THREE ESSAYS ON EXPECTATIONS AND HOUSING PRICE VOLATILITY

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Abstract

This thesis contains three empirical essays on the economics of house price dynamics.

The first essay derives a forward-looking rational expectations house price model and empirically tests its ability to explain short-run fluctuations in real house prices. A novel approach to proxying imputed rents of owner-occupied housing, as a function of housing market fundamentals, is derived and combined with a housing market arbitrage relation to derive a present value model for real house prices. Tests of the rational expectations, nonlinear cross-equation restrictions reject the joint null hypothesis of rational expectations and the asset-based housing price model for quarterly, single-detached house prices in the city of Vancouver, British Columbia, over the 1979-1991 sample period. The model fails to fully capture observed house price dynamics in two real estate booms but tracks real house prices well in less volatile times, suggesting that prices may temporarily deviate from fundamental values in real estate market upswings.

The second essay develops and applies a test of the joint null hypothesis of rational expectations, and no risk premium in the Vancouver condominium apartment market. The results show that, on average, ex post house price changes move in the opposite direction than their rational expectation under risk neutrality. This essay also documents the predictability of excess annual condominium returns using lags of annual returns and the rent/price ratio, and quarterly returns with short-term nominal interest rates. It further shows that deviations of house price changes from their (risk neutral) rational expectation are both stationary and related to the stage of the real estate price cycle.

The third essay examines whether a time-varying housing market risk premium can explain deviations in house price fluctuations from those predicted by the rational expectations hypothesis under risk neutrality. If homeowners are risk averse and housing price risk is not completely diversifiable then housing market efficiency implies that returns to housing investment should be positively correlated with a premium for bearing risk. The first part of the essay shows that, in theory, the finding of negative slope coefficients in tests of unbiased house price expectations under risk neutrality (in chapter 3) is attributable to omitted risk considerations if two conditions are satisfied: (1) the covariance between the risk premium and expected house price appreciation under risk
neutrality is negative, and; (2) the variance of the risk premium is considerably larger than the variance of expected appreciation under risk neutrality. The second part of the essay uses a conditional capital asset pricing model to investigate whether predictable returns in the Vancouver housing market are time-varying risk premia. The empirical results are inconclusive.
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Chapter 1

Introduction

This dissertation contains three empirical essays on the economics of price cycles in local housing markets. The first essay derives and tests a forward looking, rational expectations model of house price determination. The second essay develops and applies a test of the efficiency of the housing market under conditions of risk neutrality. The third essay investigates the existence of a time-varying risk premium in the housing market. This chapter discusses the motivations for the study, outlines the methods of investigation, reviews the empirical literature on rational expectations and housing market efficiency, and summarizes the contributions this thesis makes to the existing literature.

1.1 Motivation

In recent years many local housing markets in North America have undergone boom and bust cycles. These events are characterized by dramatic house price fluctuations over relatively short periods of time. For example, in Vancouver, average house prices increased from $75,500 in January 1980 to $160,000 in January 1981; an increase in one year of more than 110 percent! By July 1982 the average resale price had fallen to about $100,000; a drop of 35 percent. Beginning in mid-1987 Vancouver house prices again began to rise sharply. From $132,240 in July 1987, the average resale housing price jumped to $245,700 at its peak in February 1990; an increase of more than 86 percent.²

¹These data are the average monthly resale house price of homes sold through the Multiple Listing Service (MLS) as reported by the Real Estate Board of Greater Vancouver.
²In the period from January 1989 to February 1990 the average MLS price jumped by more than 70 percent.
Over the next year, housing prices fell almost 20 percent. Housing markets in Toronto, Boston and parts of California have also been marked by dramatic price movements over the past decade.\(^3\)

What causes such dramatic movements in house prices?\(^4\) As Shiller ([140], page 1) asks, for asset prices in general:

"...can we trace the source of movements back in a logical manner to fundamental shocks affecting the economy ...? Or are price movements due to changes in opinion or psychology, that is, changes in confidence, speculative enthusiasm... that are best thought of coming ultimately from peoples minds?"

The essays in this thesis address the first question Shiller [140] poses; can market fundamentals alone explain local house price dynamics?\(^5\) They thereby also provide indirect evidence on the role of psychology in house price fluctuations.

Some academics and housing market commentators claim that housing markets are characterized by excess speculation during real estate market upswings. Specifically, intangible expectations may lead prices to race ahead of fundamental values. Case and Shiller [22] for example, survey recent homebuyers in four metropolitan areas in the United States in an attempt to learn about the factors that influence the homebuying decision. The authors interpret the survey results as showing that the perceived degree of risk in housing, as an investment, is low, especially in hot real estate markets, and that most homebuyers have little or no knowledge of underlying market fundamentals. The

\(^3\)In Toronto the average MLS house price jumped by 85 percent over the three year period from October 1986 to October 1989. Prices subsequently fell by 20 percent over the next two years. (source: Toronto Real Estate Board). Case [20] reports that the median Boston house price (reported by the National Association of Realtors) shot up by more than 120 percent over a three year period starting in early 1984. Case and Shiller [22] and Poterba [126] provide evidence of similar house price movements in other U.S. cities. Poterba [126] also discusses boom bust episodes that have taken place in England and the Netherlands.

\(^4\)The question of what causes dramatic fluctuations in house prices has received surprisingly little academic attention. In contrast, there exists a huge literature that studies volatility in the stock and foreign exchange markets. See Shiller [140], Fama [45] and Hodrick [93] and the references contained there.

\(^5\)According to Mankiw and Weil [110] the question of "...what explains the booms and busts that occur regularly in local housing markets? ...is one of the central questions of housing economics,..." ([110], pages 579 and 573). In a recent issue of the Brookings Papers on Economic Activity, Poterba [126] writes, "Local housing booms are one of the principal challenges to understanding house price dynamics." ([126], page 170).
authors conclude that home buyers and sellers are significantly influenced by psychology during housing price booms, and therefore local boom bust episodes are driven primarily by irrational house price expectations.\textsuperscript{6}

House prices influence many important economic activities. As a result, if house price increases are due to irrational expectations or investor psychology, rather than changes in market fundamentals, there is the potential for significant resource misallocation. For example, if houses are overpriced there is an incentive to overinvest in new housing construction.\textsuperscript{7} Most housing purchases involve mortgage financing. A house’s market value is the basis for the mortgage lending decision. If house prices exceed intrinsic values then a market correction is inevitable. The resulting sharp reductions in house values may put a significant strain on the financial system. House price fluctuations, therefore, may have important real effects on local economies.

Anecdotal evidence on the real effects of rapidly rising house prices comes from conversations I had with economists at the Bank of Canada, in February of this year. I had the feeling that they believed the recent Toronto real estate boom was fuelled in part by inflated expectations of capital gains not based on market fundamentals. Moreover, this unfounded optimism further increased the general economic expansion already underway at the time, resulting in higher inflation.\textsuperscript{8} They believe that if they had had better methods to detect when a “bubble” starts they could have tightened monetary policy earlier, and more slowly.\textsuperscript{9} As a consequence, the Ontario recession may have been less severe.

\textsuperscript{6}Case [20] develops and estimates a structural model of the Boston housing market, and concludes: “Fundamentals do not seem to offer an adequate explanation for the very rapid increase in home prices in the Boston area since 1983. Recent economic theories of asset price behaviour used to explain “bubbles” in financial markets and foreign exchange markets seem to fit the housing market very well.” A “bubble” is a deviation from fundamental or intrinsic value. Poterba [126] hints that housing price booms may be due to bubbles.

\textsuperscript{7}This is a consequence of the high price elasticity of housing starts. Topel and Rosen [147] estimate that the price elasticity is about 3.


\textsuperscript{9}ARA Consultants [1] use a survey, study groups and data analysis to study the recent price boom in Toronto. They reach the opposite conclusion, namely that excess speculation was not the cause of the runup in house prices.
Case [21] puts forward a similar hypothesis. He argues that not only are housing markets driven by investor psychology, but that real estate price booms both contribute to, and significantly amplify, the magnitude of the boom and busts in economic growth of the local economy.

Periods of rapid house price increases often result in deteriorating affordability of homeownership. This is frequently associated with growing pressure for government intervention to reverse the trend in house prices. For example, in response to the most recent housing price booms in Toronto and Vancouver, Canada Mortgage and Housing Corporation (CMHC) organized ‘The Canadian Housing Conference’ in Vancouver in the fall of 1990. It brought together academics and real estate professionals to discuss ways in which to help the ‘first-time’ homebuyer, who could no longer afford to buy a home in either of these two cities.10 If house price changes result from waves of optimism and not changes in market fundamentals should the government design policies to further increase demand for owner-occupied housing?

Government intervention may be justified if housing markets are characterized by irrational expectations during boom periods. Such intervention would take the form of policies to reduce investor enthusiasm and thereby to eliminate or minimize the real effects of price cycles on local economies.11 If however, dramatic house price fluctuations are the result of fundamental shocks affecting the local economy these policies would be inappropriate. Thus, understanding the economics of boom and bust cycles is important to help guide government policy in the housing sector. As Hosios and Pesando ([100], page 58) suggest,

“...research into the behaviour of prices in housing markets improves our understanding of how these markets operate, thereby providing a better foundation for the formulation of policy.”

In addition, a better understanding of the underlying economics on the part of homeowners...
ers, investors, lenders, developers, builders and others will allow them to make decisions that ultimately reduce the impact of large house price changes on their activities. This in turn, may ultimately lower the volatility of house price movements in the future.

1.2 Expectations, Fundamentals and House Prices

This section discusses the theory that forms the basis of the empirical studies in the three essays. It examines what it means for changes in house prices to reflect shocks to market fundamentals.

The price of owner-occupied housing faced by agents is not the asset or purchase price of a dwelling unit but the cost they incur to occupy their home each period. This cost includes the sum of all operating costs minus anticipated house price appreciation. Expected capital gains lower the effective price of housing services derived through homeownership. The potentially important role played by expectations has lead researchers to treat owned housing as an asset. This implies the housing market is comprised of two separate but interrelated markets: one for the flow of housing services (a consumption good) and another for the stock of housing (an asset).

If homeowners and other investors are risk neutral then equilibrium in the market for housing stock requires that the expected return to housing is equal to the expected rate of return on alternative investments.12

One important aspect of financial asset prices is that they are generally regarded as being determined in efficient markets. An efficient market is one in which asset values fully reflect all available information.13 That is, current asset values reflect rational projections of future market fundamentals. Thus market efficiency implies that agents are forward looking and that past information cannot be used to predict future returns.

This thesis examines whether house prices reflect market fundamentals. If current house prices reflect all publicly available information about housing market fundamentals then the housing market is efficient. Thus one way to approach this is to ask, is the

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12 Much of the literature assumes risk neutrality. This is discussed in the literature review.
13 Fama [45], Hodrick [93], De Bondt and Thaler [34], Engel and Morris [38] and Froot and Thaler [61] survey the empirical literature on stock and foreign exchange market efficiency. These papers provide more detailed expositions of the concept of market efficiency.
housing market efficient?

If excess housing returns are uncorrelated over time, and it is assumed that the information set available to agents contains only the past history of housing returns then the housing market is said to exhibit weak form efficiency. If the housing market is efficient and the information set contains not only past returns but all publicly available information then future excess returns must be orthogonal to, or uncorrelated with, all variables in the current information set. In this case the market is said to exhibit semi-strong form efficiency.

1.3 Literature Review

This section provides an overview of, and identifies a number of shortcomings in, existing studies of housing market efficiency.

Linneman [104] performs the first test of the efficiency of the housing market. His data sample comprises homeowners' assessments of the values of their homes in Philadelphia in 1975 and 1978, plus the physical and locational characteristics of the homes. He uses hedonic regressions to identify homes that were undervalued in 1975 (i.e. had negative residuals in a regression of house value on physical and location characteristics), and finds that on average these homes increased in value relative to homes that were not undervalued. Thus, information contained in the hedonic regression residuals is correlated with future house price appreciation. Hence, the market is not semi-strong form efficient. Linneman [104] concludes, however, that because of transactions costs no significant arbitrage opportunities existed.

Case and Shiller [23], Meese and Wallace [115], Hosios and Pesando [99] and Poterba [126] employ time series of city-wide house price series to test the weak form version of housing market efficiency. 14 These authors all claim to find that measures of excess

\[
\text{Case and Shiller [23] use a repeat sales methodology to construct constant quality house price series for Atlanta, Chicago, Dallas and San Francisco. Hosios and Pesando [99] construct a repeat sales index for Toronto house prices. Meese and Wallace [115] derive quality adjusted house price series for a number of municipalities in the San Francisco area. They use nonparametric estimation of hedonic regression models. Poterba [126] employs U.S. metropolitan level data from a variety of sources. Most of his efficiency tests, however, use average metropolitan resale house prices as reported by the National Association of Realtors (NAR).}
\]
housing returns are correlated over time, which is evidence against weak form market efficiency. Case and Shiller [24] test, and reject, the semi-strong form of housing market efficiency. They report that current economic fundamentals have some power to predict future excess housing returns.

In addition to the traditional finance tests of market efficiency, Hamilton and Schwab [82], Meese and Wallace [115] and Poterba [126] also conduct tests of forward looking behaviour in the housing market. That is, rather than examine the implications of market efficiency for the time series properties of returns, these authors attempt to directly evaluate the proposition that house prices reflect rational projections of future market fundamentals.

Hamilton and Schwab [82] derive a measure of expected house price appreciation that assumes agents are risk neutral and have rational expectations. Using data on a large number of homes in 49 U.S. metropolitan areas they find that expectations of appreciation from the 1974-1976 period to the 1976-1979 period were systematically wrong. Specifically, past rates of appreciation could have been used to forecast future price changes.

Meese and Wallace [115] test the housing price present value model. To test their model they make a number of simplifying assumptions. These include a constant discount rate and constant rental inflation. This leads to a simple model in which current house price is a multiple of current rent. The authors compare the present value prices with actual prices and find that, “...present value prices are much lower than actual real house prices throughout our period of analysis.” ([115], page 15). Meese and Wallace [115] conclude that the present value model has little explanatory power for short-run housing price dynamics.

Poterba [126] notes that if local housing price booms are the result of rational expectations then they must be driven by expectations of future income and/or local population growth. Given the inelasticity of short-run housing supply, expectations of large short-

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15 Meese and Wallace [115] work with San Francisco and Oakland housing market data.
16 Their conclusion is based on the combination of the above results and traditional market efficiency tests.
17 He also suggests that house price “bubbles” are another alternative.
term shifts in demand fundamentals translates into sharp price changes.

Poterba [126] tests the extent of forward looking behaviour in the housing market in three ways. First, he tests for persistence in the growth rate of local incomes. If expected shifts in future income affect current house prices then future income growth must be partly predictable. He finds some persistence in metropolitan income growth rates but not enough, in his opinion, to justify observed price dynamics. In a second test, Poterba examines the power of house price changes to forecast future changes in real per capita income, in a sample of 39 U.S. metropolitan areas. The results suggest that capital gains have power to forecast future changes in income. Poterba [126] concludes that house prices appear to be partly forward looking. However, coupled with his efficiency test results, Poterba interprets his findings to imply housing market inefficiency.

Based on these studies, there is an emerging consensus that the housing market is inefficient and that housing price booms are therefore partly driven by irrational expectations. Shiller ([141], page 189) summarizes this view in the following way:

"The differences across cities in growth rates in house prices can be explained only partially in terms of a set of rational or fundamental factors. Housing prices are not set in an efficient market and are only partially forward looking. There appears to be a purely speculative component of real estate prices."

Shiller's conclusion that the housing market is inefficient is premature; it is based on a very limited amount of research. There are huge empirical literatures that test the efficiency of the markets for stocks and foreign exchange.18 Despite this vast amount of effort the efficiency question remains unanswered in both of these markets.19

In addition, available evidence suggests that other segments of the real estate market are efficient. Gau [65] tests the weak form market efficiency hypothesis in the Vancouver commercial and apartment real estate markets, using transactions-based data over the 1971-1980 period. He cannot reject the hypothesis that real estate returns follow a

19 Part of the reason is due to the joint nature of the hypothesis tests: we can never directly test for market efficiency. Some researchers treat the results as evidence of inadequate or misspecified asset pricing models. Others as evidence against market efficiency. De Bondt and Thaler [34], Engel and Morris [38] and Froot and Thaler [61] discuss the issues.
random walk for this sample. Using the same data set Gau [65] tests and fails to reject semi-strong real estate market efficiency.

Moreover, previous studies yield only weak evidence of housing market inefficiency, and only under the assumption that homeowners are risk neutral. More precisely, previous studies detect some predictability in measures of excess housing returns. I interpret existing results as weak evidence of return predictability because there are a number of shortcomings in the existing housing market efficiency literature, which the remainder of this section addresses.

The principal evidence in support of weak form inefficiency is dependence in price changes. Case and Shiller [23] and Poterba [126] detect serial correlation in annual price changes in a number of U.S. metropolitan housing markets. An increase in house prices one year indicates an increase over the next year of about one-third to half the magnitude. This does not, however, imply housing market inefficiency. An asset market is inefficient, with respect to a given information set, if excess returns are predictable. Excess returns to housing are defined as the sum of capital gains and rent minus the opportunity cost of capital (usually the risk free rate of interest rate) over the period in question. Thus serial correlation in either rents or interest rates could be driving the above results.

Researchers recognize this, but have been hampered in their attempts to construct housing return series by the lack of published quality adjusted rental data. Part of the problem is that all previous work investigates the efficiency of the market for single-family homes. Most single-family homes are owner-occupied. Rents on these homes are not observed. Federal government statistical agencies in both Canada and the United States publish indices of rents on rental dwellings, as part of their Consumer Price Indexes. Most researchers proxy single-family rents with these published rent series.

\[20\] If homeowners are risk averse then predictable excess housing returns may be evidence of a time-varying risk premium and not market inefficiency. Chapter 4 discusses this issue in detail.

\[21\] Hosios and Pesando [82] find the same pattern in Toronto house prices.

\[22\] 90 percent of the existing stock of 5.7 million occupied single-family dwelling units in Canada are owner-occupied. This figure drops slightly to 86 percent for the Vancouver Census Metropolitan Area (CMA). (Source: Table 1 in Occupied Private Dwellings: The Nation, 1991 Census of Canada, Statistics Canada catalogue 93-314.

\[23\] Meese and Wallace is an exception.
This approach suffers from a number of limitations. Rental properties may differ in quality from owner-occupied housing and thus such indices may be poor proxies. Rent indices also appear to suffer from statistical problems. For example, there is a widely held belief that the rent component of the Canadian Consumer Price Index is downward biased. Moreover, to estimate housing investment returns, rent indices must be converted into time series of rental payments to form the proxy series for single-family rents. To derive the conversion or scale factor it is assumed that the market efficiency condition holds exactly in the first period of the sample. If this assumption is not true, or the scale factor is not constant over the sample period, then subsequent results are biased.

The tests of forward looking behaviour in the housing market by Meese and Wallace [115] and Poterba [126] are ad hoc. Meese and Wallace [115] assume a constant discount rate and constant rental inflation to operationalize the present value model. Neither of these assumptions is likely to be true. They appear particular inappropriate in boom bust situations where fundamentals and discount factors are anything but constant. Poterba’s [126] tests of the degree of forward looking behaviour are not based on a model of house price determination, and are therefore difficult to interpret. Shiller [141] criticizes Poterba’s [126] tests of the power of house price changes to predict future economic activity. Because of various technical considerations Shiller suggests that Poterba’s results have no power to test the housing price present value relation.

The essays in this thesis aim to remedy these shortcomings in the existing empirical literature on rational expectations and housing market efficiency.

1.4 Dissertation Overview and Contribution

This section outlines the goal, approach and contribution of each essay in the thesis.

Essay 1:

The first essay investigates the ability of a simple asset-based housing price model, that explicitly incorporates the hypothesis of rational house price expectations, to explain

24Chapter 2 provides more details.
observed short-run fluctuations in real house prices. It derives and estimates a forward-looking rational expectations housing price model. The model is tested using a quality adjusted, quarterly single-detached house price series for the Vancouver housing market, over the 1979 - 1991 sample period.

This essay offers the following innovations. Even though the imputed rents of owner-occupied homes are not observed it may be possible to derive the stochastic process they follow. To accomplish this a method of proxying rents in terms of a parsimonious model of housing market fundamentals is derived. The model for imputed rents is combined with a housing market arbitrage condition to derive a present value model for real house prices. Together with time series models of the stochastic processes driving the fundamental variables, the present value relation yields a reduced form model that relates short-term housing price dynamics to market fundamentals. The appropriateness of the rational expectations hypothesis is tested not by single-equation regression methods but via tests of nonlinear cross equation restrictions. Thus Chapter 2 both overcomes the imputed rent data problem and provides a more rigorous test of forward looking behaviour than those of Meese and Wallace [115] and Poterba [126].

Consistent with earlier findings, the rational expectations hypothesis is rejected. The results suggest that real house prices may exceed fundamental values during real estate booms.

Essay 2:

Essay 2 develops and applies a test of the joint null hypothesis of rational expectations, or semi-strong asset market efficiency, and no risk premium in the Vancouver condominium apartment market. If the housing market behaves as an efficient asset market then excess housing returns should be uncorrelated over time and uncorrelated with all information known at the time forecasts are made. Existing studies of housing market efficiency that examine the autocorrelation structure of estimates of excess returns (Case and Shiller [23][24], Meese and Wallace [115], Hosios and Pesando [99] and Poterba [126]), and test whether any variables in the current information set have predictive power for future excess returns (Case and Shiller [24] and Meese and Wallace [115] and Poterba [126]),
yield weak evidence of housing market inefficiency.

The work in this essay differs from the existing literature in two ways. First, it applies a different testing methodology. Rather than work with excess returns I use the market efficiency condition to generate a measure of expected house price appreciation under risk neutrality. If housing market participants have rational expectations then regressions of the actual percentage changes of house prices on the expected appreciation measure yields a slope coefficient of one. This approach is shown to provide a more powerful test of the joint null hypothesis than conventional return regressions.

Second, this essay uses a new data set; The Royal Lepage Survey of Canadian House Prices, a quarterly survey of housing markets undertaken by a large Canadian real estate brokerage firm. This data set is unique in three respects. First, it provides time series of prices, rents and property taxes for a single housing type. This offers significant advantages over previous studies. It eliminates the need to proxy rents and thus does not require us to scale the rent and price data in such a way as to force market efficiency to hold at one point in time. At the same time, it is important to recognize that the data are not market transaction-based, but are estimates or appraisals. The validity of the results in this chapter therefore depend crucially on the accuracy of the data. I undertake a careful comparison of the Royal Lepage price estimates with a constant quality transaction-based price series that is available for one area. The results suggest we can be confident that the data are accurate assessments of true house prices.

Second, this data set is a time series of observations on a cross section of housing submarkets within the larger metropolitan Vancouver housing market. This allows us to test the efficiency of the housing market at a micro level. All existing studies use metropolitan-wide price and rent data. This chapter demonstrates that prices and rents in the local markets, that comprise a larger metro housing market, can behave quite differently. This implies that the use of “metro-wide” rental price indexes and house price series may provide misleading results.

Finally, the data cover a different sector of the housing market than existing studies. This is the first study, to the best of my knowledge, to employ condominium apartment market data. It is often argued that real estate markets are inefficient “based on a per-
ceived set of market imperfections," (Gau ([67], page 1)). The indivisibility or lumpiness of real estate assets and capital constraints faced by investors due to the expensive nature of the asset, are often cited forms of market imperfection, which may limit information capitalization into real estate values. Condominium apartments are generally smaller and significantly less expensive than single-family homes. This suggests that the potential lack of arbitrage due to indivisibility and capital constraints is reduced in the condominium apartment market. Thus if indivisibility and high asset value are more than just perceived market imperfections, we might expect to find that the condominium market is in some sense more efficient than the market for single-family homes.

Empirical tests find that on average, house prices move in the opposite direction than that predicted by the model. These results corroborate previous rejections of the null hypothesis of rational expectations and risk neutrality.

Essay 3:

Essays 1 and 2 of this thesis add to a growing literature that finds predictable components in excess returns to housing. All previous empirical work on housing market efficiency, including the previous two essays, assumes that housing market participants are risk-neutral, or that housing price risk is diversifiable in a portfolio of investments. If, however, homeowners are risk averse and housing price risk is not completely diversifiable then housing market efficiency implies that returns to housing investment should be positively correlated with a premium for bearing risk. This raises the possibility that earlier results are consistent with rational expectations in the presence of a time-varying housing price risk premium.

The third essay derives and estimates two econometric models of time-varying housing market risk premia. It uses a conditional capital asset pricing model (CAPM) to formulate measures of time-varying housing market risk premia. Systematic housing price risk is shown to be related to the co-movement between an unobservable market or benchmark portfolio and excess housing returns. The benchmark portfolio is a value weighted market index that contains both an unobservable housing component and aggregate common

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25 See Gau [65] [67] and Case and Shiller [16] for more on this point.
stock returns.

I test two versions of the housing price model with Vancouver housing market and Toronto Stock Exchange data. The first empirical model assumes a constant market reward-to-risk ratio. This leads to a housing market risk premium that depends on a linear combination of conditional house price volatility and the conditional covariance between Vancouver house price movements and returns to the Toronto Stock Exchange 300 composite index. The second model treats the ratios of the CAPM betas across Vancouver housing submarkets as constants and the benchmark portfolio as an unobservable or latent variable. This leads to a set of cross equation restrictions that the data must satisfy to be consistent with the constant beta version of the conditional CAPM.

The constant reward-to-risk ratio model is soundly rejected, while the latent variable model results are inconclusive.
Chapter 2

Rational Expectations, Fundamentals and Housing Price Volatility

2.1 Introduction

This chapter derives and estimates a forward-looking rational expectations housing price model. More precisely, this paper aims to answer the following question: can a simple asset-based housing price model that explicitly incorporates the hypothesis of rational house price expectations explain observed short-run fluctuations in real house prices?

A number of recent empirical studies find that house price expectations may not be rational, in the sense that information available one period can be used to predict house prices in future periods. Case and Shiller [23] [24], Hamilton and Schwab [82], Meese and Wallace [115] and Mankiw and Weil [109] all report that house price movements are positively correlated over the short-run. More importantly, information on housing market fundamentals and past price increases can be employed to forecast future excess returns. Mankiw and Weil [109] simulate the effect of demographic factors on real U.S. house prices and find that a model with myopic expectations does a much better job of matching observed house price dynamics than does an equivalent model with rational or forward-looking expectations. This implies that the price of housing does not anticipate predictable changes in demand. These findings are inconsistent with rational expectations or semi-strong market efficiency. In an efficient asset market all information
relevant to predicting future price is capitalized into current price and excess returns are unforecastable.

Previous empirical work concentrates on testing the implications of rational expectations for single-period return measures. Variants on the standard approach to testing for stock market efficiency are employed. The one-period ex post return to housing investment minus the interest rate is regressed on a constant and a group of other variables known at the beginning of the period. Under the joint null hypothesis of rational expectations and the correctly specified model, coefficient estimates on the additional variables should not be statistically significant. The true ex-post return to owner-occupied housing is the sum of capital gains and imputed rental income, which is the dividend from owner-occupied housing. But, because imputed rental income is unobservable, previous researchers employ various proxies based on indices derived from rents on rental properties. Statistical problems inherent in such proxies may be partly driving the rejections of the rational expectations hypothesis since tests based on single period returns require an estimate of actual rents. ¹

I offer the following solution. While true rents are not observed it may be possible to derive the stochastic process they follow. To accomplish this I derive a method of proxying rents in terms of a parsimonious model of housing market fundamentals. The model for imputed rents is combined with a housing market arbitrage condition to derive a present value model for real house prices. Together with time series models of the stochastic processes driving the fundamental variables, the present value relation yields a reduced form model that relates short-term housing price dynamics to market fundamentals. The appropriateness of the rational expectations hypothesis is tested not by single-equation regression methods but via tests of nonlinear restrictions across equations which are “the hallmark of dynamic rational expectations models,” (Hansen and Sargent [87]).

The model is tested using a quality adjusted, quarterly single-detached house price series for the Vancouver housing market, over the 1979 - 1991 sample period. Vancouver underwent two dramatic house price cycles over this period (see figure 2.1) and thus provides an ideal testing ground for the role of expectations in house price determination.²

¹Section 2.2, below, outlines potential statistical problems with available rental price indices.
²Hamilton and Hobden [80] derive the constant quality house price series shown in figure 2.1. Section
Empirical tests reject the joint null hypothesis of rational house price expectations, risk neutrality and the model of market fundamentals; the cross equation restrictions are not valid within the asset-based model tested here. The unrestricted reduced form of the present value model, however, does a good job of tracking real house price movements during periods of relatively stable house prices. Large prediction errors in both real estate boomssuggest that prices may, at times, deviate from those predicted by market fundamentals during more volatile times.

The remainder of the chapter proceeds as follows. Section 2.2 sets forth the basic asset-based housing price model and derives a model of imputed rental income. Section 2.3 presents an estimable form of the housing price model as well as the coefficient restrictions implied by the rational expectations hypothesis, and describes the data. Section 2.4 reports empirical results. Section 2.5 considers extensions to the basic model. Section 2.6 presents conclusions.

2.2 Theoretical Framework

This section sets out the intertemporal housing price model. A rational expectations asset market equilibrium condition between the stock and flow values of housing is derived and combined with a dynamic model of the market for owner-occupied housing services to specify a forward-looking model of housing price determination.

The price of owner-occupied housing faced by consumers is not the asset or purchase price of a dwelling unit but some measure of per period costs actually incurred. The user cost concept formally recognizes this. The user cost of owner-occupied housing over a given period of time, is defined as the sum of all costs incurred by the homeowner during that period. Let $P_t$ be the real asset price or value of a standardized unit of housing stock and assume that all operating costs excluding mortgage interest are a constant fraction $\delta$ of the value of the home. $\delta$ is comprised of property taxes, maintenance costs and depreciation. The user cost, $R_t$, is defined by

$$R_t = (\delta + r_t - g^*_t)P_t$$

(2.1)

3.2, below, provides details on the construction of the house price data series.
where $g_t^e$ is expected housing price appreciation or capital gains and $r_t$ is the real mortgage interest rate over the period. Thus, $g_t^e = \frac{P_{t+1,t}^e - P_t}{P_t}$, where $P_{t+1,t}^e$ is the value of housing expected to be realized next period. Anticipated house price appreciation lowers the effective price of housing services derived through homeownership.  

User-cost represents a per period price paid for the consumption of a flow of housing services. It is therefore a form of implicit rent received by homeowners and thus should be determined through the interaction of basic housing service supply and demand forces, or housing market fundamentals.

The link between the value of the flow of housing services, the value of the housing stock and expected appreciation can be interpreted as an asset market arbitrage condition. Rearrange the user cost to obtain

$$R_t + g_t^e P_t = (\delta + r_t) P_t$$

The left-hand side is the expected return from holding a unit of housing stock, the dividend plus expected capital gain, while the right hand-side is the cost of holding the housing asset, the sum of operating and mortgage interest costs or the return available on alternative assets if the house is purchased outright. In the absence of arbitrage opportunities the two must be equal.

Solve for house prices to give

$$P_t = \frac{R_t}{(1 + \delta + r_t)} + \frac{P_{t+1,t}^e}{(1 + \delta + r_t)}$$

The value of housing depends not only on current market fundamentals, reflected in equilibrium rents $R_t$, but also on expected capital gains (or losses).

To complete the model the way in which housing market participants form expectations of future real house prices must be specified. Under rational expectations the expectation formation mechanism is defined by

$$P_{t+1,t}^e \equiv E_t[P_{t+1} | I_t]$$

3This formulation assumes that either there is no risk premium on the returns to housing investment, or there is a risk premium that is a constant fraction of current house price (i.e. included in $\delta$). This implies either that homeowners are risk neutral or that housing price risk is diversifiable in a portfolio of investments. This is a standard assumption in the literature on housing market efficiency. Chapter 4 relaxes this assumption. Mills and Hamilton (117 Chapter 10) and Poterba (126) discuss the basic user cost of capital for housing, that is employed here.
which states that the subjective belief of economic agents of next periods price coincides with the objective conditional mean of the stochastic process generating real house prices. That is, households know the structural econometric model that generates house prices. $I_t$ is the information set available to agents in period $t$ and is comprised of all current and past publicly available information that may play a role in house price determination. Mathematically the rational expectations hypothesis specifies that

$$P_{t+1} = E_t[P_{t+1} \mid I_t] + \epsilon_{t+1}$$

where $E[\epsilon_{t+1} \mid I_t] = 0$. Forecast errors arise from the unpredictable or stochastic nature of the process generating house price realizations, and should not be predictable. Combine equations (2.3) and (2.4) to obtain

$$P_t = \frac{R_t}{(1 + \delta + r_t)} + \frac{E_t[P_{t+1} \mid I_t]}{(1 + \delta + r_t)}$$

(2.5)

which is a stochastic difference equation describing the time path of house prices under rational expectations.

Previous empirical work tests the relationship in equation (2.5). Once the expected price in period $t + 1$ is replaced by the ex-post observed price plus a random disturbance, the model can either be rearranged to derive a relationship between the percentage change in house prices and the rent-price ratio (Case and Shiller [24], Hamilton and Schwab [82], Mankiw and Weil [109]), or the coefficients implied by market efficiency can be imposed and the time series properties of the error term investigated (Meese and Wallace [115]). As noted in the introduction, this approach requires estimates for the time series of rents.

I take a different approach. If the expected rate of return is a constant, $r$, for all periods, then repeated forward substitution for $P_{t+i}$ in equation (2.5), and use of the law of iterated expectations gives

$$P_t = \frac{R_t}{(1 + r)} + \sum_{i=1}^{\infty} \left( \frac{1}{1 + r} \right)^i E_t[R_{t+i} \mid I_t] + \gamma_t(1 + r)^t$$

(2.6)

Section 2.5, below, relaxes the constant discount assumption. The law of iterated expectations states that $E_t[E_{t+j}[P_{t+j+1} \mid I_{t+j}] \mid I_t] = E_t[P_{t+j+1} \mid I_t]$. Since I do not have data on maintenance and depreciation costs, $\delta$, I omit them from this point on. This should not be a major concern.
or, \( P_t = P_t^f + B_t \), where \( P_t^f \) is the unique price sequence in which market price is the discounted present value of market fundamentals, and \( B_t = \gamma_t(1 + r)^t \), where \( \gamma_t \) is any stochastic process that satisfies the martingale property, \( E[\gamma_{t+1} | I_t] = \gamma_t \).

Positive realizations of \( B_t \) imply price paths that contain rational bubbles. That is, real house prices deviate from fundamental prices, \( P_t^f \), yet expectations of future prices are rational. Self-fulfilling behaviour on the part of market participants can generate this type of outcome. Consider the situation where all households expect house prices to appreciate and this expectation is not soundly based on expectations of fundamentals. The potential for capital gains increases the demand for housing today and pushes up the price so that the expectation does indeed turn out to be correct (i.e. rational).

Solution indeterminancies characterize rational expectations models of this type because agents decisions depend on both current market price and expected future prices. This implies that current prices are a function of future prices and future prices depend on current prices. In effect there are two endogenous variables to solve for but only one equation, hence there is no unique solution. Self-fulfilling price fluctuations are possible because of the dependence of current prices on expected future prices. A unique house price path cannot be determined without placing additional structure on the user cost-based model. Such structure comes in the form of a terminal or boundary condition. If \( \lim_{T \to \infty} E_t \left[ \left( \frac{1}{1 + r} \right)^T P_{t+T} \right] = 0 \), then rational bubbles are ruled out and the unique solution,

\[
P_t = P_t^f = \frac{R_t}{(1 + r)} + \sum_{i=1}^{\infty} \left( \frac{1}{1 + r} \right)^{i+1} E_t[R_{t+i} | I_t]
\]

is obtained. Under rational expectations, and in the absence of rational speculative price bubbles, the arbitrage condition for the time path of house prices specifies price as a function of contemporaneous market fundamentals and the entire path of future market fundamentals; the value of a house is the present value of the stream of future rents. Thus sharp movements in real house prices result from changes in real rents, either current or revisions of those expected in the future.

