# Dynamic Modelling for Foreign Exchange Rates

by

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# Abstract

Since the advent of generalized floating exchange rates in 1973, the behavior of exchange rate movements has become an extremely challenging research area. Given the importance of a good understanding of foreign exchange rate dynamics in international asset pricing theories, international portfolio management and policy-oriented questions of an open economy, it is not surprising that exchange rate economics has been the subject of many investigations during the past two decades. Recent research indicates that standard linear macroeconomic models generally fail to improve upon the simple random walk model in out-of-sample forecasting. We present an empirical study (based on over 20 years of monthly data) of several models with dynamic state structure to forecast the foreign exchange rates of seven major currencies. As part of our study, we also employ various measures and visualization techniques to evaluate the performance of our candidate models in terms of expectation and risk forecastability. In addition to the commonly used statistical measures, we compare the models in seemingly practical situations. The performance measurement methodology for risk management is based on the concept of

Value-at-Risk. The general conclusion of our study is that the performance of the forward rate model tends to be dominative. Our dynamic models are still not able to outperform the simple random walk for most of the studied exchange rates when performance is based upon the statistical measures; however, the simple random walk may not be unbeatable when performance is gauged from a practical point of view.

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# Chapter 1

# Introduction

As the Bretton Woods system fell in the early 1970s, the direct link of currencies to a gold standard with fixed parities exploded, and the international monetary system progressively evolved toward a system of floating exchange rates. Under the current system, the price of each currency is determined by the market forces, and thus, macroeconomic interdependence has markedly increased. External events, such as oil crises, trade deficits, bank failures and debt defaults, often lead to excessive exchange rate movements, which in turn, cause substantial changes in terms of trade and international competitiveness.

The inter-playing relationship between exchange rate movements and the health of an open economy makes exchange rate modeling and forecasting an interesting but yet challenging subject. A wide range of theoretically appealing models have arisen to explain the floating exchange rate experience. These models are mainly developed with the monetary theory of exchange rates. Its basic rationale is that since an exchange rate is the price of one country's money in terms of that of another, it is essential to analyze the determinants of the price in terms of the outstanding stocks of and the demand for the two monies. More explanation of the monetary theory will be given in Section 2.2. However, research indicated that these popular macrofundamental models generally failed to improve upon the naive random walk model over out-of-sample period; for example, Meese and Rogoff (1983). Murray (1997) concluded in his summary report for the Bank of Canada Conference that the application of standard linear models, relating exchange rate movements to macroeconomic fundamentals, had been shown to be incapable of explaining the foreign exchange dynamics, especially over short time periods. Some reported causes of the failure included improper functional form and time-deformation. Meese and Rose (1991) employed a non-parametric procedure to study the dependence of monthly exchange rates on key macroeconomic fundamentals for the five major OCED countries. Their findings suggested that poor performance of the models cannot be attributed to nonlinearities. Some researchers adopted a linear structure for exchange rate modeling but advocated dynamic structure for state parameters over the floating period. They believed that changes in market atmosphere, such as policy regime, introduced instability to the system of the exchange rate models, and therefore, the relationship may only be adequately mirrored by allowing changes in model parameters from time to time. Researchers worked along this direction included Wolff (1987), Schinasi and Swamy (1989), and Liu and Susko (1992). Wolff (1987) and Schinasi and Swamy (1989) studied the "time-varying" parameter (TVP) models; Liu and Susko (1992) studied a model with "parameter-shifting" signaled by an indicator variable. More detailed discussions of their studies will be presented in Section 2.3.

In our study, we adapt several dynamic models to forecast exchange rates and then compare their forecasting performance with those of the simple random walk and the forward rate model. Two of our models replicate the ideas from Wolff (1987) and Liu and Susko (1992). All of the candidate models are structured to deal with non-homogeneous relationships. They share a simple linear functional form as their basic equations for observations but differ in their setups for system equations. The linear observation equation relates exchange rate to its 1-month lagged rate and an interest rate index. Details of the model specification can be found in Section 3.2. Except for simplicity, one reason for narrowing down the list of many conventional macroeconomic fundamentals to that of only the interest rate is that, among all the variables, interest rate quotes are always closely observed by exchange market participants, and thus, it is often included as a factor in most of the macroeconomic models for explanatory purpose. In fact, this simple structure is implied in the forward rate model. The forward rate model, when combined with the international parity, is a model of relating foreign exchange rate to interest rate differential. Other reasons include poor availability and inconsistency of data as observed in the series of money supplies.

There is a subtle difference between the forward rate model and our models even though they all consider the interest rate as the only determinant for exchange rate movements. The forward rate model looks at the interest rate difference between the two countries involved, while our models try to use the interest rate variable in a broader sense. Liu and Susko (1992) first suggested that exchange rate movements may be determined by more than the two countries under investigations as international tradings are becoming more frequent, and economies are globalizing. They combined the interest rate differentials of the major trading partners in the world, the G-7 countries[1], to construct *an effective principal index.* We follow this idea to construct an effective interest rate index to determine the foreign exchange movements.

The purpose of our study is not simply to study the different candidate models but also to present the different subtleties in evaluation of models. Our evaluation method is divided into two aspects: the forecastability in expectation and the ability in risk management. In addition to the common statistics of *mean error*, *mean absolute error* and *mean squared error*, we compare the models on the basis of "buy-sell" signals generated from their forecasts of expected values. Moreover, we look at returns generated from a forward trading strategy guided by the forecasts. In the context of risk management, evaluation is based upon two portfolios with opposing positions — *long* and *short* on

[1]As a group, the G-7 countries, including United States, Canada, United Kingdom, Italy, France, Germany, and Japan, account for a large portion of total world trade and international financial flows.

the US dollar against another currency, i.e., buy in and sell off the US dollar for another currency. The monitors are the *exceedence frequency* and the *Lopez loss function.*

To demonstrate how the implementation of a dynamic system works in exchange rate forecasting, we analyze our candidate models for the following six key United States Dollar (USD) nominal bilateral exchange rates:

British Pounds (GBP)

French Francs (FRF)

German Marks (DEM)

Italian Lira (ITL)

Canadian Dollar (CAD)

Japanese Yen (JPY)

The exchange rate data are monthly prices for the value of 1 US Dollar. The first price of each of the six series refers to December 1972, the beginning of the floating experience, and the last price refers to December 1998. The bank rate is used as a proxy for the interest rate which is defined by the International Monetary Fund (IMF) as "the rate at which the monetary authorities lend or discount eligible paper for deposit money banks." All of the exchange rate and interest rate series are obtained from the International Financial Statistic (IFS) released by the International Monetary Fund (IMF) and are cited as the end-of-month quotes.

This thesis is organized as follows. We begin with a review of common foreign exchange parities, which is followed by an introduction of popular econometric models, and system equations applied in the modeling. We then discuss the two important modeling ideas in our study: the *principal effective index of interest rate differentials* and the *dynamic state structure.* The appropriateness of implementing a dynamic state structure is justified through the use of graphical tools. A detailed specification of our models is given in Chapter 3. The different subtleties in evaluation of our models and the results of model comparisons are presented in Chapter 4, along with a discussion of the nature of each measure. In Chapter 5, we conclude with reports of our general findings, and summarize some innovations and recommendations for our study.

# Chapter 2

# Review of Exchange Rate Economics and Models

This chapter gives a brief literature review of empirical models and dynamic system equations used in exchange rate determination. To be complete, we devote the first section to introduce common, simple parity conditions relating to foreign exchange rates. These relations are important in providing some of the basic building blocks for many of the current models.

## 2.1 Foreign Exchange Parity Relations

Given the complexity of multicurrency environment, it is useful to start understanding foreign exchange behavior by understanding the interlink of various domestic and foreign monetary variables. Before presenting some of the basic models for exchange rate determination in the next section, we shall recall well-known international parity conditions linking domestic and foreign monetary variables: *inflation rates*, *interest rates*, and *foreign exchange rates.* The common theoretical parity relations are:

1. the *purchasing power parity relation*, linking spot exchange rates and inflation;

2. the *international Fisher relation*, linking interest rates and inflation;

3. the *foreign exchange expectation relation*, linking forward exchange rates and expected spot exchange rates;

4. the *interest rate parity relation*, linking spot exchange rates, forward exchange rates and interest rates.

These economic concepts create a useful framework in which to analyze the international interplay of the monetary variables. A brief review of these theoretical relations is given below. At this point, it is convenient to establish the following notational conventions:

- $S_0$ is the spot exchange rate, expressed as the foreign price of one unit of the domestic currency, at the start of the period,

- $S_1$ is the spot exchange rate at the end of the period,

- $s = \frac{(S_1 - S_0)}{S_0}$ is the exchange rate movement,

- $I_F$ ($r_F$) is the inflation (interest) rate, over the period, in the foreign country,

- $I_D$ ($r_D$) is the inflation (interest) rate, over the period, in the domestic country,
- $F$ is the forward exchange rate, quoted at the start of the period for delivery at the end of the period,
- $f = \frac{(F-S_0)}{S_0}$ is the forward discount, or premium.

**Purchasing Power Parity (PPP)** states that the spot exchange rate adjusts perfectly to the price level or inflation difference between two countries. Intuitively, it says that same goods should be worth the same in different countries. There are two versions of PPP: *absolute* PPP and *relative* PPP. Absolute PPP says that the real price of a good must be the same in all countries; relative PPP focuses on the general across-the-board inflation rates in two countries and claims that the exchange rate movement should exactly offset any inflation differential. In its relative form, the PPP can be expressed as

$$\frac{S_1}{S_0} = \frac{(1+I_F)}{(1+I_D)},$$

or, by first-order approximation,

$$s = \frac{(S_1 - S_0)}{S_0} \approx I_F - I_D.$$

**International Fisher Relation** states that the interest rate differential between two countries should be equal to the expected inflation rate differential over the term of the interest rate, under the assumption of equal real interest rates across the world. As long as the expectations exist, the relation can be expressed as

$$\frac{(1+r_F)}{(1+r_D)} = \frac{(1+\mathrm{E}(I_F))}{(1+\mathrm{E}(I_D))}.$$

Again, by linear approximation, it becomes

$$r_F - r_D \approx \mathrm{E}(I_F) - \mathrm{E}(I_D).$$

**Foreign Exchange Expectations** states that the forward exchange rate is the rational, expected value of the future spot exchange rate in a risk-neutral world. It is equivalent to

$$F = \mathrm{E}(S_1) \quad \text{or} \quad f = \mathrm{E}(s).$$

**Interest Rate Parity** states that the interest rate differential must equal approximately to the forward discount, or premium, i.e., $f \approx r_F - r_D$. This equation is the first-order linear approximation of the exact relation, expressed mathematically as

$$\frac{F}{S_0} = \frac{(1+r_F)}{(1+r_D)}.$$

The interest rate parity is not an economic theory but rather a technical arbitrage condition, which must hold under the assumption of no riskless arbitrage opportunity. Even though deviations from the interest rate parity can be quite large for the closed and less-developed countries, it is verified on free money markets, and the continuing international integration of financial markets throughout the world is certain to reduce the deviations in the near future.

Except for the interest rate parity, empirical supports for the other parities are poor, especially over the short term period. In spite of the empirical invalidity, we still gain from the parity conditions a rough idea of the interrelations between the monetary variables and, more importantly, several implications about the exchange rate movements:

1. Interest rate differentials reflect expectations about currency movements. *(Foreign Exchange Expectations and Interest Rate Parity)*
2. Exchange risk reduces to inflation uncertainty. *(Purchasing Power Parity)*

## 2.2 Some Exchange Rate Models

The behavior of foreign exchange rate and its relationship with macroeconomic fundamentals, such as the monetary variables, have been subjects of many investigations. There are two common approaches used by fundamentalists[1] in foreign exchange rate determination: the *balance of payments approach* and the *asset market approach.*

[1]Some researchers resort to technical analysis, which bases predictions solely on the identification of supposedly recurring patterns in graphs of exchange rate movements, rather than on economic theory.

The "balance of payments" is a cash balance statement of a country relative to the rest of the world; it tracks all financial flows crossing the borders of a country during a given period. The balance of payments approach is traditional to foreign exchange rate determination which focuses on the relations between 'current' statement accounts and spot exchange rates. The more recent approach is the asset market approach. It has a different viewpoint from the traditional balance of payments approach. The proponents of the asset market approach argues that exchange rates are determined by expectations about the future, not 'current' trade flows — exchange rates are affected by news and expectations about future economic prospects, rather than by international flows, which have already been expected. It is this asset market approach that gives the fundamentals of our models.

The simplest model falling under the category of the asset market approach is the forward rate model. It has the usual setup as

$$\log S_t = \log F_{t-1} + \epsilon_t,$$

where $S_t$ is the spot rate at time $t$, and $F_{t-1}$ is the forward rate quoted at $t-1$ for delivery at $t$. From the interest rate parity, we can see that

$$\log F_{t-1} = \log S_{t-1} + \log(1 + r_{Ft-1}) - \log(1 + r_{Dt-1}),$$

where $r_{Ft-1}$ and $r_{Dt-1}$ are the respective foreign and domestic interest rates for the period between $t-1$ and $t$. For small $r$, a Taylor approximation leads to $\log(1+r) \approx r$. Hence the forward rate model is approximately

$$\log S_t = \log S_{t-1} + (r_{Ft-1} - r_{Dt-1}) + \epsilon_t.$$

Other well-known asset market models include the *monetary models* and the *portfolio balance models.* These econometric models are often structured in accordance with economic theories. The origins of the monetary theory can be traced back to Lord Keynes in 1924, as discussed by Frenkel (1976). The basic rationale of the monetary models is that an exchange rate is determined by the outstanding stocks of and the demand for the two monies involved since it is the price of one currency in terms of another. Usually, the models are relied on the twin assumptions of the purchasing power parity (PPP) and the existence of stable money demand functions for the domestic and foreign economies. The demand for money is most often assumed to be of the Cagan functional form, see Cagan (1956):

$$\frac{M}{P} = \kappa \exp\{-\gamma r\}\, y^{\eta},$$

where $M$ is the stock of money demanded, $P$ is the price level, $r$ is the rate of interest, $y$ is the level of real income, and $\kappa$, $\gamma$, and $\eta$ are parameters. The PPP condition is usually interpreted in the absolute manner as

$$S = \frac{P_F}{P_D},$$

where $P_F$ and $P_D$ denote the foreign and domestic price levels, respectively. The exchange rate can then be determined by substitution and manipulation of the two equations giving

$$S = \frac{M_F}{M_D}\frac{\kappa_D}{\kappa_F}\left[\frac{y_D}{y_F}\right]^\eta \exp\{-\gamma(r_D - r_F)\}.$$

To distinguish between the quantities of the foreign country and those of the domestic country, the subscripts $D$ and $F$ are imposed to represent the quantities of the domestic and foreign countries, respectively. A log transformation finally leads to

$$\log S = (\log M_F - \log M_D) + (\log \kappa_D - \log \kappa_F) + \eta(\log y_D - \log y_F) + \gamma(r_F - r_D).$$

Assuming $\kappa_D = \kappa_F$ results in the *flexible-price monetary model.* Other variants of the monetary models include the *sticky-price model* (see Dornbusch (1976)) and the *real interest differential model* (see Frankel (1979)). The *portfolio balance model* inherits the rationale of the monetary theory, but it also incorporates the "current account" of the balance of payments as a principal determinant of the exchange rates movements. For detailed discussion, see Allen and Kenen (1980), Dornbusch and Fischer (1980), and Branson (1984).

## 2.3 Dynamic System Equations

Many different system equations have been applied to the different exchange rate models to test for their empirical validity. In their earlier paper, Meese and Rogoff (1983) used the simple regression to test the exchange rate models corresponding to the flexible-price and the real interest rate differential approaches. They compared the out-of-sample performance of their models to the performance of the simple random walk model. The general conclusion is that none of the models outperforms the random walk. In a later paper, Meese and Rogoff (1984) reworked the results by imposing coefficient constraints in the estimation procedures, and they still obtained a similar conclusion.

Both techniques adopted by Meese and Rogoff share a characteristic — relations are assumed to be structurally homogeneous. However, exchange rate equations are unlikely to stay constant in the recent floating exchange rate experience. Parameter instabilities arise when there are changes in policy regime and heterogeneous beliefs by market participants. Liu and Susko (1992) studied the relation between their interest rate differential index and the exchange rate of Germany against the United States. Their results supported a 'piece-wise' linear functional pattern of which different linear forms present in different subdomains of the domain of an independent variable, and they modeled the relation by the *segmented regression*; for details, see Liu, Wu and Zidek (1997). They argued that inordinately high or low values of certain explanatory variables triggered different kinds of actions from market participants, such as governments, central banks and investors, and thereby led to instable state structure. Better out-of-sample forecasts were obtained for the period studied; however, consistency of such a model with future data was questionable.

The idea of implementing "time-varying" parameter (TVP) models as preferred method for forecasting foreign exchange rates was considered by Wolff (1987) and Schinasi and Swamy (1989). The motivation was that deformation may be due to the effect of a number of factors, such as oil crises and policy changes, that could not be explicitly included as independent variables. To accommodate potential regime shifts, Wolff (1987) adopt a random walk coefficient structure, which is a special case of the dynamic linear models (DLM)[2], for the reduced forms of the flexible-price and the real interest differential models. His examination suggested that no improvement over the the simple random walk model were obtained for most of the major currencies. Schinasi and Swamy (1989) reworked Wolff's results by using less restrictive parameter patterns, but yet not much achievements were gained.

Multiprocess mixture models, originally developed by Harrison and Stevens (1971) and taken further by Gordon and Smith (1988, 1990), are also candidates to model a relationship subject to regime shifts and outliers. Some researchers have argued that a simple, single DLM may be inappropriate to model a relationship, which involves abrupt regime changes or outliers. In the cases of rapid discontinuous shifts in parameters or sudden outlying values, the simple DLM estimates for variances are undesirable and unrepresentative in the sense that they reflect average variability over time. On the other hand, the mixture models run multiple DLM models in parallel to provide separate models for accommodating outlying observations, abrupt structural shifts in

[2]The dynamic linear model (DLM) was first suggested by Harrison and Stevens (1976) and later refined by West and Harrison (1989)

parameters, and steady states. They are more likely to capture regime shifts if they are abrupt. The mixture models have been employed to model many econometric relations, such as the relation of interest rates to weekly monies by LeSage (1992), and they may be applicable to model exchange rates.

# Chapter 3

# Candidate Models

This chapter is divided into two main sections. Before providing details of our models of exchange rates in Section 3.2, we devote the Section 3.1 to discuss the two important features of our strategy in exchange rate modeling. The first feature is the construction of an effective index; the other feature is the proposal of a dynamic state structure.

## 3.1 Modeling Features

In this section, we explain the construction of the effective principal index, and we also justify graphically the appropriateness of implementing the dynamic system.

### 3.1.1 Principal Effective Index

When studying the quarterly prices of German marks against the United States dollars, Liu and Susko pointed out the weak relation between the exchange rate and the simple "between-the-two-country" interest rate differential and urged the use of a summary index of the "many-country" interest rate differentials. They proposed a way to construct an effective principal index for the summarizing purpose.

Similar to the index used by Liu and Susko (1992), our effective interest rate index is a weighted average of the interest rate differences between the United States and the other six major industrial countries. We employ *principal component analysis* (PCA) to determine the weights assigned to the individual differentials. The procedure goes as follows. At any given month $t$, we run a PCA on the $60 \times 6$ data matrix of the six interest rate differences corresponding to the period of the five most recent years from $t$. Then the loadings of the first principal component are treated as the weights for time $t$. The resulting index is a lively variable in the sense of having high variability. Notice that the PCA is used as an explanatory tool, and the assumptions behind it are ignored.

### 3.1.2 Dynamic State Structure

In the paper of Liu and Susko (1992), a segmented regression, with the interest rate index as the segmentation variable, was suggested to model the observed V-shaped relationship between the exchange rate and the index. As have mentioned in Section 2.3, this setup indicates that there is a shift in the coefficients, and the shift is signaled by extraordinary change in the interest rate index. With our monthly data, we still obtain a similar V-shape for the same studied period; however, the segmented form does not seem to confirm with the new updated data. We believe that there could be some factors other than the index triggering the shifting, and these factors might be difficult to be incorporate as explanatory variables. Moreover, we believe that the dynamics of the parameters may be accommodated via time-series.

To explore the time series structure of the state parameters, we estimate a sequence of coefficients from a sequence of successive data sets across time. To construct the sequence of data sets, we start with a fixed window size, in which sample is presumably structurally homogeneous, and extract a sample of that size to become the first element of the data sequence; then we successively slide the fixed window, one month at a time, across the time horizon to form the whole sequence. Sequences of ordinary least square estimates of the coefficients are obtained from these successive samples. Several values for the window sizes have been tried. We provide the time-series plots for the 5-year and 10-year windows in Appendix A. For the 5-year window, the first data element is the sample corresponding to the period between December 1972 and December 1977; for the 10-year window, the first data element is the sample corresponding to the period between December 1972 and December

1982. From the plots, we could see that the dynamics of the state parameters are unlikely to be constant and in fact likely to be changing in some specific ways, possibly linearly or discontinuously, over time. The 5-year plots may suggest a random walk fashion for the state structure while the 10-year plots may suggest discontinuous shifts. There may be abrupt structural shifts in the coefficients, occurring in early-nineteenth century, for almost all of the six exchange rate series.

## 3.2 Model Specification

### 3.2.1 Introduction

As implied in the exploratory analysis, the relationship between the exchange rate and the interest rate index tends to be non-homogeneous over time. In our study, we investigate five models with different dynamic state structure. Along with two yardstick models, we analyze the following seven models:

1. Random walk model
2. Forward rate model
3. Simple regression
4. Stone's formulation for linear dynamic functional form
5. Threshold dynamic functional form/Segmented regression
6. Random coefficient regression with random walk system function

7. Multiprocess mixture model of random coefficient regression with random walk system function

Models 1 and 2 are two simple, common models for exchange rate determination. The serve as yardsticks for comparisons. The other models share a simple linear functional form for their observation equations —

$$\log S_t = \beta_0 + \beta_1 \log S_{t-1} + \beta_2\, ir_{t-1} + \epsilon_t,$$

where $S_t$ is the price of the foreign currency per 1 US dollar at time $t$, $ir_t$ is, as those constructed in Section 3.1.1, the principal index of interest rate differential at time $t$, and $\{\epsilon_t\}$ is the error sequence. Notice that the log transformation on $S_t$ is a common practice to guarantee that there will always be a zero probability of negative prices. Different setups of the system equations are assumed for the last five models. Model 3 is the simple regression model, which has constant coefficients. In models 4 and 5, the regression coefficients $\beta$'s change systematically with the calender time and the index, respectively. Models 6 and 7 assume a random walk structure for each of the coefficients. Details of the models will be given later in this chapter.

For each model, our primary focus is to predict the log price in terms of a conditional probability distribution in a *univariate* context; that is, candidate models are applied to each of the six exchange rate series separately to produce forecast distributions. The last 108 (9 years) observations of each of the exchange rate series are reserved to construct the out-of-sample predictions, which are then analyzed in different ways to evaluate the performance of the models. We compute our forecasts as follows. First, candidate models are estimated using the in-sample data from the beginning of the sample, December 1972, to December 1989, and forecasts are made for 1-month ahead. The data for January 1990 are then added to make a new in-sample data set, the models are re-estimated, and a further set of forecasts is made for the next period. This "rolling estimation and prediction" process is then repeated continually.

### 3.2.2 Detailed Description

We present below a detailed specification for the construction of the forecast probability distribution as prescribed by each of the seven models . Before the presentations, we establish the following nomenclature and notational guidelines for this and subsequent chapters.

- Exchange rate quotations and interest rate differential indices are numbered from -204 to 108 so that $t = 0$ refers to the month of December 1989.

- Forward rate quotations are numbered similarly. But they are not available from the beginning of the period. The earliest non-missing quotes belong to January 1986.

- $S_t$ and $F_t$ represent the exchange rate and forward rate at time $t$, respectively.

- $s_t = \log S_t$ is the log transformed exchange rate at $t$, and $f_t = \log F_t$ is the log transformed 1-month forward rate at $t$. Be careful not to confuse these new notations with those from previous chapters — the letters '$s$' and '$f$' DO NOT represent the the exchange rate movements and forward premiums, respectively.

- $ir_t$ is the index of interest rate differentials at $t$.

- $\boldsymbol{\beta}_t = (\beta_0, \beta_1, \beta_2)'_t$ is the state vector at time $t$; $\boldsymbol{X}_t = (1, s_{t-1}, ir_{t-1})'$ is the vector of explanatory data at time $t$.

- The nomenclature *prediction-realization pairs* refer to the ordered mathematical objects $(p_{t-1}, s_t)$ and $(P_{t-1}, S_t)$, where $p_{t-1}$ and $P_{t-1}$ are defined to be the forecast distributions, which depend only on data until $t-1$, for the realizations $s_t$ and $S_t$, respectively.

- A typewriter type style `t-1` is imposed to subscript any updating difference in the estimates of the parameters for $p_{t-1}$ or $P_{t-1}$ due to the rolling-over procedure.

### Simple Random Walk Model

This model basically serves as a yardstick based upon which the other models are compared. The model takes the form of

$$s_t = s_{t-1} + \epsilon_t,$$

where $\{\epsilon_t\}$ is assumed to be independent, identically and normally distributed with mean 0 and variance $\sigma^2$ (abbreviated as IID $\mathrm{N}(0, \sigma^2)$). The diagnostic checking of the independence and normality assumptions are provided in Appendix B. For each trial of the rolling-over process, the successive estimates $\hat{\sigma}^2_{t-1}$ are generated from the sample variance of the noise sequence $\{\epsilon_i = s_i - s_{i-1}\}$ in the in-sample period:

$$\hat{\sigma}^2_{t-1} = \frac{1}{203+t} \sum_{i=-204}^{t-1} {\epsilon_i}^2.$$

Therefore, the conditional forecast distribution $p_{t-1}$ for $s_t$ is a Gaussian distribution with mean $s_{t-1}$ and variance $\hat{\sigma}^2_{t-1}$.

### Forward Rate Model

This is another yardstick model. It has a similar structure as the random walk, $s_t = f_{t-1} + \epsilon_t$, and it also inherits the same assumptions for the noise sequence; refer to Appendix B for diagnostic justification. The successive estimates for $\sigma^2$ can be defined, with some adjustments to the missing data, analogously as

$$\hat{\sigma}^2_{t-1} = \frac{1}{46+t} \sum_{i=-47}^{t-1} (s_i - f_{i-1})^2.$$

### Simple Regression

This is a constant coefficient model represented by

$$s_t = \boldsymbol{\beta}_t' \boldsymbol{X}_t + \epsilon_t.$$

The constant coefficient vector $\boldsymbol{\beta}_t = \boldsymbol{\beta}$ is estimated by the ordinary least square (OLS) method to minimize the sum of squared error. Diagnostic plots for autocorrelation of the estimated residuals $\hat{\epsilon}_t = s_t - \hat{\boldsymbol{\beta}}' \boldsymbol{X}_t$ suggest white noise for the residual sequence, and the qq-plots suggest little deviations from normal distribution (refer to Appendix B). Therefore, $\{\epsilon_t\}$ is assumed to be IID $N(0, \sigma^2)$. The variance of the noise sequence is estimated by the sample variance of the estimated residuals. As a result, $p_{t-1}$ is a normal distribution with mean $\hat{\boldsymbol{\beta}}'_{t-1} \boldsymbol{X}_t$ and variance $\hat{\sigma}^2_{t-1}$, where $\hat{\boldsymbol{\beta}}_{t-1}$ and $\hat{\sigma}^2_{t-1}$ are estimates based on the in-sample period.

**Linear Dynamic Functional Form**

In this model, we take account of the systematic changes in structural parameters by using the calendar time as a surrogate for the underlying economic causal factors. The general form of the trend for the parameters is assumed to be linear. The model is defined by[1]:

observation equation: $s_t = \boldsymbol{\beta}'_t \boldsymbol{X}_t + \epsilon_t,$

system equation: $\boldsymbol{\beta}_t = \boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 t.$

When combining with the system equation, the observation equation can be expressed as

$$s_t = \tilde{\boldsymbol{\beta}}'_t \tilde{\boldsymbol{X}}_t + \epsilon_t,$$

where $\tilde{\boldsymbol{\beta}}_t = (\boldsymbol{\alpha}_0, \boldsymbol{\alpha}_1)'$, and $\tilde{\boldsymbol{X}}_t = (1, s_{t-1}, ir_{t-1}, t, t\, s_{t-1}, t\, ir_{t-1})'$. Notice that it reduces to the constant coefficient model if we assume that $\boldsymbol{\alpha}_1$ is a zero vector. With this representation, we can see that the model is restored to a simple regression model with five explanatory variables. The estimations for the $\beta$'s and $\sigma^2$ are the same as those for the simple regression.

[1]Singh, Nagar, Choudhry and Raj (1976) add a disturbance term to the system equation, leading to a more general class of models called the *variable mean response* (VMR) model.

## Threshold Dynamic Functional Form — Segmented Regression

This is a replication of Liu and Susko's model. It considers the effective interest rate index as a signaling indicator of structural changes in the parameters and believes that the changes are abrupt. The model can be expressed as

observation equation: $s_t = \boldsymbol{\beta}_t' \boldsymbol{X}_t + \epsilon_t$,

system equation: $\boldsymbol{\beta_t} = \boldsymbol{\beta_i}$, if $ir_{t-1} \in (\tau_{i-1}, \tau_i]$, $i = 1, \ldots, l+1$,

where $-\infty = \tau_0 < \cdots < \tau_{l+1} = \infty$. In the case where $l = 0$, it is the simple regression model. Given an integer value for $l$, the estimation procedure for the $\boldsymbol{\beta}_i$'s and $\tau_i$'s goes as follows. We estimate the coefficients on each interval $(\tau_{i-1}, \tau_i]$ by the OLS method over all possible vectors $(\tau_0, \tau_1, \ldots, \tau_{l+1})$, where $\tau_0 < \tau_1 < \ldots < \tau_{l+1}$, $\tau \in \Theta$, and $\Theta$ is the set consisting of all of the values of the interest rate differential index. For each $(\tau_0, \tau_1, \ldots, \tau_{l+1})$, we calculate the total sum of squared error over the $l+1$ intervals. This iterative procedure is implemented in C language. The estimate $(\hat{\tau}_1, ..., \hat{\tau}_l)$ is the vector giving the minimum total sum of squared error, and the corresponding OLS estimated coefficients are the estimates for the $\boldsymbol{\beta}_i$'s.

