On the Numerical Prediction and Experimental Investigation of Reciprocating Sliding Wear

By

Srinivasan S. Iyer

B.E. (Hons.) (Mechanical) University of Gauhati, India
M.A.Sc. (Mechanical) University of Ottawa

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY in MECHANICAL ENGINEERING

We accept this thesis as conforming to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA
May 2000
© Srinivasan S. Iyer
In presenting this thesis in partial fulfillment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the Head of my Department or by his or her representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Mechanical Engineering
The University of British Columbia
2324 Main Mall
Vancouver, Canada
V6T1Z4

Date

16th June 2000
This thesis is devoted to the development of an improved wear model for estimating wear-loss in the field of sliding wear. A hard cylinder sliding on a softer disc is adopted for studying the wear system. Four governing sliding wear mechanisms, namely low cycle fatigue, ratchetting, mild wear and crack growth leading to particle detachment are identified. This inference is attained by the investigation of photomicrographs of experimentally obtained sectioned wear specimens.

Surface statistical methods are used for quantifying the contacting asperities and finite element methods are used for evaluating the elastic-plastic strains. Equations are developed to predict the wear volume and the number of cycles for failure, from the history of strain-cycles and tangential work equivalence for low cycle fatigue wear and from the history of strain cycles and ratchetting depth for ratchetting wear. Through simulations, it is shown that the low cycle fatigue wear happens during plastic shakedown conditions and that ratchetting wear occurs above the ratchetting threshold.

A predictive equation is developed for mild wear, based on tangential work equivalence and Hertzian contact mechanics parameters. It is shown that mild wear occurs during the elastic shakedown state.

Applying finite element methods to linear elastic fracture mechanics, a model is developed, simulating crack growth and wear particle detachment, by assuming a surface crack. The range of mixed mode stress intensity factors for cyclic loading is evaluated and related to the crack extension in a prescribed number of cycles using a Paris type equation. The maximum tensile stress criterion is used for determining the crack turn angle during the crack propagation. A wear particle is detached from the parent surface when the crack propagates to the wearing surface. This mechanism occurs below the elastic limit, but under dry sliding and high normal loading levels.

Experiments are conducted with specialised test-rig under a variety of loading and friction conditions. The microstructures of sections of the test-worn specimens are analysed for studying the wear characteristics. Experimental values agree well with the predicted values of wear-volume and aspect ratio of wear particles, justifying the validity of the proposed sliding wear model.
Table of Contents

Abstract .......................................................... ii
Table of Contents ............................................. iii
List of Tables ................................................ x
List of Figures ............................................... xix
Notation ......................................................... xxi
Acknowledgements .......................................... xxii

1 Introduction

1.1 General Background ......................................... 1
1.2 Objective of the Thesis ..................................... 1
1.3 Problem Definition and Method of Solution .................. 2
1.4 Organization of the Thesis ................................... 5
1.5 Original Contribution in Sliding Wear Prediction ............ 6
1.6 Summary ...................................................... 7

2 Literature Review

2.1 Introduction ................................................. 8
2.2 Wear Modes – Classification of Wear ......................... 8
2.2.1 Adhesive Wear ........................................ 8
2.2.2 Abrasive Wear .......................................... 9
2.2.3 Fatigue Wear .......................................... 10
2.2.4 Delamination Wear ..................................... 10
2.2.5 Shear Fracture ....................................... 11
2.2.6 Corrosion or Oxidation ................................ 11
2.2.7 Fretting Wear ....................................... 11
2.3 Wear Modelling – A Historical Perspective of its Progress 11
2.4 Contact of Rough Surfaces ................................ 15
2.5 Mathematical Modelling of Wear ............................. 16
3. Proposed Sliding Wear Model

3.1 Introduction 26

3.2 Elastic-Plastic Response Under Cyclic Loading (RF & LCF) 26
   3.2.1 Four Regimes of Elastic-Plastic Response 26
   3.2.2 Shakedown Diagram 27
   3.2.3 Mechanism of RF 27
   3.2.4 Mechanism of LCF 28
   3.2.5 Experimental Evidence of RF & LCF 29
   3.2.6 Experimental Results and Inferences for RF & LCF 29
   3.2.7 Mathematical Background for PSL & RF 30
   3.2.8 Proposed Sliding Wear Modeling through RF & LCF 33

3.3 Mild Wear 34
   3.3.1 Sliding with Low Coefficient of Friction-Wet Sliding 34
   3.3.2 Inference From Specimens 35
   3.3.3 Elastic Shakedown Limit of Circular Contact & Mild Wear 35
   3.3.4 Proposed Model for Sliding Wear Through Mild Wear Mechanism 35

3.4 Crack Growth and Wear Particle Detachment (CGPD) 36
   3.4.1 Experimental Inference of Crack Growth & Wear Particle Formation 36
   3.4.2 Fracture Mechanics Approach to Sliding Wear 37
   3.4.3 Proposed Sliding Wear Model Through Crack Growth
4. Prediction of Sliding Wear Through LCF and RF

4.1 Introduction

4.2 Multi Asperity Contact and Contact Deformation Model
   4.2.1 General Background
   4.2.2 Characteristics and Measurement of Random (Manufactured) Rough Surfaces
   4.2.3 Measurement of Curvature of Asperities
   4.2.4 Assumptions for Modelling
   4.2.5 Calculation of ‘Normal Contact Area, A₀’
   4.2.6 Deduction of Number of Asperities in the Nominal Area of Contact

4.3 Finite Element Formulation of Contact Deformation Model

4.4 Numerical Simulation

4.5 Finite Element Mesh for Asperity Contact

4.6 Prediction of Wear Volume & Number of Cycles to Failure in LCF & RF
   4.6.1 Number of Cycles for Failure by Low Cycle Fatigue or Ratchetting Failure
   4.6.2 Prediction of Wear Volume in Sliding Wear
   4.6.3 Low Cycle Fatigue (LCF)
   4.6.4 Ratchetting Failure (RF)

4.7 Results and Discussions From Contact Deformation Analysis
   4.7.1 Applied Contact Pressure Distribution
   4.7.2 Elastic Contact Stresses
   4.7.3 Elastic-Plastic Solutions for Contact Stresses and Strain Fields
   4.7.4 Tracking of Strain Components through Cyclic Loading
4.8 Prediction of Wear Volume and Number of Cycles to Failure in RF and LCF
4.9 Importance of Finite Element Analysis in Sliding Wear
4.10 Summary

5. Prediction of Mild Wear (MW) Under Wet Sliding
5.1 Preliminary Discussion
5.2 Determination of Abrasive Groove Depth (z)
5.3 Results from Mild Wear Model, \((\mu = 0.1\) to \(0.2\))
5.4 Discussion of Results
5.5 Summary

6. Numerical Prediction of Crack Growth and Particle Detachment (CGDP) in Dry Sliding Wear
6.1 Introduction
6.2 Outline of the Proposed Work
6.3 Surface Statistics Applied to Fracture Mechanics Model
6.4 Fracture Mechanics Basis for the Sliding Wear Model (CGPD)
6.4.1 Introduction
6.4.2 The Stress Intensity Factor ‘\(K\)’
6.4.3 Crack Tip Stress Analysis
6.4.4 Determination of Mixed Mode SIFs (\(K_1\) & \(K_\Pi\)) Through Displacements at the Crack Tip
6.4.5 Prediction of Crack Turn Angle by Maximum Tensile Stress Criterion
6.4.6 Crack Growth
6.4.7 A Note on Paris Law
6.4.8 Calculation of Wear Volume for Dry Sliding Wear
6.5 Model for Crack Growth

vi
6.5.1 Development for Fracture Mechanics Model 92
6.5.2 Calculation of Hertzian Normal Pressure and Tangential Stress Distribution 93
6.5.3 Finite Element Procedure 94
6.5.4 Finite Element Analysis of Crack Growth 94

6.6 Results and Discussions for Dry Sliding Wear 95
6.6.1 Sample Calculations 95
6.6.2 Parametric Variation in c and \( \theta_0 \) (\( P = 800 \text{N}; Q = 640 \text{N}; p_0/k = 2.08 \)) 96
6.6.3 Variation in Normal Loading \( p(x) \) 98
6.6.4 Discussion of Results 101

6.7 Summary 103

7. Experimental Investigation of Sliding Wear 105
7.1 Introduction 106
7.2 Apparatus and Specimen Geometry 106
7.3 Experimental Investigation Scheme 106
7.3.1 Specimen Preparation 106
7.4 Wear Testing 107
7.5 Test Parameters 108
7.6 Micro Structure Analysis 108
7.6.1 Surface and Sub-Surface Metallography 108
7.7 Experimental Results and Discussion 109
7.7.1 Mass loss of worn specimens 109
7.7.2 Coefficient of Friction 109
7.7.3 Surface Examinations 110
7.8 Summary 111

8. Comparison of Predicted Results with Experimental Work
8.1 Introduction 112
8.2 Low cycle Fatigue (LCF) and Ratchetting Failure (RF) 112
8.3 Mild Wear (MW) 113
8.4 Crack Growth and Particle Detachment (CGPD) 114
8.5 Summary 115

9. Conclusions and Future Scope of Work 116
REFERENCES 118
FIGURES 127
List of Tables

Chapter 1
1.1 Contact loading conditions and resulting wear mechanisms 3

Chapter 4
4.1 Loading considered for the contact deformation model 50
4.2 Comparison of results for contact width and contact pressure 59
4.3 Predicted volume and number of cycles to failure 68

Chapter 5
5.1 Calculation of mild wear groove depth and volume 76
5.2 Wear volume /cycle predicted for two different friction coefficient levels 77

Chapter 6
6.1 Pressure distribution over contact area ($\mu=0.8$) 93
6.2 Crack growth and particle formation versus entry angle 97
6.3 Calculation for wear particle geometry in CGPD mechanism 104

Chapter 8
8.1 Predicted wear volume and number of cycles to particle detachment 112
8.2 Crack growth and particle formation 114
List of Figures

Chapter 1

1.1 Proposed model for predicting sliding wear of mechanical components 128

Chapter 2

2.1 Schematic diagram showing the adhesive transfer of a thin flake-like wear particle (a) and the adhesive transfer of a wedge-like wear particle (b) 129
2.2 Relationship between the critical tip angle $\theta_c$ and the hardness ratio $r$ where the normalised shear strength $f$ is zero in (a) and changes from zero to 1.0 in (b) 129
2.3 Schematic sketches of three modes of abrasive wear observed by SEM and cross-sectional profiles of grooves 130
2.4 Schematic sketches of formation of ‘filmy wear’ particles observed by SEM 130
2.5 Archard non-dimensional wear coefficient, $k$, as a function of $\psi_x$ the plasticity index in repeated sliding for various values of normalised surface separation $d/\sigma$ 130

Chapter 3

3.1 A schematic illustrating the governing mechanisms of elastic shakedown, plastic shakedown and ratchetting during cyclic loading 131
3.2 Shakedown map for line contact 131
3.3 Bauchinger effect illustrating that yield takes place at a lower value when there is reversal of stress 132
3.4 Results of ratchetting tests 132
3.5 Experimental evidence for low cycle fatigue and ratchetting failure 133
3.6 Top view of a worn specimen under lubrication depicts no crack growth or delamination (LCF) 133
3.7 Microscopic examination of worn specimens (sectioned) indicating plastic deformation and surface plasticity leading to ratchetting failure 134
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.8</td>
<td>A schematic illustrating the integration limits and shakedown depth</td>
</tr>
<tr>
<td>3.9</td>
<td>Sliding wear model predicting wear volume and number of cycles through RF and LCF</td>
</tr>
<tr>
<td>3.10</td>
<td>Magnified view of the worn specimen</td>
</tr>
<tr>
<td>3.11</td>
<td>Mass loss for dry sliding wear and sliding wear under lubrication with water : cutting fluid = 1000:1</td>
</tr>
<tr>
<td>3.12</td>
<td>Circular contact shakedown map for a kinematic hardening material</td>
</tr>
<tr>
<td>3.13</td>
<td>Sliding wear prediction through the mechanism of mild wear (MW)</td>
</tr>
<tr>
<td>3.14</td>
<td>Magnified top view of wear particle formation</td>
</tr>
<tr>
<td>3.15</td>
<td>Sectioned view of a worn specimen under dry sliding conditions</td>
</tr>
<tr>
<td>3.16</td>
<td>Value of friction coefficient for dry sliding conditions</td>
</tr>
<tr>
<td>3.17</td>
<td>Sectioned view of dry sliding specimen (500 X magnification) depicting crack growth under dry sliding conditions</td>
</tr>
<tr>
<td>3.18</td>
<td>Variation of friction coefficient with number of cycles for 60 cycle test duration</td>
</tr>
<tr>
<td>3.19</td>
<td>Sliding wear prediction through the mechanism of crack growth and particle detachment (CGPD)</td>
</tr>
</tbody>
</table>

Chapter 4

<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>Experimental mapping of rough surface using Talysurf (indicating random variation of asperity heights)</td>
</tr>
<tr>
<td>4.2</td>
<td>Height distribution $\phi(z)$ and bearing area curve given by the cumulative height distribution</td>
</tr>
<tr>
<td>4.3</td>
<td>Cumulative height distributions plotted on normal probability paper</td>
</tr>
<tr>
<td>4.4</td>
<td>Contact of a randomly rough surface with a smooth flat surface at a separation</td>
</tr>
<tr>
<td>4.5</td>
<td>Schematic illustrating the idealisation of engineering rough surfaces in contact</td>
</tr>
<tr>
<td>4.6</td>
<td>Idealisation of asperities based on Greenwood and Williamson</td>
</tr>
<tr>
<td>4.7</td>
<td>A schematic depicting the normal area of contact</td>
</tr>
<tr>
<td>4.8</td>
<td>A schematic illustrating the concept of equivalent cylindrical asperity representing 76 spherical asperities making circular contacts</td>
</tr>
<tr>
<td>4.9</td>
<td>Deformation model for cylindrical asperity in contact with a smooth half-space under plane strain conditions</td>
</tr>
</tbody>
</table>
4.10 Idealization of a single asperity contact using a single cylindrical asperity in contact with a smooth plane

4.11 Finite element modeling of a typical sliding wear problem

4.12 Motion of a single element ‘A’ on the wear track undergoing deformations in total Lagrangian and upgraded Lagrangian descriptions

4.13 Stress-strain response curve for the modeled elastic-plastic half-space

4.14 Finite element mesh employed for the simulation of contact deformation

4.15 Finite element mesh with applied boundary conditions and normal load (pressure)

4.16a A close up of contact interface

4.16b Contact elements surrounding the asperity and half-space

4.17 Illustration of cyclic loading employed in the finite element simulation of sliding wear

4.18 Figure illustrating two different elements chosen after studying the variation of strain components during cyclic loading

4.19 Normal contact pressure profiles for (a) CASE IA and (b) CASE IIA loading

4.20 Hertzian contact stress profiles and finite element values at the first sub-step of indentation

4.21a Contours of ratio of von Mises equivalent stress at the last sub-step of indentation load step to von Mises stress intensity at initial yield ($\mu = 0$; $p_0/k = 7.6$)

4.21b Contours of ratio of von Mises equivalent stress at the last sub-step of indentation load step to von Mises stress intensity at initial yield ($\mu = 0$; $p_0/k = 3.4$)

4.22a Contours of ratio of von Mises equivalent stress at the last substep of the indentation load step to yield strength of wearing material (indentation under normal pressure but for two different friction coefficient levels)

4.22b Contours of ratio of von Mises Equivalent Stress distribution at the last substep of the first load step to yield strength of wearing material (indentation under normal pressure but for two different friction coefficient levels)

4.23 Contours of strain components at the end of second load step (first quarter reciprocating sliding); ($\mu = 0$; $p_0/k = 7.6$)
4.24 Contours of strain components at the end of second load step
(first quarter reciprocating sliding); \( (\mu=0.1; \ P_0/k=7.6) \)

4.25 Contours of strain components at the end of second load step
(first quarter reciprocating sliding); \( (\mu=0.3; \ P_0/k=7.6) \)

4.26 Contours of strain components at the end of second load step
(first quarter reciprocating sliding); \( (\mu=0.5; \ P_0/k=7.6) \)

4.27 Contours of strain components at the end of second load step
(first quarter reciprocating sliding); \( (\mu=0.7; \ P_0/k=7.6) \)

4.28 Contours of strain components at the end of third load step
(after one quarter + half) reciprocating sliding cycle; \( (\mu=0; \ P_0/k=7.6) \)

4.29 Contours of strain components at the end of third load step
(after first reciprocating sliding cycle - one quarter cycle in forward direction
+ half cycle in the reverse direction); \( (\mu=0.1; \ P_0/k=7.6) \)

4.30 Contours of strain components at the end of third load step
(after first reciprocating sliding cycle - one quarter cycle in forward direction
+ half cycle in the reverse direction); \( (\mu=0.3; \ P_0/k=7.6) \)

4.31 Contours of strain components at the end of third load step
(after first reciprocating sliding cycle - one quarter cycle in forward direction
+ half cycle in the reverse direction); \( (\mu=0.5; \ P_0/k=7.6) \)

4.32 Contours of strain components at the end of third load step
(after first reciprocating sliding cycle - one quarter cycle in forward direction
+ half cycle in the reverse direction); \( (\mu=0.7; \ P_0/k=7.6) \)

4.33 Contours of strain components at the end of seventh load step
(one indentation + three reciprocating sliding cycles); \( (\mu=0.0; \ P_0/k=7.6) \)

4.34 Contours of strain components at the end of seventh load step
(one indentation + three reciprocating sliding cycles); \( (\mu=0.1; \ P_0/k=7.6) \)

4.35 Contours of strain components at the end of seventh load step
(one indentation + three reciprocating sliding cycles); \( (\mu=0.3; \ P_0/k=7.6) \)

4.36 Contours of strain components at the end of seventh load step
(one indentation + three reciprocating sliding cycles); \( (\mu=0.5; \ P_0/k=7.6) \)

4.37 Contours of strain components at the end of seventh load step
(one indentation + three reciprocating sliding cycles); \((\mu=0.7; P_0/k=7.6)\)

4.38 Contours of strain components at the end of eighth load step after removing the normal load to allow elastic rebound; \((\mu=0.3; P_0/k=7.6)\)

4.39 Contours of strain components at the end of eighth load step after removing the normal load to allow elastic rebound; \((\mu=0.7; P_0/k=7.6)\)

4.40 Figure illustrating the ratchetting phenomenon of surface element A1 for CASEI loading \((\mu=0.3; P_0/k=7.6)\)

4.41 Figure illustrating ratchetting phenomenon of sub surface element A2 for CASEI loading \((\mu=0.7; P_0/k=7.6)\)

4.42 Figure illustrating ratchetting phenomenon of surface element A1 for CASEII loading \((\mu=0.3; P_0/k=3.4)\)

4.43 Figure illustrating ratchetting phenomenon of surface element A1 for CASEII loading \((\mu=0.1; P_0/k=3.4)\)

4.44 Figure illustrating ratchetting phenomenon of surface element A2 for CASEII loading \((\mu=0.7; P_0/k=3.4)\)

Chapter 6

6.1 Contours of contact stress to yield stress ratios

6.2 Configuration of sliding wear system (hardened carbon steel cylinder in contact with SAE 410 s.s. disc)

6.3 A schematic depicting the nominal area of contact and the surface crack for the contact configuration

6.4 A schematic illustrating only one asperity in contact along the sliding direction and 76 asperities in contact along the length of the cylinder for a normal load of 800 N

6.5 A schematic illustrating the concept of equivalent cylindrical asperity representing 76 spherical asperities making circular contacts

6.6 Fracture mechanics model for calculating crack growth and predicting delamination wear under dry sliding conditions

6.7 A Schematic illustrating the angles used in the mathematical derivaiton of stresses at the crack tip
6.8 Calculation of change in displacement based on the crack tip co-ordinate system

6.9 Schematic illustrating tensile stress ($\sigma_0$) and ($\sigma_t$) in the vicinity of the crack tip

6.10 Plot of variation of maximum tensile stress ($\sigma_0$) for mode I, mode II and mixed mode as a function of $\theta$

6.11 Wear particle detachment under dry sliding from an existing crack

6.12 (a-e) Finite element mesh for the fracture mechanics model

6.13 Maximum tensile stress ($\sigma_0$) in the vicinity of the crack tip for an initial crack length of 30\(\mu\)m ($\theta_0=30\text{ deg}$)

6.14 Maximum tensile stress ($\sigma_0$) in the vicinity of the crack tip for an initial crack length of 30\(\mu\)m ($\theta_0=20\text{ deg}$)

6.15 Maximum tensile stress ($\sigma_0$) in the vicinity of the crack tip for an initial crack length of 30\(\mu\)m ($\theta_0=25\text{ deg}$)

6.16 Maximum tensile stress ($\sigma_0$) in the vicinity of the crack tip for an initial crack length of 30\(\mu\)m ($\theta_0=35\text{ deg}$)

6.17 Maximum tensile stress ($\sigma_0$) in the vicinity of the crack tip for an initial crack length of 30\(\mu\)m ($\theta_0=40\text{ deg}$)

6.18 Maximum tensile stress ($\sigma_0$) in the vicinity of the crack tip for an initial crack length of 30\(\mu\)m ($\theta_0=45\text{ deg}$)

6.19 Variation of stress intensity factors (crack length $c=15\text{ microns}$) with normal and tangential loading in first and second load steps

6.20 Variation of stress intensity factors (crack length $c=20\text{ microns}$) with normal and tangential loading in first and second load steps

6.21 Variation of stress intensity factors (crack length $c=25\text{ microns}$) with normal and tangential loading in first and second load steps

6.22 Variation of stress intensity factors (crack length $c=30\text{ microns}$) with normal and tangential loading in first and second load steps

6.23 Variation of stress intensity factors (crack length $c=35\text{ microns}$) with normal and tangential loading in first and second load steps

6.24 Variation of stress intensity factors (crack length $c=40\text{ microns}$)
with normal and tangential loading in first and second load steps

6.26 Variation of stress intensity factors (crack length c=50 microns) with normal and tangential loading in first and second load steps

202

6.27 Variation of number of cycles required for crack growth and particle formation (crack length c=25 microns) with normal and tangential loading in first and second load steps

203

6.28 Variation of number of cycles required for crack growth and particle formation (crack length c=30 microns) with normal and tangential loading in first and second load steps

204

6.29 Variation of number of cycles required for crack growth and particle formation (crack length c=35 microns) with normal and tangential loading in first and second load steps

205

6.30 Variation of number of cycles required for crack growth and particle formation (crack length c=40 microns) with normal and tangential loading in first and second load steps

206

6.31 Variation of number of cycles required for crack growth and particle formation (crack length c=45 microns) with normal and tangential loading in first and second load steps

207

6.32 Variation of number of cycles required for crack growth and particle formation (crack length c=50 microns) with normal and tangential loading in first and second load steps

208

Chapter 7

7.1 A close up view of the mounting fixture along with mounted specimens

209

7.2 Specimens used for experimental work

209

7.3 Mass loss for series A specimens

210

7.4 Mass loss for series B specimens

210

7.5 Variation of friction coefficient with number of cycles

211
7.6 Top view of worn specimen sliding under lubrication of water and cutting fluid (1000:1); normal loads (a) 200N, (b) 300 and (c) delamination wear

7.7 Top view of worn specimens under lubricated sliding (mu=0.1-0.25) at 50X magnification (Rc=35)

7.8 Top view of worn specimens under lubricated sliding (mu=0.1-0.25) at 500x magnification (Rc=35)

7.9 Top view of worn specimens under lubricated sliding (mu=0.1-0.35) at 500x magnification (Rc=45)

7.10 Top view of worn specimens under dry sliding (mu=0.5-0.8) at 50x magnification (Rc=35)

7.11 Top view of worn specimens under dry sliding (mu=0.1-0.35) at 500x magnification (Rc=35)

7.12 Top view of worn specimens under dry sliding (mu=0.1-0.35) at 500x magnification (Rc=45)

7.13 Sectioned view of dry sliding specimens (500x magnification)

7.14 Sectioned dry sliding wear specimen at 500x magnification depicting subsurface crack growth from surface cracks (300 N normal load; Rc=45)

7.15 Sectioned dry sliding wear specimen at 500x magnification depicting subsurface crack growth from surface cracks (300 N normal load; Rc=45); test specimen 1

7.16 Sectioned dry sliding wear specimen at 500x magnification depicting subsurface crack growth from surface cracks (300 N normal load; Rc=45); test specimen 2

7.17 Sectioned dry sliding wear specimen at 500x magnification depicting subsurface crack growth from surface cracks (300 N normal load; Rc=45); test specimen 3

7.18 Sectioned dry sliding wear specimen at 500x magnification depicting subsurface crack growth from surface cracks (300 N normal load; Rc=35); test specimen 4

7.19 Sectioned dry sliding wear specimen at 500x magnification depicting
<table>
<thead>
<tr>
<th>Test Specimen</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.20</td>
<td>Sectioned dry sliding wear specimen at 500x magnification depicting subsurface crack growth from surface cracks (300 N normal load; (R_c=35))</td>
</tr>
<tr>
<td>7.21</td>
<td>Sectioned dry sliding wear specimen at 500x magnification depicting subsurface crack growth from surface cracks (300 N normal load; (R_c=45))</td>
</tr>
<tr>
<td>7.22</td>
<td>Sectioned dry sliding wear specimen at 500x magnification depicting subsurface crack growth from surface cracks (300 N normal load; (R_c=45))</td>
</tr>
<tr>
<td>7.23</td>
<td>Sectioned dry sliding wear specimen at 500x magnification depicting subsurface crack growth from surface cracks (300 N normal load; (R_c=45))</td>
</tr>
</tbody>
</table>
Notation

\( A_0 \)  Nominal contact area between cylinder and flat disc, \( \mu m^2 \)

\( F(h) \)  Cumulative distribution value for asperities of height \( h \), greater than \( R_s \), (the center line average height) of the surface of the cylinder

\( H \)  Material hardness, \( Rc \)

\( K_t \)  Stress-Intensity factor for Mode I, \( MPa \sqrt{m} \)

\( K_{II} \)  Stress-Intensity factor for Mode II, \( MPa \sqrt{m} \)

\( E \)  Young’s modulus, \( Gpa \)

\( G \)  Shear modulus, \( Gpa \)

\( P_N \)  Normal load, \( N \)

\( R_c \)  Rockwell harness (c-scale)

\( R_1 \)  Radius of curvature of the first body (cylinder) in contact, \( mm \)

\( R_2 \)  Radius of curvature of the second body (flat disc) in contact, \( mm \) (infinity)

\( R \)  Radius of curvature of the actual spherical asperities, \( \mu m \)

\( R_{asp} \)  Radius of curvature of the single equivalent asperity, \( \mu m \)

\( W \)  Wear volume, \( mm^3 \)

\( a \)  Semi-contact width of one asperity of top surface (cylinder) coming in contact with the flat disc, \( \mu m \)

\( a_0 \)  Semi contact width (nominal or apparent) of the cylindrical component, \( \mu m \)

\( c \)  Length of assumed crack

\( k \)  Yield strength of material in shear, \( MPa \)

\( k \)  curvature of asperities in the equations of surface statistics in Chapter 3, \( \mu m \)

\( l_0 \)  Length of the cylindrical component in contact, \( mm \)

\( n_0 \)  Number of discrete asperity contacts in the nominal area of contact, \( A_0 \).

\( n_{oa} \)  Number of discrete asperity contacts in the axial direction of the cylinder.

\( n_{or} \)  Number of asperities in the sliding direction or radial direction of the cylinder.

\( p_0 \)  Peak asperity contact pressure, \( N/m^2 \)

\( p_s \)  Mean asperity contact pressure \( N/m^2 \)

\( \epsilon_{ij} \)  Strain tensor
\( \sigma \)  \hspace{1em} \text{Cauchy Stress tensor}

\( \mu \)  \hspace{1em} \text{Coefficient of friction}

\( \nu \)  \hspace{1em} \text{Poisson’s ratio}

\( \eta_L \)  \hspace{1em} \text{Asperity density in the axial direction of the cylinder, } 1/\text{mm}

\( \eta_R \)  \hspace{1em} \text{Asperity density in the axial direction of the cylinder, } 1/\text{mm}
Acknowledgements

This study was made possible by primary support from the Natural Sciences and Engineering Research Council (NSERC) and secondary support from the Atomic Energy Canada Limited (AECL), Chalk River, Ont., Canada. The author is also grateful for the guidance and encouragement provided by supervisors Dr. P. L. Ko, National Research Council of Canada, (NRC), Prof. H. Vaughan (UBC) and Dr. M. Gadala (UBC). The author also wishes to express his gratitude to Mr. Mark Robertson (NRC) along with R. Leung (NRC) for their technical support. The author acknowledges the support extended to him by his present employer, Babcock & Wilcox Canada during the course of this thesis. The motivation from my family, Jay and Priya and my parents, Dr. S.Hymavathy and Prof. S. Srinivasan, was instrumental in completing this work.
CHAPTER 1

Introduction

1.1 General Background

Wear is typically regarded as any process that would cause the removal of material from a surface and (or) a damage to the microstructure in the form of plastic deformation and subsurface crack propagation [Rigney 1978]. Wear depends on conditions of mechanical contact between wearing bodies, material properties and the topography of the wearing surfaces [Burwell and Strang 1953; Halling 1975]. Wear systems are found to be sensitive to small changes in the operating conditions, resulting in distinctly different wear modes. Because of this, the wear problem is a complex one, and modelling even the simplest wear processes requires continuous improvement.

The development of mathematical models to predict wear (calculation of wear volume for a prescribed loading cycle) can furnish adequate data-inputs for minimising wear in contacting machine parts in the design phase itself. Every design change can be profitably evaluated for extended and efficient service life of the product and ultimate cost control. Wear models can be similarly used in estimating the remaining service life of contacting machinery parts during in-service inspections.

1.2 Objectives of the Thesis

- Develop a mathematical model for predicting the reciprocating sliding wear of contacting mechanical components.
- Review the literature in the area of mechanical wear modelling and develop a critique of the existing research work, comparing the same with the present approach, highlighting the substantial improvements made in the present study.
Select ‘distinct sliding wear mechanisms’ through experimentation for a range of normal loading and friction levels.

Incorporate realistic material properties and accurately predict the non-linear mechanical response of wearing surfaces in sliding wear situations using finite element technique.

Develop predictive equations to calculate wear volume and the number of cycles for particle detachment based on the material response quantities, derived from finite element models.

The mathematical model is to be validated through the comparison of predicted results with those obtained from the controlled experimental observation of sliding wear.

1.3 Problem Definition and Method of Solution

The distinct wear mechanisms acting on the wear system are chosen through the direct examination of the photo-micrographs of the sectioned wear specimens obtained from experiments to facilitate modelling. Examination of these micrographs reveals the relationship between the controlling variables and the different wear mechanisms in sliding wear. The main wear mechanisms selected under normal loading and frictional conditions of sliding wear situations are:

- Sliding wear due to Low Cycle Fatigue (LCF) during plastic shakedown conditions.
- Ratchetting Failure (RF) above the ratchetting threshold.
- Above elastic limit and below elastic shakedown conditions, Mild Wear (MW) grooves result.
- Below Elastic Limit with dry sliding, severe crack growth and particle detachment (CGPD) is observed for specimens of high hardness values (Rc >45).

Table 1.1 illustrates the correlation between ‘contact conditions’ of wearing surface and the ‘resulting sliding wear mechanisms’.
Table 1.1 Contact Loading Conditions and Resulting Wear Mechanisms

<table>
<thead>
<tr>
<th>Contact Loading Conditions</th>
<th>Sliding Wear Mechanisms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Cycle Fatigue (LCF)</td>
<td>Ratcheting Wear (RF)</td>
</tr>
<tr>
<td>(line contact)</td>
<td>(line contact)</td>
</tr>
<tr>
<td>3.06 &lt; P₀/k &lt; 4.0</td>
<td>P₀/k &gt; 4.0 @u = 0</td>
</tr>
<tr>
<td></td>
<td>for 0.3 &lt; μ &lt; 0.7,</td>
</tr>
<tr>
<td></td>
<td>see Fig 3.2</td>
</tr>
<tr>
<td></td>
<td>(circular contact)</td>
</tr>
<tr>
<td></td>
<td>&lt;P₀/k = 2.87</td>
</tr>
<tr>
<td>Mild Wear (MW)</td>
<td>Crack Growth &amp; Particle</td>
</tr>
<tr>
<td></td>
<td>Detachment (CGPD)</td>
</tr>
<tr>
<td></td>
<td>P₀/k = 1.5 to 2</td>
</tr>
<tr>
<td></td>
<td>(line contact)</td>
</tr>
</tbody>
</table>

Friction Coefficient (μ)

<table>
<thead>
<tr>
<th>Ratio of Peak Hertzian Contact Pressure to Shear Yield Strength (p₀/k)</th>
<th>ESL &lt; LCF &lt; PSL</th>
<th>PSL &lt; RF</th>
<th>EL &lt; MW &lt; ESL</th>
<th>CGPD &lt; EL</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 &lt; μ &lt; 0.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.3 &lt; μ &lt; 0.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 &lt; μ &lt; 0.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5 &lt; μ &lt; 0.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ESL : Elastic Shakedown Limit
PSL : Plastic Shakedown Limit
EL: Elastic Limit

These observed wear mechanisms are related to the mechanical response of the wearing component through a mathematical model, Fig.1.1. The model employs a configuration of a cylinder sliding perpendicular to its axis on the flat smooth soft surface of a cylindrical disc. The hardness of the cylinder is assumed to be higher than the disc in order to facilitate studying the contact deformation of the disc only. Surface statistical equations are utilised to reduce the geometry of the actual wear process from the component (Cylinder sliding on a disc) to the micro asperity level in the wear model, i.e. an equivalent cylindrical asperity sliding on a smooth half-space.

Appropriate loading inputs, material properties and boundary conditions are applied to the postulated wear model, Fig.1.1. A finite element model for contact deformation is developed using the software, ANSYS 5.4 [1998] adopting a non-linear formulation to study the resulting elastic-plastic deformations/strains/stresses of the half space. The resulting elastic deformations/strains/stresses of the half-space are compared to classical

---

1 Based on circular contact to realistically model the mild abrasive wear grooves.
2 In the Cartesian system of co-ordinates, if the axis, z-o-z, being perpendicular to the paper, forms the axis of the cylinder (plane strain direction), then sliding direction is x-o-x and the normal load is applied along y-o-y.
solutions through Hertzian contact equations [Johnson 1985], to check the validity and robustness of the model.

The model branches into one of ‘low cycle fatigue failure’ (effective elastic-plastic strain forming a closed loop for a prescribed loading cycle) or ‘ratchetting failure’ (open effective strain loop of greater magnitude caused by the accumulation of incremental strain in every loading cycle). The number of cycles for low cycle fatigue wear is calculated through a power law relationship that connects the effective strain and the monotonic strain to failure under static loading. The wear volume is calculated through an equation based on tangential work equivalence. For ratchetting, a linear relationship is postulated for calculating the number of cycles for wear particle detachment based on the effective accumulative strain per loading cycle. The wear volume is based on the depth up to which ratchetting phenomenon is observed. Low cycle fatigue occurs during plastic shakedown state and ratchetting occurs beyond ratchetting threshold.

The model postulates a ‘mild wear’ situation for wet-sliding conditions. A predictive equation is developed for calculating the depth of mild wear groove by considering the real contact area of the asperities, tangential stress under contact load and material-parameters, such as the ultimate strength in shear. Mild wear is predominant during elastic shakedown state.

For situations, where the loading/friction coefficient level combination results in no plastic deformation in the wearing body (except for a very small crack tip plastic zone), but causes the near-surface cracks to grow, a linear elastic fracture mechanics model is developed. Finite element methodology is used for the simulation of wear particle detachment through crack propagation, resulting from existing micro cracks. A surface crack of known dimension and orientation is chosen because the maximum stress intensity, critical to crack growth is observed very close to the surface. A predictive equation based on Paris type law is used for relating the range of the stress intensity factors during a loading cycle and the number of cycles required for crack growth.