Equation (2.7) is analogous to both the present value model for stock valuation that

\[\text{Equation (2.7) is analogous to both the present value model for stock valuation that}\]

\[\text{\footnotesize \cite{50, 51, 52, 53}}\text{discuss the transversality condition and rational deviations of asset prices from fundamental value.}\]
has been used extensively in tests of stock market efficiency and excess volatility, and asset market models of exchange rate determination under rational expectations. It is also consistent with the predictions of recent theoretical models of urban land prices which suggest that a significant component of the price of land is a growth premium due to expected rent increases in the future.

This chapter tests the fundamental rational expectations housing price model in equation (2.7). This rules out rational bubbles as an explanation for the volatile behaviour of short-run house prices. Thus, equation (2.7) is a more restrictive model than (2.6). The latter relationship is robust to rational bubbles, and thus embodies fewer restrictions. Why test the more restrictive asset pricing model?

A comparison of house prices with data on housing market fundamentals indicates that if bubbles do exist, they are related to fundamentals. Figures 2.1 through 2.3 reveal that various measures of economic growth are highly correlated with real house prices. Thus, if prices do race ahead of those dictated by housing market fundamentals it is because agents overreact to news about fundamentals. The effect of rational bubbles is then to weaken the link between the present value of housing market fundamentals and observed price but not destroy it. Thus the data will provide evidence in support of bubbles if the model consistently underpredicts actual prices during the two housing price booms.

Moreover, since tests for bubbles are joint tests of both the null hypothesis of no self-fulfilling deviations from fundamental price and the asset market model, finding apparent evidence of bubbles in the data can always be interpreted as misspecification of the model of market fundamentals.

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6Shiller [140] provides both an introduction to and extensive survey of the stock market literature. Hoffman and Schlagenhau [96] and Finn [53] are examples of forward-looking empirical exchange rate models. See Frankel and Meese [55] for a review of empirical exchange rate models.

7See Capozza and Helsley [17] [18] for example.

8Froot and Obstfeld [60] present an intrinsic stock price bubble model in which the bubble is a deterministic function of market fundamentals (dividends). More precisely, the component of stock price not explained by the fundamental present value model is highly positively correlated with dividends. This model seems more intuitively appealing than rational bubble models which allow the unexplained component to be driven by extraneous information.

9See Flood and Hodrick [54] and Garber [63] for more on this point.
2.2.1 The Dividend to Single-Detached Housing

The asset market housing price model in equation (2.7), specifies the price of a dwelling unit as the capitalized value of current and expected future rents. Therefore, a time series of constant quality rents on single-detached homes is required to test the model. Unfortunately no such series exists. Most resale single-detached housing is owner-occupied and hence the rent is not observed. As noted by Shiller, who also offers a potential solution to this problem, ([140], page 319):

"The owner-occupant of a home earns instead an implicit rent in the form of housing services, on which there is no market valuation. The best proxy for such implicit rents that we appear to have here are rental indexes (computed from data on rental properties)."

Mankiw and Weil [109], and Case and Shiller [23] [24], for example, employ the rent component of the U.S. Consumer Price Index to proxy the value of the flow of housing services from owner-occupied housing. The rental component of the Consumer Price Index (CPI) for the Vancouver Census Metropolitan Area, reported monthly by Statistics Canada, is available over the sample period and is thus a candidate proxy for imputed rents on single-detached homes in this study.

There appear to be a number of problems with rental price indexes which call into question their reliability. Rental properties may differ in quality from owner-occupied housing. In this case indexes derived from rental properties may not be a good proxy for imputed rents. Rent indexes also seem to suffer from statistical problems. Meese and Wallace [115] find apparent inconsistencies in the published rental indexes for both Oakland and San Francisco and resort to constructing a rent series based on asking rents for two-bedroom condos pulled from the newspapers of each city. Mankiw and Weil [109] emphasize their simulation results and not their empirical tests of market efficiency because they believe there are measurement problems with the CPI rent component.

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1090 percent of the existing stock of 5.7 million occupied single-detached homes in Canada are owner-occupied. (Source: Statistics Canada, 1991 Census of Canada, Table 1 in Occupied Private Dwellings: The Nation, catalogue 93-314.

11Mills and Hamilton ([117], Chapter 10) discuss the theory behind why rents on rental properties should be closely related to imputed rents on owner-occupied housing.

12Nancy Wallace made this point in a seminar given at UBC in April 1992.
More importantly for this study, there is a strong belief that the rental component of the Canadian CPI is downward biased and thus not even a true indicator of the time path of rents for rental properties. The principal evidence in support of this claim is that average rents have historically increased at a far greater rate than the CPI rent index. There appears to be a consensus among Canadian housing market analysts that the divergence between the two series is much larger than can be attributed to quality changes.  

Direct evidence for the Vancouver housing market comes from comparing the CPI rent index for the Vancouver CMA with a time series of Vancouver rent estimates published by Royal Lepage Real Estate Services Ltd., a large Canadian real estate brokerage company. Throughout most of the 1982-1991 period, real (inflation adjusted) rents, as measured by the rent component of the Vancouver CPI, exhibit a downward trend while Royal Lepage estimates trend upwards. This reenforces the notion that the CPI rent component is likely a poor measure of quality adjusted rents for rental accommodation.

The lack of confidence in the rental component of the CPI dictates that an alternative specification for real rents on single-detached housing is required. Below, a theoretical model of price adjustment in the market for single-detached housing services is developed and an expression for changes in real imputed rents derived as a function of observable housing market fundamentals.

2.2.2 A Model of Rental Price Adjustment

This section develops a “macroeconomic-type” model of price adjustment in the market for housing services of a metropolitan housing market. A standard equilibrium supply and demand model is augmented with a price adjustment mechanism that incorporates the short-run “disequilibrium” nature of housing markets. A dynamic monocentric city model is derived to indicate which fundamental variables are of primary importance to explaining housing demand and hence should be included in the model specification. The standard stock-flow model assumptions of a homogeneous housing stock and flow of services that is proportional to the standing stock are invoked. The price of

\[ \text{Referring to Fallis [43] and Clayton et al. [28].} \]
housing services is the cost of one unit of service flow from a standardized unit of stock. These simplifying assumptions imply the demand for housing services is equivalent to the demand for housing stock.\footnote{Rosen and Smith [130] discuss the standard stock-flow model framework adopted here.}

Demand for the housing stock, $H^d_t$, is negatively related to the real price of housing services, $R$ and positively related to a vector of exogenous "shift" or nonprice variables $\mathbf{z}$. Nonprice demand factors include permanent income and demographic variables for example. Demand is specified as a linear function of service price and shift variables,

$$H^d_t = \beta_0 - \beta_1 R_t + \mathbf{z}_t^T \beta$$  \hspace{1cm} (2.8)$$

where $\mathbf{z}_t^T$ is the transpose of the vector of non-price variables positively related to housing demand and $\beta$ is a vector of corresponding coefficients.

The existing stock of housing in period $t$, $H^e_t$, is taken to be exogenous, as it is determined by decisions made by housing producers in earlier periods. In the most basic equilibrium housing market model rents are determined by the intersection of stock demand with existing supply,

$$R_t = \frac{\beta_0}{\beta_1} + \frac{\mathbf{z}_t^T \beta}{\beta_1} - \frac{H^e_t}{\beta_1}$$  \hspace{1cm} (2.9)$$

The durability of housing implies that over the short-term the existing housing stock completely dominates any net new supply, so $\Delta H^e_t \approx 0$, and the housing stock can be treated as fixed in the short-run. As a consequence, the expression for short-run rent fluctuations is written

$$\Delta R_t = \Delta \mathbf{z}_t^T \gamma$$  \hspace{1cm} (2.10)$$

where $\gamma = \beta / \beta_1$. Thus, the stock-flow model predicts that rent fluctuations are completely demand driven; supply plays no role in short-run price movements.

However, this simple equilibrium framework ignores the potential effects on housing price dynamics of periods of short-run adjustment in the supply side of the housing market, that take place in response to demand shocks. Evidence of short-run adjustment, to steady state or long-run equilibrium, includes the variability in the time path of rental market vacancy rates, the stock of newly completed but unoccupied homes and the
Multiple Listing Service sales to listings ratio for resale homes, for example. Figure 2.4 plots the quarterly stock of newly completed but unoccupied single and semi-detached dwelling units in the Vancouver metropolitan area, over the past thirteen years. This series exhibits a great deal of variation over time.

A number of studies have shown that changes in real rents on rental properties are a function of the rental apartment vacancy rate relative to its long-run or normal level. 15 Formally, real rental price adjustment is modelled by

$$\Delta \ln R_t = g(V^n - V_t) \tag{2.11}$$

where $g(.)$ is the function that relates rent changes and vacancies, $V_t$ is the actual vacancy rate and $V^n$ is the "normal" or structural vacancy rate. More general models allow $V^n$ to vary over time. Normal vacancies in the rental market arise due to search and matching on the part of households and landlords, as a result of the heterogeneous and indivisible nature of housing units and large transaction costs associated with housing decisions. 16 Vacancy rate fluctuations, about their long-run level, are the mechanism by which the housing market adjusts to changing economic conditions over the business cycle.

The important implication to draw from these models is that the effect of a demand shock on the price of housing services or real rents depends on the state of excess supply. Thus the first difference equilibrium model in equation (2.10), which predicts that demand shocks are fully transmitted into real rents, does not adequately describe the rental price adjustment mechanism. It ignores the contribution of supply-side adjustment forces working to bring the housing market into long-run or steady state equilibrium, following a demand shock. The relative levels of actual and steady state supply convey useful information for short-term price dynamics and must therefore be incorporated into any well specified model.

To model fluctuations in the price of single-detached, owner-occupied housing services I borrow from the intuition underlying the empirical work on rental markets and specify fluctuations in real imputed rents as a function of demand-side fundamentals and the

15 See Rosen and Smith [130] and Gabriel and Nothaft [62].
16 Arnott [2] develops the microeconomic foundations for the existence of vacancies in the rental housing market.
inventory of newly completed but unsold homes relative to its steady state or long-run level. The following specification is adopted:

$$\Delta R_t = \Delta z_t^\gamma - (S_{t-1} - S_{t-1}^n) + u_t$$

(2.12)

where $S_{t-1}$ is the inventory of newly completed but unoccupied single-detached dwelling units at the end of period $t - 1$ and $S_{t-1}^n$ is the time-varying steady state or long-run level of unsold inventory.\(^{17}\) All else equal, as a city grows it is expected to have a larger stock of completed but unoccupied units. $S$ plays the role the vacancy rate does in the models of rental price adjustment on rental properties outlined above.

The model of real rent fluctuations differs from the simple first-difference stock-flow model by the inclusion of unsold inventories relative to their steady state level. This term plays an important role, ensuring that the link between fluctuations in market fundamentals affecting demand and changes in rents depends critically on the state of excess supply in the housing stock.\(^{18}\)

What variables should be included in $\Delta x_t$? That is, what non-price housing demand factors are of primary importance to the local housing market in the short-run. This question is answered via a dynamic open-city growth model derived below.

2.2.3 Fundamentals, Growth and Uncertainty

This section presents a model of a growing city that highlights the relationship between the price of housing services and economic growth. The model aims to provide a sound theoretical basis for proxying the demand side component of changes in imputed rents

\(^{17}\)The existence of inventories is not inconsistent with rational expectations on the part of housing producers. Recent theoretical work by Grenadier [77] aims to explain the causes of over and underbuilding in the market for new real estate. Grenadier concludes that the length of construction time, changes in costs of adjustment and increases in demand volatility are the three primary factors that contribute to the phenomenon of overbuilding or bringing new product to a weak real estate market. More importantly, overbuilding is not necessarily the result of myopic behaviour on the part of developers but "rather the result of optimal decision making under increasingly difficult conditions," ([77], pages 5-6). I note this point because it would be quite inconsistent to test a rational expectations housing price model within a framework in which an important segment of housing market participants were assumed to be irrational.

\(^{18}\)The simple linear specification above is only one possibility. Due to the lack of a theoretical foundation for any particular functional form, a number of model specifications are entertained in the empirical estimation to ensure that any conclusions drawn are robust to the model proxying imputed rents.
of owner-occupied housing, by variables related to the strength of economic growth in a local economy.

The Model

Consider a small open monocentric (circular) urban area which is located on a flat, featureless plain. In this city a single good is produced by competitive firms for export to an expanding world market. The city is part of a small open economy so the output price is determined exogenously on world markets. All production takes place at the city centre. The opportunity cost of land is assumed to be zero.

In period $t$ the city is inhabited by $N_t$ identical households. Each household occupies one unit of housing, which is a composite good comprised of 1 unit of land plus structure. One member of each household must commute daily to the city centre to his or her place of employment. All commuting takes place in a straight line from workers’ residences to the city centre at a round trip cost of $\$T$ per unit distance. Locations are indexed by their linear distance $z$ from the city centre. All residents are risk neutral and have the identical utility function

$$U_t = W_t - R_t(z) - Tz$$

where $U_t$ is utility and $W_t$ is the real wage (employment income) in period $t$ while $R_t(z)$ is the real price or rent paid for housing services at location $z$ in period $t$. All right-hand side values are measured in terms of a numeraire nonhousing consumption good.

Households compete for locations close to the city centre to minimize commuting costs. In equilibrium all households must attain the same level of utility since they are

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19 The term open implies that this city is part of an interrelated system of urban areas and migration between cities is costless.

20 Mills [116] presents a two-period general equilibrium model in which shocks to the export sector directly impact land rent and land use in a monocentric city. Capozza and Helsley [18] examine the impact of bid rent uncertainty on land prices, development timing and city size. The model developed here borrows from the methodology of each of these papers.

21 These are standard assumptions of monocentric city models of intraurban household location. See for example Mills and Hamilton ([117], Chapter 6) and Henderson ([91], Chapter 1). The monocentric city model forms the foundation for much of modern urban economics.

22 A linear utility function is used for convenience. The results of this section are unchanged if households have Leontief preferences defined over land and a composite nonhousing consumption good, whose price does not vary spatially.
identical. Thus equilibrium is characterized by the condition \( \frac{\partial U}{\partial z} = 0 \), that is, utility is independent of location. Therefore, from (2.13), \( R_t'(z) = -T \), which is the standard spatial equilibrium condition.²³

This implies a linear bid function for housing services of the form \( R_t(z) = R_t(0) - Tz \), where \( R_t(0) \) is the price of housing at the city centre. Since the opportunity cost of land is zero, the boundary of the city is defined by the condition \( R_t(z_t^*) = 0 \), where \( z_t^* \) is the radius of the city. This allows us to solve for the value of the housing price function at the city centre as \( R_t(0) = Tz_t^* \), and thus the bid function for housing is:

\[
R_t(z) = T(z_t^* - z) \tag{2.14}
\]

Market clearing in the residential housing market requires that the supply of housing be equal to the demand. The supply of urban housing in the circular city in period \( t \) is \( \pi(z_t^*)^2 \) while the demand is simply \( N_t \), the number of households. Therefore market clearing requires that

\[
z_t^* = \left( \frac{N_t}{\pi} \right)^{1/2} \tag{2.15}
\]

Substitute this into (2.14) to obtain

\[
R_t(z) = T \left[ (N_t/\pi)^{1/2} - z \right] \tag{2.16}
\]

The flow price of housing is positively related to city-size, as measured by the number of households or employment, which are equivalent in this full employment model. Moreover, if commuting costs are relatively unchanged over time then any variation in housing prices, at each location, is entirely a result of changes in city size.

In equilibrium all households obtain equal utility so that

\[
W_t - R_t(z) - Tz = \bar{U}_t \tag{2.17}
\]

where \( \bar{U}_t \) is the exogenously determined utility available to households if they locate in a different city. Substitute for rents in (2.17) from (2.16) and solve for \( N_t \) to obtain the full employment equilibrium condition

\[
N_t = \pi \left( \frac{W_t - \bar{U}_t}{T} \right)^2 \tag{2.18}
\]

²³Again, see the textbooks by Mills and Hamilton [117] and/or Henderson [91].
Equilibrium employment is a positive function of the wage offered in this city.

Assume production of the export good requires only labour input and takes place with fixed factor proportions; $\lambda$ units of labour are required per unit output. Production sector equilibrium is characterized by zero profits so the per unit cost of production must equal the output price; $C(W_t) = \lambda W_t = q_t$, where $C(.)$ is the cost function and $q$ is the world price of the export good. This yields

$$W_t = q_t / \lambda$$

It is now easy to see how shocks to world demand for this city's output are transmitted back to the housing market. An increase in demand raises the output price which increases the demand for labour and the equilibrium wage. A rise in the wage increases the utility offered by the city above that attainable elsewhere in the country. This in turn increases employment or the number of households as newcomers are attracted to the city. The increase in population puts upward pressure on the price of housing services. House prices must rise until utility is again back to $\bar{U}$, at which time the system is back in long-run equilibrium.

**Solving the Model**

In order to derive the stochastic process for the price of housing services the process driving the price of the export good, $q_t$, must be specified. A law of motion for output price implies a process for wages which then implies a process for employment, which in turn gives the process driving rents. Assume that the future price of the export good is uncertain and follows a geometric Brownian motion, so that its behaviour is described by the stochastic differential equation

$$dq = \alpha q dt + \sigma q dy$$

where $\alpha$ and $\sigma$ are positive constants and $dy$ is a standard Brownian motion or Wiener process with zero drift. Under this specification, the percentage change in output price follows a random walk with positive drift, $\alpha$, and is log-normally distributed.

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24 Again, this simple specification is chosen for convenience. The qualitative results derived below also hold when production takes place with both labour and capital according to a Cobb-Douglas production technology.
Equations (2.19) and (2.20) imply that wages also follow a geometric Brownian motion. That is, since \( dW = (1/\lambda) dq \),
\[
dW = \alpha \left( \frac{q}{\lambda} \right) dt + \sigma \left( \frac{q}{\lambda} \right) dy = \alpha W dt + \sigma W dy
\]
(2.21)
If equilibrium utility and commuting costs are time invariant, then from (2.17),
\[
dR_t(z) = dW_t
\]
(2.22)
The equilibrium flow price of housing follows the same stochastic process as wages and the output price. To derive the stochastic relationship between flow house prices and city size linearize the equilibrium relation (2.18), that relates population and wages. To do this, assume that \( \bar{U}_t \) is constant and normalize it to zero. Taking logs of both sides of (2.18) yields
\[
\ln(N_t) = \ln(\pi) - 2\ln(T) + 2\ln(W_t)
\]
(2.23)
Therefore, \( d\ln(N_t) = 2d\ln(W_t) \). The process driving \( \ln(W_t) \) must be determined to derive the stochastic process followed by \( N_t \). Apply Ito’s Lemma to (2.21) to obtain
\[
d\ln(W) = (\alpha - \sigma^2/2) dt + \sigma dy
\]
(2.24)
and therefore \( \ln(N_t) \) follows the stochastic process
\[
d\ln(N) = (2\alpha - \sigma^2) dt + 2\sigma dy
\]
(2.25)
The logarithm of employment is governed by a standard Brownian motion with drift parameter \( (2\alpha - \sigma^2) \). And since \( d\ln R_t = d\ln W_t \) fluctuations in the real price if housing services are given by
\[
d\ln[R(z)] = d\ln(N)/2
\]
(2.26)
This result states that housing services price inflation is directly related to the percentage change in the number of households or employment growth. In this model, by definition, the population of households can only change through in-migration. The implication of the simple growth model is that imputed rents are a positive function of economic growth and that either employment growth or in-migration can be used to proxy changes in imputed rent.
2.2.4 The Fundamental Market Model

Combine the results of the dynamic growth model (equation (2.26)) with the rental price adjustment model (equation (2.12)) to obtain the empirical model of price adjustment in the market for single-detached housing services,

\[ \Delta R_t = \theta_1 M_t - \theta_2 ES_{t-1} \]

(2.27)

where \( M_t \) is net in-migration of households and \( ES_{t-1} = S_{t-1} - S^n_{t-1} \) is the excess supply of new housing at the end of period \( t-1 \) or beginning of period \( t \).\(^{25}\) The price of housing services is a linear function of two variables, one representing housing demand forces and the other housing supply considerations.\(^{26}\) Throughout the remainder of the chapter this model is referred to as the fundamental market model.

2.3 Econometric Methods and Data

This section derives the estimable form of the reduced form housing price model, outlines the econometric techniques employed to test the rational expectations hypothesis, and describes the data.

The fundamental market model (2.27) represents a proxy for fluctuations in unobservable imputed rents on owner-occupied housing, not the level of rents. As a consequence, the present value housing price relation (2.7) must be specified in first-difference form, given by

\[ \Delta P_t = \frac{\Delta R_t}{1 + r} + \sum_{i=1}^{\infty} \left( \frac{1}{1 + r} \right)^i \Delta E_t[R_{t+i} | I_t] \]

(2.28)

This specification creates two potential problems. First, differencing the model may lead to "overdifferencing". If the raw (or undifferenced) house price and rent series are either stationary or trend stationary (TS) processes then differencing is inappropriate. If, however, the levels of real house prices and rents are integrated, or difference stationary (DS), series then differencing is required for valid estimation and inference.

\(^{25}\)Section 2.3.2, below, explains the choice of immigration as the demand side variable and derives a measure of excess supply.

\(^{26}\)This is admittedly a very simple specification of real rent fluctuations. A comparison of figures 2.1 and 2.3 reveals, however, that immigration is an important factor in Vancouver housing price cycles.
A stationary series has constant unconditional first and second moments.\textsuperscript{27} If a time series can be expressed as a deterministic function of time plus a mean zero stationary process then it is trend stationary.\textsuperscript{28} Trend stationary (TS) series are nonstationary because their means depend on time. We detrend a TS series by regressing it on time and a constant. The resulting residuals are a stationary process that we can interpret as the cyclical component of the original time series.\textsuperscript{29}

Alternatively, the levels of real house prices and rents may be nonstationary due to the presence of a unit root in their autoregressive polynomials. In this case differencing is required to achieve stationarity. Series that must be first-differenced to be stationary are said to be integrated of order one, written I(1).\textsuperscript{30} Moreover, in this situation differencing is a prerequisite to meaningful inference since a regression involving the levels of nonstationary series can produce results that appear to strongly support the model specification under test, as evidenced by a large $R^2$ and high $t$-statistics, when in fact the estimation results are “nonsense” or “spurious”.\textsuperscript{31} Thus, prior to estimation it is important to determine whether the variables are trend or difference stationary. Section 3.3 accomplishes this task.

Hodrick ([93], page 37) points out a second potential problem with differencing the present value relation. He shows that differencing implies that we test a weaker version of the rational expectations hypothesis than the present value model in equation (2.7).

\textsuperscript{27}A time series is weakly, or covariance, stationary (usually just called stationary) if it satisfies the following conditions: (i) the mean is independent of time; (ii) the variance is independent of time; and (iii) the covariance between different values of the series are independent of time. For example $x_t$ is stationary if $E[x_t] = \alpha$ and $\text{Var}[x_t] = \sigma^2$, for all $t$, and $\text{Cov}[x_t, x_{t-k}] = \rho_k$, for all $t$ and $k$.

\textsuperscript{28}If $x_t$ is trend stationary (TS) then, for the linear case, $x_t = \alpha + \beta t + \epsilon_t$, where $\epsilon_t$ is a stationary random variable. $x$ has both a deterministic trend, $\beta t$, and a cyclical component, $\epsilon_t$.

\textsuperscript{29}First differencing the deterministic trend model, $x_t = \alpha + \beta t + \epsilon_t$, yields $x_t - x_{t-1} = \beta + \epsilon_t - \epsilon_{t-1}$, which eliminates the trend but results in a highly autocorrelated series. More precisely, it eliminates the trend but leads to a noninvertible moving average (MA(1)) process.

\textsuperscript{30}The simplest example of an integrated series is the random walk process, $x_t = \mu + x_{t-1} + \epsilon_t$, where $\mu$ is a constant and $\epsilon$ is white noise with variance $\sigma^2$. We can rewrite this as $(1 - L)x_t = \mu + \epsilon_t$, where $L$ is the lag operator. $x$ is said to have a unit root because the characteristic equation, $(1-z) = 0$, has a single root equal to one. After successive backwards substitution $x_t$ can be written as $x_t = x_0 + \mu t + \sum_{i=1}^{t} \epsilon_i$ and thus $E[x_t] = \mu t$ and $V[x_t] = t \sigma^2$; the variance and mean increase over time without bound. Hence, the series is nonstationary. Notice, however, that first-differencing renders the series stationary.

\textsuperscript{31}See Granger and Newbold [74] for convincing evidence of this. The cause of this phenomenon is that the variance of an integrated time series explodes over time.

32
To see this, take first differences of both sides of equation (2.5) to produce

\[ P_t - P_{t-1} = (1 + r)^{-1} (R_t - R_{t-1} + E_t[P_{t+1} \mid I_t] - E_{t-1}[P_t \mid I_{t-1}]) \]  

(2.29)

Use the law of iterated expectations to rewrite this as

\[ P_t - P_{t-1} = (1 + r)^{-1} (R_t - R_{t-1} + E_{t-1}[P_{t+1} - P_t \mid I_{t-1}]) \]  

(2.30)

This restriction is weaker than the levels model in (2.5) because it does not capture the time \( t \) restriction, \( P_t = (1 + r)^{-1} (R_t + E_t[P_{t+1} \mid I_t]) \). Hodrick [93] notes that because the first difference model provides a weaker test, strongly rejecting it implies we reject the rational expectations restrictions. Empirical tests of the first-difference model may be misleading though, if they do not reject or only provide weak evidence against the rational expectations hypothesis. That is, not rejecting the first-differenced rational expectations model does not necessarily imply we do not reject the levels specification.

2.3.1 Rational Expectations in the Asset Market Model

The fundamental rational expectations housing price model (2.28) is not estimable due to the presence of expected values of future rents on the right-hand side. To operationalize the model (i.e. derive a form which is suitable for estimation) the following steps are taken: the stochastic process generating real rents is determined and used to derive a conditional forecasting equation for future real rents. This is used to replace the expected (unobservable) future values. In order to illustrate both the solution technique and testing procedures suppose that observations on true imputed rents, \( R_t \), are available. For expositional purposes assume that \( \Delta R_t \) is a stationary fourth-order autoregressive process AR(4), which is written as

\[ \Delta R_t = \alpha_1 \Delta R_{t-1} + \alpha_2 \Delta R_{t-2} + \alpha_3 \Delta R_{t-3} + \alpha_4 \Delta R_{t-4} + u_{t,i} \]  

(2.31)

To arrive at an estimable model the univariate time series model for real rents is used to produce forecasts for the infinite sum of first-differenced expected future real rents, \( \Delta E_t[R_{t+i} \mid I_t] \) in (2.28). The expected value in period \( t + i \) is replaced by it's \( i \)-period

\[ \Delta E_t[R_{t+i} \mid I_t] \]
ahead forecast from the AR(4) model according to the following result derived by Finn ([53], page 190): If $\Delta x_t$ is a weakly stationary variable generated by an autoregressive process of order $p$, represented by

$$\alpha(L)\Delta x_t = e_t$$

(2.32)

with

$$\alpha(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \ldots - \alpha_p L^p$$

where $L$ is the lag operator, $\alpha_i$, $i = 1, \ldots, p$, is a parameter and $e_t$ is a white noise disturbance term, then

$$\sum_{i=0}^{\infty} c^i E_t \Delta x_{t+i+1} | I_t = \alpha(c)^{-1} \left[ 1 + \sum_{i=1}^{p-1} \left( \sum_{k=i+1}^{p} c^{k-i} \alpha_k \right) L^i \right] \Delta x_t + \left( \frac{c}{1-c} \right) \alpha(c)^{-1} \alpha(L) \Delta x_t$$

(2.33)

where $c$ is a scalar less than one in absolute value.

Finn's [53] result, applied to the present value pricing model (2.28), after collecting terms and simplifying, yields

$$\Delta P_t = \left[ \alpha \left( \frac{1}{1+r} \right)^{-1} \left( \frac{1+r}{r} \right) - 1 \right] \Delta R_t$$

(2.34)

$$+ \alpha \left( \frac{1}{1+r} \right)^{-1} \left[ \left( \frac{\alpha_2}{1+r} + \frac{\alpha_3}{(1+r)^2} + \frac{\alpha_4}{(1+r)^3} - \frac{\alpha_1}{r} \right) \Delta R_{t-1} + \left( \frac{\alpha_3}{1+r} + \frac{\alpha_4}{(1+r)^2} - \frac{\alpha_2}{r} \right) \Delta R_{t-2} + \left( \frac{\alpha_4}{1+r} - \frac{\alpha_3}{r} \right) \Delta R_{t-3} - \frac{\alpha_4}{r} \Delta R_{t-4} \right]$$

Given the process driving real rents, rational expectations implies a stochastic process for house prices and imposes the cross-equation restriction that $\alpha_i$, $i = 1, \ldots, 4$ are the same in both equation (2.31), the data generating process for real rents, and equation (2.34), the reduced form housing price model. Real house price movements depend on current supply and demand forces in the market for housing services, reflected in $\Delta R_t$, and a number of lags in the price of housing services, which in part reflect the stochastic process that governs $\Delta R_t$.

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33 Finn's [53] formula is an extension of the well known Hansen and Sargent ([82], page 99, equation (8)) prediction formula. See also Sargent ([134], Chapter XI, section 19).
Estimation of the rational expectations model involves joint nonlinear estimation of the system of equations (2.31) and (2.34) to account for both the nonlinearities in (2.34) and the cross equation restrictions.

To derive the restrictions imposed by the rational expectations hypothesis, compare the coefficients in (2.34) to those in an unrestricted variant of the reduced form housing price model, written as

\[ \Delta P_t = \mu_0 \Delta R_t + \mu_1 \Delta R_{t-1} + \mu_2 \Delta R_{t-2} + \mu_3 \Delta R_{t-3} + \mu_4 \Delta R_{t-4} + \nu_{1t} \quad (2.35) \]

Comparing equations (2.34) and (2.35) we see that the coefficients in the unrestricted model are nonlinear functions of the discount rate and the parameters of the time series model of rent changes, if the cross equation restrictions are satisfied. For example, the rational expectations hypothesis implies that \( \mu_0(\alpha_1, \alpha_2, \alpha_3, \alpha_4, r) = \alpha \left( \frac{1}{1+r} \right)^{-1} \left( \frac{1+r}{r} \right) - 1 \). Equation (2.34) expresses five coefficients in terms of one parameter, \( r \), not identified in the forecasting equation and thus there are 4 parameter restrictions associated with the restricted housing price model.

To test the rational expectations cross equation restrictions both the constrained system, given by (2.31) and (2.34), and the unconstrained system, given by (2.31) and (2.35) are estimated. A likelihood ratio test is used to test the cross equation restrictions. Under the null hypothesis of rational expectations, minus two times the difference in the maximized values of the log likelihood functions has a chi-squared distribution with four degrees of freedom.

In order to derive the empirical model with rents proxied by a linear combination of in-migration and the deviation of inventory from its steady state value two issues remain to be considered. First, a measure of \( S_t - S_t^a \) must be derived, and second, a method of forecasting the exogenous variables determined. Both considerations hinge upon the time series properties of the data.

2.3.2 The Data

This section briefly describes the data series used in the study. Appendix I provides a more detailed description of the data and data sources.
$P_t$, is a quarterly, constant quality time series of single-detached resale house prices in the city of Vancouver, British Columbia, divided by the all items Consumer Price Index (1986=100), excluding shelter, for the Vancouver Census Metropolitan Area. Hamilton and Hobden [80] construct a quality adjusted house price series for single-detached homes on the Westside of the City of Vancouver which covers the 1979:1 1991:9 sample period. The authors provided me with their house price series for this study. They derive the quality adjusted series from a sample of more than 60,000 transactions that took place in the city of Vancouver through over the 1979-1991 sample period. The British Columbia Assessment Authority provided the raw data.

Quality is controlled for via hedonic regression techniques. The hedonic attributes included for each house in the regressions are: lot size (square feet), age of the dwelling (years), floor space (square feet), number of bedrooms, number of bathrooms, number of fireplaces and location. Location is a dummy variable which takes on a value of 1 if the house is located on the westside of the city and 0 if it is located on the eastside. 39 percent of the sample is comprised of westside homes. Hamilton and Hobden [80] run 153 separate regressions, one for each month in the sample period, which allows the hedonic shadow prices to vary over time. They develop a time series for the asset value of a house with the mean attributes, excluding location, in the city of Vancouver over the sample period with a 1 for location.

Quarterly net in-migration, $M_t$, into British Columbia is used to capture short-run demand fluctuations in housing demand. $S_t$ is the number of newly completed but unoccupied single-detached homes for the Vancouver Census Metropolitan Area, reported by Canada Mortgage and Housing Corporation.

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34 See Hamilton and Hobden [80] for a complete description of the data base and quality adjustment methodology. $P_t$ is a 3 month moving average of Hamilton's monthly series.

35 Employment data for the Vancouver CMA is available monthly but metropolitan level data series are derived from relatively small samples and, as a consequence, short-term changes in these series may be spurious. Thus I use net-inmigration in the market fundamental model to proxy the demand for housing services.
2.3.3 Unit Root Tests

This section presents the results of unit root tests. Unit root tests are used to help determine whether the data series are trend or difference stationary. Stationarity is a prerequisite for the forecasting techniques employed below to generate predictions of future values of the exogenous variables, net in-migration and excess supply. This section also uses the unit root test results to help derive a measure of the excess inventory of new single-detached homes.

To test for the presence of a unit root in a time series \( x_t \) the model:

\[
\Delta x_t = (1 - \rho)x_{t-1} + \varepsilon_t
\]  

(2.36)

is estimated. If \( x_t \) contains a unit root (is integrated of order one) then \( \rho = 1 \) and the coefficient on the lagged \( x \) value is zero. Thus a test for a unit root is a test of the statistical significance of the coefficient estimate on \( x_{t-1} \); the null hypothesis is a unit root \( (H_0 : (1 - \rho) = 0) \), while the alternative, \( H_1 \), is \( (1 - \rho) < 0 \). Inference based on conventional \( F \) and \( t \) statistics is not valid in the presence of a unit root because they no longer have standard limiting distributions, and the ordinary least squares estimate of \( \rho \) is biased downwards. 36

The basic unit root regression is easily modified to handle more general cases. A constant term can be included in each equation, in which case the alternative hypothesis is stationarity around a constant mean, and both a constant term and time trend can be employed to determine whether a series is trend or difference stationary.

Statistical tests for unit roots have been developed by Dickey and Fuller [35] and Phillips and Perron ([122], [123]) to test the null hypotheses \( H_0 : \rho = 1 \). Critical values have been derived, using Monte Carlo methods, and tabulated by Dickey and Fuller [35]. Dickey-Fuller tests involve running the above OLS regression with lags of the dependent variable included as regressors to ensure that the errors are serially uncorrelated. The standard \( t \) statistic is calculated and compared to the appropriate Dickey-Fuller critical

\[\text{36To see that the ordinary least squares (OLS) is biased downwards, consider the AR(1) model,} \]

\[x_t = \rho x_{t-1} + \varepsilon_t\]  

The OLS estimator of \( \rho \), is \( \hat{\rho} = \frac{\sum_{t=2}^{T} x_t x_{t-1}}{\sum_{t=2}^{T} x_{t-1}^2} \). If \( \rho \) is one then we should expect successive values to be very close to one another and thus \( \hat{\rho} \) should be approximately one. But since \( V[x_t] = t \sigma_x^2 \) increases over time successive values of \( x \) are not necessarily close together. See Davidson and MacKinnon [32] for more details.
value. Rather than include lagged values of the dependent variable as regressors, Phillips-
Perron suggest a nonparametric adjustment to the standard t statistic to account for both
serial correlation and/or heteroskedasticity in the residuals. The critical values are those
tabulated by Dickey and Fuller.

Table 2.1 reports unit root test results for the level and first difference of real house
prices, net in-migration and the stock of newly completed but unsold homes. The last
column provides asymptotic critical values at the 10 percent significance level.

Both unit root test statistics are greater than the asymptotic Dickey-Fuller critical
values at the 5 percent significance level for the level of real house prices. Thus, the null
hypothesis of a unit root in real quarterly Vancouver house prices cannot be rejected.
The test statistics for the first-differenced house price series indicate that prices con-
tain a single unit root. Net provincial inmigration, the stock of newly completed but
unoccupied homes and the first difference of real house prices all appear to be stationary.

The form of nonstationarity of real house prices dictates that the housing price model
be estimated in first differences. This is consistent with the proposed model specification
in equation (2.28).

