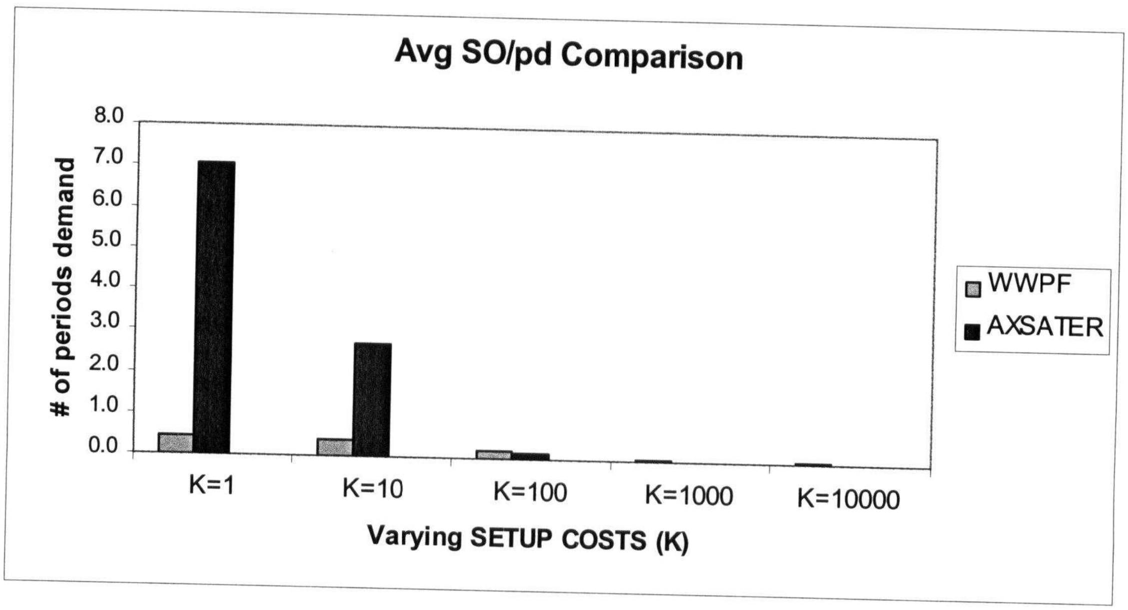

Figure 13: Average stock out levels for WWPF and Axsäter algorithms for varying setup costs

Figure 13 presents the average stock out levels for WWPF and Axsäter's heuristic for varying setup costs. Using Axsäter's heuristic, the average demand in 7 periods is not satisfied for K=1, and for K=10 average demand for about 3 periods is not fulfilled, during the 12 period Comparison zone. The reason for the poor service level in case of Axsäter's heuristic is the use of the EOQ formula for computing order quantities.

According to the formula for Economic Order Quantity, the order quantity is computed using the expression:

$Q = \sqrt{\dfrac{2K\mu}{h}}$ where,

K=Setup cost i.e. \$ per order,

h=per unit holding cost i.e. \$ per unit per period, and

μ =average demand rate.

For a setup cost in the lower range, i.e., K =1 and 10, the order quantity Q is low. The EOQ model prescribes order quantities by minimising the sum of ordering costs and the holding costs. Using Axsäter's heuristic, for a low K value, even a high value for average demand rate μ cannot result in reasonable values of order quantities. Consider an example to illustrate this point, where the values of K, h, and μ are 1,1, and 50, respectively. With these values, the order quantity using the EOQ formula would be

$$Q = \sqrt{\frac{2 \times 1 \times 50}{1}} = 10.$$

It can be seen that even for an average demand value of 50, the EOQ model would place an order for 10 units. With higher K values, the order quantities prescribed by the EOQ formula would match the average demand rate. Moreover, the EOQ model is used in context of continuous reviews so that whenever the stock-on-hand drops below the re-order level, the batch of Q units is ordered. But in our case a periodic review is considered and the algorithm needs to wait for one complete review period to order the next batch, and even then the order size is not sufficient. Therefore, in our further analysis of the individual factors of the numerical study, we do not consider the data points that correspond to K =1,10, as Axsäter's heuristic would not be able to perform adequately due to the EOQ formula.

Porteus (1985) describes a number of shortcuts and approximations for computing periodic review (s, S) parameters for stochastic, stationary demand model. Some of the shortcuts are based on EOQ model approximations and the setup cost is assumed to be relatively larger than the holding cost. But in the numerical study, the factor levels chosen for holding cost and setup cost violate this assumption. In Porteus (1985), the holding cost h is fixed as \$ 1 per unit per period, and the setup cost K is varied as \$ 0.1, 1, 10, 100, and 1000 per order. The value of setup cost K as \$ 0.1, is one-tenth of holding cost h ,fixed at \$1. Based on this fact the approximations termed as EOQROP and the NEWSBOY (Refer Porteus, 1985) methods, where setup cost should be relatively large than holding cost, do not hold good. The observation of poor service level in Axsäter as seen in Figure 12 and Figure 13, might have been observed in Porteus (1985) but it has not been reported in the paper.