The results of the predictive model, Fig. 1.1, are compared with the experimental prediction of wear particle geometry / wear volume [Present Study, Chapter 7; Knowles 94; Chen 94; Magel 90; Ko 84]. The results obtained through the predictive models are
seen to be comparable with those obtained from experimentation. Hence the predictive finite element model could satisfactorily be applied to predict the wear characteristics in different types of wear situations (various contacting geometry and material properties) that are encountered in industrial applications.

1.4 Organisation of the Thesis

In Chapter 1, a proposed wear model is introduced for studying sliding wear. Fig.1.1 illustrates schematically the wear model adopted in the present study. The organisation of the thesis work along with original contribution in the area of predicting sliding wear is presented.

In Chapter 2, existing literature work in the area of mechanical wear is reviewed and the relevance of this review to the present study is discussed. A critique is presented at the end of Chapter 2 to address the inadequacies of the existing mathematical models to correctly predict the non-linear plastic deformation / strains encountered in sliding wear. The importance of utilising finite element methodology is justified through discussions.

Chapter 3 is devoted to the presentation and explanation of the reasons behind developing the wear model with four distinct wear mechanisms, namely low cycle fatigue wear, ratcheting wear, mild wear under wet sliding conditions and crack growth and particle detachment under dry sliding conditions. Mathematical basis for the four wear mechanisms is presented.

Chapter 4 presents the surface statistical quantification of the wear system under consideration. The multi-asperity contact is quantified and an equivalent cylindrical asperity in contact with a smooth plane is modelled. Elastic-plastic contact deformation models based on finite element methodology are presented. Predictive equations are developed for low cycle fatigue wear and ratchetting wear. The numerical results based on the models and predictive equations are detailed. They are compared to Hertzian contact stresses for elastic solutions and elastic-plastic slip-line field solutions. The wear volume and the number of cycles to failure are calculated through predictive equations developed in this chapter.
Chapter 5 details the development of a predictive equation for mild wear under wet sliding conditions and the numerical results derived from the same.

Chapter 6 details the development of the fracture mechanics model for predicting crack growth and particle detachment. The mathematical basis and the finite element application are detailed. Predictive equations are derived for the number of cycles to particle detachment and wear volume. Numerical results obtained from the model and the equations are presented along with discussions. The wear particle geometry and aspect ratio are calculated for several crack orientations and loading conditions. The surface statistical methodology is adopted to quantify the multi-asperity contact as in Chapter 4.

Chapter 7 is devoted to describe the experimental study of sliding wear of the cylinder on disc configuration for a variety of normal loading, friction coefficient levels and hardness levels in the present research. A detailed study of the photomicrographs is presented to illustrate the crack growth phenomenon. Wear volume in prescribed number of loading cycles is calculated for comparison with numerical simulations.

Chapter 8 presents the comparison of predicted numerical results with experimental values. A discussion is provided on the improvements made through the present research and its implications on the numerical prediction.

Chapter 9 provides conclusions of the study and indicates directions of future research in this area.

1.5 Original Contribution in Sliding Wear Prediction

- Development of a methodology by which four distinct mechanisms of sliding wear, namely LCF, RF, MW and CGPD are identified and models/predictive equations are developed for predicting sliding wear.
- Incorporating contact elements simulates a realistic material response of the wearing component that is subsequently used for wear calculations.
- A mixed mode (Mode I and Mode II) Stress Intensity Factor solution is developed for different crack geometry in wearing components using finite element methodology, for which there is no existence of a closed form solution. The crack
propagation direction is calculated based on maximum tensile stress criterion that agrees with experimental prediction rather than assuming it to be parallel to sliding directions as in some existing studies.

- A controlled experimental work is carried out that simulates sliding wear of identical wear system and sectioned micrographs are developed for direct observation.

1.6 Summary

A general background into the phenomenon of mechanical wear is presented. The objectives behind the present study and thesis work are detailed. An investigating methodology for identifying different wear mechanisms is introduced. The mathematical model that is adopted in studying and predicting the sliding wear phenomenon through finite element technique is schematically illustrated and detailed. Four distinct wear mechanisms that are part of a single wear model for the adopted sliding wear system are presented and explained. The organisation of the thesis and the original contribution in the area of sliding wear modelling are presented.
CHAPTER 2

Literature Review

2.1 Introduction

In the sections that follow, first an account is given of the wear modes and advances made in identifying and classifying different wear mechanisms. Subsequently, a historical perspective of the developments in wear modelling research is presented. Literature in the area of Rough surface contact is reviewed and its importance to wear modelling is highlighted. Research in wear modelling is classified under three categories of analytical, numerical and empirical correlation. A critique of this available information follows in the discussion section. Finally the original contribution of the present work in wear modelling in the context of the existing literature is presented.

2.2 Wear Modes - Classification of Wear

The research work published by Archard and Hirst [1956], Hirst and Lancaster [1960], Kerridge and Lancaster [1956] and Welsh [1965] emphasised the work on dry friction and wear in the 1950’s and 1960s. A popular approach was to simply classify wear as either ‘severe’ or ‘mild’. The severe wear leads to roughening of the surfaces and failure, with the production of relatively large wear particles of size 10 \( \mu \text{m} \) - 100 \( \mu \text{m} \), or even as large as 1mm in length. In mild wear, the debris particles are much smaller in size, < 1 \( \mu \text{m} \). and leaving fine grooving on the wearing surface. Work on the scheme of classification of a more fundamental nature was continued by Burwell and Strang [1953] and a detailed treatment was put forth by Blau [1989] illustrating the diversity of wear types. The succeeding subsections present a brief review of these wear types.

2.2.1 Adhesive Wear

Adhesive wear is one of the most fundamental of wear mechanisms. It occurs when two surfaces are in dry sliding contact. Surfaces, in spite of their apparent smoothness, are microscopically rough, exhibiting asperities or surface peaks. Touching surfaces contact
only at the asperities. If the contact interface between these surfaces has enough adhesive bonding strength to resist relative sliding, a large amount of plastic deformation is introduced in the contact region resulting in wear particles in the shape of ‘flakes’ in the leading region of the contact, Fig. 2.1a, [Kayaba and Kato 1981]. Sometimes, wear particles shaped as ‘wedges’ result by crack propagation from existing micro-cracks in the trailing region of the contact as shown in Fig. 2.1b. When the crack in Fig. 2.1a or 2.1b reaches the contact interface, an adhesive wear particle is formed and adhesive transfer is completed. In the successive process of repeated sliding, these wear particles can be detached as free particles or may stay on either surface and form prows to abrade the counterface, as shown by Vingsbo and Hogmark [1980] and Chen and Rigney [1985].

2.2.2 Abrasive Wear

Hard asperities on one surface may scour, gouge, or scratch the material from the contacting softer surface. These asperities may be due to chemical processes such as oxidation or due to temperature effects, leading to precipitation of carbides. Erosive wear is also sometimes categorised as a form of abrasive wear. Alternatively, researchers, such as Engel [1978] have categorised erosion as a form of impact wear.

The necessary condition for abrasive wear to occur is the interlocking of two surfaces at contact. The inclined / curved contact face is generally formed by the indenting of a hard sharp asperity against a relatively soft flat surface. But abrasive grooving by the hard and sharp asperity is possible only when the asperity is not flattened or fractured during sliding. This requires consideration of the yield criterion of the asperity in grooving action. Theoretical analysis, based on slip line field theory, of the critical condition for the yielding of an asperity gives the relations, Fig. 2.2a and Fig. 2.2b, [Kayaba and Kato 1983], between the hardness ratio, r, of the two surfaces and the critical asperity tip angle, \( \theta_c \). The parameter \( f \) in Fig. 2.2b is the ratio of the shear strength of the interface to the shear strength of the softer surface, and Fig. 2.2a shows the relation at \( f=0 \). Plastic grooving by the asperity is possible when its tip angle, \( \theta \), is larger than the critical value \( \theta_c \) and this value changes depending on the values of hardness ratio \( r \) of two surfaces and \( f \) as shown in Fig. 2.2b.

When the hard asperity is strong enough to form a groove on the counterface during sliding, wear particles are formed in three abrasive wear modes, namely cutting mode,
wedge forming mode and ploughing mode, as shown in Fig. 2.3, [Hokkirigawa and Kato 1988].

- In the cutting mode under low friction, long and curled ribbon-like wear particles are formed, Fig. 2.3a. This is analogous to any machining process where metal is removed with the lubricant offering low friction.

- In the wedge forming mode, a wedge-like wear particle is formed at the tip of the grooving asperity as shown in Fig. 2.3b.

- In the ploughing mode, a wear particle is not generated by a single pass of sliding, and only a shallow groove is formed, Fig. 2.3c.

### 2.2.3 Fatigue Wear

Fatigue wear on a microscopic scale is associated with individual asperity contacts. Failure occurs due to the repeated deformation of the asperities of the rubbing bodies. In this process, a crack propagates through the material and may join with others to isolate pieces of material that may become detached to form a wear particle. The location of the failure can sometimes be defined by the position of maximum shear stress which, depending on the coefficient of friction, lies on or below the surface. The failure characteristics of fatigue wear are pitting and spalling. The latter is a severe case of pitting and occurs when cracks are initiated below the surface.

### 2.2.4 Delamination

Delamination wear was first described by Suh [1973] as the loss of metal in the form of flakes, caused by the formation and propagation of subsurface fatigue cracks running parallel to the surface. Delamination wear is often classified as a fatigue mechanism since it depends on the incremental propagation of subsurface micro-cracks. Cracks may initiate either due to the accumulation of dislocations or the formation of voids around inclusions in the material. The stress reversal encountered in the subsurface during sliding contact is considered responsible for the propagation of cracks. During sliding contact, an increment of damage to the material will be experienced during every asperity interaction. Eventually, when the subsurface cracks encounter surface cracks, a thin wear sheet will be lifted from the surface.
2.2.5 Shear Fracture

Shear fracture is a relatively new concept in wear mechanisms. The wear products of both delamination and shear fracture will be thin and flake-like and the wear scars will appear identical. But there is a well explicable distinction between shear fracture and delamination. The mechanism of delamination is usually considered as a brittle fracture phenomenon whereby cracks propagate beneath the surface. Shear fracture, on the other hand, is due to ductile shear causing large near surface plastic deformation.

2.2.6 Corrosion or Oxidation

Corrosive wear is usually a result of both chemical reactions and relative motion between the contacting components. Oxidation, for example, proceeds at its highest rate when a fresh surface is exposed to the surrounding environment and subsequently decreases parabolically. If the oxide product is not removed, the oxide film will actually protect the material from further wear. Stainless steels, for example, readily form such protective oxide films. The relative sliding in a wear situation will usually remove a weakly adhering oxide product. In some cases a very hard surface oxide layer is formed which may actually decrease wear. But if the hard layer gets ruptured, wear may well accelerate as a result of abrasive third-body mechanism.

2.2.7 Fretting Wear

This form of wear is common when two contacting components undergo relative oscillatory motion of very small amplitudes (100 μm – 250 μm). It is actually a combination of several wear mechanisms, including abrasion, adhesion, corrosion and fatigue. Fretting wear is usually initiated by adhesive or abrasive wear and subsequently when the wear particles are oxidised, third body wear mechanism is activated [Waterhouse and Taylor 1974].

2.3 Wear Modelling - A Historical Perspective of its Progress

More than four decades ago, Archard [1953] proposed a simple wear model relating the wear volume lost, \( W \), to the normal contact force, \( P_N \), and the hardness of the material, \( H \), as
\[ W = \frac{K P_n}{H} \]  

(2.1)

K, the wear coefficient, is defined as the "dimensionless constant of proportionality" and can vary from $10^{-1}$ to $10^{-10}$. This wear model has been and still is a widely used model for wear calculations due to its simplicity. Burwell and Strang [1952] noted a significant increase in wear rate when the nominal contact stress exceeded about one third of the yield strength of the softer material. Rabinowicz [1965] obtained a quantitative analysis of the wear rate for various sliding systems. Halling [1978] modified Archard's equation (Equation 2.1) by accounting for the surface roughness, elastic/plastic deformation of the bodies under contact loading, stress-strain relations and failure criterion. Suh et al [1979] calculated abrasive wear of contacting rough surfaces, with the assumption that the scoring agents are conical asperities with hemispherical tips. Challen and Oxley [1984] predicted theoretically the three abrasive wear modes based on a two dimensional model of the slip-line theory. Their predictions are comparable to the experimental results of sphere-on-disk tests [Hokkirigawa and Kato 1989]. Torrance [1987] developed a simple upper bound abrasion model for grinding by assuming that the abrading particles were three-dimensional pyramids.

Kragelski [1965] proposed wear was the result of fatigue. Other wear models, employing fatigue concepts were researched and developed by Halling [1975]. It was assumed that all asperity interactions, (both elastic & plastic) contribute to the fatigue process. Interfacial shear was not included in this assumption, therefore, the model was applicable only to situations where the friction coefficient was small, as in rolling. Finkin [1978] improved this model with the concept of 'damage accumulation' in the near surfaces of the wearing component. Damage accumulation of the wearing surfaces is the result of a large number of micro-cracks that exist in the sub-surface. These cracks reduce the overall stiffness of the wearing body during slided loading resulting in increased wear.

A damage accumulation model by Kimura [1981] accommodated two of the omissions of earlier models.

- First, damage of the wearing surfaces from the fabrication process was included.
• Second, the model predicts the damage accumulating in the subsurface in addition to the thickness of a wear particle.

The damage accumulation model was a purely mathematical model, which treated damage in probabilistic terms. The constants used in the mathematical formulations are therefore not well known material parameters, and only have qualitative physical significance. The appeal of this model is its ability to account for many complex wear behaviours.

Critics of damage accumulation models often argue that the models are too far removed from the reality of wear processes, since they are based on statistical treatments of asperity interactions as in the case of Halling [1975], Finkin [1978] and Kimura [1981]. Work by Alpas et al [1993], however, presents a purely mechanistic approach to damage accumulation. The critical depth for maximum damage accumulation was determined from a relationship for void growth, which was promoted by the near-surface strains but inhibited by the hydrostatic pressure of the contact. The model was able to determine the depth at which the damage rate was highest.

Another approach to this field was Suh’s delamination theory [1973] linking wear to a limiting amount of accumulated plastic strain, causing separation and crack nucleation leading to delamination wear. But in Suh’s work [1973], it is not clear how a soft dislocation-free region sub-surface can accumulate large plastic strain to initiate void nucleation; nor how a crack may propagate parallel to the sliding direction to engender laminate debris. How laminate-shaped debris is also generated in the severe wear of single-phase materials has also not been properly explained.

Jain and Bahadur [1980, 1982] developed models for fatigue wear including the effect of tangential traction. Their model determined the wear from a combination of a fatigue failure equations to calculate the number of cycles for fatigue failure, an estimate of the wear particle volume based on contact dimensions and a statistical treatment of asperity contacts.

New insight into the problem of fatigue failure emerged with the contribution of Hill [1967], Challen & Oxley [1979, 1986] and Black et al [1993] by way of their wave model of friction. Recognising the limitations in the widely held view of adhesive wear, such as the severity of the assumed fracture process and the problem of explaining subsequent particle
detachment from the harder surface, they proposed that friction was attributable to the force required to push a wave of plastically deformed softer material ahead of the hard asperity. A plane strain slip-line field was adopted to illustrate the mechanism. For small asperity angles, friction occurred without wear until a fatigue limit was reached, while at larger angles, a cutting action and chip formation ensued. Challen and Oxley’s model [1979, 1986] provided the basis for a fatigue mode of wear, in which particle formation was linked to the number of waves passed for producing a fatigue failure in the softer material. Assuming fatigue as a primary wear mechanism, they were able to predict wear coefficients comparing fairly with experiment. Using an equation similar to that of Archard [1953], Quinn and Sullivan [1980], developed predictive equations for oxidative wear. Childs [1980] presented a valuable appraisal of these developments between 1950 and 1980. Ko [1984] derived an empirical equation for wear rates based on a statistical analysis method by incorporating the values of support impact forces calculated through the experimental investigation of heat exchanger tubes under impact and sliding load.

Kapoor and Johnson [1994] considered the work done in plastic deformation as hard materials slide over softer ones. They drew attention to the shakedown concept in which initial plastic deformation can be followed by a totally elastic response. Kapoor et al [1996] developed a model for the production of laminar wear debris in which plastic ratchetting was brought about by repeated ‘pummelling’ on the softer material by the asperities on the harder counter face. Torrance [1996] assessed experimentally the low cycle fatigue in comparison to ratchetting concepts and concluded that the fracture process leading to the production of wear particles was best described by low cycle fatigue.

Investigations, premised on finite element analysis, have been conducted for evaluating crack propagation leading to wear particle formation. A review of literature shows that few studies are available in the area of subsurface crack propagation due to the Mode I and Mode II fracture process [McClintock 1977] and [Bower 1988]. Much of the development of fatigue crack growth concepts for metals has been directed at modelling surface pitting which occurs as in mixed sliding and rolling contact of gears, bearings and cams. The

1 ‘Support impact forces’ are the resultant forces of equilibrium, obtained through force sensors during actual wear tests over several cycles of contact loading between mechanical components.
fatigue life that was predicted for the first occurrence of surface pits compared favourably to experimental observations. Zhou et al [1989] presented more sophisticated contact fatigue models. Hanson and Keer [1992] developed models for determining the entire contact fatigue life. Results from the model showed that at high stresses, crack propagation is the dominant wear mechanism. The fatigue lives predicted by both models for high contact pressures, near the elastic limit are in the range of $10^5$ to $10^6$ cycles.

While the above developments have greatly advanced our understanding of wear, it remains true that the machine designers still experience problems with prediction of wear rates. Meng and Ludema [1995] have reviewed existing wear models and called for a new approach for developing wear equations that can be used by designers of machinery.

2.4 Contact of rough Surfaces

Engineering surfaces are considered rough, determined by the method of surface finishing. The effects of surface roughness on line contact pressure were analysed by Merriman and Kannel [1989]. The surface profiles are of three different textures: peaked, Gaussian, and grooved flat surfaces with the same roughness values ($R_s$). An iterative numerical procedure was then used to determine the contact pressure distribution for these three rough surfaces when pressed against a 9.4mm radius cylinder. Pressure distributions were compared graphically with the smooth-body Hertzian distribution under a low and high load and at two magnitudes of surface roughness. The Gaussian texture displayed a band of pressure variation superimposed on the Hertzian distribution. The peak texture produced much greater pressure spikes corresponding to the surface peaks, while the grooved surface had essentially the opposite effect, with zero pressures over the groove valleys. Increasing the magnitude of surface texture produced similar trends for the three different surfaces, but with a proportional increase in the pressure spikes. The most noticeable effect of increasing the load was the formation of pressure spikes on the grooved surface at the edges of the valleys. Pressure spikes ranged from two to six times the magnitude of the maximum contact pressure.
A three-dimensional analysis was performed by Yongqing and Linqing [1992], using a numerical scheme to determine the contact area, pressure distributions, and surface deformations for a flat rough surface pressed against a smooth one. Two rough surface topographies were gathered by profilometer-mapping of a real surface. The analysis produced large pressure peaks at the location of taller asperities. Magnitudes of the pressure peaks were 50 to 70 times the nominal average pressure (based on the apparent contact area). Real contact areas were only a very small fraction of the apparent area, and for light loads the real contact area was proportional to the load. Under high loads, however, the relationship between load and real contact area was far from linear. The authors noted that the interaction of asperities had a strong effect on the calculated pressures and deformations, especially at higher loads, and attributed the non-linearity of the load-versus-real contact area relationship to the interactions of asperities. [Bailey and Sayles 1991] developed a numerical procedure to determine stress distributions resulting from the contact of non-conforming rough bodies. The effects of light friction ($\mu=0.1$) were included in their model.

2.5 Mathematical Modelling of Wear

Wear modelling at the present stage can be categorised into three distinct approaches as noted below:

- Numerical treatments of wear modelling (e.g. finite element approach [Ham et al 1988, Ohame 1980 & 1985]).
- Experimental correlation of wear rates (i.e. Empirical equations of wear based on experimental studies, relating work rate with wear volume data [Magel 1991; Knowles 1994]; Statistical analysis of support reaction forces and empirical correlation of shear force component with wear rates [Ko 1984]).

The following sections provide discussions on the first two aspects of wear modelling. A review on the relevant experimental work is found in [Magel 1991, Knowles 1994]. Discussion relevant to this thesis follows.
2.5.1 Application of Contact Mechanics for Predicting Wear

Contact mechanics provides useful information on the state of near-surface stress in the sliding bodies. Hertz [1882] introduced the theory of circular contact in the absence of friction. Boundary conditions for sliding were established by Mindlin [1945]. He assumed that the shear stress in the entire contact area was proportional to the normal stress by the coefficient of friction between the sliding bodies. Stress fields beneath sliding bodies were determined by Poritsky [1950], Hamilton and Goodman [1966], and later simplified by Hamilton [1982]. These relationships describe the elastic stress field, which can be combined with failure criteria to determine the location where plasticity will first occur. For the zero friction case, the location of the maximum value of the von Mises yield parameter \( \sqrt{J_2} \), (the square root of the second invariant of the stress deviator tensor), in a circular contact, occurs below the surface at a depth of 0.5\( a \), where “\( a \)” is the circular contact radius. As the friction coefficient is increased, this location moves closer from subsurface to the surface and the value increases. At \( \mu=0.3 \), the surface value has exceeded the subsurface peak, and when \( \mu=0.5 \) there is no longer a subsurface maximum.

For the general case of ellipticity of contact areas, Sackfield and Hills [1983] and Johnson [1985] showed that circular contact can replace mild elliptical ones [axis ratio 3:1]. For flatter elliptical contacts, line contacts may be assumed instead. The error involved for yield parameter, pressure etc. is less than < 5% and calculations become much simpler.

For the simpler case of line contact of cylindrical asperities, the assumption of plane strain limits the deformation to two dimensions. Despite this fact, the results of these analyses provide useful insights on what is likely to occur in a three dimensional case.

The concept of “shakedown” assumes importance in any cyclic loading situation that produces elastic-plastic deformation in the material. This theory has been summarised by Johnson, Shercliff, and Kopalinsky [1989]. The plastic deformation, that occurs after the elastic limit has been exceeded, can lead the material to an elastic state of loading for three distinct reasons.

- First, the initial plastic deformation results in greater conformity between the surfaces and the stresses are attenuated.
- Second, protective residual stresses are established in the first few loading cycles.
Third, the material strain hardens.

Johnson et al [1989] preferred the latter two reasons for achieving shakedown for a semi-infinite solid deforming in plane strain. In their kinematic hardening model, the stress-strain behaviour of the contacting asperities determines the theoretical shakedown limit.

The high peaks of the side ridges of a groove formed by a single pass of abrasive sliding, as shown in Fig. 2.3b, have a high probability of continuous contact in repeated sliding. The well-known phenomenon of developing conformity, i.e., increase in contact area in rubbing surfaces supports this assumption. If the contact pressure between ridge and its counterface is below 5.5k of the side ridge but above the plastic shakedown limit (4k for μ= 0) where k is the yield strength in shear, thin plate-like wear particles are generated by the mechanism of plastic ratchetting [Johnson 1995]. In repeated sliding, overall plastic flow of the surface layer by ratchetting results in very thin (< 0.1 \( \mu \) m) filmy wear particle generation [Akagaki and Kato 1987], Fig. 2.4.

By introducing the concept of probability of contact of asperities on a rough surface, the Archard wear coefficient K is theoretically introduced by Kapoor et al [1995, 1996]. The coefficient K is predicted as a function of a parameter called plasticity index \( \psi \) [Johnson 1985] in the range of \( 10^{-10} \sim 10^{-2} \) as shown in Fig. 2.5, where \( \psi \) is defined as:

\[
\psi = \frac{E^*}{P_s} \frac{\sigma}{\sqrt{R}}
\]

(2.2)

Where \( E^* \) is the combined elastic modulus of the two surfaces.
\( P_s \) is the elastic shakedown limit (pressure) of the softer (wearing) surface.
\( \sigma \) is the r.m.s. of the asperity heights on the harder (non-wearing) surface.
\( R \) is the mean asperity tip radius.
\( P_s \) is a function of friction coefficient and decreases with increase in friction coefficient.

An elastic-plastic analysis for the case of a rigid cylinder in rolling contact with an isotropic (elastic perfectly plastic) semi-infinite space was presented by Merwin and Johnson [1963]. This analysis provides a plausible explanation to the cumulative “forward plastic flow” of material during rolling. Hearle and Johnson, [1987] have developed an analysis of
progressive plastic deformation in rolling and sliding based on the continuum theory of dislocations which is an improvement of the analysis of Merwin and Johnson [1963].

2.5.2 Slip Line Theory

The majority of the pioneering mathematical analyses to account for plastic behaviour in loaded mechanical components were based on the slip-line field theory by Hill [1967] and by Johnson [1985]. Using this theory, researchers [Challen and Oxley 1979, 1986] evolved a model - a rigid wedge shaped asperity sliding over a softer surface. They quantified the contributions of the mechanisms of adhesion and ploughing to sliding friction and predicted trends relating the friction coefficient as a function of surface roughness and adhesion. Assuming low cycle fatigue as a primary wear mechanism, they were able to predict wear coefficients comparable to experimentation.

Because rigid-perfectly-plastic material behaviour is assumed, the aforementioned approach of Challen and Oxley [1979; 1986], supported by Hockenhull [1991] and Black et al [1988], is appropriate for analysing contact problems associated with severe sliding as in un-lubricated sliding and heavily loaded contacts where the plastic strains are high enough for the elasticity effects to be neglected. While extensive plastic flow at and directly below the sliding surface may occur under certain conditions [Suh 1986; Kennedy 1982; Kennedy et al 1984], the elastic behaviour of most engineering materials plays a significant role in the deformation process, hence cannot be neglected in modelling.

2.5.3 Upper Bound Theorem's Application in Wear Modelling

An analytical model of friction during the steady state sliding of metals was developed by Avitzur et al [1984], based on the upper-bound theorem of plasticity. The model focussed on the energy absorbed due to the plastic deformation of the material beneath the moving slider and in the ridge, which often forms just ahead of the slider. Trends in friction predicted by the model corresponded with both experimental data and results of the slip line models of Challen [1979 & 1986]. Using a finite element model, Kennedy et al [1984] were able to predict near surface plastic deformation with some accuracy for cases involving single pass sliding.
2.5.4 Fracture Mechanics Modelling (Analytical) in Wear

Suh et al [1977] presented an overview and quantitative assessment of mechanisms for the delamination theory of wear, void nucleation and crack propagation. It was concluded in these papers that in wearing materials, when cyclically loaded by asperities making Hertzian contacts, sub-surface voids are formed in the half-space and under certain conditions they can coalesce into small surface/sub-surface cracks. These cracks may propagate in the half-space until a possible load distribution tends to cause the crack to change direction and break through the surface. This was postulated to be the dominant wear mechanism in dry sliding conditions, and an attempt was made to relate the crack propagation rate to the number of cyclic loading required for material failure.

Since the publication of Suh’s work, several researchers have attempted to use the fracture mechanics approach to the problem of delamination and wear, notably Rosenfield [1980] and Hills and Ashleby [1980]. Both approaches are similar and they compute Stress Intensity Factors for sub-surface horizontal cracks due to the applied Hertzian contact pressure. The paper by Rosenfield [1980] also considers the possibility of Mode II propagation of a crack while the crack is in a compressive field, by assigning a friction coefficient between the crack surfaces in a manner suggested by Swedlow [1976]. Related work by Sih [1980] involving fracture of plates dealt with the determination of crack tip cleavage angles using strain energy density. Tallian et al [1978] have addressed the overall problem of fatigue in rolling contact of mechanical members using a statistical methodology and subsequently a stress based approach.

In fatigue crack propagation problems, analytical/numerical methods offer an interesting alternative compared with detailed experimental study. On reviewing the existing analytical/numerical work in this area, it is found that stress intensity factors in Mode I and Mode II are calculated for subsurface horizontal cracks by Comninou [1977]; Keer et al [1980] and Sheppard et al [1987]. Hills and Comninou [1985] and Sheppard et al [1986] have analysed cracks that are perpendicular to the loading surface and computed stress intensity factors for these cracks under Hertzian contact loading. Keer and Bryant [1983] and Bower [1988] analysed cracks that are inclined to the surface and calculated the resulting stress intensity factors. In most of these analyses, a priori assumptions concerning the mode of fracture is
required, i.e., Mode I or Mode II. Some criteria for the determination of angles of crack growth under mixed mode conditions are also established [Erdogan 1963] and [Sih 1974].

2.6 Finite Element Methods for Prediction of Wear

Since the application of finite element method to contact mechanics problems, the serious difficulties associated with analytical equations were overcome. There have been many attempts to numerically calculate the responses of contacting bodies. Ham et al., [1988] used a modified finite element model to calculate the stresses, strains and the deformations produced by a repeated two-dimensional rolling-sliding contact across an elastic-perfectly plastic half-space. Later, they used a finite element model to compare the stresses; strains and deformations of an elastic-kinematic hardening-plastic representation of rail steel with those produced by elastic-perfectly plastic material. Ohame and Tsukizoe [1980] and Ohame [1986] used finite element method to simulate void nucleation and crack propagation to obtain wear particle formation in their studies of pure aluminium and pure copper plate for the contacting surfaces. They used 3-noded triangular elements to represent a dislocation cell (0.5 μm) and varied the element size as the depth of contacting surface increased (1 cm). The deformation of the aluminium pin was studied assuming the deformation of copper to be negligible. Their results, which are generally in agreement with the delamination theory of wear proposed by Suh [1977], show elastic deformation, yielding, and plastic deformation at different stages of time and that yield of the material was observed beneath the contacting surface.

A detailed analysis of stresses developed in rolling contact is available for elastic spherical contact [Johnson 1985; Hamilton 1983]. The stresses in a circular contact zone, produced by the contact of a sphere with a plane surface, for elastic contact conditions were analysed by Hamilton and Goodman [1966].

Akyuz and Merwin [1968] discussed a general methodology for solving problems of elasto-plasticity with the finite element method and Lee et al [1970 & 1972] obtained finite element solutions for plane-strain and axi-symmetric elastic contacts. The stress and deformation fields developed in a compressed half space having an elastic-constant strain hardening constitutive equation have been obtained by Dumas and Baronet [1971]. They
showed that the progression of yielding changes the contact pressure profile from an elliptical shape to an approximately flat shape. A two-dimensional plane-strain finite element analysis of rolling contact was presented by Bhargava et al [1985].

2.7 Discussion

The reviewed literature reveals that a wear model essentially relates the ‘volume of wear’ to certain ‘mechanical parameters’ for the wear system comprising of two or more bodies under contact loading with relative motion between them. These mechanical parameters could be the ‘material hardness’ of the softer of the two bodies in contact, the ‘normal contact load’ and the ‘relative sliding distance between the bodies in contact’, among others. The mechanism of wear also depends upon the interface conditions of sliding contact during a prescribed number of cycles, e.g., dry sliding contact vs. lubricated sliding contact.

Models based on contact mechanics principles [Lin and Cheng 1989; Rodkiewicz and Wang 1994] produce wear particles whose thickness is related to the maximum shear stress that occurs in the subspace at a depth 0.3a, where ‘a’ is the radius of the real area of contact [Magel 1991]. For such models the resulting wear volumes are several orders of magnitude higher than those observed in controlled experimentation [Magel 1991 Knowles 1994 and Chen 1994]. The models proposed and analysed by Challen and Oxley [1979, 1986] have an inherent assumption of perfectly plastic material for the wearing body. These approaches ignored one or more of the following factors largely responsible for modelling of the wear processes, and hence predicted results that are far from experimentation:

- The elastic stresses and strains that develop due to the contact loading were ignored when using rigid plastic material assumptions for using slip line field solutions.
- The onset of yield and the strain hardening nature of the wearing components were neglected when using elastic contact mechanics formulae for the prediction of sub-surface / surface contact failure.
- The cyclic nature of the repeated sliding was not simulated when using slip line field solutions. The slip line solutions are for one position of the asperity only. This ignores the history of stress-strain cycling of wear track and the computed stress-strain values are not realistic.
The present thesis rectifies many of these aspects as follows:

- The elastic-plastic half space is considered.
- The strain hardening nature of the material is taken into account.
- The repeated sliding is simulated.
- The stresses and strains that develop during subsequent passes take into account the entire history of loading.
- Finite element technique is used to calculate the strains in the wearing material that addresses the constrained plasticity problem. The calculated strains are more realistic than the values from the rigid perfectly plastic material assumption and slip line field theory.
- The non-linear dependence of contact loading on the contact surface profile is taken into consideration, while calculating the contact pressure profile, when surface plasticity results. Special type of elements known as ‘contact elements’ are employed in the finite element code to accurately apply the contact pressure profile at the onset of yield and during the post yield situations in the wearing material.

On reviewing the existing literature related to the prediction of crack growth,

- It is found that a complete analysis of subsurface propagation of a single crack / system of cracks under mixed mode conditions detaching as a wear particle, is not available.
- The non-proportional loading and the crack face contact conditions make it necessary to adopt a numerical iterative technique, such as the finite element analysis to study the crack growth. This methodology enables one to thoroughly understand the behaviour of the bent crack under contact loading, and subsequently the ‘kinked or crooked’ crack after initial crack growth at a specified angle. In earlier studies [Suh et al 1986], crack growth was modelled, although the resulting stress intensity factors were far below the threshold value necessary for crack growth to occur. In the same study, it was postulated that crack growth had occurred parallel to the loading surface. The direction was chosen to be the direction of the in-plane shear strain (x-y plane), but the maximum value, which would occur at a specified angle even for the shear strain failure criterion, was not taken into consideration.

The present research rectifies these aspects by proposing and simulating crack growth:

23
• A unique fracture mechanics model is developed and the crack growth simulated by allowing a single crack to grow under mixed mode crack growth conditions. This assumption is more realistic, since all formed engineering components are most likely to have defects and incursions in a microscopic scale, e.g. defects observed in rolled steel and castings). The response to contact loading would be quite different from the conventional contact mechanics calculations, which assume the material composition to be homogeneous, and elastic continuum [Suh 1977]. The present thesis rectifies the ‘simplified’ assumptions normally used in crack growth modelling by incorporating the following:

- Mixed mode stress intensity factors are calculated based on the crack tip opening/sliding displacements during sliding wear.
- The continuous growth of the crack is modelled within the finite element analysis by extending the crack in the calculated direction and subsequently calculating the stress intensity factors until the crack reaches the loading surface, and detaches as a wear particle.
- The importance of the threshold stress intensity factor is taken into consideration for crack growth.
- The present study predicts the crack turn angle, at an angle where the tensile stress is maximised, guiding the crack growth. This approach agrees with an earlier work by Keer et al [1982] for contact problems.

Based on the previous discussions in the present section, it can be concluded that wear is simulated with more realistic assumptions and through the simulation of appropriate wear mechanisms in the present research.

2.8 Summary

On reviewing the literature available on the subject of wear and wear mechanisms, it can be summarised that three distinct approaches have been used in studying this phenomenon. These approaches are the analytical, numerical and empirical investigations of the phenomenon of wear. Models that were developed based on the fundamentals of tribology are simple to use and are still widely used in the industry although they do not explain the
inherent complexities of the mechanisms of wear. Numerical models based on the axioms of elastic and elastic-plastic contact mechanics neglect the presence of defects in the materials in the form of voids and cracks in the sub-surface that alter the stress field altogether. Rough surface statistics and asperity contact are very important for wear calculations, as models based on the nominal contact geometry failed to predict wear volumes correctly. Experimental investigations are widely used for comparing the predicted values of wear volumes through numerical models.
3.1 Introduction

In this chapter, a concise presentation of the models for four distinct mechanisms of sliding wear are presented, namely

1. Low cycle fatigue failure [LCF]
2. Ratchetting failure [RF]
3. Mild Wear [MW]
4. Crack Growth and Particle Detachment [CGPD]

These wear mechanisms are identified by experimentation and the study of photomicrographs of the sections of test-worn specimens. The wear mechanisms fall into the four phases of response by an elastic-plastic material under different cyclic-loading and frictional conditions. They are due to the changes caused to the subsurface-microstructure of the material during sliding wear process. The following sections describe their relevance to the prediction of sliding wear.

