EXAMINATION AND MODELLING OF TREE FORM AND TAPER OVER TIME FOR INTERIOR LODGEPOLE PINE

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Abstract

Forest management, concerned with maintaining or increasing the output of the different forest resources, is increasingly becoming more intensive. Therefore, relevant, accurate, timely, and cost-effective forest resource information on the current inventory and future growth potential is critical. This information must also be utilized over time in the most effective way for planning purposes. Such information can be provided via forest inventories and growth and yield studies. Understanding and modelling of taper changes over time will provide some of the necessary information in an efficient manner.

The two objectives of this study were: (1) to investigate changes in tree form and taper over time as affected by changes in tree, stand and site variables, and (2) to develop a dynamic taper function for dominant and codominant trees of interior lodgepole pine (*Pinus contorta* Dougl.) based on the results of this investigation. To meet these objectives, two different sets of data were used: permanent sample plot data from Alberta and detailed stem analysis data from Interior British Columbia. The permanent sample plot data were used to develop a model to predict stand density and to select models to predict total tree height and diameter at breast height. The stem analysis data were used to examine tree form and taper changes over time, and to select, fit, and test the dynamic taper model. Instead of developing an entirely new taper model, existing taper functions were investigated as possible candidates for both objectives. Two static taper models, the simple taper equation (Husch *et al.* 1982, p. 99) and the variable-exponent taper equation by Kozak (1988), were selected.

The first objective of this study was achieved. Tree shape and taper were found to change along the stem at one time and over time with changes in tree and stand variables, such as the ratio of diameter at breast height to total tree height, crown length and crown ratio, and stand density. It was also found that trees have a simple parabolic shape at young age.
However, as trees age or increase in size, different portions of the stem take different shapes because of unequal diameter growth along the stem. Stand density and crown size appear to be the determining factors in tree shape and taper changes and any changes in these two factors will determine tree shape and taper.

The second objective of this study was also achieved. By incorporating tree, stand, and site factors into a simple static variable-exponent taper model by Kozak (1988), a dynamic taper function was developed. This dynamic taper function tracked the behaviour of very complex tree shape and taper changes over time with reasonable accuracy. The function was tested using a validation data set and it provided consistent estimates of diameter inside bark along the stem over time. The model was fitted using ordinary linear and nonlinear least squares. Feasible generalized least squares was considered, but not used, because of the difficulties in obtaining a consistent estimate of the error covariance matrix.

The dynamic taper modelling approach will be a useful tool in forest management because the taper models will enable forest managers to simulate stand development in order to achieve specific objectives. Dynamic taper modelling appears to be a feasible and practical idea, and it is recommended that dynamic taper models for other species and crown classes be developed to incorporate in individual tree growth models.
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Chapter 1

Introduction

Forest management is concerned with maintaining or increasing the output of different forest resources. Managing the forest requires the forest manager to manipulate stand variables, such as age, density, and site quality, to achieve the intended objectives. In order to do this effectively, the forest manager requires accurate and up-to-date information on the current growing stock (inventory) and future growth potential.

Numerous growth and yield methods have been developed as sources of information. Early estimates of forest growth and yield were based on yield tables. In order to provide data to construct these tables, sample plots were established over a range of age and site classes for single species, even-aged stands. The selected stands were also normally stocked (neither understocked nor overstocked). The plot data were tabulated and summarized to develop a series of alignment charts that were subsequently used to provide yield estimates by the conventional age/site index format (Spurr 1952). Unfortunately, these tables were not representative of actual stand conditions, which led to the development of stand and, later, individual tree models. Moser (1980) gave detailed historical steps in the development of growth and yield modelling.

With forest management becoming information intensive, more information is required than can be provided by stand models. Individual tree models have been developed for this purpose. All individual tree growth models incorporate equations for predicting future values of diameter outside bark at 1.3 m above ground (dbh), total tree height (height) (see Appendix A) and volume. In some models, the equations for predicting dbh and height are used together with a taper function to obtain estimates of future volume (Arney 1985).
Chapter 1. Introduction

These taper (stem profile) equations have been developed to describe how diameter or the stem cross-sectional area changes as a function of height along the stem using dbh, height, and height above ground as independent variables (e.g., Kozak et al. 1969; Ormerod 1973). When these equations are integrated, they provide estimates of stem volume for any portion of the tree. If a taper function is used as a component of a growth model to obtain the stem profile and volume of a tree in the future, an implicit relationship is defined for diameter increment along the stem. It is usually assumed that changes in dbh would adequately reflect the change in upper stem diameter. This assumption may not hold (e.g., Kramer and Kozlowski 1979; Loetsch et al. 1973).

A dynamic taper function can be defined as a taper function which gives the diameter (usually inside bark) at any point along the stem over time. Except for a few quantitative studies examining diameter growth rates at various positions along the stem throughout the tree's life and their relationship to tree taper (e.g., Arney 1974; Mitchell 1975; Clyde 1986), little has been done to model the tree form and taper change along the stem over time.

Clyde (1986) developed a dynamic taper model. She tried to improve stem profile and volume modelling by including diameter increment variation along the stem based on the first derivative of the taper model with respect to time. She found that this approach did not work well. The poor performance of Clyde's taper prediction model could be due to the fact that her height and dbh models were functions of age alone. She never included important tree, stand, and site variables known to affect tree growth in her diameter and height prediction models. Also, the taper models she used assumed that tree shape is constant over time. A dynamic taper model based on precise dbh and height growth prediction functions, along with accounting for changes in form and taper over time, should provide better predictions than Clyde's model.

The two principal objectives of this research were: (1) to investigate changes in tree form and taper over time as affected by changes in stand, tree, and site characteristics, and (2) to develop a dynamic taper function for dominant and codominant trees of interior lodgepole
pine (*Pinus contorta* var. *latifolia* Engelm.) based on the knowledge gained from completing the first objective.

In order to achieve these objectives, the following analyses were performed:

1. Based on a simple taper function (Husch *et al.* 1982, p. 99), variation in stem form and taper along the stem at one particular time and as time changes with different tree, stand, and site factors was investigated using detailed stem analysis data.

2. Using permanent sample plot (PSP) data, models for prediction of quadratic mean diameter, dbh, and height as functions of plot site index, stand density, and age were developed and tested.

3. Using the measured site, stand, and tree variables associated with stem form and taper, an equation to predict the form exponent of Kozak's (1988) taper function was selected.

4. Models for dbh, height, and the form exponent were used to refit Kozak's (1988) taper function to account for dynamic changes in form (shape) along the stem (i.e., make the static taper model dynamic). The fitting methods used included ordinary least squares and nonlinear least squares. Feasible generalized least squares was considered and is discussed, but was not used.

5. Finally, the dynamic taper model fitted using ordinary least squares and nonlinear least squares was tested by comparing it with Kozak's (1988) original taper function, fitted as both as a dynamic and a static model for predictive abilities based on mean bias, standard error of estimate, root mean square biases, fit index squared, and absolute bias using a reserved detailed stem analysis data set.

This research contributes to increasing the body of knowledge on growth modelling in the following ways:

1. A detailed study of how tree shape (form) changes over time was conducted. The benefit of this detailed study is that it has increased our understanding of this area
(basic change in tree form as the tree ages) and how various factors (site quality, stand density, crown length, etc) affect tree form.

2. Taper functions already exist and are in use for estimation of current volume. Compatibility between future and current inventory is expected if future volumes are predicted by a dynamic taper function.

3. Since the taper functions were developed using unmapped tree data, the dynamic taper function could be incorporated into a distance-independent individual tree growth model, which is relatively simple to understand, easy to calibrate, and easy to use. This should provide increased accuracy in predicting future volumes over the use of static taper functions.

Chapter Two contains a summary of the literature on growth of trees as related to tree form and taper; a review of theory about how tree form and taper change with changes in stand, tree, and site factors over time, and a discussion of how different researchers have tried to incorporate such changes during tree taper modelling are included. A description of the data used in building the taper model, the modelling process, and the methods used to compare the dynamic taper fitting techniques are provided in the third chapter. Chapters Four and Five contain the presentation and discussion of the results, respectively. Conclusions and recommendations are given in the final chapter.
Chapter 2

Background

Changes in tree form and taper are a result of changes in diameter over the stem and height growth. Therefore, in order to model tree form and taper changes, an understanding of how trees grow in diameter and height is needed. A summary of the background knowledge about tree growth, which includes the processes behind the growth of trees, the efforts of modellers to describe tree and stand growth, and a brief outline of growth models, is given in Section 2.1. The theory behind the changes in form and taper of trees and the factors that affect such changes is outlined in Section 2.2. The different taper models that have been developed for volume estimation and growth modelling are then discussed in Section 2.3. In the final section, possible alternatives to the ordinary least squares fitting methods, applicable to detailed stem analysis data used for modelling taper over time, are examined.

2.1 Tree Growth

Growth may be defined as an irreversible change in volume (or other attributes of interest), which may be accompanied by a change in form (Thomas et al. 1973). During growth, cells multiply, enlarge, and differentiate into growing parts. There is a complex interplay of many genetically determined metabolic and biophysical processes. Husch (1963) defined growth as the gradual increase in the size of an organism, population or an object over a period of time. This increase or increment consists of the difference in size between the beginning of the growth period and its termination. Biologically, growth means more than just an increase in size. For example, when a dry board is placed in water, it will swell and increase in size. However, this would not be called growth in the biological sense, since biological
growth involves both an increase in size and the formation and differentiation of new cells, tissues, and organs.

Tree growth is the increase in the sizes of individual trees and stands. Growth takes place simultaneously and sometimes independently in different parts of a tree and can be measured by many parameters (e.g., change in diameter, in height, in crown size, and in bole volume).

Tree growth is influenced by the genetic capabilities of the species concerned, interacting with the environment in which it is growing. Environmental influence is manifested through climatic factors, such as air temperature, precipitation, wind, and radiation; soil factors, such as physical and chemical characteristics, soil moisture, and soil microorganisms; and topographic characteristics such as slope, elevation, and aspect. It is the sum effect of these environmental and site factors that, for the rest of this thesis, will be referred to as the site quality. The site quality will be good if it provides favorable growth conditions and poor if it is inhibitive to the inherent growth capacity of a given species (Spurr 1952).

The total tree growth in wood and bark consists of longitudinal and radial growth of the stem, roots, and branches. Longitudinal growth results when the stem or roots are lengthened by forming new tissues at their tips. Radial growth is brought about by divisions of the cambia producing new cells, which become the new wood and bark between the old wood and old bark. Generally speaking, the pattern of growth for a tree during its entire life follows a sigmoid-shaped trend (Spurr 1952; Husch et al. 1982). That is, a tree grows slowly in all dimensions in its early years, called the youth stage, rapidly for a period of time after it becomes well established on site, called the maturity stage, with its rate of growth gradually falling off, the senescence stage, and finally becoming almost negligible at physical maturity. This growth trend is found in individual cells, tissues, organs and individuals for both plants and animals. Such “S-shaped” or sigmoid shaped curves are usually called growth curves.

The cumulative growth curves (yield) for individual trees have typical characteristics which hold for any of the dimensions of a tree. However, the exact form, shape, or position
in reference to the curve axes differs, depending on the growth variables. Plotting cumulative height, diameter, basal area, volume, or weight (mass) over age will give differing specific curves, but all will have the general sigmoid shaped trend, showing the four typical stages described above (see Spurr 1952, p. 212).

A tree is a complex system with relatively few types of structures: 1) leaves, which are the productive machines of trees, take in carbon dioxide from the air, light energy from the sun, and water from the soil and combine them in a process of photosynthesis to produce carbohydrates; 2) the stem and branches, which serve the function of supporting the leaves, transporting water and minerals to the leaves and transporting the manufactured materials (photosynthates) from leaves to the rest of the tree where they can be used for maintenance and growth; and 3) roots, which anchor the tree firmly in the ground and absorb the water and minerals from the soil.

Tree growth occurs in three dimensions: 1) the extension of each growing point forming the shoots of the crown and the roots, called primary growth; 2) the expansion of stem and root diameters (secondary growth); and 3) a combination of these processes which gives each species its characteristic aerial structure and form.

In order to understand the growth of stands and trees, it is necessary to first analyze the pattern of growth and the resulting shape of individual trees. Trees make their annual growth by extending their shoots (height increment) and by thickening of the stems and roots (diameter increment). Change in diameter can be converted into area increment along the stem. According to Pressler's growth law (Mitchell 1975), volume growth is determined by bole area increment along the stem which is a function of, or is proportional to, foliage volume. Thus, tree volume increment along the stem is the product of area and height increment.
2.1.1 Height Increment

Annual shoot growth differs with species, genotype, and climate. The magnitude of the annual height increment fluctuates, depending on the weather conditions, competition, and impact of pests and diseases. In determinate species like spruce, the duration of the annual height growth depends on the weather conditions of the growth year, and particularly on weather conditions of the previous year, especially when buds are formed (July to September) (Assmann 1970). This is called the period of shoot elongation. Tree height growth, like growth of other living organisms, follows a regular pattern in conformity with the natural laws of growth (Assmann 1970).

Height increment is influenced by a number of factors, including species type and whether the species is light demanding or shade tolerant. Light demanding species reach their maximum current annual increment earlier than shade tolerant species. Trees on good sites have rapid annual height growth until the age when culmination occurs. After this age, height growth slows down during the maturation period until senescence when the growth rate is similar for both good and poor sites. On the other hand, trees on poor sites do not show rapid early growth, but they maintain slow growth for longer periods. That is, the site trees on better sites culminate earlier than site trees on poor sites. The intrinsic constitution of individual trees and their changing positions in the social structure of the stand will cause a lot of deviations in height growth. In some species such as lodgepole pine, stand density (the available growing space) can affect height increment. Competition during the early years of growth suppresses height increment and delays the culmination age.

In conifers, particularly in lodgepole pine, height growth is known to increase with an increase in site quality and decrease with increasing stand age and rarely with density above some level. The effects of stand age and density (often represented by crown competition factor) on lodgepole pine height growth were reflected in the density-correlated site index curves developed by Alexander et al. (1967). These show that stand density is as important
a factor as age and site for determining height growth in lodgepole pine.

2.1.2 Radial Growth in Forest Trees

It is well recognized that radial growth in trees of a particular species is influenced by many factors, including climatic fluctuations, site, various stand conditions (including stand treatments), and defoliation (Larson 1963; Mott et al. 1957). Many researchers have completed descriptive studies to examine radial and longitudinal variation in diameter growth in conifers. (See reviews by Larson 1963, Gray 1956 and Assmann 1970). Duff and Nolan (1953, 1957) studied the distribution of radial increment in red pine (*Pinus resinosa* Ait.). From trees 15 to 30 years of age, records were taken of internodal lengths and the width of all annual rings at all nodes. When plotted, the assemblage of ring measurements from a single tree revealed an orderly design of ring widths which was best described in terms of three sequences (see Duff and Nolan 1953, Figure 5). Mott et al. (1957) used different names for the same descriptions (in parentheses).

**Type I (Oblique) sequence.** This is the longitudinal variation in annual diameter increment. Starting from the tip of the tree, diameter increment (growth) increases to a maximum near the crown base, that is, at the area of maximum branch development (Farrar 1961; Larson 1963). Below this point, diameter increment remains constant or decreases downwards along the clear bole and somewhere near the base of the bole, it starts to increase again giving a butt swell. This regular pattern, repeated by successive rings as they are laid down, is attributed by Duff and Nolan (1953) to nutritional gradients in the axis, arising from the distribution of foliage and incidence of light. Some random extrinsic factors can cause fluctuations in Type I sequences, but in most cases these are usually masked by the characteristic “pattern” due to intrinsic growth factors.
Type II (Horizontal) sequence. This is the radial diameter increment at a given height. Starting from the centre of the tree, this sequence increases towards a maximum in the first few rings and gradually declines in successive rings toward the bark with increasing age. Since diameter increment depends on the location along the stem and age, a better understanding of change in diameter increment can be obtained by considering spatial and temporal patterns of increment variation simultaneously.

Type III (Vertical) sequence. This sequence consists of annual rings laid down by cambium of uniform age, along the stem. It is unsystematic variation in the mean ring width (growth) of a constant number of rings from the pith towards the bark along the stem at each internode from the apex downward. This sequence is based on radial increment of the year in which the leader of the tree was formed. Thus the progression in time does not involve radial growth or cambial age, but does involve the formation of successive rings of the sequence in successive years which involves apical activity. The unpatterned growth in this sequence is associated with configuration variation, factors such as site quality and stand density, and randomly distributed variations such as wind and weather type. Tepper et al. (1968) graphically showed that the variation in the Type III sequence clearly expresses the influence of environmental factors on cambial growth (see Figures 3, 6, and 7 of Tepper et al.). Duff and Nolan (1953) also recognized that site quality and stand density have a systematic effect on radial growth, producing what is called “configuration” in the Type III sequence, and that random variations, such as weather and defoliation, produce irregular fluctuations in the growth curve.

Farrar (1961) studied the distribution of diameter growth along the stem for trees in several positions in a stand. He found that the thickness of the outermost annual ring in a healthy dominant tree in a stand of polewood size (medium-aged) timber with a medium density varied as follows:
1. The ring was narrow at the tree top, but increased in width with the descent through the crown, to a maximum near the branches with the most foliage.

2. The annual ring width decreased in the lower crown and for a major part of the bole, and widened again at the base of the tree.

3. The annual ring width of a dominant tree followed the same pattern as that of an open-grown tree, but the open-grown tree had wider rings (more growth) throughout the stem.

4. A crowded tree was characterized by less cross-sectional growth than the dominant tree, and had a maximum ring width closer to the tip, which is probably due to a smaller canopy, less thickening at the base of the tree, and a less efficient lower part of the canopy.

Fayle (1973) represented radial increment at various heights and ages graphically in a topographic map using contour lines to connect equal diameters increments. Fayle (1973), Thomson and Van Sickle (1980), and Julin (1984) constructed three-dimensional surfaces of diameter increment and area increment which enabled them to see when the conditions were not favorable for diameter growth (i.e., "troughs" and "valleys" running parallel to the height axis). Likewise, "ridges" formed when conditions were favorable. Valleys and ridges are caused mainly by disturbances such as fires, defoliation, climatic variation, silvicultural treatments and release from nearby trees (Thomson and Van Sickle 1980; Mott et al. 1957 and Stark and Cook 1957). Julin (1984) claimed that area increment represents growth more closely than diameter increment. However, area increment depends on the present diameter increment as well as the previous year's diameter (Assmann 1970). Since present total diameter is the sum of past diameter increments, area increment combines the effects of present and past disturbances and reflects cumulative size over time. This results in autocorrelation of area increment in successive time periods.
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The form of the Type II sequence at different positions along the stem changes in a fairly regular manner with increasing height, with most of the growth occurring lower on the stem. In older trees, the Type II sequences near the top of the tree tend to become flat (Clyde 1986). Another trend is that Type I sequences tend to become flatter as the tree ages, indicating that diameter increment becomes more evenly distributed along the stem. At early ages, most of the diameter increment from the longitudinal series is distributed along the upper part of the stem. In all trees examined by Clyde, diameter increment continued to increase toward the tip of the tree, without decreasing again after reaching a maximum (Type I sequence). This was in disagreement with what was reported by Duff and Nolan (1953, 1957), Farrar (1961) and Fayle (1973). Clyde attributed this result to measurement interval, length of the live crown, and site quality. She compared three species for diameter growth and found that lodgepole pine reached the maximum diameter increment at a given height fairly rapidly and then quickly decreased (Type II sequence). With white spruce (Picea glauca [Moench] Voss.) and black spruce (Picea mariana [mill] B.S.P.), diameter increment reached a maximum much later than in lodgepole pine and did not decline as rapidly after reaching the maximum. The difference was probably due to the difference in shade tolerance for the species, since lodgepole pine is shade intolerant while the spruces are shade tolerant. Also, spruce lives three to four times as long.

Smith (1980) separated radial growth along the stem (Type I sequence) into early wood and late wood. Heger (1965) postulated that temperature was an important factor governing radial growth and that differences in growth along the stem may be due to air temperature gradients, but provided no evidence. The different sizes of the early wood and latewood layers reflect the respective spring and summer environment energy gradients. Smith found that maximum growth of earlywood and latewood occurred near the base of the full-grown crown for Douglas-fir (Pseudotsuga menziesii [Mirb.] Franco.). Koch (1987) found that diameter growth in lodgepole pine was negatively correlated with stand density, but positively correlated with site quality. He noted that even on significantly better sites, high stand
densities will limit the average stand diameter to be achieved at a given age.

Differences in diameter increment among trees were observed within species in different crown classes (Duff and Nolan 1953; Mott et al. 1957; Clyde 1986). Dominant trees of all species are able to maintain growth near the level of the maximum diameter increment at a given height for a longer period of time than codominants, intermediates and suppressed individuals.

2.1.3 Tree Growth Simulation Models

Growth modelling is basically concerned with how individual trees or groups of trees change over time. Growth simulation models are mathematical relationships that quantify tree or forest development over some range of time, conditions, and treatments. The intent and objectives of a model, availability of good data, philosophy of the modeller, and many other practical considerations cause the model structure to vary from one model to another. No model can perfectly represent the real situation in nature being modelled. Therefore, nothing can be gained from proving that a model is not an exact copy of the real system. However, the model should be able to help the modeller and the user gain a better understanding of what might happen, and help the manager/user to make rational decisions.

Some of the more important reasons listed by Goudie (1987) as to why forest growth should be modelled include:

1. to help the modeller and the user to better understand the dynamic processes of tree and stand growth;

2. to provide short-term projections of inventory plots;

3. to identify critical information needs, such as determining harvest levels/allowable cut;

4. to provide means to bridge gaps in field information so that good prediction of future growth and yield can be made; and
5. to provide means of investigating the effect of applying alternative silvicultural treatments to the stand.

Growth models can be classified as stand- or individual tree-level models (Munro 1974, 1984). Stand growth simulation models are usually easy to use and provide estimates for natural stands or for certain management regimes. Individual tree growth simulation models involve modelling each tree separately, by adding increment to simulated tree boles. The increments are accumulated over time and then summed to produce stand tables or summaries. These models can either be diameter-growth or height-growth driven. Individual tree simulation models are divided into distance-independent and distance-dependent models depending on whether tree spatial data are used in modelling. This basic distinction between individual tree models relates to how competition among trees is estimated. If the estimation is based on measured or mapped distances from each subject tree to all trees within its specified zone of competition, then the model is distance-dependent (see Table 5-1, p. 100 of Davis and Johnson 1987). Common examples of these types models include: PTAEDA (Daniels and Burkhart (1975) summarized by Davis and Johnson 1987, p. 141-145), Tree and Stand Simulator (TASS) (Mitchell 1975), and FOREST (Ek and Monserud 1974). If the crown competition index is based only on the subject tree's characteristics and the aggregate stand and site characteristics, the model is distance-independent. Common examples of these models include: Prognosis (Stage 1973) updated by Wykoff (1984, 1986) and Wykoff et al. (1982), Stems and Tree Evaluation and Modelling System (STEMS) (Belcher et al. 1982), and Stand Projection System (SPS) (Arney 1985).

Most individual tree growth models handle tree growth modelling by either modelling diameter growth first and then modelling height growth (diameter-driven models), or modelling height growth first and then modelling diameter growth (height-driven models). These approaches have problems in that total and merchantable volumes have to be estimated separately using other functions, such as separate taper functions, and logical compatibility of
the height and diameter estimates is not assured. The approach used in TASS (Mitchell 1975) of modelling tree crown development and then predicting the resulting bole change over time results in a complicated and large model that is difficult to incorporate into existing inventory systems.

One possible alternative to modelling tree growth is to model taper over time (i.e., dynamic taper modelling) (Clyde 1986). The dynamic taper modelling of tree growth would result in a growth model that is based on compatible functions for prediction of diameter, height, and volume (merchantable and total). Such a tree growth model could be incorporated into an individual tree growth model that would be relatively simple to understand, easy to calibrate, have a wide application, and be less cumbersome to handle. It would also be compatible with existing inventory information, as well as being portable (require less computer power and storage).

During tree growth, trees experience change in height and diameter. However, growth in diameter for a given tree is not proportional in all sections, consequently differences in taper and form result. Modelling tree growth using taper functions requires an understanding of the theory and manner in which tree form and taper changes along the tree at one time and over time.

2.2 Variation in the Form and Taper of Trees

Stem form and taper have been studied for over a century now and still appear to be high priority subjects in forest research. There are several possible reasons for this. First, no single theory has been developed that adequately explains the variation in stem form, both within and among trees. Thus, it has not been possible to develop a satisfactory taper function that is consistently best for all estimated tree dimensions, all tree species, and uniformly acceptable over a wide range of geographic conditions. Second, and more important from a practical point of view, a taper function that can accurately predict the diameter at any
given point on the stem from a few easily measured variables is essential for estimating the volume of standing trees and constructing volume tables to different merchantable limits (Newnham 1988). Most taper models have been based only on dbh, total height, and some height above the ground; as a result, some noticeable bias still exists, particularly for butt and top sections (see Newnham 1988, Figures 10 and 11; Perez et al. 1990, Figure 2). This means that more information, either about the tree or about the stand and the site, should be taken into account in taper estimation. That is, the one or two readily measured variables are not the only essential variables for estimating the taper of standing trees.

Taper and form have been often used interchangeably. However, based on Gray’s (1956) stem profile model, \( H - h = \alpha d^2 \), (where \( H \) is the total tree height, \( h \) is any height above ground along the main stem of a tree, \( d \) is the diameter inside bark at height \( h \), and \( \alpha \) is an unknown parameter for the main stem of a tree), stem form and taper (profile) have specific meanings. Form is the characteristic shape of the solid as determined by the power index of \( d \) in Gray’s stem profile model, while taper or stem profile is the rate of decrease (narrowing) in diameter (\( d \)) over a specified length or height. Therefore, tree taper is the rate of decrease or narrowing in stem diameter with an increase in height up the stem of a given tree form. Two sections of trees of the same form can have different taper and two sections with different forms can have same average taper (i.e., the same end cross sectional areas and the same length).

According to Gray’s (1956) model, form is expected to be constant, while taper varies by tree. However, both taper and form should vary by tree. That is, form is the geometric shape of the tree stem. The shape may be regular, as for a solid of revolution, or more commonly irregular. The form may be measured by the form factor such as the ratio of the volume of the tree to that of a cylinder of equal basal cross-sectional area and height. This ratio depends on the bark thickness and the taper of the stem. The greater the taper, the smaller the form factor. The degree of taper depends on diameter increment in different parts of the stem. Even if a tree or a log has an estimated form factor of 0.5 (equal to that
of a paraboloid), concluding that the form is that of a paraboloid is unjustified, as the form is probably irregular and not constant over the entire stem.

Trees have often been considered to be composed of three sections (Spurr 1952; Husch 1963): a conical top section (which includes the crown), paraboloidal sections below the live crown, and a neiloidal butt section. Osawa (1992) divided the tree into four sections from the apex: a cylinder at the top, then a cone, a paraboloid, and a swollen base. Within these sections, various irregularities occur in tree form. These are caused mainly by: (1) an abrupt change of diameter at a node; (2) a deformity after injury to the cambium; (3) an abrupt change of diameter associated with heart rot; (4) a swelling from occlusion of branches; or (5) the influence of root swell, buttress, or stem flutes. These irregularities vary with stand density, species, site, age, and other variables (Gray 1956; Larson 1963).

Differences in tree taper can be summarized by the following statements.

1. Above the region of butt swell, the greatest taper occurs in that portion of the stem within the live crown. Both ring width and ring area increase with an increase in distance from the top of the stem, indicating that the stem is probably conical (or even neiloid) in form in lower parts of the stem below the crown (Larson 1963, 1965).

2. The maximum growth of the annual ring area occurs near the base of the crown, and the minimum occurs at some point between the maximum butt swell and the base of the crown. Both the minimum and maximum move upwards in dry years and downwards in excessively wet years (Gray 1956; Larson 1963, 1965).

3. Below the live crown, the rate of diameter growth is largely governed by the position of the tree within the crown canopy. For free-growing trees, ring area may continue to increase down the stem. For trees in the upper canopy, ring area may remain constant so that ring width will consequently decrease. Both ring width and area decline down the stem for suppressed trees (Duff and Nolan 1953; Larson 1963, 1965).
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Four theories have been derived to explain the differences in form and taper of trees. These theories will be described in detail in subsection 2.2.1. Factors affecting form and taper changes will be outlined in subsection 2.2.2.

2.2.1 Theoretical Bases for Tree Form and Taper Variation

Looking at a tree as a system with only three components, leaves, shoot system (branches and main stem), and root system, can be helpful in understanding how a tree functions, but sheds little insight about tree form and taper. The form of trees results from their design as the tallest and longest-lived plants. They have a large number of parts that are highly organized (Wilson 1984). They have refined internal anatomy for physical strength and to allow physiological processes to proceed efficiently. To try to explain the variability of tree shape (form), four theories have been put forward (Gray 1956; Larson 1963, 1965; and Assmann 1970): nutritional, water conduction, mechanistic, and hormonal.

2.2.1.1 Nutritional Theory

The nutritional theory was first proposed in 1883 by Hartig (referenced in Larson 1963). He envisioned stem growth in terms of an equilibrium between transpiration and assimilation. Transpiration was assumed to be the primary factor determining the amount of conductive tissue or early wood in stems. Therefore, a large tree with high transpirational requirements would produce a strongly tapered stem with a high proportion of early wood to satisfy these requirements. As the crown decreases, transpirational demands also would decrease and both the total amount and the proportion of early wood in the growth increment would be reduced. Suppressed trees have extremely low transpirational requirements, resulting in little or no early wood in the lower bole, and thus have less taper. Thinning increases crown size which in turn increases transpirational requirements; therefore, thinning increases early wood and taper. Pruning decreases crown size which in turn decreases transpirational requirements and decreases early wood and taper.
2.2.1.2 Water Conduction Theory

Unlike Hartig (1883, referenced in Larson 1963), Jaccard (1912, 1919 referenced in Larson 1963) held a strong quantitative and mechanistic view about water conduction. He considered the development of the crown (i.e., the organs of transpiration and sunlight absorption), and roots (i.e., the organs of water absorption) to be related and proportional in their development (growth). Thus, the tree stem size was assumed to be determined by the requirement for water conduction. This means that a cylindrical bole would be required for equilibrium water transport between crown and roots. This theory rests on the assumption that cross-sectional area growth is uniform over the branch-free bole. Butt swell was explained by comparison to a capillary system. This theory assumed that any expansion in crown size resulted in increasing the root system. These changes in crown size resulted in a change in the stem form due to an adjustment in relative water conduction area for the new crown size. This theory has a problem in that trees growing on poor sites have been found to have a very high proportion of biomass in the root system compared to trees growing on good sites (Kramer and Kozlowski 1979), yet trees on good sites may have higher taper.

2.2.1.3 Mechanistic Theory

Stems carry equal resistance to bending stresses imposed by the wind. Metzger (1894), Gray (1956) and Wilson (1984)), recognized two mechanical forces that influence the erect stem: 1) the vertical force consisting of the weight of the stem itself plus the additional weight of winter snow and ice, and 2) the horizontal force imposed upon the stem by the wind. Wind was accorded the greatest attention in developing the theory, since it was thought to determine the form and quality of growth on the stem.

The weight of the stem and subtending branches contribute to stem form (i.e., the effect of its own weight). Based on Metzger's (1894) $d^3$ rule, which says that the crown size is related to cubic diameter ($d^3$) (where $d$ is diameter at a distance $h$ from the ground), tree
form should follow static laws since the stem is a carrier of the crown and must also resist external forces applied to it. A difficulty for all stem form theories is the predictable variation in form due to wood constituents, tree age, nutrition, water supply and pressure relations of the cambium.

Gray (1956) demonstrated that the dimensions of the main stem conform to a quadratic paraboloid, $d^2/h$, where $h$ is height along the stem above the ground, and $d$ is diameter at $h$, rather than to Metzger's (1894) cubic paraboloid. A stem of this shape would be consistent with the mechanical requirements of a tree in regards to, not only horizontal wind-pressure on the crown, but also to other forces acting on the stem. Anchoring of the stem was suggested as the possible cause of butt swell. Wind was taken to be the prime factor determining the form and distribution of growth on the stem. According to the theory, the stem carries equal resistance to bending, and the force of the wind responsible for the bending action is greatest at the center of gravity or midpoint of the crown.

