Three Applications of Market Incompleteness and Market Imperfection

by

Somchai Jitsuchon

B.A. Thammasat University, Thailand, 1984
M.A. Thammasat University, Thailand, 1987

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Department of Economics

The University of British Columbia
Vancouver, Canada

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Abstract

This thesis presents two applications of the incompleteness and one application of the imperfection of the market economy. The first application, Chapter 2, studies the decision making problem of an individual seeking to accumulate an optimal amount of human capital realizing that the wage income derived from the accumulated human capital is subject to incompletely insured uncertainty. In other words, the financial market that insures against wage income risk is not fully functional. We find that the individual's inability to diversify wage income risk tends to increase the need to accumulate more human capital in order to elevate wage path and compensate for the burden of its associated risk. This is particularly true when (i) the wage income risk is positively correlated with the rate-of-return risk in the financial market, resulting in an even greater risk burden to the individual, and (ii) the individual is more risk averse. There are two possibilities that no human capital is needed. The first possibility occurs when it is optimal to work as an unskilled worker because both the burden from wage income risk and the rate of return from education are low. The second possibility is the case where the risk burden is so high that the optimal time spent in school to acquire sufficient human capital to cover the risk is so long that the discounted rate of return from education is negative. In this case, the best strategy is to invest in financial assets alone and forfeit the opportunity to earn wage income – either as an educated or as an unskilled worker – to avoid its associated risk.

Chapter 3 applies equilibrium unemployment theory with a frictional labor market to study the impact of immigration on the local labor market. Markets are imperfect in the sense that job matching takes time and recruitment is costly. We find that labor market outcomes of both the natives and existing immigrants depend crucially on how the economic surplus from successful matching is divided between the firms and the workers or, in other words, on the bargaining power of the workers. An arrival of immigrants with low bargaining power tends to benefit both the natives and the existing immigrants. A disparity between the two worker types in the matching efficiency also plays a major role. An inferior matching technology among the immigrants, interpreted here as reflecting their less established social network, lowers their wage rate and increases their unemployment rate. The

natives are more likely to benefit from additional immigration than the existing immigrants and, when they do, the overall benefit can be decomposed into "job creation spillover" effect resulting from the immigrants' low bargaining power, and "job stealing" effect resulting from the immigrants' less efficient matching. The implications on the pattern of international migration flows are also discussed.

In Chapter 4, a simple macroeconomic model is constructed and applied quantitatively to OECD countries, to analyze the effect of incomplete insurance on saving, growth and welfare in a closed economy. In this economy, precautionary saving motivated by uninsured idiosyncratic shocks raises growth rates but lowers risk-free returns. Welfare is measured by the sum of growth rates and risk-free rates of return, not growth rates alone. This welfare measure takes the negative impact of precautionary saving into consideration. Applied to the OECD data, three major results emerge: (i) the heterogeneous performance of growth and saving across the countries reflects different degrees of insurance incompleteness, (ii) since the externality of growth on productivity was very strong in the 1960's, the heavily constrained insurance market itself improves productivity by promoting growth, thereby enhancing welfare, (iii) while the externality of growth became weaker in the 1980's, the development of insurance markets lowered growth, but still contributed to a raise in welfare.

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Chapter 1

Introduction and Overview

The common theme underlying the chapters in this thesis is the maintenance that much of economic phenomena can be better explained when the classical Arrow-Debreu framework of perfect and complete market mechanism is abandoned. This is in unison with the momentum in economic literature, which has swung toward the notion that markets are either incomplete, meaning that some markets are absent or are not fully functional, or imperfect, meaning that the market auctioneers who help facilitating all the trades and exchanges without charging for their services do not really exist. Although the idea is not new, recent contributions have seen some substantial improvements of this approach in terms of clarification, specialization, rigorousness as well as technicality of the treatment of market incompleteness and market imperfection. The thesis employs these improved treatments and apply them in the studies of three different topics in economics. The market incompleteness is applied in Chapter 2 and 4 while the market imperfection is applied in Chapter 3.

Chapter 2 focuses on the effect on the level of human capital an individual decides to acquire when the uncertainty of the wage income generated from the possession of that human capital can not be fully insured. The incompletely insurability of wage income is supported by evidence and yet its study in the theoretical literature is dampened by technicality problems arising from the difficulty in obtaining analytical solutions to the optimal consumption/investment problem when insurance market for wage income risk is not fully functional. The chapter's contribution is the application of the recent advance in finding the analytical solution to this optimal control problem with constant absolute risk aversion (CARA) utility. The model in this chapter ties the amount of human capital to the time spent in school, so the optimal amount of human capital is determined by the time to quit school when the individual decides it is best to do so. We find that the individual's inability to diversify income risk tends to increase the need to accumulate more human capital in order to raise wage path which will compensate for income wage risk. This is particularly true when (i) the wage income risk is positively correlated with the rate-of-return risk in the financial market, resulting in an even greater risk burden to the individual, and (ii) the individual is more risk averse. There are two possibilities that no human capital is needed. The first possibility occurs when it is optimal to work as an unskilled worker because both the wage income risk burden and the rate of return from education are low. The second possibility occurs when the risk burden is so high that the optimal time spent in school to acquire sufficient human capital to cover the risk is so long that the discounted rate of return from education is negative. In this case, the best strategy is to invest in financial assets alone and forfeit the opportunity to earn wage income–either as an educated or as an unskilled worker–to avoid its associated risk.

Chapter 3 applies the notion of the market imperfection to study the impact of immigration on the local labor market. It combines two current themes in the economics literature: the interest in finding how immigration affects native workers, and the attempt to explain unemployment with various versions of market imperfection (such as wage rigidities, transaction cost, etc.). The model in this chapter belongs to a class of search-unemployment models, sometimes known as Diamond-Mortensen-Pissarides equilibrium unemployment theory. Markets are imperfect in the sense that job matching takes time and recruitment is costly. To single out the effect of immigration through market imperfection, we assume that natives and immigrants are different only in the matching technology and their abilities to bargain for better wage rates. We find that the labor market outcomes of both the natives and the previous immigrants workers depend crucially on both the differences between, and the levels of the workers' bargaining power. Arrivals of immigrants with low bargaining power tend to benefit both the natives and the existing immigrants. A disparity between the two worker types in the matching efficiency also plays a major role. An inferior matching technology among the immigrants, interpreted here as reflecting their less established social network, lowers their wage rate and increases their unemployment rate. The natives are more likely to benefit from additional immigration than the existing immigrants, and when they do, the overall benefit can be decomposed into the "job creation spillover" effect resulting from the immigrants' low bargaining power, and the "job stealing" effect resulting from the immigrants' less efficient matching. The model's implications on the pattern of international migration flows are also discussed.

In Chapter 4 market incompleteness is once again the theme. In this chapter, which is a joint work with Makoto Saito, the hypothesis that precautionary saving arising from incomplete insurance market can have a negative welfare effect even though it fuels growth, was put to test empirically with OECD experience during 1960-1992. A simple AK type macroeconomic model is constructed with an added assumption that the idiosyncratic part of the rate-of-return risk is not insured, which thereby generates the demand for precautionary saving and lowers the risk-free rate of returns. Since having to bear risk reduces welfare, the level of risk-free rates does carry a welfare implication. A measure of welfare is thus constructed from the model as the sum of growth rates and risk-free rates of return, not growth rates alone. Applied to the OECD data, three major results emerge: (i) the heterogeneous performance of growth and saving across the countries reflects different degrees of insurance incompleteness, (ii) since the externality of growth on productivity was very strong in the 1960's, the heavily constrained insurance market itself improves productivity by promoting growth, thereby enhancing welfare, (iii) while the externality of growth became weaker in the 1980's, the development of insurance markets lowered growth, but still contributed to a raise in welfare.

Chapter 2

Human Capital Accumulation with Incompletely Insured Wage Income

2.1 Introduction

The important role of human capital in determining economic growth and development has long been recognized[1]. Recent studies in the endogenous growth literature[2] and empirical investigations[3] also highlight the decisive contributions of human capital in the explanation of both national economic growth rates and the differences of growth among countries. Owing to its crucial role, the need to better understand how human capital is accumulated in various situations is hardly overstated. There are a great number of models developed to serve this purpose, which will be reviewed in the next section.

This chapter studies human capital accumulation within the framework of a

[1]Early modern analyses include Becker [3, 1975], Mincer [29, 1958])

[2]Lucas [21, 1988]), Romer [32, 1989], Romer [33, 1990]

[3]Barro [2, 1989], Mankiw et.al [24, 1992]

decision making problem at the individual level. The emphasis, in tandem with the theme throughout the thesis, is the effect of market incompleteness on the individual's choice of his or her level of human capital. Specifically, the market incompleteness focused on arises from the absence of a financial market that would provide complete insurance against the uncertainty of wage income. Since the only purpose an individual acquires human capital is to enjoy the more preferable income prospect it provides, the ability to insure against income risk is certainly one of the determining factors in the decision making process[4].

The thesis' focus on the impact of incompletely insured wage income risk is adequately justified. The life-cycle pattern of earning and human capital investment implied in most studies that assume a perfect insurance against wage income risk will only come close to the observed pattern if the interest rate is unrealistically high. One can interpret this as an indirect support of the incomplete market hypothesis. Moreover, an empirical test carried out by Cochrane [9, 1991] directly finds that some types of labor income risks are uninsurable.

The model developed offers additional features, by introducing other sources of income in addition to wage income. An individual may invest in the financial market to earn either the risk-free interest income or the risky income from stocks. The model's assumption that the ability to earn these financial incomes does not depend on the level of human capital opens up the possibility that no human capital is desired.

In this model human capital is acquired only in school. The level of human capital is determined by the length of time the individual spends in school. The time of quitting school is thus a direct measure of the level of human capital. The model makes use of a stochastic optimal stopping rule in a continuous-time

[4]Another source of uncertainty is the risks involved in the process of accumulating the human capital itself. A good example is the uncertainty, as well as the information asymmetry, of learning ability of each individual (especially of the young people who do most of the human capital investment). The market also most likely fails to diversify this risk.

setup. The individual always has the option of continuing the accumulation of human capital, or quitting the school and seeking employment. The optimal time of quitting school is when the marginal benefit of additional human capital is no longer greater than its marginal cost, which consists of the direct cost of the tuition fees and the opportunity cost of foregone wage income. In this aspect, the model can determine the point in time of an individual's lifetime when he or she quits school and is employed. This can not be achieved in a typical two-period model in which the individual allocates his time in the first period between working and accumulating human capital and, in some models, leisure[5].

With the formularized optimal human capital accumulation rule, we can then lay out the various conditions that govern the individuals with regard to the decision to attend school, the time to quit school, and the decision as of whether or not and when to seek employment.

We may summarize our model and its main results as follows:

- The model uses the continuous-time consumption/investment optimal control to derive value functions during and after the accumulation period of human capital (the in-school and after-school periods). The optimal time to quit school is determined with two conditions that must be satisfied at the time of the quitting – the value of quitting versus the value of continuing school are equal (a "value-matching condition") as well as their derivatives (a "smooth pasting condition").
- The human capital production is deterministic, but the process of the wage income it generates is stochastic. The uncertainty of the income process imperfectly correlates with that of the financial price process, indicating that the financial market is incapable of completely insuring wage income risk. The degree of market incompleteness affects the value of the claim to wage income.

[5] Although working during school is not an unusual practice, possibly among high school and college students, we believe our model represents a more common practice, that most students do not work while studying.

- The optimal quitting time is a deterministic constant, and depends on several factors: the rate of return from schooling, the insurance aspect of the claim to wage income, the degree of market incompleteness, the attitude toward risk, and the risk premium to financial assets. How these factors affects the level of human capital is discussed.

- The decision to attend school and work is presented in all possible sets of parameters. It is shown that school is attended only when its instantaneous rate of return is positive. Employment is sought only when the unskilled wage rate more than offsets the net unfavorable income risk.

- Allowing the starting wage to be increasing in the level of human capital lengthens the optimal time spent in school by increasing the net rate of return from schooling.

- Allowing the production of new human capital to use the existing human capital as an input opens the possibility that human capital is accumulated indefinitely.

Chapter 2 is organized as follows. Section 2.3 details the basic setup of the model. Section 2.4 shows how the value of a claim to income stream is calculated and defines the level of human capital at which the individual is indifferent between working and not working. Section 2.5 determines the optimal time to quit school, which enables comparisons of options available to each individual regarding the decision to attend school and work in Section 2.6. Section 2.7 offers two extensions of the model, one of which is the addition of the positive starting wage that is dependent on the level of human capital accumulated, and the other is an alternative human capital process. The last section, Section 2.8, contains final remarks.

2.2 Review of Some Related Literature

The literature on the models of human capital accumulation may be categorized into two broad groups: those that limit the analysis to the certainty case and those that allow uncertainty.

A benchmark model of human capital accumulation without uncertainty is developed by Ben-Porath [6, 1967], who uses the optimal control technique to describe the paths of human capital investments and earnings throughout an individual's lifetime. In that model, human capital has two uses – producing final output and producing additional human capital. The dynamics of human capital accumulation is achieved through the diminishing return in its production. During youth, when human capital is still low and commands a high marginal return, it pays to forego the earning opportunity and concentrate on accumulating more human capital. The rate of accumulation declines over time with a rising level of human capital (and a declining marginal return), and becomes zero at retirement. Ben-Porath's model was extended by Heckman [17, 1976], who allows a nonmarket benefit of human capital (human capital appears in utility function) and explicitly incorporates the individual's initial financial wealth and initial human capital level. These two models have been tested against data in a number of subsequent papers[6], using the models' implication on age-earnings profile.

The common shortcoming of these early models of optimal human capital accumulation is the absence of uncertainty. It has been recognized that investment in human capital involves a substantial uncertainty. Becker [3, 1975][7] shows that the coefficients of variation of 1939 and 1949 after-tax income of white males were large across age and education levels. One of the first papers that offers rigorous treatment of human capital accumulation under uncertainty was by Levhari and Weiss [20, 1974]. Their model is a two-period model with an uncertain future earn-

[6] See, for example, Haley [15, 1976], Brown [8, 1976], Moreh [30, 1979], and Theeuwes et.al [38, 1985].

[7] Table 9, page 182.

ing derived from the first period human capital investment. To simplify the model, they do not consider the uncertainty in nonhuman investment, and thus are able to ignore the problem of multiple risk. They show that when the marginal product of human capital is increasing with earning risk, its expected return exceeds the risk-free interest rate, that is, there is a risk premium to human capital investment. Eaton and Rosen [14, 1980] and Hamilton [16, 1987] apply the model of Levhari and Weiss to the analysis of the impact of taxation on the level of human capital investment. These two papers use two-period models and they share a result that income tax, through its impact on reducing income risk, raises the individual's demand for human capital. Eaton and Rosen therefore argue that the optimal tax rate is positive, instead of a zero tax rate with a lump-sum tax as proposed in the standard optimal taxation literature. Paroush [31, 1976] allows the uncertainty in nonhuman investment and analyzes how an individual invests his predetermined saving between the two investment opportunities. He uses the standard mean-variance analysis developed earlier in Markowitz [25, 1959] and Tobin [39, 1965]. The Levhari-Weiss's two-period model is extended into a continuous-time model by Williams [40, 1979]. Kodde [19, 1986] conducts an empirical test on the effect of income risk on the level of human capital investment and his finding contradicts the implication of all the preceding models. Specifically, Kodde finds that human capital investment increases, rather than decreases, when the individual perceives a higher income risk in the future. Snow and Warren [35, 1990] provide a possible theoretical explanation to Kodde's finding. Finally, Altonji [1, 1993] presents a model of a sequential educational decision by college students who are uncertain about their preference of fields of study and their abilities to complete college. Altonji's model only allows binary choices of education and is restricted to college education.

Because this chapter uses a continuous-time model of optimal consumption and investment with incompletely insured wage income risk, a review of the recent works in this area is in order. It is well known that closed-form solutions in this type of problem are difficult to obtain, since the control problem with both financial assets and uninsurable wage income involves nonlinear differential equations of two state variables. However, a number of recent papers offer more insight into this problem. In a continuous-time setting, Duffie, Fleming, and Zariphopoulou [10, 1993] employ a "viscosity solution" technique to prove that the value function associated with this type of problem is smooth under HARA utility. Their result ensures a simpler numerical approximation of the value function. In a similar setting as that of Duffie et.al [10, 1993], Svensson and Werner [37, 1993] are able to solve, with exponential utility, for the explicit forms of both portfolio choice and the value of a claim to wage income. Duffie and Jackson [11, 1990] provide essentially the same portfolio choice as in Svensson and Werner [37, 1993] as one of their special case, but without solving the implicit value of claim to labor income. He and Pagès [18, 1990] characterize the solution to the similar problem with a borrowing constraint.

In comparison to the previous works reviewed so far, the main contribution of the model in this chapter is the explicit treatment of how the nature and degree of market incompleteness, arising from the individual's inability to completely insure away wage income risk, affects the human capital accumulation decision. Also, the determination of the optimal human capital demonstrates for the first time how the recent progress in the valuation of nontraded risky asset can be applied. Finally, the model's recognition that uncertainty in wage income and uncertainty in financial investment may be correlated renders the decisions on the two investment opportunities interdependent to each other – a feature that is not found in other more comparable models such as that of Williams [40, 1979].

2.3 The Model Setup

An agent at time $t = 0$ is endowed with a positive initial financial wealth, $W(0) > 0$, but he does not own any human capital at birth, namely, $H(0) = 0$. He has one unit of time at his disposal, which he may spend either in a school or in a work place.

Whatever activity he devotes his time to, his labor supply is perfectly inelastic: no time sharing is allowed between two activities. A student cannot work and a worker cannot study. This assumption is purely for simplicity.

If the agent chooses to accumulate his human capital, he does so by enrolling in a school and receiving education. He pays a fixed amount Ldt during any interval dt. Once in school, his human capital increases with certainty by an amount h per unit of time spent in learning. That is[8]

$$dH(t) = hdt \tag{2.1}$$

If the agent chooses not to go to school from the beginning, or quits the school at any time, he saves the school cost Ldt but his human capital ceases to increase, namely, $dH(t) = 0$. Assume for simplicity again that once quitting school, no one is allowed to return. Furthermore, we also assume that no one, if not in school, has difficulty finding a job and earning income. The income earned depends on the agent's human capital level acquired during school.

The agent's lifetime is thus divided into two periods: in-school and after-school. Denotes these two periods by

$$\aleph_1 = [0, T] \qquad \text{and} \qquad \aleph_2 = (T, \infty),$$

where T is the time the agent quits school and immediately finds a job. During school, the individual earns no income, $Y(t) = dY(t) = 0$. Income is earned either when the agent is employed after quitting school, or if he never goes to school and starts working immediately.

Assume that the economy is subject to uncertainty that can be represented by a standard Brownian motion in R^2.

$$B(t) = \begin{bmatrix} B_1(t) \\ B_2(t) \end{bmatrix}. \tag{2.2}$$

[8]Although not a realistic feature, the human capital accumulation process is assumed to be linear for simplicity. As long as human capital increases with time spent in school, the qualitative implication of market incompleteness on the optimal human capital level, which is the focus of this chapter, will not be affected.

The property of a standard Brownian motion dictates that $dB(t)$ has zero mean and unit variance over an interval dt, namely, for $i = 1, 2$ and $s < t$,

$$E_s[dB_i(t)] = 0, \quad E_s[dB_i(t)]^2 = dt, \qquad E_s[dB_1(t)dB_2(t) = 0.$$

There are three investment opportunities in this economy: buying a risk-free bond that guarantees a risk-free interest rate, a risky stock with an uncertain dividend and capital gain and investing to increase human capital that raises wage income. The bond price Q_b is supposed to follow

$$\frac{dQ_b}{Q_b} = r dt,$$

where r is risk-free interest rate. The stock price Q follows a geometric Brownian process

$$\frac{dQ}{Q} = \alpha_q dt + S_q dB, \tag{2.3}$$

where the trend part of the process α_q captures both the instantaneous increase in the rate of return and the dividend rate paid out over the interval dt. S_q is a 1-by-2 row vector of standard deviation of the risky assets' rate of return, and is assumed to take the form

$$S_q = \begin{bmatrix} \sigma_q & 0 \end{bmatrix}.$$

In other words, the stock price is influenced only by the first element of the economy's uncertainty $B_1(t)$. The third investment opportunity involves human capital, whose return is expressed in the wage income process which is characterized by

$$dY = \begin{cases} 0 & t \in \aleph_1 \\ \alpha(hT))dt + S_y dB & t \in \aleph_2 \end{cases} \tag{2.4}$$

where B is as defined in (2.2). S_y is a 1×2 row vector of the standard deviations of the income process and, without loss of generality, can be written as

$$S_y = \begin{bmatrix} \sigma_y \rho & \sigma_y \sqrt{1 - \rho^2} \end{bmatrix},$$

with

$$S_y S_y' = (\sigma_y \rho)^2 + \sigma_y^2(1-\rho^2) = \sigma_y^2 \tag{2.5}$$

measures the variance of the income process. The parameter $\rho \in [-1, 1]$ represents the correlation coefficient between income uncertainty and the uncertainty in the rate of return of the risky assets[9]. It is obvious that, unless $|\rho| = 1$, the income process is governed by both elements of the economy's uncertainty. We can interpret $B_2(t)$, as the uninsurable idiosyncratic shocks that affect an individual's ability to earn wage income, such as the uncertainty related to permanent health conditions. These shocks are not insured through trading in the risky assets because they do not affect the stock price. However, income risk is still partly insured, as it also depends on $B_1(t)$ which is traded through the risky assets. The shock components in $B_1(t)$ can be those affecting the firms or industries in which the individual is working, so the shocks affect both the industries as well as the individual's earning. The extent to which income risk is insured therefore depends on the proportion of $B_1(t)$ relative to that of $B_2(t)$, that is, how close to unity $|\rho|$ is. The parameter ρ can thus serve as an indicator of financial market incompleteness. Many empirical studies (e.g., Cochrane [9, 1991]) found that not all person-specific shocks are insured. This means $|\rho| < 1$ in this model's context.

The drift part of the income process (2.4) is a function $\alpha(hT)$, where hT is the level of human capital at the time the individual leaves school. We assume that it has properties of $\alpha(0) = 0$ and $\alpha'(\cdot) > 0$[10]. The requirement $\alpha'(\cdot) > 0$ is natural as a person with more human capital (by staying longer in school) entertain a more rapidly rising income than those with less human capital. $\alpha(0) = 0$ means that uneducated workers will not see any deterministic wage increase, although they may possibly have a permanent income increase from the diffusion part of the process.

9

$$\rho = \frac{S_y S_q'}{\sqrt{S_y S_y'}\sqrt{S_q S_q'}}.$$

10 For reason that will become clear later, we also assume that $\alpha(\cdot)$ is not strongly convex. See Appendix 2B. In fact, we will use a linear function $\alpha(hT) = hT$ in our closed-form solution for the optimal quitting time.

Note that although the drift part $\alpha(hT)$ depends on a quitting time T, it remains constant afterward. That is, once the agent enters his after-school period of life, his income process is an Ito process with constant coefficients.

We assume that any uneducated worker can earn a nonnegative minimum wage Y_0. The income process (2.4) during the after-school period can then be written as[11]

$$Y(t) = Y_0 + \int_T^t \alpha(hT)dt + \int_T^t S_y dB(t) \qquad t \in \aleph_2. \tag{2.6}$$

To avoid triviality, we make the following two assumptions:

Assumption 1 (risky assets desirability): $\alpha_q > r$.

This assumption ensures that the risky assets is desirable to any risk-averse investor, otherwise the investor can do better by investing only in risk-free asset, and the market incompleteness will become a non-issue.

Assumption 2 (market incompleteness): $|\rho| \neq 1$.

This assumption represents this model's departure from the well-studied classical consumption/portfolio analysis such as that in Merton [27, 1971].

There is another way to measure the degree of the market incompleteness. Consider an "unhedgeable" variance in income, defined by the remaining income variance conditional on trading in the risky assets,[12]

$$\sigma_{y|q} \equiv \sigma_y^2 - \sigma_{yq}(\sigma_q^2)^{-1}\sigma_{qy},$$

where $\sigma_{yq} = S_y S_q'$ and $\sigma_{qy} = S_q S_y'$. Substituting for S_y and S_q gives $\sigma_{yq} = \sigma_{qy} = \rho\sigma_y\sigma_q$. So

$$\begin{aligned} \sigma_{y|q} &= \sigma_y^2 - (\rho\sigma_y\sigma_q)^2/\sigma_q^2 \\ &= \sigma_y^2(1-\rho^2). \end{aligned} \tag{2.7}$$

[11] Hereafter, time subscripts are sometimes omitted whenever such omissions do not cause confusion.

[12] See Svensson and Werner [37, 1993] for more details.

It is clear that the unhedgeable income variance is positive if and only if the market is incomplete.

The Agent's Problem

The agent's objective is to maximize his lifetime expected utility

$$E_0\left[\int_0^\infty e^{-\beta t} U(C(t))dt\right]. \tag{2.8}$$

Because the agent's lifetime is divided into two periods, in-school and after-school, the overall optimal control problem consists of

1. Assuming that the agent goes to school in the first place, we solve the optimal consumption/investment problem during the after-school period with a predetermined quitting time. This gives the value of human capital the individual has acquired during the school time.
2. Assuming that the agent goes to school in the first place, we solve the optimal consumption/investment problem during the in-school period, taking the solution from the after-school period as given. We can then determine the optimal time the agent quits school and becomes employed.
3. The final part involves determining the decision to go to school and to work at time $t = 0$, conditional on the additional option of not going to school.

2.4 The Value of Acquired Human Capital

This section contains the consumption/investment problem after the agent quits school and starts working with the human capital he has acquired, using an arbitrary quitting time (T) as well as financial wealth at time T, $(W(T))$, as given. Since human capital evolves deterministically according to (2.1), it is held constant during this whole period at hT and the income process is well defined by (2.6). The after-school control problem becomes

$$\max_{\{C,\pi\}} E_T\left[\int_T^\infty e^{-\beta t}U(C(t))dt\right], \tag{2.9}$$

subject to the following dynamic budget constraint, with financial wealth a state variable,

$$dW = [\pi(\alpha_q - r) + Wr - C + \alpha(hT)]dt + [\pi S_q + S_y]dB \qquad t \in \aleph_2, \tag{2.10}$$

where π is the amount of financial wealth held in risky assets.

It is well known that the explicit solution to the problem (2.9) and 2.10 is not available under a general utility $U(C(t))$. The problem is more complicated with a nontraded claim to income stream. Because a closed-form solution is needed later in the characterization of the optimal quitting time, it is necessary to restrict to the CARA (constant absolute risk aversion) utility,

$$U(C) = -\frac{e^{-\eta C}}{\eta}, \tag{2.11}$$

where η is the coefficient of absolute risk aversion. The method developed by Svensson and Werner [37, 1993] in solving for the closed-form solution to the problem under CARA[13] is used. Instead of working with the financial wealth as a state variable, Svensson and Werner use the total of financial wealth and the value of a claim to income stream

$$\hat{W} = W + F.$$

F is the value of a claim to the income stream from time T onward that the agent would be willing to hold if it was tradable. The present-valued (at time $t = 0$) value function if the agents quits school at time T takes the following form

$$J(W + F) = -\frac{e^{-\beta T + A - \eta r(W+F)}}{\eta r}. \tag{2.12}$$

A simplified derivation of the solution along the line of Svensson and Werner [37, 1993] is presented in appendix 2A. The expressions for A and F are (2A.12 and

[13] A very similar portfolio rule is independently provided by Duffie and Jackson [11] in finite-horizon model without intermediate consumptions (case number 2 of their results).

2A.11 in appendix 2A)

$$A = [r - \beta - \frac{(\alpha_q - r)^2}{2\sigma_q^2} - \frac{(\eta r)^2}{2}\sigma_y^2(1-\rho^2) + \frac{\eta r}{2}Y_0]/r,$$

and

$$F = [Y_0 + \alpha(hT) - (\alpha_q - r)(\rho\sigma_y/\sigma_q) - \eta r\sigma_y^2(1-\rho^2)]/r. \tag{2.13}$$

The value function (2.12) will serve as the boundary condition for the in-school period problem in appendix 2B.

Remarks on The Value of Human Capital

Three remarks on the value of human capital can be made. First, note that the third term in the expression for F in (2.13) is expected excess return, discounted by the risk-free interest rate, from holding an "income hedge portfolio" in the risky assets $-\rho\sigma_y/\sigma_q$. This portfolio is positive only if ρ is negative. The last term is a penalty from the remaining, unhedgeable, income uncertainty, $\sigma_y^2(1-\rho^2)$, weighted by the absolute aversion to wealth risk (ηr), and is always positive as long as markets are not complete. The addition of these two terms summarizes the effect of income uncertainty. The presence of the unhedgeable uncertainty makes the claim to the income stream F dependent on individual's utility, or more precisely, on individual's attitude toward risk. A more risk-averse person would value his entitlement to the income stream less than those who can tolerate higher risks.

