Hedging with Derivatives and Operational Adjustments under Asymmetric Information: Theory and Tests

by

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Abstract

Firms can use financial derivatives to hedge risks and thereby decrease the probability of bankruptcy and increase total expected tax shields. Firms also can adjust their operational policies in response to fluctuations in prices, a strategy that is often referred to as "operational hedging". In this paper, I investigate the relationship between the optimal financial and operational hedging strategies for a firm, which are endogenously determined together with its capital structure. This allows me to examine how operational hedging affects debt capacity and total expected tax shields and to make quantitative predictions about the relationship between debt issues and hedging policies. I also model the effects of asymmetric information about firms' investment opportunities on their financing and hedging decisions. First, I examine the case in which both debt and hedging contracts are observable. Then, I study the case in which firms' hedging activities are not completely transparent. The models are tested using a data set compiled from the annual reports of North American gold mining companies. Supporting evidence is found for the key predictions of the model under asymmetric information.

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Chapter 1

Introduction

1.1 The Development of Financial Hedging Instruments

There is little argument that the financial world today is a riskier place than it was 30 years ago. The breakdown of Bretton Woods in 1972 caused foreign exchange rate volatility to increase in the early 1970's. Around the same time, domestic interest rates and commodity prices became more volatile. The trend did not abate with time. As an illustration of the importance of risk to corporate decision making, consider some events during the year 1998, which was by no means an unusual year for financial markets. In his review of the financial markets of 1998, Smithson [43] considers high volatility to be the number one theme of the year. The year 1998 was a poor year for commodity corporations. The price of gold fell from US$415 to $278 per troy ounce; silver lost almost a dollar to around US$5 per ounce; oil lost more than 5 dollars to US$12.05 per barrel. In addition, electricity deregulation in the United States caused the price of electricity to rise dramatically. During the week of June 22, the wholesale price of electricity in the mid-west United States jumped from around $25 per megawatt hour (MWh) to $2,600 per MWh and higher. In the equity markets, The Dow Jones Industrial Average dropped by 19% on August 1, the DAX dropped by 37% on October 8 and the FTSE dropped by 25% on October 5. The most well known event in the bond markets occurred on August 7 when the Russian government defaulted on its bonds. The price of Russian government bonds fell over the year from approximately 60 cents on the dollar to 10 cents on the dollar.

These events illustrate why corporate risk management is considered by today's financial managers to be one of their most important objectives. Tufano [45] suggests that financial engineering may be as important to some firms as mechanical engineering used to be. Poor risk management can drive an otherwise healthy firm into financial distress.

To accommodate the demand for managing corporate risks, a range of financial innovations, particularly "derivatives", have been created or rediscovered. A derivative security is a security whose value depends on the values of some more basic underlying variables. Firms can trade these securities to hedge the systematic fluctuations in macroeconomic variables. The fluctuations in foreign exchange rates, interest rates, commodity prices and equity prices are four of the most commonly hedged financial risks using derivative instruments, according to Wharton/CIBC World Markets Surveys [6, 7]. In the 1998 survey, a six-page questionnaire was sent to 2000 randomly selected publicly traded firms and 154 non-financial Fortune 500 firms. Of the 399 firms that returned a completed survey, 200 reported using derivatives. Of these, foreign exchange instruments were used by 83%, interest rate instruments by 76%, commodity instruments by 56% and equity derivatives by 34%. These figures illustrate the extent to which derivatives usage has become an important activity for many firms.

1.1.1 Derivative markets

The derivatives markets have been growing rapidly over the last two decades. The growth in exchange-traded instruments has slowed in the late 90s, but over-the-counter (OTC) markets continue to thrive. Table 1.1 illustrates the size of the markets for selected financial derivative instruments. Notional amounts represent the face value of the contracts, not the market value or value at risks[1] and, as a result, may exaggerate the size of the markets. The data was taken from Bank for International Settlements (BIS) 1998 and 1997 annual reports. In the 1999 survey on the global OTC derivatives markets by BIS, the total notional amount outstanding at the end of 1998 was estimated to be US$80 trillion, while the total market value was US$3.2 trillion. The gross credit exposure, which is the gross market value after taking into account legally enforceable bilateral netting agreements, was US$1.3 trillion.

At the end of June, 1998, the notional amount of interest rate derivatives accounted for 67% of the total OTC derivatives markets, followed by foreign exchange markets with a share of 30%. Equities and commodities represented 2% and 1% of the OTC derivatives market, respectively. In market value terms, the percentages are 53%, 36%, 9% and 2% for interest rate, foreign exchange, equity and commodity instruments, respectively.

1.1.2 Hybrid securities

Firms also can combine their hedging and financing efforts by using hybrid securities. Hybrid debt securities incorporate a derivative, *i.e.* forward, option, swap

[1]Value at risks, or VaR, is defined as the minimim expected loss of a porfolio over a predeterminied period of time for a given level of probability.

Table 1.1: Markets for selected financial derivative instruments

Derivative types	Notional amounts at year-end (10^9 US$)					
	1993	1994	1995	1996	1997	1998
Exchanged-traded	7,771	8,863	9,189	9,880	12,202	13,549
Interest rate futures	4,949	5,778	5,863	5,931	7,489	7,702
Interest rate options	2,362	2,624	2,742	3,278	3,640	4,603
Currency futures	35	40	38	50	52	38
Currency options	76	56	44	47	33	19
Stock index futures	110	128	172	196	212	321
Stock index options	230	238	329	378	777	867
Over-the-counter	8,475	11,303	17,713	25,453	29,035	50,977
Interest rate swaps	6,177	8,816	12,811	19,171	22,291	...
Currency swaps	900	915	1,197	1,560	1,824	...
Interest rate options	1,398	1,573	3,705	4,723	4,920	...

etc., into a standard debt contract. A common example of a hybrid debt security is a convertible bond, which is equivalent to a straight debt contract plus an option on the equity value of the firm. Recent innovations in corporate hybrids tie principle and/or coupon payments not only to firm specific variables but also to various global economic variables such as commodity prices, interest rates, foreign exchange rates and stock market indices.

Even the most complicated type of hybrid securities can be decomposed into a standard debt or equity security and a derivative. Both hybrid debt securities and derivatives are financial hedging instruments, and each provides similar hedging benefits to firms. One may ask why firms pay financial institutions to design complicated debt contracts while there exists a well-functioning derivatives market where these firms can trade derivatives directly to hedge financial risks. One argument is that in hybrid debt contracts, the hedging instruments match perfectly with the terms of the debt contracts; whereas separate matching hedging instruments may not always be available on the market. For example, forward contracts embedded in dual-currency bonds normally have maturities of 10 years, while the liquidity in the standard foreign exchange forward market declines for maturities longer than one year and falls dramatically for maturities beyond five years. As a result, investors may be willing to pay a premium for these long maturity financial products.

Second, some indexed debt contracts are specifically designed for tax arbitrage purposes; in particular, many firms take advantage of the differences in tax treatment or regulations in different countries or markets. In one classic example, as cited by Smithson and Chew [44], US firms made 'free money' by issuing zero-coupon yen bonds in Japan and subsequently issuing dual currency bonds to hedge the residual yen exposure from the yen zero. Since the Japanese government treats income from holding a zero-coupon bond as capital gains, which are non-taxable for Japanese investors, the US firms captured this tax savings by offering a lower interest rate. Furthermore, Japanese investors were willing to pay an additional premium for these US corporate bonds because of the 10% limit on holdings of non-yen-denominated bonds of foreign corporations by Japanese pension funds. Another example is adjustable-rate convertible debt, which paid a coupon equal to the dividend rate on the firm's common stock and was convertible to common shares at the spot price at any time. This type of issue disappeared after the US Internal Revenue Service ruled it as equity for tax purposes.

A third potential reason why firms use hybrid debt securities rather than derivatives to hedge financial risks is that it might be easier for the former instruments to qualify for hedge accounting treatment. Hedge accounting avoids the increased volatility of a firm's accounting income that is induced by mark-to-market accounting. This issue has been intensified by the enforcement of the widely debated FAS 133, Accounting for Derivative Instruments and Hedging Activities, for all quarters of fiscal years beginning after June 15, 1999. This regulation is designed to unify derivatives accounting, hedge accounting and disclosure in a single statement. Although it is not clear-cut, FAS 133 may make hybrid securities even more attractive.

An example of hybrid security: commodity linked debt

Commodity linked debt is a hybrid security that ties principle and/or coupon payments to commodity prices. I use this as an example because it is close in structure to the hedging instrument discussed in this paper. The first known commodity linked financing dates back to the 19th century (see Priovos [36]), when in 1863, the government of the Confederated States of America issued a cotton linked bond to finance the war against the United States of America. In 1973, PEMEX, the state-owned Mexican oil company, issued bonds embedded with a forward contract on oil. This was the beginning of a new wave of corporate hybrid issues, and the market for hybrids flourished in the late 1980s.

Today, oil and gold are the most commonly used commodities involved in hybrid financing. Various other types of metals such as silver, nickel, zinc and copper are also used. These offerings are issued mainly by governments and by mining corporations whose major products are the underlying commodities. Banks in Canada and Australia also play active roles in these markets because of the importance of natural resources to the economies of these countries. Due to regulatory reasons, most of the issues were offered on the Euromarkets before the 1987 stock market crash. Since then, most of the commodity-linked debt issues have been private

Table 1.2: An example of commodity linked debt

Average Copper Price	**Indexed Interest Rate**
$2.00 and above	21%
1.80	20
1.60	19
1.40	18
1.30	17
1.20	16
1.10	15
1.00	14
0.90	13
0.80 and below	12

placements.

The example shown in Table 1.2, which may help to illustrate the salient features of a commodity-linked debt issue, was taken from Rawls and Smithson [38]. Magma Copper Company offered 10 year Copper Interest-Indexed Senior Subordinated Notes in November, 1988. The bond pays a quarterly coupon payment that is contingent upon the spot price of copper, as shown in Table 1.2. This debt contract embeds 40 securities with payoffs similar to options, one for each quarterly coupon payment, with maturity from 3 months to 10 years.

Commodity linked debt contracts take on very different formats. Roughly speaking, these contracts can be classified as either forward-type or option-type, depending on what their payoffs look like. In this paper, I simplify the problem by considering only forward-type contracts. The example contract shown in Table 1.2 is a typical option-type contract. Gold loans are probably the most commonly known forward-type financing tools. In these contracts, gold mining companies borrow gold and sell it immediately on the spot market. They then use the proceeds to finance exploration or mine development or to pay off old debt. When the loans mature, they pay the lending partners a certain contracted quantity of gold.

The size and total number of gold loans both increased substantially in the late 1980's. In 1987, the largest offer involved 100,000 ounces, but in 1988, the issue for the refinance of Newmont mine involved 1 million ounces. It is difficult to obtain information on private issues, but it has been estimated that the total amount of commodity linked financing was about US$300 million monthly by the beginning of the 1990s [36].

1.2 To Hedge or Not to Hedge: Theories and Evidence

To hedge or not to hedge a particular risk exposure using derivatives has long been considered one of the most difficult decisions faced by the financial managers of today's corporations. The astronomical losses experienced by some of the world's largest corporations during early 1990s made derivatives usage "a new nightmare in the boardroom", as described in the February, 1996 issue of *The Economist*. In December, 1993, Metallgesellschaft lost \$1.3bn on oil futures trading. In April, 1994, Kashima Oil lost \$1.5bn on dollar derivatives, and Procter & Gamble lost \$102m on highly leveraged interest-rate swaps. In December, 1994, Orange County lost \$1.7bn in leveraged interest-rate products. These high-profile incidents have helped to make some firms hesitant to use derivatives to hedge risks.

On the other hand, The International Swaps and Derivatives Association (ISDA) reports that privately negotiated derivatives contracts outstanding worldwide at the end of 1998 rose to \$50.997 trillion in notional principal from \$29.035 trillion in 1997. This indicates a surge in demand for derivatives for hedging purposes since major financial institutions and hedge funds reduced their risk taking positions during the year, largely in response to Russian defaults and the collapse of Asian markets.

The rapid development of financial hedging instruments also attracted intense interest from academia. The following briefly summarizes the academic research on this topic. The focus of the discussion is on the usage of derivatives and/or hybrid securities by non-financial firms to hedge corporate exposure to risks.

1.2.1 The benefits of hedging

The standard argument that a risk-averse manager-owner hedges to diversify does not explain why a modern corporation hedges. With fully developed financial markets, shareholders can hedge financial risks on their own accounts using derivatives markets. Of course, one can argue that it might be cheaper for corporations to hedge on behalf of their shareholders. General discussions on the benefits of hedging risks by corporations can be found in a large body of literature, but the seminal work by Smith and Stulz [42] is the most cited paper.

The most important benefit is the reduction in the probability of financial distress through hedging. Using the copper linked bonds offered by Magma Copper Company as an example, the firm pays a high coupon when its production generates higher profits and, more importantly, bears a smaller interest burden when a fall in copper price lowers cash inflows. This may increase firm value through two channels. First, it may decrease the expected costs of financial distress. The costs of financial distress include both the direct administrative costs during bankruptcy [20], and the indirect financial losses such as those due to anticipation of disruption in services [33]. Second, it may increase the optimal debt capacity of a firm and, as a consequence, increase expected tax shields. (see Ross [39] and Leland [24]).

Ross [39] shows that hedging might not reduce the expected costs of financial distress for an optimally levered firm because hedging increases the amount of debt a firm offers. Even when hedging does lower expected bankruptcy costs, the reduction tends to be small compared to the increase in tax savings through hedging. Graham and Smith [19] quantify the tax savings from hedging by modeling major provisions of the tax code. Their simulation results show that a 5% reduction in volatility from hedging translates into a tax saving of about 3% of taxable income. As follows from Jensen's Inequality, a necessary condition for hedging to generate tax savings is that the effective tax function for a firm be convex (see Smith and Stulz [42]). Graham and Smith [19] show that this is true in general except for local nonconvexity caused by carryback and carryforward provisions.

The empirical evidence on the relationship between leverage and hedging is mixed. A significant positive relationship is reported by Dolde [15], Samant [41], Berkman and Bradbury [4] and Haushalter [21]. Dolde studies a survey sample of 244 of Fortune 500 non-financial firms in 1992. Samant investigates the interest rate swaps usage of 354 US firms from 1992 to 1994 by using the footnotes to annual reports. Haulshalter examines the hedging activity of a survey sample of oil and gas producers from 1992 to 1994. In contrast, a significant negative relationship is documented by Allayannis and Ofek [2]. They study the foreign exchange derivatives usage of 378 of S&P 500 non-financial firms in 1993 by examining their financial statements.

Hedging can also increase the probability of using internally generated funds rather than expensive external funds to finance new investment opportunities, as modeled by Froot, Scharfstein and Stein [16]. Adam [1] extends their analysis into a two-period model and derives dynamic optimal financing/hedging portfolios. Supporting empirical evidence is documented by Geczy, Minton and Schrand [17], who examine annual reports for the foreign exchange usage of the largest 500 US industrial firms in 1991, and by Haushalter [21].

Hedging may also alleviate the agency problems associated with risky debt (Myers [31]) because both under- and over-investment problems are more severe when the firm faces the threat of financial distress. For any given level of debt, both commodity linked debt and hedging using derivatives tend to shift debt service payments to periods in which the firm is in a better financial situation, and thereby reduce shareholder-bondholder conflicts. The benefits of hedging to reduce the potential costs of underinvestment for an optimally levered firm are discussed in Ross [39]. He shows that it is ambiguous whether hedging will alleviate the agency costs of debt for an optimally levered firm. The reason is that although hedging reduces the costs of underinvestment for any given level of debt, it also increases the optimal amount of debt a firm issues.

The information advantage of hedging is discussed by DeMarzo and Duffie [13, 14]. In their 1991 paper, instead of modeling firms' hedging decisions as a signal of the manager's proprietary information, they model the case in which hedging is not observable. In this case, there exist no conflicts of interest between the manger and the shareholders. It is optimal for a firm to hedge its exposure to financial risks completely because shareholders are risk averse and can not hedge the unobservable risks on their own. In their 1995 paper, these authors study the two balancing information effects of hedging. On the one hand, hedging reduces the variability of a firm's earnings and, hence, decreases managers' wage variability. This will result in full hedging by risk averse managers when hedge accounting is in place and hedging profits are not separated from product profits. On the other hand, hedging will reduce a source of noise in firms' profits and increase the informativeness firms' profits as a signal of managerial quality if the firm's hedging activity is fully disclosed. This may discourage risk averse mangers from fully hedging because hedging may make their future income more volatile. Consequently, under certain circumstances, hedge accounting is the optimal accounting rule for shareholders. In these cases, managers do engage in full hedging, increasing the informativeness of the financial reports. More discussion on the issue of hedging disclosure can be found in Raposo [37].

Mella-Barral and Perraudin [26] show that strategic debt service by equity holders can eliminate both direct bankruptcy costs and agency costs of debt. During times of financial distress, equity holders may act strategically to force the existing bondholders to lower the interest payments on outstanding debt. However, this renegotiation can be costly. Commodity linked debt helps stakeholders avoid costly renegotiation by contracting *ex ante* on observable economic variables which impact the well-being of a firm.

Finally, Allayannis and Weston [3] examine the foreign currency derivatives usage of a sample of 720 large US nonfinancial firms between 1990 and 1995. They found a significant hedging premium of 5.7% after 1993, which suggests that there are benefits to hedge.

1.2.2 The disadvantages of hedging

In spite of the advantages of hedging discussed above, many firms with substantial exposure to hedgeable risks do not hedge at all. There are three potential explanations for why not all firms hedge that have been discussed in the literature.

One consistent finding by all the empirical studies cited here is that hedging activities exhibit economies of scale. Block and Gallagher [5], Nance, Smith and Smithson [32], Mian [28], Berkman and Bradbury [4], Geczy, Minton and Schrand [17], Allayannis and Ofek [2] and Haushalter [21] find the relationship between firm size and firm's hedging activity to be significantly positive. It may be too expensive for smaller firms to hedge using derivatives or to use indexed bonds to finance their growth, even though smaller firms' cash flows are more sensitive to price fluctuations, and, in times of financial distress, these firms have more restricted access to capital.

Another possible reason why not all firms use financial instruments to hedge their financial risks is that firms may use operational hedge as an alternative. For example, a mining firm may close or open high cost mines to adjust its overall production costs in response to commodity price changes. Petersen and Thiagarajan [34] compare the hedging behavior of two gold mining companies: American Barrick, which aggressively hedges its gold price risk, and Homestake Mining, which uses no derivatives to hedge its gold price risk but adjusts its operational policy constantly to help stabilize its cash flows. This raises the interesting question of whether operational hedging and financial hedging through derivatives are substitutes. This issue will be addressed in a later section.

