ESSAYS IN POLICY ANALYSIS AND STRATEGY:
ENTREPRENEURSHIP, JOINT VENTURING, AND TRADE

by

RICHARD JAMES AREND

B.A.Sc., The University of Toronto, 1986
M.B.A. (Honours), York University, 1989

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

in
THE FACULTY OF GRADUATE STUDIES
(Commerce and Business Administration)

We accept this thesis as conforming
to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA
July 1995
© Richard James Arend, 1995
In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the head of my department or by his or her representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

(Signature)

Department of Commerce (Policy Division)

The University of British Columbia
Vancouver, Canada

Date Aug. 24, 1995
ABSTRACT

Separate essays on entrepreneurship, joint venturing, and trade comprise this thesis.

The emergence of entrepreneurship is common in the real world but relatively less so in classical economic models. If industry incumbents are attributed with full rationality and perfect foresight, then there are few, if any, profitable opportunities left for new entrants (entrepreneurs) to exploit. This essay explains how entrepreneurs can emerge in a dynamic world when firms must choose between a technology strategy that is either statically or dynamically efficient. A model is developed which shows how such opportunities for new entry can occur when incumbents are caught in a Prisoners’ Dilemma game involving technology strategy. A relevance measure and policy implications are then explored.

Joint ventures, especially of the R&D type, are becoming increasingly important as a way to gain needed technological and market competencies. Unfortunately, many joint ventures have the characteristics of a Prisoners’ Dilemma. Firms may cooperate or defect in the venture. If contracts, side-payments, and third-party verification of the venture outcome are unavailable, then the dominant solution to the Prisoners’ Dilemma (mutual defection) results. This paper proposes the use of an ex-ante auction to obtain a Pareto-improvement for these ventures. A Pareto-improvement is assured when non-transferable costs and benefits of firms are not conditional on joint venture strategies. When this condition is not met restrictions are required to obtain the Pareto-improvement.
The problem of trade between countries that share an international open access resource is becoming significant as the world reaches the limits of critical shared resource stocks. It is modelled as a world with one primary factor, two intermediate goods, one final good (harvested from the open access resource), and two nations where it is assumed that either the trading takes place over one stage (nations are price-takers), or two stages (nations have market power). Imperfect competition and open access generated externalities affect the trading efficiency. To maximize world welfare this essay recommends subsidizing R&D where comparative advantage exists, and creating international agreements to ensure the one-stage game structure is used when trading.
# TABLE OF CONTENTS

Abstract .................................................. ii

Table of Contents ........................................ iv

List of Tables ............................................. vii

List of Figures ............................................ viii

Acknowledgement .......................................... ix

INTRODUCTION ............................................ 1

Chapter One  Essay One:  
Technological Force: the Emergence of Entrepreneurship  
Introduction .............................................. 6  
The Model .................................................. 10  
  Important Definitions .................................. 10  
  Definition of the Game ................................ 13  
  Analysis of a Specific Game ............................ 21  
  Considering an Extension to the Game ............... 32  
Conclusions ............................................... 35

Chapter Two  Essay Two:  
An Auction Solution to the Joint Venture Prisoners’ Dilemma  
Introduction .............................................. 38  
Description of an Application ............................ 43  
The Game .................................................. 50  
  The Basic Application .................................. 52  
  An Application with \( n > 2 \) Firms .................... 60  
  An Application with Asymmetric Firms ................. 61  
  An Application with Strategic Non-Transferable Costs (SNTCs) .................. 69  
  Solutions Under General SNTCs ........................ 76  
    The New Auction Solution ............................ 78  
The Contracting Alternatives ............................ 80  
Conclusions ............................................... 84
Chapter Three  Essay Three:  
Fish and Ships: Trade with Imperfect Competition and an International Open Access Resource  
Introduction  
The Assumptions, the Model and the Autarky Case  
Analysis of the Trade in a Ricardian World  
- The One Stage Game  
  Case i. One Stage Game with Unrestricted Stocks  
  Case ii. One Stage Game with Restricted Stocks  
Analysis of Trade with Production Precommitments  
- The Two Stage Game  
  Case iii. Two Stage Game with Unrestricted Stocks  
  Case iv. Two Stage Game with Restricted Stocks  
Summary and Conclusions  

Chapter Four  Overall Conclusions  

Bibliography  

Appendix Legend for Appendices One to Six  

Appendix One  Payoff Requirements Under Game Regimes  

Appendix Two  Numerical Examples of the Normal Form Game  

Appendix Three  First Period Entry Restrictions  

Appendix Four  Division of Parameter Space by Game-Type  

Appendix Five  Accounting of Possible Games  

Appendix Six  Existence of Prisoners' Dilemma  

Appendix Seven  Table of Solutions  

Appendix Eight  The Coin Flip Solution  

Appendix Nine  Alternative Solutions Under Special Futures Contracts  

Appendix Ten  Analysis of SNTC Solutions and Some Extensions  

Appendix Eleven  Equilibria with Restricted Bidding  

v
<table>
<thead>
<tr>
<th>Appendix Twelve</th>
<th>The Nash Bargaining Solution</th>
<th>176</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appendix Thirteen</td>
<td>Analysis of Case iii. Under Stackelberg Leadership</td>
<td>180</td>
</tr>
<tr>
<td>Appendix Fourteen</td>
<td>Proof of Specialization in One-Stage Game with Unrestricted Stocks</td>
<td>183</td>
</tr>
<tr>
<td>Appendix Fifteen</td>
<td>Proof that Specialization in the One-Stage Game is Greater than that in the Two-Stage Game</td>
<td>185</td>
</tr>
<tr>
<td>Appendix Sixteen</td>
<td>Concluding Proposition of Chapter Three</td>
<td>188</td>
</tr>
<tr>
<td>Table One</td>
<td>Bids and Payoffs for the Auction</td>
<td>57</td>
</tr>
<tr>
<td>---------------</td>
<td>----------------------------------</td>
<td>-----</td>
</tr>
<tr>
<td>Table Two</td>
<td>Comparative Statics of the One-Stage Game with Unrestricted Stocks</td>
<td>105</td>
</tr>
<tr>
<td>Table Three</td>
<td>Comparative Statics of the Two-Stage Game with Unrestricted Stocks</td>
<td>120</td>
</tr>
<tr>
<td>Table Four</td>
<td>Table of Solutions to the JVPD</td>
<td>162</td>
</tr>
<tr>
<td>Table Five</td>
<td>Payoffs under the Coin Flip Solution</td>
<td>165</td>
</tr>
<tr>
<td>Figure One</td>
<td>Time Line of the Model</td>
<td>14</td>
</tr>
<tr>
<td>-------------</td>
<td>------------------------------------------------------------</td>
<td>-----</td>
</tr>
<tr>
<td>Figure Two</td>
<td>The Normal Form of the Model</td>
<td>16</td>
</tr>
<tr>
<td>Figure Three</td>
<td>Prisoners' Dilemma Payoff Ordering Requirements</td>
<td>24</td>
</tr>
<tr>
<td>Figure Four</td>
<td>The Auction Solution Time Line</td>
<td>54</td>
</tr>
<tr>
<td>Figure Five</td>
<td>Time Line of the Two Stage Game</td>
<td>112</td>
</tr>
<tr>
<td>Figure Six</td>
<td>Effects of Unilateral Specialization on Elasticity</td>
<td>123</td>
</tr>
<tr>
<td>Figure Seven</td>
<td>Numerical Examples of the Normal Form Game</td>
<td>150</td>
</tr>
<tr>
<td>Figure Eight</td>
<td>Effect of Changes in Elasticity on Parameter Space Division</td>
<td>153</td>
</tr>
<tr>
<td>Figure Nine</td>
<td>Effect of Changes in Fixed Costs on Parameter Space Division</td>
<td>154</td>
</tr>
<tr>
<td>Figure Ten</td>
<td>Stackelberg Leadership in the Two-Stage Game</td>
<td>182</td>
</tr>
</tbody>
</table>
ACKNOWLEDGEMENT

Financial support from SSHRC and U.B.C. is gratefully acknowledged. I thank my supervisor Dr. Thomas Ross, and the rest of my committee Dr. Raphael Amit and Dr. William Strange. I also thank Dr. James Brander, Dr. Kenneth MacCrimmon, and others in the Policy Analysis and Strategy Division, as well as all participants in my seminars at U.B.C. for their feedback and input on this paper. I am very thankful to my family, to Ruth, and to my friends for their patience and support throughout this process. A special thanks goes to the champion Commerce Softball Team for providing me a healthy alternative to work.
INTRODUCTION

These three essays in policy analysis and strategy explore separate and important economic issues in a similar way. First, a theoretical approach is used to investigate specific aspects of entrepreneurial emergence, of inefficiencies in joint ventures, and of trade involving an international open access resource. Standard, and in most cases, classical economic and non-cooperative game theoretical assumptions are applied wherever possible in the modelling of the issues. Second, each issue entails an effect on, or by, technology. Third, each analysis provides some practical policy recommendations. And fourth, each essay provides analysis in some dynamic context.

The first essay, entitled "Technological Force: The Emergence of Entrepreneurship", explains how entrepreneurs can beat capable incumbents when new opportunities arise. The incumbents rationally decide to let the entrepreneurs beat them. This explanation and the focus on the incumbents rather than on the entrepreneurs are not the only ways in which this paper differs from the other literature.

The emergence of entrepreneurship is common in the real world but relatively less so in classical economic literature. If industry incumbents are attributed with full rationality and perfect foresight, then there are few, if any, profitable opportunities left for new entrants (entrepreneurs) to exploit. This essay explains how entrepreneurs can emerge in a dynamic world when firms must choose between a technology strategy that is either statically or dynamically efficient. A
model is developed which shows how such opportunities for new entry can occur when incumbents are caught in a Prisoners' Dilemma game involving technology strategy.

The model provides an empirically testable explanation of how industry structure changes when any technological progress occurs. Outcomes that either entail all, some, or no incumbents surviving the technological jump are all supported in the model's different regimes. The model, however, focuses on when entrepreneurs emerge regardless of whether incumbents survive or not.

Entrepreneurship is worthwhile studying because it is so important in advanced economies where it is attributed with a large share of employment and a very significant share of radical innovations. Among this essay's recommendations to promote entrepreneurship is a call for competition policy be expanded to deter collusion on technology policy by industry incumbents.

The second essay, "An Auction Solution to the Joint Venture Prisoners' Dilemma", recommends certain methods for improving economic efficiency when firms do collude in technology given that the collusion is in the form of a joint venture.

Joint ventures, especially of the R&D type, are becoming increasingly important as a way to gain needed technological and market competencies. Unfortunately, many joint ventures have the characteristics of a Prisoners' Dilemma. Firms may cooperate or defect in the venture. If contracts, side-payments, and third-party verification of the venture outcome are unavailable,
then the dominant solution to the Prisoners’ Dilemma (mutual defection) results. This paper proposes the use of an *ex-ante auction* to obtain a Pareto-improvement for these ventures. A Pareto-improvement is assured when non-transferable costs and benefits of firms are not strategic. When this condition is not met, certain restrictions must be met in order to obtain the efficiency improvement.

The auction effectively creates a competition for ownership of the joint venture that allows redistribution of the payoffs to the firms through the bids so that the firms eventually will share equally in the sum of the venture’s outcome. Under such a transformation (which makes the game dynamic) each firm’s optimal strategy becomes maximizing the total payoff, and this corresponds to the jointly efficient outcome.

Although other economic papers have used auctions and buy-out options to improve efficiency under various non-cooperative circumstances, the mechanism and application presented here is unique. Furthermore, the straightforward implementation outlined can be readily exploited by policy-makers in appropriate cases.

The third essay, *"Trade with an International Open Access Resource and Imperfect Competition"*, also contains some strategic options that policy-makers may find attractive for improving the welfare of their country, in this case, under trade.

This paper analyzes the problem of trade between countries that share an international open
access resource. Addressing this problem may be increasingly important as the world reaches the limits of critical shared resource stocks. The problem is modelled as a one primary factor, two intermediate goods, one final good (that is harvested from the open access resource), two nation world. In this world, it is assumed that either the trading game between nations takes place over one stage so that nations are price-takers, or that it takes place over two stages so that the nations take advantage of their market power. Imperfect competition and the externalities arising from the existence of the open access final good stocks affect the trading efficiency, and therefore the incentives to trade.

For example, analysis shows that no trade will occur when the open access resource stocks are restricted to a level below the combined autarkic harvesting capacity of the nations involved. Analysis also shows that, under certain conditions, one nation will favour the two-stage production and trade structure although joint welfare would be improved under the one-stage game structure.

This paper is meant to complement the trade literature, specifically the literature on strategic trade and the literature on trade of shared limited resources. While it draws from these sources, it also introduces different modelling assumptions which provide some different policy recommendations. The policy areas considered include trade, competition, and R&D funding. For example, this essay recommends that R&D funding be directed to areas where a nation has comparative advantage in order to improve that nation’s (and the world’s) welfare.
While these three essays do differ in their focus, they all provide an analysis that differs from existing literature in the area. The essays also generate some useful policy recommendations to consider. They show that formal analysis of common economic phenomena, when approached from different perspectives, yields valuable insights that are worth considering by both public and private policy-makers.
CHAPTER ONE: ESSAY ONE

TECHNOLOGICAL FORCE: THE EMERGENCE OF ENTREPRENEURSHIP

1. Introduction

The emergence of entrepreneurship is a continuous and important phenomenon in modern economies. In retrospect, it is often entrepreneurs that exploit opportunities available to an incumbent. In most classical economics literature there is no room for entrepreneurs because incumbents do not make mistakes, and because a static game is often assumed. Where the newer literature provides room for the entrepreneur, it is often because there are exogenous differences between entrepreneurs and incumbents, or because incumbents do make mistakes over time. This essay takes the view that there can be opportunities for the entrepreneur even when incumbents are perfectly rational, have perfect foresight to future technologies and have access to perfect capital markets, but are precluded from coordination amongst themselves.

So why do entrepreneurs often take advantage of opportunities that incumbents, in retrospect, could have seized? Perhaps the incumbents were focused on the short run competition amongst themselves instead of on the long run competition amongst themselves and new entrants. Consider, for example, the opportunity that arose when vacuum tubes were replaced by transistors in the 1950s. All the top incumbents failed to take advantage of this event, and it is newer firms that are at the top of the semiconductor industry today. Foster (1986) also cites other examples that show incumbents waging the competitive battle in the short run to their
detriment in the long run. Incumbents, in order to survive in the present, devote resources to achieve high current efficiency instead of using them to exploit more attractive opportunities in the future. When this occurs, future entrepreneurs whose resources are devoted to these new opportunities will have a competitive advantage over these incumbents. This essay presents a model of how such opportunities for new entry can occur when incumbents are caught in a Prisoners' Dilemma game.

Most of the literature analyzing the failure of incumbents to exploit a new opportunity focuses on differential incentives and on structural inertia. It is often the case that incumbents lose a patent race because they have different incentives than an entrant or follower. Reinganum (1985, 1989) and Beath et al (1989) provide examples of where an incumbent chooses to allow a new entrant to exploit such a foreseeable opportunity because the incumbent has less of an incentive to battle for that opportunity than the entrant. Even on in international context, Brezis et al (1993) show that if a new technology is less productive in the short-run but more productive in the long-run than an existing technology then only the follower nation will adopt it. Foster (1986), Henderson and Clark (1990), Morison (1966), and Hannan and Freeman (1984) all consider boundedly rational incumbent firms that fail to take advantage of exploitable opportunities because of an inertia which creates myopia, added cost, and delays which the entrants do not experience.

It appears that the consequences of being an incumbent are understood from the perspectives of incentive effects and inertia effects. However, the literature has not explored as fully the choices
of incumbents which take into account these adverse consequences. Consider one choice that an incumbent may have - it can structure itself to achieve a high efficiency in the short-run at a cost of high inertia or it can structure itself to achieve a higher efficiency in the long-run but at a low efficiency in the short-run. If that incumbent is not under pressure to achieve high efficiency in the short-run, it may choose to think long-term. However, in many circumstances, due to either or both competitive pressures and stakeholder pressure on the firm to be profitable in the short-term, such an incumbent is usually required to achieve short-term efficiency. As a result, most of the time, the consequential inertia and incentive effects that are described in the literature are observed.

This essay differs from this literature in that it explores the decision given the consequences. It is also different in that it does not give up the assumption of perfect rationality and foresight of the firms that is noted in some inertia literature (like Henderson & Clark (1990)); nor does it give up the certainty assumption (i.e., the ability to exploit a new opportunity is not stochastic) that is found in some patent race and leapfrog literature (like Reinganum (1989) and Beath et al (1989) respectively). Instead, this essay uses the inertia, leapfrog, switching cost, technology choice and entry game literature as a basis to compute the payoffs and consequences of the strategic game of choosing which temporal focus to pursue in a competitive environment. The choice of temporal focus is embodied in a choice of technology strategy - the choice between being statically or dynamically efficient. Where static efficiency is defined as being efficient in the short term, dynamic efficiency is defined as being even more efficient in the future but relatively inefficient in the short term.
Many authors, including Schumpeter (1934) and Klein (1977), have pointed out that it is unusual, if not impossible, to be simultaneously efficient in a static and dynamic context in a competitive, changing environment. Trade-offs must be made in reality, as in this model. The competitive environment in this model can encourage specialization and "lock-in" by firms seeking high static efficiencies which may be to their long-term detriment; or, it may encourage "flexibility" by firms seeking high dynamic efficiencies which may turn out to be to their short-term detriment. With such competition present, added pressure is put on the incumbents choosing lock-in or flexible strategies to the point where they may choose a non-optimal strategy simply as a reaction to what they feel their competition may choose. They may find themselves in a Prisoners' Dilemma, where individual incentives lead to a jointly sub-optimal outcome for the firms.

After such a sub-optimal outcome occurs, observing firms and other potential entrants may then take advantage of the situation. In the model presented below, entrants with the new technology - the entrepreneurs - emerge as a result of the Prisoners' Dilemma outcome occurring between the incumbent firms. Therefore, under the Prisoners' Dilemma game, incumbents who would like to and could exploit future opportunities for higher profits do not as it is assumed that they cannot collude amongst themselves. Instead, they rationally choose to "focus on short-term" competition leaving the entrepreneurs to exploit the future opportunities and eventually force these incumbents out of the industry. This choice of focus is the rational choice that ensures them the highest individual profits over the game when they cannot collude.
The game will be defined and then examined in the remainder of this essay. The game's outcome will be studied and an important extension considered. Analysis of the model will provide information as to how incumbents are forced out of the industry, how entrepreneurs emerge, and whether this emergence is beneficial. Section Two provides definitions of the game, an analysis of a specific example of the model, and an important extension. Section Three contains the important conclusions.

2. The Model

2.1 Important Definitions:

The emergence of entrepreneurship in this model is largely based on the technology strategy choice. For simplicity, each firm has a choice of playing one of two technology strategies in its first period of existence in the market. The two choices are "lock-in" and "flexible". These correspond roughly to choosing between static and dynamic efficiency. If a firm locks in, it can produce at a lower cost than non-locked-in competitors for its period of lock in, but it is "impaired" in its transition to a new, even-lower-cost production technology in the future compared to such a transition by non-locked-in firms and new entrants. If a firm stays flexible, it has a higher cost of production than locked-in firms for the periods it chooses to stay flexible, but it suffers no impairment to switching to new technologies when those technologies emerge. It can switch to a new technology at the same cost as a new entrant can learn that technology
(assumed a zero cost here), whereas a locked-in firm incurs some time delay or some positive additional variable cost to make the change. Furthermore, staying flexible guarantees that incumbent a "spot" in the second period as it is assumed that by its experience of flexibility in period one that it has an even lower variable cost than new entrants in the second period, but only lower by some epsilon very near zero.

In this model, it can be assumed that locked-in firms either require one period to adjust or incur higher costs to adjust. The period of adjustment is one of going through the process of "unlearning" the locked-in process. While this adjustment is taking place, it could be assumed that the firm stays at its production efficiency level or that it exits for a period if need be (in the example, it does not matter as the locked-in firm is driven out regardless).

The higher costs to adjust are incurred because it is assumed the locked-in firms must pay a positive price to "unlearn" the locked-in skills. The cost to "unlearn" as this essay describes the process, is similar to the disposal costs incurred with highly specific capital that is no longer the best technology. In this essay, capital is not necessarily the only area in which a firm incurs the "disposal" or "unlearning" cost. Other areas could be in changing processes, technologies,

---

1 Consider the technology strategy choice as a choice between an investment in either basic or specific R&D. Cohen and Levinthal (1990) show that a firm may lack knowledge diversity if it chooses to invest in specific R&D. This lack translates into a low "absorptive capacity" - a measure of a firm's ability to recognize, assimilate, and apply new external information. With low absorptive capacity, it is reasonable to assume that a firm will incur some time delay or cost to switch to a new technology.
organizational designs, corporate philosophies, or manufacturing structures\(^2\).

Now, it must be noted that the adjustment process in no way takes away from the firm being fully rational. Being fully rational allows a firm to be able to costlessly compute any strategy choice and its payoffs and other implications. It does not ensure that the firm can produce at any technique that comes along, or even be able to "think" differently in investment terms than it does in its locked-in strategy. For example, the locked-in firm is unable to efficiently invest in, or otherwise finance, another plant producing with the new technology because that locked-in firm does not know how to think in terms of that new technology and therefore cannot efficiently construct or monitor its investment. All that the locked-in firms know is that the new technology is better and by how much, but they cannot instantaneously use it.

The potential entrants and the flexible firms do not have to go through the unlearning process and can take advantage of new technologies without delay. This is how new entrants with new technology - entrepreneurs - can emerge.

\(^2\) Other support for the intuition that inflexible strategies are worse than flexible ones in the long-term in a dynamic environment are found in the areas of: retooling costs, FMS, scarce resources, the sunk-cost "fallacy", and some organizational behaviour effects. Retooling can have higher costs than tooling-up because of: firing costs, retraining or retirement costs (see Bartel & Sicherman (1993)), and costs of disposing or maintaining old-technology facilities. Flexible manufacturing systems (FMS) have shown in many circumstances (both real and theoretical) to be less costly in the long run than dedicated machines when the production environment is dynamic (see Roller & Tombak (1990)). Although perfectly rational actors may not be susceptible to the sunk-cost fallacy, real people often increase the costs of now-inefficient locked-in technology by continuing to use it (possibly due to some reputation considerations) instead of switching to better technologies. Any change in technique usually incurs some organizational change. Due to some institutional inertia and personal inertia of employees, there may be extra costs to change from a well-established condition over that from a flexible (or new) condition (see Rumelt (1994)).
2.2 Definition of the Game:

In its simplest form, the model is a two period game involving Cournot competition. There are two incumbents in the market. At time $t=0$, these incumbents must choose their technology strategies, and then play Cournot output production strategies against each other. At time $t=1$, similar decisions must be made by the remaining incumbents and any new entrants. The game ends at time $t=2$. Profits can be made in period one, between time $t=0$ and $t=1$, and in period two, between time $t=1$ and $t=2$ (see Figure One for timing). Assume that at $t=0$ a technology is available in either its flexible or lock-in forms. At $t=1$ a new technology is available in either its flexible or lock-in forms. The occurrence of the new technology was foreseen by firms at $t=0$ and $t=1$. This new technology is superior to the first technology in either form (i.e., the new technology’s lock-in form allows a lower variable cost than the old technology’s lock-in form).

Assume that the incumbents are symmetric to each other, and that the entrants are symmetric to each other. Entry is blocked at $t=0$ (for all of period one) where only two incumbents are allowed. Entry is free at $t=1$ (for all of period two). Exit is considered free throughout the game.

Without loss of generality, this model focuses on the simplest form of competition - a duopoly. The relevant player space is composed of the incumbents $\{I_1, I_2\}$. This occurs because the new entrants have their strategies and population completely defined by the model’s assumptions and
The Timing of the Simple Game:

<table>
<thead>
<tr>
<th>t=0</th>
<th>t=1</th>
<th>t=2</th>
</tr>
</thead>
<tbody>
<tr>
<td>period one</td>
<td>period two</td>
<td></td>
</tr>
</tbody>
</table>

Start of Game

Two incumbents, who decide their technology strategies and resulting production levels, exist at t=0. Only Technology C1 is available to them, in either the lock-in or the flexible forms.

At t=1 incumbents decide whether to exit. Entrants decide whether to enter. Then all remaining firms decide their own technology strategy and their resulting production output level under Cournot competition. Technology C2 is now available to them, in either the lock-in or the flexible forms. This technology is superior to the old one (i.e., C2's lock-in variable cost is lower than C1's lock-in variable cost).

The occurrence of C2 was foreseeable at t=0.

Figure One: Time Line of the Model

the play of the incumbents. The number of entrants is calculated from the inverse demand function and the production cost functions of the incumbents assuming Cournot competition. The technology strategy of the entrants will be the dominant strategy of locking-in because when they enter it is the last period in the game and there is no point to being flexible then. Thus, the entrants' strategies are completely defined by the model's assumptions and the entrants' population is completely defined by the inverse demand function and technology strategy choice of the incumbents.
Production output strategies are assumed Cournot (i.e., incumbents choose outputs knowing the market determines the price to clear the output). As the entrants’ actions and the incumbents’ output choices are fully defined, the only strategy space to consider is that of the technology strategy choice \{flexible, lock-in\} of each incumbent over each time period \{period one, period two\}. It is relatively straightforward to compute the payoffs of the two incumbents over the two periods for each technology strategy in order to analyze the game in a 2x2 (number of incumbents x number of technology strategies) normal form (see Figure Two).

While the equilibrium in production output is assumed to be Cournot, the equilibrium in the technology strategy game is assumed to be the dominant strategy solution to the normal form game where choices are made simultaneously in each period by the relevant firms.

When the incumbents are assumed to be symmetric there are only four different normal form payoffs to consider: c, d, w & s$^3$.

The normal form could take one of many well known structures such as a Prisoners' Dilemma or a Co-ordination game. When the normal form is a Prisoners' Dilemma game each incumbent is "forced" to lock-in due to the presence of its rival although it would rather pursue flexibility

---

$^3$ Asymmetric incumbents complicate the analysis by adding four more payoffs to consider. Exploring the ramifications of this complication is straightforward and a possible future extension to this game.
if it could collude with the other incumbent on strategy\(^4\). When the competitive pressures force incumbents who are capable of blocking the emergence and want to block it to rationally focus on the short-term, future opportunities are exploited by entrepreneurs instead.

Only lowest-cost firms can stay in the game in the final period as all others incur a loss. Thus, non-credible threats made by non-minimum-cost incumbents to stay in the market are ignored by potential entrants. The reason that only lowest-cost firms can stay in the game in the final

\(^4\) The analysis is more complicated when analyzing the model if the incumbents could also collude on output as well as on technology strategy choice. Efficiency would dictate that the strongest competition available (i.e., static efficiency and Cournot outputs) would lead to the highest welfare. The results of this paper are consistent with such a proposition.
period is that free entry forces out any inefficient producers. Consider the final "space" open to a firm to profitably exist in the market where the number of such spaces is calculated from the Cournot equilibrium in firm outputs. Imagine that either an efficient (lowest variable cost) firm or an inefficient (higher variable cost) firm is considering existing in that space. If either could exist and make a profit, it is logical to assume that the more efficient firm would win the space. The mechanism that would enable this to happen is a War of Attrition game waged between the two firms vying for that space taking place over a number of "sub-periods" that make up the period in question. Thus, split period two into a number of subperiods and play the War of Attrition game. Either firm is willing to stay in the market through period two's subperiods until its expected profits equal that of not being in the market in the first place. Since an efficient firm's profits are higher and losses are lower per subperiod, it will stay in longer than an inefficient firm. The inefficient firm knows this, so it exits at the beginning for a zero payoff in the second period instead of incurring a loss. The efficient firm knows this, so it enters. Note that at all times it is assumed that the firms continue to play Cournot outputs based on the number of firms in the market.

Assuming that this model contains a War of Attrition Game over its second period leads to the following proposition.
**Proposition One:** If an incumbent locks in the first period, regardless of the choice of the other incumbent, that locked-in incumbent will be forced out of the game in the second period.

**Proof:** Consider the locked-in incumbent who, in the second period, has a higher variable cost than any entrant or any flexible incumbent. Therefore, all lower cost firms that can enter will do so until there is a game between the final entrant who may enter profitably and the locked-in incumbent who wants to stay in the market. Consider this encounter in the following:

i) It may be the case that parameters are such that an "extra" entrant can enter and make a profit under Cournot competition whereas the incumbent makes a loss. In this case it is straightforward that the incumbent must exit and allow the entrant to displace it.

ii) If it is not the case above, the only other case to consider is the less straightforward one when there is a coordination game. If both the locked-in incumbent and the lower-cost firm (the entrant) both stay out of the market then they both make zero profit, if one stays out and the other is in then the outsider makes zero profit and the producer makes a positive profit (higher for the lower-cost firm), and if both produce under Cournot competition then each makes a loss (larger for the incumbent). So what is
the outcome? Consider the War of Attrition game described above occurring within period two between an efficient firm, the entrant, and the inefficient firm, the locked-in incumbent. In this case, the incumbent rationally chooses to stay out of the market in the second period.

Therefore, in either case, no locked-in incumbent exists in the second period.

It can be concluded from the preceding proposition that any incumbent who chooses to be statically efficient in any but the last period will be forced out of the market in the subsequent period. This explains how incumbents, who are capable of blocking entry and remaining in the market, can be forced out of the market by new entrants when those incumbents choose to be statically efficient at some time. In order to explore the remaining questions of how entrepreneurs emerge and whether that emergence is beneficial, a specific production-cost function and demand function are assumed. However, before answering those questions, it is important to clear up a few points regarding the play of incumbents in the general game described above.

---

5 There are other possibilities to consider if the War of Attrition is not assumed. One possibility is that a Mixed Strategy Equilibrium is "non-sensible" as it may result in neither firm entering a profitable market (and each firm making an expected zero profit in the equilibrium). An other possibility is that an ordering scheme for entry into period two could be used that is based on expected profits and experience, but it has no theoretical basis.
The points of clarification have to do with some details of the game that are not relevant to the equilibrium paths of interest. The points deal with what strategies are implemented by firms when they incur a negative profit in some period. The first point to note is that this model assumes that if a firm’s profit from competing in the second (and final) period is negative then that firm does not enter the market in that period and thus records a zero profit for that period. The second point to note is that the model assumes that if a firm’s profit in the first period is negative but its profit in the second period is positive and large enough to make the sum over the two periods positive then that firm produces in both periods. The reason for this strategy is that it is assumed that an incumbent is guaranteed a "spot" in the second period only if it is flexible in the first period (i.e., it is assumed that by its experience of flexibility in period one that it has an even lower variable cost than new entrants in the second period, but only lower by some epsilon very near zero). Thus, if that incumbent chose to stay out in the first period instead of taking the loss it would not be guaranteed a spot in the second period as there is free entry in that period and it would be on par with the "large" number of entrants vying for spots in the market at that time.

With these details cleared up, the questions of how entrepreneurs emerge and whether that phenomenon is beneficial can now be answered.
2.3 Analysis of a Specific Game:

With some loss in generality, assume a linear demand specification:

\[ P_p = A_p - B_p Q_p , \quad p = 1, 2 \]

where subscript \( p \) is the period, \( P \) is the market price, \( A \) is the price intercept set at some value greater than any of the costs for both periods, \( B \) is the demand slope, and \( Q \) is the total production of the firms present in that time period. For simplicity, assume that \( A \) and \( B \) remain constant over the two time periods.

