A STUDY OF CANADIAN RETAIL GASOLINE PRICES

by

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Abstract

This thesis presents an analysis of the pricing behaviour of firms in Canadian retail gasoline markets. The time series of retail prices for certain Canadian cities can be categorized as exhibiting one of two distinct patterns. In many cities, retail prices remain unchanged for many weeks at a time, despite frequent changes to the wholesale gasoline price. In other cities, retail prices cycle, increasing sharply, and declining more slowly. This thesis addresses questions arising from the observation of these patterns.

The first essay considers a theoretical model of price setting behavior, and asks whether the number of stations operated by each firm in a market can determine whether constant prices or price cycles are observed. Constant prices are found to exist only when firms are of similar size. On the other hand, cycle equilibria can be constructed when the firms are of similar size, but also when their sizes differ greatly. Evidence of a negative relationship between price stability and the presence of small firms is also found through an examination of a panel data set of retail prices for a number of Canadian cities.

The second essay examines the response of retail prices to wholesale price movements in the presence of a retail price cycle. A simple model based on the predictions of the theory is constructed, and estimated using data for the city of Windsor, Ontario. I find that a new cycle is initiated by a price increase whenever the distance between the previous retail price and the current wholesale price becomes sufficiently small. In addition, retail prices are found to be more responsive to wholesale prices over the increasing portion of the cycle. Finally, when the asymmetric error correction model of Borenstein, Cameron, and Gilbert (1997) is estimated, a more rapid response to wholesale price increases than to decreases is indicated. This asymmetry is shown to be consistent with my structural model, which thus provides an additional potential explanation for the regularities found in previous studies.

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Chapter 1

Introduction

This thesis presents an analysis of the pricing behavior of firms in Canadian retail gasoline markets. The time series of retail prices for certain Canadian cities can be categorized as exhibiting one of two distinct patterns. In many cities, retail prices remain unchanged for many weeks at a time, despite frequent changes to the wholesale gasoline price. In other cities, retail prices cycle, increasing sharply, and declining more slowly. As well, these patterns are not evident in the wholesale price. In this thesis, we examine a potential explanation for the existence of these patterns, and the impact that these patterns might have on analysis of retail gasoline prices.

We begin in Chapter 2 with a thorough discussion of the Canadian retail gasoline industry and a qualitative documentation of the patterns observable in retail gasoline prices. We follow this in Chapter 3 with a detailed examination of the literature relevant to this thesis. We consider two distinct bodies of literature: the existing body of empirical work studying the dynamics of retail gasoline prices, and previous theoretical approaches to analyzing price cycles. Both of these bodies of literature are considered in detail.

The remainder of the thesis consists of two individual essays concerning an explanation for the patterns in retail gasoline prices and the possible impact of the price cycle on the response of retail gasoline prices to changes in the wholesale price. Chapter 4 presents an essay in which we discuss a theoretical model that can generate both of the patterns observable in the data. We then ask the question whether the relative sizes of firms in a market influence the existence and characterization of constant price and price cycle equilibria in the theoretical model. Constant price equilibria are found to exist only when the two firms are of similar size. On the other hand, cycle equilibria can be constructed when the firms are of similar size, as well as when their sizes differ greatly. A panel data set of retail prices for a number of Canadian cities provides evidence of a negative relationship between price stability and the presence of small firms. These findings support the assertions of government documents that relate price cycles to the presence of small independent marketers.

Chapter 5 consists of an essay in which we examine the response of retail prices to wholesale price movements in the presence of a retail price cycle. A simple econometric model based on the predictions of the theory in Chapter 4 is constructed, and estimated using data for the city of Windsor, Ontario. In support of the theory, it is found that a new cycle is initiated by an increase in price whenever the distance between the previous retail price and the current wholesale price becomes sufficiently small. In addition, retail prices are found to be more responsive to wholesale prices over the increasing portion of the cycle.

Finally, when the asymmetric error correction model of Borenstein, Cameron, and Gilbert (1997) is estimated, it indicates a more rapid response to wholesale price increases than to decreases. This asymmetry is shown to be consistent with my structural model, thus providing an additional potential explanation for the regularities found in previous studies.

Chapter 2

The Canadian Gasoline Industry

The purpose of this chapter is to discuss briefly the structure of the Canadian retail gasoline industry, and to provide a qualitative discussion of the behavior of retail prices in selected markets. In addition to our own analysis, we rely heavily on discussions found in recent inquiries and government studies of Canadian gasoline pricing.

2.1 The Structure of the Industry

The Canadian gasoline industry can be broken down into two general components: crude oil and refining, and distribution and retailing. We will discuss each one in sequence. Since the focus of this thesis is on downstream retailing, the first component will be discussed only briefly.

Crude Oil and Refining

The principal suppliers of Canadian gasoline are Canadian refineries. In 1992, there were 27 refineries operating in Canada, distributed across nine provinces and territories.[1] While there has been a slight reduction of this number over time (by 1995 only 24 refineries were in operation), the number of refineries in operation has remained relatively stable. As well, there were a total of nine companies operating refineries in Canada in 1992. Of these, the major three are the national firms Imperial Oil, Shell Canada and Petro-Canada, which owned sixteen of the twenty seven refineries, the rest being operated by regional firms. These sixteen refineries represented sixty one percent of the industry's crude oil refining capacity.

Refined product produced in Canada is sold both domestically and internationally. The U.S. wholesale gasoline market is considered to be the main alternative to selling gasoline domestically, either at the wholesale or retail level, for Canadian refiners. The impact of the United States on the Canadian wholesale market is discussed in the following section.

Retailing

The gasoline sold at the retail level in Canada comes from both Canadian and American sources. A modest network of pipelines moves refined product from the refineries, concentrated largely in Alberta and Ontario, to major terminals across the country. In 1992, there were twelve major terminals in Canada, with a terminal in each major city. Gasoline from Canadian and American terminals is brought to smaller centers via truck and barge.

Because of the close proximity of most Canadian cities to the United States, gasoline at American terminals is argued to be a viable alternative to purchasing wholesale gasoline from domestic sources. Several reports have found that, as a

[1] Because the remainder of the thesis will focus on the 1989-1995 period, we use 1992 throughout as a representative year.

result, wholesale prices at Canadian terminals typically follow closely the wholesale price at the nearest American wholesale supply source.[2] These studies argue that it is reasonable to take the Canadian wholesale price at major centers to be exogenous.[3]

Retail outlets in Canada are typically divided into two broad categories: refinery brand outlets, and independent outlets. A refinery brand outlet sells the gasoline under the name of a major or regional refinery. In Canada, the three major refinery brands are Esso, Shell, and Petro-Canada. As well, there are a number of regional refinery brands, that operate a large number of stations within a specific region. Finally, independent outlets sell gasoline, but not under a refinery brand name. Independent firms are often large national or regional chains, such as Mohawk and Olco. These firms purchase wholesale gasoline at major terminals from refiners, without the right to sell the gasoline under the refiner's brand name. The price of this wholesale gasoline is referred to throughout this thesis as the unbranded rack price.

Table 2.1 provides the breakdown of stations into those belonging to each of the three outlet groups, for a city from each of seven provinces, in 1992; data are obtained from Kent Marketing Limited.[4] Note that in most of these cities, the majority of stations sell gasoline bearing the brand name of one of the three

[2]See for example Hendricks(1996), Lermer(1996), and MJ Ervin and Associates(1997).

[3]One may have reservations concerning the applicability of this idea to cities in Atlantic Canada, which are largely detached from the mainland. Industry reports suggest however that the wholesale prices in these cities will be tied to the U.S. price because of accessible ports in the major cities; see for example MJ Ervin and Associates(1997).

[4]The cities reported were chosen to represent the different geographic regions.

major firms. The main exception to this is the Atlantic region, where the market is dominated by Irving Oil, a regional refiner. A key difference across cities appears to be large variation in the presence of independent stations, ranging from 14 percent in St. John to 34 percent in Winnipeg. One should note, however, that the proportion of stations operated by refiners is not in every case a good measure of market concentration. An example of this is Winnipeg. While Winnipeg has the largest percentage of stations belonging to independents, most of these stations are operated by two large firms, so that a concentration index would take on a very high value.

An important issue concerns the different contractual arrangements that exist between the individual retail outlet and the parent company.[5] The key elements of a contractual arrangement in this industry are who determines prices, and how the retailer is compensated. According to the annual survey of gasoline retailers conducted by Octane Magazine, 38 % of Canadian stations have prices set by the parent company. However, for our purposes, this figure is misleading for several reasons. First, whether price is set by the firm or the station seems to depend highly upon whether or not the station is located in a larger urban market. Discussion with the industry finds that in large urban markets, such as most of the markets considered in this thesis, firms will typically control price directly. This is supported by Slade (1998), who reports that 64 % of stations in the Vancouver market had prices set directly by the firm. As well, the suggestion is supported by the fact that for independent brands (brands of firms that do not own their

[5]Some academic interest is paid to the choice of contractual arrangements in this industry. See for example Slade(1998).

own refineries), which are typically more concentrated in larger urban centers, the national percentage of stations at which price is set at the firm level increases considerably. Finally, in some cases where the power to set price is given to the station, the parent frequently communicates a "suggested" price to the station. The consequences, usually in the wholesale price the refiner charges the station, of not following this suggested price can be sufficiently severe to ensure that the station will set the suggested price[6]. While further study on this question is desired, we consider this evidence sufficient to conclude that at most stations in larger urban centers, the retail price at a station selling a firm's brand name is determined by the firm.

To summarize, the following emerge as key characteristics of the Canadian retail gasoline industry. Unbranded wholesale prices can be taken as exogenous. Most cities are dominated by major brands, although the presence of regional refineries and independent brands varies considerably. Finally, for most urban markets, the retail price at individual stations can be reasonably viewed as set by the parent firm.

2.2 A Qualitative Discussion of the Behavior of Prices

In this section we discuss the qualitative features of the time series behavior of retail gasoline prices in a selection of Canadian cities, as motivation for the questions addressed in this thesis. More detailed analysis is reserved for later chapters.

[6]See Stoner(1987), especially chapter 16, for a discussion of these policies.

Weekly retail prices for 19 cities for the period from January 1989 to December, 1995 are obtained from Natural Resources Canada and from the Ontario Provincial Government. Once a week, a sample of stations in each city is surveyed, and the average price across the stations is recorded. The same stations are surveyed each week, and the number of stations sampled depends on city size. Prices obtained from Natural Resources Canada are before-tax prices, while prices obtained from the Ontario Provincial Government are after-tax. Taxes are removed from these prices using tax information provided by Natural Resources Canada. The cities included in the sample are listed in Table 2.2.

The wholesale price used in this study is the unbranded rack price. An unbranded rack price represents the price paid by a small independent for gasoline, without the right to resell it under the original brand name. Larger independents (jobbers) usually obtain discounts off the unbranded rack price[7]. The extent to which the price the jobbers actually pay follows the unbranded rack over time is not certain, but it will be assumed that it is equal to the unbranded rack price less a constant. Unbranded rack prices for major Canadian cities are obtained through the National Energy Board, and are published in the Bloomberg Oil Buyer's Guide. Since rack prices for all of the cities are not available, we use for each city the rack price at the closest rack pricing point. Rack prices for Vancouver, Calgary, Edmonton, Regina, Winnipeg, Toronto, Quebec City and Montreal are utilized. While rack prices are obtained for the Atlantic provinces, these are not used since discussions with members of the provincial governments suggest that the posted prices for these markets do not follow the actual local trading

[7]British Columbia Inquiry into Gasoline Pricing, p. 14¿

price, which is the Montreal posted price plus a transportation cost. Therefore, the Montreal price was used for these cities.

Table 2.2 shows the mean, variance, and percentage of week to week price changes that are equal to zero for the retail price in each city. Table 2.3 gives the same statistics for the rack prices. We note that while the percentage of weeks during which the retail price remains unchanged varies dramatically across cities, the same statistic for the rack prices shows considerably less variation.[8] For illustrative purposes, we now divide the sample of cities into three groups: those for which the percentage of weeks in which the retail price did not change is greater than forty percent, less than twenty five percent, and between twenty-five and forty percent. We discuss each group in turn.

Group (a): In this group we place cities in which the retail price changed in less than forty percent of the weeks in the sample. These cities include Vancouver, Regina, Winnipeg, Timmins, Thunder Bay, North Bay, Halifax, St. John's and St. John. The retail and rack prices for these cities are plotted in figures 2.1 (a)-(h). In each city, we note that, except for occasional breakdowns, the retail prices remain constant for long stretches of time, changing by large amounts in periods of one or two weeks.[9] In contrast, for much of the period, the rack price changes frequently, and by small amounts.

[8]Considering other measures of price stability, such as the percentage of weeks for which the absolute price change was less than some ad hoc threshold, or the variance of percentage price changes, yields similar conclusions.

[9]The city of Regina serves as an example of a potential concern with our measure of stability, in that in between periods of constant prices, in undergoes outbursts of severe instability. However, our measure of price stability seemed more reasonable than the alternatives.

To quantify the level of rigidity in retail prices, we count for each city in group (a) the number of spells of constant prices of different lengths; these figures are given in Table 2.4. While for each city, there are a large number of spells of one or two weeks, we also observe a large number of longer spells, including many spells of three or more months. It is this characteristic of the retail prices in this group, the existence of many long spells of constant prices, that sets these cities apart from those in groups (b) and (c).

Group (b): This group consists of cities in which the retail price remaines unchanged for less than twenty-five percent of the weeks in the sample. These cities include Toronto, London, Sudbury and Windsor. Retail and rack prices for these cities are plotted in Figure 2.2(a)-(d). Here we see that although the behavior of the rack prices is similar to that observed for the Group (a) cities, for all or most of the period, retail prices display a much higher week to week variability than the rack price. In fact, retail prices seem to follow a cycle pattern, jumping up quickly within one week, and falling over one week in Toronto, and over a longer period in the other cities.

While we reserve a thorough statistical analysis of price changes in cities with cycles for Chapter 5, a closer examination of the data highlights the key features of these cycles. Table 2.5 gives for each of these cities the distributions of the lengths of runs up and down, and Table 2.6 presents the average height of the cycle, given as the average magnitude of runs up and down.[10]Table 2.5 illustrates that in all four cities the length of a run up is typically less than the length of

[10]A run is defined as a period of time over which the price series is moving in the same direction.

a run down.[11] In all four cities, the average length of a run up was between one and two weeks. As well, we notice that in all four cases, the height of the cycle is between three and four cents. Since the average retail-rack markup in Toronto is approximately 4.7 cents per litre, every cycle period, retailers are jumping prices up by an amount close to their average markup.[12]

Although there has been no published academic study of these cycles in Canadian retail gasoline prices, their existence is acknowledged by both government and the gasoline industry. For example, the North-South Gasoline Pricing Study of the Government of Ontario(1986) describes the pattern as one in which independents consistently price below major brands, forcing major brands to respond by lowering prices to maintain market share. This continues until the price level is so low that one of the firms will relent, increasing price and giving up market share in the short term. This idea is supported by the Canadian Petroleum Products Institute's presentation to the Federal Government, in which the cycle is described as one in which firms compete for market share, until the price is low enough so that such a battle is no longer profitable.

Group (c): This group consists of all cities not in Groups (a) and (b). Retail prices for these cities are plotted in Figure 2.3(a)-(e). In some of these cities, such as Montreal, Ottawa, and Quebec, prices follow a clear cycle pattern for some

[11] For Sudbury, computations use only the period 91-95.

[12] One concern may be that these cycles are not market specific, but that increases and decreases are occurring simultaneously in all markets at once. As is suggested by the differences in cycle lengths across the four cities, this does not appear to be the case. Further examination of the timing of retail price increases and decreases suggests that the cycles can be thought of as market specific.

portion of the time period. In the other cities, the price series are characterized by constant prices interspersed with periods of high volatility, and occasional cycle periods. That is, these cities are not easily classified as having either rigid prices, or prices displaying a cycle.

We conclude this section with a brief summary. Retail gasoline prices in Canadian markets display a wide variation in volatility. In many cities, there is evidence of a strong price rigidity, with prices remaining unchanged for weeks and months at a time. In contrast, prices in other cities display a cyclical pattern, with prices increasing over one or two weeks, and falling over a longer period. This cycle does not appear to be evident in the wholesale price series. Two questions emerge from this discussion of the data. One is the question of why certain cities exhibit strong price rigidities, while others display this dramatic price cycle. The other concerns the effect of these patterns on other studies of dynamic gasoline pricing.

Table 2.1: Percentage of Stations Operated by Each Type of Firm For A Selection of Cities

Firm Type	Vancouver	Edmonton	Regina	Winnipeg	Toronto	Montreal	St. John
Major	56	50	55	53	66	47	32
Regional	23	18	22	13	10	22	54
Indep	21	32	23	34	24	31	14

Table 2.2: Retail Price Summary Statistics

City	Mean	St.Dev.	Percentage of First Differences = 0
Vancouver	30.4	3.92	48
Calgary	28.6	4.68	38
Edmonton	27.7	4.85	38
Regina	28.0	4.14	53
Winnipeg	30.0	3.76	66
Toronto	27.5	4.19	2
Windsor	27.5	3.27	15
London	27.1	4.08	3
Sudbury	30.2	4.55	25
Sault St. Marie	30.0	3.39	44
Thunder Bay	31.7	3.51	45
North Bay	30.7	3.15	44
Timmins	33.3	2.36	64
Ottawa	30.0	4.21	34
Montreal	29.9	5.27	34
Quebec City	30.4	4.69	37
St. John	32.9	2.72	66
Halifax	31.4	4.55	65
St. John's	33.6	4.10	74

Table 2.3 Rack Price Summary Statistics

City	Mean	St.Dev.	Percentage of First Differences = 0
Vancouver	24.1	3.73	48
Calgary	23.2	3.84	40
Edmonton	22.8	3.86	39
Regina	23.2	3.74	43
Winnipeg	23.7	3.67	42
Toronto	22.8	2.96	17
Ottawa	22.5	3.26	9
Montreal	21.9	3.34	9
Quebec City	22.0	3.35	14

Table 2.4: Group (a): Distribution of Constant Spells (number of weeks)

City	1-2	3-4	5-6	7-8	9+	Number of Spells
Vancouver	48	8	6	3	2	67
Regina	36	6	6	0	7	55
Winnipeg	26	13	12	3	5	60
St. John	20	6	7	5	9	47
Halifax	35	14	4	2	4	59
St. John's	18	11	6	5	7	48
S.S.Marie	35	5	6	3	3	50
North Bay	39	13	2	0	4	58
Thunder Bay	36	16	14	0	1	62
Timmins	25	7	6	3	8	51

Table 2.5: Group (a): Approximate Percentage of Runs Up and Down of Different Lengths

Length	Toronto		Windsor		London		Sudbury*	
	Up	Down	Up	Down	Up	Down	Up	Down
1	86	65	58	34	86	42	76	38
2	9	23	37	21	11	36	20	22
3	3	8	2	16	4	15	4	21
4	1	2	3	6	0	7	0	9
5	0	2	0	6	0	1	0	7
6	0	0	0	3	0	0	0	2
7	0	0	0	3	0	0	0	2
8	0	0	0	1	0	0	0	0
9	0	0	0	1	0	0	0	0
10	0	0	0	3	0	0	0	0
11	0	0	0	1	0	0	0	0
12	0	0	0	1	0	0	0	0

*Evaluated from 1991-1995

Table 2.6: Group(b): Heights of Runs Up and Down

City	Height Up		Height Down	
	Mean	St. Dev.	Mean	St. Dev.
Toronto	3.5	1.74	3.5	1.75
Windsor	3.2	2.81	3.0	3.21
London	3.8	2.00	3.8	2.16
Sudbury*	2.9	2.69	3.1	1.84