Second, the role of ρ deserves more attention. Since ρ measures the correlation between the uncertainty of income and the uncertainty of financial assets, a negative value of ρ will enable the claim to income to act like insurance against risk in financial investment, and vice versa. This is reflected in a positive demand for income hedge portfolio and on a higher value of F. The opposite is true if ρ is positive.

The third remark is about how school attendance is desirable. To see whether or not an agent would wish to become educated, we compare the value function

(2.12) with a value function derived from the same problem but without stochastic income. The latter value function can be obtained, for example, from Merton [26, 1969],

$$V(W(T)) = -\frac{e^{-\beta T+\tilde{A}-\eta r W}}{\eta r}, \tag{2.14}$$

where $\tilde{A} = [r - \beta - (\alpha_q - r)^2/2\sigma_q^2]/r$. The relation between these two value functions can be written as

$$J(W+F) = e^{-D}V(W),$$

where

$$D = \eta[\frac{Y_0}{2} + \alpha(hT) - (\alpha_q - r)(\rho\sigma_y/\sigma_q) - \frac{\eta r}{2}\sigma_y^2(1-\rho^2)].$$

Since both $J(W+F)$ and $V(W)$ are negative, $J(W+F) > V(W)$ if and only if $e^{-D} < 1$, or, equivalently,

$$\frac{Y_0}{2} + \alpha(hT) - (\alpha_q - r)(\rho\sigma_y/\sigma_q) - \frac{\eta r}{2}\sigma_y^2(1-\rho^2) > 0. \tag{2.15}$$

This condition must hold if the agent should ever prefer working. The first two terms in (2.15) correspond to the deterministic parts in the value of the claim F, and the last two terms are the contributions of income uncertainty to the value of F. Since the left-hand side of (2.15) is increasing in T through the function $alpha(hT)$, the condition (2.15) is satisfied if the individual spends time in school longer than T^-, where T^- is defined by

$$\alpha(hT^-) = (\alpha_q - r)(\rho\sigma_y/\sigma_q) + \frac{\eta r}{2}\sigma_y^2(1-\rho^2) - \frac{Y_0}{2}. \tag{2.16}$$

If we assume $\alpha(hT^-)$ is a linear function hT^-, the expression for T^- can be written as

$$T^- = \left[(\alpha_q - r)(\rho\sigma_y/\sigma_q) + \frac{\eta r}{2}\sigma_y^2(1-\rho^2) - \frac{Y_0}{2}\right]/h \tag{2.17}$$

T^- thus represents a minimum time the individual needs to spend in school to accumulate human capital sufficient to make working a favorable choice, because only those who have acquired a minimum amount of human capital implied by T^-

want to become employed, while those who have less human capital will not look for job (become voluntarily unemployed) and will only invest in the financial assets. If T^- is negative, namely,

$$\frac{Y_0}{2} - (\alpha_q - r)(\rho\sigma_y/\sigma_q) - \frac{\eta r}{2}\sigma_y^2(1-\rho^2) > 0,$$

then working always yields a higher life-time utility than not working.

2.5 The Optimal Quitting Time

We now assume that the agent is currently studying in school, and try to determine the optimal quitting time. At every instant of time the agent is in school, say at time t, he has two available options: either stay in school or quit school and find a job. If he chooses to quit school, his present-valued lifetime indirect utility will be the after-school value function $J(W(t)+F(t))$ in (2.12). On the other hand, if he continues to stay in school and accumulates more human capital his present-valued lifetime indirect utility will be an in-school value function, denoted by $I(W(t),t)$. Since being in school dt longer keeps the options of quit/stay available at time $t+dt$, the in-school value function must reflect this fact. We may then write the in-school value function at time t as follow[14].

$$I(W(t),t) = \max\{J(W(t)+F(t)), E_t[\exp(-\beta\bar{t})U(C(\bar{t}))dt + I(W(t+dt),t+dt)]\},$$

where $\bar{t} \in (t, t+dt)$. The agent stays in school as long as the second argument on the right-hand side exceeds the first argument. If the first argument is larger, that is, quitting school gives higher life-time indirect utility at the time of quitting, then $I(W(t),t) = J(W(t)+F(t))$ and the value function takes the form of $J(W(\tau)+F(\tau))$ for $\tau \geq t$. Subtracting $I(W(t),t)$ from both sides

$$0 = \max\{J(W(t)+F(t)) - I(W(t),t), E_t\{\exp(-\beta\bar{t})U(C(\bar{t}))\,dt + d\,I(W(t),t)\}\},$$

[14]To be consistent with the in-school value function, we might write the after-school value function as a function of $W(t)$ and t, such as $J(W(t),t)$ However, since $J(W(t)+F(t))$ depends on time t through $F(t)$ in an explicit manner, we choose to write it as it appeared in the previous section to keep the same notation throughout Chapter 2.

where $dI(W(t),t) = I(W(t+dt),t+dt)) - I(W(t),t)$. It turns out that the second argument on the right-hand side is nothing but the right-hand side of the Hamilton-Jacobi-Bellman equation for the in-school period consumption/investment problem to be presented in the next subsection, and is thus equal to zero under optimal consumption and portfolio choice. Therefore, we must have, for all t at which time staying in school is optimal,

$$I(W(t),t)) \geq J(W(t)+F(t)). \tag{2.18}$$

At some point, the agent will find quitting school is the best decision. Denote this time as T^*. Two conditions must be satisfied at $t = T^*$. One is that the value of staying in school exactly equals the value of quitting school, namely, the above inequality (2.18) holds with equality. This condition is termed "value matching conditions". The second condition, called the "smooth pasting condition", requires that the derivatives of the two value functions be equal with respect to each and every state variable[15],

$$\left.\frac{\partial I(W(t),t;T)}{\partial t}\right|_{t=T^*} = \frac{\partial J(W(T)+F(T))}{\partial T}, \tag{2.19}$$

$$\left.\frac{\partial I(W(t),t;T)}{\partial W(t)}\right|_{t=T^*} = \frac{\partial J(W(T)+F(T))}{\partial W(T)}. \tag{2.20}$$

The detailed procedure of determining T^* is presented in appendix 2B. It is shown that the expression for the optimal quitting depends on various parameters,

$$T^* = \left[(\alpha_q - r)\rho(\sigma_y/\sigma_q) + \frac{\eta r}{2}\sigma_y^2(1-\rho^2) - \frac{Y_0}{2} + (\frac{h}{r} - L)\right]/h. \tag{2.21}$$

Using the definition of T^-, the threshold school time that separates those who want to work from those who do not, found in (2.17) , we can rewrite (2.21) as

$$T^* - T^- = (\frac{1}{r} - \frac{L}{h}). \tag{2.22}$$

[15]This solution strategy is well known in the literature of regime switching in continuous time (see, for example, Dixit [12, 1993] and [13, 1994]).

We rewrite (2.17) here for convenience of reference

$$T^- = \left[(\alpha_q - r)(\rho\sigma_y/\sigma_q) + \frac{\eta r}{2}\sigma_y^2(1-\rho^2) - \frac{Y_0}{2}\right]/h.$$

The amounts of human capital acquired after spending T^* and T^- periods of time in school are thus

$$H^* = hT^* = (\alpha_q - r)\rho(\sigma_y/\sigma_q) + \frac{\eta r}{2}\sigma_y^2(1-\rho^2) - \frac{Y_0}{2} + (\frac{h}{r} - L)$$

and

$$H^- = hT^- = (\alpha_q - r)(\rho\sigma_y/\sigma_q) + \frac{\eta r}{2}\sigma_y^2(1-\rho^2) - \frac{Y_0}{2}.$$

Remarks on the Optimal Human Capital

By examining both (2.21) and (2.22), we can draw several observations upon how the individual makes decisions on the acquisition of human capital. The followings are some of the major points that are worth emphasizing.

1. The optimal amount of human capital depends on both the economy-wide parameters and the individual's attitude toward risk.

2. The nature of the risk as well as the nature of the market incompleteness affects the individual's decision in a number of ways. First, if the unhedgeable income risk $\frac{\eta r}{2}\sigma_y^2(1-\rho^2)$ is high, the individual will need more human capital to compensate for the burden of having to bear this risk (raising mean income to compensate for higher variance).

3. Since the unhedgeable income risk depends on the attitude toward risk, η, a more risk averse person will spend time in school longer than a less averse person. We would not get this result if the market is complete, that is, $|\rho| = 1$, because all risk is hedgeable.

4. The optimal human capital level is negatively correlated to the income hedge portfolio, $-\rho\sigma_y/\sigma_q$. When the portfolio has a positive position ($\rho < 0$), the need

to accumulate human capital is less. This is because holding financial assets already provides a hedge against wage income risk, as the two risks (the wage income risk and the financial rate-of-return risk) are negatively correlated. One the other hand, a positive value of ρ increase the need to accumulate more human capital. [16]

5. The overall effect of market incompleteness thus depend on both its nature and its degree, as is reflected in the sign and magnitude of ρ. Differentiating H^* with respect to ρ gives

$$\frac{\partial H^*}{\partial \rho} = (\alpha_q - r)(\sigma_y/\sigma_q) - \rho\eta r\sigma_y^2. \tag{2.23}$$

When ρ approaches zero, markets are less complete in such a way that income risk becomes more unhedgeable. If ρ approaches zero from below (ρ becomes less negative) the benefit from insurance against wage income risk of holding financial assets becomes weaker, leaving the individual desiring more human capital. These two effects together enhance the unfavorable nature of income risk, and thus lead to an unambiguous increase in the optimal school time. On the other hand, if ρ approaches zero from above, then the rising unhedgeable risk works in the opposite direction to the increasing insurance services from holding financial assets. The net effect on the optimal quitting time depends on the parameter values as appearing in (2.23) above.

6. The equation (2.22) provides an interesting relation. The right-hand side, $(\frac{1}{r} - \frac{L}{h})$, is the "net rate of return from education". It is the difference between the present value of one additional unit of human capital acquired during school, $\frac{1}{r}$, and the school expense needed to obtain that additional unit, $\frac{L}{h}$. The left-hand side is the optimal, extra time spent in school beyond the threshold school time T^-. To understand the underlying logic of this relation, recall that a person who spends time in school less than T^- accumulates an amount of

[16] Since we do not impose nonnegativity constraint on the risky assets holding π, it is possible that the individual holds a negative amount of risky assets ('going short' on risky assets) and uses the proceeds to finance his or her accumulation of human capital.

human capital that does not make him better off working when compared to the choice of not working and only investing in financial assets. As a result, if going to school becomes an optimal choice, every individual will stay in school longer than T^-. In other words, T^- behaves like an "origin" on the time line on which the optimal quitting time is plotted. An increase in T^-, either as a result of an increase in unfavorable income risk or of a lowered unskilled wage rate, leads to an equal increase in the optimal quitting time. How far the optimal quitting time is from the "origin" T^- is determined solely by the net rate of return from education $(\frac{1}{r} - \frac{L}{h})$.

7. One very striking characteristic of T^* is that it is not only deterministic, but depends neither on the current time t nor on the financial wealth level $W(t)$. If $T^* > 0$ and the agent goes to school at time $0 \leq t < T^*$, he will always stay in school until T^* regardless of his/her financial condition during the interval $[t, T^*]$. This independence comes from the fact that the value of a claim to income $F(T)$ is a deterministic function of quitting time alone, and is thus unaffected by the evolution of uncertainty in financial investment. Moreover, the absence of nonnegativity constraints on consumption and wealth under a CARA utility (which has finite marginal utility at zero consumption) places no obstacle upon the agent in carrying out his planned quitting time T^*. If there were nonnegativity constraints on either consumption or wealth, the agent might be forced to abandon his plan and to quit school sooner than the optimal time without these constraints. The financial condition would therefore add more diversity to the level of human capital among individual in addition to the differences in their risk attitude.

8. The optimal human capital does not depend on the preference discount rate β. This is mainly because in this model accumulating human capital is an investment with risky return, the decision on which is independent of the discount rate under CARA utility[17]. To see this, note that the value of acquired

[17]Although not all utility functions generate an optimal investment rule that does not depend on discount rate,

human capital after the individual quits school at time T is a constant $F(T)$ which is independent of β. Although an individual studying in school has to wait until time T before he/she can claim this value of acquired human capital, the rate used to discount this value before time T is not the discount rate β, but rather the risk-free interest rate r (see equation 2B.13 in Appendix 2B). Consequently, the determination of the optimal quitting time T^* involves the value $F(T)$, the tuition fee L and the interest rate r, with the discount rate playing no role. An increase in discount rate, which reduces the utility value of future consumption relative to the utility value of present consumption, will only changes the consumption path (increases the entire consumption path when time is infinite as in this model). The wealth path will change accordingly, but all the investment decision rules remain unaffected[18].

2.6 Decision to Attend School and to Work

It is assumed throughout the previous section that the agent decides to go to school in the first place. In this section the study extends to include the conditions that determine whether or not the agent goes to school and whether or not he/she seeks employment.

Starting at $t = 0$, the agent has three alternatives available: (i) go to school, quit at $T^* > 0$ and find a job, (ii) start working immediately as an uneducated worker with zero human capital, (iii) neither go to school nor work. The value of each alternative can be directly calculated with the associated value functions evaluated

a wide range of utility functions possess this property; all utility functions belonging to HARA utility class do (see Merton [28, 1990] Chapters 4 and 5). Note that the absence of nonnegativity constraints on consumption and wealth is required to guarantee this result for the case of CARA utility, otherwise the portfolio rule will depend on the discount rate at the time the consumption becomes zero (see Merton [28, 1990] Chapter 6 for details).

[18]Other utility functions, such as the CRRA, yields portfolio rules that depend on wealth level. Changing β can thus affect investment decision via its affect on wealth.

at $t = 0$,

$$\begin{aligned}
I^0 &\equiv I(W(0), 0; T^*) &&= -\tfrac{1}{\eta r}\exp\left\{[A - G - \eta r F(T^*)]e^{-rT^*} + G - \eta r W(0)\right\},\\
J^0 &\equiv J(W(0) + F(0)) &&= -\tfrac{1}{\eta r}\exp\left\{A - \eta r F(0) - \eta r W(0)\right\},\\
V^0 &\equiv V(W(0), 0) &&= -\tfrac{1}{\eta r}\exp\left\{\tilde{A} - \eta r W(0)\right\}.
\end{aligned}$$

The expression for $\tilde{A}$ is given in equation (2.14)of Section 2.4. The optimal decision depends on which of the above value functions is the greatest. The following claims establish the decisive conditions.

Claim 2.6.1. I^0 is greater than (equal to) J^0 if and only if T^* is positive (non-positive).

Proof: Subtracting J^0 from I^0,

$$I^0 - J^0 = -\frac{1}{\eta r}\exp\left\{-\eta r W(0)\right\}\left\{\exp\{A - G - \eta r F(T^*)\}e^{-rT^*} - \exp\{A - G - \eta r F(0)\}e^0\right\}.$$

The comparison is reduced to that between $\Phi(T^*) = [A - G - \eta r F(T^*)]e^{-rT^*}$and $\Phi(0) = [A - G - \eta r F(0)]e^0$. Since $\Phi(\tau)$ reaches a global and unique minimum at $\tau = T^*$, $\Phi(T^*) < \Phi(0)$ if and only if $T^* \neq 0$. However, because $I(W(t), t; T^*) = J(W(t) + F(t))$ for $t \geq T^*$ by construction, $T^* < 0$ is ruled out. Finally, notice that $I^0 > J^0$ if and only if $T^* > 0$ implies $I^0 = J^0$ if and only if $T^* \leq 0$. ■

This claim determines the value of schooling, and it says that the agent decides to go to school whenever it is optimal to acquire positive human capital. The condition $T^* > 0$ is equivalent to $(\frac{1}{r} - \frac{L}{h}) > -T^-$. If both the school rate of return $(\frac{1}{r} - \frac{L}{h})$ and T^- is positive, the agent has an incentive to accumulate human capital that surpasses the threshold level, so going to school is preferred. If T^- is negative, it is still possible that optimal school time offers more human capital than the threshold level, and, if working is preferred, increases the agent's lifetime utility. If, on the other hand, the school rate of return is not high enough to offset the "attractiveness" of working immediately (reflected in the negative T^-), then school will not be attended. Note that this condition does not automatically ensure that the agent actually attends school and becomes employed afterward, since the additional option of not working at all from the beginning and receiving V^0 is not considered.

Claim 2.6.2. V^0 is greater than, less than, or equal to J^0 if and only if T^* exceeds, falls short of, or equals to $\frac{1}{r} - \frac{L}{h}$.

Proof:

$$V^0 - J^0 = -\frac{1}{\eta r}\exp\{-\eta r W(0)\}\{\exp\{\tilde{A}\} - \exp\{A - \eta r F(0)\}\}.$$

Since $\tilde{A} - A + \eta r F(0) = -hT^-$, sign$(V^0 - J^0)$ =sign(T^-). Using (2.22) the claim follows. ■

This claim is simply the recitation of the definition of T^- in Section 2.4, because $T^- = T^* - (\frac{1}{r} - \frac{L}{h})$. When it requires positive threshold human capital to compensate for income uncertainty (net of the unskilled wage rate), the agent with too low human capital will choose not to work. Again, this claim ignores the alternative of going to school , because I^0 is not considered.

Claim 2.6.3. V^0 is greater than, less than, or equal to I^0 if and only if $\frac{L}{h}$ exceeds, falls short of, or equals to $\frac{1}{r}e^{-rT^*}$.

Proof:

$$V^0 - I^0 = -\frac{1}{\eta r}\exp\{-\eta r W(0)\}\{\exp\{\tilde{A}\} - \exp\{[A - G - \eta r F(T^*)]e^{-rT^*} + G\}\}.$$

The comparison is now between $[A - G - \eta r F(T^*)]e^{-rT^*}$ and $\tilde{A} - G$. The former term is $-\eta h e^{-rT^*}/r$ from (2B.16) while the latter term is simply $-\eta L$. Since η and h are both positive, sign$[V^0 - I^0]$ =sign$[\frac{L}{h} - \frac{1}{r}e^{-rT^*}]$ and the claim is proved. ■

Unlike the condition in claim 2.6.2, when the school alternative is considered, the agent has to take into account the amount of time required to acquire the optimal amount of human capital and appropriately discounts the rate of return from schooling. To acquire one unit of human capital, the agent pays L/h. Although the present value of such unit, *at time of acquisition*, is $1/r$, the agent has to wait until time $t = T^*$ when he gets the job, to realize the benefit of the acquired human capital. The time-zero present value of one additional unit of human capital is thus $\frac{1}{r}e^{-rT^*}$, making it desirable to go to school only if $\frac{1}{r}e^{-rT^*} > L/h$.

Before proceeding to see how the three conditions in the above claims help determine the decision to attend school and work, it helps to rewrite the condition in the claim 2.6.3 regarding the comparison between V^0 and I^0. Denote the time in school beyond which V^0 is greater than I^0 by T^+. Thus T^+ solves $L/h - \frac{1}{r}e^{-rT^+} = 0$. Taking the natural logarithm of $L/h = \frac{1}{r}e^{-rT^+}$ and rearranging,

$$rT^+ = \ln(\frac{1}{r}) - \ln(\frac{L}{h}).$$

Because the natural logarithm function is concave and has slope r at $\frac{1}{r}$, we have $\ln(\frac{1}{r}) - \ln(\frac{L}{h}) > r(\frac{1}{r} - \frac{L}{h})$ whenever $\frac{1}{r} > \frac{L}{h}$[19]. Thus $T^+ > \frac{1}{r} - \frac{L}{h}$. And since $L/h - \frac{1}{r}e^{-r\tau}$ is increasing in τ, $T^* > T^+$ implies $V^0 > I^0$. If the quitting time T^* obtained from (2.21) is greater than T^+ it will no longer be an optimal quitting time because the agent can do better by investing in financial assets only without going to school and working. This is more likely when T^- is a large positive.

We will mainly consider the case where $\frac{1}{r} - \frac{L}{h} > 0$. This means the *instantaneous* rate of return from human capital investment is positive, a condition we feel is realistic in most situations. Equipped with the results from the claims 2.6.1 to 2.6.3, we can consider four possible cases. Figure 2.1 displays the graphical presentations of these four cases[20].

1. $T^* < 0 < [\frac{1}{r} - \frac{L}{h}] < T^+$. This implies $V^0 < J^0 = I^0$. In this case, the agent does not go to school but seeks employment immediately. Since $T^* < [\frac{1}{r} - \frac{L}{h}]$ implies a negative T^-, working and getting paid wage income is attractive even with

[19] One can confirm this fact by applying Taylor's expansion around $\frac{1}{r}$.

[20] To ease the presentation in these four figures, we have transformed all the value functions with $X^0 = -ln(-X^0)$, where $X = I, J, V$. This transformation does not change the qualitative comparisons, since the indirect utility functions are unique up to any positive monotonic transformations. Both the value matching condition and the smooth pasting condition are also preserved.

zero human capital ($V^0 < J^0$). Moreover, working is so attractive that being in school is considered a waste of time since it offers no better alternative than finding a job immediately ($I^0 = J^0$).

2. $0 < T^* < [\frac{1}{r} - \frac{L}{h}] < T^+$. This implies $V^0 < J^0 < I^0$. The agent goes to school, quits at time T^*, and works. Although working immediately without any human capital is still preferred to not working as in the previous case ($V^0 < J^0$) because the threshold school time (T^-) is again negative, the agent decides to postpone his working and goes to school because the accumulated human capital after T^* benefits him more ($J^0 < I^0$). In other words, the optimal quitting time is now positive.

3. $0 < [\frac{1}{r} - \frac{L}{h}] < T^* < T^+$. This implies $J^0 < V^0 < I^0$. The agent takes the same action as in the previous case, namely he goes to school and then works. The motivation is, however, different. Within this parameter set, working with zero human capital is not preferred to not working due to positive T^-. However, going to school offers a chance to acquire human capital that surpasses the threshold level of human capital ($T^* > [\frac{1}{r} - \frac{L}{h}] > 0$), without spending too much time in school ($T^* < T^+$). In other words, going to school more than offsets the unfavorable income uncertainty (net of the unskilled wage earnings). In the previous case, it enhances the already favorable income risk (net of the unskilled wage earnings).

4. $0 < [\frac{1}{r} - \frac{L}{h}] < T^+ < T^*$. This implies $J^0 < I^0 < V^0$. The agent neither goes to school nor works. He only engages in the financial investment. The claim to income stream is so unfavorable that the maximum benefit from investment in human capital fails to offset, when the time needed to reap benefit from such investment is taken into account ($T^* > T^+$).

The results in these four cases are summarized in Table 2.1 together with the other cases of non-positive school rate of return ($[\frac{1}{r} - \frac{L}{h}] = 0$ and $[\frac{1}{r} - \frac{L}{h}] < 0$). When school offers a non-positive rate of return, attending school never makes the agent better off and hence there is no incentive to accumulate human capital. The decision to work or not depends merely on whether T^- is negative or positive. One peculiarity occurs in the cases when $[\frac{1}{r} - \frac{L}{h}] < 0$ as one set of parameters renders the comparison of the three value functions impossible (case 9 in Table 2.1).

2.7 Extensions to the Basic Model

Two extensions of the model will be considered in this section. First, we allow the human capital to affect not only the individual's wage trend but also the starting wage rate. In the second extension, we use an alternative specification of human capital production, in which the production of human capital uses as inputs both the financial resource and the existing human capital itself.

Increasing Starting Wage Rate

We have modeled so far that the sure benefit from acquiring human capital is an instantaneous wage increase, represented by a higher drift of the income process $\alpha(hT)dt$. One might want to add a more realistic aspect of the human capital's benefit, that a person with higher level of human capital should be able to expect not only a more rapid increase in wage rate over time, but also a higher starting wage at the time he becomes employed. Formally, the income process will now follow

$$Y(t) = Y_0 + \bar{Y}(T) + \int_T^t \alpha(hT)dt + \int_T^t S_y dB(t), \tag{2.24}$$

for $t \geq T$. $\bar{Y}(T)$ is a deterministic positive value reflecting the starting wage rate and is a positive function of time spent in school. In the preceding sections, we implicitly set $\bar{Y}(T) = 0$. It can be easily shown that this new setting does not significantly change the model's solution. First, notice that although the wage income takes the new process as in (2.24), its differential form remains the same as in Section 2.3, namely,

$$dY(t) = \alpha(hT)dt + S_y dB(t).$$

The value of claim to income stream $F(T)$ as derived in Appendix 2A is thus still valid. Further, as the starting wage $\bar{Y}(T)$ is a deterministic constant and is earned by the agent forever from time T onward, the agent will be able to capitalize it at a value $\bar{Y}(T)/r$. This capitalized value of starting wage essentially increases the agent's wealth at time T and after, and the after-school value function can be rewritten as

$$J(W(t) + F(T)) = -\frac{1}{\eta r}\exp\{-\beta t + A - \eta r(W(t) + \bar{Y}(T)/r + F(T))\}.$$

Suppose $\bar{Y}(T)$ is assumed to be $\bar{y}T$ and $\alpha(hT) = hT$, then the above value function is exactly the same as in previous sections with h replaced by $\bar{h} = h + \bar{y} > h$. The immediate effect of positive starting wage is therefore the increase in the school rate of return $\frac{1}{r} - \frac{L}{\bar{h}} > \frac{1}{r} - \frac{L}{h}$. This leads to the increase in the optimal school quitting time T^* (from 2.21) and the increased proportion of agents going to school. Note that the effect of $\bar{Y}(T)$ on the optimal quitting time is opposite to that of the unskilled wage rate Y_0. This is because an increase in $\bar{Y}(T)$ favors going to school to acquire more human capital, while an increase in Y_0 favors working immediately.

Alternative Human Capital Production

In our basic model, the production of human capital uses only the financial resource and time. This is not entirely consistent with the conventional wisdom in the literature of human capital formation. In this literature, most models require the existing human capital to be an essential input in the creation of additional unit of human capital[21]. To accommodate this view, the following specification of human capital production is introduced,

$$dH(t) = hH(t)^{\epsilon}dt, \tag{2.25}$$

[21] Among others are Ben-Porath [6, 1967], Williams [40, 1979], Heckman [17, 1976], Lucas [21, 1988], etc.

with a given level of positive initial human capital $H(0) > 0$. The parameter ϵ measures the elasticity of human capital as an input, and we assume its value to be $0 \leq \epsilon \leq 1$. Our basic model corresponds to the case where $\epsilon = 0$, while other models, for example, Lucas's ([21, 1988]), use $\epsilon = 1$. h represents the growth parameter, when $\epsilon = 1$ human capital grows at rate h.

Because this alternative specification of human capital production does not change the solution strategy used in the basic model, both the value functions for the in-school and the after-school period keep their functional forms as in (2B.13) and (2.12). The only difference is in the value of the claim to the income steam $F(T)$, which depends on the level of accumulated human capital at time T,

$$F(T) = [Y_0 + H(T) - (\alpha_q - r)\rho(\sigma_y/\sigma_q) - \eta r \sigma_y^2(1-\rho^2)]/r. \tag{2.26}$$

Instead of a linear $H(T) = hT$, the value of $H(T)$ is now the solution to the differential equation (2.25) above with the initial condition $H(0) = 0$. The determination of the optimal quitting time is done in the same way as in Section 2.5, namely,

$$T^* = \arg\min_{\tau} \Phi(\tau) = \arg\min_{\tau}[A - G - rF(\tau)]e^{-r\tau}.$$

Using $F'(T^*) = \frac{h}{r}H(T^*)^\epsilon$, the first-order condition (2B.16 or 2.21) becomes

$$\frac{h}{r}H(T^*)^\epsilon - [H(T^*) - hT^- + L] = 0, \tag{2.27}$$

where T^- is defined as in the basic model, namely, $hT^- = (\alpha_y - r)\rho(\sigma_y/\sigma_q) + \frac{\eta r}{2}\sigma_y^2(1 - \rho^2) - \frac{Y_0}{2}$.