The third reason arises from conflicts of interest between shareholders and bondholders, or the risk shifting problem. Managers who act in the interest of shareholders may have the incentive to use derivatives to increase rather than decrease the risks a firm faces. Also, compensation contracts may motivate managers to choose hedging policies to maximize their own stakes in the firm. In some circumstances, then, derivatives usage can increase agency costs. These issues are addressed in Smith and Stulz [42] and Chang [9]. Evidence supporting this argument has been documented by Tufano [45] for the gold mining industry in North America using survey data from 1991 to 1993. Hentschel and Kothari [22], however, present evidence to suggest that firms generally use derivatives to reduce risks rather than to speculate.

This paper focuses on the tax benefits of financial and operational hedging as well as the benefits of reducing agency costs caused by underinvestment using operational adjustments. I also show that asymmetric information might be another potential reason for firms not to hedge.

1.3 Motivations for and Contributions of this Dissertation

This paper fits into the basic framework that focuses on the tradeoff between tax saving and financial distress costs. It contributes to the literature in three ways. First, I investigate the relationship between the optimal financial and operational hedges of a firm, which are determined simultaneously with the firm's capital structure. This also allows me to make quantitative predictions on the relationship between debt issues and hedging policies. Second, I model the effects of asymmetric information on firms' financing and hedging decisions. For simplicity, I examine the case in which firms can hedge only through either forward contracts or commodity linked debt offers, in which the coupon rate is a linear function of commodity price. Third, the models are tested using a data set compiled by me from the annual reports of North American gold mining companies.

There exists a large body of literature studying the relationship between financial hedging and production flexibility or production risks. Holthausen [23] and Moschini and Lapan [29, 30] are two examples. The moral of the literature is that, when there exists production flexibility or production risks, nonlinear hedging contracts, such as swaps and options, are necessary to hedge financial risks fully. The reason is that, in this case, the profit function becomes nonlinear in financial prices. Most authors use the quadratic utility maximizing framework, in which hedging reduces the variance without changing the expected value of the payoff. Therefore, a contract that matches the profit function exactly will be optimal.

Two papers, Chowdry and Howe [11] and Mello, Parsons and Triantis [27], that extend this literature into a corporate finance setting are most relevant to the modeling of operational hedging in the present paper. Both papers study multinational firms and focus on foreign exchange risks. In these two papers, operational hedging refers to opening and closing factories in the relevant countries to match the currency denomination of production costs and revenues. Chowdry and Howe [11] follow closely the setting of mean-variance maximization and conclude that operational hedging is not used when there exist no demand risks (which is assumed in both the current paper and in Mello, Parsons and Triantis [27]) because operational hedging is more costly than financial hedging. As a result, the optimal hedging portfolio is composed of forward contracts.

Mello, Parsons and Triantis [27] are the first authors to study the interaction between operational hedging and financial hedging in the presence of debt liabilities. In their paper, however, capital structure is exogenous. They define an efficient hedge as the one that would restore first-best operational policies for a given level of debt issues. They also investigate the interesting issue of whether financial hedging and operational flexibility are substitutes, *i.e.*, alternative ways of achieving the same objective. Their results suggest that this is not the case; for a given leverage ratio, firms with greater operational flexibility hedge more. This is because production flexibility increases the first best value, while financial hedging tends to move firm value to the first best.

I develop a model in which capital structure is endogenously determined together with firms' hedging and operating decisions. Under symmetric information about firms' production opportunities, this framework extends Mello, Parsons and Triantis [27] to include endogenized capital structure decisions. This allows me to examine the relationship between firms' optimal capital structure and hedging decisions. Both the hedging and the financing components of the contract can trigger bankruptcy. It is assumed that the two components have the same seniority. Cooper and Mello [12] study the importance of the priority of the debt and hedging contract.

I then develop a model in which managers have private information about their marginal production cost and use financing/hedging packages to reveal this information to the market so that the debt offers will be fairly priced. There are two components in the financing contract: a fixed, or straight, component and a varying, or hedging, component. The package can be interpreted as a linear commodity linked debt contract or a straight debt contract plus a separate forward contract with the same seniority. The information effects studied in this paper are very different from those investigated in, for example, Demarzo and Duffie [13, 14] or Raposo [37], discussed above.

I solve for the optimal financing and hedging policies with or without operational flexibility under symmetric information, as well as the fully revealing separating equilibria using both components as signals and the fully revealing separating equilibria when the hedging component is not observable. The empirical predictions derived from models in the paper are then tested by examining the hedging behavior of North American gold mining firms. Data were extracted from firms' annual reports and from COMPUSTAT.

The structure of my dissertation is as follows. In Chapters 2 and 3, I discuss the general setup of the model, solve the models under symmetric information, first with, and then without, operational flexibility. In Chapter 3, I use the solutions from Chapter 2 as boundary conditions to solve the case when there exists asymmetric information. First, I assume both components of the financing contract can be used as signals. Then, I solve the case in which only the straight component is observable and can be use as a signal of firm quality. Chapter 4 presents the empirical evidence by examining the hedging activities of North American gold mining firms.

Chapter 2

Optimal Financing and Hedging Policies under Symmetric Information

2.1 General Setup

The general setup applies to all the models derived in later sections. All agents are assumed to be risk neutral, and the probability measure used below is the risk neutral probability. At time 0, a firm faces an investment opportunity and needs to raise a fixed amount of capital, I, to finance it. If adopted, the project will produce one unit of the underlying commodity per unit time at a constant production cost c. It is assumed that the price process of the commodity follows geometric Brownian Motion,

$$\frac{dp_t}{p_t} = \mu dt + \sigma dZ_t,$$

where Z_t is a standard Brownian Motion, *i.e.* random variables dZ_t are independently normally distributed with a mean of 0 and a variance of dt.

The firm finances the project with equity and/or a debt package at time 0. It is a one time offer; the firm cannot change its debt position after time 0. In other words, there are no dynamic capital structure and hedging decisions in this paper. Any amount of the proceeds from debt financing in excess of I is paid to the stockholders as a dividend. A negative dividend means that the stockholders provide part of the funds to finance the project.

If a firm offers debt, it issues a perpetual coupon bond with the coupon rate varying with the underlying commodity price. The debt package can be interpreted as a commodity-linked bond, with coupon rate $b+hp$ per unit of time, where b is the fixed, or straight, component of the coupon rate, and h, the hedging component, determines how the coupon rate varies with the commodity price. This simplifying assumption is made not only for technical tractability but also for the fact that h and b capture the effect of capital structure on corporate hedging, one of the main issues addressed in this paper.

As will be discussed in detail later, the contract can also be interpreted as a straight perpetual coupon bond, B, plus a forward[1] contract, in which h units of commodity are sold forward each unit of time. It is also assumed that the two components have the same seniority.

I use the same continuous time contingent claim framework as in Brennan and Schwartz [8] and Mella-Barral and Perraudin [26] to evaluate the debt and equity value at time 0. Bankruptcy is determined endogenously to maximize shareholders' wealth. The equityholders can choose to inject funds to continue the production and to distribute interest payments to debtholders. The firm pays taxes on net operating income at a constant rate τ, and interest payments are tax deductible.

The interpretation of the debt contract as a combination of a straight debt contract and a forward contract remains valid in the presence of corporate taxes. Prior to the Supreme Court's 1988 decision in the "Arkansas Best" case, the gains and losses on corporate hedging transactions are taxed as ordinary income as long as the hedged item generates ordinary income as well. Between 1990 and 1993, the IRS ruled that hedging gains and losses must be taxed as capital gains, even though the item hedged was ordinary. After 1993, the IRS reverted to the policy in place prior to 1990 [35]. Therefore, in this paper, the tax treatment on the two components of the debt contract is assumed to be the same.

2.1.1 All equity financing

I consider first the case in which only equity financing is available. I assume that the firm has no production flexibility, which means that once the production process is closed it can not be reopened. I assume also that it is costless to close production. The decision to close production is made endogenously to maximize total equity value. While in production, the firm's earnings flow is $p_t - c$. Financial market equilibrium under risk neutrality requires that the value of the firm, or the equity value V_t, satisfy the following no-arbitrage condition,

$$rV_t = (1-\tau)(p_t - c) + \frac{d}{d\Delta} E_t(V_{t+\Delta}) \mid_{\Delta=0},$$

where r is the risk free interest rate. The stationarity of the cash flows involved implies that V_t depends on t only through p_t. Assume all the standard differentiability

[1]Conventionally, the forward price is set such that the initial value of the forward contract is zero. One can also create a forward contract with any forward price using, for example, a combination of call and put options. In this paper, however, the choice of forward price, $\bar{p}$, is not explicitly modeled. The forward contract is priced the same way equity and debt are priced, with default risks taken into account. The variables modeled in this paper are b, which equals $B - h\bar{p}$, and h.

conditions, then, by Ito's Lemma,

$$rV(p) = (1-\tau)(p-c) + \mu p V'(p) + \frac{\sigma^2}{2}p^2 V''(p). \tag{2.1.1}$$

Suppose that the scrap value of the firm is zero, and the firm is shut down at price p_e. Then the boundary conditions include

$$V(p_e) = 0, \tag{2.1.2}$$

and the smooth pasting condition,

$$V'(p_e) = 0, \tag{2.1.3}$$

which ensures that equityholders choose to shut down the firm at a price that maximizes the equity value, $V(p)$. The solutions should also satisfy the no-bubbles condition, *i.e.*, the equity value converges to the expected value of the perpetual cash flow if production is never shut down when the current commodity price approaches infinity.

Solving equation (2.1.1) with boundary condition (2.1.2) and (2.1.3) yields the expression for the value of the firm as a function of commodity price and marginal production cost,

$$V(p) = (1-\tau)\left\{\frac{p}{r-\mu} - \frac{c}{r} - \left[\frac{p_e}{r-\mu} - \frac{c}{r}\right]\left(\frac{p_e}{p}\right)^{-\lambda_1}\right\}, \tag{2.1.4}$$

where λ_1 is the negative root of the quadratic equation

$$\lambda(\lambda-1)\frac{\sigma^2}{2} + \lambda\mu = r, \tag{2.1.5}$$

and

$$p_e = Qc,$$

$$Q = \frac{-\lambda_1}{1-\lambda_1}\frac{r-\mu}{r} < 1. \tag{2.1.6}$$

As expressed in equation (2.1.4), the total equity value represents the expected cash flow if production is never shut down net of the value losses (gains) due to the probability of a shutdown. $(\frac{p_e}{p})^{-\lambda_1}$ can then be interpreted as the probability that production will be shut down.

2.1.2 Debt and equity financing

Assume now that both equity and debt financing are available. Denote the value of debt and equity as $L(p)$ and $V_E(p)$, respectively. The firm's bankruptcy decision is endogenized by assuming that equityholders can inject capital to cover operating losses and to make interest payments. The price level, p_b, at which bankruptcy is triggered is therefore chosen by equityholders to maximize equity value. Upon bankruptcy, debtholders take over the firm and operate it as an unlevered firm. The losses in tax shields due to bankruptcy serve as the only source of bankruptcy costs in this paper.

Financial market equilibrium requires that L and V_E obey,

$$rV_E(p) = (1-\tau)(p-c-b-hp) + \mu p V_E'(p) + \frac{\sigma^2}{2}p^2 V_E''(p), \tag{2.1.7}$$

and

$$rL(p) = b + hp + \mu p L'(p) + \frac{\sigma^2}{2}p^2 L''(p). \tag{2.1.8}$$

The boundary conditions include,

$$V_E(p_b) = 0, \tag{2.1.9}$$

$$L(p_b) = V(p_b), \tag{2.1.10}$$

the no bubbles conditions for both debt and equity values[2], and the smooth pasting condition,

$$V_E'(p_b) = 0, \tag{2.1.11}$$

which ensures that equityholders declare bankruptcy at a price maximizing equity value, $V_E(p)$.

Solving equation (2.1.7) and (2.1.8) with relevant boundary conditions (2.1.9) to (2.1.11) yields the expressions for the debt and equity values of the firm as functions of current commodity price p, the marginal production cost c, and the coupon rates b and h,

$$V_E(p) = (1-\tau)\left\{\frac{(1-h)p}{r-\mu} - \frac{b+c}{r} - \left[\frac{(1-h)p_b}{r-\mu} - \frac{b+c}{r}\right]\left(\frac{p_b}{p}\right)^{-\lambda_1}\right\}, \tag{2.1.12}$$

[2]For the value of debt, the no bubbles condition requires that the debt value converges to the expected value of the perpetual coupon payments if the firm never goes bankrupt when the current commodity price approaches infinity

$$L(p) = \left\{ \frac{hp}{r-\mu} + \frac{b}{r} - \left[\frac{hp_b}{r-\mu} + \frac{b}{r} - V(p_b) \right] \left(\frac{p_b}{p} \right)^{-\lambda_1} \right\}, \tag{2.1.13}$$

where,

$$p_b = Q\frac{b+c}{1-h}, \tag{2.1.14}$$

and $V(p_b)$ is calculated using equation (2.1.4). With b nonnegative and h between 0 and 1, p_b is at least as large as p_e. The equityholders stop injecting funds at a price level higher than the closing price of an unlevered firm. This is similar to the underinvestment problem discussed by Myers [31].

The term $(\frac{p_b}{p})^{-\lambda_1}$ in equation (2.1.12) and (2.1.13) can be treated as the probability of bankruptcy. Then, equations (2.1.12) and (2.1.13) are very easy to interpret. The equity value of the firm is the present value of the perpetual dividend stream minus the loss of that value in case of bankruptcy. The option to declare bankruptcy increases the wealth of the shareholders since they receive negative dividends at bankruptcy. The market value of debt is the present value of the perpetual coupon payments minus the difference between the loss of coupon payments and the residual firm value in case of bankruptcy.

2.2 The Optimal Financial Decisions

I now analyze the firm's financing decision under symmetric information about its investment opportunity; that is, the case in which the constant production cost c is observable by all investors at time 0. To simplify notation, denote the commodity price level at time 0 by p. Suppose that the manager acts in the equityholders' interest and chooses b and h to maximize the sum of the intrinsic equity value and dividend payments at time 0. Under symmetric information, this is equivalent to maximizing the total net present value of the firm, or

$$\max_{b,h}\{V_E(b,h,c) + L(b,h,c) - I\} = \max_{b,h}\{F(b,h,c)\}, \tag{2.2.1}$$

where $F(b,h,c) = V_E(b,h,c) + L(b,h,c)$. Using equation (2.1.12) and (2.1.13), the total firm value at time 0 is,

$$\begin{aligned} V_E(p) + L(p) = (1-\tau) &\left\{ \frac{p}{r-\mu} - \frac{c}{r} - \left[\frac{p_e}{r-\mu} - \frac{c}{r} \right] \left(\frac{p_e}{p} \right)^{-\lambda_1} \right\} \\ + \tau &\left\{ \frac{hp}{r-\mu} + \frac{b}{r} - \left[\frac{hp_b}{r-\mu} + \frac{b}{r} \right] \left(\frac{p_b}{p} \right)^{-\lambda_1} \right\}. \end{aligned} \tag{2.2.2}$$

The total value of the levered firm is therefore the sum of the value of an unlevered firm and the value of tax shields. The fact that p_b is no less than p_e

reflects the lost tax shields due to bankruptcy. The first two terms in the expression for the tax shields, which are positive, represent the present value of total tax shields from the perpetual coupon bond without bankruptcy. The negative term represents the loss of tax shields due to bankruptcy. I assume no other bankruptcy costs in the model, and the tradeoff between a higher tax shield and a higher probability of bankruptcy (and, hence, a higher probability of losing tax shields) resulting from higher b and/or h drives the optimal solutions for b and h. The value of the unlevered firm is independent of both b and h, so I need only to maximize the total tax shield. Solving the maximization problem by taking partial derivatives with respect to b and h yields two first order conditions. Combining the two equations shows that the two first order conditions can never be satisfied simultaneously, implying possible corner solutions.

In order to prevent the trivial solution in which $b = -c, h = 1$, a lower bound must be imposed on b . In that case, the equity value of the firm is zero. Debtholders provide operating cost c and are entitled to all the production of the firm. Therefore, the firm is really an all-equity firm which fully enjoys the tax advantages of debt financing. This is definitely not a feasible solution in reality. In this paper, I assume that b must be nonnegative[3].

In solving the optimization problem, the total tax shield term, denoted as TS, is regrouped. Denote the ratio p_b/p as x. Variable x is a function of both b and h, and will be shown to be a crucial variable for the problem. Also, x is perfectly correlated with $(\frac{p_b}{p})^{-\lambda_1}$, the probability of bankruptcy. Then,

$$\begin{aligned} TS &= \tau\left[\frac{hp}{r-\mu}+\frac{b}{r}-\left[\frac{hp_b}{r-\mu}+\frac{b}{r}\right]\left(\frac{p_b}{p}\right)^{-\lambda_1}\right] && (2.2.3) \\ &= \tau\left[\frac{1}{r}\left(\frac{xp}{Q}-c\right)(1-x^{-\lambda_1})\right. && (2.2.4) \\ &\quad \left.+h\frac{p}{(-\lambda_1)(r-\mu)}(x^{1-\lambda_1}-(1-\lambda_1)x-\lambda_1)\right] \\ &= \tau\left[\frac{p}{r-\mu}\left(1-\frac{Qc}{xp}\right)(1-x^{1-\lambda_1})\right. && (2.2.5) \\ &\quad \left.-b\frac{1}{(1-\lambda_1)xr}(x^{1-\lambda_1}-(1-\lambda_1)x-\lambda_1)\right], \end{aligned}$$

with $x \in [0,1]$ and $x^{1-\lambda_1} - (1-\lambda_1)x - \lambda_1$ nonnegative. Therefore, the total tax shield is increasing in h and decreasing in b for any given x. Since no possible

[3]This assumption is generally violated, however. For example, if the firm issues only a forward contract then $b = -h\bar{p}$ can be negative. Fortunately, whether the actually lower bound on b is zero or some negative constant will not affect the inferences of the models in this paper in any significant way. In the asymmetric information case in particular, this bound serves only as a boundary condition.

values for x are lost by setting b to zero, a corner solution to the above problem is obtained at $b^* = 0$. The first order condition for x is,

$$\left.\frac{\partial F}{\partial x}\right|_{b=0} = \frac{\tau p}{(r-\mu)x^2}\left[\frac{Qc}{p}[1-\lambda_1 x^{1-\lambda_1}] - (1-\lambda_1)x^{2-\lambda_1}\right] = 0. \tag{2.2.6}$$

It is shown in Appendix A that there exists a unique $x^* \in (0,1)$, with $x^* > Qc/p$ and x^* an increasing function of c, that solves equation (2.2.6). Therefore, the optimal choice of h, h^*, is strictly between 0 and 1 ($0 < h^* < 1$). It can also be shown that h is a decreasing function of c, which suggests that firms with higher production costs have lower debt capacity and also hedge less. These results are summarized in the following proposition.

