

**ARITHMETIC PROBLEM SOLVING AND SIMULTANEOUS-  
SUCCESSIVE AND PLANNING PROCESSES  
IN SIXTH GRADE CHINESE CHILDREN**

by

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## Abstract

This study explored the interrelationships among cognitive processes (planning and simultaneous and successive processing) based on Planning, Attention, Simultaneous, Successive (PASS) theory, the math problem-solving components (problem translation, problem integration, and planning) based on Mayer's (1982) model, and their underpinning math achievements. The effects of planning and simultaneous and successive processing on the comparison problem, a type of math problem specifically difficult to children and even college students, were also investigated.

One hundred Chinese sixth graders participated in the present study. The student's PASS processes were measured individually by using subtests of Kaufman Assessment Battery for Children (K-ABC) (simultaneous processing: Picture Series, Triangles; sequential processing: Number Recall, Word Order). The student's planning process was measured by Matching Numbers, a planning subtest of Cognitive Assessment System (CAS). The student's cognitive components in math problem solving were measured by a group administered math test designed by Mayer. In addition, a set of comparison problems designed by the investigator was group administered.

The results of multiple regression analyses suggested that sequential processing was significantly associated with translation problem-solving component. Both simultaneous processing and planning were significantly associated with the integration problem-solving component. Moreover, Matching Numbers and simultaneous processing were significantly associated with the problem-solving component of planning.

Students' performances in mathematical comparison problems were analyzed by a series of 2 X 2 mixed factorial ANOVAs, with the level of each PASS cognitive processing (high vs. low) and the problem type (consistent language vs. inconsistent language) as independent variables, respectively. The results showed that there were main effects of problem type and level of cognitive processing, and of the interaction among simultaneous processing and problem type, Matching Numbers and problem type. As findings of previous studies, inconsistent language (IL) comparison problems were much more difficult than consistent language (CL) comparison problems for Chinese sixth graders in this study. However, students with high simultaneous scores performed well in solving both comparison problems, whereas students with lower simultaneous scores tended to perform similar with high simultaneous students in consistent language (CL) problems but much poorer than their peers with high simultaneous processing in inconsistent language (IL) problems. Similarly, students with high Matching Numbers performed similarly in both types of problems. But those with low Matching Numbers performed significantly poorer in IL problems than their peers with high simultaneous processing do. These results can help us explain students' special difficulty with inconsistent language (IL) comparison problems.

Finally, the manifestations of PASS processes in the special groups of good and poor problem solvers in composite scores of math problem solving were compared. Students who were poor math problem solvers were poor at both subtests of simultaneous processing (Photo Series and Triangles), Matching Numbers, and Word Order, but performed similar with their peers who were good math problem solvers in Number

Recall. The profile of PASS processes in good the poor problem solvers in inconsistent language (IL) comparison problems were also compared. Poor problem solvers in IL problems were poorer at all PASS processes compared to their peers who performed well in IL problems.

It is concluded that all PASS processes (as measured by planning and simultaneous and sequential processing) involved in arithmetic word problem solving. In particular, simultaneous processing and planning are the essential cognitive processes to build up a correct problem representation, which in turn leads to successful problem-solving performance in arithmetic word problems.

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## CHAPTER 1: INTRODUCTION

### Context of the Problem

#### Needs From Educational Reform

Mathematics is one of the most important subjects in school because it is considered the basis of scientific thinking and informed living in a technologically advanced society. It provides and prepares students accessing and exploring future occupations in the fields of commerce, industry, technology and science, medicine and education among other fields. The importance and usefulness of mathematics has been widely articulated by the international mathematical education community (National Council of Teachers of Mathematics [NCTM], 1989).

However, until recently, mathematics was viewed and taught in North America as a set of isolated skills to be learned through repetitive practice. Although many students today can do mathematics, they lack the understanding of the principles underlying mathematics problems. As a result they are unable to use mathematical knowledge in their daily lives. Many students avoid mathematics courses at higher levels of education due to their beliefs that mathematical skills are innate, and that mathematics learned in school has little or nothing to do with the real world (National Council for Educational Statistics, [NCES], 1996).

Another worrisome phenomenon in math education is the well-established cross-national differences in math achievement. Studies consistently found that American students lagged behind their Asian peers (Robitaille & Garden, 1989; Stevenson, Lee, Chen, Lummins, Stigler, Liu and Fang, 1990; Stevenson & Stigler, 1992). In the third International Mathematics and Science Study (TIMSS, 1997), an important international survey was done on the outcomes of math education in the 1990s. The top four best performing countries for both the Grade four and three were all Asian countries (i.e., Singapore, Korea, Japan, and Hong Kong). The United States and Canada were not listed in the top ten countries (TIMSS, 1997). The poor performance of North American students in mathematics in recent cross-national comparisons lead researchers to question the effectiveness of the current mathematics instruction in North American schools. It was argued that mathematics needs to be reinvented as a subject of ideas and mental processes, rather than a learning of facts. Students need to be encouraged to explore patterns and seek solutions instead of passively practicing repetitive exercises and memorizing procedures and formulas (Fennema, Franke, Carpenter, & Carey, 1993).

Given the discouraging status of mathematics learning in North America, a major shift has occurred in the content and methodology of research in mathematics education. For example, Nesher (1986) argued that this shift may be attributed in part to the growing interest of cognitive science researchers in mathematical thinking and “the growing awareness of mathematics educators that remedies addressing difficulties in learning mathematics will not be found in didactic tricks, but rather should be sought in a better understanding of the cognitive processes underlying mathematical thinking” (Nesher, 1986, p. 114). Mathematics educators and researchers interested in problem solving have

been urged to examine and use the work being done currently by cognitive psychologists particularly in the area of information processing (Lester & Garofalo, 1982). Also, cognitive psychologists, including Mayer (1998), have advocated the development of the psychology of mathematical problem-solving based upon research in the area of information processing to serve as the framework for improving mathematical learning in the schools.

In general, the increasing importance of current research in cognitive psychology to mathematical education is apparent. However, the currently available cognitive studies mainly focus on basic arithmetic computation. The underlying process and mechanism of various math problem-solving processes are not clear. Thus it is more difficult to isolate the loci of performance deficits on complex math tasks (Geary, 1993). In order to get a complete picture of math problem-solving processes, both a general theory on higher level cognition and a specific theory on math problem solving are needed.

#### The PASS theory and Mayer's (1987) Model on Mathematical Problem Solving

Information processing theorists have identified a small number of elementary processes underlying all cognitive activity, although there is little agreement as exact number and nature of those fundamental processes (Palmer & Kimchi, 1986). As a result, cognitive abilities can be analyzed at many levels by deconstructing tasks into different components. Efforts have been made in this direction such as Anderson's work on memory (Anderson, 1983), Baddeley's (1986) working memory model, and the PASS theory of intelligence (Das, Naglieri, & Kirby, 1994). Based on Luria's

neuropsychological work as well as cognitive psychological findings, the PASS theory (Das et al., 1994) attempts to study the general mental processes which can be used to manipulate the information input when children attempt to solve a variety of cognitive tasks. According to the PASS theory, intelligence could be conceptualized as deriving from the cognitive processes involved in planning, attention, and information coding (Naglieri & Das, 1990).

The PASS theory and Cognitive Assessment System (CAS) (Naglieri & Das, 1997a) offer a unique approach to the understanding of human mental functioning. It is believed to “provide one of the most comprehensive accounts of behavior based upon psychological and neuropsychological theory” (Das & Varnhagen, 1986, p.122). It appears that the PASS theory of cognitive processing can provide a comprehensive theory of intelligence called for by mathematics researchers. Insight into the cognitive processes underlying mathematics abilities would also have important implications for theoretical understanding of math problem solving processes and diagnosis of students’ difficulties in mathematics, as well as for instruction and remediation.

A large body of research has been accumulated which supports the PASS theory (e.g., Naglieri & Das, 1987, 1988, 1997c; Das et al., 1994). In addition, there is a growing body of research on the relationship among the various components of the PASS theory and academic achievement (Naglieri & Das, 1987, 1997c; Cummins & Das, 1978; Kirby & Das, 1977). However, the vast majority of such research has focussed on the relationship between the PASS theory to achievement in reading areas. Only a few studies have focused on the relationship of the PASS theory and math achievement (Das, 1988; Cheng, Das & Leong, 1984; Warrick, 1989; Naglieri & Gottling, 1995; 1997; Naglieri &

Johnson, 2000, VanLuit & Naglieri, 1999). These studies mainly explored the relationships between PASS processes and students' general math achievement scores, PASS processes and computation scores. Math problem-solving is a complex task that involves various cognitive processes and components. More thorough research in this area is needed.

Understanding and solving arithmetic word problems demand the ability to access many different skills, such as language comprehension, an understanding of the described situation, the abilities to build up an appropriate problem representation and to find an equation, and computation skills for solving the problem. Mayer (1987) proposed a model on mathematical word problem solving. The model includes four cognitive components involved in solving mathematical word problems: (a) problem translation, (b) problem integration, (c) planning, and (d) execution. Mayer's (1987) model has been used to assess cognitive aspects of mathematical problem solving on the Scholastic Aptitude Test (SAT) (Bejar, Embretson & Mayer, 1987), and to explore the math problem-solving components in several cross-national studies (Mayer, Tajika, & Stanley, 1991; Tajika, Mayer, Stanley, & Sims, 1997; Cai, 1995). It was found that poor problem solving could be caused by deficiency in any one component. Based on the above description, using both the PASS theory and Mayer's (1987) model on math problem-solving components together might provide us a good opportunity to explore in detail the nature of math problem solving and students' difficulties.

A number of studies based on Mayer's (1987) model found that problem representation is a very important component for mathematics problem solving (Mayer & Hegarty, 1996). Lack of appropriate problem representation leads to failure in



mathematics problem solving. Locating students' difficulties in math problem-solving component and the deficiency of underlying cognitive processes is considered more helpful to instruction and remediation than a general math achievement score (Sternberg, 1984). This work demands a clear understanding of relationship between each math problem solving component and PASS processes, which is not available in the literature. Thus, the present study filled this gap by examining PASS processes and math problem solving components based on Mayer's (1987) model. Also, students' difficulties in math problem-solving will be analyzed in terms of the deficiencies of PASS processes. In addition, in the literature of math problem solving, the comparison problem has been found especially difficult for students from elementary schools to college (Hegarty, Mayer, & Green, 1992; Hegarty, Mayer, & Monk, 1995; Verschaffel, De Corte, & Pauwels, 1992). This study examined students' difficulties in comparison problems in terms of their deficiencies in the underlying PASS processes.

### Why Chinese Students?

Cross-cultural studies comparing the mathematical performance of students in the U.S. and in Asian countries (e.g., China, Japan, and Korea) have consistently found that Asian students outperformed American students (Robitaille & Garden, 1989; Song & Ginsburg, 1987; Stevenson & Stigler, 1992; Stevenson et al., 1990; Stigler, Lee, & Stevenson, 1990). Most of these studies involved comparisons of students' mathematics achievement scores and various cognitive processes underlying arithmetic computation. Only a few studies (e.g., Cai, 1995; Mayer, et al., 1991; Tajika, et al., 1997) explored the

cognitive components in solving arithmetic word problems. The present study extended this line of research by exploring the underlying cognitive processes, as measured by PASS processes, of arithmetic problem solving. This study aimed to identify the PASS processes underlying math problem-solving components as measured by Mayer's model for a group of sixth grade Chinese students to understand clearly the cognitive processes of this special group that performed particularly well in math achievement tests.

### Statement of the Problem

In the literature of the PASS theory, simultaneous processing and planning were found to correlate significantly with mathematics problem solving scores, whereas successive processing correlated more strongly with computation (Naglieri & Das, 1997c). However, most studies were all limited to investigations of PASS processes and general scores of mathematical achievement (either for problem solving or computation, or combining the two into one general score termed "mathematics"). The mechanism of math problem solving in terms of fundamental cognitive processes is not clear. Little is known about the relationship between PASS processes and the math problem-solving components in Mayer's (1987) model, and the contribution of PASS processes to students' difficulty in arithmetic word problems has not been explored. Moreover, students' cognitive deficits in solving comparison problems based on PASS theory have never been explored. Finally, an in depth analysis of cognitive processes for Chinese children, a special group which has been demonstrated to be very successful in mathematics achievement, has not been conducted.

## Rationale

There is a need for further investigation of the mechanism of math problem solving in terms of cognitive processes. The relationship of fundamental cognitive processes and cognitive components in mathematics problem solving need to be clarified. Further studies under both theoretical models, the PASS theory and Mayer's (1987) model of math problem solving components, might contribute significantly to our understanding of nature of math problem solving and the deficits of cognitive processes underlying students' difficulties in math problem solving. Moreover, analyzing students' performance in math comparison problems and their PASS processes can provide information to understand students' cognitive deficiencies and design effective training programs. In addition, although there are many studies on cross-national differences in math achievement between Chinese and North American students, the underlying cognitive processes in math problem solving by both groups have not been systematically investigated. Thus, this study examined the math problem-solving components and their underlying PASS processes in sixth grade Chinese children.

## Purpose of the Study

The purpose of the study was to investigate Chinese sixth graders' mathematical achievement, as measured by Mayer's (1987) model of math problem-solving components, and the underlying cognitive processes, as measured by PASS processes (planning and simultaneous and successive processing). The manifestation of PASS

processes in poor problem solvers in arithmetic word problems were also explored. In addition, this study investigated the contribution of the PASS processes (planning, simultaneous processing and successive processing) to children's problem solving in math comparison problems. The manifestation of PASS processes in poor problem solvers in the inconsistent language (IL) comparison problem, a type of problems particular difficult for students, was analyzed.

## CHAPTER 2: LITERATURE REVIEW

This chapter consists of three parts. The first part introduces the PASS theory of information processing (Das, Kirby, & Jarman, 1979; Das et al., 1994) and discusses the relevant studies. The second part is a review of the studies on mathematics word problem solving, which includes two sections. First, Mayer's (1987) model on mathematical problem solving and the relevant studies are described. Second, studies on comparison problems are reviewed. The third part integrates studies on PASS theory and Mayer's (1987) model on math problem-solving components together. Finally, the research questions and hypotheses of this study are proposed.

### The PASS Theory and the Relevant Studies

#### Introduction of the PASS Theory

Drawing upon the shift in focus within the fields of psychology and education from the examination of abilities to examination of the cognitive processes underlying abilities, Das, Kirby and Jarman (1975, 1979) and, more recently, Naglieri and Das (1990) have been advocating the reconceptualization of intelligence as cognitive processes from the traditional IQ test technology that has been dominated most of the 20th century. They suggested that this new conception should be followed up by

constructing tests based on a theory-driven, multidimensional view with cognition (Das et al., 1994). Thus, Das and his colleagues (Das et al., 1979; Naglieri & Das, 1987, 1988) have expanded Soviet neuropsychologist A. R. Luria's (1966) theory to the PASS theory (Das et al., 1994) and operationalized it to CAS (Naglieri & Das, 1997a). The PASS theory is a complete theory of cognitive processing based upon clinical neuropsychological research and cognitive psychology. As Das and his colleague (Das, et al., 1994) summarized: "We believe that it has a strong theoretical foundation, has been sufficiently operationalized, and is making significant contribution to understanding exceptionally, predicting academic and job performance, and intervention design" (p. 12).

The PASS theory consists of three parts: attention system, processing system engaging simultaneous and successive processing, and the planning system engaging in organization and monitoring of processing. Naglieri (1999) summarized the essence of each PASS process as follows:

"Planning processes provide cognitive control, utilization of processes and knowledge, intentionality and self-regulation to achieve a desired goal; attentional processes provide focused, selective cognitive activity and resistance to distraction; and simultaneous and successive processes are the two forms of operating on information" (p.11).

The four PASS systems are interdependent. Input may occur through any of the sensory receptors, and be coded on the basis of information stored in the long-term memory; the encoding is stored in the working memory, it can be maintained there, or manipulated or transferred to long-term memory. While all these occur in the processing system, the processing is guided or controlled by the planning system (Das et al., 1994). The PASS

theory proposes that planning, attention, and simultaneous and successive processing are basic building blocks of human cognition. It views intelligence as different specific abilities, which is different from the traditionally general ability approach of intelligence.

### Operationalization of the PASS Theory

Since the vast majority of research on the PASS theory has focused on the operationalization of its components, it is briefly reviewed in the following sections.

#### Investigations of Simultaneous and Successive Processing

The initial research regarding the PASS theory focused on the importance of the simultaneous and successive processes to a cognitive theory. Das (1972) first operationalized Luria's (1966) model by administering a series of tasks involving memory and reasoning to 60 educable mentally retarded (EMR) children and 60 nonretarded children. Das used principal components factor analysis to analyze the data and found that two factors accounted for the performance of both groups, and the poorer performance of the EMR children would be attributed to their selection of inferior processing. Das explained the two factors as simultaneous and successive processing.

Subsequent to this study, Das and his colleagues investigated the existence of simultaneous and successive processing across different populations. The different participants in these studies included different cultural groups (Das, 1973), different age

groups (Das & Molloy, 1975; McCallum & Merritt, 1983; Vernon, Rybe & Lang, 1978; Wachs & Harris, 1986), and different intellectual levels (Das, 1972; Jarman & Das, 1977). These studies confirmed the existence of the simultaneous and successive factors. Based on these earlier studies, Das et al., (1975, 1979) proposed the PASS theory.

In summary, many studies have focussed on operationalization of simultaneous and successive processes. These information-coding processes have been validated by factor analysis with data from a wide variety of populations. Following these earlier studies, the focus of subsequent studies was extended to studies on the PASS process of planning.

#### Research Involving the Planning Process

The tasks originally selected as measures of planning were tasks which had been demonstrated to differentiate between patients with frontal and nonfrontal lobe impairment (Luria, 1973) and emphasized the selection and implementation of efficient strategies (Ashman & Das, 1980). The results of the principal component factor analysis demonstrated the emergence of the planning factor that was orthogonal to the information coding factors (Ashman & Das, 1980). The stability of planning as a separate factor was shown in studies with different populations including adults, mildly retarded, trainable mentally retarded participants (Ashman, 1978) and noninstitutionalized moderately retarded children (Snart, O'Grady & Das, 1982).

After the general validation of the emergence of the planning factor, Das and his colleagues attempted to identify the best marker tests for planning using the factor



analysis. Through studies with college students (Das & Heemsbergen, 1983), fourth and sixth grade Chinese students (Leong, et al., 1985), and American elementary school students (Naglieri & Das, 1988; Naglieri, Prewett, & Bardos, 1989), operationalization of the planning factor was established.

### Investigations Involving the Attention Factor

Tasks designed to assess the attention component were the last to be developed. However, the attention factor is very important because theoretically both coding and planning processes depend on an appropriate level of attention. In addition, according to Naglieri and Das (1988), the assessment of this component is particularly important when dealing with problems of disorganization, hyperactivity, or impulsivity. Early studies on the emergence of the attention factor involved administering the Stroop test to large samples of children in the second, sixth and tenth grades (Price, 1987), delinquent and nondelinquent adolescents (Hunt, 1988), various groups of children including normal, learning disabled, developmentally handicapped elementary school students (Bardos, 1988), and children with Attention Deficit Hyperactivity Disorder (ADHD) (Reardon & Naglieri, 1989, cited in Das et al., 1994).

Generally speaking, there are fewer studies in the area of attention compared to the other three factors in the PASS theory. Warrick (1989) found that math achievement was best predicted by attention for Grade 3 students; however, for Grade 6 students, attention does not significantly predict math problem solving ability. Thus, the present study did not include the PASS process of attention.

### Comparing the PASS Theory with Other Models

A series of studies were conducted to compare the PASS theory with other models. For example, Kirby and Das (1978) compared the simultaneous-successive factors to the more traditional primary mental ability model (PMA) (Thurstone, 1938, cited in Kirby & Das, 1978). The PMA model involves reasoning and memory; the two batteries based on the two models were administered to 104 normal boys in the fourth grade. The results of the principal components analysis provided support for the conclusion that simultaneous processing was more than just reasoning or Level II ability, and the successive processing did not simply represent memory or Level I ability (Das, et al., 1979). Another study compared simultaneous and successive processing with the modality specific-cross modal theory (Jarman, 1978). The results of these studies indicated the stability of simultaneous-successive factors. Naglieri, et al., (1989) conducted an exploratory examination of the factorial validity of the PASS theory with the battery of tasks developed by Das and Naglieri (1988) to 112 normal fourth and fifth graders. The result indicated the appropriateness of the four factors. In another study, Naglieri, Das, Stevens, and Ledbetter (1989) conducted a confirmatory factor analysis study on a battery of PASS tasks to students in kindergarten and grade two and grades five through twelve. Again, the results supported the four-factor structure of the PASS theory. Further, they compared the PASS theory with other theoretical models (Verbal-Nonverbal, Memory-Reasoning, Verbal-Spatial-Speed, and the Null Model), and found that the PASS theory was the model that fit best.

In summary, based upon the research reviewed, the stability of the planning, simultaneous and successive processing, and attention process has been demonstrated in a

wide variety of age and culture groups, and across various levels of intellectual functioning. The operationalized tests formed the Cognitive Assessment System (CAS) (Naglieri & Das, 1997a).

### K-ABC and Math Achievements

The Kaufman Assessment Battery for Children (K-ABC) (Kaufman & Kaufman, 1983a) is a broadly investigated and frequently used test in everyday assessment practice (Obringer, 1988; Bracken, 1985). It emphasizes sequential and simultaneous processing and stresses how children solve problems rather than what type of problems they must solve. The underlying theory of this test is from Luria's cognitive approach and other neuropsychological studies. K-ABC is a well-established test for sequential and simultaneous processing.

In K-ABC, subtests of sequential and simultaneous processing were shown to be significant predictors of future school achievement. In the K-ABC Interpretation manual (Kaufman & Kaufman, 1983c), the correlations between six achievement tests and K-ABC simultaneous and successive processing scores were reported. The coefficients between mathematics scores and simultaneous-successive processing are presented in Table 1. The correlation coefficients between simultaneous-successive and reading scores in these tests are not reported here because they do not directly relate to the present study.

Table 1

The Coefficiencies Between Mathematics Test Scores and Simultaneous-Successive Processing Scores

Mathematics Tests	Simultaneous score	Successive score
SRAAS (n = 34)	.40	.41
ITBS (n = 106)	.35	.26
SAT (n = 109)	.45	.45
ITBS (n = 42)	.72	.49
CAT (n = 44)	.56	.50

*Note.* SRAAS: Science Research Associates Achievement Series; ITBS: Iowa Tests of Basic Skills; SAT: Stanford Achievement Test; CAT: California Achievement Test.

The PASS Theory and Academic Achievements

PASS Processes in Reading

Numerous studies using many different PASS tasks and achievement measures in many different populations have demonstrated that PASS processes were empirically related to achievement measures.

The vast amount of research on the application of the PASS theory to academic achievement has focused on achievement in reading. Various aspects of reading achievement have been shown to be significantly related to simultaneous and successive

processing (Cummins & Das, 1978; Kirby & Das, 1977; Naglieri & Das, 1987) and planning (Das, 1984; Naglieri & Das, 1987).

Reading can be divided into decoding and comprehension. Studies found that decoding at the early elementary grades required successive processing, while comprehension at any age required simultaneous processing (Das, et al., 1979). Some studies found that both simultaneous and successive processing were required for comprehension (Kirby & Das, 1977). Recent studies confirmed the importance of both information-coding processes and planning in comprehension (Das, Mensink, & Janzen, 1990; Das, Snart, & Mulcahy, 1982; Kirby & Gordon, 1988; Naglieri & Das, 1988).

#### PASS Processes and Math Achievement

Luria's Work. The first study in this line can be dated back to the neuropsychological research of Luria (1966, 1973). Luria found that lesions in different areas of the brain were associated with different types of difficulties in arithmetic performance. Patients with lesions in areas associated with successive synthesis (temporal lobes) experienced difficulties in problems which involve intermediate calculations to be carried out mentally and involve the memorization of the results of the previous steps. Lesions of the occipito-parietal region led to difficulties understanding relationships in the problem. Patients with frontal lobe lesions did not plan their action, but carried out disconnected operations impulsively (Luria, 1966). Luria (1966) and Das et al. (1979)

predicted that mathematics achievement might be more closely related to simultaneous processing than to successive processing due to the highly spatial nature of mathematics.

Studies on PASS Processes and Math Achievement. Some supporting data on PASS processes and math achievement came from an study conducted by Sprecht (1976, cited in Das et al., 1979). A battery of simultaneous and successive processing tasks along with vocabulary, mathematics and reading comprehension tests were administered to a sample of low-achieving high school students. The results of the factor analysis indicated that mathematics achievement demonstrated a moderate loading on the simultaneous processing factor.

Wachs and Harris (1986) administered a battery of information coding tasks to a sample of undergraduate college students and correlated the scores with the scores of the Scholastic Aptitude Test (SAT). The results demonstrated that the SAT Math scores correlated significantly with simultaneous processing, whereas, successive processing was significantly correlated with the students' grade in English.