Since the process of human capital $H(t)$ is again a monotonic positive function of time, there is no substantial change from the implications of the basic model, except that we now have a different mapping between time and the level of human capital accumulated. For example, as before, whether or not there is a positive optimal quitting time T^* that satisfies (2.27) depends on the parameter value of L, T^- and $\frac{h}{r}$. To clarify this, consider again the situation at time $t = 0$. Naturally, the individual goes to school at $t = 0$ if

$$\frac{h}{r}H(0)^{\epsilon} - [H(0) - hT^{-} + L] > 0. \tag{2.28}$$

This condition simply says that people go to school when the marginal increase in the value of human capital from staying in school one unit of time longer ($F'(H(0)) = \frac{h}{r}H(0)^{\epsilon}$) is greater than the sum of all the cost involved – the direct financial cost, L, and the opportunity cost of working immediately, $H(0) - hT^{-}$. However, the analysis here is slightly more complicated than that in the basic model. Suppose (2.28) holds, we might not have an "interior" solution for T^*, namely, we might have $T^* = \infty$. This happens when $\Phi''(T) < 0$ for all $T \geq 0$[22]. In words, human capital grows fast enough to ensure that continuing investment in human capital is the best strategy. To rule out this possibility, either additional restrictions on the parameters or an alteration of the model environment are necessary. Of course, setting $\epsilon = 0$ brings us back to the basic model in which T^* is always finite. One can also assume that the growth parameter of human capital, h in (2.25), is decreasing with time, at least after some point in time. This may capture the declining learning ability when a person passes a certain age. Alternatively, a finite lifetime is sufficient to rule out the indefinite accumulation of human capital.

2.8 Final Remarks

We have presented a model that characterizes the decision to attend school, the optimal time in school, and the decision to work for an individual who lives under uncertainty. The model is capable of identifying the effect of various factors that affect those decisions.

There are some points worth adding here. First, we have not allowed the process of human capital accumulation to be stochastic, or to be different between individuals. People possessing varying abilities to learn accumulate human capital

[22]This possibility does not arise in the basic model, since $\Phi''(T)$ is always positive.

at different rates. Uncertainty in the learning environment should also strengthen the stochastic nature of the accumulation process. Having allowed that, the optimal quitting time would have been different. We chose not to address the stochastic nature of the accumulation purely because of the technical difficulty involved. The Hamilton-Jacobi-Bellman equation for the after-school problem would have been a partial differential equation in two state variables that are much more difficult to solve.

Second, we did not discuss the role government can play in this model. There are certainly many channels through which the educational policy may be guided. For example, subsidizing school tuition fee (lowering L) will lengthen the time in school. The same effect is achieved by constantly upgrading the productivity of schooling to be better responsive to the demand in labor market (raising h).

Third, we mentioned that the absence of nonbankruptcy and nonnegative consumption constraints account for the separation of the optimal quitting time from the financial uncertainty. This is obviously not a realistic feature. An inability to borrow or to run down the financial wealth below a certain level, together with an inelastic demand for basic consumption needs, are among the most important factors that prohibit poor people from receiving education. It explains, at least partly, the persistence of high income and wealth inequality in many developing countries. Income and wealth inequality may become more serious if schooling offers a higher rate of return, because the optimal human capital will be larger (T^* is a positive function of h). There will be a greater variety of people with different educational levels created by wealth disparity. The interdependence between income and wealth disparity, and unequal educational opportunities can be thus analyzed in a model that extends the one we have presented here by allowing for nonbankruptcy and nonnegative consumption constraints.

Chapter 3

Impacts of Immigration in A Frictional Labor Market

3.1 Introduction

There is a large amount of literature on the impact of immigration on the local labor market and on the host countries' natives[1]. Most studies focus on the impact on either unemployment or earning of natives, or both, and some studies include the impact on the previous immigrants and the newly arrived immigrants themselves. Various frameworks have been used to study these issues and discussions on both negative and positive effects of immigration have been widespread. On the negative side, an influx of immigrants increases the labor supply and may substitute for certain groups of native workers, resulting in a higher unemployment and a lower wage among indigenous people. On the positive side, immigrant workers may complement the other groups of native workers in production and raise the productivity and return on education especially among skilled and higher educated

[1] Schultz [36, 1995], Borjas [11, 1994] and Borjas [12, 1994] provide comprehensive review of the overall impact of immigration.

natives[2]. Empirical studies, however, have almost universally found only marginal, if any, effect of immigration on the labor market outcomes of natives. This is the case both with the impact on employment [3] and on earning [4]. A more sizable adverse impact is found on the earning of immigrants themselves[5].

This chapter pursues an approach rather different from the conventional literature. Instead of emphasizing the importance of the inherited differences between the natives and the immigrants, such as the productivity difference arising from each individual's unequal and heterogeneous human capital, we set out to explore the implications of immigration resulting mainly from the imperfect functioning of the labor market. Specifically, we apply the job search and unemployment models sometimes known as the equilibrium unemployment theory or the Diamond-Mortensen-Pissarides search-unemployment theory[6]. The theory is based on the assumption of a frictional labor market which acknowledges the fact that job searching by unemployed workers and job recruiting by firms, or the job matching process, is not instantaneous and not without cost. In other words, the theory abandons the classical paradigm of production (or matching) coordinated by competitive market clearing price and wage.

This class of models has been widely applied to study unemployment in various settings[7]. We believe that applying the theory to study immigration issues has some

[2]Gang and Revera-Batiz [22, 1994] find a complementarity effect of unskilled immigrants on rate of return of educated workers.

[3]Among others are Altonji and Card [3, 1991], Borjas [9, 1990, pp. 92], Winegarden and Khor [38, 1991], Simon, Moore, and Sullivan cite[1993]SimonMooreSullivan93, and a more study by Pischke and Velling [32, 1997].

[4]Studies include those by Borjas [8, 1987], Borjas [9, 1990, pp. 90], Borjas, Freeman, and Katz [14, 1991], Grossman [23, 1982], Bean, Lowell, and Taylor [5, 988], Lalonde and Topel [26, 1991], Altonji and Card [3, 1991], Fix and Passel [20, 1994], Zimmermann [39, 1994].

[5]Borjas [8, 1987].

[6]Diamond [16, 1981], Diamond [18, 1982], Mortensen [29, 1982a], Mortensen [30, 1982], Pissarides [33, 1985]. The approach is presented in a unifying framework by Mortensen [31, 1989] and in a book by Pissarides [34, 1990].

[7]Among others are Howitt [25, 1985], Galor and Lach [21, 1990], Pissarides [35, 1992], Bean and Pissarides [4, 1993], Aghion and Howitt [2, 1994]. Applications within a general equilibrium setup include those by Lucas and Presscott [27, 1974], Albrecht and Axell [1, 1984], Drazen [19, 1988], Hosios [24, 1990], and McKenna [28, 1996]. Due to the positive externality inherited in the job matching process, the theory also has the potential of generating multiple equilibria and indeterminacy that is capable of explaining the persistent unemployment

advantages over the more conventional neoclassic models. For example, instead of emphasizing the analyses on utility maximization, the theory draws its main results from various structural features of the labor market. We feel that it is often the destination countries' labor market structure that influences the decision making of potential immigrants. We also believe the model developed in this chapter is more suitable when applied to immigration into countries with persistently high unemployment and strong labor unions, such as the European economies of the 1980s and the early 1990s.

Our model addresses the immigration within the framework of the equilibrium unemployment theory by introducing market heterogeneities among workers. Workers are categorized into foreign-born immigrants and domestic-born natives. Among many potential sources of market heterogeneity associated with the two groups of workers, we focus our attention to two sources we believe are interesting. The first source involves the difference in matching technology due to the inequality in the economic and social network establishment, with immigrants possessing a less efficient matching technology owing to their inferior social network. Consequently, a typical immigrant has lower probability than his/her native counterpart in getting hired. The second source of heterogeneity involves the different degree of labor organization, which results in the two worker types possessing unequal abilities to bargain for wage with the employers.

The importance of economic and social network in determining both the magnitude and direction of immigration as well as the performance of the immigrants is well documented, both within and outside economics literature[8]. Several factors influence the degree of the social network effect. One of the important factors is the number of existing, and already established, immigrants in the destination countries. A larger immigrant community offers the newly arrived immigrants more jobs

in Europe as proposed by Blanchard and Summer [7, 1988]. Discussions on multiple equilibria, as well as the conditions under which they occur can be found in Mortensen [31, 1989].

[8]For example, Borjas [13, 1995] finds that ethnic environment plays a significant role in the performance of workers.

and trade opportunities arising from production and trade of commodities specific to the community consumption preference [9].

Allowing for unequal bargaining power between the natives and the immigrants enables the model to study the reciprocal effect of labor collectivity of one worker group to the other, providing a rich implication to the pattern of the international migration flows and the welfare consideration of all workers involved.

Chapter 3 is organized as follows. The next section introduces the specification of the matching functions for the two groups of worker. Section 3.3 presents the firms' problem and their optimal hiring strategy. Section 3.4 presents the workers' problem. Wage determination is discussed in Section 3.5. Section 3.6 characterizes the steady state equilibrium and the effects of changes in the immigrants' matching efficiency and the bargaining power, while Sections 3.7 and 3.8 discuss, respectively, the impact of one-time immigrations and the model's implications for international migration flow and welfare consideration. The summary is found in Section 3.9.

3.2 The matching technology

At any point in time the labor market in the economy is characterized by a pool of unemployed workers looking for jobs, denoted by U, and a number of job vacancies waiting to be filled, denoted by V. Job creation is characterized by a Cobb-Douglas matching function

$$m(U,V) = U^{\alpha}V^{1-\alpha}, \tag{3.1}$$

with $0 < \alpha < 1$. $m(U,V)$ is the number of jobs created per unit time. The Cobb-Douglas specification implies a constant return to scale of the matching function,

[9]For example, most immigrants tend to live in close clusters of the same ethnic, thus generating trade opportunities among themselves which depends on the cluster size. It is not uncommon to see large ethnic enclaves in big cities such as the Cubans in Miami's Little Havana, the Mexicans in East Los Angeles, or the Chinese in San Francisco's Chinatown.

which is supported by empirical studies[10]. The pool of unemployed workers consists of unemployed native workers and unemployed immigrant workers,

$$U = U_n + U_i,$$

where U_n and U_i represent native unemployed and immigrant unemployed, respectively. We assume that the two types of workers have identical productivity in performing tasks required by the potential employers. However, the probability that they get hired is not identical. Job searching is an endeavor requiring personal and "network" resources with which each individual is unequally endowed. It is reasonable to assume that a typical native worker has a better chance of getting a job than a typical immigrant, due to the latter's less social establishment and less familiarity with the labor market in the new country. The assumption of identical productivity allows us to focus exclusively on differences between natives and immigrants in job search activity arising from the network effect, without resorting to the differences in their human capital.

To model the disadvantage of unemployed immigrant workers we assume that the total created jobs go to immigrants less than proportionately, namely,

$$m_i(U,V) \;=\; \delta\frac{U_i}{U}m(U,V) \;=\; \delta U_i\left(\frac{V}{U}\right)^{1-\alpha}. \tag{3.2}$$

The positive $\delta < 1$ is a parameter capturing the disadvantage of immigrants.

The job creation for native unemployed workers is given by

$$m_n(U,V) = m - m_i \;=\; (1-\delta\frac{U_i}{U})m(U,V) \;=\; (U-\delta U_i)\left(\frac{V}{U}\right)^{1-\alpha}. \tag{3.3}$$

The hiring rate of unemployed immigrant workers, m_i, has the usual property of a matching function that it increases with the pool size of the immigrant unemployed workers, namely,

$$\frac{\partial m_i}{\partial U_i} = \delta\left(\frac{V}{U}\right)^{1-\alpha}\left[1-\delta(1-\alpha)\frac{U_i}{U}\right] > 0.$$

[10]See, for example, Blanchard and Diamond [6, 1990].

The additional property of m_i, which is particular to our model, is that it decreases with the pool size of the native unemployed workers,

$$\frac{\partial m_i}{\partial U_n} = -\delta(1-\alpha)\left(\frac{V}{U}\right)^{1-\alpha}\frac{U_i}{U} < 0.$$

m_n has similar properties, namely, it increases with U_n but decreases with U_i[11],

$$\frac{\partial m_n}{\partial U_n} = \left(\frac{V}{U}\right)^{1-\alpha}\left[1-(1-\alpha)(1-\delta\frac{U_i}{U})\right] > 0,$$

$$\frac{\partial m_n}{\partial U_i} = -\left(\frac{V}{U}\right)^{1-\alpha}\left[\delta+(1-\alpha)(1-\delta\frac{U_i}{U})\right] < 0.$$

The specification of the two matching functions captures the general perception that immigration possesses a possible adverse effect on the employment prospect of native workers (especially in the models with no skill differences among workers). As only few immigrants have arranged employment before landing, most are unemployed on arrival, increasing the size of unemployed immigrants pool U_i which in turn, *ceteris paribus*, could lower the hiring rate of native workers. We will show later that this perception need not always be accurate in the equilibrium. Another important property of the two matching functions is that they increase in the total number of vacancies in the economy V, not only in the vacancies open up specifically for the respective type of worker (V_i or V_n). The underlying assumption of this specification is that the firms are free to hire either natives or immigrants and they can switch, without cost, to hire the other type of workers they do not initially intend to hire. This assumption provides another channel through which the impact of immigration is felt among native workers.

Once matching takes place, jobs are created. However, these jobs do not last indefinitely. Due to exogenous factors, each job has a probability s of termination. We require this job termination rate to be strictly positive so as to ensure the existence of a steady state unemployment in the equilibrium. Although it is likely that

[11]The properties that m_i (m_n) is increasing in U_i (U_n) and decreasing in U_n (U_i) still hold with any constant return to scale matching function $m(U,V)$.

the job termination rates are different between the two types of employed workers, we assume they are equal in order to focus our attention on the heterogeneity arising from matching technology alone.

In the steady state with unchanged labor force and zero technological progress, the unemployment levels of both native workers and immigrant workers must be constant. Let L_n and L_i denote, respectively, the total native labor force and the total immigrant labor force. The steady state requires that

$$m_i(U,V) = s(L_i - U_i), \tag{3.4}$$

$$m_n(U,V) = s(L_n - U_n). \tag{3.5}$$

The left-hand-sides of the above two equations are job creation rates, or flows per one unit time from unemployment to employment of respective type of workers, while the right-hand sides are flows from employment to unemployment. In the steady state, the flows on two sides are equal, leaving the unemployment levels unchanged.

3.3 The Firms

The firms employ workers and capital, and produce output according to a constant return to scale production function $F(K,N)$, where K is capital and N employed labors or occupied jobs. Rewriting this production function as per occupied job gives a one variable production function

$$f(k) = \frac{F(K,N)}{N}.$$

We assume $f(k)$ is increasing and strictly concave.

Due to market friction, not all the firms' demand for labor is instantly met, therefore the firms always have unfilled vacancies. The matching of a vacancy with a job seeker takes time and follows the matching functions (equations 3.2 and 3.3).

The firms may choose to hire a native or an immigrant to perform the job. Let J_n denote the value of the job occupied by a native. Suppose the firms rent capital k for each occupied job at rental r, then the dynamic optimization requires that J_n satisfies

$$rJ_n = f(k) - rk - w_n - s(J_n - O_n) + \frac{dJ_n}{dt},$$

where w_n is wage rate for natives, and O_n the capital value of a vacancy targeting natives. The right-hand side, rJ_n, is the capital cost of a filled job, which must be equal to its total benefit consisting of current profit per unit time $f(k) - rk - w_n$, expected loss from a switch to vacancy due to job termination $s(J_n - O_n)$, and a capital gain $\frac{dJ_n}{dt}$.

For a job occupied by an immigrant, its capital value follows a similar dynamic optimization equation,

$$rJ_i = f(k) - rk - w_i - s(J_i - O_i) + \frac{dJ_i}{dt}.$$

Since we restrict our model to the steady state analysis, there will be no change in both J_n and J_i, and hence zero capital gains ($\frac{dJ_n}{dt} = \frac{dJ_i}{dt} = 0$). Furthermore, following Pissarides ([33], [34]), we assume free entry and exit of firms (or, equivalently, perfect capital mobility), which eliminates the rent associated with vacancies. This means $O_n = O_i = 0$ in equilibrium. We can then rewrite the above two equations as

$$(r + s)J_n = f(k) - rk - w_n, \tag{3.6}$$

$$(r + s)J_i = f(k) - rk - w_i. \tag{3.7}$$

Because there is no productivity differential, the firms gain more from the jobs for which they pay lower wage. The difference in wages will be determined by bargaining between the firms and the two types of workers, and also on the difference in matching technology. Immigrants, owing to their less efficient matching technology, are prone to get a lower wage rate because the firms must wait longer before they meet a suitable immigrant worker. As hiring process is costly and time dependent, this translates into higher cost to the firms in hiring immigrants, which can be compensated by the firms' ability to pay a lower wage[12]. Because firms are free to hire either natives or immigrants, in the equilibrium the number of vacancies for natives and immigrants – or the number of firms looking for immigrant worker and the number of firms looking for native worker – will be adjusted such that the firms are indifferent between hiring either type of workers. Note again that the firms can switch without cost to target worker type different from the initial opening, making only the total vacancies matters from the workers' point of view.

Let V_n and V_i, respectively, denote the numbers of vacancies initially targeting natives and immigrants. In the steady state equilibrium with perfect foresight, there is no switching in the target workers and hence V_n and V_i are also the vacancies targeting natives and immigrants throughout the hiring process. Suppose that the cost of filling one job is γ per unit time. We may also think of this cost as the cost per unit time of a vacant job because the firms incur this cost only when they have a vacancy waiting to be filled. The capital value of vacancies targeting at natives and immigrants must satisfy the dynamic optimization equations,

$$\begin{aligned} rO_n + \gamma &= \tfrac{m_n}{V_n}(J_n - O_n) + \tfrac{dO_n}{dt}. \\ rO_i + \gamma &= \tfrac{m_i}{V_i}(J_i - O_i) + \tfrac{dO_i}{dt}. \end{aligned}$$

The left-hand-sides are the costs associated with the vacancies which consists of the capital cost and the recruiting cost per unit time. The right-hand-sides are the gains which consists of the expected gain when the vacancies get filled and the capital gains. Again, using zero capital gain and $O_n = O_i = 0$ we rewrite the above equations as

$$J_n = \gamma\frac{V_n}{m_n} \text{and} J_i = \gamma\frac{V_i}{m_i}.$$

We can see that in the equilibrium the market will adjust in such a way that the firms are indifferent between opening up a vacancy for a native or a vacancy for an

[12] It is not necessarily true that immigrants receive lower wage. If their bargaining power is higher than the natives', then they may command higher wage at the expense of higher unemployment rate.

immigrant, because the expected gains from the two vacancies are equal,

$$\frac{m_n}{V_n}J_n = \frac{m_i}{V_i}J_i. \tag{3.8}$$

If the probability of filling a vacancy for one type of worker is greater than the other, for example $\frac{m_n}{V_n} > \frac{m_i}{V_i}$, then the firms are willing to pay a higher wage for that type of worker according to (3.6) and (3.7). However, these hiring probabilities are also determined endogenously and are affected by both the matching technologies and the determination of wage to be discussed in the next section.

Combining equations (3.6) and (3.7) with (3.3), we obtain the "demand for labor" for each worker type.

$$(r+s)\gamma\frac{V_n}{m_n} = f(k) - rk - w_n.$$

$$(r+s)\gamma\frac{V_i}{m_i} = f(k) - rk - w_i.$$

It will become useful later to express the above two functions using the "market tightness", defined as the ratio between the number of vacancies and the number of job searchers, $X = \frac{V}{U}$, $X_i = \frac{V_i}{U_i}$, and $X_n = \frac{V_n}{U_n}$. The demands for labor thus become

$$w_n^f = f(k) - rk - (r+s)\gamma\frac{X_n}{YX^{1-\alpha}}, \tag{3.9}$$

$$w_i^f = f(k) - rk - (r+s)\gamma\frac{X_i}{\delta X^{1-\alpha}}, \tag{3.10}$$

where $Y = 1 + (1-\delta)\frac{U_i}{U_n}$. The new notations w_n^f and w_i^f are the wage rates the firms offer to the workers.

3.4 The Workers

The workers move between two states: employed and unemployed. Let E_i and N_i be the capital value of employment state and unemployment states for immigrant workers, respectively. Both capital values must also satisfy the relevant dynamic optimization equations

$$rE_i = w_i - s(E_i - N_i), \tag{3.11}$$

$$rN_i = z + p_i(E_i - N_i). \tag{3.12}$$

The interpretation is similar to the firms' equations: the right-hand-side are capital costs of E_i and N_i and the left-hand-side are respective gains. In (3.11), the net gain from occupying a job is the earned wage minus the expected loss due to job termination. The gain from being employed consists of "unemployment benefit" z and the expected gain from switching from unemployment to employment with the probability $p_i \equiv \frac{m_i}{U_i}$.

The corresponding asset pricing equations for native workers are

$$rE_n = w_n - s(E_n - N_n), \tag{3.13}$$

$$rN_n = z + p_n(E_n - N_n), \tag{3.14}$$

with $p_n \equiv \frac{m_n}{U_n}$.

The unemployment benefit z may include imputed return from any unpaid "leisure" activities, such as home improvements or recreation. Again, in order to focus on the effects of the difference in matching process, we assume this unemployment benefit to be the same for both types of workers, although they may be quite different in reality (especially when z includes unemployment insurance benefit paid by the government, for which native workers are more likely to be eligible).

If we subtract (3.12) from (3.11) we get

$$r(E_n - N_n) = w_n - z - (s + p_n)(E_n - N_n),$$

or

$$E_n - N_n = \frac{w_n - z}{r + s + p_n}. \tag{3.15}$$

Similarly,

$$E_i - N_i = \frac{w_i - z}{r + s + p_i}. \tag{3.16}$$

There are two implications from the last two equations. One is that for any unemployed worker to have any incentive to search for a job, that is, $E_n > N_n$ and $E_i > N_i$, the wages must be greater than the unemployment benefit, $w_n > z$ and $w_i > z$. Second, if paid equal wage, immigrant workers gain more from getting hired than native workers do. To see this, note that

$$\begin{aligned} p_n - p_i &= \left(1 + (1-\delta)\tfrac{U_i}{U_n}\right)\left(\tfrac{V}{U}\right)^{1-\alpha} - \delta\left(\tfrac{V}{U}\right)^{1-\alpha} \\ &= (1-\delta)(1+\tfrac{U_i}{U_n})\left(\tfrac{V}{U}\right)^{1-\alpha} \\ &> 0. \end{aligned} \tag{3.17}$$

Native workers always have higher probability of getting hired $p_n > p_i$ which implies $(E_n - N_n) < (E_i - N_i)$ for $w_n = w_i$. The reason is that, because it is more difficult for an immigrant to find a job, the gain from actually getting one is therefore higher.

3.5 Wage Determination

Since the market is frictional with positive recruiting cost, a successful job matching will create an economic rent or surplus from the saving of this cost. The question is then how this surplus is distributed between the hiring firms and the hired workers. Because the distribution of rent is directly determined by the wage rate – a higher wage means the workers get more of the surplus and the firms less of it – the important consideration is then how the wage is actually determined in the local labor market.

There are several different wage determination rules in the literature[13]. In our model, we will assume that wages are determined by Nash bargains between the

[13]These include (i) "market-clearing" wage where firms set wage equal to workers' reservation wage, and hence collect all the rent; (ii) "efficiency wage" where the workers collect some rent through their ability to shirk on the

firms and the workers. We further assume that because the immigrant workers tend to be less organized than the native workers, their bargaining power is also less. A special case is when the immigrants have no bargaining power, and the wage determination collapses to that of the "market clearing" rule.

Suppose the native workers' bargaining power can be measured by a constant parameter β_n, where $0 \leq \beta_n \leq 1$, then a Nash bargains implies[14]

$$E_n - N_n = \beta_n(J_n - O_n + E_n - N_n)$$

or, with $O_n = 0$,

$$E_n - N_n = \frac{\beta_n J_n}{1-\beta_n} = \frac{\beta_n}{1-\beta_n}\gamma\frac{V_n}{m_n}.$$

Substituting for $E_n - N_n$ from (3.15) and using again the notation of market tightness, we can write the "bargained wage" functions arising from bargaining as

$$w_n^b = z + \frac{\beta_n \gamma}{1-\beta_n}(\frac{r+s}{YX^{1-\alpha}} + 1)X_n. \tag{3.18}$$

Similarly, for the immigrants,

$$w_i^b = z + \frac{\beta_i \gamma}{1-\beta_i}(\frac{r+s}{\delta X^{1-\alpha}} + 1)X_i. \tag{3.19}$$

where $0 \leq \beta_i \leq \beta_n \leq 1$. When $\beta_i = 0$, the immigrants earn their reservation wage which is equal to unemployment benefit z and they enjoy no economic surplus.

3.6 The Steady State Equilibrium

In the steady state without economic growth all endogenous variables remain unchanged. In our model, there are five endogenous variables: U_n, U_i, V_n, V_i and capital

job under the firms' monitoring; and (ii) "insider-outsider" wage where employed workers (the insiders) enjoy some rent because the firms must incur positive turnover costs if they want to replace the employed workers with the new workers (the outsiders). The Nash bargaining used in our model is analytically similar to the insider-outsider models.

[14]One can also think of this relation as a first order condition from the maximization of weighted product of the workers' and the firms' surplus from a filled job

$$(E_n - N_n)^{\beta_n}(J_n - O_n)^{1-\beta_n}.$$

k. Since capital k is determined by $f'(k) = r$, it is therefore a question of how we determine the interest rate. We have two choices. The first choice is to follow a close economy modeling by introducing consumers' intertemporal optimization. The second choice is to fix the interest rate by assuming that the economy is small and open, and that capital is perfectly mobile across countries. We chose to follow the second choice for three reasons: (i) it allows one to focus on the effect of immigration alone without having to model the interaction between the labor market and the commodity and the capital markets, (ii) it is unlikely that immigration is in a magnitude so significant as to affect the host countries' capital accumulation and interest rate, and (iii) it simplifies the analysis.

The steady state equilibrium in this model then reduces to the four variables U_n, V_n, U_i and V_i that satisfy the following four equations

$$YX^{1-\alpha} = p_n = s\frac{(L_n - U_n)}{U_n} = s\frac{(1-u_n)}{u_n}, \tag{3.20}$$

$$\delta X^{1-\alpha} = p_i = s\frac{(L_i - U_i)}{U_i} = s\frac{(1-u_i)}{u_i}, \tag{3.21}$$

$$(\frac{r+s}{YX^{1-\alpha}} + \beta_n)X_n = \frac{(1-\beta_n)}{\gamma}\left[f(k) - rk - z\right], \tag{3.22}$$

$$(\frac{r+s}{\delta X1-\alpha} + \beta_i)X_i = \frac{(1-\beta_i)}{\gamma}\left[f(k) - rk - z\right], \tag{3.23}$$

with $U = U_i + U_n$, $V = V_i + Vn$, $Y = 1 + (1-\delta)\frac{U_i}{U_n}$, $X = \frac{V}{U}$, $X_n = \frac{V_n}{U_n}$, $X_i = \frac{V_i}{U_i}$, $u_n = \frac{U_n}{L_n}$ and $u_i = \frac{U_i}{L_i}$.

The first two equations are the steady state requirement of U_n and U_i (equations 3.5 and 3.4) divided by U_n and U_i respectively. They imply that unemployment rate among natives is always lower than unemployment rate among immigrant, because $\delta < 1$ and $Y > 1$. The last two equations combine the labor demands (equations 3.9 and 3.10) and the bargained wage (equations 3.18 and 3.19) by substituting out the wages.

We focus our attention on how the heterogeneity between the two worker types determines the effect of immigration. In our model, the heterogeneity stems from two sources, the immigrants' inferior matching technology ($\delta < 1$), and the weaker immigrants' bargaining power ($\beta_i < \beta_n$). In this section, we will first look at the effects of these two factors in shaping the pre-immigration local labor market which consists of native workers and existing immigrant workers. We then examine what happens when new one-time immigrations take place in the next section.

Table 3.1 shows the simulation results of how decreasing δ and β_i change the steady state equilibrium from the one where all workers are identical. Figure 3.1 shows these results diagrammatically. Figure 3.1 is drawn using the demands for labor and bargained wage equations (3.9), (3.18),(3.10) and (3.19), assuming that immigrant labor force is tenth time the natives. For convenience, we restate these four equations here

$$w_n^f = f(k) - rk - (r+s)\gamma\frac{X_n}{YX^{1-\alpha}} = f(k) - rk - (r+s)\gamma\frac{X_n}{p_n},$$

$$w_n^b = z + \frac{\beta_n\gamma}{1-\beta_n}(\frac{r+s}{YX^{1-\alpha}} + 1)X_n = z + \frac{\beta_n\gamma}{1-\beta_n}(\frac{r+s}{p_n} + X_n),$$

and

$$w_i^f = f(k) - rk - (r+s)\gamma\frac{X_i}{\delta X^{1-\alpha}} = f(k) - rk - (r+s)\gamma\frac{X_i}{p_i},$$

$$w_i^b = z + \frac{\beta_i\gamma}{1-\beta_i}(\frac{r+s}{\delta X^{1-\alpha}} + 1)X_i = z + \frac{\beta_i\gamma}{1-\beta_i}(\frac{r+s}{p_i} + 1).$$

The market tightnesses on the horizontal axis[15] is chosen based on a one-to-one relation between their movements and the movements in unemployment rates, namely, the unemployment rate drops when the respective market becomes tighter[16]. The intersection point of the thin lines $\bar{w}^b$ and $\bar{w}^f$ in both panels is the equilibrium that would occur if there were no difference between the two worker types, namely,

[15]In fact, all the curves can not be drawn without knowing first the equilibrium values of X and Y. In other words, the position of the curves must be determined jointly with the intersections. However, the curves' shape and position and the equilibria they imply are sufficiently accurate for qualitative discussions that follow.