PROPOSITION 1 *Under symmetric information, a levered firm offers perpetual debt with no fixed coupon payment or a straight debt contract fully hedged with a matching forward contract.*

The varying component of the coupon rate, $h^*(c)$, is a strictly decreasing function of the firm's marginal production cost c, and $0 < h^* < 1$.

The bankruptcy price of a levered firm is strictly greater than exit price of an unlevered firm and is strictly increasing in the marginal production cost of the firm.

One example of this type of financing is the use of gold loans to finance projects, where firms repay in units of gold instead of cash. The intuition behind this result is that, by hedging the price risks using commodity-linked debt or derivatives contract, a firm reduces the probability of bankruptcy for any given level of debt, and, as a result, increases tax shields and firm value. Firms with lower production costs face lower probability of bankruptcy (lower $p^*(b)$), although they offer more hedged debt (higher h^*). Numerical examples of $h^*(c)$ and $TS^*(c)$ are shown in Figures 3.2 and 3.3, respectively.

Chapter 3

The Effects of Operational Flexibility

In this section, I investigate the effects of operational flexibility on the value and debt capacity of a firm. I assume that the production process can be costlessly shut down or restarted.[1] This operational flexibility is also referred to as operational hedging. The reason is as follows. Some firms adjust their production costs by shutting down or restarting higher cost production lines (*e.g.*, mines) to stabilize their profits. The same results can also be achieved by hedging price risks using financial derivatives. The relationship between the two forms of hedging is studied in this section as well.

It is assumed that no maintenance cost is incurred when the production process is shut down. It is, therefore, never optimal for a firm to abandon the production process altogether. Instead of deciding on a price level to abandon the production process completely, the manager chooses a price level to excise the real option on operational flexibility. For an unlevered firm, this is the only decision the manager makes at time zero. For a levered firm, the manager takes operational flexibility into account when he determines the firm's financial policies.

3.1 All Equity Financed Firm with Operational Flexibility

I first solve the case of an unlevered firm. All the assumptions about the price and production processes remain the same as in last chapter. The firm's earnings flow is $p_t - c$ while in production, and it is 0 when the production is shut down.

[1] In the traditional neoclassical microeconomic paradigm, a firm faces increasing marginal production cost and is able to adjust its production level continuously until its marginal production cost equals the output price. In this extreme case, operational flexibility eliminates the role of financial hedging.

The values of the unlevered firm while in and out of production are denoted as W_t^1 and W_t^2, respectively. Financial market equalibrium requires that the two value processes satisfy the following two differential equations,

$$rW^1(p) = (1-\tau)(p-c) + \mu p W^{1\prime}(p) + \frac{\sigma^2}{2} p^2 W^{1\prime\prime}(p), \tag{3.1.1}$$

and,

$$rW^2(p) = \mu p W^{2\prime}(p) + \frac{\sigma^2}{2} p^2 W^{2\prime\prime}(p). \tag{3.1.2}$$

W_t^1 and W_t^2 should join smoothly at $p = p_c$, which is the price level at which the production process is shut down or restarted. The boundary conditions are, therefore,

$$W^1(p_c) = W^2(p_c), \tag{3.1.3}$$

and

$$W^{1\prime}(p_c) = W^{2\prime}(p_c). \tag{3.1.4}$$

Solving equations (3.1.1) and (3.1.2) with boundary conditions (3.1.3) and (3.1.4) yields the value functions,

$$\begin{aligned} W^1(p) &= (1-\tau)\left\{\frac{p}{r-\mu} - \frac{c}{r} + \frac{1}{\lambda_1-\lambda_2}\left[(\lambda_2-1)\frac{p_c}{r-\mu} - \lambda_2\frac{c}{r}\right]\left(\frac{p_c}{p}\right)^{-\lambda_1}\right\} \\ &= (1-\tau)\left\{\frac{p}{r-\mu} - \frac{c}{r}\right\} + w^1 p^{\lambda_1}, \end{aligned} \tag{3.1.5}$$

and

$$\begin{aligned} W^2(p) &= (1-\tau)\left\{\frac{1}{\lambda_1-\lambda_2}\left[(\lambda_1-1)\frac{p_c}{r-\mu} - \lambda_1\frac{c}{r}\right]\left(\frac{p_c}{p}\right)^{-\lambda_2}\right\} \\ &= w^2 p^{\lambda_2}, \end{aligned} \tag{3.1.6}$$

where $\lambda_1 < 0$ and $\lambda_2 \geq 1$ are the roots of the quadratic equation (2.1.5). The solutions also satisfy the no bubbles conditions.

In order to determine p_c, the manager maximizes the firm values W^1 and W^2. In Appendix B, I first solve for p_c by maximizing W^1, and then show that I obtain exactly the same solution $p_c = c$ if I maximize W^2 instead. This result is expected since it is assumed in the model that it is costless to shut down or restart production.

Compare this solution with that of the previous model in which the firm does not have the operational flexibility to shut down or restart the production process. The exit price p_e is less than the production cost c, but the shut down and reopen price level is exactly the same as c. In the first case, it is optimal to continue the production even when the profit is negative because of the positive probability that the price might rise above c again. In the second case, the negative profit is avoided because of the costless option to shut down and restart the production process. The value of this option, denoted by $X(p)$, is represented by the difference between the values of the two otherwise identical firms at various price levels. That is

$$X(p) = \begin{cases} (1-\tau)\left\{\frac{1}{\lambda_1-\lambda_2}\left[(\lambda_2-1)\frac{p_c}{r-\mu}-\lambda_2\frac{c}{r}\right]\left(\frac{p_c}{p}\right)^{-\lambda_1} \right. \\ \qquad \left. +\left[\frac{p_e}{r-\mu}-\frac{c}{r}\right]\left(\frac{p_e}{p}\right)^{-\lambda}\right\} & \text{if } p \geq p_c = c, \\ (1-\tau)\left\{\frac{1}{\lambda_1-\lambda_2}\left[(\lambda_1-1)\frac{p_c}{r-\mu}-\lambda_1\frac{c}{r}\right]\left(\frac{p_c}{p}\right)^{-\lambda_2} \right. \\ \qquad \left. -\frac{p}{r-\mu}+\frac{c}{r}+\left[\frac{p_e}{r-\mu}-\frac{c}{r}\right]\left(\frac{p_e}{p}\right)^{-\lambda}\right\} & \text{if } p_e \leq p \leq p_c, \\ (1-\tau)\left\{\frac{1}{\lambda_1-\lambda_2}\left[(\lambda_1-1)\frac{p_c}{r-\mu}-\lambda_1\frac{c}{r}\right]\left(\frac{p_c}{p}\right)^{-\lambda_2}\right\} & \\ & \text{if } p \leq p_e = Qc. \end{cases} \tag{3.1.7}$$

In the third region, $X(p)$ is a decreasing function of c because it equals the value of an unlevered firm with operational flexibility. In the first two regions, the value of the operational flexibility is not a simple monotonic function of c. This is because the operational flexibility is not a simple real option but is composed of a series of alternating shutdown and restart options. The exercise of one will automatically activate the other. Therefore, the value of the operational flexibility option relates to the parameters in the model and the relative positions of c and p in a very complicated way.

The properties of the function as expressed in equation (3.1.7) are dependent on particular parameter values. For the set of parameter values that I am going to use throughout the paper, $\{\tau = 0.34, \sigma^2 = 0.04, \mu = 0.04, r = 0.075\}$ and $p_0 = 10$, the value of an unlevered firm with or without operational flexibility and the value of operational flexibility are illustrated in Figure 3.1. Curve A and B represent the value of two otherwise identical firms with and without operational flexibility. Curve C is the difference between Curve A and Curve B and, therefore, represents the value of operational flexibility.

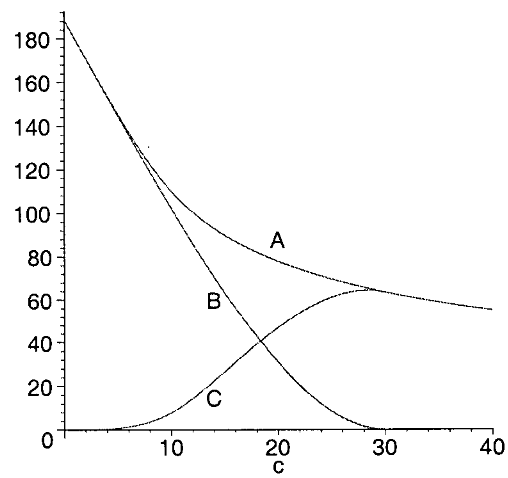

Figure 3.1: The value of operational flexibility as a function of c
Curve A represents the value of the firm with operational flexibility. Curve B represents the value of the firm without operational flexibility. Curve C = A - B, represents the value of operational flexibility.

3.2 Financial Decisions of a Levered Firm with Operational Flexibility

At time 0, the firm offers a perpetual coupon bond to finance the project. The continuous coupon rate is $b + hp_t$, where b is the fixed component and h is the varying component. Denote the value of debt and equity as $L(p)$ and $V_E(p)$, respectively. Both the bankruptcy decision, the choice of p_b at which the firm declares bankruptcy, and the operational decision, the choice of p_c, are made by the manager to maximize equityholders' wealth. Upon bankruptcy, debtholders take over the firm and operate it as an unlevered firm.

There are two scenarios that need to be considered here. If $p_c < p_b$, the firm goes bankrupt before ever exercising the option to shut down or restart production. If $p_c \geq p_b$, however, the real option is exercised when the commodity price reaches a certain threshold, p_c. In the following section, I consider the first case and find that the solution is very similar to the case in which the firm does not have operational flexibility. Then, I consider the second case in which $p_c \geq p_b$.

3.2.1 The case $p_c \leq p_b$

The Value Functions

Financial market equilibrium requires that L and V_E obey the same equations as in the model without operational flexibility (section 2.1.2) except for boundary condition,

$$L(p_b) = W^1(p_b). \tag{3.2.1}$$

Solving equation (2.1.7) and (2.1.8) with boundary conditions (2.1.9), (2.1.11) and (3.2.1) yields the expressions for the debt and equity values of the firm as functions of commodity price p, the marginal production cost c, and the coupon rates b and h,

$$V_E(p) = (1-\tau)\left\{\frac{(1-h)p}{r-\mu} - \frac{b+c}{r} - \left[\frac{(1-h)p_b}{r-\mu} - \frac{b+c}{r}\right]\left(\frac{p_b}{p}\right)^{-\lambda_1}\right\}, \tag{3.2.2}$$

$$L(p) = \frac{hp}{r-\mu} + \frac{b}{r} - \left[\frac{hp_b}{r-\mu} + \frac{b}{r} - W^1(p_b)\right]\left(\frac{p_b}{p}\right)^{-\lambda_1}, \tag{3.2.3}$$

where

$$p_b = Q\frac{b+c}{1-h}, \tag{3.2.4}$$

and $p_c = c$ is the shutting down/restarting price level for an unlevered firm. With b nonnegative and $h \in (0,1)$, which will be shown to be the case for the optimal solutions, $p_b \leq p_c$ is satisfied.

It is interesting to note that the equity value as shown in (3.2.2) is exactly the same as in the model without operational flexibility. The value of the debt, however, is different. This is because the unlevered firm has a higher value in this case due to the existence of the real option and, therefore, the residual value is higher when the firm goes bankrupt. Again, the term $(\frac{p_b}{p})^{-\lambda_1}$ in equation (3.2.2) and (3.2.3) can be interpreted to be the probability of bankruptcy. Similarly, $(\frac{p_c}{p})^{-\lambda_1}$ and $(\frac{p_c}{p})^{-\lambda_2}$ can be considered relating to the probability that the production will be shut down or restarted.

The Optimal Financial Decisions

I now analyze the firm's financing decisions. The manager chooses b and h to maximize the sum of the equity value and the dividend payment at time 0,

$$\max_{b,h}\{V_E(b,h,c) + L(b,h,c) - I\} \tag{3.2.5}$$

Using equation (3.2.2) and (3.2.3), the following expression for the total firm value at time 0 is,

$$\begin{aligned} V_E(p) + L(p) &= (1-\tau)\left\{\frac{p}{r-\mu} - \frac{c}{r} + \frac{\lambda_2}{(\lambda_2-\lambda_1)(1-\lambda_1)}\frac{c}{r}\left(\frac{p_c}{p}\right)^{-\lambda_1}\right\} \\ &\quad + \tau\left\{\frac{hp}{r-\mu} + \frac{b}{r} - \left[\frac{hp_b}{r-\mu} + \frac{b}{r}\right]\left(\frac{p_b}{p}\right)^{-\lambda_1}\right\} \\ &= W^1 + TS(b,h). \end{aligned} \tag{3.2.6}$$

TS denotes the expected value of tax shields as defined in equation (2.2.3). The total value of the levered firm is therefore the sum of the value of an unlevered firm and the expected value of total tax shields. Comparing equation (3.2.6) with equation (2.2.2) in the model without operational flexibility, the only difference is between the expressions for the unlevered firms, which does not involve either b or h. As a consequence, we obtain the same optimal solutions for b and h, $b^* = 0$ and h^* satisfies equation (2.2.6).

To discuss these results further, I repeat some of the properties I proved in the model without operational flexibility, which hold in the current model. There exists a unique $x^* = \frac{p_b^*}{p}$ strictly between 0 and 1, with $x^* > \frac{Qc}{p}$ and x^* an increasing function of c, that solves equation (2.2.6). It follows that p_b^* is an increasing function of c, and $h^* \in (0,1)$. It was also shown that h^* is a decreasing function of c.

As shown in Appendix C, there exists a maximum c that satisfies the condition $p_c \leq p_b$ for any given initial price p. This corresponds to a minimum value of h^* in the region $p_c \leq p_b$. It is also shown in the Appendix that $\frac{p_b}{p_c}$ is a decreasing function of c, which is crucial for the discussion of the cutoff point between the two regimes, $p_c \leq p_b$ and $p_c \geq p_b$.

The above results are summarized in the following proposition. Firms with lower production costs tend to issue more commodity linked debt because the probability of bankruptcy for these firms is relatively low. They also behave as if operational flexibility does not exist. This is because they declare bankruptcy when the production still generates positive profits due to their high interest obligations.

PROPOSITION 2 *Levered firms with production flexibility and a production cost lower than* $c_{max} = p\left(\frac{\lambda_2-1}{\lambda_2-\lambda_1}\right)^{\frac{1}{1-\lambda_1}}$ *go bankrupt before ever exercising their option on operational flexibility.*

These firms offer perpetual debt with no fixed coupon payment. The varying component of the coupon rate, $h^*(c)$*, is a strictly decreasing function of the firm's marginal production cost* c*, and* $\frac{1}{\lambda_2} \leq h^* < 1$.

The price at which such a levered firm declares bankruptcy is strictly increasing in the marginal production cost of the firm. The ratio of bankruptcy price, p_b*, and* *the price level at which an unlevered firm shuts down or restart its production, $p_c = c$, is a decreasing function of c.*

3.2.2 The case $p_c \geq p_b$

The Value Functions

Under this regime, the equity value has two different expressions for the region $p > p_c$, in which the firm is in production, and the region $p_b \leq p \leq p_c$, in which the firm shuts down its production. Denote the two equity values as V_E^1 for the first region and V_E^2 for the second. Financial market equilibrium requires that L, V_E^1 and V_E^2 obey the following differential equations and boundary conditions:

$$rV_E^1(p) = (1-\tau)(p - c - b - hp) + \mu p V_E^{1\prime}(p) + \frac{\sigma^2}{2} p^2 V_E^{1\prime\prime}(p), \tag{3.2.7}$$

$$rV_E^2(p) = (1-\tau)(-b - hp) + \mu p V_E^{2\prime}(p) + \frac{\sigma^2}{2} p^2 V_E^{2\prime\prime}(p), \tag{3.2.8}$$

and

$$rL(p) = b + hp + \mu p L'(p) + \frac{\sigma^2}{2} p^2 L''(p). \tag{3.2.9}$$

The boundary conditions include,

$$V_E^1(p_c) = V_E^2(p_c). \tag{3.2.10}$$

$$V_E^2(p_b) = 0, \tag{3.2.11}$$

$$L(p_b) = W^2(p_b), \tag{3.2.12}$$

and the smooth pasting conditions,

$$V_E^{1\prime}(p_c) = V_E^{2\prime}(p_c), \tag{3.2.13}$$

$$L'(p_b) = 0. \tag{3.2.14}$$

Solving equation (3.2.7) to (3.2.8) with boundary conditions (3.2.10) to (3.2.14) yields the expressions for the debt and equity values of the firm,

$$\begin{aligned} V_E^1(p) &= (1-\tau)\left\{\frac{(1-h)p}{r-\mu} - \frac{b+c}{r} + \frac{1}{\lambda_2 - \lambda_1}\left[(\lambda_2 - 1)\frac{hp_b}{r-\mu} + \lambda_2 \frac{b}{r}\right]\left(\frac{p_b}{p}\right)^{-\lambda_1} \right. \\ &\qquad \left. - \frac{1}{\lambda_2 - \lambda_1}\left[(\lambda_2 - 1)\frac{p_c}{r-\mu} - \lambda_2 \frac{c}{r}\right]\left(\frac{p_c}{p}\right)^{-\lambda_1}\right\} \\ &= (1-\tau)\left\{\frac{(1-h)p}{r-\mu} - \frac{b+c}{r}\right\} + v_E^1 p^{\lambda_1}, \end{aligned} \tag{3.2.15}$$

$$
\begin{aligned}
V_E^2(p) &= (1-\tau)\left\{\frac{-hp}{r-\mu} - \frac{b}{r} - \frac{1}{\lambda_2-\lambda_1}\left[(\lambda_1-1)\frac{hp_b}{r-\mu} + \lambda_1\frac{b}{r}\right]\left(\frac{p_b}{p}\right)^{-\lambda_2}\right. \\
&\quad \left. + \frac{1}{\lambda_2-\lambda_1}\left[(\lambda_2-1)\frac{hp_b}{r-\mu} + \lambda_2\frac{b}{r}\right]\left(\frac{p_b}{p}\right)^{-\lambda_1}\right\} \\
&= (1-\tau)\left\{\frac{-hp}{r-\mu} - \frac{b}{r}\right\} + v_E^2 p^{\lambda_2} + v_E^3 p^{\lambda_1},
\end{aligned}
\tag{3.2.16}
$$

$$
\begin{aligned}
L(p) &= \frac{hp}{r-\mu} + \frac{b}{r} - \left[\frac{hp_b}{r-\mu} + \frac{b}{r}\right]\left(\frac{p_b}{p}\right)^{-\lambda_1} \\
&\quad + (1-\tau)\frac{-\lambda_1}{(\lambda_2-\lambda_1)(\lambda_2-1)}\frac{c}{r}\left(\frac{p_b}{p}\right)^{-\lambda_1}\left(\frac{p_b}{p_c}\right)^{\lambda_2},
\end{aligned}
\tag{3.2.17}
$$

where p_b is determined by the following equation:

$$
\left(\frac{p_b}{p_c}\right)^{\lambda_2} - \lambda_2 h\frac{p_b}{p_c} - (\lambda_2-1)\frac{b}{c} = 0. \tag{3.2.18}
$$

The shutting down/restarting price level for a levered firm, p_c, is determined by maximizing equity values in both region $p > p_c$ and region $p \leq p_c$. In Appendix D, I show that $p_c = c$ satisfies both the first order condition for maximizing V_E^1 and that for maximizing V_E^2. Subsequently, I show that the second order conditions are also satisfied.