From Figure 12 it can be observed that the service level of Axsäter's heuristic improves with increase in K value. For K=100, the service level is 94%. Though Axsäter's heuristic achieves a service level same as WWPF, the cost is 20% more than that of WWPF, relative to the Baseline cost. For K=100, in absolute terms, Axsäter costs 15% more than WWPF.

For K=1000, the service level of both algorithms are almost the same and above 95%. But Axsäter's heuristic incurs 6% more in total cost than WWPF, relative to the Baseline cost. For K=1000, in absolute terms, Axsäter costs 5% more than WWPF. The value of K is fairly high and the Axsäter's heuristic is able to order sufficient quantities and performs equally good.

For K=10000, Axsäter's heuristic outperforms WWPF. WWPF costs 7% more than Axsäter and both of them achieve 99% service level (Refer Figures 10 and 12). In absolute terms, for K=10000, WWPF costs 5% more than Axsäter. In general, WWPF algorithm is reactive, i.e., lot sizing is done but the orders are released when the demand in the future periods appears to be firm. This reactive feature of WWPF algorithm allows delaying of the order quantities for the future periods without jeopardizing the lot sizing for minimization of the total cost. Due to this reactive feature, WWPF yields a good service level for K=100,1000 at a lower cost vis-à-vis Axsäter's heuristic. However, for K=10000, this reactive feature of WWPF prescribes orders in the later periods of the Comparison zone, and with more frequent ordering the algorithm ends up with higher ordering costs. The upside of the reactive feature of WWPF is that it leads to lower holding costs as the ordering is done very near to the realisation of the demand. But for setup values as high as K=10000, the effect of ordering costs on the total cost is significant and regardless of the saving on account of the holding cost, WWPF incurs a higher total cost. On the contrary, Axsäter's heuristic releases less orders as compared to WWPF algorithm, and though higher holding costs are incurred, the total cost is lower than that of WWPF algorithm.

The value of K=10000 is very high for most industrial settings and for K=1,10 is not applicable to Axsäter's heuristic. Therefore, it would be reasonable to analyse the effect of individual factors after eliminating the data points that correspond to K=1,10, and 10000. Upon doing so, the grand summary of Table 9 would look like Table 10.

Table 10: Grand Summary of performance measures after eliminating K=1,10,1000.

	Total Cost	Service Level	Avg. Stock out Level
WWPF	3522.07	96.68	0.12
AXSÄTER	3760.78	97.07	0.09
Baseline (B/L)	2584.20	100	0

The performance of both WWPF algorithm and Axsäter's heuristic is comparable and they provide service level above the 95% mark. WWPF algorithm costs 6% less than Axsäter's heuristic, and about 36% higher than the Baseline. Axsäter's heuristic costs about 46 % higher than the Baseline.

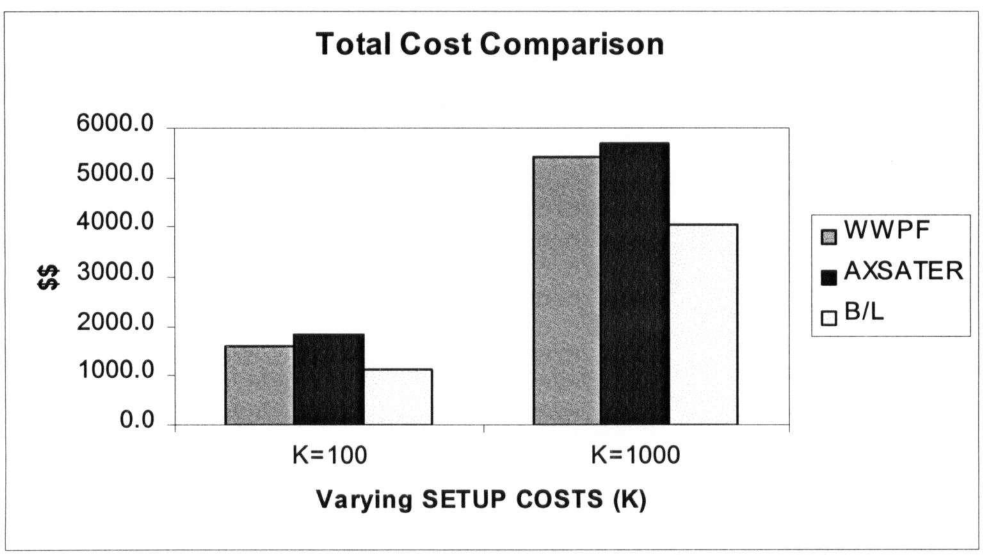

Figure 14: Total cost figures for WWPF and Axsäter for setup cost values of 100, and 1000 times of holding cost

At higher setup cost (K=100,1000), lot sizing performed by WWPF and B/L are very similar. The frequency of ordering prescribed by WWPF and B/L is almost the same. As WWPF orders based on net requirements, in turn derived from forecasts, the order quantity is more than that of the Baseline to account for safety stocks. Therefore, WWPF incurs greater cost for holding the extra units. Figure 14 shows the total cost figures for the three algorithms for setup cost values of 100 and 1000. The cost figures for WWPF and Axsäter algorithms are comparable. From figure 15, it can be seen that, for higher setup values, WWPF algorithm and Axsäter's heuristic the service level attains the 95% figure.