*Evaluated from 1991-1995

Figure 2.1: Retail and Rack Prices for Group (a) Cities

Figure 2.1(a): St. John Retail and Montreal Rack

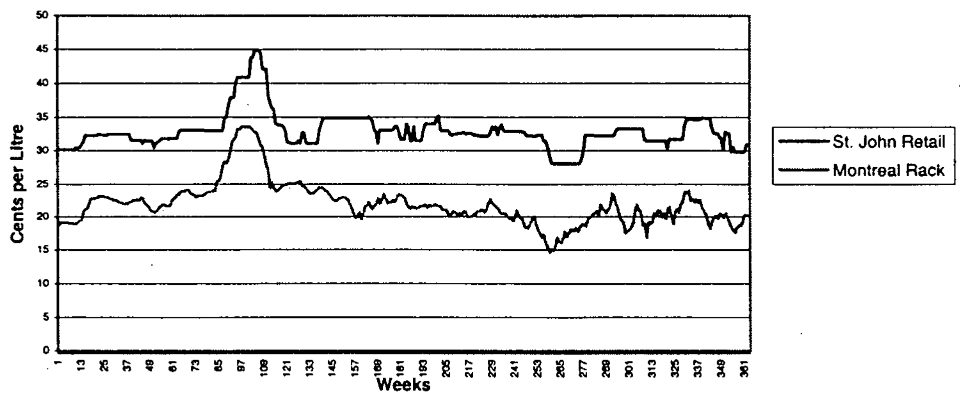


Figure 2.1(b): Halifax Retail and Montreal Rack 1989-1995

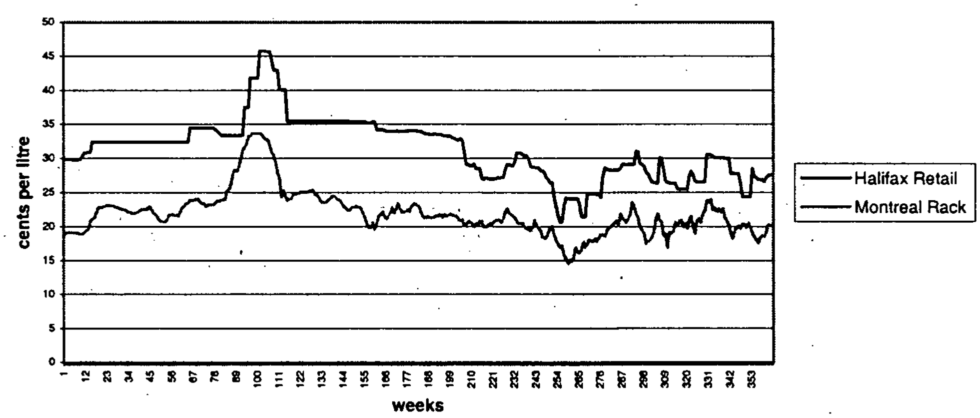


Figure 2.1(c): St. John's Retail and Montreal Rack 1989-1995

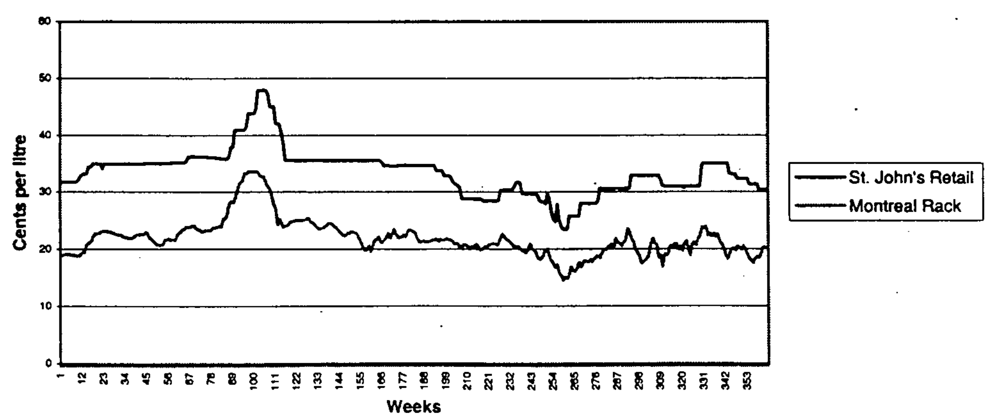


Figure 2.1(d): Timmins Retail and Toronto Rack

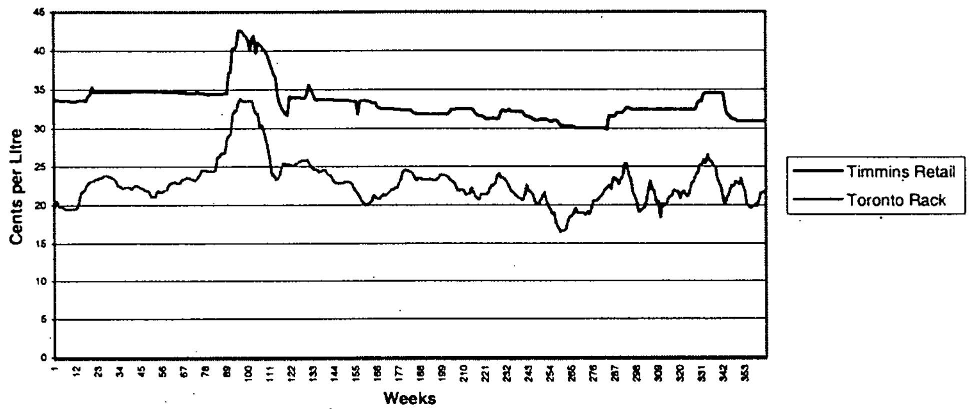


Figure 2.1(e): Thunder Bay Retail and Winnipeg Rack

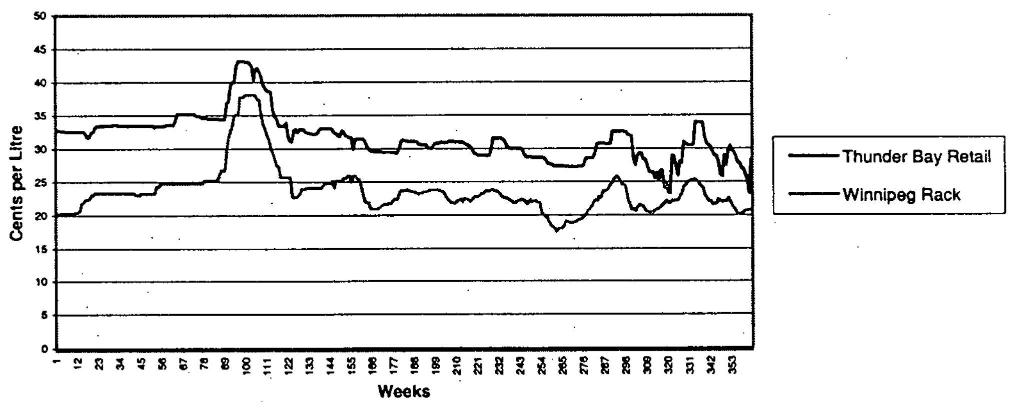


Figure 2.1(f): Sault St. Marie Retail and Toronto Rack

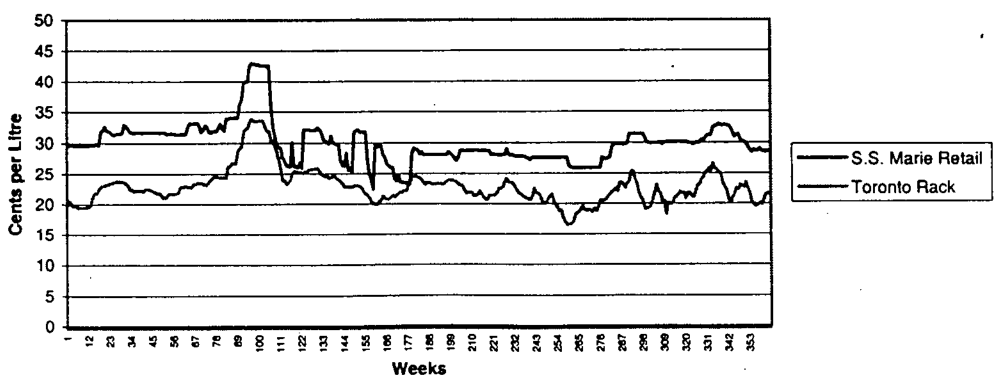


Figure 2.1(g): North Bay Retail and Toronto Rack

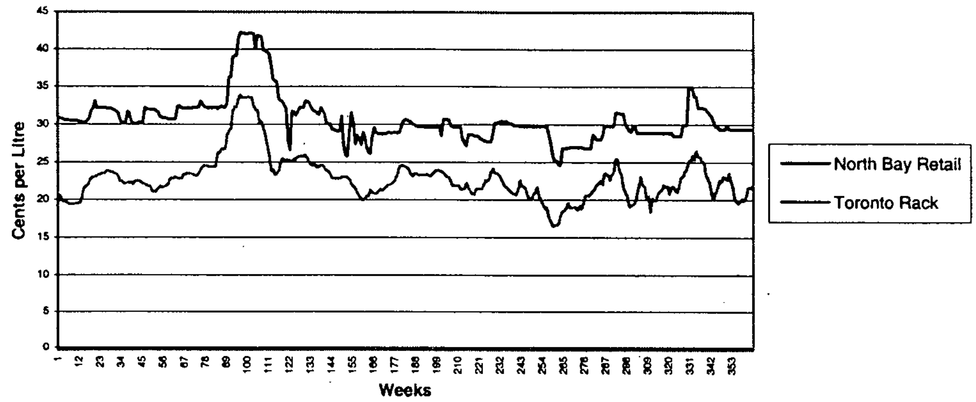


Figure 2.1(h):Winnipeg Retail and Rack Prices: 1989-1995

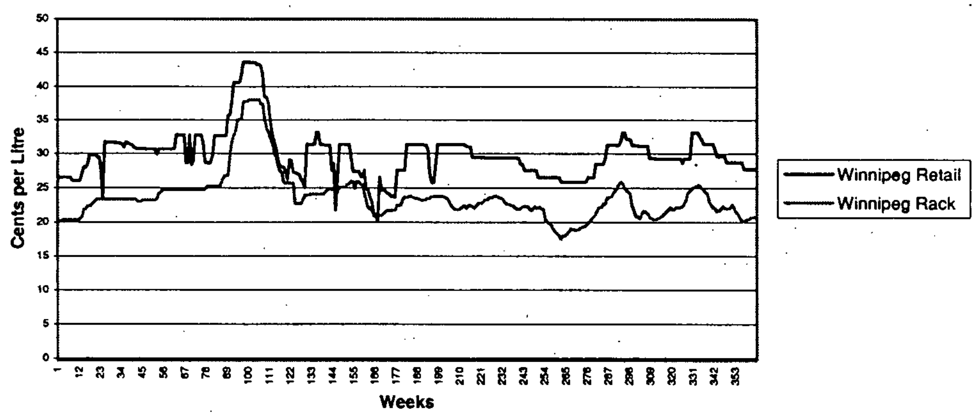


Figure 2.1(i): Vancouver Retail and Rack: 1989-1995

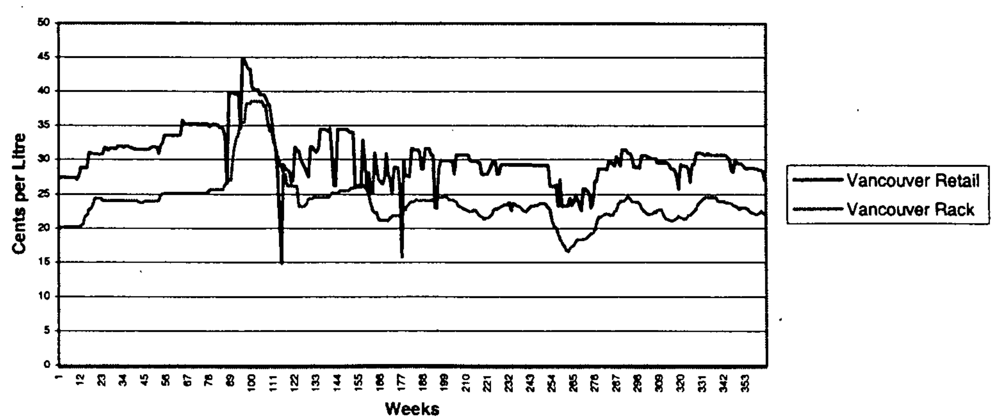


Figure 2.1(j): Regina Retail and Rack Prices: 1989-1995

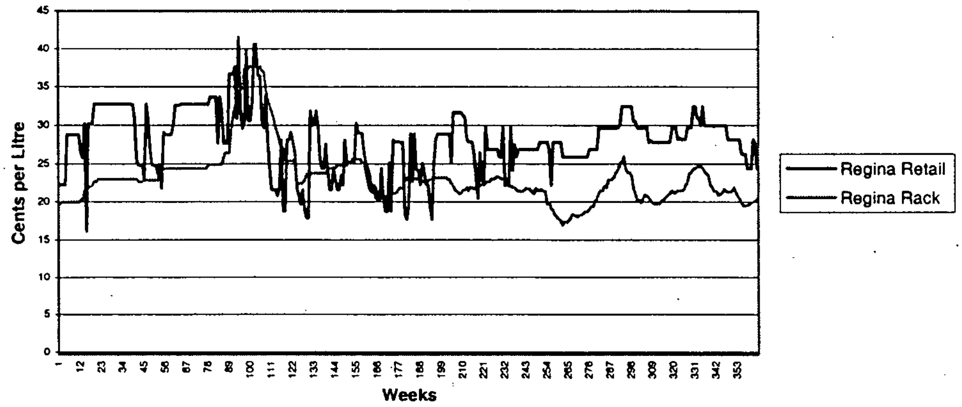


Figure 2.2: Retail and Rack Prices for Group (b) Cities

Figure 2.2(a):Toronto Retail and Rack 1989-1995

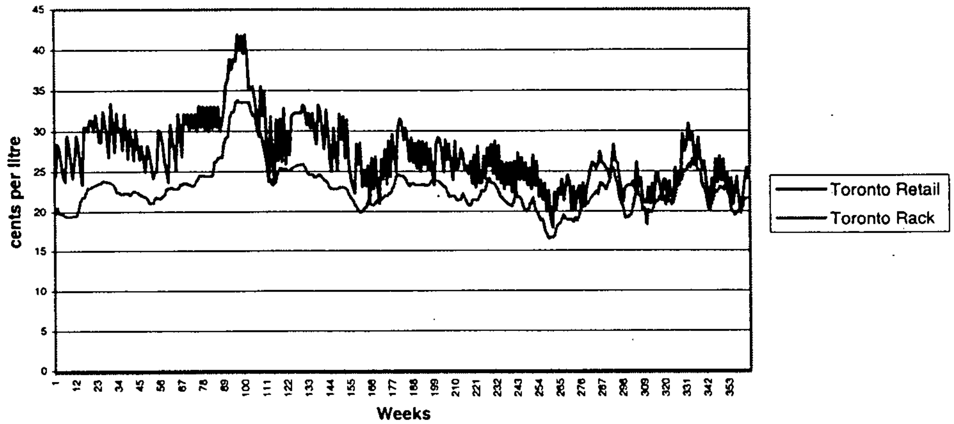


Figure 2.2(b):Windsor Retail and Toronto Rack: 1989-1995

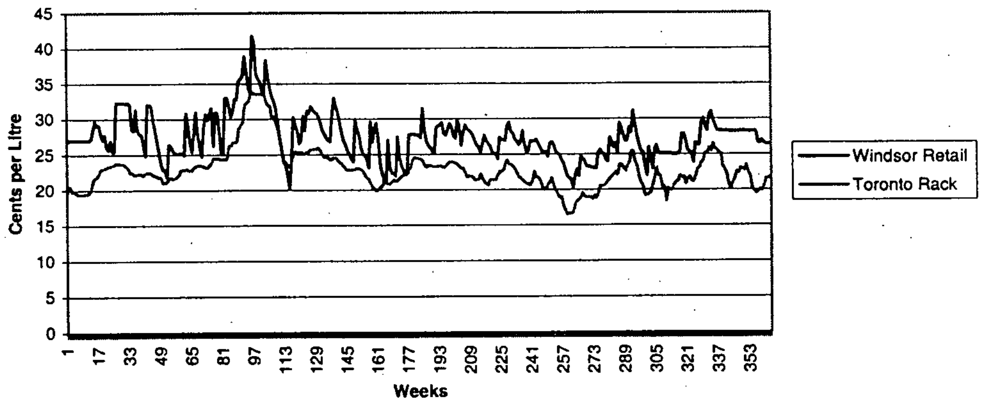


Figure 2.2(c): London Retail and Toronto Rack: 1989-1995

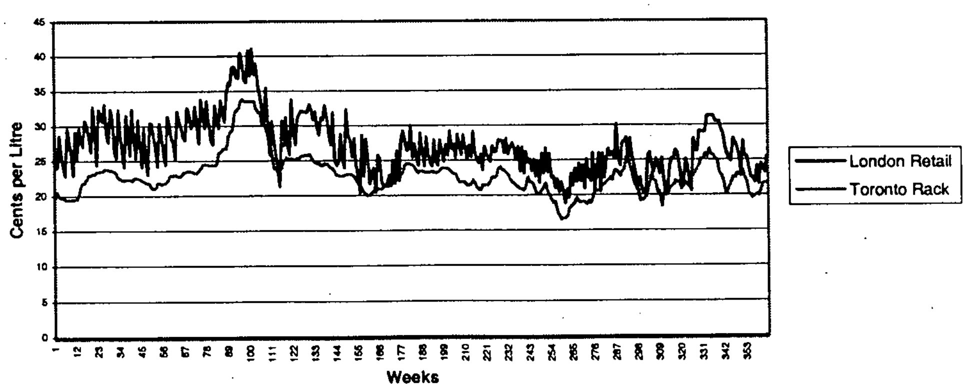


Figure 2.2(d): Sudbury Retail and Toronto Rack: 1989-1995

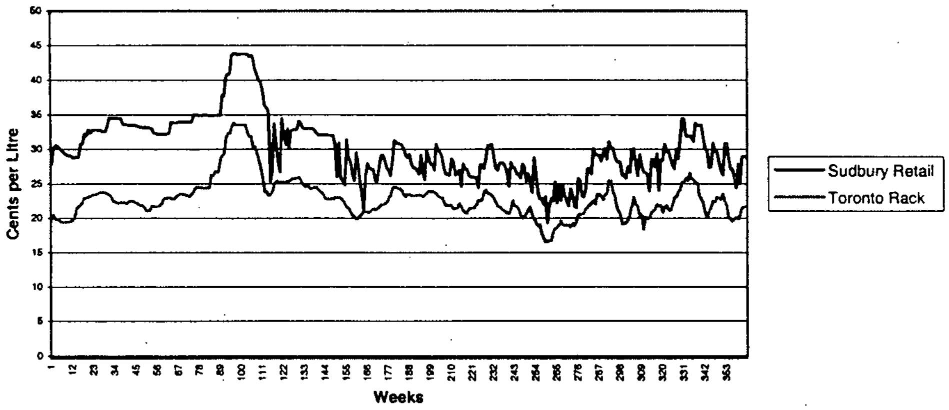
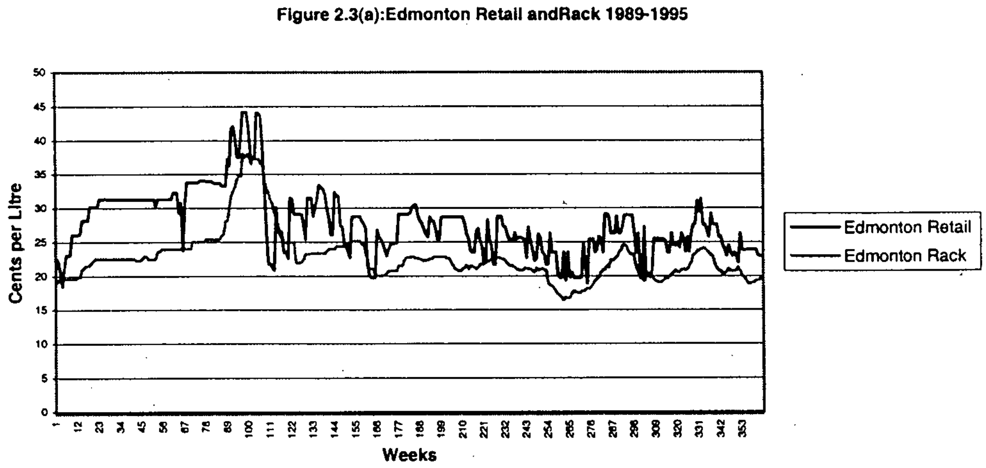
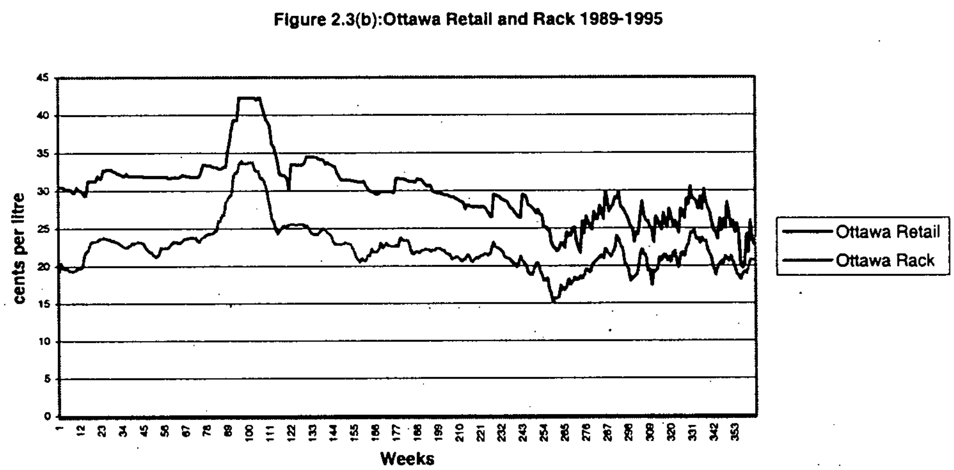


Figure 2.3: Retail and Rack Prices for Group (c) Cities





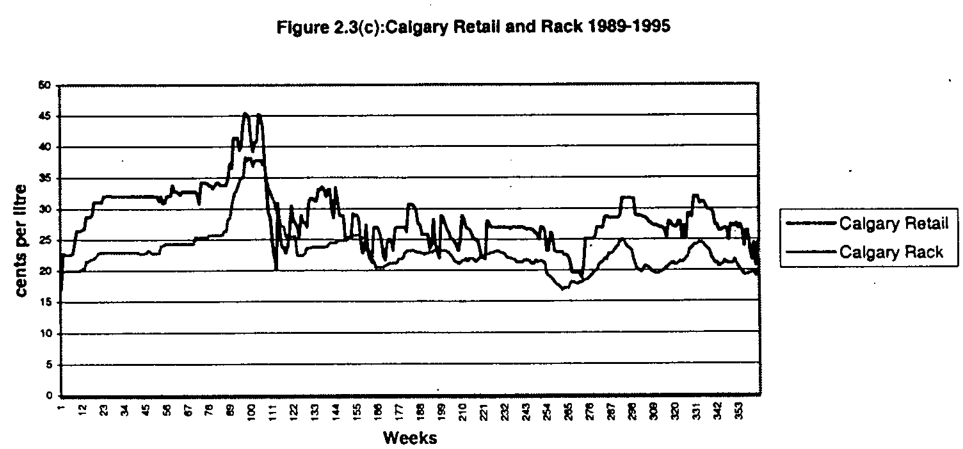


Figure 2.3(d): Montreal Retail and Rack 1989-1995

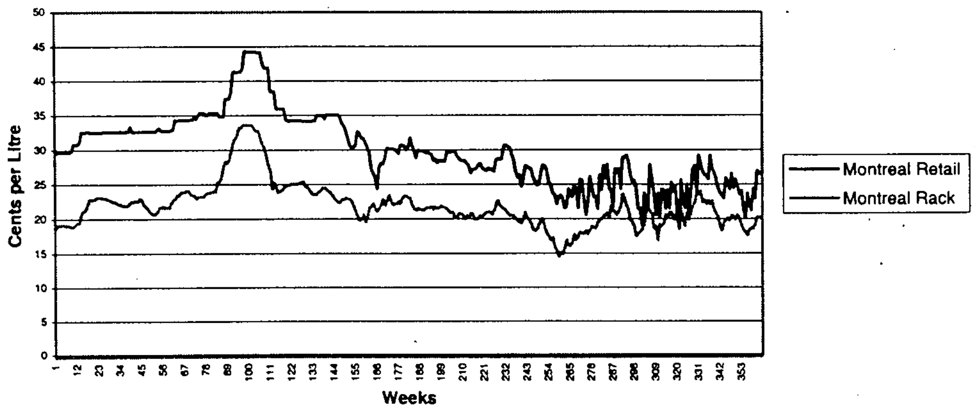
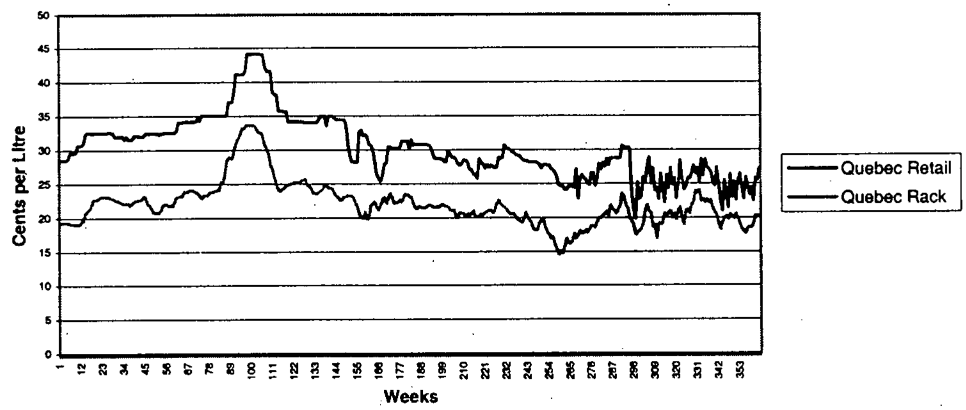


Figure 2.3(e):Quebec City Retail and Rack 1989-1995



Chapter 3

A Review of The Relevant Literature

This thesis contributes both to the empirical literature on gasoline price dynamics, and the theoretical literature on retail price cycles. Therefore, to place this thesis in its proper context, we need to consider the current state of both literatures. In the first section of this chapter, we provide a discussion of the body of empirical work considering price dynamics in retail gasoline markets, breaking this literature into two broad categories: those papers that attempt to evaluate the performance of specific theoretical models, and those that attempt to identify regularities in the behavior of retail gasoline prices, without reference to a specific theoretical model. We follow this with a brief discussion in the second section of models that provide a potential explanation for retail price cycles.

3.1 Empirical Studies of Retail Gasoline Dynamics

Often, empirical studies in industrial organization focus on specific products and markets, and occasionally, such focus becomes a subliterature in itself. This preoccupation with certain products can stem from a particular interest in these products, the availability of useful data, or features of the product and its market that make it especially suited to academic study.

Researchers study the pricing of retail gasoline potentially for all three of these reasons. First, there is public interest in the dynamics of retail gasoline pricing, resulting in a large number of government studies and commissioned studies of retail price dynamics, which in turn generate academic interest. As well, gasoline price studies benefit from the availability of good data. Publicly available price data are often weekly or monthly and city specific. The way in which gasoline prices are posted - on twenty foot high billboards - makes the collection of higher frequency station specific data straightforward, if time-consuming. High frequency volume data or other market structure data are more difficult to obtain, and are usually either purchased from a data collection firm, or else obtained through agreements with all of the firms in the market. Finally, a feature of gasoline that makes it particularly suited for study is its simple cost structure. The short-run marginal cost of a litre of gasoline for an individual station is just the wholesale price. As well, the marginal cost of a litre of wholesale gasoline is determined mainly by the price of crude oil. This basic structure makes the study of the relationship between price and marginal cost particularly simple.

In this section, we focus our survey on studies of price dynamics. This means ignoring a number of studies examining other issues in gasoline markets. These include studies of vertical separation and contractual form (see for example Shepard(1990,1993), Slade and Pinske(1998), and Slade(1998)), competition in service time(Png and Reitman(1994)), the relationship between price level and concentration (Marvel(1978)), identification of price discrimination(Shepard (1991)) and spatial price competition (Pinske, Slade and Brett(1997) and Plummer, Haining and Sheppard (1998)). As well, we will not discuss the literature which focuses exclusively on estimating demand elasticities in gasoline markets.[1]

We divide the literature into two broad categories. In the first category, we place studies examining the question of whether retail prices respond differently to increases in input prices than to decreases. In the second category, we place other studies in which the predictions of particular dynamic models (and in particular dynamic models of oligopoly) are tested using data for different retail gasoline markets. Comments on the state of the literature in each category are also provided.

3.1.1 Studies of Asymmetry

In this section, we review the literature looking for asymmetries in the response of retail gasoline prices to costs. In general, this literature attempts to determine whether retail prices respond differently to input price increases than to decreases.

[1]For a good discussion of the cumulative results in this literature, see Espey(1996).

This issue is typically not motivated by theory, but rather by public concern.[2] In recent years, many governments have investigated allegations by the public that retail gasoline prices respond more quickly to input price increases than to decreases, and that this behavior is evidence of collusion between companies. In their study of German gasoline prices in the 1970's and 1980's, Kirchgassner and Kubler(1992) argue that belief in such behavior is so common as to be considered part of modern economic folklore. There is, however, no formal model that currently exists that yields this prediction. As a result, the approach of this literature is to search for statistical support of the public allegation of asymmetry, and to speculate about the theoretical models that would generate such a pattern.

While these studies address similar questions, they employ different methodologies. In one of the first studies, Bacon(1991) considers the response of retail gasoline prices in London, England to the Rotterdam wholesale price. The model Bacon estimates is a quadratic partial adjustment model, written as

$$r_t = r_{t-1} + \alpha(A + Bt + c_{t-1} - r_{t-1})^2 + \beta(A + Bt + c_{t-1} - r_{t-1}), \tag{3.1}$$

where r_t is the retail price at time t, and c_t is the wholesale price in pounds at time t.[3]

This equation has the following interpretation. The long run equilibrium retail price is a function of the wholesale price and the exchange rate. The equation

[2]Speculation on theoretical models that might be consistent with such an asymmetry is given in, for example, Borenstein, Cameron and Gilbert (1997).

[3]Bacon also estimates a more general model, in which the total response to a change in the wholesale price is not required to be one hundred percent. For reasons of simplicity, we discuss only the restricted model.

given above specifies the manner in which the retail price converges to the long run equilibrium price, given that it is initially different from the equilibrium price. If $\alpha = 0$ then the model is the standard linear partial adjustment model, with a symmetric response to wholesale price increases and decreases. If $\alpha > 0$, then prices respond more quickly to cost increases than decreases, whereas if $\alpha < 0$ then prices respond more quickly to decreases than increases.

The data set used consists of biweekly data on retail prices net of tax and Rotterdam wholesale prices (converted to pounds), for the period from June 15 1982 to January 19 1990[4]. Bacon finds that α and β are both significant and positive, indicating that prices do in fact respond more quickly to cost increases than decreases. Specifically, he finds that for cost changes of 1 pence/litre, the mean lag of the price rise is 8.75 weeks for a rise in product prices and 9.54 weeks for a fall in product prices. The partial adjustment approach is also followed by Norman and Shin(1991) who study American prices prior to 1990. In this case, the authors find no evidence of asymmetry.

An alternative approach is provided in Borenstein *et al*(1997). The authors note that Bacon's specification puts considerable structure on the response mechanism. In particular, the response to a discrepancy between the previous price and the long-run optimum is independent of the time since the input price change. As well, the quadratic functional form imposes a specific shape to the asymmetry that has no theoretical or anecdotal motivation. Therefore, the authors consider

[4]The wholesale price the author actually uses is the lagged Rotterdam price divided by the twice lagged exchange rate. This choice of lag structure is the result of experimentation, with no immediate intuition.

as an alternative the following error correction model:

$$\Delta r_t = \theta(r_{t-1} - (\alpha_0 + \alpha_1 t + \alpha_2 c_{t-1})) + \sum_{i=0}^{n}(\beta_i^+ \Delta c_{t-i}^+ + \beta_i^- \Delta c_{t-i}^-) + \sum_{i=1}^{n}(\gamma_i^+ \Delta r_{t-i}^+ + \gamma_{t-i}^- \Delta r_{t-i}^-), \tag{3.2}$$

where $\Delta c_t^- = min\{\Delta c_t, 0\}$, $\Delta c_t^+ = max\{\Delta c_t, 0\}$ and Δr_t^- and Δr_t^+ are similarly defined. This model assumes that the current change in the retail price is a function of the distance between the previous period retail price and the long run equilibrium price level, as well as past changes in the retail price and the input price. The authors allow past retail or input price changes to have different effects depending on whether the change in question is positive or negative. Under the assumption of symmetry, $\beta_i^+ = \beta_i^-$ and $\gamma_i^+ = \gamma_i^-$ for all i.

The authors use this model to study the relationship between retail, terminal, and crude oil prices for the eastern United States for the period 1986 to 1992. The retail price used is the average of unleaded regular self-service gasoline prices in 33 United States cities east of the Rocky mountains, collected semimonthly. [5] The spot prices are prices for delivery to New York. The terminal prices are branded, and are averages of prices in the same 33 cities. The crude oil price used is the daily spot market price of West Texas Intermediate crude oil.

The authors find that retail prices do respond asymmetrically to crude oil prices. By their results, over 50 percent of the total response to a positive shock

[5] The retail series used by Borenstein *et al* is actually quite complicated. All but one of the cities were surveyed once a month, in either the first of second survey of the month. The authors construct a series of semi-monthly observations by using the average in each survey, and then include in their regressions dummies to indicate whether the observation was from the first or second survey of the month.

occurs in the first two weeks, while there is virtually no response in the first two weeks to a cost decrease. In both cases, it takes over 10 weeks for most of the response to take place. By repeating the above analysis at different points along the distribution chain, the authors find that the adjustment of spot gasoline markets to changes in crude oil prices appears to be responsible for some of the asymmetry. As well, asymmetry is identified in the response of retail prices to terminal prices, although this asymmetry accounts for less than half of the total.

This econometric model, or variations of it, has been applied to numerous data sets for various countries, with differing results. Table 3.1 provides results and key characteristics of some of these studies. The studies differ mainly by the level of aggregation (country-wide or city-wide), the frequency of data(weekly, monthly, or semi-monthly) and whether an error correction term and lagged own price changes are included. There appears to be no consistent evidence for or against asymmetry, and in one case (Kirchgassner and Kubler(1994)) the asymmetry identified is the opposite of what is expected, with prices responding more quickly to cost decreases than to increases. With the exception of Bacon, the only common feature of the studies that identify the expected asymmetry, regardless of whether it is statistically significant, is that they are all of U.S. prices. This may suggest that United States prices behave in a way that is fundamentally different from prices in other countries.

One curious result is that while Borenstein *et al* identify an asymmetry between American rack prices and crude oil prices, Hendricks does not find any asymmetry between rack and crude oil prices. This is especially curious since the rack prices Hendricks uses are in fact U.S. prices, for Buffalo, N.Y. and Anacortes, Washington. A possible explanation for this inconsistency is that the data set Hendricks employs begins after the Persian Gulf War, while the data set from Borenstein *et al* includes the war period. An informal visual inspection of the time series data suggests that the war period represents a large proportion of the variation in prices over the period. The asymmetry Borenstein *et al* find is possibly being identified largely from the response to the Gulf War.

Recently, the literature has begun to consider other forms of asymmetry. Godby *et al*, in their study of weekly Canadian retail and crude oil prices for the period 1990 to 1996, employ a threshold regression model to test whether retail prices respond differently to past crude oil price changes when the accumulated past crude oil price change is above or below an estimable threshold. The model the authors estimate is given by the following two equations:

$$\Delta r_{t,i} = \alpha_0 + \alpha_1 z_{t-1,i} + \sum_{k=0}^{n} \beta_k \Delta c_{t-k,i} + \epsilon_{i,t}, q_i < \phi, \tag{3.3}$$

and

$$\Delta r_{t,i} = \delta_0 + \delta_1 z_{t-1,i} + \sum_{k=0}^{n} \gamma_k \Delta c_{t-k,i} + \epsilon_{i,t}, q_i \geq \phi, \tag{3.4}$$

where i indicates the city, t measures time, and z is the error correction term. The authors set the lag length to 8 weeks.

Several definitions of the threshold variable were considered. The authors report results where $q_i = \Delta c_{t-k,i}$, for $i = 1, ..., 4$. As well, results are reported for a specification in which the threshold q_i is set equal to the average crude oil price change over the previous 8 weeks. In this case, the question is not whether the responses to positive changes and negative changes are different, but whether the responses are different during periods when crude oil prices are increasing or decreasing on average. The authors test the null hypothesis that all of the coefficients in the first regression equations are equal to the corresponding coefficient in the second regression equation. The number of instances in which the null hypothesis is rejected is small, and the authors conclude that there is no evidence of asymmetry in Canadian markets.

To conclude this section, we make some general comments about the asymmetry literature. This literature represents an attempt to statistically support or refute public allegations of a particular pricing pattern. There are two main problems underlying this approach. The first is that, as is occasionally acknowledged, there could be several models that might yield such an asymmetry, and little attempt is made to identify the correct theoretical model. When an asymmetry is identified, therefore, there is no obvious interpretation. A similar concern is that without a theoretical model, it is not clear precisely what asymmetry should be tested. As we have seen, the literature has modeled the asymmetry in more than one way, and it is not immediately obvious which way is preferable. Reference to a theoretical model would likely give more precise predictions about the time series behavior of retail prices, allowing us to distinguish between theories.

3.1.2 Other Studies

In this subsection we survey the recent empirical work concerning retail gasoline price dynamics, which focuses on questions other than that of asymmetric response. These studies typically identify one or more predictions of a particular theory or set of theories, and test these predictions empirically. Since these papers are often unrelated, we will discuss each separately, and in chronological order.[6] Comments on the state of the literature are then provided.

Marvel (1976)

In an early study, Marvel(1976) uses an unusual data set on gasoline prices in the United States to investigate the effect of imperfect information on the volatility of prices over time. The hypothesis the author tests is that prices will vary most at stations catering to well informed buyers and that stations serving poorly informed customers will have relatively higher prices. The argument is that changing price will act as a signal of market change, and induce new search, which will result mainly in a loss of customers. For low priced stations, new search will likely not result in a loss of customers, but rather a gain. Similarly, the author expects to see more variable prices in cities with better informed customers.

The price data the author uses consist of observations on the maximum and minimum pump price in a given period and a given city, for 23 cities over the 1964 to 1971 period. For 10 cities the observations are monthly, and for the rest only quarterly observations were available. As measures of price volatility of the high and low prices in each market, Marvel considers both the variance of the first differences of price, and also the variance of the first difference of the log of price.

The author tests the predictions in two ways. First, he establishes that the high price in a market is more stable over time than the low price, as predicted. Secondly, he regresses the price volatility of a market on proxies for the amount

[6]It is worth noting that because this literature is relatively recent, with much of the work being done during the last decade, some of the papers are currently available only in working paper form.

of information acquired by the population (the level of search), as well as other market characteristics. To proxy for the level of information acquired by consumers the author uses average gasoline purchases per car, median family income, median schooling. The intuition is that consumers will engage in a more extensive search when gasoline purchases are large, and when the opportunity cost of the search time is low (which is expected to correspond to a low annual income). As well, the author supposes that a more highly educated population engages in a more efficient search.

The author finds, as expected, that the variability of both high and low prices depend on a common set of exogenous influences, which vary with the amount of information consumers choose to obtain. In particular, the low price in the market is more volatile in markets in which search is expected to be more beneficial and less costly to the consumer. These variables, however, have no statistically significant effect on the volatility of the high price.

Slade(1987, 1992)

In these papers, the author makes use of a unique data set of daily firm level price and quantity information on gasoline stations in Vancouver, for a specific price war that took place during the summer of 1983, to examine dynamic price setting behavior. While the author has used this data set in a number of papers focusing on different issues within industrial organization, we will focus on two papers which specifically discuss price dynamics.

In Slade(1987) the author compares observed payoffs and estimated strategies to the strategies and payoffs that would obtain under different oligopoly models. To achieve these objectives, the author formulates a system of equations to be estimated, consisting of demand equations and response equations, in which a firm's daily price change is modeled as a function of the current and previous price changes of its rivals. This specification allows the author to test several distinct hypotheses. First, the author can test whether the observed response behavior conforms to the Bertrand-Nash equilibrium behavior, or to the best-response functions. Secondly, the model allows the author to test whether firms appear to respond to contemporaneous price changes by their rivals, or whether the firms respond to lagged price changes. Finally, the author can compare the profits earned by the actual strategies with those that would be earned under monopoly, Bertrand-Nash equilibrium, and other alternatives.

This analysis yields several conclusions. First, the author can reject that the strategies played are the Bertrand-Nash strategies, as well as the one-shot best responses. The author can also reject that firms only appear to respond to contemporaneous rival price changes, suggesting a model in which firms use intertemporal reaction functions. Finally, an examination of profits yields that the single period profits earned from the estimated strategies are greater than that from the Bertrand-Nash strategies, but less than one-half of the monopoly profits.

In Slade(1992), the author attempts to determine which dynamic model of tacit collusion best describes behavior in the gasoline industry. The econometric model employed is based upon an N-firm dynamic differentiated product model of price setting. The author uses this model to justify an econometric model consisting of a system of demand functions for each firm, and a corresponding set of linear reaction functions for each firm, in which a firm's change in price is a linear function of the previous change in price of all other firms. In addition, the author allows the coefficients in the reaction function to vary over time. For tractability, the author assumes two distinct strategic groups (majors and independents) and restricts parameters in the transition equations (reaction functions) to be the same over all firms in each group.

The following are the author's main conclusions:

- Firms respond asymmetrically to price increases and decreases initiated by their rivals. Rival responses to cuts by independents are stronger than responses to increases. However, rival responses to increases by majors are stronger. The result for independents supports the usual kinked-demand theory. The second result, however, is likely indicative of the different roles played by majors and independents; that is, price wars are always ended by one of the majors restoring high prices.

- The results are consistent with the notion that rather simple strategies capture the essence of station behavior. Stochastic intertemporal reaction functions that are piecewise-linear in previous period prices seem to provide reasonable approximations to station behaviour.

- The collusive price before the war was higher than the collusive price after the war, which is consistent with the war being caused by a demand shift.

Castanias and Johnson (1993)

In this note, the authors discuss the qualitative features of the time path of retail prices in Los Angeles for the 1967-1972 period, and discuss which models predict price movements that most closely resemble what is observed. The authors note that prices in Los Angeles over the given period followed a cycle, jumping up quickly over a single week, and falling more gradually. This cycle existed in retail prices despite a wholesale price that was essentially constant over the period. They discuss the time series behavior predicted by a number of dynamic models from industrial organization, including Green and Porter(1984), Rotemberg and Saloner (1986), and Maskin and Tirole(1988). The authors observe that the observed price cycle resembles the Edgeworth cycle equilibrium constructed by Maskin and Tirole (1988). In this article, no attempt is made to explain why the cycles are observed or to test the theoretical model more rigorously.[7]

Borenstein and Shepard (1996)

In this article, the authors attempt to take the models of Rotemberg and Saloner(1996) and Haltiwanger and Harrington(1991) to available data on U.S. retail gasoline prices. Rotemberg and Saloner find that highest sustainable collusive price is lower when current demand (cost) is higher (lower) than expected future demand (cost). As current demand increases relative to future demand, the gains to deviating, which depend on current demand, increase. Haltiwanger and Harrington (1991) extend this reasoning to show that, under a deterministic demand or cost cycle, the highest sustainable collusive price is lowest when demand is decreasing or cost is increasing. The purpose of this paper is to test the prediction that current price-cost margins will respond positively to expected future demand and negatively to expected future cost.

