STRENGTH MODEL AND FINITE ELEMENT ANALYSIS OF
WOOD BEAM-COLUMNS IN TRUSS APPLICATIONS

by

WILSON WAI SHING LAU

B.A.Sc., The University of Windsor, 1984
M.A.Sc., The University of British Columbia, 1987

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Abstract

A comprehensive stochastic finite element model for analyzing and predicting the load carrying capacity of beams and beam-columns is developed and presented. By incorporating the stochastic lumber properties, the model characterizes the within-member and between-member variability of the lumber properties to predict the response variability in the load carrying capacity. Applications of the model require the determination of model parameters by calibrating the model with experimental data. An extensive experimental program on a large number of members in both compact and structural sizes was conducted. In order to confirm the validity of the model, the model has been verified by comparing results obtained from additional full-size testing.

The finite element model utilizes one-dimensional beam elements incorporating large displacement but small strains. Due to the large inherent displacements and yielding of the members before reaching the ultimate load, non-linearities in both geometry and material are assumed. In addition, a new stress-strain equation is proposed. The Newton-Raphson Method was also used to iterate to the true solution at each load step.

Specific material properties of the member are modeled as one-dimensional stochastic field variables in the finite element program. As these properties may not be ergodic stationary processes, trend removal and normalization procedures were used to transform these material property processes into ergodic stationary processes. Fast Fourier transform was utilized to obtain the spectrum, transfer functions and coherence functions of these properties. Both auto- and cross-spectra were studied so that correlations between these processes were maintained during the simulations.

Realizations of the material properties were simulated by a single input - multiple output random field model. Trends were then added to these ergodic stationary processes to generate the lumber properties. The interaction relation between applied axial load and moment was also studied. Using the finite element program and the random field model, the interaction behaviour of axial load and moment was simulated and percentile statistics were determined accordingly.
Several other applications of the model and the finite element program are discussed, including the evaluation of the existing codes, axial load-slenderness curves, size, and load configuration effects.
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Chapter 1
Introduction

1.1 Background

Prefabricated nail plated light-frame wood trusses are widely used in residential, industrial and low-rise commercial buildings. Trusses were first introduced into residential structures in the 1960s as a replacement for conventional rafter framing systems. Structural design of these trusses requires a knowledge of the forces (and moments) acting in the members and connections and a knowledge of the ultimate capacity of the structural elements under the combined action of the axial forces and moments.

Engineering design of trusses requires access to structural analysis programs or design analogues capable of accurately predicting member forces and moments generated in the members and connections by the loads (dead, snow, wind and occupancy loads). The Truss Plate Institute of Canada has published procedures, the “Truss Design Procedures and Specifications for Light Metal Plate Connected Wood Trusses” (TPIC 1996), that give the design analogues and procedures for trusses. In designing the upper chords of these trusses subject to both wind and live loads, the resistance of members subjected to combined bending and compressive axial loads must be considered. In addition, bottom chords may also be subject to combined bending and tensile axial load due to load transfer from other members, as well as load induced by the member weight and other vertical loads.

Prediction of truss capacity requires knowledge of the ultimate capacity of the members under the action of a particular distribution of axial forces and bending moments induced by the applied loads. While axial forces are typically constant between panel points, the magnitude of the bending moment varies. As a result the combined applied stress at any given cross section along the length of a member also varies. In wood structural members, there is an additional complication where the tension and bending strengths also vary along the length of the member. Failure will occur at the location where the local combined stresses induced by the loads exceed the local cross-section capacity of the member. In CSA-O86.1-
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94 Engineering Design in Wood (Limit States Design), design of members subject to combined axial and bending loading utilizes a linear interaction formula, which assumes that failure of members occur when the combined stress index (CSI) exceeds unity. Design characteristic values used in the interaction formula were derived from full size tests of lumber as produced by the forest products industry. Bending, tension and compression parallel to grain tests were performed on a full length basis. These design properties are based on the strength of the weakest member section in the maximum stress zone. In truss applications, the weakest tension, bending, or compression strength regions do not necessarily occur at the location of the highest bending, tension or compression stress. As a result the truss member may carry a higher load than would be expected based on design properties assigned by the CSA-O86.1-94.

The dependence of member strength on the loading conditions is called the load configuration effect. Load configuration effects are a manifestation of the natural variation of lumber properties that occur along the length of the member. Procedures for deriving load configuration factors for bending, tension or compression members have been proposed by Barrett et al. (1992). Within member variations in strength properties also lead to size effects. For instance, structural lumber members tested at different lengths have different strengths. Generally long members are weaker than short members under the same load configuration. Statistical size effects have been evaluated for various grades, species and properties of structural lumber (Showalter et al. (1978), Lam and Varoglu (1991a,b), Madsen and Buchanan (1986), Buchanan (1984) and Barrett (1974)). Weibull weakest link concepts have been widely used to derive size adjustment factors for length and width effects for tension, compression and bending strength properties for Canadian dimension lumber (Foschi et al. 1989, Barrett et al. 1992). An alternate weak zone model (Kallsner et al., 1997) which assumes boards are made up of homogeneous clear wood and weak sections where knots occur, has been developed to derive size adjustment factors for the length effect of lumber. These studies suggest that the strength of lumber members in some truss applications could be significantly greater than predicted by current CSA-O86.1-94 design procedures. Such design procedures ignore load configuration effects in the calculation of design properties and the prediction of member capacity under combined loading.
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Buchanan (1984) applied size effect concepts in the evaluation of the behaviour of beam columns at various slenderness ratios. Buchanan (1984) showed that the capacity of timber columns subjected to combined axial and bending loads was greater than that specified by CSA-O86.1-94. The conservative design criteria in CSA-O86.1-94 also lead to an underestimation of residential truss capacities.

In response, a supplement to O86.1-94 (Supplement No.1-97) has introduced a moment capacity modification factor to account for the within member strength variation effects, which leads to conservative but more efficient designs for residential trusses based on the study done by Lau et al. (1997). However, this factor is only applicable to certain types of fully triangulated roof trusses under combined bending and axial compression. No effort has been made to address this issue for trusses outside this scope. In addition, the moment modification factor was derived from Weibull's Weakest Link theory, which assumes a homogeneous material.

This thesis will focus on developing material property profiles and structural models for predicting the ultimate strength of structural lumber members under combined bending and axial compression. The material property profiles were used as input to a finite element program to predict the stresses, strains, and deflections along the member and the ultimate load under certain failure criteria. One of the applications of this model is to study the capacity of top chord members of trusses under combined lateral and axial loading conditions. Studies were also done to derive load configuration factors and a combined stress interaction curve for these members. With the introduction of these factors, it is believed that the proposed new design format will better reflect the behavior of structural lumber members, especially in residential truss applications.

1.2 Objectives

This thesis will provide improved models for predicting the ultimate capacity of structural lumber members in truss applications. The scope of this thesis includes:

1. To develop an experimental database on basic lumber structural properties.
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2. To study the within and between member structural property variations from the established database and develop models, which can represent and simulate structural properties of dimension lumber with the correlations between properties being maintained.

3. To develop a finite element program using the simulated structural profiles as input to predict the capacity of members under combined lateral and axial loading.

4. To establish interaction curve(s) for combined axial and bending load effect which can be used in code design procedures.

5. To investigate and quantify the effects of member size and load configuration on the strength of truss members subjected to combined loads.

6. To use the material property models and the load configuration factors with CSA-O86.1 design criteria to demonstrate an approach to design of truss members under combined lateral and axial loading.

1.3 Design Codes

Trusses for residential and other small structures covered by the National Building Code of Canada, Part 9: Housing and Small Buildings (NBC 1995), are designed to match load performance requirements based on tests of conventional framing systems. As a result, member sizes specified for Part 9 trusses are generally smaller than would typically be required if the trusses were designed strictly according to the requirements of the Code for Engineering Design in Wood (CAN3-O86-M84 - Working Stress Design, 1984). Provisions introduced into the National Building Code (NBC) which permitted lighter roof trusses for residential roofs fostered the rapid expansion of the truss industry.

With the introduction of CAN3-O86.1-M89, the engineering design code was changed to the limit states design format, which is a reliability-based design method. The philosophy of reliability-based design is to design a structure with adequate reliability under some prescribed structural or serviceability demands. Designs based on this method are associated with a target reliability or probability of failure.

In the truss industry, most designers design trusses according to “Truss Design Procedures and Specifications for Light Metal Connected Wood Trusses” (TPIC,1988). The
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1988 version of TPIC procedures referred to CAN3-O86-M84 (Working Stress Design) for the allowable stresses with a load duration factor of 1.33 according to Part 9 of the NBC Code. The interaction formula specified in this procedure is working stress design-based and is almost equivalent to the one given in CAN3-O86-M84, except with the load duration factor of 1.33 rather than 1.15. With the release of NBC 1995 and CAN3-O86.1-94, the Working Stress Design version of CAN3-O86-M84 was no longer referenced in the NBC and CAN3-O86.1-94. As a result, the 33 percent increase in design properties due to load duration effects was not allowed, which significantly increased member sizes for residential trusses designed according to CAN3-O86.1-94. Subsequently, the truss industry, Canadian Wood Council, Forintek Canada Corp. and the University of British Columbia have conducted a truss research program. The objective of the research was to develop new design procedures and analogues, which are consistent with the modern limit states design philosophy without penalizing the member size.

Based on the results, a supplement to CSA O86.1-94 was released in 1997. The supplement includes a new section, which addresses the resistance of sawn lumber subject to combined bending and axial load for specific truss applications. A new interaction formula was proposed for combined bending and compressive loads, which includes a bending capacity modification factor (Lau et al., 1997). The bending capacity modification factor was developed, based on research done on the load configuration of top chord members of trusses under typical loading conditions. However, this provision is limited to fully triangulated metal plate-connected roof trusses with clear span not exceeding 12.20 m. The issue of amplified moment was not addressed.

1.4 Applications

This research contributes to the development of reliability-based design procedures for timber structures by providing new information on the distribution of strength properties of members under combined axial and bending stresses. Modification factors will also be derived to address the effect of load distribution and amplification of moment due to $P$-$\Delta$ effects. The results of this study can be incorporated into the Timber Engineering Design
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Code for structural members under combined loading or the Truss Design Procedures and Specifications for Light Metal Plated Connected Wood Trusses for the design of truss chords.
Chapter 2
Literature Survey

2.1 Introduction

This chapter reviews the historical research work on the subjects of bending, tension and compression behavior of timber, combined axial, and bending behavior of timber, and the random field models that describe the within-member variation of structural properties. It provides a report on the state-of-the-art research as ideas from some of these works have been used in developing the strength model given in this thesis. Since the material related to this topic is very broad, the present survey is limited to findings that are related closely to this thesis.

2.2 Bending Behaviour of Timber

The research of modeling the flexural behaviour of a beam dates back to 1638 when Galileo described the stress distribution of a flexural member. His finding was wrong because he assumed the neutral axis to be on the compression edge of a beam. In 1708, Parent obtained the correct elastic stress distribution of a beam but his discovery was not generally recognized until 1773 when the results were confirmed by Coulomb. The chronological development of bending behavior in the early years was summarized by Booth (1980).

The classical elastic beam theory is still widely adopted even now with the assumption that the neutral axis occurs at mid-depth of the section until failure. The stress at the extreme fiber is taken as the failure stress or modulus of rupture. This model of a simple rectangular beam is valid for an elastic isotropic material.

Testing of timber members in bending revealed that “yielding” may occur at the compression edge prior to a final tension failure. The extent of any yielding differs for clear wood and structural timber members and varies with species and grade. In general, members with clear straight grained wood without knots exhibit more yielding than members with
defects. Consequently, wood quality has a significant impact on the final failure stress distribution in a wood member. For a low strength brittle type failure, which usually occurs for a material with many knots, the modulus of rupture, determined using the extreme fiber stress, will be very close to the true maximum stress at failure. For a ductile failure, the stress distribution of a section is no longer linear and the neutral axis is not at the mid-depth any more. Therefore, the modulus of rupture no longer represents the true extreme fiber stress and it is merely an indicative stress that characterizes the stress distribution state of a member. A typical nonlinear stress-strain distribution for a wood member is shown in Figure 2.1. The corresponding stress distribution with compression yielding is illustrated in Figure 2.2.

Many researchers have modeled this non-linear stress-strain relationship at the compression side. The simplest model is the bilinear elasto-plastic stress-strain relationship in compression with the material remaining linear elastic in tension. This model was first proposed by Neely (1898) after observing the bending behavior of wood beams (see Figure 2.3(a) and Figure 2.4(a)). Dietz (1942), Bechtel and Norris (1952), Comben (1957), Ramos (1961) and Nwokoye (1975) also studied the stress-strain relationship from bending tests of wood beams. A non-linear stress-strain behavior for the compression region was observed. In general they agreed that a bilinear elasto-plastic stress-strain curve would be a good approximation. Most of them also agreed on the assumption that plane sections remained plane.

Bazan (1980) suggested a modification of the bilinear elasto-plastic stress-strain relationship. He proposed a model with a linear relationship between the stress and the strain up to the proportional limit, followed by a linear falling stress with increasing strain. In general, this model is known as the bilinear stress-strain relationship with a falling branch. The model is shown in Figure 2.3(b) which corresponds to the stress distribution shown in Figure 2.4(b). Bazan assumed that the slope of the falling branch is a variable that can be arbitrarily taken as a value, producing maximum bending moment at a prescribed neutral axis depth. Buchanan (1984) argued that this variable is a material property and suggested that it should be obtained through calibration of the model to test results. Obviously, Bazan’s model will not work for large strain since it may produce negative stresses. However, by this stress-
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strain model and considering size effect, Bazan produced reasonable predictions of bending strength of clear wood beams from the results of axial tension and compression tests.

Malhotra and Mazur (1970) suggested a stress-strain relation, which was previously proposed by Ylinen (1956), and is given by

\[ \varepsilon = \frac{1}{E} \left[ c \cdot \sigma - (1 - c) \cdot f_c \cdot \ln \left( \frac{1 - \sigma}{f_c} \right) \right] \tag{2.1} \]

where \( \varepsilon \) is strain, \( \sigma \) is stress, \( f_c \) is maximum compression stress, \( E \) is modulus of elasticity, and \( c \) is the shape parameter. The curve described by Equation (2.1) is shown as (c) in Figure 2.3.

Another model proposed by O’Halloran (1973) for clear dry wood in compression at various grain angles and grain orientations using the data of Goodman and Bodig (1971) has the form

\[ \sigma = E \cdot \varepsilon - A \cdot \varepsilon^n \tag{2.2} \]

where \( \sigma \) and \( \varepsilon \) are defined above, \( E \) is the modulus of elasticity and \( A, n \) are equation constants determined by the experimental data. This curve seems inappropriate to represent the stress-strain relationship once it passes the peak stress because of its rapid drop to negative stress.

A detailed study of the stress-strain relationship of timber with defects was done by Glos (1978) using specimens tested in compression parallel to grain. Based on experimental data, a curve with a polynomial up to the 7th power as shown in Figure 2-3 (Curve (d)) was obtained. The stress-strain relationship proposed by Glos has the format

\[ \sigma = \frac{\varepsilon / \varepsilon_1 + G_1 \cdot (\varepsilon / \varepsilon_1)^7}{G_2 + G_3 \cdot (\varepsilon / \varepsilon_1) + G_4 \cdot (\varepsilon / \varepsilon_1)^7} \tag{2.3} \]
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where

\[ G_1 = \frac{f_s}{6E \cdot (1 - f_s/f_c)} \]
\[ G_2 = 1/E \]
\[ G_3 = f_c - 7/6E \]
\[ G_4 = G_1 / f_s \]

and \( \sigma \) is the stress; \( \varepsilon \) is the strain; \( E \) is modulus of elasticity; \( f_c \) is the maximum compressive stress; \( f_s \) is the asymptotic compression for large strain and \( \varepsilon_1 \) is the strain at the maximum stress.

Glos estimated the four parameters (\( G_1 \) to \( G_4 \)) that define the curve based on his test results. From the measurable wood properties such as density, moisture content, knot area ratio and percentage compression wood, the expected values of each parameter were determined using multiple curvilinear regression techniques. The advantages of this model are that the stress does not drop to zero at large strains and that it closely represents the true shape of the stress-strain curve more accurately than other models. However, the four parameters are material property dependent and need to be calibrated for each data set. As well, most experiments are typically terminated once the maximum compression yield point has been reached and thus the stress-strain relationships for large strains are not readily available.

2.3 Axial Tension Behavior of Timber

The earliest recorded testing of wood specimens in tension parallel to grain was done by Mariotte in 1680 (Booth 1980). Performing tension tests is not as easy as compression tests because a rigid grip has to be designed that must be stronger than the specimen itself. The standard test of clear wood specified in the ASTM (1994) requires a clear wood specimen with middle region necked down to a section of 4.8x9.5 mm over a 64 mm gauge length. Generally, this will guarantee that failure occurs in the middle region. In the absence of sufficient small clear specimen tension test data, the modulus of rupture values are sometimes substituted for tensile strength. It has been shown that values obtained using this
method will be higher than the modulus of rupture (Wood Engineering Handbook, 1990), however, this is not true for structural size specimens.

With the design of more effective grips and the trend of design codes to reply on structural size tests, the current tensile design values have been based on results of in-grade tests. A summary of the Canadian in-grade test data is given by Barrett and Lau (1994).

There is a significant difference between the failure characteristics for tension strength of clear wood and timber with defects. Since defects such as knots are in general weaker than clear wood, failures will usually occur at or near these defects. Numerous studies have focused on the study of the effect of defects on tensile strength. Zehrt (1962) investigated the effect of slope of grain on small clear specimens and concluded that the strength not only depends on the perpendicular-to-grain tensile strength but also the shear strength. Dawe (1964) studied the effect of knots on the tension strength of European redwood boards and found a strong correlation between knot size and strength. Based on his study, Nemeth (1965) reported correlations between tensile strength and modulus of elasticity, while the correlation between tensile strength and density is not significant. In general, edge knots had a more profound effect on the tensile strength than center knots. This result was confirmed by Kunesh and Johnson (1972) and by Johnson and Kunesh (1975).

Several researchers have used regression techniques to correlate various properties such as knot size, flexural stiffness, slope of grain and density on tensile strength. Studies by Schneiwind and Lyon (1971), Gerhards (1972) and Heimshoff and Glos (1980) are some of the examples.

2.4 Axial Compression Behavior of Timber

The axial compression behaviour of clear wood is quite distinct as compared to tension behaviour in that the compression failure is more ductile. The short columns will exhibit a linear stress-strain relationship up to the proportional limit, which is approximately 75% of the ultimate strength, then the slope of the stress-strain curve will decrease as the columns yield. It will fail at the ultimate load with some crushing, producing compression wrinkles in the wood fibres.
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Traditionally, characteristic strength values in compression are determined using small clear specimens. ASTM (1994) specifies a clear straight-grained specimen 203mm long, 51×51 mm cross sectional area, tested to failure, to obtain the compression strength parallel to the grain. Compression strengths of clear wood of various species and grades are published in the Wood Engineering Handbook (1990).

The compression strength of lumber with defects is generally less than that of clear material of the same species. To address this issue, the traditional method of predicting the strength of lumber with defects is to reduce the clear wood strength using modification factors to account for the defects. A more rational approach, which has been adopted for the Canadian timber code, is to test members in full size (in-grade testing). This approach will yield strength values, which reflect the behaviour of a member in structural use. The test method refers to the ASTM standard D198-94 (1995), and procedures to establish the allowable properties are given in ASTM standard D1990 –91 (1995).

On the other hand, long members and short members perform differently under compression load. While short members will attain the ultimate compression strength, long members will fail in buckling. Thus, a lateral instability failure is a characteristic of slender compression members. The capacity of long columns depends on the stiffness of the member. For this reason, the design codes typically classify compression members into three categories — short, intermediate and long members, according to the slender ratios, \( C_c \), defined by

\[
C_c = \frac{L}{d}
\]  

(2.4)

where

\( L \) = length of the member, or effective length in case of different boundary conditions

\( d \) = dimension of cross-section of the member in the direction of buckling

The earliest formula for long column capacity under instability failure was given by Euler in 1744 (Timoshenko 1953), as the well-known Euler’s formula

\[
P_e = \frac{\pi^2 EI}{L^2}
\]  

(2.5)
where $P_e$ is the axial load capacity under compression for a column of length $L$ pinned at both ends; and $E$ is the modulus of elasticity.

Empirical expressions have also been developed including the secant formula, the Rankine formula, and the tangent modulus approach proposed by Engesser in 1889 (Bleich 1952). The development of the column formulae has been reviewed in detail by Buchanan (1984).

The earliest recorded testing of wood columns in North America was done by Bryson (1866), who tested different lengths of dry white pine timber. Assuming wood to be a linear elastic material, which fails when a limiting compression stress is attained, Newlin and Trayer (1925) carried out detailed analyses of clear sitka spruce columns. In Britain, the development of wood column formulae were largely contributed by Robertson (1925), in an article by Ayrton and Perry. The resulting "Perry-Robertson" formula assumed that the column remained elastic until failure, but had a small initial deviations from straightness.

Larsen (1973) made comparisons of different column formulae from different countries and recommended a formula to be adopted in the code. Burgess (1976) has concluded that the formula recommended by Larsen is essentially the Perry-Robertson formula.

Malhotra and Mazur (1970) have re-examined the "Euler-Engesser" formula and proposed a single formula to represent wood columns of all slenderness ratios. Neubauer (1973) reviewed the Rankine formula, and modified it to a simple cubic form which agreed with the results of his tests for a wide range of slenderness ratios. Chen and Atsuta (1976) summarized the development of research on beam columns loaded beyond the proportional limit into the plastic buckling region.

In the current CSA-O86 design code, the factored compressive resistance parallel to grain is represented by

$$P_r = \phi F_c A K_z K_c$$

(2.6)

where

$\phi = 0.8$

$F_c =$ factored strength in compression parallel to grain
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\( K_{zc} = \) size factor for compression for sawn lumber
\( K_c = \) slenderness factor

The slenderness factor \( K_c \) is given by

\[
K_c = \left[ 1.0 + \frac{F_c \cdot K_{zc} \cdot C_c^3}{35 \cdot E_{05} \cdot K_{SE} \cdot K_T} \right]^{-1}
\]  (2.7)

where

\( C_c = \) slenderness ratio for compression members
\( E_{05} = \) modulus of elasticity for design of compression members
\( K_{SE} = \) service condition factor for modulus of elasticity
\( K_T = \) treatment factor

2.5 Combined Bending and Axial Load

When a member is subjected to combined bending and axial load, the ultimate capacity or resistance is typically represented by an interaction curve, which shows the capacity for specific combinations of axial load \( (P) \) and bending moment \( (M) \) applied to beam-column. The classical approach of formulating this interaction relationship is to assume a straight line relation connecting the ultimate bending moment and the ultimate axial load. This theory is applicable to a fully elastic material with equal ultimate tension and compression strength. Currently a linear interaction relationship is still being used in the CSA-O86.1-94 timber design code for beam-columns, with the exception of combined loading in chord members of specific truss applications.

As we know, wood generally exhibits linear elastic behaviour up to failure for tension. For compression, wood behaves linearly up to the proportional limit. Beyond the proportional limit, wood becomes non-linear. Failure occurs in a more ductile mode. For bending, failure modes range from a brittle failure in tension with the whole section still behaving elastically to brittle failure in tension combined with yielding in compression. Thus, the interaction curve for wood can take different shapes depending on the ratio of tensile strength to compression strength and the stress-strain relationship in compression.
A curved interaction curve has been described by many researchers such as Gurfinkel (1973), Larsen and Riberholt (1981), but with very little experimental verification. Newlin (1940) performed tests on small clear specimens and suggested a parabolic equation of the following form for slender members

\[
\left(\frac{M/S}{f_r}\right)^2 + \frac{P/A}{f_c} \leq 1
\]  (2.8)

where \( M \) is the bending moment, \( S \) is the section modulus, \( f_r \) is the modulus of rupture, \( P \) is the axial load, \( A \) is the cross sectional area and \( f_c \) is the compressive stress at failure.

However, for short members, he suggested the use of a linear formula, that is

\[
\frac{M/S}{f_r} + \frac{P/A}{f_c} \leq 1
\]  (2.9)

Based on this linear formula, Wood (1950) derived formulae for columns with side loads and eccentricity.

In general, a linear relationship for combined bending and tension is assumed. This is based on the theory that the ultimate tensile stress in tension specimens and the extreme fibre tensile stress in bending specimens are the same. However, based on the brittle fracture theory, Johns and Buchanan (1982) suggested that the failure stress in an axial tension test should be less than the maximum tension stress when the same member is tested in bending. Based on this assumption, Johns and Buchanan (1982) suggested a curved relationship between bending and tension capacity for the tension region. This curved relationship was also adopted by Bradley (1981) in a study of glued laminated wood beams.

Tests of full-scale structural lumber under combined bending and tension loading were conducted by Senft and Suddarth (1970 and 1973). Their tests were carried out in such a way that the tensile stress was applied first and held constant while bending stresses were increased to failure. The test results showed that the bending strength, particularly at low tensile stress levels, does not follow the linear interaction equation that was accepted in most design codes. However, with the consideration of all the stress components, the interaction equation is still valid. This includes the incorporation of the counter-moment due to the second moment effect of the tensile load in the interaction equation. Suddarth, Woeste and
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Galligan (1978), by using the test data described above, performed a reliability analysis for wood members subjected to combined loading. By using a similar theory that deflections due to lateral loads will be reduced by axial tension forces as suggested by Senft and Suddarth, Burgess (1980) developed a model to predict the strength of tension members with lateral loads.

For long members subjected to combined bending and compression, it has been shown that the load capacity not only depends on the ultimate bending and compressive strength of the material, but also depends on the slenderness ratio of the beam-column. As a result, the interaction equation depends on the member's slenderness ratio. The interaction equation is a function of the slenderness ratio: this relationship is shown as a three dimensional plot of load vs. slenderness vs. moment given in Figure 2.5 (Johns and Buchanan 1982).

