AN ECONOMIC ANALYSIS OF THE PROPERTY/CASUALTY INSURANCE MARKET

by

MARY VIRGINIA KELLY

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Department of Commerce & Business Administration

The University of British Columbia
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Abstract

Three economic issues in property/casualty insurance are examined in this thesis. Chapter 2 explores the impact of supply side heterogeneity on the market equilibrium. Multiple period contracting and informational issues are examined in Chapters 3 and 4.

Property/casualty insurance is marketed in two manners: through agency writers and direct writers. Direct writers can sell insurance at a lower cost than agency writers. By exploiting demand side characteristics, Chapter 2 extends the traditional literature by examining the behaviour of heterogeneous insurers within a framework that admits both direct and agency writers in equilibrium. Heterogeneous travel costs are used to support this equilibrium. A second model is developed in which claim frequency heterogeneity is introduced on the demand side. It is assumed that agency writers can better discern a consumer's risk type. Characteristics of equilibria under which direct and agency writers exist are derived.

In Chapter 3, Rothschild and Stiglitz's (1976) single period insurance model is extended to multiple periods. In a multiple period framework, insurers offer a sequence of single period contracts in which future contracts are conditioned on past contract choices. For dynamic consistency, once low risks have revealed their type, future contracts must be contingent on this event. This contract structure is compared to both a sequence of one period pooling contracts and a sequence of one period separating contracts. Numerical examples illustrate the results.

In Chapter 4, learning by insurers is examined in a model in which consumers possess search costs. The presence of search costs allows inefficient insurers to remain in the market, and allows lower cost firms to earn higher profit loadings each period. Insurers, who possess differing initial valuations of a consumer's loss propensity, update the contract offered each period based on a consumer's past accident history. In a multiple period setting, consumers search for new coverage and switch insurers when the price charged by their contracting insurer exceeds the price that they are willing to pay.

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1. Introduction

This thesis examines three economic issues of property/casualty insurance. Chapter 2 analyses the structure of the North American property/casualty insurance market. In Chapter 3 dynamically consistent multiple period contracts that are contingent on past contract choice are developed. Chapter 4 ascertains the effect of bilateral information asymmetries on prices of full insurance contracts in a market in which some insurers earn positive profits.

The first model developed in Chapter 2 is a one period model of the property/casualty insurance industry. In North America, property/casualty insurance is marketed in two basic manners: agency writers distribute their products through an independent brokerage system, and direct writers sell insurance through mail order, their own sales force or exclusive agents. Direct writers have the advantage of being able to sell insurance at a much lower cost than agency insurers once the retail network is in place. Joskow (1973), Cummins and VanDerhei (1979), and Barrese and Nelson (1992) provide empirical evidence of the cost differences between direct writers and agency writers.

Much of the previous work on the economics of insurance contracts, for example Rothschild and Stiglitz (1976), Wilson (1977), Cooper and Hayes (1987) and Hosios and Peters (1989), considered insurance companies to be homogeneous. Even in papers that have assumed heterogeneity of insurers (Schlesinger and von der

Schulenburg (1991), (1993)), insurance companies were assumed to use the same distribution technology. However, the North American insurance market is characterised by non-homogeneous firms possessing one of two distribution technologies. The goal of this research is to extend the traditional literature by examining the behaviour of heterogeneous insurers within a framework that admits both direct and agency writers in equilibrium.

A one period economic model of the insurance industry that supports both types of insurers in equilibrium is constructed. This equilibrium is sustainable due to differences in consumer transactions costs and as such is similar to Posey and Yavas (1995). This market is imperfectly competitive since some insurers are price takers in equilibrium.

A second model is developed in which heterogeneity is introduced on the demand side. The claim frequency is assumed to vary across insureds. Following from Regan and Tennyson (1996), it is conjectured that agency writers can observe each consumer's risk type, but that direct writers cannot. The equilibrium conditions under which direct and agency writers exist are derived. It is shown that the existence of informational asymmetries increases the market share of the agency writer and reduces direct writer profits. Higher risk consumers are unaffected by the presence of informational asymmetries.

Typically, policyholders do not purchase property/casualty insurance only once in their lifetimes, but make annual decisions concerning insurance purchases. Chapters 3 and 4 examine some issues of insurance contracting in a multiple period framework.

Rothschild and Stiglitz' (1976) one period model of the insurance industry predicts that equilibrium exists in which consumers reveal their risk propensities by purchasing varying amounts of insurance. In Chapter 3, a dynamically consistent multiple period extension of Rothschild and Stiglitz' (1976) model is constructed in which a consumer's future contract options are contingent on her past contract purchases. As in the one period Rothschild and Stiglitz (1976) model, a consumer reveals her risk type through the amount of coverage selected in the period in which the separating contract is purchased. Since the contract purchased reveals the consumer type, future policies must be conditioned on this information and as such future contracts are contingent on past contracts. This conditioning ensures that contracts are dynamically consistent. This separating menu of contracts is designed such that insurers earn zero profits each period and no consumer has the incentive to misrepresent her type. The separation decision of low risk consumers in a multiple period world in which both the dynamically consistent separating menu of contracts and pooling contracts are offered is examined. Numerical examples are provided to assist understanding of the theoretical results.

Previous papers in the literature (Cooper and Hayes (1987), Dionne and Doherty (1994) and Watt and Vazquez (1997)) have examined the use of contracts that encourage voluntary separation in a multiple period framework but do not construct dynamically consistent multiple period contracts. Instead a sequence of single period traditional separating menus of contracts, as defined by Rothschild and Stiglitz (1976), is used. This series of contracts cannot be supported in a multiple period equilibrium because it is not re-negotiation proof and because it is not dynamically consistent.

The set of dynamically consistent contracts developed is compared to a sequence of single period separating menus of contracts. Three key results are illustrated. As has been discussed in the multiple period contracting literature, it is shown that consumers' utilities are decreased if consumers cannot pre-commit not to undertake any Pareto-improving changes in future contracts based on information revealed in previous periods. Secondly, the conditions under which consumers would prefer to pool for the entire lifetime of the contract in the world with dynamically consistent contracts are the same as for the model with the sequence of single period separating menus of contracts. And finally, differences in the resulting equilibria between the model with dynamically consistent contracts and the model with the sequence of one period separating menus of contracts arise in those situations in which low risk consumers would prefer to separate in the traditional one period Rothschild and Stiglitz (1976) model.

Bilateral information asymmetry in insurance markets is examined in Chapter 4. In this chapter, firms do not possess information on a consumer's risk propensity and consumers do not know the price charged by an individual firm. Both one and multiple period models are developed.

Traditional models in insurance, such as Rothschild and Stiglitz (1976), Wilson (1977), Kunreuther and Pauly (1985), Cooper and Hayes (1987), Hosios and Peters (1989) and

Dionne and Doherty (1994), assume that consumers possess perfect information about a firm's pricing structure. The only asymmetry in these models arises from the firm's lack of knowledge about consumers' accident propensities. However, even the most casual observer of the property/casualty insurance market will refute the statement that consumers possess perfect information concerning insurance prices. This lack of price information is not unique to the insurance industry. Pratt, Wise and Zeckhauser (1979), Maynes and Assum (1982), Mazumdar and Monroe (1990) and Grewal and Marmorstein (1994) all find that consumers undertake very little price comparison when purchasing both durable and non-durable goods.

In this chapter, as in Rothschild and Stiglitz' (1976) classic model, there are only two types of insureds, high risk and low risk consumers. Firms possess different valuations of the proportion of high risk consumers. The difference in valuations arises because firms attract distinct clienteles, or have differences in rating structures, claims handling and underwriting procedures. This results in each firm having a different valuation of the actuarial fair value of an insurance contract.

Each consumer knows the distribution of prices charged in the marketplace but not the price charged by an individual firm. In each period, the consumer incurs a cost to discover the prices charged by different firms. A one period model of price dispersion is developed. Following Pratt, Wise and Zeckhauser, if consumers possess search costs, a stable equilibrium exists in which insurers charge different prices. The lower a firm's valuation of the actuarial fair value of its contract, the greater the profit loading it can charge.

A multiple period extension of the one period model allows for learning by insurance companies since, over time, a consumer's accident experience reveals her true risk type to the insurer. Each period the consumer updates the price that she is willing to pay and firms update the prices that they charge based on a consumer's past accident history. As in the one period model, an equilibrium is constructed in which, each period, insurers charge different prices and earn different expected profits. A consumer will switch insurers if the price her contracting insurer charges exceeds the price she is willing to pay in that period. A consumer renews her policy with her contracting insurer if the price demanded is less than the consumer's reservation price. This model differs from previous models in the literature in that a consumer's decision to switch insurers is not driven solely by her accident history or risk type.

2. A One Period Model of a Spatial Insurance Market

In North America, property/casualty insurance is marketed in two basic manners. Most insurance companies distribute their products through an agency system. In this structure, independent agents represent large numbers of companies and sell the policies on commission to the public on behalf of these companies. These insurers have the advantage of incurring low start-up costs. Typically, agency writers supply independent agents with premium schedules and underwriting criteria and the agents solicit customers. The ownership of the client list rests with the agent, and agency writers are prohibited by legislation from directly soliciting the consumer's business. Companies that sell insurance through mail order, through their own sales force or through exclusive agents are called direct writers. Direct writers have the advantage of being able to sell insurance at a much lower cost than agency insurers once the retail network is in place. Commission scales for agents of direct writers, also known as exclusive agents, may also distinguish between new business and renewals, further reducing the costs of the direct writers.

If agency writers and direct writers are identical except for their distribution technology, normative economic theory concludes that the agency system should not survive in the long run if direct writers find it profitable to enter the marketplace. In this chapter, the ability of independent brokers to match consumers to agency writers supports the existence of agency writers in equilibrium.

In this chapter, two models are presented which support both direct and agency writers in equilibrium. The first embeds a symmetric information insurance market within a spatial framework and the second model expands the first to include private information. This chapter focuses in particular on the market for personal insurance coverages such as private passenger automobile insurance and homeowner's coverage.

The use of a spatial framework captures the fact that insurance consumers incur intangible costs when purchasing insurance. Insurance is a heterogeneous good; companies differ in such characteristics as payment plans, claims service and probability of insolvency. A potential cost faced by consumers is a loss in utility because the characteristics of the insurance company or the contract offered may not be exactly what are desired by the insured. This intangible cost is assumed to exist only when a consumer purchases insurance from a direct writer and not when purchasing insurance from an agency writer.

The first model characterises the conditions under which both agency writers and direct writers exist in equilibrium. In flavour, this model is similar to Posey and Yavas (1995). Direct writers operate as local monopolists with the agency writers entering between the captive markets of the direct writers to sell insurance to those consumers whom direct writers find too expensive to serve.

It is a simplification to assume that all consumers are the same, since each insurance consumer faces a different level of risk. The second half of the chapter extends the model to incorporate two types of consumers with differing claim frequencies. It is assumed that the independent agents, who serve agency writers, can differentiate between the two types of consumers and that the exclusive agents, who underwrite for direct writers, do not or cannot. This assumption follows from the empirical results presented by Regan and Tennyson (1996).

In the equilibrium with two consumer types, two types of agency writers emerge and act in perfect competition with other agency writers of the same type. One set of agency writers enters the market and sells full insurance to the low risk consumers only. The second set of agency writers sells full insurance only to the high consumer. Direct writers enter the market and offer a separating menu of contracts to all insureds. This separating menu of contracts is similar to the contracts introduced by Rothschild and Stiglitz (1976) except that, in equilibrium, direct writers earn non-negative profits.

A comparison of the symmetric information and the asymmetric information models yields the following results. The necessity of the direct writers' separating menu of contracts results in more low risk consumers preferring to purchase coverage from the agency writers than in the symmetric information model. The amount of coverage available is decreased in the direct writers' contracts designed for the low risk consumers, but this contract is offered at greater savings than if direct writers had full information. Direct writers earn less profits than if they could distinguish between consumer types. As long as it is profitable for direct writers to enter the marketplace, high risk consumers are unaffected by direct writers' inability to differentiate between consumer types.

The set-up of the chapter is as follows. In Section 2.1 some empirical evidence illustrating the cost differences of the two types of insurance distribution systems is given. Section 2.2 first discusses a key model assumption in detail before presenting the model under the conjecture of symmetric information. The symmetric equilibrium in which direct writers and agency writers co-exist is characterised. Section 2.3 extends the basic model to include asymmetric information. A summary of the primatives used and the functions defined is given in Table A-1 in Appendix A.

2.1. Empirical Evidence

An abundance of data illustrating the difference in the expenses between direct writers and agency writers exists. Joskow (1973), examines the United States property/casualty insurance industry for the years 1970 to 1971 and he states that "*expense ratios of direct writers average 10.82 percentage points less than the agency companies ceteris paribus*". Similar results over the time period 1968 through 1976 and 1978 to 1990 were reported by Cummins and VanDerhei (1979) and by Barrese and Nelson (1992) respectively. A Canadian study by Quirin *et al* (1974) notes that as the prominence of direct writers increased in the early 1960's the average commission paid to independent agents fell from 25% to a range of 8% to 15%. The data in Table 2 - 1, collected for

	Direct Writers	Agency Writers
Number of Companies	23	176
Average Expense Ratio	27.16%	38.09%
Total Net Written Premiums (thousands)	$2 241 862	$9 542 564
Average Return on Net Earned Premiums	24.1%	21.8%
Average Asset Level (thousands)	$201 651	$106 630

Table 2 - 1 - Selected Data for Direct and Agency Writers in Canada

Canadian companies in 1988 on both direct and agency writers,[1] corroborate the difference in expense ratios. These data cover personal and commercial lines excluding workers compensation and health insurance. The expense ratio is the ratio of net acquisition costs to net premiums earned and does not included loss adjustment expenses.

The data suggest that direct writers occupy an oligopoly position in the industry. Only 11.6% of insurance companies are direct writers but they wrote 19% of all premiums in 1988. These twenty-three companies include both small local firms and large federally registered companies that operate in several provinces. The seventeen national direct

[1] Data collected from Stone and Cox' Blue Chart Report 1988. No guide provides a concise listing of each insurer's distribution system. Stone and Cox' General Insurance Register provided most of the required information. Where ambiguity remained the author's best judgement was used.

writers account for 17.8% of all net written premiums. The average expense ratio for a direct writer is below that of an agency writer and the average return on net earned premiums is higher for direct writers than for agency writers, but this difference is not statistically significant.[2]

2.2. The Symmetric Information Model

In this section, the symmetric information spatial insurance model is presented. A key assumption required to derive the equilibrium is discussed. Before equilibrium conditions are derived, the behaviour of the utility maximising consumers and profit maximising insurers are described. The symmetric information equilibrium is characterised in terms of the exogenous variables.

2.2.1. Key Assumption

The main assumption used to derive the equilibrium conditions is that heterogeneous consumers incur costs when purchasing insurance from heterogeneous direct writers. This assumption extends the work of Posey and Yavas (1995). They assume that all insurance companies are identical except for the distribution system. They also assume

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[2] The test that the difference between the mean of both groups is zero was not rejected at the 5% level.

that there are only two consumer types, those with a low transaction cost and those with a high cost. In this model, there is a continuum of consumer transaction costs

The main feature of this model is the use of a spatial framework. Clapp (1985) and Schlesinger and von der Schulenburg (1991) have previously proposed the use of spatial models (such as Salop's (1979) circular city) in insurance. Both papers follow Archibald, Eaton and Lipsey's (1982) definition of location on the circumference of the circle as location in characteristic or attribute space. Clapp's (1985) use of a spatial model yields a Nash pooling equilibrium among homogeneous insurance companies within the Rothschild and Stiglitz (1976) framework. Schlesinger and von der Schulenburg (1991) develop a model of entry into the insurance market under the hypothesis that all consumers incur both switching and search costs.

The interpretation of the distance between a consumer and a direct writer follows from Archibald, Eaton and Lipsey (1982), who define the location of both the firm and consumer on the circle as their location in characteristic or attribute space. Although the insurance contract may be homogeneous, the insurance product differs between firms. Schlesinger and von der Schulenburg (1991) note that firms are differentiated by such attributes as perceived quality of service, probability of insurer insolvency, bonus/malus adjustments to premiums, convenience of claims services, availability of a local agent and method of payment. The cost to the insured from purchasing insurance from a direct writer is the loss in utility because the company does not possess the exact characteristics desired by the policyholder. It is assumed that consumers who purchase insurance from an independent agent do not face this charge because at least one of the firms represented by the agent matches the consumer's desired characteristics.

Empirical support for this interpretation has been provided by Cummins *et al* (1974), Schlesinger and von der Schulenburg (1993) and Beemer (1995). Cummins *et al*, in a survey of over 2400 insureds, find that forty percent of those surveyed think that the insurance company was the most important factor in choosing an automobile insurer. Schlesinger and von der Schulenburg (1993), in a survey of West German insurance customers, conclude that consumers do not perceive the insurance product as homogeneous and subjective assessments of satisfaction play a significant role in the decision to switch insurers. Beemer (1995) surveyed one thousand Hispanic and African Americans in five major cities. When participants were asked to list on what basis they used to select an insurance company, almost three-quarters of those surveyed were concerned with the types of payment plans offered and just fewer than one-half felt that the company's financial rating was important.

D'Arcy and Doherty (1990) comment that the insurance market may be characterised by clienteles. Some policyholders prefer the lower price of the direct writers and some policyholders are willing to pay more for the informational services of an independent agent, since independent agents can provide comparative information on many insurance companies. Berger, Cummins and Weiss (1995) note that if the cost differential between the two types of firms is indeed due to the inefficient distribution system of the agency writer then this should be reflected in the profit efficiency differentials between the two distribution systems. They examine both costs and profits of direct and agency writers. The lack of any statistical difference in profit efficiencies lead the authors to conclude that independent agents provided more or better services than exclusive agents, such as offering a greater variety of product choices or reducing policyholder search costs.

2.2.2. Demand

Let L denote the measure of a continuum of identical risk-averse consumers located uniformly about the circle where each consumer is endowed with wealth W and is assumed to have constant absolute risk aversion.[3] Consumers live in a world where there is only one time period and 2 states. With probability ρ, which is uncorrelated across consumers, the consumer suffers a loss of d. Both ρ and d are known to the consumers and the potential insurers. No adverse selection or moral hazard exists in this framework.

An individual can insure against loss either by purchasing insurance from a perfectly competitive agency writer at the price p_a or by purchasing insurance from a direct writer for p_m (where m denotes monopoly pricing). There are n identical direct writers

[3] This utility function is used so that wealth effects can be ignored. Similar results are obtained in this framework using other concave von Neumann-Morgenstern utility functions.

located symmetrically about a circle of unit circumference. If the consumer purchases insurance from the direct writer she incurs a cost of t times the distance travelled. Independent agents are located continuously about the circle and there are no such costs associated with purchasing insurance from agency firms. Furthermore, insurance companies offer only full insurance contracts, restricting each consumer's choice to full coverage or no insurance.[4] The consumer is faced with three choices: she may purchase no insurance, she may purchase full insurance from the agency writer or she may purchase full insurance from a direct writer. The consumer's preference is the option that gives her the highest expected utility. It is assumed that buying insurance from the agency writer is always preferable to no insurance.

Since agency writers act competitively, each insurer charges the expected cost per policy of $\rho d + e_a$, where e_a is the cost to the insurer of writing one policy. Therefore the consumer's utility from purchasing insurance from an agency writer is given by

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[4] Insurers are assumed to incur an additive expense when selling insurance Because of this cost involved with the purchase of insurance, the utility maximising individual would never purchase more than one policy. In the presence of a multiplicative expense loading all insureds would prefer to purchase less than full insurance (Arrow (1965), Mossin (1968), Szpiro (1985) and Borch (1990)). Eisenhauer (1993) shows that full insurance may be purchased in the presence of an expense loading if the insurer and the consumer have differing estimates of the probability of loss. Because the expense loading is additive, the utility maximising individual would always prefer to purchase full insurance or no insurance. Competition, whether realised or potential, constrains both types of insurers to offer full insurance contracts in equilibrium. The additive expense behaves as a quasi-fixed cost, since it is only incurred if insurance is purchased. Thus it is possible that this expense could be so high that consumers would not purchase insurance. Despite this drawback of the additive expense loading, the additive expense loading is a realistic representation of the costs incurred in writing a policy. As noted by Wade (1973), the use of a constant expense to cover those costs that are incurred at a constant level per policy is a dominant pricing strategy. Posey and Yavas (1995) also assume an additive expense loading.

$$V(p_a) = -e^{-\alpha(W - \rho d - e_a)}. \tag{2 - 1}$$

Let ℓ be the distance between a consumer and the closest direct writer. Under the assumption of linear distance costs, the effective cost of purchasing insurance from a direct writer is $p_m + t\ell$. Thus the consumer's utility is given by

$$V(p_m) = -e^{-\alpha(W - p_m - t\ell)}. \tag{2 - 2}$$

The consumer prefers to purchase insurance from the direct writer if $V(p_m) > V(p_a)$, a decision that depends on the location of the consumer. From (2 - 1) and (2 - 2), the location of the consumer who is indifferent between purchasing insurance from the nearest direct writer or the agency writer is

$$\ell_m = \frac{\rho d + e_a - p_m}{t}. \tag{2 - 3}$$

All consumers closer to the direct writer would prefer to purchase insurance from the direct writer. If the transaction cost, t, is very high or if there is not much difference between the prices charged by the direct and agency writers, then a consumer is more likely to buy insurance from an agency writer.

2.2.3. Supply

There are n direct writers who enter simultaneously and locate symmetrically about a circle of unit circumference. Unlike Salop's (1979) original model, it is assumed here that relocation costs are so prohibitive that once a direct writer has chosen its location, it cannot move. Each direct writer first incurs a capital cost of F and then chooses a price at which to sell insurance.

Based on the observation that agency writers have higher expenses than direct writers do, the direct writer expends $e_d < e_a$ to write a policy.[5] Agency writers are located continuously about the circle and possess ample capacity to absorb the entire market's demand at the competitive price.

High relocation costs are necessary to derive the characterisation of the model and the symmetry produces a tractable equilibrium.[6] The entry configuration of direct writers follows closely to the pattern discussed in Eaton and Wooders (1985).

Since it is the purpose of this chapter to explain the co-existence of direct and agency writers, the following analysis concentrates on equilibrium conditions that lead to the

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[5] A further restriction on e_d and e_a is $e_a - e_d < t$. This restriction ensures that one direct writer cannot enter and capture the entire market.

[6] If the symmetric entry rule is relaxed, an equilibrium in which both direct and agency writers exist can still be constructed. A heuristic argument is available from the author.

existence of both types of insurers. Before the equilibrium is characterised, the profit maximising behaviours of the direct writer acting both as a local monopolist and competing with other direct writers are described.

Lemma 2 - 1: *If a direct writer can act as a local monopolist, it sells its product at a price of* $p_m \equiv \rho d + \frac{1}{2}(e_a + e_d)$ *and earns monopoly profits of* $\Pi(p_m) \equiv \frac{L}{2t}(e_a - e_d)^2 - F$. *The length of the monopoly market is given by* $2\ell_m \equiv \frac{e_a - e_d}{t}$.

Proof: The behaviour of a profit maximising monopolist facing a demand $q_m = 2L\ell_m$, where ℓ_m has been defined in equation (2 - 3), is discussed in, for example, Tirole (1988). ■

Lemma 2 - 2: *If a direct writer competes with other direct writers in a Bertrand manner, it charges a profit maximising price of* $p_c \equiv \rho d + e_d + \frac{t}{n_c}$ *(where* c *denotes Bertrand competition) and earns profits of* $\Pi(p_c) \equiv \frac{tL}{n_c^2} - F$. *Under free entry, the number of direct writers that would exist in equilibrium is* $n_c \equiv \sqrt{tL/F}$, *and the length of each direct writer's market is* $2\ell_c = \frac{1}{n_c}$.

Proof: In this situation, agency writers do not exist and direct writers compete with

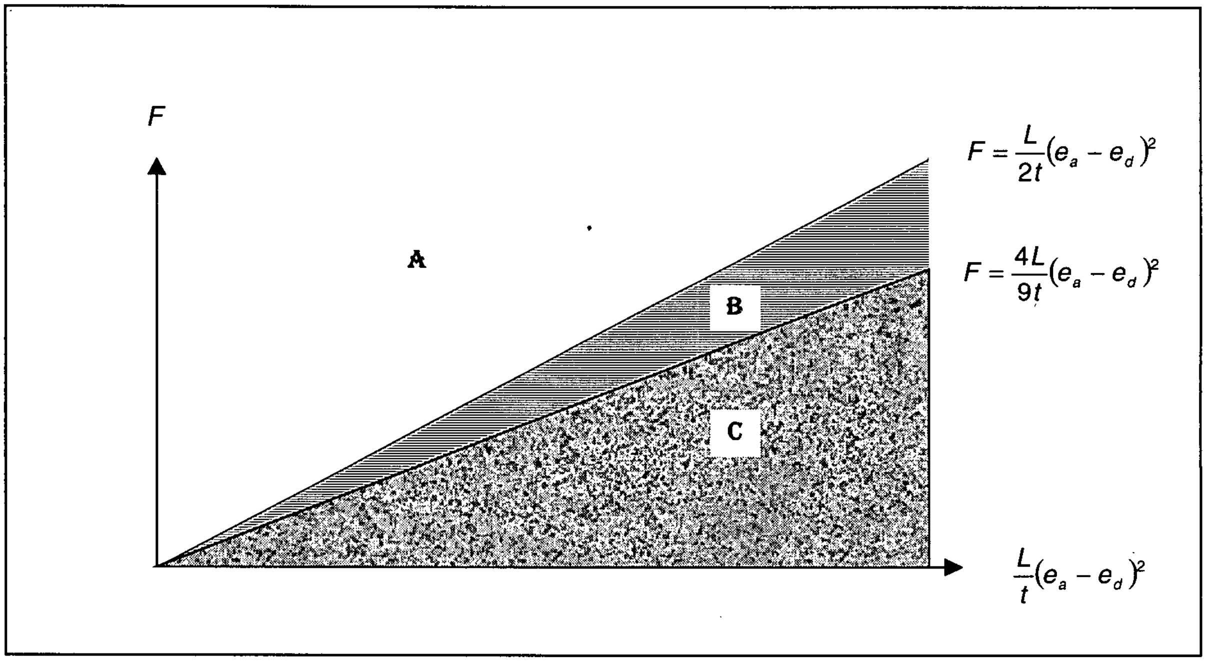

Figure 2 - 1 – Symmetric Information Equilibrium Characterised by Fixed Costs

neighbouring direct writers. The solution to this problem can also be found in Tirole (1988). ■

2.2.4. Symmetric Information Equilibrium

Given the above description of consumer and insurer behaviours, an equilibrium that supports the existence of both agency and direct writers can now be derived. This equilibrium, characterised in Theorem 2 - 1 depends on the size of the fixed cost, F, and its relationship to the other exogenous variables in this model. Figure 2 - 1 illustrates the relationships. Before the equilibrium is discussed, it is useful to derive the boundaries for the regions **A**, **B** and **C** in Figure 2 - 1. Area **A** defines a region in which no direct writer could enter and earn non-negative profits. In region **B**, non-negative profits can only be earned by direct writers acting as local monopolists, and in the area denoted as **C**, non-negative profits are assured even if direct writers compete with each other in a Bertrand fashion.