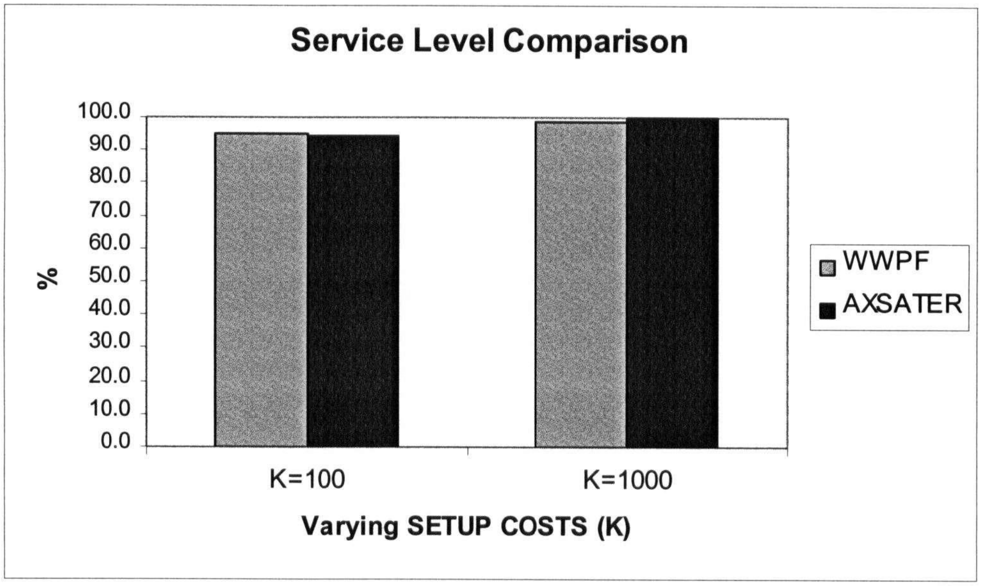

Figure 15: Service level figures for WWPF and Axsäter for setup cost values of 100, and 1000 times of holding cost

4.3 *Effect of varying degrees of nonstationary demand on algorithm performance*:

Figure 16 summarises the effect of varying slopes of nonstationary demand on the total cost measures for WWPF, Axsäter and B/L.

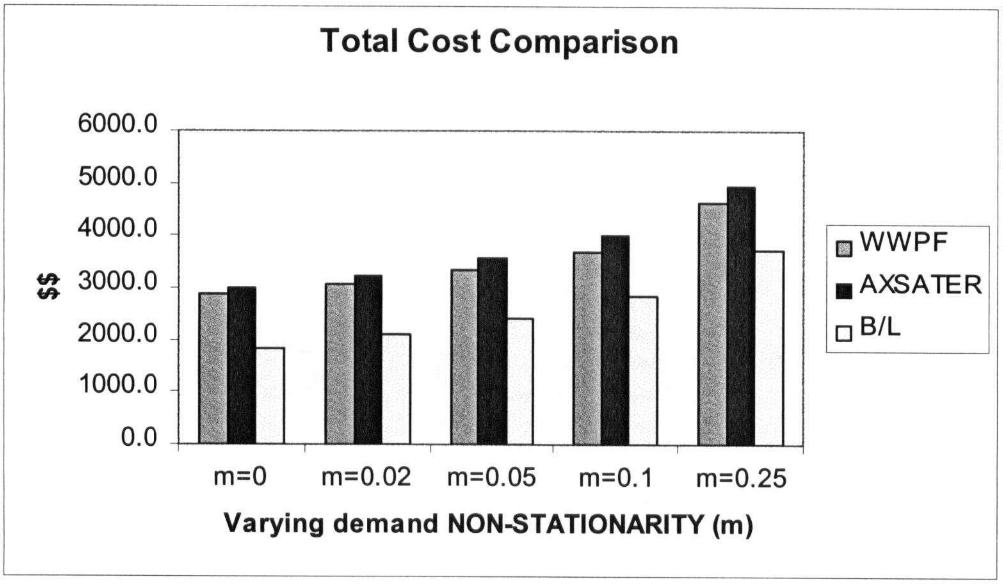

Figure 16: Total cost measures for WWPF, Axsäter, and Baseline for varying slopes of nonstationary demand

The costs for all three algorithms increase with increase in non-stationarity. The gap between WWPF cost and B/L cost decreases slightly from 55% to 24% over-cost[19] with increasing slopes, proportional to mean-intercept, of 0 to 0.25, respectively. The cost gap between Axsäter's heuristic and B/L decreases slightly from 63% to 32% over-cost with increasing slopes, proportional to mean-intercept, of 0 to 0.25, respectively. The cost incurred by Axsäter's heuristic is slightly more than that of WWPF algorithm, in the range of 3-7%. At the same time, the service level attained by Axsäter's heuristic is greater than that of WWPF algorithm, in the range of 2-3% (Refer Figure 17).