Experimental studies of beam-columns were reported by Newlin and Trayer (1925) on small clear wood members. Constant ratios of concentric axial load and transverse lateral load were used in their tests. A set of design curves were then developed based on the theory. Later, Newlin’s formula was extended by Wood (1950) and was adopted for use in the U.S. (NFPA 1982). This formula was based on the assumptions of sinusoidal deflected shape, linear elastic behaviour up to failure, and a limiting compression stress failure criterion.

The Perry-Robertson formula was developed in designs for members with combined bending and axial load. The formula also included an initial out-of-straightness term so it could be used for members with large eccentricities. Several other studies have developed alternative formulae for beam-column capacity. These include the work done by Johnston (1976), Malhotra (1982), Hammon et al. (1970), Larsen and Thielgaard (1979), Bleau (1984), Leicester (1989), and Zahn (1986, 1990, 1992).

Koka (1984) developed a finite element program for beam-column analysis using a bilinear stress-strain curve with falling branch. The model was used to estimate the ultimate load and reliability of beam-columns with lateral loads. His model incorporated a size effect but assumed homogeneous material properties within a member.
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For full size testing of commercial lumber under combined axial compression and bending, the published work includes Zahn (1982), Malhotra (1982), Bleau (1983) and Buchanan (1984).

2.6 Application of the Stochastic Process in Lumber

Many simulation models using random field process to model the performance of dimension lumber, as well as engineered wood products have been developed in recent years. Typically these models are confined to predicting mechanical properties such as the modulus of elasticity, compression strength, and tensile strength. None of the models simulate bending strength or the relations between bending, tension, compression strength in lumber.

Woeste, Suddarth, and Galligan (1979) used a regression approach to develop a procedure to simulate correlated lumber properties — ultimate tensile strength and MOE. They also used this procedure to study reliability analysis of wood structures through the Monte Carlo simulation. Kline, Woeste and Bendtsen (1986) proposed a model to generate the lengthwise variability in modulus of elasticity (MOE) of lumber. By using a second-order Markov model, serially correlated MOEs along 30-inch segments can be generated. This model was extended by Showalter, Woeste and Bendtsen (1987) to simulate within-member variation of tensile strength in lumber. Taylor and Bender (1988) presented an approach to simulate correlated, non-normal lumber properties using a transformation of the multivariate normal distribution. Their approach also preserved the marginal distributions of the correlated random variables. Using this approach, Taylor and Bender (1991) studied the spatial variability of localized MOE and tensile strength in two visual grades of Douglas fir laminated lumber and developed a stochastic model for simulating localized MOE and tensile strength. Their models can be used as input to study the reliability of engineered wood products such as the glued-laminated beams. The work by Taylor and Bender (1991) was extended by Richburg and Bender (1992) to include E-rated grades of laminated lumber.

The spatial variation of tensile strengths along the length of lumber was evaluated by Lam and Varoglu (1991a,b). A random process model using a moving average technique was used to characterize this within-member spatial variation. Xiong (1991), studied the structural performance of glulam beams by simulation using the spectral analysis and
bivariate normal distribution transformation methods. The reliability of laminated wood beams under the serviceability limit state of maximum deflection was studied by Wang and Foschi (1992) using random process representation of the within-specimen variability in modulus of elasticity. Models with different degrees of complexity, ranging from the simple beam theory to the stochastic finite element were considered in the model formulation. The modulus of elasticity was assumed to be a stationary random process. Lam and Barrett (1992) studied a method using trend removal and kriging techniques to model nonstationary within-member tensile and compressive strengths of lumber but without consideration of the correlation between MOE and strength. Lam, Wang and Barrett (1994) developed a stochastic model to simulate correlated non-stationary spatial variations of modulus of elasticity (MOE) and parallel to grain compressive strength ($f_c$). The model utilized the spectral analysis, bivariate normal transformation, trend removal, and moving average techniques to generate correlated MOE and compressive strength profiles. Simulated data was agreed with the test results. This model was extended to incorporate the gain factor model to characterize the correlation of the compressive strength and the modulus of elasticity (Wang, Lam and Barrett, 1994).

Folz (1997) studied the load-carrying capacity of glued-laminated wood beam-columns with the tensile strength and MOE which govern the response as modeled as stochastic fields. Without experimental data on the within-member variation of tensile strength and the correlation with MOE, he calibrated the tensile strength spectral process by minimizing the difference between the predicted and experimental cumulative distributions of tensile strength of glued-laminated lumber stock using a barrier-crossing approach. Based on his sensitivity studies, he concluded that the correlation of MOE and tensile strength has negligible effect on the load carrying capacity of glued-laminated beam. Thus he ignored the correlation during the simulation processes. In his model, a bilinear stress-strain relationship with a falling branch was adopted with the ultimate compressive stress modeled as a constant. No failures in compression were considered in the study, as tensile failure is usually dominant in glued-laminated beam application.
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2.7 Size Effect

The classical methodology of obtaining design values for code purposes was based on testing small clear samples. The use of small specimens to approximate full-size structural members requires the use of modification factors to account for size effect. With the recognition of the inadequacy of the small clear samples, in-grade full-size member testing was introduced. Full-size test properties of many wood products show that structural properties vary with member size, loading conditions, and failure modes—bending, tension compression. Thus there is consensus that a standard condition should be specified in the code from which the characteristic material properties would be derived, and modification factors should be given to adjust the values to different conditions (cf. ASTM 1991a, CEN 1991).

Recognizing that failure of lumber mostly occurs at the defects, brittle fracture weakest link theory has been used extensively to study size effects in various structural materials such as wood products. The classical weakest link theory was first proposed by Weibull (1939) and was first applied to wood by Bohannan (1966) to investigate the bending strength of clear wood members with different beam aspect areas. Applications of the classical brittle fracture theory for other failure modes include tension perpendicular to grain failure mode (Barrett 1974, Barrett et al. 1975, Collings 1986), and shear parallel to grain failure mode (Foschi and Barrett 1976, Collings 1986).

The weakest link model was also used to account for the variation of structural properties due to length, depth, width and volume (Buchanan 1984, Madsen and Buchanan 1986, Showalter et al. 1987, Johnson et al. 1989, Barrett and Fewell 1990, Sharp and Suddarth 1991, Madsen 1992, Lam and Varoglu 1991a, b). Experimental studies (Madsen and Nielson 1976, Buchanan 1984, Madsen 1992) show that the length effects for bending and tension members, which fails in a tensile mode, are similar (Barrett and Fewell 1990, Madsen 1992). The use of weakest link theory on lumber tested in compression is limited to lower quality material because brittle fracture theory is not applicable to ductile compression failures in the pure clear wood. However, some experimental data showed that there is an increase in maximum compression stress for lumber as the length and depth of members decreases (Buchanan 1984).
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Size factors can be derived for structure size lumber based on the in-grade test results (Barrett et al., 1992). However, these size factors may vary for different grades and species. As the size factors are closely related to the coefficient of variation, which are different for different grades, size factors would be grade dependent. Besides, different species may have different growth characteristics and defect patterns that may impact the size factors. For NLGA dimension lumber, maximum permitted knot sizes were specified in the grading rule and in general, permitted knot sizes increase as the member width increases. Thus lumber from different grades may be considered as different material with different size factors. Thus width (or depth) effects incorporate a grade effect which cannot readily be distinguished from the pure size effect.

Recognizing that bending members, especially the low quality material, fail in tension which is very similar to the pure tensile specimen, the ultimate bending strength can be predicted from the ultimate tension strength using weakest link concepts. This is especially true for low quality material because compression strength well exceed tensile strength. However, in high quality material the tension strength in general exceeds the compression strength resulting in yielding in the compression zone prior to tensile failure.

Several experimental studies were conducted to quantify length and width effects in timber (Madsen and Nielsen 1976, Madsen and Nielsen 1978, Johnson et al. 1989, Barrett and Fewell 1990, Madsen and Tomoi 1991) There are some discrepancies in the size factors obtained, especially for the depth effect. Some researchers agree that no consistent width effects were observed in their testings. Madsen (1992) also finds no consistent width effect in several studies of bending strength. Barrett and Fewell (1990) provided a summary of all the bending and tension tests conducted in North American and European lumber. Using the 5th percentile values, a significant length effect and width effect were obtained for both tension and bending.

Buchanan (1984) applied size factors to predict the maximum ultimate loads to columns and beam-column. This is based on the modified Weibull weakest link failure criteria that member strength is stress volume dependent. In order to predict the ultimate loads, several failure criterions were checked. The stress volume of the region of the column under tensile stress was checked for failure in tension. Similarly, the stress volume of the
region under compressive stress was considered to see whether yielding had already started. Yielding did not lead to immediate failure so a criteria for failure of compression was specified. Koka (1987) assumed failure in compression when the column could not develop the compression stresses required to achieve equilibrium. In addition, beam-columns and columns were also examined for failure in buckling.

2.7.1 Weakest-Link Theory (Statistical Strength Theory)

The weakest-link concept originated with Pierce (1926), who applied it to cotton yarn, and Tucker (1927), who studied concrete. Major contributions were made by Weibull (1939) who studied the strength of various materials based on the weakest-link theory. Using an exponential distribution, he showed that strength depended on the stress volume of a test specimen, assuming that all specimens consisted of statistically similar elements. It was later shown by Johnson (1953) that the Weibull distribution is asymptotic to the extreme value distribution of the smallest value in samples from any parent distribution for large sample size.

One can assume that elements of a member with reference volume \( V_r \) under a uniform tensile stress \( \sigma \) have a cumulative probability distribution \( F \) (probability of failure \( P_f \)) given by:

\[
F = P_f = 1 - e^{-\left(\frac{\sigma}{m}\right)^k}
\]  
(2.10)

where \( m \) and \( k \) are the 2-parameter Weibull model scale and shape parameters determined experimentally for this reference volume.

Using the same scale and shape parameters, the predicted cumulative distribution of an arbitrary member of volume \( V \) subjected to non-uniform stress distribution \( \sigma_n = \sigma(x,y,z) \) will be given by:

\[
F = 1 - e^{-\int_{V_r}^{V} \left(\frac{\sigma}{m}\right)^k dV}
\]

(2.11)

For the special case of uniform stress \( \sigma^* \) throughout the member, the cumulative probability is given by:
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\[ F = 1 - e^{-\frac{V_f}{V_i}(\sigma^*)} \]  \hspace{1cm} (2.12)

It is possible to define an equivalent volume \( V_e \), subject to a uniform stress \( \sigma^* \), which will have the same probability of failure as the member subject to the non-uniform stress field. The equivalent volume can be obtained by equating Eqns. (2.10) and (2.11), to yield:

\[ V_e \sigma^* = \int \sigma^* dV \]  \hspace{1cm} (2.13)

For a given load configuration, the right hand side of Eqn. (2.12) can be expressed as the product of a constant and the volume of the member. Consider the case of a simply supported beam subjected to two symmetrically placed concentrated loads spaced at a distance \( a \). The equivalent volume of the member is given by:

\[ V_e = \left[ 1 + \frac{a}{L} \cdot \frac{k}{2(k + 1)^2} \right] \cdot V = K \cdot V \]  \hspace{1cm} (2.14)

where \( V \) is the total volume of the beam, \( L \) is the beam span, and \( K \) is the constant of proportionality which depends on load configuration.

Two members of volumes \( V_1 \) and \( V_2 \) are subject to different load conditions but have the same probability of failure if

\[ 1 - e^{-\frac{V_{el}}{V_i}(\sigma_1)} = 1 - e^{-\frac{V_{e2}}{V_i}(\sigma_2)} \]  \hspace{1cm} (2.15)

where \( V_{el} \) and \( V_{e2} \) are equivalent volumes in uniform stress for the real volumes \( V_1 \) and \( V_2 \) with non-uniform stresses, and stresses \( \sigma_1 \) and \( \sigma_2 \) are reference stresses induced by the applied loads.

The equivalent volumes can then be expressed as:

\[ V_{el} = K_1 \cdot V_1 \]

\[ V_{e2} = K_2 \cdot V_2 \]  \hspace{1cm} (2.16)

where \( K_1 \) and \( K_2 \) are load configuration factors for beams 1 and 2.

Combining Eqns (2.14) and (2.15), we get:
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\[
\frac{\sigma_1}{\sigma_2} = \left[ \frac{K_2 V_2}{K_1 V_1} \right]^\frac{1}{k}
\]  
(2.17)

If the members have the same load configuration, then the constants \( K_1 \) and \( K_2 \) are equal, and the beam with larger volume has a lower strength. Then the strength-volume relationship can be represented by the following equation:

\[
\frac{\sigma_1}{\sigma_2} = \left[ \frac{V_2}{V_1} \right]^\frac{1}{k}
\]  
(2.18)

or

\[
\ln(\sigma) = b - \frac{1}{k} \cdot \ln(V)
\]  
(2.19)

The Weibull shape parameter, \( k \), can be derived by: (a) fitting the Weibull distribution to test data at fixed member sizes (Eqn. (2.10)); or (b) by comparing strengths of members subjected to different loading conditions (Eqn (2.17)); or (c) by regressing logarithms of strength against logarithms of volume or other appropriate scale factors (Eqn (2.19)).

The shape parameter, \( k \), is related to variability in strength properties. If the coefficient of variation (CV) is used to characterize property variability the shape parameter \( k \) can be approximated within 1% for typical CV values by:

\[
k = CV^{-1.085}
\]  
(2.20)

2.7.2 Anisotropic Size Effect Model

The weakest-link theory has been used extensively to study size effects in structural wood products. The Weibull weakest-link concept leads to the conclusion that the logarithm of strength properties is linearly related to the logarithm of member size, given similar member geometry and loading conditions. This type of logarithmic relationship has been used to determine size factors for member length, width and thickness. Some studies have shown that the size parameters for visually graded lumber are different for length, width and thickness due to the anisotropy of wood properties.
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Studies of thickness effects in sawn lumber (Madsen and Stinson 1982) suggest that bending strength properties increase as member thickness increases. This result is contrary to the concept of a weakest-link fracture process. Madsen and Buchanan (1986) suggested that this anomaly could result from grading rules which limit knot sizes on the wide and narrow faces of the member. These rules tend to limit the maximum size of knots for a fixed member depth independent of member thickness, even though the relative displacement of clear wood is less for thicker members.

The generalized size parameters are expressed as $S_{xy}$. The first subscript ($x$) indicates the size effect scale parameter. The second indicates the property to which the size effect applies. For example, $S_{Lb}$ represents the size parameter for length effect bending members.

The terms $S_R$ and $S_A$ are size parameters to characterize strength variation due to length and width of members using a fixed test length-to-depth ratio and due to aspect area (member length times member width) respectively.

Size effect parameters $S_R$, $S_W$, $S_L$ and $S_A$ for bending, tension and compression were derived by analyzing published data and the Canadian in-grade testing database. Length effect parameters $S_{lb}$, $S_{lh}$, $S_{lc}$ (Table 2.1) were given from published literature. The other size parameters obtained from the pooled CWC data sets are summarized in Table 2.1.

Considering all of the available published literature and research on size effects in different species groups and for different mechanical properties, a set of rounded size factors for bending, tension, and compression parameters appropriate for international acceptance was given by Barrett and Fewell (1990), and is shown in Table 2.2.

2.7.3 Load Configuration Effect

In some cases, size effect is also known as “load configuration effect” or “stress-distribution effect”. The load configuration effect describes the phenomenon that for members of given size, the maximum stress at failure tends to decrease as the highly stressed volume increased. Based on the weakest-link theory applied on homogeneous material, Madsen (1992) derived load configuration factors for different loading conditions as shown in Table 2-1. By using the weak zones method proposed by Riberholt and Madsen (1979), Czmoch et al. (1991) studied the effect of within member variability on bending strength of structural lumber. In
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their paper, characteristic values obtained from two standard test methods — Eurocode No. 5 (EC-5) and in-grade\textsuperscript{1} — are compared. It was found that the characteristic values derived from EC-5 method are significantly smaller. In addition, cumulative distribution of bending strength from different test methods were generated and could be used as input in reliability analysis. Isaksson (1999) has also studied the effects of test standard, length and load configuration. He characterized the length and load configuration effects by direct comparison and reliability analysis of the strength values of simulated boards using Riberholt’s weak zones model.

2.8 Summary

This chapter reviews the research work done in the past which forms the background and basis leading to the development of the strength model of beam-columns. Research on the applications of random field theory on lumber was reported and was shown to be limited. This chapter also reported on research of structural behaviour of lumber in bending, tension, compression and combined axial and bending loading. Size effect and the closely related load configuration effect were also defined and the historical research work on this topic was extensively described. This chapter will help the reader to understand the problem, as well as to understand the notation used for the rest of this thesis.

\textsuperscript{1} EC-5 method: The weak zone with the lowest value of bending strength is identified and then trial is undertaken to impose the load in such a way that the weakest weak zone is in the constant bending moment range. In-grade method: The tested specimen is randomly located within the board.
Chapter 2. Literature Survey

Table 2-1  Summary of bending, compression and tension size parameters (Barrett and Fewell, 1990)

<table>
<thead>
<tr>
<th>Property</th>
<th>$S_W$</th>
<th>$S_L$</th>
<th>$S_A$</th>
<th>$S_A=2S_A$</th>
<th>$S_R=S_L+S_W$</th>
<th>$S_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending</td>
<td>0.28</td>
<td>0.17</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.45</td>
</tr>
<tr>
<td>Tension</td>
<td>0.21</td>
<td>0.17</td>
<td>0.21</td>
<td>0.42</td>
<td>0.38</td>
<td>0.40</td>
</tr>
<tr>
<td>Compression</td>
<td>0.11</td>
<td>0.10</td>
<td>0.11</td>
<td>0.22</td>
<td>0.21</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Table 2-2  Size factors recommended for code harmonization (Barrett and Fewell, 1990)

<table>
<thead>
<tr>
<th>Property</th>
<th>$S_L$</th>
<th>$S_W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Tension</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Compression</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 2-3  Load Configuration factors (Madsen, 1990)

<table>
<thead>
<tr>
<th>Loading</th>
<th>Constant Moment as base</th>
<th>3rd Point Loading as base</th>
<th>UDL as base</th>
<th>Recommended values for code purposes</th>
</tr>
</thead>
<tbody>
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<td>1.04</td>
<td>1.00</td>
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<td>1.17</td>
<td>1.15</td>
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<td>1.30</td>
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<td>1.43</td>
<td>1.38</td>
<td>1.40</td>
</tr>
</tbody>
</table>
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Figure 2.1 Stress-strain relationship with compression yielding

Figure 2.2 Assumed and actual stress distribution with yielding
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Figure 2.3 Models of stress-strain relationships in compression

Figure 2.4 Stress distribution in bending members
Figure 2.5 Three-dimensional plot of axial load versus bending moment versus slenderness ratio
Chapter 3

Beam-Column Strength Model

3.1 Introduction

This chapter describes a theoretical model for predicting the strength of members under combined axial and lateral loading. MOE, tensile strength and compressive strength — the basic structural properties of this model — are treated as correlated random variables. Profiles of these variables are characterized and simulated and then used as the input to a finite element program to predict the ultimate load for a beam-column type problem. The assumptions, formulation and finite element representation of the strength model are given in this chapter, whereas the evaluation and simulation of the structural properties are depicted in Chapter 4. The finite element computer program can predict failure load of beam-columns under stability failure, tensile failure as well as compressive failure.

3.2 Formulation of the Problem

The following sections describe the different elements constituting the beam-column strength model. The model will be developed for a general beam-column type problem. Therefore it can also be used to analyze bending, tension, compression and combined bending and tension cases.

3.2.1 Assumptions

The following assumptions are made in developing the strength model:

1. Plane sections remain plane.
2. Timber stressed in tension will behave in a linear manner until brittle-type failure occurs at a limiting stress.
3. Timber stressed in compression will behave in a ductile-type manner. The material will start yielding when the stress exceeds the yield stress (proportional limit stress) and a
compression ductile-type failure is assumed to occur when the strain exceeds the limiting compressive strain.

4. Stress-strain relationship is assumed to be known and is probabilistic in nature. It is further assumed that the yield stress is not affected by the rate of loading.

5. Modulus of elasticity, yield strain, tensile strength and compressive strength are assumed to vary between boards and within a board. These structural properties are assumed to follow a random process.

6. Torsional and out of plane deformations are ignored in the model.

7. Load configuration factors associated with depth effect are incorporated to predict the ultimate load.

8. Duration of load effects and shear failures are not considered.

9. Load was considered as either live load or dead load. Dead load was applied incrementally to the load level specified in the data input and live load is applied and increased continuously until the member fails.

3.2.2 Beam-Column Kinematics

For beams under combined axial and bending loads, it is assumed that the beam undergoes large displacement but small strains. It is further assumed that plane sections remain plane. Under these assumptions, a beam element is sufficient to formulate the problem. To derive the kinematic relationship, let us consider an infinitesimal element at an arbitrary location with length $dx$ before deformation. The length of that element after deformation at the centroid axis, as shown in Figure 3.1, can be represented by

$$dl = \sqrt{\left(dx + \frac{du}{dx} \cdot dx\right)^2 + \left(\frac{dw}{dx} \cdot dx\right)^2}$$

(3.1)

Expanding by Binomial Theorem we get

$$dl = dx \left[ 1 + \frac{du}{dx} + \frac{1}{2} \left(\frac{dw}{dx}\right)^2 \right]$$

(3.2)
Chapter 3  Beam-Column Strength Model

Then the axial strain along the centroid axis can be expressed as

\[ \varepsilon_{x0} = \frac{dl - dx}{dx} = \frac{du}{dx} + \frac{1}{2} \left( \frac{dw}{dx} \right)^2 \]  

(3.3)

For strain off the centroid axis, the strain will have a term due to the rotation of the sectional plane. Referring to Figure 3.2, then fiber having a distance \( z \) from the neutral axis will have strain in the x-direction represented by

\[ \varepsilon_z = \frac{du}{dx} + \frac{1}{2} \left( \frac{dw}{dx} \right)^2 - z \cdot \frac{d^2 w}{dx^2} \]  

(3.4)

which is for the case of large rotation but small strain.

3.2.3 Stress-strain Relationship

As wood has different failure behaviors for tension and compression loading, different stress-strain relationships have to be assumed for these cases. For tension, it is postulated that the stress-strain relationship is linear to ultimate tensile stress with no plasticity behaviour. For compression, due to its ductile behaviour, a nonlinear stress-strain relationship will be used. Various studies have focused on deriving equations for modeling this nonlinearity and they were described in detail in Chapter 2. For this study, a new model was investigated. It has the form:

\[ \sigma = \frac{\varepsilon / \varepsilon_1 + G_1 (\varepsilon / \varepsilon_1)^m}{G_2 + G_3 (\varepsilon / \varepsilon_1)^n} \]  

(3.5)

where \( \sigma \) is the compressive stress; \( \varepsilon \) is the compressive strain; \( \varepsilon_1 \) is the strain at maximum stress (compressive yield strain) and \( m \) and \( n \) are parameters of the model. The model is shown graphically in Figure 3.3.
3.3 Finite Element Representation

3.3.1 Introduction

The finite element approximation method has been used extensively in solving many types of solid mechanics problems such as problems with nonlinear stress-strain relationships, geometrical nonlinearity, and complicated shapes and boundary conditions. The use of finite element method on steel and other material has been explored in the last few decades. With a non-homogeneous material like wood, the use of finite element method is becoming more extensive but still not to the same degree as other material. One of the limiting factors is computation complexity, which requires high speed and high capacity computers. With the advance of computer systems, the use of this method on highly variable materials such as wood will become more practical. The beam-column problem that we are facing is a geometrically non-linear problem with non-linear spatially dependent material properties; thus, a non-linear finite element approach is needed.

3.3.2 Element and Interpolation Functions

As mentioned earlier, the beams that we are dealing with have simple uniform rectangular cross section with linear variation of strain across the depth of cross-section. This makes the use of one-dimensional beam elements sufficient to formulate the problem. A typical 2-node beam element with its local and global coordinate system is shown in Figure 3.4. The local variable $\xi$ is related to the $x$ variable by

$$x = x_c + \Delta \cdot \xi$$  \hspace{1cm} (3.6)

where $x_c$ located at the mid depth of the member and $2\Delta$ is the length of the element. Also, the local variable $\eta$ is related to the $z$-coordinates by

$$z = \frac{h}{2} \cdot \eta$$  \hspace{1cm} (3.7)

where $h$ is the depth of the beam.
When considering the interpolation (shape) functions, one has to be certain that the continuity requirement must be satisfied. An element is deemed to be “compatible” if it provides sufficient continuity to satisfy the admissibility requirements of the Potential Energy Theorem. The beam problem, as a minimum, requires a linear interpolation function for the displacements in the $u$ direction and a quadratic interpolation function for the $w$ direction. In this study, complete cubic interpolations are used to approximate both the $u$ and $w$ displacements within an element. The use of higher order interpolation function can improve the estimation accuracy of the axial stresses and strains at less computational expense than the use of mesh refinement.