The number of segments $l+1$ can be determined by graphical means or by minimizing the information criterion $MIC(l)$ modified from the Schwartz's criterion:

$$MIC(l) := \log\Big(\frac{S(\hat{\tau}_1, ..., \hat{\tau}_l)}{n+1}\Big) + l\,\frac{c\,(\log n)^{2+\delta}}{n},$$

where $S(\hat{\tau}_1, ..., \hat{\tau}_l)$ is the minimum total sum of squared error when the number of segments is $l+1$, $n$ is the number of observations, and $c$ and $\delta$ are constant parameters. Notice that the second term assigns more penalty to more number of segments. From some graphical investigations, we believe that the 2-segmented and 3-segmented models may be appropriate, and we consider both of them in our study. This selection is further confirmed by the $MIC$ criterion for some reasonable values of $c$ and $\delta$. Diagnostic checking on the estimated residual sequences for both models approves their independence and normality; see Appendix B.

**Random Coefficient Regression with Random Walk System Function**

This model is a dynamic generalization of standard, static regression model in which the state vector evolves only through the addition of a noise term. It can be expressed as

observation equation: $s_t = \boldsymbol{\beta}_t' \boldsymbol{X}_t + \epsilon_t,$

system equation: $\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \boldsymbol{\omega}_t.$

The usual assumption of mutual independence for the error sequences is applied. The system noise $\boldsymbol{\omega}_t$ provides an additive increase in uncertainty, or, equivalently, a loss of information, about the state vector between time $t-1$ and $t$. Setting $\boldsymbol{\omega}_t = 0$ leads to the specialization of static regression. The distribution assumptions for the noises are $\epsilon_t \sim \mathrm{N}(0, V\,R_\epsilon)$ and $\boldsymbol{\omega}_t \sim \mathbf{N}(\mathbf{0}, V\,\boldsymbol{R}_\omega)$, where N and **N** denote univariate and multivariate normal distribution, respectively, and

$$R_\epsilon = 1 \quad \text{and} \quad \boldsymbol{R}_\omega = \begin{pmatrix} R_{\beta_0} & 0 & 0 \\ 0 & R_{\beta_1} & 0 \\ 0 & 0 & R_{\beta_2} \end{pmatrix}.$$

The extra term $R_\epsilon = 1$ in the observation variance is not necessary for the parameterization here, but it is included to provide some convenience when specifying the next model. The inclusion of the observation variance $V$ in the state evolution variance is a common parameterization practice adopted for the constant model, in which both of the observational and evolution variances are constant in time. The observation variance $V$ is assumed to be constant but unknown. Implementing an unknown V exempts us from specifying precise values for the observation variance, and allows the estimation procedure to provide and to update an estimate for it when new information arrives through time. Denote by $\phi$ the reciprocal of the unknown variance $\phi = V^{-1}$, and assume the following initial conditions:

$$
\begin{aligned}
(\boldsymbol{\beta}_{-204} \mid D_{-204}, \phi) &\sim \mathrm{N}(\boldsymbol{m}_{-204}, V\,\boldsymbol{C}^{\star}_{-204}),\\
(\phi \mid D_{-204}) &\sim \mathrm{G}(n_{-204}/2, d_{-204}/2),
\end{aligned}
$$

where G stands for the gamma distribution so that $\mathrm{E}(\phi \mid D_{-204}) = n_{-204}/d_{-204}$ is the reciprocal of a prior point estimate of the observation variance $V$: $\hat{V} = S_{-204} = d_{-204}/n_{-204}$. For notational convenience, we assume that the information set $D_{-204}$ contains all the future values of $ir$ so that $D_t = \{s_t, D_{t-1}\}$. The two initial, conditional variables and the two error sequences are again assumed to be mutually independent. Based on standard Bayesian theory, the distribution of $\boldsymbol{\beta}_{-204}$, conditional on $D_{-204}$ but unconditional on $\phi$, is a multivariate T-distribution with mean $\boldsymbol{m}_{-204}$ and covariance matrix $\boldsymbol{C}_{-204} = S_{-204}\,\boldsymbol{C}^{\star}_{-204}$.

With the defined model structure, we can apply a conjugate sequential updating procedure, an extended Kalman filter, to use the updated system from one time to provide a prior distribution for the next, which is revised using the observation equation to obtain a posterior distribution when new information arrives. Theorem 4.3 in West and Harrison (1989) states the following conjugate distributional analysis:

$$\begin{aligned}
(\phi \mid D_{t-1}) &\sim \mathrm{G}(n_{t-1}/2, d_{t-1}/2) \quad \text{with} \quad S_{t-1} = d_{t-1}/n_{t-1}, \\
(\boldsymbol{\beta}_{t-1} \mid D_{t-1}) &\sim \mathbf{T}_{n_{t-1}}(\boldsymbol{m}_{t-1}, \boldsymbol{C}_{t-1}), \\
(\boldsymbol{\beta}_t \mid D_{t-1}) &\sim \mathbf{T}_{n_{t-1}}(\boldsymbol{m}_{t-1}, \boldsymbol{R}_t), \\
(s_t \mid D_{t-1}) &\sim \mathrm{T}_{n_{t-1}}(f_t, Q_t), \\
(\phi \mid D_t) &\sim \mathrm{G}(n_t/2, d_t/2) \quad \text{with} \quad S_t = d_t/n_t, \\
(\boldsymbol{\beta}_t \mid D_t) &\sim \mathbf{T}_{n_t}(\boldsymbol{m}_t, \boldsymbol{C}_t),
\end{aligned}$$

where

$$\begin{aligned}
\boldsymbol{R}_t &= \boldsymbol{C}_{t-1} + S_{t-1}\,\boldsymbol{R}_\omega, \\
f_t &= \boldsymbol{X}'_t\,\boldsymbol{m}_{t-1}, \\
Q_t &= \boldsymbol{X}'_t\,\boldsymbol{R}_t\,\boldsymbol{X}_t + S_{t-1}\,R_\epsilon, \\
e_t &= Y_t - f_t, \\
n_t &= n_{t-1} + 1, \\
d_t &= d_{t-1} + S_{t-1}\,e_t^2/Q_t, \\
\boldsymbol{A}_t &= \boldsymbol{R}_t\,\boldsymbol{X}_t/Q_t, \\
\boldsymbol{m}_t &= \boldsymbol{m}_{t-1} + \boldsymbol{A}_t\,e_t, \\
\boldsymbol{C}_t &= \frac{S_t}{S_{t-1}}(\boldsymbol{R}_t - \boldsymbol{A}_t\,\boldsymbol{A}'_t\,Q_t).
\end{aligned}$$

The symbol T and $\mathbf{T}$ represent univariate and multivariate t-distribution, respectively. The forecast distribution $p_{t-1}$ for $s_t$ is a T-distribution with $n_{t-1}$ degree of freedom. The iteration across time is implemented in C language.

To complete the specification of the model, we need to assign some values for the initial quantities $\boldsymbol{m}_{-204}$, $\boldsymbol{C}^{\star}_{-204}$, $n_{-204}$ and $d_{-204}$, as well as for the matrix $\boldsymbol{R}_{\omega}$. To initialize state parameters $\boldsymbol{m}_{-204}$ and $\boldsymbol{C}^{\star}_{-204}$, and observational variance parameters $n_{-204}$ and $d_{-204}$, prior or historical information can be used. While lacking such information, we can get rough estimates from the sample data. We use the moving-window method to provide a reasonable set of prior values. By running ordinary least squares on a series of data sets formed by sliding, one observation at a time, a window of a certain size over the in-sample period, sequences of sample estimates for coefficients and variance are obtained. A window of size of five years, or, equivalently, 60 observations is applied to mimic the usual five-year business cycle. We set $\boldsymbol{m}_{-204}$ to be the mean and $\boldsymbol{C}^{\star}_{-204}$ to be the variance-adjusted covariance matrix of the sequence of coefficients; we also set $n_{-204} = 4$ and $r_{-204}$ in the way such that the prior variance estimate $S_{-204}$ is equal to the mean of variance estimates. Since the effect of the initial prior decays quite rapidly, the amount of our data is large enough to make its impact fairly unimportant in our reserved forecast period. The variance matrix of the system evolution is finally approximated. Values for the diagonal elements of $\boldsymbol{R}_{\omega}$ are obtained by optimizing a likelihood criteria based on the accuracy of the one-step-ahead predictions, as suggested by West and Harrison, over the in-sample period:

$$\sum_t \log(\Pr(s_t \mid D_{t-1})).$$

The maximum likelihood estimates of the variance parameters are obtained using the quasi-newton routine in C language.

**Multiprocess Mixture Model of Random Coefficient Regression with Random Walk System Function**

The mixture model runs, in parallel, with multiple DLM models, which are change-point models derived from a single basic DLM. Our basic model is the random walk coefficient model as specified previously. Without altering the underlying model structure, five changepoint models are derived from it by adjusting the variances of the observation and system evolutions:

$$\mathrm{var}(\epsilon_t) = V\, R_\epsilon \quad \text{and} \quad \mathrm{var}(\boldsymbol{\omega}_t) = V\, \boldsymbol{R}_\omega = V \begin{pmatrix} R_{\beta_0} & 0 & 0 \\ 0 & R_{\beta_1} & 0 \\ 0 & 0 & R_{\beta_2} \end{pmatrix}.$$

The five specifications are derived as follows:

- **M–1 Steady-State DLM:** Let $R_\epsilon = 1$ and $R_{\beta_0} = R_{\beta_1} = R_{\beta_2} = 0$. This specification creates a model in which there is no variation in coefficients over time, except those from the updating estimation due to addition of new observations.

- **M–2 Outlier DLM:** Let $R_\epsilon = 16$ and $R_{\beta_0} = R_{\beta_1} = R_{\beta_2} = 0$. This model assigns a higher probability of large $\epsilon_t$ than the steady-state model and intends to capture an outlying observation at time $t$.

- **M–3 Level-Shifting DLM:** Let $R_\epsilon = 1$, $R_{\beta_0} \geq 0$, and $R_{\beta_1} = R_{\beta_2} = 0$. A non-zero value for the disturbance to the intercept parameter reflects a shift in the level of the model.

- **M–4 Slope-Shifting DLM in Lag-1 Exchange Rate:** Let $R_\epsilon = 1$, $R_{\beta_1} \geq 0$, and $R_{\beta_0} = R_{\beta_2} = 0$. A non-zero value for the disturbance to $\beta_1$ reflects a shift in the slope term of the model.

- **M–5 Slope-Shifting DLM in Interest Rate Index:** Let $R_\epsilon = 1$, $R_{\beta_2} \geq 0$, and $R_{\beta_0} = R_{\beta_1} = 0$. Similarly, it reflects a shift in the slope term $\beta_2$ for a non-zero disturbance.

Notice that a zero for the $R_{\beta_i}$ reflects a steady state model as M–1. The parameterization for the single random walk coefficient model is applicable for the above five models. However, to distinguish between them, we impose a superscript denoting the different observation and system evolution structure stated previously, for $i = 1, \ldots, 5$:

$$\begin{aligned} s_t &= \boldsymbol{\beta}_t' \boldsymbol{X}_t + \epsilon_t^{(i)}, \\ \boldsymbol{\beta}_t &= \boldsymbol{\beta}_{t-1} + \boldsymbol{\omega}_t^{(i)}, \\ \mathrm{var}(\epsilon_t^{(i)}) &= V R_\epsilon^{(i)}, \\ \mathrm{var}(\boldsymbol{\omega}_t^{(i)}) &= V \boldsymbol{R}_{\boldsymbol{\omega}}{}^{(i)}. \end{aligned}$$

When moving from a model at time $t-1$ to another model at time $t$, the recursive procedure for the single DLM can be easily modified to handle the transition, differing only in the choice of the elements for $R_\epsilon$ and $\boldsymbol{R}_{\boldsymbol{\omega}}$. Using the same initial settings and same independence assumptions as the single DLM, we have:

$$
\begin{aligned}
(\boldsymbol{\beta}_{t-1} \mid M_{t-1}=i, D_{t-1}) &\sim \mathbf{T}_{n_{t-1}}(\boldsymbol{m}_{t-1}^{(i)}, \boldsymbol{C}_{t-1}^{(i)}),\\
(\phi \mid M_{t-1}=i, D_{t-1}) &\sim \mathrm{G}(n_{t-1}/2, d_{t-1}^{(i)}/2) \quad \text{with} \quad S_{t-1}^{(i)} = d_{t-1}^{(i)}/n_{t-1},
\end{aligned}
$$

The above distributions for $\boldsymbol{\beta}_{t-1}$ and $\phi$ depend on the model applying at $t-1$. Evolving to time $t$, they become

$$
\begin{aligned}
(\boldsymbol{\beta}_{t} \mid M_t=j, M_{t-1}=i, D_{t-1}) &\sim \mathbf{T}_{n_{t-1}}(\boldsymbol{m}_{t-1}^{(i)}, \boldsymbol{R}_{t}^{(j,i)}),\\
(\phi \mid M_t=j, M_{t-1}=i, D_{t-1}) &\sim \mathrm{G}(n_{t-1}/2, d_{t-1}^{(i)}/2) \quad \text{with} \quad S_{t-1}^{(i)} = d_{t-1}^{(i)}/n_{t-1},
\end{aligned}
$$

where $\boldsymbol{R}_t^{(j,i)} = \boldsymbol{C}_{t-1}^{(i)} + S_{t-1}^{(i)}\, \boldsymbol{R_\omega}^{(j)}$. Now, these distributions depend on the combinations of possible models applying at $t-1$ and $t$.

Given the models at $t-1$ and $t$, the forecast distribution for $s_t$ is

$$
(s_t \mid M_t=j, M_{t-1}=i, D_{t-1}) \sim \mathrm{T}_{n_{t-1}}(f_t^{(i)}, Q_t^{(j,i)}),
$$

where $f_t^{(i)} = \boldsymbol{X}_t' \boldsymbol{m}_{t-1}^{(i)}$, and $Q_t^{(j,i)} = \boldsymbol{X}_t' \boldsymbol{R}_t^{(j,i)} \boldsymbol{X}_t + S_{t-1}^{(i)} R_\epsilon^{(j)}$. The forecast distribution $p_{t-1}$ for $s_t$, which is based on information up to time $t-1$ but unconditional on the possible model combinations, is therefore the combination of the twenty-five T-distributed components. The combining is effected using discrete probability mixture:

$$
\begin{aligned}
p_{t-1}(s_t) &= \sum_{i=1}^{5}\sum_{j=1}^{5} p_{t-1}(s_t \mid M_t=j, M_{t-1}=i)\, \rho_{t-1}(j,i),\\
&= \sum_{i=1}^{5}\sum_{j=1}^{5} p_{t-1}(s_t \mid M_t=j, M_{t-1}=i)\, \pi(j)\, \rho_{t-1}(i),
\end{aligned}
$$

where $\rho_{t-1}(j,i) = \Pr(M_t = j, M_{t-1} = i \mid D_{t-1})$, $\rho_{t-1}(i) = \Pr(M_{t-1} = i \mid D_{t-1})$, and $\pi(j) = \Pr(M_t = j \mid M_{t-1} = i, D_{t-1})$. Updating the parameter distributions for newly arrived information gives

$$
\begin{aligned}
(\boldsymbol{\beta}_t \mid M_t = j, M_{t-1} = i, D_t) &\sim \mathrm{T}_{n_t}(\boldsymbol{m}_t^{(j,i)}, \boldsymbol{C}_t^{(j,i)}), \\
(\phi \mid M_t = j, M_{t-1} = i, D_t) &\sim \mathrm{G}(n_t/2, d_t^{(j,i)}/2) \quad \text{with} \quad S_t^{(j,i)} = d_t^{(j,i)}/n_t,
\end{aligned}
$$

where

$$
\begin{aligned}
e_t^{(i)} &= Y_t - f_t^{(i)}, \\
n_t &= n_{t-1} + 1, \\
d_t^{(j,i)} &= d_{t-1}^{(i)} + S_{t-1}^{(i)} \, {e_t^{(i)}}^2 / Q_t^{(j,i)}, \\
\boldsymbol{A}_t^{(j,i)} &= \boldsymbol{R}_t^{(j,i)} \boldsymbol{X}_t / Q_t^{(j,i)}, \\
\boldsymbol{m}_t^{(j,i)} &= \boldsymbol{m}_{t-1}^{(i)} + \boldsymbol{A}_t^{(j,i)} \, e_t^{(i)}, \\
\boldsymbol{C}_t^{(j,i)} &= \frac{S_t^{(j,i)}}{S_{t-1}^{(i)}} (\boldsymbol{R}_t^{(j,i)} - \boldsymbol{A}_t^{(j,i)} \boldsymbol{A}_t^{(j,i)\prime} Q_t^{(j,i)}).
\end{aligned}
$$

Notice that the estimated residual $e_t$ for $s_t$ depends only upon the possible models applying at $t-1$ since the expected coefficients $\mathrm{E}(\boldsymbol{\beta}_t \mid M_t = j, M_{t-1} = i, D_{t-1})$ and, thereby, the expected values for the conditional forecasts $\mathrm{E}(s_t \mid M_t = j, M_{t-1} = i, D_{t-1})$ do not differ across the models applying at $t$. Moreover, the degree of freedom is independent of models applying at both $t-1$ and $t$.

Running the five models in parallel could lead to a five-fold increase in the number of models through time. Considering that we have a joint model of five possible states at time $t-1$. Each model in the joint set leads to another set of the five models for time $t$, resulting in a joint model of twenty-five states. Proceeding to time $t+1$, the five models are again produced from each model. The transition of the combinatorial possible states continues repeatedly as we go through time. To avoid the expansion of the models, we invoke an approximation to collapse 25 models back to 5 models at each stage. The principle of approximating the mixtures is to assume that the effects of different models at $t-1$ are negligible for $t+1$. In other words, the mixture of 25 components are to be reduced over the possible models at $t-1$ as follows:

$$\begin{aligned}
&F_{(\boldsymbol{\beta}_t,\phi)}(\cdot \mid M_t = j, D_t) \\
&= \sum_{i=1}^{5} F_{(\boldsymbol{\beta}_t,\phi)}(\cdot \mid M_t = j, M_{t-1} = i, D_t)\ \Pr(M_{t-1} = i \mid M_t = j, D_t) \\
&= \sum_{i=1}^{5} F_{(\boldsymbol{\beta}_t,\phi)}(\cdot \mid M_t = j, M_{t-1} = i, D_t)\ \frac{\rho_t(j,i)}{\rho_t(j)}
\end{aligned}$$

Under the assumption of $f(s_t \mid D_{t-1}) > 0$, the relevant probabilities are calculated as

$$\begin{aligned}
\rho_t(j,i) &= \Pr(M_t = j, M_{t-1} = i \mid D_t) \\
&= \Pr(M_t = j, M_{t-1} = i \mid s_t, D_{t-1}) \\
&= \frac{1}{f(s_t \mid D_{t-1})} f(s_t \mid M_t = j, M_{t-1} = i, D_{t-1})\, \rho_{t-1}(j,i) \\
&= \frac{1}{f(s_t \mid D_{t-1})} f(s_t \mid M_t = j, M_{t-1} = i, D_{t-1})\, \pi(j)\, \rho_{t-1}(i), \\
\rho_t(j) &= \Pr(M_t = j \mid D_t) \\
&= \sum_{i=1}^{5} \rho_t(j,i).
\end{aligned}$$

The exact mixture for $(\boldsymbol{\beta}_t, \phi \mid M_t = j, D_t)$ is then approximated by a single

"Normal/Gamma" joint distribution to give back the required distributional form to start over the recursion. The approximation, which is designed to minimize the Kullback-Leibler divergence[2] (see Titterington, Smith, and Makov (1985)), leads to

$$
\begin{aligned}
(\boldsymbol{\beta}_t \mid M_t = j, D_t) &\sim \mathbf{T}_{n_t}(\boldsymbol{m}_t^{(j)}, \boldsymbol{C}_t^{(j)}), \\
(\phi \mid M_t = j, D_t) &\sim \mathrm{G}(n_t/2, d_t^{(j)}/2) \quad \text{with} \quad S_t^{(j)} = d_t^{(j)}/n_t,
\end{aligned}
$$

where

$$
\begin{aligned}
\frac{1}{S_t^{(j)}} &= \frac{1}{\rho_t^{(j)}} \sum_{i=1}^{5} \frac{\rho_t^{(j,i)}}{S_t^{(j,i)}}, \\
\boldsymbol{m}_t^{(j)} &= \frac{S_t^{(j)}}{\rho_t^{(j)}} \sum_{i=1}^{5} \boldsymbol{m}_t^{(j,i)} \frac{\rho_t^{(j,i)}}{S_t^{(j,i)}}, \\
\boldsymbol{C}_t^{(j)} &= \frac{S_t^{(j)}}{\rho_t^{(j)}} \sum_{i=1}^{5} \{\boldsymbol{C}_t^{(j,i)} + (\boldsymbol{m}_t^{(j)} - \boldsymbol{m}_t^{(j,i)})\,(\boldsymbol{m}_t^{(j)} - \boldsymbol{m}_t^{(j,i)})'\} \frac{\rho_t^{(j,i)}}{S_t^{(j,i)}}.
\end{aligned}
$$

To complete the specification of the model, we initialize $\boldsymbol{m}_0$, $\boldsymbol{C}_0^\star$, $n_0$ and $d_0$ the same way we do for the random walk coefficient model. We also set $\boldsymbol{\pi} = (.875, .025, .025, .025, .05)'$, reflecting the belief of steady-state for 90% of time, outlier for 5% of time, and intercept and slope shifts for equal sharing of the remaining proportion. Since we initialize only one model at the beginning, $\boldsymbol{\rho}_0 = (\Pr(M_0 = i \mid D_0),\ i = 1, \ldots, 5)'$ can be defined to be any probability vector as long as it is normalized to one. Finally, the values for

[2]The Kullback-Leibler directed divergence is a distance measure between densities, or distribution. Let $p(\boldsymbol{\theta})$ be a mixture of densities, and $p^\star(\boldsymbol{\theta})$ is the approximation for $p(\boldsymbol{\theta})$. The divergence is defined by

$$
K(p^\star) = \mathrm{E}\Big\{\log\Big(\frac{p(\boldsymbol{\theta})}{p^\star(\boldsymbol{\theta})}\Big)\Big\} = \int \log\Big(\frac{p(\boldsymbol{\theta})}{p^\star(\boldsymbol{\theta})}\Big)\, p(\boldsymbol{\theta})\, d\boldsymbol{\theta}.
$$

the diagonal elements of $\boldsymbol{R}_\omega$ in the five derived models are chosen to optimize the likelihood of one-step-ahead predictions over the in-sample period. The iterative procedure and the optimization are again performed with C code.

There is a final point to be drawn about this model: the mixture estimation procedure provides information on the posterior probabilities of outliers and structural shifts in the intercept and slope parameters. We could use these posterior probabilities to draw ex-post inferences regarding abrupt shifts in the parameters during the floating period. The posterior probabilities could be one-step-back or *smoothed* probabilities so that the probabilities over the possible models one-step back in time, at $t-1$ is

$$\begin{aligned}
\Pr(M_{t-1} = i \mid D_t) &= \sum_{j=1}^{5} \Pr(M_t = j, M_{t-1} = i \mid D_t) \\
&= \sum_{j=1}^{5} \rho_t(j, i).
\end{aligned}$$

These probabilities are plotted across time on the entire sample for each series to detect for any structural changes; see Appendix C. A shift away from the steady-state Model-1 toward another model would be indicated by an increase in the posterior probability for that model. We can observe that there are numerous shifts away from the steady-state model to the outlier model for all of the series. Confirming to our preliminary analysis, the probabilities of the steady-state model drop suddenly some time around the year of 1992.

# Chapter 4

# Performance Measures

## 4.1 Introduction

In our study, we measure the performance of each model by means of two aspects: the forecast performance in terms of expectation and risk management. The first aspect refers to how well the expected values of the forecast distributions capture the realizations of the exchange rates, and the quantification of risk is in terms of Value-at-Risk (VaR). The measurements are all based on the originally scaled exchange rates, not the log transformed ones; that is, we focus on the prediction-realization pair $(P_{t-1}, S_t)$, where $P_{t-1}$ is the forecast distribution for $S_t$.

## 4.2 Forecastability in Expectation

Within the context of measuring forecast performance in expectation, we employ two approaches to gauge the accuracy of the models: the *statistical approach* and the *return on portfolio approach.*

### 4.2.1 By Means of Statistics

The accuracy of each forecast is measured by the *percentage forecast error.* Denote the forecast formulated in $t-1$ for time $t$ or, equivalently, the expected value of the distribution function $P_{t-1}$, by $\mathrm{E}(S_t|D_{t-1})$. The percentage error is computed as

$$\varepsilon_t = \frac{S_t - \mathrm{E}(S_t|D_{t-1})}{S_{t-1}}$$

Average forecasting accuracy is then measured by the *mean error (ME)*, the *mean absolute error (MAE)*, and the *root mean-squared error (RMSE)* of the percentage error. As suggested by their names, they average the errors in real terms, absolute terms and squared terms, respectively. Each of these statistics has its usefulness and weakness. The mean error considers both direction and magnitude of the error, while the mean absolute error and the root mean-squared error consider only magnitude. However, the ME cancels out positive and negative errors; on the other hand, the MAE and RMSE consider positive and negative errors as equally important. The RMSE is more sensitive to large errors in either positive and negative directions than the MAE.

We summarize the above statistics for the six US dollar bilateral exchange rates in Table D.1. Along with the ME, MAE and RMSE, we have appended estimates of the variance of the pure percent error, the absolute percent error and the squared percent error.

The forecasting ability of the forward rate model is superior out-of-sample compared to the other models for all the studied currencies — the average forecast errors and the variance of them are smaller than those of the other models. On the basis of these measures, one could perhaps reject the segmented regression since they perform worst for five cases. However, it is interesting to note that for the GBP/USD series the 2-segmented regression seems to work well, while the 3-segmented regression works very badly. There is not much to choose between the other models.

All of the above measures are not useful to some market participants for whom the main contribution of a forecast is the generation of a correct buy and sell signal, even if the magnitude of the expected move is inaccurate. In this case, a measure counting the frequency of correct signaling is what these people are interested in. A natural standard that comes to mind immediately is to count the number of times the forecasts turn out to be on the correct sides of the realizations. We define the set of correctly signaled events as

$$\Phi_E = \{t \mid (S_t - S_{t-1})(\mathrm{E}(S_t \mid D_{t-1}) - S_{t-1}) > 0,\, t \in [1, T]\}.$$

The percentage frequency can be calculated as

$$\lambda_E = \frac{\#\Phi_E}{T},$$

where $T = 108$ is the length of the out-of-sample forecast period. Investors, especially speculators, do not always buy and sell currencies directly; sometimes, they use the forward trading vehicles. Therefore, another standard is a frequency, which counts the number of times the forecasts turn out to be on the correct sides of the forward rates. Using similar notations, we have $\Phi_F = \{t \mid (S_t - F_{t-1})(\mathrm{E}(S_t \mid D_{t-1}) - F_{t-1}) > 0,\ t \in [1, T]\}$ and $\lambda_F = \#\Phi_F/T$. The frequency statistics are summarized in Table D.2 (see Appendix D). If a model has no forecasting ability, the percent frequency would be 0.5.

The forward rate model performs reasonably well in the concept of correct "buy-sell" signaling as defined by $\Phi_E$ for most of the cases. It is in fact quite outstanding in the cases of trading British pounds and German marks for US dollars. However, the forward rate model is deficient in providing any signals for forward traders since it believes that the expected exchange rate is the forward rate. Therefore, these is no reason to compare the forward rate model to the other models based on the cardinality of $\Phi_F$. A similar situation can be found in the random walk model when the investing vehicle is direct trading of the currencies. In Table D.2, the corresponding entries are left as blanks meaning that the models tell nothing about "to buy" or "to sell" in those situations. On the basis of the cardinality of $\Phi_F$, there is no one consistently good model for all six cases; therefore, we would probably not rely on a particular model if we are forward traders of the currencies.

### 4.2.2 By Means of Portfolio Returns

Other than the descriptive statistics, another measurement method is to assume that certain strategic actions are taken systematically across time, and the ex-post financial return on the strategy is then computed. The forecastability is judged on the basis of the return on the capital invested. Assuming that we are US dollar based investors, we formulate our strategy as follows. For each month, if the forward rate of the foreign currency against the US dollar is above our forecast, we will buy a contract of the foreign currency for $1 US Dollar; if it is below, we will sell the contract. The cumulative returns in US dollars are then calculated and are summarized in Table 4.1. The returns of contracts with size of $N US dollar can be obtained by multiplying the results by N. One should notice that returns cannot be computed for the forward rate model due to its lack of signaling power as discussed in the previous section.

When viewed in terms of returns of the forward trading portfolio, the model with random walk coefficients appears to be an unfavorable model which provides poor negative returns for three cases. The 1-segmented or the simple regression, and the 2-segmented regression models are reasonable in spite of the case that the 2-segmented model performs poorly in the German market. The 3-segmented model and the mixture model are probably the best because they both give positive returns for all cases. It is clear that the simple random

Table 4.1: Return Table (in USD \$).

| | GBP/\$ | FRF/\$ | DEM/\$ | ITL/\$ | CAD/\$ | JPY/\$ |
|---|---|---|---|---|---|---|
| Random Walk | 0.057 | 0.201 | 0.026 | 0.076 | 0.108 | -0.229 |
| Forward Rate | ... | ... | ... | ... | ... | ... |
| Simple Regression | 0.229 | 0.378 | 0.129 | 0.159 | 0.031 | -0.025 |
| Linear System | 0.093 | 0.013 | 0.237 | -0.093 | -0.064 | 0.237 |
| Threshold System | | | | | | |
| 2-segment | 0.206 | 0.034 | -0.274 | 0.214 | 0.134 | 0.306 |
| 3-segment | 0.076 | 0.022 | 0.056 | 0.029 | 0.187 | 0.210 |
| R-W Coefficients | -0.186 | 0.149 | -0.103 | -0.166 | 0.106 | 0.281 |
| Mixture Model | 0.000 | 0.056 | 0.230 | 0.056 | 0.082 | 0.440 |

walk model does not guide us to decent returns.

## 4.3 Risk Control — Value-at-Risk (VaR)

To measure how well the models capture the risk component, we can employ the idea of VaR. VaR is the maximum potential loss inherent to a portfolio position with a certain probability over a pre-set horizon. At a given level of confidence $1-\alpha$, we will expect that realizations will lie beyond the VaR $\alpha \times 100\%$ of time. VaR forecast can be generated easily from the forecast distribution $P_{t-1}$ based on the following relation:

$$P_{t-1}(\mathrm{VaR}_{t-1}) = 1 - \alpha.$$

We will monitor these VaR forecasts to judge the models in terms of their ability in controlling risk.

