EQUITY FINANCE UNDER ASYMMETRIC INFORMATION

by

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M.A., Queen’s University, ON, 1994

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

in

THE FACULTY OF GRADUATE STUDIES

Department of Economics

We accept this thesis as conforming
to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

November 1999

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Date: Jan 30, 2000
Abstract

The thesis investigates the link between internal and external funds in financing new investment when asymmetric information is important. In both chapter, the entrepreneur has private information about the value of a project and, if the quality of the project is high, she tries to signal this to outside investors. The first chapter explores the tradeoff between using internal funds and raising external funds by issuing shares or bonds to finance a project. The entrepreneur can delay the project to accumulate internal funds over time from existing operations. This allows an entrepreneur with a high quality project to reduce her reliance on expensive underpriced bond or share issues. However, accumulating funds is also costly because of discounting and the risk that the project disappears. The more valuable the good project, the less the entrepreneur will delay the project, risking its loss, and so the more she relies on external financing.

When external financing is sought, the entrepreneur decides to issue bonds or shares. The greater the value of the good project, the more underpriced shares are relative to bonds. Thus an entrepreneur with a highly valuable good project chooses equity and one with a less valuable project chooses debt. Combining the two results shows that for a highly valuable good project, debt is used, and for a less valuable project, internal funds are used. External equity gets squeezed out. Aggregate data for the U.S. confirm that corporate bond issues are a more important source of funds than new share issued. Furthermore, most small firms rely on internal funds and debt, rather than external equity to finance their projects.

The second chapter provides a new theory for the underpricing of initial public offerings (IPOs). As in the first chapter, underpricing is used as a signal of quality. However, the entrepreneur is risk averse and only underprices when she cannot sell enough primary (new) shares to raise sufficient proceeds from the IPO to cover the cost of the project without diluting her position below that needed to signal a high project value. Underpricing allows the entrepreneur to maintain a high stake in the firm and still make a credible signal of quality. This allows more primary shares to be sold resulting in a net increase in proceeds.

The model predicts that underpricing should be greatest among firms that don't sell secondary shares (shares held by insiders) at the IPO and that there should be a positive relationship between the firm's capital requirement and the initial return among this group of firms only. A switching
regression framework is used. The probit model is first estimated where the probability of no secondary shares is explained by proxies for a firm's capital requirements. The initial return is then regressed on the same proxies, conditioning on whether the firm sells secondary shares or not and accounting for possible correlation between errors in the selection and regression equations. Strong support is found for the positive relationship between initial return and capital requirements for only firms without secondary share sales, as predicted.
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Acknowledgments

First of all, I would like to thank my two co-supervisors, Paul Beaudry and Kenneth Hendricks, for their many invaluable suggestions, their guidance along the way, and their help in focusing my ideas. I also thank, Raphael Amit, for his suggestions during our discussions about the practical side of IPOs. A special thanks to Margaret Slade for her comments and suggestions on the second chapter, especially those regarding testing. Thanks to the participants of the IO/Labour lunch seminar series for their questions and comments. I gratefully acknowledge the financial support of SHRCC (grant 752-96-1967) and UBC Killam Fellowship. Of course, all errors and omissions are my own.

Alan White, my friend and colleague, always good hearted, has kept me from taking my thesis, or myself, too seriously, and has made the long road of writing a thesis seem much shorter. Thanks to Yanjun Liu, Ian MacTavish, and James Townsend for listening to my ideas and sharing their thoughts. Without my parents' love and support over the years, this milestone would never have been reached. Finally, I am most indebted to my wife Christina, who has been there at every step, steadfastly supporting me in countless ways through the many months of researching and writing.
Introduction

Very few young firms tap into equity markets, especially public ones, relying almost exclusively on internal funds and debt to finance their investments. Even in aggregate, external equity is a seldom used financial source with the majority of new investment funded by internal sources. Venture capital, a private source of equity, also captures only a minute share of the market for investment funds. Modern financial theory has gone some way in explaining this pattern by incorporation informational imperfections into the financing relationship between those needing the funds and those providing them.

As Myers and Majluf (1984) first showed, if new investors worry that an issuing firm with poor investment prospects is masquerading as one with excellent ones they will discount the price they are willing to pay. A firm with a genuinely high value project should then issue riskfree debt or finance investment with internal funds. But young firms, especially, have neither sufficient accumulated internal funds nor the collateral to back large debt outlays. Risky debt is subject to the same informational problems as equity in that the lender does not know the riskiness of the firm’s project and will tend to overprice debt for firms with low-risk investments to guard against high risk borrowers.

What about the firms that do obtain funds from public equity markets? Financial economists have found several anomalies among firms going public for the first time. The price of shares sold tends to be lower than what investors are willing to pay, which amounts to a transfer in wealth from original shareholders to new shareholders, long-term returns (three years and more) seem to be lower than that for similar firms that have been public longer and many firms seem to time their initial public offerings (IPOs). We wouldn’t expect these patterns if firms operated under neoclassical assumptions with perfect capital markets
but we might if informational imperfections exist.

The thesis sets out to answer two important questions in finance. First, why is there so little reliance on external equity. Second, for those firms that go to public equity markets, why do they tend to price the offering below investors’ willingness to pay? Asymmetry of information underpins the answers to both questions. In particular, the private information about the value of the firm’s investment opportunities is held by the insiders of the firm and is not easily verifiable by those interested in investing in the firm.

The first question is addressed in chapter one of the thesis. While Myers and Majluf’s pecking order theory of capital structure goes some way in explaining the desirability of internal funds and riskfree debt over risky debt and external equity, it subsumes internal funds are accumulated rapidly compared to the arrival rate of profitable investment opportunities so that most firms have sufficient internal funds and access to riskfree debt to avoid issuing external equity and risky debt. While perhaps the case for mature firms, it is unlikely to hold for young firms. Yet young firms still avoid issuing equity to non-insiders. Part of the difficulty with the pecking order theory is that it takes internal funds as exogenous and fixed.

However, firms can and do accumulate internal funds over time waiting for profitable investment opportunities to arrive. More interestingly, once a project is available, the firm faces the decision on whether to finance the project immediately from available internal funds and covering the shortfall with a debt or equity issue or accumulate further internal funds from existing operations, assuming the firm has a positive cashflow. When sufficient internal funds have built up, the entrepreneur can undertake the project, if still available. The decision of how long to accumulate funds depends on cost of external finance arising from asymmetric information, and the value of the project. The first chapter shows that delay and internal funds tends to be a substitute for external equity and not debt. Thus issues of risky bonds will be larger than issues of risky shares.

The chapter also shows that there any scope for policy, even though the policy maker has no more information than investors and lenders. The policy maker can implement a non-linear tax by increasing the tax on successful investing firms to finance a decrease in the
tax on small firms. Even investing firms will prefer an increase in their tax rate since the
decrease in the tax rate on small firms leads to a lower cost of external finance. Essentially,
rather than burn money as a signal, which is effectively what signaling firms do when they
pay too much for financing, a non-linear tax reduces the opportunity cost of undertaking a
bad project.

The importance of information imperfections is carried over to chapter two where the
puzzle of IPO underpricing is addressed. There has been an extensive search to find an
explanation for underpricing, yet empirical studies have at best ruled out a few leaving the
question open for more research. While most explanations rely on asymmetric information,
only three papers assume that the firm has better information than investors or the under­
writer, despite the prevalence of this assumption in the capital structure literature. Other
theories have posited that the parties with private information are an exclusive group of
informed investors or the underwriters.

In my explanation for underpricing, risk aversion of the issuing entrepreneur and the lack
of internal funds drive the results. Risk aversion allows some signaling to take place through
the entrepreneur's retained stake in the firm. But the necessity to raise sufficient funds
to cover the cost of the project can force some entrepreneurs with little internal funds to
underprice the offering as an additional signal of quality. While underpricing alone reduces
the amount of proceeds, it allows the entrepreneur to reduce his stake in the firm and still
maintain a credible signal. The entrepreneur is then able to increase the number of shares
issued, and the net effect is shown to be an increase in the total amount of proceeds.

Importantly, the theory makes a clear prediction about the relation between the types
of shares issued at the IPO, (primary and secondary,) the required amount of proceeds and
the extent of underpricing.¹ Since the type of shares are observable, it is possible to test
the predictions of the model. Both simple correlations and regression analysis show strong
support for the predictions. In particular, underpricing is most acute when no secondary

¹Briefly, primary shares are shares issued by the firm, the proceeds of which go to the firm's treasury,
and secondary shares are sold by existing shareholders, in particular, the insiders of the firm.
shares are sold and the firm's capital requirements are large.

Together, both chapters suggest that many young small firms, despite having profitable investment opportunities, may significantly delay investing because of a lack of internal funds. Internal funds not only help a firm avoid using high priced external financing, they lower the cost of external financing by allowing the entrepreneur to maintain a large stake in the firm to demonstrate her belief of the high value of its projects. For those firms that do go public, having substantial internal funds can substantially lower the cost of equity by reducing the need for underpricing allowing her stake in the firm to suffice as a signal of quality.

The questions that the thesis addresses are, in several ways, fundamental to the field of finance, and will undoubtedly continue to attract research. Thus no single work is likely to give a complete answer. However, much of the existing research in capital structure has focused on large public firms, partly because of easily available data, though it seems that many of the informational problems faced by public companies will be all the more acute for young private firms. Indeed the puzzles surrounding the IPO suggest this is the case.
Chapter 1

Delay, Internal Funds and Capital Structure

Among the many decisions the entrepreneur of a young firm must make, are when to make an investment and how to finance it. Of course, investment only makes sense if it is profitable to do so, but its profitability depends in part on the firm’s cost of finance. In a neo-classical world, as Modigliani and Miller showed, there is no relationship between capital structure (the relative amounts of debt and equity) and the cost of financing. However when some of the neo-classical assumptions fail to hold, capital structure can matter.

Asymmetry of information between the insiders of the firm and outsiders is one of the important departures from the neo-classical model that has been extensively explored in the finance literature. In a static model, the effect of asymmetric information is that new shareholders include a discount in the price they are willing to pay for shares in the firm to take into account (or discourage) projects with low expected value. Thus when possible, existing shareholders will prefer to finance a project with riskfree (collaterized) debt and internal funds. However, the firm, especially a young firm, may not have sufficient collateral for its financing requirements and will have to issue risky debt that also carries an information cost. As with equity, bond issues are underpriced to discourage bad projects with high default rates from being taken (or account for low return projects with high default rates.)
Myers and Majluf's (1984) theory of a pecking order in capital structure posits that because of these information costs, internal funds will be most preferred followed by debt, or rather, low risk debt, and than high risk debt and equity. However, this preference ordering need not translate into a corresponding ranking of capital sources since it takes the availability of low risk debt and internal funds as given. Insofar, as both are in short supply, we would expect to still see a substantial external equity component.

Internal funds, however, are not entirely exogenous to the firm. When a project arrives, a firm can almost always delay undertaking it, at least for some time, and accumulate more internal funds. This chapter extends the static model of debt/equity choice to a simple dynamic setting in which the firm can delay a project and accumulate internal funds.

The benefits of accumulating internal funds are two-fold. First, it reduces the extent that the entrepreneur has to rely on underpriced bond and share issues. Second, it allows the entrepreneur to increase her stake in the project which acts as a signal of quality leading to less underpricing when she does raise external funds. The costs of building up internal funds arise from delaying the project. Delay is costly because of discounting and possible degradation in the value of the project, (e.g., competitors get a head-start, new regulations arise, or demand subsides.)

Does a pecking order arise when there are these additional costs for all sources of funds? As one might expect, the reliance on internal funds versus external funds depends on parameter values. However, a ranking does emerge with respect to debt and external equity that is endogenous and is not the result of an exogenously lower information cost of debt. That is, the size of external equity financings will be no larger, and generally smaller, than debt financings because of the entrepreneur's ability to accumulate internal funds.

The basic idea behind this result is as follows. Suppose, as in the model, there are two projects, one profitable (good) and one unprofitable (bad). Similar to Ross (1978), the decision to use debt or external equity depends on the relative distribution of returns of the
different projects. In particular, if the entrepreneur with the good project wishes to signal her project’s quality and raise funds for the project, she must offer underpriced bonds or shares. If the expected value of the good project is high relative to the bad project, debt will be more desirable than equity. In other words, for a given level of internal funds, a high expected return means that the cost of underpricing of shares needed to deter the bad type, a proportional cost on returns, is greater than that for bonds, a fixed cost in solvent states. As the expected value of the good project decreases, the cost of underpriced shares decreases and can fall below that of underpriced bonds. At the same time, the cost of raising internal funds increases with the value of the good project since discounting and risking the loss of the project are, like equity, proportional costs on returns. Taken together, only a small amount, if any, of internal funds will be accumulated for high value projects and, since debt is preferred to external equity for such projects, they will be mostly debt financed. Conversely, lower value projects will be mostly financed with retained earnings. External equity gets squeezed out.

Empirically, then, we should expect there to be more risky debt used than equity. Among small firms, debt is by far the most important external source, with most new equity being provided by the insiders themselves or their family members. In terms of the corporate sector, corporate bonds have in the U.S. been a more important source of funds than new share issues since 1945. And since the mid-1980s, there has been a surge in the issuance of corporate bonds and a net negative issuance of new shares, as many firms repurchased shares.

A policy implication also emerges from the model. Even though the government has no more information than outside investors, by using a non-linear tax schedule in which small firms are taxed at a lower rate than successful investing firms, the government can more efficiently than the market discourage small firms from undertaking bad projects. The government can do this by raising the opportunity cost of investing for small firms, to deter

---

1 Ross doesn’t consider unprofitable projects, just projects with different return distributions, though his insight is easily extended to the case of unprofitable projects.
those with bad projects from investing and thereby reducing the signaling cost to those with
good projects. Despite the increase in tax cost, an entrepreneur that hasn’t yet invested but
knows she has a good project, will prefer the non-linear tax scheme despite the higher taxes
she’ll face if her investment succeeds.

Related literature

In general, the literature on capital structure is vast. Many theories of capital structure
were developed in the 1980s and Harris and Raviv (1991) provide an extensive review. Their
initially screening of the literature turned up more than 150 theoretical papers related to
capital structure.

Elements of Myers and Majluf’s (1984), Ross (1977), Jensen and Meckling (1976) and
Leland and Pyle (1977) are all included in the present work. The first two were discussed
above. Jensen and Meckling show how agency problems can give rise to costly external
finance. In their model, managers have some discretion as to the level of pecuniary and
non-pecuniary benefits that the firm generates and the optimal level chosen by the manager
may diverge from owners if owners receive little benefit from amenities, power, etc. As the
manager’s stake in the firm is reduced through an equity issue, the divergence in interests
increases, lowering the value of the firm to shareholders other than the manager. Thus the
share price will fall when an equity issue is announced. Similar to Myers and Majluf, debt is
costly because lenders are forced to raise interest rates to account for the manager’s ability
to shift risk by switching to projects with high default rates. Leland and Pyle’s model is
the only one of these to allow the amount of internal funds to be endogenously determined.
However, their model is still static. The reason why not all internal funds are invested in the
project is because the entrepreneur is risk averse and would like to diversify her portfolio. A
high stake in the firm is then used as a signal of project quality.

Closest to my model is that of Kim, Mauer and Sherman (1998) who use a three-date
model in which the firm retains internal funds in a low return liquid asset rather than pay
dividends in order to reduce the need to rely on costly external financing in the future. The
firm optimally trades off the amount of internal funds to hold and the amount of external
financing depending on the cost of external financing, the variance of future cash flows and the profitability of future investment opportunities. However, debt is the only source of external finance and its cost is given exogenously and so they do not compare the use of debt and external equity. Martin and Morgan (1988) also examine investment in low-return liquid investment to avoid external sources but again the cost of external financing is not endogenized.

1.1 The Model

The entrepreneur begins by operating a small firm which generates a constant flow of profits $\pi$ forever. All earnings generated by the small firm are retained and invested in a safe asset with a market rate of return of $r$, the same rate at which agents discount the future. Thus the net present value of a small firm is $\pi/r$.

Rather than always operating a small firm, at any point in time the entrepreneur can choose to invest in a risky project. Such a project requires an upfront cost of $k$. There are two types of projects and the entrepreneur is endowed at time zero with one. The first type is a good ($G$) project. Such a project requires a private disutility cost $c$. This cost cannot be contracted upon but requires no monetary payment. The project's expected profit, gross of the capital outlay and any financing costs, is $v$. The second type of project is a bad ($B$) project. A bad project requires no private cost. With probability $p$ the project succeeds and yields a payoff $w$ and with probability $1-p$ the project fails and the payoff is zero. The expected value of this project (gross of any financing costs) is $pw$.

By good and bad projects, I mean profitable and unprofitable projects if the entrepreneur bore the true cost of the project, $k$. This is equivalent to the following assumption.

---

2 If $c$ is an effort cost then it cannot be avoided. That is, I assume implicitly that if $c$ is not paid the project returns no payoff.

3 The payoff can be interpreted as the present value of a stream of payments the project realizes.

4 This is a normalization. As long as the effort cost for the bad project is lower than that of the good project, all of the qualitative results continue to hold.
Assumption 1.1

\[ v - c - k > \pi/r \]
\[ pw - k < \pi/r. \]

When external finance is used, the profitability of the project from the entrepreneur's point of view will depend on the terms she obtains the financing at. To keep the distinction clear, the words *good* and *bad* distinguish the type of project and *profitable* and *unprofitable* distinguish the projects the entrepreneur would or would not undertake, taking into account the terms under which the project is financed.

A good project may, unfortunately, be lost if the entrepreneur waits too long before making the investment. For instance there may be a sudden change in demand conditions or entry of a rival. Loss occurs at a constant rate of \( \lambda \) so that the probability of still having a good project to invest in at time \( t \) is \( e^{-\lambda t} \). Once a project is lost, the game ends and the good type is left with a payoff (valued at \( t = 0 \)) of \( \pi/r \).

The entrepreneur is aware of the project type she has but an outside investor only knows the prior probability distribution of the two types. Since the focus in this chapter is on separating equilibria, it will not be necessary to give this distribution explicitly.

At the beginning of the game the entrepreneur has the option of making an offer to the market of investors, which, if the market accepts, commits the entrepreneur to undertaking a project at some date \( t > 0 \). An offer by the entrepreneur consists of a time of investment, \( t_i \geq 0 \) and a price to sell shares at \( R_i \geq 0 \) for \( i = G, B \). The offering price \( R_i \) is for a share of the firm where one share is normalized to equal the entire firm value (fractional share purchases are allowed.) The amount of capital to be raised is determined by the time of investment. Let \( I(t) \) denote the entrepreneur's external equity requirement at time \( t \). Thus

\[ I(t) = k - \frac{\pi}{r}(e^{rt} - 1). \]

Let the time at which the entrepreneur has sufficient funds to self-finance the project \((I = 0)\) be denoted by \( t^+ \).
A strategy for an entrepreneur is an offer \( \{t, R\} \). I make the convention that an offer \( \{t^+, R\} \), for any \( R \), means that no offer is made to the market. A good type that waits until \( t^+ \) will always invest, if she retains the project, and the bad type never will. Also, since any strategy for the good type where investment occurs after \( t^+ \) is strictly dominated by investment at \( t^+ \), the game effectively ends at \( t^+ \).

The market of investors, which I treat as a single player, has a constant marginal cost of capital of \( r \). After observing the entrepreneur’s offer, if one is made, it decides to either accept the offer or reject it. The market’s strategy, then, is an accept/reject decision conditional on any offer \( \{t, R\} \). Let the indicator function \( a(t, R) \) denote this strategy such that

\[
a(t, R) = \begin{cases} 
1 & \text{if } \{t, R\} \text{ is accepted} \\
0 & \text{otherwise.}
\end{cases}
\]

The market also holds beliefs about the type of project the entrepreneur has. Let \( \mu(G|t, R) \) denote the probability the market believes a good type is making the offer \( \{t, R\} \).

The entrepreneur and the market are committed to any contract that is agreed to. Commitment allows for a simple static representation of a dynamic game. However, it is not necessarily a strong restriction in that a similar equilibrium outcome to that analysed below can be achieved in a dynamic game without this commitment. I will return to this point later. Included in the contract is that the entrepreneur actually undertakes the project when the investment funds are obtained. Thus a bad type cannot simply invest the proceeds in a safe asset rather than in her project. It is in the interest of the good type to commit to undertaking the project in the prospectus (in the “use of funds” statement) to help signal her project type.

If an offer is refused, the entrepreneur can wait until \( t^+ \) to invest or can choose to never invest. The entrepreneur cannot make a new offer to the market. A failed offer carries a small fixed cost \( f \) regardless of whether the entrepreneur later invests or not. This cost can be thought of as a small amount of disutility associated with having an offer rejected.\(^5\) This

\(^5\)More generally, it can be thought of as a fixed cost of going to the market to raise funds. Under such an interpretation it should also be paid by an entrepreneur with a successful offering. Including this cost for
cost can be arbitrarily small, but needs to be present to rule out some equilibria using the

The respective payoffs to the good and bad type if an offer \( \{t, R\} \) is made are:

\[
\Pi_G(t, R, a(t, R)) = \begin{cases} 
  e^{-(r+\lambda) t} \left( (1 - I(t)/R) v - c \right) + (1 - e^{-\lambda t}) \pi/r & \text{if } a(t, R) = 1 \\
  e^{-(r+\lambda) t^+} \left( v - c \right) + (1 - e^{-\lambda t^+}) \pi/r - f & \text{if } a(t, R) = 0
\end{cases}
\]

\[
\Pi_B(t, R, a(t, R)) = \begin{cases} 
  e^{-rt} \left( 1 - I(t)/R \right) pw & \text{if } a(t, R) = 1 \\
  \pi/r - f & \text{if } a(t, R) = 0
\end{cases}
\]

First consider the good type’s payoff. If the offer is accepted, she waits until time \( t \). With
probability \( e^{-\lambda t} \) she has retained her project and invests. To raise the required
\( I(t) \) she sells
a fraction \( I(t)/R \) to the market. The higher the agreed upon price, the less of the firm she
must sell to raise the required funds. Also, the later is \( t \), the less she has to sell since her
capital requirements are smaller. Notice that the private cost, \( c \), is borne by the entrepreneur
alone. The value of successfully investing is discounted back to time zero by \( e^{-rt} \). If the
project is lost at any time \( t' < t \), she retains her accumulated funds of \( (e^{rt'} - 1) \pi/r \) plus the
value at that time of continuing to run a small firm forever, \( \pi/r \). At date \( t = 0 \), the total
discounted value is, of course, simply the value of running the small firm: \( \pi/r \). If the offer
is rejected, the entrepreneur pays the cost \( f \) and continues to run the small firm until \( t^+ \) is
reached at which point she will invest if the project has been retained.

The bad type’s payoff is similar to the good type’s. Several differences, however, are
worth pointing out. The bad type doesn’t incur the private cost and doesn’t face the loss of
a bad project. Furthermore, if the project is refused, the bad type will never invest since it
is never worthwhile to self-finance a bad project.

Making no offer, for either type of entrepreneur, gives the same payoff as having an offer
rejected except that the cost \( f \) is avoided.

For the market, given its beliefs, the current value of accepting an offer \( \{t, R\} \) is:

\[
I \left( \mu(G|t, R) v + (1 - \mu(G|t, R)) pw - R \right).
\]

all offers would not change the results qualitatively but would unnecessarily complicate the analysis.
Notice that the investor's payoff depends neither on $c$, which is borne by the entrepreneur alone, nor the market interest rate $r$, since the project's payoff occurs immediately after investment. Let $\bar{R}$ denote the maximum price an investor would ever be willing to pay:

$$\bar{R} = v.$$

Thus it is a dominant strategy for the market to refuse all offers greater than $\bar{R}$.

### 1.1.1 Equilibrium concept

Given that this is a signaling model, I use the Perfect Bayesian equilibrium (PBE) concept. As mentioned above, I also use Cho and Kreps (1987) refinement (Intuitive Criterion) to rule out equilibria that do not seem reasonable, as explained below. Finally, the focus of the analysis is on separating equilibria so I make this explicit in the following definition of equilibrium.

**Definition 1.1** An equilibrium to this game is a set of strategies $\{t^*_G, R^*_G\}, \{t^*_B, R^*_B\}, \mu^*$ and beliefs $\mu^*$ satisfying the following conditions.

1. For each type $i$, and any $\{t, R\}$,

$$\Pi_i(t^*_i, R^*_i, \mu^*) \geq \Pi_i(t, R, \mu^*)$$

2. For any $\{t, R\}$, $\mu^*(t, R) = 1$ if, and only if,

$$\mu^*(G|t, R)v + (1 - \mu^*(G|t, R))pw - R \geq 0.$$  

3. $\{t^*_G, R^*_G\} \neq \{t^*_B, R^*_B\}$ and

$$\mu^*(G|t^*_G, R^*_G) = 1$$

$$\mu^*(G|t^*_B, R^*_B) = 0$$
4. For any non-equilibrium offer \( \{t, R\} \) and any \( a \in \{0,1\} \)

\[
\mu^*(G|t, R) = \begin{cases} 
1 & \text{if } \Pi_G(t, R, 1) > \Pi_G(t^*_G, R^*_G, a^*), \ 
\Pi_B(t, R, a) < \Pi_B(t^*_B, R^*_B, a^*) \\
& \text{and } R < v
\end{cases}
\]

\[
\mu^*(G|t, R) = 0 & \text{if } \Pi_B(t, R, 1) > \Pi_B(t^*_B, R^*_B, a^*), \ 
\Pi_G(t, R, a) < \Pi_G(t^*_G, R^*_G, a^*) \\
& \text{and } R < pw
\]

\[
\mu^*(G|t, R) \in [0,1] & \text{otherwise}
\]

Condition 1 requires the entrepreneur to behave optimally as in a Nash equilibrium. That is, given the market's equilibrium strategy, there is no alternative strategy the entrepreneur prefers to her equilibrium strategy. Condition 2 is a similar optimality condition for the market and requires that the market accepts an offer if, and only if, its expected profit of doing so is non-negative. Condition 3 states that the equilibrium is separating since the equilibrium offers of the two types are different. The market's beliefs are then updated by Bayes' rule, the second part of the condition.