With a complete cubic interpolation, 4 nodal values have to be defined for each displacement and for each element. This is achieved by using the displacements and their first derivatives at each node for each displacement variable. Thus each element consists of eight degrees of freedom as represented by

$$\{\delta\} = \begin{bmatrix}
    u_i \\
    \frac{du}{dx} \bigg|_i \\
    u_j \\
    \frac{du}{dx} \bigg|_j \\
    w_i \\
    \frac{dw}{dx} \bigg|_i \\
    w_j \\
    \frac{dw}{dx} \bigg|_j
\end{bmatrix} \quad (3.8)$$

with the cubic interpolation functions as follows:
\( u(\xi) = \left( \frac{1}{2} - \frac{3}{4} \xi + \frac{3}{4} \xi^3 \right) u_i + \frac{1}{6} \left( 1 - \xi - \xi^2 + \xi^3 \right) \left( \frac{du}{dx} \right)_i \)
\( + \left( \frac{1}{2} + \frac{3}{4} \xi - \frac{1}{4} \xi^3 \right) u_j + \frac{1}{6} \left( 1 - \xi + \xi^2 + \xi^3 \right) \left( \frac{du}{dx} \right)_j \)

\( w(\xi) = \left( \frac{1}{2} - \frac{3}{4} \xi + \frac{3}{4} \xi^3 \right) w_i + \frac{1}{6} \left( 1 - \xi - \xi^2 + \xi^3 \right) \left( \frac{dw}{dx} \right)_i \)
\( + \left( \frac{1}{2} + \frac{3}{4} \xi - \frac{1}{4} \xi^3 \right) w_j + \frac{1}{6} \left( 1 - \xi + \xi^2 + \xi^3 \right) \left( \frac{dw}{dx} \right)_j \)

where \( l = 2A \). In matrix notation

\( u = N_i \cdot u_i \quad i = 1, \ldots, 4 \)
\( w = M_j \cdot w_j \quad j = 1, \ldots, 4 \)

where

\( u_i = u_i \)
\( u_2 = \left( \frac{du}{dx} \right)_i \)
\( u_3 = u_j \)
\( u_4 = \left( \frac{du}{dx} \right)_j \)
\( w_1 = w_i \)
\( w_2 = \left( \frac{dw}{dx} \right)_i \)
\( w_3 = w_j \)
\( w_4 = \left( \frac{dw}{dx} \right)_j \)
and

\[
\begin{align*}
N_1 &= \frac{1}{2} - \frac{3}{4} \xi + \frac{1}{4} \xi^3 & N_2 &= \left(1 - \xi - \xi^2 + \xi^3\right) \frac{1}{8} \\
N_3 &= \frac{1}{2} + \frac{3}{4} \xi - \frac{1}{4} \xi^3 & N_4 &= \left(-1 - \xi + \xi^2 + \xi^3\right) \frac{1}{8}
\end{align*}
\]  
(3.13)

For complete cubic interpolation for both \(u\) and \(w\), the interpolation functions are the same, that is

\[
N_i = M_i \quad i = 1, \ldots, 4
\]  
(3.14)

### 3.3.3 Virtual Work Formulation of Problem

The stiffness matrix for an element is derived from the principle of virtual work with the assumed displacement field. In matrix notation, the principle of virtual work can be expressed as

\[
\int \bar{\varepsilon}^T \sigma dV - \int \bar{\varepsilon}^T \bar{p} \, dV - \int \bar{u}^T \bar{p} \, * dS = 0
\]  
(3.15)

where the first integral is the internal virtual work, and the last two integrals representing the actual external loads moving through the corresponding virtual displacements, which is the external virtual work. In solid mechanics, the second integral usually represents the virtual work due to displacement of body force which, in general, is neglected. The last integral is the surface integral representing the virtual work due to the virtual movement of the element loads. In terms of generalized forces and assumed displacements, this integral can be discretized as

\[
\int \bar{u}^T \bar{p} \, * dS = \{\delta\}^T \cdot \{P_e\}
\]  
(3.16)

where \(\{P_e\}\) is the element generalized force vector with corresponding virtual displacement vector \(\{\delta\}\); tilde (~) represents the virtual quantity of the attached variable.
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The material law, the kinematics, and the interpolation functions can be introduced into Eqn. (3.16) to express the principle of virtual work in terms of the nodal displacements. With the assumed displacement field, the strain from Eqn. (3.4) becomes

\[ \varepsilon_x = N'_i u_i + \frac{1}{2} \left( M'_j w_j \right)^2 - z \cdot M''_j w_j \]  \( \text{(3.17)} \)

where

\[ N'_i = \frac{d}{dx} (N_i), \quad N''_i = \frac{d^2}{dx^2} (N_i), \quad M'_i = \frac{d}{dx} (M_i), \quad M''_i = \frac{d^2}{dx^2} (M_i) \]  \( \text{(3.18)} \)

In matrix notation, the displacement field can be expressed as

\[ \varepsilon_x = [B] \cdot \{\delta\} + \frac{1}{2} \{\delta\}^T \cdot [C] \cdot \{\delta\} \]  \( \text{(3.19)} \)

where

\[ [B] = \begin{bmatrix} N'_1 & N'_2 & N'_3 & N'_4 & -z \cdot M''_1 & -z \cdot M''_2 & -z \cdot M''_3 & -z \cdot M''_4 \end{bmatrix} \]  \( \text{(3.20)} \)

and

\[ [C] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M'_1 & M'_2 & M'_3 & M'_4 & M'_5 \\ 0 & 0 & 0 & M'_2 & M'_1 & M''_2 & M''_3 & M''_4 \\ 0 & 0 & 0 & M'_3 & M'_2 & M''_3 & M''_4 & M''_5 \\ 0 & 0 & 0 & M'_4 & M'_3 & M''_4 & M''_5 & M''_6 \end{bmatrix} \]  \( \text{(3.21)} \)

Then the virtual strain can be represented by

\[ \bar{\varepsilon} = [B] \cdot \{\delta\} + \{\delta\}^T \cdot [C] \cdot \{\delta\} \]
\[ = \left( [B] + \{\delta\}^T \cdot [C] \right) \cdot \{\delta\} \]
\[ = [B]_i (\{\delta\}) \cdot \{\delta\} \]  \( \text{(3.22)} \)
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Also note that

\[
\left[ \frac{dB_i}{d\{\delta\}} \right] = [C] \quad (3.23)
\]

Substituting \( \bar{\varepsilon} \) from Eqn. (3.22) into Eqn. (3.15) and using the external virtual work expression of Eqn. (3.16), we have for each element

\[
\int_{V_e} \left( [B_i(\delta)]^T \{\sigma\} \right) dV = \{\bar{\varepsilon}\}^T \{P_e\} \quad (3.24)
\]

Summing all the elements to global system yields

\[
\{\Phi(\delta)\} = \{P_e\} \quad (3.25)
\]

where

\[
\{\Phi(\delta)\} = \sum_{\text{elems} \ V_e} \int_{V_e} \left( [B_i(\delta)]^T \{\sigma\} \right) dV \quad (3.26)
\]

and

\[
\{P_e\} = \sum_{\text{elems}} \{P_e\} \quad (3.27)
\]

Eqn. (3.25) is a nonlinear equation which has to be solved using numerical methods.
3.3.4 Newton-Raphson Method

The Newton-Raphson method is commonly used to solve nonlinear and dynamic problems. Given a initial estimate, we can get closer and closer to the true solution by iterating. The Newton-Raphson method is developed based on the first order approximation of the Taylor series of the defined function. For our problem, expanding \( \{\Phi(\delta)\} \) with the current nodal displacement vector \( \{\delta_0\} \), and truncated at the first order term, we have

\[
\{\Phi(\delta)\} = \{\Phi(\delta_0)\} + \left[ \frac{\partial \Phi}{\partial \delta} \right]_{\delta_0} \cdot \{\Delta \delta\} \approx \{P_s\} \tag{3.28}
\]

or

\[
\left[ \frac{\partial \Phi}{\partial \delta} \right]_{\delta_0} \cdot \{\Delta \delta\} = \{P_s\} - \{\Phi(\delta_0)\} \tag{3.29}
\]

where the right hand side represents the unbalanced forces between the external loads and the internal stresses.

Now, we need an expression for \( \left( \frac{\partial \Phi}{\partial \delta} \right)_{\text{elem}} \) in terms of the assumed displacement field.

\[
\left( \frac{\partial \Phi}{\partial \delta} \right)_{\text{elem}} = \frac{\partial}{\partial \delta} \int_{\Omega_e} \left[ \begin{bmatrix} B_i(\delta) \end{bmatrix}^T \{\sigma\} \right] dV
\]

\[
= \int_{\Omega_e} \left[ \begin{bmatrix} \frac{\partial B_i(\delta)}{\partial \delta} \end{bmatrix}^T \{\sigma\} + \left[ B_i(\delta) \right]^T \left[ \frac{\partial \sigma}{\partial \epsilon} \right] \frac{\partial \epsilon}{\partial \delta} \right] dV
\]

\[
= \int_{\Omega_e} \left[ \begin{bmatrix} C \end{bmatrix}^T \{\sigma\} + \left[ B_i(\delta) \right]^T \left[ D_i \right] \left[ B_i \right] \right] dV
\]

\[
= \left[ K_T \right]
\]

where \( \left[ D_i \right] \) is the tangent elasticity matrix and \( \left[ K_T \right] \) is called the tangent stiffness matrix.
Then the increment of nodal values can be obtained by solving

$$\{\Delta \delta\} = [K_T]^{-1} \cdot \{\{P\} - \{\Phi(\delta_0)\}\}$$  \hspace{1cm} (3.31)

and the new nodal values can be evaluated

$$\{\delta\}_{i+1} = \{\delta\}_i + \{\Delta \delta\}$$  \hspace{1cm} (3.32)

This produces an iteration procedure which would approach the real solution, that is, an acceptable small value of the right hand expression in Eqn. (3.29) which represents the unbalanced forces.

The steps that involved to obtain the solution are as follows:

1. Guess global solution \(\{\delta\}_0\) (may start with zero load case).
2. Separate global \(\{\delta\}_0\) into local element \(\{\delta\}_e\).
3. Within an element, calculate \(\varepsilon\) from \(\{\delta\}_e\) according to Eqn. (3.19), and then \(\sigma\) and \([D_e]\)
   using the stress-strain relationship.
4. Calculate \([K_T]\) from Eqn. (3.30) and internal load vector \(\{\Phi_{elem}(\delta)\}\) according to
   \[\{\Phi_{elem}(\delta)\} = \int_{V_e} \{B_e\}' \{\sigma\} dV\]
5. Sum the \(\{\Phi_{elem}(\delta)\}\) into global system to form the global tangent stiffness matrix \([K_T]\).
6. Solve for \(\{\Delta \delta\}\) according to Eqn. (3.31) and obtain a new displacement vector using
   \(\{\delta\}_{i+1} = \{\delta\}_i + \{\Delta \delta\}\).
7. Repeat the above steps to achieve the desired accuracy.

### 3.3.5 Failure Criterion

Different failure modes have to be considered in order to predict the capacity of a beam and/or beam-column. Each failure mode and failure criterion must be checked after each load step to determine whether the beam has failed. For tensile members, bending members, and
combined tensile and bending members, the ultimate load is generally reached when the
tensile stress induced by the loads exceeds the tensile strength of the member. For columns
and beam-columns, the ultimate load is reached when either 1) the tensile stress exceeds the
tensile strength, or 2) instability failure occurs due to excessive deflection, or 3) when the
compressive strain exceeds the limiting compressive strain. The final failure mode depends
on the ratio of tensile strength to compressive strength, slenderness ratio, and loading
geometry.

To determine whether the stresses induced by the applied load exceeds the strength,
two options are available in the computer program. When no depth effect adjustment is
selected, the program will check for the stress at each Gaussian point and the stresses at the
top and bottom extreme fiber of each section. If the strength is exceeded by these stresses, the
member is considered to have failed. When depth effect is considered, an equivalent stress
integral will be determined at each section containing the Gaussian points using Weibull’s
Weakest Link theory. If the computed equivalent stress integral exceeds the strength of a
member under uniform tension, the beam is considered to have failed at that section. The
equivalent stress integral is computed as follows.

For members tested in tension, bending or combined bending and tension, the stress
may vary across a cross section. Using the Weibull Weakest Link theory, the cumulative
distribution of the tensile strength at a given section is given by:

\[ F(x) = 1 - e^{-\frac{d}{d_i} \left( \frac{x}{m} \right)^k \int_0^y dy} \]  

(3.33)

where \( x \) is the failure stress; \( y \) is the depth co-ordinate; \( d \) is the depth of the member and \( d_i \) is
the depth of the reference element; \( k \) and \( m \) are the shape and scale parameters, respectively
of the two-parameter Weibull distribution. In predicting the tensile strength, the integration
applies only to the portion of the cross section stressed in tension.

Consider the case shown in Figure 3.35(c), for axial tension, where stress is constant
over the depth, i.e., \( x = f_t \), Eqn (3.33) becomes

\[ F(x) = 1 - e^{-\frac{d}{d_i} \left( \frac{x}{m} \right)^k} \]  

(3.34)
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For the case of bending or combined bending and tension, the stress distribution are shown Figure 3.35(d) or (e), where tensile stresses vary linearly over the depth, the stress at any depth $y$ from the neutral axis is

\[ x = \frac{y}{cd} f_{tx} \]  \hspace{1cm} (3.35)

or

\[ x = r \cdot f_{tx} \]
\[ r = \frac{y}{cd} \]  \hspace{1cm} (3.36)

and Eqn (3.33) becomes

\[ F(x) = 1 - e^{-\frac{cd}{R} \left( \frac{r f_{tx}}{m} \right) x} \]  \hspace{1cm} (3.37)

where the integration is performed over the region with tensile stresses.

We can also calculate the ratio of extreme fiber stress at failure, $f_{tx}$, to the ultimate tensile stress for the same size of specimen. This can be achieved by equating the cumulative frequency of Eqns. (3.34) and (3.37), assuming the chance of failure for both cases are the same. For case (d), the ratio can be expressed as

\[ f_{tx} = \left( \frac{c}{k+1} \right)^{\frac{1}{k}} f_t \]  \hspace{1cm} (3.38)

When the neutral axis lies outside the specimen depth (as shown in Figure 3.5(e)), then the ratio becomes
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\[ f_{tx} = \frac{c}{k+1} \left[ 1 - \left( \frac{1 - 1}{c} \right)^{k+1} \right]^{-\frac{1}{k}} f_t \]  \hfill (3.39)

The extreme fiber tensile stress \( f_{tx} \) can be obtained by linear interpolation of the tensile stresses at Gaussian points as the tensile stress is linear across a section.

In the compression zone, the stress distribution of a member under axial and lateral loads will be non-linear in general as part of the member starts yielding (as shown in Figure 3.6(d) and (e)), the cumulative probability of compressive strength can be evaluated by

\[ F(x) = 1 - e^{-\frac{1}{V_1} \int_{V_c} f(x) \, dv} \]  \hfill (3.40)

where \( V_I \) is the reference volume; \( V_c \) is the volume of member under compressive stress; \( m \) and \( k \) are the scale and shape parameters respectively. The stress integral in Eqn. (3.40) can be calculated by numerically integrating the compressive stresses at Gaussian points as designated by \( \sigma^* \) in Figure 3.6(d) and (e).

For the uniaxial compression testing, which yield uniform stresses, Equation (3.40) becomes

\[ F(x) = 1 - e^{-\frac{1}{V_I} \int_{V_c} f(x) \, dv} \]  \hfill (3.41)

where \( x = f_c \) is the ultimate compressive stress under a pure compression test. Then the relationship between the ultimate compressive stress of non-uniform stress distribution and axial compressive stress can be developed by equating Eqn. (3.40) and (3.41). In order to establish the failure stress of a member under combined loading, both the tensile and compression failure modes have to be checked separately after each load increment.
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The beam may fail in an instability mode. The instability failure is identified by the inability of the column to develop the compression stresses required to achieve equilibrium. This is indicated by the singularity of the tangent stiffness matrix given in Eqn. (3.30).

The last mode of failure is the excessive compressive strain failure. This corresponds to the crushing-type of failure. In testing beam-columns or columns, for moderate tensile to compressive strength ratio, the specimens will experience some yielding prior to failure in brittle-type tension failure or crushing-type compression failure. If excessive yielding as indicated by a large compressive strain occurs, the beam is assumed to have failed in compression yielding. A limiting compressive strain of 0.04 is used in the program.

3.3.6 Computation procedures

The computation procedures include two components: simulation of the material properties and the determination of the ultimate load. The steps involved are as follows:

Step 1 - Simulation of the material properties

1. Simulate the modulus of elasticity, compressive strength, tensile strength and compressive yield strain profile of a member using a random field model.
2. Determine the modulus of elasticity, compressive strength, tensile strength and compressive yield strain at each section containing the Gaussian points.

Step 2 - Simulation of Ultimate Loads

1. Follow the steps in section 3.3.4 to obtain the tangent stiffness matrix, stress and strain at each Gaussian point.
2. If the tangent stiffness matrix is singular at any iteration, then the member is considered to have failed in stability at the prescribed load.
3. If depth effect is considered, the neutral axis is located at each section containing the Gaussian points and equivalent uniform tensile and compressive strengths are determined.
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4. Check the tensile stress at each Gaussian point or the equivalent uniform tensile strength at each section. If any tensile stress or the equivalent uniform tensile strength exceeds the corresponding tensile strength, the member is considered to have failed in tension.

5. Check the compressive strain at each Gaussian point in the compression zone against the limiting compressive strain. If the limiting compressive strain is exceeded, the member is considered to have failed in compression.

6. Repeat the simulation \( N \) times and the cumulative frequency of the ultimate load can be obtained.

3.3.7 Convergence Criteria

To obtain the solution vector at each load step we used the Newton-Raphson method to iterate the deformation increment. The iteration process terminated when convergence criterion was met. The convergence criteria is based on the comparison of the Euclidean norm with a tolerance value.

If \( \{\delta\}_0 \) and \( \{\delta\} \) represent the previous and current deformation vector respectively, then the change of deformation can be expressed as

\[
\begin{bmatrix}
\Delta \delta_1 \\
\vdots \\
\Delta \delta_n \\
\end{bmatrix} = 
\begin{bmatrix}
\delta_1 \\
\vdots \\
\delta_n \\
\end{bmatrix} - 
\begin{bmatrix}
\delta_1 \\
\vdots \\
\delta_n \\
\end{bmatrix}_0
\]

(3.42)

and the convergence criterion can be expressed as

\[
\frac{\sqrt{\sum \Delta \delta_i^2}}{\sqrt{\sum \delta_{i0}^2}} < \varepsilon \approx 10^{-4}
\]

(3.43)

where the left side is the Euclidean norm, \( \varepsilon \) is the specified tolerance.
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Figure 3.1  Large deformation of beam element

Figure 3.2  Kinematic relationship of beam element
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Figure 3.3  Proposed compression stress-strain model

Figure 3.4  Beam element in natural and global coordinates
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Figure 3.5 Tensile stress distribution in beam-column

Figure 3.6 Compressive stress distribution in beam-column
Chapter 4
Lumber Properties as Stochastic Processes

4.1 Introduction

Many random field process simulation models have been developed in recent years to model the performance of dimension lumber as well as engineered wood products (see Chapter 2 for literature review). These models are confined to the predicted properties of modulus of elasticity, compression strength and tensile strength. None of the models simulate bending strength or the correlations between modulus of elasticity and bending, tension, compression strength in lumber.

4.2 Stationary Random Processes

A random process such as earthquake ground acceleration, which has different statistics from time to time, is called a nonstationary process. On the other hand, random excitations such as ocean wave oscillations and wind pressures are stationary because they have rather small time variations in their statistics. These are examples of nonstationary and stationary random processes respectively. Specifically, a random process \( X(t) \) is called strongly stationary if all orders of its probability density functions are stationary. This means a single random variable \( X(t) \) would have the following statistics at any time \( t \)

\[
\text{mean} = \mu = E[X(t_1)] = E[X(t_2)] \tag{4.1}
\]

\[
\text{mean square} = E[X(t_1)^2] = E[X(t_2)^2] \tag{4.2}
\]

and

\[
\text{variance} = \sigma^2 = E\left\{X(t_1) - \mu\right\}^2 = E\left\{X(t_2) - \mu\right\}^2 \tag{4.3}
\]

For two random variables \( X(t_1) \) at \( t_1 \) and \( X(t_2) \) at \( t_2 \) with the time lag \( \tau = t_2 - t_1 \), the covariance and the correlation are functions of the time lag \( \tau \) only. That is, the
covariance = \( E[(X(t_1) - \mu)(X(t_2) - \mu)] \)
\[ = E[X(t_1)X(t_2)] - \mu^2 \]
\[ = E[X(t_1)X(t_1 + \tau)] - \mu^2 \] (4.4)

and the correlation function with lag \( \tau \),
\[ R(\tau) = E[X(t_1)X(t_1 + \tau)] \] (4.5)

are independent of the time \( t_i \).

Random processes may be further categorized as being either ergodic or nonergodic. Considering the properties of a stationary random process by computing time averages over specific sample functions in the ensemble, the mean value \( \mu_x(k) \) and the autocorrelation function \( R_{xx}(\tau,k) \) of the \( k \)th sample function are given by
\[
\mu_x(k) = \lim_{T \to \infty} \frac{1}{T} \int_0^T x_k(t) \, dt \] (4.6)
\[
R_{xx}(\tau,k) = \lim_{T \to \infty} \frac{1}{T} \int_0^T x_k(t)x_k(t+\tau) \, dt \] (4.7)

If the above \( \mu_x \) and \( R_{xx} \) do not differ when computed over different sample functions, the random process is said to be ergodic. For ergodic random processes, the temporal mean and autocorrelation function are equal to the corresponding ensemble averaged values.

For two stationary random processes \( x(t) \) and \( y(t) \), we can calculate their autocorrelation functions as well as their cross-correlation functions. The autocorrelation function is given in Eqn. (4.7) where the other two correlation functions are given as follows:
\[
R_{yy}(\tau) = E[y_k(t)y_k(t+\tau)] \\
R_{xy}(\tau) = E[x_k(t)y_k(t+\tau)] \] (4.8)

4.2.1 Spectral Density Functions

In general, spectral density functions can be obtained via the following methods:
Chapter 4  Lumber Properties As Stochastic Processes

(a) Fourier transform of correlation functions
(b) Finite Fourier transform of the records

The first way to evaluate the spectral density function is to take the Fourier transform of previously known correlation functions. When mean values are removed, this Fourier transform will always exist even though the Fourier transform of the original stationary random process may not exist. The Fourier transform yields a two-sided spectral density function, denoted by \( S(f) \), which is defined for \( f \) over \( (-\infty, \infty) \).

Given that the integrals of the absolute value of the correlation function are finite, that is,

\[
\int |R(\tau)| \, d\tau < \infty \quad (4.9)
\]

then the spectral density functions exist and are defined by:

\[
S_{xx}(f) = \int R_{xx}(\tau) e^{-j2\pi f \tau} \, d\tau
\]

\[
S_{yy}(f) = \int R_{yy}(\tau) e^{-j2\pi f \tau} \, d\tau \quad (4.10)
\]

\[
S_{xy}(f) = \int R_{xy}(\tau) e^{-j2\pi f \tau} \, d\tau
\]

where the inverse Fourier transforms will produce the original correlation functions

\[
R_{yy}(\tau) = \int S_{yy}(f) e^{j2\pi f \tau} \, df
\]

\[
R_{xy}(\tau) = \int S_{xy}(f) e^{j2\pi f \tau} \, df \quad (4.11)
\]

\[
R_{xx}(\tau) = \int S_{xx}(f) e^{j2\pi f \tau} \, df
\]

The equation pairs of Eqn. (4.10) and (4.11) are often called the Wiener-Khinchine relations.
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For real-value variables \(x(t)\) and \(y(t)\), the autospectral density functions are symmetrical and real while the cross-spectral density functions are in general complex-value functions of \(f\).

We can also define one-sided autospectral density functions \(G_{xx}(f)\) and \(G_{yy}(f)\), where \(f\) varies over \((0,\infty)\) by

\[
G_{xx}(f) = 2S_{xx}(f) \quad 0 \leq f < \infty \quad \text{otherwise zero}
\]

\[
G_{yy}(f) = 2S_{yy}(f) \quad 0 \leq f < \infty \quad \text{otherwise zero}
\]

(4.12)

The second method to evaluate spectral density functions is by the finite Fourier transforms. This method is used extensively nowadays as digital computer data acquisition systems are being fully developed.

Consider a pair of associated sample records \(x_d(t)\) and \(y_d(t)\) from stationary random processes \(\{x_d(t)\}\) and \(\{y_d(t)\}\). For a finite time interval \(0 \leq t \leq T\), the auto- and cross-spectral density functions can be determined by

\[
S_{yy}(f,T,k) = \frac{1}{T} Y_k^*(f,T) X_k(f,T)
\]

\[
S_{xy}(f,T,k) = \frac{1}{T} X_k^*(f,T) Y_k(f,T)
\]

\[
S_{xx}(f,T,k) = \frac{1}{T} X_k^*(f,T) Y_k(f,T)
\]

(4.13)

where

\[
X_k(f,T) = \int_0^T x_k(t)e^{-j2\pi ft} dt
\]

\[
Y_k(f,T) = \int_0^T y_k(t)e^{-j2\pi ft} dt
\]

(4.14)

and \(X_k^*(f,T), Y_k^*(f,T)\) are their complex conjugate respectively.