3.2 Elastic-Plastic Responses Under Cyclic Loading [LCF & RF]

3.2.1 Four Regimes of Elastic-Plastic Material Response

A component of elastic-plastic material under contact loading may respond in one of the four ways, Fig. 3.1.

1. Up to a load, called Elastic Limit (EL), the response of the material is perfectly elastic without yielding.
2. For loading above EL, after some yielding at first, a steady state soon results with perfectly elastic response up to a loading limit, called the Elastic Shakedown Limit [ESL]. Due to the residual stresses developed within, the material ‘shakes down’ i.e. reverts back to the elastic state. For loading above ESL, plastic deformation occurs in each cycle as in either (3) or (4), depending upon the amplitude of the load cycle.
3. A steady cyclic state of closed plastic strains, i.e. the closed strain cycles showing no accumulation of plastic strains is realised. Fatigue type failure is observed after numerous cycles of loading. This type of failure is called ‘Low Cycle Fatigue’ (LCF). The load limit for this failure is the Plastic Shakedown Limit (PSL).

4. For cycles of load-amplitude above PSL, the strain cycles are open ones, showing a steady accumulation of unidirectional plastic strains – addition of a small increment of plastic strain occurring with each cycle. These incremental strains add up to a critical value for failure after a prescribed number of cycles. This incremental process is known as ‘Ratchetting’. Failure is evidenced by the extrusion of thin slivers or flakes of wear sheets of the material, i.e. Ratchetting Failure (RF). The load above which this type of failure occurs is the ‘Ratchetting Threshold’, (R_{TH}) and is the same as the PSL for the material.

3.2.2. Shakedown Diagram

The aforementioned concepts are brought out in a map known as the shakedown diagram. The applied contact stress factor $p_0/k$ and the sliding friction coefficient form the axes of Fig.3.2. The factor ‘$p_0$’ is the peak normal Hertzian contact pressure for line contact. Curve A represents the elastic limit; below this curve, yield does not occur even during the first cyclic loading. Curve B signifies the shakedown limit for elastic-perfectly plastic material. The difference between curves A and B shows the influence of residual stresses in promoting shakedown. Curve C is for a kinematically hardening material and the difference between curves B and C shows the influence of strain hardening. It should be noted that for low friction (friction coefficient values less than 0.25), shakedown is controlled by subsurface stresses and the shakedown limit significantly exceeds the elastic limit. For high friction (friction coefficient values greater than 0.25) material yields at the surface and little protection is then provided by either residual stress or strain hardening.

3.2.3. Mechanism of RF

Ratchetting phenomenon is the result of ‘Pummelling’ of hard asperities on randomly distributed contact points on the cross section of the soft surface. After prolonged sliding each point will be contacted at least once so that all the points of the wearing surface will
be subjected to the high contact pressure. Due to this, the thin sub-surface layer of the cross-section gets compressed in the depth direction. This causes the coalescence of microvoids [Thomason 90]. With the growth of these voids, the sectional area of the specimen supporting the applied tensile stress decreases and the magnitude of the true stress increases. This promotes further void growth. This chain like process leads to instability in the specimen, which ultimately fails by the well-known cup and cone fracture called ‘necking’. This ratchetting failure is evidenced by the extrusion of thin platelets of the wear material.

3.2.4 Mechanism of LCF

From Fig. 3.1 it can be seen that LCF and RF are independent mechanisms since they occur in distinct regimes of shakedown map. If the given state of loading and frictional conditions produce no appreciable accumulation of strain components in the sub-surface / surface of the material then the material is in a state of ‘shakedown’. The closed loop of cyclic stress-strain data seen in Fig 3.1 warrants for a fatigue type failure after considerable cyclic loading. This is similar to elastic fatigue failure. The only difference is that, in elastic fatigue failure, the closed loop is replaced with a line as shown in Fig. 3.1. The number of cycles leading to failure is related to the ‘alternating strain’ of any material element on the wearing surface and is a function of all of the strain components.

This can be explained in the light of ‘Bauschinger Effect’ [Johnson W. 1962]. It states that when a metal is plastically deformed and then unloaded, residual stresses, on a microscopic scale, are left in the material. This is mainly due to the different states of stress existing in the variously oriented crystals before unloading in a specimen of polycrystalline material, i.e. metals used in engineering applications. If a metal is plastically deformed in uniform tension, unloaded, and then subjected to uniform compression in the opposite direction (or a shear stress reversal in the shear plane), due to residual system of stresses, yielding occurs at an apparently reduced stress in the reverse half-cycle. This phenomenon is present whenever there is a stress reversal, Fig. 3.3. Referring to this figure, the relevance of Bauchinger effect in plastic “shakedown” leading to low cycle fatigue wear can be interpreted as the plastic work expended in excess of elastic work during the cyclic loading that goes towards the wear process.
3.2.5 Experimental Evidence of RF & LCF

The experimental results by Coffin [1960, 1970] for LCF mode and by Benham [1961], Benham & Ford [1961] and Pisarenko et al [1981] for RF are invoked in the present section to explain the two failure modes, namely RF and LCF. For RF, a typical plot of ratchetting strain against the number of cycles up to the point rupture is given in Fig. 3.4. The curve shows three phases:

- a first phase for the straining few cycles where the ratchetting rate is large and dropping,
- a second, “steady state” phase where the ratchetting rate is almost constant (shown by the near linearity) and
- a third phase of large strain immediately preceding rupture, showing the rapid reduction in the cross-sectional area (and increase in the value of true stress) by the development of a ‘neck’ or spread of crack.

For wear studies the second phase that occurs as a steady state condition is the relevant one. To explain the difference between the RF and LCF, the experimental findings are summarised in Fig. 3.5. The abscissa is the cycles to failure and the ordinate is the normalised strain. The two distinct failure phenomena, RF and LCF are clearly evident. The number of cycles required for failure is sufficiently high in LCF phenomenon compared with RF.

3.2.6 Experimental Results and Inferences for RF & LCF

Photomicrographs of sectioned specimens (specimens exhibiting surface plastic deformation; A for LCF and B for RF) that had been test-worn as per experimental conditions were studied. The experimental conditions to simulate LCF are for specimen A (Hardness: $R_c$ 30–35; $\mu = 0.0$ to 0.1; Sliding Conditions: Wet; Applied peak Hertzian line contact pressure, $3.06 < p_0/k < 4.0$). On examining the specimen A, Fig 3.6, no severe wear in the form of delamination is seen. Sub-surface crack growth is also not seen. Since the loading is high enough, the mechanism is one that is more severe than mild wear. After 50,000 cycles of slided loading under wet conditions, grooves are formed and the failure surface contains a compacted particle layer made up of wear debris.
that had been detached from the parent surface. This is a typical fatigue type failure associated with closed loop plasticity. Kapoor [1995] has obtained similar experimental results with regard to the number of cycles of failure around 100000 cycles for specific loading conditions that simulate LCF.

On examining the specimen B, Fig.3.7, (RF: hardness: R$_c$ = 30 ~35; $\mu$ = 0.3 to 0.7; Sliding Conditions dry ; Applied peak Hertzian contact pressure p$_0$/k > 4.0), which was subjected to higher loading compared to Specimen A, much larger surface plastic deformation was observed. Incremental accumulation of plastic strains for every cycle was instrumental in imparting substantial damage to the sub-surface microstructure of the specimen. Extrusion of thin wear sheets (10 ~25 $\mu$m) was observed. Rupture and failure by Ratchetting were thus identified.

3.2.7. Mathematical Background for PSL and RF

A mathematical derivation is presented in this section based on Kinematic Shakedown Theorem proposed by Koiter [1960] to a wearing component made of elastic-perfectly plastic material. Cyclically time varying contact load (friction free) is assumed for simplicity in deriving the relationship. For shakedown to occur, the work done by the elastically imposed stresses during a cycle should be less than the plastic work expended during any kinematically admissible cycle of plastic strain, $\Delta \varepsilon_{ij}$, where

$$\Delta \varepsilon_{ij}^p = \int_{t}^{t+\Delta t} \varepsilon_{ij}^p dt$$

(3.1)

so that

$$\Omega \int_{t}^{t+\Delta t} [\sigma_{ij}^p(t) \dot{\varepsilon}_{ij}^p] dv dt \leq \int_{t}^{t+\Delta t} \int [\sigma_{ij}^p(t) \dot{\varepsilon}_{ij}^p] dv dt$$

(3.2)

where

$\sigma_{ij}^p$ is the elastic stress tensor (response of the material) due to a unit load

$\Omega$ is a numerical factor, which specifies, as multiples of a unit load, the magnitude of the load applied.

$\dot{\varepsilon}_{ij}^p$ is the strain rate tensor compatible with the assumed cycle of plastic strain.
\( \sigma_{ij}^p \) is the corresponding stresses at yield,
\( v \) is the volume of material undergoing plastic strain \( \varepsilon_{ij}^p \),
\( t \) is the time at the start of a cycle,
\( \Delta t \) is the cycle time during load.
\( \dot{\varepsilon} \) indicates time-rate derivative of \( \varepsilon_{ij} \).

For a given kinematically admissible cycle of plastic deformation the maximum value of \( \Omega \) (say \( \Omega_{SH} \)), is given by the equality sign in Eqn. 3.2

\[
\Omega_{SH} = \frac{\int_{t}^{t+\Delta t} \sigma_{ij}^p(t) \dot{\varepsilon}_{ij}^p dv dt}{\int_{t}^{t+\Delta t} \sigma_{ij}^p(t) \varepsilon_{ij}^p dv dt}
\]

(3.3)

If the applied load is such that \( \Omega > \Omega_{SH} \), then shakedown is impossible and plastic deformation must occur with every cycle of load. If all possible mechanisms are considered, one which gives the lowest value of \( \Omega_{SH} \) is the actual ‘mechanism’ by which the plastic energy is expended; any other mechanism gives an upper bound to the true shakedown limit. For the present formulation, let an ‘infinitesimally small interval’ between \( t_i \) and \( t_i+\Delta t \) during which surface yielding occurs for a particular surface element is considered. Then, using (3.1), Equation (3.3) may be expressed as

\[
\Omega_{SH} = \frac{\int \sigma_{ij}^p(t_i) \dot{\varepsilon}_{ij}^p dv}{\int \sigma_{ij}^p(t_i) \varepsilon_{ij}^p dv}
\]

(3.4)

For different material surface elements, the instant ‘\( t_i \)’ can be at different times during the loading cycle. For shakedown to occur the denominator of Equation (3.4) should be less than the numerator giving the shakedown factor ‘\( \Omega \)’ values greater than 1. This follows from the kinematic shakedown theorem that the work done by the elastically admissible stresses must be less than the plastically expended work during the cyclic loading.
In the present case, a cylindrical asperity of radius ‘RASP’ sliding on a smooth elastic-plastic half space is considered for deriving the expression for the shakedown factor $\Omega_{SH}$. Fig. 3.8 schematically represents a sliding wear set up to apply the Eqn. (3.4). It can be seen that under sliding contact, the material compresses to a depth ‘h’ in the downward y-direction and extends horizontally in the x-direction by an amount $\Delta u_x$. Let the whole system be under plane strain conditions with unit thickness in the z-direction (into the plane of paper). For volume constancy of plasticity theory, the volume compressed, and the volume extruded are equated as follows, see Fig. 3.8.

$$-lh\Delta \varepsilon_{xy}^p = (h - h\Delta \varepsilon_{xy}^p) \Delta u_x$$

(3.5)

Noting that, $\Delta u_x = \Delta \varepsilon_{xy}^p \cdot l$ and $(-\Delta \varepsilon_{xx}^p \cdot \Delta \varepsilon_{xx}^p)$ as H.O.T., Eqn. (3.5) reduces to

$$\Delta \varepsilon_{xx}^p = -\Delta \varepsilon_{yy}^p = \Delta \varepsilon_{nn}^p, s\sigma_{xy}$$

(3.6)

The limits of integration for the compressed layer are between ($-1 < x < 1$), and for the extruded layer, ($0 < y < h$). The increment in shear displacement (in-plane) in x-direction has the integration limits ($y = h; -1 < x < 1$). The incremental shear displacement in the x-direction in the shear plane is given by

$$\Delta u_x = \Delta \varepsilon_{yy}^p \cdot l$$

(3.7)

Yield will occur in the layer at the instant when $(\sigma_{xx} - \sigma_{yy})$ is a maximum and on the shear plane when $\tau_{xy}$ is maximum. These instants need not be, and indeed are not, the same. The volume of material undergoing the plastic deformation is taken to be of unit thickness in z-direction. The following equation is derived based on Eqn 3.4 and using Eqn. 3.5 – Eqn. 3.7. It should also be noted that in plane strain compression yield occurs at a stress of ‘$2k$’, and in simple shear at ‘$k$’.
\[ \Omega_{SH} = \frac{\int_{0}^{h} \int_{-l}^{l} 2k\Delta\epsilon^{p}_{\text{mm}} \, dx \, dy + \int_{-l}^{l} k\Delta u_{x} \, dx}{\int_{0}^{h} \int_{-l}^{l} \left( \sigma_{xx}^{e} - \sigma_{yy}^{e} \right)_{\text{max}} \, dx \, dy + \int_{-l}^{l} \tau_{xy}^{e} \, dx \, dy} \]

Substituting values for \( \Delta u_{x} \) from Eqn. (3.7) into Eqn (3.8) and noting that \( \left( \sigma_{xx}^{e} - \sigma_{yy}^{e} \right)_{\text{max}} \) is constant at each given depth, and \( \left( \tau_{xy}^{e} \right)_{\text{max}} \) as unvarying with \( x \), reduces Eqn. (3.8) to

\[ \Omega_{SH} = \frac{k \left( \frac{2h}{l} + 1 \right)}{h \int_{0}^{h} \left( \sigma_{xx}^{e} - \sigma_{yy}^{e} \right)_{\text{max}} \, dx + \left( \tau_{xy}^{e} \right)_{\text{max}}} \]

Noting that for line contact, \( (h/l < 1.0) \), a simple expression for \( \Omega_{SH} \) is derived as follows:

\[ \Omega_{SH} = \frac{k}{(\tau_{xy}^{e})_{\text{max}}} \]

For a mean contact pressure of \( p_{s} \) under frictionless sliding, \( \left( \tau_{xy}^{e} \right)_{\text{max}} = 0.32p_{s} \), thus the shakedown pressure is given by \( \Omega_{SH} \cdot p_{s} = 3.12 \cdot k \). The mean contact pressure, \( p_{s} \) is related to the peak Hertzian contact pressure as

\[ p_{s} = \frac{\pi}{4} p_{0} \]

Utilising Eqn. (3.11) and Eqn. (3.10), the shakedown factor can be expressed as

\[ \Omega_{SH} \left( \frac{\pi}{4} \right) p_{0} = 3.12 k \]

3.2.8. Proposed Sliding Wear Modelling through LCF and RF

Fig. 3.9 illustrates the procedure adopted in this thesis for predicting sliding wear through the mechanisms of LCF and RF. Surface statistical methodology is utilised for
the quantification of asperities in contact. An equivalent cylindrical asperity is adopted that represents the asperity distribution within the nominal area of contact. The finite element implementation of the contact deformation model is presented. The finite element model of the sliding wear system is solved using ANSYS [1998]. The mathematical background presented in the aforementioned section forms the basis for distinguishing LCF and RF for the given loading situations. The value of the shakedown factor for the unit value of applied peak contact stress ‘\(p_0/k\)’, for zero frictional conditions, if less than 4.0, will lead to LCF under plastic shakedown conditions, (Eqn.3.12). If the value is greater than 4.0, then the mechanism of RF is predicted. The contact between the asperity and the half-space is established through contact elements. The resulting material response, i.e., strains and stresses are calculated for several cycles of reciprocating sliding loading under various friction and normal load levels. The cyclic strain components of the surface elements of the half-space are tracked through load cycles. These strain values along with non-recoverable ‘plastic work per cycle’ are related to sliding wear volume through failure criteria based on LCF and RF. Predictive equations are developed for this purpose. Chapter 4 is devoted to the prediction of sliding wear volume and number cycles to failure through the mechanisms of LCF and RF.

3.3 Mild Wear

3.3.1 Sliding with low coefficient of friction – wet sliding

When a hard rough surface slides over a softer surface under wet sliding conditions, a prow of the softer surface / sub-surface material is seen to precede the driving edge of the hard asperity. Part of the prow material gets transferred as the two ridges on either side of the ploughed groove. The remaining material gets detached from the wearing surface as fine powder. This is the resulting mass loss, which evidently is very small. It is the state of multiple asperity-contact that is closer to operating conditions in practice since engineering surfaces consist of a large number of randomly distributed asperities, differing in heights and shapes and of varying dihedral angles. They present themselves at different orientations to the softer surface – some suitably inclined for mass loss, others
less so. The mild wear tracks (abrasive grooves cut by the hard asperity on the softer material) are a significant feature of this type of wear.

3.3.2. Inference from Specimens

A photomicrograph of specimen undergoing mild wear, Fig. 3.10 (Specimen C: MW: hardness: $R_c = 30 \sim 35$; $\mu = 0.1 - 0.15$; Sliding Conditions wet; Applied peak Hertzian contact pressure $p_0/k = 2.87$ to 3.0) reveals clear evidence of the formation of mild wear grooves. The specimen appears with fine grooves/scratches on the surface. Very little mass loss in the form of powder-like wear particles were seen during the observations. Fig. 3.11, shows that very distinct differences of wear mechanisms are involved in dry sliding and wet sliding. The mass loss associated with wet sliding resulting in mild wear is an order of magnitude lower than for the dry sliding conditions.

3.3.3. Elastic Shakedown Limit of Circular Contact & Mild Wear

Mild wear under wet sliding conditions is observed under loads that cause the specimens to yield a little initially and then shakedown elastically. No subsequent plastic deformations are therefore noted after the small initial yield. This phenomenon can be explained in light of Fig. 3.12. Since the mild wear grooves form due to the individual asperities of the cylindrical component in contact, the mild wear phenomenon has to be explained in the light of the circular contact shakedown map rather than a line contact shakedown map that is applicable for the reduced equivalent cylindrical asperity. The elastic shakedown occurs around $p_0/k = 2.87$. At this loading level, only one cycle of plastic deformation results along with material removal, on the surface due to the asperities cutting grooves on the surface. Subsequently the area of contact increases for every single asperity of the mechanical component in contact and the contact pressure drops in magnitude resulting only in elastic deformations. Since the loading level and friction coefficient levels were low, no crack growth was observed.

3.3.4 Proposed model for Sliding Wear through Mild Wear Mechanism

Fig. 3.13 illustrates the proposed sliding wear model based on the mechanism of Mild Wear (MW). Surface statistical equations are utilised to calculate the nominal area of contact for the contact between the cylindrical component and the flat smooth disc. The number of asperities in the nominal area of contact are subsequently calculated and
modelled as hemispheres. For the applied loading conditions \(2.87 < \frac{p_0}{k} < 3.0\); circular contact) the abrasive groove depth is evaluated by equating the tangential input work to the work required to cut the groove. A factor \(\phi\), fraction of total tangential work that goes towards mild wear is utilised in the predictive equation to calculate Mild Wear. It will be shown in Chapter 5 that after initial cycling, the system shakes down to elastic state and further cutting of mild grooves is prevented. Chapter 5 is devoted to the study and prediction of Mild Wear (MW).

3.4 Crack Growth and Wear Particle Detachment (CGPD)

During dry sliding wear situations, investigations, both theoretical and experimental have established that compressive and shear stresses dominate the sub-surface. Existing cracks extend and join the neighbouring ones, subsequently detaching as wear particles once they reach the free surface. The high levels of coefficient of friction during dry sliding impose sufficiently large shear stresses on the wear surface. These shear stresses combined with the applied normal stress subject the sliding wear system to a compressive – shear type stress distribution. This type of distribution is critical to the observed crack growth in dry sliding wear (a mixed mode fracture; (Mode I and Mode II)). The following sections describe this phenomenon in detail.

3.4.1 Experimental Inference of Crack Growth and Wear Particle Formation

During dry sliding contacts with relatively high normal loads on harder specimens, but that are still below EL, \(\frac{p_0}{k} \sim 1.5 \text{ to } 2.0; R_c 40-45; \mu = 0.5 \text{ to } 0.8\), severe wear in the form of wear particle detachment through sub-surface crack growth reaching loading surfaces was observed. Large flakes of material were seen to detach periodically from the wearing surface. Figures 3.14 to 3.15 are photomicrographs reproduced from early work by Suh [1986] & Knowles [1994]. They show examples of the appearance of worn surfaces and the growth of sub-surface fatigue cracks that ultimately reach the surface of the contacting body to form a detached wear particle. These particles are flaky and thin and long in shape, and the wear track appears to have coarse irregular edges, Fig. 3.14. Crack propagation is the important cause for severe wear, primarily under dry sliding conditions with friction coefficient levels of 0.8, Fig.3.16 & Fig.3.18. The resulting wear
is deemed to be the detachment of the softer material when these subsurface cracks reach
the surface. Figure 3.17 is a photomicrograph obtained from the present experimental
investigation\(^1\) for dry sliding wear conditions. On examining the sub-surface, the fatigue
crack growth pattern is evident and the observation substantiates growth patterns
previously observed by Suh [1986] and Knowles [1994]. It is interesting to note that
though the geometry of the harder specimen in each of the experimental studies referred
to in this section differs from each other, but that the wear trend exhibited by the wearing
specimen is very similar. They each show the propagation of surface cracks that finally
reach back to the surface to form a wear particle; also, they all exhibit a relatively high
coefficient of friction, (0.6-0.8).

The above observations lead one to infer that in dry sliding contacts, fatigue crack
growth is the result of asperity contact of the mating surfaces independent of the
configuration of the actual components in contact. Furthermore, since crack growth
depends on the asperity level contacts, a model based on contact stresses and surface
statistics of asperity distribution can be generalised for application to any macro-
geometry.

3.4.3 Fracture Mechanics Approach to Sliding Wear

The scope of the present analysis is to assume a surface crack in the elastic half-space
and study its growth for the cyclic Hertzian loading. When the crack reaches the wearing
surface, it detaches from the parent surface as a wear particle. While the location of this
crack may seem arbitrary, such specific locations exist in the structure, where, if the crack
is assumed, it is more likely to grow and detach sooner than from other locations. These
locations are determined as regions of maximum stress intensities and can be
conveniently identified using a contact deformation model. This is necessary because an
assumed crack at this location would be subject to the maximum loading more so, than it
would be, at any other location in the half-space.

---

\(^1\) A detailed experimental investigation carried out as part of this research is explained in Chapter 7. However, some photomicrographs pertaining to sub-surface crack growth are presented in this chapter for inference on crack growth and particle detachment.
3.4.4 Proposed Sliding Wear Model through Crack Growth (Delamination Wear)

For loading levels \( p_0/k < \text{EL}; \) i.e. 1.5 - 2) on hard specimens \( (R_c = 40 - 45) \), CGPD is the governing mechanism of sliding wear. Fig. 3.19 schematically illustrates how the present thesis investigates this phenomenon through a fracture mechanics model. Surface statistical equations are utilised to model the contacting asperities as an equivalent cylindrical asperity. This asperity is in contact with an elastic half-space containing an assumed surface crack. The Hertzian contact pressure and the tangential stress are translated along the sliding direction. This translation results in the deformation of the half-space. The crack face displacements are subsequently related to the stress intensity factors in the mixed mode crack problem through trigonometric functions developed and explained in Chapter 6. The crack growth is modelled using an equation similar to Paris Law that involves the calculated stress intensity factors along with experimentally fitted values for the constants in the equation. The crack is grown continuously until it reaches the wearing surface, detaching as a wear particle in the prescribed number of load steps in the FE analysis. The volume and the number of cycles for crack growth and particle detachment are presented for the parametric variation of inputs to the wear system.

3.5 Summary

From the discussions, it can be seen that the four mechanisms occur due to loading levels that fall into four distinct regimes of the shakedown map. In light of this, four sliding wear mechanisms are chosen from the study of test-worn specimens. The mechanisms of LCF and RF are distinguished and related to the material responses corresponding to contact loads that fall in distinct regimes of the shakedown map. In the following chapters 4, 5 and 6, surface statistics is used to quantify the engineering surfaces to fit in modelling using finite element methods. For mild wear situations, the asperities are modelled as hemispheres to realistically simulate the shape of the abrasive grooves. For crack growth and particle detachment mechanism that results in delamination wear, the growth of a single assumed crack is modelled until it reaches the wearing surface in a prescribed number of cycles, detaching as wear particle.
CHAPTER 4

Prediction of Sliding Wear through LCF and RF

4.1 Introduction

The discussions in Chapter 3 clearly demonstrated that the four distinct mechanisms of sliding wear fall into four distinct phases of elastic-plastic response in the shakedown map of the wearing material. This chapter presents the development and prediction of sliding wear through LCF and RF mechanisms. The following are the significant aspects of the present research in LCF and RF modelling.

1. Predictive numerical models are developed for LCF and RF mechanisms.
2. Surface statistics is utilised to quantify the engineering surfaces in contact.
3. A finite element model is developed for the contact of an equivalent cylindrical asperity (a two-dimensional representation of several asperities in contact) with an elastic-plastic half-space.
4. The contact between the asperity and the half space is established through special type of elements, known as contact elements.
5. The mechanical response quantities such as deformations, strains and stresses are calculated for several cycles of reciprocating sliding loading under various friction and normal load levels.
6. The cyclic strain components of the surface elements of the half space are tracked through the loading cycles.
7. These strain values are combined to result in effective strain that is related to the number of cycles to failure through different predictive equations for LCF and RF.
8. Sliding wear volume is calculated through tangential work equivalence for LCF.
9. The maximum depth of ratchetting, observed through FE simulation, is used for the calculation of wear volume for RF.
10. Parametric studies of the predictive models are carried out.
4.2 Multi Asperity Contact and Contact Deformation Model

4.2.1 General Background

Before developing models for contact deformation, the contact between engineering surfaces needs to be quantified. For this purpose, surface statistics is used. The equations in the following section are used to quantify the multi-asperity contact.

4.2.2 Characteristics and Measurement of random (manufactured) rough surfaces

Surface texture is most commonly measured by a profilometer that drags a stylus over a sample length of the surface of the component and reproduces a magnified trace of the surface profile as shown in Fig. 4.1a and Fig. 4.1b. A schematic representing only a section of the trace is illustrated in Fig. 4.1c. First, a datum or ‘centre-line’ is established by finding the straight line (or circular arc in the case of cylindrical components). This line is drawn such that the area enveloped by the profilometer trace above the datum is equal to the area enveloped by the trace below the datum. The average roughness is defined by

$$R_a = \frac{1}{L} \int_0^L |z(x)| \, dx$$

(4.1)

Where, \(z(x)\) is the height of the surface above the datum and \(L\) is the sampling length. The average roughness, \(R_a\), has the unit of length and is usually expressed in microns. A statistically more meaningful measure of average roughness is the ‘root-mean-square’ or standard deviation \(\sigma\) of the height of the surface from the datum or centre line, i.e.,

$$\sigma^2 = \frac{1}{L} \int_0^L z^2 \, dz$$

(4.2)

The relationship between \(\sigma\) and \(R_a\) depends, to some extent, on the nature of the surface; for a Gaussian random profile the average roughness, \(R_a\), is related to \(\sigma\) as \[Johnson 1985\]

$$\sigma = \left(\frac{\pi}{2}\right)^{0.5} R_a$$

(4.3)
If $\phi(z)$ is the probability function for the height of a particular point in the surface lying between $z$ and $z+dz$, then the probability that the height of a point on the surface greater than $z$ is given by the cumulative probability function:

$$
\Phi(z) = \int_{z}^{\infty} \phi(z) dz
$$

(4.4)

This yields a S-shaped curve identical with the bearing area curve developed by Abbott and Firestone [1933], Fig. 4.2. It has been found that many real surfaces, notably surfaces subjected to grinding operation exhibit a height distribution, which is close to the `normal' or `Gaussian' probability function [Johnson 1985]:

$$
\phi(z) = \sigma (2\pi)^{-0.5} \exp\left(\frac{-z^2}{2\sigma^2}\right)
$$

(4.5)

where $\sigma$ is the standard deviation of the asperity height distribution.

The cumulative probability is expressed as:

$$
\Phi(z) = \frac{1}{2} - \frac{1}{(2\pi)^{0.5}} \int_{0}^{\frac{z}{\sigma}} \exp\left(\frac{-v^2}{2\sigma^2}\right) dv
$$

(4.6)

When plotted on normal probability graph paper, data which follow the normal or Gaussian distribution will fall on a straight line whose gradient gives a measure of the standard deviation $\sigma$, as shown by the ground surface in Fig. 4.3a. It is convenient from a mathematical point of view to use the normal probability function in the analysis of randomly rough surfaces although there are engineering surfaces (e.g., abraded and polished) exhibiting non-Gaussian behaviour, Fig. 4.3b. In the controlled experimentation of the present research, the surface of the sliding cylinder is not polished and is one close to the surface texture represented by Fig. 4.3a. So the following calculations assume the profile of the surface to be of Gaussian.

The analysis of Greenwood and Williamson [1966] is followed for assessing the multi-asperity contact situation arising from the contact of two rough surfaces. They
assumed that the contact of the two surfaces is the same as the contact of a flat smooth surface in contact with a surface of equivalent roughness. The reason is as follows:

The elastic contact deformations of the mechanical components depend only upon the relative profile of the two surfaces, i.e. upon the shape of the gap between them before loading. The system may then be replaced without loss of generality by a flat smooth surface in contact with a body having an effective modulus $E^*$ (at the contact region) and a profile which results in the same undeformed gap (separation, $d$) between the surfaces. For two nominally flat surfaces, i.e. microscopically rough, in contact, which have root mean square roughness $\sigma_1$ and $\sigma_2$, respectively, an equivalent representation can be given by the contact of smooth flat with another plane of equivalent roughness $(\sigma_1 + \sigma_2)^{0.5}$.

The mean level of the surface is taken as the datum and the distance between the datum and the flat is referred to as the ‘separation $d$’. Fig. 4.4 illustrates the contact between a flat and an equivalent rough surface. Let the summit (asperity) heights be denoted as $z_s$, having a mean asperity height $\overline{z_s}$ or $(R_z)$ with a distribution function $\phi(z_s)$, which expresses the probability of finding a summit of height $z_s + dz_s$ (see Fig. 4.2 for the small interval corresponding to $z$ and $z+dz$). If there are $\eta_0$ asperities per unit nominal area of contact, then the number of asperities, $n_0$, in the nominal area $A_0$ at a separation ‘$d$’ is given by

$$n_0 = \eta_0 A_0 \int d \phi(z_s) dz$$

(4.7)

It is usual to express these equations in terms of normalised variables (normalised with respect to the standard deviation, $\sigma_s$, of the of the asperity height distribution of the surface) by putting $h = d/\sigma_s$ and $s = z_s / \sigma_s$. Hence Eqn. (4.7) is rewritten as

$$n_0 = \eta_0 A_0 F(h)$$

(4.8)

where $F(h)$ is the cumulative probability of $n_0$ asperities coming into contact with the flat surface, expressed as:

$$F(h) = \int_{h}^{\infty} \phi^*(s) ds$$

42
where $\phi^*(s)$ is the probability function of the height distribution of the asperities by normalising the parameters $d$ and $z_s$ with the standard deviation of the asperity heights, $\sigma_s$.

The value $n_0$ is an important parameter for the present analysis. This allows one to know the number of asperities that come into contact in the nominal area of contact. The calculation and significance of this parameter is explained in Section 4.2.5.

### 4.2.3 Measurement of Curvature of asperities

The curvature of the asperities could be calculated based on the discussions in [Thomas 1982]. A sample length, $L$ of the surface is traversed by a stylus profilometer and the height $z$ is sampled at discrete intervals of length ‘$l$’. If $z_{i-1}, z_i, z_{i+1}$ are three consecutive heights, the curvature is defined as

$$k = \frac{(z_{i+1} - 2z_i + z_{i-1})}{l^2}$$

(4.10)

The root mean square curvature of the surface is expressed as [Johnson 1985]

$$\sigma_k^2 = \left(\frac{1}{n}\right) \sum_{i=1}^{n} k^2$$

(4.11)

where the total number of heights sampled, $n$, is the ratio of sampling length ($L$) to sampling interval ($l$).

The parameter $\sigma_k$ is the property of the surface that it describes. Its values in practice depend upon both the sample length $L$ and the sampling interval, ‘$l$’ used in their measurement. The value $\sigma_k$ would later be used in the modelling of an equivalent cylindrical asperity. The radius of curvature for the equivalent cylindrical asperity would be taken as $\sigma_k$.

Hence to quantify the engineering surface contact, it is necessary to know the following parameters

- values of standard deviation of the asperity heights, $\sigma_s$,
- root mean square curvature of the asperity heights, $\sigma_k$ and
asperity density $\eta_s$ leading one to calculate the total number of asperities, $n_0$, in nominal contact area, $A_0$.

These quantities have to be deduced from the information contained in a profilometer trace of the surface. It must be kept in mind that a maximum in the profilometer trace referred to, as 'peak' does not necessarily correspond to a true maximum in the surface referred to as a 'summit'. For the trace is only an one-dimensional section, (schematically represented in Fig. 4.1c) of a two-dimensional surface (representative profilometer traces shown in Fig.4.1a and Fig.4.1b. To rectify this for the modelling in the present research, following Johnson [1985]:

- The "mean height of the summit" lies between $0.5\sigma$ and $1.5\sigma$ above the "mean level of the surface".
- The "mean summit curvature" is of the same order as the "root-mean square curvature of the surface", i.e.,

$$\overline{k}_s = \sigma_k$$

(4.12)

- By identifying peaks in the profile trace as explained above, the number of peaks per unit length of trace $n_p$ can be counted. For a regular wavy surface, the number of summits per unit area, $n_s$ is expressed as [Johnson 1985]:

$$n_s = \eta_s^2$$

(4.13)

### 4.2.4 Assumptions for Modelling

Based on the above discussions regarding the rough surface contact, the following assumptions are made towards statistical quantification of asperities in contact between two engineering surfaces. These assumptions are used in section 4.2.6 for the deduction of the number of asperities in contact

1. At some instant, the asperities on the upper surface are loaded against the asperities on the lower surface. As reciprocating sliding occurs, the contact positions will change causing these asperities to be continuously loaded and unloaded.

2. The heights of the asperities on both contacting surfaces vary randomly. This assumption is also validated from surface measurements obtained during previous
experimental investigations, Fig. 4.1a and Fig. 4.1b. Fig. 4.5a represents this assumption schematically.

3. The asperities have rounded tips, i.e.; they have a definite radius of curvature, Fig.4.5b. This assumption enables one to model surface asperities as of regular geometry such as cylinders or spheres, Fig 4.6a – Fig. 4.6b

4. The discrete contact zones are sufficiently separated so that they do not interact with each other. This assumption is necessary to avoid mathematical complexities in finite element analysis for studying the micro-mechanical response of the half-plane using finite element analysis.

The assumptions and discussions in Sections 4.2.1 – 4.2.3 enable one to model a surface of equivalent roughness (asperities having a radius of curvature) with that of a smooth flat surface at a separation d. It will also be shown in the next section that only one such asperity would come into contact in the sliding direction.

**4.2.5 Calculation of ‘Nominal Contact Area, $A_o$’**

Prior to estimation of asperities in contact within the nominal area of contact (area developed due to the contact of two loaded mechanical components) during the wear process, a preliminary calculation needs to be made regarding the nominal contact area of mechanical components that is of interest for wear studies. For the present work, a cylinder of length $(l_0 = 25.4 \text{ mm})$ and radius of $(R_1 = 5 \text{ mm})$ is chosen, so as to simulate the exact experimental conditions. This cylinder is in contact with a flat circular disc of 12.7 mm diameter and 10 mm thick, see Fig. 4.7a. The contact area between a cylinder and a flat surface is one of line contact. The contact area is calculated as follows.