The inventory of unsold homes, \( S_t \), appears to be a trend stationary process. Hence, it
can be expressed as a deterministic function of time. For the linear case, \( S_t = \alpha + \beta t + u_t \),
where \( \alpha \) and \( \beta \) are scalars, \( t \) is time and \( u \) is a stationary error term. The inventory of
newly completed but unsold homes fluctuates about the long-run trend captured by
\( \alpha + \beta t \). This implies that deviations from the long-run trend are captured by the error
term, \( u_t \), which is estimated as the ordinary least squares residual from a regression of
\( S_t \) on a constant and a time trend.

---

Unit root tests are conducted on the monthly time series of house prices to provide more observations.
Unit root test regressions for the levels data include both a constant and a time trend. Only a constant
is included in the test regression for the first differenced house price series. Immigration data is available
quarterly from 1960. Shiller and Perron [142] provide evidence that the power of unit root tests increases
significantly with the span or time frame covered by the data. Thus one should use all data available.

The finding of a unit root in real house prices is consistent with the results of earlier work. Meese
and Wallace [115] cannot reject the null hypothesis of a unit root in 14 of 16 real house price indexes
ey they develop for municipalities in the San Francisco Bay area. It is also consistent with the outcome of
unit root tests on real national U.S. house prices over the 1947-1988 sample period reported by Holland
[97].

This implies that raw (undifferenced) Vancouver house prices exhibit both a stochastic trend and
stochastic cyclical component.
2.3.4 Transformations of Market Fundamental Data

The stock of newly completed but unoccupied homes is a trend stationary process. It trends upwards over time and does not have a unit root. I use this property to derive a measure of excess supply. To model deviations in inventory about its steady state path I use residuals from a regression of $S_t$ on a time trend. Define this new variable as $ES_t = S_t - S_t^*$, to denote excess supply.

Net-inmigration is "filtered" to remove seasonal effects. A plot of net in-migration, as well as its autocorrelation function, reveal strong seasonal effects. To eliminate seasonality the series is filtered by regressing it on three quarterly dummy variables. The results indicate a significant negative coefficient on the first quarter dummy. The residuals from the least squares regression are the filtered data. Similar regressions for the price and inventory series do not reveal any significant seasonal influences and thus no prefiltering is conducted on these series.

2.3.5 Forecasting the Exogenous Variables

This section specifies the stochastic processes for the exogeneous variables, $M_t$ and $ES_t$, to determine forecasting models for each and subsequently operationalize the housing price model.

Univariate forecasting models are derived, via Box-Jenkins [9] techniques, to obtain observable expressions for the unobservable future expected values of the exogenous variables, $M_t$ and $ES_t$, in the present value model. The technique of Box and Jenkins [9] involves identification of a potential model from the auto and partial correlation functions of a series, estimation of the proposed model and diagnostic checking to ensure the

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40In order to determine the appropriate forecasting technique, univariate or multivariate, a vector autoregression (VAR) comprising net-inmigration and excess supply variables was estimated. Five lags of each variable were included in the regressions. The contemporaneous correlation of the residuals across the two equations was not statistically different from zero at conventional significance levels. A sequence of joint F-tests revealed that neither variable Granger-causes the other. The implication is that univariate forecasting models are suitable for the purpose at hand. In addition, the condition that $\Delta P_t$ does not Granger cause either $M_t$ or $ES_t$ cannot be rejected at conventional significance levels. Thus there is no feedback from the dependent variable to the explanatory variables and net-in-migration and excess supply can be considered exogeneous with respect to the first-difference of house prices. Previous research has found evidence that high levels of house prices have a negative effect on labour supply (see Case [21] and the references cited there).
model is adequate. Negative diagnostics entail a repeat of the three steps. The identification stage identifies filtered net-inmigration and the excess stock of completed but unoccupied single-family homes as AR(5) and AR(2) processes, respectively. Formally, the model specifications are given by

$$M_t = a_0 + a_1 M_{t-1} + a_2 M_{t-2} + a_3 M_{t-3} + a_4 M_{t-4} + a_5 M_{t-5} + u_{2,t}$$  \hspace{1cm} (2.37)$$

$$ES_{t-1} = b_0 + b_1 ES_{t-2} + b_2 ES_{t-3} + u_{3,t}$$  \hspace{1cm} (2.38)$$
or, in more compact notation,

$$A(L)M_t = a_0 + u_{2,t}$$

$$B(L)ES_{t-1} = b_0 + u_{3,t}$$

where

$$A(L) = 1 - a_1 L - a_2 L^2 - a_3 L^3 - a_4 L^4 - a_5 L^5$$

$$B(L) = 1 - b_1 L - b_2 L^2$$

Table 2.2 reports parameter estimates and tests of model adequacy, for each univariate model. $Q$ is the Box-Pierce-Ljung test statistic for a test of the joint significance of the first 8 sample autocorrelations of the residuals. Asymptotically $Q$ has a $\chi^2$ distribution with 8 degrees of freedom. The 5 percent critical value is 15.5 and therefore the null hypothesis of independent errors cannot be rejected in each case. This result is robust to alternative lag lengths.

When real rents are proxied by a linear combination of net-inmigration and excess inventory the estimable form of the rational expectations housing price model is

$$\Delta P_t = A \left( \frac{1}{1 + r} \right)^{-1} \left( \frac{1 + r}{r} \right) - 1 \right] \theta_1 M_t - B \left( \frac{1}{1 + r} \right)^{-1} \left( \frac{1 + r}{r} \right) - 1 \right] \theta_2 ES_{t-1}$$

$$+ A \left( \frac{1}{1 + r} \right)^{-1} \left( a_2 \frac{1}{1 + r} + a_3 \frac{(1 + r)^2}{(1 + r)^2} + a_4 \frac{(1 + r)^3}{(1 + r)^3} + a_5 \frac{(1 + r)^4}{(1 + r)^4} - a_1 \right) \theta_1 M_{t-1}$$

$$+ \left( a_3 \frac{1}{1 + r} + a_4 \frac{(1 + r)^2}{(1 + r)^2} + a_5 \frac{(1 + r)^3}{(1 + r)^3} - a_2 \right) \theta_1 M_{t-2} + \left( a_4 \frac{1}{1 + r} + a_5 \frac{(1 + r)^2}{(1 + r)^2} - a_3 \right) \theta_1 M_{t-3}$$

\footnote{Harvey [88] provides a nice exposition of the Box-Jenkins methodology.}
\[ + \left( \frac{a_5}{1+r} - \frac{a_4}{r} \right) \theta_1 M_{t-4} - \frac{a_2}{r} \theta_1 M_{t-5} \right] - B \left( \frac{1}{1+r} \right)^{-1} \left[ \left( \frac{b_2}{1+r} - \frac{b_1}{r} \right) \theta_2 ES_{t-2} - \frac{b_2}{r} \theta_2 ES_{t-3} \right] \] (2.39)

The unrestricted version is expressed as

\[ \Delta P_t = \eta_0 M_t + \eta_1 M_{t-1} + \eta_2 M_{t-2} + \eta_3 M_{t-3} + \eta_4 M_{t-4} + \eta_5 M_{t-5} - \eta_6 ES_{t-1} - \eta_7 ES_{t-2} - \eta_8 ES_{t-3} + v_{2t} \] (2.40)

A test of the rational expectations hypothesis is a test of the nonlinear restrictions across the reduced form housing price model (2.39) and the models governing the stochastic process of imputed rents, (2.37) and (2.38). For example, under the null hypothesis \( \eta_0 = A \left( \frac{1}{1+r} \right)^{-1} \left( \frac{1+r}{r} \right) - 1. \)

Equation (2.39) is referred to as 'the rational expectations' solution to the asset market model of housing price determination. Similarly, tests of the cross equation coefficient restrictions are labelled as tests of the rational expectations hypothesis. It is important to remember that rejecting these restrictions does not necessarily mean rejecting the hypothesis of rational expectations. The estimable housing price models embody the joint assumptions of rational expectations, a particular asset market model and stable processes driving the exogenous variables or no process switching. The last assumption implies that the future will be like the past so univariate time series models of the exogenous variables can be used to predict future values; the stochastic processes generating the fundamentals remain stable over time. If the cross equation restrictions are rejected then the theoretical housing price model and the processes driving fundamentals must be questioned prior to claiming house price expectations are nonrational.

### 2.4 Empirical Results

Prior to testing the rational expectations restrictions it is important to determine if the residuals in the reduced form housing price equation satisfy the classical assumptions; independent, normally distributed with zero mean and constant variance. Violations of any of the above properties prevents efficient estimation of the parameters in the rational
expectations system if a nonlinear system estimation method that assumes errors of this type is adopted. The parameters of the unrestricted house price model (2.40) are consistently estimated by ordinary least squares (OLS).

Table 2.3 presents OLS estimates of the parameters of the constant discount rate rational expectations housing price model (2.40). Real house price changes are positively related to current in-migration and negatively related to current excess supply, just as the theory predicts. None of the lagged in-migration terms are statistically significant, while the first two lags in excess supply are statistically different from zero. Thus, there is little empirical support for forward looking behaviour with respect to demand-side housing market fundamentals, in the model specification employed here. 42

Figure 2.5 illustrates the fit of the model. It shows both the actual and fitted values of real house price changes. The unrestricted constant discount rate model explains about 50 percent of the variation in real house prices, and generally tracks observed prices quite well, with the exception of the two boom and bust episodes. The model both underpredicts the magnitude of price increases and decreases, although it does a nice job of capturing the timing of peaks and troughs, especially in the boom of the early 1980s.

There is no evidence of first order autocorrelation based on the Durbin Watson (DW) statistic. A plot of the residuals versus time, however, indicates a distinct cycle throughout the sample period, which is indicative of second-order autocorrelation. The modified Box-Pierce test statistic in table 2.3 confirms the existence of AR(2) errors.

Conditional on having the true model specification, serial correlation implies a rejection of the rational expectations hypothesis because, even after accounting for market fundamentals, real house price changes have a predictable component. On the other hand, the presence of serial correlation could be due to model misspecification as a result of either one or more omitted variables, incorrect functional form or misspecified dynamics. Thus before the rational expectations hypothesis is rejected these possibilities must

42The rental component of the Vancouver CPI is identified as an AR(4) process. OLS estimation of (33), in which the CPI rental component proxies imputed rents, confirms the anecdotal evidence concerning the downward bias in CPI rents. $R^2$ is 0.087, most of the coefficient estimates are negative and none of the 4 is statistically different from zero at conventional significance levels. Either house prices have no relation whatsoever to market fundamentals or the rental component of the Vancouver CPI is a very poor proxy for imputed rents on owner-occupied single-detached homes. I claim that the latter interpretation is the correct one.
be considered.

An obvious assumption of the model to challenge is the constant discount rate. This assumption may be particularly inappropriate for the housing market given the important role of mortgage financing in housing purchases. In the next section the present value model is modified to allow for time varying expected rates of return or real mortgage rates.43

2.5 Extending the Model: Time Varying Discount Rates

A constant discount rate was assumed to derive the present value model tested above. This was done to simplify the derivation of the reduced form pricing. There is evidence, however, that expected real costs of capital, real interest rates, are not constant over the sample period. Ex-post quarterly real mortgage interest rates, calculated as nominal mortgage rates minus actual inflation, had a mean of 7.1 percent and a variance of 3.66 over the 1979-1991 sample period. Moreover, real rates ranged from a low of 1.65 to a high of over 10 percent. Thus a time-varying opportunity cost of money could be responsible for the poor diagnostics associated with the asset-based housing price model.

The important role played by mortgage financing in housing transactions reinforces this interpretation of the results. Most households require mortgage financing to purchase a home, as a result of the large purchase price and indivisible nature of the housing asset. The stock demand for housing may therefore be very sensitive to even small movements in the real mortgage interest rate.

This section modifies the present value housing price model to include time varying real mortgage rates. Real rates are approximated by the ex-post real rate; the current nominal mortgage interest rate minus the annual rate of inflation. When equation (2.7) is modified to allow for time varying expected rates of return, the present value model

43To evaluate the possibility that the simple linear functional form of the market fundamental model is the cause of the serially correlated disturbances, squared values of both $ES_{t-1}$ and $M_t$ as well as an interaction term, are added to the reduced form pricing model. The coefficient estimates are not jointly different from zero and the inclusion of these additional variables only reinforces the correlation in the residuals.
takes the form:

\[ P_t = \frac{R_t}{(1 + r_t)} + E_t \left[ \sum_{j=1}^{\infty} R_{t+j} \prod_{i=0}^{\infty} \frac{1}{(1 + r_{t+i})} \mid I_t \right] \]  

(2.41)

Since this model is nonlinear the methods used to operationalize the constant discount rate present value model cannot be employed. To test the more general model, I adopt the first-differenced form of an approximate linearized model, given by \(^{44}\)

\[ \Delta P_t = \rho_1 \sum_{i=0}^{\infty} \rho_3^i \Delta E_t[R_{t+i} \mid I_t] - \rho_2 \sum_{i=0}^{\infty} \rho_3^i \Delta E_t[r_{t+i} \mid I_t] \]  

(2.42)

where \(\rho_i, i = 1, 2, 3\) are positive constants to be estimated, and \(\rho_3 < 1\).

The market fundamental model is used to proxy \(\Delta R_t\) and the processes generating the exogenous variables are identified and employed to solve for the reduced form house price model, according to Finn’s \(^{53}\) prediction formula. In addition to the driving forces of net in-migration and the inventory of unsold homes, equations (2.37) and (2.38), real mortgage rates are identified as a random walk. \(^{45}\)

The reduced form housing price model becomes

\[ \Delta P_t = \rho_1 \theta_1 A(\rho_3)^{-1} \left\{ \left( \frac{\rho_3}{1 - \rho_3} \right) M_t + \left[ \rho_3 a_2 + \rho_3^2 a_3 + \rho_3^3 a_4 + \rho_3^4 a_5 - a_1 \left( \frac{\rho_3}{1 - \rho_3} \right) \right] M_{t-1} \right. \\
+ \left[ \rho_3 a_3 + \rho_3^2 a_4 + \rho_3^3 a_5 - a_2 \left( \frac{\rho_3}{1 - \rho_3} \right) \right] M_{t-2} + \left[ \rho_3 a_4 + \rho_3^2 a_5 \right] \\
- a_3 \left( \frac{\rho_3}{1 - \rho_3} \right) M_{t-3} + \left[ \rho_3 a_5 - a_4 \left( \frac{\rho_3}{1 - \rho_3} \right) \right] M_{t-4} - a_5 \left( \frac{\rho_3}{1 - \rho_3} \right) M_{t-5} \right\} \\
- \rho_1 \theta_2 B(\rho_3)^{-1} \left\{ \left( \frac{\rho_3}{1 - \rho_3} \right) E S_{t-1} + \left[ \rho_3 b_2 - b_1 \left( \frac{\rho_3}{1 - \rho_3} \right) \right] E S_{t-2} \right. \\
- b_2 \left( \frac{\rho_3}{1 - \rho_3} \right) E S_{t-3} \right\} - \rho_2 \left( \frac{\rho_3}{1 - \rho_3} \right) \Delta r_t \]  

(2.43)

\(^{44}\)This model comes from a first-order Taylor series expansion of (2.39). Shiller (\(^{140}\), pages 118-119) derives a linearized approximation to the modified present value model via a first-order Taylor series expansion as, \(\hat{R}_t = \sum_{i=0}^{\infty} \left( \frac{1}{(1+\hat{r})} \right)^{i+1} \hat{R}_{t+i} - \hat{r} \sum_{i=0}^{\infty} \left( \frac{1}{(1+\hat{r})} \right)^{i+1} \hat{r}_{t+i} \), where \(\hat{R}\) and \(\hat{r}\) are the expected values of real rents and interest rates, respectively, and a hat over a variable denotes the variable minus its mean. This approximation requires that the variability in real interest rates is not too large. Campbell and Shiller \(^{16}\) provide an alternative linear approximation to the present value model in terms of natural logs of real prices, dividends and the discount rate. To ensure the empirical results are robust to the form of linear approximation, the estimations to follow consider this specification as well.

\(^{45}\)Both the auto and partial correlation plots for \(\Delta r_t\) indicate that it is random. The Q test statistic is 4.82 at lag 8, which implies the residuals in the random walk model are white noise.
The unconstrained version is given by

\[ \Delta P_t = \delta_0 M_t + \delta_1 M_{t-1} + \delta_2 M_{t-2} + \delta_3 M_{t-3} + \delta_4 M_{t-4} + \delta_5 M_{t-5} \]

\[ - \delta_6 E_S_{t-1} - \delta_7 E_S_{t-2} - \delta_8 E_S_{t-3} - \delta_9 \Delta r_t + v_{2t} \]  

(2.44)

A comparison of the coefficients in (2.43) and (2.44) reveals that (2.44) expresses 10 coefficients in terms of one parameter, \( \rho_3 \), and thus there are 9 restrictions placed on the reduced form model by the rational expectations hypothesis. As outlined above, to test the rational expectations hypothesis, (2.43) and (2.44) are each estimated simultaneously with the stochastic processes for the exogeneous variables and a likelihood ratio test statistic is employed to test the nonlinear, cross equation restrictions. Prior to this (2.44) is estimated by OLS to test for model adequacy. Table 2.4 presents parameter estimates for the unrestricted model.

The coefficient estimate on real mortgage rates is negative and statistically significant. The parameter estimate on the fourth lag of in-migration is estimated with more precision and there is little change in either the parameter estimates or standard errors associated with the remaining variables. Relaxing the constant discount rate assumption results in an improved model fit, as the adjusted \( R^2 \) increases to 0.50, from 0.37. The null of independent errors cannot be rejected at conventional significance levels.

A plot of the actual and fitted values of the first-difference in prices (figure 2.5) indicates that the model still both underpredicts the magnitude of price increases and decreases associated with the two price cycles. A plot of the residuals shows that the errors associated with these two periods appear to be outliers. The Jarque-Bera [101] test statistic for normality, which has a \( \chi^2 \) distribution under the null hypothesis of normality, is 30.35. The outliers are the cause of the nonnormal errors. 46 The important question is, how are the estimation results affected if normal errors are assumed?

To evaluate this, a dummy variable which takes on the value of 1 at the peak of each boom and 0 elsewhere is added to (41) and the model reestimated by OLS. The boom dummy is positive and significant and results in a Jarque-Bera test statistic of 0.32, thus producing normal errors. More importantly, there is relatively little change in either the

46 The residuals are normally distributed once the two outliers are discarded.

45
magnitudes or standard errors of the remaining parameter estimates. To investigate the
effect of the outliers on the estimation results and tests of the cross equation restrictions,
both the restricted and unrestricted systems are estimated twice, once with the boom
dummy variable included and once without.

2.5.1 Tests of the Cross Equation Restrictions

To simplify the estimation I reduce the number of parameters to be estimated.\(^{47}\) I treat
\(\rho_1 \theta_i, \ i = 1, 2\) as single parameters, and set \(\rho_3\) equal to its theoretical value in the Taylor
series expansion in equation (2.43). If the present value relation holds then \(\rho_3 = (1 + \bar{r})^{-1}\),
where \(\bar{r}\) is the mean level of real interest rates. In our sample, \(\bar{r} = 7.1\%\), and thus I set
\(\rho_3 = 0.934\). These simplifications reduce the demands on the nonlinear algorithm, yet
retain the cross equation restrictions associated with the forecasting equations for market
fundamentals.

With these simplifications, joint nonlinear system estimation converges after 104
iterations.\(^{48}\) Table 5.1 reports the results. Parameter estimates on the processes driving
the exogenous variables are quite close in magnitude and significance to those in table
2.2. Estimates for \(\rho_1 \theta_2\) and \(\rho_2\) have the correct positive signs and are estimated quite
precisely.

To test the rational expectations cross equation restrictions we compare the values
of the log-likelihood functions from FIML estimation of the restricted and unrestricted
systems. Table 5.1 reports the values. \(LLF_R\) and \(LLF_U\) are the maximized values of the
log-likelihood functions for the restricted and unrestricted systems, respectively. Two
times the difference between the unrestricted and restricted maximized log likelihoods
yields a likelihood ratio test of the rational expectations cross equation restrictions. Table
2.5 reports that likelihood ratio test statistic is about 27. Under the null hypothesis that
the cross equation restrictions are valid this statistic follows a chi-squared distribution
with seven degrees of freedom. Once the above simplifications are made, the restricted
system involves 3 parameters not identified among the 10 parameters in the unrestricted

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\(^{47}\)Attempts to estimate equations (2.37), (2.38), (2.43) simultaneously via full information maximum
likelihood (FIML) failed to converge.

\(^{48}\)I use the nonlinear estimation routine in SHAZAM, version 7.0.
model. Hence there are 7 cross equation restrictions. The 5 percent critical value is 14.1 and thus the null hypothesis is clearly rejected.\textsuperscript{49}

2.6 Conclusions

This paper evaluates the ability of a simple, asset-based, forward-looking model of housing price determination to explain the dramatic fluctuations in Vancouver house prices, over the past twelve years. It incorporates a number of innovative features and hence both complements and extends previous empirical work. Consistent with earlier work on housing market efficiency, it rejects the rational expectations hypothesis. While contemporaneous housing market fundamentals explain a significant proportion of observed housing price volatility, there appears to be little forward looking behaviour of demand-side fundamentals on the part of housing market participants. This suggests that house prices may, at times, deviate from fundamental values.

It is important to note, however, that the above results do not necessarily constitute evidence of irrational house price expectations. The empirical tests in this chapter examine not only the null hypothesis of rational house price expectations, but a joint hypothesis which embodies a number of additional assumptions. More specifically, in addition to rational expectations this study tests the joint null hypothesis of no housing market risk premium, a simple two-variable linear model of housing market fundamentals and stable linear forecasting equations for market fundamentals that are known at all times by market participants. It also treats the housing market as a frictionless asset market. If any of these assumptions do not hold then the housing price model is misspecified and thus rejections of the cross equation restrictions are due to model inadequacy.

\textsuperscript{49}The conclusions drawn from the empirical results presented in this section are unchanged when the boom dummy is included in the model.
Figure 2.1: Real Single-Detached House Prices, Westside of the City of Vancouver, 1979:1-1991:3
Figure 2.2: Total Employment, Vancouver Census Metropolitan Area, 1979:1-1991:3
Figure 2.3: Net Inmigration, British Columbia, 1979:1-1991:3
Figure 2.4: Stock of Newly Completed But Unoccupied Single and Semi-Detached Homes, Vancouver Census Metropolitan Area, 1979:1-1991:3
Table 2.1: Unit Root Test Results

<table>
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<tr>
<th>Series</th>
<th>DF</th>
<th>PP</th>
<th>10% cv</th>
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<tr>
<td>$P_t$</td>
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<td>-0.49</td>
<td>-3.13</td>
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<tr>
<td>$\Delta P_t$</td>
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<td>-7.55</td>
<td>-2.57</td>
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<tr>
<td>$M_t$</td>
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<td>-4.81</td>
<td>-3.13</td>
</tr>
<tr>
<td>$S_t$</td>
<td>-3.93</td>
<td>-3.20</td>
<td>-3.13</td>
</tr>
</tbody>
</table>

Note: DF and PP are augmented Dickey-Fuller and Phillips-Perron unit root tests, respectively.
Table 2.2: Univariate Forecasting Models for Market Fundamental Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient 1</th>
<th>Estimate</th>
<th>t-Statistic</th>
</tr>
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<tbody>
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<td>$M_t$</td>
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<td>0.8748</td>
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<td>$a_2$</td>
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<td></td>
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<td></td>
<td>$R^2$</td>
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<td>Q(8)</td>
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<td>JB</td>
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$ES_{t-1}$

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<table>
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Note: “Diagnostics” are model misspecification tests. Q(4) and Q(8) are modified Box-Pierce statistics that test for joint significance of departures from randomness in the first 4 and 8 lags of the residuals, respectively. HET is Glejser’s [73] heteroskedasticity test, and JB is the Jarque-Bera [101] Lagrange multiplier test for normality of the residuals. 5% cv gives the 5 percent critical values.
Table 2.3: Estimation Results for the Unrestricted Constant Discount Rate Model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Est. Coefficient</th>
<th>t-Statistic</th>
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<tbody>
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<td>$M_t$</td>
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<td>$M_{t-2}$</td>
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<tr>
<td>$M_{t-3}$</td>
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<td>$M_{t-4}$</td>
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<tr>
<td>$ES_{t-3}$</td>
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<td>-1.56</td>
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$R^2$ 0.49
$\bar{R}^2$ 0.37

<table>
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<th>Diagnostics</th>
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Note: "Diagnostics" refers to misspecification tests on the residuals. DW is the Durbin Watson statistic, Q(2) and Q(4) are the modified Box-Pierce statistics that test for joint randomness in the first two and four lags of the residuals, respectively, HET is Glejser's [73] heteroskedasticity test and JB is the Jarque-Bera [101] Lagrange multiplier test for normality. 5% cv is the 5 percent critical value for each test statistic.
Figure 2.5: Actual House Price Changes Versus Fitted Values from the Estimated Unrestricted Constant Discount Rate Model
Table 2.4: Estimation Results for the Unrestricted Time Varying Discount Rate Model

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<td>$M_{t-3}$</td>
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Diagnostics:

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</tbody>
</table>

Note: "Diagnostics" refers to misspecification tests on the residuals. DW is the Durbin Watson test statistics, Q(2) and Q(4) are modified Box-Pierce tests of the joint randomness in the first two and four lags of the residuals, respectively, HET is Glejser’s [73] heteroskedasticity test and JB is the Jarque-Bera [101] Lagrange multiplier test for normality. 5% cv provides the 5 percent critical values for each statistic.
Figure 2.6: Actual House Price Changes Versus Fitted Values from Estimated Unrestricted Time-Varying Discount Rate Model
Table 2.5: Tests of the Rational Expectations Cross Equation Restrictions in the Time-Varying Discount Rate Model

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimate</th>
<th>t-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>0.6964</td>
<td>5.52</td>
</tr>
<tr>
<td>$a_2$</td>
<td>-0.0941</td>
<td>-0.60</td>
</tr>
<tr>
<td>$a_3$</td>
<td>0.0294</td>
<td>0.19</td>
</tr>
<tr>
<td>$a_4$</td>
<td>0.4590</td>
<td>2.88</td>
</tr>
<tr>
<td>$a_5$</td>
<td>-0.3365</td>
<td>-2.52</td>
</tr>
<tr>
<td>$b_1$</td>
<td>1.1293</td>
<td>15.68</td>
</tr>
<tr>
<td>$b_2$</td>
<td>-0.5879</td>
<td>-8.14</td>
</tr>
<tr>
<td>$\rho_1\theta_1$</td>
<td>0.0005</td>
<td>3.46</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>375.63</td>
<td>1.87</td>
</tr>
<tr>
<td>$\rho_1\theta_2$</td>
<td>0.0131</td>
<td>5.31</td>
</tr>
</tbody>
</table>

| $LLF_R$ | -995.06 |
| $LLF_U$ | -981.73 |
| $2*(LLF_U - LLF_R)$ | 26.66 (0.001) |
APPENDIX I: Variable Definitions and Data Sources

This appendix provides a list of variable definitions and detailed sources for all data employed in the paper. The data are quarterly and cover the 1979:1-1991:4 sample period.

$P$ : Quality adjusted real house prices, City of Vancouver. 3 month moving average of monthly house prices, $PN$, divided by the monthly all items CPI, excluding shelter costs, for Vancouver CMA.

$M$ : Filtered (to remove seasonal effects) net immigration to the Vancouver CMA. Proxied by quarterly net immigration into B.C., which is the sum of net interprovincial immigration into B.C. and net international immigration to B.C. as the province of destination.
Source: Based on data in Quarterly Demographic Statistics, Statistics Canada, catalogue 91-002 (CANSIM series D269466, D269479, D123092).

$S$ : Inventory of newly completed but unoccupied single and semi-detached dwelling units, Vancouver CMA. 3 month moving average of monthly inventory.
Source: Housing Statistics, Canada Mortgage and Housing Corporation, B.C. and Yukon Regional Office.

$N$ : Total Employment, Vancouver CMA. 3 month moving average of monthly values.

$i$ : Conventional 5 year nominal mortgage interest rate. 3 month moving average of monthly observations.
Source: Bank of Canada (CANSIM series B14024).
\( r \) :  Real mortgage interest rate. Calculated as the ex-post real interest rate: the nominal rate minus the actual annual rate of inflation, derived from the all items CPI for the Vancouver CMA. 3 month moving average of actual observations.

Chapter 3

Empirical Tests of Rational Expectations and No Risk Premium in the Housing Market

3.1 Introduction

This chapter tests whether simple measures of expected house price appreciation are unbiased predictors of future capital gains to housing. More specifically, it tests the joint null hypothesis of rational expectations, or semi-strong asset market efficiency, and risk neutrality in the Vancouver condominium apartment market.

The work in this chapter improves upon the existing literature in two ways. First, most previous tests of housing market efficiency use standard empirical finance tools. That is, researchers either examine the autocorrelation structure of estimates of excess returns or test whether any variables in the current information set have predictive power for future excess returns. If the housing market behaves as an efficient asset market then excess housing returns should be uncorrelated over time and unpredictable with information known at the time forecasts are made. This work yields weak evidence of housing market inefficiency. This chapter applies a different testing methodology. Rather than work with excess

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1 Previous studies include Hamilton and Schwab [82], Case and Shiller [23][24], Meese and Wallace [115], Hosios and Pesando [99] and Poterba [126].

2 Hamilton and Schwab [82] is an exception.

3 This interpretation of earlier results assumes that homeowners are either risk neutral or that there is no systematic risk associated with housing investment.
returns I use the market efficiency condition to generate a measure of expected house price appreciation under risk neutrality. If housing market participants have rational expectations then regressions of the actual percentage changes of house prices on the expected appreciation measure yields a slope coefficient of one.

Second, all previous work investigates the efficiency of the market for single-family homes. Most single-family homes are owner-occupied, and hence rents on these homes are not observed. Federal government statistical agencies in both Canada and the United States publish indices of rents on rental dwellings, as part of their Consumer Price Indexes. Most researchers proxy single-family rents with these published rent series. This approach suffers from a number of limitations. To estimate housing investment returns, rent indexes must be converted into time series of rental payments to form the proxy series for single-family rents. To derive the conversion or scale factor it is assumed that the market efficiency condition holds exactly in the first period of the sample. If this assumption is not true, or the scale factor is not constant over the sample period, then subsequent results are biased.

This chapter uses a new data set. Four times a year, Royal Lepage Real Estate Services Ltd., a large Canadian real estate brokerage company, reports estimates of price, monthly rents and annual property tax payments for a number of structural housing types in a variety of cities across Canada. This study tests the efficiency of the Vancouver condominium apartment market, using the Royal Lepage data. More precisely, I test the rational expectations hypothesis under conditions of risk neutrality for a standardized two bedroom condominium apartment using a time series of cross section data on eight municipalities within the Vancouver metropolitan area. The data cover the 1982-1992 sample period.

This data set is unique in three respects. First, it provides time series of prices, rents

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490 percent of the existing stock of 5.7 million occupied single-family dwelling units in Canada are owner-occupied. This figure drops slightly to 86 percent for the Vancouver Census Metropolitan Area (CMA). (Source: Table 1 in Occupied Private Dwellings: The Nation, 1991 Census of Canada, Statistics Canada catalogue 93-314.

5Meese and Wallace [115] is an exception.

6These are shortcomings in addition to the one noted in chapter 2, that rental dwellings may differ in quality from owner-occupied homes. And as a consequence, rent indexes derived from rental dwellings may not be a good approximation to imputed rents on owner-occupied, single-family rents.

7Section 3.3.1 below provides greater detail on the Royal Lepage data.
and property taxes for a single housing type. This offers significant advantages over previous studies. It eliminates the need to proxy rents and thus does not require that we scale the rent and price data in such a way as to force market efficiency to hold at one point in time. At the same time, it is important to recognize that the data are not market transaction-based, but are estimates or appraisals. The validity of the results in this chapter therefore depends crucially on the accuracy of the data. I undertake a careful comparison of the Royal Lepage price estimates with a constant quality transaction-based price series that is available for one area. The results suggest we can be confident that the data are accurate assessments of true house prices.

Second, this data set is a time series of observations on a cross section of housing submarkets within the larger metropolitan Vancouver housing market. This allows us to test the efficiency of the housing market at a more micro level. All existing studies use metropolitan-wide price and rent data. This chapter demonstrates that prices and rents in local markets that comprise a larger metro housing market, can behave quite differently. This implies that the use of "metro-wide" rental price indexes and house price series may provide misleading results.

Finally, the data cover a different sector of the housing market than existing studies. This is the first study, to the best of my knowledge, to employ condominium apartment market data. It is often argued that real estate markets are inefficient "based on a perceived set of market imperfections," (Gau ([67], page 1)). The indivisibility or lumpiness of real estate assets and capital constraints faced by investors due to the expensive nature of the asset, are often cited forms of market imperfection, which may limit information capitalization into real estate values. Condominium apartments are generally smaller and significantly less expensive than single-family homes. This suggests that the potential lack of arbitrage due to indivisibility and capital constraints is reduced in the condominium apartment market. Thus if indivisibility and high asset value are more

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8See Gau [65] [67] and Case and Shiller [16] for more on this point.
9For example, consider the following price estimates by Royal Lepage for different sized homes in the same neighbourhood on the Westside of the City of Vancouver on January 1, 1993. $260,000 for a standard two bedroom condominium (900 square feet) versus $495,000 for standard two-storey (three bedroom, 1500 square feet, single car garage) and $620,000 for an executive detached two-storey (four bedroom, 2000 square feet, two car garage).
than just perceived market imperfections, we might expect to find that the condominium market is in some sense more efficient than the market for single-family homes.

With this previously unexploited data base and different econometric testing methodology I find significant evidence against the joint null hypothesis of rational expectations and risk neutrality. More precisely, house price appreciation, on average, moves in the opposite direction than that predicted by the risk neutral rational expectations model. These results provide much stronger evidence against the joint null hypothesis of rational expectations and risk neutrality than reported in the literature to date.

Consistent with the rejection of rational expectations, tests for excess return predictability find statistically significant evidence of mean reversion in excess returns to condominium investment. These results confirm previous findings of housing market inefficiency, if there is no time-varying housing market risk premium. Unlike earlier studies however, no evidence of positive serial correlation in excess returns is found at the one year horizon. I also find that short-term interest rates are significantly negatively related to excess condominium returns in a number of the municipal housing markets. This result is consistent with work on stock return predictability.

These results do not necessarily provide evidence of housing market inefficiency. As with all tests of market efficiency, this chapter evaluates a joint hypothesis that includes rational house price expectations, no housing market risk premium and a model of equilibrium housing returns. What this chapter shows is that excess condominium returns are partly predictable. It remains on open question whether these findings are evidence of market inefficiency or time-varying risk premiums.

This chapters final contribution is to characterize the time series properties of deviations in house prices from those predicted by the risk neutral rational expectations model. I document a number of stylized facts to guide future research on house price volatility. More specifically, I find that ex post price realizations are cointegrated with their risk neutral rational expectations. Thus, while there are significant departures from market efficiency under risk neutrality in the short-run, there is a long-run equilibrium

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10 This finding is consistent with results in the empirical exchange rate literature. I discuss the parallels later in the chapter.
11 Section 3.6 discusses empirical work, and provides references, on stock return predictability.
relationship between price levels and their rational expectation.\(^{12}\) Investigation of the robustness of the rational expectations estimation and test results to subsampling indicates they are extremely sensitive to the period used in estimation. Successive subsample estimation shows that the parameters in the rational expectations model are time-varying, autocorrelated, mean reverting, and related to the stage of the real estate price cycle.

The remainder of this chapter is organized as follows. Section 3.2 sets out the theoretical rational expectations model of house price determination. Section 3.3 describes and assesses accuracy of the data. Section 3.4 addresses important econometric issues and derives estimable forms of the theoretical housing price model. Section 3.5 reports the empirical results. Section 3.6 tests for predictability in condominium returns. Section 3.7 extends the econometric investigation to characterize predictable components in excess housing returns. Section 3.8 summarizes the findings and presents concluding comments.