[19] Cost figures are referenced to B/L, so 55% means 1.55 times the B/L cost.

Figure 17 shows that with increasing slopes of nonstationary demand, the service level for both WWPF algorithm and Axsäter's heuristic remains stable. The slope value corresponding to m=0.25 is a very high figure and at this high level of nonstationarity, WWPF yields a service level of 94% and the service level for Axsäter's heuristic falls to 88%. This fall in service level may not be alarming in practical situations, as such high values of slope may not be experienced regularly. However, the stability in terms of service level indicates that WWPF is a robust algorithm. Based on the robustness of WWPF, in terms of service level at varying levels of non-stationary demand, the algorithm can be recommended for use in practice.

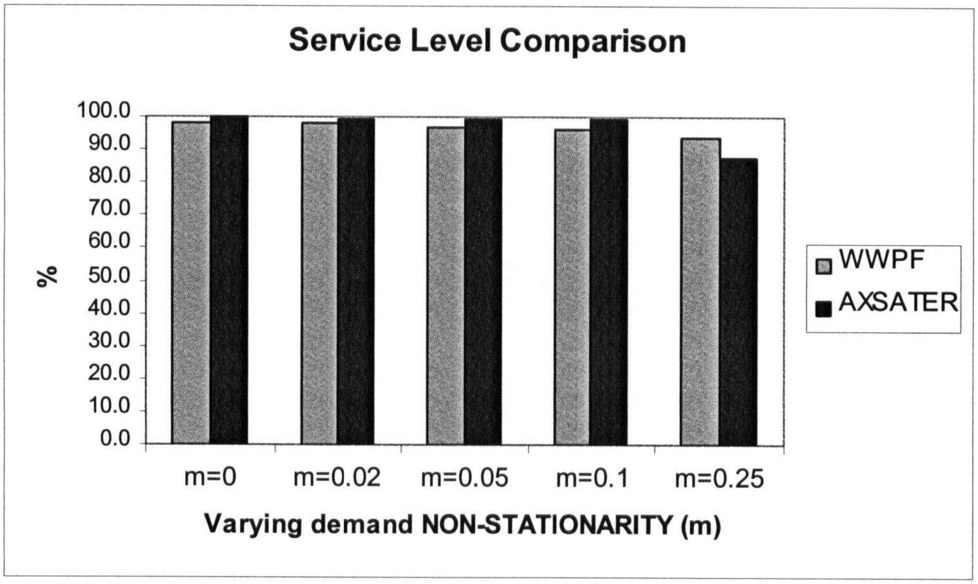

Figure 17: Service level measures for WWPF, Axsäter, and Baseline for varying slopes of nonstationary demand

4.4 Effect of varying lead times on algorithm performance:

The effect of lead-time on averaging over the other parameters viz. setup cost, mean, slope, intercept, and variance can be summarised by the following two graphs.