Das, Manos and Kanungo (1975, cited in Das et al., 1979) examined the relationship of simultaneous and successive processing to academic achievement in fourth grade children of high (N=60) and low socioeconomic status (N=60). The findings indicated that mathematics achievement was strongly predicted by a simultaneous processing task, Figure Copying, for the high SES group. Whereas for low SES children, mathematics achievement was best predicted by a successive processing task, Serial Recall, although Figure Copying was also a strong predictor. The authors concluded that although simultaneous processing may be the most efficient form of processing for mathematics, it was used less often than successive processing by the low SES group.

The low SES group demonstrated a preference for successive processing. This might relate to the different usual learning style in the groups from different socioeconomic background (More, 1990).

Das and his colleague conducted a study on simultaneous-successive processing and planning with general math achievement of Grade 4 and 6 Hong Kong Chinese children who go to Western-style schools (Cheng, et al., 1984). Factor scores for simultaneous, successive and planning processes were derived and used as predictors for standardized math achievement. The regression analysis showed simultaneous processing to be the best predictor of math achievement. However, this study did not separate composition and problem solving skills. The result was not clear in terms of the specific relationship between PASS processes and the two kinds of math achievements.

Garofalo's study (1982) has filled this vacancy and clarified the interrelationship. Garofalo (1982) examined the relationship of planning and information coding processes (simultaneous, successive processing) to mathematical abilities (including computation, problem solving and quantitative ability) for 95 grade five American students. Factor analyses found three clearly defined orthogonal factors; Problem Solving had a high loading on the simultaneous factor in contrast to successive and planning; Computation had its highest loading on the planning factor and smaller loading on simultaneous and successive factors. The author explained that problem solving required the understanding of mathematical and logico-grammatical relationships, which is a primary function of simultaneous processing. However, success in computation depends upon the planning functions of regulating and monitoring activity while completing the computations.

Another study relating planning skills to math achievement (Kirby and Ashman, 1984) found similar results. Grade five children were given several tasks to measure planning and arithmetic operations. Factor analyses revealed a significant correlation between arithmetic operation and planning.

Naglieri and Das (1987) examined the relationship of simultaneous, successive and planning processes to academic achievement in 434 students in grades two, six, and ten. The new battery of cognitive processing tasks was administered along with the Multilevel Academic Survey Test (MAST) that included MAST reading and math. The results indicated developmental change in the cognitive processes involved in computation. At the second grade level, computation was most strongly related to simultaneous processing but also demonstrated a strong relation to planning. At the sixth grade level, mathematics achievement remained most strongly related to simultaneous processing, however, significant correlations were also demonstrated with both planning and successive processing. At grade ten, planning demonstrated the strongest relationship to mathematics achievement, and mathematics achievement was also associated with both simultaneous and successive processing. These results demonstrate the importance of planning in academic achievement, especially at the development point of view.

Warrick (1989) examined the importance of PASS processes for math achievement including math concepts, computation, math problem solving, and total math in third, sixth, and ninth grade students. For the third grade, attention and simultaneous processing were the best predictor of math achievement in three of the four math areas. For the sixth grade, simultaneous processing was the best predictor of math achievement in all four areas. Meanwhile, the planning and successive also contributed



significantly to the prediction of math problem solving achievement. For the ninth grade, problem solving was best predicted by attention and planning following by simultaneous and successive processing.

According to the Cognitive Assessment System Interpretive Handbook (Naglieri & Das, 1997c), the relationship between the CAS and achievement were examined for both individually administered measures (Woodcock-Johnson-Revised Tests of Achievement) and group administered measures (Scholastic Aptitude Test). The correlations between the Full Scale and separate PASS Scale scores with the WJ-R scores ranged from .46 to .72 for Broad mathematics, .44 to .69 for Basic Mathematics Skills, .44 to .67 for Mathematics Reasoning, .35 to .63 for Calculation, .44 to .67 for Applied Problems, and .44 to .68 for Quantitative Concepts. In all, Planning processing scores correlated the highest with Calculation, Basic Mathematics, Broad Mathematics, and Applied Mathematical Problems; Simultaneous processing scores correlated highest with all the math measures except for youngest age group; Successive processing scores correlated highest with only Calculation.

The relationship between the CAS and the College Board Scholastic Aptitude Test (SAT; Donlon, 1985) suggested that Planning and Attention Scale scores were significantly related to SAT Math scores; Successive Scale scores correlated significantly with SAT Verbal scores, and Simultaneous Scale scores correlated significantly with both Verbal and Math SAT scores.

Summary. In summary, studies provided the basic data regarding the relationship of mathematics achievement and the corresponding underlying PASS processes.

However, the results of these studies have been somewhat inconclusive. It has been

shown that the relationship between mathematics achievement and the cognitive components varied depending on the type of mathematics performance examined and the type of cognitive processing task used (Garofalo, 1982). In addition, most previous research investigated only the relationship between PASS processes and students' math achievement scores or problem solving scores. A simple score is not good enough to represent the complex cognitive processes involved in mathematical word problem solving. With the new studies on math and Mayer's model on math problem-solving components, it is necessary and possible to analysis the cognitive processes underlying detail math problem-solving components.

#### Studies on Learning Disabilities and the PASS Theory

A number of studies have found that the PASS theory was very helpful for diagnosis and remediation. For example, students with poor math ability were found poor at all PASS processes (Warrick, 1989). Students with reading difficulties were found poor at Planning, Attention and Simultaneous processes compared to their normal peers, but no difference in Successive processing (Hildebrand, 1998). Wasserman and Becker (2000) summarized the clinical utility of the CAS with ADHA and learning disabilities and learning disordered as follows:

planning and attentional processes are characteristically impaired in children diagnosed with ADHD, ... a relatively weakness in simultaneous processing is associated with both verbal and quantitative difficulties in understanding relationships between items and concepts in learning disabled children, and that a

relative weakness in successive processing is associated with phonological awareness difficulties in learning disabled children (p. 5).

In general, the CAS substantially outperforms other traditionally intelligence tests such as WISC-III and Gordon Diagnostic System in terms of diagnosis and prediction, and is the first intelligence test that can correctly identifies over three fourths of children diagnosed with ADHD (Wasserman & Becker, 2000).

In addition, studies on the relationship between poor performance in specific PASS processes and scores on the WJ-R subtests suggested that different cognitive weakness in PASS processes were related to different levels of performance on the various achievement tests. For example, simultaneous and successive cognitive weakness were found highly related to low scores in reading area; whereas cognitive weakness on the Planning process were associated with lower scores in Calculation.

### Summary

Attention, planning, and simultaneous and successive processing are basic cognitive processes that are responsible for the acquisition, storage and retrieval of knowledge and planning for problem solving. All these interdependent processes may take place during perception, memory, or at conceptual level processes. As a general theory of cognition, the PASS theory is helpful to understand students' performance in all kinds of tasks, including mathematics problem solving.

## Studies on Mathematical Problem Solving

The mathematical problem solving has long been an important topic in cognitive psychology. The arithmetic word problem represents an important bridge between the child's developing computational skills and the application of these skills in real-world contexts. It is therefore important to understand how children develop problem-solving skills and to identify the sources of problem-solving difficulties. One potential impact of cognitive psychology on mathematics problem solving is the application of cognitive analysis to mathematical problems. Cognitive task analysis refers to specifying the cognitive capacities and knowledge that are required to successfully carry out a particular task, in this case, solving a mathematical word problem.

### A Brief Introduction of Mayer's Model

Mayer (1987) developed a model on math problem-solving components. Mayer (1987) assumed that the two major phases of mathematical problem solving were (1) representing the problem and (2) searching for a means to solve the problem. In order to represent a problem, a student must be able to translate each sentence of the word problem into an internal representation such as an equation or a diagram, and be able to put the elements of the problem together into a coherent whole. Cognitive research has suggested that the breakdown in linguistic comprehension, lack of schema for problem types, and the lack of the adequate simultaneous processing ability to integrate information into a coherent internal representation, are the sources of many difficulties in

problem solving (Kirby & Williams, 1991; Mayer, 1987). In order to search for a means to solve a problem, the student must also be able to plan and find an adequate algorithm and then correctly execute the algorithm. Thus, in Mayer's model, four cognitive components are involved in solving mathematical word problems: problem translation, problem integration, planning, and execution.

Mayer's model has been used to assess students' general cognitive aspects of mathematics problem solving in the Scholastic Aptitude Test (SAT) (Bejar et al., 1987). Tests based on Mayer's model can be used to identify the underlying cognitive components required for success with mathematical word problems. For example, based on Mayer's model, the four components of mathematics problem solving can be directly evaluated, respectively. Translation skills can be evaluated by asking students to recognize paraphrases of the given problem, the problem goal, and pictures or equations corresponding to a sentence in the problem. Integration skills can be evaluated by asking students to distinguish relevant and irrelevant information to solve the problem, or to represent the problem as a number sentence, equation or picture. Planning skills can be evaluated by asking students to identify sub-goals of the problem, to identify necessary operations and to draw a conclusion. Computational skills can be evaluated by asking students to identify the result of arithmetic problems. A set of mathematical tests designed by Mayer to measure each math problem-solving component, based on his model, has been used successfully in several cross-cultural studies (Cai, 1995; Mayer, et al., 1991; Tajika, et al., 1998,).

### Studies Related to the Cognitive Components in Mayer's Model

The following is a detailed review of studies related to the first three cognitive components in Mayer's model, that is, problem translation, problem integration, and planning.

#### Research on the Translation Component

According to Mayer's (1987) model, the first step in representing a problem is to translate each proposition from the problem into an internal representation, which needs linguistic and factual knowledge.

Comprehending relational statements. A number of studies suggested that the translation process could be very difficult for students, especially when the problem contained relational statements (i.e., statements that express a quantitative relation between variables).

Loftus and Suppes (1972) found that the most difficult problems were the ones that contain relational propositions. For example, "Mary is twice as old as Betty was two years ago, Mary is 40 years old, how old is Betty?" Riley, Greeno, & Heller (1983) suggested that children might have difficulty in representing relational propositions. Children in primary grades were asked to listen to and immediately repeat problems involving relational propositions. They tended to ignore the relational statements and

stated them as assignment statements. Similarly, Stern (1993) found that the symmetry of language involved in quantitative comparisons is difficult for first graders to understand.

Relational statements seem to be difficult also for adults (Soloway, Lochhead, & Clement, 1982). The study showed that college students tended to make mistakes when writing equations to represent a relational proposition (Mayer, 1982). College students were asked to read and recall eight algebra story problems. Each problem contained three types of propositions: assignments (assign a value to a variable), relations (expressed a quantitative relation between two variables), and questions (asked for a numerical value of a variable). The results indicated that students made approximately three times as many errors in recalling relational propositions than in recalling assignment propositions. It showed that some students were lack of skills to represent relations between variables.

Hegarty, et al. (1995) explored the relationship between representing relational statements and problem solving performance. They asked college students to solve 12 inconsistent language (IL) comparison problems (i.e., the keyword is inconsistent with the required operation, for example, the relational statement retains a key word "more" but the correct solution requires subtraction), and later asked them to recognize the problem they had solved from four alternatives. The three incorrect alternatives included a literal error in which the meaning of the relational statement was retained but the keyword was changed to the opposite, such as from "less" to "more", and two semantic errors in which the meaning of the relational statement was changed. Poor problem solvers made much more semantic errors than good problem solvers. In contrast, good problem solvers made much more literal errors than poor problem solvers. These results suggested that good

problem solvers were able to represent and understand the meaning of the relational statements better.

A recent study by Cai (1995) found a cross-cultural difference between Chinese and U.S. students in comprehending relational statements in mathematical problems. Cai (1995) compared Chinese and U.S. sixth graders' performance using Mayer's (1987) model of math problem-solving components. He found that Chinese students outperformed the U.S. students in both the translation component and the planning component, but scored the same in integration component. Particularly interesting, Chinese students outperformed U.S. students on all four translation questions containing a relational proposition. Cai (1995) related this result to the special linguistic characteristics of Chinese.

Teaching translation skills. Lewis (1989) has developed a two-session training program teaching students how to represent relational statements in word problems through reorganizing relational statements and representing them on a number line. Students were first trained to classify problem statements as an assignment, a relation, or a question sentence. Then, they were trained to diagram problems using a simple number-line method. A test-retest design showed that unsuccessful problem solvers improved their performance significantly after the training. Lewis (1989) concluded, "training aimed at remedying students' erroneous comprehension processes for relational statements can be successful and can result in transfer" (p.530) to more complex problems.

Brenner and his colleagues (Brenner, Mayer, Mosely, Brar, Duran, Reed, & Webb, 1997) developed a 20-day program for middle-school pre-algebra students to



practice daily experience in translating relational sentences, tables, graphs, and equations. Students who received the training showed much more improvement than those who received conventional instruction.

In summary, the results of these studies suggested that a major source of difficulty in mathematical problem solving was poor translation skills. Training students to build multiple representations of the problem in words, a diagram, or an equation can be helpful for students' mathematical problem solving.

#### Research on the Integration Component

Mayer (1999) defined the integration problem-solving component as the process "to put the statements of the problem together into a coherent representation" (p. 169). Problem integration relies on schematic knowledge of problem types during which, a coherent, integrated structure depicting the relations among the text's propositions is formed, and the simultaneous ability to integrate all information together so that the correct problem type schema in long-term memory can be activated.

Students' schemas for math word problems. According to Mayer's model, students need to possess knowledge of problem categories (schemas) to successfully solve the problems. Hinsley, Hayes, & Simon (1977) identified eighteen basic problem schemas. Recently, Mayer (1981) analyzed the story problems in typical high-school algebra textbooks and found approximately one hundred problem types. In another study (Mayer, 1982), students were asked to read and recall a series of eight story problems; the results

showed that the probability that a student can correctly recall a problem was strongly correlated to the frequency with which the problem type was represented in typical mathematical textbooks.

Building an internal representation of a problem requires more than a sentence-by-sentence translation. A solver could read and comprehend a problem's text but did not understand the problem's mathematical situation, if he or she did not have the skill to integrate the information from the text and extract a higher-level problem representation. In the previously reviewed training program by Lewis (1989), 96 undergraduate students were divided into three groups of 32 participants: (a) the diagram group received training in both translation and integration of information in statements of math comparison problems; (b) the statement group received only translation training; (c) the control group received no training but was exposed to the same problems as the other two groups. Results showed that the diagram group made significantly fewer errors than the other two groups that did not significantly differ with each other. Participants in the statement group, who learned to identify the different statement types in word problems, did not improve their comprehension of the conceptual structure of comparison problems beyond the improvement of the control group. Thus, as many theorists suggest (Kintsch & Greeno, 1985; Paige & Simmon, 1966), translation of problem statements in isolation promotes comprehension of text but not necessarily improve comprehension of a problem's mathematical conceptual relationships. The latter demands the ability to integrate information. As discussed previously in this chapter, the essence of integrating information is simultaneous processing. Therefore, in order to understand the problem, readers have to relate each element in a sentence or a paragraph together and find out

their relationships. Studies have consistently demonstrated that simultaneous processing is significantly related to students' reading comprehension.

Differences in strategy choices. Silver (1981) found that seventh graders poor at solving word problems tended to sort story problems based on their cover stories, while successful problem solvers tended to sort based on mathematical structures. Quilici and Mayer (1996) asked college students to sort twelve statistics word problems into categories based on similarity. They found that students who had no experience in statistics tended to group the problems based on surface features, while graduate students who had extensive experience in statistics tended to group the problems based on structural features. Students tended to change from sorting mainly by surface features before taking an introductory course in statistics to sorting at least partially by structural features after taking the course. Hildebrand (1998) found that students with reading difficulties displayed a profile of less well developed cognitive processing (as measured by CAS) than those of average achieving students. These students with reading difficulties adopted primarily surface level processing strategies in learning. Mayer (1999) summarized that "experienced problem solvers are more likely to focus on the structural features of problems, such as underlying principle or relation, whereas inexperienced problem solvers are more likely to focus on the surface features, such as the objects described in the problem" (p. 174).

Teaching the integration component. To make judgments about the relevance of information, a student needs to construct an integrated representation of the problem first. Low and Over (1989) found that students' performance in identifying adequacy of

information and irrelevant information highly correlated ( $r = .9$ ) with their ability to solve problems. Then Low gave students 80 minutes training in recognizing whether a word problem contained sufficient, irrelevant, or missing information, in the feedback, the teacher specified how to classify the problem. The pre- and post-test results indicated that students in the training group showed a much greater improvement than students in the conventional instruction of calculating solutions and control groups which did not receive any instruction. Thus, a greater mixture of problems in exercise would encourage students to learn how to discriminate among different types of problems. In addition, multiple choices mathematical tests including relevant and irrelevant information are a powerful tool to exam students' integration skills such as Mayer's tests in which students are asked to choose the relevant numbers needed for solving the problem.

In summary, the results of the studies reviewed above suggest that when students lack a schema for a given problem type, or lack skills to integrate information in a problem, the problem representation is more likely to be in error. Fortunately, the skills of integrating information in the problems and classifying problem types can be explicitly taught.

### Research on the Component of Planning

The third component in Mayer's math problem solving model is to "devise and monitor a plan for solving the problem" (Mayer, 1999, p. 181). Planning involves the selection of appropriate strategies and the allocation of resources. It frequently includes

setting goals, activating relevant background knowledge, monitoring progress, and evaluating results.

Mayer's study (1982) suggested that the presentation of the problem influenced the solution strategy a person will use. The subject's choice of strategy is at least partly determined by the presentation format of the problem. The recent studies on inconsistent language (IL) comparison problems showed that the key word in the statement may lead to a wrong problem representation and solution for poor problem solvers because they feel hard to overcome the interference of the key word (Hegarty et al., 1992, 1995).

Teaching the component of planning. Schoenfield (1979, 1985) attempted to teach student problem solving strategies including finding a related problem, restating the problem, and breaking the problem into subgoals. The result showed that students who practiced using these heuristics improved significantly better on the post-test than those in the control group who practiced solving problems without heuristic training. It suggests that some problem solving strategies can be explicitly taught to learners.

There are several instructional studies suggesting that metacognition in problem solving domains can be improved by direct instruction and modeling of metacognitive activities. For example, Paris and colleagues' Informal Strategies for Learning program (ISLP) (Paris, Cross, & Lipon, 1984; Jacobs & Paris, 1987) instructs children about knowledge and the use of metacognitive reading strategies in several ways. Gains during an academic school year have been particularly impressive with respect to reading awareness and evaluating the effectiveness of reading strategies. Delclos and Harrington (1991) examined fifth- and sixth-graders' ability to solve computer problems after assignment to one of three conditions. The first group received specific problem-solving

training; the second group received problem-solving plus self-monitoring training; and the third group received no training. The self-monitoring problem-solving group solved more of the difficult problems in less time than other groups did.

Recently, Naglieri and his colleagues (Naglieri & Gottling, 1995, 1997; Naglieri & Johnson, 2000) studied the effects of a cognitive strategy instruction designed to improve planning to groups of students with learning disabilities and mild mental impairments. The results showed that children with a planning weakness benefited from the instruction. Those children who were not low in planning did not show the same level of improvement. Thus, studies demonstrated that planning skills can be trained, and matching the instruction to the cognitive weakness of the child was suggested.

### Studies on Math Comparison Problems

#### Typology of Math Problems

Researchers have classified addition and subtraction word problems on the basis of semantic structure into four general categories: change, combination, comparison and equalization problems (Carpenter & Moser, 1983; Morales, Shute, & Pellegrino, 1985; Riley et al., 1983). The change problem implies that the child performed some type of action and this results in a changed (i.e., larger or smaller) collection. Combination, equalization, and comparison problems all begin with two quantities, which are either added or subtracted to find the whole or one of the parts. Although change and

combination problems are exactly the same in terms of computational demands, differences in the language presenting the problem gave them a very different meaning to children. This can influence how children represent and interpret the problems, which in turn can influence the child's conceptual understanding of what is being asked as well as the types of strategies used to solve the problem (De Corte & Verschaffel, 1987).

### Definitions of Two Types of Comparison Problems

The comparison problem contains a relational statement comparing the values of two variables. There are two types of comparison problems: consistent language (CL) and inconsistent language (IL) comparison problems. Lewis and Mayer (1987) clearly defined the two forms of comparison problems as follows:

In consistent language (CL) problems, the unknown variable (e.g., Tom's marbles) is the subject of the second sentence, and the relational term in the second sentence (e.g., *more than*) is consistent with the necessary arithmetic operation (e.g., addition). On the other hand, in inconsistent language (IL) problems, the unknown variable is the object of the second sentence, and the relational term (e.g., *more than*) conflicts with the necessary arithmetic operation (e.g., subtraction) (p. 363).

Studies have shown that both college and elementary school students have particular difficulties in solving inconsistent language (IL) comparison problems (De Corte, & Pauwels, 1992; Hegarty et al., 1992, 1995; Lewis, 1989; Lewis & Mayer, 1987; Morales, et al., 1985; Riley, et al., 1983; Stern, 1993; Verschaffel, et al., 1992).

### Theories on Difference in Comparison Problem Difficulties

In early research on young children's word problem solving, attention has turned to process models that explain why some word problems are more difficult than others are. They are briefly reviewed in the following sections.

Logico-mathematical models of word problem-solving. Several models have been proposed to explain the difficulty difference in CL and IL problems. Briars and Larkin (1984), Riley et al. (1983) and Riley and Greeno (1988) presented logico-mathematical models of word problem solving, which stressed the importance of mathematical knowledge. They assumed that mathematical knowledge develops from action-based external modeling of quantitative information to reasoning on the basis of the quantitative part-whole schema. Representing the part-whole schema includes understanding numbers as parts of each other, the commutativity and associativity, as well as the commentary relation of addition and subtraction (Resnick, 1989). Therefore, having the part-whole schema represented means to connect language about quantities with mathematical concepts.

According to the model, arithmetic word problems differ in mathematical knowledge requirements. Some problems can be modeled externally and thus require only simple procedures, whereas other problems demand the transformation of the problem text into part-whole relations. Inconsistent language (IL) comparison problems require access to the part-whole schema, combined with knowledge about the comparison of sets.



If the child can represent the texts into the three sets: compare, reference, and the difference set and their relationship, he or she can solve the IL problem by a mathematical transformation strategy. The child can infer from the second sentence in the problem the relationship between the compare set and the reference set, then either adds or subtracts the numbers. Thus, the knowledge of part-whole schema and their relationship decides whether children can transform and represent the textual information given in the word problem directly into a mathematical equation, which is believed to be the reason for the special difficulty of the IL comparison problem. But this modeling of the solution of comparison problems does not in all aspects fit with the empirical data (Riley & Greeno, 1988; Cummins, Kintsch, Reusser, & Weimer, 1988; Davis-Dorsey, Ross, & Morrison, 1991).

Text processing models. A second type of word problem-solving model (e.g., Cummins et al., 1988; Reusser, 1990) underscores the importance of text processing. According to these text-processing models, children's difficulties with word problems arise from a lack of textual understanding, which prevent them from making contact with relevant mathematical knowledge. For example, children interpret the relational statements such as "n more x than y" to indicating simple assignments such as "There are n x". Stern (1993) also proposed that an inability to understand the symmetry of language involving quantitative comparison made IL problems difficult for first graders. If the child can correctly understand the relational statement, he or she can simply use linguistic restructuring strategy to solve the problem, that is, transforming the second sentence into a consistent language (CL) sentence by exchanging the subject and object and changing the key word. This theory might be able to explain very young children's difficulties on

solving IL problems. However, it can not explain the difficulty of fifth to sixth graders and adults' difficulties in solving IL problems.

Lewis and Mayer's (1987) language consistency hypothesis. To explain the special difficulty of the IL comparison problem, Lewis and Mayer (1987) proposed the language consistency hypothesis. They believed that the information presentation in inconsistent language (IL) comparison problems that required more mental processing resulted in difficulty for children and even adults. They argued that people prefer the presenting of information in a particular order in which the unknown set is the grammatical *subject* of the second sentence. In the IL problem, the unknown set is the *object* of the relational sentence, students are assumed to mentally rearrange the relational sentence until it fits their preferred format. By doing this, the student needs to reverse the subject and the object of the relational sentence, and reverse the relational term. This additional transforming process of IL problems might put heavy demands on the student's working memory and leads to a wrong result.

#### Studies on Reasons for the Language Consistency Effect

In the 1990s, a number of studies analyzing students' problem solving errors, solution times and their eye fixation all demonstrated the existence of language consistency effects in comparison problem solving (Lewis and Mayer, 1989; De Corte, Verschaffel, & Pauwels, 1990; Hegarty et al., 1992). However, there are controversies as to the specific reasons causing this effect. Although IL problems are difficult, there are

individual differences in students' performance even as young as elementary school students. There are students who successfully solved the IL problems. Thus, researchers, including Mayer himself, started to consider the subject aspect of problem solving in addition to the general problem features such as language consistency effects as the loci of problem-solving difficulties (De Corte et al., 1990; Hegarty et al., 1992, 1995; Mayer & Hegarty, 1996).

Two types of strategies to solve mathematical word problems. One recent body of research on factors contributing to the difficulty of comparison problems emphasizes the individual difference in students' general strategies to solve math problems (De Corte et al., 1990; Hegarty et al., 1992, 1995; Mayer & Hegarty, 1996).

