OPTIMIZATION TECHNIQUES APPLIED TO THE TRADING OF NATURAL GAS FROM A RESTRICTED STORAGE FACILITY

by

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ABSTRACT

We apply operations research techniques to the trading of natural gas from a restricted storage facility. In particular, we consider a gas trading firm that has entered into a one-year contract for gas storage whose terms restrict the balance held at any time and the quantities withdrawn and injected each day. Based on price forecasts assumed to be accurate, mathematical models identify optimal trading plans each day with the objective of maximizing net trading proceeds for the year.

We conduct a two-part numerical study. First, we contrast the financial performance of a year of trades prescribed by the optimization model to the outcome according to a "naïve" strategy, which represents the strategy of a hypothetical trader unaided by such a model. Comparisons are made using perfect price information, to isolate the value of optimization, and using forecasted prices, in order to make a more practical evaluation. Second, we compare the financial outcomes of our optimal trading decisions when hedging is required and when it is not, in order to gain insight into the opportunity cost of hedging.

As expected, optimization models lead to better financial performance than naïve strategies in all instances when perfect price information is in place. When prices are forecasted and trading decisions are therefore hampered by forecast inaccuracy, the optimization model outperforms the naïve strategy on average although not in every instance. The second component of our analysis suggests that a hedging policy, while eliminating price risk, has a dramatic opportunity cost.

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CHAPTER 1
INTRODUCTION

1.1 BACKGROUND

Natural gas is a "fossil fuel." Fossil fuels originate from dead plant and animal matter deposited with the mud and silt of bodies of water millions of years ago. Heat and pressure transform the soft organic material into fossil fuels in solid form as coal, in liquid form as crude oil, and in gaseous form as natural gas; inorganic material becomes sedimentary rock. Layers of organic and inorganic material form sedimentary basins in various regions of the world. The shaded areas in the map below indicate the basins of sedimentary rock in Canada.*

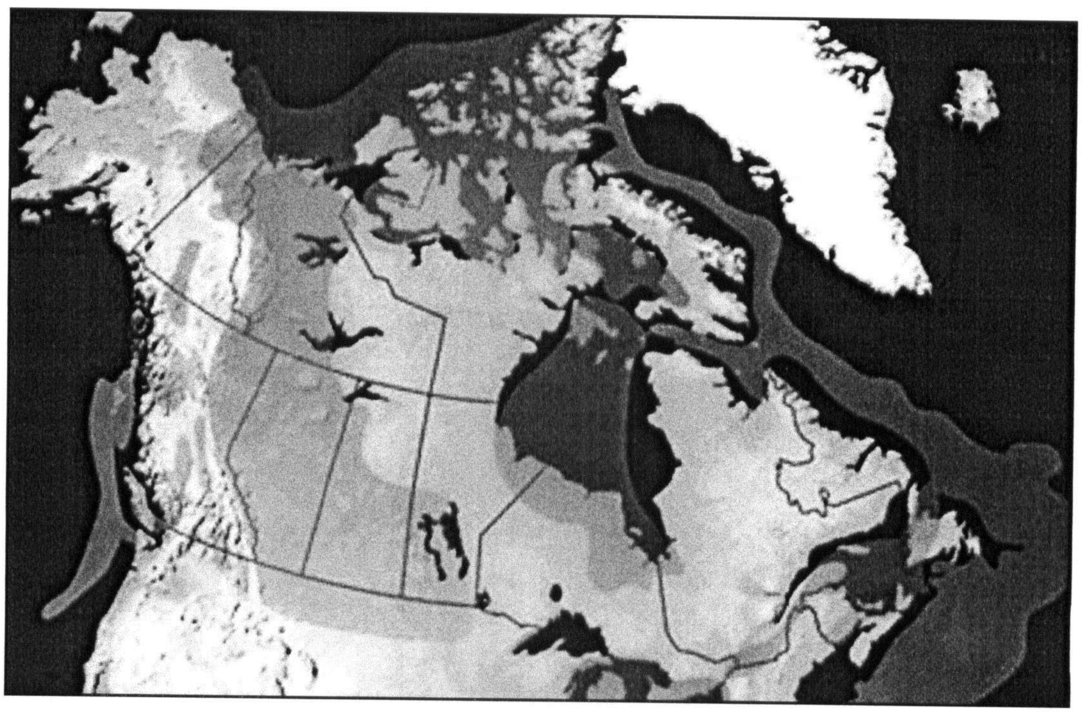

Figure 1.1
Sedimentary Basins in Canada

Every province and territory of Canada lies over at least part of one of the basins. The majority of the land area of Alberta, Saskatchewan, and the Atlantic provinces are covered by these basins. A major event in the development of Canada's fossil fuel resources was an oil strike at Leduc, Alberta, on February 13, 1947. The 1940s also saw several major finds of natural gas in Alberta. In the 1970s, natural gas and oil were discovered off the coast of Nova Scotia.

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* Map reproduced with permission from the Canadian Association of Petroleum Producers

Canada is the third-largest producer of natural gas in the world. In 2000, Canada's average daily production was 17.1 billion cubic feet (BCF), which generated about $37.8 billion in revenues. 9.7 BCF of this production was exported.*

The oil and gas industry has three major sectors: The upstream sector comprises companies that conduct and support exploration and production. The midstream sector includes pipeline owners and operators responsible for connecting the producers of oil and gas with the consumers. The downstream sector consists of oil refineries, gas distributors, oil product wholesalers, service stations, and petrochemical companies, among others.

The transmission network is the foundation of the North American natural gas market, which brings together producers, distributors, and speculators, and is depicted in the map below.†

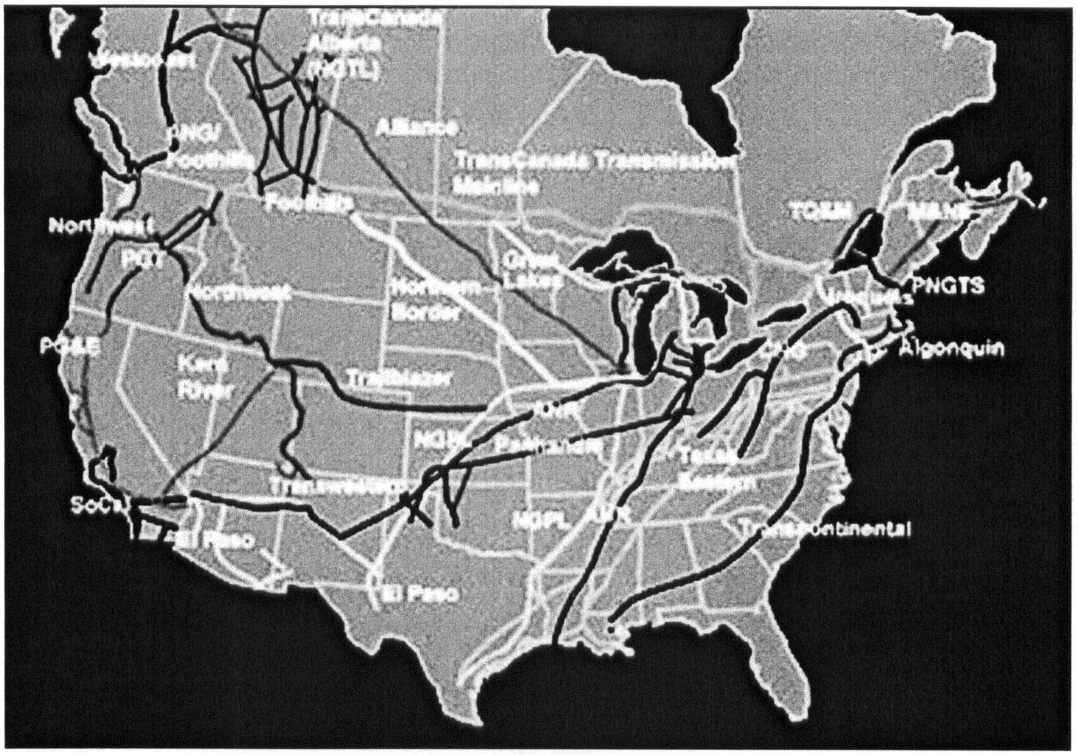

Figure 1.2
Natural Gas Transmission Pipeline Network in North America

The two sponsors of this research, BC Gas and Engage Energy, are part of the downstream segment. BC Gas Inc. is a public company based in Vancouver whose shares are traded on the Toronto Stock Exchange. The most prominent of its several subsidiaries is BC Gas Utility Ltd. Its primary function is to supply natural gas to residential and commercial consumers in British Columbia. It is the largest distributor of gas in B.C. serving 762,000 customers in more than 100 communities throughout the province. BC Gas falls under the regulatory authority of the BC Utilities Commission.

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* Canadian Association of Petroleum Producers Industry Facts and Information, Canada page, <http://www.capp.ca/default.asp?V_DOC_ID=603>

† Map reproduced with permission from the Canadian Association of Petroleum Producers

Engage Energy Canada L.P. of Calgary is a wholly owned subsidiary of Westcoast Energy Inc. of Vancouver, which is a wholly owned subsidiary of Duke Energy of Charlotte, North Carolina.* It is a gas marketing company with about 160 employees, mostly in Calgary, who are engaged in the trading and analysis of natural gas and related assets. Engage does not have obligations to gas consumers as BC Gas does. Its gas trading activities have the single purpose of generating profit as a result of the advantageous acquisition, storage, and disposition of gas in the marketplace.

Once produced at the well, natural gas is stored in huge, naturally occurring regions of porous rock beneath the surface of the earth that are surrounded by impervious rock. Gas storage companies hold parcels of land containing these formations and provide the means with which gas can be compressed and pumped into the earth for storage and extracted again for sale or use. Gas distributors and marketers such as BC Gas and Engage generally lease access to these facilities. Storage contracts run April 1–March 31 and reserve for the lessee a certain capacity for the entire contract year. The contract terms also include daily limits on the volume of gas withdrawn and injected. These limits may be fixed or functions of the current balance of gas in storage.

Gas distributors generally fill their leased storage space during periods of low customer demand and low market prices to minimize their exposure to high market prices when customer demand is high. Gas marketers generally use storage facilities to capitalize on very short term market price fluctuations, turning their gas holdings over more frequently than distributors do. Gas is bought and sold on the market in various contracts that result in delivery on the same day as the trade or anytime up to several years in the future. Such "futures" or "forward" contracts enable the trader to reduce or eliminate exposure to price risk.

A gas trader may buy a quantity of gas with the expectation that the price will rise and create an opportunity for a profitable sale at a later date. The trader takes on price risk because the future behaviour of the market price is uncertain. He or she may be compelled to sell the gas before the higher price materializes. This type of trading is sometimes termed "speculating." On the other hand, a trader may buy a quantity of gas for delivery on the same day and simultaneously sell, at a higher price, the same quantity for delivery on a future date. The trader has locked in a profit margin and has eliminated the price risk. This tactic is called "hedging."

1.2 PROBLEM DESCRIPTION

In this work, we apply optimization techniques to the trading of natural gas from a restricted storage facility, both with a hedging policy in place and without. (Hedging will be discussed in more detail in Chapter 3.) We have two objectives: first, to identify a trading decision-making model that results in a more profitable year of buying, storing, and selling natural gas than the judgement of a hypothetical trader does; second, to learn about the opportunity cost of hedging.

The question at the root of any trading decision is, "Is this price a 'good' one?" One might suppose that this can be answered adequately in terms of expectations for the price in the future. For example, one strategy might be to buy as much as possible if the price is expected to be

* At the time this document was written, Duke Energy, having recently acquired West Coast Energy, was in the process of disbanding Engage and absorbing its activities into the Duke trading operation in Calgary.

higher tomorrow than it is today and to sell as much as possible if the price tomorrow is expected to be lower. Another strategy might be to compare the price each day to a reference price such as the forecasted average price for the month, season, or year, and to buy as much of the asset as possible if the current price is lower than the reference price and sell as much as possible if the current price is higher. However, gas trading strategies that are based entirely on the current price and expectations for it fail to consider a particularly important factor: the balance currently in storage.

Consider that when several units of gas are purchased at a low price, one can be virtually certain of capturing a higher price for the first unit sold in the future, reasonably confident for the second unit, but decreasingly confident for subsequent units. Because there is a limit on the quantity of gas that can be withdrawn each day, several days may pass before the last unit from the original purchase can be sold, by which time the higher price may no longer prevail. Therefore, each unit of gas added to storage has less value than ones added previously because its expected selling price is lower. Similarly, each unit of gas withdrawn from storage has more value than ones previously withdrawn because the expected cost to replace it is higher. (Scott, Brown and Perry 1)

Our problem, then, is not simply to forecast prices and succeed at betting against them. It is to identify the best trades to make over the course of the year *given the gas in storage during that period* and based on the forecasted prices.

1.3 OUTLINE OF APPROACH

Our approach to the problem comprises the following:

- Time series analysis of real gas market price data to obtain a model for forecasting daily prices over a one year horizon,
- Formulation of mathematical models for optimal trading, and
- Numerical studies with these models in order to draw inferences.

1.3.1 Time Series Analysis

The original objective of our times series analysis was to identify a stochastic price process that was a valid representation of natural gas price behaviour as much as a year in advance. Such a model was to be used in the formulation of an optimization model that would take price uncertainty into account.

We performed our analysis on five years of actual spot prices and prompt prices from a major North American trading hub. (Spot prices, also called "day prices," are the prices for day contracts, which result in delivery on trading day.) A prompt contract results in repeated delivery everyday of the month following the trade. The analysis consisted of multiple linear regression of the current log spot price on the log spot prices of several previous days and the log prompt price. We eventually conceded that, within the scope of this project, we could not identify a stochastic price process that was supported by the data, would lead to a tractable optimization, and had error terms that could reasonably be assumed to be independent.

Temporarily relaxing our requirement for independent error terms, we tested the forecasting performance of a price model of the form described above and with sufficiently few terms to maintain the tractability of the optimization. We found that this model performed no better as a one year forecaster of spot prices than the one-month forward curves did on their own.

The one-month forward curve, or simply "forward curve," is a series of prices, effective one day, for gas delivery everyday for each of several successive, later months. For example, the forward curve on July 3 would include a price for gas delivery everyday of August, a price for gas delivery everyday of September, and so on. A prompt contract is just a particular one-month forward contract; it is the one that results in delivery during the month immediately following the current month. On July 3, for example, the August one-month forward and the prompt are the same contracts.

Thus, for the purpose of input to our optimization models, we base our price forecasts entirely on the one-month forward curves.

1.3.2 Formulation of Trading Models

We present three trading models: a model for trading in day contracts only (without hedging), a model for trading in day and prompt contracts without hedging, and a model for trading in day and prompt contracts with hedging. All of these models identify the optimal trading plan for the entire horizon based on the actual prices of the current day and the forecasted prices of future days. We formulate the first as a linear program, as a deterministic dynamic program, and as a stochastic Markov Decision Process; we formulate the second and third as linear programs only.

1.3.3 Numerical Study

Our numerical study consists of two components that align with along the lines of our major objectives. In the first component, we compare the performance of strategies based on the optimization models (the "optimal strategies") to the performance of other, hypothetical strategies that are meant to represent human judgement (the "naïve strategies"). We evaluate the competing strategies under various scenarios that incorporate different trading instruments, a hedging policy or not, and different daily withdrawal and injection limits. We present the results of 360 simulations that utilize the price data obtained from the major gas trading hub mentioned earlier.

In the second component of our analysis, assuming the optimal trading strategies, we compare the outcomes of trading day and prompt contracts when hedging is required to when it is not. Our interest here is the opportunity cost of hedging. We present the results of 90 simulations based on the same five years of actual price data as discussed above.

The presentation of our results consists of graphs, tables, and discussion. Each graph depicts the operation of a storage facility—the daily withdrawing and injecting of gas resulting from trades made—according to competing strategies. Each table presents the financial performance of the competing strategies as measured by total annual value realized from trading.

CHAPTER 2
FORECASTING PRICES

The major input to each optimal trading model is the set of daily prices forecasted for the current day to the following March 31 for each instrument being traded. In all models, our forecasts of spot and prompt prices are based entirely on the one-month forward curves. The models assume that all forecasted prices are accurate.

Our original objective was to formulate models that would take price uncertainty into account. As we are concerned with daily transactions over a one-year horizon, we required a stochastic process that was a valid representation of daily gas price behaviour at least one year in advance.

A search of the literature yielded material pertaining primarily to forecasting at a level too high to be helpful to us. For example Ger, Gonzales, Mayberry and Shamszadeh, in their work on the feasibility of capturing hydrocarbon seepage from the ocean floor off California, consider three different approaches to forecasting gas prices. In all of these approaches, the forecasting horizon ranges from five to 20 years with a price determined every half-year at best. (81–85)

Scott, Brown and Perry, in their paper that inspired our project, forecast daily prices and incorporate uncertainty as follows:

> We define a suitable set of parameters—for example, mean price path, volatility, strength of mean reversion—that we consider adequate to model market price structure to create a large sample of potential price paths. These are condensed into a probability distribution for the gas price on each day of the year … (2)

We did not use the approach of Scott, *et al.* for price modeling because to identify and estimate the parameters of the price structure at the gas hub we are studying would constitute a significant research project itself, well beyond the intended scope of this optimization research.

Thus, we attempted to develop our own price model using time series analysis. In the following we describe the data used in the time series analysis, outline our model development attempts, and explain the forecasting methods used *in lieu* of a sound stochastic price model.

2.1 PRICE DATA FROM THE HENRY HUB

The data we use in our time series analysis and later in our numerical study is from the Henry natural gas grading hub, in Louisiana. The data set consists of spot prices and one-month forward curves as observed every trading day during April 1, 1997–March 31, 2002. Observations for weekends and holidays are missing because trading does not occur on those days. One of the simplifying assumptions made in the formulation of the trading models, as discussed in Chapter 3, is that trading occurs everyday of the year. When data are not available for a particular calendar day, we assume that the prices of that day are equal to those of the most recent day for which there are data.

The spot price data consist of daily prices for which day contracts were traded, that is, the unit price for gas delivery on the same day as the trade. The following figure shows the movement of the spot price at the Henry Hub during the five-year period in question:

Figure 2.1
Spot Prices Observed at Henry Hub
April 1, 1997–March 31, 2002

The dramatic peak corresponds to the California energy shortage of late 2000–early 2001.

A one-month forward price is the current price per unit of gas under a contract that will result in delivery everyday for a particular month in the future. A one-month forward curve, or simply "forward curve," is a series of current prices for one-month forward contracts for gas delivery during each of many successive months. For example, given a current date of November 14, 2003, the forward curve would comprise about 60 prices: the unit price for gas to be delivered everyday during December 2003, the price for gas to be delivered during January 2004, and so on up to the unit price for gas to be delivered everyday during November 2008. The data set we utilize includes the forward curves that were observed on every trading day during April 1, 1997–March 31, 2002.

The following figure shows the forward curve observed every first-of-the-month during the five years of interest. (A forward curve exists in our data for every trading day; to prevent clutter, only one per month is shown.)

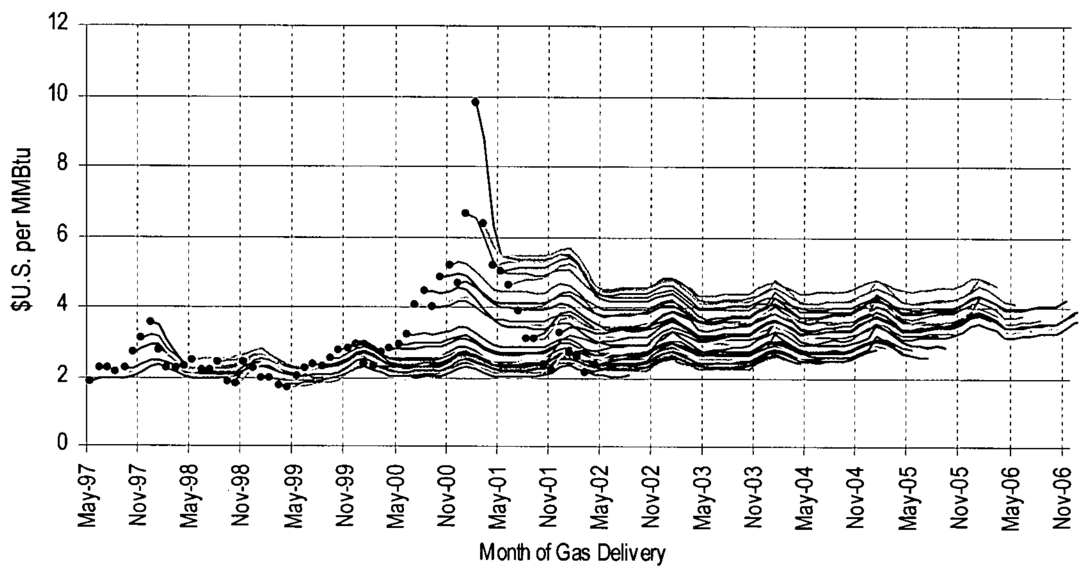

Figure 2.2
Selected Forward Curves Observed at Henry Hub
April 1, 1997–March 31, 2002
(Dots mark starts of curves.)

These plots reveal two interesting relationships. First, each heavy dot indicates the start of a curve and corresponds to the unit price for gas delivery in the month following the month of observation. An imaginary line joining these dots traces a path similar to that of the spot prices in Figure 2.1. This demonstrates that the prompt price is usually not far from the spot price.

Second, the curves are flat from May to November, peak in February, lie within a band of approximately \$2–5, and show only a slight upward trend. If the start of the curve (which reflects the current spot price according to the observation made in the paragraph above) is beyond the band, then, over several months, the curve reverts to the nearer limit of the band and stays approximately at that level.

2.2 A PROSPECTIVE STOCHASTIC PRICE MODEL

Our original objectives included formulating a stochastic optimization model. To that end, we attempt to identify a stochastic price model from the market price data described in the previous section. A price model of the following general form was assumed:

$$q_t = \alpha + \sum_{j=1}^{k} \beta_j q_{t-j} + \phi g_{m+1} + \varepsilon_t$$

where

q_t = log spot price on day t,
m = the month in which day t is and $m + 1$ = the following month,
k = number of autoregressive terms,
g_{m+1} = one-month forward log price on day t for gas delivery in the month following the month in which t is,
α, β_j, ϕ = parameters to be estimated from the Henry Hub data, and
ε_t = random error term on day t.

The number of autoregressive terms, k, is to be both low enough to lead to a tractable optimization and high enough to lead to insignificant autocorrelation among error terms when out-of-sample forecasts were made.

We estimated the parameters α, β_j, and ϕ by minimizing the sum of squared differences between actual log spot prices, q_t, in a one-year set of Henry Hub prices and the function, $\hat{q}_t(q_{t-1},\ldots,q_{t-j},g_{m+1})$, as follows:

$$\min_{\alpha,\beta_j,\phi} \sum_{t=1}^{365}(\alpha + \sum_{j=1}^{k}\beta_j q_{t-j} + \phi g_{m+1} - q_t)^2$$

Our attempts to identify a valid and useable stochastic price model were fruitless. With only a small number of autoregressive terms in the model, it was too complex to be useful for these reasons:

(a) Intractability: If we maintain a desirable level of precision when specifying the discrete sets of gas balances and prices indicated in this problem, we need to accommodate *billions of billions* of states. See further comment on "dimensional explosion" in Section 3.2.3 under "Discussion."

(b) Imprecision: Conversely, if we had forced our states to a manageable count of, say, 400,000, we would have been hampered with a truncated price range of \$1.00–5.00 U.S. per million British thermal unit (MMBtu) and \$1 precision. The Henry Hub data ranges from \$1.030 to \$10.500 with $^1/_{10}$-cent precision.

At the same time, the model was insufficiently complex to fully explain the spot price. Even as we added autoregressive terms well beyond a number that would permit a tractable optimization, autocorrelation was prevalent.

Hence, we conceded that it was impossible within the scope of this work to identify a valid price model that would support stochastic optimization models.

Last, we consider a model of the form just discussed, but excluding the random term, in order to obtain good single-point forecasts for use with deterministic optimization models. We name it "ARF(3,1)" in a tongue-in-cheek variation on the naming convention of the ARIMA time series modelling method; it is expressed algebraically as follows:

$$\hat{q}_t = \alpha + \sum_{j=1}^{3}\beta_j q_{t-j} + \phi g_{m+1}$$

Based on mean absolute percentage error (MAPE)*, this model performed approximately as well as the forward curve alone in out-of-sample tests. The following table presents the MAPE measures for April 1–March 31 and October 1–March 31 out-of-sample forecasts made based on the data of each of four years and using the two methods:

* See page 12 for a formal definition of MAPE.

Year in which Forecast Made	Date Forecast Made (Date of Last Actual Price)			
	March 31		September 30	
	ARF(3,1)	Forward Curve	ARF(3,1)	Forward Curve
1998	31%	34%	32%	37%
1999	15%	13%	14%	16%
2000	39%	38%	20%	20%
2001	85%	85%	18%	19%

Table 2.1
MAPE Forecasting Performance Measures of the ARF(3,1) Price Model and the Forward Curve

We conclude that no stochastic price model is available to support stochastic optimization models and that no autoregressive model out-performed the forward curve. Hence, as expanded on in the following section, we base our price forecasts solely on the forward curve.

2.3 FORWARD CURVES AS BASES FOR PRICE FORECASTS

We generate both "step" and "smooth" spot price forecast functions from the forward curves. Which of the two is used in particular simulations is discussed in Chapter 4. The two forecasting methods perform approximately as well as one another, as discussed in Section 2.3.3.

2.3.1 Step Forecast Function

In the step function, the forecasted spot price for day s is equal to the one-month forward price for gas delivery in the month following the month in which s is. The forward prices used in determining the forecasted spot price for all days s are those that make up the forward curve as observed on the current day, t, where $s > t$. Expressed algebraically, the step spot forecast function is the following:

$$\hat{p}_s = f_{m+1} \tag{2.1}$$

where

m is the month in which day s is, $m + 1$ is the following month, and
f_m is the currently observed one-month forward price for gas delivery in month m.

2.3.2 Smooth Forecast Function

Here we specify the forecasted spot price, $\hat{p}_s$, for day s to be the point on the curve that reflects constant daily growth from one of two "starting" prices to the one-month forward price, f_{m+1}, for gas delivery in the next month. The "starting" point of this constant growth segment of the forecast curve is either (1) the actual price of the current day, p_t, if s is in the current month, or (2) the one-month forward price, f_m, for gas delivery in the month that s is, if s is in some later month. Expressed algebraically, the smooth spot forecast function is the following:

$$
\hat{p}_s = \begin{cases} f_{m+1}^{(\frac{s-t}{d_{m+1}-t})} \, p_t^{1-(\frac{s-t}{d_{m+1}-t})} & \text{when } s \text{ is in the current month} \\ f_{m+1}^{(\frac{s-d_m}{d_{m+1}-d_m})} \, f_m^{1-(\frac{s-d_m}{d_{m+1}-d_m})} & \text{when } s \text{ is in some later month, and} \end{cases} \tag{2.2}
$$

where d_m is the first day of month m.

2.3.3 Performance of Forecast Functions

Each of the following two figures shows five pairs of forecasts, which are made as described above, against actual spot prices. The forecasts in Figure 2.3 are based on forward curves observed on successive firsts-of-April, as if each April 1 were the current day. Thus, these are one-year forecasts for the period April 1-March 31 for each of the five years.

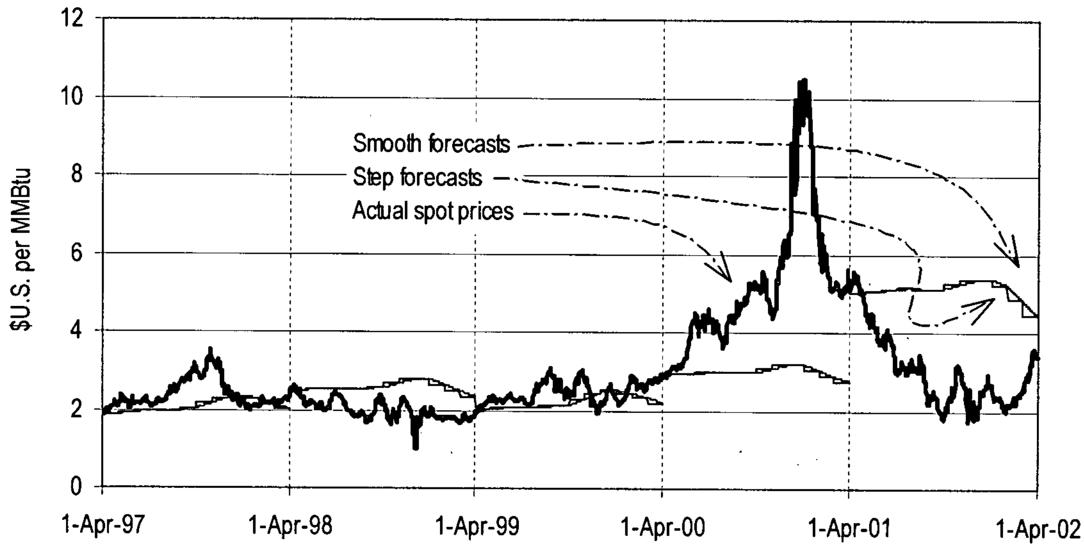

Figure 2.3
Five Pairs of Competing One-Year Forecasts Made March 31

The forecasts in Figure 2.4 are based on forward curves observed on successive firsts-of-October, as if each October 1 were the current day. These are, therefore, six-month forecasts for October 1–March 31 each year.

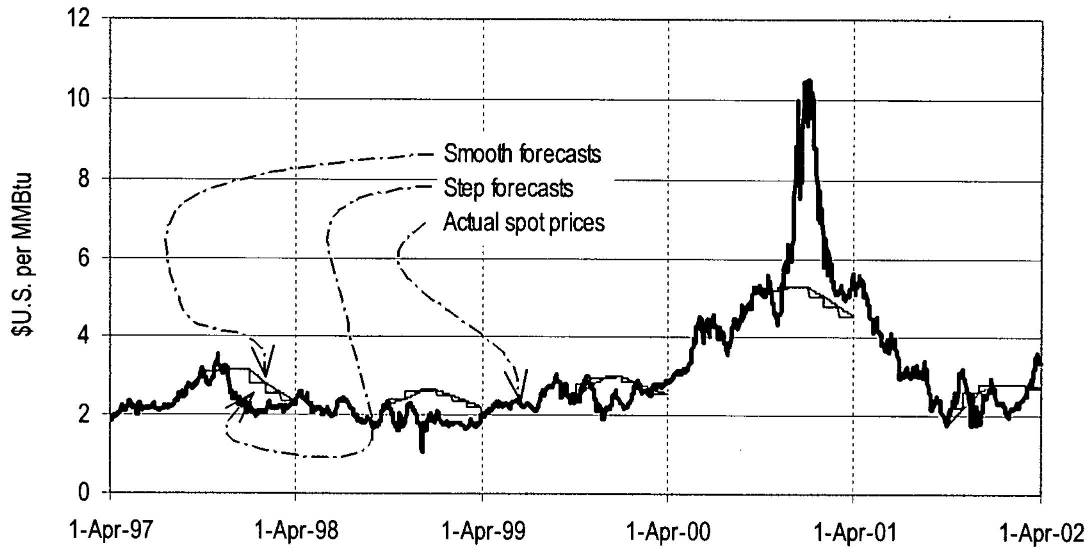

Figure 2.4
Five Pairs of Competing Six-Month Forecasts Made September 30

We use mean absolute percentage error (MAPE) to measure the performance of these forecasting methods. Each of the MAPE measures cited below is the average of the daily forecast errors arising from all days in the forecast horizon and from forecasts made on all days of the year and of all five years. Expressed algebraically, the MAPE measure is as follows:

$$MAPE = \frac{\sum_{y=1997}^{2001} \sum_{t=1}^{n_y - 1} \sum_{s=t+1}^{n_y} \frac{\hat{p}_{s,y} - p_{s,y}}{p_{s,y}}}{\sum_{y=1997}^{2001} \sum_{t=1}^{n_y - 1} \sum_{s=t+1}^{n_y} 1} \times 100\%$$

where

y = the year in which the storage contract year starts,
$\hat{p}_{s,y}$ = the forecasted price for day s of year y,
$p_{s,y}$ = the actual price on day s of year y,
n_y = the number of days in year y,
t = the day on which the forecast for the remainder of the storage contract year is made,
s = the day of the particular forecasted price, and
$s > t$.

Our performance measures of the step and smooth spot price forecasting methods calculated on the spot price and forward curve data observed at the Henry Hub during April 1, 1997–March 31, 2002 are the following:

$MAPE_{\text{step}} = 26.4\%$
$MAPE_{\text{smooth}} = 26.5\%$

Because these two numbers are virtually equal, we consider the methods interchangeable with respect to reliability. In our simulations described in Chapter 4, we use the simpler step function as often as we can and use the smooth function when the situation requires it, as will be discussed.

2.3.4 Forecasts of Prompt Prices

All optimization models involve the trading of day contracts; some incorporate prompt contracts as well. In simulations involving the latter, we forecast the future prices of forward contracts to be equal to the prices of those contracts on the current day, *t*.

CHAPTER 3
TRADING MODELS

In this section we present the formulations of the three optimization models for natural gas trading with which we perform our numerical study. These models differ from one another in the number of instruments, or types of contracts, available for trading and in whether hedging is required.

The first model reflects a situation involving only one instrument and, therefore, does not involve hedging; in particular, only day contracts are available for trading. An example of a trade would be nothing more than the purchase of ten units of gas today resulting in the receipt of those ten units later today.

The second model introduces a second instrument for trading, although hedging is not required in this situation. Under this situation, day and prompt contracts are available for trading. An example of a trade under this situation would be the sale of ten units on a day trade and the sale of one unit on a prompt contract. This trader would be required to deliver ten units of gas today and one unit everyday next month.

The third model accommodates the same two gas trading instruments, but, unlike the second model, hedging is required. An example of a trade under this scenario would be the purchase of two units on a prompt contract and the sale of 60 units on a day contract. The resulting deliveries would be 60 units withdrawn from storage today and two units injected into storage everyday next month. Note that after all of these deliveries are made, no net change in the gas balance has occurred. This is a hedged trade. (This example assumes 30 days in next month.)

All of these models take into account the best price information about the entire time horizon that is available at the time the model is solved and look for a solution for the entire horizon on that basis. Specifically, they find the optimal trades for today and the rest of the days of the year based on the actual market prices today and the forecasted market prices for the rest of the year. Uncertainty of prices is not taken into account for the most part. That is, all models are formulated using deterministic prices and all numerical analysis in Chapter 4 assumes that prices are certain. However, in Section 3.2.3 we present an additional general formulation of the first model assuming a stochastic price process.

When a practitioner's implementation of any of these models is simulated, say for days 1–365, the part of the optimal solution that is acted on is only the trade prescribed for day 1. That trade is assumed to be made, a day passes, the market prices change, and the model is solved again. An optimal trading plan for days 2–365, as identified by solving the model on day 2, is not necessarily the same plan as for days 2–365 as identified by solving the model on day 1. Apart from the possibility of multiple optimal solutions, this is because the trading plans identified on the two days are based on different market price information.

3.1 SIMPLIFYING ASSUMPTIONS

The following are the simplifying assumptions we have made. While they do not exactly reflect reality, they do not affect the concepts or the nature of the models and they do provide for clearer notation and more concise explanation. We believe that the effect on results is minimal.

(a) The true timing of cash settlement for trades is ignored in these models; we assume that the reward is received at the time the trade is made. In reality, settlement is usually made on or about the 20th of the month following the trade. In this work, we are not considering the time value of money.

(b) We assume that trading and gas delivery occur 365 days per year. In reality, delivery occurs year-round while trading occurs on only about 250 business days per year. Allowing trades only on business days could have been accomplished in either of two ways: by adding severe penalties to the prices on weekends and holidays or by constraining the primary decision variables on those days to equal zero.

(c) We assume that trading in forward contracts and the resulting deliveries align exactly with the calendar months. In reality, forward contracts are available to be traded only up until a few days before the end of the month in question and the resulting deliveries start and end a few days before or after the start of a subsequent month. To model the timing of deliveries precisely would not be conceptually more difficult but would lead to more error-prone programming and unwieldy descriptions.