To obtain the function defining the boundary between areas **A** and **B**, consider the situation where each direct writer acts as a local monopolist. From Lemma 2 - 1 the direct writer earns profits of $\Pi(p_m) \equiv \frac{L}{2t}(e_a - e_d)^2 - F$. Therefore for fixed costs greater than $\frac{L}{2t}(e_a - e_d)^2$, entry does not occur. The equation $F = \frac{L}{2t}(e_a - e_d)^2$ defines the boundary between regions **A** and **B** in Figure 2 - 1.

To derive the boundary between regions **B** and **C**, it is necessary to examine the profits accruing to direct writers competing in a Bertrand fashion. From Lemma 2 - 2, n_c direct writers each earn zero profits if the marginal consumer prefers to purchase insurance from a direct writer instead of the agency writer. This occurs if $p_c + \frac{t}{2n_c} \leq p_a$. Substituting for the number of direct writers from Lemma 2 - 2, and for prices, p_a and p_c, into this inequality yields $(e_a - e_d) \geq \frac{3t}{2n_c} \geq \frac{3}{2}\sqrt{tF/L}$ and simplifying gives

$$F \le \frac{4L}{9t}(e_a - e_d)^2 . \tag{2 - 4}$$

Thus for fixed capital costs less than $\frac{4L}{9t}(e_a - e_d)^2$, insurers find it profitable to enter even if they must compete with neighbouring direct writers. The equality of (2 - 4) defines the boundary between regions **B** and **C** in Figure 2 - 1.

To ensure the existence of agency writers in the situation $F \le \frac{4L}{9t}(e_a - e_d)^2$, restrictions on the number of direct writers existing in equilibrium are required. Theorem 2 - 1 defines the conditions under which an equilibrium that supports both direct and agency writers exists.

Theorem 2 - 1: *For* $F \in \left(\frac{4L}{9t}(e_a - e_d)^2 , \frac{L}{2t}(e_a - e_d)^2 \right]$, *there exists a symmetric equilibrium in which agency writers exist and all direct writers act as local monopolists. Furthermore the number of direct writers in this equilibrium is* $n_m \in \left(\mathrm{int}\left[\frac{t}{2(e_a - e_d)} \right], \mathrm{int}\left[\frac{t}{(e_a - e_d)} \right] \right]$. *For* $F \le \frac{4L}{9t}(e_a - e_d)^2$, *a sufficient condition so that both agency and direct writers co-exist in equilibrium is that the number of direct writers is* $n_m \in \left(\mathrm{int}\left[\frac{3t}{4(e_a - e_d)} \right], \mathrm{int}\left[\frac{t}{(e_a - e_d)} \right] \right]$.

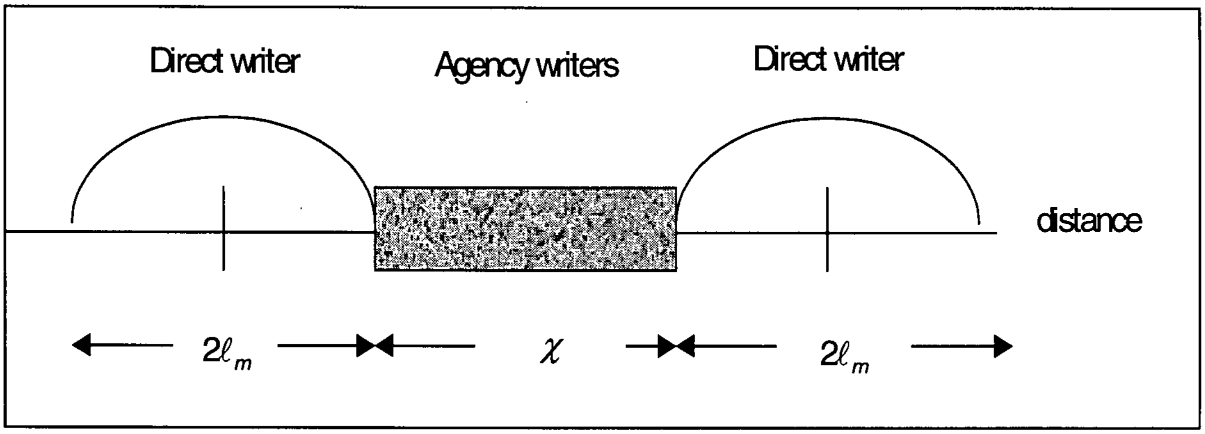

Figure 2 - 2 – Spacing between Direct Writers

Proof: Consider first the maximum number of direct writers that could operate in equilibrium. Since the circle is of unit length (the circumference of the circle is one), the maximum number of direct writers in equilibrium, $\overline{n}_m$ must be such that $2\ell_m(\overline{n}_m+1)>1>2\ell_m\overline{n}_m$, Substituting for ℓ_m from Lemma 2 - 1, yields $\overline{n}_m = \text{int}\left[\frac{t}{(e_a-e_d)}\right]$.

For $F \in \left(\frac{4L}{9t}(e_a-e_d)^2, \frac{L}{2t}(e_a-e_d)^2\right]$, entry is only profitable for direct writers if they are guaranteed a monopoly market. There will be at most $\overline{n}_m$ direct writers in equilibrium, but an equilibrium can be supported with fewer direct writers.

Consider the situation depicted in Figure 2 - 2 where there exists a distance χ between adjacent monopoly markets. A direct writer does not enter between two adjacent direct writers if the distance between adjacent monopoly markets is $\chi<2\ell_m$. Invoking symmetry yields $n_m[\chi + 2\ell_m] = 1$, therefore $n_m[4\ell_m] \geq 1$. Substituting for ℓ_m from Lemma 2 - 1, yields $n_m > \frac{t}{2(e_a - e_d)}$. The number of direct writers that will exist in equilibrium is

$$n_m \in \left(\mathrm{int}\left[\frac{t}{2(e_a - e_d)}\right], \mathrm{int}\left[\frac{t}{(e_a - e_d)}\right]\right].$$

For $F \leq \frac{4L}{9t}(e_a - e_d)^2$, the equilibrium characterised is one in which direct writers act as local monopolists and there is not sufficient space between any two adjacent direct writers so that another firm could enter and compete profitably in a Bertrand fashion with its two neighbours. For a direct writer not to be able to profitably enter between the two existing direct writers, it must be the case that $\chi + 2\ell_m < 4\ell_c$ and since the circle is of unit length, the number of direct writers in equilibrium, n_m satisfies $n_m[\chi + 2\ell_m] = 1$. Therefore $n_m[4\ell_c] \geq 1$. Substituting for $n_c = \frac{1}{2\ell_c}$ yields $n_m > \frac{3t}{4(e_a - e_d)}$. The number of direct writers that exist in equilibrium is $n_m \in \left(\mathrm{int}\left[\frac{3t}{4(e_a - e_d)}\right], \mathrm{int}\left[\frac{t}{(e_a - e_d)}\right]\right]$. ■

Therefore in equilibrium, there are n_m direct writers each earning non-negative profits and jointly providing insurance to $\frac{n_m L(e_a - e_d)}{t}$ consumers. Remaining consumers, who live between the captive markets of the direct writers, purchase insurance from the agency writers. Each direct writer charges a price of $p_m \equiv \rho d + \frac{1}{2}(e_a + e_d)$ and earns monopoly profits of $\Pi(p_m) = \frac{L}{2t}(e_a - e_d)^2 - F$.

Without the above characterisation of the number of direct writers, region B is the location of highest profit and, as such, direct writers prefer to operate in this region. Consider firms operating under conditions defined by this region and suppose that the firms have the ability to manipulate $\frac{(e_a - e_d)^2}{t}$. That is, firms can change their commission structure to alter e_d or change their marketing strategies to affect t. Since each direct writer earns a profit of $\Pi(p_m) = (e_a - e_d)^2 - F$, it has the incentive to decrease either e_d or t until the boundary condition $F = \frac{4L}{9t}(e_a - e_d)^2$ is met. If firms further decrease e_d or t, new direct writers may be able to enter between existing direct writers and earn positive profits.

If, in equilibrium, $F \leq \frac{4L}{9t}(e_a - e_d)^2$, and direct writers act as local monopolists, they too have an incentive to decrease either e_d or t subject to the restriction that $n_m \geq \frac{3t}{4(e_a - e_d)}$. If expenses are decreased too much, then new direct writers find it profitable to enter and the incumbents could be made worse off.

The market structure is also affected by exogenous changes in the number of consumers. An increase in L has the same effect on direct writers as either a decrease in e_d or t. A large increase in L can eliminate all agency writers, whereas a large decrease in L can make the market unprofitable for any direct writers. For agency writers any increase in $\frac{L(e_a - e_d)^2}{t}$ is not desirable. Agency writers have an incentive to decrease their expenses. Although they would not earn higher profits if they reduced their expense margins, lower expenses increase their probability of survival.

Using a spatial model, it is possible to characterise an equilibrium under which both direct writers and agency writers exist. However, it is a gross simplification to assume that all consumers are the same. Heterogeneity within a single rating category might occur if some traditional rating variables are banned, such as age-gender categories for private passenger automobile insurance. Within a rating category, insurance companies may not wish to underwrite all consumers, or may offer a menu of insurance contracts if price discrimination is not allowed.

2.3. Consumer Differentiation

This section embeds a Rothschild and Stiglitz (1976) insurance model within the spatial insurance framework developed in Section 2.2. To examine this extension, consider two types of policyholders with differing loss probabilities as in the Rothschild and Stiglitz (1976) model. Furthermore assume that agency writers can differentiate between the risk types but direct writers cannot. This conjecture arises from empirical results presented by Regan and Tennyson (1996).[7]

Formally, there are two types of consumers: those with probability of loss ρ^{ℓ} and those with a higher probability of loss ρ^{h}. Each consumer knows her risk type. As in the Rothschild and Stiglitz (1976) model, assume that the two consumer types are identical except for these differing loss probabilities.[8] Additionally, existing legislation does not permit price discrimination and firms are not constrained by common carrier requirements.[9]

Suppose that L still represents the measure of continuum of consumers in the market and the fraction of high risk consumers is λ. The two types of consumers are located uniformly about the circle. As in Section 2.2, consumers may purchase insurance from the nearest agency writer and incur no additional costs. Consumers who purchase insurance from a direct writer incur a cost of t times the distance travelled. This

[7] They examine the relationship between such variables as the market size, variability in losses and existence of residual markets and the market share of direct and agency writers in various commercial and personal lines. They find that agency writers have a higher market share in commercial lines and in states where the insured population appears to be more heterogeneous. Their results lead them to conclude that independent agents are better able to discern the consumer's true risk type because independent agents are more willing to undertake expensive information gathering.

[8] This is not an innocuous assumption. In reality, one might expect that a consumer's propensity of loss is correlated to both her risk aversion and her level of wealth.

[9] A common carrier requirement is a statutory provision requiring a firm to sell its produce to all whom wish to purchase it. Airlines, railroads and utilities, for example, are constrained by common carrier requirements. This model is still applicable for lines of insurance which insurers are compelled to sell, such as third party liability coverage for private passenger automobile, under the assumption that an involuntary market exists. Insurers placing business in the involuntary market are deemed to have refused business.

measure of transaction costs is the same for all consumers, although the amount paid by each consumer depends on distance between the consumer and the nearest direct writer. Define $V^{\tau}(p_j^c)$ to be utility earned by consumer type $\tau = h$ for high risk or $\tau = \ell$ for low risk, when purchasing a contract designed for type $c = h$ for high risk or $c = \ell$ for low risk, from an insurer of type $j = a$ for agency writer, $j = m$ for a direct writer acting as a local monopolist and $j = c$ for a direct writer competing with other direct writers.

As in the previous section, agency writers are located continuously about the circle and n direct writers enter and locate symmetrically about a circle of unit circumference. Each direct writer first incurs a capital cost of F and then chooses the contracts that it wishes to sell. The contracts that are offered by the direct writers depend on the contracts offered by the agency writers. As such the agency writer contracts are discussed first. The notation is consistent with the previous section.[10]

2.3.1. Agency Writer Contracts

In equilibrium two types of agency writers emerge; one type sells insurance to low risk consumers only and the second type markets coverage to high risks only. Since insurance companies are restricted from price discrimination, each agency writer can

[10] Variable names are differentiated if results from both Sections 2.2 and 2.3 are being compared. In this case variables defined in Section 2.2 are pre-subscripted with a 2.2 and variable arising from Section 2.3 are pre-subscripted with a 2.3.

only sell insurance to one risk type. At every location on the circle, there exists a "high risk" and a "low risk" agency writer. In practical terms, when a consumer enters the office of an independent agent, the agent correctly ascertains the consumer's type and places that consumer's business with the appropriate type of insurance company. It does not matter to the consumer that there are two types of agency writers. It is assumed that there are sufficiently many agency writers who underwrite both types of insureds so that there are no capacity constraints. As in Section 2.2, consumers purchasing insurance from an agency writer pay the higher additive expense of e_a. This expense is constant between the "high risk" and the "low risk" agency writers.

Lemma 2 - 3: *Low risk consumers can purchase insurance from "low risk" agency writers for a price of* $p_a^\ell = \rho^\ell d + e_a$. *High risk consumers can purchase insurance from "high risk" agency writers for a price of* $p_a^h = \rho^h d + e_a$. *The utility earned by the low risk consumer is* $V^\ell\left(p_a^\ell\right) = -e^{-\alpha\left(W - \rho^\ell d + e_a\right)}$, *and the utility earned by the low risk consumer is* $V^h\left(p_a^h\right) = -e^{-\alpha\left(W - \rho^h d + e_a\right)}$.

Proof: Given that agency writers are perfectly competitive and consumer type is known by all agency writers, the impact of the additive expense, as discussed in Section 2.2, is to make full insurance desirable by both risk types. ■

2.3.2. Direct Writer Contracts

There are two possible types of contracts that could exist in equilibrium: pooling and separating. The construction of the contracts is similar for both structures and as such only the separating menu of contracts, as formalised by Wilson (1977), will be detailed in this section. The construction of the contracts follows from Section 2.2.3. Proposition 2 - 1 details the separating menu of contracts offered by the direct writers, if each direct writer acts as a local monopolist. The contracts offered by direct writers who compete in Bertrand competition with neighbouring direct writers are given in Proposition 2 - 2.

Proposition 2 - 1: *If a separating equilibrium exists and direct writers act as local monopolists, they will offer the following separating menu of contracts. One contract is a full insurance contract at a price* $p_m^h = \rho^h d + \frac{1}{2}(e_a + e_d)$. *The second is a partial insurance contract priced at*

$$p_m^\ell = \frac{1}{2}\left(\rho^\ell d + \rho^\ell I_m\right) + \frac{1}{2}(e_a + e_d) - \frac{1}{2\alpha}\log\left[1 - \rho^\ell + \rho^\ell e^{\alpha(d - I_m)}\right]$$

for an amount of insurance I_m *that satisfies*

$$\rho^h d = \frac{\rho^\ell (d + I_m)}{2} + \frac{1}{\alpha}\log\left(1 - \rho^h + \rho^h e^{\alpha(d - I_m)}\right) - \frac{1}{2\alpha}\log\left(1 - \rho^\ell + \rho^\ell e^{\alpha(d - I_m)}\right).$$

Direct writers earn profits of

$$\Pi\left(p_m^\ell, p_m^h\right) = \frac{L}{2t}\left[\lambda(e_a - e_d)^2 + (1 - \lambda)(e_a - e_d + f(I_m))^2\right] - F\ ,$$

where for notational brevity

$$f(I_m) = \rho^\ell (d - I_m) - \frac{1}{\alpha}\log\left[1 - \rho^\ell + \rho^\ell e^{\alpha(d - I_m)}\right].$$

The size of the low risk market served by the monopolist is $2\ell^\ell_m = \frac{1}{t}\left[e_a - e_d + f(I_m)\right]$*, and for the high risks, the market is of size* $2\ell^h_m = \frac{1}{t}(e_a - e_d)$*.*

Proof: As in Lemma 2 - 1, the direct writer chooses prices of the two contracts to maximise its expected profit. In this framework, the direct writer must consider the preferences of both consumer types. As in the Rothschild and Stiglitz (1976) model, the separating menu of contract consists of two contracts, a full insurance contract, at a price p^h_m, designed for the high risk consumer and a partial insurance contract, at a price p^ℓ_m and for an amount of insurance I_m, designed for the low risk consumer.

Consider the low risk consumers. These consumers will prefer to purchase insurance from direct writers if $V^\ell(p^\ell_m) \geq V^\ell(p^\ell_a)$. Thus, the location of the marginal low risk consumer is given by

$$\ell^\ell_m = \frac{\rho^\ell d + e_a - p^\ell_m}{t} - \frac{1}{\alpha t}\log\left[1 - \rho^\ell + \rho^\ell e^{\alpha(d - I_m)}\right]. \tag{2 - 5}$$

High risk consumers prefer to purchase insurance from a direct writer if $V^h(p^h_m) \geq V^h(p^h_a)$, and as such the location of the marginal high risk consumer is given by

$$\ell_m^h = \frac{\rho^h d + e_a - p_m^h}{t}.$$

(2 - 6)

The direct writer then calculates its profits by maximising

$$\Pi(p_m^h, p_m^\ell) = 2L(1-\lambda)\pi_m^\ell \ell_m^\ell + 2L\lambda\pi_m^h \ell_m^h - F,$$

(2 - 7)

where $\pi_m^\ell = p_m^\ell - \rho^\ell I_m - e_d$ and $\pi_m^h = p_m^h - \rho^h d - e_d$ are the expected variable profits earned per policy on the partial insurance contract and the full insurance contact respectively. Solving for optimal prices yields

$$p_m^\ell = \frac{1}{2}(\rho^\ell d + \rho^\ell I_m) + \frac{1}{2}(e_a + e_d) - \frac{1}{2\alpha}\log\left[1 - \rho^\ell + \rho^\ell e^{\alpha(d - I_m)}\right]$$

(2 - 8)

and

$$p_m^h = \rho^h d + \frac{1}{2}(e_a + e_d).$$

(2 - 9)

To solve for the length of the two monopoly markets substitute for the prices from (2 - 8) and (2 - 9) into (2 - 5) and (2 - 6) respectively. Direct writer profits can be calculated by substituting monopoly prices and quantities into (2 - 7).

The direct writer then sets the amount of partial coverage, I_m, so that the high risk consumer who buys insurance from the direct writer earns greater expected utility if she purchases the contract designed for her type. That is: $V^h(p_m^h) \geq V^h(p_m^\ell)$. Substituting for prices and solving for equality yields

$$\rho^h d = \frac{\rho^\ell(d+I_m)}{2} + \frac{1}{\alpha}\log\left(1-\rho^h+\rho^h e^{\alpha(d-I_m)}\right) - \frac{1}{2\alpha}\log\left(1-\rho^\ell+\rho^\ell e^{\alpha(d-I_m)}\right).$$

Numerical routines can be used to calculate this optimal amount of indemnity. ■

On each insurance contract sold to a low risk consumer, the direct writer earns expected profits of $\pi_m^\ell = \frac{1}{2}(e_a - e_d) + \frac{1}{2}f(I_m)$. For the direct writer to sell these contracts it must be the case that this expected profit is strictly positive. Since $f(I_m) < 0$, for all reasonable parameter values,[11] the difference in expenses must exceed $f(I_m)$ in order for this contract to be sold.[12]

Direct writers earn greater expected profits per policy from high risk consumers since the difference between the variable profits is $\pi_m^\ell - \pi_m^h = f(I_m)$ which is negative. This is to be expected, for high risk consumers are offered full insurance by both direct and

[11] From Haubrich (1994), reasonable values for α range from 0.25 to 1.25. Typical loss probabilities average 10% to 15%. Typical loss amounts examined are $2000, the average size of private passenger automobile property damage claim paid in the United States for 1996 and $11 000 the average size of private passenger automobile bodily injury claim. These two figures are taken from *The Fact Book 1997: Property/Casualty Insurance*. The figure for property damage excludes claims histories from Massachusetts, Michigan, New Jersey and South Carolina. The figure for bodily injury excludes Massachusetts and all states with no-fault insurance.

[12] If this were not the case, then in equilibrium, direct writers would sell full insurance contracts to high risk consumers only and all low risk consumers would purchase insurance from "low risk" agency writers. Separation in the marketplace is achieved through choice of insurer.

agency writers, but low risks prefer the amount of insurance offered in the agency writer contracts to the amount of insurance in the direct writers' contract. In order to attract consumers, the direct writer has to be more competitive thus reducing profits earned on these contracts.

The difference in the location of the marginal low risk and high risk consumers is given by $\ell_m^\ell - \ell_m^h = f(I_m)$. Therefore, the captive high risk monopoly market is larger than the low risk monopoly market. Because of the screening menu of contracts, the direct writers are able to attract more high risk than low risk consumers. This reduction in the utility earned by the low risk consumers from the purchase of direct writers' products translates into an increased demand for coverage from agency writers.

From Lemma 2 - 1, the monopoly price under full information is ${}_{2.2}p_m \equiv \rho d + \frac{1}{2}(e_a + e_d)$.

In the asymmetric framework, the low risk consumer pays

$$p_m^\ell = \frac{1}{2}\left(\rho^\ell d + \rho^\ell I_m\right) + \frac{1}{2}(e_a + e_d) - \frac{1}{2\alpha}\log\left[1 - \rho^\ell + \rho^\ell e^{\alpha(d - I_m)}\right],$$

which is less than ${}_{2.2}p_m$. The first difference in the prices arises because of the partial insurance offered. The monopolist, under asymmetric information, bases the price charged on the average of the loss incurred and the indemnity offered. The second difference, $-\frac{1}{2\alpha}\log\left[1 - \rho^\ell + \rho^\ell e^{\alpha(d - I_m)}\right]$, arises because direct writers must compete with agency writers who offer a full insurance contract. This term is the monetary compensation for the loss in utility accruing to the low risk consumers who purchase the partial insurance contract. The effect of the screening contracts is to make less insurance available to the low risk consumers, but this contract is offered at a greater savings than if the direct writers had full information.

This loss in revenue is not recouped from the high risk consumers. Let ${}_{2.3}p_m^{\cdot}$ be the average premium collected by the direct writer. Then

$$\begin{aligned} {}_{2.3}p_m &= \lambda p_m^h + (1-\lambda)p_m^\ell \\ &= \lambda\rho^h d + \frac{1-\lambda}{2}\left(\rho^\ell d + \rho^\ell I_m\right) + \frac{1}{2}(e_a + e_d) - \frac{1-\lambda}{2\alpha}\log\left[1 - \rho^\ell + \rho^\ell e^{\alpha(d-I_m)}\right]. \end{aligned}$$

The last term is strictly negative. Even accounting for the fact that one contract is for less than full insurance, the average premium charged in the asymmetric model is less than the monopoly premium collected in the full insurance model.

From equations (2 - 3) and (2 - 5), the marginal low risk consumer is closer to the direct writer in the asymmetric information model. As already noted, because of the separating menu of contracts, fewer low risks prefer to purchase insurance from the direct writer. The agency writers' full insurance contract is more attractive despite the higher expense loading. There is no difference between equations (2 - 3) and (2 - 6) since both direct and agency writers offer full insurance contracts priced for the high risk consumer.

The final comparison is between the profits earned in the asymmetric and the symmetric information model. From Lemma 2 - 1, monopolistic direct writers earn a profit of ${}_{2.2}\Pi(p_m) = \frac{L}{2t}(e_a - e_d)^2 - F$. In the asymmetric information model, profits are given by

$_{2.3}\Pi\left(p_m^{\ell}, p_m^{h}\right)=\frac{L}{2t}\left[\lambda(e_a-e_d)^2+(1-\lambda)(e_a-e_d+f(I_m))^2\right]-F$. Since $f(I_m)<0$, profits earned in the asymmetric model are less than the profits earned in the symmetric model.

Proposition 2 - 2: *If a direct writer competes with other direct writers in a Bertrand manner it offers consumers the following menu of contracts. The first contract, designed for the high risk consumer, is a full insurance contract priced at* $p_c^h=\rho^h d+e_d+\frac{t}{n_c}$. *The second contract, designed for the low risk consumer, is a partial insurance contract for an amount of insurance* I_c *that satisfies* $\rho^h d=\rho^{\ell} I_c+\frac{1}{\alpha}\log\left[1-\rho^h+\rho^h e^{\alpha(d-I_c)}\right]$. *This contract is sold at a price of* $p_c^{\ell}=\rho^{\ell} I_c+e_d+\frac{t}{n_c}$. *Direct writers earn profits of* $\Pi\left(p_c^{\ell}, p_c^{h}\right)=\frac{tL}{n_c^2}-F$. *Under free entry, the number of direct writers that would exist in equilibrium is* $n_c=\sqrt{tL/F}$, *and the length of each direct writer's market is* $2\ell_c=\frac{1}{n_c}$.

