Three Essays on Policy Function Assignment in a Federation

by

Benoit Delage

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Abstract

The first essay explores the nature of the equilibria obtained when state governments conduct industrial policies to affect firms' location choices. The model differs from existing ones by considering industrial policy targeted at small firms. In a simple two–region, two–industry model with imperfection information, it is shown how regions attempt to attract firms from the neighbouring one, either by making cash or in–kind transfers. The model rationalizes the use of in–kind subsidies for incentive–compatibility reasons, even though they are valued less by firms than what they cost to provide. It allows to understand why regions with a smaller industrial base may pursue a more aggressive industrial policy. The model sheds some light on which industries are likely to be targeted by industrial policy, and how the means of income transfers could be selected.

The objective of the second paper is to determine under which circumstances an industrial policy that seeks to increase the number of new technologically–based firms in the economy is best assigned to the central or regional governments in a federation. Even though a decentralized industrial policy may be more flexible, it has the drawback that regions compete against each other to acquire successful firms. Because this margin is closed to a central government, it is likely to achieve a better outcome even if operating under "uniformity" constraints. The public policy implication is that this type of industrial policy should be transferred to the federal government.

The third essay presents a new rationale for intergovernmental grants in a federation

that arises strictly from the income redistribution concerns of the federal government. The central government seeks to redistribute income across agents, and behaves as a Stackelberg leader with respect to regional governments. Intergovernmental grants are needed to effect income redistribution while maintaining appropriate expenditure levels. Differentiated grants allow in some circumstances to implement a "third–best" solution when nominal prices differ across regions. They allow the federal government to affect provincial tax rate and public good provision, thus complementing the income redistribution done directly through the federal income tax system.

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Introduction

The three essays contained in this thesis study different aspects of the policy function assignment problem in a federal economy.

This thesis adopts an assumption that is common in public economics, that the state and its agents behave as benevolent dictators. This is a normative rather than a positive theory of government's behaviour. As it is well known, this is not the only possible way to model the government's action; Public Choice and the "Leviathan" point of view constitute the main alternatives, and are more positivist theories of the state. It is also a maintained assumption of this thesis that governments behave in a non–cooperative manner.

Which working assumption is chosen is crucial for the analysis of policy function assignment problems. The normative point of view can be considered biased in favour of centralization, whereas for other views there is nothing worse than an unfettered Leviathan. Also, while it is assumed throughout that governments have social welfare functions of the same type, but defined over different sets of agents, from a Public Choice approach determining the precise objective function of the government would constitute the main problem to solve.

Within the present tradition, Musgrave's classification of the functions entrusted to government — to achieve economic efficiency, to redistribute income, and to preserve the stability of the economy — take on a special meaning in an economy organised along federal, or federal–like, lines. These functions have to be assigned to the different levels of government in order to be best performed. In most cases, economists only attempt to determine which level ought to perform which function, as done by Bell [1], and, implicity, by Myers [41]. More ambitiously, economists also attempt to determine the political structure of the economy, the number of states, and whether or not a federal structure is appropriate to fullfil the required task. The book by Breton and Scott [10] represents such an effort. In this respect, the present thesis belongs to the former group.

What is to be understood by the term federal economy? For most economists, the term is quite broadly interpreted to mean any circumstance in which there are several contiguous territories, each with its own government, among which labour mobility is mostly unimpeded. For some others, the term applies only to actual federations, like Australia, Canada, and the United States of America, with two levels of governments, each having their constitutionally protected fields of jurisdiction.

The main emphasis in the literature concerned with economic efficiency appears to have been on the added distortionary impacts of taxation in federations, in terms of factor mobility, tax exporting, and underprovision of public goods. See for example the recent paper by Inman and Rubinfeld [26], and the classic paper by Flatters, Henderson, and Mieszeskowski [16]. The first two essays of this thesis are also concerned with economic efficiency problems, but more from the expenditures side. They are analyses of some specific problems of industrial policy, understood here to mean the attempts made by governments to affect the location of firms. One characteristic of the industrial policy literature is its partial equilibrium approach, whereas the emphasis is on general equilibrium effects in the related taxation literature. Relatively few of the industrial policy papers make explicit the link between the industrial policy problem and the federal structure of the economy. Exceptions are the papers by King, McAfee and Welling [30] and Taylor [51]. The case in favour of decentralization is strongest when presented from the expenditure side, and for economic efficiency reasons, because expenditure levels can be more flexibly adapted to fit local needs. Most models of industrial policy, in the sense given to the term here, are essentially rent seeking models[1]. In fact, the answer to the policy function assignment problem in most cases is truly simple: this type of industrial policy is best left to the care of the central government, which in turn does best by doing nothing. While this policy conclusion is immediate in the case of the first essay of this thesis, it is not so obviously true in the case of the second essay.

The first essay explores the nature of the equilibria obtained when state governments conduct industrial policies to affect firms' location choices. The model differs from existing ones by considering industrial policy targeted at a large number of small firms. The governments' attempts to get a single large plant, say a car assembly plant, to locate on their territory is the problem most commonly studied.

In a simple two–region, two–industry model with imperfection information, it is shown how regions will attempt to attract firms from the neighbouring one, either by making cash or in–kind transfers. Information is said to be imperfect because a government is unable to distinguish at low cost the true type of a given firm.

The model rationalizes the use of a menu of pure cash and in–kind subsidies for incentive–compatibility reasons, in some circumstances, even though the in–kind subsidies are valued less by firms than it costs the goverment to provide them. It allows one to understand why regions with a smaller industrial base may pursue a more aggressive industrial policy. It shows also that such an industrial policy has a detrimental impact on the efficient location of plants, in general. In that respect, it diverges from the usual single large plant papers, which find that location efficiency is generally maintained.

The model is broad enough to study what impact investments in infrastructure in

[1] A few of these models are described in the first essay.

the two regions may have on the industrial policy equilibrium outcome. It may also shed some light on which industries are likely to be targeted by industrial policy, and how methods of income transfers could be selected.

The main objective of the second paper is to determine under which circumstances an industrial policy that seeks to increase the number of new technologically-based firms in the economy is best assigned to the central or regional governments in a federation. A notable characteristic of the problem studied here is that it is not true that the solution is simply to assign the function to the central government, which in turn would decide, optimally, to do nothing whatsoever. Regional governments want to see such firms set up plants on their territory to benefit from the externalities they are thought to generate. Even though a decentralized industrial policy may be more flexible in using local information, it has the drawback that regions compete against each other to acquire successful firms. Because this margin is ignored by the central government — its objective function being the sum of the regional objective functions — it is likely to achieve a better outcome even if operating under "uniformity" constraints. Two types of constraints are imposed on the central government here: first, it may have poorer information about firms than regional governments do; second, it may have to operate under identical rules across regions. Nevertheless, it appears on balance that the public policy implication is that this type of industrial policy should be transferred to the federal government.

The second main function of government is to redistribute income. While it appears to be generally accepted that this function is best left to the central government, see for example Musgrave [40], paradoxically most of the literature studies the problems created when a lower level of government tries to redistribute income. See for example Pauly [45], Burbidge and Myers [11], and Wildasin [54]; exceptions are the papers by Boadway, Marchand and Vigneault [7] and Hochman and Pines [25]. The main argument in favour of centralized income redistribution revolves around labour mobility. By centralizing this function, it is possible to prevent agents from escaping redistributive taxation, and to prevent "welfare magnet" effects, or, to the contrary, a "race to the bottom" towards minimum levels of redistribution. The benefits from such centralization depend very much on the assumed behaviour of the government. The third essay of this thesis reverses somewhat the usual approach: it takes as given that the income redistribution function is best assigned to the central government, and tries to determine how to implement it in an economy with an explicit federal structure.

This essay presents a new rationale for intergovernmental grants in a federation that arises strictly from the income redistribution concerns of the federal government. The central government is the only one that seeks to redistribute income across agents, and is assumed to behave as a Stackelberg leader with respect to the regional governments. Even if their usual justifications are absent from the model, intergovernmental grants are needed to effect some income redistribution while maintaining appropriate expenditure levels. Intergovernmental grants are required because the price level varies systematically across regions, and a simple two–parameter federal income tax schedule must remain the same across regions in nominal terms, for political economy reasons. These two facts together force the federal government to supplement its own tax schedule with intergovernmental grants, in order to manipulate regional commodity tax rates and obtain an appropriate income tax schedule in real terms. Differentiated grants allow, in some circumstances, the implementation of a "third–best" solution when price levels differ across regions. They allow the federal government to affect provincial tax rate and public good provision, thus complementing the income redistribution done directly through the federal income tax system.

Chapter 1

Incentive Compatible Industrial Policy

1.1 Introduction

In recent years, there has been both a widespread adoption of free market principles in formerly regulated economies or industries, and new forms of government intervention in market economies.

The basic tenets of a free market economy have been adopted more widely throughout the world, Eastern Europe and Latin America being the most spectacular examples. In developed economies, entire industries, like air transportation, telecommunications and some utilities, have been either privatized or deregulated.

At the same time, new forms of government intervention have been adopted, the managed trade in the car industry between Japan and various other countries being a prime example. Also, it appears that new types of industrial policy are now being utilized, both by the national and sub–national governments. The creation of the Sematech consortium is an example — see Katz and Ordover [29], and *The Economist* [15].

Much less noticed has been the growth of industrial policy initiatives at the sub-national level. In a recent book, Robert Wilson [55] describes a large number of such initiatives in the United States. Some take the form of tax expenditures, like tax incentives for Research and Development, for job creation, for industrial development, etc. [55, table 4.2, page 110]. Some other initiatives are in terms of job training, direct equity injection (state–sponsored venture capital), industrial revenue bonds (firm specific or not), and technical assistance [55, table 5.1, page 150]. Enterprise zones and business incubators can be added to such a list. Not all American states offer the same mix of subsidy programmes, the programmes are targeted at different industries, and they are potentially available to a wide array of firms, large and small.

Given the bewildering number of government programmes in place, and the almost constant changes in specific details, it is relatively hard to examine their effectiveness in a systematic fashion. While there exists a vast literature on these types of government interventions from a policy perspective, relatively little has been written on the subject using formal models[1].

Economists have studied such industrial policy in two broadly different ways. Authors of theoretical papers aggregate the transfers made through these programmes into a single cash payment and concentrate their attention to the problem of attracting one large plant at a time. They mostly use a second–price auction framework, abstracting from the bargaining that occurs between firms and governments, which is an appropriate approximation of reality. Some such papers are discussed below. Authors of empirical papers, for example Michael Luger[36], sometimes group the many government programmes under a small number of headings, and build "effort indices" to represent qualitatively the extent of government intervention and to measure their relative success.

[1]From the policy perspective, one need only consult, for example, the McDonald Commission volume by André Blais [4], and the books by James Cobb [12], Wilson [55], Henry Herzog and Alan Schlottmann [23] and Jurgen Schmandt and Wilson [48] as typical, and excellent contributions to the study of industrial policy.

Each strand of the literature answers different questions. Theoretical papers identify if, and when, governments' industrial policies create distortions in the location choice of firms, and how rents are shared between firms and governments. The focus is clearly set on some notion of efficiency. Empirical papers generally measure how well governments meet their objectives, which are described either as increasing wages and reducing unemployment (Luger[36]), or in attracting new firms (Head and Ries[20]), presumably for the attached jobs and tax revenues. Head and Ries also attempt to measure the cost effectiveness of such programmes and highlight their prisoners' dilemma nature.

However, many aspects of these government programmes are not explored by either strand of the literature. First, and most importantly, no one has considered how industrial policy may differ when it targets industries made entirely of small firms, instead of a single large plant. Second, little is done to explain why in–kind subsidies are used. The use of in–kind subsidies, rather than cash transfers, represents something of a puzzle, because these are likely to be valued less by receiving firms than they cost governments to provide. Third, little has been done to explain why several subsidy programmes may coexist within a single jurisdiction at any given time. Fourth, why is it the case that firms are allowed to choose in which programmes to participate, and that often no negotiations are conducted between firms and governments? Finally, is it possible to characterise what makes a region better able to pursue such an industrial policy?

This essay tries to answers these questions within a very simple theoretical framework, with two regional governments and two industries, each made up of a large number of small firms. It takes as given that regional governments seek by their industrial policy to maximize something akin to a wage bill, net of subsidy costs. A very simple definition of industrial policy is adopted: industrial policy is an attempt by government to affect firms' production plant locations by means of direct cash transfers or in–kind subsidies[2].

The model used here bears a strong resemblance to the one used in Ravi Kanbur and Michael Keen's *Jeux Sans Frontière* [24] to study commodity tax competition[3]. In both cases, the use of a simple model is justified by the need to obtain clear results; comparative statics exercises are complicated by the fact that the player's reaction functions are discontinuous. In both cases, size asymmetry between regions has big consequences and generates qualitatively asymmetric equilibria.

If only cash subsidies are allowed in what follows, the results obtained are quite similar to Kanbur and Keen's, with one region being clearly the aggressive one in its behaviour. New in this model, the use of in–kind subsidies is justified by self–selection constraints when information is imperfect and industries truly differ. It is shown that using in–kind subsidies is preferable to using cash subsidies if the target industry values these transfers sufficiently highly, and the other industry does not. Using in–kind subsidies increases the payoff of the aggressive region in appropriate circumstances because it gives that region the ability to discriminate against firms that are, in a specific sense, infra–marginal (less likely to move): this reduces the total cost of the industrial policy. Other results obtained are as follows. First, if the aggressive region offers a menu of cash and in–kind subsidies, while the other offers only cash subsidies, it can be shown that, in some circumstances, some firms from both industries are spatially misallocated. This "cross–over" effect, each region securing some of the other's firms, is unique in the literature. In other circumstances,

[2]In general, an industrial policy is described by the type of industries targeted, their present and future conditions, by the policy instruments used to intervene (direct or indirect), by where the intervention takes place (at the border, at the industry or firm level), and by the selectivity of interventions (rule versus discretion).

[3]The present model is somewhat more complex, in that it has several potential types of subsidies and introduces a "thick" border, whereas in Kanbur and Keen, there is only one tax instrument, and the border itself is of dimension zero.

the outcome is similar to the cash–subsidy outcome, with firms from only one industry being spatially misallocated.

Second, it is found that the ability of a given region to pursue successfully such an industrial policy with a menu of subsidy programmes depends on the ratio of marginal firms gained to infra–marginal firms "bribed", and the relative costs of doing so[4]. Thus, the model predicts that a region with a smaller industrial base is better able to pursue an aggressive industrial policy, either by using only cash subsidies or a mixture of cash and in–kind subsidies. *Per se*, direct transfers made to marginal firms are always more costly with in–kind subsidies, relatively to a comparable cash–only strategy.

Finally, as a consequence of giving more choice of policy instruments to a region, the aggressive region's reaction function displays some discontinuities, which implies that sometimes there is no Nash equilibrium in pure strategies.