Assume that the cost function for firms in the industry is made up of two components: a variable cost and a fixed cost. The form of the variable cost is simply a constant marginal cost multiplied by the quantity produced by the firm. The fixed cost is a constant amount incurred by any firm which produces in a period.

The profit function for firm \( i \) derived from the costs assumed is:

\[ \Pi_{i,p} = (P_p - c_{i,p}) q_{i,p} - f_p , \quad p = 1, 2 \]

---

6 The use of a general demand specification, and the use of alternative demand specifications are possible future extensions to this model.

7 Considering \( A \) and \( B \) parameters that change over time is a possible future extension to this model.

8 Considering fixed costs that are asymmetrical or that change over time is a possible future extension to this model.
where: $f$ is a fixed cost of production incurred in each time period the firm is producing; $c_{lp}$ is the constant marginal cost of production in period $p$; and $q_{lp}$ is the firm's output in period $p$.

A positive fixed cost of production is assumed in order to obtain a finite Cournot equilibrium population. For simplicity, assume that fixed costs remain constant over time and that all producing firms incur the same fixed costs regardless of variable cost.

The Cournot reaction function for each producing-firm involved is derived from the assumed demand and cost structure as:

$$q_{lp}^* = \frac{A_p - B_p \sum q_{j\neq p} - c_{llp}}{2 B_p}; \quad j \neq i, \quad p = 1, 2$$

Assume that the parameter restriction $c_a < c_l < c_r < A$ holds, where $c_a$ is the lock-in variable cost in period two (the variable cost chosen by all new entrants and any firms which chose to be flexible in the first period), $c_l$ is the lock-in variable cost in period one, and $c_r$ is the flexible variable cost in period one. Therefore, an incumbent who chooses "flexibility" will incur variable costs of $c_r$ in period one and $c_a$ in period two, while an incumbent who chooses "lock-in" will incur a variable cost of $c_l$ in both periods.

Firms will exit when it is rational to do so. In the last period of the game if a firm sees that it will incur a loss it will exit. However, if it incurs a loss in the first period and can calculate a compensating gain in future periods, it has the ability to weather the loss and hold a debt in that

---

9 Assume that if the incumbent wanted to switch to a new technology in period two that either it must be delayed one period, or incur some cost to do so. Here, for simplicity, assume that the cost equals the difference between $c_a$ and $c_l$. The cost only needs to be positive, in general, for the results to hold.
first period in order to assure itself a position in the future period where it profits.

As assumed, the beginning of period two, at $t=1$, a new technology emerges that is superior to that used in period one. That emergence was foreseeable to each incumbent at the beginning of the game. Given that this second period is the last one in the game, the only rational choice in period two for technology strategy is to lock-in as there are no benefits from staying flexible in the last period of the game. Therefore, all flexible firms from period one and any new entrants into period two play the lock-in strategy in period two. Lastly, but most importantly, assume locked-in firms are impaired in their ability to make the transition to a new technology.

Under the general model assumptions and given the linear inverse demand specification, the profit specification and the cost restrictions the normal form can be computed for actual allowable parameter values. It can be seen that the possibility that a Prisoners' Dilemma structure will never be the result under all parameter values is zero. Appendix One provides the algebra under the assumptions outlined. It is straightforward to show either by numerical example (see Appendix Two) or by analyzing the restrictions that some allowed parameter values provide a Prisoners' Dilemma structure in the model's normal form (see Appendix Six).

Figure Three presents the normal form of the game where a Prisoners' Dilemma arises. It requires that the payoffs are ordered such that symmetric flexibility dominates symmetric lock-in, but that the individually dominant strategy is lock-in when considering either a lock-in or flexible rival. A further payoff restriction is often also included as part of the definition of a
Payoff Ordering Required to Get Prisoners' Dilemma Game
(two period game)

<table>
<thead>
<tr>
<th></th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>c, c, s, w</td>
</tr>
<tr>
<td>L</td>
<td>w, s, d, d</td>
</tr>
</tbody>
</table>

For PD Normal Form, the payoffs must be ordered as follows:
\[ w > c > d > s \] where \( d > 0 \) also assumed.
The extra condition that \( c + c > w + s \) is not required here.

Figure Three: Prisoners' Dilemma Payoff Ordering Requirements

Prisoners' Dilemma - that symmetric flexibility also dominates the swapping exploitation of a rival's flexibility (i.e., playing (F,L), (L,F), (F,L), ...). This further restriction is not applied in this model. One reason is that this model analyses only the one-shot Prisoners' Dilemma so the swapping is not an implementable option of firms or policy-makers. Another reason is that as soon as a firm locks-in it is out of the market in the next period and not available to return any swap to its partner. Lastly, this further restriction does not alter any of the conclusions or policy implications of this essay. Therefore, any mention of the Prisoners' Dilemma structure following will not assume that this further restriction need apply.
The result that a Prisoners’ Dilemma structure can arise is significant. It means that, under certain regimes, profit maximizing incumbents who would choose to be flexible if they could collude with one another choose instead to lock-in because they must compete in the short run. As a result, new entrants with new technology (i.e., entrepreneurs) are allowed to exploit the opportunity in the future that the incumbents cannot, but would have liked to.

Regardless of whether the normal form produces the Prisoners’ Dilemma game or not, entry by new firms occurs in the final period whenever an incumbent locks-in in the first period. Holding all else constant, the reduction in variable cost embodied in the new technology allows at least two low-cost firms (composed of incumbents or entrants) to produce in the final period. If one incumbent is no longer low cost, then entry must occur. The following proposition results.
Proposition Two: There is entry in period two whenever an incumbent locks in, with more entry occurring as more incumbents lock-in in the first period. There may be entry in the second period even if all incumbents are flexible in the first period.

Proof: As shown in Proposition One locked-in incumbents with higher variable costs than entrants in the second period get forced out. Thus, if one incumbent locks-in, it must be replaced by at least one entrant in period two. It is straightforward to reason, then, that if another incumbent also is locked-in it too will be replaced by at least one new entrant. Thus, the more incumbents lock-in the more entry there will be.

To get entry to occur in the second period even if both incumbents remained flexible requires the following:

\[
\text{INTEGER} \left( \frac{(A - c_n)}{\sqrt{fB}} \right) \geq 4
\]

where the operator returns the integer portion from the contents of the square brackets if it is positive, otherwise it returns zero.

As none of the parameters are fixed in value at this time, this condition is easily met. For example, if \( c_n, f, \) and \( B \) were set, an \( A \) sufficiently large will satisfy this condition and entry will occur even when both incumbents lock-in in the first
Knowing that entry will occur in each case where at least one incumbent locks-in in the first period the cases where this lock-in occurs can be defined. Lock-in can occur in three cases. It will be the dominant strategy for both incumbents to lock-in if the normal form is a case of the Prisoners' Dilemma game, or if it is the case where the lock-in is both dominant and jointly efficient for the firms. Lock-in can also occur in a Co-ordination game where a mixed strategy equilibrium entails playing the lock-in strategy with some positive probability. The case where lock-in will not occur is when the dominant and pareto optimal strategy for the incumbents is to remain flexible. No other cases are possible for the normal form to take, given the assumptions in this model. The parameter restrictions for each of these cases is given in Appendix One. It is straightforward to verify, through numerical example (see Appendix Two) or analytically, that all of these four cases are possible normal forms and that no other non-pathological case is possible (see Appendix Five).

10 Obtaining the result is not an extra-ordinary event. It is already known that:

\[ \text{INTEGER}^* \left[ \frac{(A - c_n)}{\sqrt{fB}} \right] > 3 \]

Because for \( d > 0 \):

\[ \text{INTEGER}^* \left[ \frac{(A - c_0)}{\sqrt{fB}} \right] \geq 3 \]

and \( c_n < c_0 \). Given these conditions, it is clear that the parameter values that allow entry into the second period even when both incumbents are flexible is a large subset of the parameter values that meet the necessary conditions that \( d > 0 \) and \( c_n < c_0 \).
These four cases cover the three possible outcomes that can result in an industry when a technological advance occurs. All incumbents can remain in the industry after the advance, as predicted by the flexible dominant normal form. Some but not all incumbents can remain in the industry after the advance, as predicted by the Co-ordination game normal form. Or, no incumbents can survive the advance, as predicted by the lock-in dominant and Prisoners' Dilemma normal form. Thus, the model of the effect of a technological advance on industry structure presented in this essay is robust to all possible observed outcomes.

Consider now the welfare implications of entry. When entry does occur, it may be because at least one incumbent locks-in in the first period. When this occurs at least one firm in each period is statically efficient - uses the lowest variable cost technology available. When firms are statically efficient they may end up making increased profits, but it is a certainty that the price and quantity to the consumer is improved given Cournot competition. It may then be that total firm profits, consumer surplus, and total welfare are all increased by the occurrence of entry when there is lock-in by at least one incumbent. Thus, the proposition follows.
**Proposition Three:** The profits summed over all producing companies (Total Company Profits), Consumer Surplus, and Total Welfare are all increased when entrepreneurs emerge (i.e., when both incumbents lock-in).\(^{11}\)

**Proof:**

i) The only difference in the outcome of the full game between the case where both incumbents lock-in and where both incumbents choose flexibility is in period one. Therefore, only the change in profits in period one will be analyzed (i.e., the difference in profits between the cases of (L,L) and (F,F) under Cournot output):

\[
\Delta \pi_{LL-FF} = \left(\frac{2}{9}\right) \left[ (A - c_i)^2 - (A - c_f)^2 \right] > 0
\]

as \(A > c_f > c_i\).

ii) Similar to the argument above, look only at the period one difference between (L,L) and (F,F) strategies regarding consumer surplus (the other additions to consumer surplus between these conditions cancel out):

\[
\Delta CS_{LL-FF} = \left(\frac{2}{9}\right) \left[ (A - c_i)^2 - (A - c_f)^2 \right] > 0
\]

as \(A > c_f > c_i\), where:

---

\(^{11}\) As the incumbents here are assumed to have symmetrical payoffs, only the cases where they pursue the same pure strategies will be analyzed here. The extension to the mixed strategy equilibrium in the Co-ordination game is a simple case (see footnote in Appendix Five).
\[ CS = \frac{(A - P) Q}{2}, \quad Q = \sum_i q_i \]

iii) The total change in welfare, then, is the sum of the two elements above:

\[ \Delta \text{WELFARE}_{LL-FF} = \left(\frac{4}{9} B \right) [(A - c_f)^2 - (A - c_t)^2] > 0 \]

as \( A > c_f > c_t. \)

This welfare measure does not take into account of the extra costs involved due to the emergence of entrepreneurship: the cost of two bankrupt incumbents plus the cost of setting up two replacing entrants.

Thus, if the change in the welfare is larger than the costs of having new entrants emerge then the total net change in welfare is positive. Therefore, policies that ensure the normal Prisoners’ Dilemma outcome occurs are worth pursuing.\(^{12}\)

\[ \square \]

It can be concluded that entrepreneurial entry (which occurs when both incumbents lock-in in the first period) is welfare increasing under certain restrictions. What drives this result is the

\(^{12}\) Consider a numerical example which shows the welfare benefits. If \( c_f=4, c_t=3.5, A=9.0, B=1, f=3, \) then the gross change in total welfare over both periods = 2.38, and the total welfare generated by collusive technology strategies in period one = 5.56. Therefore, the percentage change in total welfare with respect to the total welfare generated by collusive technology strategies in period one is a significant 42.8\%. 

30
presence of statically efficient firms in each period that the industry exists. When an industry contains only lowest-cost firms in each period of its existence, that industry can be defined as being "dynamically efficient". Such an industry can occur in this model. Of the cases where this does occur, perhaps the Prisoners' Dilemma normal form of the model is most interesting. Here, firms lock-in due to competitive pressures resulting in incumbents being statically efficient in the first period and locked-in entrepreneurs who fully occupy the market in the last period being statically efficient in that last period. The question of how entrepreneurs can emerge even when incumbents are capable of blocking them is thus answered in this model's Prisoners' Dilemma outcome. Further, the question of whether that phenomena is welfare improving is also thus answered.

If bankruptcy (exit) and entry costs are low, and the number of firms incurring them is low, social efficiency can improve as a result of this type of competition where entrepreneurs emerge. It makes sense that it is "socially beneficial" that some firms choose static efficiency and then are eliminated by future statically efficient firms in a path to step-by-step dynamic efficiency for the industry (and society) overall. Therefore, by having firms that are statically efficient at each period in time, dynamic efficiency is assured in the industry. The machinery behind this efficiency is competition and entrepreneurial entry.

Given that such market evolution is beneficial, there are obvious policy implications to be taken from this model. Some of the assumptions of this model can be supported in the real world when it is beneficial to do so (e.g., when the PD normal form is possible). For example, lower cost
access to technological information may enable the assumption that entrants are up to the latest technology to hold true; and some subsidization of firms may enable the assumption of low entry and exit costs to hold true. Such policies could be supported with taxes on the increased consumer surplus. Another more direct example of how policy can ensure model assumptions in the real world is through the implementation of strong Competition Law to ensure the assumption that incumbents cannot collude on technology strategies holds.

2.4 Considering an Extension to the Game:

One question to consider once this entrepreneur-creating situation is found to exist is just how prevalent is the phenomenon? To answer this question some further definition has to be given to the range (of applicability in the parameter-space) of the model. Once this is done some measure of potential entrepreneurial activity based on this model can be computed.

The further definition of the model entails putting some further restrictions on its applicable domain. First, the asymmetry of entry protection between the two time periods must be addressed. To do this the first period has to be opened up to any potentially profitable entry. Second, restrictions on the model's variables must be confirmed (they are summed up by the inequalities: \( c_n < c_i < c_f < A \)). With the range of possible variable values restricted to the model's applicable domain, it is possible to set out the restrictions that must be met to achieve the Prisoners' Dilemma (PD) and the Co-ordination (CG) game situations that may give rise to
what this essay defines as entrepreneurship.

These restrictions consist of equations that provide a specific order to the payoffs in the normal form game. The PD situation needs to meet three restrictions whereas the CG needs to meet two restrictions on the payoff ordering (see Appendix Three for details on all of the restrictions).

The "relevance measure" (an estimation of parameter-space area) can now be computed. Unfortunately, the restrictions that identify the game-type are based on Cournot competition with six degrees of freedom (assuming a linear demand function). Cournot competition entails discontinuous functions (calculating the number of entrants) and non-linearities (calculating firm profits through reaction functions). Therefore, no closed-form solution to the measure is available in even the simple two-incumbent case. Numerical methods can find the measure under a range of variable values.

The computer program [listing available upon request] simply steps by small increments through the positive real number line for a number of parameters recording the type of game outcome resulting from the parameter values. All but one parameter is fixed when the stepping occurs for the one parameter of focus. The game-type is found by evaluating the parameters against the mutually-exclusive restrictions defining each of the four game-types. The programming is straightforward working with linear inverse-demand functions and Cournot competition. From these relevant parameters and the game-type categorization, a number of results can be calculated, including: i) each game-type's share of the total parameter space; and, ii) the
Consumer Surplus, Firm Profits, and Total Welfare under each game and under each game’s possible outcome (e.g., the change in Welfare between a collusive and a competitive Prisoners’ Dilemma type game can be evaluated).

The results of numerous computer runs are of some interest. The PD situation occurs in approximately one-third of the parameter-space in this model (see Figure Eight and Nine in Appendix Four). This is a significant proportion of the parameter-space. It is unknown how realistic the ranges are for the parameters that give rise to the PD outcome. However, examination of how the parameter space is separated into the differing regimes (of the four possible normal forms) reveals a discontinuous and well spread out stratification. Therefore, it may be safe to assume that, given this spread of parameter values in the space where the PD normal form occurs, a reasonable proportion of such outcomes will occur in the real world.

Therefore, entrepreneurial emergence may arise in a significant proportion of the cases where incumbents must make the technology strategy decision outlined in this model. It is not surprising then to observe the phenomenon of incumbents, who are capable of exploiting future opportunities and would if they could collude with each other, deciding to lock-in and compete in the short run and sacrifice the long run opportunities to the new entrants with the new technology - the entrepreneurs.
3. Conclusions

This essay has attempted to show that, with slight changes in assumptions, classical economic theory does have room for entrepreneurship. Simply by adding a dynamic element to a classically-based model of competition and by assuming foreseeable innovations occur as time progresses, entrepreneurs do emerge in a substantial range of the model’s area of existence. Further, the process that is created through entrepreneurial emergence can be welfare improving. This is because by having each period’s firms choose to be statically efficient the industry achieves dynamic efficiency over all periods.

It would appear that entrepreneurs emerge in the much of the relevant parameter-space of the games defined in this essay. Entrepreneurs - new entrants with new technology - emerge in the Prisoners’ Dilemma game, in the Co-ordination game, and in the game where the "lock-in" strategy is both dominant and firm-pareto-optimal. No policy action is required to affect entrepreneurial emergence in this latter game. However, policy action can affect entrepreneurial emergence in the Co-ordination (CG) and the Prisoners’ Dilemma (PD) games. In the case of the CG, some small subsidy can be given to one or both incumbents to ensure that both do not remain flexible and block entrepreneurial emergence, if that emergence is of net benefit to society. In the case of the PD, policies that discourage collusion (ensure competition) between the incumbents (and between all later firms) will help ensure the emergence of welfare-improving entrepreneurs.
Many current competition policies only discourage collusion that has a material effect on industry competitiveness from a price perspective, while allowing some industry cooperation with respect to technology-sharing. This essay attempts to argue that competition policy should extend to the technology strategy regime as it does to the price (or quantity) fixing regime because collusion on technology decisions could be socially damaging. In either case welfare will improve under most circumstances if larger firms in the market are precluded from colluding under either regime. In the case of technology strategy this essay shows that when such collusion is restricted overall efficiency will improve. Not only will an industry be statically efficient in every period but entrepreneurs will have the opportunity to emerge, with all the favourable aspects such entrepreneurial activity has on an economy.

Although the conclusions of this essay are based on some strong simplifying assumptions like the existence of Cournot competition among small oligopolies, fully rational firms, exogenous foreseeable technologies and un-learning costs, it does offer an alternative explanation of why entrepreneurs emerge in modern economies. Even when it might appear in the real world that incumbents could have done "better" by focusing on future opportunities and blocking the entry of a lot of the entrepreneurs that ended up destroying them, some incumbents chose not to, and this essay provides one explanation. It is an alternative explanation to the incentive-difference-based patent-race and technology-adoption literature and to the organizational inertia and boundedly-rational incumbent literature. This essay has presented a model of strategic choice to explain the emergence of entrepreneurship. Although the scope of the model may be narrowed according to the relevant parameter values in the real world, some potentially useful policy
implications can be drawn from its findings.

The results presented above are robust to some generalizations of the model. For example, in the model, the industry lasted for only two periods and the new technological opportunity was a process improvement. While the extension outlined was important in that it disclosed a "relevancemeasure" of the model, other extensions are possible and straightforward. It is straightforward to consider, but less so to do the mathematics, allowing the industry to exist over further periods. This extension does not change the conclusions. The conclusions are also not changed by allowing more than two incumbents in the first period. The model is also applicable to considering the technological improvement as a product enhancement. In this case, firms that can exploit this opportunity can offer better product on the market. This translates to better value to consumers and simply parallels offering a lower price on a set product - a consequence of a process improvement.

This essay has presented an alternative explanation for why the leadership in tire cords or computer equipment or commercial aircraft has switched to entrepreneurial firms. Although explanations may be found in the leapfrogging literature like Brezis et al (1993), in the organizational structure literature like Cohen and Levinthal (1990), in the technology adoption literature like Conrad & Duchatelet (1987), in the technology life-cycle literature like Foster (1986), in the structural inertia literature like Henderson and Clark (1990), in the patent-race literature like Reinganum (1989), or in the flexible manufacturing literature like Roller and Tombak (1990), this essay provides one more story - a story from a strategic perspective.
CHAPTER TWO: ESSAY TWO

AN AUCTION SOLUTION TO THE JOINT VENTURE PRISONERS' DILEMMA

1. Introduction

This essay contributes to the literature on joint ventures by presenting an implementable method for improving the efficiency of some ventures. The essay argues that many joint ventures have the characteristics of a Prisoners' Dilemma. An inefficiency arises in such ventures because no participating firm has the incentive to carry out the jointly optimal strategy. This essay proposes the use of an ex-ante auction which makes the division of the venture's total transferable outcome uncertain until the auction winner is revealed after the venture ends. When the firms play the venture strategies before they know the division, they have an incentive to play the jointly optimal strategy. This incentive makes the outcome of some joint ventures more efficient.

According to Taylor (1989) joint ventures are very important in many technology-based industries. Roberts and Mizouchi (1989) go one step further in suggesting joint ventures are a necessity in some industries. There is a literature that supports the idea that joint ventures have the characteristics of a Prisoners' Dilemma. Parkhe et al (1993) analyze the payoff structure of interfirm strategic alliances (ISAs). Their survey of 342 senior executives experienced in ISAs supports the PD payoff structure as a representation of the ISA. Von Hippel (1987) explores
informal trading in know-how between engineers at rival and non-rival firms\(^1\). This informal type of joint venture is also found to be modelled as a Prisoners’ Dilemma.

Joint ventures can have the characteristics of a Prisoners’ Dilemma because participating firms can cooperate or defect on their actions and investments in the venture, often in a undetectable way, until the results become known. In such situations, the firms (and other affected parties like governments) would ideally like to solve the PD; that is, to obtain the Pareto-optimal cooperative outcome of the venture. There are many ways to solve the PD, depending on what further assumptions and circumstances are allowable. With minimal adjustments a straightforward solution to the one-shot PD may be available that does not change the competitive nature of the game and does not require an outside judge to verify which game strategies were played. By holding an ex-ante auction on the joint venture (JV) outcome, whose winner is not revealed until the venture is completed, the PD can be solved. The firms entering the JV will cooperate on their actions and each will be rewarded for its cooperation.

Non-cooperative game theory provides few, if any, solutions to the one-shot Prisoners’ Dilemma. Cooperative game theory provides contractual solutions (which may include side-payments and bargaining). However, there are problems associated with implementing the cooperative solutions. First, it may be expensive to negotiate a contract that is fair to both parties, and to ensure that the appropriate penalties are enforceable. Second, such cooperation

\(^1\) The know-how trading example provides a simple case of a Prisoners’ Dilemma where the auction solution collapses the original game into a trivial optimization. Only the choice of an appropriate bid is important. PD strategies are irrelevant because the winning bidder will enforce the efficient level of know-how trading between the two firms that it would then own.
might draw the attention of competition authorities worried about collusion. Third, and most importantly, the cooperative solution will require some third-party verification of strategies played, which may be expensive or impossible.

Finding third parties able to verify what strategies were played by which firms, or even what the outcome of an R&D venture was, may be impossible or too costly\(^2\), or may be undesirable due to appropriability problems\(^3\). No third-party verification of the actual strategies played is required in the ex-ante auction solution presented below. This is because the decision to enter the auction transforms the game into one in which the division of the total joint venture outcome is the focus and where knowledge of the strategies played (i.e., how the outcome was generated ex-post) is irrelevant. In effect, the auction redistributes the payoffs to the firms so that each shares equally in the sum of the JV’s payoffs. Given such a transformation, each firm’s optimal strategy then becomes maximizing the total payoff, which corresponds to the jointly efficient outcome.

Although this essay may represent the first attempt to use an auction to gain a Pareto improvement in the outcome of a PD game, there is some history of the use of an auction

\(^2\) The third party would have to have a comprehensive knowledge of either the companies or the technology or both in order to judge correctly. If the third party had to verify who defected, for example, by investing bad engineers, that third party would have to be able to judge which are good and which are bad engineers in each firm, which is a relative measure. If the third party had to verify what the outcome of the venture was, without knowing what would have been created had the venture gone on, then that third party would have to have expertise in the technology and in the competencies of the venture partners. All of this knowledge is difficult to find in a trustworthy third party. It is furthermore improbable, however, when the firms involved have an incentive to give that third party false information.

\(^3\) For example, if the output of the JV cannot be fully protected in some binding way, as through patents, then there is a concern that a third party judge would have the opportunity to appropriate some of the output.
towards similar ends in coordination games. Van Huyck, Battalio and Beil (1993) describe how an auction was used to filter entry into a coordination game. That game involved individuals applying effort in a project, and had payoffs based on differences from the mean effort put forth by all participants. Players could support beliefs (and choice) of higher effort inputs by using forward induction based upon the entry bid information. In effect, the entry auction allowed the selection of a more efficient Nash Equilibrium by allowing effective communication of information about the problem of equilibrium selection.

Beggs (1989) provides another example in which an auction can resolve an inefficiency. That auction allows the most efficient firm to signal the government to grant it the rights to a market that may have, in the absence of the auction, gone to an inefficient firm4.

One alternative solution to the one-shot Prisoners' Dilemma in the literature that is worth noting is the Lindahl process. Lindahl (1919) showed how an auction process might be used by a government to set the level of public spending. This process requires a set of auctions rather than one, and cannot guarantee an efficient outcome in general (as shown in Inman (1987)). In fact, Malinvaud (1971) described the many inadequacies of the process including the result that the outcome of the process is simply another PD, but one created by government rather than by the market.

---

4 Beggs presents a two-period model where the existence of consumer switching costs and firm start-up costs may lead to an inefficient firm winning the market. A government that holds a Vickrey auction for licenses to produce in the market will be able to ascertain and choose the most efficient firm to produce in that market.
Another solution to the one-shot Prisoners’ Dilemma that is closer to those proposed here is presented by Frohlich (1992) and based on impartial reasoning. Implementing the "You cut the cake and I choose rule" by having the players in the PD choose strategies without knowing whether they are choosing for themselves or for the other player allows mutual cooperation to occur in many cases. Frohlich explains this implementation in theory but does not explain it in practice where it is difficult to make real players in real situations uncertain about who they will become in the future (i.e., the "row" player or the "column" player). Frohlich’s solution forces some difficult constraints on the players: by forcing the players to be able to switch positions, only symmetric players are allowed who have no personal control over changing the strategy imposed on them whether they chose that strategy or not. The results of Frohlich’s solution and type of uncertainty used are similar to those of one of the solutions presented below, but that is where the similarity ends.

Still other solutions to the Prisoners’ Dilemma have been described for the long or infinite-horizon case. Axelrod (1984) writes in detail about various strategies that can be employed in the repeated PD when playing against various rival strategies (in an unknown horizon game). Guttman (1992) shows how cooperation can emerge in PD-like situations when rational players act to preserve valuable reputations over time. Similarly, Hakanson (1993) studies structural and

---

5 For example, if player i chose to defect while player j chose to cooperate and then the uncertainty about who must implement whose strategy was resolved such that i had to implement j’s strategy, it must be the case in Frohlich’s solution that i must have no control over changing that implementation (without being caught). It is not difficult to imagine instances in which this is an unreasonable assumption. The solutions presented in this essay rely on uncertainty about how payoffs are to be divided, not about who will be which player. Frohlich also fails to mention how to resolve the PD with two Nash equilibria that arises when P > (S+T)/2 (note: his notation used to describe the payoffs where P is the payoff from mutual defection, S from being the cooperator on the other’s defection, and T being the defector on the other’s cooperation).
partner selection issues regarding collaborative R&D ventures. Hakanson finds that repeated ventures where partners can build reputations can result in cooperative outcomes. Kreps et al (1982) study how rational cooperation can arise in the finitely-repeated Prisoners' Dilemma when there is incomplete information on rivals' types and strategies. Some genetic algorithm work (such as Nachbar (1992) and Linster (1994)) involves finding the evolutionary equilibrium in the repeated PD given an initially random population set where all members of the population are assumed to have limited rationality over the long horizon of the game.

In contrast, the auction solution can resolve the PD problem even in one-shot settings with there is little uncertainty. The focus here is on its application in joint ventures, and the analysis provides some guidance for business policy. The rest of this essay presents the description of an application, defines the game, outlines important assumptions, describes the various solutions, details important extensions to the model, and provides some conclusions.

2. Description of an Application

Consider a one-shot joint venture R&D project between two firms. Assume there is some value for the firms in participating in the venture (over using their resources in-house instead of in the venture). For example, complementarities are generated through combining the firms' equipment and researchers. Other reasons for joint venturing can be found in the literature. Taylor (1989) writes that R&D JVs are attractive in this globally competitive market because they will
accelerate a firm’s technological development, increase its productivity, and spread the risks of its technological projects. Roberts and Mizouchi (1989) provide an example of where R&D JVs are not only attractive, but essential: in biotechnology, research collaboration is necessary because of the high risks, large capital investments, and multiple technologies required to achieve a successful and profitable result. Ouchi and Bolton (1988) investigate how firms can structure research efforts to justify large investments that generate leaky (hard-to-appropriate) innovations. One solution they find is the joint venture structure. This structure allows the cost of the investment to be spread and the leaky property to be more quickly brought to market before it is imitated by rivals.

Given that participation in the JV is beneficial, assume that Firm A and Firm B enter into an R&D joint venture that, if fully implemented, lasts two periods. The output is a valuable new product or process. The output is incomplete but of some value at the end of the first period, and of considerably more value when completed at the end of the second period. Both firms provide the same input to the venture so that their investment costs are the same at each point in time during the joint venture.

Property rights are fully defined at each stage of the process. The JV contract is written so that each participant owns half the JV, and can prove this to a third party. However, what cannot be proven to a third party is whether the venture is at the end of the first period or the second;

---

6 Assume that either this input is monitored (and enforced) at an acceptable cost or that the input value is non-material compared to the other costs of the firms or the venture. In either case the investment costs are equal for each firm at each point in time during the venture.
whether the output is partially or fully complete. It may not be possible, feasible, or attractive to have an outside judge who has the enough knowledge of the project to ascertain at which stage the venture is if the venture is supposed to give some hard-to-imitate competitive advantage to its participants.

The venture is terminated when at least one of the firms decides to end its participation or when the end of period two occurs. Upon termination, the firms can exploit the output from the venture because each of them legally owns rights. Firms are considered fully rational and completely informed when they make their simultaneous decisions (i.e., over when to end the venture, and what to bid in the auction).

If both firms "cooperate" and work until the end of the second period, the venture is complete. Both firms would then have an equal share-holding of that output and an equal opportunity at that time to exploit that value on the market. So if the firms have the same opportunities to create wealth from the output then each should derive roughly the same welfare from the venture (the net value of which is labelled c).

Firms can also "defect" by attempting to exploit the venture prematurely by going to market with the preliminary product (or to the patent office with the infant idea) without informing the other firm which may have been planning to continue the venture. Now, if both firms defect, then they work only until the end of the first period when the output is not complete. They again split the rewards (and the venture ends at this time). The firms would obtain half shares in this lesser
total value of the joint venture (the net value of which is labelled d).

If one firm defects while the other cooperates then the defecting firm does well while the cooperating firm suffers. In this case it is assumed that the defecting firm obtains the full value of the incomplete venture output in the market (the net value of which is labelled w) as it is the only firm that is prepared to exploit the opportunity at this time. The cooperating firm is relatively unprepared to exploit the value of the output at the end of period one and it receives little or no positive benefit from the venture at all (the net value of which is labelled s). The venture ends with the defection at the end of period one so it is assumed that the cooperative firm cannot continue the research into period two with any hope of future positive returns7.

