

# LONG TERM PLANNING OF BLOCK CAVING OPERATIONS USING MATHEMATICAL PROGRAMMING TOOLS

By

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## I ABSTRACT

In the past very rudimentary methods have been used to plan and schedule the extraction of ore from block cave operations. The basic assumption of these methods has been that the movement of ore through the draw points is smooth and can be done in a specified sequence. There are many operational research tools available to allocate resources and schedule operations in an optimum way. Many of these were developed specifically for the manufacturing and service industry. Although some optimization and scheduling tools have been used in open pit mines, few have been applied in underground mining.

The principle of scheduling systems reviewed in this research is the link between strategic goals and production scheduling. Two strategic goals in particular have been formulated in this research the maximization of NPV and the optimization of the mine life in block caving. Both of these goals have required the integration of geomechanical aspects of the ore extraction, resource management, mining system and metallurgical parameters involved in the mineral extraction.

One of the main results obtained in this thesis is the integration of mine reserves estimation and development rate as a result of the production scheduling. Traditionally these parameters have been computed independent of the production scheduling. The second contribution made by this research was the formulation of link between the draw control factor and the angle of draw. This relation was built in to the actual draw function to compute schedules with high performance in draw control. Case studies have been used as an example of the application of an operational research approach to scheduling in a block caving operation.

Future research has also been outlined to approach the problem of linking the short and the long term planning, which has been resolved using mathematical programming tools. It is proposed to integrate the actual reconciliation process into the scheduling so operational information can be used to reproduce variability of the current long term planning models.

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## I INTRODUCTION

Several operations around the world are looking to apply low cost and highly productive mining methods that could work in a hard rock and highly stressed environment. Block caving has been shown to be a promising method and has gained popularity in the last years due to its low operating cost and high productivity. However very little research has been conducted or on how the mining method works and how it is planned.

Planning of block cave mines poses considerable difficulties in the areas of safety, environment, ground control and production scheduling. As the industry is faced with more marginal resources, it is becoming imperative to generate production schedules which will provide optimal operating strategies and make the industry more competitive (Chanda, 1990).

The current methodology has been focussed on formulating production schedules which often do not follow a strategic goal. For instance, is very common to hear about schedules that follow a certain production rate or a development rate, but no associated statement of a strategic goal. The weakness of this approach includes: focusing on short term production, lack of integration between mine planning procedures, and an inability to reflect long term goals into production schedules. Therefore the lack of a comprehensive methodology to plan block cave mines has led to seeking new ways of formulating the problem. This research intends to investigate whether the problem of production planning in underground mines can be formulated using available mathematical programming techniques.

Production scheduling of any mining system has a profound effect on the economics of the operation. In a marginal deposit the application of the correct scheduling mechanism might affect the life of the mine. Usually the scheduling problems are complex due to the nature and variety of the constraints acting upon the system (Denby, 1994). Although several authors such as Caccetta and Giannini (1988), Wilke et al (1984), Gershon (1987) have attempted to develop methodologies to optimize production schedules, none has satisfactorily produced a robust technique which has an acceptable level of success. One of the main

reasons for this unsuccessful history has been the failure in defining the objective function in relation to the mine planning horizons.

In this research two main planning concepts will be formulated as potential goals to optimise as part of the long term planning process. The first one is the traditional interest of mining companies to optimise economic return in such a way that all the mining, metallurgic and environmental constraints are fulfilled. The second concept developed in this research is maximization of mine life, which often has been associated with a societal goal to maintain employment levels. This research intends to demonstrate that maximizing the life of the mine could perhaps be a strategic planning goal. One of the main applications of this algorithm would be to prolong the life of an existing mine until new areas are ready to start production.

### **I.1 Objective**

The main objective of this thesis is to develop a methodology and an application that would enable mine planners to compute production schedules in block cave mining consistent with strategic goals and which satisfy all the constraints given by the mining method, the current available information related to design and mine planning, and the operational situation.

### **I.2 Description of the Problem**

The first issue with the current approach of scheduling a block cave operation is the lack of integration of the parameters that composed the mine planning process. For example the mining reserves have traditionally been computed independently of the mine schedule. However this estimation will later affect the production system through the closure of the draw points when they reach their mining reserves. Hence the decision of shutting down a draw point in a particular period of the schedule is based upon its remaining reserves and the status of the entire production system. Consequently the reserves estimation of a draw point in a block caving will depend upon the position of the draw point in the undercut sequence as well as its operational status at any given time within the life of the mine.

The second issue is that there is not a comprehensive way of computing a production schedule in block cave mining. For instance no accounts is taken of variable economic conditions and their impact on the overall schedule. Likewise there is no ability to evaluate the delay of a production area due to strategies to save production costs.

The third issue is that there is no a mathematical model to either optimise the net present value or the life of an existing operation fulfilling the mining and financial constraints. Consequently there is no way of linking strategic goals such as these with the subsequent production schedule.

### **I.3 Thesis Structure**

Chapter 2 discusses the details involved in the design and planning of block cave mining method. The algorithm to optimise the net present value based on using variable shut off grades is introduced in Chapter 3 and an algorithm to optimise the life of the mine through optimising draw control practices is introduced in Chapter 4. Chapter 5 illustrates two case studies where the proposed algorithms have been used to formulate their production schedules. Finally conclusions and recommendations are given in Chapter 6.

### **I.4 Contributions made by this thesis**

The following topics are the main contributions made by this thesis:

- The research done has developed a methodology to reconcile long term strategic goals with the mine production schedule
- Heuristic rules, defined by draw functions, have been implemented in the mathematical programming model.
- The development of an algorithm that enables mine planners to optimise net present value within a defined production schedule
- The integration of mining reserves and development rate into the net present value optimisation.

- The establishment of the concept that draw control should be a strategic decision rather than an operational goal. Based on this concept an algorithm was developed to apply proper draw control and therefore maximize the life of the mine.
- Case studies are shown that illustrate concepts developed in this research.

## II LONG TERM PLANNING IN A BLOCK CAVE MINE

### II.1 Introduction

The role of a long term mine plan is to reflect the corporate strategic objectives into the mine operation. Usually the production plan, which is the main product of the mine planning process, is understood as the business promise of the company. Therefore there are several elements in the production planning process that will affect not only the current situation of the company but also its future position in the market (Camus, 2002).

One of the main promises involved in the production plan is the quantity of metal that the operating mine will produce through its life. The manner in which this amount of metal will be produced at the mine site will bring together several factors that are part of the mine planning process such as costs, tonnage of ore, head grade, recoveries, and rate of development. Also there are significant external factors that will affect the way that the mine produces the metal such as prices, and the geology of the deposit.

Several relationships can be established among the mine planning factors, although very little has been found in the literature. For instance it is questionable whether low or high grade ore should be mined in the event of an increase in metal prices. If the company uses marginal cut off grades and it were facing an increase in the metal price, this company would decrease the cut off grade accordingly, thus decreasing the amount of metal produced for a given milling capacity. However if the price is increasing it is likely to be due to a lack of metal in the market. Then this strategy does not follow the basic economic law of supply. Examples like this can be found over and over within the mine planning process.

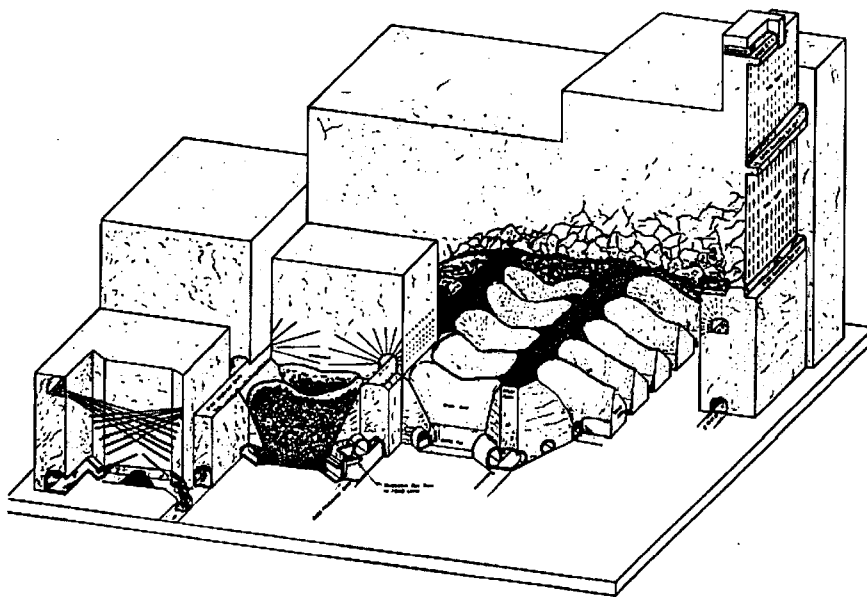
One of the main goals of long term mine planning would be to integrate internal and external mine planning factors that affect the performance of the mine operation. At the same time long term planning is responsible for coordinating strategic goals and operational activities. None of these tasks is trivial from a technical point of view. There are several issues related to information, modeling and methodology that make this step one of the most complex

within a mining system. There is no doubt that any contribution made in this regard would not only benefit the operation of the mine but also would establish the position of the company in the market.

## **II.2 Block Cave Mining Method**

Cave mining refers to all mining operations in which the ore body caves naturally after undercutting its base. The caved material is recovered using draw points (Laubscher, 1994). The size of broken material will probably dictate which ore-handling system is suitable. For fine ore fragmentation the gravity system (Grizzly) is the most feasible. For somewhat coarser material, the slusher system should be implemented. For coarse material, the mechanized system such as LHDs might be the best. In choosing the correct method other parameters should be taken into account such as, work force sophistication, labor cost, availability, capital cost of equipment and desired production rate any other factors that may be unique to the particular mine (Tobie, 1984).

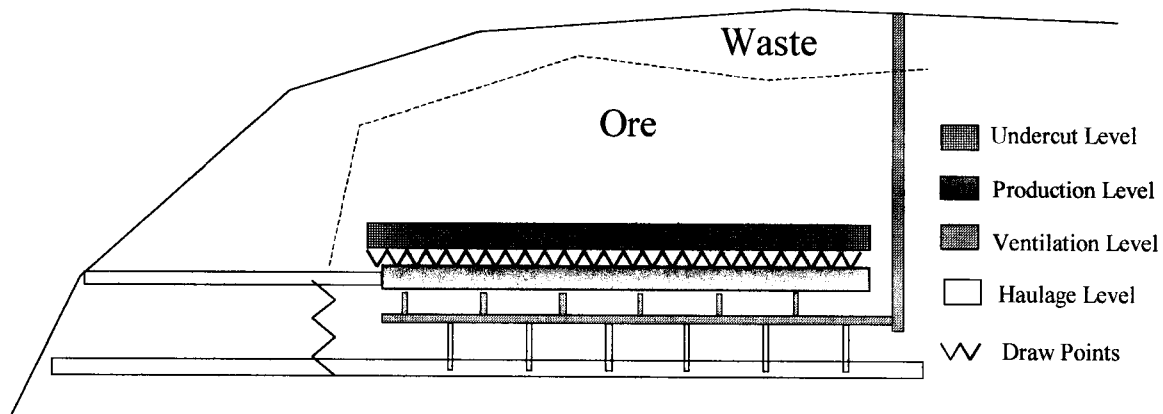
Figure 2-1 shows a three dimensional scheme of panel caving method, which is typical of the current trend in the design of block cave mines.



**Figure 2-1: Diagrammatic representation of panel caving**



Several different levels that facilitate the gravity flow and recovery of ore compose the block cave mining system (see Figure 2-2). The first level, the undercut level, facilitates the caving of the ore body. The second level, called production level, provides infrastructure to collect the broken ore through draw points. The third level, the ventilation level, contains the fans to provide air to the production level. The fourth (optional) level, the reduction level, holds the infrastructure needed to reduce the size of the fragments, using either rock hammers or small crushers. The fourth level will facilitate the flow of material from the production level to the haulage level. The fifth level, the haulage level, contains the drifts where trucks, trains or bells collect the ore coming from the production level through chutes.



**Figure 2-2: Schematic representation of block cave mining system**

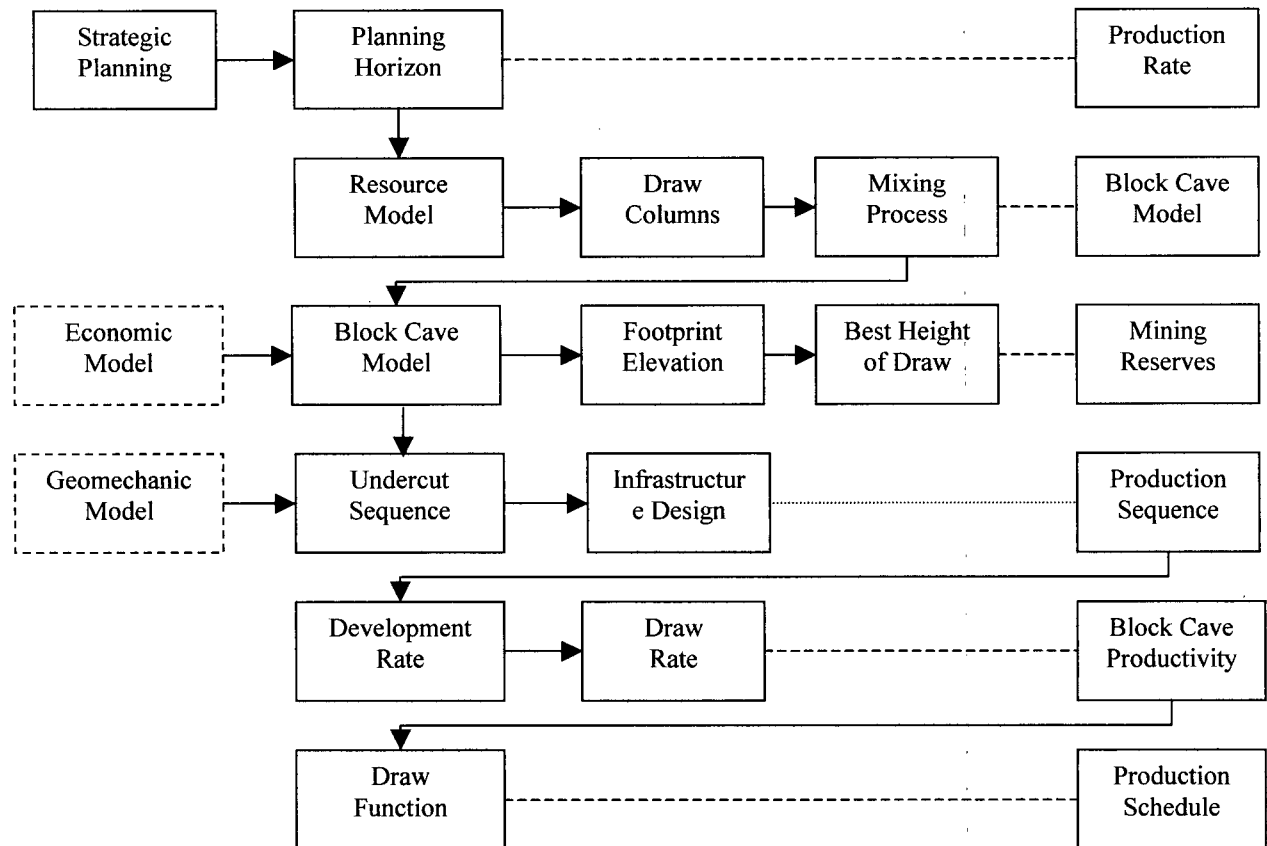
### II.3 Traditional Steps in Planning a Block Cave Mine

In block cave mining the integration of the internal mine planning factors is even more critical than in other mining methods due to the fact that there are less degrees of freedom or flexibility in terms of planning once the caving starts. It will be seen that several mine planning constraints come from geomechanical, design and operating criteria rather than from pure production considerations.

The production schedule in a block cave mine mainly defines the amount of tonnage to be mined from the draw points in every period of the plan, the grades, dilution, costs, value, etc. To compute this production schedule many decisions need to be made with regard to accessibility and infrastructure, mine and plant capacity, mining sequence, metal targets, etc.

This chapter will focus on explaining the steps that need to be followed to complete the production schedule.

Figure 2-3 shows how the actual mine planning steps are related to each other.



**Figure 2-3: Diagram of actual mine planning steps in block caving**

The strategic planning defines the goal of the project whether will be to maintain steady production, maximize return or maximize the life of the mine. The planning horizon defines the scope of the plan. Usually the scheduling resulting from a long-term plan will be less accurate than the schedule resulting from a short-term plan due to the large differences in quality and quantity of the information used in the models. The production rate defines the amount of tonnage to treat per period. Typically this is defined by the existing infrastructure such as plant and underground development. However for new mines and extensions,

production rate is a parameter that becomes a variable and it will become dependant on the amount of mineral and economic resources available.

A more detailed description of the processes involved in planning a block cave mine is presented in the following sections.

### ***II.3.1 Resource and Rock Mass Models***

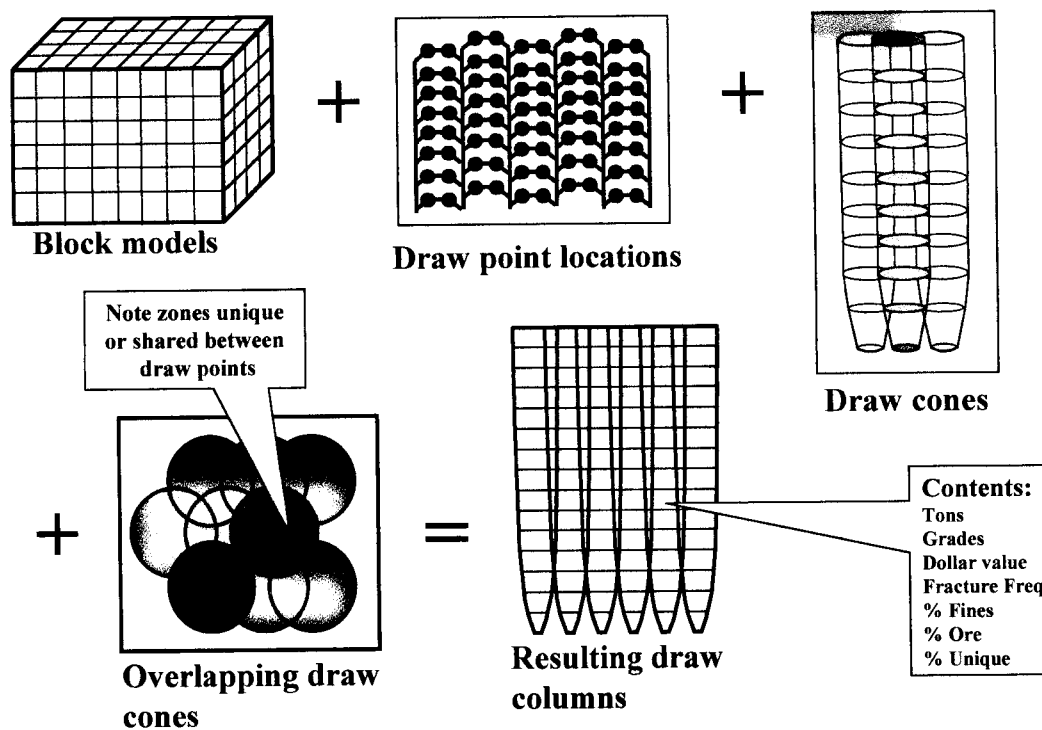
One of the first steps in planning a block cave mine will be to compute the resource model. The resource model in a block cave mine is not more than a few block models in which each block represents an attribute of the geologic deposit. Several attributes can be modeled using estimation techniques such as the polygonal method, inverse distance or Kriging. The following represents a list of the typical attributes that are modeled when planning a block cave mine

- Grades: Cu, Mo, Au, also impurities should be modeled
- Rock Types: andesites, diorites or others
- Geotechnical model: frequency of fractures per meter, RQD, RMR
- Density : very important to account for tonnage; usually this is linked with the rock type
- Fragmentation model: usually this is a result of combining some heuristic rules with the geotechnical model and rock type
- Hardness: usually this attribute is modeled using the work index indicator
- Metallurgical parameters: The trend currently is to use geometallurgical models that allow the introduction of parameters such as recovery, acid consumption, and product size in the production schedule. These parameters help to forecast the behavior of the processing plant.

### ***II.3.2 Mining Unit - Draw Column***

The second step is to find the proper mining unit that will better represent the mining method. Every draw point is simply an excavation that collects the caved material. As the mucking process takes place, a disturbance zone can be observed above the draw point,

commonly called the ellipsoid of draw (Kvapil, 1965). When the height of this ellipsoid is large enough, which is the case in block caving, then the disturbance zone is called the draw column. Thus what needs to be modeled in block cave mine are columns of material where every column represents a draw point. Flow of material along these columns will be modeled to represent the mixing that occurs within the caving zone. The following figure shows the process of transferring material from block models to draw columns. The methodology to perform these calculations has been described by Diering (2000).



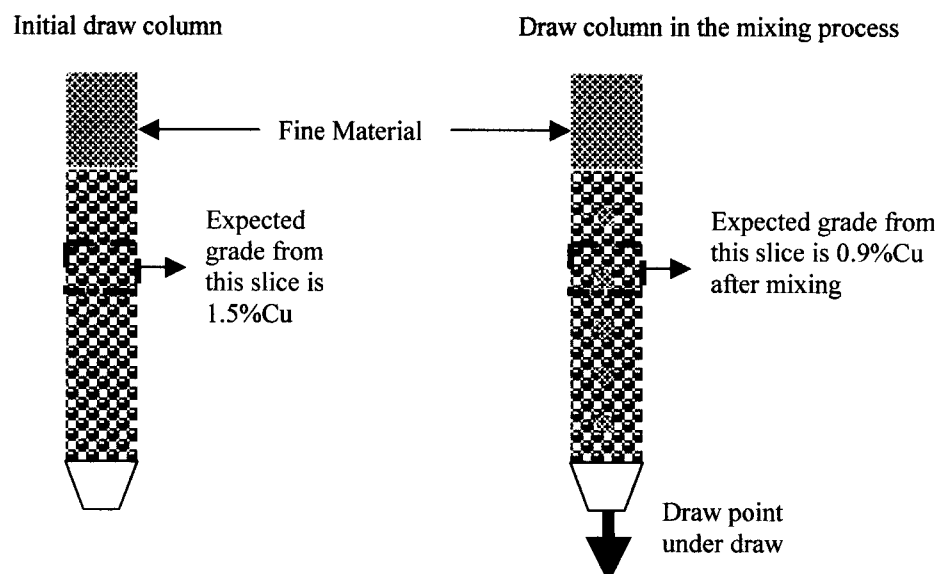
**Figure 2-4: Transferring information from the block model into the draw columns**

A key parameter of the draw columns characteristic is their diameter. According to Kvapil (1965), draw column diameter is a function of the fragmentation of the rock. Coarser material leads to larger diameters together with roughness, moisture, fragment shape, and the size of the opening where the material flows through. Although several authors such as Kvapil (1965) and Laubscher (1994) have developed empirical charts to compute the diameter of the draw column based on these parameters, no such charts or relationships are available for high stress - competent rock environments.

Once the draw column diameter has been computed, then the spacing between draw points is determined, also based on empirical charts. Kvapil (1965) recommends using the draw column diameter as the spacing between draw points while Laubscher (1994) recommends 1.5 times the draw column diameter. Susaeta (2001) recommends that the spacing between draw points be 1.6-1.7 times the diameter for high stress- competent rock environments. The main purpose here is to achieve interaction between the draw columns. It is desirable to generate horizontal movement between draw columns so there are no zones without movement between draw points that may eventually provide a point load on the production level, damaging it and ultimately producing its collapse. Interaction between draw points also acts as a barrier to the waste, located above the ore zone, retarding the entry of dilution into the draw points.

### II.3.3 Mixing Process- Dilution

The next step is to simulate the mixing process that occurs within the caved area. The main concept here is that as material is drawn down, then the finer material will move down faster inside the draw column than the coarse material. This has several implications from the planning point of view because the grade that is supposed to be at a certain location will change due to mixing with the block above it and the subsequent block. Figure 2-5 represents this phenomenon.



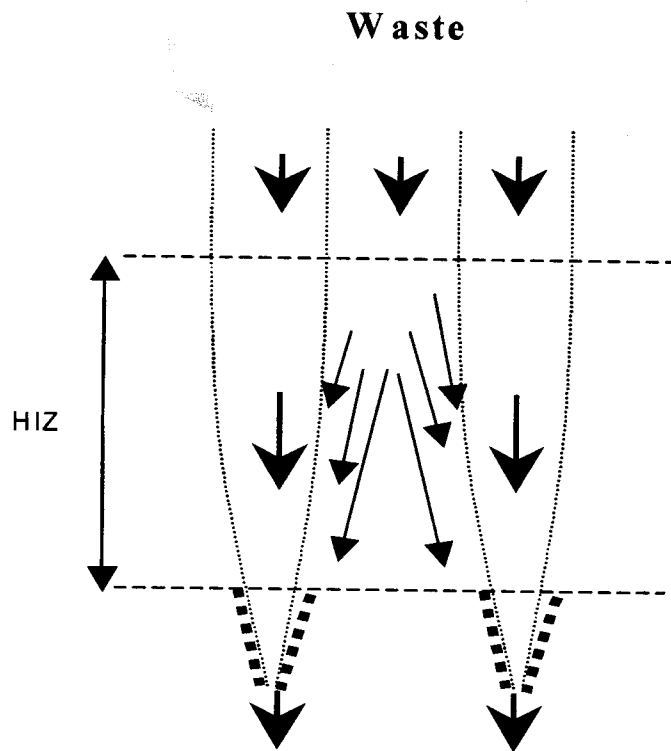
**Figure 2-5: Mixing process within the draw column**

As a result of this mixing process material that is waste can be transformed into ore if the fine material contains high grade. The opposite effect can also be observed if the fine material contains low grade, ore can be sterilized after applying mixing. Therefore the mixing algorithm will be very relevant when computing mining reserves.

Laubscher (1990) established several rules that can be applied to simulate the mixing process for an intact block so mine planning can better predict the grades to be obtained as the scheduled mining progresses. The main parameter to simulate the mixing process is the height of the interaction zone (HIZ). As material is drawn from the draw points, overlying material descends uniformly towards the zone of interaction. Once this material enters the zone of interaction, the vertical movement becomes more chaotic depending on the direction of the density gradient. Horizontal movement of this material can be expected, leading to a new component of mixing called cross mixing. Once the material enters the zone of interaction it might appear in any of the neighboring draw points. Laubscher (1981) has suggested that HIZ would be a function of the variability of fragment size within the draw column and the spacing between draw points. According to observations in the field it has been observed that the direction that material follows when it crosses the interaction zone is dependent on the differential draw that is taking place on the production level. Thus HIZ is a function of fragmentation of the rock mass, draw point spacing and the draw control practices that take place on the production level. Figure 2-6 shows the concept described above.

The greater the height of interaction, the sooner the dilution will appear in the draw point. This is quantified by a relation derived by Laubscher (1994) as follows:

$$DEP = \frac{H_c - \frac{HIZ}{s} * dcf}{H_c}$$



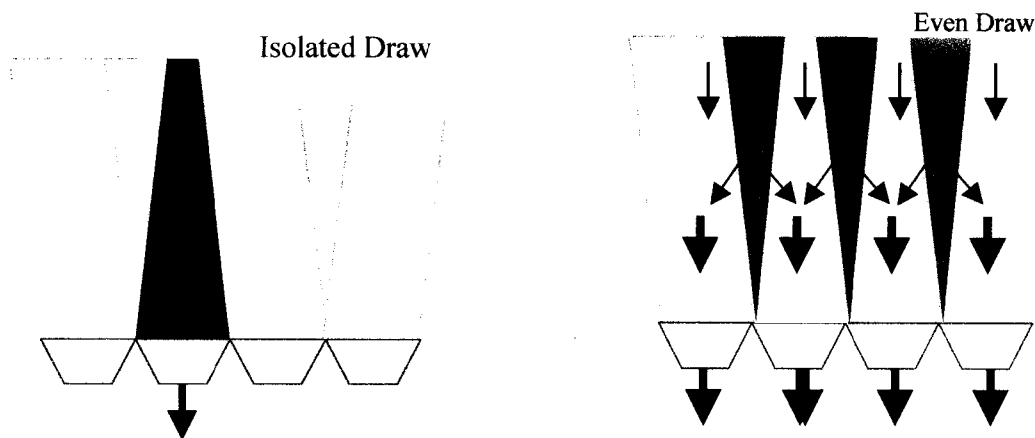
**Figure 2-6: Concept of height of interaction zone (HIZ)**

The dilution entry point (DEP) is the percentage drawn from the draw column at which the first dilution is observed,  $H_c$  is the height of the draw column.  $dcf$  measures how even the draw is performed at the production level. As the draw becomes uneven the term  $HIZ * dcf$  increases, consequently decreasing DEP. Also  $s$  represents the swell factor, the ratio between in situ density and caved density. This converts HIZ from height of caved rock into height of in situ rock to be consistent with  $H_c$ .

Draw control has been traditionally assigned as a terminology for measuring the differential draw that occurs among draw points in a period of time. According to the formulation described above the more evenly the draw points are drawn, the later the dilution appears in the draw points. Consequently, two states can be defined from a draw control point of view: isolated draw and even draw. Isolated draw means that interaction between draw points does

not take place. Even draw means that a draw point and its neighbors create a zone of interaction, which will act as a barrier for the dilution to enter. Figure 2-7 shows these two states of draw.