Stem form or taper is dependent upon variations in total height, as well as stem diameter. Density changes within limits do not usually affect height growth, but they do affect diameter. Thus, density impacts on taper as the diameter/height ratio changes. Regardless of the intricacies and subtle effects of wind and light on the diameter/height ratio, it is evident that the final tree height resulting from variations in stand closure has a profound influence on the taper of the individual tree stems.

### 2.2.1.4 Hormonal Theory

According to Larson (1963), the nutritional theory accounts for much of the variation in stem form, as well as the distribution of earlywood and latewood in the growth rings. However, various studies have shown that plant growth hormones (auxins) produced in tree apical areas, activate growth when transported to other parts of the tree. Also, growth is known to fail when hormone-producing tree parts are removed (Wilson 1984; Larson 1963). The hormonal theory bridges the gaps of the other theories. The water conduction theory holds
only for ideal stems; stem form is explained on a functional basis, not from the physiological aspect. The mechanistic theory is a functional concept also, and although this theory adequately interprets stem form within reasonable limits, it in no way provides a physiological explanation for the observed facts. However, the hormonal theory on its own cannot explain all the tree form and taper changes because hormones just carry out regulatory functions.

2.2.1.5 Comparison of Theories

Each of the stem form theories appears to explain some aspects of stem form variability. This means that they are either all applicable under certain conditions or, more likely, there are parts of these theories which hold some truth. All the theories seem to include the following two points:

1. Butt swell is very variable, but appears to have a support function.

2. Crown size, particularly crown length, is the most important single factor which determines tree form or taper. It plays a decisive role in determining the stem form.

Larson (1963) and Kramer and Kozlowski (1979) felt that the more recently developed hormonal theory does not supplant earlier developed stem form theories, but may be considered as an adjunct to them by providing the physiological basis. Kozlowski (1971) stated that the formation of wood along the stem is governed more by the physiology of the tree than its strength requirements; the fact that the stem is also mechanically efficient may be fortuitous. However, under most conditions, tree stems are known to respond to stress with, for example, the formation of the compression wood (Larson 1963).

Most of the theories that describe stem form are in qualitative terms. Only the mechanistic theory, largely developed by Metzger (1894; referenced by Larson 1963 and Assmann 1970) and subsequently modified by Gray (1956), attempts to develop a functional relationship between stem diameter and height. Because Metzger postulated that wood formation
in the stem was governed by its requirement for strength, he viewed tree form mechanically as a cantilever beam of uniform resistance to bending. That is, he described the stem as a beam of uniform resistance to bending (particularly to forces brought about by wind), with one end fastened in the soil. Such a beam would have the form of a cubic paraboloid. He was able to show that, below the centre of gravity of the crown, diameter along the tree stem, cubed ($d^3$), plotted against height ($h$) above ground along the stem was more or less a straight line. Gray claimed that the cubic paraboloidal form represented an overexpenditure of material for the strength requirements of the stem since the stem was firmly held at its base. A quadratic paraboloidal form, in which diameter squared is linearly correlated with height ($h$), would be more efficient. Newnham (1965) and Burkhart and Walton (1985) found that the $d^2$ against $h$ relationship held well for that portion of the stem between 15 and 80 percent of the total height and used it to study the variation in taper with age and thinning regime in coniferous species. It should be noted that two of the most commonly used formulae for calculating log volumes (Smalian’s and Huber’s equations (Husch et al. 1982, p. 101)) assume that the stem has the form of a quadratic paraboloid.

2.2.2 Factors Affecting Form and Taper Variation

Generally, variation in tree form and taper is caused by differences in (1) site characteristics (e.g., water, nutrient, weather, etc), and (2) tree characteristics (e.g., age, crown size, canopy position, species) and stand characteristics (e.g., density, stand age, etc).

2.2.2.1 Site Characteristics

Growth of tree crops and their productive performance over some unit of time depends on the site capability. Site influences stem form and taper through its effects on crown development (Larson 1963, 1965). Trees on poor sites (i.e., trees growing on sites deprived of nutrients or lacking water) show the greatest taper and least desirable forms (Metzger 1895, see Larson 1963, p. 9). Smith and Wilsie (1961) found that the annual increment along the stem
increased downwards (increased taper) in wet periods and decreased downwards (decreased taper) in dry periods. These difference in stem forms and taper can be traced to the well-known growth relationship with site quality. Height growth diminishes for trees of the same diameter as site quality decreases, thus increasing stem taper and changing stem form.

According to Kunz (1953 referenced in Larson 1963, p.9), trees growing on poor sites represent an exception to the general rule of taper changes. Although much of the variation in stem form can be assigned to differences in height growth, the relative distribution of diameter growth on the stem does vary widely with site quality. On good sites (i.e., sites that are sufficient in nutrients and available water) growth is concentrated in the upper crown of the lower stem classes, whereas on poor sites the growth tends to be more uniformly distributed along the bole. Newberry and Burkhart (1986) found both taper and form to decrease as the combined crown ratio-site index term decreased. In some exposed areas, wind has the effect of reducing the increase in height per unit of volume as the tree gets older and increasing the taper. Newnham (1965) used Gray’s (1956) taper function to show that site index and stand age have no significant effect on stem taper for Douglas-fir. However, this lack of effect has not been investigated thoroughly.

According to Smith (1980), trees that grow rapidly have a greater degree of taper than trees that grow slowly, due to differences in the distribution of the woody material throughout the growth ring. Young trees growing in the open on good sites will have a greater annual ring growth rate (cambial division) from the tip through the crown, and thus will have a greater degree of taper which results in a lower form factor.

Tree form is highly related to the environment (Larson 1963). Dry interior pines, compared to the wet coastal Sitka spruce (Picea sitchensis [Bong.] Carr.) and true firs (Abies spp.), tend to have short, compact, rounded, and bushy crowns. This type of form is thought to be related to the high temperature and moisture stress of their environment. Tall excurrent growth would severely expose the crown to strong, dry winds.
2.2.2.2 Tree and Stand Characteristics

In young trees, the greater height growth and the steeper slope of the Type I sequence in the upper part of the stem result in a conic form (more taper). As height growth declines, a more constant diameter increment is added over most of the stem with increasing age (i.e., the Type I sequence approaches an asymptote), except at the base and the tip. As the tree gets older, the main bole does not change as much and it becomes more cylindrical. However, the butt swell becomes more pronounced because there is still an increase in diameter increment near the base of the Type I sequence, even in older trees. In lodgepole pine, the Type I sequence becomes constant over most of the stem earlier than in spruce and other shade tolerant species, so that the main stem becomes less tapered (Larson 1963; Newnham 1965).

In addition to size and distribution of live tree crowns, species type has much to do with stem form. Some tree species have less taper and more cylindrical boles even when they are notably dominants and codominants. Such trees have high form factors (i.e., are cylindrical in shape). Gray (1956) explained the difference due to dominance by suggesting that dominance is characterized by relatively greater diameter than height growth. When two trees with the same diameter and height but different crown length are compared, the one with the longest crown will exhibit the greatest taper on the lower stem.

Some insight into the development of stem taper in lodgepole pine trees can be gained by examining the diameter increment trends using the Duff and Nolan (1953, 1957) concept developed for red pine. Koch (1987) noted that lodgepole pine is noted for its minimal taper among the conifers. Butt swell in conifers occurs over a period of time when the diameter increment at any point near the base of the tree is greater than diameter increment at another point above it. Lodgepole pine develops less butt swell and this butt swell takes longer to develop than in the spruces (Clyde 1986). Thus, more shade tolerant, longer crowned species, such as the spruces, will have a more pronounced butt swell and taper (Larson 1963; Clyde 1986) than lodgepole pine. This is mainly attributed to the influence of crown size.
There is a diversity of opinions as to the relative role of heredity in tree form (Larson 1963). However, trees of the same species growing in the same environment may vary in form and taper if some become dominants and others are suppressed. Trees of the same species growing under identical conditions are believed to have the same stem form and taper at equal relative tree heights, but forms and taper will differ at different absolute tree heights. Trees of different species may have different forms and taper. Many authors (e.g., Metzger 1896; Petterson 1927; Fischer 1954 as referenced in Larson 1963 and Gray 1956) observed that trees of a single stand with the same species also tend to show differences in form, which is mostly attributed to genetic difference. Dominant trees are taller and larger in diameter and take a more conical form than the intermediate and suppressed trees, which tend to be more parabolic. This would seem to indicate that the simple shapes assumed by the volume estimation systems (cone-paracone\(^1\)-parabola) are approximations whose performance will vary within a species (Reed and Byrne 1985).

The strong selection pressures of snow, ice, and wind force trees to adopt a conical form of growth. This excurrent tree form is an expression of strong apical control (Assmann 1970). MacDonald and Forslund (1986) examined the geometric form of five species (dominants and codominants only): balsam fir (*Abies balsamea* (L.) Mill.), black and white spruce, white birch (*Betula papyrifera* Marsh.), and trembling aspen (*Populus tremuloides* Michx.). From their analysis, they concluded that balsam fir was close to a paracone in form, black and white spruce and aspen more parabolic, and white birch more conical in form. However, from Figure 1 on page 312 of MacDonald and Forslund (1986), it appears that the forms assigned to the species result in large biases at the base and top of the tree. This shows that the shapes change from the base to the tip of the tree.

Valinger (1992) studied the effect of wind sway on stem form and crown development of Scots pine (*Pinus sylvestris* L.). He found wind to be an important factor influencing

\(^1\)A paracone is a term introduced by Forslund (1982) to mean a tree form which is neither paraboloid nor a cone but somewhere in between.
the distribution of radial growth in scots pine. Increased bending stresses imposed by wind promoted unequal diameter growth for different parts of the stem, with more diameter growth in more stressed parts of the tree. This led to different stem forms for different stem parts, with free-swaying trees having more taper and the stayed trees having less taper.

Tall open-grown trees with deep live crowns have conical shapes and high taper, whereas trees grown in dense stands (stand-grown trees) with shallow crowns and trees that have been highly pruned tend to have low taper. As the stand closes and natural competition sets in, the lower branches die and a progressively longer branch free bole is produced with a decrease in taper. According to Gray (1956), it is evident that trees become more cylindrical in form with an increase in stand density and a decrease in crown length. This is in accordance with Pressler’s growth laws (i.e., tree growth increases from the tree tip to an area of highest branch length just near the base of the crown and then equal increment is distributed to the rest of the stem) (Mitchell 1975). Suppressed trees have narrower growth rings with diameter growth decreasing from upper to lower stem parts, sometimes with missing rings. Stem form is a composite reflection of both stand density and canopy position.

Depending on the canopy position of the tree, the form and taper of the tree changes with time (Gray 1956, p. 47-50). A dominant tree, if it remains dominant, will increase its taper which likely results from change in form due to increased crown size. An intermediate or suppressed tree, if it remains in this canopy position, will have a relative decrease or no noticeable change in taper over time. However, trees are expected to change in form and taper with age, because, at a young age, the social structure of the stand is constant (i.e., no dominance or suppression before intra-specific competition begins). If a suppressed tree increases in height with time while its crown size decreases relative to height, the crown ratio decreases, giving a tree more cylindrical in form with less taper.

Newnham (1958), in his studies of form and taper of forest-grown and open-grown Douglas-fir, hemlock (*Tsuga heterophylla* [Rat.] Sarg.) and western red cedar (*Thuya plicata* Donn.), found that most open-grown trees are conical in shape (i.e, they have a conical
form) and most forest-grown trees are neiloidal in form from ground up to about 15 percent of the total height. From 16 to 80 percent of total height, forest-grown trees are quadratic paraboloidal in form, and conical in form for the last 20 percent of the total height from the ground. Using a limited subsample of hemlock, Newnham also found that taper increases throughout the lifespan of a tree as long as the tree maintains a dominanting canopy position. However, as soon as it changes canopy position (i.e., becomes dominated by the surrounding trees), its taper will begin to decrease or remain constant with increasing age depending on the new canopy position it acquires. Osawa (1992) found stand grown trees to have four distinct sections (cylinder, cone, parabola, and swollen base). However, he found a variation for open-grown and young trees which lacked the parabola and the swollen base sections respectively. Smith (1965) indicated that open-grown trees have a cylindrical form factor of about 0.33 compared to 0.40 for forest-grown trees.

Figure 4 of Baker (1950) shows the differences in depositing woody material in ponderosa pine (Pinus ponderosa Laws.), between an open-grown tree with a long live crown, and an intermediate forest-grown tree about twice the age of the first, having a live crown extending only about one-third the length of the stem. This figure demonstrates the striking difference in pattern of diameter increment along the stem between an open-grown and a forest-grown tree. Since open-grown trees often retain their live branches close to the ground (i.e., they often have nearly 100 percent crown length), the degree of taper of an open-grown tree is always greater than that of a forest-grown tree. Taper is affected by crown size which in turn depends upon whether the tree is open-grown or forest-grown.

Smithers (1961) noted that boles of open-grown lodgepole pine trees taper noticeably in an almost conical form. In extremely dense stands, the stem is whiplike and hardly thicker at ground level than at the top. He further observed that in more mature stands, density affects appearance, so that in very dense stands (for example, 25,000 stems per ha at 90 years), trees have very little taper and are rarely over 6 meters in height and 76 mm in dbh. At medium densities (2,500 to 7,500 stems per ha), the form class, defined as the ratio
between breast height diameter outside bark and the diameter inside bark at the top of the first 16-feet log (Koch 1987), is high, averaging 70 to 75 percent. In low density stands (250 to 2,500 mature trees per ha), the bole has considerably more taper and a form class of 65 to 75 percent. In even-aged forests, taper tends to decrease after canopy closure, before leveling off and, thereafter, altering very little. The change in taper at the time of crown closure follows changes in the depth of the live crown (Newnham 1965; Larson 1963).

Larson (1963, 1965) pointed out that most variations in bole form are attributed to changes in the size of the live crown, its distribution along the stem, and the length of the branch-free bole. In studying the effects of branch length on diameter growth of loblolly pine (\textit{Pinus teada} L.), Labyak and Schumacher (1954) found that the number of branchlets and the location of a single branch on the bole determined its contribution to diameter growth of the main stem. Kozlowski (1971) cited a variety of studies which verify the concept that crown size affects radial growth below the crown, but has less effect on growth within the crown. Burkhart and Walton (1985) used Gray’s (1956) taper function to find the importance of the crown in explaining tree taper. From their investigations, it can be concluded that the crown size measure (crown ratio) of loblolly pine trees in unthinned plantations is related to form and taper parameters of the models of Kozak \textit{et al.} (1969), Gray (1956), and Ormerod (1973).

The major stand treatments which will alter both tree crown size and average stand canopy closure, as well as the stand density, include thinning, pruning and fertilization. Thinning reduces stand density and allows individual trees more space to expand their crowns. For heavily thinned stands, trees grow like open-grown trees which means they have high taper and conical shapes. Pruning does the opposite (Larson 1963, p. 15-17, 1965). Pruning a tree reduces the crown size for a given height, which is similar to increasing the stand density. Thus, pruning decreases taper and makes trees look more parabolic in shape. Fertilizing a tree increases tree vigour; the tree will put on more branches, increase its crown size, and will have a higher taper as a result. However, taper was found to be only slightly
affected by heavy fertilization by Thomson and Barclay (1984). Similar results were found by Gordon and Graham (1986) working with radiata pine (Pinus radiata D. Don) and Tepper et al. (1968) working with red pine.

2.3 Tree Form and Taper Estimation

Taper estimation functions can be divided into two major divisions, static or dynamic. A static taper model is a model which predicts the diameter along the tree stem at a particular time. Whereas, a dynamic taper function is a model that predicts the changing diameter along the tree stem over time.

2.3.1 Static Tree Taper Estimation

Most of the literature on taper modelling is characterized by attempts to model static taper (e.g., Behre 1923; Kozak et al. 1969; Matte 1949; Ormerod 1973; Amidon 1984; Walters and Hann 1986). The taper of a tree can be characterized by measuring diameter at successive points along the stem. If sufficient measurements are taken, it is possible to develop average taper tables which show estimated diameter at chosen heights along the stem (Spurr 1952). The intent is to portray the actual form of the trees which can be used in the calculation of volume.

An alternative to taper tables is taper equations. Taper equations or curves are functions for estimating stem diameter at a given height from known variables such as dbh, height, distance from the tip, and crown size. While tree physiologists have been trying to discover a satisfactory theory for the complex stem form, forest mensurationists have developed mathematical functions that describe the stem profile from the ground to the tip.

Stem taper (profile) functions are used to provide (Kozak 1988):

1. predictions of inside bark diameters at any point on the stem;
2. estimates of total tree stem volume;
3. estimates of merchantable volume and merchantable height to any top diameter and from any stump height; and

4. estimates of individual log volumes.

The desirable features of any taper function are that it should be possible to directly estimate height for any stem diameter (useful for determining merchantable height to a given upper diameter limit), and that the taper function should be capable of being integrated to give a compatible volume function. If neither of these conditions is fulfilled, time-consuming iterative procedures have to be used. Taper functions which do not integrate exactly such as Kozak (1988) and Newnham (1992) may be used if they improve volume estimation. Munro and Demaerschalk (1974) discussed the advantages of compatible volume and taper functions. The usual approach is to develop the taper function first and then the volume function. However, some modellers (e.g., Demaerschalk 1973; Amateis and Burkhart 1988; and Alerndag 1988) have proceeded in the opposite direction by deriving taper functions from existing volume functions.

Different static taper functions have been developed. These can be divided into four major groups, depending on the philosophy of the modellers, as follows:

1. Simple single functions describing diameter change from ground to top (e.g., Hojer 1903 and Jonson 1911 referenced in Gray 1956 and Larson 1963; Behre 1923; Matte 1949; Kozak et al. 1969; Demaerschalk 1972 and 1973; Amidon 1984; Rustagi and Loveless 1991). These are relatively easy to fit, and generally they do not require extensive computer capabilities when applied (Alberta Forest Service 1987).

2. Different equations are used for various parts of the stem and joined in such a way that their first derivatives are equal at the points of intersections (e.g., Max and Burkhart 1976; Demaerschalk and Kozak 1977; Bennett et al. 1978; Cao et al. 1980; Walters and Hann 1986; Ormerod 1986; Flewelling and Raynes 1993; Flewelling 1993). These are
sometimes called segmented taper equations. They require special fitting approaches and more computer power than single equations.

3. Variable-form functions, which are one continuous function describing the shape of the bole, with changing exponents from ground to top to compensate for the changing shape from neiloid at the base of the tree, to paraboloid in the middle of the stem, and to a cone in the crown (e.g., Newberry and Burkhart 1986; Kozak 1988; Newnham 1988, 1992; Perez et al. 1990). These functions can be considered a modification of simple taper equations.

4. A more recent approach to taper function modelling is the use of simultaneous equations, mixed linear models in a polar co-ordinate system, and other methods (e.g., Sloboda 1977; Kilkki et al. 1978; Kilkki and Varmola 1979, 1981; Lui 1980; Lappi 1986; Ojansuu 1987; Sweda 1988).

Table 2.1 is a summary of taper functions for each of these groups. Some of these functions will be described in detail.

2.3.1.1 Simple Taper Functions

The earliest efforts to express tree taper by mathematical functions began with relatively simple formulas, similar to the taper equation given by Husch et al. (1982, p. 99):

\[ y = K \sqrt{x^r} \]  \hspace{1cm} (2.1)

where \( y \) is the radius of the stem at distance \( x \) from the tip of the tree; \( K \) is a constant for a given form (i.e., \( K = \frac{R_b}{\sqrt{H}} \) where \( R_b \) is radius at the base of the tree and \( H \) is tree total height); and \( r \) is a form exponent which changes for different geometric solids. When \( r \) is 1, a paraboloid is obtained by rotating the curve of this equation around the \( x \) axis, when \( r \) is 2, a cone is produced, when \( r \) is 3, a neiloid is produced, and when \( r \) is 0, a cylinder is
Table 2.1: A summary of some static taper models developed to date.

<table>
<thead>
<tr>
<th>Simple Taper Equations</th>
<th>Segmented Taper Equations</th>
<th>Variable-Form or -Exponent Taper Equations</th>
<th>Other Taper Models (including Mixed Linear Models, Splines, PCA(^a))</th>
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<tbody>
<tr>
<td>Hojer (1903)</td>
<td>Heijbel (1928)</td>
<td>Reed and Byrne (1985)</td>
<td>Fries (1965)</td>
</tr>
<tr>
<td>Allen (1991)</td>
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<tr>
<td>Forslund (1991)</td>
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<td>Rustagi and Loveless (1991)</td>
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<tr>
<td>Allen et al. (1992)</td>
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</tbody>
</table>

\(^a\)PCA is principal component analysis.
Figure 2.1: The geometric division of a tree stem ($D_b$ or $2R_b$) is the diameter at the tree base, $d$ is the diameter at height ($h$) above ground, and the other symbols as defined for Equation 2.1).

produced (Husch et al. 1982, Figure 8.1). However, trees are rarely cones, paraboloids, or neiloids, but generally they are a combination of all of them (see Figure 2.1).

In 1903, Hojer (quoted by Behre 1923) developed the first recorded taper equation, to express the diameter of a tree at any point on the stem based on the measurements of Norway spruce ($Picea abies$ (L.) Karst). The function was of the form:

$$\frac{d}{D} = C \log \frac{a + x}{a} + e$$

where $d$ is the diameter inside bark at height $h$ above ground or at distance $x$ from the tree tip; $D$ is dbh; $C$ and $a$ are constants; and $e$ is the error term.

In 1923, Behre used ponderosa pine data to determine whether Hojer’s (1903) equation could be improved by introducing a new term to better fit the conditions, or if a different equation could be found which would more nearly describe the average form of tree stems. As a result, a new equation was developed, which more consistently described the shape. The equation was hyperbolic in nature as follows:
\[
\frac{d}{D} = \frac{x}{a + bx} + e
\]  

(2.3)

where constants a and b vary with the form quotient, but in all cases \(a + b = 1\). Conformity to the equation is indicated if a plot of \(\frac{Dxx}{d} \) on \(x\) produces a straight line. The constants can be found graphically or by least squares.

Kozak et al. (1969) developed a taper function based on the assumption that the tree bole is a quadratic paraboloid. The equation was a second order quadratic polynomial:

\[
\left( \frac{d}{D} \right)^2 = b_0 + b_1 Z + b_2 Z^2 + e
\]  

(2.4)

subject to the restriction \(b_0 + b_1 + b_2 = 0\), where \(Z = h/H\); \(b_0, b_1\) and \(b_2\) are constants; \(h\) is height above ground to diameter, \(d\); and \(H\) is total tree height. For some of the species on which the model was tested, coastal Sitka spruce and western red cedar in British Columbia (B.C.), negative estimates of upper stem diameters were obtained. For those species, the following conditioned function was recommended:

\[
\left( \frac{d}{D} \right)^2 = b_1(1 - 2Z + Z^2) + e
\]  

(2.5)

This equation showed some systematic bias\(^2\) for both butt and tip sections of the tree for diameter estimation (see Table 3 of Kozak et al. 1969).

Amidon (1984) used bole diameter and height measurements to predict volume to a variable top diameter accurately and precisely for several conifer species. Since he did not entertain the idea of multiple bole sections, Amidon used a single equation to predict taper above breast height and estimated the volume for the section below breast height using Smalian’s formula. The taper function was:

\[
d = b_1(D \times \frac{x}{H - bh}) + b_2(H^2 - h^2)(h - bh)/H^2 + e
\]  

(2.6)

\(^2\)For this thesis, bias is defined as the difference between the actual (or measured) observation and the predicted value. If the bias is positive it means the variable (e.g., diameter) is underestimated and when negative it means the variable is overestimated.
where $d$ is the predicted diameter inside bark at height $h$; $bh$ is breast height; and $b_1$ and $b_2$ are regression coefficients to be estimated. Seven alternative models were selected for comparing diameter predictions with the above model. The hyperbolic taper function by Behre (1923) and a segmented function by Ormerod (1973) were immediately discarded because both had standard errors more than twice that of Amidon's model. The remaining five models were by Bennett and Swindel (1972), Demaerschalk (1972), Kozak et al. (1969), Max and Burkhart (1976) (a segmented model), and Bruce et al. (1968). From Table 2 of Amidon (1984), it can be seen that Amidon's model was the best, followed by the models of Max and Burkhart, Bruce et al., Demaerschalk, Bennett and Swindel, and Kozak and others.

Rustagi and Loveless (1991) developed simple, fully compatible, cubic volume and stem taper equations based on individual trees. They developed equations to predict a cylindrical form factor based on the ratios between total height above breast height and total height using varying form values (1/2, 2/3, and 3/4), and showed that it is related to the geometric theoretical sections of Figure 2.1 and therefore to stem taper. Based on Pressler's (1864) theory (referenced in Loetsch et al. 1973), the tree stem was assumed to be divided into two parts, the portion above breast height (BH), and the portion below breast height (bust section). Taper and volume equations were developed for the above breast height section and the section below breast height was approximated as being a cylinder. Modifying Ormerod's (1973) taper function, Rustagi and Loveless' taper function became:

$$d_l = (c_0 + c_1 D) \left( \frac{H - l}{H - 1.37} \right)^p + e$$

(2.7)

where $d_l$ is the diameter at height $l$ above breast height; $c_0 + c_1 D$ is a function for predicting diameter inside bark at breast height, where $c_0$ and $c_1$ are coefficients; and $p$ was the form exponent computed from form factor function ($p = b_0 + b_1 R_j$), where $R_j$ represented the height ratios of $H - l$ to $H - 1.37$ at $1/2D$, $2/3D$ and $3/4D$. Based on validation data for 20 Douglas fir trees, Rustagi and Loveless compared the performance of their taper
model with that of Kozak’s (1988) variable exponent taper function and Walters and Hann’s (1986) segmented taper model for volume prediction. From Figures 3 and 4 of Rustagi and Loveless, this taper model appears to have out-performed both Kozak’s, and Walters and Hann’s models. It was also noticed that Kozak’s taper function tracked the actual stem profile more accurately than the other two for lower form factors (less than 0.4), but as the form factor increased Kozak’s taper model became the poorest. Since Rustagi and Loveless’ compatible stem profile has no inflection points, it can model change in geometric shapes along the stem, but its form exponent is continuous, meaning that the stem form changes as height increases from the ground. This taper model has a weakness in that it assumes that all trees are cylindrical below breast height. If butt swell extends above breast height, this model will overestimate diameters in lower part of the stem above breast height and underestimate diameters for upper sections towards the tip of the stem. Another weakness is that the $R_j$ are needed and these have to be measured.

The earliest and the majority of taper functions developed to date belong to the category of simple taper models (see Table 2.1). However, the unsatisfactory predictions by these models have kept many taper modellers looking for better models to improve prediction. This has lead to development of more complicated models, such as the segmented and other types of taper functions.

2.3.1.2 Segmented Taper Functions

As pointed out by Heijbel (1928), Grosenbaugh (1966), and other researchers, stem profile may be best modelled in sections. Grosenbaugh observed that tree stems assume an infinite number of shapes and that it was difficult to develop a single, accurate equation to describe the taper of the stem. He said that each stem had a number of inflection points and the traditional conoid, paraboloid, and neiloid are merely convenient instances in a continuum of short monotonic shapes, increasing from tip to stump, with a possibility of having many inflection points along the stem. Several authors have recognized that other geometric forms
are possible. The form of a tree stem does not change abruptly from one geometric form to another; it is continuous, as has been recognized by taper modellers such as Kozak (1988) and Newnham (1988).

The introduction of computers in forest research in the early 1960s and the increased availability of appropriate software, coupled with the failure of simple taper curves to trace the multiple inflection points along the stem bole or failure to fit the swollen butt logs or both, led to the development of complex taper functions. Segmented taper models describe each of several sections of a tree bole with separate equations. The method commonly used to describe these shapes is to fit each section with a simple or polynomial equation, usually quadratic, and then mathematically provide conditions for a continuous curve at the two join points of the segments (Max and Burkhart 1976; Demaerschalk and Kozak 1977; Cao et al. 1980; Ormerod 1973, 1986). Studies by Cao et al. (1980), Martin (1981, 1984) Amidon (1984), and Walters and Hann (1986), and others, have shown that complex taper equations, such as segmented taper functions, provide better fits of the stem profile than simple taper models, especially in the high volume butt region.

Since Heijbel (1928) (referenced by Larson 1963) proposed a taper model composed of three submodel curves, one for the base, one for the clear middle part, and one for the top, several other models in this class have been developed (Ormerod 1973, 1986; Max and Burkhart 1976; Demaerschalk and Kozak 1977; Cao et al. 1980; Walters and Hann 1986; etc) (see Table 2.1). Ormerod (1973) was the first to develop a two-section taper function. He started with a simple model of the type:

\[ d = d_j \left[ \frac{H - h_j}{H - h} \right]^p + e; \quad p > 0 \]

where \( d_j \) is the measured diameter at a fixed height \( h_j \) above ground, and \( p \) is the fixed exponent. This model could not provide an adequate description of the changing tree bole, due to change in form along the length. Ormerod believed that a better description may result from the use of a modified exponent of the above equation as a step function. Therefore,
separate exponents had to be fitted for each stem section as:

\[ d = (d_j - C_i) \left[ \frac{H_i - h}{H_i - h_j} \right]^{p_i} + C_i + e; \quad p_i > 0 \]  

(2.8)

where \( H_i \) is the height at the intercept (section \( i \)); \( C_i \) is diameter at \( H_i \); \( d_j \) is the section measured diameter at fixed height \( h_j \) above ground; and \( p_i \) is the fitted exponent on the closed interval, \([h = H_{i-1}, h = H_i]\) (see Figure 1 of Ormerod for a two-section case). Ormerod showed that his taper function was more accurate than that of Kozak et al. (1969).

Perhaps, the most commonly used model in this class was developed by Max and Burkhart (1976). Their segmented polynomial taper model, developed for loblolly pine natural stands and plantations, divides the stems into three sections. Separate conditioned polynomial equations were calculated for each section. The location of the joining points was selected by the model to give the best fit to the stem profile. Max and Burkhart found that the most satisfactory equation system was a quadratic-quadratic-quadratic model of the form:

\[
\left( \frac{d}{D} \right)^2 = b_1(Z - 1) + b_2(Z^2 - 1) + b_3(a_1 - Z)^2I_1 + b_4(a_2 - Z)^2I_2 + e
\]  

(2.9)

where \( a_1 \) and \( a_2 \) are the relative distances from the top of the tree of the upper and lower joining points respectively, that is

\[
I_1 = 1, \quad 0 < Z \leq a_1
\]
\[
= 0, \quad a_1 < Z < 1
\]
\[
I_2 = 1, \quad 0 < Z \leq a_2
\]
\[
= 0, \quad a_2 < Z < 1
\]

and \( b_1, b_2, b_3, \) and \( b_4 \) are regression coefficients. The Alberta Forest Service (1987) tested 15 taper and volume functions, for accurate estimation of log and merchantable volume. The Max and Burkhart taper function was found to be the best overall and was recommended for general application in Alberta. Although this function requires nonlinear fitting methods, most statistical software packages have nonlinear regression procedures. However, a considerable amount of computing time and money may be required, particularly if initial estimates
of the parameters are not close to the actual values (Neter et al. 1985). The advantages of this function are that it can be directly integrated to give both total and merchantable tree volumes as well as log volumes and it can be transposed to directly estimate merchantable height.

Based on Ormerod’s (1973) taper model, Byrne and Reed (1986) decided to define $p$, the form exponent, as a segmented equation similar to the segmented taper functions of Max and Burkhart (1976). In order to make this form exponent correspond to the accepted theoretical tree sectional forms, a two-segment equation was used to define the coefficient $p$, depending on the relative height from the ground and the fitted coefficient, $a$, as follows:

$$
\hat{p} = \frac{3}{2} - \left(\frac{Z}{a_1}\right) - \left[1 - \left(\frac{Z}{a_1}\right)\right]I_1 + \frac{1}{2} \left[\frac{(Z - a_1)}{(1 - a_1)}\right]I_1
$$

(2.10)

where $\hat{p}$ is the predicted $p$ value; and $I_1 = 1$ if $Z \geq a_1$, otherwise $I_1 = 0$. This made $p$ continuous along the stem bole. The $p$ coefficient equation was constrained so that it was equal to $3/2$ at the tree base, then decreased to $1/2$ in the middle at a point where $a_1 = Z$, and increased again to 1 at the top. As Byrne and Reed pointed out, a linearly smooth transition from $p = 3/2$ to $p = 1/2$ without passing through $p = 1$ (for a cone) is impossible. This taper function cannot be integrated to an exact form, therefore numerical integration has to be used for volume estimation. The taper equations were calibrated using red pine and loblolly pine data. From the comparison of results of this model with that of Cao et al. (1980) and Max and Burkhart (1976), Byrne and Reed concluded that this model was more accurate and precise for prediction of diameter at any height ($h$) above ground, and total and merchantable tree volumes.