The data set used in this project consists of data on retail and wholesale prices for gasoline, as well as gasoline consumption. The sample under consideration

[7]The behavior of retail prices in this region is also noted in Comanor and Sweeny (1995), who discuss the pattern in the context of a model with loyal and non-loyal consumers.

consists of monthly observations on 43 cities for the period 1986 to 1991 inclusive.

The first model the authors estimate consists of a regression of the margin on current and expected net volume (i.e. the daily average volume in each state for each month, divided by the state mean over the sample period), the current and expected terminal price, and the change in the terminal price. The regression equation is given by:

$$MARGIN_{it} = \alpha_1 NV_{it} + \alpha_2 ENV_{it+1} + \alpha_3 TP_{it} + \alpha_4 ETP_{it+1} + \alpha_5 \Delta TP_{it} + \epsilon_{it}, \tag{3.5}$$

where $MARGIN_{it}$ equals the difference between retail and wholesale prices, and NV and ENV are the actual and expected net volume, and TP and ETP are the actual and expected terminal prices.[8] A second specification is intended to allow for a more complex lag structure in the response of gasoline prices. In this case, an error correction model is estimated, in which the margin is a linear function of current and predicted volume, a number of lags of the change in terminal prices and retail prices (with different coefficients depending upon whether the change in question was positive or negative), and an error correction term.

The authors find that in both specifications, the coefficient on expected volume is significant and positive, while the coefficient on the expected terminal price is negative in both cases, but only significant in the second specification. The authors conclude that the evidence supports the predictions of Rotemberg and Saloner (1986) and Haltiwanger and Harrington (1991).

Borenstein and Shepard (1997)

[8]Expectations are computed as the predicted level obtained from an autoregression estimated using the entire sample.

In this study, the authors develop a simple model of a gasoline market that supposes a costly adjustment of production. The authors then test two of the predictions of this theory, using two different data sets.

In the first section, the authors develop a simple dynamic model in which firms face quadratic costs to adjusting production, and can respond to cost shocks through production decisions but also through the levels of inventory holdings. The authors consider two different market structures: perfect competition, and monopoly. The model yields two predictions. First, provided that the marginal benefit of inventories is decreasing in inventory holdings, the response of price to cost shocks will be gradual and not immediate. Secondly, the response path of prices to cost shocks will be different in the case of perfect competition than in the monopoly case. However, the model does not predict which market structure will exhibit the faster response.

The authors go on to test the prediction that the rate of passthrough of cost changes into price will differ depending upon market structure using weekly data on branded and unbranded rack prices for 188 wholesale terminals in the eastern United States, for the period January 1, 1986 to November 20, 1992. In particular, the authors use time series methods to estimate the rate of pass-through of changes in the crude oil price into the rack price for each terminal. They then examine whether this passthrough rate is dependent upon market structure, by estimating models in which the passthrough rate in a market is a function of variables expected to contribute to market structure. The authors find that in markets which are expected to be more highly concentrated, wholesale prices respond more slowly to cost than in less concentrated markets.

Asplund, Eriksson and Friberg (1997)

In this study, the authors examine the response of retail gasoline prices to cost changes, using daily prices for a single Swedish retail chain, for the period from January 1980 to December 1996.

The model the authors use is a simple (S,s) monopoly model. It is assumed that a monopolist faces a fixed cost to adjusting its price. In equilibrium, the monopolist changes price only when its current price is sufficiently far from its prefered price, given current cost. When the firm adjusts its price, it sets it at the current optimal price. This implies that whether or not the firm adjusts its price in the current period will depend upon how far costs have changed since the last retail price adjustment. The authors acknowledge that their model is restrictive in several ways (most notably in that the Swedish gasoline market is not a monopoly), but argue that the notion that the firm will adjust its price only once the distance between the current and ideal prices is above some threshold is expected to generalize.[9]

The authors estimate a three equation system. The first equation is a condition determining whether a negative price change, no price change, or a positive price change is observed at time t. As well, there are equations determining the magnitude of positive and negative price changes that would be observed if a change was initiated. All three equations are assumed to be a function of the total change in the rack price since the previous retail price adjustment, as well as other variables.

[9]Models of costly price adjustment in an oligopoly framework do exist. See, for example, Slade(1998) for an analysis of a model with monopolistic competition and costly price adjustment.

The authors estimate this model following a Heckman two-stage procedure. In the first stage, an ordered probit is estimated, in which the dependent variable is equal to 1 if the price increased, 0 if it remained constant, and -1 if the price decreased. The second stage involves running separate linear regressions of positive (negative) price changes on the independent variables and the Inverse Mill's Ratio obtained from the probit equation.

The authors draw the following conclusions. First, they find, that a price adjustment is more likely to take place the further input costs have drifted since the last adjustment, and that the magnitudes of price changes in both directions are increasing in the magnitudes of the accumulated rack price change since the last retail price adjustment. However, there is evidence that the data can not be explained purely by fixed adjustment costs. In particular, there are a number of occasions in which the price change was of the opposite sign of the change in input costs, and the explanatory power of the equations is low.

To conclude this section, we make some general comments about the state of this literature. The papers discussed above find evidence in support of a wide range of different dynamic models. In fact, there appears to be no consensus on an appropriate model for analyzing the retail gasoline industry. While it is indeed possible that different models apply to different markets, one would ideally like a general model that nests markets exhibiting different forms of behavior.

Two directions of research are thus suggested. The first is empirical; since specific predictions of a wide variety of models seem to be supported in the data, the need now is for empirical work that attempts to differentiate between alternative theoretical frameworks. This is the direction taken in Slade (1987, 1992), and would likely be useful in other settings.

The second potential direction for research is theoretical. There is a need for theoretical models that can nest the diverse behavior observed in separate markets. For example, while Castanias and Johnson observe a very particular price cycle on the U.S. west coast, the Swedish prices studied by Asplund *et al* exhibit a very strong rigidity. We therefore desire theoretical work that can predict both of these patterns and provide an explanation of why the different types of behavior are observed depending on underlying market characteristics.

3.2 Theoretical Models of Retail Price Cycles

In this section we briefly discuss various models that seek to provide a theoretical explanation for price cycles. We conclude with a discussion of the features of the model developed in Maskin and Tirole(1988) which make it particularly well suited for studying the retail gasoline market.

One of the earliest models developed to explain price cycles is the cobweb model[10]. The simplest version of the cobweb model considers a single market, in which there is a one period lag in supply. It is also assumed that suppliers have naive expectations about price in the following period. In such a model, the existence of a constant (locally stable) equilibrium price requires supply to be sufficiently inelastic relative to demand. If such a condition is not met, then the two remaining possibilities are that the price converges to a two period sym-

[10]See Ezekiel (1938) for an account of the beginnings of the literature. More recent examinations of the cobweb model can be found in, for example, Hommes(1994) and Mackey (1988)

metric cycle, or that the price cycle explodes. Extensions of the simple model include supposing the supply lag is price dependent, and introducing inventories and stochastic terms. Cobweb models are typically used to explain cycles in commodity prices. For example, as Day(1994) writes "It has generally been assumed that the cycle of hog prices is one of four years and that it constitutes an almost perfect illustration of the cobweb model."

Aguirregabiria(1998) offers another potential explanation of retail price cycles, using a model of monopolistic competition in which retailers face a lump sum cost to placing orders to replenish inventories. This model generates optimal price setting rules in which a firm's price slowly increases and then decreases in a single period. As inventories decrease, the probability of running out of stock increases, decreasing price elasticity, and as a result firms increase price. Once inventories have dropped to a critical level, an order for new supplies is placed. Simultaneously, because the probability of a stock-out is lessened, price is immediately decreased.

Other models of price cycles concentrate on markets for durable goods. Conlisk, Gerstner and Sobel(1984) develop of model of a monopolist selling a perfectly durable good. The authors suppose that consumers can be divided into two groups: consumers with a high reservation price, and consumers with a low reservation price. The authors construct an equilibrium in which the monopolist periodically lowers price to sell to consumers with a low reservation price. After these consumers have been supplied, the monopolist increases price, and then gradually decreases it, until again the low reservation consumers will purchase. It is worth noting that the cyclical pattern of prices in this model seems to rely on the existence of a monopoly. In Sobel(1984), the author generalizes the model to include multiple firms, and finds that the cycle is replaced with equilibria in which a constant high price is maintained, interrupted by randomly-timed sales, in which a firm suddenly drops its price to serve the low valuation customers. In another model of a durable good, Fishman and Ferschman(1992) use search costs to generate a price cycle. In this model, consumers face a cost for each price quote they obtain in a given period. There is assumed to be a continuum of both consumers and firms. Firms set high prices and consumers defer purchasing until the accumulated demand is large enough to induce a sales period, in which firms choose randomly selected low prices, and all consumers are served.

The final story of price cycles we consider originated with Edgeworth(1897). The Edgeworth cycle arose from an attempt to break the result of marginal cost pricing in a Bertand duopoly. It was noted that if firms were assumed to have capacity constraints, then marginal cost pricing could not be sustained in equilibrium, and it was speculated that the observed equilibrium path would be a cycle, in which firms priced beneath each other, stealing each others market, until price fell sufficiently low to make setting a high price optimal.

A formal dynamic analysis of this idea, however, did not arise until Maskin and Tirole (1988). In this infinite horizon model in discrete time, two firms are assumed to compete in price. In contrast to the more traditional assumption that the two firms set prices simultaneously, it is assumed that the two firms alternate setting price, each taking its opponent's price as given when setting its own price. In this model, the authors are able to generate two types of equilibria. In the first, the equilibrium price path converges to a constant price, in which the two firms always match each other's price, sharing the market evenly in every period. In the other typical equilibrium, firms alternate setting a price just below their opponent, thus stealing the entire market in the current period, until the price is forced down to a point where it is more profitable to initiate a new cycle.[11]

In the remainder of this thesis, we use an extended version of the Maskin and Tirole model as our general framework. This model is deemed the most appropriate for several reasons. First, the Maskin and Tirole model is able to explain the asymmetric nature of the cycle, with prices rising suddenly and declining gradually. The cobweb model and inventory model in particular give very different predictions for the shape of the price cycle. Secondly, the model does not rely on unrealistic assumptions of the nature of the product (such as the durable good assumption), and market structure. Sobel(1984), for example, obtains a price cycle only in the case of a monopoly, which is clearly inappropriate. Finally, the intuition behind the cycle in the Maskin and Tirole model corresponds closely with that obtained from government and industry sources for the Canadian price cycles; prices fall as firms battle over short term market share, and then increase once the diminished prices make such a battle no longer profitable.[12]

[11]Although this model did not incorporate capacity constraints, the cycle equilibrium they construct is referred to as an Edgeworth cycle, likely because, as in Edgeworth's speculation, firms continually steal each other's market, by pricing below their opponent.

[12]Several other studies have used this general framework to study more specific questions. See for example Davis (1994) for an analysis of entry and exit in such a setting, and Wallner(1998) for an analysis of the impact of the infinite horizon assumption. In a related model, Fishman(1994) examines the impact of customer loyalty in a duopoly model with price inertia.

3.3 Conclusions

In this chapter, we survey the current state of the literature focusing on price dynamics in retail gasoline markets, as well as the theoretical literature on price cycles. The empirical literature on gasoline price dynamics is broken down into two sub-literatures. In the first we place studies which use time series methods to discuss the passthrough of cost changes into retail prices. In the second sub-literature we place those studies that attempt to test the predictions and performance of specific theoretical models using data from retail gasoline markets.

Our survey of this literature brings to light a need for both theoretical and empirical research. The fruitful direction of both theory and empirical work in this area involves developing models that can nest more than one pattern of equilibrium behavior. In both groups of literature, the need is for theoretical models that can explain different types of behavior in different markets as the result of underlying market or demand characteristics. In the case of the asymmetry literature, we desire a theory that would predict when asymmetry will and will not be present. More generally, we require theoretical models that can provide potential explanations for the wide range of pricing behavior observed in the data for different markets. Similarly, a promising direction for future empirical work is through the use of general models that allow us to say not only that the predictions of a certain model are supported by a given data set, but that the model under consideration seems to be a better explanation than the likely alternatives.

In our survey of theoretical models of price cycles, it is argued that the Maskin and Tirole (1988) alternating move duopoly model is the most reasonable framework for further analysis. This conclusion is based upon its ability to describe the shape of the cycles observed in gasoline prices, the fact that its assumptions seem reasonable in the context of retail gasoline, and that the underlying intuition it provides is consistent with that described by industry and government.

Table 3.1: Additional Studies following Borenstein *et al*'s Methodology or a Variation

Study	**Level of Aggregation**	**Frequency of Data**	**Relationships Studied**	**Approximate Time Period**	**Error Correction Term, Lagged Own Price Changes**	**Evidence of Asymmetry (+,-)**
Karrenbrock (1991)	U.S.	Monthly	Retail-Wholesale	1980's	N,N	Yes (a faster response to increases)
Hendricks(1996)	Canadian Cities (5)	Weekly	Retail-Crude, Retail-Wholesale, Wholesale-Crude	1992-1996	Y,Y	No
Duffy-Deno(1997)	Salt Lake City	Weekly	Retail-Wholesale	1989-1993	N,N	Yes(a faster Response to Increases)
Kirchgassner and Kubler(1992)	Germany	Monthly	Retail-Rotterdam Wholesale, Domestic Wholesale-Rotterdam Wholesale	1970's,1980's	Y,Y	1970's: Yes(a faster response to Decreases) 1980's: No
Balke et al (1998)	U.S.	Weekly	All pairings of Retail, Wholesale, Spot and Crude Oil	1987-1996	Y,Y	Yes in all relationships except retail-spot (a faster response to increases)

Chapter 4

The Role of Small Firms

4.1 Introduction

Recently in Canada, increasing political and public attention has been paid to the pricing of retail gasoline. One particular concern has been the existence of two different patterns in the time series of retail prices for different cities. As an example of one predominant pattern, Figure 4.1 plots the weekly after-tax retail price and wholesale price for the city of Winnipeg, for the 1990-1995 period. [1] As can be seen in the figure, the retail price in Winnipeg tends to follow the wholesale price in the long run. However, while for most of the sample the wholesale price changes by a small amount every week, the retail price often remains constant for several months at a time, responding to movements in the wholesale price in

[1]The retail price plotted is an average of the prices posted by a number of retail stations, recorded on the same morning each week. The wholesale price plotted is the unbranded rack price for Winnipeg, which is the price paid by small independent retailers, without the right to resell it under the original brand name.

large jumps occurring over a single week. This retail price behavior is typical of the behavior of retail prices in Atlantic Canada, Northern Ontario, and Western Canada.

In contrast, retail prices in other cities exhibit much more volatility than their associated wholesale prices. Figure 4.2 plots the retail price for the city of Windsor, along with the wholesale price in Toronto, for the 1990-1995 period. The wholesale price in Toronto behaves similarly to the Winnipeg wholesale price, changing by small amounts almost every week. On the other hand, the retail price in Windsor typically increases in one week by more than a fifth of the average price level, decreasing more gradually. Similar cyclical patterns can be observed in the time series of retail prices for Montreal, Quebec, and cities in southern Ontario.

Two distinct issues arise from the presence of these patterns. The first concerns identifying a model that can explain both the cycles observed in prices in central Canada, and the rigid prices observed elsewhere. A second question that arises is whether an explanation can be found for why certain cities exhibit cycles and others exhibit constant prices. In this study, we address both of these questions.

One model that generates both equilibria with constant prices and equilibria with price cycles similar to those observed in southern Ontario and Quebec is constructed in Maskin and Tirole (1988). In their model, two firms are assumed to set prices in alternating periods taking the price set by the other firm in the previous period as given. When a firm prices below its rival it steals the entire market for that period, but when it matches its rival's price, the firms share the market evenly. Restricting strategies to depend upon only the most recent price set, the authors show that two different types of equilibria exist. In one, each firm matches its rival's price along the equilibrium price path. In the other, prices repeatedly cycle. Along the equilibrium price path, firms undercut each other's price until marginal cost is reached, at which point they raise price to above the monopoly price, and the cycle is repeated.

In this paper, we extend this model by assuming that when both firms set the same price they share the market according to the number of stations that each firm operates in the market. Firms are still able to steal the entire market by pricing below their opponent. However, the immediate increase in market share realized differs between the firms, with the larger firm realizing the smaller gain. Our analysis yields two key theoretical results. First, we demonstrate that cycle equilibria exist even when one firm operates many more stations than the other firm. However, the structure of the cycle depends upon the relative sizes of the two firms. When the two firms are of similar size, a cycle equilibrium in which each firm undercuts the other can be constructed. When one firm is much larger than the other, the smaller firm still undercuts its rival over the downward portion of the cycle, but the larger firm matches the price of the smaller firm. Secondly, we establish that an equilibrium with a constant equilibrium price path cannot exist if one firm is much larger than the other. Further, we show that even for small differences in firm size, the prices that can be supported in constant price equilibria is restricted to lie strictly below the monopoly price, provided consumers are sufficiently sensitive to price differences across firms. This result is in contrast to Maskin and Tirole(1988), in which equilibria exist which support constant prices both well above and below the monopoly price.

Finally, our model suggests an explanation for the difference in price behavior across cities. In particular, it predicts that cycle equilibria are more likely to exist in cities with firms that have a small fraction of the number of stations but who have the capacity to serve a large proportion of the market. Government studies of pricing behavior in Ontario and Quebec support this prediction by associating the presence of small marketers with the existence of cycles. The "North South Gasoline Pricing Study" conducted by the Ontario Ministry of Energy discusses the relationship between major brands and jobbers, which are small chains of stations "supplied by distributors who buy gasoline from a variety of refiners":

> A recognized major brand can usually command a premium in the marketplace. To earn a share of the market, a jobber outlet must charge a somewhat lower price. If he is too successful in capturing market share, the branded dealers will react by lowering their prices. Sometimes an uneasy truce results...Sometimes, and often in certain areas, the struggle between the two kinds of dealer is constant. This behavior occurs much more frequently in southern Ontario where jobber outlets are more common.

The prediction that constant prices are associated with markets with few small firms is examined using a data set of retail prices for 19 Canadian cities for a 6-year period. In support of the theory, we find a significant positive relationship between concentration and price rigidity.

One possible concern is that there may exist other models capable of generating both cycles and constant prices, which would yield different predictions. These could include standard supergame models, cobweb models, durable good models, and inventory models. Maskin and Tirole was deemed the most appropriate model with which to work for several reasons. First, the cycle generated by the Maskin and Tirole model conforms in its shape with the observed cycle, with short periods of increasing prices followed by long periods of decreasing prices. This feature is not found in most other models of price cycles. Secondly, while still an abstraction, the Maskin and Tirole model is less offensive in its assumptions concerning market structure and the nature of the good than alternative models.

Finally, the intuition concerning the behavior of firms in Maskin and Tirole conforms closely with that given by government and industry. However, other models are less likely to provide a plausible explanation of what it is that drives price cycles. In a repeated game model, for example, one could construct an equilibrium in which firms simultaneously changed price every week to follow a cycle pattern, using punishment strategies to ensure firms would not deviate. However, this approach would fail to provide any intuition behind the behavior of firms, since the cycle would be one of many price paths supported by the threat of punishment.

The remainder of this paper will proceed as follows. Section 2 describes the model. Section 3 discusses the different types of equilibria that can occur, in the context of an example and Section 4 generalizes results described in Section 3. In Section 5 we take the prediction of the theory to the available data. Section 6 concludes.

4.2 The Model

This section describes a simple dynamic model of the retail gasoline market. There are two firms, indexed by $i = B, S$. Firm B is the major supplier in the market, operating the majority of stations in the area. Firm S is an independent retailer with a relatively small number of stations. More precisely, let θ_i be the fraction of the total number of stations in the market that are operated by firm i. Then θ_B is assumed to be strictly greater than θ_S. The number of stations operated by each firm is assumed to be exogenous and fixed throughout out analysis.

The discrete-time model has periods indexed by $t = 0, 1, 2, \ldots$. Firm B chooses a single price for all its stations in every even period, committing to that price for two periods. Firm S sets its price in odd periods. The discreteness reflects short run commitments to price on the part of firms. For retail gasoline markets, the length of the commitment is approximately one day, which implies a period length of one half of a day.

The prices that each firm can choose are restricted to a finite grid. The size of the grid is k. That is, if firm i sets a price p, and firm j wants to undercut p, the highest price it can set is $p - k$. This will ensure that an optimal undercut exists. Maskin and Tirole (1988) interpret k as the smallest amount by which a firm can undercut, which is at least one tenth of a cent. Here, we view k as the minimum amount by which a firm must undercut an opponent's price to steal its customers. For the retail gasoline market, a reasonable value for k is approximately a half of a cent. [2]

[2]Evidence provided in the North-South Retail Gasoline Pricing Study suggests that in regions where the density of stations is high, consumers will respond to price differences of less than half

The demand for gasoline in each period is the same, and given by $D(p)$ where p is the lowest price. Let c denote the cost to firm i of purchasing and transporting a litre of gasoline from the wholesale market. This is assumed to be exogenous to the firms. Define the industry operating profit function, assumed to be strictly concave, as

$$\Pi(p) := (p - c)D(p). \tag{4.1}$$

Let p^i denote firm i's pump price of a litre of gasoline in period t. Then firm i's operating profits in period t can be expressed as follows:

$$\pi^i(p^i, p^j) = \begin{cases} \Pi(p^i) & \text{if } p^i < p^j; \\ \theta_i \Pi(p^i) & \text{if } p^j = p^i; \\ 0 & \text{if } p^i > p^j, \end{cases}$$

The operating profit equation states that firm i services all of the market if its price on the grid is less than firm j's price, θ_i of the market if its price is equal to firm j's price, and none of the market otherwise[3].

The above profit equation implicitly assumes that customers can switch suppliers costlessly. If switching stations is costly to consumers, the market share of a firm in a period when prices are the same depends on how the market was

of a cent.

[3]One issue that remains unaddressed is the effect of capacity constraints. In this paper, we are interested in the role of small firms that can potentially serve the entire market, so that this ommission does not seem troublesome. Intuitively, we do not expect small capacity constraints to have a large impact on results, since what seems to be important is the relative difference in market share between matching an opponent's price and undercutting. This may, however, be a subject of future research.

divided in the previous period. For example, if firm i prices above firm j in period t and firm j matches i's price in period $t+1$, then firm j would service the entire market in period $t+1$. By contrast, equation (4.2) implies that in period $t+1$ consumers allocate themselves between the two firms according to the number of stations that each has in the market.

Our model resembles the brand loyalty model of response in Eaton and Enger(1990). Each firm has a fraction of the market that is loyal to it as long as prices do not differ substantially. In particular, firm i can steal firm j's loyal customers by undercutting firm j's price by at least k, and firm j can recapture its loyal customers by pricing within k of firm i's price. In our model, firm j is forced to at least match firm i's price to recapture its loyal customers since prices are restricted to a grid of size k. This assumption has substantive implications for the characterization of focal price equilibria, which is the focus of Eaton and Enger(1990), but is less important for the characterization of cycle equilibria, which is the primary focus of this paper.

In principle, a firm's choice in any period in which it must make a pricing decision could depend upon the entire history of play up to that period. However, we will restrict a firm's choice to depending only upon payoff relevant variables. In this case there is only one such variable, the price set by its opponent in the previous period. Therefore firm i's strategy in the game is a dynamic reaction function $R^i(p)$, which specifies the price that i sets when j's price is p. Maskin and Tirole (1997) provide a thorough discussion of the advantages of working with Markov strategies. In particular, any equilibrium within the space of Markov strategies is also an equilibrium in the space of history dependent strategies.

Each firm chooses its strategy to maximize the present discounted value of profits. The discount rate is the same for both firms and denoted by δ. Since a period is approximately one half of a day, δ is assumed to be close to one.

The solution concept is Markov Perfect equilibria(MPE). Given a pair of strategies R^B, R^S, define $V_i(p)$ as the present discounted profit of firm i in any period in which it has to choose a price, firm j's price is p and all subsequent play is determined by R^B, R^S. Similarly, define $W_i(p)$ as firm i's present discounted profit in any period in which firm j chooses price and i's price is p. Then a pair of strategies (R^B, R^S) is an MPE if for all prices $p*$, and $i = B, S$,

$$V_i(p*) = max_p[\pi^i(p, p*) + \delta W_i(p)], \tag{4.2}$$

and

$$W_i(p*) = E_p[\pi^i(p*, p) + \delta V_i(p)], \tag{4.3}$$

where $R^i(p*)$ is the optimal choice of p and the expectation is taken with respect to the distribution of $R^j(p*)$.

4.3 An Example

Maskin and Tirole have shown that the model described in the previous section typically possesses two types of equilibria: focal price and cycle equilibria. They define a focal price equilibrium as one in which the market price converges to a focal price and stays there forever, and an (Edgeworth) cycle equilibrium as one in which this does not happen. [4] This section discusses these equilibria, within

[4] See Maskin and Tirole(1988) for formal definitions.

the context of an example, for values of θ_B between one half and one.

Demand for gasoline is given by $D(p) = 1 - p$ and marginal cost is assumed to be zero. The grid size k is set at $1/10$. The prices on the grid, $0, 1/10, ...1$, are labeled $p_0, ...p_{10}$. Table 4.1 lists the prices on the grid, and the corresponding total profits multiplied by one hundred. The discount factor δ is 0.99.

The first type of equilibrium we consider is the price cycle. Suppose first that $\theta_B = 1/2$. Column (a) of Table 4.1 gives equilibrium strategies for each firm [5]. Verification that these strategies constitute an equilibrium is tedious and follows the approach given in Maskin and Tirole (1988).

In this equilibrium, each firm plays the same strategy. Once one of the firms, firm i, jumps up to p_8, firm j undercuts it in the next period by a grid size and services the entire market at price p_7. Similarly, in the following period, firm i sets its price at p_6 and steals the entire market. Each firm undercuts the other by setting the highest price at which it can claim the entire market, until one of the firms sets its price equal to p_2. At this point, it is more profitable for the other firm to respond by dropping its price down to p_0, in an attempt to restart the cycle. Note that in this equilibrium, each firm prefers the other to jump up first. Therefore, at marginal cost, in this case zero, the firms play a war of attrition, each setting its price equal to p_8 with probability approximately equal to 0.853, and continuing to price at zero with probability 0.147. Because this cycle has the property that each firm responds to prices in the set $\{p_3, p_8\}$ by undercutting by $1/10$ and stealing the entire market, it will be referred to as the undercut-undercut cycle.

[5]This equilibrium is of the class constructed by Maskin and Tirole for the general case.

Recall that we claimed the undercut-undercut cycle to be an equilibrium when each firm has exactly one half of the total number of stations in the market. In fact, it is also an equilibrium when θ_B is much larger. To see this, note that equilibrium profits are independent of θ_i. A firm's best response to a price $p > p_0$, whether on or off the equilibrium path, is either to set a higher price or undercut p. The only case where a firm's best response is to match the price set by its opponent and share the market is when $p = p_0$. In this case the one period profits are zero, independent of θ_i. This property would seem to suggest that the undercut-undercut cycle is an equilibrium for all values of θ_B. However, this turns out not to be true. As θ_B gets large, the temptation increases for firm B to respond to a price in $\{p_3, ..., p_8\}$ by matching instead of undercutting, since by matching firm B services the majority of the market, and also postpones the lower prices at the bottom of the cycle.

It can be verified that the undercut-undercut cycle is an equilibrium when the larger firm's share of the total number of stations in the market is less than or equal to 0.76. If $\theta_B = .77$, the undercut-undercut cycle is not an equilibrium because the large firm prefers to match p_3 and service only a fraction of the market rather than serve the entire market at p_2.

However, a cycle equilibria can also be constructed when θ_B is larger than 0.76. Column (b) of Table 4.1 presents equilibrium strategies for $\theta_B = .77$. This cycle differs from the undercut-undercut cycle in the following ways. First, the optimal response of a firm to a price in the downward portion of the cycle is not necessarily to undercut by a grid size. When firm B observes p_3, it matches firm S's price and services only proportion θ_B of the market. As a result, if firm S observes

p_5, it will undercut *two* grid sizes to p_3, service the entire market for the current period and proportion θ_S of the market in the subsequent period. If it were to set p_4 it would service the entire market for one period, but none of the market in the following period, which would yield lower profits. Given this response, if firm B observes price p_6, it will be better off matching, since doing so results in firm S responding with p_5, whereas undercutting to p_5 would cause firm S to respond with p_3. Secondly, firm B responds to p_0 by jumping up with probability one. When firm S sets price p_0 the best it can hope for is that firm B responds by setting p_7, which is what it would earn by jumping up itself. Therefore, firm S can set p_0 in equilibrium only if firm B responds by jumping up immediately.

As θ_B continues to increase, the number of prices to which firm B responds by matching instead of undercutting increases. Columns (c) of Table 4.1 presents an equilibrium for the case of $\theta_B = .84$. In column (c), firm B matches all prices less than or equal to p_6, but undercuts p_7. When firm S sets a price above the monopoly price, firm B realizes two distinct gains from undercutting. First, it steals the entire market. Secondly, at the lower price, total one period profits are higher since the price is nearer to the monopoly price. By matching, however, firm B delays the periods of low profits at the bottom of the cycle. At p_6 the delay effect dominates, but at p_7 the market stealing effect dominates.

Finally, for θ_B very near 1, a cycle equilibrium can be constructed in which firm B matches everywhere along the downward portion of the cycle. This equilibrium for the case $\theta_B = 0.94$ is presented in column (d). Firm B matches every price in the set $\{p_3, ..., p_8\}$, and firm S responds to every price in this set by undercutting. An equilibrium of this type will be referred to in the rest of this paper as a match-undercut cycle.

Allowing θ_B to range from 0.77 to 1, (recomputing the mixing probabilities), one finds that the cycles in cases (b),(c), and (d) are in fact equilibria for the approximate ranges $[.77, .79], [.84, .92]$ and $[.94, 1]$ respectively. These intervals do not overlap, which suggests uniqueness. For example, suppose $\theta_B \leq 0.76$, and firm B observes price p_3. Then firm B prefers to respond to p_3 by undercutting by a grid size instead of matching, given that it will take one of these two actions. Note that this result only depends on firm S responding with a price strictly below firm B's response, so that if $\theta_B \leq 0.76$, there cannot exist a cycle equilibrium in which firm B responds to p_3 by matching and firm S then sets a price less than or equal to p_2. The presence of gaps for which cycle equilibria have not been constructed is due to the coarseness of the grid.

Finally, a comment should be made concerning the payoffs in a cycle equilibrium. For each firm we compute $(1-\delta)V_i(p_0)$, which is firm i's average per period operating profit (before deducting the fixed costs), given that it initially observes price p_0. These values are given in Table 4.2. The payoffs of firm B are not monotone in θ_B. For example, at $\theta_B = .77$, B's payoffs are 7.30 per period on average, which is less than what it would earn in the cycle constructed for $\theta_B = 0.76$. This non-monotonicity is due to conflicting effects. On the one hand, an increase in θ_B may cause the length of the cycle to increase since firm B matches more frequently. When the length of a cycle increases, the low profit-periods that result from the initiation of a new cycle are less frequent, which increases the average per-period payoff. On the other hand, as the number of prices that firm B matches increases, the frequency of occasions on which it does not serve the entire market increases.