De Corte et al's study: In an early study on children's word problem solving, for the first time, De Corte et al. (1990) investigated different strategies of the high and low ability (HA and LA) students in CL and IL problems by using the eye fixation procedure. De Corte and his colleagues explored the two possible explanations of the poor performance of LA students in IL problems, that is, a rash, impulsive strategy due to an absence of a semantic processing stage or a faulty semantic analysis due to less-developed semantic schema. In the study, 20 second graders were instructed to mentally calculate 16 one-step addition and subtraction word problems and answer orally, with no time limit. Eye fixation data and the child's answer were recorded. The response time was divided into two parts: first time reading period involving problem translation and rereading period including problem representation and solution seeking. Ten high-ability (HA) and ten low-ability (LA) pupils were selected based on the total score of the 16 problems. They hypothesized that if LA children systematically use the superficial (key word)

strategies, the total time LA students spend on words will be significantly less than HA students. The study did not find supportive evidence for the systematic use of superficial strategies in LA children. Thus, they concluded that the use of superficial strategies can not be considered as the overall explanation for the lower performances of LA students on solving IL problems. The appropriate explanation is that low-ability children's failures are not the result of the absence of a semantic processing stage, but of their faulty semantic analysis.

However, this study has some methodological problems. The one-step word problems used in the study are not challenging for HA pupils, so there is no data on coping strategies of HA student on CL and IL problems. More importantly, students were asked to solve each problem by mental calculation, so the problem solving performance was confounded with text reading and the computational skills. It is hard to decide whether the longer reaction time during the stage of reread problem is due to the longer time to integrate problem representation or to calculate the answer. It is possible that LA students adopt "key-word" superficial strategy; however, their overall reaction time and reaction time in the second stage are not significantly shorter than HA due to LA students' slow calculation. Confounding integration and calculation may mask the appearance of students' different strategy choices.

Mayer and Hegarty's studies: Mayer and Hegarty further examined the language consistency hypothesis and proposed that the "key word" strategy is the reason for the poor performance on IL problems (Mayer and Hegarty, 1996; Hegarty et al., 1992, 1995). They did a series of experiments on college students using the eye movement procedure and found that poor solvers were more likely to use the key word strategy. Poor problem

solvers decided the operation based on the superficial key word of the problem, for example, add if there is a word "more", subtract if "less". Whereas good solvers were more likely to use situation strategy, deciding the operation based on text understanding and a correct problem representation.

Hegarty et al.'s study (1992) improved the above-mentioned methodological issues of De Corte et al.'s study (1990). They used more demanding two-step problems. In their study, 32 undergraduates were asked to tell how they would solve the problem but not to carry out any actual arithmetic operation, in this way, the problem representation and execution processes would not be confounded. They found that consistency language effects on response time (i.e., students spent longer time in IL than CL problems) only occurs for high-accuracy subjects. Thus, only the HA students are sensitive to the different semantic features of CL and IL comparison problems and cope with them differently. Also, they found the consistency effect occurs during the later integration and planning phases, not in the initial reading of the problem (the translation phase). Finally, they found that HA students spent more fixation time on words and key words than on the numbers in second stage of rereading the IL problems. This study concluded that low-accuracy students appeared to be using the direct translation approach, whereas high-accuracy students appeared to be using the mental model approach.

Recently, Hegarty et al. (1995) further examined their hypothesis on different strategies of HA and LA students based on eye fixation data. They proposed that the selection effect (selecting key words) observed in previous studies (Hegarty et al., 1992; De Corte et al., 1990) was symptomatic of the direct translation strategy. The absence of it is more reflective of a problem-model strategy. In their study, 38 undergraduates were

asked to tell how he or she would solve the problem but not to carry out any actual arithmetic operations. Eye movement data showed again that unsuccessful problem solvers were more likely than successful solvers to look at numbers and relational terms when they reread part of the problem. In addition, they analyzed two kinds of errors in students' recall and recognition tests. A literal error occurs when the relation between the two terms in students' recall or recognition was consistent with the presented problem, but the wording of the relational term was changed. A semantic error occurs when the relation between the two terms was reversed. They found that successful problem solvers made more literal errors and less semantic errors on both recall and recognition tasks than the unsuccessful problem solvers, which suggested that the successful problem solvers understand the problem and they are more likely to remember the situation described in the problem.

Mayer and Hegarty (1996) described in details each of the three cognitive components for two strategy users. The direct translation strategy consists of a translation process in which a problem solver mentally represents each statement in the word problem as a semantic network, and an integration process in which a problem solver extracts numbers and the key words that suggest which arithmetic operations need to be performed. The resulting solution plan is likely to be incorrect for inconsistent comparison problems. In contrast, the problem model approach consists of the same translation process but a different integration process in which the problem solver seeks to mentally construct a model of the situation described in the problem, which leads to a correct solution plan.

These studies, including eye fixations of elementary school and college students (Hegarty et al., 1992, 1995; Verschaffel et al., 1992), remembering word problems (Hegarty et al., 1995; Mayer, 1982), and learning to solve word problems (Lewis, 1989; Lewis & Mayer, 1987), all seem to provide supporting data to the two strategies hypothesis (Mayer & Hegarty, 1996). LA students systematically use key word strategy that leads to correct performance in CL problem and poor performance in IL problems.

Recently, Stern (1993) challenged Hegarty et al.'s (1992; 1995) hypothesis of systematically use of key word strategy by the following logic: if poor problem solvers solve CL problems better than IL problems because they generally used key word strategies yet did not really understand the problems, their performance in retelling the two problem types would be equivalent. Retelling problems is supposed to be able to accurately reflect students' problem representations. Because verbal storage of a problem for most problems exceeds working memory capacity, therefore, to correctly store a problem requires comprehension, one has to understand and transform the information into a problem that is less demanding of working memory. Stern conducted a series of experiments to analyze first graders' performance and their retelling protocol. He found most first graders were able to retell and understand the CL problems but were not able to retell IL problems. Thus, he made the conclusion that the general key-word strategy is not the reason for the difficulty difference in CL and IL problems.

Stern's data demonstrated that students' better performance in CL problems was not due to the usage of key word strategy because students understood CL problems and were able to correctly retell the problems. However, in logic, this does not necessarily lead to the conclusion that poor performance in IL problems is not due to key word strategy.

Actually the result indirectly supported the hypothesis that students use key word strategy for IL problems because they did not understand the IL problems and could not retell IL problems. Based on the previous studies, it seems plausible to conclude that even LA students can understand the CL problems and can correctly solve the problem based on understanding. However, they can not understand IL problems due to low working memory, and they have to use key word strategy in these IL problems.

All these studies have not directly and explicitly examined the cognitive processes of IL problem solving. The differences of cognitive processes in successful and less successful students have not been explored. There are still many cognitive factors that need to be explored, such as PASS processes and students' working memory ability.

There are a few studies relating the difficulty of IL problems to students' working memory.

Verschaffel et al. (1992) directly compared effects of the working memory load in students' performance. He gave third graders and university students one-step CL and IL problems. The results confirmed Lewis and Mayer's language consistency hypothesis only for third graders; that is, more reversal errors and longer solution times were found for IL problems than for CL problems. University students showed similar results only when they had to mentally solve more complicated two-step problems. Apparently, the effect of inconsistent language on problem solving only occurs when the comparison problems have to be processed under rather heavy cognitive demands, in which it takes more time and a longer fixation time to transform the relational sentence in IL problems. This shows that working memory load seems to be an important factor that influences performance on comparison problems. It is plausible to hypothesize that different



working memory, especially the ability to activate processing information while holding the information, is relevant to mathematics performance.

Most recently, d'Ailly, Simpson and MacKinnon (1997) tested whether and how self-referencing (i.e., using pronoun "You") affect one hundred third to fifth graders' cognitive processing in solving simple mathematical word problems. They hypothesized that self-referencing in comparison problems (for example, use of the term "You" in place of a noun) could decrease the working memory load effectively. They found that for Grade 3 children, working memory was proved to be an important factor in solving the problems. The marker test of working memory in the study was a dual task similar to the task developed by Swanson (1992).

Summary. In general, studies by Verschaffel et al. (1992), Stern (1993) and d'Ailly et al. (1997) all suggest that beyond LC and LI's problem features, students' mathematical conceptual knowledge and strategies, there might be other underlying factors influencing students' performance in IL problems. According to conceptual analysis of math problem solving components and PASS theory, these underlying factors might relate to working memory and simultaneous processing skills. Studies have found the difficulty of the IL problem located in the second stage of reading, that is, rereading and integrating information into a problem representation. Conceptually, simultaneous processing is particularly related to information integration. Thus, we hypothesize that simultaneous processing is one of the potential factors influencing students' performance in the comparison problem.

In conclusion, what the factors are that can account for students' difficulties in the IL problem remain unresolved. It needs further examination under the guidance of

advances in cognitive psychology theories. Based on recent studies (Stern, 1993; Verschaffel, 1994; d'Ailly et al., 1997), it is plausible to hypothesize that working memory and simultaneous processing are important potential factors influencing students' performance in IL problems. Studies along this line might shed some light on our understanding of the cognitive processes underlying comparison problem. However, there is not a clear body of research or theory directly addressing this issue. Fortunately, the new developments in intellectual assessment such as the PASS theory on information processing might help us understand the nature of cognitive processes involved in comparison problems, especially the process of constructing a problem representation for the IL problem.

#### The PASS Theory, Mayer's (1989) Model, and Mathematics Achievement

##### Use of the New Assessment Tools

In the past, although test scores have basically provided quantitative information regarding student performance, rarely have they reflected the qualitative or cognitive aspects of information processing (Sternberg, 1991). However, an increasing emphasis on a cognitive analysis of performance has shifted attention more to the cognitive or qualitative aspects of information processing: such as strategies for problem solving and modes of representation. The PASS theory of cognition that has been described in the previous review is an example of a new approach to examine math achievement. The

greatest advantage of such a model is that it is theory-based. It offers a theoretical basis for understanding PASS processes underlying math achievement, a solid basis for clinical diagnosis, and a rational basis for the remediation of low achievement.

The PASS theory is a domain general information processing theory applicable to various subject areas, whereas Mayer's model is a domain specific theory on mathematical word problem solving. The PASS processes are more basic level skills. Use of these skills in math problems would lead to successful performance in math problem-solving component of translation, integration and planning. Previous studies have shown that simultaneous, successive processing and planning are related to math problem solving. However, it is not clear yet how these basic cognitive processes affect each math component. We do not know the mechanism of cognitive processing in each math problem-solving component. By using these two models in this study, we can get a more complete picture with details about students' mathematical performance and its nature in aspects of the underlying cognitive processes. It can also help us understand the cognitive deficiencies of students' difficulties in math problem solving.

#### PASS Processes and Math Problem-Solving Components

There are no empirical studies that directly relate the PASS theory and Mayer's (1987) model together. However, studies on PASS theory and reading could give us some insights about the relationship between the PASS processes (planning, simultaneous and successive processing) and math problem solving components (translation, integration and planning). Conceptually, translation and integration processes actually require very similar cognitive processes to those of reading comprehension.

### Successive processing and the translation problem-solving component

The first step in mathematical word problem solving, translation, actually is a sentence comprehension task. Basic level reading has been shown to require all PASS process, especially sequential processing (Naglieri & Das, 1987; Das, et al., 1979).

Kirby and Williams (1991) stated the role of successive processing in cognition research as follows:

Within the psychology of intelligence, successive processing has been studied either as a rote memory ability or as a part of analytic reasoning. Most psychometric batteries of mental tests include measures of rote memory ... Successive processing has also featured prominently in information processing theories, though again under other names. Most theories contain a short-term or working memory, which is responsible for holding input information in serial order, to allow for further processing. ... This is presumably the same rote memory that the psychometricians test, but used in such instances for far more complex tests. The term "successive processing" is intended to capture this range of cognitive activity (p. 160).

As Kirby and Williams (1991) stated, the relationship between successive processing and reading comprehension has been manifested in literature of reading and working memory. According to Baddeley and Hitch's (1974) working memory model, much of the earlier short-term memory literature were incorporated into the concept of slave systems of their working memory model: the articulatory or phonological loop and the visuospatial

sketchpad. These slave systems refer to a temporary store (by phonological store) and a rehearsal mechanism for speech-based (by articulatory loop) and visuospatial-based information, respectively (Baddeley, 1986). Based on this working memory model, short-term memory is viewed as the part of phonological loop in working memory. Simple digit span and word span traditionally are tasks that putatively reflect the short-term memory. On the basis of conceptual analysis and research evidence, the two subtests of sequential processing in K-ABC, Number Recall and Word Order, can be argued to reflect the capacities of short-term memory or phonological loop of working memory (Kauphaus, Beres, Kaufman, & Kaufman, 1997). There is evidence that the phonological loop contributes to performance only in tasks requiring the retention of order information (Richardson, 1996). Thus it is plausible to propose tentatively that the phonological loop corresponds to the successive processing in PASS theory.

Baddeley, Gathercole, & Papagno (1998) summarized that phonological loop plays a crucial role in learning the novel phonological forms of new words. The primary purpose of phonological loop is to store unfamiliar sound patterns while more permanent memory records are being constructed. This finding is consistent with PASS model that successive processing is involved into word decoding (Das et al., 1979). Moreover, there is evidence of a relationship between phonological loop function during language acquisition and syntactic development.

Reading achievement during childhood is linked with phonological short-term memory ability. Poor readers generally do not perform well in short-term memory tasks such as digit span, serial recall of unrelated strings of words, and the repetition of non-words (Gathercole & Baddeley, 1993). Children with poor abilities to hold phonological

material temporarily may fail to develop adequate long-term representations of the words and phrases that are used to build syntactic patterns in speech (Speidel, 1989, 1993).

Correlational studies of normally developing children also support a link between phonological memory ability in young children and speech output. Word span was found to be a better predictor of performance of 2- to 6-year-old children than chronological age in an artificial grammar-learning tasks (Daneman & Case, 1981), mean length of utterance in 2- to 3-year-olds (Blake, Austin, Cannon, Lisus, & Vaughan, 1994), and 3-year-olds' spontaneous speech (Adams & Gathercole, 1995).

Engle, Cantor, and Carullo (1992) argued that short-term memory was important to reading comprehension that involves surface coding (e.g., the recall of words in a phrase, i.e. literal comprehension); whereas working memory was important for grasping the complexities in reading comprehension. Swanson (1984) found that the effect of short-term memory in predicting reading comprehension and math ability was enhanced in the sample with learning disabilities, and argued that this might be due to the fact that students with learning disabilities might relied more on surface coding than normal group.

In summary, the phonological loop, which may correspond to the successive processing in the PASS theory, plays a crucial role in syntactic learning and in the acquisition of the phonological form of lexical items. This finding in working memory is consistent with findings in PASS theory that successive processing is associated with syntactic processing of sentences. Thus, these findings all provided indirect evidences that successive processing were associated with translation problem-solving component.

### Simultaneous processing and the integration problem-solving component

During the second stage of math problem solving, integration, students were required to relate all elements in mind, and simultaneously process the information to build a problem representation. Simultaneous processing is important for the discovery of conceptual relationships between objects and events, so it is obviously involved in more advanced stages of reading, such as the integration component. Consistently, previous studies on PASS processes and reading revealed that simultaneous processing and planning were important for high level reading comprehension (Cummins and Das, 1977).

Kirby and Williams (1991) summarized that simultaneous processing was clearly demonstrated in verbal tasks that require the relating or integrating of discrete pieces of information, such as categorization and analogy. Simultaneous processing is also involved when the semantic coding of words is used to construct a new meaning in comprehending spatial or relational statements. Kirby and Williams (1991) specifically summarized the importance of simultaneous processing to math word problems as follows:

At the level of mathematical word problems, most problems contain complex verbal information that must be comprehended. In many cases, inferences, or at least relationships between sentences, must be made. Therefore, virtually all of the examples of simultaneous processing in reading are also relevant to the solution of word problems in mathematics (p.200).

In addition, the literature in working memory and reading provided us more insights on the relationships between simultaneous processing in reading. In Baddeley and Hitch's (1974) working memory model, the multiple specialized subcomponents of cognition function differently through multiple components of working memory, namely, a central executive controlling mechanism and two subsidiary or "slave" systems, called the phonological loop and the visuospatial sketchpad, which are specialized for the processing and temporary maintenance of material within a particular domain.

According to Baddeley and Logie (1999), "the central executive offers the mechanism for control processes in working memory, including the coordination of the subsidiary memory systems, the control of encoding and retrieval strategies, the switching of attention, and the mental manipulation of material held in the slave systems...

*However* (italic added), The organization of these processes remains an open question and is the subject of ongoing empirical exploration" (p. 30).

Comparing to the slave system of working memory, the central executive is a much less specified construct in the working memory model. Baddeley himself admitted that "it is not satisfactory to simply leave the central executive as a useful ragbag to contain all the phenomena that cannot be readily accounted for otherwise (Baddeley & Logie, 1999, p. 41)". Baddeley and Logie (1999) believed that "each of these components of working memory can be further fractionated if such fractionation is adequately justified empirically. ... The central executive may also, in principle, not be a unitary construct, and a fractionation into different subcomponents or subprocesses is probably necessary" (p. 32).



Despite of the unclear nature of central executive, studies have consistently found that it plays an important role in many complex cognitive activities, such as comprehension (Gathercole & Baddeley, 1993), counting (Logie & Baddeley, 1987), mental mathematics (Logie, Gilhooly, & Wynn, 1994), syllogistic reasoning (Gilhooly, 1998), and dynamic cognition and complex perceptuomotor control (e.g., a complex computer game) (Logie, Baddeley, Mane, Donchin, & Sheptak, 1989). In reading, the central executive is assumed to activate representations in long-term memory (LTM) extending up from individual words and concepts to complex schemata. Baddeley and Logie (1999) assumed that "the capacity to comprehend a particular passage will be determined both by the existing representations in LTM and by the capacity of the central executive to activate and combine such representation into a coherent mental model, which can then be consolidated into LTM" (p. 42). This process can be viewed as similar to simultaneous processing.

In summary, the working memory literature that central executive processing is associated with reading comprehension (Gathercole & Baddeley, 1993) provides indirectly evidence to the involvement of simultaneous processing in integration problem-solving component, if we assume simultaneous processing is at least one of the underlying processes of the central executive (Fan, 2000). Of course, this assumption on the simultaneous processing and the central executive is speculative and needs further empirical evidence. Studies on PASS processes and the sub-components of working memory are welcomed.

### PASS Process of Planning and Planning Problem-Solving Component

Findings of previous studies on planning and reading indicated that fast visual search group recalled more sentences, tended to master the central statements, questioned and evaluated the sentences they made, and were likely to modify their hypotheses as they went on reading (Ramey, 1985). Planning is shown to be increasingly important for reading with age increases (Naglieri & Das, 1988). Thus, it is plausible to hypothesize that PASS processes of planning skill is associated with students' translation, integration and planning components of math problem solving.

An interesting aspect is linking planning and/or attention processes of the PASS theory with the central executive component of working memory. As discussed previously, the exact nature of central executive is not clear yet. Baddeley and Logie (1999) declared that the control processes involved in the central executive are complex. More explicitly, Engle, Kane, & Tuholski (1999) claimed that "working memory capacity" is not really about storage or memory per se, but about "the capacity for controlled, sustained attention in the face of interference or distraction." (p. 104). Engle et al. (1999) further proposed that working memory equaled to controlled attention plus short-term memory. They believe that individual differences on measures of working memory capacity primarily reflect differences in capacity for controlled processing, which is the mediation of the strong relationship between working memory measures and fluid intelligence. They performed an analysis of the unique and shared variance in tasks reflecting short-term memory, working memory, and fluid intelligence. A structural model analysis revealed that the component of the working memory tasks important to

higher-order functioning was controlled attention rather than short-term memory. They summarized as follows:

When tasks demand that subjects selectively focus attention amidst external or internal sources of distraction, or that subjects shift attention according to memorized rules, or that subjects divide their attention between different stimuli or tasks, working memory capacity and psychometric  $g^F$  scores are good predictors of performance (p. 113).

According to Engle and his colleague's definition of controlled attention, conceptually the controlled attention should involve the planning and attention PASS processes. The controlled attention has been shown to be the essence of  $g^F$  and good predictors of performance in higher-order cognitive activities, therefore, planning should be associated with math problem solving, especially the problem-solving components of integration and planning.

### Summary

All above reviewed studies suggest that PASS processes might contribute to the difficulty of understanding math problem statements (e.g., in translating relational statements), of integrating information into a problem representation, and of devising an efficient solving plan. However, the relationship between PASS processes and math problem-solving components has never been explicitly explored. This study filled this gap by investigating cognitive processing (planning and simultaneous - successive processing) and math problem-solving components (translation, integration, and planning).

### Cognitive Processes in Comparison Problems

The analysis of the relationships of information coding and planning to math problem solving should be able to shed light on understanding students' special difficulty in solving inconsistent language (IL) comparison problem. This question needs to be investigated because of its theoretical importance and its educational implications.

#### Why Study Comparison Problems?

Comparison problems are chosen for this study for two reasons. First, studies consistently found that they are particularly difficult for students from elementary schools to colleges. Second, studies consistently indicated that building problem representation is the most important and hardest part in math problem solving. The comparison problem has special features that make it a good tool to study the process of problem representation formation. The two types of comparison problems have identical mathematical structures (and therefore require identical level of computations), but differ in their demands on students' representation skills, especially simultaneous processing skills and working memory. Thus, the comparison problem provides us a good tool to study children's problem representations.

### Simultaneous Processing and Comparison Problems

According to the previous review, the difficulty of the inconsistent language (IL) problem can be attributed to the integration process to build problem representation (De Corte et al., 1990; Hegarty et al., 1992; 1995). Simultaneous processing has been shown to be strongly associated with more advanced levels of comprehension and inference (Kirby & Das, 1977; Cummins & Das, 1977; Naglieri & Das, 1987). Thus, we can propose that the underlying basis of the integration process, simultaneous processing, should be one of the reasons for the individual difference of performance in IL problems. That is, some students do not have enough simultaneous processing ability or can not use this ability to construct a coherent internal representation of the problem. As summarized by De Corte (1990), studies consistently found that "low-ability children's failure are not the results of the absence of a semantic processing stage, but of their faulty semantic analysis" (De Corte et al., 1990, p.365). We can further propose that the faulty semantic analysis may be attributed to the poor simultaneous processing skills. In the IL problems, in order to correctly complete the representation process, the student must clearly understand the relationship between the two sets, such as which is the larger set and which is the small set, and remember which set is a difference set (information in the last sentence) within the limited working memory. Keeping such information in mind, at the same time, the student has to simultaneously convert the subject and object in the relational statement and change the relational term. This process definitely demands good simultaneous processing capacity. Our hypothesis in this study is that simultaneous processing is strongly associated with differences of students' performance in the two

types of comparison problems. Students with good simultaneous processing should perform similarly in both types of problems, whereas students with poor simultaneous processing should perform poorly in IL problems.

### Training of Simultaneous Processing

If students' simultaneous processing skills can be improved to a higher level, they may be able to understand the IL problem and solve it correctly as they do in the CL problem. Preliminarily, Lewis (1989)'s training study showed that we could help students effectively build the problem representation by teaching them a method for diagramming problem information. This diagramming method actually functioned as an outside memory aid. By diagramming the information, students did not need to keep the information of the problem in their working memory, they could focus on seeking the internal relationship between the problem elements and then reverse the inconsistent language sentence into a consistent one. In this way, the task demanded much less working memory. Diagramming might also reduce simultaneous processing demands by clearly and explicitly displaying the elements of the problem and therefore clarified their internal relationship.

In summary, based on all these studies, we can hypothesize tentatively that simultaneous processing correlates with students' performance differences in the two types of comparison problem. Students with higher simultaneous processing skills will be more likely to use these skills in the integration component and construct a correct problem presentation, which will result in a correct problem solution. In contrast, students

with lower simultaneous processing do not have adequate simultaneous processing ability to integrate the information and build up correct problem representations, so they are more likely to return to a lower level but easier way to solve the problem, using direct translation strategy. Even students with lower simultaneous processing skills seemed to understand that they should build up a conceptual representation, and they did tried as suggested by their longer response time in the integration phase. However, due to poor simultaneous processing, the processing overloaded their limited working memory, and they could only rely on the direct translation strategy. After trained by a more helpful strategy, for example, diagramming as an outside aid to free their simultaneous processing ability and working memory, they could improve their performance to a higher level.

### Cross-Cultural Studies on Comparison Problems

Although there are many cross-cultural studies on mathematics, there is no studies comparing North American and Asian students' performance in IL comparison problems. Cai's (1995) study is the only reported cross-cultural study involving only the consistent language (CL) comparison problem. Cai (1995) found that Chinese students outperformed U.S. students on all four translation questions involving relational propositions. Cai (1995) explained that this might relate to the more explicit expression of the comparing nature of the problem presented in Chinese.

