Essays on Quality Competition

By

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Abstract

This thesis focuses on the economics of quality differentiation. It consists of three chapters. The first chapter examines, under both Bertrand price and Cournot quantity competitions, the optimal investment policies in a vertically differentiated industry. It shows that there exists asymmetry between the optimal policies of low-quality and high-quality exporting countries with respect to subsidizing or taxing investments in quality improvement of exports. Under Bertrand competition it is optimal for the low-quality country to subsidize investments that raise the quality of its exports, while the high-quality country has an incentive to tax investments in improving the quality of its exports. Under Cournot competition, the results are reversed.

The second chapter examines two widely used non-tariff barriers (NTBs), quotas and minimum quality standard (MQS) requirements. It investigates the effects of these two NTBs in a vertically differentiated industry consisting of one foreign and one domestic firm. It shows that a quota on low-quality imports improves the home country's welfare while a quota on high-quality imports reduces, in most cases, the home country's welfare. It also finds that the imposition of an MQS on low-quality imports lowers the home country's welfare.

The third chapter examines the welfare consequences of offering a voluntary quality certification program in addition to requiring products to meet some minimum quality standard (MQS) when the quality of a product is unobservable. It considers a model of
duopolistic competition in which price and qualities are decision variables. The chapter shows that providing an option of voluntary certification at a higher quality level instead of requiring all firms to meet and obtain certification at a particular designated level (MQS) is welfare increasing. Furthermore, it shows that in this situation the optimal MQS will be lower than the optimal MQS when there is only one mandated level of certification while the higher quality level will be higher than it.
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Chapter One

Quality Differentiation in a Strategic Trade Model with Fixed Costs

1.1 Introduction

Several developments have increased the role that non-price competition plays in international trade. The increasing economic importance of Japan with its demanding consumers and its quality oriented production culture highlighted the importance of product quality in trade. The emergence of lower quality but cost competitive producers among the newly industrialised countries (NICs), has motivated producers in the developed countries to respond by differentiating their products in terms of their quality and the quality of services provided to support these products. More recently, the rapid expansion of knowledge intensive products in diverse sectors such as pharmaceuticals and computer software has intensified further competition based on differentiated product qualities rather than based solely on price competition. Indeed, in these sectors where variable costs are relatively negligible, the major determinants of product demands are the particular features that differentiate a product from others that fulfill similar functions. High quality products can command high prices while lower quality products can meet demands of those who cannot afford the product given their budget constraints. Investments in research and development (R&D), appropriate infrastructure, facilities and machinery are the most important means through which firms can improve the quality of their products. Market failures have prompted many governments to subsidize or facilitate in other ways investments in R&D in general.
Some governments adopted industrial strategies that specifically targeted product quality improvements in exporting sectors. Japan pioneered the strategy in the early 60's transforming Japanese exports from an emphasis on cheap low-quality manufactured commodities to differentiated high quality products. With a lag this strategy was imitated by Taiwan and more recently by the Republic of Korea. Among the motives of government intervention in quality enhancement were the externalities associated with country of origin reputation. Quality of imports has also become a public policy issue in importing countries. Some developing countries such as Brazil developed complex policies which were meant to ensure, on the one hand, that imports of goods (especially intermediate goods) will incorporate attributes reflecting the advantages given by the state-of-arts knowledge and technologies, while on the other hand protect local consumers and producers from exploitation by foreign producers with market power.

The fact that many countries subsidize their exports cannot be explained under traditional trade theory. Brander and Spencer (1985) were the first to show that, in a Cournot duopoly setting, an export subsidy to a domestic firm is welfare-increasing. The main feature of their "Strategic Trade Theory" was the ability of a government to shift profits from a foreign producer to a domestic producer by credibly precommitting itself to subsidizing exports of the domestic producer before any decisions were made by it or its foreign rival. In their model export subsidies were optimal for each government even though they were jointly suboptimal. Ever since then, a great number of models have been developed extending the strategic trade theory (see e.g., Brander 1995, Neary 1994, Bagwell and Staiger 1994,  

---

1 If markets are fully competitive, export subsidies should be zero in a small open economy and should be negative if the economy has monopoly power in trade.
Qiu 1994, Gruenspecht 1988, De Meza 1986, and Eaton and Grossman 1986). Most of these papers examined the symmetric case and focused on the effects of various trade policies when prices or quantities were the decision variables.

Quality-related trade policy in an imperfect competition setting has received attention only recently (see Krishna 1987; Bond 1988; Das and Donnenfeld 1987, 1989 and Chang and Kim 1989). The majority of the papers on this subject have dealt with the effects of trade restrictions on the quality of imports, and relatively little attention has been given to the case of oligopolistic competition. Also, few of the models in the quality-related strategic trade policy literature considered the effects of government subsidies on the quality of exports and domestic welfare.

In this chapter, we consider an investment subsidy (tax) game in a vertically differentiated industry in which investment levels (fixed costs) fully determine the quality of a product. The emphasis on the fixed cost of quality is based on two observations. First, fixed costs have played a very important role in the formation of many imperfectly competitive markets. Indeed, fixed costs usually serve as natural barriers to later entries and lead to oligopolies. Secondly, the quality of a product depends, in many cases, on the initial investment (which is sunk and must be committed before any production). This is the case, for example, in some high-tech industries, such as the computer hardware and software industries and the pharmaceutical industry.

The key model structure we use here is the "third-country" model of Brander and Spencer (1985), in which two firms from two different countries export all their products to a third country. However, instead of assuming that firms produce identical goods, we assume
that products differ in the level of quality, and that consumers can observe the quality of each product without any cost. Also, instead of considering export subsidies, we consider investment subsidies. We assume that governments are able to act first. Governments set investment subsidy (or tax) levels using their understanding of how subsidies (or taxes) influence firms' decisions before firms make their choices. Under the structure of vertical differentiation, we try to determine the optimal investment policy for each exporting government if each can credibly precommit itself to implement it. We consider investment subsidies under Bertrand price competition as well as Cournot quantity competition.

The main results of this chapter are as follows. There exists asymmetry between the optimal investment policy of the high-quality exporter and the optimal investment policy of the low-quality exporter. Under Bertrand competition, the optimal investment policy for the low-quality country is to subsidize the investment of its firm, while that of the high-quality country is to tax the investment of its firm. From the points of view of the exporting countries, this noncooperative Nash subsidy/tax equilibrium is suboptimal, the joint welfare of the producing countries can be improved if either country reduces the subsidy (or tax) from its noncooperative Nash equilibrium level. So, government intervention is jointly undesirable in a Bertrand setting. Under Cournot competition, however, the optimal investment policies for these two countries are reversed. It is unilaterally optimal for the low-quality country to tax the investment of its firm and the high-quality country to subsidize the investment of its firm. From the points of view of the exporting countries, the joint welfare can be improved if either the low-quality country increases its tax or the high-quality country reduces its subsidy from their noncooperative Nash equilibrium levels. In other words, in a Cournot
setting, the intervention of the low-quality country is jointly desirable while the intervention of the high-quality country is not.

The quality reaction functions are also different under different types of competition. In a Bertrand setting, products are strategic complements in the quality space. In a Cournot setting, the low-quality firm regards the high-quality product as a strategic substitute in the quality space while the high-quality firm still regards the low-quality product as a strategic complement.

The rest of the chapter is organized as follows. In the next section, we present an overview of this chapter. In section 1.3, we characterize the basic model of vertical differentiation. The subsidy games under Bertrand competition and Cournot competition are investigated in section 1.4 and section 1.5, respectively. There are basically three sub-sections in each of these two main sections. In the first sub-section, we describe the market equilibrium under price (quantity) competition without any government intervention; In the second and the third sub-sections, we examine the optimal investment policy for the low-quality country and the high-quality country, respectively. Section 1.6 consists of conclusions.

1.2 Overview

We investigate duopolistic competition when quality and price (or quantity) are both decision variables, and limit our analysis to one-dimensional quality space. Assuming that an investment in fixed cost fully determines the quality of a product and the fixed cost is the only cost, provides us with the simplest possible model capable of bringing out the main
implications. The basic features of the model used here are standard in the studies of quality differentiation with monopolistic competition (e.g., Gabszewicz and Thisse 1979, Shaked and Sutton 1982, Champsaur and Rochet 1989, and Ronnen 1991).

The competition between firms takes place in two stages. In each stage both firms make their decisions simultaneously. In stage 1, the firms decide how much to invest in quality development. In stage 2, after observing the first stage decisions, both firms decide the prices (or quantities) of their products. The two-stage structure captures the idea that a firm can usually change the product price (or quantity) fairly quickly, while a change in product quality often takes a much longer time. Consumers are quality and price takers. They need only to choose which product to buy, or choose not to buy at all, depending on the level of net benefits (surplus level) they derive if they consume the products.

This basic two-stage game is subsequently extended in several ways. First, we analyze the case where the government of the low-quality supplier is allowed to make a prior commitment to subsidize (or tax) its firm's investment in quality. Then, we analyze the case where the government of the high-quality supplier is allowed to make a prior commitment to subsidize (or tax) its firm's investment. Finally, a noncooperative investment subsidy/tax Nash equilibrium is considered. In each case we examine the subgame perfect equilibrium in the corresponding three-stage game.

1.3 The Basic Model of Vertical Differentiation

In the model, there are three countries, two of which export a good to a third country. We assume, as most researchers in this area do (Bond 1988, Das and Donnenfeld 1987, 1989
and Tirole 1988), that consumers in the third country buy either zero or one unit of the product. Other things being equal, all consumers prefer a high-quality product to a low-quality product. Let the utility that a consumer gets from consuming one unit of a product with quality $q$ be $\theta q^2$, where $\theta$ is a taste parameter for quality and is assumed to be uniformly distributed across the population of consumers $[0, 1]$. The surplus that a consumer obtains from consuming a unit of a good with quality $q$ and price $p$ can thus be derived as $U - \theta q - p$, and we assume, without loss of generality, that consumers' reservation surplus level is 0.

There are two firms (one from country 1 and the other from country 2) that can supply this product to the third country market. To identify clearly the effect of quality, we assume further that both firms are identical in many respects, the only difference between them is the fact that firm 2 is more efficient in using its investment in quality\(^2\). To produce a product with quality $q$, firm 2 requires an investment of $F(q)$ while firm 1 requires an investment of $\gamma F(q)$, where $\gamma > 1$. Such difference in efficiency often reflects the general stage of economic development of a country. Thus we may regard country 2 as the developed country and country 1 as the developing country. We assume that, for any fixed cost function $F(q)$, $\gamma$ is so large that it is always optimal for firm 1 to produce the low-quality product. Let firm

\[^2\] The results obtained with this linear utility function can be generalized to any concave utility function $u(q)$, where $u'(q) > 0$, $u''(q) < 0$.

\[^3\] The analysis of firm asymmetries in the strategic subsidies model can also be found, among others, in De Meza (1986) and Neary (1994).
i produce a good of quality $q^i$, then we have $q^2 \geq q^1 \geq 0$. To focus on the effect of the fixed cost, we assume that the marginal cost is constant, and without loss of generality, let it be zero. The fixed cost is a strictly increasing and convex function of quality for all feasible quality $q^1, q^2 \in (0, q]$; where $q$ is a sufficiently large number so that in equilibrium it will never be reached, and $F(0) - F'(0) \neq 0$. To ensure the existence of Nash equilibrium, we further assume that $F''(q) \geq 0$ (see theorem 1 in Ronnen 1991).

It is obvious that, if both firms supply products with the same quality, then they must charge the same prices and earn zero profits (here we implicitly assume the capacity is large enough). This could be one of the equilibria. We ignore this trivial case and are only interested in the case where both firms choose to supply products with different levels of quality. Assume firm 2 charges price $p^2$ for its high-quality product while firm 1 charges price $p^1$. It is obvious that if firm 1 wishes to sell any of its product, its price must be less than the price of firm 2, that is $p^1 < p^2$.

The consumer will choose the low-quality product if his (or her) taste parameter satisfies the following conditions:

$$
\theta q^1 - p^1 \geq 0, \quad \theta q^1 - p^1 \geq \theta q^2 - p^2
$$

(1a), (1b)

That is, a consumer with a taste parameter $\theta$ obtains higher (non-negative) surplus by consuming the low-quality product. Similarly, a consumer will choose the high-quality product if:

---

4 Throughout the thesis, we use superscripts to denote firms and subscripts to denote derivatives.
Chapter 1. Quality Differentiation in a Strategic Trade Model with Fixed Costs

\[ \theta q^2 - p^2 \geq 0, \quad \theta q^1 - p^1 < \theta q^2 - p^2 \] 
\[ (2a), (2b) \]

Similar to the approach in Das and Donnenfeld (1989), let \( \theta \) define the consumer for whom (1a) holds as an equality and (1b) holds as a strict inequality, and \( \hat{\theta} \) define the consumer for whom (2b) holds as an equality and (2a) as a strict inequality. Then we have:

\[ \theta - \frac{p^1}{q^1}, \quad \hat{\theta} = \frac{p^2 - p^1}{q^2 - q^1} \]

We can easily show that any consumer with a taste parameter \( \theta > \hat{\theta} \) will purchase the high-quality product, any consumer with a taste parameter \( \theta < \theta < \hat{\theta} \) will purchase the low-quality product and consumers with taste parameters \( \theta < \hat{\theta} \) will not purchase any product.

In addition, define \( \bar{\theta} = \frac{p^2}{q^2}, \quad r = \frac{q^2}{q^1} \), where \( \bar{\theta} \) is the quality-adjusted price of the high-quality product and \( r \) is the ratio of the high- to the low-quality levels. Then we have

\[ \hat{\theta} = \frac{r \bar{\theta} - \bar{\theta}}{r - 1} \]

The demand functions for the low- and high-quality products can thus be derived as follows:

\[ D^1 = \hat{\theta} - \bar{\theta} \cdot \frac{p^2 - p^1}{q^2 - q^1} \]
\[ D^2 = 1 - \hat{\theta} - 1 \cdot \frac{p^2 - p^1}{q^2 - q^1} \]

(4)

Inverting the above demand functions, we have:

\[ p^1 = (1 - D^1 - D^2) q^1 \]
\[ p^2 = (1 - D^2) q^2 - D^1 q^1 - ((1 - D^2) r - D^1) q^1 \]

(5)
1.4 Bertrand Competition under Quality Differentiation

In this section, we assume firms first choose quality levels then prices to maximize their profits. In sub-section 1.4.1 we look at the market equilibrium under price and quality competition. In the later sub-sections, we assume that a government can credibly commit itself to a certain investment policy before firms make any decisions. We examine the optimal investment policy for the low-quality and the high-quality country in sub-sections 1.4.2 and 1.4.3, respectively.

1.4.1 Market Equilibrium

We proceed by backward induction. In the second-stage price equilibrium, the low-quality firm maximizes its revenue with respect to $p_1$, taking its quality and its rival's price and quality as given, while the high-quality firm maximizes its revenue with respect to $p_2$, taking its quality and its rival's price and quality as given. That is:

\[
\begin{align*}
\max_{p_1} R_1(p_1, p_2) &= \max_{\bar{\vartheta}} [R_1(\vartheta, \bar{\vartheta}) - q_1 \vartheta (\frac{r \bar{\vartheta} - \vartheta}{r - 1} - \vartheta)] \\
\max_{p_2} R_2(p_1, p_2) &= \max_{\bar{\vartheta}} [R_2(\vartheta, \bar{\vartheta}) - q_2 \bar{\vartheta} (1 - \frac{r \bar{\vartheta} - \vartheta}{r - 1})] 
\end{align*}
\]

The first-order conditions for the revenue maximization are:

\[
\begin{align*}
R_1(\vartheta, \bar{\vartheta}) - \frac{\partial R_1}{\partial \vartheta} - \frac{r \bar{\vartheta}}{r - 1} - \frac{2 \vartheta}{r - 1} - 2 \bar{\vartheta} - 0 = 0 \\
R_2(\vartheta, \bar{\vartheta}) - \frac{\partial R_2}{\partial \bar{\vartheta}} - 1 \cdot \frac{\vartheta}{r - 1} - \frac{2 r \bar{\vartheta}}{r - 1} - 0 = 0
\end{align*}
\]
Solving these equations, we have the following results:

\[
\begin{align*}
\phi &= \frac{r - 1}{4r - 1} \\
\bar{\phi} &= 2\phi - \frac{2(r - 1)}{4r - 1} \\
\hat{\phi} &= \frac{2r - 1}{4r - 1}
\end{align*}
\]  

(8)

Checking the second-order conditions, we have:

\[
\begin{align*}
R_{11}(\phi, \bar{\phi}) &= -\frac{2r}{r - 1} < 0, & R_{22}(\phi, \bar{\phi}) &= -\frac{2r}{r - 1} < 0 \\
R_{12}(\phi, \bar{\phi}) &= \frac{r}{r - 1} > 0, & R_{21}(\phi, \bar{\phi}) &= \frac{1}{r - 1} > 0
\end{align*}
\]  

(9)

Condition (9) means that a firm's marginal revenue of price is an increasing function of its rival's price. In other words, these two products are strategic complements in the price space.

In equilibrium, the demands for the low- and high-quality products are:

\[
\begin{align*}
D^1 &= \hat{\phi} - \phi = \frac{r}{4r - 1} \\
D^2 &= 1 - \hat{\phi} = \frac{2r}{4r - 1} - 2D^1
\end{align*}
\]  

(10)

Substituting the equilibrium values of (10) in the revenue functions of both firms yields, after some simplification:

\[
\begin{align*}
R^1(q^1, q^2) &= \frac{r(r - 1)}{(4r - 1)^2} q^1 \\
R^2(q^1, q^2) &= \frac{4r(r - 1)}{(4r - 1)^2} q^2
\end{align*}
\]  

(11)
Chapter 1. Quality Differentiation in a Strategic Trade Model with Fixed Costs

We now can consider the first-stage quality competition. In the first-stage game, each firm chooses its quality to maximize its profit, taking the other firm’s quality as given.

\[
\begin{align*}
\text{Max} & \quad [\pi^1(q^1, q^2) - R^1(q^1, q^2) - \gamma F(q^1)] \\
\text{Max} & \quad [\pi^2(q^1, q^2) - R^2(q^1, q^2) - F(q^2)]
\end{align*}
\]

(12)

The Kuhn-Tucker first-order conditions are:

\[
\begin{align*}
\pi_i^t - \lambda_i &= 0 \\
\lambda_i q^t &= 0 \\
q^t &= 0 \\
\lambda_i &= 0 \\
i &= 1, 2
\end{align*}
\]

(13)

where \( \lambda_i, \ i = 1, 2 \) are Lagrange multipliers.

It follows that the first-order conditions for the low-quality firm are:

\[
\begin{align*}
\pi_1^1 - g(r) - \gamma F'(q^1) &= 0, \quad \text{if} \quad q^1 > 0 \\
\pi_1^1 - g(r) - \gamma F'(q^1) &< 0, \quad \text{only if} \quad q^1 = 0
\end{align*}
\]

(14)

The first-order conditions for firm 2 are:

\[
\begin{align*}
\pi_2^2 - f(r) - F'(q^2) &= 0, \quad \text{if} \quad q^2 > 0 \\
\pi_2^2 - f(r) - F'(q^2) &< 0, \quad \text{only if} \quad q^2 = 0
\end{align*}
\]

(15)

where \( g(r) = R^1_1(q^1, q^2) = \frac{r^2 (4r - 7)}{(4r - 1)^3} \), \( f(r) = R^2(q^1, q^2) = \frac{4r (4r^2 - 3r + 2)}{(4r - 1)^3} \).
From (14) we observe that, in equilibrium, the low-quality firm will enter the market (earn a positive profit) if and only if there is sufficient quality differentiation \( r > 7/4 \) between the low- and high-quality product. In other words, fixed costs are compatible with Bertrand competition if and only if the products are sufficiently differentiated. If \( r < 7/4 \), then we have \( \pi_1 < 0 \) and the low-quality firm will not produce. This is represented as choosing the lowest possible level \( q^1 = 0 \), and there is no strategic interaction between these two firms. As a matter of fact, when \( q^1 = 0 \), there is no demand for and supply of firm 1's product, and the high-quality firm (firm 2) is a monopoly.

However, we can prove (see theorem 1 in Ronnen 1991) that for any convex cost function \( F(q) \), where \( F'(q) > 0, F''(q) > 0 \), there exists a Nash equilibrium in which \( q^2 > q^1 > 0 \) and both firms earn positive profits.

The second-order conditions for the profit maximization are:

\[
\pi_{12} - \frac{g'(r)}{q^1} > 0; \quad \pi_{21} - \frac{r f'(r)}{q^1} > 0 \tag{16}
\]

\[
\pi_{11} = -\frac{r g'(r)}{q^1} - \gamma F''(g^1) < 0; \quad \pi_{22} = \frac{f'(r)}{q^1} - F''(g^2) < 0 \tag{17}
\]

where \( f'(r) = \frac{df(r)}{dr} = \frac{8 (5 r + 1)}{(4 r - 1)^4} < 0, \quad g'(r) = \frac{dg(r)}{dr} = \frac{2 r (8 r + 7)}{(4 r - 1)^4} > 0. \)

The stability condition is satisfied: \( \Delta = \pi_{11} \pi_{22} - \pi_{12} \pi_{21} > 0 \).

Condition (16) means that a firm's marginal revenue of quality increases with an increase in the quality of its rival's product. This amounts to saying that, under Bertrand competition, the reaction functions of quality are positively sloped. In other words, these two
products are strategic complements in the quality space.

**Proposition 1.** Under Bertrand price competition, these two products are strategic complements in the quality space.

Proposition 1 can be understood intuitively as follows. Suppose the low-quality producer increases its quality, then the low-quality product will be more similar to the high-quality product and the competition between these two goods will increase. To ease the increased competition, the high-quality producer should increase its own quality. Now suppose the high-quality producer increases its quality, then these two products will be more differentiated. To maximize its profit, the low-quality producer will react by increasing its quality, because by doing so it can charge a higher price.

The quality reaction functions can be derived as follows (see Tirole 1988):

\[
\frac{dq^1}{dq^2} = \frac{\pi_{12}}{\pi_{11}} \quad \frac{dq^2}{dq^1} = \frac{\pi_{21}}{\pi_{22}} \quad (18)
\]

We can show that the slope of the quality reaction function of firm 1 is steeper than that of the quality reaction function of firm 2 (see appendix A1).

1.4.2 The Optimal Investment Policy for the Low-quality Country

Having characterized the market equilibrium, we now turn to government intervention. In this section, we consider an investment subsidy by the low-quality country. We assume that, before firms make any decision, the government of the low-quality firm can credibly commit itself to subsidize (tax) its firm's fixed cost, and that the subsidy amount is
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Figure 1 Quality reaction functions under Bertrand price competition
\( s^1 (\gamma \, F(q^1)) \), where \( 0 < s^1 < 1 \). We ask what effect this government policy will have upon the profits of both firms and the welfare of the two exporting countries.

Obviously, a lump-sum subsidy, although it has an indirect effect on prices through quality changes, will not change directly the second-stage price competition since there is no direct interaction between subsidy and price. The subsidy, however, changes the first-stage quality competition directly. After the subsidy, the profit functions of both firms become:

\[
\begin{align*}
\max_{q^1} & \quad [\pi^1(q^1, q^2) - R^1(q^1, q^2) - (1 - s^1) \gamma \, F(q^1)] \\
\max_{q^2} & \quad [\pi^2(q^1, q^2) - R^2(q^1, q^2) - F(q^2)]
\end{align*}
\]

The first-order conditions are:

\[
\begin{align*}
\pi^1_1 - 0, & \quad g(r) - (1 - s^1) \gamma \, F'(q^1) \\
\pi^1_2 - 0, & \quad f(r) - F'(q^2)
\end{align*}
\]

To calculate the comparative static effects of the subsidy, we totally differentiate the above first-order conditions:

\[
\begin{align*}
\pi^1_{11} \, dq^1 + \pi^1_{12} \, dq^2 - \gamma \, F'(q^1) \, ds^1 \\
\pi^2_{21} \, dq^1 + \pi^2_{22} \, dq^2 - 0
\end{align*}
\]

where

\[
\begin{align*}
\pi^1_{11} - \frac{r \, g'(r)}{q^1} - (1 - s^1) \gamma \, F'(q^1) & < 0, & \pi^1_{12} - \frac{g'(r)}{q^1} & > 0 \\
\pi^2_{21} - \frac{r \, f'(r)}{q^1} & > 0, & \pi^2_{22} - \frac{f'(r)}{q^1} - F'(q^2) & < 0
\end{align*}
\]
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Solving the above equations in terms of $ds^1$, we have:

\[
\frac{dq^1}{ds^1} = \frac{-\gamma F'(q^1) \pi^2_{22} }{\Delta} > 0
\]

\[
\frac{dq^2}{ds^1} = \frac{\gamma F'(q^1) \pi^2_{21} }{\Delta} > 0
\]

where $\Delta = \pi^2_{22} \pi^1_{11} - \pi^2_{21} \pi^1_{12} > 0$

We can show that (see appendix A2) $\frac{dq^1}{ds^1} > \frac{dq^2}{ds^1}$, that is the marginal effect of this subsidy on the quality of the product supplied by firm 1 is larger than the marginal effect on the quality of the product of firm 2. In other words, the low-quality country's subsidy has a larger effect on the quality supplied by the domestic firm than on the quality supplied by the foreign firm.

The results in (22) suggest that an investment subsidy by the low-quality country enhances the quality levels chosen by both firms. A subsidy to the low-quality firm reduces its cost of quality improvement and hence induces it to supply higher quality. This makes two products more similar and increases competition. To ease the competition, the high-quality firm will also increase its quality.