For low values of θ_B only the second effect is present; the equilibrium in column (b) yields lower per period profits for firm B than the undercut-undercut cycle, since the length of the undercutting portion of the cycle is the same in each case, but in the cycle in column (b) firm B matches at certain prices. However the first effect dominates as θ_B increases , since the increase in the number of prices which firm B will match leads to a longer undercutting period.

The payoffs for firm S are also not monotone in θ_B, but for different reasons. In this case, both effects discussed above increases the payoffs to firm S. However, as θ_B increases, the amount that firm S earns in periods in which firm B matches decreases. For example, firm S earns higher payoffs when $\theta_B = 0.95$ than when $\theta_B = 0.99$, since in both cases firm B matches every price over the downward cycle, but it steals more of the market by matching in the latter case.

We turn now to the issue of the existence of focal price equilibria. Table 4.3 presents equilibrium strategies that support the monopoly price as a focal price for $\theta_B < .57$. [6] This equilibrium has a simple structure. If its opponent sets p_5 in the previous period, a firm always responds with p_5. If its opponent sets p_4, p_3, or p_2, a firm responds with p_1. If a firm observes p_1 it sets the focal price with positive probability, and matches the other firm with positive probability. Finally, if a firm observes p_0 it jumps up with probability 1, and if it observes a price above p_5, it undercuts its opponent by setting p_5.

[6] This equilibrium differs from the one Maskin and Tirole construct for the general case for a sufficiently fine grid. The coarseness of the grid in this example results in each firm playing a mixed strategy when it observes p_1. However, this would not be true with a grid size sufficiently small; in this case, a firm's response to a low price is to jump above it's opponent with probability one.

These strategies do not constitute an equilibrium when θ_B exceeds 0.57. The problem arises when firm S observes p_1. At this price, it is not possible to randomize between p_5 and p_1 so that firm B is indifferent between these options in the following period. Therefore, this focal price equilibrium is not robust to asymmetries in market share. Of course, this does not imply that there does not exist an equilibrium with p_5 as a focal price when θ_B exceeds 0.57. One may be able to support the monopoly price using other types of strategies. We address this issue in the following section.

4.4 Equilibria

In this section, we return to the general model given in section 2 and generalize two of the results that were established in the example of the previous section. Specifically, we show that the match-undercut cycle equilibrium exists for a general demand function, and we develop the necessary condition for the existence of a constant price equilibrium. Finally, we show that in the presence of asymmetry, in contrast to the findings of Maskin and Tirole for the symmetric mode, when the grid size is small any focal price must lie strictly below the monopoly price.

We begin with an analysis of cycle equilibria. Maskin and Tirole (1984) construct the following undercut-undercut equilibrium strategies for the case $\theta_B = 1/2$, for a strictly concave profit function, a sufficiently fine grid and a large discount factor:

$$R^i(p) = \begin{cases} \overline{p} & \text{for } p > \overline{p}, \\ p - k & \text{for } \overline{p} \geq p > \underline{p}, \\ c & \text{for } \underline{p} \geq p > c, \\ \left.\begin{array}{ll} c & \text{with probability } \mu(\delta) \\ \overline{p} + k & \text{with probability } (1 - \mu(\delta)) \end{array}\right\} & \text{for } p = c, \\ c & \text{for } p < c \ , \end{cases}$$

As we demonstrated for the example, because payoffs in this equilibrium never depend upon θ_B, these strategies will constitute an equilibrium as long as θ_B is small enough that firm B will not deviate by matching.

Recall from the example that as θ_B increased, cycle equilibria could be constructed in which it was optimal for firm B to match its opponent's price over certain parts of the cycle. In particular, for θ_B close to one, the cycle equilibrium had the property that firm B matched all prices over the downward portion of the cycle, while firm S undercut all prices over this range. Proposition 1 generalizes this result by constructing the match-undercut cycle equilibrium for a general demand and grid size, for the case where $\theta_B = 1$.

Proposition 1: For a small grid size the following dynamic reaction functions constitute a MPE when $\theta_B = 1$*:*

$$R^B(p) = \begin{cases} \overline{p} & \text{for } p \geq \overline{p}, \\ p & \text{for } \overline{p} > p \geq \underline{p}_B, \\ c & \text{for } \underline{p}_B > p > c, \\ \left.\begin{array}{ll} c & \text{with probability } \mu_B(\delta) \\ \overline{p}+k & \text{with probability } (1-\mu_B(\delta)) \end{array}\right\} & \text{for } p = c, \\ c & \text{for } p < c \ , \end{cases}$$

$$R^S(p) = \begin{cases} \overline{p} & \text{for } p > \overline{p}, \\ p-k & \text{for } \overline{p} \geq p > \underline{p}_S, \\ c & \text{for } \underline{p}_S \geq p > c, \\ \overline{p} & \text{for } p = c,, \\ c & \text{for } p < c \ . \end{cases}$$

The match-undercut cycle has the property that everywhere along the downward portion of the cycle, firm B prefers to match its opponent's price, while firm S undercuts by a grid size. We note that this cycle is in fact an equilibrium for θ_B less than but sufficiently near one; this finding is discussed in Appendix A.

For the case $\theta_B = 1/2$, Maskin and Tirole (1988) construct focal price equilibria for a set of focal prices containing the monopoly price and bounded away from marginal cost, and establish that a focal price must necessarily be in this range. As demonstrated for the example in the previous section, such strategies may also be equilibria for θ_B greater than one half but sufficiently small. The following proposition establishes that focal price equilibria cannot exist when θ_B is large.

Proposition 2: Suppose p^f is the focal price of a MPE, and p^m is the monopoly price.

(a) If $p^f \leq p^m$, then

$$\theta_S \geq \frac{\delta\Pi(p^f - k)}{(1+\delta+\delta^2)\Pi(p^f)}. \tag{4.4}$$

(b) If $p^f > p^m$, then

$$\theta_S \geq \frac{\delta\Pi(p^m)}{(1+\delta+\delta^2)\Pi(p^f)}. \tag{4.5}$$

In case (a), the condition ensures that firm S would not prefer to undercut by a grid size. In case (b), the condition is that firm S will not deviate by setting the monopoly price. Note that in case (a), as the grid size gets small and δ approaches one, the necessary condition becomes approximately $\theta_S \geq \frac{1}{3}$, while the condition in case (b) it approaches some number strictly greater than $\frac{1}{3}$.

This necessary condition is not sufficient and much stronger conditions would be required for sufficiency. As a first step, however, we can show that for a fine grid size, the range of prices that can potentially be focal prices when the firms differ in size is fundamentally different from the range constructed by Maskin and Tirole for equally sized firms. In particular, Proposition 3 establishes that, for a sufficiently small grid, there are no focal price equilibria with focal prices greater than or equal to the monopoly price:

Proposition 3: Suppose $\theta_B > 0.5$, and consider a price $p^f \geq p^m$ on the grid. Then for a grid size sufficiently small, p^f cannot be the focal price of an MPE.[7]

To understand the intuition of this result, consider the case where $p^f = p^m$. It can be shown that if a firm observes a price just below the monopoly price, its

[7]The proof of this proposition makes the additional assumption that the price $p < p^f$ such that $\Pi(p) = \theta_i\Pi(p^f)$ does not lie on the grid. This assumption is also made in the proof of Proposition 5 in Maskin and Tirole(1988). The assumption yields the generic case since the grid size is arbitrary and exogenous.

equilibrium response must be a lower price. However, to be optimal, this response must yield more than the firm can earn by jumping up above the opponent's price. When the focal price is equal to the monopoly price, the best possible price the firm can jump up to is the focal price itself. Therefore, the undercutting response of the firm to a deviation must yield the firm more than resetting the monopoly price.

This reasoning implies that for each firm there are prices $\underline{p}_B, \underline{p}_S$ such that *(i)* if firm i sees $\underline{p}_i + k$ it will undercut to $\underline{p}_i$ if it knows the other firm will jump up next period, and *(ii)* if firm i sees $\underline{p}_i$ it would prefer to jump up rather than undercut. It can be shown that for a sufficiently fine grid, $\underline{p}_B > \underline{p}_S$. Therefore firm B cannot respond to a deviation by undercutting, since the lowest possible price it would undercut to, assuming that firm S would jump up, is $\underline{p}_B$. But at that price firm S would still undercut. Therefore, firm B's best response must be to reset the monopoly price. As a result, S will always want to deviate from the focal price, and an equilibrium where the monopoly price is the focal price cannot exist. Proposition 3 extends this reasoning to show that as long as the two firms are of unequal size, one can always choose a grid size so that there cannot exist a focal price equilibrium in which the focal price is greater than or equal to the monopoly price.

However, this argument does not apply to focal prices strictly less than the monopoly price. The reason is that the argument discussed above relies on the existence of prices at which the optimal response is the focal price. However, when the focal price is below the monopoly price, such prices need not exist. Appendix B considers a numerical example with unit demand and asymmetric firm sizes, and constructs an focal price equilibrium in which such prices do not exist. In this equilibrium, Firm B's set of best responses to low prices includes both the monopoly (reservation) price, *and a price that is strictly above the opponent's price but strictly below the focal price.* The reason that such an example can be constructed is that when the focal price is below the monopoly price, if a firm observes a deviation its best response given that it jumps up to at least as high as the focal price, may in fact be strictly above the focal price. This increases the gains to a firm from forcing its opponent to jump up first. As a result, one can find a price $\underline{p}_B < p^f$ such that firm B will respond to prices below $\underline{p}_B$ with $\underline{p}_B$, in an attempt to force firm S to jump up to the reservation price.

The results of this section can now be summarized. A cycle equilibrium has been constructed for the extreme case of $\theta_B = 1$ for a general, strictly concave profit function, a small grid size and a discount factor near 1. This suggests that cycle equilibria are very robust to asymmetry in the number of stations. However, constant price equilibria do not exist for the entire range of θ_i. This was established in three distinct steps. First, a necessary condition was derived, which states that the large firm has to have less than approximately two thirds of the total number of stations for a focal price equilibrium to exist. Secondly, it was shown that for $\theta_B \neq \frac{1}{2}$, for a sufficiently small grid size, there is no MPE with a focal price greater than or equal to the monopoly price.

4.5 Discussion of the Data

The previous discussion shows that a constant price equilibrium can only exist in the absence of a large capacity firm with a small number of outlets. More generally, we expect to observe rigid prices only in markets with few small firms, and less stable prices in markets with many small firms. Following this suggestion, we use a panel data set for 19 Canadian cities for the period from January 2, 1990 to December 26, 1995 to look for a correlation between the presence of small firms and price stability.

In our analysis in this section, we adopt a simple atheoretic approach, because of the abstractness of the theoretical model. First, we simply examine the correlation coefficients between measures of concentration and price stability for each year in our sample. Secondly, we demonstrate that when we attempt to control for other effects suggested by the government and industry literature, this correlation persists. In particular, we suppose that price stability in city i in year t is a linear function of the concentration of the market, as well as other market and time specific characteristics. We use the estimates from the linear model to test for a significant positive relationship between price stability and concentration.

We measure price stability in city i in year t as the percentage of zero first differences in the retail price , denoted $ZEROS_{it}$[8]. A large value of $ZERO_{it}$ therefore corresponds to a small number of price changes in a particular year and city. One may instead want to use a discontinuous measure , such as a dummy variable that indicates whether prices in a particular city and year exhibit constant

[8]Other stability measures are possible. Using the average absolute weekly price change, or the fraction of changes below a threshold (e.g. one tenth of a cent) yields similar results.

prices. However, we could find no clear rule to identify a city as having constant prices. This problem arises partly since, as well as cities with very constant prices and cities with cycles, there exist some cities that have constant prices interrupted by occasional price wars. As well, the frequency of price changes can likely depend upon wholesale price movements, so that a city with a constant price equilibria would experience more price changes in periods of changing wholesale prices than in periods of stable wholesale prices. Therefore, classifying a city as experiencing stable prices if $ZERO_{it}$ is above some arbitrary threshold, seems problematic, even if a sensible threshold could be found. For these reasons, we opt to take a step back from the theory, and use a continuous measure of price stability. As a proxy for the dominance of market i by large firms in year t we use market concentration, denoted $CONC_{it}$. Thus, low concentration indicates the presence of many smaller firms, while a highly concentrated market is serviced almost exclusively by national and regional refinery brands.

Weekly retail prices for 19 cities for the period from January 10, 1990 to December 26, 1995 are obtained from Natural Resources Canada and from the Ontario Provincial Government. Once a week, a sample of stations in each city are surveyed, and the average price across the stations was recorded. The same stations are surveyed each week, and the number of stations sampled depends on city size.[9] For the prices obtained from Natural Resources Canada, before-tax

[9]The two agencies differ in the way that stations are chosen. In particular, the Ontario sample includes unbranded stations and local chains, whereas the Natural Resources Canada data does not. An informal examination of the two data sets does not reveal any systematic difference in the frequency of price changes. As well, the two data sources both collected prices for Ottawa (we use the data from Natural Resources Canada throughout our analysis). An examination of

prices are obtained. Prices obtained from the Ontario Provincial Government are after-tax. Taxes are removed from these prices using tax information provided by Natural Resources Canada. The cities included in the sample are listed in Table 4.4.[10]

Annual market structure data are obtained from Kent Marketing Ltd. and consist of the number of stations operated by each brand operating in a particular city, at some point in the year. The measure of concentration us throughout is the five firm concentration ratio, where the size of a firm is measured as the number of stations it operates. A Herfindahl index was not used due to concerns about the data for small firms. Specifically, the data is given by brand, not company. The names of second brands associated with major firms was provided by Kent Marketing. However, for small firms this information was unavailable, and proved difficult to track down via other sources due to the small size and regional nature of independents. Therefore a measure of concentration using only data for large firms was deemed more reliable.

Table 4.4 presents data on the variable $ZEROS_{it}$ for each city. The first column gives the percentage of zero first differences for the entire time period, and the next six columns give the percentages for each year. From Table 4.4, we see that

the two series for the 1990-1995 period shows no strong systematic differences in the stability of the two series. We conclude that the presence of two different data sources is not likely to affect results.

[10]Although Natural Resources Canada also provided data for the city of Charlottetown, this data was not used because of heavy retail and wholesale price regulation over the sample period. Similar regulations existed in Halifax until 1991. All results that follow remain when Halifax is dropped from the sample.

the stability of prices varies dramatically both across cities, and within cities over time. Over the entire time period, the percentage of zero first differences varies from 2 and 3 percent in Toronto and London, cities which exhibited price cycles for the entire period, to 67 and 73 percent in Winnipeg and St. John's, cities with very stable prices. As well, in many cities, the rigidity of the retail price varies strongly over time. For example, in both Ottawa and Quebec City, the percentage of first differences equal to zero drops from 69 and 73 percent respectively, to zero in both cities.

Means, minimums and maximums for concentration for each city are given in Tables 4.5. Mean concentration varies across cities within a range of approximately thirty percent. Within cities, the variation is substantially less, as there were no major market structure events that occurred during the period. Therefore, the variation in the market structure variable can be viewed as primarily cross-sectional variation.

Annual correlation coefficients between $ZEROS_{it}$ and $CONC_{it}$ for the period 1990-1995 are presented in Table 4.6. The correlations are positive for each year, and statistically different from zero at the five percent level for all years but 1990. These correlations provide initial evidence supporting the suggestion that concentration is related to price stability. In addition, the correlations are very similar in magnitude for all years but 1990, suggesting a stability in the relationship over time.

To determine whether this correlation between concentration and stability is maintained when other effects are controlled for, we conduct regression analysis using the six year panel data set.[11] In our regression analysis we control for effects as suggested by government studies of Canadian retail gasoline markets. According to government studies, the costs of operating a refinery below capacity are substantial. These costs give major brands an incentive to undercut to maintain volume in large markets, and markets near the refinery. To control for this, we use the variables POP_{it}, the population in city i in year t, and $REFINER_{it}$, which is a dummy variable which equals one if at least one refinery is located within 150 kilometres of city i in year t and zero otherwise[12]. The choice of distance was based on discussions with industry, and estimation with various distances yields similar results. POP_{it} was constructed using the 1991 and 1996 census totals, and assuming a constant annual growth.[13]

Finally, we use a measure of wholesale price volatility to control for a possible relationship between the volatility of wholesale prices and the frequency of retail price changes. The wholesale price used in this study is the unbranded rack price. A rack price represents the price paid by a small unbranded independent for gasoline, without the right to resell it under the original brand name. The large jobbers usually obtain discounts off the unbranded rack price. The extent to which the price the jobbers actually pay follows the unbranded rack over time is not

[11]Regression analysis was also performed using a cross-sectional analysis of the six-year averages. The estimated impact of concentration on stability was qualitatively similar to that found using the panel.

[12]One exception is that the Parkland Refining Ltd. refinery near Calgary is not included, because of its small size. This decision is supported by industry reports, which argue that Calgary should be viewed as having no nearby refining capacity.

[13]Models in which volume was used for market size are also estimated. Similar results are obtained as the correlation coefficient between population and volume is 0.95.

certain, but it will be assumed that the price jobber pay is equal to the unbranded rack price less a constant. Unbranded rack prices for major Canadian cities are obtained through the National Energy Board, and are published in the Bloomberg Oil Buyer's Guide. Since rack prices for all of the cities are not available, we use for each city the rack price at the closest rack pricing point. Rack prices are utilized for Vancouver, Calgary, Edmonton, Regina, Winnipeg, Toronto, Quebec City and Montreal. While rack prices are obtained for the Atlantic provinces, these are not used since as suggested by discussions with members of the provincial governments, the posted prices for these markets do not follow the actual trading price, which is the Montreal posted price plus a transportation cost. For these cities, the Montreal price was used.

Two different measures of wholesale stability are considered: the annual variance of percentage changes, and the annual standard deviation of the rack price series, denoted $STDEV_{it}$. The specification reported here use $STDEV_{it}$. However, using other measures produced similar results.

The model estimated supposes $ZEROS_{it}$ is a linear function of $CONC_{it}$, POP_{it}, $REFINER_{it}$ and $STDEV_{it}$[14] . The results of OLS estimation are given

[14] In the regression reported, we elect not to include city-specific fixed effects. This decision is based upon several factors. First is that, as previously discussed, the market concentration variation is predominantly cross-sectional, and in those cities in which there is some variation, the variation is gradual, and not sudden. That is, there are no major market-structure events off of which we would identify. Secondly, we consider the coefficient obtained without including fixed effects to be the quantity of interest, since it can be viewed as measuring the long run impact of concentration on stability. Alternatively, one could simply average the data for each city across the six years, and estimate the regression over a single cross section. This approach

in Table 7.[15] The positive relationship between concentration and stability is supported by the positive and significant coefficient on $CONC_{it}$. POP_{it} and $REFINER_{it}$ both have the expected signs, although only POP_{it} is significantly different from zero at the five percent level. The negative sign of POP_{it} supports the assertion of the government literature that firms will price more aggressively in larger markets in an attempt to keep refineries running at capacity. This suggests that demand effects may in fact be important in determining price stability, contrary to the assumptions of the theoretical model. This idea may be a valuable direction for future work.

Finally, the coefficient on $STDEV_{it}$ is not significantly different from zero, suggesting that the rigidity of retail prices is not a reflection of movements in the wholesale price. This result was maintained when the variance of percentage changes of the rack price was used as a measure of rack price volatility, either separately, or in conjunction with $STDEV_{it}$.

One possible concern with the above regression is the possibility that $CONC_{it}$ is determined endogenously. We did not feel this is a legitimate concern for two reasons. First, the concentration variable is measured in stations and not volume. Secondly, the variable consists of stations operated at any time during the year, so that exit decisions and closures would not be picked up. We did however re-estimate the model using as instruments for concentration the distance by land

yields almost identical results.

[15]A battery of heteroscedasticity tests could not reject the null of homoscedasticity. The model was also estimated using generalized least squares and assuming heteroscedasticity and first order autoregression with a parameter constant across all cross sections. Similar coefficients were obtained, and no test results were affected.

to the nearest major American wholesale terminal and the number of Canadian wholesalers with posted prices at terminals within a 200 kilometre radius. Similar results are obtained. Another possible concern is that the dependent variable is in fact restricted to lie between zero and one. In addition there exists a small mass point at zero (7 of 114 observations). Estimation of a tobit model, which allows for a mass point at zero, yielded very similar results as the linear model. Because of the exploratory nature of the analysis and the small sample size, only the results of the linear model are reported.

We conclude this section by summarizing the results. An examination of correlation coefficients for each year indicates that, in accordance with the suggestion of the theory, price stability is in part determined by downstream concentration. This conclusion is further supported by simple linear regression analysis.

4.6 Conclusions

In this paper, we extend the Maskin and Tirole (1988) alternating duopoly model to consider the case in which the firms operate different numbers of outlets in the market. We find that price cycle equilibria can be constructed for a wide range of relative firm sizes. On the other hand, the existence of constant price equilibria is much less robust to the introduction of this asymmetry. When the firms are of different sizes, the constant price that can be supported is restricted to lie below the monopoly price, in contrast with the result from Maskin and Tirole that the constant price can in fact be much larger than the monopoly price. In support of the suggestion of the theory, we find, using a panel data set of observations on

19 cities over the 1990-1995 time period, that the stability of prices is positively related to market concentration.

Several potentially fruitful directions of research exist. One shortcoming of the existing theoretical model is the assumption of a constant marginal cost. How the existence and characterization of both types of equilibria are affected by the introduction of changing costs is of interest because of the constant movement of wholesale prices. Some intuition is provided in Maskin and Tirole (1988), which discusses qualitatively the response of firms to an unanticipated permanent cost shock within a constant price equilibrium. The authors argue that the focal price observed should be the monopoly price given current cost, and that once the change occurs, firms would immediately switch to playing the strategies which support the new monopoly price. Their arguments, however, would only apply to permanent changes that occur only with a very small probability. As well, they offer no discussion of the effect of the cost change on cycle equilibria.

Secondly, the model presented here is limited in that it assumes the number of stations operated by each firm to be exogenous. Certainly, when the time period being discussed is one of days, this assumption has merit. However, an important question is whether the existence of firms of different sizes could actually result in an equilibrium in which firms choose their number of outlets. The first issue that would arise in addressing this issue is how investment in stations would be appropriately modeled. One possibility would be to suppose that firms choose the number of stations to operate each time that they set price. However, the suggestion that firms can change the number of stations they operate as frequently as they can change price is unrealistic. Therefore, a model in which firms choose their number of stations, and then play an alternating-move price setting game taking the number of stations operated by each firm as constant, may be more reasonable. This question is a likely direction of future research.

Figure 4.1: Winnipeg Before Tax Retail and Unbranded Rack Prices 1990-1995

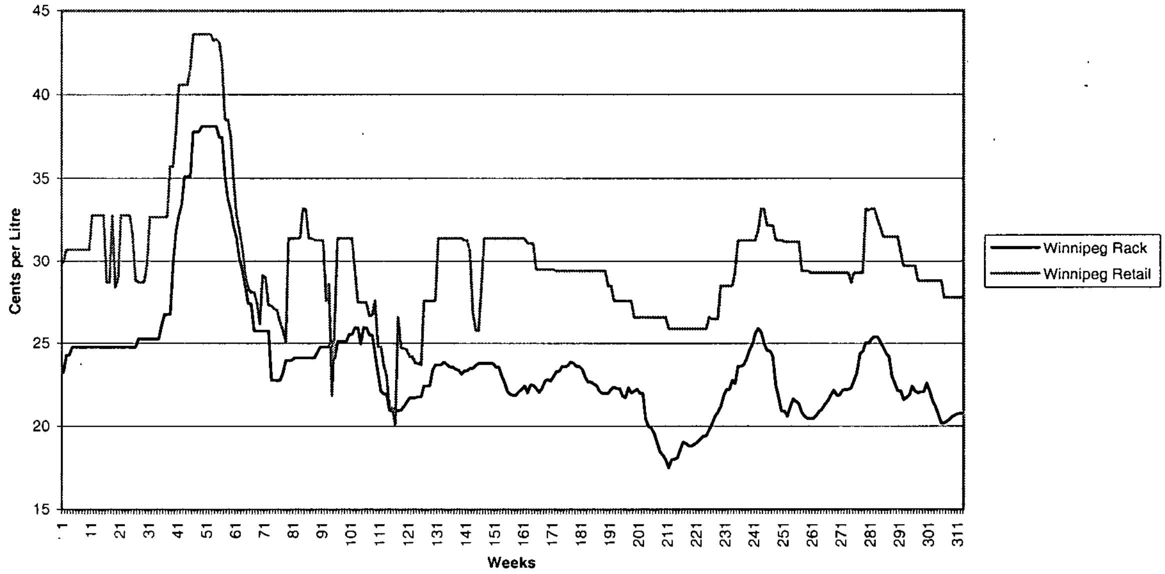


Figure 4.2: Windsor Before Tax Retail and Unbranded Rack Prices 1990-1995

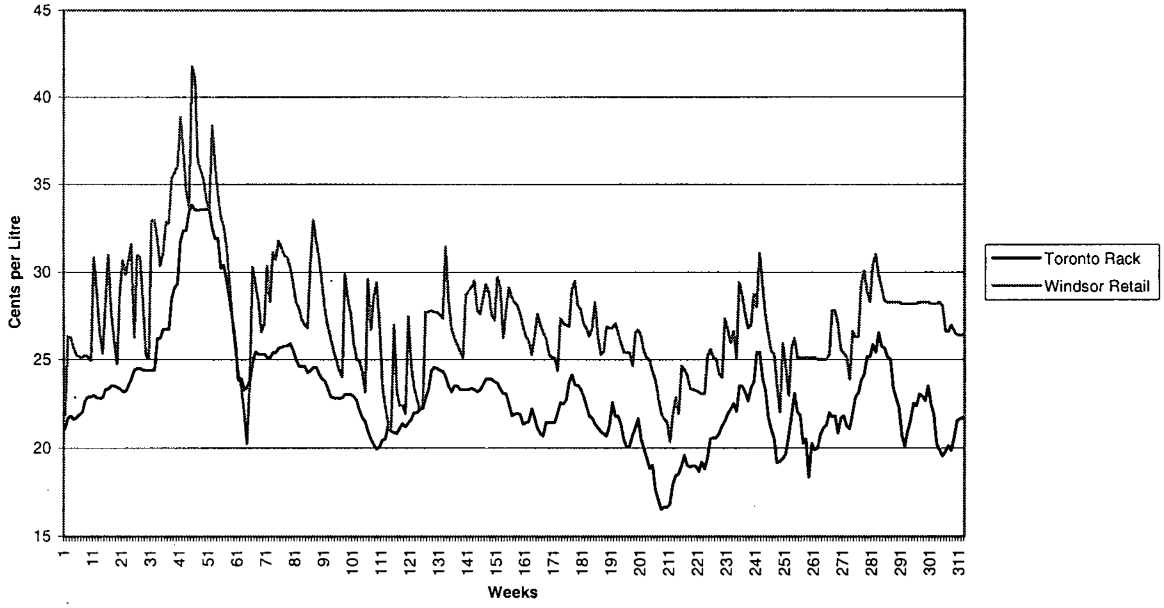


Table 4.1: Cycle Equilibria for D(p)=1-p, k=1/10

Profit	p	θ_B=0.5	θ_B=0.77		θ_B=0.84		θ_B=0.94	
		$R^i(p)$	$R^B(p)$	$R^S(p)$	$R^B(p)$	$R^S(p)$	$R^B(p)$	$R^S(p)$
0	p_{10}	p_7	p_7	p_6	p_7	p_6	p_7	p_7
9	p_9	p_7	p_7	p_6	p_7	p_6	p_7	p_7
16	p_8	p_7	p_7	p_6	p_7	p_6	p_7	p_7
21	p_7	p_6	p_6	p_6	p_6	p_6	p_7	p_6
24	p_6	p_5	p_6	p_5	p_6	p_5	p_6	p_5
25	p_5	p_4	p_4	p_3	p_5	p_4	p_5	p_4
24	p_4	p_3	p_3	p_3	p_4	p_3	p_4	p_3
21	p_3	p_2	p_3	p_0	p_3	p_0	p_3	p_0
16	p_2	p_0	p_0	p_0	p_0	p_0	p_0	p_0
9	p_1	p_0	p_0	p_0	p_0	p_0	p_0	p_0
0	p_0	p_8 w.p.=μ_a p_0 w.p.=1-μ_a	p_7	p_8 w.p.=μ_b p_0 w.p.=1-μ_b	p_7	p_8 w.p.=μ_c p_0 w.p.=1-μ_c	p_8 w.p.=μ_d p_0 w.p.=1-μ_d	p_7

Table 4.2: Cycle Equilibrium Payoffs (Approximate Values*)

θ_B	Firm B	Firm S
[0.5,0.76]	15.88	15.88
[0.77,0.79]	$5.94+11.15\theta_B$	$17.34+11.04\theta_S$
[0.84,0.92]	$18.63\theta_B$	$18.63+18.44\theta_S$
[0.94,1]	$18.97\theta_B$	$18.64+22.77\theta_S$

*Payoffs as well as interval endpoints are rounded to two decimal places

Table4. 3: Focal Price Equilibrium for the Example

Profit	p	$R^i(p)$, i=S,B
0	p_{10}	p_5
9	p_9	p_5
16	p_8	p_5
21	p_7	p_5
24	p_6	p_5
25	p_5	p_5
24	p_4	p_1
21	p_3	p_1
16	p_2	p_1
9	p_1	p_5 w.p.=(1-μ_i) p_1 w.p.=μ_i
0	p_0	p_5

where $\mu_i \cong \frac{8.91-15.75\theta_j}{8.91+15.59\theta_j}$

Table 4.4: Percentage of First Differences of Retail Price=0

City	1990-1995	1990	1991	1992	1993	1994	1995
Vancouver	47	57	30	37	73	40	45
Edmonton	32	57	23	40	17	33	23
Calgary	32	41	15	23	40	52	24
Regina	50	51	23	33	48	81	69
Winnipeg	67	67	36	60	88	75	76
Toronto	2	2	6	0	0	2	2
Ottawa	30	69	42	42	21	6	0
Montreal	26	63	60	17	8	4	2
Quebec City	33	73	60	31	13	19	0
St. John	65	76	58	56	48	83	69
Halifax	61	84	87	60	37	48	53
St. John's	73	65	83	69	54	85	82
Windsor	13	4	0	6	8	15	43
London	3	0	2	2	2	0	12
Sudbury	22	63	20	6	13	8	22
S.S.Marie	42	35	17	40	71	58	33
Thunder Bay	42	57	30	46	50	54	18
North Bay	44	29	13	37	58	63	65
Timmins	63	59	38	71	54	81	78

Table 4.5: 5-Firm Concentration by City

City	Mean	Min	Max
Vancouver	82.3	81	85
Calgary	72.2	69	76
Edmonton	65.8	64	67
Regina	75.2	71	81
Winnipeg	75.8	74	80
Toronto	77.2	74	80
Ottawa	68.2	73	81
Montreal	70.5	64	73
Quebec City	66.7	66	74
St. John	84.8	62	70
Halifax	94.0	79	89
St. John's	94.2	90	96
Windsor	68.7	92	97
London	67.0	57	77
Sudbury	71.3	62	73
S.S. Marie	74.5	71	77
Thunder Bay	74.3	71	80
North Bay	72.5	70	76
Timmins	82.0	76	88

Table 4.6: Correlation Between $ZEROS_{it}$ and $CONC_{it}$

	1990	1991	1992	1993	1994	1995
$CONC_{it}$	0.28	0.60	0.55	0.48	0.68	0.56

Table 4.7: OLS Results

	Coefficient	T-Stat
$CONC_{it}$	1.64	7.36
POP_{it}	-0.07	-3.24
$REFINER_{it}$	-7.54	-1.79
$STDEV_{it}$	1.49	0.94
$CONST_{it}$	-78.78	-5.52
R^2	0.43	

4.7 Appendix A

In this Appendix, we first prove Proposition 1. We then discuss the case where θ_B is less than but near 1. Proofs of Propositions 2 and 3 follow.