Condition 4 is the Intuitive Criterion refinement. The refinement ensures that in any equilibrium there is no deviation offer such that: 1) it makes the good type strictly better off than her equilibrium offer if it were accepted; 2) it makes the bad type strictly worse off; and 3) the market strictly prefers to accept the deviation offer if it believed the offer was made by the good type. Because such a deviation could only benefit a good type, it seems reasonable that the market should believe a good type made it and so accept the offer. If the good type expected this, she would deviate. (The restriction also applies with the role of the good and bad types reversed, though perfect Bayesian equilibria with such deviations never arise.)

The reason for the fixed cost \( f \) can now be explained more clearly. A strict application of the Intuitive Criterion requires that the bad type is strictly worse off by making the deviation offer, whether the market accepts it or not. That is, if the bad type is indifferent to never making and offer and having the deviation offer rejected (i.e., \( f = 0 \)), then the market may
conclude that the bad type made the offer expecting it to be rejected. This would prevent
the good type from deviating.

Commitment and dynamics

While the setup is that of a static game with commitment, we can think of an equivalent
dynamic game without commitment that has the same equilibrium outcome. In Appendix
II such a game is given and the main result for equity financed below is also reproduced. Here
I discuss the two reasons, which are both related to how information is released, that lie
behind this.

The first reason has to do with how much, or rather, how little information is release
by the passage of time. Suppose rather than making a commitment offer, the entrepreneur
waits until she wishes to invest and then makes an offer to undertake the project at that
moment. The entrepreneur, however, cannot make another offer if the first is refused. Thus
an entrepreneur's strategy is an offer $R$ after any history of past play $H(t)$ to raise $I(t)$. Since
only one offer is possible, when an offer is made, the only relevant information is that
no previous offer has been made so that $t$ is sufficient to describe any history.

Now, as in the commitment game, in a perfect Bayesian separating equilibrium in pure
strategies, the bad type never makes an offer. If the bad type did make an offer, which differed
from the good type, she will reveal her type. But since her project is bad, this would only
be worthwhile to her if the market, which knows her type, accepted the project and incurred
a loss. What this means is that prior to the expected offer, say at $t'$, the market's beliefs
about the type, in any equilibrium, cannot change since both types' equilibrium strategies
must be to make no offer before $t'$. Any deviation offer before $t'$ is exactly like making a
deviation offer in the static commitment game since the market's beliefs are the same.

What about if the entrepreneur waited until after $t'$ to make an offer? There are only two
possibilities. Conditional on reaching such a date, Bayes' rule is still used to update beliefs
in a perfect Bayesian equilibrium when possible. If the offer at this date is still unexpected
in that no type should have made it conditional on reaching $t$, then the market's beliefs are
not determined by Bayes' rule and so they are either arbitrary or determined by the Intuitive
Criterion. In either case, beliefs can be specified exactly as in the commitment game, and the same market strategy in response to the deviation is an equilibrium response in either game. If, on the other hand, the offer is expected from some type given that $t$ is reached, the market updates its beliefs, unlike in the static game. However, even if the good type makes the best possible offer at such a $t$ that the bad type is unwilling to make and so the market accepts it, the good type will still prefer her equilibrium payoff since the equilibrium strategies are selected to give her the highest possible payoff.

What is important for this equivalence is that the bad type never invest on her own. If her project was actually worthwhile, then in no equilibrium could she invest at a different time than the good (better) type. If she did, then as time passes the point at which she was supposed to invest, the entrepreneur would be revealed as the good type, and then the good type could make a deviation offer to invest immediately at $R$, or just under. But this would induce the bad type to mimic the good time and also wait until just slightly after her equilibrium offering time in order to get this better price. A separating equilibrium in pure strategies would not exist. In the present model, because the bad project is never profitable, it is never the case that no investment at some $t$ signals that the firm is a good type.

The second reason for the equivalence between the static and dynamic game is that because the hazard rate of project loss is constant, as time passes, the entrepreneur's assessment of which contract she prefers doesn't change. In other words, as time passes the probability of retaining a project until time $t'$ and $t$ have both increased by the same proportional amount leaving the ranking of the two payoffs unchanged. Thus there is no time inconsistency in the entrepreneur's decision at $t = 0$ in the static game.

As with commitment, the restriction of the entrepreneur's offer to be just the time and price, with the offering size only determined by the time, is also less important than it may appear. Suppose instead, that the entrepreneur could, at any date $t$, hold an offering of any size and price. If she raised more than needed, the excess would be invested in the safe asset and retained in the firm to which investors have a claim. Since investors know what rate of return these excess funds earn, there is no asymmetric information problem associated with
them and they can be correctly priced.\footnote{One could imagine offering shares for two divisions of the firm. The first set of shares are for the division of the firm that undertakes the risky project. The second set are for the division that invests in the safe asset. Since the return to the safe asset is known, the second set of shares will be correctly priced. But since the return on the safe asset equals the cost of funds to the market, the size of the second division will have no effect on overall firm value. Implicit in this is the entrepreneur's inability to burn money. If investors are not able to observe whether the extra capital is being invested in the safe asset or is rather being consumed privately by the entrepreneur, perhaps for empire building or perquisites, then raising excess funds could be used to signal firm quality.}

What if the entrepreneur raised less than $I(t)$? Barring a secondary offering, a complication not considered here, the entrepreneur would have to wait until sufficient funds had accumulated from the return on invested funds and operations of the small firm, both invested in the safe asset. Suppose this time occurs at $t'$. Because funds in the firm earn the market rate of return, for a given expected return to investors, the entrepreneur’s payoff is the same whether she obtains external funds at $t$ and then waits until $t'$ to invest or simply contract to obtain the funds at $t'$, raises $I(t')$ and invests immediately. However, since investors’ beliefs need to be specified in a game of asymmetric information, allowing the entrepreneur this flexibility adds additional complexity to the game.

### 1.2 Timing of investment

If the good type could finance the project at time zero at a price of $\bar{R}$, she will of course do so. As long as the bad type isn’t willing to accept such an offer, the separating equilibrium will be for the good type to invest at $t = 0$. Asymmetric information only has bite if the bad type is willing to mimic the good type at these low prices. Thus I make the following assumption.

**Assumption 1.2**

\[
 pw \left(1 - \frac{k}{v}\right) > \pi/r.
\]
The bad type doesn't make an offer in equilibrium as shown in the proposition below. Thus the bad type must prefer running a small firm forever to making any accepted offer, and in particular, to making the good type's offer. That is,

\[ \pi/r \geq \Pi_B(t_G^*, R_G^*, 1). \]

I refer to this as the (bad type's) incentive compatibility constraint (IC). The Intuitive Criterion ensures that the good type's payoff is maximized subject to the market's participation constraint and IC. Since there is only one offer that does this, the equilibrium outcome is unique.

**Proposition 1.1** A unique equilibrium outcome always exists. The bad type never makes an offer: \( t_B^* = t^+ \). The good type's offer \( \{t_G^*, R_G^*\} \) is characterized as follows:

1. \( t_G^* = 0 \) and \( R_G^* < R \) if

   \[ \lambda \left( v \left( \frac{\pi/r}{pw} \right) - \frac{\pi}{r} \right) \geq (\lambda + r)c \] (1.1)

2. \( t^+ > t_G^* > 0 \) and \( R_G^* < R \) if

   \[ \lambda \left( v - k - \frac{\pi}{r} \right) > (\lambda + r)c > \lambda \left( v \frac{\pi/r}{pw} - \frac{\pi}{r} \right) \] (1.2)

3. \( t^+ > t_G^* > 0 \) and \( R_G^* = R \) if

   \[ (\lambda + r)c \geq \lambda \left( v - k - \frac{\pi}{r} \right) \] (1.3)

Proof: see appendix.

The characterization of the good type's offer is divided into three cases and I will consider each in turn. However, first notice that in all cases, the good type always makes an offer and the bad type never does. Consider the good type's offer. Since the bad type would not undertake the project at time \( t^+ \), when she could self-finance the project, she also won't do it at some time slightly before \( t^+ \) when her financing requirement is very low. The offering is
so small that even at $\bar{R}$ the bad type won't undertake the project. Continuity of the payoff function (and assumption 1.2) means there is a $\hat{t} > 0$ at which the bad type is indifferent between making an offer \( \{\hat{t}, \bar{R}\} \) and never investing. That is,

$$\Pi_B(\hat{t}, \bar{R}, 1) = \pi / r.$$ 

Now since the good type can never sell shares at more than $\bar{R}$, she will always prefer to undertake the project no later than $\hat{t}$. And since $\hat{t} < t^+$, the project is partly externally financed. Notice that because $\bar{R}$ is increasing in $v$, a larger $v$ implies a larger $\hat{t}$.

If the bad type made an offer, this would reveal her type since she makes a different offer than the good type in equilibrium. But then the highest price investors would accept is $pw$ and at this price, since her project is bad, she would prefer never having made the offer.

Now consider the first case in the proposition. There is no delay and the entrepreneur raises the full amount $k$. Separation is achieved by underpricing alone. This case arises when the marginal cost of delay, at $t = 0$, outweighs its marginal benefit. To see why, refer to (1.1). The left-hand side is the marginal cost of waiting at $t = 0$ for a brief interval before undertaking the project. Over this interval, with probability $\lambda$ the good type loses her stake $\left(\pi / r\right) / (pw)$ in $v$ and gets $\pi / r$ instead. The lower is $\pi$ or the higher is $pw$, the greater the incentive the bad type has to mimic the good type and so the smaller the good type's stake must be. Notice that the cost of losing the project is not discounted. The accumulation of internal funds over this period increases the bad type's cost of investing and so allows the offering price to increase, which increases the good type's stake at the end of the interval. This increase in the good type's return exactly offsets the loss from discounting, so that neither explicitly appear in (1.1).

On the right-hand side of (1.1) is the marginal benefit of waiting. With probability $\lambda$ the project is lost and the cost $c$ is avoided. If the project isn't lost, then the cost of $c$ is discounted at the rate $r$. When the left-hand side exceeds the right-hand side, the good type prefers to undertake the project at $t = 0$ than waiting the interval. As the proof to the proposition shows, in such a case the good type will also not want to wait longer than this brief interval to undertake the project.

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In the second case, both underpricing and delay are used to separate types. The marginal cost of waiting is now less than the marginal benefit and so the good type prefers to delay the project. However, the longer she waits, the more capital she accumulates, which has two benefits. First, the amount of proceeds raised with underpriced equity decreases lowering the cost of finance. Second, the amount of underpricing decreases as the bad type’s incentive to undertake the project also decreases with the increase in retained earnings. At some point, the good type’s stake is sufficiently high that the cost of risking the loss of the project just equals the benefits. This is when the offering will take place. However, the upper bound in (1.2), ensures that this optimal time occurs before \( \hat{t} \). Thus some underpricing still occurs.

In the last case, the good type’s incentive to delay investment is so great that she will wait until \( \hat{t} \). At this point, there is no longer any underpricing so that further delay only risks the loss of the project. When this case determines the good type’s offer, a rise in \( v \) will lead to further delay since \( \hat{t} \) is increasing in \( v \).

To develop some intuition for which case will prevail, consider what happens when \( c \) and \( \lambda \) both equal zero. Under these conditions, the good type is indifferent as to when to hold the IPO. In order to satisfy the IC constraint, either equity must be underpriced for the good type or investment must be delayed. For the former, underpricing means that the entrepreneur must give up a larger share of her firm to investors to obtain the needed funds. But for the latter, the entrepreneur also gives up a share of her profits, not to investors, but to time itself. Moreover, with no private cost, underpricing and delay are both proportional cost. That is, the cost of either increases proportionally with the value of the project. This equivalence, of course, extends to the bad project. Thus regardless of how separation is achieved, the good type must give up the same amount of profits and so is indifferent between delay and underpricing.

Now suppose \( c > 0 \) but \( \lambda = 0 \). According to proposition 1.1, only the third cases is ever true and so equity will always be delayed until \( \hat{t} \). The private cost has made delay a more profitable way to separate types than underpricing. Since investors don’t pay for the private cost, selling a larger proportion of the firm to investors by underpricing does not transfer any
of the private cost to them. If instead the project is delayed, the good type discounts this cost of effort. The bad type, however, remains indifferent between underpricing and delay since no private cost is incurred. Thus delaying investment is a more profitable way for the good type to send a credible signal.

Finally, suppose that \( c = 0 \) and \( \lambda > 0 \). Now only case 1 arises and delay never occurs. Since the benefit of delay has been lost, and waiting risks losing the project, the good type will invest immediately.

More generally, when both \( c \) and \( \lambda \) are positive, delay may or may not occur depending on their relative sizes and the expected values of the two different projects. If \( v \) is small, at any time \( t < \hat{t} \), the expected marginal loss from delay is less than the marginal benefit and the good type waits until \( \hat{t} \). As the value of the good project increases, the relative importance of the private cost \( c \) decreases and the more the entrepreneur worries about losing a good project. Thus delay decreases and underpricing becomes the preferred signaling device. Once \( v \) has become sufficiently large, there is no incentive to delay at all and the optimal investment time is the corner solution \( \tau(k) = 0 \).

We have already seen that if the good type is constrained at \( \hat{t} \), then an increase in \( v \) will lead to further delay. However, this is only true at this corner solution. More generally, an increase in \( v \) will lead to a decrease in delay. When \( v \) is high, the cost of losing the project is high and delay becomes less desirable. The following proposition makes this explicit.

**Proposition 1.2** \( t^*_G \) is non-increasing with \( v \) whenever

\[
(\lambda + r)c < \lambda \left( v - k - \frac{\pi}{r} \right).
\]

Proof: Under the condition of the proposition, investment takes place before \( \hat{t} \). If \( t^*_G > 0 \), the optimal investment time is:

\[
t^*_G = \frac{1}{r} \left( \log c(\lambda + r) - \log \lambda \left( \frac{v}{pw} - 1 \right) - \frac{\pi}{r} \right),
\]
which is clearly decreasing with \( v \). If \( t^*_G = 0 \), then if \( v \) increases, the optimal offering size is unchanged since (1.1) continues to hold.

QED

These results so far only consider equity issues. The next section extends the analysis to bond issues and shows that the determinants are qualitatively similar to those for equity. The key difference, however, is that while equity is a proportion payment to investors of the observable (or contractable) part of firm value, debt is a fixed payment.

### 1.3 The model with debt

Rather than issue shares, suppose the entrepreneur could issue bonds. The strategy for the entrepreneur is now a time/bond price pair \( \{ \pi_i, D_i \} \), \( i = G, B \). The bond price, \( D_i \), is the amount a lender pays for a bond that promises to return one dollar. However, the entrepreneur is protected by limited liability and so will only reimburse bondholders to the extent that her cash position is non-negative. The time of the offering is denoted by \( \tau \) instead of \( t \) to distinguish it from an equity offering, but its interpretation and definition is equivalent to that of \( t \).

Since the project pays off immediately, outstanding bonds, when possible, are redeemed for face value the moment after they are issued. The strategy for the market is an accept/reject decision analogous to that for equity.

The current value of the good and bad project undertaken with an offer to invest at \( \tau \) at a price \( D \) are, respectively:

\[
\Pi_G(\tau, D, a(\tau, D)) = \begin{cases} 
 e^{-(\tau + \lambda)\tau} (v - c - I(\tau)/D) + (1 - e^{-\lambda\tau})\pi/\tau & \text{if } a(\tau, R) = 1 \\
 e^{-(\tau + \lambda)\tau^+} (v - c) + (1 - e^{-\lambda\tau^+})\pi/\tau - f & \text{if } a(\tau, R) = 0 
\end{cases}
\]

---

\( ^7 \)Since the outcome of the project is observable and can be contracted upon, this assumption of paying the debt off immediately is innocuous, and simplifies the analysis.
\[ \Pi_B(\tau, D, a(\tau, D)) = \begin{cases} 
  e^{-\tau\tau} (pw - pI(t)/R) & \text{if } a(\tau, R) = 1 \\
  \pi/\tau - f & \text{if } a(\tau, R) = 0
\end{cases} \]

The payoff if no offer is made is, like equity, equal to the payoff from a rejected offer plus the fixed cost \( f \). No offer is indicated by choosing \( \tau_i = t^+ \). The higher the price of a bond, the fewer bonds that need to be issued to raise a given \( I(\tau) \) and the larger the entrepreneur’s expected payoff. Implicit in the payoffs is that the good type never defaults and the bad type defaults if, and only if, her project fails. Any strategy for either type in which this is not the case is a strictly dominated by making no offer. If the good type defaults, her payoff is zero and making no offer gives a strictly higher payoff. Similarly for the bad type if she were to always be in default.

Notice how the payoffs differ here from those for equity issues. In particular, the cost of raising funds is met by making a fixed payment (when possible), rather than by making a proportional payment. This difference between debt and equity, as we will see, induces a preference ordering between the two for the good type that depends on \( \nu \) and \( w \).

The market’s expected payoff from a loan of \( I(\tau) \) at a price \( D \) is

\[ I(\tau) (\mu(G|\tau, D) + (1 - \mu(G|\tau, D))p - D). \] (1.4)

Unlike the equity investor, lenders are only concerned with the probability of being repaid and not the value of the firm. However, because default only occurs when a bad project is taken, the market’s expected payoff depends on its beliefs about the project type. Notice that now the highest price for bonds is

\[ \overline{D} = 1. \]

The equilibrium definition remains the same except that now the market's payoff for the participation constraint in condition 3 is replaced with (1.4).
1.4 Timing of investment with debt

As with equity, for the problem to be interesting the bad type must be willing to invest if she could finance the project at time zero at a debt price of $D$. At this price, debt is so underpriced (i.e., bonds are overpriced) for the bad type that the implicit subsidy of the project’s cost is enough to induce the entrepreneur to invest. Thus, equivalent to assumption 1.2, I assume:

**Assumption 1.3**

$$pw - pk > \frac{\pi}{r}.$$  

A separating equilibrium with a unique outcome also exists with debt. The following proposition gives an analogous characterization of the good type’s offer to that in proposition 1.1.

**Proposition 1.3** A unique equilibrium outcome always exists. The bad type never makes an offer: $\tau_B^* = t^+$. The good type’s offering $\{\tau_G^*, D_G^*\}$ is characterized as follows:

1. $\tau_G^* = 0$ and $D_G^* < \overline{D}$ if

$$\left(\lambda + r\right)v - \left(\lambda + r\right)w + \frac{\lambda \pi}{pr} - \frac{\lambda \pi}{r} > (\lambda + r)c$$  \hspace{1cm} (1.5)

2. $t^+ > \tau_G^* > 0$ and $D_G^* < \overline{D}$ if

$$\left(\lambda + r\right)v - \left(\lambda + r\right)w + \frac{\lambda \pi}{pr} - \frac{\lambda \pi}{r} < (\lambda + r)c < \lambda(w - k - \pi/r) + (\lambda + r)(v - w)$$  \hspace{1cm} (1.6)

3. $t^+ > \tau_G^* > 0$ and $D_G^* = \overline{D}$ if

$$(\lambda + r)c > \lambda(w - k - \pi/r) + (\lambda + r)(v - w)$$  \hspace{1cm} (1.7)
The analysis here is similar to that with equity. Consider condition (1.5). The left-hand side, as with equity, is the cost at $t = 0$ of waiting a small interval of time. By waiting, the good type now risks losing a project with a return $v$ less a repayment on bonds. The amount owed on outstanding bonds equals the amount necessary to prevent the bad type from undertaking the project given that some internal funds have accumulated over the interval. This cost is net of the good type's payoff of $\pi/r$ if the project fails. The benefit is the discounting and possible avoidance of the private cost. When the cost of this delay exceed the benefit, the good type signals using underpricing alone. As the value of the good project decreases, the cost of delay decreases and some delay is desired. When $v$ has decreased sufficiently, the optimal delay is at a corner $\hat{\tau}$, the analogous time to $\hat{t}$.

There is, however, a difference between debt and equity, which is highlighted by considering the good type's equilibrium offer when $c = 0$. Unlike equity, when bonds are issued the good type does not necessarily strictly prefer no delay. If $v$ is sufficiently smaller than $w$, (recall that while $v > pw$, nothing rules out $w > v$,) some delay is desired. This result stems from the fixed payment structure associated with debt. The cost of separating types by making high fixed payment by issuing underpriced bonds is not equivalent to separating types by making proportional payment implied by delay. A decrease in $v$ will not change the degree of underpricing or the length of delay needed to separate types since, under a debt contract, the bad type's incentive to invest hasn't changed. However, as $v$ decreases, the cost of delay decreases, since it is a proportional cost, while the cost of debt is unchanged, since the debt payment is unchanged. Thus as the good project becomes less valuable, delay becomes preferable to underpriced bonds.

Assuming investment takes place before $\hat{\tau}$, despite the difference in how debt and equity are paid out, delay continues to decrease as the value of the good project increases. The comparative static result in proposition 1.2 has its counterpart for debt:

**Proposition 1.4** $\tau^*_G$ is non-increasing with $v$ whenever

$$(\lambda + r)c > \lambda(w - k - \pi/r) + (\lambda + r)(v - w).$$
Proof: Under the condition of the proposition investment takes place before \( \hat{r} \). If \( \tau_G^* > 0 \), the optimal investment time is:

\[
\tau_G^* = \frac{1}{r} \left( \log(c + w - v)(\lambda + r) - \log \lambda \left( \frac{1}{p} - 1 \right) \frac{\pi}{r} \right),
\]

which is clearly decreasing with \( v \). If \( \tau_G^* = 0 \), then if \( v \) increases, the optimal offering size is unchanged since (1.1) continues to hold.

QED

1.5 Debt versus equity

The fixed cost of debt distinguishes it from both delay and equity underpricing as a separating device. Suppose the entrepreneur had a choice between issuing new shares or bonds. Using the equilibrium payoff to the good type for equity and debt financing calculated above, we can determine which yields a higher payoff.

Proposition 1.5 A bond issue yields a higher equilibrium expected payoff to the good type than an equity issue whenever \( v > w \) and a lower one whenever \( v < w \).

Proof: see appendix.

For a fixed \( w \) and \( p \), as \( v \) increases, the proportional payment implied by underpriced equity increases while the fixed payment implied by underpriced bonds remains unchanged. Thus for low values of \( v \), equity is preferred to debt and for higher values, debt is preferred to equity.

Now since by propositions 1.2 and 1.4 there is also a relationship between \( v \) and the optimal investment time, the preferred mode of financing and the timing or size of the issue will also be related. In figures 1.1 and 1.2 the relationship between time and the mode of finance are represented, including the possibility of corner solutions.

Consider first figure 1.1. The horizontal axis is the value of the good project and the vertical axis is time. There are four lines to consider. The horizontal line indicates \( \hat{t}_d \), the
maximum possible delay when debt is issued. It is independent of $v$ since $\bar D$ is independent of $v$. For equity, the maximum delay is increasing in $v$ and is represented by the line $\hat t(v)$. Notice that these curves cross exactly where $v$ equals $w$. Thus the upper bound for debt lies below that for equity when debt is preferred to equity, (i.e. $w > v,$) and vice versa. The curves $\tau(v)$ and $t(v)$ represent the optimal delay for debt and equity if the upper boundaries are ignored (i.e., when the market’s participation constraint is ignored). These curves have a negative slope (propositions 1.2 and 1.4), cross at $v = w$ and $\tau(v)$ lies below $t(v)$ when $v > w$ and above when $v < w$.\footnote{This curve is in fact concave, but there is no loss in generality here from representing non-linear curves with linear ones.}

The region is split into two with equity preferred to debt for $v < w$ and debt preferred for $v > w$.\footnote{To see this first note at interior solutions,
\[
e^{r\tau_0} = \frac{p(\lambda + r)(c + w - v)}{\lambda(1 - p)\pi/r}, \quad e^{r\hat t_0} = \frac{(\lambda + r)cpw}{\lambda(v - pw)\pi/r}
\]

Now compute
\[
e^{r\tau_0} - e^{r\hat t_0} = b \left( \frac{c + w - v}{1 - p} - \frac{cw}{v - pw} \right) = -b(v - w)(v - c - pw) \frac{1}{(1 - p)(v - pw)},
\]

where $b$ is a positive constant. Thus $\tau_0 > (\leq)\hat t_0$ whenever $v < (>)w$.}

Figure 1.1: Financing instrument and delay: only bonds underpriced—$w$ small.
to equity for \( v > w \) (proposition 1.5). The thick black line in the figure, the lower envelope of these curves, represents the equilibrium offering time when the good type chooses her preferred financing instrument. In this figure, \( w \) is sufficiently low that for some values of \( v \) greater than \( w \), the upper bound for debt binds. This means that the upper bound also binds whenever equity is preferred to debt. Once \( v \) is sufficiently high, an interior solution is obtained until the lower bound is eventually reached and no delay occurs. Underpricing only occurs in the shaded region where the upper constraint doesn't bind and so only debt issues are underpriced.