(d) Other than the price of gas, the only cost we consider is the cost of transmission, which is proportional to the quantity of gas and is assumed to equal 3¢ U.S. per unit withdrawn or injected*. We ignore such costs as per-trade charges and the opportunity cost of capital.

(e) Trades are assumed to be made once per day. In reality, prices change and trades can be made throughout the business day.

3.2 MODEL A: DAY CONTRACTS ONLY

We assume only day contracts are available for trading. Gas changes hands on the same day as it is purchased or sold. A purchase of one unit of gas on the current day results in the receipt of one unit of gas on the current day. We formulate the model three ways: as a linear program, as a deterministic dynamic program, and as a stochastic Markov Decision Process.

3.2.1 As a Linear Program

The linear program is formulated algebraically as follows:

* 5¢ Cdn. was the recommended assumption from BC Gas.

Decision variables

Primary:

w_t = units of gas to be sold and withdrawn on day t
i_t = units of gas to be purchased and injected on day t

Ancillary:

b_t = balance of gas in storage at the beginning of day t

where

$t' \in \{1, 2, \ldots, 365\}$ is the day on which the model is being solved,* and
$t \in \{t', t'+1, \ldots, 365\}$ is the trading day in question.

Data and Parameters

p_t = spot price per unit of gas on day t; actual price if $t = t'$, forecasted price otherwise
c = transmission cost per unit of gas
m_b = maximum gas in storage allowed by contract
m_w = maximum daily gas withdrawal allowed by contract
m_i = maximum daily gas injection allowed by contract

Objective Function

The objective of the optimization is to maximize the net trading proceeds of the entire year and is expressed algebraically as follows:

$$\underset{w_t, i_t}{\text{Max}} \left\{ \sum_{t=t'}^{365} w_t (p_t - c) - i_t (p_t + c) \right\}$$

This maximized function represents the net proceeds from all sales and purchases on all days remaining in the storage contract year including the current day. It reflects the sales proceeds from every withdrawal at the unit price on the day in question less the transmission costs minus the cost of injected gas at the unit price on the day in question plus the transmission costs.

Constraints

(a) The balance of gas at the beginning of the current day less the withdrawals of the day plus the injections of the current day equal the beginning balance of the following day.

$$b_{t+1} = b_t - w_t + i_t \quad \forall\, t = t', t' + 1, \ldots, 365$$

* In leap years, this index would range up to 366. While only non-leap years are mentioned in this document, leap years are handled appropriately in the numerical analysis.

(b) The balance of gas in storage must be zero at the beginning and at the end of the storage contract. Each day, the balance must be between zero and the maximum allowed by the storage contract.

$$b_1 = b_{366} = 0$$
$$0 \le b_t \le m_b \ \ \forall\ t = t', t' + 1, \ldots, 365$$

(c) The quantity of gas sold and withdrawn from storage each day must not exceed the maximum allowed by the storage contract:

$$w_t \le m_w \ \ \forall\ t = t', t'+1, \ldots, 365$$

(d) The quantity of gas purchased and injected into storage each day must not exceed the maximum allowed by the storage contract:

$$i_t \le m_i \ \ \forall\ t = t', t'+1, \ldots, 365$$

(e) The quantity of gas sold or purchased each day must be non-negative:

$$w_t,\ i_t \ge 0 \ \ \forall\ t = t', t'+1, \ldots, 365$$

Discussion

The primary decision variables of this linear program are the quantities of gas to be sold and withdrawn from storage and the quantities of gas to be purchased and injected into storage on each day of the year. Therefore, as examples, when the program is solved on April 1, the first day of the April 1–March 31 storage contract year, there are $2 \times 365 = 730$ primary decision variables; when it is solved on March 22 with ten days left in the storage contract year, there are 20.

This problem can be formulated as a non-linear program with half as many primary decision variables. The quantity traded would be represented by a single vector, q_t, with $q_t < 0$ reflecting a sale and $q_t > 0$ reflecting a purchase. The objective function in this case would be piecewise linear in the variables, q_t, with slope $-(p_t - c)$ when $q_t < 0$ and slope $-(p_t + c)$ when $q_t > 0$. See Figure 3.1 below.

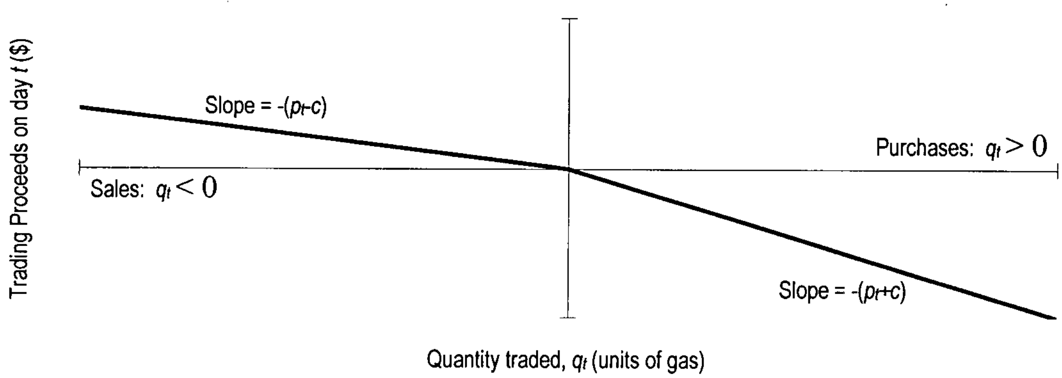

Figure 3.1
Objective Function in Non-Linear Formulation

Such a concave piecewise linear objective function can be transformed into a linear function—a non-linear program transformed into a linear one with no need for approximation—by replacing each variable q_t with two non-negative variables w_t and i_t where $q_t = i_t - w_t$. This is the approach taken with our linear programming formulation described above.

Under certain circumstances, the linear program described above is also a maximum reward network flow problem, specifically, when the storage contract parameters—the maximum balance, m_b, and the maximum daily withdrawal and injection limits, m_w and m_i—are fixed. Furthermore, if these three parameters are all integer multiples of some number, N, then the optimal trades will also be integer multiples of N. More formally, this is the integrality property of network flow problems as described by the following proposition: "The linear program $\max\{cx : Ax \le b, x \in \mathbb{R}^n_+\}$ has an integral optimal solution for all integer vectors b for which it has a finite optimal value if and only if A is totally unimodular." (Wolsey 40)

The figure below depicts this trading problem as a maximum-reward network flow problem and is followed by the remainder of the formulation.

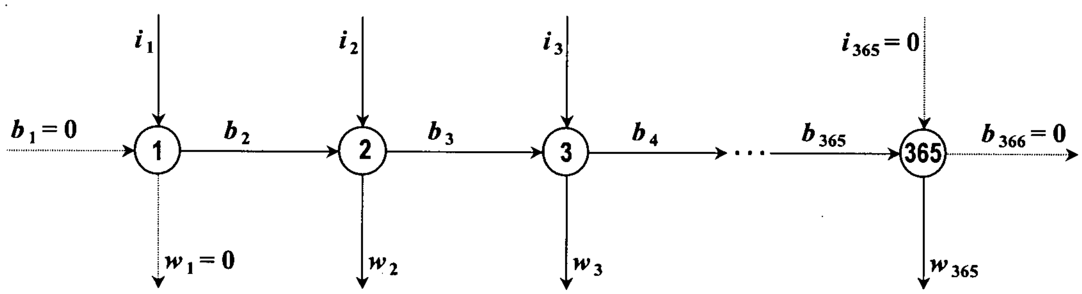

Figure 3.2
Optimal Trading Problem as a Network Flow Model

Balance equation where total inflow equals total outflow:

$$b_t + i_t = w_t + b_{t+1} \ \forall \ t \in \{1, 2, \ldots 365\}$$

Constraints arising from the storage contract:

$$0 \le w_t \le m_w,$$
$$0 \le i_t \le m_i, \text{ and}$$
$$0 \le b_t \le m_b, \ \forall \ t \in \{1, 2, \ldots 365\}$$

Starting and terminal conditions:

$$b_1 = b_{366} = 0$$

Objective function:

$$\text{Max}\left\{\sum_t^{365} w_t(p_t - c) - i_t(p_t + c)\right\} \ \forall \ t \in \{1, 2, \ldots 365\}$$

Note that the characteristics of our particular problem require the addition of two constraints to the network flow formulation. First, because "short" positions are not possible, $w_1 \equiv 0$. Second, because of the terminal condition $b_{366} = 0$, $i_{365} \equiv 0$. (A short position is created when a trader sells units of an asset he or she does not own, in hopes of the price dropping, effectively borrowing them from the market to do so.)

3.2.2 As a Dynamic Program

In this formulation, we assume that the state and action spaces are integer. This is not necessary in that different techniques than those described below are available for solving continuous variable Markov Decision Process. We address our assumption that the state and action spaces are discrete under "Discussion" at the end of this section. The formulation as a discrete time and discrete state dynamic program follows.

Decision Epochs

One trading decision is assumed to be made each day. The horizon starts on any day of the storage contract year and runs to day 366, the day after the expiration of the storage contract.

$$T = \{t', t'+1, \ldots, 366\}$$

States

States represent possible balances of gas in the storage facility, which range from 0 (empty) up to the smaller of two quantities: (1) the maximum allowed by the storage contract (full), and (2) the maximum that can be withdrawn in the remaining days of the storage contract in order for the facility to be empty at the end of the contract term. ("Empty" and "full" are used with respect to the holdings of our gas trading firm, not necessarily with respect to the entire facility which may also store the gas of many other firms.) A single unit corresponds to the smallest quantity for which a trade can be placed. The time-specific set of possible states is the following:

$$S_t = \{0, 1, \ldots, \min[m_b, m_w(365 - t)]\} \quad \forall\, t \in \{1, 2, \ldots, 365\}$$
$$S_{366} = 0$$

Actions

Available actions represent the possible trading quantities. These action sets are state- and time-specific. The largest withdrawal available given the day and the gas balance on hand is a non-positive number whose magnitude is the smaller of (1) the balance on hand, and (2) the maximum daily withdrawal allowed by the storage contract. The largest injection available, given the day and the gas balance on hand, is non-negative and the smaller of (1) the amount that would bring the resulting balance to its highest allowable level, and (2) the maximum daily injection allowed by the storage contract. The injection that would bring the resulting balance to its highest allowable level is itself based on the minimum of two numbers; it is the current balance subtracted from the minimum of (1) the number of days remaining in the storage contract times the maximum daily withdrawal allowed by the storage contract (that is, the maximum that could be completely withdrawn by the end of the storage contract leaving the facility empty), and (2) the maximum gas balance on hand allowed by the storage contract. The state- and time-specific action sets are the following:

$A_{s,t} = \{-\min[b_t, m_b], \ldots, \min\langle\min[m_w(365 - t), m_b]\text{-}b_t, m_i\rangle\}$ $\forall$ t $\in$ {1, 2, ..., 365}

No actions are taken in the final epoch, $t = 366$.

Note that since the variable a is a function of both state (balance of gas on hand) and time, the variables w, i, and r are as well. For notational simplicity, however, we omit the state subscript, s, from here on in this dynamic programming formulation.

Rewards

Periodic rewards represent net trading proceeds and depend only on the action taken, that is, the trade made; they are not affected by the state, that is, the balance of gas on hand. Rewards equal the net selling price times the quantity withdrawn less the net buying price times the quantity injected with at least one of the quantity withdrawn and the quantity injected being equal to zero.

$$r_t(a_t) = (p_t - c)w_t - (p_t + c)i_t \quad \forall \text{ t} \in \{t', t' + 1, \ldots, 365\}$$

where $a_t = (w_t, i_t)$.

State Transitions

The state transition equation, in this deterministic formulation with no uncertainty taken into account, reflects that only one state can exist next period given the current state and current action.

$$b_t - w_t + i_t = b_{t+1} \quad \forall \text{ t} \in \{t', t' + 1, \ldots, 365\}$$

Optimality Equation

The optimality equation yields the greatest possible value available on day t as a function of the gas balance. The argument of the maximization, $a_t = (w_t, i_t)$, is the optimal trade to be made on day t.

$$\begin{aligned} v_t^*(b_t) &= \max_{a_t \in A_t} \left\{ r_t(a_t) + v_{t+1}^*(b_{t+1}) \right\} \\ &= \max_{w_t, i_t} \left\{ -w_t(p_t - c) + i_t(p_t + c) + v_{t+1}^*(b_t - w_{t+1} + i_{t+1}) \right\} \\ &\forall \text{ t} \in \{ t', t' + 1, \ldots, 365\} \end{aligned}$$

Discussion

Note that if the constraint that decision variables are integer is added to the linear programming formulation in the previous section, such an integer programming formulation and the deterministic dynamic programming formulation described immediately above will yield identical results.

The integrality property of network flow problems described near the end of Section 3.2.1 provides the justification for assuming a discrete state space as we do in the dynamic program formulation immediately above and in the Markov Decision Process below. That is, when the storage parameters—the maximum balance, m_b, and the maximum daily withdrawal and injection limits, m_w and m_i—are constant and integer, the linear program becomes a network flow problem with necessarily integer solutions. In Section 3.3, we introduce a second gas trading instrument and in Section 5.2 we introduce variable contract parameters. Either of these features may lead to potentially non-integer solutions.

3.2.3 As a Markov Decision Process

Here we present a stochastic price process in general form to demonstrate a formulation of the optimal trading problem that takes price uncertainty into account. This formulation is similar to the deterministic dynamic program in many respects.

The general stochastic price process we present is the following:

$$Q_t = \alpha + \beta_1 Q_{t-1} + \beta_2 q_{t-2} + \ldots + \beta_k Q_{t-k} + \varepsilon_t$$

where

Q_t = log of the spot price P_t on day t,
$\alpha, \beta_1, \beta_2, \ldots, \beta_k$ are parameters, estimates of which would be obtained, and
ε_t are independent and identically distributed random variables.

That is, the log spot price is a linear function, with non-zero intercept, of the log spot prices of the previous k periods plus a random error term; error terms are assumed to be independent of those of previous periods.

The formulation as a finite horizon Markov Decision Process is as follows.

Decision Epochs

As in the deterministic dynamic programming formulation, one trading decision is assumed to be made each day; the storage contract year runs exactly to March 31.

$$T = \{t', t'+1, \ldots, 365\}$$

States

In this stochastic formulation, the states reflect not only the balance of gas on hand but the current spot price and the spot prices of $k - 1$ previous days. The state space therefore has $k + 1$ dimensions.

$$S_t = \{(b_t, p_t, p_{t-1}, \ldots, p_{t-k+1})\}$$

where

$b_t \in \{0, 1, \ldots, \min[m_b, m_w(365 - t)]\}$ as in the definition of the state space in the deterministic dynamic programming formulation in Section 3.2.2, and

$p_t \in \{p_L, \ldots, p_U\}$ where p_L and p_U are lower and upper bounds, respectively, on the range of possible prices

Actions

Actions represent the quantity to be traded. They are specified as they are in the deterministic dynamic programming formulation in Section 3.2.2, as follows:

$$A_{s,t} = \{-\min[b_t, m_b], \ldots, \min\langle \min[m_w(365 - t), m_b]\text{-}b_t, m_i\rangle\} \quad \forall\, t \in \{1, 2, \ldots, 365\}$$

Recall from the formulation as a dynamic program that since the variable a is a function of both state and time, the variables w, i, and r are as well. We continue to omit the state subscript, s, for notational simplicity.

Rewards

Rewards represent the periodic net trading proceeds. They also are specified as they are in the deterministic formulation, as follows.

$$r_t(a_t) = r_t(w_t, i_t) = (p_t - c)w_t - (p_t + c)i_t \quad \forall\, t \in \{t', t' + 1, \ldots, 365\}$$

State Transition Probabilities

The state transition probabilities reflect the probability of attaining a certain state in the next period—this state characterized by the gas balance on hand, the current price, and the prices of the k–1 previous periods—conditioned on the current state having existed and the current action having been taken, as in the first line below. In this problem, only prices are uncertain, since gas balances resulting from balances previously on hand and trades previously made are certain. It is therefore equivalent to state the transition probabilities as they are in the second line below. Furthermore, these transition probabilities are represented by the discrete probability distribution, π_ε, of the random error term, ε, in the stochastic price process.

$$\begin{aligned}
&\Pr\left[S_{t+1} = (b_{t+1}, p_{t+1}, p_t, \ldots, p_{t-k+2}) \mid S_t = (b_t, p_t, p_{t-1}, \ldots, p_{t-k+1}), a_t\right] \\
&= \Pr\left[P_{t+1} = p_{t+1} \mid P_t = p_t, P_{t-1} = p_{t-1}, \ldots, P_{t-k+1} = p_{t-k+1}\right] \\
&= \Pr\left[\varepsilon_{t+1} = \hat{\varepsilon}\right] \\
&\equiv \pi\left(\hat{\varepsilon}\right)
\end{aligned}$$

where

$$\hat{\varepsilon} = \log p_{t+1} - \left(\alpha + \beta_1 \log p_t + \beta_2 \log p_{t-1} + \ldots + \beta_k \log p_{t-k+1}\right), \qquad (3.1)$$

uppercase characters such as P_t and Q_t represent random variables, and the corresponding lower case characters represent specific values of the random variables.

Optimality Equation

The argument of this maximization, $a_t = (w_t, i_t)$, is the optimal trade to be made on day t. It yields the greatest available expected value on day t, which comprises the net trading proceeds of the day and the expectation, conditioned on values of ε, of maximum value to be obtained on day t+1.

$$\begin{aligned} v_t^*(s_t) &= \max_{a \in A_{s,t}} \left\{ r_t(a_t) + \sum_{s_{t+1} \in S_{t+1}} \Pr(S_{t+1} = s_{t+1} \mid S_t = s_t, a_t) v_{t+1}^*(s_{t+1}) \right\} \\ &= \max_{w_t, i_t} \left\{ \begin{aligned} &-w_t(p_t - c) + i_t(p_t + c) + \\ &\qquad \sum_{\varepsilon} \pi(\hat{\varepsilon}) v_{t+1}^*(b_t, w_t, i_t, p_t, p_{t-1}, \ldots, p_{t-k+1}, \varepsilon) \end{aligned} \right\} \end{aligned}$$

where

$s_t = (b_t, p_t, p_{t-1}, \ldots, p_{t-k+1})$, and
$\hat{\varepsilon}$ is defined by Equation 3.1 above.

Discussion

As mentioned in a previous section, the stochastic formulation renders the problem practically intractable. The dimensional explosion is due to the many variables of which the spot price is a function. If price is a function of k variables, then the state space will have k+1 dimensions: the balance of gas on hand, the current price, and the prices of k–1 previous periods. The number of possible states that would require consideration are

$$\prod_{j=1}^{k+1} n_j$$

where

n_j = number of possible values of variable j, and
$j \in \{1, 2, \ldots, k+1\}$ and corresponds to $\{b_t, p_t, p_{t-1}, \ldots, p_{t-k+1}\}$.

To demonstrate this dimensional explosion, we assume that the maximum allowable gas balance is 1,500 units; that price ranges between the minimum and maximum actually observed at the Henry Hub during April 1, 1997–March 31, 2002, specifically \$1.030–10.500 per MMBtu; that the applicable price increment is $^1/_{10}$ of a cent; and that the current price is a function of the prices of only four previous periods. Then the number of states are

$$(1500 + 1) \times [(10.500 - 1.030)/0.001 + 1]^4 = 1.2 \times 10^{19}.$$

3.3 MODELS B & C: DAY CONTRACTS AND PROMPT CONTRACTS

Under this scenario, day contracts or prompt contracts or both can be traded, where a day contract results in delivery of gas on the current day and a prompt trade results in delivery of gas everyday of the month following the month in which the trade is made. For example, buying one unit on a day contract and two units on a prompt contract today would result in delivery of one unit of gas today and two units of gas everyday next month.

3.3.1 Model B: Hedging not Required

The linear program is formulated algebraically as follows:

Decision Variables

Primary:

$w_{j,t}$ = units of gas sold on day t on contract j

$i_{j,t}$ = units of gas purchased on day t on contract j

Ancillary:

b_t = balance of gas in storage at the beginning of day t

where

$t' \in \{1, 2, \ldots, 365\}$ is the current day, the day on which the model is being solved,

$t \in \{t', t'+1, \ldots, 365\}$ is the trading day in question, and

$j \in \{1, 2\}$ corresponds to day and prompt contracts respectively.

Objective Function

$$\text{Max}\left\{\sum_{t}^{365}\left[(p_t - c)w_{1,t} - (p_t + c)i_{1,t} + (p_t - c)(e_{m+1} - e_m)w_{2,t} - (p_t + c)(e_{m+1} - e_m)i_{2,t}\right]\right\}$$

where

$p_{j,t}$ = unit price of gas per contract or instrument j on day t,

m = the month in which t is, m+1 = the following month,

e_m = the day of the horizon that is the last day of month m (e.g., e_{May} would be 61), and

c = unit cost for transmission of gas.

Constraints

(a) The beginning balance of the current day less the withdrawals of the current day plus the injections of the current day equal the beginning balance of the following day:

$$b_t - w_{1,t} + i_{1,t} = b_{t+1} \quad \forall\ t \text{ in April}$$

$$b_t - w_{1,t} + i_{1,t} + \sum_{u=d_{m-1}}^{e_{m-1}} (-w_{2,u} + i_{2,u}) = b_{t+1} \forall\ t \text{ in May–March}$$

Delivery on each April day arises from any day contract trade made on the same day. Delivery on every other day of the year arises from any day contract trade made on the same day and all prompt contract trades made on all days of the previous month.

(b) The net delivery each day (injections less withdrawals) must be within the limits of the contract:

$$-m_w \le -w_{1,t} + i_{1,t} \le m_i \quad \forall\ t \text{ in April}$$

$$-m_w \le -w_{1,t} + i_{1,t} + \sum_{u=d_{m-1}}^{e_{m-1}} (-w_{2,u} + i_{2,u}) \le m_i \ \forall\ t \text{ in May–March}$$

where

d_m = the day of the horizon that is the first day of month m (e.g., d_{June} would be 62), and
e_m = the day of the horizon that is the last day of month m (e.g., e_{June} would be 91).

Note that this constraint is expressed in terms of daily net delivery whereas constraints (c) and (d) in Section 3.2.1 are described in terms of separate daily withdrawals and injections. In the current model, withdrawals and injections can occur on the same day due to day contracts traded on that day and prompt contracts traded anytime in the previous month. Under the earlier model, which incorporates only day contracts, a withdrawal and injection cannot both occur on the same day; for each day t, at least one of the quantities wt and it is zero because the net selling price is necessarily less than the net buying price.

(c) At the start and end of the contract, the storage facility must be empty. Everyday, the gas balance must be between 0 and the maximum allowed by the storage contract:

$$b_1 = b_{366} = 0$$
$$0 \le b_t \le m_b$$

(d) Quantities bought and sold must be non-negative:

$$w_{j,t},\ i_{j,t} \ge 0$$

3.3.2 Model C: Hedging Required

Hedging is a tactic that involves locking in prices and quantities now for delivery in the future thereby eliminating price risk. Under such tactics, when gas is purchased (or sold) an offsetting sale (or purchase) is made simultaneously. The gas injections and withdrawals associated with the two transactions that make up the trade are scheduled for different dates. Thus, gas is never injected into storage without arrangements already in place for its disposal. Similarly, gas is never withdrawn from storage without arrangements in place for its replacement.

An example of a hedged trade involving only day and prompt contracts would be the purchase today of 30 units on a day contract and the sale, also today, of one unit on a prompt contract. (This example assumes next month has 30 days.) The resulting gas flow would be 30 units into storage today and one unit out of storage each day next month.

When all trades are hedged, the net change to gas inventory after all contracted withdrawals and injections are completed is zero. As a result—and this is the motivation for hedging—the trader is never compelled to buy gas at an unfavourable price in order to meet a withdrawal obligation;

similarly the trader is never compelled to sell gas at an unfavourable price in order to make room for an injection obligation or in order to empty the storage facility at the end of the contract.

The linear program of Section 3.3.1 is modified to incorporate hedging by the addition of two series of constraints. The first series of constraints requires that all quantities to be delivered as a result of the trades of each day sum to zero. This constraint locks in a favourable spread on every unit of gas bought and sold. It is also sufficient to ensure one of the terminal constraints, $b_{366} = 0$. The first additional series of constraints is the following:

(e) The net quantity of gas to be delivered as a result of each trade must be zero:

$$(-w_{1t} + i_{1t}) + (e_{m+1} - e_m)(-w_{2t} + i_{2t}) = 0 \qquad \forall\, t \text{ in current day–February 28}$$

The second series of additional constraints requires that the storage contract terms regarding gas balances, $0 \le b_t \le m_b$, be met after every trade *in the absence of any future trades*. It is no longer sufficient that all planned trades collectively meet these constraints. This restriction is imposed because the linear program identifies an optimal tentative trading plan for the period day t–March 31 based on the current price information.

It is assumed that the trader will execute the trade specified for the current day. Then, a day passes (it becomes t+1), new prices are observed, forecasts are updated, and the linear program is re-solved in order to identify a new optimal trading plan for day t+1–March 31. It is conceivable, in light of the new price information, that all future trades necessary to satisfy the storage contract terms regarding gas balances have now become unfavourable. To avoid this, that is in order for the trade on day t to be hedged, all constraints on balances must be satisfied following the completion of the trade on day t in the absence of any future trades.

This second series of additional constraints would appear to include a pair of inequalities for each trading day and each delivery day, amounting to approximately $2 \times 365 \times (1 + 30) \approx 23{,}000$ inequalities*. This number can be greatly reduced by observing that for each trade ($w_{1,t}$, $i_{1,t}$, $w_{2,t}$, $i_{2,t}$) made on day t, if balance b_{t+1} (the balance following the delivery of the day contract amount $i_{1,t} - w_{1,t}$) satisfies all constraints, and if balance $b_{d_{m+2}}$ (the balance following the last delivery of the prompt contract amount $i_{2,t} - w_{2,t}$) satisfies all constraints, then it follows that the balances for all days in between will also satisfy the constraints. This is true because prompt contracts result in repeated deliveries of the same quantity of gas everyday of the following month. Therefore, rather than adding a pair of inequality constraints for each trading day and every delivery day, it is sufficient to add a pair of inequalities for every trading day and for each of two deliveries days: the current day and the last day of the following month.

The second series of additional constraints is the following:

(f) For each trade and in the absence of all subsequent trades, the balances at the end of the trading day and at the end of the last day of the following month must be within the contractual limits:

* Two inequalities: balance ≥ 0 and balance $\le m_b$; 365 trading days per year; 1 delivery associated with each day trade and approximately 30 deliveries associated with each prompt trade

For April trades, which result in deliveries on the same day and everyday in May:

$0 \le b_t - w_{1,t} + i_{1,t} \le m_b$ and
$0 \le b_t - w_{1,t} + i_{1,t} + 31(-w_{2,t} + i_{2,t}) \le m_b, \forall\, t \in \{1, \ldots, 30\}$

For May trades, which result in deliveries on the same day and everyday in June:

$0 \le b_t - w_{1,t} + i_{1,t} \le m_b$ and
$0 \le b_t - w_{1,t} + i_{1,t} + 30(-w_{2,t} + i_{2,t}) \le m_b, \forall\, t \in \{31, \ldots, 61\}$

...

For February trades, which result in deliveries on the same day and everyday in March:

$0 \le b_{t-1} - w_{1,t} + i_{1,t} \le m_b$ and
$0 \le b_t - w_{1,t} + i_{1,t} + 31(-w_{2,t} + i_{2,t}) \le m_b, \forall\, t \in \{307, \ldots, 334\}$

The general case:

$0 \le b_t - w_{1,t} + i_{1,t} \le m_b$ and
$0 \le b_t - w_{1,t} + i_{1,t} + (e_{m+1} - e_m)(-w_{2,t} + i_{2,t}) \le m_b, \forall\, t \in \{t', t'+1, \ldots, 334\}$

where

m = the current month and
e_m = the day of the horizon that is the last day in month m (e.g., $e_{May} = 61$).

Note that the hedging requirement results in no trades during March (decision variables $w_{j,t}$, $i_{j,t} = 0 \;\forall\, t \ge 335$) since, for example, the trade of a March day contract would require hedging with a prompt contract with deliveries occurring in April. Deliveries after March 31 are infeasible. During March, the only activity that occurs is the withdrawals stemming from prompt contracts sold during February.

CHAPTER 4
NUMERICAL STUDY

We seek insight into two issues: First, the value that optimization models add to trading on top of what naïve strategies contribute, and second, the opportunity cost of hedging policies or, conversely, the upside potential of avoiding hedging.

In assessing the value added by optimization models, we simulate five years of trading using the optimal strategy and an appropriate naïve strategy. Initially we assume perfect price information in order to isolate the effect of the optimization strategy. We expect the optimal strategy to outperform the naïve in all simulations. Second, we compare outcomes of the competing strategies under forecasted prices that are updated daily before each trade. In this case, with the effect of forecast inaccuracy added to the effect of optimization, we do not expect the optimal strategy to necessary outperform the naïve, since the optimal solutions are optimal only given the information assumed at the time. Once prices become known, the trades made under the optimal strategy, in hindsight, may not be optimal.

In assessing the opportunity cost of hedging, we compare the outcomes of implementing optimization strategies with and without hedging constraints in place and using forecasted prices. We expect the hedged trading simulations always to be profitable and the unhedged simulations to be either money-losing or profitable with more extreme results than the hedged ones. On average, we expect the unhedged trading simulations to be more profitable than the hedged ones and the magnitude of that difference is what particularly interested us.

The measure of performance we use is the combined, net value of all trades realized over the storage contract year.

In each comparison of strategies—optimal versus naïve, and hedged versus not hedged—we execute 45 simulations: one for each of five years and each of nine combinations of daily withdrawal and injection limits. The reserved capacity of the storage contract is fixed. The following are the parameter values used in the simulations expressed in arbitrary units of gas volume.

Reserved capacity, or maximum balance, m_b: 1,500
Maximum daily withdrawal, m_b, and injection, m_i:
- High: 500
- Medium: 15
- Low: 1

They are correct in their relationships to one another, but the quantities in MMBtus used are confidential and therefore omitted.

We do not include any Monte Carlo simulations using randomly generated prices because of the excessive computational effort required. Simulations involving optimal strategies and forecasted prices, for example, require about an hour of computer time per year per combination of withdrawal and injection limits. The simulations we present here consumed an estimated 200–250 hours of computer time. We address the potential value of Monte Carlo simulations in Section 6.2, Future Research.

All simulations are executed using Microsoft Excel and VBA and the Frontline Systems Solver add-in for Excel.

4.1 EVALUATION OF OPTIMAL TRADING MODELS

We assess the performance of the optimal models versus the performance of other, "naïve" strategies. The naïve strategies represent the hypothetical strategies of a trader without the assistance of such mathematical models.

4.1.1 Trading with Day Contracts Only and Using Perfect Price Information

We compare the results obtained using Optimization Model A from Section 3.2.1 and using a naïve strategy; both allow the trading of day contracts only. All spot prices are assumed to be known in advance; no forecasting of prices is required.

Naïve Strategy

The naïve strategy is assumed to comprise the following:

- If the net purchase price on the current day (spot price plus unit transmission cost) is less than the average selling price for the year (average price less unit transmission cost), then buy the maximum allowable quantity.
- If the net selling price on the current day is greater than the average purchase price for the year, then sell the maximum allowable quantity.
- If neither, do not trade.

Simulation Process

Simulating the implementation of the optimal strategy, for each year and combination of withdrawal and injection limits, requires one execution of Solver with the actual price path in place. The solution is a series of 365 (w, i) pairs where at least one of w and i is zero.

Simulating the use of the naïve strategy consists of obtaining some elementary Excel spreadsheet output.

Results

The following figures depict the operation of the gas storage facility according to the naïve and optimal strategies during the five years 1997/98 through 2001/02 subject to the (medium, medium) combination of daily withdrawal and injection limits. The lower graph in each figure depicts the balance of gas in storage as a result of each strategy while the upper graph presents the price behaviour occurring at the time trading decisions are made. (Because of space constraints, we do not include gas balance graphs for all nine combinations and each year in the body of this document; rather, they are presented in the appendices.)

The tables present the net trading proceeds realized from the operation of the storage facility each year and subject to each of the nine combinations of daily withdrawal and injection limits.

1997/98:

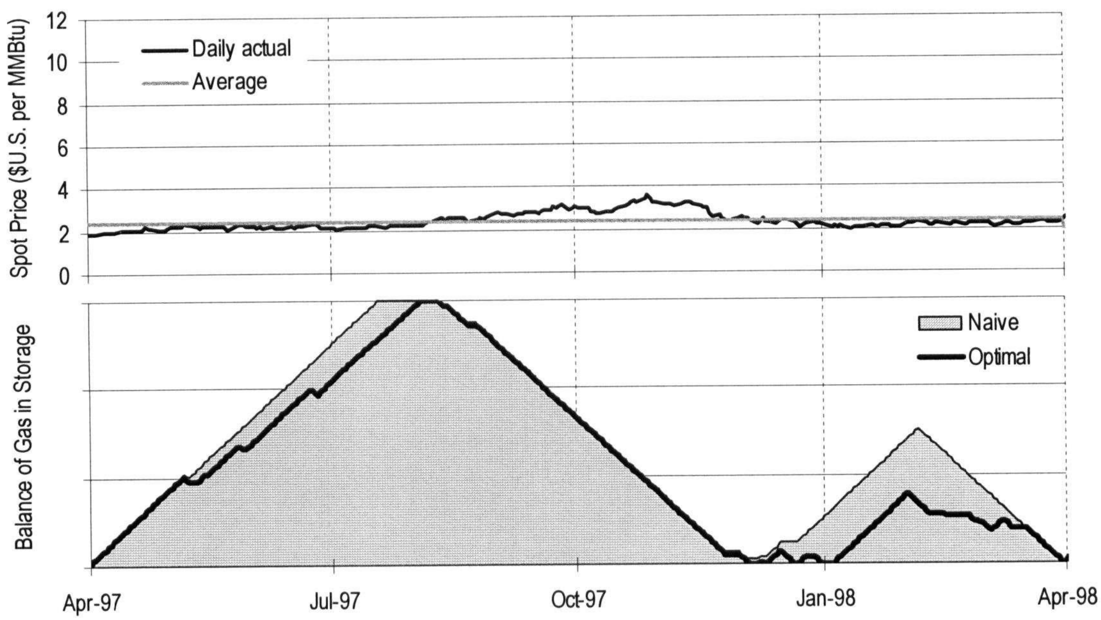

Figure 4.1
Balance of Gas in Storage Resulting from Trades for 1997/98
Day Contracts Only (No Hedging); Perfect Price Information; Medium Withdrawal and Injection Limits

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	1.67	1.21	0.10
Mid	2.40	2.09	0.09
Low	0.30	0.24	0.15

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	10.81	4.41	0.48
Mid	3.34	2.24	0.37
Low	0.34	0.27	0.16

OPTIMAL EXCEEDS NAÏVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	9.14	3.20	0.38
Mid	0.94	0.15	0.28
Low	0.04	0.02	0.02

Table 4.1
Total Annual Trading Value Realized for 1997/98 in $U.S. millions
Day Contracts Only (No Hedging); Perfect Price Information
(Boxed figures correspond to the operation depicted in the graph immediately above.)