Flewelling and Raynes (1993) developed a three segment taper function for western hemlock. This function is very complex; it requires nonlinear fitting methods, and 26 parameters have to be estimated simultaneously. No substantial improvements were noted over the simpler model by Kozak (1988) (see Table 13 of Flewelling and Raynes). Flewelling (1993)
improved on the taper function by Flewelling and Raynes by including more variables (upper stem measurements) in addition to dbh and height. However, his function seems to overestimate volume, particularly for the lower and upper segments compared to the function by Flewelling and Raynes.

In general, segmented taper models are more complicated and more difficult to fit, particularly those involving nonlinear equations. Those requiring nonlinear fitting methods may take many steps before converging or may not converge at all. Therefore, some easier-to-fit models were later developed, the variable form/exponent taper models.

2.3.1.3 Variable Form and Exponent Taper Functions

In 1985, the idea of a single model to describe tree taper resurfaced. But this time, it was known as "variable-form" taper function which described tree taper with a continuous function using a changing exponent to compensate for the changes in form of the different tree sections. Based on Forslund's (1982) idea of a paracone, Reed and Byrne (1985) used Ormerod's (1973) simple taper function and derived separate $p_i$ values for each tree depending on the tree's total height and breast height diameter. The resulting taper function was simple, and variable, that is, it accounted for form change along the tree bole. Evaluation of this taper function showed that the taper curve tended to slightly underestimate diameters in the lower stem and overestimate diameters in the upper stem.

Newnham (1988) assumed that instead of Ormerod's (1973) taper model having fixed values of $p$ for each section (see Figure 2.1 for the tree sections), the geometric form of the tree stem should vary continuously along its entire length. His model appears very simple, but the exponent $k$ ($p = \frac{1}{k}$) is complex, as shown below:

$$
\left( \frac{d}{D_{in}} \right)^k = \frac{(H - h)}{(H - 1.30)}
$$

(2.11)

where $D_{in}$ is inside bark diameter at breast height. In order to develop a model for $k$,
Newnham transformed Equation 2.11 logarithmically and calculated values for $k$. The values for $k$ varied between 0.5 and 4. In order to make $k$ continuous along the stem, he fitted six different models for $k$ based on transformations of relative height $((H-h)/(H-1.30))$, $D/H$, and $1/h$. He substituted these function for $k$ into Equation 3.11 and tested the taper models for their accuracy based on their prediction statistics. The best taper model selected for diameter estimation was:

$$d = D_{in} \times \left( S \right)^{1/[b_0 + b_1 (D/H) + b_2 S \times (D/H)^2 + b_3 (1/h)^{b_4}]} \times e$$  \hspace{1cm} (2.12)$$

where $S = (H-h)/(H-1.30)$; $D_{in}$ is inside bark diameter at breast height; $b_0$, $b_1$, $b_2$, $b_3$ and $b_4$ are coefficients to be estimated; and the last part of the exponent (fraction $1/h$) was included in the model to account for butt swell. The weaknesses of this taper model include: 1) bias at the butt and tree top (i.e., high positive values); 2) it cannot be easily used to estimate height $(h)$ at a given diameter $(d)$; 3) it is not possible to integrate directly for total and merchantable volume estimation; 4) the accuracy tests are based on predictions for $d/D_{in}$ instead of $d$, therefore, the results might be misleadingly, and 5) $D_{in}$ must be known, which is usually unavailable or difficult to measure on standing trees.

Kozak (1988) used a similar approach to Newnham (1988) in that one continuous function was used to describe the shape of the bole, with a changing exponent from ground to tree top to compensate for the various geometric forms over the tree stem. Kozak defined his model as:

$$d = DI \times M^c \times e$$  \hspace{1cm} (2.13)$$

where $M = \frac{(1-\sqrt{q})}{(1-\sqrt{q})}$, $q$ is equal to $(HI/H)$, $HI$ is the height of the lower joint point above the ground, and $DI$ is the diameter inside bark at the join point. This function provided the general shape needed to describe the change of diameter inside bark from ground to tree top. As with other variable exponent taper functions, this taper function can be viewed as a modification of the Husch et al. (1982) taper function (Equation 2.1 of this thesis). Kozak modified $X$ and $K$ in equation 2.1 to account for the sharp change in diameters near the
base and \( c \) represents \( r \) of Equation 2.1. Based on Demaerschalk and Kozak (1977), the join point ranges from 20 to 25 percent of the total height from the ground; the relative height of this join point is fairly constant within a species regardless of tree size for most of the tree species in B.C. The exponent \( c \) should vary with \( Z \) and \( D/H \). After trying different transformations of \( Z \) on the data, Kozak came up with the following function for estimating \( c \).

\[
c = b_1 Z^2 + b_2 \ln(Z + 0.001) + b_3 \sqrt{Z} + b_4 \exp(Z) + b_5 (D/H) + e
\]

(2.14)

where \( \ln \) is the natural logarithm; \( \exp \) is the exponential (i.e., \( \approx 2.71828^Z \)); and \( b_1 \) to \( b_5 \) are coefficients. Substituting the changing exponent \( c \) into the original equation, the equation becomes:

\[
d = DI \times M^{b_1 Z^2 + b_2 \ln(Z + 0.001) + b_3 \sqrt{Z} + b_4 \exp(Z) + b_5 (D/H)} \times e
\]

(2.15)

Since the diameter at the join point \( (DI) \) cannot be measured readily, Kozak developed a regression relationship for it using dbh. The equation was: \( DI = a_0 D^{a_1} a_2^D \). By rearranging the function and carrying out logarithmic transformation, the function became:

\[
\ln(d) = \ln(a_0) + a_1 \ln(D) + D \ln(a_2) + b_1 (Z^2) \ln(M) + b_2 \ln(M) \ln(Z + 0.001)
\]

\[
+ b_3 \sqrt{Z} (\ln(M)) + b_4 \ln(M) e^Z + b_5 \ln(M) (D/H) + \ln e
\]

(2.16)

This function was fit using ordinary least squares regression. It has the properties that diameter \( (d) \) equals 0 when \( h = H \), and \( d \) equals estimated \( DI \) at \( h = HI \).

Kozak’s (1988) taper model has only three independent variables \( (h, H, \text{ and } D) \) with several transformations, and as a result, the chances of multicollinearity are very high. Multicollinearity could result in the coefficient estimates from least squares being unstable and inconsistent in sign, with inflated variances. Such coefficients would be difficult to interpret. In order to reduce multicollinearity, Perez et al. (1990) tested Kozak’s model to find out if all the independent variables were needed using Oocarpa pine (\textit{Pinus oocarpa} Schiede) data from central Honduras and dropped the transformations that were not significant. The
selection of the reduced model was based on: 1) mean square error (MSE); 2) coefficient of determination ($R^2$); and 3) prediction sum of squares (PRESS) for predicted $\ln(d)$. The best model was either the one with high $R^2$ or the lowest value for one or more of the other criteria. The selected reduced model was:

$$\ln(d) = \ln(a_0) + a_1 \ln(D) + b_1 \ln(M)Z^2 + b_2 \ln(M)(Z + 0.001) + b_5 \ln(M)(D/H) + \ln e \quad (2.17)$$

The equations by Kozak, Perez et al., and by Max and Burkhart (1976) were tested by Perez et al. on an independent data set. They found that Max and Burkhart’s model had a relatively large bias in the upper sections of the tree, while the models by Kozak and Perez et al. had a very small bias. Since Newnham (1988) compared his taper model with that of Max and Burkhart and found his taper model only as accurate, it is probable that the Kozak’s taper function is more accurate than the Newnham’s taper model. It should be noted that none of these models were tested by Perez et al. for volume prediction abilities.

In 1992, Newnham acquired data from the Alberta Forest Service and refitted his 1988 taper model for jack pine ($Pinus banksiana$ Lamb.), lodgepole pine, white spruce, and trembling aspen. The comparisons of the biases in estimating $d/D_{in}$ for his models and Kozak’s (1988) taper equation seemed to show no real differences. However, Newnham assumed that diameter inside bark at breast height ($D_{in}$) is known, which is not always the case. Also, in his comparisons, he used ($d/D_{in}$) which is not a practical variable to a forest manager who is going to use the model.

### 2.3.1.4 Taper Functions based on Other Methods

Other approaches used for tree taper and form description include the use of multivariate statistical techniques, simultaneous equations, splines, mixed linear models, polar coordinates, and models developed from growth functions.

Fries (1965), Fries and Matern (1966), Kozak and Smith (1966), and Real et al. (1989) used principal component analysis (PCA) to study the form and taper of different tree
species. Kilkki et al. (1978) and Kilkki and Varmola (1979) used simultaneous equations for determining taper curves, while Liu (1980) introduced the idea of splines. In 1986, Lappi developed a system of mixed linear models for analyzing and predicting variation in the stem form of Scots pine. This concept was later used by Ojansuu (1987) and Kilkki and Lappi (1987) to model the growth of tree size and changes in stem form. The stem was defined in the polar coordinate system, pioneered by Sloboda (1977), as the logarithmic lengths of rays at different angles $d(u)$, where $u$ is the angle at which the diameter was measured, and tree size was defined as a weighted mean logarithmic dimension (mean diameter or height). Nagashima et al. (1980), Nagashima (1988), and Sweda (1988) employed the Mitscherlich growth process to produce stem taper functions. Also, Biging (1984) and Brink and von Gadow (1986) used growth and decay functions to model stem profiles.

These types of taper functions have not yet been used in many countries. One possible reason why this type of taper modelling philosophy has seen limited application is the complicated fitting process.

### 2.3.2 Dynamic Tree Taper Estimation

Ormerod (1973) stated that his taper function would be better fitted using exponents as functions of important physical and biological parameters such as dbh, total height, density, age and site, however, he claimed there were no suitable data available at the time. In his review of the available literature on tree taper and form, Sterba (1980) pointed out that the taper curve theory had reached a stage where further research would only be worthwhile if the influence of site and silvicultural treatments on relative stem taper curve were considered. Also, Kilkki and Varmola (1981) stated that the present static taper curve models looked improbable because the stem form was a result from the lifelong influence of the environment and stand characteristics. Consequently, the influence of stand density, for example, cannot easily be taken into account in static models.

When a static taper model is used as a component of an individual tree growth model,
usually only dbh and total height are predicted (projected into the future) to obtain future stem diameters and volume. However, the change in dbh over time may not necessarily be a good reflection of the change in diameters at other positions along the stem (Kramer and Kozlowski 1979). Clyde (1986) pointed out that when projecting growth, it would be ideal if taper models predicted diameter increment along the bole consistent with observed patterns of diameter growth. Unfortunately, most taper equations are empirical models with little biological basis. These models are based on cumulative dbh and total tree height rather than the growth in diameter which represents the underlying growth processes. Therefore, it should be possible to improve the existing taper functions by looking at the variation in shape along the stem, projecting diameter change along the stem, and including some important site, stand and other tree characteristics in the prediction model (Sterba 1980).

Newberry and Burkhart (1986) developed and evaluated procedures for incorporating both form and taper changes into stem profile models for loblolly pine using a modified version of Ormerod’s (1973) taper function and Gray’s (1956) taper parameter. Their variable-form stem profile equation allowed the taper and form parameters to vary as functions of dbh, crown ratio, age and site index. A two-stage fitting procedure was used to obtain a stem profile that accounted for both form and taper changes along the stem. In the first stage, the parameters of a base model were estimated for each tree. The parameters were then related to tree or stand characteristics in the second stage. From these models, they were able to determine that taper decreases as dbh decreases and form decreases with an increase in total tree height (see Newberry and Burkhart (1986) for their definition of taper and form). They concluded that variable-form taper models can be constructed if stem profile models have parameters which can be related to tree form as defined by Gray (e.g., Equation 2.1; Ormerod 1973). Their form parameter was related to several tree and stand characteristics that have a biological meaning. The taper measure was fit as a function of dbh and a product of site index and crown ratio, and the form measure was fit as a function of height to crown base, height, tree age, and a product of site index and crown ratio. This
variable-form taper model is considered to be dynamic because changes in diameter, height, crown length with time (age) will change the tree form and taper.

Clyde (1986) used the Chapman-Richards function (Pienaar and Turnbull 1973) to model dbh and height growth as functions of tree age. She substituted these models into the taper equations by Kozak et al. (1969), Max and Burkhart (1976), Bennett et al. (1978) and Amidon (1984) to make them dynamic. Clyde found that the taper models by Bennett et al. and Max and Burkhart predicted the pattern of diameter increment for the Type I sequences more reasonably than the simple models by Kozak et al. and Amidon. However, the predicted diameters for all functions were poor for ages greater than 100 years, and there were some inconsistencies between the observed and predicted Type II sequences for all of the taper models examined. She suggested that in addition to dbh and height predictions, some other variables or more complex models should be considered to obtain predictions of diameter change over time along the stem that were consistent with the growth and development of a tree.

Although the idea of dynamic taper modelling existed as early as 1973 (e.g., Ormerod 1973), it was not until 1986 that a dynamic taper model was produced. However, it should be noted that apart from the works of Clyde (1986) and Newberry and Burkhart (1986), not much has been accomplished in the area of dynamic taper modelling.

2.4 Alternative Methods for Fitting Taper Functions

Most tree taper modellers have used ordinary least squares (OLS), or nonlinear least squares (NLS) fitting procedures to obtain parameter estimates for taper functions. For example, Clyde (1986) and Kozak (1988) used OLS, Newnham (1988) used both OLS and NLS methods, to fit the taper functions. However, proper fitting of these taper models could require special techniques. Because the data used to fit these models are not identically distributed and not independent (i.e., non-iid), ordinary least squares is not efficient. If compatible
volume and taper predictions are desired, then volume and taper equations used should be fitted as a system of equations (Burkhart 1986; Reed 1987). Simultaneous fitting procedures should be used for fitting the system because they ensure numeric consistency among the component equations and more accurate and precise volume and taper estimations (Reed and Green 1984; Byrne and Reed 1986; Judge et al. 19985, p. 591-630).

Two principal sources of data for dynamic taper modelling are permanent sample plots (PSPs) and detailed stem analysis data. PSP data are characterized by repeated measurements on the same individual (a tree or plot) over time (time-series or serial measurements). The resulting data are serially correlated or have autocorrelated error terms (i.e., are not independent of each other). To use PSP data for dynamic taper modelling, measurement of diameter inside bark \((d_i)\) along the stem over time would also be needed. Since these \(d_i\)'s are taken on a single tree, they will be correlated. This type of correlation, called cross-sectional or contemporaneous correlation, is the correlation between different error terms for observations taken from the same individual at the same time. If \(d_i\)'s were available on PSP data, PSP data would be the most appropriate data for dynamic taper modelling because it would combine tree and stand information measured over time. However, such data are rarely available for modelling dynamic taper.

Detailed stem analysis data are obtained by taking diameters at several positions along the same tree at one time (called cross-sectional data). These data would have contemporaneously correlated error terms. If the cross-sectional data are also measured over time (i.e., diameters at given positions along the tree stem are taken over time), panel or longitudinal data result which generally are both contemporaneously (related over the stem) and serially correlated (related over time) (Dielman 1983, 1989). These data lack tree and stand measurements for previous time periods before felling (i.e., basal area, number of stems, crown size), which are usually available for PSP data. However, stem analysis data are the most often available for use in taper modelling.

Both PSP and stem analysis data have another characteristic. It was shown earlier that
diameter growth varies for different parts of the tree. As a result, trees will have varying cumulative diameters for the different sections along the tree stem. Therefore, error variances along the stem will likely be different (heterogeneous variances), due to the variation in diameter increments. Thus, stem analysis or PSP data used in dynamic taper model fitting can be characterized by having heterogeneous, contemporaneously correlated, and serially correlated error terms.

One model that could be used for fitting linear dynamic taper functions can be generally written as (Judge et al. 1985, p. 514):

$$Y_{ijt} = \beta_{ij0} + \sum_{k=1}^{K} \beta_{ijk} X_{ijtk} + e_{ijt}$$  \hspace{1cm} (2.18)

where $i$ corresponds to tree number; $j$ corresponds to cross-sectional measurements which are contemporaneously correlated; and $t$ corresponds to time measurements which are serially correlated. $Y_{ijt}$ is the value of the dependent variable for tree $i$, section $j$ and time period $t$; $X_{ijt}$ is the $k^{th}$ independent variable for tree $i$, section $j$, and time period $t$; $\beta_{ijkt}$ is the $k^{th}$ coefficient, associated with the $k^{th}$ independent variable, to be estimated which would vary over tree $i$, section $j$, and time period $t$; and the error terms ($e_{ijt}$) are assumed to have zero expectation, to have heterogeneous variance $\sigma_{ij}^2$, and to be contemporaneous and serially correlated.

The model for fitting nonlinear functions would be:

$$Y = f(X; \Theta) + e$$  \hspace{1cm} (2.19)

where $Y$ is a column matrix of dimension $\sum_{i=1}^{N} \sum_{j=1}^{M_i} T_{ij} \times 1$; $X$ is an $\sum_{i=1}^{N} \sum_{j=1}^{M_i} T_{ij} \times K$ matrix for the independent variables; $e$ is an $\sum_{i=1}^{N} \sum_{j=1}^{M_i} T_{ij} \times 1$ vector for error terms; $\Theta$ is an $K \times \sum_{i=1}^{N} \sum_{j=1}^{M_i} T_{ij}$ matrix of coefficients associated with the independent variables; $N$ represents the sample trees; $M_i$ equals the number of sections for a given tree $i$; $T_{ij}$ is the number of serial observations for section $j$ and tree $i$; and the other symbols are defined for Equation 2.18.
These models assume that the time periods are equal and that the model coefficients are changing with tree, section, and time. However, for stem analysis data, models with changing coefficients for different sections and trees is not practical for taper modelling because it is important that a model with one set of coefficients be developed for a population (e.g., a group of species in a region) (Kozak 1988). The models most appropriate for dynamic taper functions (linear and nonlinear) are:

\[ Y_{ijt} = \beta_0 + \sum_{k=1}^{K} \beta_k X_{ijtk} + e_{ijt} \]  \hspace{1cm} (2.20)

and

\[ Y = f(X; \theta) + e \] \hspace{1cm} (2.21)

where \( \beta_0 \) to \( \beta_k \) and \( \theta \) (a \( K \times 1 \) matrix) are coefficients to be estimated for all trees, sections, and time periods of all population; and the rest of the symbols are as defined for Equations 2.18 and 2.19. These models assume that the coefficients are constant for all trees, sections, and time periods and that any variations will be captured by the error terms.

### 2.4.1 Ordinary Least Squares and Nonlinear Least Squares

If OLS and/or NLS procedures are used with stem analysis data to estimate the coefficients \( \beta \) and \( \theta \) for the dynamic taper models, the coefficients are estimated as follows:

\[ b = (X'X)^{-1}X'Y \] \hspace{1cm} (2.22)

\[ \hat{\theta} = (F.'F.)^{-1}F.'Y \] \hspace{1cm} (2.23)

for OLS and NLS, respectively; where \( Y \) is the column vector of the dependent variable; \( X \) is a matrix of independent variables; and \( F. \) represents a matrix of the first derivatives of independent variables with respect to the parameters (see Judge et al. 1985, p. 196–201).

Equations 2.20 and 2.21 have error terms which are non-iid. If these equations are fitted using OLS and/or NLS, assuming iid error terms, the parameter estimates will be unbiased.
and consistent (Greene 1990; Kmenta 1971). However, the usual estimates of the variances will be biased as shown below for OLS.

\[
V(b - \beta) = E[(b - \beta)(b - \beta)']
= E[(X'X)^{-1}X'e'e'X(X'X)^{-1}]
= \sigma^2(X'X)^{-1}X'\Psi X(X'X)^{-1}
\]

where \( e \) is the matrix of error terms; \( \Psi \) is a positive definite matrix whose diagonal elements are not equal to one and the off-diagonal elements are not zero (see Greene 1990, p. 383 for a definition); and \( \sigma^2 \) is the common variance for the error terms. Since this new variance \( V(b) \) is different from the usual OLS variance estimate \( \sigma^2(X'X)^{-1} \), the usual OLS estimate is biased. Therefore, hypothesis tests and confidence interval construction cannot be properly conducted because the usual t and F distributions based on OLS variance estimator will be biased and misleading (Kmenta 1971, p. 247–304; Gregoire 1987). Also, the OLS variances will no longer be the lowest in the class of linear, unbiased estimators. The coefficient estimator \( b \) will no longer be the Best Linear Unbiased Estimator (BLUE). In the case of \( \theta \) for nonlinear models, when the error terms are non-iid, the usual estimator for the variance for \( \hat{\theta} \) will also be biased and \( \hat{\theta} \) will be inefficient. If statistical inferences (constructing confidence intervals and hypothesis tests) about the coefficients are important, alternative fitting methods to OLS and NLS must be selected.

With error terms \( (e_{ijt}) \) that are assumed to be contemporaneously correlated among sections for a given tree and time, serially correlated over time for a given tree and section, and with heterogeneous variances among sections of a given tree and time, generalized linear or nonlinear least squares or maximum likelihood estimates are more appropriate fitting techniques.
2.4.2 Generalized Least Squares

A Generalized Linear Least Squares (GLS) fit allows for efficient estimation of the parameters of the following model (Kmenta 1971, p. 499–508; Judge et al. 1985):

\[
\hat{\beta}_G = [X'\Omega^{-1}X]^{-1}X'\Omega^{-1}Y
\]  (2.24)

where \( \Omega \) is the variance-covariance matrix for the error terms e. The GLS estimator \( \hat{\beta}_G \) is unbiased, consistent and most efficient (Kmenta 1971). The associated generalized nonlinear least squares (GNLS) fit is consistent, and asymptotically unbiased and efficient. The GLS technique is the general approach for obtaining the BLUE of the parameters of any linear model. It is called 'generalized' because it includes other models as special cases. For example, OLS is a special case of GLS in which the variance-covariance matrix \( \Omega \) has diagonal elements with the same value and the off-diagonal elements equal to zero.

If the residuals are multivariate normally distributed, the log-likelihood for the maximum likelihood (ML) function for the sample is

\[
\ln L = -\frac{n}{2} \ln(2\pi) - \frac{1}{2} \ln |\Omega| - \frac{1}{2} e'\Omega^{-1}e
\]

where \( n \) is the sample size; and \( e \) is an \( Y - X\beta \) matrix. If \( e \) is replaced by \( Y - X\beta \), \( \ln L \) is differentiated with respect to \( \beta \), and the resulting equations are equated to zero (Kmenta 1971, p. 504), solving for \( \beta \) yields

\[
\hat{\beta}_{ML} = (X'\Omega^{-1}X)^{-1}(X'\Omega^{-1}Y)
\]

This implies that the GLS estimator is also the ML estimator when the residuals are normally distributed. Therefore, the GLS estimator has the same characteristics as the ML estimator. This also follows for GNLS when the error terms are normality distributed.

The GLS parameter estimator (Equation 2.24) assumes that the variance-covariance matrix (\( \Omega \)) of errors is known. However, in most cases (including this thesis), \( \Omega \) is unknown, so the GLS estimator is no longer a feasible choice. Instead a consistent estimator of \( \Omega \), (\( \hat{\Omega} \))
could be found and substituted into Equation 2.24. This leads to a two-step Aitken (1935) estimator or Estimated/Feasible Generalized Least Squares (EGLS or FGLS). Generally, \( \Omega \) contains \( n(n+1)/2 \) unknowns, if there are no prior restrictions on any of its elements, where \( n \) is the total number of observations. When the number of unknowns is larger than the number of observations, assumptions about the structure of \( \Omega \) must be made to restrict the number of unknown elements so that a consistent estimate can be found.

For the two-step Aitken (1935) estimator, first the ordinary least squares estimator \( \hat{b} \) and its corresponding residuals are found. These residuals are used to obtain a consistent estimate of \( \Omega \). In the second stage, the estimator \( \hat{\Omega} \) is substituted into Equation 2.24 to obtain the FGLS estimator, \( \hat{\beta}_F \) as follows:

\[
\hat{\beta}_F = [X'\hat{\Omega}^{-1}X]'X'\hat{\Omega}^{-1}Y
\] (2.25)

where \( \hat{\beta}_F \) is a column vector \( (K \times 1) \); \( K \) is the number of parameters to be estimated; \( Y \) is a column vector \( (n \times 1) \) for the dependent variable; \( n \) is the total number of observations in the sample; and \( X \) is a matrix of \( n \times K \) for the independent variables. Because FGLS is based on \( \hat{\Omega} \), a consistent estimator of \( \Omega \), \( \hat{\beta}_F \) is also consistent and is asymptotically efficient. FGLS is more efficient than OLS when \( \Omega \) is not a diagonal matrix with all diagonal elements equal. If the \( \Omega \) matrix is close to the requirement for OLS, OLS may be more efficient than FGLS in small samples. If the error terms are iid, the FGLS and OLS estimate both will be efficient.

For a nonlinear model, the appropriate estimator is the generalized nonlinear least squares (GNLS) (Gallant 1987, p. 127-139) estimator using the error covariance matrix, \( \Gamma \). When \( \Gamma \) is not known, then an estimator \( \hat{\Gamma} \), with feasible nonlinear least squares (FGNLS) is used. To find the FGNLS estimator, the two step procedure used for FGLS is followed by first fitting NLS to estimate the residuals, which are then used for estimating \( \Gamma \).

Since dynamic taper modelling involves the use of stem analysis data, the error terms are non-iid. The error covariance matrix (\( \Omega \) or \( \Gamma \)) will be characterized by heterogenous
variances among sections for a given tree and time, contemporaneous correlation among sections for a given tree and time, and serial correlation among time periods for a given tree and section. The methods of OLS and NLS for estimating parameters are inappropriate if inferences are needed. The alternative choices available are:

1. Use a combination of transformations (three transformations) to remove heteroskedasticity, and serial and contemporaneous correlations and then fit the equation using the transformed data by OLS or NLS, or

2. Use an FGLS or FGNLS estimator.

Transformations to remove heteroskedasticity and autocorrelation are available (see Judge et al. 1985), but a transformation method to remove contemporaneous correlation was not found in the literature. Instead, the use of an FGLS estimator was considered for this research.

2.4.3 Fitting Dynamic Taper Functions Using FGLS

To fit dynamic taper functions of the form in Equations 2.20 and 2.21 with the FGLS technique, some assumptions about the error terms (residuals) have to be made.

First, it would be assumed that the variances associated with the different trees vary, but trees are independent. For a given tree $i$, the following assumptions would then apply:

1. The $E[e_{ijt}] = 0$.

2. The $E[e_{ijt}e_{ij,s}] \neq 0$, for $t \neq s$. It is assumed that for a given tree and section, the first periodic measurements will be more correlated with observations taken five years later than those taken 10 years later, etc. Thus, the autocorrelation is considered to be of first order. This gives:

   (a) $e_{ijt} = \rho_{ij} e_{ij,t-1} + u_{ijt}$, where $\rho_{ij}$ is the slope of the line.
(b) \( u_{ijt} \sim N(0, \sigma^2_{u_{ijt}}) \), for each tree \( i \) and section \( j \).

(c) \( \mathbb{E}[u_{ijt}u_{ijs}] = 0 \), for all \( t \neq s \).

3. The \( \mathbb{E}[e_{ijt}e'_{ijl}] = \sigma^2_{ij} = \frac{\sigma^2_{e_{ijt}}}{1 - \rho^2_{ij}} \) (heteroscedasticity), where \( \sigma^2_{ij} \) is the variance for a given section \( j \) on tree \( i \).

4. The \( \mathbb{E}[e_{ijt}e_{il}] \neq 0 \). Instead it is equal to \( \sigma_{ijl} \) for \( j \neq l \) for tree \( i \) and time \( t \), since sections \( j \) and \( l \) are contemporaneously correlated.

5. The first value of \( e_{ijt} \) has the following property: \( e_{ij1} \sim N(0, \sigma^2_{e_{ij1}}) \).

6. In addition to these properties of the \( e_{ijt} \), for the stem analysis data, the number of serial measurements differ among the sections in a given tree. This results in some fitting problems.

To obtain consistent estimates of \( \Omega \) for the stem analysis data, when fitting dynamic taper models using FGLS for linear models, the following steps would be required:

1. OLS would be applied to pooled data for all trees, and the corresponding residuals \( (\hat{e}_{ijt}) \) for all trees by section and time period would be obtained. Since these residuals would be based on unbiased coefficient estimates, they would be consistent estimates of the actual error terms. They could be used to obtain \( \hat{\rho}_{ij} \) for a given tree and section as follows:

\[
\hat{\rho}_{ij} = \frac{\sum_{t=2}^{T_{ij}} e_{ijt} \hat{e}_{ij,t-1}}{\sum_{t=2}^{T_{ij}} \hat{e}_{ijt-1}^2}
\]  \hspace{1cm} (2.26)

2. The \( \hat{\rho}_{ij} \)'s would then be used to transform the stem analysis data as follows:

(a) The dependent variable would be transformed as:

\[
Y_{ijt}^* = Y_{ijt} - \hat{\rho}_{ij} Y_{ij,t-1}
\]  \hspace{1cm} (2.27)
(b) The independent variables would be transformed as:

\[ X_{ijtk}^* = X_{ijtk} - \hat{\rho}_{ij} X_{ij,t-1,k} \] (2.28)

where \( k \) is the \( k^{th} \) independent variable.

(c) The first observation would also be transformed, in order to improve on optimal efficiency, as suggested by Dielman (1989, p. 15) as:

\[ Y_{ij1} = (1 - \hat{\rho}_{ij}^2)^{\frac{1}{2}} Y_{ij1} \] (2.29)

and predictors as

\[ X_{ij1k}^* = (1 - \hat{\rho}_{ij}^2)^{\frac{1}{2}} X_{ij1k} \] (2.30)

(d) The residuals would be transformed as:

\[ u_{ijt} = \hat{e}_{ijt} - \hat{\rho}_{ij} \hat{e}_{ij,t-1} \] (2.31)

3. The resulting transformed values for the predictors and the response variable would be analyzed by OLS as given by the equation below:

\[ Y_{ijt}^* = \beta_1 X_{ijt1}^* + \cdots + \beta_k X_{ijtk}^* + u_{ijt} \] (2.32)

where the new estimated residuals \( (\hat{u}_{ijt}) \) are serially uncorrelated. These residuals would then be used to estimate the variances and covariances as follows:

(a) Sectional variances (e.g., section \( j \)) for \( u_{ijt} \)'s

\[ \hat{\sigma}_{u_{ij}}^2 = \frac{1}{T_{ij} - K} \sum_{t=2}^{T} \hat{u}_{ijt}^2. \] (2.33)

where \( K \) is the number of parameters to be estimated.

(b) Covariance between sections \( j \) and \( l \) for the \( u_{ijt} \)'s

\[ \hat{\sigma}_{u_{ij}} = \frac{1}{T_{ij} - K} \sum_{t=1}^{T_{ij}} \hat{u}_{ijt} \hat{u}_{ilt} \] (2.34)

This model assumes \( T_{ij} \) is equal to \( T_{il} \).
(c) Sectional variances (e.g., section $j$) for the $e_{ijl}$'s

$$
\hat{\sigma}_{ij}^2 = \frac{\hat{\sigma}_{uij}^2}{1 - \hat{\rho}_{ij}^2}
$$

(2.35)

where $\hat{\sigma}_{uij}^2$ is the variance of $\hat{u}_{ijl}$; and $\hat{\sigma}_{ij}$ is the variance of $\hat{e}_{ijl}$.