In figure 1.2, the value of \( w \) is higher. Debt issues are never constrained at the upper bound and only for those equity issues where \( v \) is small, is the length of delay constrained. All debt issues are underpriced and some equity issues are as well. If we only consider values of \( v \) for which an interior solution obtains, i.e., where some underpricing occurs, then we have the following prediction: \( t^*_G > t^*_O \).

When the entrepreneur chooses both the type of external finance and the project timing such that any issue is underpriced, equity issues are always smaller than debt issues. The intuition developed above explains this result. The greater the value of the good project, the greater the incentive to avoid delay and underprice the project. But it also means that the cost of separating with underpriced equity is increasing relative to debt. Putting the two together gives the result. Essentially, external equity gets squeezed on both sides; by debt when \( v \) is large and by internal funds (delay) when \( v \) is small.

A second prediction is that firms with large capital requirements relative to their size should have larger initial returns at an IPO than firms with small capital requirements. This prediction is examined in more detail in chapter 2, and the evidence presented there shows that indeed young firms with small book values and sales tend to underprice more than larger more established firms. As for the first prediction, some evidence is offered in the next section.
1.6 Empirical evidence

The Survey of Small Business Finance (SSBF) in 1994 gives us some idea of how little external equity is used. The survey consists of 4737 small businesses (less than 500 employees) operating in the U.S. in 1994. A reasonable 20.1 percent of firms had sought some form of equity financing in the three years prior to the survey. However, only 2.9 percent of firms actually tried to obtain financing from shareholders that didn’t already hold stock in the firm (typically the principal owners) and their families. Of these firms, 64.5 percent tried to use informal sources, 31.9 percent tried to use venture capital and 13 percent attempted to go public (some tried to use multiple sources). Only 44 percent of these firms were successful.

By contrast, 80.3 percent of firms had a line of credit, 15.7 percent had at least one capital lease, 9.2 percent used funds from a mortgage, 19.3 percent had loans for equipment, and 14.1 percent had some other types of loans. Clearly, the casual observation that most small businesses rely on debt much more than external equity is borne out in this data set.

In terms of initial public offerings (IPOs), some evidence for the difference in offering sizes of bond and share issues in given by Datta, Iskandar-Datta and Patel (1997). They examine U.S. corporate straight debt IPOs between 1976 and 1992. They report a mean
issue size of $114.9 million, which is significantly larger than the $13.8 million reported by Ritter (1991) for equity IPOs. They also find that while bond issues are not underpriced, a significant first-day excess returns does exist for firms issuing low grade bonds (rated Ba or lower), the type where we would expect that asymmetry of information is important.

We can also look for evidence in the aggregate statistics. In the U.S., between 1946 and 1998 the average net issue of corporate bonds in 1983 dollars was $34.9 billion. In contrast, it was $-10.4 billion for equity, due to the buyback of new shares in the 1980s and 1990s. Net issues for bonds and shares, respectively, prior to 1980 were $24.2 billion and $8.7 billion. Thus even before the surge in bonds and the share buybacks, bond issues were almost three times as large as equity issues. After 1980, their respective averages were, $54.1 billion and $-44.6 billion.

Figure 1.3 shows the real value of net issues of corporate bonds and shares in the U.S. from 1946 to 1998 and the deflated S&P 500 composite index. As in MacKie-Mason (1990), the values are averaged over business cycles to remove the cycles component of their variation. At no time did the size of new share issues exceed corporate bond issues.

Clearly visible in the graph is a significant increase in bond issues and share repurchases beginning in the mid-1980s. The rise in the relative (and absolute) importance of corporate bonds occurs at the same time as the deflated market index sharply rises. Inasmuch as the rise in the index can be interpreted as an overall increase in the expected value in good projects, the predictions of the model are consistent with this increase use of debt and decreased use of equity.

### 1.7 Moral hazard alone

All the results so far rely on hidden type. However, even with hidden action or moral hazard, debt issues tend to be larger than equity issues. The model with moral hazard is simpler since the market's beliefs about the project type don't arise.

Interpret $c$ now as the cost of effort needed to undertake a good project that can be
The entrepreneur has access to both projects and realizes the expected return to the good one if she doesn’t shirk on her required effort. Effort, however, cannot be monitored, and investors will consider the entrepreneur’s incentive to work hard when they decide whether to accept an IPO offering or not. Only if the entrepreneur’s stake in the firm is large will investors be convinced that the entrepreneur will work hard.

If the project is financed with an offering of size $I(t)$ and price $R$, the entrepreneur will only work hard if

$$\left(1 - \frac{I(t)}{R}\right)(v - pw) - c \geq 0. \tag{1.8}$$

Similarly, for a offering of bonds for proceeds of $I(\tau)$ at a price $D$, the entrepreneur will put in the required effort if

$$v - pw - (1 - p)\frac{I(\tau)}{D} - c \geq 0. \tag{1.9}$$

I assume that both of these constraints are violated if $t = \tau = 0$, $R = v$ and $D = 1$, since otherwise there is no information problem. Suppose prices are fixed at $R$ and $D$. There must be then an $t^* > 0$ and a $\tau^* > 0$ that satisfies both constraints with equality. (Also, both issues sizes must be greater than zero.) For offering sizes above these values, both constraints...
are violated at these prices. However, lowering either price will also cause the constraints to be violated. Thus there is no reason to underprice since it only leads to greater delay.

As long as the entrepreneur waits until min\(\{t^*, \tau^*\}\), she can set the highest possible price at the offering. Thus the choice between debt and equity comes down to which allows her to undertake the good project the earliest. Some simple algebra yields,

\[
I(\tau^*) - I(t^*) = (v - w) \frac{p(v - c - pw)}{(1 - p)(v - pw)}.
\]

Thus the entrepreneur’s preference for the mode of finance is determined exactly as before, with debt preferred to equity when \(v > w\), and vice versa.

Now consider equations (1.8) and (1.9) again. In either case, with prices at their maximum, \(t^*\) and \(\tau^*\) are decreasing in \(v\). Thus if the entrepreneur chooses her preferred financing instrument, the largest value of \(\tau^*\) and the smallest value of \(t^*\) occur when the entrepreneur is indifferent between debt and equity. Thus bond offerings are larger than equity offerings.

### 1.8 Policy implication:

**the small business tax deduction**

While moral hazard leads to a similar result as adverse selection, the policy implication here only applies to the latter. With adverse selection, the separation of the good and bad types is achieved by lowering the entrepreneur’s profits either through high priced external funds or delay. When there is delay there is an inefficiency. This section shows that a policy-maker, even though he doesn’t know firm types, can improve upon the market outcome.

Consider the small business tax deduction. Suppose the policy-maker implements a non-linear tax schedule in which a firm that doesn’t invest face a lower tax rate than one that successfully invest. Let a non-investing firm pay taxes at the rate \(x_l\) and a non-bankrupt investing firm pay taxes at the rate \(x_h\), with \(x_h > x_l\).\(^{10}\) While operating a small firm, then,

\(^{10}\)The rate \(x_h\) can be thought of as the average tax rate for an investing firm as low realized states may be taxed at the low rate. The current policy in Canada is to offer tax deductions on the first $200,000 of
the entrepreneur earns profits at the rate $\pi(1 - x_i)$ and the payoff from investing is now $(1 - x_h)\Pi_i + x_h c$, $i = G, B$—the private cost is not taxed since it is not observed.

Consider equity first. For any investment time $t$, the good type’s payoff if the IC constraint binds is

$$-ce^{-(r+\lambda)t} + e^{-\lambda t} \frac{\Pi_i}{\lambda} \left( \frac{v}{pw} - 1 \right) (1 - x_i) + \frac{\Pi_i}{r} (1 - x_i).$$

(1.10)

Suppose $\lambda > 0$ and the optimal investment time occurs before $\hat{t}_q$. The good type’s equilibrium payoff is:

$$\Pi_G(t^*, R^*, 1) = c^{\frac{-\lambda}{r}} \left( \frac{\Pi_i}{\lambda} \left( \frac{v}{pw} - 1 \right) (1 - x_i) \right)^{\frac{r+\lambda}{\lambda}} \left( \frac{\lambda}{\lambda + r} \right)^{\frac{r}{\lambda + r}} + \frac{\Pi_i}{r} (1 - x_i).$$

Thus both type’s payoffs are independent of $x_h$ and decreasing in $x_i$. The reason $x_h$ does not enter the good type’s payoff is that a high tax is equivalent to underpricing. It is simply a proportional transfer to government rather than to investors so any increase in $x_h$ is offset by an equivalent decrease in required underpricing. But notice if the policy-maker maintains a balanced budget, an increase in $x_h$ will be offset by a decrease in $x_i$. Thus under a balanced budget, an increase in $x_h$ is effectively a transfer to the small firm. The advantage for the good type of this increase in the bad type’s opportunity cost of investing is that the bad type’s incentive to mimic a good offer is reduced, allowing the good type to both reduce delay and underpricing. Thus the tax system is more effective at separating types than either underpricing or delay. However, because of the decrease in underpricing, the investment market would do worse under such a policy.

Suppose that $\lambda = 0$ so that the good type will always delay until $\hat{t}$, which is given by

$$e^{\hat{t}i} = \frac{pw(1 - x_h)(v - k - (\pi/r)(1 - x_i))}{(\pi/r)(1 - x_i)(v - pw(1 - x_h))}.$$ 

Clearly, $\hat{t}$ is decreasing in $x_h$ and increasing in $x_i$. And since when at a corner there is no underpricing, the good type will again prefer a non-linear tax scheme with a high $x_h$ and a low $x_i$. Since investors make no profits, making the tax scheme more non-linear will not business income. Large firms also receive the tax break. It is not clear who really pays for this tax break. The purpose here is only to check if there is a policy that leads to a Pareto improvement.

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effect investors and just lead to a reduction in delay. Thus in this case, a balanced-budget change in the tax system, with $x_h$ increasing and $x_l$ decreasing, is a Pareto improvement.

We can repeat this analysis for debt. The good type’s payoff for any investment time $\tau$ with the IC constraint binding is

$$
(1 - x_h)(v - c - w)e^{-(\lambda + \tau)\tau} + \left(e^{-\lambda \tau} \left(\frac{1}{p} - 1\right) + 1\right) \frac{\pi}{r} (1 - x_l).
$$

(1.11)

Now consider $\lambda > 0$ and an optimal investment time at an interior solution. The good type’s payoff is:

$$
\Pi_G(\tau^*, D^*, 1) = \left(\frac{\pi}{r} \left(\frac{1}{p} - 1\right) (1 - x_l)\right) \frac{\tau}{\lambda + \tau} \left(\frac{\lambda}{\lambda + \tau}\right)^{\frac{\tau}{r}} \left(-\frac{v - c - w}{(1 - x_h)(\lambda + \tau) - 1 + \frac{\pi}{r} (1 - x_l)}\right).
$$

Since $v - c < w$, otherwise no delay occurs, the good type’s payoff is increasing with $x_h$. Thus, for given tax rates, both types will want to increase $x_h$ and decrease $x_l$ and the policy-maker could always propose changes to keep the budget balanced. As with equity, the potential loser is the lending market since part of the benefit of the non-linear scheme is reduced underpricing.

Suppose next that $\lambda = 0$ and $v - c < w$ so that the good type waits until $\tilde{\tau}$, given by:

$$
e^\frac{\tau}{r} = \frac{(1 - x_h)p(w - k - (\pi/r)(1 - x_l))}{(1 - x_l)(1 - p(1 - x_h))\pi/r}.
$$

(1.12)

Like equity, an increase in $x_h$ and a decrease in $x_l$ reduces the amount of delay, which benefits the high type. Of course, the bad type prefers a lower tax rate. And since the lenders’ payoff remains at zero, an increase in the non-linearity of the tax system is socially desirable.

Suppose instead that delay was not desired and, in particular, $v - c > w$. Inspecting (1.11) shows that if $v$ is large, the good type will prefers both taxes to be reduced. And if a balanced budget is required, she will prefer that $x_h$ is reduced at the expense of an increase in $x_l$ so that ideally there is one uniform tax rate.\footnote{Assuming that the tax system cannot be regressive, otherwise the good type would prefer a regressive tax structure.} When $v$ is high, the good type prefers debt to both delay and equity. Equivalently, she prefers to signal her type by making a fixed payment rather than a proportional payment. Since the tax system involves a proportional
payment, despite its ability to make transfers to the bad type, debt remains more attractive if $v$ is sufficiently large.  

1.9 Conclusion

Hidden actions and hidden type can make external capital considerably more expensive than available internal capital. However, as long as a project can be delayed, the amount of internal finance is endogenously determined. The model presented here allows internal capital to be accumulated, but with a cost—discounting and possible loss of the project. Whether to delay the project or not depends on the relative costs of external versus internal financing. The higher the return to the project, the sooner it will be undertaken.

The interesting result is that since the cost of using internal funds is similar to the cost of external equity, internal funds tends to replace equity whenever an equity type contract is desirable. This is not the case with debt. When the return to the good project conditional on no default is high relative to the low project, debt is the preferred capital type. When the relative return is low, internal funds become the preferred choice. This leaves little opportunity for external equity to be used in a substantial way, consistent with its low ranking in the data as a capital source.

A policy maker, however, even without better information about project quality can potentially bring about ex ante Pareto improvements by making transfers from large to small businesses through something like a small business tax credit. What is key to such a policy is the transfer that small business receive that increases the opportunity cost of expanding the firm, helping to encourage only profitable investments. Of course, whether the current subsidy to small businesses given through the small business tax credit is sufficient or not is an empirical matter.

12If, instead, the tax was a fixed lumpsum payment, a high type in this situation would once again prefer a progressive tax system.
Appendix

Proof of proposition 1.1

If the bad type made an offer, i.e., \( t^*_B \neq t^+ \), then either condition 1 or condition 2 of definition 1.1 is violated since the market assigns probability 1 to the project being bad by condition 3. Now in any equilibrium both the IC constraint and participation constraint must be satisfied. To determine when the IPO will be conducted, we need to determine the optimal offering for the good type given the constraints. From the IC constraint, define \( \hat{R}(t) \) implicitly by

\[ \Pi_B(t, \hat{R}(t), 1) = \pi / r. \]

The best offering price satisfying the IC constraint at any time \( t \) is given by \( \hat{R}(t) \). Also define \( \hat{t} \) implicitly by

\[ \Pi_B(\hat{t}, \hat{R}, 1) = \pi / r. \]

After \( \hat{t} \) the IC constraint is strictly satisfied at the highest price \( \hat{R} \). Since the market will not accept any price over \( \hat{R} \), it is never optimal for the good type to wait past \( \hat{t} \).

Now consider the following problem:

\[ \max_{t \in [0, \hat{t}]} \Pi_G(t, R^*(t), 1). \quad (1.13) \]

Let \( t^* \) denote the solution to problem 1.13. The derivative of the objective function with respect to \( t \) is:

\[ e^{-\lambda(t)} \left[ c(\lambda + r)e^{-rt} - \lambda \left( \frac{v}{pw} - 1 \right) \frac{\pi}{r} \right]. \quad (1.14) \]

Inspecting the term in square brackets shows that either the expression is always negative or there is a \( \bar{t} > 0 \) such that it is positive for \( t < \bar{t} \) and negative for \( t > \bar{t} \). It is negative if condition 1 of the proposition holds, so \( t^* = 0 \), or otherwise (1.14) equals zero at \( \bar{t} > 0 \). To verify when \( \bar{t} \) is less than \( \hat{t} \) compute:

\[ e^{rt} - e^{r\bar{t}} = \left( \frac{\pi}{r} \left( \frac{v}{pw} - 1 \right) \right)^{-1} \left( \frac{\lambda + r}{\lambda c - (v - k - \pi / r) \lambda} \right). \]
When condition 2 of the proposition holds, \( t < \hat{t} \) and so \( t^* = \hat{t} \). (Note that by assumption 1.2, the upper bound in condition 2 is in fact greater than the lower bound.) If condition 3 holds, \( t^* = \hat{t} \). The characterization of the offer in each case in the proposition follows immediately.

Uniqueness of the separating outcome follows almost directly. Suppose that \( \{t', R'\} \) is an equilibrium offering. It must satisfy the IC and participation constraints. But if \( \{t', R'\} \) differed from \( \{t^*, R^*\} \), then by continuity of the payoff functions, there exists some small \( \epsilon > 0 \) such that the good type strictly prefers offering \( \{t^*, R^* - \epsilon\} \) to \( \{t', R'\} \) if it is accepted. But since \( \{t^*, R^* - \epsilon\} \) strictly satisfies the IC constraint, the bad type prefers making no offering then this deviation offering, and since \( R^* - \epsilon < R \), the market’s best response is to accept the deviation offering if it believes that good type made it. Thus the offering \( \{t', R'\} \) has failed the Intuitive Criterion and so \( \{t', R'\} \) cannot differ from \( \{t^*, R^*\} \).

The next step is to show that \( \{t^*, R^*\} \) is indeed part of an equilibrium. Let the market’s strategy and beliefs be:

\[
\mu(G|t, R) = \begin{cases} 
1 & \text{if } \{t^*, R^*\}, \\
0 & \text{otherwise}. 
\end{cases}
\]

\[
a(t, R) = \begin{cases} 
1 & \text{if } \{t^*, R^*\} \text{ or } R < pw, \\
0 & \text{otherwise}. 
\end{cases}
\]

Given the market’s strategy, and the definition of \( \{t^*, R^*\} \), there is no profitable deviation by either type, so conditions 1 and 2 of the equilibrium definition are satisfied. Also since \( R^* \leq v \), the participation constraint is satisfied. Finally, beliefs are determined in accordance with condition 4 of the definition.

The equilibrium survives the Intuitive Criterion since there is no deviation that the good type could make that she would strictly prefer, that would give the bad type a strictly lower payoff and that would give market a positive payoff if it believed the good type made the deviation. This establishes existence. QED
Proof to proposition 1.3

The proof is analogous to the proof of proposition 1.1. Define $\hat{\tau}$ and $D^*(\tau)$ analogously to $\hat{t}$ and $R^*(t)$. The maximization problem is

$$\max_{\tau \in [0, \hat{\tau}]} \Pi_G(\tau, D^*(\tau), 1). \quad (1.15)$$

Let $\tau^*$ denote the solution to problem 1.15. The derivative of the objective function with respect to $\tau$ is:

$$e^{-\lambda(\tau)} \left[ -(v - c - w)(\lambda + \tau) e^{-\tau} - \lambda \left( \frac{1}{p} - 1 \right) \frac{\pi}{\tau} \right]. \quad (1.16)$$

As with equity, we can define a $\bar{\tau}$ such that the solution $\tau^* = 0$ if case 1 of the proposition holds, $\tau^* = -\bar{\tau}$ if case 2 holds, and $\tau^* = \hat{\tau}$ if case 3 holds. Underpricing only occurs with the latter. (Assumption 1.3 ensures the upper bound in case 2 exceeds the lower bound.) To determine the condition for case three compute the following:

$$e^{r \hat{\tau}} - e^{r \tau} = \frac{\lambda + r}{\lambda} (w - (v - c)) - (w - k - \pi/r).$$

The rest of the proof proceeds as with equity.

QED.

Proof to proposition 1.5.

Consider any time $t \leq \min\{\hat{\tau}, \hat{t}\}$. Comparing the payoff of investing at this time at the best price for bonds and the best price for shares, such that the IC constraint holds, gives

$$(v - c - I(t)/D^*(t)) - ((1 - I(t)/R^*(t))v - c) = (v - w)\frac{pw - \pi/r}{pw}. \quad (1.17)$$

So for a given $t$, debt is preferred to equity when $v > w$ and vice versa. Now since

$$e^{r \hat{t}} - e^{r \hat{t}} = (v - w)p(pw - k - \pi/r)p(\pi/r)^{-1} \quad (1.18)$$

and $pw - k < \pi/r$, if $v > (\leq)w$, the set of possible times for a debt contract is greater (less) than for an equity contract. Thus by (1.25) and (1.18), at an equilibrium offering for bonds or share where the good type chooses the best investment time, bonds must give a higher expected payoff than shares when $v > w$, and a lower payoff when $v < w.$

QED
Appendix II

In this appendix the equivalent dynamic game is given and I show that there is an equilibrium to this game with the same characterization as that of the unique PBE satisfying the Intuitive Criterion of the commitment game. I will only consider equity as the case of debt is similar.

In order to define the game precisely I will assume time is discrete, starting at $t = 0$. In the dynamic game, the good type, at any date $t$ can make and offer to raise $I^t$ at a price $R^t$. Note that with discrete time

$$I^t = k - \pi \sum_{\tau=0}^{t-1} (1 + r) \tau = k - ((1 + r)^t - 1) \pi / r.$$ 

The market can either accept or refuse the offer. If it accepts, it provides the required funds immediately at the agreed to price, the project is undertaken and players realize their payoffs. Only one offer is possible even if it is rejected. The game ends following a rejection and the bad type gets the payoff from operating a small firm and the good type invests at $t^+$ if possible. The cost $f$ is incurred for both types following a rejection. Thus commitment is still important in this dynamic game and allows complicated issues associated with negotiation to be side-stepped. If a good project is lost, this ends the game and the entrepreneur receives the payoff from operating a small firm. The last period is $t^+$ since at this point no external funds are needed.\(^{13}\)

Since this is a dynamic game with imperfect information, strategies must specify what each player will do at any history of past play that can be reached. Let $H$ denote the set of possible histories of players’ actions and let $h^t \in H$ denote some history at date $t$. Let $a^t_e$ denote the entrepreneur’s action at date $t$, which is either an offer $R > 0$ or no offer, denoted by $\emptyset$. Let $a^t_m$ denote the market’s action at date $t$ such that $a^t_m = 1$ if the market accepts an offer $R^t$ and $a^t_m = 0$ if it rejects it: $a^t_m \in \{0, 1\}$. Let $a^t = (a^t_e, a^t_m)$ denote the vector of actions at date $t$ and so $h^t = (a^0, \ldots, a^{t-1})$.

A pure behavioural strategy for a player, denoted $s_t$, is a map from the set of histories and types to her action space: $s_e(h^t, \theta) \in \{\mathbb{R}_+, \emptyset\}$ and $s_m(h^t, a^t_e) \in \{0, 1\}$. Let $s = \{s_e, s_m\}$

\(^{13}\)The intervals are chosen so that there is an integer number of them between date 0 and date $t^+$
denote the pair of strategies. An action $a^{t}_{i}$ is simply what a player has done at date $t$, which is determined by the player's strategy of what to do at any possible history that is reached.

Since only one offer can be made, the only possible history to date $t$ is $((0, 0), \cdots, (0, 0))$. That is, if $t$ is reached, no offer has ever previously been made (and a good project has not been lost). So since there is only one possible history at every $t$ that is reached, $t$ contains all the information in $h^{t}$ and so I denote a history by just $t$. Beliefs of the market, then, are given by $\mu(\theta|t, a^{t}_{e})$.

**Payoffs**

Payoffs here are slightly different than in the commitment game since they need to be defined for any history reached. Consider any history $t$. First suppose for a strategy $s_{e}$ that conditional on reaching $t$, no offer is made until $t' \geq t$. Thus conditional on reaching $t$, the game effectively ends at $t'$ and so we need only consider the payoff for histories up to $t'$.

$$\Pi_{e}(s|G, t) = \begin{cases} (\delta \lambda)^{t'-t}((1 - I^{t'}/R)v - c) + (1 - \lambda^{t'-t})(\pi/r)(1 + r)^{t+1} & \text{if } s_{m}(t', R) = 1 \\ (\delta \lambda)^{t'-t}(v - c) + (1 - \lambda^{t'-t})(\pi/r)(1 + r)^{t+1} - f & \text{if } s_{m}(t', R) = 0 \end{cases}$$

$$\Pi_{e}(s|B, t') = \begin{cases} \delta^{t'-t}(1 - I^{t}/R)pw & \text{if } s_{m}(t', R) = 1 \\ (1 + r)^{(t+1)}\pi/r - f & \text{if } s_{m}(t', R) = 0. \end{cases}$$

where $\delta = (1 + r)^{-1}$.

Thus at any current date $t$ before $t'$, the entrepreneur evaluates the payoff of following her strategy, given the strategy of the market, at the current date. A good type, in particular, must consider the possible loss of the project between dates $t$ and $t'$. Somewhat different from the commitment game, the cost $f$ is not necessarily incurred at date 0, though this has no effect on the results.\footnote{We could assume the entrepreneur pays $(1 + r)^{t} f$ at the time the offer is refused.} If the entrepreneur strategy is such that she makes no offer and $t^{+}$ is reached then her type dependent payoffs are

$$\Pi_{e}(s|G, t) = \delta^{t^{+}-t}(v - c)$$
As in the commitment game, I implicitly rule out dominated strategies at \( t^+ \) by specifying such payoffs.