The most important observation here is that $p_c = c$ is optimal for both a levered and an unlevered firm. A firm's operating policy is, therefore, not affected by its financial choices. The shutting down and restarting thresholds coincide due to the assumption that it is costless to make production adjustments.

The Optimal Financial Decisions

I now analyze the financing decisions of a levered firm in this regime. Once again, the manager chooses b and h to maximize the sum of the equity value and the dividend payment at time 0. In the case $p_c \geq p_b$, this sum has two expressions depending on whether the firm is in production or is shut down. I will show that the decision rules have the same formulation. The manager's decision rule is

$$
\max_{b,h}\{V_E^i(b,h,c) + L(b,h,c) - I\}, i = 1, 2. \tag{3.2.19}
$$

Using equation (3.2.15) and (3.2.17), the expression for the total firm value while in production at time 0 is,

$$
\begin{aligned}
V_E^1(p) + L(p) &= (1-\tau)\left\{\frac{p}{r-\mu} - \frac{c}{r} + \frac{1}{\lambda_1 - \lambda_2}\left[(\lambda_2 - 1)\frac{p_c}{r-\mu} - \lambda_2\frac{c}{r}\right]\left(\frac{p_c}{p}\right)^{-\lambda_1}\right\} \\
&\quad + \tau\left\{\frac{hp}{r-\mu} + \frac{b}{r} - \left[\frac{hp_b}{r-\mu} + \frac{b}{r}\right]\left(\frac{p_b}{p}\right)^{-\lambda_1}\right\} \\
&= W^1 + TS(b,h).
\end{aligned}
\tag{3.2.20}
$$

Using equation (3.2.16) and (3.2.17), the total firm value while production is shut down at time 0 is given by

$$
\begin{aligned}
V_E^2(p) + L(p) &= (1-\tau)\left\{\frac{1}{\lambda_1 - \lambda_2}\left[(\lambda_1 - 1)\frac{p_c}{r-\mu} - \lambda_1\frac{c}{r}\right]\left(\frac{p_c}{p}\right)^{-\lambda_2}\right\} \\
&\quad + \tau\left\{\frac{hp}{r-\mu} + \frac{b}{r} - \left[\frac{hp_b}{r-\mu} + \frac{b}{r}\right]\left(\frac{p_b}{p}\right)^{-\lambda_1}\right\} \\
&= W^2 + TS(b,h).
\end{aligned}
\tag{3.2.21}
$$

TS denotes the expected value of tax shields as defined in equation (2.2.3), and W^1 and W^2 represent the value of an unlevered firm while it is in the state of production or shut down. The total value of the levered firm is therefore the sum of the value of an unlevered firm and the expected value of tax shields. Maximizing the total firm value is thus equivalent to maximizing the expected value of total tax shields, which has the same expression in both the producing and the shut down state. In Appendix E, I show that the maximization problem

$$
\max_{b,h}\ TS(b,h),
\tag{3.2.22}
$$

has a corner solution at $b^* = 0$. The optimal hedging policy is determined by equation (3.2.24), and the solution for the optimal choice of p_b is

$$
p_b^* = \left(\frac{\lambda_2 - 1}{\lambda_2 - \lambda_1}\right)^{\frac{1}{1-\lambda_1}} p.
\tag{3.2.23}
$$

$$
\begin{aligned}
h^* &= \frac{1}{\lambda_2}\left\{\frac{p_b^*}{c}\right\}^{\lambda_2 - 1} \\
\textit{So,} \quad &= \frac{1}{\lambda_2}\left(\frac{p}{c}\right)^{\lambda_2 - 1}\left(\frac{\lambda_2 - 1}{\lambda_2 - \lambda_1}\right)^{\frac{\lambda_2 - 1}{1-\lambda_1}}.
\end{aligned}
\tag{3.2.24}
$$

It is easy to see from equation (3.2.24) that h^* is a decreasing function of c (recall that $\lambda_2 > 1$). It is interesting to note that p_b^* is independent of c. Firms with lower production costs offer more commodity linked debt, and the effects of c and h on p_b exactly offset each other. This result is a major difference between firms in the regimes $p_b \leq p_c$ and $p_b \geq p_c$. As shown in last section, for firms in the regime $p_b \geq p_c$ which do not exercise the option on production flexibility before they go bankrupt, p_b^* is an increasing function of c.

Since p_b^* is a constant across firms in the regime $p_b \leq p_c$, $\frac{p_b^*}{c}$ is decreasing in c. The minimum c that satisfies $p_b \leq p_c$ is

$$c_{min}\big|_{\{p_c \geq p_b\}} = \left(\frac{\lambda_2 - 1}{\lambda_2 - \lambda_1}\right)^{\frac{1}{1-\lambda_1}} p. \tag{3.2.25}$$

This corresponds to a maximum value of h^* in regime $p_c \geq p_b$,

$$h_{max}\big|_{\{p_c \geq p_b\}} = \frac{1}{\lambda_2}. \tag{3.2.26}$$

The upper and lower bound for h and c for firms in regime $p_c \geq p_b$ coincide with the corresponding lower and upper bounds for firms in regime $p_c \leq p_b$. This makes the value functions and decision rules continuous across all firms. The above results for firms in regime $p_c \geq p_b$ is summarized in the following proposition.

PROPOSITION 3 *A levered firm with production flexibility and a production cost greater than c_{min} shuts down/restarts its production when the commodity price is equal to its production cost c. The price level at which the firm declares bankruptcy, p_b, is smaller than or equal to its production cost c.*

These firms offer perpetual debt with no fixed coupon payment. The varying component of the coupon rate, $h^*(c)$, is a strictly decreasing function of the firm's marginal production cost c, and the maximum value of h^* is $\frac{1}{\lambda_2}$.

The price at which such a levered firm declares bankruptcy is independent of the marginal production cost of the firm. The ratio of the bankruptcy price and the price level at which the levered firm shuts down or restart its production, $\frac{p_b}{p_c}$, is a decreasing function of c.

3.2.3 Summary

Denote $c_{min}\big|_{\{p_c \geq p_b\}} = c_{max}\big|_{\{p_c \leq p_b\}}$ as c_{cutoff}, and $h_{max}\big|_{\{p_c \geq p_b\}} = h_{min}\big|_{\{p_c \leq p_b\}}$ as h_{cutoff}. For any given initial price p, firms with different production costs c fall into two categories. Firms with $c \leq c_{cutoff}$ declare bankruptcy before exercising their options on production flexibility. These firms issue commodity linked debt with $h \geq h_{cutoff}$. Firms with $c \geq c_{cutoff}$ exercise their option on production flexibility before they declare bankruptcy. These firms issue commodity linked debt with $h \leq h_{cutoff}$.

No firms issue unhedged debt. For all firms, h^* is decreasing in c, p_b^* is non-decreasing in c and $\frac{p_b^*}{p_c^*}$ is decreasing in c. The total tax shields, TS, is also a decreasing function of c. Comparing the results from this model with those for a firm without operational flexibility sheds some insight on how operational hedging affects the total value and debt capacity of a firm. Analytical results are difficult to derive because there does not always exist an explicit expression for the optimal h. Figures 3.2 and 3.3 illustrate the differences in debt capacity and expected tax shields for each type of firm as functions of the production cost c.

Operational hedging can increase or decrease debt capacity, but it never decreases total tax shields. Therefore, production flexibility increases firm value through two channels. First, it always increases the unlevered firm value. Second, it may also increase total tax shields through its function as an operational hedge. For firms with production costs that are low relative to the current commodity price, operational hedging is irrelevant for financial decisions. Those firms issue more debt and go bankrupt before ever exercising their operational flexibility option. Therefore, operational hedging increases these firms' unlevered values without increasing their tax shields.

For firms with higher production costs, however, both value increasing effects take place. Since the operating decision $p_c = c$ is independent of the financial decisions h and b, operational hedging increases unlevered firm value no matter what their choice of h and b. For given financial choices, whether to exercise the operational flexibility option will, however, affect total tax shields. This is due to the fact that p_b is not only a function of b and h but also depends on p_c. For firms with production costs lower than c_{cutoff}, the bankruptcy threshold is the same with or without operational flexibility, and, therefore, the total tax shields are also the same. For firms with production costs higher than c_{cutoff}, however, the existence of operational flexibility decreases the threshold for bankruptcy and, hence, increases total tax shields.

It is interesting to note that when there exists operational flexibility, and when firms are actually taking advantage of this flexibility before bankruptcy, the optimal financial decisions are made such that the bankruptcy threshold is constant across all firms that fit into that category. It seems that operational flexibility allows firms to push the bankruptcy threshold as low as possible, *i.e.*, to the boundary of the two regimes. This results in a higher debt level for some firms, and a lower debt level for others, than when there is no operational flexibility.

As mentioned above, the issue of whether operational hedging and financial hedging are substitutes has been considered by several previous studies. As shown in this paper, firms always perfectly hedge the financial risk exposures in their outstanding debt issues using financial instruments. Operational hedging may increase, decrease, or leave unchanged this exposure and, hence, the financial hedging associated with it. Therefore, it is not correct to claim that they are substitutes. On the other hand, operational hedging increases total tax shields for firms that utilize it before bankruptcy. This suggests that these two forms of hedging are, to some degree, compliments.

Figure 3.2 and Figure 3.3 compare the debt capacity and total tax shields of firms that have operational flexibility with those that do not. In both figures, Curve A represents the value for a firm with operational flexibility, and Curve B represents the value of a firm without operational flexibility.

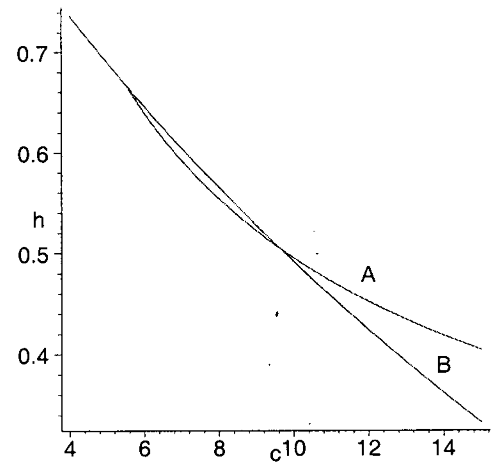

Figure 3.2: $h(c)$ as a function of c under symmetric information
Curve A represents the optimal h for a firm with operational flexibility. Curve B represents the optimal h for a firm without operational flexibility.

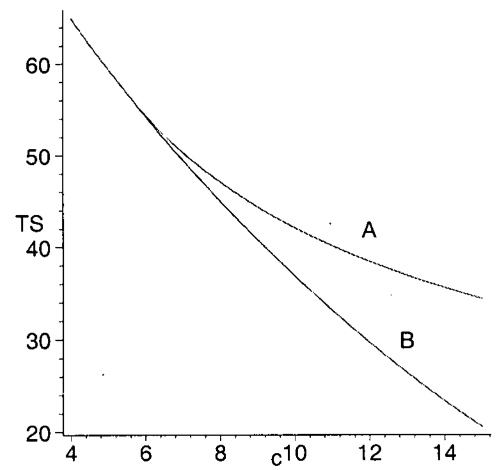

Figure 3.3: Tax shields as a function of production cost
Curve A represents the expected tax shields for a firm with operational flexibility. Curve B represents the expected tax shields for a firm without operational flexibility.

Chapter 4

The Effects of Asymmetric Information

In this chapter, I consider the case in which there exists asymmetric information about the firms' investment opportunities. For example, the manager of a gold mining firm might know more about the grade of ore or the mining technology needed for a specific mine than do other investors. It is assumed that only the manager observes the firm's marginal production cost c at time 0.

In his seminal paper, Ross [40] pointed out that firms can use their financial structure to signal firm quality. In this chapter, I investigate how asymmetric information affects firms' capital structure and hedging decisions. The observability of a firm's hedging policy and, therefore, whether it can be used as a signaling instrument, is debatable. Under the current accounting rules, firms are required to report their fiscal year-end outstanding hedging positions in the footnotes to annual reports. Firms differ greatly in their reporting practices. Some firms report all the details of their hedging activities, such as the amount of the underlying assets hedged, exercise price and expiration date for each individual contract. Others report only an aggregated average on such numbers. The enforcement of FAS 133 discussed in the introduction, however, will unify firms' reporting practices significantly. In this paper, I first examine the case in which hedging is observable and used as a signal together with the firm's capital structure. Then, I consider the case in which hedging is unobservable, and, therefore, capital structure is the only signal for firm quality.

4.1 The Effects of Asymmetric Information when Hedging is Observable

In this section, I assume that the investors infer c from the debt offer (b, h); their inference is denoted as $\hat{c}(b, h)$. Since there are two signals, b and h, and there is only one underlying attribute, c, to signal, the incentive compatibility condition alone gives rise to a family of feasible signaling schedules. The optimal signaling schedules are then determined by minimizing the signaling costs for all firm types.

4.1.1 Incentive compatibility condition

The manager's objective is still to maximize the sum of intrinsic equity value and dividend payment at time 0. That is, the manager's choice problem is

$$\max_{b,h}\{V_E(b,h,c) + L(b,h,\hat{c}(b,h)) - I\}, \tag{4.1.1}$$

subject to the full revelation condition,

$$\hat{c}(b,h) = c. \tag{4.1.2}$$

The manager evaluates the intrinsic equity value using his private information about c, but the proceeds from debt offering depend on the market's inference about c, $\hat{c}(b,h)$. The firm uses both components of the debt contract, *i.e.*, b and h, to signal the value of c. To induce the firm to signal its type truthfully, the market picks an inference schedule $\hat{c}(b,h)$, which satisfies the first order equation (4.1.3) and the full revelation condition (4.1.2). Since there exist two signals and only one variable defining firm type, the solutions to optimal b and h are not unique.

Denote $V_E(b,h,c) + L(b,h,\hat{c}(b,h))$ as $F(b,h,c,\hat{c})$. The first order condition can be written as

$$\frac{\partial F}{\partial b}db + \frac{\partial F}{\partial h}dh + \frac{\partial L}{\partial \hat{c}}d\hat{c} = 0. \tag{4.1.3}$$

Dividing both sides of the above first order equation by dc, and substituting in both the fully revealing condition (4.1.2) and the following equation,

$$\frac{d\hat{c}}{dc} = 1,$$

yields the incentive compatible condition for the firm,

$$\left.\left\{\frac{\partial F}{\partial b}\frac{db}{dc} + \frac{\partial F}{\partial h}\frac{dh}{dc} + \frac{\partial L}{\partial \hat{c}}\right\}\right|_{\hat{c}=c} = 0. \tag{4.1.4}$$

This can be written more conveniently as

$$\frac{dTS}{dc} = \frac{\partial TS}{\partial c} - \frac{\partial L}{\partial c}. \tag{4.1.5}$$

The incentive compability condition resembles the nonmimicry condition in discrete signaling problems. The incentive compability condition is only a necessary condition for a feasible signaling schedule. Any feasible condition must also satisfy the following second order condition, which resembles the single-crossing condition in the signaling model with a single signal.

$$\begin{aligned} &\frac{\partial^2 F}{\partial b^2}\left(\frac{db}{dc}\right)^2 + \frac{\partial^2 F}{\partial h^2}\left(\frac{dh}{dc}\right)^2 + 2\frac{\partial^2 F}{\partial b \partial h}\frac{db}{dc}\frac{dh}{dc} + \frac{\partial F}{\partial b}\frac{d^2 b}{dc^2} \\ &+ \frac{\partial F}{\partial h}\frac{d^2 h}{dc^2} + \frac{\partial^2 L}{\partial c \partial h}\frac{dh}{dc} + \frac{\partial^2 L}{\partial c \partial b}\frac{db}{dc} + \frac{\partial^2 L}{\partial c^2} \leq 0. \end{aligned} \tag{4.1.6}$$

Since the incentive compability condition must hold for every c, the total derivative of the LHS of first order condition (4.1.4) w.r.t. c is 0. Substituting this total derivative into equation (4.1.6), the second order condition then simplifies to

$$\frac{\partial^2 V_E}{\partial b \partial c}\frac{db}{dc} + \frac{\partial^2 V_E}{\partial h \partial c}\frac{dh}{dc} \geq 0. \tag{4.1.7}$$

I now discuss the incentive compability condition for firms in regime $p_b \geq p_c$ and regime $p_b \leq p_c$, respectively.

The Case $p_b \geq p_c$

In this case, the incentive compatibility condition is obtained by substituting equation (2.2.3), (3.2.4) and (3.2.3) into equation (4.1.5),

$$\frac{d(TS)}{dc} = \frac{1-\tau}{r}\left(\left(\frac{p_b}{p}\right)^{-\lambda_1} - \frac{\lambda_2}{\lambda_2 - \lambda_1}\left(\frac{p_c}{p}\right)^{-\lambda_1}\right) = f_1 > 0. \tag{4.1.8}$$

Compare this result with the symmetric information case. From equation (2.2.6), (A.2) and using the fact that $b^* = 0$, it follows that

$$\left[\frac{d(TS)}{dc}\right]_{sym} = \frac{\tau}{r}\frac{-\lambda_1}{1-\lambda_1}\left\{\left(\frac{p_b}{p}\right)^{*(-\lambda_1)} - \left(\frac{p_b}{p}\right)^{*(-1)}\right\} < 0. \tag{4.1.9}$$

Under symmetric information, the total tax shield decreases with the marginal production cost of the firm, since $(\frac{p_b}{p})^*$ is less than 1 and λ_1 is negative. Under asymmetric information, however, the total tax shield actually increases with the marginal production cost of a firm. The losses of tax shields for higher quality firms serve as the signaling costs for the separating equilibria in this model. It is shown in Appendix F that $p(b)$ is an increasing function of c in regime $p_b \geq p_c$. Firms with higher production costs will declare bankruptcy at a higher price level in this regime.