### 3.2 Theoretical Framework

The housing market is comprised of two separate but interrelated markets: one for the flow of housing services and another for the stock of housing capital. If investors are risk neutral then equilibrium in the market for the stock of housing requires that the expected return to housing investment equal the rate of return available on alternative investments. That is,

\[
R_t + g^*_t P_t = (i_t + pt_t + d_t)P_t \tag{3.1}
\]

where \(R\) is annual rental income, either actual if the unit is rented out or imputed if owner-occupied, \(P\) is the current asset value of the housing unit, \(i\) is the nominal return available on alternative assets (the 1 year rate on government bonds for example), \(pt\) is the annual property tax rate as a share of house value, \(d\) is the sum of depreciation and maintenance costs as a share of house value, and \(g^*_t\) is the expected annual rate of capital

\(^{12}\text{Meese and Wallace [115] report a similar finding. They find that house price changes are constrained by market fundamentals over the long-term but short-run price fluctuations are not well explained by movements in market fundamentals.}\)
gain. That is, for annual returns, with quarterly data observations,

\[ g_t^e = \frac{P_{t+4,t}^e - P_t}{P_t} \]  

(3.2)

where \( P_{t+4,t}^e \) is the expected value of housing one year from now. If the 'market' forms house price expectations rationally then \( P_{t+4,t}^e \) is defined as the mathematical expectation of the one-year ahead price conditional on all current and past information. More formally,

\[ P_{t+4,t}^e = E_t[P_{t+4} | I_t] \]  

(3.3)

where \( I_t \) is the information set available to agents in period \( t \). This condition states that the expectations of housing market participants are unbiased and all information available at time \( t \) is incorporated into the expectation of house prices one year ahead. In other words, agents have complete knowledge about the structure of the model driving house price movements, complete knowledge of the parameters and complete knowledge of all current and past values of the variables in the model. This in turn implies that realized house prices differ from expected values by only a random error,

\[ P_{t+4} = E_t[P_{t+4} | I_t] + \epsilon_{t+4} \]  

(3.4)

where \( \epsilon_{t+4} \) is a serially uncorrelated, mean zero disturbance term.

Combine equations (3.1) through (3.3) to generate an expression for the one year ahead house price prediction,

\[ E_t[P_{t+4} | I_t] = (1 + i_t + pt_t + d_t)P_t - R_t \]  

(3.5)

Therefore, under the joint null hypothesis of rational expectations, risk neutrality and the asset market equilibrium condition (3.1), house price expectations are inferred from current market data on house prices, rents and interest rates. All else equal, expected future house prices are positively related to current house prices and negatively related to current rents.

\[ \text{The theoretical setup is developed in terms of annualized data. The empirical tests, below, however, make use of both one-year and quarterly forecast horizons.} \]
Substituting equation (3.5) into (3.4) yields testable restrictions implied by rational expectations and housing market efficiency:

\[
P_{t+4} = (1 + i_t + pt_t + d_t)P_t - R_t + \varepsilon_{t+4}
\]

Thus, one way to test the hypothesis of rational house price expectations is to estimate the regression model

\[
P_{t+4} = \mu_0 + \mu_1[(1 + i_t + pt_t + d_t)P_t] - \mu_2R_t + \varepsilon_{t+4}
\]

and test the joint null hypothesis that \( \mu_0 = 0 \) and \( \mu_1 = \mu_2 = 1 \). This test specification, however, is valid only under the assumption of stationary time series. If the stochastic processes generating \( P_t \), \( R_t \), and \( (i_t + pt_t)P_t \) are nonstationary then transformations of the model, which render each variable stationary, are necessary to conduct meaningful statistical tests. The issue of nonstationarity has important implications for the modeling of house prices and is taken up following a description of the data.

### 3.3 Data and Summary Statistics

#### 3.3.1 Rent, Value and Property Tax Data

Empirical tests of housing market efficiency require house price, rent and property tax data. The *Royal Lepage Survey of Canadian House Prices* provides quarterly data on prices, monthly rents and annual property tax payments for 7 types of dwelling units in a large number of cities across Canada. The Survey reports data for four categories of single-family housing: bungalow, detached two-story, standard townhouse, senior executive; as well as a standard condominium apartment and luxury condominium apartment. Royal Lepage reports data for a wide range of neighbourhoods within major urban centres.

This study examines the efficiency of the “standard condominium apartment” market, as defined in the Royal Lepage Survey, in the Vancouver metropolitan area, over the 1982-1991 sample period. Condominiums represented 13 percent of the stock of occupied
private dwelling units in the Vancouver CMA in 1986. A standard condominium apartment is a two-bedroom carpeted unit of 900 square feet in a multi-story building with one and one-half bathrooms, 2 appliances, a small balcony and one underground parking space. The sample chosen consists of condos in the following eight locations within the Vancouver area: Burnaby, East Vancouver, Vancouver Westside, North Vancouver, West Vancouver, Richmond, Surrey and Tsawassen. Figure 3.1 shows a map of the Vancouver area which identifies the location of each municipality.

Figure 3.2 plots quarterly prices and rents for a standard condominium unit, by area. Price levels differ by area primarily due to location (distance to the downtown core and amenities). House price movements are similar in each location. Table 3.1 reports summary statistics for annual percentage changes in prices and rents.

An Assessment of Data Accuracy

Royal Lepage data are estimates. The survey reports that,

"This survey reflects Royal Lepage's estimate of "Fair Market Value" of certain housing types in each location for the value date indicated and based on both data and opinion supplied by Royal Lepage Real Estate personnel across Canada. .... For the purposes of this Survey, mortgage financing has not been taken into account in arriving at published prices and all properties have been considered as being free and clear of debt."

Data are provided by the managers of local branch offices across the country to the market research department of the head office in Toronto, which publishes the Survey. A comparable sales appraisal method, based on recent similar transactions, is combined with an analysis of current market conditions to derive the published figures. Estimated monthly rents are derived using the same procedure plus local investors and property managers are contacted and asked to provide rents by dwelling type in their local area.

14Source: 1986 Census of Canada. 1991 Census data required for this calculation is due to be released by Stats Can sometime in the next two months. This figure will certainly rise however since condominiums accounted for approximately 40 percent of all housing starts in Vancouver from 1982-1993 (Source: CMHC).

15I am grateful to Maureen Coleman, manager, Royal Lepage Real Estate Services Ltd., South Granville branch, Vancouver, for this information.
The data, therefore, are based on appraisals rather than market transactions. One potential concern is the accuracy of the estimated series. Recent work that examines the statistical properties of appraisal-based commercial real estate returns finds significant evidence of appraisal smoothing. More specifically, appraisers tend to systematically underestimate the variance of true real estate returns. This literature argues that appraisers use both current and past data on real estate values to arrive at their ‘best guess’ of current value, and thereby smooth the estimated price series over time. The explanation for why appraisers incorporate past information into current value estimates relies on both the heterogeneity and infrequent trading of commercial real estate assets. Together, these two factors imply that the appraiser has very little, or no, recent transaction data with which to construct his or her estimate of current value. As a consequence, the appraiser relies partly on the most recent ‘similar’ transactions, which likely took place some time ago. Thus the appraiser relies on lagged market data.

The presence of smoothing in Royal Lepage housing price estimates may induce measurement error which could bias the test results. I do not, however, believe there is a smoothing problem, for the following reasons. Price estimates are provided by professionals who are knowledgable about market conditions in their local market. Residential housing is much more liquid and the turnover rate higher than commercial properties, and hence the appraisal-smoothing phenomenon that affects commercial returns should not severely bias return series based on Royal Lepage price estimates. Fortunately, I have access to a transactions-based house price series for one of the municipalities, which allows me to test the above claim.

Hamilton and Hobden [80] derive a constant quality time series of single-detached house prices for the Westside of the City of Vancouver. I compare this with Royal Lepage price estimates for a “detached two story” home in Kerrisdale. Kerrisdale is a Westside neighbourhood. Thus, while the two data series do not provide value estimates for exactly the same house they both provide estimates of the price of single-detached

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16References include Giliberto [72], and Geltner ([68] [69] [70]).
17This is the same price data employed in chapter 2. The authors employ hedonic price methods to arrive at a quality adjusted house price series for a home with the mean attributes of all homes in the sample.
housing on the Westside of the City of Vancouver. A comparison should therefore help us to evaluate the extent of measurement error in the Royal Lepage data.

Figure 3.3 plots both Hamilton and Hobden’s [84] hedonic price data and Royal Lepage price estimates. Part I shows the levels data. Both series are divided by their respective first observations and multiplied by 100 to set them equal to 100 in April 1981. The two price series move closely together over most of the sample period. The correlation coefficient is 0.98. There is some divergence starting in late 1988, but the pattern of price movements remains closely aligned.

Part II shows the first differences of the logarithm of each house price series. Table 3.2 provides summary statistics. It is clear that quarterly house price appreciation, based on Royal Lepage estimates, does not diverge significantly from market-based estimates. The correlation coefficient is 0.75. Moreover, the Royal Lepage house price data exhibits greater volatility than the hedonic price series. We would expect the opposite result if Royal Lepage systematically underestimated the variance of house price changes. Thus there is no evidence of appraisal-based smoothing in the Royal Lepage single-detached house price data. Since Royal Lepage derives condominium price and rent estimates using the same procedure it seems reasonable to assume this result carries over to the condominium data.

Unlike the price data, there is no market-based series of rents with which to compare Royal Lepage rent data. However, available data indicates that a significant proportion of condominiums in the Vancouver CMA are rental units. Hamilton [81] reports that more than one-third of the condominium units in the Vancouver CMA are rental units. The relatively large rental component of the Vancouver condominium housing stock implies that market rents are observable. In addition, rent estimates are provided by knowledgeable market participants. These two factors should help to minimize the noise in the estimated monthly rental series.

3.3.2 Interest Rate Data

Equilibrium in the market for existing housing units requires that investors expect to earn a rate of return on housing investment equal to that available on alternative assets.
Given that investors are risk neutral, I use the the risk free rate of return on government bonds to measure the opportunity cost of money.

Quarterly series for both the three month and one year rates of return on alternative investments are derived from monthly series of the average nominal yield on 3 month and 1 to 3 year Government of Canada Bonds, respectively. These series are available on CANSIM, matrices B14009 and B14007 (source: Bank of Canada).

### 3.4 Empirical Model Specifications

#### 3.4.1 Econometric Issues

Prior to estimating the asset-based housing market equilibrium condition in equation (3.7), and testing the restrictions imposed by rational expectations and risk neutrality, two important econometric issues must be addressed: the overlapping nature of the data and the potential for unit roots in the time series processes of the data series. Neglect of either complication can lead to incorrect and possibly meaningless inference. This section also outlines the estimation procedures that are used.

**Overlapping Data**

The chapter uses both quarterly and annual forecast horizons to test the risk neutral rational expectations model. The data are quarterly observations, and thus the forecast horizon exceeds the sampling frequency of the data when the dependent variable is annual price change. Hansen and Hodrick [84] show that the forecast error, $\varepsilon_{t+4}$, is serially correlated in this situation. More specifically, quarterly data and one year (4 quarter) ahead predictions produce a forecast error that follows a third order moving average process [MA(3)].\(^{18}\) Ordinary least squares estimation yields consistent but inefficient parameter estimates under these conditions. Hansen and Hodrick [84] employ Hansen’s

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\(^{18}\)To see this intuitively, note that under the null hypothesis of rational house price expectations, the forecast error, $\varepsilon_{t+4}$ arises from (unanticipated) random noise or 'news' that hits the market during the 4 quarters between the time the forecast is made ($t$) and the true value is realized ($t + 4$). The next forecast error, $\varepsilon_{t+1+4}$, results from 'news' that arrives between periods ($t + 1$) and ($t + 5$). Thus the two error terms share common news elements, or overlap, for three periods, ($t + 1$) → ($t + 4$). Hence the residual autocorrelation function has a 'memory' of exactly three periods. This is characteristic of an MA(3) time series process.
Generalized Method of Moments theory to derive appropriate asymptotic standard errors.\textsuperscript{19} The empirical finance literature documents two potential problems with Hansen and Hodrick's estimator. First, it is not guaranteed to be positive definite in small samples. Second, it assumes that the error terms are conditionally homoskedastic.\textsuperscript{20} Newey-West [\textsuperscript{120}] propose the following modified covariance estimator that is robust to both of the above potential complications:

\[
\hat{\Omega} = \sum_{j=-L}^{L} \frac{1}{T} [1 - |j|/\left(1 + L\right)] \sum_{t=1}^{T} e_t x_t' x_{t-j} e_{t-j} \tag{3.8}
\]

where \( x_t \) is a vector of time \( t \) observations on the explanatory variables, \( e_t \) is the OLS residual, and \( L \) is the 'window size', which is related to the order of autocorrelation in the residuals. The window size is chosen by the econometrician. It must be set to an integer value that is larger than the order of the MA process to ensure that it accommodates the overlap induced autocorrelation. \textsuperscript{21} Most researchers now employ the Newey-West [\textsuperscript{120}] covariance matrix to account for the statistical problems created by overlapping data, and hence I do here.\textsuperscript{22}

Accounting for Common Fundamentals

Each of the eight municipal housing submarkets are part of the same larger metropolitan market, and are therefore subject to a common set of fundamental forces, such as regional employment growth, in-migration and interest rates. This leads to contemporaneous correlation across residuals in the house price equations of each municipality; the random disturbance associated with one area is correlated with the disturbance terms in all other seven markets. Ordinary least squares estimation of eight separate models is not efficient

\textsuperscript{19}See equation (4) in their paper.

\textsuperscript{20}Since the Vancouver housing market has been characterized by boom-bust periods, or tranquil followed by turbulent times this assumption may be invalid. Chapter 4 provides evidence of conditional heteroskedasticity or time-varying volatility in short-term housing price fluctuations.

\textsuperscript{21}Hansen and Hodrick's [\textsuperscript{84}] variance-covariance estimator is a matrix with ones along the main diagonal, residual autocorrelations along the first \( L \) subdiagonals and zeros everywhere else, where \( L \) is the order of the MA process (3 in this study). The Newey-West [\textsuperscript{120}] method weights the residual autocorrelations, by the term in square brackets in equation (3.8), and the weights are less than one. Thus \( L \) must be larger than 3.

\textsuperscript{22}References include Cavaglia, Verschoor and Wolf [\textsuperscript{25}], Campbell and Clarida [\textsuperscript{15}] and Green and Mork [\textsuperscript{76}].
in this situation.

To recognize the dependence across municipal housing markets, and thereby increase the efficiency of point estimates, system estimation procedures are employed. The parameters are jointly estimated by Hansen's [83] Generalized Method of Moments (GMM). The GMM coefficient estimates are identical to ordinary least squares (OLS) estimates but GMM provides heteroskedasticity-consistent and/or autocorrelation-consistent standard errors. Quarterly forecast horizon models employ White's [150] heteroskedasticity-consistent variance-covariance matrix as the GMM weighting matrix. Annual forecast horizon models use the Newey-West [120] variance-covariance matrix, which delivers heteroskedasticity and autocorrelation consistent standard errors.

**Unit Roots Tests**

The usual asymptotic theory invoked to construct hypothesis tests in standard regression models is based on the assumption that all of the underlying series are weakly or covariance stationary time series processes. If the series under study are nonstationary, due to the presence of a unit root, then the usual tests of parameter significance are invalid, and in fact can provide very misleading results. This study employs the unit root tests developed by Phillips and Perron [122] [123] to test whether the series in equation (3.7) are nonstationary.

Phillips-Perron tests for a unit root in a univariate time series, $x_t$, involve running the ordinary least squares regression

$$x_t = \mu + \beta(t - \frac{T}{2}) + \alpha x_{t-1} + u_t$$

and testing the null hypothesis $\alpha = 1$. $u_t$ is a disturbance term that may be serially correlated and/or conditionally heteroskedastic and $T$ is the sample size.

The standard $t$-statistic associated with the lagged dependent variable does not have a standard $t$-distribution under the null hypothesis of a unit root, but critical values

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23 Chapter 2 provides a detailed discussion of both the intuition underlying unit roots in economic time series and the consequences of not transforming the data to eliminate unit roots prior to estimation.

24 The time trend term can be omitted from the regression if the series does not exhibit an upward trend.
have been tabulated by Dickey and Fuller [35]. Phillips-Perron unit root tests consist of calculating the usual t-statistic, based on OLS point estimates, and then adjusting its value according to a nonparametric correction term to account for serial correlation and/or heteroscedasticity in the residuals. If the t-statistic is negative and significantly different from zero then the null hypothesis is rejected and the series is taken to be stationary (i.e. integrated of order zero, I(0)).

The first two columns of table 3.2 report unit root test results for condominium price and rent levels, \( P_t \) and \( R_t \). In general the t-statistics for both tests are well below the critical values at the 10 percent level. The hypothesis that both series contain unit roots cannot be rejected. This implies that the levels model specification in equation (3.7) is inappropriate and hence, may provide spurious results. To ensure that the estimation and test results to follow are not biased by the nonstationary nature of the price and rent data, equation (3.6) is transformed into an expression that is more likely to satisfy the stationarity assumption, yet retains the restrictions implied by rational expectations and risk-neutrality.

3.4.2 Percentage Change Model Specification

A time series that is nonstationary due to the presence of a unit root must be first-differenced to render it stationary. First-differencing the variables in (3.6), however, is not likely the appropriate solution. This approach can result in the loss of valuable long-run information and a misspecification of short-run dynamics.\(^{26}\)

\(^{25}\)Evidence from other housing market studies supports the finding of a stochastic trend or unit root in both rents and prices. Meese and Wallace [115] find a unit root in constant quality house price series and proxy rental series in the San Francisco area. Chapter two of this thesis reports a unit root in the single-detached Vancouver hedonic house price index developed by Hamilton and Hobden [80]. Furthermore, there now appears to be a consensus that the levels of many macroeconomic and financial time series are characterized by stochastic trends or unit roots. Baillie and McMahon [10] and Stock and Watson [145], and the references therein, provide evidence on the nonstationarity of spot and forward exchange rates, and other macroeconomic and financial time series, respectively.

\(^{26}\)To see this, consider a simple long-run relationship, \( y_t = cx_t \), given by economic theory. Both series are nonstationary. Assume that in steady state the levels are constant and the relation holds exactly. This means that in steady state \( \Delta y_t = \Delta x_t = 0 \). As a consequence, if a first-differenced model is estimated it would be concluded that no relationship exists between the two variables, even though in the long-run we know there is. The concept of cointegration links the long-run relationship between nonstationary variables and the short-run dynamics. Cointegration techniques are discussed in section 3.6.1 below.
As noted above, the approach I take is to transform the levels model in such a way as to yield data series that are more likely to be stationary. Subtracting current price, \( P_t \), from each side of (3.6) and then dividing both sides of the resulting expression by \( P_t \) yields the following test of the rational expectations hypothesis under risk neutrality:

\[
\frac{P_{t+1} - P_t}{P_t} = \alpha_0 + \alpha_1 (i_t + pt_t) - \alpha_2 \left( \frac{R_t}{P_t} \right) + \varepsilon_{t+1} \tag{3.9}
\]

where the different forecast horizons are now explicitly recognized with \( l = 1 \) and \( l = 4 \), for quarterly and annual forecasts, respectively.\(^{27}\) Under the joint null hypothesis of rational expectations and no risk premium \( \alpha_0 = 0 \) and \( \alpha_1 = \alpha_2 = 1 \). Unit root test results in table 3.2 indicate that the first-differenced price series are stationary. Phillips-Perron test statistics lie well above their 10 percent critical value. This is not the case for annual capital gains, as we cannot reject the null hypothesis of a unit root in six of the eight series. The latter finding is, however, inconsistent with the former.

Stationarity of quarterly price changes implies that annual price changes are stationary. To see this, notice that we can express any annual price change as the sum of the four quarterly price changes during the year. That is,

\[
(P_{t+4} - P_t) = \sum_{i=1}^{4} (P_{t+i} - P_{t+i-1}) \tag{3.10}
\]

\[= (P_{t+4} - P_{t+3}) + (P_{t+3} - P_{t+2}) + (P_{t+2} - P_{t+1}) + (P_{t+1} - P_t) \]

\(\forall t\). Since Phillips-Perron tests reject the unit root hypothesis in quarterly house price appreciation, each element in the above summation is stationary. Thus, annual price changes are stationary.

The above results illustrate the well documented low power of unit root tests to reject the null hypothesis of nonstationarity in slowly reverting stationary time series, in small samples. Shiller and Perron [142] provide evidence that the power of unit root tests depends crucially on the number of cycles covered by the data. To understand

\(^{27}\)Notice that rather than examine parameter restrictions in equation (3.9) we could simply impose them and analyze the time series properties of excess condominium returns, given by \( \varepsilon_{t+1} = \left[ P_{t+1} - P_t + R_t - i_t - pt_t \right] \). The formulation in (3.9) allows us to directly estimate the relationship between actual and expected house price changes. Furthermore, the results of sections (3.5) and (3.6.2) show that the regression-based model in (3.9) has more power to detect short-run deviations from market efficiency than does tests for autocorrelation in quarterly excess returns.

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why, consider a number of fundamental differences between stationary and integrated (nonstationary) series. Stationary, or $I(0)$, time series have constant means and variances and tend to oscillate around their mean values. An integrated series, termed $I(1)$, in contrast, has no discernable pattern and no tendency to return to a mean value. It's variance increases over time without bound. In a plot against time, an $I(1)$ series looks smooth, whereas an $I(0)$ series is choppy, bouncing about its mean.

There is only one cycle in the short span of data covered in this study. As a consequence, the annual series only crosses the horizontal axis at probable mean levels once. The first-differences of quarterly price changes, however, cross their means much more frequently. As a consequence, tests of nonstationarity in quarterly appreciation series are more powerful than those with annual price changes. Annual rates of change in condominium prices look smooth over the sample under study but would appear more erratic over a much longer period of time, when a greater number of cycles would be apparent. Thus the finding of a unit root in annual price changes is a consequence of the small sample bias of unit root tests.

The evidence is mixed on the rent-price ratio, which according to the unit root tests have a unit root in four municipalities. The unit root hypothesis is not rejected for the 3 month government bond yield but is for the one year rate. These tests likely suffer from the same small bias as do the unit root tests on annual appreciation, there simply is not a long enough span of data to produce reliable test results. On the whole, Phillips-Perron tests for unit roots in the right-hand side variables of the housing price relation in equation (3.8) are inconclusive. The individual right-hand side variables may be either stationary or nonstationary.

It is important to note that valid estimation and inference do not require the right-hand side variables in equation (3.9) to be individually stationary. They do, however, require that the sum of the nominal interest and property tax rates minus the rent-to-

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28 Standard ARMA identification techniques, however, provide informal evidence against unit roots in both rent/price ratios and nominal interest rates. The autocorrelation function of a stationary series declines quite rapidly as the lag length increases, whereas it dies out very slowly for an integrated series. In addition, the partial autocorrelation function for an $I(1)$ series exhibits a statistically significant spike at the first lag. The autocorrelation function for short-term interest rates decreases rapidly and dies out after 3 lags. None of the first autocorrelation coefficient estimates lie above 0.80. Interest rates appear to be a stationary (mean reverting) AR(2) process. Similar results are found for the rent/price ratio.
Moreover, stationarity of this linear combination of the right-hand side variables is a necessary condition for the joint null hypothesis of rational house price expectations and risk neutrality to be true.

I demonstrate these claims in the following way. Combine the right-hand side variables in the risk neutral rational expectations model in equation (3.9), to yield a single explanatory variable. That is, consider the following model:

\[
\frac{P_{t+1} - P_t}{P_t} = \theta_0 + \theta_1 \left[(i_t + pt_t) - \frac{R_t}{P_t}\right] + \nu_{t+1}
\]  

(3.11)

Under the joint null hypothesis of rational expectations and no risk premium, \(\theta_0 = 0\) and \(\theta_1 = 1\). This condition states that realized housing price appreciation equals optimally expected appreciation plus a random error. For notational convenience rewrite (3.11) as

\[
g_{t+1} = \theta_0 + \theta_1 E_t[g_{t+1} | I_t] + \nu_{t+1}
\]

(3.12)

where \(g_{t+1} = \frac{P_{t+1} - P_t}{P_t}\), and \(E_t[g_{t+1} | I_t] = (i_t + pt_t) - \frac{R_t}{P_t}\). I refer to the latter term as "expected capital gains under risk neutrality". If the joint null hypothesis of rational expectations and risk neutrality is true then we know that \(\nu_{t+1}\) is a mean zero, serially uncorrelated stationary random variable. We also know that the actual capital gains series is stationary. As a consequence, the difference, \(g_{t+1} - \nu_{t+1}\) is stationary. Thus from equation (3.12) it must be the case that, under the joint null hypothesis, the expected capital gain term does not possess a unit root (i.e. is stationary). Otherwise the relationship does not make any sense. Phillips-Perron unit root tests, in table 4.2, indicate the single expected capital gains measure, \(E_t[g_{t+1} | I_t]\), may be more likely to satisfy the stationarity requirement than its individual components.

To proceed, I maintain the assumption that the expected capital gains series do not contain unit roots, and estimate the model in (3.12). As a final point on unit roots I note that if this assumption is wrong, and expected capital gains under risk neutrality are nonstationary, the regression estimate of the slope parameter in (3.12) behaves in a predictable way. More precisely, estimates of \(\theta_1\) are driven towards zero. To see this,
consider the expression for the estimator of $\theta_1$ from ordinary least squares estimation of equation (3.11), which is given by

$$\hat{\theta}_1 = \frac{\sum (g_{t+1}) (E_t[g_{t+1} \mid I_t])}{\sum (E_t[g_{t+1} \mid I_t])^2}$$  \hspace{1cm} (3.13)

Assume that $E_t[g_{t+1} \mid I_t]$ is stationary. Take the probability limit of the right-hand side of (3.13) to characterize the limiting properties of $\hat{\theta}_1$ as follows:

$$\text{plim}(\hat{\theta}_1) = \frac{\text{cov}[g_{t+1}, E_t[g_{t+1} \mid I_t]]}{\text{var}[E_t[g_{t+1} \mid I_t]]}$$  \hspace{1cm} (3.14)

If the risk neutral measure of rationally expected house price appreciation is nonstationary then its variance increases over time without bound, which implies that

$$\lim_{t \to \infty} \text{var}[E_t[g_{t+1} \mid I_t]] = \infty$$  \hspace{1cm} (3.15)

and hence $\text{plim}(\hat{\theta}_1) = 0$. Thus, if $E_t[g_{t+1} \mid I_t]$ has a unit root we should find that we reject the rational expectations parameter restrictions. Moreover, slope coefficient estimates will be close to zero. This gives us a check on the stationarity assumption.

### 3.5 Empirical Results

Tables 3.4 and 3.5 report estimation results for the two percentage change model specification, equations (3.9) and (3.11). Part I of each table contains pooled results, which constrain all the coefficients to be the same in all eight areas. Under the joint null hypothesis of rational expectations and no risk premium, the asset market condition given by (3.6) holds in each area, and hence the parameters are independent of location.

Part I of table 3.4 shows that the estimated coefficients deviate substantially from their hypothesized values, for both annual and quarterly price changes. The estimated interest rate coefficients are the ‘wrong’ sign. Ex post condo price changes, both quarterly and annual, are negatively and significantly related to interest rates. Property taxes are a more important determinant of annual price fluctuations than suggested by rational expectations theory, but they are not statistically significant in the quarterly price change regression. The annual rent/price ratio term has the ‘correct’ sign but is not statistically
or economically different from zero while its quarterly counterpart is positively related to future quarterly price appreciation.

The remaining parts of table 3.3 relax the assumption of constant intercept (II) and constant intercept and slope parameters (III) in each municipality. Although there are location-specific differences in point estimates, the central message is unchanged. The parameter estimates are either not statistically significant and/or are much different in magnitude and opposite in sign than equation (3.9) predicts.

From table 3.4 it is evident that imposing the rational expectation parameter restrictions prior to estimation results in a much stronger rejection of the restrictions on the coefficient estimates. Slope estimates are always negative and more precisely estimated than in table 3.3.

On the whole, direct tests of the risk neutral rational expectations housing price relation provide significant evidence against it. On average, risk neutral homeowners mispredict the direction of future house price appreciation. This result holds for both quarterly and annual house price changes.

A Comparison with Foreign Exchange Efficiency Tests

Interestingly, the above results are similar to findings in empirical tests of efficiency in the market for foreign exchange. To test for efficiency of the forward exchange market researchers regress expected depreciation (percentage change in the exchange rate) on the forward premium, \[ s_{t+1} - s_t = \alpha + \beta (f_{t,1} - s_t) + \epsilon_{t, t+1} \] (3.16)

30 Different constants in each municipality may reflect location specific constant risk premiums, liquidity premiums or transction costs.
31 Forward exchange market efficiency is characterized by the condition that todays forward rate, for delivery l periods from now, is an unbiased predictor of the future l-period ahead spot exchange rate. More formally, \( f_{t,l} = E_t[s_{t+l} | I_t] \). A testable specification is given by \( s_{t+1} = \alpha + \beta f_{t,1} + \epsilon_{t, t+1} \). Under the null hypothesis \( \alpha = 0 \) and \( \beta = 1 \). Early tests of the unbiased hypothesis, which use the levels specification, generally finds support for the null hypothesis (see for example Frankel [57]). Subsequent work, however, has shown that both spot and forward rates have unit roots and thus the levels specification is invalid. More recent tests employ (3.10), which is more likely to satisfy the stationarity requirement. Hodrick[93] provides an excellent review of empirical tests of efficiency in the foreign exchange market. Froot and Thaler [61] is a recent, nontechnical summary.

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where $s$ and $f$ are the logarithm of the spot exchange rate and the forward exchange rate, respectively. Under the joint null hypothesis of rational expectations and no risk premium $\alpha = 0$ and $\beta = 1$. This relation is consistently rejected. More specifically, the slope coefficient, $\beta$, is usually found to be negative, and often statistically significant.$^{32}$ Froot and Thaler [61] report that slope coefficient estimates, $\hat{\beta}$, across 75 published studies are always less than one and frequently less than zero. The average estimated value from the 75 published estimates is -0.88.

What is the implication of similar results across different asset markets? Real estate markets are often perceived to be less efficient than other asset markets. Relative to most financial assets, real estate is heterogeneous, immobile, indivisible and illiquid. These unique attributes imply that real estate markets are characterized by high transaction costs and possibly barriers to entry. These potential market imperfections may limit investor participation, and hence information capitalization in real estate values. This implies that it may be possible for real estate investors to earn excess or abnormal returns, since markets prices do not necessarily fully reflect the present value of future cash flows.

Foreign exchange is traded in a well organized, highly liquid market, that is characterized by relatively small transaction costs, easy access to information and large numbers of buyers and sellers. Despite these potentially important differences, the relationship between actual and rationally expected asset price movements, under the maintained hypothesis of risk neutrality, is similar in housing and foreign exchange markets. One way to interpret this finding is that rejection of the risk neutral rational expectations house price model is not attributable to conventional beliefs regarding the unique characteristics of real estate assets and the markets in which they trade. That is, a common force may underly the rejections of the above models in both the housing and foreign exchange markets.

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$^{32}$ Rather than employ the forward premium as an estimate of future spot rates, a number of studies use actual appreciation expectations gathered in surveys of foreign exchange traders. Froot and Frankel [59] follow such an approach and report statistically significant negative coefficient estimates when regressing actual versus survey-based expected appreciation. Cavaglia, Verschoor and Wolff [25] report similar findings using a more recent, and different survey data set.
3.6 Tests for Return Predictability

Tests of the joint null-hypothesis of rational house price expectations and no risk premium provide significant evidence against it. Observed price changes move in the opposite direction, on average, than those predicted by the risk neutral rational expectations model. In the absence of time-varying risk premia, the results imply the Vancouver condominium market is inefficient. Thus, the above results suggest that future excess condominium returns are predictable using variables in the current information set. This section maintains the assumption of no housing market risk premium and examines the economic significance of predictable components in excess condominium returns.

3.6.1 Test Design

Expected excess returns are zero in an efficient asset market. Thus, efficiency of the Vancouver condominium market requires that

\[
E_t \left[ \frac{P_{t+1} - P_t + R_t}{P_t} - i_t \mid I_t \right] = 0
\]

That is, expected excess returns are orthogonal to all variables in the information set. There is no systematic component to forecast errors. If this condition is satisfied and the information set is assumed to contain only past excess returns, then the market is said to exhibit weak form efficiency. When \( I_t \) contains all publicly available information, including excess returns, the condition in (3.17) describes the semi-strong form of market efficiency. The semi-strong form of market efficiency is equivalent to rational expectations.

The results of the regression-based rational expectations tests provide significant evidence against (3.17) for housing returns. This implies that it may be possible to find variables in the current information set that predict future returns. That is, in regressions of the form

\[
\frac{P_{t+1} - P_t + R_t}{P_t} - i_t = \omega_0 + \omega_1 Z_t + e_{t+1}
\]

we should reject the null hypothesis that \( \omega_1 = 0 \) for some set of variables, \( Z_t \), whose values are known at time \( t \).
Choice of Forecasting Variables

A number of researchers argue that asset market inefficiency may be a result of “fads”, or trading by “noise traders”. Noise traders are market participants with irrational expectations. In a world inhabited by noise traders the demand for condominiums responds not only to news about current and future market fundamentals, but also to information completely unrelated to these variables, termed investor sentiment or noise. Market participants may follow technical trading rules and base future price projections on past returns or price changes, and thereby “jump on the bandwagon” and buy homes when prices are rising.

In such a world, short-run house prices are excessively volatile because market participants overreact to news or information on market fundamentals. In periods of rapidly rising house prices, trend chasing forces short-run house prices above fundamental value. Eventually, however, the irrational bubble collapses and prices return to their fundamental value; house prices are mean-reverting. As a consequence, while house price changes exhibit excess volatility over short-horizons, they do not over the long-term.

Econometric tests in support of the noise trader hypothesis should detect positive serial correlation in high frequency returns and negative correlation in long-horizon returns. Investor overreaction or trend chasing drives price from fundamental value in the short-run, but eventually there is a correction and prices return to fundamental value.

I test for these empirical regularities in three different ways. The first test examines the autocorrelation properties of excess quarterly returns. In the second, annual excess returns are regressed on past annual excess returns. Finally, annual excess returns are regressed on a measure of the difference between condominium prices and a measure of fundamental value.

There is considerable evidence that short-term nominal interest rates predict future excess returns on stocks. More precisely, there is a significant negative correlation between nominal interest rates and nominal excess stock returns. Based on these findings,

\[ \text{References include Shiller [141], Cutler, Poterba and Summers [30, 31], Poterba and Summers [127], and Mussa [119]. Shleifer and Summers [135] review the noise trader approach to asset pricing.} \]

\[ \text{Case and Shiller [22] and Collins et al. [27] present survey evidence that indicates homebuyers may indeed follow such extrapolative behaviour.} \]

\[ \text{For evidence on this relationship see Fama and Schwert [46], Campbell [14], Ferson [51], Breen,} \]

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I test for a relationship between 3-month nominal interest rates and excess quarterly condominium returns.

### 3.6.2 Weak Form Efficiency Tests

This section evaluates the ability of past excess housing returns to predict future housing returns. It first examines the autocorrelation structure of quarterly excess returns and then the relationship between longer horizon, annual excess returns.

#### Quarterly Returns

Ex post quarterly excess returns are calculated as rent plus capital gain divided by initial price, minus the three month risk-free rate of interest. The autocorrelation function for excess returns in each municipality is examined for patterns consistent with those noted above. Table 3.6 shows the autocorrelations over a two year period. In general, the hypothesis of random series cannot be rejected. While there is a weak tendency for returns to be positively correlated over the first year and negatively correlated over the next 4 quarters, the values are not statistically significant in a number of the municipalities. Short-term returns appear to follow a random walk.

Even though the autocorrelation properties indicate that excess quarterly returns are unpredictable from past quarterly excess returns, this is not necessarily inconsistent with earlier results. It may be a consequence of the low power of high frequency autocorrelation-based tests. As emphasized by Shiller [141] and Summers [146], tests for autocorrelation in short-horizon returns have little power to detect temporary deviations of market price from fundamental value.  

They derive simple “fads” models of stock price determination in which prices deviate from fundamental value by a slowly moving mean-reverting fad. They show that, even though the market is inefficient, short-horizon returns exhibit little autocorrelation.

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36 See also Poterba and Summers [127].

37 The intuition behind this result is as follows: Assume that the fad or temporary deviation follows an AR(1) process with slope parameter near to, but less than one. This process looks alot like a
Tests for short-term autocorrelation incorrectly accept the random walk hypothesis. Hence, absence of statistically significant correlation in quarterly returns is not necessarily inconsistent with market inefficiency.