If the payoffs order as \( w > c > d > s \) with the added condition that \( c + c > w + s \) then the game is a Prisoners’ Dilemma (with all payoffs in these conditions being net payoffs)8. It is also

---

7 This multi-stage Joint Venture game is similar in construction to the so-called centipede game, while having the payoffs of a PD. In the centipede game, first defined by Rosenthal (1982) and later by Binmore (1987) and others, each player has a choice of leaving the game or continuing it for future periods. If the players do not leave the game, each player’s payoffs increase in the future. In most centipede games, players take turns in being able to leave the game (e.g., the first player has its turn in period one, the second player in period two, etc.). When it is that player’s turn to choose, it can leave with a higher payoff than its rival at that time. If it chooses to continue the game, then the rival can choose to end the game giving the player a smaller payoff than it could have obtained one period before. However, if the rival chooses to continue the game, the player can now leave with a higher payoff than two periods before. The Joint Venture game described above does differ from the centipede game in a number of respects. First, players simultaneously choose to leave (defect) or continue (cooperate). Second, the payoffs increase to each player symmetrically and monotonically in each period, provided strategies played are symmetric.

8 In the case as described, one possible definition of the payoff is: \( s = 0 \) and \( w = d + d \) (with the payoffs being gross, that is not net the costs of investment when there are such costs).
assumed that \( c > 0 \) so that the JVPD is a socially valuable project.\(^9\)

Now consider the payoffs net the investment costs. At the end of each period both firms have made the same investment. Thus, payoffs \( w, d, \) and \( s \) all are net the same investment cost.\(^10\)

Only if both firms cooperate and the venture continues until the end of period two do the investment costs increase to both firms.\(^11\) As these investment costs have no strategic implications, without loss of generality, assume both of them are zero.

With the net payoffs forming a Prisoners’ Dilemma the usual PD outcome would be expected in this JV. Both firms would defect and only the incomplete product would come to market. This implies a social loss as well as private losses to the participating firms.

Now consider the Auction Solution. Here, the two firms place their respective shares of ownership of the venture in a trust that is held until the venture is complete. Then, the two firms bid for the two shares before the start of the venture. The outcome of the auction only becomes known when the venture is complete. The auction is of the first price sealed bid variety. The winning bidder is given all ownership rights of the venture plus its bid back less a share of the

---

\(^9\) It may also be the case that \( d > 0 \) so that the original joint venture itself is attractive to each firm. In this case the Auction Solution simply increases the efficiency of the JV. However, when \( d < 0 \), then the Auction Solution allows the JV to occur when normally it would not. Regardless of the value of \( d \), as long as the project is socially valuable, then paying the costs to implement the Auction Solution may be worthwhile not only to the participating firms but also to the government.

\(^10\) As long as the investment made by all participants is the same (regardless of strategy played up to that period in time when a new investment must be made) the investment has no strategic implications.

\(^11\) Thus, for example, payoff \( c \) is net \( i_t + i_s \), the sum of the two investment costs when those costs are assumed to be non-zero.
total bids while the losing bidder is given its bid back plus the share of the total bids collected from the winning bidder.

The auction trust has all ownership rights of the venture until the winning bidder is revealed. When the venture is terminated (i.e., made complete), only a firm which can establish ownership rights has the legal right to exploit any output from the venture. The ability to claim any payoff from the venture itself without having won the auction is therefore removed. What the auction does, in effect, is take away ownership certainty, reducing the incentive to defect on another's cooperation. The auction transforms the game into one where defection is no longer the dominant strategy as the distribution of payoffs resulting from any strategy is uncertain until the bids are revealed\(^\text{13}\).

Consider another alternative of writing a contract between firms to form a merger over the venture. The merger agreement would be written so that all profits made by either firm resulting from the venture are split evenly. This requires that each firm can be effectively audited by a third party to determine the profits generated by the venture. It is assumed that this is an unreasonable requirement in most cases not only because it would be costly to track and verify

\(^{12}\) A firm can still choose to defect under the solution and stop the venture at the end of period one and force the unsealing of the bids. Then, only if it won the auction, and so could prove ownership rights to the venture's output, could it exploit the defection by bringing the output to market then. If it lost the auction, the venture would be over and the other party could bring the output to market as it saw fit. Thus, all previous strategies are still possible under the Auction Solution.

\(^{13}\) The alternative of just having a trust hold the ownerships (without the auction) is pointless. If a trust holds the ownerships until the venture ends, a defecting firm can still prove the necessary ownership rights (although being held by the trust). The original game is unchanged under this alternative, defection remains the expected action.
all revenues and expenses of a venture but also because each firm would have an incentive to misinform the other firm in order to capture more of the profits. The auction solution entails no such requirement.

The auction creates some uncertainty over which firm will capture the "prize", and how great that prize will be. As it is structured, the auction provides the firms with incentives to maximize the size of the prize (by cooperating in the venture), and to maximize their chances of winning (by bidding up to the efficient levels).

When it solves the JVPD, implementing the Auction Solution increases the total rewards available to the firms and, in this example where mutual cooperation corresponds to a better product on the market, also to society.

---

14 The auction creates some uncertainty about allocation of the rewards where the rewards are the shares of ownership of the venture. Bidding on the ownership creates, in effect, a buyout option for each firm. Demski and Sappington (1991) provide another example of how a buyout option can result in a Pareto improvement. They analyze the use of one in a double moral hazard problem. When there is an independent business entity whose ownership is transferable, having agents that can be required to purchase it from a principal results in the agents working harder.
3. The Game

The Prisoners' Dilemma component of the game is standard. Firms have two choices: either cooperate (C) or defect (D) with the other firm. Firms choose their strategies simultaneously knowing the payoff schedule, and having complete information and full rationality. The net payoff schedule is as shown:

<table>
<thead>
<tr>
<th>Firm B</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>Firm</td>
<td>C</td>
<td>c,c</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>w,s</td>
</tr>
</tbody>
</table>

where:
1. \( w > c > d > s \)
2. \( 2c > (w+s) \) so that the jointly efficient solution is: C,C
3. \( c > 0 \) so that firms may participate in the JV under the Auction Solution

Assume that this is a one-shot game. This is consistent with the structure of many joint ventures. Investment in a joint venture will of course be a one-shot game or a finitely repeated game. In either case, when there is no uncertainty, perfectly rational firms will defect.

Now imagine the following timeline. At \( t=0 \), which is the beginning of period one, firms choose their strategies (C or D). At \( t=1 \), which is the end of period one and the beginning of
period two, the venture ends if there is any defection. If there is no defection then the venture continues until \( t=2 \) which is the end of period two. The only times that a firm can leave the venture are at \( t=1 \) and \( t=2 \). Assume that no individual opportunity costs or spillovers outside the venture exist\(^{15} \). Assume that payoffs meet the Prisoners' Dilemma structure as defined above.

If the game stays as described then defecting remains the dominant strategy. Whether the other firm cooperates or defects the payoff from defecting is greater than that from cooperating (i.e., in the case of the other firm cooperating, \( w > c \) and in the case of the other firm defecting, \( d > s \)).

Now, consider the following story: IBM® and Apple® form an R&D JV to create an operating system which will defeat Microsoft® in the marketplace. The JV produces software which the firms protect by each copyrighting\(^{16} \). They know that in some months the JV will generate an operating system capable of matching Microsoft’s. In some more months, they know that the JV will produce an operating system so good as to make Microsoft’s product unattractive to consumers. It is unlikely that a third party would be able to ascertain which operating system

---

\(^{15} \) These individual payoff flows that are outside the joint venture are non-transferable flows - one firm cannot transfer them to another firm through the ownership rights of the joint venture. The only individual opportunity costs assumed thusfar were the investment costs \( i_1 \) and \( i_2 \) which were assumed zero without loss of generality. The reason that these flows are allowable in this game is that they are of no strategic value. The investment costs are the same regardless of strategy played at each period in time in the game.

\(^{16} \) It is important that the output produced by the JV have assignable ownership rights. If the JV simply produces ideas (which are a public good between the two firms that can be kept secret from third parties), then no ownership rights can be transferred and the Auction Solution cannot work. Writing contracts to keep the integrity of trade secrets would be necessary (but not sufficient) to generate cooperation in such a setting.
is the one representing the full potential of the venture, nor any way to tell who defected if the 
operating system is the earlier one. Without seeing the second-period product, a third party 
cannot make the judgement, and if the venture ends after the first period, the third party never 
sees the second-period product. Both IBM and Apple may have an incentive to defect on each 
other and get out of the JV early. This is because if either cooperates and gets defected on, it 
is assumed the cooperator will not get to market until well after the defector has made its 
operating system the new standard. This is precisely the kind of case in which the Auction 
Solution may provide a Pareto improvement.

3.1 The Basic Application

Now consider what occurs when the Auction Solution is implemented. At $t=0$, the firms 
surrender their ownership shares to a third party. They then submit bids for ownership of all 
shares of the venture to the third party. The bids are sealed. The firms then choose and play 
their game strategies (C or D) and the venture is undertaken. When the venture is finished (at 
$t=1$ or $t=2$ depending on the strategies played) the bids are unsealed.

The firms do not know the outcome of the auction until the JVPD is finished - when the payoff 
results are attained. Otherwise, they may be inclined to change their strategies. Assume that this 
is a first price sealed bid auction. The highest bidder takes full ownership of the prize less some 
cut of the total amounts bid as its reward. The losing bidder takes the cut from the winner as 
its compensation. In the case of a tie, a coin is flipped to determine who is the winner and who
is the loser.

Assume that to run this auction costs a positive, but very small amount\textsuperscript{17}, \(\gamma\). This cost is split equally among the participating firms.

Formalizing, the ex-ante auction is defined as a first-price sealed-bid where there is no limit set on the amounts bid. The winner of the auction receives the total JVPD payoff plus its bid back less a share, \(\alpha (0 \leq \alpha \leq 1)\), of the total amounts bid. The loser receives its bid back plus the \(\alpha\)-share of the total amounts bid transferred from the winner. If there is a tie in the bidding, then a coin-flip decides the winner\textsuperscript{18}.

\textsuperscript{17} It is assumed that the cost to initiate this solution to the PD is non-trivial. This assumption eliminates one important instance of possible multiple equilibria: the equilibrium that may occur when firms do not partake in the auction and end up with the mutual defection outcome, and the equilibrium that may occur when firms do partake in the auction and still both defect. Without the \(\gamma\) the firms would have the same payoffs under either equilibrium. It is not unreasonable to consider that the extra step of partaking in the auction would be costly, so it is assumed to be in all the analysis that follows.

\textsuperscript{18} There is one further important assumption that is implicit in being able to implement the Auction Solution and that is the assumption that such a solution can be coordinated between the firms at a non-prohibitive cost. Considering that the two parties had to coordinate to be involved in the joint venture in the first place it is not an unreasonable assumption to extend their coordination a few steps further to allow the terms of the solution to be set. The agreement on the conditions of the Auction Solution comprises a third-party observable, enforceable contract between them just as the contract that defines the legal entity and ownership of the joint venture does. In either contract, all the terms are settled on beforehand.
Auction Solution Time Line:

<table>
<thead>
<tr>
<th>The two firms agree to enter into the Auction Solution at start.</th>
<th>The two firms then submit sealed bids for full ownership of the venture to the trust.</th>
<th>The venture is played out. The output is available.</th>
</tr>
</thead>
<tbody>
<tr>
<td>JV work done.</td>
<td>JV work done.</td>
<td>JV work done.</td>
</tr>
</tbody>
</table>

The two firms surrender their ownership in the venture to a trust. The firms then play their venture strategies, C or D. The bids are unsealed and the auction rules are carried out to find the winner and loser and to distribute the shares and bids accordingly.

Figure Four: The Auction Solution Time Line

\[ \text{Given: } W = \text{winning bid} \quad L = \text{losing bid} \quad V = \text{total value of JV} , \]

The winning bidder receives net payoff: \[ V - \alpha (W + L) - \frac{Y}{2} \]

The loser bidder receives net payoff: \[ \alpha (W + L) - \frac{Y}{2} \]
The original game is transformed into a new game under the Auction Solution:

Losing Bidder (bids \( L < W \))

\[
\begin{array}{cc}
C & D \\
Winning Bidder & 2c-\alpha(W+L), \alpha(W+L) & w+s-\alpha(W+L), \alpha(W+L) \\
(bids \( W > L \)) & w+s-\alpha(W+L), \alpha(W+L) & 2d-\alpha(W+L), \alpha(W+L) \\
\end{array}
\]

where: payoffs expressed are not net \( \gamma/2 \); payoffs are read winning bidder, losing bidder; and \( W \) and \( L \) change in each cell.

Assume that the auction solution uses \( \alpha = 1/4 \); at this particular value the bids of firms match the total gross value of the venture\(^{19} \). The auction pot is made up of the value of the venture payoff to Firm A plus that to Firm B. Consider what the bids will be in the general case where each firm believes the auction pot's total value is \( V \). If both firms bid \( V \) then a coin toss determines the winner, and according to the formulas, each firm receives \( V/2 \) as its "net\(^{20} \)" payoff.

Now assume it is known that one firm will bid \( V \). If the other firm bids \( V+\delta \) (where \( \delta > 0 \)) then it wins the auction pot and receives total net payoff of \( V-(2V+\delta)/4 < V/2 \) while the losing firm

---

\(^{19}\) Other values of \( \alpha \) make firms bid differently. If \( \alpha=0 \) then each firm will bid an infinite amount as it knows it will get its bid back and not have to transfer anything. In this case, the tie-breaking coin toss will give the full pot to the winner and nothing to the loser. Thus, each bidder can expect a payoff of one-half the total pot on average. If \( \alpha=1 \) then each firm will bid one quarter of what it believes will be the total value of the prize. When \( \alpha=1/4 \) each firm will bid what it believes will be the total value of the prize. Therefore, if there are financial constraints, the firms may choose to have a higher \( \alpha \) defined in the auction rules.

\(^{20}\) The "net" payoffs described in this section are not net the costs of the auction, \( \gamma/2 \).
receives a net payoff of \((2V + \delta)/4 > V/2\). If it bids \(V - \delta\) it loses the auction and receives a net payoff of \((2V - \delta)/4 < V/2\) while the winning firm receives a net payoff of \(V - (2V - \delta)/4 > V/2\).

If it bids \(V\) it receives a net payoff of \(V/2\) in either case of the coin-toss tie-breaking outcome.

Now, if it is known that one firm will bid \(W < V\) then clearly the best response is a bid of \(W + \delta < V\) in order to obtain a net payoff of \(V - (2W + \delta)/4 > V/2\). Similarly, if it is known that one firm will bid \(W > V\) then the best response is a bid of \(W - \delta > V\) in order to obtain a net payoff of \((2W - \delta)/4 > V/2\). Thus, the equilibrium bid when \(\alpha = 1/4\) is \(V\) (or the true value of the auction pot\(^{21}\)). It follows that in equilibrium each firm will bid the same and receive \(V/2\) as its certain net payoff. When \(\alpha \neq 1/4\) then bids do not mirror the total value of the auction pot but do result in an outcome which gives a certain value of \(V/2\) to each bidder as a net payoff. When \(\alpha = 0\), an expected\(^{22}\) value of \(V/2\) is the net payoff to each bidder.

With \(\alpha\) set at 1/4, and knowing that the outcome of bidding will be a tie if both firms have the same beliefs about \(V\), the different outcomes can be analyzed. Equilibrium requires that each firm has true beliefs - each firm has correct conjectures over outcomes\(^{23}\). If the outcome is mutual cooperation then each firm receives \(c\) as its certain net payoff (having both bid \(2c\) : the

---

\(^{21}\) The line of argument used to arrive at the focal bid is similar to that used to arrive at the focal price under the usual Bertrand competition case.

\(^{22}\) When \(\alpha = 0\) then each firm bids the same "infinite" amount and so their is a tie in the bids. When there is a tie in the bids a fair coin is flipped to determine the winner. Thus, each firm will have a 50% chance of winning and receiving a net payoff of \(V\), and a 50% chance of losing and receiving a net payoff of 0. Thus, each firm has an expected payoff of \(V/2\) when \(\alpha = 0\).

\(^{23}\) Thus, if each firm knows what the outcome will be then a firm may choose to cooperate even knowing that the other will defect. If it also wins the bid, it will be able to exploit the full value of the uncompleted venture output soon after \(t=1\), because it has no competition from the defecting firm at this time. This differs from what happens in the original game where the defecting firm beats the cooperative firm to the market by enough of a margin to obtain all the value of the output at \(t=1\).
total gross value of the prize pot). Similarly, if the outcome is mutual defection then each firm receives \( d \) as its certain net payoff. If the outcome is asymmetric (where one firm cooperated while the other defected) then each firm receives \( (w+s)/2 \) as its certain net payoff (see Table One for possible outcomes).

<table>
<thead>
<tr>
<th>Item \ Scenario</th>
<th>Mutual Cooperation</th>
<th>Mutual Defection</th>
<th>Single Defection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilibrium Bid</td>
<td>2c</td>
<td>2d</td>
<td>( w+s )</td>
</tr>
<tr>
<td>Net Payoff to each</td>
<td>( c-\gamma/2 )</td>
<td>( d-\gamma/2 )</td>
<td>( (w+s-\gamma)/2 )</td>
</tr>
<tr>
<td>(if optimal bid was used)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table One: Bids and Payoffs for the Auction

Mutual cooperation with bids at \( 2c \) is now a Nash Equilibrium\(^{24}\). If \( (w+s)/2 > d \) then partial cooperation is more rewarding than mutual defection. When this occurs, mutual cooperation is, in fact, the only Nash Equilibrium.

\(^{24}\) Mutual cooperation with bids at \( 2c \) is a Nash Equilibrium because any change in either the strategy choice (i.e., choosing to defect) or in the bidding choice (i.e., bidding above or below \( V \)) will decrease wealth. It is relatively straightforward to show that a case of partial cooperation (where only one firm cooperates) is not a Nash Equilibrium (under any potential equilibrium bid). The defecting firm can always improve its situation by cooperating instead. When payoff rankings are \( d > (w+s)/2 \) then mutual defection with a bid of \( 2d \) is also a Nash Equilibrium. Neither firm would change bid or strategy at this point if it had chosen to enter the auction.
The condition \((w+s)/2 > d\) may or may not hold in general\(^{25}\).

If \((w+s)/2 \leq d\) then mutual cooperation and mutual defection are both Nash Equilibria. However, a forward induction argument can be used to eliminate the mutual defection equilibrium if the \(\gamma\)-costly Auction Solution is entered into by both firms. Since the firms can assure themselves of payoff \(d\) in the original game, they would not enter the Auction Solution, play \(D\) and end up with less than \(d\) as a payoff\(^{26}\). Thus, the only strategy a firm would play if it entered the Auction Solution is \(C\) to obtain the payoff greater than \(d\) with certainty. Therefore, the Auction Solution can be used to achieve the highest joint net payoff from the Joint Venture.

In the case outlined there are no "non-transferable" costs or benefits to participating firms other than the investment costs which are of no strategic value. This means that there are no spillover benefits or opportunity costs which are individual to a firm. Thus, all payoff flows can be transferred across firms through the auction mechanism. However, in many JVPDs, non-transferable flows, such as opportunity costs that are individual to the firm, do affect the net

\(^{25}\) In a later section of this paper PD strategies are treated as investment decisions not exit decisions. Then when the condition \((w+s)/2 > d\) does hold, a characteristic of the production function is revealed: The first instance of cooperation in the venture increases the value of the total output substantially; enough to compensate for any dysfunction due to the differences in venture input choices of the two joint venture partners. For example, when firms have the choice of investing good versus bad engineers into the venture (as their PD strategies), the condition translates to the good engineers being able to re-train and be quickly complementary to the bad ones. The increase in total value due to the good engineers exceeds their added opportunity cost when all costs and benefits are summed over the two firms.

\(^{26}\) If one firm plays \(D\) while the other plays \(C\) then they each end up with \((w+s-\gamma)/2 < d\) by assumption. If both play \(D\) then they each end up with \(d-\gamma/2 < d\) by definition. Therefore, neither firm would enter the auction and play \(D\).
payoffs to each individual firm and so may be of some strategic consequence. When non-transferable flows exist and have strategic implications, the auction solution may not work as just described. This will be further detailed in Section 3.4 of this essay.

The results of the analysis can now be summarized:

Proposition One: If conditions exist on payoffs to firms participating in the Joint Venture:

i) to make net payoffs satisfy the structure of a Prisoners' Dilemma

and ii) to ensure that either:

a) there are no non-transferable flows

or b) the non-transferable flows are of no strategic consequence

then implementing an ex-ante auction (of the type described here) over the total gross payoff of the venture will result in the highest joint net payoff outcome of that venture.

Proof: As shown above, when an efficient auction is implemented the only Nash Equilibrium that survives forward induction is one of mutual cooperation in the PD itself. When this occurs, the optimal outcome of the Joint Venture is realised.
To this point the scenario has been one where there are two firms, the firms are symmetric, and the non-transferable costs are eliminated or are of no strategic value. There are some very interesting results that occur when any of these restrictions is relaxed.

3.2 An Application with \( n > 2 \) Firms

The extension to \( n > 2 \) firms is straightforward. Consider the basic case with symmetric firms. The highest bidder obtains full ownership of the venture and its own bid back less the sum of the \( \alpha \)-shares of the total bids collected. The losing bidders get their bids back plus the \( \alpha \)-share of the total bids collected transferred from the winner. If \( \alpha \) is set to equal \( 1/n^2 \) then the bids will again reflect the gross total value of the JV.

Under the Auction Solution each firm plays the cooperating strategy and bids \( nc \) for the \( n \) shares of the venture. The tie is broken by an appropriate randomizing devise. Each firm ends up with \( c \) as its net payoff. This is the dominant outcome as long as the PD structure remains as defined with \( w > c > d > s \) and \( nc > w+(n-1)s \). A forward induction argument is used when the condition \( w+(n-1)s > nd \) is violated based on a \( \gamma \)-costly auction. The mathematics for the bidding and the Nash Equilibria are derived in a straightforward manner from the base case of the Auction Solution described in the previous section.
3.3 An Application with Asymmetric Firms

With symmetric firms the basic auction described in Section 3.1 has much in common with using a simple coin-flip for ownership of the JV. The similarity is much reduced when firms are not symmetric.

A different (but consistent) approach for finding Nash equilibria is required when the firms are not symmetric as the strategy space is more complicated.

But first it is necessary to define what is meant by asymmetric firms. Assume that Firm A is better able to capitalize on the JV’s outputs so that it receives a higher gross payoff than Firm B in any of the outcome scenarios, (i.e., $c_A > c_B$, $w_A > w_B$, $d_A > d_B$ and $s_A > s_B$ with the conditions that $w_A > e_A > d_A > s_A$, $w_B > e_B > d_B > s_B$, and $c_A + c_B + e_A + e_B > w_A + s_B + w_B + s_A$ holding to maintain the PD form\(^{27}\)). Assume this is common knowledge. Assume also that the investment costs stay the same (and so are the same for each firm). Imagine that the added benefit to Firm A in each case comes from some added spillover from the venture output that can be only exploited by Firm A due to its control of certain complementary resources or such. Non-transferable payoffs are involved here (i.e., the difference in value between $c_A$ and $c_B$, between $d_A$ and $d_B$, etc..) that are not simply contingent on which PD strategy each firm played (i.e., Firm A will always obtain some non-transferable payoff if it wins the auction regardless of the

---

\(^{27}\) Thus, if both firms cooperate, Firm A gets $c_A$ and B gets $c_B$; while if both defect A gets $d_A$ while B gets $d_B$, and so on for the other outcomes. It is assumed that the increased reward that A receives over B for any of the payoffs $w,c,d$ or $s$ carries over to when A obtains ownership of the venture. For example, if A won ownership when both firms cooperated in the venture then A would receive $2c_A$ (and not $c_A + c_B$) as its gross payoff from the venture.
strategy played by Firm B).

If A wins ownership of the whole JV, its gross payoff would be $2c_A$ if A and B cooperated, $w_A + s_A$ if only A or B cooperated, and $2d_A$ if both A and B defected. Similarly, if B wins ownership of the whole JV, its gross payoff would be $2c_B$ if A and B cooperated, $w_B + s_B$ if only A or B cooperated, and $2d_B$ if both A and B defected.

Under the auction, if A wins ownership by bidding appropriately\textsuperscript{28}, the venture is transformed into the following:

<table>
<thead>
<tr>
<th>Firm</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$2c_A - c_B$, $c_B$</td>
<td>$w_A + s_A - (w_B + s_B)/2$, $(w_B + s_B)/2$</td>
</tr>
<tr>
<td></td>
<td>$w_A + s_A - (w_B + s_B)/2$, $(w_B + s_B)/2$</td>
<td>$2d_A - d_B$, $d_B$</td>
</tr>
</tbody>
</table>

The socially efficient outcome would entail Firm A obtaining full ownership rights to a venture that produces the highest joint outcome (that which arises from mutual cooperation). Specifically, the socially efficient outcome would entail Firm A winning the auction when the Auction Solution is implemented.

\textsuperscript{28} In the table shown, the appropriate bids are composed of the total venture value to B in each case, plus an infinitely small increment.
Consider now what does occur when the firms are asymmetric and there is full information. The auction works efficiently when $0 < \alpha \leq 1$ (if $\alpha = 0$, then the solution is simply a coin-flip to determine ownership of the venture). Assume $\alpha = 1/4$ as before. Now check for the optimal bid for the lower value firm, Firm B. In the general case in which it believes the auction pot’s value is $V$, if Firm B bids $V+\delta$ (where $\delta > 0$) while Firm A bids only $V$ then B wins the auction pot and receives total net payoff of $V-(2V+\delta)/4 < V/2$. If B bids $V-\delta$ while Firm A bids $V$ then B loses the auction and receives a net payoff of $(2V-\delta)/4 < V/2$. If B bids $V$ while Firm A bids $V$ also then B receives a net payoff of $V/2$ in either case of the coin-toss tie-breaking outcome. Therefore, the optimal bid for Firm B when Firm A bids $V$ and $\alpha = 1/4$ is $V$ (or the true value of the auction pot for Firm B). Now check what the optimal bid is for Firm A.

It has been shown that if Firm A also believes the auction pot’s value is $V$ then its best bid is $V$. In fact, even if Firm A believes the auction pot has a value above $V$ to itself, then a bid of $V$ will ensure it an expected reward of the average of $V$ and that higher value. If it bids $V+\epsilon$ while Firm B bids only $V$ then it wins the auction and realizes a total net payoff of $V+\beta-(2V+\epsilon)/4$ (where $\beta > \epsilon$ is the added value Firm A has of the venture outcome over that of Firm B). This is larger than the payoff from simply tying the auction, $(V+\beta)/2$. Now recheck the optimality of Firm B’s bid in light of Firm A’s optimal bid. If Firm B bids above A’s bid then B receives a total net payoff below $V/2$. If B bids below A’s bid, so that B bids $V$, then B receives a total net payoff of $V/2+\epsilon/4$. If B bids the same as Firm A at $V+\epsilon$ then B ties and

---

\[29\] Again, the "net" payoffs described in this section are not net of the auction costs, $\gamma/2$. 
receives a total net payoff if it wins the coin-flip of value \((V-\epsilon)/2\) and if it loses of value \((V+\epsilon)/2\) for an expected value of just \(V/2\). Therefore, the optimal bid for Firm B remains at \(V\).

It is conjectured that the Auction Solution will achieve the efficient outcome in which A wins the auction (with a bid of \(V+\epsilon\), where \(V\) equals \(2c_B\)), and mutual cooperation occurs. It has been shown that this outcome is possible in bidding, now it must be shown that it can occur in game strategies played.

When both firms cooperate and bid as predicted above then Firm A gets \(2c_A-c_B-\epsilon/4\) and Firm B gets \(c_B+\epsilon/4\). If B could obtain a higher payoff by defecting, it would. To rule this out requires, first, that \(w_B+s_B-c_B-3\epsilon/4 < c_B+\epsilon/4\) so that defecting and bidding to win the auction (when A is expecting cooperation) is less attractive than cooperating and losing the auction.\(^{30}\)

When this restriction is met, B is indifferent between cooperating and defecting but will bid as required\(^{31}\).

\(^{30}\) Of course, if B defected and bid to lose the auction (when A was expecting cooperation) B would still obtain \(c_B+\epsilon/4\).

\(^{31}\) Although B may be indifferent in the case described so far, some considerations that are outside the basic model, like reputation effects, may make B partial to cooperation. Even if B could not obtain a higher payoff by defecting and winning the bid, B may need to have a disincentive to defecting and losing the auction. B could obtain the same payoff if it bid the same and defected while A cooperated and bid to win. Therefore, a way to move B off the indifference point (of cooperating or defecting) may be required here. It should be noted that the indifference is generated by the fact that B knows it will lose the auction and so does not care about its strategy, as its reward is not affected by that choice directly. Therefore, what is needed is to create some uncertainty for B over its losing the auction in order to give it the incentive to cooperate. This is done simply by having A randomize its bid between \(2c_B\) and \(2c_B+\epsilon\), with a very small but positive probability on bidding just \(2c_B\). When this occurs, B wants to bid \(2c_B\) and play C for a minimum payoff of \(c_B\). If it choose either to bid less, or to play D, it would ensure itself that it would obtain a smaller payoff with positive probability. This is, in effect, a way to show that B will cooperate under the Trembling Hand equilibrium concept.
Given B's strategy, A may have an incentive to change strategies. If A defected and bid the same as before, A would obtain \( w_A + s_A - (4c_B + e) / 4 \). Thus, if \( w_A + s_A < 2c_A \) then A would not want to change strategies.

Therefore, to obtain the outcome where the firm with the highest valuation of the venture wins ownership and mutual cooperation is a Nash Equilibrium of this PD under the Auction Solution requires that two payoff rankings hold that are not guaranteed by definition of a PD:

\[
\text{Require both } 2c_B > w_B + s_B \text{ as well as } 2c_A > w_A + s_A \text{ for obtaining the desired outcome, although}
\]

\[
\text{know that } 2c_A + 2c_B > w_A + s_A + w_B + s_B \text{ given the definition of the PD.}
\]

The required payoff rankings would be met when like firms are involved. The first payoff ranking (i.e., inequality) is met when two "B" firms are involved in the JVPD. The second payoff ranking is met when two "A" firms are involved in the JVPD. The third line shows the payoff ranking that is met when two unlike firms are involved in a Prisoners' Dilemma (as is the case here - both an "A" firm and a "B" firm are involved). It should be noted that the PD characteristics do not arise due to the presence of unlike firms, but occur due to the nature of this JV.

Now, assume the required payoff rankings hold and analyze this game for equilibria. Partial cooperation (where only one firm cooperates) is never a Nash Equilibrium. At this point, the defecting firm will always want to change its strategy (and its bid) to obtain a higher payoff,
given both the payoff ordering requirements above hold.

The only other candidate Nash Equilibrium to check is that of mutual defection. If both defect and A bids $2d_B + \varepsilon$ while B bids $2d_B$ then Firm A gets $2d_A - d_B - \varepsilon/4$ and Firm B gets $d_B + \varepsilon/4$. Firms may want to change their choices at this point depending on the payoff rankings. If $2d_A > w_A + s_A$ then A has no incentive to change its choices. If $2d_B > w_B + s_B$ then B has no incentive to change its choices. When these two conditions hold, if the firms are in the auction, then this mutual defection point is another Nash Equilibrium. However, as long as $\varepsilon/4 < \gamma/2$, which will hold true for very small $\varepsilon$, B has no incentive to enter the Auction Solution and defect. As Firm A knows this, and does better by cooperating than defecting (under the payoff rankings assumed above), then A will also cooperate. As B knows this, and obtains a payoff of $c_B$ by entering and cooperating instead of $d_B$ by not entering, B will enter, and then so will A. Thus, the mutual defection Nash Equilibrium is ruled out by forward induction.