It will also be convenient to define payoffs for a particular outcome. Thus if \( s \) is a strategy such that \( s_e(\theta, t) = R \) and \( s_m(t, R) = 1 \) then define

\[
\Pi_e(R, 1|t, \theta) \equiv \Pi_e(s|t, \theta)
\]

The payoff for the market can be defined more simply. When defining an equilibrium below, to ensure payoff-maximizing behaviour we do not need to evaluate the market’s payoff of accepting a future offer, only its payoff of accepting a current offer. The payoff to the market when for some \( t < t^+ \), where \( s_e(t) = R \) for at least one type, is

\[
\Pi_m(s|t, R) = \begin{cases} 
I'(\mu(G|t, R)v + (1 - \mu(G|t, R))pw - R) & \text{if } s_m(t, R) = 1 \\
0 & \text{if } s_m(t, R) = 0
\end{cases}
\]

**Equilibrium concept for the dynamic game**

As in the commitment game, I use the concept of a PBE and focus on separating equilibria. Since this is no longer a signaling game, Cho and Kreps (1987) refinement no longer strictly applies. However, it is not unreasonable to think of an equivalent refinement for this game with the same type of restriction on beliefs. This is included in the following definition.

**Definition 1.2** An equilibrium to this game is a set of strategies \( s^* \) and beliefs \( \mu^* \) satisfying the following conditions.

1. For each type \( \theta \), alternative strategy \( s_e \) and history \( t \),

\[
\Pi_e(s^*|\theta, t) \geq \Pi_e(s_e, s_m^*|\theta, t)
\]

2. At any history \( t \) where an offer is made, \( s_m^*(t, R) = 1 \) if, and only if,

\[
\mu(G|t, R)v + (1 - \mu(G|t, R))pw - R \geq 0
\]
3. If \( t' \) is the first date such that \( s_e^*(t', G) \neq \emptyset \) and \( t'' \) is the first date such that \( s_e^*(t'', B) \neq \emptyset \) then for any \( t \in \{ t', t'' \} \),
\[
s_e^*(t, G) \neq s_e^*(t, B).
\]

4. Bayes’ rule is used to update beliefs whenever possible, even at a \( t \) that was not supposed to be reached.

5. Let \( s_e \) be an alternative strategy for which \( s_e(t', G) = R \) at some \( t' \) and \( s_e(t, G) = \emptyset \) for all \( t < t' \). Let \( s_m \) denote any market strategy for which \( s_m(t', R) = 1 \) and let \( s'_m \) be another for which \( s'_m(t', R) = 0 \). And let \( t^* \) be the date such that \( s_e^*(t^*, G) \neq \emptyset \) and \( s_e^*(t, G) = \emptyset \) for all \( t < t^* \). Then for \( t = \min\{ t', t^* \} \),
\[
\mu^*(G|t', R) = \begin{cases} 
\Pi_e(s_e, s_m|G, t) > \Pi_e(s^*|G, t), & \Pi_e(s_e, s_m|B, t) < \Pi_B(s^*|B, t) \\
\Pi_e(s_e, s'_m|B, t) < \Pi_e(s^*|B, t) & \text{and } R < v 
\end{cases}
\]
\[
\mu^*(G|t, R) \in [0, 1] \text{ otherwise}
\]

The definition of an equilibrium in the dynamic game is very similar to that of the static game. Condition 1 requires that at any \( t \) that is reached, making a deviation offer, given the market’s strategy, cannot improve the entrepreneur’s payoff. This also includes the possibility that at the date the entrepreneur was to first make an offer, none is made (a mistake or deviation), and then the entrepreneur carries on with her equilibrium strategy.

Condition 2 simply states that the market will accept any offer such that given equilibrium beliefs, its expect payoff is non-negative. Condition 3 states that the two types do not make the same offer, if they make one at all, so that separation occurs. Condition 4 is self-explanatory.

The restriction on beliefs in condition 5 is analogous to that of the static game. Consider an alternative strategy for type \( G \) such that an offer is made. First suppose this offer occurs before the equilibrium offer. The market believes the good type made the deviation if: (1) the good type’s payoff evaluated at \( t' \) from making this deviation offer that is accepted is greater
than the equilibrium payoff; (2) the bad type's payoff is strictly lower than the equilibrium payoff, regardless of the market's response; and (3) the market has a strict incentive to accept the offer if it believes the good type made it.

Second, suppose the deviation offer occurs after the date the equilibrium offer is to be made. If the same three conditions hold, with payoffs evaluated at $t^*$, the market believes the good type made the deviation offer. Since it will never be the case that the bad type prefers a deviation offer with $R < pw$, the restriction on beliefs is only important for deviations made by the good type.

The refinement does have an important restriction in how it is used. It compares deviations only with the payoff associated with the equilibrium offer. Suppose no such deviation offer existed but that the good type mistakenly didn't make her equilibrium offer. Her strategy, of course, still specifies what to do and determines her equilibrium payoff conditional on reaching any unexpected dates. One could argue that we should apply the refinement again. That is, if the market observed an offer that wasn't expected conditional on reaching a date $t$ that could not benefit the bad type, but does benefit the good type, compared to the payoff from her equilibrium strategy conditional on reaching $t$, then the market should believe the good type made this offer. However, one could also argue that if the good type's failure to make her initial equilibrium offer was a mistake, than the new deviation offer at $t$ could also be a mistake by the bad type. That is, once a mistake is thought to occur, forward induction seems less reasonable. Thus I retain the simple refinement as defined.

I now show that there is an equivalent proposition to proposition 1.1.

**Proposition 1.6** A unique equilibrium outcome always exists. The bad type never makes an offer. The good type makes an offer at $t^*_G$ with a price of $R^*_G$. The offer is characterized as follows. For critical values $c_1$ and $c_2$, determined by parameters, where $c_1 < c_2$,

1. $t^*_G = 0$ and $R^*_G < \bar{R}$ if

$$c_1 \geq -c \log(\lambda \delta)$$

\[ (1.19) \]

---

\[ ^{15}\text{If the good type makes a deviation offer that the market should accept, given condition 5, but does not, the game ends. Thus issues about beliefs in which there are multiple deviations do not arise.} \]
2. $t^+ > t^*_G > 0$ and $R^*_G < \bar{R}$ if
\[
c_2 > -c \log(\lambda \delta) > c_1 \tag{1.20}
\]

3. $t^+ > t^*_G > 0$ and $R^*_G = \bar{R}$ if
\[
-c \log(\lambda \delta) \geq c_2 \tag{1.21}
\]

Proof: The proof is similar to that for the commitment model. However, because of discrete time, explicit expressions for the boundary conditions are not possible. The main difference in the proof is the construction of the equilibrium (see the section on existence below).

If the bad type made an offer in equilibrium, the type is revealed by condition 3 of the equilibrium definition. But then either condition 1 or condition 2 of definition 1.2 is violated. That is, the offer is rejected and the bad type should not have made it, or either the bad type does worse than not making an offer or the market incurs a loss.

If the good type makes an offer, then by condition 3 it must be the case that for the first date, say $t^*$, at which the good type makes an offer, the bad type does not mimic it. Suppose $s^*_e(t^*, G) = R^*$. By condition 1,
\[
\Pi_e(s^*|t^*, B) = (1 + r)t^*+1 \pi/r \geq \Pi_e(R^*, 1|t^*, B).
\]

This is the same as the IC constraint of the commitment model. If the good type makes an offer, then the markets participation constraint must be satisfied (i.e., $R^* \leq \bar{R}$) otherwise the market will refuse the offer and the good type prefers to make no offer at $t^*$ to avoid the cost $f$.

Next I determine the optimal offer for the good type given the constraints. From the IC constraint, define $\hat{R}^t$ such that
\[
\Pi_e(\hat{R}^t, 1|t, B) = (1 + r)^{t+1} \pi/r.
\]

The best offering price satisfying the IC constraint at any time $t$ is given by $\hat{R}^t$. Also define $\hat{t}$, such that
\[
\Pi_e(\bar{R}, 1|\hat{t}, B) = \pi/r(1 + r)^{\hat{t}+1}.
\]

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After \( i \) the IC constraint is strictly satisfied at the highest price \( \bar{R} \). Since the market will not accept any price over \( \bar{R} \), it is never optimal for the good type to wait past \( i \) if the market is willing to accept \( \bar{R} \) at \( i \). Let \( \hat{R} = \bar{R} \) for each \( t \geq i \).

The good type must make an offer in equilibrium. Suppose to the contrary the good type doesn’t make an offer in equilibrium. Consider a deviation offer \( \bar{R} - \epsilon \), for some small \( \epsilon \), at \( i \). She prefers this offer to never making an offer if it is accepted. But by condition 5, \( \mu^*(G|\bar{R} - \epsilon, i) = 1 \), and by condition 2, the market will accept. So the good type makes an offer in equilibrium that is accepted. (If it wasn’t accepted, the good type would be better off not making an offer.)

Let \( t \) be the first date that the good type makes an offer when following her strategy. Now consider the following problem:

\[
\max_{t \in \{0, \ldots, \hat{t}\}} (v(1 - I^t/\hat{R}^t) - c)(\lambda \delta)^{t-t'}(1 + r)^t(1 - \lambda^{t-t})(1 + r)\pi/r, \tag{1.22}
\]

where \( t' = 0 \). The objective function is the good type’s expected payoff at \( t' \) of making an offer, if the project isn’t lost, of \( \hat{R}^t \) at \( t \geq t' \) that is accepted. Let \( t^* \) denote the solution to problem 1.22. The derivative of the objective function with respect to \( t \) is:

\[
(1 + r)^t \lambda^{t-t'} \left[ -c \log(\lambda \delta) \delta^t + \log(\lambda) \left( \frac{v}{pw} - 1 \right) (1 + r) \frac{\pi}{r} \right]. \tag{1.23}
\]

Inspecting the term in square brackets shows that either the expression is always negative or there is a \( \tilde{t} > 0 \) such that it is positive for \( t < \tilde{t} \) and negative for \( t > \tilde{t} \). It is negative at \( t = 0 \) if

\[
-c \log(\lambda \delta) \geq -c \log(\lambda \delta). \tag{1.24}
\]

Let

\[
\phi = -c \log(\lambda \delta).
\]

Since the optimal delay is increasing in \( c \) continuously, there is a \( c_1 \) such that if \( \phi = c_1 \) the good type is indifferent between investing at \( t = 0 \) and \( t = 1 \), given that \( \hat{R}^t \) is offered and accepted.
Suppose $\phi > c_1$ so that delay until at least $t + 1$ is preferred. To verify when $t + 1$ is less than $\hat{t}$ compute:

$$
\delta^t - \delta^{t+1} = (1 + r) \frac{\pi}{r} \left( \frac{v}{pw} - 1 \right) \left( \frac{1}{v - k - (1 + r)\pi/r} - \frac{\log \lambda}{c \log(\lambda \delta)} \right).
$$

As before, continuity of the optimal time in $c$ means there is a $c_2$ such that

$$
-\delta \log(\lambda)(v - k - (1 + r)\pi/r) \leq c_2 \leq -\log(\lambda)(v - k - (1 + r)\pi/r)
$$

and for $\phi = c_2$, the good type is indifferent to investing at the closest date before $\hat{t}$ and the closest date after $\hat{t}$. Thus for $\phi > c_2$, the good type's optimal investment time is $\hat{t}$ and then $R^* = v$.

**Existence**

The next part of the proof shows that there exists an equilibrium in which the good type invests at the $t^*$ determined above with an offer price of $\hat{R}^*$. This is done by construction.

Consider the following equilibrium $\{s^*, \mu^*\}$:

$$
s_e(t, G) = \begin{cases} 
\emptyset & \text{if } t < t^* \\
\hat{R}^t & \text{if } t \geq t^* 
\end{cases}
$$

$$
s_e(t, B) = \emptyset \text{ for all } t
$$

$$
\mu^*(G|t, R) = \begin{cases} 
1 & \text{if } t \geq t^* \text{ and } R = \hat{R}^t \\
0 & \text{otherwise.}
\end{cases}
$$

$$
s_m(t, R) = \begin{cases} 
1 & \text{if } (t \geq t^* \text{ and } R = \hat{R}^t) \text{ or } R \leq pw \\
0 & \text{otherwise.}
\end{cases}
$$

Given the market's strategy, any deviation by the good type before $t^*$ will lead to a rejection or selling shares at a price below $pw$, which in either case gives a lower payoff than the equilibrium payoff. Consider a deviation at any date on or after $t^*$. The good type could raise the offer, which will then be rejected. But at any $t \geq t^*$, the entrepreneur prefers
offering \( R_t \) and investing than waiting until \( t^+ \) to invest. Alternatively, \( pw \) or lower could be offered, which will be accepted. But \( R_t > pw \). The last possibility is to make no offer and wait until the next date to resume with the equilibrium strategy. But for \( t \geq t^* \), given the derivative (1.23), the good type prefers investing at \( t \) with price \( R_t \) than waiting until \( t + 1 \) to invest at price \( R_{t+1} \).

For the bad type, any deviation that does not mimic the good type’s equilibrium offer and is above \( pw \) is rejected, which gives the low type of strictly lower payoff (pays \( f \)) than not making an offer. Making the same offer as the good type leaves the bad type’s payoff unchanged because \( R_t \) is determined by the strictly binding IC constraint. Finally, any offer at or below \( pw \) cannot be profitable by definition of a bad project. Thus condition 1 of the definition is satisfied for both types. And since the good type makes an offer and the bad type doesn’t, condition 3 is satisfied.

Since the market believes that if any offer other than \( R_t \) at any \( t \geq t^* \) is made, the project is bad, it correctly rejects all such offers unless the price is below \( pw \). Accepting the offer \( R_t \) at any \( t \geq t^* \) leaves the market with a non-negative payoff, given its beliefs, and so condition 2 of the definition holds.

Last, consider beliefs. Let \( s \) be an alternative set of strategies such that at some \( t \) the good type offers \( R \leq R_t \) and \( t \neq t^* \), \( R \neq R^* \) or both. Since the sign of the derivative (1.23) does not depend on \( t' \), we have for any \( t' \leq t^* \),

\[
\Pi_e(s^*|t', G) > \Pi_e(s|t', G)
\] (1.25)

Since conditional on reaching any \( t \geq t^* \), only the good type makes an offer \( R_t \), the market’s beliefs are consistent with Bayes’ rule given that \( t \) is reached—condition 4 is satisfied. By (1.25), there is no alternative offer at any date that meets the requirements in condition 5 for the market to believe a high type made the offer. Thus conditional on reaching \( t \geq t^* \) any offer other than \( R_t \) is unexpected and so beliefs can be chosen arbitrarily.
**Uniqueness**

The final part of the proof is to show uniqueness. Suppose the equilibrium outcome is that $R'$ is offered by the good type at $t'$, it is accepted, and the bad type makes no offer. Let $s'$ be the strategies associated with this outcome. This offer must satisfy the IC and participation constraints. Suppose $R' \neq \hat{R}^*$ or $t' \neq t^*$, or both. For some small $\epsilon > 0$ consider an offer $\hat{R}^* - \epsilon$ at $t^*$. Let $s$ be a set of strategies such that $\hat{R}^* - \epsilon$ is offered by both types and accepted at $t^*$ and let $\hat{s}$ be a set of strategies such that the bad type never invests. Then since the good type’s payoff has a unique maximum at an offer $\hat{R}^*$ at $t^*$ (subject to IC and participation constraints) and by continuity of the payoff function, for $t = \min\{t^*, t'\}$

$$\Pi_e(s'|t, G) < \Pi_e(s|t, G)$$

$$\Pi_e(s'|t, B) > \Pi_e(\hat{s}|t, B) > \Pi_e(s|t, B)$$

$$\hat{R}^* - \epsilon < \bar{R}$$

But then the market must believe that $\hat{R}_e^*$ at $t^*$ is made by a good type and accepts it. The equilibrium in question, then, has failed to survive the Intuitive Criterion as defined for this game. This proves uniqueness.

QED
Chapter 2

Can Capital Requirements Explain IPO Underpricing?

During an initial public offering, the insiders of the firm must together with the underwriter specify an offering price at which shares will be sold. If everyone agreed on what the firm was worth, we should expect that the offering price multiplied by the total number of shares to be outstanding should equal the firm’s net present value. However, Ibbotson documented in 1976 that the return over the first month is unusually high, over 10 percent on average, which seemed to imply that the offering price was too low. It was soon discovered that this high average return actually occurs over just one day and typically averages close to 15 percent in the U.S. More recently, Barry and Jennings (1993) show that in their sample, on average about 90 percent of this initial return is realized in the opening transaction (see also, Hunt-McCool, Koh and Francis (1996)). This phenomena has been appropriately termed underpricing.

Underpricing has commanded a great deal of both theoretic and empirical research by financial economists. However, the empirical research has not yet been able to provide overwhelming supportive evidence for any particularly theory leaving the door open for both more theoretic and empirical investigation. This chapter does both. The first part extends existing signaling theories of underpricing in such a way that both the empirical weakness
of existing models is avoided and new predictions are derived. The second part takes these predictions to the data where they find strong support. ¹

In the simplest form of the model, underpricing arises as a signal of firm quality. The risk-averse entrepreneur, or more generally the insiders of the firm, initially owns the entire firm and wishes to undertake an IPO to both raise money to finance a risky project and to diversify his position. The entrepreneur has better information about the expected value of the risky project compared to risk-neutral investors. The project either has a high expected value or a low expected value. The entrepreneur knows which, while investors only have a probability assessment attached to each state. Investors’ willingness to pay will depend on their valuation of the firm, which provides the high type (the entrepreneur with the high value project) an incentive to distinguish himself from a low type.

Except for the firm’s capital requirement, the basic model does not otherwise differ qualitatively from Leland and Pyle’s (1978). They showed that in such a situation, the high type will keep a large stake in the firm, exposing himself to large amounts of risk, as a credible signal of firm or project quality. The credibility of the signal is a result of the low type being unwilling to keep such a large undiversified stake in a low value firm, even if he can sell a small amount of overpriced equity. However, when the firm must earn enough proceeds from the IPO to fund the project, the analysis changes.

In order to raise proceeds, the entrepreneur must issue new shares, called primary shares. The proceeds from these shares go to the firm’s treasury and can be used to finance the project. The private shares of the entrepreneur (insiders), called secondary shares, can also be sold at the IPO but the proceeds accrue to the entrepreneur and are not available to the firm. Given an offering price, to raise a certain number of proceeds for the firm, a certain amount of primary shares must be sold. However, the more that are sold, the more the entrepreneur’s stake in the firm is reduced through dilution.

¹Of course, support for one theory does not necessarily preclude other explanations as also contributing to the phenomena. Indeed the model is easily extended to show that Rock’s (1988) winner’s curse explanation can be incorporated into the signaling framework.
to offer, he must consider two constraints. On one hand, he must keep a large stake in the firm by not selling too many primary or secondary shares to signal the firm's quality. On the other, he must sell a sufficient number of primary shares to cover the cost of the project. If both constraints bind, the entrepreneur will decrease the number of secondary shares since this allows him to increase the number of primary shares, which increases the firm's proceeds, and still keep the same stake in the firm. But the constraints may bind even if no secondary shares are sold. In this case, only setting an offering price below the true inferable value of the firm will allow both constraints to be satisfied; hence, underpricing.

But it seems that decreasing the offering price will reduce the amount of proceeds and only exacerbate the firm's minimum proceeds constraint. Indeed, if only the offering price was reduced, this would be true. But a lower offering price decreases the incentive for a low type to mimic a high type, allowing the high type to reduce his stake, i.e., more primary shares can be sold. The combination of a lower price and greater number of shares sold is shown to lead to an increase in total proceeds.²

Underpricing, then, arises when the high type is constrained to earn a minimum amount of proceeds for the firm. An increase in the firm's capital requirement will cause the minimum proceeds constraint to bind tighter leading to increased underpricing. When not constrained by the proceeds requirement, a small change in the cost of the project (or the firm's internal funds) will have no effect on underpricing. Thus, if we can identify constrained and unconstrained firms, we can test how the initial return is related to the firm's capital needs in both groups. This forms the basis for the empirical test.

How then are the two groups identified? Recall, when the minimum proceeds constraint binds, the entrepreneur will not sell secondary shares since this allows him to sell more primary shares for a given stake in the firm. When the constraint doesn't bind, there is

²This is a general result and not particular to the specification of the model. Since the only incentive for a low type to mimic a high type is to be able overprice the offering, as the high type's price approaches the low type's true price, the incentive to mimic vanishes and then so does the restriction on the number of primary shares. As long as the low return project is a profitable project, the project's cost can be covered at the IPO.
no reason to not sell secondary shares. Indeed the non-binding constraint means that when sufficient funds have been raised through the selling of primary shares, the entrepreneur would still like to reduce his stake. Since no more capital is required, the entrepreneur should reduce his stake by directly selling his position rather than diluting it by raising proceeds for the firm in excess of what can be used. Thus the presence of secondary shares, observable in the data since it must appear in the prospectus, indicates that a firm is not constrained and their absence indicates that a firm is constrained.

To test these predictions I use a switching regression framework. The first step is to determine how proxies for the firm's capital requirement, such as age and sales, affect the probability that the firm is constrained or not. Following the theory section, I model this as a latent variable process where the entrepreneur decides to sell his private shares only if the firm's capital requirement falls below a critical level. A probit model is first estimated to show how these proxies for the firm's capital requirement affect the likelihood of a firm being constrained. Indeed, small young firms are more likely to face a binding constraint than their larger older counterparts.

The next step is to regress underpricing against the same proxies (plus some additional regressors) conditional on whether or not the firm is constrained. I account for the possible correlation between the error terms in the probit selection equation and the regression equations. For example, an unmeasured characteristic that makes a firm more likely to be constrained, captured in the probit error term, may also contribute to increased underpricing. I find that these proxies for the firm's capital requirement both explain underpricing in the predicted way and explain underpricing statistically much better when firms are constrained than when they are not, as predicted.

One aspect of the data the simple model is not able to explain is why there is any substantial positive initial return among unconstrained firms. Generally, one could argue there are other underpricing motivations that must also be operating. To show how this is possible, Rock's (1988) popular winner's curse explanation for underpricing (discussed below) is grafted on to the signaling model. The predictions of the simpler model remain,
but now the theory predicts that even unconstrained firms will engage in some underpricing.

Secondary offerings, lockup and moral hazard are also importantly related to the IPO. In particular, I show that when the entrepreneur's unobservable effort is important to firm value, entrepreneurs with either type of project will want to use lockup to commit to high future effort to keep the IPO price high. After the IPO, if there is no commitment, the entrepreneur has already received payment for shares at the IPO and so will engage in a secondary offering and reduce his stake in the firm further, lowering its value (since his effort will be less), which investor at the IPO will correctly account for. Thus since all types use lockup, lockup cannot be used as a signal of quality.³

Related literature

Most closely related to my model are those of Allen and Faulhaber (1989), Grinblatt and Hwang (1989), and Welch (1989).⁴ In Grinblatt and Hwang, entrepreneurs are risk averse and need to signal along two dimensions, the mean and variance of their projects. Underpricing, then, is used in conjuction with the entrepreneur's stake to signal type. The cost of the project plays no role. In contrast, in Allen and Faulhaber, and Welch, entrepreneurs are risk neutral and the purpose for the IPO is to raise funds to cover the startup cost of a project. To ensure the sorting condition holds so that types can be separated in equilibrium, both models assume that the firm's type is sometimes revealed (imperfectly in the case of Allen and Faulhaber's model) before a secondary offering. Entrepreneurs underprice their offer, the only signal available, since it allows them to sell their remaining stake in the firm at full value (with no further signaling cost) at the subsequent secondary offering.⁵

³However, without lockup, a high stake in the firm has little signaling power since the entrepreneur bears little cost if he can reduce his high stake immediately after the IPO. Thus lockup is necessary for signaling but is not a signal itself.

⁴A somewhat different approach to information revelation is given by Chemmanur (1993) in which underpricing induces investors to imperfectly learn about the firm type. Their collective information is, by their bidding in the IPO, conveyed to all investors, which is valued by a high type firm since it can expect to charge a price closer to its true value at a secondary offering.

⁵Neither paper models why risk neutral entrepreneurs need or desire a secondary offering.
All three signaling models crucially rely on a secondary offering to recoup the loss from underpricing. However, Spiess and Pettway (1997) find that the loss from underpricing is too high to be justified by a better secondary offering price and Garfinkel (1993) shows that there is little relation between underpricing and the probability of a subsequent secondary offering or open market private share sales subsequent to the IPO. Jegadeesh, Weinstein and Welch (1993) and Slovin, Sushka and Bendeck (1994) do find some support for a positive but small relationship between underpricing and the likelihood of a subsequent offering, but the first set of authors argue that other explanations for underpricing are more convincing.\footnote{Among other things, Jegadeesh et al. also show that while the price drop at the announcement of the secondary offering is less than if the firm underpriced, the effect is small.} The model developed in this chapter does not require an secondary offering to take place and so is not subject to these empirical criticisms.

There are other explanations for underpricing unrelated to signaling by the firm. Rock’s (1986) winner’s curse explanation, as mentioned above, I show can be embedded within the signaling model. This explanation posits that some informed investors have more information than both the entrepreneur and uninformed investors. However, informed investors are few in number so that uninformed investors must participate in the IPO for sufficient funds to be raised. Uninformed investors realize that if informed investors only participate when the project has a high expected value, then winning shares is more likely in a low value firm. Their expected value conditional on winning is, therefore, lower than the unconditional expected value and so the offering price, if uninformed are to participate, must also be lower than the unconditional expectation.