Although inefficiency may not be detectable in short-term correlations, the presence of temporary or fad components in asset prices imply that longer-horizon returns are strongly negatively correlated. If condo prices race ahead of fundamental value then over the long-term they are mean-reverting, and thus negatively correlated. Recent work on stock market efficiency examines the predictability of long horizon returns and finds stronger evidence of return predictability than in short-horizon returns.38

Annual Returns

To test for dependence in annual returns, ex post annual excess returns are regressed on lagged values over the past two years. The model is given by

$$r_{t+4} = \gamma_0 + \gamma_1 r_t + \gamma_2 r_{t-4} + u_{t+4}$$ (3.19)

where $$r_{t+4} = \left[ \frac{P_{t+4}+R_t - P_t}{P_t} - i_t \right]$$. Given the results of Case and Shiller [23] [24] and Hosios and Pesando [99], we might expect to find a statistically significant estimate of $$\gamma_1$$. If mean reversion is present, over a two year period, then $$\gamma_2$$ should be negative.

Case and Shiller [24] regress annual price changes on lags over the past four years. In addition to their well documented finding of positive persistence at the one year horizon ([23]), they report weak evidence of long horizon price reversals. However, the negative coefficient estimates are small in magnitude and not statistically different from zero.39

Table 3.7 reports the estimation results. Comparing the constants in parts I and II it is clear that mean excess returns are significantly different from zero and differ by (nonstationary) random walk, even though it is a stationary series. This is the same problem that plagues the power of unit root tests.

38 See Fama and French [47] and Poterba and Summers [127]. Engel and Morris [38] and De Bondt and Thaler [34] summarize the empirical work on mean reversion in the stock market.

39 Moreover, mean reversion in prices is not evidence of mean reversion in excess returns. Compare the expression for excess annual housing returns, $$\frac{P_{t+4}+R_t - P_t}{P_t} - i_t$$, with annual price appreciation, $$\frac{P_{t+4} - P_t}{P_t}$$. Case and Shiller [24] interpret mean reversion in the latter variable as evidence of mean reversion in the former. Interest rates, however, appear to be mean reverting and this implies that, in an efficient market, house prices are also mean reverting. Section 3.4.2 documents that both 3-month and one year risk-free rates follow AR(2) or low order ARMA stochastic processes (see footnote 20).
area. Current excess annual returns are negatively related to annual excess returns two years previous; returns to condominiums appear to be mean-reverting. Positive excess returns in one year predict a fall in excess annual returns, of about one fifth in magnitude, two years later. In contrast to earlier findings, there is little evidence of positive serial correlation at the one-year horizon. In fact, the one year point estimate is negative, although not statistically different from zero. Part III indicates that the conclusions are much weaker with unrestricted parameters across municipalities. There appears to be a lot of noise in the disaggregated data. In addition, given the small sample size these results must be viewed with caution, and are at best suggestive of return predictability with lagged returns.

3.6.3 Semi-Strong Form Market Efficiency

Autocorrelation-based tests of return predictability are tests of weak-form efficiency. The information set contains only current and past returns. This section tests the semi-strong form variant of market efficiency in which the information set is expanded to contain additional available information. More specifically, I examine the predictive power of a function of rent/price ratios and short-term nominal interest rates. Recent work by Fama and French [48] and Cutler Poterba and Summers [31] documents econometric predictability of stock returns using lagged dividend/price ratios.

If the Vancouver condominium market is subject to fads or investor overreaction, then during housing market upswings, price is higher than is justified by market fundamentals. Similarly price may be less than fundamental value in periods immediately following large price declines. Hence, the difference between fundamental value and observed price can be used to forecast future returns. To test the power of a measure of deviation from fundamental value to predict future returns I follow Cutler, Poterba and Summers [31] and estimate the following model:

\[ r_{t+4} = \alpha + \beta \log R_t - \log P_t + u_{t+4} \]  \hspace{1cm} (3.20)

Returns are regressed on a constant and the logarithm of the rent-price ratio. Under the null hypothesis of market efficiency \( \beta = 0 \). Cutler, Poterba and Summers [31] use the
price-dividend ratio, in testing for return predictability in stock returns, to proxy the
difference from the logarithm of a measure of fundamental value, $R_t$, and the asset price.
$\beta$ measures the fraction of the deviation of actual price from fundamental price that is
eliminated over a one-year period.

Table 3.8 presents the results. Differences between the logarithm of condo rents and
prices are statistically significant predictors of future excess returns. This finding is more
robust across the 8 municipalities than the weak-form efficiency tests. Part III shows
that in six of the eight municipalities, the measure of deviation from fundamental value
is quite large in magnitude and precisely estimated.

Table 3.9 reports the results of regressing excess quarterly condominium returns on
short-term nominal interest rates, lagged one period. Again the results are generally
consistent with empirical findings in the stock market literature. Condominium returns
are negatively related to nominal interest rates in seven of the eight housing submarkets.
The relationship is statistically significant at the 5 percent significance level in West
Vancouver and the Westside of the City of Vancouver and marginally significant in all
other areas except East Vancouver.

3.7 Characterizing Predictable Components in Excess Condominium Returns

The foregoing results firmly establish that the joint null hypothesis of rational expecta-
tions and risk neutrality is statistically inconsistent with the data. Evidence from the
percentage change specification indicates that predicted house price appreciation is very
different from the theoretical measure of expected appreciation, for both quarterly and
annual forecast horizons. A number of instruments, including lagged annual returns,
rent/price ratios and nominal interest rates, predict future excess housing returns, to
some extent.

How do we interpret the empirical results? Previous work on housing market efficiency
interprets housing return predictability as evidence of market inefficiency. I have taken
care throughout this chapter to emphasize the joint nature of the hypothesis tests. Thus
the above results are not necessarily evidence of market inefficiency. As documented
throughout the paper, the empirical results are very similar to findings in both the stock market and exchange rate efficiency literatures. Researchers in these fields generally account for rejections in tests of market efficiency with risk neutral agents in one of two ways.\textsuperscript{40}

First, there may be a risk premium in the housing market. Risk neutral rational expectations models are therefore misspecified.\textsuperscript{41} Under this interpretation, variables such as dividend yields, lagged excess returns and nominal interest rates, that have power to forecast future excess asset returns, are related to systematic risk.\textsuperscript{42} A second way to interpret the above evidence is that housing market participants are irrational.\textsuperscript{43}

Whether predictable asset returns represent evidence of market irrationality or rational time-varying risk premiums remains an unanswered question in the empirical finance literature, and I am certainly not going to resolve the problem here. However, to gain insight into the source(s) of rejection this section econometrically characterizes the deviation of condominium prices from those predicted by the risk neutral rational expectations model. It accomplishes this in two ways. The first part of the analysis tests for cointegration between observed condominium prices and their risk neutral rational expectation (equation (3.6)). I then examine the stability of parameters in the percentage change model specification (equation (3.11)) over time.

### 3.7.1 Tests for Cointegration

Statistical tests cannot reject the hypothesis that unit roots are present in the univariate stochastic processes driving prices and rents and thus standard asymptotic inference procedures are not valid in the levels variant of the rational expectations model. This implies that the rational expectations hypothesis is tested via models expressed in terms

\textsuperscript{40}See Hodrick\textsuperscript{[93]} and Fama\textsuperscript{[45]} for interpretations of predictable asset returns. Note, the two possible explanations are by no means mutually exclusive.

\textsuperscript{41}Fama\textsuperscript{[44]} shows how the existence of a time-varying foreign exchange risk premium can lead to negative slope estimates in regressions of spot exchange rate depreciation on the forward premium. In chapter 4 I derive a similar condition for the housing market.

\textsuperscript{42}Rozeff\textsuperscript{[131]} and Fama and French\textsuperscript{[48][49]} argue that time variation in dividend yields captures changes in systematic equity market risk. Ferson\textsuperscript{[50]} derives a model in which equity risk is systematically related to short-term nominal interest rates. Fama\textsuperscript{[45]} provides a review of much of this literature.

\textsuperscript{43}Section 3.6 outlined some of the current research that takes this position.
of price changes. As a consequence, the above results provide little information about whether the measure of expected house prices is an unbiased predictor of future price levels over the longer term. This section uses cointegration techniques to test for the existence of a long-run relationship between rationally expected price levels and ex post outcomes, and to characterize the short-run deviations of the Vancouver condominium from the rational expectations pricing relation.

The Theory of Cointegrated Variables

Suppose two univariate time series, \( x_t \) and \( y_t \) are I(1), or contain a unit root. In general, any linear combination, \( y_t - ax_t \), where \( a \) is a constant, is also nonstationary. There may, however, exist a linear combination, \( z_t = y_t - cx_t \), that is stationary (i.e. \( z_t \) is I(0)). In this special case, following Engle and Granger [40], \( x_t \) and \( y_t \) are said to be cointegrated, \( c \) is the cointegrating parameter, and \( z_t \) is the termed “equilibrium error”.\(^{44}\) Cointegration methods allow the relationship between nonstationary time series to be modeled. Although each individual series is I(1) and drifts upward over time with an ever increasing variance, the difference between \( y_t \) and \( cx_t \) wanders, with a constant variance about a constant mean. The existence of a cointegrating relationship provides evidence of a long-run relationship between the series, while the time series properties of the equilibrium error term describe the short-run dynamics.

Tests for cointegration between \( x_t \) and \( y_t \) consist of the following two steps. First, regress one variable on the other via ordinary least squares to obtain the least squares residuals. Second, test for a unit root in the residuals. A rejection of the null hypothesis of a unit root in the errors implies the series are cointegrated.

\(^{44}\)The concept of cointegration is developed in Engle and Granger [40]. This section relies heavily on their paper.
Cointegration Test Results

To test for cointegration between one-quarter-ahead prices and their risk neutral rational expectation, estimate the following model by ordinary least squares:\(^45\)

\[
P_{t+1} = \delta_0 + \delta_1 [(1 + i_t + pt_t)P_t - R_t] + \epsilon_{t+1}
\]  

This is simply the rational expectations relationship that says actual and expected house prices differ by a mean zero, random error term. Thus under the joint null hypothesis of rational expectations and risk neutrality \(\delta_0 = 0\) and \(\delta_1 = 1\).

Table 3.10 reports estimation and test results. The fourth data column presents the results of unit root tests for cointegration. Note that because the residuals from the cointegrating regression are an estimated time series, Dickey-Fuller [35] critical values are not valid to test for stationarity. The \(t\)-statistic has a different limiting distribution in this situation. Modified critical values are taken from Davidson and MacKinnon ([32], Chapter 20). For each municipality, the reported test statistics lie far outside the 10 percent critical region, which provides strong evidence of stationarity in the forecast errors and cointegration between actual and expected quarterly price levels.

One-quarter ahead price is highly correlated with its expected counterpart, as evidenced by the large \(R^2\)'s. The estimated cointegrating parameters are close to unity in each municipality, although this hypothesis cannot be formally tested since standard inference procedures do not apply. To further investigate the possibility that \(\alpha_1 = 1\), column 5 of table 3.10 reports the results of imposing the restrictions that \(\alpha_0 = 0\) and \(\alpha_1 = 1\) and testing for a unit root in the difference. \(^46\) In this case, standard Dickey-Fuller critical values apply because the differenced series are raw, unestimated variables. The hypothesis of stationary rational expectations forecast errors cannot be rejected. It

---

\(^{45}\) The last column of table 3.2 reports unit root tests for the expected price series. The null hypothesis of nonstationarity cannot be rejected in any of the eight municipalities. Thus both the price and expected price series are nonstationary and tests for cointegration warranted.

\(^{46}\) Stock [144] proves that if the I(1) series are cointegrated then the OLS estimates of the cointegrating parameter are "superconsistent" asymptotically because they converge at rate \(T\), where \(T\) is the sample size, rather than \(T^{1/2}\), which is the case with OLS on stationary variables. The superconsistency property arises from the uniqueness of the cointegrating parameter. The error terms are only stationary when the OLS parameter estimate is 'close' to the true parameter value. This is what motivates the present indirect test for cointegrating parameters equal to 1.
appears that irrational expectations (overreaction or noise trading for example) and/or risk premia introduce a stationary deviation into the risk neutral rational expectations pricing relation.47

Cointegration between \( P_{t+1} \) and \( E_t[P_{t+1} \mid I_t] \), and a cointegrating parameter of one, are necessary but not sufficient conditions for market efficiency.48 Efficiency requires that the residuals are uncorrelated over time. If they are not, then past information can be used to predict future returns, thus violating a condition of market efficiency. The last two columns of table 6 report Q-statistics, which test the joint significance of autocorrelation in the first 4 and 8 lags of the residuals. Under the null hypothesis of random errors, these test statistics are distributed \( \chi^2 \) with 4 and 8 degrees of freedom, respectively. Durbin Watson test statistics are reported in column 2. The null hypothesis of uncorrelated residuals is rejected in only two of the eight areas. The lack of autocorrelation detected by the Q tests in the equilibrium errors is probably a consequence of low power and not evidence in support of rational expectations. The results of the preceding section convincingly demonstrate that the rational expectations hypothesis does not hold.49

The insight gained from the error terms comes from an examination of their time series properties.

Figure 3.4 plots the equilibrium errors, \( z_{t+1} = P_{t+1} - E_t[P_{t+1} \mid I_t] \), associated with each municipality. Deviations in price from rationally predicted value are much larger in periods of volatile house prices than in more tranquil times. This finding could indicate that prices are more difficult to predict in volatile periods. However, coupled with the sound rejection of the unbiased expectations hypothesis obtained with the percentage change specification, it suggests there is more to it than that.

47Meese and Wallace [115] also find that quarterly rational expectations forecast errors are stationary (i.e. price is cointegrated with expected price), but they do not estimate the cointegrating regression to directly evaluate the model.

48Recent papers by Hakkio and Rush [79] and Barnhart and Szakmary [4] use cointegration methods to test for forward exchange market efficiency.

49This highlights the importance of ensuring small sample results are robust to a number of alternative specifications.
A Test for Common Components in the Equilibrium Error

To further help characterize the equilibrium errors this section uses the technique of principal components to extract from the correlation matrix of equilibrium errors the extent to which a single "latent" variable can account for the variation over time in the residuals across all eight submarkets. The method of principal components uses the eigenvalues associated with the correlation matrix to generate 8 orthogonal "principal components" that capture the variation in the original variables. The first principal component is the linear combination of the eight error vectors that maximizes the sample variance. It is like an index that represents the common variation in the data, and measures the extent to which the eight error terms share a common component.

The first principal component accounts for 65.4% of the variation in the correlation matrix of equilibrium errors. Hence, although there are municipality-specific components of the error terms associated with each area, one common "latent" variable accounts for almost two thirds of the variation in the equilibrium errors associated with the eight submarkets. Thus it can be concluded that the inefficiency found in each of the eight submarkets of the Vancouver metro condo market under study has primarily the same underlying source.

ARMA identification indicates that the stochastic process of the first principal component is both stationary and autocorrelated. The lag 1 autocorrelation is 0.306 and statistically different from zero at conventional significance levels (the marginal significance level for the Q statistic is 0.038). Lower frequency correlations, at lags six and seven are also quite large, -0.1256 and -0.2769, respectively. Thus in contrast to most of the individual error terms, the null hypothesis of randomness in their common component is rejected.

The results of the econometric tests up to this point imply that, while there are significant deviations of market price from fundamental value in the short-run, such deviations are eliminated over the longer term.

50 Morrison ([118] chapter 8), is a good reference for the theory of principal components.
3.7.2 Tests of Parameter Stability

The preceding sections establish that short-run deviations about the unbiased or rational expectations relation behave very differently in tranquil and boom/bust episodes. More precisely, the disequilibrium errors are much larger during market upswings and declines than in more stable periods. This suggests that our model of house price dynamics is misspecified due to structural change, omitted variable bias or regime switching for example. In this situation, inference based on full sample estimation may be misleading. This section conducts tests of parameter stability on the quarterly percentage change specification (3.11) and investigates the degree to which the preceding results depend on different subsamples.

To detect potential time periods at which structural change may take place, recursive residuals, their cumulative sum (CUSUM) and cumulative sum squared (CUSUMSQ) are examined for distinctive patterns. Recursive residuals are obtained by first estimating equation (3.11) using a small number of observations and then adding one new observation and reestimating to obtain a new residual. This process is repeated until the model is estimated over the full sample. Under the null hypothesis that the coefficients are stable over time there should be no discernible pattern to the CUSUM or CUSUMSQ.

Recursive residual-based tests indicate that structural breaks may have occurred in all municipalities in late 1986 through to late 1988. This period coincides with the beginning of the Vancouver real estate boom following Expo 1986. Chow tests confirm that structural breaks took place over this period. Hence, the hypothesis of stable slope coefficients over the whole sample period is rejected.

To examine the behaviour of the slope coefficients over time the percentage change specification (3.9) is reestimated over a number of subsamples. More specifically, a "rolling regression" procedure is employed in which the model is estimated over successive 4 year periods. For each estimation a new observation is added while the first one is dropped. The model is estimated 28 times for each area.

Figure 3.5 plots slope estimates against time for each area. The results are striking and

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51 Harvey ([89], chapter 5) provides a detailed exposition of recursive residuals, CUSUM and CUSUMSQ tests for model misspecification.
the instability of the coefficients clearly evident. The rolling regression models document a number of empirical regularities. $\alpha_{i,j}$ are not always negative, as in the full sample estimation, and follow a similar pattern in each municipality. The smaller (more negative) is the point estimate the more statistically significant it becomes, and the better the fit of the model, as evidence by the $R^2$. $\alpha_1$ becomes more positive as the mean capital gain over the estimation period increases, takes on positive values in periods surrounding the peak in quarterly capital gains, and falls sharply as prices fall. There are periods in which the null hypothesis that $\alpha_1 = 1$ cannot be rejected, but these are periods in which the model fits extremely poorly and the parameter estimate is not statistically different from zero.

Plots of auto and partial autocorrelation functions indicate that the time-varying slope estimates are stationary, but highly autocorrelated. AR(2) or low order ARMA models capture the dependence in the eight time series of point estimates.

Principal components are calculated for the correlation matrix of the point estimates of the slope coefficient, associated with each municipality. The first principal component accounts for 65 percent of the common variation. This is the same proportion of the common variation captured by the first principal component of the cointegration residuals.

On the whole, the foregoing investigation into the robustness of parameter estimates to subsampling reveals that the results are extremely sensitive to the time period covered in the estimation.52

3.8 Conclusions

This chapter uses a new data set and a different econometric testing methodology to investigate the extent to which house prices are set in an efficient asset market. The empirical results provide significant evidence against the joint null hypothesis of rational expectations and risk neutrality in the Vancouver condominium apartment market. On average, ex post house price appreciation moves in the opposite direction than predicted by the theoretical measure of expected appreciation, for both quarterly and annual fore-

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52Similar results are found when this exercise is repeated for the annual price change variant.
cast horizons. A number of instruments, including lagged annual returns, rent/price ratios and nominal interest rates, predict future excess housing returns.

These results do not necessarily indicate that the housing market is inefficient. More research is required to before we can determine whether predictable housing returns are evidence of market irrationality or rational time-varying risk premiums.

To date there is no empirical work that attempts to model predictable components in excess housing returns as time-varying risk premia. The lack of work on the pricing of risk in housing markets suggests that more attention be devoted to it prior to assuming irrationality of market participants. If housing price risk is an important consideration in explaining house price movements, then the work undertaken in this chapter suggests that a time-varying risk premium must be stationary, or transitory, in nature and relatively large in periods of rapidly changing house prices. It must also be able to explain the stochastic properties of the coefficients in the rational expectations housing price model. Thus the foregoing empirical regularities should help guide future research on the risk-return relationship in the housing market.
Figure 3.1: Map of Study Area
Figure 3.2: Condominium Rents and Prices by Municipality

I: Prices

(a) Burnaby

(b) East Vancouver
(g) Tsawassen

(h) West Vancouver
II: Rents

(a) Burnaby

(b) East Vancouver
Table 3.1: Summary Statistics for Royal Lepage Condominium Data, 1982:2-1992:1

<table>
<thead>
<tr>
<th>Area</th>
<th>Capital Gains</th>
<th>Rental Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean  SD  Min. Max.</td>
<td>Mean  SD  Min. Max.</td>
</tr>
<tr>
<td>Burnaby</td>
<td>8.55  18.17 -21.51  49.90</td>
<td>4.40  12.54 -18.23  30.01</td>
</tr>
<tr>
<td>East Van.</td>
<td>2.66  14.73 -28.77  47.00</td>
<td>2.93  14.71 -18.22  32.74</td>
</tr>
<tr>
<td>Westside</td>
<td>6.40  14.02 -31.34  36.69</td>
<td>5.90  10.81 -10.54  28.77</td>
</tr>
<tr>
<td>North Van.</td>
<td>6.31  11.77 -14.31  36.24</td>
<td>4.16  12.61 -33.65  23.84</td>
</tr>
<tr>
<td>Richmond</td>
<td>6.84  9.77 -11.28  30.22</td>
<td>6.07  9.95 -34.48  19.42</td>
</tr>
<tr>
<td>Surrey</td>
<td>7.46  14.92 -15.03  51.08</td>
<td>3.61  11.31 -15.42  31.85</td>
</tr>
<tr>
<td>Tsawassen</td>
<td>4.50  10.01 -13.98  30.22</td>
<td>2.07  3.44 -55.01  9.50</td>
</tr>
</tbody>
</table>

Note: All values are expressed as year-to-year percentage change. SD is standard deviation, Max. and Min. refer to maximum and minimum values over the sample period, respectively.
Figure 3.3: A Comparison of Royal Lepage Single-Detached House Price Estimates with a Transactions-Based Quality Adjusted House Price Series

I: Price Levels

II: Quarterly Capital Gains

<table>
<thead>
<tr>
<th>Series</th>
<th>Mean</th>
<th>SD</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Royal Lepage</td>
<td>0.90</td>
<td>6.66</td>
<td>11.12</td>
<td>12.66</td>
</tr>
<tr>
<td>Hedonic</td>
<td>1.26</td>
<td>6.17</td>
<td>12.49</td>
<td>12.66</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.75</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: All values are expressed as percentages. SD is standard deviation, Max. and Min. refer to maximum and minimum values over the sample period, respectively. Hamilton and Hobden [80] is the source of the hedonic series.

Table 3.3: Phillips-Perron Tests for Unit Roots

<table>
<thead>
<tr>
<th>Area</th>
<th>( F_t )</th>
<th>( R_t )</th>
<th>( p_t )</th>
<th>( \frac{P_{t+1} - P_t}{P_t} )</th>
<th>( \frac{P_{t+4} - P_t}{P_t} )</th>
<th>( \frac{P_t}{E_t} )</th>
<th>( E_t[\text{g}_{t,t+1}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Burnaby</td>
<td>-2.09</td>
<td>-2.44</td>
<td>-3.25</td>
<td>-4.67</td>
<td>-2.18</td>
<td>-0.85</td>
<td>-2.87</td>
</tr>
<tr>
<td>East Van.</td>
<td>-2.61</td>
<td>-2.32</td>
<td>-2.72</td>
<td>-6.18</td>
<td>-2.73</td>
<td>-3.78</td>
<td>-3.84</td>
</tr>
<tr>
<td>North Van.</td>
<td>-0.27</td>
<td>-2.53</td>
<td>-2.14</td>
<td>-6.12</td>
<td>-1.91</td>
<td>-2.30</td>
<td>-2.46</td>
</tr>
<tr>
<td>Richmond</td>
<td>-0.74</td>
<td>-5.83</td>
<td>-2.65</td>
<td>-7.05</td>
<td>-2.45</td>
<td>-2.98</td>
<td>-2.21</td>
</tr>
<tr>
<td>Surrey</td>
<td>-2.45</td>
<td>-1.52</td>
<td>-2.67</td>
<td>-4.31</td>
<td>-2.42</td>
<td>-0.63</td>
<td>-2.30</td>
</tr>
<tr>
<td>Tsawassen</td>
<td>-0.79</td>
<td>-1.49</td>
<td>-1.91</td>
<td>-5.16</td>
<td>-2.07</td>
<td>-0.22</td>
<td>-3.00</td>
</tr>
<tr>
<td>West Van.</td>
<td>-0.27</td>
<td>-2.54</td>
<td>-2.14</td>
<td>-5.86</td>
<td>-1.91</td>
<td>-3.29</td>
<td>-3.12</td>
</tr>
</tbody>
</table>


Note: The test statistics for the three-month and annual risk-free rate are -2.46 and -3.07, respectively. The 10% critical value is -2.57 in each case.
Table 3.4: Regressions of Ex Post House Price Appreciation on Variables in the Risk Neutral Rational Expectation Measure of Expected House Price Appreciation

I: Pooled Results

\[ \frac{P_{j,t+1} - P_{j,t}}{P_{j,t}} = \alpha_0 + \alpha_1 i_t + \alpha_2 p_t, t + \alpha_3 (R_{j,t}/P_{j,t}) + \epsilon_{j,t+1} \]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Annual Forecast Horizon</th>
<th>Quarterly Forecast Horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>t-Statistic</td>
</tr>
<tr>
<td>( \alpha_0 )</td>
<td>0.16098</td>
<td>2.3726</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>-2.1014</td>
<td>-3.9432</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>10.008</td>
<td>3.3964</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>-0.0004863</td>
<td>-0.01557</td>
</tr>
<tr>
<td>( LLF )</td>
<td>328.4459</td>
<td></td>
</tr>
</tbody>
</table>

II: Location-Specific Intercepts

\[ \frac{P_{j,t+1} - P_{j,t}}{P_{j,t}} = \alpha_{j,0} + \alpha_1 i_t + \alpha_2 p_t, t + \alpha_3 (R_{j,t}/P_{j,t}) + \epsilon_{j,t+1} \]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Area</th>
<th>Annual Forecast Horizon</th>
<th>Quarterly Forecast Horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Estimate</td>
<td>t-Statistic</td>
</tr>
<tr>
<td>( \alpha_0 )</td>
<td>Burnaby</td>
<td>-0.053474</td>
<td>-0.63175</td>
</tr>
<tr>
<td></td>
<td>East Van.</td>
<td>-0.063855</td>
<td>-0.79535</td>
</tr>
<tr>
<td></td>
<td>Westside</td>
<td>0.020833</td>
<td>0.28743</td>
</tr>
<tr>
<td></td>
<td>North Van.</td>
<td>-0.024822</td>
<td>-0.31690</td>
</tr>
<tr>
<td></td>
<td>Richmond</td>
<td>-0.019188</td>
<td>-0.25128</td>
</tr>
<tr>
<td></td>
<td>Surrey</td>
<td>-0.12831</td>
<td>-1.38020</td>
</tr>
<tr>
<td></td>
<td>Tsawassen</td>
<td>-0.11353</td>
<td>-1.3631</td>
</tr>
<tr>
<td></td>
<td>West Van.</td>
<td>0.020082</td>
<td>0.28659</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td></td>
<td>-1.7632</td>
<td>-3.3169</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td></td>
<td>15.772</td>
<td>4.3050</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td></td>
<td>1.5421</td>
<td>3.3548</td>
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<tr>
<td>( LLF )</td>
<td></td>
<td>347.8309</td>
<td></td>
</tr>
</tbody>
</table>
### III: Location-Specific Parameters

\[
[P_{j,t+1} - P_{j,t}] / P_{j,t} = \alpha_{j,0} + \alpha_{j,1} t + \alpha_{j,2} pt_{j,t} + \alpha_{j,3} (R_{j,t} / P_{j,t}) + \epsilon_{j,t+1}
\]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Area</th>
<th>Annual Forecast Horizon</th>
<th>Quarterly Forecast Horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>t-Statistic</td>
<td>Estimate</td>
</tr>
<tr>
<td>(a_0):</td>
<td>Burnaby</td>
<td>-0.42330</td>
<td>-1.8180</td>
</tr>
<tr>
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<td>-0.52285</td>
<td>-2.4900</td>
</tr>
<tr>
<td></td>
<td>Westside</td>
<td>-0.91204</td>
<td>-3.8626</td>
</tr>
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<td></td>
<td>North Van.</td>
<td>0.002061</td>
<td>0.13118</td>
</tr>
<tr>
<td></td>
<td>Richmond</td>
<td>-0.16657</td>
<td>-1.1006</td>
</tr>
<tr>
<td></td>
<td>Surrey</td>
<td>-0.31894</td>
<td>-1.4672</td>
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<tr>
<td></td>
<td>Tsawassen</td>
<td>0.18742</td>
<td>-1.1344</td>
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<tr>
<td></td>
<td>West Van.</td>
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<td>-2.0240</td>
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<td>-2.7722</td>
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<td></td>
<td>East Van.</td>
<td>-0.91421</td>
<td>-0.60885</td>
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<td>Westside</td>
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<td>-0.39327</td>
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<td></td>
<td>North Van.</td>
<td>-2.8741</td>
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<td>Richmond</td>
<td>-1.6810</td>
<td>-1.6974</td>
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<tr>
<td></td>
<td>Surrey</td>
<td>-1.9100</td>
<td>-1.0875</td>
</tr>
<tr>
<td></td>
<td>Tsawassen</td>
<td>-2.4289</td>
<td>-2.2042</td>
</tr>
<tr>
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<td>West Van.</td>
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<td>-3.0663</td>
</tr>
<tr>
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<td>Burnaby</td>
<td>8.6981</td>
<td>1.0303</td>
</tr>
<tr>
<td></td>
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<td>62.526</td>
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</tr>
<tr>
<td></td>
<td>Westside</td>
<td>29.971</td>
<td>2.4473</td>
</tr>
<tr>
<td></td>
<td>North Van.</td>
<td>34.428</td>
<td>2.0193</td>
</tr>
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<td></td>
<td>Richmond</td>
<td>39.405</td>
<td>2.9210</td>
</tr>
<tr>
<td></td>
<td>Surrey</td>
<td>9.8496</td>
<td>1.1719</td>
</tr>
<tr>
<td></td>
<td>Tsawassen</td>
<td>-8.994</td>
<td>-1.7079</td>
</tr>
<tr>
<td></td>
<td>West Van.</td>
<td>73.620</td>
<td>6.3317</td>
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</tr>
<tr>
<td></td>
<td>Westside</td>
<td>12.107</td>
<td>5.4244</td>
</tr>
<tr>
<td></td>
<td>North Van.</td>
<td>0.21839</td>
<td>0.12029</td>
</tr>
<tr>
<td></td>
<td>Richmond</td>
<td>0.30747</td>
<td>0.25094</td>
</tr>
<tr>
<td></td>
<td>Surrey</td>
<td>3.7733</td>
<td>3.5036</td>
</tr>
<tr>
<td></td>
<td>Tsawassen</td>
<td>2.2535</td>
<td>2.1426</td>
</tr>
<tr>
<td></td>
<td>West Van.</td>
<td>-0.40144</td>
<td>-0.41371</td>
</tr>
</tbody>
</table>

**LLF**: 390.3179

Note: t-statistics in the annual forecast horizon models are constructed using standard errors that have been adjusted to account for third order autocorrelation using the Newey-West [120] covariance matrix, with lag length set to four.
Table 3.5: Regressions of Ex Post Capital Gains on Expected Capital Gains Under Risk Neutrality

I: Pooled Results

\[
\frac{P_{j,t+1} - P_{j,t}}{P_{j,t}} = \theta_0 + \theta_1 E[(P_{j,t+1} - P_{j,t})/P_{j,t} | I_t] + \epsilon_{j,t+1}
\]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Annual Forecast Horizon</th>
<th>Quarterly Forecast Horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>t-Statistic</td>
</tr>
<tr>
<td>(\theta_0)</td>
<td>0.080919</td>
<td>4.8210</td>
</tr>
<tr>
<td>(\theta_1)</td>
<td>-0.99670</td>
<td>-4.0669</td>
</tr>
<tr>
<td>(LLF)</td>
<td>318.2137</td>
<td></td>
</tr>
</tbody>
</table>

II: Location-Specific Intercepts

\[
\frac{P_{j,t+1} - P_{j,t}}{P_{j,t}} = \theta_{0j} + \theta_1 E[(P_{j,t+1} - P_{j,t})/P_{j,t} | I_t] + \epsilon_{j,t+1}
\]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Area</th>
<th>Annual Forecast Horizon</th>
<th>Quarterly Forecast Horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Estimate</td>
<td>t-Statistic</td>
</tr>
<tr>
<td>(\theta_0)</td>
<td>Burnaby</td>
<td>0.099693</td>
<td>3.3397</td>
</tr>
<tr>
<td></td>
<td>East Van.</td>
<td>0.063496</td>
<td>2.4559</td>
</tr>
<tr>
<td></td>
<td>Westside</td>
<td>0.15935</td>
<td>6.5351</td>
</tr>
<tr>
<td></td>
<td>North Van.</td>
<td>0.11603</td>
<td>5.5567</td>
</tr>
<tr>
<td></td>
<td>Richmond</td>
<td>0.13003</td>
<td>7.4717</td>
</tr>
<tr>
<td></td>
<td>Surrey</td>
<td>0.039818</td>
<td>1.4132</td>
</tr>
<tr>
<td></td>
<td>Tsawassen</td>
<td>0.058768</td>
<td>3.4707</td>
</tr>
<tr>
<td>(\theta_1)</td>
<td>West Van.</td>
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<td>6.9356</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-2.1371</td>
<td>-5.9206</td>
</tr>
<tr>
<td>(LLF)</td>
<td>335.0166</td>
<td></td>
<td>550.2119</td>
</tr>
</tbody>
</table>
### III: Location-Specific Parameters

\[
[P_{j,t+1} - P_{j,t}] / P_{j,t} = \theta_{j,0} + \theta_{j,1} E[(P_{j,t+1} - P_{j,t}) / P_{j,t} | I_t] + \nu_{j,t+1}
\]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Area</th>
<th>Annual Forecast Horizon</th>
<th>Quarterly Forecast Horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>t-Statistic</td>
<td>Estimate</td>
</tr>
<tr>
<td>(\theta_0)</td>
<td>Burnaby</td>
<td>0.088965</td>
<td>3.2451</td>
</tr>
<tr>
<td></td>
<td>East Van.</td>
<td>0.061374</td>
<td>2.2007</td>
</tr>
<tr>
<td></td>
<td>Westside</td>
<td>0.2240</td>
<td>6.2070</td>
</tr>
<tr>
<td></td>
<td>North Van.</td>
<td>0.11199</td>
<td>4.6578</td>
</tr>
<tr>
<td></td>
<td>Richmond</td>
<td>0.11239</td>
<td>5.1907</td>
</tr>
<tr>
<td></td>
<td>Surrey</td>
<td>0.02947</td>
<td>0.94067</td>
</tr>
<tr>
<td></td>
<td>Tsawassen</td>
<td>0.056543</td>
<td>3.2019</td>
</tr>
<tr>
<td></td>
<td>West Van.</td>
<td>0.18259</td>
<td>6.5680</td>
</tr>
<tr>
<td>(\theta_1)</td>
<td>Burnaby</td>
<td>-5.0303</td>
<td>-5.3867</td>
</tr>
<tr>
<td></td>
<td>East Van.</td>
<td>-1.9556</td>
<td>-2.4885</td>
</tr>
<tr>
<td></td>
<td>Westside</td>
<td>-3.7972</td>
<td>-5.1926</td>
</tr>
<tr>
<td></td>
<td>North Van.</td>
<td>-1.9396</td>
<td>-2.7594</td>
</tr>
<tr>
<td></td>
<td>Richmond</td>
<td>-1.4422</td>
<td>-2.4035</td>
</tr>
<tr>
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<td>Surrey</td>
<td>-2.5808</td>
<td>-4.5483</td>
</tr>
<tr>
<td></td>
<td>Tsawassen</td>
<td>-1.4970</td>
<td>-2.6448</td>
</tr>
<tr>
<td></td>
<td>West Van.</td>
<td>-2.8794</td>
<td>-5.2727</td>
</tr>
</tbody>
</table>

\[ LLF \]

390.3179   554.018

**Note:** The models are estimated with iterative seemingly unrelated regression (ISUR) techniques. t-statistics in the annual forecast horizon models are constructed using standard errors that have been adjusted to account for third order autocorrelation using the Newey-West [120] covariance matrix, with lag length set to four.
Table 3.6: Autocorrelations in Excess Quarterly Condominium Returns