1998/99:

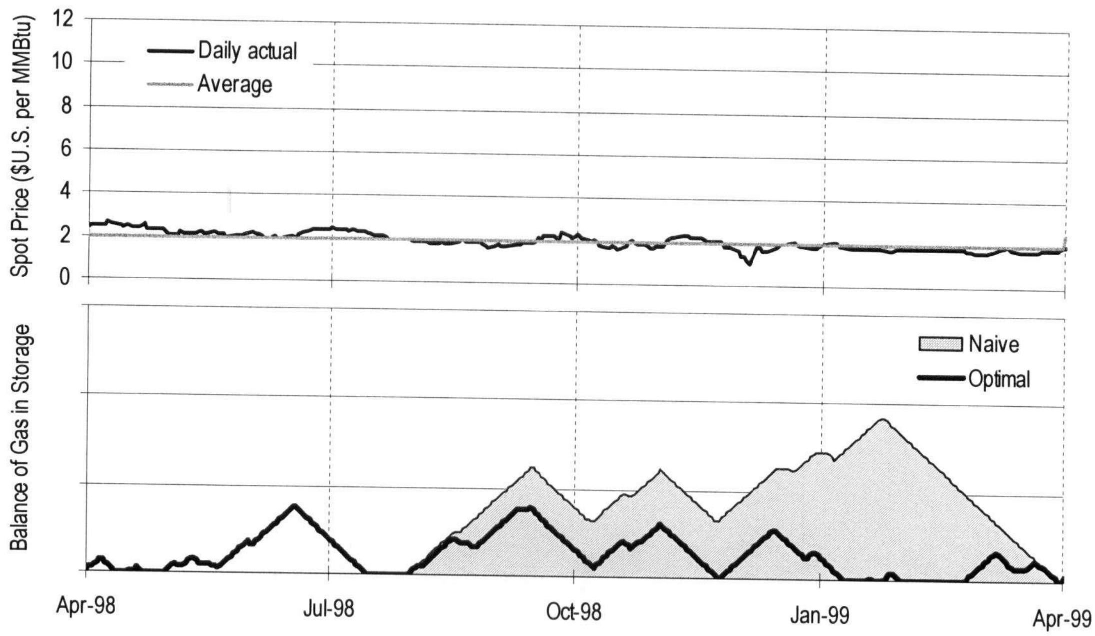

Figure 4.2
Balance of Gas in Storage Resulting from Trades for 1998/99
Day Contracts Only (No Hedging); Perfect Price Information; Medium Withdrawal and Injection Limits

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	2.34	0.94	0.08
Mid	0.12	$\boxed{0.37}$	0.06
Low	-0.02	-0.02	0.03

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	11.62	2.23	0.17
Mid	3.29	$\boxed{0.89}$	0.14
Low	0.24	0.18	0.06

OPTIMAL EXCEEDS NAÏVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	9.28	1.29	0.09
Mid	3.17	$\boxed{0.52}$	0.08
Low	0.27	0.20	0.04

Table 4.2
Total Annual Trading Value Realized for 1998/99 in $U.S. millions
Day Contracts Only (No Hedging); Perfect Price Information
(Boxed figures correspond to the operation depicted in the graph immediately above.)

1999/2000:

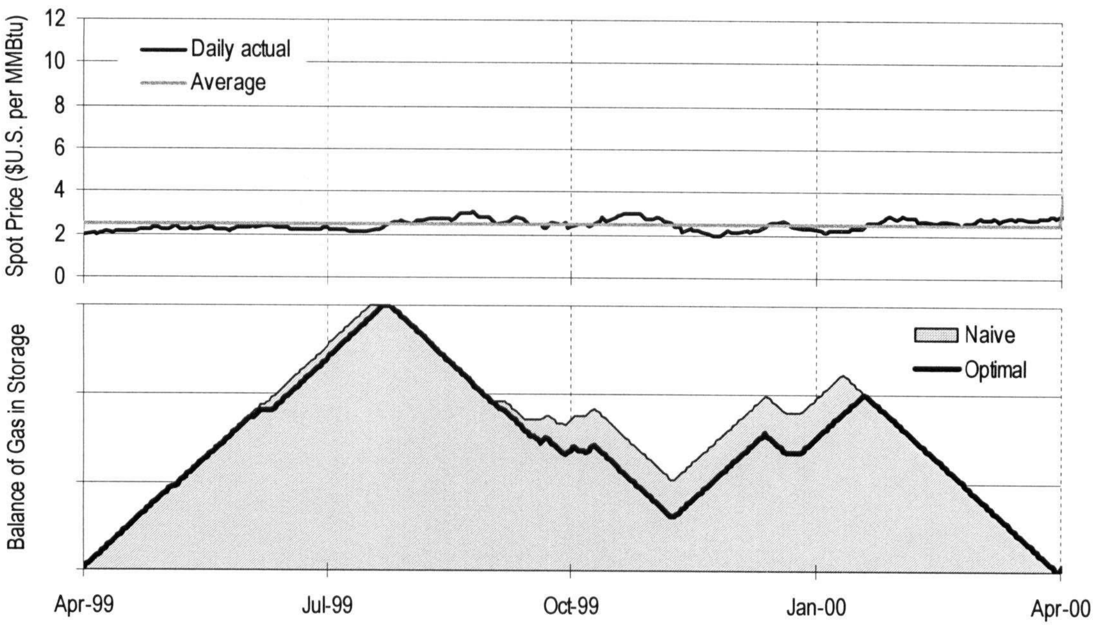

Figure 4.3
Balance of Gas in Storage Resulting from Trades for 1999/2000
Day Contracts Only (No Hedging); Perfect Price Information; Medium Withdrawal and Injection Limits

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	3.89	1.31	0.09
Mid	2.61	$\boxed{2.01}$	0.10
Low	0.34	0.23	0.14

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	13.12	4.33	0.36
Mid	4.09	$\boxed{2.10}$	0.31
Low	0.34	0.28	0.15

OPTIMAL EXCEEDS NAÏVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	9.22	3.02	0.26
Mid	1.48	$\boxed{0.09}$	0.21
Low	0.00	0.05	0.01

Table 4.3
Total Annual Trading Value Realized for 1999/2000 in $U.S. millions
Day Contracts Only (No Hedging); Perfect Price Information
(Boxed figures correspond to the operation depicted in the graph immediately above.)

2000/01:

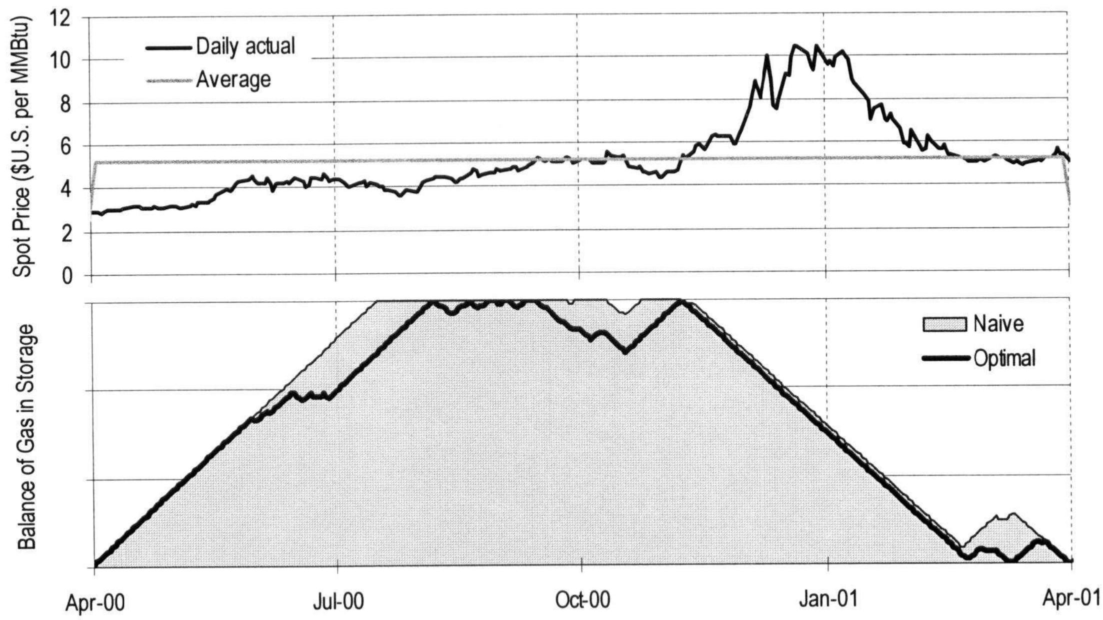

Figure 4.4
Balance of Gas in Storage Resulting from Trades for 2000/01
Day Contracts Only (No Hedging); Perfect Price Information; Medium Withdrawal and Injection Limits

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	10.61	5.58	0.49
Mid	13.37	10.98	0.57
Low	1.69	1.59	0.85

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	47.79	22.35	3.11
Mid	15.72	11.65	2.94
Low	1.71	1.60	0.89

OPTIMAL EXCEEDS NAÏVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	37.19	16.77	2.62
Mid	2.35	0.67	2.36
Low	0.02	0.01	0.04

Table 4.4
Total Annual Trading Value Realized for 2000/01 in $U.S. millions
Day Contracts Only (No Hedging); Perfect Price Information
(Boxed figures correspond to the operation depicted in the graph immediately above.)

2001/02:

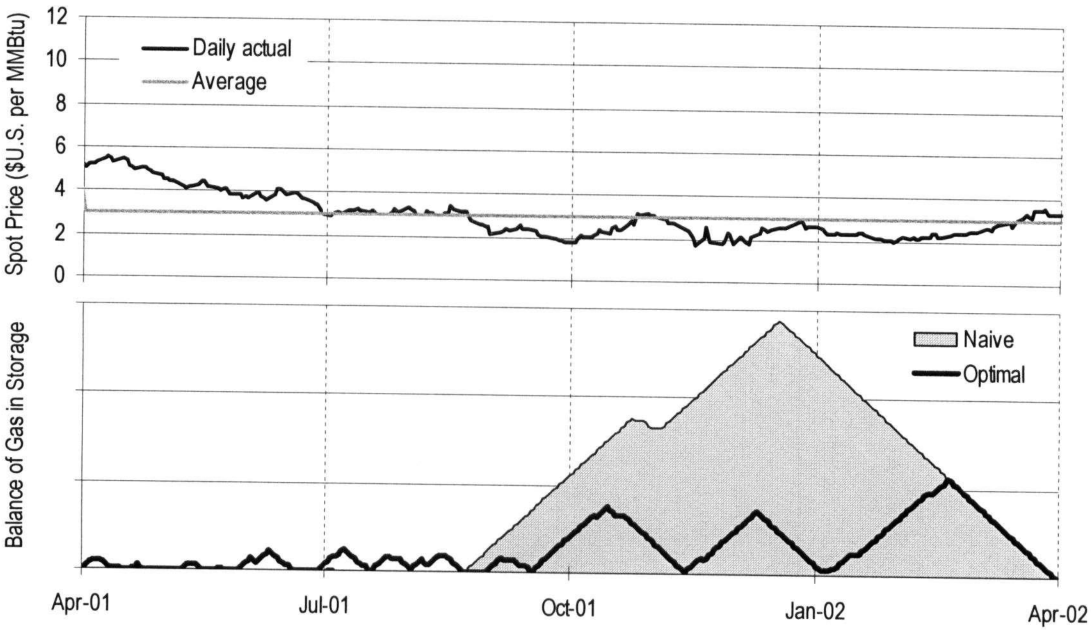

Figure 4.5
Balance of Gas in Storage Resulting from Trades for 2001/02
Day Contracts Only (No Hedging); Perfect Price Information; Medium Withdrawal and Injection Limits

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	2.23	4.19	0.30
Mid	-0.77	0.82	0.36
Low	-0.24	-0.03	0.06

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	20.21	5.89	0.55
Mid	3.72	1.89	0.43
Low	0.27	0.25	0.13

OPTIMAL EXCEEDS NAÏVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	17.98	1.70	0.25
Mid	4.49	1.07	0.07
Low	0.51	0.28	0.08

Table 4.5
Total Annual Trading Value Realized for 2001/02 in $U.S. millions
Day Contracts Only (No Hedging); Perfect Price Information
(Boxed figures correspond to the operation depicted in the graph immediately above.)

Average of 1997/98 through 2001/02:

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	4.15	2.65	0.21
Mid	3.55	$\boxed{3.25}$	0.24
Low	0.41	0.40	0.24

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	20.71	7.84	0.93
Mid	6.03	$\boxed{3.75}$	0.84
Low	0.58	0.52	0.28

OPTIMAL EXCEEDS NAÏVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	399%	196%	339%
Mid	70%	$\boxed{15\%}$	254%
Low	40%	28%	14%

Table 4.6
Average Annual Trading Value Realized 1997/98 through 2001/02 in $U.S. millions
Day Contracts Only (No Hedging); Perfect Price Information
(Boxed figures correspond to the operation depicted in the five preceding graphs.)

Observations and Discussion

The naïve strategy generally leads to balances of gas with a few and dramatic peaks and valleys during the year, whereas the optimal strategy leads to many small ones. The price tends to be above its mean or below it for long periods rather than oscillating. This induces the naïve strategy to buy gas repeatedly then sell repeatedly rather than alternating frequently as the optimal strategy does. The exception to this pattern occurs in 1998/99 when the price switches from above its mean to below more frequently than usual.

Another pattern is attributable to the behaviour of the price with respect to its mean. Once the naïve strategy starts buying, it maintains a higher balance than the optimal strategy, until the two converge to empty every March 31. Note that 1997/98, 1999/2000, and 2000/01 are years during which the prices reflect an upward trend while for 1998/99 and 2001/02 prices trend downward. In the years of downward trends, the naïve strategy starts the year trying to sell because the current price is generally above the mean. Since sales from an empty storage facility are not feasible, the facility remains empty until the current prices creep up above the mean and purchases are indicated by the naïve strategy. The early months of downward trending years are, in our simulations, the only occasions when the optimal strategy maintains a higher balance than the naïve.

Under the optimal strategy, the greatest net trading value is realized when both withdrawal and injection limits are high and the lowest value is realized when both limits are low. This is consistent with intuition since the purchase and sale of every unit is necessarily profitable and the higher limits permit more of these transactions over the course of the year.

As the limits are tightened from (high, high) to (low, low) the tightening of the withdrawal limit has greater influence toward reducing the trading value realized. A lower withdrawal limit means more days are required at the end of the year to dispose of all gas holdings. Therefore, the period during which full reserved capacity is available for profitable trades is shorter.

As we expected, the optimal strategy achieves greater net trading value in all simulations.

4.1.2 Hedging with Day and Prompt Contracts Using Perfect Price Information

We introduce the trading of prompt contracts in addition to day contracts and impose a hedging requirement. We continue to assume perfect price information as we did in Section 4.1.1. This analysis uses Optimization Model B presented in Section 3.2.2.

Naïve Strategy

The naïve strategy consists of the following:

- If the net buying price of a day contract is greater than the net selling price of a prompt contract on the current day, then buy a day contract and sell a prompt contract for the maximum allowable and exactly offsetting quantities.
- If the net selling price of a day contract is greater than the net buying price of a prompt contract on the current day, then sell a day contract and buy a prompt contract for the maximum allowable and exactly offsetting quantities.
- If neither, do not trade.

Simulation Process

Each of the 45 simulations of the optimal strategy requires one execution of Solver with the series of actual spot prices in place. The solution is $(w_{1,t}, i_{1,t}, w_{2,t}, i_{2,t})$ for all days t in April through February where $w_{1,t} = i_{2,t} = 0$ or $i_{t,1} = w_{t,2} = 0$, and if $w_{t,i} = 0$ then $i_{t,j} = 0$ for $i \neq j$.

Each simulation of the naïve strategy used one execution of Solver for each day in April 1–February 28. (Solver is not required to decide whether to buy day and sell prompt, sell day and buy prompt, or neither. But using it to determine maximum feasible quantities is more convenient than creating a spreadsheet to do so.) The solution for each day is subject to the assumption that the trades of all previous days have been made. Each simulation produces $(w_{1,t}, i_{1,t}, w_{2,t}, i_{2,t})$ for all days t in April through February with the same conditions to which the optimal output is subject.

Results

As in Section 4.1.1, results are presented by the year studied. Graphs depicting the balances of gas stored as a result of the competing strategies are accompanied by plots the price differential of day and prompt contracts to provide context within which trading decisions are induced. Net trading value realized in all scenarios are again presented in tables. Discussion of the results follows.

1997/98:

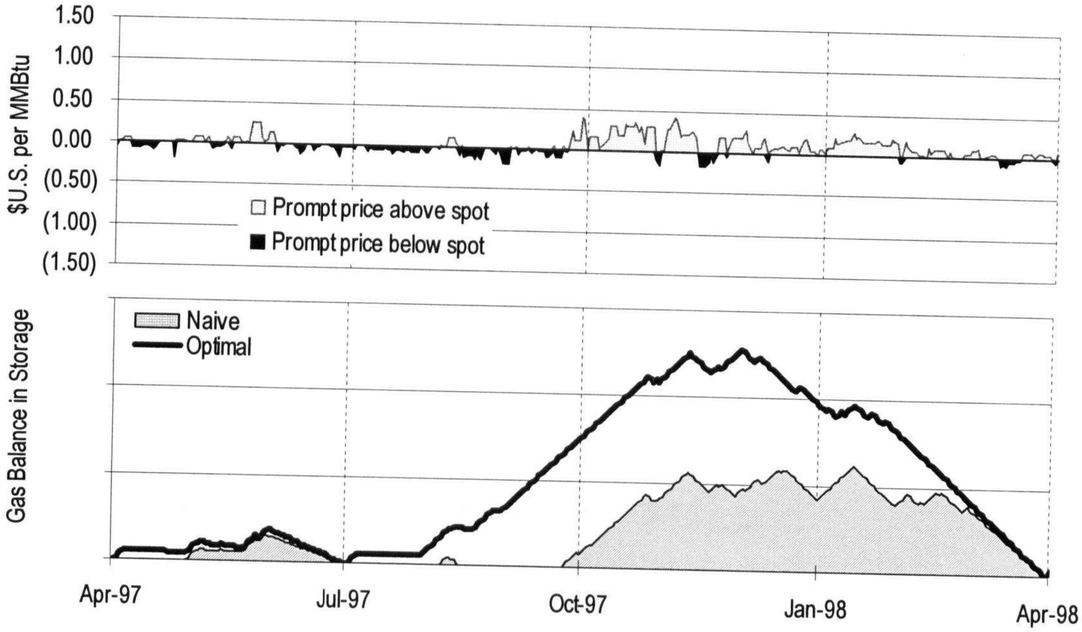

Figure 4.6
Balance of Gas in Storage Resulting from Trades for 1997/98
Day and Prompt Contracts; Hedging; Perfect Price Information; Medium Withdrawal and Injection Limits

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	3.37	0.59	0.04
Mid	0.86	$\boxed{0.54}$	0.05
Low	0.06	0.05	0.04

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	6.23	1.23	0.10
Mid	2.33	$\boxed{0.82}$	0.10
Low	0.22	0.16	0.06

OPTIMAL EXCEEDS NAÏVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	2.86	0.64	0.06
Mid	1.47	$\boxed{0.28}$	0.05
Low	0.16	0.11	0.02

Table 4.7
Total Annual Trading Value Realized for 1997/98 in $U.S. millions
Day and Prompt Contracts; Hedging; Perfect Price Information
(Boxed figures correspond to the operation depicted in the graph immediately above.)

1998/99:

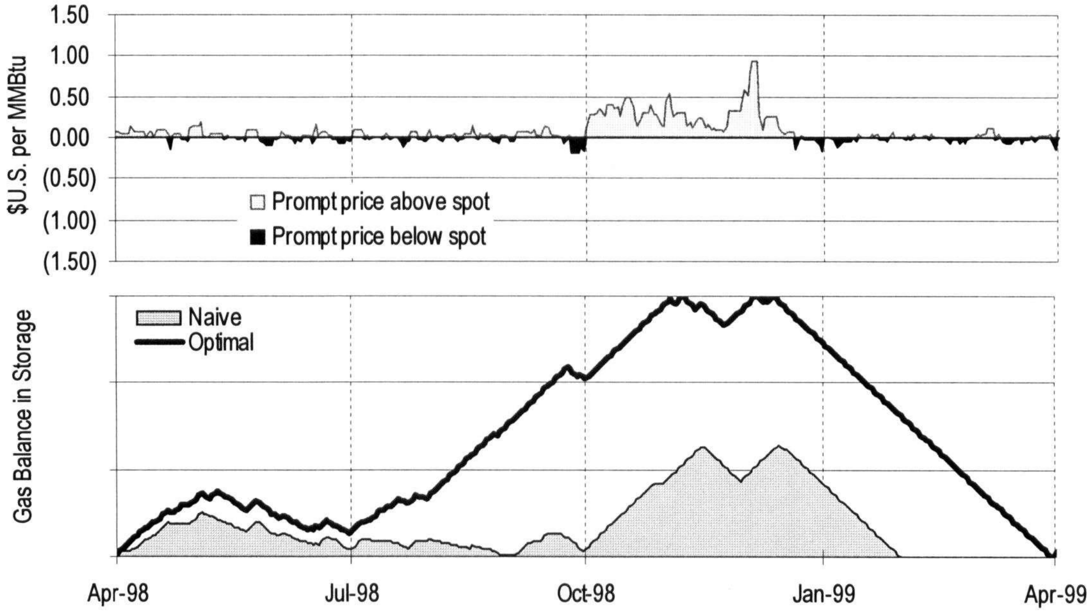

Figure 4.7
Balance of Gas in Storage Resulting from Trades for 1998/99
Day and Prompt Contracts; Hedging; Perfect Price Information; Medium Withdrawal and Injection Limits

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	2.87	1.17	0.09
Mid	1.02	$\boxed{0.80}$	0.08
Low	0.07	0.08	0.06

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	6.30	2.10	0.27
Mid	3.25	$\boxed{1.57}$	0.23
Low	0.24	0.21	0.12

OPTIMAL EXCEEDS NAÏVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	3.43	0.93	0.19
Mid	2.23	$\boxed{0.77}$	0.15
Low	0.17	0.13	0.06

Table 4.8
Total Annual Trading Value Realized for 1998/99 in $U.S. millions
Day and Prompt Contracts; Hedging; Perfect Price Information
(Boxed figures correspond to the operation depicted in the graph immediately above.)

1999/2000:

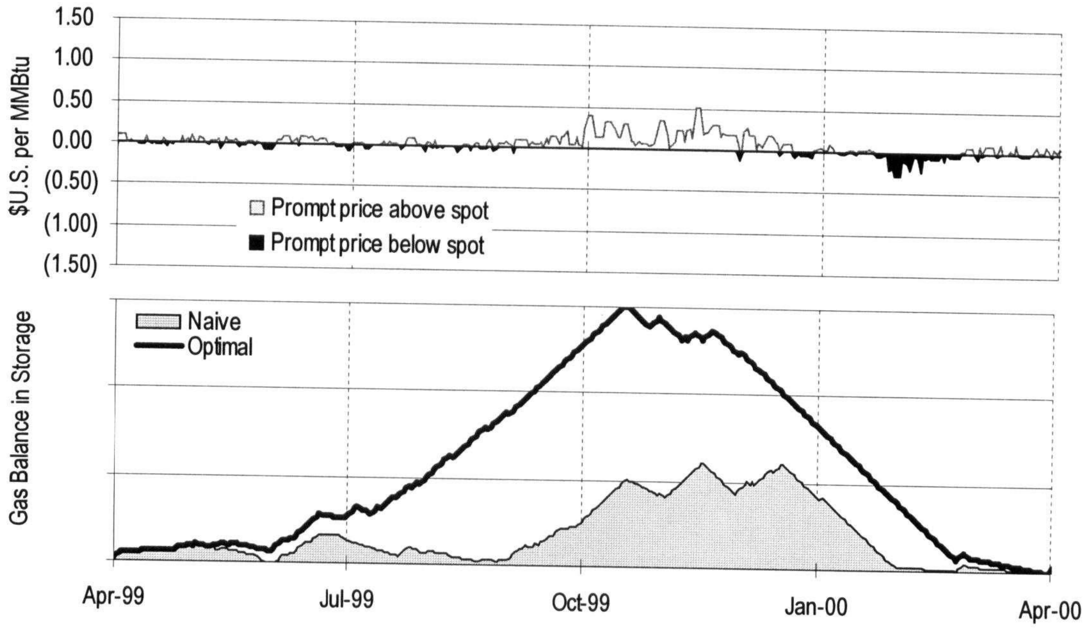

Figure 4.8
Balance of Gas in Storage Resulting from Trades for 1999/2000
Day and Prompt Contracts; Hedging; Perfect Price Information; Medium Withdrawal and Injection Limits

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	2.11	0.72	0.05
Mid	0.72	0.46	0.05
Low	0.05	0.05	0.03

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	4.54	1.61	0.19
Mid	1.73	0.86	0.15
Low	0.14	0.13	0.06

OPTIMAL EXCEEDS NAÏVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	2.43	0.89	0.13
Mid	1.01	0.39	0.10
Low	0.09	0.08	0.03

Table 4.9
Total Annual Trading Value Realized for 1999/2000 in $U.S. millions
Day and Prompt Contracts; Hedging; Perfect Price Information
(Boxed figures correspond to the operation depicted in the graph immediately above.)

2000/01:

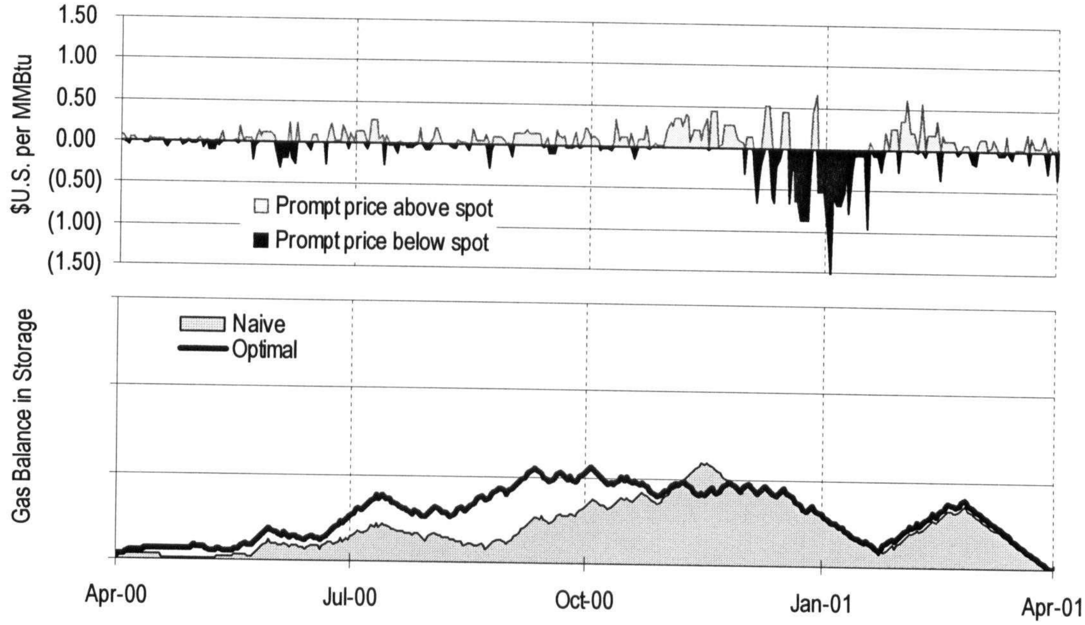

Figure 4.9
Balance of Gas in Storage Resulting from Trades for 2000/01
Day and Prompt Contracts; Hedging; Perfect Price Information; Medium Withdrawal and Injection Limits

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	12.80	1.14	0.08
Mid	1.24	0.84	0.08
Low	0.09	0.08	0.06

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	22.63	3.48	0.38
Mid	2.75	0.94	0.28
Low	0.20	0.18	0.07

OPTIMAL EXCEEDS NAÏVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	9.83	2.33	0.30
Mid	1.51	0.10	0.20
Low	0.11	0.10	0.01

Table 4.10
Total Annual Trading Value Realized for 2000/01 in $U.S. millions
Day and Prompt Contracts; Hedging; Perfect Price Information
(Boxed figures correspond to the operation depicted in the graph immediately above.)

2001/02:

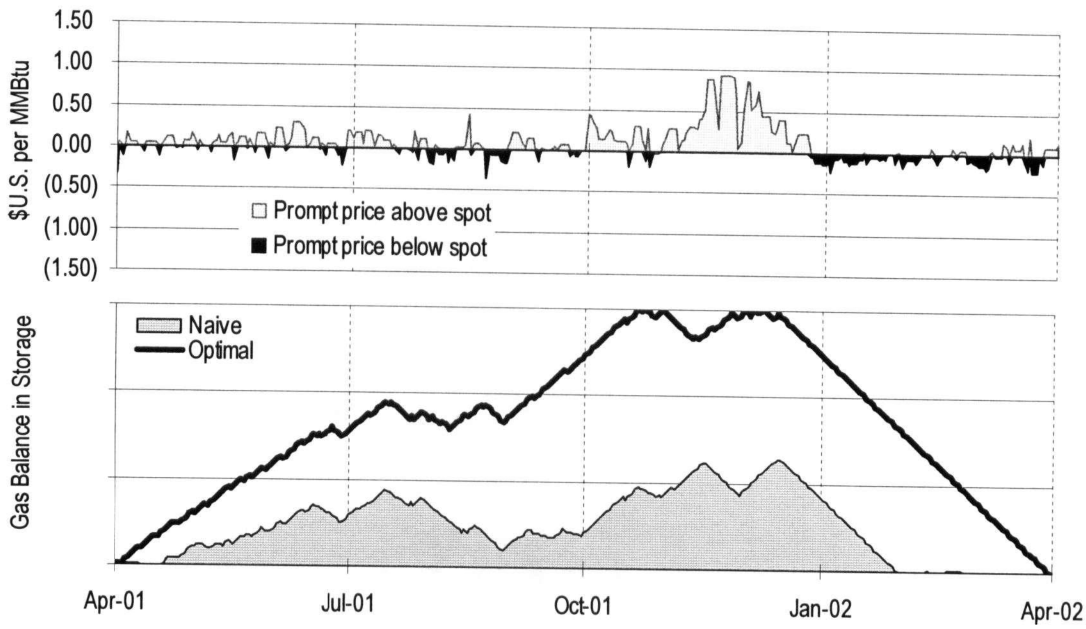

Figure 4.10
Balance of Gas in Storage Resulting from Trades for 2001/02
Day and Prompt Contracts; Hedging; Perfect Price Information; Medium Withdrawal and Injection Limits

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	6.81	1.36	0.10
Mid	1.97	$\boxed{1.05}$	0.10
Low	0.14	0.13	0.07

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	12.51	3.37	0.40
Mid	4.43	$\boxed{2.39}$	0.37
Low	0.35	0.32	0.18

OPTIMAL EXCEEDS NAÏVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	5.69	2.00	0.30
Mid	2.46	$\boxed{1.35}$	0.27
Low	0.21	0.19	0.10

Table 4.11
Total Annual Trading Value Realized for 2001/02 in $U.S. millions
Day and Prompt Contracts; Hedging; Perfect Price Information
(Boxed figures correspond to the operation depicted in the graph immediately above.)

Average of 1997/98 through 2001/02:

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	5.59	1.00	0.07
Mid	1.16	$\boxed{0.74}$	0.07
Low	0.08	0.08	0.05

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	10.44	2.36	0.27
Mid	2.90	$\boxed{1.32}$	0.22
Low	0.23	0.20	0.10

OPTIMAL EXCEEDS NAÏVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	87%	136%	272%
Mid	150%	$\boxed{78\%}$	212%
Low	177%	154%	81%

Table 4.12
Average Annual Trading Value Realized 1997/98 through 2001/02 in $U.S. millions
Day and Prompt Contracts; Hedging; Perfect Price Information
(Boxed figures correspond to the operation depicted in the five preceding graphs.)

Observations and Discussion

The results of simulations of hedging with day and prompt contracts and perfect price information show that the optimal strategy results in a balance of gas that is virtually always higher than the balance maintained under the naïve strategy. Two factors contribute to this. First, when no gas is held, as at the start of each year, and a price spread exists that indicates withdrawing now and injecting next month, the naïve strategy will always indicate "no trade" since there is no gas to withdraw. The optimal strategy, however, may indicate injecting now and withdrawing later because a loss on the immediate transaction may create an opportunity to take advantage of a larger, favourable spread on a trade in the future. Therefore, with an empty facility, there is greater probability that the optimal strategy will dictate an injection than the naïve strategy will.

Second, during the third quarter of the year (October–December) the spot and prompt prices reflect particularly large spreads favourable to buying day contracts and selling prompt contracts. In the month before the occurrence of such a spread, the optimal strategy may indicate executing the same kind of transaction, even at a loss, to "make room" for the day contracts it foresees purchasing the following month. Everything else equal, the optimal strategy will pre-empt a month of particularly big spreads by making the same trades in the earlier month as the large spreads of the next month will indicate. The optimal strategy has the foresight to see these opportunities whereas the naïve strategy does not. Since the particularly large spreads in our data are favourable to buying day contracts and selling prompts, that is, injecting now and withdrawing next month, the optimal strategy will dictate injections more often than the naïve will, resulting in a higher balance.

Note that in all years but one, the balance per the optimal strategy is greater than that of the naïve strategy and significantly so. Anomalously, 2000/01 prices reflect a late third quarter and an early fourth quarter of exceptionally large spreads in the opposite direction as those discussed above. Not surprisingly, the optimal and naïve strategies maintain balances during that year that are closer to one another; furthermore, the balance per the naïve strategy is the higher one for about two weeks in mid-December 2000.

With respect to financial performance, the optimal strategy outperforms the naïve in all years and given all combinations of withdrawal and injection limits, as expected. Under both strategies and in all years, the (high, high) limits yield the highest net trading value and the (low, low) limits yield the lowest. As limits are tightened under either strategy, lowering the injection limit has more influence on decreasing the financial performance most of the years.

4.1.3 Trading with Day Contracts Only and Using Forecasted Prices

As in Section 4.1.1, we simulate the implementation of an optimal strategy per the model described in Section 3.2.1 and a naïve strategy. In this case however, price forecasts that are updated daily are used instead of perfect price information.

Price Forecast

We use the "step" forecasting function described in Section 2.3.2 and particular by Equation 2.1.

Naïve Strategy

The naïve strategy is assumed to comprise the following:

- If the net purchase price on the current day is less than the forecasted average net selling price for the year, then buy the maximum allowable quantity.
- If the net selling price on the current day is greater than the forecasted average net purchase price for the year, then sell the maximum allowable quantity.
- If neither, do not trade.

The forecasted average price for the year as forecast at day t is the average of the actual spot prices observed on days 1 to t and the forecasted spot prices for days t+1 to 365.

Simulation Process

The simulation process for optimal strategy comprises the following:

- Initial price information comprises the actual spot price observed on April 1 and forecasts for April 2–February 28 based on the forward curve observed on April 1.
- Solver is executed once to obtain an optimal trading plan, (w_t, i_t) for all days t in April 1–February 28.
- Trade (w_1, i_1) with its delivery on April 1 is assumed to be made.
- Price information for April 2–February 28 is updated and now comprises the spot price observed on April 2 and forecasts for April 3–February 28 based on the forward curve observed on April 2.
- Solver is executed once to obtain an optimal trading plan, (w_t, i_t) for all days t in April 2–February 28.
- Trade (w_2, i_2) with its delivery on day 2 is assumed to be made.
- And so on, until Solver is executed for every day of the year.

The simulation process for the naïve strategy comprises the following:

- Initial price information is as above.

- VBA records the greatest withdrawal or injection possible or no trade, depending on the current price information, for April 1.
- Trade (w_1, i_1) with its delivery on April 1 is assumed to be made.
- Price information is updated as above.
- VBA records the greatest withdrawal or injection possible or no trade, depending on the current price information, for April 2.
- Trade (w_2, i_2) with its delivery on April 2 is assumed to be made.
- And so on, until the trade of every day has been recorded.

Results

As in the previous two subsections, our results are presented by year. Graphs depict the operation of the storage facility according to the competing strategies and subject to medium level daily withdrawal and injection limits. Tables present the financial performance of each strategy under each scenario.