The Case $p_b \leq p_c$

In regime $p_b \leq p_c$, the equity value has two different expressions, V_E^1 and V_E^2. It is straight forward to show that the two expressions lead to the same incentive compatibility condition because TS and L have the same expressions for firms in production and firms that are shut down. Substituting equation (2.2.3), (3.2.18) and (3.2.17) into equation (4.1.5),

$$\frac{d(TS)}{dc} = \frac{1-\tau}{r}\frac{-\lambda_1}{\lambda_2-\lambda_1}\left(\frac{p_b}{p}\right)^{\lambda_2-\lambda_1}\left(\frac{p}{p_c}\right)^{\lambda_2} = f_2 > 0. \tag{4.1.10}$$

Equation (4.1.8) and (4.1.10) coincide at $p_b = p_c$. Again, compare the above expression with the case under symmetric information. Differentiating equation (E.6) with respective to c, using the fact that $p_c = c$ and p_b is independent of c in regime $p_b \geq p_c$ under symmetric information, yields

$$\left[\frac{d(TS)}{dc}\right]_{sym} = \frac{\tau}{r}\frac{\lambda_1}{1-\lambda_1}\left(\frac{p}{c}\right)^{\lambda_2}\left\{\left(\frac{p_b}{p}\right)^{*(\lambda_2-1)} - \left(\frac{p_b}{p}\right)^{*(\lambda_2-\lambda_1)}\right\} < 0. \tag{4.1.11}$$

Therefore, the inference about the monotonicity of tax shields in regime $p_b \leq p_c$ is similar to that obtained in regime $p_b \geq p_c$. Under symmetric information, the total tax shield decreases with the marginal production cost of a firm. Under asymmetric information, however, the total tax shield actually increases with the firm's marginal production cost. As before, the losses in tax shields for higher quality firms serve as the signaling costs for the separating equilibria in this model. It is shown in Appendix G that $p(b)$ is an increasing function of c in regime $p_b \leq p_c$ as well. Therefore, firms with higher production costs will declare bankruptcy earlier.

4.1.2 Optimal signaling schedule

Among all the incentive compatible contracts, $\{b(c), h(c)\}$, satisfying equation (4.1.8) and (4.1.10), the optimal contracts minimize the *ex ante* expected signaling costs for all firm types. Since the signaling cost is the loss of tax shields the optimal contracts are chosen to maximize the *ex ante* expected tax shields of all firms.

Assume that firms' production costs are distributed uniformly over the interval $[\underline{c}, \bar{c}]$. The optimal contracts are determined by the following dynamic optimization problem:

$$\max_{b(c),h(c)} \int_{\underline{c}}^{\bar{c}} TS(b,h,c)dc$$
$$= \max_{b(c),h(c)} \int_{\underline{c}}^{\bar{c}} \tau\left\{\frac{hp}{r-\mu} + \frac{b}{r} - \left[\frac{hp_b}{r-\mu} + \frac{b}{r}\right]\left(\frac{p_b}{p}\right)^{-\lambda_1}\right\},$$

subject to

$$\frac{d(TS)}{dc} = \frac{\partial TS}{\partial h}\frac{dh}{dc} + \frac{\partial TS}{\partial b}\frac{db}{dc} + \frac{\partial TS}{\partial c} = f_i, \quad i = 1, 2,$$

where $f_i, i = 1, 2$ are defined as in equation (4.1.8) and (4.1.10).

As discussed previously, the incentive compability condition is similar to the nonmimicry condition in discrete type signaling problems. The incentive compability condition, together with the proper second order conditions, ensures that each firm type has no incentive to mimic the firm types $c \pm \epsilon$, where ϵ is an arbitrarily small number.

Now I consider only half of the problem, in which each firm type has no incentive to mimic the firm type $c - \epsilon$, *i.e.*, a worse type firm does not mimic a better firm arbitrarily close to it. After I obtain the optimal solution, the second order condition will ensure that a better type firm does not mimic a worse firm either. The reason for transforming the problem as described above is that the first order condition must be signed since a corner solution obtains. Rewrite the constraint into the following format:

$$\begin{aligned} &\frac{\partial TS}{\partial h} q_h + \frac{\partial TS}{\partial b} q_b + \frac{\partial TS}{\partial c} \geq f_i, \\ &\frac{dh}{dc} = q_h, \\ &\frac{db}{dc} = q_b. \end{aligned} \tag{4.1.12}$$

The Hamiltonian is

$$H = TS(b, h, c) + \eta_1 (\frac{\partial TS}{\partial h} q_h + \frac{\partial TS}{\partial b} q_b + \frac{\partial TS}{\partial c} - f_i) + \eta_2 q_h + \eta_3 q_b. \tag{4.1.13}$$

The first order conditions in addition to the given constraints are

$$\begin{aligned} &H_{q_h} = \eta_1 \frac{\partial TS}{\partial h} + \eta_2 = 0, \\ &H_{q_b} = \eta_1 \frac{\partial TS}{\partial b} + \eta_3 = 0, \\ &\dot{\eta_2} = -H_h, \\ &\dot{\eta_3} = -H_b, \\ &\eta_1 \geq 0, \ \eta_1 (\frac{\partial TS}{\partial h} q_h + \frac{\partial TS}{\partial b} q_b + \frac{\partial TS}{\partial c} - f_i) = 0. \end{aligned} \tag{4.1.14}$$

The incentive compability constraint binds at every firm type except for the worst firm type with $c = \bar{c}$. Eliminating q_b, q_h, η_2 and η_3 from the above equations yields

$$(1-\eta_1)\frac{\partial TS}{\partial h} - \eta_1\frac{\partial f_i}{\partial h} = 0, \tag{4.1.15}$$

$$(1-\eta_1)\frac{\partial TS}{\partial b} - \eta_1\frac{\partial f_i}{\partial b} = 0. \tag{4.1.16}$$

It is shown in Appendix H that the two first order conditions, as shown in equation (4.1.15) and (4.1.16), cannot hold simultaneously, and the corner solution $b^* = 0$ is obtained. The optimal signaling schedule $h(c)$ is determined by substituting $b = 0$ into the differential equations (4.1.8) and (4.1.10), which are the constraints in the dynamic optimization problem. It is also shown in Appendix H that $h(c)$ is increasing in c, in contrast to the case under symmetric information where $h(c)$ is decreasing in c. This is the most important distinction between the solution under symmetric information and that under asymmetric information, and this implication is tested empirically in Chapter 5 below. Closed form solutions do not exist and a numerical example will be given in the next section.

To ensure that the solutions in the two regimes are well connected, it is necessary to check the monotonicity of $\frac{p_b}{p_c}$ as the boundary between the two is determined by $p_b = p_c$. At $b = 0$, the expressions for $\frac{p_b}{p_c}$ can be obtained using equation (3.2.4) and (3.2.18), as follows:

$$\frac{p_b}{p_c} = \frac{Q}{1-h}, \tag{4.1.17}$$

and

$$\left(\frac{p_b}{p_c}\right)^{\lambda_2 - 1} = \lambda_2 h. \tag{4.1.18}$$

$\frac{p_b}{p_c}$ is thus increasing in c since h is increasing in c. This result is the opposite to the solution under symmetric information. In that case, $\frac{p_b}{p_c}$ is decreasing in c since h is decreasing in c. This difference is significant for the following practical reason. Assume that $\bar{c}$ is relatively large, or $\bar{c} > c_{cutoff}$, so that there exist firms in regime $p_b \leq p_c$ under symmetric information. When $\frac{p_b}{p_c}$ is decreasing in c, there are two well defined regimes in which levered firms exercise or do not exercise the option on operational flexibility depending on which regime they are in. Under asymmetric information, however, all the firms will fall into the regime $p_b \leq p_c$. Empirically, this implies that firms seldom forego their operational flexibility.

The most important observation is that firms still totally hedge their commodity price risks by using commodity linked debt with no straight component, or by hedging a straight debt contract using a matching forward contract. Therefore, information asymmetry does not distort firms' incentive to hedge the financial risk exposures of their debt contracts.

4.1.3 Summary of results

The solution under asymmetric information is determined by substituting $b = 0$ into differential equations (4.1.8) and (4.1.10), and using the boundary condition $h(\bar{c}) = h^*(\bar{c})$. When $\bar{c} \geq c_{cutoff}$, then only equation (4.1.10) is used. A numerical example is given in the following section.

The results are summarized in the following proposition.

PROPOSITION 4 *Under asymmetric information, firms with operational flexibility offer perpetual debt with no fixed coupon payment. The varying component of the coupon rate, $h^*(c)$, is a strictly increasing function of the firm's marginal production cost, c.*

The price at which such a levered firm declares bankruptcy is increasing in the marginal production cost of the firm. The ratio of the bankruptcy price and the price level at which the levered firm shuts down or restarts its production, $\frac{p_b}{p_c}$, is an increasing function of c.

If the maximum value of c is no smaller than c_{cutoff}, all firms fall into the regime $p_b \leq p_c$.

The total tax shield, TS, is increasing in c.

In this model, firms with lower production costs forgo some tax shields by offering less debt to signal that they are of better types. The loss in tax shields can also be achieved by offering more debt and, as a result, increasing the probablity of bankruptcy. However, the above results show that this would be a more costly way to signal. Increasing the debt level will both increase tax shields under good economic conditions and increase the probability of financial distress under bad economic conditions.

4.1.4 A numerical example of the optimal signaling schedule

Figure 4.1 shows a numerical example using the following parameters: $\tau = 0.34, \mu = 0.04, \sigma^2 = 0.04$ and $r = 0.075$. The initial commodity price, p, is set to 10, and the marginal production costs of the firm are assumed to be uniformly distributed between 4 and 10. The figure shows the optimal signaling schedule, $h(c)$, as a function of c. The boundary condition $h(10)$ is inferred from the solution under symmetric information. As expected, $h(c)$ is increasing in c.

4.2 The Effects of Asymmetric Information when Hedging Is Not Observable

Now I turn to the case in which there exists asymmetric information about the firms' investment opportunities, but hedging is not completely observable. This

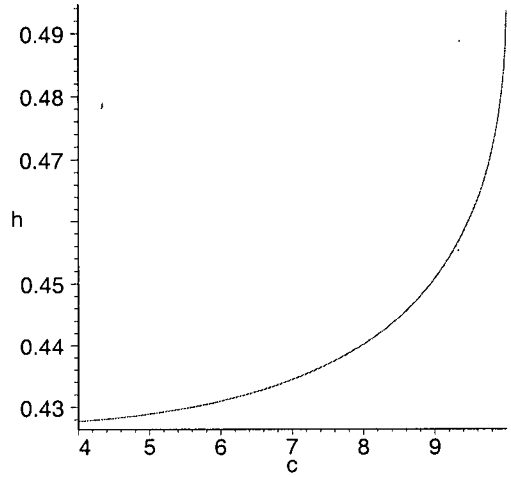

Figure 4.1: Optimal signaling schedule $h(c)$ as a function of c

assumption is motivated by the empirical observation that firms' hedging activities are not as transparent as firms' capital structure decisions. Firms are required to report their outstanding position in financial derivatives in the footnotes of their annual reports at the end of each fiscal year. Since it is relatively easy to undo their positions in the derivatives market, one can argue that firms' hedging positions are not credible signaling instruments.

In this section, I develop a model in which a levered firm uses the fixed component, b, as the signal of its underlying quality. The hedged position, h, is determined endogenously, but is not used as a signal. There are two stages in the decision process, and the problem is solved as follows. First, for given h, the signaling schedule, $b(c)$, is determined. Second, the manager of a firm makes the optimal choice of h, given that the signaling schedule will be solved as in stage 1.

It is important to note that the assumptions on the observability of b and h are not realistic, especially for contracts with derivatives. Suppose that a firm offers a financing package comprised of a perpetual coupon bond with a fixed coupon rate of B and a forward contract for h units of commodity that matures each unit of time at a forward price $\bar{p}$. It should then be assumed that B, the straight debt contract is observable, and h, the forward contract is not observable. This is different from assuming that b is observable and h is not because both the straight debt, B and part of the forward contract, $h\bar{p}$, enter the fixed component of the package. The variable component of the package is hp, and the fixed component of the package is $b = B - h\bar{p}$. I model b and h instead of B and h for the same reason discussed in footnote 1, chapter 2. It is also technically more tractible to model b and h rather than B and h. For the reasons stated above, this model serves only as a comparison with the case in which all financial policies are observable and illustrates the advantages of multiple signaling. Also, this model shows that other types of equilibria could exist depending on the information structure.

4.2.1 Incentive compatibility condition

It is assumed that the investors infer c from b; their inference is denoted as $\hat{c}(b)$. Since there is only one signal and one unknown, the incentive compatibility condition alone determines the signaling schedules. The manager's objective is still to maximize the sum of intrinsic equity value and dividend payment at time 0. That is, for any given h, the choice problem is

$$\max_{b}\{V_E(b,c) + L(b,\hat{c}(b)) - I\}, \tag{4.2.1}$$

subject to the fully revealing condition,

$$\hat{c}(b) = c. \tag{4.2.2}$$

The boundary condition is $b(\underline{c}) = 0$. Denote $V_E(b,c) + L(b,\hat{c}(b))$ as $F(b,c,\hat{c})$. The first order condition is derived as follows:

$$\frac{\partial F}{\partial b} + \frac{\partial L}{\partial \hat{c}}\frac{d\hat{c}}{db} = 0. \tag{4.2.3}$$

Substituting both the fully revealing condition (4.2.2) and the total derivative of it with respect to c,

$$\frac{d\hat{c}}{db}\frac{db}{dc} = 1,$$

into equation (4.2.3) yields,

$$\frac{\partial TS}{\partial b}\frac{db}{dc} + \frac{\partial L}{\partial c} = 0. \tag{4.2.4}$$

Any feasible solution must also satisfy the second order condition,

$$\frac{\partial^2 TS}{\partial b^2}\left(\frac{db}{dc}\right)^3 + 2\frac{\partial^2 L}{\partial b \partial c}\left(\frac{db}{dc}\right)^2 + \frac{\partial^2 L}{\partial b \partial c}\frac{d^2 b}{dc^2} - \frac{\partial L}{\partial c}\frac{d^2 b}{dc^2} \geq 0. \tag{4.2.5}$$

I now discuss the incentive compability condition for firms in regime $p_b \geq p_c$ and regime $p_b \leq p_c$, respectively.

The Case $p_b \geq p_c$

In this case, the incentive compatibility condition is obtained by substituting the expressions for TS, p_b and L, given in equations (2.2.3), (3.2.4) and (3.2.3), into equation (4.2.4),

$$\frac{\partial(TS)}{\partial b}\frac{db}{dc} + \frac{\partial TS}{\partial c} = \frac{1-\tau}{r}\left(\left(\frac{p_b}{p}\right)^{-\lambda_1} - \frac{\lambda_2}{\lambda_2 - \lambda_1}\left(\frac{p_c}{p}\right)^{-\lambda_1}\right) = f_1 > 0. \tag{4.2.6}$$

Note that the right hand side is the same as the right hand side of equation (4.1.8). The left hand side will turn out to be the same as well because, as will be shown later, the first order condition for h is $\frac{\partial TS}{\partial h} = 0$. Therefore, the incentive compatibility condition has a similar format for both the case with two signals and the current case with only one signal. The second order conditions to the incentive compatibility condition, however, are very different.

The Case $p_b \leq p_c$

In regime $p_b \leq p_c$, the equity value has two different expressions, V_E^1 and V_E^2. It is straightforward to show that the two expressions lead to the same incentive compatibility condition because TS and L have the same expressions for firms in production and for firms that are shut down. Substituting equation (2.2.3), (3.2.18) and (3.2.17) into equation (4.2.4) yields

$$\frac{\partial(TS)}{\partial b}\frac{db}{dc} + \frac{\partial TS}{\partial c} = \frac{1-\tau}{r}\frac{-\lambda_1}{\lambda_2 - \lambda_1}\left(\frac{p_b}{p}\right)^{\lambda_2-\lambda_1}\left(\frac{p}{p_c}\right)^{\lambda_2} = f_2 > 0. \tag{4.2.7}$$

Once again, the right hand side is the same as in equation (4.1.10) and the left hand side will turn out to be the same after I determine the first order condition for firms' hedging decisions in the following section.

4.2.2 The firm's hedging decision

Given that the market will infer the true production cost c from the signaling schedule determined by equation (4.2.4), the firm chooses h to maximize its total firm value, or, equivalently, the total expected tax shields. The choice problem is

$$\max_h \; TS. \tag{4.2.8}$$

The first and second order conditions for the above maximization problem are, respectively,

$$\frac{\partial TS}{\partial h} = 0, \tag{4.2.9}$$

and

$$\frac{\partial^2 TS}{\partial h^2} \leq 0. \tag{4.2.10}$$

I can now substitute equations (2.2.3), (3.2.4) and (3.2.18) into equation (4.2.9) to obtain the first order conditions for firms in regime $p_b \geq p_c$ and $p_b \leq p_c$, respectively. They are

$$\frac{\tau p}{r-\mu}\left\{1-\left(\frac{p_b}{p}\right)^{1-\lambda_1}-(1-\lambda_1)\left(\frac{p_b}{p}\right)^{1-\lambda_1}\left(\frac{1}{1-h}-\frac{c}{b+c}\right)\right\}=0, \tag{4.2.11}$$

and

$$\frac{\tau p}{r-\mu}\left\{1-\left(\frac{p_b}{p}\right)^{1-\lambda_1}-\frac{1-\lambda_1}{\lambda_2(\lambda_2-1)}\left(\frac{p_b}{p}\right)^{\lambda_2-\lambda_1}\left(\frac{p}{c}\right)^{\lambda_2-1}\frac{1}{h+\frac{b}{p_b}}\right\}=0. \tag{4.2.12}$$

Using equation (2.2.3), the first order equations (4.2.11), (4.2.12), (4.2.6) and (4.2.7), I derive the conditions under which the second order condition (4.2.10) holds for regime $p_b \geq p_c$ and $p_b \leq p_c$, respectively. In regime $p_b \geq p_c$,

$$\begin{aligned}\frac{\partial^2 TS}{\partial h^2} &= -\frac{\tau p}{r-\mu}\frac{1-\lambda_1}{1-h}\left(\frac{p_b}{p}\right)^{1-\lambda_1}\left\{1+\frac{1-\lambda_1}{1-h}-(-\lambda_1)\frac{Qc}{p_b}+\frac{1}{1-h}\left(1-\frac{Qc}{p_b}\right)\right\}\\ &< -\frac{\tau p}{r-\mu}\frac{1-\lambda_1}{1-h}\left(\frac{p_b}{p}\right)^{1-\lambda_1}\left\{1+\frac{1-\lambda_1}{1-h}\frac{b}{b+c}+\frac{1}{1-h}\left(1-\frac{Qc}{p_b}\right)\right\}\\ &< 0.\end{aligned} \tag{4.2.13}$$

Thus, the second order condition is satisfied for all parameter values in regime $p_b \geq p_c$. In regime $p_b \leq p_c$,

$$\begin{aligned}\frac{\partial^2 TS}{\partial h^2} &= -\frac{\tau p}{r-\mu}\frac{1}{p_b}\frac{\partial p_b}{\partial h}\left\{1-\lambda_1-\left(1-\left(\frac{p_b}{p}\right)^{1-\lambda_1}\right)\left(1+\frac{hp_b}{hp_b+b}\right)\right\}\\ &< -\frac{\tau p}{r-\mu}\frac{1}{p_b}\frac{\partial p_b}{\partial h}\left\{-\lambda_1-\frac{hp_b}{hp_b+b}\right\}.\end{aligned} \tag{4.2.14}$$

A sufficient, but not necessary, condition for the above expression to be negative is that $2\mu \geq \sigma^2$.