[16]X_n (X_i) moves in the same direction as $YX^{1-\alpha}$ ($\delta X^{1-\alpha}$) according to (3.22) ((3.23)). $YX^{1-\alpha}$ ($\delta X^{1-\alpha}$) in turn moves in the opposite direction to u_n (u_i) according to (3.20) ((3.21)).

the natives and the immigrants had the same matching technology and equal bargaining power. The intersections of the thick lines in panel (a) of Figure 3.1 depict the equilibria with inferior matching technology ($\delta < 1$) and the intersections of the thick lines in panel (b) depict the equilibria with weak bargaining power for immigrant workers. We will discuss the two cases separately.

Effect of Inferior matching technology

A decrease in δ lowers the rate of successful matching between the firms and the immigrants, at the expense of the increased matching rate for the natives (lower m_i and higher m_n). In the equilibrium, this also implies a lower probability that the unemployed immigrants are hired and a higher probability that the unemployed natives are hired (lower p_i and higher p_n). Declining p_i shifts the curve w_i^f down as the firms now offer a lower wage to the immigrants to compensate for the higher expected recruiting cost ($\frac{\gamma}{q_i}$) which, at any given level of market tightness, varies negatively with the matching probability p_i according to $q_i = \frac{p_i}{X_i}$ by definition.

The pressure on immigrants' wage cut is reduced with an upward shift of the curve w_i^b, which is caused by the fact that decreasing p_i raises the expected economic surplus of successful matching between the firms and the immigrants (which is equal to the job value performed by the immigrants, $J_i = \gamma\frac{X_i}{p_i}$). As a constant portion of this increased surplus belongs to the immigrant workers though wage bargaining, the firms would have to pay higher w_i for any given market tightness X_i. The new equilibrium is therefore at the intersection between the lower w_i^f curve and the higher w_i^b curve, resulting in a relatively large reduction of X_i as the firms reduce vacancies targeting immigrants (V_i) at the same time that the number of steady state unemployed immigrants (U_i) increases according to (3.21). Reducing X_i enables the firms to both bring down the recruiting cost and pay a lower wage, and it implies an increase in unemployment rate u_i. The new equilibrium wage w_i is lower than $\bar{w}$ because the immigrants do not have a complete bargaining power

($\beta_i < 1$) to prevent the wage from declining. The opposite is true with the natives. The new equilibrium market tightness among the natives X_n is higher (lower u_n) as well as the wage w_n. However, the size of changes in u_n and w_n are not as large as those of u_i and w_i because the increase in p_n is smaller than the decrease in p_i[17].

Although the immigrants clearly suffer, the natives actually benefit from the immigrants' inferior matching efficiency. In essence, the natives do not only 'steal' the jobs from the immigrants but also receive higher pay because the firms' demand for them increases. Another important aspect within this case is that the market adjustment to a declining efficiency is more on employment rates than on wage rates.

A Weaker Bargaining Power

The market adjusts rather differently to a decline in the immigrants' bargaining power. The lower panel of Table 3.1 and panel (b) of Figure 3.1 reveals this by assuming that $\delta = 1$ (which also implies $Y = \delta\frac{p_n}{p_i} = 1$) and that β_i is significantly lower than β_n. In this case, the firms will pay the same wage if the two markets are equally tight (the curves w_i^f and w_n^f coincide). However, because immigrants now demand a smaller fraction than the natives do of the economic surplus created by successful machines, the firms can negotiate to pay them a lower wage rate. The downward shift of w_i^b represents this weaker bargaining power by the immigrants. The demand for immigrants worker increases with the firms opening more vacancies toward immigrants (V_i) at the expense of the less vacancies toward the natives (V_n). The increase in V_i more than offsets the decrease in V_n, yielding a higher total vacancies V. This raises the overall matching rate $m(U, V)$ (implying an economy-wide job creation) which results in not only the increase in m_i but also in m_n. In other words, the increasing demand for the immigrant workers spills over to increase the demand for the native workers. In the new equilibrium, the probability of getting

[17] $\frac{\partial p_n}{\partial \delta} < 1 = \frac{\partial p_i}{\partial \delta}$

hired increases for both type of worker ($p_i = p_n = p$),which leads to an identical and lower unemployment rate according to (3.20) and (3.21) with $\delta = Y = 1$. Lower p_i and p_n are what cause the upward shifts of w_n^b, w_n^f and w_i^f. The market for the immigrants is tighter ($X_i > X_n$) but their wages rate is much lower ($w_i < w_n$) to reflect the lower β_i.

We draw three important observations from the above discussion. First, the native workers always benefit from the immigrants' weaker position, either from their inferior matching technology from their weaker bargaining power. The channels of the benefit are however different. For lower δ, the natives benefit from gaining a higher share of economic surplus from successful job matching, or from job stealing. For lower β_i, the natives gain from a spillover effect from job creation stimulated by the immigrants' willingness to accept lower wage rate. Second, the immigrant workers become unambiguously worse off with inferior matching efficiency, but are able to reduce the unemployment rate when they possess lower bargaining power, although at the expense of a sizable drop in the wage rate. The third observation is that the market adjustment to changes in δ is more through unemployment rates while the adjustment to changes in β_i is more through wage rates.

3.7 Impact of One-Time Immigrations

In this section, we consider the impact of a one-time immigration, which has the direct effect of increasing the number of immigrant workers in the economy. Assume that the economy has a constant native labor force L_n, and it begins with no immigrant worker, $L_i = 0$. Then we allow a one-time inflow of immigrants into the economy. Let $i = \frac{L_i}{L_n}$ be the proportion of immigrant labor force to native labor force. This proportion increases with immigration. Different labor force ratios are simulated to see the effects of pre-immigration conditions when subsequent immigrations take place.

We will assume throughout that the existing immigrants are both less efficient in job searching and possess weaker bargaining power than the natives do, namely, the pre-immigration condition is the combination of the market outcomes presented in panels (a) and (b) of Figure 3.1. Furthermore, the arriving immigrants are also assumed to be identical to the existing ones. The following proposition summarizes our findings

Proposition 3.1. There exist β_i^i and β_i^n, with $\beta_i^i < \beta_n < \beta_i^n$, such that one-time immigrations improve the labor market outcomes of both worker groups when $\beta_i < \beta_i^i$, improve only the labor market outcome of the existing immigrants when $\beta_i^i < \beta_i < \beta_i^n$, and deteriorate the labor market outcomes of both worker groups when $\beta_i^n < \beta_i$.

Proof: The formal proof of this Proposition is provided in the appendix of this chapter.

Table 3.2 presents the simulation that supports the above Proposition. The upper panel of the Table shows the simulation results when the immigrant workers have relatively weak bargaining power ($\beta_i = 0.2$), while the lower panel assumes a relatively strong bargaining power ($\beta_i = 0.8$). The value of δ is fixed at 0.8 and that of β_n at 0.5. Figure 3.2 shows how new equilibria are reached in these two cases on the wage-tightness plane.

Proposition 3.1 and the simulation state that, with fixed level of δ, the impact of immigration on both the existing immigrants and the natives depend exclusively on the immigrants' bargaining power β_i. A more favorable labor condition, namely lower unemployment rates and higher wage rates, tend to be the outcome when β_i is low and a worsening labor market outcome when β_i is high. The determinations of the critical values of β_i that determine whether either type of workers benefit from one-time immigrations is formally proof in the appendix to this chapter. The results are however better explained intuitively.

We know from the previous section that the native workers benefits from the presence in the local labor market of the immigrants who either are less efficient job searchers or possess weak bargaining power. With the arrival of the new immigrants, the local labor market now consists of three groups of workers: the natives, the existing immigrants and the newly arrived immigrants. Since the new immigrants have identical labor market characteristics as the existing immigrants, their impact on the market will strengthen the same impact the existing immigrants already possess. This is particularly true with respect to the native workers, because of the qualitatively identical labor market outcomes for the natives between those resulting from either the decreases in δ or in β_i of the existing immigrants (Table 3.1) and those resulting from the arrival of the new immigrants with low δ and low β_i (upper panel of Table 3.2). Because all workers have the same productivity and capital is assumed to be perfectly mobile across borders, the addition of new immigrants into the economy will necessarily create more jobs as there are more firms established to accommodate the larger pool of job seekers. Some of the new jobs created are occupied by the natives in the new equilibrium because of both the "job stealing" effect due to the natives' superior matching technology compared to the new immigrants, and the "job creation spillover" effect due to the immigrants's low bargaining power. The higher demand for the native workers reduces their unemployment rate and increases their wage.

The existing immigrants do not share with the natives the advantages over the new immigrants, since they are identical to the new immigrants in both δ and β_i. They therefore do not enjoy the immediate benefit from the immigration the same way the natives do. The benefit to the existing immigrants is possible, however, if the immigrations create jobs at a faster rate than the rate of increase in total labor force, namely, if the overall unemployment rate decreases. If that is the case, then the immigrants' unemployment rate will also decrease because $s\frac{(1-u)}{u} = X^{1-\alpha} = s\frac{(1-u_i)}{\delta u_i}$, where u is the overall unemployment rate. The sufficient condition for decreasing unemployment rate as a result of immigration is that the pre-immigration local labor market is characterized by a tighter market for immigrants ($X_i > X_n$). We know from the previous section and Figure 3.1 that X_i is increasing with δ and decreasing with β_i, which means a $\delta < 1$ causes X_i to be lower than X_n and the effect can be offset by lowering β_i. As a result, there exists a critical value $\beta_i^i < \beta_n$[18], such that the existing immigrants benefit from immigration only if their bargaining power is less than β_i^i. The immigrants, both the existing and the newly arrived ones, must prepare to accept much lower wage than the natives in order to ensure their employment rates in spite of their inferior matching efficiency. If β_i is greater than β_i^i, the existing immigrants will be made worse off with addition of new immigrants.

The natives will also be worse off if the immigrants have a very strong bargaining power. Let β_i^n be the critical value for which the natives stand to lose because of immigration when the immigrants possess a bargaining power greater than β_i^n. It is straightforward to see that $\beta_i^n > \beta_n (> \beta_i^i)$ as long as $\delta < 1$. From panel (a) of Figure 3.1, we see that more natives are employed because of $\delta < 1$ even if both worker groups have the same bargaining power ($\beta_i = \beta_n$). In other words, the job stealing effect is still present although the job creation spillover effect is no longer present. The bargaining power of the new immigrants must be stronger than β_n before its effect in increasing the overall unemployment rate begins to dominate the job stealing effect, and the demand for the natives workers begins to fall. That is, $\beta_i^n > \beta_n$.

Figure 3.2 displays the market adjustments to one-time immigrations when $\beta_i < \beta_i^i$ (panel (a)) and when $\beta_i > \beta_i^n$ (panel (b)). In panel (a), the labor market for the immigrants is tighter than the labor market for the natives before the new immigrations take place. The equilibria are the intersection of the thick-lines w_n^b, w_i^b, w_n^f and w_i^f. The arrival of news immigrants create more demand for both groups of workers, causing p_i and p_n to increase, and all the four curves to shift up. The

[18] β_i^i must be lower than β_n to compensate for the loss of market tightness X_i due to inferior matching technology $\delta < 1$, as shown in the panel (a) of Figure 3.1.

new equilibria are associated with tighter markets, implying lower unemployment rates, and higher wage rates. Panel (b) depicts the case where $\beta_i > \beta_i^n$, with the adjustment to the new immigration in the opposite directions to that in panel (a) because the demand for both worker types decrease.

The next proposition examines how the two critical values β_i^i and β_i^n change under different pre-immigration conditions.

Proposition 3.2. β_i^i is increasing in both the immigrants' matching efficiency (δ) and the natives' bargaining power (β_n) while β_i^n is decreasing in δ but increasing in β_n.

Proof: See the appendix to this chapter.

Table 3.3 shows the simulation results that are consistent with this Proposition.

An increase in δ implies the immigrants lose less jobs to the natives (the distance between X_i and X_n in Figure 3.1 panel (a) becomes smaller), and thus require a less weaker β_i to gain the jobs back, resulting in higher β_i^i. For a given degree of job stealing effect (a given level of $\delta < 1$), an increase in β_n reduces the gap between X_i and X_n for every β_i. This means the value of β_i^i that exactly closes the gap is also higher.

For the natives, an increase in δ reduces the advantage the natives enjoy in the form of the job stealing effect. So it will take a lower loss in the overall employment rate to neutralize this advantage, which means lower β_i^n. On the other hand, if δ is unchanged but β_n increases the full job stealing effect occur at a higher $\beta_i = \beta_n$. This implies a higher β_i^n.

In essence, the Proposition 3.2 states that (i) the host countries with higher δ, compared to those with lower δ, will see their existing immigrants more likely, and their natives less likely, to benefit from immigration and that (ii) both the natives and the existing immigrants in the host countries with higher β_n are more likely to benefit from immigration.

3.8 Implications on International Migration Flows and Welfare Considerations

Proposition 3.2 above enables us to draw some implications on migration flows as well as welfare considerations from this model. Suppose there are two destination countries available to a potential immigrant from a source country. The decision on whether or not to move and where the best destination is depends on the labor market characteristics of all the three countries involved.

Clearly if the labor market in the source country rewards their indigenous workers significantly less than at least one of the two destination countries does to its immigrant workers, the immigration into that destination country is likely. However, the potential immigrant has to also consider the prospect of getting hired in each destination. A good balance of wage rate and unemployment rate is therefore the ultimate measure upon which the choice of place to work is made. Using this criterion, our model offers the following implications on international migrations and welfare.

1. International migrations tend to originate from source countries with a less organized labor force (low β_i) and a high unemployment rate. A number of factors can account for high unemployment despite the workers' low bargaining power. Either low production productivity or high interest rates, due possibly to financial incompleteness in spite of perfect international capital mobility, can both raise unemployment and lower wage rates. Moving out of such country is therefore an improvement in welfare.

2. The natives and the existing immigrants of countries receiving immigrants with low bargaining power, such as those from the source countries described above,

are likely to gain because of the job spillover effect discussed in the last section.

3. Countries that are customarily reluctant to hire foreigners, possibly because of cultural closedness or nationalism, are less preferred destinations for potential immigrants who, if choose to immigrate, would suffer a strongly inferior matching opportunity (low δ). The immigrants would earn less and be subject to high probability of being unemployed. However, the natives of such destination countries are more likely to be better off with immigration – at least economicly if not politically – as they would earn higher wage and enjoy a lower unemployment rate because of the job stealing effect.

4. On the contrary, countries that are more friendly to foreign workers (to both legal and illegal immigrants – possibly the United States) or the countries that have a large social network among their immigrants – implying a high δ – are high on the potential immigrants' destination list. A high δ unambiguously benefit the would-be immigrants. The host country's natives are, however, less likely to benefit from the immigration.

5. A comparison of destination countries with low and high degree of their natives' bargaining power (β_n) is not as clear-cut as that between low and high δ destination. A high-β_n country offers the immigrants a higher wage rate but less chance of getting hired while a low-β_n country offers a lower unemployment rate but a relatively low wage. It thus depends on the preference function of the potential immigrants regarding the substitutability between earned income consumption and leisure.

6. A number of European countries are known to have relatively strong labor unions and rigid wage structure (high-β_n countries). For those countries that are also immigrant-unfriendly (low δ), immigration has a greater possibility of benefiting their natives, because their β_i^n is higher than other countries. They also have a better chance of attracting immigrants compared to those countries with low β_n and low δ.

These are some broad implications within the setting of our model. The overall welfare implications are harder to assess as one might want to consider the costs of social welfare the immigrants may burden the host countries. These costs are undoubtedly passed to the natives (and the existing employed immigrants).

3.9 Summary and Future Researches

In this chapter we propose a simple modification of the equilibrium unemployment theory to study the impact of immigration on unemployment and wage of both natives and immigrants. The less efficient matching technology among immigrants due to their inferior social and economic network to the natives costs them more on higher employment than on lower wage. The lower bargaining power costs the immigrants more on lower wage than on higher unemployment. The natives mostly benefit with the only exception occurs when the immigrants have substantially higher bargaining power: an unlikely characteristic of the real world labor markets.

There are several directions the model can be extended. For example, the models' results on immigration's job creation effects on the natives (both the job stealing and the job spillover effect) depend on the assumption of perfect capital mobility. Relaxing this assumption undoubtedly will lower the benefit to all parties involved as the interest rate will rise to reflect a higher demand on capital. The network effect, which is assumed to be constant here, can be made dependent on the size of the immigrant community. Furthermore, improving the network establishment can also be seen as part of the assimilation process, which may continue throughout the immigrants' lifetime and even pass on to the next generations. If we allowed for this type of assimilation, the network parameter δ would be a function of not only i but also on time and generation index with a much richer dynamics.

It should be noted that unskilled and skilled immigrants have different impact on the local labor market, a consideration that may affect the immigration policy.

Immigrants may also cause a strain in the welfare system. While immigration may create better employment and earning opportunity for natives in this model, a higher unemployment rate among immigrants put more demand on the welfare system which could result in heavier taxes on the natives. The overall implication of immigration in such cases is thus more difficult to evaluate.

Chapter 4

Precautionary Saving, Growth, and Welfare: A Cross-Country Study

4.1 Introduction

Recent studies on endogenous growth models suggest that there is no longer a monotonic relationship among saving, growth, and welfare when insurance or other financial markets are malfunctioning. In addition, the development of financial markets may have complicated effects on economic growth as well as welfare. For example[1], Devereux and Smith [14, 1994] show that, while sharing rate-of-return risk among countries leads to lower saving and slower growth, it raises welfare when risk aversion is strong. The model of Saint-Paul [38, 1992] has a similar feature. Bencivenga and Smith [4, 1991] discuss the development of financial markets in terms of the trade-off between growth-enhancing effects and saving-lowering effects. Obstfeld [34, 1992] shows that a financial market integration reduces sav-

[1] Pagano [36, 1993] surveys studies on endogenous growth models in this context.

ing, but raises growth by a portfolio shift to productive technology. Jappelli and Pagano [25, 1994] build a model where weakening liquidity constraints may lower both saving and growth with positive or negative effects on welfare. These papers indicate that the welfare implication of economic growth should be examined carefully when financial markets are evidently imperfect, or when financial markets are being deregulated. Such an attentive evaluation may lead us to putting the growth experience of developed and developing countries in a new perspective.

To address the above issue, we construct a simple endogenous growth model that quantitatively examines the effect of incomplete insurance on saving, growth, and welfare. This model is applied to the growth experience of the OECD countries. The model particularly deals with the case where permanent idiosyncratic (person-specific) shocks cannot be pooled through insurance contracts. Although asymmetric information may be the source of incomplete insurance, this model treats incompleteness exogenously rather than endogenizing it.

This setup is mainly motivated by recent empirical findings and theoretical developments. Several empirical researches suggest that a significant part of permanent idiosyncratic shocks may not be insured in markets. Mace [31, 1991] and Cochrane [8, 1991], for example, show that the assumption of complete markets is not always accepted statistically using panel data. Deaton and Paxson [12, 1994] find that the inequality of consumption grows with age within the same cohort in the U.K., the U.S., and Taiwan. As will be discussed in the text, the finding of Deaton and Paxson indicates that uninsured shocks have a permanent impact on individual wealth. In the empirical labor economics, Card [7, 1990] finds that person-specific shocks have very persistent effects on the individual wage process.

The literature on saving suggests that precautionary saving driven by persistent or permanent uninsured shocks has a great impact on macroeconomic performance, in particular aggregate capital accumulation. Modigliani [32, 1988] explicitly presents this view, while Caballero [6, 1990], Deaton [11, 1991], Kotlikoff

[28, 1988], Skinner [42, 1988], Zeldes [48, 1989], and others analyze the impact of precautionary saving on capital accumulation in various setups. The recent work by Aiyagari [1, 1994] studies the quantitative importance of precautionary saving when uninsured shocks on labor income are persistent.

In addition, the setup of permanent uninsured shocks provides a great deal of analytical convenience[2]. As shown in Constantinides and Duffie [9, 1992] for exchange economy and Saito [39, 1995] [40, 1996] for production economy, assuming permanent idiosyncratic shocks[3] leads to an extremely simple case where bond markets cannot play any role as self-insurance regardless of market frictions; consequently, this case can analyze the positive and normative implication in a simple closed-form without involving any complicated interaction between incomplete insurance and market frictions[4].

The model is a simple application of Ak type endogenous growth model (Rebelo [37, 1991]); endogenous growth is brought about by non-diminishing private returns on capital. The assumed utility function is a continuous-time version of Kreps-Porteus type non-expected utility (Kreps and Porteus [29, 1978], Epstein and Zin [17, 1989], Weil [47, 1990], Svensson [44, 1989], Duffie and Epstein [16, 1992]). This basic setup is adopted by Obstfeld [34, 1992] in the context of endogenous growth models. The major difference between this model and the previous literature is that a linear technology is driven by not only aggregate shocks, but also uninsured

[2] As the literature on incomplete insurance clarifies (e.g. Bewley [5, 1986], Lucas [30, 1990], Telmer [45, 1993], Aiyagari and Gertler [2, 1991], Heaton and Lucas [24, 1992], Aiyagari [1, 1994], Den Haan [13, 1994]), the need for precautionary saving depends on how bond markets can be substituted for insurance markets, or how bond markets can play a role as self-insurance. These papers point out that two factors may raise precautionary saving by limiting the role of bond market as self-insurance, (i) the persistence of uninsured shocks and (ii) several kinds of market frictions. Given such a complicated interaction between incomplete insurance and market frictions, the listed papers adopt a sophisticated numerical calculation technique to derive equilibrium asset prices.

[3] As Constantinides and Duffie [9, 1992] show, it is possible to construct a case where the cross-sectional distribution of wealth (consumption) is stationary in the presence of permanent idiosyncratic shocks. In Appendix 4B, we also constructs one case with a stationary cross-sectional distribution of wealth.

[4] In exchange for such an analytical convenience, this model cannot address any implications on asset pricing of trading across heterogeneous agents, which are emphasized by Aiyagari and Gertler [2, 1991], Aiyagari [1, 1994] and others.

idiosyncratic shocks[5]. In addition, the model can endogenize risk-free returns by imposing a market clearing condition on bond markets.

In a standard endogenous growth model, higher productivity leads to faster economic growth, while it raises risk-free rates because consumers have an incentive to borrow against future resources. Higher productivity improves individual welfare. In addition to this usual mechanism, our model provides another route which influences growth and welfare. When agents face uninsured permanent shocks, they save more due to precautionary reasons[6]. Such precautionary saving behavior promotes economic growth, but lowers risk-free rates. Since the volatility of future income causes precautionary saving, large precautionary saving has a negative welfare implication.

Within this environment, the welfare implication of economic growth depends on which factor is responsible for faster growth, higher productivity or stronger precautionary saving. And what is more, if economic growth has a positive externality on productivity, a trade-off emerges in terms of the impact of uninsured shocks on individual welfare. On the one hand, larger uninsured shocks reduce welfare directly as mentioned above while, on the other hand, they improve productivity due to faster economic growth, thereby enhancing welfare indirectly. In other words, the identification of the source of economic growth is crucial for quantifying the welfare implication. To this purpose, we pay attention to the sum of safe returns and economic growth. As discussed above, higher productivity and stronger precautionary behavior have the same qualitative impact on economic growth, but generate opposite effects on safe returns; the former raises risk-free rates, while the latter lowers it. In terms of welfare, for example, high growth with low returns is

[5]Using a similar framework, van Wincoop [46, 1994] allows for uninsured idiosyncratic shocks, and examines the welfare implication using the OECD data. In terms of the empirical strategy, however, his paper and the current paper differ substantially in that the former a priori assumes a certain level of person-specific risk, while the latter actually calculates the magnitude of uninsured risk for each country by exploiting the cross-country data.

[6]Although precautionary saving is driven by aggregate shocks as well, the effect on welfare and growth is marginal due to small magnitudes of aggregate shocks.

dominated by high growth with high returns.

Based on the above model, the empirical part of this chapter quantitatively evaluates the growth experience of the OECD countries. First, we calculate the size of uninsured idiosyncratic shocks for each country from observable macro data (per-capita consumption growth and interest rates). The implied magnitude of uninsured shocks may be interpreted as a measure of insurance incompleteness. The calculation result shows that individuals in each country face large idiosyncratic shocks relative to aggregate shocks, and that the magnitude of uninsured shocks is very different across countries. In addition, the cross-country difference of saving is tightly linked with the heterogeneous degree of insurance incompleteness.

Second, we show that the calculated magnitude of idiosyncratic shocks is consistent with micro evidence. For the cases of the U.K. and the U.S., the implied individual risk is quite comparable with the finding of Deaton and Paxson [12, 1994]. The relative difference in the implied magnitude among the OECD countries corresponds well to that in the proxies for the prevalence of insurance contracts (borrowed from Goldsmith [22, 1985]); countries where insurance is not so prevalent possess larger magnitudes of idiosyncratic shocks. Such consistency with micro evidence strengthens the empirical relevance of this model.

Third, we identify the source of the economic growth slowdown over the past three decades across the OECD countries. There are two major sources of lower economic growth in this model; lower productivity and less need for precautionary saving. Our calculation shows that the economic slowdown from the 1960's to the 1970's is mainly due to the (exogenous) productivity slowdown, while that from the 1970's to the 1980's is largely due to the reduced need for precautionary saving.

Finally, we quantitatively examines the welfare implication of the growth experience. According to the results, since the externality of growth on productivity is strong in the 1960's, the countries with heavily constrained insurance markets (e.g.

Japan, Greece, and Italy) improve productivity by higher growth due to strong precautionary saving, thereby enhancing welfare. On the other hand, the externality of growth becomes weaker in the 1980's; the improvement of insurance markets experienced by most countries in the 1980's reduces the need for precautionary saving and lowers economic growth, but still contributes to enhancing welfare without triggering negative welfare effects.

Chapter 4 is organized as follows. Section 4.2 presents an analytical framework and discusses its empirical and welfare implications. Using the OECD data, Section 4.3 calculates the magnitude of idiosyncratic shocks and productivity from macro data and checks its consistency with micro data, while Section 4.4 examines the welfare implication according to the proposed welfare measure (growth plus interest). Section 4.5 concludes.

4.2 Model

Suppose that many infinitely-lived consumers live in a continuous-time economy. Each agent i faces the following linear technology:

$$y(t) = [Adt + \sigma_a dB_a(t) + \sigma_h dB_i(t)]\, K(t), \tag{4.1}$$

where $y(t)$ is the output, A is the state of productivity, $K(t)$ is the capital, $dB_a(t)$ and $dB_i(t)$ are the standard Brownian motions. $dB_a(t)$ represents aggregate shocks common across agents, while $dB_i(t)$ characterizes idiosyncratic shocks or agent i-specific shocks. Though $\lim_{I\to\infty} \frac{\sum_{i=1}^{I} dB_i(t)}{I} = 0$, these idiosyncratic shocks cannot be pooled due to missing insurance markets. $dB_a(t)$ and $dB_i(t)$ are not correlated with each other. Both σ_a and σ_h, identical for all agents, measure the magnitude of these two kinds of shocks. σ_h can be interpreted as the degree of incomplete insurance. As will be clear later, both $dB_a(t)$ and $dB_i(t)$ have permanent effects on $K(t)$.

Equation (4.1) is a simple application of Ak type endogenous growth model proposed by Rebelo [37, 1991]. As he suggests, $K(t)$ may be interpreted as physical capital as well as human capital, while $y(t)$ includes not only returns on physical capital, but also returns on human capital such as wages. A is interpreted as the average expected return on both types of capital. For the moment, we treat A as an exogenous parameter. The case where economic growth has a positive externality on A will be explored at the end of this section.

Case without Externality

The individual utility function is characterized by Kreps-Porteus type non-expected utility (Kreps and Porteus [29, 1978], Epstein and Zin [17, 1989], Svensson [44, 1989], Duffie and Epstein [16, 1992]). The main feature of this type of utility is the separation of intertemporal substitution from risk aversion. This separation is very important in analyzing the precautionary saving behavior, because intertemporal substitution and risk aversion jointly influence the degree of relative precaution (see Kimball and Weil [27, 1990]).