Proposition 1: For a small grid size the following dynamic reaction functions constitute a MPE when $\theta_B = 1$:

$$R^B(p) = \begin{cases} \overline{p} & \text{for } p \geq \overline{p}, \\ p & \text{for } \overline{p} > p \geq \underline{p}_B, \\ c & \text{for } \underline{p}_B > p > c, \\ \left.\begin{array}{ll} c & \text{with probability } \mu_B(\delta) \\ \overline{p}+k & \text{with probability } (1-\mu_B(\delta)) \end{array}\right\} & \text{for } p = c, \\ c & \text{for } p < c\ , \end{cases}$$

$$R^S(p) = \begin{cases} \overline{p} & \text{for } p > \overline{p}, \\ p-k & \text{for } \overline{p} \geq p > \underline{p}_S, \\ c & \text{for } \underline{p}_S \geq p > c, \\ \overline{p} & \text{for } p = c, \\ c & \text{for } p < c\ . \end{cases}$$

Proof: The proof of this proposition follows three stages. [16] In the first stage we define the parameters of the above reaction functions. The second stage proves that the above reaction functions constitute an MPE. The third stage proves existence of the parameters.

Stage 1: In this stage we define $\underline{p}_B, \underline{p}_S, \overline{p}$, and μ_B. We proceed in four steps. First, we jointly define $\underline{p}_B, \overline{p}$, and show that for a sufficiently fine grid there must

[16] The structure of this proof follows that of Maskin and Tirole(1984).

be at least one price on the grid that lies between them. This assumption is then maintained for the remainder of the proof. In step 2, we define $\underline{p}_S$. In step 3 we claim that the value function for firm S evaluated at c is less than or equal to that of firm B. Finally, in step 4 we demonstrate that the mixing parameter μ_B is well defined[17].

(1) We define $\underline{p}_B \leq p^m, \overline{p} \geq p^m$ according to

$$\Pi(\underline{p}_B) > (1-\delta^2)V_{B,c} \geq \Pi(\underline{p}_B - k) \tag{4.6}$$

and that $\overline{p}$ is the lowest price greater than or equal to p^m such that

$$\Pi(\overline{p}) + ... + \delta^{2t-4}\Pi(\underline{p}_B) + \delta^{2t-2}V_{B,c} \geq \Pi(\overline{p}+k) + ...\delta^{2t-2}\Pi(\underline{p}_B) + \delta^{2t}V_{B,c}, \tag{4.7}$$

where

$$V_{B,c} = \begin{cases} \delta^2\Pi(\overline{p}) + \delta^4\Pi(\overline{p}-k) + ... + \delta^{2t-2}\Pi(\underline{p}_B) + \delta^{2t}V_{B,c} & \text{if } \overline{p} > \underline{p}_B \\ \delta^2\Pi(\overline{p}) + \delta^4 V_{B,c} & \text{otherwise.} \end{cases}$$

Here, the number of prices on the grid between $\overline{p}$ and $\underline{p}_B$ inclusive is $t-2$. Note that in inequality (4.7), $\overline{p} = \underline{p}_B$ would correspond to $t = 2$. To interpret these conditions, note first that provided $\underline{p}_B \geq \underline{p}_S$[18],that B is indifferent between responding to $p = c$ with $\overline{p}+k$ and c, and that $V_{B,c} = V_B(\underline{p}_B)$ then $V_{B,c} = V_B(c)$, the discounted value of future profits when subsequent play is dictated by the above strategies. It follows that $\underline{p}_B, \overline{p}$ are such that (a) upon observing price $\underline{p}_B$ firm B prefers to match as opposed to setting price equal to c, but at $\underline{p}_B - k$, B prefers to set c, and (b) upon observing $\overline{p}+k$, B prefers to set $\overline{p}$ to either of $\overline{p}+k$ or $\overline{p}-k$, assuming S will respond by undercutting by a grid size.

[17]For the remainder of the proof, the argument δ is omitted.

[18]This will be proven in stage 3.

Next we claim that for a sufficiently fine grid, $\overline{p}-\underline{p}_B > k$ by the continuity of the profit function, so that there exists at least one price between $\overline{p}$ and $\underline{p}_B$. To see this, first suppose that $\underline{p}_B = \overline{p}$. This implies that $(1-\delta^2)V_{B,c} = \frac{\delta^2\Pi(p^m)}{1+\delta^2}$. but therefore, given δ, one can find a k sufficiently small so that $\Pi(p^m - k) \geq (1-\delta^2)V_{B,c}$. Therefore, one can find a k small enough that $\underline{p}_B \neq \overline{p}$. A similar argument establishes that for a small k, there must be at least one price on the grid between $\overline{p}$ and $\underline{p}_B$. For the remainder of this proof, we assume a sufficiently fine grid.

(2): Defining $\underline{p}_S$: given $\underline{p}_B$ and $\overline{p}$, define $\underline{p}_S$ by

$$\Pi(\underline{p}_S) > (1-\delta^2)V_{S,c} \geq \Pi(\underline{p}_S - k), \tag{4.8}$$

where

$$V_{S,c} = \begin{cases} \delta^2\Pi(\overline{p}-k) + \ldots + \delta^{2r-2}\Pi(\underline{p}_S) + \delta^{2r}V_{S,c} & \text{if } \underline{p}_B \leq \underline{p}_S \\ \delta^2\Pi(\overline{p}-k) + \ldots + \delta^{2t-2}\Pi(\underline{p}_B - k) + \delta^{2t}V_{S,c} & \text{otherwise,} \end{cases}$$

where $r \leq t$. Note that if S is indifferent between responding to $p = c$ with $\overline{p}$ or with c, $V_{S,c} = V_S(c)$, which is the continuation payoffs to S upon observing c, given that all subsequent play follows the specified strategies. $\underline{p}_S$ is then the lowest price such that upon observing $\underline{p}_S + k$, S prefers to say $\underline{p}_S$ than c.

(3): Claim (1): $V_{S,c} < V_{B,c}$, where $V_{B,c}$ and $V_{S,c}$ are as defined above. The proof of Claim (1) will be reserved for Stage 3, since it relies on techniques introduced in Stage 2. Claim (1) implies that $\underline{p}_S \leq \underline{p}_B$ by the definitions of these parameters, which in turn implies that $V_{B,c}$ is equal to $V_B(c)$, the continuation payoffs of firm B upon observing $p = c$, given that all subsequent play follows the given strategies.

(4) Finally, we establish that μ_B is well defined, given the parameters defined previously. Define $V_{S,\underline{p}_B} = V_{S,c}$ if $\underline{p}_B = \underline{p}_S$, and $V_{S,\underline{p}_B} = \Pi(\underline{p}_B - k) + \delta^2 V_{S,c}$ otherwise.

As well, define

$$V_{S,\overline{p}+k} = \Pi(\overline{p}) + \ldots + \delta^{2t-4}\Pi(\underline{p}_B) + \delta^{2t-2}V_{S,\underline{p}_B} \tag{4.9}$$

and

$$V_{S,\overline{p}} = \Pi(\overline{p}-k) + \ldots + \delta^{2t-2}\Pi(\underline{p}_B) + \delta^{2t}V_{S,\underline{p}_B}. \tag{4.10}$$

Provided that play follows $R^B(\cdot)$ and $R^S(\cdot)$ and given that upon observing $p = c$ S is indifferent between setting $\overline{p}$ and c, $V_{S,\underline{p}_B}$, $V_{S,\overline{p}}$, and $V_{S,\overline{p}+k}$ equal $V_S(\underline{p}_B)$, $V_S(\overline{p})$ and $V_S(\overline{p}+k)$ respectively, and can be interpreted as the continuation payoffs to firm S.

Recall that firm B chooses μ_B to make S indifferent between setting $p = c$ and $p = \overline{p}$ upon observing B's price equal to c. We now establish that there in fact exists a $\mu_B \in (0,1)$ such that $\delta^2 V_{S,\overline{p}} = \mu_B \delta^4 V_{S,\overline{p}} + (1-\mu_B)\delta^2 V_{S,\underline{p}+k}$. This result follows since

$$\delta^2 V_{S,\overline{p}} > \delta^4 V_{S,\overline{p}} \tag{4.11}$$

and

$$\delta^2 V_{S,\overline{p}} < \delta^2 V_{S,\overline{p}+k}. \tag{4.12}$$

To show the latter we note that from the definition of $\overline{p}$,

$$\Pi(\overline{p}) + \ldots + \delta^{2t-4}\Pi(\underline{p}_B) + \delta^{2t-2}V_B(c) > \Pi(\overline{p}-k) + \ldots + \delta^{2t-6}\Pi(\underline{p}_B) + \delta^{2t-4}V_B(c). \tag{4.13}$$

Note that $V_{S,\underline{p}_B} < V_B(c)$. If $\underline{p}_B = \underline{p}_S$ this follows directly from the claim. If $\underline{p}_B > \underline{p}_S$ then this follows since by Claim (1) $V_{S,\underline{p}_B} < \Pi(\underline{p}_B) + \delta^2 V_B(c) < V_B(c)$. Therefore, if the expression $V_B(c)$ is replaced with $V_S(\underline{p}_B)$ in the above, the inequality will still hold, so that $V_{S,\overline{p}} < V_{S,\overline{p}+k}$ as required.

Stage 2: Stage 2 proves that these strategies constitute an MPE.

(1) From the definitions of $\underline{p}_B$ and $\underline{p}_S$ the following are true. If B sees $p \in (c, \underline{p}_B)$, it prefers to say c than any price in $(c, p]$. Similarly, if S observes $p \in (c, \underline{p}_S]$, it prefers to say c than any price in (c, p). If B observes $\underline{p}_B$ it prefers to match than set a price in $[c, \underline{p}_B)$, and if S observes $\underline{p}_S + k$ it prefers to set $\underline{p}_S$ than any price in $[c, \underline{p}_S)$. Finally, if the set $(\underline{p}_S, \underline{p}_B]$ is nonempty, then upon observing $p \in (\underline{p}_S, \underline{p}_B]$ S prefers to set $p - k$ than any price below $p - k$.

(2) Neither firm will ever deviate to a price less than c. This follows since the payoff to the deviation is less than or equal to $\delta^2 V_i(c)$, which is strictly less than the payoff realized by setting c.

(3) Next we suppose $\underline{p}_B < p^m$. We show that for $p \in (\underline{p}_B, p^m]$, setting $p - k$ is preferred to setting a price strictly less than $p - k$ for $i = S$, and setting p is preferred to stetting a price strictly less than p for $i = B$.

First consider $i = B$. We first show that at p firm B will set p instead of $p - k$. If firm B matches it gets

$$\Pi(p) + \delta^2 \Pi(p-k) + \ldots + \delta^{2q} \Pi(\underline{p}_B) + \delta^{2q+2} V_B(c), \tag{4.14}$$

where q is an arbitrary counter. If it sets $p - k$ it receives

$$\Pi(p-k) + \delta^2 \Pi(p-2k) + \ldots + \delta^{2q-2} \Pi(\underline{p}_B) + \delta^{2q} V_B(c). \tag{4.15}$$

Comparing, we get the condition that

$$\Pi(p) - \Pi(p-k) + \delta^2(\Pi(p-k) - \Pi(p-2k)) + \ldots \delta^{2q}(\Pi(\underline{p}_B) - (1-\delta^2)V_B(c)) \geq 0, \tag{4.16}$$

which clearly holds (with strict inequality) by the definition of $\underline{p}_B$ and the fact that p is less than the monopoly price. It follows that B prefers to set p than $p - jk$, for all integers j greater than 1 and such that $p - jk \geq \underline{p}_B$. Specifically, upon observing p B will not deviate to $p - jk$ since upon observing $p - (j-1)k$ it would prefer to say $p - (j-1)k$ than $p - jk$. Finally, B prefers p to setting c since it prefers setting p to $\underline{p}_B$.

The proof for $i = S$ is similar. Here we need to show that firm S would respond to p with $p - k$ as opposed to $p - 2k$. The condition becomes

$$\Pi(p-k) - \Pi(p-2k) + \ldots \delta^{2q-2}(\Pi(\underline{p}_B) - (1-\delta^2)V_S(\underline{p}_B))) \geq 0. \tag{4.17}$$

This is clearly satisfied (again with strict inequality). Arguments against setting prices $< p - k$ follow those used in the case of firm B.

(4): Here we demonstrate that neither firm will deviate to a price below the response specified in the strategies, upon observing a price between the monopoly price and $\overline{p}$. Suppose $p^m < \overline{p}$ and $p \in (p^m, \overline{p}]$, and consider the case of firm B. Again, we need to establish that

$$\Pi(p) - \Pi(p-k) + \delta^2(\Pi(p-k) - \Pi(p-2k)) + \ldots \delta^{2q}(\Pi(\underline{p}_B) - (1-\delta^2)V_B(c)) \geq 0. \tag{4.18}$$

Therefore, the result follows, with strict inequality, from the definition of $\overline{p}$.

Next, consider firm S and $p \in (p^m, \overline{p} + k]$. We need to show that

$$(\Pi(p-k) - \Pi(p-2k)) + \ldots \delta^{2q}(\Pi(\underline{p}_B) - (1-\delta^2)V_S(\underline{p}_B)) \geq 0. \tag{4.19}$$

Since $V_S(\underline{p}_B) < V_B(c)$ by Claim 1, this follows, with strict inequality, from above.

(5) Here we show that if $R^B(p) > p$ then either $R^B(p) = \overline{p} + k$ or $R^B(p) = c$. Similarly, if $R^S(p) \geq p$ then either $R^S(p) = \overline{p}$ or $R^S(p) = c$. First note that firm B will never respond to p with a price $p* \geq p$ and strictly between $\overline{p} + k$ and c. If firm B raises its price above that of the other firm to such a $p*$, its discounted profit is $\delta^2 V_B(p*)$, where $R^S(\cdot)$ and $V_B(\cdot)$ are both nondecreasing for $i = B, S$ over the specified range. Similarly, S will never respond to p with a price greater than p and strictly between $\overline{p}$ and c. As well, since $W_B(p)$ is the same for all $p \geq \overline{p} + k$, and $W_S(p)$ is the same for all $p \geq \overline{p}$, firm B will never set its price above that of S to a price strictly above $\overline{p} + k$, and firm S will never set its price above or equal to firm B to a price strictly above $\overline{p}$.

(6) Finally, consider $p > \overline{p}$. We show that at p, firms undercut to $\overline{p}$. That Firm B has no prefered deviation follows from the definition of $\overline{p}$. Now we show that firm S will not undercut to $\overline{p} + k$. If S says $\overline{p}$ she gets

$$\Pi(\overline{p}) + \delta^2 V_S(\overline{p}). \tag{4.20}$$

If she says $\overline{p} + k$ she receives

$$\Pi(\overline{p} + k) + \delta^2 V_S(\overline{p}). \tag{4.21}$$

Since $\overline{p} \geq p^m$, the result follows from the concavity of the profit function.

Stage 3 In stage 3, we prove Claim (1), show that $\overline{p}$ and $\underline{p}_B$ jointly exist, and that given $\overline{p} - \underline{p}_B \geq 2k$, $\underline{p}_S$ is well defined.

First, we prove Claim (1): $V_{S,c} < V_{B,c}$.

Proof: Suppose instead that $V_{S,c} > V_{B,c}$. This proof considers two distinct cases.

Case 1: $\underline{p}_S > \underline{p}_B$. Recall that we defined $V_{B,c}$ by

$$V_{B,c} = \delta^2 \Pi(\overline{p}) + \ldots + \delta^{2t} \Pi(\underline{p}_B) + \delta^{2t+2} V_{B,c}. \tag{4.22}$$

By definition of $\overline{p}$,

$$V_{B,c} > \delta^2\Pi(\overline{p} - k) + ... + \delta^{2t-2}\Pi(\underline{p}_B) + \delta^{2t}V_{B,c}. \tag{4.23}$$

Now, in Case 1,

$$V_{S,c} = \delta^2\Pi(\overline{p} - k) + ... + \delta^{2r-2}\Pi(\underline{p}_S) + \delta^{2r}V_{S,c}, \tag{4.24}$$

where $2 \leq r < t$ (r=1 would correspond to $\overline{p} = \underline{p}_S$, which implies $V_{S,c} = \delta^2 V_{S,c}$, which in turn yields a contradiction). It follows that

$$V_{B,c} > \delta^2\Pi(\overline{p} - k) + ... + \delta^{2r-2}\Pi(\underline{p}_S) + \delta^{2r}V_{B,c}. \tag{4.25}$$

Therefore,

$$V_{S,c} - V_{B,c} < \delta^{2r}(V_{S,c} - V_{B,c}). \tag{4.26}$$

However, this yields a contradiction since $\delta < 1$.

Case 2: $\underline{p}_S \leq \underline{p}_B$. If $\underline{p}_S < \underline{p}_B$ then the contradiction follows from the definitions of $\underline{p}_S$ and $\underline{p}_B$. If $\underline{p}_S = \underline{p}_B$, then using a similar argument to Case 1, we can show that $V_{S,c} - V_{B,c} < \delta^{2t}(V_{S,c} - V_{B,c})$, which can only be true if $V_{S,c} \leq V_{B,c}$. This concludes the proof of Claim 1.

Next, we prove that $\underline{p}_B$ and $\overline{p}$ jointly exist. Consider an arbitrary $V* \in [0, \frac{\delta^2\Pi(p^m)}{(1-\delta^2)}]$. Define $\underline{p}_B(V*)$ by

$$\Pi(\underline{p}_B(V*)) \geq (1-\delta^2)V* > \Pi(\underline{p}_B(V*) - k) \tag{4.27}$$

and define $\overline{p}(V*)$ as the lowest price $\geq p^m$ such that

$$\Pi(\overline{p}(V*)) + ... + \delta^{2t-2}V* > \Pi(\overline{p}(V*) + k) + ... \delta^{2t}V*. \tag{4.28}$$

Note that for $V*$ in the specified range, both $\overline{p}$ and $\underline{p}_B$ are well defined since $\Pi()$ was assumed strictly concave (even though it is possible that they both equal p^m).

Next, note that $\overline{p}(V*)$ is equivalently defined by

$$\Pi(\overline{p}(V*)) + ... + \delta^{2t-2}V* \geq \Pi(\overline{p}(V*) + k) + ... + \delta^{2t}V* \tag{4.29}$$

and

$$\Pi(\overline{p}(V*)) + ... + \delta^{2t-2}V* > \Pi(\overline{p}(V*) - k) + ... + \delta^{2t-4}V* \tag{4.30}$$

if such a price $\overline{p}(V*)$ exists, else $\overline{p}(V*) = p^m$. To see this, note that the only way this could not hold is if the above inequalities are true for $\overline{p}(V*)$, but also for some $\tilde{p}(V*) < \overline{p}(V*)$, where $\tilde{p}(V*) \geq p^m$. In negation, suppose $\overline{p}(V*) > p^m$ and such a $\tilde{p}(V*)$ exists. Then

$$\Pi(\tilde{p}(V*)) + ... + \delta^{2s}V* \geq \Pi(\tilde{p}(V*) + k) + ... + \delta^{2s+2}V* \tag{4.31}$$

where $s < t$ measures the number of prices between $\tilde{p}(V*)$ and $\underline{p}(V*)$. But since the profit function is strictly decreasing over the range of prices above the monopoly price, the above inequality implies

$$\Pi(\tilde{p}(V*) + k) + ... + \delta^{2s+2}V* \geq \Pi(\tilde{p}(V*) + 2k) + ... + \delta^{2s+4}V* \tag{4.32}$$

This can be repeated, by adding one higher price to each side of the inequality, until we arrive at

$$\Pi(\overline{p}(V*) - k) + ... + \delta^{2s+2}V* > \Pi(\overline{p}(V*)) + ... + \delta^{2s+4}V* \tag{4.33}$$

which yields a contradiction, proving the equivalence of the two definitions.

Define $V(V*)$ as

$$V(V*) = \begin{cases} \delta^2\Pi(\overline{p}(V*)) + \ldots + \delta^{2t-2}\Pi(\underline{p}_B(V*)) + \delta^{2t}V* & \text{if } \overline{p}(V*) > \underline{p}_B(V*) \\ \delta^2\Pi(\overline{p}(V*) + \delta^4 V* & \text{otherwise.} \end{cases}$$

Note that even though $\overline{p}(V*)$ and $\underline{p}_B(V*)$ are discontinuous in $V*$, $V(V*)$ is in fact continuous in $V*$. For example, when $\underline{p}_B(V*)$ jumps up (down) the continuation payoffs at $\underline{p}_B(V*) + k$ ($\underline{p}_B(V*)$ remain unchanged by the definition of $\underline{p}_B(V*)$. Therefore, when $\underline{p}_B(V*)$ jumps, $V(V*)$ is also continuous. By similar reasoning, $V(V*)$ is continuous at points of discontinuity of $\overline{p}(V*)$.

Next, we follow a fixed point argument. Specifically, we need to show that $V(\cdot)$ maps the interval $[0, \delta^2\Pi(p^m)/(1-\delta^2)]$ into itself. Clearly, if $V* \geq 0$, $V(V*) \geq 0$. Now suppose $V* \leq \delta^2\Pi(p^m)/(1-\delta^2)$. Then by the definition of $V(V*)$, we can claim

$$V(V*) \leq \delta^2\Pi(p^m) + \ldots + \delta^{2t}\Pi(p^m) + \delta^{2t+2}V*, \tag{4.34}$$

which implies $V(V*) < \delta^2\Pi(p^m)/(1-\delta^2)$ since $V* \leq \delta^2\Pi(p^m)/(1-\delta^2)$. Therefore, since $V()$ is continuous and maps the interval $[0, \delta^2\Pi(p^m)/(1-\delta^2)]$ into itself, it has a fixed point over this interval by Brouwer's fixed point theorem. Therefore, the parameters $\underline{p}_B$ and $\overline{p}$ jointly exist.

Finally, we show that $\underline{p}_S$ is well defined. First suppose $\underline{p}_S = \underline{p}_B$. By the arguments in the proof of Case 1 of Claim 1 we have that either $\underline{p}_S = \underline{p}_B$ satisfies condition (4.9), or else $\Pi(\underline{p}_S - k) \geq (1-\delta^2)V_{S,c}$. That is, if $\underline{p}_S = \underline{p}_B$ does not satisfy the definition of $\underline{p}_S$, it must be that firm S prefers to set $\underline{p}_S - k$ than c, upon observing $\underline{p}_S$. If (4.9) is satisfied then we are done. If not, then it must be that $\underline{p}_S < \underline{p}_B$. This implies $(1-\delta^2)V_{S,c} = \frac{\delta^2\Pi(\overline{p}-k)+\ldots+\delta^{2t+2}\Pi(\underline{p}_B-k)}{1+\delta^2+\ldots\delta^{2t+2}}$. Notice that $\Pi(\underline{p}_B - k) \geq (1-\delta^2)V_{S,c}$ is satisfied. We then set $\underline{p}_S$ as the lowest price such that $\Pi(\underline{p}_S) \geq (1-\delta^2)V_{S,c}$. Q.E.D.

The case of θ_B less than but near 1: To see that the strategies constructed in Proposition (1) are an equilibrium for θ_B sufficiently near 1 (with the mixing probability recomputed), we simply note that for all deviations that become more attractive relative to the prescribed strategy as θ_B falls, and since payoffs are continuous in θ_B, the relevant condition for the case of $\theta_B = 1$ holds with strict inequality. Therefore, there exists a θ_B sufficiently near one so that these deviations will remain not preferred.

(1) Take $\underline{p}_B, \underline{p}_S$, and $\overline{p}$ as defined in proposition 1.

(2) Note that since(4.11) and (4.12) both hold with strict inequality in Proposition 1, it follows that they both hold for θ_B sufficiently near 1. Therefore, for θ_B sufficiently large, the mixing parameter μ_B is well defined.

(3) Checking Firm B's deviations: It suffices to check that upon observing a price $p \in [\underline{p}_B, \overline{p}]$, matching is optimal for B (all other cases follow immediately from the arguments in Proposition 1). However, recall that in Proposition 1(a), the payoff to matching $p \in [\underline{p}_B, \overline{p}]$ was always strictly greater than that from undercutting to a price $\in (c,p)$. It therefore follows that for θ_B sufficiently large, undercutting is not an optimal deviation. Note that as k gets small, the necessary size of θ_B will approach 1.

(4) Checking Firm S deviations: First, we note that Firm S will never match. Consider first $p \in (\underline{p}_B, \overline{p}]$. We require that $(1+\delta)(\theta_S)\Pi(p) + \delta^2 V_S(p) \leq V_S(p)$, or $(1+\delta)(\theta_S)\Pi(p) \leq (1-\delta^2)V_S(p)$. Clearly, there exists θ_S sufficiently small to

ensure that this inequality holds, since $(1-\delta^2)V_S(p) > 0$. That firm S will not match prices $\in (c, \underline{p})$ or above $\overline{p}$ follows similarly.

Secondly, we verify that firm S will never undercut a price in $[\underline{p}_S + k, \overline{p}]$ by more than a single grid size. This follows since 4.17 and 4.19 hold with strict inequality. Therefore, there exists a θ_S sufficiently small so that these conditions will still hold.

Third, we note that upon observing c, S prefers to jump to $\overline{p}$ than to any price above or below $\overline{p}$, for θ_B sufficiently large. That S will not jump to above $\overline{p}$ follows since the response of B to observing $\overline{p} + k$ and $\overline{p}$ are the same. To see that for large θ_B firm S will not deviate to a price below $\overline{p}$ follows since in the case where $\theta_B = 1$ this deviation was strictly not optimal. Finally, we note that all other deviations follow from the arguments given in Proposition 1.

Proposition 2: Suppose p^f is the focal price of a MPE, and p^m is the monopoly price.

(a) If $p^f \leq p^m$, then

$$\theta_S \geq \frac{\delta\Pi(p^f - k)}{(1+\delta+\delta^2)\Pi(p^f)}. \tag{4.35}$$

(b) If $p^f > p^m$, then

$$\theta_S \geq \frac{\delta\Pi(p^m)}{(1+\delta+\delta^2)\Pi(p^f)}. \tag{4.36}$$

Proof:

(a): One available deviation to a firm when it sees p^f is to undercut one grid size, and the return to the focal price at the next turn. The payoff to this deviation is at least

$$\Pi(p^f - k) + \delta^3 \frac{\theta_i \Pi(p^f)}{1-\delta}. \tag{4.37}$$

Requiring that this be less than the equilibrium path payoff yields the result. (b): In this case, consider the deviation of responding to p^f by saying p^m and then returning to p^f at the next available opportunity. The payoff to this deviation is at least

$$\Pi(p^m) + \delta^3 \frac{\theta_i \Pi(p^f)}{1-\delta}. \tag{4.38}$$

Again, requiring that this be less than the equilibrium path payoff yields the result.

Proposition 3: Suppose $\theta_B > 0.5$, and consider a price $p^f \geq p^m$ on the grid. Additionally, suppose the price $p < p^f$ such that $\Pi(p) = \theta_i \Pi(p^f)$ does not lie on the grid. Then for a grid size sufficiently small, p^f cannot be the focal price of an MPE.[19]

Proof: Suppose in negation that $p^f \geq p^m$. Define $\underline{p}_S, \underline{p}_B$ as the lowest price the corresponding firm would be set instead of returning to the focal price, upon observing a price just below the focal price, and assuming the other firm will respond by jumping up to the focal price. After some manipulation, these definitions can be given by

$$\frac{1+\delta}{\delta\theta_i}\Pi(\underline{p}_i) \geq \Pi(p^f) > \frac{1+\delta}{\delta\theta_i}\Pi(\underline{p}_i - k) \tag{4.39}$$

for $i = B, S$. Suppose the grid is sufficiently fine that $\underline{p}_S < \underline{p}_B$. As well, suppose that p^f is the focal price of an MPE.

[19] This proof is inspired by the proof of Proposition 5 in Maskin and Tirole(1988).

Claim 1: $p > p^f$ implies $R^i(p) = p^f$. To see this, suppose not and let $p*$ be the lowest price $> p^f$ such that for some i $p* > p^f$ and $p(> p^f) \in R^i(p*)$. Because p^f is a focal price $p \in [p^f, p*]$. (Lemma C of Maskin and Tirole(1988)). First suppose $p < p*$. Since $p > p^f$, then

$$\frac{\Pi(p)}{1+\delta\theta_i} \geq \Pi(p^f), \tag{4.40}$$

which contradicts p^f being no less than the monopoly price. Next, suppose $p = p*$. The best possible scenario for firm i is that firm j responds by setting $p*$ as well. Then this implies that

$$(1-\delta^2)V_i(p*) \leq (1+\delta)\theta_i\Pi(p*), \tag{4.41}$$

which can be rewritten

$$(1-\delta)V_i(p*) \leq \theta_i\Pi(p*). \tag{4.42}$$

This yields a contradiction, because if firm i responded to $p*$ with p^f it would receive an average per period profit of $\theta_i\Pi(p^f)$.

Claim 2: if $p* < p^f$ and $p \in R^i(p*)$ then $p \leq p^f$. Proof of Claim: Suppose not. Then by Claim 1,

$$\delta^2 V_i(p^f) \geq \delta\theta_i\Pi(p^f) + \delta^2 V_i(p^f), \tag{4.43}$$

which yields a contradiction.

Claim 3: If $p* < p^f$ and $p > p*$ is a realization of $R^i(p*)$, then $p = p^f$. Proof of Claim 3: By claim 2, $p \leq p^f$. Clearly, $W_i(p) \geq W_i(p^f)$. Now if $p \neq p^f$ then

$$\theta_i\Pi(p^f) + \delta W_i(p^f) \geq \Pi(p) + \delta W_i(p), \tag{4.44}$$

which implies $\theta_i \Pi(p^f) > \Pi(p)$. Now, since $R^j(p) \leq p^m$ and $V_i(p)$ is nondecreasing and p^f is a realization of $R^i(p^f)$

$$W_i(p) \leq \Pi(p) + \delta V_i(p^f) \tag{4.45}$$

which can be rewritten as

$$\frac{\theta_i}{1-\delta}\Pi(p^f) \leq \Pi(p) + \delta\theta_i\Pi(p^f) + \delta^2\frac{\theta_i}{1-\delta}\Pi(p^f) \tag{4.46}$$

which implies $\theta_i \Pi(p^f) \leq \Pi(p)$ which yields a contradiction.

Claim 4: for all $p \leq \underline{p}_B R^B(p) = p^f$. Proof: By setting a price less than or equal to p, the best B can do is $(1+\delta)\Pi(\underline{p}_B - k) + \delta^2 \frac{\theta_B}{1-\delta}\Pi(p^f)$, if S jumps up to p^f in response. However, if B says p^m, she receives $\frac{\delta\theta_B}{1-\delta}\Pi(p^f)$, which is larger by the definition of $\underline{p}_B$.

Claim 5: Define $\tilde{p}$ as the highest price $< p^f$ such that $\Pi(\tilde{p}) < \theta_B \Pi(p^f)$. Suppose $p \in (\underline{p}_S, \tilde{p}]$. Then $R^S(p) \leq p$. Proof: follows from Claim Three and the definition of $\underline{p}_S$.