### 4.3.1 Exceedence Frequency

When monitoring the VaR forecasts at a given level of confidence, we are faced with the difficulty that any realized VaR values are not observed after the event. Therefore, we can only carry out the monitoring by checking whether the forecasts are consistent with subsequently realized values. The most obvious way is to investigate how frequently losses are realized in excess of the VaR forecasts over a certain period to time. If a confidence level of $1 - \alpha$ is chosen, excessive losses are expected to occur $\alpha \times 100\%$ of time. We summarize the frequency of the exceedence events for $\alpha = .05$ over the out-of-sample forecast period in Table 4.2. We look at pairs of portfolios with opposing positions, *long* (L) US dollar and *short* (S) US dollar against the other currency, to get rid of any bias due to selection of a particular portfolio position.

To tell whether the realized frequency of excessive losses is sufficiently different from the chosen frequency, we can apply the likelihood ratio (LR) test suggested by Kupiec (1995). The construction of the test statistic is based on the point that the probability of observing $N$ failures in a sample of size $T$ is governed by a binomial process. Denote the length of the forecast period by $T$ and the frequency of losses in excess of VaR over the $T$ periods by N, the test statistic is simply

$$-2\log[(1-\alpha)^{T-N}\alpha^{N}] + 2\log[(1-N/T)^{T-N}(N/T)^{N}],$$

which follows a chi-squared distribution with one degree of freedom. The re-

Table 4.2: Frequency Table of Excessive Losses.

| | GBP | | FRF | | DEM | | ITL | | CAD | | JPY | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | L | S | L | S | L | S | L | S | L | S | L | S |
| Random Walk | ∘2 | 4 | 3 | 3 | ∘2 | 4 | 5 | 8 | 4 | 7 | 7 | 5 |
| Forward Rate | •1 | 4 | ∘2 | 4 | •1 | 4 | 3 | 3 | ∘2 | 8 | 5 | 5 |
| Simple Regression | ∘2 | 4 | 3 | 3 | ∘2 | 4 | 5 | 7 | 4 | 7 | 6 | 4 |
| Linear System | ∘2 | 5 | 4 | 3 | •1 | 5 | 4 | 14• | 3 | 10∘ | 6 | 4 |
| Threshold System | | | | | | | | | | | | |
| 2-segment | ∘2 | 5 | ∘2 | 6 | 3 | 4 | 6 | 8 | 5 | 8 | 7 | 4 |
| 3-segment | ∘2 | 6 | 3 | 6 | 4 | 5 | 6 | 9 | 5 | 7 | 7 | 4 |
| R-W Coefficients | 3 | 4 | 3 | 4 | ∘2 | 4 | 3 | 7 | 4 | 7 | 9 | 4 |
| Mixture Model | 3 | 7 | 3 | 7 | 3 | 5 | 6 | 8 | 5 | 8 | 9 | 3 |

sult of testing the null hypothesis $H_0 : p = \alpha$ is given in Table 4.2. The significance of rejection is represented by "∘" for 90% confidence and a "•" for 95%. Notice that this likelihood ratio test depend on the assumption of independence of the failure events across time. There are other tests designed to monitor VaR forecasts; for example, the *Crnkovic-Drachman VaR Percentile Test* (Crnkovic and Drachman (1995)), the *Christoffersen's Interval Forecast Test* (Christoffersen (1996)) and the *Size of Excessive Losses Test* (Zangari (1995)). We should point out that the hypothesis testing guides the validity of a model but not the choice between models, i.e., we may not be able to choose among models that are all accepted in the tests.

The occurrence of the exceedence events is more severe in the short position than in the long position for all models and for all series, except for the Japanese Yens which is the reverse of the situation. This means that our models are more capable to estimate risk when investors are in the short position of US dollars. One would reject the model with the linear state system as being under-estimating the risk, at least for the situations of *long* Italian Liras and *long* Canadian dollars. Most of the rejections of the hypothesis are due to conservative in the sense of always over-estimating risk — fewer exceedence counts than expected. The Mixture model is the only model which is not rejected in any cases.

### 4.3.2 Lopez Loss Function

All the statistical tests have a common drawback: due to the fact that we are looking at classes of extreme observations when talking about risk management, the tests tend to have low power in classifying a bad model as bad, especially when the data set is small. Lopez (1996) suggests getting around the problem of low power by using a forecast evaluation standard, rather than by testing hypotheses. In his criterion, the accuracy of a model in terms of risk control is gauged by how well it scores on a loss function.

In the formulation of the loss function, instead of looking at the events above a certain confidence level, we look at the events wherein the predicted movement exceeds a certain absolute size, parameterized by a percentage change in the prices of the foreign currency per 1 US dollar. In other words, we start by specifying a tolerance percentage change in the prices of the foreign currency per 1 US Dollar, which is taken as the VaR, and then predict the probability of a loss in excess of the VaR from the forecast distribution $P_{t-1}$. The loss function is set up to evaluate the goodness of these probability forecasts against realized outcomes in the following way:

$$QPS = 2\sum_{t=1}^{T} \frac{(P^f_{t-1} - I_t)^2}{T},$$

where $P^f_{t-1} = P_{t-1}(\text{VaR})$ is the forecasted probability that an excessive loss over the VaR will happen at time $t$, $I_t$ is the indicator variable that takes the value of one if the event actually occurs and zero otherwise, and $T$ is the length of the forecast period. The loss function is so named as QPS since it is the quadratic probability score due originally to Brier (1950). It is expected that a better model would generate lower score. It should be clear that the Lopez score function directly evaluates the performance of a model for large price changes.

For better visualization of performance, the above scoring measure is plotted for each model and each exchange rate series as a function of increasingly percentage change in prices. We have one further variation in the presentation of this performance measure — the long position in US dollar, the short position in US dollar and the average of them. To account for loss events, a change means a drop in the price of US dollar for the long position, and it refers to a rise for the short position. The plots can be found in Appendix E. There is a serious caveat with regard to low number of exceedence events when focusing on high percent price changes; therefore, we have only plotted the percentile levels when there is at least five exceedence events beyond a certain VaR value.

Based on the Lopez score lines, we define the following criterion for comparing two models:

**Definition:** If the Lopez scores of Model 1 are higher than those of Model 2 for all percentile levels, Model 2 is better than Model 1 under the Lopez loss measure.

Therefore, the acceptance of a model is based on performance of the model over the entire range of percentile levels.

When looking at the cases of *long* US dollars, we observe that the forward rate model dominates the other models. However, when considering those of the short position, the Lopez score lines corresponding to the forward rate model tangle with those of the other models. In the cases of selling US dollars for British pounds and Italian Liras, it twists with the score lines of the 2-segmented model and the 3-segmented model, respectively. In the case of German marks, it twists with the score line of the linear system model. In other cases, the score lines seem to be all tangled up. However, the forward rate model is still dominating in average sense because it is too outstanding in the cases of the long position.

## 4.4 Discussion

In this section we discuss the details of what the drawbacks of the various performance measures are and how they complement each other. The measures we consider in our study are: the means of pure, absolute and squared percentage error, the "buy-sell" signaling frequency, the returns of a forward trading portfolio, the frequency of excessive loss, and the Lopez loss scores.

It is obvious that counting the number of correct "buy-sell" signals is not very informative in the sense of ignoring the size of losses and gains. In fact this is always the drawback of frequency measures. Although the ME, MAE and RMSE do capture the size of errors in different respects, they all consider the positive and negative errors as equally important. Their usefulness as a performance measure arises a lot of controversy since it is believed that investors tend to be more concerned about the amount of losses than gains. When computing returns on portfolios guided by the forecasts of the models, we have considered both "side" and "size" at the same time. But conclusions drawn from comparing the returns may depend on the particular choice of portfolio structure. One could probably need to consider a lot of portfolios to make a general conclusion.

The need for monitoring losses leads to the concept of Value-at-Risk. Various measures have been derived to help investors to choose between models when they are adverse to risk of big losses. The exceedence frequency is one of the candidates. However, it is a relatively coarse measure since it is not sensitive in distinguishing models which have the same exceedence count but a different degree of exceedence. To overcome this problem, some researchers have resorted to express exceedence as a function of the probability density of the event. While incorporation of the likelihood does help in establishing the degree of exceedence of the events, it still fails as a valid comparative measure between models. This is because the set of exceedence events is model dependent — the exceedence events referred to by a specific choice of confidence level do not correspond to the same set of events for different models. Therefore, differing sets of exceedence events at the same confidence level imply differing theoretical likelihood of exceedence, making direct comparison fallacious. As a result, researchers can only test the expected losses with realized losses to assess the validity of the model. However, the validity of a model does not infer the order of the model in comparison with the others. Moreover, the hypothesis test for validation has low power in classifying out bad models, especially if sample size is small.

As already remarked, the Lopez loss measure presents a score of all movements exceeding a certain percentile level rather than all movements exceeding a certain confidence level, and it is exempt from any statistical testing. The X-axis of our score plots depends only on the data and the corresponding scores refer to the same set of events, making comparison of models meaningful. Moreover, its formulation encourages good forecasts since the score depends on the forecasted probability of both the exceedence and non-exceedence events.

# Chapter 5

# Conclusions

Our concerns about the effect of changes in policy and market atmosphere on the relation between exchange rates and interest rate indices initiate a motivation to model the dynamics of the state structure. In our study, we propose several models with different dynamic state structure to determine the behavior of monthly bilateral nominal exchange rates between the United States and six major economies for the period from January 1973 to December 1998. As part of our study, we also present various measurement methodology in evaluation of the models.

In general, the simple forward rate model performs outstandingly better than our candidate models under most of the measurements. It is even better than the simple random walk. Perhaps this can be attributed to market efficiency in some manner: the forward exchange rates have already incorporated all relevant information available at the time of quotations, at least in the chosen forecast period — this century. However, this is in contradiction with many of the recent research in market rationality. A common finding of autocorrelation of forecast errors in the regression between the exchange rate movement and the forward premium is not consistent with the properties of an efficient market. A justification brought out by some researchers for this finding is that exchange rates tend to exhibit jumps. However, the segmented regression and the mixture model, which try to find out when the jumps take place, perform worse than the forward rate model in forecasting. This may perhaps be due to the fact that they are not capable to identify the jumps correctly since the nature of the event is identified only after further realizations are observed.

For market practitioners who do forward trading, the forward rate model however is deficient in informing them because it implicitly assumes that the forward exchange rate is the best predictor of the future spot rate. Therefore, traders who want to do well over the forward market may need to look for other models.

In views of the statistical measures, the *mean error*, the *mean absolute error* and the *root mean-squared error*, we conclude that not much improvements over the random walk could be achieved from our dynamic models. However, the ex-post returns of the simulated forward trading portfolio are not good if investors follow the signals generated from the random walk model.

The random walk seems to be beaten in this sense.

Even when the dynamic macroeconomic models fail to forecast better than a random walk, the dynamics of the state parameters of the models is unlikely to stay constantly over the floating experience. Preliminary analysis has already suggested sudden changes in relation in early-nineteenth centuries, and the non-homogeneous relation is further supported by the one-step-back or smoothed probabilities provided from the mixture model. Certainly, following up testing into the future is necessary to make any statements with a degree of uncertainty.

It is a difficult task to choose a consistently best model with the variety of measurement methods. The Lopez score lines tend to provide a good basis for comparisons for three reasons. First, it allows for visualization of model performance over an adequately large range of price movements and discrimination of model behavior for both large and small movements. Second, it is sensitive to degree of loss amount. Third, it is exempt from any hypothesis testing with usually low power. However, we do not admit to any particular measure to guide investors to choose among models, and we believe the selection should be somehow dependent on the objectives of the investments.

Finally, we would like to end our thesis with a discussion of some further research that could be done. It should be note that the *principal compo-*

*nent analysis* is not the only way to summarize information, another possible method is the *canonical correlation analysis.* The resulting canonical index would be highly correlated with the exchange rates, leading to possibly better forecasts if used in place of the principal index. Another extension would be to analyze our models in a multivariate context; that is, one could generalize the models to handle multivariate data, and measure the multivariate models statistically and, in conjunction, on a portfolio of the risk factors weighted equally.

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# Appendix A

# Exploratory Graphs for Dynamic State Structure

*The graphs are intended to assist in exploring the structure of the coefficients for the exchange rate to the interest rate index relation over time. Series of sample coefficients are estimated from the moving-window method with window sizes of five years and ten years for the six key USD exchange rates; see Section 3.1.2. In each figure, the graphs on the left-handed column correspond to the 5-year window in which the time span is from December 1977 to December 1998 (252 months), while the right-handed column refers to the 10-year window in which the time span is from December 1982 to December 1998 (194 months). The slope parameters for the lag-1 exchange rate and the interest rate index are denoted by* $\beta_1$ *and* $\beta_2$*, respectively.*

Figure A.1: Time Series Plots of Sample Coefficients for the Series GBP/USD.

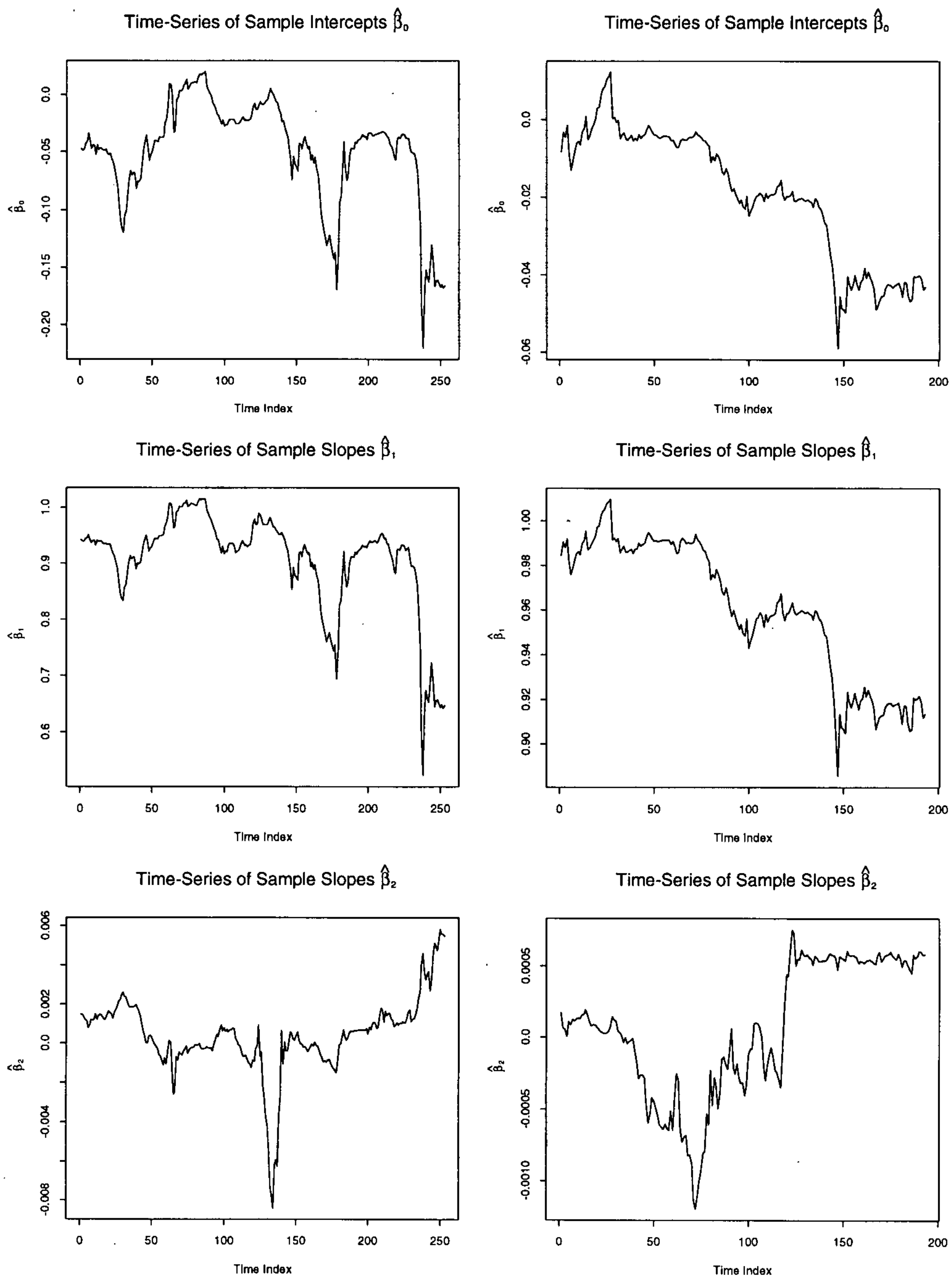


Figure A.2: Time Series Plots of Sample Coefficients for the Series FRF/USD.

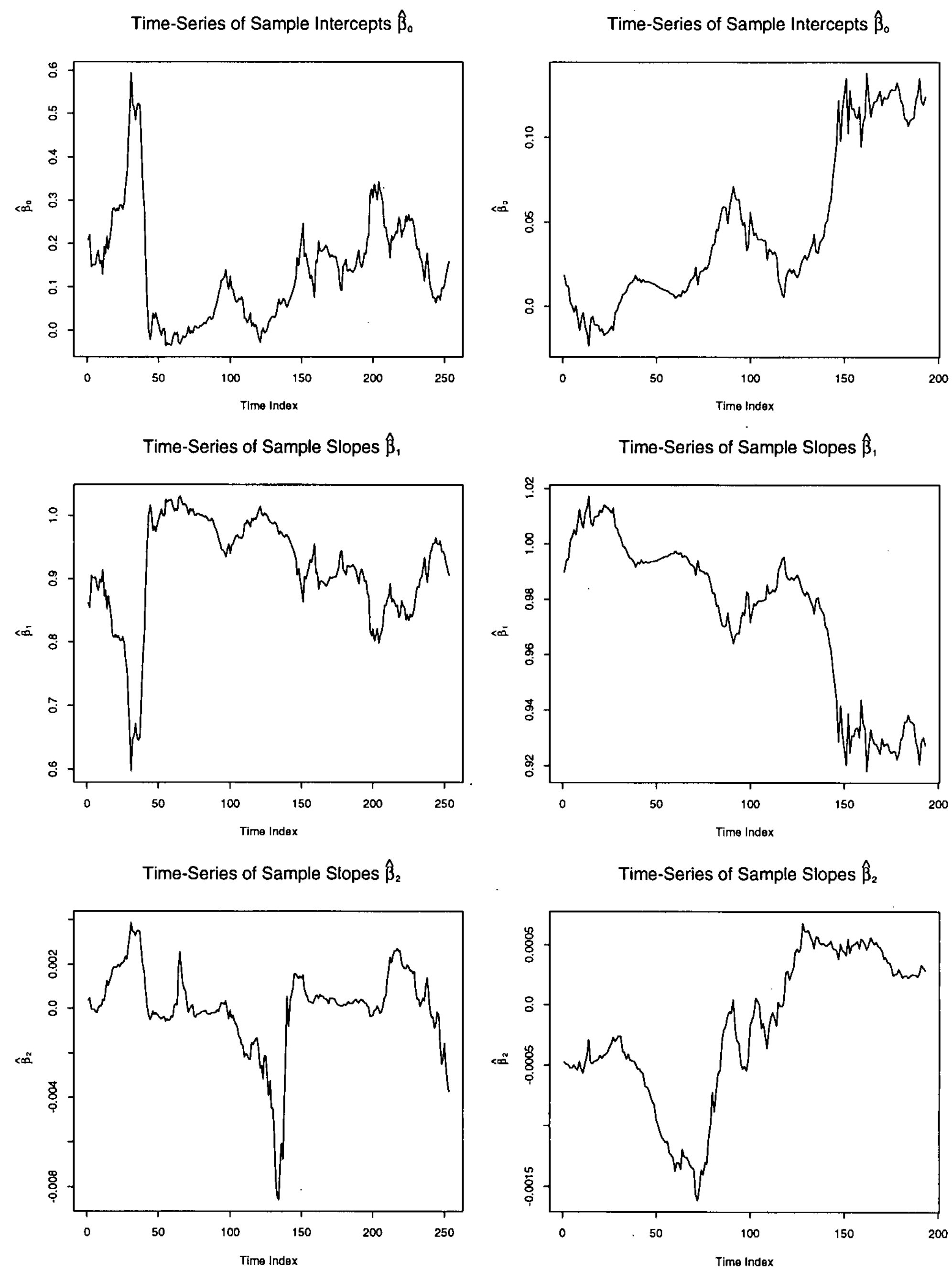


Figure A.3: Time Series Plots of Sample Coefficients for the Series DEM/USD.

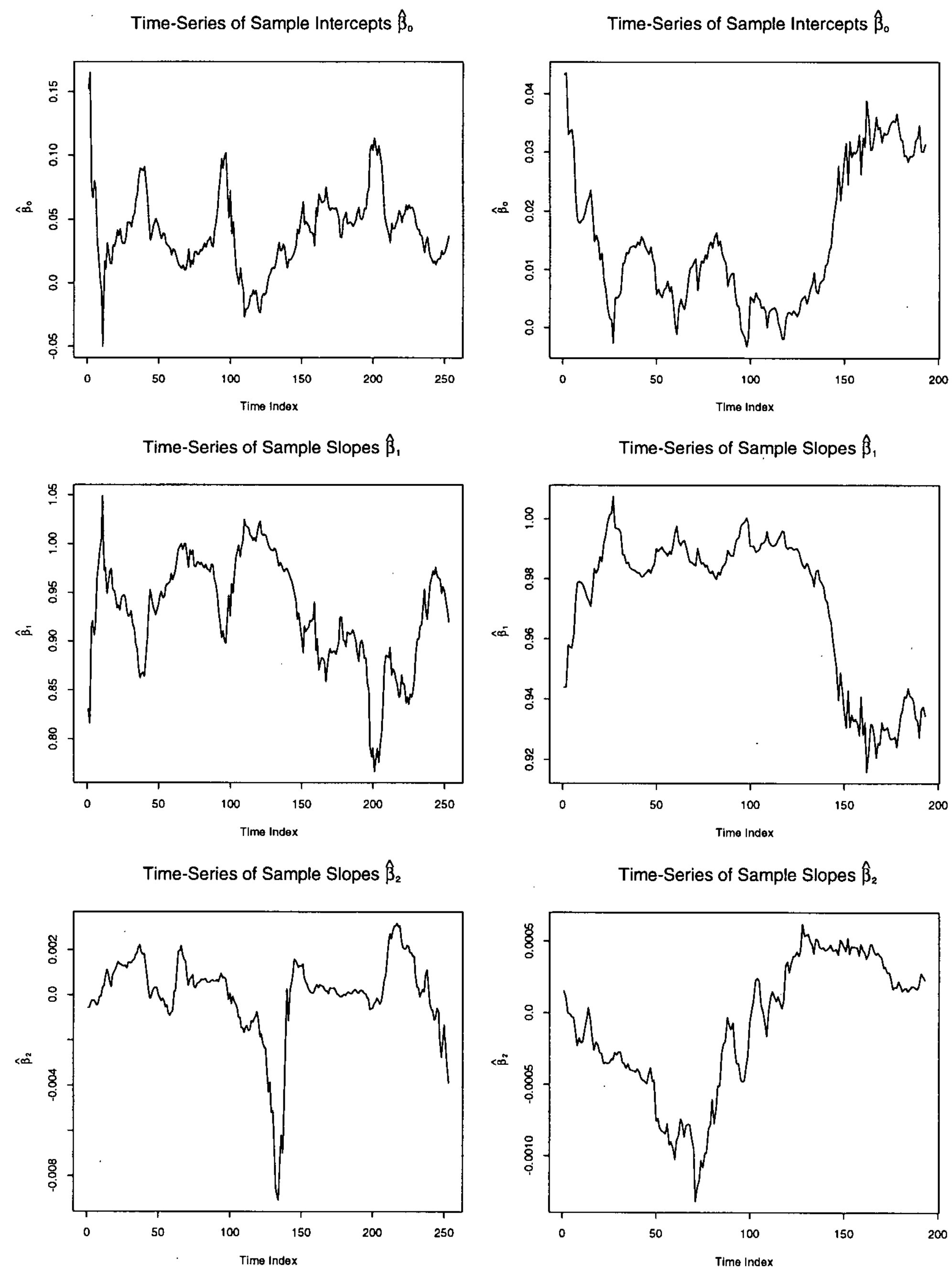


Figure A.4: Time Series Plots of Sample Coefficients for the Series ITL/USD.

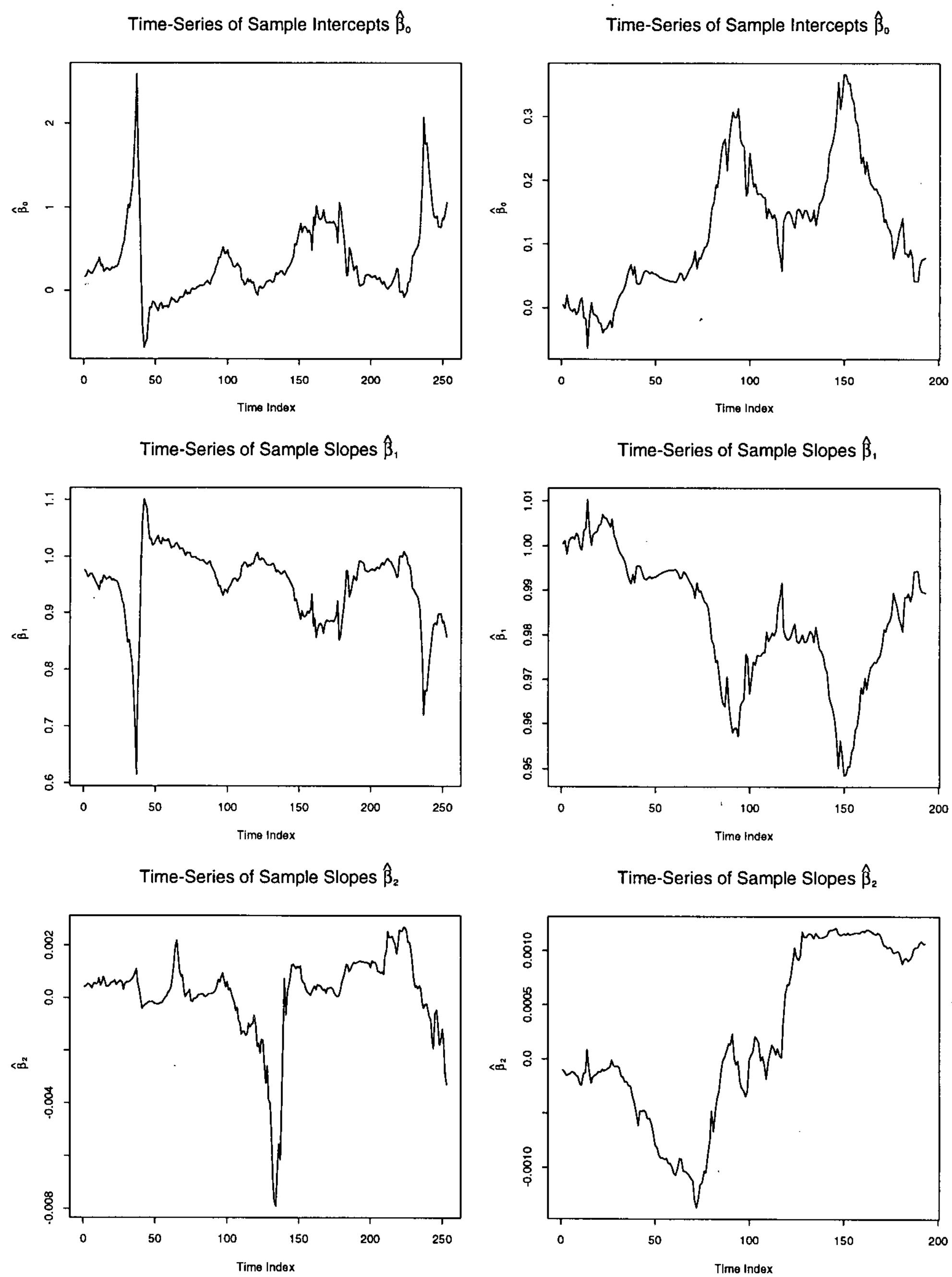


Figure A.5: Time Series Plots of Sample Coefficients for the Series CAD/USD.