Proof: The proof is similar to the proof of Lemma 2 - 2, where direct writers compete with neighbouring direct writers. Let $\left(p_c^{\ell}, I_c\right)$ and $\left(p_c^{h}, d\right)$ be the two contracts offered by the direct writer, and let $\left(p_{c+}^{\ell}, I_{c+}\right)$ and $\left(p_{c+}^{h}, d\right)$ and $\left(p_{c-}^{\ell}, I_{c-}\right)$ and $\left(p_{c-}^{h}, d\right)$ be the contracts offered by its closest neighbours. The direct writer chooses the contracts to maximise its profits subject to the contract choices of its neighbours. The marginal low risk consumer who is indifferent between the direct writer and its nearest neighbour is located at

$$\ell_c^\ell = \frac{1}{2t}\left(p_{c+}^\ell - p_c^\ell\right) + \frac{1}{2n_c} + \frac{1}{\alpha}\log\left[\frac{1-\rho^\ell + \rho^\ell e^{\alpha(d-l_{c+})}}{1-\rho^\ell + \rho^\ell e^{\alpha(d-l_c)}}\right] \quad .$$

(2 - 10)

In the same manner, the location of the marginal high risk consumer is

$$\ell_c^h = \frac{1}{2t}\left(p_{c+}^h - p_c^h\right) + \frac{1}{2n_c} \quad .$$

(2 - 11)

The profit function of the direct writer is given by

$$\begin{aligned}\Pi\left(p_c^h, p_c^\ell\right) &= L\lambda\left\{\frac{1}{2t}\left(p_{c+}^h + p_{c-}^h - 2p_c^h\right) + \frac{1}{n_c}\right\}\left(p_c^h - \rho^h d - e_d\right) \\ &+ L(1-\lambda)\left\{\frac{1}{2t}\left(p_{c+}^\ell + p_{c-}^\ell - 2p_c^\ell\right) + \frac{1}{n_c} + \frac{1}{2\alpha t}\log\left[\frac{2\left(1-\rho^\ell\right) + \rho^\ell e^{\alpha(d-l_{c+})} + \rho^\ell e^{\alpha(d-l_{c-})}}{2\left(1-\rho^\ell\right) + 2\rho^\ell e^{\alpha(d-l_c)}}\right]\right\} * \\ &\left(p_c^\ell - \rho^\ell l_c - e_d\right) \quad .\end{aligned}$$

(2 - 12)

Maximising profits with respect to the two contract prices and invoking the symmetry of direct writers (they all offer identical contracts) yields insurance prices of

$$p_c^\ell = \rho^\ell l_c + e_d + \frac{t}{n_c} \quad ,$$

(2 - 13)

and

$$p_c^h = \rho^h d + e_d + \frac{t}{n_c} \; .$$

(2 - 14)

The partial amount of insurance, I_c, is chosen so that the high risk consumer has no incentive to misrepresent her type, that is $V^h(p_c^h) \geq V^h(p_c^\ell)$. Substituting for prices p_c^ℓ and p_c^h from (2 - 13) and (2 - 14) respectively into the equality of utilities yields an amount of insurance, I_c, which satisfies $\rho^h d = \rho^\ell I_c + \frac{1}{\alpha}\log\left[1 - \rho^h + \rho^h e^{\alpha(d - I_c)}\right]$.

Symmetry arguments are used to solve for the two market lengths. Specifically (2 - 10) reduces to $2\ell_c^\ell = \frac{1}{n_c}$ and (2 - 11) reduces to $2\ell_c^h = \frac{1}{n_c}$. Direct writer profits can be calculated by substituting prices into (2 - 12) and invoking symmetry arguments. These profits are set equal to zero to solve for the number of direct writers under free entry. ■

In a model in which only direct writers exist, the demand for a direct writer's product, the numbers of direct writers, the profits earned by each direct writer and the maximum number of direct writers in equilibrium are not affected by the asymmetric information. The results in Proposition 2 - 2 are comparable to the results stated in Lemma 2 - 2.

From Lemma 2 - 2, the price charged by direct writers is ${}_{2.2}p_c \equiv \rho d + e_d + \frac{t}{n_c}$. Define the average price charged in the asymmetric model to be

$$
\begin{aligned}
{}_{2.3}p_c &= \lambda p_c^h + (1-\lambda)p_c^\ell \\
&= \left(\lambda \rho^h d + (1-\lambda)\rho^\ell I_c\right) + e_d + \frac{t}{n_c}
\end{aligned}
$$

The term in the brackets represents the average indemnity paid and is comparable to

ρd in the symmetric information model. Therefore this average premium corresponds directly to the premium ${}_{2.2}p_c$ charged in the symmetric information model.

Unlike the monopoly contracts in Proposition 2 - 1, in these contracts the location of the marginal low risk and high risk consumer is the same. The difference arises in the monopoly model because of the difference in the amounts of insurance offered by the direct and agency writers. Under Bertrand competition, all direct writers offer the same separating menu of contracts and so the location of the marginal low risk and high risk consumer is the same. Only if t, the transaction cost, differed between consumer types would the locations of the marginal consumers be different.

The amount of indemnity that is offered in the partial insurance contract under Bertrand competition is the same as the amount of indemnity that is offered in the standard Rothschild and Stiglitz (1976) partial insurance contract in a model in which homogeneous insurers earn zero profits. A numeric comparison of the amount of insurance offered in both the monopoly and competitive contracts is given in Table 2 - 2. For these examples, ρ^{ℓ} is set at 10% and λ is set at 0.5. If the high risk probability of loss is 12%, the competitive contract offers 2.3% more coverage than the monopoly contract. If the high risk probability of loss is 15%, the competitive contract offers 6.2% more coverage.

α	0.40	0.40	0.40	0.40	0.40	0.40	0.80	0.80	0.80	0.80	0.80	0.80
ρ^h	12%	12%	12%	15%	15%	15%	12%	12%	12%	15%	15%	15%
d	1000	2000	4000	1000	2000	4000	1000	2000	4000	1000	2000	4000
I_m	950	1906	3817	885	1774	3551	953	1908	887	887	1776	3553
I_c	972	1950	3905	939	1884	3773	975	1953	3908	942	1886	3775

Table 2 - 2 – Comparison of the Indemnity Amounts in the Monopoly and Competitive Contracts

2.3.3. Asymmetric Information Equilibrium

Now that the types of contracts offered by both direct and agency writers have been defined, it is possible to construct an equilibrium that supports both types of writers. Specifically, only conditions under which both agency and direct writers underwrite both high and low risk consumers are considered.[13] As in Section 2.3.2, for notational brevity,

[13] Within the framework of this chapter, equilibria can be constructed in which partial separation can be achieved through the choice of insurance company. Two equilibrium configurations which are not pursued are:

- "Low risk" agency writers underwrite all low risk consumers and high risk consumers purchase insurance from the insurer type that maximises their utilities.
- Direct writers underwrite all high risk consumers and low risk consumers purchase insurance from the insurer type that maximises their utilities.

define $f(I) \equiv \left(\rho^{\ell}(d - I) - \frac{1}{\alpha}\log\left[1 - \rho^{\ell} + \rho^{\ell} e^{\alpha(d-I)}\right] \right)$, where $I = I_c$ or $I = I_m$.

Theorem 2 - 2: *Given that the separating menu of contracts defined in Proposition 2 - 1 is supportable, the equilibrium is characterised by the following:*

- *A continuum of "high risk" agency writers who operate between the direct writers' captive high risk markets and who offer the contracts defined in Lemma 2 - 3 to the high risk consumers.*
- *A continuum of "low risk" agency writers operating between the captive low risk markets served by the direct writers and offering the contracts defined in Lemma 2 - 3 to low risk consumers.*
- *For fixed costs* $F \in \left(\frac{4L}{9t}(e_a - e_d + f(I_c))^2, \frac{L}{2t}\left[\lambda(e_a - e_d)^2 + (1-\lambda)(e_a - e_d + f(I_m))^2\right] \right]$ *and for* $e_a - e_d > -f(I_m)$, *direct writers act as local monopolists providing the contracts detailed in Proposition 2 - 1 to both low risk and high risk consumers. The number of direct writers in equilibrium is given by* $n_m \in \left(\text{int}\left[\frac{t}{2(e_a - e_d)}\right], \text{int}\left[\frac{t}{(e_a - e_d)}\right] \right]$.
- *For* $F \leq \frac{4L}{9t}(e_a - e_d + f(I_c))^2$ *, sufficient conditions so that both direct and agency writers co-exist and underwrite both high and low risks is that the number of direct writers satisfies* $n_m \in \left(\text{int}\left[\frac{3t}{4(e_a - e_d + f(I_c))}\right], \text{int}\left[\frac{t}{(e_a - e_d)}\right] \right]$ *and* $e_a - e_d > -4f(I_c)$.

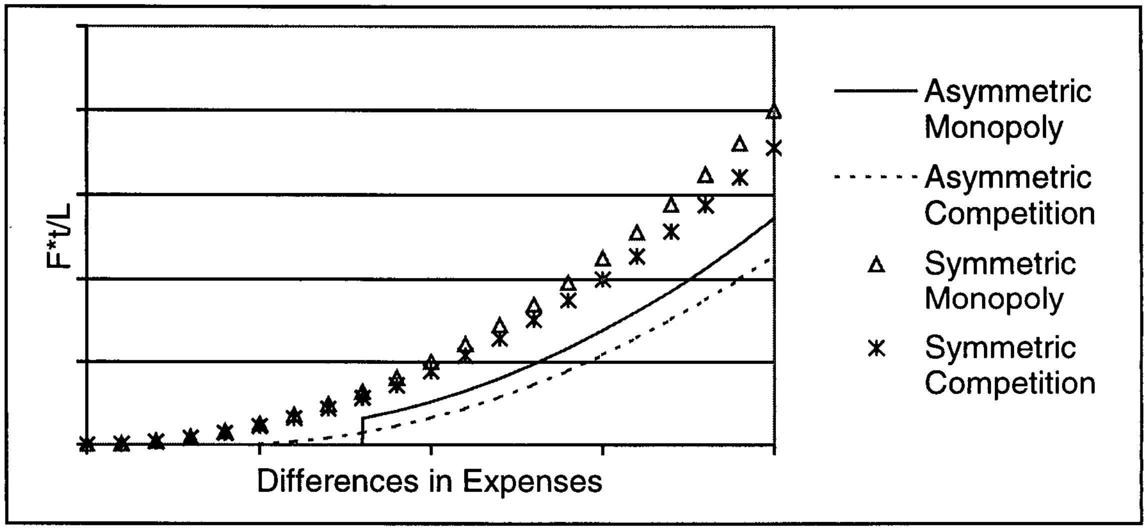

Figure 2 - 3 – Asymmetric Information Equilibrium Characterised by Fixed Costs

Proof: This proof follows from the proof for Theorem 2 - 1. Consider Figure 2 - 3, which is comparable to Figure 2 - 1 in the symmetric case.[14]

The bottom two lines give the break-even points for which direct writers can enter the market-place and earn non-negative profits in the asymmetric information framework. The solid black line, labelled Asymmetric Monopoly, represents the locus of points for which direct writers can enter and earn zero profits acting as monopolists. To define this equation note that from Proposition 2 - 1 direct writers earn profits of

[14] For comparison purposes, the results contained in Figure 2 - 1 are shown in Figure 2 - 3. The line dividing areas **A** and **B**, in Figure 2 - 1, the Symmetric Monopoly break-even locus, is given by the triangle markings in Figure 2 - 3 and the line dividing areas **B** and **C** Figure 2 - 1, the Symmetric Competition break-even locus, is given by the asterisk markings in Figure 2 - 3.

$$\Pi\left(p_m^{\ell}, p_m^{h}\right) = \frac{L}{2t}\left[\lambda\left(e_a - e_d\right)^2 + \left(1-\lambda\right)\left(e_a - e_d + f\left(l_m\right)\right)^2\right] - F .$$

The break-even profit condition is represented in Figure 2 - 3 by

$$\frac{tF}{L} = \frac{\lambda}{2}\left(e_a - e_d\right)^2 + \frac{\left(1-\lambda\right)}{2}\left(e_a - e_d + f\left(l_m\right)\right)^2 .$$

The kink in the line arises from this added restriction on the asymmetric model that $\left(e_a - e_d\right)$ must exceed $\left|f\left(l_m\right)\right|$. The difference in expenses must be high enough so that each policy earns non-negative profits. For all combinations of $\frac{tF}{L}$ and $e_a - e_d$ laying above and to the left of this line, entry does not occur since expected profits are negative in this area. Acting as local monopolist, direct writers can earn non-negative profits in the region below and to the right of this locus of points.

To ensure that direct writers sell policies to both high and low risk consumers, the maximum number of writers that can exist in equilibrium, $\overline{n}_m$, satisfies $2\ell_m^h\left(\overline{n}_m + 1\right) > 1 > 2\ell_m^h\overline{n}_m$.[15] Substituting for ℓ_m^h from Proposition 2 - 1, yields $\overline{n}_m = \text{int}\left[\frac{t}{e_a - e_d}\right]$.

[15] If the low risk market size is used in place of the high risk market size, a situation arises in which areas on the circle not served by direct writers would exist between captive low risk markets, but not high risk markets. This would produce the second equilibrium configuration discussed in footnote 13.

For $F \in \left(\frac{4L}{9t}(e_a - e_d + f(l_c))^2, \frac{L}{2t}\left[\lambda(e_a - e_d)^2 + (1-\lambda)(e_a - e_d + f(l_m))^2\right]\right]$, direct writers will enter only if they are guaranteed not to compete with other direct writers. Under the assumption of prohibitive relocation costs, the minimum number of direct writers that can exist in equilibrium satisfies $n_m\left[4\ell_m^h\right] \geq 1$. Substituting again for ℓ_m^h yields $n_m \geq \frac{t}{2(e_a - e_d)}$. Therefore, if direct writers can only operate profitably as monopolists, in equilibrium the number of direct writers is $n_m \in \left(\operatorname{int}\left[\frac{t}{2(e_a - e_d)}\right], \operatorname{int}\left[\frac{t}{(e_a - e_d)}\right]\right]$.

As in the proof for Theorem 2 - 1, the asymmetric competition break-even locus of points, as given by the dashed line in Figure 2 - 3, is calculated by examining the profits accruing to the direct writers competing in a Bertrand fashion. The size of the high risk and low risk market is the same for each direct writer. To ensure that Bertrand competition holds, the low marginal consumer must prefer to purchase insurance from the direct writer.[16] The marginal low risk consumer buys insurance from a direct writer instead of an agency writer if

$$\begin{gathered} V^\ell\left(p_m^\ell\right) \geq V^\ell\left(p_a^\ell\right) \\ -e^{-\alpha\left(W - p_c^\ell - t/2n_c\right)}\left[1 - \rho^\ell + \rho^\ell e^{\alpha(d - l_c)}\right] \geq -e^{-\alpha\left(W - p_a^\ell\right)}. \end{gathered}$$

Substituting for the prices p_a^ℓ from Lemma 2 - 3 and p_a^ℓ from Proposition 2 - 1 yields

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[16] Otherwise, it is possible to construct the first equilibrium configuration that is discussed in footnote 13.

$n_c \geq \dfrac{3t}{2(e_a - e_d + f(l_c))}$. Substituting for the equilibrium number of direct writers from Proposition 2 - 2 and rearranging terms gives

$$\frac{tF}{L} \leq \frac{4}{9}(e_a - e_d + f(l_c))^2 \ . \tag{2 - 15}$$

Thus for fixed capital costs which satisfy this inequality and for differences in expenses that exceed $|f(l_c)|$, insurers find it profitable to enter even if they must compete with neighbouring direct writers. The equality of (2 - 15) defines the line labelled Asymmetric Competition in Figure 2 - 3.

To ensure the existence of agency writers in this situation, a restriction on the number of direct writers is required. For $F \leq \dfrac{4L}{9t}(e_a - e_d + f(l_c))^2$, as in Theorem 2 - 1, the equilibrium characterised is one in which direct writers act as local monopolists and there is not sufficient space between any two adjacent direct writers so that another firm could enter and compete profitably in a Bertrand manner with its two neighbours. For a direct writer not to be able to profitably enter between the two existing direct writers, it must be the case that $n_m \lfloor 4\ell_c^h \rfloor \geq 1$. Substituting for ℓ_c^h from Proposition 2 - 2 yields $n_m \geq \dfrac{3t}{4(e_a - e_d + f(l_c))}$. However, it is possible that the size of the market served by a Bertrand competitor could be significantly less than the size of the monopoly market. An additional constraint is required so that a monopoly situation can be supported: it must be the case that $2\ell_m^h < 4\ell_c^h$. Otherwise, two direct writers could locate within an existing direct writer's market and earn non-negative profits under Bertrand competition. Substitution for ℓ_m^h from Proposition 2 - 1 and ℓ_c^h from Proposition 2 - 2 gives $e_a - e_d > -4f(I_c)$.

Therefore, in the case where direct writers could earn non-negative profits if they competed with neighbouring direct writers, a monopoly situation can be supported if $e_a - e_d > -4f(I_c)$ and if the number of direct writers in equilibrium is such that

$$n_m \in \left(\operatorname{int}\left[\frac{3t}{4(e_a - e_d + f(I_c))} \right], \operatorname{int}\left[\frac{t}{(e_a - e_d)} \right] \right].$$ ■

The effect of the screening contracts on direct writer profits is dramatically displayed in Figure 2 - 3. This impact on profits affects the structure of the industry. Under the asymmetric information assumption, if direct writers face fixed costs in the range

$$\left(\frac{L}{2t}\left[\lambda (e_a - e_d)^2 + (1-\lambda)(e_a - e_d + f(I_m))^2 \right], \frac{L}{2t}(e_a - e_d)^2 \right],$$

they cannot enter the market and earn non-negative profits, whereas entry would occur under the assumption of symmetric information. The inability of direct writers to effectively screen consumers increases the possibility of a market place in which only agency writers exist. The better screening mechanisms developed by agency writers do not increase their profits, but do increase both the market-share of the agency writers and the likelihood that they are the only type of insurer operating.

The number of direct writers that can be supported in equilibrium if only a monopoly situation is sustainable is the same in both the asymmetric and symmetric case since the size of the high risk market is the same in both cases. In the asymmetric information case, there will be more "low risk" agency writers writing low risk consumers than if direct writers had full information. The effect of the screening contract offered by the direct writers is to increase the market-share of the "low risk" agency writers.

The overall impact of the direct writer's inability to differentiate between consumer types is that, in equilibrium, direct writers sell fewer low risk insurance contracts at a lower price and subsequently earn less profits than if they had full information about risk types.

As long as it is profitable for direct writers to enter the market, high risk consumers are unaffected by the direct writer's screening contracts. Some low risk consumers are made better off – they purchase less insurance from the direct writers than if the direct writers had full information, but they receive this coverage at greater savings.

3. Dynamically Consistent Contracts Contingent on Previous Contract Choice

Typically, policyholders do not purchase property/casualty insurance only once in their lifetimes, but make annual decisions concerning insurance purchases. In most personal and commercial lines of insurance, contracts are renewed annually and relationships between insurers and policyholders often include significant past history. To reflect this reality, one period insurance models have been expanded into a multiple period framework.

Papers in the literature (Cooper and Hayes (1987), Dionne and Doherty (1994) and Watt and Vazquez (1996)) have examined the use of single period contracts that encourage voluntary separation in a multiple period framework but these papers do not construct dynamically consistent multiple period contracts. Instead, they make use of a sequence of traditional one period separating menus of contracts and either impose a constraint which restricts the consumer's ability to re-negotiate a contract once information has been revealed or do not discuss the fact that these one period contracts are not optimal in a multiple period setting.

A sequence of single period traditional separating menus of contracts cannot be supported in a multiple period equilibrium for two reasons. First of all, the contracts are not dynamically consistent. Once consumer type has been revealed, low risk consumers want future contracts to be conditioned on this revelation. Secondly, the single period contracts violate multiple period incentive compatibility constraints. In the one period model, the amount of indemnity in the partial insurance contract is chosen so that high risk consumers receive no gains in that period from misrepresenting their risk type. In the multiple period framework, the amount of indemnity in the partial insurance contract must be such that the utility of future insurance coverage accruing to the high risk consumer is the same whether or not she correctly reveals her risk type.

Rothschild and Stiglitz' (1976) one period model of the insurance industry predicts that equilibrium exists in which consumers reveal their risk propensities by purchasing varying amounts of insurance. A dynamically consistent multiple period extension of Rothschild and Stiglitz' (1976) model can be constructed in which a consumer's future contract options are contingent on her past contract choice. As in the one period model, a consumer reveals her risk type through the amount of coverage bought in the period when the separating contract is purchased. Since the contract purchased reveals the consumer type, future policies must be conditioned on this information. This conditioning ensures that contracts are dynamically consistent.

The purpose of this chapter is to correct for dynamic inconsistencies in the current literature. In this chapter, a dynamically consistent multiple period separating menu of contracts is constructed. This menu of contracts is designed such that insurers earn zero profits each period and no consumer has the incentive to misrepresent her type. In this multiple period framework, until a consumer reveals her type, she may purchase either one of the contracts in the separating menu of contracts or a single period pooling contract satisfying Wilson's (1977) anticipatory equilibrium. After a consumer reveals her risk type, she receives full insurance priced for her risk type for the remaining periods.

The use of experience rating is not examined in this chapter. If contract choice fully reveals consumer type, then experience rating will not be observed in equilibrium since its only function would be to shift risk from risk neutral insurers to risk averse consumers. However, if contract choice is not fully revealing, for example if there exists a continuum of consumer types and a finite number of contract choices, then experience rating may be desired by both parties. As noted by Watt and Vazquez (1996), experience rating is also desirable when consumers possess less than perfect knowledge about their own risk types.

The separation decision of low risk consumers in a multiple period world in which both the dynamically consistent separating menu of contracts and pooling contracts are offered is examined. The decision to separate is a function of the number of periods remaining in the model, the loss probabilities of the different consumer types, the mix of consumer types in the economy and the size of potential loss. The decision to separate is <u>not</u> a function of the total number of periods in the model.

Numerical examples are provided to assist understanding of the theoretical results. It is shown that over a reasonable range of parameter values, there is no equilibrium amount of indemnity that can be offered by insurers that will prevent high risk consumers from misrepresenting their loss type several periods before the last period. Therefore, in many situations, perfectly competitive insurers offer only pooling contracts. Even when both pooling and separating contracts are feasible, utility-maximising low risk consumers may never wish to reveal their type.

This set of dynamically consistent contracts is compared to a series of single period separating menus of contracts. Three key results are illustrated. As has been discussed in the multiple period contracting literature, the inability of low risk consumers to commit not to re-negotiate contracts is expensive. If low risk consumers could credibly promise not to demand that future contracts be conditioned on risk type once type has been revealed, they earn higher utility.

It is shown that the conditions under which consumers would prefer to pool for the entire lifetime of the contract in the world with dynamically consistent contracts are the same as in the single period model. In a world in which only pooling contracts are seen in equilibrium, it is impossible to discern whether or not consumers and firms are behaving myopically or in a dynamically consistent manner.

Differences in consumer behaviour between the model with the dynamically consistent contracts and the model with the sequence of one period Rothschild and Stiglitz (1976) contracts arise in those situations in which low risk consumers would prefer to separate in the one period Rothschild and Stiglitz (1976) model. In a world in which only the sequence of one period separating menu of contracts is offered, separation would occur every period over the lifetime of the sequence of contracts. In a world in which the dynamically consistent contracts are offered, low risk consumers would prefer to pool over a portion of the lifetime of the sequence of contracts. Thus the two equilibria differ radically.

Section 3.1 reviews of some of the multiple period contract structures in the literature. In Section 3.2, Wilson's (1977) pooling contracts, Rothschild and Stiglitz' (1976) single period contracts and dynamically consistent multiple period separating menu contracts are constructed and analysed. In Section 3.3, implications of the results derived in Section 3.2 are discussed. A summary of the primitives defined and the functions derived can be found in Appendix A, Table A-2.

3.1. The Structure of Separating Contracts in the Literature

Rothschild and Stiglitz' (1976) model provides great insight into informational problems surrounding a one period insurance model. Cooper and Hayes (1987), Dionne and Doherty (1994) and Watt and Vazquez (1996) have incorporated a separating menu of contracts into a multiple period model. All three papers use the contract structure given in the single period Rothschild and Stiglitz (1976) framework.

Cooper and Hayes (1987), in a two period framework, introduce two models in which both low and high risk consumers are offered one period separating menu of contracts in the first period. Those consumers, who have revealed themselves to be high risk in the first period, receive full insurance priced for their risk type in the second period. Those consumers, who have revealed themselves to be low risk in the first period, receive a partial insurance contract in the second period. The amount of indemnity and the price of the contract depend on the accident history.

Cooper and Hayes' (1987) contracts cannot be supported in equilibrium because they are not dynamically consistent. Once consumers have revealed their type in the first period, they would prefer full insurance contracts to the experience rated contracts. In the presence of full information in the second period, experience rating provides no benefit to consumers.

Two types of contracts are offered in the two period model presented by Dionne and Doherty (1994). One contract is a dynamically consistent contract that consists of a pooling contract in the first period and a separating menu of contracts conditioned on loss experience in the second period. This set of contracts is compared to a sequence of two single period separating menus of contracts. The authors note that this sequence of contracts is not dynamically consistent. The use of the traditional one period separating menu of contracts in the second period as the "outside" or alternative contract places restrictions on the prices and amounts of indemnity offered in the second period experience rated contracts.