(d) Covariances between sections $j$ and $l$ for the $e_{ijl}$'s

$$
\hat{\sigma}_{ijl} = \frac{\hat{\sigma}_{uijl}}{1 - \hat{\rho}_{ij}\hat{\rho}_{il}}
$$

(2.36)

After estimating $\rho_{ij}$, then $\hat{P}_{ij}$ a consistent estimator of $P_{ij}$ (see Kmenta 1971, p. 511–514 for definition of $P_{ij}$) would be estimated as:

$$
\hat{P}_{ijl} = \begin{bmatrix}
1 & \hat{\rho}_{il} & \hat{\rho}_{il}^2 & \cdots & \hat{\rho}_{il}^{T-1} \\
\hat{\rho}_{ij} & 1 & \hat{\rho}_{il} & \cdots & \hat{\rho}_{il}^{T-2} \\
\vdots & \vdots & \vdots & & \vdots \\
\hat{\rho}_{ij}^{T-1} & \hat{\rho}_{ij}^{T-2} & \hat{\rho}_{ij}^{T-3} & \cdots & 1
\end{bmatrix}
$$

(2.37)

These $\hat{P}_{ij}$ are substituted into $\hat{\Phi}_i$, a consistent estimator of $\Phi_i$. $\hat{\Phi}_i$ would be estimated as given below:

$$
\hat{\Phi}_i = \begin{bmatrix}
\hat{\sigma}_{i1}^2 \hat{P}_{i11} & \hat{\sigma}_{i12} \hat{P}_{i12} & \cdots & \hat{\sigma}_{i1M_{i1}} \hat{P}_{i1M_{i1}} \\
\hat{\sigma}_{i21} \hat{P}_{i21} & \hat{\sigma}_{i2}^2 \hat{P}_{i22} & \cdots & \hat{\sigma}_{i21} \hat{P}_{i2M_{i2}} \\
\vdots & \vdots & \ddots & \vdots \\
\hat{\sigma}_{ij1} \hat{P}_{ij1} & \cdots & \hat{\sigma}_{ijl} \hat{P}_{ijl} & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
\hat{\sigma}_{iM_{i1}} \hat{P}_{iM_{i1}} & \hat{\sigma}_{iM_{i2}} \hat{P}_{iM_{i2}} & \cdots & \hat{\sigma}_{iM_{iM_{i}}} \hat{P}_{iM_{iM_{i}}}
\end{bmatrix}
$$

(2.38)

where $\hat{\sigma}_{ij}^2$ is the estimated variance for section $j$ on tree $i$; $\hat{\sigma}_{ijl}$ is the estimated covariance between sections $j$ and $l$ where $j \neq l$.

After obtaining the $\hat{\Phi}_i$'s and the calculated variances and covariances (using Equations 2.35 and 2.36), $\hat{\Omega}$ would be obtained by substitution as follows:
where $\mathbf{0}$ is a matrix of zeros. For a given section $j$ on a particular tree $i$, the error terms are correlated over time.

After $\Omega$ is estimated, the FGLS estimator would be obtained as follows:

$$
\hat{\beta}_F = [X'\hat{\Omega}^{-1}X]^{-1}X'\hat{\Omega}^{-1}Y
$$

The resulting coefficient estimate ($\hat{\beta}_F$) will be consistent and asymptotically efficient if the assumptions made about $\Omega$ are correct, but might not be efficient for small sample sizes (Greene 1990). Since taper models involve the use of large quantities of data, the asymptotic properties of FGLS (Kmenta 1971) would apply. This means that the coefficient estimates for a dynamic taper model would be asymptotically efficient when fitted using FGLS compared to the inefficient coefficient estimates resulting from the fit by OLS. Also, the resulting standard errors for the coefficient estimates will be consistent estimates. As well, the coefficients will be asymptotically normally distributed. For the nonlinear models, the resulting FGNLS coefficient estimates would be consistent and asymptotically normal.
Chapter 3

Methods

Most existing taper functions are static, which means they are only able to predict tree shape at a particular time. Common to all these static taper models is that they tend to slightly underestimate diameters in some parts of the stem and overestimate diameters in other parts. This indicates that: (1) either the individual trees are not the exact shapes assumed or that these shapes are not constant over time, and (2) the models used are either misspecified or some important variables are not included in the models.

The central objectives of this research were to examine changes in tree shape along the stem at one time and over time using a static taper function, and then to develop a dynamic taper function based on the relationships found between the form exponent of Kozak’s (1988) variable-exponent taper equation and the site, tree, and stand variables.

The data used in the study will be described in detail in Section 3.1. The methods used to determine the factors that influence tree shape are then described in section 3.2. Finally, the methods used to develop and test the dynamic taper model are discussed in Section 3.3.

3.1 Data Preparation

In order to meet the objectives of the study, permanent sample plot (PSP) and detailed stem analysis data for interior lodgepole pine were obtained. The PSP data were used to develop models to predict stand density, tree dbh, and height. PSP data were used to develop height and dbh prediction models, because the stem analysis data lacked a stand density measure at previous time periods before felling and also, the stem analysis data set was small, only 135 trees in 50 plots. The stem analysis data were used to examine the changing form exponent
over time, and to develop, fit, and test the dynamic taper model. The stem analysis data were used instead of PSP data for these purposes because they provided diameter inside bark measurements along the stem over time, which were absent from the PSP data.

### 3.1.1 Permanent Sample Plot Data

The PSP data were provided by the Alberta Forest Service, Timber Management Branch. The “Permanent Sample Plot Field Procedures Manual” (Alberta Forest Service 1990) gives a good description of the data collection procedures used by the Alberta Forest Service for their PSP measurements. Only plots with more than 80 percent pine by basal area were used for this study. All plots which had noticeable incidences of natural (e.g., windthrow, disease) or man-made (e.g., cutting) interference during the remeasurement periods were deleted. The data were from Western Alberta, which is dominated by lodgepole pine; jack pine, a closely related species, occurs in Eastern Alberta.

Remeasurements in each plot were planned for every five years for conifers less than 80 years old or greater than 130 years old, and at every 10 years for stands between 80 and 130 years old. However, the actual remeasurement schedule varied from three to seven years. The plots were established between 1960 and 1965. The number of remeasurements varied from one to four.

Plot sizes varied from 0.1 to 0.34 ha depending on density. In each plot, all standing trees (dead and live) $\geq 9.1$ cm dbh were tagged. Dbh was measured to the nearest 0.1 cm using a metal diameter tape. A minimum of 30 heights per species was measured using a clinometer and a 30 or 50 m measuring tape. Crown length was measured on three trees per plot for recent remeasurements (starting 1983) only. To measure plot age, three trees adjacent to the plot were felled, and cuts were made at stump and breast height. Both stump (0.3 m above ground) and breast height ages were measured by counting the number of rings on both sides of the cut disks and averaging them.

For each plot, the following variables were calculated:
Chapter 3. Methods

1. number of stems per hectare (SPH);

2. average plot age at breast height, calculated by averaging the ages for three trees felled near the plot;

3. average height, calculated by averaging all dominant and codominant tree heights in the plot;

4. average dbh, calculated in a similar manner to average height;

5. site index (SI), the average height of all dominant and codominant trees in the plot at a reference age, was calculated using average plot age and height. The following function provided by Alberta Forest Service (1985, p. 3-5) for calculating SI at 50 years was used:

\[
SI = 1.3 + 10.9408 + 1.6753(H - 1.3) - 0.3638(\ln(AGE))^2 + 0.0054(AGE)(\ln(AGE)) + 8.82281(\frac{(H-1.3)}{AGE}) - 0.2569(H - 1.3)(\ln(H - 1.3))
\]  

(3.41)

where \(SI\) is the estimated site index (m) at 50 years breast height age, \(H\) is the mean height per plot for the dominant and the codominant trees, \(AGE\) is the mean breast height age per plot, and \(\ln\) is the natural logarithm;

6. basal area per hectare (BA), the cumulative cross-sectional area for all measured trees at 1.3 m above ground;

7. quadratic mean diameter (QD), the dbh for the tree of average basal area, calculated as described in Davis and Johnson (1987, p.80-81);

8. relative density (RD), calculated as the ratio of basal area per hectare to the square root of the quadratic mean diameter (Davis and Johnson 1987, p. 81); and

9. stand density index (SDI), the number of trees per unit area (ha or acre) that a stand would support at a standard average dbh (Husch et al. 1982).
For calculating average dbh, BA, SPH, QD, RD, and SDI, all trees in the plot which had dbh's $\geq 9.1$ cm were used. After calculating the above stand statistics, only dominant and codominant trees and the associated stand level statistics were selected for use in this thesis. This was done because the stem analysis data to be used in this research were composed of only the dominant and codominant trees.

The PSP tree data were checked for outliers\(^1\) by plotting pairs of variables. No obvious outliers were found. The PSP data consisted of 1908 trees on 613 plots. These 1908 trees were stratified into five-centimeter dbh and five-meter height classes and within each class, thirty percent of the data was randomly selected for validation purposes (574 trees on 185 plots) (Table 3.2). Seventy percent of the data (1334 trees on 428 plots) was used for developing models to predict QD at 50 years, dbh, and height. The rationale for data splitting was to validate the fitted models, because the usual t and F tests would not be applicable when using the dependent PSP data.

### 3.1.2 Stem Analysis Data

The stem analysis data were collected by Dr. Q. Wang, who was a graduate student at the time in the Department of Forest Sciences, under the supervision of Dr. Karl Klinka. The data were obtained from North of Burns Lake, South of Anaheim Lake, and East and Southeast of Prince George in the Interior of B.C. Physiographically, these areas occur within the Sub-Boreal Spruce and Sub-Boreal Pine-Spruce Biogeoclimatic Zones of B.C. (Meidinger and Pojar 1991). Stands selected for sampling were dominated by lodgepole pine (greater than 80 percent of the crown cover), even-aged, fully stocked, and relatively free of disturbance after establishment.

Fifty 0.04 ha fixed area plots were selected. For each plot, the elevation, slope, aspect, soil parent materials, and other features were recorded. The dbh of each tree greater than 1.0 cm

---

\(^1\) An outlier can be loosely defined as an observation which in some sense is inconsistent with the rest of observations or which disproportionately influences the conclusions drawn from the data set.
Table 3.2: Summary statistics for the permanent sample plot data.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>dbh (cm)</td>
<td>23.32 (23.38)</td>
<td>6.2 (6.4)</td>
<td>8.4 (8.6)</td>
<td>47.0 (48.8)</td>
</tr>
<tr>
<td>H (m)</td>
<td>19.31 (19.24)</td>
<td>3.97 (4.13)</td>
<td>7.2 (8.2)</td>
<td>30.8 (29.3)</td>
</tr>
<tr>
<td>AGE (years)</td>
<td>76.7 (76.8)</td>
<td>25.2 (25.7)</td>
<td>16 (17)</td>
<td>162 (160)</td>
</tr>
<tr>
<td>SI (m)</td>
<td>14.6 (14.4)</td>
<td>2.7 (2.7)</td>
<td>6 (6)</td>
<td>23 (23)</td>
</tr>
<tr>
<td>BA (m²/ha)</td>
<td>37.96 (37.10)</td>
<td>9.35 (10.2)</td>
<td>9.1 (9.09)</td>
<td>62.7 (62.0)</td>
</tr>
<tr>
<td>SPH</td>
<td>2696 (2647)</td>
<td>2184 (2089)</td>
<td>256 (256)</td>
<td>18228 (17199)</td>
</tr>
<tr>
<td>QD (cm)</td>
<td>11.6 (11.4)</td>
<td>3.1 (3.1)</td>
<td>5 (5)</td>
<td>21 (22)</td>
</tr>
</tbody>
</table>

- The dbh and height (H) statistics are based on individual tree information, whereas all other statistics were based on plot information.
- The statistics in brackets are for the validation data set (574 trees or 185 plots) and the unbracketed statistics are for the model development data set (1334 trees or 428 plots).

was measured. Then, for each plot, two or three dominant and codominant trees (totalling 147 trees) were destructively sampled. After felling, height to live crown and total tree height were measured on these trees. Trees were sectioned at 0.3 m, 0.6 m, and 1.3 m above ground, and at subsequent one metre intervals after felling. For small trees (trees with dbh ≤ 8.0 cm), subsequent sections after 1.3 m above ground were cut at 0.5 m intervals. The number of sections per tree varied from nine to 23. Disks were cut off the top of each section. These disks were returned to the laboratory and measured for annual radial increment width using a light microscope with a horizontal bar upon which the disk was placed. This was connected to a microcomputer with a program written in FORTRAN for recording annual increment measurements for the average radius on each disk. The procedure for measurement was:

1. the widest diameter \( D_1 \) and narrowest diameter \( D_2 \) inside bark were measured;

2. the geometric mean diameter \( D_a \) was calculated as \( D_a = \sqrt{D_1 \times D_2} \), and converted to radius as \( R = \frac{D_a}{2} \).
3. a ruler was rotated around the cut disk with the zero at the pith, until a radius equal to the average was located. This was marked on each disk and the disk was placed on the horizontal bar in such a way that it was directly below the microscope during measurement; and

4. annual radial increments were measured along the transit from the pith outwards.

For each plot, the following variables were calculated using the same methods as for the PSP data: SPH, average breast height age (AGE), SI, average height, average dbh, QD, BA, RD, and SDI. Site index was calculated by first averaging height and breast height age for the dominant and codominant pine trees in each plot. It should be noted that these plots were assumed to have come from even-aged stands. For plots with breast height ages between 48 and 52 years, SI was approximated as the average height of dominant and codominant trees per plot, while for other ages, Equation 3.41 was used to predict plot site index at a reference age of 50 years at breast height. For each tree that was felled, crown length (CL) was calculated as the difference between total tree height and height to live crown, and crown ratio (CR) was calculated as the ratio of crown length to total tree height.

To prepare the pine data for analysis, diameter inside bark for each section was calculated by summing up all the annual increment widths for the section. The age at the time of felling of each section was the ring count from the pith to the bark. Diameter inside bark was also calculated over each 5 year period, starting from the bark. Tree height was calculated for each 5 year age using a method developed by Carmean (Carmean 1972; Dyer and Bailey 1987; Newberry 1991). Dbh ($D$) for each 5 year period was calculated using a bark factor ($k$) of 0.9 (i.e., $D = d_{in}/0.9$) (Husch et al. 1982, p. 104–108). Husch et al. (p. 105) stated that this lower-stem bark factor ($k$) ranged from 0.87 to 0.93 varying with species, age, and site. However, the majority of the variation is accounted for by species. Therefore, it was reasonable and convenient to assume a constant bark factor for lodgepole pine and a value of 0.9 was used because the trees used were still young. Therefore, they were assumed to
have a lower bark thickness than mature trees.

After reconstructing trees at five-year periods, a plot of diameter inside bark (d) against height above ground (h) was produced for each tree by time period. All together, over 1000 graphs were produced. From these graphs, 11 trees were eliminated, because of suspected measurement error. These trees had more than one sectional diameter well out of line with the rest (i.e., outliers). Also, one tree without stump height (0.3 m above ground) diameter measurements was removed, since this is an important measurement for taper function development. Therefore, only 135 trees remained, representing 887 5-year periodic measurements (tree-measures) with 8584 sectional measures.

In order to examine the variation in form and taper along the stem at a particular time and as time changes, a representative measure had to be used. The form exponent for the simple taper equation (Equation 2.1) was selected. This form exponent (r) was selected because it can be associated directly with the known shapes of the stems (Figure 2.1). The form exponents were calculated by rearranging Equation 2.1 as follows:

\[ y_{ijt} = K_{ijt} \sqrt{x_{ijt}^r} = \frac{R_{bit}}{H_{it}} \sqrt{x_{ijt}^r} \]  
\[ (3.42) \]

where \( y_{ijt} \) is the radius at distance \( x_{ijt} \) from the tree tip for tree \( i \), section \( j \) and for 5 year period \( t \) (hereafter called time \( t \)); \( x_{ijt} \) is the distance from the tip for a given section on any sample tree (i.e., \( x_{ijt} = H_{it} - h_{ijt} \), where \( H_{it} \) is total height for tree \( i \) and for time \( t \) and \( h_{ijt} \) is the height of measured radius \( y_{ijt} \) from the point of germination); \( K_{ijt} \), the measure of taper, is equal to \( \frac{R_{bit}}{H_{it}^{r_{ijt}}} \); \( r_{ijt} \) is the form exponent for tree \( i \), section \( j \), and time \( t \); and \( R_{bit} \) is the radius at the base for tree \( i \) and time \( t \).

Rearranging equation 3.42 yields:

\[ \frac{y_{ijt}}{R_{bit}} = \sqrt{\frac{x_{ijt}^r}{H_{it}^{r_{ijt}}}} = \left[ \frac{x_{ijt}}{H_{it}} \right]^{r_{ijt}/2} \]

By multiplying the numerator and denominator of the equation by two, \( 2y_{ijt} \) becomes diameter \( (d_{ijt}) \) at a given height and \( 2R_{bit} \) becomes diameter \( (D_{bit}) \) at the base (i.e., at stump
height). The shape of the stem from the point of germination to the stump height was assumed to be a cylinder, and to carry negligible volume. This assumption simplifies volume calculation, but might introduce some bias in the calculated volumes. The above equation also can be written as:

$$\frac{d_{ijt}}{Db_{it}} = \left[ \frac{x_{ijt}}{H_{it}} \right]^{\frac{r_{ijt}}{2}} = \left[ \frac{H_{it} - h_{ijt}}{H_{it}} \right]^{\frac{r_{ijt}}{2}} = \left[ 1 - \frac{h_{ijt}}{H_{it}} \right]^{\frac{r_{ijt}}{2}}$$

Solving for $r_{ijt}$, the equation becomes:

$$r_{ijt} = \frac{2 \times \ln \left[ \frac{d_{ijt}}{Db_{it}} \right]}{\ln \left[ \frac{x_{ijt}}{H_{it}} \right]}$$

where $\ln$ is the natural logarithm.

In order to calculate periodic (five-years) form exponents for every tree $(i)$ and section $(j)$, the following periodic measurements were needed: total tree height $(H_{it})$, diameter inside bark $(d_{ijt})$ to a given height $(h_{ijt})$, distance from tree top $(x_{ijt})$ ($x_{ijt} = H_{it} - h_{ijt}$), and the diameter inside bark at the tree base $(Db_{it})$. At the base of the tree (in this case stump height), $r_{ijt}$ was undefined; therefore, it was not included in the sample for $r$ calculation. The values for $r_{ijt}$ ranged from 0.7 to 5.1. The calculated $r_{ijt}$ values were plotted against $h_{ijt}$ for all sample trees to check for obvious outliers.

The dynamic taper model developed as part of this research was based on Kozak’s (1988) taper function (see Equation 2.15). Therefore, in addition to calculating the exponent $(r)$ of Equation 2.1, the exponent $(c)$ of Kozak’s equation was calculated for every tree by section and time. Kozak’s taper function is similar to Equation 2.1, except that Kozak used a different base diameter (diameter at the join point) rather than $Db_{it}$, and modified the base for the form exponent. If a subscript for the tree, section, and time are included, Kozak’s taper function becomes:

$$\tilde{d}_{ijt} = DI_{it} \times M_{ijt}^{c_{ijt}}$$

where $DI_{it}$ is the diameter at the lower join point of tree $i$ at time period $t$ and $M_{ijt} = \ldots$
This model is nonlinear, but can be intrinsically linear depending on the assumptions about the associated error terms. In order to come up with an appropriate join point, $\frac{d}{D}$ was plotted against $\frac{h}{H}$ for the last time period (time of felling) for a subset of 20 randomly selected trees. From the plots, the visual join points ($q$) ranged between 22 to 27 percent of total tree height. Therefore, an approximate value of 25 percent was used. This agrees with the results by Demearschalk and Kozak (1977), Kozak (1988), and Perez et al. (1990), who found that the value ranged from 15 to 30 percent of total height for most species and that any value used within this range would not affect the results greatly.

Based on $q = 0.25$, the form exponent for Kozak's (1988) model was calculated for each tree by section and time period as:

$$c_{ijt} = \frac{\ln \left[ \frac{d_{ijt}}{D_{ijt}} \right]}{\ln M_{ijt}}.$$  

Since $DI_{it}$ was not measured during data collection, interpolation was used to calculate $DI_{it}$ at $HI_{it} = 0.25H_{it}$. Although a tree is not a straight line, linear interpolation was selected because the length of the sections were very short. The equation used for interpolation was:

$$\frac{d_{i2t} - d_{i1t}}{d_{i2t} - DI_{it}} = \frac{h_{i2t} - h_{i1t}}{h_{i2t} - HI_{it}},$$

which was rearranged to give:

$$DI_{it} = d_{i1t} + \left[ \frac{(d_{i2t} - d_{i1t})(h_{i2t} - HI_{it})}{h_{i2t} - h_{i1t}} \right],$$

where $d_{i1t}$ is the diameter below $DI_{it}$; $d_{i2t}$ is the diameter above $DI_{it}$; $h_{i1t}$ and $h_{i2t}$ are height above ground corresponding to diameters $d_{i1t}$ and $d_{i2t}$ respectively; and $HI_{it}$ is the height of $DI_{it}$ above ground.

The stand measures and the crown size measure included with the stem analysis data could only be collected at the time of felling. As a result, stand density and crown size were not available for previous time periods. To overcome these problems, two options were
available. The first option was to develop a taper model without stand density and crown size measures. However, it can be seen from Section 2.2.2 that stand density and crown size are very important factors which are expected to influence tree form changes. The second option was to develop prediction models for stand density and crown size measures using PSP data and then use these models to predict density and crown size for each plot of stem analysis data over time. Unfortunately, the PSP data did not have enough measured crown lengths for dominant and codominant trees. As a result, no measure of crown size was used. However, crown size measures such as crown length are highly correlated with stand density. Therefore, using predicted stand density would somewhat account for crown size changes.

The model for predicting stand density for each plot of the stem analysis data was based on the PSP information which included site index, average plot age, and average plot height for each PSP. The data consisted of 613 plots which were divided into two sets: the model development data set (70%) and the validation data set (30%). In order to model stand density, an appropriate measure of average stand density had to be selected. The stand density measures considered included: SPH, BA, QD, RD, and SDI. (See Davis and Johnson 1987, p. 79–84 for a discussion of the different stand density measures). Crown competition factor (see Davis and Johnson 1987, p. 86 for definition) was not considered because both the PSP and stem analysis data lacked crown width.

For a given stand age and site quality, SPH is a good measure of stand density. BA is more commonly used than SPH because it relates directly to stand volume. However, if SPH and BA are combined, they give a better measure of stand density by indicating the size and number of the trees together. QD, RD, and SDI combine both SPH and BA. Each of these stand density measures was graphed against plot age and site index using the PSP data. These graphs were used to identify which stand density measure was most highly correlated with these variables. In addition, a graph of the form exponent $r$ at the time of felling against each of the five measures of stand density was obtained. QD was found to be more highly correlated with plot breast height age, site index, and the form exponent than
RD and SDI.

From the graphs of QD against SI and AGE, some transformations \(AGE^2, \frac{1}{AGE}, SI^2,\) \(\ln AGE, \ln SI\) and interactions of these variables \((SI \times AGE, SI^2 \times AGE)\) were selected. As a result, a total of nine variables were used to find the best model to predict QD. The RSQUARE procedure in SAS (SAS Institute, Inc. 1985) was used to select the best possible models. Four models were selected based on mean squared error (MSE), multiple coefficient of determination \((R^2)\) and adjusted \(R^2\) \((R^2_a)\), and Mallow’s statistic \((C_p)\) (see Neter et al. 1985, p. 423-427 or Judge et al. 1985, p. 863-6 for definitions of \(R^2_a\) and \(C_p\)). Two of the four models were linear and the other two were intrinsically linear and were transformed to linear models using a natural logarithmic transformation. The four models selected were:

1. \(\hat{QD} = b_0 + b_1(SI \times AGE) + b_2SI^2 + b_3\frac{1}{AGE}\)

2. \(\hat{QD} = b_0 + b_1(SI \times AGE) + b_2SI^2 + b_3\frac{SI}{AGE}\)

3. \(\hat{QD} = SI^{b_1} \times AGE^{b_2} \times \exp(b_3 + b_4SI + b_5SI^2)\)

This model was transformed into:

\[\ln(\hat{QD}) = b_1 \ln(SI) + b_2 \ln(AGE) + b_3 + b_4SI + b_5SI^2\]

4. \(\hat{QD} = b_0 \times SI^{b_1} \times AGE^{b_2}\)

This model was transformed into:

\[\ln(\hat{QD}) = \ln b_0 + b_1 \ln(SI) + b_2 \ln(AGE)\]

where \(b_i\) are the coefficients to be estimated.

All the selected linear models were fit using the REG procedure in SAS (SAS Institute, Inc. 1985) (OLS). In addition, models 3 and 4 were fit using the NLIN (nonlinear) procedure in SAS. The models were evaluated for their predictive abilities based on the validation data.
set (185 plots). Model selection was based on the fit and prediction statistics. The fit statistics included $R^2$, the standard error of estimate (SEE), and the PRESS statistic (see Draper and Smith 1981, p.325–327). The prediction statistics used for model evaluation included the fit index (FI), estimated SEE ($S\hat{EE}$), root mean square error (RMSE), and mean and absolute biases. These prediction statistics were based on the differences (residuals) between the observed and the predicted values, that is:

$$\hat{e}_i = Y_i - \hat{Y}_i,$$

where $\hat{e}_i$ is the residual or difference between the measured and the predicted value for a given observation $i$, $Y_i$ is the observed value, and $\hat{Y}_i$ is the predicted value. The five prediction statistics used in model evaluation were calculated as follows:

1. Fit Index (FI) or Estimated Coefficient of Determination:

$$FI = 1 - \frac{\sum_{i=1}^{n} \hat{e}_i^2}{\sum_{i=1}^{n} (Y_i - \bar{Y})^2}$$

where $\bar{Y}$ is the mean value of the dependent variable; and $n$ is the number of observations in the validation data set.

The value of estimated $R^2$ (FI), like that of $R^2$, indicates the degree of goodness-of-fit of an equation. A higher value for a model indicates a better fit.

2. Estimated standard error of estimate ($S\hat{EE}$) (Spurr 1952):

$$S\hat{EE} = \left[ \frac{\sum_{i=1}^{n} \hat{e}_i^2}{n - K} \right]^{0.5}$$

where $K$ is the number of coefficients to be estimated.

A model with high $S\hat{EE}$ would be a poorer model than a model with a small $S\hat{EE}$. The $S\hat{EE}$ is a good indicator of the spread of the actual observations ($Y_i$) around the predicted values ($\hat{Y}_i$) (Spurr 1952). However, with a large number of observations, FI and $S\hat{EE}$ will rank models identically.
3. Mean Bias (Bias) (sometimes called Mean Differences):

\[ \text{Bias} = \frac{\sum_{i=1}^{n} \hat{e}_i}{n} \]

Bias is a good measure of the accuracy of the model's prediction abilities since it reveals how well or poorly the model represents the actual observations on average. If Bias is small, this indicates that the model predicts well for an independent data set. However, large negative and large positive biases could cancel each other out and resulting in a small value for Bias.

4. Mean square bias or error (MSB) and the root mean square bias or error (RMSE)

\[ \text{MSB} = \frac{1}{n} \sum_{i=1}^{n} \hat{e}_i^2 \quad \text{and} \quad \text{RMSE} = \sqrt{\text{MSB}} = \left[ \frac{1}{n} \sum_{i=1}^{n} \hat{e}_i^2 \right]^{0.5} \]

Neter et al. (1985) stated that if the MSB is fairly close to the MSE based on the regression fit to the model-building data set, then the error mean square MSE for the selected regression model is not gravely biased and gives an appropriate indication of the predictive abilities of the models. They stated that if MSB is much larger than MSE, then MSB should be relied upon to indicate the predictive ability of the selected model.

5. Mean Absolute Deviations (Residuals) (MAD):

\[ \text{MAD} = \frac{\sum_{i=1}^{n} |\hat{e}_i|}{n} \]

where \(||\) is the symbol for absolute value.

MAD overcomes the problem of large positive and large negative biases cancelling out, like MSB, but without squaring the values. Small values of MAD would indicate good predictive ability of the selected model for the validation data set.
Table 3.3: The fit and prediction statistics for the quadratic mean diameter (cm) prediction models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Fit(^a) (n=428 plots)</th>
<th>Prediction (n=185 plots)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(R^2) (SEE) (PRESS)</td>
<td>(FI) (SEE) (RMSE) (Bias) (MAD)</td>
</tr>
<tr>
<td>1</td>
<td>.7869 2.06 1832.671</td>
<td>.7337 2.11 2.08 -0.10 1.61</td>
</tr>
<tr>
<td>2</td>
<td>.7867 2.06 1835.162</td>
<td>.7330 2.11 2.09 -0.10 1.61</td>
</tr>
<tr>
<td>3</td>
<td>.7902(.8127) 2.05(0.35)  -(7.916)</td>
<td>.7431 2.07 2.05 -0.08 1.61</td>
</tr>
<tr>
<td>4</td>
<td>.7808(.8079) 2.09(0.14)  -(8.060)</td>
<td>.7245 2.15 2.13 0.02 1.64</td>
</tr>
</tbody>
</table>

\(^a\)The fit statistics in brackets were calculated based on the natural logarithmic transformed models (Models 3 and 4) fitted using OLS.

\(^b\)\(R^2\) is the coefficient of multiple determination; \(SEE\) is the standard error of estimate in cm; \(PRESS\) is the predicted sum of squares residuals (Draper and Smith 1981, p. 325–327); \(FI\) is the fit index; \(SEE\) is the estimated standard error of estimate in cm; \(RMSE\) is the root mean square error in cm; \(Bias\) is the mean bias in cm; and \(MAD\) is the mean absolute bias in cm.

From the fit and prediction statistics (Table 3.3), it can be seen that Equation 1 was slightly better than Equation 2 (linear models) in terms of fit and prediction statistics. Overall, Equation 3 (a nonlinear model) had the best fit and prediction statistics. Based on these results, Equation 3 was selected for predicting plot quadratic diameter. The selected model was:

\[
\hat{QD} = SI^{2.826423} \times AGE^{0.690036} \times \exp[-4.290671 - (0.360959SI) + (0.007929SI^2)] \quad (3.45)
\]

This model was then conditioned for predicting quadratic mean at a given (reference) age, similar to the procedure used to develop anamorphic site index curves. Without prior information about stand densities for the plots, it was necessary to bring all the plots to a common age (to standardize density for the various ages of plots). Fifty years at breast height was selected as the reference age. The predicted quadratic mean diameter at 50 years breast height age (\(\hat{QD}_{50}\)) can be defined as the expected quadratic mean diameter for a given stand growing on a given site at that age. As an example, Figure 3.2 shows \(\hat{QD}_{50}\) estimates...
for a site index of 20 m (see Appendix B, Figures B.16 and B.17 for site indices of 10 and 15 m).

Predicted quadratic mean diameter at breast height age 50 years ($\hat{Q}D_{50}$) was calculated for each stem analysis plot using the following model:

$$\hat{Q}D_{50} = QD - [(SI^{2.826423} \times \exp(X1)) \times (AGE^{0.690036} - 50^{0.690036})]$$

(3.46)

where $X1 = -4.290671 - 0.360935SI + 0.007929SI^2$.

The stem analysis data with all the associated measured and calculated variables (Table 3.4) were stratified into five-centimeter dbh and five-meter height classes and then, for each class, the observations were randomly divided into model development and validation data subsets. Seventy percent of the data (612 tree-measures, representing 5916 sectional measurements over time) were used for model development; model assessments were carried out using the remaining data as a validation data set (275 tree-measures, representing 2668 sectional measures over time). These data were considered to be representative of the whole
Table 3.4: Summary statistics for the stem analysis data.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>dbh (cm)</td>
<td>8.84 (8.82)(^a)</td>
<td>4.76 (4.60)</td>
<td>1.0 (1.2)</td>
<td>23.5 (21.9)</td>
</tr>
<tr>
<td>H (m)</td>
<td>8.48 (8.06)</td>
<td>4.44 (4.14)</td>
<td>1.6 (2.0)</td>
<td>22.8 (20.9)</td>
</tr>
<tr>
<td>CL (m)(^b)</td>
<td>6.99 (7.20)</td>
<td>1.91 (1.67)</td>
<td>2.8 (4.3)</td>
<td>12.7 (12.2)</td>
</tr>
<tr>
<td>CR</td>
<td>.582 (.624)</td>
<td>.1320 (.1494)</td>
<td>.342 (.378)</td>
<td>.943 (.947)</td>
</tr>
<tr>
<td>AGE(^c) (years)</td>
<td>21.6 (22.7)</td>
<td>13.8 (13.6)</td>
<td>1 (2)</td>
<td>61 (57)</td>
</tr>
<tr>
<td>SI(^d) (m)</td>
<td>17.1 (16.1)</td>
<td>3.9 (4.1)</td>
<td>9 (9)</td>
<td>24 (24)</td>
</tr>
<tr>
<td>BA (m(^2)/ha)</td>
<td>21.55 (22.84)</td>
<td>12.12 (12.55)</td>
<td>2.66 (4.09)</td>
<td>53.24 (53.24)</td>
</tr>
<tr>
<td>SPH</td>
<td>3,702 (3,851)</td>
<td>2,814 (3,032)</td>
<td>350 (350)</td>
<td>10,925 (10,925)</td>
</tr>
<tr>
<td>(QD_{50}) (cm)</td>
<td>14.3 (14.0)</td>
<td>4.7 (5.0)</td>
<td>6 (6)</td>
<td>25 (25)</td>
</tr>
</tbody>
</table>

\(^a\)The stem analysis data were divided into a model-development data set (612 tree-measures, the values without brackets) and a model-validation data set (275 tree-measures, the values in brackets).