Draw control practices would affect the performance of the entire mine. For instance isolated draw will accelerate the dilution process, consequently affecting the life of the mine. In addition to the dilution behaviour there is also a geomechanical response to the isolated draw due to compaction of draw points that are not drawn for a period of time. These inactive zones will collapse, losing production area that can rarely be recovered in the future.



**Figure 2-7: Isolated and even draw states**

Traditionally draw control practices have been left up to the operators of the mine, considering that this is a concern of operation more than planning. However this research intends to promote the concept of draw control being part of the mine planning process. Consequently draw control will be presented as a desired goal of this process.

#### **II.3.4 Draw control factor**

It has been said before that the draw control factor ( $dcf$ ) is an indicator of how even the draw is performed among active draw points at a given horizon of time. This indicator was first introduced by Laubscher(1981). The following formula has been traditionally used to compute the draw control factor



$$dcf_i = \frac{\left( \frac{\sum_{k=1}^K (d_i - d_k^i)^2}{K} \right)^{0.5}}{\frac{1}{K+1} \sum_{k=1}^{K+1} d_k}$$

where

$d_i$  is the tonnage drawn from draw point  $i$  in a period of time.

$d_k^i$  is the tonnage drawn from draw point  $k$ , a neighbor of draw point  $i$  in the same period of time.

$K$  is the number of neighbors of draw point  $i$ .

Note that the denominator includes the draw point  $i$  in computing the average amount of drawn material from the  $K+1$  draw points. A value of  $dcf$  close to unity indicates even draw; a value significantly different from unity indicates isolated draw. Also  $dcf$  is a function of time. For example an acceptable draw control factor for a period of one year can be 3; however for one week it could be 10. Therefore the regularity of a draw point and its neighbors to stay in a draw status will dictate the influence of it on the overall gravity flow process. For example if a specific draw point that has been overdrawn,  $d_i \gg d_k$  and the uniformity index would be close to 1, erroneously reflecting good draw control. Consequently the following formula has been proposed to compute the draw control factor based on the overall performance instead of on a draw point by draw point basis.

$$dcf_i = \frac{\text{Max}\{d_k\}}{\text{Min}\{d_k\}} \text{ where } 1 \leq k \leq K+1 \text{ including draw point } i.$$

### II.3.5 Economic boundary

The next step is to define which part of the ore body can be economically mined. This is a non trivial question because the search for the best combination of production level and draw height of each draw point will likely involve trying millions of combinations. Traditionally

trial and error techniques have been used to determine the optimum production level and the corresponding draw height. Figure 2-8 illustrates the process of finding the best elevation for the production level.

Assuming that the elevation of the best production level is known then the evaluation of the economic boundary becomes a simple problem. Usually the first step is to compute the value of the slices that compose the draw column of the mixed model. The data involved are metal prices, mining and processing costs, selling costs and overhead costs to cover the development of the draw point.

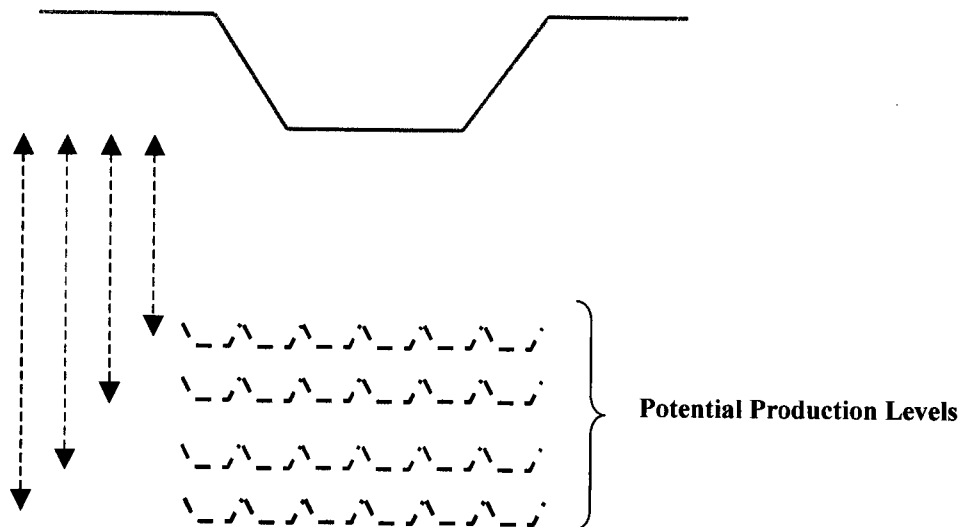


Figure 2-8: Designing the location of the production level and its height of draw

Slice	Current Grade	Current Value	Cum Value
1	1.2	5	5.00
2	1	2.5	7.50
3	0.8	0	7.50
4	0.75	-0.625	6.88
5	0.7	-1.25	5.63
6	1	2.5	8.13
7	0.7	-1.25	6.88
8	0.8	0	6.88
9	0.6	-2.5	4.38
10	0.7	-1.25	3.13
11	0.4	-5	-1.88
12	0.5	-3.75	-5.63
13	0.6	-2.5	-8.13
14	0.4	-5	-13.13

RF  
CM

12.5  
10

← BHOD technique

← Marginal cut off grade

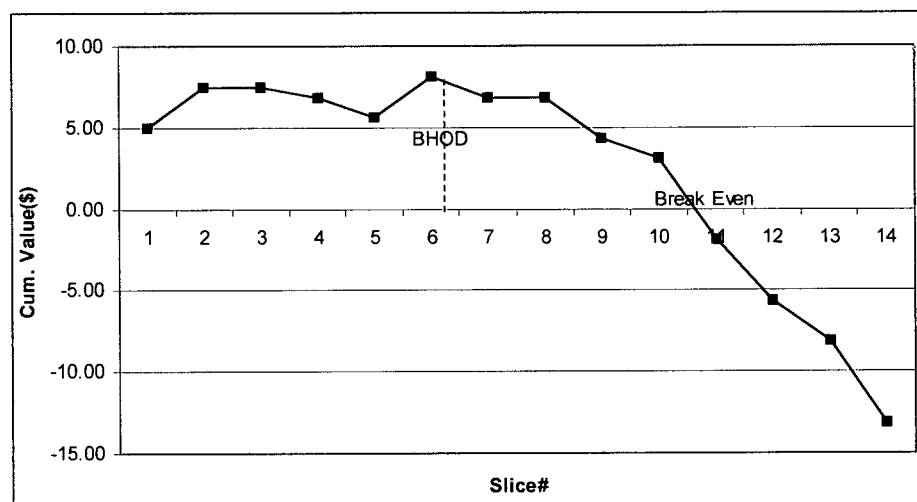
**Table 2-1: Calculating best height of draw (BHOD)**

Table 2-1 shows the calculation for one draw column of copper. Note that the first slice is located at the bottom of the production level. RF represents the revenue factor, which is the metallurgical recovery times the price reduced by the selling cost. CM represents the mining cost and the processing cost. Table 2-1 also shows the cumulative value along the draw column

Traditionally the concept of marginal cut off grade has been used to define mining reserves particularly in open pit mines. This algorithm computes the cumulative value along draw the draw column. Once the cumulative value reaches zero the height of draw is set. In the above example slice 10 would be the height of draw using the marginal cut off grade concept.

There are two reasons why this method is not applicable in block caving. First this method is mining material that does not add value such as slices 7, 8, 9, 10. Second, it does not include the development cost, resulting very large columns that are not feasible in block caving.

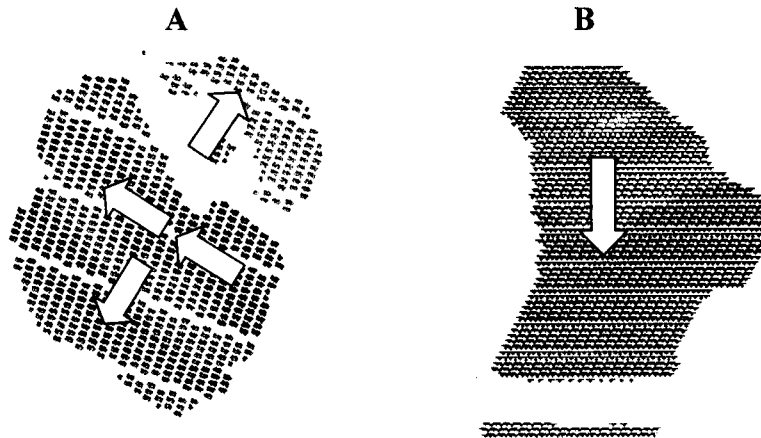
The second traditional method, developed by Diering (2000) finds the maximum cumulative value along the draw column. This height is called the best height of draw (BHOD). In the above example slice 6 would be the BHOD. The following figure shows how the marginal cut off grade and BHOD operate for one draw point example.

**Figure 2-9: Comparison between different methodologies to estimate mining reserves**

Usually the break even cut off grade methodology will define more reserves than the BHOD technique. Therefore the criterion to be used to define draw height has a significant influence on the mining reserves, and therefore in the life and economics of the project.

### ***II.3.6 Undercut sequence***

The undercut sequence in a block cave mine defines where to start the caving within the layout and how to progress. The direction and geometry of the cave front are also involved in defining the sequence of a block. Bartlet (1992) recommends starting the caving where the weak rock is located, so the hydraulic ratio can be reached earlier in the life of the mine and the time to recover the investment is shortened. This approach also allows the mine to reach a steady production earlier since adequate fragmentation is achieved. Another method consists on starting where the high grade ore is located leading to early payback of the investment or higher net present value. However this latter approach might prove to be a very expensive approach due to the potential need to excessive secondary blasting to achieve production targets. There is no a consensus in this matter. However the main goal here is to have a strong economic model that incorporates most of the variables involved in these decisions. Figure 2-10 represents the geometry followed in two different sequences at two different mines. Layout A starts production in the center of the footprint moving through the life of the mine towards the boundaries. This decision was mainly made based on rock mass characteristics where the center of the layout presented weaker rock. Layout B starts production in one extreme of the footprint moving through the life of the mine towards the opposite side of the footprint. The decision in this case was mainly made based on economic value.



**Figure 2-10: Representation of undercutting sequence for different mining layouts**

The direction of the cave front will be always chosen to be perpendicular to the main geological structures, so that more blocks are produced and a better fragmentation is achieved. The geometry of the cave front will be defined according to stand practice. However currently there is a consensus to implement a concave front instead of a flat cave front.

### ***II.3.7 Development rate***

The development rate defines the minimum number of draw points that have to be developed in order to reach production targets. The development rate is coupled with the productivity of the draw point or draw rate, which ultimately defines the mine production capacity. Usually the development rate acts as a constraint for the mine productivity and it is defined by the amount of activities that can happen inside the mine without affecting the logistics of the operation.

### ***II.3.8 Rate of Draw***

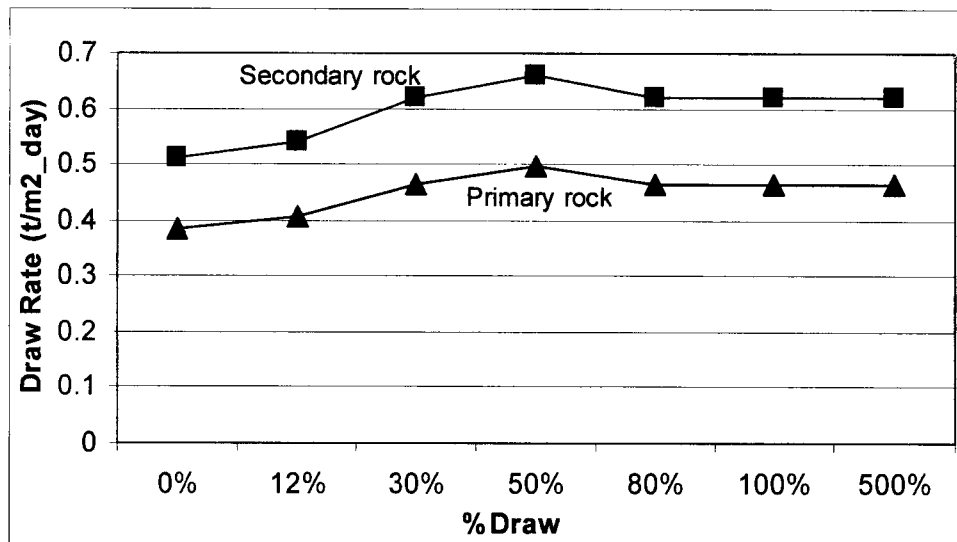
Rate of draw defines the maximum amount of tonnage that can be drawn from a draw point in a period of time. This parameter usually is the most important one in terms of defining

mine capacity. Traditionally the rate of draw has been defined as tonnage/day per draw point, if every draw point can be identified by its basal area then draw rate is better defined as tonnes/day/area (few mines also use inches/day/area). The main parameters that define the rate of draw are as follows:

- *Fragmentation.* Smaller fragmentation allows higher draw rates, since there is no much secondary breakage
- *Stresses.* Highly stressed rock is likely to burst if it is exposed to high draw rates. Seismicity has a very important role in deep hard rock mining. There are a few mines around the world that constrain the draw rate to avoid rock bursting (Dunlop 1995)
- *Equipment.* A larger bucket allows higher draw rates
- *Mine design.* The layout geometry as well as the length of the ore passes will contribute significantly to the feasible draw rate. For instance if the layout is a herringbone offset, it will be impossible for the LHD to load facing the muck pile (Esterhuizen, 1992). If the ore passes are too short, then the LHD will have to interrupt its cycle once it has finished within a drift to move to the next drift.
- *Secondary breakage.* If the secondary breakage system works efficiently it will facilitate a higher draw rate. The interrelation between secondary breakage and LHD productivity is something that is not very well understood and it will affect all mines in a high stress, hard rock environment.

Figure 2-11 shows the typical curve used to define the draw rate during the life of a draw point. The age of a draw point is defined by the height of draw. Consequently a percentage of the height of draw identifies every stage on the life of a draw point. Usually the X axis of the draw rate curve represents this measurement (% of draw) and the Y axis the draw rate.

This research promotes the concept of having a minimum draw rate as well as the maximum one. The minimum draw rate allows even draw and also contributes to avoiding compaction of draw points.



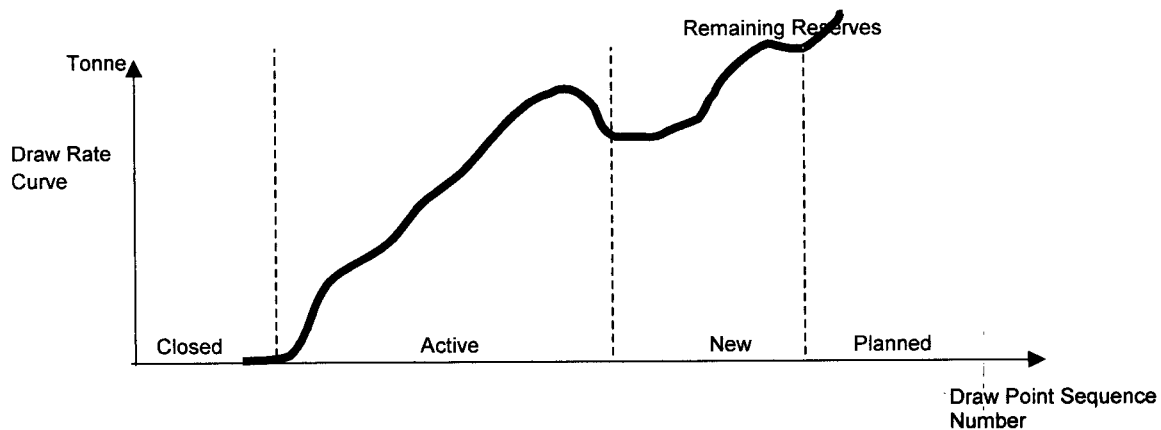
**Figure 2-11: Common draw rate curves (Third Panel Andina - Codelco, personal communication)**

It is important to notice that the rate of draw is a function of the planning horizon. For instance the rate of draw of a draw point for daily planning may vary within the range 1.4-2.5  $t/m^2/day$ , while the rate of draw for planning on an annual basis usually lies in the range 0.2-0.8  $t/m^2/day$ . The correct rate of draw should be chosen to correspond with the planning horizon.

### ***II.3.9 Production strategy - Draw function***

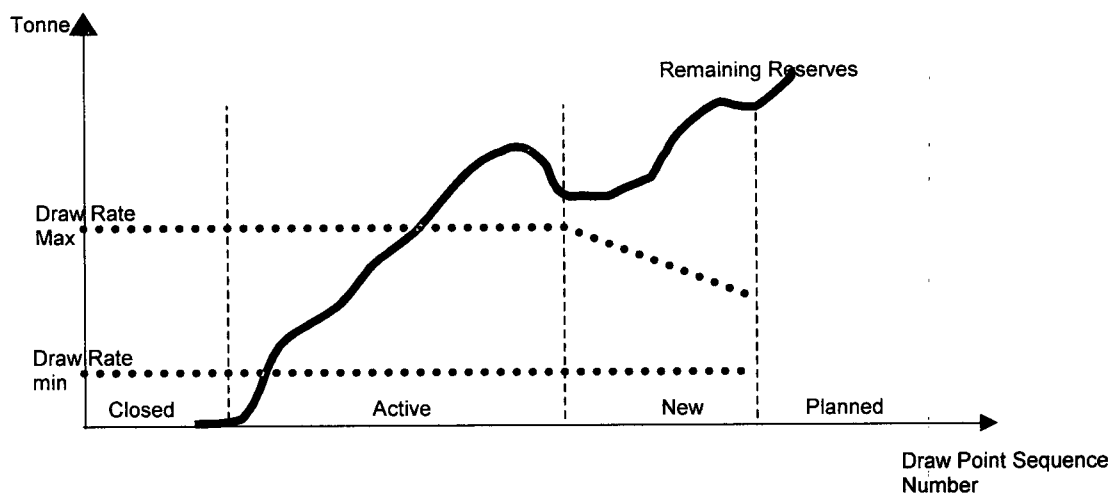
The concept of draw function is intended to relate the parameters that have been defined above, such as sequence, %of draw, draw rate, reserves remaining, draw point status. This function will define the overall draw control policy as well the production dynamics, specifying the amount of tonnage to draw from every draw point in a period of time. The draw function concept was developed by Diering (2001, personal communication). The relevance of the draw function in this research is that it provides the feasible region to achieve the optimized schedule.

The X axis of the draw function curve represents the sequence number which is given by the undercut sequence. The lower the number the earlier the draw point starts production. The Y axis represents tonnages either as a constraint or as a result of the analysis. The second attribute to be plotted is the draw point status at a certain period of the production plan schedule. There will be closed or extinguished draw points, active draw points, new draw points (which is in relation with the development rate) and future or planned draw points. Figure 2-12 shows the remaining reserves of the draw points.



**Figure 2-12: Schematic representation of the production schedule at any given period**

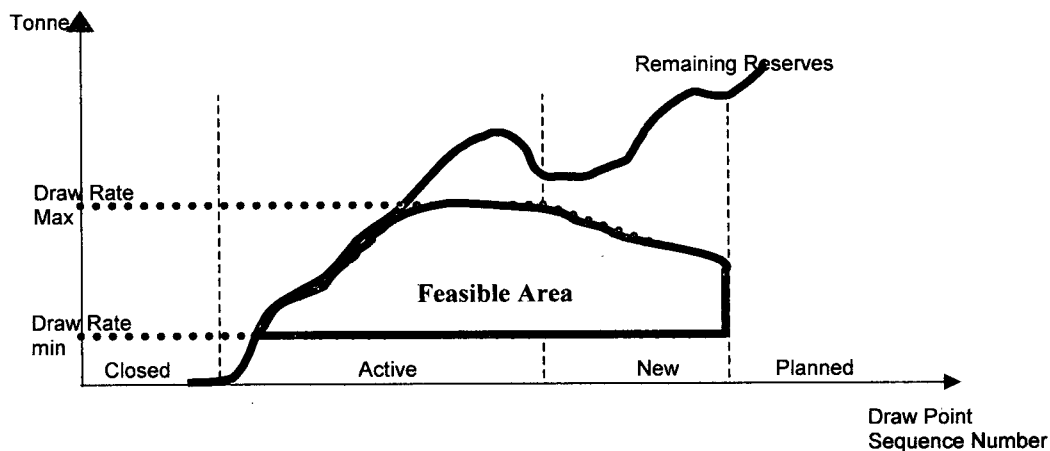
The following step will add draw rate constraints. These constraints will affect the maximum and minimum amount of tonnage to be drawn from the active and new draw points.





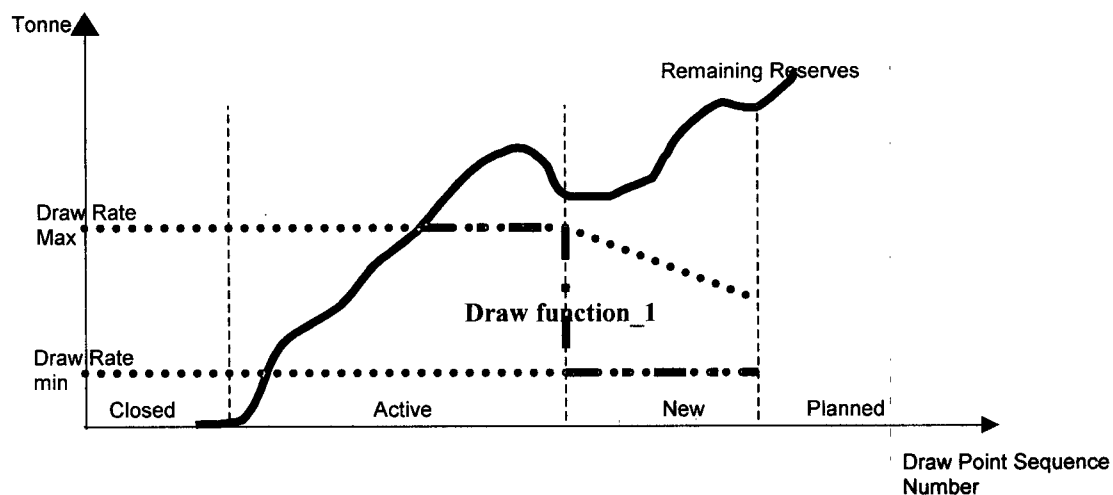
**Figure 2-13: Schematic representation of the actual constraints for the production schedule**

Finally the feasible area for any production strategy to be applied is shown as follows.



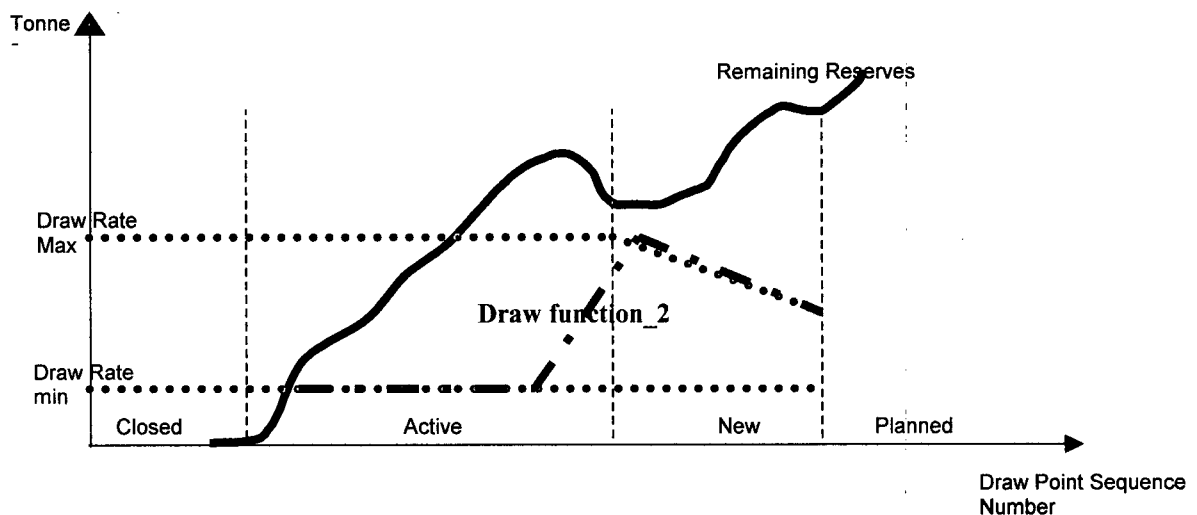
**Figure 2-14: Schematic representation of the feasible area of production at any given time within the production schedule**

The chosen strategy to be followed within the feasible area should be contributing towards the objective function and the final objective of the production plan. For instance Figure 2-15 shows a feasible production strategy strategy.



**Figure 2-15: Schematic representation of a draw function at any given time within the schedule**

Strategy number one shows that is mining most of the production from the old draw points drawing the new draw points to the minimum allowable. This strategy attempts to recover the maximum of the reserves that belong to the old draw points and slowly incorporating production from new areas. This strategy is the most inexpensive one because eventually it would not need to incorporate that many new draw points to reach the production target. However because usually old draw points have low grade (owing to dilution) the grade will be not as high as it had mined the new draw points to a higher grade. Therefore this strategy is contributing to minimize the operating cost of the mine. A different strategy can be seen in Figure 2-16, this strategy looks to mine as little as possible from the old areas in order to reach a higher overall grade drawing the new areas to their maximum capacity. This strategy may be followed to maximize the value of the plan, mining the high grade early instead of later. Note that the way in which the transition between the minimum and the maximum draw rate takes place will affect the draw control behavior. In this scenario collapses and problems with dilution are likely if this transition is not managed properly.



**Figure 2-16: Schematic representation of a draw function at any given time within the schedule**

Finally by adding up the production strategies followed in every period of the life of the mine the production schedule can be drawn. This research will focus on defining the correct production strategies per period to achieve an overall objective function. The way of approaching this problem has been through the use of mathematical programming, which provides efficient tools to define which sequence of production strategies should be followed in order to achieve a specified objective function.

#### **II.4 Proposed Mine Planning Model for Block Cave Mining**

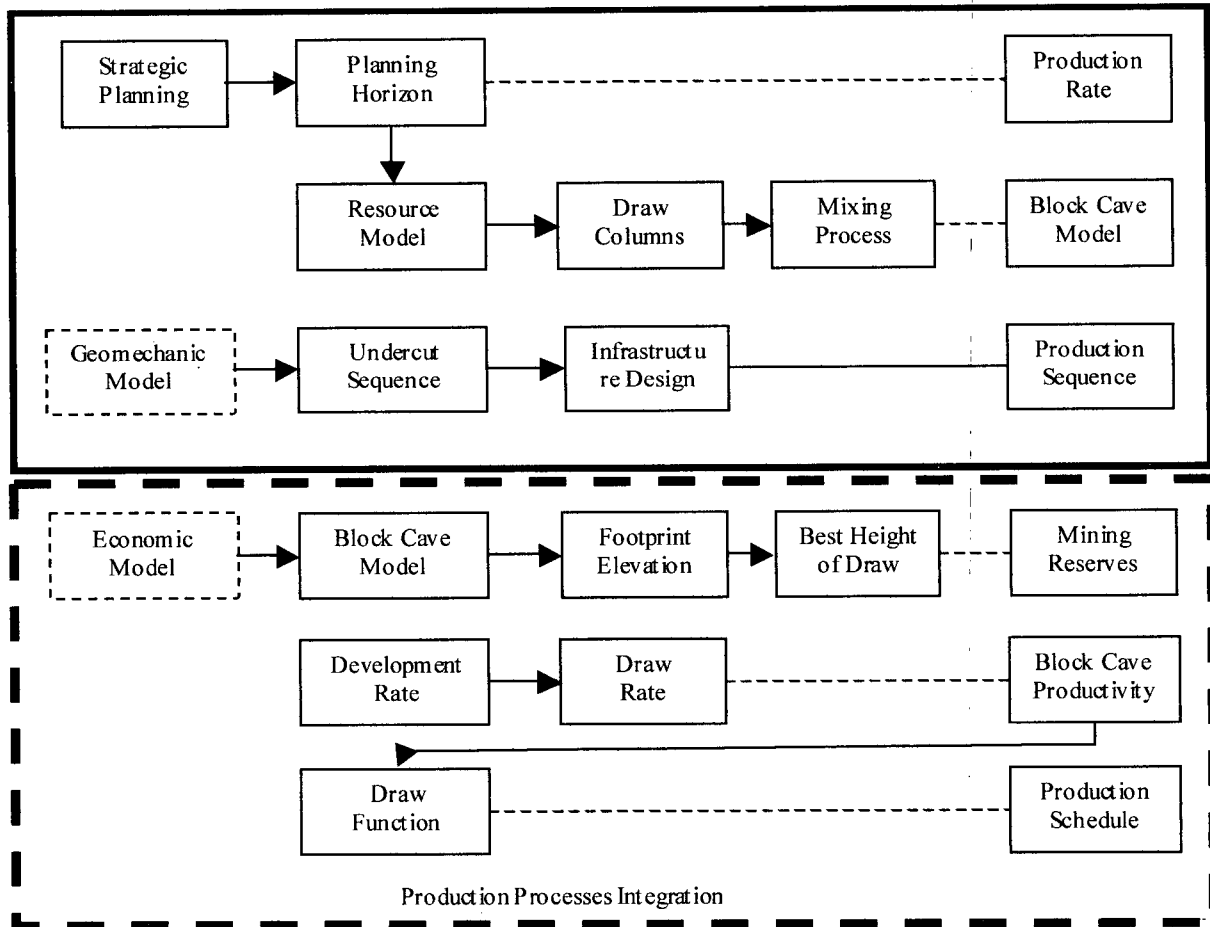
The current methodology used to compute production schedules in block caving has been presented. There are several issues with this approach, some are related to the methodology and others related to the information used to justify some of the decisions involved in the process. The issues used to justify the research in this thesis are the following:

- Lack of linkage between strategic goals and production schedules: Often strategic goals are forgotten at the moment of scheduling a mine. Usually this is due to the inability to represent strategic goals as mathematical functions that will render the goal into production plans. This is extremely important because these plans will later affect the operation of the mine. Actual production plans typically follow the objective of reaching the production rate or geomechanical conditions. This translates into adding considerable pressure into operations without achieving the business objective, which frequently differs from the current approach.
- No integrated mine planning process: The current model follows a disintegrated approach, which is the result of the history of mine planning. A decade ago production schedules were computed almost manually with very little computing. Consequently the planning process became very disaggregated in which every step was computed in isolation from the rest. Several examples of disintegration are found in the current approach. However a critical one is the relation between mining reserves and production scheduling. Consequently the current method to compute mining reserves are unrealistic because usually they conflict with the constraints found in the production scheduling

- No clear scheduling methodology: Although before was presented a model to schedule block cave mines was presented, in reality the mine planning steps are followed according to the criteria of the mine planner. Then the rational used to incorporate parameters in the mine planning model is not clear. Typically draw rate for example has been always defined based on experience and not on the actual productivity of the draw point. Another important parameter is the undercut sequence, which often is computed based on geotechnical more than economical considerations.