Let the modulii of Elasticity of the two contacting surfaces be $E_1$ & $E_2$. Their Poisson’s ratios are given as $\nu_1$ and $\nu_2$. For the flat surface the radius, $R_2 = \infty$. Based on the Hertzian formulae for contact, the equivalent modulus of elasticity and equivalent radius of contact for the two bodies in contact are given as

$$\frac{1}{E} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$$

(4.14)
The semi contact width for the cylindrical component in contact with a flat surface, Fig 4.7b, is given as follows

\[
\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}
\]

and

\[R_1 = \infty\]

(4.15)

The contact width \((2a_o)\), once multiplied by the length of the cylindrical component in contact, \(l_0\) would give the nominal area of contact.

\[A_0 = 2a_0 l_0\]

(4.16)

The asperity density of the surface can be estimated from the 'talysurf' measurement of engineering surfaces and is expressed as \(n_s\). Then following Eqn.4.8, and substituting for \(A_o\) from Eqn.4.17, the number of asperities in the nominal area of contact that come into contact with a flat can be expressed as (from Eqn. (4.8))

\[n_o = 2n_s F(h)a_0l_0\]

(4.17)

To model the idealised asperity, \(R_{asp}\), is taken as the average radius of curvature, \(k_s\) of the asperities (from Eqn. (4.12)). This information is obtained from the sample of the cylindrical component used in the experimentation of the present research. The average roughness, \(R_a\) and root mean square roughness, \(\sigma\), are also measured from the same surface. These summit properties of interest (average roughness of the surface, \(R_a = 0.3\) \(\mu\)m, standard deviation of roughness, \(\sigma = 0.5586\) \(\mu\)m and the average radius of curvature of the asperities, \(R_{asp} = R = 1\) mm) are subsequently used for deducing the number of asperities. Similar values are obtained in the experimental work of Magel [1990]. In his work, a probability density distribution of the radius of curvatures of the asperities was
given ranging from 100 μm to 1200 μm. The experimental values for the asperity heights were between 0.03 μm and 1.2 μm, with most of the asperities having heights between 0.3 - 0.4 μm.

4.2.6 Deduction of number of asperities in the nominal area of contact

Substituting the values from the above paragraph in Eqn. (4.9), the value of \( F(h) = 0.296 \) is obtained. Substituting the value of \( F(h) \) into Eqn. (4.18) along with values of \( l_0 = 25.4 \text{ mm} \) and \( a_0 = 0.203 \text{ μm} \), (see equations 4.14 - 4.17), with the asperity density of the surface, \( \eta_s \), as 100 per mm\(^2\) (from experimental work of the present research), the number of asperities in contact is obtained. It should be noted that the average roughness is used for the lower integration limit of \( 'd' \) in Eqn (4.7), i.e. it is postulated that all the asperities having heights greater than or equal to mean asperity height (average roughness, \( R_a \)) would come into contact with a flat surface. This assumption had been previously used in the estimation of the cumulative probability value, \( F(h) \) and subsequently used to calculate the wear characteristics [Chen 1994] and found to correlate well with the wear prediction.

The value of \( 'd' = R_a = 0.3 \text{ μm} \) is normalised with respect to \( \sigma = 0.5586 \), the standard deviation per Eqn. (4.9) so that the assumed Gaussian function would have a standard deviation of unity. Substituting the value of \( F(h) = 0.296 \) in Eqn. (4.21) the number of asperities in contact is obtained as \( n_0 = 76 \). Also it is important to know the number of asperities in the axial direction of the cylindrical component that comes into contact \( n_{oa} \), and in the radial direction of the cylindrical component, \( n_{or} \), in the nominal area of contact, \( A_0 \). The calculations show that the number of asperities in the sliding direction \( n_{or} \), is approximately equal to 1 and the number of asperities in the axial direction of the cylinder, \( n_{oa} \), are 76. This estimated configuration is illustrated in Fig. 4.8. These 76 asperities are seen to fall in one line within the nominal area of contact.

It is very tedious to analyse the 3D contact mechanics loading applicable for the elastic/plastic deformation analysis. The semi-contact width is relatively small compared with the sliding distance, \( l \). These two reasons allow one to model all the asperities (spherical or elliptical in 3-D) as one single equivalent cylindrical asperity under the assumption of plane strain (i.e., in the axial direction of the cylindrical component). The
equivalent cylindrical asperity in contact with the half-space develops a line contact (see Fig. 4.8 for illustration). The transfer of contact forces across the interface to the wearing body is still accurate, since FE analysis ensures the equilibrium of forces and moments. Since the radius of curvature of the equivalent asperity is still maintained as that of the spherical asperities, the real area of contact is also calculated with accuracy. The individual pressure spikes that are associated with actual rough surface contact could produce very localised yielding on the wear track. To model this behaviour would warrant an enormous computing time and memory. Therefore the approach of equivalent cylindrical asperity under plane strain conditions is chosen for representing the individual asperities.

It should be noted that several such equivalent cylindrical asperities could exist depending on the nature of the asperity density distribution within the nominal contact area $A_0$, (see Fig. 4.7). But normally, the observed nominal contact width is very small compared with the sliding length of the specimen. The contact width is of the order of $200\mu m$ and sliding length in experimentation being $2000 \mu m$. Through calculations it has been observed that atmost, only one row of asperities could come into contact in the direction of sliding.

4.3 Finite Element Formulation of Contact Deformation Model

The primary notion behind the FE modelling is to simulate the effect of sliding the equivalent asperity (cylindrical in shape) on a smooth plane (top surface of the half-space) under normal loading. In this section, the finite element model development of the wearing region of the half-space, subjected to contact loading by the equivalent cylindrical asperity is presented. Figs. 4.9-4.11 illustrate the model of the line contact between the equivalent cylindrical asperity and the elastic-plastic half-space in 2D (plane-strain) established through the quantification procedure explained in Section 4.2. Fig.4.9 illustrates the translation of the applied load on the elastic-plastic half-space, consequent to the sliding of the equivalent cylindrical asperity on the half-space. Fig.4.10. For modelling purposes, the applied normal load, $P_N$, is usually known from experimental tests, [present experimental work].
Fig. 4.11 describes the normal pressure applied on the ‘truncated’ equivalent asperity. Fig. 4.12 describes the sliding wear problem schematically for a single element on the wear track, Element A. The elastic-plastic strains acting on this element are shown in the original un-deformed configuration. The element formulations in both total and updated Lagrangian configurations are shown in Fig.4.12. The normal strains during the cyclic loading, \( \varepsilon_{xx} \) and \( \varepsilon_{yy} \), the shear in plane strain, \( \varepsilon_{xy} \), are illustrated on a single element ‘A’. The deformations, strains and the stresses resulting from the application of contact loading are tracked throughout the history of sliding. The stresses before yielding are due to elasticity effects. The onset of yield is simulated using von Mises criterion for yield. The post yield stresses are calculated using a flow rule and the constitutive equations are described in ANSYS [1998].

The contact between the equivalent cylindrical asperity and the half-space is established through special purpose elements known as “contact elements” in the finite element program, ANSYS [ANSYS Elements Manual 1998].

4.4 Numerical simulation

The single hard asperity of a rough surface is idealised as a cylinder of radius \( r = 1000 \) \( \mu \text{m} \) with a length of 25.4 mm in the direction of plane strain, (z-direction). The asperity is assumed to be in contact with an elastic-plastic semi-infinite half space, (5.1 mm x 2.54 mm).

The equivalent cylindrical asperity is allowed to undergo only elastic deformation to simulate the effect of a hard asperity (i.e., no yielding). The simulation is carried out with the material properties of steel type SAE 410 for the elastic-plastic half space [ASME B&PV Code 1992].

- Young’s Modulus, \( E = 200 \) GPa
- Poisson’s ratio, \( \nu = 0.3 \)
- Yield strength \( \sigma_y = 474 \) MPa
- Tangent modulus, \( H' = 137 \) MPa

\(^1\) The equivalent asperity is truncated on top to facilitate the speeding up of computer time. Appropriate changes are made to the applied pressure distribution to account for this change, (i.e. area change from \( \pi R_{asp} l_0 \) changed to \( 2*R_{asp} l_0 \)).
Fig. 4.13 illustrates the material stress-strain curve, which utilises the above parameters. The elastic-plastic half space is assumed to follow the material model of ‘kinematic hardening’ after the yielding. The asperity is modelled to deform elastically with type SAE 410 s.s. Properties of E and v. The model was analysed for several values of friction coefficients ranging from (\( \mu = 0, \mu = 0.1, \mu = 0.3, \mu = 0.5 \) and \( \mu = 0.7 \)). The effect of the friction coefficient on the resulting stresses and strains was analysed and compared with experimental results. Two different normal loading cases are analysed (one case low enough to be within the plastic shake down limit and the other case, a very severe loading that could take the wear system beyond plastic shake down to ratchetting).

The flat surface of the equivalent cylindrical asperity, Fig. 4.11, is loaded in the normal direction by a pressure, P, of 69 MPa (for CASE I loading) on the top surface of the asperity resulting in a maximum Hertzian contact pressure, ‘\( p_0 \)’ of 2.068 GPa. Similarly to simulate a lower normal loading level that would result in plastic shakedown (below ratchetting level) the top surface of the truncated asperity was loaded by a pressure of P = 14 MPa. This loading results in a maximum Hertzian contact pressure, \( p_0 \) of 0.93 GPa.

| Table 4.1. Loading considered for the contact deformation model |
|---|---|---|---|---|---|
| Normal loading | Friction coefficient | CASE I | CASE IB | CASE IC | CASE ID | CASE IE |
| CASE I | \( p_0 = 2.068 \) GPa; \( p_0/k = 7.6 \) | CASE IIA | CASE IIB | CASE IIC | CASE IID | CASE IIE |
| CASE II | \( p_0 = 0.93 \) GPa; \( p_0/k = 3.4 \) | CASE IIA | CASE IIB | CASE IIC | CASE IID | CASE IIE |

These loadings are chosen to simulate plastic ratchetting (CASES IA – IE & CASES IIC - IIE) and plastic shake down (CASE IIA and IIB) in the half-space through reciprocating sliding of the asperity, see Fig. 3.2 and discussion in Section 3.2, Chapter 3. The factor ‘\( k \)’ is the shear yield strength of the material, usually taken as 0.5773 \( \sigma_y \) (Von-Mises criterion). The value of \( k \) for the present simulation works out to 273 MPa. It should be noted that for low friction (friction coefficient values less than 0.25), shakedown is controlled by subsurface stresses and the shakedown limit significantly exceeds the
elastic limit. For higher friction (friction coefficient values greater than 0.25) material yields at the surface and little protection is then provided by either residual stress or strain hardening. The two normal loading cases presented in Table 4.1, correspond to \( p_0/k = 7.6 \) (CASE I) and \( p_0/k = 3.4 \) (CASE II).

4.5 Finite Element Mesh for Asperity Contact

Under the plane strain assumption, the FE mesh of the present contact problem is illustrated in Fig. 4.14 and shown with the boundary conditions and applied pressure in Fig. 4.15. Fig. 4.16a – Fig. 4.16b show the close up view of the FE mesh in the contact region. The mesh for the elastic-plastic half space consists of 2010 four node quadrilateral plane-strain elements comprising of 2160 nodes (designated as ‘Plane 42’ in ANSYS). The truncated cylindrical asperity is modelled with 399 elements comprising of 450 nodes. A linear 2x2 gauss quadrature integration scheme was used [Zienkiewicz 1996]. The size of the elements near the surface on both the asperity and the half-space was kept as 7 \( \mu \text{m} \times 7 \mu \text{m} \) (see Fig. 4.16a). This element size is increased progressively outside the contact zone (of the order of 70 \( \mu \text{m} \) to 100 \( \mu \text{m} \)). Further, contact interface elements [ANSYS 1998] are used between the nodes of the elements forming the asperity and nodes of the elements forming the elastic-plastic half-space to detect contact between the surface of the half-space and the cylindrical asperity, see Fig. 4.16b). The lines connecting the nodes of the asperity and the elastic-plastic half-space forming contact elements are not shown in Fig. 4.16b.

The contact elements have the capability to allow the users of the program to track the position of the sliding asperity at any given time (i.e., during the load steps of indentation and sliding). This feature is also necessary so that the program can distinguish between the two distinct regions of the finite element model, namely, the hard cylindrical asperity and the half-space [ANSYS 5.4 1998]. The tracked nodes are identified as being in contact or separation by measuring the relative positions of the nodes pertaining to two surfaces in the normal direction to the surfaces. The relative positions of the tracked nodes of the asperity and the half-space result in quantities known as ‘gaps’ [ANSYS 5.4 1998]. If the gap is positive then it is assumed that contact is not established. If the gap is
negative, then it is assumed that contact has been established between the asperity and the half-plane and the contact pressure would be applied on the contact area.

A normal contact force is applied when the contact is established. This is derived from a quantity known as the normal stiffness, $K_N$. This is an input parameter for the contact problem. Its magnitude does not affect the material response due to the applied loading. The normal stiffness value, $K_N$, is multiplied by the ‘gap’ to calculate the restoring force required by the program to keep the meshes from overlapping. Immediately, an equal and opposite value of the normal force is calculated and applied on the nodes to maintain force balance in the element. This procedure is repeated for every element that comes into contact.

Since equilibrium is maintained due to the application of these forces, the applied load is not altered and is transferred to the elastic plastic half-space by these elements. If the normal stiffness value, $K_N$, were much lower than the necessary value for compatibility, then the meshes would overlap. If too high in value, the program would take more time per time step to converge. By trial and error for individual problems, a suitable value is identified. For the present problem, a normal stiffness value of $3.447 \times 10^{11}$ N/m is assigned to the contact elements to avoid penetration of either surface during loading.

The half space is constrained in the x- and y-directions on the bottom, the left and the right hand sides, Fig.4.15. The hard asperity is constrained against moving in the x-direction for the first load step, involving only normal loading. This restriction is removed for the subsequent sliding load steps. The boundaries of the half space are constrained so that the displacements would vanish on the constrained boundaries and stresses would vanish on the unconstrained boundaries. A non-linear static analysis is performed for each load step and the loaded asperity is translated across the top face of the half space.

The entire loading history is broken into several load steps and sub-steps to facilitate convergence of the solution. A “full Newton-Raphson method” (see ANSYS [1998] for explanation) is adopted for the solution until the “force convergence tolerance value” of 0.001 is met for each equilibrium iteration pertaining to each sub-step of every load step. Each load step is further divided into “sub-load-steps or sub-steps”. The first load step
that involves only the indentation of the asperity on the half-space is performed using 20 sub-steps, where only the normal loading is applied and the asperity is constrained so that there is no sliding movement in the x-direction (sliding direction), Fig. 4.17. The incremental increase of normal pressure to its peak value on the truncated asperity is referred to as the first indentation load step. Subsequently, keeping the normal load on the asperity, an incremental sliding of the hard asperity over the half space in the forward direction (+x –direction) is effected. This is referred to as a half-sliding load cycle. The total displacement applied over this load step is 0.3175 mm. Subsequently the sliding of asperity in reverse direction (-x –direction) is effected. A total displacement value of 0.635 mm is applied. This is referred to as another sliding half-cycle. Both the half-cycles constitute a full reciprocating sliding cycle. Two more reciprocating sliding cycles with a sliding length of 0.635 mm are subsequently effected. Fig. 4.17 illustrates the loading sequence schematically for the three reciprocating loading cycles for the asperity. Throughout this sliding, the normal load is kept on the asperity. At the end of the three reciprocating loaded sliding cycles the normal load is removed to allow elastic rebound of the lower half space to occur. The application of the sliding displacement is further broken into sub-steps for each of these sliding load cycles. Forty sub-steps per half-sliding cycle were required to obtain convergent solutions at every load step.

It should be noted that simulation of several reciprocating loading cycles is possible at the expense of computer time and the computer memory availability. From the finite element results, however, it was shown that three reciprocating cycles were enough to simulate and show the required plastic ratchetting phenomenon and closed loops of alternating strain cycles as the mechanical response of material elements. This information is presented in the next section for parametrically varying friction coefficients and normal loads. Subsequently, the equations for LCF and RF were used with the derived strain data and plastic work expended per cycle for predicting sliding wear.

4.6 Prediction of Wear Volume and Number of Cycles to Failure in LCF & RF

In order to produce a wear particle in the proposed configuration, simulation of cyclic
plastic deformation alone is not enough. From the knowledge of fatigue failure of ductile metals, it can be inferred that the number of cycles for fatigue failure can be related to the ranges of strain components. For LCF mechanism, a power law that relates the effective strain to the number of cycles leading to failure is developed. For ratchetting wear, the number of cycles to failure may be estimated by dividing a ‘critical value of strain’ (the strain to failure in monotonic tension for the given material) by the ratchetting strain per cycle. Low cycle fatigue wear and ratchetting wear are independent mechanisms for a particular location in the material under given loading and friction coefficient levels, say at element A1 in Fig.4.18. The input loading and interface friction coefficient are responsible for the given mechanism of failure at a particular location. The following section describes calculation of wear particle dimensions and the number of cycles needed to detach a particle from the parent surface.

4.6.1 Number of cycles for failure by Low Cycle Fatigue or Ratchetting failure

**LCF**

In this thesis the following predictive equation is used. The number of cycles to failure is related to the equivalent strain, $\varepsilon_{\text{eqv}}$, by using the relation:

$$ N_f = \left( \frac{2C}{\Delta \varepsilon_f} \right)^{1/n} $$

(4.19a)

where $\Delta \varepsilon_f = \Delta \varepsilon_{\text{eqv}}$ is the range of alternating effective plastic strain

$n$ is approximately 0.5 (derived from experiments Kapoor [1994]).

The factor $C$ is a strain related to the failure strain in static loading.

The component strains are used to calculate the effective or equivalent strain $\varepsilon_{\text{eqv}}$, which is given as follows [ANSYS 5.4 1998]

$$ \varepsilon_{\text{eqv}} = \frac{1}{\sqrt{2}} \left[ (\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2 + \frac{3}{2} (\varepsilon_{xy}^2 + \varepsilon_{yz}^2 + \varepsilon_{xz}^2) \right]^{0.5} $$

(4.19b)

Since for the plane strain case, the strains in the $z$-direction are zero, Eqn. (4.19b) reduces to
\[ \varepsilon_{eq} = \frac{1}{\sqrt{2}} \left[ 2\varepsilon_x^2 + 2\varepsilon_y^2 + 2\varepsilon_x\varepsilon_y + \frac{3}{2} \varepsilon_{xy}^2 \right]^{0.5} \]  

(4.19c)

The equivalent strain is often used for combining the component strains. This is a convenient mathematical quantity representing the combined effect of the axial and shear strains. Although it is difficult to attribute a physical meaning to it, the combination of several strain components to calculate the equivalent strain is widely used in literature for experimental and analytical studies of sliding wear [Kapoor 1994].

RF

Ratchetting failure (RF) will take place when the total accumulated strain reaches a critical value. The number of cycles to failure by ratchetting \( N_r \) is then given as

\[ N_r = \varepsilon_c / \Delta \varepsilon_c \]  

(4.20)

where \( \varepsilon_c \) is the strain at failure under monotonic loading,

\( \Delta \varepsilon_c \) is the maximum incremental equivalent strain observed through the cyclic loading. This is similar to \( \Delta \varepsilon_{nf} \) except that the material is past plastic shake down and is in steady state ratchetting because of the severity of the loading.

4.6.2 Prediction of Wear Volume in Sliding Wear

Having calculated the number of cycles to failure based on any one of the failure mechanisms explained in Section 4.6.1, the next procedure is to calculate the volume of wear during the cyclic loading. Wear volume is an important tribological parameter that could be compared with the experimental work involving standard wear tests. In this thesis, equations based on tangential work equivalence are derived for calculating wear volume through low cycle fatigue. For ratchetting failure, the depth to which the material ratchets is calculated from the finite element results. This depth is used with the sliding length to calculate the wear volume.

For the calculation of wear volume, \( V \), the total volume of material removed in \( N \) reciprocating cycles over a sliding distance of ‘1’ per reciprocating cycle, under normal load, \( P_n \), with friction coefficient \( \mu \), the total tangential work input to the wear system during sliding, \( W_t \), can be expressed as:
\[ W_E = \mu P_N Nl \]  

(4.21)

Only a fraction of this total work goes towards plastic deformation. Let this fraction be expressed as \( r \). The tangential work done on the system causing irreversible plastic deformation is then

\[ W_E = r \mu P_N Nl \]  

(4.22)

where \( r \) could easily be calculated from processing the finite element results. This is done by calculating the elastic energy \( \left( \int \sigma_y \varepsilon_y \right) \) for the entire finite element domain. By knowing the total energy and the elastic energy recovered, the energy for plastic deformation can be computed. This is an output quantity in ANSYS 5.4 [1998].

4.6.3 Low Cycle Fatigue (LCF)

The cumulative effective strain over \( N_f \) cycles to failure (described in the previous section) is expressed as

\[ \varepsilon_{cuf} = N_f \Delta \varepsilon_f \]  

(4.23)

Substituting for \( N_f \) from Eqn. (4.19) and setting \( 1/n = D \);

\[ \varepsilon_{cuf} = \left( \frac{2C}{\Delta \varepsilon_f} \right)^D \Delta \varepsilon_f \]  

(4.24)

Let \( k \) be the yield strength of the wearing half-space in shear. The plastic work (i.e. total work - recoverable elastic work) required to produce volume, \( V \), of worn material is then expressed as

\[ W_{LCF} = k \varepsilon_{cuf} V \]  

(4.25)

Substituting values for \( \varepsilon_{cuf} \) from Eqn (4.24), the required plastic work to produce unit volume of worn material (over \( N_f \), reciprocating sliding cycles) is expressed as

\[ W_{LCF} = k 2^{D-1} C^D \Delta \varepsilon_f^{1-D} V \]
The volume of wear can be obtained for $N_f$ cycles of loaded sliding as the ratio of the available work to the required work to produce the wear particle

$$V = \frac{\mu P_N N_l}{k^2 \alpha C^D \Delta \varepsilon_f^{1-D}}$$  

(4.27)

By rearranging Eqn (4.27), the following expression is obtained:

$$\frac{V}{P_N L} = \frac{r \mu}{k^2 \alpha C^D \Delta \varepsilon_f^{1-D}}$$  

(4.28)

$L = N_r \times 1$, (the number of reciprocating cycles of loading x the reciprocating sliding length).

The above Equation can be seen to obey the 'basic laws of wear. Comparing Eqn. (4.28) with the Archard's equation for wear ($V = K P_N L / 3H$) [Archard 1953], and substituting for $k = H / (3 \times 3^{1/2})$ as suggested by Tabor [1951], the following expression for Archard's coefficient is obtained.

$$K = \frac{9 \times 3^{1/2} r \mu}{2^D C^D \Delta \varepsilon_f^{1-D}}$$  

(4.29)

4.6.4 Ratchetting Failure (RF)

Wear volume through ratchetting failure is calculated by simply identifying the depth of the half space to which the elements ratchet. This depth is further used in calculations along with the number of calculated cycles to deduce the wear volume. The number of cycles to failure is calculated through Eqn. (4.20). The wear volume is the product of sliding length ($l$) * maximum depth of ratchetting ($d$) * plane-strain dimension ($l_0$).

4.7 Results and discussions from the analysis of Contact Deformation

Elastic-Plastic finite element solutions of the surface and subsurface stress and deformation fields resulting from indentation and sliding of the asperity on the half-space
at five different friction coefficient levels and two different normal pressure loading levels are presented and analysed in this section. The emphasis has been on the interpretation of results, which must reveal the significance of the normal load, the number of reciprocating load cycles, and friction coefficient level on the resulting deformation behaviour.

4.7.1 Applied contact pressure distribution

Fig 4.19a - Fig 4.19b show the variation of contact pressure as a function of load. During the first indentation load step, for CASE 1A loading, Fig 4.19a shows that the pressure profile deviates from the Hertzian profile. Fig 4.19b depicts the same for CASE IIA loading. The contact area increases with the incremental indentation load during the load step and the contact pressure decreases, i.e., the initial pressure distribution changes significantly with the onset of plastic deformation. This phenomenon significantly differs from analytical contact pressure calculations, using Hertz contact equations. In particular, increasing the load levels flattens the pressure distribution and the simultaneous increase in contact width. This is more significant for the higher normal loading case, (CASE IA) than for the lower normal loading level (CASE IIA). This is expected, since the higher normal load results in a larger area of conformity between the contact surfaces. These results are in agreement with previous plane-strain indentation analyses [Akyuz and Merwin 1968; Dumas and Baronet 1971; Komvopoulos 1989].

The following discussion along with Table 4.2 proves the aforementioned behaviour from the finite element results. To illustrate this, CASE 1A and CASE IIA are chosen. The last sub-step in the indentation load step is chosen for both cases, where the applied pressure on the truncated asperity is 69 MPa (CASE 1A) and 14 MPa for CASE IIA. This applied pressure, \( P \), is listed in the first column. The values of maximum Hertizian contact pressure, \( p_0 \), and contact width, \( 2a \) are derived from the results of the FE analysis. The appropriate values, shown in Table 4.2 are compared with data corresponding to ‘equivalent Hertzian profiles’ calculated elastically for the same load. The table shows that even for the lighter normal pressure of 14 MPa, there is a substantial difference
between the finite element and Hertzian profiles for an equivalent load. As the applied pressure increases to 69 MPa, the differences are significant.

Table 4.2 Comparison of results for contact width and contact pressure

<table>
<thead>
<tr>
<th>Applied pressure</th>
<th>Finite element results (Elastic-Plastic profile)</th>
<th>Results from Equivalent Hertzian profile (Elastic elliptic profile)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P, MPa</td>
<td>( p_0 ), GPa</td>
<td>( 2a ), ( \mu )m</td>
</tr>
<tr>
<td>69</td>
<td>0.81</td>
<td>120</td>
</tr>
<tr>
<td>14</td>
<td>0.57</td>
<td>40</td>
</tr>
</tbody>
</table>

From the second sliding cycle on, relatively uniform contact pressure distributions of much larger radius and lower maximum pressure result. This is due to the increased interfacial conformity resulting from the residual deformation of the half space. Kral [1993] found similar results in the elastic-plastic indentation analysis.

Several previous elastic-plastic analyses of indentation and sliding have been based on the assumed elastic elliptical contact pressure distribution [Kulkarni et al 1990 & 1991; Bhargava et al 1985a & 1985b]. In view of the foregoing and the results of previous analyses [Hardy et al 1971; Komvopoulos 1989], it is apparent that when plasticity effects are dominant, such assumptions may influence the accuracy of the solution. The use of contact elements for obtaining realistic solutions is therefore justified. Another advantage of using contact elements to calculate the applied contact force is that prior knowledge of contact pressure distributions is not needed for elastic-plastic contact problems.

Any analysis of repeated indentation and/or sliding of elastic-plastic material properties must therefore account for the variation in the pressure profile resulting from the increased conformity after plastic deformation. It should also be pointed out that for the cases of rolling or sliding under very light loads, the changes in the contact geometry may not be as critical as in indentation / sliding under heavy loads (\( p_0/k \sim 4.0 \) and above).

4.7.2 Elastic contact stresses

The finite element solutions for the elastic contact stresses (normalised with respect to peak Hertzian contact pressure, \( p_0 \)), at the corresponding sub-step, just before the yielding of the half-space, are plotted against the depth (normalised with respect to the semi-
contact width ‘a’, in Fig. 4.20 for case 1A. For case 1A, yielding starts at $p_0 = 0.2$ GPa.

In the same figure, three analytically obtained Hertzian contact stresses, $\sigma_{yy}/p_0 = -0.5; \sigma_{xx}/p_0 = -0.01; \sigma_y = 0.25$ for $y/a = 2.0$ are also plotted (see summarised equations in [Johnson 1985]). The FE solutions for elastic regime appear to be in good agreement with the Hertzian solutions.

The difference between the value of maximum elastic deformation in the global $y$-direction and the calculated Hertzian value is 0.96% which is quite reasonably accurate using the finite element program. The elastic solutions of stresses from the finite element analysis, which when compared to the Hertzian solutions in Fig.4.20 also validate the behaviour of contact elements. In view of the favourable comparisons with analytical results, it may be concluded that the finite element mesh and modelling assumptions are appropriate for simulating the indentation and subsequent sliding of a half-space by the hard asperity.

**4.7.3 Elastic-Plastic solutions for contact stresses and strain fields**

Fig. 4.21a shows the equivalent elastic-plastic stress contours (iso-stress lines) at the last sub step of the indentation load step, normalised with respect to von Mises equivalent stress at initial yield, i.e., when contact pressure $p_0 = 0.2$ GPa. It should be noted that the normal loading is still in effect on the asperity. Similarly Fig. 4.21b illustrates the contours of the same ratios for CASE IIA. Fig.4.22 illustrates the same for indentation with different values of interface friction.

The figures clearly indicate the evolution of the plastic deformation and its effect on the surface and subsurface elastic-plastic contact stresses. Contour values greater than ‘1.0’ represent the regions prone to subsequent yielding with the incremental loading. The information obtained, by comparing the two load cases of Fig. 4.21a and Fig. 4.21b, clearly indicates the effect of increase in normal loading on the yielding of the half-space, i.e. CASE 1A over CASE IIA.

By comparing the Fig. 4.21a (CASE 1A), with Fig.4.22a, the effect of friction coefficient on the plastic deformation during indentation process can be seen. Fig. 4.22b explains this phenomenon further, by showing the nature of the plastic zone extrusion towards the surface as the friction coefficient is increased. The increase in the friction
coefficient for the same normal loading level, increases the yielding of the material. It should be noted that the asperity is not allowed to slide. This effectively restricts the tangential movement of the nodes of the asperity. But the nodes of the elastic-plastic half-space can freely move in tangential direction. Hence the effective flattening of plastic zone results for higher loading and higher coefficient of friction.

The sub-surface maximum of 2.112 (ratio of elastic-plastic equivalent stress to von Mises equivalent stress at initial yield), Fig. 4.21a is attained for higher loading compared with a maximum value of 1.167 obtained in CASE IIA loading as depicted in Fig. 4.21b. Similarly, the increase in the friction coefficient from 0 to 0.3 increases the maximum to 3.522. These results are expected, in view of the realistic stress-strain response of the material by considering material hardening, as depicted in Fig.4.13. With regard to the perfectly plastic material assumption alone, this increase would not happen and the material would continue to yield to limit the elastic-plastic stress values to the yield strength of the material. This phenomenon could be simulated, however, due to the strain hardening nature of the adopted material model. This is one of the distinct advantages of adopting a rather simple finite element solution, which is closer to experimental simulations.

The development of a plastic zone for different frictional conditions under sliding loading are illustrated in Fig. 4.23 - Fig. 4.39 for CASE IA loading. These figures illustrate the total strain components of various parametric conditions through the cyclic loading (reciprocating sliding). Fig. 4.23 - 4.27 show the effect of the friction coefficient on the plastic deformation of the half-space at the end of the second load step (see Fig. 4.17). It is seen that the increase in the friction coefficient increases the plastic deformation of the half-space, Fig. 4.23 - Fig. 4.27.

In the aforementioned set of figures
(a) pertains to the contours of the strain component in x-direction, $\varepsilon_x$.
(b) pertains to the contours of the strain component in y-direction.
(c) depicts the shear strain contours and
(d) represents the contours of a calculated “equivalent strain”.

By comparing (a) through Fig. 4.23- Fig. 4.27, it can be observed that the axial strain
in x-direction, $\varepsilon_{xx}$ increases with the increase in the coefficient of friction. i.e. the surface maximum ("red coloured contours" of axial strain increases from 0.027 for $\mu = 0$, (see, Fig. 4.23(a)) to 0.1 for $\mu = 0.7$, (see, Fig. 4.27(a)). For intermediate friction coefficients, (see Fig. 4.23a through Fig. 4.27a), the increase is clearly seen through the zone of maximum strain contours. Also the maximum is identified as 'Mx' in the figures. It is also of interest to see the formation of a 'bulge' or 'wave' in front of the sliding asperity. The nature of this type of surface deformation is clearly seen in the experimental works of Magel [1990]. The axial strain, $\varepsilon_x$ is tensile in the trailing edge of the slider whereas it assumes a maximum compressive (most negative) value identified as 'Mn' in Fig. 4.23a - Fig. 4.27a and through the "blue coloured zone".

By comparing figures Fig.4.23b - Fig. 4.27b, it can be observed that the maximum shown in red attains a value of 0.019 for $\mu = 0.0$. It increases with the increase in the friction coefficient (seen by comparing the red zones through the set of figures Fig. 4.23b - Fig. 4.27b). For the friction coefficient level of 0.7, it reaches a maximum value of 0.188. The maximum occurs at the leading edge of the asperity unlike the axial strain in x-direction, where the maximum occurred at the trailing edge. The wave formation, is the same as shown in (a) set of these figures.

Fig. 4.23(c) to Fig.4.27(c), illustrate contours depicting the distribution of shear strain. The shear strain (in plane) is seen to increase from 0.03 for $\mu = 0$ (Fig. 3.23 (c)) to 0.688 (a rather high value compared to the axial strains due to the high coefficient of friction; Fig. 4.27(c)). The accumulation of shear strain is predominant with high friction coefficient values. Magel [1990], in his studies obtained photomicrographs of sectioned wear specimens subjected to similar loading conditions, depicting similar nature of shear strain accumulation, Fig.3.7. The equivalent strain is expressed in Fig. 4.23(d) - Fig. 4.27(d). Comparing set (d) figures could see the effect of the friction coefficient on the increase of equivalent strain.

As expected, since it is a function of strain components, the equivalent strain increases with the increase in friction coefficient levels. The wave that formed ahead of the sliding asperity is seen in experimental studies using talysurf mapping of the worn surface [Knowles 1994; Magel 1990 and Chen 1994]. Also Challen and Oxley [1986] have
simulated similar surface plastic deformation in their analysis of wear problems. Black et al [1988], through experimental work, have demonstrated the same kind of surface plastic deformation seen through the present finite element studies. Lee and Kobayashi [1970] also observed similar nature of plastic zone development in their plane strain and axisymmetric analysis.

The present work incorporates the realistic strain hardening nature of the material and simulates the same kind of plastic deformation seen through experimentation. In addition, the strain component values are calculated for the cyclic loading. These strains are in line with the values obtained through experimental simulations of the aforementioned researchers.

The following sections describe the cyclic loading effect on the strain components. To this end, the strain contours at the end of third load step (see, Fig. 4.17) and the 7th and 8th load step are illustrated and discussed as below.

Fig. 4.28 (a-d) - Fig. 4.32(a-d) depict the values of strains, when the asperity completes one full reciprocating sliding, i.e., third load step. The strain components are seen to increase with the friction coefficients by comparing (a), (b), (c) and (d) of the set of figures that pertains to the third load step. The trend remains the same as discussed for the forward sliding. The value for $\varepsilon_x$ reaches a maximum of 0.1 at the trailing edge of the slider for $\mu = 0.7$. $\varepsilon_y$ reaches a maximum of 0.189 at the leading edge of the sliding asperity (see Fig. 4.32 (a-d)).

Fig. 4.33(a-d) - Fig. 4.37 (a-d), at the end of three reciprocating load cycles, indicate very similar increases in the values of the strain components during the cyclic loading. In all the cases accumulation of individual strain components through cyclic loading are evident. These figures indicate the plastic deformation of the surface seen through the controlled experimentation of wear involving several cycles of sliding. The wave formation at the edges of the sliding length is a typical phenomenon seen in experimentation [Magel 1990]. With the increase in the value of coefficient of friction, the height of the wave above surface level increases. The wave formation and its variance, with respect to the parametric variables such as the friction coefficient, are in line with the results reported by Challen and Oxley [1979 &1986] using their slip-line...
field solutions. During sliding, the wave or bulge formed ahead of the slider is pushed forward until the end of sliding. The subsurface / surface plastic zone also traverses this path at a near surface depth of 5 μm - 10 μm during the forward sliding (second load step). It subsequently reaches the surface and free surface plastic deformation results. For CASE I, the wave of material that formed ahead of the sliding asperity grows in size as sliding proceeds. After three cycles, when the normal load is lifted to simulate elastic recovery, the height of the wave above the surface reference level reaches a peak value.