1997/98:

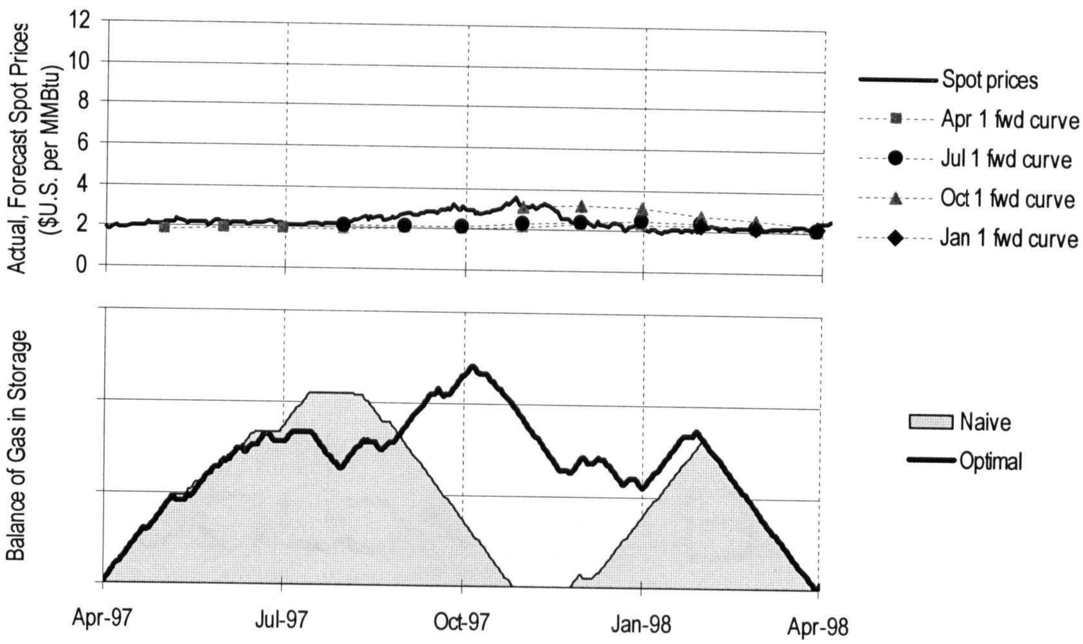

Figure 4.11
Balance of Gas in Storage Resulting from Trades for 1997/98
Day Contracts Only (No Hedging); Forecasted Prices; Medium Withdrawal and Injection Limits

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	1.44	0.82	0.06
Mid	2.02	1.40	0.06
Low	0.30	0.25	0.10

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	1.55	2.48	0.46
Mid	1.97	0.58	0.05
Low	0.33	0.20	0.07

OPTIMAL EXCEEDS OR (FALLS SHORT OF) NAIVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	0.11	1.66	0.40
Mid	(0.05)	(0.83)	(0.02)
Low	0.03	(0.05)	(0.03)

Table 4.13
Total Annual Trading Value Realized for 1997/98 in $U.S. millions
Day Contracts Only (No Hedging); Forecasted Prices
(Boxed figures correspond to the operation depicted in the graph immediately above.)

1998/99:

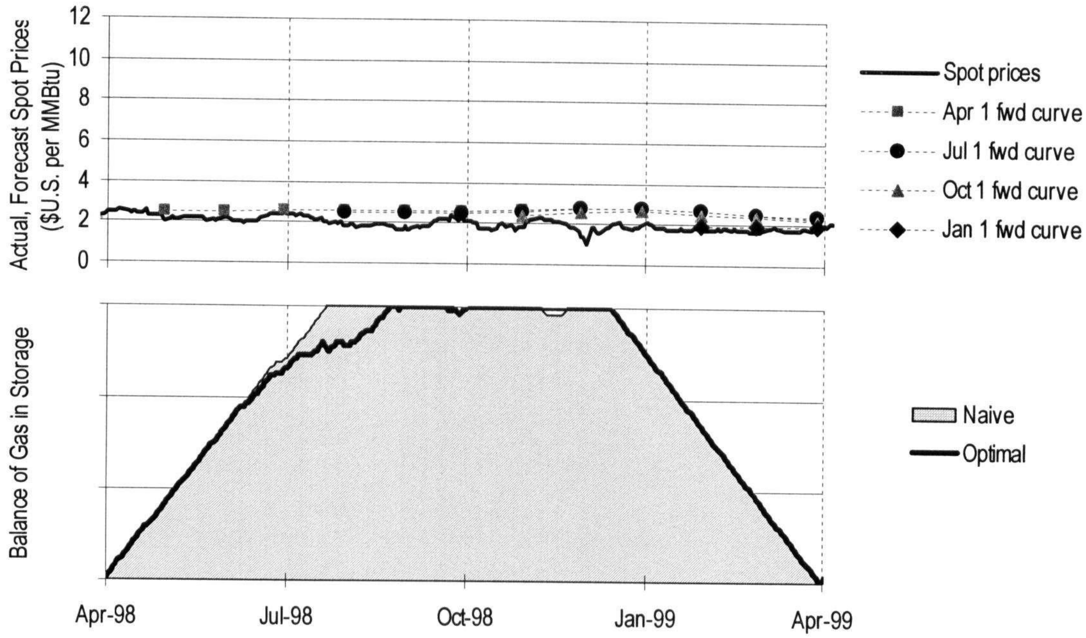

Figure 4.12
Balance of Gas in Storage Resulting from Trades for 1998/99
Day Contracts Only (No Hedging); Forecasted Prices; Medium Withdrawal and Injection Limits

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	(0.89)	0.96	0.14
Mid	(2.09)	$\boxed{(1.39)}$	(0.05)
Low	(0.36)	(0.39)	(0.12)

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	(1.19)	(0.66)	(0.02)
Mid	(2.11)	$\boxed{(1.26)}$	(0.11)
Low	(0.34)	(0.37)	(0.11)

OPTIMAL EXCEEDS OR (FALLS SHORT OF) NAIVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	(0.30)	(1.62)	(0.16)
Mid	(0.02)	$\boxed{0.14}$	(0.06)
Low	0.02	0.03	0.00

Table 4.14
Total Annual Trading Value Realized for 1998/99 in $U.S. millions
Day Contracts Only (No Hedging); Forecasted Prices
(Boxed figures correspond to the operation depicted in the graph immediately above.)

<u>1999/2000:</u>

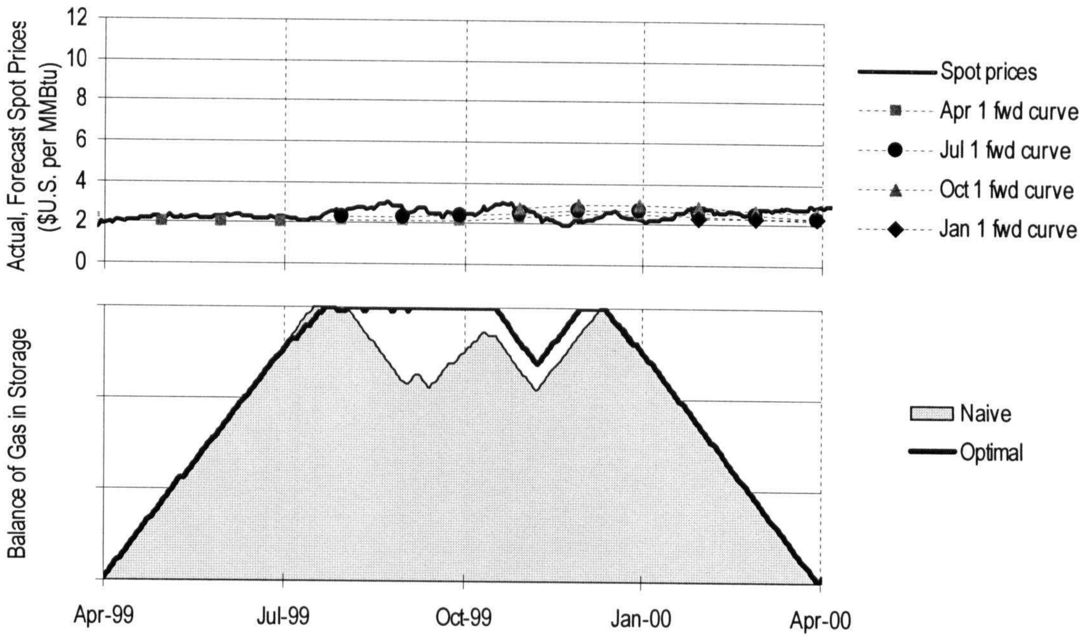

Figure 4.13
Balance of Gas in Storage Resulting from Trades for 1999/2000
Day Contracts Only (No Hedging); Forecasted Prices; Medium Withdrawal and Injection Limits

NAIVE STRATEGY

Daily Withdrawal Limit		Daily Injection Limit: High	Mid	Low
	High	4.29	1.81	0.14
	Mid	2.23	1.64	0.14
	Low	0.33	0.22	0.12

OPTIMAL STRATEGY

Daily Withdrawal Limit		Daily Injection Limit: High	Mid	Low
	High	3.18	0.94	0.05
	Mid	2.03	1.35	0.08
	Low	0.34	0.18	0.07

OPTIMAL EXCEEDS OR (FALLS SHORT OF) NAIVE

Daily Withdrawal Limit		Daily Injection Limit: High	Mid	Low
	High	(1.12)	(0.87)	(0.09)
	Mid	(0.20)	(0.29)	(0.06)
	Low	0.01	(0.04)	(0.05)

Table 4.15
Total Annual Trading Value Realized for 1999/2000 in $U.S. millions
Day Contracts Only (No Hedging); Forecasted Prices
(Boxed figures correspond to the operation depicted in the graph immediately above.)

2000/01:

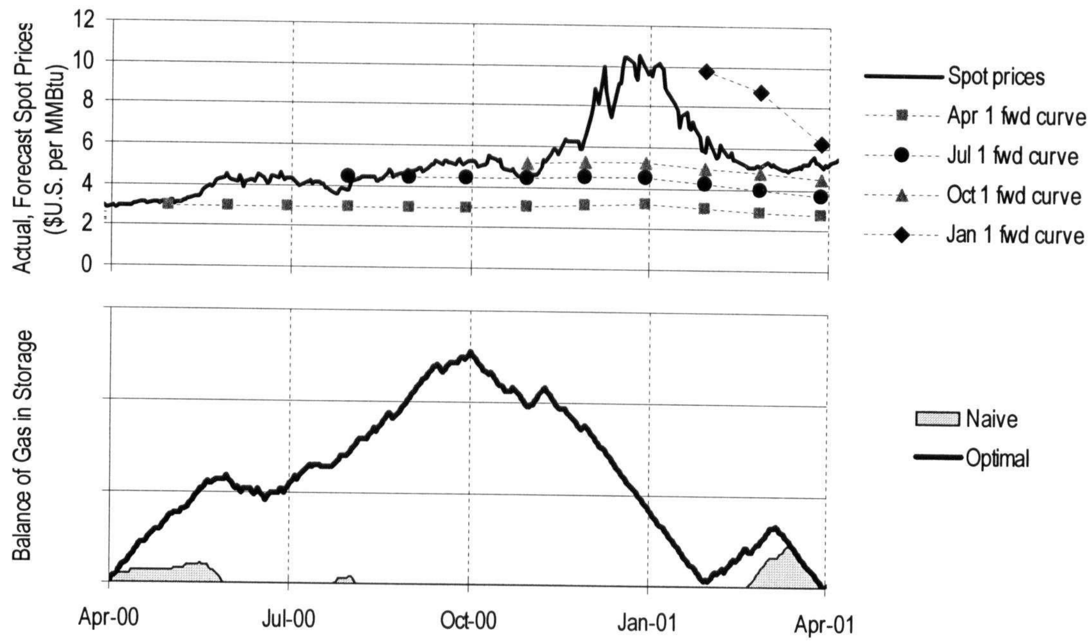

Figure 4.14
Balance of Gas in Storage Resulting from Trades for 2000/01
Day Contracts Only (No Hedging); Forecasted Prices; Medium Withdrawal and Injection Limits

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	5.21	0.40	0.03
Mid	6.66	$\boxed{0.28}$	0.03
Low	1.69	0.41	0.02

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	24.96	4.14	0.32
Mid	9.90	$\boxed{9.11}$	0.74
Low	1.48	0.95	0.65

OPTIMAL EXCEEDS OR (FALLS SHORT OF) NAIVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	19.74	3.74	0.29
Mid	3.23	$\boxed{8.83}$	0.70
Low	(0.20)	0.54	0.63

Table 4.16
Total Annual Trading Value Realized for 2000/01 in $U.S. millions
Day Contracts Only (No Hedging); Forecasted Prices
(Boxed figures correspond to the operation depicted in the graph immediately above.)

2001/02:

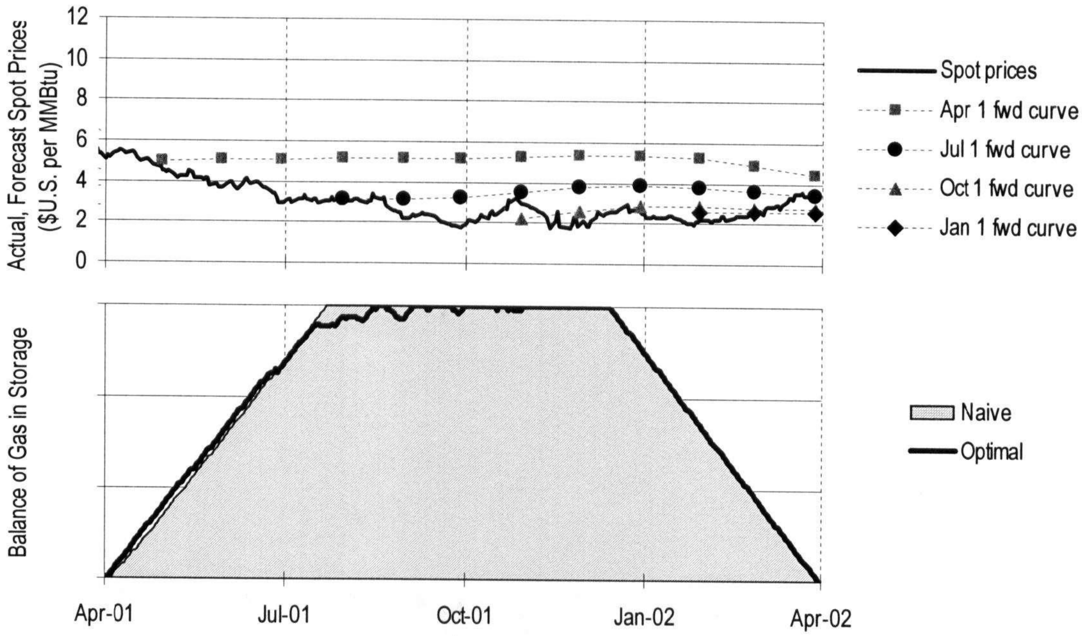

Figure 4.15
Balance of Gas in Storage Resulting from Trades for 2001/02
Day Contracts Only (No Hedging); Forecasted Prices; Medium Withdrawal and Injection Limits

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	(6.19)	(2.50)	0.11
Mid	(8.14)	(4.59)	0.12
Low	(1.54)	(1.62)	(0.37)

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	(1.00)	(2.95)	(0.14)
Mid	(9.54)	(4.66)	(0.42)
Low	(1.84)	(2.58)	(0.40)

OPTIMAL EXCEEDS OR (FALLS SHORT OF) NAIVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	5.19	(0.45)	(0.25)
Mid	(1.39)	(0.07)	(0.54)
Low	(0.30)	(0.97)	(0.02)

Table 4.17
Total Annual Trading Value Realized for 2001/02 in $U.S. millions
Day Contracts Only (No Hedging); Forecasted Prices
(Boxed figures correspond to the operation depicted in the graph immediately above.)

Average of 1997/98 through 2001/02:

Note that in Table 4.18 immediately below, the differences between the average annual trading values obtained under the competing strategies are expressed in millions of dollars in the right-most matrix rather than as percentages as in earlier, similar tables. Such percentage differences are not meaningful when one of the input numbers is negative as is the case below.

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	0.77	0.30	0.09
Mid	0.14	$\boxed{(0.53)}$	0.06
Low	0.08	(0.23)	(0.05)

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	5.50	0.79	0.13
Mid	0.45	$\boxed{1.02}$	0.07
Low	(0.00)	(0.32)	0.06

OPTIMAL EXCEEDS OR (FALLS SHORT OF) NAIVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	4.72	0.49	0.04
Mid	0.31	$\boxed{1.56}$	0.01
Low	(0.09)	(0.10)	0.11

Table 4.18
Average Annual Trading Value Realized 1997/98 through 2001/02 in $U.S. millions
Day Contracts Only (No Hedging); Forecasted Prices
(Boxed figures correspond to the operation depicted in the five preceding graphs.)

Observations and Discussion

The balance maintained over the course of the year as a result of the optimal strategy is nearly always above the balance associated with the naïve strategy. Only in the first half of the second quarters of some years does the naïve strategy result in a higher balance. In 2000/01, the year with the very dramatic spike in prices during the winter months, the optimal strategy results in balances that constantly and dramatically exceed the balances resulting from the naïve strategy.

That same year, the optimal strategy outperforms the naïve strategy in financial terms in all withdrawal and injection limit scenarios except one; in all other years, the naïve strategy outperforms the optimal in most scenarios. However, when the naïve strategy is the better financial performer, it is only slightly better; when the optimal strategy was the better one, it is significantly better. In terms of average value over all five years, the optimal strategy outperforms the naive strategy under seven of nine possible combinations of withdrawal and injection limits.

Comparing these results to those presented in Section 4.1.1, as we do in table 4.19 immediately below, reveals the severe cost of inaccurate forecasting.

OPTIMAL STRATEGY PERFECT PRICE INFO

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	20.71	7.84	0.93
Mid	6.03	3.75	0.84
Low	0.58	0.52	0.28

OPTIMAL STRATEGY FORECASTED PRICES

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	5.50	0.79	0.13
Mid	0.45	1.02	0.07
Low	0.00	-0.32	0.06

VALUE LOST TO FORECAST INACCURACY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	-73%	-90%	-86%
Mid	-93%	-73%	-92%
Low	-101%	-162%	-80%

Table 4.19
Average Annual Trading Value Realized 1997/98 through 2001/02 in $U.S. millions
Optimal Strategy; Day Contracts Only (No Hedging); Perfect Price Information vs. Forecasted Prices

4.1.4 Hedging with Day and Prompt Contracts and Using Forecasted Prices

Here we assess the performance of the optimizing strategy as compared to the naïve when two instruments are available, hedging is required, and prices must be forecasted.

Price Forecast

We use the "smooth" forecasting function described in Section 2.3.2 and particular by Equation 2.2.

Naïve Strategy

The naïve strategy used for this analysis is identical to that described in Section 4.1.2, when two instruments were available and hedging was required but perfect price information was assumed. Because the naïve strategy is a myopic one, that is, it considers only the information pertaining to the current day, and because price information for the current day is identical whether the information for the subsequent days is forecasted or "perfect," the results for the naïve strategy are the same as in Section 4.1.2.

Simulation Process

The simulation of the optimal strategy is identical to the process described in Section 4.1.2 except that each trade identified comprises quantities sold and purchased under day and prompt contracts rather than under day contracts only.

Results

As in the previous three subsections, our results are presented by year. Graphs depict the operation of the storage facility according to the competing strategies and subject to medium level daily withdrawal and injection limits. Tables present the financial performance of each strategy under each scenario.

1997/98:

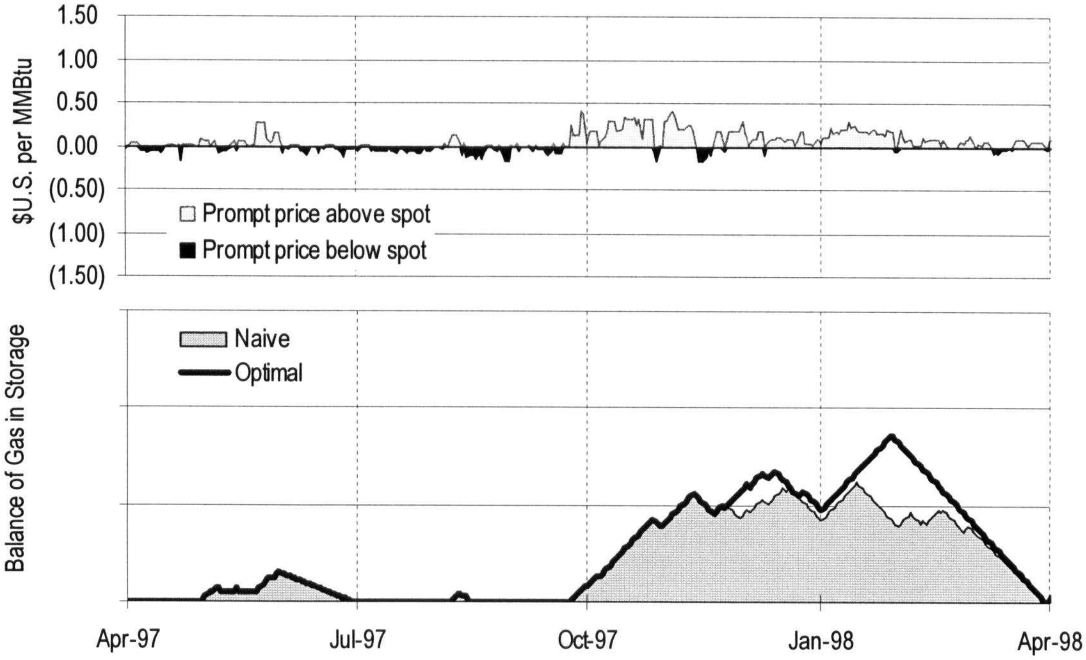

Figure 4.16
Balance of Gas in Storage Resulting from Trades for 1997/98
Day and Prompt Contracts; Hedging; Forecasted Prices; Medium Withdrawal and Injection Limits

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	3.37	0.59	0.04
Mid	0.86	0.54	0.05
Low	0.06	0.05	0.04

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	3.71	0.61	0.04
Mid	1.55	0.65	0.05
Low	0.10	0.10	0.05

OPTIMAL EXCEEDS OR (FALLS SHORT OF) NAIVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	0.34	0.02	0.00
Mid	0.69	0.10	0.00
Low	0.04	0.05	0.01

Table 4.20
Total Annual Trading Value Realized for 1997/98 in $U.S. millions
Day and Prompt Contracts; Hedging; Forecasted Prices
(Boxed figures correspond to the operation depicted in the graph immediately above.)

1998/99:

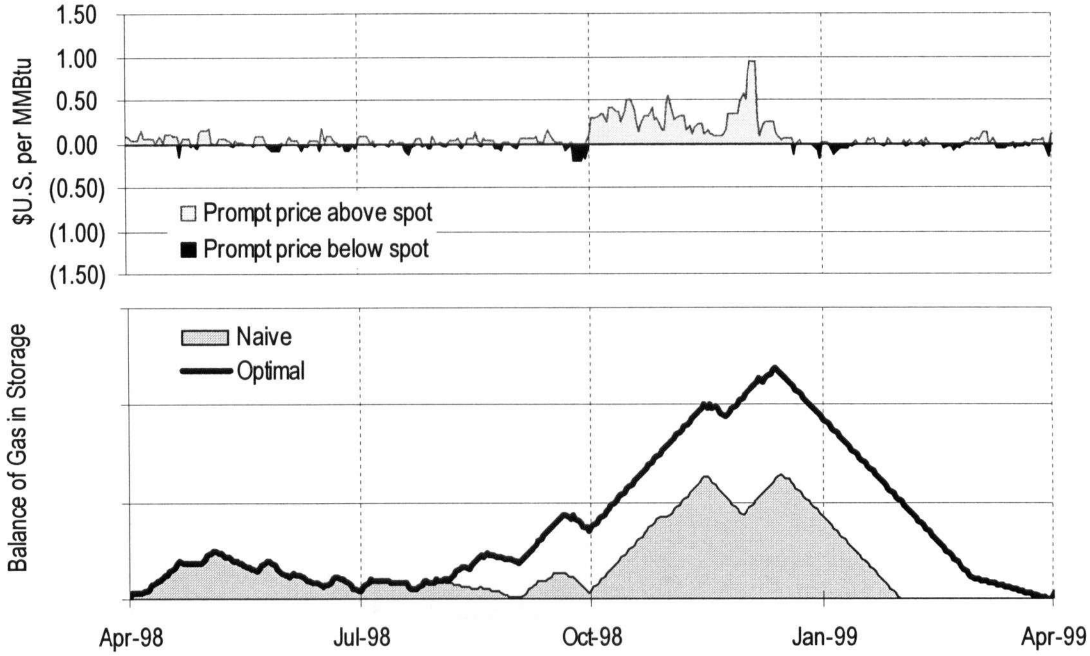

Figure 4.17
Balance of Gas in Storage Resulting from Trades for 1998/99
Day and Prompt Contracts; Hedging; Forecasted Prices; Medium Withdrawal and Injection Limits

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	2.87	1.17	0.09
Mid	1.02	0.80	0.08
Low	0.07	0.08	0.06

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	2.83	1.26	0.09
Mid	1.60	1.26	0.09
Low	0.12	0.13	0.09

OPTIMAL EXCEEDS OR (FALLS SHORT OF) NAIVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	(0.04)	0.09	0.01
Mid	0.58	0.46	0.01
Low	0.05	0.05	0.03

Table 4.21
Total Annual Trading Value Realized for 1998/99 in $U.S. millions
Day and Prompt Contracts; Hedging; Forecasted Prices
(Boxed figures correspond to the operation depicted in the graph immediately above.)

1999/2000:

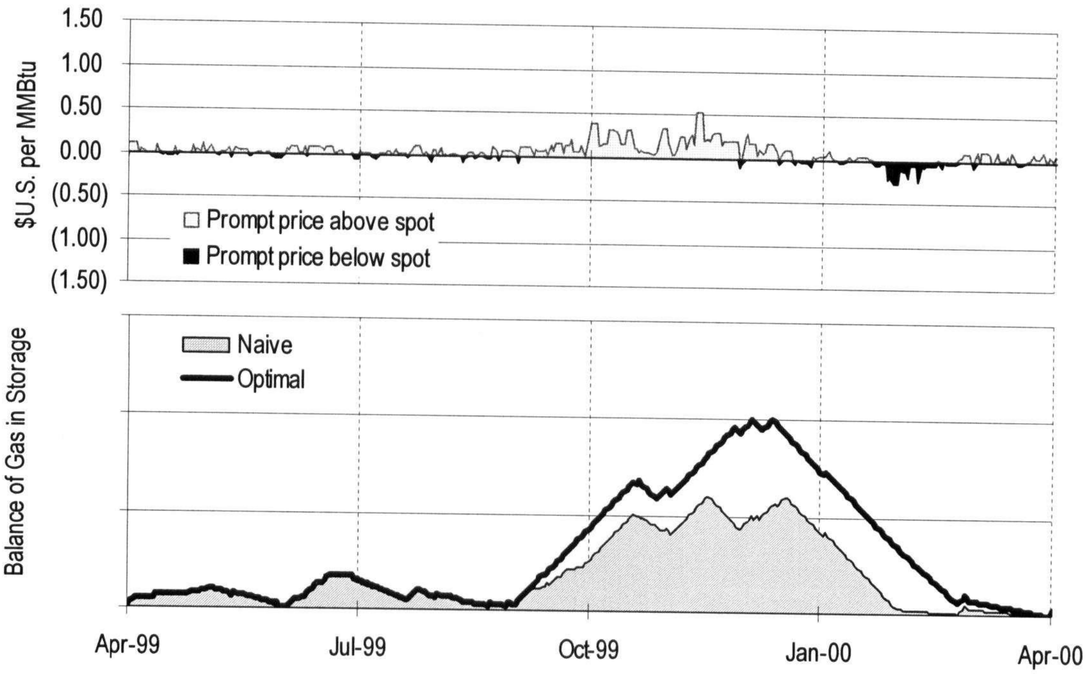

Figure 4.18
Balance of Gas in Storage Resulting from Trades for 1999/2000
Day and Prompt Contracts; Hedging; Forecasted Prices; Medium Withdrawal and Injection Limits

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	2.11	0.72	0.05
Mid	0.72	$\boxed{0.46}$	0.05
Low	0.05	0.05	0.03

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	2.43	0.85	0.06
Mid	1.34	$\boxed{0.64}$	0.06
Low	0.12	0.11	0.05

OPTIMAL EXCEEDS OR (FALLS SHORT OF) NAIVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	0.32	0.12	0.01
Mid	0.62	$\boxed{0.17}$	0.01
Low	0.07	0.06	0.01

Table 4.22
Total Annual Trading Value Realized for 1999/2000 in $U.S. millions
Day and Prompt Contracts; Hedging; Forecasted Prices
(Boxed figures correspond to the operation depicted in the graph immediately above.)

2000/01:

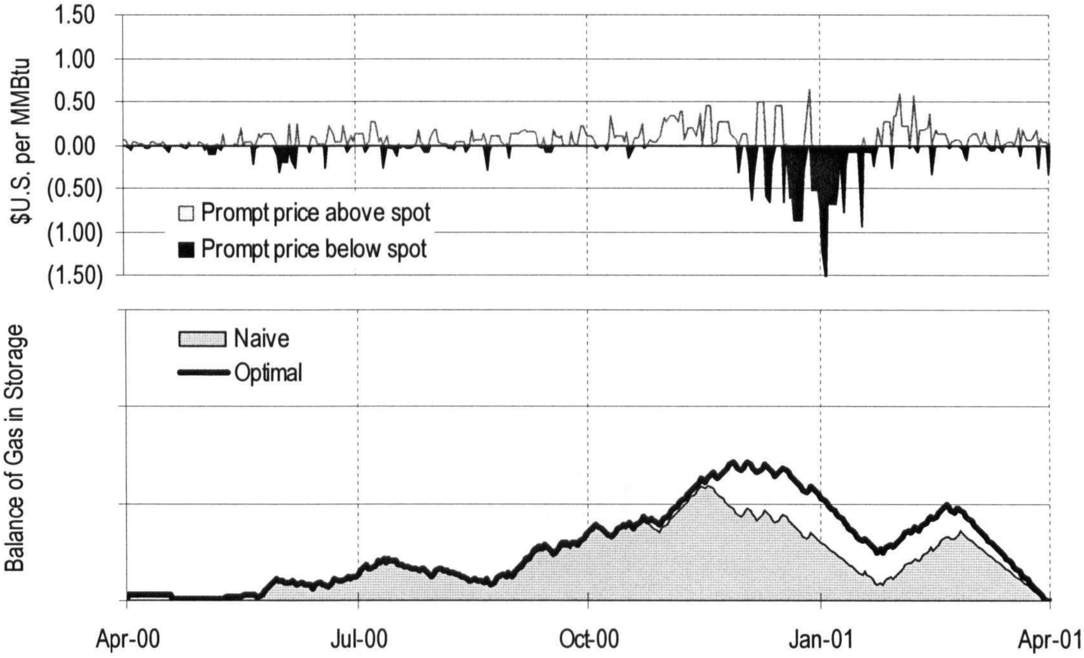

Figure 4.19
Balance of Gas in Storage Resulting from Trades for 2000/01
Day and Prompt Contracts; Hedging; Forecasted Prices; Medium Withdrawal and Injection Limits

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	12.80	1.14	0.08
Mid	1.24	$\boxed{0.84}$	0.08
Low	0.09	0.08	0.06

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	13.47	1.16	0.08
Mid	0.95	$\boxed{0.80}$	0.08
Low	0.07	0.08	0.06

OPTIMAL EXCEEDS OR (FALLS SHORT OF) NAIVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	0.67	0.02	0.00
Mid	(0.29)	$\boxed{(0.04)}$	0.00
Low	(0.01)	(0.00)	(0.00)

Table 4.23
Total Annual Trading Value Realized for 2000/01 in $U.S. millions
Day and Prompt Contracts; Hedging; Forecasted Prices
(Boxed figures correspond to the operation depicted in the graph immediately above.)

2001/02:

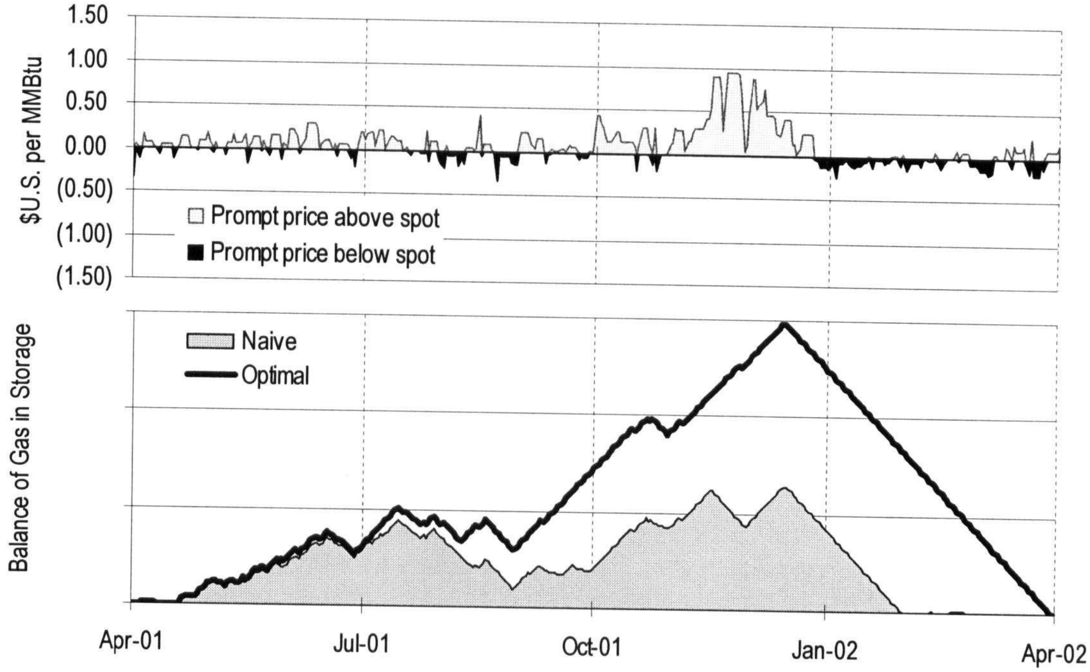

Figure 4.20
Balance of Gas in Storage Resulting from Trades for 2001/02
Day and Prompt Contracts; Hedging; Forecasted Prices; Medium Withdrawal and Injection Limits

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	6.81	1.36	0.10
Mid	1.97	1.05	0.10
Low	0.14	0.13	0.07

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	7.75	1.68	0.12
Mid	3.02	2.07	0.15
Low	0.24	0.23	0.15

OPTIMAL EXCEEDS OR (FALLS SHORT OF) NAIVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	0.93	0.32	0.02
Mid	1.06	1.03	0.05
Low	0.10	0.10	0.07

Table 4.24
Total Annual Trading Value Realized for 2001/02 in $U.S. millions
Day and Prompt Contracts; Hedging; Forecasted Prices
(Boxed figures correspond to the operation depicted in the graph immediately above.)

Average of 1997/98 through 2001/02:

NAIVE STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	5.59	1.00	0.07
Mid	1.16	0.74	0.07
Low	0.08	0.08	0.05

OPTIMAL STRATEGY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	6.04	1.11	0.08
Mid	1.69	1.08	0.09
Low	0.13	0.13	0.08

OPTIMAL EXCEEDS OR (FALLS SHORT OF) NAIVE

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	8%	11%	12%
Mid	46%	47%	20%
Low	57%	67%	47%

Table 4.25
Average Annual Trading Value Realized 1997/98 through 2001/02 in $U.S. millions
Day and Prompt Contracts; Hedging; Forecasted Prices
(Boxed figures correspond to the operation depicted in the five preceding graphs.)

Observations and Discussion

In the graphs above, we see that the balances resulting from the optimal strategy and medium-level withdrawal and injection limits are always above the balances the naive strategy creates. (This is also true for all other combinations of withdrawal and injection limits except two, as shown in the graphs in the appendices.) The two strategies trade almost identically as long as the prompt-spot price differential switches from positive to negative frequently. When the balances of the two strategies diverge due to the optimal strategy injecting on more days than the naïve, the behaviour of the prices at that time reveals many weeks of continually positive differentials, meaning that the spread indicates injection now and withdrawal next month.

The optimal strategy generally outperforms the naïve in terms of net annual trading value realized. The instances when it does not are in 1998/99 when the withdrawal and injection limits are both high, and five out of nine scenarios in 2000/01, the year of the dramatic spike in prices. Considering the average annual net trading value, the optimal strategy outperforms the naïve under all combinations of withdrawal and injection limits by 8–67%.

In this analysis, we give up value due to forecast inaccuracy, but the loss is not as severe when we are hedging as when we are not as in Section 4.1.3.