Figure 2-17 shows the proposed changes to the actual mine planning process shown before as the traditional steps in planning a block cave mine.

There are two proposed changes to the actual planning model. The first one will be to embed mining reserves estimation and development rate within the production scheduling process. This first change will address the lack of integration of the existent model, providing a more comprehensive estimation of mining reserves and development rate by fulfilling planning and operational constraints. The second change to the actual model is the incorporation of a mathematical methodology to compute production schedules. This change will facilitate the integration of information and also provide the tools to link the strategic goals with the production strategies in the operation of the mine.



**Figure 2-17: Proposed integration to the actual block cave mine planning process**

## II.5 Mathematical Programming in Block Cave Production Scheduling

As shown in the previous sections, the problem of scheduling a block cave mine is a matter of finding the goal that better represents the strategic planning vision subject to several mine design, geotechnical and operational constraints. Mathematical programming (MP) contains all the tools needed to formulate the block cave scheduling in a comprehensive manner. The block cave scheduling should always pursue a goal, which can be represented by an objective function. The actual constraints in the production schedule also can be represented explicitly as constraints to the optimisation. Several attempts have been made to use this technique in scheduling open stope mining (Trout, 1990) where the decision of mining a stope is represented by integer decision variables. Smith (1998) applied mathematical programming to open pit scheduling where the traditional trial and error technique is replaced by a

scheduler capable of optimizing the feed to the mill within a current mining phase of the open pit. However very few applications of MP have been made to block cave mining. The first attempt to use MP in block cave scheduling was made by Chanda (1990) who implemented an algorithm to write daily orders. This algorithm was developed to minimize the variance of the milling feed in a horizon of three days. Guest (2000) made another application of MP in block cave long term scheduling. In this case the objective function was explicitly defined to maximize draw control behavior. However the author stated that the implicit objective was to optimize NPV. There are two problems with this approach. The first one is that maximizing tonnage or mining reserves will not necessarily lead to maximum NPV. The second problem is the fact that draw control is a planning constraint and not an objective function. The objective function in this case would be to maximize tonnage, minimize dilution or maximize mine life.

The approach proposed in this research is to use MP to solve the production scheduling problem. There will be two objective functions defined as the main goal of mine planning: maximize NPV and maximize mining reserves or life of the mine. Also this research will show that there is a trade off between these two goals. Finally a potential method to reconcile these two strategic goals by modifying the current draw functions used to optimize NPV will be mentioned.

## **II.6 Objective functions in Block Cave Production Scheduling**

In this research two objective functions will be developed and be optimized along with the production schedule. The first one is the maximization of NPV by explicitly defining the mathematical expression of NPV to maximize and discussing the problems found in trying to solve this MP. The second proposed objective function is to minimize dilution entry by controlling the profile of the caving back. This second algorithm will indirectly represent the maximization of the life of the mine. Even though every mining company should always look to maximize its value, sometimes it is found that prolonging the life of the mine will facilitate the delivery of metal according to long term contracts. For example it is found that while a new sector of a particular mine gets prepared there is a need for the operating sector

to prolong its life as much as it can so there is no a gap in production that could eventually end up in breaking long term metal contracts. This last argument can sound as a patch to cover lack of planning. However due to the tremendous uncertainty in block cave mining, situations like this can be expected fairly often.

### III NET PRESENT VALUE MAXIMIZATION IN BLOCK CAVE MINING

One of the main goals of planning a block cave mine is to maximize the value of the mine. Commonly the net present value (NPV) is maximized because it will produce the maximum return to the shareholders. Several operations have recognized this strategy as the main driver for the mine planning process (Guest, 2000). However the way that the concept is applied differs from one mine to the other. For instance, in diamond mining maximizing NPV is a synonymous with minimizing the amount of dilution. In porphyry copper mining the strategy is to mine high grade ore early in order to maximize NPV. The following formulation considers the second approach to maximize NPV, understanding the minimization of dilution as a different objective function.

To maximize the NPV in a block cave mine there must be extra capacity that allows a choice of which draw point will be drawn and which will not. Usually the draw points with higher value will be drawn. These draw points that are not drawn in a period should be closed to avoid compaction and other operational problems. The present grade at which draw points are closed is usually called shut off grade. Therefore there is a direct relation between the definition of these shut off grades and NPV optimization.

The following formulation assumes that there is just one mine in production at the time ignoring possible blending between mines. A different problem can be set up to maximize NPV in an operation with multiple sources (mines) and destinations (plants). It is also assumed that the undercut sequence is known. Another assumption would be the size of the layout, which should be fixed, and defined as part of the previous planning steps. The formulation will therefore maximize NPV of the current mine production schedule.

#### III.1 Mathematical Formulation of NPV Maximization

The objective function in this case is composed by the tonnage to be drawn from the draw points and a binary variable that indicates whether a specific draw point has a production call in a particular period of the schedule or not. A set of parameters specifying grades



metallurgical recoveries and costs are also included in the formulation of the objective function. the more profitable draw points will be chosen to be part of the production call in order to optimise the NPV. The binary variable will lead into an integer variable that accounts on the number of draw points to open in a particular period of the schedule. Consequently tonnage and draw point status are part of the variables of the problem. The following set of constraints also is included in the formulation of the problem:

- **Development rate**; the development rate controls the maximum feasible number of draw points to be open at any given time within the schedule horizon. This constraint is usually based on the geometry of the ore body and the existent infrastructure of the mine, which typically will define the number of accesses available to the mining faces
- **Undercut sequence**; the undercut sequence will define the order in which the draw points will be open. This constraint usually acts on the draw point status activating those that are at the front of the production face.
- **Draw point status**; the activation and the closure of a draw point have to be related so the solution is consistent with the actual operation of the mine.
- **Maximum opened production area**; the total number of active draw points at any given time within the production schedule has to be constraint according to the size of the ore body, available infrastructure and equipment availability. A large number of active draw points might lead into serious operational problems such as exceeding optimum haulage distance or extremely maintenance of draw points.
- **Draw rate**; the draw rate will control two aspects of the block cave the first one is the actual flow of muck at the draw point which leads into controlling the caving back and the draw control factor through constraining the maximum and the minimum rate of draw.
- **Period Constraints**; the period constraints forces the mining system to produce the desires production usually keeping it within a range that allows flexibility for potential operational disturbances.

Note that in this formulation mining reserves are not part of the constraints breaking the traditional paradigm of computing mining reserves previously to run the production schedule. In this case the mining reserves will be part of the result of the schedule.

### III.1.1 Objective function

The first step is to compute the profit to be earned per period of the mine schedule. This can be computed as follows

$$P_t = \sum_{i=1}^I d_{it} a_{it} \left[ \sum_{k=1}^K (G_{kit} RF_{kt} MR_k) \right] - MC_{it} - PC_{it}$$

where:

$P_t$  is the profit to be earned in period  $t$ .

$d_{it}$  is tonnage to be drawn from draw point  $i$  in period  $t$ .

$a_{it}$  is a binary variable =  $\begin{cases} 1 & \text{if drawpoint } i \text{ is drawn in period } t \\ 0 & \text{otherwise} \end{cases}$

$s_{it}$  is a binary variable =  $\begin{cases} 1 & \text{if draw point } i \text{ is opened in period } t \\ 0 & \text{otherwise} \end{cases}$

$c_{it}$  is a binary variable =  $\begin{cases} 1 & \text{if draw point } i \text{ is closed in period } t \\ 0 & \text{otherwise} \end{cases}$

$G_{kit}$  is the diluted grade of element  $k$  from draw point  $i$  in period  $t$ .

$RF_{kt}$  is the revenue factor of element  $k$  in period  $t$ .

$MR_k$  is the metallurgical recovery of element  $k$ .

$MC_{it}$  is the mining cost of draw point  $i$  in period  $t$ .

$PC_{it}$  is the processing cost of draw point  $i$  in period  $t$ .

The objective function is to maximize the NPV over all periods  $1 \leq t \leq T$

$$MAX \left[ \sum_{t=1}^T \frac{P_t - v_t DV_t}{(1 + \delta)^t} \right]$$

where:

$\delta$  is the discount rate of one period

$v_t$  is the total number of new draw points to open in period  $t$ .

$DV$  is the development cost of opening a new draw points

### III.1.2 Definition of constraints

#### i. Development rate

$$\sum_{i=1}^I s_{it} = v_t \quad \forall t, 1 \leq t \leq T$$

$v_t \leq \text{New}_t \quad \forall t, 1 \leq t \leq T$ . Note that  $v_t$  is an integer variable and  $\text{New}_t$  is an upper bound for the problem. By construction  $v_t \geq 0$ .

#### ii. Undercut sequence

$$\sum_{k=1}^t s_{ik} \geq s_{i+1t} \text{ guarantees that draw points are opened in sequence.}$$

$$\sum_{t=1}^T s_{it} \leq 1 \text{ guarantees that every draw point is opened just once.}$$

#### iii. Draw point status

To avoid the closure of the draw point before it is opened the following constraint needs to be formulated

$$\sum_{k=1}^t s_{ik} \geq \sum_{k=1}^{t-1} c_{ik}.$$

Also to avoid that a draw point is opened twice the following needs to be added to the set of constraints

$$\sum_{t=1}^T c_{it} \leq 1.$$

The following constraint will link  $a_{it}$  with  $c_{it}$

$$a_{it} = \sum_{k=1}^t s_{ik} (1 - \sum_{k=1}^t c_{ik})$$

If draw point  $i$  has not been opened  $\sum_{k=1}^t s_{ik} = 0$ , which means  $a_{it} = 0$ . Thus the draw point  $i$  will not be part of the schedule in period  $t$ . If  $\sum_{k=1}^t s_{ik} = 1$ , then  $a_{it}$  can either be 1 or 0 depending on the need to use draw point  $i$  in period  $t$  of the schedule. If the draw point  $i$  is not drawn in period  $t$ , then this draw point is closed automatically ( $c_{it} = 1$ )

**iv. Maximum opened production area**

$\sum_{i=1}^I a_{it} \leq A_t$  This constraint controls the maximum open area at any given period of the schedule.  $A_t$  should be given as an input to the algorithm.

**v. Draw rate**

$$d_{it} \leq TU_{it} a_{it}$$

$$d_{it} \geq TL_{it} a_{it}$$

where  $TU_{it}$  and  $TL_{it}$  represent the maximum and minimum draw rate of draw point  $i$  in period  $t$ . This constraint can also be written as  $TL_{it} = TU_{it} / dcf_t$  where  $dcf_t$  represents the desired draw control factor to be used in period  $t$  of the schedule.

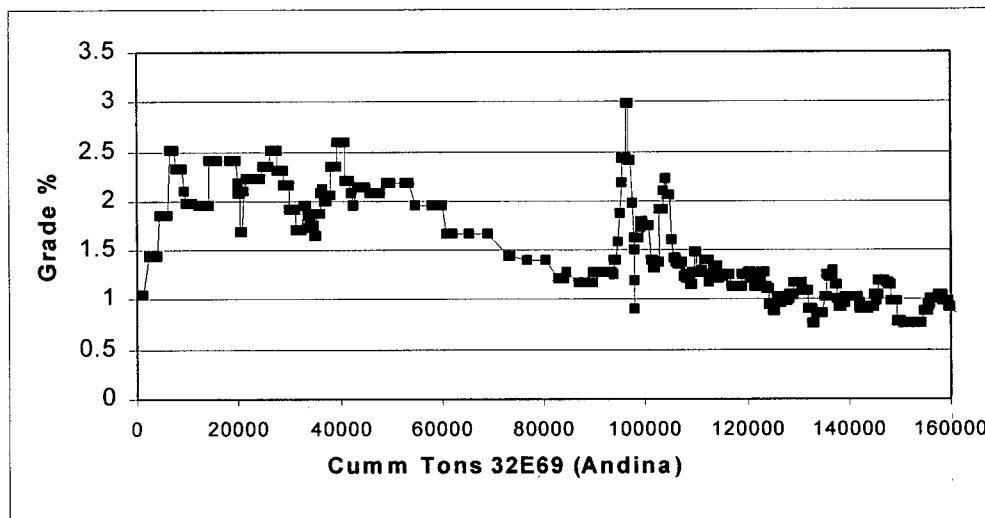
**vi. Period Constraints**

$$\sum_{i=1}^I d_{it} \leq TTU_i$$

$$\sum_{i=1}^I d_{it} \geq TTL_i$$

where  $TTU_i$  and  $TTL_i$  represent the maximum and minimum feasible production targets for the mine.

The objective function in the above problem is non linear since the relation between tonnage and grade along a draw column is non linear. Generally in a draw column tonnage ( $V_{it}$ ) and grade ( $G_{kit}$ ) are coupled by a non linear function given by the initial block model and the mixing process. The following figure shows the resultant relationship between  $V_{it}$  and  $G_{kit}$ .



**Figure 3-1: Grade – tonnage relationship within a draw point ( extracted from Third Panel Andina)**

Note also that in the above formulation there is no relation constraining the total tonnage drawn from a draw point. Therefore mining reserves are not included in the above formulation, since it is believed that these reserves should be an output of the optimization rather than an input.

The above formulation is a mixed integer problem, since it contains integer and real variables. Similar problems in mining have been solved (Guest 2000; Chanda, 1990; Trout, 1995). The objective function is also non linear and the interior point algorithm (Terlaky,

1996) has been used to solve the problem. Consequently the solution algorithm employs an iterative technique to find the optimum solution.

### **III.2 Opportunity Cost Concept in Block Cave Mining**

Maximizing NPV in a block cave mine means choosing which draw points should be drawn in every period, closing those that do not add value. This is equivalent to finding the right mechanism to shut down draw points in a manner that will facilitate NPV maximization. The mechanism used in this research to link NPV maximization with the ability of the draw point to add value. A draw point will stay active, at any given period of the schedule, if it has enough remaining value to pay the financial cost of delaying production from newer draw points that may have a higher remaining value. This cost is a financial cost and is typically called opportunity cost. This concept was first introduced to mining by Lane (1988). The application of the opportunity cost to open pit in mining has demonstrated significant improvements in profitability of mining companies (Four X reference manual, 1998).

The opportunity cost is a fictitious cost that takes into account the delay in earning the future value of the deposit by sending material from a current active area to the mill. In other words it charges to the active draw points an extra cost for the right to use the mill before the draw points that are next in the undercut sequence. Therefore the opportunity cost is a function of the mill capacity as well as the remaining value of the deposit. This opportunity cost should be shared in proportion to the contribution of the draw points to the overall mine throughput.

The opportunity cost can be computed as follows. Assume that the value of the mine is  $V$ , including the infrastructure, processing plant and the ore body. This would be, in a sense, capital which incurs two penalties as a result of being tied up in the operation. One is the interest that could have been earned were it deployed elsewhere, which might be written as  $\delta V$ , where  $\delta$  represents the corporate cost of capital per period. The second penalty is the increase in cost as a consequence of declining metal prices (or the decrease in cost as prices increase), such as metal prices. This component of the opportunity cost is defined as  $-dV / dt$ . Therefore the opportunity cost can be written as

$$OC = \left[ \delta V - \frac{dV}{dt} \right]$$

Note that this cost is a function of time required to process a certain amount of tonnage. Therefore this time will be a function of the plant capacity.

An iterative technique is used to compute the opportunity cost. A starting value is assumed and a complete production plan computed, step by step until resources are exhausted (Lane 1988). The residual present value should be zero; if it is not, the starting value is adjusted and the process is repeated.

It is first necessary to compute the remaining value of the draw point in every period of the schedule. If this value is not high enough to pay the opportunity cost, then the draw point will be closed otherwise it will remain active. Consequently, at the end of the scheduling run for every draw point, the total tonnage drawn will be computed and this will become very close to what the mining reserves are.

There are two main applications of this concept in planning a block cave operation. The first one is the estimation of draw point reserves using opportunity cost. The second is the introduction of opportunity cost in computing the production schedule.

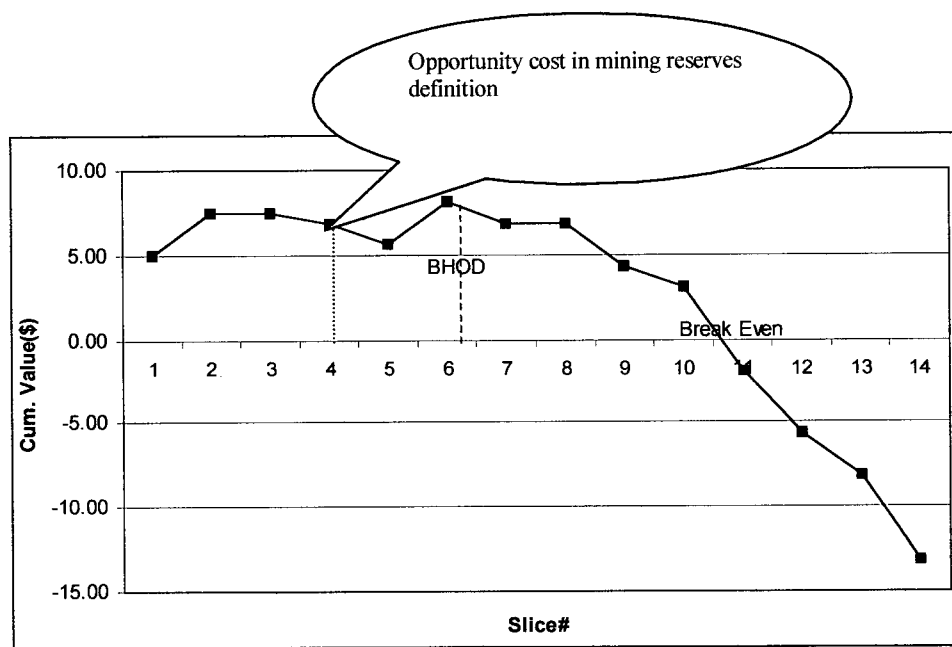
### **III.3 Mining Reserves Definition Using the Opportunity Cost Concept**

It is interesting to note that none of the methods used to compute mining reserves in block caving incorporates the mining sequence or the production strategy in the calculation. Then, for example, if the mine intends to produce 12 million tonnes of ore per year, it will report the same amount of reserves if it were to produce 20 million tonnes. This is clearly not correct because the mining reserves should be a function of the actual mine or plant capacity whichever is smaller. An analogy can better illustrate this concept. For example a store that contains a fixed capacity in terms of volume will carry those goods that provide the highest

benefits, it will not carry goods that would contribute marginally to the business. The same concept applies in mining, where either the mine or plant represents a fixed capacity and the economic ore is evaluated as a function of this capacity. Therefore mining reserves will be those ores that contribute the highest economic value to the operation subject to the available mine or plant capacity. At the moment both the marginal cut off grade and BHOD are computed independent of the rest of the mine planning factors. An important part of this research will investigate at this aspect of current practice. Mining reserves must be coupled with the rest of the decisions and parameters involved in planning a block cave mine.

One way of linking the undercut sequence with the definition of mining reserves is through the incorporation of opportunity cost. This cost basically will introduce the time dimension to the definition of reserves, which at the moment is static. By introducing the concept of opportunity cost, every tonne from a particular draw column will pay an extra cost to have the right to be incorporated as part of the mining reserves. Therefore the total amount of reserves per draw point will be reduced according to its position on the undercut schedule. For instance, a draw point early in the life of the mine the reduction in reserves perhaps might be 40% due to the cost of delaying those draw points located later on the undercut sequence. Similarly a draw point that is positioned at the end of the life of the mine may be reduced 5% because the opportunity of the remaining reserves is little. Figure 3-2 shows the effect of introducing the opportunity cost concept in the definition of mining reserves.





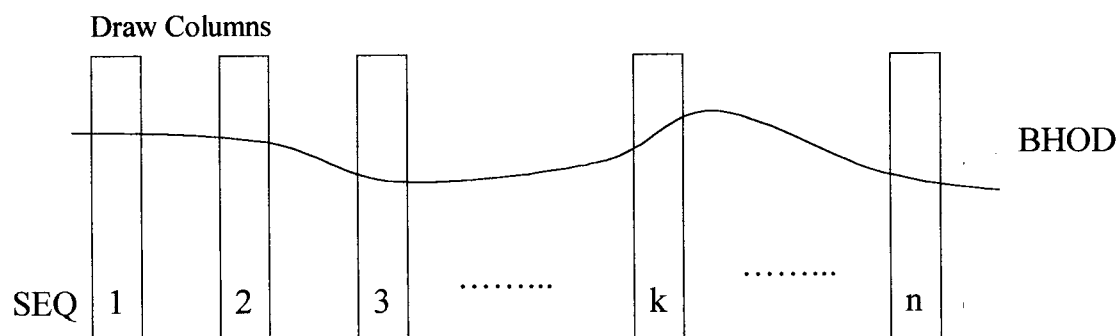
**Figure 3-2: Effect of introducing opportunity cost in defining mining reserves**

### III.3.1 Algorithm to introduce opportunity cost into mining reserves definition

The following notation is used to describe the opportunity cost algorithm in the selection of mining reserves.

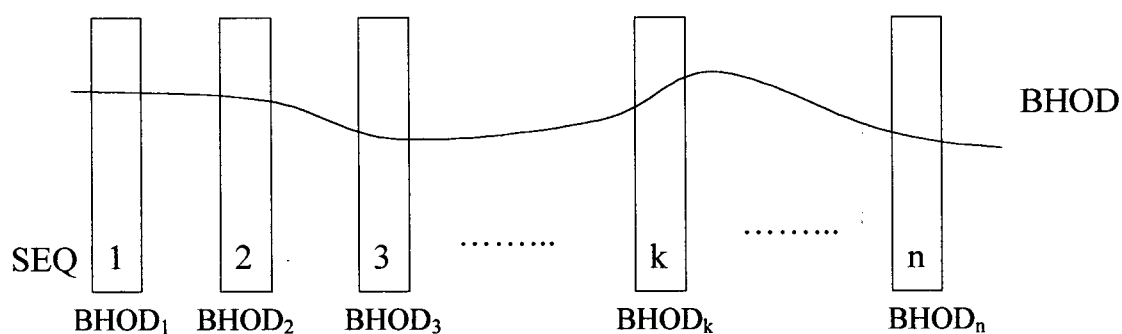
- $j$  = Draw point sequence
- $BHOD_j$  = Best Height of draw of draw point  $j$
- $BT_j$  = Best Tonnes of the draw point  $j$ . Computed from  $BHOD_j$
- $C_j$  = Profit of drawing  $BT_j$  from draw point  $j$  including the opportunity cost
- $V_j$  = NPV of the current extraction (draw points  $j+1$  to  $N$ )
- $\tau_j$  = Time to extract the current draw point  $j$
- $\delta$  = Discount ratio per period
- $F_j$  = Opportunity cost of draw point  $j$
- $M$  = Mine capacity of one period of time in terms of tonnes/time

The first step is to sort the draw points within the economic layout according to the undercut sequence. Figure 3-3 shows the procedure.



**Figure 3-3: Draw points sorted by undercut sequence**

- 1) Assuming that the initial opportunity cost is zero.  $F_j=0$  where  $1 \leq j \leq N$
- 2) Compute  $BHOD_j$  for  $1 \leq j \leq N$ . This follows the regular calculation of BHOD

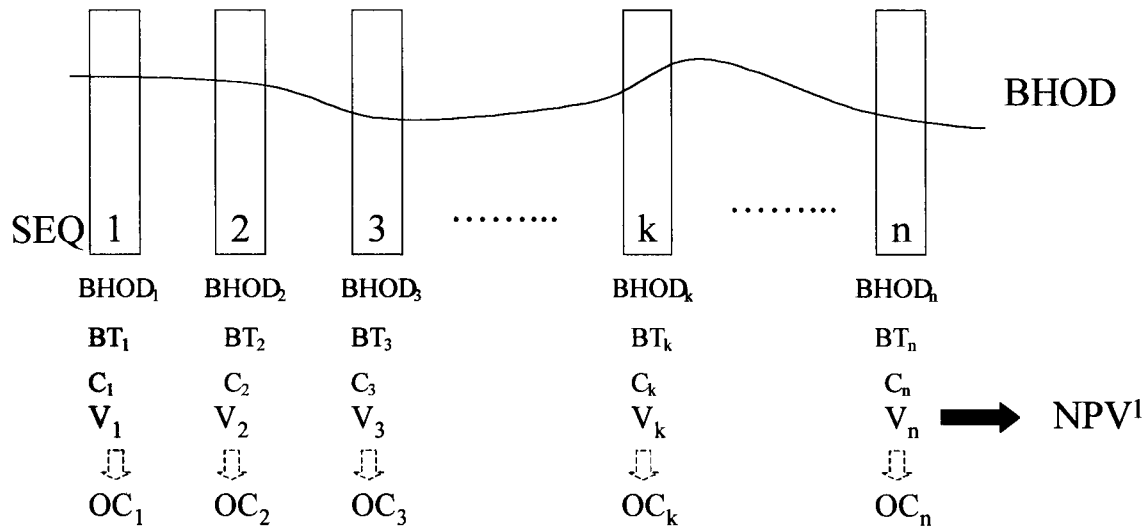


**Figure 3-4: Surface representation of current BHOD calculation**

- 3) Compute  $BT_j$  from  $BHOD_j$ . This transforms draw heights in tonnage to be mined per draw point
- 4) Compute  $\tau_j = \frac{BT_j}{M}$  which is the time that takes to mine draw point  $j$
- 5) Compute  $C_j$  from  $BT_j$ , the profit to be earned by drawing draw point
- 6) Compute actual NPV. Is this NPV equal to the previously calculated NPV, if yes exit, else go 7

- 7) Compute  $V_j = \sum_{k=j+1}^I \frac{C_k}{(1+\delta)^k}$  which represents the remaining value of the deposit
- 8) Compute opportunity cost associated with draw point  $j$ .  $OC_j = \delta V_j \tau_j$
- 9) Add opportunity cost to the production cost (\$/t)
- 10) Go to step 2

The following represents a sequence of calculations to find the optimum draw heights.



**Figure 3-5: Calculation of opportunity cost**

The output of the above algorithm will be as follows:

- The optimum draw heights
- The overall optimised NPV
- The opportunity cost per draw point
- The shut off grade per draw point

Note that this algorithm computes the undercut sequence that yields the best height of draw which therefore gives the mining reserves. This is a significant result because it changes the traditional way of computing reserves and also incorporates scheduling parameters in the definition of height of draw.

Figure 3-4 shows an example of 10 draw points. The draw columns are composed of copper with 15 slices in each of them.

Slice										
15	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
14	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
13	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
12	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
11	0.1	0.1	0.1	0.8	0.8	0.8	0.8	0.1	0.1	0.1
10	0.1	0.1	0.1	0.8	0.8	0.8	0.8	0.1	0.1	0.1
9	0.1	0.1	0.1	0.8	0.8	0.8	0.8	0.1	0.1	0.1
8	0.7	0.7	0.7	0.8	0.8	0.8	0.8	0.1	0.1	0.1
7	0.7	0.7	0.7	1.3	1.3	1.3	1.3	0.9	0.9	0.9
6	0.7	0.7	0.7	1.3	1.3	1.3	1.3	0.9	0.9	0.9
5	2	2	2	1.3	1.3	1.3	1.3	0.9	0.9	0.9
4	2	2	2	1.3	1.3	1.3	1.3	2.3	2.3	2.3
3	2	2	2	1.3	1.3	1.3	1.3	2.3	2.3	2.3
2	2	2	2	1.3	1.3	1.3	1.3	2.3	2.3	2.3
1	2	2	2	1.3	1.3	1.3	1.3	2.3	2.3	2.3
Sequence	1	2	3	4	5	6	7	8	9	10

**Figure 3-6: Sand box grade model**

Using a mining cost of \$8 per block and a revenue factor of \$12/ unit of percentage (metallurgical recovery, metal price and smelting costs), the cumulative value along the draw column was computed. The results are shown in the following table.