**Proposition 2.** Under Bertrand competition, a small investment subsidy to the low-quality product

(a) raises the quality of the product of the high-quality firm;

(b) increases the competition between the two firms;

(c) lowers the quality-adjusted prices of both products;

(d) increases the number of consumers for both products;
(e) increases the profit of the low-quality firm; and

(f) reduces the profit of the high-quality firm.

Proof.

(a). See results in (22).

(b). The lower the ratio of high-quality to low-quality, the more severe the competition is.

\[
\frac{dr}{ds} - \left( \frac{dq^2}{ds} - r \frac{dq^1}{ds} \right) \frac{1}{q^1} = - \frac{r \gamma F'(q^1) F'(q^2)}{\Delta} \frac{1}{q^1} < 0 \tag{23}
\]

(c). The quality-adjusted prices of the low-quality and the high-quality products are

\[
\bar{q} - \frac{r - 1}{4 r - 1}, \quad \bar{q} - 2 \bar{q}, \text{ respectively.}
\]

\[
\frac{d\bar{q}}{ds} - \frac{3}{(4 r - 1)^2} \frac{dr}{ds} < 0, \quad \frac{d\bar{q}}{ds} - 2 \frac{d\bar{q}}{ds} < 0 \tag{24}
\]

(d). The comparative statics of the demand functions are:

\[
\frac{dD}{ds} - \frac{1}{(4 r - 1)^2} \frac{dr}{ds} > 0, \quad \frac{dD^2}{ds} - 2 \frac{dD}{ds} > 0 \tag{25}
\]

(e). Totally differentiating \( \pi^1 \) with respect to \( s^1 \) gives,

\[
\pi^1_{s^1} = \pi^1_q q^1_s + \pi^1 q^2_s \gamma F(q^1) > 0
\]

where

\[
h(r) = \frac{2 r - 1}{(4 r - 1)^3}
\]

(f). Similar to (e), we have,
Proposition 3. The low-quality country has a unilateral incentive to offer an investment subsidy to its firm.

Proof.

The welfare function of the low-quality country is

\[ w^1(s^1) - \pi^1(q^1, q^2, s^1) - \gamma F(q^1) \]

Differentiating it by \( s^1 \) yields, at \( s^1 = 0 \),

\[ w_{s^1}^1 - \pi_{s^1}^1 - \gamma F(q^1) - h(r) q^2_1 > 0 \]

The optimal subsidy can be derived by letting \( w_{s^1}^1 = 0 \), from which we have

\[ s^1 = \frac{\pi^1_q q^2_1}{\gamma F'(q^1) q^1_2} > 0 \]

Similar to the result of Brander and Spencer (1985), the low-quality country's subsidy not only increases its firm's profit, it also increases the welfare of that country. The subsidy, although merely a transfer of capital, changes strategically the positions of the two firms, from which they start the competition.

Figure 2 shows graphically the effects of a low-quality country's subsidy. Before the subsidy, the market equilibrium is at point A. A small investment subsidy to the low-quality firm shifts its quality reaction function to the right, ending at a new Nash equilibrium point B. As a result, the low-quality country (country 1) will move to a higher isowelfare contour (moving from curve 11 to curve 12) while the high-quality country (country 2) will move to
Figure 2 The effects of the low-quality country's investment subsidy under Bertrand price competition
a lower isowelfare contour (moving from curve 21 to curve 22). The net gain of the low-quality country's welfare (firm 1's profit less the subsidy) is the difference between isowelfare contour 12 and 11; and the net loss of the high-quality country's welfare (firm 2's profit) is the difference between isowelfare contour 21 and 22.

**Proposition 4.** The optimal investment subsidy maximizes country 1's rent earned from exports by moving the low-quality firm to what would have been the Stackelberg leader-follower point in the quality space with no subsidy.

Proof.

Suppose there is no investment subsidy and the low-quality firm is a Stackelberg leader in the quality space. The first-order condition for a profit maximum, taking account of the reaction function of the high-quality firm, is

$$
\frac{\partial \pi_1}{\partial q_1} \cdot \frac{\partial \pi_2}{\partial q_2} = 0
$$

With a given subsidy, the low-quality firm's profit is \( \pi_1 \cdot s_1 \cdot F'(q_1) \). The first-order condition in the first stage quality competition is: \( \pi_1 \cdot s_1 \cdot F'(q_1) = 0 \). Taking the subsidy at the optimal level \( s_1 = \frac{\pi_2}{\gamma F'(q_1) q_2} \), we obtain the same first-order condition as (30). This amounts to saying that Stackelberg leader-follower behaviour without an investment subsidy gives rise to the same results as Nash behaviour with the optimal subsidy.

This proposition is essentially the same as proposition 3 in Spencer and Brander (1983). Figure 3 shows the new equilibrium (point O) when the low-quality country offers an optimal investment subsidy to its firm.
Figure 3  The effects of the low-quality country's optimal investment subsidy under Bertrand price competition
1.4.3 The Optimal Investment Policy for the High-quality Country

Let the investment subsidy to the high-quality firm be $s^2 F(q^2)$. Again, it is obvious that there is no direct effect upon the second-stage price competition. After the subsidy, the profit functions of both firms are:

$$\begin{align*}
\max_{q^1} \quad & [\pi^1(q^1, q^2) - R^1(q^1, q^2) - \gamma F(q^1)] \\
\max_{q^2} \quad & [\pi^2(q^1, q^2) - R^2(q^1, q^2) - (1 - s^2) F(q^2)]
\end{align*}$$

The first-order conditions are:

$$\begin{align*}
\pi^1_1 - 0, & \quad g(r) - \gamma F'(q^1) \\
\pi^2_1 - 0, & \quad f(r) - (1 - s^2) F'(q^2)
\end{align*}$$

Total differentiation yields:

$$\begin{align*}
\pi^1_{11} dq^1 + \pi^1_{12} dq^2 - 0 \\
\pi^2_{21} dq^1 + \pi^2_{22} dq^2 - F'(q^2) ds^2
\end{align*}$$

where

$$\begin{align*}
\pi^1_{11} - \frac{r g'(r)}{q^1} - \gamma F''(q^1) < 0, & \quad \pi^1_{12} - \frac{g'(r)}{q^1} > 0 \\
\pi^2_{21} - \frac{r f'(r)}{q^1} > 0, & \quad \pi^2_{22} - \frac{f'(r)}{q^1} - (1 - s^2) F''(q^2) < 0
\end{align*}$$

By Cramer's rule, we have

$$\begin{align*}
\frac{dq^1}{ds^2} - \frac{F'(q^2) \pi^1_{12}}{\Delta} > 0 \\
\frac{dq^2}{ds^2} - \frac{F'(q^2) \pi^1_{11}}{\Delta} > 0
\end{align*}$$
where \( \Delta - \pi_{11}^1 \pi_{22}^2 - \pi_{12}^1 \pi_{21}^2 > 0. \)

Notice that here we have \( \frac{dq}{ds^2} > \frac{dq}{ds^2}, \) that is the marginal effect of this subsidy on the quality of the product supplied by firm 2 is larger than the marginal effect on the quality of the product of firm 1. In other words, the high-quality country's subsidy has a larger effect on the quality supplied by the domestic firm than on the quality supplied by the foreign firm.

The result that an investment subsidy to the high-quality firm also increases the qualities of both products can be understood intuitively as follows. A subsidy to the high-quality firm reduces its cost of quality, this leads it to supply an even higher quality product. The increased distance between these two products along the quality spectrum eases the competition and allows the low-quality firm to increase its quality and charge a higher price.

**Proposition 5.** Under Bertrand competition, a small investment subsidy to the high-quality firm

(a) raises the quality of the product of the low-quality firm;

(b) reduces the competition between the two firms;

(c) increases the quality-adjusted prices of both products;

(d) reduces the number of consumers for both products; and

(e) increases the profits of both firms.

Proof.

(a). See the results in (34).

(b). The higher the ratio of high-quality to low-quality, the less severe the competition is.

\[
\frac{dr}{ds^2} - \left( \frac{dq}{ds^2} - r \frac{dq}{ds^2} \right) \frac{1}{q^1} \frac{\gamma F'(q^2) F'(q^1)}{\Delta q^1} > 0
\]

(35)
(c). The quality-adjusted prices of the low-quality and the high-quality products are

\[ \theta = \frac{r - 1}{4 r - 1}, \quad \bar{\theta} = 2 \theta, \] respectively.

\[ \frac{d\theta}{ds^2} = \frac{3}{(4 r - 1)^2} \frac{dr}{ds^2} > 0, \quad \frac{d\bar{\theta}}{ds^2} = 2 \frac{d\theta}{ds^2} > 0 \quad (36) \]

(d). The comparative statics of the demand functions are:

\[ \frac{dD_1}{ds^2} - \frac{1}{(4 r - 1)^2} \frac{dr}{ds^2} < 0, \quad \frac{dD_2}{ds^2} - 2 \frac{dD_1}{ds^2} < 0 \quad (37) \]

(e). Totally differentiating \( \pi^1(q^1, q^2, s^2), \pi^2(q^1, q^2, s^2) \) with respect to \( s^2 \) at \( s^2 = 0 \) gives:

\[ \pi^1_{s^2} = \pi^1_{q^1} q^1 + \pi^1_{q^2} q^2 - h(r) q^3 > 0 \]

\[ \pi^2_{s^2} = \pi^2_{q^1} q^1 + \pi^2_{q^2} q^2 + \partial \pi^2/\partial s^2 = -4 r^2 h(r) q^1 F(q^2) \quad (38) \]

Using the conditions that \( F(0) - F'(0) = 0, F''(q) \leq 0 \) and \( r > 7/4 \), we have

\[ \pi^2_{s^2} > -4 r^2 h(r) \frac{F'(q^2)}{r F'(q^2)} \cdot F(q^2) \]

\[ > -\frac{(F'(q^2))^2}{2 F'(q^2)} \cdot F(q^2) - \frac{2 F(q^2) F'(q^2) - (F'(q^2))^2}{2 F'(q^2)} > 0 \]

Unlike the results in Proposition 2, an investment subsidy to the high-quality firm increases both firms' profits.

Note that when the low-quality firm is subsidized it raises the quality of its product more than the high-quality firm which is not subsidized, thus the differentiation along the quality spectrum between the firms is reduced, resulting in lower profit for the high-quality
In this case where the high-quality firm is subsidized its increase in quality is larger than the increase in quality of the unsubsidized firm. Thus differentiation increases and the low-quality firm enjoys more profit. The high-quality firm, because of the convexity of the fixed cost function, is paying more for the quality improvement than before thus some of the gains that accrue from the subsidy dissipates. As a result, although a high-quality country's subsidy increases firm's profit, it reduces the overall welfare of that country.

**Proposition 6.** The high-quality country has a unilateral incentive to tax its firm.

Proof.

The welfare function of the high-quality country is $w^2(s^2) - \pi^2(q^1, q^2, s^2) - s^2 F(q^2)$. At $s^2 = 0$,

$$w^2_2 = \pi^2_2 - F(q^2) - \pi^2_1 q^1 - 4 r^2 h(r) q^1 < 0 \quad (40)$$

Thus, to increase domestic welfare, $s^2$ must be negative. In other words, the high-quality country must tax its firm rather than subsidize it.

The optimal investment tax can be calculated by letting $w^2_2 = 0$, from which we have

$$s^2 = \frac{\pi^2_1 q^1}{F'(q^2)} - \frac{\pi^1_1}{F'(q^2)} < 0 \quad (41)$$

Using the quality reaction functions and isowelfare contours, we can better interpret the above results.

In Figure 4, a high-quality country's subsidy shifts the quality reaction function of the firm.
high-quality producer upward, the new Nash equilibrium is now at point C. As a result, country 1 will move to a higher isowelfare contour (moving from curve 11 to curve 13) while country 2 will move to a lower isowelfare contour (moving from curve 21 to curve 23). The overall welfare loss of the high-quality country is the difference between isowelfare contours 21 and 23.

To increase domestic welfare, the high-quality country should impose tax rather than subsidize its firm's investment. A small investment tax gives firm 2 a strategic advantage and makes country 2 move to a higher isowelfare contour while making country 1 move to a lower isowelfare contour. As a result, both firm 2 and country 2 are better off while firm 1 and country 1 are worse off.

**Proposition 7.** The optimal investment tax maximizes country 2's rent earned from exports by moving the high-quality firm to what would have been the Stackelberg leader-follower point in the quality space with no tax.

Proof. The proof is very similar to that of proposition 4.

**Proposition 8.** The noncooperative Nash subsidy/tax equilibrium (where both countries are allowed to intervene) is characterized by an investment subsidy in the low-quality country and an investment tax in the high-quality country.

Proof.

The welfare function of country $i$ is $w_i(s^1, s^2) - x_i^0(q^1, q^2, s^1, s^2) - F(q^i)$ where $i = 1, 2$. The noncooperative Nash investment subsidy/tax equilibrium is characterized by: $\frac{\partial w_i}{\partial s^1} = 0, \frac{\partial w_i}{\partial s^2} = 0$, from which we have:
Figure 4  The effects of the high-quality country's investment subsidy under Bertrand price competition
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\[ s^1 = \frac{\pi_1^1 q_2^2}{\gamma F'(q^1) q_1^1} - \frac{\pi_2^1}{\gamma F'(q^1)} dq^2 > 0, \quad s^2 = \frac{\pi_1^2}{F'(q^2)} dq^1 < 0 \]

Note that the values of \( \pi_i^j, dq^i/dq^j \) and \( F'(q^i) \) (where \( i, j = 1, 2 \)) are different from those in (29) and (41).

Although each country has a unilateral incentive to intervene in the market, their joint welfare can be increased if either or both reduce the intervention from the Nash subsidy/tax equilibrium levels.

**Proposition 9.** The noncooperative Nash subsidy/tax equilibrium is suboptimal.

**Proof.** We prove proposition 9 by showing that the joint welfare of the exporting countries can be increased by reducing the subsidy or tax in either country. The joint welfare \( J \) is

\[ J(s^1, s^2) = w^1(s^1, s^2) + w^2(s^1, s^2) \]

\[ J_1 = w^1_1 + w^2_1 - 0 + w^2_2 - \frac{d (\pi^2 - s^2 F(q^2))}{ds^1} \]

\[ - \frac{\pi^1_2 dq^1}{ds^1} + \frac{\pi^2 dq^2}{ds^1} - s F'(q^2) - \frac{\pi^1 dq^1}{ds^1} - (\frac{\pi^1_2 \pi^1_{12}}{F'(q^2) \pi^1_{11}}) F'(q^2) dq^2 \]

\[ - \frac{\pi^1_2 F'(q^1)}{\Delta} \frac{\pi^1_{12} \pi^1_{21} - \pi^1_{22} \pi^1_{11}}{\pi^1_{11}} - \frac{\gamma \pi^1_2 F'(q^1)}{\pi^1_{11}} < 0 \]

where we have used the facts that \( \pi^2_1 < 0 \) and \( \pi^1_{11} < 0 \). Similarly,

\[ J_2 = \frac{\pi^1_2 F'(q^2)}{\Delta} \frac{\pi^1_{12} \pi^1_{21} - \pi^1_{11} \pi^1_{22}}{\pi^2_{22}} - \frac{\pi^1_2 F'(q^2)}{\pi^2_{22}} > 0 \quad (44) \]
1.5 Cournot Competition under Quality Differentiation

In this section, we assume that firms choose qualities and quantities rather than qualities and prices. We first characterize in sub-section 1.5.1 the market equilibrium without any government intervention; in sub-sections 1.5.2 and 1.5.3 we examine the optimal investment policy for the low- and high-quality country, respectively.

1.5.1 Market Equilibrium

We start with the second-stage quantity game. In the second-stage, for any given quality pair \((q^1, q^2)\), firms choose quantities to maximize their revenues. Using the inverted demand functions of (5), we have:

\[
\begin{align*}
\max_{D^1} & \quad [R^1(D^1, D^2) - p^1 D^1 - (1 - D^1 - D^2) q^1 D^1] \\
\max_{D^2} & \quad [R^2(D^1, D^2) - p^2 D^2 - ((1 - D^2) r - D^1) q^1 D^2]
\end{align*}
\]

(45)

The first- and second-order conditions are:

\[
\begin{align*}
R^1_{1c}(D^1, D^2) - & \quad 1 - 2 D^1 - D^2 = 0 \\
R^1_{2c}(D^1, D^2) - & \quad r - D^1 - 2 r D^2 = 0
\end{align*}
\]

(46)

\[
\begin{align*}
R^1_{11}(D^1, D^2) - & \quad 2 < 0 , \quad R^1_{21}(D^1, D^2) - 2 r < 0 \\
R^1_{12}(D^1, D^2) - & \quad 1 < 0 , \quad R^1_{22}(D^1, D^2) - 1 < 0
\end{align*}
\]

(47)

Not surprisingly, products are strategic substitutes in the quantity space.
Solving (46) leads to the following equilibrium solutions:

\[
D^1 = \frac{r}{4r - 1}, \quad D^2 = \frac{2r - 1}{4r - 1}
\]  

(48)

Thus we have the following quality-adjusted prices:

\[
\hat{\theta} = \frac{r}{4r - 1}, \quad \hat{\theta} = \frac{2r - 1}{4r - 1}, \quad \hat{\theta} = \frac{2r - 1}{4r - 1}
\]  

(49)

Finally, the corresponding revenue functions are given by:

\[
R^{1c}(q^1, q^2) = \frac{r^2}{(4r - 1)^2} q^1
\]

\[
R^{2c}(q^1, q^2) = \frac{(2r - 1)^2}{(4r - 1)^2} q^2
\]  

(50)

We now look for the solutions of the quality game. Firms will choose their quality specifications to maximize their profits:

\[
\text{Max}_{q^1} [\pi^{1c}(q^1, q^2) - R^{1c}(q^1, q^2) - \gamma F(q^1)]
\]

\[
\text{Max}_{q^2} [\pi^{2c}(q^1, q^2) - R^{2c}(q^1, q^2) - F(q^2)]
\]  

(51)

The first-order conditions of this problem become:

\[
\pi^{1c}_1 - m(r) - \gamma F'(q^1) = 0
\]

\[
\pi^{2c}_2 - n(r) - F'(q^2) = 0
\]  

(52)

where

\[
m(r) = R^{1c}_1(q^1, q^2) - \frac{(4r - 1) r^2}{(4r - 1)^3}, \quad n(r) = R^{2c}_2(q^1, q^2) - \frac{(16 r^3 - 12 r^2 - 4r - 1)}{(4r - 1)^3}
\]
From (52) we note that fixed costs are compatible with Cournot competition for any pair of quality specifications \((q^1, q^2)\). However, in equilibrium, firms supply products of different qualities.

Lemma 1. In Cournot equilibrium, \(q^2 > q^1 > 0\)

Proof. See appendix A3.

The second-order conditions are:

\[
\pi_{11}^{le} = - \frac{r m'(r)}{q^1} - \gamma F^*(q^1), \quad \pi_{22}^{2e} = - \frac{n'(r)}{q^1} - F^*(q^2) < 0
\]

\[
\pi_{12}^{le} = \frac{m'(r)}{q^1} < 0, \quad \pi_{21}^{2e} = - \frac{r n'(r)}{q^1} > 0
\]

(53)

where \(m'(r) = \frac{dm(r)}{dr} = - \frac{2 (8 r + 1)}{(4 r - 1)^4} < 0, \quad n'(r) = \frac{dn(r)}{dr} = - \frac{8 (r - 1)}{(4 r - 1)^4} < 0.\)

\[
\Delta^e = \pi_{11}^{le} - \pi_{22}^{2e} = \pi_{12}^{2e} = \pi_{21}^{le} \quad \frac{r m'(r)}{q^1} F^*(q^2) - \frac{n'(r)}{q^1} F^*(q^1) + \gamma F^*(q^1) F^*(q^2)
\]

Lemma 2. In Cournot equilibrium, \(\pi_{11}^{le} < 0, \Delta^e > 0.\)

Proof. See appendix A4.

Condition (53) means that, under Cournot competition, these two products are no longer complete strategic complements, as in the case of Bertrand competition.

Proposition 1'. Under Cournot quantity competition, from the low-quality firm's point of view, the high-quality product is a strategic substitute to its product in the quality space, while from the high-quality firm's point of view, these two products are still strategic complements in the quality space.
Figure 5  Quality reaction functions under Cournot quantity competition
1.5.2 The Optimal Investment Policy for the Low-quality Country

Assuming the government of the low-quality country can credibly commit to give a subsidy \( s^1 F(q^1) \), where \( 0 < s^1 < 1 \), to its firm before firms enter the competition, then we have very similar profit functions and first-order conditions as those in (19) and (20).

Following similar steps to those in sub-section 1.4.2, we can reach the following comparative static results:

\[
\frac{dq^1}{ds^1} = -\frac{\gamma F'(q^1) \pi_{22}}{\Delta^c} > 0
\]

\[
\frac{dq^2}{ds^1} = \frac{\gamma F'(q^1) \pi_{21}}{\Delta^c} > 0
\]

where

\[
\pi_{11} = -\frac{r'}{q^1} - (1 - s^1) \gamma F^*(q^1) < 0, \quad \pi_{12} = -\frac{m'(r)}{q^1} < 0
\]

\[
\pi_{21} = -\frac{r}{q^1} n'(r) > 0, \quad \pi_{22} = -\frac{n'(r)}{q^1} F^*(q^2) < 0
\]

\[
\Delta^c = \pi_{11} \pi_{22} - \pi_{12} \pi_{21} > 0
\]

**Proposition 2'.** Under Cournot competition, a small investment subsidy to the low-quality firm

(a) increases the quality of the product of the high-quality firm;

(b) increases the competition between the two firms;

(c) lowers the quality-adjusted prices of both products;

(d) increases the number of consumers for both products;

(e) increases the profit of the low-quality firm; and

(f) reduces the profit of the high-quality firm.

Proof.
The proofs of (a), (b), (c) and (d) are straightforward and hence is omitted here. We only prove (e) and (f). Totally differentiating $\pi^{1e}$ with respect to $s^1$ gives, at $s^1 = 0$

$$\pi^{1e}_s = \pi^{1e}_1 q_s^1 + \pi^{1e}_2 q^2_1 + \partial \pi^{1e}/\partial s^1$$

$$- k(r) q^2_1 + \gamma F(q^1) - k(r) \frac{\gamma F'(q^1) \pi^{2e}_2}{\Delta^e} + \gamma F(q^1)$$

$$> \frac{r F'(q^1) k(r)}{F^*(q^1) + \gamma F(q^1)} - \frac{\gamma (F'(q^1))^2}{2 F^*(q^1)} + \gamma F(q^1)$$

$$- \gamma \frac{2 F(q^1) F^*(q^1) - (F'(q^1))^2}{2 F^*(q^1)} > 0$$

(55)

where $k(r) = \pi^{1e}_2 - \frac{2 r}{(4 r - 1)^3}$, we have also used the results in appendix A4.

Similarly, we have

$$\pi^{2e}_s = \pi^{2e}_1 q_s^1 + \pi^{2e}_2 q^2_1 + l(r) q^1_1 + 0 < 0$$

(56)

where $l(r) = \pi^{2e}_1 - \frac{4 r^2 (2 r - 1)}{(4 r - 1)^3}$.

The results in proposition 2' are the same as those in proposition 2. However, under Cournot competition, a low-quality country's subsidy to its firm, although increasing the firm's profit, reduces the overall welfare of that country.

**Proposition 3'**. The low-quality country has a unilateral incentive to tax its firm.

Proof.

Let the welfare of the low-quality country be $w^{1e}(s^1) = \pi^{1e}(q^1, q^2, s^1) - s^1 \gamma F(q^1)$, we have at $s^1 = 0$,

$$w^{1e}_{s^1} = \pi^{1e}_{s^1} - \gamma F(q^1) - k(r) q^2_1 < 0$$

(57)
Thus, to increase welfare, \( s^1 \) must be negative. In other words, the low-quality country must tax its firm rather than subsidize it.

Similar to the result in sub-section 1.4.2, the optimal tax level in this case is:

\[
\begin{align*}
    s^1 &= \frac{\pi_2^1 q_2^1}{Y F'(q^1)} - \frac{\pi_2^2 \pi_{21}}{Y F'(q^1) \pi_{22}^2} < 0
\end{align*}
\]  

(58)

Graphically, under Cournot competition, a low-quality country's subsidy shifts the quality reaction function of firm 1 upward. As a result, both countries move to the lower isowelfare contours (see Figure 6). So the low-quality country's optimal trade policy is to tax its firm. A small tax applied to the low-quality firm makes both firms, as well as both exporting countries, better off.

**Proposition 4'.** The optimal investment tax maximizes country 1's rent earned from exports by moving the low-quality firm to what would have been the Stackelberg leader-follower point in the quality space with no tax.

Proof. The proof is the same as that for proposition 4.

1.5.3 The Optimal Investment Policy for the High-quality Country

In this sub-section, we consider the optimal investment policy of the high-quality country. Let the investment subsidy be \( s^2 F(q^2) \), where \( 0 < s^2 < 1 \). Using a similar approach, we can derive the effects of this subsidy upon the quality that each firm chooses as follows:
Figure 6 The effects of the low-quality country's investment subsidy under Cournot quantity competition
Chapter 1. Quality Differentiation in a Strategic Trade Model with Fixed Costs

\[ \frac{dG_1}{ds^2} - \frac{F''(q^1)}{\Delta^e} \pi_{12}^{le} < 0 \]

\[ \frac{dG_2}{ds^2} - \frac{F''(q^2)}{\Delta^e} \pi_{11}^{le} > 0 \]

(59)

where

\[ \pi_{11}^{le} = - \frac{r m'(r)}{q^1} - F''(q^1) < 0, \quad \pi_{12}^{le} = \frac{m'(r)}{q^1} < 0 \]

\[ \pi_{21}^{2e} = - \frac{r n'(r)}{q^1} > 0, \quad \pi_{22}^{2e} = \frac{n'(r)}{q^1} - (1 - s^2) F''(q^2) < 0 \]

\[ \Delta^e = \pi_{11}^{le} \pi_{22}^{2e} - \pi_{12}^{le} \pi_{21}^{2e} > 0 \]

Under Cournot competition, the high-quality country's subsidy to its firm has opposing effects upon the qualities that both firms would supply. This result differs from the results in the previous three sub-sections. Under Cournot competition, a subsidy by the high-quality country will increase the quality of its domestic firm but reduce the quality of the low-quality firm. This is because, from the low-quality firm's point of view, these two products are strategic substitutes in the quality space.