Watt and Vazquez (1996), in a multiple period competitive setting, compare a sequence of one period separating menus of contracts to a series of experience rated pooling contracts in which insurer's beliefs about consumer types are revised using Bayesian updating. As in Dionne and Doherty (1994), the set of separating menus of contracts serve as a benchmark set of contracts. The authors state that this set of separating menus of contracts is the only feasible set of contracts under the assumption that insureds are not bound to multiple period contracts.

In these last two papers, the sequence of one period separating menus of contracts is sustainable in equilibrium because they are not dynamically consistent. After low risk consumers have revealed their types, they want future contracts conditioned on this information. Only if first period contracts were completely uninformative could single period Rothschild and Stiglitz (1976) contracts be offered in the second period. Thus it is unclear if the experience rated contracts constructed satisfy Wilson's (1977) anticipatory equilibrium. The proper "outside" or alternative contract that should be used in these papers is the dynamically consistent separating menu of contracts constructed in this paper.

3.2. Multiple Period Contract Design

In this section, underlying model assumptions are given. Rothschild and Stiglitz' (1976) one period separating menu of contracts and Wilson's (1977) pooling contract are defined. The condition under which low risk consumers would prefer the one period pooling contract to the one period separating menu of contracts is defined. Dynamically consistent contracts are then introduced and the optimal separation decision of a low risk consumer is defined.

3.2.1. Model Assumptions

The basic structure of the economy is as follows. Consumers live in a world where there are multiple time periods and two states of the world in each period. There is no moral hazard in this framework; consumers cannot affect their loss probabilities.

As in the original Rothschild and Stiglitz (1976) model, there exist risk-averse consumers who differ only by their risk propensity. Low risk consumers, who represent $1-\lambda$ of the population, will, in any one period, incur a loss of size d with probability ρ^{ℓ}. High risk consumers, who represent λ of the population, will, in any one period, incur a loss of size d with probability $\rho^{h} > \rho^{\ell}$. For simplicity, this probability of loss is uncorrelated across consumers and across time periods. Also, neither consumers nor firms discount future returns. Consumers are endowed with initial wealth W. This level of wealth is significantly large such that consumers face no wealth constraints over the entire timeframe.

All consumers possess constant absolute risk aversion. This assumption is extremely useful. First, it allows for the insurance purchasing decision of a consumer to be examined separately from her investment and consumption decisions. As long as changes in investment and consumption are uncorrelated with potential losses, then a consumer's insurance purchasing decision can be examined in isolation. Secondly, the use of the negative exponential utility function ensures the period by period time consistency of the model. Under constant absolute risk aversion, the optimal amount of insurance that is purchased at time $i+1$ is not affected by the consumer's loss experience during period i. Therefore, the optimal separation decision of the low risk consumer can be defined before insurance is purchased for the first time.

In each time period, an individual can insure against a loss by purchasing a one period insurance contract from a perfectly competitive insurer. It is assumed that consumers receive greater utility from purchasing insurance than from foregoing insurance. Insurers offer repeated contracts to consumers, which is consistent with reality. Dynamically consistent contracts are contingent only upon past contract choice and not past accident history. In each period, the moves are as shown in Figure 3 - 1.

The two probabilities of loss in each period, ρ^h and ρ^ℓ are known to the potential insurers, as is the size of the potential loss, d, and the proportion of high risk consumers in the population, λ. Each consumer knows if she is high or low risk, but this information cannot be observed by any of the insurance companies. In selling insurance each period, the perfectly competitive insurers incur an additive expense e to write a single policy. The key implication of the additive expense, as discussed in Chapter 2, is to make full insurance desirable in the presence of symmetric information.

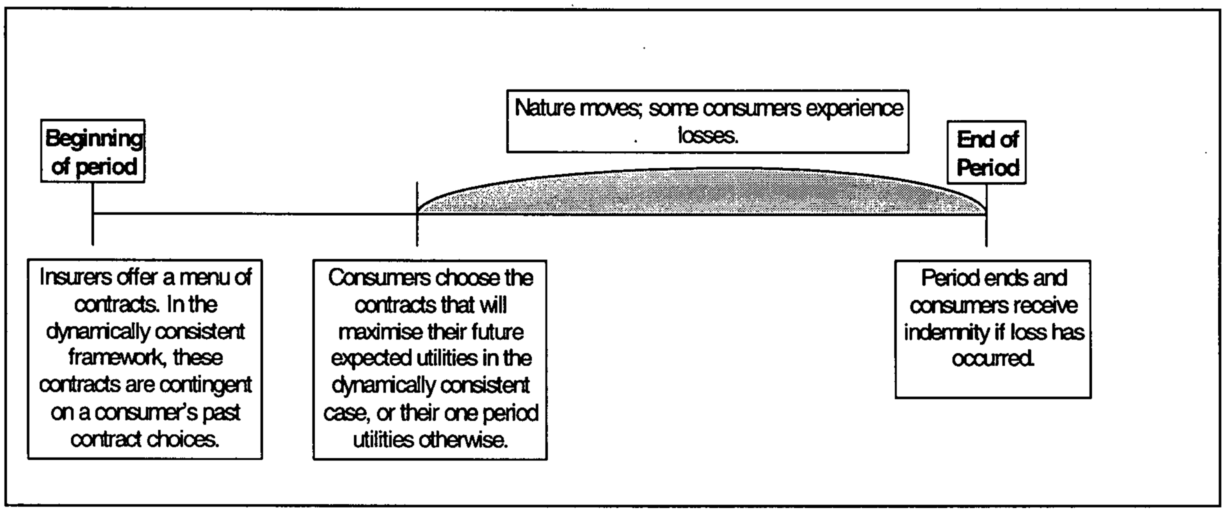

Figure 3 - 1 – Ordering of Movement within a Period

Since all consumers maximise the present value of future utility, the contracts offered in equilibrium do not have to earn insurers zero expected profits each period, but insurers must earn zero expected profit over the entire association between the consumer and the firm. However, to simplify the analysis, only contracts that earn zero expected profits each period are examined.

In the dynamically consistent framework, it is assumed that contracting insurers observe the past policy choices of their consumers while rival insurers only know if a consumer is new to the insurance market. Rival insurers are constrained to offering full insurance priced for the high risk type to all consumers who switch insurers since any other contract offering has the potential to earn negative expected profits for the rival insurer. Because of this, only high risk consumers would ever have the incentive to switch insurance companies.

3.2.2. Single Period Contract Structure

The following lemmas give Rothschild and Stiglitz' (1976) one period separating menu of contracts and a pooling contract that satisfies Wilson's (1977) anticipatory equilibrium. The condition under which low risk consumers prefer each period to pool instead of separate is also defined.

Lemma 3 - 1: *The optimal single period separating menu of contracts, following from Rothschild and Stiglitz (1976), is given by two contracts. The first is a full insurance contract priced at* $p^h = \rho^h d + e$. *The second contract is for a level of coverage* I *at a price* $p^\ell = \rho^\ell I + e$, *where* I *is the solution to* $\rho^h e^{-\alpha((1-\rho^\ell)I-d)} + (1-\rho^h)e^{\alpha\rho^\ell I} = e^{\alpha\rho^h d}$.

Proof: The separating menu of contracts consists of two policies: the first contract, which is designed for the high risk consumer, is a full insurance contract priced at the expected cost of insuring a high risk consumer for one period. The second policy is a partial insurance contract priced at the expected cost of insuring a low risk consumer. The level of indemnity is chosen so that there is no incentive for the high risk consumers to misrepresent their type. ■

Lemma 3 - 2: *A pooling contract that satisfies Wilson's (1977) anticipatory equilibrium criterion is given by a level of coverage* I^0 *and a price* $p^0 = \rho^0 I^0 + e$, *where* I^0 *is given* *by* $I^0 \equiv d - \frac{1}{\alpha}\log\left[\frac{\rho^0(1-\rho^\ell)}{\rho^\ell(1-\rho^0)}\right]$*, and where* ρ^0*, the pooled probability of loss, is defined by*

$\rho^0 \equiv \lambda\rho^h + (1-\lambda)\rho^\ell$.

Proof: From Wilson (1977), the pooling contract offered is one that maximises the utility of the low risk consumer subject to a single period zero expected profit constraint on insurers. ■

Rothschild and Stiglitz (1976) note that a pooling contract would dominate the separating contracts if the cost of separation is too high and if the cost of pooling is small. Mathematically, the inequality which identifies the condition under which a pooling equilibrium exists is defined in Lemma 3 - 3.

Lemma 3 - 3: *The separating menu of contracts characterised in Lemma 3 - 1 does not hold in equilibrium if* $(1-\rho^\ell)\left[e^{\alpha\rho^\ell I} - e^{\alpha\rho^0 I^0}\right] + \rho^\ell e^{\alpha d}\left[e^{-\alpha(1-\rho^\ell)I} - e^{-\alpha(1-\rho^0)I^0}\right] \geq 0$*, where* I^0 *and* ρ^0 *have been defined in Lemma 3 - 2.*

Proof: To show the necessity of the condition, suppose the pooling contract (ρ^0, I^0) breaks the equilibrium defined by the menu of contracts given in Lemma 3 - 1. For this to occur, the utilities earned by both the low risk consumer and the high risk consumer must be greater if they choose the pooling contract than if they choose a contract from the separating menu of contracts. Substituting for the prices of the separating contracts and $e^{\alpha\rho^h d}$ as defined in Lemma 3 - 1 into consumers' utility functions yields

$$\begin{aligned}(1-\rho^h)\left[e^{\alpha\rho^\ell l} - e^{\alpha\rho^0 l^0}\right] + \rho^h e^{\alpha d}\left[e^{-\alpha(1-\rho^\ell)l} - e^{-\alpha(1-\rho^0)l^0}\right] \geq 0 \\ (1-\rho^\ell)\left[e^{\alpha\rho^\ell l} - e^{\alpha\rho^0 l^0}\right] + \rho^\ell e^{\alpha d}\left[e^{-\alpha(1-\rho^\ell)l} - e^{-\alpha(1-\rho^0)l^0}\right] \geq 0\end{aligned} \quad .$$

(3 - 1)

Because $\rho^h > \rho^\ell$, the top inequality will be automatically satisfied if the bottom inequality holds. Since there exists a pooling contract which earns the insurer non-negative profits and is preferred to the separating contracts by both types of consumers, the separating menu of contracts does not exist in equilibrium. ■

From Lemma 3 - 3, it is clear that if a sequence of single period contracts is offered in the multiple period framework, there are only two possible sequences of contract structures that would exist. Either consumers prefer to pool every period or consumers prefer to separate every period. If consumers prefer to pool every period, then the expected utility earned by the low risk consumer is

$$\begin{aligned}\tilde{V}_0 &= -e^{-\alpha(W - n\rho^0 l^0 - ne)} \sum_{j=o}^{n} \binom{n}{j} (\rho^\ell)^j (1-\rho^\ell)^{n-j} e^{\alpha j(d - l^0)} \\ &= -e^{-\alpha(W - n\rho^0 l^0 - ne)} \left(1 - \rho^\ell + \rho^\ell e^{\alpha(d - l^0)}\right)^n,\end{aligned}$$

(3 - 2)

where the tilde on the utility function denotes that this is the utility earned when contracts are not dynamically consistent. The consumer is endowed with an initial wealth of W, and in each of the n periods pays $\rho^0 l^0 + e$ to purchase the pooling contracts. The summation represents the expected utility earned from the random losses over the n periods.

Alternatively, the low risk consumer could purchase a contract from the separating menu of contracts for each of the n periods. In this case her expected utility would be

$$\begin{aligned}\tilde{V}(n) &= -e^{-\alpha(W-n\rho^{\ell}I-ne)}\sum_{j=o}^{n}\binom{n}{j}(\rho^{\ell})^{j}(1-\rho^{\ell})^{n-j}e^{\alpha j(d-I)} \\ &= -e^{-\alpha(W-n\rho^{\ell}I-ne)}\left[1-\rho^{\ell}+\rho^{\ell}e^{\alpha(d-I)}\right]^{n}.\end{aligned}$$

The first term, $-e^{-\alpha(W-n\rho^{\ell}I-ne)}$, represents the utility from the original endowment less the cost of purchasing the partial insurance contract each period. The summation represents the expected utility earned from the random losses over the n periods. If (3 - 1) is satisfied then $\tilde{V}_0 > \tilde{V}(n)$.

3.2.3. Dynamically Consistent Contracts

In the dynamically consistent framework once a consumer has already revealed her risk type, *ex-ante* competition, whether realised or potential, restricts insurers to offering full insurance priced for each risk type. That is a low risk consumer would pay $\rho^{\ell}d+e$ and a high risk consumer would pay $\rho^{h}d+e$ to purchase full insurance. Because consumers maximise their utility over the entire time frame, they would never choose a stream of contracts that promised less than full insurance once separation has occurred.[16] This

[16] This, of course, depends on the assumption that consumers choose policies that maximise utility over all future periods. Kunreuther and Pauly (1985) note that there has not been much empirical verification of the degree of foresight that policyholders possess, although they conjecture that policyholders behave myopically. Non-myopic behaviour of consumers is a necessary condition in a framework in which contracts are contingent on past contract choice.

precommitment of the insurer to a series of contracts is standard in multiple period insurance models (see, for example, Cooper and Hayes (1987), Dionne (1983) and Dionne and Doherty (1994)).

In equilibrium, low risk consumers will maximise their expected utilities by separating $k=k^{*}$ periods before the last period, where k is defined as follows:

- $k=0$ implies that consumers never separate. All consumers purchase the single period pooling contract as defined in Lemma 3 - 2 for each of the n periods.
- $k=1$ implies that consumers separate in the last period. They never receive full insurance contracts, and they purchase the single period pooling contract as defined in Lemma 3 - 2 in each of the first $n-1$ periods.
- For any value of $k>1$, insureds purchase the single period pooling contract given in Lemma 3 - 2 for each of the first $n-k$ periods and single period full insurance contracts priced for their risk type for the last $k-1$ periods. In period $n+1-k$, low risk consumers purchase the partial insurance contract from the separating menu of contracts offered by the insurance company.

Figure 3 - 2 shows the stream of policies purchased if consumers separate k periods before the last period. Because high risk consumers are subsidised in the pooling contracts, they would never choose to separate. Therefore the low risk consumers make the separation decision. As such, only the expected utility earned by the low risk consumer is examined.

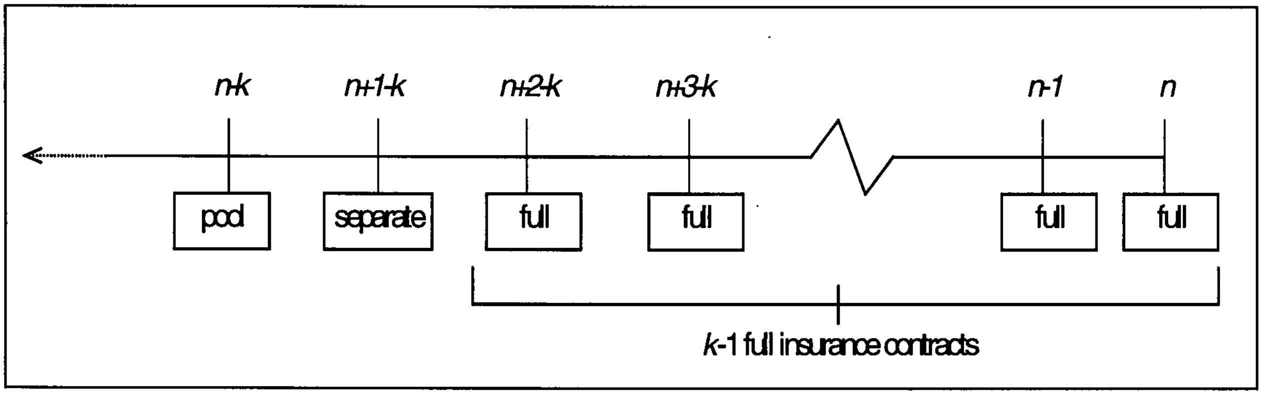

Figure 3 - 2 – Stream of Contracts if Separation Occurs in Period k

Figure 3 - 3 shows the possible paths for consumers in a three-period world. The number of paths grows exponentially as the number of periods increases. In an n period model, there are $3*2^n-2$ possible outcomes. Because consumers possess negative exponential utility functions, low risk consumers *a priori* select the period in which they wish to separate. Since insurance companies know the preferences of insureds, they too know *a priori* in which period separation will occur. Therefore, even if there are $3*2^n-2$ possible outcomes, in equilibrium, most of these paths will never be observed. If insureds separate k periods before the last period, 2^{n-k+1} outcomes are possible.

If consumers have not yet revealed their risk type, insurers offer one of two types of contracts. The first contract is the pooling contract given in Lemma 3 - 2. The second set is a menu of contracts; one policy is a full insurance contract priced at the expected cost of insuring a high risk consumer and the second is a partial insurance contract

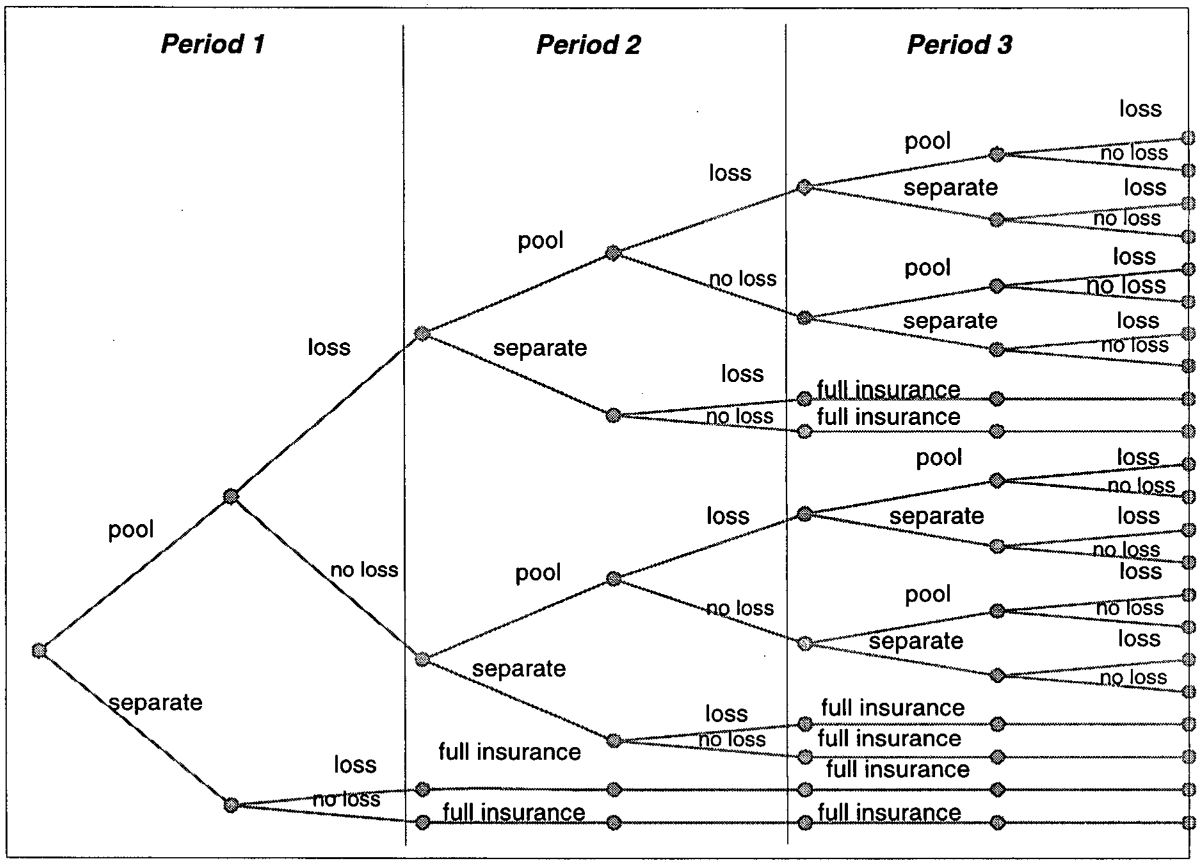

Figure 3 - 3 – Possible Outcomes in a 3 Period Model

priced at the expected cost of insuring a low risk consumer. The separating menu of contracts is defined in Proposition 3 - 1.

Proposition 3 - 1: *A separating menu of contracts for period* k *is given by two contracts. The first is a full insurance contract priced at* $p^h = \rho^h d + e$. *The second contract is for a level of coverage* l_k, *where* l_k *solves* $\rho^h e^{-\alpha\left(\left(1-\rho^\ell\right)l_k - d\right)} + \left(1-\rho^h\right)e^{\alpha\rho^\ell l_k} + e^{\alpha(k-1)\rho^\ell d} = e^{\alpha k \rho^h d}$ *at a price of* $p_k^\ell = \rho^\ell l_k + e$.

Proof: The separating menu of contracts consists of two policies: the first contract, designed for the high risk consumer, is a full insurance contract priced at the expected cost of insuring a high risk consumer for one period. The second policy, designed for the low risk consumer, is a partial insurance contract priced at the expected cost of insuring a low risk consumer.

The level of indemnity is chosen so that there is no incentive for a high risk consumer to misrepresent her type. If the high risk consumer truthfully reveals her type, she receives full insurance coverage for k periods. The incremental utility[17] earned over the last k periods is

$$-e^{\alpha k\left(\rho^h d+e\right)}. \tag{3 - 3}$$

Alternatively, she could dissemble as to her risk type. She would purchase the partial insurance contract designed for the low risk consumer in period $n+1-k$, and for the remaining $k-1$ periods receive full insurance priced at the cost of insuring a low risk consumer. In this case, her incremental expected utility is

$$-\rho^h e^{-\alpha\left(\left(1-\rho^\ell\right)I_k-d-e\right)}-\left(1-\rho^h\right)e^{\alpha\left(\rho^\ell I_k+e\right)}-e^{\alpha(k-1)\left(\rho^\ell d+e\right)}. \tag{3 - 4}$$

A high risk consumer will be indifferent between the two contracts in this menu when the

[17] This ignores the expected utility gained from the initial wealth less the $n-k$ periods of pooling.

expected utilities of the two contract choices are equal. Equating functions (3 - 3) and (3 - 4)) gives

$$\rho^h e^{-\alpha\left(\left(1-\rho^\ell\right)I_k-d\right)}+\left(1-\rho^h\right)e^{\alpha\rho^\ell I_k}+e^{\alpha(k-1)\rho^\ell d}=e^{\alpha k\rho^h d},$$

(3 - 5)

which can be solved for I_k to yield the equilibrium level of insurance. ■

The amount of partial insurance that is offered in the separating menu of contracts is a function of the period in which separation occurs, the size of the potential loss and the risk propensities of the two types of insureds. The amount of indemnity is not a function of the proportion of high risk insureds in the economy.

The relationship between the size of the indemnity offered in the partial insurance contract and the period of separation is shown in Figure 3 - 4. The points plotted in Figure 3 - 4 display the optimal amount of indemnity over a range of risk aversion coefficients for differing values of ρ^h with $\rho^\ell = 10\%$. The size of loss is chosen to be 1000. As in Chapter 2, the range of α, the risk aversion coefficient, corresponds to the range suggested by Haubrich (1994).

In the four scenarios with $\rho^h = 15\%$, if the low risk consumers wish to separate more than six periods before the end, there is no positive amount of indemnity that can be offered by insurers that will prevent the high risk consumers from misrepresenting their risk types. In the remaining four scenarios with $\rho^h = 12.5\%$, there is no positive amount

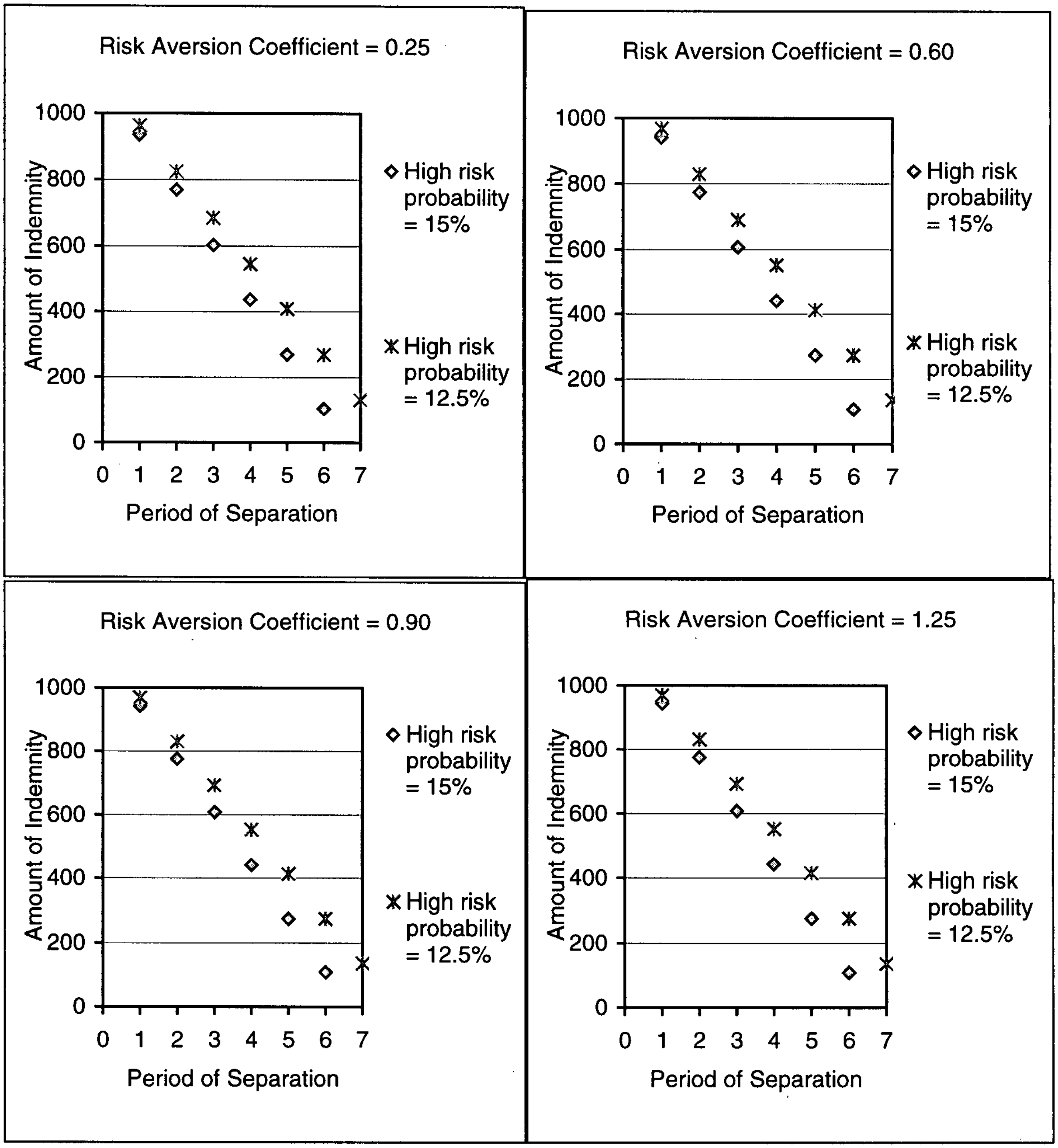

Figure 3 - 4 – Relationship between Size of Partial Indemnity and Period of Separation

of indemnity that can be offered more than seven periods from the end. The value of indemnity offered in the partial insurance contract, I_k, that satisfies

$\rho^h e^{-\alpha((1-\rho^\ell)I_k - d)} + (1-\rho^h)e^{\alpha\rho^\ell I_k} + e^{\alpha(k-1)\rho^\ell d} = e^{\alpha k\rho^h d}$, the incentive capability constraint, is strictly negative. The benefit to the high risk consumers of misrepresentation is so great that there exists no level of indemnity that will entice high risk consumers to correctly reveal their risk type. Exact indemnity amounts for these eight scenarios can be found in Appendix B.