For a ensemble of records, the ensemble one-sided spectral density functions can be determined by taking the expectation of the transformed records as follows:
Chapter 4  Lumber Properties As Stochastic Processes

\[ G_{xy}(f) = 2 \lim_{T \to \infty} \frac{1}{T} E\left[X_k^*(f, T)Y_k(f, T)\right] \]

\[ G_{xx}(f) = 2 \lim_{T \to \infty} \frac{1}{T} E\left[|X_k(f, T)|^2\right] \]  \hspace{1cm} (4.15)

\[ G_{yy}(f) = 2 \lim_{T \to \infty} \frac{1}{T} E\left[|Y_k(f, T)|^2\right] \]

If a random process is stationary, the characteristic of the system can be described by a unit impulse response function \( h(\tau) \) and a transfer function \( H(f) \). The unit impulse response function is defined as the system output at time \( \tau \) to a unit impulse input applied at time zero. It relates the input \( x(t) \) and output \( y(t) \) by the convolution integral

\[ y(t) = \int_{-\infty}^{t} h(\tau)x(t-\tau)\,d\tau \]  \hspace{1cm} (4.16)

The unit impulse response function also relates to the transfer function \( H(f) \) by Fourier transform

\[ H(f) = \int_{0}^{\infty} h(\tau) e^{-j2\pi f\tau}\,d\tau \]  \hspace{1cm} (4.17)

The unit impulse response function and transform function are generally complex-value. The transfer function \( H(f) \) can be conveniently expressed in terms of the gain factor \( |H(f)| \) and the phase factor \( \phi(f) \) as follows:

\[ H(f) = |H(f)| e^{-j\phi(f)} \]  \hspace{1cm} (4.18)

4.3 Normalization of Data

Most of the stochastic theories apply only to stationary ergodic processes. The MOE and other structural properties considered in this study are non-ergodic and non-stationary random processes. However, these profiles can be transformed to ergodic stationary processes by using normalization procedures.
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4.3.1 Trend Removal Techniques

Lam, Wang and Barrett (1994) have used the trend removal techniques to remove trends in modulus of elasticity (MOE) and compressive strength profiles. Consider \( \{u_n\} \), which represents a random process sampled at an interval of \( \Delta x \) with \( N \) values sampled. A polynomial of degree \( k \) can be fitted to the data with the following predicted values:

\[
\bar{u}_n = \sum_{m=0}^{k} b_m (n\Delta t)^m \quad m = 0, 1, \ldots, k \quad n = 1, 2, \ldots, N
\]  

(4.19)

where \( b_m \), the coefficients of the polynomial, can be obtained by optimizing the squared discrepancies between the data values and the polynomial using the least squares method. This process yields the following formula

\[
\sum_{m=0}^{k} b_m \sum_{n=1}^{N} (n\Delta t)^{m+l} = \sum_{n=1}^{N} u_n (n\Delta t)^l \quad l = 0, \ldots, k
\]  

(4.20)

For linear trend \((k=1)\), the parameters \( b_0 \) and \( b_1 \) are given as follows:

\[
b_0 = \frac{2(2N+1)\sum_{n=1}^{N} u_n - 6\sum_{n=1}^{N} n \cdot u_n}{N(N-1)}
\]  

(4.21)

\[
b_1 = \frac{12\sum_{n=1}^{N} n \cdot u_n - 6(N+1)\sum_{n=1}^{N} u_n}{\Delta t \cdot N(N-1)(N+1)}
\]

where the standard error \((\sigma)\) is given by

\[
\sigma = \sqrt{\frac{\sum (u_n - b_0 - b_1 (n\Delta t))^2}{N-2}}
\]  

(4.22)

In this study, a linear trend has been assumed for MOE and compressive strength profiles but zero order trend is assumed for the tensile strength profiles. The non-ergodic and non-stationary random process of MOE \((x(t))\), compressive strength \((y_1(t))\) and tensile strength
(\(y^2(t)\)) can be normalized into ergodic stationary random processes by the following transformations:

\[
\begin{align*}
x^*_n(t) &= (x_n(t) - \bar{x}_n) / \sigma_x \\
y^*_n(t) &= (y_{1n}(t) - \bar{y}_{1n}) / \sigma_{y_1} \\
y^*_2n(t) &= (y_{2n}(t) - \bar{y}_{2n}) / \sigma_{y_2}
\end{align*}
\]

(4.23)

where \(\bar{x}_n(t)\), \(\bar{y}_{1n}(t)\) and \(\bar{y}_{2n}(t)\) are the predicted MOE, compressive strength and tensile strength values from the polynomial trends respectively; \(x^*_n(t)\), \(y^*_n(t)\) and \(y^*_2n(t)\) are their corresponding ergodic, stationary random processes. The predicted value of tensile strength \(\bar{y}_{2n}(t)\) in the above equation is the mean of each board.

By re-arranging the equation, the MOE, compact compressive strength and tensile strength can be expressed as

\[
\begin{align*}
x_n(t) &= \bar{x}_n + x^*_n(t) \cdot \sigma_x \\
y_{1n}(t) &= \bar{y}_{1n} + y^*_n(t) \cdot \sigma_{y_1} \\
y_{2n}(t) &= \bar{y}_{2n} + y^*_2n(t) \cdot \sigma_{y_2}
\end{align*}
\]

(4.24)

which will be used to re-generate the processes in the simulation process.

### 4.3.2 Weibull's Transformation Techniques

Linear transformation assumes the process to be normally distributed around the regression line. In general, lumber properties may not be necessarily normally distributed. Taylor and Bender (1989) proposed a method for transforming non-normal variables into normal variates.

The transformation between the normal distribution \(\mathcal{N}(y)\) and the non-normal distribution \(F(x)\) is depicted in Figure 4.1. The following is the procedure for the transformation:

1. For a given structural property sample \(x\) with \(N\) readings within a board \((x_1, x_2, ..., x_N)\), the values are ranked and cumulative distribution is determined.
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2. An appropriate probability distribution such as the 2-parameter or 3-parameter Weibull distribution can then be fitted to the data.

3. Assuming the 3 parameter Weibull distribution was used in step 2, for a specific reading \( x_i \), the corresponding cumulative probability \( u_i \) can be determined by:

\[
u_i = 1 - e^{-\left(\frac{x_i - x_0}{m}\right)^k}\]  \hspace{1cm} (4.25)

4. For the cumulative probability \( u_i \), the normal variate \( y_i \) is given by

\[
y_i = \Phi^{-1}(u_i)
\]

\[
y_2 = \Phi^{-1}(u_2)
\]

\[
y_N = \Phi^{-1}(u_N)
\]

where \( \Phi(x) \) is the standard normal cumulative probability function. The set of \( y_i \) obtained will then be standard normally distributed with zero mean and unit standard deviation.

5. Repeating steps 1 and 2 for each board, a set of Weibull parameters can be obtained.

6. Finally, the ensemble means and correlation matrix can be determined for these Weibull parameters.

To simulate a lumber property in real space, a set of standard normal variable values \( \{y_i, i=1,2,...,N\} \) are first simulated. Then a set of Weibull parameters \( m, k, \) and \( x_0 \) are simulated using multivariate normal distribution method. Following the reverse path of Figure 4.1 the corresponding non-normal variable values \( \{x_i, i=1,2,...,N\} \) can be evaluated according to

\[
x_i = F^{-1}(u_i)
\]

\[
x_2 = F^{-1}(u_2)
\]

\[
x_N = F^{-1}(u_N)
\]

which represents the simulated data in real space.

4.4  Single-Input/Multiple-Output Model

This section is concerned with the theory and applications of input/output relationships for the material model. It is assumed that records are from ergodic stationary random processes with zero mean values and that systems are constant-parameter linear systems.
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Since MSR lumber can be non-destructively graded to obtain an MOE, it is advantageous to have MOE as the input to the model, whereas the other structural properties are the output of the model.

Consider a linear system with single stationary (ergodic) random input \( x(t) \) producing \( r \) measured outputs \( y_i(t) \) with extraneous noise \( n_i(t) \) as shown in Figure 4.2. The individual output \( y_i(t) \) is given by the convolution integral

\[
y_i(t) = \int_0^\infty h_i(\tau) x(t - \tau) d\tau + n_i(t) \quad i = 1,2,\ldots,r
\]

(4.28)

where \( h_i(\tau) \) is the unit impulse response function of the \( i \)th path.

Taking expected values of the product \( y_i(t)y_j(t+\tau) \) yields the input/output autocorrelation function

\[
R_{y,y}(\tau) = \int_0^\infty \int h(\alpha) h(\beta) R_{xx}(\tau + \beta - \alpha) d\alpha d\beta + R_{n,n}(\tau)
\]

(4.29)

where \( R_{n,n}(\tau) \) is the autocorrelation function of the extraneous noise \( n_i(t) \).

It can be shown that direct Fourier transforms of Eqn. (4.28) yields the one-sided spectral density function \( G_{y,i}(f) \) which is related to the input spectral density function \( G_{xx}(f) \) by the transfer function as shown below:

\[
G_{y,i}(f) = |H_i(f)|^2 G_{xx}(f) + G_{n,n}(f)
\]

(4.30)

where \( G_{n,n}(f) \) is the one-sided spectral density function of the noise \( n_i(t) \).

The cross-correlation function between the input \( x(t) \) and any of the outputs \( y_i(t) \) is given by

\[
R_{x,y}(\tau) = \int_0^\infty h_i(\alpha) R_{xx}(\tau - \alpha) d\alpha
\]

(4.31)

and the corresponding cross-spectral density function is

\[
G_{x,y}(f) = H_i(f) G_{xx}(f)
\]

(4.32)
The cross-correlation function between any two of the output records \( y_i(t) \) and \( y_j(t) \) where \( i \neq j \) is given by:

\[
R_{y_iy_j}(\tau) = E[y_i(t)y_j(t+\tau)]
\]

\[
= \int_0^\infty \int_0^\infty h_i(\alpha)h_j(\beta) E[x(t-\alpha)x(t+\tau-\beta)]d\alpha d\beta
\]

\[
= \int_0^\infty \int_0^\infty h_i(\alpha)h_j(\beta) R_{xx}(\tau+\alpha-\beta)d\alpha d\beta
\]

and the associated cross-spectral density function is given by:

\[
G_{y_iy_j} = H_i^*(f)H_j(f)G_{xx}(f)
\]

We can also calculate the coherence functions \( \gamma_{y_iy_j}^2(f) \) between the input \( x(t) \) and the output \( y_i(t) \) when the auto- and cross-spectral density functions are known.

\[
\gamma_{y_iy_j}^2(f) = \frac{|G_{y_iy_j}(f)|^2}{G_{xx}(f)G_{y_iy_j}(f)}
\]

For linear systems, the coherence function can be interpreted as the fractional portion of the mean square value at the output \( y(t) \) that is contributed by the input \( x(t) \). It has to satisfy the inequality

\[
0 \leq \gamma_{y_iy_j}^2 \leq 1
\]

The auto- and cross-spectral density functions can be obtained by fast Fourier transform (FFT) analysis of the recorded samples. The transfer functions \( H_i(f) \) can then be determined by

\[
H_i(f) = \frac{S_{y_iy_i}(f)}{S_{xx}(f)}
\]

and the cross-spectral density functions can be determined from Eqn.(4.34).
4.5 Simulating Multivariate Random Processes

This section presents an overview of the basic theory of digital simulation of a general random process which will be applied throughout this study to develop spatially stochastic material information for application within a finite element analysis. The presentation will be restricted to one-dimensional stochastic fields.

The essential feature of this approach is that a random process can be represented by a series of cosine functions with weighted amplitudes, evenly spaced frequencies, and random phase angles. It has been proved that the process simulated is ergodic as $N \to \infty$ (Shinozuka 1987; Shinozuka and Deodatis 1991) and this method of simulation has recently been applied successfully to a variety of engineering problems (Shinozuka and Astill 1972; Shinozuka and Wen 1972; Wang et al. 1994).

Consider a set of $m$ homogeneous Gaussian 1-dimensional processes $y_j^0(t)$ $(j = 1, 2, \ldots, m)$ with zero means and cross-spectral density matrix $S^0(\omega)$ defined by

$$S^0(\omega) = \begin{bmatrix}
S_{11}^0(\omega) & S_{12}^0(\omega) & \cdots & S_{1m}^0(\omega) \\
S_{21}^0(\omega) & S_{22}^0(\omega) & \cdots & S_{2m}^0(\omega) \\
\vdots & \vdots & \ddots & \vdots \\
S_{m1}^0(\omega) & S_{m2}^0(\omega) & \cdots & S_{mm}^0(\omega)
\end{bmatrix}$$

(4.38)

where $S_{jk}^0(\omega)$ is the Fourier transform of the cross correlation $R_{jk}^0(\tau)$.

It can be shown that $S^0(\omega)$ is non-negative definite and Hermitian as

$$S_{jk}^0(\omega) = \overline{S_{kj}^0(\omega)}$$

(4.39)

where $\overline{S_{kj}^0(\omega)}$ is the complex conjugate of $S_{kj}^0(\omega)$ due to the fact that $R_{jk}^0(\tau) = R_{kj}^0(-\tau)$.

In general, the random process can be simulated and represented by the following series

$$y_j(t) = \sum_{m=1}^{N} \sum_{i=1}^{N} A_{jm} \cos[\omega_i \cdot t + \theta_{jm}(\omega_i) + \phi_{mi}]$$

(4.40)

where
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$A_{jm}$ are the weighted amplitudes,

$\phi_{mi}$ are independent random phase angle, uniformly distributed between 0 and $2\pi$,

$\omega_i = i\Delta\omega$,

$\Delta\omega = 2\pi/T$,

and

$$\theta_{jm}(\omega_i) = \tan^{-1}\left(\frac{\text{Im} S_{jm}(\omega_{mi})}{\text{Re} S_{jm}(\omega_{mi})}\right) \quad \text{(4.41)}$$

For bivariate homogeneous random processes, derivation of the weighted amplitude has been given by Folz (1997) and Eqn. (4.37) becomes

$$y_1(t) = \sum_{i=1}^{N} \sqrt{2\Delta\omega S_{y_1y_1}(\omega_i)} \cos[\omega_i t + \phi_{1i}]$$

$$y_2(t) = \sum_{i=1}^{N} \sqrt{2\Delta\omega S_{y_2y_2}(\omega_i) \gamma_{y_1y_2}^2(\omega_i)} \cos[\omega_i t + \theta_i + \phi_{2i}]$$

$$+ \sum_{i=1}^{N} \sqrt{2\Delta\omega S_{y_2y_2}(\omega_i) [1 - \gamma_{y_1y_2}^2(\omega_i)]} \cos[\omega_i t + \phi_{2i}] \quad \text{(4.42)}$$

where

$$\gamma_{y_1y_2}^2(\omega_i) = \frac{|S_{y_1y_2}(\omega_i)|^2}{S_{y_1y_1}(\omega_i) S_{y_2y_2}(\omega_i)} \quad \text{(4.43)}$$

and

$$\theta_{12}(\omega_i) = \tan^{-1}\left(\frac{\text{Im} S_{y_2y_2}(\omega_i)}{\text{Re} S_{y_1y_2}(\omega_i)}\right) \quad \text{(4.44)}$$

Similar derivation can be done for 3-variable random processes. For brevity of presentation, the following formulae are given for the weighted amplitudes without proof:
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\[ A_{11} = \sqrt{2S_{Y_1Y_1}(\omega_1)}\Delta \omega \]
\[ A_{21} = \sqrt{2\Delta \omega S_{Y_2Y_1}(\omega_1)}\sqrt{\gamma_{Y_1Y_2}^2(\omega_1)} \]
\[ A_{22} = \sqrt{2\Delta \omega S_{Y_2Y_2}(\omega_1)}\sqrt{1 - \gamma_{Y_1Y_2}^2(\omega_1)} \]
\[ A_{31} = \sqrt{2\Delta \omega S_{Y_3Y_1}(\omega_1)}\sqrt{\gamma_{Y_1Y_3}^2(\omega_1)} \]
\[ A_{32} = \sqrt{2\Delta \omega S_{Y_3Y_2}(\omega_1)}\left[ \frac{\sqrt{\gamma_{Y_2Y_3}^2(\omega_1)} - \sqrt{\gamma_{Y_1Y_2}^2(\omega_1)}\gamma_{Y_1Y_3}^2(\omega_1)}{\sqrt{1 - \gamma_{Y_1Y_2}^2(\omega_1)}} \right] \]
\[ A_{33} = \sqrt{2\Delta \omega S_{Y_3Y_3}(\omega_1)}\left[ 1 - \gamma_{Y_1Y_3}^2(\omega_1) - \frac{\sqrt{\gamma_{Y_2Y_3}^2(\omega_1)} - \sqrt{\gamma_{Y_1Y_2}^2(\omega_1)}\gamma_{Y_1Y_3}^2(\omega_1)}{1 - \gamma_{Y_1Y_2}^2(\omega_1)} \right] \]

Now consider the general case for multivariate homogeneous processes with mean zero and cross-spectral density matrix as given in Eqn. (4.38). Suppose one can find a lower matrix \( H(\omega) \) which satisfies the equation

\[ S^0(\omega) = H(\omega)H(\omega)^T \]  \hspace{1cm} (4.46)

where \( S^0(\omega) \) is the specified target cross-spectral matrix, then \( f_j(x) \) \((j=1,2,...,m)\) can be simulated by the following filtering technique:

\[ f_j(x) = \sum_{k=1}^{m} \int_{-\infty}^{x} h_{jk}(x-\tau)\eta_k(\tau)\,d\tau \] \hspace{1cm} (4.47)

where \( h_{jk}(x) \) is the Fourier transform of \( H_{jk}(\omega) \),

\[ h_{jk}(x) = \int_{-\infty}^{\infty} H_{jk}(\omega)\exp(-i\omega x)\,d\omega \]

where \( \eta_k(x) \) is an independent normalized white noise component.

To find the matrix \( H(\omega) \), we can assume that \( H(\omega) \) is a lower triangular matrix of the form:

\[ H(\omega) = \begin{bmatrix} H_{11}(\omega) & 0 & 0 & \cdots & 0 \\ H_{21}(\omega) & H_{22}(\omega) & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ H_{m1}(\omega) & H_{m2}(\omega) & \cdots & \cdots & H_{mm}(\omega) \end{bmatrix} \]  \hspace{1cm} (4.48)
By substituting this matrix into Eqn. (4.46), elements of $H(\omega)$ can be solved

$$H_{kk}(\omega) = \left[ \frac{D_k(\omega)}{D_{k-1}(\omega)} \right]^{1/2}, \quad k = 1, 2, \ldots, m$$

(4.49)

where $D_k(\omega)$ is the $k$th principal minor of $S^0(\omega)$ with $D_0$ being defined as unity, so

$$H_{jk}(\omega) = H_{kk} \frac{S^0_{12\ldots k-1, j}}{D_k(\omega)}, \quad k = 1, 2, \ldots, m, \quad j = k + 1, \ldots, m$$

(4.50)

where

$$S^0_{1,2\ldots k-1,j} = \begin{vmatrix} S^0_{11} & S^0_{12} & \cdots & S^0_{1,k-1} & S^0_{1,k} \\ S^0_{21} & S^0_{22} & \cdots & S^0_{2,k-1} & S^0_{2,k} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ S^0_{k-1,1} & S^0_{k-1,k-2} & \cdots & S^0_{k-1,k-1} & S^0_{k-1,k} \\ S^0_{j1} & S^0_{j2} & \cdots & S^0_{j,k-1} & S^0_{jk} \end{vmatrix}$$

(4.51)

is the determinant of the submatrix by deleting all elements except the $(1,2,\ldots k-1,j)$th row and $(1,2,\ldots k-1,k)$th column of $S^0(\omega)$.

Once $H_{jk}(\omega)$ is computed using equations (4.49) and (4.50), the multivariate stationary random processes $f_j(t)$ can be simulated efficiently by the following series:

$$f_j(t) = \sum_{m=1}^{N} \sum_{j=1}^{m} \left| H_{jm}(\omega_i) \right| \sqrt{2\Delta \omega} \cos(\omega_i t + \theta_{jm}(\omega_i) + \phi_{mi})$$

(4.52)

where

$$\left| H_{jm}(\omega_i) \right|$$

is the absolute value of element $jm$ of matrix $H_{jm}(\omega_i)$:

$$\theta_{jm}(\omega_i) = \tan^{-1}\left( \frac{\text{Im} H_{jm}(\omega_i)}{\text{Re} H_{jm}(\omega_i)} \right)$$

(4.53)

The above equations will simulate multivariate random processes with auto- and cross-spectral density functions approaches to the target ensemble of average spectral density functions as $N \to \infty$. 

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The simulation process is schematically shown in Figure 4.3.

4.6 Concluding Remarks

This chapter summaries the method used to simulate spatially correlated multivariate structural properties of lumber. The simulated profiles can be implemented into finite element analysis using a Monte Carlo algorithm to predict the ultimate strengths of beams and beam-columns. The simulated stationary random processes represented by cosine series as trend removal and normalization methods are used to convert the data into ergodic stationary processes before simulations. The random structural properties under consideration are MOE, compressive strength, tensile strength and compressive yield strain.
Chapter 4  Lumber Properties As Stochastic Processes

Figure 4.1 Transformation between normal and non-normal distributions

\[ F(x) \rightarrow \Phi(x) \]

\[ u_i \rightarrow u_j \]

\[ x(t) \rightarrow y_i(t) \]

*\( m(t) \)*

\[ H(x) \rightarrow \Sigma \]

\[ y_j(t) \]

*\( y(t) \)*

Figure 4.2 Single-input / multiple-output system

\[ x(t) \rightarrow H_1(t) \rightarrow \Sigma \rightarrow y_1(t) \]

*\( n_1(t) \)*

(Compresive Strength)

\[ x(t) \rightarrow H_2(t) \rightarrow \Sigma \rightarrow y_2(t) \]

*\( n_2(t) \)*

(Tensile Strength)

\[ x(t) \rightarrow H_3(t) \rightarrow \Sigma \rightarrow y_3(t) \]

*\( n_3(t) \)*

(Compressive Yield Strain)

Figure 4.2 Single-input / multiple-output system
Chapter 4  Lumber Properties As Stochastic Processes

Figure 4.3  Flowchart of structural properties simulation model
Chapter 5
Experiments

5.1 Objectives

A set of comprehensive experiments has been conducted to develop a database for basic input to the material model, for calibration of the model, and for verification of the model. The aims of this experimental study are summarized as follows:

1. To characterize the randomness of the within-board and between-board material properties of 1650f-1.5E MSR lumber. The properties that need to be characterized are modulus of elasticity (MOE), tensile strength, compressive strength and compressive yield strain as well as the correlation between them.

2. To establish the failure locus in the plane of bending moment (M) versus axial force (P). Emphasis will be put more on the sections where bending stress exceeds compressive stress since that is the region of greatest interest in the design of wood frame structures and truss chord members. Experimental results will be compared to simulated values to verify the strength models.

3. To study the non-linearity of the stress-strain relationship for uniaxial compression tests so that the nonlinear stress distribution across a section under combined moment and axial load can be derived.

4. To verify the material model under different load configurations. Effects due to stress distribution (depth effect) will be incorporated into the model and thus can be verified.

5.2 Material

Approximately 1800 pieces of nominal 38mm x 89 mm (nominal 2 x 4) 5.486 m (18-foot) length MSR 1650f-1.5E boards were sampled and conditioned to obtain a moisture content between 10% and 15%. Approximately 750 pieces of them were shipped to Forintek Canada
Chapter 5 Experiments

Corp. and the flatwise MOE of each piece was determined non-destructively using a Cook Bolinders AG-SF stress grading machine. Simple beam bending theory was used to calculated the MOE profile from the measured load profile.

After the MOE profiles of the lumber were determined, they were divided into 15 groups of 50 pieces by a matched groups technique based on the average flatwise MOE. Four of the groups were further merged into two groups of 100 specimens for the third-point bending test and the long span pure compression test. Thus a total of 13 subgroups were produced.

Another 100 pieces were selected randomly from the original 1800 pieces for the within-member property tests. These boards were also evaluated by the Cook Bolinders AG-SF stress grading machine at UBC.

The groups and the sample sizes of the two groups are as follows:
1) Within-board properties testing group (Group A, n=100).
2) Full-size testing group (Group B with 13 subgroups, n=750).

5.3 Pre-testing Preparations

Each specimen of Group A went through the following procedures before testing:

1. Using a Cook Bolinders AF-SF stress grading machine, the flatwise modulus of elasticity were measured along the length of the specimen. Boards with excessive twists, bows and warping were discarded.
2. The location and knot information along each board were recorded according to a “Forintek” knot coding system (Abbott 1990). For practical purposes, knots which are less than 10 mm were ignored. A knotgroup is defined as a group of knots where all knots appear within a length equal to the depth of the lumber.
3. Moisture content values were checked at various points along the board. Boards which were too dry or excessively moist were conditioned.
Chapter 5 Experiments

Each specimen of Group B also went through items 1 and 3 of the above procedures. In addition, for the specimens tested in third point bending, the edgewise bending stiffness values were measured by a static bending test method.

5.4 Within-board Structural Properties Tests (Group A material)

Two types of test were conducted — the Compact Compression Test (Test A1) and the Multiple Failures Tension Test (Test A2). The 100 pieces of group A boards were divided randomly into two groups (70 pieces for Compact Tension Test and 30 pieces for Compact Compression Test). For the compression test, the boards were cut into 29 segments with each segment of 152 mm (6 inches) long. The first 500 mm from both ends of each board were discarded since no MOE data was available from the Cook Bolinder machine for these sections. These specimens were then numbered sequentially starting from the same end that was fed into the Cook Bolinder machine so that the MOE values for each specimen within the board could be referenced to the board and specimen numbers. The specimen number has a format of XXXYY where XXX represents the board number and YY represents the specimen number within the board (from 1 to 29).

These short 150 mm specimens were then tested in compression parallel to grain using the MTS 810 structural test system until failure. A load cell of 250 kN with a loading rate of 0.457 mm/min (0.018 in/min) was used which produced failure in about 2 minutes. The stress-strain relationship during the loading period were recorded through the data acquisition system so that stress-strain non-linearity could be studied.

For the Tension Test, each board was first tested at full span — 4267 mm (14 feet) gauge length with a 610 mm (2 feet) grip length at each end. After the first failure, the unbroken sections of the board were re-tested in tension with the longest feasible gauge length. Successive tests were performed until it was not possible to conduct further testing on the same piece. The minimum gauge length of the test machine is 610 mm (2 feet) with a minimum grip length of 610 mm (2 feet) for each grip. Thus the shortest span the machine can test is 1830 mm (6 feet). All the tension tests were done with the Metriguard 403 Tension
Chapter 5 Experiments

Proof Tester at UBC Wood Mechanics Laboratory. Locations and lengths of failure were recorded so that the correlation of tensile strength and Cook-Bolinder MOE could be studied.

5.5 Combined Axial and Lateral Loads Full-Size Tests (Group B Material)

After the boards have been E-rated by the Cook-Bolinder machine, they were assigned randomly to different subgroups by the matching MOE method as mentioned before. The subgroups and the type of tests are designated and described below.

5.5.1 Test B1 – Third Point Bending Test

For the third-point bending tests, the test span was 2286 mm (90 inches) with two loads spaced at 762 mm (30 inches) apart as shown in Figure 5.1. Prior to testing the specimens, the boards were cut into two equal length and were labeled A and B with A corresponding to the end of the specimen where the grade stamp was located. Therefore in total 200 specimens were available for testing in third-point bending for the 100 boards assigned to these group. Both specimens were tested with the load applied to the same edge of the board.