OPTIMAL STRATEGY PERFECT PRICE INFO

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	10.44	2.36	0.27
Mid	2.90	1.32	0.22
Low	0.23	0.20	0.10

OPTIMAL STRATEGY FORECASTED PRICES

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	6.04	1.11	0.08
Mid	1.69	1.08	0.09
Low	0.13	0.13	0.08

VALUE LOST TO FORECAST INACCURACY

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	-42%	-53%	-70%
Mid	-42%	-18%	-62%
Low	-43%	-34%	-19%

Table 4.26
Average Annual Trading Value Realized 1997/98 through 2001/02 in $U.S. millions
Optimal Strategy; Day and Prompt Contracts; Hedging; Perfect Price Information vs. Forecasted Prices

4.2 THE OPPORTUNITY COST OF HEDGING

In this analysis we look for insights into the opportunity cost of hedging. We know that when hedging is employed, losses will never be incurred and that annual profits depend on market expectations for magnitude of price fluctuations over the course of the year. On the other hand, when hedging is not employed, losses may be incurred. Over several years, the annual net value obtained when not hedging may be negative or positive and when positive, would be greater than the annual profits made in the same year under a hedged strategy. In short, we anticipate that a hedging strategy, the expected value obtained and the variance of it will be lower than under a no-hedging strategy.

This analysis consisted of a series of simulations of the optimal model described in Section 3.2.2 (and analysed in Section 4.1.4) with all hedging constraints in place and a series with all hedging constraints relaxed (constraints (e) and (f) in Section 3.3.2). Each series comprises 45 simulations of the five years and nine combinations of withdrawal and injection limits, with day and prompt contracts available, and using forecasted prices.

The simulation process is identical to that described in Section 4.1.4.

Results

As in the previous four subsections, our results are presented by year. Graphs depict the operation of the storage facility according to the competing, hedging and no-hedging strategies and subject to medium level daily withdrawal and injection limits. Tables present the financial performance of each strategy under each scenario.

1997/98:

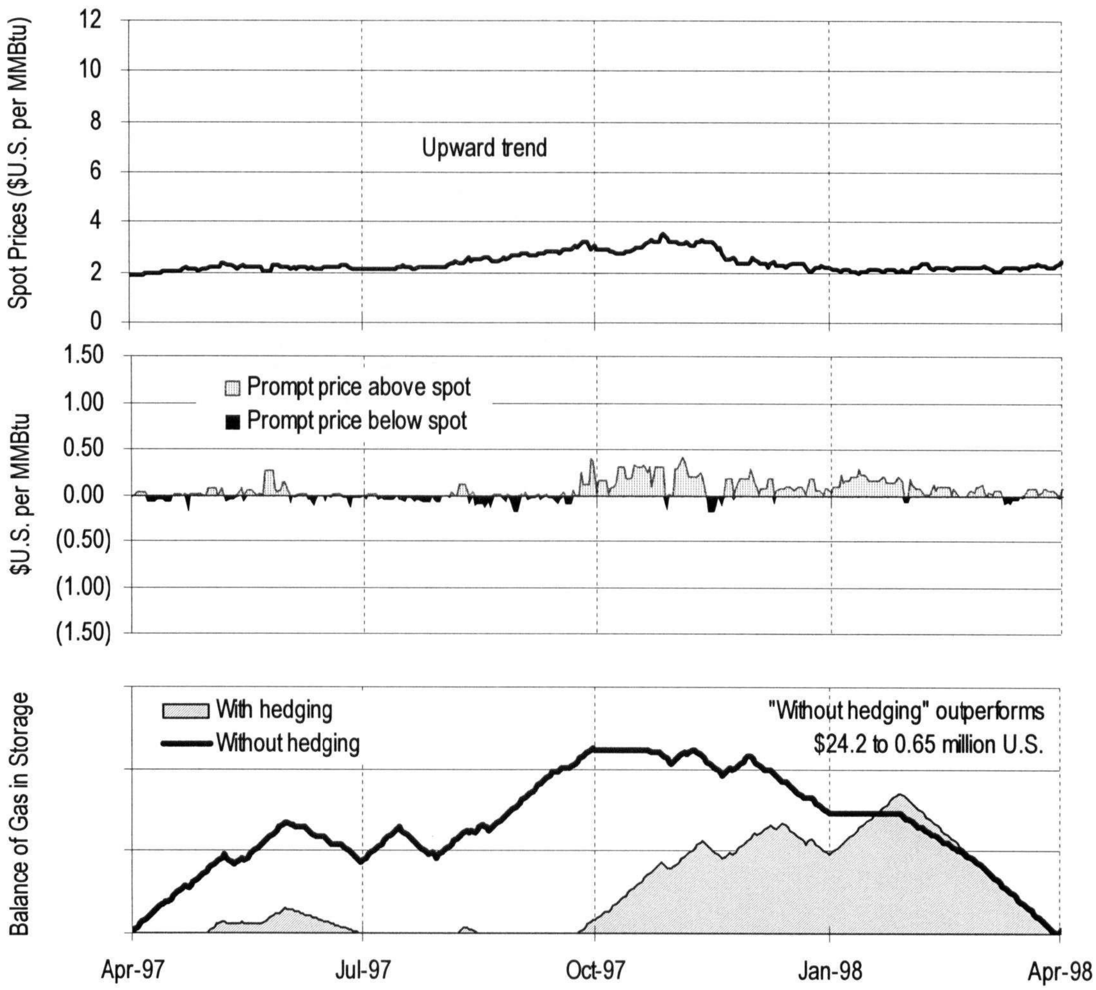

Figure 4.21
Balance of Gas in Storage Resulting from Optimal Hedged and Unhedged Trades for 1997/98
Day and Prompt Contracts; Forecasted Prices; Medium Withdrawal and Injection Limits

WITH HEDGING

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	3.71	0.61	0.04
Mid	1.55	$\boxed{0.65}$	0.05
Low	0.10	0.10	0.05

WITHOUT HEDGING

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	769.6	23.6	1.9
Mid	24.6	$\boxed{24.2}$	1.9
Low	2.0	2.0	1.8

GAINED OR (FOREGONE) BY HEDGING

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	-765.9	-23.0	-1.9
Mid	-23.0	$\boxed{-23.6}$	-1.9
Low	-1.9	-1.9	-1.7

Table 4.27
Total Annual Trading Value Realized With Hedging and Without for 1997/98 in $U.S. millions
Day and Prompt Contracts; Forecasted Prices
(Boxed figures correspond to the operation depicted in the graph immediately above.)

1998/99:

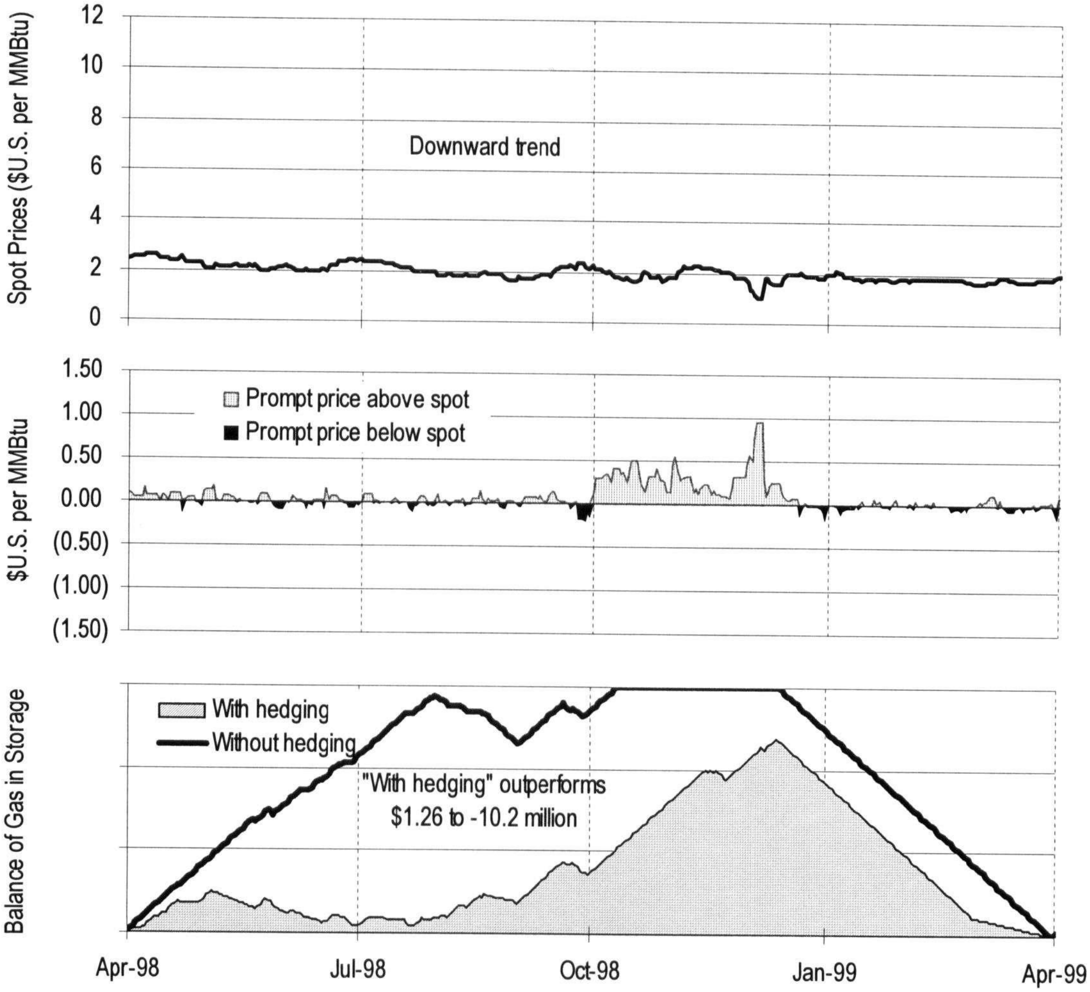

Figure 4.22
Balance of Gas in Storage Resulting from Optimal Hedged and Unhedged Trades for 1998/99 Day and Prompt Contracts; Forecasted Prices; Medium Withdrawal and Injection Limits

WITH HEDGING

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	2.83	1.26	0.09
Mid	1.60	1.26	0.09
Low	0.12	0.13	0.09

WITHOUT HEDGING

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	-336.8	-9.8	-0.7
Mid	-10.9	-10.2	-0.7
Low	-1.0	-1.0	-0.8

GAINED OR (FOREGONE) BY HEDGING

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	339.7	11.0	0.8
Mid	12.5	11.5	0.8
Low	1.1	1.2	0.9

Table 4.28
Total Annual Trading Value Realized With Hedging and Without for 1998/99 in $U.S. millions Day and Prompt Contracts; Forecasted Prices
(Boxed figures correspond to the operation depicted in the graph immediately above.)

1999/2000:

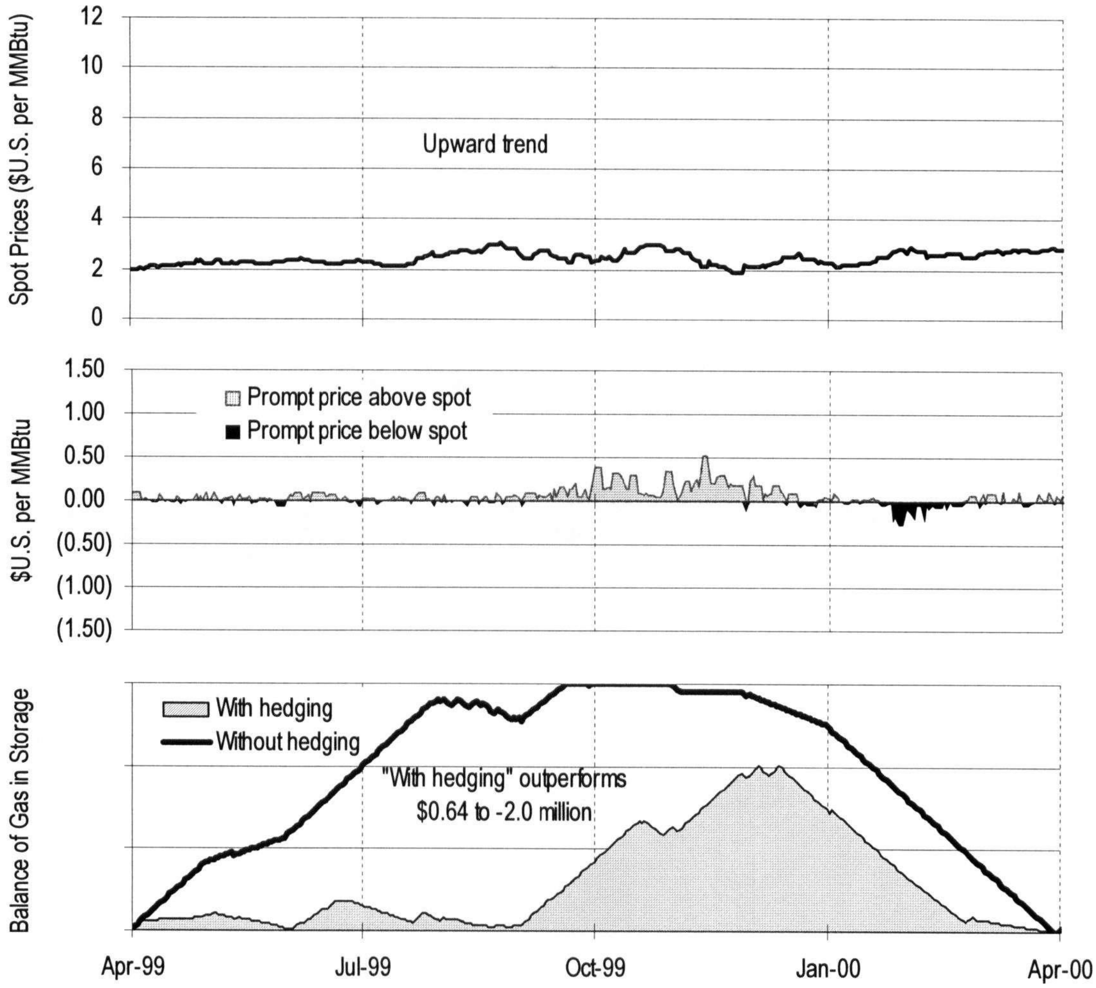

Figure 4.23
Balance of Gas in Storage Resulting from Optimal Hedged and Unhedged Trades for 1999/2000
Day and Prompt Contracts; Forecasted Prices; Medium Withdrawal and Injection Limits

WITH HEDGING

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	2.43	0.85	0.06
Mid	1.34	$\boxed{0.64}$	0.06
Low	0.12	0.11	0.05

WITHOUT HEDGING

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	-116.7	-1.0	0.1
Mid	-1.3	$\boxed{-2.0}$	0.1
Low	0.1	0.0	-0.1

GAINED OR (FOREGONE) BY HEDGING

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	119.1	1.9	0.0
Mid	2.6	$\boxed{2.7}$	0.0
Low	0.0	0.1	0.1

Table 4.29
Total Annual Trading Value Realized With Hedging and Without for 1999/2000 in $U.S. millions
Day and Prompt Contracts; Forecasted Prices
(Boxed figures correspond to the operation depicted in the graph immediately above.)

2000/01:

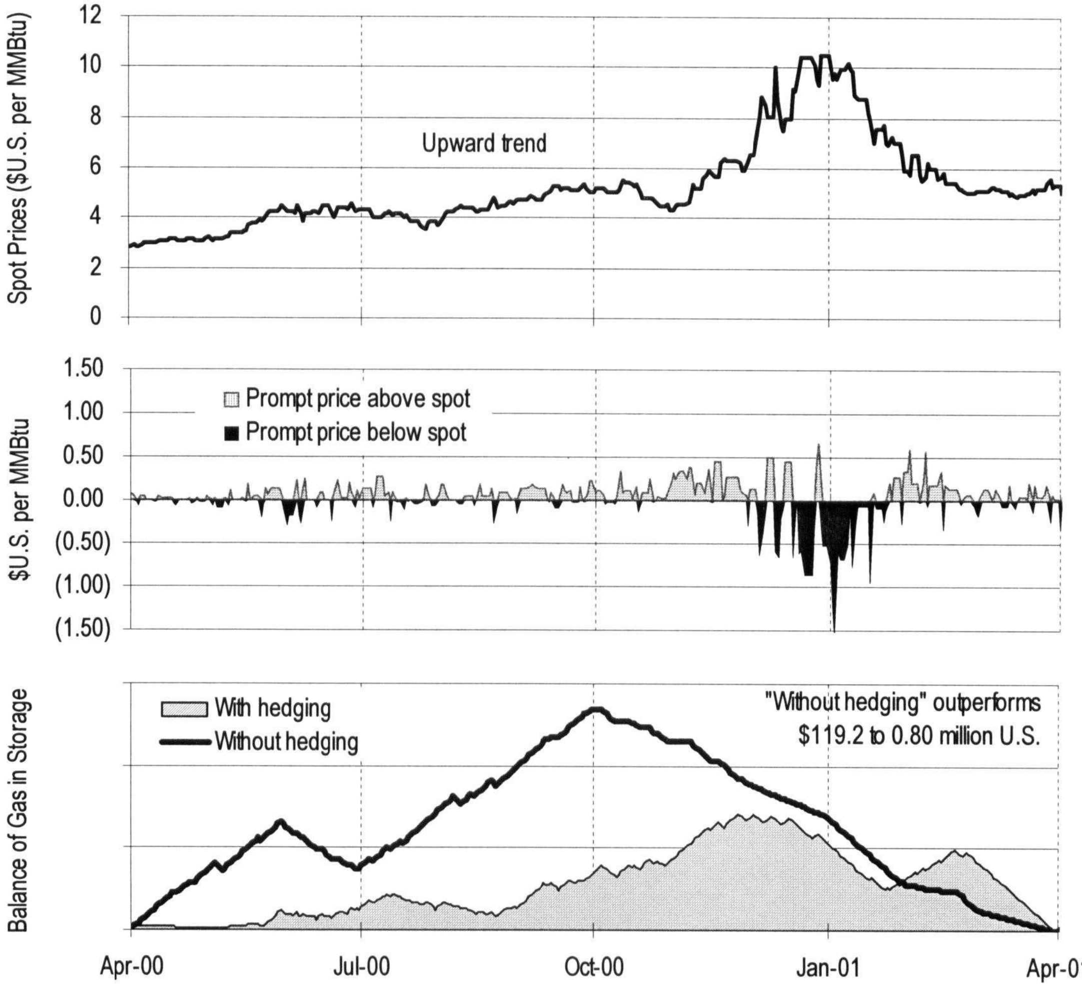

Figure 4.24
Balance of Gas in Storage Resulting from Optimal Hedged and Unhedged Trades for 2000/01 Day and Prompt Contracts; Forecasted Prices; Medium Withdrawal and Injection Limits

WITH HEDGING

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	13.5	1.16	0.08
Mid	0.95	0.80	0.08
Low	0.07	0.08	0.06

WITHOUT HEDGING

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	4,008	115.7	8.3
Mid	121.2	119.2	8.3
Low	9.1	9.0	8.5

GAINED OR (FOREGONE) BY HEDGING

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	-3,995	-114.6	-8.3
Mid	-120.3	-118.4	-8.3
Low	-9.0	-8.9	-8.5

Table 4.30
Total Annual Trading Value Realized With Hedging and Without for 2000/01 in $U.S. millions Day and Prompt Contracts; Forecasted Prices
(Boxed figures correspond to the operation depicted in the graph immediately above.)

2001/02:

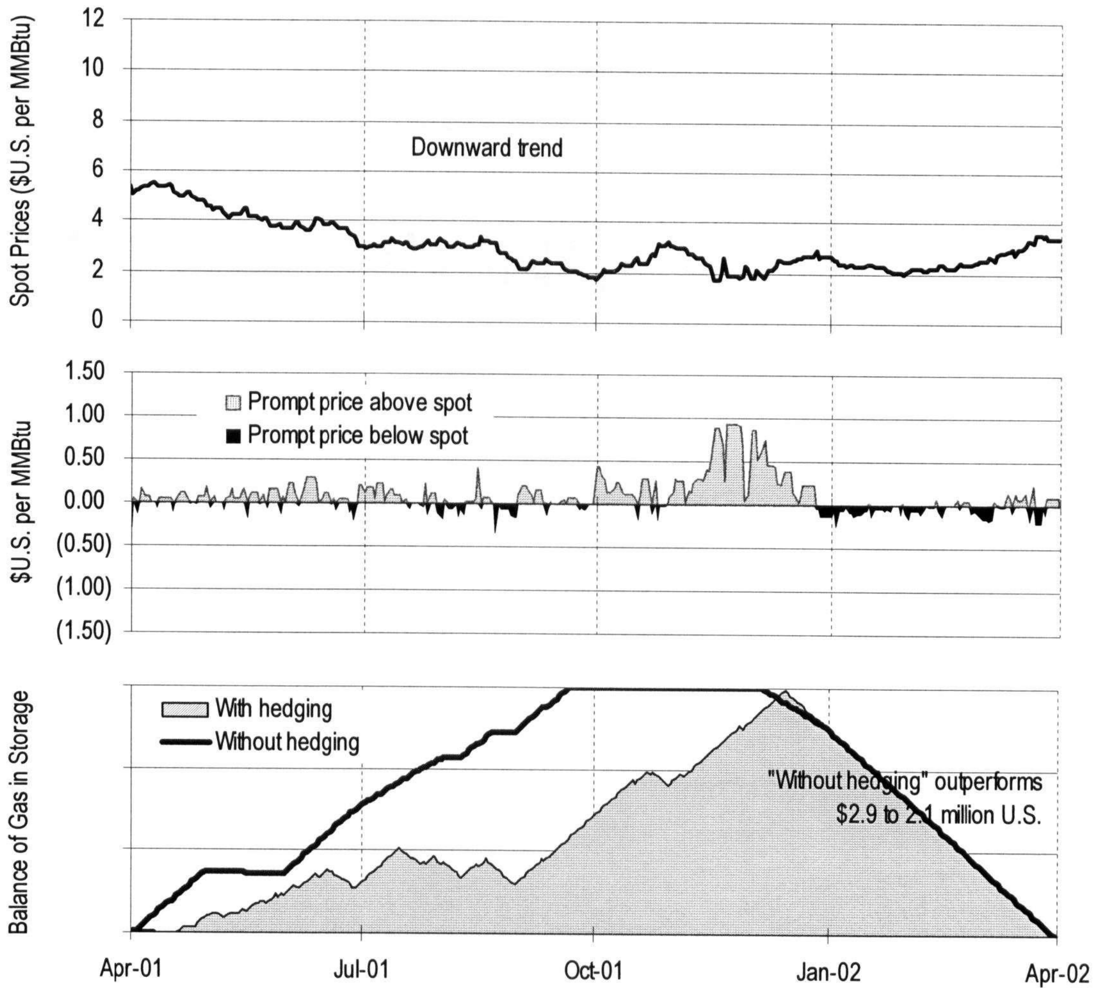

Figure 4.25
Balance of Gas in Storage Resulting from Optimal Hedged and Unhedged Trades for 2001/02 Day and Prompt Contracts; Forecasted Prices; Medium Withdrawal and Injection Limits

WITH HEDGING

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	7.75	1.68	0.12
Mid	3.02	2.07	0.15
Low	0.24	0.23	0.15

WITHOUT HEDGING

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	260.9	5.9	0.4
Mid	-2.5	2.9	0.2
Low	-1.3	-1.2	0.0

GAINED OR (FOREGONE) BY HEDGING

Daily Withdrawal Limit	Daily Injection Limit: High	Mid	Low
High	-253.2	-4.2	-0.3
Mid	5.6	-0.8	0.0
Low	1.5	1.5	0.1

Table 4.31
Total Annual Trading Value Realized With Hedging and Without for 2001/02 in $U.S. millions Day and Prompt Contracts; Forecasted Prices (Boxed figures correspond to the operation depicted in the graph immediately above.)

Average of 1997/98 through 2001/02:

WITH HEDGING

Daily Withdrawal Limit		Daily Injection Limit: High	Mid	Low
	High	6.04	1.11	0.08
	Mid	1.69	1.08	0.09
	Low	0.13	0.13	0.08

WITHOUT HEDGING

Daily Withdrawal Limit		Daily Injection Limit: High	Mid	Low
	High	917.0	26.9	2.0
	Mid	26.2	26.8	2.0
	Low	1.8	1.7	1.9

FOREGONE BY HEDGING

Daily Withdrawal Limit		Daily Injection Limit: High	Mid	Low
	High	-99%	-96%	-96%
	Mid	-94%	-96%	-96%
	Low	-93%	-93%	-96%

Table 4.32
Average Annual Trading Value Realized With Hedging and Without
for 1997/98 through 2001/02 in $U.S. millions
Day and Prompt Contracts; Forecasted Prices

Observations and Discussion

As expected, the financial performance achieved with the no-hedging strategy varies much more widely than the performance with the naïve strategy. We transform the results under the no-hedging strategy from dollars to multiples of the value obtained using the hedging strategy. Only four out of 45 simulations yield multiples in the interval [0, 1] that reflects that not hedging is profitable but less so than hedging. In 24 simulations, multiples greater than one and as high as almost 300 times are observed; 17 simulations yield negative multiples (losses), the most severe being –120 times.

The following figure plots the financial performance of the no-hedging strategy as a multiple of the performance by the hedging strategy for the same years and combinations of withdrawal and injection limits for all 45 simulations:

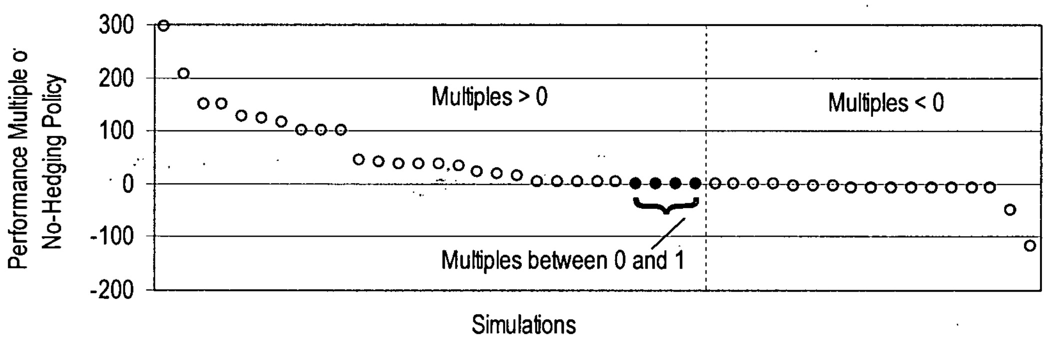

Figure 4.26
Performance of Optimal No-Hedging Strategy Expressed as
Multiples of Net Annual Trading Value Realized with Hedging Policy
All Simulations

In order to understand the factors that may influence the performance of the competing strategies, we consider three parameters of the simulations: daily withdrawal limit, daily injection limit, and price trend during the year. Only the latter seems to influence whether the hedging policy or no-hedging policy is the better performer. Of 45 simulations, the no-hedging policy outperforms the naïve strategy in about $^2/_3$. Of each group of 15 simulations that are subject to high, medium, or low daily withdrawal limits, about $^2/_3$ yield better performance with the no-hedging policy; the same is the case for the three groups of 15 simulations each that are subject to high, medium, and low daily injection limits.

Whether the spot price trend for the year is upward or downward is apparently influential. In years with upward price trends, the no-hedging policy outperforms the hedging policy in about $^7/_9$ of the simulations; in years with downward trends, it is the better policy in only $^1/_3$ of the simulations.

On average, the value foregone due to hedging ranges from 93% to 99% depending on the combination of daily withdrawal and injection constraints. From the opposite point of view, the net trading value realized when hedging is not required is, on average, 34 times the value realized when hedging is required.

CHAPTER 5
VARIATIONS ON TRADING MODELS

We present two variations on the optimal gas trading models described in preceding sections that incorporate features in which BC Gas and Engage Energy are specifically interested. The simplifying assumptions cited in Section 3.1 also apply to these models. Only the formulations are included; we do not perform any numerical analysis with them.

5.1 MODEL D: ROLLING HORIZON AND MULTIPLE MARKETS

This is a variation of Model A presented in Section 3.2; it accommodates only day contracts and therefore does not consider hedging. It varies from that model in two ways. The first variation is that the time horizon is a rolling year that does not wind down as days pass. This reflects, for example, a storage contract that is to be continually renewed and that does not require zero gas holdings at the expiration of each year. Any balance is possible anytime as long as it is between zero and the maximum allowed under the storage contract. Therefore, all solutions and values generated by the model are most meaningful expressed as functions of the balance of gas on hand. The second variation allows gas to be bought and sold at any of several price points.

We formulate this model as a deterministic dynamic program and include descriptive text only where the model is significantly different from Model A.

Decision Epochs

$$T = \{1, 2, \ldots, 365\}$$

States

The state space of this model is no longer time specific. The balance of gas on hand can be between empty and the maximum allowed under the storage contract, inclusive, on any day.

$$S = \{0, \ldots, m_b\}$$

Actions

The action space of this model also is no longer time specific. On any day, possible trades range from a withdrawal in the amount of the balance of gas on hand or the maximum allowable daily withdrawal, whichever is less, to an injection in the amount of the room left in the reserved capacity or the maximum allowable daily injection, whichever is less.

$$A_s = \{-\min[b_t, m_w], \ldots, \min[m_b - b_t, m_i]\}$$

Rewards

Because several gas price points are now accessible, the market price from which the transmission costs are subtracted to yield the net selling price is the highest of available prices. The price to which transmission costs are added to yield the net buying price is the lowest available. The new reward function is represented algebraically as follows:

$$r_t(a_t) = r_t(w_t, i_t) = (\max[p_{1,t}, \ldots, p_{n,t}] - c)w_t - (\min[p_{1,t}, \ldots, p_{n,t}] + c)i_t$$

where

$j \in \{1, 2, \ldots, n\}$, and
$p_{j,t}$ = price at market j at time t.

State Transitions

$$b_t - w_t + i_t = b_{t+1} \quad \forall\, t \in \{1, 2, \ldots, 365\}$$

Optimality Equation

The optimality equation yields the greatest possible value available on day t as a function of the gas balance, through selection of w_t and i_t.

$$\begin{aligned} v_t^*(b_t) &= \max_{a \in A_s} \left\{ r_t(a_t) + v_{t+1}^*(b_{t+1}) \right\} \\ &= \max_{w,i} \left\{ (\max[p_{1,t}, \ldots, p_{n,t}] - c)w_t - (\min[p_{1,t}, \ldots, p_{n,t}] + c)i_t + v_{t+1}^*(b_t - w_t + i_t) \right\}, \\ &\forall\, t \in \{t', \ldots, 365\} \end{aligned}$$

In this model reflecting a perpetually renewing storage contract under which we are not compelled to dispose of all gas holdings by the end of the horizon, we assume that the value of gas on hand at the end of the horizon, when $t = 366$, is equal to the proceeds of a sale of the entire balance not subject to daily withdrawal limits. Hence,

$$v_{366}^*(b_{366}) = (\max[p_{1,366}, \ldots, p_{n,366}] - c)b_{366}$$

5.2 MODEL E: SEVERAL TRADING INSTRUMENTS, VARIABLE WITHDRAWAL AND INJECTION LIMITS, AND TRADING PRIOR TO THE STORAGE CONTRACT

This is a variation of Model C presented in Section 3.3.2. It is a model for the hedged trading of several instruments and varies from Model C in three ways.

The first variation is that five instruments are available for trading rather than two. The instruments accommodated in Model E are described in Table 5.1.

Instrument	Delivery Obligations
Day	• Traded quantity is delivered on trading day (as described in previous models)
Rest-of-month	• Traded quantity of gas is delivered everyday from the day following the trade to the last day of the same month
Prompt	• Traded quantity is delivered everyday of the month following the month in which the trade occurs (as described in previous models)
Summer strip	• Traded quantity is delivered everyday starting on the first of the month following the month in which the trade occurs or April, whichever is later, and ending on the following October 31
Winter strip	• Traded quantity is delivered everyday starting on the first of the month following the month in which the trade occurs or October, whichever is later, and ending on the following March 31

Table 5.1
Delivery Obligations Associated with Trading Instruments

The second variation allows for trading as early as the February 1 immediately prior to the April 1 start of the storage contract term. Since no withdrawals or injections can occur until April 1, only summer and winter strips can be traded during that first February and only summer and winter strips and prompts can be traded during that first March. (From here on, "FebruaryX" and "MarchX" are used to refer to the two months prior to the start of the storage contract term in order to distinguish them from the February and March at the end of the term.)

The third variation is that the daily withdrawal and injection limits are variable. In this case the daily limits are functions of balance of gas in storage although the exact functional form is proprietary information.

Assuming that these daily limit functions are linear, we formulate this model as a linear program and include descriptive text only when the model is significantly different from Model C. Table 5.2, below, provides a cross-reference for certain index values used in the formulation.

Month	Days in the Month	Day Number in the Horizon: First day of the Month	Day Number in the Horizon: Last day of the Month	
FebruaryX	28	1	28	
MarchX	31	29	59	
April	30	60	89	
May	31	90	120	
June	30	121	150	
July	31	151	181	Up to 214 Delivery Days in a Summer Strip
August	31	182	212	
September	30	213	242	
October	31	243	273	
November	30	274	303	
December	31	304	334	
January	31	335	365	Up to 151 Delivery Days in a Winter Strip
February	28	366	393	
March	31	394	424	

Table 5.2
Important Period Lengths And Day Numbers in the 14-Month Horizon

Decision Variables

There is no change from Model C except for the set of possible values for index j.

Primary:
$w_{j,t}$ = units of gas sold in the form of instrument j on day t
$i_{j,t}$ = units of gas purchased in the form of instrument j on day t

Ancillary:
b_t = balance of gas in storage at the beginning of day t

where

$t' \in \{1, 2, \ldots, 424\}$ is the current day, the day on which the model is being solved*
$t \in \{t', t'+1, \ldots, 424\}$ is the trading day in question
$j \in \{1, 2, 3, 4, 5\}$ corresponding to day, rest-of-month, prompt, summer strip, and winter strip contracts, respectively.

Data And Parameters

There is no change from Model C.

p_t = spot price per unit of gas on day t; actual price if $t = t'$, forecasted price otherwise

* In non-leap years, there are 424 days from February 1 of one year to March 31 of the following year.

c = transmission cost per unit of gas
m_b = maximum gas in storage allowed by contract
m_w = maximum daily gas withdrawal allowed by contract
m_i = maximum daily gas injection allowed by contract

Objective Function

$$\underset{w_{j,t},i_{j,t}}{\text{Max}} \left\{ \sum_{j=1}^{5} \sum_{t=1}^{365} \left[(p_{j,t} - c) n_{j,t} w_{j,t} - (p_{j,t} + c) n_{j,t} i_{j,t} \right] \right\}$$

where

$n_{j,t}$ = number of future delivery days associated with instrument j traded on day t.

This objective function differs from that of Model C in two ways: first, by summing over a set of trading instruments, index j, and second, in its specification of the number of days of delivery, $n_{j,t}$, associated with each forward contract, j, where $j \in \{2, 3, 4, 5\}$. (When $j = 1$, the instrument in question is a day contract and results in exactly one day of gas delivery.) Model C is able to use $e_{m+1} - e_m$ because its forward contracts always result in a month of daily deliveries. The forward contracts of Model E result in anywhere from one to 214 daily deliveries depending on the type of contract and day it is traded. Conceptually, however, the objective functions for Models C and E are the same.

Constraints

There is no conceptual difference between the constraints of this model and the constraints of Model C. The differences are due to the greater number of instruments and the various future delivery periods associated with them.

(a) The beginning balance on the current day less the withdrawals plus the injections of the day equal the beginning balance on the following day:

$$b_t - w_{1,t} + i_{1,t} + \sum_{u=d_m}^{t-1} (-w_{2,u} + i_{2,u}) + \sum_{u=d_{m-1}}^{e_{m-1}} (-w_{3,u} + i_{3,u}) + \sum_{u=1}^{e_{m-1}} (-w_{4,u} + i_{4,u}) = b_{t+1},$$

$\forall\, t$ in April–October,

$$b_t - w_{1,t} + i_{1,t} + \sum_{u=d_m}^{t-1} (-w_{2,u} + i_{2,u}) + \sum_{u=d_{m-1}}^{e_{m-1}} (-w_{3,u} + i_{3,u}) + \sum_{u=1}^{e_{m-1}} (-w_{5,u} + i_{5,u}) = b_{t+1},$$

$\forall\, t$ in November–March,

where

m = the month in which t is, $m - 1$ = the previous month,
d_m = the day of the horizon that is the first day of month m (e.g., $d_{\text{April}} = 60$), and
e_m = the day of the horizon that is the last day of month m (e.g., $e_{\text{April}} = 89$).