\(^b\)CL is crown length and CR is crown ratio (the ratio of crown length to height). These are tree variables based on the final felling data. The number in brackets are for the validation data set of 41 trees and the numbers outside the brackets are for the development data set of 94 trees.

\(^c\)AGE is the tree breast height age (years).

\(^d\)SI (site index), BA (basal area per ha), SPH (number of stem per ha), and \(QD_{50}\) (quadratic mean diameter at age 50 years) are plot statistics based on 30 plots while the rest are individual tree statistics.

population. Figure 3.3 and Table 3.5 show the distribution of the fit and validation data sets by age class and by height and dbh. The rationale for data splitting is discussed in section 3.3.6.

3.2 Examination of Tree Form and Taper Variation

Tree form and taper change along the stem continuously (Kozak 1988; Newnham 1988) at one time and over time (Larson 1963; Clyde 1986). These changes in form and taper are processes which depend on changes in other factors such as site, stand, and tree variables (Section 2.2.2). Instead of developing an entirely new taper model, existing static taper models (see Chapter 2) were investigated as possible candidates. Two static taper functions
Figure 3.3: Fit and prediction data distribution by breast height age classes (years). The age classes represent age class ranges: age class 1 represent 1-5 years, age class 2 represent 6-10 years, age class 3 represent 11-15 years, ..., age class 12 represent 55-60 years, and age class 13 represent 61 years and over.

Table 3.5: Number of tree-measures by height (m) and dbh (cm) classes.

<table>
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<th>HEIGHT</th>
<th>2.0</th>
<th>4.0</th>
<th>6.0</th>
<th>8.0</th>
<th>10.0</th>
<th>12.0</th>
<th>14.0</th>
<th>16.0</th>
<th>18.0</th>
<th>20.0</th>
<th>22.0</th>
<th>TOTAL</th>
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<td>DBH</td>
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</tbody>
</table>

TOTAL 55(22) 95(48) 107(52) 94(41) 87(47) 77(26) 34(17) 32(12) 16(7) 12(3) 3 612(275)

Note: The numbers in brackets are the values for the validation data set.
were selected, the simple taper equation (Equation 2.1) and the variable-exponent static equation by Kozak (1988) (Equation 2.15). Equation 2.1 was used because the values of $r$ can be associated with known stem shapes (Figure 2.1). Kozak’s (1988) static taper function was selected for use in this research because it was shown to be consistent and precise in predicting $d$ along the stem and total tree volume (Newnham 1992; Perez et al. 1990) and it relates well to Equation 2.1. However, this taper function has a form exponent that cannot be directly related to the theoretical tree shapes shown in Figure 2.1. Therefore, the simple static equation was selected to examine the relationship between the calculated values for the form exponent and the actual tree geometric shapes.

To meet the first objective of this research, changes in tree form and taper over time were examined in relation to changes in tree, stand, and site variables using all the stem analysis data (887 tree-measures). The analysis involved the following steps:

1. The form exponent ($r$) was plotted against measured and calculated stand, tree, and site variables at specific relative heights (10 percent height, 50 percent height, and 80 percent height) to identify important variables that appeared to be related to form and taper changes. The variables examined included $QD_{50}$, AGE, CL and CR based on the final felling data, height, dbh, the dbh over height ratio ($D_n/H_n$, hereafter referred to as the D/H ratio), and SI.

2. Six trees with a range of ages and sizes were selected arbitrarily to demonstrate the variation in tree form and taper. The selected trees were categorized as large and old (tree number two in plot 16 (16.2)), large and young (tree number one in plot 46 (46.1)), small and old (tree number two in plot 25 (25.2)), and small and young (tree two in plot 43 (43.2)). Two additional trees were included, which were assumed to be of middle age and size (tree number two in plot 13 (13.2) and tree number one in plot 42 (42.1)).

3. For each of these six trees, the variation in form ($r_{it}$) and taper were examined as
follows:

(a) Plots of $r_{ijt}$ against breast height age at the three relative heights on the stem (0.1$H_{it}$, 0.5$H_{it}$, and 0.8$H_{it}$) were drawn. These plots were intended to show how tree form varied with time at different positions on the stem.

(b) Plots of $r_{ijt}$ against relative height were obtained. These plots were intended to show how tree form changed along the tree for a given age (measurement period).

(c) Relative diameter (the ratio between diameter inside bark at a given height ($d_{ijt}$) and dbh ($D_{it}$), i.e., $d_{ijt}/D_{it}$, hereafter referred as $d/D$) was plotted against relative height (height at a given diameter ($h_{ijt}$) to height, i.e., $h_{ijt}/H_{it}$, hereafter referred as $h/H$). This was intended to show tree taper variation as a function of relative tree height.

(d) Finally there was a need to see how tree shape changed along the stem during growth. Therefore, three-dimensional plots of $r_{ijt}$ against $h_{ijt}$ and AGE were prepared for the same six trees.

3.3 Dynamic Taper Model Development

Model development was carried out in three steps: (1) finding the factors that affect the form exponent of taper function, (2) finding models for predicting total tree height, breast height diameter, and the selected variables correlated to the form exponent, and (3) substituting these models and the variables into Kozak's (1988) static taper model to make it dynamic and refitting the model. The fitted dynamic taper model was then assessed for its predictive abilities for diameter inside bark along the stem and total tree volume.

Turning a static taper equation into a dynamic taper function involves specifying a function capable of predicting $d_{ijt}$ over time at given heights ($h_{ijt}$) above the ground. However, in order to predict $d_{ijt}$ dynamically, variables used to predict $d$ which change with time such
as the form exponent of Kozak's (1988) function \( c \), height, and dbh would have either to be measured, which is most times not possible, or predicted over time.

As part of the dynamic taper model development and testing process (objective two of this research), prediction models for height, dbh, and the variables highly correlated with \( c \) were determined. The models built had to be biologically meaningful and to give good predictions. The biological criteria included making sure that variables which are known to be biologically correlated with the variable being modelled were included in the models. Such variables included SI and age for the height model, age and density for the dbh model, and D/H ratio, age, and density for the form exponent. The statistical criteria used to assess the models were \( F_I \), \( S\hat{E}E \), RMSE, mean and mean absolute biases (Bias and MAD) (see Section 3.1.2 for definitions), and the characteristics of the residuals (i.e., plotting residuals against predicted values and against the independent variables used).

Since height, dbh, and the form exponent could not be measured over time, they had to be predicted. Therefore, models had to be developed based on either the stem analysis or PSP data, for predicting height and dbh. The stem analysis data set would have been better, but it lacked stand density measured over time and was small. For that reason, the PSP data were used to select the best models for predicting height and dbh, and then the selected models were fitted using only the measurements of the stem analysis taken at time of felling (final felling measurements).

### 3.3.1 Height Prediction Model

Height growth in lodgepole pine depends on the site quality, tree age, and stand density when trees are still young and often at extreme densities (Section 2.1.1). A literature search was carried out for height prediction models based on these variables. Many models were readily available which predict height as a function of site index and age, commonly called site index curves. Other models, often incorporated in growth models, predicted height as a function of dbh. Two models were selected, one linear and the other nonlinear.
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The selected linear model was proposed by Alexander et al. (1967). This model included breast height age, site index, and stand density as independent variables. Crown competition factor (CCF) was used as the measure of stand density. However, for this research \( QD_{50} \) was used instead of CCF (see Section 3.1.2). The height prediction model was therefore:

\[
\hat{H}_{kmo} = b_0 + b_1 AGE_{km} + b_2 AGE_{km}^2 + b_3 (SI_k \times QD_{50k}) + b_4 (AGE_{km} \times SI_k) + b_5 (AGE_{km}^2 \times SI_k)
\] (3.47)

where \( \hat{H}_{kmo} \) is the predicted total tree height; \( k \) is the PSP number; \( m \) is the measurement period; \( o \) is the tree number in plot \( k \); \( AGE_{km} \) is the average plot age at period \( m \); \( SI_k \) is the site index for plot \( k \); and the \( b_0 \) to \( b_5 \) are the coefficients to be estimated.

In addition to using the above model with \( QD_{50} \), a reduced model (without a measure of stand density) was also tried. The reduced model was:

\[
\hat{H}_{kmo} = b_0 + b_1 AGE_{km} + b_2 AGE_{km}^2 + b_3 (SI_k \times AGE_{km}) + b_4 (AGE_{km}^2 \times SI_k)
\] (3.48)

where \( b_0 \) to \( b_4 \) are parameters to be estimated and the other symbols are as defined above.

The selected nonlinear model was developed by Goudie (1984; as referenced in Goudie et al. 1990). This top height prediction model is based only on site index and breast height age without a measure of stand density. In other words, Goudie’s model assumes that density does not affect height growth of dominant and codominant trees. The model is a conditioned logistic function as shown below:

\[
\hat{H}_{kmo} = 1.3 + (SI_k - 1.3) \times \left[ \frac{1.0 + \exp(c_0 - (c_1 \ln(50)) - (c_2 \ln(SI_k - 1.3)))}{1.0 + \exp(c_0 - (c_1 \ln(AGE_{km})) - (c_2 \ln(SI_k - 1.3)))} \right]
\] (3.49)

where \( c_0 \) to \( c_2 \) are the parameters to be estimated using NLS and the rest of the symbols are as defined above.

The linear prediction models were fit by OLS, using the REG procedure in SAS (SAS Institute, Inc. 1985) and the nonlinear model was fit by NLS, using the NLIN procedure in
SAS. For all fits, the PSP model development data composed of 1334 trees was used. The parameter estimates given by Goudie (1984; as referenced in Goudie et al. 1990) were used as the starting values for the nonlinear model. In order to ensure that the parameter estimates resulting from the NLS fit were optimal (the global minimum residual sum of squares), the three methods available in SAS NLIN procedure were used with multiple starting values. These were the Newton-Gauss method which uses Taylor series, the Marquardt method which uses the updating formula method, and the Secant or “Doesn’t Use Derivative” (DUD) iterative method (see SAS Institute, Inc. 1985, p. 585–6).

The models could not be compared based on the t and F tests because they were of different types. Also, the error terms would not be iid because the observations are expected to be serially correlated. However, the prediction abilities of the models would not be affected because the coefficients estimated using OLS are unbiased and consistent (Kmenta 1971). Model selection was based on comparison of the fit statistics and the prediction statistics based on the 574 trees in the validation data (185 plots). The prediction statistics used for model evaluation included the $R^2$, $SEE$, $FI$, $S\hat{E}E$, RMSE, Bias and MAD (see Section 3.1.2, p. 69–70 for definitions).

One problem with the height prediction models was that they were based on plot age instead of tree age. This meant that the selected model would predict average plot height instead of individual tree height. If average plot measures from the PSP data were used to develop the height prediction model, the resulting taper model would be predicting average plot tree form and taper. As a result, individual tree characteristics would be lost and the dynamic taper model would give biased predictions for individual tree diameters along the stem. To alleviate this problem, the PSP data were used for model selection and the final felling stem analysis data were used for model calibration. The resulting model was:

$$\hat{H}_{it} = 1.3 + (SI_k - 1.3) \times \left[ \frac{1.0 + \exp(c_0 - (c_1 \ln(50)) - (c_2 \ln(SI_k - 1.3))}{1.0 + \exp(c_0 - (c_1 \ln(Age_{it})) - (c_2 \ln(SI_k - 1.3))} \right]$$

(3.50)

where $\hat{H}_{it}$ is the predicted tree height; $i$ is the tree number; $t$ is the five-year measurement
period for the stem analysis data; \( SI_k \) is plot site index at 50 years breast age; \( Age_{it} \) is tree breast height age; \( c_0 \) to \( c_2 \) are the parameters to be estimated using NLS; and the rest of the symbols are as defined previously. After calibration, the height prediction model was used to predict heights for each tree over time.

3.3.2 Prediction Model for Diameter at Breast Height

Diameter growth is highly correlated with stand density, AGE, SI, and a crown size measure such as CL (Section 2.1.2). However, CL or crown diameter, found by Sprinz and Burkhart (1987) to be an important variable in dbh prediction, was not used in the dbh prediction model because CL was only measured at the time of felling for the stem analysis data and crown diameter was not measured at all.

The growth and yield literature was searched for an existing dbh prediction model. However, most models predict dbh increment. Alternatively, the relationships between the dependent variable dbh, with possible predictor variables were examined by plotting dbh against each predictor variable (\( \dot{H} \), \( AGE \), \( SI \), \( QD_{50} \)), and transformations and interactions of these variables. These plots enabled assessing relationships, if any, that existed between dbh and the variables. The RSQUARE procedure in SAS (SAS Institute, Inc. 1985) was used to select a few candidate models based on minimum MSE, highest \( R^2 \) and \( R^2_a \), and lowest \( C_p \). Three linear models were selected for further testing:

\[
\begin{align*}
\dot{D}_{kmo} &= b_0 + b_1 \dot{H}_{kmo} + b_2 QD_{50k} + b_3 QD_{50k}^2 + b_4 (QD_{50k} \times AGE_{km}) + b_5 (QD_{50k} \times AGE_{km}^2) \\
\dot{D}_{kmo} &= b_0 + b_1 \dot{H}_{kmo}^2 + b_2 SI_k + b_3 (SI_k \times QD_{50k}) + b_4 QD_{50k} + b_5 (QD_{50k})^2 + b_6 (QD_{50k} \times AGE_{km}) + b_7 (QD_{50k} \times AGE_{km}^2) \\
\dot{D}_{kmo} &= b_0 + b_1 \dot{H}_{kmo} + b_2 SI_k + b_3 SI_k^2 + b_4 QD_{50k} + b_5 QD_{50k}^2 + b_6 (QD_{50k} \times AGE_{km}) + b_7 QD_{50k} \times AGE_{km} + b_8 AGE_{km}
\end{align*}
\]
where $\hat{H}_{km}$ is the predicted dbh; and $b_0$ to $b_8$ are the coefficients to be estimated. All trees in the same PSP have the same SI, and $QD_{50}$. $AGE_{km}$ represented the plot age at measurement period $m$.

These models were fit using OLS by the REG procedure in SAS using the PSP fit data. The models were then tested on the validation data set for their predictive abilities. The statistics used for model evaluation included $R^2$, $SEE$, PRESS statistic for the fit data, and $FI$, $S\bar{E}E$, RMSE, Bias and MAD for the validation data (see Section 3.1.2, p. 69–70 for definition of these statistics).

The dbh prediction models had the same problem as the height prediction models in that they were based on plot predicted height and plot age instead of individual tree height and age. This meant that any model selected would predict average plot dbh, not individual tree dbh. To alleviate this problem, the PSP data were used for model selection and the final felling stem analysis data were used for model calibration. The model that was calibrated was:

$$\hat{D}_{it} = b_0 + b_1\hat{H}_{it} + b_2\hat{H}_{it}^2 + b_3SI_k + b_4SI_k^2 + b_5\hat{Q}D_{50k} + b_6\hat{Q}D_{50k}^2 + b_7(\hat{Q}D_{50k} \times Age_{it}) + b_8Age_{it}$$

(3.54)

where $\hat{D}_{it}$ is the predicted dbh for tree $i$ and measurement period $t$ for the stem analysis data; $b_0$ to $b_8$ are the coefficients to be estimated; and the rest of symbols are as already defined.

### 3.3.3 Relative Height Function

In a number of top sectional measurements, the value for $h_{ijt}$ was higher than the predicted height (i.e., $h_{ijt}$ at the tip of tree is greater that $\hat{H}_{it}$). As a result:

1. both $1 - \sqrt{Z_{ijt}}$ and $\frac{h_{ijt}}{\hat{H}_{it}}$ were no longer bounded by zero and one.
2. when $M_{ijt} = 1 - \sqrt{Z_{ijt}}$ became negative, the logarithm was not defined.
3. whenever the form exponent of the Kozak’s (1988) function \((c)\) was less than one, the result of \(M_{ijt}^c\) was undefined.

In order to correct this problem, predicted values for \(Z_{ijt}\) were constrained between zero and one using a logistic function as follows:

\[
\hat{Z}_{ijt} = \frac{1}{1 + \exp\left[ f_1 \right]} \tag{3.55}
\]

where \(\hat{Z}_{ijt}\) is the predicted relative height; and \(f_1\) is a function of the independent variables selected.

In order to find out the most important independent variables for the above model, the RSQUARE procedure in SAS (SAS Institute, Inc. 1985) was used to construct a linear function for \(f_1\) using the following 11 independent variables: \(h_{ijt}, h_{ijt}^2, \hat{H}_{it}, \frac{1}{\hat{h}_{ijt}}, (\frac{1}{h_{ijt}})^2, \hat{H}_{it}^2, \frac{1}{\hat{H}_{it}}, \ln(\hat{H}_{it}), h_{ijt} \times \hat{H}_{it}, (h_{ijt} \times \hat{H}_{it})^2, h_{ijt}^2 \times \hat{H}_{it}\). Two models were selected:

1. \(f_1(h_{ijt}, H_{it}) = b_0 + b_1 h_{ijt} + b_2 h_{ijt}^2 + \frac{b_3}{h_{ijt}} + \frac{b_4}{h_{ijt}^2} + b_5 \hat{H}_{it} + b_6 \ln \hat{H}_{it} + b_7 (h_{ijt} \times \hat{H}_{it}) + b_8 (h_{ijt} \times \hat{H}_{it})^2 + b_9 (h_{ijt}^2 \times \hat{H}_{it})\)

2. \(f_1(h_{ijt}, H_{it}) = b_0 + b_1 h_{ijt} + b_2 h_{ijt}^2 + \frac{b_3}{h_{ijt}} + \frac{b_4}{h_{ijt}^2} + b_5 \ln \hat{H}_{it} + b_6 (h_{ijt} \times \hat{H}_{it}) + b_7 (h_{ijt} \times \hat{H}_{it})^2 + b_8 (h_{ijt}^2 \times \hat{H}_{it})\)

where \(b_0\) to \(b_9\) are the coefficients to be estimated.

The two linear models for \(f_1\), were substituted into Equation 3.55. The following two logistic models resulted.

\[
\hat{Z}_{ijt} = \frac{1}{1 + \exp \left( b_0 + b_1 h_{ijt} + b_2 h_{ijt}^2 + \frac{b_3}{h_{ijt}} + \frac{b_4}{h_{ijt}^2} + b_5 \hat{H}_{it} + b_6 \ln \hat{H}_{it} + b_7 (h_{ijt} \times \hat{H}_{it}) + b_8 (h_{ijt} \times \hat{H}_{it})^2 + b_9 (h_{ijt}^2 \times \hat{H}_{it}) \right)} \tag{3.56}
\]

\[
\hat{Z}_{ijt} = \frac{1}{1 + \exp \left( b_0 + b_1 h_{ijt} + b_2 h_{ijt}^2 + \frac{b_3}{h_{ijt}} + \frac{b_4}{h_{ijt}^2} + b_5 \ln \hat{H}_{it} + b_6 (h_{ijt} \times \hat{H}_{it}) + b_7 (h_{ijt} \times \hat{H}_{it})^2 + b_8 (h_{ijt}^2 \times \hat{H}_{it}) \right)} \tag{3.57}
\]
These two functions were fitted with the DUD option in the NLIN procedure of SAS (SAS Institute, Inc. 1985) to data generated using the measured heights from the stem analysis fit data for which the $Z_{ijt}$ values were between 0 and 1. The coefficient estimates from the RSQUARE procedure for $f_1$ were used as starting values for the logistic nonlinear models. Model selection was based on the fit ($R^2$, $SEE$) and prediction statistics ($FI$, $S\hat{E}E$, RMSE, Bias, and MAD).

### 3.3.4 Variables Selected for Estimating the Form Exponent

In order to find the variables correlated with $c_{ijt}$, graphical analysis was carried out. Plots of $c_{ijt}$ against the variables found to be correlated with $r_{ijt}$ ($H_{it}$, $D_{it}$, age, $Q\hat{D}_{50}$ and transformations of these variables) were constructed. In addition, $c_{ijt}$ values were plotted against all the variables used by Kozak (1988) in the form-exponent.

Using the knowledge gained from objective one and these graphs, all the variables found to be correlated with $r$, and transformations of these variables, were used to find the best model for predicting $c_{ijt}$. The variables selected were: SI, Age, $Q\hat{D}_{50}$, $\hat{D}_{it}$, $\hat{H}_{it}$, $h_{ijt}$, $\hat{D}_{it}/\hat{H}_{it}$ ratio, $\hat{Z}_{ijt}^2$, $\sqrt{\hat{Z}_{ijt}}$, and $1/\hat{Z}_{ijt}$. Not all transformations of the variables used by Kozak (1988) were used. The variable $\ln(\hat{Z}_{ijt} + 0.001)$ was dropped, because $\ln(Z_{ijt})$ is undefined at the tree base; Kozak added an arbitrary constant of 0.001 to obtain a value at the tree base. The variable $\exp(\hat{Z}_{ijt})$ was only marginally correlated with $r_{ijt}$ and also was dropped before the analysis. The RSQUARE procedure in SAS (SAS Institute, Inc. 1985, p. 711–724) was used to find the best subset model based on remaining variables.

The two best subset models for predicting $c_{ijt}$, based on minimum MSE, highest $R^2$ and $R_a^2$, and low $C_p$ were:

\[
\hat{c}_{ijt} = b_1 \hat{Z}_{ijt}^2 + b_2 \sqrt{\hat{Z}_{ijt}} + b_3 \frac{\hat{D}_{it}}{\hat{H}_{it}} + b_4 \frac{1}{\text{Age}_{it}} + b_5 Q\hat{D}_{50k}^2 + b_6 \frac{1}{\hat{H}_{it}} + b_7 \frac{1}{\hat{h}_{ijt}} \tag{3.58}
\]
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and

\[
\hat{c}_{ijt} = b_0 + b_1 \hat{Z}_{ijt}^2 + b_2 \sqrt{\hat{Z}_{ijt}} + b_3 \frac{\hat{D}_{it}}{\hat{H}_{it}} + b_4 \frac{1}{\text{Age}_{it}} + b_5 \hat{Q} D_{50k} + b_6 \ln \hat{H}_{it} + b_7 \frac{1}{\hat{h}_{ijt}} + b_8 \sqrt{\hat{h}_{ijt}} \quad (3.59)
\]

where \(\hat{c}_{ijt}\) is the predicted form exponent; \(i\) is the tree number; \(j\) is the section number on tree \(i\); \(t\) is measurement period; and \(b_0\) to \(b_8\) are coefficients to be estimated. For both equations, when \(h_{ijt} = 0\) at ground level, a small constant such as the one used by Kozak (1988) would be added to \(h_{ijt}\) to become \(h_{ijt} + 0.001\) so that the models are defined.

3.3.5 Dynamic Taper Model Assembly and Fitting

The two selected models for predicting \(c_{ijt}\) and the predicted values for \(\hat{Z}_{ijt}, \hat{D}_{it},\) and \(\hat{H}_{it}\) were incorporated into the Kozak (1988) taper function (Equation 3.44) by substituting these for the variable-exponent \((c)\), \(Z_{ijt}, D_{it}\), and \(H_{it}\) respectively. For comparison purposes, Kozak's (1988) taper function was fit using \(\hat{Z}_{ijt}, \hat{H}_{it},\) and \(\hat{D}_{it}\) as independent variables. This dynamic version of Kozak's (1988) model was included in order to compare the new dynamic taper models with a dynamic taper model obtained simply by using predicted height and dbh. The static model was included since it represented the best predictions possible using measured height and dbh that is standard for current inventory. These four taper models (three dynamic and one static) are presented below.


\[
\hat{d}_{ijt} = a_0 \times D_{it}^{a_1} \times z_{it}^{a_2} \times \left[ \frac{1}{1-\sqrt{q}} \right]^{b_1 Z_{ijt}^2 + b_2 \ln(Z_{ijt} + 0.001) + b_3 \sqrt{Z_{ijt}} + b_4 e^{Z_{ijt}} + b_5 D_{it}}{b_6 H_{it}}
\]

2. Kozak's (1988) dynamic taper model (Model 2) (variables used: \(\hat{D}_{it}, \hat{H}_{it},\) and \(\hat{Z}_{ijt}\)).

\[
\hat{d}_{ijt} = a_0 \times \hat{D}_{it}^{a_1} \times \hat{Z}_{ijt}^{a_2} \times \left[ \frac{1}{1-\sqrt{q}} \right]^{b_1 Z_{ijt}^2 + b_2 \ln(Z_{ijt} + 0.001) + b_3 \sqrt{Z_{ijt}} + b_4 e^{Z_{ijt}} + b_5 D_{it}}{b_6 H_{it}}
\]

3. New dynamic taper model (Model 3A).

\[
\hat{d}_{ijt} = a_0 \times \hat{D}_{it}^{a_1} \times \hat{D}_{it}^{a_2} \times \left[ \frac{1}{1-\sqrt{q}} \right]^{b_1 \sqrt{Z_{ijt} + b_2 \hat{D}_{it} \frac{\hat{H}_{it}}{\text{Age}_{it}} + b_4 \frac{1}{\text{Age}_{it}} + b_5 Q D_{50k} + b_6 \frac{1}{\hat{H}_{it}} + b_7 \frac{1}{\hat{h}_{ijt} + 0.001}}{b_8 \sqrt{\hat{h}_{ijt}}}
\]
4. New dynamic taper model (Model 3B).

\[
\hat{d}_{ijt} = a_0 \times \hat{D}_{it}^{e_1} \times a_2 \hat{D}_{it} \times \left[ \frac{1 - \sqrt{Z_{ijt}}}{1 - \sqrt{q}} \right]^{b_0 + b_1 \hat{D}_{it}^{e_2} + b_2 \hat{D}_{it}^{e_3} + b_3 \hat{D}_{it}^{e_4} + b_4 \ln \hat{H}_{it} + b_5 Q \hat{D}_{it}^{e_5} + b_6 \ln \hat{H}_{it} + b_7 \hat{H}_{it} + b_8 \hat{H}_{it} + b_9 + b_{10}}
\]

where \(a_0\) to \(a_2\) and \(b_0\) to \(b_{10}\) are coefficients to be estimated; and \(q\) is given as \(0.25 H_{it}\). For Models 2, 3A and 3B, when \(h_{ijt} = H_{it}\), \(\hat{d}_{ijt}\) will equal \(D_{it}\), since \(M_{ijt} = \frac{1 - \sqrt{h_{ijt}}}{1 - \sqrt{q}} = 1.0\). At this point, \(\hat{d}_{ijt}\) is equal to \(a_0 \times \hat{D}_{it}^{e_1} \times a_2 \hat{D}_{it}^{e_2}\), which corresponds to predicted \(D_{it}\). Therefore, at this point, the predicted form exponent has no impact on the \(\hat{d}_{ijt}\), since \((M_{ijt})^{e_3}\) for any exponent equals 1.0. The predicted value of \(d_{ijt}\) will only be affected by the measured or predicted \(D_{it}\) and measured or predicted \(H_{it}\), since \(H_{it}\) is some fixed proportion of \(H_{it}\). The static model (Model 1) has the same properties, except that the measured values for dbh and height are the inputs instead of the predicted values.

These taper functions are intrinsically nonlinear in the parameters, if they are assumed to have additive error terms. However, if it is assumed that they have multiplicative error terms, as Kozak (1988) did, then the taper models will be intrinsically linear, because they can be expressed in linear forms by logarithmic transformation. Therefore, the four taper functions can be fit using either OLS or NLS, depending on the assumptions made about the error terms.

During the model fitting process, the above dynamic taper equations were assumed to have either (1) additive error terms and were fitted using NLS or (2) multiplicative error terms and were transformed using natural logarithms and fitted using OLS. This would result in eight sets of fit statistics, two sets for each taper model. However, before the actual model fitting process began, the two new dynamic taper functions (3A and 3B) were transformed and fitted using OLS (REG procedure in SAS Institute, Inc. 1985) based on the stem analysis model development data set. They were then evaluated based on the validation stem analysis data set. The fit and prediction statistics used for evaluation included \(R^2\), SEE, PRESS, FI, \(S\hat{E}\), RMSE, Bias, and MAD. The model with the better prediction statistics was selected and labelled as Model 3.
In addition to these three models, a test using the coefficients of Model 1 (static), but with predicted height and dbh was included. Commonly, growth and yield models use an existing taper function and simply input predicted height and dbh (e.g., Arney, 1985). The static model with measured height and dbh was labelled Model 1a and the static model with inputted predicted height and dbh was labelled Model 1b.

Therefore, eight taper models were examined. Four models were dynamic (two linear and two nonlinear), two were static (one linear and the other nonlinear), and two were the static models with inputs of predicted height and dbh. For clearer identification of the models, “N” was added to the model number to indicate a nonlinear fit and “L” was added to the model number to indicate a linear fit.

Using the above taper models, the following comparisons were therefore possible:

1. Model 1a with Models 2 and 3. The static model (Model 1a) would be expected to perform better than Models 2 and 3 because it is based on measured height and dbh. Model 1a is the standard model used for current inventory taper predictions.

2. Model 1b with Models 2 and 3. Model 1b is the standard for use in growth and yield predictions of tree taper. Model 1b should perform better than Model 2 if the current procedure used in growth and yield is accurate. However, if Model 2 performs better than Model 1b, then the current procedure used in growth and yield should be questioned. Models 2 and 3 are expected to perform better than Model 1b because these dynamic taper functions were fitted as a system.

3. Model 2 with Model 3. Model 3 is expected to perform better than Model 2 because Model 3 includes new variables which were found be correlated with tree form, and Model 3 was developed specifically for modelling taper over time.
3.3.5.1 Optimization of $q$

As stated previously, $q$ (the percent height at which the join point occurs) was found to vary between 22 and 27 percent. An attempt was made to find the best value which minimized the variation around the taper function. During model fitting using NLS, the DUD iterative method in NLIN was used with the OLS coefficient estimates as the starting values. Model 3N was fit by trying to optimize the coefficient estimates as well as the value of $q$. The $q$ value was limited to between 0.00 and 0.35 and incremented by 0.005. The procedure took over 45 minutes of CPU time to converge. The optimum value obtained was very small ($q=0.0003$). This value was too small to make any impact on the model; therefore, two more NLS runs were made for model 3N. One run was based on $q=0.00$ and the other on $q=0.25$. The results showed no difference for the coefficients using $q=0.00$ versus the optimum value $q=0.0003$. However, for $q=0.25$, there were some differences. Optimization of $q$ was tried for Kozak's (1988) taper model (model 2N) and the results showed that scaling was not necessary.

For all further analyses using models 3L and 3N, no scaling was assumed (i.e., $q = 0.0$). However, for models 1aL, 1aN, 1bL, 1bN, 2L, and 2N the scaling factor of $q=0.25$ was maintained, to agree with Kozak (1988).

3.3.5.2 Fitting the Taper Models using OLS and NLS

After transforming the data using the natural logarithm, models 1 to 3 were fit using OLS. Based on the findings from the optimization process for $q$, model 3L was fit using no scaling factor ($q=0.0$). All taper functions were fit using the REG procedure in SAS (SAS Institute, Inc. 1985, p. 655–709) using the stem analysis fit data. The resulting coefficient estimates were used in the evaluation of the OLS fitting method. These coefficient estimates are unbiased and consistent, but are inefficient (Kmenta 1971) because of the error term characteristics outlined in Section 2.4. Graphical analyses of the residuals were performed
to check for lack of fit.