If we can test and compare American and Chinese students' simultaneous and successive processing, and planning, in relation to mathematical achievement, the result

might be able to shed light on and explain the well-documented cultural difference in math achievement. We can hypothesize that Asian students' higher math score in two math tests (computation and problem solving) should be correlated respectively to their relatively higher cognitive processing. If they are good at only computation, we can predict that it might be due to their better successive processing. Geary et al., (1993) found that Chinese students adopted a more advanced counting strategy due to their better short-term memory, which is mediated by their one-syllable number words in Chinese. If Chinese students are good at problem solving, it might be due to their superior simultaneous processing and /or planning skills. Cross-cultural studies on cognitive styles provided preliminary data to support this hypothesis. For example, Gardner (1986) conducted a validity study of K-ABC to Cantonese, English and Punjabi speaking third graders in Vancouver, Canada. The findings that Cantonese speaking students' high scores on the Triangles (the highest simultaneous loading) and the much lower scores on the Number Recall subtest (the highest sequential loading) supported the conclusion that the Cantonese group was much stronger in simultaneous processing than sequential processing compared to the English and Punjabi children (Gardner, 1986). This might relate to the fact that the Chinese characters used in students' daily life are structurally different from English and need to be processed as a whole.

### Summary

Studies increasingly suggest that children's simultaneous-successive processing and planning related to their mathematical problem solving. However, systematic studies on children's cognitive processes in mathematical problem solving have not yet been done from the perspectives of PASS theory.



Despite of this, there are a number of studies on individual differences in math problem solving indirectly related to this topic. Hembree (1992) did a meta-analysis to 487 research reports (1920s - 1980s) on math problem solving. Four main regions that related to math problem solving were identified. First, studies examined the relationship between math problem solving and a series of mental abilities including high-order thinking skills and a structure of mental abilities such as creative thinking, critical thinking, memory, perception, reasoning, skill with analogies, skill with inferences and spatial ability. Skill of forming analogies and skill at general reasoning achieved the strongest link with problem solving measures. Second, the relationships between problem solving performance and standardized measures of IQ, verbal achievement (reading and vocabulary), and math achievement (computation, math concepts, reasoning and vocabulary) were also positive and statistically significant. However, none of the links seemed stable across school grade level. The strongest relations with problem solving were found for basic skills in math, the weakest relations appeared for traditional IQ measures. Third, differences in math problem solving performance were studied with regard to demographic variables of gender, ethnicity and socioeconomic status (SES). No difference in performance was found between females and males in Grade 1-8. However, males performed better in high schools and showed a greater advantage in colleges. Students in the majority group scored significantly better than minority groups. High SES students outperformed their peers of low SES. Finally, a number of studies examined relations between problem solving performance and measures of various sub-skills involved with the problem-solving processes (e.g., comprehending the problem). The results consistently found that successful problem solvers are those students who

comprehend better, translate from English to mathematical symbolization more easily, select correct operations more often, and judge related information more correct. The more students perceived similarity on the basis of surface details or context, the lower their problem solving scores. Particularly interesting, to identify activities that may affect problem-solving performance, students were asked to think aloud while solving the problem. The result revealed that drawing a correct diagram to represent a problem was the most effective way to benefit problem-solving performance. This result is consistent with the finding of Lewis's training study (1989). Diagrams can function as an external memory aid to reduce the working memory load and the demand of simultaneous processing.

The findings of this meta-analysis showed that some basic mental abilities and the various sub-skills involved with word problem solving were important factors influencing math problem solving performance. Based on the previous review, we can view PASS processes as basic level mental abilities that might be related to math problem solving. Mayer's math problem-solving components can be viewed as a series of sub-skills of math problem solving. Conforming to the findings of Hembree's meta-analysis (1992), we hypothesize that PASS processes and Mayer's components significantly relate to math problem solving performance.

Similarly, Geary (1994) suggested the direction of future studies as follows:

it is very likely that individual differences in the ability to mentally translate and represent the meaning of arithmetical and algebraic word problems (Lewis & Mayer, 1987) along with the ease with which the associated schemas develop are important sources of performance differences in mathematical reasoning.

Individual-difference studies that explicitly examine these skills, in concert with arithmetical processing and working memory skills, for their relation to performance on mathematical reasoning are needed to fill in the gaps in our understanding of this area (p. 147).

This study is an attempt in this direction. Individual differences in mathematical problem solving were examined in concert with cognitive components of problem solving (translation, integration and planning), computation skills, and general cognitive processing related to working memory (planning, simultaneous and successive processing). The findings is helpful for our understanding of the cognitive processes underlying math problem-solving so that more effective instructional and remedial programs can be designed in the future.

### Research Questions

The purpose of this study was to examine the children's simultaneous-successive processing and planning and its attribution to mathematical problem solving. This proposed study aimed to clarify the relationship between PASS processes (planning and simultaneous-successive processing) and math problem-solving components (translation, integration, and planning). In addition, the effects of PASS processes on students' performance in the two types of comparison problems (CL and IL) were examined to explain students' difficulty. Moreover, cognitive profiles of PASS processes in the special poor problem solvers in problem solving and in IL problems were analyzed. A particular

group, Chinese sixth graders, which has been consistently found to be prestigious in mathematics achievement, was chosen in this study.

Six research questions derived from this literature review are as follows:

1. What is the relationships between PASS processes (planning and simultaneous-successive processing) and arithmetic computation?
2. What is the relationships between PASS processes (planning and simultaneous-successive processing) and arithmetic problem solving?
3. What is the relationships between PASS processes (planning and simultaneous-successive processing) and Mayer's math problem-solving components (translation, integration, and planning), respectively?
4. How are PASS processes manifested in the special group of poor arithmetic problem solvers?
5. What is the relationships between PASS processes (planning and simultaneous-sequential processing) and students' performance in the two types of comparison problems (CL and IL)?
6. How are PASS processes manifested in the special group of poor problem solvers in inconsistent language (IL) comparison problems?

### Research Hypotheses

Based on the research questions presented in the previous section, it is hypothesized that:

1. Successive processing and planning are significantly associated with children's computation skills.

2. All PASS processes (planning, simultaneous and successive processing) are significantly associated with children's arithmetic problem solving performances. In addition, simultaneous processing is the best predictor of math problem solving.

3. Successive processing is significantly associated with students' performance in translation problem-solving component.

4. Simultaneous and planning are significantly associated with integration problem-solving component.

5. Planning and simultaneous processing are significantly associated with planning problem-solving component.

6. Poor arithmetic problem solvers perform much poorer in all PASS processes than their peers who can successfully solve the arithmetic word problems.

7. For Chinese sixth graders in this study, inconsistent language (IL) comparison problems are significantly harder than consistent language (CL) comparison problems.

8. There are significant main effects of problem type and simultaneous processing level on students' performance in comparison problems. Students with high simultaneous processing skill significantly outperform their peers with lower simultaneous processing skill. And there is a main effect of the interaction between problem type and simultaneous processing level. Students with high simultaneous processing perform similarly in the two types of problems, whereas students with lower simultaneous processing perform significantly poorer in IL problems than in CL problems.

9. There are significant main effects of problem type and sequential processing level on students performance in comparison problems. Students with high sequential processing outperform those with low sequential processing. However, the interaction between sequential processing level and problem type is not significant. The performance difference in the two types of comparison problems are similarly for both students groups.

10. There are significant main effects of problem type and planning level on students' performance in comparison problems. Students with high planning skills perform significantly better than those with low planning skills. And there is a significant interaction between planning level and problem type. The performance differences in the two types of comparison problems are small for students with high planning skills, whereas the performance differences are large for those with poor planning skills, that is, students with poor planning skills perform significantly poor in IL problems than in CL problems.

11. Poor problem solvers in IL problems perform significantly poorer in all PASS processes than their peers who can solve IL problems successfully.

## CHAPTER 3: METHODOLOGY

This chapter first describes the participants and design, then describes in detail measures of the three PASS processes, the three math problem-solving components, and students' performance in comparison problems. Finally, the procedures is briefly described.

### Participants

The participants in this study were 100 grade six students from two suburban "common" schools in Xi'an, China.

### General Background Information

Xi'an is a large, industrialized city and cultural center in Northwest China. Xi'an covers a total area of over 9700 square kilometers, and there are seven districts and six counties under the jurisdiction of the municipal government. It has a population of over 5.2 million, which is representative of the population of most of the large cities in China.

The school system in Xi'an consists of two types of schools: a few key schools that are highly selective and offer a high-quality educational program; then, many common schools that recruit students by family location -- they have a much more representative population than the selective "key" schools. Two "common" schools participated in this study. One school [601 Elementary School] is in the southwest of

Xi'an. About half of the children are from farm families, about one-quarter from worker families and the remainder are from professional/intellectual families (at least one parent has a university degree). Two intact classes in grade six participated in this study. All 77 children whose parents gave consent for participation in this study were recruited; no other exclusive criteria were used. The author excluded one child's data from data analysis because the mathematics teacher identified the child as having learning difficulties. The other 24 children were recruited from another school. This school [Xi'an Steel Factory Affiliated Elementary school] is located in the western suburb of the city. Most children come from families of factory workers and merchants. They were selected by the math teacher from a sixth grade class, the criterion is randomly selecting from high, medium and low achievement groups.

#### Grade Level of the Participants

Grade six students were chosen in this study for the following reasons. First, this study attempts to examine children's problem solving in mathematical word problems, and mathematical word problems are included and emphasized mainly in the higher grades in elementary schools in China. Second, this study will use a set of mathematical word problems adapted from a cognitive component test originally designed by Mayer. The test was designed and previously used for grade five and grade six children. In addition, Cai (1995) has used this test to measure the mathematical achievement of American and Chinese Grade six students, and found that the test is age appropriate. Thus, this study recruited sixth graders as participants.



### Sample Size

There are two ways to decide the minimum number of participants in this study. One is to simply decide according to a rule of thumb: select the number that represents the largest number of scores the dependent variables will generate, which is three in this case (translation, integration and planning). Then, multiply this number by a highly conservative number 20 (there are 20 subjects per variable). Thus, this study requires at least 60 participants.

Another way to determine the number of participants more accurately is to do a power analysis. Power is a function of sample size, the magnitude of the effects of the independent variables, and the alpha level. In this study, .80 was selected as the level of power. Then the next step is to decide the magnitude of the effects of the cognitive processing based on findings from previous studies that examined mathematical achievement and cognitive processing. For example, correlation coefficients between simultaneous processing and various mathematics achievement scores ranged from .34 to .72; correlation coefficients between successive processing and mathematical achievement scores ranged from .22 to .50. Thus, the average correlation coefficient between simultaneous processing and mathematics, successive processing and mathematics would be around .50 and .40, respectively (Kaufman & Kaufman, 1983c; Cheng et al., 1984; Garofalo, 1982). Finally, one can look at the Table of Power of Significance Test of  $r$ , at  $\alpha = .05$  (Two Tailed), for power = .80, when population  $r = .40$ ,  $n = 46$ ; when  $r = .50$ ,  $n = 28$ . Thus, 30 to 50 participants would be enough for the power level of .80 at the  $\alpha$  level of .05. To be more conservative, we doubled the size, and so the sample of this study was set at 100.

## Design

The present study included three parts. The first part was a correlational design (see Table 2). The data were analyzed by computing zero-order correlations between cognitive processing (simultaneous-successive processing and planning) and two levels of achievement measures: score level and cognitive components level. The score level of mathematics achievement included students' final scores in mathematical problem solving and computation; the cognitive components level of mathematics achievement involved the three cognitive components of solving mathematical word problems based on Mayer's model, translation, integration and planning. Then factor analysis of the five subtests of PASS theory and multiple regression on each math problem-solving component with PASS processes as independent variables were conducted.

The second part of the study was a  $2 \times 2$  mixed factorial design (see Table 3). It investigated the effects of cognitive processing (as measured by PASS processes) and problem type (CL vs. IL) on children's performance in the two types of comparison problems. Based on students' scores of each cognitive processing, participants were assigned into high and low level. All groups of students received tasks of consistent language (CL) and inconsistent language (IL) comparison problems. The independent variables were cognitive processing level and problem type. Cognitive processing level (high vs. low) was a between-subject variable. Problem type (CL vs. IL) was a within-subject variable. The dependent variable was performance in the two types of comparison problems. The mixed factorial ANOVA on each math problem-solving component was

conducted, respectively, with cognitive processing level and problem type as independent variables.

Table 2

Correlational Design

<b>Cognitive Processing</b>	<b>Mathematics Achievement</b> (dependent variables)		
<b>PASS processes</b> (independent variables)	<b>Score Level</b>	<b>Cognitive Processing Level</b>	
	Computation -tion	Problem Solving	Translation Tion
<b>Simultaneous Processing</b>			
Photo Series			
Triangles			
<b>Successive Processing</b>			
Number Recall			
Word Order			
<b>Planning</b>			
Matching Numbers			

In the last part, the manifestations of PASS processes for good and poor problem solvers in the composite scores of Problem Solving and in performance in IL comparison problems were analyzed, respectively.

Students were first assigned into groups of good and poor problem solvers based on their performance in the composite scores in Problem Solving. A MANOVA on all

PASS processes was analyzed with the group of good and poor problem solvers as the independent variable (see Table 4).

Then students were assigned into groups of good and poor IL problem solvers based on their performance in IL problems. A MANOVA on all PASS processes was analyzed with the group of good and poor IL problem solvers as the independent variable (see Table 4).

Table 3

Mixed Factorial Design for Part 2

Level of PASS processes <i>(Between-subject)</i>	Type of Comparison problems <i>(Within-subject)</i>	
	Consistent language (CL)	Inconsistent language (IL)
High Level Group		
Low Level Group		

Table 4

MANOVA Analyses on PASS Processes for Good and Poor Problem Solvers

Performance Group <i>(independent variable)</i>	PASS processes <i>(dependent variables)</i>		
	Planning	Simultaneous	Sequential
Good Problem Solvers			
Poor Problem Solvers			

## Measures

### Measures of Cognitive Processing

Many tasks in CAS (Naglieri & Das, 1997a) assessing simultaneous and successive processing involve verbal materials; for Chinese participants of the present study, usage of them will involve many translation issues. Because of this consideration, the similar tasks of the Kaufman Assessment Battery for Children (K-ABC) (Kaufman & Kaufman, 1983a) were chosen as the tests of simultaneous and successive processing in the present study.

#### Simultaneous Processing

The Photo Series and Triangles subtests of the Kaufman Assessment Battery for Children (K-ABC) (Kaufman & Kaufman, 1983a), were used to assess simultaneous processing in the present study.

Photo series. The Photo Series task measures the child's ability to organize a randomly placed array of photographs illustrating an event in the proper time sequence. The task was composed of a total of 17 items. Each item used four to ten cards. The investigator put all the cards in order as the number shown on the back of each card. The child was asked to put each card back in the investigator's hand one by one, in the proper time sequence. The cards given back always faced the child. Administration began with the first item and was discontinued if the child failed every item in one unit before

reaching the stopping point. There was no time limit for this task and the child could correct their response at any time. The child got 1 point for each item only when he or she put back all the picture series in correct order. The child's total score in Photo Series was the number of total items he or she correctly finished. As Kamphaus and his colleagues emphasized, "the most crucial aspect of solving Photo Series items correctly involves developing a sense of the whole series of pictures and how they connect to one another" (Kamphaus et al., 1997, p. 350).

Triangles. The Triangles subtest measures the child's ability to assemble several identical rubber triangles (blue on one side, yellow on the other) to match a picture of an abstract design. It measures nonverbal concept formation. The child has to figure out the relationship among the triangles and mentally integrate the components of the design to "see" the whole structure so that they can reproduce the abstract design.

The investigator showed the child the abstract design in the K-ABC easel-kit, then the child was asked to use the rubber triangles to produce the same design as that in the kit. The design in the easel-kit was available to the child during the whole process. There were a total of 18 items in the subtest; this study used only Item 10-18 that was appropriate for children age 8 - 12½. Each item was allowed to be completed in 2 minutes. Administration began with the first item and was discontinued if the child failed every item in one unit before reaching the stopping point. The child got 1 point for each item only when he or she correctly reproduced the matching design within the time limit. The total score of Triangles for the child was the total number of correct responses calculated by the ceiling score minus the number of errors.

### Successive Processing

The Number Recall and Word Order subtests of the Kaufman Assessment Battery for children (K-ABC) (Kaufman & Kaufman, 1983a) were used to assess successive processing in the present study. Both tasks emphasize "the arrangement of stimuli in sequential or serial order for successful problem solving" (Kamphaus et al., 1997, p. 350).

Number recall. Number Recall measures the child's ability to repeat in sequence a series of numbers spoken by the investigator. K-ABC's Number Recall only includes forward span, which has been shown by Das, Kirby and Jarman (1975, 1979) to be a consistently strong measure of sequential or successive processing.

The investigator read a series of numbers. The child was asked to repeat them right away. There were a total of 19 items, 2 numbers to 7 numbers per item were used. Administration began with the first item and was discontinued if the child failed every item in one unit before reaching the stopping point. The child got 1 point only when he or she completely repeated each item in correct order.

Word order. Word Order "ranks behind Number Recall as the premier measure of sequential processing. The task required a child to touch a series of pictures in the same sequence as they were named by the examiner" (Kamphaus et al., 1997, p. 368). The task included two parts: the first part was as above described, the second part had an additional interference activity of naming color. After the investigator read a word series, the child was asked to name as soon as possible the color of a series of circles in the easel-kit in five seconds. Then the child was asked to point to pictures in the same sequence as the investigator named them at the beginning. It is primarily an adaptation of

the auditory-vocal clinical test used by Luria (1966) to measure higher cortical functions of the left temporal lobe. Das, Kirby and Jarman (1979) also used a similar interference activity for the successive processing task in their test battery.

### Planning

Planning was assessed using the Matching Numbers subtest from the Cognitive Assessment System (CAS) (Naglieri & Das, 1997a). The subtest of Matching Numbers was chosen because it involved only numbers, while other planning subtests in CAS all involved English letters that might be unfamiliar to Chinese students. Thus, the test of planning is referred to as "Matching Numbers" hereafter so that it can be clearly differentiated from Mayer's planning task.

Matching Numbers. The Matching Numbers is a 4-page paper and pencil task. Each page consists of 1 item. Each item contains 8 rows of numbers with 6 numbers per row. Numbers increase in digit length with row. Children were required to underline the two same numbers in each row. In this study, item 2 through 4 appropriate for children age 8-17 were administered. The test score for each item was the ratio score of the total amount of time in seconds to complete each item and the number of item correctly completed, based on the Ratio Score Conversion Table included in the Record Form of CAS. The total score of this subtest was the sum of the ratio scores for all items.



## Reliability and Validity of Subtests in This Study

### Simultaneous and Sequential Processing Tasks of K-ABC.

#### Reliability.

The internal consistency, or homogeneity, of scores on each subtest has been examined by using the split-half method, a procedure that evaluates the degree to which each score represents measurement of an uni-dimensional, homogeneous ability or trait. Overall, the obtained split-half reliability coefficients showed very good internal consistency for the K-ABC subtests across the entire age range, as mean values of .80 and above were obtained for 12 of 16 subtests (Kaufman & Kaufman, 1983c). For age group 11-0 to 11-11 ( $n = 200$ ), the split-half reliability coefficients for the Number Recall is .77, Word Order .75, Triangles .84, Photo Series, .81. For age group 12-0 to 12-5 ( $n = 100$ ), the coefficients are .83 for Number Recall, .76 for Word Order, .79 for Triangles, .86 for Photo Series. Thus, all items used in this study have a good internal consistency (split-half reliability coefficients are .75 or above).

#### Validity.

Principal factor analysis of the K-ABC Mental Processing subtests offered strong support for the construct validity of the K-ABC (Kaufman & Kaufman, 1983c). Factor analysis showed that the most effective measures of Simultaneous Processing were Photo Series and Triangles (Their factor loadings in Simultaneous Processing for age 10 children ( $N=200$ ) are .75 and .69, respectively). The most effective measures of Sequential Processing were Number Recall (Factor loading: .92) and Word Order (Factor loading: .69) (Kaufman & Kaufman, 1983c). In a study on constructive validity of K-ABC, selected confirmatory factor analysis solutions for the K-ABC (based on

standardization data) revealed that for age 11 (N=200), Number Recall (.58) and Word Order (.57) were the best marker tests for sequential processing. Triangles (.71) and Photo Series (.66) were the best marker tests for simultaneous processing (Kaufman & Kaufman, 1983c). Thus, this study selected Photo Series and Triangles as measures of simultaneous processing; Number Recall and Word Order as measures of sequential processing.

Reliability of the Planning Subtests of CAS. According to Naglieri and Das (1997c), the average reliability for the Planning of standard Battery PASS Scales is .88. The internal reliability of the subtests was obtained by using the split-half method and the Spearman-Brown formula. The internal reliability coefficients of the subtest of Matching Numbers for children age 11-12 is: .75, .78; The test-retest reliabilities (Stability) for subtest of Matching Numbers across age groups over time is also good (.73). These reliabilities are good enough as suggested by Das and his colleagues (Das et al., 1994).

#### Validity of CAS.

Content validity. The PASS theory is based on Luria's (1980) three functional unit model for identifying the important processes involved in human cognitive competence. The subtest and items of CAS were developed based on the PASS theory. Therefore, they have good content validity.

Convergent and discriminant validity. CAS subtests have shown an appropriate increase with age. Subtests from each of the PASS Scales typically correlate the highest with the scales on which they are assigned and lower on the scales on which they are not included.

Criterion-related validity. A representative sample of 1600 children aged 5 - 17 were administered both the CAS and several WJ-R achievement tests (Woodcock-Johnson Revised (WJ-R) tests of Achievement). The results showed that the PASS cognitive processes are related to achievement as measured by the WJ-R tests of achievement. Wasserman and Becker (2000) reported that the classification accuracy of CAS compares favorably to use of the WISC-III as well as the Gordon Diagnostic System in identifying children with ADHD. The results showed that the CAS was the best predictor among leading intelligence tests of reading and math academic skills.

In general, various studies have provided evidence that tasks used to operationalize simultaneous, successive, and planning processes have functioned similarly despite wide differences in culture, language, and socioeconomic status. In particular, the Planning subtest has been found to correlate with other planning tasks in several studies (Ashman & Das, 1980; Naglieri & Das, 1988; Naglieri, Bardos, & Prewett, 1989).

### Measures of Mathematics Achievement

#### Measures of Computation and Problem Solving

The mathematical achievement measures were mainly adapted from the computation and problem-solving booklets designed by Mayer (Mayer et al., 1991). One question in Mayer's original computation booklet involves negative numbers. In the pilot

study, it was found difficult for the Chinese sixth grader children because the computation of negative numbers has not been taught at their schools, thus the investigator removed it from the official computation booklet. The computation booklet consists of 14 multiple-choice arithmetic computation problems that Mayer selected from a test derived by Stevenson et al. (1986). The problem-solving test consists of 18 multiple-choice word problems designed by Mayer (1991), which tapped the three cognitive components translation, integration, and planning, respectively. Both tests are multiple choice tests. The instructions and practice problems were presented in the first page of each test booklet. Table 5 shows the example items of each booklet in English. This test has been used in three cross-cultural studies on mathematical achievement between Asian and American students (Cai, 1995; Mayer et al., 1991; Tajika et al., 1997).

Table 5

Four Cognitive Processes Involved in Mathematical Problem Solving

Cognitive process (and test)	Example test item
Translation process (6 items on problem-solving test)	Which number sentence is correct?  John has 5 more marbles than Peter.  a. John's marbles = 5 + Peter's marbles  b. John's marbles + 5 = Peter's marbles  c. John's marbles + Peter's marbles = 5  d. John's marbles = 5

(To be continued)

(Continued)

Cognitive process (and test)	Example test item
Integration process (6 items on problem-solving test)	<p>Which numbers are needed to solve this problem?</p> <p>Liu Wei has 3 Yuan. He bought a book for .95 Yuan, a pencil for .20 Yuan, and a notebook for .45 Yuan. How much money did he spend?</p> <p>a. 3, 0.95, 0.20, 0.45</p> <p>b. 0.95, 0.20, 0.45</p> <p>c. 0.95, 0.45</p> <p>d. 3</p>
Planning process (6 items on problem-solving test)	<p>Which operations should you carry out to solve this problem?</p> <p>If it costs 50 cents per hour to rent roller skates, what is the cost of using the skates from 1:00 p.m. to 3:00 p.m.?</p> <p>a. subtract, then multiply</p> <p>b. subtract, then divide</p> <p>c. add, then divide</p> <p>d. multiply only</p>
Execution process (14 items on computation test)	<p><math>62.3 - 37.8 =</math></p> <p>a. 24.5</p> <p>b. 25</p> <p>c. 25.5</p> <p>d. none of these</p>

Note. Correct answers, respectively, are (a), (b), (a), and (a).

In his cross-cultural study on mathematics achievement of American and Chinese students, Cai (1995) reported that the reliability estimate (KR-20) for the computation tasks are .65 for the Chinese students, and the reliability estimate (KR-20) for the component tasks are .81 for the Chinese students. Thus, the mathematical tests used in the present study have good reliability.

### Measures of Problem Solving in Comparison Problems

The investigator designed an assessment of problem solving in consistent language (CL) and inconsistent language (IL) comparison problems, which is very similar to the measures used in the studies of Verschaffle et al., (1994, 1992), De Corte and Verschaffle (1990), and Hegarty et al., (1992, 1995).

In previous studies, measures usually included some target problems (CL and IL comparison problems) and some filler problems. For example, Verschaffel (1994) gave fifth graders 9 one-step arithmetic problems, including 1 warm-up question, 4 target questions (2 CL, 2 IL) and 4 filler questions. In another study, Verschaffel (1992) gave 15 grade three children a set of 26 questions, including 16 target comparison problems (8 CL, 8 IL) and 10 other computation questions as filler items. Similarly, De Corte and Verschaffel (1990) studied the performance of 20 second graders in comparison problems. The test material included 16 target questions and 4 filler items. Thus, in the present study, the author designed a set of questions including some comparison problems as target problems and some filler items to remove the response pattern.