The specimens were centered in the test frame and loads were applied by a hydraulically controlled UBC Bending Test machine at a controlled displacement rate of approximately 30 mm/min which produced failure in about one to two minutes. A yoke was used to measure deflections at mid-span. Before loading the specimens to destruction, the specimens were E-rated by loading the specimens to approximately 1780 N (400 lb). The MOE was determined by the applied load and the corresponding mid-span deflection. The MOE obtained can be regarded as the average edgewise MOE of the piece.

After testing the members to failure, the members were inspected to determine the failure location and failure code. The “Forintek” failure coding system (Abbott 1990) was used in this exercise. Full cross-section blocks were cut near the failure defects to estimate the specific gravity and the moisture content of the piece.
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5.5.2 Test B2 – Laterally Supported Long Span Compression Test

One hundred boards were selected for testing in full-size compression. Two specimens were cut from each board producing a total of 200 specimens. They are labeled in the same way as bending specimens with the end containing the grade stamp marked as “A”.

The specimens were tested with spans of 2286 mm (90 inch) and were laterally supported in both the strong and weak axes to prevent buckling as shown in Figure 5.2. This produced a yield-type failure. These tests were done by a compression testing machine at Forintek Laboratory. The compression strength was obtained from the quotient of the ultimate compressive load and the actual cross-sectional area of the specimen.

After the tests, blocks were cut from the section close to the failure location and were used to determine the moisture content and specific gravity. In addition, the failure location and the failure code were recorded for each board.

5.5.3 Test B3 – Eccentric Compression Tests

In these tests, the specimens were cut at randomly generated preset position within the board. The gauge length was 2286 mm. Each end of the test specimen fitted snugly into a steel “boot” through which axial compression was applied at a pre-determined eccentricity about the strong axis as shown in Figure 5.3. Lateral support was provided along the member so that buckling about the weak axis was prevented. Linear variable differential transducers (LVDT) were used at four locations along the member to measure the deflection. To assure that the specimen would bend away from the applied eccentricity side of the neutral axis, an initial deflection was administered by bringing the cross-head of the lateral load down to a prescribed value. Then the axial load was applied in the displacement control mode until failure. Failure mode and failure codes were determined by inspecting the failed specimens. Moisture content and specific gravity were determined from the blocks obtained close to the failed section.
Chapter 5 Experiments

5.5.4 Test B4 – Single Span Combined Bending and Compression Tests

In this series, members were tested under combined axial compression and lateral bending load.

A specimen of length 2286 mm was cut from each board. The location of the specimens within the board was determined randomly. Specimens were then inserted into the steel boots. The mid-span actuator’s piston rod was then lowered to produce a mid-span deflection of approximately 10 mm. Then the axial load was applied under the load controlled mode until the prescribed load level. Five axial load levels are tested — 4 kN, 8 kN, 12 kN, 25 kN, 25 kN and 40 kN. The lateral load was then applied at third-points under a displacement controlled mode until failure, while the axial load was maintained at the prescribed load level. LVDTs were installed at four locations along the members to obtain the deflections as the test proceeded. Lateral supports was provided along the member so that buckling about the weak axis was prevented. The dimension of the specimen and the location of LVDTs are shown in Figure 5.4. The experimental setup and the grip details are shown in Figure 5.5 and Figure 5.6 respectively.

After the tests, specimen blocks were cut near the failed section to determine the moisture content and specific gravity. Failure location, failure mode and failure codes were also recorded for each specimen.

5.5.5 Test B5 – Double Span Combined Bending and Compression Tests

The test span of the double span tests is 4472 mm. Each specimen was cut from the center section of the board with both ends trimmed. Then it was inserted into the end grips of the testing apparatus. The specimen was supported at the mid-span and at the ends. Eight LVDTs were installed along the span to measure the deflections. Four points of loading with two at the third-points of each span were applied by a actuator. Axial compression load was applied by a horizontal actuator located at the right side of the test frame. The configuration and experimental setup of this test is depicted in Figure 5.7 and Figure 5.8 respectively.
The cross-head of the lateral actuator was lowered to produce a deflection of 10 mm at the mid-span to ensure a desired failure mode. Then axial load was applied and maintain at a prescribed level. Three prescribed axial load levels were used in the experiments—4, 8 and 12 kN. Finally the lateral load was increased until failure. The experimental groups and the sample size of each group are summarized in Table 5.1.
## Table 5-1 Summary of Experimental Tests

<table>
<thead>
<tr>
<th>Group</th>
<th>Test Mode</th>
<th>N</th>
<th>Total Span/Gauge Length (mm)</th>
<th>No. of Span</th>
<th>Eccentricity</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>Compact Compression</td>
<td>754</td>
<td>152</td>
<td>1</td>
<td>Nil</td>
</tr>
<tr>
<td>A2</td>
<td>Multiple Tension</td>
<td>67</td>
<td>Vary</td>
<td>1</td>
<td>Nil</td>
</tr>
<tr>
<td>B1</td>
<td>Third-point Bending</td>
<td>200</td>
<td>2286</td>
<td>1</td>
<td>Nil</td>
</tr>
<tr>
<td>B2</td>
<td>Full-size Compression</td>
<td>200</td>
<td>2286</td>
<td>1</td>
<td>Nil</td>
</tr>
<tr>
<td>B3-1</td>
<td>Eccentric Compression</td>
<td>50</td>
<td>2286</td>
<td>1</td>
<td>0 mm</td>
</tr>
<tr>
<td>B3-2</td>
<td>Eccentric Compression</td>
<td>50</td>
<td>2286</td>
<td>1</td>
<td>5 mm</td>
</tr>
<tr>
<td>B3-3</td>
<td>Eccentric Compression</td>
<td>50</td>
<td>2286</td>
<td>1</td>
<td>75 mm</td>
</tr>
<tr>
<td>B3-4</td>
<td>Eccentric Compression</td>
<td>50</td>
<td>2286</td>
<td>1</td>
<td>200 mm</td>
</tr>
<tr>
<td>B4-1</td>
<td>Bend+Comp (C= 4 kN)</td>
<td>50</td>
<td>2286</td>
<td>1</td>
<td>Nil</td>
</tr>
<tr>
<td>B4-2</td>
<td>Bend+Comp (C= 8 kN)</td>
<td>50</td>
<td>2286</td>
<td>1</td>
<td>Nil</td>
</tr>
<tr>
<td>B4-3</td>
<td>Bend+Comp (C= 12 kN)</td>
<td>50</td>
<td>2286</td>
<td>1</td>
<td>Nil</td>
</tr>
<tr>
<td>B4-4</td>
<td>Bend+Comp (C= 25 kN)</td>
<td>25</td>
<td>2286</td>
<td>1</td>
<td>Nil</td>
</tr>
<tr>
<td>B4-5</td>
<td>Bend+Comp (C= 40 kN)</td>
<td>25</td>
<td>2286</td>
<td>1</td>
<td>Nil</td>
</tr>
<tr>
<td>B5-1</td>
<td>Bend+Comp (C= 4 kN)</td>
<td>50</td>
<td>4472</td>
<td>2</td>
<td>Nil</td>
</tr>
<tr>
<td>B5-2</td>
<td>Bend+Comp (C= 8 kN)</td>
<td>50</td>
<td>4472</td>
<td>2</td>
<td>Nil</td>
</tr>
<tr>
<td>B5-3</td>
<td>Bend+Comp (C= 12 kN)</td>
<td>50</td>
<td>4472</td>
<td>2</td>
<td>Nil</td>
</tr>
</tbody>
</table>
Chapter 5 Experiments

Figure 5.1  Test B1 – Third point bending test

Restraints to prevent buckling in both strong and weak axes

Figure 5.2  Test B2 – Laterally supported compression test
Chapter 5  Experiments

Figure 5.3  Test B3 – Eccentric compression test

Figure 5.4  Test B4 – Single span combined axial and lateral loads test
Figure 5.5  Test B4 – Single span combined axial and lateral loads test setup

Figure 5.6  Grip details
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Figure 5.7 Test B5 – Double span combined axial and lateral loads test

Figure 5.8 Test B5 – Double span combined axial and lateral loads test setup
Chapter 6
Experimental Results and Model Calibrations

6.1 Introduction

This chapter summarizes and discusses the results of the experiments described in the previous chapter. Based on these results, lumber structural property models were calibrated and model parameters were derived. The four lumber properties investigated herein are modulus of elasticity, compressive strength, tensile strength and compressive yield strain. Spectral analysis method was utilized to obtain the auto- and cross-spectral representations of these properties and from these, models of the lumber strength and elastic properties were characterized. Coherence and transfer functions were also determined by method outlined in Chapter 4 and lumber properties profiles were generated accordingly.

The lumber structural property models were verified by comparing the statistics of the simulated profiles with the experimental results. The goodness-of-fit of these comparisons were based on visual inspection. Alternatively, statistical goodness-of-fit tests such as the chi-squared test can be performed on these comparisons. As the level of confidence of these tests is also a subjective matter, sometimes it would be better to determine the goodness-of-fit by visual inspection. Sensitivity analyses were also performed on certain model parameters to study the effect of varying these parameters.

The spectral representations of these structural properties were obtained by performing the Fast Fourier transform as described in chapter 4. Once the model was calibrated, spatial structural properties in real space were simulated and implemented in a finite element program to predict the capacity of beams and beam-columns.

Key to the material model in spatial analysis is the discretization of the continuous random fields into a denumerable of random variables which includes a cosine series with random phase angles and amplitudes. Details of the discretization process are given in chapter 4 and can also be found in Shinozuka 1987 and Shinozuka and Deodatis 1991.
Chapter 6  Experimental Results and Model Calibrations

6.2  Modulus of Elasticity

Random field theory was used to characterize the within member spatial variation of MOE and calibrate the MOE power spectral density function models. Simulations of MOE profiles were performed using the trend removal techniques, multivariate normal generation method and random process simulation procedures as given in Shinozuka and Jan (1972).

6.2.1  Spectral Representation of Modulus of Elasticity

The flatwise MOE of each piece was evaluated non-destructively by the Cook Bolinders AG-SF stress grading machine. With the board supported flatwise by two rollers, 900 mm apart, a prescribed deflection was imposed at center point of the section and the induced force was measured. The process is repeated every 2 mm along the length of the piece. Using the simple beam theory, the flatwise MOE for each of this 900mm section were determined. Assigning this MOE value to the mid-point of the section, a MOE profile was determined from the deflection measurements excluding a 500 mm section from each end. For a 5.486 m (18 foot) long board, a maximum of 2243 readings of 2 mm interval can be obtained.

In what follows let $x(t)$ denote the flatwise MOE profile along a board which is non-ergodic and non-stationary. For a generic test record, $x_n(t)$, such processes may not be ergodic and stationary as shown in Figure 6.1. However, the process can be converted to an ergodic and stationary random process, $x^*_n(t)$, by the normalization method given in chapter 4.

Figure 6.2 shows the ensemble mean and standard deviation of MOE denoted by $\bar{x}(t)$ and $\bar{\sigma}_x(t)$ respectively. It is apparent that the ensemble mean MOE profile shows a distinct trend whereas only a minor trend can be observed for the ensemble standard deviation of MOE. Trend exists in MOE profiles can be due to inherent growth characteristics or machine systematic error. Linear trend removal techniques were then applied to the MOE profiles to remove the trend and convert the processes into ergodic stationary processes. Figure 6.3 shows the plot of the ensemble normalized MOE profile $\bar{x}^*(t)$ and its associated standard deviation profile $\bar{\sigma}^*(t)$, versus the location along the board. No apparent trend can be observed for both statistics.
Chapter 6  Experimental Results and Model Calibrations

Now we are ready to see how the Fast Fourier Transform techniques (FFT) can be used to calculate an estimate for the spectral density of a MOE process. Suppose we have a continuous record of MOE which lasts from $t=0$ to $t=T$. In what follows it is necessary to assume that we are analyzing a periodic record, in which $x_n^*(t)$ constitutes a single cycle. From this record we can generate the discrete time series $\{x_r\}$, $r=0, 1, 2, 3, \ldots, (N-1)$, with a sampling rate $\Delta = T/N$. We can then calculate the DFTs $\{X_k\}$ of the discrete MOE series $\{x_r\}$ by

$$X_k = \frac{1}{N} \sum_{r=0}^{N-1} x_r e^{-(2\pi r/N)} k = 0,1,2,\ldots,(N-1) \quad (6.1)$$

Then the two-sided auto-spectral density function of MOE, $S_{xx}(k)$, can be obtained from the product of

$$S_{xx}(k) = X_k^*X_k$$

where $X_k^*$ is the complex conjugate of $X_k$. \quad (6.2)

Because of the aliasing, the Fourier coefficients for frequencies $S_{xx}(k)$ calculated by the DFT are only correct up to

$$\omega_k = \frac{2\pi k}{N\Delta} = \frac{\pi}{\Delta} \quad (6.3)$$

or

$$f = \frac{1}{2\Delta} \quad (6.4)$$

The frequency $1/2\Delta$ Hz is called the Nyquist frequency (or sometimes the folding frequency) and is the maximum frequency that can be estimated from records with spacing equal to $\Delta$.

Figure 6.4 shows the ensemble one-sided power spectral density of MOE, $G_{xx}(k)$, obtained from performing FFT of the data. It is clear that there are two dominant spectral frequencies in the plot around 2 rad/m and 21 rad/m. Similar patterns have been observed by Wang et al (1994). These dominant frequencies can be attributed to the growth characteristics of the wood such as knots or any other defects. In general, wood is weaker or less stiff at these inherent growth characteristics. The primary dominant frequency of 2 rad/m corresponds to a period of a 3.0 m match with the rate of occurrence of big knots or defects observed from the tested material. The secondary dominant frequency of 21 rad/m may be
explained by the occurrence of the smaller knots which are generally spaced at 300 mm interval for this grade. This explanation is confirmed by the disappearance of this peak in higher grade lumber such as 2400f-2.0E (Wang et al 1994).

In order to facilitate the simulation process, it is advantageous to characterise the power spectral density data with a model. Two models were evaluated in this study — Wang's model and Shinozuka's model.

Wang's model has the form of

\[
G(\omega) = \frac{\alpha}{N\pi} \sum_{i=1}^{N} \left[ \frac{1}{\alpha^2 + (B_i + \omega)^2} + \frac{1}{\alpha^2 + (B_i - \omega)^2} \right] - \frac{1}{2\beta\sqrt{\pi}} \exp \left( \frac{-\omega^2}{4\beta^2} \right) \quad -\infty < \omega < \infty
\]

(6.5)

with the constraint function

\[
\beta = \frac{N\sqrt{\pi}}{4\alpha} \left/ \sum_{i=1}^{N} (\alpha^2 + B_i^2)^{-1} \right.
\]

It also has the corresponding autocorrelation function \( R(\tau) \) as

\[
R(\tau) = \frac{2}{N} \exp(-\alpha|\tau|) \sum_{i=1}^{N} \cos(B_i\tau) - \exp(-\beta^2\tau^2) \quad -\infty < \tau < \infty
\]

(6.6)

where \( N \) is the number of cosine terms.

Figure 6.4 shows the fitted curve of Wang's model (with number of terms \( N = 9 \)). It is apparent that the model fits the data very well except around the second dominant frequency.

Shinozuka's model (1972) has been used by Folz (1997) to model the load carrying capacity of laminated wood beams and has the form of

\[
S(\omega) = \sum_{i=1}^{N} \frac{1}{(2i)!} \sigma^2 b_{2i} \omega^{2i} e^{-b_{2i}\omega}
\]

(6.7)

Figure 6.5 shows the fitted curve for this model with the number of terms equal to six. Similar to Wang's model, it fits well with the data except for the region around the secondary dominant frequency of 20 rad/m.

It is of interest, at this point, to study the effect of the sampling resolution on the power spectral density functions. The MOE profiles were originally sampled at interval \( \Delta = 2 \) mm. However, in the compact compression test, the specimen has a length of 152.4 mm (6
Chapter 6 Experimental Results and Model Calibrations

inch). Spectral density functions of MOE corresponding to this sampling resolution, \( \Delta = 152.4 \text{mm} \), could be established by reducing the sampling resolution. Two approaches have been used to get the reduced sample records — using the average of the MOE values within the 152.4 mm region (within specimen average MOE) and using the minimum MOE value within the region (within specimen minimum MOE). The results are shown in Figure 6.6 with the power spectral density of the original resolution noted as data. Since the number of data points are reduced significantly, the cutoff frequency is now 20 rad/m. Comparing the three curves within the 20 rad/m, the within-specimen average MOE shows a curve similar to the within-specimen minimum MOE power spectral density. However, reduction in the sampling points indicates increase in the magnitude of peaks of the curve besides reducing the cutoff frequency.

6.2.2 Autocorrelation Function of MOE

The autocorrelation function of MOE shows the spatial correlation of MOE along the board. It has a value of 1 at lag equal to zero which means the MOE at a position is fully correlated to itself whereas at other locations it has a value between zero and one.

Autocorrelation function can be calculated from the power spectral density function by

\[
R_r = \sum_{k=0}^{N+L-1} S_k e^{j(2\pi kr/N+L)}\] (6.8)

or calculated directly from the data

\[
R_r = E[x_n^*(t) \cdot x_n^*(t+x)]\] (6.9)

Figure 6.7 shows the ensemble autocorrelation function of MOE calculated by the two methods described before. The two curves agree each other with small lag (\(<1\text{m})\) but shows some difference at a higher lag. It is clear from the figure that the autocorrelation calculated directly has more noise at higher lag. This is expected as each data point on the curve was determined by averaging the MOE values for a given lag and there is a lower number of data points on the higher lag.
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Figure 6.8 and 6.9 show the ensemble autocorrelation functions of within-specimen average and minimum MOE using the direct calculation method. Also shown in the figures are the fitted curve using Wang’s model as given in equation 6.6.

Knowing the parameters of the autocorrelation function, the corresponding power spectral density functions can be obtained from the Wiener-Khinchine transform pair as given in equation 6.5.

6.2.3 Simulations of MOE

For a one dimensional single variable stationary stochastic process \( x^*(t) \) with mean value equal to zero and one-sided power spectral density function \( G_{xx}(\omega) \), the process can be simulated by the following series

\[
x^*(t) = \sum_{n=0}^{N-1} A_n \cos(\omega_n t + \phi_n)
\]

where

\[
A_n = \sqrt{2G_{xx}(\omega_n)\Delta\omega}
\]

and \( \phi_n \) are independent random phase angles which are uniformly distributed over the interval \((0,2\pi)\). For bivariate and multivariate random process simulation, the procedures are given in Section 4.5.

To obtain the MOE process, we need to add the trend to the normalized stationary processes. Assuming the MOE process \( x(t) \) has a linear trend with the estimation of MOE denoted by \( \bar{x}(t) \) where

\[
\bar{x}(t) = b_1 + b_2 t
\]

then the normalized ergodic stationary random process \( x^*(t) \) is related to the MOE process by

\[
x(t) = \bar{x}(t) + x^*(t) \cdot \sigma_x
\]

where \( \sigma_x \) is the standard error of the linear trend of MOE.

From MOE test data, the ensemble mean \( \mu_b \) and covariance matrix \( \Sigma_b \) of the trend coefficients and standard error can be determined from the trend removing process and are given in Table 6.1. Know these coefficients, sets of random trend parameters and standard
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errors can be generated by the multivariate normal generation process as given in Wang et al. (1994).

Figure 6.10 to Figure 6.12 shows the cumulative probability distributions of within-board mean MOE, minimum MOE, and standard deviation of MOE. In each figure, cumulative probability functions of 100 simulated boards generated by different models are given together with the data MOE. Wang’s model 1 assumes that $\alpha$ and $B$; are constant in the spectral density function model, whereas Wang’s model 2 assumes they are random normal variables which are different from board to board. In both cases, $N=5$ was used in the simulation. Shinozuka’s model of power spectral density function with $N=5$ as given in equation 6.7 was also used to simulate the MOE process in which the cumulative probability function was determined. In addition, 100 boards of MOE profiles were also simulated from the power spectral density functions obtained directly from the Fast Fourier transform of the data.

Inspecting Figure 6.10, curves from different models agree very well with the data. Wang’s model 1 gives the best fit to the data, especially at the lower tail, whereas Wang’s model 2 gives slightly higher percentile values than the data for all levels. In general, Shinozuka’s model gives a reasonable fit to the data at all levels.

For the within-board minimum MOE, most models give slightly higher values at the lower tail but they all agree reasonably well with the data. The within-board minimum MOE value is the minimum value of all MOE values simulated within a board.

The cumulative probability functions of within-board standard deviation of MOE are given in Figure 6.12. Most models give a better fit to the data at lower tail than the upper end. Overall, they give reasonable estimates of the cumulative probability distribution of standard deviation of MOE.

6.3 Compact Compressive Strength (CCS)

Twenty-six boards were cut into 152 mm (6 inch) specimens and were tested in compression to determine the parallel to grain compressive strength. At each end of the board, 500 mm of wood was discarded; therefore, in total twenty-nine specimens were cut from each board, forming a total of 754 specimens. The details of the tests were given in Chapter 5.4.
6.3.1 CCS Test Results and Model Calibration

Assigning the compressive strength to the center location of the specimen, a profile consisting of 29 data points was formed for each board. Two sample records of compressive strength profile were shown in Figure 6.13 and 6.14 with the x-axis zero to the center location of the first specimen.

An apparent linear trend was observed in the data, but it may have a different sign for the slope for different boards. The within-board mean and a linear regression line were also shown in the figures. It is clear that a linear trend is sufficient to model the trend of the within-board compressive strength. The means and the covariance matrix of the linear trend model is given in Table 6.2.

Figure 6.15 shows the ensemble mean and standard deviation of the compression strength along the board. Even though the ensemble compressive strength and standard deviation do not show significant trends, the trend appears in individual records. Thus trend removing techniques were applied to individual records to normalize the data and convert them into ergodic stationary processes.

6.3.2 Spectral Representation of Compact Compressive Strength

The random field model has been applied successfully to model the within-member compressive strength (Wang et al. 1994). In this study, boards of \( L = 5.486 \text{ m} \) (18 ft.) were tested with specimen length \( (\Delta) \) of 152.4 mm (6 inch). This yielded a cutoff frequency \( (\omega_c) \) of 20.6 rad/m and \( \Delta\omega = 1.288 \text{ rad/m} \).

After the compressive strength profiles were normalized, Fast Fourier transform was applied to obtain the spectral representation of the compressive strength. Figure 6.16 shows the ensemble power spectral density functions with two smoothing resolutions \( (N=3 \text{ and } N = 5) \). Smoothing the spectrum has been proven to improve the accuracy of the spectrum estimates (Newland, 1984) in which the spectrum estimate is obtained by averaging adjacent estimates.

Wang et al.(1994) have shown that for small numbers of data points within a record the spectral density function can be estimated from its corresponding autocorrelation function, where the autocorrelation function can be calculated from the following formula...
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\[ R_{y_i y_i}(\tau) = E[y_i(t) y_i(t+\tau)] \]  \hspace{1cm} (6.14)

where \( E[\cdot] \) denotes expected value.

To simulate compressive strength profiles, models were fitted to the autocorrelation function data and the corresponding power spectral density function was determined and used for simulation. Figure 6.17 shows the ensemble autocorrelation of compact compressive strength data with the fitted curves of Wang’s model \((N=9\text{ and } 19)\) obtained by the least square minimization method. The equation of the autocorrelation function is given in equation 6.6. Apparently, the autocorrelation model with 9 terms seem to fit the data better.

Shinozuka’s model can also be applied to the autocorrelation. For the power spectral density model of

\[ G(\omega) = \sum_{i=1}^{N} \frac{1}{(2i)!} \sigma^2 b_i^{2i+1} \omega^{2i} e^{-b_i \omega} \]  \hspace{1cm} (6.15)

it has the following autocorrelation functions \( R_n(\tau) \) for different number of terms \( N \)

\[ R_1(\tau) = \frac{1}{N'} \sigma^2 b_1^4 \frac{(b_1^2-3\tau^2)}{(b_1^2+\tau^2)^3} \]

\[ R_2(\tau) = R_1(\tau) + \frac{\sigma^2}{N'} \left\{ b_2^6 \frac{(b_2^4-10b_2^2\tau^2+5\tau^4)}{(b_2^2+\tau^2)^5} \right\} \]  \hspace{1cm} (6.16)

\[ R_3(\tau) = R_2(\tau) + \frac{\sigma^2}{N'} \left\{ b_3^8 \frac{(b_3^6-21b_3^4\tau^2+35b_3^2\tau^4-7\tau^6)}{(b_3^2+\tau^2)^7} \right\} \]

where \( N' \) is the number of sample records.

Figure 6.18 shows the ensemble autocorrelation function fitted with Shinozuka’s model with \( N=1,2 \) and 3. It can be seen that Shinozuka’s model only fit to the data with lag less than 1 m.

After calibrating the model to fit the autocorrelation function, the power spectral density functions can be established from the obtained parameters using equations (6.5) and (6.15). Figure 6.19 shows the power spectral density functions of compressive strength of the test data and different models investigated. It seems that Wang’s model with \( N = 9 \) best fitted to the data.
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After obtaining the power spectral density functions of both MOE and compressive strength, the gain factor $|H_{y_i}(\omega)|$ can be determined from the following equation

$$G_{\alpha,\gamma}(\omega) = |H_{x_i}(\omega)|^2 \cdot G_{\alpha,\gamma}(\omega)$$

(6.17)

where $G_{\alpha,\gamma}(\omega)$ and $G_{\alpha,\gamma}(\omega)$ are the power spectral density functions of MOE and compressive strength respectively.

6.3.3 Simulations of Compressive Strength Profiles

Using the power spectral density of the data or the fitted models, profiles of compressive strength can be simulated following equations 6.10 to 6.13 using the spectral analysis approach coupled with the trend removal techniques.