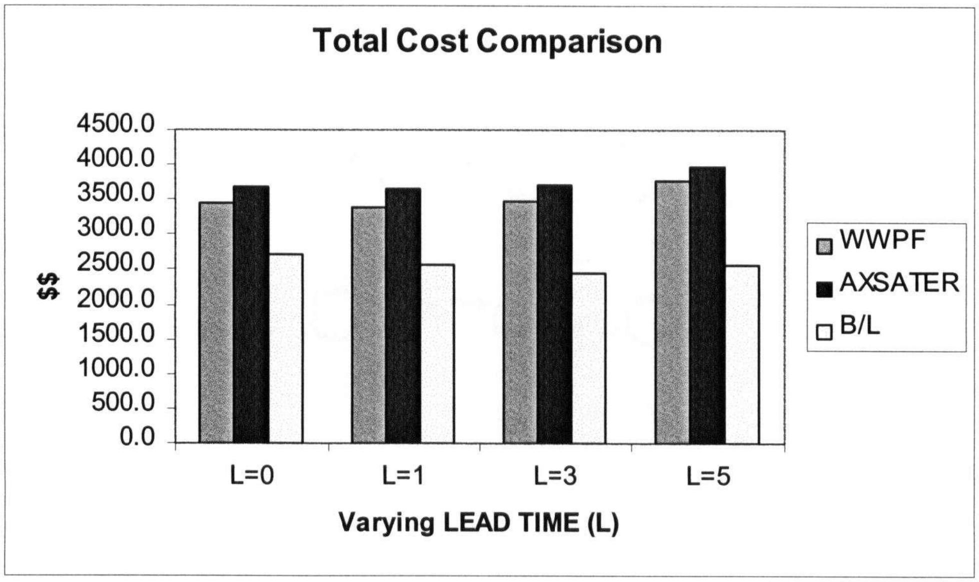

Figure 18: Total cost measures for WWPF, Axsäter, and B/L for varying lead times

Figure 18 presents the total cost measures for WWPF, Axsäter, and B/L algorithms for varying lead times. WWPF cost increases with increase in lead time, the cost difference is 10% as the L varies from 0 to 5 periods. Axsäter's heuristic seems incur similar increase in cost values, and the cost difference is 7% as the L varies from 0 to 5 periods. This can be explained by the fact that both algorithms have to incur greater holding costs with increase in lead times. The Baseline exhibits a slight decrease in cost as lead time varies from 0 to 3 periods, and then the cost increases moderately for a lead time of 5 periods. It seems that greater holding costs increase the Baseline cost at high lead times.

Figure 19 gives the service level measures for WWPF and Axsäter algorithms for varying lead times. The service level for both algorithms is above the 95% level. Therefore, both WWPF algorithm and Axsäter's heuristic are performing well in terms of both total cost and service level for varying lead times.

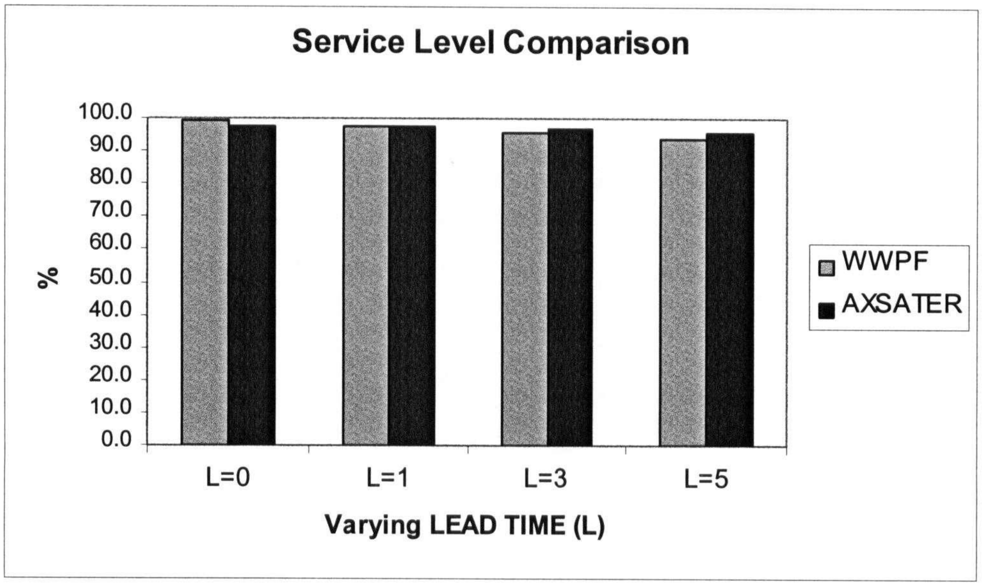

Figure 19: Service level measures for WWPF, Axsäter, and B/L for varying lead times

4.5 Effect of varying mean demand level on algorithm performance:

The mean demand level is varied by varying the intercept of the nonstationary mean. Figure 20 shows the total cost measures for the three algorithms with varying mean demand levels. The total cost for WWPF is more than that of Axsäter's heuristic, at lower demand levels. With increase in demand level WWPF gains some cost improvements w.r.t. Axsäter's heuristic. WWPF cost decreases from 71% to 27% over B/L cost, with increase in mean from 2 to 60, respectively.

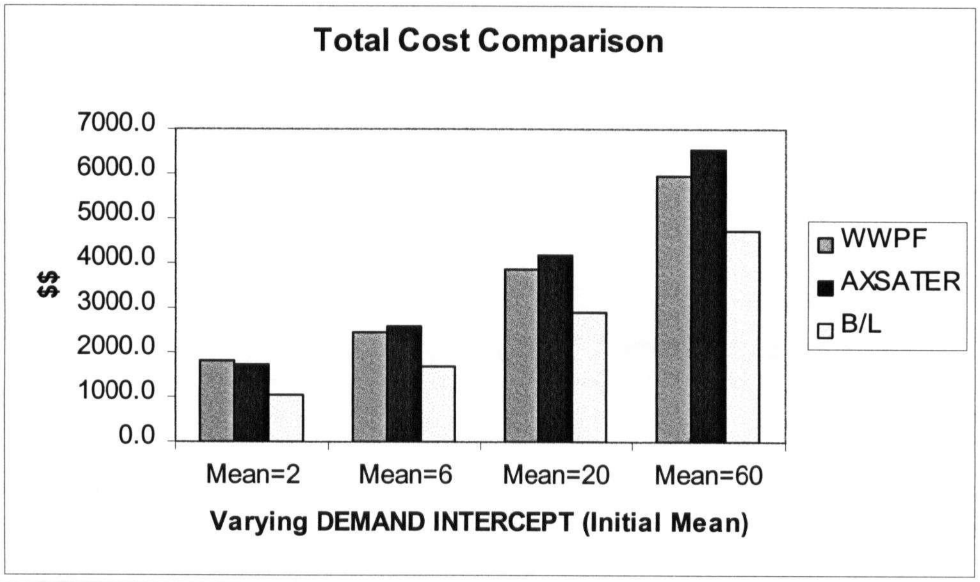

Figure 20: Total cost measures for WWPF, Axsäter, and B/L algorithms for varying mean demand levels