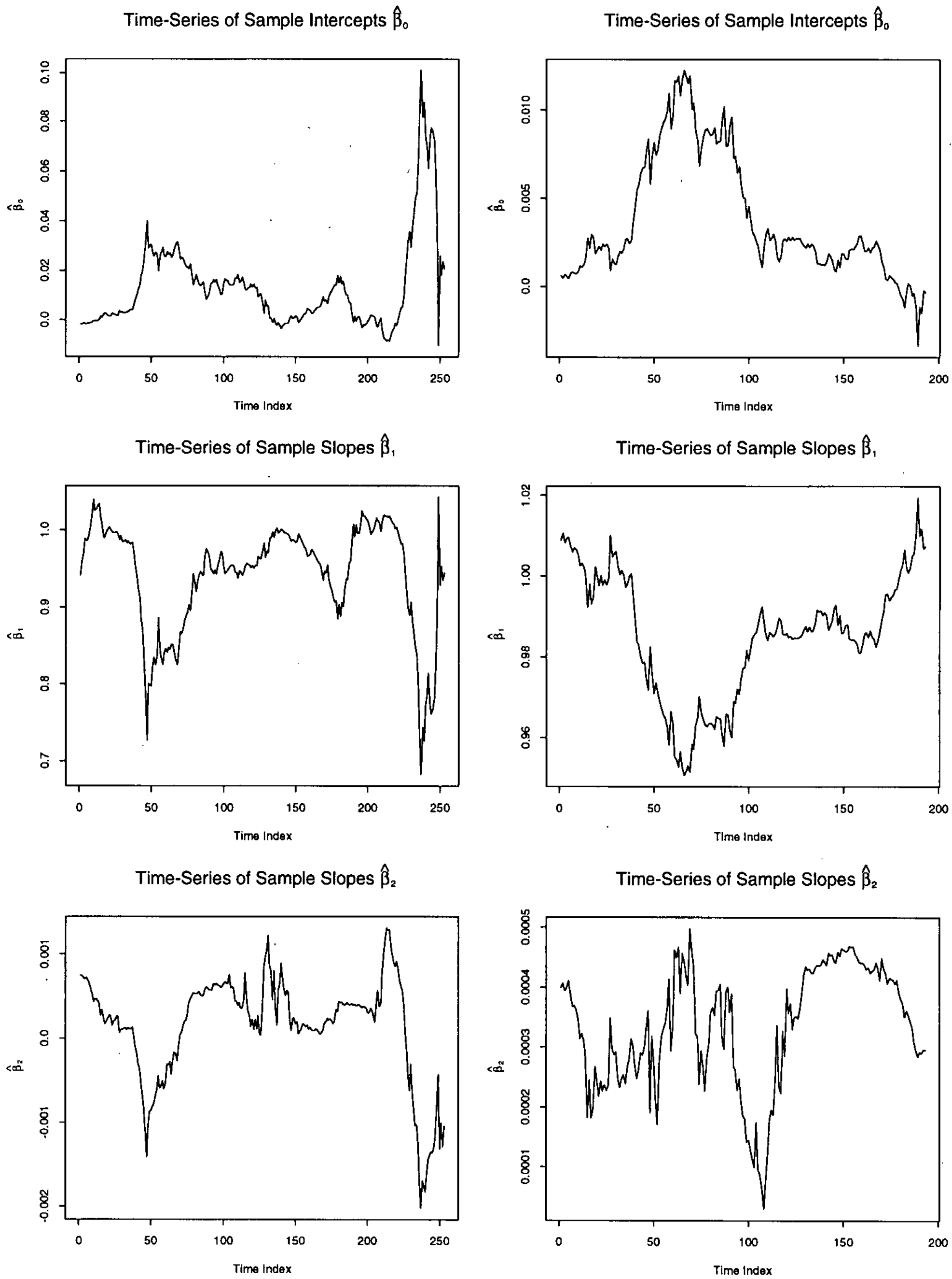
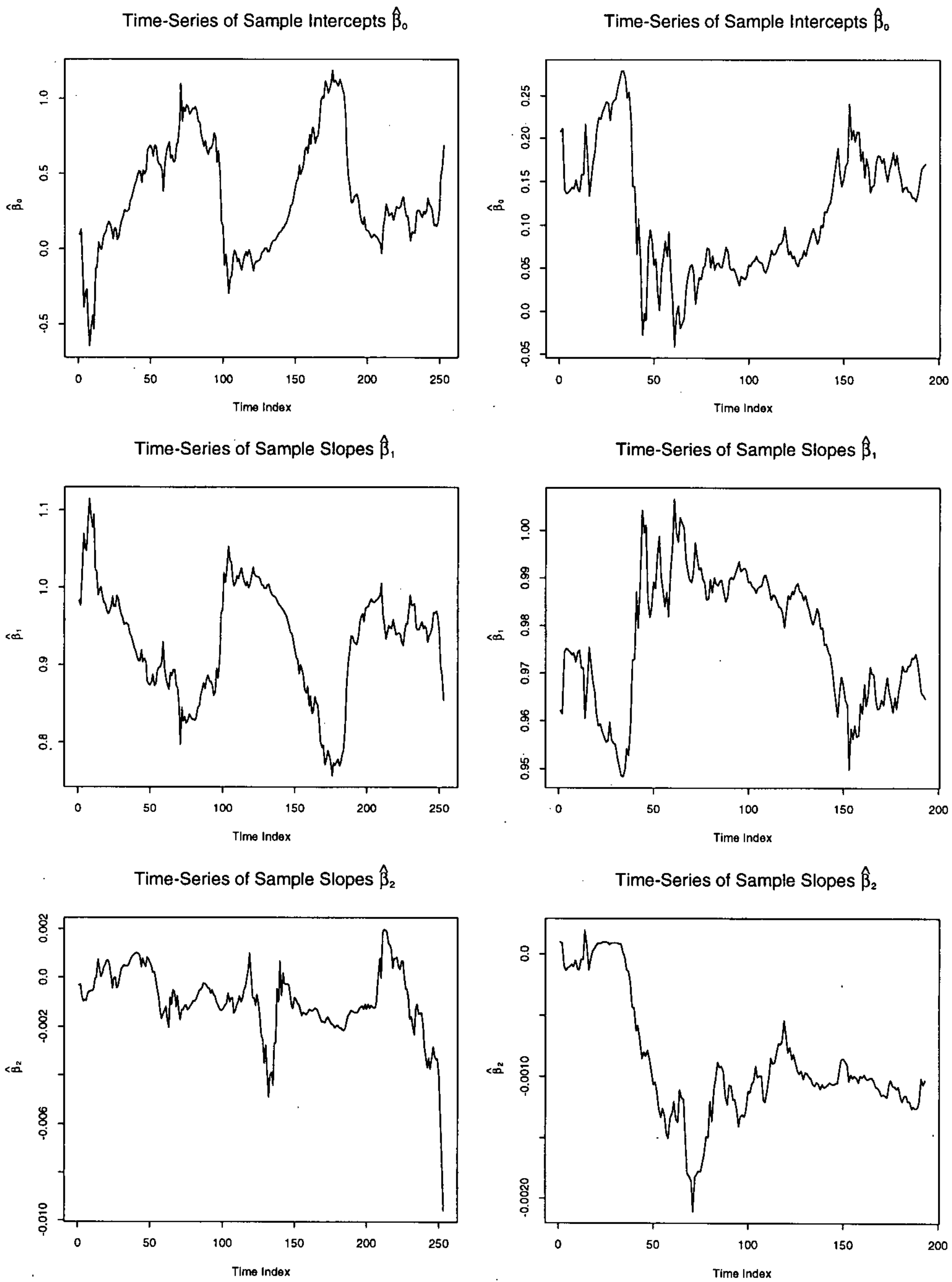


Figure A.6: Time Series Plots of Sample Coefficients for the Series JPY/USD.

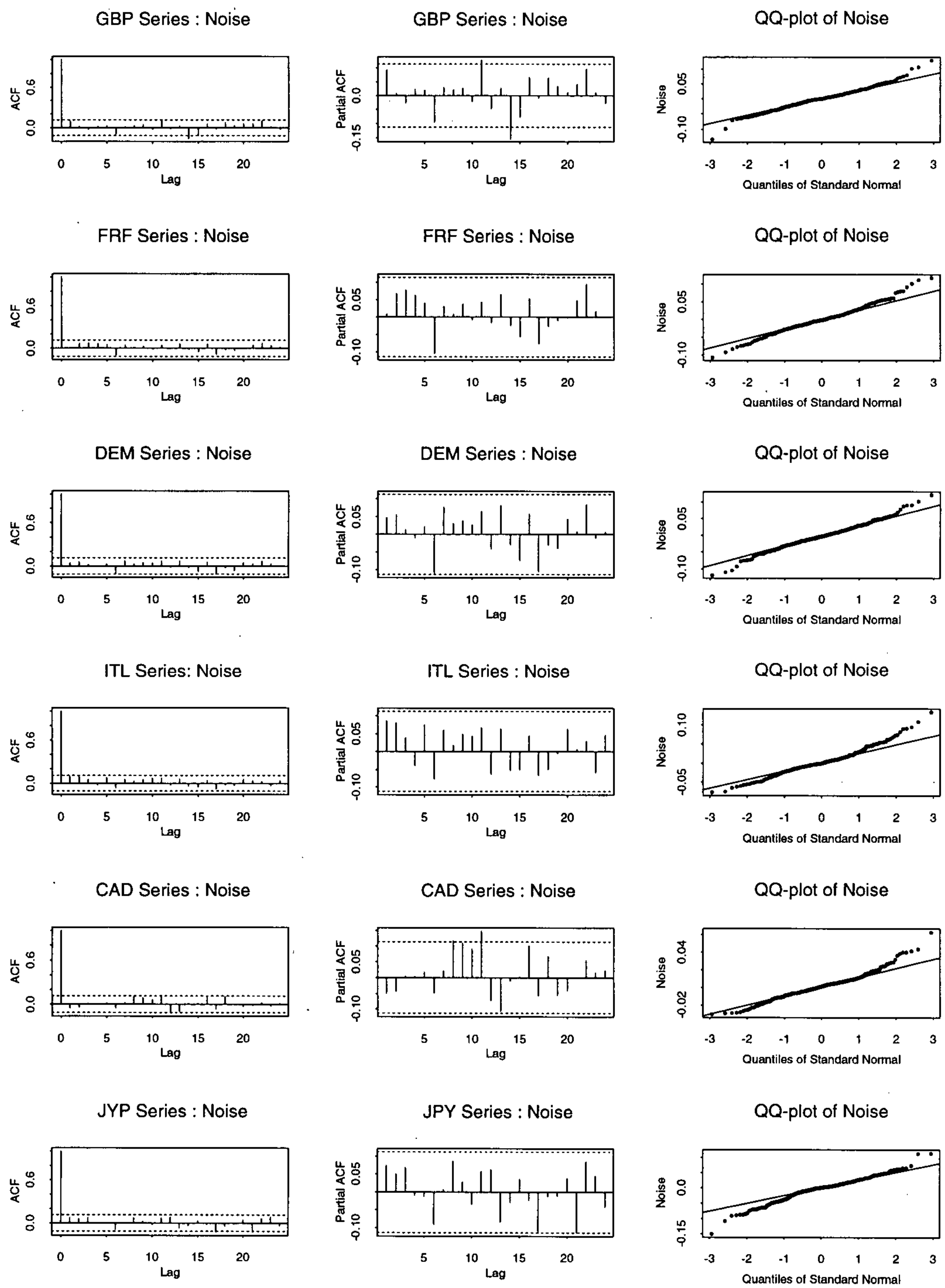


# Appendix B

# Diagnostic Checking on Residuals

*The diagnostic plots justify the independence and normality assumptions of the estimated error sequences for the models. The autocorrelation (acf) and partial autocorrelation (pacf) plots check whether the sequences are white noises, and the qq-plots check the normality of the sequences. For each model, we perform the diagnostics up to the second moment. There seems to be no strong evidence against the two assumptions.*

Figure B.1: Diagnostic Plots for the Noises of the Simple Random Walk



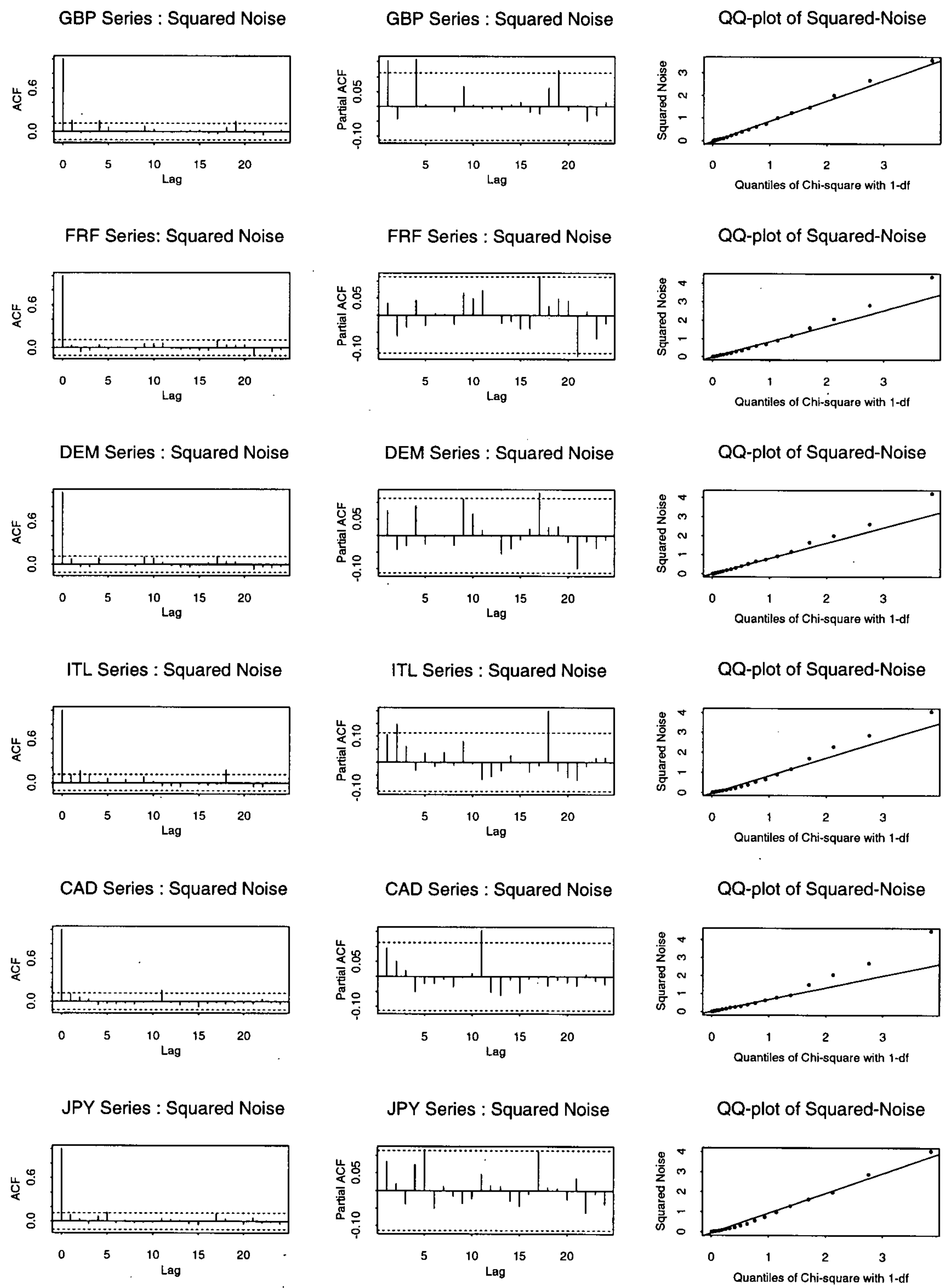
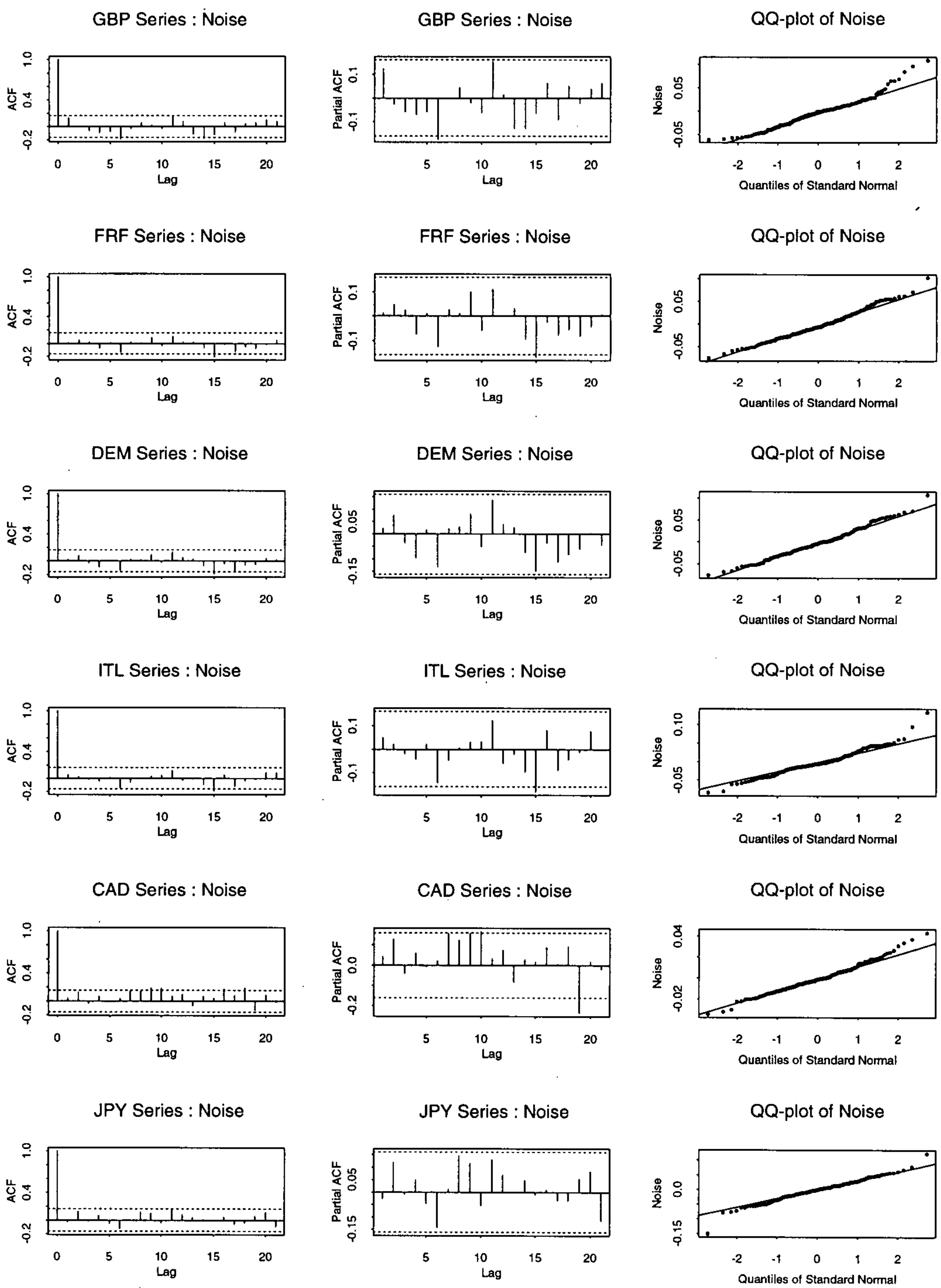

GBP Series : Squared Noise
ACF
Lag
GBP Series : Squared Noise
Partial ACF
Lag
QQ-plot of Squared-Noise
Squared Noise
Quantiles of Chi-square with 1-df
FRF Series: Squared Noise
ACF
Lag
FRF Series : Squared Noise
Partial ACF
Lag
QQ-plot of Squared-Noise
Squared Noise
Quantiles of Chi-square with 1-df
DEM Series : Squared Noise
ACF
Lag
DEM Series : Squared Noise
Partial ACF
Lag
QQ-plot of Squared-Noise
Squared Noise
Quantiles of Chi-square with 1-df
ITL Series : Squared Noise
ACF
Lag
ITL Series : Squared Noise
Partial ACF
Lag
QQ-plot of Squared-Noise
Squared Noise
Quantiles of Chi-square with 1-df
CAD Series : Squared Noise
ACF
Lag
CAD Series : Squared Noise
Partial ACF
Lag
QQ-plot of Squared-Noise
Squared Noise
Quantiles of Chi-square with 1-df
JPY Series : Squared Noise
ACF
Lag
JPY Series : Squared Noise
Partial ACF
Lag
QQ-plot of Squared-Noise
Squared Noise
Quantiles of Chi-square with 1-df

Figure B.2: Diagnostic Plots for the Noises of the Forward Rate Model



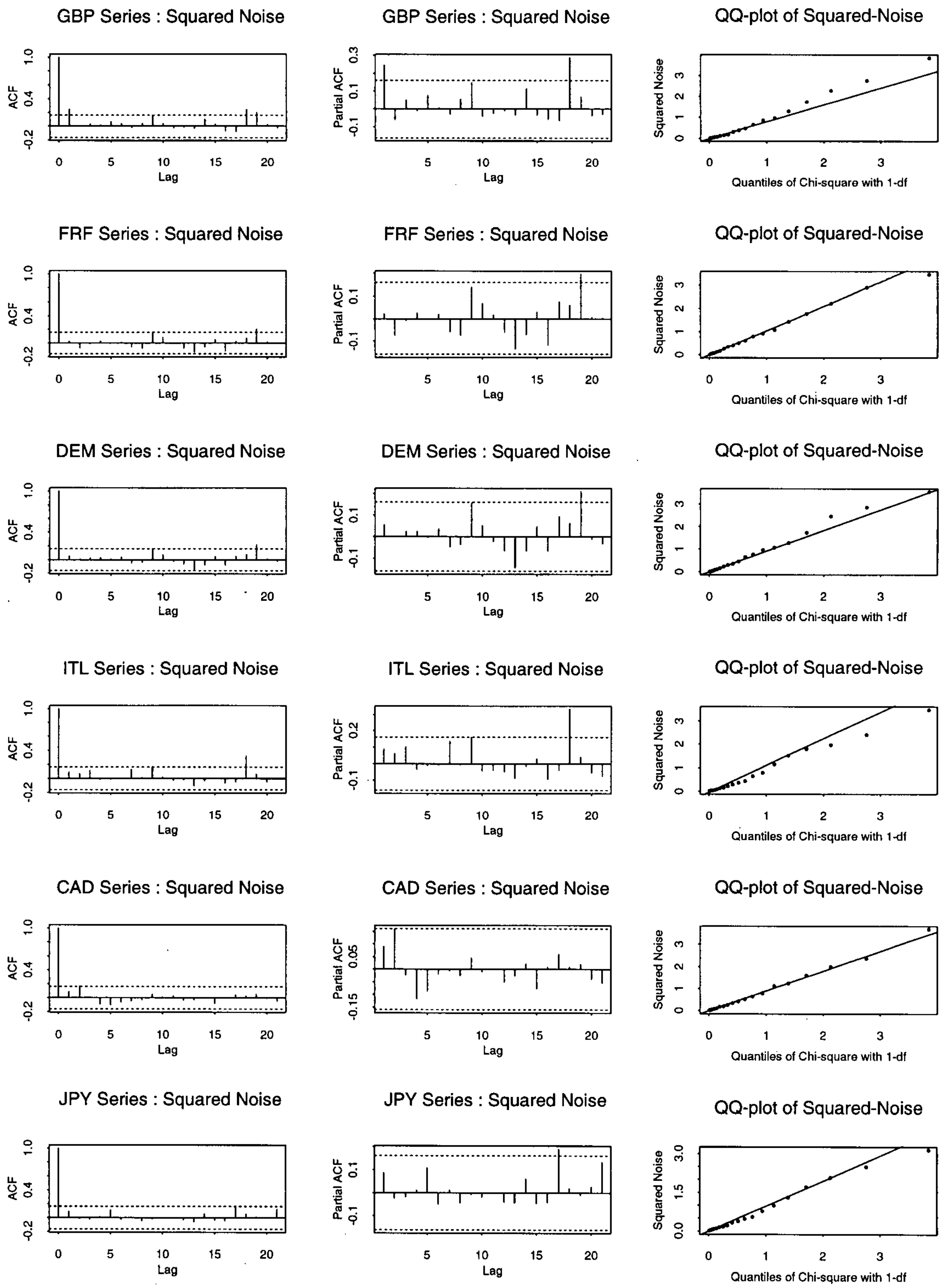
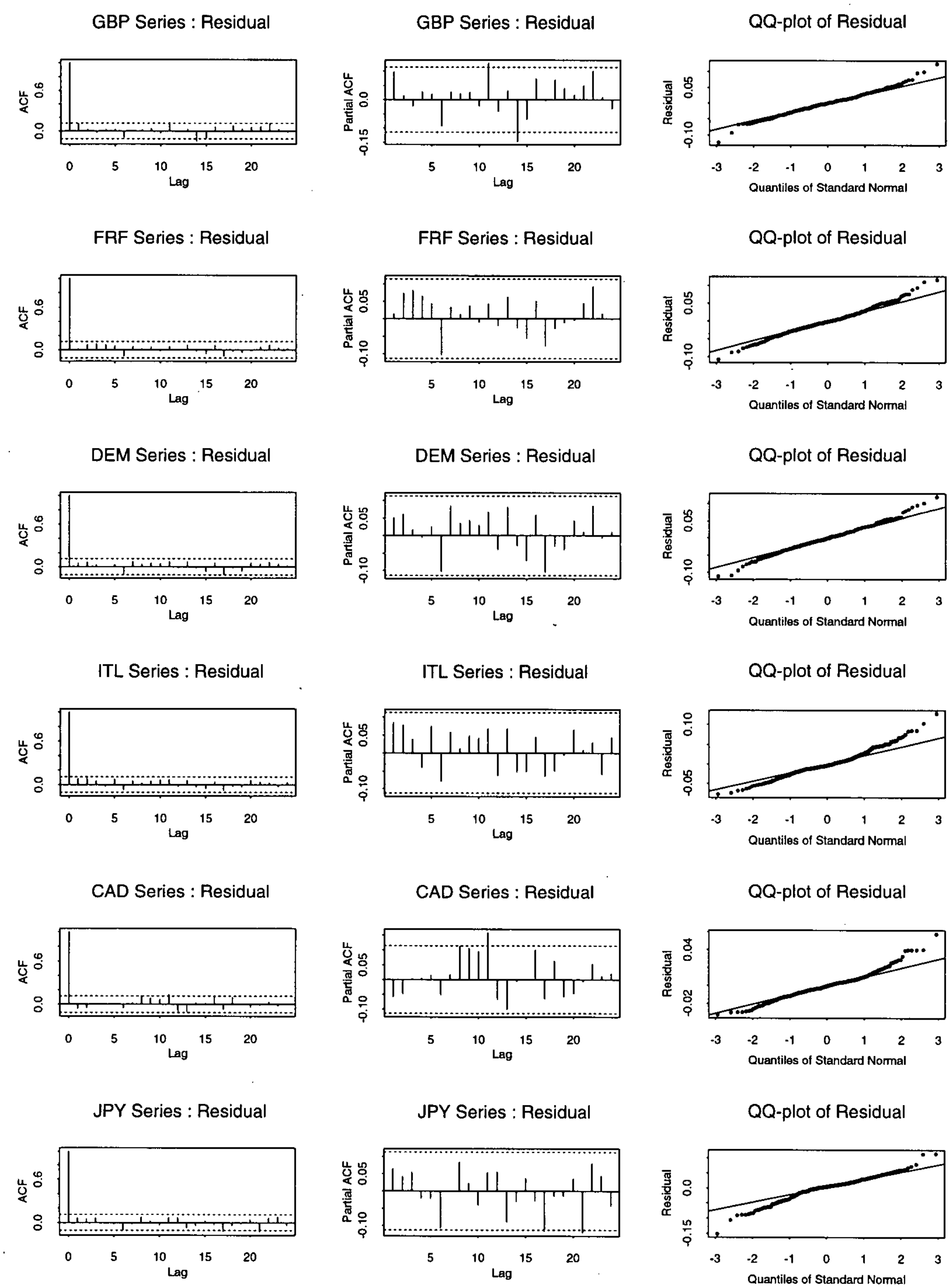

GBP Series : Squared Noise
GBP Series : Squared Noise
QQ-plot of Squared-Noise
FRF Series : Squared Noise
FRF Series : Squared Noise
QQ-plot of Squared-Noise
DEM Series : Squared Noise
DEM Series : Squared Noise
QQ-plot of Squared-Noise
ITL Series : Squared Noise
ITL Series : Squared Noise
QQ-plot of Squared-Noise
CAD Series : Squared Noise
CAD Series : Squared Noise
QQ-plot of Squared-Noise
JPY Series : Squared Noise
JPY Series : Squared Noise
QQ-plot of Squared-Noise
ACF
Partial ACF
Squared Noise
Lag
Quantiles of Chi-square with 1-df