After fitting the three taper models using OLS, the estimated coefficients were used as starting values in the NLS fitting. NLS was applied to the three models using the procedure NLIN with the DUD method in SAS (SAS Institute, Inc. 1985, p. 586) on the stem analysis fit data. The initial coefficient estimates were varied slightly and the model refit until the coefficients estimates stabilized. This process ensured getting the global minimum values for the sum of squared residuals. The resulting coefficient estimates are consistent and asymptotically normal, but might not be efficient (Judge et al. 1985, p. 198–201). Graphical analyses of residuals were performed to check lack of fit.

3.3.5.3 Fitting the Taper Models using FGLS

The stem analysis data used for dynamic taper model fitting was characterised as having residuals which were non-iid. If the dynamic taper models were fitted using OLS or NLS, the coefficient estimates would be unbiased and consistent, but would not be BLUE and the usual estimates of the variances for the coefficients would be biased and inconsistent.

Knowing that the OLS and NLS fitting methods for estimating parameters would result in biased estimates of the variances for the coefficient estimates, the use of FGLS and FGNLS was considered in order to carry out statistical inferences. The FGLS and FGNLS methods are slightly different, but the general procedure in both methods is the same and only one method, the FGLS for linear models, will be described in detail.

Assuming that there are no differences in parameters (regression coefficients) for the different measurement periods, sections and trees, the FGLS parameter estimator is the most appropriate method. Each of the transformed dynamic taper models can be written as:

\[ Y_{ijt} = \beta_0 + \beta_1 X_{ijt1} + \beta_2 X_{ijt2} + \cdots + \beta_{10} X_{ijt10} + e_{ijt} \]  
(3.60)
or simplified as

\[ Y_{ijt} = \beta_0 + \sum_{k=1}^{K} X_{ijtk} \beta_k + e_{ijt} \]  

(3.61)

where \( Y_{ijt} \) is the natural logarithmic transformation of \( d_{ijt} \); \( X_{ijtk} \) is the \( k^{th} \) independent variable, \( k = 1, 2, \ldots, K \); \( K \) is the number of independent variables used in the model; \( i \) is the tree number; \( j \) is the section number; \( t \) is the measurement period; \( \beta_0 \) to \( \beta_{10} \) are the coefficients to be estimated from the data; and \( e_{ijt} \) is the logarithmic error term associated with the predicted \( \ln(d_{ijt}) \) for the sample tree \( i \), section \( j \), and time period \( t \). In matrix form the above equation becomes:

\[ \mathbf{Y} = \mathbf{X}\beta + \mathbf{\epsilon} \]  

(3.62)

where \( \mathbf{Y} \) is the column vector of dependent variable (\( \ln(d_{ijt}) \)); \( \mathbf{X} \) is a matrix for the \( K \) independent variables depending on the model (these variables included predicted \( D \) and the product of \( \ln M \) and the selected variables for \( c \)); \( \beta \) is the column vector of model parameters to be estimated; and \( \mathbf{\epsilon} \) is the column vector of random error terms (\( e_{ijt} \)).

The vector \( \mathbf{Y} \) and the matrix \( \mathbf{X} \) appear as:
For the analysis, all the $d_{ij}$ measurements on all sample trees of the model development data set for all time periods were pooled together ($5916 \times 1$). Different trees were assumed to have been randomly selected in the field and, therefore, were assumed to be independent of one another. However, these trees came from different plots in different areas, they had different crown sizes and growth rates and, therefore, were assumed to have differing variances. The matrix $(X)$ for the independent variables is a $\sum_{i=1}^{N} \sum_{j=1}^{M_i} T_{ij} \times K$ matrix ($5916 \times 11$). $\hat{\beta}_F$ is a $K \times 1$ ($11 \times 1$) column vector shown below:

$$\hat{\beta}_F = \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \vdots \\ \hat{\beta}_K \end{bmatrix}, \quad \text{and} \quad \hat{\epsilon} = \begin{bmatrix} \hat{\epsilon}_{111} \\ \hat{\epsilon}_{112} \\ \vdots \\ \hat{\epsilon}_{NMT_{ij}} \end{bmatrix},$$

where $\hat{\epsilon}$ is an $\sum_{i=1}^{N} \sum_{j=1}^{M_i} T_{ij} \times 1$ ($5916 \times 1$) vector.
To fit the three taper functions using FGLS, the components of $\hat{\Omega}$ would be obtained by first calculating the $\hat{\rho}_{ij}$'s (Equation 2.26), the components of $\hat{P}_{ij}$, followed by estimating $\Phi_i$ (Equation 2.38). Finally, $\hat{\Omega}$ would be substituted into Equation 2.40 to get $\hat{\beta}_F$. This fitting process would follow the following steps:

1. The first stage would begin by pooling all the observations and running OLS using Equation 3.60. Since the OLS coefficient estimates ($b$) are unbiased and consistent (Kmenta 1971), these would be used to estimate the residuals ($e_{ijt}$) for each tree as follows:

$$
\hat{e}_{ijt} = Y_{ijt} - \hat{Y}_{ijt}
$$

where $\hat{Y}_{ijt}$ is the predicted value using OLS.

2. Using these residuals, a consistent estimate of $\rho_{ij}$, assuming first order autocorrelation, would be calculated for each section using Equation 2.26.

However, the following problems were recognized in estimating the $\rho_{ij}$'s.

(a) For all trees, the last section had only one serial measurement. Therefore, calculation of $\hat{\rho}_{ij}$ was not possible. Since it contained only one measurement, correlation was assumed to be perfect and would be assigned a value of one.

(b) For every tree, there were some sections with only two serial measurements. This meant that the calculated $\hat{\rho}_{ij}$ values may be unreliable.

(c) Using $\hat{\rho}_{ij} = 1$ for the last sections for each tree meant these measurements would not be transformed. As a result, these sections would be lost.

To solve these problems, the possible solutions would have been:

(a) If the time series measures are very few, it may be preferable to impose the restriction of a common $\rho$ (Baltagi 1986), even if it is violated in the population,
in order to improve the small sample performance of the FGLS estimator (Greene 1990, p.474). This means that a single $\rho$ value for each tree ($\rho_i$) would be used. From the equation for calculation of $\rho$ (see Equation 12.32 of Kmenta 1971, p. 512), sections with only one periodic measurement would not be used to estimate $\rho$ using Equation 2.26. As a result, the estimated value for tree $i$ will be biased.

(b) Use the econometric methods for fitting panel data, i.e., use varying coefficient models (see Judge et al. 1985, p 415–425, for discussion of these models). However, as already noted in Section 2.4, such models are not practical for taper modelling.

(c) Another option would be to pool the sections together for all trees and try to find models that predict the variation in $\rho_i$ for individual trees or $\rho_j$ for each section along the stem as functions of tree, stand, and site variables as:

$$\hat{\rho}_i = \text{function(tree, stand, and site variables)}$$

or

$$\hat{\rho}_j = \text{function(tree, stand and site variables)}$$

Pooling sections together over trees to calculate a common $\rho_i$ for each tree, as suggested by Baltagi (1986), or pooling all observations for all sections for all trees together for calculating a common $\rho_j$, are possible. However, these trees were sampled from different forests in different areas, meaning that they have experienced different growing conditions even for trees of the same age, and also, some trees were at different growth stages at the different locations. It is very tenuous to assume that all trees experienced the same growing conditions at different ages. Therefore, pooling the data together in this manner might result in poor estimates. Also, based on Gertner's (1985) study, serial correlation might not have been a serious problem for the nonlinear models, because the remeasurement period was five years. Gertner found the relative efficiency of NLS to FGNLS to increase with the measurement interval. For a measurement interval
of five years, the relative efficiency of NLS compared to FGNLS was 99 percent; Gertner said that NLS is almost equivalent to FGNLS for measurement intervals of five or more years.

3. Since $\hat{\rho}_{ij}$ is a consistent estimator of $\rho_{ij}$ (Kmenta 1971), the original data would be transformed using this estimate and Equations 2.27 to 2.30.

4. The resulting estimated residuals $(u_{ij})$ (Equation 2.31) would be serially uncorrelated and would be used to estimate sectional variances and covariances using Equations 2.33 to 2.36. However, following problems were identified at this stage.

(a) For all sections with one serial measurement, the variance could not estimated. Also, for sections with only two serial measurement, the estimated variances based on two observations could be unreliable.

(b) For a given tree, sections had unequal numbers of serial measurements (i.e., $j$ decreases from sections near the ground to one serial measurement for the last section). From Equations 2.33 and 2.36 for calculating covariances, some observations would not be utilized (from larger sections with more observations). Therefore, the estimated covariances between such sections could be unreliable.

Some possible solutions to these problems include:

(a) Pooling the data over sections for all trees and then calculating common estimates for variances and covariances. However, these trees were samples from different forests in different areas, and the variances among sections vary considerably.

(b) Pooling the data for all sections of each tree and then calculating a single variance for each tree. This choice also does not make much sense, because it is apparent from graphs of $d_{ijt}$ versus $h_{ijt}$ that the variances for the different sections of a tree decrease as $h_{ijt}$ increases.
Another option would be to pool the data together for each section over all trees and try to find models that predict the variation in $\sigma_j^2$ along the stem as functions of tree, stand, and site variables:

$$\hat{\sigma}_j^2 = \text{function(tree, stand, and site variables)}.$$ 

The resulting coefficient estimates using FGLS would be asymptotically efficient if a consistent estimate of $\Omega$ could be obtained. Since the fitted model would involve the use of a large quantity of data, the asymptotic properties of FGLS (Kmenta 1971) would also apply. This means that the coefficient estimates and their variances would be better than those estimated using OLS and NLS. However, with the problems noted in obtaining the variance-covariance matrix, $\hat{\Omega}$, estimated coefficients could be greatly biased and even less efficient than OLS or NLS estimates.

Therefore, FGLS was not used. If the model to be developed is to be used for prediction, as the dynamic taper model would be, then it is more important to have unbiased coefficient estimates. OLS and NLS fit provide such estimates.

### 3.3.6 Dynamic Taper Model Testing and Evaluation

Model validation involves determining the quality of model predictions. As used by many modellers, model validation refers to the process of assessing, in some sense, the degree of agreement between the model and the real system being modelled (Reynolds et al. 1981). Snee (1977) and Neter et al. (1985) suggested the following ways of validating a model:

1. Examining the model's performance on the model-building data set or self-validation. This indicates how good the model is for predicting within the confines of the data set.

2. Comparing the model predictions and coefficients with theoretical expectations, earlier empirical results, and simulation results. This is not possible if there is no prior information available.
3. Collecting new data to check the model and its predictive ability. This would be the best choice, but it may not be possible or may be very expensive, making it infeasible.

4. Using part of the data collected as an independent data set to check the model’s prediction (data splitting) or the techniques of cross-validation, jacknifing, and bootstraping (Gong 1986).

The validation technique most used by taper modellers is to split the data into two sets of varying proportions. Newnham (1988, 1992) and Kozak (1988) used one-half of their data for validation, Perez et al. (1990) and Byrne and Reed (1986) used one-third, while Max and Burkhart (1976) used one-fourth. The data for validation can either be systematically selected (Newnham 1988, 1992) or randomly selected (Kozak 1988; Byrne and Reed 1986; Perez et al. 1990; Max and Burkhart 1976). As stated previously, the PSP and the stem analysis data were split randomly into two sets: the model development (70%) and the validation or prediction data sets (30%). Only a small proportion of data (30%) was retained for model validation because with only 135 trees, a bigger proportion had to be used for model fitting.

The process of model selection should be based on both practical and statistical considerations. For practical and theoretical purposes, a taper model should be easy to apply, coefficient estimates should be easily derived, the variables used should be easy to measure, and the model should be able to track the real system being modelled. In this case, the model selected should be able to take into account the variation in tree form over time from ground to tree tip.

Model selection for this research involved comparing the measured $d_{ijt}$, and calculated total tree volume ($V_{it}$) with $\hat{d}_{ijt}$ and $\hat{V}_{it}$, respectively, as generated by each fitted taper function. The accuracy and precision of the predictions for each taper equation depended on how well the fitted taper equation was able to track the tree profile. Unfortunately, the fitted taper models could not be integrated to an exact form therefore, numerical integration
was used for volume prediction.

Comparison of the four taper models for each of the two fitting techniques (NLS and OLS) was based on fit and prediction statistics. The fit statistics used were $R^2$, SEE, and PRESS (for linear models only), while the prediction statistics used were Bias, RMSE, MAD, $\hat{S}$EE, and FI. It should be noted that the predicted values used in calculating the prediction statistics were based on $d_{ij}$ rather than $\ln d_{ij}$, which was used in the fitting process, in order to facilitate comparison of the linear and nonlinear models and to better relate to the real system being modelled.

When a model is logarithmically transformed, its predicted values will systematically underestimate the actual values (Baskerville 1972; Flewelling and Pienaar 1981). Therefore, the logarithmically transformed taper models had to be corrected for this bias. According to Flewelling and Pienaar (1981), if the number of observations minus the number of coefficients (DF) to be estimated is greater or equal to 30, and the variance estimate (MSE) for the error term of the logarithmic model is less than 0.5, the bias will be less than one percent. For this research, the smallest DF was 5907 and the largest MSE was 0.066. With such large DFs and small error variances, the biases would be very small. Nevertheless, Flewelling and Piennar's (1981) correction factor (CF) of $\exp(\frac{1}{2} \text{MSE})$ was used for the OLS fitted models. This CF value was applied by calculating the predicted value of $d_{ij}$ as follows:

$$\hat{d}_{ij} = \exp(\frac{1}{2} \text{MSE}) \times \exp(\ln(\hat{d}_{ij}))$$  \hspace{1cm} (3.64)

All the fit and prediction statistics identified above were used in selecting and evaluating the developed dynamic taper model, because no single criterion is best. A model can perform better using one measure and poorer using another; therefore, using any one criterion alone might not give a true picture of the model.

Graphical analysis also was used as part of model evaluation process. Graphs of Bias in $\hat{d}_{ij}$ for all trees and for three dbh classes along the stem were plotted. As well, graphs for
predicted and observed tree $V_{it}$ over time were plotted for the large dbh trees. Predicted total volumes for each tree were determined by numerical integration of the taper functions. The observed (actual) estimates of the volume for each tree were calculated using Smalian's formula (Husch et al. 1982, p. 101–103). The purpose of these graphical analyses was to determine which regions along the stem were more biased for a given tree or group of trees.
Chapter 4

Results

This chapter is divided into three sections. Section 4.1 presents the results of investigating the influence of tree, stand, and site factors on the variation of tree form and taper over time (study objective one). Also, individual tree taper and form variation with age at different stem positions and with age and height above ground are presented as three-dimensional plots. Section 4.2 presents the selected models for predicting total tree height, dbh, relative height, and the form exponent (c), along with the fit and prediction statistics (study objective two). Also, the dynamic taper models developed are given. Section 4.3 presents the results of assessing the dynamic taper models for their prediction abilities for $d_{ijt}$ and total tree volume.

4.1 Variation in Tree Form and Taper

Examinations of the changes in the form exponent ($r$) of Equation 2.1 with age, dbh, height, D/H ratio, CL, CR, $QD_{50}$, and SI were performed using both final felling and periodic measurements for the stem analysis data. Since the relationship between $r$ and these variables was expected to vary over the tree stem, three positions on the tree stem were chosen (0.1H, 0.5H, and 0.8H).

4.1.1 The Variation of Tree Form and Taper with Tree, Stand, and Site Factors

Tree form is known to vary with some tree, stand, and site variables. In this subsection, the results of investigating the relationships between the form exponent ($r$) and total tree height, dbh, D/H ratio, AGE, SI, $QD_{50}$, CL and CR are presented. Graphs of these relationships
were based on the stem analysis data composed of trees measured intervals of five years (887 tree-measures), except for CL and CR, which were based on the final felling data (135 trees).

4.1.1.1 Total Tree Height

At the base of the tree, \( r \) increased with increase in total height (Figure 4.4A). The variability in \( r \) is greater over the height range than the variability of \( r \) over the dbh range (Figure 4.4B). At 0.5\( H \) or 0.8\( H \), the value of \( r \) was constant throughout the height range (Figures 4.5A and 4.6A). However, the relationship of \( r \) with height may be confounded by changes in dbh and stand density as the trees grow.

4.1.1.2 Diameter at Breast Height

As trees increased in dbh, the value of \( r \) increased (Figures 4.4B, 4.5B, and 4.6B). The most notable changes in the shape coefficient (\( r \)) occurred at the base of the tree (Figure 4.4B). Dbh is an important factor in the prediction model for \( r \), particularly for tree butt sections.

4.1.1.3 Dbh to Height Ratio

The trend of the form exponent (\( r \)) with D/H ratio indicated a steep slope at the base of the tree (Figure 4.4C). The value of \( r \) also rose with D/H ratio at the middle and top positions of the tree stem, but the slope was less steep (Figures 4.5C and 4.6C). The use of D/H ratio as a variable to indicate changes in \( r \) may prove more important for predicting form exponent changes than use of dbh and height separately.

4.1.1.4 Age at Breast Height

For a young tree, no real shape differences were apparent with change in time for any point on the tree stem (Figures 4.4D, 4.5D, and 4.6D). However, for older trees shape changes were evident at the base and top of the tree (Figures 4.4D and 4.6D). At the tree base, the
Figure 4.4: Form exponent ($r$) by (A) Height (m), (B) Dbh (cm), (C) D/H ratio (cm/m), (D) Age (years) at breast height, (E) SI (m), and (F) $QD_{50}$ (cm) at 0.1H above ground.
Figure 4.5: Form exponent (r) by (A) Height (m), (B) Dbh (cm), (C) D/H ratio (cm/m), (D) Age (years) at breast height, (E) SI (m), and (F) $QD_{50}$ (cm) at 0.5H above ground.
Figure 4.6: Form exponent (r) by (A) Height (m), (B) Dbh (cm), (C) D/H ratio (cm/m), (D) Age (years) at breast height, (E) SI (m), and (F) $QD_{50}$ (cm) at 0.8H above ground.
form exponent changed from a conical shape ($r$ of 2) to a neiloid shape ($r$ of 3) and to higher values (up to about 5), representing concave shapes. At the tree top, there seemed to be some trend from a paraboloid ($r$ of 1) to a conical shape.

The large variability of $r$ with age for all three positions, similar to that shown for height, indicates that the trends of individual trees were confounded by changes in other tree and stand variables. Although no obvious trend of $r$ with age was shown in Figures 4.4D, 4.5D, and 4.6D, the age variable will likely be important once other variables such as dbh are included in the prediction equations.

Six representative trees were selected and the form exponent or shape variation with age was analysed for the same relative height positions (Figures 4.7 and C.18 to C.22 of Appendix C). From these individual tree trends, it can be seen that the form exponent ($r$) increased with tree age, particularly for the base section. The increase was more pronounced for large trees (trees 16.2 and 46.1; Figures C.18 and C.22, respectively) than for small trees (trees 25.2 and 43.2; Figures C.19 and C.21, respectively). For all the trees at young ages, the form exponent was almost the same at all relative heights; however, the $r$ value at the base increased faster with increasing age, compared to the other relative heights which maintained almost the same value. This means that the trees maintained almost the same shape from ground level to tree top as they grew.

At young ages, trees had similar shapes at the three positions (Figures 4.7 and C.18 to C.22) with an $r$ value ranging from 1 to 1.5 (more or less paraboloid in shape). However, as the trees grew, they tended to assume different shapes for the different positions; this might be confounded by changing stand density. At 0.1H, trees tended to become conic and then neiloid if grown in less dense stands (Figure C.18, for tree 16.2; and Figure C.20, for tree 42.1), while at 0.5H and 0.8H trees continued to be more or less parabolic.
Figure 4.7: Form exponent (r) by breast height age (years) for tree 13.2, dbh=15.6 cm, height=16.8 m, and AGE=54 years, $QD_{50}=9$ cm, at three relative heights (0.1H, 0.5H, and 0.8H) above the ground.

4.1.1.5 Site Index

Although the relationship of r with SI was quite variable for all positions on the stem (Figures 4.4E, 4.5E and 4.6E), there appeared to be some increase in the form exponent with increasing SI. This relationship was clouded by changes in dbh due to differences in density, tree breast height age, and changes in the crown length (Figures 4.8A, 4.8C, and 4.8E).

4.1.1.6 Predicted Quadratic Mean Diameter at Age 50

The relationship of r with $QD_{50}$ was not very strong (Figures 4.4F, 4.5F, and 4.6F). There was a stronger relationship at 0.1H than at 0.5H and 0.8H. It should be noted that the data contained only dominant and codominant trees and this will have an impact on the results. Other measures of stand density, such as basal area per hectare, number of stems per ha, stand density index, and relative density, were examined, but none showed any stronger relationship than $QD_{50}$. Basal area per hectare does not necessarily reflect changes in tree
size or average competition, which might have resulted in the lack of a strong relationship with $r$. Also, the number of stems per hectare does not reflect the tree size. Stand density index and relative density showed stronger relationships with $r$ than basal area per ha and number of stems per ha, but weaker than $QD_{50}$.

4.1.1.7 Crown Length and Crown Ratio

For dominant and codominant trees, the relationship between $r$ and crown length was stronger at the tree base (0.1H) (Figure 4.8A) than for 0.5H and 0.8H (Figure 4.8C and 4.8E). As the tree crown increased in length, the base became more swollen, changing shape from a cone ($r=2$) to a neiloid ($r=3$) or even higher $r$ values. The relationship of $r$ with crown ratio varied greatly at the base of the tree (Figures 4.8B, 4.8D, and 4.8F). A strong relationship occurred at the middle and top of the tree, with an increase in crown ratio, corresponding to an increase in the form exponent. As the crown ratio increased, the shape of the upperbole of the tree changed from a paraboloid shape ($r=1$) to a conical shape ($r=2$). Crown length appeared to be a good indicator of butt swell, whereas crown ratio was a good indicator of stem change within the crown.

4.1.2 Variation in Form and Taper within Individual Trees over Time

The form and taper of individual trees varies with many factors, as shown above. In this subsection, the results of examining the variation in tree shape and taper for specific relative heights ($h/H$) over time are given. Also, 3-dimensional graphs of $r$ versus age and height above ground are presented.

4.1.2.1 Form and Taper Variation along the Stem

The variation in form and taper along the stem for a particular tree over time is demonstrated in Figures 4.9 and 4.10 (see also Figures D.23 to D.32 of Appendix D). Figure 4.9 for tree
Figure 4.8: Form exponent (r) by crown length (m) and crown ratio at 0.1H (A and B), 0.5H (C and D), and 0.8H (E and F) for the final felling data.
13.2 showed that at a young age (14 years), the tree had a similar shape (paracone) from the ground to the top. The shape began to change as the tree grew. At older ages (e.g., 54 years), the tree tended to have different shapes at different relative heights, although they tended to maintain the same shape in the upper part of the stem (above approximately 60 percent of the height). Similarly, tree 16.2 (Figure D.23), at 13 years, also appeared like a paracone in shape from base to top. As it grew, the lower parts began to change in shape, curving inwards (concave-shaped), and eventually at age 53 years, the tree had a different shape at the base. In contrast, the upper parts remained unchanged in shape or tended to curve outwards (parabolic or conical). This is characteristic of the trees growing in low density stands (trees 16.2 and 46.1; Figures D.24 and D.32).

From these figures, it can be seen that trees have roughly the same shape from ground to top at very young ages (parabolic or conic, $r$ of 1 or 2). As age increases, the shape at the base changes for larger dbh trees to a neiloid ($r$ of 3) or even larger $r$ values (Figure D.23 and Figure D.31). Smaller dbh trees tended to maintain almost the same shape ($r$ between 1 and 2) throughout their lifetime (Figures D.25 and D.29).

4.1.2.2 Form Variation with Tree Age and Height

Tree shape changes with age at a given height above ground are presented in Figures 4.11 and E.33 to E.37 (Appendix E). From these three-dimensional plots, it is shown that at young ages, the tree shape (form) was relatively constant over the height of tree (see also Figures 4.4D, 4.5D, 4.6D, and Appendix E), with an $r$ of less than 1.8. As the tree grew in height and increased in age, differences in form started to appear and differentiation between the base and upper stem parts began. However, for small dbh trees growing in dense stands (e.g., Figures E.34 and E.36), no major ridges and valleys occurred compared to large dbh trees growing in more open stands (Figures E.33 and E.37).
Figure 4.9: Form exponent ($r$) by relative height for tree 13.2, dbh=15.6 cm, height=16.8 m, age=54 years, and $QD_{50}=9$ cm, for different measurement periods.
Figure 4.10: Relative diameter by relative height for tree 13.2, dbh=15.6 cm height=16.8 m, age=54 years, and $Q' D_{50}=9$ cm, for different measurement periods.
Figure 4.11: Form exponent ($r$) by height (m) above ground and breast height age (years) for tree 13.2, dbh=15.6 cm, height=16.8 m, age=54 years, and $Q'D_{50}=9$ cm, for all measurement periods.
4.2 Dynamic Taper Model Building

This section presents the results of the dynamic taper modelling process and evaluation (objective two). The models for predicting height, dbh and relative height are presented in Subsections 4.2.1, 4.2.2 and 4.2.3, respectively. This is followed by presentation of the dynamic taper models in Subsection 4.2.4. Results of the evaluation of the dynamic taper models are presented in Section 4.3.

4.2.1 Height Prediction Model

The total tree height prediction models that were fit and tested included two linear models and one nonlinear model (Equations 3.47 to 3.49). For all three models, plotting the residuals against the predicted height, and against each of the independent variables, indicated no obvious violations of the assumptions of linear and nonlinear least squares. However, it is known that the PSP data used to fit these models are dependent, because the same tree was measured more than once (possible presence of serial correlation).

Because of this dependence, the models could not be evaluated based on the t and F statistics, since the usual estimated variances associated with the parameter estimates using OLS are biased (Kmenta 1971). Although the data were not independent, the prediction abilities of the models were not affected, because the coefficient estimates are unbiased and consistent. Therefore, the three models were evaluated only for their prediction abilities based on the fit and prediction statistics.

The linear models had almost the same fit statistics ($R^2$, SEE, and PRESS, Table 4.6), with the values for Equation 3.47 being slightly lower than for Equation 3.48. The prediction statistics are also better for Equation 3.47, except for the mean bias. Therefore, Equation 3.47 was deemed to be better than Equation 3.48 in terms of both fit and prediction.

Equation 3.47 also had slightly better fit statistics (higher $R^2$ and lower SEE) than Equation 3.49 (Table 4.6). However, for the prediction statistics, which are important for
Table 4.6: Fit and prediction statistics for the linear and nonlinear height (m) prediction models using PSP data.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Fit (n=1334 trees)</th>
<th>Prediction (n=574 trees)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R^2$</td>
<td>SEE</td>
</tr>
<tr>
<td>Linear</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.47</td>
<td>.9337</td>
<td>1.0236</td>
</tr>
<tr>
<td>3.48</td>
<td>.9331</td>
<td>1.0283</td>
</tr>
<tr>
<td>Nonlinear</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.49</td>
<td>.9324</td>
<td>1.0329</td>
</tr>
</tbody>
</table>

$^a R^2$ is the coefficient of multiple determination; SEE is the standard error of estimate in m; PRESS is the predicted sum of squares residuals (Draper and Smith 1981, p. 325–327); $F$ is the fit index; $S\hat{E}E$ is the estimated standard error of estimate in m; Bias is the mean bias in m; RMSE is root mean squared error in m; and MAD is the mean absolute bias in m.

Comparing models fitted using different techniques, the models had similar $F$ values of about 0.939, with the nonlinear model having a slightly lower $S\hat{E}E$ than the linear model. Since the two models had a similar $F$, the model with the lower $S\hat{E}E$ and RMSE would be more accurate. However, Equation 3.47 was slightly less biased (Bias = 0.05 m) compared to the nonlinear model (Bias = 0.07 m). Both models had almost the same MAD values. The plots of the residuals against the predicted height for the fit data showed no particular problems for either model. Thus, the linear and the nonlinear models could not easily be separated based on their fit and prediction statistics; instead, other measures or factors had to be taken into account.

Other factors considered when selecting the height prediction model included:

1. The number of parameters to estimate. The nonlinear model requires fewer variables to be measured (only two) and fewer parameters to be estimated (only three). Whereas, the linear model requires an additional variable (stand density) to be measured which
could prove to be costly. Also, the linear model has six parameters to be estimated.

2. The growth of trees is a nonlinear process and the growth in height of trees is a function of available nutrients and sunlight for a given site, species, and age. For a given species and site, this nonlinear process is a function of age. Therefore, a linear function would be a mere approximation of this growth process and such a function would be useful mostly for predictions within the range of the fit data. In terms of forecasting (i.e., making predictions beyond the fit data range), nonlinear models are expected to work well and are known to be more flexible than linear models (Payandeh 1983). Given the fact that the calibration data range is very small, choosing a more flexible model was considered to be beneficial.

Based on the above information and the prediction statistics in Table 5.6, the nonlinear model (the conditioned logistic function by Goudie (1984, referenced in Goudie et al. 1990)) was selected. The model selected was:

\[
\hat{H}_{kmo} = 1.3 + (SI_k - 1.3) \left[ \frac{1.0 + \exp[8.5368 - 0.9000 \ln(50) - 1.4900 \ln(SI_k - 1.3)]}{1.0 + \exp[8.5368 - 0.9000 \ln(AGE_{km}) - 1.4900 \ln(SI_k - 1.3)]} \right]
\]

(4.65)

It should be noted that this model was based on the 1334 dominant and codominant trees in PSP model development data set only.

The selected model was recalibrated using the final felling stem analysis data (135 trees). The resulting fit showed that some coefficients changed values, but the changes were small, and no coefficient changed signs. The final form of the model was:

\[
\hat{H}_{it} = 1.3 + (SI_k - 1.3) \left[ \frac{1.0 + \exp[8.2660 - 1.1414 \ln(50)] - 1.1743 \ln(SI_k - 1.3)]}{1.0 + \exp[8.2660 - 1.1414 \ln(Age_{it}) - 1.1743 \ln(SI_k - 1.3)]} \right]
\]

(4.66)
Table 4.7: Fit and prediction statistics for the dbh (cm) prediction models using PSP data.

<table>
<thead>
<tr>
<th>Equation Number</th>
<th>Fit (n=1334 trees)</th>
<th>Prediction (n=574 trees)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R^2$, SEE, PRESS</td>
<td>FI, $SEE$, Bias, RMSE, MAD</td>
</tr>
<tr>
<td>3.51</td>
<td>.9123, 1.8417, 4552.4135</td>
<td>.8783, 2.2279, 0.1207, 2.2162, 2.0403</td>
</tr>
<tr>
<td>3.52</td>
<td>.9114, 1.8525, 4616.8883</td>
<td>.8828, 2.1902, 0.1073, 2.1692, 2.0372</td>
</tr>
<tr>
<td>3.53</td>
<td>.9216, 1.7433, 4084.3257</td>
<td>.8844, 2.1771, 0.0927, 2.1600, 2.0350</td>
</tr>
</tbody>
</table>

$R^2$ is the coefficient of multiple determination; SEE is the standard error of estimate in cm; PRESS is the predicted sum of squares residuals (Draper and Smith 1981, p. 325–327); FI is the fit index; $SEE$ is the estimated standard error of estimate in cm; Bias is the mean bias in cm; RMSE is the root mean square bias or residuals (cm); and MAD is the mean absolute bias in cm.

4.2.2 Prediction Model for Diameter at Breast Height

Three models for predicting dbh (see Section 3.3.3) were fit using the PSP data set and tested using the validation PSP data set. Comparison of the prediction models was based on the fit and prediction statistics.