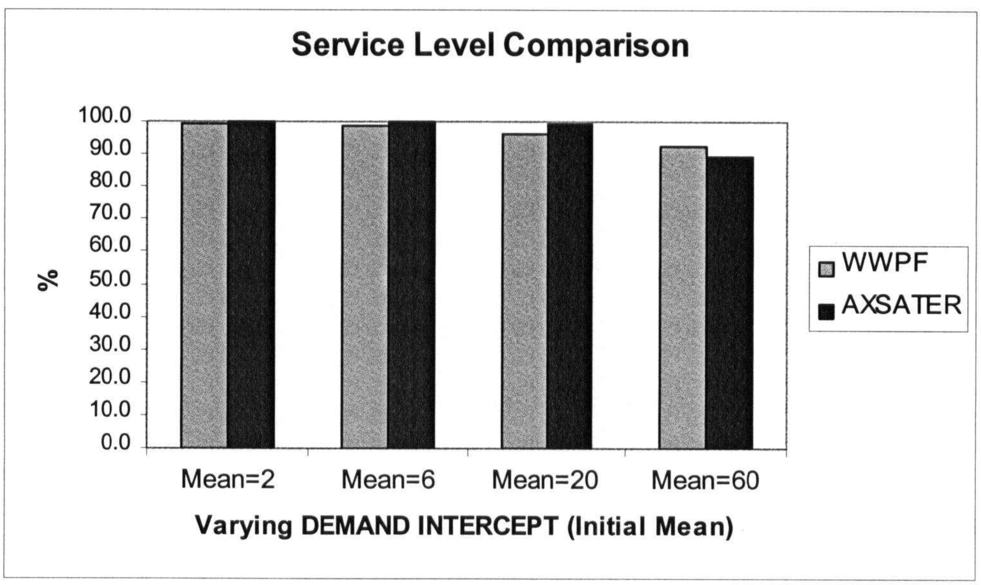

Figure 21: Service level measures for WWPF, Axsäter, and B/L algorithms for varying mean demand levels

Figure 21 shows the service levels for WWPF and Axsäter algorithms. Both algorithms have comparable service levels, above 95% range. Axsäter's heuristic adds safety stock to account for the forecast inaccuracy and the amount is added to the re-order level, while the order quantity is based on the updated demand rate. On the contrary, for WWPF safety stock is added to the order quantity. As the actual demand is being generated using a simulation, it is possible to generate zero demand values at low mean-intercept values (Mean=2). At higher mean-intercept values, though the actual demand figures vary depending on the non-stationary parameters, the demand in a single period does not drop to zero. In these cases where the demand is zero, WWPF orders are more than required and incur greater holding costs. On the contrary, Axsäter's heuristic takes care of trend, but an order is not placed until the Inventory position falls below the re-order level. This feature in Axsäter's heuristic results in lower cost as compared to that of WWPF, at low demand values. However, the trend changes as the mean-intercept level increases, and Axsäter's heuristic incurs more cost than WWPF algorithm.

4.6 Effect of varying demand variances on algorithm performance:

Figure 22 summarises the effect of varying variances on the total cost measures of WWPF, Axsäter and B/L algorithms. The variance in demand has a significant effect at lower slope values, but when all factors are combined and averages are computed, variance does not affect the performance of the algorithms significantly. For higher slope values, even with high variance parameters, an upward trend is the dominating effect.