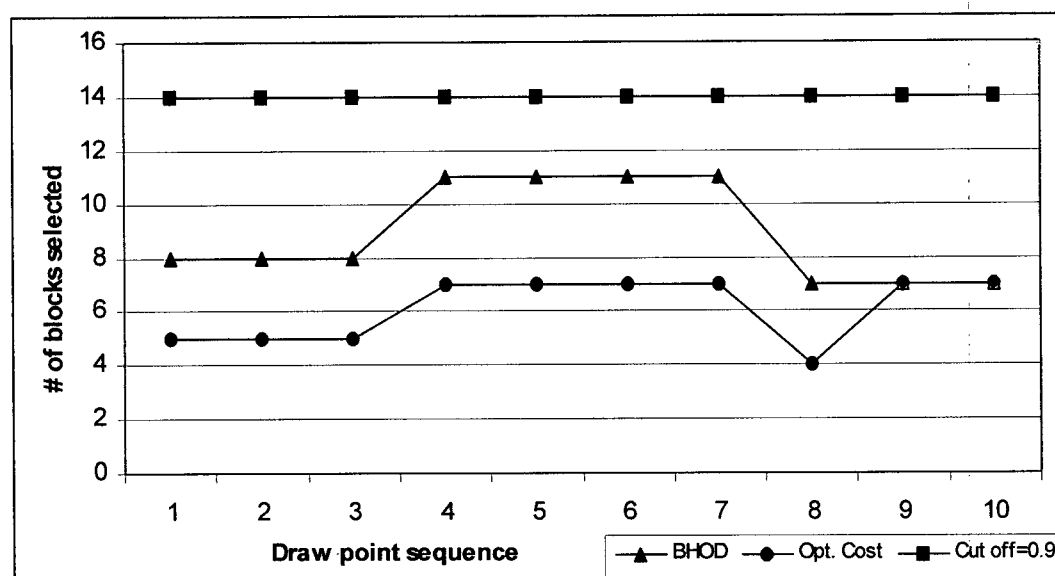
Slice										
15	33.6	33.6	33.6	32.4	32.4	32.4	32.4	32.4	32.4	32.4
14	40.4	40.4	40.4	39.2	39.2	39.2	39.2	39.2	39.2	39.2
13	47.2	47.2	47.2	46	46	46	46	46	46	46
12	54	54	54	52.8	52.8	52.8	52.8	52.8	52.8	52.8
11	60.8	60.8	60.8	59.6	59.6	59.6	59.6	59.6	59.6	59.6
10	67.6	67.6	67.6	58	58	58	58	66.4	66.4	66.4
9	74.4	74.4	74.4	56.4	56.4	56.4	56.4	73.2	73.2	73.2
8	81.2	81.2	81.2	54.8	54.8	54.8	54.8	80	80	80
7	80.8	80.8	80.8	53.2	53.2	53.2	53.2	86.8	86.8	86.8
6	80.4	80.4	80.4	45.6	45.6	45.6	45.6	84	84	84
5	80	80	80	38	38	38	38	81.2	81.2	81.2
4	64	64	64	30.4	30.4	30.4	30.4	78.4	78.4	78.4
3	48	48	48	22.8	22.8	22.8	22.8	58.8	58.8	58.8
2	32	32	32	15.2	15.2	15.2	15.2	39.2	39.2	39.2
1	16	16	16	7.6	7.6	7.6	7.6	19.6	19.6	19.6
Sequence	1	2	3	4	5	6	7	8	9	10

**Figure 3-7: Sand box cumulative profit along the draw column**

A comparison of the traditional way of computing mining reserves and the introduction of opportunity cost into this definition is shown in figure 3-8.

Table 3-1 summarises the results of using different methodologies to compute mining reserves. It is relevant to observe that the maximum NPV obtained by the application of the

opportunity cost concept is the largest. It is also important to note that the NPV maximization reduces the amount of blocks to be mined, which shortens the life of the mine.



**Figure 3-8: Height of draw comparison using different techniques to compute mining reserves**

Sequence	BHOD	Opt. Cost	Cut off=0.9
1	8	5	14
2	8	5	14
3	8	5	14
4	11	7	14
5	11	7	14
6	11	7	14
7	11	7	14
8	7	4	14
9	7	7	14
10	7	7	14
<b>Blocks</b>	<b>89</b>	<b>61</b>	<b>140</b>
<b>NPV</b>	<b>327.6</b>	<b>404.9</b>	<b>175.2</b>

**Table 3-1: NPV comparison using different method to compute mining reserves**

The presented algorithm had first been introduced by De la Huerta (1994) and it has been extensively used at Codelco's mines in Chile. Nevertheless there are several assumptions that make this algorithm inapplicable in block caving mines. Some of the assumptions are:

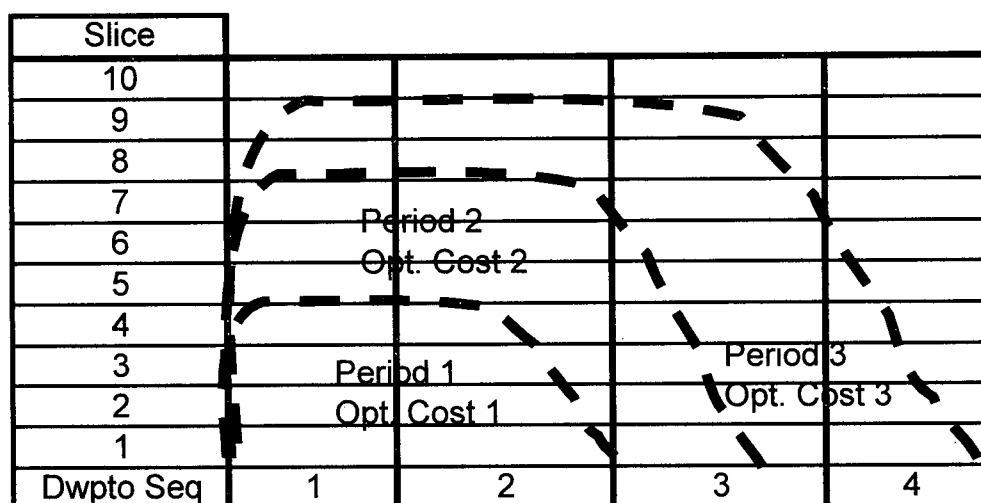
- 1) A constant economic scenario is assumed within a period

- 2) The opportunity cost is unique for the entire column, since it does not change by slice.
- 3) The resulting draw heights are usually not smoothed. This is a problem for posterior draw control
- 4) Draw rate per draw point is not a constraint. This means that every draw point is drawn until it is exhausted
- 5) It is not coupled with the production schedule. Thus just a few of the constraints of the real problem are used.

The application of opportunity cost in the long-term production schedule is slightly different to the application used to select mining reserves. Probably the major difference is related to using the draw rate as a constraint to define the increments of tonnage of every draw point in a period of the schedule. Another difference is that every slice can have a different opportunity cost, depending on the location of the slice in the production plan, due to the fact that the draw rate curve constrains the number of slices included from a draw point in a specific period. Therefore every period of the production schedule has a singular opportunity cost, which represents the cost of delaying the current NPV in one period.

#### **III.4 NPV Optimization by Introducing the Opportunity Cost in Computing the Production Schedule**

To apply the opportunity cost in the computation of the production. Mining reserves are computed in every period of the mine schedule. The best height of draw calculation will shut those draw points that do not have enough remaining value in their draw column. Then only those draw points that contain the highest value will be remain opened. Consequently the active draw points will be drawn according to their draw function to drive the NPV to its maximum point. The opportunity cost is introduced in the cost structure as a fictitious cost.



**Figure 3-9: Variation of opportunity cost as the schedule progresses**

Figure 3-9 shows how a draw point may perhaps have a different opportunity cost, depending on when the draw point is drawn within the schedule. The lines in the figure represent how the draw profile progresses along with the production schedule. There are draw points being drawn in more than one period and those draw points will have a different opportunity cost depending on when they are drawn.

In any given period of the schedule BHOD is run to determine the uneconomic draw points and close them. As result of this integration, mining reserves will fulfil the constraints that define the scheduling problem. This methodology breaks the paradigm of first computing mining reserves prior to run production schedule by computing reserves as a result of the mine schedule.

The draw function used to optimize NPV draws the maximum allowable tonnage from the actual active area at any given time within the schedule. If there is extra capacity the newest draw points will be set as idle. If there is not enough capacity to achieve the production target, the production strategy will bring the maximum allowable according to the draw rate curve.

#### **III.4.1 Algorithm to determine optimum mine schedule**

The following steps show how the algorithm works

1. Compute  $BT_i$  for  $1 \leq i \leq I$ .  $BT_i$  should pay at least the mining costs and the opportunity cost of period  $t$  ( $OC_t$ ). For  $t=1$  the opportunity cost would be zero.
  - If  $(BT_i + V_{it}) < MINT_i$  Then  $BT_i = MINT_i - V_{it}$  where  $MINT_i$  is the minimum allowable tonnage to be drawn from draw point  $i$ . This guarantees that the in situ draw column will be broken without leaving pillars behind the active zone.
  - If  $BT_i = 0$  then  $c_{it} = 1$ . If the draw point does not have enough remaining value it is closed.
  - If  $BT_i = F(BT_j)$  where  $F$  is a function of the neighbors  $j$  of draw point  $i$ . This procedure regulates the height of draw, for draw control purposes. Usually this process is called draw height smoothing.
2. Incorporate new draw points according to the given undercut sequence, add  $New_t$  to the production plan to reach production target.
3. Assign tonnage to the draw points according with the maximum and minimum draw rate. If there is an extra capacity the newest draw points are flagged as idle status. If there is not enough capacity the tonnage target constraint is broken. In this step the draw function is used to compute the correct tonnage according to the previously specified constraints.
4. Deplete assigned tonnages from the draw column model and update the model.
5.  $t = t + 1$ , return to 1

Once an entire schedule has been run, the output of the first iteration would be as follows:



Period	Planned	Active	Closed	New	Idle	Current tons	Remaining	Requested	\$Value
1	7	3	0	3	0	183050	2816557	183050	16.0
2	5	5	0	2	2	183050	2633507	183050	16.0
3	3	7	0	2	4	183050	2450457	183050	10.0
4	1	8	1	2	6	183050	2267407	183050	3.3
5	0	8	2	1	5	183050	2084357	183050	5.4
6	0	7	3	0	4	183050	1901307	183050	7.0
7	0	7	3	0	4	183050	1718257	183050	7.6
8	0	7	3	0	4	183050	1535207	183050	5.8
9	0	7	3	0	4	183050	1352157	183050	3.0
10	0	5	5	0	3	183050	1169107	183050	4.3
11	0	5	5	0	2	183050	986057	183050	7.8
12	0	4	6	0	1	183050	803007	183050	9.1

**Table 3-2: Data after first iteration of the production schedule**

Table 3-2 is explained as follows:

Period	Period number from 1 to $T(t)$
Planned	Number of draw points planned in period $t$ ( $Planned_t$ )
Active	Number of draw points active in period $t$ ( $Active_t$ )
Closed	Number of draw points closed in period $t$ ( $Closed_t$ )
New	Number of draw points new draw points in period $t$ ( $New_t$ )
Idle	Number of draw points without extraction in period $t$ ( $Idle_t$ )
Current tons	Tonnage drawn in period $t$ ( $d_t$ )
Remaining	Remaining Reserves at period $t$ ( $Limit_t$ )
Requested	Tonnage target for period $t$ ( $TTU_t$ )
\$Value	Average revenue per ton in period $t$ ( $\$Value_t$ )

There are several parameters that need to be computed after this first iteration. A list of parameters as well as the procedures as follows:

- $Cum\_new_t$ : represents the cumulative number of draw points open up to date

$$Cum\_new_t = \sum_{k=1}^t v_k$$

- $Used\_new_t$ : represents the number of draw points used in the current period. It can be read as the optimum number of draw point to be opened.

$$Used\_new_t = Cum\_new_t - Idle_t - \sum_{k=1}^{t-1} Used\_new_k$$

- Revenue per period:  $R_t = \$Value \cdot d_t$
- Development cost per period:  $DV_t = v_t \cdot DV$ ,  $DV$  being the development cost per draw point
- Profit per period:  $P_t = R_t - DV_t$
- Remaining value: represents the remaining value of the mine at any given time within the production schedule

$$V_t = \sum_{k=t+1}^T \frac{P_k}{(1+\delta)^{k-t}}$$

- Opportunity cost per period: This cost will be represented in units of currency per ton and it represents the financial cost of tightening the actual value of the deposit and the fixed costs to the actual production strategy

$$OC_t = \frac{(V_t + f_t)\delta}{TTU_t}$$

where  $f_t$  represents the fixed costs of the mine in period  $t$ .

In this research  $f_t$  has been taken as zero. However this is not strictly true, because when the mine is not producing there are still costs to cover such as manpower, reclamation costs, maintenance, etc.

An example of the above calculations is shown in Table 3-3.

cum-new	Used-new	Revenue	Development	Profit	Vt	Oct
3	3	2,921,512	0	2,921,512	9,144,726	5.00
5	0	2,921,512	0	2,921,512	7,137,687	3.90
7	0	1,836,735	0	1,836,735	6,014,720	3.29
9	0	606,151	0	606,151	6,010,041	3.28
10	2	980,556	0	980,556	5,630,489	3.08
10	1	1,281,015	0	1,281,015	4,912,523	2.68
10	0	1,395,480	0	1,395,480	4,008,296	2.19
10	0	1,057,184	0	1,057,184	3,351,941	1.83
10	0	540,757	0	540,757	3,146,379	1.72
10	1	783,117	0	783,117	2,677,899	1.46
10	1	1,435,534	0	1,435,534	1,510,156	0.82
10	1	1,661,171	0	1,661,171	0	0.00

a	0.1
DV	0
Cap	183,050
NPV	10,969,306

**Table 3-3: Calculations needed to perform the optimisation**

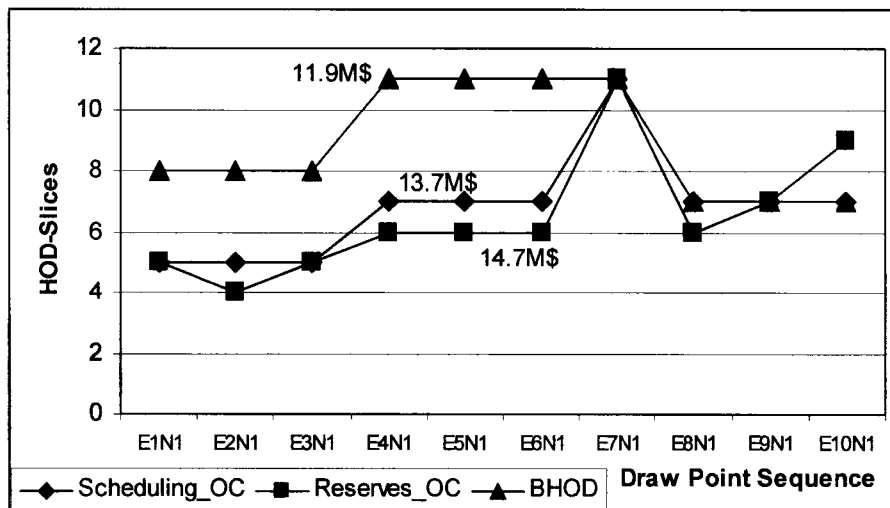
Note that the above example shows the column development with 0s. This is because DV was set as 0 in this example.

- 1) Is the current NPV close to the last NPV?. If yes stop. Otherwise return to 1) this time including the opportunity cost ( $OC_t$ ) in the mining cost structure according with the period of the schedule. This means that the BHOD will be reduced this time due to the effect of the opportunity cost.

Every iteration will produce a different opportunity cost per period. Thus  $OC_t^k$  would represent the opportunity cost of period  $t$  for the iteration  $k$ . Denoting the optimum iteration  $z$ ,  $OC_t^z$  would represent the optimum opportunity cost policy that would produce the optimum set of tonnage and grades per period.  $V_{IN}^z$  would represent the optimum reserves (tonnage) to extract from the draw point  $i$ . Consequently the above algorithm integrates the production schedule and the mining reserves optimisation in a single task within the production planning process. This result is fairly significant considering the fact that traditionally these two processes have been computed independently of each other.

An example of the above algorithm is shown in the following figure for the same 10 draw points that have been used in this chapter.

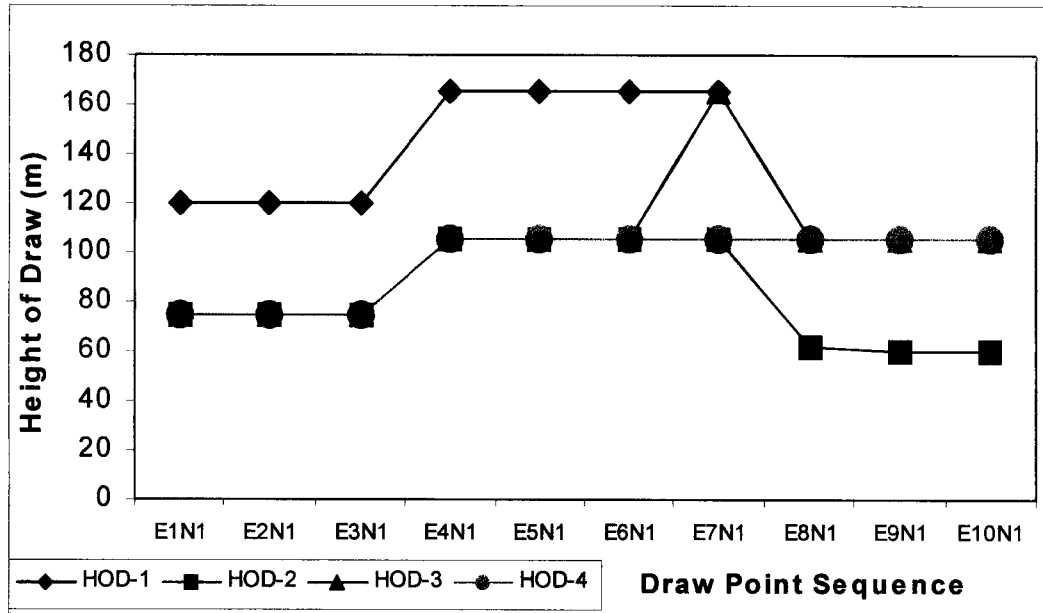
Figure 3-10 shows the different height of draw as a function of the method used to compute mining reserves. It is noticed that the uses of opportunity cost in the production schedule reduces the amount of ore compared to the definition of reserves given by the traditional BHOD. However compared with the reserves definition using opportunity cost the amount of ore is larger. This effect is due to the smoothing process on the heights of draw produced by draw control constraints defined within the production schedule algorithm. The smoothing is due to the fact that all the draw points should fulfill a certain draw rate and a ratio between its minimum and maximum value



**Figure 3-10: Comparison between methods to compute mining reserves in Block Caving**

The resulting NPV however decreases compared with that computed based on reserves optimization. Again this effect is due to the draw function that governs the tonnage relations among draw points. Then the difference in NPV between Reserves\_OC and Scheduling\_OC should be understood as the cost of having proper draw control practices. As the draw control practices are relaxed within the scheduling, this cost may become very low.

Figure 3-11 shows progressively how the algorithm converges to the maximum NPV with respect to heights of draw.



**Figure 3-11: Comparison between methods to compute mining reserves in Block Caving**

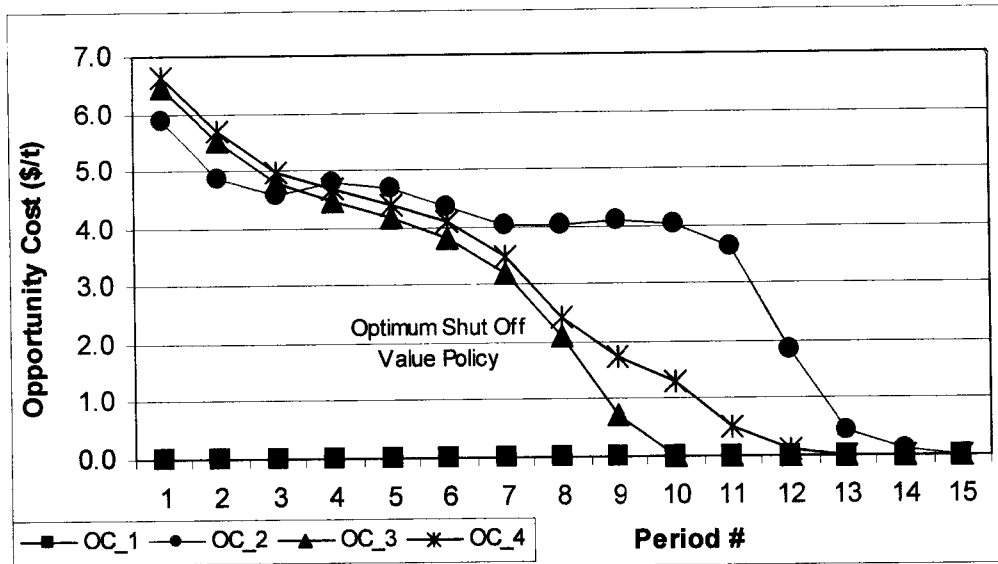
One of the main results that can be derived from the Figure 3-11 is the fact that draw point number 7 is overdrawn in the optimum strategy. Usually this would be an inappropriate draw control practice. Although the ratio between the height of draw of draw point 7 and its neighbours is 1.6 (160/100), this usually is acceptable since the maximum acceptable ratio is 3. If a smoother height of draw profile is desired then the draw function should be changed to overcome this. The following table shows a more detailed report of the optimisation process.

Height of Draw (m)	HOD-1	HOD-2	HOD-3	HOD-4
E1N1	120	75	75	75
E2N1	120	75	75	75
E3N1	120	75	75	75
E4N1	165	105	105	105
E5N1	165	105	105	105
E6N1	165	105	105	105
E7N1	165	105	165	105
E8N1	105	62	105	105
E9N1	105	60	105	105
E10N1	105	60	105	105
<b>NPV (\$ 000)</b>	<b>12,409</b>	<b>13,396</b>	<b>13,676</b>	<b>13,671</b>
<b>Grade (%Cu)</b>	<b>1.34</b>	<b>1.69</b>	<b>1.53</b>	<b>1.58</b>
<b>Reserves (tons)</b>	<b>2,865,670</b>	<b>1,721,805</b>	<b>2,160,732</b>	<b>2,029,028</b>

**Table 3-4: Detailed optimization output report**

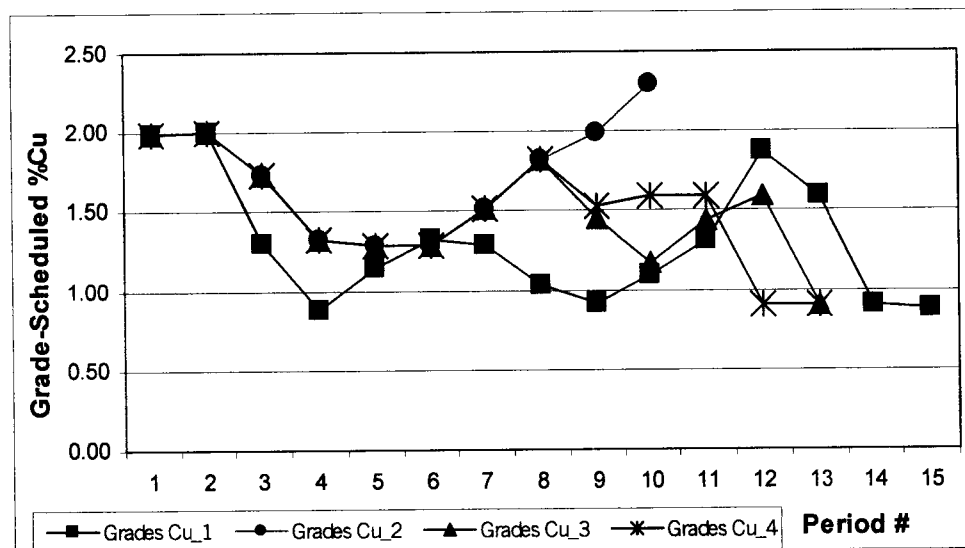
In the above table HOD-1 represents the base case scenario and the HOD-3 is the final optimised heights of draw (or mining reserves). There is a 10% of improvement between the base case scenario and the optimised scenario. The amount of NPV improvement would be a function of how constrained the problem is and if there is extra capacity to select the draw points to be drawn in every period. Since in a fully constrained scenario where the production capacity is not reached due to draw rate or minimum tonnage constraints, a very small amount of improvement can be expected because the whole layout will be kept open to try to reach the mine production capacity. Consequently no draw points will be closed in advance therefore there will be no flexibility to vary shut off grade within the production schedule.

Figure 3-12 shows the evolution of the opportunity costs throughout the production schedule optimisation. Note the draw points shut off value is higher in the first periods of the mine schedule. This is because of the remaining value of the mine at the beginning of the schedule is higher than at the end of its life. As a result of applying opportunity cost in the production schedule the life of the mine is shortened although its NPV is maximized. Also it is interesting to note that the value of the opportunity cost does not decrease in a linear manner, which means that it is not possible to initially guess the geometry of this curve. In fact the shape of the opportunity cost curve would reflect the grade distribution in the deposit. Then the shape of the opportunity cost curve would reflect any potential benefit on changing undercutting sequence. For instance if the opportunity cost curve has any picks means that those areas should be mined in early in the sequence. Therefore a right undercut sequence would produce a smooth opportunity cost curve.



**Figure 3-12: Evolution of opportunity cost through the production schedule for different iterations**

Every iteration of the NPV maximization algorithm computes a different production schedule that forecasts tonnages, grades, area to open, etc. Figure 3-13 represents the output of these schedules as the iterations progress.



**Figure 3-13: Production schedule outputs for NPV optimization**

Note that the optimum schedule (Grades Cu\_3) gives the smoothest grade profile during the life of the mine. This has several implications from the operations point of view since it is

always desire to minimize the grade variations between periods. Another characteristic of this figure is the fact that this algorithm does not always maximize the grade; it maximizes the value of the deposit. For example in period 10 the optimum strategy says that the average grade has to be 1.2 %Cu (Grades Cu\_3). However scheduled Grades Cu\_4 says that in that specific period the grade should be 1.55 %Cu. The problem here is that iteration 4 looks at only the current status and it does not see that by maximizing the grade in period 10 the grade in period 12 drops below 1%. Basically this is because in period 10 several drawpoints were shut and there was not enough remaining ore to keep the grade up until the end of the life of the mine. This demonstrates that this algorithm links the periods and responds in a dynamic way to the changes of grade and general mining constraints occurring during the mine schedule.

Another interesting result is seen in the development rate. The algorithm also gives the development rate because it has been coupled with the problem through the draw function. Table 3-5 shows one of the outputs of the algorithm that estimates the number of draw points needed to be open per period.

Development rate	Used-new_1	Used-new_2	Used-new_3	Used-new_4
1	3	3	3	3
2	0	0	0	0
3	0	2	2	2
4	1	1	1	1
5	1	0	0	0
6	1	1	1	1
7	0	1	1	1
8	0	1	1	1
9	0	1	0	0
10	2	0	1	1
11	0	0	0	0
12	2	0	0	0
13	0	0	0	0
14	0	0	0	0
15	0	0	0	0

**Table 3-5: Development rates as a result of the optimization process**

As expected, the first iteration gives the most inexpensive plan in terms of development of the new area. Note that the second iteration opens a draw point in period 9 which will generate an extra development cost, which is why this iteration does not give the optimum strategy. The third iteration does not open the draw point in period 9, sacrificing the grade but saving costs that might end up affecting the overall NPV.



The development cost can be incorporated into the optimization through the calculation of the profit per period which contains the revenue minus the development cost and fixed costs (which have been considered to be 0 in this research). Then when the opportunity cost is calculated, the development cost is included. The effect will be a reduction in the opportunity cost per period allowing mining to a greater height of draw.

One important simplification made to the problem has been the definition of production targets per period as hard constraints, so that the overall mining throughput is not a variable. Usually if the production target is known and the draw rates are known then the number of required active draw points can be estimated using the draw function. If the number of active draw points is not enough, then new draw points will be opened to meet the required production. This simplification assumes that the chosen tonnage target is known and facilitates NPV optimization. Traditionally the method of introducing this variable in the optimization of NPV is by trial and error. Consequently, several production targets are simulated and the one that provides the maximum NPV is chosen.

Until now this research has assumed the metal price is constant throughout the production schedule. This is not realistic and there is the need to introduce this factor into the scheduling process. The following section will discuss the introduction of a variable metal price into the optimization process.

#### ***III.4.2 The effect of variable price on the NPV optimization***

In the previous section a complete description on how to optimize the NPV associated with the production schedule of a mine was presented. However, no account was taken of prices and their effect on the NPV optimization. This section describes a method to deal with quantifying the impact of introducing a variable metal price on the overall mine schedule.

Until now the formula used to compute the opportunity cost has been

$$OC_t = \frac{\delta V_t}{TTU_t}$$

where the fixed costs have been omitted. The above formula does not take into account the fact that a deposit can lose or gain value if economic conditions change from one period to another. For example if we are facing a rise in metal prices, then it may be more appropriate to wait for prices to recover. On the other hand, while waiting for increased prices the deposit is losing value in delaying its operation. Thus there is thus a trade off between the incremental value gained by economical external changes and by opportunity cost.

There are always two costs associated with any investment decision. The first one relates to loss of the opportunity to invest the same amount in a better alternative. There will be always an alternative to invest. The second relates to economic change and its effect on the value of the investment. In order to account for economic change it is necessary to add a new term into the opportunity cost formulation (Lane 1988). The following formulation shows the entire opportunity cost:

$$OC = \tau \left[ \delta V - \frac{dV}{dt} \right]$$

The component  $dV / dt$  takes into account the increment or decrement of value of the deposit due to economic change.  $\tau$  has units of 1/tonnes representing the amount of tonnage to be mined in one period of time, usually  $\tau$  will be equal to 1/mine capacity.