The rest of the essay is organized as follows. Section 1.2 reviews related literature. Section 1.3 contains a description of the problem studied in this essay, and presents the objective functions of the agents, the properties of the targeted industries, and how to incorporate all these in a simple model. Next, in section 1.4 the problem is solved in its simplest form, when regions compete by giving cash subsidies only. Studying the cash subsidy game establishes the basic logic of the problem at hand. It shows that size asymmetry across industries plays a crucial role in determining the outcome of the game, and the extent of firms' misallocation.

Section 1.5 first describes how a menu of subsidy programmes may be selected. The problem is solved when only one region uses in–kind subsidies, with the other region constrained to pay only cash subsidies. The problem is decomposed into several parts, to maintain clarity. Different configurations of firm allocations are possible, depending

[4]A firm is said to have been bribed if it locates where it would have in a *laisser–faire* solution. Otherwise, it is said to have been acquired. Bribed firms receive large information rents.

on the parameter values. Any movement of firms is inefficient. In some cases, firms of both industries are found in both regions, which is described as giving a two–way or "cross–over" flow of firms. In other cases, firms flow only one way. Section 1.6 studies two equilibrium concepts, depending on whether or not a government must commit itself to using only one or the other type of industrial policy. In–kind subsidies are preferred to cash subsidies in the absence of commitment, for some subsets of the parameter space such that there is either a one or two–way flow of firms. With commitment, in–kind subsidies are preferred by the aggressive region for a subset of the parameter space that generates a one way flow of firms.

Section 1.7 extends the basic model. The problem becomes more complex when even slight modifications are made to the basic framework. More precisely, parameters representing other possible types of industrial policy are introduced (for example, industry–specific infrastructure improvements), in the simplest possible fashion, and their impact on the existence of Nash equilibrium in pure strategies is studied, both for the cash–subsidy and in–kind subsidies problem. Section 1.8 discusses how the model breaks down when industries comes to resemble single large firms. Section 1.9 contains concluding remarks.

1.2 Some Models of Industrial Policy

The introduction alluded to some theoretical papers, which are briefly discussed here. The paper by Black and Hoyt provides the basic framework. The papers by King, McAfee and Welling, King and Welling, and Biglaiser and Mezzetti can be seen as variations on this basic theme. As such, they are presented rather elliptically. Finally, Taylor's paper is discussed in some detail. The common points of all these models are discussed next, followed by a motivation of model used in this paper.

1.2.1 Basic Framework

In "Bidding for Firms", Dan Black and William Hoyt [3] consider the problem of two competing cities, Louisville and Indianapolis, bidding to attract some large firms by offering cash payments. The rationale for wanting more firms in the city is to spread the fixed cost of providing public goods. By attracting a large firm, the city widens its tax base; for firms this has the beneficial effect of reducing the gross wages demanded by workers because the tax rate goes down with an increase in the tax base.

Firms differ in their preference for the cities; that is, everything else equal, some firms would always locate in Louisville, some others in Indianapolis. Black and Hoyt are only interested in location efficiency, and they obtain the result that, with perfect information about the preferences of each firm, bidding by cities always leads to an efficient location of firms. With incomplete information about a firm's preferences, location–efficiency occurs only if a large firm is equally likely to prefer either city. The outcome of the bidding process is similar to the one obtained with a second–price auction. It can also be shown that it is preferable to attract firms by giving subsidies, not by changing the level of public goods provided.

1.2.2 Variations on the Basic Framework

First–Stage Investment

King, McAfee and Welling[30], henceforth KMW, present a model where a firm and two regions play a two–stage game. In the first period, the firm may incur some sunk costs to set up a plant in one of two regions, and obtain a region–specific return. The region–specific rate of return for the firm depends on the infrastructure levels in the regions, which are known to all, and also has a random component. At the beginning of period two, the total rate of return obtained where the investment has been made is revealed to all. If the firm receives a bad draw, it may decide to move to the other region. If it were to do so, it would have to incur the sunk cost anew, with its rate of return in the second region uncertain *ex ante*. Knowing the structure of this game and the possibility firms have of relocating in the second period, the two regions bid for the firm in the first period by offering some tax package, keeping in mind that bidding is still possible in the second period.

KMW are mainly interested in the location–efficiency properties of the equilibrium obtained, and in the distribution of rents. They show that no location inefficiency is created by this bidding process. Again, something similar to a second–price auction obtains. The overall process is complicated by the two–period framework, but essentially that is what happens.

KMW expand this basic model in the following way. Before the bidding process starts, regions invest in infrastructure. If regions start with identical levels of infrastructure, KMW determine that no pure symmetric equilibrium exists. The asymmetric equilibrium involves one region (called region A) investing more in infrastructure[5]. The second region invests also, to improve its chance of obtaining a transferring firm in period 2, following a bad draw. Again, KMW show that location–efficiency obtains in this more general model.

The Commitment Problem

King and Welling [31] consider a variant of the basic Black and Hoyt problem in a two period setting, and concentrate on the commitment problem. Both the firm and regional governments may face such a problem if they want to change their course of action in the second period, once sunk costs have been incurred by the firm and some subsidies paid out by the government. Their model involves some matching aspect between regions and the firm, with the quality of the match uncertain *ex ante*. Again,

[5] They do not attempt to explain which region would become region A.

they concentrate on how the expected surplus is divided between the firm and the regions, and on how the lack of means to commit affects the rent distribution and, possibly, location efficiency.

Political Considerations

Biglaiser and Mezzetti [2] offer another variant of this type of model. In this case, there are two periods, one firm, and n ($\geq$ 2) states. The value of the bid put up by each state is chosen by its governor, whose interest may differ from that of his constituents. In effect, a winning bid provides information on the governor's ability to attract peripheral business, or to reduce transition cost. The governor wants to reveal his ability to improve his chances in the second period reelection.

Thus, the political process may explain why some firms receive "too much" money. The divergence of interests between the governor and his constituents also means that a firm may end up in an inefficient location, even when one takes into account the value of information generated in the process.

1.2.3 A Different Approach

Taylor[51] uses an optimal control setting to examine the tendency toward rent dissipation that is a by–product of industrial policy. In his model, a single firm is up for grabs, with known benefits in terms of tax revenue, workforce training opportunities, attraction of upstream and downstream firms, and other such forms of benefits, with a known total value in money terms. Regional planners know that to win this prize, they must offer a given infrastructure level. Starting from identical lower infrastructure levels regions race to achieve the target level by investing at some given rate, between now and some chosen future time. The cost function for investment is convex, which means that there are some gains to going slowly. They race because it is a case of winner takes all, and that they know that at each moment, there is some probability that another region will achieve the target level before them[6].

The amount of rent dissipation can be estimated by comparing the discounted net value of the optimal plan, in the absence of competition, for the region which has the highest starting level of infrastructure (if there is one), with the benefits of the winning region, minus the costs incurred by all the regions. In the presence of competition, the target level is achieved faster, so that the present value of the gross benefits is higher. However, the sum of investment costs is also higher. Rent dissipation occurs if the increase in cost more than offsets the increase in benefits. It is possible to get cases with more than 100% rent dissipation if the number of competing regions is large enough and the process lasts a long time.

1.2.4 Common Themes

Even though each of these papers explores a different aspect of the problem, they have a lot in common. First, regions fight over a single large plant. Second, the approach taken is purely a partial equilibrium one, considering the sometimes ill-defined surplus to be had from acquiring a firm. Third, the bid or transfer made to the firm always takes the same form, mostly in cash. King and Welling discuss the repercussions of the commitment problem on the division of the rent, without exploring how governments could choose some form of transfer that would solve this problem, at least partially. For instance, the gift of infrastructure equipment, as in Taylor, has the interesting property of not being portable. Finally, location efficiency considerations play an important role in the analysis done. In most of these papers, the industrial policy does not change the ultimate location of production plants from what they would have been under *laisser-faire*.

[6]It seems odd to formulate the problem in this way, with a probability distribution, since everyone does start from a common point and faces the same cost function. It allows Taylor to use optimal control techniques, and maybe to claim some more generality for his model, if regions had different starting points or cost functions. However, he does not explore these cases explicitely.

1.3 The Model

This section contains two parts. The first describes the problem to be solved in this essay in some details, what are the underlying assumptions and the logic underlying incentive–compatible industrial policy. The second part presents the basic settings of the model.

1.3.1 A New Approach

The type of industrial policy discussed in this essay is strikingly different from the ones discussed in the section above.

There are four agents, or groups of agents, in this model; one government for each region, A and B, and firms which belong to two different industries. The fact that this model deals with *industries* is the crucial difference between it and the other models. That the solution technique differs, too, follows logically.

Industries are defined as follows: if regions A and B are truly different, in terms of their existing industrial structures, geographical locations, quality of infrastructure, mere size, and other such qualities, then in a *laisser–faire* world new firms electing to locate in one or the other region are likely to be somewhat different[7]. Firms going to region A (B) in such a *laisser–faire* world are said to belong to industry 1 (2). The term industry is something of a misnomer, for they constitute an heterogeneous group according to any standard industrial classification (SIC). However, they are likely to share some characteristics in terms of input mix, labour–skill intensity and capital intensity. Also, they may differ in their need to be close to suppliers of specific (service or physical) inputs.

[7]This "fact" is generally well accepted in an international trade context, when one talks about two different countries; it is less likely to be so when one compares regions within a country. One needs to think of regions sufficiently different, like Ontario and New Brunswick to see that it is not that hard to accept.

Governments share a simple objective function: they want their industrial policy to "generate" as many jobs as possible. While this objective function may appear ill–advised to many economists, it has strong seductive powers for politicians, as witnessed by their electoral promises and daily utterances. Moreover, analysts often evaluate the success of states' industrial policy programmes in similar terms; for example, Luger [36] does it by measuring the impact policies had on wages and unemployment levels and growth rates. Finally, it can be argued, as do Charles de Bartolomé and Mark Spiegel [14, page 240], that "because all labor income is earned by state residents whereas most capital income earned in the state is owned by individuals who live out of state, ...", seeking to increase the number of firms present in the region appears to be a satisfactory way to increase the capital stock of the region and maximize the welfare of the region's residents. Regional governments use subsidy schemes as the unique instruments of their industrial policy.

How much firms prefer one region over the other is likely to vary from firm to firm, and in general is known only to themselves. In an ideal world, politicians would find ways to identify firms' preferred location together with the intensity of their preferences, and offer differentiated cash incentives to attract them. But in fact governments do not know how strongly a firm prefers a given region, nor can they easily assess to which industries, as defined above, a firm belongs.

Consider the first type of private information, the degree of attachment to a region. Because this model discusses small firms, the idea of private information can be taken quite literally. The firm's desire to stay in a region may depend on something as ethereal as the managers/owners' desire to stay close to their relatives. It may come from the need to be located close to preferred suppliers or customers, or to maintain a close relationship with the financial backers of the company.

What about the other type of private information? If there were only two types of industries, in the usual sense, it would be hard to argue that a government is unable to assess quickly to which one a firm belongs. However, in this model governments would need to observe the firm's technology because a firm's factor's intensity determines to which industry it belongs. After it had acquired this information, the government would then offer the firm participation in a specific subsidy programme, and bar participation in others. The calculus to be done then, is to evaluate the gains, after having incurred the information gathering costs, stemming from the more narrowly targeted subsidy programmes. This may not be worth the effort, so that rational ignorance is a better policy.

Finally, even if ignorance turns out not to be a better policy, it may be difficult to act upon the gathered information. Writing out in a bill of law how much subsidies each (SIC) industry may obtain could just be too complex. It may also become untenable from a political point of view, because some firms would seek to be reclassified from one programme to another, while others would directly lobby for increased payouts for their industries[8].

Thus, an industrial policy must cater to firms at two margins; on one (extensive) margin, the region may wish to induce firms that would naturally locate in the other region to location instead in its region; on the other (infra) margin, the region will wish its own natural firms to locate there, without claiming the subsidy designed to induce the other region's firms to relocate. The problem is analogous to that faced by a monopsonist who, to attract new workers, must increase the wage to all workers. The cost of a new worker is thus spread over two margins. Any policy that allows

[8]Consider this quote as an illustration: "In a world with imperfect information, it does not seem reasonable to recommend any particular set of subsidies and taxes for specific industries. Instead, such recommendations are likely to lead to a situation where more heavily taxed firms and industries raise legitimate questions about the underlying models that form the basis for the particular system of subsidies and taxes. As a result, these programs are steadily extended to more firms, with the cost of the programs rising accordingly"(Michael Wasylenko [53, page 29]). Obviously, this essay tries to show that the opening sentence pessimism is not warranted.

an inefficient wage–benefits mix to attract marginal workers, but that allows smaller inframarginal payments to existing workers, may be desirable: it reduces the total wage bill associated with hiring a given additional number of workers.

Thus, a simple escape route for the government that seeks to implement an industrial policy targeted at small firms is to offer only a small number, here two, of subsidy programmes that are freely available to all, and to let the firms' owners self–select into one or the other programme. Resisting rent–seeking pressure is easier when programmes are mutually exclusive but freely available to all. One programme, relatively generous, is targeted at firms it wants to attract, and a second one is made just good enough to be chosen by firms for which its region already is the preferred location. One may think of the relation between these two programmes in terms of a participation constraint, for the marginal firms, and an incentive compatibility constraint, for the inframarginal firms. The government's ability to get firms to self–select between programmes depends on the degree of dissimilarity between industries.

To obtain such self–selection properties, at least two types of transfers must be used in the programmes. Thus it could be cash subsidies on one hand, and in–kind transfers on the other. These in–kind transfers could take the form of labour training, land gifts, or subsidized access to industrial parks. Because every firm attaches the same value to a cash transfer, it cannot be used alone, but only in combination with at least one other form of subsidy[9].

It could be argued that the offers take the form of labour training subsidies, land gift, etc., for two other reasons. The first is to solve partially the commitment problem. On both sides, very precise forms of transfer help solve the commitment problem because they are awarded early and are not portable. Land is immobile and workers are not very mobile. Machinery is somewhat more mobile, while cash is perfectly so.

[9]Those transfers are assumed never to change the input mix chosen by the firm.

That is why these last two types of transfers are not usually used. The second reason is to please a wider constituency: workers who benefit from the training, and landowners.

But what ultimately matters for the firm is the cash equivalent of these subsidies, and governments are well aware of that. It is a contention of this essay that the main function played by the specifics of the programmes is to induce firms' self–selection, not to solve a commitment problem.

1.3.2 Basic Setting

This section contains a detailed description of the model. More precisely, it presents a description of the industries, the firms' objective function, and the governments' objective function.

The sequence of movement in the game is as follows: first, governments choose their industrial policy; taking this industrial policies as given, firms then choose where to locate. The industrial policy chosen by a government can be thought as having two components: the instruments used, whether cash–only or a mixture of cash and in-kind subsidies, and the level of financial support offered to firms through those instruments. Section 1.6 explores whether it matters or not that the government can commit itself to using only one type of instrument.