The amount of partial insurance offered if separation occurs k periods from the end, increases as the probability of loss faced by the low risk consumers increases and decreases as the probability of loss faced by the high risk consumers increases. As the two types become more dissimilar, the benefit to the high risk consumer from misrepresentation increases. The partial indemnity offered in the period of separation decreases to compensate for this.

From equation (3 - 5) define the implicit function

$$G(\alpha, \rho^\ell, \rho^h, d, I_k, k) = \rho^h e^{-\alpha((1-\rho^\ell)I_k - d)} + (1-\rho^h)e^{\alpha\rho^\ell I_k} + e^{\alpha(k-1)\rho^\ell d} - e^{\alpha k\rho^h d} \qquad (3 - 6)$$

The relationship between I_k and ρ^ℓ can be derived by straightforward differentiation of (3 - 6). Specifically $\dfrac{\partial I_k}{\partial \rho^\ell} = \dfrac{I_k e^{\alpha k\rho^\ell I_k}\left(\rho^h e^{\alpha(d-I_k)} + 1 - \rho^h\right) + (k-1)d\ e^{\alpha(k-1)\rho^\ell d}}{e^{\alpha k\rho^\ell I_k}\left[\rho^h(1-\rho^\ell)e^{\alpha(d-I_k)} - \rho^\ell(1-\rho^h)\right]} > 0$. The relationship between I_k and ρ^h is easiest to show through numerical computation.

As shown in Figure 3 - 4, the amount of partial insurance offered is a decreasing function of k, the number of periods remaining in the world. The earlier low risk consumers separate, the lower the amount of insurance offered in the partial insurance contract. Since the benefit from misrepresentation to the high risks increases as the number of periods until the end increases, the partial indemnity offered in the contract designed for low risk consumers in the period of separation must decrease to discourage misrepresentation. Differentiation of (3 - 6) yields

$$\frac{\partial I_k}{\partial k} = \frac{-d\left[\rho^h e^{\alpha k \rho^h d} - \rho^\ell e^{\alpha(k-1)\rho^\ell d}\right]}{e^{\alpha k \rho^\ell I_k}\left[\rho^h\left(1-\rho^\ell\right)e^{\alpha(d-I_k)} - \rho^\ell\left(1-\rho^h\right)\right]} < 0. \tag{3 - 7}$$

The relationship between I_k and α is much more complex. For small values of d, I_k first decreases and then increases in α.[18] For larger and more realistic values of d, the relationship is monotonic, as is shown in Figure 3 - 5. The curves display the optimal amount of indemnity over a range of risk aversion coefficients for differing values of ρ^h and d. The low risk probability of loss is set at 10%. The period of separation is selected to be 4; the graphs depict the relationship between I_4 and α. The range of α and the high risk loss probabilities are the same as those used to construct Figure 3 - 4. As discussed in Chapter 2, the values of d utilised, \$2000 and \$11 000, are the average sizes of private passenger automobile property damage and bodily injury claims paid in the United States for 1996, respectively.

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[18] The smaller values of d are unrealistic in this model and so graphs of this relationship are not included.

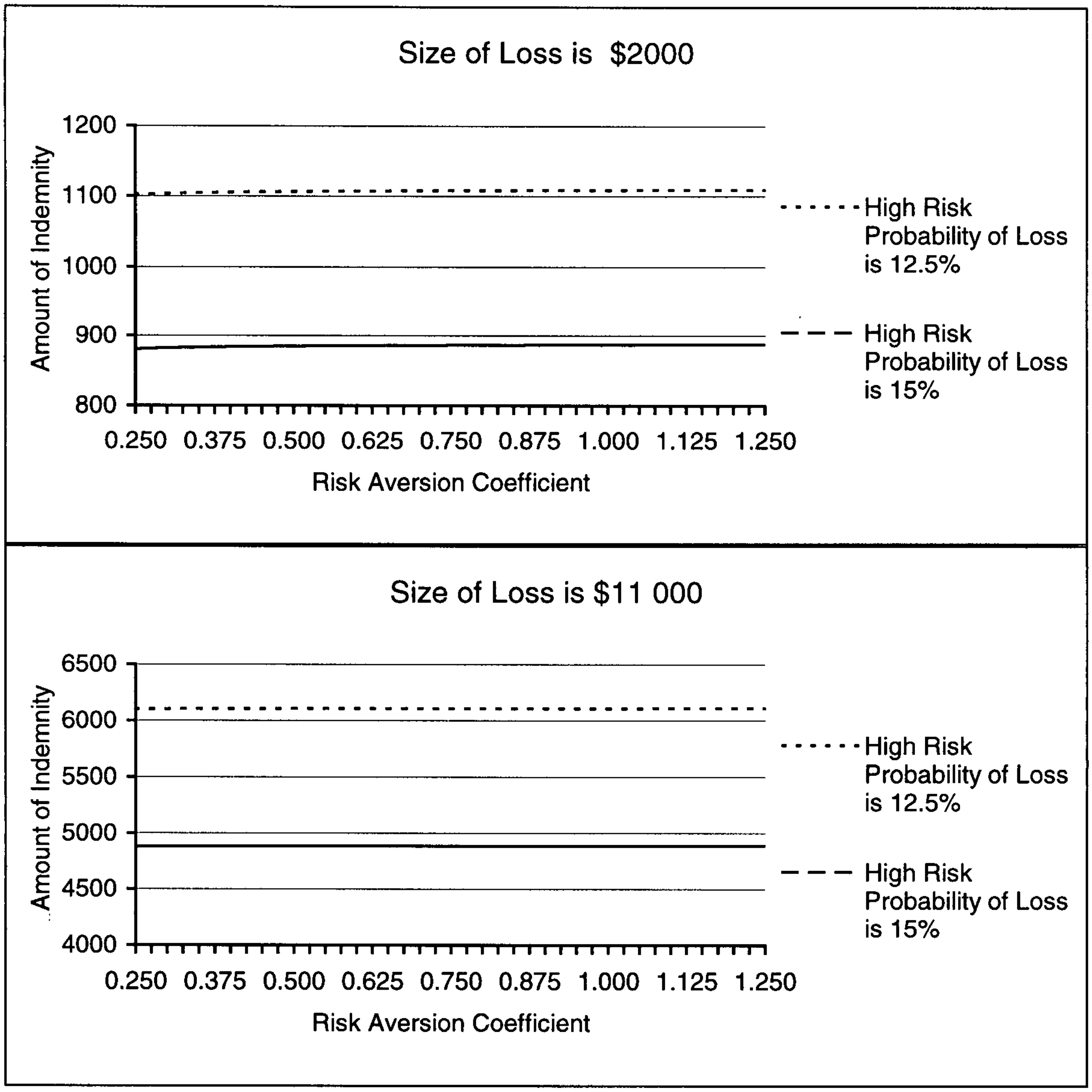

Figure 3 - 5 – Relationship between the Indemnity and the Risk Aversion Coefficient

The amount of partial insurance is an increasing function of the loss sizes. This can be shown numerically. As is observable from (3 - 5), the amount of partial insurance is independent of the number of periods for which the pooling contracts were purchased. The amount of partial insurance is also independent of the proportion of high risk consumers.

Now that the possible contracts that could be offered by perfectly competitive insurers have been defined, the expected utility accruing to the low risk consumer and her subsequent maximisation problem can be developed. The expected utility earned by a low risk consumer who pools for the entire n periods, as given in equation (3 - 2), is $V(0) \equiv V_0 = -e^{-\alpha\left(W - n\rho^0 l^0 - ne\right)}\left(1 - \rho^\ell + \rho^\ell e^{\alpha\left(d - l^0\right)}\right)^n$. The absence of a tilde on V_0 implies that this is the utility earned if policyholders behave non-myopically and contracts are dynamically consistent. Pooling contracts reveal no information about consumer type, so $V_0 \equiv \tilde{V}_0$.

Alternatively, the low risk consumer could decide to separate k periods, where $k \geq 1$, before the last period. In this case, the utility earned by the low risk consumer is

$$\begin{aligned} V(k) &= -e^{-\alpha(W-ne)} e^{\alpha(k-1)\rho^\ell d}\left[\rho^\ell e^{-\alpha\left(\left(1-\rho^\ell\right)l_k - d\right)} + \left(1-\rho^\ell\right) e^{\alpha\rho^\ell l_k}\right]\left\{e^{\alpha(n-k)\rho^0 l^0} \sum_{j=o}^{n-k}\binom{n-k}{j}\left(\rho^\ell\right)^j\left(1-\rho^\ell\right)^{n-k-j} e^{\alpha j\left(d-l^0\right)}\right\} \\ &= -e^{-\alpha(W-ne)} e^{\alpha(k-1)\rho^\ell d} e^{\alpha\rho^\ell l_k}\left[1-\rho^\ell+\rho^\ell e^{\alpha\left(d-l_k\right)}\right]\left\{e^{\alpha(n-k)\rho^0 l^0}\left[1-\rho^\ell+\rho^\ell e^{\alpha\left(d-l^0\right)}\right]^{n-k}\right\} \quad . \end{aligned} \tag{3 - 8}$$

In the last line of the function the first term, $e^{-\alpha(W-ne)}$, represents the utility from the original endowment less the expense loading that must be paid every period. The second term, $e^{\alpha(k-1)\rho^\ell d}$, is the contribution from the $k-1$ periods of full insurance. The next two terms, $e^{\alpha\rho^\ell l_k}\left[1-\rho^\ell+\rho^\ell e^{\alpha\left(d-l_k\right)}\right]$, express the addition to expected utility arising from the period in which separation occurs and the term in the curly brackets arises from the $n-k$ periods in which the consumer purchased pooling contracts.

The maximisation problem of the low risk consumer is given in Theorem 3 - 1.

Theorem 3 - 1: *To maximise expected utility, low risk consumers choose the optimal period of separation, k^*, where k^* solves $k^* = \underset{k}{\operatorname{argmax}}(V_0, V(k))$. An approximation to k^* is given by $\bar{k}^*$, where $\bar{k}^*$ is the solution to*

$$\left[Q(\bar{k}^*)\right]^{1-\rho^\ell}\left(1-\rho^h+\rho^h Q(\bar{k}^*)\right)=\rho^h e^{\alpha(\bar{k}^*\rho^h-\rho^\ell)d}-\rho^\ell e^{\alpha(\bar{k}^*-2)\rho^\ell d}.$$

$Q(\bar{k}^*)$ is given by

$$\frac{\left(a\left((\rho^\ell)^2(1-\rho^h)-\rho^h(1-\rho^\ell)^2\right)+\alpha(1-\rho^\ell)(1-2\rho^h)\rho^\ell b(\bar{k}^*)\right)}{2(1-\rho^\ell)\rho^\ell\rho^h\left(a-\alpha b(\bar{k}^*)\right)}$$
$$-\frac{\sqrt{\left\{a\left((\rho^\ell)^2(1-\rho^h)-\rho^h(1-\rho^\ell)^2\right)+\alpha(1-\rho^\ell)(1-2\rho^h)\rho^\ell b(\bar{k}^*)\right\}^2+4(1-\rho^\ell)^2(\rho^\ell)^2(1-\rho^h)\rho^h\left(a-\alpha b(\bar{k}^*)\right)^2}}{2(1-\rho^\ell)\rho^\ell\rho^h\left(a-\alpha b(\bar{k}^*)\right)}$$

and a and $b(\bar{k}^*)$ are defined as follows: $a \equiv \alpha\left(-\rho^\ell d+\rho^0 l^0\right)+\log\left[1-\rho^\ell+\rho^\ell e^{\alpha(d-l^0)}\right]$ and $b(\bar{k}^*)=\dfrac{d\left[\rho^h e^{\alpha\bar{k}^*\rho^h d}-\rho^\ell e^{\alpha(\bar{k}^*-1)\rho^\ell d}\right]}{e^{\alpha\bar{k}^*\rho^h d}-e^{\alpha(\bar{k}^*-1)\rho^\ell d}}$. To ascertain the true value of k^*, the expected utility calculated at the two integer values of k on either side of $\bar{k}^*$ must be compared.

Proof: Since k is an integer, function (3 - 8) is not differentiable in k. Define the continuous function $\bar{V}(k)=V(k)$, for all $k>0$, which is continuously differentiable in k and identically equal to $V(k)$ for integral values of k. Figure 3 - 6 displays, for selected parameter values, the relationship between the functions $V(k)$ and $\bar{V}(k)$.[19] In the

[19] These graphs actually display a monotonic transformation of the two functions.

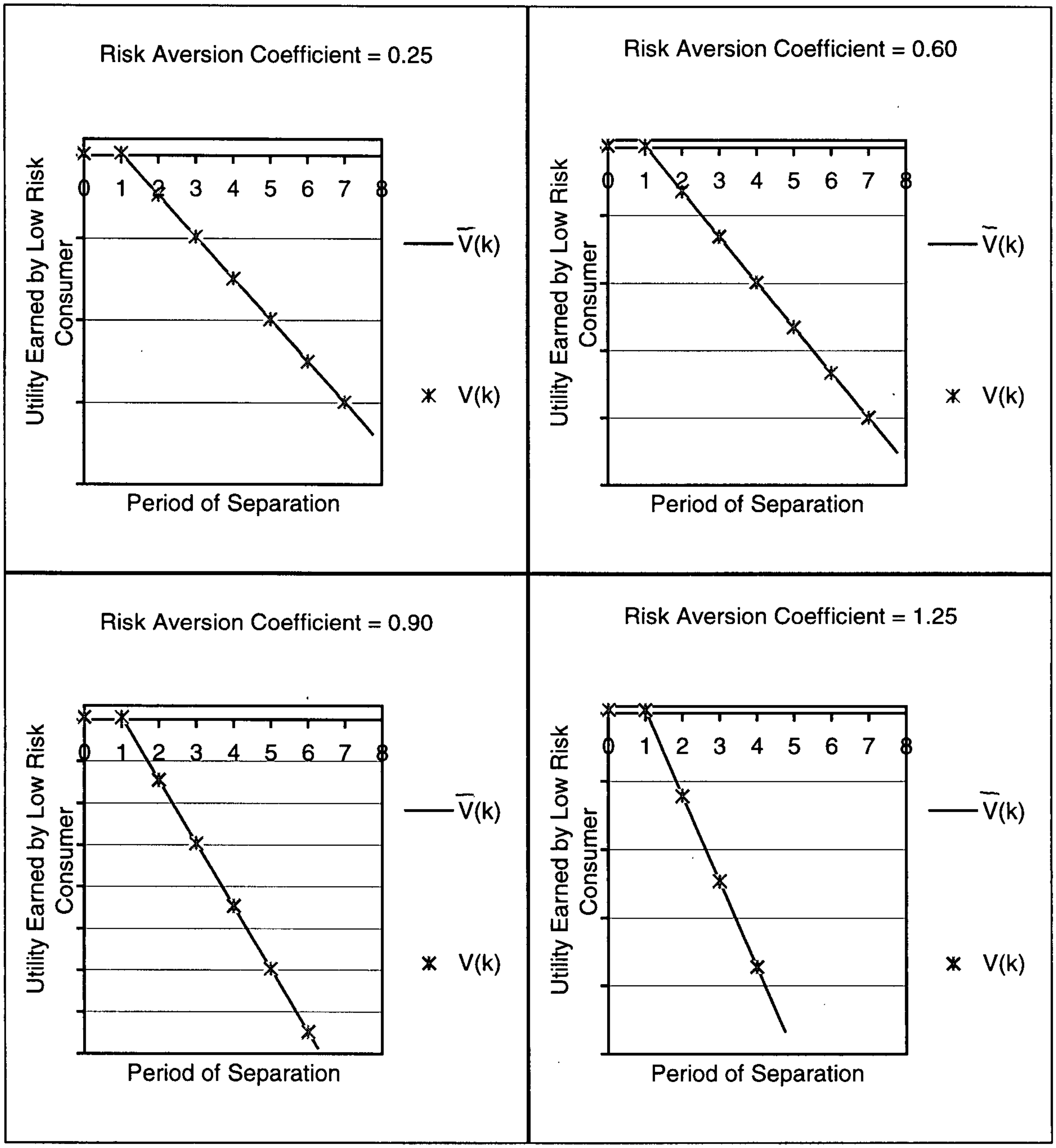

Figure 3 - 6 – Relationship Between $V(k)$ and $\overline{V}(k)$

graphs, $\rho^h = 12.5\%$, $\rho^\ell = 10\%$, $\lambda = 98\%$ and $d = 1000$. The function $\overline{V}(k)$ possesses a unique maximum (see Appendix B for details) and so the value of k which maximises $\overline{V}(k)$ will also maximise $V(k)$. If $\overline{k}^*$ is less than one, both $V(k=1)$ and V_0 must be examined to ascertain the true value of k^*.

Maximising $\overline{V}(k)$ with respect to k, collecting terms and setting the derivative equal to zero yields the equation

$$\begin{aligned}\frac{\partial \overline{V}(k)}{\partial k} &= \left(1-\rho^{\ell}\right)\left[\alpha\left(-\rho^{\ell} d+\rho^{0} I^{0}\right)+\log\left(1-\rho^{\ell}+\rho^{\ell} e^{\alpha\left(d-I^{0}\right)}\right)\right]-\alpha\rho^{\ell}\left(1-\rho^{\ell}\right)\left[1-e^{\alpha\left(d-I_{k}^{*}\right)}\right]\left.\frac{\partial I_{k}}{\partial k}\right|_{I_{k}=I_{k}^{*}, k=\overline{k}^{*}} \\ &+\rho^{\ell}\left[\alpha\left(-\rho^{\ell} d+\rho^{0} I^{0}\right)+\log\left(1-\rho^{\ell}+\rho^{\ell} e^{\alpha\left(d-I^{0}\right)}\right)\right] e^{\alpha\left(d-I_{k}^{*}\right)} \\ &\equiv 0 \quad .\end{aligned} \tag{3 - 9}$$

For notational convenience, define $a=\left[\alpha\left(-\rho^{\ell} d+\rho^{0} I^{0}\right)+\log\left(1-\rho^{\ell}+\rho^{\ell} e^{\alpha\left(d-I^{0}\right)}\right)\right]$, which is constant with respect to k. Then (3 - 9) can be rewritten as

$$\frac{\partial \overline{V}(k)}{\partial k}=a\left(1-\rho^{\ell}+\rho^{\ell} e^{\alpha\left(d-I_{k}^{*}\right)}\right)-\alpha\rho^{\ell}\left(1-\rho^{\ell}\right)\left[1-e^{\alpha\left(d-I_{k}^{*}\right)}\right]\left.\frac{\partial I_{k}}{\partial k}\right|_{I_{k}=I_{k}^{*}, k=\overline{k}^{*}}=0 \quad .$$

Using (3 - 7), and substituting for $e^{\alpha\rho^{\ell} I_{k}}$ from (3 - 5) yields

$$\left.\frac{\partial I_{k}}{\partial k}\right|_{I_{k}=I_{k}^{*}, k=\overline{k}^{*}}=b\left(\overline{k}^{*}\right)\frac{\left(1-\rho^{h}\right)-\rho^{h} e^{\alpha\left(d-I_{k}^{*}\right)}}{\rho^{\ell}\left(1-\rho^{h}\right)-\rho^{h}\left(1-\rho^{\ell}\right) e^{\alpha\left(d-I_{k}^{*}\right)}},$$

for $b\left(\overline{k}^{*}\right)=\dfrac{d\left[\rho^{h} e^{\alpha\overline{k}^{*}\rho^{h} d}-\rho^{\ell} e^{\alpha\left(\overline{k}^{*}-1\right)\rho^{\ell} d}\right]}{e^{\alpha\overline{k}^{*}\rho^{h} d}-e^{\alpha\left(\overline{k}^{*}-1\right)\rho^{\ell} d}}$. Substituting back into (3 - 9) yields

$$\begin{gathered}\left(1-\rho^{\ell}\right)\rho^{\ell}\left(1-\rho^{h}\right)\left(a-\alpha b\left(\overline{k}^{*}\right)\right)+e^{\alpha\left(d-I_{k}^{*}\right)}\left(a\left(\left(\rho^{\ell}\right)^{2}\left(1-\rho^{h}\right)-\rho^{h}\left(1-\rho^{\ell}\right)^{2}\right)+\alpha\left(1-\rho^{\ell}\right)\left(1-2\rho^{h}\right)\rho^{\ell} b\left(\overline{k}^{*}\right)\right) \\ -e^{2\alpha\left(d-I_{k}^{*}\right)}\left(1-\rho^{\ell}\right)\rho^{\ell}\rho^{h}\left(a-\alpha b\left(\overline{k}^{*}\right)\right)=0\end{gathered}$$

which is a quadratic equation in $e^{\alpha\left(d-I_{k}^{*}\right)}$. Using the quadratic formula yields

$$
\begin{aligned}
e^{\alpha(d-l_k^*)} &= \frac{\left(a\left((\rho^\ell)^2(1-\rho^h)-\rho^h(1-\rho^\ell)^2\right)+\alpha(1-\rho^\ell)(1-2\rho^h)\rho^\ell b(\bar{k}^*)\right)}{2(1-\rho^\ell)\rho^\ell\rho^h\left(a-\alpha b(\bar{k}^*)\right)} \\
&\quad + \frac{\sqrt{\left\{a\left((\rho^\ell)^2(1-\rho^h)-\rho^h(1-\rho^\ell)^2\right)+\alpha(1-\rho^\ell)(1-2\rho^h)\rho^\ell b(\bar{k}^*)\right\}^2+4(1-\rho^\ell)^2(\rho^\ell)^2(1-\rho^h)\rho^h\left(a-\alpha b(\bar{k}^*)\right)^2}}{2(1-\rho^\ell)\rho^\ell\rho^h\left(a-\alpha b(\bar{k}^*)\right)} \\
&\equiv Q(\bar{k}^*)
\end{aligned}
$$

which is an equation in both k^* and l_k^*. Substituting for $e^{\alpha(d-l_k^*)} = Q(\bar{k}^*)$ in (3 - 5) gives

$$
\left[Q(\bar{k}^*)\right]^{-\rho^\ell}\left(1-\rho^h+\rho^h Q(\bar{k}^*)\right)=\rho^h e^{\alpha(\bar{k}^*\rho^h-\rho^\ell)d}-\rho^\ell e^{\alpha(\bar{k}^*-2)\rho^\ell d}.
$$

(3 - 10)

This can be solved numerically for $\bar{k}^*$. Once $\bar{k}^*$ has been calculated, the monotonic properties of $\bar{V}(k)$ imply that the optimal value k^* is one of the two integer values closest to $\bar{k}^*$. ■

The relationship between the utility earned by the low risk consumer and the period of separation is shown in Figure 3 - 7.[20] As in previous figures, the low risk probability of loss is set at 10%, d is set at 1000 and the proportion of high risk consumers in the economy is 98%. The diamond markings denote the utility of the low risk consumer if she separates in that period. The asterisk denotes $\bar{k}^*$, the approximation to the optimal period of separation developed in Theorem 3 - 1, and triangle marking denotes the optimal period of separation, k^*. In the two top graphs, with $\alpha = 0.25$, low risk consumers maximise their expected utility by separating in the last period in the multiple period world. In the two

[20] The graphs actually display a monotonic transformation of the negative exponential utility function.