Fig.4.38 (a-d) - Fig. 4.39 (a-d) indicate the residual strain system after the removal of the applied normal load (i.e., the applied pressure of 69 MPa for CASE IA loading) for two different friction coefficient values. By analysing the contours, through cyclic loading, the accumulation of strain can be seen at any point of the half-space close to the surface.

So far all of the figures depicting the strain contours were shown only for the CASE I type loading level. CASE II loading levels produced very similar plastic deformations but of a lesser magnitude. For reasons of brevity in presenting the figures for describing the nature of variation of strain components, figures depicting the strain components pertaining to the CASE II loading levels were not presented along with CASE I. The strain values tracked throughout the loading history for CASE II, however, are available for the sliding wear analysis. For friction coefficient levels of 0.1 and 0.0 (frictionless), for CASE II loading, the wearing material is not seen to accumulate strain components, but is seen to reach a state of plastic shake down. This is expected, in view of the discussions pertaining to PSL and RF in Section 3.2, Chapter 3. The tracked strain components for CASE II loading are presented in the next few paragraphs along with relevant discussions.

4.7.4 Tracking of strain components through cyclic loading

In section 4.7.3, it was seen that the sub-surface and surface of the half space yield considerably due to the sliding of the asperity on the surface. It is of interest to study the effect of this sliding at specific locations during the cyclic loading. For this purpose, specific locations are chosen as illustrated in Fig.4.18. The strain components for these locations are tracked through the loading cycles. These locations are positioned at one of
the extreme ends of the sliding length (elements A1 - A2). These positions are chosen because from Figs 4.23 - 4.39, it was seen that the maximum values of strains (all strain components) occur at or near the edge of the sliding length for all loading conditions. Since tracking every single element along the wear track at a given depth is not feasible, these elements are taken as representative elements for a given depth in the half space.

Once the failure by RF or LCF occurs in these elements, then the component of interest is a “worn component”. Subsequent analysis with the initial configuration is not of concern as the wear would have already altered the geometry. This is true even in experimentation, where worn out specimens, after a prescribed number of loading cycles are analysed and not re-used. Any failure criterion, evaluated on the basis of the strains obtained through these elements would therefore be the bounding case for the entire wear track. The nature of cyclic strain of a surface element needs to be shown as either an open cycle, thus leading to ratchetting, or a closed cycle, leading to low cycle fatigue. The strain cycles shown in Fig. 4.40 - Fig. 4.44 are bi-axial and lead to accumulation of plastic strain in the material. To illustrate this phenomenon further, consider a case where the strain cycle response of the material to the applied loading cycle consists of

- accumulating axial strain in the x-direction of magnitude $\Delta \varepsilon_x$ per strain cycle.
- accumulating shear strain of magnitude $\Delta \varepsilon_{xy}$ per strain cycle.
- accumulating axial strain in the y-direction of magnitude $\Delta \varepsilon_y$ per strain cycle.

Because of plane-strain assumption, the strains, $\Delta \varepsilon_z$, $\Delta \varepsilon_{xz}$ and $\Delta \varepsilon_{xy}$ are zero.

The effective or equivalent strain is determined from the strain tensor for each element. The variation of the component strains and the equivalent strain through the cyclic loading for two representative elements, (elements A1 and A2 described in Fig. 4.18) are plotted in Fig. 4.40(a-d) - Fig. 4.44 (a-d).

In Fig 4.40(a-d) - 4.44(a-d), the figures represent time in x-axis and strain in y-axis. The quantity ‘time’ is a convenient quantity used to represent the load steps from the indentation load step through to the sliding load steps, i.e. the indentation load step occurs between time = 0.0 to 1.0 and the forward sliding load step occurs between time = 1.0 and 2.0 etc. This terminology is commonly used in solving non-linear structural problems.
Fig. 4.40(a) depicts the accumulation of $\varepsilon_x$ for the surface element Al throughout the loading history for CASE IC. (b) refers to $\varepsilon_y$. (c) refers to $\varepsilon_{xy}$ and (d) refers to $\varepsilon_{eqv}$. Points marked as 1, 2, 3, etc. refers to the end points of the load steps. The first reciprocating cycle occurs between 2-3. The second reciprocating cycle occurs between 4-5. The third reciprocating cycle occurs between 6-7. Referring to (a), it is seen that during the indentation load step, up to the point marked as ‘1’ which is happening at half the sliding distance away, the surface element experiences a mild compressive strain in the x-direction. This is due to the nodal movements that push this element toward surrounding elements creating an axial compression in the x-direction. During 1-2, the asperity moves away in the forward direction having negligible effect on the element. Between 2 & 3, the asperity approaches this element, travels over it slightly and stops at the end of the load step to reverse the direction. This load reversal that happens at the edge of the sliding length, induces very high strains and the response strain cycle could be seen from Fig. 4.48a for the axial component. Between points 3 and 4 the asperity moves away. But during 4-5 and 6-7 the element experiences two strain cycles.

Fig. 4.40(a-d) represents the ratchetting of CASE IC. All of these four figures represent the cyclic nature of the strain response of the element. The applied load cycle produces very severe strains on the element when it reverses the direction and moves away after the forward sliding. The accumulation of strains differs between the second cycle and the first and third cycle. This could be explained by the ‘little’ protective effect of the residual stress system induced during the first cycle. This residual system counteracts against the applied load and attempts shake down, but the applied loading during the third cycle is high enough to overcome this and the ratchetting continues along the same lines seen in the first cycle.

Fig. 4.41(a-d) illustrates the closed loop of no accumulation of strains for CASE IE loading. This is the bounding case that identifies the maximum depth to which ratchetting takes place. During the post-processing phase of the analysis, no strain
accumulation is seen either in this element or below the element A2 for CASE IE. Fig. 4.42(a-d) depicts the ratchetting of element A1 in CASE IIC loading. Referring to Fig. 4.42(a-d), the aforementioned discussions apply to this case as well.

Fig. 4.43 (a-d) illustrates no ratchetting of even the surface element for CASE II loading where closed loops of strain cycles form. This signifies the plastic shakedown condition (see Fig. 3.2, Chapter 3). This particular situation warrants the use of LCF for wear. Fig. 4.43 presents the bounding case for CASE II loading for ratchetting. For friction coefficient levels $>0.1$, for CASE II loading, ratchetting was the governing mechanism. Only for $\mu = 0.0$ and $0.1$ the strain cycle is one of closed loop and there is no accumulation of strain seen for these two cases. The increase in the axial and the shear components of the strains for different coefficients of friction and normal loading levels are thus clearly evident from the component strains tracked through the loading cycles.

Bower and Johnson [1989] have investigated the plastic deformation which occurs when a hard cylinder rolls and slides over an elastic-plastic surface under a load which exceeds the elastic limit with a coefficient of sliding friction $\mu > 0.25$. Under these rather severe conditions a thin near surface layer becomes plastic.

The present research, however, had an additional complication to the perfectly plastic material that was assumed and analysed by Bower and Johnson [1989], namely the strain hardening effect of the half-space. The shear strain is also an open cycle, and found to be increasing with the number of cycles of loading. In addition, the accumulation is different at different depths of the half-space, i.e., an element, A2 just below A1 accumulates different amounts of strain per cycle than A1. These reasons necessitate the plots outlined in Figs. 4.40-4.44.

The figures clearly distinguish and warrant different types of failure mechanisms based on the loading level and the friction coefficient levels. This can be seen in the light of the shakedown map shown in Fig. 3.2, Chapter 3. The map shows that for all loading values of $p_0/k = 7.6$ greater surface plastic deformation results for all friction coefficient levels. For $p_0/k = 4.0$, the map indicates that only for friction coefficient levels greater than or equal to 0.3, surface plastic deformation would result but could be protected by the residual stress system developed within the first two cycles. The present research
shows that the developed residual system may not counteract the applied loading to the extent expected using such simple analytical models. For the present research, the surface ratchetting continues for friction coefficients greater than 0.1 even for CASE II loading.

The figures also give a clear picture of the depth to which the surface plastic deformation and plastic ratchetting would affect. This could be seen in light of the figures depicting no ratchetting for sub-surface elements, A2. The following section deals with the calculation of the number of cycles to failure using ratchetting failure and low cycle fatigue mechanisms.

4.8 Prediction of Wear Volume and Number of Cycles to Failure in RF and LCF

Eqn. (4.19a) is used for predicting the number of cycles to failure in LCF mode for CASE IIA and CASE IIB. Eqn. 4.20 is used for predicting failure in RF mode for CASE IA - CASE IE & CASE IIC – CASE IIE. The wear volume for LCF mode is calculated using Eqn.(4.27). For RF, the wear volume is predicted as explained in Section 4.6.4. The results are summarised in Table 4.3. The actual comparison with experimental work is described in Chapter 8.

Table. 4.3 Predicted Volume and Number of cycles to Failure

<table>
<thead>
<tr>
<th>CASES</th>
<th>Applied Pr., P, MPa</th>
<th>Contact Pressure, $p_0$, GPa</th>
<th>Friction Coefficient</th>
<th>Mechanism of wear</th>
<th>$\Delta \varepsilon_f$ or $\Delta \varepsilon_i$</th>
<th>Number of cycles to failure</th>
<th>Wear Volume, mm$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CASE IA</td>
<td>69</td>
<td>2.068</td>
<td>0.0</td>
<td>Ratchetting</td>
<td>0.01639</td>
<td>76</td>
<td>N/A</td>
</tr>
<tr>
<td>CASE IB</td>
<td>69</td>
<td>2.068</td>
<td>0.1</td>
<td>Ratchetting</td>
<td>0.054</td>
<td>23</td>
<td>0.226</td>
</tr>
<tr>
<td>CASE IC</td>
<td>69</td>
<td>2.068</td>
<td>0.3</td>
<td>Ratchetting</td>
<td>0.1101</td>
<td>11</td>
<td>0.226</td>
</tr>
<tr>
<td>CASE ID</td>
<td>69</td>
<td>2.068</td>
<td>0.5</td>
<td>Ratchetting</td>
<td>0.1454</td>
<td>9</td>
<td>0.226</td>
</tr>
<tr>
<td>CASE IE</td>
<td>69</td>
<td>2.068</td>
<td>0.7</td>
<td>Ratchetting</td>
<td>0.21</td>
<td>6</td>
<td>0.226</td>
</tr>
<tr>
<td>CASE IIA</td>
<td>14</td>
<td>0.93</td>
<td>0.0</td>
<td>Low cycle fatigue</td>
<td>0.001</td>
<td>810000</td>
<td>N/A</td>
</tr>
<tr>
<td>CASE IIB</td>
<td>14</td>
<td>0.93</td>
<td>0.1</td>
<td>Low cycle fatigue</td>
<td>0.00197</td>
<td>673100</td>
<td>0.026</td>
</tr>
<tr>
<td>CASE IIC</td>
<td>14</td>
<td>0.93</td>
<td>0.3</td>
<td>Ratchetting</td>
<td>0.001869</td>
<td>669</td>
<td>0.1</td>
</tr>
<tr>
<td>CASE IID</td>
<td>14</td>
<td>0.93</td>
<td>0.5</td>
<td>Ratchetting</td>
<td>0.0352</td>
<td>355</td>
<td>0.1</td>
</tr>
<tr>
<td>CASE IIE</td>
<td>14</td>
<td>0.93</td>
<td>0.7</td>
<td>Ratchetting</td>
<td>0.02688</td>
<td>46</td>
<td>0.1</td>
</tr>
</tbody>
</table>

4.9 Importance of Finite Element Analysis for Sliding Wear

The significant differences through FE analysis to the existing knowledge in the field of contact deformation modelling and wear volume prediction are:
• Surface statistics is used with actual experimental observations to arrive at an accurate representation of the surfaces in contact. The information regarding the number of asperities are calculated in this manner (Section 4.2-4.3) are further used with FE analysis (Section 4.4) to predict the material response quantities such as stresses and strains due to contact loading.

• The detailed FE simulation of the sliding takes into account the complete stress-strain history of loading and the residual stresses on completing every load cycle. (i.e., the first load step induces residual stresses into the half-space, which is considered along with the next load step of sliding). This could be considered as an improvement on the methodology adopted by Challen and Oxley [1979] for the calculation of stresses and strains and subsequently the wear particle detachment. The important contribution of the elastic stresses and strains are included in the present analysis whereas slip-line theory adopts a rigid plastic material ignoring the loading history and residual stress effect.

• The simulation of strain cycles through the FE analysis predicted results for volume and number of cycles that are close to the experimental observations [Kapoor 1994; Kapoor and Johnson 1994; Challen and Oxley 1979]. A detailed comparison is presented in Chapter 8.

• The parametric study conducted through the FE analysis in Section 4.7 clearly illustrated the combined effect of normal loading and the friction coefficient on the cycles of strain. The mechanisms of wear (low cycle fatigue and ratchetting failure) discussed in [Kapoor 1994] and Johnson and Kapoor [1994] are simulated successfully through cyclic loading.

4.10 Summary

Surface statistics is used to quantify two engineering surfaces in contact. The equivalent representation consisted of a cylindrical asperity in contact with a smooth half-plane. The half-space was given elastic-plastic material properties whereas the asperity was given elastic material properties to simulate yielding of only the half space. The cylindrical asperity was further loaded in the normal direction and was subjected to
reciprocating sliding. Various levels of interface friction coefficients were used to study its effect on the resulting deformation. A materially and geometrically non-linear finite element formulation using the updated Lagrangian method was used for the solution process. The resulting stresses and strains were plotted to analyse the behaviour of the elastic-plastic half space under reciprocating sliding. The strains were tracked through the loaded sliding to calculate the accumulated effective strain over the cyclic loading. The elastic solutions compare favourably with classical solutions. The need for FE analysis in contact problems is highlighted. The elastic-plastic solutions are presented systematically. The resulting values were used in two different predictive wear mechanisms, namely Low Cycle Fatigue (LCF) and Ratchetting Failure (RF) to simulate sliding wear. Actual comparisons with experimental results are detailed in Chapter 8.
CHAPTER 5

Prediction of Mild Wear (MW) under Wet Sliding

5.1 Preliminary discussion

When wet sliding occurs at elastic shakedown regime, see, Fig. 3.12, ‘mild abrasive wear’ could be the active sliding wear mechanism (see discussions in Chapter 3). The worn surfaces appear to have been polished with many fine scratch marks in the direction of the sliding (see Section 3.3, Chapter 3). Since the mild wear happens in elastic shakedown conditions, LCF and RF are not involved. The low friction coefficient level under wet sliding conditions eliminates the CGPD mechanism since the resulting SIF is below the threshold value for any appreciable crack growth to occur. A new wear model that is different from the LCF, RF and CGPD should therefore be considered. Presently, a model based on the equivalence of tangential work input into the wear system is proposed. This tangential work is equated to the energy required for the material to fail, based on the ultimate shear strength of the material, to calculate the depth of a wear groove.

5.2 Determination of abrasive groove depth ($z$)

In the present approach a formulation based on the energy equivalence is considered. Let $E_w$ be the specific energy, i.e., the energy required to produce a wear groove of unit volume, and expressed as (Work done = force x distance):

$$E_w = \sigma_{ult \_shear} \times area \times length$$

(5.1)

where $\sigma_{ult \_shear}$ is the ultimate strength of the material evaluated in shear.

Since the above equation deals with unit area and unit sliding length it can be written as

$$E_w = (\sigma_{ult \_shear})$$

(5.2)

It is assumed that the wear groove has a cross-sectional area, $A$, that is semi-circular in
shape and whose radius is equal to the depth ‘z’ of the wear groove. The volume of the half-cylindrical wear groove can thus be expressed as

\[ V = \frac{\pi z^2}{2} (l) \]

(5.3)

Next, let \( W \) be the frictional work expended by the sliding asperity contacts, i.e., as \( q(x) \) translates over the contact surface it performs an amount of frictional work that equals \( W_T \). It is further assumed that \( q(x) \) does not change with the translation as the contact area is very small for asperity contacts. The real contact area due to a loaded single asperity is \( \pi a^2 \), where ‘\( a \)’ is the radius of the circular contact area given as [Johnson 1985]

\[ a = \sqrt[3]{\frac{3P_N R_{sph}}{4E^*}} \]

(5.4)

\( P_N \) is the normal load
\( R_{sph} \) is the radius of curvature of the asperity and
\( E^* \) is the equivalent modulus of elasticity for the contacting components.

During MW, it is recognised that the deformation is elastic-plastic in nature, The observed plasticity on the surface, however, is minimal in the case of MW. The end results for wear volume and wear groove depth, also appear to be closer to the experimental conditions using the elastic relationship than using purely plastic deformation for the first few cycles of loading. The area of contact is established, therefore, assuming an elastic contact conditions rather than purely plastic flow conditions in Eqn 5.4.

Let the average value of the tangential stress distribution be represented as \( q_{0-avg} \). The average load distribution for a single asperity can be expressed as

\[ f_{0-ave} = (\pi * a^2) * q_{0-avg} \]

(5.5)

The amount of frictional work imparted to the opposite surface by a single asperity in one cycle can be expressed as
\[ W_T = (\pi a^2) * q_{0-\text{avg}} * \ell \]

(5.6)

For \( N \) cycles the total sliding length \( 'L' \) can be expressed as the product of \( N \) and \( 'l' \), i.e.,

\[ L = N \times l \]

(5.7)

Substituting the sliding length value for \( N \) cycles from Eqn. (5.7) into (5.6), the work done in \( N \) cycles can be obtained as follows

\[ W_T = (\pi a^2) * (q_{0-\text{avg}}) * (N) * (l) \]

(5.8)

Equating the work necessary to produce a groove with volume \( V \) to the energy expended for this process, the following energy balance equation can be written

\[ (E_w)(V) = W_T \]

(5.9)

Substituting the values for \( E_w, V, \) and \( W \) into the above equation, the following expression is developed

\[ (\sigma_{sh - \text{shear}}) * \left( \frac{\pi a^2}{2} \right) * (l) = (\pi a^2) * (q_{0-\text{avg}}) * (N) * (l) \]

(5.10a)

In the energy balance equation, it is assumed that all of the input energy goes toward forming the abrasive groove, and hence the wear volume. In reality this is not the case. Only a fraction of energy goes towards forming the groove. This fraction can be expressed as \( \phi \). This factor has to be incorporated into the applied energy to the wear system. This factor can be easily calculated from Fig. 3.12, the circular contact shakedown map. Since the applied loading \('p_0/k'\) is very close to the elastic shakedown limit for the range of traction coefficients \((0.1 \sim 0.2)\) in wet sliding, (e.g., for a traction coefficient of \(0.1 \sim 0.2\), the peak contact pressure / shear yield strength of the chosen material is \((1539/525 = 2.93)\). This is in excess of the shakedown factor, \( 'p_{0s}/k' \) for a traction coefficient \( \mu = 0.2 \), by \((2.93 - 2.875) / 2.9 = 0.02\). Thus, the difference between
the applied \( p_0/k \) and the elastic shakedown \( p_0 s/k \) is the amount of load available for further plastic deformation / wear groove formation. This non-dimensional ‘% difference’ is expressed as ‘\( \phi \)’ and incorporated into the equation.

\[
(\frac{\sigma_{ult \_shear}}{2})^* (l) = (\pi a^2)^* (q_{0 \_avg})^* (N)^* (l)^* (\phi)
\]

(5.10b)

where the non-dimensional fractional energy \( \phi = \frac{P_{0 \_P}}{P_{0 \_S}} \frac{k}{k} \)

Re-arranging Eqn. 4.28b, the depth of the wear groove, \( z \), can be expressed as

\[
z = \sqrt{\frac{2a^2 q_{0 \_avg} N \phi}{\sigma_{ult \_shear}}}
\]

(5.11)

The relationship also satisfies the dimensionality of the thickness on the L.H.S. of Eqn. (5.11), and can be viewed as a product of material strength and loading conditions. The abrasive groove depth is evaluated for the first cycle. Subsequently the contact pressure is calculated. If the contact pressure is above the elastic shakedown limit for circular contact, again a new set of values for ‘\( \phi \)’, \( p_0 \) are evaluated and the calculation is repeated for the subsequent ‘\( z \)’. If the contact pressure falls below the elastic shakedown limit, owing to the increase in contact area due to the formation of the groove and the conformal contact in the first cycle, then only elastic deformation results. It will be shown in Section 5.3 that this is the case for the range of loading and friction coefficients analysed in the present thesis. The evaluated value of ‘\( z \)’ is the limiting value and any subsequent sliding only results in elastic deformation.

### 5.3 Results from Mild Wear Model, (\( \mu = 0.1 \) to 0.2)

The input parameters to the MW model are chosen so that they are consistent with the experimentation explained in Chapter 7 for comparison purposes. A 10 mm (diameter) cylinder, which is made to slide against the 19 mm dia x 5 mm thick disc, under wet
surface conditions. A weak solution of cutting fluid and water (1:1000) is used to keep the surfaces wet. In the MW model two values of $\mu$, first 0.1 and then 0.2 are considered.

The nominal semi-contact width derived is 74 $\mu$m. The number of asperities in contact between the cylindrical component and the disc is 38 for 16 N/mm applied loading on a cylindrical component of length 12.7 mm. The asperities in the wet sliding model are modelled as hemispheres that form circular contact areas. The asperities are of radius $r = 1$ mm. The radius of the contact circle, based on the real area of contact is $a = 3.298 \times 10^{-5}$ m (vide Eqn. 5.4). The normal contact stress is calculated to be 1539 MPa. The fraction of energy factor $\phi$ is calculated to be 0.01.

The depth of the mild wear groove is further evaluated using Eqn. 5.11. The tangential stress, tangential traction and the depth of the groove are all detailed in Tables 5.1 and 5.2. Table 5.1 details the calculation involved in a spreadsheet format and the results are summarised in Table 5.2. The increase in the depth of the wear groove is observed for increase in several normal loading cases when the equation to predict the depth of mild wear groove is applied. The elastic shake down sets in immediately after the first cycle. It is observed that after one cycle, the contact pressure drops below the elastic shakedown limit due to conformal contact. No more grooving takes place, therefore, due to subsequent cycling. For the mild wear case, this could be the reason why no appreciable mass loss was seen experimentally. The depth of the predicted groove along with the total volume pertaining to 38 grooves and the mass loss pertaining to these in the very first cycle of ‘run in’ are given in the following table. Subsequent cycles produce only elastic deformation. This observation is significant since no appreciable mass loss was seen even after 720 cycles in experimentation (Chapter 7).
Table 5.1 Calculation for the mild wear groove depth 'z' and wear volume

<table>
<thead>
<tr>
<th>Calculation for the mild wear groove depth</th>
<th>200 N load</th>
<th>250 N load</th>
<th>300N load</th>
<th>200 N load</th>
<th>250 N load</th>
<th>300N load</th>
</tr>
</thead>
<tbody>
<tr>
<td>equivalent E, psi</td>
<td>15953589.56</td>
<td>15953589.56</td>
<td>15953589.56</td>
<td>15953589.56</td>
<td>15953589.56</td>
<td>15953589.56</td>
</tr>
<tr>
<td>equivalent R, inches</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>number of asperities in half inch of cylindrical component</td>
<td>38</td>
<td>38</td>
<td>38</td>
<td>38</td>
<td>38</td>
<td>38</td>
</tr>
<tr>
<td>Normal load/asperity, lbf</td>
<td>1.2</td>
<td>1.5</td>
<td>1.8</td>
<td>1.2</td>
<td>1.5</td>
<td>1.8</td>
</tr>
<tr>
<td>coefficient of friction</td>
<td>mu=0.1</td>
<td>mu=0.1</td>
<td>mu=0.1</td>
<td>mu=0.2</td>
<td>mu=0.2</td>
<td>mu=0.2</td>
</tr>
<tr>
<td>semi contact width based on spherical body contacting a flat plane</td>
<td>0.001299</td>
<td>0.001401</td>
<td>0.001488</td>
<td>0.001299</td>
<td>0.001401</td>
<td>0.001488</td>
</tr>
<tr>
<td>a, the semi-contact width in inches</td>
<td>33.0</td>
<td>35.6</td>
<td>37.8</td>
<td>33.0</td>
<td>35.6</td>
<td>37.8</td>
</tr>
<tr>
<td></td>
<td>3.30022E-05</td>
<td>3.6E-05</td>
<td>4E-05</td>
<td>3.3E-05</td>
<td>3.6E-05</td>
<td>3.8E-05</td>
</tr>
<tr>
<td>in microns</td>
<td>in meters</td>
<td>in microns</td>
<td>in meters</td>
<td>in microns</td>
<td>in microns</td>
<td>in microns</td>
</tr>
</tbody>
</table>

To calculate the average contact pressure

| load per asperity, lbf                  | 1.2         | 1.5         | 1.8         | 1.2         | 1.5         | 1.8         |
| contact area, circular, in sq.in        | 5.3E-06     | 6.2E-06     | 7.0E-06     | 5.30595E-06 | 6.17E-06    | 6.95E-06    |
| contact area, circular, in sq.m         | 3.4E-09     | 3.98E-09    | 4.49E-09    | 3.42E-09    | 3.98E-09    | 4.49E-09    |
| average contact pressure, p_avg, psi    | 223185.4496 | 240103.4597 | 255151.1002 | 223185.4496 | 240103.4597 | 255151.1002 |
| p_avg in ksl                            | 223.19      | 240.10      | 255.15      | 223.19      | 240.10      | 255.15      |
| p_avg in MPa                            | 1538.9      | 1655.5      | 1759.3      | 1538.9      | 1655.5      | 1759.3      |

z, the groove depth for the first cycle of sliding, (Eqn.5.11), in meters

| volume after first cycle, m^3            | 2.5         | 4.7         | 5.9         | 3.6         | 6.7         | 8.4         |
| volume after first cycle, mm^3           | 1.0E-14     | 3.5E-14     | 5.5E-14     | 2.0E-14     | 7.0E-14     | 1.10E-13    |
| volume after first cycle, mm^3 3 for 38 grooves | 1.0E-05     | 3.5E-05     | 5.51E-05    | 2.01E-05    | 7.02E-05    | 0.00011     |
| Subsequent area of contact after 1 cycle of loading | 0.00038     | 0.00133     | 0.0021      | 0.00076     | 0.0027      | 0.0042      |
| A sq. in                                | 2.46276E-05 | 4.6074E-05  | 5.77043E-05 | 3.48287E-05 | 6.51585E-05 | 8.16062E-05 |
| A sq. m                                 | 1.6E-08     | 2.97E-08    | 3.7E-08     | 2.2E-08     | 4.2E-08     | 5.26E-08    |
| po, MPa                                 | 331.3       | 221.3       | 212.1       | 234.2       | 156.5       | 150.0       |
Table 5.2 Wear volume/cycle predicted for two different friction coefficient levels

Normal load (16 N/mm resulting in 400 N on a cylinder of axial length 1”).

<table>
<thead>
<tr>
<th></th>
<th>( P_n = 200 \text{ N} )</th>
<th>( P_n = 250 \text{ N} )</th>
<th>( P_n = 300 \text{ N} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu = 0.1 )</td>
<td>2.5</td>
<td>4.7</td>
<td>5.9</td>
</tr>
<tr>
<td>( \mu = 0.2 )</td>
<td>3.6</td>
<td>6.7</td>
<td>8.4</td>
</tr>
<tr>
<td>Cylindrical groove / 1 cycle, mm(^3)</td>
<td>1e-05</td>
<td>3.5e-05</td>
<td>5.5e-05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2e-05</td>
<td>7.02e-05</td>
</tr>
<tr>
<td>Total volume of wear/cycle, mm(^3) in N(_s) cycles (for 38 wear grooves)</td>
<td>0.00038</td>
<td>0.0013</td>
<td>0.0021</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0008</td>
</tr>
</tbody>
</table>

5.4 Discussion of Results

Under MW conditions, Fig. 3.2, the residual stresses induced during the first cycle of loading counteract in the subsequent loading, causing the system shakedown to an elastic state in addition to the conformable contact. The abrasive groove that occurs during the first cycle is by plastic deformation in the micro-scale and the groove is the resultant of both material removal and material displacement. Once the groove is formed, conformable contact is established between the asperity and the contact area. Consequently, the peak Hertzian contact pressure falls to 104 MPa from 1538 MPa for the first cycle of loading (for 200N load; \( \mu = 0.1 \)), Table 5.1. This is well below the elastic limit for the material. As a result, no further plastic deformation is possible. The same trend is seen for different loading conditions and friction coefficient levels in Table 5.1.

The factor \( \phi \) plays the role of accounting for the energy required for elastic deformation. Since most of the energy goes towards the elastic deformation, the resultant volume is very small. The same trend is observed experimentally for mild wear conditions (Chapter 7). Comparison between the experimental work and the numerical simulation is given in Chapter 8.
5.5 Summary

An analytical model for Mild Wear (MW) that occurs during Elastic Shakedown conditions based on tangential work equivalence is developed. A factor that establishes the amount of energy required for elastic deformation is introduced. The abrasive wear groove depth and volume during the first cycle of loading is calculated. It is shown through actual simulation that conformal contact is established after one cycle of loading leading to elastic state (Elastic Shakedown conditions), thus preventing further wear.
CHAPTER 6

Numerical Prediction of Crack Growth and Particle Detachment (CGPD) in Dry Sliding Wear

6.1 Introduction

As mentioned in Chapter 3, for loading conditions below EL ($p_0/k \sim 1.5$ to 2) under high friction ($0.5 < \mu < 0.8$), the wearing material exhibits only elastic response, with surface crack growth ending in particle detachment. The examined photomicrographs of the test-worn specimens testify to this fact. They also reveal that the crack growth and propagation are from the existing cracks in the material and these cracks finally re-emerge at the wearing surface terminating in particle detachment (Chapter 3). Since this phenomenon occurs entirely in the elastic regime, the application of Linear Elastic Fracture Mechanics (LEFM) to the study of dry sliding wear becomes important. The fracture mode in the complex contact loading conditions of sliding wear is one of a mixed mode fracture process. The analysis of the same is more complicated than of fracture models describing the simple Mode I fracture usually analysed in the engineering industry.

It is also of interest to show that below the EL regime, LCF, RF and MW are not present. The mechanism of shear fracture, (see Section 2.2.5, Chapter2) is not simulated since the crack growth happens below EL and in the elastic regime. For softer wearing specimens, however, shear fracture may have to be considered. Under high friction coefficient levels, however, RF would be predominant in such cases. To identify the zone of maximum stress intensity, a FE analysis was made for the homogeneous half-space (elastic-plastic properties). For loading level of $p_0/k = 2; \mu = 0.8$, a contact deformation model is analysed to assess the existence, if any, of surface plastic deformation. Fig. 6.1a illustrates the contours of ratio of equivalent stress to yield strength at the last indentation load step. All of the values are below 1.0 indicating no plastic deformation. Similarly Fig. 6.1b indicates the total strain contours depicting a maximum of 0.001096 which is in the elastic range. Fig. 6.1c indicates that after three
reciprocating sliding cycles, the maximum value is still at 0.001246, showing no plastic deformation. Finally Fig. 6.1d indicates that the zone of maximum stress intensity is identified at the extreme edge of the sliding length. This zone is used for locating a crack for CGPD simulation. Since it is clear that no plastic deformation is taking place, as the stress ratios are all less than one and the strain is below the strain level for yield (0.2% offset for yield), the application of LEFM for these loadings is justified.

6.2 Outline of the Proposed Work

Previous studies in this area (Chapter 2) were based on a priori assumption relating to the mode of fracture for the surface crack. No efforts were made to address the near-threshold values of the Stress Intensity Factors arising from the loading. The present thesis addresses these points by incorporating a single surface crack and predicting its propagation under cyclic loading, leading to wear particle formation.

The wear system is prepared for analysis by the quantification of the asperities by surface statistical methods and evolving ‘equivalent contacting surfaces’. The system is constrained to simulate experimental conditions (Chapter 7). The input parameters for wear system are ascribed to the flat and the equivalent cylindrical asperity. A finite element model is developed by incorporating a single crack and solved for the resulting deformations. The stress intensity factors are extracted as explained from the mathematical base of fracture mechanics, (the crack face displacements in both opening and sliding are related to the stress intensity factors in Mode I and II). The change in the stress intensity factors for a given load cycle is evaluated and related to crack growth through a proposed Paris type law.

This model can predict the wear volume with reasonable accuracy under different operating conditions in Hertzian loading and frictional conditions and material properties for various values of crack entry angle and initial crack length. The following sections describe the present work in this context by way of a detailed parametric study. The contribution of the present thesis in the area of development of a predictive dry sliding wear model in comparison with the existing research work and the experimental work (Chapter 7) are highlighted.
6.3 Surface Statistics Applied to Fracture Mechanics Model

Section 4.2 of Chapter 4 (Eqns. 4.1- 4.18) described the necessary equations for calculating the number of asperities in contact, \( n_0 \). The idealisation of an ‘equivalent cylindrical asperity’ representing the contact of \( n_0 \) asperities within the nominal contact area, \( A_0 \) was also described. The surface statistical approach in calculating the number of asperities does not change in the fracture mechanics model since the nominal contact area and the number of asperities remain the same. The only difference is that the half-space, has an embedded surface crack and deforms elastically, while in Chapter 4 there is no crack and the deformations are elastic-plastic in nature.

The nominal contact area, number of asperities in contact, and the idealisation of an equivalent cylindrical asperity are calculated for the loading (\( p_0/k = 2.0 \)), and are used for the fracture mechanics model. These calculations show that at least one asperity would come into contact in the sliding direction (radial direction of the cylinder) and 76 asperities would come into contact in the axial direction of the cylinder. Fig. 6.2 to Fig. 6.5 schematically illustrates the asperity contact configuration of the wear system. The surface crack, along with the equivalent cylindrical asperity representation is shown. Since plane strain approach is adopted for the representation, the surface crack faces extend for the entire axial length of the ‘equivalent cylindrical asperity (in z-direction)’ for the analysis purposes.

For mathematical modelling, a cylinder of diameter 10 mm and axial length 25.4 mm is used as the top sliding body. The cylinder is ground finished, (i.e., asperities have a local radius of curvature of approximately 1 mm). The cylinder slides over the flat surface of a disc of diameter 19 mm and thickness 5 mm. The lower surface is assumed to be an elastic semi-infinite, nominally smooth half-space (average roughness value \( Ra = 1 \mu m \)) with an inherent surface defect in the form of a crack oriented at a shallow angle, \( \theta_0 \) as shown in Fig. 6.6a – Fig.6.6b.

6.4 Fracture Mechanics Basis for the Sliding Wear Model (CGPD)

6.4.1 Introduction

The stress-field conditions very close to the crack tip [point at a distance \( r \) from crack tip, as \( r \to 0 \)] are the controlling factors that govern the propagation of a crack, from its
inception to its eventual termination at the wearing surface with the detachment of a wear-particle.

### 6.4.2 The Stress Intensity Factor ‘K’

The SIF, $K_1$ or $K_{II}$ is the controlling parameter of the crack tip field since it characterises completely the crack tip conditions in LEFM. The stresses, strains and displacements at the crack tip are all proportional to $K$. Its evaluation explains their distribution at the crack tip. When the material fails at some critical combination of stress and strain, fracture must occur and the value of $K$ at this stage is the critical SIF $K_c$, (i.e. at failure $K = K_c$). Below $K_c$, and up to the threshold value of the stress intensity factor, $K_{TH}$, sub-critical crack growth governs the failure mode.

This is the region of interest in wear studies since the SIFs in wear studies have been observed to be in this region (Chapter 2). Sometimes more than one mode may contribute to failure. The material is then said to fracture under a mixed mode. The effective SIF, $K_{eff}$ (in the proportionate mixture of the $K$'s for the involved modes) will govern the fracture process. $K$ is also independent of the specimen size and is assumed to be a material property.