There are two industries in the model, each made up of a large number of firms. To which industry a firm belongs is private information, as discussed in section 1.3.1. Firms within an industry are characterized by two elements: everything else equal, they have the same preferred location; also, subsidies offered by governments enter their profit function in the same way within an industry and differently across industries. The size of industry 1 is normalized to one. Industry 2 has N $(N \geq 1)$ firms. N is a real number.

Firms want to maximize profits. Within the context of this model profits depend on location, and on location only, in two ways. First, firms within an industry differ in the value they assign to being at their preferred location. For industry 1 (2), the monetary value of this preference for region A (B) is described by a parameter α (β), which is distributed according to a cumulative distribution function $F(\alpha)$ ($H(\beta)$), between 0 and $\overline{\alpha}$ (0 and $\overline{\beta}$). As mentioned, this preference for a region differs across firms and is private information.

Second, when regional governments offer subsidies, firms' profits vary according to the subsidies available in their chosen location.

Governments may offer cash or in–kind subsidies. Cash subsidies, b, enter the firms' profit function linearly. Two types of in–kind subsidies can be offered, called θ and γ[10]. Subsidies are indexed by industry and region. t_i represents the value assigned by industry i to a unit of θ, while g_i is the value of a unit of γ.

To simplify, the following assumptions are made:

Assumption 1.1 *Firms value subsidies less than their cost of provision: thus both t_i and g_i are less than one.*

Assumption 1.2 *the benefit of a given θ–subsidy is at least as large for a type 2 firm than for a type 1 firm: $t_2 \geq t_1$.*

Assumption 1.3 *the marginal rate of substitution of θ–subsidy for γ–subsidy in firm profits is larger for type 2 than for type 1 firms: $\frac{t_2}{g_2} > \frac{t_1}{g_1}$.*

Figure 1.1 represents iso–profits curves for a typical firm of each industry in the γ–θ space.

[10]For concreteness, one may think of θ as labour training, and γ as access to subsidized industrial parks.

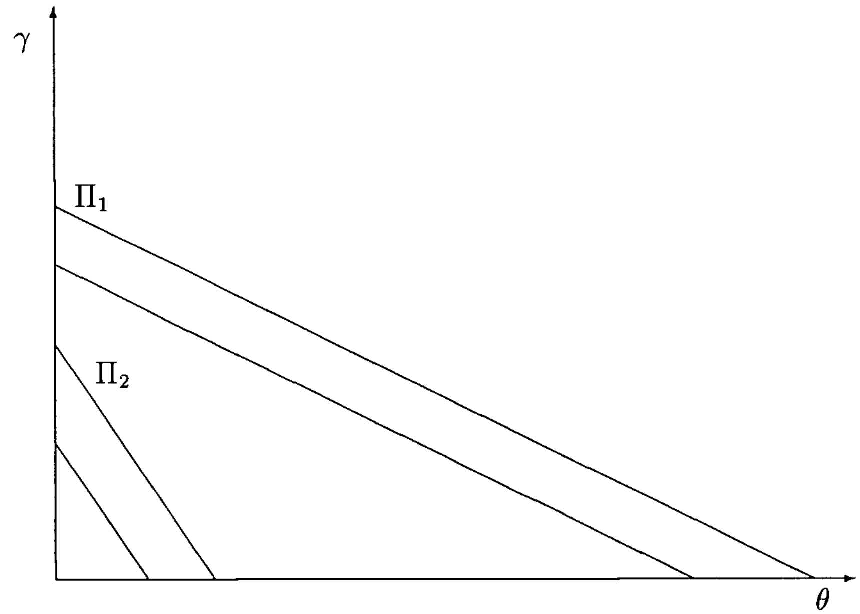

Figure 1.1: Iso–profit curves in θ–γ space.

Thus, each type 1 firm has a profit function[11]:

$$\Pi_1(\alpha,\theta,\gamma) = \begin{cases} \alpha + t_1\theta_{1A} + g_1\gamma_{1A} + b_{1A} & \text{in region A} \\ t_1\theta_{1B} + g_1\gamma_{1B} + b_{2B} & \text{in region B;} \end{cases} \tag{1.3.1}$$

while for type 2 firms:

$$\Pi_2(\beta,\theta,\gamma) = \begin{cases} t_2\theta_{2A} + g_2\gamma_{2A} + b_{1A} & \text{in region A} \\ \beta + t_2\theta_{2B} + g_2\gamma_{2B} + b_{2B} & \text{in region B.} \end{cases} \tag{1.3.2}$$

Thus, firms have simply to choose where to locate and which subsidy programme to accept when more than one is offered in a region.

Governments' objective functions are simple: each wants to maximize the benefits in terms of jobs of its industrial policy, net of the subsidy costs. The full problem

[11]Having chosen to represent the profit function as strictly separable in its α, θ, and γ components has serious implications. Separability between α and (θ, γ) means that from a revelation mechanism point of view, two different types – two different α's – cannot be distinguished from each other by any combination of subsidies, whereas this might be possible if the profit function had the form: $z(\alpha,\theta) + g_1\gamma$.

That the profit function is separable and linear in in–kind subsidies is of lesser consequence. It helps get sharp and easily interpretable results.

to be solved by a government, say government A, is as follows: it offers two subsidy triplets, one intended for each type of firm, $(\theta_{1A}, \gamma_{1A}, b_{1A})$ and $(\theta_{2A}, \gamma_{2A}, b_{2A})$, to maximize its benefits from having firms on its territory, net of subsidy costs, given self–selection constraints, and non–negativity constraints for subsidies. Remember that the self–selection constraints arise from the inability, or unwillingness, of governments to identify to which industry any specific firm belongs. Thus, government A solves the problem[12]:

$$\max_{\theta_{1A},\gamma_{1A},b_{1A},\theta_{2A},\gamma_{2A},b_{2A}}$$

$$\tilde{\mathcal{L}}_A = (1 - F(\hat{\alpha}))\,[L_1 - \theta_{1A} - \gamma_{1A} - b_{1A}] + NH(\hat{\beta})\,[L_2 - \theta_{2A} - \gamma_{2A} - b_{2A}]$$
$$+\lambda_1\,[\Pi_1(\theta_{1A}, \gamma_{1A}, b_{1A}) - \Pi_1(\theta_{2A}, \gamma_{2A}, b_{2A})] + \lambda_2\,[\Pi_2(\theta_{2A}, \gamma_{2A}, b_{2A}) - \Pi_2(\theta_{1A}, \gamma_{1A}, b_{1A})]$$
$$+\eta_1\theta_{1A} + \eta_2\gamma_{1A} + \eta_3 b_{1A} + \eta_4\theta_{2A} + \eta_5\gamma_{2A} + \eta_6 b_{2A}.$$

L_1 and L_2 represent respectively the monetary value associated with the employment provided by each type 1 and type 2 firm. The terms $1 - F(\hat{\alpha})$ and $H(\hat{\beta})$ are the shares of type 1 and type 2 industries found in equilibrium in region A, for $\hat{\alpha}$ and $\hat{\beta}$ defined by

$$\hat{\alpha} = t_1(\theta_{1B} - \theta_{1A}) + g_1(\gamma_{1B} - \gamma_{1A}) + b_{1B} - b_{1A};$$
$$\hat{\beta} = t_2(\theta_{2A} - \theta_{2B}) + g_2(\gamma_{2A} - \gamma_{2B}) + b_{2A} - b_{2B}.$$

These last two equations could be interpreted as participation constraints. That is, given the subsidies offered, industry 1 firms for which $\alpha < \hat{\alpha}$ have higher profits in region B than in region A, even taking into account their preference for region A, and are thus willing to accept region B's offer. The Lagrange multipliers λ_1 and λ_2 are associated with the incentive–compatibility constraints that firms from each industry must not be worse off by choosing the subsidy programme the government designed

[12]Here and throughout the paper, the problem is presented from government A's point of view.

for them. Finally, non–negativity constraints apply: cash and in–kind subsidies b, θ, and γ have to be greater than or equal to zero[13].

In general, the game played involves each government using an objective function such as the one above. A direct attempt to solve the problem all at once is complex and tedious. Instead, some smaller problems are solved to illustrate what the key issues involved are. The case when only cash subsidies are used is presented in the next section. It allows clear understanding of the mechanics of the problem. The use of in–kind subsidies is introduced later, in section 1.5. Finally, in section 1.7 some friction in the form of minimum bid differentials are introduced in the model.

1.4 Cash Competition

The problem to be solved when only cash subsidies are used by each region is relatively simple. Broadly speaking, a regional government can choose to overbid or underbid the other region. The value of doing so depends on the relative number of firms to be had, at the margin, as compared to all the infra–marginal firms that have to be bribed. Thus, it depends essentially on the parameter N. It is instructive to study cash competition between regions because it presents the logic of the model clearly, and constitutes the basis on which more complex problems are built in latter sections. Reaction functions for the two regional governments are constructed in the first section below. The next section presents the some propositions for this cash game. These propositions make clear the asymmetric nature of the problem at hand: the behaviour of a regional government depends on the relative size of its "home industry".

[13]The non–negativity constraints means that discriminatory taxation of firms is not possible.

1.4.1 Reaction Functions

To make the task easier, specific distribution functions for the degree of attachment firms have for a region are used from this point on.

Assumption 1.4 *The degrees of attachment for a region, α and β, have uniform distribution functions over the unit line segment* $[0-1]$.

Assumption 1.5 *The monetary value of the employment created by firms is the same across regions and industries, and is larger than one, $L > 1$.*

Assumption 1.4 brings the model close to that of Kanbur and Keen. Assumption 1.5 rules out asymmetric outcomes strictly due to different values of employment.

Then, the regional government may solve its industrial policy problem, when it pays cash subsidies,

$$V_A(b_A, b_B, N, L) = \max_{b_A} (1 - F(\hat{\alpha}))(L - b_A) + NH(\hat{\beta})(L - b_A),$$

given that

$$\hat{\alpha} \equiv b_B - b_A \geq 0, \qquad \hat{\beta} \equiv b_A - b_B \geq 0.$$

Taking as given the bid paid by region B, there are three possible regimes that region A may choose. It may choose to overbid region B and acquire some type 2 firms, that is to obtain $\hat{\beta} \geq 0, \hat{\alpha} = 0$; it may choose to preserve exactly the *laisser–faire* allocation of firms, $\hat{\beta} = 0, \hat{\alpha} = 0$; or it may choose to underbid region B and let go some of its type 1 firms, $\hat{\beta} = 0, \hat{\alpha} \geq 0$. This exhausts the possibilities. Which regime is preferred depends on the maximized value of the objective function.

For the case when $b_A \geq b_B$ the objective becomes:

$$\max_{b_A} V_A = (1 + N(b_A - b_B))(L - b_A). \tag{1.4.1}$$

From the first–order condition, one obtains the reaction function:

$$b_A = \frac{L - 1/N}{2} + \frac{b_B}{2}. \tag{1.4.2}$$

Similarly, for $b_A < b_B$, the objective is:

$$\max_{b_A} V_A = N(1 - (b_B - b_A))(L - b_A), \tag{1.4.3}$$

from which one obtains the reaction function

$$b_A = \frac{L - 1}{2} + \frac{b_B}{2}. \tag{1.4.4}$$

Simple computation using the value function, spelled out in appendix A.1, leads to the following form for the overall reaction function for region A:

$$b_A = \begin{cases} \frac{L-1/N}{2} + \frac{b_B}{2} & \text{for } b_B \leq L - 1/\sqrt{N} \\ \frac{L-1}{2} + \frac{b_B}{2} & \text{for } b_B \geq L - 1/\sqrt{N} \end{cases} \tag{1.4.5}$$

Similar computations, again to be found in appendix A.1, lead to the following reaction function for region B:

$$b_B = \begin{cases} \frac{L-N}{2} + \frac{b_A}{2} & \text{for } b_A \leq L - N \\ b_A & \text{for } L - N \leq b_A \leq L - \sqrt{N} \\ \frac{L-1}{2} + \frac{b_A}{2} & \text{for } b_A \geq L - \sqrt{N} \end{cases} \tag{1.4.6}$$

It is to be noted here that these reaction functions are in general discontinuous. The reason for this is quite simple. For low values of b_A, it is profitable for region B to bid away some firms from region A, even if it is costly to pay subsidies to all its infra–marginal firms. For larger values of b_A however, it becomes increasingly expensive to pay subsidies to infra–marginal firms; this leads to a switch at some point, when region B's government decides to underbid region A, letting go some of its own firms at the margin, rather than pay high subsidies to every asking firm.

The results for the cash competition model are presented in increasing order of complexity. First, the purely symmetric scenario is considered. Then some asymmetries, either in industry size or common factors of attachment to a region, are introduced.

1.4.2 Equilibrium Outcome to the Cash–only Competition

First, consider the basic case, when both industries are of the same size:

Proposition 1.1 *For perfectly symmetric industries, there exists a single Nash equilibrium for the cash subsidy game in which no firm relocates.*

Proof: Symmetric industries means that $N = 1$. Thus, one can look only at the problem of one region, knowing that the same obtains for the other region. From the possible values of the reaction function 1.4.5, it is easily seen that

$$b_A = \frac{L - 1 + b_B}{2}, \tag{1.4.7}$$

thus, one can solve for

$$b_A = \frac{L - 1 + b_B}{2}, \qquad b_B = \frac{L - 1 + b_A}{2}, \tag{1.4.8}$$

and obtain that $b_A^* = b_B^* = L - 1$. This implies that $V_A = V_B = 1 < L$. □

This proposition shows in a stark way the inherent inefficiency of this type of competition between regions. That is, the solution is efficient from a locational point of view, firms being located in the equilibrium outcome in the same region as they would be in the *laisser–faire* solution. It is inefficient from the narrow point of view of governments because they make transfers to firms without gaining anything, as it is also the case in the papers discussed above. The equilibrium outcome is thus clearly inferior to the cooperative one because, in the cooperative solution, both governments would get a surplus of L instead of 1. Figure 1.2 illustrates. The game is a prisoners' dilemma in its basic form because paying a subsidy is a dominant strategy for each government. But with both industries being perfectly symmetric, it is not possible for either government to gain an upper hand on the other one.