We follow the formulation of Svensson [44, 1989]. Each agent i maximizes the below life-time utility ($V(K_i(t))$) by investing her wealth in both the above risky technology and risk-free assets:

$$V(K_i(t)) = \left[c_i(t)^{\frac{\epsilon-1}{\epsilon}}dt + \exp(-\rho dt)\left(E_t[V(K_i(t+dt)^{1-\gamma}]\right)^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}\right]^{\frac{\epsilon}{\epsilon-1}} \tag{4.2}$$

subject to

$$dK_i(t) = \left[(1-x_i(t))r(t)dt + x_i(t)(Adt + \sigma_a dB_a(t) + \sigma_h dB_i(t)) - \mu_i(t)dt\right]K_i(t), \tag{4.3}$$

where $c_i(t)$ is the consumption level, $x_i(t)$ is the share of risky assets, ρ is the discount rate, γ (> 0) is the degree of relative risk aversion, ϵ (> 0, $\epsilon \neq 1$) is the elasticity of intertemporal substitution, E_t is the conditional expectation operator, $r(t)$ is a risk-free rate, and $\mu_i(t)$ is the marginal propensity to consume out of wealth. The depreciation rate of the capital is assumed to be zero for simplicity. When $\epsilon\gamma$ is equal to one, the above formulation reduces to the expected utility function. As equation (4.3) clearly indicates, both aggregate and idiosyncratic shocks have permanent effects on the capital.

In this setup, there is no trade of the risk-free bond market at equilibrium since the bond market plays no role as self-insuring permanent idiosyncratic shocks (see Saito [39, 1995])[7]. Hence, the market equilibrium condition is

$$x_i(t) = 1 \quad \forall i, t. \tag{4.4}$$

Once equation (4.4) holds, risk-free returns do not change over time due to a constant condition of bond market clearing.

Combining the optimal portfolio strategy derived by Svensson [44, 1989] with equation (4.4), we obtain the equilibrium risk-free rate as follows:

$$r(t) = r = A - \gamma(\sigma_a^2 + \sigma_h^2). \tag{4.5}$$

The optimal consumption rule is characterized as below given a constant risk-free rate:

$$\mu_i(t) = \mu = \epsilon(\rho - r) - \frac{\gamma + \gamma\epsilon}{2}(\sigma_a^2 + \sigma_h^2) + A. \tag{4.6}$$

For an equilibrium path to exist,

$$\mu > 0. \tag{4.7}$$

See Appendix 4B for the detailed derivation.

Under the optimal consumption rule (4.6), the individual consumption growth follows

$$\frac{dc_i(t)}{c_i(t)} = \epsilon(r - \rho)dt + \frac{\gamma + \gamma\epsilon}{2}(\sigma_a^2 + \sigma_h^2)dt + \sigma_a dB_a(t) + \sigma_h dB_i(t). \tag{4.8}$$

[7]The reason for this is that agents cannot borrow permanently to compensate a permanent wealth reduction due to negative shocks. Such a permanent borrowing is not allowed under no-Ponzi-game conditions. Similarly, if agents lend permanently in response to positive permanent shocks, this goes against optimality.

The constancy of μ leads to the equality between consumption growth and wealth growth. Since there are no dynamics in the above equilibrium path, we will drop time-subscripts from now on.

In equation (4.5), $\gamma(\sigma_a^2 + \sigma_h^2)$ can be interpreted as risk premia on the capital. In the right-hand side of equation (4.8), the first term ($\epsilon(r - \rho)$) represents the intertemporal substitution motive, while the second term ($\frac{\gamma+\gamma\epsilon}{2}(\sigma_a^2 + \sigma_h^2)$) represents the precautionary saving motive due to both aggregate shocks and idiosyncratic shocks. Notice that $\gamma + \gamma\epsilon$ is the degree of relative precaution defined by Kimball and Weil [27, 1990] in the context of non-expected utility.

Per capita aggregate consumption ($C(t)$) follows

$$dC = \frac{\sum_{i=1}^{I} dc_i}{I} = \left[\epsilon(r-\rho) + \frac{\gamma+\gamma\epsilon}{2}(\sigma_a^2 + \sigma_h^2)\right] C dt + \sigma_a C dB_a + \sigma_h \frac{\sum_{i=1}^{I} c_i dB_i}{I}. \tag{4.9}$$

The last term of the above right-hand side approaches zero as I goes to an infinity thanks to the law of large numbers ($c_i dB_i$ has a finite variance $c_i^2 dt$). Hence,

$$\frac{dC}{C} = \left[\epsilon(r-\rho) + \frac{\gamma+\gamma\epsilon}{2}(\sigma_a^2 + \sigma_h^2)\right] dt + \sigma_a dB_a. \tag{4.10}$$

One important observation is that the aggregate consumption process is different from the individual consumption process (4.8) due to the absence of idiosyncratic disturbances. We will later exploit this difference in order to infer the magnitude of idiosyncratic risk (σ_h).

Combining equation (4.10) with equation (4.5) leads to

$$E\frac{dC}{C} = \left[\epsilon(A-\rho) + \frac{\gamma(1-\epsilon)}{2}(\sigma_a^2 + \sigma_h^2)\right] dt, \tag{4.11}$$

$$Var\frac{dC}{C} = \sigma_a^2 dt. \tag{4.12}$$

In this model, the average aggregate saving rate is defined as $S = E\frac{\sum_{i=1}^{I}(y_i - c_i)}{\sum_{i=1}^{I} y_i}$, where y_i is the individual income from risky investment. Using equation (4.1) and

(4.11), and approximating up to the first order, S is derived as

$$S \approx \frac{\epsilon(A-\rho) + \frac{\gamma(1-\epsilon)}{2}(\sigma_a^2 + \sigma_h^2)}{A}. \tag{4.13}$$

If $\sigma_a^2 = \sigma_h^2 = 0$, then S is the saving rate of typical Ak models.

This simple endogenous growth model has a clear prediction regarding the relationship among expected economic growth ($E\frac{dC}{C}$), risk-free rates (r), and the underlying parameters. When capital productivity is large (high A), interest goes up (equation (4.5)) and the economy grows fast (equation (4.11)). When each individual faces large idiosyncratic shocks (large σ_h^2), on the other hand, the economy still grows fast when ϵ is smaller than one (a very small value of ϵ is empirically confirmed, see Hall [23, 1988]), but interest goes down. The latter prediction is the consequence of precautionary saving.

The above observation suggests that the factor which causes economic growth should be identified in evaluating individual welfare, since high productivity and large uninsured shocks have the same effect on economic growth, but have the opposite effect on welfare. In other words, the welfare evaluation relying on only economic growth may be quite misleading. This insight is confirmed as follows.

Substituting equation (4.5) and (4.10) to the value function ($V(K_i)$) formulated by Svensson ([44, 1989]), we obtain

$$V(K_i) = \mu^{\frac{1}{1-\epsilon}} K_i = \left[\frac{1}{1+\epsilon}\left((1-\epsilon)(g_c + r) + 2\epsilon\rho\right)\right]^{\frac{1}{1-\epsilon}} K_i, \tag{4.14}$$

where g_c is equal to the expected consumption growth, or $\frac{E\frac{dC}{C}}{dt}$. See Appendix 4B for the detailed derivation.

The above equation (4.14) implies that given K_i, $V(K_i)$ is increasing in $g_c + r$ under a given set of parameters[8]. Restating this, given the wealth level, the sum of economic growth (per capita consumption growth) and risk-free returns can serve as an exact measure of individual welfare. From equation (4.5) and (4.11), we obtain

[8] In the case where $\epsilon = 1$, this relation still holds, because $lnV(K_i) = \frac{1}{2\rho}(g_c + r) - \frac{1}{2} + ln\rho + lnK_i$.

$g_c + r = (\epsilon+1)A - \epsilon\rho - \frac{\gamma+\gamma\epsilon}{2}(\sigma_a^2 + \sigma_h^2)$. Therefore, a reduction in σ_h^2 is welfare-improving. Later, the welfare effect of a reduction in σ_h^2 will be modified in the presence of externality.

At an intuitive level, one may think that the above-constructed welfare measure penalizes high economic growth by considering low interest caused by precautionary saving. When precautionary saving is absent, economic growth and interest rates move in the same direction (see equation (4.10)); consequently, adding interest to economic growth may not change the welfare ordering relative to the growth ranking. When precautionary saving motive is strong, however, the growth ranking and the welfare ordering based on the proposed measure may differ substantially.

As equation (4.14) indicates, adopting $g_c + r$ as the welfare measure has several practical advantages. First, $g_c + r$ contains all relevant information regarding the technological opportunity represented by equation (4.1). Second, the measure does not depend on the degree of relative risk aversion (γ). Third, if the elasticity of intertemporal substitution is very small, then the impact of the discount rate (ρ) on individual welfare becomes negligible.

Given these features, we may have two possible cases where the proposed measure is reliable in the context of a cross-country comparison. The first case is that ϵ is rather small for all countries. Then, the welfare ranking based on g_c+r may be robust to the heterogeneities in the underlying parameters γ and ρ among countries. The second case is that either technological factors or insurance incompleteness (A, σ_a, and σ_h) is mainly responsible for the heterogeneous macroeconomic performance among countries. In the latter case, $g_c + r$ contains all relevant information regarding the welfare evaluation. In this paper, we will pursue the second case to evaluate the growth experience of the OECD countries. Section 4.3 will show that the cross-country difference in macroeconomic performance really reflects the heterogeneous technological opportunity and insurance incompleteness rather than the heterogeneous preference.

Case with Externality

To conclude this section, we construct a simple case where growth (capital accumulation) itself has a positive externality on productivity. Suppose that A is characterized as

$$A = \alpha g_c + \beta. \tag{4.15}$$

Since no individual consumer considers the externality of growth on productivity, the above equilibrium without externality still carries over to the case with externality. Combining equation (4.15) with equation (4.11), we obtain

$$g_c = \frac{1}{1-\alpha\epsilon}\left[\epsilon(\beta-\rho) + \frac{\gamma(1-\epsilon)}{2}(\sigma_a^2 + \sigma_h^2)\right], \tag{4.16}$$

where $\alpha\epsilon < 1$[9].

In this case, the effects of uninsured shocks on welfare are twofold. One is, as mentioned earlier, directly reducing welfare, while the other is indirectly increasing welfare through the above externality triggered by high economic growth. Given equation (4.14) and (4.16), it is easy to show that when $\alpha > 1$, the latter effect dominates the former effect[10]. In other words, if the externality is strong ($\alpha > 1$), the net effect of lowering uninsured shocks is to reduce welfare. The magnitude of α is, accordingly, critical in evaluating the welfare consequence of the development of financial markets. In the next section, we will estimate α from the cross-country data.

4.3 Idiosyncratic Shocks and Productivity

This section and the next section interpret the growth experience of the OECD countries along the model developed in the previous section. In so doing, the

[9]The assumption that $\alpha\epsilon < 1$ is maintained throughout this chapter.

[10]The derivation leads to $\frac{d(g_c+r)}{d\sigma_h^2} = \frac{(\alpha-1)(\epsilon+1)}{2(1-\alpha\epsilon)}$.

empirical procedure relies on two important assumptions: identical preference and closed economy (the absence of international capital mobility). While these two assumptions are often made implicitly or explicitly in cross-country studies, it may be worthwhile commenting upon these assumptions here.

By assuming the first, we attribute the heterogeneous macroeconomic performance to the cross-country difference in technological opportunities and insurance incompleteness. To demonstrate the plausibility of this assumption, we carefully examine whether the magnitude of A (measures of productivity) and σ_h (the degree of insurance incompleteness) implied by this model under identical preference parameters (ρ, ϵ and γ) is consistent with other empirical evidence. In particular, the reasonability of the implied σ_h is crucial in assessing the empirical validity of the first assumption because the effect of large σ_h on economic growth and saving is qualitatively similar to that of either small ρ or large ϵ. For example, a country with large σ_h may be observationally equivalent to a country with negative ρ at aggregate levels. If any convincing identification of σ_h were not available from non-macroeconomic data, σ_h would be a meaningless free parameter.

Whether the second assumption is plausible has been controversial since the issue was explicitly raised by Feldstein and Horioka [19, 1980] in 1980. In particular, when and how international capital markets were integrated is quite important for our empirical exercise. According to the existing literature (Feldstein and Bacchetta [18, 1991], Frankel [20, 1990], et al.), while the international integration had been rather restricted by 1980, financial markets have been integrated to some extent during the 1980's. Even now, however, the trade of contingent claims such as equities and insurance contracts is surprisingly limited across borders (French and Poterba [21, 1991], et al.).

If the international integration has not been extended to the trade of contingent claims, most implications of our model may carry over since non-contingent claims can only play a very limited role as self-insuring permanent country-specific or person-specific shocks. In this sense, the model may be robust with respect to the current stage of the financial integration.

Nevertheless, one may not completely rule out the case where our empirical investigation may be subject to the effect of the recent integration of capital markets. In particular, such a potential problem may be serious for the time-series comparison between the 1960's to 1970's and the 1980's. To overcome this time-series problem, we borrow the idea of Barro and Sala-i-Martin [3, 1990] which regards the aggregated OECD countries as a hypothetically-constructed closed economy by arguing that capital inflows and outflows are canceled out across countries. In what follows, we will pay attention to not only the cross-country comparison, but also the time-series behavior of the aggregated OECD countries.

Size of Idiosyncratic Shocks

To examine the empirical prediction, we use four variables for each OECD country, the average and variance of per capita consumption growth, the average risk-free rate (real short term interest rate), and the average saving rate. The data construction is described in Appendix 4A, while the basic statistics are available upon request. The full sample periods is 1960 to 1992 and the three sub-sample periods are the 1960's, the 1970's, and the 1980's respectively.

To construct the aggregated economy from the OECD countries, each of the above four variables are averaged with the weight of each country's consumption share relative to the total consumption of the OECD countries[11]. Hereafter, the aggregated economy is called by the 'average OECD'. Table 4.1 (first panel) reports the basic statistics of the average OECD.

Equation (4.5), (4.10 or 4.11) and (4.12) can impose the structural restriction

[11]The consumption share is adopted from Summers and Heston [43, 1988]. The share used here is averaged over sample periods.

on $E\frac{dC}{C}$, $Var\frac{dC}{C}$ and rdt. We replace the expectation and variance of consumption growth and risk-free rates in these equations by the first and second sample moments of the observed variables for both the full sample period and the three sub-sample periods. As a natural set of preference parameters, we set the discount rate (ρ) equal to 0.02, the degree of risk aversion (γ) in the range of 3 to 5, and the elasticity of intertemporal substitution (ϵ) equal to one third. Unless the result is very sensitive to the value of γ, we report only the case of $\gamma = 3$.

In this model, it is easy to infer the magnitude of idiosyncratic risk from macroeconomic data because σ_h shows up in the drift term of the aggregate consumption, but not in the diffusion term. From equation (4.10) and (4.12), we obtain

$$\sigma_h^2 = \frac{2}{\gamma + \gamma\epsilon} g_c + \frac{2\epsilon}{\gamma + \gamma\epsilon}\left[-r + \rho\right] - \frac{Var\frac{dC}{C}}{dt}. \tag{4.17}$$

The above equation can determine the magnitude of idiosyncratic shocks (σ_h) from the observed moments given a set of preference parameters. From equation (4.5), the magnitude of A is identified given σ_h^2 and σ_a^2 ($= \frac{Var\frac{dC}{C}}{dt}$). Table 4.2.1 and 4.2.2 report the implied σ_h and the ratio of σ_h to σ_a, while Table 4.3 reports the implied value of A[12]. The existence condition (4.7) can be satisfied for all countries except Portugal and Spain of the second sub-sample of the 1970's.

Under the above setup of preference parameters, the implied magnitude of idiosyncratic shocks is three to seven times as large as that of aggregate shocks, while the magnitude is very different from one country to another. In addition, the magnitude tends to decline over time (in particular from the 1970's to the 1980's) in most countries. In the case of $\gamma = 3$, for example, the implied σ_h of the average OECD is 0.142 in the 1960's and 0.136 in the 1970's, but falls to 0.090 in the 1980's (Table 4.1: second panel, Table 4.2.1). Are these features of the implied σ_h consistent with micro or other evidence ?

We first examine the empirical plausibility of the implied σ_h. Although larger

[12] The implied A does depend on ϵ, but not on γ.

σ_h tends to make the cross-sectional consumption distribution more disperse, the economy-wide measure of consumption inequality may not serve as a direct measure for σ_h. Let us see why. As shown in Appendix 4C, the time $t+z$ consumption inequality within the cohort which starts at time t is characterized by

$$Var\left[\ln\frac{c_i(t+z,t)}{C(t+z,t)}\right] = \sigma_g^2 + \sigma_h^2 z, \tag{4.18}$$

where $c_i(t+z,t)$ is the time $t+z$ consumption of the individual i belonging to the cohort which starts at time t, and $C(t+z,t)$ is the time $t+z$ per capita consumption of the same cohort. σ_g^2 is the cross-sectional variance of consumption distribution when this cohort starts, or equal to $Var\left[\ln\frac{c_i(t,t)}{C(t,t)}\right]$. Given this consumption distribution within the cohort, the economy-wide measure of consumption inequality is represented by

$$Var\left[\ln\frac{c_i(t)}{C(t)}\right] = \sigma_g^2 + \frac{1}{q}\sigma_h^2, \tag{4.19}$$

where $\frac{1}{q}$ is life expectancy at birth (q is the instantaneous probability of death, see Appendix 4C for the detailed derivation of equation (4.19)).

As equation (4.19) implies, the economy-wide measure of consumption inequality is a function of not only σ_h, but also q and σ_g. Hence, it may be hard to infer σ_h from this measure. For example, the fact that the economy-wide measure of consumption inequality increases from the 1970's to the 1980's in the U.S. (e.g. documented by Cutler and Katz [10, 1991]) does not directly contradict our finding that the implied σ_h of the U.S. decreases during the same periods. The fact that the consumption distribution of the whole economy is less dispersed in Japan is not necessarily inconsistent with our finding that the implied σ_h of Japan is very large.

As Deaton and Paxson [12, 1994] propose, a more plausible measure of σ_h is the growth of consumption inequality within the same cohort. Equation (4.18) implies that the cross-sectional variance of logarithmic individual consumption increases by σ_h^2 every year. Deaton and Paxson estimate how fast consumption inequality grows with age from cohort data of the U.K., the U.S. and Taiwan. According to their estimate, σ_h is 0.101 for the U.K. (1969 to 1990) and 0.083 for the U.S. (1980 to 1990)[13]. Their estimated values for these two countries are quite comparable to the value implied by our model; from Table 4.2.1 ($\gamma = 3$), we observe that the implied σ_h is 0.129 in the 1970's and 0.102 in the 1980's for the U.K. and 0.083 in the 1980's for the U.S. At least for these two countries, the model can generate very plausible values of σ_h.

Due to the limited availability of cohort data, a direct measure of σ_h for other countries can not be obtained; yet indirect measures of σ_h across countries may be available. If the cross-country difference of the implied σ_h is associated with that of broad measures on prevalence of insurance contracts, then the implied σ_h may be reasonably interpreted as a measure of insurance incompleteness.

Along this idea, we find one plausible measure from Goldsmith [22, 1985] (Table 54 on p. 148). Goldsmith reports the ratio of insurance and pension (including social security claims) relative to the whole financial assets up to 1978 for the thirteen countries of our sample. As Figure 4.1 and 4.2 show, the countries with wide prevalence such as U.K., U.S., Sweden and Australia accompany smaller σ_h, while the countries with low prevalence such as Japan and Italy are with larger σ_h. The only exception to this tendency is France. The correlation coefficient between the prevalence measure and the implied σ_h is statistically significant[14].

The overall reduction in σ_h, in particular from the 1970's to the 1980's, can be interpreted as the consequence of financial deregulation. In most OECD countries, the regulation of insurance markets has been relieved significantly in the 1980's. In particular, a broad range of financial institutions is allowed to participate in insurance markets. See OECD [35, 1992] for the institutional description. The

[13] Deaton and Paxson calculate the cross-sectional variance of ln(consumption) from the cohort data. Then, the calculated variance is regressed on the age. The coefficient on the age (corresponding to σ_h^2 of our model) is 0.0069 (t-statistics: 30.29) for the U.S. and 0.0102 (t-statistics: 22.95) for the U.K.

[14] The correlation coefficient is -0.813 (the standard error: 0.205) in Figure 4.1 and -0.619 (s.e.: 0.277) in Figure 4.2. When France is removed, the correlation coefficient is -0.871 (s.e.: 0.125) in Figure 4.1 and -0.742 (s.e.: 0.237) in Figure 4.2.

above cross-sectional and time-series observation provides some convincing evidence to regard the implied σ_h as a measure of insurance incompleteness.

Productivity

According to Table 4.3, the implied productivity (A) declines dramatically from the 1960's to the 1970's across countries, then recovers modestly from the 1970's to the 1980's (notice that the implied A does not depend on γ at all); the implied A of the average OECD is 7.6 % in the 1960's, falls to 4.5 % in the 1970's and recovers to some extent (5.4 %) in the 1980's. This time-series pattern of A corresponds to that of the total factor productivity of the traditional growth accounting, so-called the post-1973 productivity slowdown (Denison [15, 1985], Jorgenson [26, 1990], and Shigehara [41, 1992], et al.).

One important issue is on the interaction between productivity and uninsured shocks through the positive externality of economic growth. The following linear relationship among A, σ_a^2, and σ_h^2 (derived from equation (4.15) and (4.16)) can serve to identify this externality using the cross-country data:

$$A = \frac{\alpha}{1-\alpha\epsilon}\left[\epsilon(\beta-\rho) + \frac{\gamma(1-\epsilon)}{2}(\sigma_a^2+\sigma_h^2)\right] + \beta + \xi, \tag{4.20}$$

where ξ is a country-specific random part.

The estimation procedure is as follows. A, σ_a^2, and σ_h^2 are calculated for each country given preference parameters. Then, treating A, σ_a^2, and σ_h^2 as cross-sectional data, we can estimate α and β of equation (4.20) by nonlinear least squares method. Since A, σ_a^2, and σ_h^2 are determined simultaneously, we also estimate equation (4.20) by instrumenting σ_h^2 using the previously-used Goldsmith's measure for both the 1960's and the 1970's data[15]. Because the magnitude of σ_a^2 is very small relative to that of σ_h^2, whether σ_a^2 is instrumented does not matter very much in estimating α

[15] Since any other relevant instrumental variables are not available here, we cannot test over-identification restrictions.

and β.

Table 4.4 reports the estimation result for the three sub-sample periods[16]. The estimated α is significantly greater than one in the 1960's, but it declines since then. In both the 1970's and the 1980's, the estimated α is not significantly greater than one; if α is less than one, the net effect of uninsured shocks on welfare is negative. According to the above estimation result, as will be discussed in detail later, heavily constrained insurance markets enhance individual welfare in the 1960's, while the reduction in idiosyncratic risk in the 1980's slows down economic growth, but does not lead to lowering welfare.

Contribution to Growth

This subsection examines how much productivity and insurance incompleteness matter in determining economic growth. Using equation (4.11), we can evaluate the contribution of these factors to economic growth. The first term in the bracket ($\epsilon(A-\rho)$) represents the contribution of productivity, while the second term ($\frac{\gamma(1-\epsilon)}{2}(\sigma_a^2+\sigma_h^2)$) captures the effect of the precautionary saving caused by either aggregate shocks or uninsured idiosyncratic shocks.

As Table 4.5.1 clearly shows, the precautionary saving due to idiosyncratic shocks mainly drives up consumption growth. This precautionary saving contributes to around 40 % to 70 % of average economic growth, while productivity improvement is responsible for 30 % to 60 % of consumption growth. The precautionary saving due to aggregate shocks does not play any active role in determining economic growth.

Let us look at the source of the slowdown in economic growth over the past three decades. As Table 4.1 (first panel) indicates, the consumption growth declines over time across the OECD countries; 3.9 % in the 1960's, 2.7 % in the 1970's and 2.0

[16]Given relatively small sample sizes, we use a demanding level of significance at 0.5 %.

% in the 1980's for the average OECD. Table 4.5.2 calculates how much of changes in the productivity and changes in the need for precautionary saving contribute to such economic slowdown. In each column, the first numbers indicate changes from the 1960's to the 1970's, while the second numbers imply changes from the 1970's to the 1980's.

Across countries, the productivity slowdown is mainly responsible for the slowdown in economic growth from the 1960's to the 1970's. In the average behavior (the last line of Table 4.5.2), the productivity slowdown can explain 1.1 % out of 1.2 % decline in economic growth. On the other hand, a decline in the implied σ_h (the reduced need for precautionary saving) is mostly accountable for the economic slowdown from the 1970's to the 1980's. On the average, the reduced need for precautionary saving can explain most decline in consumption growth, while a modest improvement of productivity contributes to economic growth.

Relation to Saving Rates

This model can potentially impose another restriction on macroeconomic data, that is one on saving rates using equation (4.13). Rigorously speaking, however, testing this restriction is subject to one measurement problem. Within this model, saving includes not only physical investment, but also investment in human capital. The conventionally-used saving rates are, accordingly, defined too narrowly from a viewpoint of this model and may not be adopted for a rigorous quantitative examination. In this subsection, we use the conventional measure of saving rates in testing only qualitative implications of this model.

According to equation (4.13), when the elasticity of intertemporal substitution (ϵ) is less than one, a decrease in insurance incompleteness (σ_h) leads to a reduction in saving rates. We examine this qualitative implication using one of the conventional measure of saving rates (the net national saving rate). As Figure 4.3 shows, the average saving rate is highly positively correlated with the magnitude of the implied σ_h for the full sample periods; the correlation coefficient between saving rates and σ_h is statistically significant (Table 4.6). For the sub-sample periods, the correlation coefficient is strong in both the 1960's and the 1970's, while the coefficient becomes weaker in the 1980's.

As pointed out before, the implied σ_h declines significantly from the 1970's to the 1980's in most countries (see Table 4.1: second and third panels, Table 4.2.1 and 4.2.2). During the same periods, the net national saving rate declines as well; the average saving rate is 14.7 % in the 1960's and 13.5 % in the 1970's, but falls to 9.2 % in the 1980's (Table 4.1: first panel). A decrease in the net national saving rate actually corresponds to a change in the implied σ_h across countries. We find that countries with larger changes in the implied σ_h accompany larger changes in the saving rate. For example, Greece, Denmark, Belgium and Ireland experience a significant reduction in both the average saving rate and the implied σ_h, while Switzerland, the U.K., the U.S., and Finland undergo a relatively small change in these two variables. The exception to this tendency includes Norway and Netherlands where, in these countries, the saving rate is still high regardless of a significant reduction in σ_h. The behavior of these two countries is mainly responsible for a weaker correlation between the implied σ_h and the average saving rate in the 1980's[17].

4.4 Welfare Ranking

In this section, we will evaluate the welfare implication of the growth experience of the OECD countries according to the welfare measure constructed in Section 4.2 or the sum of economic growth and interest rates ($g_c + r$). This welfare measure

[17]If Norway and Netherlands are excluded from the sample, then the correlation coefficient between idiosyncratic shocks and saving rates becomes statistically significant (the correlation coefficient is 0.552 and its standard error is 0.172).

can compare welfare in terms of both the cross-country and the time-series when the resident in each country holds one unit of capital at the very beginning of the sample periods. In other words, this measure abstracts wealth effects from other effects. For example, if the full sample is used, the measure represents the individual welfare which is evaluated as of 1960 given the wealth level.

Table 4.7 compares the welfare ranking with the growth ranking for the full sample. The former is dramatically different from the latter. The preceding empirical investigation can provide several reasonable explanations for this difference. Some high-growth countries are ranked lower according to the proposed measure. Portugal and Spain are, for example, ranked significantly lower. In these countries, a relatively large σ_h triggers economic growth, but lowers welfare. Japan and Italy are, on the other hand, ranked highly in both the growth and welfare ranking. These countries have extremely large σ_h, but benefit from the externality of high growth on productivity. Some middle-growth or low-growth countries improve ranking under the proposed measure. Belgium and Canada belong to this example. France, the U.K. and the U.S. are also ranked moderately higher. These countries receive less welfare losses due to smaller σ_h.

Table 4.1 (first panel) reports the time series of the welfare measure of the average OECD for the three sub-sample periods. Again, there are interesting contrasts between the growth rate itself and the constructed welfare measure. Although the economic growth continues to slow down over the past three decades in most countries, the welfare measure does not show such a monotonic pattern. From the 1960's to the 1970's, the welfare measure declines (from 5.4 % to 1.5 %), but from the 1970's to the 1980's, the welfare measure recovers itself (from 1.5 % to 4.8%).

The time-series pattern of the welfare measure reflects the source of economic slowdown. Since the productivity slowdown is responsible for the 1960's to 1970's economic slowdown, welfare deteriorates during these periods. On the other hand, the reduced need for precautionary saving causes the 1970's to 1980's slowdown.