Most previous studies on elementary students mainly involved one-step comparison problems; whereas, studies on college students mainly involved two-step

comparison problems. For example, Hegarty and Mayer (1992, 1995) used two-step consistent language (CL) and inconsistent language (IL) comparison problems in their studies on college students. For college students, one-step problems have been shown to be too easy to reveal the cognitive processes of solving comparison problems (Verschaffel et al., 1992). In addition, the main part of class instruction and the practice of solving mathematical word problems for sixth graders in China involve various two-step questions. Thus, in order to avoid the ceiling effect, in the present study, two-step comparison problems were used. The measure consisted of 16 target comparison problems, including 8 consistent language (CL) problems and 8 inconsistent language (IL) problems. Each type of problem included 4 additions and 4 subtractions. The format and structure of the comparison problems were very similar to those used in previous studies (Hegarty et al., 1992, 1995; Verschaffel, 1994; Verschaffel et al., 1992; De Corte et al., 1990). Each target problem consisted of three sentences, presented in four lines, as shown in Table 6. There were four cover stories for target problems as given in the rows of Table 6. The questions involved common names of objects in Chinese students' everyday context.

To avoid stereotyped responses, the investigator combined the problem-solving booklet and the 16 comparison problems into one booklet. The 18 problem-solving tasks tapping cognitive components were used as filler problems in the present study. The order of problem type (CL and IL) and the order of the four cover stories were counterbalanced. Thus, the whole mathematical test included two booklets: computation test and word problems. The latter consisted of problem-solving tasks and comparison problems. For

the English version of the mathematics computation test, see Appendix A. For a complete English version of problem-solving tasks and comparison problems, see Appendix B.

Table 6

Consistent Language and Inconsistent Language Comparison Problems

Consistent (Less)	Inconsistent (Less)
At store A, a box of candy costs 1.13 Yuan.	At store A, a box of candy costs 1.13 Yuan.
Candy at store B costs 5 cents less per box than store A.	This is 5 cents less per box than candy at store B.
If Xiao Wang wants to buy 5 boxes of candy,	If Xiao Wang wants to buy 5 boxes of candy,
How much will he pay at store B?	How much will he pay at store B?
At store A, workers earn 10.00 Yuan per hour.	At store A, workers earn 10.00 Yuan per hour.
Workers at store B earn 50 cents less per hour than workers at store A.	This is 50 cents less per hour than workers at store B.
If Da Wei works for 8 hours,	If Da Wei works for 8 hours,
How much will he earn at store B?	How much will he earn at store B?
At a grocery store at school, a pencil costs 0.20Yuan;	At a grocery store at school, a pencil costs 0.20Yuan;
In a supermarket, a pencil costs 2 cents less than pencil at the grocery store.	This is 2 cents less than a pencil at a supermarket.
If Xiao Ming want to buy 4 pencils,	If Xiao Ming want to buy 4 pencils,
How much will he pay at the supermarket?	How much will he pay at the supermarket?



(Continued)

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In store Dafa, apple costs 0.70 Yuan per pound.	In store Dafa, apple costs 0.70 Yuan per pound.
In Xiaoli's store, apple costs 20 cents less per pound than store Dafa.	This is 20 cents less per pound than Xiaoli's store.
If you want to buy 12 pounds of apples, How much will you pay at Xiaoli's store?	If you want to buy 12 pounds of apples, how much will you pay at Xiaoli's store?

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Note. The remaining problems were identical to these except that *more* was substituted for *less* in the second line of each problem.

#### Reliability of Comparison Problems

The internal consistency of comparison problems was analyzed and good reliabilities were obtained. The cronbach alphas for the total comparison problems, consistent language (CL) comparison problems (total 8 items) and inconsistent language (IL) comparison problems (total 8 items) were .80, .66 and .79, respectively.

#### Chinese Translations

The contents of the selected K-ABC tests were primarily pictures and numbers, so the translation task was very limited; it only included direct translation of the instructions and numbers and names of the pictures in K-ABC easel-kit of measures of successive

processing (Number Recall and Word Recall). Contents of Matching Numbers subtest of CAS also involved only numbers, thus only the instruction was needed to be translated into Chinese. The investigator completed these simple translations.

The translation of the Computation test into Chinese is straightforward and just involves translating instruction. The investigator completed it. The Chinese version of the problem-solving test was adapted from Cai (1995). Cai (1995) translated Mayer's cognitive component test into Chinese in a cross-culture study on math achievements. According to Cai, change of personal names, object names, terminology, and contexts into appropriate words for Chinese students would not affect the mathematical difficulty of the tasks. The investigator also consulted with the two mathematics teachers in the two schools in Xi'an regarding the appropriateness of the Chinese version of the tests; they both indicated that the questions were clearly described and that students were expected to be able to solve them.

### Pilot Study

A pilot study was conducted to examine the appropriateness of the test. The three sets of test were administered to 10 randomly selected students. The students all showed that they understood the instructions. Only one question involving negative numbers has been shown to be too difficult; the investigator removed it from the formal test because it was beyond the content that students had been taught at school when the test was administered.

## Procedure

### Setting

The tests were administered to grade six students from three math classrooms in two schools of Xi'an, China. A letter with a consent form was sent to parents explaining the purpose and the procedures of the study. All parents agreed that their child could participate in the study. One student with learning difficulties identified by the teacher also participated in the study; however, her data was removed from the final analysis.

The tests started first at the 201 Affiliated Elementary School. After finishing data collection at this school, testing at the Xi'an Steel Factory Affiliated Elementary School followed.

In both schools, the tests of cognitive processing were individually administered to all students first in a quiet room at school. The K-ABC easel was put on the table, and the investigator and the child sat at adjacent sides of the table, so that the investigator could see the child's side of the easel. The mathematical tests were group administered to the intact class in the classroom. The testing occurred as an extra-curricular activity in an afternoon; it took one class time period (45 minutes). At the end, the two math teachers were asked to rate students' general math problem-solving abilities.

## Procedure

### Tests of Cognitive Processing

The child was asked background information before the test started. The information included the child's name, gender, birth data, and parents' education level. Then the formal tests started with instruction and sample problems. According to Naglieri (1999), in this study, the cognitive processing tests were conducted in the order of planning, simultaneous processing and sequential processing. The investigator administered the tests according to the Administrative and Scoring Manual for the Kaufman Assessment Battery for Children (Kaufman & Kaufman, 1983b) and Cognitive Assessment System Administration and Scoring Manual (Naglieri & Das, 1997b). Testing time for the cognitive processing battery was approximately fifty minutes. Students' answers were recorded during the test administering.

### The Math Tests

The mathematical tests included tests using two booklets: the computation booklet and the problem-solving booklet (including math problem-solving component problems and comparison problems). Students were allowed 30 minutes for the problem-solving booklet first, and then 15 minutes for the computation booklet. Each booklet started with one page of instruction and practice problem samples. For each question, students were asked to choose the correct answer from the four given choices. Prior to each test, the investigator gave the instruction and practice problems in Chinese to the intact class (see Appendix A and Appendix B). The instruction and procedures were identical to those used by Mayer et al. (1991). The investigator was present during the whole testing

process and was available to answer questions. Two mathematical teachers helped classroom management while the test was administered. They were required not to give any hints about how to solve the problems.

### Data Coding

#### Cognitive Processing Tests

According to the Administrative and Scoring Manual for the Kaufman Assessment Battery for Children (Kaufman & Kaufman, 1983b), raw scores of each item were calculated. Then a composite score of each cognitive processing (simultaneous, successive processing and planning) was calculated by averaging the scores of the two corresponding subtests.

Based on Cognitive Assessment System Administration and Scoring Manual (Naglieri & Das, 1997b), the scores of Matching Numbers were calculated.

#### Mathematical Achievement Tests

The score of computation is the total number of correctly answered items in the computation test. The maximum score is 14. The score of each problem-solving component is the total number of correctly answered items in the corresponding six items. The maximum score of each cognitive component problem is 6. The score of Problem Solving is the average of scores in the cognitive component test: translation, integration and planning. Thus, the maximum score of the composite score of Problem Solving is 6.

Tests of Comparison Problems

The score of consistent language (CL) and inconsistent language (IL) comparison problems is the total number of correctly answered items. The maximum score is 8.

## CHAPTER 4: RESULTS

This chapter presents the results of the present study into three sections:

(a) PASS processes and math problem-solving components: descriptive statistic analyses; correlational and regression analyses; and MANOVA analyses.

(b) PASS processes and students' performance in consistent language (CL) and inconsistent language (IL) comparison problems: contributions of each PASS process on students' performance were analyzed by a series of ANOVA analyses on students' performance, with level of each PASS process and problem type as independent variables. Profile of PASS processes in good and poor problem solvers in comparison problems were explored by a MANOVA analysis.

(c) Summary of the results.

Analyses were performed using SPSS 7.5 (1977). An alpha level of .05 was used for all statistical tests.

### PASS Processes and Math Problem-Solving Components

#### Demographic Statistics

##### Demographic Data

Data regarding the distribution of the sample by school, sex, parent education level (PEL) are presented in Table 7. As shown in Table 7, the sample consisted of 76 participants from two Grade 6 math classes in 601 Elementary School and 24 participants

from a Grade 6 math class in Xi'an Steel Factory affiliated School, resulting in a total sample of 100 participants. The mean age of the sixth grader students was 11 years 8 months ( $SD = 5.91$  months).

The sample represented an approximately equal distribution of males ( $N = 52$ ) and females ( $N = 48$ ). 47% of the participants were from professional / intellectual families (at least one of the parents had received college level education), 59% of the students were from lower education families (neither parents had college level education).

Table 7

Demographic Data

	Gender		PEL		School	
	Male	Female	Low	High	601	Factory
Number	52	48	59	47	76	24
% of Total Sample	52 %	48 %	59 %	47 %	76 %	24 %

Demographic Variables and Descriptive Data

The cognitive processing tasks included three parts: sequential, simultaneous, and planning tasks. The simultaneous processing tasks included Triangles (TR) and Photo Series (PS); the sequential processing tasks included Number Recall (NR) and Word Order (WO); the planning task was Matching Numbers (MN).

The composite score for each cognitive process was formed by averaging the scores of all included relevant variables for each participant. Two composite scores were formed to represent the simultaneous processing and sequential processing. The



composite scores were constructed as follows: Sequential Processing = Number Recall, Word Order; Simultaneous Processing = Triangles, Photo Series.

The math problem-solving components included three parts: translation, integration, and planning. Each component was measured by six items in the test designed by Mayer (Mayer et al., 1991), respectively.

The student's math achievements were measured using computation and problem-solving tasks. Computation score was the total score in the computation booklet. Problem-solving score was a composite score, which is an average of the students' scores in the three math problem-solving components. A complete description of all tasks in this study and the method of scoring for each task were provided in Chapter III. Because this study only used one task for the cognitive process of planning (i.e., Matching Numbers), it is referred to as "Matching Numbers" hereafter so that it can be clearly differentiated from Mayer's planning task which was one of the math problem solving components.

Table 8 presents the raw score means and standard deviations of the cognitive processes (simultaneous, sequential processing, and planning), math problem-solving components (translation, integration, and planning), and math achievement scores (computation and problem-solving) for total sample and by gender. Table 9 represents raw score means and standard deviations of cognitive processing, math problem-solving components, and math achievements by parental education level (PEL). Table 10 represents raw score means and standard deviations of cognitive processing, math problem-solving components, and math achievements by school.

Table 8

Raw Score Means (M) and Standard Deviations (SD) of PASS Processes, Math Components, and Math Achievement Tests for Total Sample and by Gender

Tasks	Total Sample ( <u>n</u> = 100)		Males ( <u>n</u> = 52)		Females ( <u>n</u> = 48)	
	<u>M</u>	<u>SD</u>	<u>M</u>	<u>SD</u>	<u>M</u>	<u>SD</u>
<b>Cognitive Processing</b>						
Number Recall	17.70	1.47	17.73	1.60	17.67	1.34
Word Order	14.75	1.71	14.98	1.70	14.50	1.71
Sequential processing	16.23	1.29	16.36	1.37	16.08	1.20
Triangles	16.89	1.26	17.00	1.14	16.77	1.39
Photo Series	14.84	1.57	14.87	1.51	14.81	1.65
Simultaneous Processing	15.87	1.19	15.93	1.15	15.79	1.23
Matching Numbers	12.51	3.01	12.63	2.94	12.38	3.11
<b>Math Components</b>						
Translation	5.44	1.01	5.40	1.03	5.48	.99
Integration	4.95	1.28	4.98	1.39	4.92	1.16
Planning	5.19	1.07	5.21	1.09	5.17	1.06
<b>Math Achievement</b>						
Computation	12.90	1.52	12.52	1.90	13.31	.78
Problem Solving	5.19	.89	5.20	.94	5.19	.83

Table 9

Raw Score Means (M) and Standard Deviations (SD) of PASS Processes, Math Components, and Math Achievement Tests by PEL

Tasks	High Education		Low Education	
	(n = 41)		(n = 59)	
	<u>M</u>	<u>SD</u>	<u>M</u>	<u>SD</u>
<b>Cognitive Processing</b>				
Number Recall	17.73	1.38	17.50	1.76
Word Order	15.30	1.91	14.53	1.38
Sequential processing	16.28	1.33	16.06	1.17
Triangles	17.05	1.01	16.89	1.21
Photo Series	15.28	1.24	14.56	1.68
Simultaneous Processing	15.95	1.07	15.58	1.50
Matching Numbers	12.85	3.21	12.27	2.86
<b>Math Components</b>				
Translation	5.68	.57	5.47	.97
Integration	5.30	1.04	4.81	1.39
Planning	5.53	.85	4.94	1.22
<b>Math Achievement</b>				
Computation	13.05	1.28	12.67	1.99
Problem Solving	5.50	.67	5.07	.92

Table 10

Raw Score Means (M) and Standard Deviations (SD) of PASS Processes, Math Components, and Math Achievement Tests by School

Tasks	Xi'an Steel Factory			
	601 Elementary School		Elementary School	
	(n = 76)		(n = 24)	
	<u>M</u>	<u>SD</u>	<u>M</u>	<u>SD</u>
<b>Cognitive Processing</b>				
Number Recall	17.62	1.57	17.96	1.12
Word Order	14.93	1.72	14.17	1.61
Sequential processing	16.28	1.78	16.06	1.17
Triangles	16.97	1.11	16.63	1.66
Photo Series	14.93	1.50	14.54	1.77
Simultaneous Processing	15.95	1.07	15.58	1.50
Matching Numbers	12.43	3.18	12.75	2.42
<b>Math Components</b>				
Translation	5.58	0.79	5.00	1.44
Integration	5.07	1.24	4.58	1.38
Planning	5.25	1.07	5.00	1.06
<b>Math Achievement</b>				
Computation	12.87	1.65	13.00	1.02
Problem Solving	5.30	0.82	4.86	1.02

### Gender, PEL, and School Differences

From Table 8, the means by gender appear to be similar for all variables. The correlations between each variable and Gender, PEL and school are presented in Table 11. T tests were further employed to examine whether there is a significant gender difference between male and female students in all task performance. The results are presented in Table 12.

As shown in Table 11, gender correlated significantly with only Computation. Results presented in Table 12 further showed that female group ( $M = 13.31$ ,  $SD = .78$ ) appeared to perform significantly better than male group ( $M = 12.52$ ,  $SD = 1.90$ ) in computation test ( $t(98) = -2.8$ ,  $p < .05$ ). There were no gender differences on all PASS processes and all math problem-solving components. This result is different with the finding in previous study that significant gender differences were found in Planning and Attention processes (Warrick & Naglieri, 1993). This study used only one subtest of Planning process and involved only one age group of Chinese students. With more Planning and especially attention subtests and various age groups involved, we might be able to find more gender differences in PASS processes and various math scores.

As shown in Table 11, PEL was significantly correlated with Word Order, Photo Series, all three math problem-solving components, and Problem Solving. School significantly correlated with only a few variables, namely Translation and Problem Solving. Although the effects of PEL and School on students' math achievements are not the main purposes of this study, these data can provide some insights about our understanding of the social factors influencing students' cognitive performance. It is

particularly interesting that school was significantly correlated with PEL. It makes sense considering that many students in 601 Elementary Schools came from families that at least one of the parents had received college level education, whereas most students in Xian Steel Factory came from families both parents had never entered college.

In summary, as shown in Table 11 and 12, male and female students performed significantly different only in computation test ( $t(98) = -2.8, p < .05$ ), in which female students outperformed male students. This result is consistent with previous findings that girls are slightly better at computation than boys during the elementary and middle school years (Hyde, Fennema, & Lamon, 1990). Students from different PEL families performed significantly different in Word Order, Photo Series, Triangles, Simultaneous Processing, Planning, and Problem Solving. Students from families that their parents had received higher level of education outperformed their peers from lower education families. School did not have a significant effect on most variables, and the only exception was Problem Solving. The Problem Solving scores of students from 601 Elementary schools were significantly better than those of students from Xi'an Steel Factory Affiliated School.

Table 11

Correlations Between Each Variable and Gender, PEL, and School

Tasks	Gender	PEL	School
	<u>R</u>	<u>r</u>	<u>R</u>
<b>Cognitive Processing</b>			
Number Recall	-.022	-.032	.099
Word Order	-.141	-.241*	-.192
Sequential processing	-.106	-.178	-.071
Triangles	-.091	-.089	-.119
Photo Series	-.017	-.242*	-.107
Simultaneous Processing	-.060	-.207*	-.134
Matching Numbers	-.043	-.096	.045
<b>Math Components</b>			
Translation	.038	-.202*	-.246*
Integration	-.025	-.240*	-.162
Planning	-.021	-.271**	-.100
<b>Math Achievement</b>			
Computation	.262**	-.095	.037
Problem Solving	-.006	-.301**	-.211*
PEL	-.176	1.00	.421**
School	.069	.421**	1.00

Note. \*  $p < .05$ , \*\*  $p < .01$ .

Table 12

T-Test of Students' Performances by Gender

Tasks	<i>t</i> value	Degrees of Freedom	probability
<b>Cognitive Processing</b>			
Number Recall	0.22	98.00	.829
Word Order	1.41	98.00	.162
Sequential processing	1.05	98.00	.295
Triangles	0.91	98.00	.367
Photo Series	0.17	98.00	.867
Simultaneous Processing	0.59	98.00	.556
Matching Numbers	0.43	98.00	.669
<b>Math Components</b>			
Translation	-0.37	98.00	.711
Integration	0.25	98.00	.804
Planning	0.21	98.00	.835
<b>Math Achievement</b>			
Computation	-2.8	68.59	.007**
Problem Solving	0.06	98.00	.950

Note. \*  $p < .05$       \*\*  $p < .01$ .



## PASS Processes and Math Problem Solving Components

### Correlational Matrix

Intercorrelations among the three cognitive processing, the three math components, and math achievement scores were examined and presented in Table 13.

As shown in Table 13, the two subtests of sequential processing were significantly correlated ( $r = .314, p < .01$ ), and the two subtests of simultaneous processing were also significantly correlated ( $r = .404, p < .01$ ).

In addition, all cognitive processing tests were correlated significantly with Translation. Thus, it is clear that all three PASS processes were significantly related to students' performances in all three math problem-solving components, namely Translation, Integration, and Planning at  $\alpha$  level of .01. Computation score significantly correlated with one sequential (Number Recall) and one simultaneous subtest (Triangles). Finally, Problem Solving score significantly correlated with all cognitive processing variables at  $\alpha$  level of .01. These results are consistent with the hypotheses of this study.

Table 13

Correlations Between PASS Processes, Math components, and Math Achievement (N = 100)

Tasks <sup>a</sup>	1	2	3	4	5	6	7	8	9	10
1. Number Recall	--	.314**	.232*	.198*	-.192	.355**	.313**	.152	.298**	.346**
2. Word Order		--	.249*	.177	-.182	.263**	.265**	.291**	.186	.344**
3. Triangles			--	.404**	-.225*	.316**	.383**	.322**	.368**	.434**
4. Photo Series				--	-.221*	.269**	.534**	.404**	.171	.521**
5. Matching Numbers					--	.268**	.334**	.446**	.201*	.441**
6. Translation						--	.416**	.362**	.323**	.724**
7. Integration							--	.522**	.253*	.849**
8. Planning								--	.327**	.790**
9. Computation									--	.376**
10. Problem Solving										--

Note. \*  $p < .05$ . \*\*  $p < .01$ .

That is, all three PASS processes were involved in understanding each sentence in arithmetic word problem. Students' abilities to integrate each elements of the information into a coherent problem representation were associated with all three cognitive processing, especially simultaneous processing. Their ability to plan to solve the problem also was associated with all three cognitive processing. Consistently, students' problem-solving performance was associated with all the three processes. Their computation performance was associated with both coding processes (Sequential and simultaneous processing) and planning. These findings were consistent with findings from other studies that successive and simultaneous processing was associated with reading comprehension (Das & Cummins, 1982; Kirby, 1982; Kirby & Robinson, 1987; Leong, 1985). Based on the finding of this study, the association between students' math reasoning performance and planning, successive and simultaneous processing found in other studies (Garofalo, 1982; Cheng et al., 1984; Kaufman & Kaufman, 1983c; Warrick, 1989; Naglieri & Das, 1987) was further clarified by the fact that the three PASS processes were involved in the components of math problem solving.

#### Principal Components Factor Analysis of Subtests of K-ABC and Matching Numbers

This study used four subtests of K-ABC and one planning subtest of CAS as measures of PASS processes. Although the structure of these subtests was consistent with the PASS theory, whether the subtests in this study actually tap the three processes of the PASS theory needs to be verified. Thus, factor analyses were performed to further examine the structure of the five cognitive processing tests. Principal components factor

analysis has been used extensively in previous studies on the PASS theory (Das et al., 1994; Naglieri & Das, 1997c). Thus, principal components factor analysis was employed in this study. The investigator hypothesizes that there are three factors underlying the five sub-tests, namely sequential, simultaneous processing, and planning, based on the theories and the similarity of tasks in this study and tasks involving in the previous studies on PASS theory (Naglieri, Bardos, & Prewett, 1989).

Both the orthogonal and oblique solutions for the principal components factor analyses were performed. The orthogonal solution results in uncorrelated factors, and allows comparison with previous researches using this method. However, the oblique technique keeps correlations between factors and is more consistent with the interactive nature of the theoretical model. Table 14 and 15 presents the results of the both orthogonal and oblique solutions of the principal components factor analysis. As shown, the results demonstrate that factor structures obtained from two methods are very similar. The findings of both orthogonal and oblique solutions for the principal components factor analyses suggest that the three obtained factors can be identified as Simultaneous (Triangles, Photo Series); Sequential (Word Order, Number Recall); and Planning (Matching Numbers). Thus, the appropriateness for usage of these five subtests to measure sequential, simultaneous processing and planning as hypothesized is verified. In addition, factor analysis provides the basis for data reduction through either summated scales or factor scores. With the confirmed results of factor analysis, in the further analysis, we will be confident to combine the sub-tests within each factor into a single variable that can represent the original sub-tests: sequential and simultaneous processing.

Table 14

Varimax Solution for the Principal Component Factor Analysis (N=100)

Cognitive	Factor I	Factor II	Factor III
Processing	Simultaneous	Sequential	Planning
Triangles	<b>.794</b>	.054	.111
Photo Series	<b>.849</b>	.216	.075
Word Order	.124	<b>.807</b>	.047
Number Recall	.121	<b>.783</b>	.107
Matching Numbers	.140	.120	<b>.982</b>

Table 15

Oblique Solution for the Principal Component Factor Analysis (N=100)

Cognitive	Factor I	Factor II	Factor III
Processing	Simultaneous	Sequential	Planning
Triangles	<b>.796</b>	-.082	.023
Photo Series	<b>.873</b>	.100	-.020
Word Order	.067	<b>.822</b>	-.027
Number Recall	-.001	<b>.791</b>	.037
Matching Numbers	.002	.006	<b>.997</b>

Principal components factor analysis was further used to determine the factor loadings of the three math components. In addition, separate factor analyses were

performed with the five cognitive processing tasks and each of the three math components (Translation, Integration, and Planning) respectively. Garofalo (1982) and Warrick (1989) have used this technique previously in their studies to determine the factor loadings of math tests in relation to the components of the PASS theory.

Table 16 and 17 presents the results of the principal components factor analysis (orthogonal and oblique solutions) for the cognitive processing and math problem-solving component of Translation. The results of both analyses demonstrated significant loading of sequential processing and translation, which was consistent with conceptual analysis and our hypothesis.