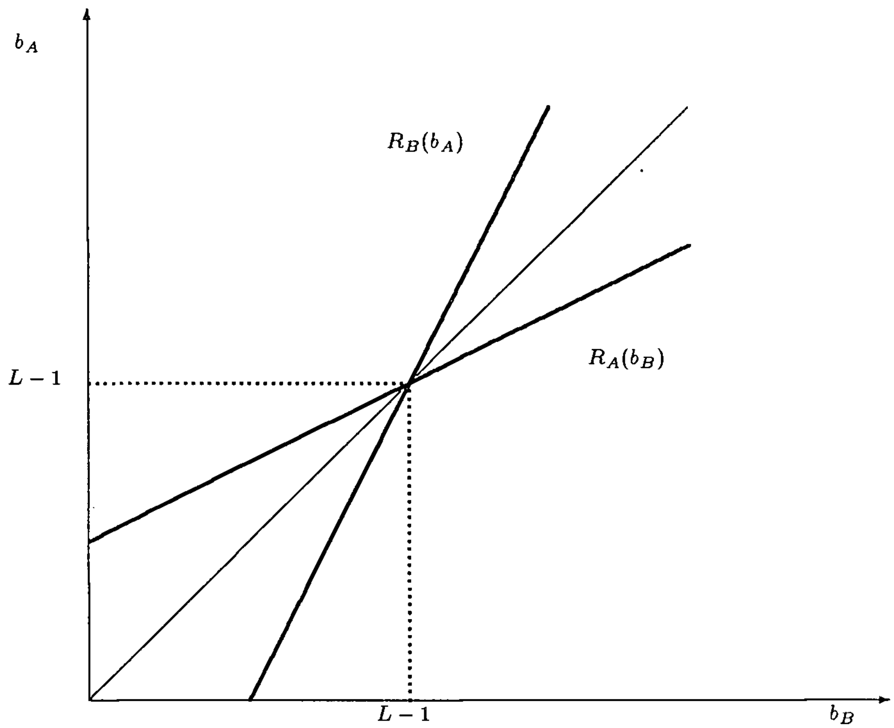

Figure 1.2: Equilibrium solution for perfectly symmetric industries. The location of firms remain identical to the *laisser–faire* solution. It is an illustration of the prisoners' dilemma, because equilibrium payoffs are equal to 1, instead of L in the *laisser–faire* solution.

It is likely to be the case in general that industries are of different sizes. Size difference can be very important for the equilibrium that obtains, as the following proposition shows:

Proposition 1.2 *For industry 2 larger than industry 1, $N > 1$, there exists a single Nash equilibrium to the cash subsidy game in which region A outbids region B and attracts firms of both types in equilibrium.*

Proof: See appendix A.2. □

Figure 1.3 illustrates this result. As a direct consequence of proposition 1.2, one obtains the following result.

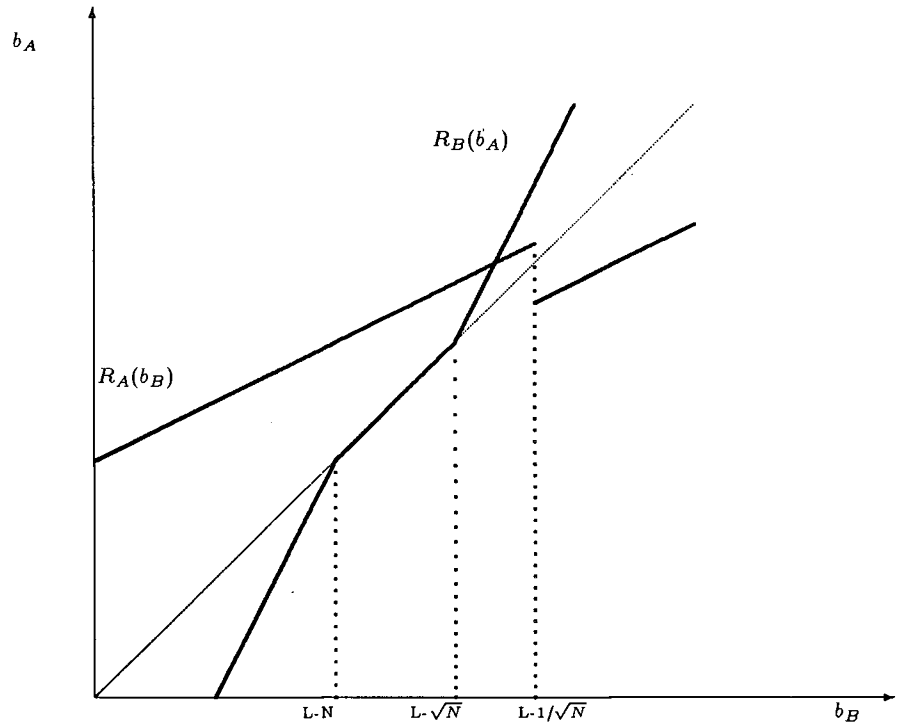

Figure 1.3: Equilibrium solution for industries of different sizes. The smaller region A takes some type 2 firms away from region B. The share of firms taken away depends on the degree of asymmetry between industries, N.

Proposition 1.3 *As the relative size of industry 2 increases, the proportion of type 2 firms going to region A increases at a decreasing rate in equilibrium.*

Proof: from the proof above, equilibrium bids b_A and b_B are such that

$$b_A - b_B = \frac{N-1}{3N}.$$

□

The last three propositions are related to propositions found in Kanbur and Keen [24][14]. They illustrate the asymmetry that exists in the behaviour of small and large regions. Large regions are unwilling to go aggressively after firms from a small region because

[14]There is no strict proposition–to–proposition parrallel that can be made between the two papers. Nevertheless, their propositions 2 and 3 are quite similar, in terms of qualitative outcomes, to propositions 1.1, 1.2, and 1.3.

the subsidies to be paid to a large number of infra–marginal firms are too costly. The problem can be thought of in terms of ratio: the small region faces a relatively more favorable ratio of extensive–margin to infra–margin firms, a ratio that increases as N increases. For the large region, changes in N has no impact on its ratio; the number of firms at its margin and infra–margin change in exactly the same proportion when N changes, because both the margin and infra–margin are made of type 2 firms.

The basic logic presented in this section carries through even if firms use more than just cash subsidies. It may help to explain the observed behaviour of small provinces in Canada that pursue aggressive industrial policy targeting entire industries, while a large province like Ontario refrains from doing so.

1.5 In–kind Subsidy Competition

The preceding section illustrated what happens when regions compete by using cash subsidies. In this section, the problem is studied when one of the two governments, that of region B, is constrained to using only cash subsidies, while region A's government may choose between cash and a mixture of cash and in–kind subsidies. This constraint imposed on region B is reasonable because the smaller region A is the one that benefits from being aggressive, as has been seen in the preceding section, and using in–kind subsidies is profitable only to the extent that it allows a region to discriminate against infra–marginal firms, which can be done, in the present context, only by the aggressive region.

Because of its complexity, the problem is decomposed into several steps. First, the ideal mixture of cash and in–kind subsidies is determined in the simplest case, when region B is assumed to be perfectly passive. This illustrates how the in–kind subsidies work, and is done in section 1.5.1. Second, section 1.5.2 presents a special case in which region B uses a cash subsidy, and for which a mixture of cash and in–kind subsidies is at least as good for region A as using only cash subsidies. This establishes the basis for more extensive analysis of the model, by showing that circumstances exist in which region A might rationally choose a mixture of cash and in–kind subsidies, rather than simply cash alone, in the face of a cash subsidy policy of region B. Next, in section 1.5.3, the equilibrium outcomes are characterized in a situation where region B uses a cash subsidy and region A is constrained to use a mixture of cash and in–kind subsidies. This allows a complete description of the various equilibrium configuration that would be possible if in–kind subsidies were used by region A. However, one would not expect to observe all these outcomes in practice, because region A might prefer to use cash only, rather than a mixture of cash and in–kind subsidies, for particular parameter configurations. In section 1.6 this issue is examined, by considering the choice by region A of either cash or a mixture of cash and in–kind policies.

1.5.1 Basic Set–up

Given the basic set–up of the model, one begins by asking how regional government A might select its policy instruments in the absence of any use of subsidy by government B. The next proposition establishes the simplest possible case in which using in–kind subsidies is profitable for region A[15].

Suppose that region A wants to acquire any arbitrary number of type 2 firms, $NH(\hat{\beta})$. The following proposition shows that region A never wants to make a transfer using γ–type subsidies.

Proposition 1.4 *Suppose that region B offers no subsidies. Then, the cost–minimizing way for region A to attract $NH(\hat{\beta})$ type 2 firms is to offer an in–kind subsidy $\theta_{2A} = \hat{\beta}/t_2$ to type 2 firms, and to maintain incentive–compatibility for type 1 firms by offering a cash subsidy $b_{1A} = t_1\hat{\beta}/t_2$.*

[15]The full problem to be solved by government A has been presented above in section 1.3.2.

Proof: See appendix A.3. □

Intuitively, the subsidy that is relatively least attractive to the type 1 firm is chosen by region A to attract type 2 firms, in order to minimize the payoff to type 1 firms that is needed to ensure incentive compatibility. If $t_1 > t_2$ then a θ–subsidy is absolutely more attractive to firm 1 than firm 2, and cash should be used to attract type 2's to A. If $t_2 \geq t_1$, then θ–subsidy should be given to firm 2. Firm 1 should be given cash, because that is the cheapest way to bring its profits to a point where the subsidy to firm 2 is incentive–compatible.

The logical second step for region A's government is to choose the optimal level of $\hat{\beta}$. Using the uniform distribution for $H(\hat{\beta})$, region A's government can optimize its share of industry 2 by maximizing:

$$\max_{\beta} \left[L - \frac{t_1 \beta}{t_2} \right] + N\beta \left[L - \frac{\beta}{t_2} \right]. \tag{1.5.1}$$

The first–order condition is:

$$\frac{-t_1}{t_2} - \frac{N\hat{\beta}}{t_2} + N \left[L - \frac{\beta}{t_2} \right] = 0; \tag{1.5.2}$$

this leads to an optimal share of industry 2:

$$\hat{\beta} = \frac{t_2 L - t_1/N}{2}. \tag{1.5.3}$$

The first two terms on the left–hand side of 1.5.2 represent the costs of increasing A's share of industry 2 in terms of having to pay larger transfers to infra–marginal industry 1 firms (the first term), and industry 2 firms that have been obtained with a slightly lower transfer (the second term). The third term represents the net gain in payoff for the marginal type 2 firm acquired. This can be compared to the solution that would obtain when using cash only:

$$\hat{\beta}_c = \frac{L - 1/N}{2}. \tag{1.5.4}$$

The difference between the two is

$$\hat{\beta} - \hat{\beta}_c = \frac{NL(t_2 - 1) + 1 - t_1}{2N}. \tag{1.5.5}$$

The numerator of 1.5.5 is used repeatedly later on in this essay. It shows how using in–kind subsidies may reduce the cost of having an industrial policy that seeks to affect the location of firms. The basic point is that firms at the infra–margin must also receive a benefit. In–kind subsidies then have a role: even though they are more costly than cash in attracting new firms at the extensive margin, they can reduce the total cost of the subsidy programme by reducing the expenditures that must be made at the infra–margin.

To keep the discussion as simple as possible, the following terminology is adopted. First, giving in–kind subsidies is an inefficient way to transfer money ($t_2 < 1$); the closer the t_2–parameter is to one, the more "transfer–effective" will the subsidy be said to be. Second, the difference between the values of t_1 and t_2 will be referred to as the "discriminating power" of using in–kind subsidies. When region B's government is passive, then ideal circumstances for the use of in–kind subsidies arise when in–kind subsidies are highly transfer–effective and have strong discriminating power[16]. In fact, the next proposition shows that using cash and in–kind subsidies can never be preferred to using cash–only subsidies if the former have no discriminating power, that is if firms of both industries value the in–kind subsidies the same.

Proposition 1.5 *Suppose that region B offers no subsidies. With firms of both industries valuing in–kind subsidies the same, region A always prefers to use cash–only subsidies over in–kind subsidies.*

Proof: see appendix A.4. □

[16]Remember that t_1, t_2, N and L are all considered parameters of the problem, not subject to choice by either government. However, to lighten the following discussion, the conditional tense is not used while enumerating all the different possible cases.

Thus, some discriminating power is a necessary condition for in–kind subsidies to be preferred to cash–only subsidies when region B is totally passive. In the extreme case of discriminating power, for $t_1 = 0$, the two industries are independent from each other as far as the in–kind subsidy industrial policy is concerned: there are no infra–marginal costs to be incurred, so using even relatively ineffective (low value of t_2) in–kind subsidy payments to get type 2 firms to establish their plants in region A can have a much higher payoff than using cash subsidies.

1.5.2 First Comparison Between Strategies

The previous section constrained region B to be passive. Henceforth that constraint is relaxed by allowing region B to compete with a cash subsidy. This section demonstrates that it can be more profitable to use in–kind rather than cash subsidies by describing a specific parameter set such that region A is not made worse off when it uses in–kind subsidies[17]. This tests that the analysis is worth pursuing. The problem is complicated by the presence of four parameters of unknown value, t_1, t_2, N, and L. To deal with this the argument takes on the following indirect structure: first, a general expression comparing cash and in–kind subsidy profitability is specified, and is then decomposed into terms that work in favour and against using in–kind subsidies. A proposition is offered for a restricted case, which in turns allows one to infer the range of possible solutions for the general case. The following two sections describe what happens more generally.

If region A uses in–kinds subsidies, its reaction function, when $t_2\theta_A \geq b_B$, in terms of the actual value of payments to type 2 firms[18], is:

$$t_2\theta_A = \frac{t_2L - t_1/N}{2} + \frac{b_B}{2}, \tag{1.5.6}$$

[17]Section 1.6 establishes in which situation using in–kind subsidies is strictly preferred.

[18]The cash payment to type 1 firms is assumed to satisfy $\bar{b}_A = t_1\bar{\theta}_A$, to maintain incentive–compatibility. This assumption is relaxed in the next section.

while if it uses cash, it is

$$b_A = \frac{L - 1/N}{2} + \frac{b_B}{2}. \tag{1.5.7}$$

It is easy to observe that, first, the slope of the two reaction functions is the same, and, second, that the relative position of the intercept depends on the sign of the expression $(1 - t_1) - (1 - t_2)NL$. If it is positive, then the reaction function with in–kind subsidy has a greater intercept.

Comparing the value of the objective functions for both strategies, one gets that the in–kind strategy is preferred if

$$[1 - F(\bar{\alpha})]\left[L - t_1\bar{\theta}_A\right] + NH(\bar{\theta}_A)\left[L - \bar{\theta}_A\right] \geq \left[L - \tilde{b}_A\right] + NH(\tilde{b}_A)\left[L - \tilde{b}_A\right].$$

The terms with a tilde $(\tilde{b}_A)$ refers to cash equilibrium, while the ones with the bar $(\bar{\theta}_A, \bar{\alpha})$ refer to the mixture of cash and in–kind subsidies equilibrium. The expression above can be rewritten as

$$\begin{aligned} (1 - t_1)\bar{\theta}_A + \left[1 + NH(\tilde{b}_A)\right]\left[\tilde{b}_A - \bar{\theta}_A\right] \\ +N\left[H(\bar{\theta}_A) - H(\tilde{b}_A)\right]\left[L - \bar{\theta}_A\right] - F(\bar{\alpha})\left[L - t_1\bar{\theta}_A\right] \ \geq 0. \end{aligned} \tag{1.5.8}$$

The first term represents the gain from discriminating against type 1 firms, whose payouts cost a fraction $(1 - t_1)$ less than to type 2 firms. This term is always positive, and is increasing in b_B.