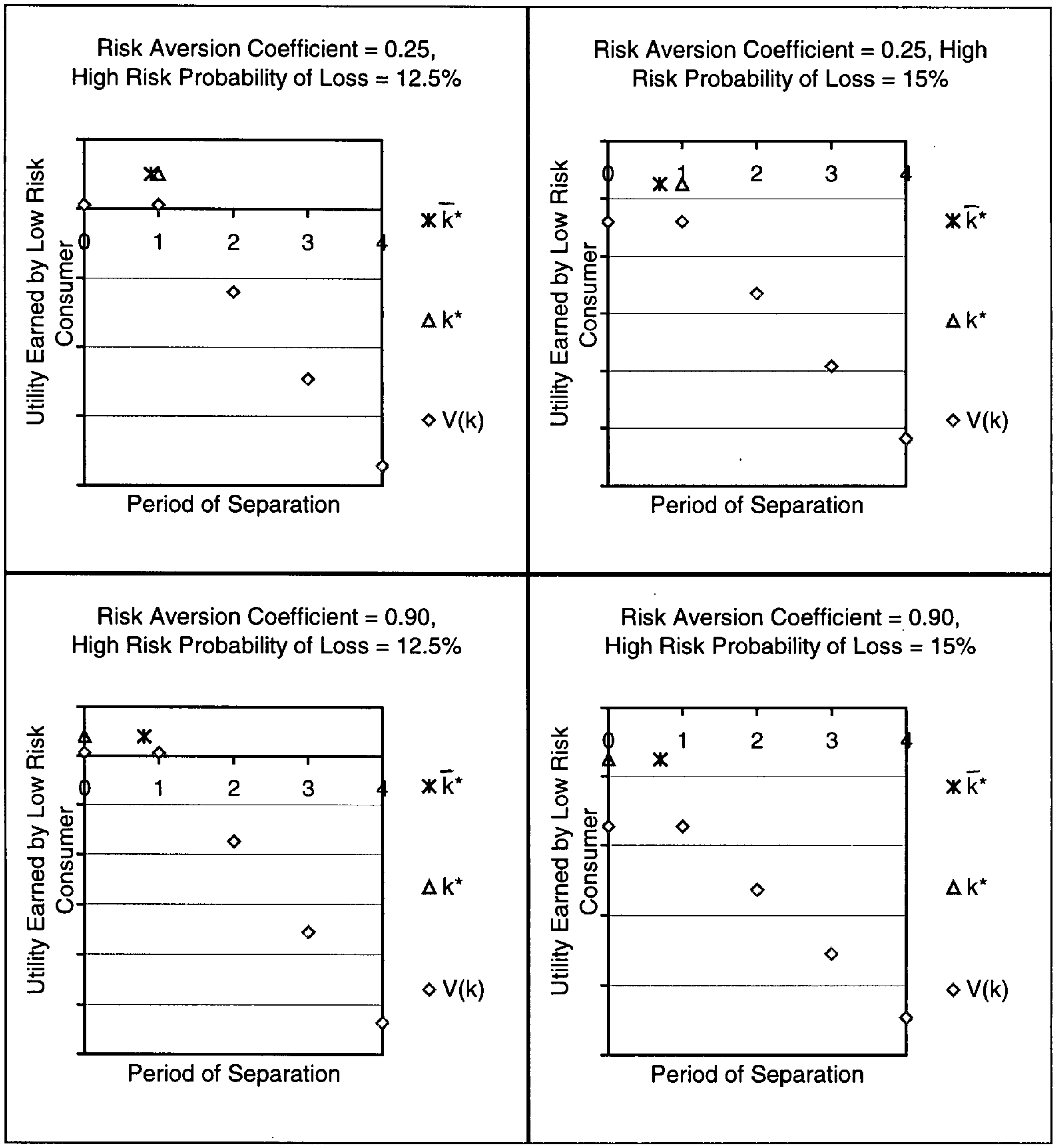

Figure 3 - 7 – Utility Earned by Low Risk Consumer and Optimal Period of Separation

bottom graphs, with $\alpha = 0.90$, low risk consumers maximise their expected utility by pooling for the entire length of the contract.

Equation (3 - 10) is independent of n, the number of periods in the model. The separation decision depends on the number of periods remaining in the model and not on the number of periods for which insurance has already been purchased.

In this section, it is assumed that the cost to the insurer of writing a policy does not change after the first period. However a major portion of the expense associated with a policy accrues from the initial underwriting. To reflect this contracting insurers may wish to lower the expense loadings charged in subsequent periods. In a perfectly competitive market if future expense loadings can be reduced between the first and subsequent periods, this will not affect the behaviour of low risk consumers who have no incentive to switch insurers in any period. In this model with constant expenses, high risk consumers are currently indifferent to switching after type has been revealed. If contracting insurers lower their expense loading in subsequent periods, then the high consumers also have no incentive to switch insurers after type is revealed.

3.3. Discussion

In this section, the utility earned by the low risk consumer under the dynamically consistent contract structure is compared with the utility earned by low risk consumers if they purchased a sequence of one period contracts. The similarities between resulting equilibria under the dynamically consistent contract structure and the ratchet effect observed in monopoly regulation are also discussed.

Scenario	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
α	0.25	0.25	0.90	0.90	0.25	0.25	0.25	0.25
p^h	12.5%	15%	12.5%	15%	7.5%	7.5%	7.5%	7.5%
p^ℓ	10%	10%	10%	10%	5%	5%	5%	5%
λ	0.98	0.98	0.98	0.98	0.50	0.60	0.80	0.90
d	1000	1000	1000	1000	100	100	100	100
Utility earned from repeated purchase of 1 period pooling contract.								
$-\log\left(-\tilde{V}_0\right)$	1.0	-47.9	3.6	-172.7	-6.87	-7.24	-7.98	-8.35
Utility earned from repeated purchase of 1 period separating contract.								
$-\log\left(-\tilde{V}(n)\right)$	1.8	-46.8	1.8	-176.8	-6.80	-6.80	-6.80	-6.80
Utility earned from optimal separation with dynamically consistent contracts.								
k^*	1	1	0	0	1	1	1	1
$-\log\left(-V\left(k^*\right)\right)$	1.1	-47.8	3.6	-172.7	-6.85	-7.16	-7.78	-8.09

Table 3 - 1 – Utility Earned Under Three Contract Choices

3.3.1. Optimal Behaviour in Multiple Period Situations

Table 3 - 1 calculates expected utilities earned by the low risk consumers under the three contract choices: pool for all periods, purchase a sequence of one period Rothschild and Stiglitz (1976) separating menu of contracts for all periods and purchase the dynamically consistent set of contracts. In the dynamically consistent framework, the optimal period of separation, k^*, is also given. As in the previous illustrations, the utility stated here is a transformation of the negative exponential utility function. The

parameter values used in the first four scenarios correspond to the values used in Figure 3 - 7. The effect of proportion of high risk consumers, λ, on the expected utility earned by the low risk consumer is demonstrated in the last four scenarios.

From the last four scenarios developed, it is evident that as λ increases, the gains from separation also increase in the dynamically consistent framework. In Scenario 5, with $\lambda = 0.50$, there is very little difference in the low risk consumer's utility is she pools for all n periods or if she separates optimally in the last period. By separating in the last period, she increases her utility by 0.16%. When $\lambda = 0.90$, optimally separating one period before the last period results in an increase in utility of 3.09% over pooling for the entire n periods.

As in most multiple period asymmetric information models, early revelation of information is expensive. This has been noted in Figure 3 - 4; the earlier low risks separate, the less the amount of indemnity received in the separating contract. Dionne (1983) discusses the use of experience rating by insurance companies as an imperfect sorting mechanism because it is too costly to directly observe risk types. Dewatripont (1989), in an examination of labour markets, has shown a loss of utility results if future contracts must account for information revealed in previous contracts. The introduction of dynamically consistent contracts generally results in a loss of utility. This can be seen in Table 3 - 1. It is evident that in the situations in which low risk consumers would prefer to separate (Scenarios 1, 2, 5, 6, 7 and 8), low risk consumers would receive higher utility if they could purchase the repeated one period Rothschild and Stiglitz

(1976) contract.

In Scenarios 3 and 4, the low risk consumers maximise their expected utility by pooling with high risk consumers in both the world with dynamically consistent contracts and in the model with the sequence of single period separating menus of contracts. The utility earned is the same in both models because the contracts offered each period (the one period Wilson (1977) pooling contract) are the same under both frameworks. Because of this, in a world in which only pooling contracts are seen in equilibrium, it is impossible to discern whether or not consumers and firms are behaving myopically or in a dynamically consistent manner.

Differences in consumer behaviour arise in those situations in which low risk consumers would prefer to separate in the one period Rothschild and Stiglitz (1976) model. In a world in which only the sequence of one period separating menus of contracts is offered, separation would occur every period. In Scenarios 1, 2, 5, 6, 7 and 8, if consumers are given a choice between a sequence of one period pooling contracts and a sequence of one period separating menu of contracts, they would choose the separating menu of contracts every period. In these scenarios, if dynamically consistent contracts were offered, low risk consumers would prefer to pool for every period except the last period.[21] Thus the two equilibria differ radically.

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[21] Using the negative exponential utility function, it is difficult to choose parameter values that would entice the low risk consumers to separate more than one period before the end if firms offer dynamically consistent contracts.

3.3.2. Pooling Equilibria and the Ratchet Effect

The ratchet effect arises in the regulation of monopolists. The regulator wishes to offer a contract to the firm that will entice the firm to expend effort to reduce costs. Unfortunately, the regulator does not know if the firm is efficient or inefficient. In a multiple period setting, an efficient producer is unwilling to expend effort and thus reveal its type because this jeopardises future rents. The regulator infers from a low cost production period the ability of the firm to repeat this performance in the future. Recognising this, the firm has the incentive to maintain a low profile and mimic a high cost producer (for a more detailed discussion, see, for example, Laffont and Tirole (1993)).

The are similarities between this regulation problem and the multiple period insurance problem. First of all, it is true that both the efficient firm's profits and the low risk consumer's utility could be increased if the regulator or insurance company could commit not to use any past information. In the insurance case, low risk consumers would purchase either a sequence of pooling contracts or single period Rothschild and Stiglitz (1976) contracts every period. In the regulatory model this would lead to the efficient firm expending effort every period.

Typically, both models display a pooling equilibrium in early periods. Laffont and Tirole note that if the efficient firms heavily discount the future, then separation occurs. In the insurance framework, if consumers heavily discount the future (which corresponds to not many periods until the end), then separation will also occur.

In both models the more efficient/lower risk types prefer not to reveal their types, but for different reasons. In the monopoly problem, the more efficient firm hides its type and mimics the inefficient firm to collect rents. In the pooling contract of the insurance model, the low risk consumers are actually subsidising the high risk consumers in the pooling contract. Low risk consumers do not separate because the cost of separation is too high. This cost is driven by the incentive compatibility constraint placed on the high risk consumers.

4. The Two Way Street: Bilateral Information Asymmetry in Insurance Markets

Most models of insurance information asymmetry, for example Rothschild and Stiglitz (1976), Wilson (1977), Kunreuther and Pauly (1985), Cooper and Hayes (1987), Hosios and Peters (1989) and Dionne and Doherty (1994), are "one-way models"; only one direction of information asymmetry is addressed. Even Dionne and Doherty's (1992) survey article on adverse selection only considered one-way information asymmetry. The insured possesses private information about her loss probabilities that is not directly observable by insurers. In the other direction, these papers assume that all information concerning the prices charged and the contracts offered by various insurance companies is common knowledge.

However even a casual observer of the property/casualty insurance market knows that consumers do not possess perfect information about insurance companies. The following three questions, as presented by Harris Schlesinger in his 1997 presidential address to the American Risk and Insurance Association, clarify this point.

1. *What is your annual auto insurance premium?*
2. *If you decided to switch insurers, what is the premium for your next best alternative?*
3. *How much would your auto insurance increase if you reported a single car accident with damages $400 above your deductible?*

It seems likely that most consumers will know the answer to the first question. Some consumers may know the answer to the second question, but very few consumers would know the answer to the third question. This lack of knowledge about the insurance industry has been previously noted in the literature. Cummins *et al* (1974), in a survey of insurance customers, find that one-half of those questioned did not obtain a second price quote for their homeowners or automobile insurance.

This lack of price information is not unique to the insurance market. Maynes and Assum (1982), and Grewal and Marmorstein (1994) find that consumers undertake relatively little price comparison shopping for durable goods. Mazumdar and Monroe (1990) note the same aspect in the market for non-durable goods. Pratt, Wise and Zeckhauser (1979) sampled thirty-nine products and services and recorded substantial differences among quoted prices. They also noted a large positive relationship between the standard deviation of prices and the mean price for each good and service.

Some of the cost differences observed in the insurance marketplace may arise because of heterogeneity of even seemingly identical insurance companies. As discussed in Chapter 2, a consumer may prefer a certain company because of the characteristics of either the insurance product or the company. Consumers incur search costs because they must become informed about the company and its product. In this chapter, it is assumed that the insurance product is perfectly homogeneous, but that price differences arise because insurers have different valuations of their expected liabilities that result in different prices. These differences arise because the proportion of high and low risk consumers insured differs across insurance companies. This differentiates this work from previous models in the literature in that price differences are not driven by the heterogeneity of the insurance product.

The models developed in this chapter incorporate both the consumer's lack of information about the price of the insurance product and the firm's lack of information about a consumer's true risk type and as such are referred to as bilateral information asymmetry models. One period and multiple period models are presented. In the models introduced, all insureds possess private information about their risk propensity and possess some information about the distribution of prices in the insurance marketplace. It is assumed that consumers incur a search cost to ascertain the cost of insurance from a new insurer. This cost varies across the population of consumers and is uncorrelated with a consumer's risk type. Insurance companies have knowledge of their own pricing structure, the distribution of prices charged by other insurers and the distribution of search costs across the population of insureds. It is assumed that insurers cannot observe a consumer's risk type.

A one period model of price dispersion in the marketplace is developed. Following Pratt, Wise and Zeckhauser (1979), if consumers possess search costs, a stable equilibrium exists in which insurers charge different prices. In the equilibrium constructed, all insurance companies offer a pooling contract to consumers and for convenience it is assumed that contracts are for full insurance.[22] The presence of search costs restricts a consumer from canvassing the market until she has found the least expensive insurer. This equilibrium supports both efficient and inefficient insurers.[23] The amount of expected profit that can be earned by any given firm increases as the range of prices charged in the market increases.

The one period model is extended to a multiple period framework. In the extension, insurers use Bayesian updating to revise their beliefs about an individual's risk type. Based on its calculation of a consumer's probability of loss, the firm picks the price at which to sell insurance to maximise the profit earned from each contract. Each period, a consumer updates the price that she is willing to pay for insurance. She switches insurers if the price the firm is charging exceeds the price she is willing to pay that period. This work differs from previous literature in that the decision to switch insurance companies is not strictly based on either a consumer's risk type or her accident history, but instead on the relationship between changes in a firm's pricing structure and changes in a consumer's reservation price over time. In switching insurers, she will incur another search cost. This search cost remains constant over time. As in the one period model, the presence of such costs prevents the consumer from automatically

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[22] This assumption follows from Watt and Vazquez (1996). Optimal levels of insurance are straightforward to calculate but add an unnecessary level of complication to the model. The simplest explanation for this assumption is that regulatory bodies require all insureds to carry full insurance.

[23] The term efficiency is misleading. All insurers have the same cost of distribution, but some insurers underwrite a lower proportion of high risk consumers which allows them to sell insurance at a lower price than those firms who underwrite more high risk consumers.

seeking out the lowest price insurer each time period.

The set-up of the chapter is as follows. Section 4.1 discusses some basic assumptions for both the one and multiple period models. The one period model and examples are presented in Section 4.2. The conditions that support a multiple period extension are given in Section 4.3. As in Chapters 2 and 3, a summary of the primitives used and the functions derived in this chapter can be found in Appendix A, Table A-3.

4.1. Basic Assumptions

The underlying behaviours of consumers and insurers in both the one period and multiple period frameworks are described in this section. The basic structure of the economy developed in this chapter follows from Chapters 2 and 3.

4.1.1. Demand

In this world, there exist L consumers who differ by their risk propensity and search costs, where L represents the measure of consumers in the economy. All consumers possess negative exponential utility functions with risk parameter α. As discussed in Chapter 3, this assumption is extremely useful in the multiple period case since it allows for the insurance purchasing decision of a consumer to be examined separately from her investment and consumption decisions. Additionally, under the assumption of negative exponential utility, a consumer's decision to search the market for a better price for insurance is independent of the number of searches she has already undertaken. Low risk consumers will, in any one period, incur a loss of size d with probability ρ^{ℓ}. The rest of the population is high risk and will, in any one period, incur a loss of size d with probability $\rho^{h} > \rho^{\ell}$. It is assumed that this loss probability is uncorrelated across consumers, and, in the multiple period models, across time periods. Consumers are endowed with initial wealth W, which is significantly large such that consumers face no wealth constraints over the entire timeframe. Each consumer knows if she is low or high risk, but this information cannot be observed by the insurance company. In the multiple period model, consumers do not discount future cash flows.

In each time period, an individual can insure against loss by purchasing a one period insurance contract. It is assumed that consumers receive higher utility from purchasing insurance than from foregoing insurance. In each period, insurers offer pooling contracts. In the multiple period model, contracts offered each period are contingent on a consumer's past accident history. Consumers with the same accident history receive the same contract. In the initial period, firms offer one contract to all consumers and in subsequent periods, firms offer a single contract to all consumers with the same accident history.[24]

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[24]Unlike the Rothschild and Stiglitz (1976) separating menu of contracts which allows consumers to voluntarily reveal their risk type through contract choice, in both the one and multiple period models, consumers are never given a choice of contracts. In the multiple period framework, the updating procedure used by firms will eventually separate consumers by risk type and each consumer will receive a full insurance contract priced for her risk type.

Before insurance is purchased, consumers conduct a price search to find an acceptable price for insurance. Each consumer has a search cost that is constant over time but differs across consumers and is uncorrelated with risk type. Differences in search costs will lead different consumers to purchase insurance at different prices: consumers with higher search costs are willing to pay more for insurance than are consumers with lower costs. A consumer's reservation price is the maximum price that she is willing to pay for insurance. All consumers possess perfect information about the distribution of prices, but do not have any prior information on the price charged by any specific insurer. This assumption ensures the optimality of a myopic search strategy. Using this strategy, a consumer will choose the first contract whose price is less than or equal to her reservation price.

In the multiple period framework, a consumer updates her reservation price each period. If the price being offered by the consumer's current insurer falls below her reservation price, then she will not undertake a search that period. Switching between insurers will occur in the multiple period framework when a consumer believes that her current insurer is charging too much for the single period coverage.

4.1.2. Supply

Insurance companies are assumed to be identical except for their valuation of the proportion of high risk consumers among first time insurance purchasers. Firms possess different valuations of this proportion for several reasons. Companies attract different clienteles that would lead to different population mixes in their portfolios. Differences in rating structures, claims handling and underwriting procedures will also lead to differing valuations of the proportion of high risk consumers in their portfolios of first time insurance purchasers.

All firms know that there are two types of consumers; high risk consumers with probability of loss ρ^h and low risk consumers with a probability of loss ρ^ℓ. Suppose that that λ^k is company k's proportion of the initial portfolio that is high risk. The *a priori* belief about a random insured's probability of loss is $\rho^k = \lambda^k \rho^h + (1-\lambda^k)\rho^\ell$ which has been previously defined in Chapters 2 and 3 as the pooled probability of loss. In the multiple period setting, insurers use a consumer's accident history to update this probability.

Each insurance company knows its own expected marginal cost of providing insurance, $\rho^k d$, the true distribution of sellers' prices in every period and the search strategies of consumers. Based on this information, each insurer sets a price to maximise its profits. It is also assumed that insurers incur no expenses in selling the policy.[25] In the multiple period model, firms do not discount future cash flows.

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[25] This differentiates these models from those presented in Chapters 2 and 3. Since firms are assumed to have the same distribution system, ignoring expenses does not significantly affect the models.

4.2. One Period Model

The derivation of the price that is charged by a profit maximising insurer in a one period world and some examples of functions that can be supported in equilibrium are given in this section.

In a one period model potential consumers and insurance companies behave as defined in Sections 4.1.1 and 4.1.2 respectively. Define $G(p)$ to be the cumulative distribution function representing the underlying distribution of prices at which insurance is sold. The probability of receiving a price quote p from a single search in the marketplace is simply $g(p)$. This distribution is a function of both the distribution of search costs across consumers and the distribution of insurer valuations of the proportion of high risk consumers underwritten by each firm. Let $q(\lambda)$ be the distribution of λ across insurance companies.

It is known by both consumers and insurers that insurance prices range between p^L and p^U. Since all consumers would prefer insurance to no insurance, all consumers are willing to pay at least p^L for coverage. It may be the case that firms are willing to sell insurance at a price lower than p^L, but if all consumers are willing to pay at least p^L to purchase a policy, then profit maximising insurers would never sell for less than this price.

The insurance process places a minimum bound on p^L, in that the lowest price at which firms are willing to sell insurance must be at least $\rho^\ell d$. In the one period world, if all firms have a valuation of λ that exceeds zero then p^L exceeds $\rho^\ell d$. In the multiple period framework, the price charged to consumers who have excellent driving records will approach $\rho^\ell d$ over time.

If the marginal consumer will pay at most p^U to purchase insurance, then firms cannot charge higher than this amount and still attract consumers. A firm that wishes to charge a price higher than p^U would not enter the marketplace. If the firm's cost per policy is less than p^U, Proposition 4 - 1 gives the optimal price that should be charged.

Consider a consumer with a search cost, s. She samples prices in the marketplace until her total expected utility from undertaking another search equals her total utility from not searching. Denote p as the maximum price at which the expected utility from undertaking one more search equals utility from not undertaking one more search. Then

$$-\int_{p^L}^{p} e^{-\alpha(W-s-x)} g(x)dx - \int_{p}^{p^h} e^{-\alpha(W-s-p)} g(x)dx = -e^{-\alpha(W-p)}$$

$$\int_{p^L}^{p} e^{\alpha(x-p)} g(x)dx + 1 - G(p) = -e^{-\alpha s}, \qquad (4 - 1)$$

and rearranging terms gives $s(p) = p - \frac{1}{\alpha}\log\left[\int_{p^L}^{p} e^{\alpha x} g(x)dx + e^{\alpha p}[1 - G(p)]\right]$. As noted by

Pratt, Wise and Zeckhauser (1979), if consumers know the distribution of prices exactly then a myopic search strategy is optimal. Consumers would never look more than one search ahead and they would never return to a previous price quotation.

The consumer's wealth W incorporates the cost of any past searches. The total expected utility from searching has two components: $-\int_{p^L}^{p} e^{-\alpha(W-s-x)} g(x)dx$ is the contribution to total expected utility from securing a new price that is less than p and $-\int_{p}^{p^h} e^{-\alpha(W-s-p)} g(x)dx$ is the contribution to total expected utility from finding and rejecting a price that exceeds p.

As can be seen from (4 - 1), the reservation price p is a function of the search cost and the distribution of insurer prices. The relationship between the reservation price and the search cost is positive, specifically $\frac{\partial p}{\partial s} = \frac{e^{-\alpha s}}{\int_{p^L}^{p} e^{\alpha(x-p)} g(x)dx}$. The higher a customer's search cost, the higher the price she is willing to pay for insurance. From (4 - 1), a consumer with zero search costs will canvas the marketplace until she has secured a price of p^L for coverage. Substituting for $\int_{p^L}^{p} e^{\alpha(x-p)} g(x)dx = e^{-\alpha s} - (1 - G(p))$ in $\frac{\partial p}{\partial s}$ yields $\frac{\partial p}{\partial s} = \frac{e^{-\alpha s}}{e^{-\alpha s} - (1 - G(p))}$. A dollar increase in search costs leads to more than a dollar increase in the reservation price.

Suppose there are N insurers in the marketplace. Using their knowledge of both (4 - 1), and the distribution of search costs across the population, $h(s)$, insurance companies can compute the probability that a random buyer has a reservation price p. This probability is given in the reservation function $f(p)$, which can be written as

$$f(p) = h(s(p))s'(p)$$

$$= h\left(p - \frac{1}{\alpha}\log\left[\int_{p^L}^{p} e^{\alpha x} g(x)dx + e^{\alpha p}[1 - G(p)]\right]\right) * \frac{\int_{p^L}^{p} e^{\alpha x} g(x)dx}{\int_{p^L}^{p} e^{\alpha x} g(x)dx + e^{\alpha p}[1 - G(p)]}.$$

(4 - 2)

For example, assume that insurance prices in the marketplace follow a uniform distribution over the support $[p^L, p^U]$ and that the distribution of search costs over the consumers is exponential with parameter $\theta > 0$. Then $g(p) = \dfrac{1}{p^U - p^L}$ for $p^L \le p \le p^U$ and $h(s) = \theta e^{-\theta s}$ for $s > 0$. From (4 - 2), for $p^L \le p \le p^U$, the consumers' reservation function is

$$f(p) = \theta e^{-\theta p} \frac{\left[e^{\alpha p}\left(\alpha(p - p^L) - 1\right) + e^{\alpha p^L}\right]}{e^{\alpha p}\left(\alpha(p - p^L) - 1 + \alpha^2(p^U - p)\right) + e^{\alpha p^L}} \left[\frac{e^{\alpha p}\left(\dfrac{p - p^L}{\alpha} - \dfrac{1}{\alpha^2} + p^U - p\right) + \dfrac{e^{\alpha p^L}}{\alpha^2}}{\left(p^U - p^L\right)}\right]^{\theta/\alpha}.$$

Using this reservation function $f(p)$, insurers then pick the price at which they will sell insurance. Proposition 4 - 1 gives the equilibrium profit maximising price for an insurance company with an expected marginal cost of $\rho^k d$.