Table 16

Varimax Solution for the Principal Component Factor Analysis of PASS Processes and Math Problem-Solving Component of Translation (N=100)

Cognitive Processing	Factor I Simultaneous	Factor II Sequential	Factor III Planning
<b>Number Recall</b>	<b>.782</b>	.081	.112
<b>Word Order</b>	<b>.760</b>	.116	-.011
<b>Translation</b>	<b>.538</b>	.297	.348
Photo Series	.063	<b>.844</b>	.115
Triangles	.226	<b>.785</b>	.078
Matching Numbers	.104	.119	<b>.959</b>

Table 17

Oblique Solution for the Principal Component Factor Analysis of PASS Processes and Math Problem-Solving Components of Translation (N=100)

Cognitive Processing	Factor I Simultaneous	Factor II Sequential	Factor III Planning
<b>Number Recall</b>	<b>.802</b>	.052	.033
<b>Word Order</b>	<b>.786</b>	-.007	-.099
<b>Translation</b>	<b>.478</b>	-.184	.278
Photo Series	.093	<b>-.879</b>	.010
Triangles	.094	<b>-.795</b>	-.037
Matching Numbers	-.023	.026	<b>.985</b>

Table 18 and 19 presents the results of the principal components factor analysis (orthogonal and oblique solutions) for the cognitive processing and math problem-solving component of Integration. The results of both analyses demonstrated significant loading of simultaneous processing and integration, which was consistent with conceptual analysis and our hypothesis.

Table 18

Varimax Solution for the Principal Component Factor Analysis of PASS Processes and Math Problem-Solving Component of Integration (N=100)

Cognitive	Factor I	Factor II	Factor III
Processing	Simultaneous	Sequential	Planning
Number Recall	.161	<b>.766</b>	.112
Word Order	.129	<b>.812</b>	.044
<b>Integration</b>	<b>.722</b>	.222	.297
<b>Photo Series</b>	<b>.860</b>	.019	.074
<b>Triangles</b>	<b>.711</b>	.223	.016
Matching Numbers	.156	.113	<b>.967</b>

Table 19

Oblique Solution for the Principal Component Factor Analysis of PASS Processes and Math Problem Solving Component of Integration (N=100)

Cognitive	Factor I	Factor II	Factor III
Processing	Simultaneous	Sequential	Planning
Number Recall	.022	<b>.773</b>	.039
Word Order	-.011	<b>.833</b>	-.032
<b>Integration</b>	<b>.688</b>	.088	.211
<b>Photo Series</b>	<b>.905</b>	-.130	-.021
<b>Triangles</b>	<b>.718</b>	.116	-.081
Matching Numbers	.004	.004	<b>.984</b>



Table 20 and 21 presents the results of the principal components factor analysis (orthogonal and oblique solutions) for the cognitive processing and math problem-solving components of Planning. As hypothesized, Planning was loaded with Matching Numbers.

Table 20

Varimax Solution for the Principal Component Factor Analysis of PASS Processes and Math Problem-Solving Component of Planning (N=100)

Cognitive Processing	Factor I Simultaneous	Factor II Sequential	Factor III Planning
Photo Series	<b>.829</b>	.041	.196
Triangles	<b>.780</b>	.228	.090
Number Recall	.138	<b>.821</b>	.017
Word Order	.101	<b>.758</b>	.205
<b>Matching Numbers</b>	.032	.131	<b>.883</b>
<b>Planning</b>	.388	.102	<b>.737</b>

Table 21

Oblique Solution for the Principal Component Factor Analysis of PASS Processes and Math Problem-Solving Component of Planning (N=100)

Cognitive	Factor I	Factor II	Factor III
Processing	Simultaneous	Sequential	Planning
Photo Series	<b>-.846</b>	-.080	.072
Triangles	<b>-.787</b>	-.132	-.051
Number Recall	-.285	<b>.838</b>	-.085
Word Order	-.041	<b>.759</b>	.125
<b>Matching Numbers</b>	.020	.049	<b>.914</b>
<b>Planning</b>	.121	-.012	<b>.706</b>

In general, correlational and factor analyses found preliminary support to our hypotheses about the relationship between cognitive processing based on PASS theory and the math problem-solving components based on Mayer's model. Specifically, all three PASS processes correlated with all three problem-solving components. Furthermore, the most important variable for Translation was Number Recall, the subtest of sequential processing. The most important process for Integration was Photo Series, the subtest of simultaneous processing. Also, Matching Numbers was the underlying process for the Planning component. These results supported our hypotheses derived from conceptual analysis of PASS theory and Mayer's model, that is, sequential processing is the underlying process for Translation component; Simultaneous processing is the underlying

process for Integration component; and finally Matching Numbers is the underlying process for Planning component.

### Multiple Regression Analyses

To further test our hypotheses, multiple regression analyses was conducted on math problem-solving components (Translation, Integration and Planning) respectively, with PASS processes (Sequential, Simultaneous processing, and Matching Numbers) as independent variables. The assumptions of normality, Linearity, homoscedasticity, independence of residuals, and multicollinearity were checked and they were not severely violated.

Translation score was analyzed by multiple regression with the five PASS processes (Number Recall, Word Order, Photo Series, Triangles, and Matching Numbers) as independent variables. The results are presented in Table 22. Number Recall appeared to be the only significant predictor for translation. Items in Translation test in this study were basically sentence comprehension tasks. This result was consistent with findings from other studies that sequential or successive processing is associated with reading comprehension, especially reading comprehension in sentence level such as single sentence comprehension. (Das & Cummins, 1982; Kirby, 1982; Kirby & Robinson, 1987; Leong et al., 1985).

Table 22

Summary of Multiple Regression Analysis for PASS Processes Predicting Math Problem-Solving Component of Translation (N=100)

Variable	B	SE B	$\beta$
<i>Sequential</i>			
Number Recall	.162	.067	.236*
Word Order	.061	.057	.104
<i>Simultaneous</i>			
Triangles	.128	.082	.160
Photo Series	.069	.065	.107
Matching Numbers	.048	.032	.144

Note.  $R^2 = .230$ .      adjusted  $R^2$  = .190.

\*  $p < .05$ .      \*\*  $p < .01$ .

Integration score was analyzed by multiple regression with all five sub-tests of PASS processes (Number Recall, Word Order, Photo Series, Triangles, and Matching Numbers) as independent variables. One outlier was found and deleted. The summary of the results is presented in Table 23. Photo Series and Number Recall were found to be the significant predictors for Integration component of math problem solving. Photo Series, the subtest of simultaneous processing, was the best predictor of integration. This result supported our hypothesis and was consistent with the findings of the significant relationship between math and simultaneous processing from other studies (Dash et al., 1985; Cheng, et al., 1984; Kaufman & Kaufman, 1983c; Garofalo, 1982; Naglieri & Das,

1987). In addition, as predicted, Matching Numbers was also significantly related to integration, although to a less extent. This result is in keeping with findings from previous studies that planning is important for math achievement (Das, 1984; Naglieri & Das, 1987).

Table 23

Summary of Multiple Regression Analysis for PASS Processes Predicting Math Problem Solving Component of Integration (N=99)

Variable	B	SE B	$\beta$
<i>Sequential</i>			
Number Recall	.125	.076	.144
Word Order	.065	.065	.086
<i>Simultaneous</i>			
Triangles	.130	.093	.128
Photo Series	.327	.073	.400**
<i>Planning</i>			
Matching Numbers	.074	.036	.174*

Note.  $R^2 = .388$ . Adjusted  $R^2 = .356$ .

\*  $p < .05$ .      \*\*  $p < .01$ .

Multiple regression was completed for math problem solving component of Planning with four subtests of PASS processes (Word Order, Photo Series, Triangles, and Matching Numbers) as independent variables. Number Recall was not included because it was found not significantly correlated with Planning in the previously mentioned

correlation analyses (see Table 13). The results are presented in Table 24. Planning actually measured planning of the necessary operation to solve the problem based on a correct problem representation. The student has to decide which computation he or she needs to do in certain sequence so that he or she can finally solve the problem. We hypothesize that Matching Numbers should be related to Planning score. Also, a correct representation of the problem is necessary for a good plan, although it is not sufficient. As predicted, Matching Numbers and Photo Series were significant predictors for Planning score.

Table 24

Summary of Multiple Regression Analysis for PASS Processes Predicting Math Problem Solving Component of Planning (N=100)

Variable	B	SE B	$\beta$
<i>Sequential</i>			
Word Order	.099	.055	.158
<i>Simultaneous</i>			
Triangles	.086	.080	.102
Photo Series	.177	.063	.260**
<i>Planning</i>			
Matching Numbers	.120	.031	.337**

Note.  $R^2 = .334$ . adjusted  $R^2 = .306$ .

\*  $p < .05$ .      \*\*  $p < .01$ .

As shown in Table 22, 23, and 24, results supported the three main hypotheses of this study. As we hypothesized, sequential processing was significantly associated with translation component; simultaneous processing and Matching Numbers were significantly associated with integration component, and Matching Numbers and simultaneous processing were significantly associated with the component of Planning. These findings were consistent with theory and findings from other studies on PASS and reading comprehension. Particularly interesting, simultaneous processing was found to be the best predictor for Integration and Planning; sequential processing was the best predictor for Translation. In the present study, items in Translation were simple sentence comprehension tasks that demand mainly sequential processing and less simultaneous processing. Integration and Planning tasks in the present study both required relating every pieces of information together for the coherent problem representation, so they were significantly associated with simultaneous processing.

#### PASS Processes and Math Achievement Scores

##### Computation.

The computation test was relatively too easy for the participants in this study and there was a ceiling effect, although it has been proved age appropriate in other cross-cultural studies with Chinese students (Cai, 1995). Thus, advanced statistic analysis such as multiple regression can not be performed. However, simple correlational analysis can provide us some useful information on the underlying process of computation.

Table 13 shows correlations among PASS processes and computation score. Number Recall, Triangles, and Matching Numbers were significantly correlated with computation. Interestingly, it was Triangles rather than Photo Series that was correlated with computation. It makes sense if we consider the fact that both tests involve spatial cognition, for example, computation includes borrowing, carry over and so on. The finding that Number Recall and Matching Numbers was related to computation was in keeping with findings from other studies that complicated computation process involve planning and successive processing (Das et al., 1979; Garofalo, 1982).

#### Problem Solving.

Math problem solving is a complicated process. Despite the above clear picture of the association between PASS processes and each math problem solving component, the dynamic involvement of PASS processes may be different when the three problem solving component are considered separately as above and when student use them together in the actual problem solving process. For example, to grade one students, Successive processing and Translation skills may be the most important factors to differentiate good and poor math achievers, because understanding basic verbal expressions of math problems is the main task for this age group. However, for sixth graders, Translation skill may be well mastered. Simultaneous process and Integration component, Matching Numbers and Planning component may be the hardest part.

In order to clarify the contribution of PASS processes to the overall Problem Solving performance for Chinese sixth graders, students' composite Problem Solving



score were analyzed by multiple regression, with the T score of PASS processes as independent variables. The results are presented in Table 25.

Table 25

Summary of Multiple Regression Analysis for PASS Processes Predicting Math Problem Solving Composite Score (N=100)

Variable	B	SE B	$\beta$
<i>Sequential</i>			
Number Recall	.042	.024	.141
Word Order	.044	.024	.149
<i>Simultaneous</i>			
Triangles	.049	.025	.165
Photo Series	.100	.025	.339**
<i>Planning</i>			
Matching Numbers	.082	.023	.275**

Note.  $R^2 = .470$ . adjusted  $R^2 = .441$ .

\*  $p < .05$ .      \*\*  $p < .01$ .

As shown in Table 25, the results supported our hypothesis that for sixth graders, Simultaneous (Photo Series) and Planning (Matching Numbers) are the main factors associate with general problem solving performance. Although Triangles was not significant; however, it was close to the  $\alpha$  level of .05 ( $p = .055$ ).

## Manifestation of PASS Processes in Poor Problem Solvers

### MANOVA Analyses.

In order to understand students' problems with arithmetic word problems more clearly, profile of PASS processes for students performed poorly in arithmetic word problems is necessary.

Students' composite scores in Problem Solving were converted to T scores (mean=10, SD=3). Two groups of problem solvers were selected from the whole sample based on the composite Problem Solving T scores. Those who fell in the upper and lower fourths of the distribution of T scores of Problem Solving were defined as good ( $N = 22$ ) and poor problem solvers ( $N = 25$ ). Teachers' ratings of students' general math problem-solving ability were consistent with this selection. Good problem solvers performed all correct in Problem Solving tests. Poor problem solvers have at least four or more errors in Problem Solving tests. The descriptive data are presented in Table 26. A MANOVA analysis was performed on PASS processes, with performance group (good vs. poor) as independent variable. The results of MANOVA analysis are presented in Table 27 and Table 28.

Table 26

T scores of PASS Processes for Good and Poor Problem Solvers

	Poor Problem Solvers		Good Problem Solvers	
	(n = 25)		(n = 22)	
	Mean	SD	Mean	SD
Number Recall	8.58	4.44	10.43	2.34
Word Order	8.34	2.95	10.60	3.19
Sequential Processing	8.09	3.79	10.64	2.51
Triangles	7.41	3.69	10.69	2.03
Photo Series	7.63	3.31	11.52	2.09
Simultaneous Processing	7.06	3.49	11.37	1.73
Matching Numbers	8.14	2.62	10.99	2.59

Table 27

Summary of MANOVA Analysis for Good and Poor Problem Solvers (N = 47)

Effects	Value	F	Hypothesis df	Error's df	p
Pillai's Trace	.519	8.836	5.000	41.000	.000
Wilks' Lambda	.481	8.836	5.000	41.000	.000
Hotelling's Trace	1.078	8.836	5.000	41.000	.000
Roy's Largest Root	1.078	8.836	5.000	41.000	.000

Table 28

Test of Between-Subjects Effects on PASS Processes for Good and Poor Problem Solvers(N = 47)

	SS	Df	MS	F
Number Recall	40.08	1	40.08	3.07
Word Order	59.76	1	59.76	6.37*
Sequential Processing	76.14	1	76.14	7.20*
Triangles	126.17	1	126.17	13.72**
Photo Series	177.61	1	177.61	22.49**
Simultaneous processing	217.66	1	217.66	27.56**
Matching Numbers	95.08	1	95.08	14.02**

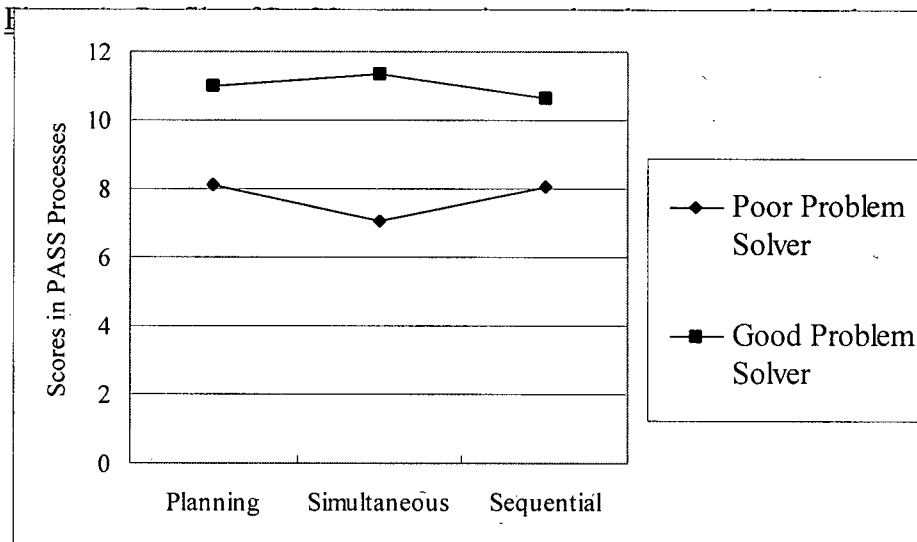
Note. \*  $p < .05$ .      \*\*  $p < .01$

As shown in Table 27 and Table 28, poor problem solvers performed significantly worse than good problem solvers did in four PASS processes except Number Recall. In which, Word order is significant at the  $\alpha$  level of .05, and all other three processes are significant at the  $\alpha$  level of .01. Number Recall is a task of STM; Word Order is a more complicated task similar to the dual tasks measuring working memory in the literature. It demanded cognitive processes more than just short-term memory. Thus, the results demonstrated that poor problem solvers were poorer than good problem solvers in simultaneous processing, planning, and working memory. Hildebrand (1998) has found similar results in adolescents with reading difficulties. In the present study, short-term memory for poor and good problem solvers was not significantly different. This result is

in keeping with previous findings from studies on memory performance of good and poor arithmetic problem solving (Passolunghi, Cornoldi, & Liberto, 1999).

Profiles of PASS Processes for Good and Poor Problem Solvers.

To explicitly describe the profiles of PASS processes for good and poor problem solvers in Problem Solving composite scores, all T scores of PASS processes (i.e., T scores of Matching Numbers and T scores of composite scores of Sequential and Simultaneous processing, see Table 26) were depicted in Figure 1.



## PASS Processes and Performance in Comparison Problems

This section first focusses on exploring the contributions of PASS processes on students' performances in comparison problems, especially in inconsistent language (IL) comparison problems. Second, the manifestation of the PASS processes in a special sample of poor problem solvers in IL problems is examined. Finally, profiles of PASS processes for poor and good IL problem solvers are provided.

### Contributions of PASS Processes on Students' Performance in Comparison Problems

Studies have consistently found that the comparison problem, especially the inconsistent language (IL) comparison problem, is particularly difficult for students from elementary school to college (Riley, Greeno, & Heller, 1983; Lewis & Mayer, 1987; Hegarty et al., 1992, 1995; Mayer & Hegarty, 1996; Verschaffel et al., 1992). There are many controversies on the reasons for the special difficulty of the inconsistent language (IL) comparison problem compared to the consistent language (CL) comparison problem. A number of eye fixation studies on low and high performance groups found that students performed significantly different only in the second phase of problem-solving (i.e., rereading the statements in the problem), rather than the first part (reading all the sentences for the first time). In the second phase of problem-solving, students have to integrate all the information from the statements into a coherent problem representation, which is very difficult for the low performance students. The CL problem contains simple

information that does not need to be modified or manipulated in order to find out a correct relationship between variables. Whereas, in the second sentence of the IL problem, the key word in the statement is inconsistent with the actual required computation. In order to build up a correct problem representation based on finding out a correct conceptual relationship between the variables in the problem, the student has to manipulate the information and convert the key word of the IL problem. Previous analyses have revealed that simultaneous processing was especially important for Integration component of arithmetic problem solving. Thus, Simultaneous processing should contribute to students' performance in the IL comparison problem, the problem that demands much more mental resources and simultaneous processing than the CL problem.

### Descriptive Analyses

This section addresses the question that which PASS process is associated with performance difference in the two types of comparison problems. That is, why IL problem is particularly difficult for students who can successfully solve CL problems? According to the first part of this chapter, both simultaneous processing and planning are the most important processes underlying performance in Integration component, which leads to building up correct problem representations. Studies have shown that the special difficulty of IL problems locates in integration process. Thus, the investigator hypothesizes that simultaneous processing and planning are associated with students' performance difference in the two types of comparison problems. Moreover,

Simultaneous processing has a significant interaction with problem type. These hypotheses are summarized as the hypothesis 7 and 8 previously described in chapter 2.

These hypotheses were tested by conducting a series of 2x2 mixed ANOVAs on students' performance in comparison problems, with level of each PASS process (high vs. low) and problem type (CLvs.IL) as independent variables. The problem type is a within subject variable, and the level of PASS process is a between subject variable.

Students were assigned to high and low ability groups based on the T scores of their composite score of simultaneous processing, sequential processing, and planning (i.e., Matching Numbers), respectively. The high and low PASS process groups were defined as those who fell in the upper and lower fourths (up and bottom 25% percentile) of the distribution of scores of each PASS process, respectively. The sample size of high level group of simultaneous and successive processing and planning group is 47, 34, 30, respectively. The sample size of low level group of simultaneous and sequential processing and planning is 38, 28, 22, respectively. The descriptive statistics for these groups are presented in Table 29.



Table 29

Descriptive Data of Problem Solving for High and Low Groups (N=100)

Problem Type	Cognitive Processing Level	Problem Solving (Simultaneous groups)			Problem Solving (Sequential groups)			Problem Solving (Planning groups)		
		Mean	SD	N	Mean	SD	N	Mean	SD	N
CL	Low	6.89	1.74	38	7.04	1.35	28	7.05	1.50	22
	High	7.45	.88	47	7.35	.95	34	7.60	.67	30
IL	Low	5.05	2.73	38	5.39	2.47	28	5.23	2.35	22
	High	6.89	1.22	47	6.74	1.83	34	6.83	1.44	30

Correlation Matrix

Correlation coefficients of simultaneous, sequential processing, Matching Numbers, and performance scores in CL and IL problems are presented in Table 30.

Table 30

Correlation Matrix of PASS Processes and Performance in CL and IL Problems (N=100)

	Cognitive Processing		
	Sequential	Simultaneous	Matching Numbers
CL	.083	.192	.134
IL	.307**	.393**	.315**

Note. CL: consistent language comparison problem. IL: inconsistent language comparison problem.

\*  $p < .05$ . \*\*  $p < .01$

Correlational analyses showed that students' three PASS processes were not associated with CL performance. All students mastered CL problems very well. However, students' three PASS processes all were associated with IL performance at  $\alpha$  level of .01.

#### Analyses of Variance (ANOVA).

We test hypothesis 7 and 8 (see Chapter 2) here together.

A 3X2 mixed ANOVA was conducted on students' performance in comparison problems, with problem type (CL vs. IL) and level of simultaneous processing (high vs. low) as independent variables. The results are presented in Table 31 and Figure 2.

Table 31

#### Analysis of Variance for Performance in Comparison Problems with High and Low Levels of Simultaneous Processing (N=100)

Source	SS	Df	MS	F	Sig.
Between Subjects					
Simultaneous	60.164	1	60.164	15.594**	.000
<u>S</u> within-group error	320.224	83	3.858		
Within Subjects					
P_Type	60.277	1	60.277	29.200**	.000
P_Type x Simultaneous	17.453	1	17.453	8.455**	.005
P_Type x Simultanoues	171.335	83	2.064		
within-group error					

Note. S = subjects. P-Type = Problem type

\*  $p < .05$ . \*\*  $p < .01$

As shown in Table 31, at  $\alpha$  level of .01, there were significant main effects of simultaneous group and problem type, and a significant interaction between simultaneous processing and problem type. Inconsistent language (CL) comparison problem was significantly harder than consistent language (IL) comparison problem. In addition, students with different simultaneous processing performed significantly different in two types of comparison problems. The group with high simultaneous processing performed significantly better than the low simultaneous group in both types of problems. Furthermore, the performances difference in the two types of problems (CL vs. IL) for the high simultaneous group was significantly smaller than that for the low simultaneous group. In another word, the student in high simultaneous group performed relatively similar in the two types of problems; whereas, students in low simultaneous group performed significantly different in the two types of problems. Their performances in IL problems were much poorer than in CL problems. IL problems were much harder for them to solve.

The results of the present study demonstrated that, first, for Chinese students, same as findings with Caucasian students and European students, inconsistent language (IL) comparison problems were much harder than consistent language (CL) comparison problems. Second, Students with high simultaneous processing performed significantly better than those with low simultaneous processing did. Third, the performance difference in the two types of problems for high simultaneous group was very small, whereas it was much bigger for low simultaneous group. They performed much worse in the IL problem that demands high simultaneous processing.

In summary, simultaneous processing demonstrated to be a very important factor influencing students' performance in comparison problems. Particularly, students' low simultaneous processing contributed to their poor performance in the inconsistent language (IL) comparison problem that demanded high simultaneous processing due to its specific linguistic characteristics.

Hypothesis 9 was tested and the results are presented in Table 32. A 3X2 mixed ANOVA was conducted on students' performance in both types of comparison problems, with problem type and level of sequential processing as independent variables

Table 32

Analysis of Variance for Performance in Comparison Problems with High and Low Levels of Sequential Processing (N=100)

Source	SS	Df	MS	F	Sig.
Between Subjects					
Sequential	21.147	1	21.147	5.799*	.019
<u>S</u> within-group error	218.796	60	3.647		
Within Subjects					
P_Type	39.231	1	39.231	17.406**	.000
P_Type x Sequential	8.069	1	8.069	3.580	.063
P-Type x Sequential within-group error	135.229	60	2.254		

Note. S = subjects. P-Type: problem type

\*  $p < .05$ . \*\*  $p < .01$

As shown in Table 32, there were significant main effects of sequential processing and problem type. However, their interaction was not significant. Inconsistent language (IL) comparison problems were significantly harder than consistent language (CL) problems, and students with high sequential processing performed significantly better than those with low sequential processing. However, the difference of performance in the two types of problems was similar for different sequential groups.

Finally, hypothesis 10 was tested and the results are presented in Table 33. A 3X2 mixed ANOVA was conducted on students' performance in both types of comparison problems, with problem type and level of Matching Numbers as independent variables.

Table 33

Analysis of Variance for Performance in Comparison Problems with High and Low Levels of Matching Numbers Scores (N=100)

Source	SS	Df	MS	F
Between Subjects				
Matching Numbers	29.625	1	29.625	9.565**
<u>S</u> within-group error	154.865	50	3.097	
Within Subjects				
P_Type	52.401	1	42.401	26.071**
P_Type x Sequential	7.017	1	7.017	4.314*
P_Type x Sequential	81.320	50	1.626	
within-group error				

Note. S = subjects. P-Type: problem type

\*  $p < .05$ . \*\*  $p < .01$

As shown in Table 33, there were significant main effects of Matching Numbers and problem type. Inconsistent language (IL) comparison problems were significantly difficult than consistent language (CL) comparison problems. High Matching Numbers group performed significantly better than low Matching Numbers group. In addition, the interaction effect of Matching Numbers and problem type was also significant. The performance difference in the two types of problems for high planning group was similar; however, that for low planning group was significantly different. Students with lower Matching Number scores performed much worse in IL problems.