Some additional explanations are lawsuit avoidance put forward by Ibbotson (1975), Baron’s (1982) principal-agent framework where the underwriter has private information, and an ownership dispersion argument of Booth and Chua (1996). Support for law-suit avoidance is provided by Tinic (1988) but evidence against is given by Drake and Vetsuypens (1993). Evidence against Baron’s model is provided by Muscarella and Vetsuypens (1989), who show that in their sample underpricing still arises among IPOs of self-underwritten
The rest of the chapter proceeds as follows. The basic model is presented in the next section and shows in what way the distinction between primary and secondary share is important. In addition, the full information case is briefly examined and a numerical example is introduced, and continued in appropriate subsequent sections, to help fix ideas. The effect of adverse selection under a non-binding minimum proceeds constraint is considered in section 2.2 and the case when the constraint binds and underpricing arises is dealt with in section 2.3. Section 2.4 shows how Rock's winner's curse model can be incorporated to explain positive initial returns among unconstrained firms. The possibility of moral hazard arising from unobserved effort is briefly discussed in section 2.5 followed by a more detailed consideration within the context of lockup. The rest of the paper consists of testing the theory. The data is introduced and some preliminary results are given in section 2.7 and then the switching regression model is given in section 2.8. Results and specification tests for the regression model are given in section 2.9. A brief conclusion then follows. All proofs are found in the appendix.

2.1 The model

There are two reasons for taking a firm public. First, the entrepreneur may need to raise funds to cover the cost of a new project. The IPO is a way to tap into new equity. Second, the IPO allows the entrepreneur to sell some of his stake in the firm and diversify his portfolio. Both these motivations for going public are present in the model.

The project

To undertake the project the entrepreneur needs to raise $K$ dollars of capital to cover its start-up cost. We can think of $K$ as the rental cost of physical capital for the duration of the project, which must be paid when the project begins.\footnote{Alternatively, the capital is firm specific and has no outside market value so that the cost $K$ is sunk. Furthermore, $K$ is net of any assets already in place and paid for prior to the IPO.} The entirety of the cost must
be covered by the proceeds from the IPO. The project lasts one period and generates an end-of-period payment of

$$\mu_i + \epsilon_i,$$

where,

$$\epsilon_i = \sigma z_i,$$

$i = H, L$, $\mu_H > \mu_L$, $\sigma > 0$, and $z_H$ and $z_L$ are random variables with zero means and unit variances. The subscripts $H$ (high) and $L$ (low) indicate which of the two possible project types the entrepreneur has. The high type project has a higher expected value than the low type project. It is common knowledge that the probability of having a high type project is $h \in (0, 1)$. The entrepreneur always knows which type of project he has. We will consider below the case where investors, those that buy shares at the IPO, know the project type and the case where they only know the distribution of projects. The discount rate is normalized to zero.

Preferences

The entrepreneur's preferences are defined over terminal wealth $w$ and are represented by a strictly concave utility function, $U(\cdot)$, which exhibits non-increasing absolute risk aversion. That is, the entrepreneur dislikes risk and is willing to pay a risk premium, which is decreasing in his expected wealth, to avoid it. More formally, let $r(w)$ denote the Arrow-Pratt measure of absolute risk-aversion, i.e.,

$$r(w) = -\frac{U''(w)}{U'(w)},$$

then $r(w) \geq 0$ and $r'(w) < 0$. Non-increasing absolute risk aversion ensures that the entrepreneur with the high type project is willing to tolerate at least as much risk, i.e., requires a smaller risk premium to take on a given level of risk, than an entrepreneur with a low type project. Unlike entrepreneurs, investors are assumed to be risk neutral.\(^8\)

\(^8\)Rather than an end-of-period payment, $\mu_i + \epsilon_i$ can be thought of as the expected present gross discounted value of the cash flow the project generates.

\(^9\)More realistic would be to assume investors are also risk-averse but that they have optimally diversified
2.1.1 Primary and secondary shares

Two different types of common shares can be issued at the public offering: primary and secondary.\textsuperscript{10} Primary shares are issued by the firm and the proceeds from their sale belong to the firm. That is, these proceeds belong to each post-IPO shareholder in proportion to his or her stake in the firm. Secondary shares are issued by the entrepreneur, or more generally, insiders in the firm, directors and large pre-IPO stakeholders, and proceeds from their sale belong to the individual issuers and not the firm. Thus secondary share do not contribute to the firm's available post-IPO cash. We will later see why this distinction is important.

To be precise, let $n_i^e$ denote the number of shares the type $i$ entrepreneur holds prior to the IPO.\textsuperscript{11} Since the number of pre-IPO shares is arbitrary, I normalize $n_i^e$ to equal one. At the IPO the type $i$ entrepreneur sells $n_i(\geq 0)$ primary shares and retains $\gamma_i \in [0,1]$ shares so that $1 - \gamma_i$ secondary shares are sold. Denote the type $i$ entrepreneur's post-IPO stake in the firm as $\alpha_i$. Thus,

$$\alpha_i = \gamma_i / N_i.$$  

Clearly the entrepreneur increases his stake in the firm if he reduces his sale of secondary shares. But also notice that an increase in primary shares leads to an increase in $N_i$, the total post-IPO number of shares outstanding, which dilutes the entrepreneur's ownership of the firm. Thus the entrepreneur can achieve the same stake in the firm with different mixes of primary and secondary share sales. Of course, that does not mean that the value of the firm is necessarily independent of the mix of primary and secondary share sales. We will portfolios. This complication, however, is not important for the paper's main insights and so I use the simpler risk neutrality assumption.

\textsuperscript{10}Only common, and not preferred, shares are considered here since the model abstracts from debt and priority of shareholders' claims on the firm in default states is then not relevant. That is not to say that the distinction is unimportant, but only that introducing debt and a hierarchy of claims on the firm would make the model very complex and obfuscate the main results. For the same reason, I also abstract from claims on the firm more complex than common equity, such as derivatives (calls, puts, warrants, etc.).

\textsuperscript{11}I sometimes will refer to the entrepreneur as having a type instead of holding a project of a certain type, only a semantic distinction is intended.
consider how the entrepreneur chooses the relative proportions of primary and secondary shares in sections 2.2 and 2.3.

The value of the type \( i \) firm at the end of the period is

\[
\pi_i = \mu_i + \xi_i + n_i p_i - K, \tag{2.1}
\]

where \( p_i \) is the offering price per share and \( \xi_i \) is a realization of \( \varepsilon_i \). Only proceeds from primary shares, \( n_i p_i \), and not those from secondary share sales, enter the firm’s profits. These proceeds are earned and the capital cost is paid at the beginning of the period while the return \( \mu_i + \xi_i \) is earned at the end of the period. (Recall that the discount rate is normalize to zero.)

In order to finance the project, sufficient proceeds must be raised to cover the capital cost:

\[
n_i p_i \geq K. \tag{2.2}
\]

Thus, if for the moment we take \( p_i \) as independent of \( n_i \), then (2.2) places a lower bound on \( n_i \).

How is \( p_i \) determined? The offering price is chosen by the firm, together with the underwriter, and cannot exceed the investors’ willingness to pay.\(^{12}\) If the project type is known, then investors will be willing to pay:

\[
p_i = E\pi_i = \frac{\mu_i + n_i p_i - K}{N_i},
\]

since at this price they expect to break-even. That is, investors pay \( p_i \) for a share at the beginning of the period and receive for every share they own, \( \pi_i/N_i \) at the end of the period. Solving for this full information (FI) price \( p_i \) gives:

\[
p_i^{FI} = \mu_i - K. \tag{2.3}
\]

The type \( i \) entrepreneur receives \( (1 - \gamma_i)p_i \) for the secondary shares he sells at the beginning of the period and receives \( (\gamma_i/N_i)\pi_i \) at the end of the period. So with a stake \( \alpha_i \) in

\(^{12}\)Here, demand is assumed to be non-stochastic and perfectly elastic at the investors’ expected value of the firm conditional on their information set at the IPO. This abstraction allows us to focus on the role of information about the project type in determining the offering price rather than demand conditions.
the firm, a price of \( p_t^{FI} \) and \( N_t \) total shares outstanding after the IPO, the entrepreneur’s monetary payoff is:

\[
(1 - \gamma_i) p_t^{FI} + \alpha_i \pi_i. \tag{2.4}
\]

Substituting (2.1) and (2.3) into (2.4) yields:

\[
\mu_i + \alpha_i \hat{\pi}_i - K. \tag{2.5}
\]

Thus, at least under full information, since only \( \alpha_i \), and not \( \gamma_i \) or \( n_i \), explicitly appears in (2.5), the mix of primary and secondary shares is only relevant insofar as (2.2) must be satisfied. Moreover, since the entrepreneur dislikes risk, \( \alpha_i \) will be set to zero to avoid it—the entrepreneur sells his entire stake.

The minimum size of the primary offer is determined by (2.2) and (2.3):

\[
n_t^{FI} \geq \frac{K}{\mu_i - K}. \tag{2.6}
\]

Issuing more primary shares than what is needed to cover the start-up cost has no effect on total payoffs to either investors or the entrepreneur. Cash not used to cover the project cost is simply kept in the firm (or, rather, invested in a safe asset) and then returned to each investor in accordance with their contribution (see (2.1)). For the entrepreneur, the increase in the value of the firm is irrelevant since he has no claim on the firm. However, even if he kept some stake in the firm, the benefit of a higher per share price from the increase in cash will be exactly offset by the dilution of his shares. To see this, simply take the expectation of (2.5); type i’s expected payoff does not depend the number of shares outstanding. Thus, regardless of whether the entrepreneur sells his entire claim on the firm or not, there is no reason to issue more primary shares than those needed to cover the project’s cost. We will need this assumption later and so label it here:

**Assumption 2.1** The entrepreneur never sells more primary shares than needed to cover the cost of the project. That is, \( p_i n_i = K, \ i = H, L. \)

Hence, using (2.6), it follows that \( n_H^{FI} > n_L^{FI} \), and \( n_t^{FI} \) is increasing in \( K \); a firm with low value project must sell more primary shares to cover a given start-up cost than a firm with
high value project and the greater the cost the more primary shares either type of firm must sell.

2.1.2 Example

While the above analysis is not complicated, the simple example introduced here will later prove useful to fix ideas about the effect of adverse selection introduced in the following section. Parameterize the model as follows: $\mu_H=$15 million, $\mu_L=$10 million, $K=$5 million, $\sigma=$4 million and $\gamma=25\%$. Assume that distribution of $\epsilon_i, i = H, L$, are normal and the entrepreneur’s utility function is negative exponential so that we can use the following certainty equivalent:

$$U(w_i) = E(w) - (1/2)\text{var}(w),$$

where the coefficient of absolute risk aversion is one.

For the purposes of the example, let the number of shares outstanding just before the IPO, $n^*_0$, be 1,000,000 rather than 1 (a more realistic number). The entrepreneur with the better project will set an offering price of $10 per share, and will sell all one million secondary shares as well as 500,000 primary shares to cover the cost of the project. The entrepreneur with the other type of project will set a price of $5 per share and sell all one million secondary shares as well as one million primary shares. The utilities in monetary terms (which the certainty equivalent permits) of the high and low types are $10 million and $5 million, respectively.

2.2 Adverse selection

In this section, investors no longer know which type of project the entrepreneur has. They only know the distribution of project types, though they may change their beliefs about which project the entrepreneur has after observing the offering price and the number and type of shares issued.

The private information of the entrepreneur gives rise to an adverse selection problem:
the low type would like to pass himself off as the high type to get the high offering price. Investors’ account for the low type’s incentive to mimic the high type by refusing to pay any more than the pooled value of the firm, \( h \pi_H + (1 - h) \pi_L \), if they receive no credible information beyond the initial distribution of types about which type of firm is selling shares. In the example, the pooled price is only $6.25, a considerable discount for the high type.

The high type, however, may be able to make an offer that is a credible signal of the project quality in that the low type has no incentive to mimic the offer. For this to be possible, the high type’s contract will need to be sufficiently costly to the low type. For instance, an increase in the high type’s stake (greater exposure to risk) or decrease in his offering price, or both, makes the high type’s offer less attractive. To determine what the high type will do, I model this situation as a signaling game.

*Strategies and the equilibrium concept*

The players’ strategies need to be given explicitly. The type \( i \) entrepreneur makes an offer \( \{ \gamma_i, n_i, p_i \} \) to investors who can either accept or reject the offer. If the offer is accepted, each player receives a payoff as described in the previous section. If it is rejected, investors retain their initial wealth, and the entrepreneur is unable to undertake the project and receives a reservation utility \( \bar{U} \). As we will see, sometimes only \( \alpha_i \), and not \( \gamma_i \) or \( n_i, i = H, L \), are payoff relevant. In such cases \( \{ \alpha_i, p_i \} \) will be used as a reduced form representation of the entrepreneur’s offer.

The perfect Baysian equilibrium (PBE) concept is the most common for signaling games. A PBE requires that each player’s action is optimal, given all other players’ actions, and that investors’ prior beliefs regarding the project’s unknown type, are, after observing the entrepreneur’s offer, updated using Bayes rule whenever it applies. The problem with the PBE concept for this game is due to the flexibility in specifying beliefs whenever Bayes rule doesn’t apply – whenever the entrepreneur makes an offer that he was not expected to ever make, regardless of his type.

For instance, suppose we have found a PBE but that there exists an alternative offer that the high type strictly prefers to his equilibrium offer and that investors, if they believe that
the project was the high type, are better off if they accept rather than reject it. Further suppose that the low type strictly prefers his equilibrium offer to this alternative offer. Then it seems reasonable that the high type could state in the prospectus (the so-called announcement): “The deviation offer is not a mistake and that only if my project type was truly high would it be beneficial to make this deviation.” Investors could infer, since they know the parameters of the game, that the entrepreneur’s statement is credible and so would accept the offer, justifying the entrepreneur’s deviation.

Though perhaps unreasonable, equilibria of this type can be supported as PBE: no such announcement is made and investors assume that such a deviation was made by a low type, a mistake perhaps, and refuse the offer making the deviation unprofitable for the high type and the original equilibrium is sustained. The Intuitive Criterion, an equilibrium refinement developed by Cho and Kreps (1986), rules out precisely these types of unreasonable equilibria and so I use it, in conjunction with PBE, as the relevant equilibrium concept.\(^{13}\)

**Signaling project type**

As a simple starting point, assume \(K = 0\) so that no primary shares need to be sold and the type \(i\) entrepreneur only specifies \(\{\alpha_i, p_i\}\) in the prospectus. This allows us to ignore the minimum proceeds constraint, (2.2). While we don’t need \(K\) to equal zero to ignore (2.2), it is conceptually simpler. The case of \(K > 0\) will be considered shortly.

To simplify notation, let type \(i\)’s utility from making the offer \(\{\alpha, p\}\) be given by \(U_i(\alpha, p)\). Let \(\phi(\alpha, p) \in [0, 1]\) be the probability that investors believe the firm to be the high type when the offer \(\{\alpha, p\}\) is made. Now suppose \(\{\alpha_i, p_i\}, i = H, L,\) are PBE offers. Then

\[
EU_H(\alpha_H, p_H) \geq EU_H(\alpha_j, p_j) \text{ for } p_j \leq E(\pi|\phi(\alpha_j, p_j))/N_j \tag{2.7}
\]

\[
EU_L(\alpha_L, p_L) \geq EU_L(\alpha_j, p_j) \text{ for } p_j \leq E(\pi|\phi(\alpha_j, p_j))/N_j \tag{2.8}
\]

\[
EU_i(\alpha_i, p_i) \geq U \tag{2.9}
\]

\(^{13}\)Allen and Faulhaber (1987) and Welch (1987) also use the Intuitive Criterion.
\[ p_H \leq E(\pi|\phi(\alpha_H, p_H) = 1)/N_H \] (2.10)

\[ p_L \leq E(\pi|\phi(\alpha_L, p_L) = 0)/N_L, \] (2.11)

for any \( \{\alpha_j, p_j\} \), and where \( \phi(\alpha_j, p_j) \in [0, 1] \).

The incentive compatibility constraints (2.7) and (2.8) ensures that each type of entrepreneur weakly prefers his equilibrium contract to any other contract that investors would accept. The entrepreneur’s willingness to participate follows from (2.9) and the investors’ willingness to accept the equilibrium offers is given by (2.10) and (2.11).

When the equilibrium is separating, the firm’s type is revealed by the contract so that \( \phi(\alpha_H, p_H) = 1 \) and \( \phi(\alpha_L, p_L) = 0 \). With the PBE concept, for contracts other than \( \{\alpha_H, p_H\} \) and \( \{\alpha_L, p_L\} \), those off the path-of-play, beliefs are arbitrary and can be set to zero, which will support the greatest number of different separating outcomes. However, the Intuitive Criterion requires for any \( \{\alpha_k, p_k\} \), that \( \phi(\alpha_k, p_k) = 1 \) if \( EU_L(\alpha_L, p_L) > \max\{EU_L(\alpha_k, p_k), \bar{U}\} \), \( EU_H(\alpha_H, p_H) < EU_H(\alpha_k, p_k) \) and \( p_k < E\pi_H/N_H \). In words, investors believe that the firm is the high type if \( \{\alpha_k, p_k\} \) is offered where the low type does strictly worse with this offer than with \( \{\alpha_L, p_L\} \), the high type does strictly better than with \( \{\alpha_H, p_H\} \), and investors are strictly better off if they accept this offer from a high type. Thus for \( \{\alpha_i, p_i\}, i = H, L \), to be PBE contracts surviving the Intuitive Criterion, they must be chosen such that no offer like \( \{\alpha_k, p_k\} \) exists.

There exist a unique pair of equilibrium contracts that satisfy the above restrictions and survive the Intuitive Criterion. Also, even if \( K > 0 \), provided it is not too large, the minimum proceed constraint is also satisfied. When the minimum proceeds constraint doesn’t bind, the mix between \( n_i \) and \( \gamma_i \), for a given \( \alpha_i \) has no effect on the entrepreneur’s payoff (see (2.5)). Thus, \( n_i \) is determined according to Assumption 2.1 and an offer of \( \{\alpha_i, p_i\} \) is then equivalent to an offer \( \{\gamma_i, K/p_i, p_i\} \).

**Proposition 2.1** When \( K \leq \mu_H(1 - \alpha_H^*) \equiv K \), there is a unique separating equilibrium outcome denoted \( \{\alpha_L^*, p_L^*\}, \{\alpha_H^*, p_H^*\} \) such that:

1. \( \alpha_L^* = 0 \) and \( \alpha_H^* \in (0, 1) \)
2. \( p_L^* = \mu_L - K, \ p_H^* = \mu_H - K \)

3. \( \gamma_L^* = 0 \) and \( \gamma_H^* < 1 \) unless \( K = \mu_H(1 - \alpha_H^*) \), in which case \( \gamma_H^* = 1 \).

Proof: see appendix.

The first point states that the high type retains a stake in the project to signal its quality, as in Leland and Pyle (1977). The second point shows that both types of firms are priced at their true expected value. The intuition for why only the high type's stake, and not the share price, is used for signaling is as follows. To remain indifferent between contracts, the low type needs to be compensated with a larger increase in the selling price for a given increase in \( \alpha \) than the high type for two reasons. First, non-increasing absolute risk aversion means the cost of risk from a greater stake is at least as high for the low type as the high type. Second, an increase in \( \alpha \) means that the low type sells less of his firm at the high type's price and keeps a larger stake in his low value firm. While the high type also reduces the amount of firm he sells, he keeps a higher stake in his high value firm. Thus, it is more costly for the low type to increase his stake in the firm, greater cost of risk and lower expected profits, than for the high type. The cost, then, of a greater stake relative to a lower price is less for the high type than the low type, (the sorting condition), so underpricing does not arise.\(^{14}\)

The third point is that secondary shares will be sold, except in the unlikely event that the minimum proceeds constraint just binds at the equilibrium high type offer. This is straightforward implication of assumption 1, which states that the entrepreneur will sell secondary shares if there is slack in the minimum proceeds constraint.

Finally, the restriction on \( K \) can be deduced as follows. First, recall that the proceeds raised by the high-type firm are equal to

\[
p_H n_H = (\mu_H - K) n_H.
\]

To maximize proceeds for a given \( \alpha_H, n_H \) needs to be set as high as possible. But

\[
n_H = \frac{\gamma_H}{\alpha_H - 1},
\]

\(^{14}\)Grinblatt and Huang (1989) show this result in the context of their model.
so that $n_H$ is maximized when $\gamma_H = 1$. Thus the maximum amount of proceeds is given by

$$(1/\alpha_H - 1)(\mu_H - K),$$

which must be at least as large as $K$. Solving for $K$ gives the restriction.

Example continued

We can determine the size of the high type's stake in the example. The high type chooses $\alpha_H$ as small as possible, since he dislikes risk, such that the low type will not copy. This amounts to keeping the low type just indifferent between his own equilibrium contract and the high type's contract:

$$\alpha_H \mu_L + (1 - \alpha_H) \mu_H - K - 0.5\alpha_H^2 \sigma^2 = \$5\text{ million}.$$ 

Some simple algebra gives $\alpha_H = 53.8\%$. The high type's payoff is now only $7.69$ million instead of $10$ million.

Clearly, if the high type only sells secondary shares, the capital cost will not be covered since no proceeds for the firm are raised. Under assumption 1, the high type sells 500,000 primary shares to cover the cost of the project. Thus 1.5 million shares, the 1 million shares initially held by the entrepreneur plus the 500,000 new shares, will be outstanding after the IPO. Now since

$$\alpha_H = \gamma_H \times 1,000,000 \div 1,500,000 = 0.538,$$

$\gamma_H$ must be 0.806. Thus the high type must sell 193,612 ($= (1 - 0.806) \times 1,000,000$) secondary shares in addition to 500,000 primary shares.

When primary shares are sold, the high type's stake in the firm is reduced through dilution so that the number of secondary shares that need to be sold to achieve a given stake is reduced. For example, if no capital were needed, no primary shares would be sold and the high type, with an undiluted position in the firm, would need to sell 462,422 secondary shares, over twice as many as before, to achieve the same stake in the firm.

What if the high type lowered the offering price from $10$ to $9$ dollars as part of the signal? Given the price, $\alpha_H^*$ solves:

$$\alpha_H^* (\mu_L - K) + (1 - \alpha_H^*) K - 0.5\alpha_H^* \sigma^2 = \$5\text{ million},$$

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where I have used \( n_H p_H = K \). This gives \( \alpha_H^* = 50\% \), a slight decrease from the level above. The decrease in the high type's stake means the cost of risk is reduced, but it took a 10% drop in the price to achieve a 7% drop in his stake. Furthermore, the marginal benefit of reduced risk from increasing \( \alpha_H^* \) by one percentage point, when \( \alpha_H^* = 53.8\% \), is $8,600, while the marginal cost of a one percentage point drop in the price from $10 per share, is $10,000. Thus the net effect of the one dollar price drop is a decrease in the high type's payoff: from $7.69 to $7.5 million. This reflects the inefficiency of underpricing as a signal.

### 2.3 Binding minimum proceeds constraint

When \( K \) is sufficiently large, the restriction in proposition 2 will be violated. For instance, continuing with the example, if no secondary shares are sold, then the maximum amount of proceeds the high type can raise without diluting his stake below the required level of 53.8%, is $8.59 million \( (= (1/\alpha_H - 1)n_H p_H) \), more than covering the $5 million capital cost. But what happens if the capital cost of the project is higher, say $7 million? The linearity of mean-variance utility implies that the high type still needs to retain 53.8% stake in the firm. However, now the full information price is $8 per share, not $10, so that the maximum the high type can raise is only $6.87 million, $130,112 short of the required amount.

Suppose that the high type lowered his offering price from $8 to $6 per share. Clearly, if the same number of primary shares are sold, firm proceeds will decrease. However, decreasing the offering price allows the high type to reduce his stake in the firm and still credibly signal the project. Recall that underpricing, pricing below the inferable value of the firm, is ruled out in proposition 1 because it is inefficient, compared to share retention, and not because it is ineffective. With an offering price of $6 per share, the high type need only retain a 45.3% stake in the firm allowing him, if he sells no secondary shares, to sell over 1.2 million primary shares without excessively diluting his position. At $6 per share, this would raise $7.25 million, more than enough to cover the capital cost. Indeed the entrepreneur, under assumption 1, should sell 1.17 million primary shares to raise the $7 million and then sell
18,717 secondary shares to achieve the desired 45.3% stake in the firm. The high type's payoff is now $5.26 million rather than the full information value, at the higher capital cost, of $8 million.

Is the $6 share price optimal? Since underpricing is inefficient, it is in the high type's interest to minimize it. If he decreases the number of secondary shares offered, the number of primary shares can be increased, without changing his stake in the firm. With this increase in the firm's proceeds, the high type can reduce the number of primary shares, increasing his stake, and consequently allowing the share price to increase towards its full information level. Thus, no secondary shares should be sold.

In the example, when the high type sells no secondary shares, $7.25 million could be raised for the firm with a $6 share price (see above). So there is room to decrease the number of primary shares sold, which increases the high type's stake (less dilution), which in turn allows him to increase the share price. If the high type chooses a $7 offering price, then his required stake to signal his type is 50%. With no secondary shares sold, 1 million primary shares are sold, raising exactly the $7 million needed for the project's cost. The high type entrepreneur's total payoff increases from $5.26 million to $5.5 million. If the price were raised above $7, the high type would have to retain such a large stake in the firm that even without selling secondary shares, proceeds would fall short of the project's cost.