There are a few parameters that need to be defined to formulate the integration of  $dV / dt$  into the optimization algorithm, as follows:

$RF_t$  is the revenue factor per period.

$MC_t$  is the mining cost per period including the processing cost.

$R_t$  is the revenue earned in period  $t$ .

$R'_t$  is the revenue that will be earned if the project is delayed in one period of time. It takes economic parameters from  $t+1$  period.

$P_t$  is the profit earned in period  $t$ .

$P'_t$  is the profit using  $R'_t$ .

$V_t$  is the future value of the mine at the end of year  $t$ .

$W_t$  is the future value of the mine at the end of year  $t$  using economic parameters of year  $t+1$ .

$G_t$  is the grade given by the production schedule.

The calculations proceed as follows:

$$R_t = d_t(G_t \cdot RF_t - MC_t)$$

$$R'_t = d_t(G_t \cdot RF_{t+1} - MC_{t+1})$$

$$P_t = R_t - DV_t$$

$$P'_t = R'_t - DV_t$$

$$V_t = \sum_{k=t+1}^T \frac{P_k}{(1+\delta)^k}$$

$$W_t = \sum_{k=t+1}^T \frac{P'_k}{(1+\delta)^k}$$

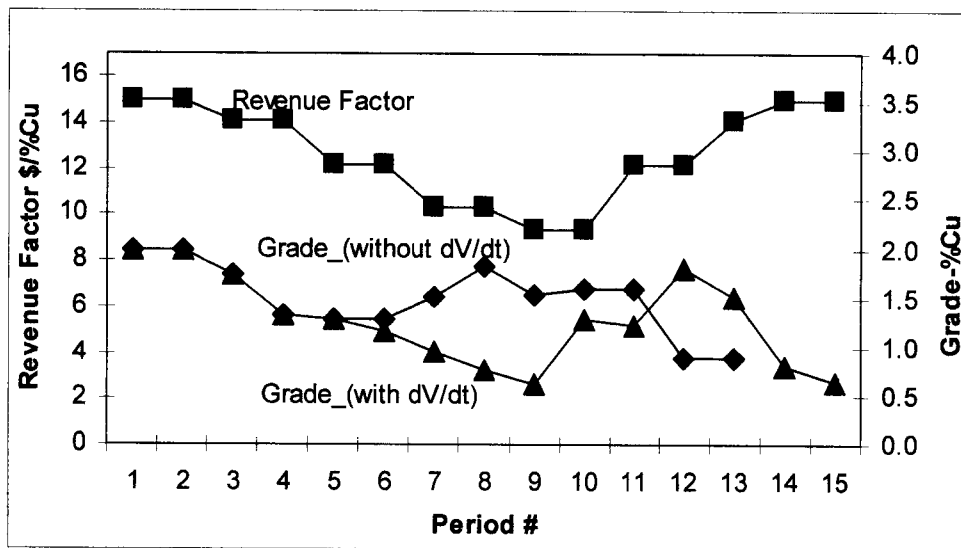
Then the factor  $\frac{dV}{dT} = W_t - V_t$  is known and the opportunity cost per period can be computed as follows:

$$OC_t = \frac{\delta V_t - (W_t - V_t)}{TTU_t}$$

Figure 3-14 shows how the revenue factor can vary through the life of the mine together with the opportunity costs, with and without economic variability. The calculations were based on only one iteration of the algorithm. It is interesting to note that the opportunity cost without incorporating economic variability does not bear any relation with the revenue factor or metal price. In contrast, the opportunity cost including economic variability has a direct correlation with the revenue factor.

Looking at Figure 3-14, it is possible to conclude that when prices decrease the grade will decrease accordingly, which means that less value would be needed for the draw column to stay active in production. Consequently, a lower grade will be mined and therefore less

metal will be produced. This appears to be appropriate in economic terms: if the price of goods decreases because there is a reduction in its demand, then the offer should decrease according to the actual demand. The reverse scenario is also valid from the graph. When prices are increased, due to increased demand in the market, then the opportunity cost also will increase. Consequently draw columns with low grade remaining will be shut and the overall grade of the period will increase. Therefore the metal production will increase in response to the increased demand.



**Figure 3-14: Opportunity cost curve with and without incorporating economic changes**

One important result of introducing economic changes on the opportunity cost is that it makes the algorithm more efficient in terms of optimization. Since the opportunity cost now has been coupled with metal prices, eventually the algorithm will allow mining larger columns. Therefore the overall reserves may perhaps increase as a result of reducing the shut off grade in periods when prices are low. Table 3-6 shows a comparison between NPV optimization through opportunity cost with and without economic changes.

It is clear that the impact of the change in value due to economic change is significant. The algorithm without the differential of value with time does not reproduce a realistic scenario because it does not account for the relation between shut off grade and metal price. On the

contrary, the algorithm with the differential of value on time does couple price and grade, which does avoid any contradiction with the fundamental economic theory.

Draw Point Name	HOD_OC	HOD_OC-dV/dT
E1N1	75	75
E2N1	75	75
E3N1	75	75
E4N1	105	216
E5N1	105	191
E6N1	105	205
E7N1	105	186
E8N1	105	172
E9N1	105	126
E10N1	105	122
<b>NPV 000\$</b>	<b>14,965</b>	<b>15,047</b>
<b>Reserves</b>	<b>2,029,028</b>	<b>3,093,550</b>

**Table 3-6. Heights of draw comparison using a with and without economic changes algorithm**

Although the above result is significant from the theoretical point of view, in reality there are no reliable tools to predict the metal price. Thus evaluating the applicability of the above result is not feasible since it depends on future metal price behaviour. However, through simulation (Smith, 1999) it is possible to reproduce a set of metal price scenarios. The NPV optimization should then be run for each of them. Then a statistical analysis could be performed on the results of the optimization, producing a distribution of production schedules.

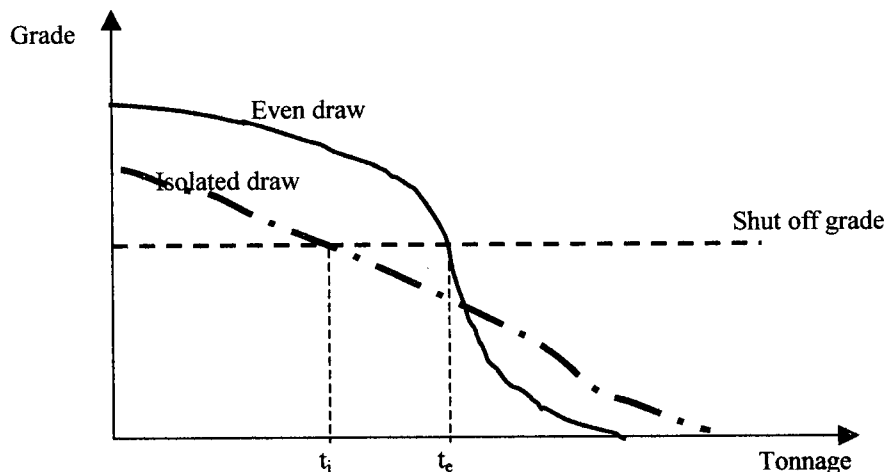
#### IV MINE LIFE OPTIMIZATION IN BLOCK CAVE MINING

Several mines around the world are looking to maximize the life of their operations. Some of the arguments for this approach are: delay the mine closure, so enough cash can be produced to commit to the mine closure, provide social benefits, facilitate enough time to undercut an underlying deposit. Therefore there is a need to define an algorithm that will enable mine planners to compute production plans that can drive the operation towards maximum mine life.

In this research maximizing the life of the mine has been understood as maximizing the quality of the draw profile throughout the life of the mine. This means maximize the uniformity of the draw during the life of the mine. By drawing uniformly at least three effects can be observed at the mine, such as less dilution at the draw points (Laubscher, 1994), reduced stresses (Bartlett, 2000) and finer fragmentation at the draw points. The relation between even draw and percentage of dilution entry has been researched extensively by Laubscher (1994) and Kvapil (1965). The main result in this area is that as the draw becomes more even, the dilution entry at the draw point is retarded. Therefore more tonnage can be drawn without dilution. The relationship between stresses behaviour and isolated draw condition has been observed clearly in the field specifically at Premier mine in South Africa (Bartlett, 2000) and Andina mine in Chile. Typically where as the draw becomes isolated the frequency of collapses increases. Isolated draw is usually practiced to increase production and reach production targets. This situation translates into compaction occurring at these draw points, consequently stresses are transmitted to the production level leading to the collapse of the production drift. Finally the even draw condition relates to fragmentation through the draw rate or draw point productivity. Usually to perform even draw it is necessary to reduce the effective draw rate which leads to leaving the caved material a longer time within the muck pile overlying the production level. Consequently every piece of rock will be exposed a longer time to interact with the rest of the muck pile, increasing friction and therefore the particle size will be reduced significantly.

Although the relationship between draw control practices and mine life has been accepted within the mining community, very little has been done to translate this concept into

production plans. Traditionally draw control has been left to the operators, which usually are driven by a different objective which is to maximize short-term production. This approach is a problem in all mining methods, not only in block caving. However it is believed that in block caving the practice of an erroneous draw strategy can easily end with the early sterilization of the mine due to dilution or more typically due to stresses that damage the production level. Figure 4-1 shows the effect of different draw status on the grade of a draw point. It can be observed from this figure that by applying an isolated draw condition the tonnage of the draw point may perhaps increase by diluting reserves. However the resulting grade along the draw column would decrease due to the dilution effect, assuming that waste cap contains a lower grade than the ore draw column. Eventually this grade might be below the shut off grade, which will lead into either a non economic draw point or a short height of draw ( $t_i < t_e$ ). Consequently this draw point will be closed early in its life, therefore shortening the overall life of the mine.



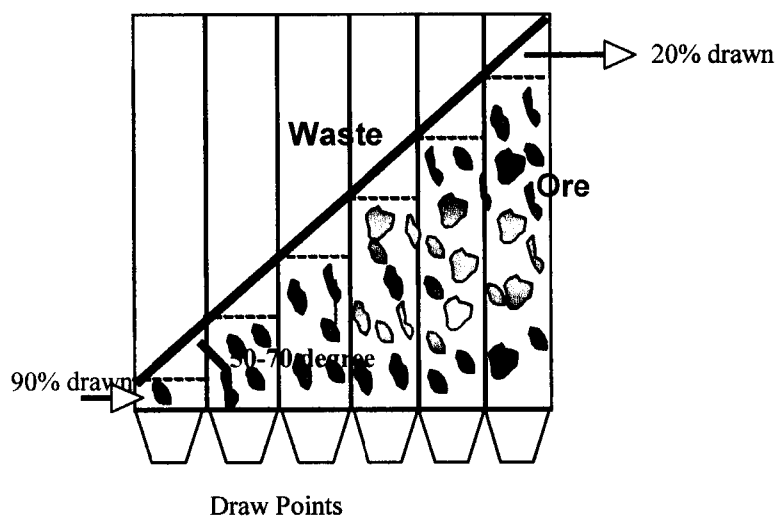
**Figure 4-1: Scheme of dilution behaviour for different draw strategies**

In this research draw control is not regarded as an operational practice or a given factor, but rather as a variable that can be driven or managed in the long term plan as a result of an optimization algorithm. There are at least two ways of introducing draw control into the performance of production scheduling. The first one is to control the draw profile forcing it

to follow a specific draw angle. The second one is to apply tonnage constraints on the active draw points to reach a uniform draw condition.

#### IV.1 Draw angle and even draw condition

The angle of draw concept was first introduced for panel caving mines where there is a moving caving front that opens a production area as needed to reach production targets. By having an angle of draw, then the contact between ore and waste is controlled thus reducing the likelihood of encountering waste in new open draw points. The concept is shown in the Figure 4-2.

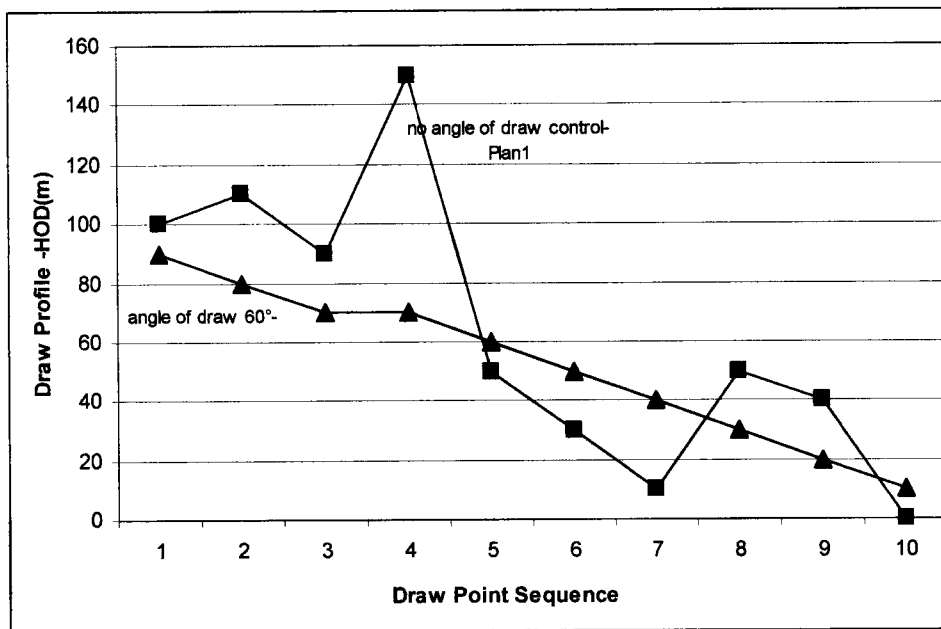


**Figure 4-2: Angle of draw concept**

The angle of draw can also be represented as the relation between the amount of new area to open and the current height of draw. Figure 4-3 shows the heights of draw during one period of the production schedule using two different long-term strategies. Plan 1 does not use an angle of draw to constrain the draw profile for a given period of the schedule, so that a lower dilution entry point (DEP) can be expected. Plan 2 uses an angle of draw of 60 degrees to constrain the draw profile and a later DEP can be expected. Steeper draw angles exhibit a lower DEP than flatter angles, according to operational experience. However a flatter draw angle generally leads to more expensive plans in that more area needs to be developed to produce the same tonnage. Thus the fact of opening more area needs to be considered on the



final economic evaluation of this method. The intent will be to balance the amount of area to open to achieve a correct draw performance with the capital cost involved in this decision.



**Figure 4-3: Height of draw for different long term strategies**

#### ***IV.1.1 Mathematical formulation***

The mathematical formulation for the angle of draw optimization is based on minimize the deviation of the actual draw profile given by the actual height of draw and a plane or surface that will represent the desired angle of draw. This angle needs to be calibrated for every mine because it might be the case where an angle of draw of 70 degrees represents a DEP equal to 70% and for a different mine the same angle might represent a DEP of 40%.

Therefore the relationship between angle of draw and DEP is not unique and it will depend on the gravity flow parameters such as differential fragmentation, shape of fragments, angle of repose of material, caving rate. The variable involved in the objective function are the tonnage to be drawn from the draw points and status of the draw point that shows whether a draw point is active or not. The following constraints are also part of the problem formulation:

- **Development rate**; the development rate controls the maximum feasible number of draw points to be open at any given time within the schedule horizon. This constraint is usually based on the geometry of the ore body and the infrastructure of the mine, which typically will define the number of access points available to the mining faces.
- **Undercut sequence**; the undercut sequence will define the order in which the draw points will be open. This constraint usually acts on the draw point status activating those that are at the front of the production face.
- **Draw point status**; the activation and the closure of a draw point have to be related so the solution is consistent with the actual operation of the mine.
- **Maximum opened production area**; the total number of active draw points at any given time within the production schedule has to be constrained according to the size of the ore body, available infrastructure, and equipment availability. A large number of active draw points might lead serious operational problems such as exceeding optimum haulage distance or extreme maintenance of draw points.
- **Draw rate**; the draw rate will control in this case just the actual flow of muck at the draw point which leads to controlling the caving back. The draw control factor in this case is left out of the formulation for being redundant with the objective function.
- **Period Constraints**; the period constraints force the mining system to produce the desired production, usually keeping it within a range that allows flexibility for potential operational disturbances.
- **Mining Reserves**; the mining reserves constraint will avoid exceeding the maximum draw height so no waste material is incorporated into the schedule. The definition of ore and waste in this case is based upon the marginal cut off grade or the traditional best height of draw calculation. Therefore variations in the shut off grade during the schedule are not expected.

#### IV.1.1.1 Objective function

$$\min \left\{ \sum_{t=1}^T \sum_{i=1}^I a_{it} \{V_{it} - AT_{it}\}^2 \right\}$$

where:

$AT_{it}$  is the height of draw targets per period to achieve the overall draw angle. Note that this target will be a function of the number of new draw points to open per period, which is a decision variable.

$V_{it}$  is the height of draw at the end of period  $t$  for draw point  $i$ .

$d_{it}$  is the tonnage to be drawn from draw point  $i$  in period  $t$ .

$a_{it}$  is a binary variable =  $\begin{cases} 1 & \text{if drawpoint } i \text{ is drawn in period } t \\ 0 & \text{otherwise} \end{cases}$ .

$s_{it}$  is a binary variable =  $\begin{cases} 1 & \text{if draw point } i \text{ is opened in period } t \\ 0 & \text{otherwise} \end{cases}$

$c_{it}$  is a binary variable =  $\begin{cases} 1 & \text{if draw point } i \text{ is closed in period } t \\ 0 & \text{otherwise} \end{cases}$

#### IV.1.1.2 Definition of constraints

##### i. Development rate

$$\sum_{i=1}^I s_{it} = v_t \quad \forall t, 1 \leq t \leq T$$

$v_t \leq \text{New}_t \quad \forall t, 1 \leq t \leq T$ . Note that  $v_t$  is an integer variable and  $\text{New}_t$  is an upper bound for the problem. By construction  $v_t \geq 0$ .

##### ii. Undercut sequence

$$\sum_{k=1}^t s_{ik} \geq s_{i+1t} \text{ guarantees that draw points are opened in sequence.}$$

$$\sum_{t=1}^T s_{it} \leq 1 \text{ guarantees that every draw point is opened just once.}$$

##### iii. Draw point status

To avoid the closure of the draw point before it is opened the following constraint needs to be added.

$$\sum_{k=1}^t s_{ik} \geq \sum_{k=1}^{t-1} c_{ik} .$$

Also to avoid opening a draw point twice, the following constraint needs to be added.

$$\sum_{t=1}^T c_{it} \leq 1 .$$

The following constraint will link  $a_{it}$  with  $c_{it}$

$$a_{it} = \sum_{k=1}^t s_{ik} (1 - \sum_{k=1}^t c_{ik})$$

If draw point  $i$  has not been opened  $\sum_{k=1}^t s_{ik} = 0$ , which means  $a_{it} = 0$ . Thus the draw point  $i$  will

not be part of the schedule in period  $t$ . If  $\sum_{k=1}^t s_{ik} = 1$ , then  $a_{it}$  can be either 1 or 0 depending on

the desirability of using draw point  $i$  in period  $t$  of the schedule. If the draw point  $i$  is not drawn in period  $t$  then this draw point is closed automatically ( $c_{it} = 1$ )

**iv. Maximum opened production area**

$\sum_{i=1}^I a_{it} \leq A_t$  This constraint controls the maximum open area at any given period of the

schedule.  $A_t$  should be given as an input to the algorithm.

**v. Draw rate**

$d_{it} \leq a_{it} TU_{it}$  where  $TU_{it}$  represents the maximum draw rate of draw point  $i$  in period  $t$ . Note that the draw control factor is not included in the formulation because it is considered to be redundant with the objective function.

**vi. Period Constraints**

$$\sum_{i=1}^I d_{it} \leq TTU_i$$

where  $TTU_i$  and  $TTL_i$  represent the maximum and minimum feasible

$$\sum_{i=1}^I d_{it} \geq TTL_i$$

**vii. Mining Reserves**

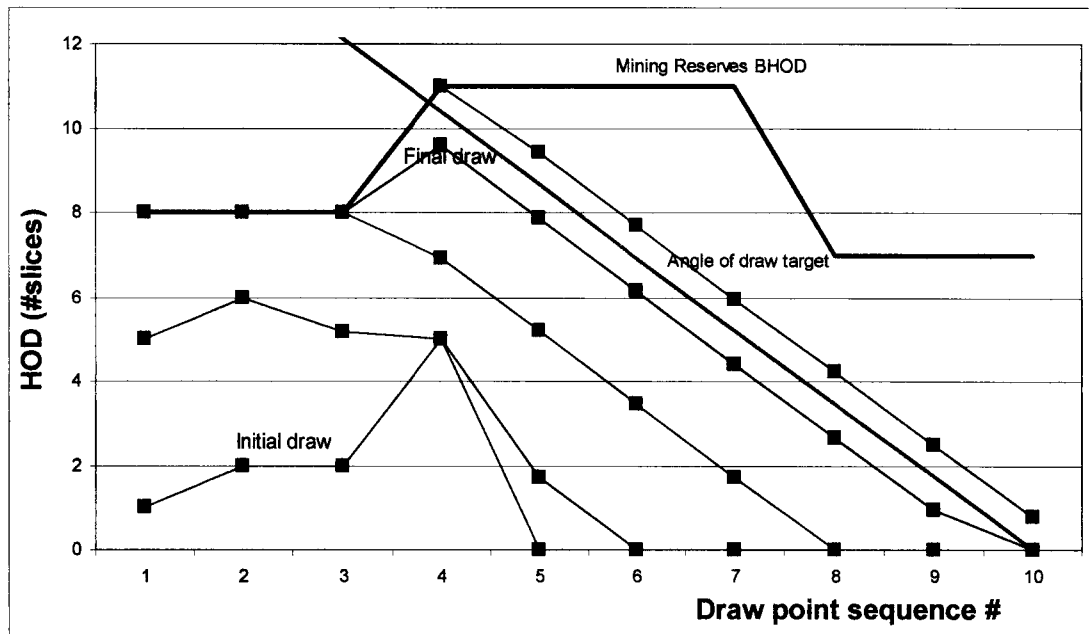
$a_{it} (BT_i - V_{it}) \geq 0$  where  $BT_i$  are the reserves computed using best height of draw and  $V_{it}$  represents the tonnage drawn from draw point  $i$  until the beginning of period  $t$ . This constraint controls that no draw point exceeds the best height of draw or the mining reserves.

To solve this non- linear programming problems two assumptions were made. First the integer variable, number of draw points to opened per period, was relaxed and the draw control factor constraint was eliminated. The first assumption is reasonable from the operational point of view since there is very little flexibility in this variable due to existing contracts and mine infrastructure. The second assumption was that the draw control factor would pursue the same objective of a target draw angle to achieve even draw. Consequently having the draw angle as a target for the objective function in addition to a draw control factor constraint might become a redundant formulation. Figure 4-4 shows how the algorithm behaves for an example involving 10 draw points.

The second assumption was that the draw angle would be an equivalent target for the objective function. In other words a particular draw angle would result on even draw and maximize the mine life subject to the constraints. In this case having the draw control factor as a constraint would be redundant.

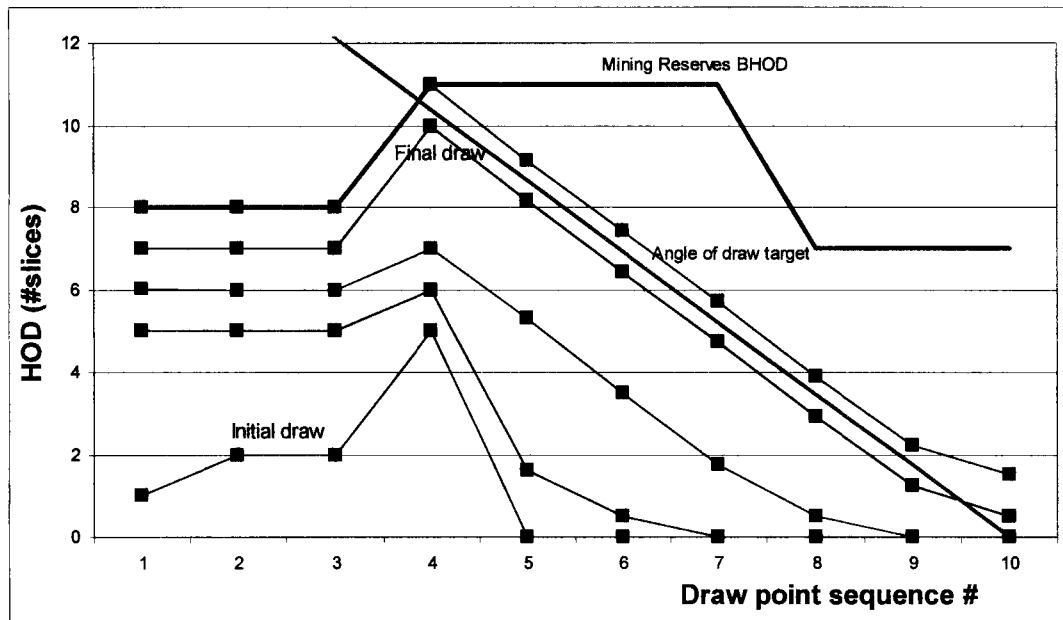
Figure 4-4 shows that in the first two years of the schedule there was no control on the draw profile. After the second year of the schedule the angle of draw concept was used as the long term planning goal. It is also shown that the overall draw profile follows the long term target

that has been defined by following the desired angle of draw. The sequence for draw point closure depends upon the mining reserves defined for each draw point. However closer inspection of the production schedule shows that draw point number 4 does not get any draw in periods 3 and 4 of the schedule. This is due to the relaxation of the draw control constraint. From a mathematical point of view this is a reasonable solution since it idles the draw points that are far advanced in their draw. However from the block caving point of view this solution is not acceptable because no draw point can be left for two years without draw due to the compaction that would transmit high stresses to the production level. Therefore the current solution needs to be improved to address this issue.



**Figure 4-4: Evolution of draw profile within the production schedule as a result of following an angle of draw strategy**

Figure 4-5 represents the solution of the algorithm running with the draw control constraint. It can also be seen in this graph that the objective function is well defined, because the algorithm brings the overall draw profile close to the target and all the constraints are satisfied. The problem with this result is that all draw points are still open at the end of the life of the mine. This actually is feasible in a block cave operations but not in a panel caving operation where the final layout perhaps might contain a few thousand draw points.



**Figure 4-5: Draw profile for the angle of draw strategy including the draw control constraint.**

The following table shows the production schedule associated with the above heights of draw profile.

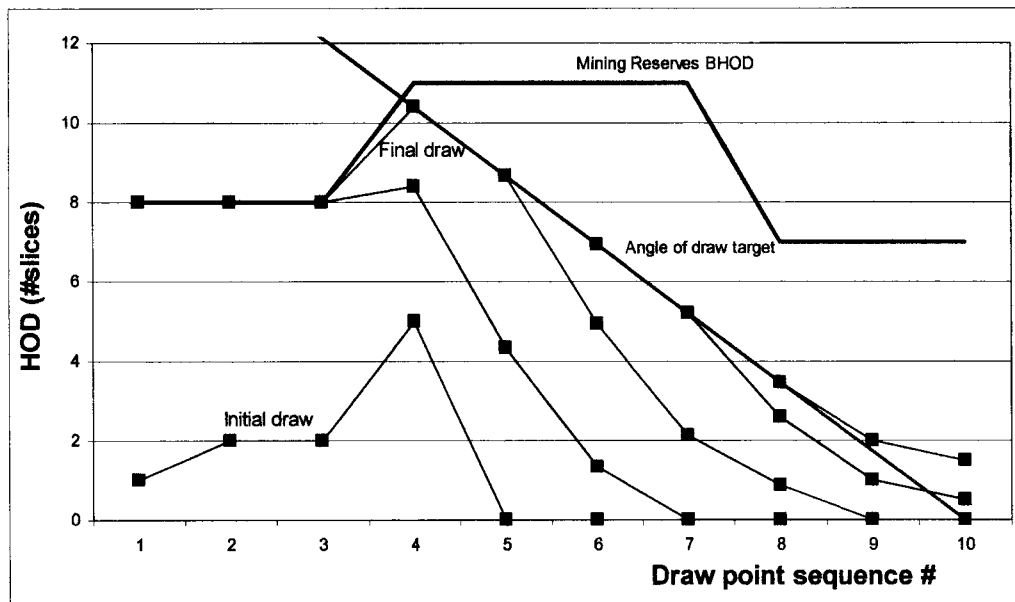
draw/period	1	2	3	4	5	6	7	8	9	10
1	0.0	1.0	2.0	2.0	2.0	0.8	0.1	0.1	0.1	0.1
2	2.0	0.0	2.0	2.0	1.4	0.4	0.1	0.1	0.1	0.1
3	0.0	2.0	1.7	0.9	1.0	1.5	0.7	0.1	0.1	0.1
4		5.0	0.1	0.1	0.1	0.3	1.3	1.2	1.1	1.0
5		0.0	0.2	1.0	1.0	1.5	1.3	1.2	1.1	1.0
6		0.0		0.1	0.4	1.5	1.3	1.2	1.1	1.0
7		0.0			0.1	0.2	1.3	1.2	1.1	1.0
8		0.0				0.1	0.1	0.9	1.1	1.0
9		0.0					0.1	0.1	0.2	1.0
10		0.0						0.1	0.1	0.1
Total	2	8	6	6	6	6	6	6	6	6
TTUt	6	6	6	6	6	6	6	6	6	6
TTLt	5	5	5	5	5	5	5	5	5	5

**Table 4-1: Production schedule following the angle of draw strategy including draw control constraints**

The next change to the algorithm was the introduction of a deterministic “life of the draw points” as a fixed parameter in the optimization. Then the opening and the closure of a draw point have to be specified in advance to compute the production schedule. This assumption

is not completely unrealistic since it is an acceptable practice to not keep draw points open for more than a specific period of time, for instance 4 years. Another improvement to the model was to avoid the intermediate surfaces or targets so that the resulting plan minimizes the difference with the final draw profile target, simplifying the objective function considerably. Figure 4-6 shows the resulting draw profile applying these new improvements on the current algorithm.