Claim 6: Suppose $p \in [\underline{p}_B, \tilde{p}]$. Then $R^S(p) < p$. Proof: Suppose not. Define $p** \in [\underline{p}_B, \tilde{p}]$ as the lowest price in the specified range such that $p** \in R^S(p**)$. First, note that $p** > \underline{p}_B$ for small k. This follows since by claim 4, B responds to both $\underline{p}_B$ and $\underline{p}_B - k$ with p^f. Therefore if S observes $\underline{p}_B$, it would prefer to set $\underline{p}_B - k$ to $\underline{p}_B$ for small k.

Next we claim that if $p < p**$, then $R^B(p) = p^f$. Suppose not. Then there exists at least one price $p \in (\underline{p}_B, p**)$ (which is now nonempty by assumption) such that $R^B(p)$ not equal to p^f. Let $p*$ be the lowest price in this set. Clearly $R^B(p*) \geq \underline{p}_B$. Now,

$$V^B(p*) < \Pi(p*) + \delta^3 W^B(p^f), \tag{4.47}$$

since the one-period profit from the response to $p*$ is strictly less than $\Pi(p*)$, the response by S to $R_B(p*)$ is strictly below $R_B(p*)$, and B's response is then to jump to the focal price. If firm B sets p^f, he gets $\frac{\delta}{1-\delta}\theta_B\Pi(p^f)$. So

$$V^B(p*) \geq \frac{\delta}{1-\delta}\theta_B\Pi(p^f) \tag{4.48}$$

which implies after some manipulation that

$$(1+\delta)\theta_B\Pi(p^f) < \Pi(p*) \tag{4.49}$$

which yields a contradiction, since $\Pi(p*) < \theta_B\Pi(p^f)$.

Next we claim that for k small, $p** \notin R^S(p**)$, which contradicts the initial assumption. Suppose S observes $p**$. If S says $p** - k$ it gets

$$(1+\delta)\Pi(p** - k) + \delta^2 V_S(p^f). \tag{4.50}$$

If S says $p**$, it gets at most

$$(\theta_S + \delta)\Pi(p**) + \delta^2 V_S(p^f), \tag{4.51}$$

since the best S can hope for is that B jumps up. Clearly, for k small, we get a contradiction.

Claim 7: If $p \in [\underline{p}_B, \tilde{p}], R^B(p) = p^f$. Proof: In claim 6 it was proved that if $p \in [\underline{p}_B, p**]$, $R_B(p) = p^f$. Since there does not exist such a price $p**$ we can say $R_B(p) = p^f$ for $p \in [\underline{p}_B, \tilde{p}]$.

Claim 8: Firm S prefers to respond to p^f with $\tilde{p}$ than with p^f, for k sufficiently small. Proof: If S sets p^f it gets

$$(1+\delta)\theta_S\Pi(p^f) + \delta^2\frac{\theta_S\Pi(p^f)}{(1-\delta)}, \tag{4.52}$$

while if she sets $\tilde{p}$ she gets approximately

$$(1+\delta)\theta_B\Pi(p^f) + \delta^2\frac{\theta_S\Pi(p^f)}{(1-\delta)} \tag{4.53}$$

for k sufficiently small. Therefore if $\theta_B > 1/2$ we arrive at a contradiction.

4.8 Appendix B

Here we present an example of a focal price equilibrium for $\theta_B > \theta_S$, in which for some prices $p < p^f$, there exists $p' \in R_B(p)$ such that $p' \in (p, p^f)$.

Suppose the firms face unit demand; that is, the demand is equal to one if price is less than or equal to one, and zero if price is greater than one. Suppose $\delta = .999$, $\theta_B = 0.53$, and $k = 1/200$. Then the following strategies constitute a MPE:

$$R^B(p) = \begin{cases} 1 & \text{for } p > 1, \\ 1 & \text{with probability } \mu_B \text{ if } p = 1, \\ p^f & \text{with probability } 1-\mu_B \text{ if } p = 1, \\ p^f & \text{for } p \in [p^f, 1), \\ \underline{p}_B & \text{for } p \in (\underline{p}_S, p^f), \\ \underline{p}_B & \text{with prob } \alpha_B \text{ if } p = \underline{p}_S, \\ 1 & \text{with prob } 1-\alpha_B \text{ if } p = \underline{p}_S, \\ \underline{p}_B & \text{for } p < \underline{p}_S\ , \end{cases}$$

$$R^S(p) = \begin{cases} 1 & \text{for } p > 1, \\ 1 & \text{with probability } \mu_S \text{ if } p = 1, \\ p^f & \text{with probability } 1-\mu_S \text{ if } p = 1, \\ p^f & \text{for } p \in [p^f, 1), \\ p-k & \text{for } p \in (p*, p^f), \\ \underline{p}_S & \text{for } p \in (\underline{p}_B, p*], \\ \underline{p}_S & \text{with prob } \alpha_S \text{ if } p = \underline{p}_B, \\ 1 & \text{with prob } 1-\alpha_S \text{ if } p = \underline{p}_B, \\ \underline{p}_S & \text{for } p \in (\underline{p}_S, \underline{p}_B)\ , \\ 1 & \text{for } p \leq \underline{p}_S\ , \end{cases}$$

where $p^f = 0.69, p* = 0.65, \underline{p}_B = .205$, and $\underline{p}_S = .09$. μ_B μ_S are set so that each firm is indifferent between matching the reservation price and setting the focal price. Similarly, α_B is chosen so that S is indifferent between responding to a price greater than $\underline{p}_S$ with $\underline{p}_S$ or with the reservation price, and α_S is such that B is indifferent between responding to a price less than $\underline{p}_B$ with $\underline{p}_B$ and the reservation price. That is, $\mu_j, j = B, S$ solves

$$0.69 + \frac{\delta(.69\theta_j)}{1-\delta} = \theta_j + \mu_i(\theta_j\delta + \delta^2(.69 + \frac{\delta(.69\theta_j)}{1-\delta})) + (1-\mu_i)(\frac{\delta^2(.69\theta_j)}{1-\delta}), \tag{4.54}$$

which yields $\mu_B \cong 0.651828208$ and $\mu_S \cong 0.651778833$. α_B solves (approximately)

$$\begin{aligned} 0.648067204 =& (1-\delta^2)(1+\delta)(0.09) + (1-\alpha_B)\delta^2(1-\delta^2)(0.69 + \delta\frac{0.47(0.69)}{1-\delta}) \\ &+ \delta^2\alpha_B(0.648067204) \end{aligned} \tag{4.55}$$

or $\alpha_B \cong .00192502$. Similarly, α_S solves (approximately)

$$\delta W_B(1) = (1-\alpha_S)(\delta(.205) + \delta^2(.69) + \delta^3 \frac{(.53)(.69)}{1-\delta}) + \alpha_S \delta^3 W_B(1) \tag{4.56}$$

Which yields $\alpha_S \cong .005814918$.

Our method of verifying that there are no optimal deviations involves computing $W_i(p)(1-\delta)$ and $V_i(p)(1-\delta)$ for each firm and for every p. It is then straightforward to verify that neither firm has a prefered deviation at any price. The following can be computed.

$$(1-\delta^2)V^B(p) \cong \begin{cases} 0.732622105 & \text{for } p > 1, \\ 0.731682575 & \text{for } p \in (0.69, 1], \\ 0.7310343 & \text{for } p = 0.69, \\ 0.7310329 & \text{for } p \in (0.205, 0.69), \\ 0.730840296 & \text{for } p = 0.205, \\ 0.730623105 & \text{for } p \in (0.09, 0.205)\ , \end{cases}$$

$$(1-\delta^2)V^S(p) \cong \begin{cases} 0.65066204 & \text{for } p > 1, \\ 0.649006734 & \text{for } p \in (0.69, 1], \\ 0.6482757 & \text{for } p = 0.69, \\ (1-\delta^2)(p-k) + 0.646771554 & \text{for } p \in (0.65, 0.69), \\ 0.646771717 & \text{for } p \leq 0.65, \end{cases}$$

$$
\delta(1-\delta^2)W^B(p) \cong \begin{cases} 0.730219941 & \text{for } p > 1, \\ 0.730623105 & \text{for } p = 1, \\ 0.729572961 & \text{for } p \in (0.69, 1), \\ 0.730303265 & \text{for } p = 0.69, \\ 0.729571565 & \text{for } p \in (0.65, 0.69), \\ 0.729162589 & \text{for } p \in (0.205, 0.65), \\ 0.730623105 & \text{for } p = 0.205, \\ 0.729162589 & \text{for } p \in (0.09, 0.205)\ , \\ (1-\delta^2)\delta p + 0.730219941 & \text{for } p \leq 0.09, \end{cases}
$$

$$
\delta(1-\delta^2)W^S(p) \cong \begin{cases} 0.647709369 & \text{for } p > 1, \\ 0.648067204 & \text{for } p = 1, \\ 0.646979796 & \text{for } p \in (0.69, 1), \\ 0.647627424 & \text{for } p = 0.69, \\ 0.646771717 & \text{for } p \in (0.205, 0.69), \\ 0.646964128 & \text{for } p = 0.205, \\ \delta p(1-\delta^2) + 0.646771717 & \text{for } p \in (0.09, 0.205)\ , \\ 0.647887294 & \text{for } p = 0.09, \\ \delta p(1-\delta^2) + .646771717 & \end{cases}
$$

As well, suppose firm i sets $p < p_j$. Then

$$\Pi(p)+\delta(1-\delta^2)W^B(p) = \begin{cases} 0.732622105 & \text{for } p = 1, \\ (1-\delta^2)p + 0.729572961 < .731571961 & \text{for } p \in (0.69, 1), \\ 0.731682575 & \text{for } p = 0.69, \\ (1-\delta^2)p + 0.729571565 < .730950875 & \text{for } p \in (0.65, 0.69), \\ (1-\delta^2)p + 0.729162589 < .730461939 & \text{for } p \in (0.205, 0.65), \\ 0.7310329 & \text{for } p = 0.205, \\ (1-\delta^2)p + 0.72916589 < .729572384 & \text{for } p \in (0.09, 0.205)\ , \\ (1-\delta^2)(1+\delta)p + 0.730219941 & \text{for } p \leq 0.09, \end{cases}$$

$$\Pi(p)+\delta(1-\delta^2)W^S(p) = \begin{cases} 0.650066204 & \text{for } p = 1, \\ (1-\delta^2)p + 0.649797960 < .648978793 & \text{for } p \in (0.69, 1), \\ 0.649006734 & \text{for } p = 0.69, \\ (1-\delta^2)p + 0.646771717 < 0.64815027 & \text{for } p \in (0.205, 0.69), \\ 0.647373923 & \text{for } p = 0.205, \\ \delta p(1-\delta^2) + 0.646771717 < 0.647590897 & \text{for } p \in (0.09, 0.205)\ , \\ 0.6480672014 & \text{for } p = 0.09, \\ (1+\delta)p(1-\delta^2) + .64677177 < 0.648067204 & \end{cases}$$

Finally, to check all deviations, we require the following numbers. If B matches 0.205 it gets 0.730840296. If B matches 0.69 it gets 0.731034299. If S matches 0.69 it gets 0.648275699.

To verify that neither firm has an preferred deviation upon observing a price p set by its opponent is now a simple matter of referring the value functions presented above.

4.9 Appendix C: Numerical Computations for the Example

Note: All numbers given are of course approximate. Eight decimal places are given in most cases. For each case, the value function evaluated at each price and for each firm is given in Table 4.8.

Case 1(a)

$$(1-\delta^2)V(p) = \frac{24\delta^2 + 24\delta^4 + 16\delta^6}{1+\delta^2+\delta^4+\delta^6} = 15.87779227. \tag{4.57}$$

Mixing Probabilities: solving $15.87779227 = (1-\mu)\delta^2(16.25555423)+\mu\delta^2(15.87779227)$ yields $\mu = 0.146596185$.

Checking Deviations: The only deviations that cannot be checked by consulting the table are those that involve matching. Note: we only need to consider firm B, since firm S will clearly never deviate by matching.

Firm B: $(1)p = 8$. We require $16(1-\delta^2)\theta_B + \delta^2(16.20016941) \leq 16.25555423$, which is approximately rewritten $\theta_B \leq 1.186$.

$(2)p = 7$. We require $21(1-\delta^2)\theta_B + \delta(16.15922769) \leq 16.20016941$, which is approximately rewritten $\theta_B \leq 0.86746$.

(3) $p = 6$. We require $24(1-\delta^2)\theta_B + \delta(16.04180125) \leq 16.15922769$, which is approximately rewritten $\theta_B \leq 0.914$.

(4) $p = 5$. We require $25(1-\delta^2)\theta_B + \delta(15.9797242) \leq 16.04180125$, which is approximately rewritten $\theta_B \leq 0.76397$.

(5) $p = 4$. We require $24(1-\delta^2)\theta_B + \delta(15.88022422) \leq 15.9797242$, which is approximately rewritten $\theta_B \leq 0.87$.

(6) $p = 3$. We require $16 \geq 21\theta_B$. Approximately rewritten as $\theta_B \leq 0.761904761$.

Case 1(b)

$$(1-\delta^2)V_B(p) = \frac{\theta_B 24\delta^2 + 24\delta^4 + 21\theta_B\delta^6}{1+\delta^2+\delta^4+\delta^6} = 5.938500288 + 11.151859552\theta_B. \tag{4.58}$$

$$(1-\delta^2)V_S(p) = \frac{(1+\delta\theta_S)24\delta^2 + 24\delta^4 + 21(1+\delta\theta_S)\delta^6}{1+\delta^2+\delta^4+\delta^6} = 17.33779732 + 11.040340920\theta_S. \tag{4.59}$$

From the table, all deviations are immediately not optimal except the following:

Firm S:

$p = 6$: S prefers 5 to 4 if $17.56120271 + 11.010795539\theta_S \geq 17.41067515 + 11.234359914\theta_S$ or $\theta_S \leq 0.6755$. To prefer 5 to 6 requires $17.682483 + 11.264504560\theta_S - 0.4776(1-\theta_S) \leq 17.56120271 + 11.010795539\theta_S$ or $\theta_S \leq 0.4778798$.

$p = 5$: We require $21(1+\delta\theta_S) \geq 24$ or $\theta_S \geq 0.1443$.

$p = 0$: We require that S doesn't prefer to jump up to $p = 6$. This requires $7.33779732 + 11.040340920\theta_S \geq 0.4728240\theta_S + 17.21222483 + 10.798699660\theta_S$ or $\theta_S \leq 0.54$.

Firm B:

$p = 3$: For B not to prefer setting $p = 0$ requires $5.820324132 + 11.347837520\theta_B \geq 5.938500288 + 11.151859552\theta_B$ or $\theta_B \geq 0.603$. For B not to prefer setting $p = 2$ requires $21\theta_B \geq 16$ or $\theta_B \geq 0.7619$.

$p = 4$: For firm B not to prefer matching, requires $6.23226673 + 10.929937520\theta_B \geq 6.182099682 + 11.122015555\theta_B - 0.4776(1-\theta_B)$ or $\theta_B \leq 0.79699$.

$p = 5$: For firm B not to prefer saying $p = 3$ we require $6.182099682 + 11.122015555\theta_B \geq 6.238226673 + 10.929937320\theta_B$ or $\theta_B \geq 0.610324$. For firm B not to prefer saying $p = 5$ requires $24 \geq 25\theta_B$ or $\theta_B \leq 0.96$.

$p = 6$: For firm B not to prefer saying $p = 5$ requires $6.059075898+11.378287440\theta_B \geq 0.4975 + 5.704499682 + 11.122015555\theta_B$ or $\theta_B \geq 0.557704668$. For Firm B not to prefer saying $p = 4$ requires $6.059075898 + 11.378287440\theta_B \geq 6.182099682 + 11.122015555\theta_B$, or $\theta_B \geq 0.4777$.

$p = 7$: For Firm B not to prefer matching requires $6.536675898+10.900687440\theta_B \geq 0.41790\theta_B + 5.938500288 + 11.151859570\theta_B$ or $\theta_B \leq 0.894$.

$p = 8$: For firm B not to say 6 requires $6.356400788 + 11.151859520\theta_B \geq 6.536675898 + 10.900687440\theta_B$ or $\theta_B \geq 0.72726$.

$p = 9, 10$: We require $21\theta_B \geq 16$, or $\theta_B \geq 0.7619$.

Finally, there clearly exists a μ such that B is indifferent between saying 7 and 0 since $V_B(0) \geq \delta^2 V_B(0)$ and $5.938500288+11.151859520\theta_B \leq (0.9801)(6.356400258+ 11.151859520\theta_B)$.

Case 1(c)

$$(1-\delta^2)V_B(p) = \frac{\theta_B(24\delta^2 + 25\delta^4 + 24\delta^6 + 21\delta^8)}{1 + \delta^2 + \delta^4 + \delta^6 + \delta^8} = 18.628915050\theta_B. \tag{4.60}$$

$$(1-\delta^2)V_S(p) = \frac{(1+\delta\theta_S)(24\delta^2 + 25\delta^4 + 24\delta^6 + 21\delta^8)}{1 + \delta^2 + \delta^4 + \delta^6 + \delta^8} = 18.62891505(1+\delta\theta_S). \tag{4.61}$$

All deviations follow from the table except the following:

Firm B:

$p = 4$: Not preferring to undercut to $p = 3$ requires $18.782045226\theta_B \geq 18.676099664\theta_B + 0.4179(1 - \theta_B)$, or $\theta_B \geq 0.8$.

$p = 5$: Not preferring to undercut to $p = 4$ requires $18.905782256\theta_B \geq 18.782045226\theta_B + 0.4776(1 - \theta_B)$ or $\theta_B \geq 0.79$.

$p = 6$: Not preferring to undercut to $p = 5$ requires $19.007157748\theta_B \geq 18.905782256\theta_B + .4975(1 - \theta_B)$ or $\theta_B \geq 0.830724385$.

$p = 7$: Not preferring to match requires $19.007157748\theta_B = 0.4776(2 - \theta_B) \geq 19.046815505\theta_B$ or $\theta_B \leq 0.92333$.

$p \geq 8$: Not preferring to deviate to $p = 6$ requires $19.04681505 + (1 - \theta_B).4179 \geq .4766(1 - \theta_B) + 19.007257748\theta_B$ or $\theta_B \geq 0.6$.

Firm S: $p = 0$: We need to verify that upon observing $p = 0$, firm S does not prefer to jump up to a price other than $p = 8$. Firm S has no incentive to jump up above $p = 8$, as payoffs would be unaffected. Jumping to $p = 6$ will not be optimal provided $18.62891505(1 + \delta\theta^S) \geq 18.90578256(1 - \delta^2)(1 + \delta\theta_S) + 0.4728224\theta_S$, or $\theta_S \leq 0.25$. Similarly, jumping to $p = 5$ will not be optimal provided $18.62891505(1 + \delta\theta^S) \geq 18.782042(1 - \delta^2)(1 + \delta\theta_S) + 0.4925225\theta_S$, or $\theta_S \leq 0.84$. That S won't jump up to a price less than $p = 5$ follows. (All other deviations for S can be seen to be not optimal from the table.)

Finally, we show that μ is well defined. This follows since $V_B(0) > \delta^2 V_B(0)$ and $18.628915505\theta_B \leq \delta^2(19.04681505)\theta_B$.

Case 1(d)

$$(1 - \delta^2)V_B(p) = \frac{\theta_B(21\delta^2 24\delta^4 + 25\delta^6 + 24\delta^8 + 21\delta^{10})}{1 + \delta^2 + \delta^4 + \delta^6 + \delta^8 + \delta^{10}} = 18.971020073\theta_B. \tag{4.62}$$

$$(1-\delta^2)V_S(p) = \frac{\delta\theta_S 21 + (1+\delta\theta_S)(24\delta^2 + 25\delta^4 + 24\delta^6 + 21\delta^8)}{1+\delta^2+\delta^4+\delta^6+\delta^8}$$

$$= 22.769440015\theta_S + 18.62891505. \tag{4.63}$$

Checking Deviations:

Firm B: (note only deviations that don't immediately follow from the table are presented).

$p = 4$. For B not to prefer setting $p = 3$ we require $19.110670061\theta_B \geq 19.011397742\theta_B + 0.4179(1-\theta_B)$ or $\theta_B \geq 0.81$.

$p = 5$. For B not to prefer setting $p = 4$ we require $19.227868826\theta_B \geq 19.110670061\theta_B + 0.4776(1-\theta_B)$, or $\theta_B \geq 0.80296$.

$p = 6$: For B not to prefer setting $p = 5$ we require $19.322833369\theta_B \geq 19.227868826\theta_B + 0.4975(1-\theta_B)$, or $\theta_B \geq 0.839711441$.

$p = 7$: For B not to prefer setting $p = 6$ we require $19.35620093\theta_B \geq 19.322833369\theta_B + 0.4776(1-\theta_B)$, or $\theta_B \geq 0.934682577$.

$p = 8$: For B not to prefer matching requires $19.35620093\theta_B + 0.4179(1-\theta_B) \geq 0.31184\theta_B + 18.971020013\theta_B$, which clearly holds.

$p = 9, 10$ For B not to prefer setting $p = 8$ requires $19.35620093\theta_B + 0.4179(1-\theta_B) \geq 0.31184\theta_B + 18.971020013\theta_B$, which again clearly follows.

Firm S: Note S will clearly never match from the table, nor will it undercut by more than a grid size. As well, upon observing $p > 9$, setting $p = 7$ is prefered to any price ≥ 8, since B's response is the same, and $(1+\delta\theta_S)21 > 16$.

We need still to verify that S's best jump up is to $p = 7$. For S not to prefer jumping to $p = 6$ we require $18.62891505 + 22.769440015\theta_S \geq 0.4728224\theta_S +$

$\delta^2(18.90578256 + 22.790334650\theta_S)$ or $\theta_S \leq 2.47$.

For S not to prefer jumping to $p = 5$ we require $18.62891505 + 22.769440150\theta_S \geq 0.4875997508\theta_S + \delta^2(18.78204566 + 22.750545550\theta_S)$, or $\theta_S \leq 14$.

Finally, we need to verify that μ is well defined. Clearly $V_S(0) > \delta^2 V_S(0)$. We need to show that $18.66778343 + 22.316328470\theta_S \geq 18.62891505 + 22.769440150\theta_S$ or $\theta_S \leq 0.086$.

Focal Price Equilibrium

To aid in checking deviations, we make the following claims:

(1) Neither firm will jump above the price of the other firm to a price $p > 5$. This clearly follows, since setting $p = 5$ yields at least $\delta\frac{25\theta_i}{1-\delta} > \delta^2\frac{25\theta_i}{1-\delta}$.

(2) Neither firm will jump above the price of the other firm to a price $p \in 2, 3, 4$. Follows since setting p in this range yields $\delta^2 V_i(1) \leq V_i(1)$ which is at least what would be obtained by following the specified strategies.

(3) Given that firm i sets a price strictly below that of the other firm, it prefers to set $p = 1$ rather than $p = 0$. Follows since $\delta\frac{\theta_i 25}{1-\delta} + (1 - \theta_i)9 \geq \delta^2 > \frac{\theta_i 25}{1-\delta}$.

Now we check deviations: $p = 4$. Following the specified strategy yields $\delta\frac{\theta_i 25}{1-\delta} + (1 - \theta_i)9$. Setting $p \in 2, 3$ yields no more than $21 + \delta^3\frac{\theta_i 25}{1-\delta}$. For this not to be optimal requires $s1 \leq (1 - \theta_i)9 + 25\delta\theta_i + 25\delta^2\theta_i$ or $\theta_i \geq 0.2981$. Setting $p = 4$ yields $24\theta_i + \delta^3\frac{\theta_i 25}{1-\delta}$, so that this deviation is not optimal. Clearly, setting $p = 1$ is prefered to $p = 5$. The rest of the deviations follow from (1), (2), and (3) above.

$p = 2, 3$. Follows from arguments used in the previous case.

$p = 1$. We need to verify that i is indifferent between setting $p = 5$ and $p = 1$. i.e. we require

$\delta\frac{\theta_i 25}{1-\delta} = \theta_i 9 + \mu_j\theta_i 9\delta + \mu_j\delta^2(\delta\frac{\theta_i 25}{1-\delta}) + (1 - \mu_j)9\delta + \delta^2(1 - \mu_j)(1 + \frac{\delta}{1-\delta})\theta_i 25$, which yields approximately $\mu_j = \frac{8.91-15.75\theta_i}{15.5925\theta_i+8.91}$. So $\mu_j > 0$ if $\theta_i \leq 0.56$. (approximately).

$p = 5$: If say $p = 5$ receive $\frac{25\theta_i}{1-\delta}$. If say $p =$ get $24 + \delta^3 \frac{25\theta_i}{1-\delta}$. We require $24 \leq 74.2575\theta_i$ or $\theta_i \geq 0.32$.

$p > 5$: Clearly.

Table 4.8: Value Functions for the Examples:

	(a)	(b)		(c)		(d)	
	$(1-\delta^2)V^i(p)$	$(1-\delta^2)V^B(p)$	$(1-\delta^2)V^S(p)$	$(1-\delta^2)V^B(p)$	$(1-\delta^2)V^S(p)$	$(1-\delta^2)V^B(p)$	$(1-\delta^2)V^S(p)$
p_{10}	16.25555423	$6.356400288+ 11.151859520\theta_B$	$17.68982483+ 11.264504560\theta_S$	$0.4179(1-\theta_B)+ 19.046815050\theta_B$	$19.00715748(1+.99\theta_S)$	$0.4179(1-\theta_B)+ 19.35620930\theta_B$	$19.04681505+ 22.76944033\theta_S$
p_9	16.25555423	$6.356400288+ 11.151859520\theta_B$	$17.68982483+ 11.264504560\theta_S$	$0.4179(1-\theta_B)+ 19.046815050\theta_B$	$19.00715748(1+.99\theta_S)$	$0.4179(1-\theta_B)+ 19.35620930\theta_B$	$19.04681505+ 22.76944033\theta_S$
p_8	16.25555423	$6.356400288+ 11.151859520\theta_B$	$17.68982483+ 11.264504560\theta_S$	$0.4179(1-\theta_B)+ 19.046815050\theta_B$	$19.00715748(1+.99\theta_S)$	$0.4179(1-\theta_B)+ 19.35620930\theta_B$	$19.04681505+ 22.76944033\theta_S$
p_7	16.20016941	$6.536675898+ 10.900687440\theta_B$	$17.68982483+ 11.264504560\theta_S$	$0.4776(1-\theta_B)+ 19.007157480\theta_B$	$19.00715748(1+.99\theta_S)$	$19.35620930\theta_B$	$19.00715749+ 22.80963099\theta_S$
p_6	16.15922769	$6.059075898+ 11.378287440\theta_B$	$17.56170221+ 11.010795390\theta_S$	$19.007157480\theta_B$	$18.90578256(1+.99\theta_S)$	$19.32283369\theta_B$	$18.90578256+ 22.79033465\theta_S$
p_5	16.04180125	$6.182099682+ 11.122015550\theta_B$	$17.41067515+ 11.234359140\theta_S$	$18.905782560\theta_B$	$18.78204520(1+.99\theta_S)$	$19.22786826\theta_B$	$18.78204526+ 22.75054551\theta_S$
p_4	15.97972420	$6.238226673+ 10.929937520\theta_B$	$17.41067515+ 11.234359140\theta_S$	$18.782045260\theta_B$	$18.67609964(1+.99\theta_S)$	$19.11067061\theta_B$	$18.67609964+ 22.73004949\theta_S$
p_3	15.88022422	$5.820324132+ 11.347873752\theta_B$	$17.33779732+ 11.040340920\theta_S$	$18.676099640\theta_B$	$18.62891505(1+.99\theta_S)$	$19.01139742\theta_B$	$18.62891505+ 22.76944015\theta_S$
p_2	15.87779227	$5.938500288+ 11.151859520\theta_B$	$17.33779732+ 11.040340920\theta_S$	$18.628915050\theta_B$	$18.62891505(1+.99\theta_S)$	$18.97102073\theta_B$	$18.62891505+ 22.76944015\theta_S$
p_1	15.87779227	$5.938500288+ 11.151859520\theta_B$	$17.33779732+ 11.040340920\theta_S$	$18.628915050\theta_B$	$18.62891505(1+.99\theta_S)$	$18.97102073\theta_B$	$18.62891505+ 22.76944015\theta_S$
p_0	15.87779227	$5.938500288+ 11.151859520\theta_B$	$17.33779732+ 11.040340920\theta_S$	$18.628915050\theta_B$	$18.62891505(1+.99\theta_S)$	$18.97102073\theta_B$	$18.62891505+ 22.76944015\theta_S$

Chapter 5

Price Cycles and the Response to Wholesale Prices

5.1 Introduction

In recent years, there has been considerable public and government interest concerning the competitiveness of retail gasoline markets. In particular, there has been concern that retail gasoline prices in various markets respond more rapidly to cost increases than to decreases, and that this behavior is suggestive of market power. The allegation typically made by private citizens is that when costs increase, firms respond quickly, maintaining the original price-cost margin, but that when cost falls, the previous retail price provides a natural focal price for the firms, so that high prices are maintained in the short term. In Canada, this question has been most recently addressed in studies commissioned by the Competition Bureau (Hendricks (1996), Lermer (1996)), which find that for a subset of

Canadian markets retail prices do not respond more rapidly to crude oil price and wholesale price increases than to decreases. A large academic literature looking for such an asymmetry in the behavior of retail gasoline prices in various countries has also developed. Representative studies include Borenstein *et al*(1997) and Bacon(1991).

These studies typically assume a long run relationship between the retail price and cost, and estimate the convergence path of the retail price to the long run level following a shock. However, for Canadian retail gasoline prices the existence of specific patterns in the time series for different cities potentially complicates analysis. Retail prices in cities across Canada typically exhibit one of two patterns. In cities in the west or in Atlantic Canada, retail prices are often constant for many months at a time, changing by large amounts in a single week. On the other hand, many cities in Ontario and Quebec exhibit cycles in the retail price not present in the wholesale price or the crude oil price. These cycles consist of two phases: an undercutting phase, and an increasing phase. Over the undercutting phase, which lasts from one week to greater than a month, firms slowly lower prices until the bottom of the cycle, which is typically near marginal cost, is reached. At this point, the increasing phase is initiated, in which price increases in a single week by much larger amounts, often by as much as four or five cents in a week, until the top of the cycle is obtained. In earlier work, in a discussion of the Canadian retail gasoline industry, show that these cycles are consistent with equilibrium.

In this article, we examine the response of retail prices to wholesale prices in the presence of a price cycle. Specifically, we study weekly retail gasoline prices for the city of Windsor for the period from November 27, 1989 to September

25, 1994. Over this period, prices cycle as described above with a cycle height of approximately four cents. Prices increase by more than two cents per week on average, and decrease by less than one cent per week on average. As well, while all but two increasing periods take only one or two weeks, over half of the undercutting phases observed in the data take strictly longer than two weeks.

We opt for a case study of a single city, as opposed to examining all cities with price cycles, for two distinct reasons. First is that, while a number of cities exhibit cycles, only four have cycles for a time period long enough to make a study feasible. Montreal, Quebec city and Ottawa all have cycles, but only for the last two years of the available data. Secondly, the weekly frequency of the data makes an examination of other cities infeasible. For example, in Toronto, the cycle has a period of approximately two weeks. Therefore, we would be unable to study the behavior of prices over the different parts of the cycle using only weekly data; higher frequency data (e.g. daily) would be required. On the other hand, the stark asymmetry of the cycle, as observable in weekly price data, makes prices in Windsor particularly well suited for study.