The second term represents the cost differential of using in–kind instead of cash subsidies, for the proportion of firms obtained on the cash reaction function. The expression $\tilde{b}_A - \bar{\theta}_A = ((t_1 - t_2) + (t_2 - 1)Nb_B)/(2Nt_2)$ is always negative, by assumptions 1.1 and 1.2, so the entire second term is negative. Moreover, the entire second term is decreasing in b_B.

The third term represents the net gain by firms, obtained under the in–kind industrial policy, $(L - \bar{\theta}_A)$, multiplied by the difference in the number of type 2 firms obtained under the two policies, $H(\bar{\theta}_A) - H(\tilde{b}_A)$. This term has the same sign as $(1-t_1) - (1-t_2)NL$. $(L - \bar{\theta}_A)$ is decreasing in b_B, while $H(\bar{\theta}_A) - H(\tilde{b}_A)$ does not depend on b_B.

The fourth term represents the potential loss of type 1 firms associated with the lower incentive–compatible payments made to type 1 firms. $F(\bar{\alpha})$ is equal to zero, by definition, if $t_1\theta > b_B$. Otherwise, this term is decreasing b_B. This fourth term reveals an important weakness of the in-kind subsidy strategy. Being forced to make transfers to one's own firms, for incentive compatibility reasons, has the incidental benefit for region A that it increases the minimum payment region B must make before attracting the very first type 1 firm. If region A chooses to use the in–kind subsidy strategy, it leaves its own firms more vulnerable to being raided than they would be if it chose to use cash subsidies only; if region A chooses the cash subsidy, and it pays enough to acquire some type 2 firms, then it also pays enough to keep all the type 1 firms. Use of in–kind subsidies may allow a two–way flow of firms in equilibrium as will be seen below.

To recapitulate, if region A uses in–kind subsidies, it may or may not end up with more type 2 firms. The gains from using in–kind subsidies come strictly from the ability given to region A to discriminate against its own natural clients, type 1 firms, but may come at the cost of losing some of them to region B. The ability it has in discriminating may well push region A to bid consistently more for type 2 firms, but it has to be remembered that, in itself, giving in–kind subsidies θ_A is an ineffective way to transfer money.

If $(1-t_1) > (1-t_2)NL$, then the first and third terms of 1.5.8 are positive; the in-kind "reaction function" being higher for any value of b_B, there are two gains from using them: discriminating against infra–marginal firms, and obtaining a larger fraction of the type 2 firms. The only costs come from using an ineffective type of transfer.

Conversely, if $(1-t_1) < (1-t_2)NL$, using in–kind subsidies is less likely to be prefereable because only the first term of 1.5.8 is positive; using in–kind subsidies is detrimental both because it is more costly to subsidize type 2 firms, and as a consequence fewer of them would transfer from region B to region A.

In general, to complete the evaluation of 1.5.8 would require computation of the equilibrium strategies in the cash–only and the mixture of cash and in–kind subsidies. This is done in section 1.6. For the moment, the special case for which $(1-t_1) = (1-t_2)NL$ is examined. In that case, the cash only and cash and in–kind "reaction functions" overlap perfectly. The third term of 1.5.8 becomes equal to zero. Consider in addition, the case when region A obtains exactly the same mix of type 1 and type 2 firms with either of the potential strategies, for any given value of b_B; this occurs only if $(1-t_1) = (1-t_2)NL$ and $t_1\theta_A \geq b_B$. In this scenario, all benefits of the in–kind strategy are the result of discriminating against type 1 firms. The next proposition describes the set of possible values for t_1, t_2, N, and L such that:

- the allocation of firms across regions for the equilibrium in which region A uses in–kind (for type 2 firms) and cash (for type 1 firms) subsidies, and region B uses cash subsidies, would remain exactly the same as compared to the equilibrium where both regions use cash subsidies; and,

- region A would be no worse off by using a mixture of in–kind and cash subsidies, i.e., the value of its total payout of subsidies is not higher than in the strictly cash subsidy equilibrium.

Proposition 1.6 *Keeping the firms' allocation across regions constant under the two regimes, region A is not worse off in the cash and in–kind subsidy equilibrium, if t_1 and t_2 are contained in the set defined by:*

$$t_2 \geq 1 - \frac{(N-1)}{1+2N-5NL+NL(3NL-N)},$$

$$t_1 = 1 - (1 - t_2)NL.$$

Proof: See appendix A.5 . □

This proposition shows that the minimum values of t_1 and t_2 required for region A to prefer using in–kind subsidies depend strongly on the relative size of the two industries. It gives a more precise meaning to the vague expression "in–kind subsidies being an effective way to transfer money". On one hand, when the two industries are almost of the same size, $N \rightarrow 1$, a preference for in–kind subsidies requires that t_2 be really close to 1 because it is "pushed" from below by the twin requirements that $t_2 \geq t_1$, and $t_1\theta_A \geq b_B$. At the limit, for $N = 1$, the proposition is true only if $t_1 = t_2 = 1$. The concept of in–kind subsidies becomes empty in this case.

On the other hand, when the two industries diverge widely in size, the in–kind transfer parameter t_2 needs to converge toward one, to become more effective, because an ever increasing fraction of total payouts is made to type 2 firms, which receive money in a wasteful way. In the limit, as N becomes really large, incentive–compatible payouts to type 1 firms become relatively negligible, and the waste associated with in–kind transfers must decrease (i.e., t_2 increase) if region A is not to prefer paying cash to everyone. Remember that by proposition 1.3 the fraction of type 2 firms going to region A increases as N increases.

For intermediate values of N, there exists a set of values for t_1, t_2 that meet the requirements of proposition 1.6. For these values, in–kind subsidies reduce the cost of the industrial policy. In the best circumstances, when the in–kind subsidies have strong discriminating power, region A is able to reduce its payment to type 1 firms down to b_B.

Figures 1.4 and 1.5 illustrate the proposition. Figure 1.4 illustrates how the minimum value of t_2 respecting the first condition contained in proposition 1.6 changes as

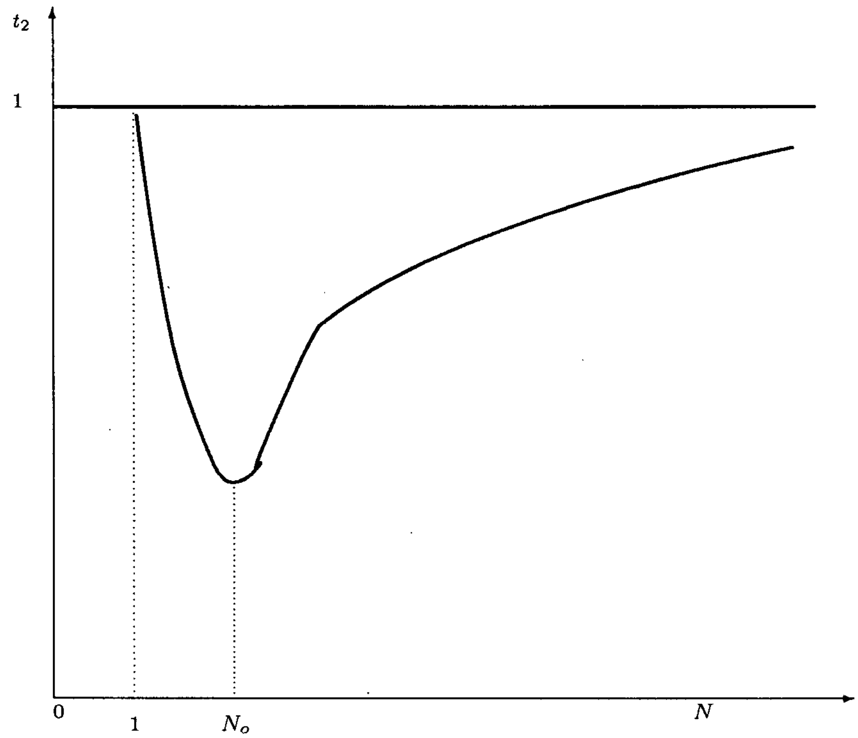

Figure 1.4: Minimum required value of the t_2–parameter as the asymmetry, N, between industries increases. This minimum value is high for large N because a large fraction of total transfers are made to infra–marginal type 2 firms in a transfer–ineffective way.

N increases. In figure 1.5, the bold line represents t_2 as a function of t_1 (given N and L), from the second condition contained in proposition 1.6; the dotted line represents t_2 as a function of t_1, from the equality contained in proposition 1.6. Values of (t_1, t_2) along the line segment a, b satisfy the conditions of the proposition. These two curves shift up as the relative size of industry 2 increases, if N is larger than value N_o that gives the minimum of the "J–curve" in figure 1.4.

1.5.3 Equilibrium When Region A Uses In–kind Subsidies

In this section the equilibrium outcomes are characterized when region A uses only a mixture of cash and in–kind subsidies; region B uses cash only. The restriction that region A use only a cash and in–kind subsidy mixture is relaxed in section 1.6, where

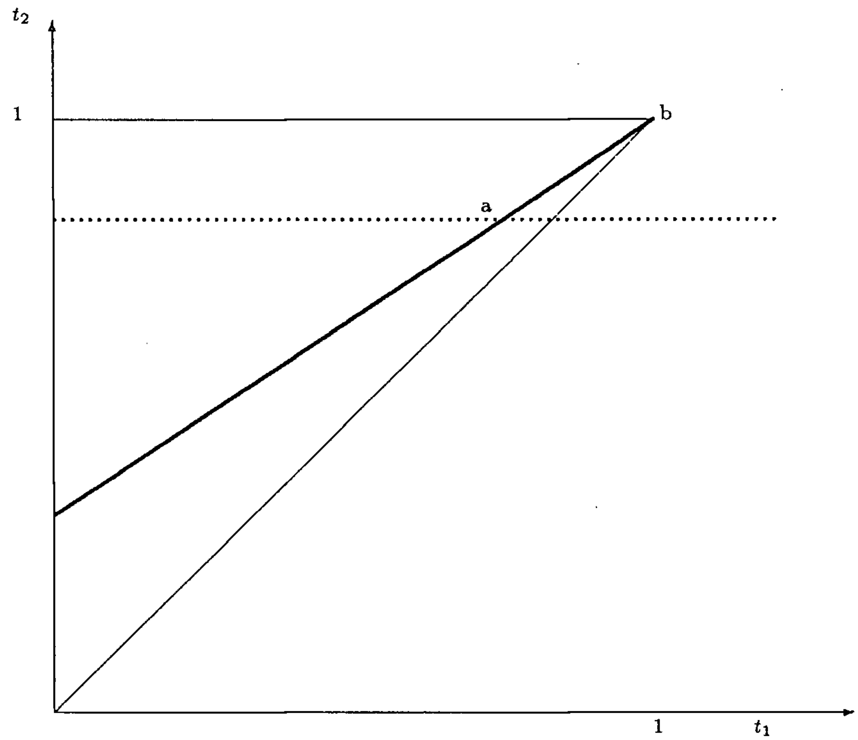

Figure 1.5: The set of (t_1, t_2) values for which proposition 1.6 applies is limited to the segment a,b. Both the bold and the dotted lines move up, as the asymmetry (N) between industries increases, provided that N is already larger than the minimum point of the "J–curve" shown in figure 1.4.

A is allowed to choose between the two strategies. The analysis of this section is of interest in its own right, and is a necessary preliminary ot that of section 1.6.

Equilibrium outcomes depend in important ways on the underlying values of the parameters t_1, t_2, L and N. Recall that, for any configuration of these values there is a well–defined solution to the pure cash game. In particular, for $N > 1$, region A overbids region B in equilibrium, and takes away a share of type 2 firms. What are the possible in–kind subsidy solutions? First, keeping N and L constant, for both parameters t_1, t_2 sufficiently large, region A may get to keep all type 1 firms, and obtain some type 2 firms. This resembles the solution obtained using cash subsidies. This first case, being closest to what has been seen before, is studied first in section 1.5.3. Second, for lower values of t_1, region A may get to take away some type 2 firms, at the cost of losing some type 1 firms to region B. Thus, it is possible to have solutions for which some firms of both industries are mislocated. If this feature occurs, the solution is said to have a cross–flow of firms: some firms from both industries are located other than they would have been in the *laisser–faire* solution. There are two sub–cases here. In one, region A offers type 1 firms no more than the strictly incentive–compatible cash payment. In the other, it offers type 1 firms strictly more than the incentive–compatible cash payment. The two cases are examined in turn in section 1.5.3. Finally, it can also be the case that an in–kind offer by region A is not sufficient to get any type 2 firms to locate there, and that it also loses some type 1 firms to region B. In this last case, there is no rationale for using in–kind subsidies, and the underlying cash subsidy solution is preferred. All these cases are tackled in turn, their union spanning all possible combinations of (t_1, t_2), for given values of N, L.

One–way Flow of Firms

Consider the first scenario in which region A gets to keep all type 1 firms and gets also some type 2 firms. This can occur in the following circumstances: the incentive compatible payment made to type 1 firms by region A must be no lower than region B's equilibrium payment, $b_A = t_1\theta_A \geq b_B$, and the payment made to type 2 firms must also be high enough to attract some of those firms, $t_2\theta_A \geq b_B$. The following proposition describes when this can happen.

Proposition 1.7 *Given the assumptions that the government of region A uses a mixture of cash and in–kind subsidies, and that region B restricts itself to using only cash subsidies; then, the government of region A keeps all type 1 firms, and gets a share* $[(t_2 - 1)NL + N - t_1]/3$ *of type 2 firms for a set of parameter values* t_1, t_2, N *and* L

described by the following inequality:

$$[(1+t_2)(t_1-t_2)+t_2(t_1-1)]\,NL+(t_2-t_1)(N+t_1)+(Nt_2-t_1t_1)>0;$$

Proof: see appendix A.6 . □

Under the circumstances described by this proposition, the allocation of firms resembles what obtains when cash subsidies are used. It can be noted that for the limit case for which both t_1 and t_2 parameters equal 1, then the share of type 2 firms obtained is $(N-1)/3$, exactly as when only cash subsidies are used.