The only Nash Equilibrium that survives forward induction is that of mutual cooperation (under the payoff ranking assumptions above). Therefore, the Auction Solution is an efficient method of solving the PD under certain restrictions in the case of asymmetric firms. It should be noted, however, that these restrictions are not automatically satisfied by the definition of the game as it has been presented. When these restrictions are not satisfied then mutual cooperation is no

---

32 The conditions would be met if it is assumed that the added spillover to Firm A is non-decreasing as the venture outcome becomes more valuable (becomes more complete). Thus, the added spillover when there is at least one defection is the same as when there are two as the venture is terminated at the end of period one in either case. It is assumed that that added spillover is not greater when there are no defections. Therefore, this gives the following conditions: $w_A + s_A - w_B - s_B = d_A + d_B - d_B$ and $c_A + c_B - c_B \geq d_A + d_A - d_B$. Thus, a monopoly on a more valuable product is worth more than one on a less valuable (incomplete) product. From the two
longer a Nash Equilibrium because at least one firm has an incentive to defect on the other's cooperation.

A case in which these conditions are satisfied arises when the difference between Firm A's payoff and Firm B's is constant over the outcome space: \(e_A = e_B + \alpha\), \(w_A = w_B + \alpha\), \(d_A = d_B + \alpha\) and \(s_A = s_B + \alpha\). The original PD game definition now becomes \(4c_B > 2(w_B + s_B)\). Now the Auction Solution for these asymmetric firms is applicable as the two restrictions are met. The solution will also work under similar restrictions when it is not known by how much the higher-valuing firm values the venture (as such information has not been required by the solution presented here thus far)\(^{33}\).

Now consider the effects of a less complicated solution technique on the case of asymmetric firms. This solution is an extreme member of the Auction Solution set; it occurs when \(\alpha\) is set to zero. This amounts to a coin-flip to determine ownership of the venture. It shall be termed the Coin Flip Solution. The firms again place their shares of ownership in trust to be held until

\[2c_A > w_A + s_A\] and \(2c_B > w_B + s_B\). When these conditions are met, the Payoff orderings required under the general case of asymmetric firms are met, and the Auction Solution achieves efficiency.

\(^{33}\) Consider another case of asymmetric firms, where each firm draws from a distribution (i.e., the same distribution for both players) of spillover capabilities which affect payoff evaluation. Each firm knows the distribution but not the other firm's draw. Although this is a case for future work, one possible outcome to consider would be that each firm would (usually) bid its true valuation of the JV output and receive a certain payoff of one-half its own true valuation of the total auction pot plus one-quarter of the absolute value of differential in the values (when \(\alpha = 1/4\)). This outcome is more efficient than using a simple coin flip to determine ownership of the venture (or when \(\alpha = 0\)). The coin flip results in less expected total payoff, as it is not the best firm which wins the auction each time:

\[
\text{Auction joint payoff is } \frac{3}{2} \frac{V_A + V_B}{2} \text{ versus } \frac{V_A + V_B}{2} \text{ for the coin flip.}
\]
the venture ends. Before the venture begins (or upon completion) a fair coin is flipped and its result left secret until the venture ends whereupon the outcome of the flip is revealed. At that time the full ownership is handed over to the winning firm. Under this solution each firm, in effect, will have an expected payoff of half its valuation of total ownership of the venture.

As in the original Auction Solution when firms are symmetric and risk neutral they will participate in the Coin Flip Solution and cooperate when both $c > (w+s)/2$ (which is met by definition) and $(w+s)/2 > d$. When $(w+s)/2 < d$, then a forward induction argument can be used to eliminate the Nash Equilibrium of mutual defection\(^4\).

This solution appears to be less complicated than the Auction Solution (as bids are not submitted or counted). However, it does not come up with the optimal solution in the case of asymmetric firms. Only with expected probability of 1/2 will the ownership of the venture go to the firm that values it more highly.

The Coin Flip Solution is more formally presented in Appendix Eight and shown to be dominated by the Auction Solution when firms are risk averse.

\(^4\) If a firm enters and defects then it either gets (in expected value terms) $d-\gamma/2$ or $(w+s-\gamma)/2$ which are both less than $d$ which it can obtain with certainty by not entering the Coin Flip Solution. Again, it has been assumed that this technique for resolving the PD entails a cost. The cost, $\gamma$, can be assumed to be zero in which case forward induction does not eliminate the partake-in-the-solution-and-defect equilibrium.
3.4 An Application with Strategic Non-Transferable Costs (SNCTC)

The case of asymmetric firms provides a natural introduction to SNCTC as these are in essence asymmetries of a different type. While the asymmetry just described arises due to differences in payoffs to each firm independent of who played which PD strategy, the asymmetry that arises due to SNCT is due only to who played which PD strategy (i.e., who defected and who cooperated). In the SNCT case, the asymmetry arises due to cost differences between playing D versus playing C and so it does matter to each firm who defected and who cooperated.

The non-transferable amounts are extra costs and benefits that cannot be transferred between participating firms by redistributing the shares of the venture. For example, the opportunity cost that a participating firm incurs by sending an engineer to work in the venture is an investment cost that it will not be directly reimbursed no matter who ends up with ownership of the venture. While the valuable new product or process can be transferred in ownership rights to the final owner of the shares, the non-transferable extras (like the opportunity costs or added spillover benefits) cannot. These extras are in essence outside the venture itself but do affect the net payoffs of the participating firms.

Now, consider the case of two symmetric firms participating as equal partners in an R&D Joint Venture. They can cooperate by investing in the JV the time of good engineers and machines.

---

35 Analysis of any and all combinations of the three derivations from the basic application (as described in Sections 3.2, 3.3 and 3.4) is not done in this essay but left for future work.

36 The firm will be indirectly compensated for its investment when it receives whatever reward it gets from the venture. However, the choice of the quality of the investment alone does not guarantee proportionally greater rewards for a firm. This is partly because a firm’s marginal cost and benefit are not those of the venture.
at cost $i_c$ or they can defect by investing in the JV the time of bad engineers and machines at cost $i_b$ where $i_c > i_b$ (assume $i_b = 0$ without loss of generality). It is implied by this inequality that the opportunity cost of sending bad resources is less than that of sending good ones. Further, assume that verification by a third party of which investments were made is too costly$^{37}$. The difference in opportunity cost between investing the time of good engineers and bad ones represents a private saving to the defecting firm while the cost of the defection is not totally private.

Assume that the output of the JV increases with better investment so that mutual cooperation results in the highest gross JV output ($P_{cc}$), single defection the second highest ($P_{cd}$), and mutual defection the lowest ($P_{dd}$; such that $P_{cc} > P_{cd} > P_{dd} > 0$). Now define the net payoff per firm from mutual cooperation as $c = (P_{cc}/2) - i_c$; the net payoff from mutual defection as $d = P_{dd}/2$; the net payoff from defecting while the other cooperates as $w = P_{cd}/2$; and the net payoff from cooperating while the other defects as $s = (P_{cd}/2) - i_c$. In order to be a PD, payoffs need to be structured: $w > c > d > s$ and $2c > w + s$. (In this case, $d > 0$ is assumed so that firms willingly participate in the JV.)

---

$^{37}$ Verification by a third party of whether a firm cooperated or defected may be too costly or unattractive to participating firms for a number of reasons. For example, it may be very difficult without intimate knowledge of a firm which of its engineers are relatively good versus bad. Also, it may be unattractive to firms who are engaged in a venture that produces valuable patents or trade secrets to have outside parties become aware of them before they can be fully protected.
The original game:

\[
\begin{array}{c|cc}
 & C & D \\
\hline
\text{Firm } & (P_{CC}/2)-i_C, (P_{CC}/2)-i_C & (P_{CD}/2)-i_C, P_{CD}/2 \\
\text{A} & P_{CD}/2, (P_{CD}/2)-i_C & P_{DD}/2, P_{DD}/2 \\
\end{array}
\]

is transformed into the a new game under the Auction Solution:

\[
\begin{array}{c|cc}
 & C & D \\
\hline
\text{Winning Bidder} & P_{CC}-i_C-(b_A+b_B)/4, (b_A+b_B)/4-i_C & P_{CD}-i_C-(b_A+b_B)/4, (b_A+b_B)/4 \\
\text{Losing Bidder} & P_{CD}-(b_A+b_B)/4, (b_A+b_B)/4-i_C & P_{DD}-(b_A+b_B)/4, (b_A+b_B)/4 \\
\end{array}
\]

where: \( b_A \) is Firm A's bid and \( b_B \) is Firm B's bid and these bids can differ in each cell; \( \alpha=1/4 \); these payoffs are not net the auction set-up cost \( \gamma/2 \); and the winning bidder has a higher bid than the losing bidder.

The game is a Prisoners' Dilemma with only one Nash equilibrium: mutual defection. This general description of a JVPD involving SNTCs differs from the first JVPD description that did not involve SNTCs. Under this new JVPD the simple Auction Solution alone will not work. This is proven by example.
Consider the venture as defined above with the auction technique as defined previously and with $\alpha$ set to $1/4$. To prove that the Auction Solution fails to elicit mutual cooperation it is sufficient to show that mutual cooperation is \textit{never} a Nash Equilibrium. Assume that mutual cooperation is the conjectured outcome at bids $b_A$ and $b_B$ (for Firm A and B respectively). If a firm defected instead and bid $P_{CD}$, it would ensure itself a payoff of at least $w > c$ when the other firm remained cooperative, regardless of that other firm's bid. If the cooperative firm bid $P_{CC}$ or more and won, the defecting firm would receive at least $(P_{CC}+P_{CD})/4$, or $(w+c+c)/2$, which is larger than $c$. If the cooperating firm bid less than $P_{CC}$ but greater than or equal to $P_{CD}$ and won, the defecting firm would receive at least $P_{CD}/2 = w > c$. If the cooperating firm bid any less it would lose the auction, and the defecting firm would receive at least $P_{CD}/2 = w > c$. Therefore, each firm has an incentive to at least change strategy at this point of mutual cooperation. Mutual cooperation cannot be a Nash Equilibrium under the Auction Solution as defined.

Mutual cooperation is no longer a Nash Equilibrium as it was when there were no SNTCs. This is because with SNTCs, a firm that defects can receive a higher payoff than the firm that cooperates in equilibrium. When there are no SNTCs, both firms receive the same amount in equilibrium so it does not pay to defect. When SNTCs exist, the defecting firm can assure itself of payoff $w$ (with an appropriate bid) and therefore obtain more than the cooperating firm regardless of which firm wins the auction. The difference in payoffs between the cooperating and defecting firm is composed of the SNTCs.
It has already been shown that the mutual cooperation point (under any bids) is not a Nash Equilibrium. It is also straightforward to show that mutual defection under the Auction Solution is not an equilibrium. If bids are above or below \( P_{DD} \) then one firm can do better by changing its bid to approach \( P_{DD} \). For example, if bids were tied at \( P_{DD} + \epsilon \) then Firm A could reduce its bid to \( P_{DD} \) and increase its payoff by \( \epsilon/4 \). Thus, it is sufficient to prove that mutual defection with bids of \( P_{DD} \) is not an equilibrium. If \( w+s > 2d \) then each firm has an incentive to change its choices to cooperate and bid \( P_{DD} + \epsilon \) in order to obtain a payoff of \( w+s-d-\epsilon/4-\gamma/2 \) which is assumed to be larger than \( d-\gamma/2 \). If \( w+s \leq 2d \) then mutual defection with bids of \( P_{DD} \) is a Nash equilibrium if the firms entered the Auction Solution. However, they would not enter the Auction Solution as they could increase wealth by \( \gamma/2 \) through not entering.

All that is left is to show that partial cooperation is not a Nash equilibrium under the Auction Solution. If bids are above or below \( P_{CD} \) then one firm can do better by changing its bid to approach \( P_{CD} \). For example, if bids were tied at \( P_{CD} - \epsilon \) then either firm could change its bid to \( P_{CD} \), win the auction, and increase its payoff by \( \epsilon/4 \). Therefore, assume that the bidding is consistent with the strategic outcome; bids focus on \( P_{CD} \). At this point, the cooperating firm receives \( s \) only and, therefore, has an incentive to defect instead and bid a minimum of \( P_{DD} \) to ensure itself a minimum payoff of \( d > s \). Thus, partial cooperation is not a Nash Equilibrium.

Therefore, there are no pure strategy Nash equilibria under the Auction Solution (regardless of the bids played). The only possible Nash equilibria to the JVPD left to explore then are: not entering the auction; or, entering the auction and playing mixed strategies and bids. When
payoffs are ordered such that no mixed strategy Nash equilibria entail both firms receiving more than \( d \) as a net payoff, then not entering the auction is the only Nash Equilibrium\(^3\). If payoffs are ordered otherwise, then firms are better off entering the Auction Solution and playing mixed strategies and bids. As playing mixed strategies implies cooperating with positive probability, the Auction Solution does involve a Pareto improvement (under the payoff ordering assumed)\(^4\).

However, the (near) jointly efficient outcome can be restored under the existence of SNTCs with a *redefined* auction. If the choice of bids is restricted to the three focal amounts then the Auction Solution can be efficient. The focal amounts are those defined by the three possible venture outcome payoffs: \( 2c, w+s, 2d \) (these particular amounts are based on symmetric firms and \( \alpha = 1/4 \)). Now assume that firms can only choose (and be held to only) one of these three bids when participating in the auction.

Analyzing this new case involves checking all possible strategy combinations. The firms can play pure or mixed bids (the bids that are mixed correspond to the two possible outcomes that could

\(^3\) For example, consider mixed Nash equilibria when one firm always defects while the other randomizes cooperating and defecting, and assume that the firms bid efficiently based on their beliefs and knowledge. This case defines the worst possible situation where the firms have a chance to be more profitable by entering the auction (i.e., if both firms randomized cooperating and defecting then the chances of higher payoffs would generally be greater). Straightforward calculations reveal that for both firms to receive at least \( d \) as their net payoff requires the necessary condition that \( w+s > 2d \). This may be considered as one of the payoff orderings that must be met in order to have a Nash equilibrium that involves partaking in the Auction Solution.

\(^4\) As the choice of bids is infinite, it is generally complicated to find the Nash Equilibrium in mixed strategies and bids when the firms do enter the solution. However, the following example shows that a Pareto improvement can occur at such equilibria: Assume that the payoffs are ordered such that: \( w+s-\gamma > 2d \) and also \( c+d = w+s \). There is a Nash Equilibrium in mixed strategies when firms cooperate half the time and always bid \( w+s \). Each firm's expected payoff is then \( (w+s-\gamma)/2 \) which is a Pareto improvement under the assumptions.
occur when one firm assumes the other will play a certain strategy\(^{40}\). The analysis (see Appendix Eleven for details) gives a condition on \(i_C\) to ensure mutual cooperation and mutual bids of \(2c\):

\[
2c - (w + s) > 4i_C
\]

When this condition holds (which is not guaranteed by PD definition or by the assumptions) then mutual cooperation is a Nash Equilibrium. It is the only Nash Equilibrium if a further condition on \(i_C\) holds which ensures that a firm will cooperate on another firm's defection:

\[
(w + s) - 2d > 4i_C
\]

However, when \(2d - (w+s) + i_C/3 > 0\) then mutual defection (with consistent bids of \(2d\)) is another Nash Equilibrium if the firms enter the solution. This equilibrium can be ruled out by forward induction. No firm will partake in the solution\(^{41}\) and defect as it would ensure itself a dominated payoff\(^{42}\).

\(^{40}\) For example, if Firm A is trying to find its best responses assuming that Firm B will cooperate, then it has to assume that B will randomize bids between \(2c\) and \(w+s\).

\(^{41}\) Firms will only enter the auction under forward induction if there is some possibility that they could improve their payoffs, i.e., if \(c > d + \gamma/2\).

\(^{42}\) For example, by not entering the solution, it would get a payoff of \(d\) instead of a possible payoff of \(d - \gamma/2\) by entering the solution.
Thus, under a slightly redefined Auction Solution the Pareto-optimal outcome of the JVPD can be obtained (given the parameters meet certain restrictions) when SNTCs are present.

3.5 Solutions Under General SNTCs

There is still a way to solve the PD even when SNTCs are present but the parameters do not meet the required restrictions. It requires certain further assumptions and an additional mechanism. The mechanism required is a special futures-like contract on each participant’s stock. The assumptions required are for near perfect stock markets and near perfect accounting disclosures.

The special futures-like contract would be created and given only to participants in the joint venture. It is issued at the time the joint venture is announced and only if the JVPD Solution is in effect. Its maturity date is the completion of the joint venture. The contract entitles the holder to the difference in the worth of the stock between the maturity date and the issue date.

---

43 Note that the simple coin toss for ownership version of this solution will not work. The bidding space cannot be restricted when \( \alpha \) is set to zero, so that \( \alpha \) value is ruled out in the redefined Auction Solution.

44 The mechanism and assumptions are possible in this case where Prisoners’ Dilemma participants are companies and not people (as there is a working market for stocks of companies but not for people as of yet).

45 When the difference in worth is positive, the holder has the incentive to redeem the contracts. When the difference is negative, the issuer can force the holder to redeem the contracts and pay the difference. The conditions on the contracts are not as restrictive as they may first appear. The issuer can offer to buy these from the holder at any time. This means that after the strategy has been chosen and before the maturity date, a firm may be able to get back its contracts if it had other (privately known) projects to do and did not want the other firm profiting from the increase in contract value.
The assumption that the market is near perfect means that the only difference in the stock price that the special futures contract will capture is that company's net payoff from the joint venture (which would include the opportunity cost of the company and any spillovers that the company would capture). Therefore, all information regarding the initiation of the venture would be private to the firms involved and no inside information or other leaks are assumed to occur that would allow the market to anticipate the transaction and alter the value of the special futures contracts. The assumption that the accounting disclosure is near perfect ensures that the stock prices will accurately reflect the net value of participation in the joint venture; therefore, no information is hidden from stockholders.

The mechanism and assumptions described allow previously non-transferred payoffs of the venture to now be transferred. For example, now opportunity costs of participants can be transferred by having the other participants hold each others' special futures contracts.

When the payoffs of participation in the joint venture are transferrable or are made transferrable then the game is transformed into one where maximizing joint payoffs is in line with maximizing individual payoffs.

There are many different types of solutions (see Appendix Nine for details) to the SNTC JVPD when the special futures contracts are available, including a new Auction Solution.
3.5.1 The New Auction Solution

It is assumed that participating in the joint venture generates some costs or benefits which accrue to the individual firms above those accounted for in the net value of the joint venture itself. Those costs and benefits must be addressed as they are non-transferable (as is assumed in the cases of SNCTs). It is assumed that these individual costs and benefits can be made transferable by issuing special futures contracts as described above. The special futures contracts cover the full stock of each participant and are put into the auction pot (along with the shares of the ownership of the Joint Venture) to make up the total prize.

Thus, at the same time that the rights to ownership of the Joint Venture are placed in trust so are the special futures contracts. The rules (and costs) of the original Auction Solution apply. When \( \alpha \) set at 1/4, each firm will bid the true value, \( V \), of the new total auction pot.

With \( \alpha \) set at 1/4 in a \( \gamma \)-costly auction, and knowing that the outcome of bidding will be a tie if both firms have the same beliefs about \( V \), the different outcomes can be analyzed. Assume that in equilibrium each firm has true beliefs. Therefore, in equilibrium, each firm holds the same beliefs and those beliefs correspond to the outcome being analyzed. If the outcome is mutual cooperation then each firm receives \( c \) as its certain net payoff\(^{46}\) (having both bid \( 2c \) - the total value of the prize pot). If the outcome is mutual defection then each firm receives \( d \) as its certain net payoff. If the outcome is asymmetric (where one firm cooperated while the other defected) then each firm receives \( (w+s)/2 \) as its certain net payoff.

\(^{46}\) These payoffs are not net the cost of setting up the auction, \( \gamma/2 \).
As with the solutions presented before mutual cooperation is a Nash Equilibrium\(^{47}\), and the only one that survives forward induction, given certain conditions. If \((w+s)/2 > d\) then mutual cooperation is the only Nash Equilibrium. If \((w+s)/2 \leq d\) then mutual cooperation and mutual defection are both Nash equilibria. A forward induction argument based on the added cost of the Auction Solution can be used to eliminate the mutual defection equilibrium if the Auction Solution is entered into by both firms\(^{48}\). Therefore, the New Auction Solution can be used to achieve the highest joint net payoff from the Joint Venture if SNTCs and special futures contracts are present.

The solution considered thus far in this section (and in Appendix Nine) entails \(\gamma\)-transactions costs. It is a Pareto-improving application if the reward to each firm net the solution set-up cost, \(\gamma\), is greater than \(d\) (which has been assumed to hold).

In Appendix Ten, this solution is compared to the others (from Appendix Nine) and some extensions to these solutions are analyzed to show their robustness.

\(^{47}\) As in section 3.1 it is the case that neither firm would have any incentive to change either its bid or its PD strategy at this point.

\(^{48}\) If \(\alpha = 0\) then a forward induction argument, based on expected values versus certain values if players are risk-averse, can be used to eliminate the mutual defection equilibrium instead.
4. The Contracting Alternatives

However, it is difficult to judge the merits of these solutions without an analysis of the usual possible alternatives used to solve such dilemmas. This analysis is carried out in the following section.

There are other alternatives to the solutions presented above that may also provide an efficiency improvement to the JVPD but have different implementation requirements and costs. The Coin-Flip Solution (see Appendix Eight) may be unattractive due to the risks (of losing) involved. The Auction and Transference Solutions (see Appendix Nine) may appear to be quite complicated. However, all of these solutions hold one main advantage over the contracting alternatives in that they do not require any third-party verification of the PD strategies (i.e., C or D) played. Each firm enters into these solutions knowing that the JVPD outcome is what is at stake and that how it resulted is irrelevant to their payoffs. The payoffs to the firms are not changed by knowing who cooperated and who defected ex-post. Therefore, no third-party verification is desired nor required in any of these three solutions.

The contracting alternatives, however, do require the third-party verification. The payoffs to the firms are based on proving who cooperated and who defected ex-post. The alternatives that require this third-party verification are full contracting, and side payment arrangements. These alternative solutions can provide a Pareto-improvement to the JVPD if their implementation costs are low enough. Once again, if the payoff to each firm net implementation costs is greater than
d, then these solutions are attractive. However, the costs of these contracting solutions may differ from the costs of the Coin-Flip, Transference and Auction Solutions as third-party verification is now required.

Under contracting where, for example, the two firms would define ex-ante the arrangement to generate the mutual cooperation outcome, ex-post third-party verification of the strategies played is essential. This is because each party has an incentive to maximize its own payoff and no incentive to maximize the joint payoff. The contract may specify some penalty for breaking the terms sufficient to ensure, under perfect ex-post strategy verification by a third party, an adequate incentive to play the cooperative strategy. Thus, two third-party verifications are required (at a minimum) as each firm must be able to verify that its own play was correct to the other so as not to have a penalty imposed upon it. The efficient penalties involved would be simple transfers between affected parties. If both defected, there would be no transfer. If only one firm defected then there would be a transfer to the cooperating firm of a sum sufficient to offset its loss from a normal PD outcome (i.e., transfer = d - s) plus perhaps some additional amount to make it better off than the defecting party. In any case the contracting alternative requires some additional third-party verification and possible material expense and complication compared to the Coin Flip, Transference and Auction Solutions.

There are also two main side-payment arrangements to consider as alternative solutions. The first arrangement is one in which one firm offers some side payment, $\delta$, to the other firm if it plays C in the game. Consider that the offer is only "enforceable" on the offering firm: if the offeree
can verify that it played C, then the offerer’s payment is enforced. The offeree is not forced to play C; and if it does not play C, the offeree is not forced to pay δ. Compared to the contract, there are no penalty terms and only one third-party verification is required.

Now consider the side-payment requirements, the firms’ actions and the outcome. When the offerer initiates the scheme, it does so to maximize its payoff. Inspection of the normal form of the original game shows that this occurs when it plays D and the other plays C. Thus, the offerer pays the offeree δ to play C so it can play D and receive w-δ as its net reward. In order for it to be better off than in the original game this reward must be greater than d. For the offeree, it will only accept the offer if it does better than in the original game and if it does better than breaking the offer contract; in either case it plays C if its net payoff of δ+s is greater than d. These conditions require: w-d > δ > d-s or that w+s > 2d. This requirement is not a restriction of the PD game. Therefore, this solution may not be available in all JVPD situations. If it were, however, who would want to be the offerer and who the offeree would depend on the values of the two net rewards, w-δ and δ+s. The payoff to each would be equal when the offer is equal to δ=(w-s)/2 (which would be the case under a Nash Bargaining Solution between the parties). The payoff to each firm is then (w+s)/2 (and this is better than d when the restriction of w+s > 2d is met). Under this side-payment arrangement the total gross welfare generated is w+s which is less than the gross under the other Solution alternatives (e.g., the Auction Solution). However, this alternative is relatively simple but does require one third-party verification. If the restriction on payoffs is met and the costs to implement this solution lead to net payoffs to each firm greater than d then implementing this side-payment arrangement is
beneficial.

The second side-payment arrangement occurs with similar one-sided "enforceability" but also requires that payoffs be transferable. A further Pareto-improvement is possible in this scenario. Consider one firm offering $\delta$ to the other firm if that other firm plays C and transfers to the offerer its share of the JVPD net payoff. In order for the offerer to maximize its payoff in this scenario, it will also play C to maximize the total payoff to be split in this scenario. The result is that the offerer receives a payoff of $2c-\delta$ while the offeree receives $\delta$. The offerer will have an incentive to offer only if its payoff is better than in the original game - if $2c-\delta > d$. The offeree will only have an incentive to accept if its payoff is better than in the original game - if $\delta > d$ - and if it cannot do better by breaking the offer contract by defecting knowing the offerer will cooperate - if $\delta > w$. These requirements place a restriction on the offer that $2c-d > \delta > w$. This places a restriction on the PD payoffs that $2c > w+d$ which is not a requirement of the PD game definition. If, however, this restriction is met then by inspection it can be seen that the offeree does better than the offerer. Therefore, no one may want to offer and another Prisoners' Dilemma results. If, however, this arrangement is implemented then the total gross welfare generated is $2c$ which is as high as that generated by either the Transference, Coin-Flip or the Auction Solution. However, this arrangement, while somewhat straightforward, does require costs in the form of one third-party verification and some contracting. If it can be implemented and the net payoff to each firm is greater than $d$ then this arrangement is an attractive alternative to consider.
While these alternative solutions require some third-party verification, the Auction Solution does not\(^49\). The reason that this is important is that the cost of third-party verification may be restrictive. The cost of the act of third-party verification may also be a substantial as the judgement on whether a firm cooperated or defected may be based on a relative measure, relative to the firm itself. In these cases, then, where third-party verification is very costly relative to the other set-up costs (and relative to the payoffs generated from the joint venture), these contracting alternatives are inferior to the Auction Solution (original or New).

5. Conclusions

This essay has presented implementable solutions to a JVPD - the Auction Solution (original and New and those other Solutions found in Appendix Nine). Even when the payoffs are not completely transferable, an appropriately constructed Auction Solution may result in a Pareto-improvement to the Joint Venture under certain parameter ordering restrictions and the availability of certain futures markets. Therefore, policies that enable the Solutions to be implemented are encouraged when any of these Solutions is the best way to solve the dilemma.

The Auction Solution has many advantages. The auction mechanism is simple to understand, legal, and requires few resources (just a machine to hold bids, compare them, and then distribute the shares and bids). It allows the optimal bidder (the one with the highest valuation of the

---

\(^49\) Neither does the Transference or the Coin Flip Solution; see Appendix Nine for details.
venture) to obtain ownership. The Auction Solution may also be more acceptable under competition law. It does not have the same "overly-cooperative" appearance as contract-based scenarios (like the side-payment solution); after all, the auction is a competitive one. The Auction Solution also appears to have the most flexibility for obtaining the cooperative outcome to any joint venture that can be represented by a Prisoners' Dilemma, especially if it entails strategic non-transferable costs.

The solutions presented in this essay entail some strong assumptions to be workable. It is assumed that in most cases that there are no non-transferable costs involved in the JV, and that many JVs exhibit the form of a Prisoners' Dilemma. As well, it is assumed that it is relatively easy and inexpensive to implement the solutions.

Future work may include relaxing some of the assumptions. It may also include experimental trials of the solutions on subjects. Perhaps even empirical work may be attempted where buyouts of the JV by one of the partners will signal that an auction took place.

Even taking the assumptions as given, the model has presented some worthwhile solutions to possible dilemmas. The Auction Solution has even been shown to be a viable method that would enable some JVs to be undertaken that would not be otherwise (i.e., when $c > 0$ but $d < 0$).

There may be even further benefits to consider when the cooperative outcome to the JVPD is attained. Kamien et al (1992) study welfare effects of some Research Joint Venture types. They
find that cooperative Research Joint Ventures (cooperating in the R&D decisions) result in the highest social welfare compared to independent or competitive R&D ventures. The finding reveals that there may be additional economic benefits to obtaining the cooperative (C,C) outcome in R&D Joint Ventures above that specified by the participants in the venture itself. There may also be positive consumer surplus effects and other spillovers to consider as, for example, occurs under the original venture scenario when the venture output is allowed to be completed and offered to the market.
1. Introduction

This essay considers the problem of trade between two countries that share a very valuable open access resource. Addressing this problem may be increasingly important as the world approaches the limits of some key shared resource stocks. The problem is modelled both as a one-stage trading game where nations are price-takers and a two-stage trading game where nations assume market power. The one-stage game yields the familiar Ricardian outcome when the amount of the open access resource is not too low. The two-stage game yields a different outcome (i.e., division of welfare) and so can be considered as an alternative way of modelling the problem. It also offers a potentially attractive method of structuring trade for the nation who stands to benefit from the new division of welfare.

In the absence of market failures, such as those caused by externalities and imperfect competition, trade is welfare-improving for the participants. However, the gains in welfare generated by trade may be substantially decreased by losses resulting from the negative

---

1 The two-stage game may explain to some extent why nations who trade do not obtain the Ricardian result. It may also explain why trade may be unattractive when there are non-zero costs which are not compensated for by expected Ricardian gains from trade. The typical Ricardian result would entail at least one of the two nations specializing fully in the production of one of the intermediate goods. This type of trading is outlined in most economics texts such as Krugman and Obstfeld (1991).
externality arising from an open access resource. When the open access resource is scarce enough that its successful exploitation is measured by a nation's relative fitness to harvest that resource, trade may alter such relative fitness to the detriment of at least one nation. Similarly, when nations exercise market power in imperfectly competitive markets, the welfare gains from trade may be substantially decreased. When both sources of market failure are present, gains may be severely decreased.

The models presented in this essay differ from those in the existing literature in a number of ways. First, the models (of this essay) focus on trade in a unique type of intermediate goods. These intermediate goods are inputs to a production function that allows harvesting (and subsequent consumption) of a final good whose stock is internationally open access. Second, one of the models presented in this essay is a two-stage game where the production decision is temporally separated from the trade decision. This allows the nations to manipulate price rather than take it as given. Third, differing levels of the open access resource stock - from severely restricted to unlimited - are analyzed in the models. The levels markedly influence trade.