For example, the closing balance on April 6 is made up of the starting balance of the day, deliveries of day trades made that day, deliveries of rest-of-month trades made on all earlier April days, deliveries of prompt trades made on all MarchX days, and deliveries of summer strip trades made on all FebruaryX and MarchX days. No deliveries from winter strip trades occur in April.

Recall that since the storage contract starts on April 1, only future contracts with delivery starting on or after April 1 are traded prior to April 1. Hence, daily balance constraints are not needed for FebruaryX and MarchX.

(b) The net delivery (injections less withdrawals) on each day must be within the limits of the contract:

$$-m_{t,w}(b_t) \le -w_{1,t} + i_{1,t} + \sum_{u=d_m}^{t-1} (-w_{2,u} + i_{2,u}) + \sum_{u=d_{m-1}}^{e_{m-1}} (-w_{3,u} + i_{3,u}) + \sum_{u=1}^{e_{m-1}} (-w_{4,u} + i_{4,u}) \le m_{t,i}(b_t),$$

$\forall$ t in April–October,

$$-m_{t,w}(b_t) \le -w_{1,t} + i_{1,t} + \sum_{u=d_m}^{t-1} (-w_{2,u} + i_{2,u}) + \sum_{u=d_{m-1}}^{e_{m-1}} (-w_{3,u} + i_{3,u}) + \sum_{u=1}^{e_{m-1}} (-w_{5,u} + i_{5,u}) \le m_{t,i}(b_t), \text{ and}$$

$\forall$ t in November–March,

(c) At the start and end of the contract, the balance of gas on hand must be zero. Everyday, balances must be between empty and the maximum balance allowed under the contract:

$b_1 = b_{425} = 0$, and
$0 \le b_t \le m_b \ \forall t \in \{t', t'+1, \ldots, 424\}$.

(d) Certain instruments are unavailable at times because they would result in delivery before the start of or after the end of the storage contract. Quantities bought and sold must be non-negative:

$w_{j,t}, i_{j,t} = 0, \forall j = 1$ and $\forall t$ in FebruaryX–MarchX;
$w_{j,t}, i_{j,t} = 0, \forall j = 2$ and $\forall t$ in FebruaryX–MarchX or the last day of any month;
$w_{j,t}, i_{j,t} = 0, \forall j = 3$ and $\forall t$ in March;
$w_{j,t}, i_{j,t} = 0, \forall j = 4$ and $\forall t$ in October–March; and
$w_{j,t}, i_{j,t} = 0, \forall j = 5$ and $\forall t$ in March.
$w_{j,t}, i_{j,t} \ge 0, \forall j \in \{1, 2, 3, 4, 5\}$ and $\forall t \in \{t', t'+1, \ldots, 424\}$.

(e) The net quantity of gas to be delivered as a result of each trade must be zero:

$$(-w_{1,t} + i_{1,t}) + n_{2,t}(-w_{2,t} + i_{2,t}) + n_{3,t}(-w_{3,t} + i_{3,t}) + n_{4,t}(-w_{4,t} + i_{4,t}) + n_{5,t}(-w_{5,t} + i_{5,t}) = 0,$$

$\forall t \in \{t', t'+1, \ldots, 424\}$.

(f) For each trade and in the absence of all subsequent trades, the balances following the last delivery from each contract traded must be within the contractual limits:

for FebruaryX trades which include only summer and winter strips,

$0 \le 0 + 214(-w_{4,t} + i_{4,t}) \le m_b$, and
$0 \le 0 + 214(-w_{4,t} + i_{4,t}) + 151(-w_{5,t} + i_{5,t}) \le m_b$,
$\forall t \in \{1, \ldots, 28\}$;

for MarchX trades which include only prompts and summer and winter strips,

$$
\begin{aligned}
&0 \le 0 + 30(-w_{3,t} + i_{3,t}) \le m_b, \\
&0 \le 0 + 30(-w_{3,t} + i_{3,t}) + 214(-w_{4,t} + i_{4,t}) \le m_b, \text{ and} \\
&0 \le 0 + 30(-w_{3,t} + i_{3,t}) + 214(-w_{4,t} + i_{4,t}) + 151(-w_{5,t} + i_{5,t}) \le m_b, \\
&\quad \forall\, t \in \{29, \ldots, 59\};
\end{aligned}
$$

for April trades which include all instruments,

$$
\begin{aligned}
&0 \le b_t + (-w_{1,t} + i_{1,t}) \le m_b, \\
&0 \le b_t + (-w_{1,t} + i_{1,t}) + (89 - t)(-w_{2,t} + i_{2,t}) \le m_b, \\
&0 \le b_t + (-w_{1,t} + i_{1,t}) + (89 - t)(-w_{2,t} + i_{2,t}) + 31(-w_{3,t} + i_{3,t}) \le m_b, \\
&0 \le b_t + (-w_{1,t} + i_{1,t}) + (89 - t)(-w_{2,t} + i_{2,t}) + 31(-w_{3,t} + i_{3,t}) + 184(-w_{4,t} + i_{4,t}) \le m_b, \text{ and} \\
&0 \le b_t + (-w_{1,t} + i_{1,t}) + (89 - t)(-w_{2,t} + i_{2,t}) + 31(-w_{3,t} + i_{3,t}) + 184(-w_{4,t} + i_{4,t}) \\
&\qquad + 151(-w_{5,t} + i_{5,t}) \le m_b, \\
&\quad \forall\, t \in \{60, \ldots, 89\};
\end{aligned}
$$

and so on.

Generally, the j^{th} constraint in the group of five—there are five instruments traded in this model—requires that the balance following the last daily delivery arising from the trade of the j^{th} instrument be between 0 and m_b, where $j \in \{1, 2, 3, 4, 5\}$ corresponding to day, rest-of-month, prompt, summer strip, and winter strip contracts, respectively; the elements of the set reflect the sequence in which the five series of daily deliveries arising from a trade involving all five instruments complete. The general form of this series of constraints is the following:

$$
\begin{aligned}
&0 \le b_t + (-w_{1,t} + i_{1,t}) \le m_b, \\
&0 \le b_t + (-w_{1,t} + i_{1,t}) + (e_m - t)(-w_{2,t} + i_{2,t}) \le m_b, \\
&0 \le b_t + (-w_{1,t} + i_{1,t}) + (e_m - t)(-w_{2,t} + i_{2,t}) + (e_{m+1} - e_m)(-w_{3,t} + i_{3,t}) \le m_b, \\
&0 \le b_t + (-w_{1,t} + i_{1,t}) + (e_m - t)(-w_{2,t} + i_{2,t}) + (e_{m+1} - e_m)(-w_{3,t} + i_{3,t}) + \\
&\qquad (273 - \max[e_m, 59])(-w_{4,t} + i_{4,t}) \le m_b, \text{ and} \\
&0 \le b_t + (-w_{1,t} + i_{1,t}) + (e_m - t)(-w_{2,t} + i_{2,t}) + (e_{m+1} - e_m)(-w_{3,t} + i_{3,t}) + \\
&\qquad (273 - \max[e_m, 59])(-w_{4,t} + i_{4,t}) + (424 - \max[e_m, 273])\,(-w_{5,t} + i_{5,t}) \le m_b, \\
&\quad \forall\, t \in \{t', t' + 1, \ldots, 393\}.
\end{aligned}
$$

CHAPTER 6
CONCLUSION

6.1 SUMMARY

This paper discusses deterministic optimization models for the trading of natural gas from a restricted storage facility. We consider trading with one instrument and two, and with and without a hedging policy in place. Our numerical study first compares the performance of each optimization model to an appropriate "naïve" strategy representing the hypothetical strategy of an unaided trader. Second, it compares the outcomes of applying the optimization models when hedging is required to when it is not, as a means of gaining insight into the opportunity cost of a hedging policy.

When perfect price information is used, the optimization model outperforms the naïve strategy in every simulation. On average, it did so by 14–399% depending on whether only day contracts were being traded or day and prompt contracts were hedged and on the combination of daily withdrawal and injection limits in place.

When price forecasts replace perfect information, either the optimization model or the naïve strategy may be the better performer from one simulation to another. When the optimization model performs better, however, it does so by a greater margin. Considering the average annual value realized over the five years, the optimization model performs better than the naïve strategy in all scenarios when a hedging policy is in place and in all but two when a no-hedging policy is in place.

As expected, the relative performance of the optimization models drops when perfect price information is replaced by forecasts. Based on the average results of the five years, when a hedging policy is in place, 18–70% was lost and with a no-hedging policy, 73–162%.

Our comparison of the financial performance of the optimization model when a hedging policy is in place to when it is not suggests that a hedging policy has a dramatic opportunity cost. Averaged over the five years and everything else unchanged, the hedging policy results in 93–99% less net trading value realized than the no-hedging policy. From the opposite perspective, the no-hedging policy realizes 34 times the value that the hedging policy does.

6.2 FUTURE RESEARCH

Price uncertainty is the issue most deserving further examination. In Chapter 2, we establish that it is very difficult to identify a stochastic process that (1) fully explains price, except for "white noise" that is entirely random and independent from day to day, and (2) leads to a tractable optimization. Consider, however, a price model that, admittedly, does not fully explain price and which yields dependent daily error terms that are nevertheless assumed to be random and independent. Despite these compromises, such a model could conceivably suggest trades that lead to better overall financial performance than our deterministic models do, by taking price uncertainty into account. Thus, additional effort would be well spent researching the combination of a small number of variables that partially explain price and together yield the greatest improvement in financial performance.

Another issue is that the numerical study of our deterministic models would be enhanced by a series of Monte Carlo simulations. The comparisons made in Chapter 4 between optimal and naïve strategies and between hedging and no-hedging policies give compelling results. However, a large number of simulations using randomly generated year-long price paths would provide better long-run estimates of the value of optimization models and the opportunity cost of hedging. As discussed, a price model for use in a stochastic optimization, requiring valid price projections one year in advance, is not available. However, a random model that is a reasonable forecaster one day in advance may be both attainable and adequate as a random price generator for Monte Carlo simulations.

Last, we suggest an application of optimization techniques beyond the scope of this work. Consider the reserved storage capacity that is assumed to be in place when our trading models are applied. Such storage capacity is traded in the marketplace as gas itself is. The optimization models described in this paper could be expanded and applied to the optimal acquisition of storage capacity by, for example

- determining the marginal value of each of the major storage contract parameters: reserved capacity, the maximum daily withdrawal quantity, and maximum daily injection quantity;

- determining the highest level of a particular storage parameter that can be utilized, given the other two; and

- determining the optimal combination of values for these storage contract parameters.

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APPENDIX A

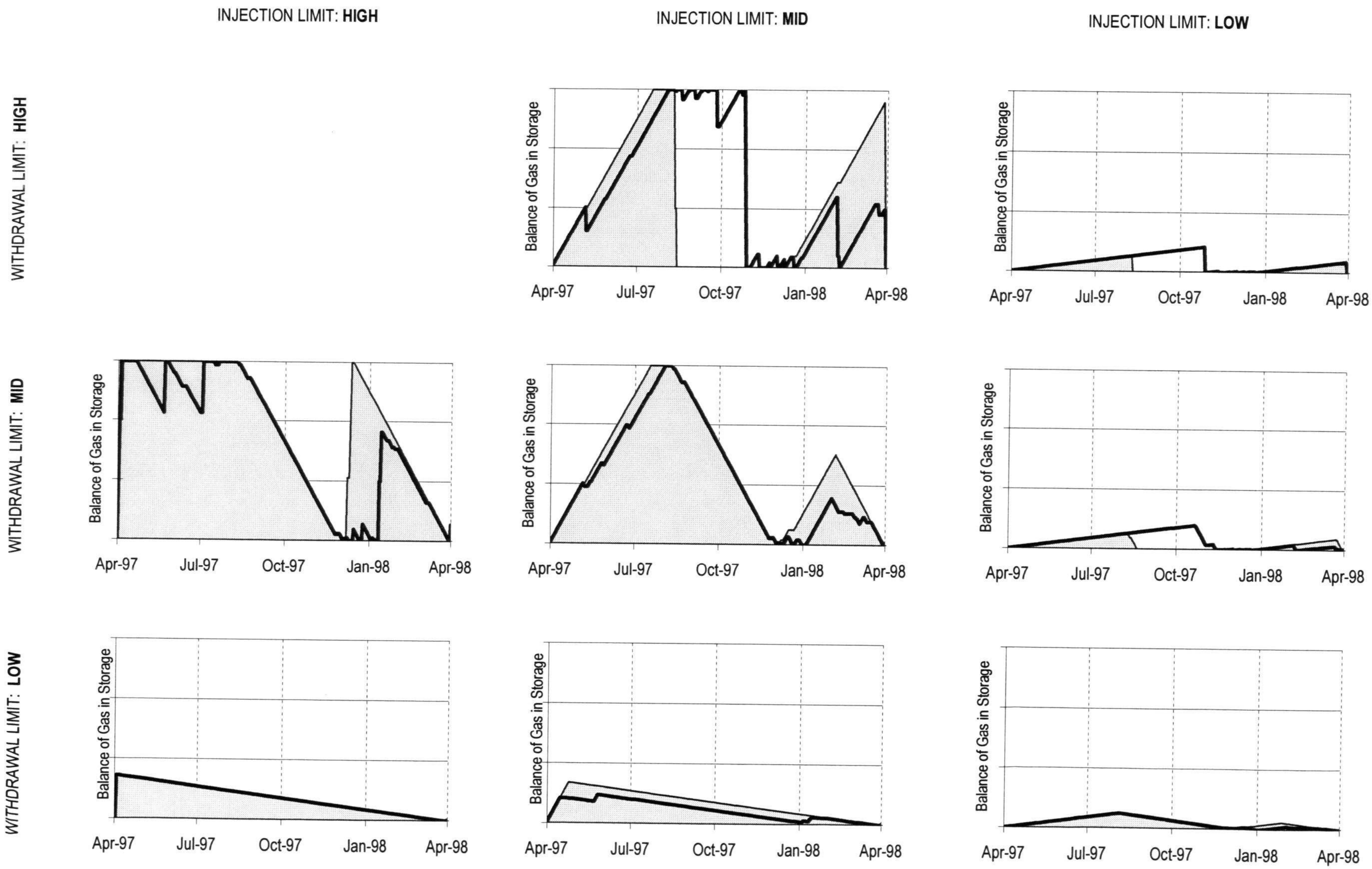
GRAPHS OF BALANCES OF GAS IN STORAGE
PERTAINING TO SECTION 4.1.1

ALL COMBINATIONS OF WITHDRAWAL AND INJECTION LIMITS AND ALL YEARS

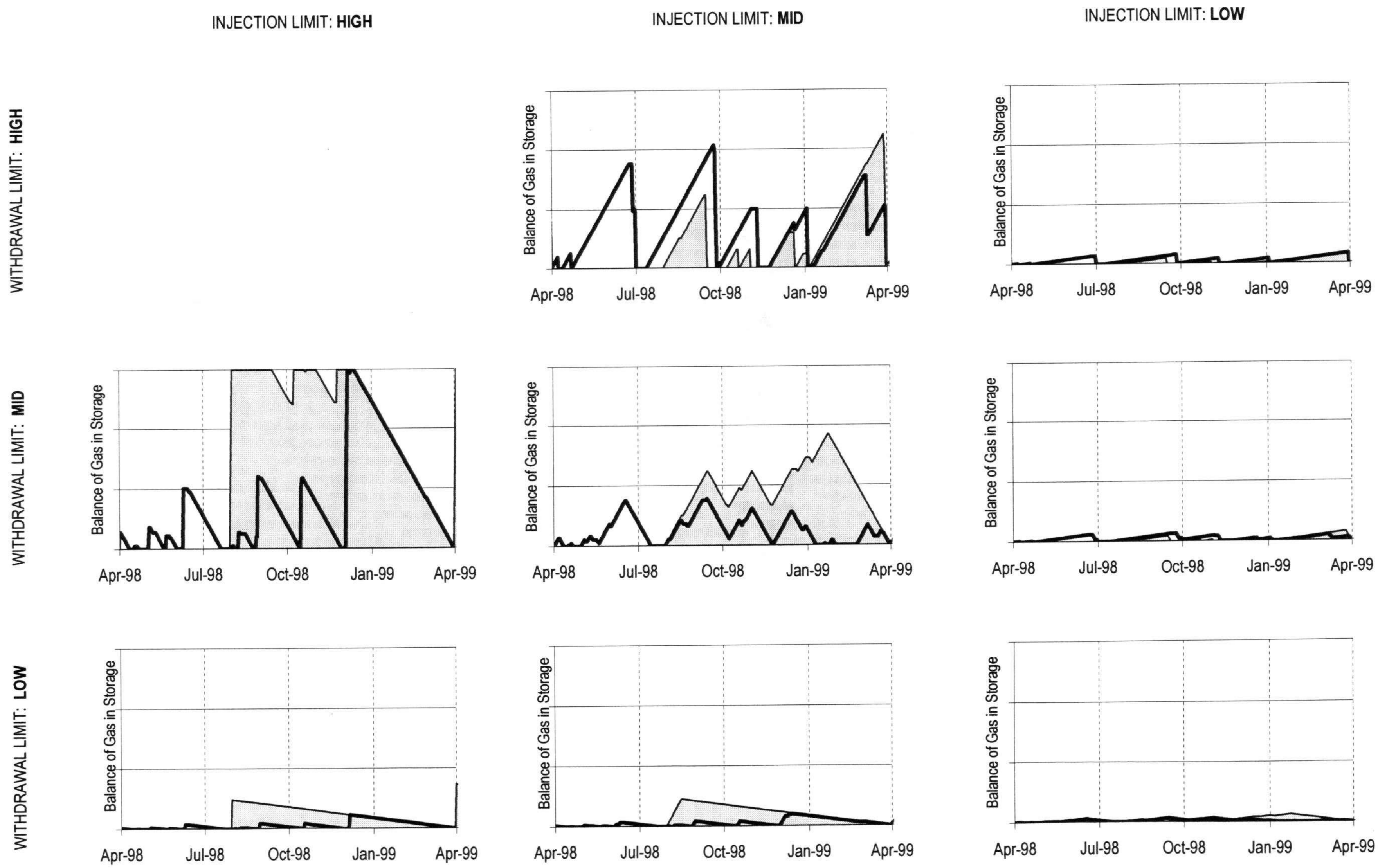
OPTIMAL AND NAÏVE STRATEGIES
DAY CONTRACTS ONLY, PERFECT PRICE INFORMATION

On each graph, the shaded area reflects "naïve" operation; the heavy line reflects "optimal" operation. No graphs for (high, high) limits are included; these relaxed limits lead to such frequent withdrawals and injections that the graphs are illegible.

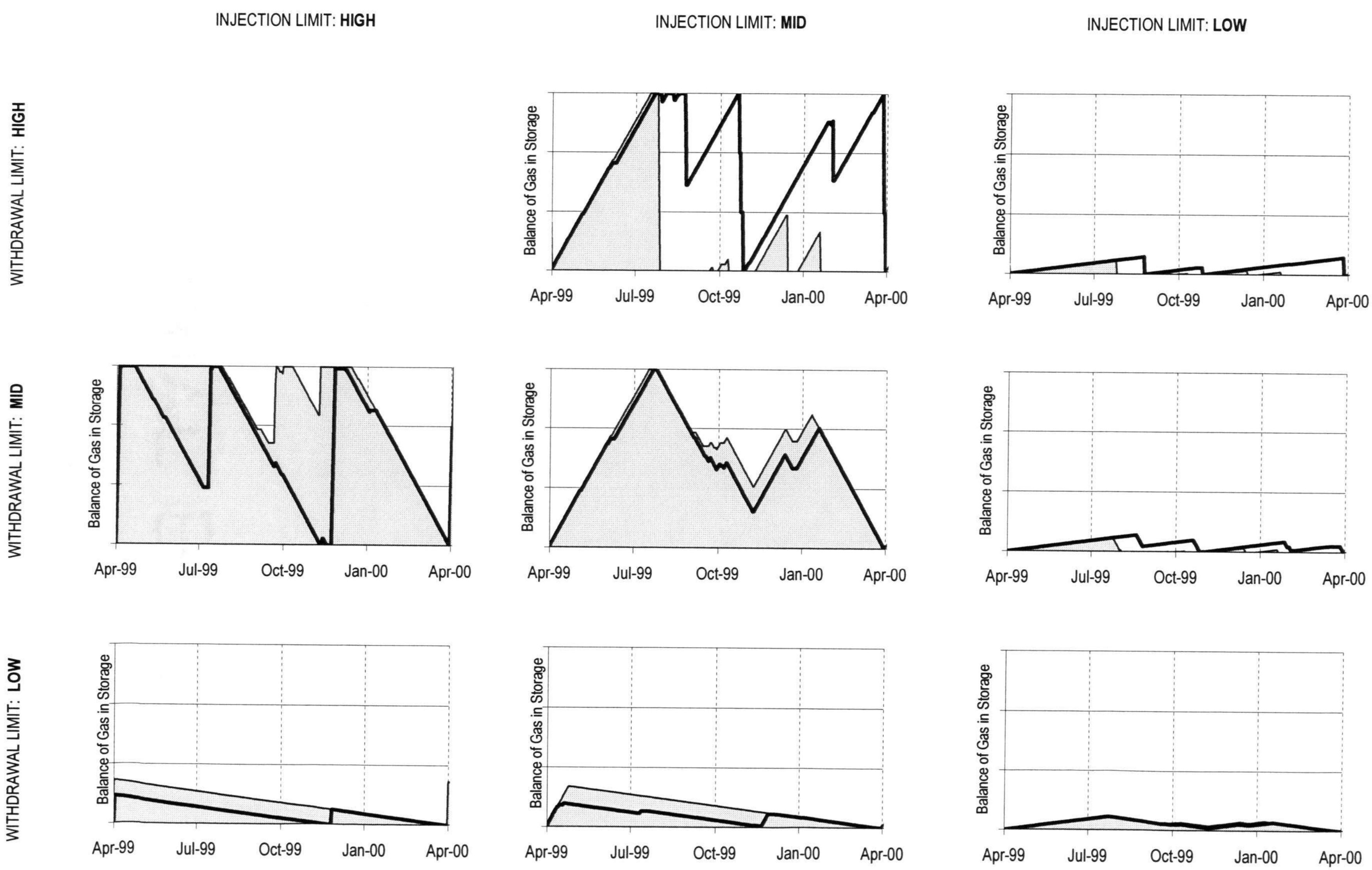
OPTIMAL VERSUS NAÏVE, DAY CONTRACTS ONLY (NO HEDGING), PERFECT PRICE INFORMATION



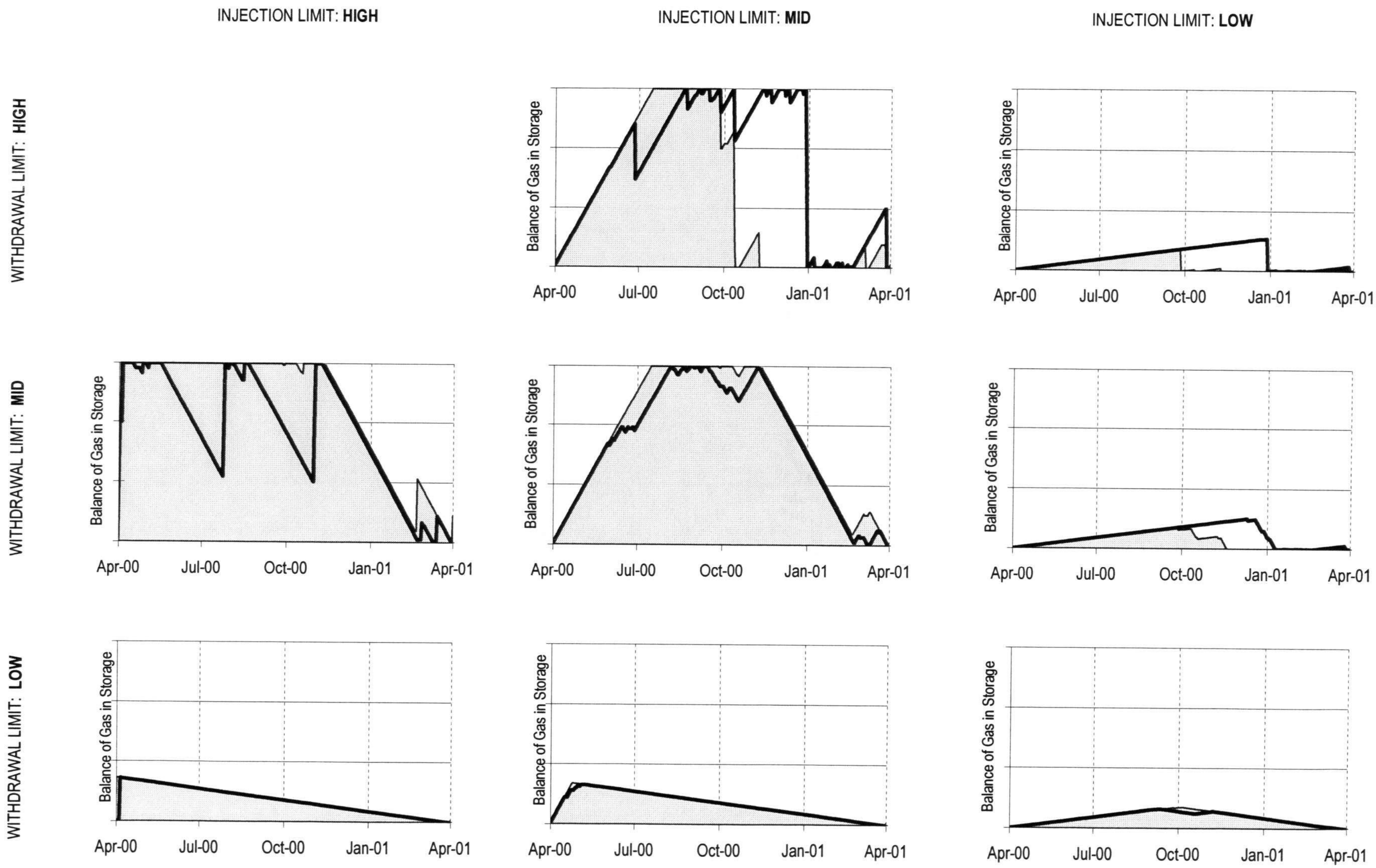
OPTIMAL VERSUS NAÏVE, DAY CONTRACTS ONLY (NO HEDGING), PERFECT PRICE INFORMATION



OPTIMAL VERSUS NAÏVE, DAY CONTRACTS ONLY (NO HEDGING), PERFECT PRICE INFORMATION



OPTIMAL VERSUS NAÏVE, DAY CONTRACTS ONLY (NO HEDGING), PERFECT PRICE INFORMATION



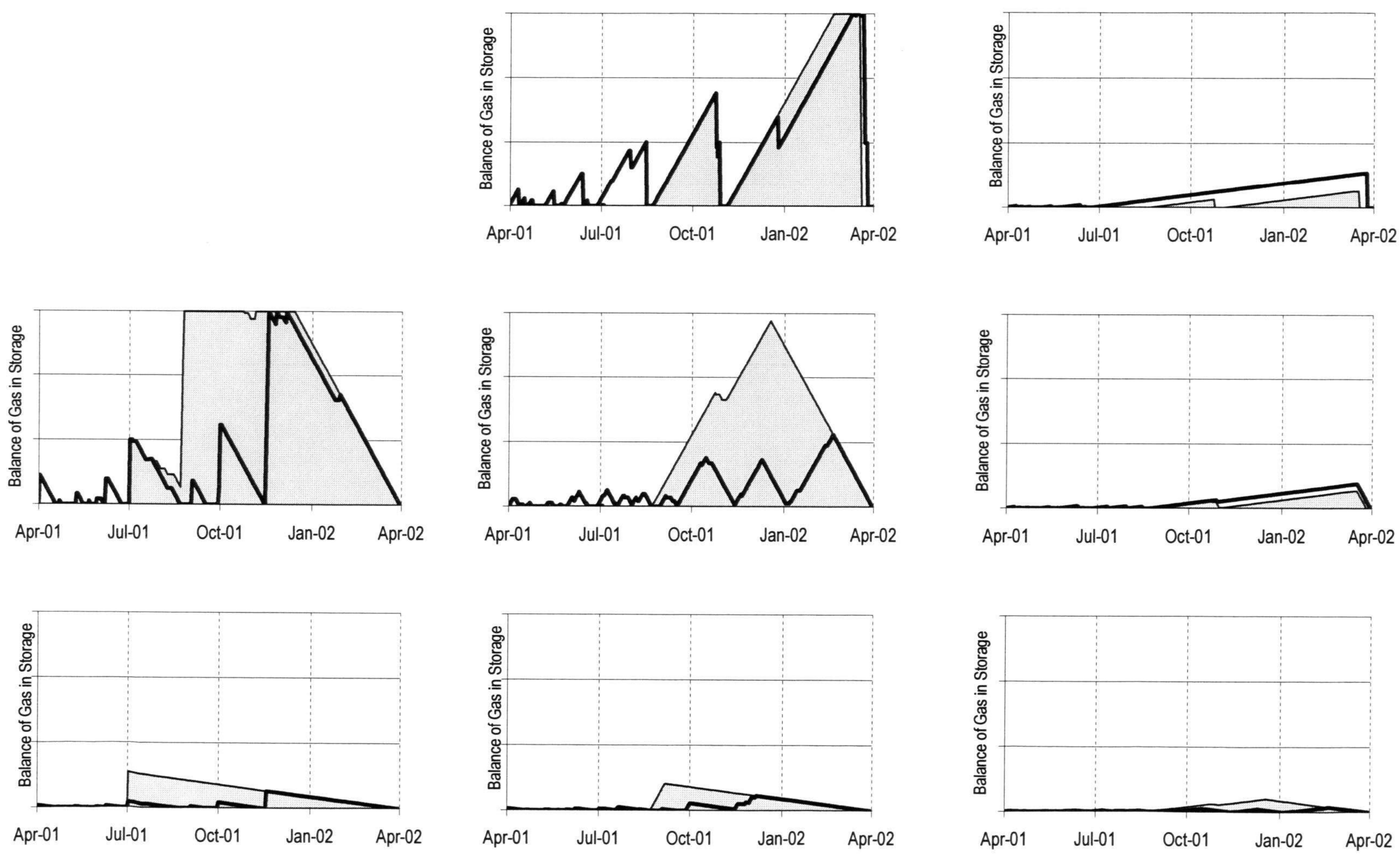
OPTIMAL VERSUS NAÏVE, DAY CONTRACTS ONLY (NO HEDGING), PERFECT PRICE INFORMATION

INJECTION LIMIT: **HIGH** | INJECTION LIMIT: **MID** | INJECTION LIMIT: **LOW**

WITHDRAWAL LIMIT: **HIGH**

WITHDRAWAL LIMIT: **MID**

WITHDRAWAL LIMIT: **LOW**



APPENDIX B

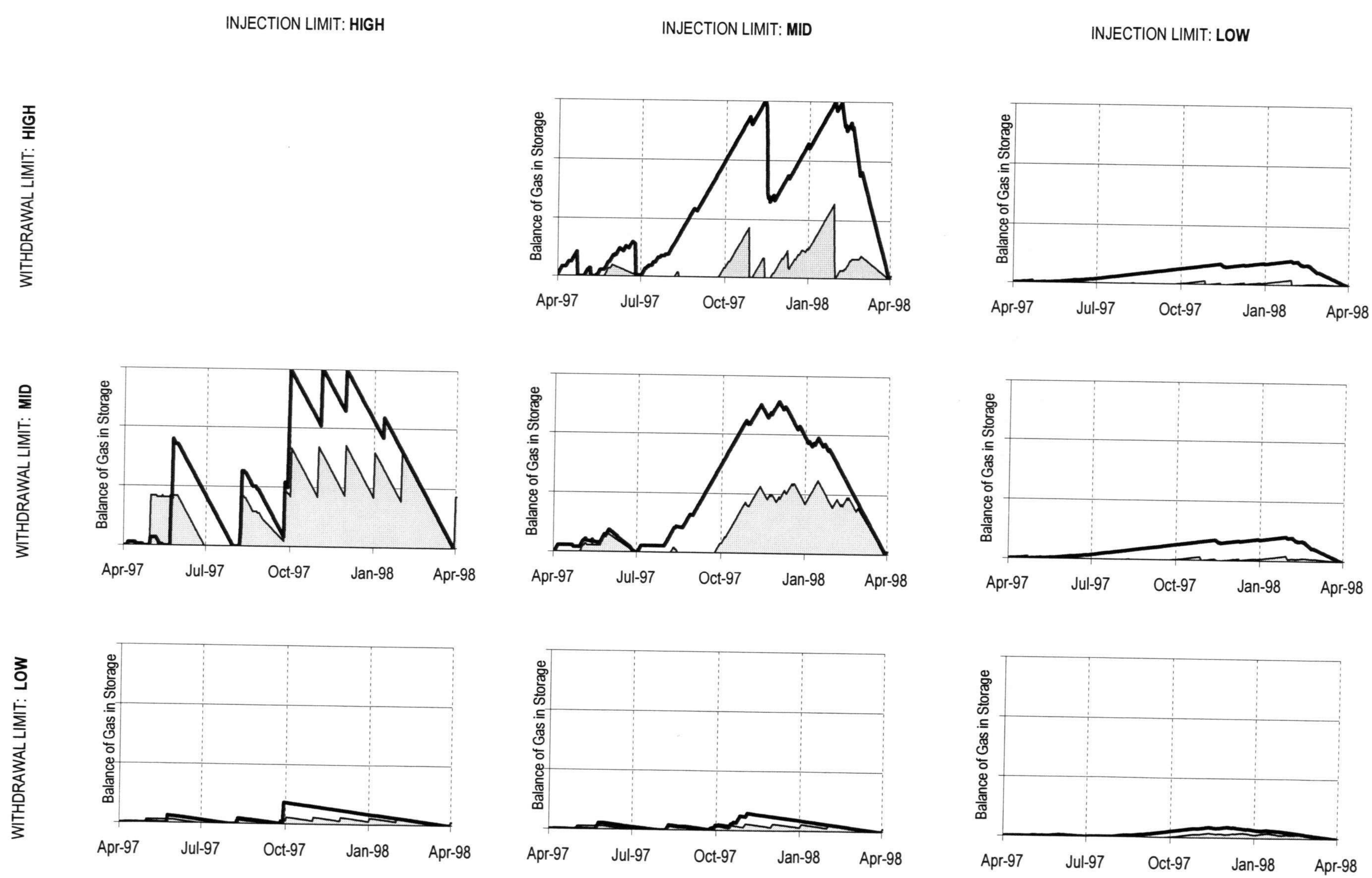
GRAPHS OF BALANCES OF GAS IN STORAGE
PERTAINING TO SECTION 4.1.2

ALL COMBINATIONS OF WITHDRAWAL AND INJECTION LIMITS AND ALL YEARS

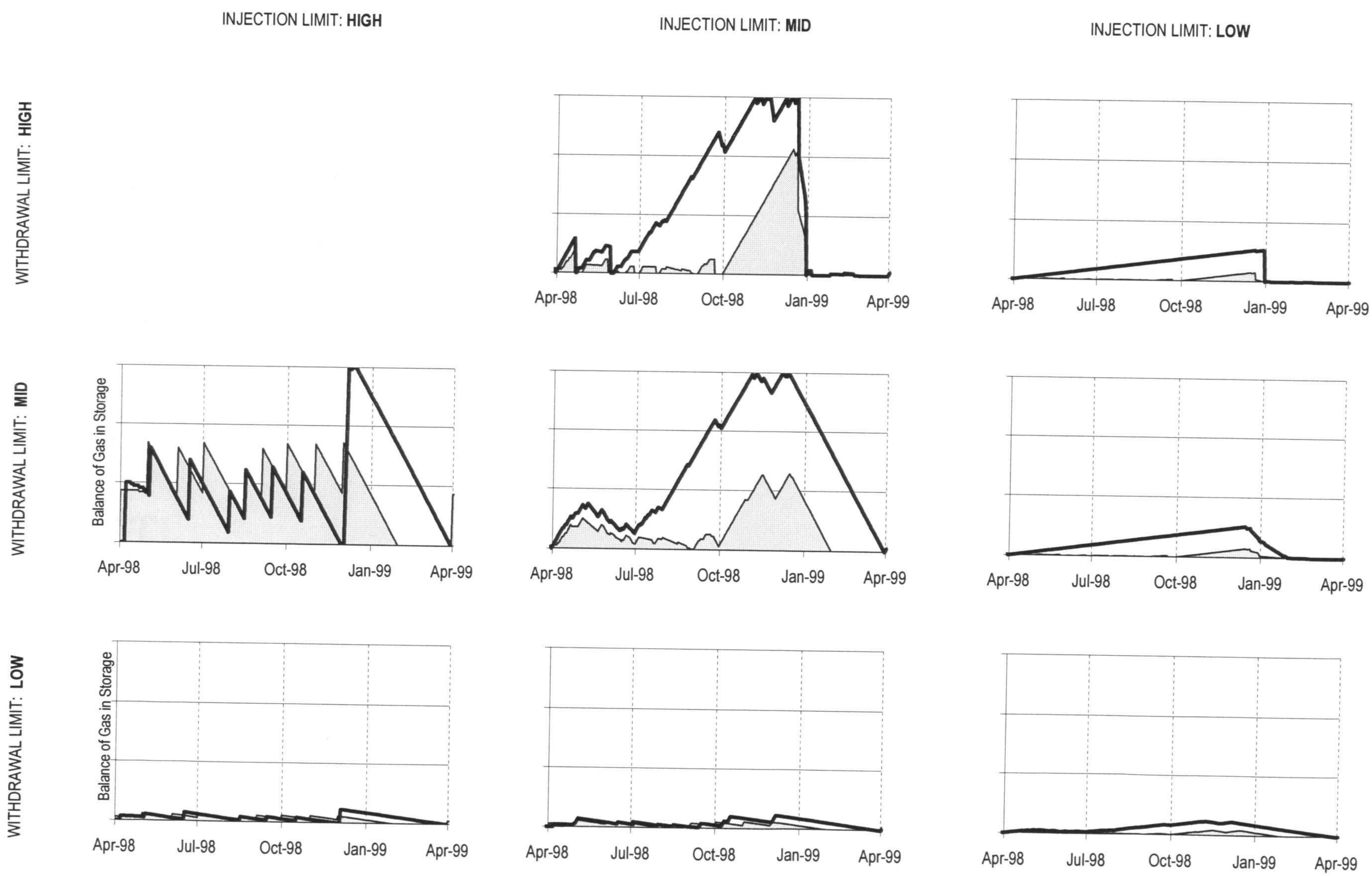
OPTIMAL AND NAÏVE STRATEGIES
DAY AND PROMPT CONTRACTS, HEDGING, PERFECT PRICE INFORMATION

On each graph, the shaded area reflects "naïve" operation; the heavy line reflects "optimal" operation. No graphs for (high, high) limits are included; these relaxed limits lead to such frequent withdrawals and injections that the graphs are illegible.