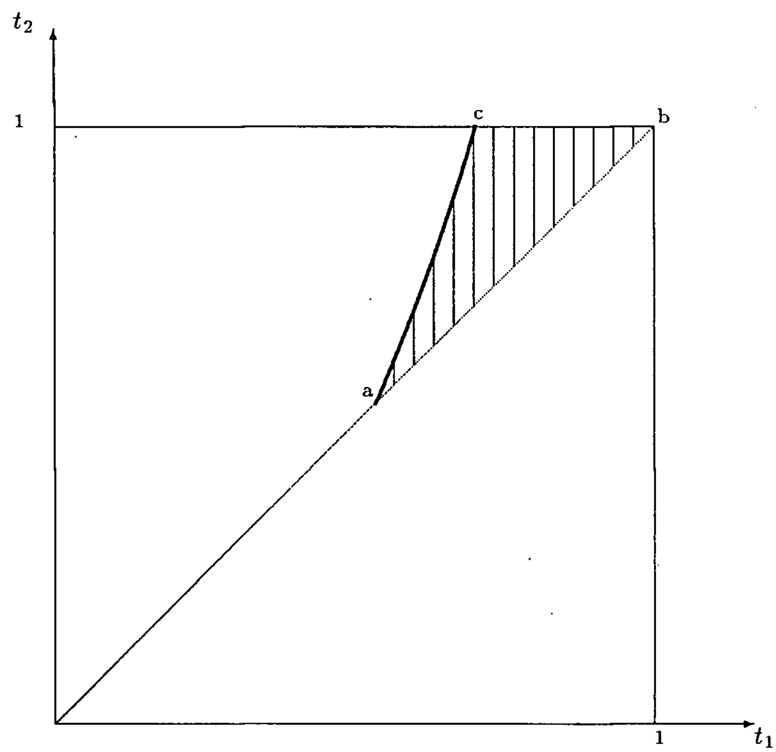

Figure 1.6: An illustration of proposition 1.7. Values of (t_1, t_2) such that $b_A = t_1\theta_A \geq b_B$ and $t_2\theta_A \geq b_B$ are represented by the triangle *a-b-c*. This triangle is delimited by the 45 degree line and the constraint derived from the inequality in proposition 1.7. The constraint *a-c* shifts toward the left as N increases, expanding the set of admissible values.

The inequality in the proposition determines the mimimum value of t_1 as a function of the other three parameters, t_2, N and L, such that the incentive–compatible payment made to type 1 firms is at least as high as region B's cash payment. Figure 1.6 illustrates the region for which the proposition holds[19]. The limit case $t_1 = t_2$, where the constraint above crosses the 45 degree line, moves toward the left as N increases. In the limit case, for $N = 1$, the constraint crosses the 45 degree line at $t_1 = t_2 = 1$. The proposition describes what is the minimum level of transfer–effectiveness that is required, by indicating the minimum value of t_1 for any value of t_2. Remember that by assumption, $t_2 \geq t_1$. Note also that in–kind subsidies need not have any discriminating power for this type of solution to exist. This is true because A is restricted to use in–kind subsidies. Anywhere along the 45 degree line in this set *a-b-c*, region A could replace its in–kind offer to type 2 firms by the cash offer targeted at type 1 firms and save $(1 - t_2)\theta_A$ per firm in the process. It is seen in section 1.6.1 that region A will in fact prefer cash only to a mixture of cash and in–kind subsidies when the discriminating power of the in–kind subsidies is very weak (t_1 close to t_2).

Cross–flow of Firms

Other type of firms allocations across regions are also possible. It may also be the case that firms from both industries are found in both regions — cross–flow of firms — if A's optimal offer of in–kind subsidies to type 2 firms is higher than region B's cash offer, and its cash offer to type 1 firms is lower than region B's. That is, $t_2\theta_A > b_B$ and $b_A < b_B$. There are two cases. Either A makes an incentive–compatible offer to type 1 firms that is just adequate, $b_A = t_1\theta_A$; or it offers more than incentive–compatibility warrants, $b_A > t_1\theta_A$.

To illustrate, consider the limit case when industry 1 does not value θ–type subsidies, $t_1 = 0$. Then, no cash offer need be made to type 1 firms to ensure incentive–compatibility. Yet, given that region B offers positive cash subsidies, it may be rational

[19]Most figures in this and the next sections have been drawn with $N = 2$ and $L = 1.5$ as underlying values.

for A to offer some positive cash payments to type 1 firms to prevent too many of them from moving to region B. Proposition 1.8 below describes this case and, more generally, the situation when region A wants to offer more than just incentive–compatible cash payments to type 1 firms. This proposition applies for relatively low values of t_1, given the other three parameters, t_2, N, L.

For intermediate values of t_1, in the sense of being in between the ones described implicitly in propositions 1.7 and 1.8, region A offers no more than incentive–compatible cash payments to type 1, $b_A = t_1\theta_A$, but that this amount is nevertheless lower than region B's optimal cash payment. This is found in the circumstances described by proposition 1.9.

The proper way to determine whether or not region A's government wants to offer type 1 firms more than strictly incentive–compatible cash payment is to solve a constrained maximization problem, which is done in appendix A.7. There, it is shown that the sign of the following expression is key to determining which solution is relevant, because it determines the value of the Lagrange multiplier associated with the incentive–compatibility problem for type 1 firms, $b_A \geq t_1\theta_A$:

$$\begin{aligned} &\frac{\left[3t_1(1+t_2)+2Nt_1(1+2t_2)-t_2(6+5N+Nt_2)\right]NL}{X_1} \\ &\qquad\qquad -\frac{N\left[t_1(1+2N)-t_2(4+5N)\right]}{X_1}, \end{aligned} \tag{1.5.9}$$

$$\text{for } X_1 = (4N+3)(t_1)^2 - Nt_1 + 4Nt_2 + 3N^2t_2 - Nt_1t_2. \tag{1.5.10}$$

If expression 1.5.9 is greater than zero, then $b_A = t_1\theta_A$ (proposition 1.8), while $b_A > t_1\theta_A$ can occur only if 1.5.9 is negative.

The next proposition seeks to show what occurs when the incentive–compatibility constraint is not binding and there is a cross–flow of firms. This means that in–kind subsidies offered to type 2 firms by region A are worth at least as much as the cash subsidies offered by region B, $t_2\theta_A > b_B$; that B's cash offer is as least as great as that offered by A to type 1 firms: $b_B > b_A$; and that the cash offered to type 1 firms by region A is both at least as high as the value these firms put on the in–kind subsidies offered to type 2 firms, and not greater than the value put on in–kind subsidies by type 2 firms: $t_2\theta_A \geq b_A \geq t_1\theta_A$.

Proposition 1.8 *Given the assumptions that the government of region A uses a mixture of cash and in–kind subsidies, and that region B restricts itself to using only cash subsidies; and provided that parameter values are such that the expression 1.5.9 is negative; then firms of both type 1 and type 2 are found in both regions if parameters* t_1, t_2, N *and* L *are within a subset described by the following inequality:*

$$t_2 \geq 1 - \frac{(1+2N}{(}3 + 2N)L.$$

Proof: See appendix A.7 . □

The constraint in the proposition statement guarantees that the offers made to firms by the two governments are such that some type 2 firms prefer region A's offer, $t_2\theta_A \geq b_B$, and that some type 1 firms prefer region B's cash offer to region A's cash offer, $b_B \geq t_1\theta_A$. It also guarantees that the in–kind subsidy offer made to type 2 firms is preferred by these firms to the cash payments targeted at type 1 firms, $t_2\theta_A \geq b_A$. This constraint is not a function of the t_1–parameter because region A offers at least as much to type 1 firms as the incentive–compatible payments, $b_A \geq t_1\theta_A$. If the t_2–parameter is lower than required by the proposition, region A is unable to attract any type 2 firms by using in–kind subsidies.

Figure 1.7 illustrates the proposition in (t_1, t_2) space. The constraint of the proposition statement is represented by the horizontal line. The second boundary is defined where expression 1.5.9 equals zero. This boundary rotates on the 45 degree line as industries become more asymmetric in size. Because region A offers more to type 2

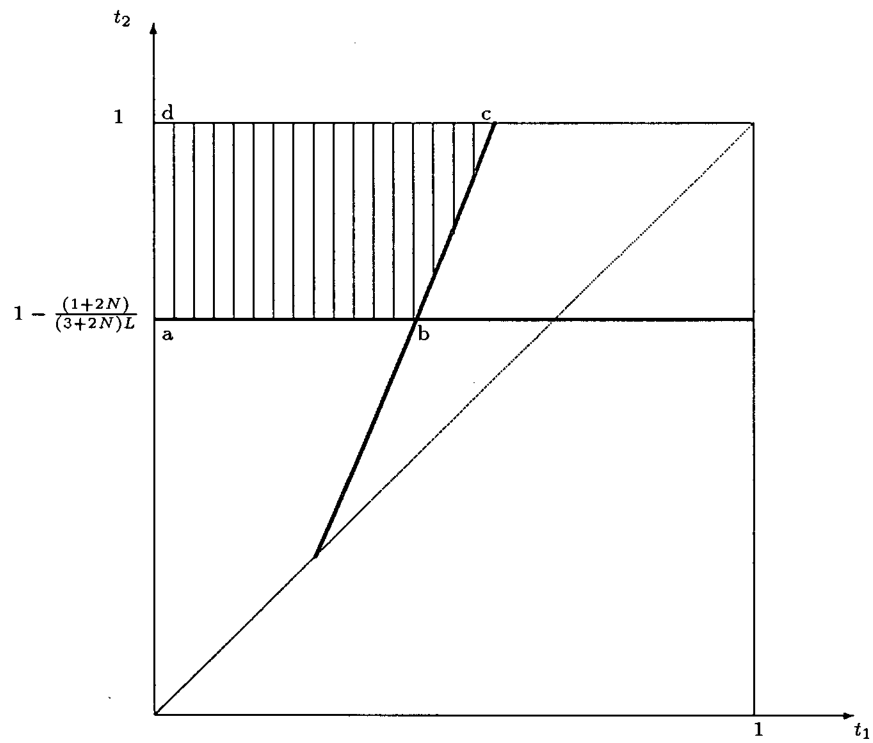

Figure 1.7: An illustration of proposition 1.8. The set of (t_1, t_2) values such that 1)$t_2\theta_A \geq b_B$, 2) $b_B \geq t_1\theta_A$, and 3) $b_A \geq t_1\theta_A$, is represented by *a-b-c-d*. It is contained between the horizontal line, for $t_2 = 1 - \frac{(1+2N)}{(3+2N)L}$, and $t_2 = 1$, and to the left of the pairs (t_1, t_2) such that 1.5.9 is equal to zero, line *b-c*. As N increases, the expression 1.5.9 rotates to the left, anchored on the 45 degree line.

firms as N increases, the set *a-b-c-d* of (t_1, t_2) values meeting the constraints of this proposition shrinks, as the boundary *b-c* moves leftward.

The next proposition resembles the last one but for the fact that the expression in 1.5.9 is positive. In this case, the government of region A finds it optimal to pay no more than the minimum incentive compatible payment to type 1 firms. This case differs from the one described by proposition 1.7 because type 1 firms are found in both regions, as region B offers larger cash subsidies to all firms than region A offers to type 1 firms.

Proposition 1.9 *Given the assumptions that the government of region A uses a mix-*

ture of cash and in-kind subsidies, and that region B restricts itself to using only cash subsidies; provided that parameters are such that expression 1.5.9 is positive; then, firms of both type 1 and type 2 are found in both regions when government A uses a mixture of cash and in-kind subsidies and government B uses only cash subsidies, if parameters t_1, t_2, N and L are within a subset described by the following inequalities:

$$t_2\left\{\frac{[N+3t_1+2Nt_2](1+N)L-[(N)^2+t_1(2+3N)]}{X_1}\right\}$$
$$-\left\{\frac{(N[2t_2(1+N)+2t_1(t_1+t_2)+N(t_2)^2]+3(t_1)^2)L}{X_1}\right.$$
$$\left.-\frac{[t_1(t_1+Nt_2)+2N[((t_1)^2+Nt_2)]}{X_1}\right\}\geq 0.$$

and

$$\frac{(N[2t_2(1+N)+2t_1(t_1+t_2)+N(t_2)^2]+3(t_1)^2)L}{X_1}$$
$$-\frac{[t_1(t_1+Nt_2)+2N[((t_1)^2+Nt_2)]}{X_1}$$
$$-t_1\left\{\frac{[N+3t_1+2Nt_2](1+N)L-[(N)^2+t_1(2+3N)]}{X_1}\right\}\geq 0$$

for X_1 defined by 1.5.10.

Proof: See appendix A.7 . □

The two inequalities in the proposition statement are hard to interpret in general. They represent the constraints $t_2\theta_A \geq b_B$ and $b_B \geq t_1\theta_A$, respectively. The expressions are somewhat messy because regions compete on both margins and the intensity of the competition on the type 1 firm margin depends on the value of t_1, which is not the case for proposition 1.8. Figure 1.8 illustrates the proposition. Note first that the constraint from 1.5.9 give the same boundary as in figure 1.7, and that it represents the minimum value of t_1 for any set of t_2, N, and L such that only incentive compatible payments are made to type 1 firms. The two other constraints represent the inequalities found in the proposition statement. They represent the limiting values of the subisidy offers $t_2\theta_A$, $t_1\theta_A$, and b_B such that firms flow both ways. Quite obviously, these two

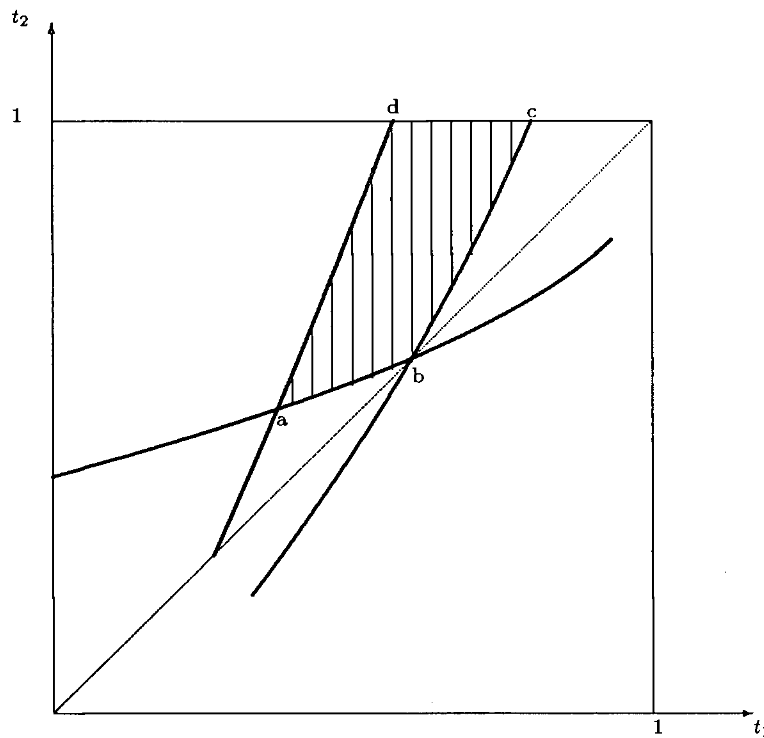

Figure 1.8: An illustration of proposition 1.9. The set of (t_1, t_2) values such that 1)$t_2\theta_A \geq b_B$, 2) $b_B \geq t_1\theta_A$, and 3) $b_A = t_1\theta_A$, is represented by *a-b-c-d*. The set is bounded by on the left by expression 1.5.9, line *a-d*; on the right, line *b-c*, by the constraint that $b_B \geq t_1\theta_A$; and below, line *a-b*, by the constraint $t_2\theta_A \geq b_B$, as presented in the statement of proposition 1.9.

constraints cross at the 45 degree line, where $t_1 = t_2$. The flatter of the two constraints is $t_2\theta_A \geq b_B$ and is the relevant one for relatively low values of t_2, while the steeper constraint matters for higher values of t_2. This is sensible because the higher t_2 is, the higher would be the in–kind subsidies offered by region A to type 2 firms. As the asymmetry between industries increases, the two constraints move down and to the left, keeping a common pivotal point on the 45 degree line. Again, this comes from the fact that region A bids more aggressively as the asymmetry increases. Thus, the set *a-b-c-d* of admissible values (t_1, t_2), inside the three constraints, such that firms of both types are found in both regions expands as N increases.