The resulting production schedule is shown in Table 4-2. One of the first conclusions coming from the mine life optimization is that a greater area needs to be opened to achieve the desired production rates. For instance on the above plan from year 7 until the end of the life of the mine production rates have been reduced considerably, basically due to the nature of the objective function. By extending the life of the draw points, better performance can be expected. A second way of improving the production rates is by increasing the draw rates. Although this is a traditional way of ramping up production, it is not recommended due to the effect upon the profile of the caving back and the stability of the production level.



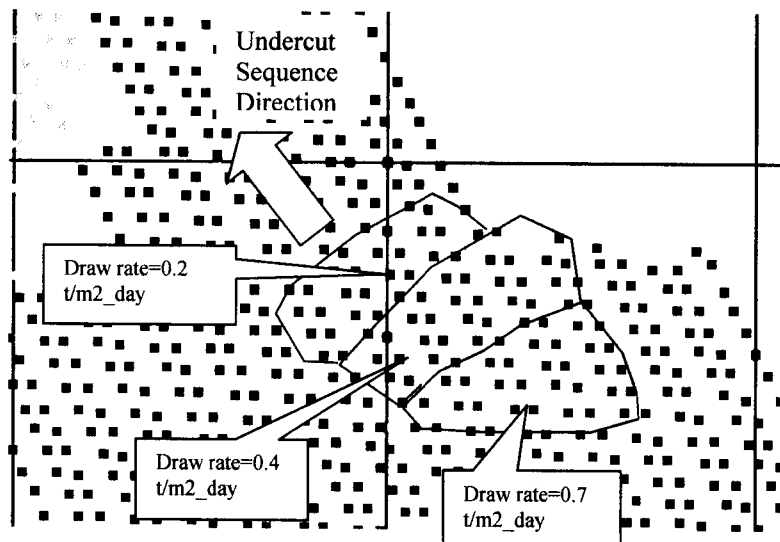
**Figure 4-6: Height of draw profile without intermediate surfaces and a fixed life for draw points**



draw/period	1	2	3	4	5	6	7	8	9	10
1	0.0	1.0	3.5	3.5	0.0	0.0	0.0	0.0	0.0	0.0
2	2.0	0.0	3.0	3.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	2.0	3.0	3.0	0.0	0.0	0.0	0.0	0.0	0.0
4		5.0	1.7	1.7	2.0	0.0	0.0	0.0	0.0	0.0
5		0.0	2.2	2.2	2.2	2.2	0.0	0.0	0.0	0.0
6		0.0	0.0	1.3	1.7	1.9	2.0	0.0	0.0	0.0
7		0.0	0.0	0.0	1.1	1.1	1.1	2.0	0.0	0.0
8		0.0	0.0	0.0	0.0	0.9	0.9	0.9	0.9	0.0
9		0.0	0.0	0.0	0.0	0.0	0.5	0.5	0.5	0.5
10		0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.5	0.5
Total	2.0	8.0	13.4	14.7	7.0	5.9	4.4	3.9	1.9	1.0
TTLt	6.0	6.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0
TTLt	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0

**Table 4-2: Production schedule without intermediate surfaces and with a fixed draw point life**

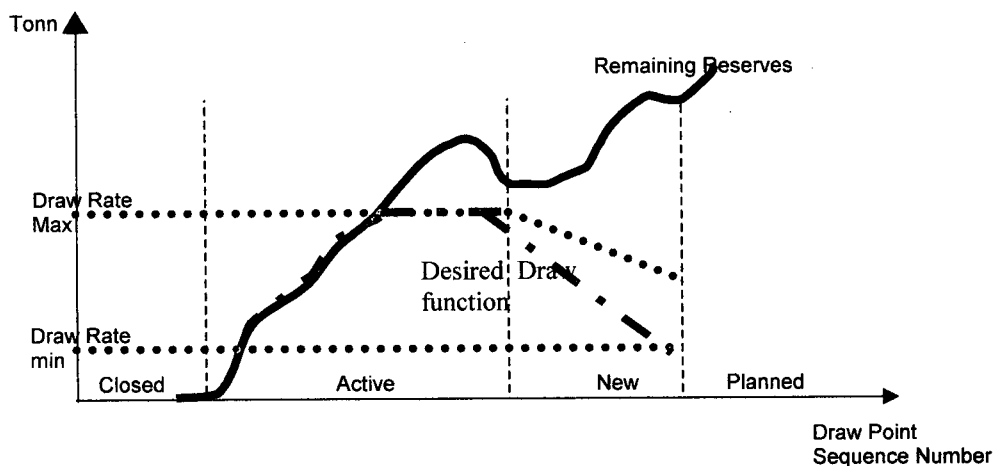
By analyzing the production schedules shown in Tables 4-1 and 4-2, it is noted that in general new draw points are drawn slower than old draw points. This means that applying a draw function that would draw the draw points near to the front cave slower than the ones that are located at the back of the active zone, it is possible to reach a desired draw profile. In particular, by calibrating this new draw function it is possible to represent a draw profile that follows a desired angle of draw. Figure 4-7 shows this concept.



**Figure 4-7: Representation of a differential draw rate to reach an angle of draw objective**

To conceptualize the above ideas a draw function that will pursue the angle of draw strategy must be defined. This function will be embedded into the production schedule algorithm so

the draw profile matches the long term strategy. Figure 4-8 shows the shape of the draw function that can represent the above concept.



**Figure 4-8: Concept of a draw function to pursue the angle of draw as a long term goal**

In summary the optimization algorithm has been used to define a heuristic function that will represent the objective function that minimizes the deviation of the resultant draw profile with respect to the desired angle of draw plane. The shape of this function can be derived from linking the draw rate, rate of development and draw control constraints. This relationship will not be an explicit equation but rather it will be derived by a calibration process.

Although the above method has been successfully implemented, it was necessary to develop an algorithm capable of optimizing a more comprehensive objective function that could actually integrate the local draw control (a draw point and its neighbors) into the objective function.

## IV.2 Uniform draw optimisation

An alternative method to reach an even draw condition is by minimizing the variability of the tonnage drawn per period. Since minimizing draw variability is directly related to the concept of uniform draw, the overall dilution will be minimized and therefore the dilution

entry point retarded. Consequently this will result in extending the life of the mine. One of the first problems with this approach is to find the correct estimator of variability. In this research the standard deviation of tonnages drawn from the active draw points at any given period of the schedule has been chosen to quantify this variability. Then, by minimizing this variability an even draw condition can be expected. This formulation guaranties at least that draw will be controlled at any given time in the schedule.

#### ***IV.2.1 Mathematical Formulation***

The objective function in this case minimizes the deviation of tonnages drawn from the active draw points at any given period of the schedule. The variables of the problem are the tonnages to be drawn from the draw points and the draw point status. The draw point status variable will linked to the number of new draw points to incorporate in the schedule per period, adding an integer variable to the problem. The following constraints are included in the formulation of the problem:

- ***Development rate***; the development rate controls the maximum feasible number of draw points to be open at any given time within the schedule horizon. This constraint is usually based on the geometry of the ore body and the existent infrastructure of the mine, which typically will define the number of accesses available to the mining faces
- ***Undercut sequence***; the undercut sequence will define the order in which the draw points will be open. This constraint usually acts on the draw point status activating those that are at the front of the production face.
- ***Draw point status***; the activation and the closure of a draw point have to be related so the solution is consistent with the actual operation of the mine.
- ***Maximum opened production area***; the total number of active draw points at any given time within the production schedule has to be constraint according to the size of the ore body, available infrastructure and equipment availability. A large number of active draw points might lead to serious operational problems such as exceeding optimum haulage distance or extreme maintenance of draw points.
- ***Draw rate***; the draw rate will control two aspects of the block cave the first one is the actual flow of muck at the draw point which leads into controlling the caving back

and the draw control factor through constraining the maximum and the minimum rate of draw.

- **Period Constraints;** the period constraints forces the mining system to produce the desired production usually keeping it within a range that allows flexibility for potential operational disturbances.
- **Mining Reserves;** the mining reserves constraint will control not to exceed the maximum draw height so no waste material is incorporated into the schedule. The definition of ore and waste in this case is based upon the marginal cut off grade or the traditional best height of draw calculation. Therefore it is not expected variations on the shut off grade during the schedule.

#### IV.2.1.1 Objective function

$$\text{MIN} \left\{ \sum_{t=1}^T a_t \sqrt{\frac{1}{a_t - 1} \sum_{i=1}^I a_{it} \left( d_{it} - \bar{d}_t \right)^2} \right\}$$

where:

$d_{it}$  is the tonnage to be drawn from draw point  $i$  in period  $t$ .

$\bar{d}_t$  represents the average of tonnages scheduled in period  $t$ .

$a_t$  is the total number of draw points active in period  $t$ .

$a_{it}$  is a binary variable =  $\begin{cases} 1 & \text{if drawpoint } i \text{ is drawn in period } t \\ 0 & \text{otherwise} \end{cases}$

$s_{it}$  is a binary variable =  $\begin{cases} 1 & \text{if draw point } i \text{ is opened in period } t \\ 0 & \text{otherwise} \end{cases}$

$c_{it}$  is a binary variable =  $\begin{cases} 1 & \text{if draw point } i \text{ is closed in period } t \\ 0 & \text{otherwise} \end{cases}$

#### IV.2.1.2 Definition of constraints

##### i. Development rate

$$\sum_{i=1}^I s_{it} = v_t \quad \forall t, 1 \leq t \leq T$$

$v_t \leq \text{New}_t \quad \forall t, 1 \leq t \leq T$ . Note that  $v_t$  is an integer variable and  $\text{New}_t$  is an upper bound for the problem. By construction  $v_t \geq 0$ .

### ii. Undercut sequence

$$\sum_{k=1}^t s_{ik} \geq s_{i+1t} \text{ guarantees that draw points are opened in sequence.}$$

$$\sum_{t=1}^T s_{it} \leq 1 \text{ guarantees that every draw point is opened just once.}$$

### iii. Draw point status

To avoid the closure of the draw point before it is opened the following constraint needs to be formulated

$$\sum_{k=1}^t s_{ik} \geq \sum_{k=1}^{t-1} c_{ik} . \text{ Also to avoid that a draw point is opened twice the following needs to be}$$

$$\text{added to the set of constraints } \sum_{t=1}^T c_{it} \leq 1 .$$

The following constraint will link  $a_{it}$  with  $c_{it}$

$$a_{it} = \sum_{k=1}^t s_{ik} (1 - \sum_{k=1}^t c_{ik}) \text{ If draw point } i \text{ has not been opened } \sum_{k=1}^t s_{ik} = 0, \text{ which means } a_{it} = 0. \text{ Thus}$$

the draw point  $i$  will not be part of the schedule in period  $t$ . If  $\sum_{k=1}^t s_{ik} = 1$ , then  $a_{it}$  can be either

1 or 0 depending on the desire of using draw point  $i$  in period  $t$  of the schedule. If the draw point  $i$  is not drawn in period  $t$  then this draw point is closed automatically ( $c_{it} = 1$ )

### iv. Maximum opened production area

$\sum_{i=1}^I a_{it} \leq A_t$  This constraint controls the maximum open area at any given period of the schedule.  $A_t$  should be given as an input of the algorithm.

**v. Draw rate**

$d_{it} \leq TU_{it} a_{it}$  where  $TU_{it}$  and  $TL_{it}$  represent the maximum and minimum draw rate of draw point  $i$  in period  $t$ . This constraint can also be written as  $TL_{it} = \frac{TU_{it}}{dcf_t}$  where  $dcf_t$  represents the desire draw control factor to be used in period  $t$  of the schedule.

**vi. Period Constraints**

$\sum_{i=1}^I d_{it} \leq TTU_t$   
 $\sum_{i=1}^I d_{it} \geq TTL_t$  where  $TTU_t$  and  $TTL_t$  represent the maximum and minimum feasible

**vii. Mining Reserves**

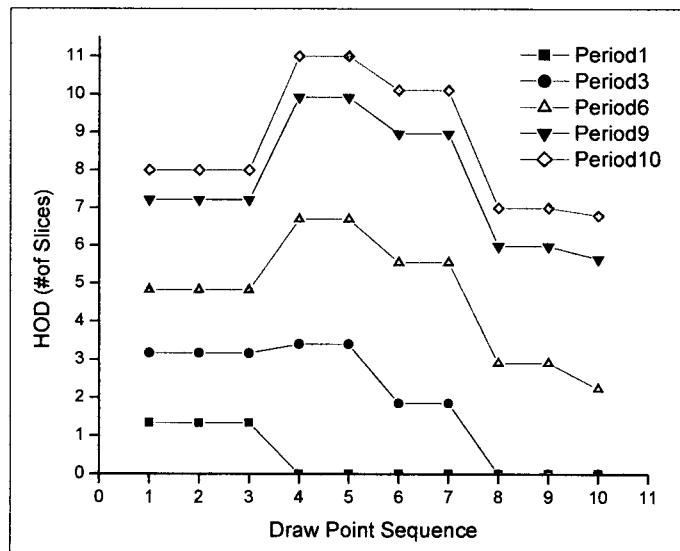
$a_{it} (BT_i - V_{it}) \geq 0$  where  $BT_i$  are the reserves computed using best height of draw and  $V_{it}$  represents the tonnage drawn from draw point  $i$  until the beginning of period  $t$ . This constraint controls that no draw point exceeds the best height of draw or the mining reserves.

The set of constraints is the same as the presented before to optimize the angle of draw. Figure 4-9 shows the results of using this algorithm.

The overall draw profile produces a reasonable angle of draw even though no such target had been set up. This objective function seems to be much more realistic than the angle of draw which constraint the whole operation leaving very little of flexibility to deal with operational problems. The second advantage of the above method is that changing the uniformity index

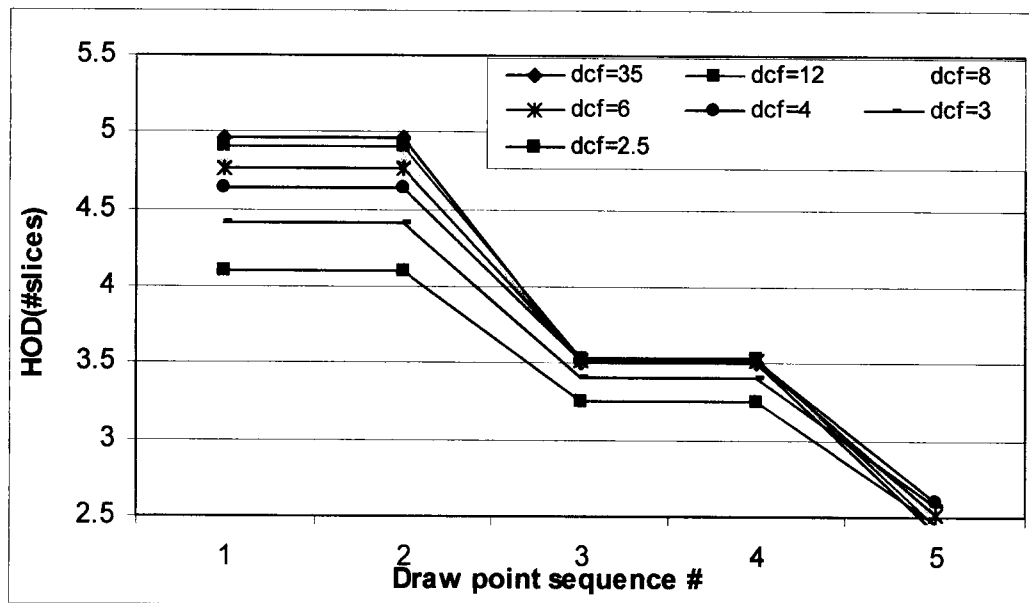
can still control the resulting angle of draw. Since a higher uniformity index (maximum tonnages divided by minimum tonnages) provides more flexibility, and a steeper angle of draw can be expected.

Figure 4-10 shows the relation between angle of draw and uniformity index. This is a very interesting result from the point of view of a potential relationship between uniformity index and angle of draw. Perhaps the uniformity index can be included as part of the draw function in future applications. By integrating the uniformity index in the draw function there is no need for an explicit angle of draw to be pursued in the optimization process.



**Figure 4-9: Draw profile for a draw control optimization algorithm**

Another interesting point is the fact that the overall draw profile converges to the angle of draw plane regardless of which objective function is used. This has two immediate results. First, the angle of draw concept has been validated as a proper draw control technique because by applying it the resulting production schedule minimizes the tonnage deviation within a period of the schedule. Second, there is considerable benefit to be gained from the integration of draw control practices in the NPV optimization.



**Figure 4-10: Relation between uniformity index and overall draw profile as an angle of draw**



## V APPLICATION OF THE PROPOSED METHODOLOGIES: CASE STUDIES

This chapter will discuss two real applications of the algorithms described in the previous chapters. The first mine is the third panel of Andina mine that belongs to Codelco – Chile. The second example is adapted from an actual mine that will be named X in the following. It is important to note that these two mines are actually in production and using the following results to perform their actual production schedules.

### **V.1 Third Panel Andina- Codelco Chile**

#### ***V.1.1 Introduction***

Andina mine is located 220 km Northwest of Santiago, Chile at 3800 m above the sea level in the Andes mountains. This mine has been in production since 1978. The mining methods used currently are panel caving and open pit.

Block caving began in 1978 in the first panel which was exhausted by the end of 1985. Production then began from the second panel which was depleted by the end of 1995. The third panel was constructed by the end of 1994 and began production in 1995. This case study concentrates on the production plan supporting the operation of this third panel.

The main access to the mine is via a horizontal tunnel, which is 6350 m long from the surface to the production level. A 2000m long ramp, 12% slope, connects to the horizontal tunnel which provides access to the rest of the mine.

The equipment consists of 12 underground trucks each with a capacity of 50 tonnes, and ten 6 yd<sup>3</sup> LHDs. Ten hammers are located underneath the production level.

The production plan horizon is 25 years with steady production at the third year reaching the mine capacity of 35,000 tonnes per day. The development rate was estimated to be 15,300 m<sup>2</sup> per year.

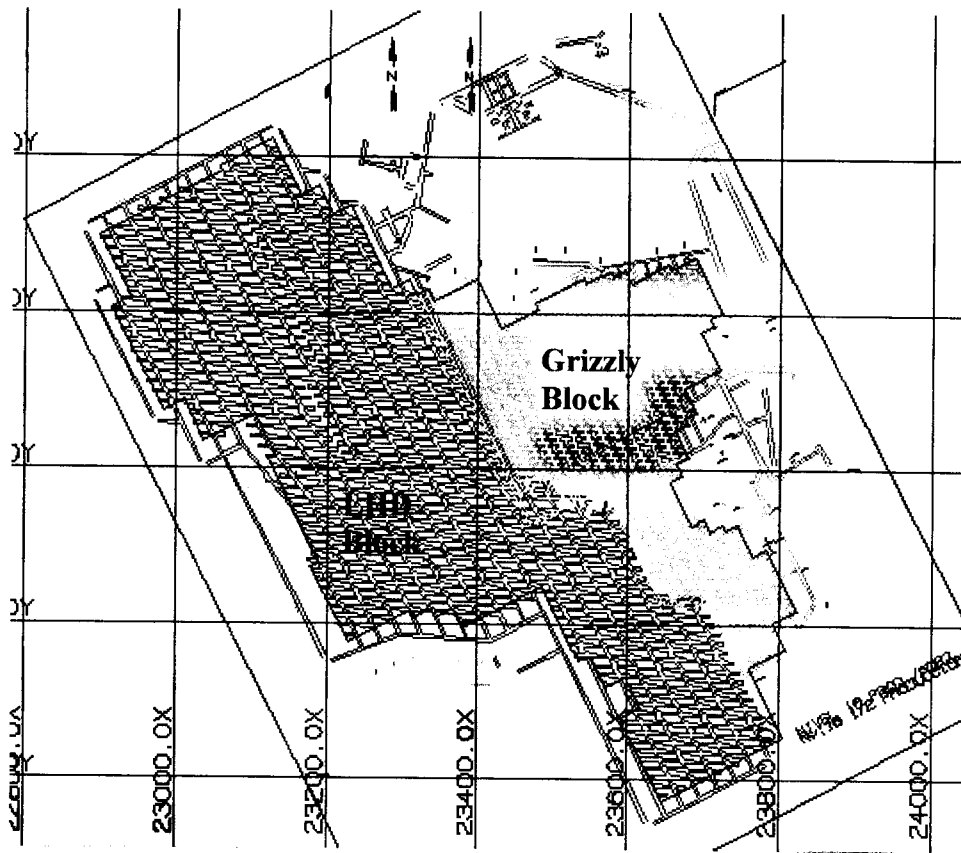
The capital cost was estimated to be \$150 million and the operating cost is estimated at \$2.5 /ton.

### ***V.1.2 Mining System***

The mining system at the Andina mine was designed based on the geotechnical properties of the rock mass. A main fault defined two zones from a geotechnical point of view, the first one comprising weaker rock than the second one. Therefore the third panel mine was mainly divided into two blocks to represent the geotechnical variability (Figure 5-1). In weak rock mass, a grizzly manual system was built and on the panel containing stronger rock mass a mechanized LHD was used. The LHD block will be the subject of the following case study because it is in its early stage and several design and planning considerations might be changed.

The layout was designed using a design similar to El Teniente, which has demonstrated an excellent performance in terms of stability and operations point of view (Esterhuinzen, 1992).

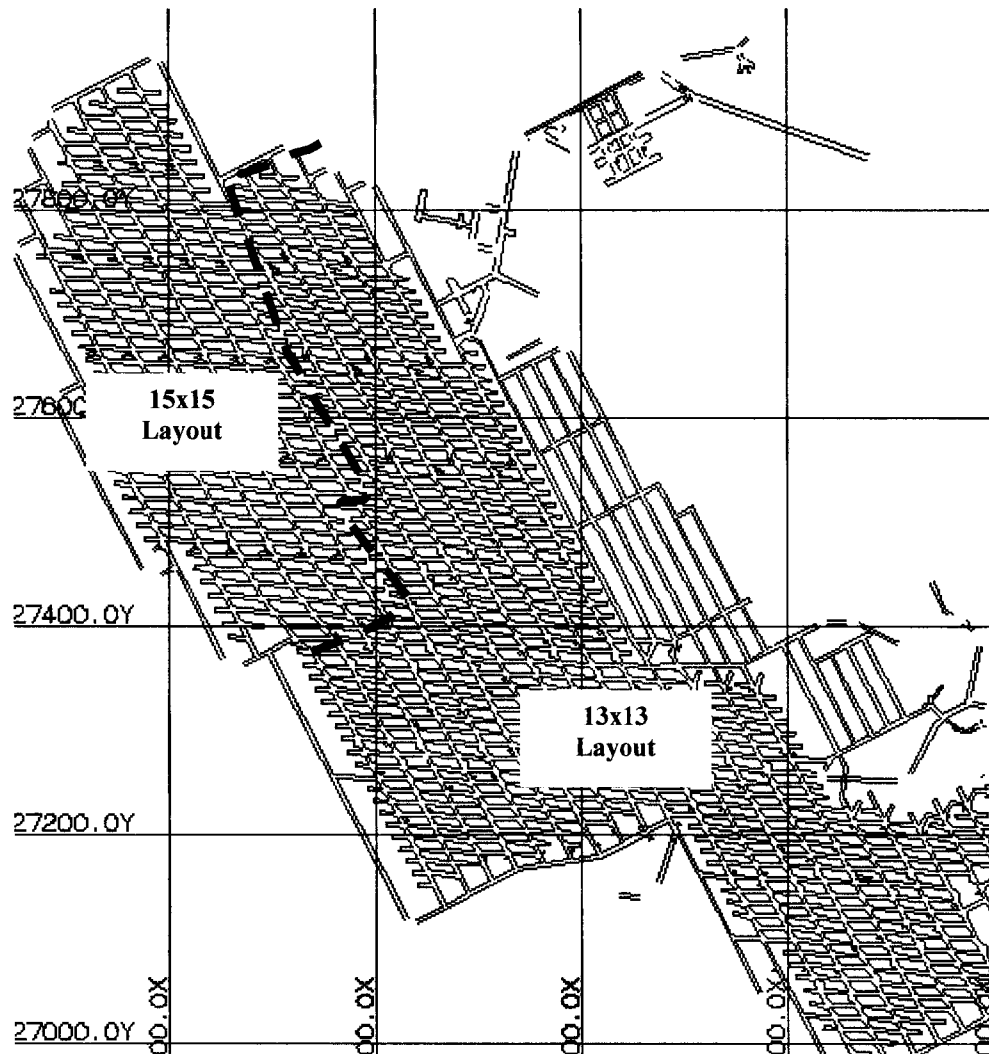
Draw point spacing was established based on the initial fragmentation study considering that the south part of the layout was in a medium rock mass (RMR=50) and the north part of the layout was located in more competent rock mass (RMR=60). Therefore the spacing at the north part of the layout was 15m and 12.5 at the south part (Figure 5-2). Interaction between draw points has been achieved and it can be demonstrated by the first six years of the mine operation.



**Figure 5-1: Third Panel Andina production level layout**

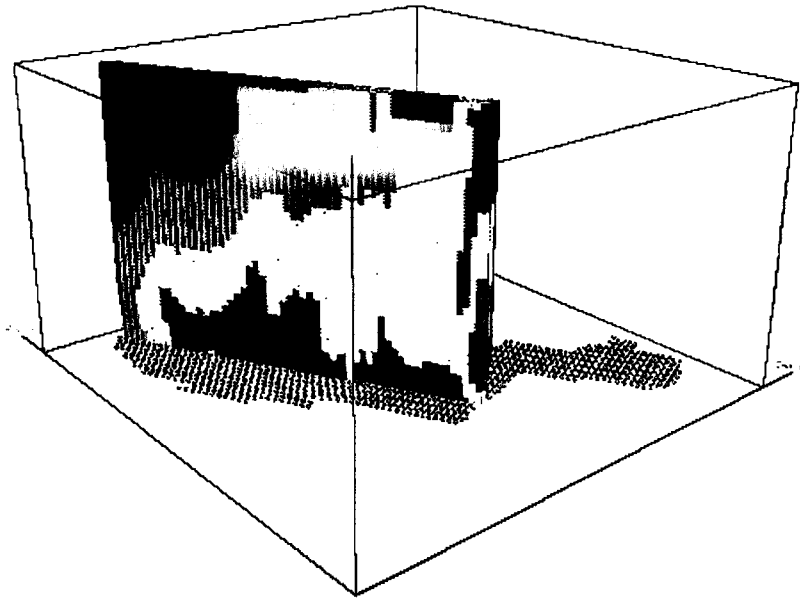
### ***V.1.3 Block Cave Model***

The diameter of the draw columns was designed yield interaction with at least 1 draw point. This means that horizontal flow of rock can be modeled if there is evidence of this behaviour. The diameter for the 12.5x12.5 layout was chosen to be 16m and 17.5m for the 15x15 layout. This will provide enough overlap between draw points to allow the application of horizontal mixing.



**Figure 5-2: Production layout in the LHD sector of the Andina Third Panel mine**

Figure 5-3 shows one production drift of the actual layout showing the copper grade after mixing and prior to the mixing that would occur as the mining progresses. The height of interaction simulated was 160m with five iterations. The draw control factor has already been included in the estimation of the height of interaction.



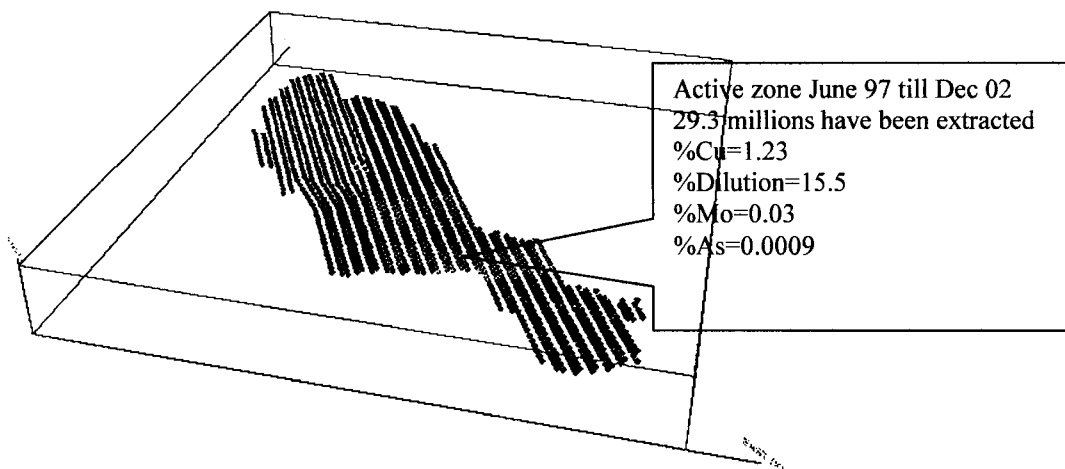
**Figure 5-3: Draw columns of one production drift showing the copper grade model**

#### ***V.1.4 Economic boundary of the layout***

This step was not considered as part of the study even though it is a fundamental step in planning a block cave mine. Several optimization tools might be used to define the economic boundary of a deposit, however this was not part of the scope of this study. In this case the economic boundary was taken from the actual mine design, defined by Andina.