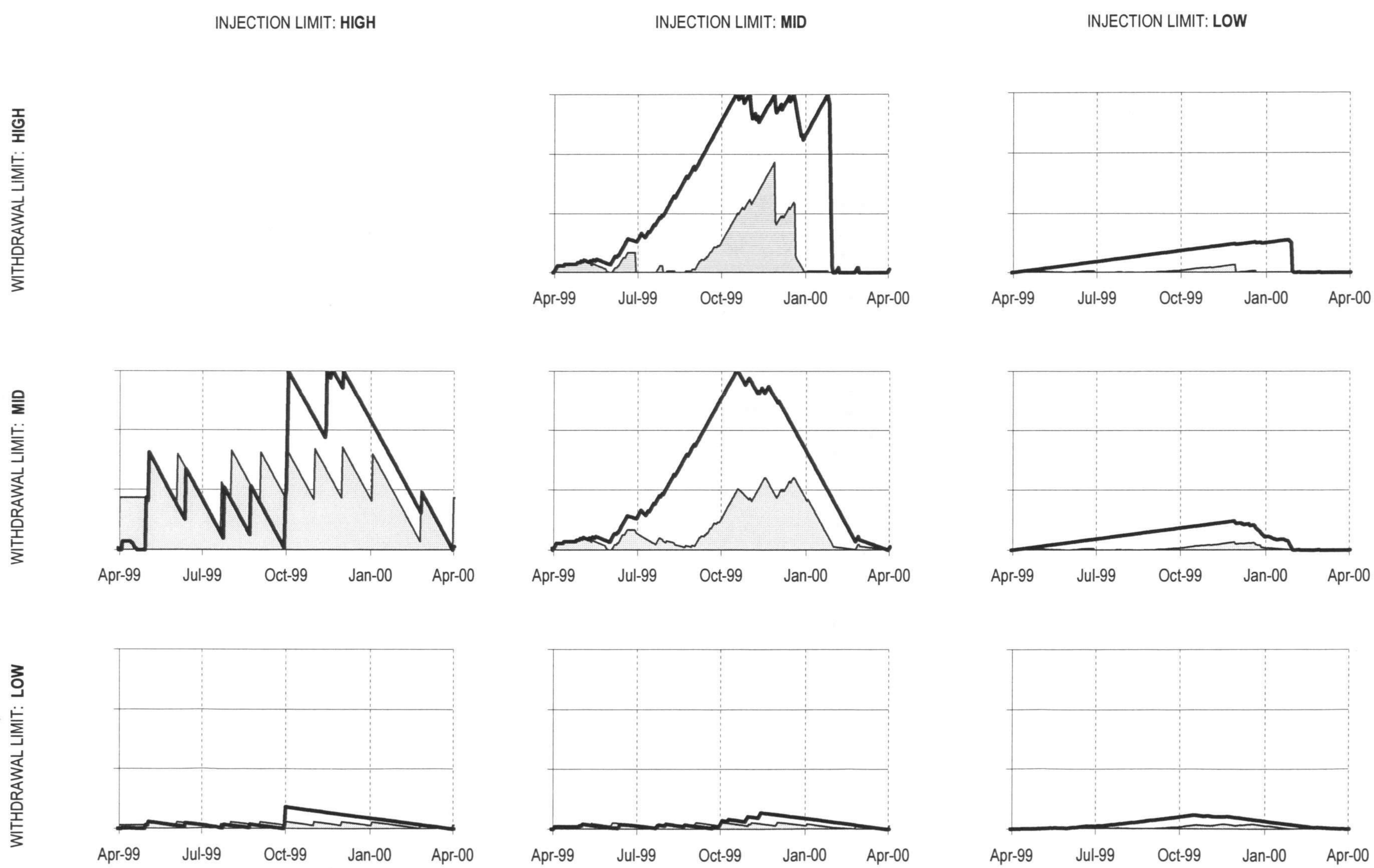
OPTIMAL VERSUS NAÏVE, DAY AND PROMPT CONTRACTS, HEDGING, PERFECT PRICE INFORMATION



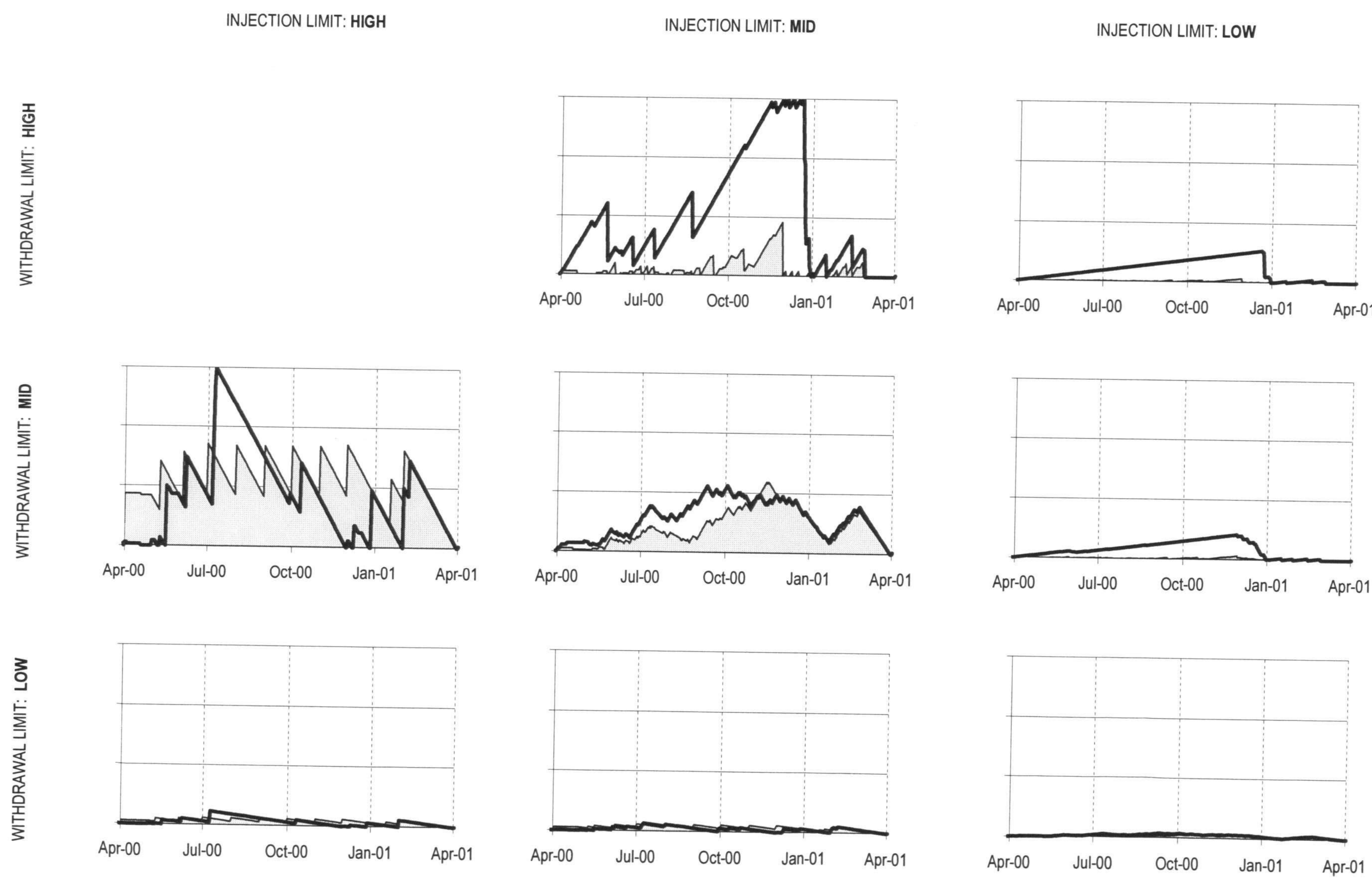
OPTIMAL VERSUS NAÏVE, DAY AND PROMPT CONTRACTS, HEDGING, PERFECT PRICE INFORMATION



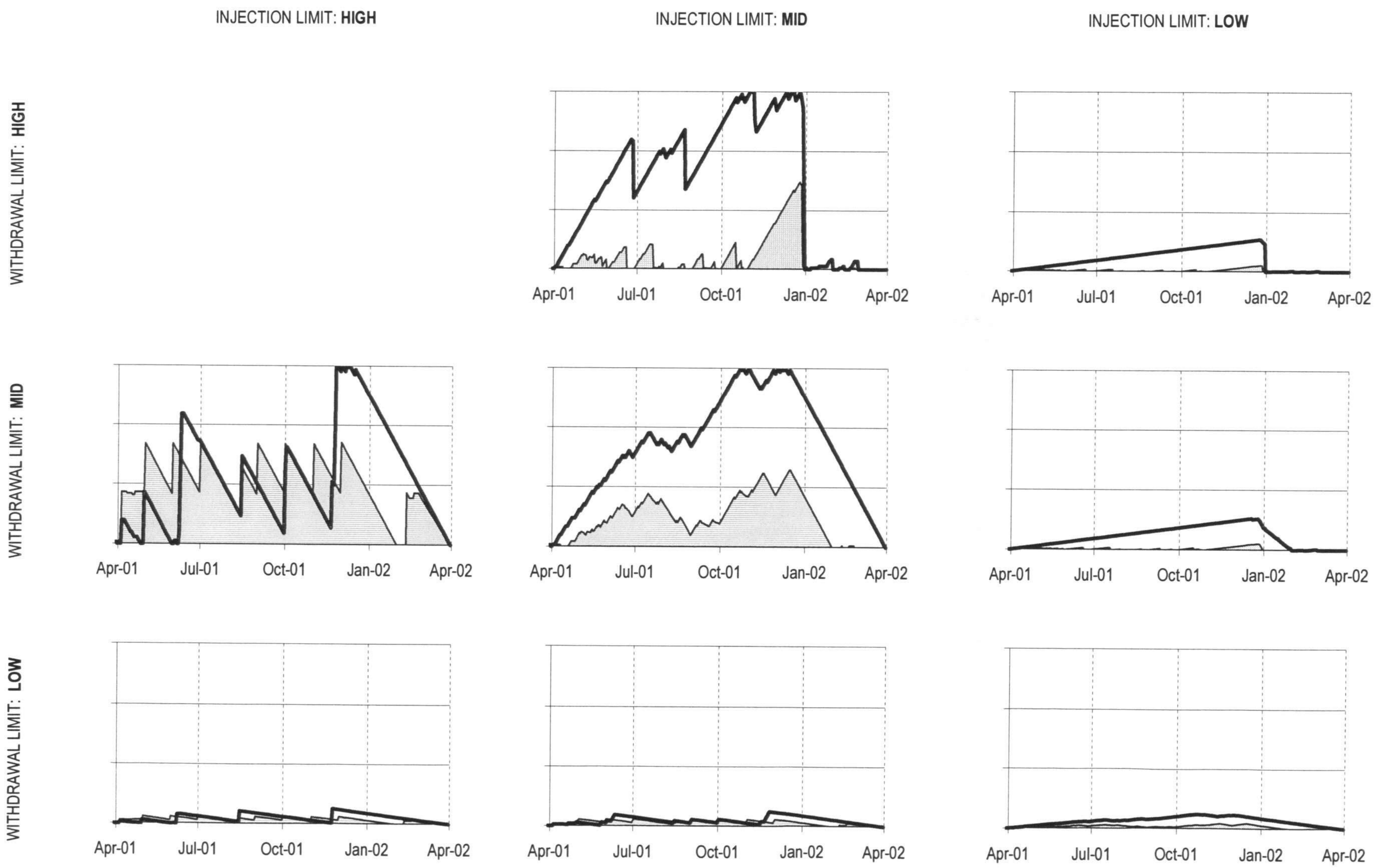
OPTIMAL VERSUS NAÏVE, DAY AND PROMPT CONTRACTS, HEDGING, PERFECT PRICE INFORMATION



OPTIMAL VERSUS NAÏVE, DAY AND PROMPT CONTRACTS, HEDGING, PERFECT PRICE INFORMATION



OPTIMAL VERSUS NAÏVE, DAY AND PROMPT CONTRACTS, HEDGING, PERFECT PRICE INFORMATION



APPENDIX C

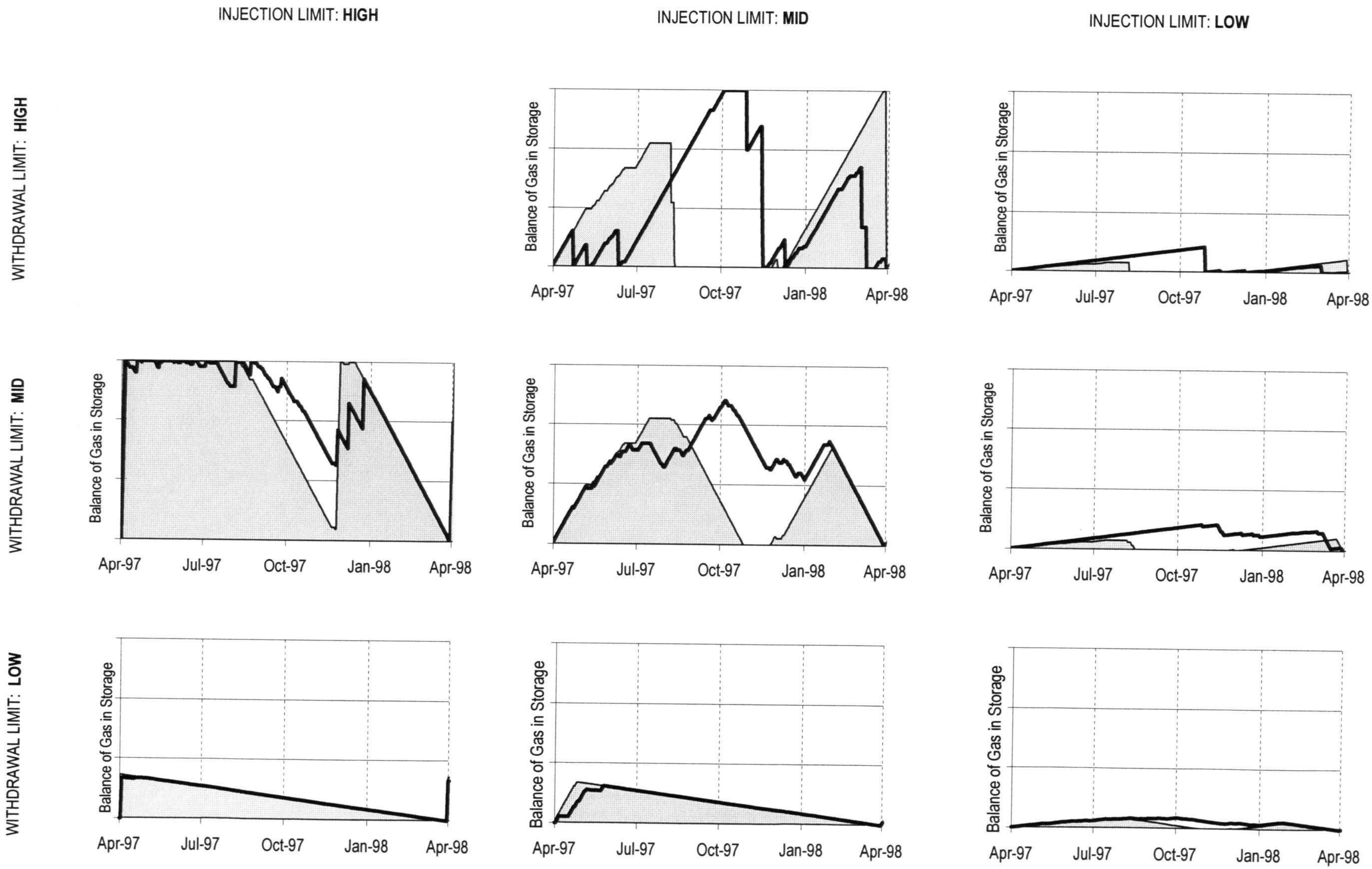
GRAPHS OF BALANCES OF GAS IN STORAGE
PERTAINING TO SECTION 4.1.3

ALL COMBINATIONS OF WITHDRAWAL AND INJECTION LIMITS AND ALL YEARS

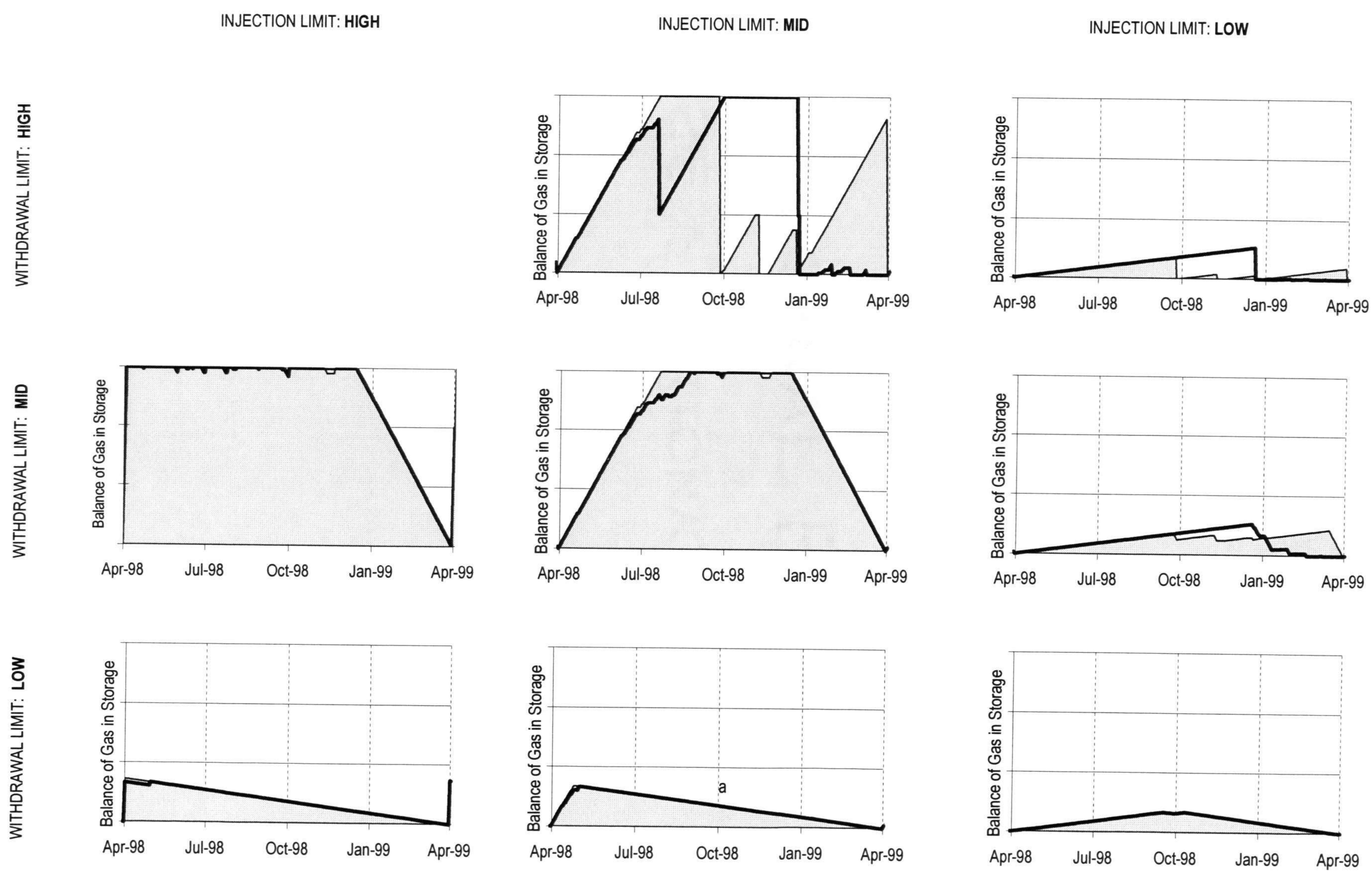
OPTIMAL AND NAÏVE STRATEGIES
DAY CONTRACTS ONLY, FORECASTED PRICES

On each graph, the shaded area reflects "naïve" operation; the heavy line reflects "optimal" operation. No graphs for (high, high) limits are included; these relaxed limits lead to such frequent withdrawals and injections that the graphs are illegible.