#### Summary of the results.

In summary, all four hypotheses were supported by the results of the present study. Consistent with findings of previous studies, inconsistent language (IL) comparison problems were significantly harder than consistent language (CL) comparison problems for Chinese sixth graders. As hypothesized, all PASS processes significantly influenced students' performance in comparison problems. However, the difference of performance in CL and IL problems was mediated by only the factors of students' simultaneous processing and planning (i.e., Matching Numbers). High simultaneous and high planning students performed similarly in both types of comparison problems. Whereas, students in low simultaneous group performed significantly poorer in IL problems than in CL problems. This finding has important implications for diagnosing students' difficulties in math problem solving and designing remediation programs.

### Profile of PASS Processes for Poor Problem Solvers in IL Problems

To further understand students with poor performance in IL problems, a thorough exploration of their cognitive profiles seems to be necessary. Manifestation of PASS processes in this special sample of poor problem solvers in IL problems may provide us useful insights about these students' specific problems in solving arithmetic word problems and the nature of their cognitive deficiencies.

Descriptive Analyses. Previous analyses revealed that simultaneous processing was the underlying process for students' special difficulty in IL problem. To further understand the cognitive deficiencies of poor problem solvers, their PASS profiles are provided here.

The raw scores of students' performance were converted to T scores (Mean = 10, SD = 3). Poor problem solvers in IL problems were identified by lower fourths (e.g., 25% percentile) of the distribution of T scores of IL performance. The sample size is 26. Good problem solvers are those got full score in IL problems. The sample size is 32. The descriptive data are presented in Table 34.

Table 34

T Scores of PASS Processes for Good and Poor Problem Solvers in IL Problems (N=58)

	Poor Problem Solvers		Good Problem Solvers	
	(n = 26)		(n = 32)	
	Mean	SD	Mean	SD
Number Recall	9.20	3.59	10.67	1.90
Word Order	8.49	2.94	10.88	2.98
Sequential Processing	8.54	3.31	10.96	2.43
Triangles	8.98	3.02	10.93	1.93
Photo Series	8.47	3.08	10.66	2.35
Simultaneous Processing	8.45	3.20	10.93	1.84
Matching Numbers	8.61	2.36	10.77	2.66

MANOVA Analyses.

A MANOVA was conducted on T scores of PASS processes, with group of problem solvers group as independent variable. The results are presented in Table 35 and Table 36.



Table 35

Summary of MANOVA Analysis on PASS Processes for Good and Poor IL ProblemSolvers (N = 58)

	Value	F	Hypothesis df	Error df	p
Pillai's Trace	.381	6.410	5.000	52.000	.000
Wilks' Lambda	.619	6.410	5.000	52.000	.000
Hotelling's Trace	.616	6.410	5.000	52.000	.000
Roy's Largest Root	.616	6.410	5.000	52.000	.000

Table 36

Univariate ANOVA on PASS Processes for Good and Poor IL Problem Solvers (N = 58)

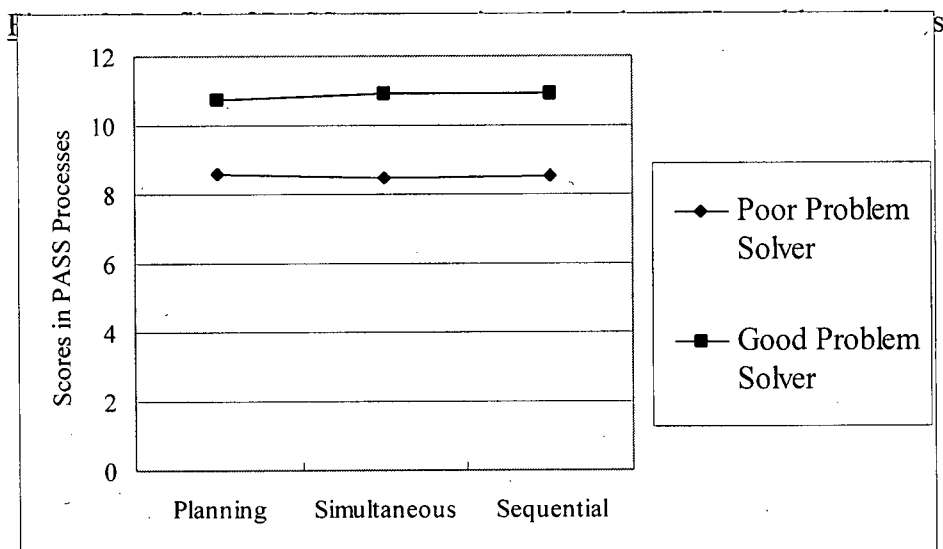
	SS	Df	MS	F
Number Recall	31.12	1	31.13	4.02*
Word Order	81.95	1	81.95	9.32**
Sequential Processing	84.15	1	84.15	10.30**
Triangles	54.43	1	54.43	8.86**
Photo Series	69.32	1	69.32	9.48**
Simultaneous Processing	88.51	1	88.51	13.75**
Matching Numbers	66.845	1	66.845	10.435**

Note. \*  $p < .05$ .      \*\*  $p < .01$

As shown in Table 35 and Table 36, poor problem solvers performed significantly worse than good problem solvers in all five PASS processes. In which, only Number

Recall is significant at  $\alpha$  level of .05, all other four processes were significant at  $\alpha$  level of .01. Number Recall is a task of STM; Word Order is a task similar to the dual tasks measuring working memory in the literature. Thus, it seems that poor problem solvers were poorer than good problem solvers in all PASS processes, especially in simultaneous processing, planning and working memory. Poor problem solvers' short-term memory (as measured by Number Recall) was also poorer than good problem solvers, although to a less extent. These results are consistent with findings in other studies that there were differences in short-term memory between mathematical disabled children and a control group (Shafir & Siegel, 1994; Siegel & Linder, 1984; Siegel & Ryan, 1989).

Figure 2 explicitly shows the profile of PASS processes for poor and good problem solvers in IL problems.



## CHAPTER 5: DISCUSSION AND CONCLUSION

The intent of this study was to investigate the relationships among cognitive processes (as measured by PASS processes of planning and sequential and simultaneous processing), math problem solving components based on Mayer's (1987) model (as measured by components of Translation, Integration, and Planning), and math problem solving performance (as measured by students' computation and composite problem solving scores). In this chapter, the findings of the study are discussed within the context of the two main research questions concerning: (a) PASS processes and math problem solving components, and (b) Pass processes and performance in inconsistent language (IL) comparison problems.

### Pass Processes and Math Problem Solving Components

This section focusses on the following two main questions: (a) what are the relationships between PASS processes and math problem-solving components based on Mayer's model? And (b) what are the manifestation of PASS processes in a special sample of poor arithmetic problem solvers.

### PASS Processes and Math Problem Solving Components

Based on the conceptual analyses of PASS processes and each math problem solving components in Mayer's model, the investigator proposed that sequential processing is the underlying process of the translation component of math problem

solving, whereas simultaneous processing and planning (as measured by Matching Numbers) are the underlying processes of the integration and planning components of math problem solving.

### Translation Component and Sequential Processing

According to Mayer's (1987) model, translation component in math problem solving implies the process that students' understand each sentence of the math word problem. From this definition, translation component can be viewed as a reading comprehension process at the sentence level. Naglieri and Das (1997c) described successive processing as "a mental process by which the individual integrates stimuli into a specific serial order that forms a chain-like progression" (p.5). Naglieri (1999) further explained that "successive processing has strong sequential components and is involved in the syntax of language" (p. 18).

The results of this study showed that sequential processing was significantly associated with students' performance in the Translation component. First, both subtests of sequential processing (i.e., Number Recall and Word Order) had significant correlations with Translation ( $r = .355^{**}$ ,  $r = .263^{**}$ , respectively). Second, principal components factor analysis of PASS processes and Translation showed that Translation was loaded with the two subtests of sequential processing; see Table 10 and Table 11. Finally, multiple regression analyses indicated that Number Recall is the only PASS process significantly associated with Translation score ( $p < .05$ ). These results conclusively showed that sequential processing is the underlying process of Translation component. They are in keeping with recent research findings of sequential processing

and reading comprehension (Cummins & Das, 1977; Naglieri & Das, 1987) and finding on phonological STM tasks and reading (Gathercole & Baddeley, 1993).

### Integration Component and Simultaneous Processing

Mayer (1999) defined the integration component in math problem solving as the ability "to put the statements of the problem together into a coherent representation" (p. 169). Conceptually relating each piece of information together is the essence of simultaneous processing. Luria (1970) defined simultaneous processing as "a mental process by which the individual integrates separate stimuli into a single whole or group (cited in Naglieri & Das, 1997c, p.4). Naglieri (1999) further explained the nature of simultaneous processing as "that the person must see how all the separate elements are interrelated in a conceptual whole. Simultaneous processing has strong spatial and logical dimensions for both nonverbal and verbal (e.g., grammar) content" (p.17).

In general, the results of this study showed that simultaneous processing was significantly associated with students' performance in the Integration component of math problem solving. First, both subtests of simultaneous processing (i.e., Triangle and Photo Series) significantly correlated with Integration ( $r = .383^{**}$ ,  $r = .534^{**}$ , respectively); see Table 7. Second, principal components factor analysis of PASS processes and Integration showed that Integration loaded with the two subtests of simultaneous processing; see Table 12 and Table 13. Finally, multiple regression analysis indicated that Photo Series is the most significant predictor of Integration ( $p < .01$ ), although Matching Numbers also predicted Integration to a less extent ( $p < .05$ ).

These results are consistent with previous results of other studies that simultaneous processing is primarily involved in high level reading comprehension

(Cummins & Das, 1978; Kirby et al., 1996), and planning becomes increasingly important for reading and math achievement with age (Naglieri & Das, 1987).

A good example of the involvement of simultaneous processing in mathematics problems is reading comprehension in paragraph level of math Comparison problems. In Comparison problems, students must keep the early information in the first two sentences in mind and must integrate them with question statement occurring much later in order to find out the relationship between the two variables. Obviously, simultaneous processing is the key element for successful solving of this problem. Simultaneous processing is even more important for inconsistent language (IL) Comparison problems, in which the relationship between the two variables is not explicitly shown from the surface. The student must infer the meaning of a pronoun ("this") and convert the position of subject ("this") and object in the second sentence; see Appendix B. This process is very complex to younger students. It demands simultaneous processing and planning skills.

The integration component of math word problem solving in this study was measured indirectly by questions requiring students to select necessary information for solving the problem. Mayer designed this test based on the logic that if students can correctly integrate all information into a coherent problem representation by using simultaneous processing, they should be able to identify relevant and irrelevant information to the problem. As expected, in the present study, students' simultaneous processing was found to be the most significant predictor of their performance in integration component. Matching Numbers also predicted students' performance to a less extent. Thus, this study's results and literature on the PASS processes and reading together conclusively demonstrate that simultaneous processing and planning are

involved in high level reading comprehension, which is very important for setting up a correct problem representation for arithmetic word problems.

### Planning Component and Matching Numbers

Mayer (1999) defined Planning component of math problem solving as the process to "devise and monitor a plan for solving the problem" (p. 181). Planning involves setting goals, selecting appropriate strategies, allocating resources, and monitoring process. Similarly, Naglieri and Das (1997c) stated that "planning is a mental process by which the individual determines, selects, applies, and evaluates solutions to problems" (p.2). Naglieri (1999) further pointed out that planning is a complex process that may involves attention, simultaneous, successive processes as well as knowledge. The essence of it includes control the impulse to act without careful considerations.

The results of this study showed that Matching Numbers and Photo Series were significantly associated with students' performance in the Planning component ( $\alpha < .01$ ). First, Matching Numbers and Photo Series significantly correlated with Planning component ( $r = .446$ ;  $r = .404$ ; see Table 7). Second, principal components factor analysis of PASS processes and Planning component showed that Planning component loaded with Matching Numbers (see Table 14 and Table 15). Finally, multiple regression analysis indicated that Matching Numbers is the most significant predictor of Planning ( $\beta = .337$ ,  $p < .01$ ). Photo Series is also significantly associated with Planning to a less extent ( $\beta = .260$ ,  $p < .01$ ). Consistently, the literature of the PASS theory shows that planning is an important ability for many higher level cognitive activities, such as reading comprehension (Naglieri & Das, 1988) and math reasoning (Ashman & Das, 1980; Garofalo, 1982; Kirby & Ashman, 1984). Planning has been found especially important

to math achievement with age level increases (Warrick, 1989; Naglieri & Das, 1987). The involvement of simultaneous processing is expected if we consider the fact that correctly devising a plan to solve the problem depends on the ability to set up a correct problem representation.

In addition, according to the research on working memory, controlled attention has been proved to be important for higher-order level cognition and fluid intelligence. If we assume planning process is involved in controlled attention, planning process should be associated with math problem solving. This point has been proven by the results of this study.

#### Manifestation of PASS Processes in a Special Group of Poor Problem Solvers

Mathematical disabilities (MD) in children are widely reported (Badian, 1983). The cognitive and neuropsychological studies on cognitive deficits have mainly focussed on arithmetic computation, and few studies have been conducted on cognitive deficits in math problem solving performance (Geary, 1993). This is partly because that the nature of the cognitive process underlying different problem solving components is not clearly addressed. Passolunghi et al (1999) studied working memory and inhibition process in children's arithmetic problem solving for a group of students with difficulties in math problem solving. Their results showed that math problem solving ability was related to the ability to reduce the memory accessibility of nontarget and irrelevant information, which should conceptually correspond to the construct of selective attention in PASS theory or controlled attention in Engle et al.'s (1999) working memory theory. The present study followed this line of research by examining the cognitive processing of a special



group of students with difficulties in math problem solving. The manifestation of PASS processes (planing and simultaneous and sequential processes) manifested in a special group of students with difficulties in arithmetic problem solving were explored.

#### Planning in good and poor math problem solvers

According to Das et al., (1994), planning is involved in higher level cognitive processing and associated with school achievement including math and reading. That is, planning directs, regulates and evaluates problem-solving behavior. Naglieri and Gottling (1997) pointed out that "effective use of problem solving strategies is particularly problematic for students with learning difficulties (Das et al., 1994) and especially important in mathematics, where careful analysis and systematic execution of procedures is required" (p. 513). Results of the present study indicated that students with arithmetic problem solving difficulties exhibited poorer planning skills than good problem solvers. This study provides empirical data to support Naglieri and Gottling's (1997) suggestion that "poor planning processes should be considered as another important influence on mathematical performance, along with other variables, such as slow rate of execution (Kirby & Becker, 1988), deficit reading skills, and working memory limitations (Kirby & Williams, 1991)" (p.519).

In addition, previous research has shown that reading disabled children were significantly lower than the normal group in planning (Bardos, 1988; Das, Snart, & Mulcahy, 1982; Ramey, 1985; Hildebrand, 1998). However, this study and Warrick (1989)'s study are the only studies that compared planning process in good and poor math problem solvers.

### Simultaneous and Sequential Processing in Good and Poor Math Problem Solvers

Previous studies on PASS processes and reading disabilities revealed that students with reading difficulties demonstrated significantly poorer performance on simultaneous processing than their normal peers, whereas no differences between groups were found on successive processing (Hildebrand 1998).

In the present study, students with math problem solving difficulties showed significantly poor performance than their good problem solver peers on both simultaneous and sequential processing, although the magnitude of difference on sequential processing is less than that of simultaneous processing. In addition, although the two sequential processing tasks were moderately correlated ( $r = .314$ ), see Table 7, students with problem solving difficulties and their good problem solver peers performed differently on Word Order and Number Recall. The performance difference of the two groups on Number Recall is not significant, whereas that on Word Order is significantly at  $\alpha$  level of .01. This might relate to the nature of the two tasks.

For example, on Number Recall, the participant is required only to repeat the list of numbers given by the investigator. This only demands STM. Whereas, on Word Order, the participant listens to a list of words, then has to name as quickly as possible the colour of a series of circles in another page in 10 seconds. Then he or she is asked to point to the figures according to the list of words reported. This test of delayed memory is an actual dual task similar to working memory tasks. Previous studies have found working memory ability was associated with math problem solving and computation (Geary, 1993; Logie et al., 1994; Siegel & Ryan, 1989; Swanson et al., 1993). Consistently, in this study, poor math problem solvers were lower in Word Order than good problem solvers. Passolunghi

et al. (1999) compared the memory performance of groups of children who were poor and good at arithmetic problem solving, and found that the two groups performed similarly on short-term memory tests. Thus, it is not unexpected that students with math problem solving difficulties performed poorly on Word Order but not on Number Recall.

### Summary

According to the results of this study, students with math problem solving difficulties demonstrated poorer planning and simultaneous and sequential processing skills compared to their peers who are better math problem solvers. The performance difference of the two groups of students, however, is not significant in Number Recall, which measured short-term memory.

### PASS Processes and Math Comparison Problems

This section addresses the research question on relationships between PASS processes and performance in math comparison problems, especially inconsistent language (IL) comparison problems. There were two main questions regarding (a) what is the contribution of PASS processes on comparison problem performance, and (b) what is the manifestation of PASS processes on poor problem solvers in IL problems?

### PASS Processes and Comparison Problem Performance

The comparison problem has long been demonstrated as one of the most difficult type of math word problem for students from elementary school to college (Lewis &

Mayer, 1987; Hegarty et al., 1992; 1995). The inconsistent language (IL) comparison problem is especially much harder for students. There are controversies in terms of the reason for this difficulty. The reasons include lack of part-whole schema (Riley & Greeno, 1988); extra processing caused by language inconsistency (Lewis & Mayer, 1987); students' different strategies (conceptual understanding or key word strategy) (Hegarty et al., 1992, 1995); and lack of understanding of the symmetry of language about quantitative comparison and working memory load (d'Ailly et al., 1997). However, none of these studies have systematically examined the underlying processes of comparison problem solving. This study has filled this gap by exploring students' PASS processes and their performance in Comparison problems.

#### Problem Representation and Simultaneous Processing

The conceptual analysis of cognitive processes involved in the two types of comparison problem reveals clearly that the two types of problem demand different level of integration processing. Studies have located the performance difference in the two types of comparison problems in the second phase of problem solving: integrating and planning (Verschaffel et al., 1992; Hegarty et al., 1992; 1995). The results of this study have demonstrated that simultaneous processing and Matching Numbers is the main underlying process of the components of integration and planning, respectively. Thus, if the integration ability is the key factor influencing the difficulty of the problems, we can infer that simultaneous processing should associate with the difference of students' performance in the two types of comparison problems. If planning also contribute to difference in comparison problems, Matching Numbers should be associated with performance.

In this study, the contributions of PASS processes on students' performance in the two types of comparison problems were examined statistically using a series of 2x2 mix ANOVAs on students' performance, with level of each PASS processes (high vs. low) and problem type (IL vs. CL) as independent variables. The results showed that IL problem is significantly difficult for this sample of 6th grade Chinese students, same as the result for American and European students (Hegarty et al., 1992, 1995; Stein, 1993; Verschaffel et al., 1992). Simultaneous and Matching Numbers were associated significantly with students' performance in comparison problems. Moreover, they both showed significant interaction with problem type. That is, the performance difference between IL and CL problems was similar for good problem solver group, whereas it was significantly bigger for poor problem solver group. In another word, poor problem solvers performed similar to good problem solvers in CL problems, but they did significantly poorly compared to their peers did in IL problems. Sequential processing also influence students' performance, but its influence is similar to both groups of students. Thus, the inconsistent language (IL) comparison problem is especially difficult for students with low simultaneous and / or planning skills.

#### Strategy Choices, Controlled Attention, and Inhibition Process: Planning or Attention?

A main explanation of the difficulty of comparison problems is students' different strategies in inconsistent language (IL) comparison problems (Mayer & Hegarty, 1996; Hegarty et al., 1992; 1995). In which, good problem solvers were found focus more on variables and key word; whereas poor problem solvers focus solely on key words and this surface processing lead them to poor performance. Studies on planning process in PASS theory found that strategy choice were related to planning process. Thus, if different

strategies were the main reason for performance difference in comparison problems, Matching Numbers should be associated with performance. The present study did find supportive results for this hypothesis. Matching Numbers had a significant effect on performance in comparison problems. Despite that we used only one sub-test (Matching Numbers) of CAS to represent the planning process of the PASS theory, we still found supportive results. Thus, we can conclude that planning is involved in the performance of comparison problems and students' strategy choices. This is consistent with the findings from other studies that different groups of problem solvers use different strategies (Geary et al., 1993).

From the view of cognitive processes, there may be some overlap between the explanations of the difficulty of IL problems in terms of strategy choices and simultaneous processing/working memory. They probably are two theories trying to explain same phenomena from different angles. Geary et al. (1993) found that working memory influences kindergarten children's use of different counting strategies. Chinese students developed more advanced counting strategies than American students because of their higher working memory span which may be mediated by their one-syllabus Chinese number words. American students adopted less advanced counting strategies that fit their working memory level. Thus, different working memory leads to different problem-solving strategies. Based on this logic, when inconsistent language (IL) comparison problem demands large working memory or simultaneous processing due to its complicated semantic structure, students with poor simultaneous/working memory may face a limit of available mental resources. As a coping method, they might use the "key

word” strategy intentionally or use it without any controlled planning. Thus, simultaneous processing may influence students’ strategy choices.

Another interesting aspect is to view it from the perspective of inhibition. Recent studies on working memory has focused on the mechanism of inhibition (Conway, Tuholski, Shisler, & Engle, 1998; Engle et al., 1999). One main function of controlled attention is to suppress or inhibit irrelevant and misleading information such as the “key word” in Comparison problems. R. Engle, M. Kane, and S. Tuholski (1999) described situations demand controlled attention such as “when there is value in maintaining some task information in the face of distraction and interference; ... when there is value in suppressing or inhibiting information irrelevant to the task; ... when controlled, planful search of memory is necessary or useful” (p. 104). Thus, “working memory capacity reflects the ability to apply activation to memory representations, to either bring them into focus or maintain them in focus, particularly in the face of interference or distraction” (Engle et al., 1999, p. 104). This definition and the above listed situations seem to reflect the functions of planning and attention processes in the PASS theory. Planning subtests in CAS measure the ability to design a strategy to solve the problem effectively. Attention subtests measure the ability to maintain focus especially in face of distraction (Naglieri, 1999). Conceptually, these processes should correspond to Engle et al.’s (1999) construct of controlled attention. Currently, the definition and nature of controlled attention is still quite vague. There is no clear operational definition of controlled attention available except the standard working memory span tasks. It will be helpful to develop other measures on controlled attention so that the exact nature of it can be further clarified. The PASS theory and CAS might be a suitable choice for this purpose. CAS is a well

established measure for simultaneous processing, planning, and attention processes, the processes proposed to correspond to the main parts of controlled attention or central executive (Fan, 2000).

In addition, inconsistent language (IL) comparison problem may be a very good tool to examine controlled attention. Its complicated semantic structure demands higher level of controlled processing. The key word can be misleading if the student processes the problem statements in the literal level rather than seeking for conceptual understanding. In order to overcome the interference from the key words, the student has to intentionally suppress the automatically activated information by the key words and focus attention on the conceptual relation between the variables. This process demands planning and attention. The results of this study strongly supported the planning part of this hypothesis, although there was only one test of planning used. Because this study did not include attention tests, it is not clear whether attention was included in comparison problem solving. Further studies including both factors and more subtests are welcomed.

#### Manifestation of PASS Processes in Poor Problem Solvers in IL Problems

The profile of scales of CAS has been used to diagnose children's various problem, such as ADHD, reading disable, math calculation problem (Naglieri, 1999). In a set of clinical investigation of students diagnosed having learning disabilities or learning disorders, results show that relative weakness in simultaneous processing tend to be associated with reduced performance on Quantitative Concepts and Reading Vocabulary, test involve understanding the relationships between separate elements. Whereas, relative weakness in successive processing are associated with reduced Word Attack performance, a measure of auditory processing and phonological awareness. And to a less



extent, associated with reading in general, broad reading, basic reading skills, and reading comprehension (Wasserman & Becker, 2000). Particularly, according to Wasserman and Becker (2000):

planning and attentional processes are characteristically impaired in children diagnosed with ADHD. A relative weakness in simultaneous processing is associated with both verbal and quantitative difficulties in understanding relationships between items and concepts in learning disabled children, and a relative weakness in successive processing is associated with phonological awareness difficulties in learning disabled children" (p.5).

Thus, it may be an effective way to identify children's problem with Comparison problems by their PASS profiles.

A profile analysis for comparison problem performance was conducted in this study. The results showed that students with difficulties in IL problems were significantly lower in all five PASS processes. Except Number Recall was significant at  $\alpha$  level of .05, all other four processes were significant at  $\alpha$  level of .01. These results are in keeping with the above mentioned findings in other studies. Although only simultaneous processing and planning significantly contributed to the special difficult of IL problems, the results demonstrated that students who are poor at IL problems perform poorly at all five PASS processes.