It is easy to show that in any equilibrium satisfying equation (4.2.6), (4.2.11), (4.2.7) and (4.2.12), b is a decreasing function of c. As shown before, $\frac{\partial TS}{\partial b} \leq 0$

if $\frac{\partial TS}{\partial h} = 0$. Since $\frac{\partial TS}{\partial c}$ is negative, the monotonicity of b is obvious from equation (4.2.6) and (4.2.7). Other properties of the equilibria are difficult to derive due to the complexity of the model. A numerical example of the signaling schedule is shown in the following section, and $h(c)$ is shown to be increasing in c.

The loss in tax shields, which is the signaling cost in this model, results from the fact that better firms are forced to use non-negative b, the straight component of debt, to signal their type. Given that the market interprets its type from b, the firm's optimal response is to choose a smaller, possibly negative, h. This will decrease its total debt commitment and, as a result, decrease the probability of bankruptcy.

4.2.3 A numerical example of the signaling schedule

A numerical example using the following parameters: $\tau = 0.34, \mu = 0.04, \sigma^2 = 0.04$ and $r = 0.075$, is discussed in this section. The initial commodity price, p, is set to 10, and the marginal production costs of the firm are assumed to be uniformly distributed between 4 and 10. The boundary conditions are $b(10) = 0$, and $h(10)$ as inferred from the solution under symmetric information. Figure 4.2 and Figure 4.3 show the optimal hedging schedule, $h(c)$, and the optimal signaling schedule, $b(c)$, as functions of c. As expected, $b(c)$ is decreasing in c. In this example, and in all other examples using different parameters that were considered, $h(c)$ is always increasing in c.

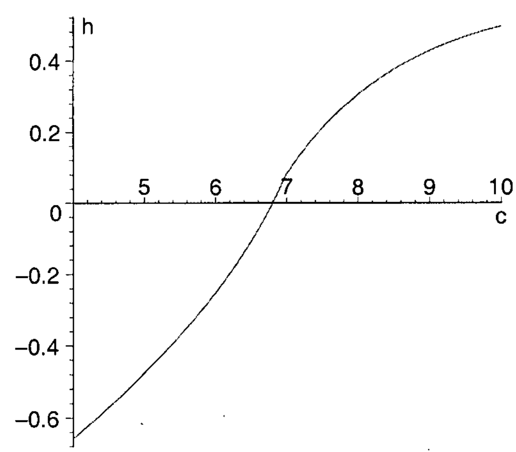

Figure 4.2: The optimal hedging schedule $h(c)$ as a function of c

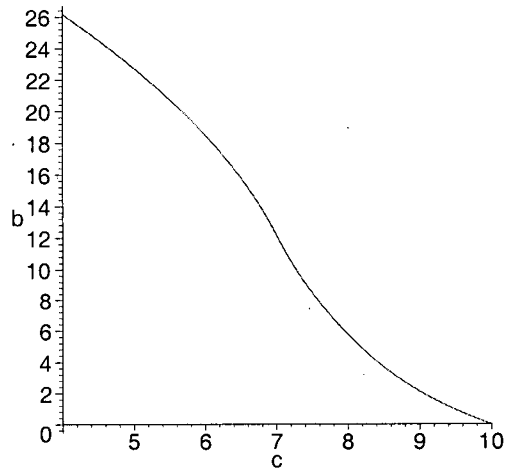

Figure 4.3: The optimal signaling schedule $b(c)$ as a function of c

4.3 Discussion of the Use of Multiple Signals in Reducing Signaling Costs

In this section, the above results are compared with the case in which both b and h are observable and can be used as signals. Under symmetric information, firms prefer to use perfectly hedged commodity linked debt or derivatives contracts and, as a result, set the fixed component of the coupon rates to zero. When there exists asymmetric information, however, firms' optimal financial decisions depend on what information can be disclosed precisely to the public. In the case with both debt and hedging observable and available as signals, it is still optimal for firms to hedge perfectly their debt exposures using commodity linked debt or derivatives contracts. In the case in which hedging is not observable completely, however, firms are forced to use debt as the only signal.

To compare the signaling costs in the two cases, calculate the total tax shields, TS, defined by equation (2.2.3) of all firm types using the signaling schedules shown in Figure 3.2 (the symmetric information case), Figure 4.1 (the two signal case) and Figures 4.2 and Figure 4.3 (the one signal case). As shown in Figure 4.4, the option to use both components of the contracts as signals lowers the signaling costs in the separating equilibria. In Figure 4.4, Curves A, B and C represent the value of tax shields under symmetric information, asymmetric information with two signals and asymmetric information with one signal, respectively. Curves B and C are upward sloping and Curve A is downward sloping, as expected. The three curves coincide at maximum production cost as the solution under symmetric information serves as the boundary condition for the solutions under asymmetric information.

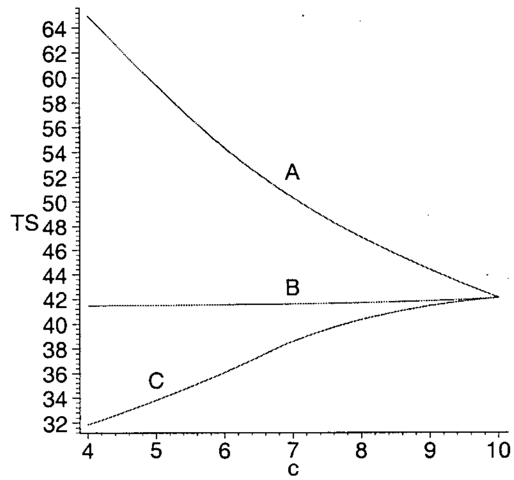

Figure 4.4: Value of total tax shields as functions of c
Curve A represents the value of total tax shields under symmetric information. Curve B represents the value of total tax shields under asymmetric information with two signals. Curve C represents the value of total tax shields under asymmetric information with one signal.

Under symmetric information, firms' hedged debt positions, h^*, are decreasing in their production costs, c. Under asymmetric information, however, h^* is increasing in c. Since the solutions for h^* obtained in different models join at $c = \bar{c}$, firms hold larger hedging positions under symmetric information than they do under asymmetric information. In the two-signal case, firms do not offer any unhedged debt. In the one-signal case, in contrast, firms are forced to use unhedged debt as the signaling instrument. Asymmetric information is, therefore, another potential reason why firms do not fully hedge their financial risk exposures. Higher signaling costs are incurred in the case when hedging is not completely observable.

Chapter 5

Empirical Implications and Tests

5.1 Empirical Implications

The models derived in the last two chapters provide testable implications. In this paper, I assume that firms produce 1 unit of the commodity per unit time. Therefore, all variables should be normalized by total production. I assume also that firms' revenue is variable due to the financial risks involved in the price process, and firms are differentiated by their production costs. Natural candidates for firms with this type of production process are firms in the mining industry, precious metals or oil and gas. Foreign exchange risks affect firms' revenue in ways similar to commodity price risks. The interpretation of c, however, becomes ambiguous in the foreign exchange case. Data collected for North American gold producing firms are used to test the empirical implications of the paper.

A time period of one year is considered since data are extracted from firms' annual reports. Three crucial variables from the models are h, b and c. The normalized h, denoted as H, is the percentage of production hedged for any given year t. Most firms report an average of total cash cost, denoted as CC, per troy ounce of gold for each fiscal year, calculated according to the standard outlined by *The Gold Institute*. CC is used as a proxy for production cost c. The proxy for b, however, is not as straightforward.

Most authors use long-term debt, defined as the portion of long term debt that matures in more than a year, as a proxy for firms' debt liabilities in their empirical tests. In this paper, the relevant variable, b, represents the UNHEDGED portion of debt payment due DURING year t. Variable b is not directly observable because a forward contract consists of a fixed component as well. Suppose that a firm offers a debt package comprised of a straight zero coupon bond with a face value B and a forward contract on h ounces of gold which both mature in year t. The relationship between b and B can be represented by $B = h\bar{p} + b$[1], where

[1]For the case of hedging using commodity linked bonds, B simply represents the sum of the values of the straight and commodity linked bonds.

$\bar{p}$ represents the forward price in a particular contract. Although data on B and h are readily available, the estimation of b is practically impossible because firms often use nonliear contracts such as put and call options, where the interpretation of the forward price is not feasible. A proxy such as the spot price of gold p could be used for $\bar{p}$, and $p = \bar{p} + error$. However, the error term is likely to be too large to make the statistical analysis meaningful.

The models make specific predictions about the relationship between h and c, b and c. The relationship between h and b can also be predicted in most cases. But, due to the difficulty in estimating b, the main prediction I test here is the relationship between h and c. The model under symmetric information predicts that $H = h$ decreases in $CC = c$, while the model under asymmetric information predicts that $H = h$ increases in $CC = c$.

By assuming $\bar{p}$ to be constant for all firms, I can also make predictions about the relationship between H and B and about the relationship between B and CC. The empirical implications of the models for the relationships among hedge ratios, debt ratios and cash costs are summarized in Table 5.1. The predictions for the relationship between debt ratio and cash costs are shown in Table 5.2. The symbol 'x' represents no correlation because b is predicted to be a constant. the question mark, '?', indicates that the correlation can not be signed by a particular model[2].

Table 5.1: Empirical predictions for the relationship between hedge ratio H and b, H and B, and H and CC

H	b	B	CC
Sym. Info.	x	+	-
Two-Signal	x	+	+
One-Signal	-	?	+

5.2 Empirical Tests

The data types used in testing corporate hedging decisions fall into two categories: Survey data or annual report (or financial statement) data. Only in recent years, since more detailed and accurate reporting practices have been adopted by more and more firms, has the second type of data become available. In this paper, I use annual report data for North American gold mining firms to test the empirical

[2]The relationship between B and CC observed in the data should be interpreted with caution. The assumption about a firm's capital structure decision is highly simplified in this paper, and riskless debt is not considered in the models. The predictions with respect to the relationship between B and CC are made assuming that $\bar{p}$ is a fixed constant for all firms, which is likely violated.

Table 5.2: Empirical predictions for the relationship between b and CC, and B and CC

	b and CC	B and CC
Sym. Info.	x	-
Two-Signal	x	+
One-Signal	-	?

predictions of the models. As mentioned above, one reason to choose this set of data is that the production cost c has a natural accounting counterpart for these firms. But, more importantly, gold firms face common financial price risks, *i.e.* gold price fluctuations, and most gold firms do not have a diversified business portfolio. Therefore, these firms fit very well into the production type assumed in this paper.

One drawback of this data set is that most gold firms have gold mines all over the world and, therefore, they also face substantial foreign exchange and political risks. Firms can hedge foreign exchange risks by using foreign currency denominated debt or foreign currency derivatives. Since some currencies, Canadian dollars for example, are highly correlated with commodity prices, the hedging of foreign exchange risks and commodity price risks can interact with each other. Agnico-Eagle is an example of a firm which actively hedges its foreign exchange exposure, but has no outstanding contracts for commodity risks hedging. It is, however, beyond the scope of this paper to discuss the impact of foreign exchange hedging on the tests.

Tufano [45] also uses data for North American gold mining firms to test a variety of corporate hedging theories. My data set is different from his in two important aspects. The first, and the most important, difference is that Tufano uses survey data compiled by Ted Reeve, a Canadian equity analyst who covers precious metals firms, while I use annual report data. Secondly, his sample period ranges from 1991 to 1993, while mine ranges from 1993 to 1998. Since both studies cover some common regression variables, the results will be compared and discussed later in this section.

5.2.1 Data description

The primary criterion for a firm-year to be included in the data set is that the firm's annual reports available on SEDAR[3] for Canadian firms and EDGAR[4] for US firms. Canadian gold firms are listed under the category 'Gold and precious metals' on SEDAR. US gold firms are listed under the category 'Gold and Silver Ores' on EDGAR, with SIC code 1040.

Construction of the Test Variables

In the footnotes to the annual report of fiscal year $t - 1$, relevant information can be found about the firms' hedging programs in place. Instead of aggregating firms' total hedging positions for the subsequent three years as in Tufano [45], I use only the information about hedging contracts which mature in year t. *Ex post* production in year t is then used as a proxy for expected production of year t at the end of year $t - 1$. Tufano uses firms' estimated production from the surveys instead. Table 5.3 shows the typical format in which a firm reports its hedging activities in its annual report. The information regarding commodity linked debt issues can be found in the footnotes as well.

A spot deferred forward contract is a forward sale which will accrue contango until the intended delivery date of the contract. Contango is the difference between the spot price and the higher forward price. The rate at which contango accrues will be determined by interest rates less gold lease rates existing at the time of each rollover. It is, therefore, similar to a rolling forward contract. The four types of contracts shown in Table 5.3 are the most commonly used hedging instruments by firms in the sample. Of the 61 firm-years that have nonzero hedge ratios, 52 use forward and/or spot deferred contracts, 30 use option contracts and 11 use gold loans.

A measure of the equivalent amount of gold hedged in each contract is then constructed as in Tufano [45]. The equivalent amount hedged is calculated as the product of the underlying amount hedged and the delta of the contract. The delta of a derivative security is defined as the rate of change of its price with respect to the price of the underlying asset. Some simplifying assumptions are made due to lack of information on some details of the hedging contracts. Put and call options are assumed to be European options. It is assumed also that one twelfth of the

[3]SEDAR is the System for Electronic Document Analysis and Retrieval, the electronic filing system for the disclosure documents of public companies and mutual funds across Canada. All Canadian public companies and mutual funds are generally required to file their documents in the SEDAR system.

[4]EDGAR, the Electronic Data Gathering, Analysis, and Retrieval system, performs automated collection, validation, indexing, acceptance, and forwarding of submissions by companies and other who are required by law to file forms with the US Securities and Exchange Commission (SEC).

Table 5.3: The hedging positions outstanding at Dec 31, 1996 for Placer Dome Inc

Expiration Date	1997	1998	1999	2000	2001	2002+
Spot deferred contracts						
Amount hedged (000's oz)	26	89	257	677	384	290
Averaged price ($/oz)	502	427	408	407	405	401
Fixed forward contracts						
Amount hedged (000's oz)	558	733	740	243	163	-
Averaged price ($/oz)	449	464	500	501	465	-
Put options purchased						
Amount hedged (000's oz)	62	-	-	-	-	-
Averaged price ($/oz)	440	-	-	-	-	-
Call options sold						
Amount hedged (000's oz)	93	-	-	-	-	-
Averaged price ($/oz)	500	-	-	-	-	-

underlying amount of each contract matures at the end of each month unless stated otherwise in the annual report.

Time series of gold prices, US and Canadian T-bill rates and gold lease rates are needed to calculate the value of delta for each contract. I obtained gold price and T-bill rate time series from DATASTREAM. Delta is estimated using the following formula for dividend paying European call and put options, respectively:

$$\Delta_{call} = e^{-q(T-t)}N(d_1), \tag{5.2.1}$$

and

$$\Delta_{put} = e^{-q(T-t)}[N(d_1) - 1], \tag{5.2.2}$$

where q is gold lease rate and the standard Black-Scholes notation applies. Implied volatility is estimated using the closing prices from daily data over the most recent 90 days.

The one year gold lease rate series was obtained from KITCO. It is around $1 - 3\%$ for the time period considered in this paper. Gold loans and fixed or spot deferred forwards have a delta of -1. The delta for the put and call options in Table 5.3 are -0.9739 and 0.0000, respectively. The delta for the put is very close to -1 because it is well in-the-money, and the delta of the call is almost 0 because the contract is well out-of-money. The differences in the value of delta for different contracts show why it is important to adjust the hedged amounts using delta. The equivalent amount hedged for its 1997 production by Placer Dome Inc is, then, $26 + 558 + 62 * 0.9739 + 93 * 0.0000 = 644$ thousand ounces.

The hedge ratio for year t, H, is computed by dividing the equivalent amount hedged by the total amount of gold produced in year t as reported in its annual report. For example, the production for Placer Dome in 1997 is 2,563 thousand ounces, so the hedge ratio for 1997 is 25.1%.

The Gold Institute is an international trade association of companies that mine and refine gold, manufacturers of gold products, and the world's leading bullion banks and gold dealers. As stated on its website, its member mining companies produce more than 75% of all North American gold and account for 22% of world gold production. To facilitate comparisons among companies in the gold industry, *the Gold Institute* developed the *Gold Production Cost Standard.* According to this standard, total cash costs include all operating costs (including overhead) at mine sites, royalties and production taxes but exclude reclamation, depreciation and amortization. Total cash cost per ounce of gold produced, CC, is used as a proxy for production cost c.

The current portion of long-term debt outstanding at the end of year $t-1$, defined as the portion that matures within year t, was extracted from COMPUSTAT. This variable must be normalized before it can be used as the proxy for B for year t. As suggested by the basic setup of the models in this paper, it is appropriate to normalize by the product of production and spot gold price. Denote total assets as TA. Denote long-term debt as LTB. Define $Cash$ as working capital plus current portion of long-term debt. That is, $Cash$ represents working capital with the current portion of long-term debt not included in current liabilities. There are two hypotheses regarding $Cash$. The riskless portion of debt should be positively related to $Cash$, while firms with less $Cash$ should hedge more. These three variables, TA, LTB and $Cash$, were also obtained from COMPUSTAT. Both LTB and $Cash$ are normalized by the product of production and the spot price of gold. Variables TA and LTB are included in some of the tests to make the results comparable to related studies. Another normalization, dividing by TA, was also investigated, but the results (not reported) were not changed significantly.