#### ***V.1.5 Production schedule base case against NPV optimization scenario***

The LHD sector of Andina Third Panel mine has been in production since 1997 which means that 29 millions tonnes will have been drawn from the active area by the end of December 2002.



**Figure 5-4: Active area on the LHD sector from April 1997 to Dec 2001**

- Mining Reserves (traditional approach)

The first step was to set the economic parameters to calculate the mining reserves using the actual best height of draw criterion.

Item	Units	Value
Mining and Processing Costs	\$/t	9
Metallurgical recovery	%	85
Metal Price Copper	\$/lb	0.85
Smelting and Refining costs	\$/lb	0.2
<b>Overall revenue factor</b>	<b>\$/%Cu</b>	<b>12.18</b>

**Table 5-1: Parameters to compute mining reserves for the base case scenario**

The above overall revenue factor does not include credits for molybdenum nor penalties for arsenic.

Mining reserves were calculated after depleting the initial years of extraction (June 1997 untill December 2002). These reserves were constrained to the minimum height of 80m which represents one third of the in situ draw column. This consideration has tremendous implications in the overall performance of the caving process because if the economic

reserves of a draw point were less than 80m, then the draw height would not be enough to propagate the caving up to surface, creating bridges or pillars that could affect the local stability of the layout.

Table 5-2 shows the mining reserves resulting from applying the BHOD methodology with the above parameters.

Tonnage(tonnes)	%Cu	%Mo	%Pb	Wi (Kwh/tonne)	%As
178,702,800	0.989953	0.029476	0.00149	16.0076	0.008068

**Table 5-2: Mining reserves for LHD sector after Dec 2001**

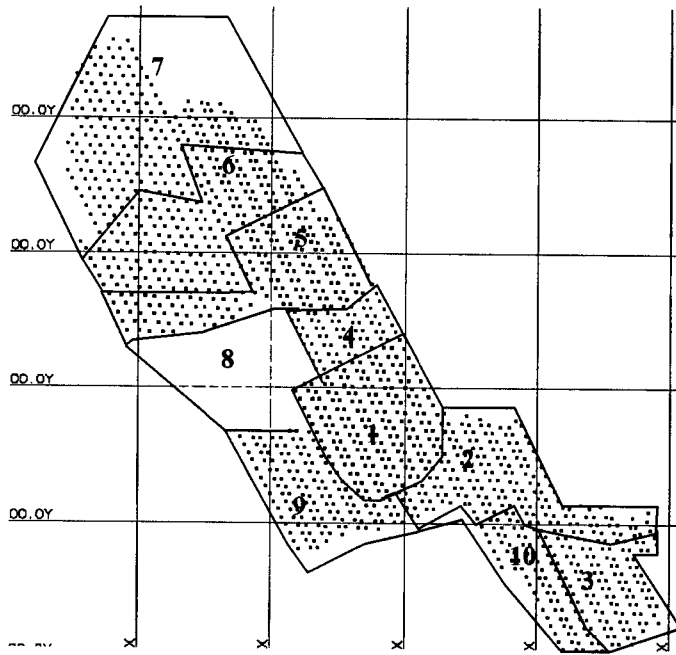
Finally the reserves for the LHD sector of third panel Andina mine are estimated to be 178 millions with an average copper grade of 0.99%Cu.

- Undercutting Sequence

The undercutting sequence was chosen by maximizing the economic value of the deposit. This means production will start where the high grades are and will move toward lower grades. The direction of the undercutting front was determined to be the one that produces most of the unstable blocks, so fragmentation will not be problem when production starts. Typically the criterion is to use a cave front direction perpendicular to the main geologic structures. The following figure 5-5 shows a map with the undercutting sequence

- Draw Rate

The draw rate to be used in this case study was computed to reflect the actual history of the third panel mine. A statistical analysis was performed on the raw data to estimate the shape of the draw rate curve. Table 5-3 represents some of the results obtained.



**Figure 5-5 Undercutting sequence for LHD sector**

%Drawn	Avg	Std Dev.	Min	Max	Q3	# Sample
0.12	3,180	3,847	64	19,210	3,015	123
0.3	2,953	3,611	31	32,817	3,196	1916
0.5	2,963	2,403	14	45,418	3,703	1474
0.8	3,195	2,829	43	59,371	3,953	1476
>0.8	2,964	2,169	16	27,503	3,681	828

**Table 5-3: Analysis of historical tonnage depleted from LHD sector since 1997**

The time frame for the above data is one month. Thus it will help to forecast long term draw rate. Q3 is the third quartile of the tonnage so that 75% of the sample is below it. Assuming 85% of the draw points are available due to maintenance, the draw rates in Table 5-4 are computed based on the above data. The units of draw rate are in tonnes/m<sup>2</sup>/day

The draw rate of LHD13x13 was computed using historical data. LHD15x15 is computed by multiplying by (13x13)/(15x15) to adjust for area.

% Draw	LHD13x13	LHD15x15
--------	----------	----------



0.00001	0.51	0.38
0.12	0.54	0.40
0.3	0.62	0.46
0.5	0.66	0.49
0.8	0.62	0.46
1	0.62	0.46

**Table 5-4: Draw rates LHD sector**

- *Desired production rates*

The production rates as well as the development rates were given by Andina personnel.

Table 5-5 shows the tonnage target as well as the number of draw points to be opened per period of the schedule.

Period	TONNES	New
Yr_2002	11,000,000	120
Yr_2003	11,000,000	120
Yr_2004	11,000,000	120
Yr_2005	11,000,000	120
Yr_2006	11,000,000	120
Yr_2007	11,000,000	120
Yr_2008	11,000,000	120
Yr_2009	11,000,000	120
Yr_2010	11,000,000	120
Yr_2011	11,000,000	120
Yr_2012	11,000,000	120
Yr_2013	11,000,000	120
Yr_2014	11,000,000	120
Yr_2015	11,000,000	120
Yr_2016	11,000,000	120

**Table 5-5: Production targets for LHD sector**

The TONNES column represents the desired production and NEW the number of draw points to be opened per period.

- *Resulting schedule*

The maximization of NPV has been applied within the actual mine planning scenario. This means that the optimized scenario keeps the definition of undercut sequence, production rates and draw rates. Then the optimized schedule will change shut off grades, development rates and life of the mine in conjunction with the production schedule.

The economic parameters used in the optimization were a discount rate of 10% given by Codelco, a development cost per draw point of \$60,000 and a mine capacity of 11 million tonnes per year. Four iterations were needed to converge to the optimized schedule. Table 5-6 summarizes the main results of the optimization.

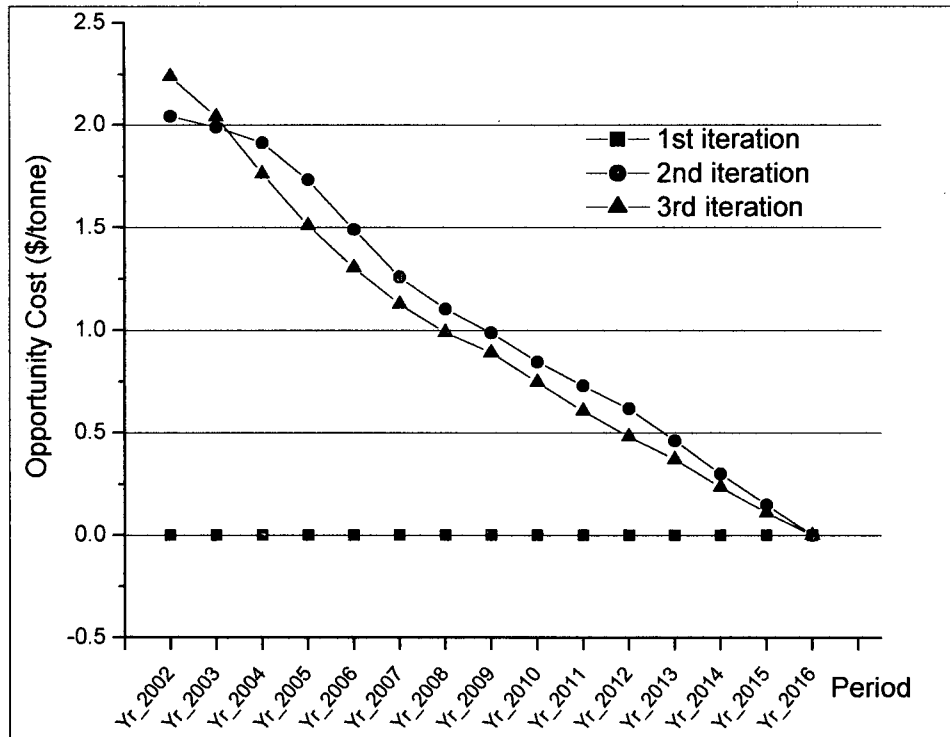
Item	Base Case	Optimized Case
NPV(\$)	219,486,71	261,428,012
Net DV costs (\$)	44,055,822	37,775,780
Cu %	0.9800	1.0007
Mo %	0.0292	0.0291
Pb %	0.0016	0.0016
Wi (Kwh/tonne)	16.0461	16.0022
Reserves (tonne)	237,007,056	208,440,656

**Table 5-6: Production schedule optimization results**

As shown the algorithm increases the current NPV by 19.1% and reduces the mining reserves by 12%. Likely this result will decrease the life of the mine by two to three years. Note that the optimized scenario shows a smaller work index than the base case scenario. This will in fact increase NPV even higher because the processing cost probably will be less than in the base case scenario.

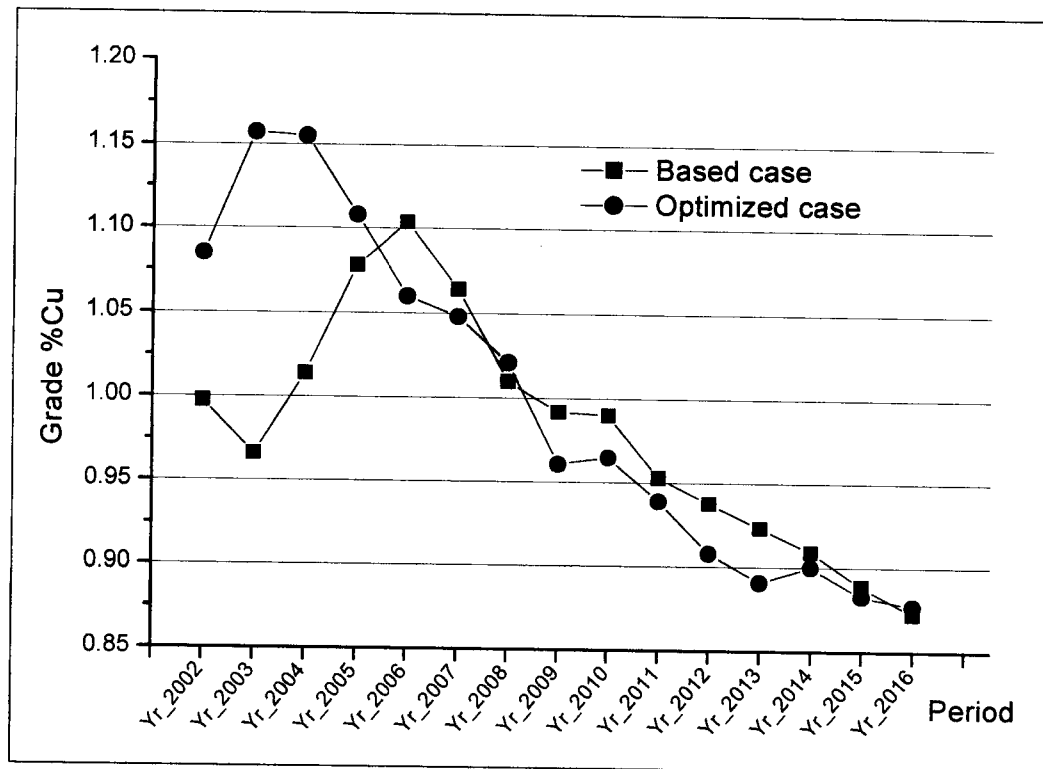
A comparison between copper grade of optimized case and base case scenario is shown in Figure 5-6. Note that the optimized case shows an increment on copper grade during the first year of the schedule. This is usually the main effect of introducing a variable shut off grade in the production schedule algorithm. The highest grade is mined first and gradually this will decrease as the deposit is depleted.

Figure 5-6 shows the value of the opportunity costs for different iterations and the optimized case. Note that the shape of the opportunity cost curve is non linear and it represents how variable the deposit is.



**Figure 5-6: Evolution of the opportunity cost through the iterations of the optimization process**

The tonnage for the above schedule was constant at 11 million tonnes per year. After simulating this first stage production schedule it was desired to change the mine throughput to seek new ways of optimizing the actual scheduling changing the mine – mill capacity.



**Figure 5-7: Copper content comparison between optimized schedule and base case scenario**

Table 5-7 shows summarized the effects of optimizing NPV varying the mine capacity.

Scenario	Base Case	Optimized Scenario	%Improvement
11M	219,486,741	261428012.2	19.1%
14M	266,845,510	284,438,308	6.6%
16M	287,947,545	289,239,241	0.4%

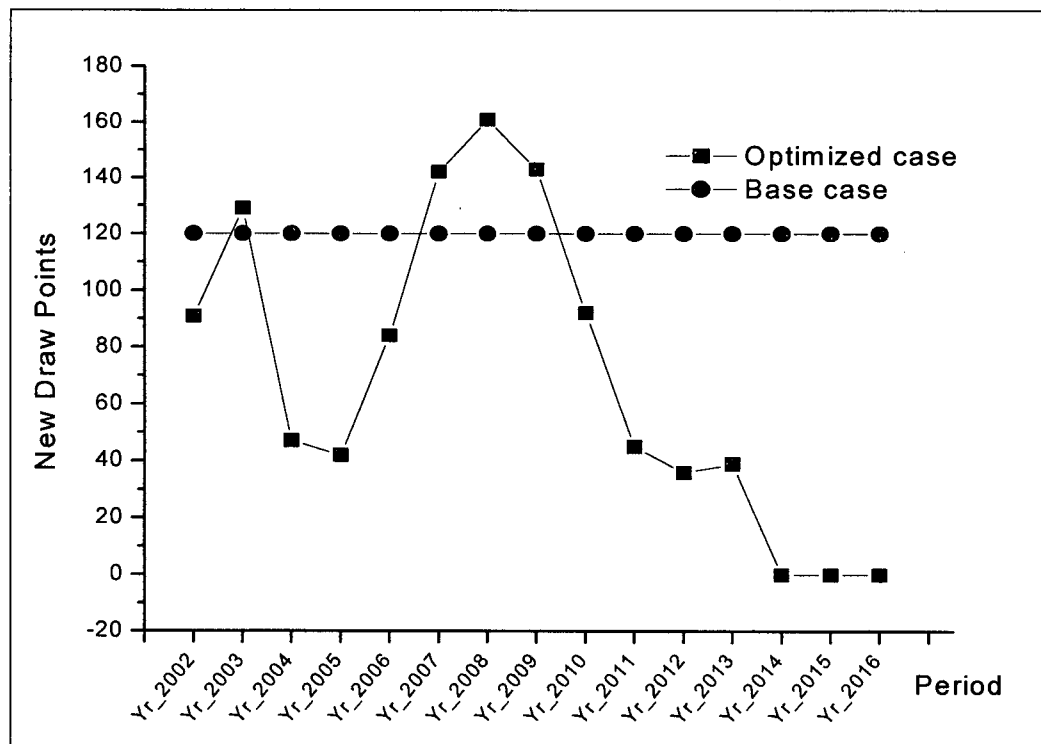
**Table 5-7. Effect of mine throughput in the overall optimization process**

As was discussed in Chapter 4, to maximize NPV block caving there has to be enough flexibility to mine those draw points with the lowest remaining value. For example from Table 5-7 the 16M scenario tonnes the optimized scenario gives just 0.4% of improvement on the top of the base case scenario. The reason for this behaviour is because neither the development rate nor draw rate were changed. Thus the production target was not reached due to early closing of draw points with marginal remaining value.

Therefore by modifying the development rate to, say, 140 draw points per year it would be feasible to optimize the 16M scenario. However for contract reasons and also from a logistical point of view, the maximum number of draw points to open per period was set as 120. Then it was decided to stay in the 11M scenario.

- **Development rate**

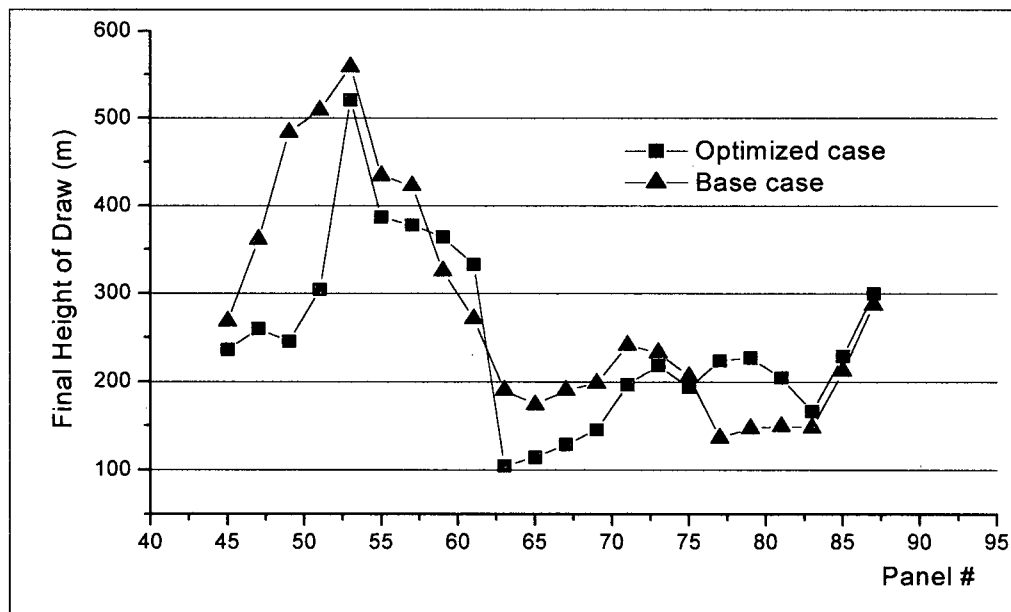
A comparison between the base case and the optimized scenario development rates is presented in Figure 5-8. As shown the development rate for the optimized scenario goes beyond 160 draw points. This is unacceptable from an operational point of view that defines the maximum number of draw points to develop per year as 140 and the minimum as 60. Therefore the above development rate curve has to be adjusted to those requirements. However the shape of this curve should be kept to maintain the convergence of the optimization.



**Figure 5-8: Comparison of the development rate for the base case and optimized scenario**

- **Heights of Draw**

A useful way to compare different production schedules is plotting the overall height of draw at the end of the life of the mine by production drift or panel. This will report how evenly the draw is performed for a specified schedule. Usually in block caving it is desirable that the shape of this graph to be smooth so that high stresses as well as early dilution are avoided. One of the main constraints given by Andina mine was that the minimum height of draw in operations is 80m. At this height it is expected that the entire draw column will be broken so that abutment stresses and early dilution are avoided. For the actual layout 80m of draw column represents 40 kton for 13x13 and 50 kton for 15x15 layout. Figure 5-9 compares the height of draw at each panel for the base case and the NPV\_11M scenario.



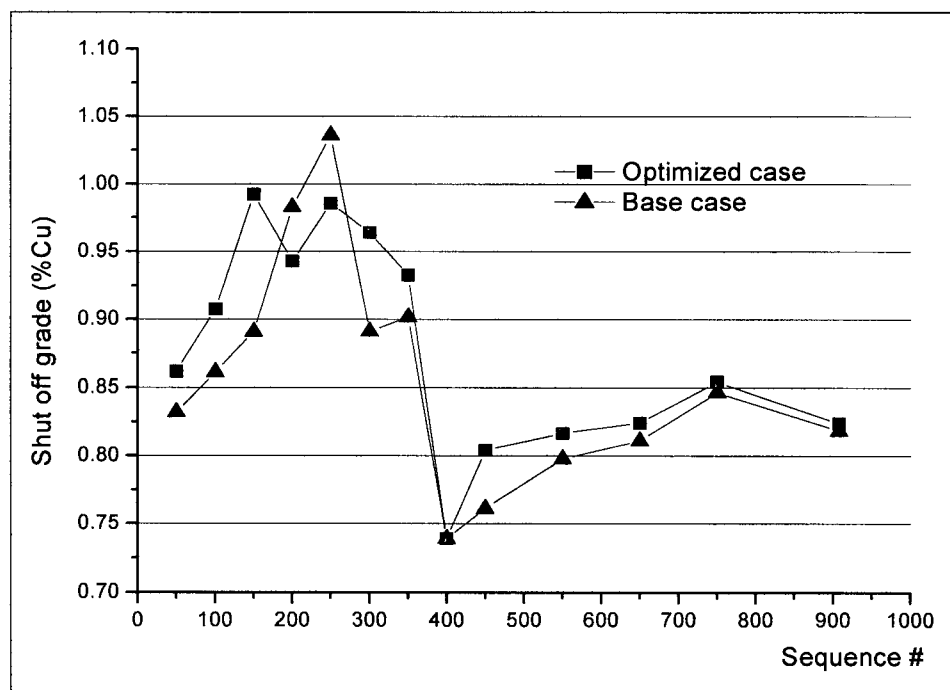
**Figure 5-9: Height of draw comparison between NPV\_11M versus base case schedule**

According to the above figure there might be potential overdraw from panel 53 and underdraw from panel 63. These two panels should be reviewed to ensure that no high stresses will be produced as a result of applying the optimization algorithm. Despite these two problems, the overall height of draw after NPV optimization looks fairly similar to the

base case scenario. Therefore this result should be acceptable from the draw control point of view at this stage in the planning process.

- *Shut off grades*

As expected the shut off grade for the optimized case will be higher than that used in the base case. This is probably one of the most notorious effects of introducing the opportunity cost in the definition of ore. This difference between shut off grades of the base case and optimized case should not be constant through the life of the mine. Rather this difference should decrease as the sequence number increases. Figure 5-10 represents the application of this concept at Andina mine.



**Figure 5-10: Shut off grades comparison between base case and optimized scenario**

From the above figure is possible to note that the first draw points in the sequence have a shut off grade higher than the base case scenario. As the sequence increases the difference becomes smaller. Therefore this result agrees with the theory of variable shut off grades.

### ***V.1.6 Effects of variable economic conditions on the overall schedule***

The second requirement from the mine was to include the variability of the economic parameters into the optimisation. Usually the main parameter to study is the metal price and its effects on the production schedule. The introduction of changing metal prices is an easy task for a base case scenario because for every single period of the schedule the depleted tonnages will be evaluated according to the defined price. The following table shows how the economic parameters are changed as the schedule progresses.



Step	Period	Price (\$/lb)	Met. Recovery	Smelting (\$/lb)	Rev. factor (\$/%)	Mining Cost
1	Ano_2002	1	0.85	0.20	14.9872	9
2	Ano_2003	1	0.85	0.20	14.9872	9
3	Ano_2004	0.95	0.85	0.20	14.0505	9
4	Ano_2005	0.95	0.85	0.20	14.0505	9
5	Ano_2006	0.85	0.85	0.20	12.1771	9
6	Ano_2007	0.85	0.85	0.20	12.1771	9
7	Ano_2008	0.75	0.85	0.20	10.3037	9
8	Ano_2009	0.75	0.85	0.20	10.3037	9
9	Ano_2010	0.7	0.85	0.20	9.367	9
10	Ano_2011	0.7	0.85	0.20	9.367	9
11	Ano_2012	0.85	0.85	0.20	12.1771	9
12	Ano_2013	0.85	0.85	0.20	12.1771	9
13	Ano_2014	0.95	0.85	0.20	14.0505	9
14	Ano_2015	1.1	0.85	0.20	16.8606	9
15	Ano_2016	1.1	0.85	0.20	16.8606	9

**Table 5-8: Economic parameters of the production schedule**

The above table shows the metal price (copper), the metallurgic recovery, the smelting and refining cost, revenue factor and mining and processing cost per period. Only the metal price was varied in this case study because the other parameters will be linked with a geo-metallurgic model that is under development.

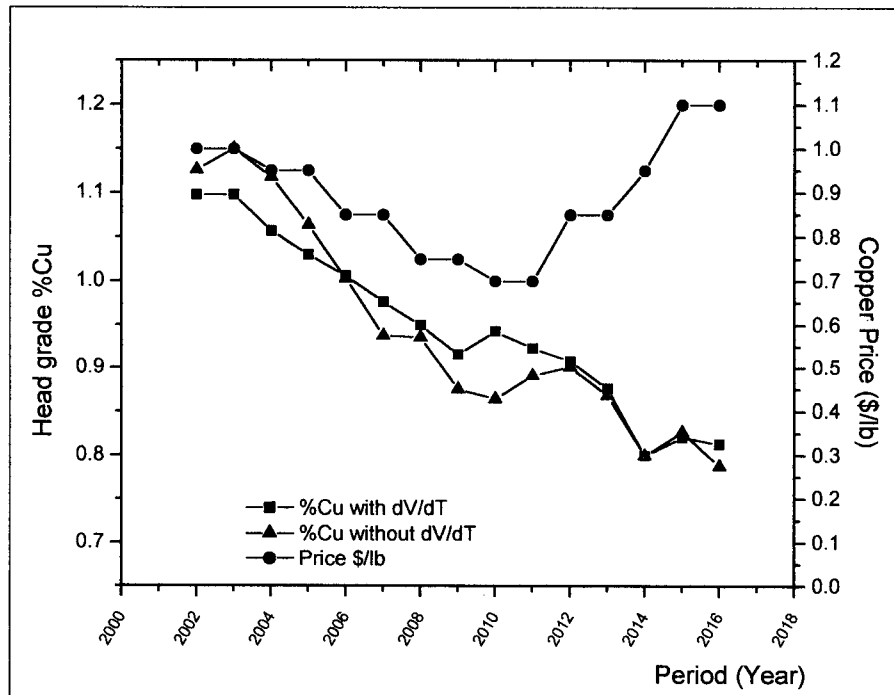
Once the economic parameters are introduced in the schedule the next step is to measure the effect of the variable metal price on the calculation of the opportunity cost. In section 3.5.2 it was mentioned that the opportunity cost has two components, the first one is the capital cost of an investment and the second one is the financial cost due to variable economic conditions in the market. This principle also applies to mining. For instance the economic value of a deposit will vary upon economic conditions. By introducing the term  $-dV/dT$  in the calculation of opportunity cost the variable economic conditions will be taken into account. The following table shows the effect of introducing  $-dV/dT$  in the calculation of opportunity cost. The base case scenario represents a scenario evaluated with variable price but without using variable shut off grade optimisation

	<b>Base Case</b>	<b>Optimized Case</b>
Closed dpts	943	895
Reserves (Mt)	289.7	296.2
NPV (M\$)	254.1	270

**Table 5-9: Comparison between base case scenario and optimised case varying metal price**

According to the above table the amount of optimisation is less than 3% in terms of NPV. However the number of draw points used to achieve those NPVs is fairly different: 943 for base case and 895 for the optimised case. For the optimised case there are still 50 draw points without draw by the end of year 2015. Consequently the amount of reserves grows by 6.5 million tonnes. Although these results perhaps may look irrelevant from a long term perspective, what is really interesting is how the production schedule behaves with the variable metal price in terms of grade forecast. The following figure shows the base case scenario and the optimised scenario and their relation with the metal price.

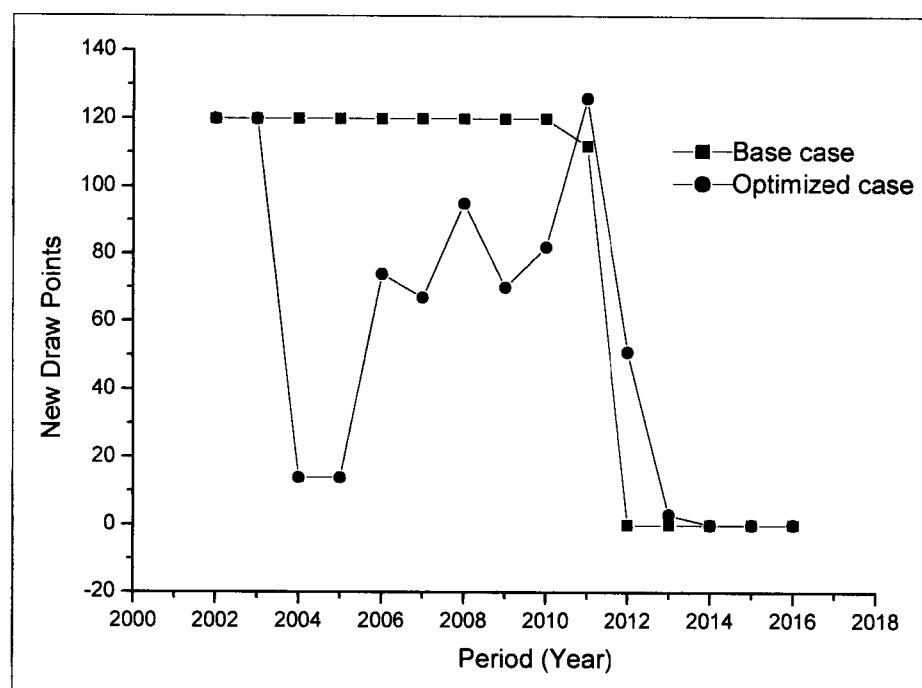
Note from the above figure that for the base case scenario the copper grade behaves regardless the metal price. For the optimized case (with  $dV/dT$ ) the result is different from a perspective that when the metal price decreases the grade also decreases accordingly. For example from the above figure when copper price goes beyond \$0.80/lb the optimized case proposes to use 0.85%Cu, while for the base case the grade is 0.95%Cu. This has significant implications from a strategic planning point of view. In fact the declaration made by the president of Codelco last year (2001) about reducing the copper production in 2002 was made based upon this analysis.