### Summary

All PASS processing influence students' performance in comparison problems, and simultaneous and planning particularly contribute to students' special difficulty in inconsistent language (IL) comparison problems. Students who perform poor at solving

inconsistent language (IL) problems are much poor in all five PASS subtests than their peers who perform well in IL problems. This study is the first to explore math comparison problems in the cognitive processes level, and the first to link PASS processes with math comparison problems.

### Conclusions

The purpose of this study was to examine the relationships between PASS processes and math problem solving components, PASS processes and math comparison problems. The findings of this study have indicated that sequential processing was associated with translation component of math problem solving; simultaneous processing and planning were associated with integration component of math problem solving; and planning and simultaneous processing were associated with planning component of math problem solving.

Thus, cognitive processes, as measured by PASS processes, are the underlying processes of math problem solving. This finding further provided support for the previous findings of the relationship between PASS processes and math reasoning achievement (Naglieri & Das, 1987; Garofalo, 1982).

Students with math problem solving difficulty displayed a profile of cognitive processing that was discrepant from their peers who performed well in arithmetic word problems. Poor math problem solvers' skill in planning, their competence in processing information integrally, and their working memory were less developed than their peers who performed well in math word problems.

Simultaneous and planning, rather than sequential processing, are found contribute to students' special difficulty in inconsistent language (IL) comparison problems. Students performed poorly in IL problems displayed poorer PASS processing in planning and simultaneous and sequential processing than their peers who performed well in IL problems did.

### Implications for Educational Practice and Future Research

This study was the first to examine math problem solving components in the perspective of PASS processes. The findings clarify the relationships between each math problem solving component and its underlying PASS processes. This information is very helpful for our understanding of nature of math problem solving. The findings of poor math problem solvers are particularly important for theoretical understanding of math problem solving deficits and intervention programs. Naglieri and his colleagues (Naglieri & Gottling, 1995, 1997; Naglieri & Johnson, 2000) have found that instruction on planning for students who are poor at computation can be helpful, computation performance for students with a cognitive weakness in Planning on the CAS improved the most. This study provides useful information on students' problems in arithmetic word problems. Further intervention program based on the results of this study may be successful to help students improve their math problem solving. Especially, instructional programs in simultaneous and planning may be particularly helpful with students' difficulties in IL problems.

This study is the first to link PASS processes with components of math problem solving and students' performance in comparison problems. Only two subtests of

Simultaneous and Sequential Processing on the K-ABC and one subtest of Planning on CAS were used in the present study. More empirical studies using the CAS certainly are needed to verify the findings. Attention process is not included in this study, future studies with attention are warranted in order to provide better and more thorough explanation of students' difficulties in arithmetic word problems.

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## Appendix A

### Computation Tasks

Name: \_\_\_\_\_  
                    First                    middle                    last

School: \_\_\_\_\_

This test has 14 mathematical questions for you to solve. For each problem there will be four possible answers labeled a, b, c, d. Your job is to circle the letter next to the correct answer. If none of the answers is correct then circle the letter d. Here's a sample problem for you to try. Circle one of the letters.

---

$$5 \times 3 =$$

a.  $\frac{3}{5}$   
b. 8  
c. 15  
d. none of these

---

The correct answer is 15 so you should circle the letter c.  
(The fraction  $\frac{3}{5}$  is just another way to write  $\frac{3}{5}$  )

When I say "START" you should turn the page and begin working on the problems. You will have 15 minutes. If you finish one page go on to the next page. Keep working until I tell you to stop. Check over your answers if you finish early.

**DO NOT TURN THIS PAGE UNTIL I SAY "START"**

- (1)  $42 \div 6 =$
- a. 6
  - b. 7
  - c. 8
  - d. none of these
- 

- (2)  $24 \div 3 =$
- a. 4
  - b. 6
  - c. 8
  - d. none of these
- 

- (3)  $6 \overline{)432}$
- a. 66
  - b. 72
  - c. 70R12
  - d. none of these
- 

- (4)  $\begin{array}{r} 198 \\ \times 4 \\ \hline \end{array}$
- a. 400
  - b. 762
  - c. 792
  - d. none of these
- 

- (5)  $5 \overline{)3281}$
- a. 556
  - b. 656
  - c. 656R1
  - d. none of these
- 

GO ON TO THE NEXT PAGE

(6)

$$\begin{array}{r} 45 \\ \times 26 \\ \hline \end{array}$$

- a. 71
  - b. 1170
  - c. 1350
  - d. none of these
- 

(7)

$$102.9 + 56 =$$

- a. 56.9
  - b. 108.5
  - c. 158.9
  - d. none of these
- 

(8)

$$62.3 - 37.8 =$$

- a. 24.5
  - b. 25
  - c. 25.5
  - d. none of these
- 

(9)

$$\frac{24}{36} =$$

- a.  $\frac{2}{3}$
  - b.  $\frac{3}{4}$
  - c.  $\frac{12}{18}$
  - d. none of these
- 

(10)

$$\frac{3}{4} - \frac{1}{6} =$$

- a.  $\frac{2}{3}$
  - b.  $\frac{2}{6}$
  - c.  $\frac{7}{12}$
  - d. none of these
- 

GO ON TO THE NEXT PAGE

(11)

$$\frac{7}{8} \times \frac{18}{20} =$$

- a.  $25/28$
  - b.  $63/80$
  - c.  $126/20$
  - d. none of these
- 

(12)

$$\frac{5}{11} \div \frac{2}{3} =$$

- a.  $3/8$
  - b.  $10/33$
  - c.  $15/22$
  - d. none of these
- 

(13)

$$3 \div 5 =$$

- a.  $1/15$
  - b.  $3/5$
  - c.  $5/3$
  - d. none of these
- 

(14)

$$4 \overline{) 7}$$

- a. .57
  - b. 1.30
  - c. 1.75
  - d. none of these
- 

GO ON TO THE NEXT PAGE

## Appendix B

### Problem-Solving Questions

Name: \_\_\_\_\_  
                     First                    middle                    last

School: \_\_\_\_\_

This test has 34 mathematical questions for you to solve. For each problem there will be four possible answers labeled a, b, c, d. Your job is to circle the letter next to the correct answer. The questions do not ask you to compute anything, so you should not do any arithmetic. Here's a sample problem to try. Circle a letter.

---

**Which numbers are needed to solve this problem?**

Marbles come in bags of 5 marbles each and each bag costs 25 cents. You want to buy 10 marbles. How many bags of marbles should you buy?

- a. 5, 25, 10
- b. 5, 25
- c. 5, 10
- d. 10

---

You need to use only 5 and 10 so you should circle letter c. Now try this problem.

---

**Which operations should you carry out to solve this problem?**

There are 12 hats and 24 children. How many children will not get hats?

- a. add, then subtract
- b. divide, then subtract
- c. divide only
- d. subtract only

---

The correct answer is to subtract 12 from 24 so you should circle d. Now try this.

---

**Which number sentence is correct?**

Apples come in crates of 72 apples each. There are 6 crates.

- a. the total number of apples =  $72 \times 6$
- b. the total number of apples  $\times 6 = 72$
- c. the total number of apples  $\times 72 = 6$
- d. the total number of apples = 72

---

You should circle the latter a. If you multiply the number of apples in each crate (72) by the number of crates (6) times you will find the total number of apples.

Remember that you never have to compute a solution; just answer the question. When I say "START" you should turn the page and begin working on the problems. You will have 30 minutes. If you finish one page go on to the next page. Keep working until I tell you to stop. Check over your answers if you finish early.



**DO NOT TURN THIS PAGE UNTIL I SAY "START"****(1) Which numbers are needed to solve this problem?**

A package of 5 pencils costs 59 cents. Li Xiang bought 3 packages and gave the cashier \$2. How many pencils did he buy?

- a. 5, 59, 3, 2
- b. 59, 3, 2
- c. 5, 59, 3
- d. 5, 3

**(2) Which operations should you carry out to solve this problem?**

At a grocery store at school, a pencil costs 0.20 Yuan;

In a supermarket, a pencil costs 2 cents more than pencil at the grocery store.

If Xiao Ming want to buy 4 pencils,

How much will he pay at the supermarket?

- a. Subtract only
- b. add only
- c. subtract, then multiply
- d. add, then multiply

**(3) Which operations should you carry out to solve this problem?**

The 200 children at a school are going on a bus trip. Each bus holds 50 children. How many buses are needed?

- a. divide, then add
- b. subtract only
- c. multiply only
- d. divide only

**(4) Which operations should you carry out to solve this problem?**

At store A, a box of candy costs 1.13 Yuan.

Candy at store B costs 5 cents less per box than store A.

If Xiao Wang wants to buy 5 boxes of candy,

How much will he pay at store B?

- a. Add, then multiply
- b. subtract only
- c. add only
- d. subtract, then multiply

**(5) Which number sentence is correct?**

Huang Xia and Li Na have 20 books altogether.

- a. Huang Xia's books = Li Na's books + 20
- b. Huang Xia's books + 20 = Li Na's books
- c. Huang Xia's books + Li Na's books = 20
- d. Huang Xia's books = Li Na's books

**(6) Which operations should you carry out to solve this problem?**

In store Dafa, apple costs ¥ 0.70 per pound.

In Xiaoli's store, apple costs 20 cents per pound more than store Dafa.

If you want to buy 12 pound apples,

How much will you pay at Xiaoli's store?

- a. Subtract, then multiply
- b. Add, then multiply
- c. Subtract only
- d. Add only

**(7) Which numbers are needed to solve this problem?**

Chen Qiang's home is 8 blocks from his school. School starts at 8:00. He left home at 7:42 and arrived at school at 7:54. How long did it take her to get there?

- a. 8, 8:00, 7:42, 7:54
- b. 8:00, 7:42, 7:54
- c. 8:00, 7:54
- d. 7:42, 7:54

**(8) Which operations should you carry out to solve this problem?**

At store A, workers earn 10.00 Yuan per hour.

This is 50 cents less hour than workers at store B.

If Da Wei works for 8 hours,

How much will he earn at store B?

- a. Add only
- b. Subtract, then multiply
- c. Add, then multiply
- d. Subtract only

**(9) Which operations should you carry out to solve this problem?**

There are 30 students in a class, including 12 boys and 18 girls. The teacher asks them to get into groups of 3. How many groups are there?

- a. add, then multiply
- b. divide, then divide
- c. divide only
- d. subtract only

**(10) Which operations should you carry out to solve this problem?**

At store A, workers earn 10.00 Yuan per hour.

Workers at store B earn 50 cents more hour than workers at store A.

If Da Wei works for 8 hours,

How much will he earn at store B?

- a. Subtract only
- b. Add only
- c. Subtract, then multiply
- d. Add, then multiply

**(11) Which number sentence is correct?**

Chen Liang and Zhang Da ate 12 candies altogether.

- a. number of candies Chen Liang ate = number of candies Zhang Da ate + 12
- b. number of candies Chen Liang ate + 12 = number of candies Zhang Da ate
- c. number of candies Chen Liang ate + number of candies Zhang Da ate = 12
- d. number of candies Chen Liang ate = number of candies Zhang Da ate

**(12) Which operations should you carry out to solve this problem?**

At a grocery store at school, a pencil costs 0.20 Yuan;

In a supermarket, a pencil costs 2 cents less than pencil at the grocery store.

If Xiao Ming want to buy 4 pencils,

How much will he pay at the supermarket?

- a. Add, then multiply
- b. Subtract only
- c. Add only
- d. Subtract, then multiply

**(13) Which numbers are needed to solve this problem?**

Liu Wei has 3 Yuan. He bought a book for .95 Yuan, a pencil for .20 Yuan, and a Notebook for .45 Yuan. How much money did he spend?

- a. 3, 0.95, 0.20, 0.45
- b. 0.95, 0.20, 0.45
- c. 0.95, 0.45
- d. 3

**(14) Which operations should you carry out to solve this problem?**

At store A, a box of candy costs 1.13 Yuan.

This is 5 cents more per box than candy at store B.

If Xiao Wang wants to buy 5 boxes of candy,

How much will he pay at store B?

- a. Subtract, then multiply
- b. Add, then multiply
- c. Subtract only
- d. Add only

**(15) Which operations should you carry out to solve this problem?**

Twelve candies come in each bag at the store. You buy 3 bags on Monday, 2 bags on Wednesday, and 1 bag on Friday. How many candies do you have?

- a. add, then multiply
- b. add, then divide
- c. add only
- d. divide only

**(16) Which operations should you carry out to solve this problem?**

In store Dafa, apple costs 0.70 Yuan per pound.

This is 20 cents per pound less than Xiaoli's store.

If you want to buy 12 pound apples,

How much will you pay at Xiaoli's store?

- a. Add only
- b. Subtract, then multiply
- c. Add, then multiply
- d. Subtract only

**(17) Which number sentence is correct?**

Zhao Min has 5 more marbles than Zhou Xiang.

- a. Zhao Min's marbles =  $5 + \text{Zhou Xiang's marbles}$
- b. Zhao Min's marbles + 5 = Zhou Xiang's marbles
- c. Zhao Min's marbles + Zhou Xiang's marbles = 5
- d. Zhao Min's marbles = 5

**(18) Which operations should you carry out to solve this problem?**

In store Dafa, apple costs 0.70 Yuan per pound.

In Xiaoli's store, apple costs 20 cents per pound more than store Dafa.

If you want to buy 12 pound apples,

How much will you pay at Xiaoli's store?

- a. Subtract only
- b. Add only
- c. Subtract, then multiply
- d. Add, then multiply

**(19) Which numbers are needed to solve this problem?**

Recess at Eastern Elementary School starts at 10:00 and is over at 10:20. Students go home for lunch starts at 12:15. If it is 9:40 right now, how many minutes are there before recess?

- a. 10:00, 10:20, 12:15, 9:40
- b. 10:00, 12:15
- c. 10:00, 10:20, 12:15
- d. 10:00, 9:40

**(20) Which operations should you carry out to solve this problem?**

At store A, workers earn 10.00 Yuan per hour.

Workers at store B earn 50 cents less hour than workers at store A.

If Da Wei works for 8 hours,

How much will he earn at store B?

- a. Add, then multiply
- b. Subtract only
- c. Add only
- d. Subtract, then multiply

**(21) Which operations should you carry out to solve this problem?**

If it costs 50 cents per hour to rent roller skates, what is the cost of using the skates from 1:00 p.m., to 3:00 p.m.?

- a. subtract, then multiply
- b. subtract, then divide
- c. add, then divide
- d. multiply only

**(22) Which operations should you carry out to solve this problem?**

At a grocery store at school, a pencil costs 0.20 Yuan;

This is 2 cents more than a pencil at a supermarket.

If Xiao Ming want to buy 4 pencils,

How much will he pay at the supermarket?

- a. Subtract, then multiply
- b. Add, then multiply
- c. Subtract only
- d. Add only

**(23) Which number sentence is correct?**

Wang Feng weighs 6 more kg than his brother Wang Bin.

- a. the weight of Wang Feng = 6 + weight of Wang Bin
- b. the weight of Wang Feng + 6 = weight of Wang Bin
- c. the weight of Wang Feng + weight of Wang Bin = 6
- d. the weight of Wang Feng = 6

**(24) Which operations should you carry out to solve this problem?**

At store A, a box of candy costs 1.13 Yuan.

This is 5 cents less per box than candy at store B.

If Xiao Wang wants to buy 5 boxes of candy,

How much will he pay at store B?

- a. Add only
- b. Subtract, then multiply
- c. Add, then multiply
- d. Subtract only

**(25) Which numbers are needed to solve this problem?**

It takes Zhang Jin 15 minutes to walk 3 blocks to school. Chen Deming lives 4 blocks from school and he needs 5 more minutes than Zhang Jin to walk to school. How long does it take for Chen Deming to walk to school?

- a. 15, 3, 5, 4
- b. 15, 5, 4
- c. 15, 3
- d. 15, 5

**(26) Which operations should you carry out to solve this problem?**

At store A, a box of candy costs 1.13 Yuan.

Candy at store B costs 5 cents more per box than store A.

If Xiao Wang wants to buy 5 boxes of candy,

How much will he pay at store B?

- a. Subtract only
- b. Add only
- c. Subtract, then multiply
- d. Add, then multiply

**(27) Which operations should you carry out to solve this problem?**

You need to bring enough cookies so everyone at the class party can have 2 cookies each.

There are 20 people at the party. Cookies come in boxes of 10 cookies each. How many boxes should you bring?

- a. divide, then add
- b. multiply, then divide
- c. divide only
- d. multiply only

**(28) Which operations should you carry out to solve this problem?**

In store Dafa, apple costs 0.70 Yuan per pound.

In Xiaoli's store, apple costs 20 cents per pound less than store Dafa.

If you want to buy 12 pound apples,

How much will you pay at Xiaoli's store?

- a. Add, then multiply
- b. Subtract only
- c. Add only
- d. Subtract, then multiply

**(29) Which number sentence is correct?**

Cai Xiaoqin is 12 years old. This is 3 years older than Sun Liping.

- a.  $\text{Cai Xiaoqin's age} + 3 = \text{Sun Liping's age}$
- b.  $\text{Cai Xiaoqin's age} = \text{Sun Liping's age} + 3$
- c.  $\text{Cai Xiaoqin's age} + \text{Sun Liping's age} = 3$
- d.  $\text{Sun Liping's age} = 12 + 3$

**(30) Which operations should you carry out to solve this problem?**

At store A, workers earn 10.00 Yuan per hour.

This is 50 cents more hour than workers at store B.

If Da Wei works for 8 hours,

How much will he earn at store B?

- a. Subtract, then multiply
- b. Add, then multiply
- c. Subtract only
- d. Add only

**(31) Which numbers are needed to solve this problem?**

Mr. Li spent 8 Yuan for 2 packages of large nails. Mr. Zhang spent 4 Yuan more than Mr. Li and bought 6 packages of small nails. How much did Mr. Zhang spend?

- a. 8, 2, 4, 6
- b. 8, 4, 6
- c. 8, 4
- d. 4

**(32) Which operations should you carry out to solve this problem?**

At a grocery store at school, a pencil costs 0.20 Yuan;

This is 2 cents less than a pencil at a supermarket.

If Xiao Ming want to buy 4 pencils,

How much will he pay at the supermarket?

- a. Add only
- b. Subtract, then multiply
- c. Add, then multiply
- d. Subtract only

**(33) Which operations should you carry out to solve this problem?**

On five tests in your math class your scores are 98, 63, 72, 86, and 100. What is your average score?

- a. Add, then multiply
- b. Add, then divide
- c. Divide only
- d. Multiply, then subtract

**(34) Which number sentence is correct?**

An apple costs 10 cents. This is 5 cents more than the cost of a banana.

- a. cost of an apple = cost of a banana + 5
- b. cost of an apple + 5 = cost of a banana
- c. cost of an apple + cost of a banana = 5
- d. cost of a banana = 10 + 5

--- END ---

## **Appendix C**

### **The Letter of Contact to the Principals**



Name, Principal  
\_\_\_\_\_Elementary School

Oct. 16, 1998

Dear

I am a graduate student in the Department of Educational Psychology and Special Education, the University of British Columbia, Canada. I am writing to request the support of your school to recruit participants for a study for my Masters thesis. The study will be conducted under the supervision of Dr. Arthur More. The project has been approved by the University of British Columbia's Ethical Review Committee.

The project explores the underlying cognitive processes in mathematics achievement. Researchers do not yet understand the nature of the mathematics learning. Especially the cognitive processes underlying solving the mathematical word problems has not been clearly uncovered. The results of the study will provide a better understanding of the interrelationships between mathematics achievement (both computation and problem solving) and cognitive processes. Teachers will find the results useful.

For a detailed description of the study, please see the enclosed Outline of Research Methodology.

If your school is able to agree to participate, students will be recruited by sending home letters of information and consent to their parents. We would like to collect the following data from each student involved in the study:

- Group mathematics achievement test (computation and problem solving)
- Marker tests of cognitive processes (simultaneous, successive and planning). This tests will be administrated by me on an individual basis.

Students will be given consent form to indicating whether or not they would like to participate in the study. If they do choose to take part, they have the right to withdraw from this study at any time. If they do not take part or if they decide to withdraw, their standing at the school will **not** be affected in any way. The answers to the tests will be viewed only by the researchers and used for the research purposes. Data will be kept on file by the investigators in a locked cabinet during data coding and analysis. Completed data will be destroyed after verification of coding and scoring. There are no expected risks with the proposed procedures.

I will call your office recently to discuss the project in detail. Thank you very much for your time and consideration. I look forward to speaking with you.

Respectfully yours,

---

Aimei Fan, Co-investigator,  
Department of Educational and Counselling Psychology, and Special Education  
Phone number (home): (604) 228-2367

**Project Title: mathematics achievement and simultaneous-successive processing and planning of Chinese Sixth Graders**

**Principal Investigator:** Arthur J. More,

Department of Educational and Counselling Psychology, and Special Education

The University of British Columbia

Tel: 822-2338

**Co-investigator:** Aimei A. Fan,

Department of Educational and Counselling Psychology, and Special Education

The University of British Columbia

Tel: 228-2367

**Outline of Research Methodology**

The purpose of the project is to explore the relationship between math achievement (computation and problem solving skill) and its underlying cognitive processes (simultaneous, successive, and planning). Researchers have not yet understood the nature of mathematical problem solving and differences in the underlying cognitive processes which may contribute to individual difference in math achievement. The objective of this study: first, to examine the relationship between the cognitive processing (simultaneous, successive processing and planning) and math achievement (computation and problem solving); second, to examine the relationship between the three cognitive processes (simultaneous, successive and planning) and math in the terms of the more detail cognitive components in math problem solving; third, to further examine the difference in cognitive processing and students' performance in a special difficult math problem: complex problems.

The study will extend the studies in math (Mayer, 1991) and studies in simultaneous-successive processing and planning (Garofalo, 1983; Leong, Cheng & Das, 1984; Kirby & Ashman, 1984) by using two kinds of math tests (computation and problem solving) and three cognitive processes (simultaneous, successive, and planning). The two math tests are adapted from a test developed by Mayer (Mayer, 1991). The three cognitive processing tests are adapted from the K-ABC and CAS (Das, 1994).

Participants will be approximately 100 sixth graders in China. Students will receive letters with a consent form, describing the project, assuring them confidentiality and anonymity. No person will be intentionally excluded from participating in the study except those who do not give consent or those who have learning difficulties.

**Procedure:** Aimei Fan will give a brief introduction of the study and procedure. Each participant will be tested individually for the three marker tests of simultaneous, successive and planning. Then all students will be tested in-group for two math booklet as an intact class. The administration time will be approximately 45 minutes for the math tests and 50 minutes for the processing tests.

**Design:** This study will be comparative. Correlational analyses will also be conducted to analyze the relationship between math achievement and the underlying cognitive processing.

## **Appendix D**

### **Informed Consent Form**

## Informed Consent Form

**Project:** Cross-cultural comparison in math achievement and simultaneous-successive processing and planning

**Principal Investigator:** Arthur J. More, Ph.D., Department of Educational and Counselling Psychology, and Special Education, U.B.C., (822-2338)

**Co-investigator:** Aimei A. Fan, Department of Educational and Counselling Psychology, and Special Education, U.B.C., (228-2367)

**Dear Parent or Guardian,**

We are writing you to describe a research project at .....  
Elementary School and to invite you to participate in the project. We are interested in children's math and problem solving skills.

This study is being completed by Aimei Fan, in partial fulfillment of the requirements for the Masters of Arts in Human Learning, Development and Instruction at the University of British Columbia, under the supervision of Dr. Art More.

Dr. More and I are pleased to get the cooperation from the principal and teachers, we hope to also get your support. We hope that the study will provide information important for researchers and teachers, who are interested in better understanding of the cognitive processes in math problem solving. Hopefully it will benefit for teachers and your child that the result might provide some strategies of math learning and benefit developing programs to enhance students' math learning.

### Project Summary

Our study attempts to examine the thinking processes of math (computation and problem solving). First, your child will be given will be given three groups of cognitive processing tasks to solve in about fifty minutes. Then in another day, your child will participate in the group testing together with the intact class. Your child will be asked to solve two math booklets (computation and problem solving) within 45 minutes (15 minutes for computation booklet, 30 minutes for word problem booklet) in one afternoon class of the extra-curriculum activity. The answers will be treated anonymously and will be used for research purposes only.

Please sign below indicating whether or not you would like your child to participate in the study. If you do choose to take part, you have the right to let your child withdraw from this study at any time. If you do not want your child to take part in the study, your child's standing at the school will **not** be affected in any way.

If now or at any time in the future you have questions about the research, please feel free to contact us at the above numbers. If you have any concerns about your child's treatments or rights as a research participant, you may contact Dr. Richard Spratley, Director of the University of British Columbia Office of Research Services and Administration, at 822-8598.

Thank you very much for your interest and cooperation.

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Aimei A. Fan, M.Sc.

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Arthur J. More, Ph.D.

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### **Parent Assent Form**

I have read the project summary. I understand the nature of the involvement for those students who agree to participate. I am aware that my child's participation in this study is entirely voluntary and that I may withdraw consent for my child's participation from this study at any time without jeopardy to her/his standing at school.

I have received a copy of this consent form for my own records.

I **do consent / do not consent** (circle one) to my child's participation in this study.

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(Signature)

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(Date)