**Proposition 5'.** Under Cournot competition, a small investment subsidy to the high-quality firm

(a) reduces the quality of the product of the low-quality firm;

(b) reduces the competition between the two firms;

(c) increases the quality-adjusted prices of both products;

(d) decreases the number of consumers for both products;

(e) reduces the profit of the low-quality firm; and

(f) increases the profit of the high-quality firm.
Proof.

We omit the straightforward proofs of (a), (b), (c) and (d) and only prove (e) and (f).

Totally differentiating $\pi^{1c}, \pi^{2c}$ with respect to $s^2$ gives:

\[ \begin{aligned}
\pi^{1c}_s &= \pi^{1c}_1 q^{1c}_s + \pi^{1c}_2 q^{2c}_s - k(r) q^{2c}_s < 0 \\
\pi^{2c}_s &= \pi^{2c}_1 q^{1c}_s + \pi^{2c}_2 q^{2c}_s + \partial \pi^{2c}/\partial s^2 \\
&= l(r) q^{1c}_s + 0 + F(q^2) > 0 
\end{aligned} \]  

(60)

**Proposition 6'**. The high-quality country has a unilateral incentive to subsidize its firm.

The proof is very easy and hence is omitted here. This proposition and proposition 5' can also be verified in Figure 7.

Similar to the result in sub-section 1.4.3, the optimal subsidy level is:

\[ s^2 = \frac{\pi^{2c}_1 q^{1c}_s}{F'(q^2) q^{2c}_s} - \frac{\pi^{2c}_1 \pi^{1c}_2}{F'(q^2) \pi^{1c}_{11}} > 0 \]  

(61)

**Proposition 7'**. The optimal investment tax maximizes country 2's rent earned from exports by moving the high-quality firm to what would have been the Stackelberg leader-follower point in quality space with no tax.

Proof. The proof is the same as that for proposition 4.

**Proposition 8'**. Under Cournot competition, the noncooperative Nash subsidy/tax equilibrium is characterized by an investment tax in the low-quality country and an investment subsidy in the high-quality country.

The proof is very similar to that for proposition 8 and is omitted here.
Figure 7  The effects of the high-quality country's investment subsidy under Cournot competition
Although each country has a unilateral incentive to intervene in the market, the consequences of such interventions in terms of their joint welfare are different. The joint welfare can be increased if the low-quality country increases its intervention (tax) or the high-quality country reduces its intervention (subsidy) from their noncooperative Nash equilibrium levels.

**Proposition 9'.** The joint welfare of the producing countries can be increased by increasing the low-quality country's tax and/or reducing the high-quality country's subsidy from the noncooperative Nash subsidy/tax equilibrium.

Proof. The joint welfare $J^e$ is $J^e(s^1, s^2) = w^{1e}(s^1, s^2) + w^{2e}(s^1, s^2)$.

$$J^e_{s^1} = w^{1e}_{s^1} + w^{2e}_{s^2} = 0 + w^{2e}_{s^1} - d \frac{\pi^{2e} - s^2 F(q^2)}{ds^1}$$

$$= \pi^{1e}_{s^1} \frac{dq^1}{ds^1} + \pi^{2e}_{s^2} \frac{dq^2}{ds^1} - s^2 \frac{dF(q^2)}{ds^1}$$

$$= \pi^{2e} \frac{dq^1}{ds^1} \left( - \frac{\pi^{1e}}{F'(q^2) \pi^{1e}_{s^1}} \right) F'(q^2) \frac{dq^2}{ds^1}$$

$$= \gamma \frac{\pi^{2e}_{s^1} F'(q^2)}{\Delta^e} \frac{\pi^{1e}_{s^1} \pi^{2e}_{s^2} - \pi^{2e}_{s^1} \pi^{1e}_{s^2}}{\pi_{s^1}} < 0$$

where we have used the facts that $\pi^{2e} < 0$, $\pi^{1e} < 0$. Similarly,

$$J^e_{s^2} = \frac{\pi^{2e}_{s^2} F'(q^2)}{\Delta^e} \frac{\pi^{1e}_{s^1} \pi^{2e}_{s^2} - \pi^{2e}_{s^1} \pi^{1e}_{s^2}}{\pi^{2e}_{s^2}} < 0$$

1.6 Conclusions

Investment subsidies (such as preferential loans) have been used by many countries to promote their exports. Some governments targeted their investment subsidies to increase
the non-price competitiveness of their exports. This was the case in the 1960's in Japan and more recently in Taiwan and Korea (Wade 1991). By offering exporting firms investment subsidies to improve the quality of their products, governments in these countries attempted to convert reputation for low cost and low quality to one of high quality exports. The normative implications of export-promoting investment subsidies received relatively little attention (Matthews and Ravenhill 1993, Smith 1994). No previous research was published on the normative consequences of export-promoting investment subsidies targeted at non-price competition.

This chapter builds a simple economic model to consider the optimal investment policies in a vertically differentiated industry with a fixed cost of quality. It examines a subsidy/tax game under both Bertrand price competition and Cournot quantity competition. The chapter shows that there exists asymmetry between the optimal policies of low- and high-quality country with respect to subsidizing or taxing investments in quality improvements. Under Bertrand competition it is optimal for the low-quality country to subsidize investments that raise the quality of its exports, while the high-quality country has an incentive to tax investments in quality improvement. This is because Bertrand competition tends to lead to over-differentiation in the quality spectrum and thus the governments attempt to reduce quality differentiation. Under Cournot competition, the results are reversed, the low-quality country should tax investments that improve the quality of its exports while the high-quality country is better off subsidizing investments in quality improvement. In this case because firms tend to over-produce when they perceive higher demand the governments intervene to increase differentiation thus reducing competition.
The model in this chapter is very simple and future research may consider the following extensions: 1. Adding a marginal cost of quality in addition to the fixed cost; 2. Considering multi-dimensional quality space rather than a uni-dimensional space; 3. Using a more general utility function in which consumers are allowed to buy more than one unit of a product; 4. Investigating other types of trade policies such as unit subsidies/taxes; 5. Examining a repeated game rather than a one-shot game.
Chapter Two

Trade Restrictions in a Vertically Differentiated Industry

2.1 Introduction

During the last two decades, both international trade theorists and practitioners have witnessed a gradual but fundamental change in the nature of world protectionism. While several rounds of GATT negotiations have succeeded in reducing tariffs to very low levels, many governments have now resorted to non-tariff barriers (NTBs) to protect their domestic industries from foreign competition. The instruments of this so called "new protectionism" are diverse and include, among others, voluntary export restraints, import quotas, technical standards, government procurement schemes and discriminatory customs valuation procedures. The characteristics that these instruments share are that they tend to be opaque, discriminatory, directed largely against Japan and the newly-industrialised countries (NICs), and focused mostly on the manufacturing industries. The effects of these NTBs are quite different from those of tariffs and thus need careful analysis.

Another important new feature of international trade is the increasing importance of non-price competition. International competitors have been placing an increasing weight on strategies of product differentiation which rely on product reliability, durability, availability of service facilities and other product related attributes that consumers increasingly value.

Most of the economic literature has been focusing on price and quantity competition. The theory of quality competition has remained somewhat less developed despite of the
increasing importance of non-price competition (Shaked and Sutton 1982, Shapiro 1982, 1983 and Ronnen 1991). Krishna (1987) was one of the first to analyze the desirability of quality control in international competition. In her model, quality and quantity are the prime decision variables of a monopolistic foreign supplier. She found that a crucial determination of the direction of the effects of tariffs, quotas and quality control is "the valuation of increments in quality by the marginal consumer, relative to that of all consumers on average". Assuming a specific type of utility structure, Das and Donnenfeld (1987) also examined the effects of different trade policies on decisions about quality, price and quantity of a foreign monopoly. They demonstrated that "a quota and a specific tariff are equivalent, and both dominate an ad-valorem tariff in terms of welfare". Das and Donnenfeld (1989) investigated the effects of NTBs such as quantity and quality limitations in an oligopolistic industry consisting of foreign and domestic firms. Assuming the marginal cost to be a convex function of quality, they found that "the effects of trade policy hinge on the location of the quality produced by the firms in the quality spectrum."

Without exception, the cost of quality in all these models was assumed to be variable. This simplification is rather restrictive. Casual observation suggests that the introduction of products of different quality usually employs different technologies and machinery, thus requiring different levels of investment. This is especially true in industries with large-scale production, such as the steel and automobile industries, as well as in the knowledge-intensive industries such as pharmaceuticals and software. Decisions about the desirable quality levels depend largely on the fixed rather than the marginal costs. Indeed, unit variable costs are often negligible (e.g., as is the case with many pharmaceutical products) or are constant (i.e.,
In this chapter we examine, in a setting of oligopolistic competition, the effects of some NTBs on product quality and domestic welfare. Following the model of Das and Donnenfeld (1989), we investigate an industry consisting of one foreign and one domestic firm and analyze quality restrictions (minimum quality standards) and quantity restrictions (import quotas). The significant difference between our model and that in Das and Donnenfeld (1989) is the treatment of the costs of quality. In their model, the cost of quality is only reflected in the unit cost, while in our model we also incorporate a fixed cost of quality (i.e., investments in infrastructure, capacity and R&D). As we have argued above, the fixed costs in many industries associated with improvements of quality are significant in comparison to the variable costs.

Our results confirmed Das and Donnenfeld (1989)'s conclusion that the imposition of minimum quality standards (MQS) on the low-quality imports lowers home country's welfare. In our model, however, some ambiguities with respect to import quotas are resolved, we conclude that: 1. A quota on the low-quality imports improves home country's welfare; 2. A quota on the high-quality imports reduces, in most cases, home country's welfare, and only in some extreme cases a quota on the high-quality imports improves home country welfare. We also found that the optimal quantity and quality of a firm are positively correlated while a firm's optimal quantity and quality are negatively correlated to its rival's optimal quantity and quality, respectively.

This chapter is organized as follows. The concept of vertical differentiation is introduced at the very beginning. The main analysis is presented in sections 2.3 and 2.4. In
section 2.3, we assume zero marginal cost, i.e., fixed cost is the only cost. We derive the main results and insights of the chapter in this simplified model; in section 2.4, we introduce, in addition to the fixed cost of quality, a variable cost, which is an increasing concave function of quality. We find that the results obtained in section 2.3 remain valid in this more general setting. There are basically four sub-sections in each of these two main sections: In the first sub-section we look at the market equilibrium without any trade policy. In the second and third sub-section, we assume that the foreign firm produces the low-quality product and examine the effects of an MQS and a quota respectively. In the fourth sub-section, we assume that the foreign firm is the high-quality producer and investigate the effect of a quota on the imports of the high-quality product. Section 2.5 provides the concluding remarks.

2.2 The Basic Model

In the model, there are two firms, one domestic and the other foreign, producing a product differentiated along one quality scale. The only consumer market is domestic. We assume, as most researchers in this area do (Bond 1988, Tirole 1988, Das and Donnenfeld 1987, 1989), that consumers in the domestic country buy either zero or one unit of the product. Other things being equal, all consumers prefer a high-quality product to a low-quality product. Let the utility that a consumer gets from consuming one unit of a product with quality $q$ be $\theta q$, where $\theta$ is a taste parameter. The taste parameter $\theta$ for quality is assumed to be uniformly distributed across the population of consumers $[0, 1]$. The surplus that a consumer obtains from consuming a unit of a good with quality $q$ and price $p$ can thus be derived as $U = \theta q - p$. We assume, without loss of generality, that consumers' reservation
surplus level is 0.

To identify clearly the effect of quality, we assume further that both firms are identical in most respects, the only difference between them is the fact that their products have different qualities. Let firm 2 be the high-quality producer with quality \( q^2 \) while firm 1 be the low-quality supplier with quality \( q^1 \), that is \( q^2 > q^1 \), it is obvious that the domestic firm can be either the high-quality producer (firm 2) or the low-quality producer (firm 1).

Throughout the paper, we assume that the fixed cost is an increasing and convex function of quality \( F(q) \) for all feasible quality \( q^1, q^2 \in (0, \bar{q}] \), where \( \bar{q} \) is a sufficiently large number so that in equilibrium it will never be reached, and \( F(0) - F''(0) = F''(0) - 0 \), \( F'''(q) \geq 0 \).

If we define \( p^2, p^1, \theta, \delta, r \) in the same ways as those in chapter one, then we have the demand functions for the low- and high-quality products as follows:

\[
D^1 = \delta - \theta - \frac{p^2 - p^1}{q^2 - q^1} \frac{p^1}{q^1} \\
D^2 = 1 - \theta - 1 - \frac{p^2 - p^1}{q^2 - q^1} 
\]

Inverting the above demand functions, we have:

\[
p^1 = (1 - D^1 - D^2) q^1 \\
p^2 = (1 - D^2) q^2 - D^1 q^1 - ((1 - D^2) r - D^1) q^1 
\]

2.3 Zero Marginal Cost

In this section, we assume zero marginal cost and derive the main results of the
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First, in sub-section 2.3.1, we look at the market equilibrium without any government intervention. Assuming the foreign firm is the low-quality supplier, we examine, in sub-sections 2.3.2 and 2.3.3, the effects of an MQS and a quota respectively. In sub-section 2.3.4, we assume the foreign firm is the high-quality producer and investigate the effect of a quota on the high-quality imports.

### 2.3.1 Market Equilibrium

Each firm chooses its quality and quantity to maximize its own profit, assuming the rival's quality and quantity are given:

\[
\begin{align*}
\text{Max}_{D^1, q^1} [\pi^1 & - p^1 D^1 - F(q^1) - (1 - D^1 - D^2) q^1 D^1 - F(q^1)] \\
\text{Max}_{D^2, q^2} [\pi^2 & - p^2 D^2 - F(q^2) - ((1 - D^2) r - D^1) q^1 D^2 - F(q^2)]
\end{align*}
\]  

(66)

The first- and second-order conditions are:

\[
\begin{align*}
\pi^1_{D^1} & = 0, \quad 1 - 2 D^1 - D^2 = 0 \\
\pi^2_{D^2} & = 0, \quad (1 - D^1 - D^2) D^1 - F'(q^1) = 0
\end{align*}
\]  

(67) and (68)

\[
\begin{align*}
\pi^1_{q^1} & = 0, \quad (1 - D^1 - D^2) D^1 - F'(q^1) = 0 \\
\pi^2_{q^2} & = 0, \quad (1 - D^2) D^2 - F'(q^2) = 0
\end{align*}
\]  

(69) and (70)
\[
\begin{align*}
\pi_{D1}^1 D1 - 2 &< 0, \quad \pi_{q1}^1 q1 - F^*(q^1) < 0 \\
\pi_{D2}^2 D1 - 2 q^2 &< 0, \quad \pi_{q2}^2 q2 - F^*(q^2) < 0
\end{align*}
\] (71)

The market equilibrium \((D^1, q^1, D^2, q^2)\) is characterized by equations (67)-(70).

From the first-order conditions (68), (70) we notice that, in equilibrium, the quality of the high-quality product depends only upon its own quantity while the quality of the low-quality product depends on both its own quantity and the total quantity. Solving equations (67), (69) simultaneously, we obtain the following equilibrium results:

\[
D^1 = \frac{r}{\Delta}, \quad D^2 = \frac{2r - 1}{\Delta}
\] (72)

where \(\Delta = 4r - 1\)

To obtain the relationship between the optimal quantity and quality of a product, we totally differentiate the first-order conditions (67)-(70):

\[
\begin{align*}
\frac{dD^2}{dq^2} &= -2 \frac{dD^1}{dq^1} \\
(1 - 2D^1 - D^2) \frac{dD^1}{dq^1} - D^1 \frac{dD^2}{dq^2} - F^*(q^1) dq^1 &= 0 \\
(1 - 2D^2) dq^2 - 2q^2 \frac{dD^2}{dq^1} - q^1 \frac{dD^1}{dq^1} - D^1 dq^1 &= 0
\end{align*}
\] (73)

The relationship between the optimal quantity and quality of the low- and high-quality products can be derived, respectively, as follows:

\[
\begin{align*}
\frac{dD^1}{dq^1} &= \frac{r^2 \Delta F^*(q^2)}{2r - \Delta^3 q^2 F^*(q^2)} \\
\frac{dq^1}{dD^1} &= \frac{2r}{\Delta F^*(q^1)}
\end{align*}
\] (74)
\[
\frac{dD^2}{dq^2} = \frac{2 \Delta F^*(q^1)}{2 \Delta^3 q^1 F^*(q^1)} - \frac{1}{\Delta F^*(q^2)} \tag{75}
\]

Defining \(x = 2 r - \Delta^3 q^2 F^*(q^2)\), \(y = 2 r^2 - \Delta^3 q^1 F^*(q^1)\), then we have

\[\text{Lemma 1. } x = 2 r - \Delta^3 q^2 F^*(q^2) < 0, \quad y = 2 r^2 - \Delta^3 q^1 F^*(q^1) < 0.\]

Proof.

\[
x = 2 r - \Delta^3 q^2 F^*(q^2) < 2 r - 3 \Delta^2 q^2 F^*(q^2)
\[
< 2 r - 3 \Delta^2 F^*(q^2) - 2 r - 3 \Delta (2 r - 1) 2 r < 0
\]

\[
y = 2 r^2 - \Delta^3 q^1 F^*(q^1) < 2 r - 3 \Delta^3 < 0
\]

where we have used the facts that \(r > 1\), \(F^*(q) > F'(q)\) and equations (68) and (70).

Combining the results of (74), (75) and lemma 1, we obtain the following proposition.

**Proposition 1.** In equilibrium, the optimal quantity of a product is positively correlated with its optimal quality.

We can show proposition 1 graphically in figure 8 and figure 9. Figure 8 and 9 can be understood intuitively as follows: When a firm increases its quality it can reduce the fixed cost per unit by increasing the quantity produced and thus realizing economies of scale. Higher quality, ceteris paribus, increases the demand for its product and since supply of additional units is costless for a given quality the firm will increase the quantity produced. Similarly, increases of quantity produced reduce the fixed cost per unit associated with quality
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Figure 8 The relationship between the optimal quantity and quality of the low-quality product

Figure 9 The relationship between the optimal quantity and quality of the high-quality product
improvement and since investment in quality becomes cheaper a higher investment in quality than in the initial equilibrium is indicated.

Note that these positive correlations between optimal quality and quantity can be reversed when one assumes that the significant costs of quality are variable. For example, Das and Donnenfeld (1989) found a negative correlation assuming convex variable costs of quality. In their case increases in quality increase the marginal costs thus pushing prices upward and quantities demanded downward. The higher quality, however, shifts the demand for the product upward. The combined effect of these opposing forces, because of the convexity of the marginal costs of quality, is to reduce the quantity in the new equilibrium.

We can show (see appendix B1) that the slope of AA (A′A′) is steeper than the slope of BB (B′B′).

2.3.2 MQS

Requiring imported products to meet an MQS is a common practice, yet the effect of an MQS upon the domestic welfare has received little theoretical investigation. As a matter of fact, Das and Donnenfeld (1989) is the only paper that addressed this issue in an international oligopolistic setting.

Assuming that the domestic government sets an MQS level a little bit higher than the market equilibrium level, we find the comparative statics and welfare implications of this specific MQS.

After the imposition of the MQS, $q^1$ is no longer an endogenous variable. Thus, to obtain the comparative static results, we need only to differentiate the first-order conditions
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(67), (69) and (70):

\[
\begin{align*}
\frac{dD^2}{dq^2} - 2 \frac{dD^1}{dq^1} \\
(1 - 2D^2) dq^2 - 2q^2 dD^2 - q^1 dD^1 - D^1 dq^1 &= 0 \\
(1 - 2D^2) dD^2 - F(q^2) dq^2 &= 0
\end{align*}
\] (78)

Substituting the results of (72) and solving these equations in terms of \(dq^1\), we have:

\[
\begin{align*}
\frac{dD^2}{dq^1} &= \frac{2 r^2 \Delta F^*(q^2)}{2 r - \Delta^3 q^2 F^*(q^2)} \\
\frac{dq^2}{dq^1} &= \frac{2 dD^2}{dD^1} - \frac{1}{2} \frac{dD^2}{dq^1} \\
\frac{dD^1}{dq^1} &= -\frac{1}{2} \frac{dD^2}{dq^1}
\end{align*}
\] (79)

Proposition 2. An MQS constraint on low-quality imports

a. increases the volume of imports;

b. decreases the domestic supply;

c. downgrades the quality of the domestic product; and

d. reduces the total sales in the market.

Proof. The proofs of a - c can easily be shown from (79) and lemma 1, so here we only prove d.

\[
\frac{d(D^1 + D^2)}{dq^1} - \frac{dD^1}{dq^1} + \frac{dD^2}{dq^1} - \frac{1}{2} \frac{dD^2}{dq^1} - \frac{1}{2} \frac{dD^2}{dq^1} < 0
\]

Intuitively, an increase in the quality of the low-quality product will increase its sales.

Higher quality increases the demand for the product and since the only costs involved are fixed the exporter will increase its volume so as to enjoy the economies of scale. The increased sales of the low-quality product will reduce the market share of the high-quality
product. The shrinking demand for the high-quality product increases its fixed cost per unit (making quality relatively more expensive). To compensate for this the high-quality firm will lower its quality. Lowering the quality and lowering the price as a consequence allows it to capture some of the demand lost.

In Das and Donnenfeld (1989), forcing the low-quality producer to increase the quality of its exports results in a shrinkage of the total market and a more intense competition between the low-quality exporter and the high-quality domestic producer. To compensate for the reduced differentiation the domestic producer must increase its quality (and price). In this case, there are no economies of scale and thus no special inducement to reduce quality when quantities are lower as was the case in our model.

Using the envelop theorem, the effects of the MQS on the foreign firm's profit \( \pi^f \) and the domestic firm's profit \( \pi^d \) can be obtained as follows:

\[
\frac{d\pi^f}{dq^1} - \frac{d\pi^1}{dq^1} \frac{\partial \pi^1}{\partial q^1} \frac{\partial q^1}{\partial D^1} \frac{\partial q^2}{dq^1} \frac{\partial D^2}{dq^1} \frac{\partial D^2}{dq^1} \frac{\partial D^2}{dq^1} > 0 \tag{81}
\]

\[
\frac{d\pi^h}{dq^1} - \frac{d\pi^2}{dq^1} \frac{\partial \pi^2}{\partial q^1} \frac{\partial q^1}{\partial D^1} \frac{\partial q^2}{dq^1} \frac{\partial D^2}{dq^1} \frac{\partial D^2}{dq^1} \frac{\partial D^2}{dq^1} < 0 \tag{82}
\]

The Consumers' Surplus (CS) is:

\[
CS = q^1 \int_{\theta_0}^{\theta_1} \theta d\theta + q^2 \int_{\theta_0}^{\theta_1} \theta d\theta - p^1 D^1 - p^2 D^2 - q^1 \int_{1-D^1}^{1-D^2} \theta d\theta + q^2 \int_{1-D^1}^{1-D^2} \theta d\theta - p^1 D^1 - p^2 D^2 + q^1 \frac{(D^1)^2}{2} + q^2 \frac{(D^2)^2}{2} \tag{83}
\]
Define the domestic welfare function $w$ as the sum of the Consumers' Surplus and the domestic firm's profit, then the effect of the MQS on domestic welfare is:

$$\frac{dW}{dq} - \frac{dCS}{dq} + \frac{d\pi^h}{dq}$$  \hspace{1cm} (84)

We can prove (see appendix B2) that $\frac{dCS}{dq} > 0$, $\frac{dW}{dq} < 0$. Combining these comparative static results leads to the following proposition.

**Proposition 3.** An MQS constraint on low-quality imports

a. increases the foreign firm's profit;

b. decreases the home firm's profit;

c. increases the Consumers' Surplus; and

d. reduces the overall welfare of the home country.

In many models, when some policy affects CS and firm's profit in two opposite ways, the effect upon CS usually dominates the effect upon firm's profit. This, however, does not happen here because in our model the increase in CS is limited by the facts that, after the MQS is imposed, fewer consumers participate in the market (d in proposition 2), fewer consumers enjoy the high-quality products (b in proposition 2) and even those who do enjoy the high-quality product can buy only a lower quality than before the imposition of MQS (c in proposition 2).

2.3.3 A Quota on the Low-quality Imports.

Direct quantitative restrictions on international trade such as quotas have become particularly widespread. In this section, we consider the case in which a quota is set upon the
low-quality imports, in the next section, we consider the case in which a quota is set upon the high-quality imports. Note that unlike the MQS which can only be used to restrict low-quality imports, quotas are more flexible.