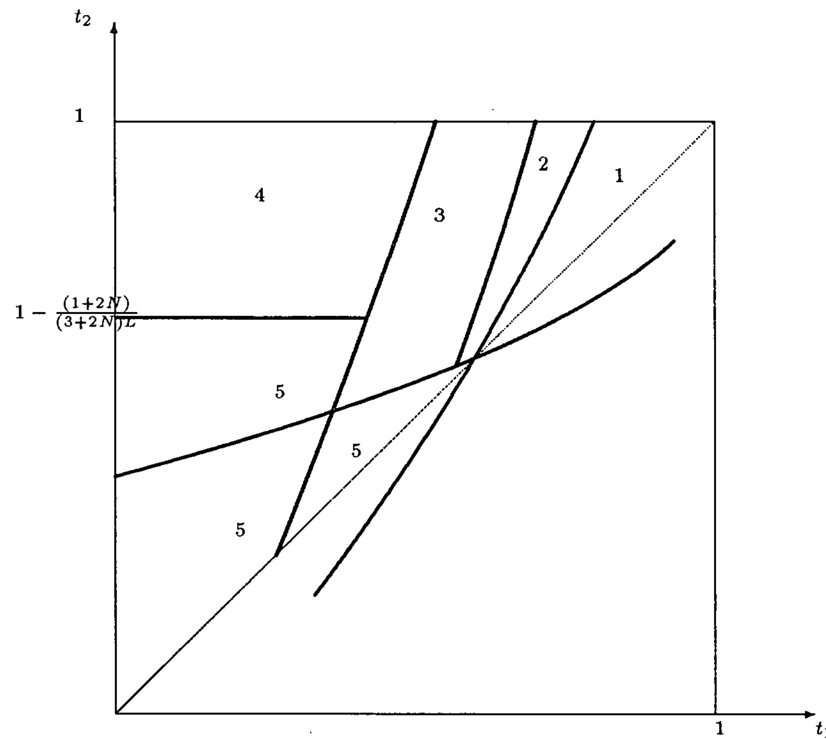

Figure 1.9: The set $1 \geq t_2 \geq t_1 \geq 0$ can be partitioned in five regions, each of which is associated with a specific type of in–kind subsidy solution, except for region 5 where no such solution exists.

Note that this set may overlap with the set described in proposition 1.7. Figure 1.9 illustrates the last three propositions simultaneously. The space can be divided into several regions. In region 1, proposition 1.7 applies and region A gets firms of both types in equilibrium, while region B obtains no type 1 firms. In region 2, there are multiple equilibria, with the outcomes of propositions 1.7 and 1.9 being possible. In region 3 only proposition 1.9 applies. In region 4, proposition 1.8 applies. What about region 5? There, government A does not offer high enough in–kind subsidies to get any type 2 firms, as proposition 1.8 shows. Thus, the use of in–kind subsidies is of no benefit to region A, and for this subset it unequivocally prefers to use only cash subsidies.

1.6 Cash versus In–kind Equilibrium Comparison

Section 1.5 has shown how firms of both industries are allocated across regions when a mixture of cash and in–kind subsidies is used by region A's government. It is also known that underlying any point in the t_1, t_2, N and L space there exists an equilibrium solution to the cash subsidy game studied in section 1.4. Thus, it is important to develop the insight of propostion 1.6 by showing that using a mixture of cash and in–kind subsidies is more than just a theoretical possibility and that it is preferable for region A to using cash–only subsidies for some subset of the parameter space. The present section shows that there exists such subsets of parameter values. Thus, it demonstrates that it is an equilibrium outcome to use wasteful ways to subsidize marginal firms and get a higher payoff because a menu of subsidy programmes allows region A to discriminate against infra–marginal firms.

Two different notions of equilibrium are explored in this section. Which one is relevant depends on how much flexibility governments might have in the timing of the announcement of the parameters of their industrial policy. It has been said in the introduction that the industrial policy examined here constitutes a case of "legislated competition". This means that regional governments introduce bills of law to declare how much and what types of payments are going to be made to firms. If regional government A announces simultaneously the rate and the type of subsidy payments to be made, then one must think in terms of a non–commitment equilibrium. The question to be answered by government A is: given an exact payment b_B by region B, is it preferable to offer the optimal cash/in–kind subsidy pair, $\{b_A^{ik}(b_B), \theta_A(b_B)\}$, or the optimal cash–only payment, $b_A^c(b_B)$? It is a non–commitment equilibrium because region A can change its policy instrument of choice, cash versus a mixed cash/in–kind subsidies, as easily as it can change its subsidy rates. This first type of equilibria corresponds to the usual notion of a Nash equilibrium. Below, it is shown that this type of equilibria can be found for subsets of sets described by propositions 1.7, 1.8 and 1.9.

It is also possible to imagine that for a variety of administrative and political reasons the government would find it desirable to commit first to using either a cash–only or a mixed cash/in–kind policy, and then entering into a bidding game. In this two–stage scenario the government of region A announces in stage one whether mixed cash/in–kind or cash–only subsidies are going to be used, and engages in the bidding game with region B at stage two. This second type of equilibrium can be thought as a Subgame Perfect Nash Equilibrium (SPNE). The comparison here requires the computation of equilibrium payofffs that would arise when region A uses either mixed cash/in–kind or simply cash subsidies, and then compare the respective payoffs to decide which equilibrium region A prefers. It is shown below that for a subset of the set described by proposition 1.7, using mixed cash/in–kind subsidies is an optimal strategy in the subgame perfect sense.

1.6.1 Simple Nash Equilibrium

The comparison required to show that mixed cash/in–kind subsidies are preferred in the usual Nash sense is quite simple. Begin with the equilibrium strategies when region A uses mixed cash/in–kind subsidies; these are described in section 1.5.3. Then, given this equilibrium value of region B's cash payment, compute what A's best cash–only responses would be. If A's payoff from the best cash/in–kind strategy is larger than from the best cash–only strategy, then the cash/in–kind equilibrium of section 1.5.3 remains an equilibrum in this broader game. Otherwise it does not. In section 1.4, it has been seen that, when $N > 1$, region A's cash reaction function is:

$$b_A^c = \frac{NL - 1 + Nb_B}{2N}, \tag{1.6.1}$$

while the surplus function is:

$$V_A^c = (L - b_A^c)(1 + N(b_A^c - b_B)). \tag{1.6.2}$$

Substitute into these two expressions the relevant value of b_B from the cash/in–kind equilibrium, and compare region A's payoff for the cash competition conditional on this value of b_B with A's payoff for the cash/in–kind competition at b_B. This exercise tests whether, at an in–kind equilibrium, region A would prefer to switch to cash payments, away from the cash/in–kind equilibrium.

One–way Flow of Firms

The case described by proposition 1.7 is the most interesting to explore first because it constitutes the simplest possible departure from proposition 1.6, which shows that region A can get at least as high a payoff when it uses in–kind subsidies. It is also in these circumstances that the strongest case can be made for using in–kind subsidies, because, as has been seen, region A benefits from discriminating against infra–marginal firms without losing any of them to region B. Also, it is sensible to start by studying this case because in the limit, when $t_1 = t_2 = 1$, the solution becomes perfectly equivalent to paying only cash subsidies. The following proposition obtains:

Proposition 1.10 *Assume that the condition underlying proposition 1.7 holds; then, using in–kind subsidies constitutes a simple Nash equilibrium in pure strategies if the following inequality is satisfied:*

$$\begin{aligned} &\frac{\left[2t_2(1-t_1) + (t_2 - t_1)\right] NL + t_1(2t_1 + N)}{3Nt_2} \\ &+ \left[\frac{(t_2-1)NL + N - t_1}{3}\right]\left[\frac{(t_2-1)NL + N + 2t_1}{3Nt_2}\right] \\ &\qquad > \frac{1}{N}\left[\frac{(1-t_2)NL + t_1 + 3 + 2N}{6}\right]^2. \end{aligned}$$

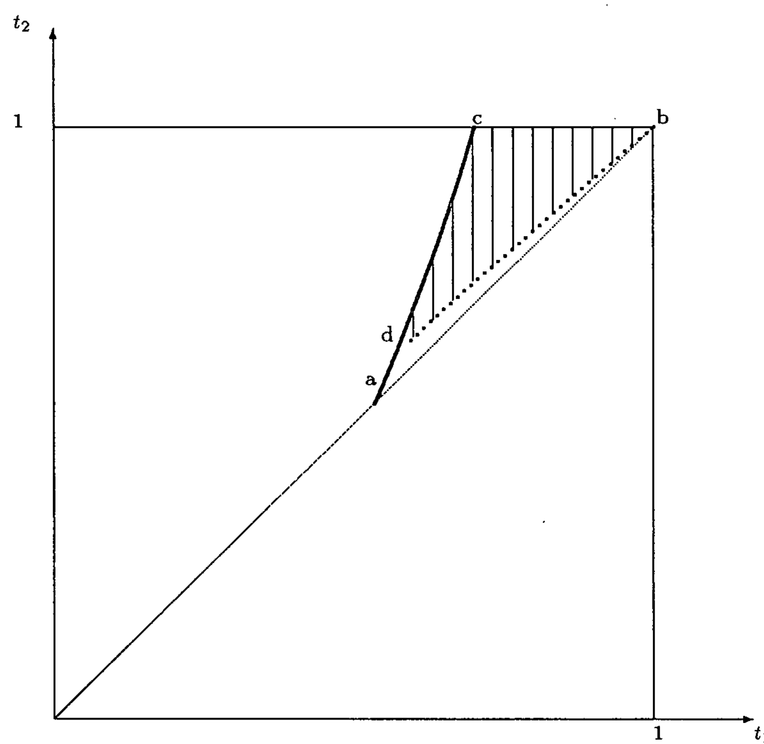

Figure 1.10: An illustration of a Nash equilibrium with one–way flow of firms. The set of (t_1, t_2) values for which proposition 1.7 holds, *a-b-c*, has been reduced to a smaller set *d-b-c*. A minimum level of discriminating power is required to make the mixed cash/in–kind subsidy solution constitute an equilibrium when A also has the possiblity of using cash–only subsidies. This added constraint is represented by the dotted line.

Proof: see appendix A.8. □

The left–hand side of the inequality contained in the proposition is obtained by substituting into the surplus function the equilibrium values of $\{b_A^{ik}(b_B), \theta_A(b_B)\}$ and b_B, while the right–hand side is the surplus function that obtains for the cash subsidy game at the same level of b_B. Figure 1.10 illustrates the proposition. The dotted curve above the 45 degree line shows that a minimum level of discriminating power must exist to ensure that mixed cash/in–kind subsidies remain in place. That is, the cost of in–kind subsidies comes from using an ineffective form of transfer at the extensive margin, and this ineffectiveness must be compensated by large enough savings to A at the infra–margin; this is achieved if low enough subsidies can be paid to type 1 firms, which requires a minimum level of discrimination.

The proposition shows that mixed cash/in–kind subsidies are preferable to cash–only subsidies for some subset of parameter values when firms flow only in one direction. In that case, the allocation of firms across regions does not change drastically from the mixed cash/in–kind strategy to the cash–only strategy. This is trivially true for type 1 firms, since they all remain in region A in both cases. The share of type 2 firms obtained by region A is $[(t_2 - 1)NL + N - t_1]/3$ when using the mixed cash/in–kind subsidies, as compared to $(N-1)/3$ with cash-only subsidies. Therefore, for region A the only trade–off to be made is between the difference in the fraction of (marginal) type 2 firms obtained versus the savings made at the infra–margin.

Cross–flow of Firms

In the next two cases, this is not true anymore. Firms flow both ways in the cases described by propositions 1.8 and 1.9. In the underlying cash equilibrium, region A captures some type 2 firms and it keeps all type 1 firms. Thus, firms flow in one direction only. Nevertheless, it is possible to prefer to use the mixture of cash and in–kind subsidies in a Nash equilibrium, as the next proposition shows.

Proposition 1.11 *Assume that the conditions underlying proposition 1.8 hold; then, using mixed cash/in-kind subsidies constitutes a Nash equilibrium if the following inequality is satisfied:*

$$\left[\frac{(1-t_2)NL+4+5N}{6(1+N)}\right]^2 + \frac{N}{t_2}\left[\frac{[(3+2N)(t_2-1)]\,L+1+2N}{6(1+N)}\right]^2$$
$$> \left[\frac{N(1-t_2)L}{6(1+N)} + \frac{3+4N+2N^2}{6N(1+N)}\right]\left[\frac{N^2(1-t_2)L}{6(1+N)} + \frac{(N-1)2N+3(1+2N)}{6(1+N)}\right].$$

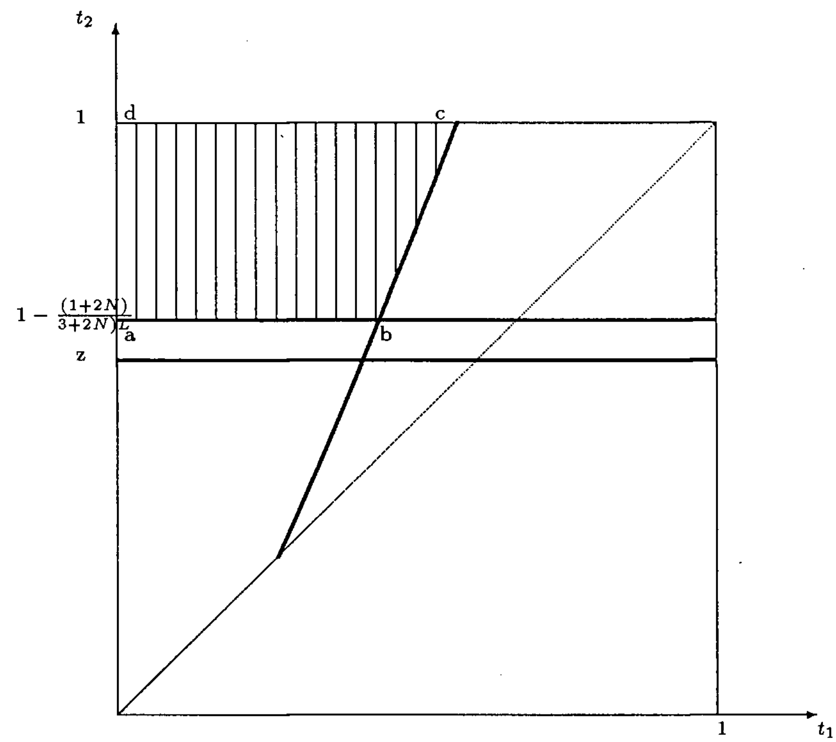

Figure 1.11: One type of Nash equilibrium with cross–flow of firms (See Figure 1.7 for comparison). The inequality contained in the statement of proposition 1.11 is represented by the horizontal line labeled z. This constraint moves up (and may become binding) as the asymmetry between industries, N, increases.