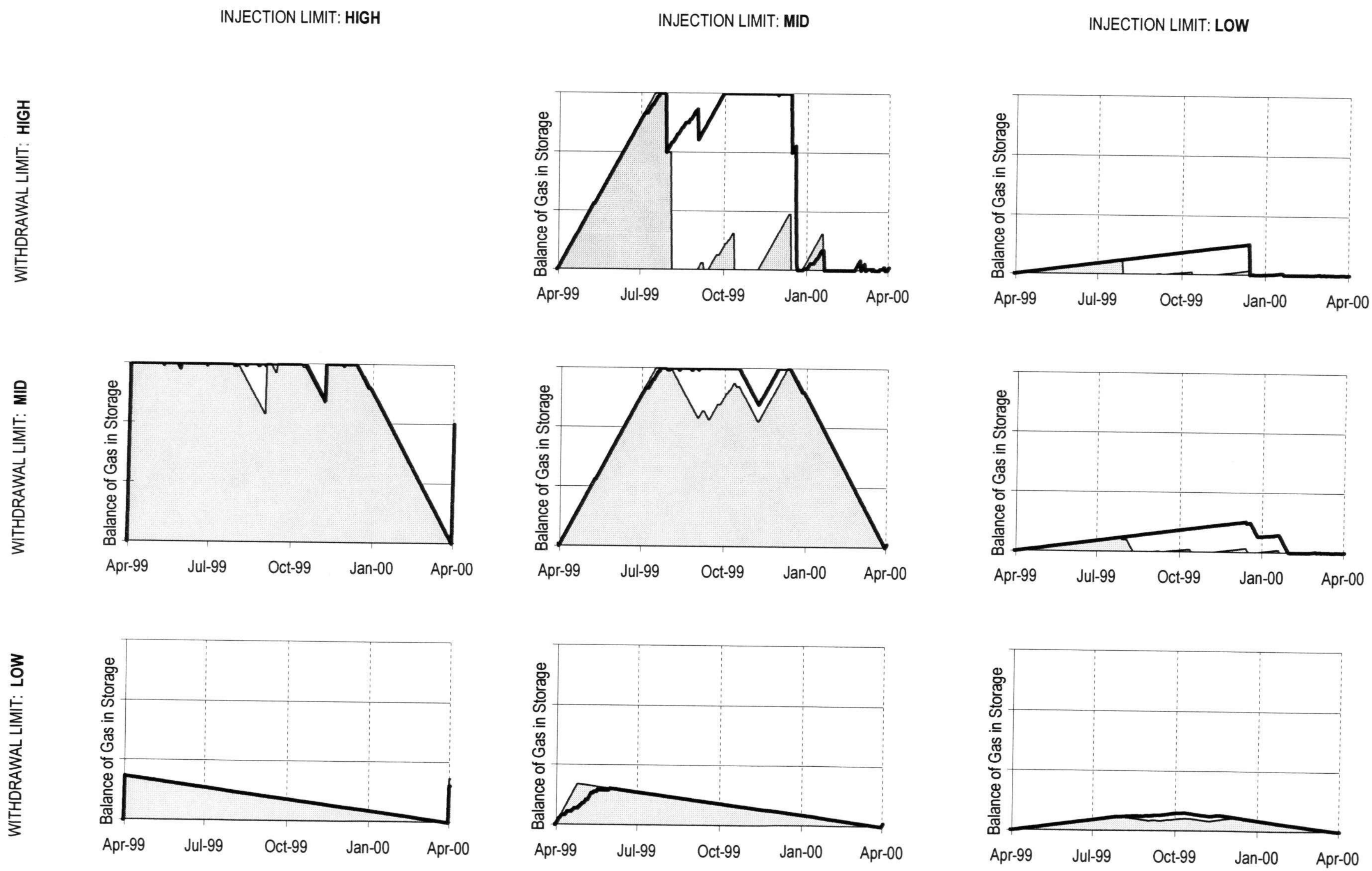
OPTIMAL VERSUS NAÏVE, DAY CONTRACTS ONLY (NO HEDGING), FORECASTED PRICES



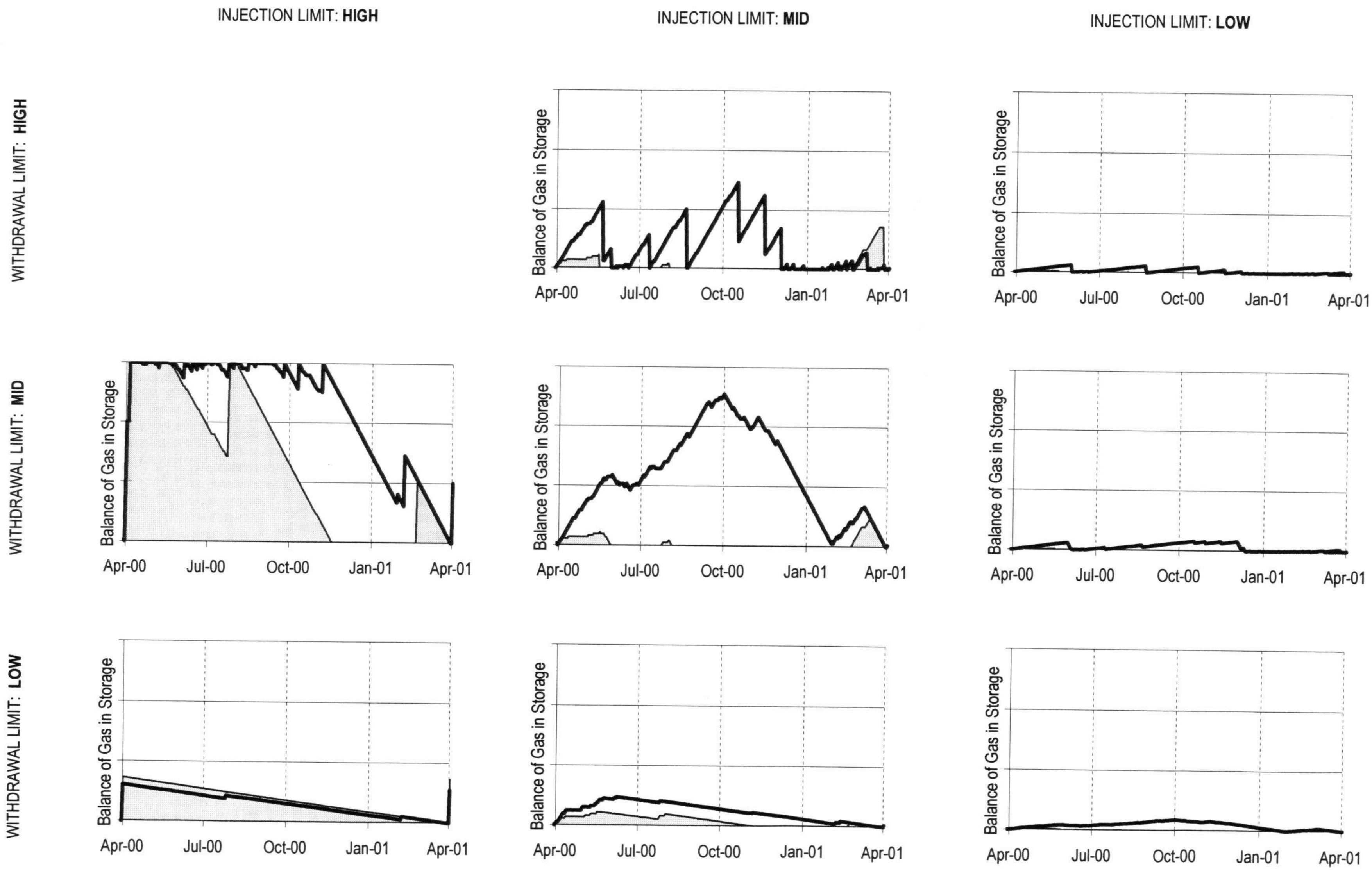
OPTIMAL VERSUS NAÏVE, DAY CONTRACTS ONLY (NO HEDGING), FORECASTED PRICES



OPTIMAL VERSUS NAÏVE, DAY CONTRACTS ONLY (NO HEDGING), FORECASTED PRICES



OPTIMAL VERSUS NAÏVE, DAY CONTRACTS ONLY (NO HEDGING), FORECASTED PRICES



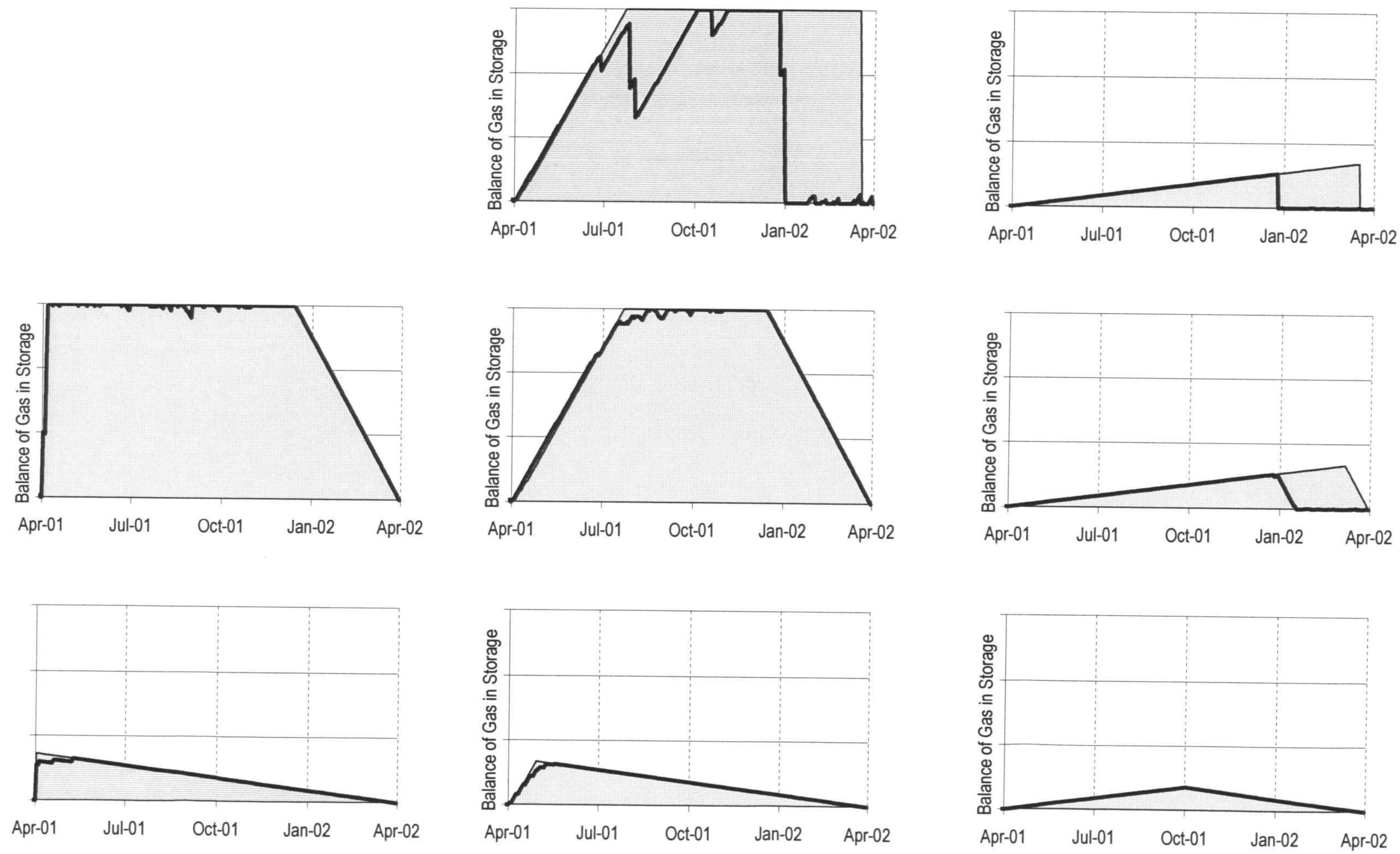
OPTIMAL VERSUS NAÏVE, DAY CONTRACTS ONLY (NO HEDGING), FORECASTED PRICES

INJECTION LIMIT: **HIGH** | INJECTION LIMIT: **MID** | INJECTION LIMIT: **LOW**

WITHDRAWAL LIMIT: **HIGH**

WITHDRAWAL LIMIT: **MID**

WITHDRAWAL LIMIT: **LOW**



APPENDIX D

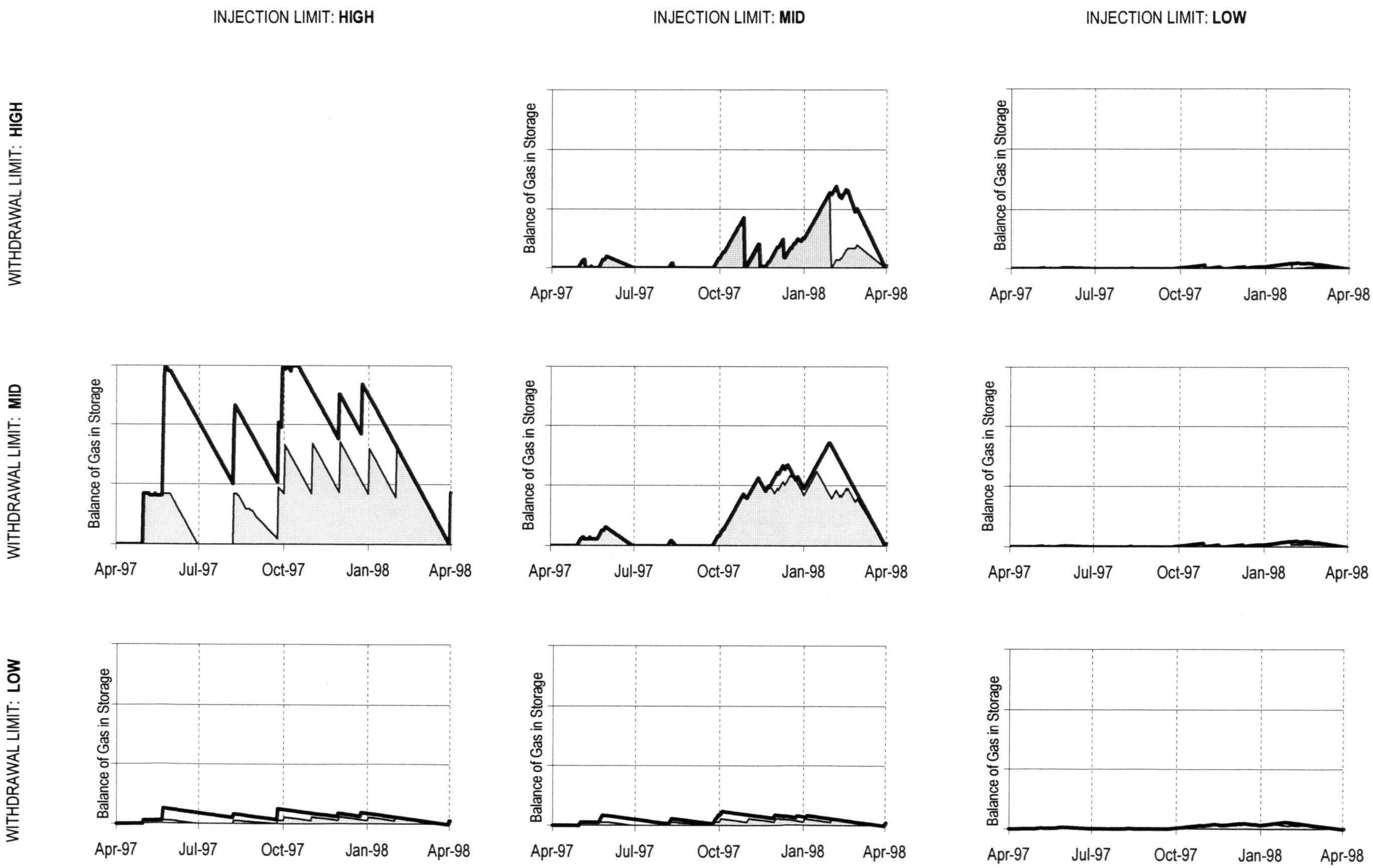
GRAPHS OF BALANCES OF GAS IN STORAGE
PERTAINING TO SECTION 4.1.4

ALL COMBINATIONS OF WITHDRAWAL AND INJECTION LIMITS AND ALL YEARS

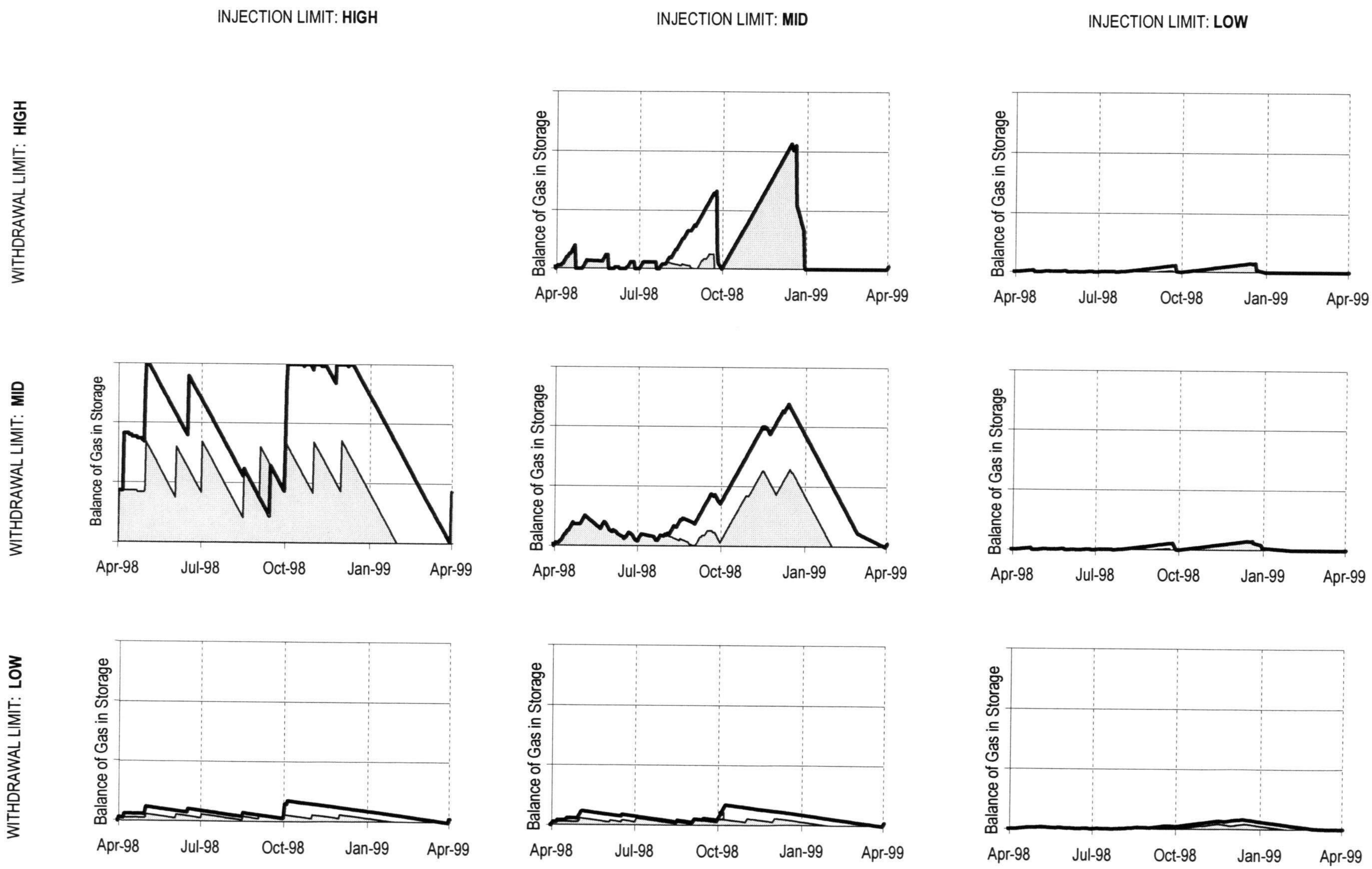
OPTIMAL AND NAÏVE STRATEGIES
DAY AND PROMPT CONTRACTS, HEDGING, FORECASTED PRICES

On each graph, the shaded area reflects "naïve" operation; the heavy line reflects "optimal" operation. No graphs for (high, high) limits are included; these relaxed limits lead to such frequent withdrawals and injections that the graphs are illegible.

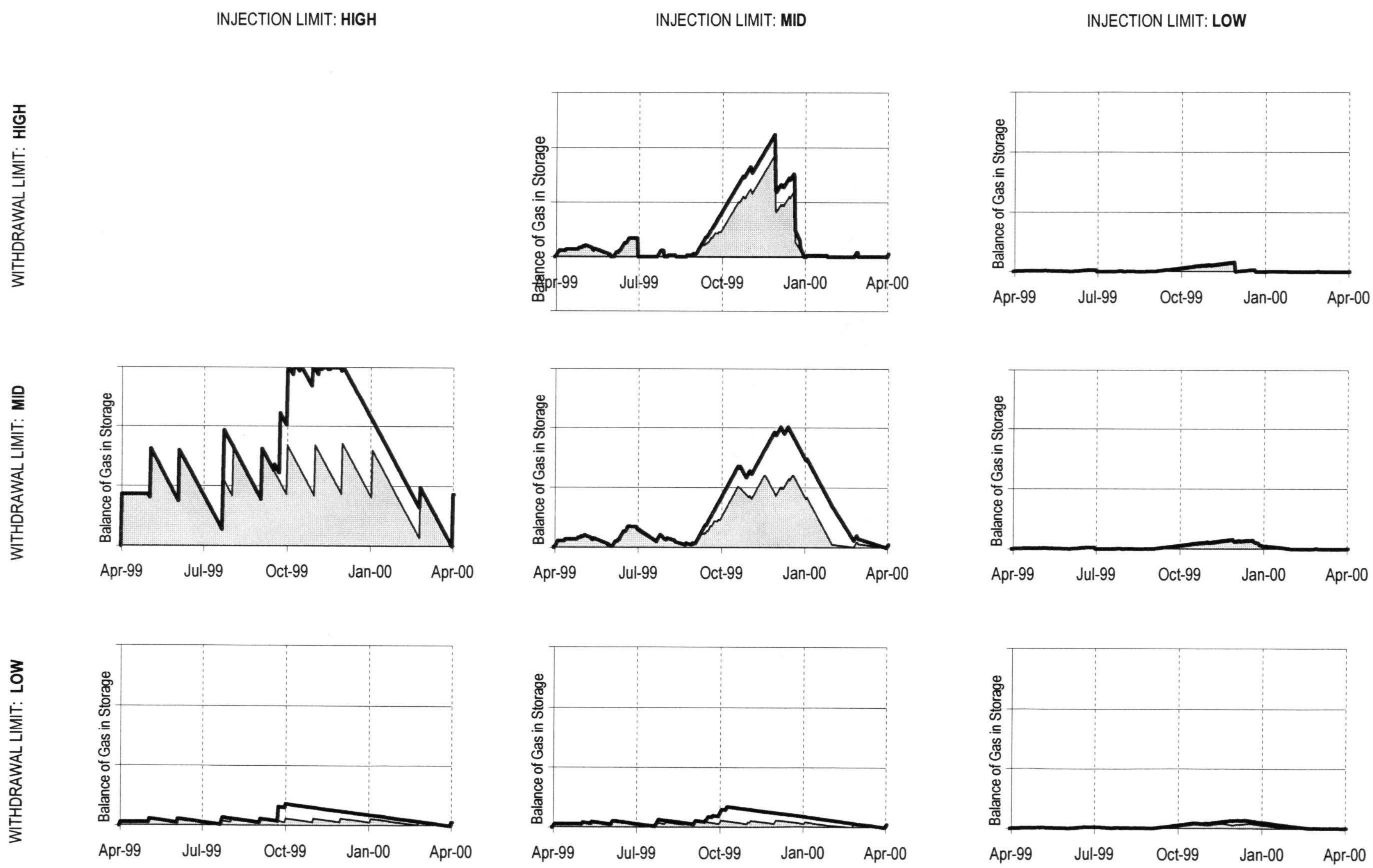
OPTIMAL VERSUS NAÏVE, DAY AND PROMPT CONTRACTS, HEDGING, FORECASTED PRICES



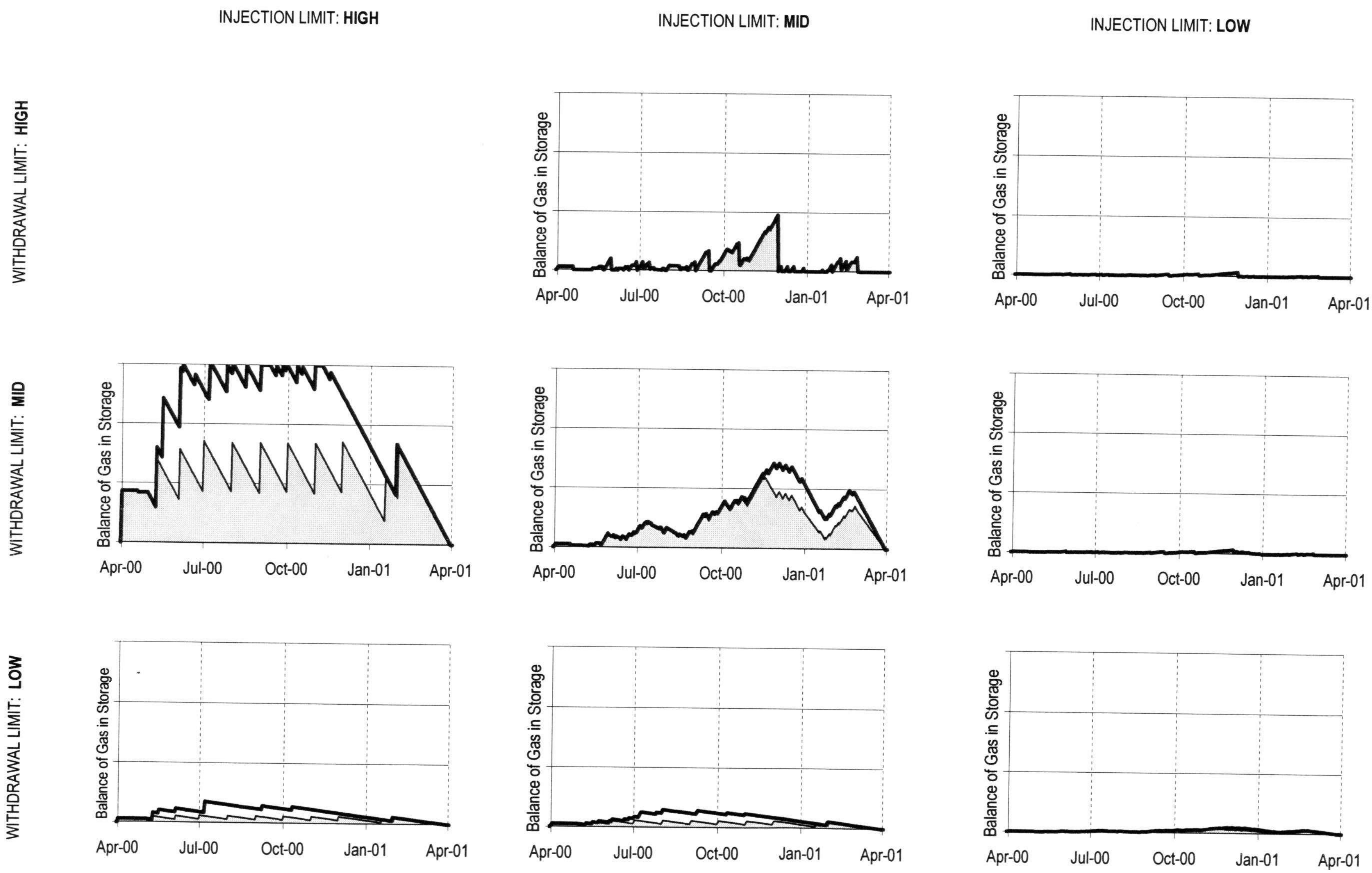
OPTIMAL VERSUS NAÏVE, DAY AND PROMPT CONTRACTS, HEDGING, FORECASTED PRICES



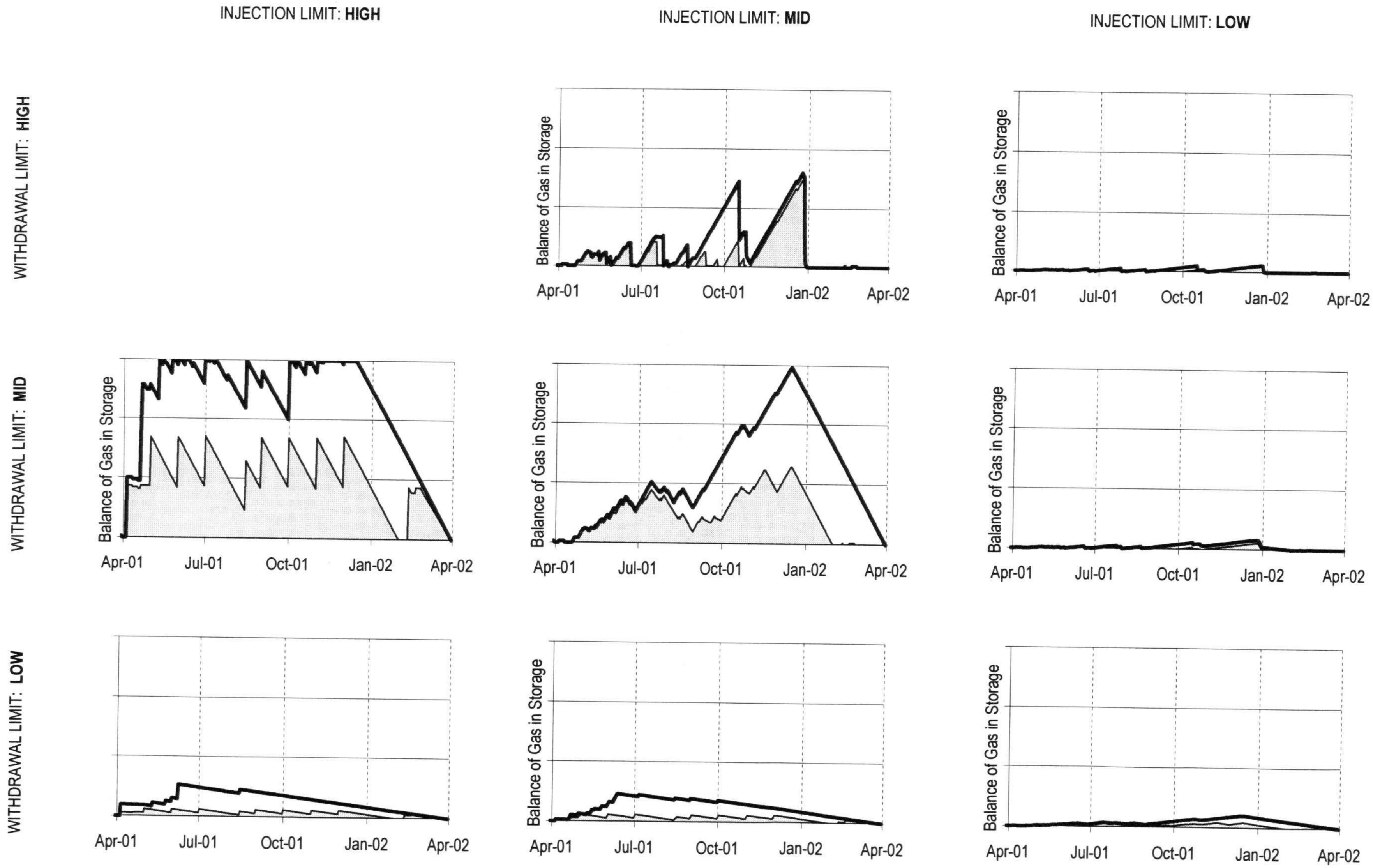
OPTIMAL VERSUS NAÏVE, DAY AND PROMPT CONTRACTS, HEDGING, FORECASTED PRICES



OPTIMAL VERSUS NAÏVE, DAY AND PROMPT CONTRACTS, HEDGING, FORECASTED PRICES



OPTIMAL VERSUS NAÏVE, DAY AND PROMPT CONTRACTS, HEDGING, FORECASTED PRICES



APPENDIX E

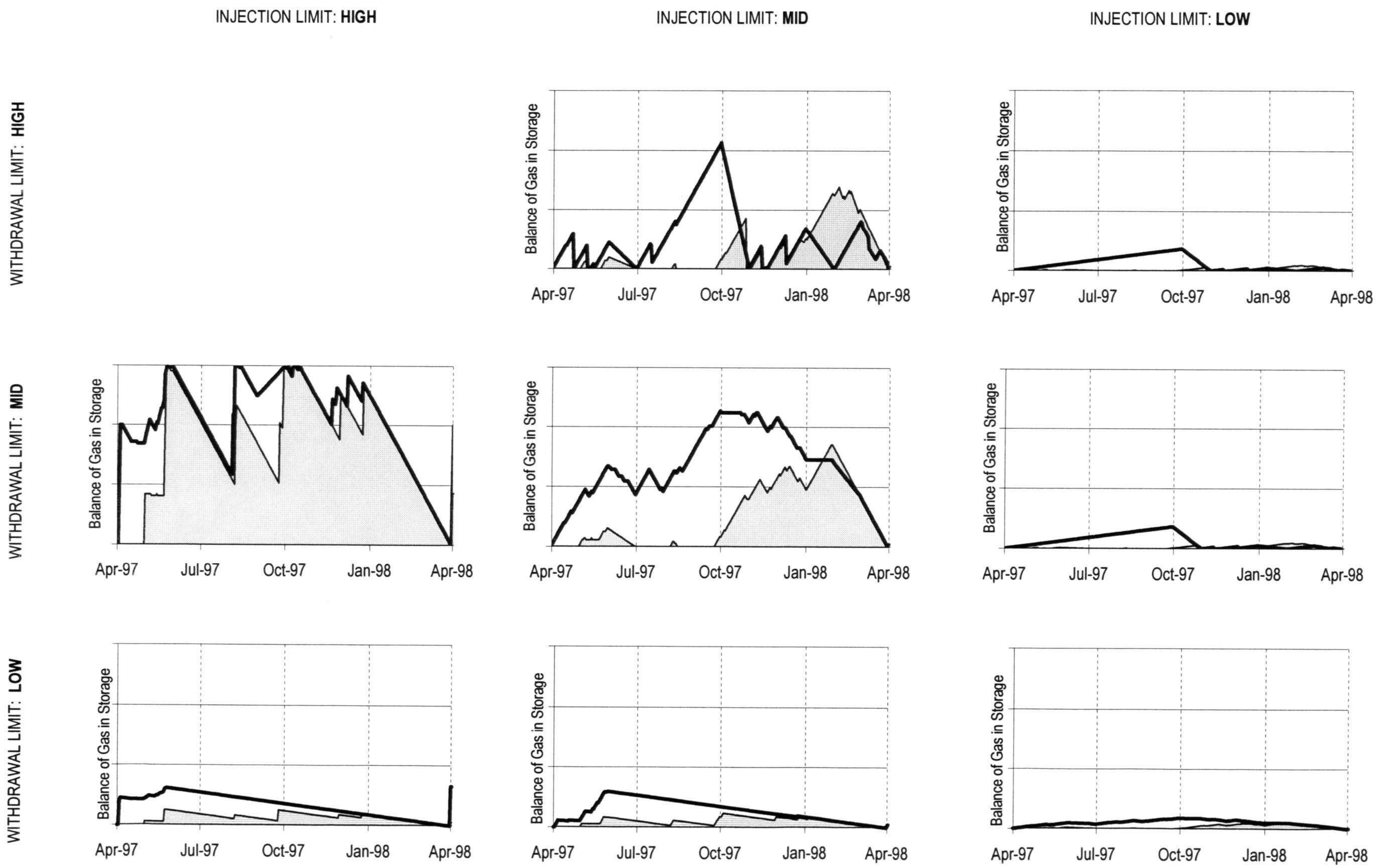
GRAPHS OF BALANCES OF GAS IN STORAGE PERTAINING TO SECTION 4.2

ALL COMBINATIONS OF WITHDRAWAL AND INJECTION LIMITS AND ALL YEARS

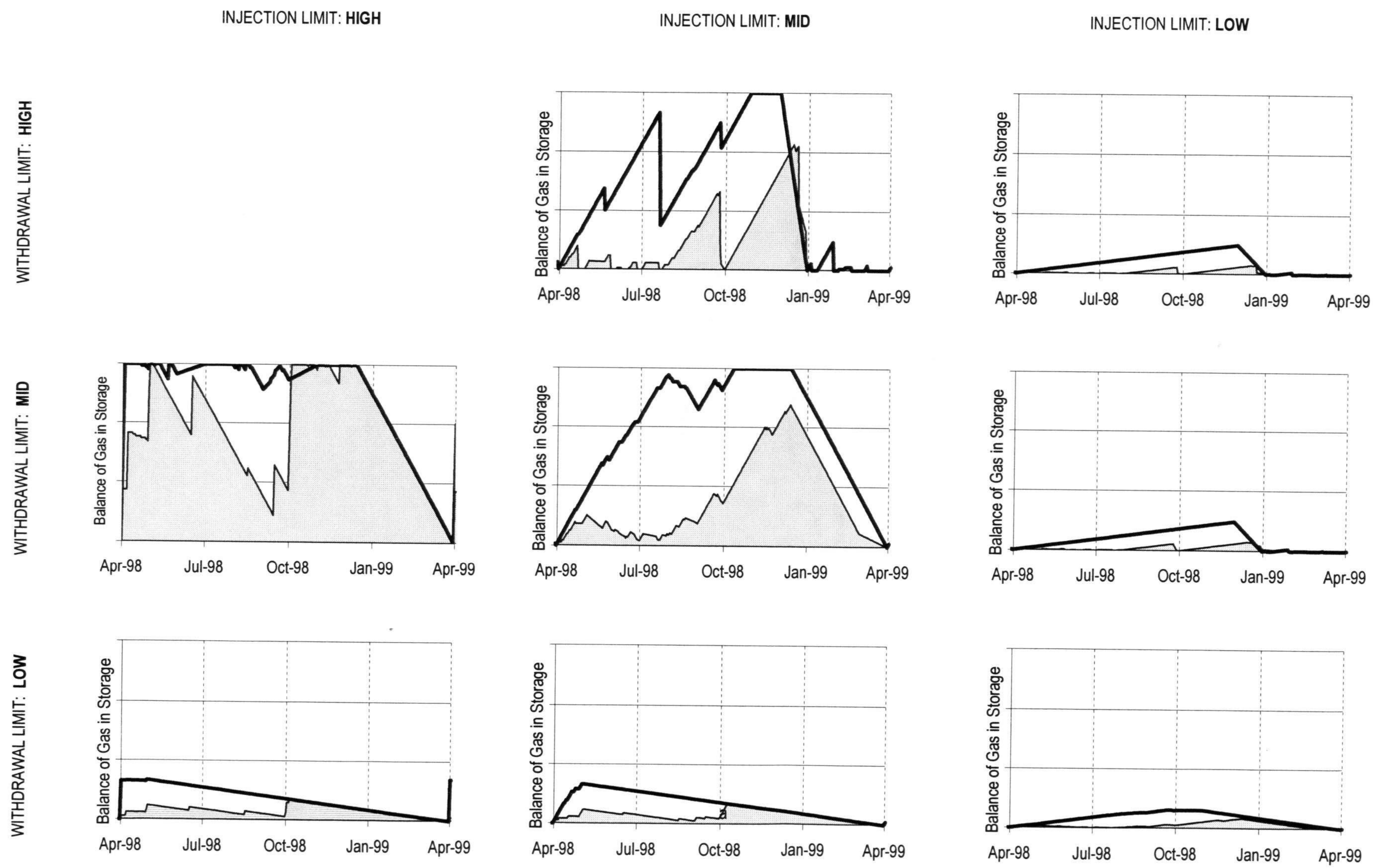
HEDGING AND NO-HEDGING STRATEGIES OPTIMIZATION MODELS, FORECASTED PRICES

On each graph, the shaded area reflects "no-hedging" operation; the heavy line reflects "hedging" operation. No graphs for (high, high) limits are included; these relaxed limits lead to such frequent withdrawals and injections that the graphs are illegible.

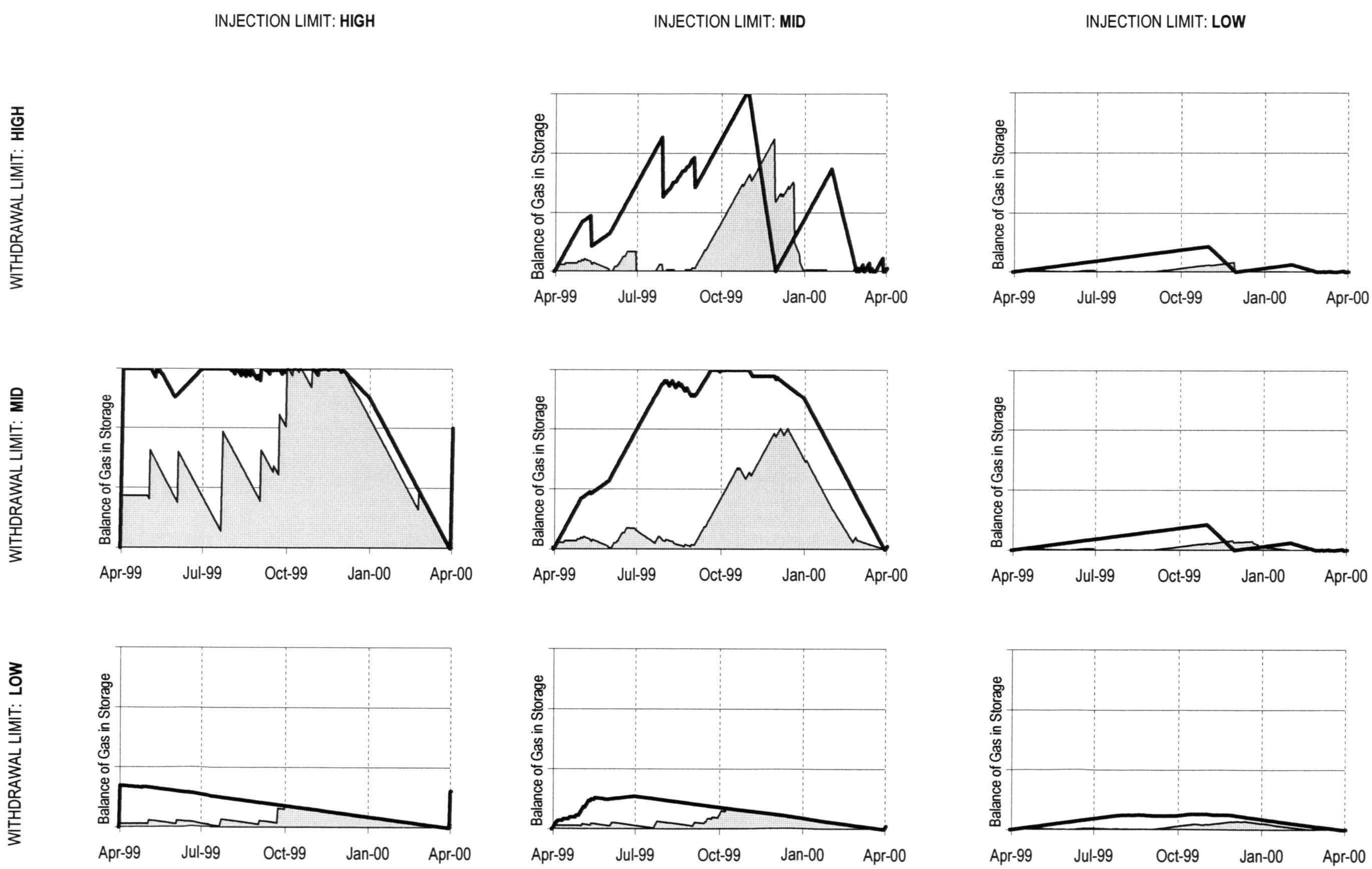
HEDGING VERSUS NOT HEDGING, OPTIMAL STRATEGY, DAY AND PROMPT CONTRACTS, FORECASTED PRICES



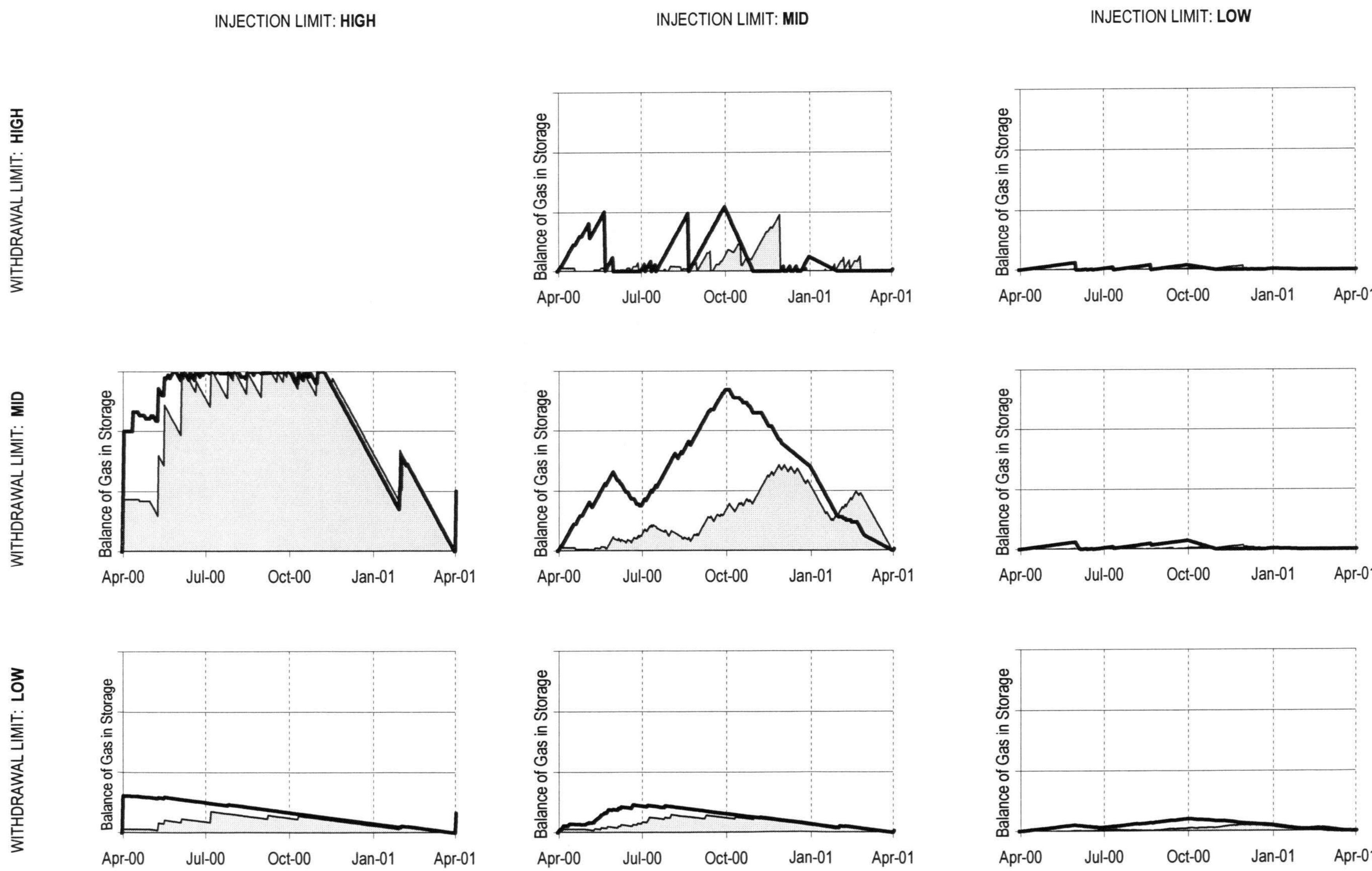
HEDGING VERSUS NOT HEDGING, OPTIMAL STRATEGY, DAY AND PROMPT CONTRACTS, FORECASTED PRICES



HEDGING VERSUS NOT HEDGING, OPTIMAL STRATEGY, DAY AND PROMPT CONTRACTS, FORECASTED PRICES



HEDGING VERSUS NOT HEDGING, OPTIMAL STRATEGY, DAY AND PROMPT CONTRACTS, FORECASTED PRICES



HEDGING VERSUS NOT HEDGING, OPTIMAL STRATEGY, DAY AND PROMPT CONTRACTS, FORECASTED PRICES