Figure B.3: Diagnostic Plots for the Residuals of the Simple Regression Model



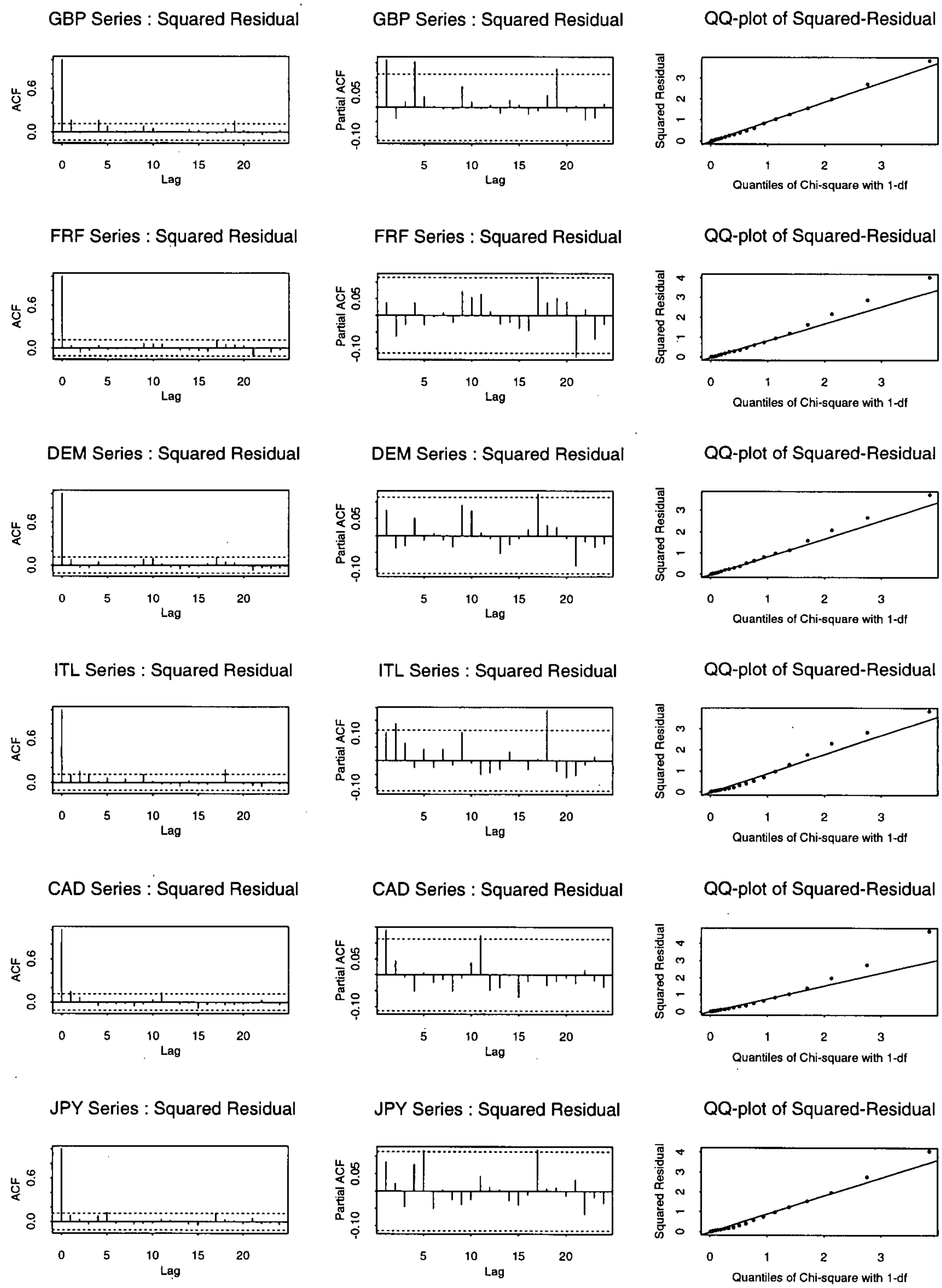
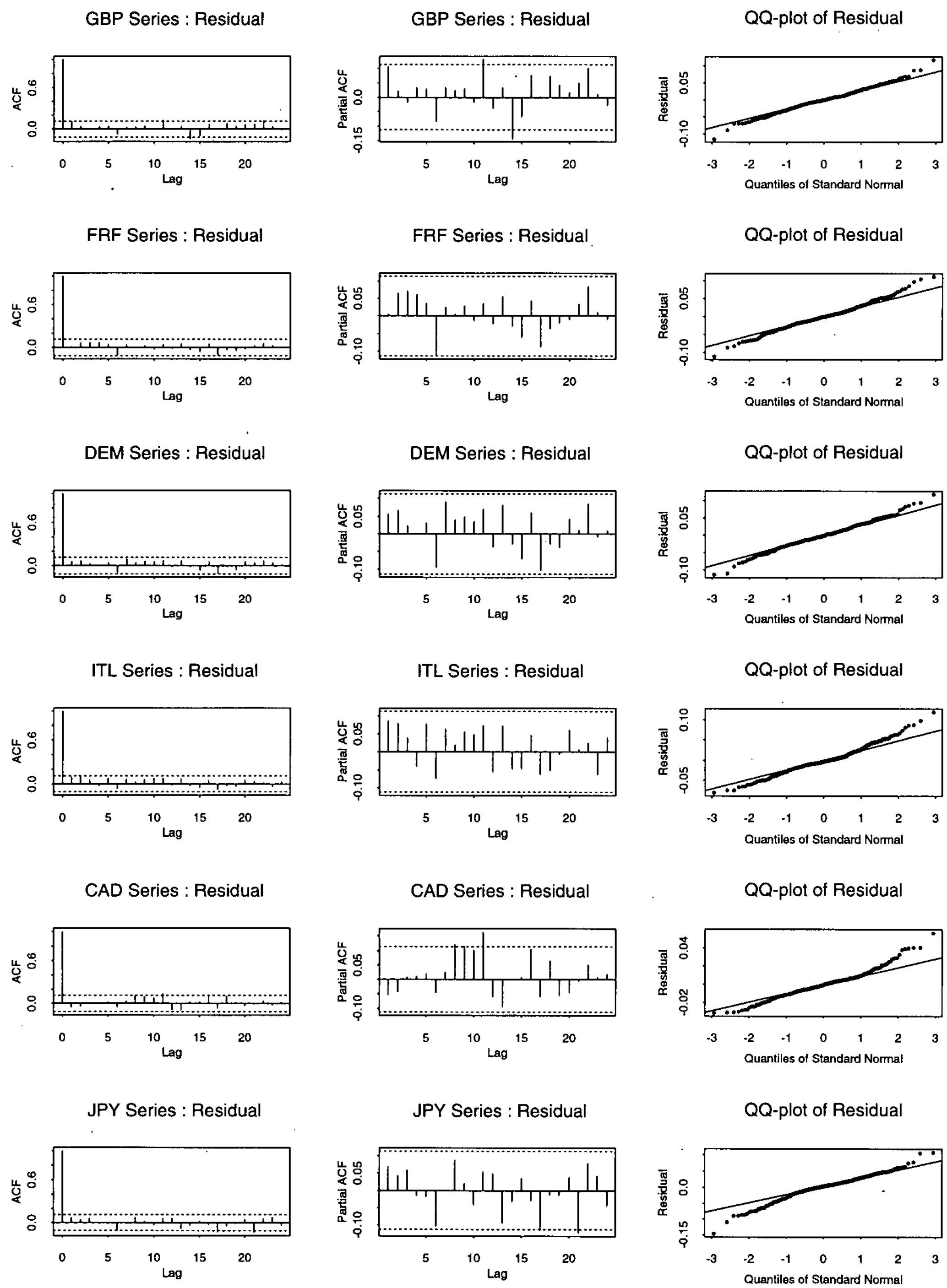
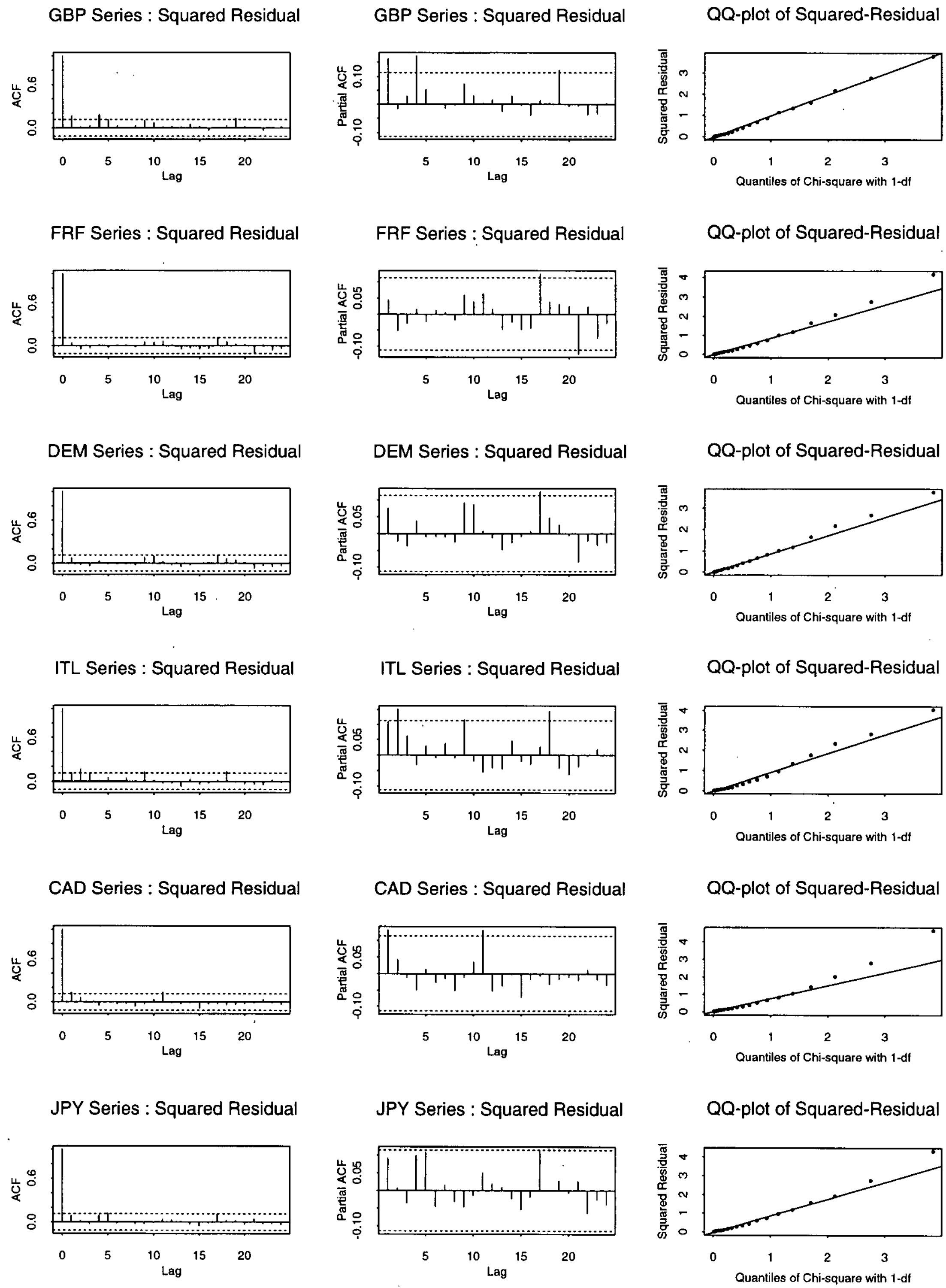

GBP Series : Squared Residual
GBP Series : Squared Residual
QQ-plot of Squared-Residual
FRF Series : Squared Residual
FRF Series : Squared Residual
QQ-plot of Squared-Residual
DEM Series : Squared Residual
DEM Series : Squared Residual
QQ-plot of Squared-Residual
ITL Series : Squared Residual
ITL Series : Squared Residual
QQ-plot of Squared-Residual
CAD Series : Squared Residual
CAD Series : Squared Residual
QQ-plot of Squared-Residual
JPY Series : Squared Residual
JPY Series : Squared Residual
QQ-plot of Squared-Residual
ACF
Partial ACF
Squared Residual
Lag
Quantiles of Chi-square with 1-df

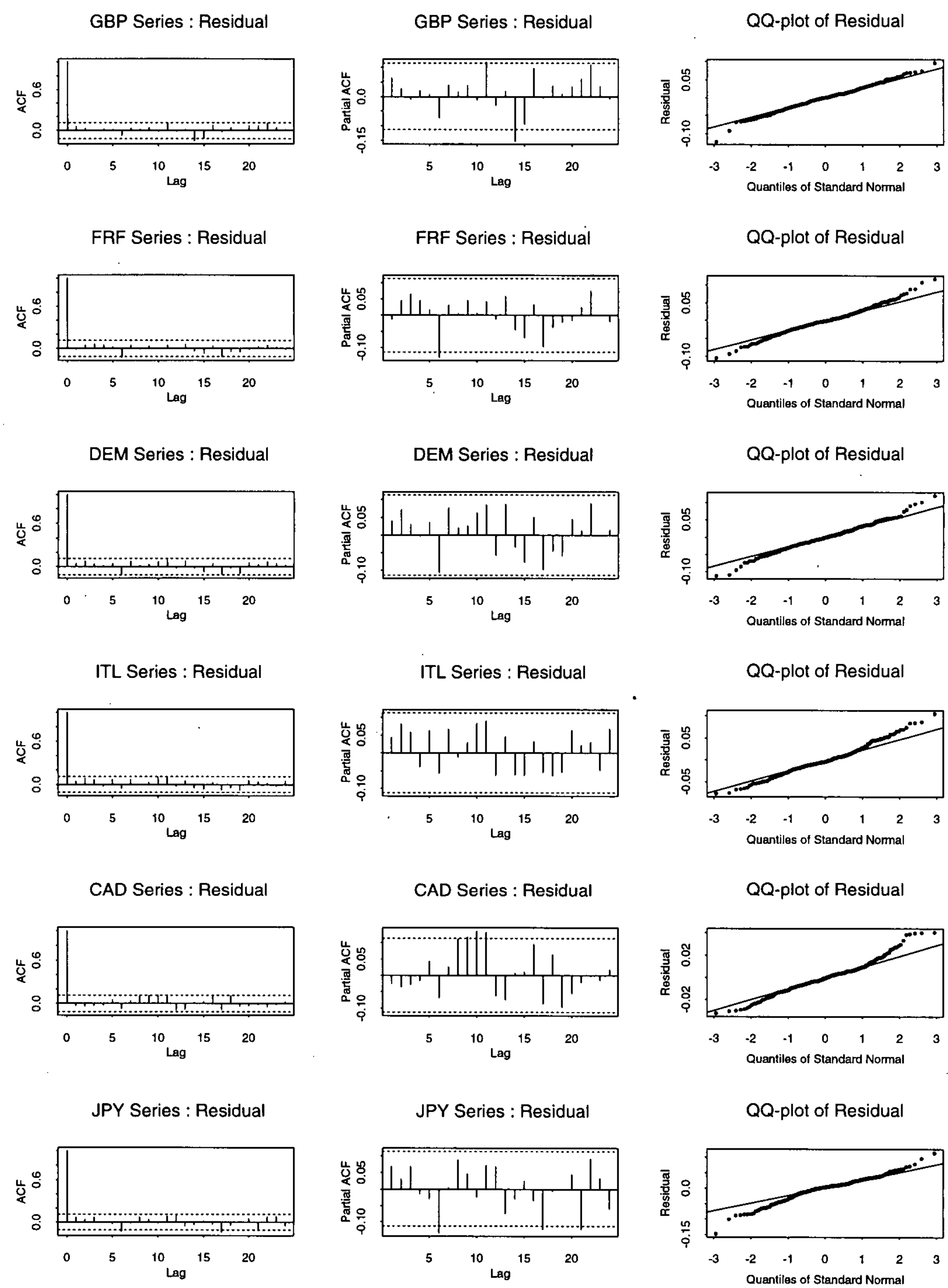
Figure B.4: Diagnostic Plots for the Residuals of the Linear System Model





GBP Series : Squared Residual
ACF
0.0
0.6
0
5
10
15
20
Lag
GBP Series : Squared Residual
Partial ACF
-0.10
0.10
5
10
15
20
Lag
QQ-plot of Squared-Residual
Squared Residual
0
1
2
3
Quantiles of Chi-square with 1-df
FRF Series : Squared Residual
ACF
FRF Series : Squared Residual
Partial ACF
-0.10
0.05
QQ-plot of Squared-Residual
Squared Residual
4
DEM Series : Squared Residual
DEM Series : Squared Residual
QQ-plot of Squared-Residual
ITL Series : Squared Residual
ITL Series : Squared Residual
QQ-plot of Squared-Residual
CAD Series : Squared Residual
CAD Series : Squared Residual
QQ-plot of Squared-Residual
JPY Series : Squared Residual
JPY Series : Squared Residual
QQ-plot of Squared-Residual

Figure B.5: Diagnostic Plots for the Residuals of 2-Segmented Regression



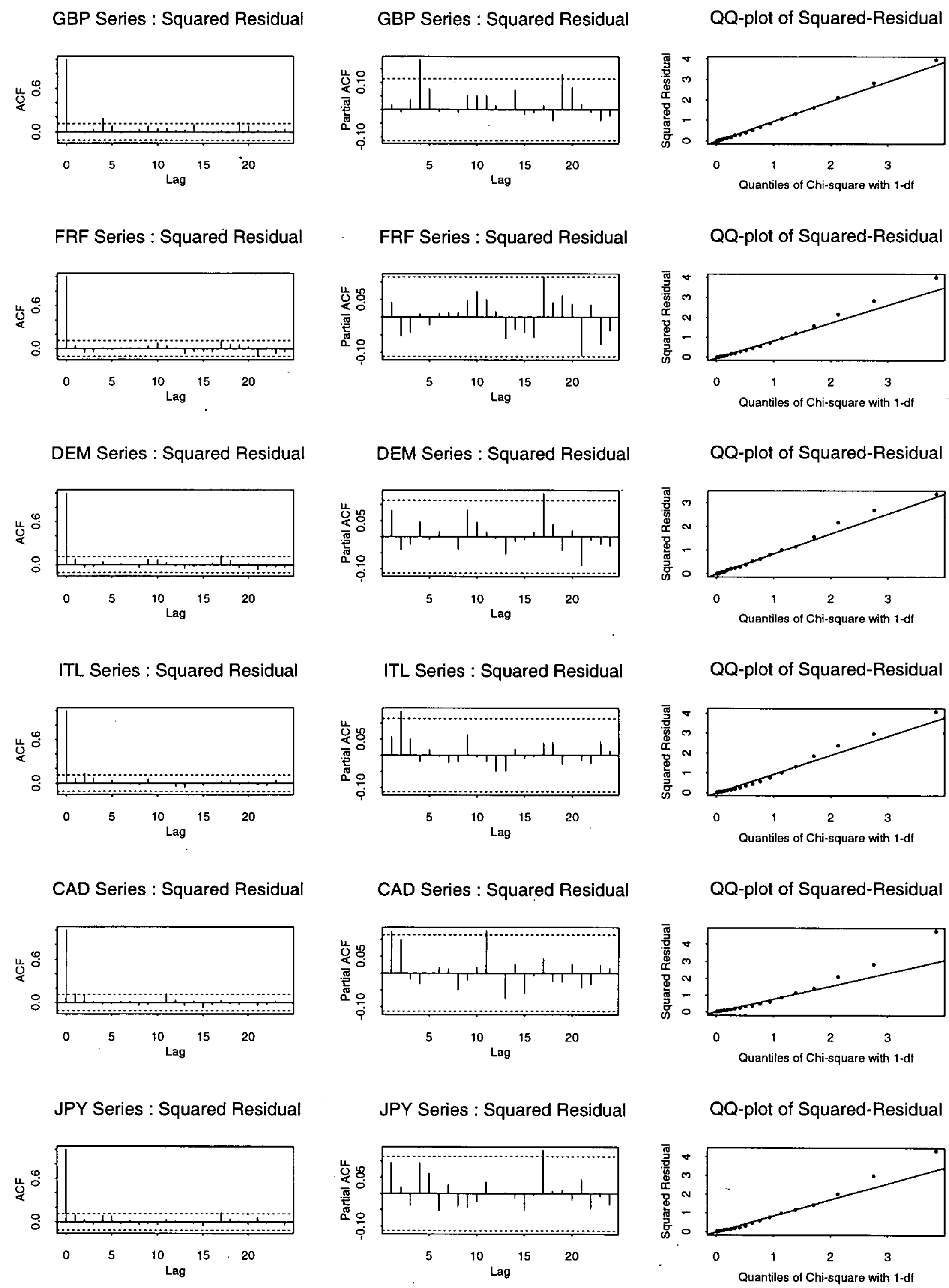
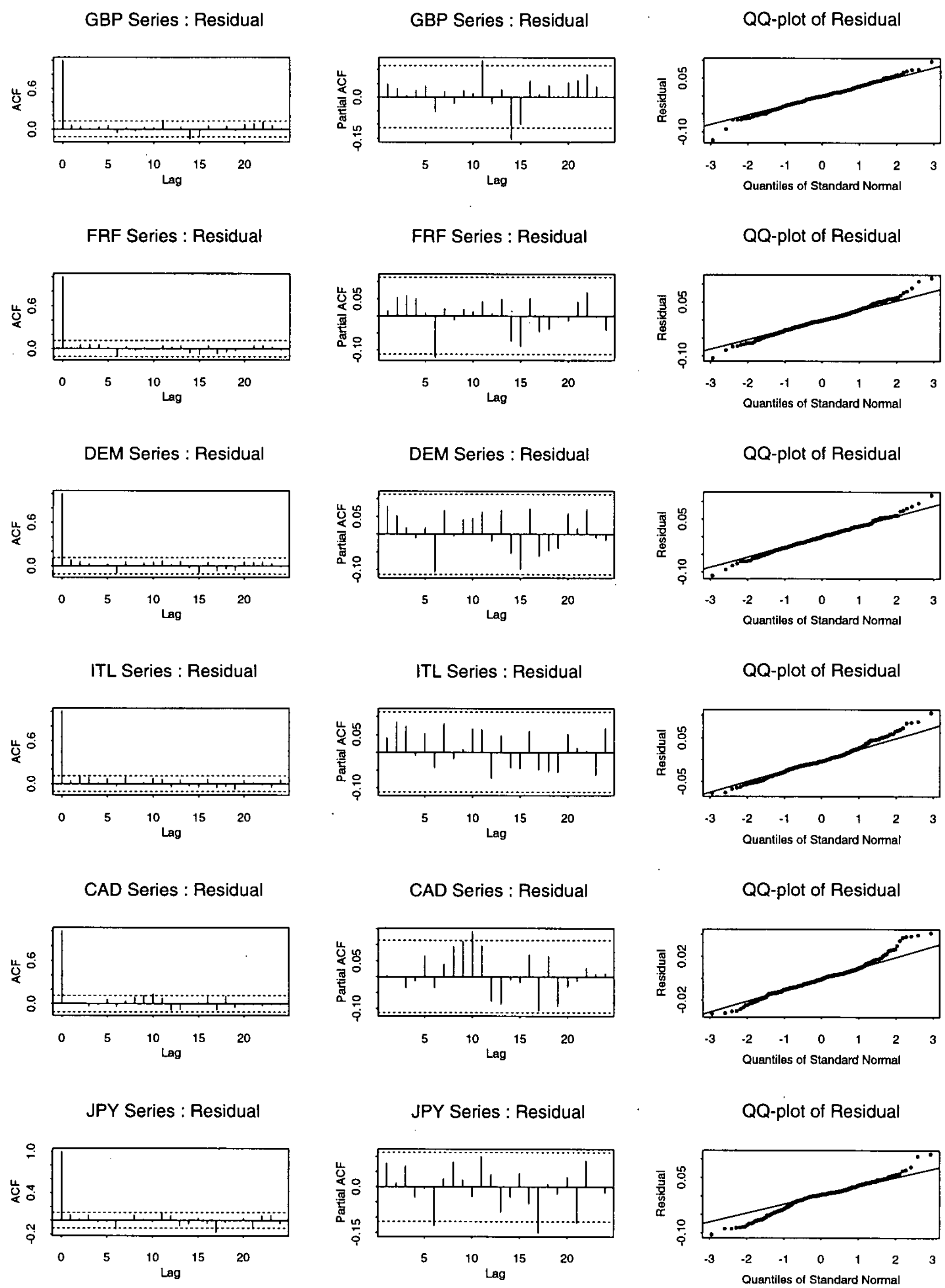
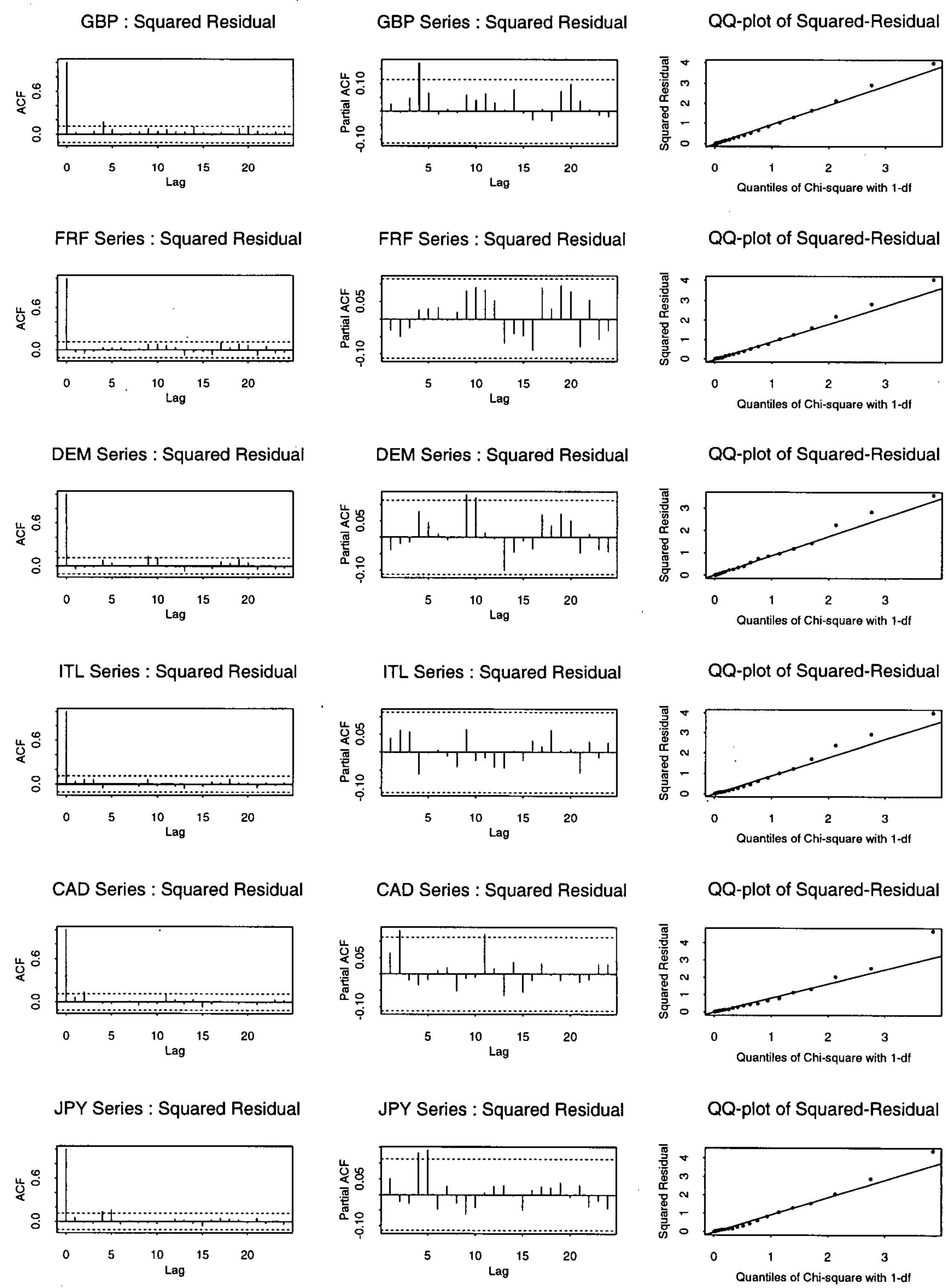

GBP Series : Squared Residual
GBP Series : Squared Residual
QQ-plot of Squared-Residual
FRF Series : Squared Residual
FRF Series : Squared Residual
QQ-plot of Squared-Residual
DEM Series : Squared Residual
DEM Series : Squared Residual
QQ-plot of Squared-Residual
ITL Series : Squared Residual
ITL Series : Squared Residual
QQ-plot of Squared-Residual
CAD Series : Squared Residual
CAD Series : Squared Residual
QQ-plot of Squared-Residual
JPY Series : Squared Residual
JPY Series : Squared Residual
QQ-plot of Squared-Residual
ACF
Partial ACF
Squared Residual
Lag
Quantiles of Chi-square with 1-df

Figure B.6: Diagnostic Plots for the Residuals of 3-Segmented Regression




GBP : Squared Residual
GBP Series : Squared Residual
QQ-plot of Squared-Residual
FRF Series : Squared Residual
FRF Series : Squared Residual
QQ-plot of Squared-Residual
DEM Series : Squared Residual
DEM Series : Squared Residual
QQ-plot of Squared-Residual
ITL Series : Squared Residual
ITL Series : Squared Residual
QQ-plot of Squared-Residual
CAD Series : Squared Residual
CAD Series : Squared Residual
QQ-plot of Squared-Residual
JPY Series : Squared Residual
JPY Series : Squared Residual
QQ-plot of Squared-Residual
ACF
Partial ACF
Squared Residual
Lag
Quantiles of Chi-square with 1-df

# Appendix C

# Plots of Posterior Probabilities of the Models

*The graphs plot the one-step-back or smoothed probabilities of the models across time for the six exchange rate series. Notice that the probabilities for the model of slope shifting in the interest index (M–5) are missing for all six series because the optimization procedure of the mixture model does not suggest any chance of its occurrence. There are other missing models in some of the series for the same reason.*

Figure C.1: One-Step-Back Probabilities of the Models for the Series GBP/USD.

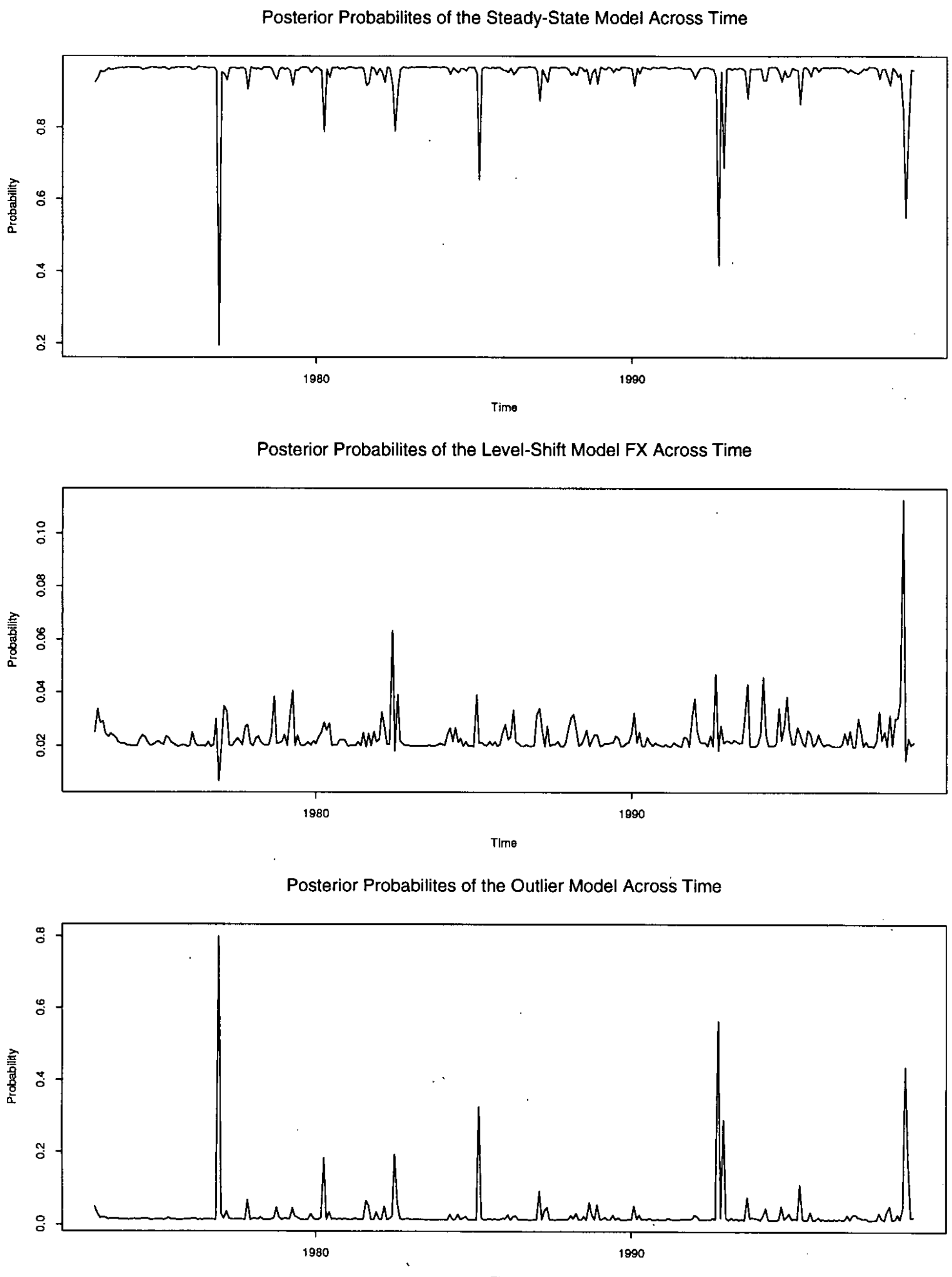


Figure C.2: One-Step-Back Probabilities of the Models for the Series FRF/USD.