Equation 3.53 had the best fit and prediction statistics (highest $R^2$ and FI, lowest SEE, $SEE$, PRESS, Bias, and MAD, Table 4.7) compared to Equations 3.51 and 3.52. Therefore, it was selected for use in the dynamic taper model development. However, it is apparent from the fit and prediction statistics that none of the three models predicted dbh very well. The dbh prediction model selected was:

$$\hat{D}_{km0} = -4.6810 + 0.4210\hat{H}_{km0} - 0.0072\hat{H}_{km0}^2 + 0.0993SI_k + 0.0381SI_k^2 - 0.2333\hat{Q}D_{50k}$$

$$+ 0.0054\hat{Q}D_{50k}^2 + 0.0035(\hat{Q}D_{50k} \times AGE_{km}) + 0.1380AGE_{km} \quad (4.67)$$

As was the case for height prediction, the above model was recalibrated using the final felling stem analysis data. The resulting model with new coefficient estimates was:
\[ \hat{D}_{it} = -1.7567 + 1.2433\hat{H}_{it} - 0.0268\hat{H}^2_{it} + 0.4662SI_k - 0.0197SI^2_k - 0.3104QD_{50k} \]
\[ + 0.0137QD^2_{50k} + 0.0128(QD_{50k} \times Age_{it}) - 0.0760Age_{it} \]  

(4.68)

### 4.2.3 Relative Height Prediction Model

For prediction of relative height, the two prediction models (see Section 3.3.4) were fit using NLS for the model development stem analysis data set and tested on the validation data. The t or F tests were not used because the observations in the stem analysis data set were dependent. Therefore, comparison of relative height prediction models was based on the fit and prediction statistics.

Equation 3.56 was better than Equation 3.57 in terms of prediction statistics (lowest $S\hat{E}E$ and highest FI, Table 4.8) and in terms of bias (lowest Bias and MAD). The plot of the residuals against predicted relative height showed no obvious lack of fit. Therefore, Equation 3.56 was selected as the model for predicting relative heights. The fitted logistic function used for prediction of relative heights was:

\[ \hat{Z}_{ijt} = \frac{1}{1 + \exp(-(3.128023 + 0.338347h_{ijt} - 2.067061h^2_{ijt} + \frac{0.069242}{h_{ijt}} + \frac{0.332756}{h^2_{ijt}} +
\[ 0.179957\hat{H}_{it} - 2.528776\ln\hat{H}_{it} - 0.017853(h_{ijt} \times \hat{H}_{it}) + 0.000179(h_{ijt} \times \hat{H}_{it})^2
\]
\[ -0.006469(h^2_{ijt} \times \hat{H}_{it}))})] \]  

(4.69)

### 4.2.4 Dynamic Taper Function Selection

Using logarithmic transformations and an OLS fit, Model 3B (with 11 estimated coefficients) fitted the data better than Model 3A (with nine estimated coefficients) (Table 4.9). However, Model 3A had better prediction statistics (higher FI, lower $S\hat{E}E$, Bias, RMSE, and MAD) than Model 3B. Since the dynamic taper function will be used mainly for predictive purposes,
Table 4.8: The fit and prediction statistics for the relative height prediction logistic models.

<table>
<thead>
<tr>
<th>Equation</th>
<th>$R^2$</th>
<th>SEE</th>
<th>FI</th>
<th>$SE$</th>
<th>Bias</th>
<th>RMSE</th>
<th>MAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.56</td>
<td>0.9904</td>
<td>0.0295</td>
<td>0.9829</td>
<td>0.0398</td>
<td>0.0067</td>
<td>0.0397</td>
<td>0.0230</td>
</tr>
<tr>
<td>3.57</td>
<td>0.9880</td>
<td>0.0331</td>
<td>0.9795</td>
<td>0.0437</td>
<td>0.0078</td>
<td>0.0436</td>
<td>0.0261</td>
</tr>
</tbody>
</table>

*Fit statistics are based on 5916 sectional measures and the prediction statistics are based on 2668 sectional measures.*

*R$^2$ is the coefficient of multiple determination; SEE is the standard error of estimate; PRESS is the predicted sum of squares residuals (Draper and Smith 1981, p. 325–327); FI is the fit index; $SE$ is the estimated standard error of estimate in cm; Bias is the mean bias; RMSE is root mean squared error; and MAD is the mean absolute bias.*

Model 3A was selected as more appropriate than Model 3B and will hereafter be referred to as Model 3.

When Model 3 was fitted using NLS (Section 3.3.5.1) allowing the coefficient $q$ to vary, it was found that this coefficient was not necessary and it was eliminated. Therefore, Model 3 became:

$$
\hat{d}_{ijt} = a_0 \times \hat{D}^{a_1}_{it} \times \left[ 1 - \sqrt{Z_{ijt}} \right]^{b_1 + b_2 \sqrt{Z_{ijt}} + b_3 \frac{D_{it}}{h_{it}^{q/2}} + b_4 \frac{1}{h_{it}} + b_5 QD_{it}^2 + b_6 \frac{1}{h_{ijt}} + b_7 \frac{1}{h_{ijt}^{0.001}}} \quad (4.70)
$$

This model no longer has the same properties as Kozak's (1988) model (Equation 2.15), since it is not conditioned to pass through the Kozak's join point of $.25H_{it}$. However, the model is conditioned in such a way that when $h_{ijt} = H_{it}$, then $\hat{d}_{ijt} = 0$, and when $h_{ijt} = 0$, then $\hat{d}_{ijt} = a_0 D^{a_1}$, which is the predicted diameter inside bark at tree base.

4.2.5 Taper Models Fitted

The selected taper models (dynamic Models 2 and 3 and static Model 1a) were fitted using OLS and NLS as described in Section 3.3.5. Since the fitting techniques resulted in different
Table 4.9: Fit and prediction statistics for the two dynamic taper functions for diameter inside bark (d_{ijt}) (cm) using OLS.

<table>
<thead>
<tr>
<th>Model Type</th>
<th>Fit(^a)</th>
<th>Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(R^2)^b</td>
<td>SEE PRESS</td>
</tr>
<tr>
<td>3A</td>
<td>.9068(.9722)</td>
<td>.2554(.7807)</td>
</tr>
<tr>
<td>3B</td>
<td>.9152(.9657)</td>
<td>.2436(.8672)</td>
</tr>
</tbody>
</table>

\(^a\)The fit statistics were based on 5916 sectional measures while the prediction statistics were based on 2668 sectional measures. The fit statistics are based on ln\(d_{ijt}\) and values in brackets were estimated using the antilogarithm of ln\(d_{ijt}\).

\(^b\)\(R^2\) is the coefficient of multiple determination; SEE is the standard error of estimate in cm; PRESS is the predicted sum of squares residuals (Draper and Smith 1981, p. 325–327); FI is the fit index; SSEE is the estimated standard error of estimate in cm; Bias is the mean bias in cm; RMSE is root mean squared error in cm; and MAD is the mean absolute bias in cm.

coefficient estimates (Table 4.10), it likely that they have different predictive abilities for both diameter inside bark over the tree stem and total tree volume.

4.3 Taper Function Evaluation

Using the coefficient estimates in Table 4.10, the three taper models were evaluated for \(d_{ijt}\) and \(V_{it}\) predictions. Both linear and nonlinear fits were evaluated.

4.3.1 Model Evaluation for Predicting Diameter Along the Tree Stem

The OLS fitted models (Models 1aL, 2L, and 3L) were corrected for underprediction since a logarithmic transformation was used (Equation 3.64). A correction factor (CF) of 1.00647, 1.03283, and 1.03212 were used for Models 1aL, 2L, and 3L respectively.

From the fit and prediction statistics (Table 4.11), it can be seen that the static taper models (Models 1aL and 1aN) outperformed all the other taper models in all respects, as expected, since measured dbh and height were used. They had both the best fit and the best
Table 4.10: Coefficient estimates for the static and two dynamic taper models fitted using OLS and NLS.\(^a\)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1aL</td>
<td>1aN</td>
<td>2L</td>
</tr>
<tr>
<td>(a_0)</td>
<td>1.60636</td>
<td>1.38712</td>
<td>1.47846</td>
</tr>
<tr>
<td>(a_1)</td>
<td>0.66631</td>
<td>0.77700</td>
<td>0.64360</td>
</tr>
<tr>
<td>(a_2)</td>
<td>1.01998</td>
<td>1.00916</td>
<td>1.03060</td>
</tr>
<tr>
<td>(b_1)</td>
<td>0.35906</td>
<td>0.00999</td>
<td>-1.54544</td>
</tr>
<tr>
<td>(b_2)</td>
<td>-0.04343</td>
<td>0.07509</td>
<td>0.44487</td>
</tr>
<tr>
<td>(b_3)</td>
<td>0.22627</td>
<td>-0.97381</td>
<td>-4.69504</td>
</tr>
<tr>
<td>(b_4)</td>
<td>-0.02953</td>
<td>0.52304</td>
<td>2.62135</td>
</tr>
<tr>
<td>(b_5)</td>
<td>0.23267</td>
<td>0.30002</td>
<td>0.09211</td>
</tr>
<tr>
<td>(b_6)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(b_7)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

\(^a\)In the table, L represents linear models; N represents nonlinear models; and \(\text{"a"}\) represents models assessed using measured height and dbh. It should be noted that Models 1bL and 1bN have the same coefficient estimates as Models 1aL and 1aN respectively.
prediction statistics. When the same fitted equations (Models 1bL and 1bN) were assessed by inputting predicted height and dbh, they performed poorer than the dynamic taper models (Models 2 and 3), particularly in the nonlinear form (poorer prediction statistics). Comparing the dynamic taper functions based on the fit statistics, Model 3L was a better dynamic taper function for fitting the data for diameter inside bark than Model 2L (higher $R^2$, lower SEE and PRESS). Also, Model 3N was a better model than Model 2N (lower $R^2$ and SEE). However, based on the prediction statistics, Model 2L was a better function (all prediction statistics are lower) than model 3L. For the functions fitted using NLS, Model 2N was more biased (higher Bias and MAD), but more precise (lower $S\hat{E}E$ and RMSE) than Model 3N, even though the FI values were equal. The NLS fitted models were better than OLS fitted models (lower prediction statistics). However, based on these statistics, a clear choice between Models 2 and 3 cannot be made.

Since the statistics in Table 4.11 represent the overall fit and prediction of the models and fitting methods, they do not indicate which model is more or less biased along the stem. In order to effectively assess the taper functions and the two fitting methods (OLS and NLS), the mean biases in diameter prediction were assessed for a given percent height above ground using the validation stem analysis data (Table 4.12). Such analyses are very important for selecting a good taper function.

The static taper models (Models 1aL and 1aN) had lower biases than the other models (Table 4.12) with Model 1aN having the lowest biases along the stem. However, for the static model assessed using predicted height and dbh (Model 1b) was more biased when OLS- and NLS-fitted than Model 2. For section $0.9H_g$, Model 1b had especially high biases. For the dynamic taper functions, the NLS-fitted functions had lower biases for the stem sections near the ground, than the OLS-fitted models, but very high biases for upper stem sections, particularly for the last section ($0.9H_g$). Kozak's (1988) taper model altered using predicted dbh, height, and relative height (Model 2L), had very low biases compared to the dynamic taper model (Model 3L), except at the 90 percent of height. Model 2L had the lowest biases
### Table 4.11: Fit and prediction statistics for the three taper functions for diameter inside bark ($d_{ijt}$) (cm) using OLS and NLS.

<table>
<thead>
<tr>
<th>Model Number</th>
<th>Fit&lt;sup&gt;b&lt;/sup&gt;</th>
<th>Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R^2$</td>
<td>SEE</td>
</tr>
<tr>
<td>1aL</td>
<td>.9816</td>
<td>.1136</td>
</tr>
<tr>
<td>1aN</td>
<td>.9917</td>
<td>.4270</td>
</tr>
<tr>
<td>1bL</td>
<td>.9816</td>
<td>.1136</td>
</tr>
<tr>
<td>1bN</td>
<td>.9917</td>
<td>.4270</td>
</tr>
<tr>
<td>2L</td>
<td>.9049</td>
<td>.2580</td>
</tr>
<tr>
<td>2N</td>
<td>.9805</td>
<td>.6531</td>
</tr>
<tr>
<td>3L</td>
<td>.9097</td>
<td>.2514</td>
</tr>
<tr>
<td>3N</td>
<td>.9809</td>
<td>.6468</td>
</tr>
</tbody>
</table>

<sup>a</sup>In the table, L represents linear models; N represents nonlinear models; “a” represent the static models assessed using measured height and dbh; and “b” represent static models assessed using predicted height and dbh.

<sup>b</sup>The fit statistics were based on 5916 sectional measures and the prediction statistics were based on 2668 sectional measures. These fit statistics, $R^2$, SEE, and PRESS for the linear models (Models 1L, 2L, and 3L) are based on the natural logarithm transformed ln $d_{ijt}$ values.

<sup>c</sup>$R^2$ is the coefficient of multiple determination; SEE is the standard error of estimate in cm; PRESS is the predicted sum of squares residuals (Draper and Smith 1981, p. 325–327); FI is the fit index; $\hat{S}\bar{E}$ is the estimated standard error of estimate in cm; Bias is the mean bias in cm; RMSE is root mean squared error in cm; and MAD is the mean absolute bias in cm.
Table 4.12: Mean biases for all taper functions fitted with OLS and NLS for diameter inside bark (cm).\textsuperscript{a}

<table>
<thead>
<tr>
<th>Percent Height Above Ground</th>
<th>No. of Sections</th>
<th>Mean bias in sectional diameter inside bark (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Kozak (1988)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1aL</td>
</tr>
<tr>
<td>&lt;5.0%</td>
<td>364</td>
<td>-0.01</td>
</tr>
<tr>
<td>10.0%</td>
<td>268</td>
<td>0.09</td>
</tr>
<tr>
<td>20.0%</td>
<td>218</td>
<td>0.03</td>
</tr>
<tr>
<td>30.0%</td>
<td>201</td>
<td>0.01</td>
</tr>
<tr>
<td>40.0%</td>
<td>187</td>
<td>0.05</td>
</tr>
<tr>
<td>50.0%</td>
<td>183</td>
<td>0.13</td>
</tr>
<tr>
<td>60.0%</td>
<td>221</td>
<td>0.21</td>
</tr>
<tr>
<td>70.0%</td>
<td>243</td>
<td>0.17</td>
</tr>
<tr>
<td>80.0%</td>
<td>322</td>
<td>0.16</td>
</tr>
<tr>
<td>90.0%</td>
<td>428</td>
<td>0.05</td>
</tr>
<tr>
<td>100.0%</td>
<td>33</td>
<td>0.00</td>
</tr>
</tbody>
</table>

\textsuperscript{a}In the table, L represents linear models; N represents nonlinear models; "a" represents the static models assessed using measured height and dbh; and "b" represents static models assessed using predicted height and dbh.

followed by Models 3N and 2N; while Models 3L, 1bN, and 1bL had the highest biases, particularly for $0.1H_\text{ut}$. 

Further analyses were needed in order to judge the taper models and the fitting techniques for different tree sizes, because it is the large size trees that are important for a forest manager. The data were further divided into three dbh classes to check the models' prediction abilities for different dbh classes. The three dbh classes were: (1) 0.1 to 10.0 cm, (2) 10.1 to 20.0 cm, and (3) greater than 20.0 cm. Models 1b, 2, and 3 were more biased than Kozak's (1988) static model (Models 1a) (Figure 4.12). Generally, all models predicted $d_{ijt}$ for lower sections (sections less than 10 m above ground) with little bias (Bias of less than or equal
to 0.3 cm). For upper sections, all models were biased with a maximum of 1.5 cm at 19.3 m above ground. However, only 4 trees were available in the validation data set for this height, therefore this could be the reason why the Bias is high. This upper section biases occurred particularly for the OLS-fitted dynamic models (Models 2L and 3L). The NLS-fitted functions were less biased for all sections (maximum Bias of 0.7 cm for Model 1bN for 19.3 m above ground). It is impossible to determine the predictive abilities for $d_{ij}$ by the taper functions for different tree sizes from Figure 4.12, since trees of all sizes are included. Breaking the data into diameter classes was necessary to determine which function was a better predictor of $d_{ij}$ for large dbh classes which contain more volume.

For small trees (dbh class 1), Kozak's (1988) static taper functions (Model 1aL and 1aN) had low biases (Bias less than ±0.2 cm) compared to the dynamic taper functions which had higher biases (Bias less than ±0.4 cm) for sections less than 10 m above ground (Figure 4.13). Also, Models 1bL and 1bN had lower biases compared to Models 2 and 3, except for the last section (14.3 m above ground) for OLS-fitted models. For NLS-fitted models, all the models had similar biases (i.e., they were all less biased, Bias < 0.2), except for the last section. However, as tree size increased, the differences in biases increased.

For medium trees (dbh class 2), Kozak’s (1988) static taper function (Model 1aN) still had the lowest Bias (Figure 4.14). However, Model 1aL was not different from Model 2 (both were equally biased, but in opposite directions) for sections less than 10 m above ground. Model 1bL was had the highest Bias. Models 2L and 3L were the best linear models except for the section at 0.6 m above ground where they were poorer than model 1aL. For the NLS-fitted taper functions, the dynamic taper functions showed lower biases than Model 1aN up to 13.3 m above ground; for the upper sections (above 13.3 m), all models were equally biased. At the butt (0.3 m and 0.6 m above ground), Model 1bN was more biased than Models 2N. This indicates that the use of predicted height and dbh without refitting the model results in higher biases. Both Models 2N and 3N had lower biases than the OLS-fitted functions (Bias of ±0.5 cm for sections less than 10 m above ground and Bias < 1.0 cm
Figure 4.12: Mean biases for diameter inside bark (cm) prediction at various heights above ground for trees of all sizes (275 trees). The section lengths between 0.3 m and 1.3 m are 0.6 m above ground and the other unmarked section lengths are a metre apart.
Figure 4.13: Mean biases for diameter inside bark (cm) prediction at various heights above ground for trees of dbh class 1 (dbh \leq 10.0 \text{ cm} \text{ for 180 trees}). The section lengths between 0.3 m and 1.3 m are 0.6 m above ground and the other unmarked section lengths are a metre apart.
for sections more than 10 m above ground). However, for sections more than 10 m above ground Models 1aN, 2N, and 3N seemed to be more biased than the OLS-fitted functions except for Model 1bL which had equally high biases.

All the taper models were most biased when predicting $d_{ijt}$ for large dbh trees (dbh class 3, Figure 4.15). Of the OLS-fitted models, Model 1aL was more biased for sections less than 9 m above ground than the other models except Model 2L, but it was less biased for sections more than 9 m above ground. Model 1bL had lower biases than all the other models for sections less than 9 m above ground, but it was the worst for sections more than 9 m above ground. Model 3L performed better than Model 2L for both lower and upper sections, but not for the middle sections. Apart from a few middle sections, Model 1bL was more biased than Model 3L. Overall, Model 2L performed the poorest.

For the NLS-fitted models, Model 2N had slightly lower biases than Model 1aN for sections below 12 m above ground; however, for the upper sections (sections more than 12 m above ground) Model 1aN had the lowest biases of all the NLS models. Model 1bN performed consistently poorer than Model 2N for all sections along the stem, confirming the idea that the use of predicted height and dbh without refitting the model will result in more biased models, while Model 3N was consistently less biased than Model 2N for all sections above ground. Model 1bN gave had the poorest predictions.

Comparing the OLS-fitted and the NLS-fitted taper models for sections less than 10 m above ground, the NLS-fitted models were less biased (Bias < 0.5 cm compared to Bias > -2.5 cm). For upper sections (sections above 10 m), Models 1aL and 1aN had the lowest biases. Kozak’s (1988) taper models refitted using OLS, whether static or dynamic (Models 1aL, 1bL, and 2L), were characteristically biased for lower sections, particularly the first three sections (Figures 4.14 and 4.15). These models overpredicted $d_{ijt}$ for lower sections and underpredicted it for upper sections. For overall bias, however, these overpredictions and underpredictions for $d_{ijt}$ cancel each other out, and Model 2L appears less biased than Model 2N (Table 4.12).
Figure 4.14: Mean biases for diameter inside bark (cm) prediction at various heights above ground for trees of dbh class 2 (10.0 < dbh ≤ 20.0 cm for 91 trees). The section lengths between 0.3 m and 1.3 m are 0.6 m above ground and the other unmarked lengths are a metre apart.
Figure 4.5: Mean biases for diameter inside bark (cm) prediction at various heights above ground for trees of dbh class 3 (dbh > 20.0 cm for 4 trees). The section lengths between 0.3 m and 1.3 m are 0.6 m above ground and the other unmarked section lengths are a metre apart.
Overall, large dbh trees are the most important because they contain more volume. Also, lower sections contain a large proportion of the total tree volume. Therefore, a taper model that has good predictions for lower sections for large dbh trees would be the best. Based on these results, Model 3N qualifies as the best model.

In order to select the best dynamic taper function, selection criteria cannot be based on the prediction of $d_{ij}$ alone. Trees are important for their volume, and a taper function that is less biased for volume prediction (particularly for the lower sections which contain the majority of the total tree volume) is more desirable. Therefore, the dynamic taper functions were further assessed for predicting total tree volume.

### 4.3.2 Evaluation of the Taper Functions for Total Tree Volume Prediction

Kozak’s (1988) refitted static taper models (Models 1aL and 1bL) had the best overall prediction statistics for volume (Table 4.13). The OLS-fitted functions had higher FI values than their counterpart NLS-fitted functions, except for Model 2L. Kozak’s dynamic taper function fitted with OLS (Model 2L) was the most biased for all dbh classes, except for trees with dbh less than 8.0 cm where it was equal to all other models. For small dbh classes (dbh ≤ 8.0 cm), all the models had very low biases, and were not distinguishable. As dbh increased, the OLS-fitted models became more biased than the NLS-fitted models, except for the last dbh class, where all models had high biases. Model 1a is better than Models 2 and 1b, particularly for OLS fitting. Model 1b had lower biases than Model 2, except for large dbh classes (18.1–22.0 cm). Model 3 is better than Model 2, particularly for the OLS fitting. Model 1bN had the poorest predictions for the largest dbh class.

All models had larger biases for the four largest trees (20.1–22.0 cm). All the OLS-fitted models had particularly high biases for both the lower and upper sections, with Model 2L having the highest. Since large dbh trees contain more volume than small trees, a taper function that has lower biases for such trees is desirable. It should be pointed out that mean biases for all trees can be influenced by large biases for a few individual trees.
Table 4.13: Mean biases for volume ($m^3$) prediction using all taper functions for different diameter classes.\(^a\)

<table>
<thead>
<tr>
<th>Dbh Class</th>
<th>No. of Trees</th>
<th>Mean biases in volume (cubic metres)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1-2.0</td>
<td>8</td>
<td>0.0002</td>
<td>0.0003</td>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0003</td>
<td>0.0003</td>
<td>0.0003</td>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0002</td>
</tr>
<tr>
<td>2.1-4.0</td>
<td>38</td>
<td>0.0001</td>
<td>0.0003</td>
<td>0.0000</td>
<td>0.0002</td>
<td>0.0003</td>
<td>0.0003</td>
<td>0.0002</td>
<td>0.0005</td>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0002</td>
</tr>
<tr>
<td>4.1-6.0</td>
<td>43</td>
<td>0.0001</td>
<td>0.0001</td>
<td>-0.0005</td>
<td>-0.0004</td>
<td>0.0002</td>
<td>-0.0002</td>
<td>0.0007</td>
<td>-0.0002</td>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0002</td>
</tr>
<tr>
<td>6.1-8.0</td>
<td>39</td>
<td>-0.0001</td>
<td>-0.0003</td>
<td>-0.0008</td>
<td>-0.0009</td>
<td>0.0005</td>
<td>-0.0005</td>
<td>0.0011</td>
<td>-0.0002</td>
<td>0.0011</td>
<td>-0.0002</td>
<td>0.0011</td>
</tr>
<tr>
<td>8.1-10.0</td>
<td>52</td>
<td>0.0009</td>
<td>0.0002</td>
<td>0.0002</td>
<td>-0.0005</td>
<td>0.0017</td>
<td>0.0002</td>
<td>0.0022</td>
<td>0.0005</td>
<td>0.0022</td>
<td>0.0005</td>
<td>0.0022</td>
</tr>
<tr>
<td>10.1-12.0</td>
<td>29</td>
<td>0.0015</td>
<td>0.0001</td>
<td>0.0012</td>
<td>-0.0000</td>
<td>0.0027</td>
<td>0.0009</td>
<td>0.0021</td>
<td>0.0008</td>
<td>0.0021</td>
<td>0.0008</td>
<td>0.0021</td>
</tr>
<tr>
<td>12.1-14.0</td>
<td>24</td>
<td>0.0017</td>
<td>0.0004</td>
<td>0.0025</td>
<td>0.0014</td>
<td>0.0021</td>
<td>0.0020</td>
<td>0.0025</td>
<td>0.0016</td>
<td>0.0025</td>
<td>0.0016</td>
<td>0.0025</td>
</tr>
<tr>
<td>14.1-16.0</td>
<td>21</td>
<td>0.0032</td>
<td>0.0025</td>
<td>0.0007</td>
<td>0.0003</td>
<td>-0.0039</td>
<td>0.0004</td>
<td>-0.0028</td>
<td>-0.0009</td>
<td>-0.0028</td>
<td>-0.0009</td>
<td>-0.0028</td>
</tr>
<tr>
<td>16.1-18.0</td>
<td>11</td>
<td>0.0020</td>
<td>0.0034</td>
<td>-0.0001</td>
<td>0.0017</td>
<td>-0.0126</td>
<td>0.0007</td>
<td>-0.0038</td>
<td>0.0005</td>
<td>-0.0038</td>
<td>0.0005</td>
<td>-0.0038</td>
</tr>
<tr>
<td>18.1-20.0</td>
<td>6</td>
<td>0.0075</td>
<td>0.0143</td>
<td>0.0054</td>
<td>0.0130</td>
<td>-0.0238</td>
<td>0.0094</td>
<td>0.0010</td>
<td>0.0113</td>
<td>0.0010</td>
<td>0.0113</td>
<td>0.0010</td>
</tr>
<tr>
<td>20.1-22.0</td>
<td>4</td>
<td>-0.0009</td>
<td>0.0137</td>
<td>0.0106</td>
<td>0.0256</td>
<td>-0.0390</td>
<td>0.0196</td>
<td>0.0080</td>
<td>0.0245</td>
<td>0.0080</td>
<td>0.0245</td>
<td>0.0080</td>
</tr>
<tr>
<td>22.1+</td>
<td>0</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Total</td>
<td>275</td>
<td>0.0010</td>
<td>0.0010</td>
<td>0.0005</td>
<td>0.0006</td>
<td>-0.0010</td>
<td>0.0008</td>
<td>0.0010</td>
<td>0.0009</td>
<td>0.0010</td>
<td>0.0009</td>
<td>0.0010</td>
</tr>
<tr>
<td>FI</td>
<td></td>
<td>0.9971</td>
<td>0.9953</td>
<td>0.9963</td>
<td>0.9926</td>
<td>0.9856</td>
<td>0.9947</td>
<td>0.9950</td>
<td>0.9939</td>
<td>0.9950</td>
<td>0.9939</td>
<td>0.9950</td>
</tr>
</tbody>
</table>

\(^a\)In the table, L represents linear models; N represents nonlinear models; "a" represents the static models assessed using measured height and dbh; "b" represents static models assessed using predicted height and dbh; FI is the fit index; and No. of trees represent tree-measures.
All models had poor predictions for older trees (51–57 years old) (Table 4.14). Model 1aL had the smallest Bias while Model 1bN had the highest. Model 1bL had lower biases than Model 2L, but Model 1bN had higher biases than Model 2N. Model 3 had lower biases than model 2, particularly Model 3L which was the best dynamic model for volume prediction. The biases associated with the last three age classes were exceptionally high. Therefore, further assessment was necessary for the taper models.

When predicted volumes for large dbh trees were plotted against age (Appendix F), the differences in volume predictions by the static and dynamic taper models for the two fitting methods became more visible. These figures show that the static model was superior to the dynamic models and the NLS-fitted models were better than the OLS-fitted models. For the small tree (tree 25.1, Figure F.40), all taper functions (static and dynamic) had similar predictions, except Model 1b which consistently underestimated tree volume for all ages. The static taper models (Models 1aL and 1aN) had more accurate volume predictions for the big trees (Figures F.38, F.39, and F.41 for trees 14.1, 17.1 and 41.1, respectively) as expected. Of the OLS-fitted models, Model 2 had the poorest volume predictions; it consistently overestimated tree volume. Model 1b consistently underestimated tree volume as tree aged. Models 1aL, 1bL, and 2L showed poorer volume predictions for tree 41.2 than Model 3L. For the NLS-fitted taper functions, all models slightly underestimated tree volume, with Model 1bN having the highest underestimates. Model 2L had the highest biases, overestimating volume for the larger trees. However, at young ages (trees 14.1 and 17.1, Figures F.38 and F.39, respectively), all models had similar predictions for tree volume. From these results, it is apparent that Models 2L performed the poorest, followed by Models 1bL and 1bN. The best model was Model 1a followed by Model 3 fitted by either OLS or NLS.
Table 4.14: Mean biases in volume ($m^3$) prediction for all taper functions by 3-year age classes.\(^a\)

<table>
<thead>
<tr>
<th>Age Class</th>
<th>No. of Trees</th>
<th>Mean biases in volume (cubic metres)</th>
<th>Kozak (1988)</th>
<th>New Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Static</td>
<td>Dynamic</td>
<td>Dynamic</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1aL</td>
<td>1aN</td>
<td>1bL</td>
</tr>
<tr>
<td>1-3</td>
<td>18</td>
<td>0.0003</td>
<td>0.0004</td>
<td>0.0005</td>
</tr>
<tr>
<td>4-6</td>
<td>16</td>
<td>0.0002</td>
<td>0.0004</td>
<td>0.0001</td>
</tr>
<tr>
<td>7-9</td>
<td>28</td>
<td>0.0005</td>
<td>0.0006</td>
<td>0.0005</td>
</tr>
<tr>
<td>10-12</td>
<td>22</td>
<td>0.0002</td>
<td>0.0002</td>
<td>-0.0001</td>
</tr>
<tr>
<td>13-15</td>
<td>23</td>
<td>-0.0002</td>
<td>-0.0005</td>
<td>0.0004</td>
</tr>
<tr>
<td>16-18</td>
<td>28</td>
<td>-0.0004</td>
<td>-0.0005</td>
<td>-0.0003</td>
</tr>
<tr>
<td>19-21</td>
<td>15</td>
<td>-0.0000</td>
<td>-0.0002</td>
<td>-0.0005</td>
</tr>
<tr>
<td>22-24</td>
<td>22</td>
<td>0.0004</td>
<td>-0.0001</td>
<td>-0.0003</td>
</tr>
<tr>
<td>25-27</td>
<td>16</td>
<td>0.0012</td>
<td>0.0006</td>
<td>0.0003</td>
</tr>
<tr>
<td>28-30</td>
<td>12</td>
<td>0.0003</td>
<td>-0.0005</td>
<td>-0.0008</td>
</tr>
<tr>
<td>31-33</td>
<td>18</td>
<td>0.0023</td>
<td>0.0014</td>
<td>0.0012</td>
</tr>
<tr>
<td>34-36</td>
<td>7</td>
<td>0.0067</td>
<td>0.0069</td>
<td>0.0034</td>
</tr>
<tr>
<td>37-39</td>
<td>14</td>
<td>0.0018</td>
<td>0.0006</td>
<td>0.0001</td>
</tr>
<tr>
<td>40-42</td>
<td>9</td>
<td>0.0058</td>
<td>0.0063</td>
<td>0.0037</td>
</tr>
<tr>
<td>43-45</td>
<td>7</td>
<td>-0.0002</td>
<td>-0.0008</td>
<td>-0.0003</td>
</tr>
<tr>
<td>46-48</td>
<td>12</td>
<td>0.0029</td>
<td>0.0039</td>
<td>0.0007</td>
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<tr>
<td>49-51</td>
<td>4</td>
<td>0.0025</td>
<td>0.0117</td>
<td>0.0087</td>
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<tr>
<td>52-54</td>
<td>3</td>
<td>0.0031</td>
<td>0.0027</td>
<td>-0.0034</td>
</tr>
<tr>
<td>55-57</td>
<td>1</td>
<td>0.0051</td>
<td>0.0221</td>
<td>0.0269</td>
</tr>
<tr>
<td>58+</td>
<td>0</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

\(^a\)In the table, L represents linear models; N represents nonlinear models; "a" represent the static models assessed using measured height and dbh; "b" represents static models assessed using predicted height and dbh; and No. of trees represent tree-measures.
Chapter 5

Discussion

This chapter is divided into two sections. First, the variation of tree form and taper for dominant and codominant lodgepole pine trees is discussed. Individual tree taper and form variation with age at different heights along the stem is also discussed. Second, the dynamic taper functions that were fit are discussed along with their predictive abilities for diameter inside bark ($d_{ib}$) and total tree volume.