One hundred boards were simulated and the within-board mean, minimum and standard deviation of the compressive strength were determined. Cumulative distributions of these statistics are shown in Figure 6.20 to 6.22.

Figure 6.20 shows the within-board mean compressive strength of the data and different models. As can be seen from the figures, both Wang’s models and Shinozuka’s model show remarkable fit to the data even though the power spectral density functions show significant difference. However, the cumulative probabilities of the within-board minimum compressive strength shows a difference for different models as can be depicted from Figure 6.21. In general, Shinozuka’s model gives smaller percentile values whereas Wang’s models show a better fit to the data. Cumulative probabilities of within-board standard deviation show more variation than the other two statistics. In Figure 6.22, Wang’s model with $N = 9$ gives the best fit to the data, while Shinozuka’s model gives much higher percentile values at all levels except for the lower tail.

6.4 Tensile Strength Modeling and Simulations

Modeling of the tensile strength profile is different from modeling the compressive strength or MOE as it is very difficult to get many test data points within a board. In order to maximize the number of data points within a board in the analysis, multiple failure tension
tests as described in Chapter 5.4 were performed on 69 boards. However, due to the requirement of minimum grip length and long failure section, a maximum of 5 breakages were achieved during the tests. This is much less than the minimum data points required to perform the Fourier Transform. As a result, a regression approach was used to simulate the tensile strength profiles from which spectral power density functions were derived.

6.4.1 Test Results of Tension Tests

There were a total of 69 boards. A minimum of three breakages were achieved for all boards. For fifty-six boards a fourth breakage could be obtained and only 6 boards could take the fifth breakage. The cumulative distributions of first, second, third, fourth and fifth breakage are shown in Figure 6.23 to 6.27. In all but seven cases the first breakage has a value higher than one of the other breakages, which implies that the boards may have suffered some damage during the first breakage.

6.4.2 Regression of Tensile Strength with Other Lumber Properties

Before we can simulate the tensile strength profiles, we need to characterize the within-member and between-member variations. Since direct measurement of the tensile strength profiles is not feasible, prediction of tensile strength profiles can only be done by studying the relationships with other known variables. Simulation of tensile strength from other lumber properties using a regression approach has been studied by numerous researchers including Woeste et al. (1979) and Taylor and Bender (1989). In order to see which variables are significantly related to tensile strength, multiple regression analyses were done on tensile strength with other variables at or near the failure section. Variables studied include average MOE, knot area ratio (KAR), margin knot area ratio (MKAR), top margin knot area ratio (TMKAR) and bottom margin knot area ratio (BMKAR) within one depth distance from the failure location. Forward selection and backward elimination methods were used to determine the relative significance of the variables at a significant level of 0.05.

Both forward selection and backward elimination methods produced the same set of variables. Table 6.3 summarizes the result of the analysis. MOE and KAR are the two most
important tensile strength predictors according to the regression analysis. The model coefficient of determination ($r^2$) is 0.4548 with the following regression equation is

$$\bar{T} = 0.3381 - 0.0276 \cdot MOE - 0.2093 \cdot KAR$$  \hspace{1cm} (6.18)

Looking at the partial $r^2$ shows that MOE (partial $r^2 = 0.4548$) is highly significant while KAR only contributes 0.017 to the model partial coefficient of determination. Plots of the tensile strength versus each independent variable are shown in Figure 6.28 and 6.29. Clearly, MOE has a more distinct trend with tensile strength than the KAR.

The above analyses were done when all the tensile failures were considered with a total of data points of 271. Similar analyses were done on just the first breakage tensile strength. The results are summarized in Table 6.4. Both forward selection and backward elimination methods selected the same set of variables — MOE and BMKAR. It yields the following regression formula

$$\bar{T} = -19.6261 + 4.3008 \cdot MOE - 5.0111 \cdot BMKAR$$  \hspace{1cm} (6.19)

The coefficient of determination of the model ($r^2 = 0.6068$) is larger than the previous analysis, indicating that the first breakage tensile strength is more reliable than the subsequent breakages. Figures 6.30 and 6.31 show the plots of tensile strength versus MOE and BMKAR respectively.

### 6.4.3 Simulations of Tensile Strength Profile

Once the regression model is characterized, prediction of tensile strength at a particular location along the board can be achieved by knowing the MOE profile and the knot area information near that location. Using the information collected for the tension test specimens (67 boards), we can simulate the tensile strength for each of these boards test in tension. The procedures are as follows:

1. Determine the predicted tensile strength profiles using equation 6.19 for a 2 mm interval using the Cook-Bolinder flatwise MOE measurements and knot information.
2. Add to each of these values a random error which is normal distributed around the predicted values.
3. Using a moving average approach to correlate the adjacent predicted values.
The random errors in step 2 are generated following the procedures for prediction of new observations given in Neter et al. (1990). For a given predicted value $\tilde{y}$, it has a variance of

$$s^2(\tilde{y}) = \text{MSE} \left[ 1 + X_h'(X'X)^{-1}X_h \right]$$  \hspace{1cm} (6.20)

where

$$X = \begin{bmatrix}
1 & x_{11} & x_{12} & \cdots & x_{1,p-1} \\
1 & x_{21} & x_{22} & \cdots & x_{2,p-1} \\
\vdots & \vdots & \vdots & & \vdots \\
1 & x_{n1} & x_{n2} & \cdots & x_{n,p-1}
\end{bmatrix} \quad X_h = \begin{bmatrix}
1 \\
x_{h1} \\
x_{h2} \\
\vdots \\
x_{h,p-1}
\end{bmatrix}$$  \hspace{1cm} (6.21)

$X$ is the data set of the independent variables with $n$ data points and $p$ is the degree of the polynomial of the multiple regression; $X_h$ is the set of independent variables for which we want to predict the new observation; and $\text{MSE}$ is the mean square error.

As a random error was added to each simulated tensile strength value, adjacent predicted values were independent, whereas in reality they should be correlated. Moving average techniques were used here to correlate the adjacent predicted values. The weight function used in this analysis is given in Figure 6.32. The function has the shape of a trapezoid with ramps up to quarter point of the window at both ends. Two sizes of window, $n = 10$ and $n = 25$ have been investigated and a sample of simulated tensile strength profile is given in Figure 6.33. In general, the larger the window size, the less variation is expected for the simulated profiles. The window size was optimized to give the best fit of the first breakage tensile strength. The first breakage tensile strength of the simulated profile is defined as the minimum strength value within the board. It has been found with $n = 10$ gives the best fit to the data as shown in Figure 6.23. Second and subsequent breakage tensile strengths were also estimated by finding the minimum predicted value of the simulated profile within the test span of subsequent tests. The results of the cumulative distributions of the predicted second and subsequent breakage tensile strength are given in Figure 6.24 to 6.27. As observed from the figures, the predicted curves fit reasonably well with the data at lower tails but fall below it at higher levels.
6.4.4 Spectral Representations of Tensile Strength

Spectral representations of these simulated tensile profiles can be done by using the fast Fourier transform as given in previous sections. Since the compression strengths were determined with specimen lengths of 152.4 mm (6 inch), it is advantageous to have the same interval for the tensile strength profiles. The tensile strength profiles were originally simulated from the MOE profiles with an interval of 2 mm. To determine the tensile strength profile for 152.4 mm (6 inch), we divided the simulated boards into 6 inch sections and determined the minimum value within each section. Using these 6 inch interval tensile strength values, power spectral density function was established and is shown schematically in Figure 6.34. A zero order trend was assumed for the tensile strength profiles as a higher order trend could not be verified due to insufficient information.

Based on the power spectral density functions and the zero order trend, 100 boards of tensile strength profile were simulated and the cumulative distribution of the within-board minimum tensile strength was determined and is shown in Figure 6.35. It is apparent that it fits well with the data.

6.5 Stress-Strain Relationship

During the compact compression tests, the applied load and the cross-head displacement were recorded until failure. From these data, stress-strain curves were established. A few examples of the plot of compressive stress versus compressive strain are shown in Figures 6.36 to 6.39. The stress-strain curves remained linear until the specimen started to yield. From these, it behaved nonlinearly and the slope decreased and dropped to zero at maximum stress. Beyond that point, the stress decreased as the strain increased, and it is expected that the stress will approach zero as strain approaches infinity.

A proposed model of the stress-strain relationship is shown in Figure 6.40 which has the following form

$$\sigma = \frac{\varepsilon_1 + G_1 (\varepsilon / \varepsilon_1)^\nu}{G_2 + G_3 (\varepsilon / \varepsilon_1)^\nu} \quad (6.22)$$
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where $\sigma$ is the compressive stress; $\epsilon$ is the compressive strain; $\epsilon_1$ is the strain at maximum stress (compressive yield strain); $f_c$ is the maximum stress; and $E$ is the modulus of elasticity.

The proposed model has the following properties:

1. $\sigma = 0$ when $\epsilon = 0$.
2. $\sigma \to 0$ as $\epsilon \to \infty$.
3. $\frac{d\sigma}{d\epsilon} = E$ when $\epsilon = 0$.
4. $\frac{d\sigma}{d\epsilon} = 0$ when $\epsilon = \epsilon_1$.

This yields the parameters $G_1$, $G_2$ and $G_3$ in terms of the experimental determined constants

$$G_1 = \left( \frac{n}{n-m} \right) \left( \frac{f_c}{\epsilon_1 E} + \frac{1}{n} - 1 \right)$$

$$G_2 = \frac{1}{\epsilon_1 E}$$

$$G_3 = \frac{1+mG_1}{nf_c}$$

Figures 6.36 to 6.39 also show the fitted curves of the proposed model. The curves were calculated using equation 6.22 with $f_c$, $\epsilon_1$ and $E$ determined from the curves; $m$ and $n$ were determined from minimizing the residual squares between the data and the model using non-linear optimization techniques. There are two methods for determining the parameters $m$ and $n$ — either obtaining a set of $m$ and $n$ for each specimen or one set of $m$ and $n$ for all the specimens. The latter method was used in subsequent analyses. The results of both methods are shown in Table 6.5.

6.6 Compressive yield strain

The strain at maximum compressive stress (denoted as compressive yield strain) is a random variable which varies from board to board and also along the board. As the stress-strain relationship is required in the finite element analysis, the yield strain was also modeled as a random variable and is one of the outputs ($y_3$) in the multiple input-multiple output model. As the compressive yield strains were collected during the compact compression tests, a value of
compressive yield strain was determined for each 152.4 mm (6 inch) specimen. Plots of the cumulative probability distribution of the within-board mean, minimum, and standard deviation compressive yield strains are given in Figures 6.41 to 6.43 respectively.

From the compressive yield strain profiles, ergodic, stationary processes were determined by using the linear trend removing techniques and the normalization procedures (details given in Chapter 4.3). Both the normal distribution normalization procedures and the Weibull distribution normalization were studied in this exercise. After the data were normalized, FFT was used to obtain the power spectral density function. Then 100 boards of compressive yield strain profile were simulated and the boards’ statistics were shown in Figure 6.41 to 6.43 for both normalization procedures.

From Figure 6.41, it is noted that the 3-P Weibull, 2-P Weibull and the Normal distribution normalization procedures produced reasonable results and the simulated cumulative distributions fitted the data very well. However, investigating the within-member minimum compressive yield strain cumulative distribution curves (Figure 6.42) reveal that both the 2-P Weibull and Normal distribution procedures are inappropriate to simulate the within-member compressive yield strain, while the 3-P Weibull normalization procedures produce cumulative distribution that fits the data well.

Figure 6.43 shows the cumulative probability of the within-board standard deviation of the compressive yield strain of the data and the simulated boards. It is apparent that the normal distribution normalization procedure is the best in this aspect, whereas the 3-P Weibull underestimates the standard deviation at all levels.

The data curves do not show much information in the lower tail as the sample size of the board statistics is only 26. If more boards had been available for this investigation, a better comparison could have been made, since the within-board mean and minimum compressive yield strain are more critical than the standard deviation in the finite element analysis. The 3-P Weibull normalization was adopted to simulate the profiles in subsequent analyses.
6.7 Cross Spectrum and Coherence Functions Evaluations

Consider the cross-spectral density function between two stationary (ergodic) Gaussian random processes $x(t)$ and $y(t)$. Specifically, given a pair of sample records $x(t)$ and $y(t)$ of unlimited length $T$, the one-sided cross-spectrum is given by

$$G_{xy}(f) = \lim_{T \to \infty} \frac{2}{T} \mathbb{E}[X^*(f,T)Y_i(f,T)]$$

(6.24)

where $X(f,T)$ and $Y_i(f,T)$ are the finite Fourier transforms of $x(t)$ and $y_i(t)$, respectively — that is

$$X(f) = X(f,T) = \int_0^T x(t)e^{-j2\pi ft} dt$$

(6.25)

$$Y_i(f) = Y_i(f,T) = \int_0^T y_i(t)e^{-j2\pi ft} dt$$

(6.25)

It follows that a preliminary estimate of the cross-spectrum for a finite record length $T$ is given by

$$\tilde{G}_{xy}(f) = \frac{2}{T} \mathbb{E}[X^*(f)Y_i(f)]$$

(6.26)

which has a resolution bandwidth of

$$B_e \approx \Delta f = 1/T$$

(6.27)

meaning that spectral components are only estimated at the discrete frequencies

$$f_k = k/T \quad k = 0,1,2,...$$

(6.28)

In practice, in order to increase accuracy, the ensemble cross spectrum was determined from $n_d$ different records, each of length $T$, and averaging the results to obtain a smooth estimate of the cross-spectrum

$$\hat{G}_{xy}(f) = \frac{2}{n_d T} \sum_{k=1}^{n_d} X_k^*(f)Y_{ik}(f)$$

(6.29)

The auto-spectrum can also be determined readily with minor changes to the formula.
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\[ \hat{G}_{xx}(f) = \frac{2}{n_d T} \sum_{k=1}^{n_d} X_k^*(f) X_k(f) \]  
\[ \hat{G}_{yy}(f) = \frac{2}{n_d T} \sum_{k=1}^{n_d} Y_k^*(f) Y_k(f) \]  

(6.30)

After determination of the auto- and cross-spectra, the transfer functions can also be determined by

\[ \hat{H}_i(f) = \frac{\hat{G}_{yi}(f)}{\hat{G}_{xx}(f)} \]  

(6.31)

where \( \hat{H}_i(f) \) is the transfer function of the input variable \( x \) and the output variable \( y_i \) as shown in Figure 4.2. The absolute value of \( \hat{H}_i(f) \) is commonly known as the gain factor. The gain factors \( |\hat{H}_i(f)| \) of MOE and three output variables — compressive strength, tensile strength, and compressive yield strain shown in Figures 6.44, 6.45 and 6.46 respectively.

Coherence functions can also be estimated by the following equation

\[ \hat{p}_{xy}^2(f) = \frac{\left| \hat{G}_{xy}(f) \right|^2}{\hat{G}_{xx}(f) \hat{G}_{yy}(f)} \]  

(6.32)

Figure 6.47 shows the coherence function of MOE and compressive strength. Two smoothing schemes were investigated. From the figure, it is seen that MOE and compressive strengths displayed a relatively strong coherence of 0.55 over the frequency range from about 1 to about 12 rad/m. Above this range, the coherence function decreases. Comparing the two curves with different numbers of smooth points, the curve with \( N=5 \) is smoother and it has slightly smaller values than the curve with \( N=3 \).

The coherence function of flatwise MOE and tensile strength is depicted in Figure 6.48. It shows an average coherence of around 0.3 to 0.4. It remained relatively constant in the studied frequency range. Similar to the coherence curve of MOE and compressive strength, the curve with the number of smooth points equal to 5 is smoother and has slightly smaller values.

Figure 6.49 gives the coherence function of MOE with compressive yield strain. It shows a decrease of coherency as the frequency increases. The average coherence is around
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0.45 and the behavior of the two curves with different numbers of smooth points is similar to
the previous two figures.

The coherence function is an indication of the contribution of individual input
variables to the output variable. It has a value between zero and one. If the input and output
variables are highly correlated, their coherence has a value close to 1 and vice versa. From
this exercise, it was found that MOE has the largest average value of coherence function (\( \gamma^2 =
0.55 \)) with compressive strength indicating that MOE is more closely correlated to
compressive strength than to other output variables.
### Table 6-1 Mean and Covariance Matrix of Trend Parameters of MOE

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<th>Parameter</th>
<th>$b_0$</th>
<th>$b_1$</th>
<th>$\sigma$</th>
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</thead>
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<tr>
<td>Mean ($\mu_b$)</td>
<td>1.10E+01</td>
<td>-1.02E-04</td>
<td>5.50E-01</td>
</tr>
<tr>
<td>Covariance ($\Sigma_b$)</td>
<td>1.2040E+00</td>
<td>-1.4832E-04</td>
<td>4.9748E-02</td>
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<tr>
<td></td>
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<td></td>
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<td>-1.6788E-05</td>
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### Table 6-2 Mean and Covariance Matrix of Trend Parameters of Compression Strength

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<tr>
<th>Parameter</th>
<th>$b_0$</th>
<th>$b_1$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean ($\mu_b$)</td>
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<td>-0.00013</td>
<td>3.4785</td>
</tr>
<tr>
<td>Covariance ($\Sigma_b$)</td>
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<td>2.2345</td>
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<td></td>
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<td>1.08E-06</td>
<td>-0.00032</td>
</tr>
<tr>
<td></td>
<td>2.2345</td>
<td>-0.00032</td>
<td>1.4050</td>
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### Table 6-3  Regression of Tensile Strength with Non-Destructive Lumber Properties (All breakages included)

<table>
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<tr>
<th></th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F</th>
<th>Prob&gt;F</th>
</tr>
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<tbody>
<tr>
<td>Regression</td>
<td>2</td>
<td>9822</td>
<td>4911</td>
<td>119.71</td>
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</tr>
<tr>
<td>Error</td>
<td>268</td>
<td>10995</td>
<td>41.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>270</td>
<td>20818</td>
<td></td>
<td></td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter Estimate</th>
<th>Standard Error</th>
<th>Type II Sum of Squares</th>
<th>F</th>
<th>Prob&gt;F</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERCEP</td>
<td>-14.7438</td>
<td>3.7246</td>
<td>642.88</td>
<td>15.67</td>
<td>0.0001</td>
</tr>
<tr>
<td>MOE</td>
<td>4.4015</td>
<td>0.3134</td>
<td>8093.24</td>
<td>197.26</td>
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<tr>
<td>KAR</td>
<td>-15.4564</td>
<td>5.2583</td>
<td>354.49</td>
<td>8.64</td>
<td>0.0036</td>
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</table>

<table>
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<tr>
<th>Variable Entered</th>
<th>Number In</th>
<th>Partial $r^2$</th>
<th>Model $r^2$</th>
<th>F</th>
<th>Prob&gt;F</th>
</tr>
</thead>
<tbody>
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<td>MOE</td>
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<td>0.4548</td>
<td>0.4548</td>
<td>224.40</td>
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<td>KAR</td>
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<td>0.0170</td>
<td>0.4718</td>
<td>8.6402</td>
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### Table 6-4  Regression of Tensile Strength with Non-Destructive Lumber Properties (First breakage only)

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<tr>
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<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F</th>
<th>Prob&gt;F</th>
</tr>
</thead>
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<tr>
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<td>2158</td>
<td>1079</td>
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<tr>
<td>Error</td>
<td>66</td>
<td>1388</td>
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<td>Total</td>
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<td>3546</td>
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<table>
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<th>Parameter Estimate</th>
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<th>Type II Sum of Squares</th>
<th>F</th>
<th>Prob&gt;F</th>
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<tr>
<td>MOE</td>
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<td>0.4366</td>
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<tr>
<td>BMKAR</td>
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<td>2.1463</td>
<td>114.65</td>
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</table>

<table>
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<tr>
<th>Variable Entered</th>
<th>Number In</th>
<th>Partial $r^2$</th>
<th>Model $r^2$</th>
<th>F</th>
<th>Prob&gt;F</th>
</tr>
</thead>
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<td>0.5762</td>
<td>91.0994</td>
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</tr>
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<td>BMKAR</td>
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<td>0.0323</td>
<td>0.6085</td>
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### Table 6-5 Stress-Strain Model parameters

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<th>Parameters fit to individual board</th>
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<th>n</th>
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<td>Mean</td>
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<td>2.6805</td>
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<tr>
<td>Standard Deviation</td>
<td>0.5594</td>
<td>0.5259</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters fit to all boards</th>
<th>m</th>
<th>n</th>
</tr>
</thead>
<tbody>
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<td>Parameters</td>
<td>1.5790</td>
<td>2.3300</td>
</tr>
</tbody>
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Figure 6.1  Cook-Bolinder flatwise MOE profile

Figure 6.2  Ensemble mean and standard deviation of flatwise MOE
Chapter 6  Experimental Results and Model Calibrations

Figure 6.3  Ensemble normalized MOE mean and standard deviation

Figure 6.4  Ensemble power spectral density of flatwise MOE
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Figure 6.5 Ensemble power spectral density function with Shinozuka's model

Figure 6.6 Ensemble power spectral density of flatwise MOE (resolution study)
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Figure 6.7  A sample of autocorrelation function of flatwise MOE

Figure 6.8  Ensemble autocorrelation function of flatwise MOE (Wang's model using within-specimen average MOE)
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Figure 6.9  Ensemble autocorrelation function of flatwise MOE (Wang's model using within-specimen minimum MOE)

Figure 6.10  Cumulative probability of within-board mean flatwise MOE
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Figure 6.11  Cumulative probability of within-board minimum flatwise MOE

Figure 6.12  Cumulative probability of within-board standard deviation of flatwise MOE

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Figure 6.13   Compact compression strength profile with regression line (Specimen No. 1803410f)

Figure 6.14   Compact compression strength profile with regression line (Specimen No. 1803420f)
Figure 6.15  Ensemble compact compression strength profile

Figure 6.16  Power spectral density function of compression strength
Figure 6.17  Ensemble autocorrelation function of compression strength with Wang's Model fitted

Figure 6.18  Ensemble autocorrelation function of compression strength with Shinozuka's model fitted
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Figure 6.19 Power spectral density functions of compression strength models

Figure 6.20 Cumulative probability of within-board mean compression strength
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Figure 6.21  Cumulative probability of within-board minimum compression strength

Figure 6.22  Cumulative probability of within-board standard deviation of compression strength
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Figure 6.23  Cumulative probability of within-board minimum tensile strength (first breakage tensile strength)

Figure 6.24  Cumulative probability of second breakage tensile strength
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Figure 6.25  Cumulative probability of third breakage tensile strength

Figure 6.26  Cumulative probability of fourth breakage tensile strength
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Figure 6.29  Plot of tensile strength versus knot area ratio

Figure 6.30  Plot of within-board minimum tensile strength versus Cook Bolinder flatwise MOE
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Figure 6.31  Plot of within-board minimum tensile strength versus bottom marginal knot area ratio

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Figure 6.34  Generated power spectral density function of tensile strength

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Figure 6.35  Cumulative probabilities of data and simulated tensile strength

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Figure 6.37  Plot of compressive stress versus compressive strain (Spec. No. 37707)

Figure 6.38  Plot of compressive stress versus compressive strain (Spec. No. 35820)
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Figure 6.42 Cumulative probability of within-board minimum compressive yield strain
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Figure 6.43  Cumulative probability of within-board standard deviation of compressive yield strain

Figure 6.44  Gain factor of flatwise MOE and compressive strength
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Figure 6.45  Gain factor of flatwise MOE and tensile strength

Figure 6.46  Gain factor of flatwise MOE and compressive yield strain
Figure 6.47  Coherence function ($\gamma^2$) of flatwise MOE and compressive strength

Figure 6.48  Coherence function ($\gamma^2$) of flatwise MOE and tensile strength
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Chapter 7
Model Verifications

7.1 Introduction

To verify the ability of the strength model to predict the load-capacity of beam-columns, simulation results were compared with experimental data. These comparisons considered two aspects — by comparisons with the statistics of the original and model predicted lumber properties data, and by comparisons with the predicted model and test results of full-size members.

Simulations of the lumber property profiles were achieved by the random generation of correlated ergodic stationary profiles using the single input-multiple output model as outlined in Section 4.4 and by adding trends to these profiles as depicted in Section 4.3.

7.2 Comparisons of Cumulative Probability Distributions

To confirm the stochastic characteristics of the lumber properties information of the model, cumulative probability distributions of the within-board mean, minimum, and standard deviation were compared between the experimental and simulated profiles. Four lumber properties were evaluated, including Cook-Bolinder flatwise MOE, compression strength, tensile strength, and compressive yield strain. In the case of tensile strength, only the cumulative probability distribution of the within-board minimum value was compared because the physical limitation of the number of breakages per board prevented other statistics from being obtained directly from the experiments.

7.2.1 Flatwise MOE

The cumulative probability distributions of within-board mean flatwise MOE from the Cook Bolinder evaluations and the simulated results are presented in Figure 7.1. The simulation profiles are based on a sample size of 100 boards. Linear trend and five smoothing
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points in the power spectral density function averaging were also used. The simulations are in very good agreement with the Cook-Bolinder flatwise MOE data, especially at probability levels between 0.20 and 0.70.

Figure 7.2 shows the cumulative probability distributions of the within-board minimum from the Cook-Bolinder tests and simulations; visual inspection confirms good agreement between the two distributions, especially in the lower tail with a probability level less than 0.50.

The cumulative probability distributions of the within-board standard deviation of the flatwise MOE are depicted in Figure 7.3. It can be observed from Fig. 7.3 that, for a probability level less than 0.5, the simulation underestimates the within-board standard deviation of flatwise MOE, whereas above 0.5, it overestimates the values. However, the deviations are relatively small and, in general, the two cumulative probability distributions are in agreement.

7.2.2 Compressive Strength

The compressive strength model was calibrated with the compact compression test data. A linear trend was generated and added to the ergodic stationary process simulated using the single-input multiple-output model. One hundred replications with data point intervals of 152.4 mm (6 inch) were simulated. The number of smoothing points used in averaging the power spectral density function during the calibration stage was five. Cumulative probability distributions of the within-board mean compressive strength are shown in Figure 7.4. From Fig. 7.4 it can be seen that the simulation results fit the data very well over all probability levels.