As discussed before, the externality of economic growth on productivity becomes weaker in the 1980's (α of equation (4.15) is not greater than one); consequently, slower economic growth caused by weaker precautionary saving does not have a negative impact on welfare. Up to the end of the 1980's, the welfare loss due to the productivity slowdown from the 1960's to the 1970's is offset to a large extent by the welfare gain due to the reduced need for precautionary saving from the 1970's to the 1980's[18].

4.5 Conclusion

This study sheds light on the importance of the need for precautionary saving, when we quantitatively evaluate the growth experience of the OECD countries. As demonstrated in Section 4.3, the empirically plausible magnitude of idiosyncratic shocks is accountable for a large portion of the economic growth of the individual country, and these shocks can consistently explain the cross-country difference as well as the time-series of saving and economic growth. As Table 4.1 summarizes, the growth slowdown from the 1960's to the 1970's is caused by the productivity slowdown, while that from the 1970's to the 1980's is largely due to the reduced need for precautionary saving. The latter growth slowdown or slower capital accumulation is consistent with a drastic decrease in the saving rate from the 1970's to the 1980's.

In addition, the interaction between productivity improvement and incomplete insurance through the externality of growth is analyzed. We find that the externality of growth on productivity is very strong in the 1960's. The heavily constrained insurance market itself improves welfare through the strong externality during these

[18]The exception to this tendency includes Netherlands, Greece, Denmark, and Germany. In these countries, a reduction in σ_h accompanies a decrease in A from the 1970's to the 1980's; consequently, the welfare of these countries has not been improved by the end of the 1980's. The externality of slower growth still seems to work to lower productivity in those countries.

periods. The externality, on the other hand, becomes weaker in the 1980's; accordingly, the reduced need for precautionary saving lowers economic growth, but can contribute to the welfare improvement in the 1980's without having a negative impact on productivity.

One may extend this study in several directions. First, the incompleteness of insurance markets addressed in this study and the liquidity constraints analyzed by Jappelli and Pagano [25, 1994] have very similar qualitative implications on saving, growth, interest, and welfare in the context of endogenous growth models. As discussed by Jappelli and Pagano, saving and growth may be promoted by liquidity constraints (down payment constraints) which keep agents from allocating resources intertemporally[19]. Reduction in loan demand brings down the interest rates. If the externality of growth on productivity is strong, liquidity constraints may have a chance to enhance welfare. In this respect, their model shares some common features with the model in this chapter. Jappelli and Pagano also present empirical evidence in favor of liquidity constraints using the OECD data.

A second direction may be to analyze the effect of integrated financial markets across countries. Since bond markets play a very limited role in insuring permanent idiosyncratic shocks as mentioned before, the integration of bond markets may not change the effect of uninsured shocks on growth and welfare. The integration of insurance markets or equity markets, however, may have a significant impact on macroeconomic performance. In particular among the European countries, insurance markets will be integrated under a certain standard. This kind of integration would alter growth, saving, and welfare of these countries[20].

[19]In Jappelli and Pagano [25, 1994], liquidity constraints do matter not because they limit the role of bond markets as self-insurance, but because they keep agents from transferring resources from tomorrow to today in certainty contexts.

[20]The simulation of Obstfeld [34, 1992] and van Wincoop [46, 1994] shows that the availability of contingent claims across countries would improve welfare dramatically.

Chapter 5

Concluding Remarks

The three substantive chapters of this thesis have shown that certain aspects of either market incompleteness or market imperfection give rise to economic phenomena that would not take place under the Arrow-Debreu perfect and complete market economy. How individuals respond to these aspects, and their consequences at the economy-wide level have been discussed at length. The quantitative importance of market incompleteness and market imperfect was most clearly shown in Chapter 4 where, according to our model, precautionary saving arising from insurance incompleteness accounted for more than half of economic growth in OECD countries during the three decades of 1960's to early 1990's.

In most cases, departure from perfect and complete markets implicates departure away from the first-best Pareto optimality. The recent past has seen some development in many areas towards more complete and more perfect markets. Examples are the financial innovations, both locally and globally, that led to the openings of markets that never existed before. This development was probably responsible for the generally negative contributions of change in idiosyncratic risk in the per-capita con consumption growths in OECD in the last decade of our study. We can only hope that the financial innovations will eventually reduce the uninsured wage income risk which is the focal point of our study of human capital accumulation in Chapter 2. On the other hand, developments in other areas may be slower, or even not progressing at all. The immigrants' inferior matching technology, an important assumption of the model in Chapter 3, is more likely to be persistent since immigration always means relocations of people to the new places they do not grow up in. The disadvantage of immigrants in finding jobs may also stems from cultural reasons that withstand changes. The wage bargaining, although may also be persistent or even more prevalent with more labors getting educated, does not in itself a symbol of market imperfection. Rather, it results from the existence of recruiting cost and non-instantaneous matching, both of which may improve with progress in labor matching technology.

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Appendices to Chapter 2

Appendix 2A Derivation of the After-School Value Function

In this appendix, we apply the method developed by Svensson and Werner [37, 1993] in deriving the after-school value function under CARA with nontradable wage income. Svensson and Werner introduce an intuitive way of incorporating the presence of stochastic wage income stream into the standard Merton's consumption/portfolio problem that encompasses both complete and incomplete market cases[1]. They do so by defining a "comprehensive wealth" as a sum of financial and human wealth,

$$\hat{W} \equiv W + F,$$

where F is value of claim to income stream the agent would be willing to hold if such claim were traded. This definition of claim to income stream allows the standard methods of asset pricing theory in characterizing F.

Since financial wealth consists of risky and risk-free assets, the comprehensive wealth would evolve according to

$$d\hat{W} = [(\alpha_q - r)\pi + (\alpha_f - r)F + r\hat{W} - C]dt + [\pi S_q + FS_f]dB, \tag{2A.1}$$

where α_f and S_f are constant rate of return and (1×2 row vector of) standard

[1] In Merton [27, 1971], the problem with wage income is characterized only under complete market, namely, the claim to wage income is tradable. See Merton [27, 1971], footnote 20 and references therein.

deviation associated with F, both are yet to be determined such that F is willingly held. The next step is to derive the value function that would be obtained if the agent maximized his expected lifetime utility from time $t = T$ onward (as in equation 2.9) taking $\hat{W}$ as state variable and (2A.1) as the dynamic budget constraint. Since the time horizon is infinite, the Hamilton-Jacobi-Bellman equation can be written in current-value form, namely,

$$0 = \max_{(C,\pi,F)} \Big\{ U(c) - \beta\tilde{J} + [\pi(\alpha_q - r) + F(\alpha_f - r) + r\hat{W} - c]\tilde{J}_{\hat{w}} + \tfrac{1}{2}[\pi^2 S_q S_q' + 2\pi F S_q S_f' + F^2 S_f S_f']\tilde{J}_{\hat{w}\hat{w}} \Big\}, \tag{2A.2}$$

where $\tilde{J}(\hat{W})$ is the current-valued value function that solves the above HJB equation. Since we may think of F as one risky asset, its inclusion to the problem will not affect the standard form of value function $\tilde{J}(\hat{W})$ when utility is exponential. That is,

$$\tilde{J} = -\frac{e^{A-\eta r\hat{W}}}{\eta r}, \tag{2A.3}$$

with $-\tilde{J}_{\hat{W}\hat{W}}/\tilde{J}_{\hat{W}} = \eta r$ denotes the constant absolute aversion to wealth risk, which equals to constant absolute aversion to consumption risk times risk-free interest rate.

For the purpose of solving for F, it suffices at this point to use only the first-order conditions for the HJB equation (2A.2) with respect to π and F,

$$\tilde{J}_{\hat{W}}(\alpha_q - r) + \tilde{J}_{\hat{W}\hat{W}}(\pi S_q S_q' + F S_q S_f') = 0, \tag{2A.4}$$

$$\tilde{J}_{\hat{W}}(\alpha_f - r) + \tilde{J}_{\hat{W}\hat{W}}(S_f S_q' \pi + S_f S_f' F) = 0. \tag{2A.5}$$

Rearranging (2A.4) gives

$$\pi = \frac{(\alpha_q - r)}{\eta r \sigma_q^2} - \frac{S_q S_f' F}{\sigma_q^2}, \tag{2A.6}$$

here we have used $S_q S_q' = \sigma_q^2$ and $-\tilde{J}_{\hat{W}\hat{W}}/\tilde{J}_{\hat{W}} = \eta r$. The first term is the usual mean-variance efficient portfolio in standard consumption/portfolio problems, while the second term, $-S_q S_f' F/\sigma_q^2$, represents the demand for risky assets to hedge against income risk, an "income hedge portfolio", and will be shown to be positive if $\rho \in (-1, 0)$.

The next step makes use of the fact that one can characterize the price process of claim to wage income stream, F, in two ways. First, let Q_f denotes price to claim F. By definition,

$$\frac{dQ_f}{Q_f} = \alpha_f \, dt + S_f \, dB_t. \tag{2A.7}$$

The other characterization of the price of F is obtained by the fact that holding a claim to wage income for an interval dt enables the agent to receive a "dividend" dy. Hence, the price process of F also follows

$$\frac{dQ_f}{Q_f} = \frac{dy}{F}. \tag{2A.8}$$

Equating (2A.7) with (2A.8) and substituting for dy yields

$$F(\alpha_f - r) = \alpha(hT) - Fr \tag{2A.9}$$

and

$$FS_f = S_y. \tag{2A.10}$$

Multiplying $F/\tilde{J}_{\hat{W}}$ through (2A.5) and use $-\tilde{J}_{\hat{W}\hat{W}}/\tilde{J}_{\hat{W}} = \eta r$ and (2A.10) we can rewrite (2A.5) as

$$F(\alpha_f - r) = \eta r(S_y S_q' \pi + S_y S_y').$$

Substituting π in (2A.6) and using $S_y S_q = \rho \sigma_y \sigma_q$ and $S_y S_y' = \sigma_y^2$,

$$\begin{aligned} F(\alpha_f - r) &= \eta r[(\alpha_q - r)\rho(\sigma_y/\eta r \sigma_q) + \sigma_y^2(1-\rho^2)] \\ &= (\alpha_q - r)\rho(\sigma_y/\sigma_q) + \eta r \sigma_y^2 (1-\rho^2). \end{aligned}$$

Equating the above expression with (2A.9) gives the value of F as appeared in Svensson and Werner's paper,

$$F = \left[\alpha(hT) - \rho(\alpha_q - r)(\sigma_y/\sigma_q) - \eta r \sigma_y^2(1-\rho^2)\right]/r.$$

However, because our model includes an unskilled wage rate Y_0 in the income process (2.6) which is not present in Svensson's and Werner's model, the value of F will be slightly different. Since positive Y_0 only raises the path of income process at every point in time (shifting the trend up) without affecting its diffusion part, we can capitalize it at the risk-free interest rate r and add to the value of the claim to income stream, that is,

$$F = \left[Y_0 + \alpha(hT) - \rho(\alpha_q - r)(\sigma_y/\sigma_q) - \eta r \sigma_y^2(1-\rho^2)\right]/r. \tag{2A.11}$$

Plugging (2A.10) into (2A.6) yields optimal holding of financial risky assets π.

$$\pi = \frac{(\alpha_q - r)}{\eta r \sigma_q^2} - \rho(\sigma_y/\sigma_q).$$

Substituting both F and π into Hamilton-Jacobi-Bellman equation through the postulated value function (2A.3) gives the closed-form value function with

$$A = \left[r - \beta - \frac{(\alpha_q - r)^2}{2\sigma_q^2} - \frac{(\eta r)^2}{2}\sigma_y^2(1-\rho^2) + \frac{\eta r Y_0}{2}\right]/r. \tag{2A.12}$$

The after-school current-valued value function is thus

$$\tilde{J} = -\frac{1}{\eta r}\exp\left(\left[r - \beta - \frac{(\alpha_q - r)^2}{2\sigma_q^2} - \frac{(\eta r)^2}{2}\sigma_y^2(1-\rho^2) + \frac{\eta r Y_0}{2}\right]/r - \eta r(W+F)\right) \tag{2A.13}$$

Since $\tilde{J}$ is independent of time, it satisfies the transversality condition

$$\lim_{t\to\infty} e^{-\beta t}\tilde{J}(\hat{W}(t)) = 0.$$

Finally, deflating $\tilde{J}(W(T)+F(T))$ to the time-zero present value gives the after-school value function $J(W(T)+F(T))$ as appeared in the text,

$$J(W(T)+F(T)) = e^{-\beta T}\tilde{J}(W(T)+F(T)).$$

Appendix 2B The Determination of the Optimal Quitting Time

The procedure of determining the optimal quitting time, T^* is broken down into three steps.

Step one. Fix a quitting time at an arbitrary $t = T$ and form a finite-horizon consumption/investment problem during the in-school period with the following boundary condition

$$I(W(T), T; T) = J(W(T) + F(T)),$$

where $I(\cdot, \cdot; T)$ and $J(\cdot, \cdot)$ are both time-zero present-valued value functions during the in-school and after-school period, respectively. The parameter T appearing as the third argument of the in-school value function reflects its dependence on the fixed quitting time T.

Step two. Characterize a set of *admissible quitting times* defined by

$$\hat{T} = \{T \geq 0; \quad I(W(t), t; T) \geq J(W(t) + F(t)) \text{ for all} t \leq T\}. \tag{2B.1}$$

Step three. Choose within the set of admissible quitting times the optimal quitting time.

Step one: solving for the in-school value functions

We now form a finite-horizon consumption/investment problem during the in-school period $\aleph_1 = [0, T]$, taking a quitting time T as given, and the value function (2.12) as the problem's boundary function. The agent solves

$$\max_{\{C,\pi\}} \quad E_0 \left[\int_0^T e^{-\beta t} U(C(t)) dt + J(W(T) + F(T)) \right], \tag{2B.2}$$

subject to

$$dW = [\pi(\alpha_q - r) + rW - C - L]\, dt + \pi S_q dB \qquad t \in \aleph_1. \tag{2B.3}$$

Again, we maintain the exponential utility (2.11).

Let $I(W(t),t;T)$ denote the present-valued value function for problem (2B.2), namely,

$$I(W(t),t;T) = \max_{\{C,\pi\}} E_t \left[\int_t^T e^{-\beta t} U(C(t))dt + J(W(T) + F(T)) \right].$$

The operator E_t is the expectation operator conditional on the set of information before and up to time t. The associated Hamilton-Jacobi-Bellman equation is

$$0 = \max_{\{C,\pi\}} \left\{ e^{-\beta t} U(C) + I_t + [\pi(\alpha_q - r) + rW - C - L] I_W + \frac{1}{2}\pi^2 \sigma_q^2 I_{WW} \right\}, \tag{2B.4}$$

while the boundary condition is

$$I(W(T),T;T) = J(W(T) + F(T)). \tag{2B.5}$$

The subscripts denote the obvious partial derivatives. With CARA utility, we posit a trial solution

$$I(W(t),t;T) = -\frac{1}{\eta r}\exp(-\beta t + B(t;T) - \eta r W), \tag{2B.6}$$

$B(t;T)$ is to be determined. The parameter T is included in the above value function to recognize that the problem is parameterized via the boundary condition (2B.5).

The first-order conditions with respect to consumption and portfolio choice in (2B.4) are

$$e^{-\beta t - \eta C^*} - I_W = 0. \tag{2B.7}$$

$$(\alpha_q - r) I_W + \pi^* \sigma_q^2 I_{WW} = 0. \tag{2B.8}$$

Solving for C^* and π^* gives

$$C^* = -\frac{\log(e^{\beta t} I_W)}{\eta}, \tag{2B.9}$$

$$\pi^* = -\frac{(\alpha_q - r)}{\sigma_q^2}\left(\frac{I_W}{I_{WW}}\right). \tag{2B.10}$$

From the trial solution (2B.6) we have,

$$\begin{aligned} I_t &= (B'(t;T) - \beta)I, \\ I_W &= -\eta r I, \\ I_{WW} &= (\eta r)^2 I. \end{aligned} \tag{2B.11}$$

Substituting (2B.9), (2B.10), and (2B.11) into the Hamilton-Jacobi-Bellman equation and dividing through by I yields

$$0 = r - \beta + B'(t;T) - rB(t;T) + \eta r L - \frac{(\alpha_q - r)^2}{2\sigma_q^2}.$$

This becomes a first order linear ordinary differential equation in $B(t;T)$, with a solution

$$B(t;T) = (B_0 - G)e^{rt} + G, \tag{2B.12}$$

where

$$G \equiv \left[r - \beta + \eta r L - \frac{(\alpha_q - r)^2}{2\sigma_q^2}\right]/r.$$

Next, we equate the after-school value function $J(W(T)+F(T))$ with the in-school value function (2B.6) with $B(t;T)$ substituted out using (2B.12). This gives

$$A - \eta r F(T) = B(T;T) = (B_0 - G)e^{rT} + G.$$

B_0 can then be solved for

$$B_0 = [A - G - \eta r F(T)]e^{-rT} + G.$$

Consequently, the in-school value function becomes

$$I(W(t), t; T) = -\frac{1}{\eta r}\exp\{-\beta t + [A - G - \eta r F(T)]e^{-r(T-t)} + G - \eta r W(t)\}. \tag{2B.13}$$

Step two: admissible quitting times

The in-school value function (2B.13), which depends on an arbitrary choice of quitting time T, was derived to satisfy only the Hamilton-Jacobi-Bellman equation (2B.4) and the boundary condition (2B.5). To satisfy the fundamental equation of optimal stopping (2.5), it must also be true that quitting at any time before T is not optimal[2]. Specifically, (2.18) must hold at any $t \leq T$.[3]

It turns out that not all quitting times satisfy the above condition. Denote the set of all quitting times that satisfy this condition by $\hat{T}$ as in (2B.1). We make the following claim.

Claim 2B.1. $\hat{T} = [0, T^*]$, where T^* solves

$$\min_{\tau} \; [A - G - \eta r F(\tau)] e^{-r\tau}.$$

Proof. Substituting $I(W(t), t; T)$ and $J(W(t) + F(t))$ into (2.18) and after some manipulations we have

$$[A - G - \eta r F(T)] e^{-r(T-t)} + G - \eta r W(t) \leq A - \eta r F(t) - \eta r W(t).$$

Note that the existence of financial wealth, $W(t)$, do not affect the comparison of $I(\cdot, \cdot; T)$ with $J(\cdot)$. We can thus drop it from the above condition. This simplifies the problem substantially because we can then consider only the effect of time variable t.

[2]This requirement also ensures that the *continuation interval* $[0, T]$ is connected. Nonconnected continuation intervals may arise in general problems of optimal stopping but we will show later that this is not the case in our model. If it were, then our assumption that no one is allowed to come back to school after quitting would eliminate such consideration and T would be defined as the quitting time at the end of the first continuation interval.

[3]In general, when the value functions depends on twostate variables, the *continuation region* must be defined in those two state variables. These are $W(t)$ and t in this model. However, as the proof in claim 2B.1 will show, comparisons of the two value functions do not need the financial wealth $W(t)$, we are therefore able to reduce the continuation region into a continuation interval of time alone.

The above inequality can be rearranged as

$$[A - G - \eta r F(T)]e^{-rT} \leq [A - G - \eta r F(t)]e^{-rt}.$$

Let $\Phi(\tau) \equiv [A - G - \eta r F(\tau)]e^{-r\tau}$. It can be easily shown that $\Phi(\tau)$ has a global minimum under our regularity assumptions on $\alpha(H)$ (see Appendix 2C) . Let $T^* = \arg\min_\tau \Phi(\tau)$, then $\Phi(t) \geq \Phi(T^*)$ only if $0 \leq t \leq T^*$[4].■

Step three: the optimal quitting time

The determination of the optimal quitting time makes use of the solution strategy of "free boundary problem", since the boundary condition (2B.5) changes with changes in the quitting time T. In general, the solution is determined by two conditions: a "value matching condition" and a "smooth pasting condition", both are evaluated at the time of quitting. The value matching conditions for t and $W(t)$ are satisfied automatically when we solve for $I(W(t), t; T)$, because we use the value function $J(W(T) + F(T))$ as the boundary condition. The smooth pasting conditions require that at the time of quitting the derivatives of the two value functions be equal with respect to each variable, namely,

$$\left.\frac{\partial I(W(t), t; T)}{\partial t}\right|_{t=T} = \frac{\partial J(W(T) + F(T))}{\partial T}, \tag{2B.14}$$

$$\left.\frac{\partial I(W(t), t; T)}{\partial W(t)}\right|_{t=T} = \frac{\partial J(W(T) + F(T))}{\partial W(T)}. \tag{2B.15}$$

The derivatives on the left-hand side of the above two equations are evaluated at $t = T$. However, as we have shown earlier that financial wealth $W(t)$ enters into both value functions in a similar way that it cancels out, we will not have to consider the smooth pasting condition for $W(t)$. Formally, note that at $t = T$, $\frac{\partial I(W(t), t; T)}{\partial W} = -\eta r I(W(T), T; T)$ and $\frac{\partial J(W(T)+F(T))}{\partial W} = -\eta r J(W(T) + F(T)$. But since $I(W(T), T; T) =$

[4] The set $\hat{T}$ starts from zero because of this section's assumption that the agent decides to go to school at time $t = 0$.

$J(W(T) + F(T))$, the two partial derivatives are always equal. It then remains that only the smooth pasting condition with respect to time determines the optimal quitting time.

Rather than using (2B.14), we will use an equivalent but more intuitive approach for determining the optimal quitting time. Note that whenever the in-school value function satisfies the optimal stopping equation (2.5) for a fixed quitting time T, it also automatically satisfies

$$I(W(t), t; T) = \max_{\{C,\pi;T\}} E_t \left[\int_t^{\infty} e^{-\beta s} U(C(s)) ds \right].$$

If we fix the current time t and let the quitting time T vary, we are essentially comparing different school plans available to the agent at time t. It can be easily shown that choosing the educational plan, or choosing the quitting time, that maximizes his present-valued in-school value function yields the smooth pasting condition (2B.14) as the first-order condition (Appendix 2D shows this). Since maximizing $I(W(t), t; T)$ with respect to T is more intuitive, we will present our problem in this way.

Using the expression in (2B.13) for $I(W(t), t; T)$, we find that for any current time in the set of admissible quitting times, $t \in \hat{T}$, the quitting time that yields maximum in-school value function is $T = T^*$ because T^* minimizes $\Phi(T) \equiv [A - G - \eta r F(T)]e^{-rT}$ and thus maximizes $I(W(t), t; T)$ given t. Differentiating $\Phi(T)$ and setting to zero gives

$$\Phi'(T^*) = -r[A - G - \eta r F(T^*)]e^{-rT^*} - \eta r F'(T^*)e^{-rT^*} = 0. \tag{2B.16}$$

Substituting for A, G, F, F' and assuming that $\alpha(hT) = hT$, we have

$$hT^* + \frac{Y_0}{2} - (\alpha_q - r)\rho(\sigma_y/\sigma_q) - \frac{\eta r}{2}\sigma_y^2(1 - \rho^2) - (\frac{h}{r} - L) = 0. \tag{2B.17}$$

Or

$$T^* = \left[(\alpha_q - r)\rho(\sigma_y/\sigma_q) + \frac{\eta r}{2}\sigma_y^2(1-\rho^2) - \frac{Y_0}{2} + (\frac{h}{r} - L) \right] /h. \tag{2B.18}$$

Appendix 2C

This appendix shows that T^* globally minimizes $\Phi(\tau) = [A - G - \eta r F(\tau)]e^{-r\tau}$. The first-order condition of $\Phi(\tau)$ with respect to τ is

$$\Phi'(\tau) = -r[A - G - \eta r F(\tau)]e^{-r\tau} - \eta r F'(\tau)\, e^{-r\tau} = 0. \tag{2C.1}$$

Rearranging,

$$A - G - \eta r F(\tau) + \eta F'(\tau) = 0. \tag{2C.2}$$

Let T^* solves the above equation. Now differentiating the first-order condition (2C.1) to check for second-order condition,

$$\begin{aligned} &r^2 e^{-r\tau}[A - G - \eta r F(\tau) + \eta F'(\tau)] - re^{-r\tau}[\eta F''(\tau) - \eta r F'(\tau)] \\ &= re^{-r\tau}\eta[rF'(\tau) - F''(\tau)] = re^{-r\tau}h[r\alpha' - \alpha'']. \end{aligned}$$

We have used (2C.2) in the first equality and the expression for F from (2A.11) in the second equality. A sufficient condition for a global minimum of $\Phi(\tau)$ is that $\alpha(H)$ is concave, linear, or not too strongly convex. In the main text, we have assumed that $\alpha(H)$ is linear.

Appendix 2D

This appendix shows how the necessary condition for maximizing $I(W(t), t; T)$ with respect to T is equivalent to the smooth pasting condition (2B.14). Differentiating $I(W(t), t; T^*)$ with respect to t gives

$$\frac{\partial I(W(t), t; T^*)}{\partial t} = \{-\beta + r[A - G - \eta r F(T^*)]e^{-(T^*-t)} - \eta r W'(t)\} I(W(t), t; T).$$

At $t = T^*$, the above derivative becomes

$$\left.\frac{\partial I(W(t), t; T^*)}{\partial t}\right|_{t=T^*} = \{-\beta + r[A - G - \eta r F(T^*)] - \eta r W'(T^*)\} I(W(T^*), T^*; T^*).$$

Differentiating $J(W(T^*), F(T^*))$ with respect to T^* gives

$$\frac{\partial J(W(T^*), F(T^*))}{\partial T^*} = \{-\beta - \eta r F'(T^*) - \eta r W'(T^*)\} J(W(T^*), F(T^*)).$$

Equating the two derivatives and use the value matching condition $I = J$, we have

$$r[A - G - \eta r F(T^*)] = -\eta r F'(T^*),$$

which is the same condition as (2B.16) in the text.

Appendix to Chapter 3

In this appendix we restate and give the proofs to Proposition 3.1 and 3.2.

Proposition 3.1. There exist β_i^i and β_i^n, with $\beta_i^i < \beta_n < \beta_i^n$, such that one-time immigrations improve the labor market outcomes of both worker groups when $\beta_i < \beta_i^i$, improve only the labor market outcome of the existing immigrants when $\beta_i^i < \beta_i < \beta_i^n$, and deteriorate the labor market outcomes of both worker groups when $\beta_i^n < \beta_i$.

Proof: We first show that $X_i > X_n$ when $\beta_i < \beta_i^i$ and $X_i < X_n$ when $\beta_i > \beta_i^i$. Dividing the steady state condition (3.22) by the condition (3.23) and rearranging for the ratio $\frac{X_n}{X_i}$, then equate it to unity

$$1 = \frac{X_n}{X_i} = \frac{1-\beta_n}{1-\beta_i}\frac{Y}{\delta}\left[\frac{r+s+\beta_i\delta X^{1-\alpha}}{r+s+\beta_n Y X^{1-\alpha}}\right] \tag{3A.1}$$

Further, note that X is a weighted average of X_i and X_n according to

$$X = \frac{U_i}{U}X_i + \frac{U_n}{U}X_n. \tag{3A.2}$$

When (3A.1) holds, we will have $X_n = X_i = X$. For given values of β_n and δ, let β_i^i be the value of β_i that fulfills (3A.1)

$$1 = \frac{1-\beta_n}{1-\beta_i^i}\frac{Y}{\delta}\left[\frac{r+s+\beta_i^i\delta X^{1-\alpha}}{r+s+\beta_n Y X^{1-\alpha}}\right] \tag{3A.3}$$

Because the last term in (3A.1) is increasing in β_i, any value of β_i that is lower than β_i^i will make $\frac{X_n}{X_i} < 1$ (or $X_i > X_n$), and any value of β_i that is higher than β_i^i will make $\frac{X_n}{X} > 1$ (or $X_i < X_n$). Also, as $\delta < 1$ and $Y > 1$, it is clear that $\beta_i^i < \beta_n$.

Since all immigrants arrive unemployed, the new equilibrium will be associated with a higher ratio of unemployed immigrants to the total unemployed, $\frac{U_i}{U}$. Because this ratio is the weight of X_i in (3A.2), when $X_i > X_n$ the arrival of new immigrants will leads to a higher overall market tightness X according to (3A.2). The opposite is true, namely, X decreases with one-time immigrations when $X_i < X_n$. Since u_i moves in the opposite direction to X according to the steady state condition of u_i (3.21), u_i must then decreases with immigration when $\beta_i < \beta_i^i$ and increases when $\beta_i > \beta_i^i$. Lemma 3A shows that w_i moves in the opposite direction to u_i, enabling us to conclude that the existing immigrants are better off when $\beta_i < \beta_i^i$.