### 6.4.3 Crack Tip Stress Analysis

Under general in-plane loading, following Williams [1957] the following relationships are derived for crack tip stresses.

\[
\sigma_{rr} = \frac{1}{4\sqrt{r}} \left[ s_1 \left( 5\cos^2 \frac{\theta}{2} - \cos^3 \frac{3\theta}{2} \right) + t_1 \left( 5\sin^2 \frac{\theta}{2} - 3\sin^3 \frac{3\theta}{2} \right) + 4s_2 \cos^2 \theta + \alpha(r^{1/2}) + \ldots \right]
\]

(6.1a)

\[
\sigma_{\theta\theta} = \frac{1}{4\sqrt{r}} \left[ s_1 \left( 3\cos^2 \frac{\theta}{2} + \cos^3 \frac{3\theta}{2} \right) + t_1 \left( 3\sin^2 \frac{\theta}{2} + 3\sin^3 \frac{3\theta}{2} \right) + 4s_2 \sin^2 \theta + \alpha(r^{1/2}) + \ldots \right]
\]

(6.1b)

\[
\tau_{r\theta} = \frac{1}{2r} \left[ s_1 \left( \sin \frac{\theta}{2} + \sin \frac{3\theta}{2} \right) - t_1 \left( \cos \frac{\theta}{2} + 3\cos \frac{3\theta}{2} \right) + 2s_2 \sin 2\theta + \alpha(r^{1/2}) + \ldots \right]
\]

(6.1c)
Note that the constants $s_i$ in Eqn. (6.1) are multiplied by ‘cosine’ terms and $t_i$ are multiplied by ‘sine’ terms. Thus the stress function contains symmetric and anti-symmetric components w.r.t. $\theta = 0$. When the loading is symmetric (Mode I) about $\theta = 0$, $t_i = 0$, while $s_i = 0$ for the special case of pure anti-symmetric loading (Mode II). It is convenient and customary to rename the constants $s_1$ and $t_1$ in Equations (6.1a) – (6.1c) by Mode I ($K_1$) and Mode II ($K_{II}$) stress intensity factors respectively,

$$ s_1 = -\frac{K_1}{\sqrt{2\pi}} \quad ; \quad t_1 = \frac{K_{II}}{\sqrt{2\pi}} $$

(6.2)

The stress intensity factor defines the amplitude of the crack tip singularity. All stress and strain components at points near the crack tip increase in proportion to $K$, provided the crack is stationary. The stress relationships would further be utilised for identifying the principal tensile stress direction under mixed mode loading. The value of $K_1$ and $K_{II}$ are evaluated by knowing the Crack Tip Opening Displacements (CTOD) and Crack Tip Sliding Displacements (CTSD). The next section details the procedure for this. On evaluating the constants $K_1$ and $K_{II}$ the identification of principal tensile stress direction from Eqn. (6.1a) – Eqn.(6.1c) would follow.

**6.4.4 Determination of Mixed Mode SIFs ($K_1$ & $K_{II}$) through Displacements at the Crack Tip**

The displacement field surrounding the crack tip for a two dimensional plane strain fracture mechanics problem can be expressed as [Sih 1980]:

$$ u = \frac{\kappa}{4G} \sqrt{\frac{r}{2\pi}} \left((2\lambda - 1)\cos \frac{\theta}{2} - \cos \frac{3\theta}{2}\right) - \frac{\kappa}{4G} \sqrt{\frac{r}{2\pi}} \left((2\lambda + 3)\sin \frac{\theta}{2} + \sin \frac{3\theta}{2}\right) + H.O.T $$

(6.3)

$$ v = \frac{\kappa}{4G} \sqrt{\frac{r}{2\pi}} \left((2\lambda - 1)\sin \frac{\theta}{2} - \sin \frac{3\theta}{2}\right) - \frac{\kappa}{4G} \sqrt{\frac{r}{2\pi}} \left((2\lambda + 3)\cos \frac{\theta}{2} + \cos \frac{3\theta}{2}\right) + H.O.T $$

(6.4)

where $u$ & $v$ are the displacements in a local co-ordinate system as shown in Fig. 6.8.
\[ \lambda = 3 - 4\nu, \]
\[ \nu \text{ being the Poisson's ratio.} \]
\[ r, \theta \text{ are the co-ordinates in the local cylindrical co-ordinate system adopted Fig. 6.7.} \]
\[ G \text{ is the shear modulus.} \]

\[ K_I \text{ & } K_{II} \text{ are stress intensity factors in Mode I & Mode II fractures.} \]

Fig. 6.8 illustrates the crack faces and a local Cartesian co-ordinate system at the crack tip. The nodes (or points on both the crack faces) are represented as I, J, K, L and M. The node I is the common node representing the crack tip.

The stress intensity factors under the applied asperity contact loading are calculated by knowing the values of the displacements of the crack faces to the applied loading, i.e., the displacement field (response of the structure) surrounding the crack tip. A Finite element procedure ANSYS 5.4 [1998], is utilised for calculating the displacement field surrounding the crack tip. The displacements at the crack tip are subsequently related to the SIFs. The nodal displacements of the crack-faces are related to the stress intensity factors from the FE model. The 2D displacements, \( u \) and \( v \) of any of the nodes on the crack-faces are further related to the stress intensity factors \( K_I \) & \( K_{II} \). This is achieved by evaluating the general expressions for \( u \) and \( v \) at \( \pm 180^\circ \). On evaluating the expressions (Eqn. 6.3 and Eqn. 6.4) at \( \pm180^\circ \) and calculating for the absolute changes in \( u \) and \( v \),

\[ |\Delta u| = \frac{k_{II}}{G} \sqrt{\frac{r}{2\pi}} (1 + \lambda) \]

(6.5)

\[ |\Delta v| = \frac{k_I}{G} \sqrt{\frac{r}{2\pi}} (1 + \lambda) \]

(6.6)

Fig. 6.8 further explains this schematically. For the case of no symmetry between the top and bottom surface nodes of the crack faces, the expressions for \( K_I \) & \( K_{II} \) are rearranged to read as

\[ K_I = \sqrt{2\pi} \frac{G}{1 + \lambda} \left| \frac{\Delta V}{\sqrt{r}} \right| \]

(6.7)
where \( \Delta u \) & \( \Delta v \) are the motions of one crack face w.r.t. other given by the relative displacement of the nodes on both the surfaces in 2D.

**Numerical Determination of \( K_I \) and \( K_{II} \)**

From general analytical theory it is known that near the tip of the crack, the crack opening displacements between two corresponding points distant \( r \) from the crack tip are of the form

\[
\frac{|\Delta v|}{r} = A + B\sqrt{r} + H.O.T
\]  
\[\text{(6.9)}\]

\[
\frac{|\Delta u|}{r} = C + D\sqrt{r} + H.O.T
\]  
\[\text{(6.10)}\]

With reference to Fig.6.4b, \( |\Delta v| \) and \( |\Delta u| \) are evaluated at two corresponding positions shown. These four quantities with corresponding known values of \( 'r' \) are then used numerically in Eqns. (6.9) and (6.10) to determine the values of \( A, B, C \) and \( D \). Only \( A \) and \( C \) are used of since the comparison of Eqn. (6.7) and Eqn. (6.8) with Eqn (6.9) and (6.10), shows that

\[
K_I = \sqrt{2\pi} \frac{G}{1+\lambda} A
\]  
\[\text{(6.11)}\]

\[
K_{II} = \sqrt{2\pi} \frac{G}{1+\lambda} C
\]  
\[\text{(6.12)}\]

Thus \( K_I \) and \( K_{II} \) are evaluated numerically.

The FE approach gives values of the stress intensity factors in Mode I and Mode II. It will be shown later in the FE analysis results and discussions that the Mode II stress intensity factor is the predominant one for the initially assumed crack of specified length and orientation for the first load step. A small value of Mode I stress intensity factor however accompanies this Mode II stress intensity factor. This fact is significant because it shows that the crack faces slide relative to each other as well as open to a small extent.
relative to each other during the first load step. Subsequently the crack is grown and a
second load step is effected on the ‘kinked’ or bent crack. This time, the Mode I stress
intensity factor that accompanies this Mode II stress intensity factor increases
significantly and signifies a typical mixed mode fracture. The crack propagation problem
is thus one of a mixed mode crack growth problem with Mode II dominating during the
first load step compared with Mode I. The formulation and the discussions are presented
in the following sections for the general case of Mixed mode fracture, i.e., Mode I and
Mode II resulting in $K_I$ and $K_{II}$ respectively.

6.4.5 Prediction of Crack Turn Angle by Maximum Tensile Stress Criterion

Normally for Mode I type fracture, the extension continues in the same direction as
the original crack orientation. But for pure Mode II fracture, experimental results
[ Vaughan 1998] reveal that the crack propagation deflects from the original direction, i.e.
occurs at an angle to the initial orientation of the crack. This angle will be termed as
'Crack Turn Angle (CTA)' in the following sections. The 'CTA' coincides with the
angle at which the tensile stress ahead of the crack tip attains a maximum value.
Analytical treatment of this phenomenon, that is critical to wear particle detachment
follows.

Fig. 6.9 illustrates the half-space containing the initially assumed crack along with the
local co-ordinate system at the crack tip. The various stress components for the chosen
element are shown. The element orientation, where the stress component $\sigma_0$ attains its
maximum is the critical angle at which the crack is expected to propagate in the sub­
surface. Eqns. (6.1a) – (6.1c) are rewritten by substituting the value for $s_1$ and $t_1$ from
Eqn.(6.2), for the following applicable crack tip stresses in a plane ($r$-$\theta$ plane)
perpendicular to the plane of crack faces, i.e. $r$ - $z$ plane.

$$\sigma_r = \frac{K_I}{\sqrt{2\pi r}} \left[ -\frac{5}{4} \cos \left( \frac{\theta}{2} \right) + \frac{1}{4} \cos \left( \frac{3\theta}{2} \right) \right] + \frac{K_{II}}{\sqrt{2\pi r}} \left[ \frac{5}{4} \sin \left( \frac{\theta}{2} \right) - \frac{3}{4} \sin \left( \frac{3\theta}{2} \right) \right]$$

(6.13a)

$$\sigma_\theta = \frac{K_I}{\sqrt{2\pi r}} \left[ -\frac{3}{4} \cos \left( \frac{\theta}{2} \right) - \frac{1}{4} \cos \left( \frac{3\theta}{2} \right) \right] + \frac{K_{II}}{\sqrt{2\pi r}} \left[ \frac{3}{4} \sin \left( \frac{\theta}{2} \right) + \frac{3}{4} \sin \left( \frac{3\theta}{2} \right) \right]$$

(6.13b)
\[ \tau_{r\theta} = \frac{K_I}{\sqrt{2\pi r}} \left[ -\frac{1}{4} \sin \left( \frac{\theta}{2} \right) - \frac{1}{4} \sin \left( \frac{3\theta}{2} \right) \right] - \frac{K_{II}}{\sqrt{2\pi r}} \left[ \frac{1}{4} \cos \left( \frac{\theta}{2} \right) + \frac{3}{4} \cos \left( \frac{3\theta}{2} \right) \right] \tag{6.13c} \]

The above relations are the sums of symmetric and anti-symmetric parts. For pure Mode II conditions, \( K_I \) is zero and only the anti-symmetric part remains.

**Pure Mode II condition**

The stresses \( \sigma_\theta \) and \( \tau_{r\theta} \) vanish on the crack faces at \( \theta = +180^\circ \) and \( \theta = -180^\circ \). At \( \theta = 0^\circ \), i.e., immediately in front of the crack, the stresses \( \sigma_\theta \) and \( \sigma_r \) vanish. The only non-zero stress is the shear stress, \( \tau_{r\theta} \) given by

\[ \tau_{r\theta} = \frac{-K_{II}}{\sqrt{2\pi r}} \tag{6.14} \]

At the crack tip it is the tensile stress \( \sigma_{90} \) that opens the crack, regardless of the orientation of the crack. Eqn. 6.13b shows that \( \sigma_{\theta} \) is greatest when \( \cos \frac{\theta}{2} + 3 \cos \frac{3\theta}{2} = 0 \), which is also the condition from Eqn (6.13c) that \( \tau_{r\theta} = 0 \). The root of \( \cos \frac{\theta}{2} + 3 \cos \frac{3\theta}{2} = 0 \) is \( \cos \theta = 1/3 \), that is \( \theta = 70.5^\circ \). The corresponding value of \( \sigma_{\theta} \) is

\[ \sigma_{\theta_{\text{max}}} = \frac{2K_{II}}{\sqrt{6\pi r}} \tag{6.15} \]

The root \( \theta = 180^\circ \) gives \( \sigma_{\theta} \) a local minimum. The crack turn angle pertaining to pure Mode II fracture is 70.5° (as seen from the above discussions). It is also well known that a pure Mode I fracture would propagate at 0° (i.e., no crack turning takes place).

**Mixed Mode Fracture (Mode I and Mode II)**

It is best illustrated by plotting Eqn (6.13b), for different values of \( \theta \), Fig.6.10. \( K_{II} \) and \( K_I \) are selected from actual FE runs. Also pure Mode I and Mode II are plotted for \( K_I = 6.0 \text{ MPa m}^{0.5} \) and \( K_{II} = 6.0 \text{ MPa m}^{0.5} \) along with mixed mode fracture for comparison. The maximum value occurs at (70.5 deg) for pure Mode II. For pure Mode I condition, the maximum value of hoop stress occurs at 0° as expected. For mixed mode fracture...
with $K_1 = 0.16 \text{ MPa m}^{0.5}$ and $K_{II} = 3.54 \text{ MPa m}^{0.5}$ (values from actual FE runs) the hoop stress distribution for mixed mode is plotted. Through such simulations, it is also observed that $K_{II}$ is the dominant mechanism in the equation for turning the crack from its original path. For mixed mode fractures, containing Modes II, the latter predominating in the first load step, a crack turn angle between $0^\circ$ and $70.5^\circ$ and in the neighbourhood of $65^\circ$ is expected. For the second load step, however, even if Mode I increase significantly, Mode II still governs the crack turn. So, for the mixed mode case of the present analysis, the crack turn angle is in the neighbourhood of $60^\circ - 70^\circ$ depending upon the resulting proportion or mix of Mode I and Mode II stress intensity factors for a particular orientation and loading.

**6.4.6 Crack Growth**

For a growing crack under a constant amplitude cyclic stress intensity, if the miniscule plastic zone at the crack-tip is contained by the surrounding elastic singularity zone, the conditions at the crack tip are defined by the current value of ‘$K$’ at every instant. The crack growth depends upon $\Delta K = (K_{\text{max}} - K_{\text{min}})$. The influence of the plastic zone is implicit in $\Delta K$, since the zone is controlled by $K_{\text{max}}$ and $K_{\text{min}}$. For crack growth mechanisms, the cracking rate can be correlated to $K$ and failure condition can be computed if the fracture toughness $K_c$ of the material is known.

Crack growth in metals is usually given by the following relationship due to Paris and Erdogan [1961] as

$$\frac{dc}{dN} = C(\Delta K)^m$$

(6.16)

where $dc/dN$ is the crack growth per cycle

$\Delta K$ is SIF range, $(K_{\text{max}} - K_{\text{min}})$

$C$ and $m$ are material constants to be found experimentally.

A log-log plot of $dc/dN$ against $\Delta K$ gives a sigmoidal curve showing three distinct stages. At intermediate $\Delta K$ levels, the curve is near linear and obeys Paris law. This growth rate deviates, though, from linear trends at high and low $\Delta K$ values. For high $\Delta K$ values, the crack growth rate $dc/dN$ accelerates as $K_{\text{max}}$ approaches $K_{\text{critical}}$, the fracture toughness of
the material. At the lower end, \( \frac{dc}{dN} \) approaches zero at a threshold of \( \Delta K \). The constant ‘m’ was given as 4 by Paris [1961] but found to vary from 2 to 7 for different materials [Anderson 1995].

Many researchers have developed equations to model the whole or part of the curve. These attempts have been on empirical basis however. Much of this work has been for the simple opening type (i.e) Mode I, compared to other Modes including mixed modes. Almost all modelling has followed the lines of Paris Law. This is because the \( \frac{dc}{dN} \) vs \( \Delta K \) relation has been recognised as a power relation and is conveniently described in the form of Paris law and satisfactory experimental curve fit is obtained with relevant values of \( C \) and \( m \) dictated by the current situation.

The study of the growth of fatigue cracks under cyclic loading for a combination of Modes I & II was pioneered along the above lines. Early experiments established the fact that the mode II component of the applied load plays a predominant role in accelerating crack growth and also that the law for crack growth can be conveniently expressed in the form

\[
\frac{dc}{dN} = C(\Delta K_{eff})^m
\]  

(6.17)

where \( c \) and \( m \) can have varying values depending on the material and operating conditions.

Various expressions have been arrived at for \( \Delta K_{eff} \), the effective mixed mode SIF. The one given by Tanaka [1974] has been adopted as suitable for the present study in view of the preponderant effect of \( K_{II} \) value over \( K_I \) observed at this work. The relation between \( \Delta K_{eff} \) and the mixed mode factors is given as

\[
\Delta K_{eff} = \left[ \Delta K_I^4 + 8\Delta K_{II}^4 \right]^{1/4}
\]  

(6.18)

So much so, the contribution of \( K_I \) becomes negligible and \( \Delta K_{eff} \) becomes equal to \( 8^{1/4}\Delta K_{II} \). If the numerical coefficient of \( \Delta K_{II} \) is absorbed in the constant \( C \) of the formula, the \( \frac{dc}{dN} \) & \( \Delta K \) relation reduces to
\[
\frac{dc}{dN} = C[\Delta K]_n
\]  
(6.19)

This relation is similar to Paris Law. \( C = 1.99 \times 10^{-10} \) and \( m = 3 \) [ASME B & PV Code 1992 ed.] are material constants.

An integration procedure involving the change in the stress intensity factor is used to calculate the number of cycles needed for this. This is achieved by first normalising the stress intensity factor with a nominal stress intensity factor based on the relationship

\[
K_0 = \left(\sqrt{p_0^2 + q_0^2}\right)\sqrt{\pi c}
\]  
(6.20)

where \( p_0 \) and \( q_0 \) are the peak contact pressure and the corresponding tangential stress respectively for each loading case. The normalised stress intensity factor is further used in the following integral relationship to derive the number of cycles required crack extensions in either load step 1 or 2.

\[
N = \frac{\int_{c}^{c_1} \frac{dc}{(\pi)^{1.5}(1.99 \times 10^{-10})(\beta)(c)^{1.5}}} 
\]

(6.21)

where \( N \) is the number of cycles required for the crack extension from \( c \) to \( c_1 \).

\( c \) is the original crack length,

\( c_1 \) is the extended crack length for the first load step

\[
\beta = \frac{F_0^3 \cdot R_0^3}{5K_0^7/K_0^7}
\]

\[
F_0 = \delta K_{II} / K_0 \quad \text{and} \quad R_0 = \sqrt{p_0^2 + q_0^2}.
\]

On evaluating the integral in Eqn. (6.21), the following relationship for the integration procedure is obtained.

\[
N_1 = \frac{-9 \times 10^8}{\beta} \left[ \frac{1}{c_1^{0.5}} - \frac{1}{c^{0.5}} \right]
\]  
(6.22)

In the above relationship, the quantity within the bracket will be negative since \( c_1 \) is greater than \( c \), thus making the overall result a positive quantity for the number of cycles.
A similar expression for the second load step is derived by replacing \( c_1 \) with \( c_2 \), and \( c \) with \( c_1 \). The expression for the second load step is detailed as

\[
N^2 = \frac{-9 \times 10^8}{\beta} \left[ \frac{1}{c_2^{0.5}} - \frac{1}{c_1^{0.5}} \right]
\]

(Eqns. 6.22 and Eqn. 6.23)

Eqns. 6.22 and Eqn. 6.23 are automated within a program to predict the number of cycles required for each load step depending upon the stress intensity factors obtained for that particular load step and the crack extension.

6.4.7 A Note on Paris Law

Paris law, though empirical, still compares favourably with experiment, especially for Mode I propagation. It is used, therefore, whenever possible. Current crack tip conditions are dependent, however, upon prior loading history. If this is taken into account the crack growth analysis becomes highly complicated. As a result such analyses are only approximate in variable amplitude loading. Again, this equation does not incorporate the threshold value of \( K \) and does not explain the equation becoming asymptotic when \( K_{\text{max}} = K_{\text{critical}} \). Even under conditions unfavourable to it the equation gives approximate but reliable results. This also serves as a base for relevant modifications to suit various experimental conditions. Though a lot of equations with \( C \) and \( m \) varying according to the situation have been developed, the preference for its first use is still widely established.

6.4.8 Calculation of Wear Volume for Dry Sliding Wear

Knowing the extension in crack length per cycle, \( dc/dN \), and the crack turn angle for mixed mode fracture, the crack is grown for a lumped increment of \( N \) cycles per load cycle. Fig 6.11 illustrates this. Subsequently the loading is applied on the bent crack and the stress intensity factors and crack turn angles are once again calculated for the bent crack. The whole process is automated within ANSYS 5.4 [1998] and the program is run until the crack reaches the free surface (two load steps). Once the crack propagation vector reaches the surface of the half-space, a wear particle is assumed to be detached from the half space. The loci of the crack tip determine the cross sectional area of the wear particle, and due to the assumption of the plane strain conditions, the third
dimension is the plane strain direction (out of plane). The volume of the wear particle can be expressed as
\[ V_p = \{ \text{cross-sectional area of the quadrilateral in } r\theta \text{ plane formed by crack growth} \} \times \{ l_0 \}. \]

Further, based on long duration test results for dry sliding wear, the wear volume obtained from the experiments is compared with the single wear particle volume based on the FE approach. The volume of the single wear particle could be multiplied by an average estimate for the number of cracks in the wear track to predict the total wear volume.

6.5 Model for Crack Growth

6.5.1 Development for Fracture Mechanics Model

The scope of the present analysis is to assume a crack in the elastic half-space and study its growth for cyclic Hertzian loading. The Hertzian loading results from the sliding of the asperities of the hard surface over the elastic half-space. The asperity sliding is simulated with a hard cylinder of Carbon Steel hardened to Rc = 55, sliding on a soft-flat disc made of Type SAE 410 s.s. hardened to Rc = 33. The cylinder is of diameter 10 mm and axial length 25.4 mm and having a ground finish (i.e. asperities have a local radius of curvature of 1 mm). This is the top sliding body. The lower (soft) surface, assumed to be elastic semi-infinite nominally smooth (average roughness R_a \approx 1 \mu m) is the flat surface of the disc of diameter 19 mm and thickness 5 mm. An inherent defect in the form of a crack oriented at a shallow angle \theta_0 as shown in Fig. 6.6a is assumed. The material properties of Carbon steel and stainless steel SAE 410 are assigned to the cylinder and the disc, respectively.

Since interfacial friction, according to contact mechanics, influences the locale of the maximum shear, this parameter also included in the present study, the system is analysed for various values of high friction (0.6 \& 0.8). Such friction coefficients are common in dry sliding conditions (see Chapter 3 for discussions), under which the system is subjected to various normal loading in two steps for each loading, keeping the loading range below EL (p_0 / k \sim 1.5 to 2 and to a maximum of 2.4). Because of the high shear yield strength of the wearing disc material, 0.62 GPa, the value of the applied normal
pressure, \( p_0 \) is set around 1 to 1.49 GPa. This loading range is set to facilitate the comparison of computational results with those of the present experimental study (Chapter 7) and other experiments of longer duration [Knowles 1994].

### 6.5.2 Calculation of Hertzian Normal pressure and Tangential stress distribution

The cylinder is pressed against the disc with a vertical force \( P_N \) and then subjected to an oscillatory sliding motion with a corresponding overall frictional force \( F \), where \( F = \mu P_N \). The overall vertical force \( P_N \) brings the local surface asperities modelled as an equivalent cylindrical asperity into contact over a contact width “2a”.

This results in a local normal contact stress \( p(x) \) over the contact surface with a corresponding tangential or shear stress distribution \( q(x) \), where \( q(x) = \mu p(x) \), \( \mu \) being the global friction coefficient. The relationship between the normal force \( P_N \) and the contact width “2a” is non-linear, and is found from the Hertzian contact equations. The numerical values of \( p(x) \) over the contact width 2a for the particular values of \( P_N \) used in the investigation (that is, 400N, 500N, 600N, 700N and 800N) are given in Table 6.1.

<table>
<thead>
<tr>
<th>( P_N ) (N)</th>
<th>( x ) (( \mu )m)</th>
<th>( p(x) ) (GPa)</th>
<th>0</th>
<th>1.9</th>
<th>3.8</th>
<th>5.7</th>
<th>7.7</th>
<th>9.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>400N</td>
<td></td>
<td></td>
<td>1.05</td>
<td>0.84</td>
<td>0.77</td>
<td>0.67</td>
<td>0.50</td>
<td>0.41</td>
</tr>
<tr>
<td>500N</td>
<td></td>
<td></td>
<td>0.84</td>
<td>0.72</td>
<td>0.61</td>
<td>0.50</td>
<td>0.39</td>
<td>0.31</td>
</tr>
<tr>
<td>600N</td>
<td></td>
<td></td>
<td>0.72</td>
<td>0.61</td>
<td>0.50</td>
<td>0.39</td>
<td>0.31</td>
<td>0.24</td>
</tr>
<tr>
<td>700N</td>
<td></td>
<td></td>
<td>0.61</td>
<td>0.50</td>
<td>0.39</td>
<td>0.31</td>
<td>0.24</td>
<td>0.17</td>
</tr>
<tr>
<td>800N</td>
<td></td>
<td></td>
<td>0.50</td>
<td>0.39</td>
<td>0.31</td>
<td>0.24</td>
<td>0.17</td>
<td>0.10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( P_N ) (N)</th>
<th>( x ) (( \mu )m)</th>
<th>( q(x) ) (GPa)</th>
<th>0</th>
<th>1.9</th>
<th>3.8</th>
<th>5.7</th>
<th>7.7</th>
<th>9.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>400N</td>
<td></td>
<td></td>
<td>0.84</td>
<td>0.72</td>
<td>0.61</td>
<td>0.50</td>
<td>0.39</td>
<td>0.31</td>
</tr>
<tr>
<td>500N</td>
<td></td>
<td></td>
<td>0.72</td>
<td>0.61</td>
<td>0.50</td>
<td>0.39</td>
<td>0.31</td>
<td>0.24</td>
</tr>
<tr>
<td>600N</td>
<td></td>
<td></td>
<td>0.61</td>
<td>0.50</td>
<td>0.39</td>
<td>0.31</td>
<td>0.24</td>
<td>0.17</td>
</tr>
<tr>
<td>700N</td>
<td></td>
<td></td>
<td>0.50</td>
<td>0.39</td>
<td>0.31</td>
<td>0.24</td>
<td>0.17</td>
<td>0.10</td>
</tr>
<tr>
<td>800N</td>
<td></td>
<td></td>
<td>0.39</td>
<td>0.31</td>
<td>0.24</td>
<td>0.17</td>
<td>0.10</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Table 6.1 Pressure distribution\(^1\) over contact area (\( \mu = 0.8 \))

---

\(^1\) Pressure distribution at 800N is the upper bound for EL for \( \mu = 0.8 \).
Note that the pressures are symmetric about \( x=0 \). Thus for a load of 600N, the contact width is 23.4 \( \mu \text{m} \) and \( p(x) \) and \( q(x) \) are distributed over the region \(-11.7 < x < 11.7\) and over the axial length of the cylinder, 25.4 mm. The surface shear distribution \( q(x) \) is given by \( q(x) = \mu p(x) \) for each case.

### 6.5.3 Finite Element Procedure

The FE method is used in the analysis of the half-space with the assumed surface crack for varying loads and friction conditions for different crack lengths and orientations. For plane strain conditions, the present crack growth is modelled with the FE mesh as shown in Figs. 6.12a – 6.12e. Fig. 6.6a represents the idealised cylindrical asperity in contact with a smooth half-space with a surface crack. The Hertzian contact pressure distribution and the sliding directions are represented in Fig.6.6b. The transition of the applied normal and tangential contact stress is illustrated. The initially assumed crack length, \( c \) and orientation, \( \theta_0 \) are also shown. As mentioned in 6.5.1 the boundary conditions of the discretized semi-infinite half-space are simulated as in the experiment (Chapter 7) of the present work.

An array of 6 node iso-parametric triangular elements is used to model the semi-infinite medium with the 30 \( \mu \text{m} \) crack embedded into it at 30° to the free surface. The stresses and hence the SIFs at the crack tip have \( (1/Vr) \) singularity (see Eqns. 6.13 a-c). This is modelled in the finite element program, ANSYS by placing the mid-side nodes of the elements at quarter the distance from one of the end nodes surrounding the crack tip, [ANSYS 1998]. The boundaries of the discretized semi-infinite half space, are constrained to simulate the conditions used in sliding experiments, i.e., the lower surface is constrained in \( y \)-direction and the edge of one of the sides of the flat disc specimen is constrained in \( x \)-direction. The exact conditions of the experimental work of this thesis are thus simulated in the mathematical model.

### 6.5.4 Finite Element Analysis of Crack Growth

A linear elastic static analysis of the FE model is performed by applying and translating the \( p(x) \) and \( q(x) \) distributions on the loading surface of the disc. For each load step (each translated position of the applied normal and tangential contact stress distributions), the stress intensity factors are evaluated as per (Eqn. 6.11 - Eqn.6.12). The resulting stress intensity factors are calculated at two positions. Firstly, the translated
loading on the sliding surface is in a position that gives the maximum value of either Mode I or Mode II stress intensity factor, $K_{i_{\text{max}}}$ ($i = I$ or II). Secondly, when the translated loading is at a considerable distance, i.e., at the end of the sliding length, the resulting stress intensity factor, $K_{i_{\text{min}}}$ ($i = I$ or II) is very low and taken as 0.0 MPa$\sqrt{m}$. Since $\delta K_{i}$ equals $(K_{i_{\text{max}}}-K_{i_{\text{min}}}; i = I$ or II), the change in the stress intensity factor, $\delta K_{i}$ ($i = I$ or II) is related to the sub-critical crack growth through Eqn. (6.22) and Eqn. (6.23).

The change in the stress intensity factor, $\delta K_{i}$ ($i = I$ or II), is further checked with the threshold value of the stress intensity factor, $K_{\text{TH}}$, for the given material (6.0 MPa$\sqrt{m}$ for steel). The threshold stress intensity factor value for Mode I is adopted for the mixed mode crack growth problem in the present study. If the value of $K_{i}$ ($i = I$ or II) is greater than the threshold value, then the initially assumed crack length c is extended for a short distance $dc = c/2$ through an angle, $\text{CTA} = d\theta_{1}$. The number of cycles required for this extension, $N_{1}$ is calculated.

After the crack is extended in the direction of the crack turn angle, the crack becomes 'kinked' or 'bent' crack. The crack tip then has a new set of co-ordinates. This bent crack is further analyzed as before by translating the set of normal and tangential contact stresses on the loading surface until the maximum and minimum values are found for the stress intensity factors. The analysis of this bent crack is termed as load step 2. The extension, and the number of cycles required for this extension, are calculated as before with a new crack turn angle for the second load step, $d\theta_{2}$.

### 6.6 Results and Discussions for Dry Sliding Wear

#### 6.6.1 Sample Calculation

(\text{Normal load } P_{0}=600N; \text{ tangential load } F_{0}: 480N (p_{0} / k \sim 2.08) )

For the initial calculations, an initial crack orientation angle of 30° and a crack length of 30 $\mu$m is specified but these values are changed in the parametric study described later. An average global friction coefficient, $\mu=0.8$ is used for dry sliding wear. As described in Section 6.5, a specific surface crack is introduced into the flat surface of a half-plane. The normal contact stress, $p(x)$ and the shear stress distribution $q(x)$ are now allowed to slide along the surface passing over the crack. A finite element scheme is
used as the tool to investigate the possibility that the crack will grow and also turn from its original orientation of $30^\circ$ to the free surface. Under the plane strain assumption, the present contact problem is modelled with the finite element mesh shown (see Fig. 6.12a-Fig.6.12e).

A linear elastic analysis is performed by applying, and, translating the $p(x)$ and $q(x)$ stress distributions, on the loading surface of the disc (the wearing surface). Table 6.2 gives the results of the finite element analysis for the 30 µm crack at different initial orientations (entry angles). The quantities $\delta K_i$ ($i = I$ (0.297 MPa m$^{0.5}$) or II (6.4 MPa m$^{0.5}$), $c/2$ (15 µm), and a definite crack extension used in (Eqn. 6.22 – Eqn.6.23 are summarised in Table 6.2. The CTA s are represented as the quantity ‘$d\theta_j$’ ($j = 1$ (69 deg) or 2 (65 deg) corresponds to load step 1 or 2). After two load steps the crack breaks through the free surface detaching as a wear particle. It is seen from Table 6.2 that Mode II stress intensity factor is predominant during the first load step in mixed mode fracture. In the second load step, the low value of Mode I that accompanied the Mode II stress intensity factor during the first load step increases. Figs. 6.13(a-b) (graphical output from FE analysis for two load steps) illustrate the location of the actual maximum tensile stress in the vicinity of the crack tip to determine the crack turn angles for the two load steps. The number of cycles is calculated in load steps 1 and 2, are 105 and 18. Table 6.2 in conjunction with Figs. 6.11 (a-b) fully explains the crack extension of 15 µm for $N_1 = 108$ cycles in the first load step and crack extension of 2.27 µm for $N_2 = 18$ cycles for the second load step.

**Discussion for One Complete Load Cycle ($c = 30$ µm; $\theta_0 = 30$ deg)**

As the loading $p(x)$ and $q(x)$ are translated on the wearing half space, Fig.6.11, containing the crack, the SIFs at the crack tip vary. When the loading are far away from the crack, the SIFs are very low (almost zero, being the $K_{i \text{min}}$). As the translated loading approach the crack, the SIFs increase. The loading are translated by an amount equal to ‘a’ where a is the semi-contact width of the equivalent cylindrical asperity (13.5 µm in the present case). When the profile is between $(-3a, -a)$, the opening of the crack faces on the wearing surface, $K_I$ attains a low value of 0.164 MPa m$^{0.5}$ and $K_{II}$ attains a value of 5 MPa m$^{0.5}$. On translating these loadings further toward the crack by a distance ‘a’, the applied distribution is between $(-2a, 0)$. At this position, $K_{II}$ reaches its maximum of 6.4
MPa m$^{0.5}$ and $K_1$ attains a value of 0.297 MPa m$^{0.5}$. This is the governing position for the loading cycle as $K_{II}$, the predominant SIF attains its maximum at this position. This value is summarized in Table 6.2. On translating the loading further, to the new coordinate position over the top of the crack to the other side of the crack, (0, 2a), the value of $K_{II}$ falls to 2.0 MPa m$^{0.5}$. The value of $K_1$ increases, however, to 2.1 MPa m$^{0.5}$. These values are below the threshold for any crack growth to occur. As the translated Hertzian profile moves further away, mode II SIF continues to fall more rapidly than Mode I SIF. Once the translated loading is past (3a, 5a), both SIFs attain low values (typically less than 1.0 MPa m$^{0.5}$ and does not have any bearing on the crack growth. This discussion shows that the position just before the crack is the governing position (-2a,0) for crack growth in load step 1.

In load step 2, where the crack tip reaches close to the surface, the return half-cycle of the reciprocating loading governs the maximum. The only position of significance for crack growth is between (2.2a, 4.2a) from the crack opening on the surface. The discussion presented here for one complete loading cycle for load step 1 and 2 are for the sample calculation. The same is true for all other crack lengths and orientations and the SIFs attained their respective maximums and minimums in the same way as the (c = 30 μm; $\theta_0$ = 30 deg) case.