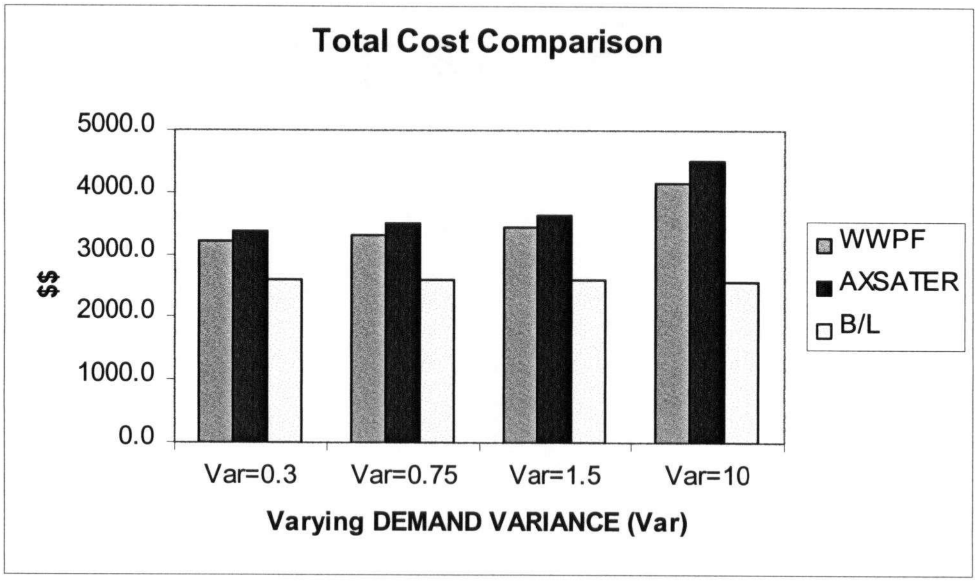

Figure 22: Total cost measures for WWPF, Axsäter, and B/L algorithms for varying demand variance levels

As seen from figure 22, the total cost for both algorithms increases with increase in variance. The increase in cost of both WWPF and Axsäter is significant when the variance parameter increases from 1.5 to 10. The gap between the WWPF and B/L, and Axsäter and B/L widen with increase in variance. Both WWPF and Axsäter have to plan for sufficient safety stock to account for the high volatility in demand figures, attributable to the high variance parameter. However, B/L does not suffer from these rapid changes. Both algorithms display stability in terms of cost and service level, and therefore are suitable for volatile demand environment.

Figure 23 summarises the effect of varying demand variances on the service levels of WWPF and Axsäter algorithms. The service levels for both algorithms do not show any significant drop at higher variance levels, and the algorithms are above the 95% service level.

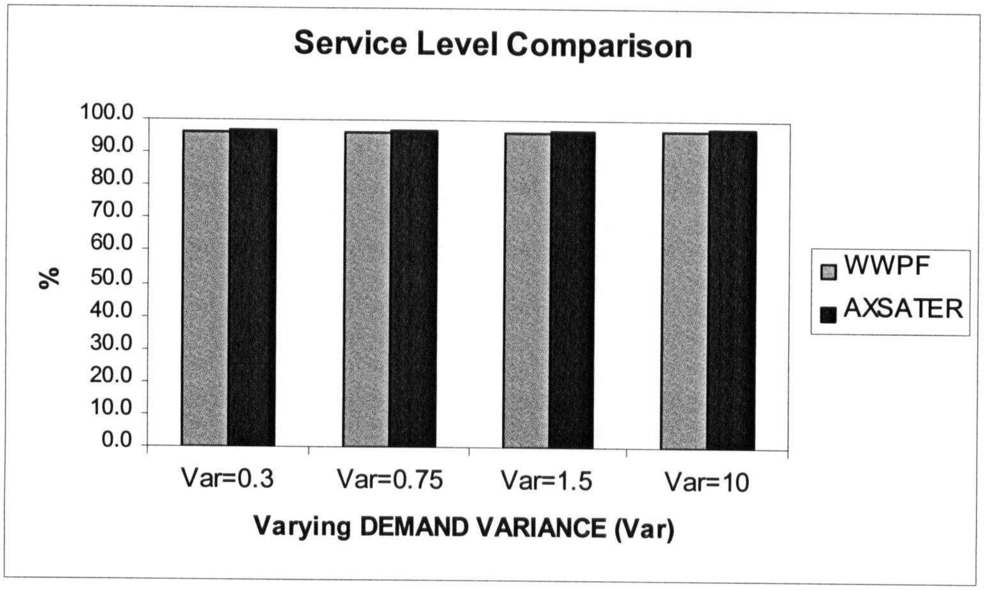

Figure 23: Service level measures for WWPF, Axsäter, and B/L algorithms for varying demand variance levels

CHAPTER 5: CONCLUSIONS & FURTHER RESEARCH

Based on the results of the numerical study discussed in Chapter 4, this chapter presents the highlights of comparison between WWPF algorithm and Axsäter's heuristic. The scope for future research is discussed in the later section.

5.1 *Comparison between WWPF and Axsäter algorithms*:

The salient points of comparison between WWPF algorithm and Axsäter's heuristic have been summarised in Table 11.