<table>
<thead>
<tr>
<th>Area</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Burnaby</td>
<td>0.35</td>
<td>0.15</td>
<td>0.22</td>
<td>0.13</td>
<td>-0.07</td>
<td>-0.04</td>
<td>-0.11</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.034)</td>
<td>(0.028)</td>
<td>(0.042)</td>
<td>(0.071)</td>
<td>(0.115)</td>
<td>(0.143)</td>
<td>(0.207)</td>
</tr>
<tr>
<td>East Van.</td>
<td>0.14</td>
<td>0.10</td>
<td>0.09</td>
<td>-0.33</td>
<td>-0.15</td>
<td>-0.03</td>
<td>-0.08</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>(0.358)</td>
<td>(0.529)</td>
<td>(0.654)</td>
<td>(0.132)</td>
<td>(0.141)</td>
<td>(0.215)</td>
<td>(0.274)</td>
<td>(0.341)</td>
</tr>
<tr>
<td>Westside</td>
<td>0.00</td>
<td>-0.21</td>
<td>-0.06</td>
<td>0.03</td>
<td>0.02</td>
<td>-0.08</td>
<td>-0.17</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(0.983)</td>
<td>(0.365)</td>
<td>(0.536)</td>
<td>(0.695)</td>
<td>(0.815)</td>
<td>(0.855)</td>
<td>(0.763)</td>
<td>(0.840)</td>
</tr>
<tr>
<td>North Van.</td>
<td>-0.01</td>
<td>0.14</td>
<td>-0.11</td>
<td>0.16</td>
<td>0.22</td>
<td>0.11</td>
<td>0.19</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>(0.973)</td>
<td>(0.637)</td>
<td>(0.691)</td>
<td>(0.597)</td>
<td>(0.396)</td>
<td>(0.444)</td>
<td>(0.360)</td>
<td>(0.463)</td>
</tr>
<tr>
<td>Richmond</td>
<td>-0.04</td>
<td>0.24</td>
<td>-0.10</td>
<td>-0.09</td>
<td>0.10</td>
<td>-0.09</td>
<td>0.27</td>
<td>-0.07</td>
</tr>
<tr>
<td></td>
<td>(0.809)</td>
<td>(0.260)</td>
<td>(0.362)</td>
<td>(0.458)</td>
<td>(0.530)</td>
<td>(0.599)</td>
<td>(0.287)</td>
<td>(0.355)</td>
</tr>
<tr>
<td>Surrey</td>
<td>0.39</td>
<td>-0.06</td>
<td>-0.20</td>
<td>-0.16</td>
<td>0.13</td>
<td>0.11</td>
<td>-0.06</td>
<td>-0.17</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.028)</td>
<td>(0.028)</td>
<td>(0.034)</td>
<td>(0.045)</td>
<td>(0.063)</td>
<td>(0.095)</td>
<td>(0.089)</td>
</tr>
<tr>
<td>Tsawassen</td>
<td>0.20</td>
<td>0.10</td>
<td>0.02</td>
<td>0.02</td>
<td>-0.01</td>
<td>0.02</td>
<td>-0.03</td>
<td>-0.14</td>
</tr>
<tr>
<td></td>
<td>(0.184)</td>
<td>(0.323)</td>
<td>(0.517)</td>
<td>(0.681)</td>
<td>(0.805)</td>
<td>(0.888)</td>
<td>(0.936)</td>
<td>(0.900)</td>
</tr>
<tr>
<td>West Van.</td>
<td>0.014</td>
<td>0.09</td>
<td>0.01</td>
<td>0.14</td>
<td>0.04</td>
<td>-0.11</td>
<td>-0.04</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>(0.359)</td>
<td>(0.551)</td>
<td>(0.754)</td>
<td>(0.700)</td>
<td>(0.807)</td>
<td>(0.822)</td>
<td>(0.887)</td>
<td>(0.892)</td>
</tr>
<tr>
<td>Average</td>
<td>0.131</td>
<td>0.069</td>
<td>-0.016</td>
<td>-0.013</td>
<td>0.049</td>
<td>-0.014</td>
<td>-0.004</td>
<td>-0.029</td>
</tr>
</tbody>
</table>

Note: Figures in parentheses below each lag are the marginal significance levels associated with the Q-statistics for tests of joint significance in the autocorrelations up to and including that lag.
Table 3.7: Regressions of Excess Annual Condominium Returns on Lagged Excess Returns

### I: Pooled Results

\[ r_{jt,t+4} = \gamma_0 + \gamma_1 r_{j,t} + \gamma_2 r_{j,t-4} + u_{j,t+4} \]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimate</th>
<th>t-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_0 )</td>
<td>-0.032806</td>
<td>-0.82198</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>-0.04632</td>
<td>-0.49090</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>-0.09461</td>
<td>-1.1642</td>
</tr>
</tbody>
</table>

\( LLF = 245.8160 \)

### II: Location-Specific Intercepts

\[ r_{jt,t+4} = \gamma_{jt,0} + \gamma_1 r_{j,t} + \gamma_2 r_{j,t-4} + u_{j,t+4} \]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Area</th>
<th>Estimate</th>
<th>t-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_0 ): Burnaby</td>
<td>0.2022</td>
<td>4.9382</td>
<td></td>
</tr>
<tr>
<td>East Van.</td>
<td>0.0539</td>
<td>1.6511</td>
<td></td>
</tr>
<tr>
<td>Westside</td>
<td>0.057622</td>
<td>2.2520</td>
<td></td>
</tr>
<tr>
<td>North Van.</td>
<td>0.10396</td>
<td>4.3097</td>
<td></td>
</tr>
<tr>
<td>Richmond</td>
<td>0.080986</td>
<td>3.5249</td>
<td></td>
</tr>
<tr>
<td>Surrey</td>
<td>0.18962</td>
<td>5.2503</td>
<td></td>
</tr>
<tr>
<td>Tsawassen</td>
<td>0.081687</td>
<td>3.5983</td>
<td></td>
</tr>
<tr>
<td>West Van.</td>
<td>0.041445</td>
<td>1.9297</td>
<td></td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>-0.0545</td>
<td>-0.63850</td>
<td></td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>-0.21739</td>
<td>-3.1293</td>
<td></td>
</tr>
</tbody>
</table>

\( LLF = 283.3792 \)
### III: Location-Specific Parameters

\[ r_{j,t+4} = \gamma_{j,0} + \gamma_{j,1} r_{j,t} + \gamma_{j,2} r_{j,t-4} + u_{j,t+4} \]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Area</th>
<th>Estimate</th>
<th>t-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_0 )</td>
<td>Burnaby</td>
<td>0.15505</td>
<td>3.7256</td>
</tr>
<tr>
<td></td>
<td>East Van.</td>
<td>0.055298</td>
<td>1.5617</td>
</tr>
<tr>
<td></td>
<td>Westside</td>
<td>0.065979</td>
<td>2.5931</td>
</tr>
<tr>
<td></td>
<td>North Van.</td>
<td>0.089512</td>
<td>4.4303</td>
</tr>
<tr>
<td></td>
<td>Richmond</td>
<td>0.072482</td>
<td>3.229</td>
</tr>
<tr>
<td></td>
<td>Surrey</td>
<td>0.32663</td>
<td>8.6554</td>
</tr>
<tr>
<td></td>
<td>Tsawassen</td>
<td>0.055081</td>
<td>2.3238</td>
</tr>
<tr>
<td></td>
<td>West Van.</td>
<td>0.040529</td>
<td>1.8964</td>
</tr>
</tbody>
</table>

| \( \gamma_1 \) | Burnaby       | 0.16155  | 1.5011      |
|                | East Van.     | 0.016048 | 0.12362     |
|                | Westside      | -0.18664 | -1.8064     |
|                | North Van.    | 0.076531 | 0.82118     |
|                | Richmond      | 0.081461 | 0.82628     |
|                | Surrey        | -0.57279 | -4.7960     |
|                | Tsawassen     | 0.32534  | 3.411       |
|                | West Van.     | -0.065002| -0.43851    |

| \( \gamma_2 \) | Burnaby       | -0.053865| -0.74628    |
|                | East Van.     | -0.27632 | -2.4877     |
|                | Westside      | -0.34619 | -4.6176     |
|                | North Van.    | 0.0044739| 0.04932     |
|                | Richmond      | -0.13277 | -1.1742     |
|                | Surrey        | -0.64373 | -6.6993     |
|                | Tsawassen     | -0.066272| -0.71577    |
|                | West Van.     | -0.15106 | -1.2727     |

\[ LLF \quad 316.0368 \]

Note: The models are estimated with iterative seemingly unrelated regression (ISUR) techniques. t-statistics are constructed using standard errors that have been adjusted to account for third order autocorrelation using the Newey-West [120] covariance matrix, with lag length set to four.
Table 3.8: Regressions of Annual Condominium Returns on Lagged Dividend Yields

I: Pooled Results

\[ r_{j,t+4} = \lambda + \phi \log(R_t/P_t) + \nu_{j,t+4} \]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimate</th>
<th>t-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>0.42736</td>
<td>6.8981</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.15582</td>
<td>6.8727</td>
</tr>
<tr>
<td><strong>LLF</strong></td>
<td>307.7511</td>
<td></td>
</tr>
</tbody>
</table>

II: Location-Specific Intercepts

\[ r_{j,t+4} = \lambda_j + \phi \log(R_t/P_t) + \nu_{j,t+4} \]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Area</th>
<th>Estimate</th>
<th>t-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda ):</td>
<td>Burnaby 0.79758</td>
<td>6.6807</td>
<td></td>
</tr>
<tr>
<td></td>
<td>East Van. 0.75933</td>
<td>5.9246</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Westside 0.878662</td>
<td>6.0268</td>
<td></td>
</tr>
<tr>
<td></td>
<td>North Van. 0.81485</td>
<td>6.0677</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Richmond 0.83191</td>
<td>6.0477</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Surrey 0.75368</td>
<td>6.5184</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tsawassen 0.75402</td>
<td>6.1927</td>
<td></td>
</tr>
<tr>
<td></td>
<td>West Van. 0.87970</td>
<td>5.7812</td>
<td></td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.30576</td>
<td>5.8549</td>
<td></td>
</tr>
<tr>
<td><strong>LLF</strong></td>
<td>283.3792</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
III: Location-Specific Parameters

\[ r_{j,t+4} = \lambda_j + \phi_j \log(R_t/P_t) + \nu_{j,t+4} \]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Area</th>
<th>Estimate</th>
<th>t-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda ):</td>
<td>Burnaby</td>
<td>2.2244</td>
<td>4.8338</td>
</tr>
<tr>
<td></td>
<td>East Van.</td>
<td>0.91722</td>
<td>3.5129</td>
</tr>
<tr>
<td></td>
<td>Westside</td>
<td>2.4798</td>
<td>5.7417</td>
</tr>
<tr>
<td></td>
<td>North Van.</td>
<td>0.68129</td>
<td>3.0401</td>
</tr>
<tr>
<td></td>
<td>Richmond</td>
<td>0.14493</td>
<td>0.57941</td>
</tr>
<tr>
<td></td>
<td>Surrey</td>
<td>1.3427</td>
<td>8.8870</td>
</tr>
<tr>
<td></td>
<td>Tsawassen</td>
<td>0.23158</td>
<td>0.98793</td>
</tr>
<tr>
<td></td>
<td>West Van.</td>
<td>1.0003</td>
<td>4.3991</td>
</tr>
<tr>
<td>( \phi ):</td>
<td>Burnaby</td>
<td>0.94146</td>
<td>4.6175</td>
</tr>
<tr>
<td></td>
<td>East Van.</td>
<td>0.37166</td>
<td>3.4402</td>
</tr>
<tr>
<td></td>
<td>Westside</td>
<td>0.88752</td>
<td>5.6655</td>
</tr>
<tr>
<td></td>
<td>North Van.</td>
<td>0.25223</td>
<td>2.8179</td>
</tr>
<tr>
<td></td>
<td>Richmond</td>
<td>0.036971</td>
<td>0.37902</td>
</tr>
<tr>
<td></td>
<td>Surrey</td>
<td>0.578686</td>
<td>8.2994</td>
</tr>
<tr>
<td></td>
<td>Tsawassen</td>
<td>0.079545</td>
<td>0.78758</td>
</tr>
<tr>
<td></td>
<td>West Van.</td>
<td>0.34826</td>
<td>4.3490</td>
</tr>
</tbody>
</table>

\[ LLF = 349.0214 \]

Note: The models are estimated with iterative seemingly unrelated regression (ISUR) techniques. \( t \)-statistics are constructed using standard errors that have been adjusted to account for third order autocorrelation using the Newey-West [120] covariance matrix, with lag length set to four.
Table 3.9: Regressions of Excess Quarterly Condominium Returns on Lagged Nominal Interest Rates

\[ r_{j,t+1} = a_{0,j} + a_{1,j} i_{t} + e_{j,t+1} \]

<table>
<thead>
<tr>
<th>Area</th>
<th>( \hat{a}_1 )</th>
<th>t-Statistic</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Burnaby</td>
<td>-3.42</td>
<td>-1.62</td>
<td>0.06</td>
</tr>
<tr>
<td>East Van.</td>
<td>0.57</td>
<td>0.32</td>
<td>0.00</td>
</tr>
<tr>
<td>Westside</td>
<td>-7.51</td>
<td>-3.00</td>
<td>0.18</td>
</tr>
<tr>
<td>North Van.</td>
<td>-3.09</td>
<td>-1.78</td>
<td>0.07</td>
</tr>
<tr>
<td>Richmond</td>
<td>-2.39</td>
<td>-1.65</td>
<td>0.06</td>
</tr>
<tr>
<td>Surrey</td>
<td>-1.45</td>
<td>-0.76</td>
<td>0.01</td>
</tr>
<tr>
<td>Tsawassen</td>
<td>-2.07</td>
<td>-1.59</td>
<td>0.06</td>
</tr>
<tr>
<td>West Van.</td>
<td>-4.98</td>
<td>-3.67</td>
<td>0.25</td>
</tr>
</tbody>
</table>
Table 3.10: Tests for Cointegration Between Actual and Expected One Quarter Ahead House Prices

<table>
<thead>
<tr>
<th>Area</th>
<th>$\delta_1$</th>
<th>$R^2$</th>
<th>DW</th>
<th>PP Unit Root Tests</th>
<th>Q(4)</th>
<th>Q(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>($\delta_0$, $\delta_1$)</td>
<td>($\delta_0 = 0$, $\delta_1 = 1$)</td>
<td></td>
</tr>
<tr>
<td>Burnaby</td>
<td>0.94318</td>
<td>0.9657</td>
<td>1.331</td>
<td>-4.5565</td>
<td>-4.4292</td>
<td>8.34</td>
</tr>
<tr>
<td></td>
<td>(33.939)</td>
<td>(33.939)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>East Van.</td>
<td>0.85969</td>
<td>0.8767</td>
<td>1.410</td>
<td>-4.6536</td>
<td>-6.0185</td>
<td>7.58</td>
</tr>
<tr>
<td></td>
<td>(17.556)</td>
<td>(17.556)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Westside</td>
<td>0.80480</td>
<td>0.8764</td>
<td>1.407</td>
<td>-6.8943</td>
<td>-5.3627</td>
<td>2.52</td>
</tr>
<tr>
<td></td>
<td>(15.728)</td>
<td>(15.728)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>North Van.</td>
<td>1.0157</td>
<td>0.9485</td>
<td>2.179</td>
<td>-7.2836</td>
<td>-5.9755</td>
<td>4.89</td>
</tr>
<tr>
<td></td>
<td>(28.004)</td>
<td>(28.004)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Richmond</td>
<td>0.98416</td>
<td>0.9580</td>
<td>2.327</td>
<td>-7.7140</td>
<td>-6.6759</td>
<td>5.40</td>
</tr>
<tr>
<td></td>
<td>(30.684)</td>
<td>(30.684)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surrey</td>
<td>0.91984</td>
<td>0.9551</td>
<td>1.346</td>
<td>-4.3981</td>
<td>-5.2458</td>
<td>12.55</td>
</tr>
<tr>
<td></td>
<td>(31.181)</td>
<td>(31.181)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tsawassen</td>
<td>0.96768</td>
<td>0.9399</td>
<td>1.771</td>
<td>-5.6617</td>
<td>-4.6681</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>(25.739)</td>
<td>(25.739)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>West Van.</td>
<td>0.91425</td>
<td>0.9562</td>
<td>1.528</td>
<td>-6.5627</td>
<td>-5.5903</td>
<td>2.26</td>
</tr>
<tr>
<td></td>
<td>(29.004)</td>
<td>(29.004)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5% cv</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9.488</td>
<td>15.507</td>
</tr>
<tr>
<td>10% cv</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-3.04</td>
<td>-2.57</td>
</tr>
</tbody>
</table>

Note: DW is the Durbin-Watson statistic, PP are Phillips-Perron Unit Root Tests for nonstationarity in the residuals and Q(4) and Q(8) are modified Box-Pierce statistics that test for the significance of joint randomness in the first 4 and 8 lags of the residuals, respectively. Standard t-statistics appear in parentheses. These statistics do not have t-distributions, however, due to unit roots in the level series.
Figure 3.4: Rational Expectations Forecast Errors

(a) Burnaby

(b) East Vancouver
Figure 3.5: Estimated Slope Coefficients from Rolling Regressions

(a) Burnaby

(b) East Vancouver
(g) Tsawassen

(h) West Vancouver
Chapter 4

Is There a Time-Varying Risk Premium in the Housing Market?

4.1 Introduction

This chapter formulates and estimates two econometric models of time-varying housing market risk premia. Chapter 3 tests rational expectations housing price models which assume homeowners are risk neutral. Under the joint null hypothesis of unbiased house price expectations and risk neutrality, regressions of ex post house price appreciation on its rational expectation should yield a slope coefficient of one. Empirical tests, however, find statistically significant negative parameter estimates and thereby reject the joint null hypothesis. This chapter aims to answer the following question: can a time-varying housing market risk premium explain deviations in house price fluctuations from those predicted by the rational expectations hypothesis under risk neutrality? More precisely, it derives and tests conditional capital asset pricing models for housing, in which nonzero expected excess housing returns reflect time-varying housing market risk.

There is now a considerable empirical literature that rejects the simple market efficiency hypothesis in the markets for common stock and foreign exchange, (i.e. the joint null hypothesis of rational expectations and no risk premium).¹ It is widely accepted that there are predictable excess returns on both stocks and foreign exchange. A number

¹Shiller [140], Fama [45], De Bondt and Thaler [34] and Engel and Morris [38] review and provide references to the stock market literature. Boothe and Longworth [8], Hodrick [93], Baillie and McMahon [10] and Froot and Thaler [61] do the same for the empirical exchange rate literature.
of researchers interpret these findings as evidence of time-varying risk premia.\textsuperscript{2} Expected returns to assets that are subject to systematic risk should be nonzero. If the systematic risk moves over time then so do rational expectations of future returns.\textsuperscript{3}

Chapters 2 and 3 of this thesis contribute to a growing literature that detects predictable components in excess returns to housing. All previous empirical work on housing market efficiency assumes that housing market participants are risk-neutral, or that housing price risk is diversifiable in a portfolio of investments.\textsuperscript{4} If, however, agents are risk averse, and housing price risk is not completely diversifiable, then housing market efficiency implies that returns to housing investment should be positively correlated with a premium for bearing risk.\textsuperscript{5} This raises the possibility that earlier results are consistent with rational expectations in the presence of a time-varying housing price risk premium.

The first part of this chapter shows that, in theory, the finding of negative slope coefficients in tests of unbiased house price expectations under risk neutrality (i.e. the results of Chapter 3) can be attributed to omitted variable bias if two conditions are satisfied:

\textsuperscript{2}This is only one interpretation. Hodrick ([93], chapter 4) provides a good discussion of the schools of thought that have emerged within the finance profession on the causes that underly rejections of simple market efficiency tests.

\textsuperscript{3}Modeling predictable returns as time-varying risk premia is an active area of research in finance at the present time. Existing attempts have met with mixed success. See for example Hansen and Hodrick [85], Fama [44], Hodrick and Srivastava [94], Mark [111][112], Domowitz and Hakkio [37], Cumby [29], Froot and Frankel [59], and Hopper [98] on foreign exchange risk premiums, and Gibbons and Ferson [71], Ferson [50], Campbell [14], Harvey [90], Fama and French [48][49] and Ferson, Foerster and Keim [52] on pricing time-varying stock market risk.

\textsuperscript{4}In contrast, research on housing tenure choice and tenure transition timing explicitly incorporates risk aversion and portfolio considerations. For example, Plaut [124] notes that households who own their homes typically hold highly undiversified portfolios dominated by a single risky (housing) asset (see also Henderson and Ioannides [92]). He derives a theoretical model of the timing of tenure transition (from renting to owning) in which “part of the “cost” of purchasing housing is generally the assumption of large portfolio risk, stemming from the limited capability of households to diversify portfolios once housing is purchased” ([124], page 321). He shows that risk averse households delay transition longer the higher the volatility of house prices and the stronger the correlation between housing and stock returns. Rosen, Rosen and Holtz-Eszkin [129] empirically investigate the effects of house price uncertainty on tenure choice and find that the proportion of U.S. households that are homeowners varies inversely with house price uncertainty.

\textsuperscript{5}Gyourko and Keim [78] find that common factors drive returns on both real estate (residential and nonresidential) and non-real estate related assets. In addition, recent evidence suggests that the variation in stock and bond returns over time is predictable and the result of changes in business cycle conditions (see for example Campbell [14] and Fama and French [49]). Together these findings imply that returns to housing investment may be related to the business cycle and thus housing price risk is not completely diversifiable.
(1) the covariance between the risk premium and expected house price appreciation under risk neutrality is negative, and; (2) variation through time in the risk premium is considerably larger than the variation in expected appreciation under risk neutrality.

The second part of the paper uses a conditional capital asset pricing model (CAPM) to derive a measure of time-varying housing market risk premia. Systematic housing price risk is shown to be a positive function of comovement between an unobservable market or benchmark portfolio and excess housing returns. The benchmark portfolio is a value weighted market index that contains both an unobservable housing component and aggregate common stock returns.

I test two versions of the housing price model with Vancouver housing market and Toronto Stock Exchange data. The first empirical model assumes a constant market reward-to-risk ratio. This leads to a housing market risk premium that depends on a linear combination of conditional house price volatility and the conditional covariance between Vancouver house price movements and returns to the Toronto Stock Exchange 300 composite index. The data convincingly rejects this risk premium model. The second model treats the ratios of the CAPM betas across Vancouver housing submarkets as constants and the benchmark portfolio as unobservable. This leads to a set of cross equation restrictions that the data must satisfy to be consistent with the model. Tests of the cross equation restrictions offer some support for the CAPM risk premium model. Another interpretation of the results is that the Vancouver housing market is subject to “fads”.

The rest of this chapter proceeds as follows. Section 4.2 establishes the conditions under which omitted variable bias, due to neglect of a time-varying risk premium, leads to negative coefficient estimates in tests of rational expectations housing price models that assume risk neutrality. Section 4.3 presents a CAPM model of housing price risk. Section 4.4 derives the econometric housing market risk models and presents empirical results. Section 4.5 gives concluding comments.
4.2 Time-Varying Risk and Omitted Variable Bias

This section investigates the effect of omitting a relevant explanatory variable on parameter estimates in tests of the rational expectations hypothesis which assume investors are risk neutral. It combines statistical analysis of omitted variable bias with the stylized facts about deviations in price from fundamental value documented in chapter 3 to examine the theoretical relevance of omitted risk considerations as the cause of rejections of the risk neutral rational expectations housing price model.

Suppose that homeowners are risk averse and housing price risk is not completely diversifiable. That is, there is a systematic component to housing price risk. Under these conditions, predictable variation in expected excess returns to housing reflect time variation in compensation for bearing risk. That is,

\[ E\left[ \frac{P_{t+1} - P_t + R_t}{P_t} - i_t - p_t \mid I_t \right] = \rho_t \]  

where \( \rho_t \) is a time-varying risk premium, that is positively related to expected appreciation.\(^6\) The risk premium is a function of time \( t \) information. Moreover, \( \rho_t \) must be both autocorrelated and stationary if it is to explain the deviations of market price from fundamental value found in tests for cointegration and mean reversion in chapter 3, which assume risk neutrality. These two properties illustrate how serial correlation in rational expectations forecast errors and predictability of returns with past information, under risk neutrality, could be due to a time varying risk premium.

From (4.1), the true rational expectations housing price model is given by\(^7\)

\[ \frac{P_{t+1} - P_t}{P_t} = \left[ (i_t + p_t) - \frac{R_t}{P_t} \right] + \rho_t + \varepsilon_{t+1} \]  

For notational convenience rewrite equation (4.2) as

\[ g_{t+1} = g^e_{t+1} + \rho_t + \varepsilon_{t+1} \]  

where \( g_{t+1} = \frac{P_{t+1} - P_t}{P_t} \) is ex post actual capital gains and \( g^e_{t+1} = \left[ (i_t + p_t) - \frac{R_t}{P_t} \right] \) is optimally expected capital gains in the absence of a risk premium (i.e. under risk neutrality).

---

\(^6\)Section (4.3) derives \( \rho_t \) from a representative household’s optimal portfolio choice problem.

\(^7\)It is easy to see that taking conditional expectations of both sides of (4.2) yields (4.1).
Chapter 3 omits the premium variable and tests the rational expectations hypothesis via inference on the parameters of the following regression model:\footnote{This is equation (3.11) from Chapter 2.}

\[ g_{t+1} = \alpha_0 + \alpha_1 g_{t+1}^e + u_{t+1} \]  

(4.4)

Under the joint null hypothesis of rational expectations and no time-varying risk premium, \( \alpha_1 = 1 \) and \( u_{t+1} \) is serially uncorrelated. The model in equation (4.3), however, is the true representation of the rational expectations housing price relation, and hence estimates of \( \alpha_1 \) from (4.4) are biased. To examine both the magnitude and direction of the bias, consider the least squares slope coefficient estimate from the misspecified model in (4.4), which is given by\footnote{The variables on the right-hand side of (4.5) should be expressed as deviations from their means, but to simplify notation are not. This does not affect the results below.}

\[ \hat{\alpha}_1 = \frac{\sum g_{t+1}^e g_{t+1}^e}{\sum (g_{t+1}^e)^2} \]  

(4.5)

Substitute the expression for ex post capital gains from (4.3) into (4.5) to obtain

\[ \hat{\alpha}_1 = \frac{\sum g_{t+1}^e (g_{t+1}^e + \rho_t + \epsilon_{t+1})}{\sum (g_{t+1}^e)^2} \]  

(4.6)

Expand the right-hand side of (4.6), simplify and take the probability limit of both sides to characterize the limiting properties of \( \hat{\alpha}_1 \) as follows:\footnote{The derivation of (4.7) uses the property that \( E[\sum g_{t+1}^e \epsilon_{t+1}] = E[\sum \rho_t \epsilon_{t+1}] = 0 \). That is, market efficiency requires that the forecast errors (or unanticipated capital gains) are orthogonal to all time \( t \) information. Section 4.2 of Chapter 3 provides details on probability limits (\( plims \)).}

\[ plim(\hat{\alpha}_1) = 1 + plim \left[ \frac{\sum g_{t+1}^e \rho_t}{\sum (g_{t+1}^e)^2} \right] \]  

(4.7)

Thus, if the random quantity on the right side of (4.7) converges to a nonzero value, the slope estimate, \( \hat{\alpha}_1 \), is biased. The condition in equation (4.7) states that the in the limit \( \hat{\alpha}_1 \) converges to a constant that equals its true value under the null hypothesis of rational expectations, which is one, plus a term that reflects omitted variable bias. The bias turns out to be the probability limit of the least squares coefficient estimate in a regression of expected capital gains under risk neutrality on the risk premium. With this result we can provide a more intuitive expression for the condition in equation (4.7),
as follows:\footnote{This result requires that all variables are weakly or covariance stationary. I assume this condition holds. Chapter 3 provides evidence to support this assumption.}

\[
plim(\hat{\alpha}_1) = 1 + \frac{\text{cov}[g_{t+1}^e, \rho_t]}{\text{var}[g_{t+1}^e]} \tag{4.8}
\]

which reveals that the sign of the bias is a function of both the sign of the covariance between expected capital gains under risk neutrality and the risk premium, and the relative magnitudes of this covariance and the variance of expected house price appreciation under risk neutrality. More precisely, \(\text{cov}[g_{t+1}^e, \rho_t] < 0\) is a necessary condition to reconcile the negative coefficient estimates documented in chapter 3 with the rational expectations hypothesis.\footnote{Notice that this condition implies that \(\text{cov}[(i_t + pt_t - R_t/P_t), \rho_t] < 0\). Expand this expression to yield: \(\text{cov}[i_t, \rho_t] + \text{cov}[pt_t, \rho_t] - \text{cov}[R_t/P_t, \rho_t] < 0\). There are a number of different combinations of both signs and magnitudes of the three covariance terms that lead to a negative covariance between expected capital gains under risk neutrality and the risk premium.}

Sufficiency, however, requires further that the covariance term be larger than the variance of expected appreciation with no risk premium, \(g_{t+1}^e\).

To further qualify the economic conditions under which an omitted time-varying risk variable may explain deviations from unbiased house price expectations under risk neutrality, I present two alternative expressions for the bias statistic in (4.8). First, to express the true rational expectations house price model in terms of volatilities take variances of both sides of expression (4.3) to give

\[
\text{var}[g_{t+1}] = \text{var}[g_{t+1}^e] + \text{var}[\rho_t] + 2\text{cov}[g_{t+1}^e, \rho_t] + \text{var}[\varepsilon_{t+1}] \tag{4.9}
\]

Solve (4.9) for \(\text{cov}[g_{t+1}^e, \rho_t]\), substitute the resulting expression into the bias formula (4.8) and simplify to yield

\[
plim(\hat{\alpha}_1) = \frac{1}{2} + \frac{\text{var}[g_{t+1}] - \text{var}[\rho_t] - \text{var}[\varepsilon_{t+1}]}{\text{var}[g_{t+1}^e]} \tag{4.10}
\]

which expresses the estimation bias in terms of the sample variances of the individual variables. It indicates that the variance of the risk premium must be ‘large’ relative to the variances of both actual and expected house price changes to produce a negative slope point estimate.

Finally, use the definition of the correlation coefficient to establish a bound for the relative magnitudes of the variance components. By definition

\[
\text{corr}[g_{t+1}^e, \rho_t] = \frac{\text{cov}[g_{t+1}^e, \rho_t]}{\sqrt{\text{var}[g_{t+1}^e]\text{var}[\rho_t]}} \tag{4.11}
\]
and therefore

\[
\text{cov}[g_{t+1}^*, \rho_t] = \text{corr}[g_{t+1}^*, \rho_t] (\text{var}[g_{t+1}^*])^{0.5} (\text{var}[\rho_t])^{0.5}
\]  \hspace{1cm} (4.12)

Hence, a third way in which to specify the bias condition combines (4.8) and (4.12) to yield

\[
\text{plim}(\hat{\alpha}_1) = 1 + \frac{\text{corr}[g_{t+1}^*, \rho_t] (\text{var}[\rho_t])^{0.5}}{(\text{var}[g_{t+1}^*])^{0.5}}
\]  \hspace{1cm} (4.13)

The absolute value of \(\text{corr}[g_{t+1}^*, \rho_t]\) is bounded by 1, and thus this expression allows us to derive a condition on the relative magnitudes of the variances of variables of interest, which must be satisfied if a time-varying risk premium is to rescue the unbiased hypothesis. The minimum value of the correlation term is negative 1, and

\[
\text{corr}[g_{t+1}^*, \rho_t] = -1 \Rightarrow \text{plim}(\hat{\alpha}_1) = 1 - \left( \frac{\text{var}[\rho_t]}{\text{var}[g_{t+1}^*]} \right)^{0.5}
\]  \hspace{1cm} (4.14)

To obtain a negative point estimate for \(\alpha_1\) it must be the case that \(\text{var}[\rho_t] > \text{var}[g_{t+1}^*]\). In general, however, it is more likely that \(\text{corr}[g_{t+1}^*, \rho_t] > -1\), and hence the variation in the risk premium must be “considerably larger” than the variance of expected appreciation net of premium.

In summary, this section derives three complementary characterizations of the bias in the estimated slope coefficient of the risk neutral version of the rational expectations housing price model which is misspecified due to an omitted variable. The conditions given in equations (4.8), (4.10) and (4.14) establish that if the finding of negative point estimates in chapter 3 is due to neglect of housing market risk considerations, then two conditions must hold: first, a large proportion of the variation in house price fluctuations must be attributable to variation in risk premiums, and second, the risk premium must be negatively correlated with expected appreciation under risk neutrality.\(^{13}\)

\(^{13}\)The above two conditions on the variance of the risk premium and the covariance of the risk premium with expected appreciation are identical to results derived by Fama [44] for the efficiency of the forward exchange market. Fama aims to explain rejections of forward exchange market efficiency tests by neglect of risk, as I am doing in this chapter. Unlike in the housing market, a direct measure of expected appreciation under rational expectations is available in the foreign exchange market, namely the forward exchange rate. Early tests of rational expectations in this market assume risk neutrality. But Fama [44] recognizes that the forward rate encompasses both expectations of future spot rates and a risk premium. Most researchers ignore the risk premium. He decomposes the forward rate into two unobservable components, the spot rate and a time varying risk premium and uses least squares parameter estimate formulas to derive the conditions which must be satisfied by the risk premium. My work differs from
4.3 Modeling Housing Market Risk Premiums

The preceding analysis establishes that under certain conditions a time-varying housing market risk premium is a theoretically plausible explanation for earlier rejections of tests of the joint null hypothesis of rational house price expectations and risk neutrality. This section presents an asset-based housing price model that explicitly incorporates risk. It aims to both put economic significance to the risk variable, $\rho_t$, and guide empirical model specifications.