Cumulative probability distributions of the within-board minimum compressive strength are given in Figure 7.5. In general, the simulation results tend to underestimate the within-member compressive strength slightly, but overall the results are in good agreement.

Cumulative probability distributions of the within-board standard deviation of compressive strength are depicted in Figure 7.6. The simulation results slightly overestimate the standard deviation of compressive strength. However, based on the small number of sample size per board in the test data (N = 29), it is a very good fit.
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7.2.3 Tensile Strength

Verification of the tensile strength model is achieved by regenerating the cumulative probability distribution of the within-board minimum tensile strength. This is different from other lumber properties for which the model was calibrated from compact specimen values. The tensile strength model is regression based and calibrated using the cumulative probability distribution of the within-board minimum tensile strength and the MOE profiles. Figure 7.7 shows the cumulative probability distributions of within-board minimum tensile strength from the experimental tests and the simulation results. As can be observed from the figure, the two cumulative probability distributions are in good agreement over all probability levels. A sample size of 100 boards was used in the simulation.

7.2.4 Compressive Yield Strain

The compressive yield strain profiles were simulated using the single input-multiple output model coupled with the Weibull distribution transformation techniques. Details of the transformation were given in Section 4.3.2. The sample size of the simulation was 100.

The cumulative probability distributions of the within-board mean compressive yield strain are presented in Figure 7.8. As can be seen from the figure, the simulation results fit the data very well. Cumulative probability distribution of the within-board minimum compressive yield strain is shown in Figure 7.9. The simulated cumulative probability distribution underestimates the within-board minimum compressive yield strain for a probability level less than 0.3 but overestimates it for a probability level greater than 0.4. However, considering that only 29 data points per board were used to create the experimental cumulative probability distribution, the simulated results are acceptable. Figure 7.10 shows the cumulative probability distribution of the within-board standard deviation of compressive yield strain from the test data and from the simulations. The simulated cumulative probability distribution has lower values than the test data for all probability levels. This indicates that the within-board variation of the simulated profiles is smaller than the test data profiles. This may be explained by the fact that frequencies higher than 20 rad/m could not be obtained due to the small number of data points per board (N=29). This results in the simulated profiles...
having the higher frequency components being ignored which would lead to a reduction in the standard deviation of the profiles.

### 7.3 Coherence Function Verifications

Besides possessing the stochastic characteristics of each lumber property of the data, the developed strength model should also retain the correlation relationship between these lumber properties. Using a sample size of 100 and 5 smoothing points, profiles were simulated and the coherence functions between lumber properties in the normalized space were generated and studied. Figure 7.11 shows the coherence functions \( \gamma^2 \) of MOE and compressive strength from the test data and from the simulation results. As can be seen from the figure, the simulated coherence function has slightly less variability than the test data. Nevertheless, they are in good agreement.

The coherence functions between MOE and tensile strength and between MOE and compressive yield strain are depicted in Figure 7.12 and 7.13, respectively. The simulated coherence functions agree closely with the coherence functions of the data.

### 7.4 Full Size Compression, UTS and MOR Tests

Besides re-generating the statistics of the test data from which the model was derived, the strength model could also be verified by comparing predicted strength values with test results of other tests. Full-size tests that evaluate three elementary structural properties — Compression, Tension, and Modulus of Rupture (MOR) — were conducted and verifications compared the predicted strength values from the model and the test results. The details of these tests were given in Chapter 5.

#### 7.4.1 Full Size Compression Test

The full size compression tests consisted of boards, which were tested under pure compression with lateral support in both the major and minor axes. One hundred pieces of 2\( \times \)4 5.486 m (18 feet) long boards were randomly selected; from these pieces, 200 specimens were obtained with two equal length specimens cut from each board. The specimens were
labeled with the board number, suffixed with "A" and "B", corresponding to the two specimens in each board.

In predicting the compression strength, stationary ergodic profiles were first simulated using the single input-multiple output model as described before. Then the profiles were transformed back to true structural profiles by adding the trends to reverse the normalization procedures. The profiles were then divided into two to give the profiles for Specimens "A" and "B", which were then used as input material properties in the finite element analyses to predict the ultimate load of the member. For compression tests, failures were considered to have occurred if 1) the applied compressive stress exceeded the compressive strength at one or more of the Gaussian points or the top or bottom extreme fiber of the member, or 2) if the compressive strain at any Gaussian point or top or bottom extreme fiber exceeded 0.02. The ultimate strength was determined by dividing the ultimate load by the cross-sectional area

\[
\sigma = \frac{Ultimate\ Load}{Area} \quad (7.1)
\]

Six finite elements with 25 (5×5) Gaussian points per element were adopted for the analyses.

Cumulative probability distributions of the compressive strength of the "A" specimens and the "B" specimens are shown in Figures 7.14 and 7.15 respectively. As can be seen from the figures, although the simulated cumulative probability distribution underestimates the compressive strength slightly between the probability levels of 0.15 to 0.80, it fits the data very well in general. If both "A" and "B" specimens were pooled together, the cumulative probability distributions are given in Figure 7.16. As can be observed, the two cumulative probability distributions are in good agreement.

Statistics of the data and simulated compressive strength are presented in Table 7.1. From the table, there is not much difference between the statistics of "A" specimens and "B" specimens and also the pooled data set. The correlation coefficient of "A" specimens and "B" specimens is 0.25, whereas the strength model prediction is 0.22. The difference in the predicted and actual 5\textsuperscript{th} percentile values of all tested specimens is 3.39\%. 

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7.4.2 Full Size Tension Test

Data for the full size tension tests were readily available by considering only the first breakage of the multiple failures tension test (Test A2) as described in Section 5.4. Comparisons of the cumulative probability distributions obtained from the data and simulated profiles were discussed in Section 6.4.4 and were shown schematically in Figure 6.35. It is apparent that the simulations produce remarkable agreement with test data.

7.4.3 Full Size MOR Test

It would be beneficial to test whether the strength model could predict ultimate values for tests, which are different in nature from the tests from which the model was derived. In this exercise, the results of full-size third-point bending tests were compared with the simulated results. Details of the experiment setup and procedures were given in Section 5.5.1.

Similar to the full-size compression test, the boards were cut into two sections to produce two specimens per board (labeled as Specimen “A” and “B”). One hundred boards were randomly selected, and from these boards, two hundred specimens were prepared and tested.

The modulus of rupture (MOR) was determined by

\[ \text{MOR} = \frac{M_{\text{max}}}{S} \]  

(7.2)

where \( M_{\text{max}} \) is the maximum moment and \( S \) is the section modulus.

For the simulated profiles, the ultimate load was obtained by increasing the applied load until either 1) the tensile strength was exceeded, 2) the compressive strength was exceeded, or 3) the compressive strain exceeded 0.02 at any Gaussian point or extreme fiber.

As the lumber properties were derived based on tests during which the members were under uniform stress across the section, the stresses calculated from the finite element program have to be corrected for depth effect before comparing them with the test data. Depth effect was derived from Eqn. 2.13, assuming a shape factor of 10 for compression and 6 for tension, which are consistent with the in-grade database.

Twelve beam elements with 25 Gaussian points per element were used in the finite element analyses and 100 boards were simulated.
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Cumulative probability distributions for Specimen “A”, “B” and “All” are depicted in Figures 7.17, 7.18 and 7.19 respectively. As can be seen from the figures, they show good agreement between the simulated boards and test data. A summary of the statistics of both the test data and simulated boards are shown in Table 7.2. Note that the correlation coefficient between Specimen “A” and “B” of the simulated boards has a value of 0.2982, which is comparable with the test results ($r^2 = 0.2437$).

7.5 Eccentric Compression Tests

As described previously, two hundred boards were tested at four different eccentricities, with 50 boards in each sample. The eccentricities were 0 mm, 5 mm, 75 mm and 200 mm. These boards were laterally supported along the full length of the board about the weak axis so that buckling about the weak axis was prevented.

There are several ways to present the results of the test. One way is by plotting the interaction of the applied axial load versus the mid-span moment. The mid-span moment was calculated as the product of the axial load and the maximum deflection from the line of thrust. During the experiments, LVDTs were installed at four locations to measure the deflections. Using these four deflection values, a sinusoidal curve was fitted. The maximum deflection was calculated accordingly.

Plots of the applied axial load versus mid-span moment at failure from the eccentric beam tests, third-point bending tests, and pure compression tests are given in Figure 7.20. The third-point bending MOR test values were converted to the case of constant moment test values by applying the load configuration factor given in Table 2-3 and are shown as data points along the moment axis (x-axis). Results from the full length compression tests with lateral support in both the major and minor axes were also shown in the figure along the axial load axis (y-axis). Each point on the curve represents the combination of axial load and mid-span moment at each failure. For each end eccentricity, the data points show some scatter and deviation from a radial line because the measured deflection under load is different in each board. In order to calculate the percentile values of each data set, the data points were normalized into a radial line. The normalization procedures include first determining the mean of mid-span moment and axial load. Use this mean value, a radial line was determined.
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so that all the data points would fall on this line. For each point on the plot, the distance from
the origin was determined. Maintaining this distance, the point was normalized such that it
fell on the radial line with the same distance from the origin as the unnormalized situation.
Plotting of the normalized axial load versus mid-span moment is shown in Figure 7.21. For
each normalized eccentricity data set, non-parametric 5th, 50th and 95th percentile values were
determined based on the distance of the point from the origin. Figure 7.22 shows the
percentile values of the various tests with linear and parabolic interaction trends. As can be
observed from the figure, the linear trend fits the data better except at low axial loads and 5th
percentile level. The current design code adopted both a linear and quadratic interaction
equations depend on the application of the member. The quadratic interaction equation has
the form of

\[
\left( \frac{P}{P_0} \right)^2 + \frac{M}{M_0} = 1 \quad (7.3)
\]

where \( P_0 \) and \( M_0 \) are the failure load under pure axial load and pure moment respectively.

It can be observed from the figure that the curves corresponding to equation (7.3) are
overestimating when compared to the data, except for the 5th percentile curve at low axial
load levels. This indicates that equation (7.3) is inadequate to model the trend of the
interaction relationship.

The statistics of the interaction relationship from the eccentric compression tests, pure
compression tests and third-point MOR tests are shown in Tables 7.3, 7.4 and 7.5
respectively. The corresponding statistics obtained from simulation are shown in Tables 7.6,
7.7 and 7.8.

Using the strength model as developed in previous chapters, the interaction curves of
axial load and mid-span moment were generated by simulating virtual eccentric compression
tests and constant moment tests. The eccentric compression test is exactly the same testing
configuration as Test B3, whereas constant moment tests are members with the same length
tested in pure constant moment along the member. The results of the interaction of the failure
loads are shown in Figure 7.23. When compared to the test data as shown in Figure 7.20, the
simulated members with no eccentricity would fail at slightly lower axial loads compared to
the actual test results. As can be observed from the figure, the data points show an average
axial load around 45 kN. An explanation of this behaviour is that 45 kN is the approximate buckling load of the member under compression or small eccentric compression. The program assumed that the member would fail once this buckling load is reached. In reality, the tested members may not fail immediately at this buckling load due to various reasons. One reason is that some members may have inherent bows or twists that act as arches, which help to prevent the members from buckling. In addition, once the member starts bending, the edges of the grips might have touched the members, which reduced the effective length of the members. Also, the test members might not be perfectly simply supported as there was friction between the lateral support rollers and the members. This may help in preventing the members from failing at the buckling loads.

A plot of these simulated points with the fitted curves of the test data percentile values are presented in Figure 7.24. The test data of the pure compression test under full lateral support are also shown for reference purposes. From now on, these pure compression test data points are referred to as material compression strength.

A comparison of the percentile values of the simulated boards and the test data are shown in Figure 7.25. The simulated data show a mean buckling failure load of 44.92 kN. A higher failure axial load can only be reached if additional restraint is available to the buckling axis or shorter members are considered. The simulated members show good agreement with the test results at median level but have less variation than the test data. This is indicated by the higher 5\textsuperscript{th} percentile values and lower 95\textsuperscript{th} percentile values. In addition, at the 5\textsuperscript{th} percentile level, the simulated data shows an increase of failure moment as the axial load increases. This behaviour does not exist in the 50\textsuperscript{th} percentile and the 95\textsuperscript{th} percentile values. This behaviour of increased axial load as the applied moment increases has also been reported by other researchers. Buchanan (1984) has explained this as a transition point of compression failure to tension failure for weak members.

7.6 Combined Axial and Lateral Loading Tests

As described in Section 5.5.4 and 5.5.5, testing was carried out in single span and double span members, with a prescribed axial compression applied first, followed by increments of lateral load until failure.
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7.6.1  Single Span Test

For the single span tests, the prescribed axial load levels were 4, 8, 12, 25 and 40 kN. Fifty specimens were tested for the 4, 8 and 12 kN load cases, and twenty-five specimens for the 25 and 40 kN load cases. Axial load was applied in increments until the prescribed load level was reached, then the lateral load was applied and increased in a displacement control mode until failure, while keeping the axial load at the prescribed level. Deflection along the member was measured by several LVDTs so that the maximum deflection could be calculated. A plot of the interaction of axial load and mid-span moment at failure is shown in Figure 7.26. Each point in the figure represents the failure load combination of a member. Theoretically, the points should be in bands of horizontal lines. However, this behaviour can only be observed at low prescribed axial load levels. The axial loads also show some scatter for the 25kN and 40 kN axial load cases. This can be explained by the fact that for lower axial load testing, the members failed mostly in a material failure mode, while at higher axial load level, the members failed in stability mode or a combined material and stability mode. As a member would begin to fail in a stability mode, a large lateral deflection would result. In this situation, the axial load might not be fully maintained at all times. Thus the axial load at failure may be somewhat higher or lower than the prescribed level (in most cases it would be lower).

Another interesting aspect of Figure 7.26 is that for the prescribed axial load level of 40 kN, the data points show an increasing curve up to about 40 kN. The explanation of this behaviour is that as the members were loaded up to 40 kN, some members failed in buckling before the prescribed axial load level of 40 kN was reached. For those members that did not fail after the targeted axial load was attained, they were failed by the imposition of the lateral load and are represented by a point along the 40 kN line in the figure. Therefore, the axial load level of 40 kN shows two distinct data grouping representing two failure modes. As a result, the percentile values for this group was not included in the further study.

Figure 7.27 shows the 5th, 50th and 95th percentile values of each test group. Also shown is the constant moment and pure compression material test statistics. The three fitted lines show the trends of the interaction behaviour of the data points. As can be seen from the figure, the moment capacity at the 5th percentile level increases as the axial compression is
increased from zero, and then decreases to zero at higher levels of axial compression load. This “nose” trend was more prevalent here than for the eccentric compression tests.

Using the developed strength model, the capacity of a beam-column for different axial load and moment combinations is shown in Figure 7.28. A sample size of 100 boards was used for each group. The experimental test data are also shown in the figure for comparison and are represented by hollow symbols, whereas the simulated results are shown in solid symbols.

It can be observed that the simulated results for the low axial load levels agree with the test data very well. At higher axial load levels, the results show more variability and deviation compared to the test results, due to the large deflection during stability failures. For the 40 kN load level, the simulated results show two data regions corresponding to two failure modes. According to Euler’s formula, a column is assumed to fail in buckling once the Euler’s buckling load is reached. At the same time, the lateral deflection will go to infinity. Hence in the axial load–lateral moment plot, the failure would be represented by a point along the axial load axis. On the other hand, some deflections might have occurred prior to the stability failures of the board in the experiments.

A plot of the experimental and simulated interaction percentile values is presented in Figure 7.29. Due to small sample size for each test group, three-parameter Weibull percentile estimates were determined and plotted for the experimental data, whereas non-parametric estimates were shown for the simulated results. As can be seen, the simulated and experimental results agree reasonably well at 4 kN, 8 kN, 12 kN, and 25 kN axial load levels. For the group of 25 kN axial load, the experimental percentile values are higher. This can be explained by the fact that the experimental 5th percentile and 95th percentile values may not be too accurate, as only 25 specimens were tested in this group.

Figures 7.30 to 7.33 show the cumulative distribution of the failure moment for the axial load levels of 4 kN, 8 kN, 12 kN and 25 kN respectively. For the 4 kN, 8 kN and 12 kN load levels, the simulated results show good agreement at the lower and higher tails. While the model tends to overestimate the failure moment in the middle probability range, the discrepancy is acceptable. For the 25 kN test group, the simulated results have smaller values
than the test results for probability levels above 10%; however, in general they are in good agreement.

### 7.6.2 Double Span Tests

The experimental setup of the double span tests is shown in Figure 5.5. The total span is 4472 mm. Three axial compression load levels were tested — 4, 8 and 12 kN. In order to obtain the proper shape of deflection, the lateral load was first applied to induce an initial deformation. While keeping the specimen in the deformed shape, the axial load was applied until the prescribed level was reached. Then the lateral load was increased until the specimen failed.

Most of the members failed in tension at or near the mid-support where the moment was a maximum. Some yielding was observed in compression near the mid-support prior to failure. Once the member failed at the mid-support, it was followed by a second failure at the mid-span of either the left or the right span.

To verify the predicting capacity of the model, the cumulative probability distributions of the ultimate load from the model and from the data were compared. Figures 7.34 to 7.36 show the cumulative probability distributions for axial load levels of 4, 8 and 12 kN respectively. As can be seen from the figures, that the predicted failure loads agree with the experimental results for axial load level of 4 kN but underestimate for the other two load cases. It indicates that there are some other mechanisms that help to carry the loads but are not modelled by the strength model. These could be the inherent bows or twists of the specimens, or the test members might not be perfectly simply supported due to friction, that help the members to carry the applied loads. Besides, considering a small sample size for each axial load level test, the test results are more readily subjected to experimental errors.

### 7.7 Summary

This chapter has brought together the test results of Chapter 6 and the strength model of Chapter 4. The calibration of the strength model was done only with the compact specimen tests, with the exception of the tensile strength portion. The calibrated strength model was used to predict the capacity of beam-columns under different loading conditions. It has been
shown in this chapter that the strength model gives reasonably accurate predictions for the experimental tests conducted. Discrepancies between the predicted results and the test results are generally small based on visual inspections. On the basis of this chapter’s findings, the strength model will be used in the next chapter to study the behaviour of beam-columns under different loading conditions.
Table 7.1 Statistics of Data and Simulated Full-Size Compression Strength (in MPa)

<table>
<thead>
<tr>
<th></th>
<th>“A” Specimen</th>
<th>“B” Specimen</th>
<th>All Specimen</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Simulation</td>
<td>Data</td>
</tr>
<tr>
<td>Count</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Mean</td>
<td>30.701</td>
<td>31.914</td>
<td>31.311</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>3.643</td>
<td>3.221</td>
<td>5.549</td>
</tr>
<tr>
<td>COV</td>
<td>11.866%</td>
<td>10.09%</td>
<td>17.723%</td>
</tr>
<tr>
<td>Maximum</td>
<td>42.148</td>
<td>40.843</td>
<td>68.644</td>
</tr>
<tr>
<td>5th Pctl</td>
<td>25.000</td>
<td>27.000</td>
<td>25.197</td>
</tr>
<tr>
<td>50th Pctl</td>
<td>30.490</td>
<td>31.694</td>
<td>30.460</td>
</tr>
<tr>
<td>95th Pctl</td>
<td>36.881</td>
<td>37.069</td>
<td>39.105</td>
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</table>

Table 7.2 Statistics of Data and Simulated Full-Size MOR Strength Values (in MPa)

<table>
<thead>
<tr>
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<th>“A” Specimen</th>
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<td></td>
<td>Data</td>
<td>Simulation</td>
<td>Data</td>
</tr>
<tr>
<td>Count</td>
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<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Mean</td>
<td>48.835</td>
<td>50.002</td>
<td>48.188</td>
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<tr>
<td>COV</td>
<td>22.343%</td>
<td>17.849%</td>
<td>26.388%</td>
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<tr>
<td>Maximum</td>
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<td>66.029</td>
<td>81.240</td>
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<tr>
<td>5th Pctl</td>
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<td>25.201</td>
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<tr>
<td>50th Pctl</td>
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<td>51.789</td>
<td>48.045</td>
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<tr>
<td>95th Pctl</td>
<td>65.118</td>
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#### Table 7.3 Statistics of Eccentric Compression Test Results

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<th>e = 75 mm</th>
<th>e = 200 mm</th>
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</thead>
<tbody>
<tr>
<td>Moment (kN.m)</td>
<td>Axial Load (kN)</td>
<td>Moment (kN.m)</td>
<td>Axial Load (kN)</td>
</tr>
<tr>
<td>Count</td>
<td>19</td>
<td>49</td>
<td>47</td>
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<tr>
<td>Mean</td>
<td>0.968</td>
<td>1.362</td>
<td>2.046</td>
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<tr>
<td>5th Pctl</td>
<td>0.724</td>
<td>0.940</td>
<td>1.421</td>
</tr>
<tr>
<td>50th Pctl</td>
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<tr>
<td>95th Pctl</td>
<td>1.468</td>
<td>1.936</td>
<td>2.500</td>
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</table>

#### Table 7.4 Statistics of Full Size Compression Test Results

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<th>Moment (kN.m)</th>
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</thead>
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<td>Mean</td>
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</tr>
<tr>
<td>5th Pctl</td>
<td>0</td>
</tr>
<tr>
<td>50th Pctl</td>
<td>0</td>
</tr>
<tr>
<td>95th Pctl</td>
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</table>

#### Table 7.5 Statistics of Third-Point Bending Test Results

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<th>Moment (kN.m)</th>
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<td>Mean</td>
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<tr>
<td>5th Pctl</td>
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<tr>
<td>50th Pctl</td>
<td>2.128</td>
</tr>
<tr>
<td>95th Pctl</td>
<td>2.889</td>
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</table>
### Table 7.6 Statistics of Simulated Eccentric Compression Test Results

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<th>e = 0 mm</th>
<th>e = 5 mm</th>
<th>e = 75 mm</th>
<th>e = 200 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Mean</td>
<td>1.944</td>
<td>44.920</td>
<td>2.286</td>
<td>2.276</td>
</tr>
<tr>
<td>5th Pctl</td>
<td>0.333</td>
<td>1.476</td>
<td>1.879</td>
<td>1.644</td>
</tr>
<tr>
<td>50th Pctl</td>
<td>0.842</td>
<td>3.120</td>
<td>2.311</td>
<td>2.353</td>
</tr>
<tr>
<td>95th Pctl</td>
<td>7.725</td>
<td>3.311</td>
<td>2.664</td>
<td>2.710</td>
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</table>

### Table 7.7 Statistics of Simulated Full Size Compression Test Results

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<th></th>
<th>Moment (kN.m)</th>
<th>Axial Load (kN)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>200</td>
</tr>
<tr>
<td>Mean</td>
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<td>107.801</td>
</tr>
<tr>
<td>5th Pctl</td>
<td>0</td>
<td>88.076</td>
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<tr>
<td>50th Pctl</td>
<td>0</td>
<td>108.000</td>
</tr>
<tr>
<td>95th Pctl</td>
<td>0</td>
<td>125.380</td>
</tr>
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</table>

### Table 7.8 Statistics of Simulated Third-Point Bending Test Results

<table>
<thead>
<tr>
<th></th>
<th>Moment (kN.m)</th>
<th>Axial Load (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count</td>
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<tr>
<td>Mean</td>
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</tr>
<tr>
<td>5th Pctl</td>
<td>1.279</td>
<td>0</td>
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<tr>
<td>50th Pctl</td>
<td>2.178</td>
<td>0</td>
</tr>
<tr>
<td>95th Pctl</td>
<td>2.736</td>
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Figure 7.1  Cumulative Probability Distribution of Within-board Mean MOE (Data and Simulation)

Figure 7.2  Cumulative Probability Distribution of Within-board Minimum MOE (Data and Simulation)
Figure 7.3  Cumulative Probability Distribution of Within-board Standard Deviation of MOE (Data and Simulation)

Figure 7.4  Cumulative Probability Distribution of Within-board Mean Compressive Strength (Data and Simulation)
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Figure 7.5  Cumulative Probability Distribution of Within-board Minimum Compressive Strength (Data and Simulation)

Figure 7.6  Cumulative Probability Distribution of Within-board Standard Deviation of Compressive Strength (Data and Simulation)
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Figure 7.7 Cumulative Probability Distribution of Within-Board Minimum Tensile Strength (Data and Simulation)

Figure 7.8 Cumulative Probability Distribution of Within-Board Mean Compressive Yield Strain (Data and Simulation)
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Figure 7.9  Cumulative Probability Distribution of Within-Board Minimum Compressive Yield Strain (Data and Simulation)

Figure 7.10  Cumulative Probability Distribution of Within-Board Standard Deviation of Compressive Yield Strain (Data and Simulation)
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Figure 7.11 Coherence Function ($\gamma^2$) of MOE and Compressive Strength (Data and Simulation)

Figure 7.12 Coherence Function ($\gamma^2$) of MOE and Tensile Strength (Data and Simulation)
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Figure 7.13  Coherence Function ($\gamma^2$) of MOE and Compressive Yield Strain (Data and Simulation)

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Figure 7.18 Cumulative Probability Distribution of Data and Simulated MOR (Specimen "B")
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Figure 7.20 Test results of Eccentric Compression
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Figure 7.21  Normalized Values of Eccentric Compression Tests

Figure 7.22  Percentile results of Eccentric Compression Tests
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Figure 7.23 Simulation Results of Eccentric Columns

Figure 7.24 Data and Simulated Results of Eccentric Compression Tests
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Figure 7.25 Data and Simulation Percentile Results of Eccentric Compression Tests

Figure 7.26 Test Results of Combined Lateral Bending and Axial Compression
Figure 7.27  Percentile Results of Combined Lateral Bending and Axial Compression Test

Figure 7.28  Data and Simulation Results of Combined Lateral Bending and Axial Compression
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Figure 7.29 Data and Simulation Percentile Results of Combined Lateral Bending and Axial Compression

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Figure 7.31 Cumulative Distribution of Failure Moment of Combined Lateral Bending and Axial Compression Tests (Axial Load = 8 kN)

Figure 7.32 Cumulative Distribution of Failure Moment of Combined Lateral Bending and Axial Compression Tests (Axial Load = 12 kN)
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Figure 7.33 Cumulative Probability Distribution of Failure Moment of Combined Lateral Bending and Axial Compression Tests (Axial Load = 25 kN)

Figure 7.34 Cumulative Probability Distribution of Total Ultimate Lateral Load of the Two-span Combined Bending and Compression Tests (Axial Load = 4 kN)
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Figure 7.35  Cumulative Probability Distribution of Total Ultimate Lateral Load of the Two-span Combined Bending and Compression Tests (Axial Load = 8 kN)

Figure 7.36  Cumulative Probability Distribution of Total Ultimate Lateral Load of the Two-span Combined Bending and Compression Tests (Axial Load = 12 kN)
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Applications

8.1 Introduction

The finite element strength model developed in this thesis has been calibrated and verified. For its successful introduction into design practice, it is importance that the developed model be applicable for a wide range of applications — especially in predicting the capacity of beam-columns under different length and loading conditions.