In the first part of our analysis, we document the qualitative features of the price cycle in Windsor. We then develop and estimate a simple econometric model of price cycles, based on the theoretical discussion of price cycles in gasoline markets given in chapter 4. In particular, we suppose that the average retail price falls until a lower bound is reached, at which point firms increase prices and a new cycle begins. The theory suggests that the lower bound, or trough of the cycle, should be determined by the rack price. During the undercutting phase, the theory predicts that a firm's retail price change is determined primarily by the size of the change required to steal a rival's customers, and that price decreases should therefore be insensitive to cost. However, the magnitude of price increases are expected to depend upon the rack price. These predictions are supported by the data.

Finally, we compare the results from our model to those obtained by estimating the alternative model most often used in the literature. The most influential article within this literature is Borenstein *et al* (1997), which studies the response of retail gasoline prices to wholesale prices and crude oil prices in the United States. In this article the authors take a descriptive approach, specifying a very general dynamic model, in which the current change in the retail price is supposed to depend differently on a past retail or wholesale price change according to whether this change was positive or negative. The authors find that retail prices respond more rapidly to wholesale price increases and crude oil price increases than to decreases. Estimating the model of Borenstein *et al* on the data for Windsor identifies a similar asymmetry. This asymmetry is consistent with the behavior identified by the model developed in our paper. Retail prices are found to be more sensitive to the wholesale price when it is high than when it is low, because when the wholesale price is high, the retail price is more likely to increase than decrease, and prices are found to be more sensitive to cost during the increasing phase of the cycle than during the undercutting phase.

Our findings have a clear implication for policy. Typically, when retail prices are found to respond more rapidly to wholesale price increases than to decreases, this asymmetry is taken to suggest the presence of oligopolistic co-ordination or collusion.[1] In contrast, we identify a similar asymmetry that is based upon a very different model. Retail price decreases are less sensitive to cost than increases, because over the undercutting phase of the cycle, firms are behaving very aggressively, setting prices not to reflect cost, but so as to steal their opponent's customers. When cost goes down, the resulting price change will be insensitive to this cost change, not because the firms are colluding, but because firms are more likely to be in the undercutting phase of the cycle, in which their price changes are determined largely by the sensitivity of consumers to price differences across stations. That is, retail prices will be less sensitive to cost decreases because the cost decrease will likely lead to a battle between the firms over market share, during which their pricing behavior will not be reflective of costs. Therefore, using evidence of asymmetry as a motivation for policy aimed at reducing collusion may be misguided, as the asymmetry may in fact be resulting from very aggressive pricing behavior.

The remainder of this paper will proceed as follows. Section 2 briefly discusses the data set and the qualitative features of the retail price cycle. Section 3 outlines a theoretical model of retail price cycles, while an econometric model is developed in section 4. Estimation results from the econometric model are discussed in section 5. In Section 6, we estimate the model in Borenstein *et al* using the data for Windsor. Section 7 discusses the relationship between the results from our model and that of Borenstein *et al.* Section 8 concludes.

[1]Occasionally other explanations are also considered. See for example Borenstein *et al* (1997).

5.2 Data Set and Preliminary Analysis

The purpose of this section is to document the qualitative features of the retail and wholesale price data for the city of Windsor, for the period November 27, 1989 to September 26, 1994. Throughout, we define p_t and r_t as the retail and wholesale prices at time t, and Δp_t and Δr_t as the corresponding first differences. The 253 observations on the nominal before-tax retail price and wholesale price are plotted in Figure 5.1, and summary statistics for these two series are given in Table 1. The retail and wholesale gasoline price data used in this study were obtained from the Ontario provincial government. The before-tax retail price series consists of a weekly average of the price charged by a sample of Windsor stations. The wholesale price we use is the Toronto unbranded rack price, which is the published price charged at Toronto terminals to large-scale purchasers of unbranded wholesale gasoline. The unbranded rack price is considered the best available measure of the marginal cost of a unit of gasoline. [2] Rack prices for Windsor are unavailable but examination of other rack prices in Ontario, Quebec, and the North-Eastern United States reveals almost no differences between the series.

[2] According to the report by Imperial Oil Limited, submitted to the 1996 British Columbia Inquiry into Gasoline Pricing, "Independent non-major branded retailers (such as Supersave, Superstore, and 7-11) usually purchase their gasoline based on rack prices." A measure of an intermediate price that would be relevant for stations selling major brand gasoline was not available. However, Lermer, in his 1996 report to the Competition Bureau of Industry Canada, notes that the price at which independents purchase gasoline is the opportunity cost of the gasoline that a the refiner delivers to its wholly owned and wholly managed stations, and to dealers at owned and leased stations.

The key feature of the data is the difference in the week to week behavior of the retail and rack prices. The rack price series is relatively smooth, with upward and downward movements appearing to be symmetric. Eighteen percent of the first differences are equal to zero, 43 percent are positive, and 39 percent are negative. As well, the mean positive first difference is equal to 0.45, while the mean negative first difference is 0.44, suggesting that the magnitude of positive and negative changes are not different on average (see Table 5.1). Finally, we examine the distribution of the length of runs up and down. Define a run up as a length of time over which the series is strictly increasing, and define a run down similarly. Table 5.2 provides the number of runs up and down of various lengths for the rack price series. From Table 5.2, we see that the distribution of runs up and down appear to be similar. The percentages of runs up and down that are of one or two weeks are 60 and 62 respectively and the percentages of runs up and down of length strictly greater than one month are 10 and 18 respectively. Using a Komolgorov-Smirnov test of the equality of the distributions, we cannot reject the null for any reasonable level of significance.[3] From this descriptive analysis, we conclude that the rack price series is characterized by a large number of zero first differences, positive and negative changes that are of approximately the same magnitude, and runs up and down that appear to be drawn from the same population.

[3] Let $S_U(l)$ and $S_D(l)$ be the sample distributions of runs up and runs down. We wish to test the null hypothesis that the length of runs up or down are generated by the same distribution function, against the alternative that $F_U(l) \leq F_D(l)$ for all l. The one sided test statistic is $D = max(S_U(l) - S_D(l)) = 0.09$, which is less than the five percent critical value of 0.22, so that we cannot reject the null hypothesis of symmetry.

In contrast, the most noticeable feature of the retail price data is the existence of cycles that are not observed in the wholesale price series(see Figure 5.1). In particular, retail prices jump up quickly, and fall back down slowly over time. The retail price series increases in 27 percent of the weeks in the sample, decreases for 68 percent of the weeks, and is constant for only 4 percent of the sample. An examination of the summary statistics of the first difference of the retail series highlights the importance of the fluctuations as well as the asymmetry of the cycle. These statistics are given in Table 5.1. The mean positive retail price change is 2.38 cents, implying that, on average, retail price increases are ten percent of the mean before-tax retail price. On the other hand, the mean negative change, in absolute value, is 0.98 cents, or less than four percent of the average price level, and approximately one third of the average price increase. Clearly, the magnitudes of both increases and decreases in the retail price greatly exceed those of the wholesale price, suggesting that retail price changes are not determined only by wholesale price changes.

The asymmetric pattern in retail prices can be further illustrated through an examination of the distribution of runs up and down. The number of occurences of runs up and down of various lengths are provided in Table 5.2. We find that 95 percent of runs up are of 2 weeks or less, and that there are no runs up longer than one month. In comparison, only 48 percent of runs down are two weeks or less, and one quarter of the runs are strictly longer than one month. Using a Komolgorov-Smirnov test, we clearly reject the null hypothesis that runs up and down are drawn from the same distribution, in favour of the alternative that the distributions are different.[4]

To conclude, the key feature of the data is the existence of an asymmetric cycle in the retail price series that is not present in the wholesale price. In particular, retail prices increase suddenly, on average by ten percent of the average price level. Prices then take as long as several months to decline, before another cycle is initiated.

5.3 The Theoretical Model

The econometric model we estimate is based on the stylized theoretical model of a retail gasoline market discussed in earlier work, which is an extension of the alternating move, infinite horizon, price setting duopoly model of Maskin and Tirole (1988). In this section, we briefly outline the model and discuss both the results of chapter 4 and the predictions of the theoretical model that can be taken to the data.

In the model, two firms, indexed S and B, have fractions θ_S and θ_B of the stations in the market. It is assumed that $\theta_B > \theta_S$. The model is of discrete time with an infinite horizon, and time is indexed by $t = 0, 1, ...$ Here, t is assumed to be a portion of a day. Firm B chooses a single price for all its stations in every even period, committing to that price for two periods. Firm S sets its price in odd periods. Gasoline is assumed to be a homogeneous good, and c denotes the

[4] We wish to test the null hypothesis that the length of runs up or down are generated by the same distribution function, against the one-sided alternative. The one sided test statistic is 0.47, which exceeds the five percent critical value of 0.20, so that we reject the null hypothesis of symmetry.

cost to firm i of purchasing and transporting a litre of gasoline from the wholesale market. Cost is assumed to be exogenous to the firms.

The demand faced by firm i depends on its size, as well as the prices of the two firms. In particular, firm i services all of the market if its price is less than firm j's price by at least some constant k, proportion θ_i of the market if both firms set the same price, and none of the market otherwise.[5]

In the paper, price cycle equilibria are constructed for different relative firm sizes. For cases where the firms are of approximately the same size, the price cycle equilibrium that exists is that constructed by Maskin and Tirole, in which firm i responds to its opponent's price according to the following reaction function:

$$R^i(p) = \begin{cases} \overline{p} & \text{for } p > \overline{p}, \\ p - k & \text{for } \overline{p} \geq p > \underline{p}, \\ c & \text{for } \underline{p} \geq p > c, \\ \left.\begin{array}{ll} c & \text{with probability } \mu(\delta) \\ \overline{p} + k & \text{with probability } (1 - \mu(\delta)) \end{array}\right\} & \text{for } p = c, \\ c & \text{for } p < c\ , \end{cases}$$

where $\underline{p}$ and $\overline{p}$ are functions of marginal cost and demand parameters. In this equilibrium, firms undercut each other in a battle for market share, until one of the firms sets a price sufficiently low. At this point, a war of attrition begins, in which each firm randomizes between setting price equal to marginal cost, and setting $\overline{p} + k$. Once a firm sets $\overline{p} + k$, a new sequence of falling prices begins. As

[5]In the paper, the prices that each firm can choose are restricted to a finite grid with a grid size equal to k. That is, if firm i sets a price p, and firm j wants to undercut p, the highest price it can set is $p - k$. This assumption ensures that an optimal undercut and overcut exist.

an example, Figure 5.2 plots a simulated time path for the average price when demand is given by $1-p$, $c=0$, and k is $1/10$.

As the relative size of the firms vary, similar cycle equilibria can be constructed. These cycles differ according to whether the large firm chooses to undercut or match its opponent's price at points along the downward portion of the cycle. For example, if θ_B is sufficiently near one, a cycle equilibrium can be constructed in which firm B always matches the price of the small firm as long as this price is above a lower bound.

For the purposes of taking this model to data, the following are key features of the equilibrium price path discussed above. For every price p set by its opponent, a firm can choose either to price below its opponent, serving the entire market, or to price above its opponent, temporarily relinquishing market share, in order to push prices up. The former option is optimal provided the opponent has set a price above marginal cost. Therefore, the cycle is naturally divided into regimes, the undercutting regime, in which firms price below each other in a battle over market share, and an increasing regime, in which firms cease battling for the market and instead choose to restore temporarily high prices. The undercutting regime is expected to apply until the retail price has fallen sufficiently low, relative to the wholesale price. Since firms lower prices gradually, but increase prices by large jumps, the cycle appears asymmetric, with prices falling for longer periods and by smaller increments than they rise.

Finally, the theory provides some intuition as to the role of costs on price changes over different portions of the cycle. One drawback of the theoretical model is that it assumes a wholesale price that is constant over time. We are aware of no attempts to generalize the theory in this direction, and such a generalization would likely prove difficult. However, the nature of the cycle suggests that we should expect prices in the undercutting regime to be insensitive to costs, while prices in the increasing regime may be more sensitive to costs. Suppose, for example, that there is an unanticipated increase in the wholesale price. If the current price lies above marginal cost, it is reasonable to expect that firms would continue to undercut each other until the new lower bound is reached. In addition, because undercuts are chosen as the amount required to steal the market, the amount by which the price falls should not be expected to reflect cost. However, if the marginal cost increase has resulted in price being below cost, we expect that a new increasing regime would result. In this case the size of the increase will likely depend on the distance between the current price and the new top of the cycle.

To conclude this section, we summarize the main suggestions of the theoretical model. First, it predicts that whether the retail price increases in a given week will depend upon whether last period's retail price is above or below a lower bound, which is an increasing function of the wholesale price. Secondly, retail price increases and decreases are expected to be different functions of the position of the cycle, and those variables that contribute to the sensitivity of consumers to price differences across firms. Decreases in the retail price are expected to be insensitive to the wholesale price, whereas increases may be more sensitive.

5.4 An Econometric Model

This section presents an econometric model based upon the discussion in the previous section. The model assumes that changes in the retail price are generated by two different underlying processes: one that generates positive price changes, and the second that generates price decreases. The first subsection presents the equations that determine which regime will apply and the magnitude of price changes in each regime. The second subsection presents the choice of explanatory variables for each equation in the model.

5.4.1 The Equations

Define p_t as the average retail price at time t, and Δp_t as $p_t - p_{t-1}$. The theoretical discussion in the previous section suggests the following simple switching regression model:

$$\underline{p}_t = z_t'\gamma + \eta_t, \tag{5.1}$$

$$I_t = \begin{cases} 1 & \text{if } p_{t-1} \leq \underline{p}_t, \\ 0 & \text{if } p_{t-1} > \underline{p}_t, \end{cases} \tag{5.2}$$

$$ln(|\Delta p_t|)^U = X_{Ut}'\beta_U + \nu_{Ut}, \tag{5.3}$$

$$ln(|\Delta p_t|)^D = X_{Dt}'\beta_D + \nu_{Dt}, \tag{5.4}$$

and

$$ln(|\Delta p_t|) = \begin{cases} ln(|\Delta p_t|)^U & \text{if } I_t = 1, \\ ln(|\Delta p_t|)^D & \text{if } I_t = 0, \end{cases} \tag{5.5}$$

where X_{Ut}, X_{Dt} and z_t contain explanatory variables, and ν_{Ut}, ν_{Dt} and η_t are distributed as trivariate normal with mean zero and covariance matrix Ω.

This model describes a simple price cycle. $\underline{p}_t$ represents the lower bound, or the bottom of the cycle[6]. I_t is a variable that equals one if the retail price increases at time t, and zero if it decreases. Expression (5.2) says that the series decreases as long as the retail price from the previous period is greater than the current lower bound. Once the lower bound is reached, the series increases. After a single increase, the retail price resumes decreasing, provided the lower bound has not increased to above the price from the previous period. The magnitudes of retail price increases and decreases are determined by two different equations. For every set of circumstances, there is a price decrease that would be set if the firms decided to compete over market share, pushing the average price down, and a price increase that would result if the firms decided to temporarily relinquish market share so as to push prices up. The magnitude of a retail price decrease is given by $ln(|\Delta p_t|)^D = X'_{Dt}\beta_D + \nu_{Dt}$, while the size of a price increase is determined by $ln(|\Delta p_t|)^U = X'_{Ut}\beta_U + \nu_{Ut}$. The purpose of the lognormal specification is to maintain logical consistency across equations.[7] If the dependent variables were specified as Δp_t, then the assumption of normal errors implies that the equation could in fact predict positive price changes over the decreasing part of the cycle. In addition, the lognormal assumption seems reasonable upon examination of the distributions of positive and negative price changes. Finally, the observed price change at time t is given by (5.5).

[6]We assume that firm's choose price for period t at the beginning of the period, so that the lower bound will depend on time t observations

[7]See Cragg (1971) for discussion.

One concern about the model as stated is that, from our descriptive analysis of the price cycle, we know that in the data the increasing portion of the cycle frequently takes more than one week. This suggests that the assumption in the theory that the time period is portion of a day may not be appropriate, and that a more realistic assumption would be a time period of closer to a week. Because of this, the assumption that upon reaching the lower bound the retail price increases the entire height of the cycle in a single period seems inappropriate, and that in fact due to the averaging, we observe the price at a point when some firms have increased price and others have not. To this end, we introduce a modification to the econometric model. The model currently assumes that once the retail price begins to fall, it continues to decrease until it reaches the lower bound. Similarly, we assume that once the retail price begins to increase, it keeps increasing until it has passed an upper bound, representing the top of the cycle. This would arise if during a week some firms have increased their price, but others have not yet followed. Because of data limitations, we restrict the distance between the top and bottom of the cycle to be equal to a constant, which we label α.[8] Condition (5.2) can then be rewritten as

$$I_t = \begin{cases} 1 & \text{if } p_{t-1} \leq \underline{p}_t + \alpha I_{t-1}, \\ 0 & \text{if } p_{t-1} > \underline{p}_t + \alpha I_{t-1}, \end{cases} \tag{5.6}$$

That is, if the retail price increased in the previous period, it will continue to

[8] A model allowing the height of the cycle to depend upon other explanatory variables was also estimated. The hypothesis that the coefficients on all additional variables were equal to zero could not be rejected at any reasonable significance level. See Slade (1997) for an application in a different context, in which similar lower and upper bounds are estimated, allowing all coefficients to differ.

increase until it reaches the lower bound plus the constant α, which represents the height of the cycle.

Finally, we introduce a transformation of the condition determining whether the retail price will increase or decrease. Combining (5.1) and (5.4) yields the condition that the retail price will increase provided $p_{t-1} - z_t'\gamma - \alpha I_{t-1} \leq \eta_t$ and decrease otherwise, where η_t is normal with mean zero and unconditional variance σ^2. This can be rewritten, by dividing both sides by σ and reversing the inequality, as $z_t'\tilde{\gamma} + \tilde{\alpha} I_{t-1} - \tilde{\delta} p_{t-1} \geq \epsilon_t$, where ϵ is unconditionally distributed standard normal, and $\tilde{\gamma} = \gamma/\sigma$, $\tilde{\alpha} = \alpha/\sigma$, and $\tilde{\delta} = 1/\sigma$.

The model which we choose to estimate is expressed by a condition determining when prices increase or decrease, and equations determining the size of the price change:

$$I_t = \begin{cases} 1 & \text{if } z_t'\tilde{\gamma} + \tilde{\alpha} I_{t-1} - \tilde{\delta} p_{t-1} \geq \epsilon_t, \\ 0 & \text{if } z_t'\tilde{\gamma} + \tilde{\alpha} I_{t-1} - \tilde{\delta} p_{t-1} < \epsilon_t, \end{cases} \tag{5.7}$$

$$ln(|\Delta p_t|)^U = X_{Ut}'\beta_U + \nu_{Ut}, \tag{5.8}$$

$$ln(|\Delta p_t|)^D = X_{Dt}'\beta_D + \nu_{Dt}, \tag{5.9}$$

and

$$ln(|\Delta p_t|) = \begin{cases} ln(|\Delta p_t|)^U & \text{if } I_t = 1, \\ ln(|\Delta p_t|)^D & \text{if } I_t = 0, \end{cases} \tag{5.10}$$

where ν_{Dt}, ν_{Ut} and ϵ_t are trivariate normal with mean zero and covariance matrix Σ, where

$$\Sigma = \begin{pmatrix} \sigma_U^2 & 0 & \sigma_{U\epsilon} \\ 0 & \sigma_D^2 & \sigma_{D\epsilon} \\ \sigma_{U\epsilon} & \sigma_{D\epsilon} & 1 \end{pmatrix}.$$

We suppose that ν_{Dt} and ν_{Ut} are uncorrelated, as the covariance can not be identified. $\sigma_{U\epsilon}$ and $\sigma_{D\epsilon}$ can be identified provided that appropriate restrictions are made. These restrictions are discussed in the following section.

5.4.2 Explanatory Variables and the Estimation Procedure

The choice of explanatory variables in the lower bound equation is based on the theoretical model and anecdotal evidence. As suggested by the theory, the present wholesale price is included.[9] Dummy variables for the second, third and fourth quarters (Q_2, Q_3, and Q_4) are included to pick up seasonal demand and cost effects and a linear time trend is included to pick up the effect of any omitted trend. We also need to control for the Gulf war. The Gulf war crisis consisted of two distinct events: the sudden jump in crude oil prices on the occasion of the invasion (in the first week of August 1990) and the equally sudden and dramatic decrease in crude oil prices once it became apparent that the war would be short(beginning in the first week of January 1991).[10] Since it is possible that the response of

[9]We also consider including lagged costs, and first differences of costs, in the lower bound function. Testing down on the coefficients on past costs or first differences in the lower bound equation, all lags are removed.As well, no results were affected by the inclusion of past cost terms.

[10]See, for example, the Petro Canada Annual Reports for 1990 and 1991 for discussion.

product prices to each event may have been fundamentally different[11] we include two separate dummy variables: $BEFORE_t$, which equals 1 for the months of August to December 1990, and $AFTER_t$, which equals 1 for the months of January to May of 1991. The beginning and ending dates used in these dummies are based on approximate dates given in the Petro-Canada Annual Reports.

Due to the lack of strong theoretical predictions concerning the magnitude of price changes, we take a more descriptive approach with the second stage equations. Explanatory variables are chosen on the basis of the theory and anecdotal evidence. First, we control for the position of the current price in the cycle. This is defined by the distance between the previous retail price and the current lower bound. We anticipate that when prices are near the top (bottom) of the cycle, price increases (decreases) will be smaller. To maintain generality, we include all of the variables in the lower bound equation rather than the estimated bound itself.[12] This method allows for the possibility that these variables have effects other than through the lower bound. In particular, the size of price changes is also a function of the sensitivity of consumers to price differences across firms. In the theoretical model, this is clearly true of price decreases, most of which are equal to the minimum decrease required to attract the entire market. Price increases are also a function of this sensitivity, although the exact effect is less clear. The quarterly dummies are expected to pick up the effects of seasonal changes in the nature of

[11] Sorenson and Dougher (1991) document and discuss the fact that in the U.S. the rack price jumped above the spot price of gasoline at the time of the invasion, but fell below after the drop in crude oil prices.

[12] A more structural model would include the predicted lower bound, continuous on the direction of the price change. This is a likely direction of future work.

traffic on consumer sensitivity. Likewise, any trends in consumer sensitivity will be captured by the time trend. The war period may have had an affect on the pricing policies of firms, and thus the war dummies are also included in the price change equations. Finally, as in other econometric models of gasoline prices, we allow for the possibility that retail price changes reflect current and past changes in wholesale prices. Since one of the purposes of this paper is to examine how the response of retail prices to wholesale prices differs according to whether prices are increasing or decreasing, we consider specifications that include current and past first differences in cost.

One concern regards the explanatory variables in the price change equation and the assumptions about the covariance matrix Ω. As discussed above, we expect the magnitude of price changes to depend upon the distance between the previous retail price and the bottom of the cycle, measured by $p_{t-1} - \underline{p}_t$, or $p_{t-1} - z_t'\gamma + \eta_t$. If we estimate the reduced form equation (i.e. by including in the price change equations the variables in z_t), then the residuals in the price change equations, ν_{Ut} and ν_{Dt} will contain η_t. Therefore, ν_{Ut} and ν_{Dt} are clearly expected to be correlated with η_t. This introduces an identification problem. To identify the correlation between the errors, we require a variable which determines whether the retail price increases or decreases, but not by how much. In this model, that variable is the term αI_{t-1}. This exclusion restriction allows us to identify the model.

We estimate the above model using a two stage procedure, as outlined in Maddala(1983, section 8.3). In the first stage, we estimate equation (7), which is simply a probit. Dividing the estimates of $\tilde{\gamma}$ by $\tilde{\delta}$ yields estimates of the original parameters γ. The delta method is used to obtain approximate standard errors.

In the second stage, we estimate the parameters in (5.8)and (5.9). We run two separate regressions, one over all positive retail price changes, and the other over all negative price changes. Note that, conditional on the retail price increasing, the expectation of ν_{Ut} is non-zero since $\sigma_{U\epsilon} \neq 0$. Under the assumption of normality, and defining $w_t'\psi = z_t'\tilde{\gamma} + \tilde{\alpha}I_{t-1} - \tilde{\delta}p_{t-1}$,

$$E(\nu_{Ut}|w_t'\psi \geq \epsilon_t) = -\sigma_{U\epsilon}\frac{\phi(w_t'\psi)}{\Phi(w_t'\psi)}. \tag{5.11}$$

Here, $\phi(\cdot)$ and $\Phi(\cdot)$ are the standard normal density and cumulative distribution functions. Similarly,

$$E(\nu_{Dt}|w_t'\psi < \epsilon_t) = \sigma_{D\epsilon}\frac{\phi(w_t'\psi)}{1-\Phi(w_t'\psi)}. \tag{5.12}$$

Denoting $\hat{\psi}$ as the first stage estimate of ψ, the inverse Mill's ratio is defined as

$$IMR_t = \begin{cases} \frac{\phi(w_t'\hat{\psi})}{\Phi(w_t'\hat{\psi})} & \text{if } I_t = 1, \\ -\frac{\phi(w_t'\hat{\psi})}{1-\Phi(w_t'\hat{\psi})} & \text{otherwise.} \end{cases} \tag{5.13}$$

We include IMR_t in each second stage regression to control for the fact that the conditional expectations of ν_{Ut} and ν_{Dt} are non zero. The estimated coefficients on IMR_t from the regressions over positive and negative price changes are estimates of $-\sigma_{U\epsilon}$ and $\sigma_{D\epsilon}$. Finally, we follow the procedure outlined in Maddala(1983) to construct consistent estimates of the covariance matrices of the parameter estimates for each regression.

One difficulty that arises is the small number of zero first differences, for which the log transformation does not exist. We consider two ways of dealing with these observations. One method consists of dropping the zero observations from the sample, and estimating the first and second stages using the remaining observations. In the second method, we estimate two separate first stage probits, one in which the binary variable equals one if the series is strictly increasing, and one in which the binary variable equals one if the series is strictly decreasing. The inverse Mill's ratio from the first is used in the second stage regression involving positive changes, and the inverse Mill's ratio from the second in the regression involving negative changes. The results we report here are computed using the first approach, although both approaches give the same qualitative results.

5.5 Results

5.5.1 The Probit Equation

The results from the probit are presented in Table 5.3. Several results are noteworthy. The coefficient on I_{t-1} is significant with the expected positive sign, implying that once the retail price begins to increase it will continue to increase until an upper bound is reached. [13] The gulf war dummy variables are not jointly significant at the five percent level. A joint test of the hypothesis that the seasonal dummies are equal to zero does not reject the null at the five percent level, implying that seasonal demand movements do not change the probability of a price increase. The coefficient on r_t is positive and significant, suggesting that the lower bound

[13] More precisely, the interpretation of the positive coefficient on I_{t-1} is the following. It is more likely that p_{t-1} is below the top of the cycle than below the bottom. Therefore, *conditional on p_{t-1} and all other right hand side variables*, the probability that the price will increase is higher if it has increased in the previous period.

function depends positively on the rack price.

As an aid to interpreting the probit results, we compute the lower bound function. Recall that the coefficient on p_{t-1} is equal to $-1/\sigma$, and the estimated coefficient vector $\tilde{\gamma}$ is equal to γ/σ. Making this transformation allows us to write the estimated lower bound function:

$$\underline{p}_t = \underset{(3.96)}{4.67} + \underset{(0.17)}{0.82} r_t + \underset{(.006)}{0.001} t +$$
$$+ \underset{(1.65)}{2.44} BEFORE_t - \underset{(1.26)}{1.97} AFTER_t + \underset{(0.89)}{1.63} Q_{2t} + \underset{(0.92)}{1.44} Q_{3t} + \underset{(0.99)}{0.003} Q_{4t}.$$

Approximate standard errors obtained using the delta method are given in parenthesis. A one cent increase in the level of the rack price, holding its first difference constant, increases the lower bound by almost a cent, suggesting that the retail-rack margin is a reasonable measure of the current position of the cycle. Finally, the implied estimate of the height of the cycle is approximately 2.90 cents.

One concern with the approach taken is the assumption that the errors terms in the probit equation are independent. It is well documented that when lagged dependent variables are included, the parameter estimates will not be consistent under the presence of serial correlation. [14]. However, a test for serial correlation applied directly to the probit estimated above is precluded by the dropping of the occasional observation for which the first difference equals zero. To examine the possibility of serial correlation, we re-estimated the probit equation over the entire sample, assigning the zero first differences first to one group and then to the other. The null hypothesis of no serial correlation was then tested using the errors from each of the two probits. The test for serial correlation employed here is a test of

[14] See, for example, Maddala and Nelson (1975)

the conditional moment restriction

$$T^{-1}\sum_{t=1}^{T} E(\epsilon_t|I_t, z_t)E(\epsilon_{t-1}|I_{t-1}, z_t) = 0, \tag{5.14}$$

where I_t is the indicator variable that equals one if the series increases at time t. In computing the test statistic, the generalized errors were used for the expectations of the errors (see Gourieroux, Monfort, Renault and Trognon (1987) or Pagan and Vella (1989) for discussion). Define $m_t = \tilde{\epsilon}_t\tilde{\epsilon}_{t-1}$, where $\tilde{\epsilon}_t$ is the generalized error at time t. Define M to be the T X 1 vector with components m_t, and define D as the T X p matrix whose tth row consists of the derivatives of the log likelihood with respect to each of the p parameters, evaluated at the tth observation. Then the test statistics takes the form $MRT = i'M(M'M - M'D(D'D)^{-1}D'M)^{-1}M'i$, where i is the T X 1 unit vector.[15] The test statistic is distributed $\chi^2(1)$ under the null. In the two specifications estimated, the test statistic was equal to 1.9 and 5.6. Since the five percent critical value is 3.84, the evidence against serial correlation is somewhat mixed. However, we conclude that there is no strong evidence of serial correlation in the generalized errors.

5.5.2 2nd Stage Regressions

In this subsection we present our estimates of the retail price change equations. The results from both regressions are reported in Table 5.4. Column (1) gives the results for negative price changes, and column (2) reports results for positive price

[15]See Hall, Henry and Pemberton(1990) on the use of this test statistic in the presence of lagged dependent variables.

changes.[16] In each regression, the number of lags of cost changes was determined using a testing down approach (for details see Appendix). In both regressions, no first differences of the wholesale price were included in the final specification. This suggests that current and lagged rack price changes do not play an important role in determining the size of a retail price change, conditional on the inclusion of the current rack price.

The coefficients on the variables p_{t-1} and r_t are significantly different from zero in both regressions. Therefore, a model in which retail prices fall by an amount independent of wholesale price movements or the current position in the cycle is rejected. In both regressions, the hypothesis that the coefficients on p_{t-1} and r_{t-1} sum to zero cannot be rejected at the five percent level. This implies that retail price changes in both regimes are decreasing functions of the retail-rack margin. In other words, the amount by which retail prices fall is greater at the top of the cycle than at the bottom; likewise, the size of a retail price increase is greater at the bottom of the cycle.

The effect of the margin on the magnitude of price changes may have several

[16]One possible concern is that the residuals in each regime are serially correlated. Unfortunately, we are not aware of an good test under these circumstances. A runs test of the null hypothesis of randomness yields test statistics of 0.82 and -1.46 for the regressions on positive and negative changes. Compared to a 5 percent critical value of 1.96, we are unable to reject the null hypothesis that the residuals are randomly ordered. In addition, the estimated autocorrelations in the residuals are -0.15 and 0.15 using all observations (-0.15 and 0.27 using only adjacent observations, i.e. dropping observations separated by more than one week). These numbers are considered to be sufficiently small so that any effect of serial correlation, if it is present, would be negligible.

interpretations. The first is that the amount by which individual firms change price is indeed a function of the retail-rack margin. For price increases, this explanation supports the theory in that the amount by which a firm increases its price is a function of the distance between the current price and the top of the cycle, which would be a function of the wholesale price. However, the explanation that the amount by which individual firms decrease price is a function of the position in the cycle cannot be derived directly from the theoretical model.