Proposition 4 - 1: *The price charged by an insurer with an expected marginal cost of* $\rho^k d$ *is* $p^k = \frac{G(p^k)}{f(p^k)} \int_{p^k}^{p^U} \frac{f(x)}{G(x)} dx + \rho^k d$, *where* $f(p)$, *as derived in (4 - 2), is the reservation function of the consumers and* $G(p)$ *is the cumulative distribution function of insurance prices in the marketplace.*

Proof: If there are N insurance companies in the marketplace, then there are $NG(p^k)$ firms selling at or below the price p^k. Assume each of these insurers has the same probability of securing a sale to consumer when the reservation price is p^k. The distribution of reservation prices is given by the function $f(p^k)$ defined in (4 - 2) and so there are $Lf(p^k)$ consumers who will pay at most p^k for coverage. The expected quantity sold by an insurer at a price p^k is $Q(p^k) = \frac{L}{N} \int_{p^k}^{p^U} \frac{f(x)}{G(x)} dx$. On each policy sold, the insurance company earns expected profits of $\pi(p^k) = p^k - \rho^k d$. The firm's expected profit is $\Pi(p^k) = \left(\frac{L}{N} \int_{p^k}^{p^U} \frac{f(x)}{G(x)} dx \right)(p^k - \rho^k d)$. Maximising this function with respect to the price, p^k, yields

$$p^k = \frac{G(p^k)}{f(p^k)} \int_{p^k}^{p^U} \frac{f(x)}{G(x)} dx + \rho^k d .$$

(4 - 3) ■

Given that buyers respond optimally, no seller could increase expected profits by charging another price. This distribution of prices, $G(p)$, is an equilibrium distribution of prices when all buyers behave according to the reservation function given by (4 - 2).

The price charged by the insurance company is influenced by two factors. It is a function of the actuarial fair value for insurance $\rho^k d$ plus a loading $\frac{G(p^k)}{f(p^k)} \int_{p^k}^{p^U} \frac{f(x)}{G(x)} dx$ that arises due to consumers' positive search costs. All insurance companies with the same valuation of the proportion of high risk consumers will charge the same price in equilibrium.

The distribution of equilibrium prices has a direct correspondence to the distribution of valuations of the proportion of high risk consumers across insurance companies. Rewriting (4 - 3) yields

$$\lambda^k(p^k) = \frac{1}{d(\rho^h - \rho^\ell)} \left[p^k - \rho^\ell d - \frac{G(p^k)}{f(p^k)} \int_{p^k}^{p^U} \frac{f(x)}{G(x)} dx \right].$$

(4 - 4)

If $Q(\lambda)$ is the cumulative distribution function of λ, then, from (4 - 4), the distribution of equilibrium prices must satisfy

$$G(p)=Q\left(\frac{1}{d(\rho^h-\rho^\ell)}\left[p-\rho^\ell d-\frac{G(p)}{f(p)}\int_p^{p^U}\frac{f(x)}{G(x)}dx\right]\right), \tag{4 - 5}$$

for prices between the support $[p^L,p^U]$. Substituting for $f(x)$ from (4 - 2), into (4 - 5) yields

$$G(p)=Q\left(\frac{1}{d(\rho^h-\rho^\ell)}\left[p-\rho^\ell d-\frac{G(p)\displaystyle\int_p^{p^U}\frac{h\left(x-\frac{1}{\alpha}\log\left[\int_{p^L}^{x}e^{\alpha y}g(y)dy+e^{\alpha x}[1-G(x)]\right]\right)*\int_{p^L}^{x}e^{\alpha y}g(y)dy}{G(x)*\left(\int_{p^L}^{x}e^{\alpha y}g(y)dy+e^{\alpha x}[1-G(x)]\right)}dx}{h\left(p-\frac{1}{\alpha}\log\left[\int_{p^L}^{p}e^{\alpha x}g(x)dx+e^{\alpha p}[1-G(p)]\right]\right)*\frac{\int_{p^L}^{p}e^{\alpha x}g(x)dx}{\int_{p^L}^{p}e^{\alpha x}g(x)dx+e^{\alpha p}[1-G(p)]}}\right]\right).$$

This is a fixed-point problem.

Since $G(p)$ is a cumulative distribution function, $G(p^U)=Q\left(\frac{p^U-\rho^\ell d}{d(\rho^h-\rho^\ell)}\right)=1$. If λ is distributed over the range $[0,1]$, then $Q(1)=1$. This implies that $p^U=\rho^h d$, so that the most inefficient producer earns zero profits in equilibrium. It is also the case that $G(p^L)=Q\left(\frac{1}{d(\rho^h-\rho^\ell)}\left[p^L-\rho^\ell d-\frac{G(p^L)}{f(p^L)}\int_{p^L}^{p^U}\frac{f(x)}{G(x)}dx\right]\right)=0$. If $Q(0)=0$, then the maximum profits that can be earned by an insurer is $p^L-\rho^\ell d=\frac{G(p^L)}{f(p^L)}\int_{p^L}^{p^U}\frac{f(x)}{G(x)}dx$. If $f(p^L)$ is not zero, then the minimum price charged in the marketplace is the actuarial fair value of insuring a low risk consumer.

4.2.1. Examples

Two examples of the single period model are given. In both cases, the lowest price charged in the marketplace, p^L, is a function of both the high and low risk probability of loss. Specifically, the lowest price charged in the marketplace is positively correlated to the high risk probability of loss so that the greater the heterogeneity in loss probabilities, the greater the profit that can be earned by more efficient firms.

Example 1: Suppose that the support of prices in this economy is such that $p^L = \frac{1}{2}\left(\rho^\ell d + \rho^h d\right)$ and, as discussed earlier, $p^U = p^h d$. Assume that insurer prices follow a uniform distribution over this support, so that $G(p) = \frac{p - p^L}{p^U - p^L}$. Furthermore assume that the distribution of λ across all insurers follows a uniform distribution between zero and one, that is $Q(\lambda) = \lambda$. A reservation function that satisfies (4 - 5) is given by $f(p) = \frac{2\left(p - p^L\right)}{\left(p^U - p^L\right)^2}$.[26] The price charged by an insurer whose proportion of high risk consumers is λ^k is $p^k = \frac{p^U + \rho^k d}{2}$.

[26] This assumes that there exists an appropriate distribution of search costs $H(s)$ such that the relationship between the reservation function and the distribution of insurer prices is feasible.

Example 2: Assume that the support of prices is such that $p^L = \rho^\ell d + \frac{1}{\delta}\left(e^{\delta\left(\rho^h d - p^L\right)} - 1\right)$ and $p^U = \rho^h d$. Assume that the distribution of prices in the marketplace is given by $G(p) = \frac{\delta\left(p - p^L\right) - e^{\delta p^U}\left(e^{-\delta p} - e^{\delta p^L}\right)}{\delta\left(p^U - p^L\right) - 1 + e^{\delta\left(p^U - p^L\right)}}$.[27] Furthermore assume that the distribution of λ across all insurers follows a uniform distribution between zero and one. A reservation function satisfying (4 - 5) is $f(p) = \frac{\left(\delta\left(p - p^L\right) + e^{\delta\left(p^U - p^L\right)}\right)e^{\delta p} - e^{\delta p^U}}{\frac{1}{\delta}\left(e^{\delta\left(p^U - p^L\right)} - 1\right)\left(e^{\delta p^U} - e^{\delta p^L}\right)}$.[28] The profit maximising price charged by an insurance company whose proportion of high risk consumers is λ^k is $p^k = \frac{1}{\delta}\left(e^{\delta\left(p^U - p\right)} - 1\right) + \rho^k d$.

Figure 4 - 1 is an illustration of these examples. For reference purposes, the actuarial fair value of the coverage for this insurer is provided in each graph. In the top graph, it is assumed that the size of loss is \$2000,[29] and that 20% of the insurer's portfolio is high risk. It is assumed that the probability of loss faced by the low risk consumer is $\rho^\ell = 10\%$. The high risk probability of loss ranges from 12% to 20%, and so the maximum price charged in the market ranges from \$240 to \$400. For Example 2, the parameter value $\delta = 0.003$ was chosen. Numerical testing (not included) suggests that

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[27] Note that $G\left(p^L\right) = 0$, $G\left(p^U\right) = 1$ and $G'(p) > 0$, so that this is a proper distribution function.

[28] As in the previous example, this assumes that there exists an appropriate distribution of search costs, $H(s)$.

[29] As discussed in Chapters 2 and 3, \$2000 is approximately the average size of loss for private passenger automobile property damage coverage in the United States for 1996.

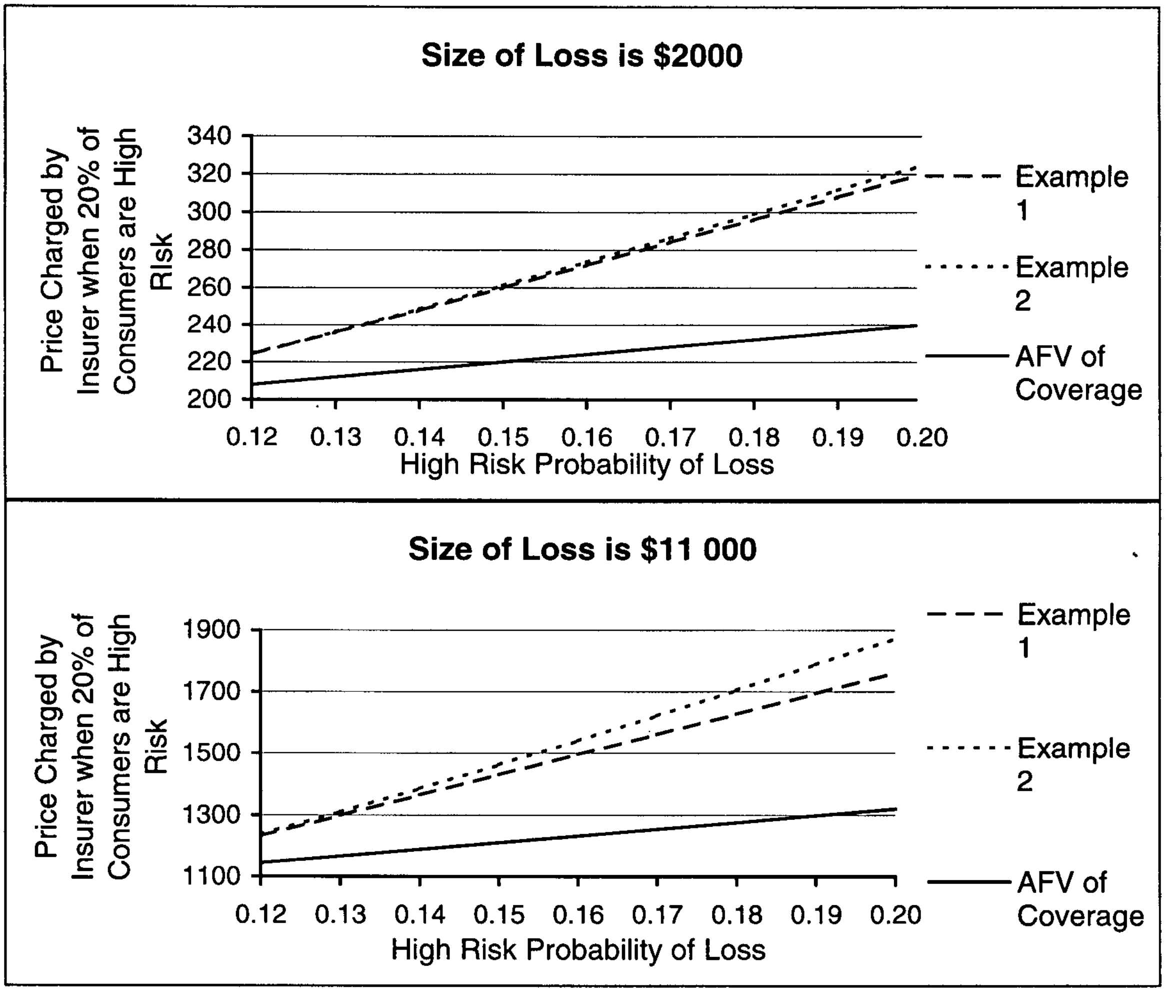

Figure 4 - 1 – Prices Charged by an Insurer when 20% of Its Consumers are High Risk.

the prices derived for Examples 2 are relatively insensitive to changes in δ. In the bottom graph, the size of loss is taken to be $11 000,[30] with all other values remaining the same. For Example 2, the parameter value $\delta = 0.00075$ was chosen.

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[30] As discussed in Chapters 2 and 3, $11 000 is approximately the average size of loss for private passenger automobile bodily injury coverage in the United States for 1996.

The graphs illustrate that the higher the probability of loss faced by the high risk consumer, the larger the expected profit earned by the insurance company. If there is not much difference between the loss frequency of the high and low risk consumer, then insurance companies are much more competitive, earning smaller profits.

4.3. Multiple Period Model

The one period model presented in Section 4.2 is extended in this section. In the multiple period model, firms offer a sequence of one period pooling contracts. At the beginning of each period, the contracting insurer offers each of its consumers a contract based on both its valuation of λ (which is constant over time) and the consumer's accident history. The consumer decides either to purchase this contract or to search for less expensive coverage. The ordering of movement within each period is displayed in Figure 4 - 2.

No new firms enter the marketplace to compete for existing customers and the number of consumers in each cohort also does not change over time. Because each contract is priced independently of other contracts, whether or not firms solicit first time consumers in subsequent periods will not affect the equilibrium prices for any existing cohort. Additional assumptions about the behaviour of consumers and firms are given in Sections 4.3.1 and 4.3.2 respectively. Conditions that support an equilibrium are detailed in Section 4.3.3.

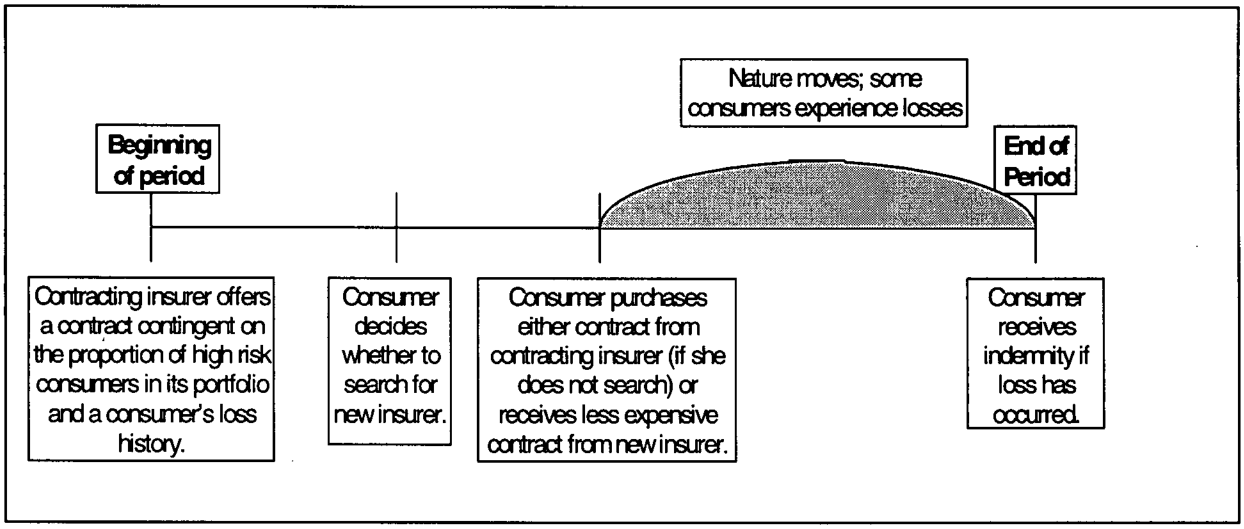

Figure 4 - 2 – Ordering of Movement within a Period

4.3.1. Demand

As in many multiple period insurance models, consumers are assumed to behave myopically, see for example, Kunreuther and Pauly (1985), Hosios and Peters (1989), and Fombaron (1997). Consumers are only concerned about maximising their one period utility. As discussed in Chapter 3, Kunreuther and Pauly (1985) note that this is not an unrealistic assumption in that there is very little empirical support that insurance consumers maximise utilities over the long run. The following question illustrates this point.

> *When you collect price quotes for insurance coverage, do you collect information on more than the current year's prices?*

Each period an individual insures against a possible loss by purchasing a one-period insurance contract. This contract pools all consumers with the same accident history. As in the one period model, before the first period, each consumer conducts a search to find an acceptable price for insurance. In each subsequent period, she only conducts a price search if the price charged by her current insurer exceeds her reservation price for that period. As in the single period framework, the amount that a consumer is willing to pay in any period is a function of her search cost and the prices being charged in the marketplace.

Consider a consumer who has reported j claims in the first i periods. Let $g_{ij}(p)$ be the distribution of prices in marketplace charged to consumers with this accident history. Then following from (4 - 1), the price, p_{ij}, she is willing to pay is

$$\int_{p_{ij}^L}^{p_{ij}} e^{\alpha(x-p_{ij})} g_{ij}(x)dx + 1 - G_{ij}(p) = e^{-\alpha s} \quad , \tag{4 - 6}$$

where p_{ij}^L is the lowest price found in the marketplace for consumers who have reported j claims in the first i periods.[31]

From (4 - 1) and (4 - 6), the relationship between the price, p, that the consumer is

[31] The value of p_{ij}^L, as in Section 4.2, depends on the functional forms of both the distribution of prices and the distribution of search costs across consumers.

willing to pay to purchase insurance in the initial period and p_{ij} is

$$\int_{p^L}^{p} e^{\alpha(x-p)} g(x)dx - G(p) = \int_{p_{ij}^L}^{p_{ij}} e^{\alpha(x-p_{ij})} g_{ij}(x)dx - G_{ij}(p).$$

Let $f_{ij}(p)$ be the reservation function of an individual who has reported j losses in i periods. Then, from (4 - 2)

$$f_{ij}(p) = h\left(p - \frac{1}{\alpha}\log\left[\int_{p_{ij}^L}^{p_{ij}} e^{\alpha x} g_{ij}(x)dx + e^{\alpha p_{ij}}\left[1 - G(p_{ij})\right]\right]\right) * \frac{\int_{p_{ij}^L}^{p_{ij}} e^{\alpha x} g_{ij}(x)dx}{\int_{p_{ij}^L}^{p_{ij}} e^{\alpha x} g_{ij}(x)dx + e^{\alpha p_{ij}}\left[1 - G_{ij}(p)\right]}. \tag{4 - 7}$$

The updated reservation function reflects the new distribution of prices in the marketplace.

4.3.2. Supply

As in Kunreuther and Pauly (1985), and Watt and Vazquez (1996), insurers revise their beliefs about a consumer's true risk type through Bayesian updating. The Bayesian contract offers a gradual convergence to full coverage priced for a consumer's risk type as type is revealed over time. However when loss probabilities are small, this convergence is likely to be very slow. The use of Bayesian updating also does not allow a consumer to be "punished" for extremely bad experience.

Let ρ_{ij}^{k} denote insurance company k's updated probability that an insured who has reported j accidents in the past will have a claim in period i, where $i=0$ denotes the first period. Lemma 4 - 1 gives the updated probabilities.

Lemma 4 - 1: *Based on the insurer's proportion of high risk consumers in its portfolio of first time insurance purchasers, λ^k, if a consumer, in period i, has reported j claims the firm's estimate of the consumer's loss probability is ρ_{ij}^k where*

$$\rho_{ij}^k = \frac{\lambda^k(\rho^h)^{j+1}(1-\rho^h)^{i-j} + (1-\lambda^k)(\rho^\ell)^{j+1}(1-\rho^\ell)^{i-j}}{\lambda^k(\rho^h)^j(1-\rho^h)^{i-j} + (1-\lambda^k)(\rho^\ell)^j(1-\rho^\ell)^{i-j}}.$$

Proof: The updating procedure used by the insurance companies is based on the original belief that a customer chosen at random has initial loss probability $\rho_0^k = \lambda^k\rho^h + (1-\lambda^k)\rho^\ell$. Bayesian updating of this probability gives the probability ρ_{ij}^k. ■

This probability, ρ_{ij}^k, is an increasing function of j for a fixed i; as the number of accidents increases, the insurer's estimate of a consumer's risk propensity also increases. The insurer's estimate of the probability of loss can be written as

$$\rho_{ij}^k = \rho^\ell + \frac{\lambda^k(\rho^h-\rho^\ell)(\rho^h)^j(1-\rho^h)^{i-j}}{\lambda^k(\rho^h)^j(1-\rho^h)^{i-j} + (1-\lambda^k)(\rho^\ell)^j(1-\rho^\ell)^{i-j}}$$

(4 - 8)

or

$$\rho_{ij}^k = \rho^h - \frac{(1-\lambda^k)(\rho^h-\rho^\ell)(\rho^\ell)^j(1-\rho^\ell)^{i-j}}{\lambda^k(\rho^h)^j(1-\rho^h)^{i-j} + (1-\lambda^k)(\rho^\ell)^j(1-\rho^\ell)^{i-j}}.$$

(4 - 9)

Since $\rho^h > \rho^\ell$, from (4 - 8) and (4 - 9), ρ_{ij}^k is bounded above and below by ρ^h and ρ^ℓ respectively. As $i - j \to \infty$, then ρ_{ij}^k approaches ρ^ℓ. If the number of accidents is very large (as $j \to i$ and $i \to \infty$), then the insurer's estimate of the probability of loss approaches ρ^h.

Suppose that in the $i-1^{st}$ period, the insurer's estimate of the loss probability is $\rho_{i-1,j-1}^k$. In period $i-1$, an accident may or may not be reported. If an accident has been reported, then the insurer's updated belief at the start of the i^{th} period about the probability of loss is ρ_{ij}^k. If no accident was reported then the updated probability of loss is $\rho_{i,j-1}^k$. Insight into the insurance company's updating process is gained if these two possible i^{th} period probabilities are rewritten as

$$\rho_{ij}^k = \rho_{i-1,j-1}^k + \left(\frac{1}{\rho_{i-1,j-1}^k}\right)\left[\frac{\lambda^k(1-\lambda^k)(\rho^h)^{j-1}(1-\rho^h)^{i-j}(\rho^\ell)^{j-1}(1-\rho^\ell)^{i-j}(\rho^h-\rho^\ell)^2}{\left(\lambda^k(\rho^h)^{j-1}(1-\rho^h)^{i-j}+(1-\lambda^k)(\rho^\ell)^{j-1}(1-\rho^\ell)^{i-j}\right)^2}\right]$$

(4 - 10)

and

$$\rho_{i,j-1}^k = \rho_{i-1,j-1}^k - \left(\frac{1}{1-\rho_{i-1,j-1}^k}\right)\left[\frac{\lambda^k(1-\lambda^k)(\rho^h)^{j-1}(1-\rho^h)^{i-j}(\rho^\ell)^{j-1}(1-\rho^\ell)^{i-j}(\rho^h-\rho^\ell)^2}{\left(\lambda^k(\rho^h)^{j-1}(1-\rho^h)^{i-j}+(1-\lambda^k)(\rho^\ell)^{j-1}(1-\rho^\ell)^{i-j}\right)^2}\right].$$

(4 - 11)

Typically ρ^h is significantly less than one-half and since the probabilities are bounded above by ρ^h, then $1-\rho_{i-1,j-1}^k$ is significantly larger than $\rho_{i-1,j-1}^k$. Since the term in the square brackets is the same in (4 - 10) and (4 - 11) then $\left|\rho_{ij}^k - \rho_{i-1,j-1}^k\right| > \left|\rho_{i,j-1}^k - \rho_{i-1,j-1}^k\right|$.

		j - number of reported					
		0	**1**	**2**	**3**	**4**	**5**
i - number of periods	**0**	11.00%					
	1	10.96%	11.36%				
	2	10.91%	11.31%	11.80%			
	3	10.87%	11.25%	11.73%	12.29%		
	4	10.83%	11.20%	11.67%	12.22%	12.79%	
	5	10.79%	11.15%	11.61%	12.15%	12.72%	13.27%

Table 4 - 1 – Updating of Probabilities in the First Six Periods

The reporting of a loss provides more information to the insurer about a consumer's type than the event that no loss has been reported.

This information effect is illustrated in Table 4 - 1, which gives the insurer's updated loss probabilities for the first six periods. In the table, the insurer's valuation of the initial proportion of high risk consumers in its portfolio is 0.20 and the two loss probabilities are $\rho^h = 15\%$ and $\rho^\ell = 10\%$. If a consumer reports no accidents in the first five periods, the company's estimate of her probability of loss falls to 10.79%, only 0.21% less than the original estimate of her probability of loss. However the firm's estimate of a consumer's loss probability increases dramatically when a consumer reports an accident. If a consumer has an accident in the first period, the firm's estimate of her loss probability increases from 11% to 11.36%.[32] The greater a firm's proportion of high risk

[32] This rigidity in the downward movement of probabilities is reflected in the claims rated scale or bonus/malus scale used by most property/casualty insurance companies. For example,

consumers, the greater the increase in its estimate of a consumer's loss probability when an accident occurs and the smaller the decline in loss probabilities resulting from reporting no accidents.

The distribution of prices in the marketplace charged to consumers who have reported j claims in i periods is given by $G_{ij}(p)$. As in the one period model, this distribution is a function of the distribution of search costs across consumers, $h(s)$, and the distribution of λ across insurers, $q(\lambda)$.

4.3.3. Multiple Period Equilibrium

Using the updated reservation function of consumers, insurers pick a price at which they

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consider the experience rating mechanism used by the Insurance Corporation of British Columbia to price private passenger third party liability coverage. Movement within the claims rated scale is as follows. In the scale, 25 represents the base (zero years of no at-fault claims reported). For each year of no reported at-fault claims, the insured moves down one level. If an insured submits an at-fault claim, she moves up three levels, where higher levels face higher premiums.