**Underpricing**

For efficient signaling, the high type's problem is to find a contract that maximizes his utility and that satisfies (2.8), the low type's incentive compatibility constraint, and (2.2), the minimum proceeds constraint:

\[
\max_{\gamma_H, n_H, p_H \leq \mu_H - K} EU_H(\alpha_H, p_H) \tag{2.12}
\]

subject to

\[
EU_L(\alpha_H, p_H) \leq EU_L(0, \mu_L - K) \tag{2.13}
\]

\[
p_H n_H \geq K \tag{2.14}
\]
\[ \alpha_H \equiv \gamma_H / (1 + n_H) \] (2.15)

For a given \( \alpha_H \), setting \( \gamma_H = 1 \) allows \( n_H \) to be as large as possible which relaxes the minimum proceeds constraint. But since \( n_H \) and \( \gamma_H \) don’t appear in the objective function or incentive compatibility constraint, the high type’s payoff, if the binding minimum proceeds constraint relaxes, must increase. Using the definition of \( \alpha_H \), total proceeds from primary share issues are:

\[
\left( \frac{1}{\alpha_H} - 1 \right) p_H,
\]

which is decreasing in \( \alpha_H \) and increasing in \( p_H \). Thus when \( p_H \) is at its maximum value, \( \mu_H - K \), proceeds can only be increased by decreasing \( \alpha_H \). So when the minimum proceed constraint binds, \( \alpha_H < \alpha_H^* \).

With \( \gamma_H = 1 \) there remains two choice variables, \( \alpha_H \) and \( p_H \), and two constraints. Substituting (2.14) into both the objective function and (2.13), gives

\[
\max_{\alpha_H} EU(\alpha_H(\mu_H + \epsilon_H))
\]

subject to

\[
EU_L(\alpha_H(\mu_L + \epsilon_L)) \leq EU(\mu_L - K).
\]

Decreasing absolute risk aversion, as shown in lemma 2.1 of the appendix, implies that \( EU(\alpha_H(\mu_H + \epsilon_H)) \) is increasing whenever \( EU_L(\alpha_H(\mu_L + \epsilon_L)) \) is. Moreover, both are concave in \( \alpha_H \) and increasing as \( \alpha_H \) approaches zero (implying \( n_H \) approaches infinity). Also, as \( \alpha_H \) approaches zero the constraint is strictly satisfied. Thus since \( \alpha_H < \alpha_H^* < 1 \), there exists some \( \alpha_H \in (0, \alpha_H^*) \) such that the constraint holds exactly. The price is then given by (2.14).

The following proposition summarizes the results.

**Proposition 2.2** When the minimum proceeds constraint is violated with the contract \( \{ \alpha_H^*, p_H^* \} \), the unique separating equilibrium contract for the high type, denoted \( \{ \alpha_H^{**}, p_H^{**} \} \) is characterized as follows:

1. \( \alpha_H^{**} < \alpha_H^* \)
2. \( p_H^{**} < \mu_H - K \)

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3. $\gamma_H = 1$

The low type continues to offer $\alpha_L^* = 0$ and $p_L^* = \mu_L - K$ as his equilibrium contract.

Proof: see appendix.

Point 2. states that the high type underprices. Because the equilibrium is separating, investors know how much the firm is worth and are willing to trade the high type firm at a price of $p_H^* = \mu_H - K$. Thus, those investors that buy shares in the firm from the underwriter at $p_H^*$ can immediately sell their shares in the market at $p_H^*$. This translates into a positive immediate return: $R_H = p_H^*/p_H^* - 1 > 0$.

Points 2. and 3. together show that when a firm underprices it also sells no secondary shares, $\gamma_H^* = 1$. However, from proposition 2.2, point 3, we know that when a firm doesn’t underprice, some secondary shares are sold. This is an important testable implication of the model:

Secondary shares are sold if, and only if, the firm doesn’t underprice.

We have already seen another testable implication in the example: an increase in $K$ from $5$ million to $7$ million led to underpricing. Moreover, a further increase in $K$ would have led to even greater underpricing. The following proposition generalizes this result.

**Proposition 2.3**

\[
\frac{\partial R_H}{\partial K} = 0 \quad \text{if } K < \bar{K}
\]
\[
\frac{\partial R_H}{\partial K} > 0 \quad \text{if } K > \bar{K}
\]

Proof: see appendix.

It is also possible to consider how changes in the variance in the project’s cash flow affect underpricing. Unlike the change in the capital cost, an increase in variance of the high type’s project will have no effect on his equilibrium contract. Underpricing is completely determined by the low type’s incentive compatibility constraint and the minimum proceeds
constraint, neither of which depends on the variance in the high type project. In the above analysis, however, the variance of the two types of projects are assumed to be the same. None of the results would qualitatively change if the variance in the low value project was higher than that for the high value project. If, on the other hand, it was sufficiently lower, the single crossing condition, which allows the high type to signal his project credibly, would not hold. The following result examines the effect of a change in \( \sigma \), an equal change in the variance of both types of projects, though keep in mind, it is only the change in the low value project's variance that matters.

**Proposition 2.4** There exists a \( \sigma^2 \) such that

\[
\frac{\partial R_H}{\partial \sigma^2} = 0 \quad \text{if } \sigma^2 > \sigma^2 \\
\frac{\partial R_H}{\partial \sigma^2} < 0 \quad \text{if } \sigma^2 < \sigma^2
\]

Proof: see appendix.

With the rise in \( \sigma^2 \), the incentive to mimic the high type by keeping a stake in the firm is reduced. This allows the high type to reduce his stake and subsequently increase the number of primary shares sold. The resulting relaxation of the minimum proceeds constraint leads to a decrease in underpricing.

In the example, if \( \sigma \) decreased from $4 million to $2 million, and \( K \) stayed at $5 million, the high type would have to price at $6.45 per share rather than $10 per share, only slightly higher than the pooling rate of $6.25 per share. If \( \sigma \) were $7 million, then the high type could increase his price to $9.13 a share. And with \( \sigma = $8 \) million, no underpricing is necessary.

**Multiple types**

Underpricing also arises when there are multiple types but the nature of the equilibrium can be quite different. For instance, suppose that in addition to the low and high types, there is a very high type. If the high type faces a binding incentive compatibility constraint with the low type and minimum proceeds constraint, then, if entrepreneurs are not too risk
averse, the very high type will face the same constraints. Thus the high and the very high types must pool on the same contract. There will still be a positive initial return, but now the jump in price will be to the expected pooled value of the high and very high types rather than just to the expected value of the high type.

2.4 Winner's curse

As mentioned in the introduction, another popular explanation for underpricing put forward by Rock (1988) is the winner's curse. This explanation posits that the more informed party is not the entrepreneur, but rather a group of outside investors. However, this group is not sufficiently large to provide enough capital to cover the project's costs, so some uninformed investors must also participate. If informed investors only submit requests for shares when they see that the firm is a high type, uninformed investors are more likely to receive shares (or a greater number of shares) when the firm is of low value rather than high value. Thus winning shares as an uninformed investor indicates that the firm is more likely of low value. Anticipating the winner's curse, these investors will only participate if they pay no more than their expected value of the firm conditional on winning, which is less than the unconditional expectation. Hence, there is underpricing.

This idea of a winner's curse can be incorporated into the signaling model above in a simple way. One advantage of its inclusion is that some underpricing among unconstrained firms can be explained. Anticipating the empirical results, the initial return among unconstrained firms is still positive, not zero as predicted by the simple signaling model.

For both signaling and winner's curse to matter, three levels of information are needed. Uninformed investors only know that a firm is a high type with probability $h$, as before. An entrepreneur has better, but now not perfectly certain, information about his firm's quality. Type will continue to refer to the entrepreneur's state of knowledge. The entrepreneur believes that the chance of having a high return project is either good (G) or bad (B). The former type of entrepreneur is an optimist and the latter, a pessimist. The probability that an optimist has a high type project is $h_G > h$ and the probability a pessimist has a high
type project is $h_B < h$. Finally, informed investors know exactly which type of project the entrepreneur has.

There is a total of $M$ investors, of which a fraction $x$ of them are informed. Investors are willing (or able) to purchase at most one share. There are, however, not enough informed investors to cover the cost of a high type project. That is,

$$xM < \frac{K}{\mu_H - K}.$$  

On the other hand, there are enough uninformed investors to cover the cost of a low type project:

$$(1 - x)M > \frac{K}{\mu_L - K}.$$  

As in Rock (1988), assume that when there are more bidders than shares available, each individual investor has an equal probability of getting a share.

Suppose, for the moment, the uninformed investors have the same information as the entrepreneur. The probability of winning a share if the project is low return is

$$1 - \frac{\gamma_i + n_i}{(1 - x)M},$$

and the probability of winning a share if the project is high return is

$$\frac{1 - \gamma_i + n_i}{M},$$

for $i = G, B$. Clearly the probability of winning is higher when the project is a low type.

Using Bayes rule, we can compute the updated probability that a firm has a good project if an uninformed investor wins a share:

$$h_iW = \frac{1 - \gamma_i + n_i}{\frac{\gamma_i + n_i}{M}h_G + \frac{1 - \gamma_i + n_i}{(1 - x)M}(1 - h_G)} = \frac{(1 - x)h_G}{1 - h_Gx} < h_G, \ i = G, B.$$  

Thus if an uninformed investor knew what the entrepreneur knew, she would be willing to pay

$$p^w_i = \frac{h_iW\mu_H + (1 - h_iW)\mu_L + n_i\mu_i - K}{N_i}.$$  

This is only a normalization to ensure that uninformed investors sometimes need to participate.
Solving for $p_i$ gives
\[ p_i^{wc} = h_iW \mu_H + (1 - h_iW) \mu_L - K, \] (2.16)
which, since $h_iW < h_i$, is clearly less than $p_i^{FI}$.

We can now reintroduce the adverse selection problem by assuming the uninformed investors do not have the entrepreneur’s private information. The analysis is just as in the above sections except that now, full information prices, for either type project, cannot exceed the level implied by (2.16). When the capital constraint is sufficiently binding, the price the good type offers will be even below the winner’s curse price. For less capital constrained firms, the offering price will be higher and underpricing will be reduced. For a sufficiently low $K$, the maximum price of $p_G^{wc}$ will be achieved. Informed investors that do not get a share at the IPO will purchase shares in the aftermarket if the firm is a high type and will not purchase otherwise. The lack of demand by informed investors in the latter case reveals that the firm has a low type project and the post-IPO price will fall to reflect it. However, since $p_G^{wc}$ is below $p_G^{FI}$, the initial return will be positive on average.\(^\text{16}\)

The winner’s curse model is consistent with the signaling model in that the predictions from the latter carry through. The only important difference is that the winner’s curse implies underpricing will still occur among unconstrained firms, but to a lesser degree than constrained firms.

### 2.5 Moral hazard

One problem with the above analysis, as the example illustrates, is that we typically don’t observe entrepreneurs selling their entire stake in the firm. Insofar as the firm’s productivity depends on the entrepreneurs unobserved effort, either type of entrepreneur has some incentive to maintain a stake in the firm. The following section on lockup shows precisely how moral hazard operates. In this section, a heuristic description is given.

\(^{16}\)It is also the case that if the winner’s curse problem is important even entrepreneurs that think they have a bad chance of having a good project will underprice.
Suppose for the moment the entrepreneur's project type is known to investors. After an IPO, where \( \{a, p\} \) has been offered and accepted, the entrepreneur chooses his effort level. The entrepreneur only claims a fraction of the firm's profits but must bear all the cost of effort. So when he chooses his effort level he fully takes into account the total cost of effort but only accounts for his share of the benefits. The greater his stake, the greater his effort and the higher the expected profits. Investors anticipate the dependence of the entrepreneur's effort on his stake and adjust their willingness to pay accordingly.

A zero stake in the firm is no longer optimal. The entrepreneur would put no, or the minimum, effort into the firm and the offering price would reflect the low expected value of the firm. A higher stake, while it does expose the entrepreneur to risk, encourages more ex post effort which translates into both a higher expected firm value and a higher offering price. So even low type entrepreneurs may keep a sizeable stake in their firms.

Because the entrepreneur has a desire to retain a stake in the firm, the minimum proceeds requirement must be considered even when absent adverse selection. Indeed, just as in section 2.3, the proceed requirement may bind forcing the entrepreneur to decrease his stake in the firm.\(^{17}\) Also as in section 2.3, when the constraint binds, the entrepreneur will not sell secondary shares since proceeds from them do not contribute to covering the capital cost but further reduce the entrepreneur's stake in the firm from its optimal level. Thus, the relationship between \( K \) and the presence of secondary share sales is the same as with adverse selection. However, with moral hazard alone, underpricing doesn't arise.

If the return to effort is a concave function and the cost of effort is a convex function, the results from sections 2.2 and 2.3 continue to hold when there is both adverse selection and moral hazard. To signal his type, the high type still retains a larger stake than is optimal and if \( K \) is sufficiently large, underpricing will arise.

The comparative static results, however, are weakened slightly: when \( K \) is large and the high type is constrained, a small increase in \( K \) may lead to less rather than more underpricing.

\(^{17}\)The resulting firm value, and offering price, because of the entrepreneur's decreased stake, decreases the amount of proceeds but by less than the increase from selling more shares so that total proceeds increase.
The reason is that the higher $K$ induces the high type to lower his stake and the offering price, as in proposition 2.3 above, but the lower stake reduces the value the firm trades at in the after-market, since the market expects less effort from the entrepreneur. When the high type’s stake is low because $K$ is large, the decrease in the after-market price may be larger than the decrease in the offering price, so that underpricing decreases. However, for a smaller $K$, but with the high type still constrained, the change in the offering price is greater than the change in the after-market price and underpricing increases with $K$, as in proposition 2.3.

2.6 Lockup

An important implicit assumption so far is that following the IPO the entrepreneur does not immediately engage in a secondary offering. If he were to do so he would bear risk for such a short period that there would be almost no cost to keeping a high initial stake in the firm. Thus, a high type would be unable to separate himself from a low type.

However, entrepreneurs do have the option of writing a lockup contract with their underwriters that commits insiders to not sell their shares for a specified period of time. In the data set used in the empirical section, lockups average about a year and can last over two years, a considerable amount of time to be wed to the firm. But if the entrepreneur can write such a contract, if the firm is constrained why not just choose a long lockup period to signal firm quality and refrain from expensive underpricing? That is, the longer the lockup period, the greater the entrepreneur’s exposure to risk, which is analogous to holding a large stake in the firm except the lockup doesn’t restrict the number of primary shares sold.

Lockup, however, can have limited use as a signaling device. Lockup may be used to commit the entrepreneur to work hard for the firm in the future increasing the value of the firm to IPO investors, and thus, increasing the value the entrepreneur receives for shares at the IPO. This incentive for lockup would hold not just for high types but low types as well. A present commitment is, however, only important if their is a time inconsistency problem in that what the entrepreneur considers to be an optimal future stake at the IPO is not what
he considers to be an optimal stake after the IPO.

The reason why such a problem arises is related to the change in the entrepreneur’s stake prior to the IPO, where he owns the entire firm, to after the IPO, where he is only a part-owner. For the moment, ignore the effect of risk. Because of moral hazard, the entrepreneur’s effort is increasing in his stake in the firm, which he and his investors know. As discussed in the previous section, this leads the entrepreneur to keep some stake in the firm to induce some effort and increase the value of the firm. However, the entrepreneur’s decision over how much of the firm to retain, and, hence, how much effort to commit to, depends on his current stake in the firm. When it is large, he will not want to sell too large a stake since if he did, he would bear the cost of the reduced price in the secondary offering. When his stake is small, a further sell off has little effect on his wealth since his stake in the firm is small. Thus the smaller his stake after the IPO, the greater is his incentive to sell a large portion of his remaining shares.

Now back up to before the IPO. The entrepreneur owns the entire firm so he has a strong incentive to commit to a large future stake in it, which is equivalently a commitment to high future effort, to keep both the price of shares at the IPO and the future value of the firm high. That is, he would like to convince IPO investors that if there was a secondary offering, his stake would remain high. Following the IPO, his stake is reduced and, given investors have already paid their money, he is no longer interested in keeping such a high stake at a secondary offering, preferring to reduce his exposure to risk. Without commitment, investors will correctly predict the entrepreneur’s true actions at a secondary offering, and offer him less at the IPO. Hence, if possible, the entrepreneur would like to commit to restrict his future ability to sell his stake by either limiting his ability to sell shares at a secondary offering or extending the period between an initial and secondary offering.

The incentive to sign a lockup applies to high and low types. That means, low types will want to lockup their shares, to increase the value of a low return firm at the IPO, which reduces or eliminates lockup as a signal. That is, the low type already has an incentive to maintain a future stake when moral hazard matters, so a long lockup announced by a high
type has little or no value in signaling firm quality. The next subsection formalizes this argument.

2.6.1 A two period extension with moral hazard

There are now two periods to consider. At the beginning of period 1, the IPO is undertaken. At the beginning of period 2, the entrepreneur conducts a secondary offering in which only secondary shares are sold. (There is no reason to sell primary shares since no further capital is needed.) We will consider both the case when the entrepreneur can at the IPO commit to selling a certain number of his remaining shares during a secondary offering and when he cannot.

Some modifications to the above model are necessary in this two period setting. Variables will now be time subscripted but the type subscripts are dropped since the adverse selection problem is not immediately important. Adverse selection will be considered towards the end of the section. The project now lasts two periods and generates a final period payment of

$$\mu + e_2 + \epsilon_1 + \epsilon_2,$$

where,

$$\epsilon_i = \sigma z_i,$$

$$i = 1, 2, \sigma > 0, \text{ and } z_1 \text{ and } z_2 \text{ are random variables with zero means and unit variances.}^{18}$$

The entrepreneur’s unobservable effort in period 2 is denoted by $e_2$. It has a dollar cost of $(1/2)e_2^2$, which is borne only by the entrepreneur. At the cost of more notation, effort in period one could also be included with no qualitative change to the results.

At the end of period 1, investors and the entrepreneur learn the realization of $\epsilon_1$ before deciding on how to structure a secondary offering. When there is no commitment, the entrepreneur offers at the beginning of period two, $1 - \gamma_2$ of his remaining $\gamma_1$ shares for a price $p_2$. When there is commitment, the entrepreneur specifies $\gamma_2$ at the IPO, which does not condition on $\epsilon_1$.

---

$^{18}$Independence of the random variables is not needed for the proposition of this section.
Learning the realization at the end of the first period complicates the entrepreneur's decision in that investor's willingness to pay and his willingness to keep a stake in the firm, depend on first period results. In particular, insofar as absolute risk aversion is decreasing, the higher the realization of $z_1$, the lower the cost of second period risk (assuming the two error terms are not strongly negatively correlated) and the higher his desired second period stake. Thus maintaining flexibility in the size of the secondary offering is valuable to the entrepreneur since it allows him to condition on new information. If this information is verifiable, the entrepreneur could commit to a schedule of secondary offering sizes that condition on the information.\footnote{In reality, lockup is not just a restriction on the number of shares insiders can sell at a secondary offering, but when a secondary offering can take place. However, the principle is the same.}

Since there is no adverse selection first and second period prices with no commitment will be:

$$p_1 = \frac{\mu + \epsilon_2(\alpha_1) + np_1 - K}{N}$$

$$p_2 = \frac{\mu + \epsilon_2(\alpha_1, \alpha_2) + np_1 - K + \epsilon_1}{N}.$$

Because of moral hazard, effort levels, as we shall see, depend on the entrepreneur's stake. However, IPO investors (when there is no commitment) only know $\alpha_1$ when deciding whether to accept the offer or not. Investors at the secondary offering (SO), know the entrepreneur's ex-SO stake as well. Notice also that $\epsilon_1$ enters the second period price since it is observed.

The entrepreneur's final payment, over which his utility is defined, is

$$(1 - \gamma_1)p_1 + (1 - \gamma_2)\gamma_1p_2 + \frac{\gamma_1}{N}\gamma_2(\mu + np_1 - K + \epsilon_1 + \epsilon_2) = \frac{1}{2}\epsilon^2.$$

Recall that

$$\alpha_1 = \frac{\gamma_1}{N}.$$

To keep the notation consistent, define

$$\alpha_2 \equiv \gamma_2.$$
Thus the entrepreneur’s final stake in the firm is $\alpha_1 \alpha_2$. Since the entrepreneur’s effort decision is made after the secondary offering the entrepreneur’s choice of effort is easily shown to be

$$e_2 = \alpha_1 \alpha_2.$$  

The entrepreneur’s effort is increasing in his final stake.

Consider the case in which there is no commitment. The entrepreneur is able to freely choose $\alpha_2$ at the secondary offering. Using the above prices, gives after some manipulation:

$$\max_{\gamma_2} E\{U(\mu - K + (1 - \alpha_1)e_2(\alpha_1) + \alpha_1(\alpha_1 \alpha_2) + \alpha_1 \epsilon_1 + \alpha_1 \alpha_2 \epsilon_2 - (\alpha_1 \alpha_2)^2 / 2)|\epsilon_1}\}.$$  

This gives:

$$\alpha_2(\epsilon_1) = 1 - \frac{-E\{U'(\cdot)\epsilon_2|\epsilon_1\}}{E\{U(\cdot)|\epsilon_1\}} < 1. \tag{2.17}$$

Thus the entrepreneur will hold a secondary offering. The more risk averse, the more shares will be sold at the secondary offering. (Note that $\alpha_1$ appears in the argument of $U$ so inspection does not readily indicate how $\alpha_2$ depends on $\alpha_1$.) By lemma 2.1, under increasing absolute risk aversion, the greater is $\epsilon_1$ the greater will be $\alpha_2$. Equivalently, as the Arrow-Pratt coefficient of absolute risk aversion approaches a constant, $\alpha_2$ will approach a constant so that the new information has a vanishingly small effect on the entrepreneur’s optimal $\alpha_2$.

Because $\alpha_2$ depends on $\alpha_1$ and in turn, effort depends on their product, the first period price will indeed depend on $\alpha_1$ as indicated above. When the entrepreneur can commit to an $\alpha_2$ level at the IPO, the IPO price will depend directly on $\alpha_2$ rather than just indirectly through $\alpha_1$. That is, under commitment, the prices will be:

$$p_1 = \frac{\mu + e_2(\alpha_1, \alpha_2) + np_1 - K}{N}$$

$$p_2 = \frac{\mu + e_2(\alpha_1, \alpha_2) + np_1 - K + \epsilon_1}{N}.$$  

The advantage of commitment is that when choosing $\alpha_2$, the entrepreneur takes into account how it directly affects $p_1$. When there is no commitment, the choice of $\alpha_2$ occurs after shares at the IPO have already been purchased at a price $p_1$. Since the entrepreneur doesn’t take into account the loss in value a low $\alpha_2$ imposes on IPO investors, he chooses
too low an \( \alpha_2 \). Of course, IPO investors realize this and are only willing to pay a low price, compared to the commitment case. The disadvantage of commitment to a single level for \( \alpha_2 \) is that the entrepreneur cannot respond to new information about the firm’s value by adjusting the size of the secondary offering. As stated above, as risk aversion becomes constant, the value of information decreases and the advantage of commitment dominates. The result is state formally in the next proposition.

**Proposition 2.5** *If \( r(w) \) is almost constant, the entrepreneur will prefer to commit to have no secondary offering at all.*

Proof: We wish to show:

\[
\max_{\alpha_1, \alpha_2} EU(\alpha_1, \alpha_2) > \max_{\alpha_1, \alpha_2} \{ \max \{ \max \{ U(\alpha_1, \alpha_2) | \epsilon_1 \} \} \}
\]

This is equivalent to showing:

\[
\max_{\alpha_1, \alpha_2} EU(\alpha_1, \alpha_2) - \max_{\alpha_1, \alpha_2} \{ \max \{ \max \{ U(\alpha_1, \alpha_2) | \epsilon_1 \} \} \} > \max_{\alpha_1, \alpha_2} \{ \max \{ U(\alpha_1, \alpha_2) | \epsilon_1 \} \} - \max_{\alpha_1, \alpha_2} \{ \max \{ U(\alpha_1, \alpha_2) \} \}.
\]

But by lemma 2.1, \( \alpha_2(\epsilon) \) given by (2.17) approaches a constant as \( r(w) \) approaches a constant. Thus, by continuity, the right-hand side of the inequality approaches zero. Thus we need only show that the left-hand side is positive. Clearly it is non-negative. The first order conditions to the first program are:

\[
EU'(\cdot)(\alpha_2 - \alpha_1 \alpha_2^2 + \epsilon_1) = 0
\]

\[
EU'(\cdot)(\alpha_1 - \alpha_1^2 \alpha_2 + \alpha_1 \epsilon_2) = 0,
\]

which gives \( \alpha_2 = 1 \). So there is no secondary offering. Note if \( \alpha_2 < 1 \) then at least one first order condition will not hold, (i.e. if the second holds the first does not.) Thus the solution to the first left-hand side program differs from the solution to the second left-hand side program (see 2.17). Indeed, if in the first program \( \alpha_2 \) were to be less than one, then \( \alpha_1 \) would be constrained at one which, given the binding constraint, must yield strictly less utility than the unconstrained maximum. Hence, the left-hand side is strictly positive.
Thus as long as not too much information is revealed about the firm quality or, as in the proposition, it has little effect on the entrepreneur's decision, there will be a commitment to not engaging in a secondary offering, at least for some period. Of course, as time passes more and more information is revealed and, moreover, as the firm expands, the important of the IPO insiders' effort contribution to the firm is likely to decrease. As some point, the entrepreneur may wish the commitment to no secondary offering to end. But if this point is sufficiently far in the future, it may be difficult for the entrepreneur and underwriter to write a credible commitment that extend long after this point anyway. In such a case, the role of lockup as a signaling device is eroded. That is, since a low type is willing to lockup shares through most of the possible commitment period, the high type cannot signal quality through lockup.