Summary Statistics

There are a total of 29 firms in the sample; 14 are Canadian and 15 are US firms. The total number of firm-years is 81, 29 of which are Canadian and 52 of which are US. This asymmetry in firm-years between Canadian and US firms is due to the fact that EDGAR and SEDAR started at different times. The time period of the sample ranges from 1993 to 1998, with some firms having data for all five years and some having data for only a single year. There are 6 firms which did not have any hedging program in place for any of the years covered in the sample. There is one negative observation, with $H = -56.6\%$. About 25% of the firm-years have a zero hedge ratio. There are also five observations with H greater than 100%, the highest being 241%. Table 5.4 shows the distribution of the hedge ratio, H, among

Table 5.4: Distribution of hedge ratios

Hedge Ratio $H(\%)$	Percentage of Firm-years	Number of Firm-years
<0	1.2	1
Exactly 0	24.7	20
0.1-10	14.8	12
10-20	12.3	10
20-30	8.6	7
30-40	4.9	4
40-50	4.9	4
50-60	3.7	3
60-70	4.9	4
70-80	6.2	5
80-90	2.5	2
90-100	4.9	4
>100	6.2	5
		81
Mean 35.0		
Median 17.0		
Standard deviation 48.3		
Minimum -56.6		
Maximum 241.0		

firm-years.

The firm characteristics, B, LTB, TA and CC, defined in the previous section, are then compared between two groups segmented by the levels of their hedge ratios. The first group includes the firm-years with H exactly 0 and the one observation with negative H. The single firm-year with a negative hedge ratio is also listed separately in the table. The firm-years with positive hedge ratios form the second group. The t-statistics and p-value of the differences of means between the two groups are reported in Table 5.5. The group of firm-years with positive hedging positions have significantly greater total assets and more current debt. They also have higher total cash costs, but this difference is not statistically significant.

The correlations among different variables are shown in Table 5.6. As with previous studies, it appears from this sample that hedging exhibits economies of scale; larger firms tend to hedge more. The hedge ratio, H, is positively correlated with cash costs, CC, as suggested by the models under asymmetric information. The current portion of long-term debt, B, however, only marginally correlated

Table 5.5: Summary statistics of subgroups

	Statistics of Characteristics Conditional on Hedge Ratio						
	Negative (N=1)	None (N=20)		Hedging (N=60)		Tests of Differences	
	Value	Mean	Std. Dev.	Mean	Std. Dev.	t-ratio	p-value
H	-56.6%	0%	0%	48.1%	49.3%		
TA (m$)	590.4	308.7	454.6	816.0	1,065.7	2.99	**0.00**
CC ($/oz)	261	228	62	244	57	1.00	0.32
B (%)	4.98	3.2	9.0	12.5	36.1	1.85	**0.07**
LTB (%)	297.2	55.5	96.6	78.9	94.6	0.99	0.33
Cash (%)	-134.6	121.6	229.7	68.6	80.3	-1.08	0.29

with cash costs. There exist strong positive correlations among hedge ratio, H, and current portion of long-term debt, B, and long-term debt, LTB.

Table 5.6: The cross sectional correlation coefficients

Correlation Coefficient	H	B	LTB	CC	TA	Cash
Hedge Ratio H	1.00	0.19	0.19	0.20	0.11	-0.12
Current Debt B		1.00	0.33	-0.04	-0.09	0.19
Long-term Debt LTB			1.00	0.13	0.02	0.21
Cash Cost CC				1.00	-0.26	0.20
Total Assets TA					1.00	-0.18
Cash						1.00

5.2.2 Regression results

OLS regressions

To begin the analysis, I first examine the relationship between hedge ratio and the independent variables using Ordinary Least Squares. The regression results, with t-statistics computed using both OLS and White's heteroscedastic-consistent standard errors, are shown in Table 5.7. Consistent with the predictions under the asymmetric information model, the coefficient on cash cost is significantly positive. The hedge ratio has a statistically significant positive relationship with debt ratio as well.

Using the Jarque-Bera test, the hypothesis that the OLS residuals are normally distrubuted can be rejected (p-value $< 0.1\%$). This suggests that the OLS

Table 5.7: Determinants of hedge ratio: OLS estimations

$H_{i,t} = \beta_0 + \beta_1 * TA_{i,t} + \beta_2 * B_{i,t} + \beta_3 * CC_{i,t} + \beta_4 * Cash_{i,t} + e_{i,t}$			
	Coefficient	Standard p-value	White's p-value
Intercept	-0.0757	0.753	0.674
TA	0.0002	0.200	0.168
B	0.0275	**0.037**	**0.005**
CC	0.0035	**0.052**	**0.010**
Cash	-0.2304	0.160	0.125
$R^2 = 0.1308$		Adjusted R^2=0.0851	

regression model is misspecified.

Tobit Regressions

As noted in Tufano [45], there are three main issues in applying standard regression techniques to analyze a data set like the one shown in Table 5.4. First, the sample size is relatively small. Second, the sample is composed of a set of unbalanced panel data, where both heteroskedasticity and autocorrelation are likely because firm values are correlated from year to year. Third, hedge ratios for 20 out of 81 firm-years are exactly 0[5] which indicates censorship of the data set. This censorship could be caused by the possible negative impact on a firm's reputation by using derivatives to increase rather than reduce risk exposures. Instead, firms could use other alternatives, such as increasing exploration activities, to achieve higher exposures to gold price fluctuations. Tobit regressions are also used in this paper to analyze the censored data.

Rigorous treatment of autocorrelation in the estimation of panel data Tobit models is prohibited by the small sample size. Instead, I analyze the data year by year following Tufano [45] and Haushalter[21]. In the six-year period covered by the data set, the number of firms in each year is 1, 5, 7, 14, 26 and 28 from 1993 to 1998, respectively. Only the results for the last two years are reported.

The effects of heteroskedasticity are accounted for by using a Gibbs sampling Tobit regression method [25, 18, 10]. The Maximum Likelihood Estimation results assuming homoskedasticity are also presented. The statistical package developed

[5] Unlike the survey data in Tufano [45] and Haushalter [21], where all hedge-ratios are non-negative, one negative hedge-ratio is observed in the data. The models under asymmetric information in this paper do predict possible negative hedge-ratios. In the regression results reported here, I include the data point by using 0 as the hedge ratio and ignoring the negative value observed. I also run all the regressions excluding the data point and include the data point by using the negative value in a variation of Tobit model in the simple case without heteroskedasticity. Neither affects the regression results and, therefore, are not reported.

Table 5.8: Determinants of hedge ratio: pooled sample

$H_{i,t} = \beta_0 + \beta_1 * TA_{i,t} + \beta_2 * B_{i,t} + \beta_3 * CC_{i,t} + \beta_4 * Cash_{i,t} + e_{i,t}$				
	Maximum Likelihood Tobit Estimates		Bayesian Heteroskedastic Tobit Gibbs Estimates	
	Coefficient	p-value	Coefficient	p-value
Intercept	-0.6814	**0.05**	-0.5522	**0.04**
TA	0.0001	**0.05**	0.0001	**0.02**
B	0.5501	**0.01**	0.4934	**0.02**
CC	0.0037	**0.01**	0.0029	**0.00**
Cash	-0.1365	**0.02**	-0.1041	**0.06**

by LeSage [25] is used in this paper. The main prediction I wish to test here is the relationship between hedge ratio, H, and cash cost, CC. I am also interested in the relationship between H and capital structure variables, B and LTB, and total assets TA. The estimation of the determinants for B should be interpreted with caution for reasons noted in Footnote 2.

Table 5.8 shows the regression results of tests for the determinants of hedge ratios for the pooled sample. From Table 5.6, the correlation between the current portion of long-term debt, B, and total assets, TA, and cash costs, CC, are relatively low. The tobit regression is also performed by excluding TA and/or $Cash$. The results are not reported because they are essentially the same as those in Table 5.8. Table 5.9 and Table 5.10 show the regression results for the segmented samples for fiscal year 1997 and 1998 respectively.

For the pooled sample, the regression coefficient on cash cost is significantly positive, as predicted by the models under asymmetric information. Firms with higher production costs tend to hedge more. Also consistent with the prediction of the two-signal model under asymmetric information, the coefficient on current debt ratio, B, is significantly positive. The coefficient on firm size, TA, is also significantly positive, suggesting the existence of economies of scale. The coefficient on $Cash$ is strongly negative, which indicates that firms with more cash and, hence, lower probability of financial distress, hedge less.

To test the robustness of the results from the pooled sample, segmented data sets for fiscal year 1997 and 1998 are used to eliminate the effects of possible autocorrelation within each firm. However, there are only 26 and 28 data points in the segmented data sets, with 7 and 8 of them censored, respectively. The estimates are likely to be imprecise because five parameters need to be estimated in each regression. Therefore, it is not surprising that only one coefficient in the regression for 1998 data set is significant, and the significance disappears after the adjustment of heteroskedasticity. For year 1997, however, the coefficients on firm

Table 5.9: Determinants of hedge ratio: 1997

$H_i = \beta_0 + \beta_1 * TA_i + \beta_2 * B_i + \beta_3 * CC_i + \beta_4 * Cash_i + e_i$				
	Maximum Likelihood Tobit Estimates		Bayesian Heteroskedastic Tobit Gibbs Estimates	
	Coefficient	p-value	Coefficient	p-value
Intercept	-0.6255	0.48	-0.6776	**0.09**
TA	0.0002	0.13	0.0002	**0.05**
B	0.0275	0.96	0.0617	0.93
CC	0.0035	0.11	0.0037	**0.03**
Cash	-0.2304	0.11	-0.2369	**0.06**

Table 5.10: Determinants of hedge ratio: 1998

$H_i = \beta_0 + \beta_1 * TA_i + \beta_2 * B_i + \beta_3 * CC_i + \beta_4 * Cash_i + e_i$				
	Maximum Likelihood Tobit Estimates		Bayesian Heteroskedastic Tobit Gibbs Estimates	
	Coefficient	p-value	Coefficient	p-value
Intercept	0.0639	0.92	0.0658	0.91
TA	0.0001	0.39	0.0001	0.33
B	0.4330	**0.07**	0.4288	0.14
CC	0.0006	0.82	0.0003	0.91
Cash	-0.0746	0.29	-0.0633	0.43

Table 5.11: Determinants of debt ratio

$B_{i,t} = \beta_0 + \beta_1 * TA_{i,t} + \beta_2 * Cash_{i,t} + \beta_3 * CC_{i,t} + e_{i,t}$				
	Maximum Likelihood Tobit Estimates		Bayesian Heteroskedastic Tobit Gibbs Estimates	
	Coefficient	p-value	Coefficient	p-value
Intercept	0.0716	0.77	0.0356	0.70
TA	-0.0000	0.63	-0.0000	0.55
Cash	0.0496	0.24	0.0018	0.31
CC	-0.0006	0.54	-0.0001	0.84

size, cash costs and $Cash$ are all statistically significant, with the same signs as the estimated coefficients for the pooled sample. These results suggest that the strong relationships shown in Table 5.8 cannot be the results of autocorrelation alone.

Table 5.11 shows the regression results of the test for determinants of debt ratio using the pooled sample. I am mainly interested in the relationship between debt ratio and cash costs. Variable $Cash$ is included in the hope of capturing the effects of riskless debt. As shown in the table, however, none of the correlation coefficients are significant. Although this potentially could be interpreted as supporting evidence for the one-signal model under asymmetric information, I would be reluctant to draw any conclusions for reasons stated in Footnote 2.

To compare the results in this paper with those of Tufano [45][6], I also use long-term debt, LTB instead of B in the regressions for the pooled sample. The long-term debt ratio is used by most other authors in testing the relationship between hedging and capital structure. The test results are reported in Table 5.12. The strong positive relationship between H and CC persists. Hedge ratio also has a positive relationship with long-term debt ratio, which, however, is not statistically insignificant after the adjustment for heteroskedasticity. The effects of economies of scale is again evident. These results contrast the findings of Tufano [45]. He finds that neither cash cost nor firm size has a statistically significant effect on hedge ratio. Consistent with Tufano [45], a significant negative correlation is found between hedge ratio and $Cash$.

In summary, the empirical results support some of the key predictions by the two-signal model under asymmetric information. The strong positive relationships between hedge ratios and cash costs and between hedge ratios and current debt ratios are documented in almost all of the tests performed, including those not reported in the paper.

[6]Tufano uses different variables to proxy for long-term debt ratio, Cash and firm size.

Table 5.12: Determinants of hedge ratio: pooled sample with HLTB

$H_{i,t} = \beta_0 + \beta_1 * TA_{i,t} + \beta_2 * HLTB_{i,t} + \beta_3 * CC_{i,t} + \beta_4 * Cash_{i,t} + e_{i,t}$				
	Maximum Likelihood Tobit Estimates		Bayesian Heteroskedastic Tobit Gibbs Estimates	
	Coefficient	p-value	Coefficient	p-value
Intercept	-0.5622	**0.10**	-0.4655	**0.08**
TA	0.0001	0.12	0.0001	**0.04**
LTB	0.1455	**0.04**	0.0903	0.18
CC	0.0030	**0.02**	0.0025	**0.01**
Cash	-0.1303	**0.03**	-0.1034	**0.08**

Conclusions

Firms can hedge financial price risks using derivatives or hybrid securities. Firms can also adjust their production costs by shutting down or restarting higher cost production lines to stabilize their profits. In this paper, I develop a model in which capital structure is endogenously determined together with firms' hedging and operating decisions. Under symmetric information about firms' production opportunities, I find that firms never issue unhedged debt, and firms with lower production costs offer more hedged debt and have higher tax shields.

In this case, I find that operational flexibility increases firm value through two channels. First, operational flexibility always increases the unlevered firm value. Second, it may also increase total tax shields through its function as an operational hedge. For firms with production costs that are low relative to the current commodity price, operational hedge is irrelevant for financial decisions. Those firms issue more debt and go bankrupt before ever exercising their operational flexibility option. Therefore, operational hedging increases these firms' unlevered values without increasing their tax shields. For firms with higher production costs, however, both value increasing effects take place. For these firms, the existence of operational flexibility decreases the threshold for bankruptcy and, hence, increases total tax shields. Operational hedging may increase, decrease or leave unchanged a firm's debt capacity.

I then develop models in which managers have private information about their marginal production cost and use financing/hedging packages to reveal this information to the market so that the debt offers will be fairly priced. First, I investigate the case in which both the straight and the hedging components of the debt contract are observable and are used as signals. Then, I turn to the one-signal case in which only the straight component is observable. Under asymmetric information, the amounts of unhedged debt issued and the total tax shields are increasing in firms' production costs.

Since the solutions for the optimal amount of hedged debt obtained in different models join at the maximum value of production cost, firms hold larger hedging positions under symmetric information than they do under asymmetric information. In the two-signal case, firms do not offer any unhedged debt. In the one-signal case, in contrast, firms are forced to use unhedged debt as the signaling instrument, and the optimal amount of unhedged debt is a decreasing function of firms' production costs. Asymmetric information is, therefore, another potential reason why firms do not fully hedge their financial risk exposures. Higher signaling costs are incurred in the case when hedging is not completely observable.

Data collected from firms' annual reports and COMPUSTAT for North American gold producing firms are used to test the empirical implications of the paper. As with previous studies, it appears from this sample that hedging exhibits economies of scale and firms with more cash hedge less. The empirical results support some of the key predictions by the two-signal model under asymmetric information. Strong positive relationships between hedge ratios and cash costs and between hedge ratios and current debt ratios are documented in almost all of the tests performed.

Although the empirical evidence supports some of the key predictions by the two-signal model, it might also be consistent with other models and hypotheses. However, more rigorous tests will not become feasible until a larger data set is available.

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Appendices

A Proof of Propositon 1

Proof: Totally differentiating equation (2.2.6) with respect to c yields

$$\frac{dx}{dc} = \frac{\frac{1}{c}x^2}{(2-\lambda_1)x - \frac{Qc}{p}(-\lambda_1)x}. \tag{A.1}$$

Since x is greater than $\frac{Qc}{p}$, the right hand side of equation (A.1) is positive. Totally differentiating x with respect to c and substituting into equation (A.1), it follows that

$$x^2 p\frac{dh}{dc} = \frac{\frac{Qc}{p}(-\lambda_1)x - (1-\lambda_1)x^2}{(2-\lambda_1)x - \frac{Qc}{p}(-\lambda_1)x}. \tag{A.2}$$

The right hand side is negative, so $\frac{dh}{dc}$ is negative. □

B Solution of the Optimal Operating Policy of an Unlevered Firm under Symmetric Information

Taking the first derivative of (3.1.5) with respect to p_c yields the first order condition for the maximization problem,

$$\max_{p_c} W^1, \tag{B.1}$$

which is essentially the same as maximizing

$$w^1(p_c) = \frac{1}{\lambda_1 - \lambda_2}\left[(\lambda_2 - 1)\frac{p_c}{r-\mu} - \lambda_2\frac{c}{r}\right](p_c)^{-\lambda_1}. \tag{B.2}$$

The first and second order conditions are

$$\frac{1-\tau}{\lambda_1-\lambda_2}\left[(1-\lambda_1)(\lambda_2-1)\frac{p_c}{r-\mu} + \lambda_1\lambda_2\frac{c}{r}\right](p_c)^{-\lambda_1} = 0, \tag{B.3}$$

and

$$\frac{1-\tau}{\lambda_1-\lambda_2}\frac{1}{p_c}\frac{(1-\lambda_1)(\lambda_2-1)}{r-\mu}(p_c)^{-\lambda_1}<0. \tag{B.4}$$

Therefore,

$$p_c=\frac{\lambda_1\lambda_2}{(1-\lambda_1)(1-\lambda_2)}\frac{r-\mu}{r}c=c. \tag{B.5}$$

The second equality holds since λ_1 and λ_2 are the solutions to the quadratic equation (2.1.5). Therefore, the firm shuts down its production when the commodity price reaches the level of production cost. This exact match is due to the assumption that it is costless to shut down or restart the firm's production.