This essay explores trade in a world with one primary factor\(^2\), two intermediate goods, one final good, and two nations. The one primary factor, production time (labour), has only one valuable use - to produce the two (intermediate) goods. These two intermediate goods are the only inputs to a production function that generates a level of harvesting capacity of a stock of an internationally open access resource. The utility of each nation is a function of the amount of

\(^2\) The common fishing ground can also be considered a primary factor. However, it does not enter into the algebra of the model.
the open access (final) good it harvests. Analysis of the model reveals that when the stock of the open access (final good) resource falls below the combined autarkic harvesting capacity of the nations, no trade occurs. Analysis also reveals that when the stock of the open access (final good) resource is above the combined harvesting capacity of the nations were they to trade under the assumption of unlimited stocks, trade does occur and both nations increase welfare over autarky levels. Further analysis reveals that when the open access resource stock level lies between these two states, trade still occurs but with different divisions of the welfare gains over autarky depending on whether the one-stage or two-stage game is implemented.

The literatures on the economics of open access resources, on the economics of renewable resources, and on trade involving renewable (and non-renewable and exhaustible) and open access resources is all related to the present study. Brander and Taylor (1994) provide an excellent review of the literatures of the limited, renewable resource problem and related trade effects. Their paper analyzing interaction of trade and open access renewable resources presents a strong case against trade when the increased resource exploitation that results from trade causes renewable stocks to fall below a critical level for sustainability. The resource literature related to inefficiencies arising from open access resources starts with Gordon (1954). Kemp and Long (1984) review the related trade literature, focusing mainly on non-renewable resources. Somewhat recent literature on inefficiencies generated by open access resources include Bolle (1980), Khalatbari (1977), Sinn (1984), Reinganum and Stokey (1985), and Mason and Polasky (1994).

Brander and Taylor (1994) explain the important differences between resources that are common property (collectively owned) and open access (unrestricted access). Open access resources create market failures.
Markusen (1976) is the closest in subject to this essay. He considers both the market failures created by open access resources and by imperfect competition. He models the two usual externalities arising from open access resources: the intertemporal and the interdependent. The intertemporal externality arises because, in a multi-period analysis, the future stocks of the resource depend on the current stocks. This essay does not consider this intertemporal externality in the present analysis (although it may be considered in future work) in order to simplify the model and focus directly on the interdependence externality. The interdependence externality is simply the "commons" problem in which one nation decides upon a harvest rate without considering the effects on the other nations. Markusen’s model is a two primary factor, two final good, two nation world where one primary factor is the open access resource. The differences of Markusen’s model to the current essay lead him to the different conclusions that countries producing from an open access resource can influence each other both in terms of available future stocks and willingness to trade.

There is also some literature on resolving these externalities that may be of interest. For example, Samuelson and Messick (1986) found that experimental subjects will attempt to resolve inefficiencies generated by the commons externality (and probably the intertemporal one as well). Their subjects often elected a superordinate authority in order to divide the use of the commons more efficiently.

The intertemporal externality is simply assumed away in this paper. The paper only looks at the present time period (only one harvest stage) and ignores the future. Such an approach is reasonable under a number of alternative assumptions: very high discount rates, very high resource stocks, or some restriction on the harvest such as banning any harvesting of stocks under a certain size (e.g., as in fish, where the banned fish - the babies and propagating adolescents - replenish the stocks to the same level every period with the babies becoming the next adolescents and the adolescents the next harvested adults).

One main difference that makes Markusen’s conclusions different is that in his model (and in much other literature on this topic) competition for goods coming out of the open access stocks takes place before trade. This allows one nation to specialize in the resource intensive good, in general, thus relieving much of the competitive effects. In this essay’s model, competition is forced to be downstream of trade ensuring that the full effects of competition are considered in trade.
The literature on the economics of trade with imperfect competition (non-monopoly) starts with Brander and Spencer (1985). They find that nations which can, do attempt to shift economic rent to the domestic economy, for example, by subsidizing certain domestic industries. This individual maximizing behaviour by all nations results in investments made to redistribute rents rather than maximize the total rents available. The outcome is an inefficient equilibrium. The two-stage model of this essay yields a similar result when the open access resource stock is unlimited.

The remainder of this essay is divided into four main sections: The main assumptions, the model and the autarky case are outlined in the next section. That is followed by the analysis sections of the one-stage game and the two-stage game, and then by a summary and conclusions. In the analysis sections, four cases are examined altogether: the one-stage game without and with restrictions on the open access goods stock, and the two-stage game without and with restrictions on the open access goods stock. This ordering of these cases provides, respectively, the benchmark case (of Ricardian-like trade), the effects of the open access goods externality, the effects of imperfect competition, and the combined effects of both the open access resource externality and imperfect competition.

2. The Assumptions, the Model and the Autarky Case

In this section, the major assumptions are formalized in order to construct a model of the
situation. Production functions are presented (and utility functions are discussed) for optimization in the analysis sections that follow.

Consider two nations and a common resource. Nations A and B can produce fishing nets and fishing boats according to their own technological and labour endowments. The populations of both nations (which are assumed to be equal) consume only one final good - fish. Although the water boundaries may be owned by either nation, the stock of uncaught fish (and the fishing grounds) are a common (open access) resource. The "final" consumption good - the fish - is assumed to be produced through the use of the intermediate goods, nets and boats, on the common fishing ground.

Production time is the only primary factor in production of the intermediate goods. Time has no other uses of value. Similarly, the intermediate goods have no other value except in their use for harvesting fish. The wage here is determined by giving all workers equal proportions of the final good harvested (that is, if it is assumed that they all worked to their required productivity level in either intermediate good). This situation, as described, is basically a one primary factor, two intermediate goods, one final good, two nation world.

---

7 The caught fish are not common property. They are owned by whomever catches them.

8 Another example of the common resource of two nations would be an oil field that can be accessed from either nation. In this case, the shared resource would be non-renewable.

9 This may not be the most general description of the model. In the most general description, each nation has one time and two productivity endowments (where "time" may represent a bundle of necessary production inputs). The two productivities represent the manufacture of any two goods. Utility is represented by the consumption of the final good (fish in this case). When the stock of the final good is unlimited then the consumption can be represented by the production function given the two intermediate goods alone. When the stock is restricted then relative fitness matters and the relative harvesting capacity can be interpreted as a potential utility function (i.e.,
Each nation is endowed with \( H \) units (e.g., hours) of production time. Nation A can produce \( r_{1A} \) nets or \( r_{2A} \) boats per unit of production time. Nation B can produce \( r_{1B} \) nets or \( r_{2B} \) boats per unit of production time. It is assumed, without loss of generality, that Nation A has the comparative advantage in nets so when the nations trade, A gives up nets in exchange for boats from B.

The nations are fully rational and have complete information at each stage in the game.

The harvesting capacity of the final good is summarized by a production function based on the assumptions of the model outlined thus far. The most general tractable functional form is assumed for the production function. The function is the log-transformation of the Cobb-Douglas form. The intermediate goods are the inputs. These intermediate goods are combined with the open access resource (i.e., the common fish stocks) to produce the final good (i.e., caught fish).

\[ \text{it represents consumption if the other nation does not exist). Each nation ends up consuming the actual harvest level when the other nation does interact. Under these more general descriptions, this model can be considered in a less specific manner if need be.} \]
Define each nation’s “harvesting capacity” to be $Y_k$ $(K = A, B)$ where:

$$Y_A = a \ln[r_{1A} (H - x_A) - t] + (1 - a) \ln[r_{2A} x_A + p t]$$  \hspace{1cm} (3-1)$$

$$Y_B = a \ln[r_{1B} (H - x_B) + t] + (1 - a) \ln[r_{2B} x_B - p t]$$  \hspace{1cm} (3-2)$$

where:  
$t$ = quantity of good one exchanged  
$p$ = price of good two for good one  
$1 > a > 0$  
where $\frac{a}{1 - a}$ is the weighting measure of good one vs. good two  
where $a$ is the technology parameter reflecting the relative importance of input one into the production of fish  

$x_A$ = time Nation A spends making good two  
$x_B$ = time Nation B spends making good two  
$H$ = amount of production time available for each nation  
$r_{1A}$ = Nation A's productivity for good one  
$r_{2A}$ = Nation A's productivity for good two  
$r_{1B}$ = Nation B's productivity for good one  
$r_{2B}$ = Nation B's productivity for good two
Now define each nation’s fish production function (representing utility) to be $Z_K (K = A, B)$ where:

$$Z_A = \text{Min}\left[Y_A, \left(\frac{Y_A}{Y_A + Y_B}\right) S\right]$$

$$Z_B = \text{Min}\left[Y_B, \left(\frac{Y_B}{Y_A + Y_B}\right) S\right]$$

*where: $S = \text{available fish stocks.}$*

Thus, the harvesting capacity function specified above represents the amount of the final good a nation consumes when fish stocks are unlimited. When fish stocks are limited then a nation’s relative fitness matters (as measured by the ratio of their harvesting capacity to the combined harvesting capacity of both nations). As well, consumption of fish (utility) will also depend on the level of available fish stocks.

The functional form of the harvesting capacity contains only one primary factor, the production time, divided between work on intermediate goods one and two. These are the only two inputs to the production of the harvesting capacity. Each is necessary for harvesting capacity as long as a does not equal zero or one. This form exhibits decreasing returns to scale (DRS) in inputs when they are increased and the original harvesting capacity value is larger than $\text{Ln}[2]$ units$^{10}$. 

---

$^{10}$ Constant returns to scale (CRS) occur in inputs occur when they are increased and the original harvesting capacity is $\text{Ln}[2]$ units. The same results for both of these cases (i.e., DRS and CRS) occur in terms of the primary factor, $H$, when it is additionally assumed that when $H$ is increased $x_K (K = A, B)$ is increased proportionally (i.e., the proportional division of the time $H$ is unchanged when the increase occurs).
This is assumed to be the case in the rest of the essay.

The optimization of any reasonable utility function (one that is either CRS or DRS in this context to give non-increasing marginal utility) will correspond with that of the harvesting capacity function because the utility is a function of the harvesting capacity.

Again, the only time that the harvesting capacity does not equal the fish production is when the combined harvesting capacity of the nations exceeds the final goods stocks. Thus, two levels of final resource stock will be analyzed in this essay. The first level is labelled *unrestricted*. At this level, stocks are in excess of the combined equilibrium harvesting capacity of the nations when those nations are trading under the assumption of unlimited resource stocks. The second level is labelled *restricted*. At this level, stocks lie below the combined equilibrium harvesting capacity of the nations when those nations are trading under the assumption of unlimited resource stocks (i.e., free trade levels).
Thus, define the following levels of fish stocks:

\[
S_L = \text{the level of fish stocks depleted under autarky} \\
S_H = \text{the level of fish stocks depleted under free trade}
\]

where:

- \( S \geq S_H \) for unrestricted fish stock levels
- \( S_H > S \) for restricted fish stock levels
- \( S_H > S_L \)

For the upper range of restricted stocks, \( S \), such that \( S_H > S > S_L \) the nations still have an incentive to trade (to increase their harvesting capacity over the autarkic levels). However, the nations cannot reach the equilibrium trade level they enjoyed when stocks were unrestricted. The restricted stocks are divided up depending on a nation’s relative fitness, which is determined by its relative harvesting capacity as defined in the fish production function, \( Z_K \). The outcomes of the games when stocks are restricted differ from those when stocks are unrestricted. Games with restricted stocks have payoffs (final goods consumption measures) that directly include the harvesting capacity of the other nation(s).

Now consider the equilibrium of the model under autarky. Each nation will optimize its production division decision based on the no-trade assumption. The actual amount of available fish stocks is irrelevant because by optimizing its harvesting capacity, a nation also optimizes its relative fitness when stocks are restricted. Therefore, Nation A will optimize (3-1) with respect to \( x_A \), and Nation B will optimize (3-2) with respect to \( x_B \), taking \( p \) and \( t \) equal to zero. First (and second) order conditions for maxima give the following:
\[
\frac{\partial Y_A}{\partial x_A} = 0 \Rightarrow x_A^* = H (1 - a) \\
\Rightarrow Y_{A\text{autarky}} = a \ln[r_{1A} H a] + (1 - a) \ln[r_{2A} H (1 - a)]
\]

\[
\frac{\partial Y_B}{\partial x_B} = 0 \Rightarrow x_B^* = H (1 - a) \\
\Rightarrow Y_{B\text{autarky}} = a \ln[r_{1B} H a] + (1 - a) \ln[r_{2B} H (1 - a)]
\]

3. **Analysis of the Trade in a Ricardian World - The One Stage Game**

In this section, the one-stage game is defined and analyzed under different levels of the open access (final) good stock. Nations have no influence over price in a one-stage game and so a Ricardian outcome is expected where nations make production and trade decisions based on comparative advantage.

In the one-stage game the nations simultaneously choose their production division (A chooses \(x_A(p)\), B chooses \(x_B(p)\)) and terms of trade (A and B each offer their own \(t(p)\) function) in the one stage. In the trading equilibrium the price is where the \(t(p)\) function of Nation A intersects with that of Nation B. This is equivalent to a Walrasian determination of the equilibrium price.

**Case i. One Stage Game with Unrestricted Stocks**

This is the benchmark case. The nations trade only when there exists some comparative advantage between them - when \(r_{2A}/r_{1A} \neq r_{2B}/r_{1B}\). The equilibrium trading price will always fall
on or between these two production ratios. However, both nations will not be indifferent between trading and not trading only when the price of good two in terms of good one (i.e., the amount of good two bought for one unit of good one) falls between $r_{2A}/r_{1A}$ and $r_{2B}/r_{1B}$.

Two kinds of equilibria are possible for this case: 1) full specialization by both nations; and 2) full specialization by only one nation. This is the standard Ricardian result - that at least one nation fully specializes production in the trading equilibrium. Although this result is accepted and may be found in most economics (of trade) textbooks like Krugman and Obstfeld's (1991), it is proven for this particular model in Appendix Fourteen.

In order to find the equilibrium production divisions ($x_A$ and $x_B$) and terms of trade ($p$ and $t$), first optimize equations (3-1) and (3-2) for trade level, $t$. It has been assumed that each nation specializes its production where it has the comparative advantage. Nation A specializes in good one, Nation B in good two (as it was assumed that $r_{1A}/r_{2A} > r_{1B}/r_{2B}$ previously). Optimization reveals:

\[ \text{Nation A's optimal } t = \frac{1}{p} \left[ (1 - a) p r_{1A} (H - x_A) - a r_{2A} x_A \right] \quad (3-3) \]

\[ \text{Nation B's optimal } t = \frac{1}{p} \left[ - (1 - a) p r_{1B} (H - x_B) + a r_{2B} x_B \right] \quad (3-4) \]

In a competitive equilibrium, the market clearing price is such that the two desired trade levels are equal.
If it is assumed that both nations fully specialize then $x_A = 0$ and $x_B = H$. At these values, the other equilibrium values become:

$$p = \frac{a r_{2B}}{(1 - a) r_{1A}}, \quad t = (1 - a) \frac{r_{1A} H}{r_{1A} r_{2A}}$$

Since it is not acceptable to simply assume that both nations fully specialize, the equilibrium production division decisions of each nation are determined through optimization.

Equations (3-1) and (3-2) are used to optimize for values of $x_A$, $x_B$, and $t$ independently (i.e., Nation A optimizes its choice of $x_A$ and $t$ over equation (3-1) and Nation B optimizes its choice of $x_B$ and $t$ over equation (3-2)). From the previous analysis on equations (3-3) and (3-4), the price is determined from the market clearing condition (that the desired trade levels are equal in equilibrium). The optimization over the production decision for each nation give the following conditions:

$$x_A^* = \frac{(r_{1A} r_{2A} H - r_{2A} t) (1 - a) - a p r_{1A} t}{r_{1A} r_{2A}}$$  (3-5)

$$x_B^* = \frac{(r_{1B} r_{2B} H + r_{2B} t) (1 - a) + a p r_{1B} t}{r_{1B} r_{2B}}$$  (3-6)

At least one nation will not satisfy its marginal conditions for an interior solution of its production decision at the values assumed, instead attaining its corner solution (i.e., its full specialization level with its first derivative increasing in that direction and second order
conditions to verify that the level is near a maximum). For example, when the intermediate goods are equally weighted (when \( a = 1/2 \)) and each nation has an absolute advantage in one good then neither nation satisfies its marginal conditions for an interior solution but instead each nation hits its full specialization corner solution (i.e., \( x_a = 0 \) and \( x_b = H \)). Thus, the equilibrium values first assumed (i.e., full specialization by both nations) do correspond with the computed equilibrium under certain conditions.

When stocks are unrestricted (with the price, as determined endogenously by equilibrium conditions, falling between comparative advantage ratios), the nations will shift production to their area of comparative advantage and trade to an efficient level. The general result is that each nation increases its harvesting capacity over the autarky level, where harvesting capacity corresponds to consumption (given unlimited final good stocks):

\[
\text{Full Specialization Trade } Y_A = a \ln[r_{1A} a H] + (1 - a) \ln[r_{2B} a H]
\]

\[
\text{Full Specialization Trade } Y_B = a \ln[r_{1A} (1 - a) H] + (1 - a) \ln[r_{2B} (1 - a) H]
\]

\[
\text{Full Specialization Trade } p = \left(\frac{a}{1 - a}\right) \frac{r_{2B}}{r_{1A}}
\]

\[
\text{Full Specialization Trade } t = (1 - a) r_{1A} H
\]

where, for obtaining Full Specialization Trade the following is required:

\[
r_{1A} \geq \frac{a}{1 - a} r_{1B} \quad [fs1]
\]

\[
r_{2B} \geq \frac{1 - a}{a} r_{2A} \quad [fs2]
\]

where [fs1] together with [fs2] gives:

\[
\frac{r_{1A}}{r_{1B}} \geq \frac{a}{1 - a} \geq \frac{r_{2A}}{r_{2B}}
\]
Combining the two conditions for full specialization by both nations reveals that comparative advantage is not the only requirement. First order conditions for full specialization by both nations not only require that there be comparative advantage but also that where each nation has its comparative advantage that it must also have sufficiently high relative advantage as well (as the two conditions above - [fs1] and [fs2] - show\(^{11}\). The first condition must be satisfied for B to fully specialize when A does, the second for A to fully specialize when B does). Thus, if the goods are equally weighted and comparative advantage exists, it is still possible that full specialization by both nations will not occur in the trading equilibrium. This matches the standard Ricardian result where at least one nation will specialize if there is comparative advantage, but for both nations to fully specialize entails further requirements on relative productivities\(^{12}\).

The gains in harvesting capacity for each nation, as measured by the difference in capacity between when the two nations are trading under full specialization (by both nations) and when the nations are in autarky, is:

\[\text{Gains} = \text{Capacity}_{\text{full specialization}} - \text{Capacity}_{\text{autarky}}\]

---

\(^{11}\) The conditions required for full specialization by both nations in either the one-stage or two-stage game are formulated in the same way through first order conditions. In order for A to specialize fully given that B already is requires that: \(\frac{\partial Y_A}{\partial x_A} \leq 0\) at \(x_A=0\) and \(x_B=H\). Similarly, for B to specialize fully given A already is requires that: \(\frac{\partial Y_B}{\partial x_B} \geq 0\) at \(x_A=0\) and \(x_B=H\). The partial derivatives are evaluated using optimal \(p\) and \(t\) values as defined by (3-3), (3-4), or \(p^*, t^*\) from the two-stage game analysis that follows; and checking that second order conditions are satisfied for maxima.

\(^{12}\) See Appendix Fourteen for proof that it is the case that at least one nation fully specializes when comparative advantage exists (and both nations want to trade).
\[ \Delta Y_A = (1 - a) (\ln[r_{2b} a H] - \ln[r_{2a} (1 - a) H]) > 0 \]
\[ \Delta Y_B = a (\ln[r_{1a} (1 - a) H] - \ln[r_{1b} a H]) > 0 \]

under the conditions specified above

Re-arranging, the condition for the absolute gains from trade of nation B to be greater than those of nation A is:

\[ a \ln[r_{1a} H (1 - a)] + (1 - a) \ln[r_{2a} (1 - a) H] > a \ln[r_{1b} a H] + (1 - a) \ln[r_{2b} a H] \]

If \( a = 1/2 \), so that the goods are equally weighted, then the left hand side of the inequality corresponds to A’s autarkic harvesting capacity and the right hand side to B’s. The initially inferior nation (B in this case) would obtain a greater (relative and absolute) increase in harvesting capacity when trading occurs. This is not always the case, but the initially superior nation can obtain the greater gain only when the weighting (a) favours its comparative advantage.

One nation is indifferent to trade when conditions are such that only one nation fully specializes in equilibrium. In this case the nation that is not fully specializing obtains its autarky level of final goods whether it trades or not (assuming that side payments are ruled out), and so may choose not to trade. The equilibrium price in the case of only one nation fully specializing is the non-fully-specializing nation’s internal price of the intermediate goods (i.e., the slope of its production possibilities frontier). This makes that nation indifferent to costless international trade because it can achieve the same result internally. For example, A does not fully specialize under
the assumption that B does, when:

\[ r_{2B} = \left( \frac{1 - a}{a} \right) r_{2A} \kappa \quad \text{where } 1 > \kappa > 0 \]

Equilibrium for this condition, assuming that B does fully specialize at the equilibrium conditions, gives:

\[ x_A^* = H (1 - a) (1 - \kappa), \quad p = \frac{r_{2A}}{r_{1A}}, \quad t = (1 - a) H r_{1A} \kappa \]

\[ Y_A(x_A^*, p, t) = \text{Autarky } Y_A \]

Nation A only obtains its autarky level of fish under these conditions. Since such a nation would be indifferent between carrying out the trade that only benefits the other nation, it may well choose not to trade\(^{13}\). This result, again, corresponds with the standard Ricardian result of what occurs when only one nation fully specializes in equilibrium.

Only trade under bi-nation full specialization for the one-stage game will be considered in the analysis that follows (the other case of unilateral full specialization with trade is considered a special case and left for future work). When both nations fully specialize, both nations obtain gains from trade.

Comparative statics are used to determine the effects of certain parameter changes on bi-nation

\(^{13}\) However, this indifference can be broken in favour of trade by the nation who benefits more when the trade occurs. Side payments are not possible in a one-stage world; but, the enticing nation (which wants to trade) could soften its terms of trade offered in order to break the other nation’s indifference. Simply by asking for epsilon less in exchange, the indifference can be broken and both nations can be better off (with the enticing nation taking most of the gains from trade).
full specialization equilibrium harvesting capacity, $\partial Y$, and on gains from trade, $\partial \Delta Y$. The gains from trade are measured by the change in a nation’s harvesting capacity over autarky (where harvesting capacity equals consumption when final stocks are unrestricted). These comparative statics correspond with intuition in general:

<table>
<thead>
<tr>
<th>Partial Derivative</th>
<th>$\partial Y_A$</th>
<th>$\partial \Delta Y_A$</th>
<th>$\partial Y_B$</th>
<th>$\partial \Delta Y_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\partial r_{1A}$</td>
<td>+</td>
<td>0</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$\partial r_{2A}$</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\partial r_{1B}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>$\partial r_{2B}$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>$\partial a$</td>
<td>+:low $a$</td>
<td>+:low $a$</td>
<td>-:high $a$</td>
<td>+:low $a$</td>
</tr>
<tr>
<td></td>
<td>sign[$r_{1A} - r_{2B}$]</td>
<td>?:mid $a$</td>
<td>sign[$r_{1A} - r_{2B}$]</td>
<td>?:mid $a$</td>
</tr>
<tr>
<td></td>
<td>for other $a$</td>
<td>-:high $a$</td>
<td>for other $a$</td>
<td>-:high $a$</td>
</tr>
<tr>
<td></td>
<td>see (cs1)</td>
<td></td>
<td>see (cs2)</td>
<td></td>
</tr>
</tbody>
</table>

Table Two: Comparative Statics of the One-Stage Game with Unrestricted Stocks
\[
\frac{\partial \Delta Y_A}{\partial a} = \frac{1}{a} + \ln[r_{2A} (1 - a) H] - \ln[r_{2B} a H] \tag{cs1}
\]

\[
\frac{\partial \Delta Y_B}{\partial a} = -\left(-\frac{1}{1 - a}\right) + \ln[r_{1A} (1 - a) H] - \ln[r_{1B} a H] \tag{cs2}
\]

When a nation increases its productivity where it has a comparative advantage (i.e., A in good one making \( r_{1A} \) increase, or B in good two making \( r_{2B} \) increase) only the other nation obtains an increase in its gains from trade (although both increase harvesting capacity over the levels before the productivity increase) in general. The nation that experiences the increased productivity cannot specialize further if it is already fully specialized so it will not obtain any increase in harvesting capacity relative to autarky capacity. The other nation does obtain an increase in its gains from trade as it now gets more of the other good in exchange for its own, as price adjusts in its favour. This adjustment occurs because price is based on the relative abundance of the two goods. The adjustment does not favour the nation that experiences a productivity gain (where that nation is currently specializing) as the relative abundance of that good would increase making it worth less.

When a nation increases its productivity where it does not have a comparative advantage (i.e., A in good two making \( r_{2A} \) increase, or B in good one making \( r_{1B} \) increase) then only that nation has a decrease in its gains from trade (although neither nation changes harvesting capacity over the levels before the productivity increase) in general. The nation that experiences the increased
productivity does not change its specialization if the increase does not change the comparative advantage between the nations (no change is assumed). Price and production under trade is unaffected so the nation that does not experience the gain in productivity has no change to its gains from trade. However, the nation that does experience the gain in productivity increases its autarkic harvesting capacity and so decreases its gains from trade as a result.

When $a$ is at extreme values (high or low), there is not room for much further specialization by at least one nation as that nation moves from autarky production division to full specialization under trade. As a result, the gains from trade (when nations fully specialize under trade) decrease as the weighting factor (of production inputs not consumer tastes$^{14}$) moves toward more extreme values. Thus, decreasing $a$ further when it is already small decreases the gains from trade, as does increasing $a$ more when it is already large. When $a$ is in the intermediate values the other parameters determine whether either nation increases its gains from trade. The nation with the comparative advantage in the good that is weighted by $a$ always obtains an increase in autarky welfare (and the opposite for the other nation) when $a$ increases. However, both nations either increase or decrease their consumption level under trade depending on whether the specialization productivity in the good weighted by $a$, $r_{1a}$, is greater or less than that weighted by $(1 - a)$, $r_{2b}$. Regardless of whether there is an increase in welfare under trading (when both nations specialize fully), a nation’s gains from trade do not necessarily always increase as well.

---

$^{14}$ Changes in $a$ affecting welfare and gains from trade are worth noting. This weighting factor could change as technology changes the input requirements of the production function. If $a$ is considered this way, and not simply as a taste parameter, the results may be more meaningful.
Overall, the analysis of the one-stage game with unrestricted stocks matches the standard Ricardian results found in most economic (trade) texts, such as Krugman and Obstfeld’s (1991). Nations are price-takers and, in equilibrium, at least one fully specializes when nations trade to each one’s benefit.

Case ii. One Stage Game with Restricted Stocks

When the resource stock is restricted to a level at or below the combined autarkic harvesting capacity of the nations, $S_L$, then the nations do not trade at any price.

At these low stock levels, at least one nation will not want to trade because the game is zero-sum. Since, in this case, trade cannot increase the total size of the catch but can only alter the shares going to each country, there is no opportunity for mutual gains from trade. There is only one price (per trade amount considered) that leaves the relative harvesting capacity unchanged (from autarky levels) under trade. However, this is not the equilibrium price (based on either Ricardian or autarky production) in general because the conditions for a price that does not alter the relative gains from trade does not correspond with the conditions for an equilibrium price\(^{15}\) except in pathological cases. Thus, assume that the pathological case does not hold. Therefore, trade will not take place when the level of common stock is at or below $S_L$.

\(^{15}\) The equilibrium price is determined by the intersection of the optimized $t(p)$ offer curves of the two nations where optimization is through the first order condition. In the function assumed, this gives an equilibrium price which is determined without having to evaluate any natural logarithms. The price that maintains the relative harvesting capacity of the two nations when they trade is determined by the relative harvesting capacities. These involve evaluating natural logarithms. It is therefore reasonable to assume that the equilibrium price will not usually equal the relative-harvesting-capacity-maintained price.
When the common resource stock is above $S_L$ then an equilibrium with trading can exist. As in the case of unrestricted stocks, the nations trade only when the trading price of the intermediate goods lies between their comparative advantage ratios.

Equations (3-5) and (3-6) (or their corner solution counterparts) still have to be satisfied in order to give the optimal production division for each nation. However, now it is not possible to ensure that equations (3-3) and (3-4) are satisfied. Nevertheless, they will still want to trade as gains from trade are available to be split.

A new requirement defining the trading level is introduced when the fish stocks are restricted. In equilibrium there is assumed to be no excess harvesting capacity. Thus, in equilibrium, the combined harvesting capacity will equal available fish stocks. Nations will not trade to an excess of harvesting capacity because by doing so the relative fitness of at least one nation will decline. The nations also will not trade to a shortage of harvesting capacity because each would still stand to gain if they traded more. This equilibrium condition defines the terms of trade:

$$\text{Equilibrium } (t, p) \text{ is such that: } Y_A + Y_B = (\text{restricted fish stock level})$$

where: $Y_A$ defined by (3-1), $Y_B$ defined by (3-2)

Solving the system requires terms of trade, $(t, p)$, such that demand, as characterized by the total available harvesting capacity of the fish stocks, equals supply, as characterized by the available open access fish stocks:
\[ Y_A + Y_B = S \quad \text{at } (t^*, p^*) \]

where:

- \( x_A \) of \( Y_A \) = \( \text{Max}[0, x_A^* \text{ defined by (3-5)}] \)
- \( x_B \) of \( Y_B \) = \( \text{Min}[H, x_B^* \text{ defined by (3-6)}] \)

\[
p^* = \left( \frac{a}{1 - a} \right) \left( \frac{r_{2A} x_A + r_{2B} x_B}{r_{1A} (H - x_A) + r_{1B} (H - x_B)} \right)
\]

\( S_H > S > S_L \)

A simple numerical example may be helpful. Assume that \( r_{1A} = 2, r_{2A} = 1, r_{1B} = 1, r_{2B} = 3/2, \)
\( a = 1/2, H = 10, \) and \( S = 3.887 \) units. These values give the following equilibrium solution:

- \( x_A = 3.95, x_B = 6.22, p = 0.837, \) and \( t = 1.568. \) Nation A gains 0.051 units of harvest over its
autarky level while nation B gains 0.067 units over its autarky level.

Equilibrium specialization when fish stocks are restricted will never be greater than (and will usually be less than) the equilibrium specialization when fish stocks are unrestricted because the amount traded in equilibrium will always be less.

Also, in equilibrium with restricted stocks, neither (3-3) nor (3-4) is satisfied in general\(^{16}\). However, each nation does gain in harvesting capacity over its autarky state. The inferior nation in autarky, Nation B, may gain a larger relative and absolute amount in equilibrium (as it does in the numerical example above).

---

\(^{16}\) Strictly speaking an altered form of (3-3) and (3-4) would continue to hold. This altered form would recognize the discontinuity implied by the exhaustion of the available fish stocks (and the potential for further gains from trade).
When $S$ is restricted to a level between $S_L$ and $S_H$, then the equilibrium of the one-shot game involves: production specialization not greater than (and usually less than) free trade levels; trade to a level that is efficient for the resource stocks available; and an increase in harvest to each nation from autarky levels.