Table 11: Highlights of comparison between WWPF algorithm and Axsäter's heuristic

WWPF ALGORITHM	AXSÄTER'S HEURISTIC
Total cost is slightly less than Axsäter for reasonable values of setup costs (K=100,1000). Gap between WWPF and B/L decreases at high setup costs.	Low service level (SL) for low setup costs i.e. K =1,10. Ordering based on EOQ formula not applicable for low setup costs.
Performs equally good as Axsäter's heuristic. Robust algorithm at high levels of nonstationarity. Gap between B/L and WWPF cost decreases with increase in non-stationarity	Cost is slightly more than that of WWPF algorithm. Service falls at exceptionally high levels of nonstationarity. Gap between B/L and WWPF cost decreases with increase in non-stationarity
Average over all combinations: Cost 42 % higher than B/L SL 94.2 %	Average over all combinations: Cost 36% higher than B/L SL 74%
Worst case performance: Cost 160% higher than B/L SL 85 %	*Worst case performance*: Cost 64% higher than B/L SL 18 %
WWPF applicable to Seasonal demand by using Holt-Winters plus WW	Axsäter's heuristic, in its present form, handles only trend but it can be modified for seasonality

Moreover, WWPF uses exponential smoothing with trend as the forecasting model but in essence the algorithm is independent of the forecasting method. Other forecasting models such as Moving average, ARIMA, regression model, etc. can be incorporated in WWPF for improved modeling of the demand characteristics. In its present form, Axsäter's heuristic uses an algorithm for updating the average demand rate based on the exponential smoothing with trend model. If any other forecasting model needs to be incorporated, then an alternate method of updating average demand rate would be required for Axsäter's heuristic.

For reasonable values of K, i.e., 100 and 1000, both algorithms provide good service level at reasonable costs. Both WWP and Axsäter algorithms replicate "real-world" inventory practice and can be recommended as inventory control heuristics for production planning and purchasing functions in manufacturing and retail industries.

5.2 *Limitations of the current research:*

The results of the current research are based on certain approximations. Firstly, adding safety stock for a periodic review system with lost sales is a key issue. The way safety stock is added to both algorithms resembles the formula for a backorder environment and the approach is not optimal for a lost sales environment. The computation of safety stock for both algorithms may have a significant effect on the results; hence it would be desirable to obtain optimal safety stock values. However, the approximation was followed, as it was simple to understand. Moreover, in practice, it is a challenge to model the shortage costs. Therefore, it is the objective of most practitioners to aim for higher fill rate and reduce the backlogging of sales.

Secondly, Axsäter's heuristic has been primarily proposed for continuous review inventory systems and we selected the adaptive (s, S) transformation of Axsäter's heuristic for comparison with WWPF algorithm. The use of the EOQ formula in Axsäter's heuristic is not reasonable for low setup costs. If at all the EOQ formula is used for making ordering decisions, the Axsäter's heuristic would need a modification to amplify the order quantities. The EOQ model minimises the total cost, sum of ordering and holding costs, and assumes a constant demand rate. However, Axsäter's heuristic uses this EOQ model for an increasing demand, which may lead to higher than expected holding costs. This could partly explain the higher costs incurred by Axsäter's heuristic as compared to WWPF algorithm. To test the efficacy of WWPF algorithm, we will have to compare it with other methods such as, the myopic heuristic proposed by Bollapragada and Morton (1999), and Askin's heuristic (1981).

Finally, the results obtained from the numerical study may be sensitive to the length of the planning horizon. Ideally, the planning horizon, for conducting an exhaustive numerical study the partitioned zones must be long enough. Bollapragada and Morton (1999) consider a 70-period horizon, which mitigates the end-of-horizon effects. Similarly, the numerical study should have been done for a longer planning horizon, but because of the computational complexity and the limited programming capabilities with MS Excel and VBA, we could not conduct an extended numerical study. A longer planning horizon would be possible using efficient coding languages such as C++, but the objective in the current research was to build a tool on MS Excel, a widely used application.

5.2 *Further Research*:

The scope of future work related to WWPF and Axsäter algorithms is discussed in this section. Some of the following extensions could have been included in the current research to provide a holistic view, but they were not addressed due to time constraints.

5.2.1 Incorporation of seasonality:

The current work evaluates the effectiveness of WWPF and Axsäter for nonstationary demand data with trends. However, a next logical extension of this work would be to incorporate trend and / or seasonality in the nonstationary demand parameters and then evaluate the effectiveness of WWPF. WWPF can handle trend and / or seasonality by incorporating exponential smoothing with trend and seasonality[20] and rest of the algorithm would be unaltered. Axsäter's heuristic in its present form cannot handle seasonality but with modification, adaptive (*s, S*) parameters can be computed for seasonal demand.

5.2.2 Design of Forecasting methods for better inventory control:

Forecasting methods have usually been considered as statistical tools to fit data and then predict the future based on events in the past. However, fitting data accurately does not necessarily imply that the inventory parameters have been decided optimally. Traditionally, sales forecasting models have been judged based on the accuracy of fitting data. However, the efficacy of the process does not lie in just predicting the right numbers. On the contrary, for enhanced supply chain efficiency we need to design and integrate forecasting and inventory models so that the total cost of inventory is minimised.

The smoothing parameters in exponential smoothing models are chosen so as to minimise the MSE (mean square error) or MAPE (mean absolute percent error). But how about optimizing the smoothing parameters so that the total cost of inventory is minimised!

[20] Also known as Holt-Winters forecasting method

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