**Figure 5-11: Production schedule and its relation with the variable metal price**

The following figure shows the development rate resulting from the optimization. As a result of this is possible to say that given a low price the amount of development has to be reduced and as the prices improve the development is reactivated. This clearly shows that the grade as well as other mine planning components of the production schedule depend significantly on price.

Note from the above figure that as the price rises ( after 2012) the development rate is increased to provide enough flexibility to the mining system. This is exactly what is needed in situations when the metal price is high because draw points will be shut early in their life in order to just keep those with the highest grade in production.



**Figure 5-12: Resultant development rate after optimising the production schedule**

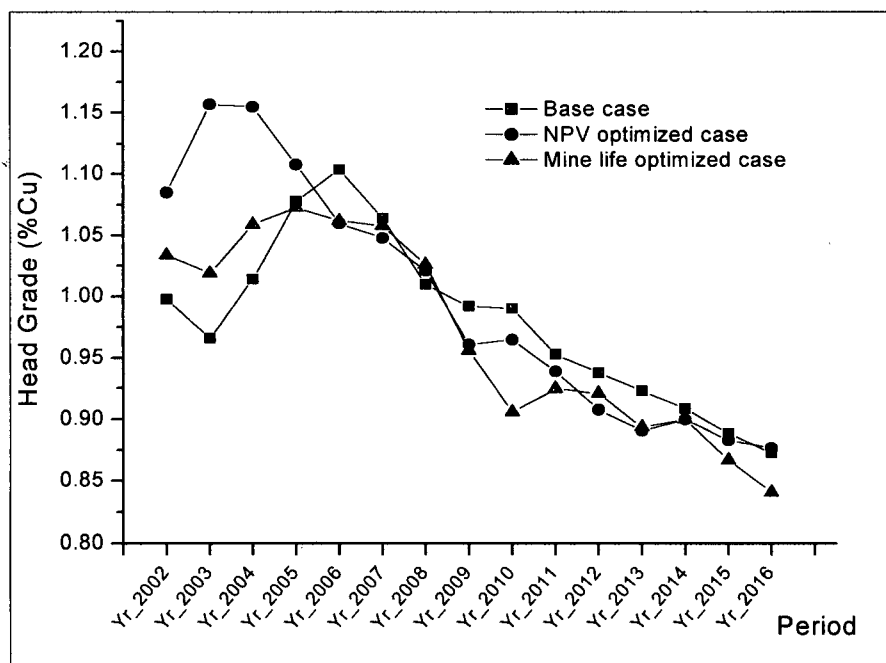
### *V.1.7 Production schedule mine life optimization*

Another requirement from the mining company was to run a schedule that reproduces a scenario of mine life optimization. The algorithm developed in section 4.1 was used to optimize the life of the mine through pursuing an angle of draw as the main goal. The production schedule was run using the draw function shown in Figure 4.5. The following table shows the main results of the schedule as well as a comparison of this method with the base case and the NPV optimization scenarios.

	Base Case	NPV	Mine Life
NPV (\$)	219,486,741	261,428,012	217,282,113
Net DV costs (\$)	27,856,477	37,775,780	44,055,822
%Cu	0.9800	1.0007	0.96762253
%Mo	0.0292	0.0291	0.02908003
%Pb	0.0016	0.0016	0.00164489
Wi (Kwh/tonne)	16.0461	16.0022	16.0638299
Reserves (tonnes)	237,007,056	208,440,656	245,432,624

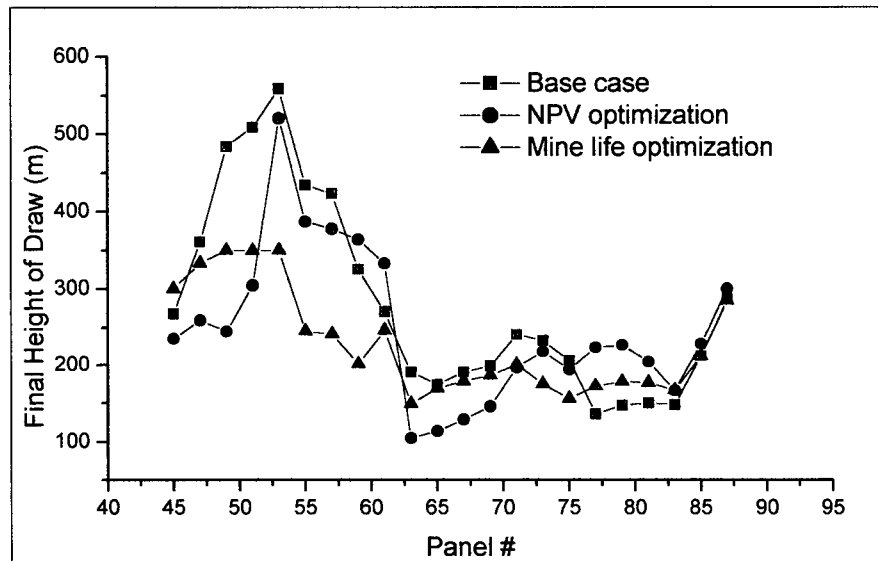
**Table 5-10: Mine life optimization comparison with NPV optimization and base case scenario**

From the above table is possible to note that there is a difference of about \$45million between mine life optimization and NPV optimization. This difference can be understood as the cost of performing proper draw control. Another difference is in terms of mining reserves, the mine life optimization produces 37 million tonnes more reserves, which can be depleted in about 4 years. Consequently there is a trade off between mine life optimization and NPV optimization from the net present value point of view. The following figure shows the grade profile for the different schedule scenarios.



**Figure 5-13: Grade profile for different production schedule scenarios**

It is possible to note from the above figure that for the NPV optimization scenario the grade in the first year is raised as much as possible to maximize the early return. Figure 5-13 shows the resulting height of draw for the different schedule scenarios.



**Figure 5-14: Resultant height of draws for different schedule scenarios**

Following the logic the smoothest height of draw is produced by the mine life optimisation schedule. This is expected because by construction the main goal of the angle of draw optimisation algorithm is to reproduce a smooth surface of height of draws through the life of the mine. Then the above figure demonstrates that the algorithm is behaving as it should.

### V.1.8 Discussion

The decision of whether to use the NPV optimised schedule or the mine life optimised schedule will depend upon the strategic planning at the corporate level. There are several factors to take into account such as metal availability, long term contracts, social aspects, and safety aspects. Then the decision has to be made on a case by case basis. Nevertheless it is always recommended to produce an analysis like this case study to quantify the pros and the cons of each scenario.

Although up to now there is no a formal formulation to reconcile the mine life optimisation with the NPV optimisation, there is one way of approaching the problem. For the mine life optimisation schedule a special draw function was used to reproduce the effect of following a draw angle as the schedule progresses. Then by introducing this draw function in the NPV

optimisation algorithm it might be possible to reconcile these two factors. Therefore the opportunity cost will be used to control the draw points closure while the draw profile will be controlled by the draw function. The value for the optimised scenario is expected to be lower, however this value will fulfil the desired draw control performance. It may be necessary to add more variables into the draw function such as neighbouring draw control and a mechanism to check for isolated draw points. These two enhancements have been proposed and are under development at this.

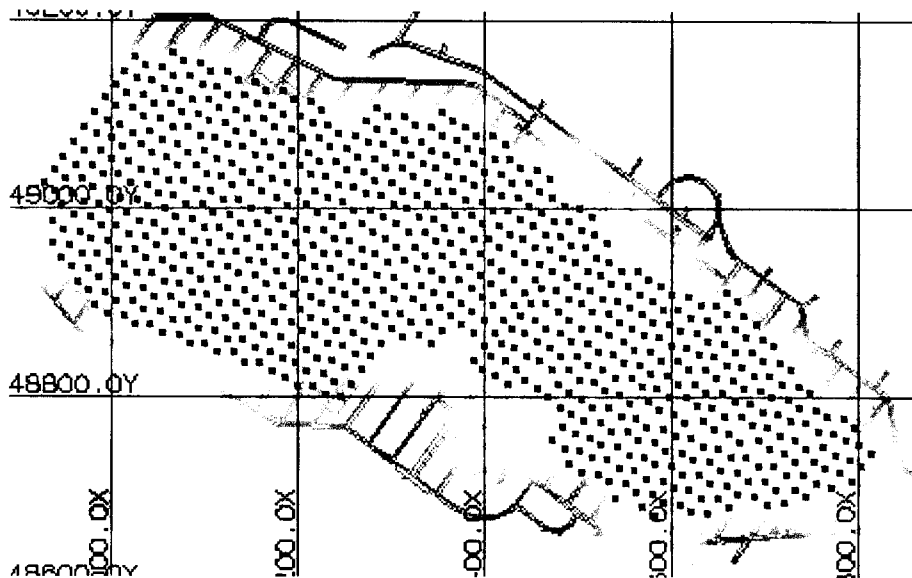
## **V.2 X mine**

Due to confidentiality agreements all the numbers presented in this section have been normalized. Nevertheless the trend of the study as well as the main results are still considered to be valid from a theoretical point of view.

### ***V.2.1 Mining System***

The mining system is panel caving with mechanized LHD. The mine layout corresponds to the Off Set model (Esterhuinzen, 1992). This design was chosen based on operational considerations rather than geomechanical ones. The following figure shows the mine layout specifying position of ore passes and material-handling infrastructure

The draw point spacing was established to achieve interaction between draw columns. The spacing between draw points belonging to the same draw bell is 15m and the spacing between neighbour draw points is 18m. These spacing gives a draw point area of 270m<sup>2</sup>.



**Figure 5-15: Layout of the production level**

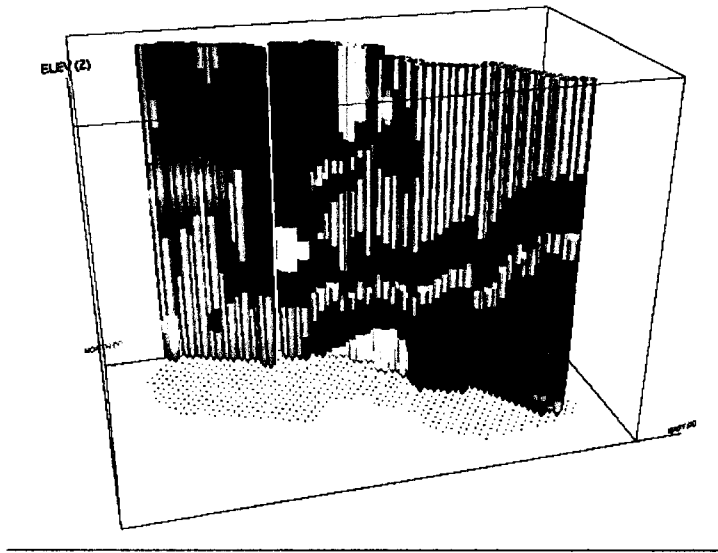
### ***V.2.2 Block Cave Model***

The diameter of the draw columns was designed to have interaction of at least 1 draw point. This means that horizontal flow of rock can be modeled if it is evidence of this behaviour. The diameter for the draw column was designed to be 14.5 m.

The following picture represents one production drift of the actual layout showing the copper grade.

Figure 5-16 shows a model that has been mixed previously to simulate that mixing that would occur as the mining process progresses. The height of interaction simulated was 90m with 2 iterations, the draw control factor has already been included in the estimation of the height of interaction.





**Figure 5-16: Draw columns of one draw bell line showing a diluted grade model**

### ***V.2.3 Economic boundary of the layout***

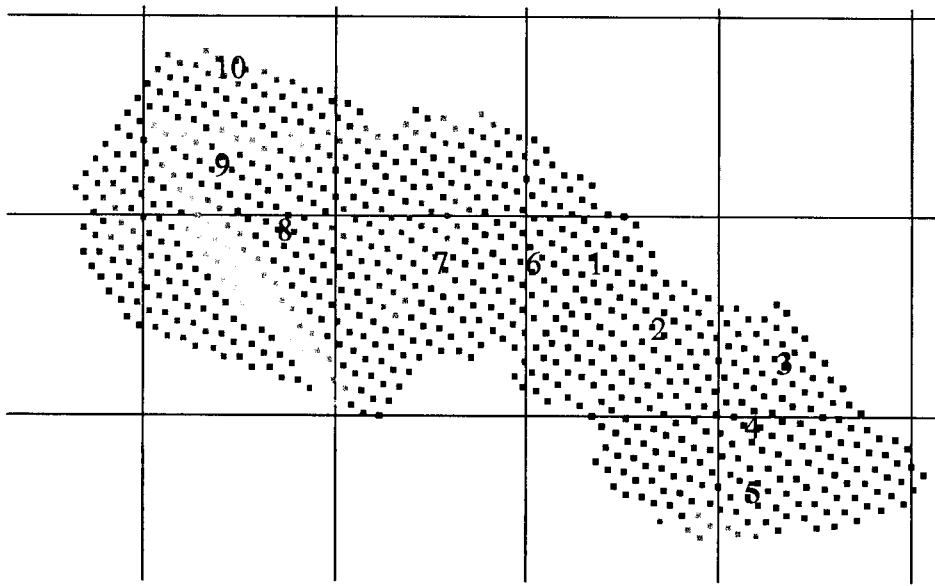
The economic boundary of the layout was given by personnel of mine X. No further calculations related to economic boundary were made.

### ***V.2.4 Production schedule base case scenario versus optimised scenario***

The base case scenario is represented by a throughput requirement of 35,000 tonnes of ore per year. The maximum development rate was set to be 110 new draw points per year. The base case scenario at this mine represents the optimisation of its life where the main goal is to reproduce an angle of draw.

- Undercutting Sequence

The undercutting sequence was designed based on geotechnical data and following geomechanic considerations. The main criterion used was to avoid abutment stresses and facilitate fragmentation until reaching hydraulic ratio. The following picture shows the macro undercutting sequence



**Figure 5-17: Undercutting sequence at X mine**

- **Draw Rate**

The draw rate was specified to follow a 6 inches per day in zones under high stresses and 10 inches per day in zones where there is enough opened area to dissipate stresses concentration.

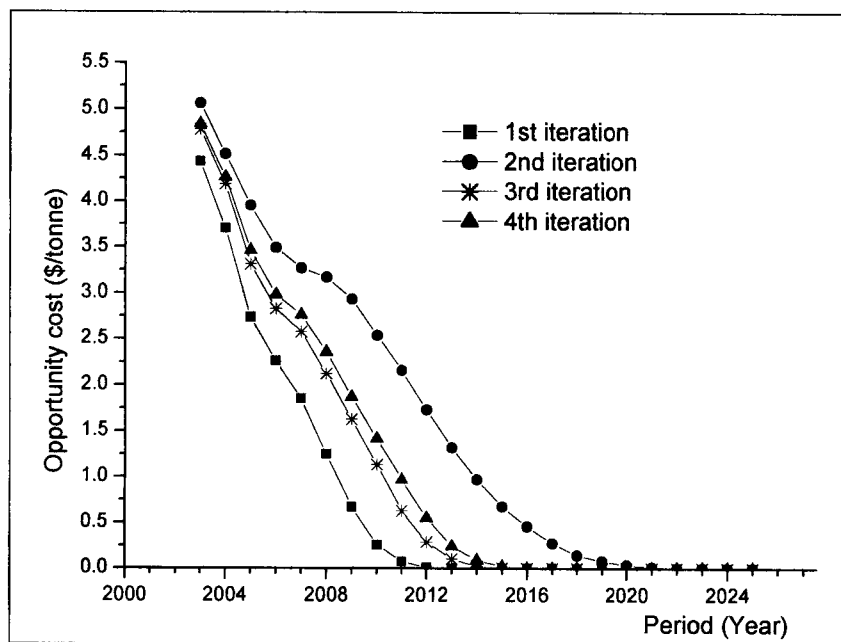
- **Resulting schedule**

Table 5-11 summarizes a comparison between the optimized scenario and the base case scenario. The NPV was optimized about 5% and the mining reserves reduced in one year of production. Despite the small amount of NPV optimization this result is still considerable interesting because the proposed schedule does neither change the undercut sequence, the draw rates nor the production rates. Another important observation is that the mean value of the schedule per ton represented by the attribute value is optimized in almost 13%.

	Base case	NPV Optimization
NPV	341,295,501	357,858,971
Cost	32,053,601	30,316,387
Tons	105,158,467	95,007,024
Value	4.27	4.81
Cu	0.58	0.64
G2	0.41	0.42
G3	3.15	3.53
Yrs	2,017.00	2,016.00

**Table 5-11: Summary of results after applying the NPV optimization algorithm**

The optimization process was carried out by applying 5 iterations. The following figure shows the resulting opportunity cost after 5 iterations.

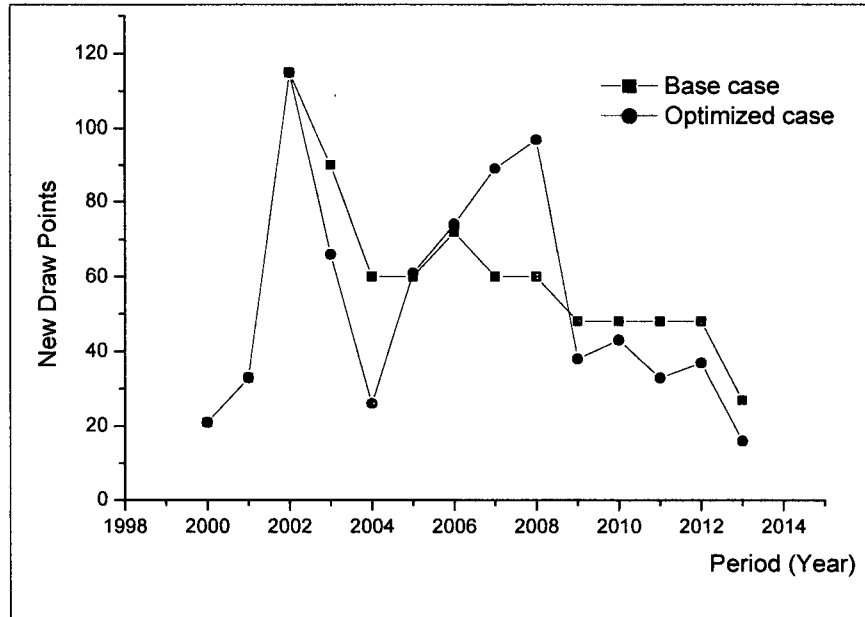


**Figure 5-18: Opportunity cost for different iterations of the NPV optimization algorithm**

Note that the above figure shows a clear convergence to the optimum set of opportunity costs or shut off grades. As discussed for the Andina case study the shape of this curve is non linear and it depends on the variability of the grade within the deposit.

- *Development rate*

The development rate is one of the results of the optimization algorithm. The following figure shows a comparison between the optimized case and the base case.



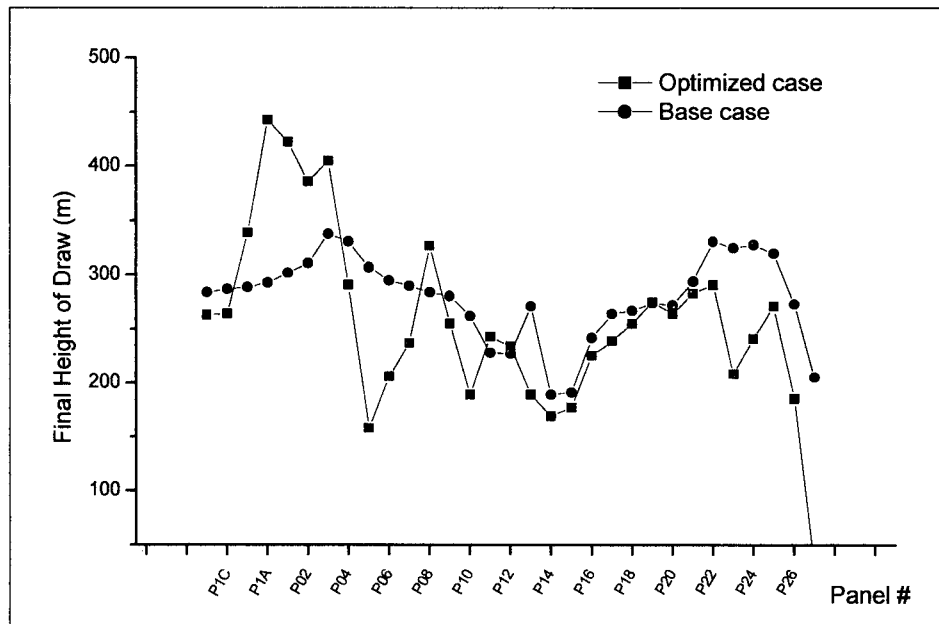
**Figure 5-19: Development rate for the base case versus optimized scenario**

The above proposed development rate needs to be smoothened so the rate does never get beyond 40 draw points per period. This was an extra constraint given by the operators when the propose schedule was presented. Due to the fact that this is a constraint for this particular mine it is believed that these modifications should be part of a second instance of reviewing and smoothing and not part of the optimization algorithm.

- **Heights of Draw**

Figure 5-20 shows the resulting height of draw at the end of the life of the mine. The original base case scenario of this mine was computed using the angle of draw algorithm, this is the reason why the height of draw representing the base case scenario are adequately smoothened. Although the height of draw representing the NPV optimization algorithm are more irregular it is believed that the effect of this behaviour should be quantified and finally evaluated against the potential NPV improvement. Another conclusion from the above figure is to question about mining panel 5,6 and 7. Perhaps the undercut sequence can be

changed so that panel 1D is mined towards panel 8. Therefore panels 5,6 and 7 will be mine by the end of the life of the mine. This will be a topic of a different evaluation and it is out of the scope of this case study



**Figure 5-20: Resultant heights of draw after optimizing NPV**

- *Effects of variable economic conditions on the overall schedule*

In this case the company does not reproduce a profile of prices due to strategic considerations. Therefore they operate regardless the metal price concentrating in the variables or mine planning parameters that they can control. As a result of this no evaluation was applied on this regard.

At the moment after evaluating this case study mine X is pursuing further analysis to present the modified schedule to the directory and finally decide whether to use the concept of variable shut off grade as a mine planning technique or not. In the second part of this study several undercut sequences will be evaluated to liberate the optimization from this constraint.

## VI CONCLUSIONS AND RECOMMENDATIONS

### VI.1 Summary

This thesis discusses in detail the design and planning of the block cave mining method. It then introduces an algorithm aimed to optimise the net present value based on introducing the opportunity in the production scheduling. An algorithm is also presented aimed to optimise the life of the mine by adjusting draw control practices. These methodologies have been applied to two case studies where the proposed algorithms have been used to formulate production schedules. This has enabled an analysis and evolution of the proposed methodology

### VI.2 Conclusions

The main conclusions of this research can be summarized as follows:

#### *VI.2.1 Development of mine planning concepts*

The introduction of the draw function as the main driver of the mine planning schedule in block caving has allowed the introduction of a series of heuristic rules to the determination of optimum production schedules. This mine planning tool would also allow mine planners to introduce more variables and parameters in order to evaluate different scenarios. For instance the remaining value of the deposit can also be plotted as another attribute of the layout. An optimization algorithm could eventually use that to draw the maximum value for any given period of the schedule. Another useful concept would be to add the draw control constraint so it can be graphically evaluated in the feasibility region, as the schedule progresses. There are also other ways to improve the draw function. However, this is believed to be a novel way of translating the block cave constraints into a graphical scenario.

Traditionally mining reserves have been computed before running the actual production schedule. Basically because it is part of computing the ultimate economic layout. However

in this research it is proposed to separate the definition of the economic layout from the definition of mining reserves. For instance in defining the economic layout a development cost per draw point has to be added into the cost structure. However in computing mining reserves this cost must be taken out because if a draw point is part of the layout it has already paid for its development. This research shows that there are several benefits of introducing the mining reserves estimation as part of the production schedule. One of the most important is that it couples the definition mining reserves with the block cave constraints such as draw rate, minimum height of draw, neighboring draw control. Another benefit of linking reserves definition with the production schedule is that it uses the best height of draw mechanism to shut draw points. Consequently the mining reserves will fulfill any one of the main constraints defined by the production scheduling.

Another enhancement to the traditional approach to block cave mining is the introduction of the development rate as a variable in the production scheduling. Traditionally this has been taken as a known parameter in the production schedule, however it is linked fully with the production decisions. The number of active draw points at any given time is a parameter that should be controlled closely in scheduling a block cave mine. Therefore development rate was included in this research as a variable in the scheduling process.

Draw control was introduced as a strategic goal of long term planning. Traditionally draw control has been left to the operators of the mine considering it to be an operational issue more than a planning issue. This research intended to demonstrate that draw control has an important impact on the life of the mine and its performance. Consequently an algorithm developed to optimize draw control as a main driver to maximize the life of the mine.

### ***VI.2.2 The introduction of opportunity cost into mine schedules***

The introduction of the variable shut off grade concept has successfully driven the net present value optimization. Although the introduction of the opportunity cost has reduced the amount of mineable reserves, it has successfully selected those reserves that produce the higher contribution to the maximization of the value of the operation. The opportunity cost

drops when metal prices decrease and increase when prices increase. Consequently the shut off grade drops when facing a scenario of low prices. The effect of reducing shut off grade is higher draw columns are mined, typically reducing the amount of metal produced. Mines should produce less metal when facing scenarios of low prices. This conforms to the basic laws of supply and demands. However this result has significant implications from a strategic planning point of view.

### ***VI.2.3 The introduction of draw angle in production schedule***

The optimization of the angle of draw successfully produced the smoothest height of draw and thus reduced the likelihood of experiencing early dilution or collapses. In this case the model helped to understand the problem and a draw function was designed to partially fulfill the requirements defined within the set of constraints. More research needs to be done in this area to design a draw function that would deal with draw control neighboring adjustments.

### **VI.2.4 The method to integrate NPV and mine life optimization**

Using the correct draw function along with the iterations of NPV optimization an adequate draw control can be performed. This draw function needs to have the uniformity index build in so it performs an angle of draw as it optimizes NPV. Hence the resultant schedule will optimize NPV and at the same time will satisfy all the main rules of the mining method

## **VI.3 Recommendations**

It was demonstrated in this research that within the current planning methodologies it is impossible to reconcile NPV maximization and mine life maximization. A way to approach this problem was specified without specifying details about the design of the proper draw function. A different draw function needs to be designed to take into account draw control within a cluster of draw points and maximum heights of draw. It will also be necessary to develop different indicators, which quantify the value of the schedule in a more holistic way,



integrating environmental and social aspects of production scheduling. These indicators should be quantifiable so that some of the algorithms developed in this thesis can be used.

There is still at the moment a lack of integration among mine planning parameters concerning the dynamics of caving. For instance draw rate has been taken in this research as a known parameter. However, there are several factors that ultimately will interfere in the definition of draw rate. For example, draw rate usually is associated with fragmentation but it has been found lately that it also depends on the behaviour of the rock mass and the likelihood of inducing air gaps. These two factors have not been included into the definition of the mathematical problem of scheduling. There are other mine planning parameters that are a function of the behaviour of the rock mass. Future research will need to define all these relationships so a more comprehensive mine planning model can be established.

At the moment the algorithm developed in this research to optimise NPV assumes several parameters to be constant, such as development cost, mining costs, draw rates. One important parameter is metallurgic recovery. It has been found in the literature (Camus, 2002) that there are significant opportunities, from an optimisation point of view, to examine the metallurgical recovery as a function of the mine throughput. Since metallurgical recovery is a function of the mill product size and this is a function of throughput, it is possible to link everything together in an algorithm that will optimise mine throughput and NPV at the same time. An algorithm to implement this concept still under development and it is believed it will change some traditional concepts of block cave mining design.

At this moment there are several parameters in the production schedule methodology that have been assumed to be constant, mainly because there are currently no planning tools to introduce the probabilistic behaviour of these parameters into the process of planning a block cave mine. These parameters include draw rates, grades, undercut sequence, development rate and air gaps. Also monitoring and instrumentation data in such mines does not tend to be integrated as part of the long term model, mainly because there are no tools to statistically couple this information with long term models. As a result the mine planning process tends to be fragmented and unrealistic. From a cost point of view, it has been observed that this is

due to the uncertain information available used to plan block cave mines. This usually results in inaccurate estimation of the active area actually needed to meet production targets. As a result of this either an under draw or an over draw is performed in the operation. Research needs to be done to develop techniques that will allow mine planners to integrate short and long term information in a more comprehensive system. This would be a “cave management system” which would be based upon a probabilistic rather than a deterministic approach to planning a block cave mine.

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