Next, we give the proof that the natives benefit from immigration when $\beta_i < \beta_i^n$ where $\beta_i^n > \beta_n$. First note that for every $\beta_i < \beta_i^i$ the natives' unemployment rate u_n must decreases with immigration because it moves, according to (3.20), in the opposite direction to $YX^{1-\alpha}$ which increases because both Y and X increases from the above discussion. We will have to show that u_n continues to decrease even after β_i exceed β_i^i (that is, after u_i starts to rise). Specifically, we will show that $YX^{1-\alpha}$ continues to increase until β_i reaches $\beta_i^n > \beta_n$. For this purpose, it will be sufficient to show that at $\beta_i = \beta_n$, u_n is still falling.

In stead of continuing the proof by exploring what happens when i increases, we follow an alternative route. When $\beta_i = \beta_n$, the only difference between the immigrants and the natives is in $\delta < 1$. If the economy begins without any immigrants, the arrival of immigrants with $\delta < 1$ will produce exactly the same outcomes as another economy in which its immigrant workers initially have $\delta = 1$ (namely, they are identical with that economy's native workers) and subsequently experience a drop in their matching efficiency to $\delta < 1$. The second situation is as depicted in panel (a) of Figure 3.1. The outcomes in these two situations will be exactly the same as long as the fraction of immigrant workers after the immigration in the first economy is the same as the fraction of immigrant workers in the second economy (same i). We proceed to show that the natives benefit when $\beta_i = \beta_n$ in the second situation.

Before δ decreases, the immigrants and the natives are identical as far as their labor characteristic is concern. That means $X_i = X_n = \bar{X}$ where

$$\left[\frac{r+s}{\bar{X}^{1-\alpha}} + \beta_n\right]\bar{X} = \frac{(1-\beta_n)}{\gamma}\left[f(k) - rk - z\right].$$

After δ decreases to $\delta < 1$, X_n and X_i are determined by (3.22) and (3.23) at $\beta_i = \beta_n$, respectively,

$$(\tfrac{r+s}{YX^{1-\alpha}} + \beta_n)X_n = \tfrac{(1-\beta_n)}{\gamma}\left[f(k) - rk - z\right], \tag{3A.4}$$

$$(\tfrac{r+s}{\delta X^{1-\alpha}} + \beta_n)X_i = \tfrac{(1-\beta_n)}{\gamma}\left[f(k) - rk - z\right]. \tag{3A.5}$$

Since $YX^{1-\alpha} > \bar{X}^{1-\alpha}$ and $\delta X^{1-\alpha} < \bar{X}^{1-\alpha}$, the above two equations imply that $X_n > \bar{X} > X_i$. In other words, X_n increases which means u_n decreases. Consequently, for u_n to increase, a β_i that is greater than β_n is needed to bring down $YX^{1-\alpha}$ through lowering X. We thus complete the proof of Preposition 3.1.

Lemma 3A: Changes in wage rates for both natives and immigrants are always in the opposite direction to changes in their respective unemployment rates.

Proof: Differentiating (3.6) and (3.18) with respect to i gives

$$\frac{dw_n}{di} = -(r+s)\frac{dJ_n}{di}$$

and

$$\frac{dw_n}{di} = \frac{\beta_n}{1-\beta_n}(r+s+p_n)\frac{dJ_n}{di} + \frac{\beta_n}{1-\beta_n}J_n\frac{dp_n}{di}.$$

Substituting for $\frac{dJ_n}{di}$ and using $\frac{dp_n}{di} = -\frac{s}{u_n^2}\frac{du_n}{di}$ from the differentiation of (3.20),

$$\left(1 + \frac{p_n\beta_n}{r+s}\right)\frac{dw_n}{di} = -\frac{s\beta_n J_n}{u_n^2}\frac{du_n}{di}.$$

Therefore

$$sign\left|\frac{dw_n}{di}\right| = -sign\left|\frac{du_n}{di}\right|.$$

The exactly same procedure applies to the immigrants, that is,

$$sign\left|\frac{dw_i}{di}\right| = -sign\left|\frac{du_i}{di}\right|.$$

This proves the Lemma.

Proposition 3.2. β_i^i is increasing in both the immigrants' matching efficiency (δ) and the natives' bargaining power (β_n) while β_i^n is decreasing in δ but increasing in β_n.

Proof: The right-hand side of (3A.3) is decreasing in both δ and β_n, but is increasing in β_i^i. Using the implicit function theorem, this implies

$$\frac{d\beta_i^i}{d\delta} > 0, \qquad \frac{d\beta_i^i}{d\beta_n} > 0.$$

Now we rewrite (3A.4) and (3A.5) with β_i equals β_i^n instead of β_n,

$$\begin{aligned}(\tfrac{r+s}{YX^{1-\alpha}} + \beta_n)X_n &= (\tfrac{r+s}{\bar{X}} + \beta_n)X_n = \tfrac{(1-\beta_n)}{\gamma}\left[f(k) - rk - z\right], \\ (\tfrac{r+s}{\delta X1-\alpha} + \beta_i^n)X_i &= \tfrac{(1-\beta_i^n)}{\gamma}\left[f(k) - rk - z\right].\end{aligned}$$

Here $YX^{1-\alpha} = \bar{X}$ because at $\beta_i = \beta_i^n$ the natives by definition must neither benefit nor lose from immigration. An increase in δ results in a lower Y and thus lowers $YX^{1-\alpha}$, which can be brought up back by an increase in $X^{1-\alpha}$ through a decrease in β_i^n. An increase in β_n has the opposite effect. It raises $YX^{1-\alpha}$, which can be brought down back by a decrease in $X^{1-\alpha}$ through an increase in β_n. Thus,

$$\frac{d\beta_i^n}{d\delta} < 0, \qquad \frac{d\beta_i^n}{d\beta_n} > 0.$$

Appendices to Chapter 4

Appendix 4A Data Appendix to Chapter 4

In this appendix we describe, in some detail, the macroeconomic data used in Section 4.3 and Section 4.4. All data are gathered from the following two sources: the International Financial Statistics (provided by IMF, hereafter the IFS dataset) and the National Account Statistics (compiled by OECD). From these datasets, we construct three variables, the growth rate of per capita real aggregate consumption, the real short-term interest rate, and the net national saving rate.

Both the per capita real aggregate consumption and the real risk-free rate are constructed from the IFS dataset. To get the former, we divide the nominal aggregate consumption by both the consumer price index and the total population. Although consumption deflators (which are not available in the IFS dataset) are conventionally used instead of consumer price indices, the real consumption growth based on the consumer price index is only marginally different from those provided by the conventional method. The calculated consumption growth is comparable to that from the OECD National Account Statistics[5].

The official discount (bank) rate (evaluated at the end of each year) is used as

[5] We carefully compared the sample moments between the IFS dataset and the OECD National Account Statistics for both the full and sub-samples. We could not find any significant differences except for simple mis-entries of data.

the proxy for nominal safe interest rates. Those nominal rate is converted to the real rate using the consumer price index. The main reason for using these series is that they have relatively long sample periods for all of the OECD countries; other money market returns such as treasury bill rates and commercial paper rates have extremely limited sample periods (which often start from the 1980's) for most countries. For the case of the U.K., we use treasury bill rates since 1980, because the bank rate system was suspended in 1981.

The saving rate is obtained from the OECD National Account Statistics. The definition of the saving rate used in this chapter is the net national saving rate. This saving rate is defined as the ratio of the net national saving (private saving plus net income transfer from foreign countries minus government deficits) to the net national product. This concept of saving rate is adopted in Modigliani [33] and Jappelli and Pagano [25].

We calculate growth and interest at annual rates. With a few exceptions, all data are available from 1960 to 1992. The exceptions include: Ireland's 1992 real interest rate, Australia's 1960-1969 real interest rates, Sweden's 1960-1963 net national saving rates.

Appendix 4B Derivation of Equation (4.5), (4.6) and (4.14)

Svensson [44] derives the following optimal portfolio and consumption rules of the optimization problem (4.2) subject to the budget constraint (4.3) under constant investment opportunities:

$$\mu = \epsilon \left[\rho - \frac{\epsilon - 1}{\epsilon} \left(r + \frac{1}{2\gamma} \frac{(A - r)^2}{\sigma_a^2 + \sigma_h^2} \right) \right], \tag{4B.1}$$

$$x = \frac{A - r}{\gamma(\sigma_a^2 + \sigma_h^2)}. \tag{4B.2}$$

From the above two equations, we obtain

$$\mu = \epsilon(\rho - r) + xA + (1-x)r - \frac{\gamma + \gamma\epsilon}{2}x^2(\sigma_a^2 + \sigma_h^2). \tag{4B.3}$$

Substituting the bond market clearing condition (equation (4.4)) into equation (4B.2) and (4B.3) leads to equation (4.5) and (4.6).

Svensson also derives a closed-form of the value function (4.2) as

$$V(K) = \mu^{\frac{1}{1-\epsilon}}K.$$

From equation (4.10) and (4B.3) with $x = 1$, we get

$$\mu = A - g_c.$$

Substituting equation (4.5) into the above yields

$$\mu = r + \gamma(\sigma_a^2 + \sigma_h^2) - g_c.$$

From equation (4.10), we derive

$$\gamma(\sigma_a^2 + \sigma_h^2) = \frac{2}{1+\epsilon}\left[g_c - \epsilon(r - \rho)\right]$$

Combining the last two equations leads to

$$\mu = \frac{1}{1+\epsilon}\left[(1-\epsilon)(g_c + r) + 2\epsilon\rho\right],$$

then to equation (4.14).

Appendix 4C Derivation of Equation (4.18) and (4.19)

Let us consider the case where the instantaneous probability of death is equal to q, and a new cohort enters the economy such that the total population is constant. Suppose that the wealth accidentally left by the previous cohort is distributed to a newly entering cohort through either bequests or inheritance taxes. The initial wealth distribution of a new cohort ($Var[ln\frac{c_i(t,t)}{C(t,t)}]$) is assumed to be σ_g. Since each consumer takes the probability of death into consideration, the marginal propensity to consume increases by ϵq. The process of $c_i(t+z,t)$ is, then, analogous to equation (4.8):

$$\frac{dc_i(t+z,t)}{c_i(t+z,t)} = \left[\epsilon(r-\rho-q) + \frac{\gamma+\gamma\epsilon}{2}(\sigma_a^2+\sigma_h^2)\right] dz + \sigma_a dB_a(t+z) + \sigma_h dB_i(t+z). \tag{4C.1}$$

Like in equation (4.9), the per capita aggregate consumption within the cohort $(C(t+z,t))$ follows

$$\frac{dC(t+z,t)}{C(t+z,t)} = \left[\epsilon(r-\rho-q) + \frac{\gamma+\gamma\epsilon}{2}(\sigma_a^2+\sigma_h^2)\right] dz + \sigma_a dB_a(t+z) \tag{4C.2}$$

Applying Ito's lemma to equation (4C.1) and (4C.2), we derive

$$d\ln\frac{c_i(t+z,t)}{C(t+z,t)} = -\frac{\sigma_h^2}{2}dz + \sigma_h dB_i(t+z). \tag{4C.3}$$

Integrating equation (4C.3) from time t to time $t+z$ leads to

$$\begin{aligned}\ln\frac{c_i(t+z,t)}{C(t+z,t)} &= \ln\frac{c_i(t,t)}{C(t,t)} + \int_t^{t+z}\left[d\ln\frac{c_i(\tau,t)}{C(\tau,t)}\right]d\tau \\ &= \ln\frac{c_i(t,t)}{C(t,t)} + \int_t^{t+z}(-\frac{\sigma_h^2}{2}d\tau) + \int_t^{t+z}\sigma_h dB(\tau).\end{aligned} \tag{4C.4}$$

Given equation (4C.4), the current variance of the consumption distribution of the cohort which starts z years ago is equal to equation (4.18) or

$$Var\left[\ln\frac{c_i(t,t-z)}{C(t,t-z)}\right] = \sigma_g^2 + \sigma_h^2 z.$$

In the above setup, the age distribution of the entire society follows the exponential distribution (its density function is $q\exp(-qz)$). The economy-wide measure of consumption inequality is, accordingly, the equation (4.18) integrated by the population density, or equation (4.19):

$$\begin{aligned}Var\left[\ln\frac{c_i(t)}{C(t)}\right] &= \int_0^\infty\left\{Var\left[\ln\frac{c_i(t,t-z)}{C(t,t-z)}\right]q\exp(-qz)\right\}dz \\ &= \sigma_g^2 + \frac{1}{q}\sigma_h^2.\end{aligned} \tag{4C.5}$$

Table 2.1: Optimal Decisions Under Various Parametric Possibilities

i. Cases with $\frac{1}{r} - \frac{L}{h} > 0$

Case	Parameter Ranges	Value Functions	Actions
1	$T^* < 0 < [\frac{1}{r} - \frac{L}{h}] < T^+$	$V^0 < J^0 = I^0$	No School, Work
2	$0 < T^* < [\frac{1}{r} - \frac{L}{h}] < T^+$	$V^0 < J^0 < I^0$	School, Work
3	$0 < [\frac{1}{r} - \frac{L}{h}] < T^* < T^+$	$J^0 \leq V^0 < I^0$	School, Work
4	$0 < [\frac{1}{r} - \frac{L}{h}] < T^+ < T^*$	$J^0 < I^0 \leq V^0$	No School, No Work

ii. Cases with $\frac{1}{r} - \frac{L}{h} = 0$

Case	Parameter Ranges	Value Functions	Actions
5	$T^- = T^* < 0$	$V^0 < J^0 = I^0$	No School, Work
6	$T^- = T^* = 0$	$V^0 = J^0 = I^0$	No School, No Work
7	$0 < T^* = T^+$	$J^0 < I^0 < V^0$	No School, No Work

iii. Cases with $\frac{1}{r} - \frac{L}{h} < 0$

Case	Parameter Ranges	Value Functions	Actions
8	$T^* < T^{--}$	$V^0 < J^0 = I^0$	No School, Work
9	$T^{--} < T^* \leq (\frac{1}{r} - \frac{L}{h})$	NA	NA
10	$(\frac{1}{r} - \frac{L}{h}) < T^* \leq 0$	$J^0 = I^0 < V^0$	No School, No Work
11	$0 < T^*$	$J^0 < I^0 < V^0$	No School, No Work

Note: T^{--} in cases (iii) is defined similarly to T^+ (see text).

Table 3.1: Implications of Decreased Immigrants' Matching Efficiency and Bargaining Power

Implications of Decreased Immigrants' Matching Efficiency (δ)		
	Immigrants	Natives
Unemployment level (U)	+	-
Unemployment rate (u)	+	-
Vacancies (V)	+	-
Market tightness (X=V/U)	-	+
Matching Rate (m)	-	+
p = m/U	-	+
q = m/V	-	+
Wage (W)	-	+
Surplus from Matching	+	-
Implications of Decreased Immigrants' Bargaining Power (β_i)		
	Immigrants	Natives
Unemployment level (U)	-	-
Unemployment rate (u)	-	-
Vacancies (V)	+	-
Market tightness (X=V/U)	+	+
Matching Rate (m)	+	+
p = m/U	+	+
q = m/V	-	+
Wage (W)	-	+
Surplus from Matching	+	-

Note:
(i) Assuming immigrants accounting for 10 percent of the labor force.
(ii) Other Parameter Values: α=0.5, γ=5, r=0.1, s=0.03, z=5

Table 3.2: Impact of One-time Immigrations on Labor Market

Weak Immigrants' Bargaining Power (β_i = 0.2, β_n=0.5)			
	Immigrants	Natives	Overall
Unemployment level (U)	+	-	+
Unemployment rate (u)	-	-	-
Vacancies (V)	+	-	+
Market tightness (X=V/U)	+	+	+
Matching Rate (m)	+	+	+
p = m/U	+	+	+
q = m/V	+	+	-
Wage (W)	+	+	
Surplus from Matching	-	-	
Strong Immigrants' Bargaining Power (β_i = 0.8, β_n=0.5)			
	Immigrants	Natives	Overall
Unemployment level (U)	+	+	+
Unemployment rate (u)	+	+	+
Vacancies (V)	+	+	+
Market tightness (X=V/U)	-	-	-
Matching Rate (m)	+	-	+
p = m/U	-	-	-
q = m/V	-	-	+
Wage (W)	-	-	
Surplus from Matching	+	+	

Note:
(i) Other Parameter Values: δ=0.8, α=0.5, γ=5, r=0.1, s=0.03, z=5

Table 3.3: Changes in Unemployment Rates ($\frac{du_i}{di}$, $\frac{du_n}{di}$)

Varying δ, Fixed β_n=0.5					
β_i	δ				
	0.2	0.4	0.6	0.8	1.0
0.0	(+,-)	(-,-)	(-,-)	(-,-)	(-,-)
0.1	(+,-)	(-,-)	(-,-)	(-,-)	(-,-)
0.2	(+,-)	(-,-)	(-,-)	(-,-)	(-,-)
0.3	(+,-)	(-,-)	(-,-)	(-,-)	(-,-)
0.4	(+,-)	(+,-)	(-,-)	(-,-)	(-,-)
0.5	(+,-)	(+,-)	(+,-)	(+,-)	(0,0)
0.6	(+,-)	(+,-)	(+,-)	(+,-)	(+,+)
0.7	(+,-)	(+,-)	(+,-)	(+,+)	(+,+)
0.8	(+,-)	(+,-)	(+,-)	(+,+)	(+,+)
Fixed δ=0.8, Varying β_n					
β_i	β_n				
	0.3	0.4	0.5	0.6	0.7
0.0	(-,-)	(-,-)	(-,-)	(-,-)	(-,-)
0.1	(-,-)	(-,-)	(-,-)	(-,-)	(-,-)
0.2	(-,-)	(-,-)	(-,-)	(-,-)	(-,-)
0.3	(+,-)	(-,-)	(-,-)	(-,-)	(-,-)
0.4	(+,-)	(+,-)	(-,-)	(-,-)	(-,-)
0.5	(+,+)	(+,-)	(+,-)	(-,-)	(-,-)
0.6	(+,+)	(+,+)	(+,-)	(+,-)	(-,-)
0.7	(+,+)	(+,+)	(+,+)	(+,-)	(+,-)
0.8	(+,+)	(+,+)	(+,+)	(+,+)	(+,-)
Note: Other Parameter Values: α=0.5, γ=5, r=0.1, s=0.03, z=5					

Table 4.1: Summary Table of Average OECD (per year)

Basic Statistics				
	1960-92	1960-69	1970-79	1980-89
Average Consumption Growth	0.0271	0.0393	0.0272	0.0198
Average Real Interest	0.0114	0.0143	-0.0126	0.0279
Average Net National Saving Rate	0.1206	0.1470	0.1354	0.0923
Welfare Measure	0.0385	0.0536	0.0147	0.0477
Prediction, $\rho = 0.02, \gamma = 3, \epsilon = 1/3$				
	1960-92	1960-69	1970-79	1980-89
Productivity (A)	0.0564	0.0760	0.0446	0.0536
Aggregate Shocks (σ_a)	0.0252	0.0185	0.0251	0.0228
Idiosyncratic Shocks (σ_h)	0.1198	0.1422	0.1357	0.0897
Prediction, $\rho = 0.02, \gamma = 5, \epsilon = 1/3$				
	1960-92	1960-69	1970-79	1980-89
Productivity (A)	0.0564	0.0760	0.0446	0.0536
Aggregate Shocks (σ_a)	0.0252	0.0185	0.0251	0.0228
Idiosyncratic Shocks (σ_h)	0.0914	0.1095	0.1039	0.0680

Table 4.2.1: Magnitude of Idiosyncratic Shocks (per year)
$\rho = 0.02$, $\gamma = 3$, $\epsilon = 1/3$

Country		1960-92	1960-69	1970-79	1980-89
United States	σ_h	0.1013	0.1197	0.1105	0.0826
	σ_h/σ_a	4.4	7.8	4.6	3.0
United Kingdom	σ_h	0.0975	0.0902	0.1289	0.1017
	σ_h/σ_a	3.7	8.6	4.5	3.2
Austria	σ_h	0.1328	0.1426	0.1549	0.1117
	σ_h/σ_a	7.2	9.4	7.0	7.7
Belgium	σ_h	0.1107	0.1297	0.1425	0.0454
	σ_h/σ_a	5.7	10.6	7.3	2.4
Denmark	σ_h	0.0995	0.1268	0.1206	0.0588
	σ_h/σ_a	3.0	3.6	3.5	1.8
France	σ_h	0.1247	0.1608	0.1320	0.0957
	σ_h/σ_a	6.3	12.3	5.6	9.2
Germany	σ_h	0.1298	0.1561	0.1489	0.0877
	σ_h/σ_a	4.7	4.2	8.6	5.6
Italy	σ_h	0.1530	0.1782	0.1812	0.1049
	σ_h/σ_a	5.9	7.0	6.3	6.2
Netherlands	σ_h	0.1264	0.1697	0.1499	0.0301
	σ_h/σ_a	4.2	5.2	10.9	1.2
Norway	σ_h	0.1152	0.1421	0.1432	0.0595
	σ_h/σ_a	3.9	7.5	5.3	1.4
Sweden	σ_h	0.1019	0.1247	0.1281	0.0811
	σ_h/σ_a	4.6	12.8	6.0	3.6
Switzerland	σ_h	0.1180	0.1457	0.1291	0.0928
	σ_h/σ_a	6.1	10.6	5.8	7.6
Canada	σ_h	0.0928	0.1100	0.1324	0.0621
	σ_h/σ_a	3.9	7.1	7.0	2.5
Japan	σ_h	0.1594	0.2064	0.1657	0.1129
	σ_h/σ_a	4.9	11.7	5.2	8.2
Finland	σ_h	0.1296	0.1564	0.1522	0.1219
	σ_h/σ_a	3.4	3.7	4.3	9.0
Greece	σ_h	0.1384	0.1640	0.1585	0.0983
	σ_h/σ_a	4.4	7.9	4.2	5.1
Ireland	σ_h	0.1140	0.1316	0.1476	0.0555
	σ_h/σ_a	3.5	5.3	4.4	1.5
Portugal	σ_h	0.1546	0.1615		0.1125
	σ_h/σ_a	3.7	5.1		3.6
Spain	σ_h	0.1470	0.1862		0.0764
	σ_h/σ_a	4.6	6.4		2.8
Australia	σ_h	0.0886	N.A.	0.1340	0.0392
	σ_h/σ_a	4.9		7.2	2.1
Average OECD	σ_h	0.1198	0.1422	0.1357	0.0897
	σ_h/σ_a	4.7	7.7	5.4	3.9

Note: The calculated μ is negative for Portugal and Spain in 1970's.

Table 4.2.2: Magnitude of Idiosyncratic Shocks (per year)
$\rho = \mathbf{0.02}$, $\gamma = \mathbf{5}$, $\epsilon = \mathbf{1/3}$

Country		1960-92	1960-69	1970-79	1980-89
United States	σ_h	0.0771	0.0922	0.0842	0.0615
	σ_h/σ_a	3.3	6.0	3.5	2.2
United Kingdom	σ_h	0.0736	0.0696	0.0981	0.0762
	σ_h/σ_a	2.8	6.7	3.4	2.4
Austria	σ_h	0.1022	0.1100	0.1192	0.0860
	σ_h/σ_a	5.5	7.2	5.4	5.9
Belgium	σ_h	0.0849	0.1001	0.1097	0.0331
	σ_h/σ_a	4.4	8.2	5.6	1.8
Denmark	σ_h	0.0742	0.0957	0.0909	0.0406
	σ_h/σ_a	2.2	2.7	2.6	1.3
France	σ_h	0.0958	0.1242	0.1012	0.0738
	σ_h/σ_a	4.8	9.5	4.3	7.1
Germany	σ_h	0.0990	0.1186	0.1148	0.0673
	σ_h/σ_a	3.6	3.2	6.6	4.3
Italy	σ_h	0.1174	0.1371	0.1392	0.0806
	σ_h/σ_a	4.6	5.4	4.9	4.8
Netherlands	σ_h	0.0960	0.1298	0.1158	0.0171
	σ_h/σ_a	3.2	4.0	8.4	0.7
Norway	σ_h	0.0873	0.1094	0.1096	0.0381
	σ_h/σ_a	2.9	5.8	4.1	0.9
Sweden	σ_h	0.0777	0.0964	0.0983	0.0611
	σ_h/σ_a	3.5	9.9	4.6	2.7
Switzerland	σ_h	0.0905	0.1125	0.0990	0.0714
	σ_h/σ_a	4.7	8.2	4.4	5.9
Canada	σ_h	0.0703	0.0847	0.1019	0.0455
	σ_h/σ_a	3.0	5.4	5.4	1.9
Japan	σ_h	0.1218	0.1595	0.1267	0.0871
	σ_h/σ_a	3.8	9.0	4.0	6.3
Finland	σ_h	0.0975	0.1182	0.1157	0.0940
	σ_h/σ_a	2.6	2.8	3.3	7.0
Greece	σ_h	0.1054	0.1264	0.1204	0.0751
	σ_h/σ_a	3.3	6.1	3.2	3.9
Ireland	σ_h	0.0859	0.1007	0.1124	0.0356
	σ_h/σ_a	2.6	4.1	3.4	0.9
Portugal	σ_h	0.1168	0.1235		0.0848
	σ_h/σ_a	2.8	3.9		2.7
Spain	σ_h	0.1120	0.1431		0.0565
	σ_h/σ_a	3.5	4.9		2.0
Australia	σ_h	0.0677	N.A.	0.1031	0.0281
	σ_h/σ_a	3.7		5.6	1.5
Average OECD	σ_h	0.0914	0.1095	0.1039	0.0680
	σ_h/σ_a	3.6	5.9	4.1	3.0

Note: The calculated μ is negative for Portugal and Spain in 1970's.

Table 4.3: Magnitude of Productivity (per year)
$\rho = 0.02$, $\epsilon = 1/3$

Country	Implied A			
	1960-92	1960-69	1970-79	1980-89
United States	0.0454	0.0619	0.0359	0.0499
United Kingdom	0.0492	0.0518	0.0333	0.0723
Austria	0.0611	0.0735	0.0638	0.0477
Belgium	0.0667	0.0721	0.0649	0.0604
Denmark	0.0450	0.0641	0.0424	0.0254
France	0.0624	0.0824	0.0565	0.0494
Germany	0.0655	0.0904	0.0650	0.0428
Italy	0.0784	0.0986	0.0633	0.0720
Netherlands	0.0612	0.0922	0.0585	0.0327
Norway	0.0490	0.0627	0.0364	0.0419
Sweden	0.0372	0.0651	0.0315	0.0355
Switzerland	0.0394	0.0594	0.0323	0.0351
Canada	0.0560	0.0613	0.0610	0.0610
Japan	0.0805	0.1374	0.0544	0.0591
Finland	0.0640	0.0979	0.0536	0.0629
Greece	0.0671	0.1181	0.0567	0.0374
Ireland	0.0558	0.0751	0.0457	0.0436
Portugal	0.0491	0.0634		0.0507
Spain	0.0592	0.0976		0.0538
Australia	0.0514	N.A.	0.0362	0.0642
Average OECD	0.0564	0.0760	0.0446	0.0536

The calculated μ is negative for Portugal and Spain in 1970's.

Table 4.4: Nonlinear Least Squares Estimations of Equation (4.20)
$\rho = 0.02$, $\gamma = 3$, $\epsilon = 1/3$

sample period	method	estimated α	estimated β	no. of sample
1960-69	non-IV	1.281	0.0250	19
		(0.087)	(0.0032)	
	IV	1.312	0.0235	12
		(0.088)	(0.0040)	
1970-79	non-IV	0.999	0.0185	18
		(0.136)	(0.0050)	
	IV	1.495	0.0039	13
		(0.303)	(0.0079)	
1980-89	non-IV	0.733	0.0368	20
		(0.405)	(0.0081)	

(i) The null hypothesis is that there is no positive externality from growth to productivity, namely, the test is that α is one-sided away from zero.

(ii) The number in the parenthesis is the heteroscedasticity-robust standard error of estimated parameters.

(iii) Non-IV implies nonlinear least squares estimation without instrumental variables, while IV is one with instrumental variables.

(iv) Goldsmith measure is available for the U.K., the U.S., Canada, Sweden, Belgium, Switzerland, Norway, Denmark, Germany, France, Italy, Japan, and Australia.