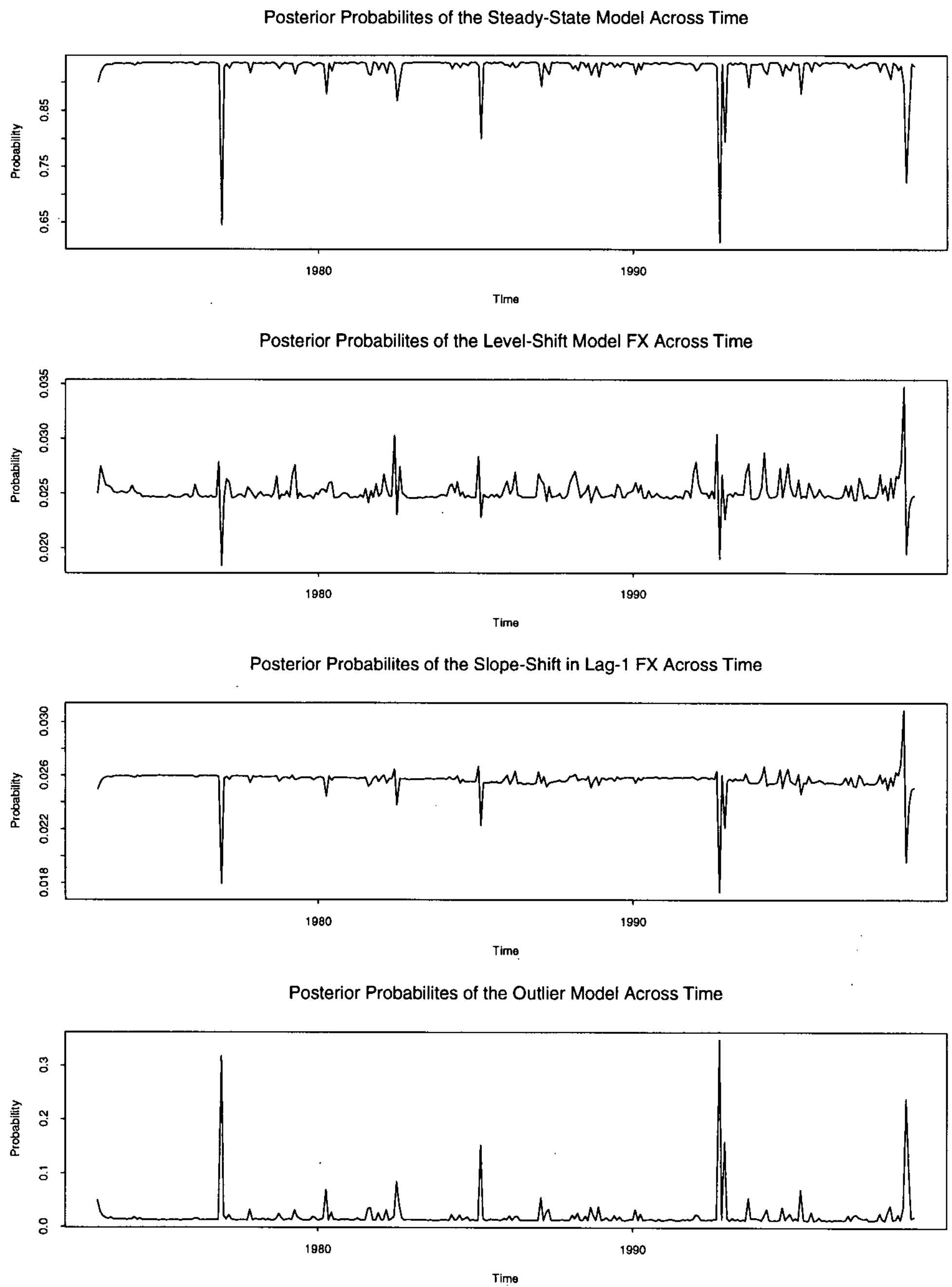


Figure C.3: One-Step-Back Probabilities of the Models for the Series DEM/USD.

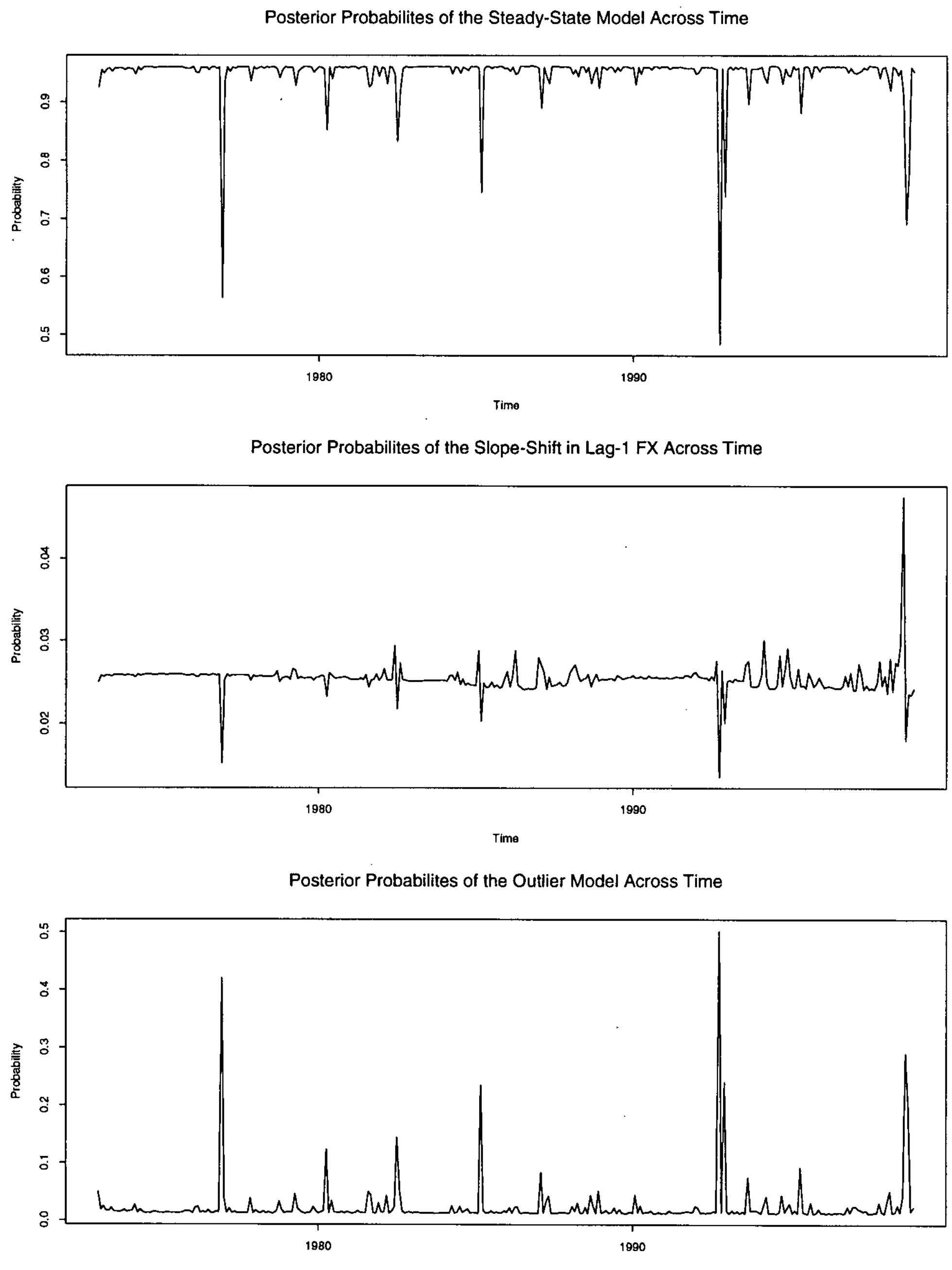


Figure C.4: One-Step-Back Probabilities of the Models for the Series ITL/USD.

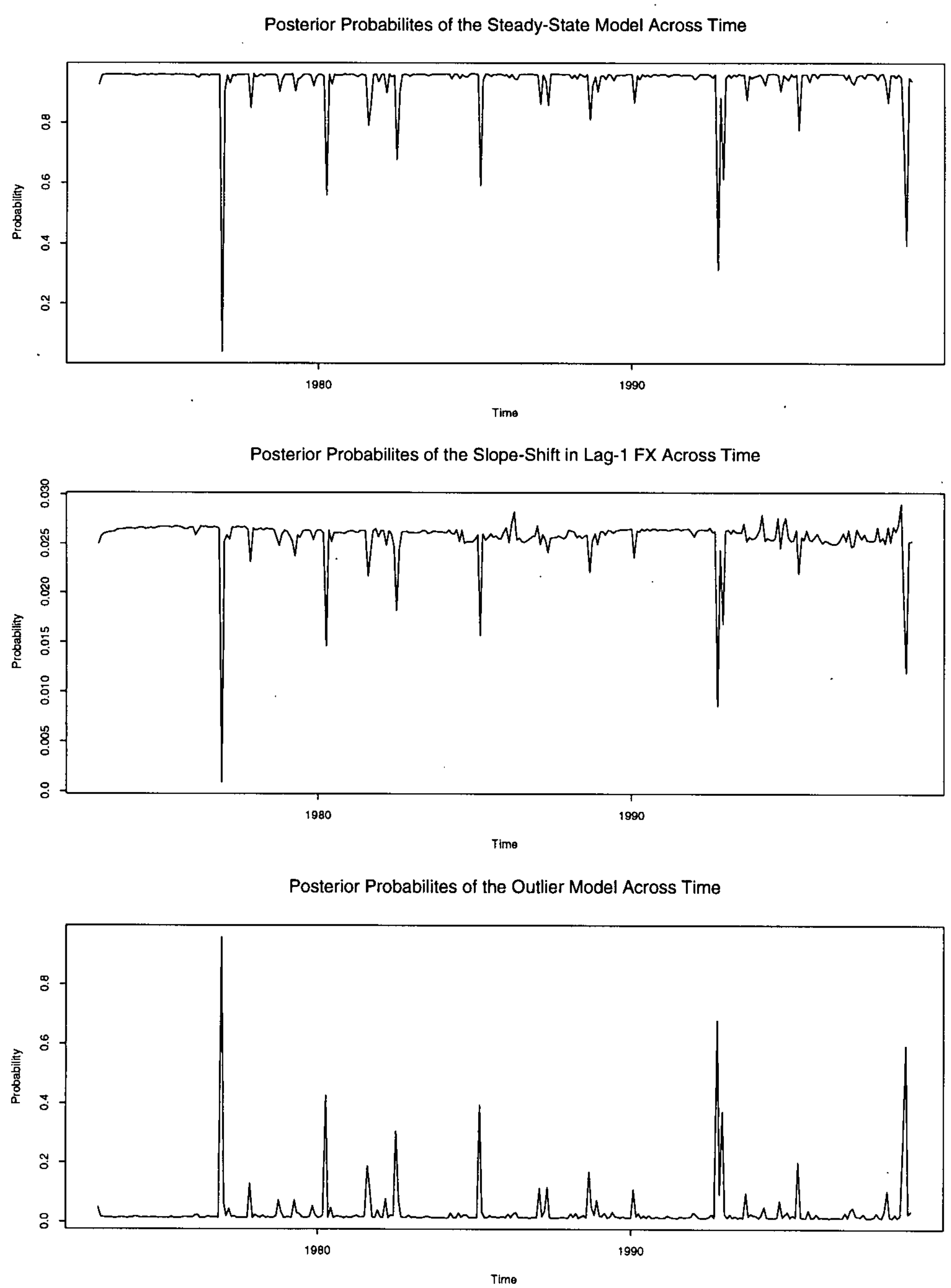


Figure C.5: One-Step-Back Probabilities of the Models for the Series CAD/USD.

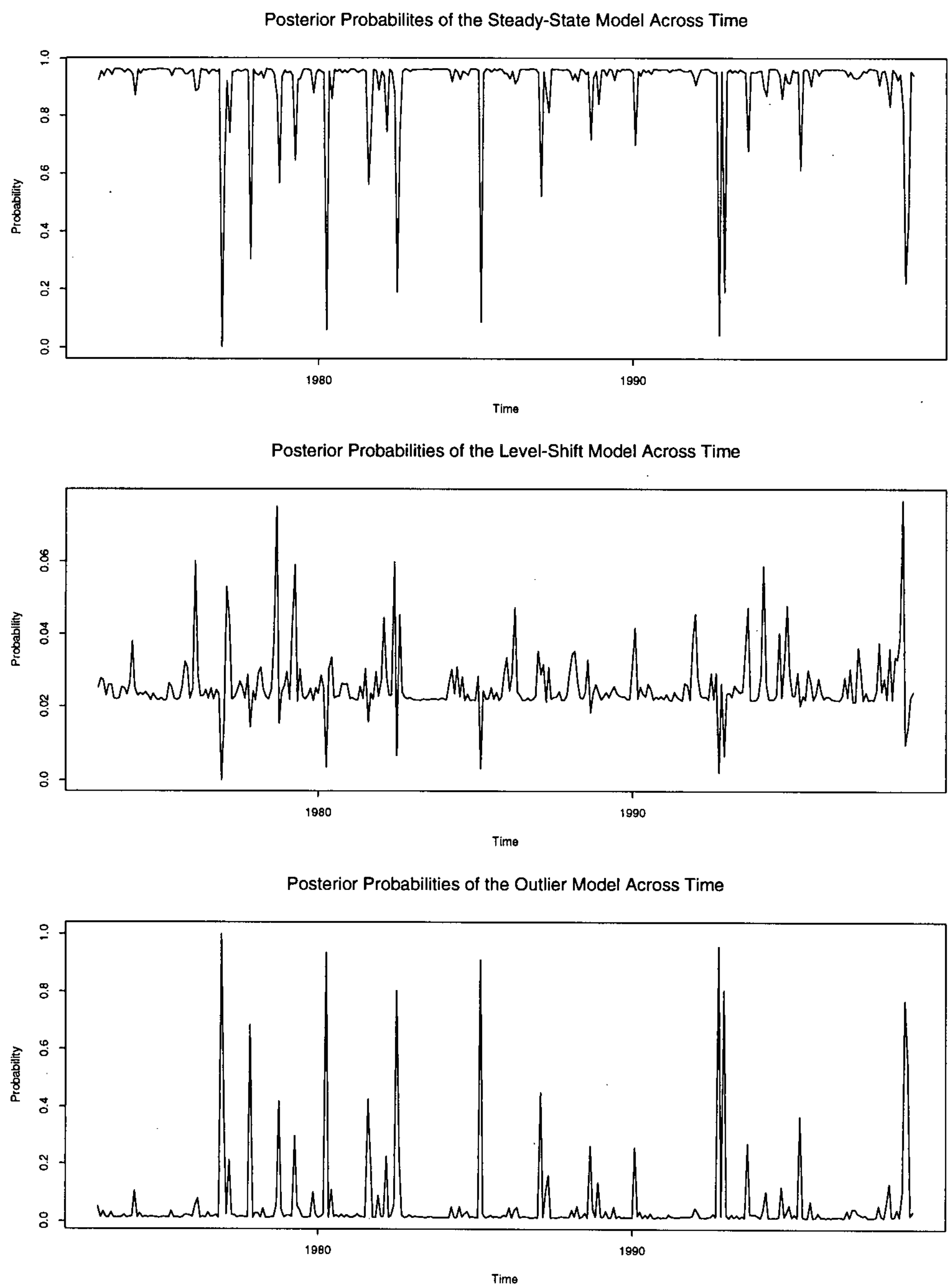
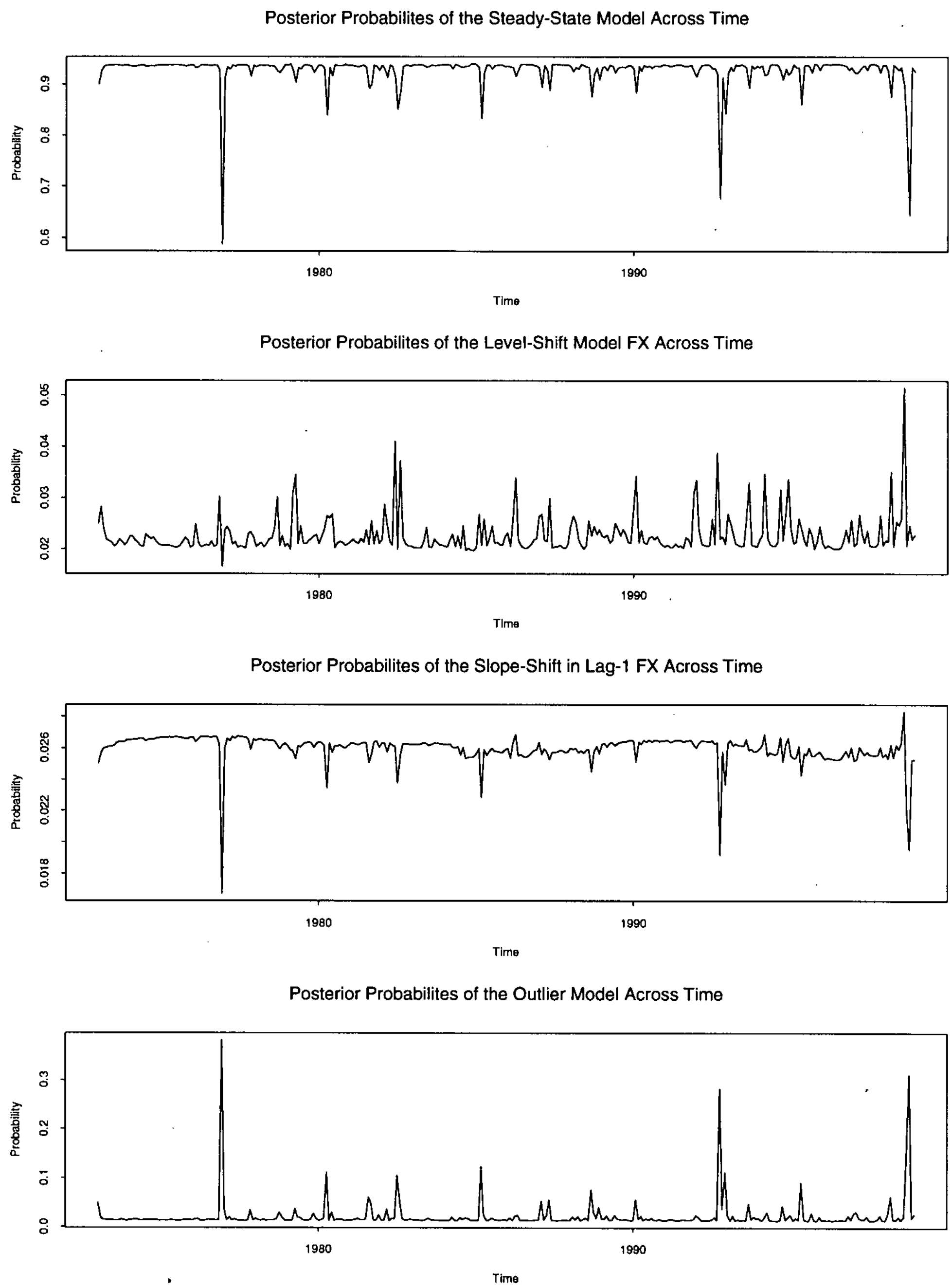


Figure C.6: One-Step-Back Probabilities of the Models for the Series JPY/USD.



# Appendix D

# ME, MAE, RMSE and Frequency Tables

Table D.1: Out-of-Sample Performance in terms of ME, MAE, RMSE of Percentage Errors.

| | ME | MAE | RMSE |
|---|---|---|---|
| | GBP/USD | | |
| Random Walk | 0.01301 (3.08552) | 2.093 (2.258) | 3.071 (4.941) |
| Forward Rate | -0.2154 (2.8118) | 1.943 (2.035) | 2.807 (4.315) |
| Simple Regression | 0.05633 (3.05781) | 2.082 (2.231) | 3.044 (4.845) |
| Linear System | 0.1740 (3.0948) | 2.015 (2.267) | 3.085 (4.921) |
| Threshold System | | | |
| 2-segment | 0.03298 (2.92039) | 2.052 (2.069) | 2.907 (4.247) |
| 3-segment | 0.5678 (6.4695) | 2.600 (5.946) | 6.464 (18.414) |
| R-W Coefficients | 0.02974 (3.1069) | 2.108 (2.273) | 3.093 (4.977) |
| Mixture Model | 0.2325 (3.2032) | 2.175 (2.354) | 3.197 (5.205) |
| | FRF/USD | | |
| Random Walk | 0.01915 (3.07602) | 2.320 (2.007) | 3.062 (4.333) |
| Forward Rate | -0.1578 (2.9387) | 2.282 (1.846) | 2.929 (3.860) |
| Simple Regression | -0.007013 (3.077480) | 2.326 (2.002) | 3.063 (4.363) |
| Linear System | 0.04012 (3.11576) | 2.346 (2.038) | 3.102 (4.221) |
| Threshold System | | | |
| 2-segment | 0.2541 (3.2025) | 2.424 (2.095) | 3.198 (4.392) |
| 3-segment | 0.1008 (3.1523) | 2.396 (2.038) | 3.139 (4.371) |
| R-W Coefficients | 0.08085 (3.10218) | 2.323 (2.046) | 3.089 (4.440) |
| Mixture Model | 0.1504 (3.1317) | 2.353 (2.059) | 3.121 (4.477) |
| | DEM/USD | | |
| Random Walk | 0.0328 (3.0876) | 2.296 (2.052) | 3.073 (4.435) |
| Forward Rate | -0.07737 (2.94462) | 2.181 (1.968) | 2.932 (4.036) |
| Simple Regression | -0.1982 (3.0973) | 2.334 (2.034) | 3.089 (4.372) |
| Linear System | 0.5362 (3.1462) | 2.355 (2.143) | 3.177 (4.746) |
| Threshold System | | | |
| 2-segment | -0.1110 (3.2600) | 2.499 (2.083) | 3.247 (4.517) |
| 3-segment | -0.1566 (3.2242) | 2.442 (2.098) | 3.213 (4.557) |
| R-W Coefficients | 0.03059 (3.08326) | 2.323 (2.015) | 3.069 (4.354) |
| Mixture Model | 0.1366 (3.0937) | 2.332 (2.026) | 3.082 (4.394) |

| | ME | MAE | RMSE |
|---|---|---|---|
| | ITL/USD | | |
| Random Walk | 0.2957 (3.2689) | 2.266 (2.364) | 3.267 (5.041) |
| Forward Rate | 0.01759 (3.10106) | 2.180 (2.196) | 3.087 (4.825) |
| Simple Regression | 0.2221 (3.2698) | 2.277 (2.347) | 3.262 (5.003) |
| Linear System | 0.8054 (3.2693) | 2.290 (2.460) | 3.352 (5.074) |
| Threshold System | | | |
| 2-segment | 0.1502 (3.5048) | 2.335 (2.609) | 3.492 (5.595) |
| 3-segment | 0.2989 (3.5821) | 2.446 (2.624) | 3.578 (5.591) |
| R-W Coefficients | 0.1995 (3.3161) | 2.297 (2.390) | 3.307 (5.272) |
| Mixture Model | 0.03843 (3.48126) | 2.407 (2.505) | 3.465 (5.522) |
| | CAD/USD | | |
| Random Walk | 0.2672 (1.3101) | 1.017 (0.863) | 1.331 (1.772) |
| Forward Rate | 0.2138 (1.2954) | 0.9984 (0.847) | 1.307 (1.719) |
| Simple Regression | 0.2086 (1.3167) | 1.018 (0.855) | 1.327 (1.728) |
| Linear System | 0.3869 (1.3342) | 1.061 (0.892) | 1.383 (1.807) |
| Threshold System | | | |
| 2-segment | 0.2111 (1.3147) | 1.011 (0.859) | 1.325 (1.740) |
| 3-segment | 0.1817 (1.3196) | 1.012 (0.861) | 1.326 (1.756) |
| R-W Coefficients | 0.1962 (1.3341) | 1.023 (0.873) | 1.342 (1.753) |
| Mixture Model | 0.2227 (1.3484) | 1.031 (0.891) | 1.360 (1.798) |
| | JPY/USD | | |
| Random Walk | -0.1371 (3.5317) | 2.573 (2.410) | 3.518 (5.080) |
| Forward Rate | 0.01848 (3.38960) | 2.445 (2.336) | 3.374 (5.113) |
| Simple Regression | 0.1151 (3.5125) | 2.544 (2.412) | 3.498 (5.153) |
| Linear System | 0.2862 (3.5077) | 2.534 (2.431) | 3.503 (5.161) |
| Threshold System | | | |
| 2-segment | -0.1738 (3.5992) | 2.592 (2.491) | 3.587 (5.372) |
| 3-segment | 0.06024 (3.66427) | 2.610 (2.560) | 3.648 (5.437) |
| R-W Coefficients | -0.1154 (3.529) | 2.546 (2.435) | 3.515 (5.147) |
| Mixture Model | -0.1490 (3.531) | 2.551 (2.434) | 3.518 (5.146) |

Table D.2: Percent Frequency of Correct Buy and Sell Signals.

| | $\lambda_E$ | $\lambda_F$ | $\lambda_E$ | $\lambda_F$ | $\lambda_E$ | $\lambda_F$ |
|---|---|---|---|---|---|---|
| | GBP/USD | | FRF/USD | | DEM/USD | |
| Random Walk | ··· | 49.1 | ··· | 53.7 | ··· | 52.8 |
| Forward Rate | 64.8 | ··· | 53.7 | ··· | 62.0 | ··· |
| Simple Regression | 51.9 | 55.6 | 47.2 | 56.5 | 44.4 | 53.7 |
| Linear System | 56.5 | 50.9 | 50.9 | 49.1 | 54.6 | 55.6 |
| Threshold System | | | | | | |
| 2-segment | 51.9 | 54.6 | 56.5 | 52.8 | 38.0 | 46.3 |
| 3-segment | 49.1 | 53.7 | 58.3 | 51.9 | 44.4 | 52.8 |
| R-W Coefficients | 54.6 | 48.1 | 56.5 | 52.8 | 49.1 | 50.9 |
| Mixture Model | 47.2 | 52.8 | 52.8 | 51.9 | 49.1 | 54.6 |
| | ITL/USD | | CAD/USD | | JPY/USD | |
| Random Walk | ··· | 56.5 | ··· | 56.5 | ··· | 50.9 |
| Forward Rate | 56.5 | ··· | 54.6 | ··· | 59.3 | ··· |
| Simple Regression | 48.1 | 57.4 | 50.0 | 50.9 | 59.3 | 52.8 |
| Linear System | 55.6 | 60.2 | 42.6 | 51.9 | 58.3 | 57.4 |
| Threshold System | | | | | | |
| 2-segment | 54.6 | 60.2 | 51.9 | 54.6 | 60.2 | 56.5 |
| 3-segment | 54.6 | 57.4 | 57.4 | 55.6 | 59.3 | 57.4 |
| R-W Coefficients | 50.9 | 56.5 | 51.9 | 50.9 | 57.4 | 58.3 |
| Mixture Model | 48.1 | 56.5 | 48.1 | 53.7 | 59.3 | 61.1 |

# Appendix E

# Plots of Lopez Loss Scores against Percentage Changes in Prices

*The labels in the legend represent:*

- RW — Simple Random Walk
- FR — Forward Rate Model
- SR — Simple Regression
- LSY — Linear System Model
- TSR2 — Threshold 2-Segmented Regression
- TSR3 — Threshold 3-Segmented Regression
- RWC — Random Walk Coefficient Model
- MRWC — Mixture of Random Walk Coefficient Models

Figure E.1: Lopez Scores for Short USD against GBP.

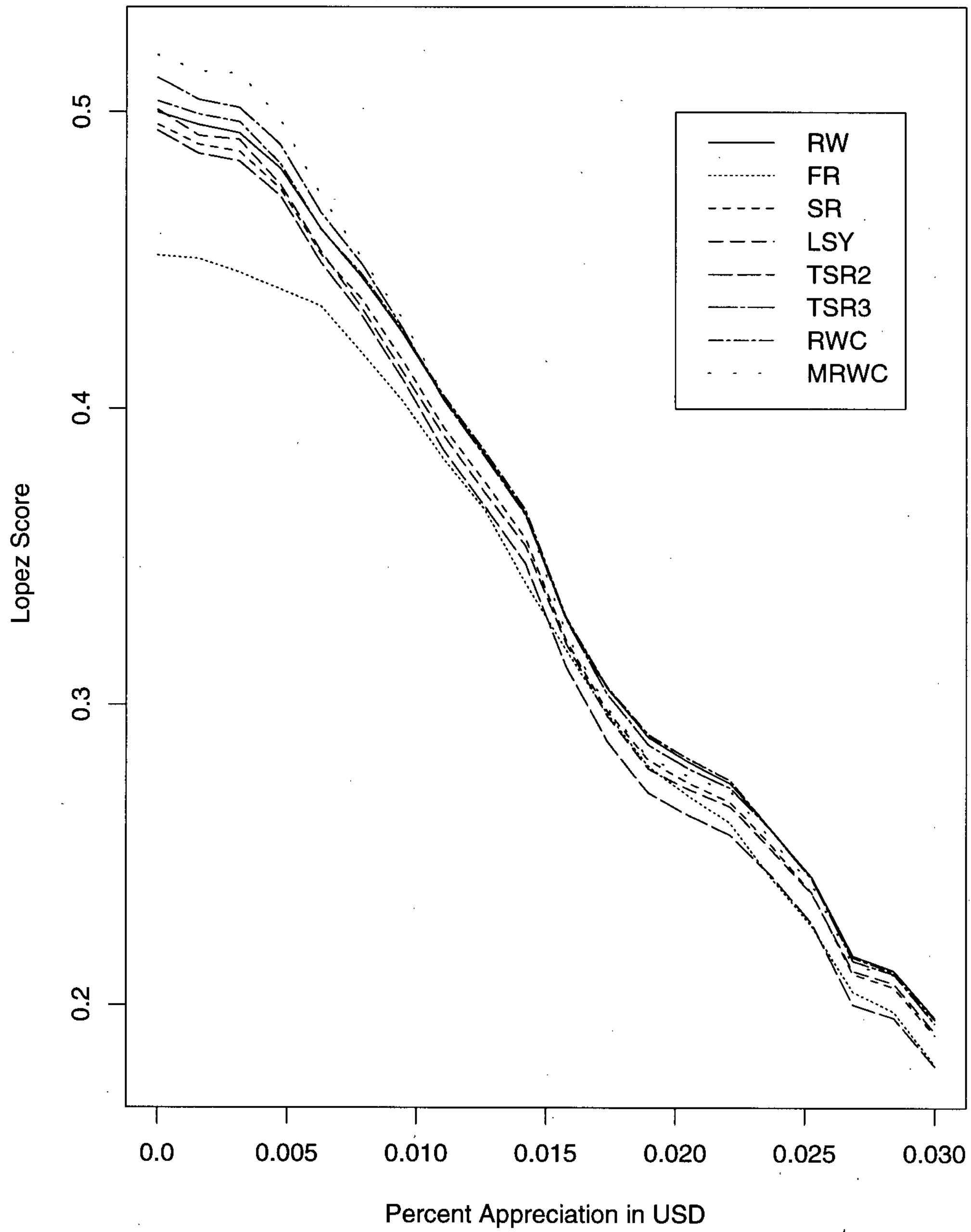


Figure E.2: Lopez Scores for Long USD against GBP.