To show that the same conclusion is obtained by maximizing W^2 or $w^2(p_c)$, I exploit the symmetry between (B.2) and (B.6). Note that

$$w^2(p_c)=\frac{1}{\lambda_1-\lambda_2}\left[(\lambda_1-1)\frac{p_c}{r-\mu}-\lambda_1\frac{c}{r}\right](p_c)^{-\lambda_2}, \tag{B.6}$$

or $w^1(p_c)$ equals $-w^2(p_c)$ if I switch λ_1 and λ_2 in the expression for $w^2(p_c)$. Therefore, the first order condition for maximizing $w^2(p_c)$ yields the same value for p_c and the second order condition is also negative.

C Proof of Propositon 2

Proof: To determine the maximum value of c, I substitute $x=\frac{c}{p}$ into equation (2.2.6). It follows that

$$\left(\frac{c}{p}\right)^{\lambda_1-1}=\frac{\lambda_2-\lambda_1}{\lambda_2-1}, \tag{C.1}$$

or

$$c_{max}|_{\{p_c\leq p_b\}}=\left(\frac{\lambda_2-1}{\lambda_2-\lambda_1}\right)^{\frac{1}{1-\lambda_1}}p. \tag{C.2}$$

This corresponds to a minimum value of h^* which satisfies the condition $p_c\leq p_b$,

$$h_{min}|_{\{p_c\leq p_b\}}=\frac{1}{\lambda_2}. \tag{C.3}$$

To check the monotonicity of $\frac{p_b}{p_c}$, substitute $b=0$ into the expression for p_b in equation (3.2.4),

$$\frac{p_b}{p_c}=\frac{\lambda_1}{\lambda_1-1}\frac{r-\mu}{r}\frac{1}{1-h}. \tag{C.4}$$

It is clear that $\frac{p_b}{p_c}$ is a decreasing function of c since h is decreasing in c. □

D Solution of the Optimal Operating Policy in Regime $p_c \geq p_b$ under Symmetric Information

The solution technique is similar to, but more involved than, that used in Appendix B. It is necessary to write out the expressions for v_E^1, v_E^2 and v_E^3, which were derived from the four boundary conditions (3.2.10), (3.2.11), (3.2.13) and (3.2.14).

$$\begin{aligned} v_E^1 &= \frac{1-\tau}{\lambda_2-\lambda_1}\left\{\left[(\lambda_2-1)\frac{hp_b}{r-\mu}+\lambda_2\frac{b}{r}\right](p_b)^{-\lambda_1}-\left[(\lambda_2-1)\frac{p_c}{r-\mu}-\lambda_2\frac{c}{r}\right](p_c)^{-\lambda_1}\right\} \\ &= v_E^3+\frac{1-\tau}{\lambda_2-\lambda_1}\left\{-\left[(\lambda_2-1)\frac{p_c}{r-\mu}-\lambda_2\frac{c}{r}\right](p_c)^{-\lambda_1}\right\} \end{aligned} \tag{D.1}$$

$$\begin{aligned} v_E^2 &= -\frac{1-\tau}{\lambda_2-\lambda_1}\left\{\left[(\lambda_1-1)\frac{hp_b}{r-\mu}+\lambda_1\frac{b}{r}\right](p_b)^{-\lambda_2}\right\} \\ &= -\frac{1-\tau}{\lambda_2-\lambda_1}\left\{\left[(\lambda_1-1)\frac{p_c}{r-\mu}-\lambda_1\frac{c}{r}\right](p_c)^{-\lambda_2}\right\}, \end{aligned} \tag{D.2}$$

$$v_E^3 = \frac{1-\tau}{\lambda_2-\lambda_1}\left\{\left[(\lambda_2-1)\frac{hp_b}{r-\mu}+\lambda_2\frac{b}{r}\right](p_b)^{-\lambda_1}\right\}. \tag{D.3}$$

The following maximization problem

$$\max_{p_c}\ V_E^1 \tag{D.4}$$

is equivalent to maximizing the coefficient in front of p^{λ_1}, or

$$\max_{p_c}\ v_E^1, \tag{D.5}$$

and

$$\max_{p_c}\ V_E^2 \tag{D.6}$$

is equivalent to

$$\max_{p_c}\ \{v_E^2 p^{\lambda_2}+v_E^3 p^{\lambda_1}\}. \tag{D.7}$$

Using the two different expressions for v_E^2 in equation (D.2), I compare the first derivatives of v_E^1, v_E^2 and v_E^3.

$$\frac{dv_E^3}{dp_c} = -\frac{dp_b}{dp_c}\frac{1-\tau}{\lambda_2-\lambda_1}\left\{\frac{(\lambda_1-1)(\lambda_2-1)}{\lambda_2\lambda_1}\frac{hp_b}{r-\mu}+\lambda_2\lambda_1\frac{b}{r}\right\}(p_b)^{-\lambda_1-1} \tag{D.8}$$

$$\begin{aligned}\frac{dv_E^2}{dp_c} &= \frac{dp_b}{dp_c}\frac{1-\tau}{\lambda_2-\lambda_1}\left\{\frac{(\lambda_1-1)(\lambda_2-1)}{\lambda_2\lambda_1}\frac{p_c}{r-\mu}-\lambda_2\lambda_1\frac{c}{r}\right\}(p_c)^{-\lambda_2-1}\\ &= \frac{dp_b}{dp_c}\frac{1-\tau}{\lambda_2-\lambda_1}\left\{\frac{(\lambda_1-1)(\lambda_2-1)}{\lambda_2\lambda_1}\frac{hp_b}{r-\mu}+\lambda_2\lambda_1\frac{b}{r}\right\}(p_b)^{-\lambda_2-1}\\ &= -\frac{dv_E^3}{dp_c}p_b^{\lambda_1-\lambda_2}\end{aligned} \tag{D.9}$$

$$\begin{aligned}\frac{dv_E^1}{dp_c} &= \frac{dv_E^3}{dp_c}+\frac{dp_b}{dp_c}\frac{1-\tau}{\lambda_2-\lambda_1}\left\{\frac{(\lambda_1-1)(\lambda_2-1)}{\lambda_2\lambda_1}\frac{p_c}{r-\mu}-\lambda_2\lambda_1\frac{c}{r}\right\}(p_c)^{-\lambda_1-1}\\ &= \frac{dv_E^3}{dp_c}+\frac{dv_E^2}{dp_c}p_c^{\lambda_2-\lambda_1}\\ &= \frac{dv_E^3}{dp_c}-\frac{dv_E^3}{dp_c}\left(\frac{p_c}{p_b}\right)^{\lambda_2-\lambda_1}\end{aligned} \tag{D.10}$$

It follows from the above three equations that at $p_c = c$,

$$\frac{dv_E^1}{dp_c} = \frac{dv_E^2}{dp_c} = \frac{dv_E^3}{dp_c} = 0 \tag{D.11}$$

Therefore, the first order conditions for maximizing both V_E^1 and V_E^2 are satisfied at $p_c^* = c$. Now, I check the second order conditions for the two maximization problems. First, I calculate the second derivatives for v_E^1, v_E^2 and v_E^3, respectively, by totally differentiating equation (D.8), (D.9) and (D.10). It follows that

$$\frac{d^2v_E^2}{dp_c^2} = \frac{dp_b}{dp_c}\frac{1-\tau}{\lambda_2-\lambda_1}\frac{(1-\lambda_1)(\lambda_2-1)}{r-\mu}p_c^{-\lambda_2-1} < 0, \tag{D.12}$$

$$\frac{d^2v_E^3}{dp_c^2} = -\frac{d^2v_E^2}{dp_c^2}p_b^{\lambda_2-\lambda_1} > 0, \tag{D.13}$$

and

$$\frac{d^2v_E^1}{dp_c^2} = \left\{1-\left(\frac{p_c}{p_b}\right)^{\lambda_2-\lambda_1}\right\}\frac{d^2v_E^3}{dp_c^2} < 0. \tag{D.14}$$

It is easy to show that $\frac{dp_b}{dp_c}$ is positive at $b = 0$ and $h > 0$ by differentiating equation (3.2.18) with respect to p_c for fixed b and h.

I prove that $p_c^* = c$ is a global maximum by showing that the signs of the second order derivatives $\frac{d^2V_E^1}{dp_c^2}$ and $\frac{d^2V_E^2}{dp_c^2}$ are negative. The signs of the second order conditions are determined as follows:

$$\frac{d^2V_E^1}{dp_c^2} = \frac{d^2v_E^1}{dp_c^2}p^{\lambda_1} < 0, \tag{D.15}$$

and

$$\begin{aligned}\frac{d^2V_E^2}{dp_c^2} &= \frac{d^2v_E^2}{dp_c^2}p^{\lambda_2} + \frac{d^2v_E^3}{dp_c^2}p^{\lambda_1} \\ &= \frac{d^2v_E^2}{dp_c^2}p^{\lambda_2}\left\{1 - \left(\frac{p_b}{p}\right)^{\lambda_2-\lambda_1}\right\} < 0.\end{aligned} \tag{D.16}$$

E Solution of the Optimal Financing Policies in Regime $p_c \geq p_b$ under Symmetric Information

Differentiating equation (3.2.18) with respect to h and b respectively yields,

$$\frac{\partial p_b}{\partial b} = \frac{1}{\lambda_2}\frac{p_b}{hp_b + b}, \tag{E.1}$$

and

$$\frac{\partial p_b}{\partial h} = \frac{1}{\lambda_2 - 1}\frac{p_b^2}{hp_b + b}. \tag{E.2}$$

Differentiating $TS(b,h)$ with respective to b and h yields,

$$\frac{1}{\tau}\frac{\partial TS}{\partial b} = \frac{1}{r}\left\{1 - \left(\frac{p_b}{p}\right)^{-\lambda_1}\right\} + \frac{1}{\tau}\frac{\partial TS}{\partial p_b}\frac{\partial p_b}{\partial b}, \tag{E.3}$$

and

$$\frac{1}{\tau}\frac{\partial TS}{\partial h} = \frac{1}{r-\mu}\left\{1 - \left(\frac{p_b}{p}\right)^{1-\lambda_1}\right\} + \frac{1}{\tau}\frac{\partial TS}{\partial p_b}\frac{\partial p_b}{\partial h}. \tag{E.4}$$

Combining equation (E.3) and (E.4) yields

$$\frac{\partial TS}{\partial b} - \frac{\partial TS}{\partial h}\frac{\lambda_2 - 1}{\lambda_2}\frac{1}{p_b} = \frac{\tau}{r(\lambda_1 - 1)}\frac{p}{p_b}\left\{\left(\frac{p_b}{p}\right)^{1-\lambda_1} - (1-\lambda_1)\left(\frac{p_b}{p}\right) - \lambda_1\right\} < 0. \tag{E.5}$$

The above equation states that $\frac{\partial TS}{\partial b}$ and $\frac{\partial TS}{\partial h}$ cannot be zero simultaneously. This implies that I obtain a corner solution for at least one of the independent variables. As assumed before, b is restricted to be nonnegative. If $\frac{\partial TS}{\partial h}$ is set to zero, then $\frac{\partial TS}{\partial b}$ must be negative according to equation (E.5). Therefore, $b^* = 0$. Substituting the optimal b into TS,

$$\frac{1}{\tau}TS(b=0,x) = \frac{p}{\lambda_2(r-\mu)}\left(\frac{p}{p_c}\right)^{\lambda_2-1}\left\{x^{\lambda_2-1} - x^{\lambda_2-\lambda_1}\right\}, \tag{E.6}$$

and h^* is determined by the following maximization problem:

$$\max_x \left\{x^{\lambda_2-1} - x^{\lambda_2-\lambda_1}\right\}. \tag{E.7}$$

The first and second order equations with respect to $x = \frac{p_b}{p}\Big|_{b=0}$ are

$$\left\{(\lambda_2-1)x^{\lambda_2-2} - (\lambda_2-\lambda_1)x^{\lambda_2-\lambda_1-1}\right\} = 0, \tag{E.8}$$

and

$$-x^{\lambda_2-2}(\lambda_2-\lambda_1)(1-\lambda_1)x^{-\lambda_1} < 0. \tag{E.9}$$

Therefore, the optimal solution is

$$p_b^* = \left(\frac{\lambda_2-1}{\lambda_2-\lambda_1}\right)^{\frac{1}{1-\lambda_1}} p, \tag{E.10}$$

and substituting p_b and $b=0$ into equation (3.2.18) yields

$$\begin{aligned} h^* &= \frac{1}{\lambda_2}\left\{\frac{p_b^*}{c}\right\}^{\lambda_2-1} \\ &= \frac{1}{\lambda_2}\left(\frac{p}{c}\right)^{\lambda_2-1}\left(\frac{\lambda_2-1}{\lambda_2-\lambda_1}\right)^{\frac{\lambda_2-1}{1-\lambda_1}}. \end{aligned} \tag{E.11}$$

F The Monotonicity of p_b in Regime $p_b \geq p_c$ in the Two-signal Model

Substituting equation (3.2.2) and (3.2.4) into the second order condition (4.1.7) yields

$$\begin{aligned} &(1-\tau)\frac{-\lambda_1}{r}\frac{1}{p_b}\left\{\frac{p_b}{p}\right\}^{-\lambda_1}\left\{\frac{p_b}{1-h}\frac{dh}{dc} + \frac{Q}{1-h}\frac{db}{dc}\right\} \\ &= (1-\tau)\frac{-\lambda_1}{r}\frac{1}{p_b}\left\{\frac{p_b}{p}\right\}^{-\lambda_1}\left\{\frac{\partial p_b}{\partial h}\frac{dh}{dc} + \frac{\partial p_b}{\partial b}\frac{db}{dc}\right\} \geq 0. \end{aligned} \tag{F.1}$$

This implies that

$$\begin{aligned}&\frac{\partial p_b}{\partial h}\frac{dh}{dc}+\frac{\partial p_b}{\partial b}\frac{db}{dc}\\&=\frac{dp_b}{dc}-\frac{\partial p_b}{\partial c}\geq 0.\end{aligned} \tag{F.2}$$

Since $\frac{\partial p_b}{\partial c}$ is positive, it must be that $\frac{dp_b}{dc}$ is positive. Therefore, for any feasible signaling schedule, p_b is an increasing function of c.

G The Monotonicity of p_b in Regime $p_b \leq p_c$ in the Two-signal Model

Substituting equation (3.2.15), or equivalently (3.2.16), and (3.2.18) into the second order condition (4.1.7) yields

$$\begin{aligned}&(1-\tau)\frac{-\lambda_1\lambda_2}{r}\frac{1}{p_b}\left\{\frac{p_b}{p}\right\}^{-\lambda_1}\left\{\frac{1}{\lambda_2}\frac{dh}{dc}+\frac{p_b}{\lambda_2-1}\frac{db}{dc}\right\}\frac{\partial p_b}{\partial c}\\&=(1-\tau)\frac{-\lambda_1}{r}\frac{1}{p_b}\left\{\frac{p_b}{p}\right\}^{-\lambda_1}\left\{\frac{p_b}{p_c}\right\}^{\lambda_2}\left\{\frac{\partial p_b}{\partial h}\frac{dh}{dc}+\frac{\partial p_b}{\partial b}\frac{db}{dc}\right\}\geq 0.\end{aligned} \tag{G.1}$$

Following the same line of argument as in Appendix F, this shows that for any feasible signaling schedule, p_b is an increasing function of c.

H Corner Solution in the Two-signal Model

Here I show that the two first order conditions given by equations (4.1.15) and (4.1.16) cannot hold simultaneously, and the corner solution $b^* = 0$ is obtained. It is necessary to point out that $f_i, i = 1, 2$, as defined in equation (4.1.8) and (4.1.10), are dependent on h and b only through p_b. Therefore,

$$\begin{aligned}\frac{\partial f_i}{\partial h}&=\frac{\partial f_i}{\partial p_b}\frac{\partial p_b}{\partial h},\\\frac{\partial f_i}{\partial b}&=\frac{\partial f_i}{\partial p_b}\frac{\partial p_b}{\partial b}.\end{aligned} \tag{H.1}$$

For firms in both regime $p_b \geq p_c$ and regime $p_b \leq p_c$, the partial derivatives of p_b with respect to h and b are related as follows,

$$\frac{\partial p_b}{\partial b}=\frac{\partial p_b}{\partial h}\frac{\lambda_2-1}{\lambda_2}\frac{1}{p_b}. \tag{H.2}$$

Multiplying the right hand side of equation (4.1.15) with $\frac{\lambda_2-1}{\lambda_2}\frac{1}{p_b}$, and subtracting from the right hand side of equation (4.1.16) yields

$$\begin{aligned}&(1-\dot{\eta}_1)\frac{\tau}{r(\lambda_1-1)}\frac{p}{p_b}\left\{\left(\frac{p_b}{p}\right)^{1-\lambda_1}-(1-\lambda_1)\left(\frac{p_b}{p}\right)-\lambda_1\right\},\\&=(1-\dot{\eta}_1)\Delta\end{aligned} \tag{H.3}$$

where Δ is negative. If the first order conditions (4.1.15) and (4.1.16) hold simultaneously, $1-\dot{\eta}_1$ must be zero, which does not satisfy either of the two equations. Therefore the two first order conditions cannot hold simultaneously, which implies a corner solution for at least one of the variables. If $1-\dot{\eta}_1$ is positive, the derivative of the Hamiltonian with respect to b is always negative if we set the Hamiltonian with respect to h to zero. Therefore, to show that $b=0$ is the corner solution, I need to show that the expression in equation (H.3) is negative, or $1-\dot{\eta}_1$ is positive.

Using equation (4.1.8) and (4.1.10), it is obvious that $\frac{\partial f}{\partial h}$ and $\frac{\partial f}{\partial b}$ are both positive. First order condition (4.1.14) states that η_1 is nonnegative. Therefore, from equation (4.1.15), the sign of $1-\dot{\eta}_1$ is the same as $\frac{\partial TS}{\partial h}$. I now determine the sign of $\frac{\partial TS}{\partial h}$ in regime $p_b \geq p_c$ and regime $p_b \leq p_c$, respectively.

At $b=0$, to satisfy the second order equation (F.1) and (G.1), $\frac{dh}{dc}$ must be positive. Under symmetric information, the optimal solution h^* is a decreasing function of c. If I set the boundary condition as $h(\bar{c})=h^*(\bar{c})$, then at any other value of c, it must be true that $h(c)<h^*(c)$.

Recall that $TS|_{b=0}$ is a concave function in h, and that $\frac{\partial TS}{\partial h}\big|_{b=0,h^*}=0$, it follows that $\frac{\partial TS}{\partial h}\big|_{b=0,h<h^*}>0$. Therefore, $1-\dot{\eta}_1$ is positive and the first order condition with respect to b, equation (4.1.16), is always negative when the first order condition with respect to h, equation (4.1.15) is set to 0. Thus, a corner solution obtains.