If a quota is set at the level just marginally below the market equilibrium \( D^1 \) obtained in sub-section 2.3.1, then the effects of the quota on other decision variables can be represented by the following equations:

\[
\begin{align*}
(1 - 2D^1 - D^2)\,dD^1 - D^1\,dD^2 - F^*(q^1)\,dq^1 & = 0 \\
(1 - 2D^2)\,dq^2 - 2q^2\,dD^2 - q^1\,dD^1 - D^1\,dq^1 & = 0 \\
(1 - 2D^2)\,dD^2 - F^*(q^2)\,dq^2 & = 0
\end{align*}
\]

(85)

From which we have:

\[
\begin{align*}
\frac{dD^2}{dD^1} & = \frac{\Delta^2 q^1 F^*(q^1) F^*(q^2)}{r^2 F^*(q^2) \cdot F^*(q^1) - 2 \Delta^2 q^2 F^*(q^1) F^*(q^2)} \\
\frac{dq^1}{dD^1} & = \frac{q^1 dq^2}{dD^2} \frac{dD^1}{dD^1} - \frac{r}{\Delta D^1} \frac{dD^2}{dD^1} \\
\frac{dq^2}{dD^1} & = \frac{dD^1}{dD^2} \frac{dD^1}{dD^2} - \frac{1}{\Delta F^*(q^2)} \frac{dD^1}{dD^1}
\end{align*}
\]

(86)

Lemma 2. \( \frac{dD^2}{dD^1} < 0 \).

Proof. Since the numerator is positive, to prove lemma 2 we need only to prove

\[
z = r^2 F^*(q^2) \cdot F^*(q^1) - 2 \Delta^2 q^2 F^*(q^1) F^*(q^2) < 0
\]

where we have used the facts that \( r > 1, q F^*(q) > F^*(q) \) and (68).

Proposition 4. A quota on low-quality imports
a. reduces the quality of imports;

b. upgrades the quality of domestic products;

c. expands the sales of domestic product; and

d. reduces the total sales in the market.

Proof. Proving a, b and c is straightforward using lemma 2 and (86). Here we only prove d.

\[
\frac{d(D^1 \cdot D^2)}{dD^1} = 1 + \frac{dd^2}{dD^1} = \frac{r^2 F''(q^2) + F''(q^1) - (2r - 1) q^1 F'(q^1) F'(q^2)}{r}
\]

Since \(r < 0\), to prove \(\frac{d(D^1 \cdot D^2)}{dD^1} > 0\), we need only to prove the numerator is negative.

We divide the proof of d into two cases: 1. The fixed cost function is not very convex (less than cube), that is \(F'''(q) > 0, F''''(q) < 0\); 2. The fixed cost function is very convex: \(F''''(q) > 0\).

Case 1. In this case, \(F'''(q) > 0, F''''(q) < 0\). If we define \(g(q) = F''(q)\), then we have \(g'(q) > 0, g''(q) < 0\) and \(\frac{d(g \cdot g(q) - g(q))}{dq} = q g''(q) < 0\). Since \(g(0) = 0\), we conclude that \(g \cdot g'(q) - g(q) < 0\) for all \(q > 0\). This is equivalent to saying that \(\frac{d(q F''(q) - 2 F'(q))}{dq} \leq 0\), from which we have \(q F''(q) \leq 2 F'(q)\) and \(\frac{F'(q^2)}{F'(q^1)} < \frac{(q^2)^2}{(q^1)^2} - r^2\).

We next prove that in equilibrium \(r > 1.675\). That is, if the fixed cost function is not very convex, then in equilibrium firms supply highly differentiated products. Using the first-order conditions (68) and (70), we have:

\[
r^2 > \frac{F'(q^2)}{F'(q^1)} = \frac{2r(2r - 1)}{r^2}
\]
Solving the above inequality, we have $r > 1.675$.

\[
r^2 F^*(q^2) + F^*(q^1) - (2 - r - 1) \Delta^2 q^1 F^*(q^1) F^*(q^2) < (r^2 + 1) F^*(q^2) - (2 - r - 1) r^2 F^*(q^2) - (2 r^3 - 2 r^2 - 1) F^*(q^2) < -2.787 F^*(q^2) < 0
\]

where we have used (68) and the facts that $F^*(q) q > F'(q)$, $r > 1.675$.

Case 2. If $F(q)$ is very convex (more than cube), then we have $F^*(q) q > 2 F'(q)$. Checking the numerator, we have:

\[
r^2 F^*(q^2) + F^*(q^1) - (2 - r - 1) \Delta^2 q^1 F^*(q^1) F^*(q^2) < (r^2 + 1) F^*(q^2) - (2 - r - 1) 2 r^2 F^*(q^2) - (4 r^3 - 3 r^2 - 1) F^*(q^2) < 0
\]

Thus, we have proved d in proposition 4.

As we have noted before, the presence of economies of scale will induce the domestic firm to increase quality when it increases quantity since quality becomes relatively cheaper. Conversely, the low-quality producer will reduce quality since with fewer units (a quota) the costs of quality per unit will increase. These economies of scale effects are not present when the costs of quality are variable, as was the case in Das and Donnenfeld (1989). In their model, the reduced competition that the high-quality domestic firm faces allows it to reduce its quality and thus reduce differentiation and costs while increasing its sales.

The effects of the quota on the firms' profits and domestic welfare are obtained as follows:

\[
\frac{\partial \pi_f}{\partial D} + \frac{\partial \pi_1}{\partial D} + \frac{\partial \pi_1}{\partial q} \frac{dq}{dq} + \frac{\partial \pi_1}{\partial D} \frac{dq^2}{dq^2} + \frac{\partial \pi_1}{\partial D} \frac{dq^2}{dq^2} \frac{dq^2}{dq^2} > 0
\]

(92)
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\[
\frac{dx^h}{dD_1} - \frac{dx^2}{dD_1} - \frac{\partial x^2}{\partial q^1} \frac{dq^1}{dD_1} - \frac{\partial x^2}{\partial D} \frac{dD}{dD_1} < 0 
\]

(93)

\[
\frac{dW}{dD_1} - \frac{dCS}{dD_1} - \frac{dn^h}{dD_1} 
\]

(94)

We can prove (see appendix B3) that \( \frac{dCS}{dD_1} > 0, \frac{dW}{dD_1} < 0 \).

**Proposition 5.** A quota on low-quality imports

a. reduces foreign firm's profit;

b. increases domestic firm's profit;

c. decreases Consumers' Surplus; and

d. increases the welfare of the home country.

Consumers' Surplus is affected by two opposite forces. It is affected negatively by the reduction of competition (products are more differentiated, from a and b in proposition 4) and lower sales (d in proposition 4). It is affected positively by the increase in the quality of the high-quality product (b in proposition 4) and the increase in its consumption (c in proposition 4). Because of the economies of scale, the former force dominates the latter force and the overall CS is decreased.

From the results of sub-sections 2.3.2 and 2.3.3 we conclude that, if a low-quality product is imported, then the imposition of a quota is welfare increasing while the MQS constraint is welfare reducing. This result is different from the result in Das and Donnenfeld's model. In their model, the effect of a similar quota on the low-quality imports has an
ambiguously effects on domestic welfare.

2.3.4 A Quota on the High-quality Imports.

In this sub-section, we assume that the foreign firm supplies the high-quality product and a quota is set at a level that is marginally lower than the market equilibrium $D^2$. We show below the effects that this quota has upon the sales and quality of the domestic product, the quality of the foreign product, the profits of both firms and domestic welfare.

Differentiating the first-order conditions of (67), (68) and (70), we have:

\[
\begin{align*}
\frac{dD^2}{dD^1} &= 2 \frac{dD^1}{dD^1} \\
(1 - 2D^1 - D^2) \frac{dD^1}{dD^2} - D^1 \frac{dD^2}{dD^1} - F'(q^1) dq^1 &= 0 \\
(1 - 2D^2) \frac{dD^2}{dD^2} - F'(q^2) dq^2 &= 0
\end{align*}
\]

(95)

Solving the above equations in terms of $D^2$ and using (72), we reach the comparative static results as follows:

\[
\begin{align*}
\frac{dD^1}{dD^2} &= - \frac{1}{2} < 0 \\
\frac{dq^1}{dD^2} &= - \frac{r}{\Delta F'(q^1)} < 0 \\
\frac{dq^2}{dD^2} &= \frac{1}{\Delta F'(q^2)} > 0
\end{align*}
\]

(96)

This leads to the following proposition.

**Proposition 6.** A quota on high-quality imports

a. reduces the quality of the imports;

b. upgrades the quality of domestic product;

c. increases the domestic supply; and
d. reduces the overall sales in domestic market.

Note that while in our model the quota leads to a downgrade of the quality of the imports, in Das and Donnenfeld (1989) the opposite result is obtained.

The effects of the quota on firms' profits and domestic welfare can be obtained using a similar approach:

\[
\frac{d\pi^h}{dD^2} - \frac{d\pi^1}{dD^2} \cdot \frac{\partial \pi^1}{\partial d_1} \cdot \frac{dq^1}{dD^2} \cdot \frac{\partial \pi^1}{\partial d_1} \cdot \frac{dD^1}{dD^2} < 0 \quad (97)
\]

\[
\frac{d\pi^f}{dD^2} - \frac{d\pi^2}{dD^2} \cdot \frac{\partial \pi^2}{\partial d_2} \cdot \frac{dq^2}{dD^2} \cdot \frac{\partial \pi^2}{\partial d_2} \cdot \frac{dD^1}{dD^2} - q^1 D^2 \frac{dD^1}{dD^2} - D^1 D^2 \frac{dq^1}{dD^2} > 0 \quad (98)
\]

\[
\frac{dW}{dD^2} - \frac{dCS}{dD^2} \cdot \frac{d\pi^h}{dD^2} \quad (99)
\]

We can prove (see appendix B4) that the sign of \( \frac{dCS}{dD^2} \) is positive while the sign of \( \frac{dW}{dD^2} \) can be either negative or positive. However, in most cases (as long as the fixed cost function \( F(q) \) is not extremely convex) we have \( \frac{dW}{dD^2} > 0 \).

Proposition 7. If the foreign firm supplies the high-quality product, a quota on the imports reduces the profit of the foreign firm while it increases the profit of the domestic firm. The welfare effect is unclear but in most cases a quota on the high-quality imports reduces domestic welfare.

Comparing the results in this sub-section with those in sub-section 2.3.3, we conclude that a quota on the high-quality imports usually has a different welfare implications than a
quota on the low-quality imports. The asymmetric effects of a quota on different quality imports has yet to receive more theoretical and empirical investigation.

2.4 Marginal Cost is Concave in Quality

In this section, instead of assuming zero marginal cost, as we did in the previous section, we assume that the marginal cost of production $c(q)$ is an increasing concave function of quality, i.e., $c'(q) > 0$, $c''(q) < 0$. Note that this assumption includes the case of a linear marginal cost. We further assume that the fixed cost of quality is more than twice as large as the marginal cost, that is $F(q) > 2c(q)$ for all $q \in (0, \bar{q}]$ and $c(0) = 0$. This is a reasonable assumption in many industries (e.g., the pharmaceutical industries).

The outline of this section is the same as that in section 2.3: In sub-section 2.4.1, we look at the free trade equilibrium. Assuming that the foreign firm is the low-quality firm, we examine the effects of an MQS and a quota in sub-section 2.4.2 and sub-section 2.4.3, respectively. In sub-section 2.4.4 we investigate the effect of a quota on the imports of the high-quality product.

We find that all the propositions obtained in the previous section remain valid in this more general model.

2.4.1 Market Equilibrium

Each firm chooses its quality and quantity to maximize its own profit, assuming the rival's quality and quantity are given:
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\[
\begin{align*}
    \text{Max} & \quad [\pi^1 - (p^1 - c(q^1)) D^1 - F(q^1) ] \\
    & \quad - ((1 - D^1 - D^2) q^1 - c(q^1)) D^1 - F(q^1) ] \\
    \text{Max} & \quad [\pi^2 - (p^2 - c(q^2)) D^2 - F(q^2) ] \\
    & \quad - ((1 - D^2) q^2 - D^1 q^1 - c(q^2)) D^2 - F(q^2) ]
\end{align*}
\]

The first- and second-order conditions are:

\[
\begin{align*}
    \pi^1_{D^1} &= 0 : \quad (1 - 2 D^1 - D^2) q^1 - c(q^1) = 0 \quad (101) \\
    \pi^1_{q^1} &= 0 : \quad (1 - D^1 - D^2 - c'(q^1)) D^1 - F'(q^1) = 0 \quad (102) \\
    \pi^2_{D^2} &= 0 : \quad (1 - 2 D^2) q^2 - D^1 q^1 - c(q^2) = 0 \quad (103) \\
    \pi^2_{q^2} &= 0 : \quad (1 - D^2 - c'(q^2)) D^2 - F'(q^2) = 0 \quad (104)
\end{align*}
\]

\[
\begin{align*}
    \pi^1_{D^1 q^1} &= -2 q^1 < 0 , \quad \pi^1_{q^1 q^1} = - F'(q^1) - D^1 c''(q^1) \\
    \pi^2_{D^2 q^2} &= -2 q^2 < 0 , \quad \pi^2_{q^2 q^2} = - F'(q^2) - D^2 c''(q^2) \quad (105)
\end{align*}
\]

From the first-order conditions (101), (103), we obtain the following equilibrium values:

\[
\begin{align*}
    D^1 &= - \frac{r}{\Delta} - \frac{r}{\Delta} \frac{c(q^2)}{q^2} - \frac{2 r}{\Delta} \frac{c(q^1)}{q^1} \\
    D^2 &= - \frac{2 r - 1}{\Delta} - \frac{2 r}{\Delta} \frac{c(q^2)}{q^2} - \frac{1}{\Delta} \frac{c(q^1)}{q^1} \quad (106)
\end{align*}
\]

\textit{Proposition 1'}. In equilibrium, the optimal quantity of a product is positively correlated with
its optimal quality.

We can prove proposition 1' in a similar way to that in section 2.3.

Lemma 3. \[ D^2 \geq \frac{2(r - 1)}{r} D^1, \quad c'(q^2) \leq c'(q^1) \leq \frac{c(q^1)}{q^1} < \frac{r(46r - 15)(18r - 1)}{1024\, \Delta^2 (3r - 1)} \]

Proof. See appendix B5.

Corollary 1. \[ D^1 > \frac{r(46r - 15)}{16\, \Delta (3r - 1)} > \frac{c(q^1)}{q^1}, \quad D^2 > \frac{(2r - 1)(47r - 16)}{16\, \Delta (3r - 1)} > \frac{c(q^2)}{q^2}. \]

Lemma 4. In equilibrium, \( r < 4 \).

Proof. See appendix B6.

If we define \( a = 1 - 2D^1 - D^2 - c'(q^1), \quad b = 1 - 2D^2 - c'(q^2) \), then we have

Lemma 5. \( b > a > 0 \).

Proof. See appendix B7.

Lemma 6. \( \pi_{11}^1 < 0, \pi_{22}^2 < 0 \)

Proof.

\[
\pi_{11}^1 = -F'(q^1) - D^1 c'(q^1) < -\frac{F'(q^1) - D^1 c'(q^1)}{q^1} < -\frac{aD^1}{q^1} \leq 0 \\
\pi_{22}^2 = -F'(q^2) - D^2 c'(q^2) < -\frac{F'(q^2) - D^2 c'(q^2)}{q^2} < -\frac{bD^2}{q^2} < 0
\]

where we have used the results of lemma 5 and equations (102) and (104).

In the next two sub-sections, we assume that the domestic firm supplies the high-quality product while the foreign firm supplies the low-quality product.

2.4.2 MQS
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Assuming that the domestic government sets the MQS level a little bit higher than the market equilibrium level $q^1$ obtained in sub-section 2.4.1, we try to find the comparative statics and welfare implication of this MQS.

With the setting up of the MQS, $q^1$ is no longer an endogenous variable, so we need only to differentiate the first-order conditions (101), (103) and (104) to obtain the comparative static results.

\[ \begin{align*}
   a dq^1 - 2 q^1 dD^1 - q^1 dD^2 &= 0 \\
   b dq^2 - 2 q^2 dD^2 - q^1 dD^1 - D^1 dq^1 &= 0 \\
   b dD^2 + \pi_{22} dq^2 &= 0
\end{align*} \]  

(107)

Solving the above equations in terms of $dq^1$, we have:

\[ \begin{align*}
   \frac{dD^2}{dq^1} &= \frac{(2 D^1 \cdot a) (- \pi_{22})}{2 b^2 + \Delta q^1 \pi_{22}} \\
   \frac{dq^2}{dq^1} &= \frac{dD^2}{dq^1} + \frac{b}{\pi_{22}} dq^1 \\
   \frac{dD^1}{dq^1} &= \frac{1}{2} \frac{dD^2}{dq^1} + \frac{a}{2 q^1}
\end{align*} \]  

(108)

Lemma 1'. $A - 2 b^2 + \Delta q^1 \pi_{22}^2 < 0$.

Proof. See Appendix B8.

Proposition 2'. An MQS constraint on low-quality imports

a. increases the volume of imports;

b. decreases the domestic supply;

c. downgrades the quality of domestic product; and

d. reduces the total sales in the market.
Proof. The proofs of a, b and c are straightforward by using lemma 1' and (108). Here we only prove d.

\[
\frac{d(D^1 \times D^2)}{dq} = \frac{dD^1}{dq} \times \frac{dD^2}{dq} - \frac{(2D^1 - (4r - 2)a)(-\pi_2\pi_2^2)q^1 + 2a b^2}{2A q^1}
\]

For any \( r > 1 \), we can easily show that \( a < \frac{c(q^1)}{q^1} < 0.1 D^1 \). Noticing that \( r < 4 \), we have \( 2D^1 - (4r - 2)a > 0.6 D^1 > 0 \).

So, the numerator: \( (2D^1 - (4r - 2)a)(-\pi_2\pi_2^2) q^1 + 2a b^2 \) is positive.

Since \( A < 0 \), we have \( \frac{d(D^1 \times D^2)}{dq} < 0 \).

As the unit cost is a concave function of quality while the fixed cost of quality is convex, there still exists large economies of scale. As a result, the quantity produced by the firm is positively related to its quality level.

The effects of the MQS on firms' profits are:

\[
\frac{d\pi^f}{dq} = \frac{d\pi^1}{dq} \times \frac{\partial \pi^1}{\partial q^1} \times \frac{d\pi^1}{dq^1} + \frac{\partial \pi^1}{\partial D^1} \times \frac{d\pi^2}{dq} + \frac{\partial \pi^1}{\partial D^2} \times \frac{d\pi^2}{dq^1} + \frac{\partial \pi^1}{\partial D^2} \times \frac{d\pi^2}{dq^1} - D^1 q^1 \frac{dD^2}{dq^1} > 0 \quad (110)
\]

\[
\frac{d\pi^h}{dq} = \frac{d\pi^2}{dq} \times \frac{\partial \pi^2}{\partial q^1} \times \frac{d\pi^2}{dq^1} + \frac{\partial \pi^2}{\partial D^1} \times \frac{d\pi^2}{dq^1} + \frac{\partial \pi^2}{\partial D^2} \times \frac{d\pi^2}{dq^1} + \frac{\partial \pi^2}{\partial D^2} \times \frac{d\pi^2}{dq^1} - D^1 D^2 - q^1 D^2 \frac{dD^1}{dq^1} < 0 \quad (111)
\]

The CS in this section is derived in an identical way as in (83):

\[
CS = q^1 \int_{1-D^1 - D^2}^1 \theta \ d\theta + q^2 \int_{1-D^2}^1 \theta \ d\theta - p^1 D^1 - p^2 D^2 - q^1 \frac{(D^1)^2}{2} - q^2 \frac{(D^2)^2}{2} - q^1 D^1 D^2 \quad (112)
\]
The effect of the MQS on domestic welfare ($w$) is:

$$\frac{dw}{dq^1} = \frac{dCS}{dq^1} + \frac{d\pi^h}{dq^1}$$

We can prove (see appendix B9) that $\frac{dCS}{dq^1} > 0$, $\frac{dw}{dq^1} < 0$.

Proposition 3'. An MQS on low-quality imports

a. increases the foreign firm's profit;

b. decreases the home firm's profit;

c. increases the Consumers' Surplus; and

d. reduces the overall welfare of the home country.

The argument behind this is similar to that for proposition 3 in section 2.3.

2.4.3 A Quota on the Low-quality Imports.

We now consider a case where the domestic government sets up a quota rather than an MQS to restrict the low-quality imports. Suppose a quota is set marginally lower than the market equilibrium. then we have:

$$a \left( dD^1 - D^1 dq^2 \right) + \pi^1 dq^1 - 0$$
$$b \left( dq^2 - 2q^2 dD^2 - q^1 dD^1 - D^1 dq^1 \right) = 0$$

Solving the above equations in terms of $dD^1$ leads to the following comparative statics:
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\[
\frac{dD^2}{dD^1} = -\frac{\pi^1_{11} \pi^2_{22} q^1 - \pi^2_{22} D^1 a}{b^2 (\pi^1_{11}) + (D^1)^2 (\pi^2_{22}) - 2 q^2 \pi^1_{11} \pi^2_{22}}
\]

\[
\frac{dq^1}{dD^1} = -\frac{a}{\pi^1_{11}} - \frac{D^1}{\pi^1_{11}} \frac{dD^2}{dD^1}
\]

\[
\frac{dq^2}{dD^1} = -\frac{b}{\pi^2_{22}} \frac{dD^2}{dD^1}
\]

(115)

Lemma 2'. \( B = b^2 (\pi^1_{11}) + (D^1)^2 (\pi^2_{22}) - 2 q^2 \pi^1_{11} \pi^2_{22} < 0 \)

Proof. See appendix B10.

Proposition 4'. A quota on low-quality imports

a. reduces the quality of imports;

b. upgrades the quality of domestic products;

c. expands the sales of domestic product; and

d. reduces the total sales in the market.

Proof. Proving a, b and c is straightforward and hence is omitted here. Here we prove d. A quota on the low-quality imports reduces the total sales in the market.

\[
\frac{d(D^1 \cdot D^2)}{dD^1} = -\frac{b^2 \pi^1_{11} - D^1 (D^1 \cdot a) \pi^2_{22} - (2 r - 1) q^1 \pi^1_{11} \pi^2_{22}}{B}
\]

We can prove that the numerator \( n = -b^2 \pi^1_{11} - D^1 (D^1 \cdot a) \pi^2_{22} - (2 r - 1) q^1 \pi^1_{11} \pi^2_{22} \) is negative (see appendix B11). Since \( B < 0 \), we conclude that \( \frac{d(D^1 \cdot D^2)}{dD^1} > 0 \).

The effects of the quota on the firms' profits and domestic welfare are obtained as follows:
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\[
\frac{d\pi^h}{dD^1} - \frac{d\pi^2}{dD^1} \cdot \frac{\partial \pi^2}{\partial D^1} \cdot \frac{dq^1}{dq^1} \cdot \frac{\partial q^2}{dD^1} \cdot \frac{\partial \pi^2}{dD^1} \cdot \frac{dD^2}{dD^1} < 0
\] (117)

\[
\frac{d\pi^f}{dD^1} - \frac{d\pi^1}{dD^1} \cdot \frac{\partial \pi^1}{\partial D^1} \cdot \frac{dq^1}{dq^1} \cdot \frac{\partial q^2}{dD^1} \cdot \frac{\partial \pi^1}{dD^1} \cdot \frac{dD^2}{dD^1} \cdot \frac{dD^2}{dD^1} > 0
\] (118)

\[
\frac{dW}{dD^1} \cdot \frac{dCS}{dD^1} \cdot \frac{d\pi^2}{dD^1}
\] (119)

Appendix B12 shows that \(\frac{dCS}{dD^1} > 0, \frac{dW}{dD^1} < 0\).

**Proposition 5'.** A quota on low-quality imports

a. reduces foreign firm's profit;

b. increases domestic firm's profit;

c. decreases Consumers' Surplus; and

d. increases the welfare of the home country.

2.4.4 A Quota on the High-quality Imports.

In this case, the domestic firm is assumed to be the low-quality supplier while the foreign firm is assumed to be the high-quality producer. A quota is set up against the high-quality imports at a level marginally lower than the market equilibrium \(D^2\) obtained from (101)-(104). The comparative statics are:
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\[
a dq^1 - 2 q^1 D^1 - q^1 D^2 = 0
\]
\[
a D^1 - D^1 D^2 + \pi_{11} dq^1 = 0
\]
\[
b dD^2 + \pi_{22} dq^2 = 0
\]

(120)

From which we have:

\[
\begin{align*}
\frac{dD^1}{dD^2} &= \frac{a D^1 - q^1 \pi_{11}}{a^2 + 2 q^1 \pi_{11}} \\
\frac{dq^1}{dD^2} &= \frac{(a + 2 D^1) q^1}{a^2 + 2 q^1 \pi_{11}} \\
\frac{dq^2}{dD^2} &= \frac{b}{-\pi_{22}} > 0
\end{align*}
\]

(121)

Lemma 7. \( C - a^2 \cdot 2 \cdot q^1 \pi_{11} < 0 \).

Proof. See appendix B13.

Proposition 6'. A quota on high-quality imports

a. reduces the quality of imports; 

b. upgrades the quality of domestic product; 

c. increases the domestic supply; and 

d. reduces the overall sales in the domestic market.