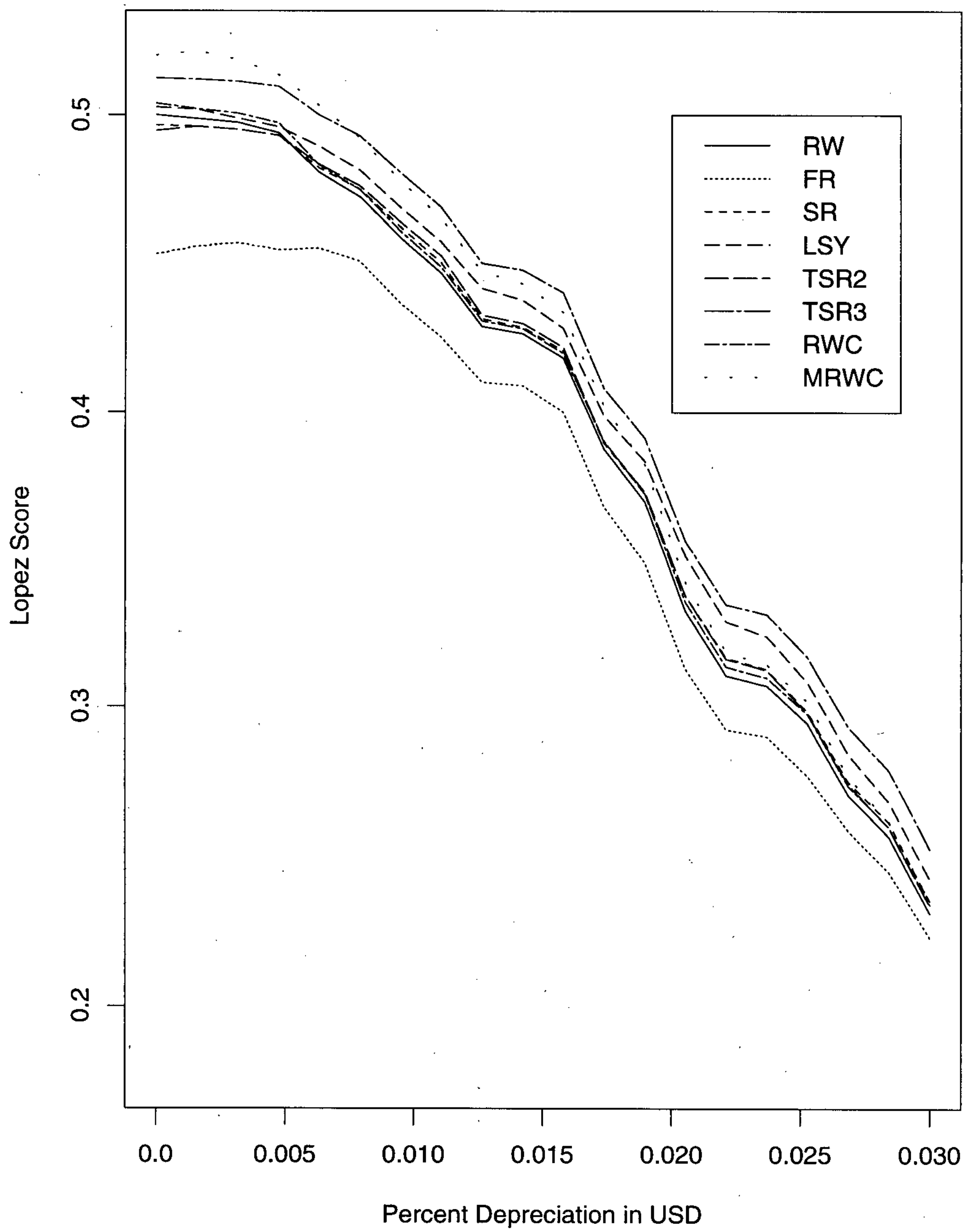


Figure E.3: Average Lopez Scores over Opposing Positions in USD against GBP.

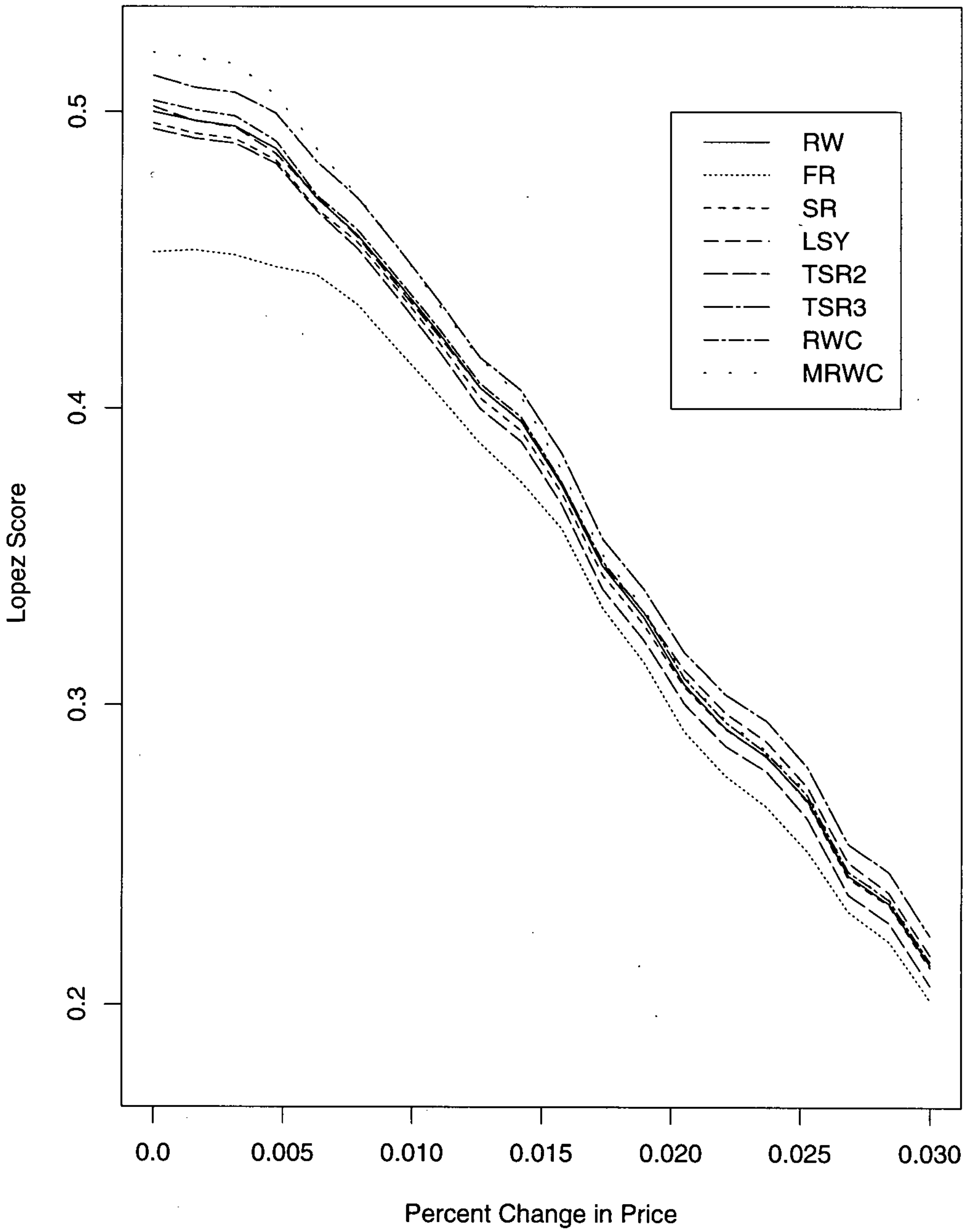


Figure E.4: Lopez Scores for Short USD against FRF.

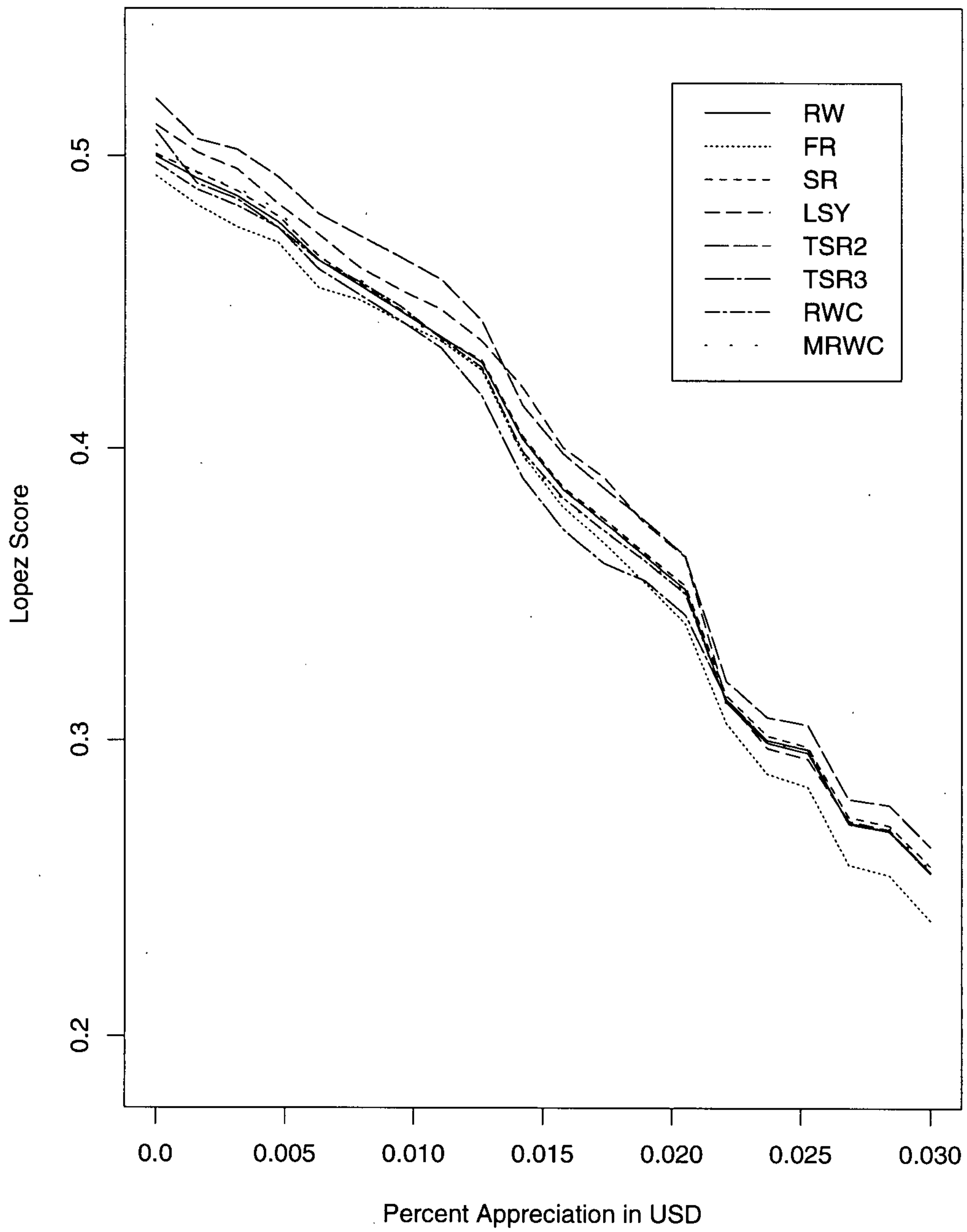


Figure E.5: Lopez Scores for Long USD against FRF.

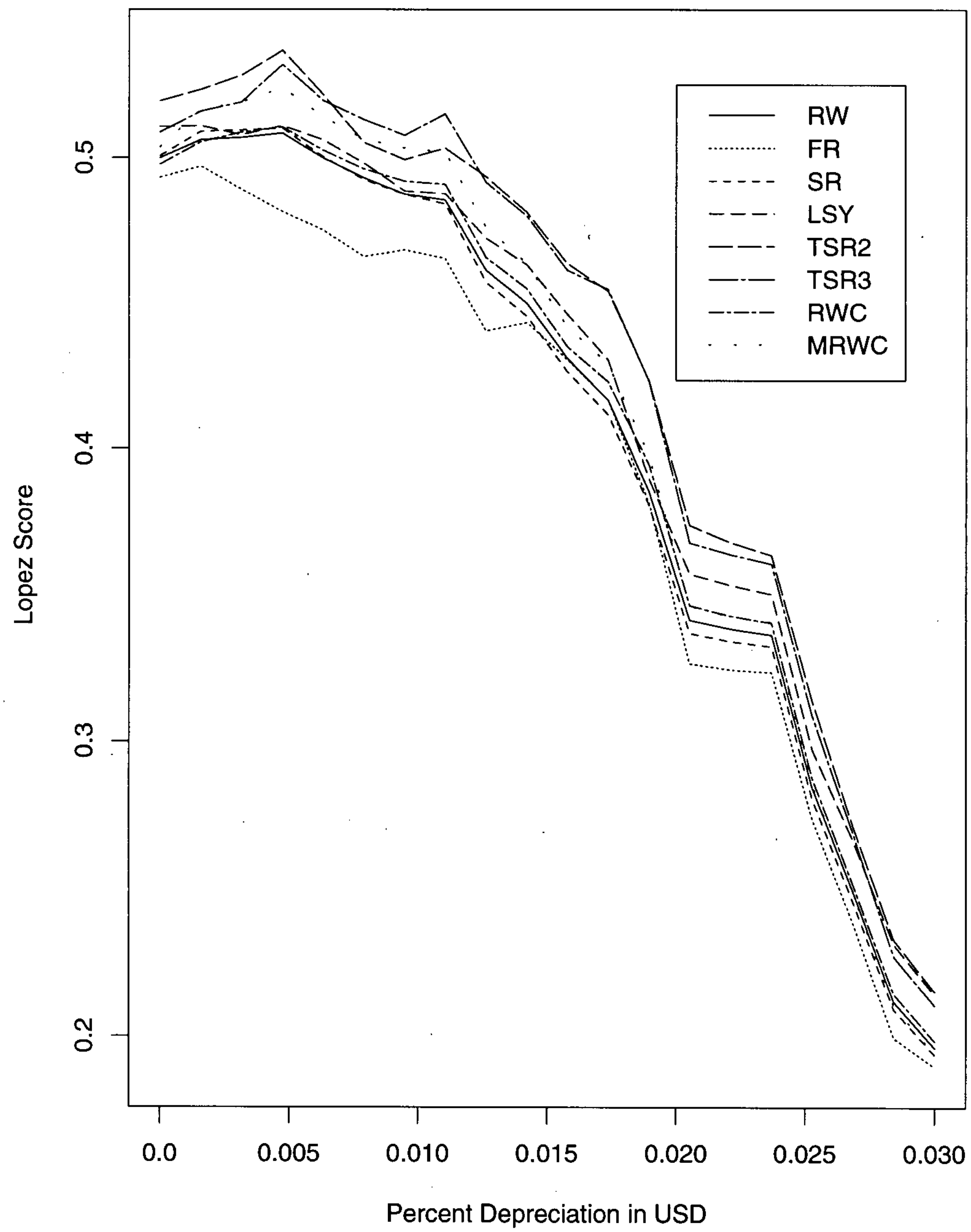


Figure E.6: Average Lopez Scores over Opposing Positions in USD against FRF.

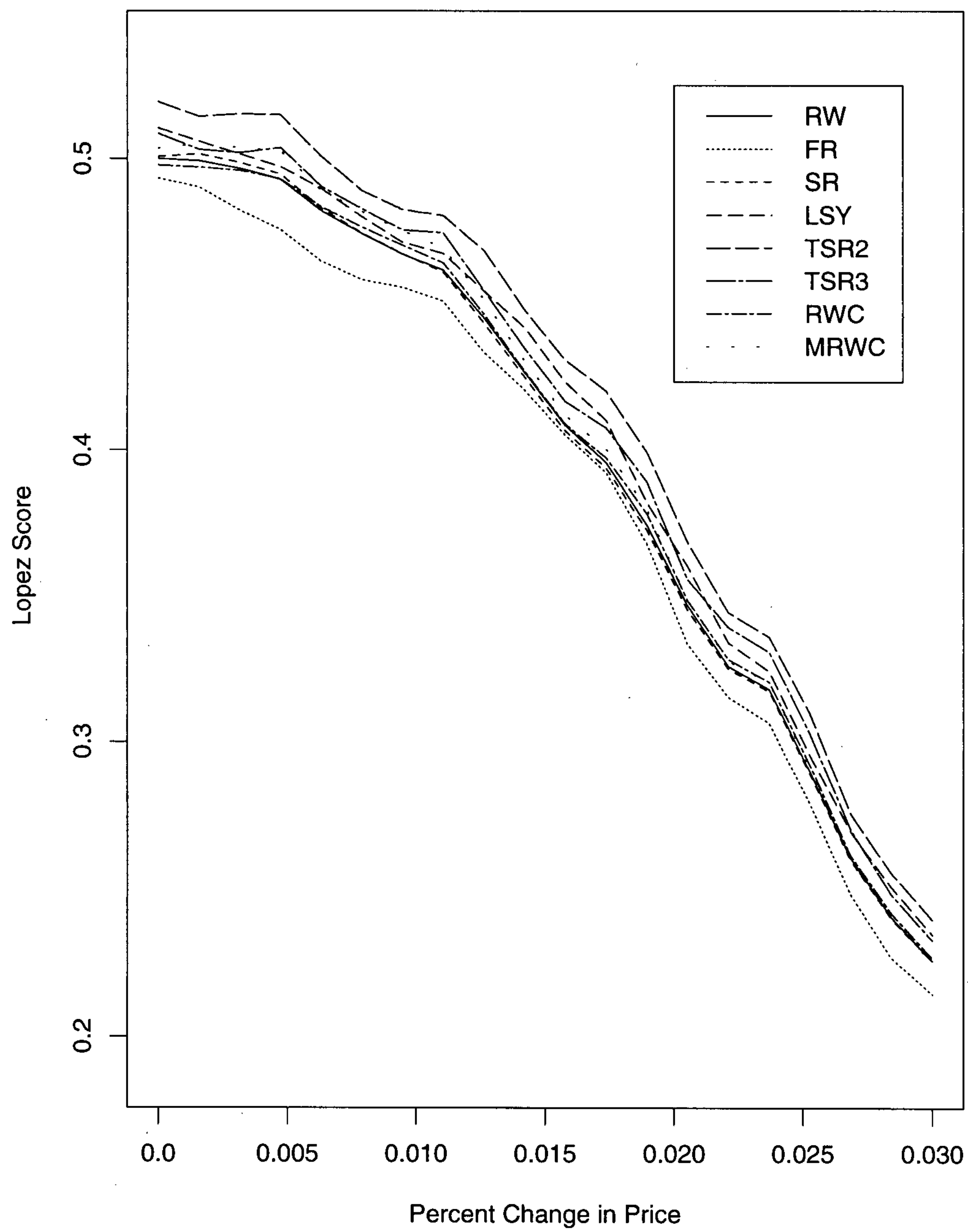


Figure E.7: Lopez Scores for Short USD against DEM.

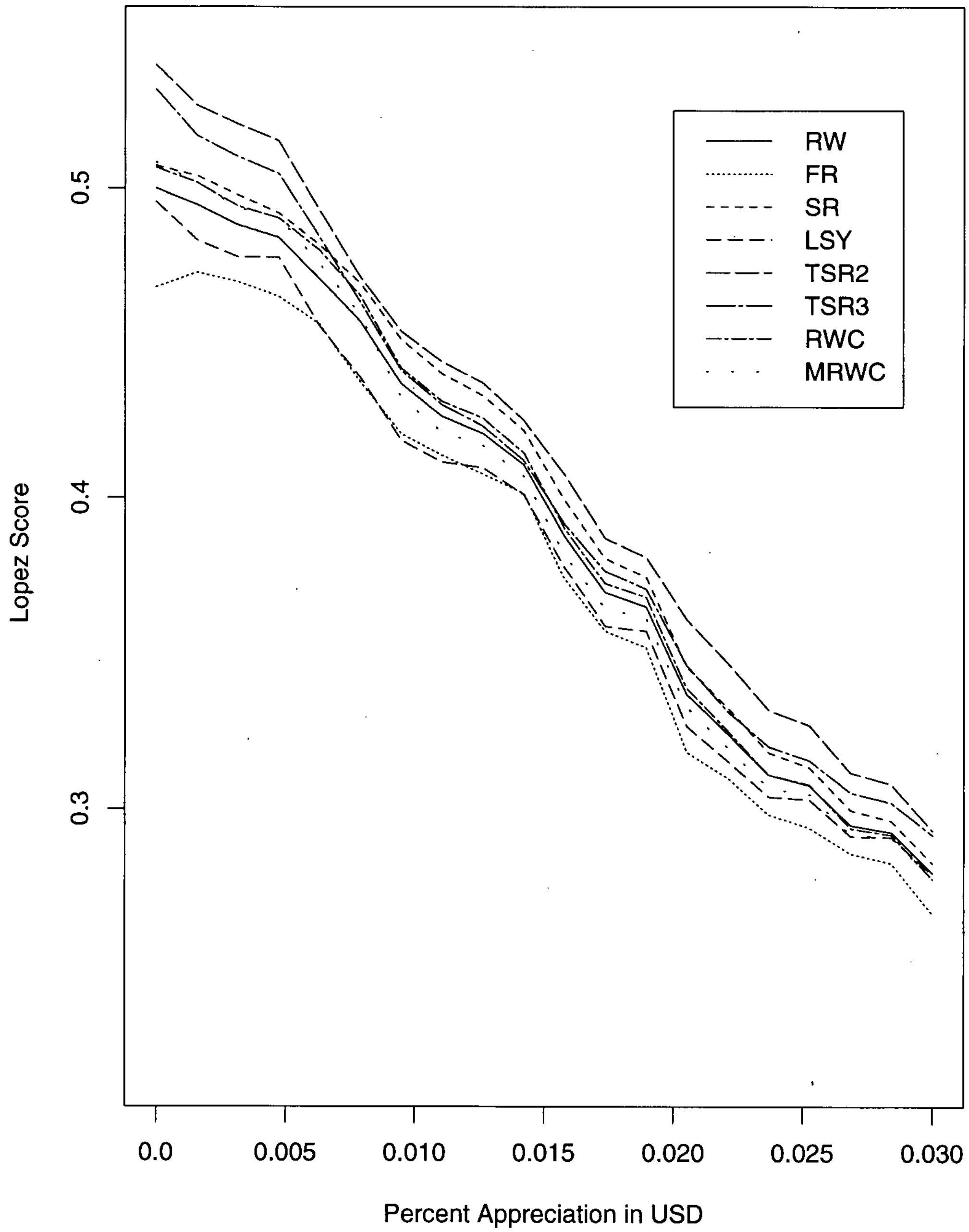


Figure E.8: Lopez Scores for Long USD against DEM.

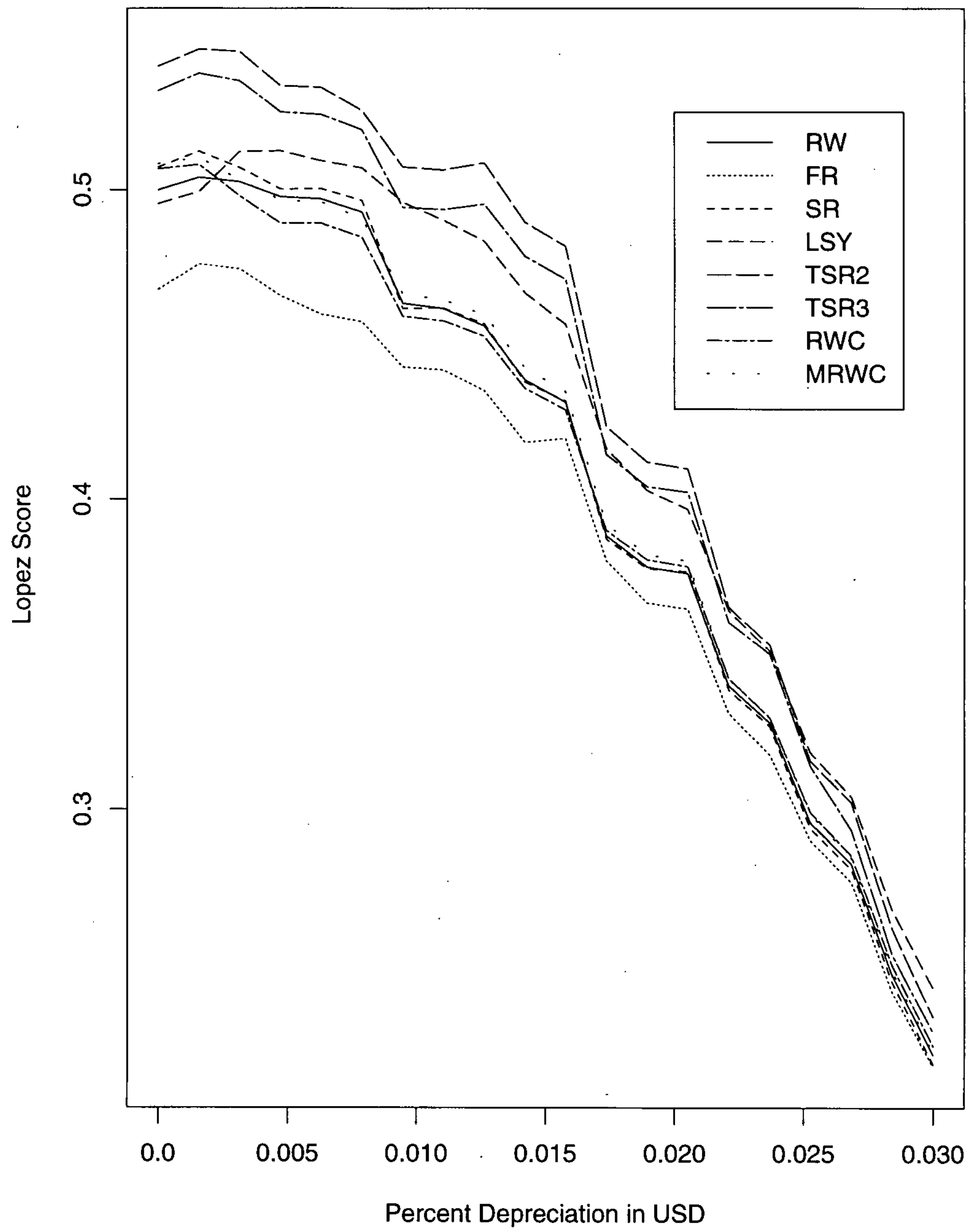


Figure E.9: Average Lopez Scores over Opposing Positions in USD against DEM.

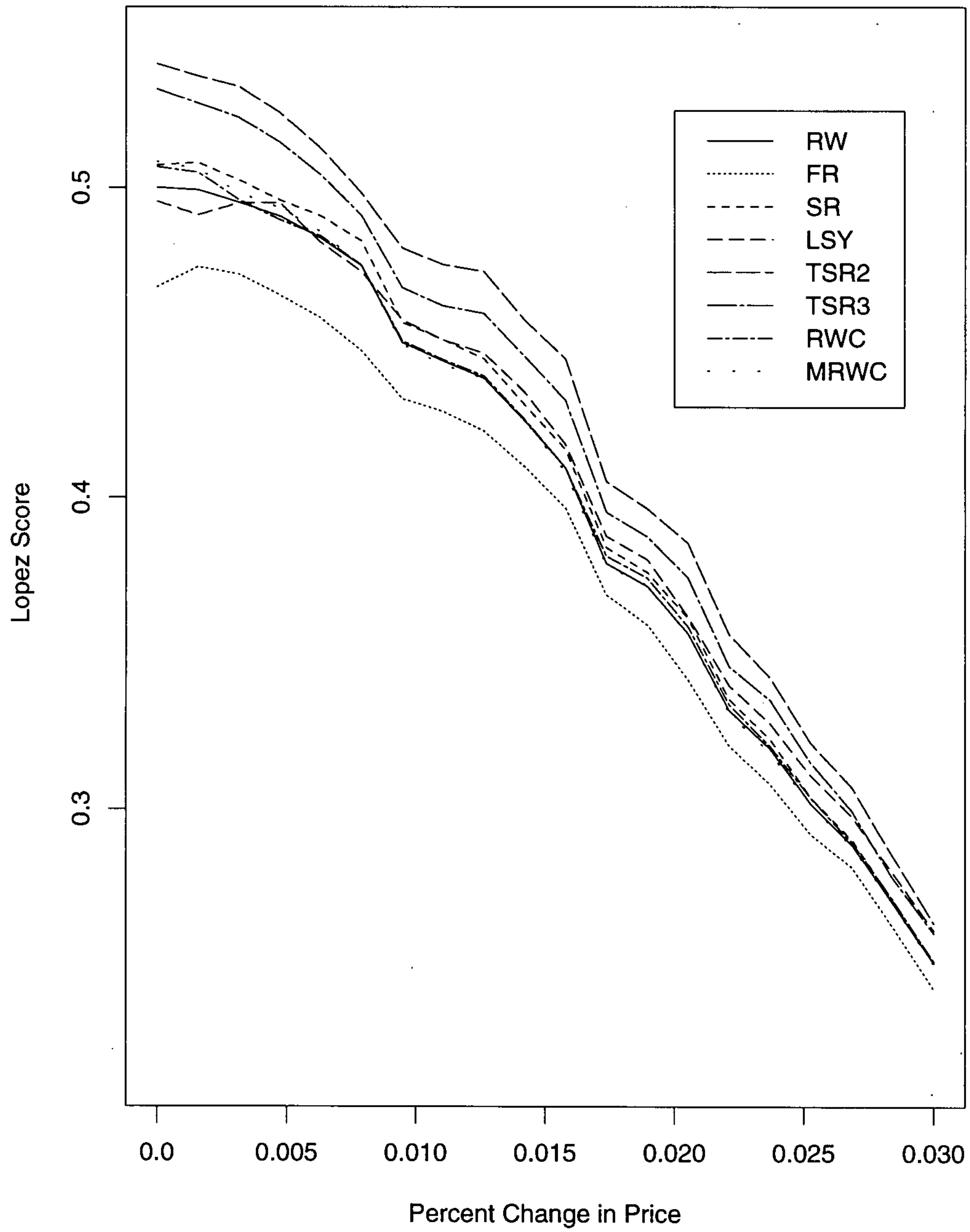


Figure E.10: Lopez Scores for Short USD against ITL.