**Table 6.2 Crack growth and particle formation versus entry-angle**

(crack length 30 μm, Normal Force $P_N = 800$N shear force ; $F = 640$ N)

<table>
<thead>
<tr>
<th>Crack entry-angle (deg.)</th>
<th>$\Delta K_{I}$ MPa.m$^{0.5}$</th>
<th>$\Delta K_{II}$ MPa.m$^{0.5}$</th>
<th>$d_{c1}$ &amp; $d_{c2}$ μm</th>
<th>$d_{th}$ deg.</th>
<th>$N_c$ cycles</th>
<th>Aspect ratio$^*$</th>
<th>Mass of ptcl.$^{**}$, x 1e-2 mg</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>$N_c=116$</td>
<td>N/A</td>
<td>no particle</td>
</tr>
<tr>
<td>20</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>$N_c=116$</td>
<td>N/A</td>
<td>no particle</td>
</tr>
<tr>
<td>25</td>
<td>0.252</td>
<td>6.15</td>
<td>15</td>
<td>70</td>
<td>$N_c=6$</td>
<td>2.91</td>
<td>4.9</td>
</tr>
<tr>
<td>30</td>
<td>0.297</td>
<td>6.4</td>
<td>15</td>
<td>69</td>
<td>$N_c=105$</td>
<td>-</td>
<td>no particle</td>
</tr>
<tr>
<td>35</td>
<td>2.31</td>
<td>7.51</td>
<td>5.7</td>
<td>65</td>
<td>$N_c=18$</td>
<td>2.42</td>
<td>6.04</td>
</tr>
<tr>
<td>40</td>
<td>0.33</td>
<td>7.03</td>
<td>15</td>
<td>69</td>
<td>$N_c=78$</td>
<td>-</td>
<td>no particle</td>
</tr>
<tr>
<td>45</td>
<td>3.57</td>
<td>6.86</td>
<td>8.9</td>
<td>62</td>
<td>$N_c=40$</td>
<td>2.1</td>
<td>7.18</td>
</tr>
<tr>
<td>50</td>
<td>0.36</td>
<td>7.47</td>
<td>15</td>
<td>69.5</td>
<td>$N_c=65$</td>
<td>-</td>
<td>no particle</td>
</tr>
<tr>
<td>55</td>
<td>3.63</td>
<td>6.26</td>
<td>11.9</td>
<td>59</td>
<td>$N_c=53$</td>
<td>1.9</td>
<td>8.3</td>
</tr>
<tr>
<td>60</td>
<td>0.396</td>
<td>7.92</td>
<td>15</td>
<td>69</td>
<td>$N_c=55$</td>
<td>-</td>
<td>no particle</td>
</tr>
</tbody>
</table>
** Ratio of (length of wear particle/ maximum depth of wear particle).

** Volume of wear particle x density of wear particle

6.6.2 Parametric Variation in c and $\theta_0$ ($P = 800N; Q = 640 N; p_0/k = 2.08$)

Fig.6.14 and Fig.6.15 (a-b) – Fig.6.18 (a-b) illustrate the location of maximum tensile stress in the vicinity of the crack tip for the initially assumed crack length of 30 $\mu$m for different orientations (@ different crack entry angles). Since the crack turn angle is sensitive only to the loading direction, and the proportion of Mode I and Mode II in a particular computer run, the change in the magnitude of the applied load (keeping the proportion constant) does not affect the value of the crack turn angle. Thus, Fig. 6.14 (a-b) - Fig. 6.18 (a-b) therefore hold good for any applied loading provided the ratio of tangential to normal stress remains the same as 0.8.

In the set of figures, contours of hoop stress are values in ‘r’ and ‘$\theta$’ co-ordinates in the cylindrical system of co-ordinates. The symbol ‘MX’ signifies the location of the maximum crack stress surrounded by high values of the tensile hoop stress. The surrounding red contours are iso-stress lines of having high tensile stress value. It can be observed from all of these figures, that for load step 1, the tensile maximum occurs at a consistent value of 69 deg. $\sim$ 70 deg. This is expected, since the amount of the Mode I stress intensity factor is very low in each case (see Table 6.2).

The crack faces for the subsequent load step are generated to pass through the maximum tensile stress location. In almost all of the cases it can be observed that the location lies between 69 $\sim$ 70 deg. Further, the top and bottom faces of the cracks are extended by an amount equal to 15 $\mu$m.

For the initial case of $c = 30 \mu m$ at different orientations, the stress intensity factors and the number of cycles are derived for load steps 1 and 2. These are summarized in Table. 6.2. The stress intensity factor is seen to increase with the increase in crack entry angle. This is due to the aligning of the crack faces along the direction of the resultant of applied forces $P_N$ and $F$, achieving a maximum effectiveness of the applied loading in sliding the faces relative to one another. A marginal effect of opening of the crack faces is seen. This is an important result. It proves that the crack faces are not in contact and even though a predominant Mode II is observed, the crack faces are still apart. As a
result there is no effect of the friction coefficient of the crack faces in the first load step. This result was possible due to the incorporation of the contact elements into the crack faces thereby detecting contact if any. This result is unlike other studies, [Bhargava et al. 1987, Keer et al. 1982 and Lamacq et al. 1997] among others, where a priori assumption of the modes of fracture was made even before the analysis. It resulted in disproportionate amounts of $K_I$ and $K_{II}$ especially for mixed mode fracture. This caused them to assume a coefficient of friction of the interface. But the actual nature of contact of crack faces was not investigated and the coefficient of friction was used to inhibit the growth of microcracks. The present study advances the existing knowledge in this field by actually simulating the deformation of the crack faces and proving clearly through the existence of the mixed mode fracture that the crack faces do open in addition to sliding. Thus even though the crack face friction was assumed, it did not have any effect in the growth of the cracks. Also Keer et al. [1991] have investigated a crack entry angle of 90 deg for only one load step without considering the effect of threshold value. The present study addressed these assumptions by incorporating contact elements and calculating the SIF in correct proportion. In the present study, it is proven from Table, 6.2, that beyond the crack entry angle value of 45 deg, the stress intensity factors pertaining to load step 2 would fall below the threshold value, thus inhibiting the growth of cracks. This gives the bounding value for the initial crack entry angle as 45 deg. The stress intensity factors are seen to increase in the second load step for shallow crack entry angles, i.e., 25deg, 30 deg etc. But as the crack entry angle is increased the SIF in the second load step drops and falls below the threshold value of 6.0 MPa m$^{0.5}$ for an entry angle of 45 deg. This is an important result since 45 deg forms the bounding case for the crack entry angle. Increased values of Mode I compared to load step 1 were also seen in the second load step. From Figures 6.13 (a-b) and Fig. 6.15 – 6.18 (a-b), the particle detachment was captured from the actual ANSYS output and shown in figures numbered as (b). The opening of the crack faces could be clearly seen. This is, however, a deformation plot and is enlarged graphically for clarity. The actual crack opening and sliding displacement numbers are obtained from the program to calculate the stress intensity factors. It was found that a very small (two to three elements) length, of the bent crack faces, far away from the crack tip, come into contact during the second load step. Its effect is found to be negligible however as the deformations of the crack faces close to the crack tip are that
govern the stress intensity factors. It was also found that except for these two or three elements, the entire “bent crack face length” is stress free, thus signifying the ineffectiveness of the crack face friction coefficient. This result is the consequence of the presence of contact elements that accurately detect the contact between the crack faces. This signifies the importance of these elements for modeling the crack faces. Any other type of modelling with a priori assumption would produce erroneous results in the case of a mixed mode fracture.

6.6.3 Variation in Normal Loading p(x)

In the parametric study considered so far, the applied normal and tangential stress have remained fixed (for the distributions pertaining to 800N normal load in Table 6.1) but the crack geometry has changed. In this section, the applied loadings are changed with the following observations:

1. A change in the magnitude of the applied load (keeping the direction and the proportion of normal and shear the same) will not affect the crack turning angle for a given crack length and initial orientation (crack entry) angle. This is a consequence of the theory of linear elastic fracture mechanics. A wear particle will therefore be created with the same dimensions as above as long as the threshold stress intensity factor is still exceeded.

2. The Paris law is a non-linear relationship connecting the stress intensity factor and the crack growth for a specified number of cycles. The number of cycles required to create a wear particle will then change as the load changes, even though the size and shape of the particle does not change. The changes in the overall normal load $P_N$ from 800N through the values 700, 600, 500 and 400N respectively have been considered. The corresponding values of the tangential load are in each case $0.8P_N$ (Table 6.1).

For each of the loading summarized above, the analysis described in Sections 6.6.1–6.6.3 are performed for predicting the crack growth and wear particle formation. Each run has a specific initially assumed crack length and initially assumed crack orientation, (e.g., 30μm at 30°). Eight different crack lengths, (15μm, 20μm, 25μm, 30μm, 35 μm, 40 μm, 45 μm and 50 μm) are analyzed for six different orientations (15°, 20°, 25°, 35°, 40°, 45°), in addition to the 30° discussed earlier for 30 μm crack length. As expected, higher loads create higher stress intensity factors.
The resulting stress intensity factors are summarised by parametric plots in Figs. 6.19 (a-d) - Figs. 6.26 (a-d) for different crack lengths. Each sub-plot (i.e. a, b etc. in Fig 6.19 to Fig. 6.26) describes the variation of the stress intensity factors for two different load steps and modes I & II for different orientations are described as individual lines in the plots (15° - 45°). The mixed mode stress intensity factors are in line with expectations. The predominance of Mode II is clearly seen through the figures. The accompanying Mode I values are presented to illustrate the mixed mode nature of the problem and the ineffectiveness of the crack face friction coefficient in the results and clearly demonstrate the Mode II and accompanying Mode I stress intensity factors.

Figs. 6.27 (a-b) - Figs. 6.32(a-b) illustrate the effects of initial crack length and crack orientation on the number of cycles required for generating wear particles of the volume specified in Fig.6.34. Each of these four figures signifies a different initial crack length assumption, i.e. 25 µm, 30 µm etc. For the case of 15 µm and 20 µm initial crack length assumptions, for all orientations, and for the entire loading range considered, the resulting stress intensity factors are below the threshold value. The crack extension and the number of cycles for the extension are not applicable. From 25 µm onwards, the results are presented. It can be seen from the plots (Fig. 6.28 - 6.32), that as the loading increased, the number of cycles required for crack extension decreased. The crack extension for the second load step, i.e., the extension of the bent crack, is not a fixed amount as in the case of load step 1. This is consequent of the fact that the extension reaches the surface, even before an extension of half-the-crack length could be performed. The actual value is parametrically illustrated for different orientations in the spreadsheet Table 6.3 (on page 104).

6.6.4 Discussion of results

The number of cycles also decreases with the increase in the initial orientation of the angle. The non-linear nature of Paris law is evident from the curves depicting the number of cycles vs. the applied loading. The results are along expected lines and the number of cycles required is in line with the experimental observations of the present research and Knowles [1994] and Ko [1984] for similar situations. The results agree closely with the crack growth pattern seen in the experimentation within 700 cycles of loading.
numerical simulation, then, through the finite element analysis is validated. The results are also in agreement with the general nature of the crack growth laws. The results of mass loss, aspect ratio and the maximum depth of the particle, crack turn angles and the stress intensity factors are all presented in Fig. 6.33 and Table 6.3. The experimental work performed as part of the present research also validates the shape of the wear particle and aspect ratio, (Chapter 7).

In the experimental studies of dry sliding wear, surface cracks ranging from 15 to 50 μm in length and oriented at 15° to 45° in steps of 5° from the surface have been observed from worn specimens of an earlier study, [Knowles 1994 and Magel 1990]. The present Finite element analysis incorporated these parameters as initial input conditions to the model and performed calculations for the wear particle generation. The characteristics, such as the aspect ratio and volume of the estimated particles shown in Fig. 6.33 for a specific initially assumed orientation and length, appear to be realistic. This conclusion can be reached by comparing the aspect ratios of the wear particles, and the thickness of the particles with those obtained from experimentation (Knowles 1994; Magel 1990; Present study).

Even though the particle formation model is only two-dimensional, the estimated wear volumes based on the particle size and surface statistics are within the range of the experimental wear results obtained. The model shows, as expected, that the formation of the particle and the number of cycles required are affected by the original crack orientation and length. Fig. 6.33 and the accompanying table show that the calculated particle volume, maximum thickness of the particle and the number of cycles needed to form a particle all increase with increasing initial crack length and crack orientation. The results of Fig. 6.26(a-b) - Fig. 6.32(a-b) show that the number of cycles needed to generate a particle decrease as the applied contact loading is increased and that the rate of increase also decreases with increasing contact stresses. It was observed that below a normal force value of 400N, the stress intensity factor obtained is less than the threshold level of 6.0 MPa m°.5 and the crack ceases to propagate. Thus this value is the lower bound value in the analysis for normal loading. The upper bound value of 800N is from the values normally used for experimental wear studies [Knowles 1994 & Magel 1990].

The parametric study produced very similar trends in the change in particle volume, thickness and aspect ratio compared to the experimental work [Knowles 1994 and Present...
There may be several discontinuities in the sliding length and these may contribute to several wear particles during one pass. An approximate and useful calculation would be to compare the numerical results obtained above with those of long range experimental values for the same loading [Knowles 1994]. Referring to his experimental work, the volume obtained for a test similar to the present model, gives an average wear volume of $0.0000384 \text{ mm}^3$ to $0.0001253 \text{ mm}^3$ per cycle (upper and lower limits). A $30 \mu\text{m}$ crack with an orientation of $30^\circ$ would give rise to a volume of (based on the lower limit of the experiment) $0.015 \text{ mm}^3$. The numerical results obtained for the same are $0.008 \text{ mm}^3$ in $102+18 = 120$ cycles (see Table 6.3 and Fig.6.33), of volume loss pertaining to $30 \mu\text{m}$ and $30^\circ$. The numerical models predict the results reduced by a factor of $2$. If such an analysis is made for all orientations, the predictions are consistent and the factor falls between $2$ and $5$. This is probably due to the fact that only one crack was considered in the FE analysis, and was observed as to whether this crack could grow and form a wear particle. In reality, for the $2 \text{ mm}$ sliding length, $2 \sim 5$ such surface cracks may exist in the “wear track”. The factor can be incorporated into the wear calculation as a coefficient that is an estimate of the number of cracks present in the wear track. If the volumes obtained in the numerical models are multiplied by a factor of $2$ (lower bound) to $5$ (upper bound), the actual wear volume from the long range experiments could then be simulated quite accurately.

### 6.7 Summary

The wear mechanism of CGPD is described. The SIFs are evaluated as function of change in the displacements of the crack faces. A power law relating the SIFs and the number of cycles to failure is developed. The CTA for each situation is calculated based on FE results. The results agree favourably with the classical solutions where applicable. Through parametric variation of the crack geometry and applied loading, the mechanism of CGPD is established for different dry sliding wear situations. Two load steps are effected in FE analysis to detach the wear particle in the prescribed number of cycles. The volume of the wear particle and the number of cycles for particle detachment is calculated and the results are discussed in context with the experimental work carried out in Chapter 7.
### Table 6.3 Calculation for wear particle geometry

For notation, see Fig. 6.33

<table>
<thead>
<tr>
<th>c</th>
<th>d1</th>
<th>d0</th>
<th>dmt</th>
<th>dmt2</th>
<th>tin</th>
<th>t2</th>
<th>CDt</th>
<th>AE</th>
<th>FD</th>
<th>AD</th>
<th>vo</th>
<th>mass mg</th>
<th>AR</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>12.5</td>
<td>35</td>
<td>69</td>
<td>62</td>
<td>34</td>
<td>14,333</td>
<td>7,346</td>
<td>10.37</td>
<td>145.79</td>
<td>112.35</td>
<td>-2.813</td>
<td>0.000256</td>
<td>7.561</td>
</tr>
<tr>
<td>30</td>
<td>15</td>
<td>25</td>
<td>70</td>
<td>69</td>
<td>45</td>
<td>12,673</td>
<td>2.07</td>
<td>10.61</td>
<td>172.3</td>
<td>78.216</td>
<td>-0.951</td>
<td>0.000252</td>
<td>2.6360</td>
</tr>
<tr>
<td>30</td>
<td>15</td>
<td>30</td>
<td>69</td>
<td>65</td>
<td>39</td>
<td>14,993</td>
<td>5.557</td>
<td>11.66</td>
<td>194.8</td>
<td>119.81</td>
<td>-3.835</td>
<td>0.000311</td>
<td>1.7820</td>
</tr>
<tr>
<td>30</td>
<td>15</td>
<td>35</td>
<td>69</td>
<td>62</td>
<td>34</td>
<td>17.2</td>
<td>8.816</td>
<td>12.44</td>
<td>211.38</td>
<td>161.79</td>
<td>-4.051</td>
<td>0.000369</td>
<td>8.8633</td>
</tr>
<tr>
<td>30</td>
<td>15</td>
<td>40</td>
<td>69.5</td>
<td>59</td>
<td>29.5</td>
<td>19,275</td>
<td>11.89</td>
<td>13.06</td>
<td>221.55</td>
<td>203.48</td>
<td>1.8072</td>
<td>0.000427</td>
<td>11.987</td>
</tr>
<tr>
<td>35</td>
<td>17.5</td>
<td>25</td>
<td>70</td>
<td>69</td>
<td>45</td>
<td>14,785</td>
<td>2.415</td>
<td>12.38</td>
<td>234.51</td>
<td>106.46</td>
<td>-1.295</td>
<td>0.00034</td>
<td>6.2469</td>
</tr>
<tr>
<td>35</td>
<td>17.5</td>
<td>30</td>
<td>69</td>
<td>65</td>
<td>39</td>
<td>17,492</td>
<td>6.484</td>
<td>13.6</td>
<td>265.14</td>
<td>163.08</td>
<td>-5.22</td>
<td>0.000423</td>
<td>8.0803</td>
</tr>
<tr>
<td>35</td>
<td>17.5</td>
<td>35</td>
<td>69</td>
<td>62</td>
<td>34</td>
<td>20,068</td>
<td>10.28</td>
<td>14.51</td>
<td>287.72</td>
<td>220.21</td>
<td>-5.513</td>
<td>0.000502</td>
<td>10.341</td>
</tr>
<tr>
<td>35</td>
<td>17.5</td>
<td>40</td>
<td>69.5</td>
<td>59</td>
<td>29.5</td>
<td>22,488</td>
<td>13.67</td>
<td>15.23</td>
<td>301.59</td>
<td>276.97</td>
<td>2.9889</td>
<td>0.000881</td>
<td>13.890</td>
</tr>
<tr>
<td>35</td>
<td>17.5</td>
<td>45</td>
<td>69</td>
<td>59</td>
<td>24</td>
<td>24,785</td>
<td>17.62</td>
<td>15.99</td>
<td>306.25</td>
<td>338.66</td>
<td>19.185</td>
<td>0.000664</td>
<td>17.176</td>
</tr>
<tr>
<td>40</td>
<td>20</td>
<td>25</td>
<td>70</td>
<td>69</td>
<td>45</td>
<td>16,891</td>
<td>2.76</td>
<td>14.15</td>
<td>306.3</td>
<td>139.05</td>
<td>-1.691</td>
<td>0.000444</td>
<td>7.0301</td>
</tr>
<tr>
<td>40</td>
<td>20</td>
<td>30</td>
<td>69</td>
<td>65</td>
<td>39</td>
<td>19,961</td>
<td>7.41</td>
<td>15.55</td>
<td>346.3</td>
<td>213.7</td>
<td>-8.418</td>
<td>0.000552</td>
<td>7.6349</td>
</tr>
<tr>
<td>40</td>
<td>20</td>
<td>35</td>
<td>69</td>
<td>62</td>
<td>34</td>
<td>22,933</td>
<td>11.75</td>
<td>16.58</td>
<td>375.79</td>
<td>287.63</td>
<td>-7.201</td>
<td>0.000656</td>
<td>11.916</td>
</tr>
<tr>
<td>40</td>
<td>20</td>
<td>40</td>
<td>69.5</td>
<td>59</td>
<td>29.5</td>
<td>25,701</td>
<td>15.86</td>
<td>17.41</td>
<td>392.87</td>
<td>361.75</td>
<td>3.3906</td>
<td>0.000759</td>
<td>15.826</td>
</tr>
<tr>
<td>40</td>
<td>20</td>
<td>45</td>
<td>69</td>
<td>59</td>
<td>24</td>
<td>28,273</td>
<td>20.14</td>
<td>18.27</td>
<td>400.42</td>
<td>442.34</td>
<td>25.058</td>
<td>0.000867</td>
<td>20.295</td>
</tr>
<tr>
<td>45</td>
<td>22.5</td>
<td>25</td>
<td>70</td>
<td>69</td>
<td>45</td>
<td>19,009</td>
<td>3.105</td>
<td>15.92</td>
<td>387.67</td>
<td>175.99</td>
<td>-2.141</td>
<td>0.000562</td>
<td>3.3970</td>
</tr>
<tr>
<td>45</td>
<td>22.5</td>
<td>30</td>
<td>69</td>
<td>65</td>
<td>39</td>
<td>22.49</td>
<td>8.336</td>
<td>17.49</td>
<td>438.29</td>
<td>260.58</td>
<td>-6.629</td>
<td>0.000698</td>
<td>16.292</td>
</tr>
<tr>
<td>45</td>
<td>22.5</td>
<td>35</td>
<td>69</td>
<td>62</td>
<td>34</td>
<td>25.8</td>
<td>13.22</td>
<td>18.66</td>
<td>475.61</td>
<td>364.03</td>
<td>-9.114</td>
<td>0.000831</td>
<td>15.206</td>
</tr>
<tr>
<td>45</td>
<td>22.5</td>
<td>40</td>
<td>69.5</td>
<td>59</td>
<td>29.5</td>
<td>28,913</td>
<td>17.84</td>
<td>19.59</td>
<td>498.5</td>
<td>457.84</td>
<td>4.2912</td>
<td>0.000961</td>
<td>17.645</td>
</tr>
<tr>
<td>45</td>
<td>22.5</td>
<td>45</td>
<td>69</td>
<td>59</td>
<td>24</td>
<td>31,804</td>
<td>22.66</td>
<td>20.56</td>
<td>506.25</td>
<td>559.83</td>
<td>31.715</td>
<td>0.001098</td>
<td>22.832</td>
</tr>
<tr>
<td>50</td>
<td>25</td>
<td>25</td>
<td>70</td>
<td>69</td>
<td>45</td>
<td>21,121</td>
<td>3.458</td>
<td>17.68</td>
<td>478.6</td>
<td>217.27</td>
<td>-2.634</td>
<td>0.000693</td>
<td>3.7251</td>
</tr>
<tr>
<td>50</td>
<td>25</td>
<td>30</td>
<td>69</td>
<td>65</td>
<td>39</td>
<td>24,869</td>
<td>9.292</td>
<td>19.43</td>
<td>541.3</td>
<td>322.82</td>
<td>-10.65</td>
<td>0.000863</td>
<td>9.5438</td>
</tr>
<tr>
<td>50</td>
<td>25</td>
<td>35</td>
<td>69</td>
<td>62</td>
<td>34</td>
<td>28,660</td>
<td>14.69</td>
<td>20.73</td>
<td>587.18</td>
<td>449.42</td>
<td>-1.125</td>
<td>0.001025</td>
<td>14.773</td>
</tr>
<tr>
<td>50</td>
<td>25</td>
<td>40</td>
<td>69.5</td>
<td>59</td>
<td>29.5</td>
<td>32,126</td>
<td>19.82</td>
<td>21.76</td>
<td>615.43</td>
<td>565.24</td>
<td>5.2977</td>
<td>0.001186</td>
<td>19.028</td>
</tr>
<tr>
<td>50</td>
<td>25</td>
<td>45</td>
<td>69</td>
<td>59</td>
<td>24</td>
<td>35,341</td>
<td>25.18</td>
<td>22.84</td>
<td>625.62</td>
<td>691.15</td>
<td>38.154</td>
<td>0.001555</td>
<td>25.369</td>
</tr>
</tbody>
</table>

A.R. Aspect Ratio
Experimental Investigation of Sliding Wear

7.1 Introduction

An experimental investigation of sliding wear of stainless steel specimens (type SAE 410) vs. a hardened carbon steel specimen was carried out using a Cameron-Plint reciprocating wear test rig in the Tribology Laboratory at the National Research Council of Canada in Vancouver. The results of these experiments are used for comparison with the predictions reported in Chapters 4-6. The reciprocating sliding rig used for the experiments is shown in Fig. 7.1. For the wear tests, a hard cylinder was pressed against a softer smooth disc, and then subjected to a reciprocating sliding motion. This loading configuration was maintained for all the operating conditions in the experimental work. A description of the important components of the apparatus, along with the procedure used for wear testing is explained in the following sections.

Two different disc specimens, made of stainless steel SAE type 410 hardness levels Rc35 and Rc45, were used. The moving cylindrical specimen, made of carbon steel, was hardened to Rc55. The test conditions include two numbers of cyclic duration, 60 and 720 cycles; two normal loads, 200 N and 300N; and two friction bands, 0.1<μ<0.2 and 0.5<μ<0.8. These bands were achieved by using a lubricant made of cutting fluid and water (1:1000) for μ between 0.1-0.2, whereas dry sliding gave friction levels between 0.5 and 0.8. Two specimens were used for each combination of the above operating parameters resulting in a total of 32 tests. Mass loss for all wear tests was obtained by measuring the specimens before and after the tests, using a sensitive electronic balance. Frictional (tangential) force was measured by a force transducer, and was recorded on a chart recorder. Once the wear test was completed, the specimens were cleaned and the wear tracks were photographed at different levels of magnification for further analysis. The specimens were then sectioned, polished, and etched in order to examine the
subsurface conditions created during the wear process. The experimental methods and the results of the subsequent experimental investigation are now described.

7.2 Apparatus and specimen geometry

The apparatus is mounted on two separated cast iron work-benches, on one of which is mounted the stationary specimen and its assembly including a specimen trough and a force transducer. Mounted on the other bench is an electric motor, which drives a “Scotch yoke mechanism”. The scotch yoke mechanism converts the rotary motion of the electric motor into reciprocating motion using an adjustable eccentric cam. The rotary speed of the electric motor is reduced by means of a gearbox located between the electric motor and the scotch yoke mechanism. The speed of the d.c. motor can be varied and, if necessary, a gearbox with a different set of gear ratio can be used to further vary the speed range. A self-aligning mounting assembly, which houses the cylindrical specimen, is attached to the open end of the reciprocating shaft. The normal load is applied to the dynamic specimen assembly by way of a cylindrical roller attached on the top of the assembly, thus ensuring the applied load reacts normally onto the specimens. The loading arrangement is activated using a lever mechanism. The stationary specimen trough is mounted on a series of flexures enabling the friction force at the interface of the stationary and dynamic specimens to be monitored by a force transducer, which is pressed against the trough in the direction of motion, Fig.7.1. The measured force is recorded on chart as well as in a computer. The motor controller starts and automatically stops the motor after a pre-set number of cycles.

7.3 Experimental investigation scheme

7.3.1 Specimen Preparation

Stationary Disc Specimens

The specimens were cut from a SAE 410 stainless steel rod 22.2 mm (0.875 in.) in diameter with a hardness Rc 35. They were ground to a surface finish value of $R_a = 20-30 \mu m$. Sixteen of these specimens were heat treated to a higher hardness level Rc 45. All of the specimens were then cleaned in an ultrasonic bath with soap, water and
ethanol, and finally dried in air. They are then polished to a surface finish value of $R_s = 15\mu$, then $6\mu$, and finally $1\mu$. During this polishing procedure, the specimens were cleaned in the ultrasonic bath after each step extreme care was taken to ensure that the surface was void of contaminated particles. On completing the polishing operation, the specimens were stored in a dessicator until needed.

**Dynamic Cylindrical Specimens**

A cylinder from the carbon steel bar was machined to 19mm diameter and then cut to a length of 12.7 mm. This cylinder was much harder than the discs, so experienced negligible wear during the tests. This one cylinder was used for all the tests but was rotated to a fresh contact surface for each test.

### 7.4 Wear testing

A single cleaned disc, hardness value Rc35, was first weighed using a sensitive electronic balance. This stationary specimen was then inserted into the specimen bracket and held in place by two screws, which hold the specimen only by contact pressure exerted through its flange along the edges of the circular disc specimen, Fig.7.2. The cylindrical specimen was similarly weighed using the balance and its initial mass reading was recorded separately in Table.7.1. The cylindrical specimen was mounted onto its mounting bracket as shown in Fig.7.1-b. As described in an earlier section, the cylindrical specimen is an integral part of the reciprocating arm that is driven by the scotch yoke mechanism. A normal load was then applied, through the lever mechanism and the roller on top of the mounting bracket, onto the specimens.

The whole experimental rig was then initialised to run a specified number of cycles, e.g., 60 or 720 loading cycles. The chart recorder was calibrated to give a value of 50N/div. along the y-axis and 6 cycles /div. for the x-axis. The motor speed was set to give a cycling rate at 0.1 Hz using a 200:1 gear-box. The electronic counter was programmed to automatically start and stop after a pre-set number of loading cycles for a given test. After the test, the specimens were removed and cleaned in the ultrasonic bath following the procedures described in the earlier section. Both specimens were weighed and their values were recorded in Table 7.1, which also listed the mass losses, i.e., the
differences in specimen weights before and after the test. The specimens were then stored for surface examinations later. This experimental procedure was repeated for different operating conditions in order to study the effects of various parameters on the wearing process.

7.5 Test parameters

The test program was designed to study the effects of normal load, number of cycles, surface hardness and interface friction coefficient. Therefore the test parameters include two friction levels, 0.1-0.2 and 0.6-0.8; two load levels, 200N and 300N; two numbers of test cycles, 60 and 720 cycles; and two hardness levels, Rc35 and Rc45. This requires 16 tests to cover each of these variables once. When the number for an identical set of repeated tests was added, the total number of tests was 32. For the tests that were decided to study the wear mechanisms during low friction sliding, the trough that housed the stationary disc specimen was filled with a water based fluid containing 1000 parts of water to 1 part of cutting fluid. This light lubrication resulted in lowering the coefficient of friction to 0.1-0.2 from 0.6-0.8 for dry sliding with relatively heavy normal loads.

7.6 Microstructure analysis

7.6.1 Surface and Sub-Surface Metallography

The worn disc specimens were examined with a scanning electron microscope. Initially the wear tracks on the surface were examined under magnifications of 50x and 500x. They revealed the nature of wearing process under different operating parameters. These photographs are described in the following sections along with the operating conditions for the tests.

Later, the specimens were sectioned along a plane parallel to the wear track in order to examine for evidence of subsurface crack growth and wear particle formation. The specimens were cut using a diamond cutting wheel and then were mounted on epoxy by moulding under high temperature and pressure. The sectioned surface was polished to $R_a = 0.25 \mu$. and etched. Again, these sections were examined in the SEM under high magnifications.
Experimental Results and Discussion

7.7.1 Mass loss of worn specimens

The main objective of the experimental tests is to investigate the wear processes and the wear mechanisms involved when a hardened cylinder slides against a softer flat surface under various loading and frictional conditions. Owing to the relatively small number of cycles used in these tests, the resultant mass losses are small, particularly under sliding conditions with diluted cutting fluid for lubricant. The mass loss results listed in Table 7.1 are used to provide an assessment of the effects of these various parameters along with the long duration tests run by Knowles [1994] for further comparison with numerical prediction.

In general, the results are in agreement with the established trends, which show that mass loss increases with increased normal load and larger number of sliding cycles under both dry and lightly lubricated conditions. Significantly higher mass losses were observed under dry sliding conditions than under lightly lubricated conditions. This latter observation helped to confirm the results of the metallographic examinations (will be discussed in a later section), which show that two different wear mechanisms were involved in the dry and lightly lubricated sliding conditions. Fracture and delamination can result in severe wear with substantial mass losses whereas mild abrasive wear results in low mass losses.

Figures 7.3 and 7.4 show respectively the mass loss results of the softer (Rc35) series-A and harder (Rc45) series-B specimens. The results show that the mass losses of the harder specimens, after 720 cycles, were of an order of magnitude higher than their softer counterparts under high load and dry sliding conditions. On the other hand, the harder specimens showed negligible mass losses under mild abrasive wear conditions. It is noted that the harder discs were re-heat-treated to increase their hardness from Rc35 to Rc45, the process might have created more micro-cracks on the surface and sub-surface resulting in early particle formation and delamination.

7.7.2 Coefficient of Friction

Fig. 7.5 shows friction coefficient vs. load cycle plots for the two series of tests. The results clearly show two distinctive bands of friction coefficients; one from 0.6 to 0.8 due
to dry sliding, and the other between 0.1 and 0.2 due to sliding in diluted cutting fluid. The coefficient of friction appears to be consistent throughout the test duration. Tests with the lower normal load, 200N, seem to show slightly lower coefficients of friction under both dry and lightly lubricated conditions; this is particularly apparent in the series of tests with harder discs; the coefficients of friction dropped to 0.6 from 0.8, and to 0.1 from 0.17.

7.7.3 Surface examinations

Fig. 7.6 (a-b) shows a series of photomicrographs of the worn areas of specimens from sliding in diluted cutting fluid. In almost all the cases, very fine scratch marks in the direction of sliding can be observed on the worn surfaces indicating that a mild form of abrasive wear is the predominant wear mechanism under these load and sliding conditions. Figure 7.6c shows another series of photomicrographs from dry sliding tests. Figures 7.7 to 7.12 show some more photomicrographs for similar situations. These photos are under different operating conditions on series A and series B specimens. They all depict delamination wear under dry sliding conditions and mild wear under wet sliding conditions. The micrographs reveal more severely damaged scars, evidence of delamination and material transfer can be observed. Similar observations have been reported in many early studies, in particular, Knowles [1994] and reported the results in his thesis and later in a published paper [Ko, Knowles and Vaughan, 1996].

**Sectioned view of specimens**

The severely worn specimens were further examined by sectioning the specimens along their wear scars. Figures 7.13a-7.13d show photomicrographs from the 200N and 300N tests with Rc35 and Rc45 discs. The softer Rc35 disc showed several sub-surface micro-cracks, Figs. 7.13 (a & b) whereas fairly extended near surface cracks can be clearly seen on the harder Rc45 (Figs. 7.13 c & d). These micro-cracks are likely to have been initiated from the surface, then nucleated through the near surface layer and returned to the surface forming a wear sheet or wear particle, Figure 7.13(c). Figures 7.14 - 7.19 show close-up views of the cross-sections of several specimens. Figures 7.20 – 7.23 show the enhanced sectioned specimens that were seen in Figures 7.14 - 7.19. These figures illustrate the surface and sub-surface micro-cracks and the nearly detached particles. It is
noted that the shapes of cross-sections of several detached particles are shaped as quadrilaterals indicating the crack path turned twice before reaching the surface.

7.8 Summary

An experimental study for the reciprocating sliding wear was performed using a standard wear testing apparatus. Mass loss and tangential force for the wearing process were measured to quantify the wear process. Surface and sectional metallographic examination was conducted. Two different wear mechanisms pertaining to all operating conditions were identified, i.e., abrasive mild wear for lubricated and dry sliding conditions and severe delamination wear for dry sliding conditions.
CHAPTER 8

Comparison of Predicted Results with Experimental Work

8.1 Introduction

The numerical model for sliding wear, illustrated in Fig.1.1, containing four distinct mechanisms, namely LCF, RF, MW, and CGPD was utilised to predict wear under a variety of situations in Chapters 4, 5 and 6. These predictive mechanisms of the model, yielded results for wear volume and the number of cycles to failure that are normally encountered in sliding wear testing. Comparison with experimental data was made in the individual chapters. In the sections that follow, a summary of the numerical results and the comparison with experimental observations are given. Experimental comparisons are made wherever possible in the present research (Chapter 7). In addition, experimental data generated at the National Research Council of Canada in Vancouver for sliding wear situations and experimental observations available from literature are also used for comparison.

8.2 Low Cycle Fatigue (LCF) & Ratchetting Failure (RF)

Table 8.1 presents the results of wear volume and number of cycles to failure predicted by the ‘LCF’ and ‘RF’ mechanisms. The actual numerical simulations were carried out in Chapter 4.