When resource stocks are restricted to a level at or below the combined autarkic harvesting capacity, then nations do not trade. Each nation divides its production to maximize its autarky capacity (i.e., its fitness level in this case). Fish stocks (and utility) are then distributed based on each nation’s fish production function (i.e., $Z_A$ and $Z_B$). The price taken is irrelevant because no trade occurs, and harvesting amounts decrease as the fish stock level decreases.

4. Analysis of the Trade with Production Precommitments - The Two Stage Game

In the two-stage game, the nations simultaneously choose their production division (A chooses $x_A$, B chooses $x_B$) in the first stage, and their terms of trade (each nation offers its own $t(p)$ function) in the second stage (see Figure Five for timing). The nations have the rationality to work backwards from the second stage to the first to optimize their production decisions knowing how that will affect the amount that can be traded and so the trading price (which is determined by the Walrasian auctioneer) as well. The method of determining the trading price

17 The sequential decision making of the two-stage game may reflect reality better than the one-stage game for the following reasons: There is less possibility of reaching a non-trading equilibrium in the two-stage game as nations would always attempt to trade after the first stage, if it were possible, regardless of their previous beliefs about trading. Also, the production processes of the nations may be sequential and not simultaneous.
is known beforehand. It is assumed to be the Walrasian (market-clearing) price based on the relative abundance of the two intermediate goods after production of these goods is complete\textsuperscript{18}. This allows the nations to affect price through their production decisions - their production pre-commitments (i.e., they do not take price as given as they did in the one-stage game).

---

\textbf{Production and Trade Time Line for the Two Stage Game:}

<table>
<thead>
<tr>
<th>Time</th>
<th>Intermediate goods production occurs.</th>
<th>Final goods production occurs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>Trade occurs.</td>
</tr>
<tr>
<td>1</td>
<td>Production plan set for each nation's intermediate goods.</td>
<td>Ex-post agreement occurs.</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>Final consumption occurs.</td>
</tr>
</tbody>
</table>

Figure Five: Time Line of the Two Stage Game

---

\textsuperscript{18} This Walrasian market clearing solution also corresponds to a Nash Bargaining Solution that uses threat points based on end-of-first-period states. Nations bargain over a harvesting capacity proxy in the solution. See Appendix Twelve for details.
Case iii. Two Stage Game with Unrestricted Stocks

In comparison to the one-stage game, the nations will always attempt to manipulate the trading price of the intermediate goods to their own advantage. This is possible in the two-stage game because the terms of trade allow influence through previous actions (those which occur in the previous stage). It is not surprising that this two-stage process, where nations alter their behaviour in an attempt to influence price, will have different equilibrium outcomes than the one-stage game where the nations were price-takers.

Now consider the case where the nations are in stage two, having made their production decisions, and are optimizing for their own trading levels. Nation A would optimize equation (3-1) for $t$ in terms of $p$ and the other parameters, and taking $x_A$ as given. Nation B would optimize equation (3-2) for $t$ in terms of $p$ and the other parameters, taking $x_B$ as given. The two $t(p)$ schedules would then be used by the market (a Walrasian auctioneer) to find the common $t$ (and the equilibrium $p$). Solving for the equilibrium $p$ this way gives:

$$p^{**} = \left(\frac{a}{1 - a}\right) \left(\frac{r_{2A} x_A + r_{2B} x_B}{r_{1A} (H - x_A) + r_{1B} (H - x_B)}\right)$$  \hspace{1cm} (3-7)

The equilibrium $p$ is simply the ratio of the amount of good two available to the amount of good one available (where good one and good two are the intermediate goods). The amounts are measured at the beginning of stage two and weight-adjusted to reflect the relative importance of the two goods. (For example, if good one were much more important than good two as expressed by the weighting - a high a value - then price would go up so that a nation exchanging
good one for good two would get a lot more good two in return.)

The equilibrium trade level, \( t \), can be defined in terms of the other variables by back-substituting for \( p \) in the optimal \( t \) equations formulated from either (3-1) or (3-2). The resulting equilibrium trade level is:

\[
\begin{align*}
\hat{t} = (1 - a) \left( \frac{r_{1A} r_{2B} x_B (H - x_A) - r_{1B} r_{2A} x_A (H - x_B)}{r_{2A} x_A + r_{2B} x_B} \right)
\end{align*}
\]  

(3-8)

The equilibrium \( t \) decreases as the importance of the good two (i.e., the good traded for by A) decreases. The importance of good two is measured both by its weighting factor, \( 1 - a \), and by the reciprocal of its relative abundance at the beginning of stage two (where the relative abundance is \( r_{2A} x_A + r_{2B} x_B \)).

With all stage-two equilibrium variables defined in terms of stage-one variables, nations can consider how to optimize their stage-one (i.e., production division) decisions. Both \( t \) and \( p \) can be substituted into (3-1) and (3-2) to eliminate all choice variables but \( x_A \) and \( x_B \). Each nation optimizes its production decision knowing the exchange amount and rate that will occur. The equilibrium corresponds to mutual best responses (satisfying each nation's optimal trade and production decisions)\(^{19}\).

\(^{19}\) It should be noted that the analyses of Cases i. and iii. do not require the open access resource assumptions. The harvesting capacity functions (3-1) and (3-2) can be considered as utility functions directly. The analyses of Cases i. and iii. give results for a one primary factor, two final good, two nation world where utility is a function of the two final traded goods directly.
Equation (3-1) is maximized over $x_A$ while (3-2) is maximized over $x_B$ (given that the $p^{**}$ and $t^{**}$ definitions have already been substituted into these equations). The maximizations result in a system of two equations (the first order conditions for (3-1) and (3-2)) in two unknowns ($x_A$ and $x_B$). The solution of the system is ($x_A^{**}$, $x_B^{**}$). The second order conditions at this solution are used to verify that the maximum has been found

Unfortunately, the system of equations does not have a simple, easily interpretable closed form solution. The system is a set of two "cubic" (i.e., fourth order) equations. One of the cube roots to each of the equations gives the correct equilibrium productivity decision for each nation. This seemingly simple system cannot be solved algebraically in general.

Therefore, a representative numerical example is focused on for the analysis that follows (i.e., the example represents a comprehensive testing of the parameter space using computer simulations).

---

20 The solution must also meet the additional requirement that the production division is feasible.
(i.e., that: $H \geq x_A^{**} \geq 0$, $H \geq x_B^{**} \geq 0$, and $|t| < r_{1A} x_A + r_{1B} x_B$).

21 These fourth order equations are of the form:

\[ c_1 x_A^3 + c_2 x_B^3 + c_3 x_A^2 x_B + c_4 x_A x_B^2 + c_5 x_A^3 + c_6 x_B^3 + c_7 x_A^2 x_B^2 + c_8 x_A x_B^2 + c_9 x_B^3 = 0 \]

\[ d_1 x_A^3 + d_2 x_B^3 + d_3 x_A^2 x_B + d_4 x_A x_B^2 + d_5 x_A^3 + d_6 x_B^3 + d_7 x_A^2 x_B^2 + d_8 x_A x_B^2 + d_9 x_B^3 = 0 \]

where $c_i$ and $d_i$ are determined by the productivity, weighting, and production time parameters.

22 The problem in solving the system analytically arises because there are non-real (i.e., imaginary) parts to deal with in cubic roots that are left in algebraic form. These imaginary parts do not easily reduce in general. Furthermore, solving the system itself translates to solving an equation of greater than fourth order, and no closed form algebraic solutions exist for such equations.
Consider again the following example:

\[
H = 10 \quad a = \frac{1}{2}
\]

\[
r_{1A} = 2 \quad r_{2A} = 1 \quad r_{1B} = 1 \quad r_{2B} = \frac{3}{2}
\]

The solution to the system of equations for this case is:

\[
x_{A}^{**} = \frac{5 \left(139189560 - 57995650 \sqrt{3}\right)}{53355998} \approx 3.6302
\]

\[
x_{B}^{**} = \frac{5 \left(-38 + 58 \sqrt{3}\right)}{46} = 6.7890
\]

\[
t^{**} = 4.2739 \quad p^{**} = 0.8660
\]

The second order conditions are satisfied at these values.

The resulting harvesting capacities at this equilibrium solution are larger than those under the autarkic conditions:

\[
\text{Autarky } Y_{A} = 1.956 \quad \text{Equilibrium } Y_{A} = 2.064
\]

\[
\text{Autarky } Y_{B} = 1.812 \quad \text{Equilibrium } Y_{B} = 1.941
\]

It can be seen that the equilibrium solution does not correspond to the most efficient division of production among nations. The nations do not do as well, generally, under this two-stage game as under the one-stage game where the Ricardian outcome results. When nations trade under conditions of imperfect competition, inefficiencies may arise as Brander and Spencer (1985) have shown.

When nations believe that they can alter price they will make (production) decisions to obtain
the best terms of trade for themselves. By attempting to maximize their own welfare in this way they may not maximize the joint welfare. When the equilibrium of individual maximization (of harvesting capacity) in the two-stage game does not correspond with that of the one-stage game, potential Pareto improvements are available. Movement towards these improvements can be considered to be a Prisoners' Dilemma. If both nations defect they remain at the two-stage game equilibrium. If both nations cooperate they can move towards the one-stage game equilibrium and both take welfare gains (i.e., gains in harvesting capacity).23

The Ricardian outcome and joint-maximization outcome can both result in much greater welfare gains from trade (as measured by the number of fish caught) in general (the Ricardian example is shown in Case i.).

The two-stage equilibrium can result in full specialization as in the one-stage game, but the conditions required are more restrictive than those of the one-stage game. First order equations are used to define the conditions, and second order equations to ensure that the full specialization equilibrium is the maximum corner solution. For nation A to fully specialize while B is fully specialized requires:

23 If one nation cooperates while the other defects then the cooperating nation loses harvesting capacity while the defecting nation gains. The cooperating nation loses because it specializes more, which benefits both nations, but is no longer at its best response point to the defecting nation, and by definition must experience a loss in welfare. The defecting nation gains although it is not at its best response to the new position to which the cooperating nation has moved. The defecting nation benefits through the increased specialization of the cooperating nation regardless because the increased specialization is a (strategic) complement to any level of specialization of the defecting nation. The result is a Prisoners' Dilemma where the dominant strategy is to remain at the two-stage game equilibrium point, although that is not jointly efficient.
and for nation B to fully specialize while A is fully specialized requires:

\[ r_{2B} \geq \left( \frac{1 - a^2}{a^2} \right) r_{2A} \]

These restrictions require that the nation producing a certain good needs to have a higher relative advantage in that good than under the one-stage game to allow that nation to fully specialize in that good in the trading equilibrium. The stronger conditions on the two-stage game over the one-stage game in this example of full specialization by both nations in the trading equilibrium implies an apparently general conclusion for any trading equilibrium: For a given set of parameters (productivities, weighting, and productivity time allotment), the trading equilibrium of the two-stage game will always involve less specialization than the one-stage game unless it is the case that both equilibria allow full specialization by the nations. This conclusion has been reached based on extensive numerical analysis of the two-stage game equilibria\(^{24}\).

When the Nash equilibrium does not align with the jointly optimal outcome, a policy-maker would be interested in finding how to change some conditions to improve efficiency. For

\(^{24}\) As well, a mathematical proof for a special case can be found in Appendix Fifteen to support this hypothesis.
example, consider the two-stage game if the first stage had a Stackelberg leader. An increase in harvesting capacity for both nations results with a Stackelberg leader when the equilibrium harvesting capacity of the one-stage game is larger than that of the non-Stackelberg two-stage game. Although both nations gain, the Stackelberg follower gains more than the leader, as measured from the simultaneous Nash equilibrium. This occurs because the increases in specialization are strategic complements (see Appendix Thirteen for details).

There are other ways to alter the equilibrium besides changing the rules of the game (e.g., by making one firm a Stackelberg leader). To explore these other ways, it is useful to know how changes in key parameters affect the outcome of the trade. Comparative statics provides some insight.

25 Modelling the game as Stackelberg may be appropriate in some cases. For example, if it is overlapping generations who are trading instead of nations, it may be the case that the older generation will have to lead.
Comparative statics align with intuition generally\textsuperscript{26}:

<table>
<thead>
<tr>
<th>Partial Derivative</th>
<th>$\partial Y_A$</th>
<th>$\partial \Delta Y_A$</th>
<th>$\partial Y_B$</th>
<th>$\partial \Delta Y_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\partial r_{1A}$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$\partial r_{2A}$</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\partial r_{1B}$</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$\partial r_{2B}$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$\partial a$</td>
<td>-:low $a$</td>
<td>+:low $a$</td>
<td>-:low $a$</td>
<td>+:low $a$</td>
</tr>
<tr>
<td></td>
<td>+:mid $a$</td>
<td>?:mid $a$</td>
<td>-:mid $a$</td>
<td>?:mid $a$</td>
</tr>
<tr>
<td></td>
<td>+:high $a$</td>
<td>-:high $a$</td>
<td>+:high $a$</td>
<td>-:high $a$</td>
</tr>
</tbody>
</table>

Table Three: Comparative Statics of the Two-Stage Game with Unrestricted Stocks

When Nation A increases its comparative advantage by increasing its productivity in good one (i.e., $r_{1A}$ increases) then both nations obtain a welfare-increase under trade as well as positive gains from trade. This coincides with the intuition that such an increase in efficiency allows both nations to shift production further towards where they add most value. The same effect occurs when Nation B increases its comparative advantage (by increasing productivity in good two.

\textsuperscript{26} The analysis that follows is based solely on numerical analysis, as analytical solutions were intractable. The computer simulations so far completed covered a wide range of parameter values, and so there is some confidence that the comparative statics shown are representative of the general case.
making \( r_{2B} \) increase). These productivity increases may benefit the nation experiencing the increase in productivity in the two-stage game where they do not in the one-stage game, in terms of gains from trade, because they allow both nations in the two-stage game to more fully specialize production (where the nations do not specialize further in the one-stage game as they are already assumed to be fully specialized).

When Nation A increases its productivity so as to close the comparative advantage gap with Nation B (i.e., \( r_{2A} \) increases) then it decreases its own gains from trade as well as those of Nation B. However, Nation A does increase its own welfare in general (i.e., unless it is already fully specializing) because it produces intermediate good two more efficiently. Nation A’s more efficient production where B adds value (i.e., in good two) increases A’s price elasticity for \( t \) (see Figure Six). All else being equal, higher prices for each trade amount increase A’s welfare and decrease B’s welfare, and consequently B’s gains from trade. The same effect occurs when Nation B closes the comparative advantage gap with A through B’s more efficient production of good one (i.e., when \( r_{1B} \) increases). The finding is in agreement with the intuition that closing the comparative advantage gap (the reason for trade here) will adversely affect at least one nation (holding all else constant). The finding runs against an intuition that any increase in productivity of one nation will benefit all nations in a trading world. The nation experiencing the productivity increase decreases its welfare gains (over autarky) for two reasons: it specializes

---

27 Consider the case where the nations are "e-close" in comparative advantage terms before the change in productivity. If nations trade then they experience a wealth greater than that in autarky. Now consider what occurs when one nation gets the productivity increase that eliminates the comparative advantage. Now the nation with the increased productivity will still have a wealth greater than that of the pre-change autarky. However, no trade will occur. The other nation will experience a decrease in wealth over the pre-change (trading) case as it then reverts to its autarky wealth.
less in response which decreases the other nation’s specialization (as specialization is a strategic complement in the two-stage game) resulting in less trade and less gains from trade over autarky; and the productivity gain increases its autarky welfare thus decreasing further any gains from trade. The two-stage game outcome differs from that of the one-stage game where the nation not experiencing such a productivity increase incurs no change in gains from trade. This is because it was assumed that nations do not change production decisions as a result of the increase whereas they do in the two-stage game.

The preceding analysis has an intuitive implication for government policy regarding where to direct R&D subsidies. Subsidies that increase the production efficiency in any area will increase a nation’s welfare (under trade or autarky) in a two-stage game model. However, only subsidies directed at increasing productivity where a nation enjoys comparative advantage will increase a nation’s gains from trade when nations trade under a two-stage game model. Furthermore, only subsidies in this area are non-detrimental to the other nation, and so will not initiate (welfare-decreasing) retaliatory action.

---

28 Analysis of comparative statics in the one-stage yields a similar recommendation - direct R&D funding where the nation has a comparative advantage in order to increase welfare. Investing in the productivity may also be better than investing in ways to make the final good harvesting capacity technology mix be more in a nation’s favour (i.e., Nation A investing so that a increases). The regions were such an investment is favourable is restricted by parameters that are outside a nation’s control.

29 It should be noted that these recommendations for R&D subsidy direction are opposite what they would be when stocks are restricted. When nations are in direct competition with each other on how to "divide the pie", each would want to increase its own relative fitness at the detriment of the other nation. To do so entails strengthening a nation’s autarky harvesting capacity and this translates into subsidizing R&D in the area of competitive disadvantage. However, a nation would not want to go so far as to decrease the size of the pie by altering fitness to the point where there are excess resources that go unharvested (i.e., when $SH > S > SL$).
The only parameter left to comment on is $a$. Although it is reasonable to expect that the productivities of the nations will change over time, it may be less reasonable to expect that the technology mix that creates harvesting capacity will change over time. If it does then the following analysis is of use. The changes in gains from trade with respect to changes in the weighting factor of the production inputs, $a$, correspond to those of the one-stage game in general. When $a$ is at extreme values (high or low), there is not room for much further specialization by at least one nation. As a result, the gains from trade when it shifts production further decrease as the weighting factor moves to more extreme values. Thus, decreasing $a$
further when it is already small decreases the gains from trade, as does increasing $a$ more when it is already large. However, at these extreme values, the harvesting capacity under trade increases for both nations as the values become more extreme. This is because the gains in capacity from the weighting that increasingly focuses on one intermediate good more than compensates for the losses in capacity from the decreased trade arising from decreased differences in production specialization amongst nations. When $a$ is in the intermediate values, the other parameters determine whether either nation increases its gains from trade, as is the case in the one-stage game. However, in general, the nation with the comparative advantage in the good that is weighted by $a$ obtains an increase in welfare (and the opposite for the other nation) when $a$ increases.

As it is noted that in many cases the trading equilibrium of the one-stage and the two-stage games differ, it is not surprising that the division of welfare gains and rate of change of welfare gains with respect to some key parameters also differs across the games. It may even be the case that one nation may obtain a greater harvesting capacity in the two-stage game than in the one-stage game although it is never the case that both nations would do so at once (see Appendix Sixteen for a proof). Therefore, without the possibility of side-payments in the one-stage game, the individual-welfare-improving strategic trade policy of one nation’s government is to attempt to implement the two-stage game construction instead of the one-stage construction.
Case iv. Two Stage Game with Restricted Stocks

Once again, the two-stage game outcomes can differ from the one-stage game outcomes when the nations trade with $S < S_{II}$. The difference is again due to each nation’s attempt to influence price without regard to its effects on the other nation’s welfare in the two-stage game.

The outcome of the two-stage game is the same as the outcome of the one-stage game when fish stocks, $S$, are restricted to a level (equal to or) below $S_L$. No trade takes place when only two nations are involved. If nations were to trade at least one of the nations would decrease its relative fitness, as relative fitness is affected by trade in general. Since a decrease in relative fitness translates into a decrease in welfare in this now zero-sum game, at least one nation will not have an incentive to trade when excess harvesting capacity already exists in the two nation world. As a result, no trade will occur\(^{30}\).

When $S > S_L$, nations trade and the individual influence of each nation begins to show. The results of these influences generate the differences in outcome between the one and two-stage games.

\(^{30}\) However, when more than two nations are involved coalitions may trade (in either the one or two-stage game). The case of coalition trades in the two-stage game when stocks are severely restricted provides an example. When relative fitness is a proxy for harvesting capacity, less fit nations may find mutual trade an effective means of increasing their own harvesting capacities at the expense of a more fit nation. The less fit nations form a trading coalition (or trading block) that excludes the more fit nation. The more fit nation is not be able to align itself with any other nation as its gains from trading in any coalition is not great enough to form a credible commitment to completing a trading agreement with any potential partner. The net result is that the less fit nations increase their harvesting capacities through trading amongst themselves to the detriment of the nations outside their coalition.
The equilibria that are analyzed below are considered (as before in the previous sections) to be Nash equilibria - they are best responses to best responses. Therefore, the nations obtain their highest welfare in these equilibria based on the responses of the other nation. As a result, all the equilibria studied below meet the same restrictions for best responses: \( x_b^e \leq x_b^{**} \), \( x_A^e \geq x_A^{**} \), \( S = Y_A + Y_B, \ t^e \leq t^{**}, \) and \( p^e = p^{**} \) (where superscript \( e \) are equilibrium values, and superscript \( ** \) are equilibrium values assuming \( S = S_B \)).

The analysis that follows is based on numerical (computer simulation) study only. The numerical example previously described in Case iii. will be used as to represent the outcome of several computer runs through many areas of the parameter space.

The amount of fish stocks, \( S \), analyzed next can vary between \( S_L \) and \( S_H \). In the numerical example, \( S_L = 3.768 \) and \( S_H = 4.005 \) units of fish. The space between these levels is divided into three regions for analysis:

1. Region One:

In this first region, nations have a choice of producing to autarky levels and then trading to a level less than or equal to \( t^{**} \), or producing to levels which satisfy trading at \( t^{**} \). The resulting price and division of fish will differ between the two choices in general because they are generated from two different sets of restrictions. This implies that one nation will be worse off by moving away from autarky production. Thus, it will choose to remain at autarky production.
This forces the other nation to remain at autarky production as well because a move to unilateral specialization decreases its elasticity and thus its welfare. The choice of production, therefore, is forced to be autarkic in this region.

Fish stocks in this region are bounded below by $S_L$ and above by the highest level of combined harvesting capacity that can be attained through trade given that the nations produced intermediate goods under autarky (i.e., they trade in stage two given they made production decisions based on autarky in stage one). This upper bound will be identified as $S_I$:

$$S_I = a \left( \frac{1}{r_{2A} + r_{2B}} \ln \left( \frac{a H R_1}{r_{2A} + r_{2B}} \right) + \frac{1}{r_{1A} + r_{1B}} \ln \left( \frac{a H R_2}{r_{1A} + r_{1B}} \right) \right)$$

$$+ (1 - a) \left( \frac{1}{r_{1A} + r_{1B}} \ln \left( \frac{(1 - a) H R_1}{r_{1A} + r_{1B}} \right) + \frac{1}{r_{1A} + r_{1B}} \ln \left( \frac{(1 - a) H R_2}{r_{1A} + r_{1B}} \right) \right)$$

where:

$$R_1 = r_{1A} r_{2A} + r_{1B} r_{2A} (1 - a) + a \left( r_{1A} r_{2B} + r_{1B} r_{2B} \right)$$

$$R_2 = r_{1B} r_{2B} + r_{1A} r_{2B} (1 - a) + a \left( r_{1A} r_{2A} + r_{1B} r_{2A} \right)$$

Since both nations are producing to autarky levels, the price is easily determined as:

$$p^t = \frac{r_{2A} + r_{2B}}{r_{1A} + r_{1B}}$$

where: $\frac{r_{2A}}{r_{1A}} < p^t < \frac{r_{2B}}{r_{1B}}$

which falls between the internal prices of the two nations.

The trade amount is determined by the fish stock level, $S$, given the known price and intermediate good levels. The trade amount is just enough to ensure that the post-trade combined
harvesting capacity just equals the available fish stocks. (More trade would be detrimental to one nation under the relative fitness function assumption. Less trade would leave obtainable extra fish stocks wasted.)

The highest amount that can be traded in this Region is restricted by the efficient trading level as defined by $t^*$. In the numerical example, this occurs when $t = 2$ at a price of $p = 5/6$. At this point Nation A’s fish harvest is 1.988, Nation B’s is 1.855, and combined fish production is 3.843 units. In this region, it is Nation B, the nation with the lower autarkic harvesting capacity, that forces the production to autarkic levels.

2. Region Two:

In this second region, the nations again have a choice on how to produce and trade. They can both specialize and trade to the efficient level for the fish stocks available or only one can specialize and let trade obtain the efficient level. The nation that can remain unspecialized does better under the latter choice. Thus, it chooses the production and trade structure in this region. The other nation has an incentive to specialize unilaterally nonetheless as it gains enough from its division of the increased fish stocks that are available when it specializes to more than offset

---

31 As in Case ii, the equilibrium is efficient in the sense that no resources are wasted (i.e., all the open access resource stock is used). This efficient equilibrium occurs in spite of the interdependence externality and related market failure generated by the existence of the open access resource stock. So, although there is a non-priced effect and social marginal benefits do not equal social marginal costs at the equilibrium, it is efficient (in an important sense) nonetheless.
its losses from its decreased elasticity arising from its unilateral specialization\textsuperscript{32}.

Only a nation with a higher autarky harvesting capacity can remain at its autarky production level in this region. Increasing its own specialization in this region results in losses from decreased elasticity that outweigh its gains from its division of any available fish stocks. The reason for this asymmetry between the non-identical nations can be found in the Region One price equation. In Region Two, where the nations are close to autarky productions of intermediate goods, the price is in the favour of the nation with the lower autarkic harvesting capacity. At this price, only the favoured nation can compensate for losses due to unilateral specialization through an increase in fish stocks whose division is a function of the trading price.

In this region, fish stocks are bounded below by $S_i$ and above by the maximum combined harvesting capacity attained when the nations trade but one nation remains at autarkic production. This upper bound will be identified as $S_j$. At $S_j$ price will be $p^*$, trade level will correspond both with $t^*$ and the efficient level to make $Y_A + Y_B = S_j$, and one nation's production decision will correspond to its optimum, $x_j = x_j^{**}$, while the other nation will remain at its autarkic production level. It is useful to separate this region from where both nations remain at autarkic production and from where neither nation does, as this region differs in equilibrium decisions made by the nations.

\textsuperscript{32} When nations have identical autarky harvesting capacities, this region disappears as neither nation is able to remain at its autarky level and have the other nation experience an incentive to unilaterally specialize.
In the numerical example, Nation B will increase its specialization in production by changing $x_B$ from 5 to any level up to 6.164 (which corresponds to $x_B^{**}$ at these conditions). At this point, which signifies the upper bound of Region Two, the combined harvesting capacity is 3.892 units.

3. Region Three:

In this third region, both nations specialize, but only one specializes to its optimal level, $x_f^{**}$. The nations will again have a choice again between different production and trade methods to obtain the efficient level of trade in this region so that combined harvesting capacity equals available fish stocks. As in the other regions, one nation will be able to "force" a choice that is better for it while the other nation will have an incentive for accepting the choice and fulfilling the trade requirements (while it obtains less welfare than under the other choice nonetheless).

In this region, as in the second region, it is the nation with the higher autarky harvesting capacity that forces the choice. Under this choice, the other nation produces to its optimal level, $x_f^{**}$, while the forcing nation remains at a specialization less than that called for by its $x_k^{**}$ level. This allows it a better elasticity (and consequently welfare) than under the other choice. The other nation has no incentive to deviate from its own (optimal) production level so there is no need to analyze the incentives of that nation under this equilibrium.

In this region, fish stocks are bounded below by $S_f$ and above by $S_h$. The equilibrium in this region is characterized by: price equalling $p^{**}$, trade equalling $t^{**}$ and satisfying the condition
that \( S = Y_A + Y_B \); and \( x_j = x_j^{**} \) for (at least) one nation.

In this Region both nations specialize (to some extent) in production where each has its comparative advantage. Just as in Region Two, in the numerical example, Nation A has the advantage.

Once the upper bound to Region Three has been reached neither nation has the incentive to specialize further. The gains from being able to obtain excess fish (if there were any) through further unilateral specialization are more than offset by the losses resulting from a decreased elasticity. (As was noted in Case iii., however, Stackelberg leadership does result in harvesting capacity gains.)

The analysis of the two-stage game under the three Regions of restricted stocks shows differences to the analysis of the one-stage game under similar restrictions. A notable difference is that the stronger nation (in autarky, Nation A in the example,) can choose the form of production and trade structure in most regions of the two-stage game. In effect, the nation with the higher autarky capacity can influence the division of open access resource stocks to its own advantage. It cannot do this in the one-stage game because nations do not act as to affect price as they are considered price-takers.

As a numerical example, consider the case when fish stocks are restricted to a level of 3.887 units, like in the example in Case ii. In the two-stage game, nation A gains 0.059 units of fish.
while B gains 0.060 units. In contrast to the one-stage game, nation A obtains a larger division of the gains from trade here\(^ {33} \). Therefore, when the stocks are restricted, the government of the nation with the greater autarky wealth may choose to intervene and attempt to control production and trade so as to structure trade between the two nations as a two-stage game\(^ {34} \).

5. Summary and Conclusions

This essay has analyzed trade when nations can exchange intermediate goods that allow them access to a final open access good. In a one-stage game when the open access resource stocks are not restricted, the nations are price-takers and a Ricardian world is modelled where very efficient (at or near jointly-optimal) trading results. When that resource is restricted to a level below that of combined autarkic harvesting capacity, no trade takes place (in a two nation world). When the resource level falls between these two points, trade occurs so that no resources are wasted. When trade occurs, both nations benefit (i.e., increase welfare over autarky).

---

\(^ {33} \) See Appendix Sixteen for a proposition and proof on this matter. It does show that it can be the case that one nation may find it attractive to implement the two-stage game rather than the one-stage game although this is jointly inefficient.

\(^ {34} \) When stocks are restricted the "size of the pie" is constant and the division of that pie is of concern. As mentioned, the nation that can influence terms of trade to obtain a bigger division will. This is of no concern, however, from a world point of view. What is of concern is when a nation does better under the two-stage game than under the one-stage game when the total stocks harvested are larger under that one-stage game. This scenario is possible - that one nation does better under the two-stage game but not both. In this case, the pie can be larger, so an international policy is required to obtain a Pareto-improvement. The policy consists of an agreement to partake in the one-stage game with a price mutually set that ensures that each nation does better than under the two-stage game. Under these circumstances, one nation effectively threatens the other into improved terms of trade for itself, and the world is better off regardless.
An alternative model is proposed where this production and trade game takes place over two stages rather than one. The second stage allows the possibility of influencing the trade terms and trade price. Nations try to influence the trade terms through their first-stage production decisions. When the open access resource is unrestricted, the nations' actions to influence the trade terms results in an outcome that is less efficient than the Ricardian outcome. When the resource is restricted to a level below the combined autarkic harvesting capacity, no trade takes place. When the resource level falls between these two points, trade occurs so that no resource is wasted. In this region, a nation can affect the form of the production and trade structure in order to gain a larger division of the excess resource stocks (than it would have in the one-stage game).

The analysis of these games has produced some interesting policy recommendations. Among them are: 1) R&D subsidies should be directed at industries where a nation has comparative advantage in order to achieve both an individual and joint welfare increase; and 2) a government should attempt to control production and trade so as to structure trade as a two-stage game (whether open access resource stocks are restricted or not) if this structure increases its nation's own welfare, regardless of the possibility of decreased joint welfare. Since there is no international enforcement against the latter policy, trade associations such as GATT may want to consider regulations against it (unless decreased "fish" production is actually welfare improving due to positive temporal effects of increased present stocks).
Future work may consider the temporal effects of the existence of the internationally open access resource more explicitly in the existing model framework. Other future work would relax some of the strong assumptions about alternative uses of factor endowments and intermediate goods, and the number of factors and nations involved. An analysis of representative firms within the nations is also possible in the future. Perhaps with the addition of these elements, a number of this model's conclusions could be tested empirically.
CHAPTER FOUR: OVERALL CONCLUSIONS

The three essays use standard economic tools to model some specific processes in the areas of entrepreneurship, joint venturing, and trade. The processes are then examined for inefficiencies. Once the inefficiencies are understood, policy recommendations are offered to increase efficiency where possible. In the first essay, a policy is offered that would preclude the solution of a Prisoners' Dilemma among incumbents through collusion when such collusion is to the detriment of consumers and new entrants. In the second essay, a method is presented to solve a Prisoners' Dilemma between partners in a joint venture when cooperative completion of that venture is beneficial both for the firms involved and for society as well. In the third essay, policies are offered that would eliminate inefficient arms-race-type investments in order to increase world welfare.