Consider a representative risk averse household that faces a static portfolio choice problem. The household aims to optimally allocate its initial wealth endowment between housing, stocks and a risk-free asset. Formally, the problem is to choose the quantities of housing and shares to maximize expected utility subject to a wealth constraint. More precisely, the household solves the following problem:

$$\max_{H,S} \ E[U(\tilde{W})]$$

subject to:

$$\tilde{W} = (W_0 - P_H H - P_S S)(1 + i) + RH + P_H(1 + \hat{g})H + P_S(1 + \hat{r}_S)S$$

where $U(.)$ is a concave utility function, $\tilde{W}$ is expected end of period wealth, $P_H$ is the price per unit of housing, $\hat{g}$ is the random rate of change of house prices, $H$ is the quantity (number of units) of housing, $i$ is the risk-free rate of interest, $P_S$ is the price.

\[1\] Fama's in that there is no observed measure of appreciation expectations in the housing market, both it and the risk premium must be modeled and thereby inferred from data.

\[14\] This section utilizes a highly simplified one-period model to highlight the economic intuition of the housing market risk premium. The results, however, carry over to more complicated environments. The connection with more general intertemporal asset pricing models is discussed later in the chapter.

\[15\] Think of stock holdings as the number of units in a mutual fund.

\[16\] This framework assumes the household's consumption decisions are independent of it's investment decisions. This implies that we can ignore the consumption side of the maximization problem in deriving the housing price relation below. To see this suppose that in addition to the portfolio choice problem, the household chooses housing consumption, $h$, and other consumption goods, $c$, to maximize the utility function $V(h, c) + E[U(\tilde{W})]$ (this analysis borrows from Henderson and Ioannides [92]). Assume $c$ is the numeraire (i.e. has a price of one). With $c$ and $Rh$ added to the budget constraint, the first order conditions with respect to housing and other goods yield the optimal consumption rule, $V_h/V_c = R$, or the ratio of the marginal utilities equals the price ratio. This result is independent of optimal asset allocations, and thus does not affect these.
per unit of shares in a stock market index (i.e. a mutual fund), $S$ is the number of shares purchased, and $\tilde{r}_S$ is the random stock return, which includes dividends. The first order conditions are

\[
\begin{align*}
\frac{dE[U'({\tilde{W}})]}{dH} &= E[U'({\tilde{W}})(-P_H(1 + i) + R + P_H(1 + \tilde{g}))] = 0 \quad (4.16) \\
\frac{dE[U'({\tilde{W}})]}{dS} &= E[U'({\tilde{W}})(-P_S(1 + i) + P_S(1 + \tilde{r}_S))] = 0
\end{align*}
\]

which imply

\[
E \left[ U'({\tilde{W}}) \left( \frac{R}{P_H} + \tilde{g} \right) \right] = E[U'({\tilde{W}})]i 
\quad (4.17)
\]

The optimal allocation rule is to invest in an asset up to the point at which the marginal opportunity cost equals the expected marginal benefit where both variables are measured in expected utility terms. For housing the marginal cost is the risk-free rate while the marginal benefit is a function of rent and expected capital gains. Rewrite the optimal solution for housing investment in the following way:

\[
\frac{E[U'({\tilde{W}})\tilde{g}]}{E[U'({\tilde{W}})]} = i - \frac{R}{P_H} \quad (4.18)
\]

which yields a housing price model similar in appearance to the one tested in chapter 3, except for the presence of the expected utility terms. Use the definition of the covariance between two random variables to decompose the numerator of the left-hand side of (4.18) into two components as follows:

\[
E[U'({\tilde{W}})\tilde{g}] = cov[U'({\tilde{W}}), \tilde{g}] + E[U'({\tilde{W}})]E[\tilde{g}] \quad (4.19)
\]

Combine equations (4.18) and (4.19) to characterize expected housing price appreciation

\[\text{Concavity of the utility function is a consequence of risk aversion. I assume that rents paid during the period are known in advance but that dividends received at the end of the period are uncertain. As a consequence, the random variables are housing price appreciation and stock returns.}\]

\[\text{Sufficiency for a maximum requires } E[U''({\tilde{W}})] < 0, \text{ which is guaranteed by the concavity of the utility function. Thus the first-order conditions are both necessary and sufficient for a maximum.}\]

\[\text{The covariance between two random variables } X \text{ and } Y \text{ can be expressed as } cov[X, Y] = E[XY] - E[X]E[Y].\]
under rational expectations in the following manner: \(^{20}\)

\[
E[\hat{g}] = i - \left( \frac{R}{P_H} \right) - \frac{\text{cov}[U'(\tilde{W}), \hat{g}]}{E[U'(\tilde{W})]} \quad (4.20)
\]

Risk aversion implies that positive expected excess returns to housing investment, \((E[\bar{g}] + \frac{R}{P_H} - i)\), reflect a premium for risk bearing. Risk is measured by the co-movement of house price appreciation and the marginal utility of wealth. In general, a risky asset is one with returns that tend to be low in periods when wealth is low and high when wealth is high. If housing appreciation and wealth are positively correlated, the payoffs from housing tend to be larger during periods when households place relatively less utility value on wealth, and thus housing investment is relatively risky. \(^{21}\) The size of the risk premium is a function of homeowner's attitudes toward risk, their initial asset position and the stochastic properties of house price appreciation. \(^{22}\)

It follows from (4.20) that the housing price risk premium is zero if market participants are risk neutral, or if the covariance between house price appreciation and marginal utility is zero. If either of these two conditions are satisfied then the housing price model collapses to the user cost model tested in Chapters 2 and 3. To see this, note that risk neutrality implies the marginal utility of wealth, \(U'(\tilde{W})\), is constant and hence the covariance term is zero.

**4.3.1 Housing Price Risk in the Traditional CAPM**

Tests of asset pricing models in the empirical finance literature generally examine the risk-return relationship within the context of the Sharpe \([137]\) and Lintner \([105]\) version of the capital asset pricing model (CAPM). This version of the CAPM models an asset's risk by the covariance of its return with the return on a “market” or “benchmark” portfolio of

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\(^{20}\)Repeating these steps for the first order condition for stocks yields an analogous expression for expected stock returns: \(E[\bar{r}_S] = i - \frac{\text{cov}[U'(\tilde{W}), \bar{r}_S]}{E[U'(\tilde{W})]}\).

\(^{21}\)The discussion of risk here is based on Breeden's \([11]\) Consumption Capital Asset Pricing Model (CCAPM) which relates risk to uncertainty about future consumption. While this section derives risk in terms of the marginal utility of wealth, it is implicitly assumed that wealth and consumption are directly related and it therefore makes no difference which one is used in this context.

\(^{22}\)Household's attitude towards, or aversion to, risk is measured by the concavity of their utility function \((U'')\). Section 4.4 illustrates the role \(U''\) plays in the housing price risk premium.
assets. It assumes there exists a portfolio of assets whose payoffs are perfectly negatively correlated with the marginal utility of wealth or consumption, so that

\[ U'(\tilde{W}) = -\gamma \tilde{r}_M \]  

(4.21)

where \( \tilde{r}_M \) is the return on the market portfolio. This definition implies that

\[ \text{cov}[U'(\tilde{W}), \tilde{g}] = -\gamma \text{cov}[\tilde{r}_M, \tilde{g}] \]  

(4.22)

and,

\[ E[U'(\tilde{W})] = -\gamma E[\tilde{r}_M] \]

Furthermore, since households are assumed to be utility maximizers, an equilibrium condition analogous to (4.20) for housing, must be satisfied for the market portfolio of assets, and thus:

\[ E[\tilde{r}_M] = i + \gamma \frac{\text{var}[\tilde{r}_M]}{E[U'(\tilde{W})]} \]  

(4.23)

Substitute both (4.22) and (4.23) into equation (4.20) to establish the equilibrium relationship between expected returns to housing investment and the expected return to the market portfolio,

\[ E[\tilde{g}] = i - \frac{R}{P_H} + \beta (E[\tilde{r}_M] - i) \]  

(4.24)

where:

\[ \beta = \frac{\text{cov}[\tilde{g}, \tilde{r}_M]}{\text{var}[\tilde{r}_M]} \]

Equation (4.24) is the CAPM security market line for housing. Beta is a measure of housing price risk. More precisely, beta is the responsiveness of housing price movements to returns on the market portfolio. In this framework, risk depends only on the covariance of house price changes with the returns to the market portfolio and the variance of the market portfolio. It remains to specify the market portfolio.

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23 In principle the CCAPM-based model in (4.20) is testable. The difficulty is that we require a measure of the marginal utility of wealth. One way to measure this variable is to parameterize the functional form of the representative household’s utility function and gather data on aggregate consumption. In practice however there is the question of what is the relevant measure of consumption to use. Furthermore, empirical tests of the risk premium models below, employ monthly data and publicly available consumption data is not reported at this frequency. Econometric testing of the the CCAPM is an active area of current research. See for example Breeden, Gibbons and Litzenberger [12].

24 This derivation of the traditional CAPM from the consumption-based CAPM follows Blanchard and Fischer ([5], Section 10.1).

25 More formally, \( \beta = \frac{\text{cov}[\tilde{g}, \tilde{r}_M]}{\text{var}[\tilde{r}_M]} \) is the estimated slope coefficient in a regression of \( \tilde{g} \) on \( \tilde{r}_M \).
4.3.2 The Market Portfolio

In theory the "CAPM Index" or market portfolio is a well-diversified portfolio of all risky assets in the economy, and therefore includes (at least) stocks, bonds, real estate and human capital. In practice, however, researchers in empirical finance usually proxy the market portfolio by an aggregate stock market index such as the Standard and Poor 500 or the Toronto Stock Exchange 300. Roll [128] criticizes such tests of the CAPM and shows that the ranking of the risk-return tradeoff of different stock portfolios is a function of the proxy for the market portfolio. More recent empirical examinations, however, conclude that the use of various broad stock market indexes yield consistent performance rankings for portfolios of stocks. 26

Roll's [128] critique is applicable to tests of the risk-return tradeoff in commercial real estate, the majority of which also utilize aggregate stock market indexes to represent the market portfolio. While aggregate stock market returns may approximate the market index for stocks, it may be particularly problematic for inference concerning real estate risk, because real estate assets themselves are omitted from the market portfolio. Liu et al. [106] investigate whether the omission of real estate assets from the market portfolio proxy biases measures of systematic risk of unsecuritized commercial real estate. They find that the composition of the market portfolio is important. In particular, the systematic risk of real estate is significantly higher when the market portfolio includes real estate asset returns. 27

The above discussion motivates a specification of the market portfolio which includes both stocks and real estate. Consider a world in which market participants can choose to invest in a risk-free asset (a government bond), stocks and houses. The market index is therefore comprised of stocks and housing. However, agents only purchase homes within the metropolitan area in which they live. This assumption appears to be a reasonable

26Brealey and Myers et al. ([10], pages 183-186) discuss some of this literature. They also devote considerable attention to the question of what is the relevant index for Canadian stocks? Their assessment of the literature leads them to conclude that the risk-return relationship in the Canadian equity market is unique and that the TSE 300 is sufficient for ranking the portfolio performance of Canadian stocks. There is no need to combine it with international or U.S. market indexes.

27In addition, Webb [148] finds that different proxies for the market portfolio lead to dramatically different optimal portfolio asset allocations in a CAPM framework.
approximation to reality for three reasons. First, most homes, especially single-family dwelling units, are owner-occupied, and the owners of these homes obviously live where they invest. Second, home equity is the major form of real estate investment for most households. Finally, housing units are immobile and thus housing markets are localized. As a consequence, location specific information is a necessary input into the investment decision process. High information costs discourage investment in far away locales.

The return on the market portfolio is a weighted average of the returns to housing and the stock market of the form

\[ \tilde{r}_M = \lambda \tilde{r}_H + (1 - \lambda) \tilde{r}_S \]  \hspace{1cm} (4.25)

where \( \lambda \) is the value weight of housing in the market portfolio, which is assumed constant over time, and \( \tilde{r}_H \) is the nominal, rent inclusive return to housing. Assume there are \( J \) distinct housing submarkets within the larger metropolitan area in which the representative agent lives. \( \tilde{r}_H \) is itself an unobservable value weighted index of aggregate returns in the metro-wide housing market. Think of this as the equivalent of a value weighted aggregate stock market index for housing in a single large metropolitan area. This formulation of the market portfolio implies the risk associated with a housing asset located in any of the \( J \) submarkets comes from two sources: the common economic forces driving the metropolitan housing sector and the co-movement with the stock market.

Substitute the above market portfolio decomposition into the CAPM relation, (4.24), and manipulate the resulting expression to express the housing price model as

\[ E[\tilde{g}_j] = i - \left( \frac{R_j}{P_{Hj}} \right) + \left[ \frac{\lambda \text{cov}[\tilde{g}_j, \tilde{g}] + (1 - \lambda) \text{cov}[\tilde{r}_S, \tilde{g}]}{\text{var}[\tilde{r}_M]} \right] (E[\tilde{r}_M] - i) \]  \hspace{1cm} (4.26)

where \( \tilde{r}_M = \lambda \tilde{g} + \lambda \frac{R}{P_e} + (1 - \lambda) \tilde{r}_S \), and the term in square brackets in (4.26) is the housing price beta for location \( j = 1, \ldots, J \). Housing price risk in submarket \( j \) is a function of the covariance of house price changes in that location with a measure of market-wide price movements and the covariance of stock returns with housing price appreciation. This

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28Chawla [26] reports that home equity made up 39 percent of household wealth in Canada in 1984. In contrast equity in other real estate accounted for only about 7 percent of wealth. This suggests that only a minority of people invest in real estate other than their principal residence.
model forms the basis for empirical tests of the risk-return relationship in the housing market.

### 4.4 Econometric Models and Empirical Results

The above analysis uses a static portfolio choice framework to derive a measure of housing price risk. In a dynamic framework in which a representative agent maximizes a time-separable, state-independent utility function, the asset pricing relation in equation (4.26) holds at every point in time. This yields a conditional CAPM for housing, which is written as:  

$$E[g_{j,t+1} \mid I_t] = i_t - \left( \frac{R_{jt}}{P_{Hjt}} \right) + \beta_{Hjt} (E[r_{M,t+1} \mid I_t] - i_t)$$

(4.27)

where:

$$\beta_{Hjt} = \frac{\lambda \text{cov}[g_{t+1}, g_{j,t+1} \mid I_t] + (1 - \lambda) \text{cov}[r_{S,t+1}, g_{j,t+1} \mid I_t]}{\text{var}[r_{M,t+1} \mid I_t]}$$

and:

$$r_{M,t+1} = \lambda g_{t+1} + \lambda \frac{R_t}{P_{Ht}} + (1 - \lambda)r_{S,t+1}$$

and $I_t$ is the information set available to market participants at time $t$. All moments in the conditional CAPM are conditioned on this information set. Everything in the risk premium term of (4.27) is time-varying. Expected conditional CAPM risk premiums vary over time as a result of variation in the conditional betas and/or the market return. Time variation in betas results from fluctuations in any of the following: the conditional covariance of submarket and metro-wide house price changes, the conditional covariance between submarket price appreciation and aggregate stock market returns, and the conditional variance of returns to the market portfolio.

Because the market portfolio is unobservable, a testable model requires additional structure. That is, we must combine the conditional CAPM theory with “statistical assumptions” about the properties of the conditional first and second moments in equation

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29 The one-period, asset-based housing price models presented above are equivalent to the first-order conditions (Euler equations) for an intertemporal optimization problem. Scott [136] presents a version of the Lucas [107] asset pricing model, in which a representative infinitely-lived agent chooses housing, stocks and nonhousing consumption goods to maximize a time invariant utility function which is separable in consumption goods and the service flow of housing. He derives a house price relation almost identical to equation (4.20) above.
It is important to note, therefore, that the sections below test the joint hypothesis that the asset pricing model is correct and that the particular statistical assumptions made to operationalize the model are valid. The following passage from Hodrick's ([93], page 18) review of foreign exchange efficiency tests summarizes the implications of the joint nature of market efficiency tests:

"... as with other financial markets any test of market efficiency is a joint test of several composite hypotheses. Hence, it is impossible to develop a direct test of the hypothesis that the foreign exchange market is efficient. All that can be done is to present various statistical hypotheses regarding what one means by market efficiency and test these specifications by placing additional assumptions on the statistical properties of the data. Rejection of the null hypothesis is consequently not necessarily identified with market inefficiency."

I test the empirical implications of two different versions of the conditional housing market CAPM. In the first model I assume the reward-to-risk ratio of returns on the market portfolio is constant. The market reward-to-risk ratio is the conditional expectation of excess returns to the market portfolio divided their conditional variance. This yields a housing market risk premium that is a function of conditional heteroskedasticity in the housing and stock markets. Tests of this model specification require statistical models of conditional second moments.

The second approach involves two statistical assumptions. It assumes that the conditional housing submarket CAPM betas are constant over time and that the conditional expected excess returns of the market portfolio are a linear function of an observable set of instrumental variables. Together, these two assumptions lead to a set of cross equation restrictions on the coefficients that relate expected excess returns in each housing submarket to the expected excess returns on the benchmark portfolio.
4.4.1 A Constant Market Reward-to-Risk Ratio Model

The first empirical model treats the conditional CAPM market 'reward-to-risk ratio', defined as,

\[
\frac{E[r_{M,t+1} \mid I_t] - i_t}{\text{var}[r_{M,t+1} \mid I_t]}
\]
as a constant. With this assumption the conditional CAPM housing price model in (4.27) reduces to:

\[
E[r_{H,j,t+1} \mid I_t] = \mu (\lambda \text{cov}[g_{t+1}, g_{j,t+1} \mid I_t]) + (1 - \lambda) \text{cov}[r_s,t+1, g_{j,t+1} \mid I_t])
\]

(4.28)

where \( \mu \) is the constant reward-to-risk ratio. Housing price risk in submarket \( j \) is a function of time variation in the conditional covariances between house price appreciation in \( j \) and both market-wide house price fluctuations and aggregate stock market returns.

Assume that price changes in each submarket are proportional to unobserved market price changes, such that

\[
g_{j,t+1} = \omega_j g_{t+1}
\]

(4.29)

where \( \omega_j, j = 1, \ldots, J \) are time invariant factors of proportionality. Pick any location, say \( j^* \) and take conditional variances of both sides of (4.29), with \( j = j^* \), to show that

\[
\text{var}[g_{j^*,t+1} \mid I_t] = \omega_{j^*}^2 \text{var}[g_{t+1} \mid I_t]
\]

(4.30)

or,

\[
\text{var}[g_{t+1} \mid I_t] = \frac{1}{\omega_{j^*}^2} \text{var}[g_{j^*,t+1} \mid I_t]
\]

In addition to providing a convenient way in which to derive an estimable model, there are conditions under which \( \mu \) can be given economic content. To simplify the exposition consider a market portfolio with only housing. If conditional house price changes are conditionally normally distributed then \( \mu \) is interpreted as the product of the Arrow-Pratt coefficient of relative risk aversion and the proportion of the household's investment portfolio held in housing. To see this use the property that the normal distribution is closed under addition, and thus if \( \tilde{g} \) is normally distributed then so is \( \tilde{W} \). Rubinstein[133] proves that for two jointly normally distributed random variables, \( X \) and \( Y \), \( \text{cov}[X, f(Y)] = E[f'(Y)\text{cov}[X,Y]] \), where \( f(Y) \) is a continuous and differentiable function. Apply this result to the housing price relation, (4.20), and simplify to derive a mean-variance relation of the form:

\[
E[\tilde{g}] = i - \left( \frac{R}{P_H} \right) - \frac{E[U''(\tilde{W})]}{E[U(\tilde{W})]} \sigma^2_{g^*}
\]

where \( U'' \) is the second derivative and \( \sigma^2_g \) is the variance of percentage changes in house prices. Rewrite this as \( E[\tilde{g}] = i - \left( \frac{R}{P_H} \right) + \theta \left( \frac{E[U''(\tilde{W})]}{E[U(\tilde{W})]} \right) \sigma^2_g \) where \( \theta = -\tilde{W} \frac{E[U''(\tilde{W})]}{E[U(\tilde{W})]} \) is the Arrow-Pratt coefficient of relative risk aversion. This exercise carries over to a market portfolio with both housing and stocks if they are jointly conditionally normally distributed.
Use this expression to eliminate the unobservable conditional covariance of market-wide and location $j$ house price changes in equation (4.28) and thus rewrite the constant reward-risk ratio CAPM as:

$$E[r_{H,j,t+1} \mid I_t] = \mu \left( \frac{\omega_j}{\omega_j^2} \text{var}[g_{j,t+1} \mid I_t] + (1 - \lambda)\text{cov}[r_{S,t+1}, g_{j,t+1} \mid I_t] \right)$$  \hspace{1cm} (4.31)

If this model accurately describes predictable components in expected excess housing returns, then either conditional house price volatility or the conditional covariance between housing and stock returns, or both, must vary over time. Estimating (4.31), therefore, requires time series models of these conditional second moments.

### 4.4.2 Modeling Conditional Second Moments

This section investigates the statistical properties of Vancouver house prices and Canadian stock returns, using monthly data over the 1979:1 to 1991:9 sample period, and derives univariate time series models of the conditional second moments in model (4.31). House price data for this study comes from two sources: Royal Lepage and Hamilton and Hobden [80]. I use the Toronto Stock Exchange composite (300) index to represent aggregate Canadian stock returns.

I first characterize the stochastic properties of housing price volatility. I then test for a time-varying conditional covariance between Vancouver housing price appreciation and returns to the TSE 300. Finally, this section uses the monthly house price data to derive an estimated quarterly conditional variance series. This allows us to test the constant market reward to risk ratio model on both the monthly single-detached and quarterly condominium housing markets.

### Conditional Volatility in Monthly House Prices

Figure 4.1 plots the first difference of the log of Hamilton and Hobden’s [80] monthly single-detached house price series for the Westside of Vancouver over the 1979:2-1991:9

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31Detailed descriptions of the housing market data employed in this chapter are found in the data sections of chapters 1 and 2.

32The TSE 300 composite index (1975=1000) is available on CANSIM (matrix B4237, source: Bank of Canada). I also use the TSE 300 composite dividend yield (CANSIM matrix B4245, source: Bank of Canada) in the latent variable models derived in section 4.4.4.
sample period. The city of Vancouver has seen two housing price cycles over the past 12 years, one in the late 1970s/early 1980s and the other in the late 1980s/early 1990s. Figure 4.1 reveals that housing prices exhibit increased volatility during periods within and surrounding the two price cycles. Price changes are much larger during volatile periods. This suggests that the variance of the percentage change in monthly house prices is time dependent and a function of the economic environment. \(^{33}\)

Figure 4.1 also shows that price changes exhibit serial dependence in the form of a cyclical pattern. Therefore, prior to modeling the conditional variance of month-to-month house price movements we must ensure that any dependence in the mean of the price change process is eliminated. Otherwise, we may incorrectly attribute serial correlation in the mean to the conditional variance process. To further investigate this, figure 4.3 plots the sample autocorrelation function of the first differenced price series. There are statistically significant autocorrelations at lags 1 through 3, as the values of the autocorrelation function lie outside the approximate 95 percent confidence interval (the solid lines at + or - 0.24) at these lags. The modified Box-Pierce Q-test statistic for randomness in the first 12 lags of the first-differenced series is 102.52.\(^{34}\) Under the null hypothesis of uncorrelated errors the Box-Pierce statistic has a \(\chi^2_{12}\) distribution. The 5 percent critical value is 21.03 and thus the null hypothesis is clearly rejected. \(^{35}\)

The sample autocorrelation and partial correlation (not shown) functions indicate that a fourth-order autoregressive model, or AR(4), specification is required to capture the dependence in the first-difference of the log of monthly house prices. I estimate an AR(4) model and find statistically significant parameters on lags 1, 2 and 4 and no evidence of serial correlation in the residuals. The plot of the squared residuals, in figure 4.4, indicates that conditional heteroskedasticity persists, even after accounting for serial

\(^{33}\)A plot of the squared values of the percentage change in monthly house prices, in figure 4.2, provides additional evidence of time-varying housing market volatility. It also indicates there is persistence in volatility, as large price changes tend to cluster together.

\(^{34}\)The modified Box-Pierce Q statistic for serial correlation up to lag \(j\) is given by the expression \(T \sum_{\tau=1}^{j} r^2(\tau)\), where \(r^2(\tau)\) is the sample autocorrelation at lag \(\tau\). See Harvey [88] for details.

\(^{35}\)It is difficult to determine if the statistical significance of low order autocorrelations in the monthly price change process have economic content or are simply a statistical phenomenon. Hamilton and Hobden [80] report that one potential problem with the assessment authority data is an administrative lag, of between one and two months, in recording the data. This may explain why I find statistically significant autocorrelations at these lags. I discuss this point in more detail later in the chapter.
dependence in the mean.

Alternating periods of turbulence (large unanticipated price movements) and relative tranquility (small residuals) is characteristic of the behaviour of many financial time series, such as stock returns and foreign exchange rates. A large number of empirical finance studies find that Engle’s [39] Autoregressive Conditional Heteroskedasticity (ARCH) model, or subsequent extensions, characterize time-varying second moments in short-run asset price movements. ARCH models specify today’s conditional variance as a linear function of past squared values of forecast errors in the mean process of the asset return series. The appeal of this class of models is that they statistically characterize the observation that “large” surprise, or unexpected, asset price changes in one period, tend to be followed by relatively large price movements, either up or down, over the next “few” periods. Thus, in ARCH models the second moments are related through time. Serial correlation in the squared residuals from a univariate time series for the conditional mean of house price changes is evidence of ARCH.

ARIMA identification techniques indicate that the squared AR(4) model residuals exhibit substantial serial dependence. It appears that either a twelfth order moving average, an eighth order autoregressive process, or low order ARMA model describes the time series properties of the squared residuals. The autocorrelation function exhibits statistically significant spikes at lags one and two, and every second lag thereafter up until lag 8. This distinct pattern suggests that some form of dependence remains in the mean of the price change process. An obvious candidate is seasonality. There is little change in the time series behaviour of the squared residuals, however, when the AR(4) model includes 11 monthly dummy variables.
Attempts to estimate an AR(4)-ARCH(8) model by nonlinear maximum likelihood either fail to converge or produce negative variance estimates, depending on the algorithm used, and whether analytic or numeric derivatives are used. ARCH(4) through ARCH(7) models yield similar results. Only for an ARCH(3) specification do I find sensible parameter estimates for the conditional variance equation. The left-hand side of Table 4.1 reports maximum likelihood estimates of the parameters of an AR(4)-ARCH(3) model for the first-differenced log of monthly house price series. The first two autoregressive parameters and two of the three ARCH parameters are statistically significant at conventional significance levels. This suggests that the significance of the third and fourth lags of house price changes in the linear AR(4) model, which assumes constant conditional variance, may be spurious, since explicit allowance for time-varying conditional variances in the housing price process dramatically reduces both the coefficient estimates and their significance.

The ARCH(3) specification, however, fails to fully capture the nonlinear dependence in the AR(4) residuals. Q-tests reveal significant departures from randomness in the squared standardized residuals, at higher lags. Despite this finding, the statistical significance and relatively large magnitudes of low order ARCH slope parameters implies that ARCH-type effects likely contribute to time-varying price volatility. But there appears to be more to conditional heteroskedasticity in house price changes than simple ARCH effects.

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41 Engle [39] shows that a Lagrange multiplier test for ARCH(q) is undertaken by regressing the squared residuals, from OLS estimation of the original linear model, on a constant and q lags of the squared residuals and calculating the statistic, $LM_{ARCH_q} = T \cdot R^2$, where $T$ is number of observations. $LM_{ARCH_q}$ has a $\chi^2$ distribution with q degrees of freedom under the null hypothesis of homoskedasticity. Bollerslev [6] generalizes the ARCH model to allow for a more parsimonious parameterisation of long order ARCH processes.

42 Bollerslev's [6] generalized ARCH model also fails to converge.

43 There is some theory to support this conjecture. The presence of ARCH implies the residuals from the AR(4) model are neither independent nor identically distributed. Standard tests for serial correlation, however, are based on these assumptions and conditional heteroskedasticity may therefore bias Q-statistics against the null hypothesis of no serial correlation. This in turn may lead to incorrect inference regarding the statistical significance of sample autocorrelations. Diebold [36] shows that the presence of ARCH implies that the usual standard errors employed to assess statistical significance of the sample autocorrelations is too small. That is, the true 95 percent confidence interval is larger in the presence of ARCH. In addition Q-statistic no longer has a limiting $\chi^2$ distribution.
ARCH models parameterize conditional volatility solely as a linear function of past volatility. This is a restrictive model, which, as noted above, is simply a statistical characterization of volatility clustering in asset price movements.\textsuperscript{44} In an attempt to obtain a better model of housing price volatility I incorporate additional explanatory variables into the conditional variance equation. This allows for a more flexible parameterization of conditional heteroskedasticity in house price changes than the simple ARCH model.

Recent studies on the economic interpretation of the ARCH effect in stock returns report that volatility is related to macroeconomic uncertainty and that nominal interest rates are significant determinants of stock market volatility.\textsuperscript{45} Since interest rates are arguably the most important short-term macroeconomic variable for the housing sector, I examine the relationship between monthly interest rates and housing price volatility in a regression of the squared residuals, $e_t^2$, from the estimated linear AR(4) model on the one-month nominal interest rate lagged one period:

$$e_t^2 = \alpha_0 + \alpha_1 i_{t-1} + u_t$$ (4.32)

OLS estimation yields $\hat{\alpha}_0 = -0.00071$ (-2.16), $\hat{\alpha}_1 = 0.15$ (4.35), and $R^2 = 0.11$, where heteroskedasticity consistent t-statistics, calculated using White's [150] procedure, appear in parentheses. The statistically significant slope coefficient implies that nominal interest rates are important predictors of future housing market volatility. The negative coefficient, however, indicates that nominal rates by themselves do not fully capture the dynamics of housing market volatility, since it permits negative variance values.

After experimenting with a number of conditional variance specifications that include both ARCH and interest rate influences I constructed a measure of housing price volatility using the one month lag of housing price volatility (an ARCH(1) effect) plus the one month lag of the short-term nominal interest rate and its squared value, so that

$$\sigma_t^2 = b_0 + b_1 (\varepsilon_{t-1})^2 + b_2 i_{t-1} + b_3 (i_{t-1})^2 + u_t$$ (4.33)

\textsuperscript{44}Hodrick ([93], page 110) suggests that the ARCH model is an inappropriate way in which to model conditional expected volatility in asset prices because the ARCH process constrains the conditional variance to have its largest values after the largest residuals in the mean process occur. If big forecast errors are a result of the resolution of uncertainty then conditionally volatility is smaller after they occur, not larger, as the ARCH model implies.

\textsuperscript{45}See Campbell [14] and Breen, Glosten and Jagannathan [13] for example.
The right-hand side of table 4.1 presents estimation results for the AR(4) model with the above conditional variance specification. As in the ARCH(3) model the coefficient estimates on $\gamma_3$ and $\gamma_4$ in the mean equation are small in magnitude and not statistically different from zero. The ARCH(1) effect is large, precisely estimated and has the correct positive sign. Lagged nominal interest rates are negatively related to housing market volatility while their squared value is positively correlated with volatility, after accounting for ARCH(1) and interest rate level influences. There is no correlation in either the raw or squared residuals.

The presence of both the nominal interest rate and its square in the variance equation implies that both the level and rate of change of the conditional variance of house price fluctuations are functions of the level of nominal interest rates. To see this, take the first derivative of equation (4.33) and replace the coefficients in the resulting expression by their maximum likelihood point estimates from table 4.1. The result is

$$\frac{\partial \sigma_i^2}{\partial i_{t-1}} = -0.46 + 65.8 i_{t-1}$$

which means that:

$$i_{t-1} > 0.006991 \quad \Rightarrow \quad \frac{\partial \sigma_i^2}{\partial i_{t-1}} > 0$$

This condition states that an upward movement in interest rates increases expected housing price volatility when interest rates exceed 8.38 percent on an annualized basis.\(^{46}\) Short-term nominal rates hit 19 percent in 1980 and rose above 12 percent in 1990, and therefore exceed 8.38 percent during both of Vancouver’s housing price booms.

Figure 4.5 plots the estimated conditional variance of monthly house prices. The largest movements in price volatility coincide with the periods of dramatic expansion and contraction in the Vancouver housing market. Section 4.3, below, uses this measure of expected volatility to test the relationship between expected returns and volatility in the single-detached housing market of the Westside of the City of Vancouver.

\(^{46}\) A 0.699 percent monthly interest rate translates into an 8.38 percent annual rate.
Conditional Covariance of House Prices and Stock Returns

The other important second moment in the CAPM beta is the conditional covariance between house prices and aggregate stock returns. This section tests for time variation in the conditional covariance between the percentage change in monthly single-detached house prices on the Westside of Vancouver and returns on the TSE 300.

By definition:

\[
\text{cov}[r_{S,t+1}, \Delta p_{t+1} | I_t] = E[(r_{S,t+1} - E[r_{S,t+1} | I_t]) (\Delta p_{t+1} - E[\Delta p_{t+1} | I_t])]
\]  

(4.35)

In order to estimate the conditional covariance between Vancouver house prices and TSE returns we need to model conditionally expected returns to each. Following Gibbons and Ferson [71] and Harvey [90] I condition on an observable subset of variables or instruments in the unobservable information set \(I_t\) and assume that the conditional expectations of housing and stock returns are linear in the instruments. This implies the following relationships:

\[
E[\Delta p_{t+1} | Z_t] = \sum_{i=1}^{I} \tau_{iH} z_{it}
\]

(4.36)

\[
E[r_{S,t+1} | Z_t] = \sum_{i=1}^{I} \tau_{iS} z_{it}
\]

where \(z_i, i = 1, \ldots, I\), are elements of \(Z\), which is an observable subset of the information set \(I\).\(^{47}\) Substitute these expressions into (4.37) to reexpress it as:

\[
\text{cov}[r_{S,t+1}, \Delta p_{t+1} | Z_t] = E \left[ \left( r_{S,t+1} - \sum_{i=1}^{I} \tau_{iS} z_{it} \right) \left( \Delta p_{t+1} - \sum_{i=1}^{I} \tau_{iH} z_{it} \right) | Z_t \right]
\]

(4.37)

I use four instrumental variables to model expected returns: the first lag of the percentage change in house prices, the first lag of the return to the TSE composite index, and one period lags of nominal monthly short-term interest rates and TSE dividend yields. The choice of instruments follows previous empirical work on predicting stock and housing returns, including Chapter 3 of this thesis.

Table 4.2 reports ordinary least squares estimation results for the conditional return equations. Wald tests of the null hypothesis of joint insignificance of the instrumental

\(^{47}\)By the law of iterated expectations \(E[E[r_{S,t+1} | I_t] | Z_t] = E[r_{S,t+1} | Z_t]\). Hence, even though the true information set is not observable, tests using elements in \(Z\) have statistical power as long as the instruments have explanatory power in predicting returns.
variables do not reject in both cases. Lagged TSE returns are significant predictors of Vancouver house price changes and lagged house price changes appear to have explanatory power for stock returns. Interest rates are negatively related to both dependent variables and high dividend yields this month predict high stock returns next month. The four instrumental variables together account for almost a third of the variation in monthly house price appreciation. The explanatory power of the model for stock returns is weaker but consistent with results in Campbell [90] and Ferson, Foerster and Keim [52]. Overall, the empirical results indicate that the chosen instruments have predictive power and thus tests of the conditional covariance between stock and house price fluctuations with these instruments have statistical power.

The estimated residuals form the basis for a test of the null hypothesis that the conditional covariance between stock and house prices is constant. From equation (4.37) it is evident that the product of the return regression residuals is an estimate of the conditional covariance. Under the null hypothesis of a constant conditional covariance, the product of the residuals from the regressions of house price appreciation and stock returns on the instrumental variables should be uncorrelated with the instrumental variables. Thus one way to test the constant conditional variance hypothesis is to regress the product of the residuals on a constant and the predetermined instruments. The estimated slope coefficients should be zero if the null is true.

A regression of the product of the residuals on a constant and the four instruments yields an $R^2 = 0.009$. A Wald test of the joint statistical significance of the four regressors produces a value of 1.19, which has a p-value of 0.88. None of the coefficients on the four instruments are statistically significant. The null hypothesis of a time-varying conditional covariance between monthly aggregate Canadian stock returns and Vancouver single-detached house prices is rejected. Moreover, we cannot reject the null hypothesis that the conditional covariance is zero. 

---

48 Cumby [29] and Campbell [90] use this procedure.

49 The t-statistics are all less than one in magnitude. Note the standard errors, and thus the Wald test statistic, are calculated using the Newey-West [120] covariance matrix and are therefore robust to both heteroskedasticity and autocorrelation in the residuals.

50 An alternative way to test this is to use Hansen's [83] Generalized Method of Moments (GMM) estimation and a test of the overidentifying restrictions. This involves the following steps: Define the product of the residuals to be $e_{\Delta P_{ts},t+1}$. If the null hypothesis is true then $E[e_{\Delta P_{ts},t+1} | Z_t] = 0$
A Measure of Conditional Quarterly House Price Volatility

This section first uses the monthly house price data to generate a quarterly measure of housing price volatility. It then models the time series properties of the resulting series via standard ARIMA methods and thereby derives quarterly conditional forecasts of the variance of Vancouver house prices. I use the time series model of conditional quarterly volatility to test the constant market reward-risk ratio risk premium model with the quarterly Royal Lepage condominium data set.

I estimate the quarterly variance of house price changes, $\sigma^2_{qt}$, as the sum of the squared residuals, $\hat{e}^2$ within the quarter, from a second-order autoregression for monthly price changes:\footnote{This procedure follows French, Schwert and Stambaugh [56] who estimate the monthly variance of stock returns as the sum of squared daily returns plus an adjustment for first order autocorrelation in stock returns. The authors argue that since the volatility of stock returns is not constant a more precise estimate of monthly volatility is obtained from using only price changes within that month, rather than employing monthly price changes directly.}

$$\sigma^2_{qt} = \sum_{i=1}^{3} \hat{e}^2_i$$

(4.38)

where $i$ indexes the three months within each quarter.

Visual inspection of the autocorrelation and partial autocorrelation functions (not shown) suggests that $\sigma^2_{qt}$ follows a fourth order autoregressive process, given by:

$$\sigma^2_{qt} = \psi_0 + \psi_1 \sigma^2_{q,t-1} + \psi_2 \sigma^2_{q,t-2} + \psi_3 \sigma^2_{q,t-3} + \psi_4 \sigma^2_{q,t-4} + \epsilon_{qt}$$

(4.39)

Table 4.3 reports estimation results for the above model. The parameters at lags one and four are statistically significant at conventional significance levels, the model provides a reasonable fit and the hypothesis that the disturbances are random cannot be rejected.