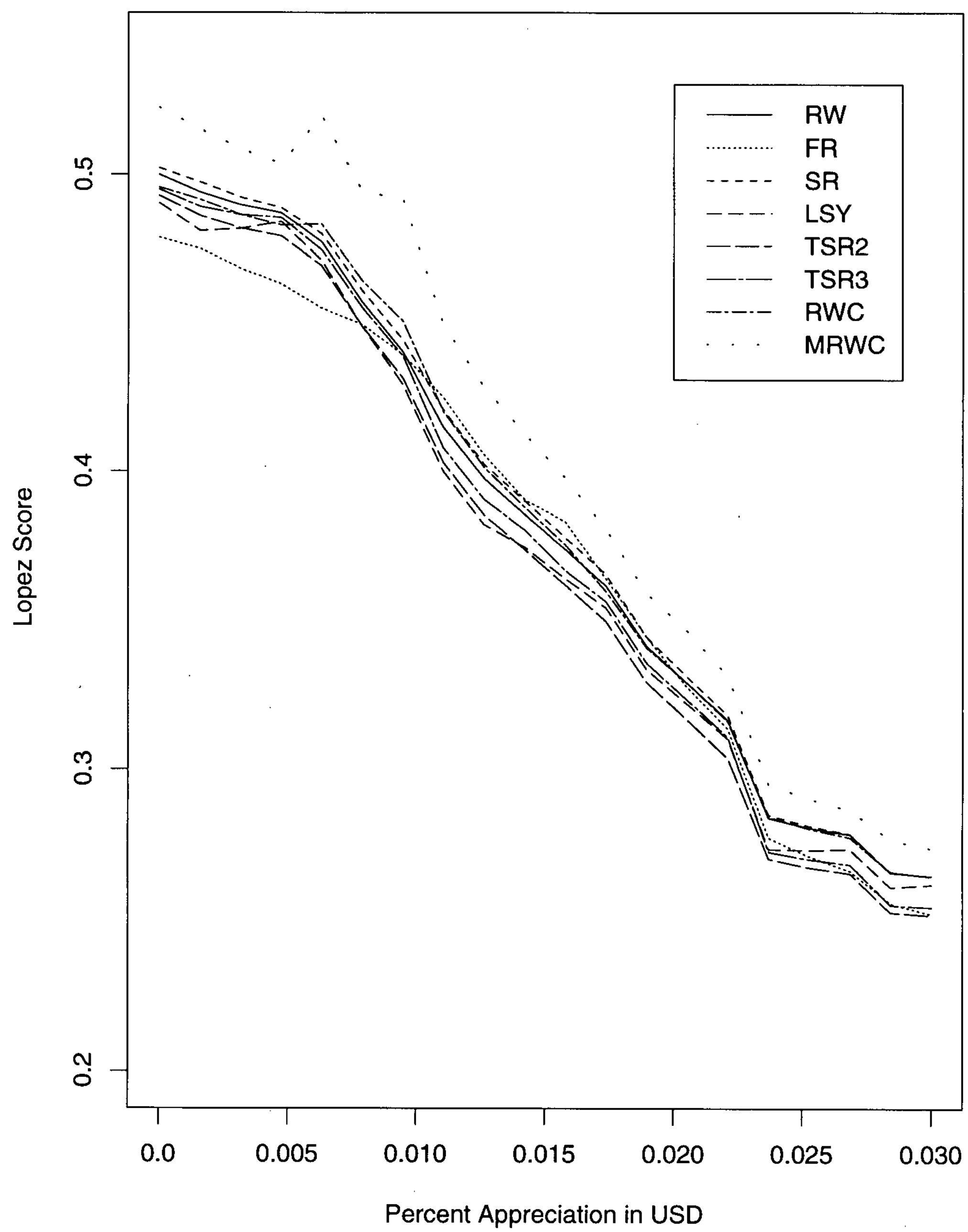


Figure E.11: Lopez Scores for Long USD against ITL.

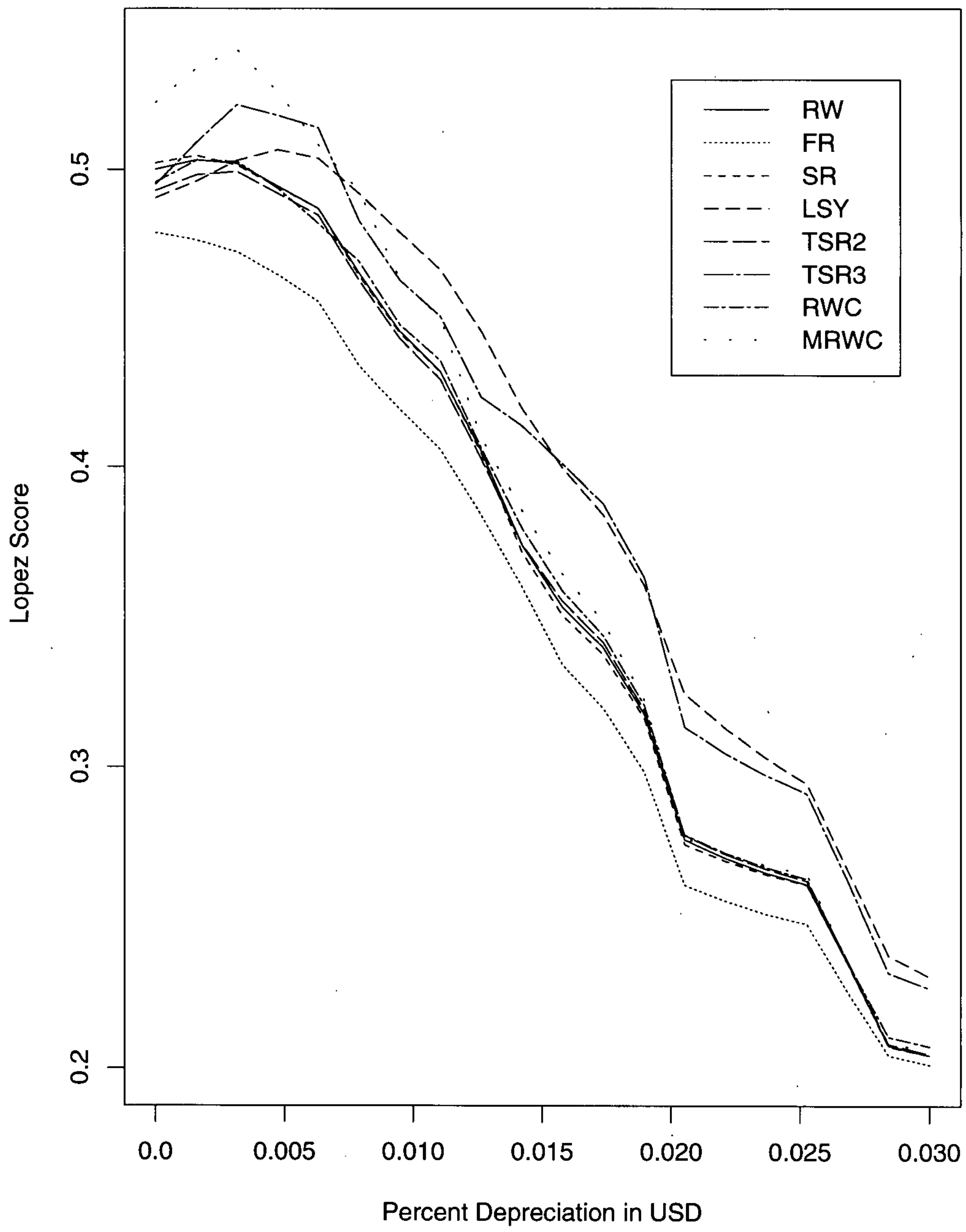


Figure E.12: Average Lopez Scores over Opposing Positions in USD against ITL.

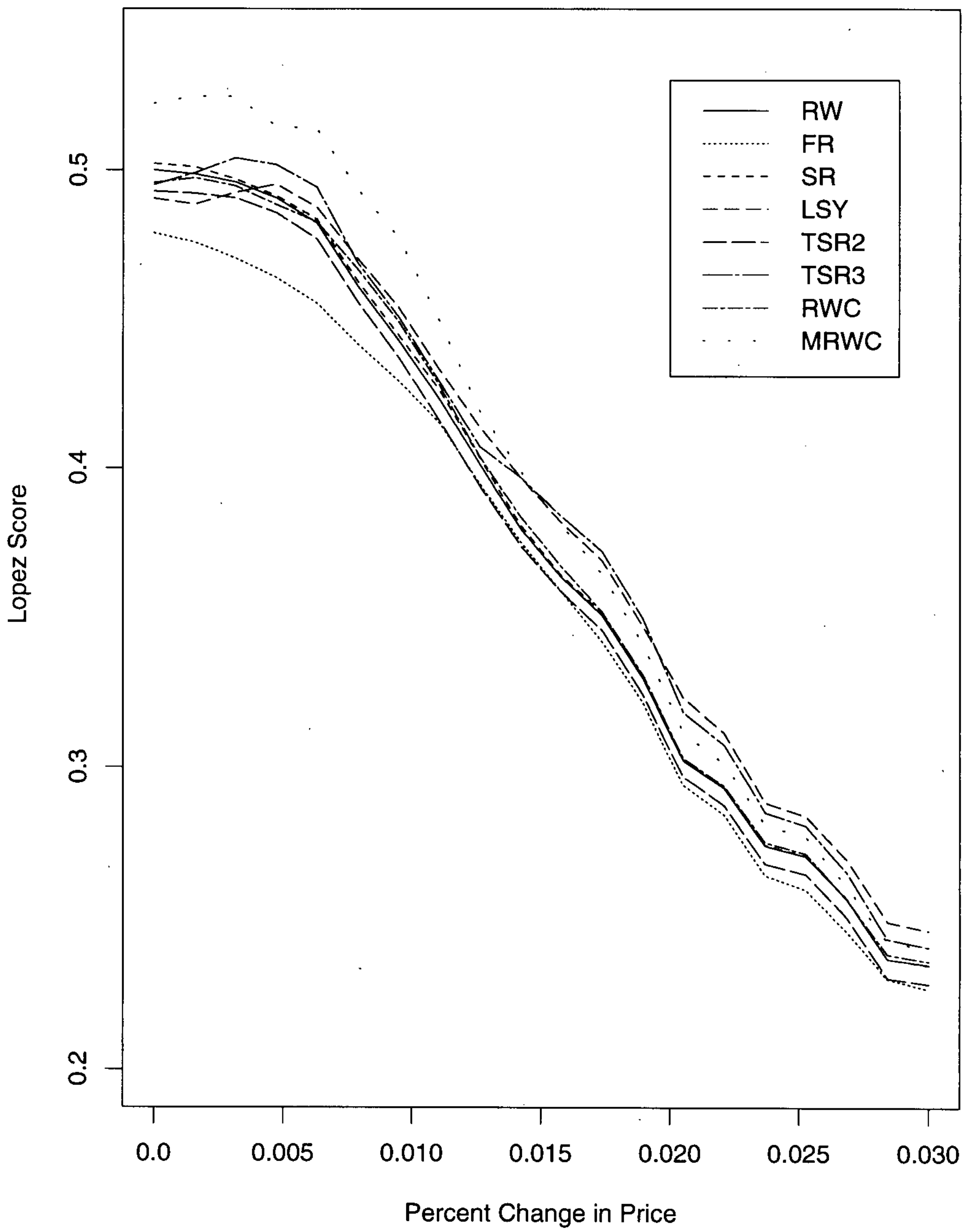


Figure E.13: Lopez Scores for Short USD against CAD.

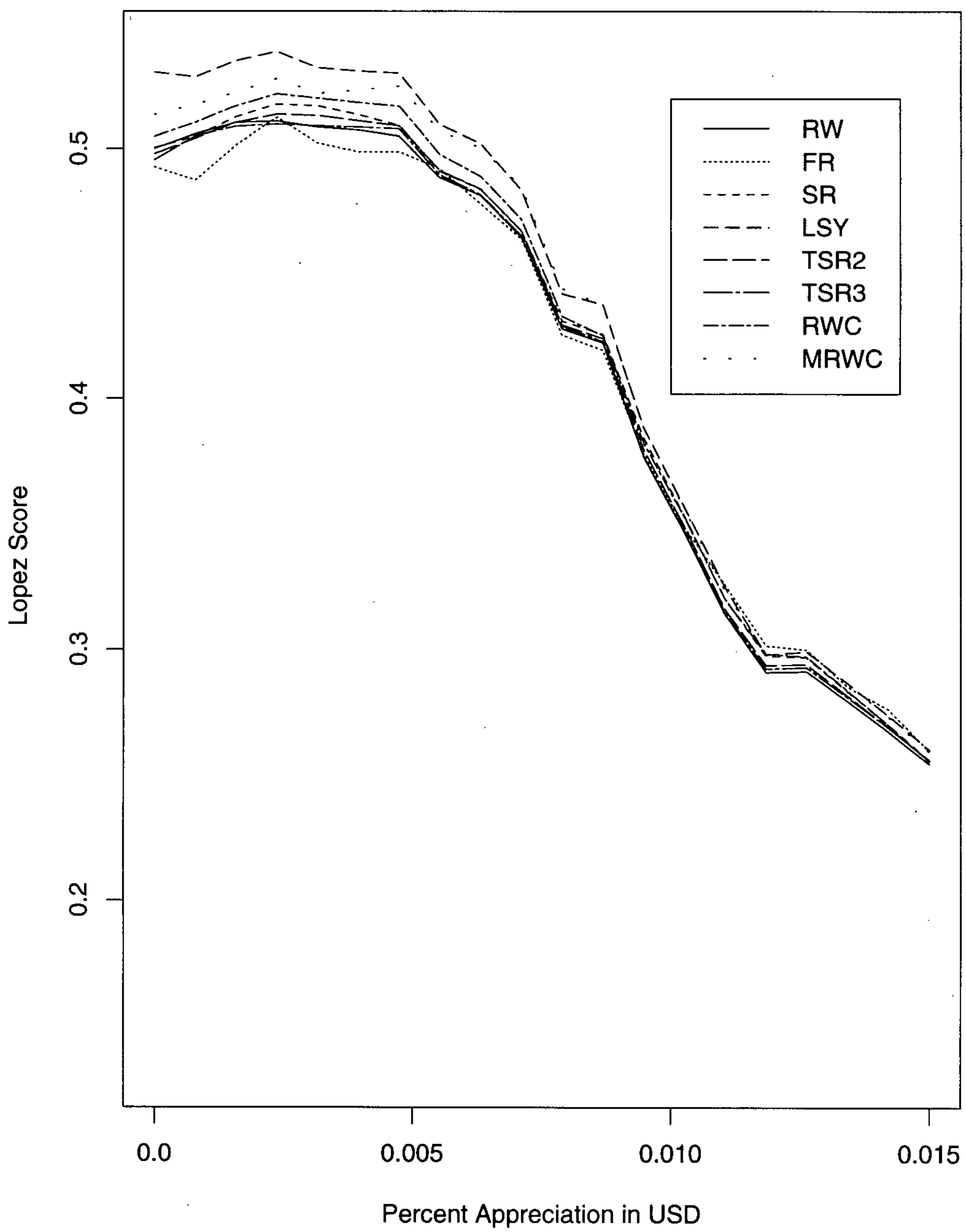


Figure E.14: Lopez Scores for Long USD against CAD.

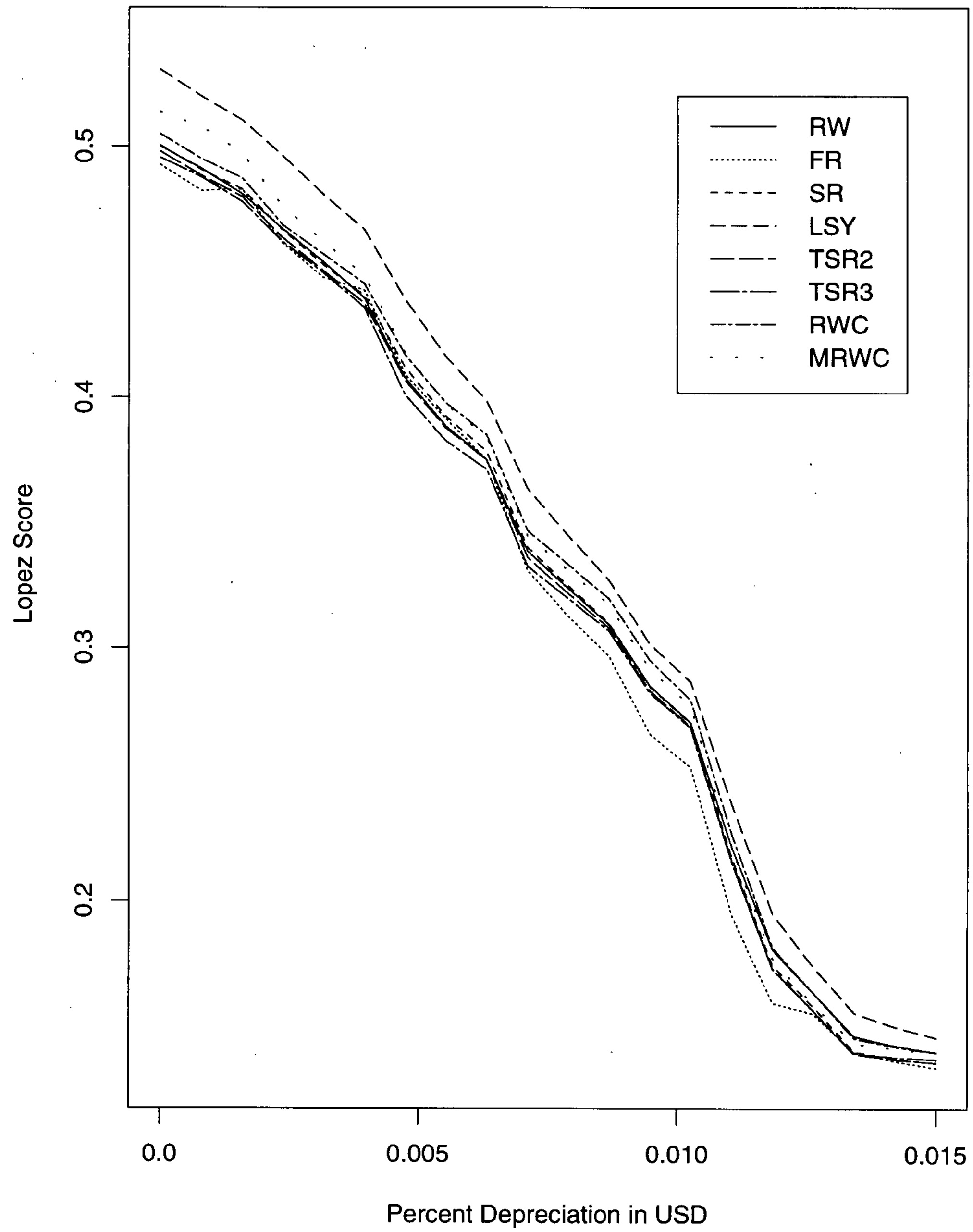


Figure E.15: Average Lopez Scores over Opposing Positions in USD against CAD.

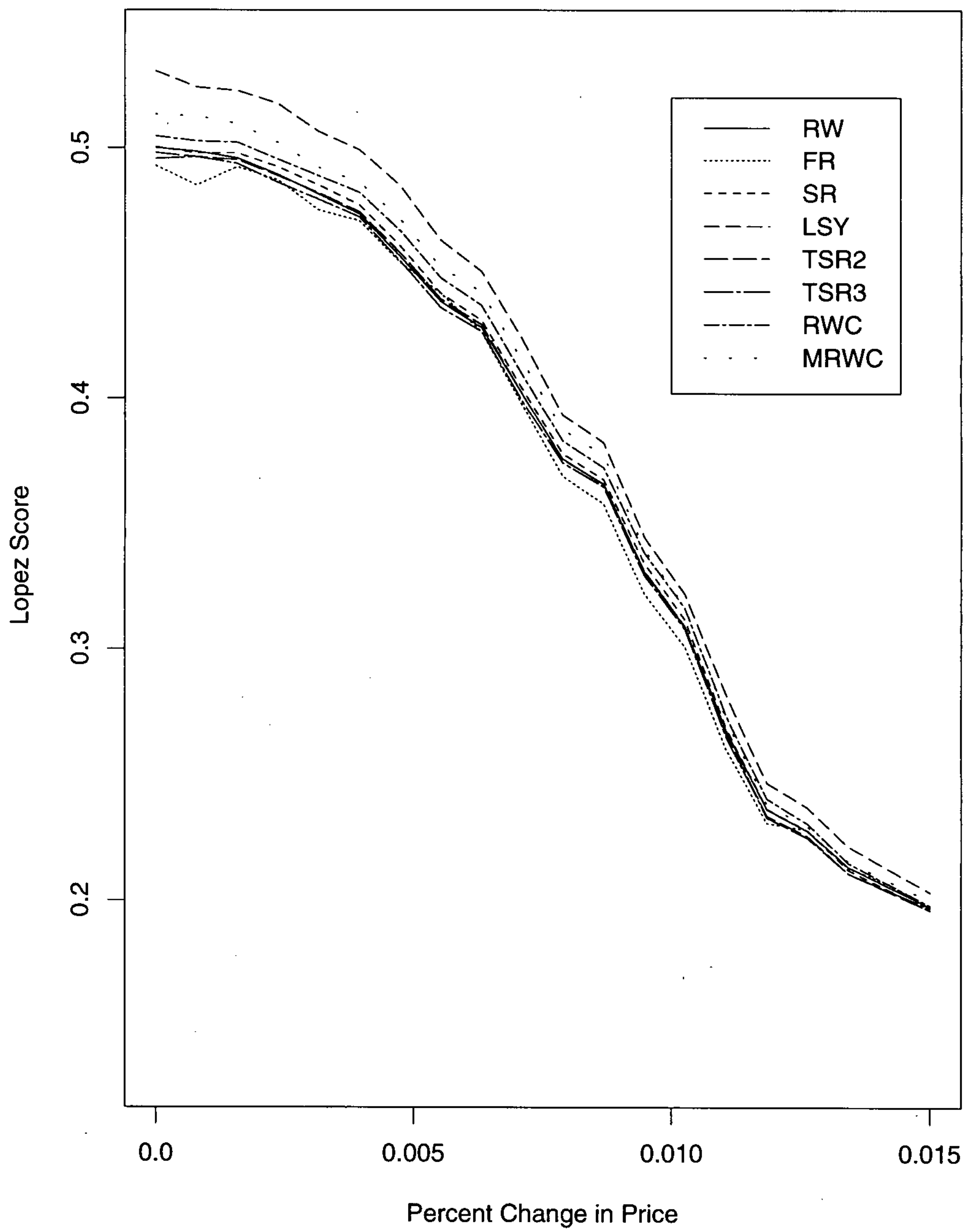


Figure E.16: Lopez Scores for Short USD against JPY.

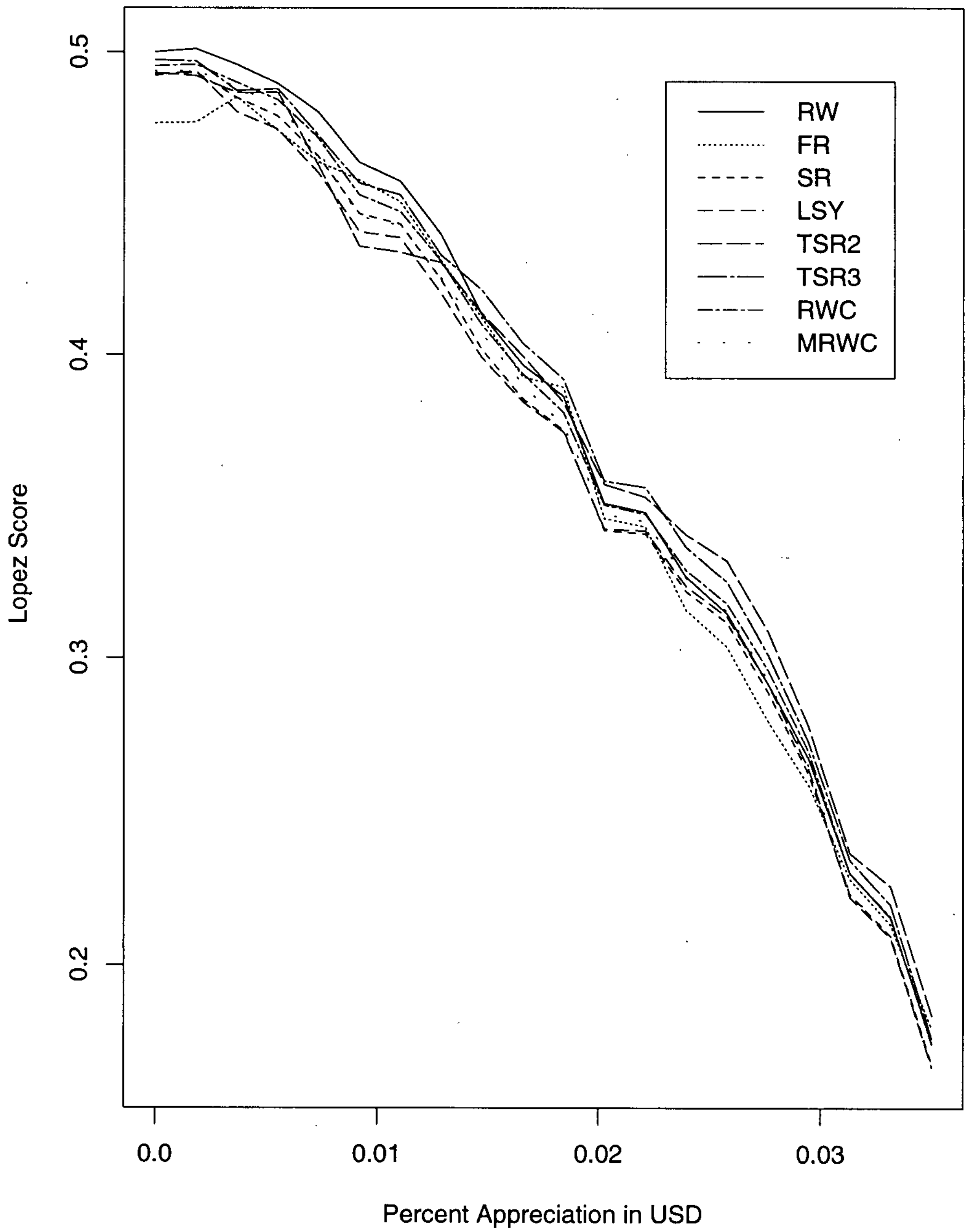


Figure E.17: Lopez Scores for Long USD against JPY.

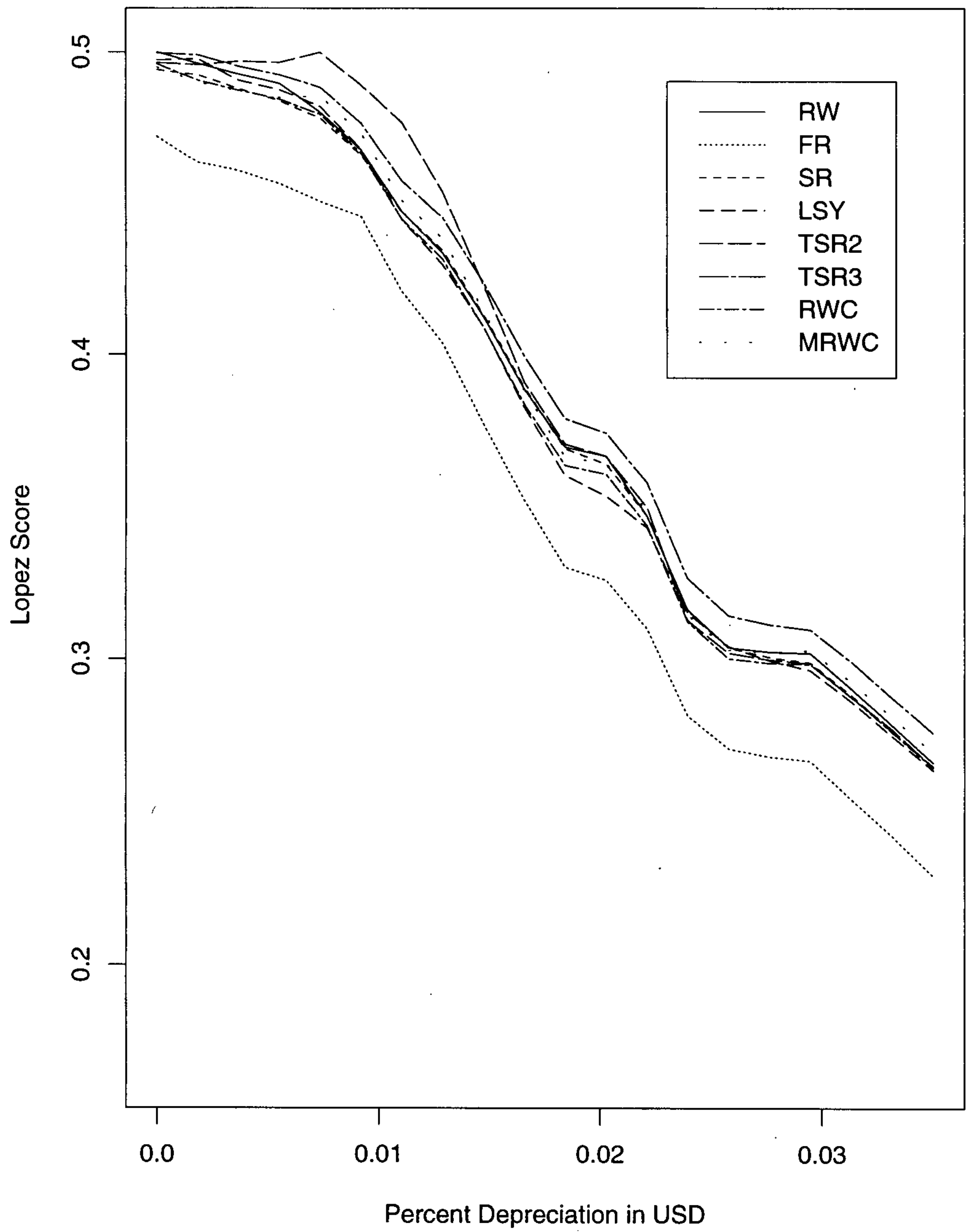


Figure E.18: Average Lopez Scores over Opposing Positions in USD against JPY.