The policies that are generated by the three essays may be worth considering because they are based on different analyses than found in the existing literature. The first essay uses slightly altered classical economic assumptions, game theory, and an incumbent’s perspective in order to explain the emergence of entrepreneurship. Such an approach is somewhat novel to the field. The second essay provides solutions to the one-shot Prisoners’ Dilemma that are markedly different from any found in the existing literature. As well, the specific application to R&D joint ventures is unique. The third essay models the production and trade process in two periods instead of the standard one period found in most Ricardian trade literature. The essay also analyzes trade for a wide range of open access resource levels, unlike some trade literature. As
a result of the distinctive nature of the essays, their recommendations may provide new insight into important problems.

The essays, although somewhat unique in their fields, do share some common ground amongst themselves. First, they all involve some element of dynamics - they all model actions that take place over more than one period of time. In fact, elements of all the essays transform the standard static model of each problem into a more dynamic one. For example, the first essay adds a second period to the competitive environment and this allows the possibility of new opportunities to be available to entrepreneurs. Second, all three essays involve technology at some level. Essay one uses a technological advance to spur changes in the industry structure. Essay two includes technological complementarities to create the joint venture Prisoners’ Dilemma. Essay three assumes trade based on comparative advantage in production technologies. So, although the three essays cover disparate topics, they share some common modelling elements.

Now, consider the conclusions that are drawn from each essay separately.

The first essay, "Technological Force: the Emergence of Entrepreneurship", shows that, with slight changes in assumptions, classical economic theory does have room for entrepreneurship. Simply by adding a dynamic element to a classically-based model of competition and by assuming foreseeable innovations occur as time progresses, entrepreneurs do emerge in a substantial range of the model’s area of existence. Further, analysis reveals that the outcome
created by entrepreneurial emergence can be welfare improving.

It would appear that entrepreneurs emerge in the much of the relevant parameter-space of the games defined in this essay. However, policy action can affect entrepreneurial emergence in the Co-ordination (CG) and the Prisoners’ Dilemma (PD) games. In the case of the CG, some small subsidy can be given to one or both incumbents to ensure that both do not remain flexible and block entrepreneurial emergence, if that emergence is of net benefit to society. In the case of the PD, policies that discourage collusion (ensure competition) between the incumbents (and between all later firms) will help ensure the emergence of welfare-improving entrepreneurs. Many current competition policies only discourage collusion that has a material effect on industry competitiveness from a price perspective, while allowing some industry cooperation with respect to technology-sharing. This essay argues that competition policy should extend to the technology strategy regime as it does to the price (or quantity) fixing regime because collusion on technology decisions could be socially damaging.

This essay also answers an important question: Why, even when it might appear in the real world that incumbents could do "better" by focusing on future opportunities and blocking the entry of a lot of the entrepreneurs that will end up destroying them, some incumbents choose not to. The essay provides an alternative explanation to the incentive-difference-based patent-race and technology-adoption literature and to the organizational inertia and boundedly-rational incumbent literature. The essay's explanation is based on a model of rational strategic choice to explain the emergence of entrepreneurship.
The second essay, "An Auction Solution to the Joint Venture Prisoners' Dilemma", presents implementable solutions to a JVPD. Even when the payoffs are not completely transferable, an appropriately constructed Auction Solution may result in a Pareto-improvement to the Joint Venture under certain parameter ordering restrictions and the availability of certain futures markets. Therefore, policies that enable the Solutions to be implemented are encouraged when any of these Solutions is the best way to solve the dilemma.

The Auction Solution presented has many advantages. The auction mechanism is simple to understand, legal, and requires few resources (just a machine to hold bids, compare them, and then distribute the shares and bids). It allows the optimal bidder (the one with the highest valuation of the venture) to obtain ownership. The Auction Solution may also be more acceptable under competition law. It does not have the same "overly-cooperative" appearance as contract-based scenarios (like the side-payment solution); after all, the auction is a competitive one. The Auction Solution also appears to have the most flexibility for obtaining the cooperative outcome to any joint venture that can be represented by a Prisoners' Dilemma, especially if it entails strategic non-transferable costs.

The third essay, "Fish and Ships: Trade with Imperfect Competition and an International Open Access Resource", analyzes trade when nations can exchange intermediate goods that allow them access to a final open access good. In a one-stage game when the open access resource stocks are not restricted, the nations are price-takers and a Ricardian world results. When that resource is restricted to a level below that of combined autarkic harvesting capacity, no trade will take
place. When the resource level falls between these two points, trade occurs so that no resources are wasted. When trade occurs, both nations benefit (i.e., increase welfare over autarky).

An alternative model is proposed where this production and trade game takes place over two stages rather than one. The second stage allows the possibility of influencing the trade terms and trade price. Nations try to influence the trade terms through their first-stage production decisions. When the open access resource is unrestricted, the nations' actions to influence the trade terms in their favour results in an outcome that is less efficient than the Ricardian outcome. When the resource is restricted to a level below the combined autarkic harvesting capacity, no trade takes place. When the resource level falls between these two points, trade occurs so that no resource is wasted. In this region, a nation can affect the form of the production and trade structure in order to gain a larger division of the excess resource stocks.

The analysis of these games has produced some interesting policy recommendations. Among them are: 1) R&D subsidies should be directed at industries where a nation has comparative advantage in order to achieve both an individual and joint welfare increase; and 2) a government should attempt to control production and trade so as to structure trade as a two-stage game (whether open access resource stocks are restricted or not) if this structure increases its nation's own welfare, regardless of the possibility of decreased joint welfare. Since there is no international enforcement against the latter policy, trade associations such as GATT may want to consider regulations against it (unless decreased "fish" production is actually welfare improving due to positive temporal effects of increased present stocks).
These three essays offer some new and potentially valuable insights in their specific areas of economics. When analyzing a problem in any area, the ability to view (and model) an issue from different perspectives allows improved solutions to be generated (and implemented). It is hoped that these essays bolster that ability.
BIBLIOGRAPHY


Rochet, J-C. 1987. "Some Recent Results in Bargaining Theory". 


INSEAD working paper of September 14, 1994.


APPENDICES

Appendix Legend for Appendices 1 - 6:

\[ A = \text{intercept of inverse-demand function} \]

\[ B = \text{slope of inverse-demand function} \]

\[ c_r = \text{flexible variable cost in period one} \]

\[ c_l = \text{lock-in variable cost in period one} \]

\[ c_a = \text{lock-in variable cost in period two} \]

\[ f = \text{fixed costs per period for producing firms} \]

\[ A > c_r > c_l > c_a > 0 \]

\[ P = A - B Q : \text{the inverse-demand function} \]

\[ Q = \text{sum of all firms productions (q_i’s)} \]

\[ \pi_{FFL,LLL}^L \]

on profit functions: the superscript is the third-incumbent’s strategy attempted in the first period; subscripts are incumbent strategies attempted where the third-incumbent strategy is listed rightmost of the three per period (listed period one, period two); and F denotes flexible strategy while L denotes lock-in strategy
Appendix One: Payoff Requirements Under Game Types

Assume the game is of the form:

\[
\begin{array}{ccc}
\text{I1} & \text{F} & \text{L} \\
\text{I2} & & \\
\end{array}
\]

where: I1 indicates incumbent one, I2 indicates incumbent two, F denotes choosing to be flexible in period one, L denotes choosing to lock-in in period one, and payoffs are symmetric and denoted in each cell as I1’s payoff, I2’s payoff.

1. For the Prisoners’ Dilemma outcome:

\[ w > c > d > s. \] As well, \( d > 0 \) is assumed to allow incumbents to be profitable under the dominant strategy outcome. The ordering and the positive profit condition define a Prisoners’ Dilemma outcome where lock-in, L, is the dominant strategy played by the incumbents. As a result, new entrants do emerge in period two.

The conditions translate to the following:

\[
d > 0 \Rightarrow \frac{(A - c_f)^2}{9B} - f > 0
\]

\[
w > c \Rightarrow \frac{(A + c_f - 2c_f)^2}{9B} - f > \frac{(A - c_f)^2}{9B} - f + \frac{(A - c_n)^2}{B(n + 1)^2} - f
\]
\[ c > d = \frac{(A - c_l)^2}{9B} - f + \frac{(A - c_n)^2}{B(n + 1)^2} - f > \frac{(A - c_j)^2}{9B} - f \]

\[ d > s = \frac{(A - c_j)^2}{9B} - f > \frac{(A + c_l - 2c_j)^2}{9B} - f + \frac{(A - c_n)^2}{B(n + 1)^2} - f \]

Note: it is implicitly assumed above that no incumbent who chooses to lock-in in period one can profitably remain in the market in period two (see Proposition One of Chapter One).

2. For the Mixed Strategy outcome of the Co-ordination Game, the ordering \( w > c \) and \( s > d \) is required. As well, \( d > 0 \) is assumed to hold as before. See above for mathematics of restrictions.

3. The "Co-ordination Game" where players want to play \( L \) if other plays \( L \) and \( F \) if the other plays \( F \) is ruled out because if \( c > w \) then it is impossible for \( d > s \) (see Appendix Five for details).

4. A Pure Strategy Equilibrium outcome occurs when the dominant strategy corresponds to the individually efficient firm strategy. There are two possible:

1) when \( c > d \) and \( c > w \) then \( F \) is the dominant and individually efficient firm strategy. No new entrant with new technology (a technology not also implemented by an incumbent) emerges;
2) and when $d > c$ and $d > s$ then $L$ is the dominant and individually efficient firm strategy. It is ensured by Proposition One of Chapter One that entrepreneurs emerge in period two in this case.
Appendix Two: Numerical Examples of the Normal Form Game

Numerical Examples of the Normal Form Game:

Common assumptions for all following games:
Fixed costs (f = 3), Variable costs for flexible incumbent (cf = 4),
Variable costs for locked-in incumbent (ci = 3.5),
Variable costs for locked-in new entrant (cn = 1)
Inverse demand function (P = A - B*Q, where B = 1)

<table>
<thead>
<tr>
<th>Prioners' Dilemma Outcome</th>
<th>Mixed Strategy Outcome</th>
<th>Pure Strategy Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = 9.1</td>
<td>A = 9.3</td>
<td>A = 10</td>
</tr>
<tr>
<td>I2</td>
<td>I2</td>
<td>I2</td>
</tr>
<tr>
<td>flexible</td>
<td>flexible</td>
<td>flexible</td>
</tr>
<tr>
<td>lock-in</td>
<td>lock-in</td>
<td>lock-in</td>
</tr>
<tr>
<td>0.78</td>
<td>1.04</td>
<td>1.04</td>
</tr>
<tr>
<td>0.78</td>
<td>1.21</td>
<td>1.21</td>
</tr>
<tr>
<td>0.78</td>
<td>0.66</td>
<td>0.66</td>
</tr>
<tr>
<td>1.00</td>
<td>1.27</td>
<td>2.29</td>
</tr>
<tr>
<td>0.36</td>
<td>0.61</td>
<td>1.55</td>
</tr>
<tr>
<td>0.25</td>
<td>0.61</td>
<td>1.55</td>
</tr>
</tbody>
</table>

Note: read payoffs in cells as: Player One payoff
Player Two payoff

Figure Seven: Numerical Examples of the Normal Form Game
Appendix Three: First Period Entry Restrictions

1. Restrictions that ensure both incumbents are profitable when they both choose either flexible, F, or lock-in, L, technology strategies in period one are:

i) In the case of both incumbents locking-in in period one:

\[ \pi_{LLL}^L > 0 \Rightarrow \frac{(A - c)^2}{9 B} - f > 0 \]

ii) In the case of both incumbents being flexible in period one:

\[ \pi_{FF,FF}^F > 0 \Rightarrow \frac{(A - c)^2}{9 B} - f + \frac{(A - c_n)^2}{B (n + 1)^2} - f > 0 \]

where:

\[ n = \text{INTEGER}^*[\frac{(A - c_n)}{\sqrt{f B}} - 1] \geq 2 \]

where: \text{INTEGER}^* operator returns the largest integer composition of its contents if positive, otherwise it returns zero.

2. Restrictions that ensure no "third incumbent" in the first period are:

i) For the all incumbents locked-in case:

\[ \pi_{LLL,LLL}^L < 0 = \frac{(A - c)^2}{16 B} - f < 0 \]

ii) For the case where third incumbent is flexible, and the others lock-in:
For the case where the third incumbent and one other is flexible, while the remaining incumbent locks-in:

\[ \pi_{LFF,LLL}^F < 0 \Rightarrow \frac{(A + c_i - 2 c)^2}{16 B} - f + \frac{(A - c_n)^2}{B (n + 1)^2} - f < 0 \]

For the case where all incumbents are flexible in period one:

\[ \pi_{FFF,LLL}^F < 0 \Rightarrow \frac{(A - c_f)^2}{16 B} - f + \frac{(A - c_n)^2}{B (n + 1)^2} - f < 0 \]

For the case where third incumbent locks-in and the two other incumbents choose to be flexible:

\[ \pi_{FFL,LLL}^L < 0 \Rightarrow \frac{(A + 2 c_f - 3 c)^2}{16 B} - f < 0 \]

For the case where the third incumbent and one other incumbent locks-in while the remaining incumbent chooses to be flexible:

\[ \pi_{FLL,LLL}^L < 0 \Rightarrow \frac{(A + c_f - 2 c)^2}{16 B} - f < 0 \]

which is subsumed in v) above.

Note: it is implicitly assumed in 1. and 2. above that no incumbent who chooses to lock-in in period one can profitably remain in the market in period two (see Proposition One of Chapter One).
Appendix Four: Division of Parameter Space by Game-Type

Parameter Space Game Division
into PD, CG, FD or LD games

Note: Game-Type Legend:

PD = Prisoners' Dilemma normal form
CG = Co-ordination game normal form
FD = F-dominant Strategy normal form
LD = L-dominant Strategy normal form

Figure Eight: Effect of Changes in Elasticity on Parameter Space Division

Frequency (Percentage)

Type of Game 1=PD, 2=CG, 3=FD, 4=LD

B=0.5  B=3.0  B=5.5
Parameter Space Game Division
into PD, CG, FD or LD games

B=1, Cm=1; Cf, Cl, A varying.

Figure Nine: Effect of Changes in Fixed Costs on Parameter Space Division
Appendix Five: Accounting of Possible Games

There are four different payoffs possible in a two by two normal form game with two symmetric players, and they have been labelled w, c, d, s. Consider the non-pathological cases when inequalities between pairs of these payoffs occur (note: pathological cases occur when two or more of these payoffs are equal). There are 4-choose-2 cases to consider:

c > s, w > d, c > w, c > d, w > s, w > c, s > d, s > w, d > e, d > s, s > c, d > w

The last two of these inequalities cannot hold given the actual construction of the payoffs.

\[ c - s = \frac{(A - c)^2 - (A + c_i - 2 c_f)^2}{9 B} \]

Note that the contents of the two round brackets must both be positive. Examine which of these contents is larger:

\[(A - c_f) - (A + c_i - 2 c_f) = c_f - c_i > 0 \text{ as } c_f > c_i \]

Therefore \( c > s \) by construction of the payoffs.

A similar examination can be done for \( d > w \):

\[ w - d = \frac{(A + c_f - 2 c_p)^2 - (A - c_p)^2}{9 B} \]
This simplifies to:

\[
\frac{(c_f - c_i) (2A - 3c_i + c_f)}{9B} > 0 \text{ as } A > c_f > c_i
\]

Therefore \( w > d \) by construction of the payoffs.

Consider now all the possible ways of ordering the four payoffs. There are 4-factorial ways to do so:

<table>
<thead>
<tr>
<th>Ordering</th>
<th>Game Type</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w &gt; c &gt; d &gt; s )</td>
<td>Prisoners' Dilemma</td>
<td>1</td>
</tr>
<tr>
<td>( w &gt; c &gt; s &gt; d )</td>
<td>Co-ordination Game</td>
<td>2</td>
</tr>
<tr>
<td>( w &gt; d &gt; c &gt; s )</td>
<td>L Dominant</td>
<td></td>
</tr>
<tr>
<td>( w &gt; d &gt; s &gt; c )</td>
<td>ruled out as ( c &gt; s ) by construction</td>
<td></td>
</tr>
<tr>
<td>( w &gt; s &gt; c &gt; d )</td>
<td>ruled out as ( c &gt; s ) by construction</td>
<td></td>
</tr>
<tr>
<td>( w &gt; s &gt; d &gt; c )</td>
<td>ruled out as ( c &gt; s ) by construction</td>
<td></td>
</tr>
<tr>
<td>( c &gt; w &gt; d &gt; s )</td>
<td>Co-ordination Game (ruled out)</td>
<td>3</td>
</tr>
<tr>
<td>( c &gt; w &gt; s &gt; d )</td>
<td>F Dominant</td>
<td></td>
</tr>
<tr>
<td>( c &gt; d &gt; w &gt; s )</td>
<td>ruled out as ( w &gt; d ) by construction</td>
<td></td>
</tr>
<tr>
<td>( c &gt; d &gt; s &gt; w )</td>
<td>ruled out as ( w &gt; d ) by construction</td>
<td></td>
</tr>
<tr>
<td>( c &gt; s &gt; w &gt; d )</td>
<td>F Dominant</td>
<td></td>
</tr>
<tr>
<td>( c &gt; s &gt; d &gt; w )</td>
<td>ruled out as ( w &gt; d ) by construction</td>
<td></td>
</tr>
<tr>
<td>( d &gt; c &gt; w &gt; s )</td>
<td>ruled out as ( w &gt; d ) by construction</td>
<td></td>
</tr>
<tr>
<td>( d &gt; c &gt; s &gt; w )</td>
<td>ruled out as ( w &gt; d ) by construction</td>
<td></td>
</tr>
</tbody>
</table>
Formally, a Prisoners' Dilemma game may also include the restriction on payoffs that

\[ 2c > w + s \]

to ensure that the Pareto-optimal outcome for the players is to play \( F \) in each game and is not to take turns playing \( L \) on the other's playing \( F \). Players are eliminated in the next period after they play \( L \), therefore, they are not available in the next period to play \( F \) and return the exploitative favour. As well, only agreements that would not adversely affect market forces could be used to enforce such mutual exploitation. The ordering of \( w > c > d > s \) reveals that incumbents could increase their individual (and joint) welfare if they could agree to collude. The further restriction that \( 2c > w + s \) does not affect the other entry and welfare effects presented in this paper.
2. This Co-ordination game lies on the asymmetric diagonal. When its rival incumbent plays \( F \), the incumbent wants to play \( L \); when that rival incumbent plays \( L \), the incumbent wants to play \( F \). As \( w > s \) in this game, both incumbents want to play \( L \) to the other's play of \( F \). Hence, there is a co-ordination problem between the incumbents. This may be overcome through some form of collusion. This essay suggests that a policy that would help coordinate the outcome would be valuable. A policy that provides either (or possibly both\(^1\)) incumbents an incentive to play \( L \) would provide an increase in gross welfare.

\[ \Delta \pi_{LL-FF} = \left( \frac{2}{9} \right) \left[ (A - c_f)^2 - (A - c_l)^2 \right] > 0 \]

\[ \Delta \pi_{FL-FF} = \left( c_f - c_l \right) \left( 2A + 3c_f - 5c_l \right) \left( \frac{1}{9B} \right) > 0 \]

\[ \Delta CS_{LL-FF} = \left( \frac{2}{9} \right) \left[ (A - c_f)^2 - (A - c_l)^2 \right] > 0 \]

\[ \Delta CS_{FL-FF} = \left( c_f - c_l \right) \left( 2A - \frac{c_l}{2} - \frac{3c_f}{2} \right) \left( \frac{1}{9B} \right) > 0 \]

\(^1\) It is known that there is an increase in welfare when the \((F,F)\) outcome is displaced by either the \((L,L)\) outcome or the \((F,L)\) (or \((L,F)\)) outcomes:
3. This Co-ordination game is ruled out by construction of a set of payoff relationships.

When \( c > w \) it is impossible for \( d > s \):

\[
c > w \rightarrow \frac{(A - c_n)^2}{B(n + 1)^2} - f > \frac{(c_f - c_f) (4 A - 4 c_f)}{9 B}
\]

\[
d > s \rightarrow \frac{(c_f - c_f) (4 A - 4 c_f)}{9 B} > \frac{(A - c_n)^2}{B(n + 1)^2} - f
\]

In order to satisfy these two conditions then:

\[4 A - 4 c_f > 4 A - 4 c_i \rightarrow c_i > c_f \quad \text{BUT KNOW} \quad c_f > c_i\]

Therefore, this Co-ordination game is ruled out by construction of the payoffs.
Appendix Six: Existence of Prisoners' Dilemma

It can be proven that the possibility a Prisoners' Dilemma normal form will never occur under all parameter values is zero.

Given that:

(1) \( A > c_r > c_l > c_a > 0 \) : the parameter ordering in this model

(2) \( n \geq 2 \) in period one : non-negative profits for the two incumbents given the lowest symmetrical payoff [here, assuming the PD where \( c > d \), non-negative profits must be available to incumbents under the \((d,d)\) scenario]

The Prisoners' Dilemma normal form then requires:

(3) \( d > 0 \) : non-negative profits for incumbents under the Nash equilibrium outcome

(4) \( w > c \)

(5) \( c > d \)

(6) \( d > s \)

\[(3) \Rightarrow \frac{(A - c_l)}{9 B} > f \quad \text{while} \quad (2) \Rightarrow \frac{(A - c_a)}{\sqrt{B} f} - 1 \geq 2\]

The fixed cost factor can be defined in terms of other parameters to satisfy both conditions:

\[\text{set } f = \frac{(A - c_l)^2}{16 B} \Rightarrow (3) : 4 > 3 , (2) : 3 \geq 2\]

Now compare (4) to (6). It can be seen that both place a restriction of the same form on the payoffs. However, the restriction of (4) is subsumed in the restriction of (6). Therefore, with

160
f set, only two more inequalities need to be satisfied: (5) and (6). Consider (5) and (6). Satisfying both inequalities reduces to satisfying one of them and the condition that:

\[ 2A > 3c_f - c_i \text{ which is } > 0 \text{ as } c_f > c_i \]

The inverse demand function intercept, \( A \), can then be defined in terms of another factor in the game as: \( A = 4c_f \). This satisfies the condition above.

Now consider satisfying (6). Substituting in the definitions of \( A \) and \( f \), this reduces to:

\[
\left\{ \text{INTEGER} \right\} \left[ \frac{4(4c_f - c_n)}{(4c_f - c_i)} \right]^2 > \frac{(16)(9)(4c_f - c_n)^2}{(64)(c_f - c_i)(3c_f) + (9)(4c_f - c_i)^2}
\]

Define \( c_i \) in terms of another factor in the game as: \( c_i = \frac{c_f}{2} \). Substitute this definition into the preceding equation. Now, in order to satisfy all the conditions to obtain a Prisoners’ Dilemma normal form as described above, one final equation needs to be satisfied through one parameter-ratio:

\[
\text{INTEGER} \left[ 4.57 - 1.14 \frac{c_n}{c_f} \right] > 2.70 - 0.68 \frac{c_n}{c_f}
\]

Given that \( c_f > c_n \), the range of the ratio \( c_n/c_f \) can be from 0 to 1. The LHS of the equation then has the range 4 to 3 with a corresponding range to the RHS of 2.7 to 2.0. It can be seen that the equation is satisfied under all of the range without having to define the parameters further.

Therefore, given any \( c_f \) value, a large range of parameter values can be found that will satisfy the Prisoners’ Dilemma normal form. Thus, the Prisoners’ Dilemma normal form subset of the full parameter set is non-empty.
**Appendix Seven: Table of Solutions**

<table>
<thead>
<tr>
<th>Solution Type</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ex-Ante Auction</strong></td>
<td>Pareto-optimal outcome is possible. Possible to use when some payoffs are non-transferable. No third-party strategy verification is required. Ownership goes to the optimal firm when firms are asymmetric.</td>
<td>Requires payoffs be transferable to work; or some restrictions on transferable versus non-transferable payoffs (so may require special futures contracts).</td>
</tr>
<tr>
<td><strong>Coin-Flip</strong></td>
<td>Pareto-optimal outcome is possible. Relatively simple to implement. No third-party strategy verification required.</td>
<td>Requires payoffs be transferable to work; or payoffs are made transferable through special futures contracts. Results in expected value payoffs only. There is a loser who ends up with nothing.</td>
</tr>
<tr>
<td><strong>Full Contract</strong></td>
<td>Payoffs need not be transferable. Pareto-optimal outcome is possible. Relatively simple to implement.</td>
<td>Minimum of two third-party strategy verifications required. Need a legal system.</td>
</tr>
<tr>
<td><strong>Side Payment</strong></td>
<td>Payoffs need not be transferable. Pareto-improvement is possible. Relatively simple to implement.</td>
<td>Minimum of one third-party strategy verification required. Need a legal system. Entails restrictions on payoffs.</td>
</tr>
<tr>
<td><strong>Side Payment</strong> (no payoff transfers)</td>
<td>Pareto-optimal outcome is possible. Relatively simple to implement.</td>
<td>Minimum of one third-party strategy verification required. Need a legal system. Entails transferability and restrictions on payoffs. Each player has no incentive to initiate the solution.</td>
</tr>
<tr>
<td><strong>Side Payment</strong> (payoffs transferable)</td>
<td>Pareto-optimal outcome is possible. Relatively simple to implement.</td>
<td>Minimum of one third-party strategy verification required. Need a legal system. Entails transferability and restrictions on payoffs. Each player has no incentive to initiate the solution.</td>
</tr>
<tr>
<td><strong>Transference</strong></td>
<td>Pareto-optimal outcome is possible. Relatively simple to implement. No third-party strategy verification required.</td>
<td>It is a merger. Requires special futures contracts. The joint venture product is split up. Not possible to use when any payoffs are non-transferable.</td>
</tr>
</tbody>
</table>

Table Four: Table of Solutions to the JVPD
Appendix Eight:  The Coin Flip Solution

Although the Coin Flip Solution may not choose the optimal winner in the case of asymmetric firms, it is nonetheless an interesting alternative solution to consider. It is a solution which is very simple to implement but has some penalties if firms are at all risk-averse.

In order to analyze this Solution consider the basic application to two symmetric firms as described in the Auction Solution base application. Here, the two firms agree to partake in the solution, then they put their shares in the trust and have the trust flip the coin without revealing the winner at that time. The firms then play their venture strategies. The outcome of the venture is then realised. The outcome of the coin flip is revealed with all shares going to the winner and nothing going to the loser. With a fair coin it can be seen that each firm foresees an expected value of one-half the full value of the venture outcome. Each firm must consider that this reward is dependent on its venture strategy as well as on that of the other firm. Thus, each firm can see pursuit of the same goal - to maximize the venture outcome - as being in its own self-interest.

Consider the strategies that firms would play to maximize the value of the venture outcome (see Table Five for details\(^2\)). If one firm believed that the other would cooperate then it would also cooperate in order to maximize the venture outcome because \( c > (w+s)/2 \). Thus, mutual cooperation is a Nash Equilibrium in this scenario.

\(^2\) Net payoffs referred to in this Appendix are not net the Solution set-up cost, \( \gamma/2 \), unless specified.
Now consider what occurs when one firm believes that the other will defect. If the expected value of \((w+s)/2\) is greater than the expected value of \(d\) then the firm should cooperate to maximize the venture outcome. If payoffs are ordered as such (although this is not part of the definition of a PD) then mutual cooperation is the only Nash Equilibrium. If, however, the payoffs are \((w+s)/2 < d\) then a forward induction argument can be used to eliminate the second Nash Equilibrium of mutual defection given that a \(\gamma\)-costly Coin Flip solution was entered. Consider risk-neutral firms. If the firms do not enter into the Coin Flip Solution they guarantee themselves \(d\) as their actual net payoff whereas if they enter and defect then they receive either \(d-\gamma/2\) or \((w+s-\gamma)/2\) as their payoff, which are both less than \(d\) by assumption or definition. If the firms are at all risk-averse then a similar argument occurs because then the certain payoff by not entering the solution dominates the expected payoff (of \(d\)) of entering the Coin Flip solution and defecting.

Thus, the Coin Flip Solution can solve the JVPD in certain domains. Additionally, it involves less complication than the Auction Solution. But it has one major drawback besides being inefficient in the asymmetric firm case. It deals strictly with expected values. When firms are at all risk-averse (with respect to the payoff values\(^3\)) then these expected rewards are less than the certain rewards offered by the Auction Solution. If, in the extreme case, the firms are so risk-averse to make the expected value of \(c\) worth less than the certain value of \(d\) then the Coin Flip Solution may not be attractive at all (while the Auction Solution still would be).

\(^3\) Risk aversion here is defined in terms of the payoff values. Under the Coin Flip Solution, the firms risk the possibility of ending up with a zero or negative payoff (if they lose the coin flip). In comparison, under the Auction Solution each player ends up with the same payoff (whether in terms of money or ownership of the venture) with certainty.
<table>
<thead>
<tr>
<th>Item \ Scenario</th>
<th>Mutual Cooperation</th>
<th>Mutual Defection</th>
<th>Single Defection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Payoff if win the toss</td>
<td>2c</td>
<td>2d</td>
<td>w+s</td>
</tr>
<tr>
<td>Net Payoff if lose the toss</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Expected Net Payoff</td>
<td>c</td>
<td>d</td>
<td>(w+s)/2</td>
</tr>
</tbody>
</table>

Table Five: Payoffs under the Coin Flip Solution

There is one other solution available that is as seemingly simple as the Coin-Flip Solution but without the problems associated with the all-or-nothing outcome, and that is to have a trust divide all proceeds of the JV as they come in. For this to work, however, a number of conditions must be satisfied. A PD must exist without SNTCs that is different from the one described in the basic application in essay two. Unfortunately, this would entail the JV itself generating revenues. In the context of a R&D JV it is possible to generate licensing fees, for example, but then it is improbable that a PD exists because one JV partner would have to have the ability to take more than its fair share of those revenues under the original ownership structure. The case where such a trust could exist under the original application is tackled early in essay two, and the argument is made that any such institution which gives the partners ownership with certainty cannot work.

---

4 In the usual case a JV would create an output (i.e., a new manufacturing process) to be used by the partners in production of their goods. There is no method for one partner to take more than the other in this situation, so a PD does not exist and the trust is useless.
Appendix Nine: Alternative Solutions Under Special Futures Contracts

1. The Transference Solution

Assume the basic case described above under the SNTC section. Consider what occurs when each firm is given the special futures contracts of the other firm in exchange for its own. Specifically, consider the case where each firm is given contracts covering half the stock of the other firm at a transaction cost of $\gamma/2$. Each firm will now receive half of its individual net payoff plus half of the other firm’s. The game is transformed into one where mutual cooperation is a Nash Equilibrium. If both firms cooperate, each receives $c/2 + c/2 = c$. If both firms defect, each receives $d/2 + d/2 = d$. If one firm defects while the other cooperates, the defector receives $w/2 + s/2 = (w+s)/2$ and the cooperator receives $s/2 + w/2 = (w+s)/2$.