Proof. Here we only need to prove d.

\[
\begin{align*}
\frac{d(D^1 \cdot D^2)}{dD^2} &= \frac{a^2 \cdot a D^1 \cdot q^1 \pi_{11}}{a^2 + 2 q^1 \pi_{11}} \\
&> \frac{a^2 \cdot a D^1 - (1 - D^1 - D^2 - 2 c'(q^1)) D^1}{C} - \frac{a^2 \cdot a D^1 - (a + D^1 - c'(q^1)) D^1}{C} \\
&> \frac{(1 D^1)^2 - 0.9 (D^1)^2}{C} > 0
\end{align*}
\]

The effects of the quota on firms' profits and the domestic welfare are:
\[
\frac{d\pi^h}{dD^2} - \frac{d\pi^1}{dD^2} + \frac{\partial \pi^1}{\partial q^1} \frac{dq^1}{dD^2} + \frac{\partial \pi^1}{\partial q^2} \frac{dq^2}{dD^2} - \frac{\partial \pi^1}{\partial D^1} \frac{dD^1}{dD^2}
\]
\[- - \quad D^1 q^1 < 0 \]

\[
\frac{d\pi^f}{dD^2} - \frac{d\pi^2}{dD^2} - \frac{\partial \pi^2}{\partial q^1} \frac{dq^1}{dD^2} + \frac{\partial \pi^2}{\partial q^2} \frac{dq^2}{dD^2} + \frac{\partial \pi^2}{\partial D^1} \frac{dD^1}{dD^2}
\]
\[- - q^1 D^2 \frac{dD^1}{dD^2} - D^1 D^2 \frac{dq^1}{dD^2} > 0 \]

\[
\frac{dW}{dD^2} = \frac{dCS}{dD^2} + \frac{d\pi^h}{dD^2}
\]

We can prove (see appendix B14) that the sign of \(\frac{dCS}{dD^2}\) is always positive while the sign of \(\frac{dW}{dD^2}\) can be either negative or positive. However, in most cases (as long as the fixed cost function \(F(q)\) is not extremely convex) we have \(\frac{dW}{dD^2} > 0\).

**Proposition 7'.** If the foreign firm supplies the high-quality product, a quota on the imports reduces the profit of the foreign firm while it increases the profit of the domestic firm. The welfare effect is unclear but in most cases a quota on the high-quality imports reduces domestic welfare.

### 2.5 Concluding Remarks

This chapter examines two widely used NTBs, namely quotas and MQS requirements, in a vertically differentiated industry consisting of one foreign and one domestic firm. Assuming that quality improvement involves convex fixed costs and concave variable costs, our analysis shows that: 1. A quota on the low-quality imports improves the home country's welfare; 2. A quota on the high-quality imports reduces, in most cases, the home country's
welfare, and only in some extreme cases a quota on the high-quality imports improves home country welfare. We also find that the optimal quantity and quality of a firm are positively correlated, while a firm's optimal quantity and quality are negatively correlated to its rival's optimal quantity and quality, respectively.

If the variable costs, however, are convex instead of concave, then the results depend on the relative importance of the fixed and variable costs. If the economies of scale effects (the effect of the convexity of fixed costs) dominate the effects of the convexity of marginal costs, then the above results remain valid; if, on the other hand, the effects of the convexity of marginal costs dominate the economies of scale effects, then a quota has an ambiguous effect on domestic welfare.

Combining our results with those in Das and Donnenfeld (1989) we can conclude that, for convex fixed costs and either convex and concave variable costs, the imposition of an MQS on the low-quality imports lowers the home country's welfare.
Chapter Three

The Economics of Certifying Unobservable Qualities

3.1 Introduction

Asymmetries between buyers and sellers about product qualities are important causes of market failures. Such asymmetries can imply that only poor quality products are attracted to the market although consumers are willing to pay for a higher quality and suppliers can offer the desired quality (see Akerlof 1970). The qualities of a product can be defined in terms of all those attributes that the consumer values and is willing to pay for. Some of these attributes ("type 1") can be observed by consumers at the point of purchase (e.g., colour, stiffness). There are no asymmetries in information between buyers and sellers with respect to such attributes. Information about some other attributes ("type 2") can be obtained by buyers only after consuming the goods (e.g., product durability and reliability). There are, however, attributes that cannot be discerned by consumers even after consumption ("type 3"). For example, producers may make long term health claims about their products that cannot be verified by consumers. Other examples include consumption of "politically correct" or socially beneficial products. This type of product differentiation by consumers is becoming more important with the evolution of "post materialistic" value systems. This is especially true with regard to the value people place on environmental protection. Many consumers have interests in the environmental impacts caused by the production, packaging and disposal of the products they purchase and are willing to pay more for those products that are more
environmental friendly or "green", i.e., those with a lesser negative impact on the environment. Products produced through processes that have different impacts on the environment are often indistinguishable. Thus there is an asymmetry in information between the consumer and the producer. In some of these cases though actions of consumers are not likely to change the global "stock" of the attribut, be it justice or a healthy environment, "doing the right thing" increases the utility of the individual.

Learning can reduce market failures resulting from asymmetries of information of type 2. Through repeated purchases or sequential consumption consumers can learn (at least partially) about the quality of the product and share this information. Future demand will reflect the improvements in information about quality differences among competing products reducing the extent of the market failure over time (see Schmalensee 1978, Shapiro 1982, 1983, Milgrom and Roberts 1986 and Allen 1994).

In addition, for products in this category, a high quality producer may use promotional strategies including temporary lower prices to encourage consumers to sample the product and learn about its superior quality thus permitting future higher prices (see Nelson 1974). Higher prices, however, may serve under certain conditions as signals of higher quality. This is the case when there exist information asymmetries between consumers and the choices of informed consumers affect market prices and provide signals to the uninformed consumers (see, e.g., Chan and Leland 1982, Cooper and Ross 1984). Consumers can verify the signal through consumption. These mechanisms for correcting market failure do not work effectively in consumption decision involving unobservable attributes (type 3) since the consumer cannot validate through consumption claims made by sellers and sellers may increase prices to signal
Similarly, suppliers of products with type 2 attributes can overcome market failures with respect to asymmetries of information about quality by using their reputation, advertising or warranties to signal higher quality. These "supply" oriented strategies to correct for the market failure are not available or are not as effective in signalling information with respect to unobservable attributes.

Correcting for market failure resulting from asymmetries of information about unobservable attributes (type 3) must involve either a research effort by the consumer or the use of credible third parties that can provide the information to consumers at or before the point of purchase. Since there are economies of scale in the generation of such information, the establishment of such "third party" organizations is a precondition for the emergence of a market for products with higher unobservable quality. Typically, such organizations generate the information and certify the product. Sellers apply and pay for the certification and use product labels or advertising to inform consumers.

The importance of unobservable attributes for differentiating products has increased significantly with the growth of the environmental movement. Caincross (1992) reports that in a survey of U.S. consumers 53% of the respondents reported that in the year previous to the survey they had decided not to buy a product because they were worried about its effects on the environment. Consumers, however, are both confused by and distrust claims by sellers about the environmental quality of the products they sell. The distrust is not entirely without merit as Kangun et al., 1991 found that 58% of the advertising of environmental product attributes in their sample contained at least one misleading deceptive claim. The growth of
environmental and other third party credible organizations to assess and certify environmental impact claims of products is facilitating the emergence of separate markets for "environmental friendly" products. The first significant effort of environmental certification began in West Germany in 1977 with the Blue Angel program which now certifies 3,000 products in 52 countries.

The forest products industry provides a current example of a significant transformation of a traditional commodity market into a market with products differentiated by their environmental friendliness. Recently for example, the American Forest and Paper Association identified wood products environmental certification as one of the top issues facing the industry (AFPA 1994). There are several competing types of certification programs in the forest products sector. Several environmental groups or coalition of environmental groups have developed organizations to certify or approve certifiers of forest products. For example, World Wildlife Fund of Nature (WWF) led a coalition of environmental groups in establishing in 1994 the Forest Stewardship Council (FSC), a council that approves certifiers. The Rainforest Alliance have established perhaps the oldest wood certification program "Smart Wood". The competition between different programs of environmental groups' certification is based on alterative degrees of strictness of criteria of qualification and provides consumers with alterative gradation of desired quality of forestry and production practices in terms of their impacts on the environment. Industry groups and governments have also responded to consumers' demands for differentiated products by extending the activities of the International Standard Organization (ISO) to forest certification. An international effort is now underway to articulate the criteria and processes of environmental forest certification.
Though only about 0.5 percent of the international traded forest products are now certified (Baharuddin and Simala, 1994), the rate of growth of certification services and of certified products available to the market is high and there is evidence that a significant market for environmentally friendly forest products exists. "Two willingness-to-pay studies have found that there is a consumer demand for certified wood products. A WWF study found that consumers say they are willing to pay 13.6% more on average for wood products originating from sustainable forests....Another study found that 19% of educated consumers with relatively high incomes claim they are willing to pay more for certified wood products" (Ozanne and Vlasky 1996). Clearly the welfare consequences of certification will also depend on the costs of certification.

In this chapter we focus on the certification of unobservable quality, such as environmental forest certification. We extend previous analyses by (1) considering a multi-dimensional quality (attribute) space, (2) adding an unobservable quality, in addition to the observable quality (type 1), as a decision variable, and (3) assuming a multi-attribute utility function. We investigate the welfare consequences of offering a voluntary certification service in addition to requiring industry to meet some minimum quality standards (MQS).

The chapter shows that providing an option of certifying a higher quality level instead of requiring all firms to meet and obtain certification at a particular designated level (MQS) is welfare increasing when there are no externalities involved. Furthermore, it shows that in this situation the optimal MQS will be lower than the optimal MQS when there is only one mandated level of certification while the higher quality level will be higher than it.

The rest of the chapter proceeds as follows. In the next section, we present the model.
Section 3.3 considers quality certification when consumers' utility functions are additive while section 3.4 examines quality certification when consumers' utility functions are multiplicative. Section 3.5 gives the conclusions.

3.2 The Model

The basic model is duopolistic competition with prices and qualities being the decision variables. We assume that to achieve a higher quality the firm has to invest in fixed assets. (e.g., to mitigate the effects of environmental demands Annual Allowable Cut will be permanently reduced, thus a larger forest land base will be needed to produce the same level of timber). The certification costs are assumed to be proportional to the value of the fixed assets (e.g., the larger forests will also be more expensive to certify since they involve larger area). The choices of unobservable quality levels to be certified are made by the government while the firms make decisions about their prices and quality levels. The choice of unobservable quality level by a firm is restricted by the certification options made available to it by the government. The government must decide how many certification levels (processes) to establish (in this model, either one or two) and what level of quality each certification must assure.

The model is analyzed as a four-stage game. At the first stage, the government agency sets up the certification level or levels, with the full anticipation of the decisions that will be made by firms in the later stages of the game. At the second stage, after observing the decision of the government, each firm simultaneously chooses to apply for one of the certification levels available for the unobservable quality. At stage 3, firms choose their levels
of observable quality simultaneously. At the fourth stage, firms simultaneously announce their prices after knowing all the information about quality levels of both products.

The solution concept employed here is sub-game equilibrium and it is solved by backward induction.

Two primary multi-attribute utility functions are used in this paper. In section 3.3, consumers' utility functions are assumed to be additive and satisfy the condition of mutual independence (Fishburn 1965, Pollak 1967). In section 3.4, consumers' utility functions are assumed to be multiplicative and satisfy the condition of mutual independence (Pollak 1967, Keeney 1968, 1974). While the additive utility function allows complete compensation between quality attributes, the multiplicative utility function suggests that some balance must exist between the quality attributes of a product. Thus, for example, a consumer cannot be easily compensated by improving the reliability or other functional properties of a product when the product may cause significant damage to the environment. Similarly, a product which is completely environmentally friendly but has no satisfactory functional quality may have little value.

It is also assumed throughout the chapter that consumers either buy one unit of the product (from either of the two firms) or make no purchases at all. If we denote $y$ as the level of the observable quality and $c$ as the level of the unobservable quality, then we have, in section 3.3, a net surplus function $U(y, c, p) - \theta (y + c) - p$, while in section 3.4 the function becomes $U(y, c, p) - \theta y c - p$, if a consumer consumes one unit of a product with qualities $y, c$ and pays price $p$, and 0 otherwise. In both sections, the taste parameter $\theta$ is assumed to be uniformly distributed across the population of consumers $[0, 1]$. In this chapter we assume
that all the benefits from consuming a higher quality product accrue to the consumer and there are no externalities involved.

Since the qualities of many products depend largely on the start-up cost (which are sunk costs), for simplicity, we assume that the unit production cost is zero while the fixed costs of production depend on the quality level set for each quality type independently. So the total cost of production is \( TC_i(y, c) = C_i(y) \cdot F_i(c) \), where \( i = 1, 2 \) and both \( C_i(y) \) and \( F_i(c) \) are convex functions, \( C_i'(y) > 0 \), \( C_i''(y) \geq 0 \); \( F_i'(c) > 0 \), \( F_i''(c) \geq 0 \) and \( F(c) \) is more convex than \( C(y) \). To focus on the effects of different quality levels on firms' profits, it is further assumed throughout the chapter that firms are identical in most respects (e.g., they have the same technology and efficiency). The only difference between them is that their products have different qualities. Only the unobservable quality needs to be certified and the certification cost \( K(c) \) is assumed to be proportional to the fixed cost of the unobservable quality, i.e., \( K(c) = \alpha F(c) \), where \( \alpha > 0 \).

Let \( Q \) be the total-quality of a product, then \( Q \) is either \((y + c)\) in section 3.3 or \((y + c)\) in section 3.4. It is obvious that, if both firms supply products with the same total-qualities, then they must charge the same prices and earn zero profits, this is certainly one of the equilibria. We ignore this trivial case and only consider the case where the total-quality is different. Without loss of generality, we assume that firm 2 supplies the high total-quality \((Q^2)\) product with price \( p^2 \) while firm 1 supplies the low total-quality \((Q^1)\) with price \( p^1 \). It is obvious that if firm 1 wants to attract any demand, its price must be less than that of firm 2, that is \( p^1 < p^2 \).

If we define \( a_1, \delta \) in similar ways to those in chapter one, then we have:
We can easily show that consumers with taste parameters $\theta > \hat{\theta}$ will purchase the high total-quality product, consumers with taste parameters $\hat{\theta} < \theta < \hat{\theta}$ will purchase the low total-quality product and consumers with taste parameters $\theta < \hat{\theta}$ will not purchase any product.

In addition, define $\bar{\theta} = \frac{P^2}{Q^2}$, $r = \frac{Q^2}{Q^1}$, where $\bar{\theta}$ is the quality-adjusted price of the high total-quality product and $r$ is the ratio of the high total-quality to the low total-quality. Then we have $\hat{\theta} = \frac{r \bar{\theta} - \theta}{r - 1}$.

The demand functions for the low and high total-quality products can be derived as follows:

\[ D^1 - \hat{\theta} - \bar{\theta} = \frac{P^2 - P^1}{Q^2 - Q^1} \]
\[ D^2 - 1 - \hat{\theta} - 1 = \frac{P^2 - P^1}{Q^2 - Q^1} \]  
(127)

3.3 Additive Consumer Utility Functions

In this section, we assume a consumer's utility function is additive, i.e., $Q \cdot y + c$. The net surplus function then becomes $U(y, c, p) = \theta (y + c) - p$. We start the solution with stage 4.

Stage 4. The Choices of the Prices

In the price game (stage 4), the low total-quality firm maximizes its revenue with respect to $p^1$, taking its qualities and its rival's price and qualities as given, while the high
total-quality firm maximizes its revenue with respect to \( p^2 \), taking its qualities and its rival's price and qualities as given. That is:

\[
\begin{align*}
\text{Max } R^1(p^1, p^2) & = \text{Max } \left[ R^1(\theta, \bar{\theta}) - Q^1 \left( \frac{r \bar{\theta} - \theta}{r - 1} \right) \right] \\
\text{Max } R^2(p^1, p^2) & = \text{Max } \left[ R^2(\theta, \bar{\theta}) - Q^2 \left( 1 - \frac{r \bar{\theta} - \theta}{r - 1} \right) \right]
\end{align*}
\]

The first-order conditions for the revenue maximization are:

\[
\begin{align*}
R^1(\theta, \bar{\theta}) & = \frac{\partial R^1}{\partial \theta} - \frac{r \bar{\theta}}{r - 1} - 2 \frac{\bar{\theta}}{r - 1} - 2 \frac{\theta}{r - 1} = 0 \\
R^2(\theta, \bar{\theta}) & = \frac{\partial R^2}{\partial \theta} - \frac{\theta}{r - 1} - 2 \frac{r \bar{\theta}}{r - 1} = 0
\end{align*}
\]

Solving these equations, we have the following results:

\[
\begin{align*}
\theta & = \frac{r - 1}{\Delta_1} \\
\bar{\theta} & = 2 \theta - \frac{2 (r - 1)}{\Delta_1} \\
\hat{\theta} & = \frac{2 r - 1}{\Delta_1}
\end{align*}
\]

where \( \Delta_1 = 4 r - 1 \)

Checking the second-order conditions, we have:

\[
\begin{align*}
R^1_{11}(\theta, \bar{\theta}) & = - \frac{2 r}{r - 1} < 0, & R^2_{22}(\theta, \bar{\theta}) & = - \frac{2 r}{r - 1} < 0 \\
R^1_{12}(\theta, \bar{\theta}) & = \frac{r}{r - 1} > 0, & R^2_{21}(\theta, \bar{\theta}) & = \frac{1}{r - 1} > 0
\end{align*}
\]

In equilibrium, the demands for the low and high total-quality products are:
Substituting these equilibrium values into the revenue functions of both firms yields, after some simplification:

\[
\begin{align*}
D^1 &= \frac{\hat{\theta} - \theta}{\Delta_1} \\
D^2 &= 1 - \frac{2r}{\Delta_1} - 2D^1
\end{align*}
\]  

(132)

The Consumers' Surplus (CS) is:

\[
CS = Q^1 \int_{\theta}^{\hat{\theta}} d\theta + Q^2 \int_{\theta}^{\hat{\theta}} d\theta - p^1 (\hat{\theta} - \theta) - p^2 (1 - \hat{\theta}) - \frac{r^2 (4r + 5)}{2\Delta_2} Q^1
\]  

(134)

We can now move to the third-stage: the observable quality game.

Stage 3. The Choices of the Observable Qualities

In the third-stage game, each firm chooses its observable quality to maximize its profit, taking the other firm's observable quality and both unobservable qualities as given.

\[
\begin{align*}
\max_{y^2} & \quad [\Pi - R(c^2) - F(y^2, c^2) - \frac{4r(r - 1)}{\Delta_2^2} (y^2 \cdot c^2) - C(y^2) - F(c^2)] \\
\max_{y^1} & \quad [\pi - R(c^1) - F(y^1, c^1) - \frac{r(r - 1)}{\Delta_1^2} (y^1 \cdot c^1) - C(y^1) - F(c^1)]
\end{align*}
\]
The Kuhn-Tucker first-order conditions are:

\[
\begin{align*}
\pi_i^t & + u_i^t = 0 \\
 u_i^t y_i^t & = 0 \\
y_i^t & = 0 \\
u_i^t & = 0 \\
i & = 1, 2
\end{align*}
\]

where \( u_i^t, i = 1, 2 \) are Lagrange multipliers.

It follows that the first-order conditions for the high total-quality firm are:

\[
\begin{align*}
\Pi_2 - f(r) - C'(y_2) & = 0, \quad \text{if} \quad y_2 > 0 \\
\Pi_2 - f(r) - C'(y_2) & < 0, \quad \text{only if} \quad y_2 = 0
\end{align*}
\]

The low total-quality firm's first-order conditions become:

\[
\begin{align*}
\pi_1 - g(r) - C'(y_1) & = 0, \quad \text{if} \quad y_1 > 0 \\
\pi_1 - g(r) - C'(y_1) & < 0, \quad \text{only if} \quad y_1 = 0
\end{align*}
\]

where

\[
\begin{align*}
f(r) = & \frac{dR^2}{dy^2} = \frac{4 r (4 r^2 - 3 r + 2)}{(4 r - 1)^3}, \\
g(r) = & \frac{dR^1}{dy^1} = \frac{r^2 (4 r - 7)}{(4 r - 1)^3}
\end{align*}
\]

From (138) we notice that the low total-quality firm will produce a positive level of the observable quality if and only if, in equilibrium, there is a sufficient total-quality differentiation \((r > 7/4)\) between these two products. If \( r \leq 7/4 \), the low total-quality firm will choose \( y_1 = 0 \). The assumption of \( C''' > 0 \) ensures the existence of a positive \( y_1 \) in the equilibrium (see theorem 1 in Ronnen 1991).

Using the second-order conditions, we can easily confirm that the reaction functions of the observable quality are positively sloped.
Lemma 1. A producer will obtain a high total-quality if and only if it has a high observable quality. That is, $Q^2 > Q^1 \Rightarrow y^2 > y^1$.

Proof.

1. $Q^2 > Q^1 \Rightarrow y^2 > y^1$. Since $f(r)$ is larger than $g(r)$ for all $r$, using the first-order conditions, we have $C'(y^2) > C'(y^1)$. Noticing that $C'(y) > 0$, we conclude $y^2 > y^1$.

2. $y^2 > y^1 \Rightarrow Q^2 > Q^1$. If not, that is $Q^2 < Q^1$, we have $r \cdot \frac{Q^1}{Q^2} > 1$, the first-order conditions become $f(r) - C'(y^1) - g(r) - C'(y^2)$, from which we conclude $y^1 > y^2$, a contradiction.

Lemma 1 says that, the choice of the unobservable quality has no direct effect on the ratio of the total-qualities. It is the choice of the observable quality (the last choice of quality) that has a decisive influence on the total-quality of a product.

Stage 1 and 2. The Government's Decision about Certification Options and Firms' Choice of the Unobservable Qualities

There are two cases with respect to the decision of the unobservable quality. In one case the government offers two different certification levels of the unobservable quality (the higher one is optional while the lower one is compulsory) and each firm applies for a different level; in the other case the government offers no choice and requires both firms to obtain the certification at the MQS level.

Case One. Each firm applies for a certification of a different quality level.

In this case, we assume that government sets up two different certification levels for the unobservable quality and each firm applies for a different one. Suppose the high total-
quality firm obtains certification \( c^2 \), while the low total-quality firm obtains certification \( c^1 \) where \( c^1 \neq c^2 \).

We first consider the effects of a marginal change of the unobservable qualities upon the decision of the observable qualities. To do so, we totally differentiate the first-order conditions of (137) and (138):

\[
\begin{align*}
C^* (y^2) \frac{dy^2}{dr} - df(r) \frac{(dc^2 \cdot dy^2)(c^1 \cdot y^1) - (dc^1 \cdot dy^1)(c^2 \cdot y^2)}{(c^1 \cdot y^1)^2} \\
C^* (y^1) \frac{dy^1}{dr} - dg(r) \frac{(dc^2 \cdot dy^2)(c^1 \cdot y^1) - (dc^1 \cdot dy^1)(c^2 \cdot y^2)}{(c^1 \cdot y^1)^2}
\end{align*}
\]

where \( df(r) \cdot \frac{df(r)}{dr} - \frac{8 (5 r + 1)}{(4 r - 1)^4} < 0 \), \( dg(r) \cdot \frac{dg(r)}{dr} - \frac{2 r (8 r - 7)}{(4 r - 1)^4} > 0 \)

Solving the above equations in terms of \( dc^1, dc^2 \), we have

\[
\begin{align*}
\frac{dy^2}{dr} &= \frac{df(r) C^* (y^1) (dc^2 - r dc^1)}{\Delta} \\
\frac{dy^1}{dr} &= \frac{dg(r) C^* (y^2) (dc^2 - r dc^1)}{\Delta}
\end{align*}
\]

where \( \Delta = r dg(r) C^* (y^2) - df(r) C^* (y^1), Q^1 C^* (y^1) C^* (y^2) > 0 \).

Lemma 2. In equilibrium, the level of the observable quality of a firm's product is negatively correlated to the level of its unobservable quality while it is positively correlated to the level of the unobservable quality of its rival's product.

Proof. From (140), we have
Chapter 3. The Economics of Certifying Unobservable Qualities

\[
\begin{align*}
\frac{dy^2}{dc^2} - \frac{df(r) C^*(y^1)}{\Delta} < 0, & \quad \frac{dy^2}{dc^1} - \frac{df(r) C^*(y^1)}{\Delta} > 0 \\
\frac{dy^1}{dc^2} - \frac{dg(r) C^*(y^2)}{\Delta} > 0, & \quad \frac{dy^1}{dc^1} - \frac{dg(r) C^*(y^2)}{\Delta} < 0
\end{align*}
\]
(141)

Since in this section consumers' utility functions are assumed to be additive: \( U(y, c) = u(y + c) \), these two types of quality of a product are perfect substitutes, so the observable quality is negatively correlated to the unobservable quality; From chapter one, we have proved that, under Bertrand competition, the quality (total-quality) reaction functions are positively sloped, that is \( \frac{dQ^2}{dc^2} > 0 \) \( \frac{dQ^1}{dc^1} > 0 \). Since \( \frac{dQ^2}{dc^2} - 1 \cdot \frac{dy^2}{dc^2} > 0 \), we have \( \frac{dy^2}{dc^2} - \frac{dQ^2}{dc^2} > 0 \); Also, since \( \frac{dQ^1}{dc^1} - 1 \cdot \frac{dy^1}{dc^1} > 0 \), we have \( \frac{dy^1}{dc^1} - \frac{dQ^1}{dc^1} > 0 \). Thus, we conclude that the level of the observable quality of a firm's product is positively correlated to the level of the unobservable quality of its rival's product.

Also notice,

\[
\left| \frac{dy^1}{dc^1} - \frac{dy^1}{dc^2} \right| - \frac{dg(r) C^*(y^2) (r - 1)}{\Delta} > 0, \quad \left| \frac{dy^2}{dc^1} - \frac{dy^2}{dc^2} \right| - \frac{df(r) C^*(y^1) (r - 1)}{\Delta} < 0,
\]

that is the choice of the observable quality of the low total-quality product is more affected by the choice of its unobservable quality than by the rival's choice of unobservable quality, while the choice of the observable quality of the high total-quality product is less affected by the choice of its unobservable quality than by the rival's choice of unobservable quality.