2.7 Empirical implementation

Underpricing in the data is identified simply as a positive initial return on the first day of trading. Clearly, even in a world of perfect information and certainty we should expect some, albeit quite small, positive first day return if investors are impatient. Furthermore, variability in demand makes the initial return (and returns over any other time interval) stochastic. Indeed, in the data, negative initial returns are not rare. A positive initial return will not correctly "identify all cases of underpricing. Nevertheless, they should be highly correlated and so, if the model is true, the initial return should be positively related to capital requirements only for firms that are constrained. If, for example, the firm's age and pre-IPO sales are taken as proxies for capital requirement, younger firms with low sales, which have more need for external capital, should more likely be constrained (identified by a lack of secondary shares sold) and have a greater initial return.

To carry out such a test one might just simply regress the initial return against proxies for capital requirement and risk over two subsets of firms—those with and those without
secondary share sales. However, there is some concern that the error terms in the regressions
do not have zero means because the regressions are conditional on whether the firm is
constrained or not. For instance, if a firm is relatively old and has large sales but is still
constrained then the unmeasured component of its capital requirement must be large. But
then the expected error in the underpricing regression may also be large (not zero) and if
this is not taken into account, the results will be biased and inconsistent.

To correct for this potential problem I propose a switching regression model. First a
selection equation is estimated where the probability that a firm is constrained or not depends
on its measured characteristics, such as age and sales, and its unmeasured characteristics,
i.e., the error term. Then the regressions for the two sub-samples are run, but the possible
correlation between the error term in the selection equation and in each regression equation
is accounted for. The details of the specification are left to section 2.8.

But before proceeding to the regression results, I check to see if the average initial return
for firms without secondary share sales is higher than that for firms with secondary share
sales. I also check to see if the average initial return among constrained firms is higher in­
creases with capital requirements and if among unconstrained firms, there is little correlation
between capital requirements and the initial return (proposition 2.3). But first consider the
data set.

2.7.1 Data

The data set was constructed by Ritter, which he has made available on the Internet.\textsuperscript{20} The
subset that I use is the same as in Ritter (1991), and a detailed examination of the data
can be found there. There are 1,526 initial public offerings included in the subset out of a
total of 2,476 offerings between 1975 and 1984 in the United States. The subset includes
only firms taken public by an investment banker with an offer price of $1.00 per share or
more and gross proceeds over one million 1984 dollars. Unit offerings are excluded and the
company must have been listed on the CRSP daily Amex-NYSE or NASDAQ tapes within

\textsuperscript{20}Internet address: http://linux.agsm.ucla.edu/ipo/ipodata/ritter.html
six months of the offer date.\textsuperscript{21}

While the restricted data set excludes many of the smallest firms, of interest here, it is more complete and it also contains monthly returns over 3 years for many of the firms that are not available in the larger data set. Additional variables that are useful are revenue (SA) and book value (BOOK) the year prior to the IPO, the offering date, the founding date of the firm, the 3 digit SIC code, the per share offering price (OP), the number of shares sold (SH) and the number of shares sold by insiders (IN). Because of large right-hand tails of the distributions of SA, AGE, and BOOK, I use logarithms of these variables rather than levels, indicated by a pre-appended L. Nominal variables were deflated using the all-items CPI.

Two results of Ritter's investigation are particularly pertinent here. Ritter observes that firms classified as financial institution have very high mean and median ages relative to other industries at the time of their IPO and also tend to be very large as measured by annual sales. Most of these firms went public due to a regulatory change in 1982.\textsuperscript{22} He also remarks that oil and gas firms tended to be the youngest and smallest firms.\textsuperscript{23} We will need to control for these firms to see how influential they are.

The firm's age (AGE) at the time of the IPO was calculated as follows. The offering date was converted to a decimal year and the founding date was assumed to be half way through the founding year unless this gave the firm a negative age. In such cases the age was set equal to a quarter of a year.\textsuperscript{24}

\subsection*{2.7.2 Initial results}

The proportion of firms with no secondary sales in the entire sample is just over one half at 56.6 percent. In figure 2.1, the average initial return conditional on whether or not the firm sells secondary sales is shown. Consider the "all other" category where financial services

\textsuperscript{21}See Ritter (1991) for additional details.
\textsuperscript{22}See Masulis (1987).
\textsuperscript{23}See Ritter (1984) for an analysis of the hot issue market in 1980. A substantial part of the underpricing in the data set can be attributed to these oil and gas firms.
\textsuperscript{24}Note that for the log specification, $\text{LAGE}=\log(\text{AGE}+1)$. Similarly, $\text{LSA}=\log(\text{SA}+1)$.
Figure 2.1: Initial returns
and oil and gas industries are excluded. The average initial return is twice as high when firms are constrained as compared to when they aren't. The difference is significant at less than 1 percent, (the standard deviation of the difference is 1.4 percent). In the oil industry, where underpricing for this sample period was extremely high, the difference is even more dramatic. Just under 78 percent of oil and gas firms were constrained and among these firms the initial return was 38 percent but only 9 percent for unconstrained firms.

The initial return for financial institutions, however, is low regardless of whether the firm is constrained or not and the difference is not statistically significant. Despite the low initial return among financial institutions, only 10.4 percent of these firms sold secondary shares at their IPO. While this is not consistent with the theory, it seems to be better evidence that the regulatory changes that prompted financial institutions to go public are important.

These results continue to hold when the tails of the distributions are considered. For example, over 70 percent of firms that do not have a positive initial return have secondary sales but only 17 percent of firms that have a positive initial return of 50 percent or more have secondary sales (not shown in figures).

The next two figures show how underpricing is related to the firm's capital requirement when the firm is constrained. Financial institutions and oil and gas firms are included. (Excluding them has no qualitative effect.) Consider figure 2.2. To the left of the center line firms are constrained and to the right they are unconstrained. Light bars indicate that the firm's sales, book value, or age is below its median value and dark bar indicate that it is above its median value. For example, the first bar represents the average underpricing among constrained firms with sales above the median and the next dark bar represents the average for constrained firms with sales below the median.

The initial return is nearly three times as high among constrained firms with high capital requirements, (i.e., low sales, book or age). However, among unconstrained firms, the difference is considerably smaller, (though it is significant in all three categories at the 5 percent, but not 1 percent, level.) In figure 2.3 the median initial return, rather than the average, is shown. Outliers are clearly not driving the results of the other figures, as the pattern in
Figure 2.2: Initial returns by proxies for capital requirements
Figure 2.3: Initial median returns by proxies for capital requirements

Figure 2.2 is clearly reproduced in figure 2.2. Furthermore, the median initial return among unconstrained firms is very low at around 3 percent.

The higher initial return among unconstrained firms with high capital requirements compared to constrained firms with low capital requirements may be indicating that there is a small classification problem. The difference, however, is only statistically significant at conventional levels for the sales variable. Possibly some entrepreneurs may have personal financial obligations that require them to sell some of their private shares even though the firm faces a binding capital requirement constraint. However, the difference is very small when medians are compared indicating that the misclassification is likely restricted to only a few firms.

The story the figures tell is as follows. The initial return averages around 10 percent for firms that are unconstrained and changes very little as their capital requirements increase so long as the constraint continues not to bind. Increase the firm's capital requirement to
the point at which the constraint begins to bind and the initial return starts to increase. Increasing it further and the constraint binds tighter leading to yet a higher initial return.

We can check that constrained firms have larger offerings than unconstrained firms. Consider the ratio of primary share proceeds to primary proceeds plus book value as a relative measure of the offering size. Excluding the financial industry, for constrained firms the average ratio is 0.84 whereas for unconstrained firms it is 0.52, and the difference is significant at less than 1 percent.\footnote{If we use total proceeds (including those from secondary sales) in the numerator, then the ratio is 0.89 for unconstrained firms, but the difference with constrained firms is not statistically significant.}

Finally, there is the issue of lockup. To what extent is lockup used as a signal, diminishing the need for underpricing compared to the extent it is used as a commitment device? Lockup data is only available for the years 1983 and 1984. However, this data is available in the larger data set which includes the small firms. Of the 1485 sample points in this range, 769 have missing data leaving 716 sample points. The average lockup period for the retained sample is 203 days.
Among firms with no IPO insider share sales, the average lockup is 250 days while for those with insider share sales, the average is 154 days. The difference is significant at less than the one percent level. The data seem to suggest that when firms are constrained they do use lockup as a signaling device to reduce underpricing. However, the fact that unconstrained firms also have a long lockup period is not inconsistent with the need for lockup to make share retention a useful signal and to commit the relevant insiders to exert high effort.

It is interesting to compare means for different size firms among those that are constrained with those that are not. In figures 2.4 and 2.5 firms are grouped by book value into quartiles and the average mean lockup period is reported. In figure 2.4, firms are constrained and in figure 2.5 they are unconstrained. The first three quartiles in figure 2.4 have the longest average lockup period but there is no statistical difference in their levels. It seems that lockup is used in a limited way to avoid underpricing, but the most constrained firms are not able to use a longer lockup than less constrained firms. For the highest quartile there is a statistically significant (at less than the one percent level) decrease in the lockup period, and
it is less, but not significantly less, than the lowest quartile for unconstrained firms. Again, this may reflect some misclassification of constrained and unconstrained firms. In figure 2.5, there is no evidence that the lockup length differs by book value. None of the differences in mean lockup is significantly different from zero at usual levels.

2.8 A switching regression model

The results of the previous section line up very well with the models predictions. But the different explanatory variables can be controlled for simultaneously in a regression framework, and we still have not considered how variance affects underpricing. However, as mentioned above, it may not be legitimate to simply use ordinary least squares applied to the two subsamples because unmeasured firm characteristics are likely correlated with the firm’s state – constrained or unconstrained. To correct for this potential problem, a reduced form empirical model corresponding to the theory section is given.

Let \( K_i^* \) denote a critical level of \( K \) for firm \( i \), where \( i \) now only indexes the firm and not the firm type, such that if \( K \) exceeds this level, the capital requirement constraint binds and no secondary shares are sold. The critical value is determined according to \( K_i^* = \theta_i Z_i + \eta_i \), where \( Z_i \) is a vector of observable firm characteristics and \( \theta_i \) is a vector of parameters. The error term captures what is not observable to the researcher. Assumptions about \( \eta \) will be made shortly. Let \( y_i \) be an indicator of whether secondary shares are sold or not, where \( y_i = 1 \) if there are no sales, indicating a binding capital requirement constraint, and \( y_i = 0 \) otherwise. Thus

\[
y_i = 1 \quad \text{if } K_i \geq K_i^* \tag{2.18}
\]
\[
y_i = 0 \quad \text{otherwise.} \tag{2.19}
\]

The indicator \( y_i \) tells us if the firm is constrained or not. Underpricing, then, is related to firm characteristics and whether or not the firm is constrained. That is,
\[ R1 = \beta_1'X_i + \nu_{1i} \quad \text{if } y_i = 1 \]
\[ R1 = \beta_2'X_i + \nu_{2i} \quad \text{if } y_i = 0, \]

where \( R1 \) is the initial return. The relevant firm characteristics are given by the \( X_i \) vectors and the associated parameters are given by the \( \beta_j \) vectors, where \( j \) denotes the regime. Note that the set of explanatory variables is the same in both regimes. I assume that the error terms are uncorrelated with the explanatory variables and are multivariate normal with the following covariance matrix.\(^{26}\)

\[
\Omega = \begin{pmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{1\eta} \\
\sigma_{21} & \sigma_{22} & \sigma_{2\eta} \\
\sigma_{\eta1} & \sigma_{\eta2} & 1
\end{pmatrix}
\]

### 2.8.1 Proxies for \( K \) and \( \sigma^2 \)

First consider \( K \). It can be calculated as

\[
K = (SH - IN) \times OP,
\]

which is simply the proceeds from the IPO from primary shares. However, this is an endogenous regressor since it contains \( IN \). In a regression framework, IV estimation would be appropriate, but this variable enters the probit selection equation.

To correct for this problem only BOOK value is used rather than \( K \) or \( k \), (where \( k = K + \text{BOOK} \).) This is equivalent to re-specifying (2.18) as

\[
K^*_i - K_i = \theta'Z_i + \eta_i
\]

\(^{26}\)Normally, \( \sigma_{\eta}^2 \) is not identified and so set to 1, as shown above. Here, however, if the coefficient on \( K^i \) is restricted to equal 1, then \( \sigma_{\eta}^2 \) is identified. But in the final specification given below, this restriction is dropped so \( \sigma_{\eta}^2 \) is finally not identified. Also, \( \sigma_{12} \) does not appear in the likelihood function of an endogenous switching regression, which this model is, and so is not estimable. The empirical validity of the assumptions on the error terms is considered below.
and excluding $k$ from $Z$, (though BOOK is still included in $Z$.) Similarly, only BOOK is used in the underpricing equations so that the assumption that the error terms are uncorrelated with the explanatory variables can be maintained.

Now consider the measure of riskiness of the project. We ideally want the measure of volatility for the firm with a low-type project and not that of an underpricing firm to test proposition 2.4. However, each firm observed is only of one type so that we do not observe both the initial return of the high type firm and the variance of the low type firm. Only if there is high correlation between the variance of different types, can we use the variance in returns of the observed firm as a proxy for that of the unobserved low type firm. Inasmuch as this is not the case, we should expect variance to be positively associated with underpricing as it is then a proxy for capital requirements.

One simple approximation for the volatility the entrepreneur faces I derive from the Sharpe-Litner CAPM, or rather the market model. Let the time $t$ return to the firm in excess of a risk-free rate be denoted by $R_{it}$ and let the market excess return be denoted by $R_{mt}$. Then we have

$$R_{it} = \alpha_0 + \beta_i R_{mt} + \xi_{it},$$

where $\xi_{it}$ is iid with mean zero and variance $\sigma^2_{\xi_{it}}$. The total variance in $R_{it}$, for $t > 1$, conditional on time zero information, is $\beta^2_i \sigma^2_M + \sigma^2_{\xi_{it}}$, where $\sigma^2_M$ is the market variance and $\beta^2_i \equiv \beta^2_M$. In the results below, I let the coefficients on the undiversifiable component of variance (undiversifiable from an investor's point of view), $\beta^2_i \times \sigma^2_M$ and the diversifiable component, $\sigma^2_{\xi_{it}}$ differ. That is, $\beta^2_M$ and $\sigma^2_{\xi_{it}}$ enter all the equations as separate regressors, (where the subscripts $i$ have been omitted for convenience, as with the other explanatory variables.) The advantage of this approach is that a possible multicollinearity problem between variance and the capital requirement proxies doesn’t arise when $\beta^2_M$ is used. For example, the correlation coefficient for LSA and $\sigma^2_{\xi_{it}}$ is -0.4 but is only -0.12 for LSA and $\beta^2_M$. Because the diversifiable component of variance is much

27 If the CAPM model was valid, $\alpha_0$ would equal zero.

28 Note that since market variance is the same for all firms, $\sigma^2_M$ is captured in the coefficient on $\beta^2_M$. 

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larger in magnitude than the undiversifiable one for most of the firms, it turns out that the
effect of diversifiable risk is swamped by undiversifiable risk when the two component are
added together.

To calculate the return volatility the S&P 500 composite index (Jan 1968-Aug 1998)
is used as the market portfolio and the 3 month t-bill is used as the risk-free asset. The
market return period was matched up with the same return period for firm returns using
the description of how firm returns were calculated in Ritter (1991).

Another measure of volatility that I use is the number of months for which return data
on a firm exists (MNTHS). The likely reasons a firm exits the data set early are that it
failed, delisted as a public firm, or was acquired. Firms with high volatility are more likely
to end up in one such category, especially the first and last, than low volatility firms if we
assume that those states that would lead to such change in status, very good or very bad
performance, are more likely for firms with high volatility. Also, firms with few resources,
those facing binding capital requirement constraints, are also more likely to exit.

2.8.2 Empirical Specifications

The specification that is implemented based on (2.21) is:

\[
\text{Constrained: } R_l = \beta_1' X_i + \nu_{1i} \text{ iff } -\theta' Z_i \geq \eta_i \tag{2.22}
\]

\[
\text{Unconstrained: } R_l = \beta_0' X_i + \nu_{0i} \text{ iff } -\theta' Z_i < \eta_i \tag{2.23}
\]

The variables in \(X\) and \(Z\) are LSA, LAGE, LBOOK, MNTHS, VAR, BETA2. The first
three variables should capture the firms capital requirements. Young firms with low sales
and small book values are expected to have the largest need for capital. Thus the coefficients
(without the negative signs) on these variables should be positive in the probit equation. The
signs of the coefficients on VAR and BETA2 are ambiguous because of their possible dual
roles as proxies for liquidity constrained firms and for the variance in low type projects.
None of the explanatory variables should be significant in the regression equation for unconstrained firms. On the other hand, for constrained firms, the coefficients on LSA, LAGE and LBOOK should all carry negative signs and should be, at least, jointly significant. The effect and significance of the variance terms is again ambiguous.

As discussed above, negative initial returns are present in the data and are likely explained, at least in part, as the result of demand uncertainty. The theory makes no allowance for overpricing so I control for it by interacting a negative initial return dummy with all the explanatory variables. These variables are indicated by an "X" suffix. Of course, some firms with positive initial returns are also likely misclassified as underpricing firms, when in fact other reasons explain the positive initial return, but, without being able to identify these firms, I proceed as if all firms, once negative returns are controlled for, are correctly classified.

The results in the following section are based on a standard two stage estimation approach applied to the above model. The selection equation is estimated as a probit and then the regressions are estimated using weighted OLS, including the inverse Mill's ratio. Weighted OLS is necessary to correct of the heteroscedasticity introduced by the correlation in errors between regime equations and the selection equation. It is also straightforward to obtain consistent estimates of the identifiable components of the covariance matrix $\Omega$, as well as the covariance matrix for the parameters. See Maddala (1983) or Lee and Trost (1978) for details.

### 2.9 Results

The model was estimated excluding financial institutions because of their atypical characteristics as evidence above. However, I retain the oil and gas firms, but include an oil and gas dummy, DOIL, to control for any fixed effect. Some outliers in the explanatory variables also seemed to have an undue effect on the results. The reported results exclude 14 observations for which sales were greater than $500 million, 1 observation where book value was over $200 million, and 21 observations where BETA2 was greater than 16. The data with high betas tended to have few return observations making the estimated betas suspect. Including these
observations led to an increase in the point estimate of the coefficient on BETA2, and its
significance, and a decrease in the significance of the other coefficients.

The first set of results are reported in table 2.1. LSA and LAGE have positive and
significant coefficients as predicted. The coefficient on LBOOK, however, is negative but not
significant, though this is partly due to multicollinearity with LSA and LAGE. When LSA
and LAGE are omitted, the coefficient on LBOOK is positive and highly significant. MNTHS
has a positive coefficient indicating that firms that exited the sample early were more likely
constrained while VAR carries a positive and significant sign. The effect of an increase in
variance seems to be to increase the likelihood of being constrained. The coefficients on the
market beta and its square are not significant. The results of the probit equation supports
the predicted positive relationship between capital requirements and secondary share sales,
which is consistent with the presence of moral hazard.

The regression results for constrained firms are in table 2.2 and for unconstrained firms
in table 2.3. Comparing the two, the adjusted coefficient of determination is 0.318 for con­
strained firms but only 0.119 for unconstrained firms. In addition, none of the coefficients
in the unconstrained regression are significant while 7 coefficients are significant in the con­
strained regression at a level of 10 percent and 4 are significant at the 5 percent level or
better.

The coefficients on the proxies for capital requirements are significant and carry negative
signs as expected. Somewhat surprising is that despite LBOOK's wrong sign in the probit
equation, in the regression equation, it is significantly negative. The positive coefficient on
BETA2 indicates that risk is positively associated with underpricing. Somewhat surprising
is the negative, though insignificant, coefficient on VAR given the positive coefficient on
BETA2. This is precisely the reverse result of the probit equation, where VAR carries a
positive significant coefficient and BETA2 carries a negative insignificant coefficient.29 When

29One could argue that with free entry into an industry, firms with high market BETAs should also have
projects that have higher expected payoffs to offset the additional risk, explaining BETA's lack of significance
in the constrained regression equation.
<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>LBOOK</td>
<td>-0.111</td>
<td>0.125</td>
</tr>
<tr>
<td>LSA</td>
<td>0.236***</td>
<td>2.88E-02</td>
</tr>
<tr>
<td>LAGE</td>
<td>0.243***</td>
<td>4.60E-02</td>
</tr>
<tr>
<td>BETA2</td>
<td>0.166E-01</td>
<td>3.44E-02</td>
</tr>
<tr>
<td>BETA</td>
<td>0.191E-01</td>
<td>8.53E-02</td>
</tr>
<tr>
<td>MNTHS</td>
<td>0.281E-01***</td>
<td>7.72E-03</td>
</tr>
<tr>
<td>IND3</td>
<td>-0.331*</td>
<td>0.190</td>
</tr>
<tr>
<td>VAR</td>
<td>-7.960***</td>
<td>2.11</td>
</tr>
<tr>
<td>OVRPR</td>
<td>-5.829</td>
<td>4.170</td>
</tr>
<tr>
<td>LBOOKX</td>
<td>0.147</td>
<td>0.270</td>
</tr>
<tr>
<td>LSAX</td>
<td>0.163*</td>
<td>8.82E-02</td>
</tr>
<tr>
<td>LAGEX</td>
<td>0.150E-01</td>
<td>0.107</td>
</tr>
<tr>
<td>BETA2X</td>
<td>-0.508E-01</td>
<td>0.124</td>
</tr>
<tr>
<td>BETAX</td>
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<td>0.325</td>
</tr>
<tr>
<td>MNTHSX</td>
<td>-0.236E-03</td>
<td>2.19E-02</td>
</tr>
<tr>
<td>IND3X</td>
<td>0.859**</td>
<td>0.43</td>
</tr>
<tr>
<td>LVARX</td>
<td>9.328**</td>
<td>4.73</td>
</tr>
<tr>
<td>CONSTANT</td>
<td>-3.103*</td>
<td>1.88</td>
</tr>
<tr>
<td>LF</td>
<td>-653</td>
<td></td>
</tr>
<tr>
<td>N0</td>
<td>632</td>
<td></td>
</tr>
<tr>
<td>N1</td>
<td>733</td>
<td></td>
</tr>
</tbody>
</table>

*significant at 10%, ** significant at 5%, *** significant at 1%

Table 2.1: Probit
<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>LBOOK</td>
<td>-9.74E-02**</td>
<td>4.42E-02</td>
</tr>
<tr>
<td>LSA</td>
<td>-1.06E-02***</td>
<td>3.96E-03</td>
</tr>
<tr>
<td>LAGE</td>
<td>-3.01E-02*</td>
<td>1.75E-02</td>
</tr>
<tr>
<td>BETA2</td>
<td>1.25E-02*</td>
<td>7.69E-03</td>
</tr>
<tr>
<td>BETA</td>
<td>-8.54E-03</td>
<td>2.04E-02</td>
</tr>
<tr>
<td>MNTHS</td>
<td>9.86E-04</td>
<td>2.22E-03</td>
</tr>
<tr>
<td>IND3</td>
<td>2.41E-01***</td>
<td>4.54E-02</td>
</tr>
<tr>
<td>VAR</td>
<td>-0.257</td>
<td>0.347</td>
</tr>
<tr>
<td>OVRPR</td>
<td>-2.21</td>
<td>1.53</td>
</tr>
<tr>
<td>LBOOKX</td>
<td>1.05E-01</td>
<td>9.26E-02</td>
</tr>
<tr>
<td>LSAX</td>
<td>1.41E-02*</td>
<td>7.81E-03</td>
</tr>
<tr>
<td>LAGEX</td>
<td>3.66E-02</td>
<td>3.42E-02</td>
</tr>
<tr>
<td>BETA2X</td>
<td>-1.16E-02</td>
<td>1.56E-02</td>
</tr>
<tr>
<td>BETAX</td>
<td>6.02E-03</td>
<td>4.08E-02</td>
</tr>
<tr>
<td>MNTHSX</td>
<td>-3.90E-04</td>
<td>6.01E-03</td>
</tr>
<tr>
<td>IND3X</td>
<td>-2.76E-01***</td>
<td>0.102</td>
</tr>
<tr>
<td>LVARX</td>
<td>2.81E-02</td>
<td>0.755</td>
</tr>
<tr>
<td>CONSTANT</td>
<td>1.95*</td>
<td>0.722</td>
</tr>
<tr>
<td>$\sigma_{\nu_1}$</td>
<td>-1.12E-02</td>
<td>6.54E-02</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.318</td>
<td></td>
</tr>
<tr>
<td>N0</td>
<td>733</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>19.98</td>
<td></td>
</tr>
</tbody>
</table>

*significant at 10%, ** significant at 5%, *** significant at 1%

Table 2.2: Constrained firms
<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>LBOOK</td>
<td>-2.08E-03</td>
<td>3.71E-02</td>
</tr>
<tr>
<td>LSA</td>
<td>-1.14E-02</td>
<td>1.85E-02</td>
</tr>
<tr>
<td>LAGE</td>
<td>-1.30E-02</td>
<td>5.12E-02</td>
</tr>
<tr>
<td>BETA2</td>
<td>-6.43E-03</td>
<td>9.41E-03</td>
</tr>
<tr>
<td>BETA</td>
<td>1.51E-02</td>
<td>3.20E-02</td>
</tr>
<tr>
<td>MNTHS</td>
<td>1.17E-03</td>
<td>6.26E-03</td>
</tr>
<tr>
<td>IND3</td>
<td>2.29E-03</td>
<td>1.07E-01</td>
</tr>
<tr>
<td>VAR</td>
<td>6.95E-01</td>
<td>0.535</td>
</tr>
<tr>
<td>OVRPR</td>
<td>-4.41E-01</td>
<td>9.36E-01</td>
</tr>
<tr>
<td>LBOOKX</td>
<td>1.45E-02</td>
<td>7.46E-02</td>
</tr>
<tr>
<td>LSAX</td>
<td>7.05E-03</td>
<td>5.94E-02</td>
</tr>
<tr>
<td>LAGEX</td>
<td>8.68E-03</td>
<td>2.63E-02</td>
</tr>
<tr>
<td>BETA2X</td>
<td>3.67E-03</td>
<td>5.61E-02</td>
</tr>
<tr>
<td>BETAX</td>
<td>-1.34E-02</td>
<td>1.75E-01</td>
</tr>
<tr>
<td>MNTHSX</td>
<td>-1.85E-03</td>
<td>5.69E-03</td>
</tr>
<tr>
<td>IND3X</td>
<td>2.97E-03</td>
<td>0.191</td>
</tr>
<tr>
<td>VARX</td>
<td>-6.41E-01</td>
<td>2.157278</td>
</tr>
<tr>
<td>CONSTANT</td>
<td>0.318</td>
<td>1.160581</td>
</tr>
<tr>
<td>( \sigma_{\nu_1} )</td>
<td>1.81E-02</td>
<td>3.15E-01</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.119</td>
<td></td>
</tr>
<tr>
<td>N0</td>
<td>632</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>5.69</td>
<td></td>
</tr>
</tbody>
</table>

*significant at 10%, ** significant at 5%, *** significant at 1%

Table 2.3: Unconstrained firms
the total variance,

\[ TVAR = \beta_2 \times \sigma_m^2 + VAR \]

is used, it behaves very similarly to VAR; positive and significant in the probit equation and insignificant in both regression equations. It seems that unlike proposition 2.4, firms with high variance projects are more likely to be constrained and to underprice, which suggests that VAR is a better proxy for capital requirements than for variance in low type returns.