5.1 Variation in Tree Form and Taper

Understanding the variation in tree shape and taper is very important. Since trees with the same dbh and height can have a different form and taper, they can have different volumes. For example, a parabolic shaped tree will have more volume than a cone shaped tree with the same dbh and height. Also, implications for management options exist (i.e., pruning or thinning to get different tree shapes).

Tree form and taper were found to increase with increases in height, dbh, D/H ratio, CL and age, particularly for lower sections, and with an increase in CR for middle and upper sections. SI showed no clear relationship with tree form and taper changes. For $QD_{50}$, the relationship was strong for the tree base, but relatively poor for the middle and upper sections.

Change in tree shape as tree height increases likely is only associated with changes in other factors that are related to height, as height on its own should not affect changes in tree shape. However, large diameter trees tend to be tall in most cases, because trees put on increment in height and diameter together (Sections 2.1.1 and 2.1.2). This is probably the
major factor affecting tree shape changes. As trees grow, they increase in diameter along the stem, including dbh. Trees change shape because diameter increment is not uniform along the whole tree stem, a factor confirmed by Clyde (1986) and Larson (1965). This was shown in this research by increases in the shape coefficient ($r$) for the base and crown sections of the tree.

The variation in tree shape with height and dbh was indicated by a high linear correlation between $r$ and the D/H ratio for all trees. Tree shape increased with an increase in the D/H ratio. An increase in the D/H ratio, which was used as a measure of taper by Kozak (1988) and Marshall (1991), seems to be an indicator of tree shape changes, particularly for the butt sections of the trees. An increase in the D/H ratio indicates that the tree will be putting on more radial growth than terminal growth in a relative sense. As a result, the tree will change shape. Therefore, the use of the D/H ratio as a variable to indicate changes in tree shape is justifiable. Kozak (1988) and Newnham (1988, 1992) included D/H ratio in their models to account for butt swell.

For young trees, no real shape differences over the stem were apparent, irrespective of where the trees were growing. This seems to suggest that at young ages, factors that affect tree shape and taper changes have not yet begun to act, and that the small shape differences that may occur are only indicative of genetic variability, agreeing with Larson (1963) and Newnham (1965). However, as trees grow, different sections start changing into different shapes. At the base, tree shape changes from a parabolic shape towards a neiloid; at the top, tree shape seems to change more slowly from parabolic to conic. Young trees had a form exponent ranging from 0.8 to 1.5, which agrees with Forslund's idea of a paracone for trees at young age. Also, a look at variation in form for individual trees with age at different heights showed that tree form coefficient tended to increase over time. Different parts of the trees increased in $r$ at different rates, which gave trees different shapes for different sections. For old trees, shape changes were particularly evident at the base and top (within the crown). This was witnessed by the presence of humps for lower sections of the trees at older ages on
the 3-D graphs, compared to valleys for upper sections; no humps and valleys were present for the same trees at young ages.

One possible reason why trees have uniform shapes at young ages could be that trees are putting their growth efforts into height to achieve good crown positions for sunlight rather than on radial increment. Also, all trees have similar crown size at this age, and they have not yet differentiated into crown classes. As age increases, growth occurs unequally along the stem, resulting in different shapes at various points along the stem. Radial growth is undoubtedly correlated with age, but it is not caused by age; rather, growth is caused by the accumulation of photosynthetic products. This reasoning agrees with the existing theory about tree shape (Figure 2.1), and the literature review by Larson (1963, 1965) and Gray (1956).

Site index was found to have no clear relationship with the shape or taper of trees. Tree height growth is related to site productivity. Therefore, site index was expected to affect change in shape of trees, as pointed out by Larson (1963). Trees growing on good sites are supposed to add on more height growth than trees in growing on poor sites. Also, trees growing on good sites will put on more diameter growth along the stem than trees growing on poor sites. As a result, they will have different shapes for different sections of the tree. The lack of a clear relationship between $r$ and site index for this study might have been caused by confounding due to differences in density and tree age, as well as changes in the crown length. No relationship of tree shape and taper with site index also was found by Newnham (1965) and Burkhart and Walton (1985).

The relationship of $r$ with $\hat{QD}_{50}$ was not very strong; however, there was a better linear relationship at 0.1H than at 0.5H and 0.8H. This means that form and taper of trees, particularly for butt sections, increases with $\hat{QD}_{50}$. High $\hat{QD}_{50}$ values are associated with lower stand densities for a given stand age. These results agree with the findings of Larson (1963), Gray (1956), Newnham (1988) and Valinger (1992) that tree form and taper decreases with increasing stand density. As noted by Smithers (1961), trees that grow in less dense stands
(e.g., tree 46.1) were almost conical, and trees in very dense stands (e.g., tree 25.2) were.
whiplike and hardly had any difference in shape and taper from the ground to the top. Trees
put on different amounts of diameter increment along the stem depending on where they
are growing (Clyde 1986). If a tree is growing in a dense stand, it will put on less diameter
growth compared to a tree growing in less dense stand, because the amount of shared re-
sources (water, mineral nutrients and sunlight) will be more limiting for a tree growing in a
dense stand. Trees growing in dense stands will have less butt swell also, because they will
be sheltering each other from wind and providing support to each other. As a result, trees
growing in a dense stand will be less tapered than trees growing in less dense stands.

Crown length, like dbh, was found to be highly related to tree shape, but mainly for
the lower tree sections. Among individual trees, trees with longer crowns (e.g., tree 46.1
with a CR of 0.81) had bigger shape changes from the top to the base than trees with short
crowns (e.g., tree 25.2 with a CR of 0.46). When tree crowns increased in length, the tree
base became more swollen and changed in shape from a cone to a neiloid and even more
convex shapes. These results agree with Larson (1963, 1965) and the literature review in
Section 2.2 of this thesis, in that crown length is very important in determining tree shape
and might be a strong contributor to tree butt swell. Gray (1956) showed that trees become
more cylindrical in form with an increase in stand density and a decrease in crown length.
For this study, an increase in stand density and a decrease in crown length resulted in similar
results to Gray’s, but trees tended to have a parabolic form. Trees growing in less dense
stands had large crowns (e.g., trees 16.2 and 46.1) compared to trees growing in very dense
stands (e.g., tree 25.2).

The relationship between tree shape ($r$) and tree crown ratio varied greatly at the base
of the tree. However, a strong relationship existed between $r$ and the middle (0.5H) and top
(0.8H) sections of the trees. An increase in crown ratio corresponded to an increase in the
form exponent (change in shape). As crown ratio increased, the shape of the upper bole of
the tree changed from a parabolic to a conical shape.
Thus, crown length seems to be a good indicator of changes at the base of the tree (butt swell), whereas crown ratio is a good indicator of stem change within the crown. Therefore, a model with D/H and crown ratio should be able to capture the variations in taper and form in both the lower and upper ends of the tree stem.

From the results of this study and the literature review, it appears that the main factors controlling tree form and taper are stand density and tree crown size. Stand density controls the amount of canopy a given tree will possess and, as a result, it controls the form and taper of the trees. It was observed that older trees with small crowns growing in dense stands (e.g., tree 25.2: age of 48 years, $Q\overline{D}_{50}$ of 6 cm or 8175 trees per ha and a crown ratio of 0.48) do not necessarily have differentiated shapes for different sections along the stem. However, young trees with large crowns and growing in less dense stands (e.g., tree 46.1: age of 20 years, $Q\overline{D}_{50}$ of 22 cm or 425 trees per ha and a crown ratio of 0.81), have drastic changes in shape along the stem. Age does not appear to be the controlling factor for tree shape changes, since no obvious trend of $r$ with age was found for all trees. However, age should be an important variable in a prediction model for diameter inside bark along the tree stem because other factors, that do affect tree form and taper changes (e.g., crown size and stand density), change over time.

Stand treatments such as thinning, pruning and fertilization, will affect tree shape and taper by altering both tree crown size and stand density. For example, thinning will reduce stand density and, in the process, will allow individual trees more space to expand their crowns. If stands are heavily thinned, trees will grow like open-grown trees with long crowns, which means that they will have high taper and more conical shapes. However, when trees are pruned, crown sizes are reduced for a given height, which should be similar to increasing the stand density. As a result, pruning will decrease taper and makes trees appear more parabolic in shape. Therefore, thinning trees heavily would not be appropriate if the aim is to produce less tapered trees with high volume recovery after sawing, unless heavy thining is accompanied with heavy pruning to reduce the crown size. Fertilizing may increase tree
growth vigour; the tree puts on more branches and increases crown size, resulting in increased diameter growth along the stem. However, a fertilized tree will also increase in height growth. With increased height and diameter growth, the resulting tree may not show a large change in taper because the D/H and CR ratios might not have changed. This could be the reason Gordon and Graham (1986), Tepper et al. (1968), and Thomson and Barclay (1984) found taper to be only slightly affected by heavy fertilization.

Because the crowns of dominant trees extend above the general level of the canopy and receive light from above and the sides, they are better able to put on more diameter growth and increase in taper, change shape, and have butt swell. Crowns of codominant trees form the general level of the stand canopy and receive sunlight from above only; therefore, they put on less diameter growth than dominant trees, resulting in less tapered and more uniformly shaped (paracone shaped) trees with little butt swell. These dominant and codominant trees will have higher CR and D/H ratios than intermediate and suppressed trees. Crowns of intermediate trees are within the general canopy, competing with crowns of other trees, making the crowns smaller and receiving minimum sunlight from above. Diameter growth along the stem is much lower than for dominant and codominant trees. This means that intermediate trees would be expected to have essentially uniform shapes from ground to tree top, minimum taper, and little or no butt swell. Suppressed trees, which are completely deprived of direct sunlight, would have very small crowns and diameter growth along the stem would be minimum because they would be putting all their energy into height growth to gain access to direct sunlight. This would result in trees with virtually no taper (CR and D/H ratios very small) and the shape would be uniformly cylindrical from the ground to the top (MacDonald and Forslund 1986; Clyde 1986; Gray 1956).

The form and taper of trees also varies with species (Clyde 1986; Koch 1987), depending on whether the species is shade tolerant or intolerant. For lodgepole pine, a shade intolerent species, diameter growth would quickly reach a maximum and then decline rapidly, compared to shade tolerant species such as spruce (i.e, it would have a low D/H ratio) (Clyde 1986 and
Assmann 1970). This could be due to the species' inability to maintain high levels of growth with increased competition. As a result, shade intolerant species would have relatively simple taper and would have less differentiated shapes (Clyde 1986; Koch 1987). In contrast, shade tolerant species would maintain diameter growth much longer, even when shaded by adjacent tree crowns. As a result, these species would have more taper (high D/H ratios) than shade intolerant species and would be more differentiated into different shapes from ground to top than shade intolerant species.

In summary, it was found that tree shape and taper change along the stem at one time, and over time, change with changes in tree and stand variables. The tree and stand variables which were important included D/H ratio, CL, CR, and $QD_{50}$. AGE was found to be important also, but its importance could be due to other factors which change over time. It also was found that trees have a simple shape (i.e., they are all parabolic in shape from ground to top) at young ages. However, as they increase in size, different portions of the stem change into different shapes, because of unequal growth in diameter along the stem. Thus, to model taper over time, a process that will either capture or mimic the changes in stand density, crown size and D/H ratios as the trees grow should be involved.

5.2 The Dynamic Taper Functions

Modelling taper over time should provide a taper function that is able to predict accurate $d_{ijt}$ and total tree volume over time. Such a taper function could be incorporated into a tree growth model that will be relatively simple to understand, easy to calibrate, have wide application, and be less cumbersome to handle. The dynamic taper modelling process used in this study involved predicting height and dbh over time, predicting relative height, finding the correct model form for the form exponent, and, finally, putting these models together into a dynamic model. This model was fit using two common fitting techniques, OLS and NLS.

The new dynamic taper model that was developed (Model 3) took into account the fact
that tree shapes and taper change continuously along the entire length of the tree at one time, and over time at the same position, by incorporating models which predict dbh, height, and form changes over time. These changes in tree shape and taper along the stem were expressed as functions of tree and stand variables which change over time. As a result, the dynamic taper function was able to predict, with reasonable accuracy, the change in diameter inside bark along the stem bole and in total tree volume over time.

Kozak's (1988) static taper function (Model 1a), had better \( d_{ijt} \) and volume prediction than the same function converted into a dynamic model (Model 2). This was expected, because Model 1a is based on measured dbh and height. This means that very good prediction models for height and diameter would improve the predictive abilities of any fitted dynamic taper function. The most probable reason why Model 2 was slightly more biased than Model 1a for large dbh classes is that the dbh prediction model was poor. Model 2 had better predictions (lower biases) for \( d_{ijt} \) and \( V_{it} \) than Model 1b as expected. Model 1b represents the process used in growth and yield modelling. This means that the process currently used in growth and yield modelling of using an existing fitted taper function and inputting predicted height and dbh for projecting stand growth and yield and future inventory can result in biased projections. The new technique of fitting the system as a dynamic process improved the predictions for \( d_{ijt} \) and \( V_{it} \) and would provide better estimates for future growth and yield and inventory projections.

Model 3 was marginally superior to Model 2 in predicting \( d_{ijt} \) and obviously superior in predicting volume. The poorer performance of Model 2 could be attributed to the two variables, \( \ln(Z_{ijt} + 0.001) \) and \( \exp(Z_{ijt}) \) in Model 2 that were excluded from Model 3. The variable \( \ln(Z_{ijt} + 0.001) \) was dropped, because \( \ln(Z_{ijt}) \) is undefined at the tree base; Kozak added an arbitrary constant of 0.001 to obtain a value at the tree base. The addition of this constant does not affect the predictive abilities of taper function, but it does shift the whole function. The variable \( \exp(Z_{ijt}) \) was only marginally correlated with \( r_{ijt} \) and might inflate small values. Also, Model 3 had additional variables, AGE and \( QD_{50} \), added to the
The fact that differences between Model 2 and 3 were small was surprising because tree form and taper were found to be correlated with $QD_{50}$, particularly for lower tree sections. However, it could be attributed to the limited taper variability of the pine data used, or to the fact that the inclusion of the $\bar{D}/\bar{H}$ in the Model 2 might have already accounted for most of the variation in taper. Also, since height was a function of age and site index and dbh was a function of height, age and density, these variables (site index, age, and $QD_{50}$) were already accounted for in the taper model to some extent. One possible conclusion is that the tree form exponent (i.e., the shapes of trees) might not be changing as fast as the trees grow. This would mean that shape change is very marginal after trees reach a certain age. Another possibility is that the form exponent, which is a function of crown size and distance from crown, might already be well modelled and trying to add any more variables will not make any further improvements. It is possible that additional improvements could be made by including tree and stand variables when predicting the base (the model for base diameter) of Model 3.

The NLS-fitted dynamic taper models were less biased than the associated OLS-fitted taper models. The differences in predictions for the different fitting methods were most apparent for the static Kozak (1988) taper model (Models 1a) and the dynamic version of this model (Model 2). The associated OLS-fitted taper models were poorer in predicting both $d_{ijt}$ and $V_{it}$ than the same models fitted using NLS.

The dynamic taper models (Models 2 and 3) had predictions for $d_{ijt}$ and volume which were surprisingly similar to the predictions of Model 1a. This good performance of the dynamic taper models, particularly Model 3, could be attributed to two reasons:

1. The data set used (the stem analysis data) was composed of only dominant and codominant trees. These data had less variation than if suppressed and intermediate trees were included. Also, the age range of the data was narrow (1-61 years). Therefore,
the form and taper variation among trees might have been small, resulting in a more accurate dynamic taper function.

2. Lodgepole pine is known to have relatively simple taper (Koch 1987; Clyde 1986). As a result, a very complicated model was not required for this species, compared to species that have large taper such as Douglas-fir, western red cedar, and spruces.

Even though Model 3 showed surprising good predictions for both $d_{ij}$ and $V_{it}$, there are areas that could be improved. Such areas include:

1. The height and dbh prediction models were selected using a large data set (PSP data), but fit using a very small data set (final felling stem analysis data). Therefore, these models might have coefficients which are unreliable for use beyond the range of the data used.

2. According to the literature on tree growth, dbh growth depends on density and crown size, and the form and taper of trees is just a reflection of the crown status of trees. Therefore, crown length or crown ratio could have been important variables in the dynamic taper model; these variables were not available for time periods previous to that of the time of felling. However, the D/H ratio is known to be highly correlated with crown length (Burkhart and Walton 1985, Newnham 1988). Since the D/H ratio was included in the model for predicting taper over time, it can be assumed that it did account somewhat for lack of a crown size measure. Also, the D/H ratio is known to be a good measure of taper (Kozak 1988, Marshall 1991).

3. The dynamic taper models developed were based on only dominant and codominant trees, which are less affected by stand density than intermediate and suppressed trees. These models should not be used for data that include intermediate and suppressed trees without re-calibration, as they may produce biased results. However, the use
of $Q_{D50}$, and the D/H ratio might somewhat account for the variation due to crown position.

4. The dynamic taper models could be improved when fitted using NLS by incorporating functions for height and dbh predictions for optimization as well. However, this could delay or completely prevent the fitting algorithm from reaching optimum solutions.

These problems are not all difficult to solve. For example, the height and dbh models could be refitted based on a data set with a wider range. The problem of lack of crown size measurements cannot easily be solved unless there exists data that have crown size measured over time (PSP data with upper stem measurements). Such data also could improve the dbh model because crown diameter was found to be important for dbh prediction (Sprinz and Burkhart 1987). The use of dominant and codominant trees is not a big problem since this can be solved by calibrating the model with a data set composed of all crown classes. Also, the presence of $Q_{D50}$ and the D/H ratio might account for crown class variations.

Based on Clyde's (1986) findings, a dynamic taper modelling approach to growth modelling seemed to be an infeasible idea. However, from the results of this study, dynamic taper modelling does appear feasible. Reasons for this difference could include the following.

1. The static taper models used by Clyde assumed that tree form did not change along the stem over time. Therefore, her dynamic taper models would not account for the changes in stem form over time. Such a deficiency might have caused the poor results.

2. The prediction models for height and dbh used by Clyde were functions of age alone. She never included other tree, stand, and site variables such as site index for height and dbh prediction and stand density and height for dbh prediction that were found to be important in this study.
Chapter 6

Conclusions and Recommendations

Forest management has become more intensive and reached a stage where acquiring the best information, in the most efficient way, is very important. Much of the necessary information is provided by forest inventories and growth and yield projections. Both forest inventories and growth and yield projections require accurate and current information for both tree and stand volume. Understanding and modelling taper changes over time will help improve the quality of the information provided.

In this study, several aspects of variation in form and taper of trees along the stem over time were examined for dominant and codominant lodgepole pine trees from the interior of British Columbia. The objectives of this research were twofold: (1) to investigate the variation in tree form and taper over time as affected by changes in stand, tree, and site characteristics; and (2) to use the knowledge gained from objective one to develop a dynamic taper function. The goal of developing this dynamic taper function was to be able to accurately predict the tree shape and taper changes over time given certain stand conditions.

The first objective of this study has been achieved. The impact of various factors on tree shape and taper changes along the stem at one time and over time was studied. It was found that lodgepole pine trees have a simple shape (parabolic) from ground to top, a statement made by Forslund (1982) without confirmation. At the same time, the results demonstrated that taper and form changes over time were different at different heights along the stem. The pattern of variation in tree taper and form was different for different ages, crown sizes, and stand densities. Trees growing in less dense stands were more tapered at a given height than trees growing in more dense stands. Since stand density and crown
size influence the distribution of diameter growth along the stem, it is possible for a forest manager to manipulate stand density and tree crown length in order to produce trees with desired characteristics.

Using the knowledge gained from objective one, the second objective of this study was achieved because two dynamic taper functions were developed which were able to accurately predict diameter along stem and tree volume over time. These dynamic taper models were not fit using a technique that would account for the non-iid characteristics of the stem analysis data used because of a number of problems involved with using such fitting techniques. Nevertheless, the dynamic taper models gave precise predictions for \( d_{ijt} \) along the stem, especially for lower sections which contain more volume, and for total tree volume, when fitted using either OLS and NLS. These models took into account the fact that the geometric shapes of the tree stem changes continuously along its entire length and over time at the same position. One of the dynamic taper models (Model 3) expressed change in tree shape along the stem as a function of tree and stand variables which change over time.

Model 3 has the following advantages:

1. It gives consistently accurate estimates of diameter along the stem over time, particularly for lower sections, and total tree volume over time. This model compares favourably with Kozak's (1988) static taper model (Model 1a), used for current inventory. It is also an improvement over the static model (Model 1b) with inputs of predicted height and dbh, presently used for growth and yield.

2. Since it includes age and predicted quadratic mean diameter at age 50 (a stand density measure) in its form exponent model, this model is good for tracking tree shape and taper changes over time. This could be the reason that it was found to be less biased for \( d_{ijt} \) and \( V_{it} \) than Model 2.

3. Like Kozak’s (1988) model (Model 1a), it takes into account the continuous variation in form along the stem (a factor that was confirmed by objective one), and thus requires
a single taper function. In addition, tree form changes over time were modelled.

4. It is easy to fit using the standard fitting methods (OLS and NLS).

However, Model 3 has some problems.

1. It had poorer predictions than desired for upper stem diameter, particularly for the two sections before the tip.

2. It is not easily integrated to obtain volume (merchantable or total) directly. Therefore, potentially time consuming numerical methods have to be used. Also, it is not possible to transpose and obtain the height directly, as with Max and Burkhart's (1976) taper function. This means height has to be obtained by iterative methods. However, this is becoming less of a problem with the ever increasing computing power available.

Apart from the few problems noted above, the developed dynamic taper model was able to take into account form variation along the stem over time. The future of the dynamic taper modelling philosophy looks brighter than was portrayed by Clyde (1986).

Probably the greatest promise in the dynamic approach to modelling tree taper is the fact that it allows visualization of how tree shape and taper will vary given a set of stand conditions. This would allow a forest manager to simulate the manipulation of the stand density and crown length through thinning and pruning in order to achieve specific tree shape and volume objectives.

Also, this study is a contribution to increasing to body of information on growth modelling in the following ways.

1. A detailed study was conducted on how tree shape and taper changes over time for dominant and codominant lodgepole pine. This increased understanding of the basic changes in tree form and taper over time and how various factors (site quality, stand density, crown length, etc) affect tree form and taper.
2. The dynamic taper functions developed, which incorporated site, stand, and tree information, can be used to estimate current and future volume and taper of individual trees. These models could be incorporated easily into an individual tree growth model for volume prediction.

By incorporating tree, stand, and site factors into a simple variable-exponent taper model by Kozak (1988), a dynamic taper function was developed. This dynamic taper function was able to track the behaviour of very complex tree shape and taper changes over time with reasonable accuracy. The dynamic taper modelling approach will be a useful tool in forest management because dynamic taper models (e.g., Model 3) can be developed that will enable forest managers to simulate stand development in order to achieve specific objectives.

This study has provided the basis for further investigation into the variation of tree shape and taper over time and dynamic taper modelling for B.C. forests. However, some complications were encountered. These led to the following suggestions for further investigations into the process of dynamic taper modelling.

1. There is a possibility of applying FGLS or FGNLS for fitting dynamic taper functions, if the modeller is willing to make simplifications concerning serial correlation and heteroskedasticity by pooling data. The error variances and covariances, and the measures of serial correlation, could be assumed to be functions of tree, stand, and site variables, in order to remove the problem of undefined values.

2. Dynamic taper models could be improved by including a crown size measure, because crown length and crown ratio were found to be highly correlated with tree shape. This would require upper stem measures of trees in PSPs along with crown size over time, rather than stem analysis data for fitting the model. Alternatively, crown size could be monitored for trees on PSPs and then these trees could be felled at a later date for stem analysis.
3. Although the new dynamic taper model (Model 3) had good predictions for $d_{ijt}$ and volume, some improvements could be achieved if the prediction model for base diameter ($a_0D^{a_1}$) was improved by adding some tree, stand, and site variables.

4. Dynamic taper models could be extended to other species and crown classes (i.e., include intermediate and suppressed trees). These models may not be as good as those developed for this thesis, since lodgepole pine has relatively little taper, compared to species like Douglas-fir, western red cedar, and spruces.

5. Additional work could be done to examine the effect of other stand density measures, and how specific silvicultural treatments such as thinning and pruning directly affect taper and form changes along the stem over time. The dynamic taper functions could then be re-examined and modified if necessary.

6. The accuracy of any dynamic taper model depends on the precision of the models employed for predicting height and dbh. Another model for dbh prediction could be selected and fitted, preferably a nonlinear model, since the dbh model used for this study was not as accurate as desired.

The current procedure used in growth and yield of inputting predicted height and dbh into an existing taper function was found to result in biased predictions for $d_{ijt}$ and $V_{it}$. Therefore, the dynamic taper modelling approach should be used because it was found to provide more accurate predictions for $d_{ijt}$ and $V_{it}$ than the method currently used.

This research opens a new approach to estimating volume for growth and yield models and projecting future forest inventories. The dynamic taper modelling approach could replace the process currently used. Such a modelling approach would enable forest managers to simulate the effects of various silvicultural operations on tree shapes over time. Dynamic taper modelling is a feasible and practical idea, and the development of dynamic taper models for other species and crown classes is encouraged.
Literature Cited


Fries, J. 1965. Eigenvector analyses show that birch and pine have similar form in Sweden and British Columbia. For. Chron. 41: 135–139.


Appendix A

Glossary of Variables Used

$H$ or height is total tree height in metres.

$D$ or dbh is diameter inside bark at breast height in centimetres.

$d$ is diameter in centimetres inside bark at height $h$ above ground.

$h$ is height above ground in metres.

BH is breast height (1.3 metres above ground).

$V$ is total volume in cubic metres.

Age is tree age at breast height in years.

CL is crown length in metres.

CR is crown ratio or crown length to total tree height ratio.

SI is site index or height of site trees (average height of dominant and codominant trees) in metres at age 50 years.

QD is quadratic mean diameter in centimetres.

$QD_{50}$ is quadratic mean diameter at age 50 years in centimetres.

SDI is the stand density index.

RD is the relative density.

SPH is the number of stems per hectare.

BA is basal area in square metres per hectare.

AGE is the mean breast height age per plot in years.
Appendix B

Quadratic Mean Diameter Curves
Appendix B. Quadratic Mean Diameter Curves

Figure B.16: Quadratic mean diameter (cm) for site index 15 m by breast height age.

Figure B.17: Quadratic mean diameter (cm) for site index 10 m by breast height age.
Appendix C

Tree Form Variation with Tree Breast Height Age along the Stem
Appendix C. Form Variation with Tree Age

Figure C.18: Form exponent (r) by breast height age (years) for tree 2 in plot 16 (16.2), dbh=22.2 cm, height=21.1 m, Age=53 years, and $QD_{50}=18$ cm at three relative heights (0.1H, 0.5H, and 0.8H) above the ground.

Figure C.19: Relative diameter by breast height age (years) for tree 2 in plot (25.2), dbh=8.3 cm, height=9.3 m, Age=46 years, and $QD_{50}=6$ cm at three relative heights (0.1H, 0.5H, and 0.8H) above the ground.
Figure C.20: Form exponent \( (r) \) by breast height age (years) for tree 1 in plot 42 (42.1), dbh=13.9 cm, height=13.0 m, Age=21 years, and \( QD_{50}=18 \) cm at three relative heights (0.1H, 0.5H, and 0.8H) above the ground.

Figure C.21: Form exponent \( (r) \) by breast height age (years) for tree 2 in plot 43 (43.2), dbh=5.5 cm, height=5.1 m, Age=14 years, and \( QD_{50}=12 \) cm at three relative heights (0.1H, 0.5H, and 0.8H) above the ground.
Figure C.22: Form exponent \( r \) by breast height age (years) for tree 1 in plot 46 (46.1), \( \text{dbh} = 17.0 \) cm, height = 9.7 m, Age = 20 years, and \( Q_{D_{50}} = 22 \) cm at three relative heights (0.1H, 0.5H, and 0.8H) above the ground.
Appendix D

Tree Form and Taper Variation along the Stem Against Relative Tree Height
Figure D.23: Form exponent $(r)$ by relative height for tree 16.2, dbh=22.2 cm, height=21.1 m, Age=53 years, and $QD_{50}=18$ cm for different measurement periods.

Figure D.24: Relative diameter by relative height for tree 16.2, dbh=22.2 cm, height=21.1 m, Age=53 years, and $QD_{50}=18$ cm for different measurement periods.
Appendix D. Form and Taper Variation along the Stem

Figure D.25: Form exponent ($r$) by relative height for tree 25.2, dbh=8.3 cm, height=9.3 m. Age=46 years, and $QD_{50}=6$ cm for different measurement periods.

Figure D.26: Relative diameter by relative height for tree 25.2, dbh=8.3 cm, height=9.3 m. Age=46 years, and $QD_{50}=6$ cm for different measurement periods.
Figure D.27: Form exponent \((r)\) by relative height for tree 42.1, \(\text{dbh}=13.9 \text{ cm}, \text{height}=13.0 \text{ m}, \text{Age}=21 \text{ years}, \text{and } QD_{50}=18 \text{ cm} \) for different measurement periods.

Figure D.28: Relative diameter by relative height for tree 42.1, \(\text{dbh}=13.9 \text{ cm}, \text{height}=13.0 \text{ m}, \text{Age}=21 \text{ years}, \text{and } QD_{50}=18 \text{ cm} \) for different measurement periods.
Figure D.29: Form exponent (r) by relative height for tree 43.2, dbh=5.5 cm, height=5.1 m. Age=14 years, and $QD_{50}=12$ cm for different measurement periods.

Figure D.30: Relative diameter by relative height for tree 43.2, dbh=5.5 cm, height=5.1 m. Age=14 years, and $QD_{50}=12$ cm for different measurement periods.
Figure D.31: Form exponent ($r$) by relative height for tree 46.1, dbh=17.0 cm, height=9.7 m, Age=20 years, and $QD_{50}=22$ cm for different measurement periods.

Figure D.32: Relative diameter by relative height for tree 46.1, dbh=17.0 cm, height=9.7 m, Age=20 years, and $QD_{50}=22$ cm for different measurement periods.
Appendix E

Tree Form Variation with Height Above Ground and Breast Height Age
Figure E.33: Form exponent (r) by height (m) above ground and breast height age (years) for tree 16.2, dbh=22.2 cm, height=21.1 m, Age=53 years, and $QD_{50}=18$ cm for all measurement periods.

Figure E.34: Form exponent (r) by height (m) above ground and breast height age (years) for tree 25.2, dbh=8.3 cm, height=9.3 m, Age=46 years, and $QD_{50}=6$ cm for all measurement periods.
Appendix E. Form Variation with Height and Age

Figure E.35: Form exponent ($r$) by height (m) above ground and breast height age (years) for tree 42.1, dbh=13.9 cm, height=13.0 m, Age=21 years, and $QD_{50}=18$ cm for all measurement periods.

Figure E.36: Form exponent ($r$) by height (m) above ground and breast height age (years) for tree 43.2, dbh=5.5 cm, height=5.1 m, Age=14 years, and $QD_{50}=12$ cm for all measurement periods.
Figure E.37: Form exponent $(r)$ by height (m) above ground and breast height age (years) for tree 46.1, dbh=17.0 cm, height=9.7 m, Age=20 years, and $Q_D_{50}=22$ cm for all measurement periods.
Appendix F

Volume Predictions per Tree for all Taper Functions
Figure F.38: Total tree volume ($m^3$) by breast height age (years) for tree 14.1 (dbh=21.9 cm, height=20.9 m, $Q_{D_{50}}=18$ cm, and $S_l=20$ m).
Figure F.39: Total tree volume ($m^3$) by breast height age (years) for tree 17.1 (dbh=21.2 cm, height=20.6 m, $QD_{50}=16$, and SI=18).
Figure F.40: Total tree volume (m$^3$) by breast height age (years) for tree 25.1 (dbh=8.1 cm., height=9.4 m, $QD_{50}$=6 cm. and $SI=10$ m).
Figure F.41: Total tree volume (m³) by breast height age (years) for tree 41.2 (dbh=20.1 cm, height=10.4 m, $QD_{50}=25$ cm, and SI=21 m).