A second possible explanation relies on the fact that the price data used is an average across firms, and not firm-specific. Consider first price increases, and suppose that some firms (e.g. independent brands) do not increase price in the same week as major brands, but in fact respond with a lag. If the presence of major brands in the sample is greater than that of independents, then we would see a larger increase in the first week when the margin is small, than in the second week when the margin is high. Similar reasoning can be applied to price decreases. If some firms stop undercutting at a higher price than others, then even though each firm decreases its own price by an amount independent of the margin, the decrease of the average price would be margin-dependent.

Next, we address the relative sensitivity of retail price changes to the retail-wholesale margin. To examine this issue, we compute the partial derivative of the unconditional predicted price increase and decrease for different values of the explanatory variables. In particular, setting all dummy variables equal to zero, time equal to the sample mean of 126, and the rack price equal to the sample mean of 23.2, we compute the partial derivatives with respect to cost, for different values of the previous retail price. Table 5.5 provides these partial derivatives as p_{t-1} takes on values from 23.7 to 26.7, which are approximately equal to the expected lower and upper bounds of the cycle. Here we see that the impact of a rack price change on a price increase is everywhere greater than the effect on a price decrease. In particular, the effect of a wholesale price change is an approximately equal change to the retail price on the upward portion of the cycle, while the effect on changes on the price-cutting portion of the cycle only one-tenth of the size.

Other notable results from the second stage regressions include the following. The inverse Mills ratio is not statistically significant in either regression, implying that there is no correlation between the error terms in the price change equations and the error term in the probit equation. The main effect of the war seems to be in a substantial increase in the rate at which prices fell in the latter half of the war period. The size of price decreases fell over time, while no such effect was found in price increases. Finally, the hypothesis that the coefficients on all seasonal dummy variables equal zero cannot be rejected at the five percent level.

The main results of the estimation can now be summarized. In support of the theory, the probability that a new cycle will begin at time t is decreasing in the distance between last period's retail price and this period's wholesale price. The change in the retail price is also decreasing in this margin, regardless of whether the series is currently on the upward or downward portion of the cycle. The effect of the margin on retail price decreases allows us to reject a model suggested in government documents in which price decreases are not a function of the position of the cycle or of wholesale price movements. However, the impact of the wholesale price on the retail price is greater when the series increases than when it is on the undercutting portion of the cycle.

5.6 Estimation of Asymmetric Response Model

In this section, we estimate the model presented in Borenstein *et al*(1997), using the data set described in the previous section. The results from this model will be compared to the results we obtain from our structural model in a later section.

Borenstein *et al* estimate an asymmetric error correction model in which the change in the retail price is a linear function of past retail price changes, past and current cost changes, and an error correction term. Although the authors assume that retail and rack prices in their data set are both I(1) and cointegrated, in this study, both series are assumed to be trend stationary. Unit root test results justifying this assumption are discussed in the Appendix. Despite the different assumptions, we estimate their model for purposes of comparison to the structural model presented previously.

In their model, the authors extend the standard error correction model by allowing the coefficients on past and present cost and retail price changes to differ depending on whether the change is positive or negative. Following their specification, we estimate the following reduced form error correction model:

$$\Delta p_t = \alpha_0 + \alpha_1 p_{t-1} + \alpha_2 r_{t-1} + \textstyle\sum_{i=0}^{n}(\beta_i^+ \Delta r_{t-i}^+ + \beta_i^- \Delta r_{t-i}^-)$$
$$+ \sum_{i=1}^{n}(\gamma_i^+ \Delta p_{t-i}^+ + \gamma_{t-i}^- \Delta p_{t-i}^-) + \alpha_3 t + \sum_{j=2}^{3} Q_j + \alpha_4 BEFORE_t + \alpha_5 AFTER_t. \tag{5.15}$$

Here, $\Delta p_t^+ = max\{0, \Delta p_t\}$, $\Delta p_t^- = min\{0, \Delta p_t\}$, and Δr_t^+, Δr_t^- are similarly defined. Following Borenstein *et al* we control for seasonality by including seasonal dummy variables, Q_{2t}, Q_{3t}, and Q_{4t} which equal one in the second, third, and fourth quarters. We include a linear time trend to ensure that our results are not due to an omitted trend. Finally, in contrast to Borenstein *et al*, we include the dummy variables $BEFORE_t$ and $AFTER_t$ to control for the Gulf war period.[17]

The lag length is chosen using the testing down procedure discussed in the Appendix, and a lag length of 1 week is used. Results for the final specification are given in Table 5.6. The reduced form coefficients imply a long-run relationship between the retail and wholesale prices given by $p_t = 6.3 + 0.93r_t$, setting time and all dummy variables equal to zero. Supposing the econometric model to be appropriate, this suggests that, in the long run a one cent change in the wholesale price leads to a retail price change of 0.93 cents. As well, the null hypothesis of complete passthrough cannot be rejected.

The regression results in Table 5.6 suggest that the immediate response of the retail price to wholesale price changes is greater for increases than decreases. The coefficient on current wholesale price increases is equal to 1.20 and not significantly different from one, whereas the coefficient on wholesale price decreases is equal to 0.24 and not significantly different from zero. Furthermore, the joint hypothesis that $\beta_0^+ = 1$ and $\beta_0^- = 0$ is not rejected. However, the evidence against symmetry is weakened by the lack of precision of the estimate of the coefficient on negative changes. The p-value from a test of the null hypothesis that $\beta_0^+ = \beta_0^-$ against the two tailed alternative is 0.09, so that the null is rejected at the ten percent level, but not at the five percent level.

As in Borenstein *et al* we compute the cumulative response paths to positive and negative changes in the wholesale price (see Borenstein *et al* for the com-

[17]The regression equation was also estimated excluding the war dummies, and similar qualitative results were obtained.

putation procedure). These paths are plotted in Figure 5.3. As can be seen, the cumulative response to a one cent change in the wholesale price exceeds one by the first week after the change for both positive and negative wholesale price changes. This analysis suggests that retail prices respond quickly to wholesale price changes, corresponding to Borenstein *et al* and Hendricks (1996). Further discussion of these results is deferred to the following section, when we compare them to the results of the structural model.

5.7 Comparison of Results

In this section we compare the predictions of the two models estimated above. To compare the results of our model to those of Borenstein *et al*, we first examine their relative ability to predict the observed price changes, and the occurrence of positive and negative price changes. In our model, the predicted retail price change is computed according to:

$$E[\Delta p_t] = E[\Delta p_t|\Delta p_t > 0]Pr[\Delta p_t > 0]$$
$$+E[\Delta p_t|\Delta p_t < 0]Pr[\Delta p_t < 0].$$

Note that all predictions are also conditional on the exogenous and predetermined variables at time t, but for notational convenience these arguments are suppressed.[18]

First, we consider the relative performance of the two models. For each model we compute the in-sample mean absolute error and root mean squared error in

[18]For discussions of analytic predictors such as the one used here, in non-linear models, see for example Mariano and Brown(1983) and Arvin-Rad(1997).

the prediction of current retail price changes.The mean absolute errors for the Borenstein *et al* model and our model are 1.22 and 1.28 respectively, for a ratio of 0.95, and the root mean squared errors are 1.72 and 1.9, for a ratio of 0.91. This suggests that while the performance of the two models is very similar, the model of Borenstein *et al* has a slight advantage in prediction. This difference is likely a result of several factors. First, one would expect the Borenstein model to have an advantage, as it allows for a very general functional form and minimizes the sum of squared errors, whereas our model does not. As well, the difference may be the result of the logarithmic functional forms assumed for the price change equations. As an additional means of comparison, we compute for each model the percentage of correct predictions of the direction of change. Note that for our model, this comparison utilizes only the probit equation. The percentage of correct predictions for the Borenstein *et al* model and our model are 66 and 76 percent respectively, indicating that our model is better at predicting the direction of change. We conclude that the performance is approximately the same for the two models.

Recall that the purpose of this paper is to examine the response of retail prices to rack prices, in the presence of a price cycle. From our estimation of the Borenstein *et al* model, it was found that the coefficient on positive rack price changes was substantially larger than the coefficient on negative price changes. This can be interpreted as saying the partial derivative of the expected price change with respect to the current wholesale price is greater when the current wholesale price is greater than the wholesale price from the previous period. For the purpose of comparison, we now compute for our model the partial derivative of the expected retail price change with respect to the current wholesale price.

In our model, a change in the rack price will affect each of the four terms on the right hand side of the equation defining the predicted retail price change. The partial derivative of the predicted retail price change with respect to the current rack price change is clearly positive, since the predicted price change in each regime is increasing in the rack price, as is the probability of a positive retail price change. However, the second partial derivative is not signable in general and we therefore consider an example [19]. As in the example considered previously, we set the rack price at time $t-1$ equal to the series mean (note that this value will not directly enter into computations). We set $t = 126$ and set all dummy variables equal to zero. The retail price at time $t-1$ is set equal to 25.2, which is approximately half way between the lower and upper bounds evaluated at the wholesale price at time $t-1$. The first difference of the rack price is allowed to vary from -1.5 to 1.5 cents per litre, which is approximately equal to the range observed in the data. This implies that the current rack price takes on values from 21.7 to 24.7.

The partial derivatives of the predicted retail price change with respect to the cost change at different points for this example are given in Table 5.7, along with the corresponding partial derivatives from the model of Borenstein *et al.* From Table 5.7 we see that, from our model, the predicted price change is clearly convex in the current wholesale cost change.[20] The slope of the predicted price change varies from 0.62 to 2.04, corresponding to the increase from 0.24 to 1.20 estimated from the Borenstein model. We note that the fact the slope from our model is

[19]Qualitative results remain the same when different examples are considered.

[20]This curvature could, in theory, be due to the assumed functional forms; however, when we replace the expected change in each regime with linear approximations, similar results are found.

greater than that from the Borenstein model at all points is a result of the specific example considered. For different examples, while the convexity is maintained, the ranking of slopes across the two models does not remain.

The convexity discussed above can be understood by examining the effect of the wholesale price on both the probability of a price increase, and the magnitude of price changes. First, we recall that the probability of a price increase was found to be increasing in the current cost level. Moreover, while both price increases and decreases are increasing in the cost level, the effect along the upward portion of the cycle is much larger. While the curvature of the predicted price change function is complicated by the assumed functional forms, we note that even if the probability of an increase and the functions determining the magnitude of price changes were linear the convexity would remain.

This example demonstrates a key relationship between the two models. The asymmetry found in Borenstein *et al* is assumed to be between positive and negative rack price changes. However, this asymmetry can also result from a model in which retail prices respond to a greater degree to rack price changes when on the upward portion of the retail price cycle than on the downward portion. That is, asymmetries between the responses over different portions of the price cycle can be mistaken for asymmetries in the response to positive and negative rack price changes.

5.8 Conclusions

In this article, we examine the relationship between retail and wholesale gasoline prices in a specific market in which a retail price cycle is observed. Based on theoretical foundations, we estimate a model in which the retail price is determined by two different regimes, depending upon whether the series is above or below an estimated lower bound. In one regime, retail prices always increase, while in the other they decrease. We find that the probability of a retail price increase is decreasing in the distance between retail and wholesale prices, and increasing in the first difference of the wholesale price. As well, both retail price increases and decreases are decreasing in the retail-wholesale margin, although the effect of the margin is greater on price increases than on decreases.

An important implication of this study is that, in the estimation of other models, the dynamics described above may appear to be an asymmetric response to wholesale price increases and decreases. In particular, if retail price increases are more responsive to the wholesale price than decreases, and if the probability of a retail price increase is increasing in the rack price, then the expected retail price change may be convex in the current wholesale price. This convexity would imply that the retail price is more sensitive to rack price increases than decreases. An examination of our results demonstrates that this is in fact the case. This result provides an alternative explanation behind the results of Borenstein *et al.* In particular, it suggests that the asymmetry in the retail-rack relationship may not be a result of production smoothing or sticky downward price adjustment in an oligopoly, but rather from the existence of a retail price cycle.

A further implication is that cross-sectional variations in the asymmetry of the response to wholesale price increases and decreases may be a function of the rate at which firms undercut each other in the downward portion of the cycle. For example, in a city in which a cycle exists with a period of approximately two weeks, one would expect that weekly retail price increases and decreases would *not* be different functions of the wholesale price, since the retail price declines by the entire height of the cycle in a single week. This in turn would imply that there would be no asymmetry in the response to positive and negative rack price changes. One example of this is the city of Toronto, which for much of the 1990's has experienced a retail price cycle in which prices increase one week and decrease the next. Indeed, Hendricks(1996) finds that for the period 1992-1996 there is no evidence of asymmetry in Toronto. A formal cross-sectional analysis of this proposition would require a panel data set of weekly prices for several cities exhibiting retail price cycles simultaneously, and is therefore not currently feasible.

A possible direction for future research is to compare the response of retail prices to rack prices in cities with price cycles to the dynamics in cities in which cycles are not exhibited. For example, Asplund *et al* (1997) discuss the relationship between wholesale and retail gasoline prices in Sweden, analyzing a data set in which retail prices changed in only 250 out of 5693 daily observations. Similar rigidity is observable in cities in Atlantic Canada and northern Ontario. Whether or not the two different patterns would lead to observable differences in the speed or asymmetry of adjustment could potentially be addressed in a cross-sectional analysis of available Canadian data.

Figure 5.1: Windsor Retail and Toronto Rack Nov 27 1989-Sept 26 1994

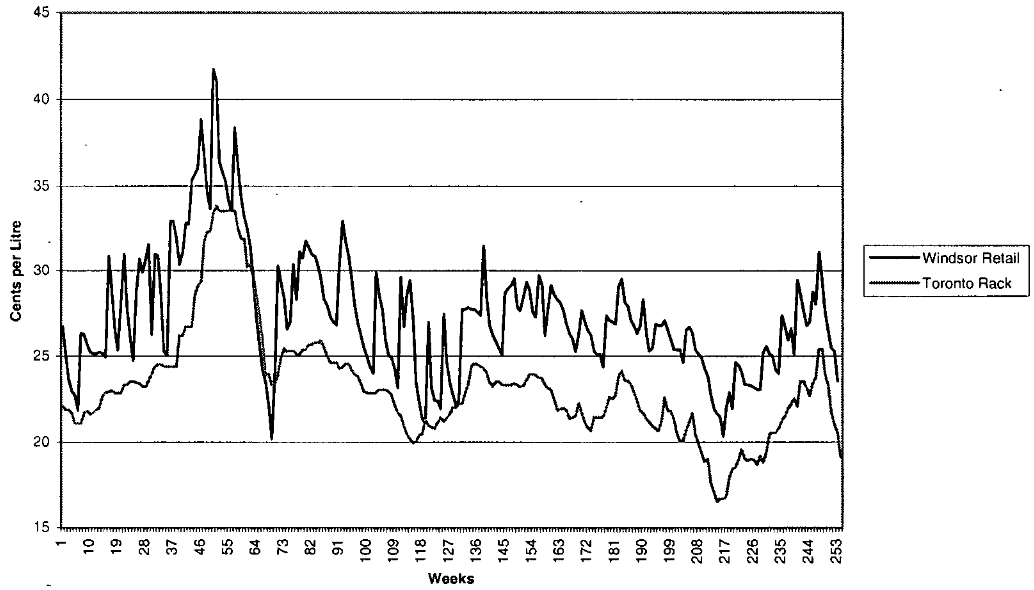


Figure 5.2: An Example

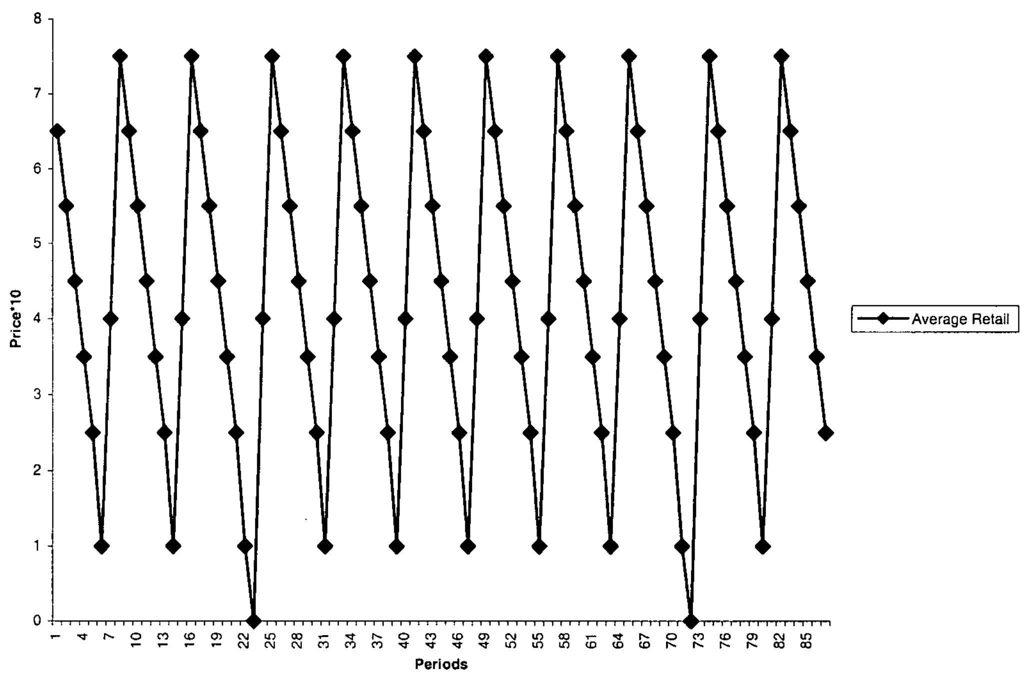


Figure 5.3: Cumulative Adjustments

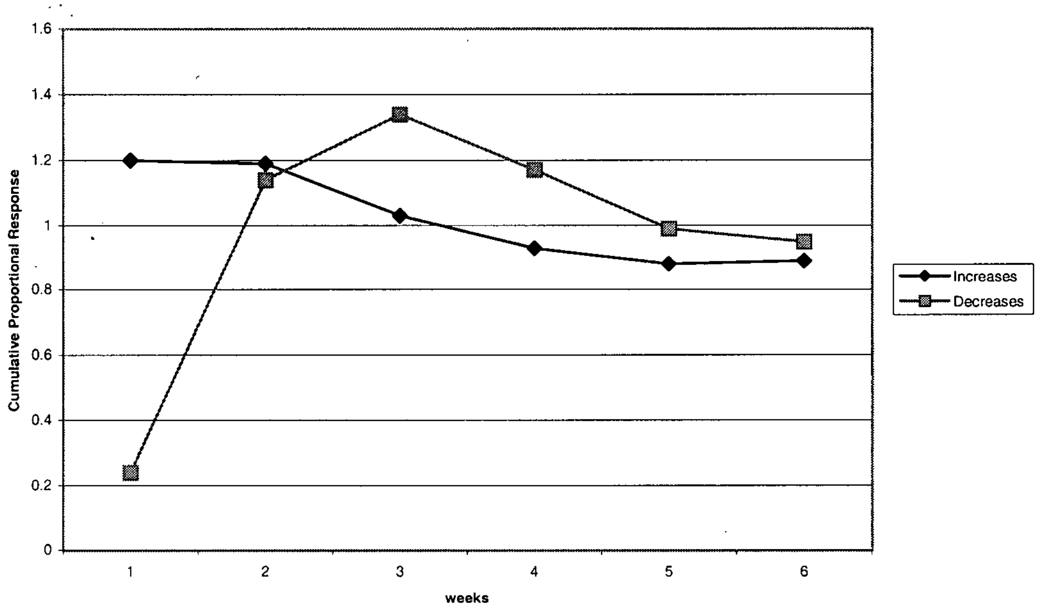


Table 5.1: Summary Statistics

Variable	Mean	Standard Deviation
p_t	27.4	3.6
r_t	23.2	3.3
$\Delta p_t (>0)$	2.38	2.0
$\Delta p_t (<0)$	-0.98	0.9
$\Delta r_t (>0)$	0.45	0.44
$\Delta r_t (<0)$	-0.44	0.37

Table 5.2: Runs Up and Down

	Rack Price		Retail Price	
Length	Up	Down	Up	Down
1	19	18	23	15
2	6	6	19	10
3	7	5	0	10
4	6	3	2	4
5	2	0	0	3
6	2	3	0	4
7	0	1	0	1
8	0	1	0	1
9	0	1	0	0
10	0	1	0	2
11	0	0	0	1
12	0	0	0	1
Total	42	39	44	52

Table 5.3: Probit Results

Variable	Coeff.	T-stat.
p_{t-1}	-0.336	-5.673
$BEFORE_t$	0.832	1.508
$AFTER_t$	-0.672	-1.551
I_{t-1}	0.968	4.021
Q_{2t}	0.548	1.885
Q_{3t}	0.485	1.552
Q_{4t}	0.002	0.003
CONSTANT	1.574	1.136
r_t	0.275	3.601
t	0.0003	0.218
Likelihood Ratio	49.3	
Test Statistic	(9 df)	

Table 5.4: 2nd Stage Regressions(dependent variable $\ln(|\Delta p_t|)$)

		(1)		(2)	
Variable		Coeff.	T-stat.	Coeff.	T-stat.
p_{t-1}		0.155	3.000	-0.251	-2.919
r_t		-0.147	-2.333	0.285	2.740
t		-0.003	-2.364	-0.003	-1.350
Q_{2t}		0.087	0.409	-0.231	-0.545
Q_{3t}		0.275	1.297	-0.340	-0.739
Q_{4t}		0.202	1.010	-0.053	-0.105
IMR_t		0.238	0.589	-0.190	-0.430
CONSTANT		-1.123	-1.180	1.216	0.604
$BEFORE_t$		0.183	0.431	-0.325	0.456
$AFTER_t$		0.925	2.946	-0.494	-0.959

Table 5.5: Partial Derivatives of Price Increases and Decreases w.r.t. the Rack Price

Equation	P_{t-1}=23.5	P_{t-1}=24.5	P_{t-1}=25.5	P_{t-1}=26.5
Up	1.90	1.48	1.16	0.90
Down	0.07	0.07	0.08	0.10

Table 5.6: Asymmetric Error Correction Model (Dependent Variable $=\Delta p_t$)

Variable	Coefficient	T-stat.
p_{t-1}	-0.59	-8.96
r_{t-1}	0.54	6.28
Δp^{+}_{t-1}	0.05	0.51
Δp^{-}_{t-1}	0.37	2.61
Δr^{+}_{t}	1.20	3.43
Δr^{-}_{t}	0.24	0.59
Δr^{+}_{t-1}	0.09	0.27
Δr^{-}_{t-1}	0.40	0.95
Q_{2t}	0.36	1.11
Q_{3t}	0.37	1.08
Q_{4t}	0.11	0.31
$BEFORE_t$	0.28	0.43
$AFTER_t$	-1.07	-2.02
CONSTANT	3.40	2.32
T	-.00004	0.02

Table 5.7: Partial Derivatives of Predicted Price Change with respect to r_t

r_t	Our Model	Borenstein
21.7	0.62	0.24
21.95	0.69	0.24
22.2	0.76	0.24
22.45	0.84	0.24
22.7	0.94	0.24
22.95	1.04	0.24
23.2	1.15	
23.45	1.27	1.20
23.7	1.40	1.20
23.95	1.54	1.20
24.2	1.70	1.20
24.45	1.86	1.20
24.7	2.04	1.20

5.9 Appendix

Determination of Lag Lengths

A testing down procedure was used for the determination of lag lengths. Starting with $l = 8$ lags, lags were progressively deleted until the null hypothesis that the coefficients on the lags of length l are equal to zero is rejected using a 5 percent significance level, and t or Wald tests. A similar testing up procedure selected the same models in all cases.

Unit root tests

To test for unit roots, we Use an Augmented Dickey Fuller test, and choose the lag length as the highest significant lag of either the autocorrelation function or the partial autocorrelation function. For both series, a lag length of three weeks was used, although for different lag lengths the same results obtained. The t-statistics for the null hypothesis of a unit root against the alternative of a deterministic trend are -3.68 and -3.36 for the retail and rack price series respectively. The five percent and ten percent critical values are -3.41 and -3.13, so that the null hypothesis is rejected in favour of the alternative of no unit root at the five-percent level for the Windsor retail price, and at the ten percent level for the Toronto Rack price. In addition, the decision to treat the Toronto rack price as trend stationary was based on the following arguments. First, when the data set was expanded to include all weeks from January 1989 to December 1995 the null hypothesis of a unit root is rejected at the five percent level. Secondly, the rack series for the original sample was regressed on a dummy variable which equals one over the Persian Gulf war period. An Augmented Dickey-Fuller test on the residual of this regression rejects the null of a unit root. Finally, a simple comparison of the standard deviation of the series shows that for the first 122 weeks, the standard deviation is 3.6, while for the entire sample it falls to 3.3.

Prediction

Define $\bar{z}_t$ as the vector containing z_t, I_{t-1}, and p_{t-1}. We first consider the predictor of negative price changes, conditional on a negative change being observed. Our predictor is based upon the following(time subscripts are suppressed):

$$E[e^{X_D\beta_D+\nu_D}|\epsilon > \phi\bar{z}] = \frac{e^{X_D\beta_D}}{1-F(\phi\bar{z})}\int_{-\infty}^{\infty}\int_{\phi\bar{z}}^{\infty} e^{\nu_D} f(\epsilon,\nu_D)d\epsilon d\nu_D \tag{5.16}$$

$$= \frac{e^{X_D\beta_D}}{1-F(\phi\bar{z})}\int_{\phi\bar{z}}^{\infty} E[e^{\nu_D}|\epsilon]g(\epsilon)d\epsilon \tag{5.17}$$

$$= \frac{e^{X_D\beta_D}}{1-F(\phi\bar{z})}\int_{\phi\bar{z}}^{\infty} e^{(\rho\sigma_D\epsilon)}e^{(1-\rho^2)\sigma_D^2/2}g(\epsilon)d\epsilon \tag{5.18}$$

$$= \frac{e^{X_D\beta_D}}{1-F(\phi\bar{z})}e^{(1-\rho^2)\sigma_D^2/2}\int_{\phi\bar{z}}^{\infty} e^{\rho\sigma_D\epsilon}g(\epsilon)d\epsilon \tag{5.19}$$

$$\tag{5.20}$$

where ρ is the correlation between ϵ and ν_D, and $g(\cdot)$ is the marginal density of ϵ. It remains to evaluate the integral in the last expression. Using the formula given in Johnson and Kotz(p. 129) for the evaluation of a truncated lognormal random variable we find that

$$\int_{\phi\bar{z}}^{\infty} e^{\rho\sigma_D\epsilon}g(\epsilon)d\epsilon = e^{\rho^2\sigma_D^2/2}[1-\Phi(\phi\bar{z}-\rho\sigma_D)] \tag{5.21}$$

where $\Phi(\cdot)$ is the cumulative standard normal function. The expected magnitude of positive price changes is similarly derived.

The second term in the expression on page 139 can be written as

$$-e^{X_D\beta_D}e^{\sigma_D^2/2}[1-\Phi(\phi\bar{z}-\rho\sigma_D)]. \tag{5.22}$$

Suppose that variable x_D^i is in both X_D and $\bar{z}$. Then the partial derivative of the above expression with respect to x_D^i is given as

$$-e^{X_D\beta_D}e^{\sigma_D^2/2}[\beta_D^i[1-\Phi(\phi\bar{z}-\rho\sigma_D)]-\psi(\phi\bar{z}-\rho\sigma_D)\phi^i] \tag{5.23}$$

where ψ is the standard normal density function, and β_D^i and ϕ^i are the coefficients on x_D^i.

The derivative of the first term of the expression from page 139 is similarly derived.

Chapter 6

Concluding Remarks

This thesis is an examination of retail gasoline pricing in Canadian markets. Focus is provided by the observations, which we discuss in Chapter 2, that retail gasoline prices in Canadian markets tend to follow one of two distinct patterns, and that neither pattern is observable in the wholesale price. In particular, we note that while in many cities, retail prices are noticeably more rigid than wholesale prices, in other markets retail prices follow a cyclic pattern not observed in wholesale prices. The questions addressed in the remainder of the thesis are motivated by this initial observation.

Chapter 3 reviews the relevant literature, which we divide into two groups: empirical studies of retail gasoline price dynamics, and theoretical models of price cycles. We find in our survey of the empirical literature that the retail price patterns we consider have gone largely unexamined in the academic literature. Our survey of the theoretical literature on price cycles leads us to choose the Maskin and Tirole alternating move duopoly model of price competition as our general framework for the thesis. This decision is based upon the qualitative features of the cycle, the fact that the model can generate both cycles and constant prices, and the fact that this model makes more reasonable assumptions concerning market structure and the nature of the product than do competing alternatives.

In Chapter 4 we consider why we observe constant prices in some cities and cyclic prices in others. To this end, we provide theoretical justification for the argument made by government literature that prices will remain constant only in markets in which there are few firms with a small number of stations but a large per-station capacity. In particular, we demonstrate in the theoretical model that when one firm operates many more stations than its rival, a constant cost equilibrium can not be maintained. In contrast, a cycle equilibrium can be constructed in this circumstance, and also when the two firms are similarly sized. Finally, an initial examination of available price, cost, and market structure data finds a positive correlation between price stability and concentration. This correlation is considered to be consistent with the suggestion that prices will be more stable in markets in which their are few small firms.

In Chapter 5, we address the question of how retail prices respond to wholesale prices in a city with a price cycle. We formulate a model based on the predictions of the theory, consisting of three equations. The first equation determines whether a price increase or decrease is observed. The second equation determines the size of a price increase that firms would choose to implement if increasing price was determined optimal. The final equation determines the size of a price decrease that would be implement if undercutting behavior were considered optimal. This model is then estimated using weekly data for the city of Windsor, Ontario, for a period of approximately five years.

We find, as expected, that price increases are more sensitive to cost than are price decreases. In addition, we find that the retail price is more likely to increase when the price from the previous period is near the wholesale price than when it is much above the wholesale price. We show that behavior is consistent with the observation that the retail price is more sensitive to the wholesale price when the wholesale price is high than when it is low. This behavior is in turn consistent with the finding of numerous studies of the response of retail prices to wholesale prices that retail prices are more responsive to wholesale prices after an increase in the wholesale price than after a decrease.

The work presented in this thesis is considered to represent initial steps towards answering a large array of pertinent questions. In particular, several directions for ongoing research emerge from the work presented above.

The theoretical model used throughout as a general framework can be viewed as short-term, in that it does not allow firms to choose the number of stations they operate. One may, however, wish to examine the effect of endogenizing the number of stations operated by each firm on the types of equilibria that can be observed. As well, we might use the model to determine whether firms will invest in large or small numbers of stations, and whether an asymmetric division of stations could be observed in equilibrium.

Another question that this thesis leaves unaddressed concerns how the theoretical model would be effected by the introduction of a wholesale price that is non-constant over time. In Chapter 5, our empirical model was constructed from intuition based on the properties of the cycle under the assumption of constant costs. However, a formal theoretical treatment of varying costs is desired before a more in-depth study, for example using micro data, is attempted. Similarly, the long run desire to take a variation of the Maskin and Tirole model to micro data motivates the possible extension of the model to more than two firms.

Finally, this thesis does not attempt to measure the way in which retail prices respond to wholesale prices in cities with rigid retail prices. Without a theoretical model that incorporates time-varying wholesale prices, one would likely need to follow the approach of chapter 5, and base the empirical approach on the intuition that emerges from the constant cost model. Such a project is a viable direction of future research.

Chapter 7

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