Now we are in a position to examine the decision of the unobservable quality. As assumed before, the exact levels \( (c^1, c^2) \) of the unobservable quality are set up (chosen) by the government while firms only have freedom to choose from these two levels.
From the government's point of view, the objective of setting up the two certification levels \( c^1, c^2 \) is to maximize the social welfare function \( W \), which is defined as

\[
W = CS + \Pi + \sum_i \alpha F(c^i)
\]

After some calculation, we have:

\[
W = r (6 r - 2) Q^2 + r (3 r - 2) Q^1
- C(y^1) - C(y^2) - (1 + \alpha) F(c^1) - (1 + \alpha) F(c^2)
\]

where \( \alpha F(c) \) is the certification cost.

The optimal levels the government will set for the unobservable quality certification are characterized by the following first-order conditions:

\[
\frac{dW}{dc^2} = \frac{\partial W}{\partial r} \frac{\partial W}{\partial c^2} + \frac{\partial W}{\partial y^1} \frac{\partial W}{\partial y^2} + \frac{\partial W}{\partial y^2} \frac{\partial W}{\partial c^2} = 0
\]

\[
\frac{dW}{dc^1} = \frac{\partial W}{\partial r} \frac{\partial W}{\partial c^1} + \frac{\partial W}{\partial y^1} \frac{\partial W}{\partial y^1} + \frac{\partial W}{\partial y^2} \frac{\partial W}{\partial c^1} + \frac{\partial W}{\partial c^1} \frac{\partial W}{\partial c^1} = 0
\]

By straightforward calculation, the above first-order conditions become:

\[
\frac{dW}{dc^2} = \frac{3 r^2}{2} \frac{dy^1}{dc^2} + \frac{8r^3 - 6r^2}{2} \frac{dy^2}{dc^2} + \frac{24r^3 - 18r^2}{\Delta_1^3} \frac{dy^2}{dc^2} \frac{1}{\Delta_1} - (1 + \alpha) F'(c^2) = 0
\] (145)

\[
\frac{dW}{dc^1} = \frac{3 r^2}{2} \frac{dy^1}{dc^1} + \frac{8r^3 - 6r^2}{2} \frac{dy^2}{dc^1} + \frac{20r^3 - 17r^2}{\Delta_1^3} \frac{dy^2}{dc^1} \frac{1}{2 \Delta_1} - (1 + \alpha) F'(c^1) = 0
\] (146)

Lemma 3. It is socially optimal to have the firm with the high total-quality product obtain a high unobservable quality certification.

Proof. See appendix C1.
Combining the results in lemma 1, lemma 3 amounts to saying that it is socially optimal to have firm with the high (low) total-quality supply both the high (low) observable and the high (low) unobservable quality. By supplying both high (low) observable and unobservable quality, products are further differentiated and thus maximum product differentiation occurs. There are several forces at play here. The quality adjusted prices are higher which leads to losses in Consumers' Surplus. These losses are, however, more than compensated by increases in firms' profits.

It is interesting to compare the optimal choices made by the government about the unobservable quality levels to be certified with the optimal choices of firms had they been allowed to choose the unobservable quality without constraints. If firms can choose freely their unobservable quality levels (rather than just picking up one of the two points set up by the government), then each firm will choose its unobservable quality so as to maximize its profit (taking into account the consequences of this choice on the choice of the observable qualities).

\[
\frac{d\Pi}{dc^2} = \frac{\partial \Pi}{\partial r} \frac{\partial r}{dc^2} + \frac{\partial \Pi}{\partial y^2} \frac{\partial y^2}{dc^2} = 0
\]

\[
\frac{d\pi}{dc^1} = \frac{\partial \pi}{\partial r} \frac{\partial r}{dc^1} + \frac{\partial \pi}{\partial y^1} \frac{\partial y^1}{dc^1} = 0
\]

After some simplification, the above first-order conditions become:

\[
F'(e^2) - f(r) = \frac{4 \ r^2 \ (2 \ r - 1)}{(4 \ r - 1)^3} \frac{\partial y^1}{dc^2}
\]

\[
F'(e^1) - g(r) = \frac{(2 \ r - 1)}{(4 \ r - 1)^3} \frac{\partial y^2}{dc^1}
\]

(148)
Lemma 4. The firm supplying the high total-quality product has an incentive to choose a high unobservable quality while the low total-quality firm will offer a low unobservable quality product. That is, if $Q^2 > Q^1$, we must have $c^2 > c^1$.

Proof. Noticing that $d g(r) C^*(y^2) < \Delta$, $d f(r) C^*(y^1) < \Delta$, we have $\frac{d y^1}{d c^2} < 1$, $\frac{d y^2}{d c^1} < r$.

$$F'(c^2) - F'(c^1) > f(r) - g(r) - \frac{4r^2 (2 r + 1)}{\Delta^3_1} - \frac{(2 r + 1) r}{\Delta^3_1} - \frac{r (r - 1) (4 r - 7)}{\Delta^3_1} > 0$$

Although it is optimal, from both the government's and firms' points of view, to have the high (low) total-quality firm supply the high (low) unobservable quality, the optimal levels of the unobservable quality chosen by the government are usually different from those chosen by the firms. Only in very few cases (if any, when $a$ takes some specific values) $c^1$ or $c^2$ obtained from (145), (146) and (148) will be the same.

From (140), we have $\frac{d r}{d c^2} - \frac{Q^1 C^*(y^1) C^*(y^2)}{\Delta} > 0$, $\frac{d r}{d c^1} - \frac{r Q^1 C^*(y^1) C^*(y^2)}{\Delta} < 0$.

That is, the larger the $c^2$, the larger the $r$ and the total-quality $Q^2$ will be; on the other hand, the larger the $c^1$, the smaller the $r$ and the larger the total-quality $Q^1$ will be.

Case Two. There is only one certification level and both firms must obtain this certification.

In this case, the ratio of the high total-quality to the low total-quality is $r - \frac{c \cdot y^2}{c \cdot y^1}$, the first-order conditions with regard to $y^1, y^2$ are virtually the same as those in case one. Using a similar approach, we can prove that lemma 1 remains valid.

Similar to case one, we can get the comparative static results as follows:
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\[
\frac{dy}{dc} - \frac{df(r) C^*(y^1)}{\Delta} (r - 1) > 0
\]

\[
\frac{dy}{dc} - \frac{dg(r) C^*(y^2)}{\Delta} (r - 1) < 0
\]

\[
\frac{dr}{dc} - \frac{(r - 1) Q^1 C^*(y^1) C^*(y^2)}{\Delta} < 0
\]

where \( \Delta = r \frac{dg(r) C^*(y^2) + Q^1 C^*(y^1) C^*(y^2) - df(r) C^*(y^1)}{\Delta} > 0 \)

We can derive the social welfare function in an identical way as in (143),

\[
W = r \frac{(6 r - 2)}{(4 r - 1)^2} (e - y^2) + \frac{r (3 r - 2)}{(4 r - 1)^2} (e - y^1) - C(y^1) - C(y^2) - 2 (1 + \alpha) F(e)
\]

The optimal certification level is characterized by the following equation:

\[
\frac{dW}{dc} - \frac{\partial W}{\partial r} \frac{dr}{dc} + \frac{\partial W}{\partial y^1} \frac{dy^1}{dc} + \frac{\partial W}{\partial y^2} \frac{dy^2}{dc} + \frac{\partial W}{\partial e} = 0
\]

After some simplification, we have:

\[
2 (1 + \alpha) F'(e) - \frac{3 r^2}{2} \frac{dy^1}{dc} + \frac{8 r^4 - 6 r^2 - 3 r + 1}{\Delta_1^3} \frac{dy^2}{dc} + \frac{68 r^3 - 53 r^2 + 10 r + 2}{2 \Delta_1^3} = 0
\] (153)

**Proposition 1.** If both firms must obtain certifications, it is socially optimal to set two different certification levels than just one certification level, i.e., case one dominates case two.

**Proof.**

To prove proposition 1, we need only to prove that it is welfare improving to marginally increase (decrease) the certification level from the optimal level \( c \) obtained in
case two for the high (low) total-quality supplier. This is equivalent to proving that 
\( F'(e^2) > F'(e), \quad F'(e^1) < F'(e) \) at the point \( c^1 - c^2 - c \). This is shown in appendix C2.

### 3.4 Multiplicative Consumer Utility Functions

In this section, consumers' utility functions are assumed to be multiplicative, the preferences for the different qualities of a product are assumed to be independent. Thus, we have \( Q - y c \) and the net surplus function becomes \( U(e, y, p) = y e - p \). Again, there are two cases with regard to the number of certification levels of the unobservable quality.

**Case One. Each firm buys a different certification.**

In this case, the ratio of the high total-quality to the low total-quality is \( r \frac{e^2 y^2}{e^1 y^1} \). Again, we proceed by backward induction.

**Stage 4. The Choices of the Prices**

In a similar fashion to the derivation for the case of the additive utility function in (128)-(134), we can obtain the following profit functions:

\[
\Pi = R(e^2) - F(y^2, e^2) - \frac{4 r (r - 1)}{(4 r - 1)^2} (y^2 e^2) - C(y^2) - F(e^2)
\]

\[
\pi = R(e^1) - F(y^1, e^1) - \frac{r (r - 1)}{(4 r - 1)^2} (y^1 e^1) - C(y^1) - F(e^1)
\]

**Stage 3. The Choices of the Observable Qualities**

In this stage, each firm chooses its observable quality to maximize its profit, taking
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the rival's observable quality and both unobservable qualities as given. The first-order conditions with regards to firms' choices of the observable quality are:

\[ \Pi_2 - c^2 f(r) - C'(y^2) = 0 \]
\[ \Pi_2 - c^1 g(r) - C'(y^1) = 0 \]

where \( f(r), g(r) \) are defined in a same way as before.

Note that here we encounter the requirement of minimum quality differentiation \((r > 7/4)\). In other words, under the assumption of multiplicative utility function, fixed costs are compatible with Bertrand competition if and only if the products are sufficiently differentiated.

Lemma 1'. A firm will produce a high total-quality if and only if it chooses to produce a high observable quality. That is, \( Q^2 > Q^1 \Rightarrow y^2 > y^1 \).

Proof.

1. \( Q^2 > Q^1 \Rightarrow y^2 > y^1 \). From the first-order conditions, we have

\[ y^2 C'(y^2) - y^2 (c^2 f(r)) - Q^2 f(r) > Q^1 f(r) > Q^1 g(r) - y^1 C'(y^1) \]

Since \( (y C'(y))^' > 0 \), we conclude that \( y^2 > y^1 \).

2. \( y^2 > y^1 \Rightarrow Q^2 > Q^1 \). If not, that is \( Q^2 < Q^1 \), then we have \( r - \frac{Q^1}{Q^2} > 1 \), the first-order conditions become \( c^1 f(r) - C'(y^1), c^2 g(r) - C'(y^2) \). Since \( y^2 > y^1, c^2 y^2 < c^1 y^1 \), we must have \( c^1 > c^2 \). Thus, \( C'(y^1) - c^1 f(r) > c^1 g(r) > c^2 g(r) - C'(y^2) \) which leads to \( y^1 > y^2 \), a contradiction.

We next examine the comparative static effects of \( c^i \) upon \( y^j \), where \( i = 1, 2; j = 1, 2 \).

Totally differentiating the first-order conditions, we have:
\[
g(r) \, dc^1 \cdot c^1 \, dg(r) \, y^2 \, dc^2 \cdot c^2 \, dy^2 \cdot r \left( c^1 \, dy^1 \cdot y^1 \, dc^1 \right) \cdot C^*(y^1) \, dy^1 \\
\]
\[
f(r) \, dc^2 \cdot c^2 \, df(r) \, y^2 \, dc^2 \cdot c^2 \, dy^2 \cdot r \left( c^1 \, dy^1 \cdot y^1 \, dc^1 \right) \cdot C^*(y^2) \, dy^2
\]

Solving the above equations in terms of \(dc^1, dc^2\), we obtain:

\[
\begin{align*}
\frac{dy^2}{dc^2} &= \frac{y^2 \left( f(r) \, dg(r) \, r \, c^1 \cdot f(r) \, C^*(y^1) \, y^1 \cdot r \, df(r) \, C^*(y^1) \, y^1 \right) \, dc^2}{\Delta} \\
\frac{dy^1}{dc^1} &= \frac{- r^2 \, df(r) \, y^1 \left( g(r) \, c^1 \cdot C^*(y^1) \, y^1 \right) \, dc^1}{\Delta} \\
\end{align*}
\]

where \(\Delta = r \, dg(r) \, C^*(y^2) \, y^2 \, c^1 \cdot y^1 \, y^2 \, C^*(y^1) \, C^*(y^2) - r \, c^2 \, df(r) \, y^1 \, C^*(y^1) > 0\).

Lemma 2'. In equilibrium, the observable quality of the high total-quality firm is positively correlated to both its own and its rival's unobservable quality; while the observable quality of the low total-quality firm is positively correlated to its rival's unobservable quality, its relationship with its own unobservable quality is ambiguous.

Proof. From (157) we have the following results:

\[
\begin{align*}
\frac{dy^2}{dc^2} &= \frac{y^2 \left( f(r) \, dg(r) \, r \, c^1 \cdot f(r) \, C^*(y^1) \, y^1 \cdot r \, df(r) \, C^*(y^1) \, y^1 \right) > 0}{\Delta} \\
\frac{dy^1}{dc^1} &= \frac{- r^2 \, df(r) \, y^1 \left( g(r) \, c^1 \cdot C^*(y^1) \, y^1 \right) > 0}{\Delta} \\
\frac{dy^1}{dc^2} &= \frac{y^2 \, dg(r) \left( f(r) \, c^2 \cdot C^*(y^2) \, y^2 \right) > 0}{\Delta} \\
\frac{dy^1}{dc^1} &= \frac{y^1 \left( - r \, df(r) \, g(r) \, c^2 \cdot C^*(y^2) \, y^2 \, dg(r) \, r \cdot C^*(y^2) \, y^2 \, g(r) \right) < 0}{\Delta}
\end{align*}
\]
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Since the utility function in this section is assumed to be multiplicative: \( U(y, c) = 0 \ y \ c \), the observable and the unobservable quality of a product are complements. So the observable quality of the firm is positively correlated to its unobservable quality.

After some straightforward calculation we have \( \frac{dy^2}{dc^2} - \frac{dy^1}{dc^1} > 0 \), that is, contrary to the results in the previous section, the choice of the observable quality of the high total-quality product is more affected by the choice of its unobservable quality than by the choice of the rival's unobservable quality. However, we also have \( \frac{dy^1}{dc^1} - \frac{dy^1}{dc^2} > (>) 0 \), that is, we don't know which unobservable quality has a dominant effect on the choice of the observable quality of the low total-quality product.

Stage 1 and 2. The Government's Decision about Certification Options and Firms' Choices of the Unobservable Qualities

In stage 1, the government sets up the optimal levels of the unobservable quality to be certified so as to maximize the social welfare. The social welfare function is:

\[
W = \frac{r (6 r - 2)}{(4 r - 1)^2} c^2 y^2 - \frac{r (3 r - 2)}{2 (4 r - 1)^2} c^1 y^1 - C(y^1) - C(y^2) - (1 - \alpha) F(c^1) - (1 - \alpha) F(c^2) - (4 r - 1) \]

The first-order conditions with respect to \( c^1, c^2 \) are somewhat different from those in previous section.

\[
\frac{dW}{dc^1} = \frac{3 r^2}{2 \Delta^2} c^1 \frac{dy^1}{dc^1} + \frac{8 r^3 - 6 r^2 - 3 r + 1}{\Delta^2} \frac{dy^2}{dc^2} + \frac{24 r^3 - 18 r^2 - 5 r + 1}{\Delta} y^2 - (1 - \alpha) F'(c^2) = 0
\]

\[
\frac{dW}{dc^2} = \frac{3 r^2 c^2}{2 \Delta^2} \frac{dy^1}{dc^1} + \frac{8 r^3 - 6 r^2 - 3 r + 1}{\Delta^2} \frac{dy^2}{dc^2} + \frac{20 r^2 - 17 r + 2}{\Delta} y^1 - (1 - \alpha) F'(c^1) = 0
\]

(161)
Lemma 3'. It is socially optimal to have the firm with the high total-quality product obtain a high unobservable quality certification.

Proof. See appendix C3.

Case Two. There is only one certification level and both firms must certify their products.

The ratio of the high total-quality to the low total-quality in this case becomes

\[ r = \frac{c y^2}{c y^1} \cdot \frac{y^2}{y^1} \]. Note that in this case \( r \) does not depend directly upon the level of the unobservable quality.

The comparative static effects are:

\[
\begin{align*}
\frac{dy^1}{dc} &= c \frac{dg(r)}{\Delta} (\frac{y^2 f(r)}{y^1} + \frac{g(r)}{r} - y^2 C^*(y^2)) > 0 \\
\frac{dy^2}{dc} &= \frac{y^2 f(r)}{\Delta} (c \frac{dg(r)}{\Delta} r - y^1 C^*(y^1)) - y^1 g(r) \frac{r}{\Delta} C'(y^2(')) > 0
\end{align*}
\]

where \( \Delta = r \frac{dg(r)}{\Delta} c y^2 C^*(y^2) + y^1 y^2 C^*(y^1) \frac{C(y^1)}{\Delta} - r \frac{df(r)}{\Delta} c y^1 C^*(y^1) > 0 \)

The social welfare function in this case becomes

\[
W = \frac{r (6 r - 2)}{(4 r - 1)^2} (c y^2) + \frac{r (3 r - 2)}{2 (4 r - 1)^2} (c y^1) - C(y^1) - C(y^2) - 2 (1 + a) F(c)
\]

The optimal certification level is characterized by the following equation:

\[
\frac{dW}{dc} = \frac{\partial W}{\partial r} \frac{\partial r}{dc} + \frac{\partial W}{\partial y^1} \frac{\partial y^1}{dc} + \frac{\partial W}{\partial y^2} \frac{\partial y^2}{dc} + \frac{\partial W}{\partial c} = 0
\]

From which, we have:

\[
2 (1 + a) F'(c) - \frac{3 r^2}{2 \Delta_1^2} \frac{dy^1}{dc} - \frac{8 r^3 - 6 r^2 - 3 r + 1}{\Delta_1^3} \frac{dy^2}{dc} - \frac{12 r^2 - r - 2}{2 \Delta_1^2} y^2
\] (165)
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**Proposition 1'.** If both firms must obtain certifications, it is socially optimal to set two different certification levels of the unobservable quality, i.e., case one dominates case two.

Proof. To prove proposition 1', we need only to prove that it is welfare improving to marginally increase (decrease) the certification level from the optimal level \( c \) obtained in case two for the high (low) total-quality supplier. This is equivalent to proving that \( F'(c^2) > F'(c) \), \( F'(c^1) < F'(c) \) at the point \( c^1 = c^2 = c \). This is shown in appendix C4.

3.5 Conclusions

The chapter explores the consequence of the introduction of a voluntary certification scheme of an unobservable quality of products in addition to requiring firms to meet an MQS. It shows that: 1. The introduction of such a scheme as opposed to just imposing an MQS may improve social welfare when no externalities are created from the consumption of higher quality products; 2. The MQS can be set at a lower level than the one that would have been set if only one level of certification was considered. This normative assertion which is straightforward for our first example (that of certifying health claims where all benefits of quality accrue to the consumer) may be more controversial when applied to our second example of environmental certification. If we assume that the consumption of the "greener" products will not affect the quality of the environment (as some critics argue) then there are no externalities involved and our normative conclusions are correct. Alternatively, it is possible that there is an impact on the environment and the government identifies some optimal level of environmental quality. It can use environmental certification as a means to achieve its goal using voluntary actions (softer means than direct regulation). If the consumers
internalize their social responsibilities and "doing the right thing" is utility generating and if such voluntary actions exceed or meet the targets of environmental quality set by the government then the normative implications of the model remain valid. If, however, the average environmental quality achieved through the certification scheme is lower than the one deemed socially optimal by the government, then there is no other choice than direct government intervention to achieve its target. This may involve regulation or the use of the price mechanism (taxes or subsidies) to affect producer and consumer choices.

The analysis also shows that when the government introduces two certification levels for the unobservable quality so as to maximize social welfare (assuming no externalities) the firms will specialize. One will focus on the high end of the quality spectrum of both the observable and unobservable attributes while the other firm will focus on the lower end. Not surprisingly, if the firms could choose certification levels so as to maximize their profits, they would also specialize in overall quality to benefit from the reduction in competition that is due to differentiation. The levels of quality certification will not necessarily be the ones that a social welfare-maximizing government would have chosen.
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Appendix A

Proofs for Chapter One

A1. Under Bertrand competition, the slope of the low-quality's reaction function is steeper than that of the high-quality reaction function.

\[
\frac{1}{dq^1/dq^2} \left( - \frac{\pi_{11}}{\pi_{12}} + \frac{\pi_1}{q^1} \right) + \frac{r g'(r) + F'(q^1) q^1}{g'(r)} > r > \frac{r^2 f'(r)}{f(r)} > \frac{r^2 f'(r)}{r f'(r) - F'(q^2) q^2} \cdot \frac{dq^2}{dq^1}
\]

A2. \( \frac{dq^1}{ds^1} > \frac{dq^2}{ds^1} \):

\[
\frac{dq^1}{ds^1} - \frac{dq^2}{ds^1} = \frac{F'(q^1)}{\Delta} \left( - \frac{\pi_2}{\pi_2 \pi_2} - \frac{\pi_{21}}{\pi_{21}} \right) - \frac{F'(q^1) (F'(q^2) q^2 + r (r - 1) f'(r))}{q^1} \frac{F'(q^1) (F'(q^2) q^2 + r (r - 1) f'(r))}{q^2 \Delta}
\]

\[
> \frac{F'(q^1)}{q^2 \Delta} \left( 4 r (16 r^3 - 26 r^2 + 19 r) \right) > 0
\]

A3. \( q^2 > q^1 > 0 \).

\[
F'(q^2) - F'(q^1) - \gamma F'(q^1) - n(r) - m(r) \cdot \frac{12 r^3 - 13 r^2 + 4 r - 1}{(4 r - 1)^3} > 0
\]

So we have \( F'(q^2) > F'(q^1) \). Since \( F'(q) > 0 \), we conclude that \( q^2 > q^1 \).

A4. \( \pi_{11}^{1c} < 0, \Delta^c > 0 \).

We separate the proofs into two cases according to the convexity of the fixed cost...
function.

Case 1. $\frac{F'(q^2)}{F'(q^1)} < r^2$. In this case, by using the first-order conditions of (52), we can easily show that $r > 1.3833$.

\[
\pi_{11}^e - \frac{r}{q^1} m'(r) - \gamma F^e(q^1) < \frac{r}{q^1} m'(r) - \gamma F^e(q^1)
\]
\[
< \frac{2 r (8 r + 1)}{(4 r - 1)^4} - \frac{(4 r + 1) r^2}{(4 r - 1)^3} - \frac{r (16 r^3 - 17 r - 2)}{(4 r - 1)^4} < 0
\]

\[
\Delta^e - \frac{r}{q^1} m'(r) F^e(q^2) - \gamma \frac{n'(r)}{q^1} F^e(q^1) + \gamma F^e(q^1) F^e(q^2)
\]
\[
> [ \frac{r}{q^1} m'(r) + \gamma F^e(q^1) ] F^e(q^2) - \pi_{11}^e F^e(q^2) > 0
\]

Case 2. $\frac{F'(q^2)}{F'(q^1)} > r^2$, in this case we have $q F^e(q) > 2 F'(q)$, then

\[
\pi_{11}^e - \frac{r}{q^1} m'(r) - \gamma F^e(q^1) < \frac{r}{q^1} m'(r) - \gamma 2 F'(q^1)
\]
\[
< \frac{2 r (8 r + 1)}{(4 r - 1)^4} - \frac{2 (4 r + 1) r^2}{(4 r - 1)^3} - \frac{2 r (16 r^3 - 9 r - 1)}{(4 r - 1)^4} < 0
\]

\[
\Delta^e - \frac{r}{q^1} m'(r) F^e(q^2) - \gamma \frac{n'(r)}{q^1} F^e(q^1) + \gamma F^e(q^1) F^e(q^2)
\]
\[
> [ \frac{r}{q^1} m'(r) + \gamma F^e(q^1) ] F^e(q^2) - \pi_{11}^e F^e(q^2) > 0
\]
Appendix B

Proofs for Chapter Two

Section 2.3 Zero Marginal Cost

B1. The slope of AA (A'A') is steeper than that of BB (B'B').

Proof. To prove the slope of AA is steeper than that of BB, we need to prove that

\[
\frac{dD}{dq} < \frac{1}{dq1dD}.
\]

This equals to proving \(2 \frac{r^3 F'(q^2) - F'(q^1)}{\Delta^3 q^2 F'(q^2) - 2 r} < 0\):

\[
2 \frac{r^3 F'(q^2) - F'(q^1)}{\Delta^3 q^2 F'(q^2) - 2 r} < 2 \frac{r^3 F'(q^1)}{\Delta^3 q^2 F'(q^2) - 2 r}
\]

\[
= \frac{F'(q^2) q^2 r^2 (4 r - 3)}{\Delta^2} 
\]

\[
< \frac{2 r (2 r - 1) r^2 (4 r - 3)}{\Delta^2} \frac{2 r^6}{\Delta^2} \frac{2 r^3}{\Delta^2} < 0.
\]

To prove that the slope of A'A' is steeper than that of B'B', we need to prove that

\[
\frac{dD}{dq} < \frac{1}{dq2dD}.\]

This equals to prove \(X - 2F'(q^1) - F'(q^2) (\Delta^3 q^2 F'(q^2) q^1 - 2 r^2) < 0\). We separate the proofs into two cases: a. The fixed cost is not a very convex function of quality \(F''''(q) < 0\); b. The fixed cost is a very convex function of quality \(F''''(q) > 0\).