In 1971, the following transition rules were legislated for private passenger third party liability coverage sold by all companies operating in Belgium. The bonus/malus scale consists of 18 classes, with higher classes paying more than lower classes. Individuals start in base class 6. For each claim free year, the consumer moves down by one class. Policyholders are moved up two classes if they report one accident in a given year and are moved up by three classes for each additional claim reported during the same year. An examination of this tariff structure in 1988 (Lemaire (1988)) led to the proposal that the base class be changed to 10 and stricter penalties be applied. Policyholders are still moved down one class for each year no claim is reported, but they are moved up three classes if they report one accident in a given year and are moved up by four classes for each additional claim reported during the same year.

In both rating structures, because of the low probabilities of loss, the reporting of an at-fault claim reveals more information to the insurer than the absence of such a claim.

wish to sell insurance. The profit maximising price, as given in Lemma 4 - 2, is similar to the price charged by the insurer in the one period model.

Lemma 4 - 2: *The profit maximising price charged by an insurer with a marginal cost of $\rho_{ij}^{k}d$ *to a consumer who has experienced j accidents in i periods and has a reservation function given by (4 - 7) is $p_{ij}^{k} = \dfrac{G_{ij}(p_{ij}^{k})}{f_{ij}(p_{ij}^{k})}\displaystyle\int_{p_{ij}^{k}}^{p_{ij}^{U}}\frac{f_{ij}(x)}{G_{ij}(x)}dx + \rho_{ij}^{k}d$. $G_{ij}(\cdot)$ is the cumulative distribution function of insurance prices in the marketplace at time i for consumers who have reported j accidents and p_{ij}^{U} is the maximum price charged to these consumers.*

Proof: Follows from the proof of Proposition 4 - 1. ■

As in the single period model, the distribution of equilibrium prices has a direct correspondence to the distribution of valuations of the proportion of high risk consumers in the initial portfolio across insurance companies. From Lemma 4 - 1,

$$\rho_{ij}^{k} = \frac{\lambda^{k}(\rho^{h})^{j+1}(1-\rho^{h})^{i-j} + (1-\lambda^{k})(\rho^{\ell})^{j+1}(1-\rho^{\ell})^{i-j}}{\lambda^{k}(\rho^{h})^{j}(1-\rho^{h})^{i-j} + (1-\lambda^{k})(\rho^{\ell})^{j}(1-\rho^{\ell})^{i-j}}.$$

Substituting for ρ_{ij}^{k} into $p_{ij}^{k} = \dfrac{G_{ij}(p_{ij}^{k})}{f_{ij}(p_{ij}^{k})}\displaystyle\int_{p_{ij}^{k}}^{p_{ij}^{U}}\frac{f_{ij}(x)}{G_{ij}(x)}dx + \rho_{ij}^{k}d$ and rearranging terms yields

$$\lambda^k(p)=\frac{(\rho^\ell)^j(1-\rho^\ell)^{i-j}\left[\frac{G_{ij}(p)}{f_{ij}(p)}\int_p^{p_{ij}^U}\frac{f_{ij}(x)}{G_{ij}(x)}dx-p+\rho^\ell d\right]}{c_0\left[\frac{G_{ij}(p)}{f_{ij}(p)}\int_p^{p_{ij}^U}\frac{f_{ij}(x)}{G_{ij}(x)}dx-p+\rho^\ell d\right]-c_1}.$$

(4 - 12)

where $c_0=(\rho^\ell)^j(1-\rho^\ell)^{i-j}-(\rho^h)^j(1-\rho^h)^{i-j}$ and $c_1=d\left((\rho^h)^{j+1}(1-\rho^h)^{i-j}-(\rho^\ell)^{j+1}(1-\rho^\ell)^{i-j}\right)$. As in the single period model, if $Q(\lambda)$ is the cumulative distribution function of λ, then, from (4 - 12), the distribution of equilibrium prices must satisfy

$$G_{ij}(p)=Q\left(\frac{(\rho^\ell)^j(1-\rho^\ell)^{i-j}\left[\frac{G_{ij}(p)}{f_{ij}(p)}\int_p^{p_{ij}^U}\frac{f_{ij}(x)}{G_{ij}(x)}dx-p+\rho^\ell d\right]}{c_0\left[\frac{G_{ij}(p)}{f_{ij}(p)}\int_p^{p_{ij}^U}\frac{f_{ij}(x)}{G_{ij}(x)}dx-p+\rho^\ell d\right]-c_1}\right),$$

(4 - 13)

for prices between the support $[p_{ij}^L,p_{ij}^U]$. Substituting for $f_{ijj}(p)$ from (4 - 7) yields the fixed-problem in the multiple period case. The single period fixed-point problem given in (4 - 5) is a special case of (4 - 13). When $i=j=0$, $c_0=0$ and $c_1=(\rho^h-\rho^\ell)d$, and so (4 - 13) reduces to (4 - 5).

Since $G(p)$ is a proper cumulative distribution function, from (4 - 13), $G(p_{ij}^U)=Q\left(\frac{(\rho^\ell)^j(1-\rho^\ell)^{i-j}[-p+\rho^\ell d]}{c_0[-p+\rho^\ell d]-c_1}\right)=1$. If λ is distributed over the range $[0,1]$, then $Q(1)=1$. Substituting for c_0 and c_1 yields $p_{ij}^U=\rho^h d$. The maximum price charged any consumer, regardless of her accident history, remains constant over time and is the actuarial fair value of providing insurance to a high risk consumer. The minimum price that exists in the marketplace is $p_{ij}^{L} - \rho^{\ell} d = \frac{G_{ij}(p_{ij}^{L})}{f_{ij}(p_{ij}^{L})} \int_{p_{ij}^{L}}^{p^{U}} \frac{f_{ij}(x)}{G_{ij}(x)} dx$. As in the single period model, if $f_{ij}(p_{ij}^{L})$ is not zero, then the minimum price charged in the marketplace is the actuarial fair value of insuring a low risk consumer.

5. Future Work and Conclusions

Avenues of possible future work are discussed in Section 5.1. Key results and conclusions from Chapters 2, 3 and 4 are given in Section 5.2.

5.1. Future Work

Future work on the models presented in Chapter 2 can be classified into two areas: technical details and welfare implications. Technical details involve the examination of the various underlying assumptions of the models. One conjecture in the chapter is that insurance consumers possess negative exponential utility functions. The sensitivity of the results of Chapter 2 to this functional form should be examined.

Regan and Tennyson have conjectured that agency writers discriminate between consumer types better than direct writers can. The model in Section 2.3 provides another test of this hypothesis. If agency writers can better discriminate between risk types, then their portfolios should display more homogeneity with respect to both the amount of insurance purchased and the accident frequencies across consumers.

The sections in this chapter have dealt with the normative issues of the economics of the structure of the insurance market. Future work will also involve the calculation of welfare effects to consumers of admitting both agency and direct writers. From Section 2.2, one possible hypothesis to examine is whether it is welfare increasing or more cost effective for the social planner (or the insurance regulator) to abolish agency writers and to offer some sort travel subsidy to those risks located too far from the direct writers. A key assumption in the derivation of the models in Section 2.3 is the absence of a common carrier requirement for the direct writers. The welfare implications of such a requirement, or its absence, need to be addressed.

In Chapter 3, the functional form of the utility function is necessary to separate the insurance purchasing decision from investment and consumption decisions. However, it is difficult to encourage separation in the dynamically consistent framework under the assumption of negative exponential utility. Embedding the insurance purchasing decision model within a larger framework which accounts for both a consumer's investment and consumption decisions would produce a much richer model and allow for the examination of other utility functions. Within such a framework it would be possible to examine the relationship between the consumer's separation decision and her discount factor as opposed to considering the number of periods left in the model.

Several avenues of future work arise from Chapter 4. In this chapter, it is assumed that consumers know the distribution of prices in the marketplace. This assumption is unrealistic and could be relaxed. This would necessitate the introduction of a Bayesian search strategy, such that after each price sampled, the consumer updates her belief about the distribution of prices in the economy. A myopic search strategy is not optimal in this case since a previously sampled price may be lower than the most recent sampled price. The consumer's search serves two purposes; not only is she looking for the lowest price but she is also collecting information on the distribution of prices in the marketplace. The relationship between this more sophisticated search strategy on both the insurers' profit maximisation prices in the one and multiple period needs to be developed.

In the multiple period model presented in Section 4.3, conditions under which an equilibrium can be supported are given. An example of the evolution of the optimal pricing structure over time would provide insight into the equilibrium conditions. However given the complexity of the relationships between the distributions of consumer search costs, insurer valuations of the proportion of high risk consumers and prices charged in the marketplace, it is unclear as to whether or not an example can be constructed.

If an example could be constructed, simulation could b used to examine the statistical relationship between the accident frequency and the number of times a consumer switches insurance companies. Also of interest is the relationship between the number of times a consumer switches insurance companies and such variables as the consumer's original reservation price, the firm's original offer prices and a consumer's risk type. The relationship between the length of time between switches and these variables could also be explored.

As in Chapter 2, there are welfare issues that can be examined. From Section 4.2, the greater the range of prices that consumers are willing to pay, which is directly correlated with the range of search costs incurred, the greater the possible profits that can be earned by insurance company. The effect of reducing search costs on the market structure need to be examined. This reduction in search costs can be brought about by greater consumer education.

5.2. Conclusions

Chapter 2 of the thesis examines some of the economic implications of a one period insurance market in which firms possess different technologies. The first model presented provides an economic explanation for the co-existence of agency and direct writers in the property/casualty insurance market. By exploiting the difference in accessibility between the two types of insurance distribution systems, a symmetric equilibrium is constructed in which direct writers act as local monopolists and where regions between their captive markets are served by agency writers. In order to characterise the equilibrium, the model relies on a prohibitive relocation costs for the direct writers.

This model is extended to include two types of consumers, under the assumption that agency writers can differentiate between risk types but direct writers cannot. An equilibrium is constructed in which agency writers offer full insurance contracts to both high and low risk consumers and direct writers, who earn positive profits, offer a separating menu of contracts. The presence of the screening contracts increases the utility of some of the low risk consumers who can purchase insurance at greater savings than in the symmetric information model and does not affect the utility of the high risk consumers.

The effect of the asymmetric information is to reduce the profits accruing to the direct writers.

The models in Chapter 2 differ from the previous literature in the amount of heterogeneity assumed to exist across consumers. These models allow for a continuum of consumer transaction costs and allow for differences in consumer risk types. The second innovation in this chapter is the introduction of the relationship between the insurance company's ability to discern risk types and its distribution technology.

In Chapter 3, the use of a separating menu of contracts in a multiple period framework is examined. Previous papers have embedded the traditional one period separating menu of contracts, as given by Rothschild and Stiglitz (1976), in a multiple period world. This one period sequence of contracts however is not dynamically consistent and does not satisfy the multiple period incentive compatibility constraints. The dynamically consistent contract introduced in this chapter corrects these shortcomings. The contract structure is such that insurers earn zero profits each period and no consumer has the incentive to misrepresent her type.

The separation decision of low risk consumers in a multiple period world in which both the dynamically consistent separating menu of contracts and pooling contracts are offered is examined. Numerical examples are provided to assist understanding of the theoretical results. It is shown that the cost of separation is so high that low risk consumers are better off pooling with high risk consumers for most of their lifetimes. If consumers pool for their entire lifetime, then it is impossible to know whether consumers and firms are behaving myopically or in a dynamically consistent manner. Finally, a comparison of the utility earned by consumers if they purchase the dynamically consistent menu of contracts or a series of one period Rothschild-Stiglitz menu of contracts confirms the fact that the inability of consumers to commit not to rewrite future contracts once information has been revealed is expensive.

The effect of search costs in both one period and multiple period frameworks is the focus of Chapter 4. In these models, the consumer is assumed to know her probability of loss, the price she is willing to pay for insurance and the distribution of prices charged in the marketplace. She does not know the price charged by any specific firm. Insurance companies know the consumers' reservation function and the distribution of prices charged in the marketplace. They do not know each consumer's risk type and each firm underwrites a different proportion of high risk consumers in its portfolio.

In the one period model, firms offer a pooling contract to all consumers. Each firm picks a price to maximise its profits. The presence of consumer search costs allows most firms to earn positive expected profits on each contract sold. Firms with a lower valuation of the expected marginal cost of providing coverage earn greater expected profits than do firms whose contracts have a higher actuarial fair value. This model differs from previous models in the literature in that it does not require true heterogeneity among insurance companies and their products to generate market imperfections.

In the multiple period model, insurance companies use Bayesian updating to incorporate a consumer's accident history into its prices. Firms offer a pooling contract to all consumers with the same accident history. Each period a consumer updates the price she is willing to pay for insurance. The firm picks the profit maximising price to sell insurance based on the consumer's accident history, the prices that are being charged in the marketplace and its proportion of high risk consumers in its original portfolio. If the firm's price is greater than what the consumer is willing to pay, then the consumer will switch insurance companies. This model differs from previous models in the literature in that all consumers may have the incentive to switch insurance companies at some point, regardless of their accident history or their underlying risk type.

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Appendix A. Summary of Functions and Notations

Primitives		Reference
$-e^{-\alpha W}$	All consumers possess negative exponential utility.	
p_a	Price charged by perfectly competitive agency writers in symmetric information model.	
$e_a > e_d$	Agency writers incur a greater additive expense than do direct writers.	
p_a^{ℓ}, p_a^{h}	Prices charged by two types of perfectly competitive agency writers in asymmetric information model.	Lemma 2-3
Derived Values and Functions		**Reference**
ℓ_m	Location of indifferent consumer when direct writer acts as local monopolist.	$(2-3)$
$p_m, \Pi(p_m)$	Price charged and profits earned by a direct writer acting as a local monopolist.	Lemma 2-1
$p_c, \Pi(p_c)$	Price charged and profits earned by a direct writer competing with neighbouring direct writers.	Lemma 2-2
${}_{2.2}n_m$	Number of direct writers operating in equilibrium in symmetric information model.	Theorem 2-1
(p_m^{ℓ}, I_m) (p_m^{h}, d) $\Pi(p_m^{\ell}, p_m^{h})$	Contracts offered and profits earned by a direct writer acting as a local monopolist in asymmetric information model.	Proposition 2-1
$\ell_m^{\ell}, \ell_m^{h}$	Locations of indifferent low risk and high consumers when direct writer acts as a local monopolist in asymmetric information model.	$(2-5)$, $(2-6)$
(p_c^{ℓ}, I_c) (p_c^{h}, d) $\Pi(p_c^{\ell}, p_c^{h})$	Contracts offered and profits earned by a direct writer competing with other direct writers in asymmetric information model.	Proposition 2-1
$\ell_c^{\ell}, \ell_c^{h}$	Locations of indifferent low risk and high consumers when direct writer competes with other direct writers in asymmetric information model.	$(2-10)$, $(2-11)$
${}_{2.3}n_m$.	Number of direct writers operating in equilibrium in asymmetric information model.	Theorem 2-2

Table A - 1 - Primitives and Derived Functions for Chapter 2

Primitives		Reference
$-e^{-\alpha W}$	All consumers possess negative exponential utility.	
e	All firms incur an additive expense.	
	All firms are perfectly competitive.	
Derived Values and Functions		**Reference**
(p^{ℓ}, l) (p^{h}, d)	One period Rothschild-Stiglitz (1976) menu of contracts.	Lemma 3-1
(p^{0}, l^{0})	One period Wilson (1977) pooling contract.	Lemma 3-2
$\tilde{V}_0 = V(0)$	Utility earned by consumer, acting either myopically or non-myopically, if she pools for all n periods.	$(3-2)$
$\tilde{V}(n)$	Utility earned by consumer, acting myopically, if she purchases a separating contract for all n periods.	
(p_k^{ℓ}, l_k) (p^{h}, d)	In dynamically consistent model, separating menu of contracts offered in period k.	Proposition 3-1
$V(k)$	Utility earned by consumer in dynamically consistent framework if she separates in period k.	$(3-8)$
$k^{*}, \bar{k}^{*}$	Optimal period of separation, and approximation to the optimal period of separation.	Theorem 3-1

Table A - 2 - Primitives and Derived Functions for Chapter 3

Primitives		Reference
$Q(\lambda)$	Distribution of firm valuations of the proportion of high risk consumers in the portfolio of first time insurance consumers.	
$h(s)$	Distribution of search costs across all consumers.	
$-e^{-\alpha W}$	All consumers possess negative exponential utility.	
ρ_{ij}^{k}	Firms revise probabilities using Bayesian updating.	Lemma 4-1
Derived Values and Functions		**Reference**
$f(p)$	Consumer's reservation function.	$(4-2)$
p^{k}	Optimal price charged by insurer with valuation λ^{k} in single period model.	Proposition 4-1
$G(p)$	Distribution of prices charged by sellers in single period model.	$(4-5)$
$f_{ij}(p)$	Consumer's reservation function in multiple period model.	$(4-7)$
p_{ij}^{k}	Optimal price charged by insurer with valuation λ^{k} in multiple period model.	Lemma 4-2
$G_{ij}(p)$	Distribution of prices charged by sellers in multiple period model.	$(4-13)$

Table A - 3 - Summary of Primitives and Derived Functions for Chapter 4

Appendix B. Chapter 3 Results

α	0.25	0.25	0.60	0.60	0.90	0.90	1.25	1.25
ρ^h	15%	12.5%	15%	12.5%	15%	12.5%	15%	12.5%
k	I_k	I_k	I_k	I_k	I_k	I_k	I_k	I_k
0								
1	936.01	962.98	940.93	968.37	942.10	969.66	942.76	970.37
2	769.35	824.09	774.26	829.48	775.44	830.77	776.09	831.48
3	602.68	685.20	607.60	690.59	608.77	691.88	609.42	692.60
4	436.01	546.31	440.93	551.70	442.10	552.99	442.76	553.71
5	269.35	407.42	274.26	412.82	275.44	414.10	276.09	414.82
6	102.68	268.54	107.60	273.93	108.77	275.21	109.42	275.93
7		129.65		135.04		136.32		137.04

In all scenarios $d = 1000$ and $\rho^{\ell} = 10\%$.

Table B - 1- Indemnity Offered in the Separating Contract

Table B - 1 gives the amount of indemnity offered in the partial insurance contract in the period of separation for various parameter values. These values correspond to the graphs in Figure 3 - 4.

From this table, it is evident that the risk aversion coefficient has only a small effect on the amount of indemnity offered in the separating menu of contract, and this effect is greater the smaller the risk aversion coefficient. This relationship can be seen graphically in Figure 3 – 5. More dramatic is the effect of the difference in the loss probabilities on the amount of indemnity offered.

Proposition B - 1: *The function* $\overline{V}(k)$, *as defined in Theorem 3 – 1, possesses a unique maximum with respect to the variable* k.

Proof: From *Theorem 3 – 1,* $\overline{V}(k)$ is defined by

$$\overline{V}(k) = -e^{-\alpha(W-ne)} e^{-\alpha(k-1)\rho^{\ell} d} e^{\alpha\rho^{\ell} l_k} \left[\rho^{\ell} e^{\alpha(d-l_k)} + 1 - \rho^{\ell}\right] \left\{ e^{\alpha(n-k)\rho^{0} l^{0}} \left[\rho^{\ell} e^{\alpha(d-l_0)} + 1 - \rho^{\ell}\right]^{n-k} \right\}$$

To show that this function possesses a unique maximum, it is necessary to show that the second derivative of $\overline{V}(k)$ is strictly negative. Differentiating $\overline{V}(k)$ twice with respect to k and simplifying yields:

$$\frac{\partial^2 \overline{V}(k)}{\partial k^2} = \overline{V}(k) \left\{ \left[-a + \alpha\rho^{\ell} \frac{\partial l_k}{\partial k} * \frac{e^{\alpha(d-l_k^{*})} - 1}{e^{\alpha(d-l_k^{*})} + \frac{\rho^{\ell}}{1-\rho^{\ell}}} \right]^2 + \alpha\rho^{\ell} \frac{\partial^2 l_k}{\partial k^2} * \frac{e^{\alpha(d-l_k^{*})} - 1}{e^{\alpha(d-l_k^{*})} + \frac{\rho^{\ell}}{1-\rho^{\ell}}} \right.$$

$$\left. - \alpha^2 \rho^{\ell} \frac{\partial l_k}{\partial k} * \frac{e^{\alpha(d-l_k^{*})}}{\left(1-\rho^{\ell}\right)\left(e^{\alpha(d-l_k^{*})} + \frac{\rho^{\ell}}{1-\rho^{\ell}}\right)^2} \right\},$$

where a has been previously defined as $a = \left[\alpha\left(-\rho^{\ell} d + \rho^{0} l^{0}\right) + \log\left(1 - \rho^{\ell} + \rho^{\ell} e^{\alpha(d-l^{0})}\right)\right]$.

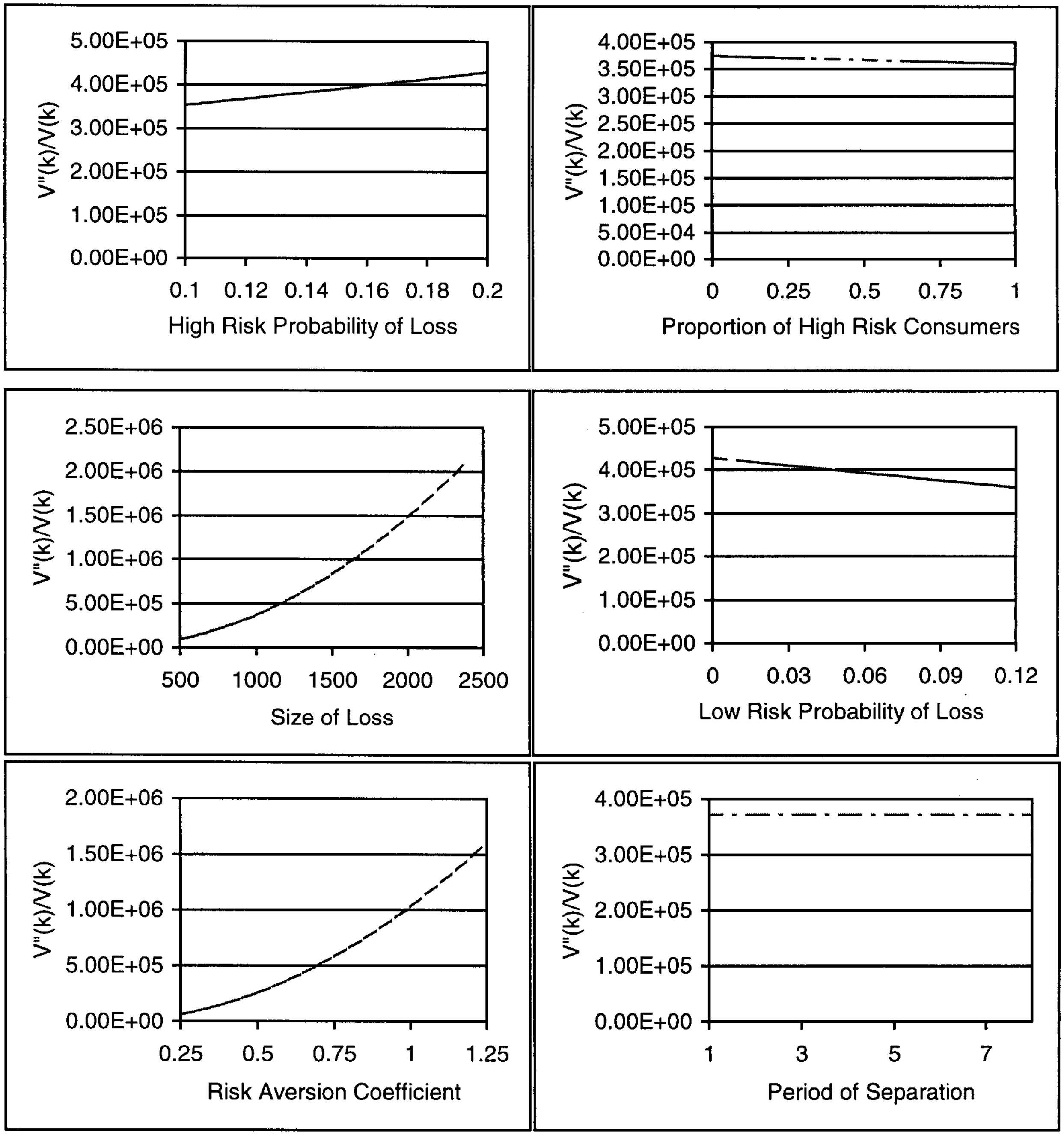

Figure B - 1 – Profiles of $\frac{\overline{V}''(k)}{\overline{V}(k)}$ with respect to Underlying Parameters.

Since $\overline{V}(k)$ is strictly negative, then $\dfrac{\overline{V}''(k)}{\overline{V}(k)}$ must be strictly positive for $\overline{V}''(k)$ to be less than zero. Unfortunately, the terms in the curly bracket cannot be signed, and as such numerical methods are needed to ascertain the sign of $\dfrac{\overline{V}''(k)}{\overline{V}(k)}$.

Profile graphs of $\dfrac{\overline{V}''(k)}{\overline{V}(k)}$ with respect to the underlying variables, $\alpha, \lambda, \rho^h, \rho^\ell, d$ and k, are given in Figure B - 1. All the graphs are strictly positive over a moderate range of the underlying variables. Values for the variables in each graph that were not examined were set at $\alpha = 0.60$, $\lambda = 0.25$, $\rho^h = 0.12$, $\rho^\ell = 0.10$, $k = 4$ and $d = 1000$. In the profile graph with respect to α, the range of α examined corresponds to the range suggested by Haubrich (1994). The proportion of high risk consumers in the population, λ, was examined over the entire range from zero to one. In the profile graph with respect to the high risk's probability of loss, the range extends from the low risk's probability of loss upward to 20%. The range examined for the low risk probability of loss extended from zero to ρ^h. Due to computation constraints d was examined over a range of relatively small values. And finally, the range examined for k, the number of periods before the end in which separation occurs, is from one to eight. For values greater than eight, insurers do not offer a separating menu of contracts.