Table 4.5.1: Contribution of Productivity and Precautionary Saving to Per-capita Consumption Growth, 1960 to 1992 (per year)
$\rho = 0.02$, $\gamma = 3$, $\epsilon = 1/3$

Country	Consumption Growth	Contribution: Productivity	Contribution: Precautionary Saving: Aggregate Shocks	Contribution: Precautionary Saving: Idiosyncratic Shocks
	g_c	$\frac{\epsilon(A-\rho)}{g_c}$	$\frac{\frac{\gamma-\epsilon\gamma}{2}\sigma_a^2}{g_c}$	$\frac{\frac{\gamma-\epsilon\gamma}{2}\sigma_h^2}{g_c}$
U.S.	0.0193	0.0085 (0.4399)	0.0005 (0.0277)	0.0103 (0.5324)
U.K.	0.0199	0.0097 (0.4878)	0.0007 (0.0354)	0.0095 (0.4767)
Austria	0.0317	0.0137 (0.4325)	0.0003 (0.0108)	0.0176 (0.5567)
Belgium	0.0282	0.0156 (0.5518)	0.0004 (0.0132)	0.0123 (0.4350)
Denmark	0.0193	0.0083 (0.4315)	0.0011 (0.0564)	0.0099 (0.5121)
France	0.0301	0.0141 (0.4700)	0.0004 (0.0131)	0.0155 (0.5169)
Germany	0.0328	0.0152 (0.4627)	0.0008 (0.0231)	0.0169 (0.5142)
Italy	0.0436	0.0195 (0.4472)	0.0007 (0.0152)	0.0234 (0.5376)
Netherlands	0.0306	0.0137 (0.4483)	0.0009 (0.0300)	0.0160 (0.5218)
Norway	0.0238	0.0097 (0.4057)	0.0009 (0.0370)	0.0133 (0.5573)
Sweden	0.0166	0.0057 (0.3456)	0.0005 (0.0291)	0.0104 (0.6253)
Switzerland	0.0208	0.0065 (0.3116)	0.0004 (0.0180)	0.0139 (0.6704)
Canada	0.0212	0.0120 (0.5667)	0.0006 (0.0268)	0.0086 (0.4065)
Japan	0.0466	0.0202 (0.4324)	0.0010 (0.0223)	0.0254 (0.5453)
Finland	0.0329	0.0147 (0.4464)	0.0014 (0.0429)	0.0168 (0.5106)
Greece	0.0359	0.0157 (0.4379)	0.0010 (0.0276)	0.0192 (0.5345)
Ireland	0.0260	0.0119 (0.4595)	0.0011 (0.0405)	0.0130 (0.5001)
Portugal	0.0353	0.0097 (0.2744)	0.0017 (0.0487)	0.0239 (0.6769)
Spain	0.0357	0.0131 (0.3662)	0.0010 (0.0289)	0.0216 (0.6049)
Average OECD	0.0271	0.0121 (0.4465)	0.0006 (0.0221)	0.0144 (0.5314)

The number in the parenthesis is the contribution share of each factor.

Table 4.5.2: Change in Contribution of Productivity and Precautionary Saving to Per-capita Consumption Growth 1960's to 1970's and 1970's to 1980's (per year)
$\rho = 0.02$, $\gamma = 3$, $\epsilon = 1/3$

Country	Consumption Growth		Contribution					
			Productivity		Precautionary Saving			
					Aggregate Shocks		Idiosyncratic Shocks	
	$\Delta[g_c]$		$\Delta[\epsilon(A-\rho)]$		$\Delta[\frac{\gamma-\epsilon\gamma}{2}\sigma_a^2]$		$\Delta[\frac{\gamma-\epsilon\gamma}{2}\sigma_h^2]$	
U.S.	-0.0104	-0.0005	-0.0086	0.0047	0.0003	0.0002	-0.0021	-0.0054
U.K.	0.0030	0.0069	-0.0062	0.0130	0.0007	0.0002	0.0085	-0.0063
Austria	0.0007	-0.0172	-0.0032	-0.0054	0.0003	-0.0003	0.0037	-0.0115
Belgium	0.0013	-0.0198	-0.0024	-0.0015	0.0002	-0.0000	0.0035	-0.0183
Denmark	-0.0088	-0.0169	-0.0072	-0.0057	-0.0000	-0.0001	-0.0015	-0.0111
France	-0.0167	-0.0111	-0.0086	-0.0024	0.0004	-0.0004	-0.0084	-0.0083
Germany	-0.0118	-0.0219	-0.0085	-0.0074	-0.0011	-0.0001	-0.0022	-0.0145
Italy	-0.0105	-0.0195	-0.0117	0.0029	0.0002	-0.0005	0.0011	-0.0218
Netherlands	-0.0184	-0.0298	-0.0112	-0.0086	-0.0009	0.0004	-0.0063	-0.0216
Norway	-0.0081	-0.0141	-0.0088	0.0019	0.0004	0.0010	0.0003	-0.0170
Sweden	-0.0099	-0.0085	-0.0112	0.0013	0.0004	0.0001	0.0009	-0.0098
Switzerland	-0.0133	-0.0075	-0.0090	0.0009	0.0003	-0.0003	-0.0046	-0.0081
Canada	0.0054	-0.0134	-0.0001	0.0000	0.0001	0.0002	0.0054	-0.0137
Japan	-0.0421	-0.0140	-0.0277	0.0016	0.0007	-0.0008	-0.0151	-0.0147
Finland	-0.0166	-0.0063	-0.0148	0.0031	-0.0005	-0.0011	-0.0013	-0.0083
Greek	-0.0213	-0.0229	-0.0205	-0.0065	0.0010	-0.0010	-0.0018	-0.0155
Ireland	-0.0048	-0.0191	-0.0098	-0.0007	0.0005	0.0003	0.0045	-0.0187
Australia	N.A.	-0.0071	N.A.	0.0094	N.A.	-0.0000	N.A.	-0.0164
Average OECD	-0.0121	-0.0074	-0.0105	0.0030	0.0003	-0.0001	-0.0018	-0.0104

(i) In each column, the first number is the change from 1960's to 1970's, while the second number is that from 1970's to 1980's.

(ii) Since the calculated μ is negative for Portugal and Spain in 1970's, the result of these two countries is not reported.

Table 4.6: Saving Rate and Idiosyncratic Shock
$\rho = 0.02$, $\gamma = 3$, $\epsilon = 1/3$

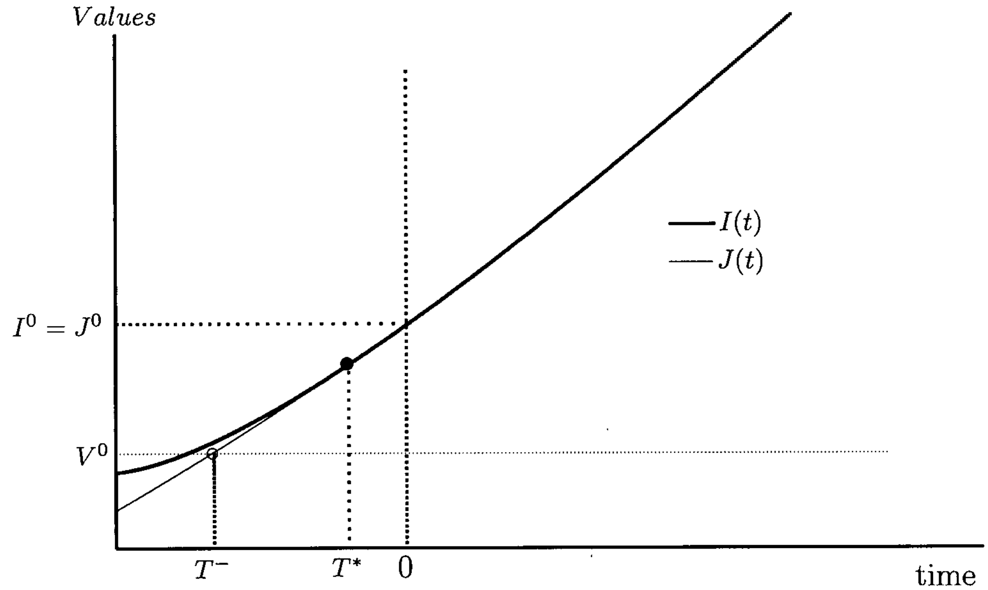
Period	Correlation Coefficient between Net National Saving Rates and σ_h
1960-92	0.7999
(s.e.)	(0.1502)
1960-69	0.7936
(s.e.)	(0.1522)
1970-79	0.6462
(s.e.)	(0.2401)
1980-89	0.3658
(s.e.)	(0.2212)

The number in the parenthesis is the standard error of correlation coefficient.

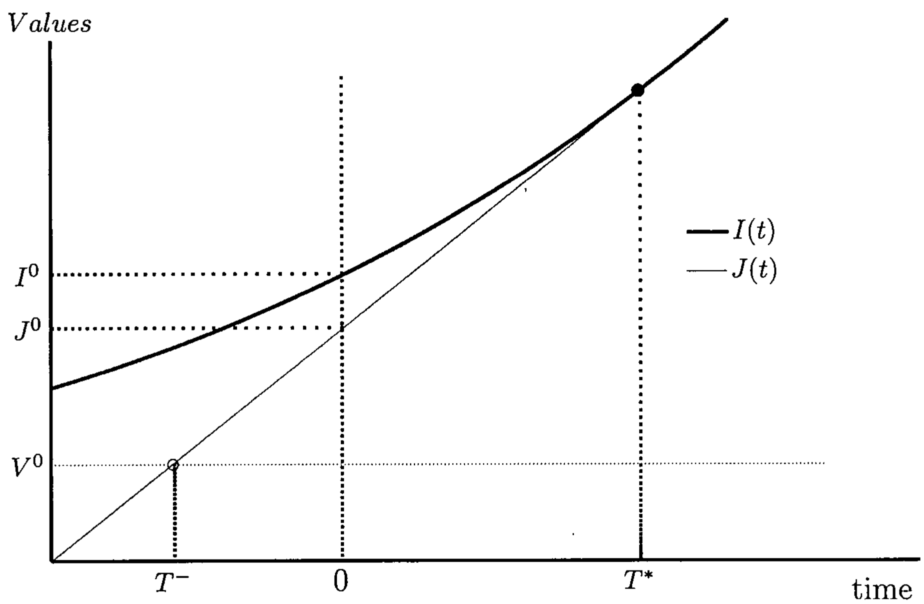
Table 4.7: Welfare Measure (1960-92, Growth + Interest, per year)

Rank	Country	Growth	Rank	Country	Growth+ Interest
1	Japan	0.0466	1	Belgium	0.0569
2	Italy	0.0436	2	Italy	0.0498
3	Greece	0.0359	3	Canada	0.0497
4	Spain	0.0357	4	Japan	0.0477
5	Portugal	0.0353	(5	Australia	0.0454)
6	Finland	0.0329	6	Germany	0.0454
7	Germany	0.0328	7	France	0.0447
8	Austria	0.0317	8	Greece	0.0425
9	Netherlands	0.0306	9	Finland	0.0423
10	France	0.0301	10	Netherlands	0.0411
11	Belgium	0.0282	11	Ireland	0.0397
12	Ireland	0.0260	12	Austria	0.0389
13	Norway	0.0238	13	United Kingdom	0.0385
14	Canada	0.0212	14	United States	0.0323
15	Switzerland	0.0208	15	Denmark	0.0314
16	United Kingdom	0.0199	16	Norway	0.0303
17	Denmark	0.0193	17	Spain	0.0270
18	United States	0.0193	18	Sweden	0.0212
(19	Australia	0.0186)	19	Switzerland	0.0173
20	Sweden	0.0166	20	Portugal	0.0075
	Average OECD	0.0271		Average OECD	0.0385

Figure 2.1: Values of Three Alternative Decisions

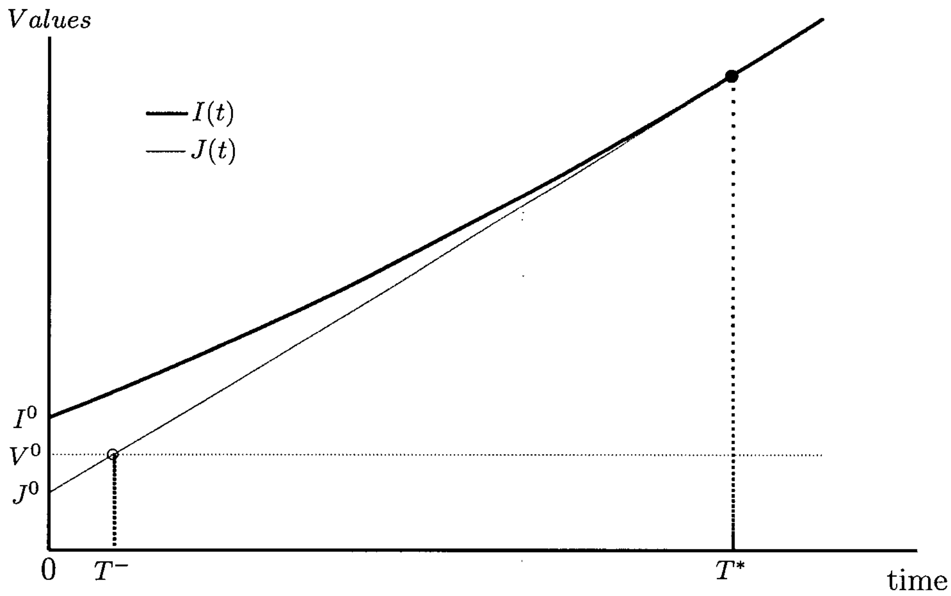


(a) Case 1: No school, works immediately

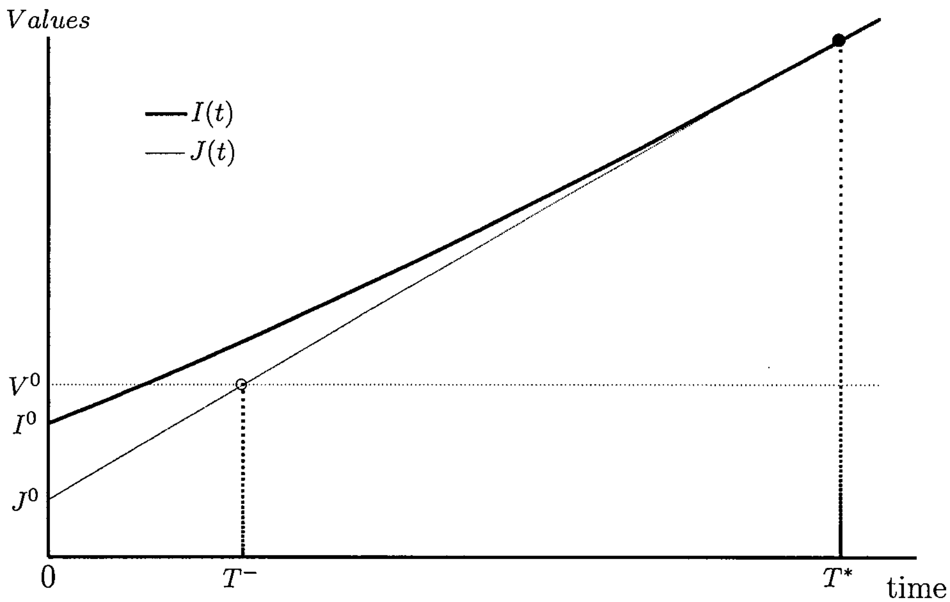


(b) Case 2: Goes to school, then works

Figure 2.1: Values of Three Alternative Decisions (continued)

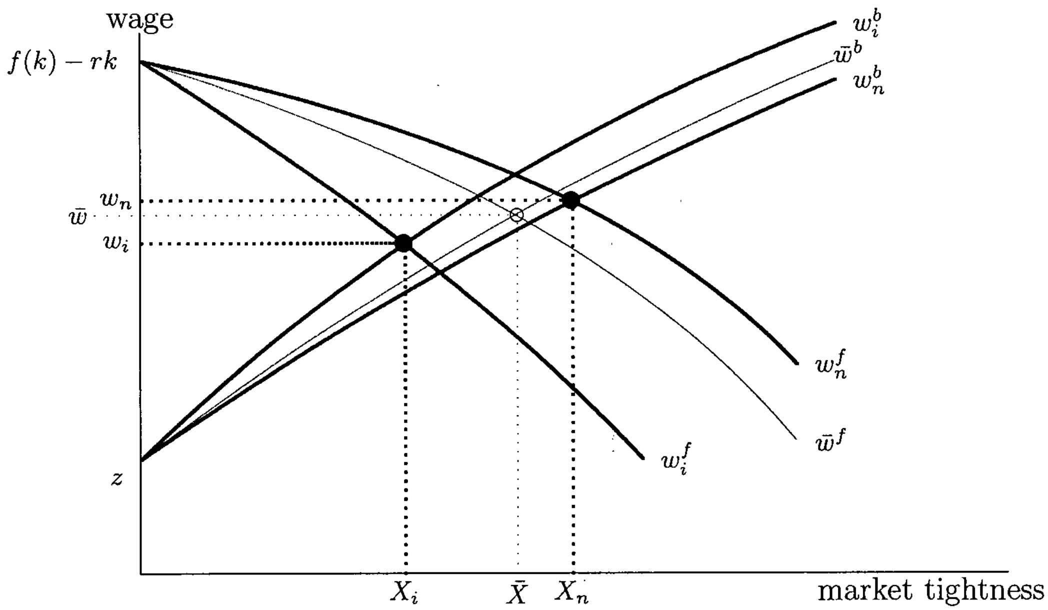


(c) Case 3: Goes to school, then works

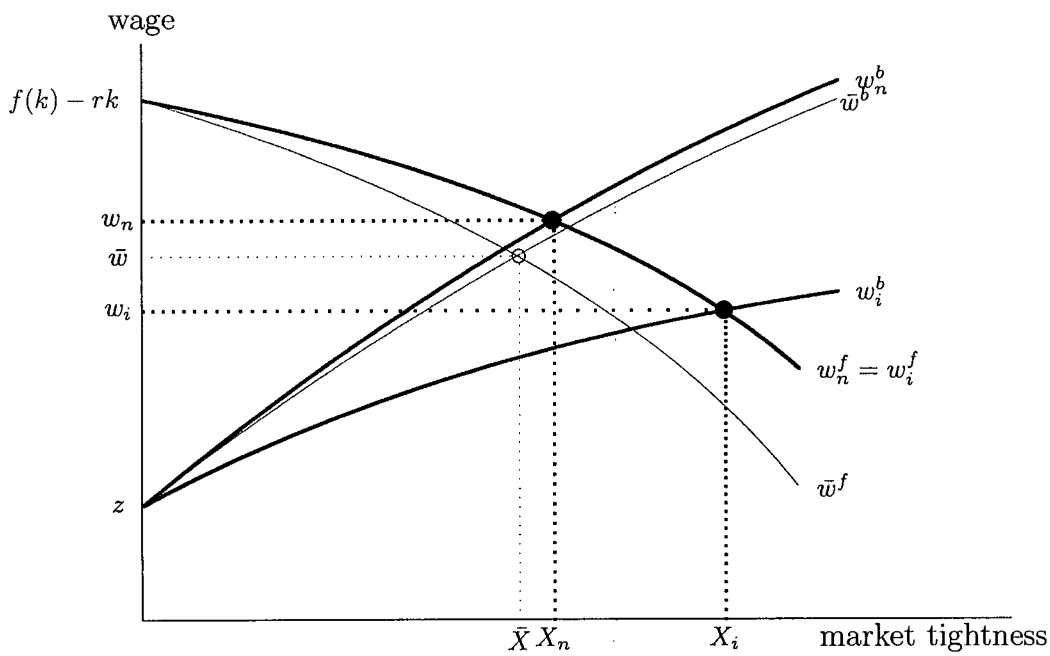


(d) Case 4: No school, no work

Figure 3:1: Labor Market Equilibria

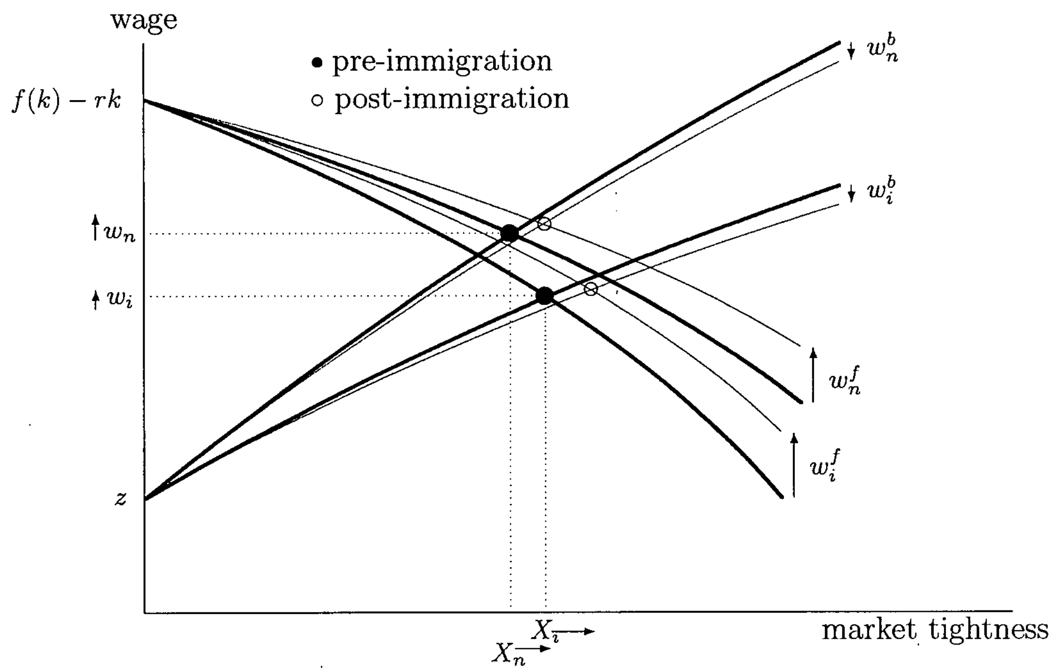


(a) Inferior Matching Technology $(\beta_i = \beta_n,\ \delta < 1)$

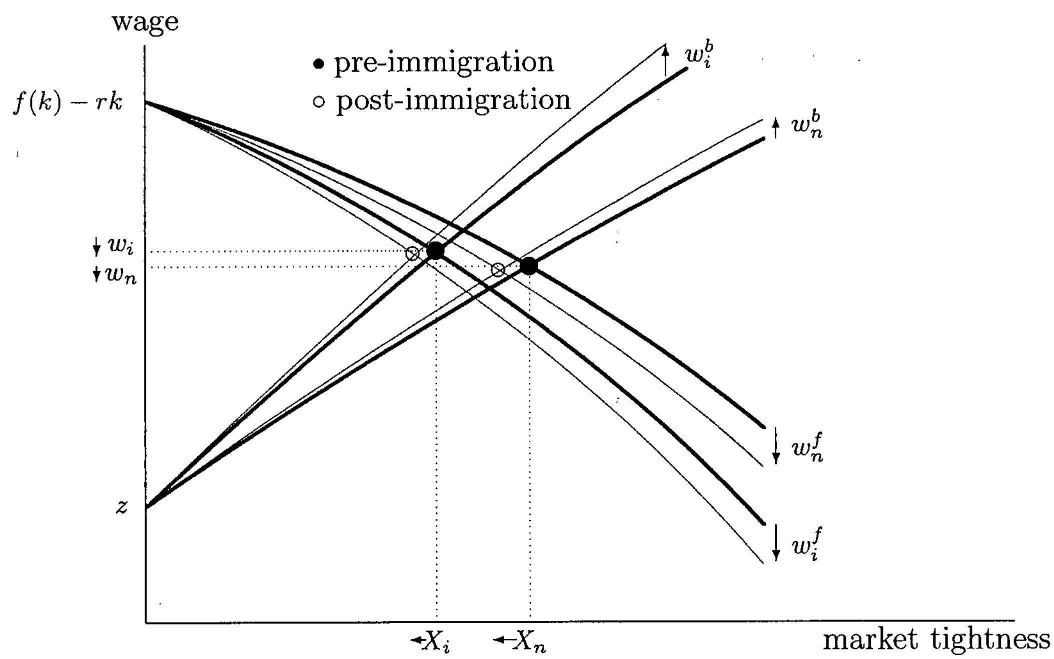


(b) Unequal Bargaining Power $(\beta_i < \beta_n,\ \delta = 1)$

Figure 3.2: Impact of One-Time Immigrations



(a) Weak Immigrants' Bargaining Power ($\beta_i < \beta_i^i, \delta < 1$)



(b) Strong Immigrants' Bargaining Power ($\beta_i > \beta_i^n, \delta < 1$)

Figure 4.1: Idiosyncratic Shocks and Prevalence of Insurance

1960-1969, ρ=0.02, γ=3, ϵ=1/3

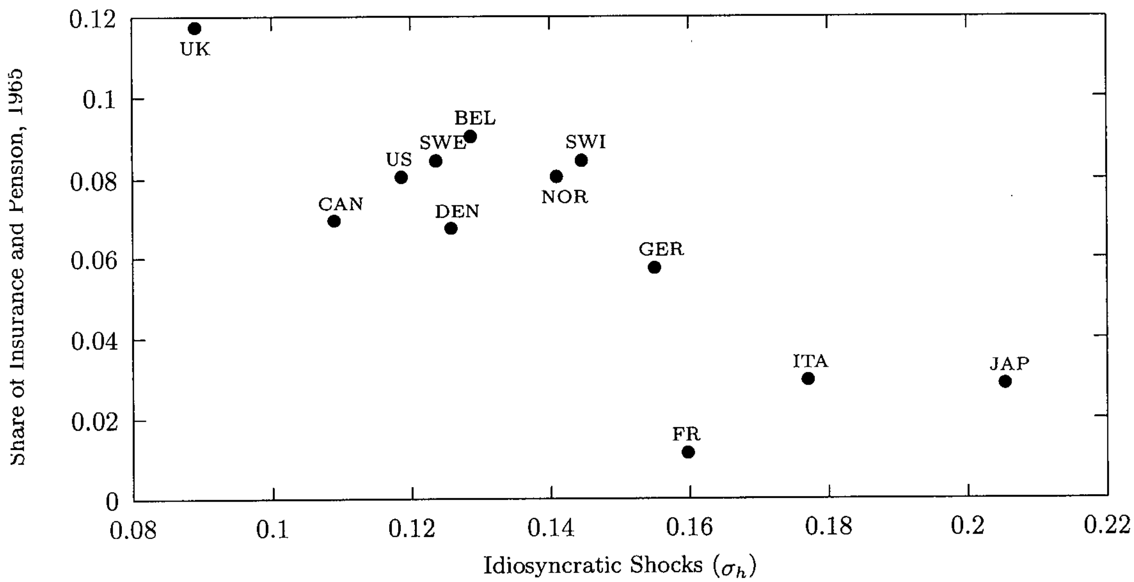


Figure 4.2: Idiosyncratic Shocks and Prevalence of Insurance

1970-1979, ρ=0.02, γ=3, ϵ=1/3

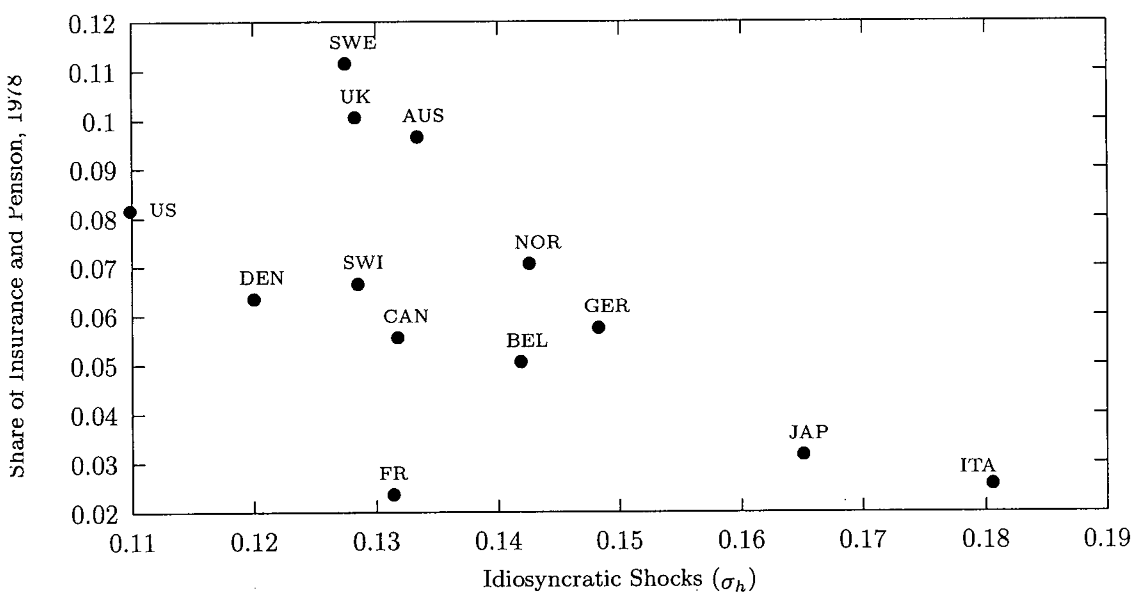


Figure 4.3: Idiosyncratic Shocks and Saving Rates
1960-1992, ρ=0.02, γ=3, ϵ=1/3