Case a. In this case we have, from the proof of proposition 4, \(r > 1.675\).

\[
X < 2 F'(q^1) - F'(q^2) (\Delta r^2 - 2 r^2) - 2 F'(q^1) - F'(q^2) (4 r - 3) r^2
\]

\[
< [2 - (4 r - 3)] F'(q^1) < -1.7 F'(q^1) < 0.
\]
Case b. In this case, we have $q F^*(q) > 2 F'(q)$.

\[ X < 2 F^*(q_1) - F^*(q_2) (\Delta \frac{2}{2} r_2^2 - 2 r_4^2) - 2 F^*(q_1) - 2 r^2 (4 r - 2) F^*(q_2) < 2 - 4 r^2 F^*(q_1) < 0 \]

B2. $\frac{dCS}{dq} > 0$, $\frac{dW}{dq} < 0$:

\[
\frac{dCS}{dq} = \frac{r (5 r - 2)}{2 \Delta^2} \cdot \left[ \frac{(2 r - 1)^2}{2 \Delta^3 F^*(q^2)} + \frac{4 r^2 - 3 r - 1}{2 \Delta} q^1 \right] \frac{dD^2}{dq} \\
- \frac{r (5 r - 2)}{2 \Delta^2} \cdot \frac{(2 r - 1)^2}{2 \Delta^3 F^*(q^2)} \cdot \frac{2 r^2 \Delta F^*(q^2)}{2 \Delta^2 (2 r - \Delta^3 q^2 F^*(q^2))} \\
- \frac{2 r^2 (5 r - 2) + (2 r - 1)^2}{2 \Delta^2 (2 r - \Delta^3 q^2 F^*(q^2))} \cdot \frac{2 r^2 (4 r^2 - 3 r + 1)}{2 \Delta^2 (2 r - \Delta^3 q^2 F^*(q^2))} < 0
\]

where we have used (70), (72) and the facts that $r > 1$, $x - 2 r = \Delta^3 F^*(q^2) < 0$.

\[
\frac{dW}{dq} = \frac{dCS}{dq} + \frac{d\pi^b}{dq} \\
= \frac{r^2}{2 \Delta^2} \left[ \frac{(2 r - 1)^2}{2 \Delta^3 F^*(q^2)} \cdot \frac{q^2}{2} \right] \frac{2 r^2 \Delta F^*(q^2)}{2 \Delta^2 (2 r - \Delta^3 q^2 F^*(q^2))} < 0
\]

B3. $\frac{dCS}{dD} > 0$, $\frac{dW}{dD} < 0$:

\[
\frac{dCS}{dD} = - q^1 (D^1 + D^2) \cdot \left[ \frac{(D^1)^2}{2} D^1 D^3 (\frac{D^1}{F^*(q^1)}) \right] \cdot \frac{(D^2)^2}{2} (1 - 2 D^2) \cdot \frac{q^2 D^2}{2} \cdot \frac{q^1 D^1}{2} \frac{dD^2}{dD^1} \\
- q^1 \frac{3 r - 1}{\Delta} - q^2 (5 r - 2) F^*(q^2) - 2 r (2 r - 1)^2 F^*(q^1) + q^1 \frac{2 r^2}{\Delta^2} 4 r F^*(q^l) F^*(q^2) \\
- \frac{\Delta}{\Delta} \frac{q^2 r^2 F^*(q^2)}{2} + (4 r^2 - 2 r - 1) q^1 F^*(q^l) \cdot (8 r - 4) \Delta^2 q^1 q^2 F^*(q^l) F^*(q^2) \\
\frac{2 r}{\Delta^2} \geq \frac{2 \Delta x}{\Delta} \cdot \frac{2 r^2 q^2 F^*(q^2)}{1 - r} - q^1 F^*(q^l) \cdot 2 \frac{r [2 r - 1] (7 r - 4) - 2 r - 1]}{\Delta} \geq 0
\]

where we have used the facts that $x - r^2 F^*(q^2) \cdot F^*(q^l) - 2 q^2 \Delta^2 F^*(q^l) F^*(q^2) < 0$, $r > 1$. 
Appendix B. Proofs for Chapter Two

\[
\frac{dW}{dD} = -q^1 D^1 \cdot \left[ \frac{(D')^2}{2} \left( -\frac{D}{F'(q)} \right) + \frac{(D^2)q}{2} \frac{(1 - 2 \cdot D)}{F'(q)} \cdot q^2 D^2 + q^1 D^1 \right] \frac{dD^2}{dD} \\
- q^1 \frac{F''(g)}{2 \Delta} \cdot \frac{(2 \cdot r - 1)^2}{2 \Delta^3 F'(q)} \cdot \frac{2 \cdot r^2}{q^1} \frac{dD^2}{dD} \\
- q^2 \cdot \frac{F''(q)}{2 \Delta} \cdot \frac{(4 \cdot r^2 - 2 \cdot r - 1)}{q^1} F'(q) < 0
\]

B4. \( \frac{dCS}{dD^2} > 0, \frac{dW}{dD^2} (>) < 0: \)

\[
\frac{dCS}{dD^2} = q^2 D^3 + q^1 D^1 \cdot \left[ \frac{(D')^2}{2} + \frac{q^2 (D^2)q}{2} \frac{D^1}{dD^2} + \frac{q^1 (D^1 + D^2)}{dD^2} \frac{dD^2}{dD} \right] \frac{dD^2}{dD} \\
- \frac{2 \cdot r \cdot q^2}{\Delta} \cdot \frac{r^2 (5 \cdot r - 2) F'(q)}{2 \Delta^3 F'(q)} \cdot \frac{1}{2} \frac{(3 \cdot r - 1)}{F'(q)} \frac{dD^2}{dD} \\
+ \frac{q^1 \cdot (4 \cdot r^2 - 3 \cdot r - 1)}{2 \Delta^3 F'(q)} \frac{q^1}{F'(q)} F''(q^2) - \frac{r^2 (5 \cdot r - 2) F'(q)}{2 \Delta^3 F'(q)} \cdot \frac{2 \cdot r - 1}{F'(q)} F''(q^2) > 0
\]

We next prove that the numerator is positive. To do so, we separate the proofs into two cases: Case 1. The fixed cost function is not very convex (less than cube), that is \( F'''(q) > 0, F''''(q) < 0; \) Case 2. The fixed cost function is very convex: \( F''''(q) > 0. \)

Case 1. In this case, we have in equilibrium \( r > 1.675. \) Using the fact that \( q F'(q) > F'(q) \)
and the first-order condition (68), we have the numerator:

\[
q^1 \cdot (4 \cdot r^2 - 3 \cdot r - 1) \Delta \frac{F'(q)}{F''(q)} F''(q^2) - \frac{r^2 (5 \cdot r - 2)}{F'(q)} \cdot \frac{(2 \cdot r - 1)^2}{F''(q^2)} \\
> (4 \cdot r^2 - 3 \cdot r - 1) \frac{r^2 F''(q^2)}{F'(q)} - \frac{r^2 (5 \cdot r - 2)}{F'(q)} \cdot \frac{(2 \cdot r - 1)^2}{F''(q^2)} \\
- (4 \cdot r^2 - 8 \cdot r - 3) F''(q^2) > 0
\]

Case 2. In this case we have \( q F'(q) > 2 \cdot F'(q). \) Checking the numerator:

\[
q^1 \cdot (4 \cdot r^2 - 3 \cdot r - 1) \Delta \frac{F'(q)}{F''(q)} F''(q^2) - \frac{r^2 (5 \cdot r - 2)}{F'(q)} \cdot \frac{(2 \cdot r - 1)^2}{F''(q^2)} \\
> 2 \cdot (4 \cdot r^2 - 3 \cdot r - 1) \frac{r^2 F''(q^2)}{F'(q)} - \frac{r^2 (5 \cdot r - 2)}{F'(q)} \cdot \frac{(2 \cdot r - 1)^2}{F''(q^2)} \\
- (8 \cdot r^2 - 11 \cdot r + 4) F''(q^2) > 0
\]

Combining the results in both cases, we have \( \frac{dCS}{dD^2} > 0. \)
If the fixed cost function takes the following form: \( F(q) = 0.01\alpha q^a \), then we have

\[
\frac{dW}{dD^2} > 0 \quad \text{if} \quad \alpha \leq 7.36, \quad \frac{dW}{dD^2} < 0 \quad \text{if} \quad \alpha > 7.36.
\]

From the above result we conclude that a quota on high-quality imports reduces, in most cases, domestic welfare. Only in some extreme cases in which the fixed cost function is very convex, a quota on high-quality imports can increase domestic welfare.

Section 2.4 Marginal Cost is Concave in Quality

B5. Lemma 3. Since \( c(0) = 0, c'(q) > 0, c''(q) < 0, q^2 > q^1 \), we can easily prove that \( c'(q^2) \leq \frac{c(q^2)}{q^2} \leq \frac{c(q^1)}{q^1} \) and \( c''(q^2) \leq c''(q^1) \leq \frac{c(q^1)}{q^1} \). Using the results of (106), we can show by straightforward calculation that \( rD^2 > (2r - 1)D^1 \).

We next prove that \( \frac{c(q^1)}{q^1} < \frac{r}{16(3r - 1)} \):

Since \( F'(q^1) > 2 \frac{F(q^1)}{q^1} > 4 \frac{c(q^1)}{q^1} \), from (102) and \( rD^2 > (2r - 1)D^1 \), we have

\[
(1 - D^1 - \frac{2r - 1}{r}D^1)D^1 \geq (1 - D^1 - D^2)D^1 \]

\[
> (1 - D^1 - D^2 - c'(q^1))D^1 - F'(q^1) > 4 \frac{c(q^1)}{q^1}
\]

Noticing that

\[
\max_{D^1} (1 - D^1 - \frac{2r - 1}{r}D^1)D^1 = \frac{r}{4(3r - 1)}
\]

So, \( \frac{r}{4(3r - 1)} > 4 \frac{c(q^1)}{q^1} \) from which we have \( \frac{c(q^1)}{q^1} < \frac{r}{16(3r - 1)} \).
This result is obtained at the point \( D^1 = \frac{r}{2 (3 r - 1)} \). However, since
\[
D^1 = \frac{r}{\Delta} \cdot \frac{\Delta q^2}{q^2} \cdot \frac{e(q^2)}{q^2} > \frac{r}{\Delta} \cdot \frac{2 r - 1}{q^1} \cdot \frac{e(q^1)}{q^1} > \frac{r}{16 \Delta (3 r - 1)} \cdot \frac{(46 r - 15)}{2 (3 r - 1)}
\]
we have
\[
4 \frac{e(q^1)}{q^1} < \frac{(1 - D^1 - \frac{2 r - 1}{\Delta} D^1) D^1}{r} \cdot \frac{(1 - \frac{3 r - 1}{\Delta} D^1) D^1}{r}
\]
\[
< \frac{r}{\Delta} \cdot \frac{16 \Delta (3 r - 1)}{16 \Delta (3 r - 1)} \cdot \frac{r (18 r - 1)}{256 \Delta^2 (3 r - 1)} (46 r - 15)
\]
from which we conclude that \( \frac{e(q^1)}{q^1} < \frac{r (18 r - 1) (46 r - 15)}{1024 \Delta^2 (3 r - 1)} \).

B6. Lemma 4. In equilibrium \( r < 4 \):

Since \( F'''(q) > 0 \), we have \( \frac{F'(q^2)}{F'(q^1)} > \frac{q^2}{q^1} \cdot r \). Defining \( f = \frac{F'(q^2)}{F'(q^1)} \) and using the first-order conditions (102) and (104), we have \( f = \frac{F'(q^2)}{F'(q^1)} = \frac{(1 - D^2 - e'(q^2)) D^2}{(1 - D^1 - D^2 - e'(q^1)) D^1} > r \). Also, using the facts that
\( D^2 < \frac{2 r - 1}{\Delta} \), \( D^1 < \frac{r}{\Delta} \) and corollary 1, we have:
\[
f < \frac{1 - D^2 - e'(q^2)}{1 - D^1_{\text{max}} - D^2 - e'(q^2)_{\text{max}}} \frac{D^2_{\text{max}}}{D^1_{\text{min}}} \frac{1 - D^2}{1 - r/\Delta - r/(16 (3 r - 1)) - D^2} \frac{16 (3 r - 1) (2 r - 1)}{r (46 r - 15)}
\]
Noticing that \( \frac{1 - x}{m - x} < \frac{1 - y}{m - y} \) for all \( 0 < y < m < 1 \) and \( x < y \), we have
\[
f < \frac{1 - D^2_{\text{max}}}{1 - r/\Delta - r/(16 (3 r - 1)) - D^2_{\text{max}}} \frac{16 (3 r - 1) (2 r - 1)}{r (46 r - 15)}
\]
\[
< \frac{1 - (2 r - 1)/\Delta}{1 - r/\Delta - r/(16 (3 r - 1)) - (2 r - 1)/\Delta} \frac{16 (3 r - 1) (2 r - 1)}{r (46 r - 15)}
\]
\[
= \frac{32 (3 r - 1)}{44 r - 15} \frac{16 (3 r - 1) (2 r - 1)}{r (46 r - 15)}
\]
From \( r < f < \frac{32 (3 r - 1)}{44 r - 15} \cdot \frac{16 (3 r - 1) (2 r - 1)}{r (46 r - 15)} \), we conclude \( r < 3.982 < 4 \).

Since \( 1 < r < 4 \), we have \( \frac{a}{D^1} < \frac{c(q^1)}{q^1} \cdot \frac{(18 r - 1)}{64 \Delta} < 0.089 \), \( \frac{c(q^2)}{q^2} \cdot \frac{D^2}{16 \Delta (3 r - 1)} > 0.3229 \), \( b < 1 - 2 D^2 < 0.3652 < 1.1 D^2 \).

**B7. Lemma 5.** \( b > a \geq 0 \).

\[
\begin{align*}
    a - 1 - 2 D^1 &- D^2 - c'(q^1) - \frac{c(q^1)}{q^1} \\
    b - 1 - 2 D^2 &- c'(q^2) - \frac{1}{\Delta} \cdot \frac{4 r}{\Delta} \cdot \frac{c(q^2)}{q^2} - \frac{2}{\Delta} \cdot \frac{c(q^1)}{q^1} - c'(q^2) \\
    > &\frac{1}{\Delta} (1 - 2 \cdot \frac{c(q^1)}{q^1}) > \frac{1}{15} (1 - 2 \cdot 0.031) > 0.06 > \frac{c(q^1)}{q^1} > a \geq 0
\end{align*}
\]

where we have used the fact that \( r < 4 \).

**B8. Lemma 1'.** \( A < 0 \)

\[
A = 2 b^2 + \Delta \cdot \frac{q^1}{q^1} \cdot \frac{q^2}{q^2} < 2 [b^2 - 1.5 D^2 (b + D^2 - c'(q^2))] < 0
\]

where we have used the facts that \( D^2 > c'(q^2), \ 1.5 D^2 > 1.1 D^2 > b \).

**B9. Proposition 2'.** \( \frac{dCS}{dq^1} > 0, \frac{dW}{dq^1} < 0 \)
Appendix B. Proofs for Chapter Two

We separate the proofs into two cases.

**Case 1.** The fixed cost function is not very convex (less than cube). In this case we can show, using a similar approach to the proof of proposition 4, that \( r > 1.62 \),

so \( D^2 > \frac{(2 \cdot r - 1)}{16} \frac{47 \cdot r - 16}{(3 \cdot r - 1)} \) \( .398 \), \( b < 1 - 2 \cdot D^2 < .204 < .513 \cdot D^2, \ D^1 < .724 \cdot D^2. \)

\[
\frac{dCS}{dq^1} > \frac{D^1 (1 - D^2 - 2 \cdot c'(q^2)) D^2 (- 4 \cdot D^2 - D^1)}{2 \cdot A}
+ \frac{2 \cdot D^1 \cdot b \cdot (D^1 + 2 \cdot D^2) \cdot (D^2)^2 \cdot a \cdot b \cdot (2 \cdot D^1 + D^3) \cdot (D^2)^2}{2 \cdot A}
+ \frac{D^1 \cdot 9 \cdot D^2 \cdot b \cdot D^2 \cdot (- 4 \cdot D^2 - D^1)}{2 \cdot A}
+ \frac{2 \cdot D^1 \cdot 5.13 \cdot D^2 \cdot 5.13 \cdot D^2 \cdot (D^1 + 2 \cdot D^2) \cdot (D^2)^2}{2 \cdot A}
+ \frac{1 \cdot D^1 \cdot 5.13 \cdot D^2 \cdot 2 \cdot 5.13 \cdot D^2 \cdot 7.24 \cdot D^2 \cdot D^2 \cdot (D^2)^2}{2 \cdot A}
+ \frac{D^1 \cdot (- 1.37 \cdot (D^2)^3 - .373 \cdot D^1 \cdot (D^2)^2 \cdot 4 \cdot b \cdot (D^2)^2 - b \cdot D^1 \cdot D^2)}{2 \cdot A} > 0
\]

where we have used the facts that \( a < \frac{c(q^1)}{q^1} < .1 \cdot D^1, c'(q^2) < .1 \cdot D^2, A < 0. \)

**Case 2.** The fixed cost function is very convex, i.e., \( F'''(q) > 0 \). In this case we have

\( q \cdot F''(q) > 2 \cdot F'(q). \)
Appendix B. Proofs for Chapter Two

\[ \frac{dCS}{dq} > \frac{D^1 (2 - 2 D^2 - 3 c'(q^2)) D^2 (-4 D^2 - D^1)}{2 A} \]
\[ \cdot \frac{2 D^1 b (b (D^1 + 2 D^2) + (D^2)^2)}{2 A} \cdot a b (2 b (D^1 + D^2) + (D^2)^2) \]
\[ > \frac{D^1 (1.9 D^2 + 2 b) D^2 (-4 D^2 - D^1)}{2 A} \]
\[ \cdot \frac{2 D^1 b (1.1 D^2 (D^1 + 2 D^2) + (D^2)^2)}{2 A} \cdot \frac{1 D^1 b (2.1 D^2 (D^1 + D^2) + (D^2)^2)}{2 A} \]
\[ = \frac{D^1}{2 A} (-7.6 (D^2)^3 - 1.28 b (D^2)^2 - 1.9 D^1 (D^2)^2 + 0.42 b D^1 D^2) > 0 \]

where we have also used the facts that \( a < \frac{c(q^1)}{q^1} < .1 D^1, c'(q^2) < .1 D^2, A < 0. \)

\[ \frac{dW}{dq} \cdot \frac{(D^1)^2 + D^1 q^1 (\pi_{22})^2 D^2 + (D^1)^2}{dA} \cdot \frac{(D^1 q^1 + D^2 q^2) D^2 + (D^2 q^2 + D^1 q^1) D^2}{dA} \]
\[ = \frac{(D^1 a) D^1 (2 b^2 \Delta q^1 \pi_{22})^2 - [(2 D^2 q^2 + D^1 q^1) (-\pi_{22}) + (D^2 q^2 + D^1 q^1)] D^2}{2 A} \]
\[ < \frac{[- \Delta D^1 (D^1 + a) + (2 D^1 + a) (2 r D^2 + D^1)] q^1 (-\pi_{22}) + (\Delta D^1) q^1 (-\pi_{22})}{2 \Delta A} \]
\[ < \frac{[- \Delta D^1 (D^1 + a) + (2 D^1 + a) \Delta D^1)] q^1 (-\pi_{22})}{2 \Delta A} < 0 \]

where we have used the fact that \( r D^2 > (2 r - 1) D^1, A < 0. \)

B10. Lemma 2'. \( b \cdot b^2 (-\pi_{11}) - 2 q^2 (\pi_{11} \pi_{22}) + (D^1)^2 (-\pi_{22}) < 0. \)

\[ B \cdot b^2 (-\pi_{11}) - 2 q^2 (\pi_{11} \pi_{22}) + (D^1)^2 (-\pi_{22}) \]
\[ = (-\pi_{11}) [b^2 - 0.8 q^2 (-\pi_{22})] + (-\pi_{22}) [(D^1)^2 - 1.2 q^2 (-\pi_{11})] \]
\[ < (-\pi_{11}) [b^2 - 0.8 D^2 (b + D^2 - c'(q^2))] + (-\pi_{22}) [(D^1)^2 - 1.2 r D^1 (a + D^1 - c'(q^1))] \]
\[ < (-\pi_{11}) [b - 0.8 D^3] b - 0.72 (D^3)^2 + (-\pi_{22}) [(D^1)^2 - 1.2 r D^1 0.9 D^1] \]
\[ < (-\pi_{11}) [0.3 D^2 1.1 D^2 - 0.72 (D^3)^2] - 0.08 (D^1)^2 (-\pi_{22}) < 0 \]
where we have used the facts that $c'(q^2) < 0.1 \ D^2$, $c'(q^1) < 0.1 \ D^1, \ b < 1.1 \ D^2$.

B11. $n = - b^2 \pi^2_{21} - D^1 (D^1 \cdot a) \pi^2_{22} - (2 \ r - 1) \ q^1 \pi^1_{11} \pi^2_{22} < 0$

We separate the proofs into two cases.

Case 1. The fixed cost function $F(q)$ is not very convex: $\frac{F''(q^2)}{F'(q^1)} < r^3$. Using a similar approach to the proof of proposition 4, we have $r > 1.335$. Substituting $r > 1.335$ into $D^1, c'(q^1)$, where $i = 1, 2$, we have $D^2 > .374 > 1.48 \ b$.

\[
n = -(1.16 - 1) q^1 \pi^1_{11} + \pi^2_{22} - b^2 \pi^1_{11} - D^1 (D^1 \cdot a) \pi^2_{22} - 0.84 \ q^1 \pi^1_{11} \pi^2_{22}
\]

\[
< \pi^1_{11} [(1.16 - 1) (D^2, b - c'(q^2)) D^2, b^2] + \pi^2_{22} [0.84 \ q^1 \cdot a - c'(q^1)) D^1 - D^1 (D^1 \cdot a)]
\]

\[
< \pi^1_{11} [0.41 \ 3.45 \ b^2 - b^2] \cdot D^1 \pi^2_{22} [0.84 \ 1.335 \ 0.9 \ D^1 - D^1] < 0
\]

Case 2. In this case we have $q F^*(q) > 3 F'(q)$.

Using corollary 1, we can easily show that for any $r > 1, b < 1.5 D^1$. We separate the proofs into two sub-cases:

Sub-case 1. $|\pi^1_{11}| < |\pi^2_{22}|$.

\[
n < (2 \ r - 1) [3 (1 - D^1 - D^2 - c'(q^1)) D^1 - D^1 c'(q^1)] \pi^2_{22} - b^2 \pi^2_{22} - D^1 (D^1 \cdot a) \pi^2_{22}
\]

\[
- [(3 - 3 D^1 - 3 D^2 - 4 c'(q^1)) D^1 - 1.5 D^1 b - D^1 (D^1 \cdot a)] \pi^2_{22}
\]

\[
- [3 - 3 D^1 - 3 D^2 - 4 c'(q^1) - 1.5 \cdot 3 D^2 - 1.5 c'(q^2) - D^1 - 1 + 2 D^1 + D^2 + c'(q^1)] D^1 \pi^2_{22}
\]

\[
- [0.5 - 2 D^1 + D^2 - 3 c'(q^1) + 1.5 c'(q^2)] D^1 \pi^2_{22} < 0
\]

Sub-case 2. $|\pi^1_{11}| \geq |\pi^2_{22}|$. 


\[ n < (2 \cdot r - 1) \left( 3 \left( 1 - D^2 - c'(q^2) \right) - c'(q^2) \right) D^2 \pi_{11}^1 - b^2 \pi_{11}^1 - D^1 D^1 (D^1 + a) \pi_{11}^1 \]

\[ - \left[ (3 - 3 D^2 - 4 c'(q^2)) D^2 - 1.5 D^1 b - D^1 (D^1 + a) \right] \pi_{11}^1 \]

\[ - \left[ 3 - 3 D^2 - 4 c'(q^2) - 1.5 \cdot 3 D^2 + 1.5 c'(q^2) - D^1 - 1 \right] D^1 D^2 + c'(q^2) D^1 D^1 \pi_{11}^1 \]

\[ - [0.5 + D^1 + D^2 + c'(q^1) - 2.5 c'(q^2)] D^1 \pi_{11}^1 < 0 \]

B12. \( \frac{dCS}{dD^1} > 0, \frac{dW}{dD^1} < 0. \)

\[ \frac{dCS}{dD^1} - (D^1 q^1 + D^2 q^1) \left( \frac{D^1 b}{2} + D^1 D^2 \right) \frac{dW}{dD^1} - \left( D^1 q^1 \right) \frac{dW}{dD^1} - \left( D^2 q^1 \right) \frac{dW}{dD^1} \frac{dW}{dD^1} - \frac{L}{2 B} \]

where \( L \) is the numerator and take the following form:

\[ L = (- \pi_{11}^1) \left[ 2 b (D^1 + D^2) + (D^1 q^1) b q^1 \right] + \]

\[ (- \pi_{22}^2) \left[ (D^1 q^1) - 2 a r D^2 - 2 a r D^1 + 2 D^1 a) D^1 q^1 \right] + \]

\[ (\pi_{11}^1 \pi_{22}^2) \left[ - (2 r D^2 + 2 (2 r - 1) D^1) (q^1) \right] + \]

\[ [2 D^1 D^2 a b^2 + (D^1 q^1) a b^2 + (D^2 q^1) D^1 a b] \]

We can prove \( L < 0 \) and separate the proofs into two cases.