GMM estimation chooses parameter estimates to minimize the quadratic form: $\psi = \hat{e}_{\Delta p_{rs}}^T W Z^T e_{\Delta p_{rs}}$ where $W$ is a symmetric weighting matrix. The presence of conditional heteroskedasticity and possibly autocorrelation dictates that we use Newey and West's [120] covariance matrix for $W$. Hansen [83] shows that the number of observations times the minimized value of the above quadratic form, $T\psi_{\text{min}}$, is distributed chi-squared with degrees of freedom equal to the number of orthogonality conditions. $T\psi_{\text{min}}$ is known as the test for overidentifying restrictions or Hansen's J-test. It tests the adequacy of the model. A large value indicates that the error terms are correlated with the instruments and thus the model is misspecified. For our purposes a large value of $T\psi_{\text{min}}$ implies a rejection of the null hypothesis of a constant conditional covariance between stock returns and house price changes. GMM estimation of yields a value of 1.11 for the $\chi^2$ test of the overidentifying restrictions. This is well below the 5 percent critical value of 9.49. The estimated constant is 4.36*10^{-5} with an asymptotic t-statistic of 0.62. For a practical overview to GMM and its applications see Ogaki [121].

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Thus quarterly volatility is predictable from its own past. The fitted values from (4.47) form the estimates of conditional forecasts of quarterly housing price volatility. Figure 4.6 plots the predicted quarterly variance of the percentage change in Vancouver house prices. Thus I use quarterly measures of volatility derived from single-detached house prices to proxy volatility in condominium prices. It seems reasonable to assume that prices in both sectors are highly correlated.

Summary

The preceding section documents two important facts: Vancouver house prices exhibit time-varying volatility that is partly predictable from past information, and the conditional covariance between aggregate Canadian stock returns and the monthly percentage change in Vancouver house prices is not statistically different from zero. The next section combines these results with the constant market reward to risk ratio model in equation (4.31), and tests the model with data from the Vancouver housing market.

4.4.3 Constant Market Reward-to-Risk Ratio: Results

Based on the above analysis of conditional second moments I can set \( \text{cov}[r_{S,t+1}, g_{j,t+1} | I_t] \) to zero in the constant risk-reward model specification in equation (4.30). The housing price model simplifies to:

\[
E[r_{Hj,t+1} | I_t] = \frac{\mu_j}{\omega_j^2} \text{var}[g_{j,t+1} | I_t]
\]

Risk is proportional to conditional variance. I use the measures of conditional house price volatility derived in section 4.6 to test this relationship on both the monthly single-detached house price series and the Royal Lepage condominium data set. Equation (4.40) implies a positive, linear relationship between ex-post housing returns and risk. It is important to emphasize that tests of this model evaluate the composite null hypothesis of rational expectations, risk averse agents, and a constant conditional CAPM market portfolio reward to risk ratio. For tests with the Royal Lepage cross sectional time series

\[52\]This model specification is similar to tests of the relationship between expected returns and expected volatility in aggregate U.S. stock market returns in French, Schwert and Stambaugh [56].
the null hypothesis also includes the assumption that house prices in each municipality move in proportion to the unobservable metropolitan-wide housing component of the market portfolio.

**Monthly Single-Detached Westside Housing Market Data**

Table 4.4 reports estimation results for monthly single-detached house prices. In part (a) nominal monthly excess returns are regressed on a constant and the expected conditional variance of house price changes. The conditional variance measure comes from the variance model in (4.32). The coefficient estimate on the risk variable is negative but not statistically different from zero. The low $R^2$ indicates a poor model fit.

Table 4.4(b) moves nominal interest rates from the left-hand side of the pricing relationship and includes it as a regressor. This yields tests similar to those of Chapter 3, but with a risk premium added as a regressor. The model provides a much improved fit and both of the explanatory variables are statistically significant. The risk parameter is positive but the coefficient estimate on nominal interest rates has the 'wrong' sign. It should be plus one under the null hypothesis of rational expectations.

These results suggest that nominal interest rates play an important role in housing price dynamics. The level of short-term nominal rates is significantly negatively related to future house price movements. In addition, interest rates are used to forecast future conditional house price volatility, which is strongly related to price fluctuations.

**Quarterly Cross Sectional Condominium Apartment Market Data**

Table 4.5 examines the co-movement between expected quarterly risk and expected condominium capital gains under risk neutrality. The omitted variable bias analysis in section 4.2 shows that a risk premium interpretation of previous results requires a negative correlation between these two quantities. Table 4.5 reports positive covariances.

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53Excess returns do not include a dividend yield, but with monthly data I suspect this is not a big concern.

54Note, that because $\hat{\sigma}$ is a generated regressor ordinary least squares standard errors underestimate the true variation in parameter estimates on the risk variable. Thus the true $t$-statistic is likely less than 0.81 in magnitude.
Consistent with this finding, regressions of excess quarterly condominium returns on expected volatility, in table 4.6, produce negative estimates on the risk parameter. The point estimates are statistically significant at conventional significance levels in about half of the municipalities.

In table 4.7 the model specification is the same as in Chapter 3, plus the risk variable. Coefficient estimates on expected capital gains remain negative and almost all are statistically different from zero. The values of the risk parameters increase in magnitude, and become positive in Richmond and the Westside of Vancouver, and less negative in the remaining areas. Of particular note are the Westside results. Condos here show a positive relationship between appreciation and risk, as proxied by housing market volatility, that is statistically significant at the 10 percent level.

Summary

The above results are inconsistent with the joint hypothesis under test. More precisely, the presence of the conditional CAPM constant market reward-to-risk risk premium does not reverse earlier findings of a negative relationship between actual and expected house price appreciation. These findings may be evidence of housing market irrationality. On the other hand, the empirical results in this section may imply that the market reward-to-risk ratio is not constant as assumed. This in turn implies that conditional house price volatility is not an adequate measure of housing price risk.

4.4.4 Latent Variable Models

To derive a testable conditional CAPM, the preceding analysis assumes that conditional expected excess returns on the benchmark portfolio move in proportion to their conditional variance. This approach eliminates the unobservable market portfolio from the model and specifies a relationship between conditional excess housing returns and conditional house price volatility. This in turn requires statistical models for time-varying conditional second moments.

This section follows a different approach. Rather than assume a constant market reward-to-risk ratio this section treats housing submarket betas as constants. It therefore
assumes that time-varying expected excess returns are driven by changes in expected excess returns to the market portfolio.

With constant betas equation (4.27) implies a set of equilibrium relationships of the form:

\[ E[r_{Hj,t+1} | I_t] = \beta_{Hj} E[r_{M,t+1} | I_t] \]  

(4.41)

where \( r_{Hj} \) and \( r_M \) denote excess returns, above the risk-free rate, for housing in each area and the market or benchmark portfolio, respectively, \( j = 1, \ldots, J \) indexes housing submarkets that comprise the larger metro housing market, and the beta parameters are time invariant. Expected excess returns to housing investment in each area are positively related to the expected area-specific risk premium, which is a linear function of the expected excess return on a common benchmark portfolio. More precisely, the model states that movements in conditional expected excess housing returns are proportional to movements in the expected return to the market portfolio, where the constant of proportionality is the area-specific beta.

I use the single latent variable methods of Gibbons and Ferson [71], Hansen and Hodrick [85] and Campbell [14] to test the constant beta version of the conditional CAPM for housing.\(^{55}\) Latent variable models treat the market portfolio as an unobservable or latent variable and allow us to test for housing market risk premium within a CAPM framework without having to specify the market portfolio.\(^{56}\)

I first set up the empirical model and illustrate the test procedures. Following this I discuss in more detail the intuition behind the latent variable approach and the interpretation to be given of the results of this section.

Combine the CAPM relation for location \( j = 1 \) with that for any of the other \( J - 1 \)

\(^{55}\)Gibbons and Ferson [71] and Hansen and Hodrick [85] independently pioneered the latent variable approach to testing asset pricing models. Campbell [14] and others have since refined the econometric tests associated with it.

\(^{56}\)Gibbons and Ferson [71] argue that this implies latent variable tests are free from Roll's [128] criticism about the testability of the CAPM, due to the unobservability of the market portfolio. Wheatley [149] however, notes that the authors replace the specification of the CAPM market portfolio with a distributional assumption about the market portfolio return, which is untestable. Thus the method does not overcome the problem of unobservability of the market portfolio. This has important implications for the interpretation to be given to latent variable tests, which I discuss in more detail below.
areas, which eliminates the market portfolio and thereby yields:

\[ E[r_{H1,t+1} \mid I_t] = \frac{\beta_{H1}}{\beta_{Hj}} E[r_{Hj,t+1} \mid I_t] \]  

(4.42)

for all \( j = 2, \ldots, J \). The ratios of expected excess returns across different areas are constant and the constants of proportionality are given by the ratio of respective CAPM betas. Thus the CAPM implies a cross section relationship between expected excess housing returns in different areas that does not involve expected returns on the market portfolio. To derive testable implications, latent variable models assume that expected returns to the market portfolio are a linear function of an observable subset of variables or instruments, \( X_t \), in the unobservable information set \( I_t \). This implies, \( E[r_{M,t+1} \mid X_t] = \sum_{i=1}^{K} \phi_i x_{it} \), where \( x_i, \ i = 1, \ldots, K \), are instrumental variables and \( \phi_i \), are fixed parameters. Conditionally expected housing returns in each submarket \( j \) are linear functions of the observable instruments:

\[ E[r_{Hj,t+1} \mid I_t] = \beta_{Hj} \sum_{i=1}^{K} \phi_i x_{it} \]  

(4.43)

for all \( j = 1, \ldots, J \). Equations (4.42) and (4.43) together form a linear system of equations of ex post excess returns to housing under the single latent variable CAPM. Express this in matrix notation as follows:

\[ r_{t+1} = Ax_t + v_{t+1} \]  

(4.44)

where \( v_{t+1} \) is a vector of mean zero, random errors, that are contemporaneously correlated and may be conditionally heteroskedastic. The system in (4.44) implies a set of cross equation restrictions of the form \( a_{ji} = \beta_j \phi_i \), where \( a_{ji} \) is a typical element of \( A \). Since the betas are unobservable and only the relationship between them is of interest to identify the restrictions set the beta for submarket one equal to one. That is, assume \( \beta_{H1} = 1 \), to normalize the vector of beta coefficients, and for expositional purposes assume \( J = 8 \).
The restricted system in (4.44) is then written as:

\[
\begin{bmatrix}
    r_{1,t+1} \\
    r_{2,t+1} \\
    r_{3,t+1} \\
    r_{4,t+1} \\
    r_{5,t+1} \\
    r_{6,t+1} \\
    r_{7,t+1} \\
    r_{8,t+1}
\end{bmatrix}
= 
\begin{bmatrix}
    \phi_1 & \phi_2 & \phi_3 \\
    \beta_2 \phi_1 & \beta_2 \phi_2 & \beta_2 \phi_3 \\
    \beta_3 \phi_1 & \beta_3 \phi_2 & \beta_3 \phi_3 \\
    \beta_4 \phi_1 & \beta_4 \phi_2 & \beta_4 \phi_3 \\
    \beta_5 \phi_1 & \beta_5 \phi_2 & \beta_5 \phi_3 \\
    \beta_6 \phi_1 & \beta_6 \phi_2 & \beta_6 \phi_3 \\
    \beta_7 \phi_1 & \beta_7 \phi_2 & \beta_7 \phi_3 \\
    \beta_8 \phi_1 & \beta_8 \phi_2 & \beta_8 \phi_3
\end{bmatrix}
\begin{bmatrix}
    x_{1t} \\
    x_{2t} \\
    x_{3t}
\end{bmatrix}
+ 
\begin{bmatrix}
    v_{1,t+1} \\
    v_{2,t+1} \\
    v_{3,t+1} \\
    v_{4,t+1} \\
    v_{5,t+1} \\
    v_{6,t+1} \\
    v_{7,t+1} \\
    v_{8,t+1}
\end{bmatrix}
\tag{4.45}
\]

where note the “housing” subscript, \( H \), is omitted to simplify notation. A test of the latent variable CAPM model requires a parameterization for the linear model of expected returns to the benchmark portfolio, estimation of the restricted model in equation (4.45) and a test of the nonlinear parameter restrictions.

Discussion

The latent variable models combine CAPM asset pricing theory with two statistical assumptions to formally test the hypothesis that risk premia for housing in different areas of a single metropolitan area move in proportion to a single latent variable.\(^{57}\)

The two assumptions are that submarket betas are constants and that the conditional expected returns of the benchmark portfolio are a linear function of observable variables or instruments. Latent variable models derive their power from the assumption that expected returns to the benchmark portfolio are changing over time and are correlated with observable instruments. The cross equation restrictions measure the extent to which returns in the different submarkets move together and in proportion to the benchmark portfolio. Thus, as Campbell [14] and Campbell and Clarida [15] note, latent variable tests have two interpretations. First, they can be viewed as tests of a specialized version of the CAPM, in which assets have constant betas on a single, unobservable benchmark portfolio. Alternatively, they are an atheoretical, statistical tool that characterizes the extent to which asset returns, in some sense, move together. It is important to keep these

\(^{57}\)Chapter 3 reports that the first principal component accounts for a significant amount amount of variation in deviations of house price movements from those predicted by the risk neutral rational expectations across municipalities. This section formally tests to determine if this single driving force is consistent with common time-varying risk premium in a CAPM framework.
two different interpretations in mind when attempting to draw conclusions from latent variables model results.

4.4.5 Latent Variable Models: Results

This section presents estimation and hypothesis test results for the latent variable model of time-varying expected excess quarterly returns for the Vancouver condominium market over the 1982:2-1992:4 sample period.

I employ three instruments to predict one quarter ahead excess returns on the market portfolio. They are net provincial inmigration, $IN$, the three month T-bill rate, $i$, and a measure of excess supply in the new housing market, $ES$. Data availability, economic theory and the need to keep the list small for computational purposes govern the choice of this particular set.\(^{58}\) The urban growth model in chapter 2, as well as the empirical results there, provide evidence on the importance of net in-migration and excess new housing supply in housing market dynamics. The ability of nominal short-term interest rates to forecast future movements in stock, bond and foreign exchange returns is documented in Fama and Schwert\[46\], Campbell\[14\], Breen, Glosten and Jagannathan\[13\] for example. Results in chapter 3 and in table 4.2 in this chapter show that nominal interest rates forecast future housing price movements.

The model used to proxy the latent return on the market portfolio is specified as:

$$r_{H,t+1} = \phi_0 + \phi_1 IN_t + \phi_2 ES_t + \phi_3 i_t + u_t$$  \hspace{1cm} (4.46)

Table 4.8 reports estimation results, and diagnostic test statistic values for each of the eight condominium submarkets. The $R^2$'s indicate a good model fit in each submarket. Parameter estimates have their expected signs. Hypothesis tests generally reject the null hypothesis that coefficients on the three instruments are jointly zero. There is weak evidence of serial correlation in some areas and strong evidence of nonnormal and heteroskedastic residuals.

To identify the parameters in the restricted model, normalize the beta for Burnaby

\(^{58}\) Preliminary regressions included lagged returns and the lagged dividend yield from the TSE 300. The coefficients on these two variables were not statistically significant in regressions of the latent variable model instruments on one-period ahead excess condominium returns.
at one. Table 4.9 reports GMM estimation results of (4.45).\textsuperscript{59} The instruments are the three predetermined variables and the weighting matrix is the Newey-West [120] covariance matrix, with lag length set to 2, which accounts for both autocorrelation and heteroskedasticity in the return residuals. The coefficients are all estimated precisely and all take on reasonable values. Burnaby has the largest systematic housing price risk, followed by the westside of Vancouver city. North Vancouver has the lowest.\textsuperscript{60}

Hansen's [83] test of overidentifying restrictions has a 0.08 p-value. Thus we reject the latent variable model restrictions at the 5 percent significance level but we cannot reject at the 10 percent level of significance. This makes the test results difficult to interpret. They are roughly consistent with the notion that excess condominium returns move in proportion to a single latent variable comprised of housing market fundamentals (i.e. immigration, excess supply and interest rates). One way to interpret the results, therefore, is as evidence in support of the hypothesis that predictable excess condominium returns reflect time-varying risk premia.

One the other hand, another way to interpret the latent variable model results is as evidence of "fads", as discussed in Chapter 3, in the Vancouver housing market. In this interpretation, the latent variable model suggests that all areas are subject to "fads" that are systematically related to housing market fundamentals. Different values for beta may imply that the extent of market inefficiency differs across housing submarkets.

4.5 Conclusion

This chapter examines whether a time-varying housing market risk premium can explain deviations in house price fluctuations from those predicted by the rational expectations hypothesis under risk neutrality. To accomplish this, it formulates and estimates two econometric models of time-varying housing market risk premia.

The first empirical model assumes a constant market reward to risk ratio. This leads to a housing market risk premium that depends on a linear combination of conditional house price volatility and the conditional covariance between Vancouver house price

\textsuperscript{59}These are iterated GMM results.

\textsuperscript{60}Note we cannot attribute any significance to magnitudes of individual beta coefficients. They are all measured relative to the beta for Burnaby
movements and returns to the Toronto Stock Exchange 300 composite index. Empirical
tests cannot reject the hypothesis of zero conditional covariance between Vancouver house
price changes and TSE 300 returns. This reduces the model to a relationship between
conditional means of excess returns and conditional variances of house price changes.
This model is convincingly rejected. There is little evidence of a positive relationship
between housing price changes and volatility. In fact, the two tend to be negatively
related.

The second model examines the extent to which the cross section of excess condo-
minium returns move in proportion to a single latent variable. This can be viewed as
a specialization of the conditional CAPM, which assumes that the ratios of the CAPM
betas across Vancouver housing submarkets are constants and treats the benchmark port-
folio as unobservable. This leads to a set of cross equation restrictions on the CAPM
betas across municipalities.

The empirical results are somewhat supportive of the latent variable model restrictions
when the market or benchmark portfolio is assumed to contain only housing assets. A
single latent variable, comprised of housing market fundamentals, captures most of the
variation in returns across submarkets over time. However, once stocks are included in
the market portfolio the cross equation restrictions are rejected.

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Figure 4.1: First Difference in the Log of Monthly House Prices
Figure 4.2: Squared Values of the First Difference in the Log of Monthly House Prices
Figure 4.3: Sample Autocorrelation Function for the First Difference in the Log of Monthly House Prices
Figure 4.4: Squared Residuals from Estimated Fourth Order Autoregressive [AR(4)] Model for the First Difference in the Log of Monthly House Prices
Figure 4.5: Estimated Conditional Variance of Monthly Single-Detached House Prices
Figure 4.6: Estimated Conditional Quarterly House Price Volatility
Table 4.1: Nonlinear Maximum Likelihood Estimation Results for AR(4) Univariate Time Series Models with Time-Varying Second Moments, of the First Difference in the Log of Monthly House Prices

\[ \Delta p_t = \gamma_0 + \sum_{i=1}^{4} \Delta p_{t-i} + \varepsilon_t \]

\[ \sigma_i^2 = a_0 + \sum_{i=1}^{3} a_i \varepsilon_{t-i}^2 \]

\[ \sigma_i^2 = b_0 + b_1 \varepsilon_{t-1}^2 + b_2 \varepsilon_{t-1}^2 + b_3 \varepsilon_{t-1}^2 \]

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<td>0.00017</td>
<td>3.16</td>
<td>$b_0$</td>
<td>0.0017</td>
<td>1.48</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.4945</td>
<td>2.96</td>
<td>$b_1$</td>
<td>0.6728</td>
<td>3.618</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.1417</td>
<td>1.31</td>
<td>$b_2$</td>
<td>-0.4582</td>
<td>-1.66</td>
</tr>
<tr>
<td>$a_3$</td>
<td>0.3154</td>
<td>2.28</td>
<td>$b_3$</td>
<td>32.904</td>
<td>1.97</td>
</tr>
<tr>
<td>LLF</td>
<td>337.98</td>
<td></td>
<td>LLF</td>
<td>345.07</td>
<td></td>
</tr>
<tr>
<td>Diagnostics*</td>
<td></td>
<td></td>
<td>Diagnostics*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q(4)$</td>
<td>4.71 (0.32)</td>
<td></td>
<td>$Q(4)$</td>
<td>4.28 (0.37)</td>
<td></td>
</tr>
<tr>
<td>$Q(8)$</td>
<td>10.34 (0.24)</td>
<td></td>
<td>$Q(8)$</td>
<td>9.59 (0.30)</td>
<td></td>
</tr>
<tr>
<td>$Q^2(4)$</td>
<td>2.71 (0.61)</td>
<td></td>
<td>$Q^2(4)$</td>
<td>5.22 (0.27)</td>
<td></td>
</tr>
<tr>
<td>$Q^2(8)$</td>
<td>14.04 (0.08)</td>
<td></td>
<td>$Q^2(8)$</td>
<td>11.60 (0.17)</td>
<td></td>
</tr>
<tr>
<td>$Q^2(12)$</td>
<td>27.05 (0.00)</td>
<td></td>
<td>$Q^2(12)$</td>
<td>13.31 (0.35)</td>
<td></td>
</tr>
</tbody>
</table>

*Diagnostics are misspecification tests carried out on the standardized residuals, $u_t = \varepsilon_t / \hat{\sigma}_t$, where $\varepsilon$ is the vector of regression residuals and $\hat{\sigma}$ is the vector of estimated conditional standard errors. $Q(q)$ and $Q^2(q)$ are the modified Box-Pierce statistics which test for joint significance of serial correlation in the first $q$ lags of the standardized residuals and squared standardized residuals, respectively. Marginal significance levels for the test statistic values appear in parentheses. LLF is the maximized value of the log-likelihood function.
Table 4.2: Regressions of Monthly House Price Appreciation and TSE 300 Index Returns on the Instrumental Variables

\[ y_{j,t+1} = \tau_0 + \tau_1 \Delta p_t + \tau_2 r_{St} + \tau_3 psdw_t + \tau_4 i_t + u_{j,t+1} \]

<table>
<thead>
<tr>
<th>( y_{j,t+1} )</th>
<th>( \tau_0 )</th>
<th>( \tau_1 )</th>
<th>( \tau_2 )</th>
<th>( \tau_3 )</th>
<th>( \tau_4 )</th>
<th>( R^2 )</th>
<th>Q(12)</th>
<th>HET</th>
<th>W</th>
<th>JB</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta p_{t+1} )</td>
<td>0.03</td>
<td>0.33</td>
<td>0.14</td>
<td>-0.01</td>
<td>-2.53</td>
<td>0.27</td>
<td>44.26</td>
<td>19.08</td>
<td>27.67</td>
<td>15.86</td>
</tr>
<tr>
<td></td>
<td>(2.28)</td>
<td>(3.57)</td>
<td>(3.09)</td>
<td>(0.01)</td>
<td>(-1.67)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r_{s,t+1} )</td>
<td>-0.03</td>
<td>0.23</td>
<td>0.05</td>
<td>2.63</td>
<td>-6.63</td>
<td>0.09</td>
<td>11.65</td>
<td>11.04</td>
<td>16.59</td>
<td>132.52</td>
</tr>
<tr>
<td></td>
<td>(-0.90)</td>
<td>(1.45)</td>
<td>(0.70)</td>
<td>(2.49)</td>
<td>(-2.64)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5% cv 21.03 9.49 9.49 5.99

Note: t-statistics, constructed with heteroskedasticity-consistent standard errors (see White [150]), appear in parentheses. Q(12) is the modified Box-Pierce test for randomness in the first 12 lags of the model residuals, JB is the Jarque-Bera [101] Lagrange multiplier test for normality, HET is the Glejser [73] test for heteroskedasticity, and W is a Wald test of the joint significance of the four explanatory variables, which is robust to heteroskedasticity.
Table 4.3: AR(4) Model for Estimated Conditional Quarterly House Price Volatility

\[ \sigma_{q,t}^2 = \psi_0 + \psi_1 \sigma_{q,t-1}^2 + \psi_2 \sigma_{q,t-2}^2 + \psi_3 \sigma_{q,t-3}^2 + \psi_4 \sigma_{q,t-4}^2 + \epsilon_t \]

<table>
<thead>
<tr>
<th>( \hat{\psi}_0 )</th>
<th>( \hat{\psi}_1 )</th>
<th>( \hat{\psi}_2 )</th>
<th>( \hat{\psi}_3 )</th>
<th>( \hat{\psi}_4 )</th>
<th>( R^2 )</th>
<th>Q(4)</th>
<th>Q(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00064</td>
<td>0.2637</td>
<td>0.1225</td>
<td>-0.0382</td>
<td>0.3422</td>
<td>0.26</td>
<td>3.98</td>
<td>5.89</td>
</tr>
<tr>
<td>(1.41)</td>
<td>(1.90)</td>
<td>(0.81)</td>
<td>(-0.26)</td>
<td>(2.45)</td>
<td></td>
<td>(0.41)</td>
<td>(0.66)</td>
</tr>
</tbody>
</table>

Note: Q(4) and Q(8) are the values of the modified Box-Pierce test for serial correlation in the model residuals up to lag 4 and 8, respectively. The parentheses below the coefficient estimates contain t-statistics, while those below the Q-statistics report marginal significance levels.
Table 4.4: Regression of Monthly House Price Appreciation and Excess Returns on Expected Conditional Volatility

(a) Excess Monthly Housing Returns

\[ \Delta p_t - i_t = \alpha_0 + \alpha_1 \hat{\sigma}^2_{t-1} + \epsilon_t \]

<table>
<thead>
<tr>
<th>( \hat{\alpha}_0 )</th>
<th>( \hat{\alpha}_1 )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0035</td>
<td>-3.1903</td>
<td>0.014</td>
</tr>
<tr>
<td>(1.06)</td>
<td>(-0.81)</td>
<td></td>
</tr>
</tbody>
</table>

(b) Monthly House Price Appreciation

\[ \Delta p_t = \alpha_0 + \alpha_1 \hat{\sigma}^2_{t-1} + \alpha_2 i_{t-1} + \epsilon_t \]

<table>
<thead>
<tr>
<th>( \hat{\alpha}_0 )</th>
<th>( \hat{\alpha}_1 )</th>
<th>( \hat{\alpha}_2 )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.082</td>
<td>11.703</td>
<td>-8.350</td>
<td>0.19</td>
</tr>
<tr>
<td>(4.92)</td>
<td>(3.09)</td>
<td>(-4.47)</td>
<td></td>
</tr>
</tbody>
</table>

Note: t-statistics in parentheses are calculated using the Newey-West [120] variance-covariance estimator with lag length set equal to two. The standard errors are therefore robust to heteroskedasticity and second order autocorrelation found to characterize \( \Delta p_t \).
Table 4.5: Comovements in Expected House Price Appreciation Under Risk Neutrality and Expected Conditional House Price Volatility

<table>
<thead>
<tr>
<th>Area</th>
<th>corr</th>
<th>$g_{t+1}^e, \hat{\sigma}_{st}^2$</th>
<th>cov</th>
<th>$g_{t+1}^e, \hat{\sigma}_{st}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Burnaby</td>
<td>0.25</td>
<td>0.17*10^{-5}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>East Van.</td>
<td>0.14</td>
<td>0.77*10^{-6}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Westside</td>
<td>0.23</td>
<td>0.14*10^{-5}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>North Van.</td>
<td>0.30</td>
<td>0.17*10^{-5}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Richmond</td>
<td>0.37</td>
<td>0.19*10^{-5}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surrey</td>
<td>0.17</td>
<td>0.16*10^{-5}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tsawassen</td>
<td>0.25</td>
<td>0.16*10^{-5}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>West Van.</td>
<td>0.25</td>
<td>0.15*10^{-5}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4.6: Regression of Excess Quarterly Condominium Returns on Predicted Conditional House Price Volatility

\[ r_{Hj,t+1} = \beta_0,j + \beta_1,j \sigma_{t,j}^2 + e_{j,t+1} \]

<table>
<thead>
<tr>
<th>Area</th>
<th>( \hat{\beta}_1 )</th>
<th>t-statistic</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Burnaby</td>
<td>-30.93</td>
<td>-2.57</td>
<td>0.18</td>
</tr>
<tr>
<td>East Van.</td>
<td>-18.24</td>
<td>-1.51</td>
<td>0.08</td>
</tr>
<tr>
<td>Westside</td>
<td>-3.16</td>
<td>-0.16</td>
<td>0.00</td>
</tr>
<tr>
<td>North Van.</td>
<td>-12.26</td>
<td>-2.21</td>
<td>0.05</td>
</tr>
<tr>
<td>Richmond</td>
<td>-4.18</td>
<td>-0.57</td>
<td>0.01</td>
</tr>
<tr>
<td>Surrey</td>
<td>-10.95</td>
<td>-1.92</td>
<td>0.03</td>
</tr>
<tr>
<td>Tsawassen</td>
<td>-7.24</td>
<td>-1.55</td>
<td>0.04</td>
</tr>
<tr>
<td>West Van.</td>
<td>-14.47</td>
<td>-2.76</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Note: t-statistics are calculated with heteroskedasticity-consistent standard errors using White's [150] procedure.
Table 4.7: Regressions of Ex Post Condominium Price Appreciation on Expected Capital Gains Under Risk Neutrality and Expected Conditional Price Volatility

\[ g_{j,t+1} = \alpha_0 + \alpha_1 g_{t+1}^2 + \alpha_2 \sigma_{g,t}^2 + u_{j,t+1} \]

<table>
<thead>
<tr>
<th>Area</th>
<th>( \hat{\alpha}_1 )</th>
<th>( \hat{\alpha}_2 )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Burnaby</td>
<td>-3.52</td>
<td>-20.42</td>
<td>0.19</td>
</tr>
<tr>
<td>East Van.</td>
<td>-2.18</td>
<td>-11.97</td>
<td>0.07</td>
</tr>
<tr>
<td>Westside</td>
<td>-8.65</td>
<td>25.34</td>
<td>0.22</td>
</tr>
<tr>
<td>North Van.</td>
<td>-1.29</td>
<td>-7.72</td>
<td>0.04</td>
</tr>
<tr>
<td>Richmond</td>
<td>-3.61</td>
<td>5.42</td>
<td>0.08</td>
</tr>
<tr>
<td>Surrey</td>
<td>-1.75</td>
<td>-3.88</td>
<td>0.02</td>
</tr>
<tr>
<td>Tsawassen</td>
<td>-0.74</td>
<td>-3.41</td>
<td>0.02</td>
</tr>
<tr>
<td>West Van.</td>
<td>-3.49</td>
<td>-4.33</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Note: Iterative seemingly unrelated estimation is employed to account for contemporaneous correlations in the residuals across municipalities. t-statistics, constructed with heteroskedasticity-consistent standard errors (see White [150]), are shown in parentheses.
Table 4.8: Regressions of Excess Quarterly Condominium Returns on Latent Variable Model Instruments

\[ r_{ij,t+1} = \delta_0 + \delta_1 IN_t + \delta_2 i_t + \delta_3 ES_t + u_{i,t+1} \]

<table>
<thead>
<tr>
<th>Area</th>
<th>( \hat{\delta}_1 )</th>
<th>( \hat{\delta}_2 )</th>
<th>( \hat{\delta}_3 )</th>
<th>( R^2 )</th>
<th>DW</th>
<th>Q(4)</th>
<th>HET</th>
<th>W</th>
<th>JB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Burnaby</td>
<td>0.5501</td>
<td>-5.95</td>
<td>-0.8214</td>
<td>0.24</td>
<td>1.67</td>
<td>2.42</td>
<td>4.97</td>
<td>21.57</td>
<td>167.03</td>
</tr>
<tr>
<td></td>
<td>(2.06)</td>
<td>(-2.02)</td>
<td>(-1.86)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>East Van.</td>
<td>0.3758</td>
<td>-1.56</td>
<td>-0.9876</td>
<td>0.19</td>
<td>2.08</td>
<td>6.71</td>
<td>17.97</td>
<td>6.92</td>
<td>5.63</td>
</tr>
<tr>
<td></td>
<td>(1.54)</td>
<td>(-0.58)</td>
<td>(-2.47)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Westside</td>
<td>0.4615</td>
<td>-9.24</td>
<td>-0.7950</td>
<td>0.28</td>
<td>2.42</td>
<td>3.63</td>
<td>9.57</td>
<td>6.27</td>
<td>23.23</td>
</tr>
<tr>
<td></td>
<td>(1.47)</td>
<td>(-2.67)</td>
<td>(-1.54)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>North Van.</td>
<td>0.5106</td>
<td>-5.14</td>
<td>-0.6227</td>
<td>0.19</td>
<td>2.29</td>
<td>2.70</td>
<td>10.95</td>
<td>26.25</td>
<td>77.37</td>
</tr>
<tr>
<td></td>
<td>(2.50)</td>
<td>(-2.29)</td>
<td>(-0.19)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Richmond</td>
<td>0.5297</td>
<td>-4.78</td>
<td>-0.5322</td>
<td>0.33</td>
<td>2.61</td>
<td>10.59</td>
<td>11.85</td>
<td>25.16</td>
<td>4.71</td>
</tr>
<tr>
<td></td>
<td>(3.27)</td>
<td>(-2.68)</td>
<td>(-1.99)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surrey</td>
<td>0.4117</td>
<td>-4.58</td>
<td>-0.7244</td>
<td>0.23</td>
<td>1.35</td>
<td>6.19</td>
<td>6.91</td>
<td>30.45</td>
<td>38.06</td>
</tr>
<tr>
<td></td>
<td>(1.85)</td>
<td>(-1.88)</td>
<td>(-1.98)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tsawassen</td>
<td>0.4467</td>
<td>-2.17</td>
<td>-0.5797</td>
<td>0.31</td>
<td>2.34</td>
<td>3.36</td>
<td>28.62</td>
<td>25.35</td>
<td>34.41</td>
</tr>
<tr>
<td></td>
<td>(3.33)</td>
<td>(-1.48)</td>
<td>(-2.62)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>West Van.</td>
<td>0.3725</td>
<td>-7.69</td>
<td>-0.2418</td>
<td>0.41</td>
<td>2.35</td>
<td>3.99</td>
<td>2.80</td>
<td>15.67</td>
<td>17.14</td>
</tr>
<tr>
<td></td>
<td>(2.30)</td>
<td>(-4.31)</td>
<td>(-0.91)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5% cv 9.49 7.81 7.81 5.99

Note: \( \hat{\delta}_1 \) and \( \hat{\delta}_1 \) must be multiplied by \( 10^{-5} \) and \( 10^{-3} \), respectively to get the true parameter estimates. DW is the Durbin-Watson statistic, Q(4) is the modified Box-Pierce test for randomness in the first four lags of the model residuals, JB is the Jarque-Bera [101] Lagrange multiplier test for normality, HET is the Glejser [73] test for heteroskedasticity, and W is a Wald test of the joint significance of the three explanatory variables, which is robust to heteroskedasticity.
Table 4.9: Generalized Method of Moments Estimation of and Hypothesis Tests for the Single Latent Variable Model of Condominium Returns

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimate</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\phi}_1$</td>
<td>$0.35 \times 10^{-5}$</td>
<td>4.55</td>
</tr>
<tr>
<td>$\hat{\phi}_2$</td>
<td>-1.03</td>
<td>-4.60</td>
</tr>
<tr>
<td>$\hat{\phi}_3$</td>
<td>$-0.11 \times 10^{-3}$</td>
<td>-7.59</td>
</tr>
<tr>
<td>$\beta_{uv}$</td>
<td>0.85</td>
<td>4.43</td>
</tr>
<tr>
<td>$\beta_{ws}$</td>
<td>0.96</td>
<td>3.82</td>
</tr>
<tr>
<td>$\beta_{nv}$</td>
<td>0.46</td>
<td>5.84</td>
</tr>
<tr>
<td>$\beta_r$</td>
<td>0.71</td>
<td>5.97</td>
</tr>
<tr>
<td>$\beta_s$</td>
<td>0.50</td>
<td>5.14</td>
</tr>
<tr>
<td>$\beta_t$</td>
<td>0.51</td>
<td>7.90</td>
</tr>
<tr>
<td>$\beta_{uv}$</td>
<td>0.52</td>
<td>4.58</td>
</tr>
<tr>
<td>$T^{\psi_{min}}$</td>
<td>31.42 (0.08)</td>
<td></td>
</tr>
</tbody>
</table>

Note: The beta for Burnaby is normalized to 1. $T^{\psi_{min}}$ is Hansen’s [83] J-test of overidentifying restrictions. It has a $\chi^2$ distribution with 22. The marginal significance level is in parentheses.