Of some interest in the significant oil and gas dummy, IND3, in both table 2.1 and table 2.3. Oil and gas companies at the time were more likely to be constrained and the extent of underpricing among such constrained firms was greater than for firms in other industries. Other regressions (results not shown) do indicate some industry differences, but no clear pattern was found. Furthermore, year dummies were included but not significant year effect seemed to be in the data. While 1980 was characterized by above average initial return, this was mostly due to oil and gas firms and is picked up by the IND3 dummy (see Ritter (1984)).

The magnitude of the coefficients on the proxies for capital constraints, especially that for LBOOK, are economically important. For instance, an increase in book value from zero to its average of about $6 million would lead to over a 100 percent increase in the initial return. (We can be 95 percent confident that the effect would be at least 9.4 percent.) Similar calculations for sales and age give increases in the initial return of 18.2 and 7.4 percent, respectively.

It seems that the covariances in the error terms between the selection equation and both regression equations is not important. It does, however, carry a negative sign which is consistent with the positive signs on LBOOK, LSA, LAGE. That is, when the error term is large and negative in the probit equation, indicating that the firm's unmeasured capital requirement is large, the error in the constrained regression equation is more likely to be positive reflecting the tighter capital requirement constraint. However, it is insignificant.
2.9.1 Specification tests

Homoscedasticity and normality are assumed in the switching regression model. However, as typical with micro data sets, these assumptions may not hold. Two types of tests for heteroscedasticity and one for non-normality are found in table 4. Following Pagan and Vella (1989) and Davidson and MacKinnon (1984), I first use artificial regressions to construct simple tests. The tests simply involve regressions such as

$$
\Phi_{+}^{-1/2}(1-\Phi_{+})^{-1/2}(y_{i} - \hat{\Phi}) = \Phi_{+}^{-1/2}(1-\Phi_{+})^{-1/2}\frac{\partial \hat{\Phi}_{i}}{\partial \gamma} + \eta_{i},
$$

where the hat denotes an estimate, $\Phi_{+} = \Phi(\beta^{\prime}X_{i}, \gamma^{\prime}Z_{i})$, is the standard normal cdf with density $\phi_{i}$, $\eta$ is a spherical error, and $\gamma$ equals zero under the null. For heteroscedasticity, $\Phi = \Phi(\beta^{\prime}X_{i}(1+\gamma Z_{i}))$ is used where $Z_{i}$ is one of the regressors in $X_{i}$. And for normality, $\Phi = \Phi(\beta_{i}X_{i} + \gamma_{i}(\beta_{i}X_{i})^{2} + \gamma_{2}(\beta_{i}X_{i})^{3})$, as suggest by Pagan and Vella.

The simplicity of the artificial regression has a cost as noted by Pagan and Vella in that the estimate of the variance of the test statistic may be quite imprecise giving rise to a divergence between nominal and actual test sizes. As an alternative, I use the following

<table>
<thead>
<tr>
<th>Test statistic $\chi^{2}$ (1 df)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOOK</td>
</tr>
<tr>
<td>SA</td>
</tr>
<tr>
<td>AGE</td>
</tr>
<tr>
<td>VAR</td>
</tr>
<tr>
<td>BETA</td>
</tr>
<tr>
<td>BETA2</td>
</tr>
<tr>
<td>MNTHS</td>
</tr>
</tbody>
</table>

* significant at 1%

Table 2.4: LM test for heteroskedasticity
specification for heteroscedasticity, found in Harvey (1976),

$$\sigma^2_{v,i} = (exp(\gamma Z_i))^2.$$  

With this specification it is straightforward to construct an LM test by the method of scoring.

The results show that the data is neither homoscedastic nor normal. LBOOK, LSA and LAGE seem to be the most important source of the heteroscedasticity problem.

The poorly behaving error term may mean that the coefficient estimates are likely biased and inconsistent. However, if the covariance between the error terms, i.e., $\sigma_{v1}$ and $\sigma_{v2}$, are not important then we can estimate the two regressions with ordinary least squares and ignore the covariance terms. White's (1980) heteroscedastic consistent covariance matrix for the coefficients can then be estimated. The results for the two regressions are in tables 2.6 and 2.7.

There is no significant qualitative change in the results except that the adjusted $R^2$ is reduced to 0.202 for the constrained regression from 0.318. Thus unless the covariance terms
are more important than indicated in table 2.1, which we cannot detect because of the heteroscedastic and non-normal errors, the conclusions from the switching regression model hold up quite well. Furthermore, the evidence of section 2.7.2 is consistent with the findings of this section.

2.10 Conclusion

The first part of the paper has shown that underpricing can arise from adverse selection even when no secondary offering is ever expected to be made. According to the theory, underpricing occurs when risk averse entrepreneurs with high value projects cannot signal their project type in the usual way, by keeping a high stake in the firm, since this reduces proceeds so much that the project cost is no longer covered. While underpricing alone does reduce proceeds, by allowing the entrepreneur to reduce his stake and still signal his firm's type net proceeds increase. The implication is that firms that have the most need for capital will also be most likely to underprice.

In addition, when the entrepreneur is constrained he will not sell any of his private shares since this does not help relax the capital requirement constraint, which is the cause of costly underpricing. This was important for testing the theory since it allowed us to identify constrained and unconstrained firms by the entrepreneur's choice to sell private shares or not.

The data strongly support the models main predictions. First, constrained firms tend to be young and have low sales and few assets. They also tend to display higher variance in the returns than unconstrained firms, consistent with the presence of multiple types. These same characteristics also explain underpricing among constrained firms in the predicted way but have much weaker explanatory power for unconstrained firms. This evidence points to a signaling explanation for underpricing which does not require subsequent secondary offerings. Adverse selection seems to be important factor affecting young firms' ability to raise capital.
<table>
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<th>Term</th>
<th>Estimate</th>
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| $\bar{R}^2$ | 0.202 |
| N0          | 733   |
| F           | 11.88 |

*significant at 10%, ** significant at 5%, *** significant at 1%

Table 2.6: Constrained firms using heteroskedastic consistent estimates
<table>
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<tr>
<th>Variable</th>
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<td>F</td>
<td>5.74</td>
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</tbody>
</table>

*significant at 10%, ** significant at 5%, *** significant at 1%

Table 2.7: Unconstrained firms using heteroskedastic consistent estimates
Appendix

Lemma 2.1 Let \( p = p(\alpha) \) such that

\[
\frac{dp}{d\alpha} = \frac{-\partial EU / \partial \alpha}{\partial EU / \partial p},
\]

and let \( p_H = p(\alpha_H) \) and \( p_L = p(\alpha_L) \). Then

\[
EU_H(\alpha_H, p_H) - EU_H(\alpha_L, p_L) > EU_L(\alpha_L, p_L) - EU(\alpha_L, p_L)
\]

iff \( \alpha_H > \alpha_L \).

Proof: We can equivalently show that

\[
\int_{\mu_L}^{\mu_H} \int_{\alpha_L}^{\alpha_H} \frac{d^2 EU(\cdot)}{d\alpha d\mu} d\alpha d\mu = \int_{\mu_L}^{\mu_H} \int_{\alpha_L}^{\alpha_H} \left( \frac{\partial^2 EU}{\partial \alpha \partial \mu} + \frac{\partial^2 EU}{\partial \mu \partial p} \frac{dp}{d\alpha} \right) d\alpha d\mu
\]

\[
= \int_{\mu_L}^{\mu_H} \int_{\alpha_L}^{\alpha_H} \frac{\partial (\frac{\partial EU/\partial \alpha}{\partial EU/\partial p})}{\partial \mu} d\alpha d\mu > 0 \quad \text{iff} \quad \alpha_H > \alpha_L.
\]

Thus it suffices to show that

\[
\frac{\partial (\frac{\partial EU/\partial \alpha}{\partial EU/\partial p})}{\partial \mu} = -\left( 1 - \alpha + \frac{\partial (\frac{EU'|E(\cdot)}{EU(E(\cdot))})}{\partial \mu} \right) < 0.
\]

Let the probability of a realization of \( \epsilon_i \), where \( \epsilon_i > \epsilon_j \) whenever \( i > j \), be denoted by \( p_i \) and let \( U_i \) be the associated utility with this realization. Expanding the last term of the second expression gives:

\[
\frac{\partial (\frac{E'U'}{EU(\cdot)})}{\partial \mu} = \sum_i \sum_j \rho_i \rho_j \epsilon_i \epsilon_j U_i' U_j' - \sum_i \sum_j \rho_i \rho_j \epsilon_i r_j U_i' U_j'
\]

\[
= \sum_i \sum_{j > i} \rho_i \rho_j U_i' U_j'(r_i - r_j)(\epsilon_i - \epsilon_j) < 0,
\]

where the inequality follows from \( r_i > r_j \) for \( i < j \) because of decreasing absolute risk aversion. QED
Proof of Proposition 2.1

Assume that minimum proceed constraint does not bind, given \( K \), and then we’ll verify that this is the case later. In any separating equilibrium the low type will offer \( \{0, \mu_L - K\} \) since for any other offer \( \{\alpha_L, p_L\} \) that investors would accept, knowing that he is the low type, he does worse.

Consider next the following program:

\[
\max_{\alpha_H \in [0,1], \mu_H < \mu_H - K} EU_H(\alpha_H, p_H)
\]

subject to

\[
EU_L(0, \mu_L - K) \geq EU_L(\alpha_H, p_H).
\]

I show that there is a unique solution with \( p^*_H \), denoting the optimal \( p_H \), equal to \( \mu_H - K \).

Suppose to the contrary, \( p^*_H < \mu_H - K \). The first order necessary condition are:

\[
E(U^H(\cdot)(\mu_H - K + p + \epsilon)) - \lambda E(U^L(\cdot)(\mu_L - K - p + \epsilon)) = 0,
\]

\[
EU^H(\cdot) - \lambda EU^L(\cdot) = 0,
\]

where \( \lambda \) is the Lagrange multiplier. Using the second condition to substitute out \( \lambda \) in the first yields:

\[
(\mu_H - \mu_L) \left( \frac{E(U'_H(\cdot)\epsilon)}{EU'_H(\cdot)} - \frac{E(U'_L(\cdot)\epsilon)}{EU'_L(\cdot)} \right) = 0.
\]

But by Lemma 2.1, the second term on the left-hand side is positive so the equality cannot hold. Hence, \( p^*_H - \mu_H - K \).

Now given \( p^*_H = \mu_H - K \), the constraint is violated at \( \alpha_H = 0 \) and strictly satisfied at \( \alpha_H = 1 \). Furthermore, the right-hand side of the constraint is strictly decreasing with \( \alpha_H \). Thus there is a unique \( \{\alpha^*_H, p^*_H\} \) that solves the program.

Next I show that any separating PBE satisfying the Intuitive Criterion must solve the above program implying the equilibrium is unique. Again the proof is by contradiction. Suppose, then, that there exists a contract \( \{\alpha_H, p_H\} \) that does not solve the program but is a PBE contract satisfying the Intuitive Criterion. Consider a deviation offer.
\( \{ \alpha_H^*, p_H^* \} = \{ \alpha_H^*, P_H^* - \iota \} \), where \( \iota \) is a small positive number. Since the objective function in the above program is continuous in \( p \), for \( \iota \) sufficiently small, \( EU_H(\alpha_H^*, p_H^*) > EU_H(\alpha_H, p_H) \) and the incentive compatibility constraint is strictly satisfied by \( \{ \alpha_H^*, p_H^* \} \). But then this is a deviation offer that makes the low type strictly worse off than his equilibrium offer and the high type strictly better off than his equilibrium offer. Moreover, conditional on only a high type making the offer, it is a best response for investors to accept it. Thus the equilibrium with underpricing fails the intuitive criterion.

Now for \( \{ \{ \alpha_H^*, p_H^* \}, \{ \alpha_L^*, p_L^* \} \} \) no such deviation offer can be constructed. Moreover, specifying

\[
\phi(\alpha_k, p_k) = \begin{cases} 1 & \text{if } \{ \alpha_k, p_k \} = \{ \alpha_H^*, p_H^* \} \\ 0 & \text{otherwise} \end{cases}
\]

it is easily verified that (2.7)-(2.11) are satisfied so that \( \{ \alpha_H^*, p_H^* \} \) and \( \{ \alpha_L^*, p_L^* \} \) are indeed equilibrium contracts.

Finally, it is verified in the text that with \( \gamma_H^* = 1 \), and \( p_H^* = \mu_H - K \), the minimum proceed constraint is not binding when \( K \leq \mu_H(1 - \alpha_H^*) \).

**Proof of proposition 2.2**

Analogous to the proof to proposition 2.1, the Intuitive Criterion requires that any equilibrium contract \( \{ n_H^*, \gamma_H^*, p_H^* \} \) solves the following program:

\[
\max_{n_H \geq 0, \gamma_H \in [0,1], p_H \leq \mu_H - K} EU_H(n_H, \gamma_H, p_H)
\]

subject to

\[
EU_L(0, \mu_L - K) \geq EU_L(n_H, \gamma_H, p_H)
\]

\[
n_H p_H \geq K.
\]

It is easy to see that \( \gamma_H^* \) must equal one. Let \( H(\gamma) = \{ n | n \geq 0 \text{ and } \alpha \in [0,1] \text{ for a given } \gamma \} \). Clearly \( H(\gamma) \subset H(1) \) for any \( \gamma \in [0,1] \). But the objective function and constraints only

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depend on $p_H$, $\alpha_H$ and $n_H$ and not directly on $\gamma_H$. Thus setting $\gamma_H$ equal to one increases the choice set for $n_H$ and so relaxes the last constraint.

The equivalent problem then is

$$\max_{\alpha_H \in [0,1], p_H \leq \mu_H - K} EU_H(\alpha_H, p_H)$$

subject to

$$EU_L(0, \mu_L - K) \geq EU_L(\alpha_H, p_H)$$

$$\frac{1 - \alpha_H}{\alpha_H} p_H \geq K.$$

Suppose $p_H^{**} = \mu_H - K$. Now note that

$$EU_H(\alpha_H, \mu_H - K) \text{ and } \frac{1 - \alpha_H}{\alpha_H} (\mu_H - K)$$

are strictly decreasing in $\alpha_H$. Thus, if the minimum proceeds constraint is violated at $\{\alpha_H^*, p_H^*\}$, then $\alpha_H^{**} < \alpha_H^*$ and so

$$EU_H(\alpha_H^{**}, \mu_H - K) > EU_H(\alpha_H^*, \mu_H - K).$$

But this contradicts $\{\alpha_H^*, p_H^*\}$ as a solution to the unconstrained maximization program. Hence, $p_H^{**} < \mu_H - K$.

Next I show a unique solution exists. Clearly, the high type can mimic the low type’s contract, which satisfies the minimum proceeds constraint since $\mu_L > K$. However, by the single crossing condition from Lemma 2.1, there is some $\{\alpha_H, p_H\}$ arbitrarily close to $\{0, \mu_L - K\}$ that the high type strictly prefers to the low types contract and that the low type strictly dislikes to his own contract. Since the contracts are arbitrarily close, the high type contract will still satisfy the minimum proceeds constraint.

The last step is to show that the solution is unique. Substituting out $p_H$ with the capital requirement constraint reduces the program to:

$$\max_{\alpha_H \in [0,1]}EU_H(\alpha_H(\mu_H + \epsilon))$$

subject to

$$EU_L(\alpha_H(\mu_L + \epsilon)) \leq EU_L(0, \mu_L - K).$$
Since the objective function is always greater than the left-hand side of the constraint, the constraint determines $\alpha_{H}^{*}$. The left-hand side of the constraint is strictly concave in $\alpha_{H}$ and increasing at $\alpha_{H} = 0$. Also, the constraint hold strictly at $\alpha_{H} = 0$. Hence, there is a unique $\alpha_{H}^{**}(< \alpha_{H}^{*})$ satisfying the constraint.

**Proof of Proposition 2.3**

From Proposition 2.1 we know that $p_{H} = \mu_{H} - K$ whenever $K \leq \bar{K}$, giving the first condition of the proposition. Now since proceeds are decreasing in $\alpha_{H}$, an increase in $K$ requires a decrease in $\alpha_{H}^{*}$ when the minimum proceeds constraint is binding. But to maintain incentive compatibility with the low type, $p_{H}^{*}$ must decrease, giving the second condition of the proposition. QED

**Proof of Proposition 2.4**

Suppose the minimum proceeds constraint is not binding. Then it is easily verified that an increase in $\sigma$ will reduce the low type’s incentive to copy the high type, i.e., relax the incentive compatability constraint, allowing $\alpha_{H}^{*}$ to decrease. That is,

$$\frac{\partial EU_{L}(\alpha_{H}, p_{H})}{\partial \sigma} = \alpha_{H} E(U'_{L}(\cdot)z_{L}) < 0.$$ 

This relaxes the non-binding minimum proceeds constraint allowing the offering price $p_{H}^{*}$ to remain at $\mu_{H}^{*}$. No underpricing occurs.

Now suppose the minimum proceeds constraint is binding so that there is underpricing. Fix $\alpha_{H}^{**}$. A small increase in $\sigma$ still relaxes the low type’s incentive compatibility constraint so that the price, $p_{H}^{**}$ can increase. The increase in the offering price means $R_{H}$ decreases. Also, since this increases proceeds, $\alpha_{H}^{**}$ can then be raised slightly, towards $\alpha_{H}^{*}$. QED
IPOs and young firms:
the case of oil and gas industry

In the first chapter I established why under asymmetric information debt and internal funds tend to be preferred to external equity finance. Nevertheless, a small percentage of firms do rely on external equity finance from public markets. Among these firms that go public, underpricing is likely to decrease as the firm’s capital requirements decrease. The second chapter highlighted the importance of the entrepreneur’s aversion to risk in making the connection of underpricing not only to age (or, more generally, capital requirements,) but also to the composition of shares sold at the IPO. Indeed, evidence for signaling was derived from the empirical support for these predicted relationships.

We could imagine merging the two models together. That is, allowing the entrepreneur in chapter 1 to be risk averse. Underpricing could be reduced through keeping a large stake in the firm. Now when the firm has almost no capital, some underpricing is expected for the reasons explained in chapter 2. The entrepreneur may choose to instead delay the IPO and accumulate more capital to reduce or avoid underpricing, as in chapter 1. The more valuable the good project, the greater the incentive to undertake the project early and underprice. And such firms should issue bonds not shares.

However, it is the presence of young firms with large capital requirements that issue equity in the data set that allowed the chapter two model to be tested. Why is there this small fraction of firms that raise substantial capital in equity markets? Is there something

\[30\text{Note that if the entrepreneur sells some of her bonds she also reduces risk so that not issuing too much debt also can be used to signal type.}\]
particular about them?

A useful way to think about these questions is in terms of a concrete example. As we saw in chapter two, firms in the oil and gas industry were unusually young. They also raised substantial capital relative to their book values, compared to other industries, and had, on average, very high initial returns at their IPOs. Ritter (1984) has shown the “hot issue” market in the 15-month period beginning in January 1980, was driven by these oil and gas firms. The main impetus for these firms to go public was the huge increases in oil prices driven by the Iranian revolution and the Iran/Iraq war. In 1979, the crude oil price was just over 25 dollar per barrel (in 1995 dollars). It increased dramatically to just under $40 the next year and then further increased to its highest point in over 100 years of about $53 per barrel.

This high price of crude encouraged entry into the industry as attested to by the surge in the number of oil and gas IPOs just after the hot issue market. Furthermore, since the high prices were driven by supply restrictions in the middle east because of conflict there, it was unclear how long such high prices would last. Indeed, following steady price declines after 1981, a change in Saudi Arabia’s petroleum policy in late 1985 led to more than a 50 percent fall in prices between December 1985 and April 1996. Profits from US oil and gas production fell from its high in 1980 to a low in 1986 by a staggering 96 percent. Even by 1982, profits had fallen over 30 percent from the year before, after holding steady in 1981 (Owen, 1997).

The high price of crude in the early 1980s and the uncertainty surrounding its future level encouraged not only rapid entry into the industry, but a desire to operate as many wells as early as possible. That is, small existing operators and new entrants had strong incentives for making significant investments with little delay. The high price was equivalent to a high expected project value and the future uncertainty in price, eventually realized in a price collapse, could be partly reflected in an expected high degradation or project loss rate. But according to chapter one, wouldn’t this suggest a reliance on debt and not external equity?—not necessarily.
We have to consider two related factors. First, the asymmetric information that is important is related to the quality of a firm’s exploration activity and its evaluation of wells. For instance, it may be very difficult to verify how many viable wells a firm has, or for that matter, how much of a price decrease can occur for operation of a well to remain profitable. Second, bankruptcy in the industry would be most likely, especially for firms with marginal wells, if there was a sudden collapse in prices. But such a decrease would mean the market value of drilling equipment and oil rigs would also plunge as demand dried up, so that the collateral value of equipment in securing debt was likely low. While this would have increased the riskiness of debt for all firms, insofar as it was more important for small operators with questionable expansion opportunities, the spread between the default rates of a bad firm and a good firm would be large. The potentially high default rate for bad firms would make it difficult to secure debt at reasonable rates.

Since there was such an incentive to move quickly, some small operators were not willing to accumulate the funds necessary for expansion by delaying entry. The high price of debt made equity markets relatively attractive. However, even though most did not sell any secondary shares, presumably as a signal of the quality of the firms’ prospects, investors would not be willing to pay full price. Investors realized that the dilution in an entrepreneur’s stake resulting from raising the large amount of needed funds, would encourage those firms poorly prepared to expand and with wells of marginal value to also invest. The entrepreneur that really had a profitable investment opportunity, had to underprice the offering. For natural resource firms with sales under $500,000, the initial return during the hot market was 116.5 percent (Ritter, 1984). Interestingly, despite the incentive to invest early, much of the volume of natural resource IPOs occurred after the hot issues market, implying that many oil and gas firms preferred to still wait and accumulate some funds to reduce underpricing.

More generally, IPOs among young firms with large capital requirements should occur when, on one hand, lenders will only make sizeable loans at very high interest rates, if at all, fearful that they are subsidizing high risk ventures with little collateral, and, on the other, the firm needs the capital now to fund a potentially very valuable expansion or risk losing
it altogether. The latest wave of IPOs among Internet related companies, with seemingly huge yet largely uncertain profitability, little collateral with potentially high default rates, and strong incentives to be early players, seem to fit this pattern. Indeed, in November of 1998, the highest initial return ever occurred. The Globe.com, an Internet service provider, saw its initial price jump by over 800 percent at the opening bell. While most firms focus on retained earnings and debt as their dominant source of investment capital, it is the young IPOs, the potentially highly profitable but also highly risky firms, which generate the most excitement.
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