Case 1. The fixed cost function \( F(q) \) is not very convex (less than cube): \( \frac{F'(q^2)}{F'(q^1)} \leq r^2. \)

Using a similar approach to the proof of proposition 4, we have \( r > 1.62, b < 0.52 D^2. \)

We prove \( L < 0 \) in three steps:

a. \( (\pi_{11}^1 \pi_{22}^2) [r D^2 + (2 r - 1) D^1] (q^1)^2 > (- \pi_{11}^1) [2 b (D^1 + D^2) + (D^1 q^1)] b q^1 \)
Appendix B. Proofs for Chapter Two

\[(\pi_1^{11} \pi_2^{22}) [r D^2 \cdot (2 r - 1) D^1] (q^1)^2 \cdot (- \pi_1^{11}) [2 b (D^1 \cdot D^2) \cdot (D^2)^2] b q^1\]

\[> (- \pi_1^{11}) q^1 \cdot [(D^2 \cdot \frac{2 r - 1}{r} D^1) (1 - D^2 \cdot 2 c'(q^2)) D^2 - (2 b^2 D^1 + 2 b^2 D^2 + b (D^2)^2)]\]

\[> (- \pi_1^{11}) q^1 \cdot [(D^2 + 1.382 D^1) (9 D^2 \cdot b) D^2 - (2 b^2 D^1 + 2 b^2 D^2 + b (D^2)^2)]\]

\[> (- \pi_1^{11}) q^1 \cdot [((0.9 D^2 \cdot b) (D^2)^2 - (2 b^2 D^2 + b (D^2)^2))\]

\[\cdot (1.382 D^1 D^2 b - 2 0.52 D^2 D^1 b)] > 0\]

b. \((\pi_1^{11} \pi_2^{22}) [r D^2 (q^1)^2] > 2 D^1 D^2 a b^2 + a b^2 (D^1)^2 + a b D^1 (D^2)^2\)

\[(\pi_1^{11} \pi_2^{22}) [r D^2 (q^1)^2 - (D^2)^2 (D^1 + a - c'(q^1)) (D^2 - b - c'(q^2)) D^1\]

\[> [D^1 (D^2)^2 (0.9 D^1 \cdot a) (0.9 D^2 \cdot b)]\]

\[0.81 (D^1)^2 (D^2)^2 + a b D^1 (D^2)^2 + 0.9 a D^1 (D^2)^2 \cdot 0.9 b (D^2)^2 (D^2)^2\]

\[> a b D^1 (D^2)^2 + a b^2 D^1 D^2 + a b^2 D^1 D^2 + a b^2 (D^1)^2\]

c. \((\pi_1^{11} \pi_2^{22}) [(2 r - 1) D^1 (q^1)^2] > (- \pi_2^{22}) [(D^1)^2 - 2 a r D^2 + 2 D^1 a - 2 D^1 r a) D^1 q^1]\)

\[(\pi_1^{11} \pi_2^{22}) [(2 r - 1) D^1 (q^1)^2] > (- \pi_2^{22}) (2 r - 1) (D^1 + a - c'(q^1)) (D^1)^2 q^1\]

\[> (- \pi_2^{22}) q^1 [2.24 0.9 (D^1)^2] > (- \pi_2^{22}) q^1 [(D^1)^2]\]

\[> (- \pi_2^{22}) [(D^1)^2 - 2 a r D^1 + 2 D^1 a - 2 D^2 r a) D^1 q^1]\]

Combining the results in a, b, c, we have proved that \(L < 0\).

Case 2. The fixed cost function \(F(q)\) is very convex: \(\frac{F''(q^2)}{F'(q^1)} > r^2\), from which we have \(q F''(q) > 2 F'(q)\). Noticing that for any \(r > 1\), we have \(1.1 D^2 > b\). Again, we take three steps to prove \(L < 0\).

a. \((\pi_1^{11} \pi_2^{22}) [r D^2 \cdot (2 r - 1) D^1] (q^1)^2 > (- \pi_1^{11}) [2 b (D^1 \cdot D^2) + (D^2)^2] b q^1\)
Appendix B. Proofs for Chapter Two

\[ (\pi_{11}^2 \pi_{22}^2) [r D^2 \cdot (2 r - 1) D^1] (q^4)^2 - (\pi_{11}^2) [2 b (D^1 \cdot D^2) \cdot (D^2)^2] b q^1 > (- \pi_{11}^2) q^1 \frac{2 - 1}{r} D^1 \cdot (2 - 2 D^1 \cdot 3 c'(q^4)) D^2 - (2 b^2 D^1 \cdot (2 b \cdot D^2) D^2 b)\]

\[ > (- \pi_{11}^2) q^1 [(1.9 D^2 \cdot 2 b) (D^2)^2 - (2 b^2 D^2 \cdot b (D^2)^2)] \]

\[ + D^1 D^2 (1.9 D^2 \cdot b) - 2 1.1 D^2 D^1 b] > 0 \]

b. \( (\pi_{11}^2 \pi_{22}^2) [r D^2 (q^4)^2] > 2 D^1 D^2 a b^2 \cdot a b^2 (D^1)^2 \cdot a b D^1 (D^2)^2 \)

\[ (\pi_{11}^2 \pi_{22}^2) [r D^2 (q^4)^2] - (D^2)^2 (2 D^1 \cdot 2 a - e'(q^4))) (2 D^2 \cdot 2 b - e'(q^4)) D^1 \]

\[ > D^1 (D^2)^2 (1.9 D^1 \cdot 2 a) (1.9 D^2 \cdot 2 b) \]

\[ - [3.61 (D^1)^2 (D^2)^3 + 4 a b D^1 (D^2)^2 \cdot 3.8 a D^1 (D^2)^3] \]

\[ > a b D^1 (D^2)^2 \cdot a b^2 D^1 D^2 \cdot a b^2 D^1 D^2 \cdot a b^2 (D^1)^2 \]

\[ c. (\pi_{11}^2 \pi_{22}^2) [(2 r - 1) D^1 (q^4)^2] > (- \pi_{22}^2) [((D^1)^2 - 2 a r D^2 \cdot 2 D^1 a - 2 D^1 r a) D^1 q^1] \]

\[ (\pi_{11}^2 \pi_{22}^2) [(2 r - 1) D^1 (q^4)^2] > (- \pi_{22}^2) (2 r - 1) (2 D^1 \cdot 2 a - e'(q^4)) (D^1)^2 q^1 \]

\[ > (- \pi_{22}^2) q^1 [1.9 (D^1)^2] > (- \pi_{22}^2) [((D^1)^2 - 2 a r D^1 \cdot 2 D^1 a - 2 D^2 r a) D^1 q^1] \]

Combining the results in a, b, c, we have proved that \( L < 0 \).

\[ \frac{dW}{dD^1} \cdot \frac{dx^h}{dD^1} \cdot \frac{dCS}{dD^1} - D^1 q^1 \cdot \frac{(D^1)^2}{2} \frac{dq^1}{dD^1} \cdot \frac{(D^2)^2}{2} \frac{dq^2}{dD^1} \cdot (D^1 q^1) \frac{dD^2}{dD^1} \]

\[ \cdot \frac{[2 b D^1 \cdot (D^2)^2]}{2 B} \cdot \frac{b q^1 (- \pi_{11}^2)}{2 B} \cdot \frac{D^1 q^1 [(D^1)^2 \cdot 2 a r (D^2 \cdot D^1) \cdot 2aD^1] D^1 q^1 (- \pi_{22}^2)}{2 B} \]

\[ \cdot \frac{[r D^2 - (2 r - 1) D^1] (q^4)^2 (- \pi_{11}^2) (- \pi_{22}^2)}{B} \cdot \frac{(b^2 (D^1)^2 a \cdot (D^2)^2 a b D^1)}{2 B} < 0 \]

The inequality is because each item in the numerator is positive while the denominator is negative.
B13. Lemma 7. \[ C - a^2 + 2 q^1 \pi_{11}^1 < 0 \]

\[
C - a^2 + 2 q^1 \pi_{11}^1 < a^2 - 2 \left( F'(q^1) - D^1 \epsilon'(q^1) \right) D^1 - a^2 - 2 D^1 (a - D^1 - \epsilon'(q^1)) < 0
\]

B14. \[
\frac{dCS}{dD^2} > 0, \quad \frac{dW}{dD^2} (>0) < 0.
\]

\[
\frac{dCS}{dD^2} - \left[ \frac{(D^1)^2}{2} + D^1 D^2 \right] \frac{dq^1}{dD^2} \cdot (D^1 q^1 + D^2 q^1) \frac{dD^1}{dD^2} + \frac{(D^2)^2}{2} \frac{dq^2}{dD^2} \cdot D^2 q^2 \cdot D^1 q^1
\]

\[
- \left[ \frac{(D^1)^2}{2} + D^1 D^2 \right] \frac{(a + 2D^1) q^1}{C} \cdot (D^1 \cdot D^2) q^1 \cdot a D^1 - q^1 \pi_{11}^1 \cdot \frac{(D^2)^2}{2} \frac{b}{\pi_{12}^2} \cdot D^1 q^1 \cdot D^2 q^2
\]

\[
> \frac{D^1 (D^1 \cdot 2 D^2) (a + 2 D^1) q^1}{2 C} \cdot q^1 (D^1 \cdot D^2) (a D^1 - q^1 \pi_{11}^1) \cdot q^1 (D^2 r \cdot D^1) C
\]

\[
- \frac{(q^1)^2 \pi_{11}^1 ((2r - 1) D^2, D^1) \cdot a(D^1, D^2) q^1 D^1}{C} \cdot a^2 q^1 (D^2 r \cdot D^1) \cdot \frac{D^1 (D^1, 2 D^2) (a + 2 D^1) q^1}{2 C}
\]

We separate the proofs into two cases: Case 1. The fixed cost is not very convex: \[ \frac{F'(q^2)}{F'(q^1)} < r^{1.8} \]; Case 2. The fixed cost is very convex, that is \[ \frac{F'(q^2)}{F'(q^1)} \geq r^{1.8} \].

Case 1. In this case we can easily show \( r > 1.737 \). So we have

\[
D^2 > \frac{2 r - 1}{r} \frac{D^1 > 1.42 D^1}{C}
\]

\[
e'(q^1) < \frac{r (46 r - 15) (18 r - 1)}{1024 \Delta^2 (3 r - 1)} < \frac{18 r - 1}{64 (4 r - 1)} D^1 < 0.08 D^1.
\]
Case 2. In this case, we have $q F''(q) > 1.8 F'(q)$. Since $\frac{c(q^1)}{q^1} < 0.089 < \frac{1}{11}$, from (106), we have $D^1 > \frac{9r - 1}{11 (2r - 1)} D^2$. 

$$\frac{dCS}{dD^2} > - \frac{q^1 D^1 (a \cdot D^1 - c'(q^1)) ((2r - 1) D^2 \cdot D^1) + (D^1)^2 q^1 (D^1 + 2 D^2)}{C}$$

$$\cdot \frac{2 a^2 q^1 (D^2 r \cdot D^1) + 2 a D^1 q^1 (D^1 \cdot D^2) + a D^1 q^1 (D^1 + 2 D^2)}{2 C}$$

$$> q^1 - \frac{D^1 (1.8 D^1 - 2.8 c'(q^1)) ((2r - 1) D^2 \cdot D^1) + (D^1)^2 (D^1 + 2 D^2)}{C}$$

$$> q^1 (D^1)^2 - 92 (2.474 D^2 \cdot D^1) \cdot D^1 \cdot 2 D^2 \cdot 0.1 D^1 > - \frac{0.21 q^1 (D^1)^3}{C} > 0$$
where we have used the facts that \( a < 0.1 \ D^1 \), \( C < 0 \).

Similarly,

\[
\frac{dW}{dD^2} - \left( \frac{(D^1)^2}{2} \cdot D^1 D^2 \right) \frac{da}{dD^2} + \left( D^1 q^1 + D^2 q^1 \right) \frac{dD^1}{dD^2} + \left( \frac{(D^2)^2}{2} \right) \frac{dD^2}{dD^2} + D^2 q^2
\]

\[
\frac{(q^1)^2}{C} \left( (2 r - 1) D^2 - D^1 \right) \frac{a}{2} \left( D^1 \right)^2 + 4 a D^1 D^2 \frac{2}{2} \left( (D^1)^3 \right) + 4 (D^1)^2 \frac{2}{2} q^1
\]

\[
\frac{a^2 D^2 q^2}{C} \cdot \frac{(D^2)^2 b}{2 (- \pi_{22}^2)} (>) < 0
\]

If \( F(q) \), \( c(q) \) take the same form as \( F(q) = 0.01 q^a \), \( c(q) = 0.01 q^5 \), then we have

\[
\frac{dW}{dD^2} > 0 \quad \text{if} \quad a < 7.19, \quad \frac{dW}{dD^2} < 0, \quad \text{if} \quad a > 7.19
\]

From the above result we conclude that a quota on the high-quality imports, in most cases, reduces domestic welfare. Only in some extreme cases in which the fixed cost function is extremely convex, a quota on the high-quality imports can increase domestic welfare.
Appendix C

Proofs for Chapter Three

C1. Lemma 3. It is socially optimal to have the firm with the high total-quality product supply the high unobservable quality.

Proof. From (145) and (146), we have

\[
(1 + a) \left( F'(e^2) - F'(e^1) \right) = \frac{3 r^2}{2 \Delta_1} \left( \frac{\delta y_1}{dc} - \frac{\delta y_2}{dc} \right) + \frac{8 r^3 - 6 r^2 - 3 r + 1}{\Delta_1} \left( \frac{\delta y_2}{dc} - \frac{\delta y_1}{dc} \right)
\]

\[
+ \frac{24 r^3 - 18 r^2 + 5 r + 1}{2 \Delta_1} - \frac{20 r^3 - 17 r^2}{2 \Delta_1}
\]

\[
= m \left( C^*(y_2) \right) r \left( 40 r^3 - 10 r^2 + 7 r + 2 \right) + C^*(y_1) \left( 16 r^3 - 24 r^2 + r - 14 \right) + \left( 28 r^3 - 19 r^2 + 10 r \right) \left( C^*(y_1) - C^*(y_2) \right)
\]

\[
> m \left( C^*(y_1) \right) \left( 16 r^3 - 24 r^2 + r - 14 \right) + \left( 28 r^3 - 19 r^2 + 10 r \right) \left( C^*(y_1) - C^*(y_2) \right)
\]

\[
> m \left( C^*(y_1) \right) \left( 16 r^3 - 24 r^2 + r - 14 \right) + \left( 28 r^3 - 19 r^2 + 10 r \right) f(r) > 0
\]

where \( m = \frac{1}{2 \Delta_1} \).

So we have \( F'(e^2) > F'(e^1) \), from which we conclude \( e^2 > e^1 \).

C2. Proposition 1. If both firms must obtain certifications, it is socially optimal to set two different certification levels of the unobservable quality, i.e., case one dominates case
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two.

Proof. As argued before, to prove proposition 1 we need only to prove the fact that 
\( F'(e^2) > F'(e) \) and \( F'(e^1) < F'(e) \) at points \( e^1 - e^2 - e \) where \( F'(e^2), F'(e^1) \) and \( F'(e) \) are obtained from (145), (146) and (153), respectively. 

\[
2 F'(e^2) - 2 F'(e) = \frac{3 r^2}{2 \Delta_1^2} \left( 2 \frac{dy_1}{dc} - \frac{dy_1}{dc} \right) + \frac{8 r^3 - 6 r^2 - 3 r + 1}{\Delta_1^3} \left( 2 \frac{dy_2}{dc} - \frac{dy_2}{dc} \right)
\]

- \( m \left( 3 r^2 (r + 1) \right) \frac{dg(r) C^*(y^2)}{\Delta_1} + 2 \left( 8 r^3 - 6 r^2 - 3 r + 1 \right) \left( r + 1 \right) \frac{df(r) C^* (y^1)}{\Delta_1} \)

\[
\quad \cdot \left( 28 r^3 - 19 r^2 + 7 r + 1 \right) \left( r \frac{dg(r) C^* (y^2)}{\Delta_1} - \frac{df(r) C^* (y^1)}{\Delta_1} \right) + Q^1 C^* (y^1) C^* (y^2))
\]

\[
> m C^* (y^1) \left( 2 F'(e^2) - 2 F'(e) \right) > 0
\]

Thus, we have \( F'(e^2) > F'(e) \) from which we have \( e^2 > e \).

Similarly, \( 2 F'(e) - 2 F'(e^1) \):

\[
2 F'(e) - 2 F'(e^1) = \frac{3 r^2}{2 \Delta_1^2} \left( 2 \frac{dy_1}{dc} - \frac{dy_1}{dc} \right) + \frac{8 r^3 - 6 r^2 - 3 r + 1}{\Delta_1^3} \left( 2 \frac{dy_2}{dc} - \frac{dy_2}{dc} \right)
\]

- \( m \left( 3 r^2 (r + 1) \right) \frac{dg(r) C^*(y^2)}{\Delta_1} + 2 \left( 8 r^3 - 6 r^2 - 3 r + 1 \right) \left( r + 1 \right) \frac{df(r) C^* (y^1)}{\Delta_1} \)

\[
\quad \cdot \left( 28 r^3 - 19 r^2 + 7 r + 1 \right) \left( r \frac{dg(r) C^* (y^2)}{\Delta_1} - \frac{df(r) C^* (y^1)}{\Delta_1} \right) + Q^1 C^* (y^1) C^* (y^2))
\]

\[
- 2 F'(e) - 2 F'(e^1) > 0
\]

So, we have \( F'(e) > F'(e^1) \) from which we have \( e > e^1 \).

C3. Lemma 3'. To prove \( e^2 > e^1 \), we need only to prove \( F'(e^2) - F'(e^1) > 0 \). From (161),
\[ F'(c^2) - F'(c^1) = \frac{3r^2}{2\Delta_1} c^1 \left( \frac{d^1_c}{dc^2} - \frac{d^1_r}{dc^1} \right) + \frac{8r^3 - 6r^2 - 3r + 1}{\Delta_1} c^2 \left( \frac{d^2_c}{dc^2} - \frac{d^2_r}{dc^1} \right) \]
\[ + \frac{24r^3 - 18r^2 + 5r + 1}{\Delta_1} y_2 - \frac{20r^3 - 17r^2}{2\Delta_1} y_1 \]
\[ - \frac{3r^2}{2\Delta_1} \left[ y_2 d_g(r) f(r) c^2, y_2 d_g(r) C^r(y^2) y_2, y_1 r d_f(r) g(r) c^2, y_1 C^r(y^2) y_2 d_g(r) r - y_1 C^r(y^2) y_2 g(r) \right] \]
\[ + \frac{8r^3 - 6r^2 - 3r + 1}{\Delta_1} c^1 \left[ y_2 d_f(r) y_2 d_g(r) r c^1, y_2 d_f(r) y_2 C^r(y^1) y_1, y^2 r d_f(r) C^r(y^1) y_1 \right] \]
\[ + r^2 \frac{d_f(r) y_1 g(r) c^1 + r^2 \frac{d_f(r) y_1 C^r(y^1) y_1}{f(r)} + \frac{24r^3 - 18r^2 + 5r + 1}{\Delta_1} y_2 - \frac{20r^3 - 17r^2}{2\Delta_1} y_1}{f(r)} \]
\[ > m \left( C^r(y^2) c^1 y_1 y^2 g(r) \left( \frac{d_g(r) r}{g(r)} \right) (40r^3 - 10r^2 + 7r + 2) - 3r^2 (4r - 1) \right) \]
\[ + C^r(y^1) c^1 y_1 y^2 f(r) (2 (8r^3 - 6r^2 - 3r + 1) [1 + (1 - \frac{r y_1}{y^2}) \frac{d_f(r)}{y^2}] - (28r^3 - 19r^2 - 10r + 2) \frac{d_f(r)}{f(r)} \]
\[ + r c^1 c^2 y_2 d_g(r) f(r) (3r (4r - 1) + 2 (8r^3 - 6r^2 - 3r + 1)) \]
\[ + \frac{d_f(r)}{d_g(r) f(r)} (3r^2 (4r - 1) + 2r (8r^3 - 6r^2 - 3r + 1)) \frac{y_1}{y^2} \]
\[ + (28r^3 - 19r^2 + 10r + 2) (y_1)^2 y^2 C^r(y^1) C^r(y^2) \]
\[ > m \left( C^r(y^2) c^1 y_1 y^2 g(r) \left( \frac{2 (8r + 7)}{\Delta_1 (4r^3 - 10r^2 + 7r + 2)} - 3r^2 \Delta_1^2 (4r - 7) \right) \right) \]
\[ + C^r(y^1) c^1 y_1 y^2 f(r) \left( \frac{8r \left( 128r^6 - 304r^5 + 240r^4 + 32r^2 + 31r - 2 \right)}{\Delta_1^4 f(r)} \right) \]
\[ + r c^1 c^2 y_2 d_g(r) f(r) (16r^3 - 9r + 2) [1 + \frac{r d_f(r) g(r)}{d_g(r) f(r)}] \]
\[ + (28r^3 - 19r^2 + 10r + 2) r (y_1)^2 y^2 C^r(y^1) C^r(y^2) \]
\[ > m \left( C^r(y^2) c^1 y_1 y^2 g(r) (-12r^3) + (C^r(y^2) c^1 y_1 y^2 g(r) (28r^3 - 19r^2 + 10r + 2)) \right) \]
\[ = m \left( C^r(y^2) c^1 y_1 y^2 g(r) (16r^3 - 19r^2 + 10r + 2)) > 0 \]

where \( m = \frac{1}{2 \Delta_1 \Delta} \). We have used the facts \( y^2 > y_1 \), \( y_1 C^r(y^1) > C^r(y^1) \cdot c^1 g(r) \) and \( r > 1 \).
C4. Proposition 1'. If both firms must obtain certifications, it is socially optimal to set two different certification levels of the unobservable quality, i.e., case one dominates case two.

Proof. Similar to the proof of proposition 1, we need only to prove \( e^2 > e > e^1 \).

\[
2 \ F'(e^2) - 2 \ F'(e) = \frac{3}{2} \Delta_1 \ c \left( \frac{\partial g(r)}{\partial r} \right) - \frac{8}{6} \Delta_1 \ c \left( \frac{\partial C^*(y^2)}{\partial y} \right) + \frac{8}{r^3} \Delta_1 \ c \left( \frac{\partial C^*(y^1)}{\partial y} \right) \\
\times 2 \ \ \frac{24 r^3 - 18 r^2 + 5 r + 1}{2 \ \Delta_1} y^2 - \frac{12 r^2 - r - 2}{2 \ \Delta_1} y^2 \\
\times \frac{3}{2} \Delta_1^2 c \left( \frac{\partial g(r)}{\partial r} \right) - \frac{8}{6} \Delta_1 \ c \left( \frac{\partial C^*(y^2)}{\partial y} \right) + \frac{8}{r^3} \Delta_1 \ c \left( \frac{\partial C^*(y^1)}{\partial y} \right) \\
\times \frac{8 r^3 - 6 r^2 - 3 r - 1}{\Delta_1^3} \Delta \\
\times \frac{8 r^3 - 6 r^2 - 3 r - 1}{\Delta_1^3} \Delta \\
\times \frac{48 r^3 - 56 r^2 - 27 r - 2}{\Delta_1^3} y^2 \ C^*(y^1) \ y^1 \ C^*(y^2) \ y^2 \\
\times \left[ 576 r^4 + 592 r^3 - 292 r^2 + 347 r + 28 \right] \Delta_1^4 \\
\times \left[ 64 r^6 - 112 r^5 + 108 r^4 - 81 r^3 + 63 r^2 + 23 r - 2 \right] \Delta_1^4 \\
\times \left[ 128 r^5 - 32 r^4 + 24 r^3 - 31 r - 42 \right] \Delta_1^6 \\
\times \left[ 48 r^3 - 56 r^2 + 27 r + 2 \right] \ C^*(y^1) \ C^*(y^2) \ (y^1)^2 y^2 > 0
\]

The inequality is because each item in the square bracket is positive. Similarly,
\[
2 \frac{F'(c) - 2 F'(c^1)}{2} = \frac{3 r^2}{2 \Delta_1} c \left( \frac{dy}{dc} - 2 \frac{dy}{dc^1} \right) + \frac{8 r^3 - 6 r^2 - 3 r + 1}{\Delta_1^3} c \left( \frac{dy}{dc} - 2 \frac{dy}{dc^1} \right)
\]

\[
+ \frac{12 r^3 - r^2 - 2 r}{2 \Delta_1^2} y^1 - \frac{20 r^3 - 17 r^2}{\Delta_1^3} y^1
\]

\[
- \frac{3 r^2}{2 \Delta_1} y^1 g(r) \frac{df}{dc} - y^1 g(r) C^*(y^2) y^2 + c \frac{dg(r)}{dc} y^2 f(r) + 2 y^1 \frac{dg(r)}{dc} y^2 f(r) C^*(y^2) y^2
\]

\[
- \frac{8 r^3 - 6 r^2 - 3 r + 1}{\Delta_1^3} c \frac{y^2 f(r)}{c} \frac{dg(r)}{dc} r + y^2 f(r) C^*(y^1) y^1 + r^2 y^1 g(r) c \frac{df(r)}{dc} + 2 r^2 y^1 \frac{df(r)}{dc} C^*(y^1) y^1
\]

\[
- \frac{48 r^3 - 56 r^2 - 27 r + 2}{\Delta_1^3} y^2 C^*(y^1) y^1 C^*(y^2) y^2 - 2 F''(c^2) - 2 F''(c) > 0
\]