If a firm believes that the other will cooperate, its best reply is cooperation as $c > (w+s)/2$ by definition of the Prisoners’ Dilemma. Thus, mutual cooperation is now a Nash Equilibrium. If a firm believes that the other will defect, then its best reply will depend on the payoff from defecting versus cooperating. If $(w+s)/2 > d$ then cooperating is the best reply. In this case the only Nash Equilibrium of the game is mutual cooperation and the Prisoners’ Dilemma is solved. If, however, $d \geq (w+s)/2$ then defecting is the best reply (or an indifferent reply in the case of an equality). Mutual defection becomes a second Nash Equilibrium to the game. However, this Equilibrium can be eliminated by a forward induction argument. The payoff from mutual defection ($d > d-\gamma/2$) can be achieved by not entering the Transference Solution and just carrying out the original venture. By choosing to enter the solution, the only rational strategy to play is cooperation. The only Nash Equilibrium which remains after the forward induction
argument is made in either payoff ordering case is mutual cooperation. Thus, the Transference Solution solves the Prisoners' Dilemma.

This solution is just a simple merger limited to the venture itself. As it is known that a merger solves the PD it is no surprise that this solution technique does so as well.

2. The New Coin Flip Solution

Now, consider the next simplest form of the solution to the Joint Venture Prisoners' Dilemma (JVPD) under SNTCs: a γ-costly coin toss for ownership of the sum of each (of the two) participant's net values of the completed venture. The unbreakable agreement to perform the coin toss must be made before the venture is started, and the coin toss outcome revealed only after the venture is completed. Otherwise investment strategies may be misplayed if the agreement is made after the venture is started, or strategies may be changed if the outcome of the coin toss is known before the venture is completed. For example, if the loser knew it was the loser before the venture was finished it would choose to invest its least costly resources into the venture at that time.

SNTCs are made transferable by issuing special futures contracts as described in the Transference Solution. However, in this case the special futures contracts cover the full stock of each participant, and are put into the prize pot (along with the shares of the ownership of the Joint Venture) to make up the prize.
When the New Coin Flip Solution is implemented, the original game of certain payoffs is now transformed into one of expected payoffs. As before, in any outcome each firm will receive an expected payoff of half of the sum of the each participant's net payoff of the completed joint venture (opportunity costs and spillover benefits accounted for). Thus, if both firms cooperate, each receives an expected payoff of \( c - \gamma / 2 \). If one firm defects while the other cooperates, each firm receives an expected payoff of \( (w+s-\gamma)/2 \). If both firms defect, each firm receives an expected payoff of \( d - \gamma / 2 \).

As in the case of the Transference Solution, the best reply to cooperation is cooperation and to defection depends on the relative values of \( d \) to \( (w+s)/2 \). Thus, if \( (w+s)/2 > d \) then the only Nash Equilibrium is mutual cooperation. If \( d \geq (w+s)/2 \) then mutual cooperation and mutual defection are both Nash Equilibria. However, if the forward induction argument is again considered then the only Nash Equilibrium which results is that of mutual cooperation. Therefore, the New Coin Flip Solution results in an outcome that is jointly efficient.

Although the New Coin Flip Solution is viable, it is not very realistic. Risk averse stakeholders would probably not allow companies to partake in its implementation. Fortunately, there is an alternative which generates similar or further efficiency-improvements (in certain regimes when agents are risk averse). This alternative is the ex-ante auction, and it is somewhat more complicated to administer (see Section 3.5.1 of Chapter Two for details).
Appendix Ten: Analysis of SN TC Solutions and Some Solution Extensions

1. Comparison of SN TC Solutions

Why consider the New Auction Solution when the Transference and the New Coin-Flip Solutions are less complicated? The New Auction Solution can handle some cases of non-transferable costs whereas the other solutions cannot. The New Auction Solution has an advantage over the New Coin-Flip Solution because it deals with certain payoffs instead of expected payoffs; this is valuable when stakeholders are risk-averse. The New Auction Solution is more competitive than either a simple coin-flip or a simple exchange of special futures (because the exchange of ownership is based on a competition - an auction); this is of value when anti-trust considerations are important. The New Auction Solution is better than the Transference Solution when the output of the Joint Venture itself is difficult to split up or has less value when split up. The New Auction Solution does not split up the output whereas the Transference Solution does. There is one other important reason why the New Auction Solution may be a superior one but lies in an extension of the model to be explored next.

2. Extensions to the SN TC Solutions

Now consider some extensions to the model of SN TC. First, consider what occurs when the number of players increases from two upwards. Second, consider what occurs when some non-transferable costs enter the model.
2.1 Extension to More Than Two Firms

Just consider the three Solutions - Transference, New Coin-Flip and New Auction - in this analysis. To accommodate $n$ (where $n > 2$) participants in the Joint Venture under Transference simply allow each player to exchange $1/n$ of its own special futures for $1/n$ of each of the other’s. After the exchange each firm owns $1/n$ of each of the $n$ firm’s special futures (including its own). Then each firm becomes, in effect, an equal partner in the net payoff to the all participants in venture. Mutual cooperation may then be achieved as described before.

Under the New Coin-Flip Solution $n$ players are accommodated by using a fair $n$-outcome generator instead of one simple coin-toss. Again mutual cooperation may then be achieved as described before.

Under the New Auction Solution the $n$ participants must follow the same procedures as the two participants did before. The only change necessarily required is to use some fair $n$-outcome generator in case of a tie in the bids (which is the equilibrium case). The other change that may be desired is to alter $\alpha$ to allow bids to reflect the total value. To do this, it is necessary to have $\alpha = 1/n^2$. Mutual cooperation may then be achieved as described before.

This extension gives one more reason to use the New Auction Solution over the Transference Solution and the New Coin Flip Solution. If a firm’s own special futures have some control premium then the New Auction Solution is most efficient in re-issuing those contracts. The

---

5 Control premium here means that there exists some value over the face value to controlling the futures contracts to the firm issuing that stock itself. For example, these contracts may entail some persuasive valuable power inside the company, or some voting influence, or be of some importance to the board of the company as a signal.
auction winner can give each firm back its special futures as compensation (for example, consider that this is required under the rules of partaking in the Auction Solution). Under Transference firms can exchange their special futures to gain their own back. These ex-post exchanges differ in their number depending on whether the New Auction or the Transference Solution is used. Under the auction, only n - 1 deals need to be done (the winner deals with everyone else). Under Transference, \([n(n - 1)]/2\) deals need to be done (which is a greater number of deals when \(n > 2\)). Thus, if there is some cost to these ex-post transactions then the New Auction Solution is superior, all else being equal. Under the New Coin Flip Solution, of course, no contracts are returned to the losers, so their control premium is lost.

2.2 The Non-Transferable Costs Extension

The Transference, New Coin-Flip and New Auction Solutions are based on the JVPD payoffs being transferable through the special futures contracts. This requirement leads to an interesting result regarding the symmetry of the participating firms. When the JVPD payoffs are considered completely transferable the result is that the players have symmetry forced upon them. The players have the same payoffs, the same strategy space, and the same rationality. Therefore, they are symmetric.

Relaxing this symmetry requirement provides a practical extension to the models. Under the fully transferable case, if the net payoffs from participating in the joint venture initially differ among firms under similar outcomes then the three Solutions provide the same net payoffs to

\[\text{to the market.}\]
each firm in equilibrium regardless. Such a result may not seem fair to the firm who initially was gaining more than its rivals under any particular outcome. Although the treatment of asymmetrical players is left for future work, it appears to be possible to achieve fairer outcomes from the Solutions by altering the amount of special futures given by each participant so as to reflect the asymmetry. For example, a firm that was gaining more from the venture output than its partners could, through negotiating a "fair" agreement (i.e., where all parties receive payoffs only according to their own potential), only have to give up a fraction of special futures contracts that the other firms do. If firm A generated twice the value from the venture's output as B, a fair requirement may be to have A only give B half the special futures contracts that B gives A so each exchanges goods of the same total value. Leaving formal analysis of this symmetry consideration for future work, the extension to non-transferable payoffs is now explored.

When the JVPD net payoffs are not completely transferable some of the three Solutions may not apply. First, consider symmetric players with the same non-transferable payoffs\textsuperscript{6}. These payoffs will weigh on a firm's evaluation of its net payoffs under each scenario in the dilemma. It is assumed that the non-transferable payoffs are worse for cooperation than for defection.

Section 3.4 of Chapter Two provides the basis for how this case is handled. The non-transferable payoffs are simply SNTCs of that section. Only under certain parameter restrictions will a near Pareto-optimal solution be available. Only a redefined New Auction Solution will be applicable;

\textsuperscript{6} For example, if the principals involved in the participating firms achieve some personal level of satisfaction from cheating and some personal dissatisfaction when cheated on then these cannot be transferred even by the special futures contracts.
the other two solutions will not work in this case.

A further, more complicated extension than the one just explored could include non-symmetric opportunity costs (or benefits). It is hypothesized that there would be a significant range of these games whose outcome could be improved upon by the implementation of an appropriately defined Solution.
Appendix Eleven: Equilibria with Restricted Bidding

Firms can only bid either 2c, w+s, or 2d under restricted bidding. They can play mixed bids of this choice set. The following analysis reveals how each firm will choose its best mix and strategy.

Consider Firm A's best bid mix when it assumes Firm B will play C and bid 2c with x probability and w+s with (1 - x) probability. First, consider A cooperating and bidding 2c with probability y and w+s with probability (1 - y). The calculation simplifies to:

\[
\text{Max}_{y} \quad (y - x) \left( \frac{c}{2} - \frac{w + s}{4} \right) + c - i_c \rightarrow y = 1
\]

The optimal choice is to bid 2c with certainty, bidding consistent with strategy played and belief of the other firm’s strategy played.

Similarly, when A considers defecting with the same beliefs on B as above, A’s optimal bid will be consistent with its play: A will bid w+s.

The next step is then to compare the payoffs generated by each strategy and choose the dominant one. Assuming B plays C with certainty, and bids 2c with probability x and w+s with probability (1 - x) gives Firm A the following payoff comparison between cooperating and defecting (assuming it bids optimally):
$$\left(1 - x\right) \left(\frac{c}{2} - \frac{w + s}{4}\right) + c - i_c \ vs. \ \left(\frac{c}{2} - \frac{w + s}{4}\right)x + \frac{w + s}{2}$$

The required restriction on $i_c$ (see Section 3.4 of Chapter Two) can then be taken from this comparison to ensure that mutual cooperation is a Nash Equilibrium under the restricted bids auction.

Similar analysis can also be done when it is assumed that B defects to find the condition on $i_c$ (see Section 3.4 of Chapter Two) that makes mutual defection a Nash Equilibrium under the restricted bids auction.
Appendix Twelve: The Nash Bargaining Solution

There is another literature which is relevant to the models presented in this paper - the bargaining literature. Although the Walrasian market clearing solution is assumed to be the mechanism that determines the terms of trade (the price) in both the one and two-stage game, that solution corresponds to a bargaining solution. Whereas a Walrasian auctioneer may not appeal to some as a realistic mechanism, bargaining may be more widely accepted (and is analyzed here). Furthermore, the bargaining mechanism provides an alternative exposition of how different parameters and strategies can affect terms of trade, for example, through moving the threat points (i.e., bargaining positions) of the nations involved.

The relevant bargaining literature begins with Nash’s (1950 and 1953) work on the bargaining solution. Rochet (1987) provides a comprehensive review of initial and recent developments in the field since Nash. Osborne and Rubinstein’s (1990) text presents a good analysis of basic bargaining and extensions. Recent papers focusing specifically on international exchange include Chan (1988) and Rogoff (1990). Rogoff finds: 1) that when nations bargain over utility (including risk aversion effects) the nation with the higher threat point gains more from the trade; and 2) that nations will change investment patterns in order to enhance their bargaining positions. Shogren (1992) finds evidence against Rogoff’s first claim. Shogren’s experiments reveal that negotiated agreements usually split rewards evenly (85% of the time). This trade paper assumes a Walrasian market clearing solution which corresponds to the Nash Bargaining-type Solution outlined below.
Assume that the nations bargain over the division of the two intermediate goods. The product of the two intermediate goods stocks of a nation is used as the proxy for harvesting capacity for that nation. The negotiation process results in an equal gain in this harvesting-capacity-proxy. The gain is measured from a nation’s threat point (or harvesting-capacity-proxy evaluated at the end of stage one - when the intermediate goods have been produced).

The proxy is used rather than the actual harvesting capacity because the Nash Bargaining Solution only applies (in general) when the function representing the welfare-generating-goods being bargained over meets an invariance criterion (i.e., if the amount of welfare-generating-goods is linearly transformed then the new bargaining solution is the linear transformation of the old bargaining solution). The harvesting-capacity-proxy (i.e., the product of good one and good two) is such an invariant function. The Cobb-Douglas function (or the log of that Cobb-Douglas function) does not represent such an invariant function as non-linear transformations are involved (i.e., roots or logs).

Consider the following representation of the Nash Bargaining Solution to the negotiations over trading of harvesting-capacity-proxy elements:

\[
\text{Max}_{t_1, t_2} : [(g_{1A} - t_1)(g_{2A} + t_2) - g_{1A} g_{2A}] [(g_{1B} + t_1)(g_{2B} - t_2) - g_{1B} g_{2B}]
\]

where: \( g_{ij} = \text{Nation } J \text{'s product of good } i \text{ just before trade} \)

\( t_i = \text{the amount of good } i \text{ traded among nations} \)

If the only factor of welfare-generating-goods, \( H \), is linearly transformed then the bargaining solution \((t_1, t_2)\) defined by the maximization above is as well:
Consider \( H' = \alpha H + \beta \),
this can be written as \( H' = \gamma H \) instead without loss of generality.
The solution of the Maximization above is:

\[
\begin{align*}
t_1 &= \left( \frac{g_{1A} + g_{1B}}{g_{2A} + g_{2B}} \right) t_2, \\
t_2 &= \frac{g_{1A} g_{2B} - g_{2A} g_{1B}}{2 (g_{1A} + g_{1B})} \\
\end{align*}
\]
as \( g_U = H \) multiplied by some productivity-type constants it can be seen by inspection that if the transformation is done then

\( H' = \gamma H \Rightarrow g_{u}' = \gamma g_{U} \Rightarrow t_1' = \gamma t_i \)
so the invariance criterion is met.

However, by inspection, it can be seen that if the actual harvesting capacity function were used instead of the proxy then the invariance criterion would not be met. Thus, the proxy is used in determining the final terms of trade bargained to. The threat points are clearly defined as the pre-trade product of the two intermediate goods each nation has.

The harvesting-capacity-proxy assumed here needs to be adjusted when the importance of good one and good two differ (i.e., when \( a \neq 1/2 \)). A weight adjustment is factored into the amount of good two exchanged for good one when \( a \neq 1/2 \). The Nash Bargaining Solution (NBS) equation for the price negotiated, \( p \), becomes:
\[ p \text{ such that: } W_{A\text{after trade}} - W_{A\text{pre-trade but after stage one}} = W_{B\text{after trade}} - W_{B\text{pre-trade but after stage one}} \]

where: \( W \) is the harvesting-capacity-proxy (proxy for \( Y \))

\[
W_{A\text{after trade}} = (r_{1A} (H - x_A) - t) (r_{2A} x_A + p \frac{(1 - a)}{a} t)
\]

\[
W_{A\text{pre-trade but after stage one}} = (r_{1A} (H - x_A)) (r_{2A} x_A)
\]

\[
W_{B\text{after trade}} = (r_{1B} (H - x_B) + t) (r_{2B} x_B - p \frac{(1 - a)}{a} t)
\]

\[
W_{B\text{pre-trade but after stage one}} = (r_{1B} (H - x_B)) (r_{2B} x_B)
\]

with \( \frac{1 - a}{a} \) as the weighting adjustment when \( a \neq \frac{1}{2} \)

It is interesting to note that the resulting equilibrium from the negotiation process outlined above falls between two other Nash Bargaining-type Solutions: The first of which consists of the nations sharing equally in harvesting capacity gains (not proxy gains) from trade over the autarkic threat points. The second of which is based on same weight-adjusted intermediate good harvesting-capacity-proxies as outlined above. However, the nations share equally in harvesting-capacity-proxy gains measured from their autarkic threat points (i.e., not their threat points just before trade, that is, after stage one in the two stage game). On and between the two Nash Bargaining-type Solutions is where experimental bargaining outcomes have been found to lie. Thus, the equilibrium concept assumed above corresponds well with experimental evidence such as that presented in Roth, Malouf and Murnighan (1981).
Appendix Thirteen: Analysis of Case iii. Under Stackelberg Leadership

Consider the two-stage game if the first stage had a Stackelberg leader. The leader increases its own harvesting capacity by increasing production where it has a comparative advantage to a level over that dictated by the optimal (simultaneous) solution. For example, if Nation A unilaterally committed to an $x_A > x_A^*$ and Nation B produced a best response to this, Nation A would increase its harvesting capacity (as would Nation B).

When a nation does increase specialization as a Stackelberg leader, the follower Nation does so as well in a best response. The follower, however, obtains a larger increase in harvesting capacity than the leader. The follower profits more because it has the final opportunity to alter price and trade level through its own production decision based on that of the leader. The follower specializes less than if it were the leader, and so influences price to its advantage. Both nations gain when there is a Stackelberg leader, but the equilibrium joint harvesting capacity still may fall below that of the one-stage game regardless of which nation is the leader, as the following numerical example shows (based on the numerical example thus far assumed in the third essay):

---

7 This phenomenon only occurs for increases up to the efficient division of production for the world (e.g., Nation B has $x_B = H$).
When Nation A leads: $Y_A = 2.074$  
$\Delta Y_A$ versus autarky $= 0.118$  
$Y_B = 2.006$  
$\Delta Y_B$ versus autarky $= 0.194$

When Nation B leads: $Y_A = 2.113$  
$\Delta Y_A$ versus autarky $= 0.157$  
$Y_B = 1.950$  
$\Delta Y_B$ versus autarky $= 0.138$

Now consider the situation depicted in Figure Ten. The best response curve of nation 1 in the diagram, R1, shows the specialization decision that maximizes 1’s welfare for every specialization decision of nation 2 (and similarly for R2). The constant utility curve for each nation (shown as a broken line) peaks on that nation’s best response line (i.e., nation 1’s constant utility curve is horizontal when it touches nation 1’s best response curve). With respect to the two-stage game in this paper, more specialization translates to nation A decreasing $x_A$ towards 0 and nation B increasing $x_B$ towards H, and utility translates to harvesting capacity.

When there is no Stackelberg leader, the two-stage game equilibrium is where the two best response curves meet, at N. When nation 1 is the leader, the equilibrium shifts out to the point S1 where the constant utility curve of 1 is tangent to 2’s best response curve. At S1 nation 1 is more specialized than at N, as is nation 2. However, because of the (less than 45 degree) slope of 2’s best response curve, nation 2 does not increase specialization as much as 1. The converse is true at S2 when nation 2 is the Stackelberg leader.

Now consider the terms of trade as determined by the Nash Bargaining Solution to the harvesting-capacity-proxy as outlined in Appendix Twelve. When a nation increases its specialization, it decreases is proxy threat-point (or position to leverage the terms of trade in the
Walrasian case) before trading. Thus, when a nation is the Stackelberg leader, and so increases its specialization more than the follower, it can expect to reap less of the gains from trade than the follower as it has a weaker threat point (i.e., lower price elasticity of trade). Of course, both nations do benefit from the leadership as both specialize more, given specialization is a strategic complement.
Appendix Fourteen: Proof of Specialization in One-Stage Game with Unrestricted Stocks

Hypothesis: At least one nation fully specializes its production when nations trade in a one-stage game under the assumption of unrestricted open access (final) good stocks.

Proof: In order for Nation A to fully specialize given any production decision of Nation B requires that the following first order condition holds true (with the second order condition satisfied for a maximum):

\[
\frac{\partial Y_A}{\partial x_A} \bigg|_{x_B} \leq 0 \quad \Rightarrow \quad x_A^* \geq \frac{r_{2A} H (1 - a) (r_{1B} + r_{1A})}{r_{1A} r_{2B} a + r_{1B} r_{2A} (1 - a)}
\]

Similarly, in order for Nation B to fully specialize given any production decision of Nation A requires that the following first order condition holds true (with the second order condition satisfied for a maximum):

\[
\frac{\partial Y_B}{\partial x_B} \bigg|_{x_A} \geq 0 \quad \Rightarrow \quad x_B^* \leq \frac{r_{2B} H (r_{1A} (1 - a) - r_{1B} a)}{r_{1A} r_{2B} (1 - a) + r_{1B} r_{2A} a}
\]

If \( x_A^* \) as defined by (3-5) is greater than or equal to zero, then all that is needed to have B fully specialize is for the optimal production division response of A to the conditions assumed to be less than or equal to the production division required by the first order condition outlined above, or that: \( x_A^* - x_A^* \succeq 0 \). Recall that more specialization for A means a smaller production division, \( x_A \). Therefore, the
condition where the optimal response of A allows full specialization by B is:

\[
a H (1 - a) \left( r_{1A} r_{2B} - r_{2A} r_{1B} \right) \left[ r_{1A} r_{2A} + a \left( r_{1A} r_{2B} - r_{1B} r_{2A} \right) \right] \\
\geq 0
\]

\[
\frac{r_{1A} r_{2A}}{(r_{1A} r_{2A}) (r_{1A} r_{2B} (1 - a)) + a r_{1B} r_{2A}}
\]

(*)

This is true given that:

1 > a > 0, \hspace{1em} r_{u} > 0, \hspace{1em} H > 0, \hspace{1em} \frac{r_{1A}}{r_{2A}} > \frac{r_{1B}}{r_{2B}}.

Similarly, if \( x_{b}^{*} \) as defined by (3-6) is less than or equal to \( H \), then all that is needed to have A fully specialize is for the optimal production division response of B to the conditions assumed to be greater than or equal to the production division required by the first order condition outlined above, i.e., \( x_{b}^{*} - x_{b}^{*} \geq 0 \). Recall that more specialization for B means a larger production division, \( x_{b} \). Therefore, the condition where the optimal response of B allows full specialization by A is: exactly the same as (*) above.

Thus far it has been proven that if \( x_{A} > 0 \) in equilibrium then B fully specializes and if \( x_{B} < H \) in equilibrium then A fully specializes. The only conditions that violate either of these inequalities entail at least one nation fully specializing by definition, as required.
Appendix Fifteen: Proof that Specialization in the One-Stage Game is Greater than that in the Two-Stage Game

Hypothesis: Unless it is the case that both nations are fully specializing in the two-stage game, it will always be that the two-stage game involves less specialization than the one-stage game.

Proof: Consider the following representative case where Nation B is fully specialized but Nation A is not; A has \( x_A = e > 0 \). Thus, first (and second) order conditions for Nation A to maximize its welfare when B fully specializes must give \( x_A^* = e \):

\[
\frac{\partial Y_A}{\partial x_A} = 0
\]

In the one-stage game this condition translates into the following equality:

\[
r_{2B} = r_{2A} \left[ \frac{1 - a}{a} - \frac{e}{Ha} \right]
\]

In the two-stage game, the resulting equality is more complicated:

\[
r_{2B} = r_{2A} \left[ \frac{1 - a^2}{2a^2} - \frac{e(1 + a)}{2Ha^2} \right. \\
+ \frac{\sqrt{e^2(1 - a)(1 + 3a) + H^2(1 - a^2)^2 - 2eH(1 - a)[(1 + a) + a(1 - a)]}}{2Ha^2} \]

185
e can realistically range from 0, when A fully specializes, to \( H(1-a) \), when A decides for autarky production. Now consider a value for \( e \) that falls in between these two limits:

\[
e = \frac{H(1-a)}{k} \text{ where } k > 1.
\]

Substituting this identity into the equality for the one-stage game gives:

\[
r_{2B} = r_{2A} \left( \frac{k-1}{k} \right) \left( \frac{1-a}{a} \right)
\]

Substituting the identity into the equality for the two-stage game gives:

\[
r_{2B} = \frac{r_{2A}}{2 \ a^2} \left[ \frac{(1-a^2)(k-1)}{k} \right.
+ \left( \frac{1-a}{k} \right) \sqrt{(1-a)(1+3a) + k^2(1+a)^2 - 2k(1+2a-a^2)} \]

Now compare the multipliers on \( r_{2A} \) under each game. The multiplier of the two-stage game is larger than that of the one-stage game when:

\[
(1-a)(k-1) + \sqrt{(1-a)(1+3a) + k^2(1+a)^2 - 2k(1+2a-a^2)} > 0
\]

This holds true for all \( 1 > a > 0 \) and \( k > 1 \) as the first product is larger than zero by inspection and the terms in the square root bracket can be factored as follows:
\[(k - 1)^2 + a [2(k - 1)^2] + a^2 [(k + 3)(k - 1)] > 0.\]

Thus, for a given level of specialization, \(e\), for Nation A, the multiplier in the two-stage game is larger than that of the one-stage game. It can be seen that for this example, a larger multiplier means a smaller \(e\) means more specialization (i.e., consider the equality of the one-stage game and the definition of \(e\)). Therefore, less specialization means that the multiplier is less than that required by the equality condition (for the game in question) as defined above. Now if the same parameters are used in both games, the ratio of \(r_{2B}\) to \(r_{2A}\) must be fixed so the multiplier must be fixed. Consider a case where the multiplier satisfies the equality condition for the one-stage game. It is known that the multiplier will therefore be smaller than that required for the condition of the two-stage game. Therefore, it necessarily follows that less specialization occurs in the two-stage game than in the one-stage game (unless it is the case that the two-stage game involves full specialization by both nations).
Appendix Sixteen: Concluding Proposition of Chapter Three

Proposition: The two-stage game can result in greater welfare than the one-stage game for one nation but not for both nations.

Proof: There are a number of different aspects to consider in supporting this proposition. First, it must be shown that the two-stage game can result in greater welfare than the one-stage game for one nation. This must be shown for both the unrestricted and the restricted final good stocks case. However, because it has been shown that the two-stage game cannot be solved algebraically in general, only numerical examples will be provided as support for this part of the proposition. The examples shown will not be pathological ones, but simply representative of a sizeable set of examples.

Second, it must be shown that the combined welfare of the two nations under the two-stage game is never greater than that under the one-stage game. This is accomplished in two steps: 1) show that production specializations of nations are strategic complements; and, 2) show that it is never the case that there is more specialization in the two-stage game than in the one-stage game.
Example with unrestricted stocks:

Assume:  
\[ H = 10 \quad a = \frac{1}{3} \quad r_{1A} = 2 \quad r_{2A} = 1 \quad r_{1B} = 1 \quad r_{2B} = \frac{3}{2} \]

One-Stage Game Outcome:  
\[ x_A = 1.67 \quad x_B = 10 \]
\[ p = 0.50 \quad t = 10 \quad Y_A = 1.90 \quad Y_B = 2.30 \]

Two-Stage Game Outcome:  
\[ x_A = 5.74 \quad x_B = 8.43 \]
\[ p = 0.91 \quad t = 3.57 \quad Y_A = 2.00 \quad Y_B = 2.04 \]

Nation A's welfare under the two-stage game is better than under the one-stage game.

Example with restricted stocks:

Assume:  
\[ H = 10 \quad a = \frac{1}{2} \quad r_{1A} = 2 \quad r_{2A} = 1 \quad r_{1B} = 1 \quad r_{2B} = \frac{3}{2} \]
also assume fish stocks are restricted at 3.8867 units

One-Stage Game Outcome:  
\[ x_A = 3.95 \quad x_B = 6.22 \]
\[ p = 0.837 \quad t = 1.568 \quad Y_A = 2.0074 \quad Y_B = 1.8793 \]

Two-Stage Game Outcome:  
\[ x_A = 5.00 \quad x_B = 6.00 \]
\[ p = 1.000 \quad t = 2.500 \quad Y_A = 2.0149 \quad Y_B = 1.8718 \]

Nation A's welfare under the two-stage game is better than under the one-stage game.

The first part of the proposition has been proven. Also note that the examples are consistent with the second part of the proposition.

Before proving the difficult part of the second half of the proposition, the non-controversial cases can be eliminated. The two games will result in the same combined welfare for the two nations when either: i) the stocks are restricted to a level below \( S_N \) of the two-stage game; or ii) the nations are both fully
specializing in both games.

It is therefore left to prove that combined welfare of the two-stage game is less than that of the one-stage game whenever final good stocks are unrestricted in the two-stage game sense, but not in the one-stage game sense (in general).

Unless it is the case that in both games there is full specialization by both nations, it has been proven (in Appendix Fifteen) that more specialization occurs in the one-stage game than in the two-stage game. Therefore, if specializations are strategic complements, then the final good stocks level that separates restricted from unrestricted must be higher for the one-stage game than the two-stage game. If this is indeed the case, then the one-stage game must result in greater combined welfare than the two-stage game whenever final goods stocks are unrestricted in the two-stage game sense.

Thus, all that is left to prove the second half of the proposition is to show that the specializations are strategic complements (under both games).

For the one-stage game the following conditions hold:

\[
\frac{\partial x_A^*}{\partial x_B} = -\frac{r_{1B}}{r_{1A}} (1 - a) + \frac{r_{2B}}{r_{2A}} a < 0
\]

\[
\frac{\partial x_B^*}{\partial x_A} = -\frac{r_{1A}}{r_{1B}} (1 - a) + \frac{r_{2A}}{r_{2B}} a < 0
\]
The first condition shows that Nation A's best response is to specialize further (i.e., decrease $x_A$) whenever B increases its specialization; and the second condition shows that Nation B’s best response is also to specialize further when A increases specialization. Therefore, specializations are strategic complements (i.e., both nations increase welfare when specializations increase under best responses).

Proving that specializations are strategic complements in the two-stage game is a bit more complicated as no algebraic closed form solutions are available for best responses, in general. The proof is done indirectly as in Appendix Fifteen using a representative algebraic example.

First, generalizable but specific values are assumed for production specializations: $x_A = e > 0$ and $x_B = H z$ where $z < 1$. Now it is further assumed that $e$ is Nation A’s best response to B at this value of $x_B$. A condition on the parameters is generated when this is assumed:

$$\left. \frac{\partial Y_A}{\partial x_A} \right|_{x_A = e > 0} = 0 \rightarrow x_A^* = e > 0 \rightarrow r_{2B} = r_{2A} K$$

where: $K$ is a multiplier like those of Appendix Fifteen

Now consider the effect on the multiplier, $K$, as $z$ increases (i.e., $x_B$ increases), where $K$ is a function of $z$ and other parameters. From Appendix Fifteen, it is known that a smaller $e$ (i.e., increased specialization by A) requires a larger
multiplier. Some algebraic manipulation shows the following relationship between the multiplier, \( K \), and \( z \):

\[
\frac{\partial K}{\partial z} < 0
\]

Thus, as \( B \) specializes more, \( z \) increases and \( K \) decreases. When \( K \) decreases, it is easier to meet the condition for \( x_A^* = e \). For a given set of parameters, the multiplier, \( K \), is set by the parameters and not by the condition for \( x_A^* = e \). For a given set of parameters, as \( x_B \) increases, \( x_A^* \) decreases because the condition on \( K \) is decreasing and now a condition which was once too high (and involved a smaller \( x_A \)) can be satisfied by the multiplier.

Therefore, specializations are also strategic complements in the two-stage game. This completes the proof of the second half of the proposition.