In this chapter, the model will be used to develop interaction relations of axial load and moment for different $L/d$ (length-to-depth) ratios for beam columns, as well as studying the effects of size and stress distribution on the performance of lumber.

The computed interaction relationship will be compared with the current code allowable loads so that new design methods for beams under combined bending and axial loads can be considered.

It is beyond the scope of this study to expand into the development of reliability-based design methods for beam columns, as more extensive tests and studies would be needed. However, a major contribution of this study is to model and produce information on the within-member variation of lumber properties so that reliability-based design methods can eventually be developed using the model developed in this thesis.

8.2 Current Codes for Combined Bending and Axial Load

For members subjected to combined bending and axial compression, the Canadian code CSA-O86.1-94 specifies two different interaction equations, depending on the application.

For general sawn lumber, the code specifies a linear interaction between the axial load capacity of a concentrically loaded column and the moment capacity in bending. The formula is:
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\[ \frac{P_f}{P_r} + \frac{M_f}{M_r} \leq 1.0 \]  \hspace{1cm} (8.1)

where

\( P_f \) = factored compressive axial load
\( M_f \) = factored bending moment, taking into account end moments and amplified moments due to axial loads in laterally loaded members
\( P_r \) = factored compressive load resistance parallel to grain
\( M_r \) = factored bending moment resistance.

For sawn lumber in truss chord applications of fully triangulated metal plate-connected roof trusses as given in Clause 5.5.13.1, the code specifies a parabolic relation with the axial load terms squared. The interaction equation takes on the following form:

\[ \left( \frac{P_f}{P_r} \right)^2 + \frac{M_f}{K_M M_r} \leq 1.0 \]  \hspace{1cm} (8.2)

where

\( K_M \) = bending capacity modification factor which accounts for the stress distribution effect of the members.

At present, the code provides no specific guidance on how to calculate the amplified moment due to the column deflection. The consensus in the truss industry is to use the amplification factor adopted in other design codes such as the NDS (National Design Specification for Wood Construction) code adopted in the U.S. or the Canadian code for steel design.

In the steel code, the interaction of axial load \( P \) and bending moment \( M \) is given by

\[ \frac{P}{P_u} + \frac{F \cdot M}{M_u} \leq 1 \]  \hspace{1cm} (8.3)

where the moment amplification factor, \( F \), in the equation is given by

\[ F = \frac{1}{1 - P / P_e} \]  \hspace{1cm} (8.4)

where \( P \) is the applied axial load and \( P_e \) is the Euler buckling load given by equation 2.5.
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In the NDS code, the amplification factor is similar to the steel code except that it is described in terms of stresses. Furthermore, a safety factor is also incorporated into Euler buckling load for column buckling about the x-axis:

\[
f_{bx} = \frac{1}{1 - \frac{f_c}{F_{cEx}}}
\]

where

- \( f_c \) = actual (computed) compressive stress
- \( F_{cEx} \) = Euler-based elastic buckling stress

\[
F_{cEx} = \frac{K_{cE}E'_x}{(l_e/d_x)^2}
\]

- \( K_{cE} \) = Euler buckling coefficient for columns
  - 0.3 for visually graded lumber
  - 0.384 for MEL
  - 0.418 for products with less variability such as MSR lumber and glulam
- \( E'_x \) = modulus of elasticity associated with the x-axis modified by some factors.
- \( l_e \) = effective unbraced length of column

When the bending moment is about the x-axis, the general interaction formula for the common case of an axial compressive force combined with a bending moment given by the NDS is

\[
\left( \frac{f_c}{F'_c} \right)^2 + \left( 1 - \frac{1}{1 - \frac{f_c}{F_{cEx}}} \right) \frac{f_{bx}}{F'_{bx}} \leq 1.0
\]

where

- \( F'_c \) = allowable column stress
- \( f_{bx} \) = actual (computed) bending stress about x axis
- \( F'_{bx} \) = allowable bending stress about x axis

The above interaction formulae for a beam-column takes into account a number of factors including column buckling, lateral torsional buckling, and the \( P-\Delta \) effect. Compared with the linear interaction formula adopted in the previous NDS code, the new NDS formula.
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adopts a squared term in the axial compressive stress ratio. It has been shown by many researchers (Zahn 1991, Buchanan 1984) that the linear interaction equation is conservative and a squared term typically reflects the failure envelope of beam-columns.

8.3  Interaction Curves simulated by Eccentric Columns

This section presents the failure interaction relationship of axial compression and bending moment by simulating eccentric columns of different lengths and eccentricities using the strength model developed in previous chapters.

During the simulation, the axial load of the eccentric column was increased until the member fails either in the material or by instability. The mid-span moment was then calculated using

\[
\text{Mid-span } M = P(e + e')
\]

where \( e \) is the applied eccentricity and \( e' \) is the maximum deflection at failure load. One hundred replications were used for each eccentricity length combination.

Figure 8.1 shows the median of the interaction relation for different length to depth (\( l/d \)) ratios. As can be seen from the figure, the failure axial load increases as the \( l/d \) ratio decreases. Besides, a clear "nose" behaviour is observed near the maximum mid-span moment for low axial load levels. The difference in the curves at low axial load level for different \( l/d \) ratios is small as the columns are governed by strength criteria. Above the nose, the columns fail in the stability mode and there are large differences among the curves. The simulated curves agree with the curves obtained by Buchanan (1984).

The failure envelope can also be expressed by the interaction of axial load and end-moment. The end-moment is defined as the product of failure axial load and end eccentricity. In practice, this may be more useful than the interaction of axial load and mid-span moment as it can be determined more easily. Figure 8.2 shows the interaction curves of axial load and end-moment for different \( l/d \) ratios of eccentric columns. The shapes of the curves are quite different from Figure 8.1. There is a rapid decrease in axial load as the end moment increases from zero. The axial load decreases less rapidly as the moment increases for larger \( l/d \) ratios. The "nose" pattern exists only for \( l/d = 7 \) (short columns).
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For a given slenderness ratio \((l/d)\) and axial load level, the horizontal distance between these two curves (Figure 8.1 and 8.2) represents the actual amplification of moment due to the \(P-\Delta\) effect in the member. A plot of the moment amplification and \(P/P_e\) for different slenderness ratios is presented in Figure 8.3. In general, the moment amplification increases as the \(P/P_e\) or the \(l/d\) ratios increase. It has a minimum value of 1 which means there is no moment amplification.

Figure 8.4 shows the same data points with comparison of the moment amplification factor as specified in equation 8.4. As can be observed, the curve fits the data quite well. In order to see the impact of using the simplified moment amplification formula (Eqn. 8.4) for calculating the amplified moments, the end-moments of Figure 8.2 were multiplied by \(1/(1-P/P_e)\) to estimate the mid-span moments commonly computed by the designers. The predicted (solid lines) and the actual (dotted lines) axial load vs. mid-span moment interactions are shown in Figure 8.5. The \(1/(1-P/P_e)\) moment amplification factor gives good predictions of amplified moments for \(l/d\) around 17 (moderate slenderness ratio). For shorter columns, the formula tends to predict larger amplified moments than the actual mid-span moments. However, for highly slender columns, the formula underestimates the amplified moments, especially near the “nose” of the interaction curves.

The current timber design codes (CSA O86.1-94) adopted a parabolic shape for the interaction of combined compression and bending for truss applications. Figure 8.6 shows the simulated (solid lines) and code (dashed lines) allowed interaction curves. As can be seen from Figure 8.6, the parabolic interaction (Eqn. 8.2) is quite conservative, especially near the “nose” and for high \(l/d\) ratios.

Results reveal that the code design procedures are conservative in the interaction equation but not in the moment amplification calculation using \(1/(1-P/P_e)\) as the moment amplification factor. It is beneficial to compare the actual code allowable interaction equation of axial load and mid-span moment with the simulated results. To derive the code allowable interaction curves, the moments of the parabolic interaction equation in Figure 8.6 were divided by the factor \(1/(1-P/P_e)\) to obtain the code “allowable” interaction equation of axial load and end-moment. Then the end-moments were multiplied by the actual moment amplification (given in Figure 8.3) to get the mid-span moments corresponding to the code.
allowable interaction. The results of these calculations are shown in Figure 8.7. As can be seen from the figure, the actual interaction of axial load and mid-span moment correspond to the code's parabolic interaction curve and the $1/(1-P/P_e)$ moment amplification factor is conservative at all axial load levels and $l/d$ ratios. In general, the curves are more conservative for short columns than for long columns. One way to improve the interaction equation is to use a different exponent for the axial load ratio term so that the interaction equation can get closer to the actual interaction equation.

8.4 Axial Load-Slenderness Curves

Another view of the results of the simulated eccentric columns is the axial load-slenderness plot. Figure 8.8 shows the plot of axial load versus the slenderness ($l/d$). As can be expected, the axial load at failure decreases as the slenderness increases. In addition, the axial load decreases as the eccentricity increases. This trend agrees with the results from other research (Buchanan 1984). Many formulae that relate the axial load and slenderness have been developed in the past. No attempt has been made here to compare all these formulae, or to derive new formulae. Nevertheless, the results of these findings can be used in future studies when a new axial load-slenderness formula is needed.

8.5 Interaction Curves simulated by Combined Axially and Laterally Loaded Beams

Interaction curves of axial load and mid-span moments computed by simulating beams under combined axial and lateral loading are shown in Figure 7.29. During the simulations, the axial load was applied first, followed by load increments in the lateral loads. Failure envelopes for different axial load levels were established, and which were shown to be comparable with the test results.

Combined axial tension and lateral bending tests can also be simulated in the same manner. In this exercise, the failure of the member was checked at each load step. Four failure modes are possible — failure in tension by exceeding the tensile strength, failure in
compression by exceeding the compressive strength, failure in yielding by exceeding the allowable compressive strain, and failure in buckling.

Combining the results with Figure 7.29, the interaction curve for both compression and tension axial loads versus mid-span moment appears as shown in Figure 8.9. The $l/d$ ratio for this study is approximately 26 (intermediate column). An important feature of this figure is the much larger strength variability in tension than in compression. The same behaviour has been observed by Buchanan (1984). This is not surprising as the cumulative distribution of tensile strength shows more variability than the compressive strength counterpart.

8.6 Size Effect Study

Size effect studies have been performed by many researchers and a comprehensive literature survey is provided in Section 2.7. The size effect that is exhibited in lumber properties is a very complex mechanism, as lumber is not a homogeneous material; rather there are many discrete inherent defects within the boards. One way to quantify this size effect is to perform experiments to study the effect of lumber properties due to the change in length and width. From the results, size factors can be derived. However, this is a rather time consuming and expensive practice as many length, depth and material combinations have to be tested. It would be more economical to study the size effect by simulation using the developed strength model. Only limited experimental testing would be required to calibrate and verify the simulated results.

As lumber properties are expressed in one-dimensional profiles, boards of different widths cannot be simulated in this study; thus, width effect cannot be quantified. However, the length effect can be studied by simulating specimens of different lengths.

In this study, the length effect of three basic lumber properties — bending, tension and compression — were characterized. Five length-to-depth ratios — 7, 13, 17, 26 and 36 — and a sample size of 200 were simulated for each case. The statistical results of the simulated specimens are shown in Table 8.1. In general, the longer the member, the smaller the strength value.
Analyses of the results followed the procedures of Barrett, Lam and Lau (1995), which assumed an exponential size factor relating the strength and the length as given in the following equation:

\[
\frac{\sigma_1}{\sigma_2} = \left( \frac{L_2}{L_1} \right)^{S_L}
\]

(8.8)

where \(\sigma_1\) and \(\sigma_2\) are the strength values with length \(L_1\) and \(L_2\) respectively; \(S_L\) is the length size effect.

Taking the logarithm of the equation will transform the equation into a linear form with the size factor as the negative of the slope of the equation.

\[
\log(\sigma) = b - \frac{1}{k} \cdot \log(L)
\]

(8.9)

Figure 8.10 shows the log-log plot of the MOR versus length of member. The MOR were evaluated using a third-point bending test. Three percentile levels were studied here — the 5\(^{th}\), 50\(^{th}\) and 95\(^{th}\) percentiles. It is apparent that the data show linear trends in all percentile levels. The length size effect factors derived are tabulated in Table 8.2.

Figure 8.11 shows the plot of tensile strength versus length of member. Similar to the findings for MOR length effect, the figure shows a negative slopes of linear trends for the three percentile levels investigated. The length parameters derived are also shown in Table 8.2.

A plot of the compressive strength versus the length of member is shown in Figure 8.12. Linear curves were also fitted to the data to derive the length effect in compression. As can be seen from the figure, the data also exhibits a linear trend for each percentile level. The derived length size effect factors are given in Table 8.2.

Examining the length size effect factors in Table 8.2, it is clear that the size factors of tensile strength and MOR are very close (approximately 0.20). In general, the compressive strength has smaller length factors, which agrees with findings by other researchers (Barrett, Lam and Lau (1995), Barrett and Fewell (1990)).
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8.7 Load Configuration Effect

The load configuration effect, sometimes known as the stress distribution effect, was also studied briefly in this thesis. Using the strength model and the finite element program developed herein, the ultimate capacity of members under 6 different loading conditions were predicted. With a sample size of 100 and the depth effect being considered, the mean MOR was calculated for each case. The ratio between the mean MORs of different loading conditions give the load configuration factors. In general, the load configuration factors are based on either the third-point bending MOR or the uniformly distributed loading (UDL) MOR values.

Table 8.3 shows the results of the simulation. The factors are compared with the values reported by Madsen (1990). As can be seen, the factors are quite close in some cases, such as with the constant moment and third-point bending, while they are quite different in other cases. The discrepancy may be due to the difference in material used to derive these factors. Madsen used size factors derived from tests of visually graded lumber whereas in this study, the strength model is based on machine graded lumber which believed to have less variations in structural properties. The discrepancy may also be due to the strength model itself. Madsen assumes a uniform Weibull type material in deriving these factors, whereas in this study, a stochastic random field model was used.
### Table 8.1 Statistics of Simulated Strength for Different $l/d$ Ratios

<table>
<thead>
<tr>
<th>L/d =</th>
<th>7</th>
<th>13</th>
<th>17</th>
<th>26</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
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<tr>
<td>Mean</td>
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<td>48.633</td>
<td>49.981</td>
<td>41.558</td>
<td>35.874</td>
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<tr>
<td>5th Pctl</td>
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<td>49.238</td>
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<td>25.759</td>
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<tr>
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<td>50.365</td>
<td>42.843</td>
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<tr>
<td>95th Pctl</td>
<td>74.934</td>
<td>67.384</td>
<td>58.963</td>
<td>55.385</td>
<td>55.298</td>
</tr>
</tbody>
</table>

**Tensile Strength (MPa)**

<table>
<thead>
<tr>
<th>L/d =</th>
<th>7</th>
<th>13</th>
<th>17</th>
<th>26</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>Mean</td>
<td>41.876</td>
<td>37.236</td>
<td>35.745</td>
<td>34.774</td>
<td>30.224</td>
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<tr>
<td>50th Pctl</td>
<td>42.835</td>
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<tr>
<td>95th Pctl</td>
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<td>53.853</td>
<td>55.864</td>
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**Compressive Strength (Mpa)**

<table>
<thead>
<tr>
<th>L/d =</th>
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<th>13</th>
<th>17</th>
<th>26</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>Mean</td>
<td>36.442</td>
<td>36.055</td>
<td>33.863</td>
<td>32.651</td>
<td>31.721</td>
</tr>
<tr>
<td>5th Pctl</td>
<td>33.294</td>
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<td>28.700</td>
<td>27.782</td>
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<tr>
<td>50th Pctl</td>
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<td>95th Pctl</td>
<td>50.349</td>
<td>48.298</td>
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<td>41.387</td>
<td>40.919</td>
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### Table 8.2 Size Length Effect by Simulations

<table>
<thead>
<tr>
<th>Property Level</th>
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<th>Tensile Strength</th>
<th>Compressive Strength</th>
</tr>
</thead>
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<tr>
<td>5th Pctl</td>
<td>0.1514</td>
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<tr>
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<td>0.1707</td>
<td>0.0888</td>
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<td>95th Pctl</td>
<td>0.2023</td>
<td>0.1196</td>
<td>0.1406</td>
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</table>

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### Table 8.3 Load Configuration Factors by Simulations

<table>
<thead>
<tr>
<th>Loading</th>
<th>Constant Moment as base</th>
<th>3rd Point Loading</th>
<th>UDL</th>
</tr>
</thead>
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<td><img src="image1" alt="Constant Moment" /></td>
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<td>0.87</td>
<td>0.92</td>
</tr>
<tr>
<td><img src="image2" alt="Point Loading" /></td>
<td>(1.00)*</td>
<td>(0.87)</td>
<td>(0.84)</td>
</tr>
<tr>
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</tr>
<tr>
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<td>(1.15)</td>
<td>(1.00)</td>
<td>(0.96)</td>
</tr>
<tr>
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<td>1.09</td>
<td>0.94</td>
<td>1.00</td>
</tr>
<tr>
<td><img src="image6" alt="Point Loading" /></td>
<td>(1.20)</td>
<td>(1.04)</td>
<td>(1.00)</td>
</tr>
<tr>
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<td>1.01</td>
<td>1.06</td>
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<tr>
<td><img src="image8" alt="Point Loading" /></td>
<td>(1.40)</td>
<td>(1.22)</td>
<td>(1.17)</td>
</tr>
<tr>
<td><img src="image9" alt="Point Loading" /></td>
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<td>(1.55)</td>
<td>(1.35)</td>
<td>(1.30)</td>
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<td>1.15</td>
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<td><img src="image14" alt="Point Loading" /></td>
<td>(1.65)</td>
<td>(1.43)</td>
<td>(1.38)</td>
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</tbody>
</table>

*Values in parenthesis are from Madsen (1992).*
Figure 8.1  Fiftieth percentile of axial load versus mid-span moment interaction equation for different l/d ratios by eccentric column simulations

Figure 8.2  Fiftieth percentile of axial load versus end moment interaction equation for different l/d ratios by eccentric column simulations
Figure 8.3  
Moment amplification due to P-Δ effect

Figure 8.4  
Moment amplification due to P-Δ effect with fitted curve
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Figure 8.5  Plot of axial load versus mid-span moment with predicted amplified moments from end moments

Figure 8.6  Plot of axial load versus mid-span moment with interaction curve from current code
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Figure 8.7  Plot of axial load versus code allowable mid-span moment interaction curve with simulated results

Figure 8.8  Plot of axial load versus slenderness
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Figure 8.9 Interaction curve by simulating combined bending and axial load members

Figure 8.10 Length size effect in MOR
Figure 8.11  Length Size Effect in Tensile Strength

Figure 8.12  Length Size Effect in Compressive Strength
Chapter 9
Conclusions

9.1 Summary
A comprehensive stochastic finite element model for analyzing and predicting the load carrying capacity of beams and beam-columns is developed and presented herein. By incorporating stochastic lumber properties, the model is capable of characterizing the within-member and between-member variability of the lumber properties to predict the response variability in the load carrying capacity. However, applications of the model require the determination of model parameters by calibrating the model with experimental data. In order to warrant the validity of the model, the model has been verified as well.

The finite element model utilizes one-dimensional beam elements under large displacement but small strains. It is assumed that plane sections remain plane during the course of load application. Due to the large displacements inherent in the problem, non-linearity in geometry is assumed. In addition, the beams may also yield before reaching the ultimate load, thus requiring non-linearity in the stress-strain relationship. A new stress-strain equation has been proposed and has been shown to fit the data very well. As the problems are material and geometrical non-linear, load has to be applied incrementally; hence, the Newton-Raphson Method was used to iterate to the true solution at each load step.

The ultimate load is determined by checking for failure at the end of each load step. For beam, column and beam-column problems, the members can fail in either of several modes, depending on the loading and boundary conditions. In the finite element program, four failure criteria have been incorporated, including failures in excessive tensile stress, excessive compressive stress, excessive compressive strain, and instability. For the first three failure modes, the stresses and strains computed by the finite element method are compared with the strength values, whereas the instability failure is indicated by singularity of the tangent stiffness matrix.
Some of the material properties in the finite element program are modeled as one-dimensional stochastic field variables. They include the modulus of elasticity, tensile strength, compressive strength and compressive yield strain. As these properties may not be ergodic stationary processes, trend removal and normalization procedures were used to transform these processes into ergodic stationary processes. After the processes were normalized, the Fast Fourier transform was utilized to obtain the spectrum of these properties. Auto and cross spectra were studied so that correlations between these processes were maintained during the simulations. Realizations of each material property were simulated by a summation of sinusoidal functions, each with a phase angle, and weighted according to the power spectral density function. All of these spectral density functions, except for the tensile strength property, were calculated from compact specimen tests. Transfer functions and coherence functions were determined as well. As the tensile strength of small compact specimen cannot be determined experimentally, fictitious tensile strength profiles were simulated by the regression method from the MOE profiles, and then power spectral density functions were obtained accordingly. While simulating the tensile strength profiles, moving averaging techniques were utilized to characterize the within-member correlation of the lumber properties. Calibration was performed to minimize the residual of the within-member minimum tensile strength cumulative probability distribution so that a valid tensile strength model was obtained.

A single input – multiple output model was used to simulate the lumber properties of a member. MOE was formulated as the input to the model while compressive strength, tensile strength, and compressive yield strain was formulated as the outputs. After the processes were simulated, trends were then added to these ergodic stationary processes to form the realizations of the lumber properties. The statistics of the realizations were compared with the test results and good agreement was obtained.

To verify the model, testing of full-size specimens was performed, which included the third-point bending MOR test, the long span tension test, the long span compression test with lateral support, the eccentric compression test and the combined bending and compression test. The statistics of these test results were compared to the statistics generated by simulations using the developed model. The cumulative probability distributions and the
coherence functions were compared and good agreement was obtained between the test results and simulations.

The interaction relations of mid-span moment and axial load were generated from the test results of eccentric compression and combined bending and compression tests. Percentile values of the interaction curves were established. Using the finite element program and the Monte Carlo simulation method, the interaction behaviour of axial load and mid-span moment was generated and percentile statistics were determined accordingly. The simulated statistics show reasonably good agreement with the test results in the cumulative probability distribution and interaction equation comparisons.

The interaction of mid-span moment and axial tension was also determined to complete the interaction plot of both axial compression and tension. This was done by simulating combined bending and tension members as no experiments were performed in this study.

Several other applications of the model and the finite element program were shown, including the evaluation of the existing codes. It has been shown that even though the moment amplification factor is underestimated in some situations, the design procedures are still conservative as the interaction equation given in the codes is generally overestimating.

Other applications of the model include the generation of axial load-slenderness curves, a size effect study and a load configuration factor study. The size effect of lumber has been studied extensively in the past. No attempt was made here to quantify the size factors for all lumber applications; however, a brief study was conducted to demonstrate the capacity of the strength model and the program. Length size factors were determined for MOR, tensile strength, and compressive strength for the tested material (2×4 S-P-F); they are comparable with results obtained by Barrett et al. (1995). Load configuration factors have been briefly studied and simulation results were compared with Madsen’s results (Madsen, 1990). Good agreement was observed for constant moment and third-point loading cases but are quite different for other cases. The could be explained by the difference in material and the strength model from which these factors were derived.
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9.2 Further Areas of Research

The research presented here can be extended to study the behaviour and response variability of members under tension and combined bending and tension tests. This requires the support by experimental data.

As there are some discrepancies between the model predictions and the experimental data for cases with moment reversal, it is advised that localized bending strength test values should be obtained to calibrate the model and further studies should be carried out.

Modification of the current design codes in the sections regarding the design of beam-columns can be achieved by further study of the response variability of the interaction of beams under combined bending and compression loading. Experimental tests on other species and sizes should be done to verify the validity and generality of the model. From that, design interaction formulae can be deduced.

Reliability analyses of beams and beam-columns can also be done with the developed finite element program so that the variability of the load-carrying capacity can be quantified in terms of reliability levels. This also requires the testing of other species, sizes, and grades to calibrate the model.

Size effect and load configuration effect have been briefly studied in this thesis. The length size effect has been predicted by simulation using the established finite element program. The research can be extended to study the width effect by testing members of different widths. This will complete the size effect analysis. On the other hand, load configuration factors have been derived to demonstrate the usefulness of the program as an analysis tool. Further research can be done with the program to compare testing procedures of different standards so that design values can be harmonized or converted.

It would be of interest to extend the model to a 2-dimensional stochastic random field model so that members with irregular shape or under complex loading can be analyzed. This will also benefit the study of width size effect and load configuration factor. However, the formulation of a two-dimensional model requires a failure criterion and material properties in two-dimensions. Some two-dimensional failure criteria have been established in the past, such as the Tsai-Wu model, the von Mises model and the Tresca model; however, lumber
material properties and their correlations in two-dimensions is very difficult to obtain. This may require further theoretical developments and model calibrations.
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