Table 8.1 predicted wear volume and number of cycles to particle detachment

<table>
<thead>
<tr>
<th>CASES</th>
<th>Applied Pressure (\text{P}_r, \text{P}_c), MPa</th>
<th>Contact Pressure (\text{P}_0), GPa</th>
<th>Friction Coefficient</th>
<th>Mechanism of wear</th>
<th>(\Delta \varepsilon_f) or (\Delta \varepsilon_r)</th>
<th>Number of cycles to failure</th>
<th>Wear Volume, mm(^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CASE IA</td>
<td>69</td>
<td>2.068</td>
<td>0.0</td>
<td>Ratchetting</td>
<td>0.01639</td>
<td>76</td>
<td>N/A</td>
</tr>
<tr>
<td>CASE IB</td>
<td>69</td>
<td>2.068</td>
<td>0.1</td>
<td>Ratchetting</td>
<td>0.054</td>
<td>23</td>
<td>0.226</td>
</tr>
<tr>
<td>CASE IC</td>
<td>69</td>
<td>2.068</td>
<td>0.3</td>
<td>Ratchetting</td>
<td>0.1101</td>
<td>11</td>
<td>0.226</td>
</tr>
<tr>
<td>CASE ID</td>
<td>69</td>
<td>2.068</td>
<td>0.5</td>
<td>Ratchetting</td>
<td>0.1454</td>
<td>9</td>
<td>0.226</td>
</tr>
<tr>
<td>CASE IE</td>
<td>69</td>
<td>2.068</td>
<td>0.7</td>
<td>Ratchetting</td>
<td>0.21</td>
<td>6</td>
<td>0.226</td>
</tr>
<tr>
<td>CASE IIA</td>
<td>14</td>
<td>0.93</td>
<td>0.0</td>
<td>Low cycle fatigue</td>
<td>0.001</td>
<td>810000</td>
<td>N/A</td>
</tr>
<tr>
<td>CASE IIB</td>
<td>14</td>
<td>0.93</td>
<td>0.1</td>
<td>Low cycle fatigue</td>
<td>0.00197</td>
<td>673100</td>
<td>0.026</td>
</tr>
<tr>
<td>CASE IIC</td>
<td>14</td>
<td>0.93</td>
<td>0.3</td>
<td>Ratchetting</td>
<td>0.001869</td>
<td>669</td>
<td>0.1</td>
</tr>
<tr>
<td>CASE IID</td>
<td>14</td>
<td>0.93</td>
<td>0.5</td>
<td>Ratchetting</td>
<td>0.0352</td>
<td>355</td>
<td>0.1</td>
</tr>
<tr>
<td>CASE IIE</td>
<td>14</td>
<td>0.93</td>
<td>0.7</td>
<td>Ratchetting</td>
<td>0.02688</td>
<td>46</td>
<td>0.1</td>
</tr>
</tbody>
</table>
It is seen from Table 8.1 that the number of cycles to detach a wear particle through the mechanism of plastic ratchetting range from 6 to 76 depending on the level of friction coefficient for CASE I loading. Kapoor [1994] obtained similar values in his experimental work for comparable loading levels. Challen and Oxley [1986] obtained between 3 and 39 cycles to failure corresponding to ratchetting strains of 0.73 to 0.064. On comparing these values with those in Table 8.1, the RF mechanisms of this thesis predicts the ratchetting cycles between 6 and 76 corresponding to ratchetting strains of 0.21 to 0.01639.

The wear groove depth obtained for LCF is 0.014 mm for CASE I loading, which compares very well with Magel [1990] who obtained experimental values for particle depths up to 0.02 mm for similar loading levels ($p_0/k \sim 7$). The number of cycles calculated through experimentation for severe wear with surface plasticity in Knowles [1994] were 10 and 20 cycles for similar load levels (CASE I). For CASE II loading, the calculated depth is 0.007 mm which is normally observed in the experimental studies [Knowles 1994; Magel 1990]. The value of 0.026 mm$^3$ obtained for wear volume in such large number of cycles seems reasonable for the fatigue type failure and agrees with the experimental work of Knowles [1994].

The macro contact geometry used in the above experimentation was one of sphere on flat as against cylinder on flat configuration used for the present simulation. Their contact configuration had very similar nominal contact area and asperity density distribution, however, as obtained in the present study. The loading levels were normalised to the shear yield strength for comparison purposes. As a result, the predictive model is seen to calculate the wear volume and wear particle geometry correctly for the case of RF and LCF.

8.3 Mild Wear

The mild wear model described in Chapter 5 predicts the depth of grooves that form by the action of abrading asperities. These asperities quickly attain conformal contact as explained in Chapter 5 leading to ESL. The wear volumes predicted for one cycle leading to conformal contact are summarised in Table 5.2. These results were seen to
compare well with the very low values of mass loss seen in Fig. 7.3 and 7.4 of Chapter 7 for wet sliding conditions.

8.4 Crack Growth and Particle Detachment (CGPD)

The results of the experimental work carried out in Chapter 7 are compared with the numerical prediction of wear particle geometry/wear volume through CGPD. The Wear volume for the detached particles from experimentation were calculated based on the micrographs of the sectioned specimens for the depth of the particles. The volume of wear particles and the aspect ratios of the predicted results are calculated as explained in Chapter 6. The following table details the predicted results for particle volume, aspect ratio and the number of cycles for particle generation in one particular loading condition ($p_0/k = 2$). Several orientations of crack geometry and loading conditions were seen to produce very similar results validating the robustness of the model as seen from Table 6.3 (Chapter 6).

Table 8.2 Crack growth and particle formation

<table>
<thead>
<tr>
<th>Crack entry-angle (deg.)</th>
<th>$d_{c1}$, $d_{c2}$ μm</th>
<th>$d_0$, deg.</th>
<th>$N_1$, cycles</th>
<th>Aspect ratio*</th>
<th>Volume, mm$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>N/A</td>
<td>70</td>
<td>$N_2$=N/A</td>
<td>N/A</td>
<td>-</td>
</tr>
<tr>
<td>15</td>
<td>N/A</td>
<td>N/A</td>
<td>$N_2$=N/A</td>
<td>N/A</td>
<td>no particle</td>
</tr>
<tr>
<td>20</td>
<td>N/A</td>
<td>70</td>
<td>$N_2$=N/A</td>
<td>N/A</td>
<td>no particle</td>
</tr>
<tr>
<td>25</td>
<td>2.27</td>
<td>69</td>
<td>$N_1$=116</td>
<td></td>
<td>0.0032</td>
</tr>
<tr>
<td>30</td>
<td>5.7</td>
<td>65</td>
<td>$N_2$=18</td>
<td>2.42</td>
<td>0.004</td>
</tr>
<tr>
<td>35</td>
<td>8.9</td>
<td>62</td>
<td>$N_1$=78</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>40</td>
<td>15</td>
<td>69.5</td>
<td>$N_2$=53</td>
<td>1.9</td>
<td>0.00545</td>
</tr>
<tr>
<td>45</td>
<td>N/A</td>
<td>59</td>
<td>$N_2$=N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

* ratio of (length of wear particle/ maximum depth of wear particle).
N/A - No particle formation since the SIF was below the threshold value.
$dc_1$ & $dc_2$ are crack extensions.
$d_0$ is the crack turn angle.
$N_1$ and $N_2$ are the number of cycles required for the extension.
The predicted volume is halved, since the dimension in plane strain direction correspond to 12.7 mm in the experimental studies.
The shapes of the particles were quadrilateral as seen in experimental work, Chapter 7. The length and width of the particles were also measured and the volumes of the particles (shaped as a quadrilateral) were calculated. The volume was in the range of 0.0010 mm$^3$ to 0.004 mm$^3$. The aspect ratios of the particles were seen to be around 2 to 3. The results obtained through the predictive models agreed well with those obtained from experimentation for mass loss, number of wear cycles required to produce the wear volume and the geometry of wear particles. Thus the finite element models could therefore satisfactorily be applied to predict the wear characteristics in similar types of wear situations that are encountered in industrial applications.

8.5 Summary

Results from the numerical simulations are summarised for the mechanisms of LCF, RF, MW and CGPD. The wear volume and the number of cycles to failure are compared with the experimental work that is carried out as part of the present research in addition to the data already available. The wear particle aspect ratios, wear volumes, the number of cycles to failure compare well with the experimental work validating the sliding wear model developed in this thesis.
CHAPTER 9

Conclusions and Future Scope of Work

A predictive model for reciprocating sliding wear is developed and shown to calculate the characteristics of wear particles observed through experimentation. This model branches into one of several mechanisms, LCF, RF, MW or CGPD depending on the type of contact configuration and loading conditions. The finite element technique is used for simulating the mechanisms of LCF, RF, and CGPD. Classical equations are developed for predicting MW. This model can be expanded to study the fretting wear of tubes in the nuclear steam generator. The exact geometry (asperity level) and material combinations need to be input to the model and the results could thus be compared to the experimental values.

The numerical and the experimental studies reveal that there are more than one wear mechanism that come into play for the same contact configuration, at different regions of the shakedown map. This observation was also made in previous experimental studies and it was an important numerical modelling consideration in the present thesis while branching off into the four distinct wear mechanisms.

The CGPD approach for modelling particle detachment is a unique approach in this thesis. The crack growth was accurately modelled and the CTA was predicted. In other studies, the crack growth was not continuously modelled, thus leading one to speculate after the calculation of SIF in the first load step. The finite element analysis showed that the formation of particles and their characteristics are sensitive to the applied load, initial crack length and crack orientation. The particle size increased with the increase in loading, crack length and angle of orientation from the surface. The modelled wear particles provide reasonable wear volume prediction in comparison with the present and past experimental results of similar materials. The mild wear model produced grooves in the first cycle and showed that the contact system subsequently shakes down elastically. The finite element program and wear particle geometry prediction algorithms are automated within a computer. Once knowing the input parameters of a particular wear
system, a designer can then branch into the appropriate wear mechanism and the volume of wear, hence the mass loss could be estimated through the predictive model developed in the present thesis, Fig. 1.1.

Important future considerations:

1. The numerical models are 2D models under plane strain conditions. The present model could be extended to 3D analysis with minimal modifications to the FE algorithms. The asperities could be modelled as spheres and hence the elliptical planform observed under experimental conditions could be simulated. ANSYS [1998] is capable of handling this aspect of modelling, however, the hardware platforms on which the program is run, need to be updated for memory capacity and CPU speed.

2. The sliding wear model comprising the four mechanisms is a static analysis model, i.e. no dynamic effects are considered. For industrial problems with significant dynamic loading, this aspect needs to be addressed. This could be accomplished by incorporating the dynamic effects into the finite element simulation. ANSYS [1998] is quite capable of handling this aspect.

3. On the experimental side, the specimens could be polished using an automated polisher to save time. Further, the effect of temperature on the specimens could be investigated by running the tests in an autoclave under different temperature and pressure to simulate the pressure vessel industry problems of tube to tube support contact.
REFERENCES


2. ASME B&PV Code, Section XI, Section II, Section III, 1992 Ed. with 1994 Addenda and Appendices.


47. Greenwood, 1984


101. Longuet-Higgins 1957a &b


114. Nayak, 1971


143. Tanaka, K., “Fatigue Crack Propagation from a Crack Inclined to the Cyclic Tensile Axis”,
144. Tallian, T.E., Chiu, Y.P., and Van Amerongen, E., “Prediction of Traction and Microgeometry
Effects on Rolling Contact Fatigue Life”, ASME Journal of Lubrication Technology, Vol.100,
1978, pp.156-165.
pp.45-54.
150. Vingsbo O. and Hogmark, S., Wear of Steels, Fundamentals of Friction and Wear of Materials,
152. Welsh, N.C., The Dry Wear of Tool Steel. I. The General Pattern of Behavior; II. Interpretation of
Delhi.
Figures
Inputs to FE model for Contact deformation

1. Geometry of wearing components
2. Material properties of wearing components
3. Boundary conditions
4. Normal loading and friction coefficient
5. Contact stiffness (normal and tangential) to avoid overlap of finite element meshes

Contact mechanics equations
Compare elastic portion of the contact analysis from FE model with Hertzian equations [Johnson 1985]

Are the loads and friction levels high enough to cause surface / near surface plastic deformation?

YES  NO

Is the effective strain cycle above plastic shakedown causing ratchetting?

YES  NO

Ratcheting Failure mechanism for wear (RF)
Low Cycle Fatigue mechanism for wear in plastic shakedown conditions (LCF)
Abrasive Mild Wear (in Elastic shakedown Conditions) (MW)

Second FE model and crack growth analysis through LEFM

Output from FE analysis

Is the stress intensity factor in Mode I and / or Mode II above the threshold value?

YES  NO

Crack Growth in a direction perpendicular to the maximum tensile stress and wear through Particle Detachment (CGPD)

Fig. 1.1 Proposed model for predicting sliding wear of mechanical components
Fig. 2.1 Schematic diagram showing the adhesive transfer of a thin flake-like wear particle (a) and the adhesive transfer of a wedge-like wear particle (b) [Kayaba and Kato 1981].

Fig. 2.2 Relationship between the critical tip angle $\theta_c$ and the hardness ratio $r$. $f$ is zero in (a) and changes from zero to 1.0 in (b) [Kayaba and Kato 1983].
Fig. 2.3 Schematic sketches of three modes of abrasive wear observed by SEM and cross-sectional profiles of grooves: (a) cutting mode, steel pin on brass plate; (b) wedge forming mode, steel pin on stainless steel plate; (c) ploughing mode, steel pin on brass plate [Hokkirigawa and Kato 1988].

Fig. 2.4 Schematic sketches of formation of 'filmy wear' particles observed by SEM. (a) Hard ball with ground flat sliding on softer ground surface. Sliding perpendicular (b) and parallel (c) to soft asperities [Alagaki and Kato 1987].

Fig. 2.5 Archard non-dimensional wear coefficient, $K$, as a function of $\psi$, the plasticity index in repeated sliding for various values of normalized surface separation $d/s$ [Kapoor et al., 1995].
Fig. 3.1 A schematic illustrating the governing mechanisms of elastic shakedown, plastic shakedown and ratcheting during cyclic loading.

Fig. 3.2 Shakedown map for line contact. Below Curve A elastic deformation results. Curve B is the shakedown limit for perfectly plastic material. Curve C is the shakedown limit for kinematically hardening material. [Kapoor et al 1996]
Fig. 3.3  Bauchinger effect illustrating that yield takes place at a lower value when there is reversal of stress.

Fig. 3.4  Results of ratchetting tests: Load controlled, mild steel from Benham and Ford [1961]; strain controlled, copper from Coffin [1960 & 1970].
Fig. 3.5 Experimental evidence for low cycle fatigue and ratcheting failure [Kapoor 1994].

Fig. 3.6 Top view of a worn specimen under lubrication depicts no crack growth or delamination [Knowles 1994].
Fig. 3.7. Microscopic examination of worn specimens (sectioned) indicating sub-surface plastic deformation and surface plasticity leading to delaminated wear sheets [Magel 1990].

Fig. 3.8 A Schematic illustrating the integration limits and shakedown depth.
Surface Statistics Equations for Quantification of Rough Surfaces

Equivalent cylindrical asperity

FE model (2D) for contact of equivalent asperity with a smooth half-plane

Compare elastic contact stresses with Hertzian equations [Johnson 1985]

Output from FE analysis

1. Deformations (u_i)
2. Strains (e_{ij})
3. Stresses (\sigma_{ij})

Computer Database / Results / Plots

Is p_{ij}/k < 4.0

YES

Plastic Shakdown

WEAR MECHANISM LCF

\Delta e_i = f(e_{ij})

N^f = \frac{E_t}{AE^f}

\Delta e_i = \frac{e_i}{N_i}

N^f = \frac{2E_t}{AE^f}\n
V = f(tangential work)

NO

WEAR MECHANISM is RF

\Delta e_r = f(e_{ij})

N_r = \frac{e_r}{\Delta e_r}

V = d \cdot L \cdot b

\Delta e_i is the Steady state accumulation of ratchetting strain per cycle

N_r is the number of cycles to failure in RF mode

V is wear volume, function of tangential work

\Delta e_i is the alternating strain per cycle

N_f is the number of cycles to failure in LCF mode

V is the wear volume, function of tangential work

Fig. 3.9 Sliding wear model predicting wear volume and number of cycles through RF and LCF.
Fig. 3.10 Magnified view of the worn specimen illustrating mild wear [Knowles 1994].

Fig. 3.11 Mass loss for dry sliding wear and sliding wear under lubrication with water : cutting fluid = 1000:1). The curves above clearly show two distinct regimes depending upon the interface friction coefficient levels.
Fig. 3.12 Circular contact shakedown map for a kinematic hardening material [Knowles 1994]

Fig. 3.13 Sliding wear prediction through the mechanism of Mild Wear (MW).

Surface Statistics Equations for Quantification of Rough Surfaces

For $2.87 < p/k < 3.0; \mu = 0.1$

Asperities are modelled as hemispheres within the nominal area of contact

$z = f(t, \text{tangential work, } \phi)$

$\phi = \text{fraction of total tangential work done during mild wear}$

After two cycles of Elastic Shakedown Conditions Wear grooves are in a state of elastic state, i.e., below EL.
Fig. 3.14 Magnified top view of wear particle formation illustrated in Fig. 4.3 [Suh 1986].

Fig. 3.15 Sectioned view of a worn specimen under dry sliding conditions [Knowles 1994].
Fig. 3.16 Value of friction coefficient for dry sliding conditions (Knowles, G., (1994)).

Fig. 3.17 Sectioned view of dry sliding specimen (500 X magnification) depicting crack growth under dry sliding conditions [present study].
Fig. 3.18 Variation of friction coefficient with number of cycles for 60 cycle test duration. (Lubricated and dry sliding conditions are represented separately [Present study].)

Fig. 3.19 Sliding wear prediction through the mechanism of Crack Growth and Particle Detachment (CGPD).
Fig. 4.1 Experimental mapping of rough surface using Talysurf (indicating random variation of asperity heights). (a) Longitudinal Talysurf trace and (b) Transverse Talysurf trace.
Fig. 4.2  Height distribution $\phi(z)$ and 'bearing area' curve given by the cumulative height distribution $\Phi(z)$ [Johnson 1985].
Fig. 4.3 Cumulative height distributions plotted on normal probability paper: solid circle - surface heights; cross - peak heights; triangle - summit heights. (a) Bead-blasted aluminium, (b) mild steel abraded and polished, [Williamson 1967, 1968].
Fig. 4.4 Contact of a randomly rough surface with a smooth flat surface at a separation [Johnson 1985].

(a) Engineering rough surface of second body in contact

Radius of curvature of individual asperity, $R_i$

(b) Idealization of asperities shown in (a) with regular geometry having radii of curvature

Fig. 4.5 Schematic illustrating the idealization of engineering rough surfaces in contact.
Idealized asperities as cylinders of radii $r_j$

(a) Regular geometry such as cylinders (2D) are used for further simplification [Greenwood & Williamson 1966]

Idealized asperities as cylinders of radii $r$, with equivalent roughness in contact with a smooth plane

(b) Asperities having equivalent roughness in contact with a smooth plane

Fig. 4.6 Idealization of asperities based on Greenwood and Williamson [1966].
Fig. 4.7a. A cylindrical component in contact with a wearing disc.

Fig. 4.7b A schematic depicting the nominal area of contact.
Axial direction of the cylindrical component (Direction of plane strain)

Only one spherical (3D) asperity come in contact within the nominal contact width ($2a_0$)

76 spherical (3D) asperities are in contact along the length of nominal contact area ($l_o$)

Wearing disc (SAE 410 s.s., Elastic-Plastic properties)

Fig. 4.8a. A schematic illustrating one asperity in contact along the sliding direction and 76 asperities in contact along the length of the cylinder (plane strain direction).

An equivalent cylindrical asperity of length $l_o$ and radius $R_{asp}$ representing all the 76 (3D) spherical asperities

Wearing disc (SAE 410 s.s., Elastic-Plastic properties)

Fig. 4.8b. A schematic illustrating the concept of equivalent cylindrical asperity representing 76 spherical asperities making circular contacts.
(a) Model

(b) Normal pressure distribution ($\mu = 0 \Rightarrow$ no tangential stress).

(c) Normal pressure and tangential stress distributions ($\mu > 0$).

Fig. 4.9 Deformation model for cylindrical asperity in contact with a smooth half-space under plane strain conditions. The normal pressure and tangential stress.
Fig. 4.10  Idealization of a single asperity contact using a single cylindrical asperity in contact with a smooth plane (top face of the half-space).

Fig. 4.11  Finite element modeling of a typical sliding wear problem. The rectangle is the elastic-plastic half space. The idealized equivalent cylindrical asperity is loaded with contact pressure on the top of the truncated asperity.
Fig. 4.12. Motion of a single element ‘A’ on the wear track undergoing deformations in Total Lagrangian and updated Lagrangian descriptions.

Fig. 4.13. Stress-Strain response curve for the modeled elastic-plastic half-space.

- $E = 200$ GPa.
- $Y = 474$ MPa.
- $H' = 137$ MPa.
Fig. 4.14 Finite element mesh employed for the simulation of contact deformation.

Fig. 4.15 Finite element mesh with applied boundary conditions and normal load (pressure).
Fig. 4.16b Contact elements surrounding the asperity and half-space (these elements detect contact and apply the normal and tangential forces to the nodes of the asperity).
Fig. 4.17 Illustration of cyclic loading employed in the finite element simulation of sliding wear.

Fig. 4.18 Figure illustrating two different elements chosen after studying the variation of strain components during cyclic loading. The strain is during cyclic loading for these elements.
Fig. 4.19  Normal contact pressure profiles for (a) CASE IIA and (b) CASE IIIA loading. The friction coefficient, $\mu = 0$ for both cases. Elements shown with contours form "contact width (2a)" for the given loading.

Fig. 4.20. Hertzian contact stress profiles and finite element values at the first sub-step of indentation.
(a). \( \mu = 0 ; P_0 / k = 7.6 \).

Fig. 4.21a. Contours of Ratio of von Mises stress intensity at the last substep of the indentation load step to von Mises stress intensity at initial yield.

(b). \( \mu = 0 ; P_0 / k = 3.4 \).

Fig. 4.21b Contours of Ratio of von Mises equivalent stress at the last sub-step of the indentation load step to von Mises equivalent stress at initial yield.

(\( \mu = 0.3 ; p_0 / k = 7.6 \))

Fig. 4.22a. Contours of ratios of von Mises equivalent stress at the last sub step of the indentation load step to von Mises equivalent stress at initial yield.
Fig. 4.22b: Von Mises equivalent stress ($\sigma_v$) distribution at the last substep of the first load step (indentation under same normal pressure, but for two different friction coefficient levels)
(a) Contours of $\varepsilon_{xx}$

(b) Contours of $\varepsilon_{yy}$

(c) Contours of $\varepsilon_{xy}$

(d) Contours of equivalent strain $\varepsilon_{eqv}$

Fig.4.23. Contours of strain components at the end of second load step (first quarter reciprocating sliding; $\mu = 0$; $P_0 / k = 7.6$).
Fig. 4.24. Contours of strain components at the end of second load step (first quarter reciprocating sliding; \( \mu = 0.1; P_0 / k = 7.6 \)).
Fig. 4.25. Contours of strain components at the end of second load step (first quarter reciprocating sliding; $\mu = 0.3$; $P_0 / k = 7.6$).
Fig. 4.26. Contours of strain components at the end of second load step (first quarter reciprocating sliding; $\mu = 0.5$; $P_0 / k = 7.6$).
Fig. 4.27. Contours of strain components at the end of second load step (first quarter reciprocating sliding; $\mu = 0.7$; $P_0 / k = 7.6$).
Fig. 4.28. Contours of strain components at the end of third load step (after one reciprocating sliding cycle; $\mu = 0; P_0 / k = 7.6$ ).
Fig. 4.29. Contours of strain components at the end of third load step (after first reciprocating sliding cycle; $\mu = 0.1; P_0 / k = 7.6$).
Fig. 4.30. Contours of strain components at the end of third load step (after first reciprocating sliding cycle; $\mu = 0.3; P_0/k = 7.6$).
Fig. 4.31. Contours of strain components at the end of third load step (after first reciprocating sliding cycle; $\mu = 0.5$; $P_0 / k = 7.6$).
Fig. 4.32. Contours of strain components at the end of third load step (after first reciprocating sliding cycle; $\mu = 0.7$; $P_0 / k = 7.6$).
Fig. 4.33. Contours of strain components at the end of seventh load step (one indentation + three reciprocating sliding cycles; \( \mu = 0.0; P_0 / k = 7.6 \).
Fig. 4.34. Contours of strain components at the end of seventh load step (one indentation + three reciprocating sliding cycles; $\mu = 0.1, P_0/k = 7.0$).
Fig. 4.35. Contours of strain components at the end of seventh load step (one indentation + three reciprocating sliding cycles; (μ = 0.3; P₀/k = 7.6).
Fig. 4.36. Contours of strain components at the end of seventh load step (one indentation + three reciprocating sliding cycles; \(\mu = 0.5; P_0 / k = 7.6\)).
Fig. 4.7. Contours of strain components at the end of seventh load step (one indentation + three reciprocating sliding cycles; $\mu = 0.7$, $P_0/K = 7.6$).
(a) Contours of $\varepsilon_{xx}$

(b) Contours of $\varepsilon_{yy}$

(c) Contours of $\varepsilon_{xy}$

(d) Contours of equivalent strain $\varepsilon_{eq}$

Fig. 4.38 Contours of strain components at the end of eighth load step after removal of normal load to allow elastic rebound; $\mu = 0.3$, $P_0 / k = 7.6$. 
Fig. 4.39. Contours of strain components at the end of eighth load step after removal of normal load to allow elastic rebound.

(a) Contours of $\varepsilon_{xx}$

(b) Contours of $\varepsilon_{yy}$

(c) Contours of equivalent strain $\varepsilon_{eq}$

($\mu = 0.7, P_0/k = 7.6$)
Fig. 4.40. Figure illustrating the ratcheting phenomenon of surface element A1 for CASE 1 loading ($\mu = 0.3$, $\mu_v/k = 7.6$).
Fig. 4.41 Figure illustrating no ratcheting phenomenon of sub surface element A2 for CASE I loading $(\mu = 0.7; p_0/k = 7.6)$. 
Fig. 4.42 Figure illustrating the ratcheting phenomenon of surface element A1 for CASE II loading ($\mu = 0.3, P_0, k = 3.4$).
Fig. 4.43 Figure illustrating no ratcheting phenomenon of surface element A1 for CASE II loading ($M = 0.1$, $R_i/R_o = 3.4$)
Fig. 4.44 Figure illustrating no ratcheting phenomenon of sub-surface element A2 for CASE II loading ($\mu = 0.7, p_0/k = 3.4$).
Fig. 6.2 Configuration of sliding wear system (Hardened carbon steel cylinder in contact with SAE 410 s.s. disc.). The disc is assumed to have a defect in the form of a surface crack oriented at an angle to the loading surface.

Fig. 6.3 A schematic depicting the nominal area of contact and the surface crack for the contact configuration shown in Fig. 4.16.
Axial direction of the cylindrical component (Direction of plane strain)

Only one spherical (3D) asperity come in contact within the nominal contact width ($2a_0$)

Wearing disc (SAE 410 s.s., Elastic properties)

76 spherical (3D) asperities are in contact along the length of nominal contact area ($l_0$)

Fig. 6.4 A schematic illustrating only one asperity in contact along the sliding direction (see Appendix A2 for calculation of number of asperities in contact, ($n_{r0} = 0.6$; $n_{e0} = 76$)) and 76 asperities in contact along the length of the cylinder (plane strain direction) for a normal load of 800 N.

An equivalent cylindrical asperity of length $l_0$ and radius $R_{asp}$ representing all the 76 (3D) spherical asperities

Wearing disc (SAE 410 s.s., Elastic properties)

Fig. 6.5 A schematic illustrating the concept of equivalent cylindrical asperity representing 76 spherical asperities making circular contacts.
Fig. 6.6 (a&b) Fracture mechanics model for calculating crack growth and predicting delamination wear under dry sliding conditions.

(a) Model

(b) Normal pressure and tangential stress distributions
Fig. 6.7  Illustration the local co-ordinate system adopted for crack tip displacement calculations.

Fig. 6.8  Schematic explaining the calculation of change in displacement based on the crack tip co-ordinate system.

Fig. 6.9  Schematic illustrating tensile stress (σ₀) and (σ r) in the vicinity of the crack tip.
CASE I

\[ r := 1 \quad K_{I} := 0.16 \quad K_{II} := 3.54 \]
\[ t := -180 \text{-deg.}, -170 \text{-deg.}, 180 \text{-deg} \]

\[ S(t) := \frac{1}{4 \sqrt{r}} \left[ \left( \frac{K_{I}}{\sqrt{2}} \left( -3 \cos \left( \frac{t}{2} \right) - \cos \left( \frac{3t}{2} \right) \right) \right) + \left( \frac{K_{II}}{\sqrt{2}} \left( 3 \sin \left( \frac{t}{2} \right) + 3 \sin \left( \frac{3t}{2} \right) \right) \right) \right] \]

\[ K_{II}a := 6.0 \quad K_{II}a := 6.0 \]

\[ S_{1}(t) := \frac{1}{4 \sqrt{r}} \left[ \left( \frac{K_{II}a}{\sqrt{2}} \left( -3 \cos \left( \frac{t}{2} \right) + \cos \left( \frac{3t}{2} \right) \right) \right) \right] \]

\[ S_{2}(t) := \frac{1}{4 \sqrt{r}} \left[ \left( \frac{K_{II}a}{\sqrt{2}} \left( 3 \sin \left( \frac{t}{2} \right) - 3 \sin \left( \frac{3t}{2} \right) \right) \right) \right] \]

Fig. 6.10 Plot of variation of maximum tensile stress for mode I (S1(t)) and mode II (S2(t)) and mixed mode (S(t)) as a function of angle.
Fig. 6.11 Wear particle detachment under dry sliding from an existing crack
Fig. 6.12a Fracture Mechanics model for dry sliding wear.

Fig. 6.12b Finite Element mesh for the fracture mechanics model with an embedded crack.
Fig. 6.12c Magnified view of the fracture mechanics model with embedded crack. The crack faces are modelled with contact elements to detect contact between the crack faces and prevent the meshes from overlapping.

Fig. 6.12d Crack faces modelled with contact elements to prevent overlapping of meshes and detecting contact between them during the second load step after crack growth.

Fig. 6.12e Elements for simulating stress singularity at the crack tip.
Fig. 6.13 Maximum tensile stress ($\sigma_0$) in the vicinity of the crack tip for an initial crack length of 30 µm.
Fig. 6.14 Maximum tensile stress ($\sigma_{th}$) in the vicinity of the crack tip for an initial crack length of 30 $\mu$m.
Fig. 6.15 Maximum tensile stress ($\sigma_b$) in the vicinity of the crack tip for an initial crack length of 30 $\mu$m.
Fig. 6.16 Maximum tensile stress ($\sigma_0$) in the vicinity of the crack tip for an initial crack length of 30 $\mu$m.
Fig. 6.17 Maximum tensile stress ($\sigma_0$) in the vicinity of the crack tip for an initial crack length of 30 $\mu$m.
Fig. 6.18 Maximum tensile stress ($\sigma_0$) in the vicinity of the crack tip for an initial crack length of 30 $\mu$m.
Fig. 6.19 Variation of stress intensity factors (crack length c=15 microns) with normal and tangential loading in first and second load steps.
Fig. 6.20  Variation of stress intensity factors (crack length c=20 microns) with normal and tangential loading in first and second load steps.
Fig. 6.21  Variation of stress intensity factors (crack length $c=25$ microns) with normal and tangential loading in first and second load steps.
Fig. 6.22 Variation of stress intensity factors (crack length c=30 microns) with normal and tangential loading in first and second load steps.
Fig. 6.23  Variation of stress intensity factors (crack length $c=35$ microns) with normal and tangential loading in first and second load steps.
Fig 6.24 Variation of stress intensity factors (crack length c = 40 microns) with normal & tangential loading in first and second load steps.
Fig. 6.25 Variation of stress intensity factors (crack length c=45 microns) with normal and tangential loading in first and second load steps.
Fig. 6.26 Variation of stress intensity factors (crack length $c=50$ microns) with normal and tangential loading in first and second load steps.
Fig. 6.27  Variation of number of cycles required for crack growth and particle formation (crack length c=25 microns) with normal and tangential loading in first and second load steps.
Fig. 6.28  Variation of number of cycles required for crack growth and particle formation (crack length c=30 microns) with normal and tangential loading in first and second load steps.
Fig. 6.29  Variation of number of cycles required for crack growth and particle formation (crack length \( c = 35 \) microns) with normal and tangential loading in first and second load steps.
Fig. 6.30 Variation of number of cycles required for crack growth and particle formation (crack length c=40 microns) with normal and tangential loading in first and second load steps.
Load step 1
Fig. 6.31 Variation of no. of cycles required for crack growth and particle formation (original crack length, c = 45 microns) with normal & tangential loading in first and second load steps.
Fig. 6.32 Variation of no. of cycles required for crack growth and particle formation (original crack length, c = 50 microns) with normal & tangential loading in first and second load steps.
Original crack length = $AB (c)$
Original crack entry angle = $\theta_0$ deg.

Aspect Ratio = $\frac{AD}{h1}$

Volume = (Area $ABCD \text{ mm}^2 \times 25.4 \text{ mm}$)

Crack extension (Load Step 1) = $BC (d_c)$

Crack extension (Load Step 2) = $CD (d_{c2})$

Crack turn angles = $d_{\theta_1}$ & $d_{\theta_2}$

Fig. 6.33 Wear particle dimensions
Fig 7.1 A close-up view of the mounting fixture along with specimens.

Fig. 7.2 Specimens used for experimental work.
Fig. 7.3 mass loss from series-A specimens (Rc = 35)

Fig. 7.4 mass loss from series-A specimens (Rc = 45)
Fig. 7.5  Variation of friction coefficient with number of cycles
Fig. 7.6  Top views of worn specimens under wet and dry sliding (500X magnification).

(a) normal load -300N - (mild wear grooves under wet sliding conditions)
(b) normal load -200N - (mild wear grooves under wet sliding conditions)
(c) normal load -200N - (dry sliding depicting edge of a delaminated wear particle)
Fig. 7.7  Top view of specimens under lubricated sliding ($\mu = 0.1 \text{ to } 0.35$) at 50X magnification.

(a) Rc35, load = 200N, 720 cycles

(b) Rc35, load = 300N, 60 cycles

(c) Rc35, load = 300N, 720 cycles
Fig. 7.8 Top view of worn specimens under lubricated sliding ($\mu = 0.1-0.35$) at 500X magnification.
Fig 7.9  Top view of worn specimens under lubricated sliding ($\mu = 0.1 - 0.35$) at 500X magnification.
Fig. 7.10 Top view of worn specimens under dry sliding ($\mu = 0.5 - 0.8$) at 50X magnification.
(a) Rc35, load=200N, 60cycles

(b) Rc35, load=200N, 720cycles

(c) Rc35, load=300N, 60cycles

(d) Rc35, load=300N 720cycles

Fig. 7.11 Top view of worn specimens under dry sliding ($\mu = 0.5-0.8$) at 500X magnification.
Fig. 7.12 Top view of worn specimens under dry sliding ($\mu = 0.5 - 0.8$) at 500X magnification.
Fig. 7.13 Sectioned view of dry sliding wear specimens (500X magnification).
Fig. 7.14 Sectioned dry sliding wear specimen at 500x magnification depicting subsurface crack growth from surface cracks (300 N normal load; Rc 45).

Fig. 7.15 Sectioned dry sliding wear specimen at 500x magnification depicting subsurface crack growth from surface cracks (300 N normal load; Rc 45).
Fig. 7.16 Sectioned dry sliding wear specimen at 500x magnification depicting subsurface crack growth from surface cracks (300 N normal load; Rc 45).

Fig. 7.17 Sectioned dry sliding wear specimen at 500x magnification depicting subsurface crack growth from surface cracks (300 N normal load; Rc 45).
Ref. No. 2A1240X

Fig. 7.18 Sectioned dry sliding wear specimen at 500x magnification depicting subsurface crack growth from surface cracks (300 N normal load; Rc 35).

Ref. No. 2A1240X – Specimen 2

Fig. 7.19 Sectioned dry sliding wear specimen at 500x magnification depicting subsurface crack growth from surface cracks (300 N normal load; Rc 35)
Fig. 7.20 Sectioned dry sliding wear specimen at 500x magnification depicting subsurface crack growth from surface cracks (300 N normal load; Rc 45).
Fig. 7.21 Sectioned dry sliding wear specimen at 500x magnification depicting subsurface crack growth from surface cracks (300 N normal load; Rc 45).
Fig. 7.22 Sectioned dry sliding wear specimen at 500x magnification depicting subsurface crack growth from surface cracks (300 N normal load; Rc 45).
Fig. 7.23 Sectioned dry sliding wear specimen at 500x magnification depicting subsurface crack growth from surface cracks (300 N normal load; Rc 45).