Proof: see appendix A.9. □

Figure 1.11 illustrates the result. It differs from figure 1.7 only because a second horizontal line has been added, to represent the inequality found in the proposition statement. Only when in–kind subsidies are sufficiently effective (t_2 sufficiently high) can they be preferred to cash subsidies.

Finally, in the last case described in section 1.5.3, it is also possible to prefer a mixed cash/in–kind subsidies at the Nash equilibrium if the payments made to type 1 firms are sufficiently high.

Proposition 1.12 *Assume that the conditions underlying proposition 1.9 hold; then, using mixed cash/in–kind subsidies constitutes a Nash equilibrium if the following in-*

equality is satisfied:

$$
\begin{aligned}
&(L - t_1\theta_A) + N(t_2\theta_A - b_B)(L - \theta_A) > (L - b_A^c)\left[1 + N(b_A^c - b_B)\right],\\
&\textit{for} \quad \theta_A = \frac{[N + 3t_1 + 2Nt_2]\,(1+N)L - [(N)^2 + t_1(2+3N)]}{X_1};\\
&b_B = \frac{\left(N\left[2t_2(1+N) + 2t_1(t_1+t_2) + N(t_2)^2\right] + 3(t_1)^2\right)L}{X_1}\\
&\qquad - \frac{[t_1(t_1+2N) + 2N\left[(t_1)^2 + Nt_2\right]]}{X_1};\\
&b_A^c = \frac{NL - 1 + Nb_B}{2N},
\end{aligned}
$$

and for $X_1 = (4N+3)(t_1)^2 - Nt_1 + 4Nt_2 + 3N^2t_2 - Nt_1t_2$.

Proof: see appendix A.10. □

Figure 1.12 illustrates the proposition[20]. In this case, it is shown that the subset of admissible values shrinks, and that both a high level of transfer effectiveness and discriminating power are required before mixed cash/in–kind subsidies are preferred to cash–only subsidies. Having a high level of discriminating power guarantees that savings made at the infra–margin are high enough, even though it also means that some type 1 firms are lost.

Sources of Savings and Existence of Equilibrium

The last three propositions have established that there exist some parameter values such that using mixed cash/in–kind subsidies instead of using cash–only subsidies constitutes a Nash equilibrium. Conversely, these propositions have also shown that for some other subsets, in–kind subsidies are not part of equilibrium. This does not automatically mean that cash subsidies dominate when in–kind subsidies do not. The next two propositions explore in more detail the underlying structure of Nash equilibrium

[20]The underlying parameter values are $N = 3, L = 1.5$. Most other graphs have $N = 2, L = 1.5$ as parameter values; for these values, the graph would have been hard to read because the equilibrium subset remains essentially unchanged, as compared to the one illustrating proposition 1.9.

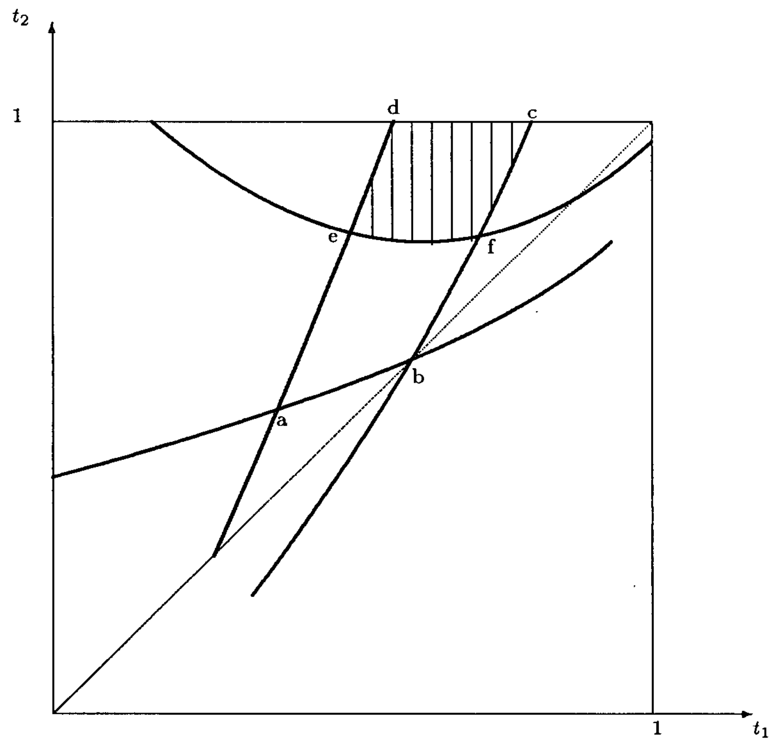

Figure 1.12: The second type of Nash equilibrium with cross–flow of firms. The set of (t_1, t_2) values has shrunk from *a-b-c-d* to *e-f-c-d.* Thus, high levels of transfer–effectiveness and discriminating power are required for mixed cash/in–kind subsidies to be preferred to cash–only subsidies within this region of the parameter space.

when firms flow in one direction only, as in proposition 1.10. Proposition 1.13 makes it clear that government A benefits from using in–kind subsidies only because this allows it to discriminate against infra–marginal firms. Next, proposition 1.14 shows that there exist some circumstances for which no Nash equilibrium in pure strategies can be found.

Logically, both propositions are closely related to proposition 1.6, because they both use the same decomposition of the cash versus in–kind solutions.

To generalize proposition 1.6, consider a set of parameter values $\bar{t}_1, \bar{t}_2, \bar{N}$ and $\bar{L}$

respecting its second condition: $\bar{t}_1 = 1 - (1 - \bar{t}_2)\bar{N}\bar{L}$. Define Δ as

$$\Delta \equiv \frac{((1 - \bar{t}_2)\bar{N}\bar{L} + t_1 - 1)}{2}. \tag{1.6.3}$$

This parameter Δ represents half the increment in the share of type 2 firms that region A gets by using cash instead of in–kind subsidies. Its value is equal to zero for $\bar{t}_1, \bar{t}_2, \bar{N}$ and $\bar{L}$, and it increases for t_1 above $\bar{t}_1$. Using Δ, inequality 1.5.8, which represents the difference in payoffs between the mixed cash/in–kind strategy and the cash–only strategy, may be rewritten as[21]:

$$(1 - t_1)\theta^*_A + \left[1 + \bar{N}(\frac{\bar{N} - 1 + \Delta}{3\bar{N}})\right]\left[(\bar{t}_2 - 1)\theta^*_A + \frac{\Delta}{\bar{N}}\right] \\ -\bar{N}\frac{\Delta}{\bar{N}}\left[\bar{L} - \theta^*_A\right] \geq 0. \tag{1.6.4}$$

From the expression above, it appears straightforward to obtain answers to the following two questions: first, is it ever possible to choose to pay in–kind subsidies so as to transfer a lesser amount of money than is transferred in the cash solution, to type 2 firms? Second, does taking into account the possibility of using either only cash or a mixture of cash and in–kind subsidies create difficulties in terms of obtaining a Nash equilibrium in pure strategies? The following two propositions answer these questions.

Proposition 1.13 *At a Nash equilibrium involving mixed cash/in–kind subsidies, it can never be true that region A pays type 2 firms the same amount or less in in–kind subsidies, than it would pay if it were to use a cash–only strategy given region B's equilibrium value bid, b_B.*

Proof: See appendix A.11. □

The proof proceeds by contradiction. The contradiction shows that it is impossible to pay the same amount in in–kind subsidy to type 2 firms as one would have paid

[21]Note that the fourth term of 1.5.8 is equal to zero for the one–way flow case examined here.

in the equivalent (same b_B) cash strategy and still prefer the in–kind subsidy strategy. That is, the neighborhood around the parameter set defined by proposition 1.6 is not large enough to include an equilibrium such that government A pays a lesser amount both to marginal and infra–marginal firms. This shows that the in–kind subsidy strategy generates benefits only from the fact that it allows discrimination against infra–marginal firms.

In similar fashion, and using proposition 1.13, it can be shown that Nash equilibrium in pure strategies may not exist in some circumstances.

Proposition 1.14 *Consider a set of parameter values satisfying proposition 1.6; there exist values $\bar{t}_1 < t_1 < \bar{t}_2$, such that Nash equilibrium in pure strategies does not exist. For t_1 high enough, a sufficient condition for no PSNE to obtain is : $L \geq 2(\frac{1}{3} + \frac{2}{3N})$.*

Proof: See section A.12 . □

To understand intuitively this result, consider the following: if $\Delta = 0$, then the intercepts of the mixed cash/in–kind subsidy "reaction function" and the cash–only "reaction function" are the same, and both have an identical slope. Thus, the equilibrium share of type 2 firms obtained by region A would be the same in both cases. Yet, region A has to decide whether it prefers to use mixed cash/in–kind subsidies or cash–only subsidies for any given value b_B. Using in–kind subsidies involves making a trade–off between savings made at the infra–margin (i.e., because of discrimination against type 1 firms) and waste at the extensive margin (using transfer–ineffective in–kind subsidies for the share of industry 2 obtained). The relative size of these two groups changes when different bids by region B are considered, and so does the attractiveness of this trade–off. Given that region A offers more to type 1 firms than region B, its share of type 1 firms is equal to 1. Region A's share of type 2 firms is $N(t_2\theta_A - b_B)$ when it uses mixed cash/in–kind subsidies or $N(b_A - b_B)$ when it

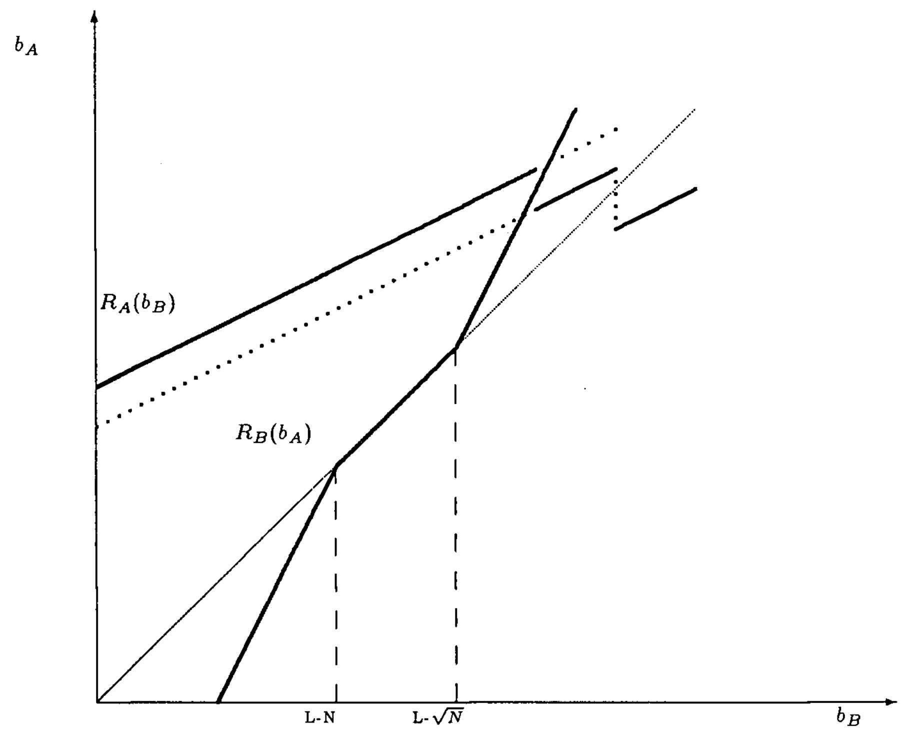

Figure 1.13: An illustration of proposition 1.14 showing parameter values such that no Nash equilibrium in pure strategies exist. Region A's reaction function, $R_A(b_B)$, is pieced from segments of the mixed cash/in–kind subsidies "reaction function" (the one with the lower intercept) and the cash–only "reaction function"; it is represented by the discontinuous bold line, while region B's reaction function, $R_B(b_A)$, is continous.

uses cash–only subsidies (here, A's share of type 2 firms would be the same, given the $\Delta = 0$ assumption). Close to the intercept, when b_B is low, the relative share of type 2 firms is large and the cost of using transfer–ineffective in–kind subsidies is high. Thus, cash–only subsidies would be used for low values of b_B. Closer to the 45 degree line, for b_B large and close in value to region A's bid, the relative share of type 2 firms is low and savings made at the infra–margin become relatively more important. Thus, mixed cash/in–kind subsidies would be used for larger values of b_B. Region A's reaction function is *not* discontinuous, if $\Delta = 0$, but region A does switch from one subsidy regime to the other.

Figure 1.13 illustrates proposition 1.14. If $\Delta > 0$, for $t_1 > \bar{t}_1$ (see equation 1.6.3), then the mixed "reaction function" has a lower intercept than the cash "reaction function". It is labeled $R_A(b_B)$, and is represented by the bold segments. The dotted lines represent segments of the respective pure cash and pure in-kind "reaction functions" that are not part of the true reaction function. Thus, the switch from the cash-only subsidy regime to the mixed cash/in-kind subsidy regime, as b_B increases, creates a gap through which region B's reaction function can sneak in, leading to the absence of a Nash equilibrium in pure strategies. On the other hand, it has to be the case that the value of t_1 satisfying the proposition is smaller than $\bar{t}_2$, $t_1 < \bar{t}_2$, because if the mixed cash/in-kind subsidy regime had no discriminating power, then the cash-only subsidy regime would be preferred for any value of region B's bid, b_B, as proposition 1.5 shows.

1.6.2 Subgame Perfect Nash Equilibrium

As noted, there exists a second type of equilibrium comparison that can be made, where there is the possibility of commitment. The type of subsidy regime to be used is announced first, and the subsidy rates are announced in a second stage. The thought experiment involves solving the problem backward. First, the equilibrium payoff associated with mixed cash/in-kind subsidies is computed, as in section 1.5.3, and then it is compared with the cash-only subsidy payoff found in section 1.4 for the same set of t_1, t_2, N and L parameter values. Given this computation, region A's government announces which type of subsidy regime it wants to offer firms, choosing the type associated with the highest equilibrium payoff. Quite clearly, the thought process just described corresponds to the usual notion of a Subgame Perfect Nash Equilibrium. Also, as a consequence, there is always a pure strategy equilibrium once the commitment is made, so non-existence problem of the type found in proposition 1.14 do not occur here.

Only for the case when region A keeps all its type 1 firms and gains some type 2 firms is using mixed cash/in–kind subsidies shown to be more profitable than using cash–only subsidies in a subgame perfect sense. Thus, in the case illustrated by proposition 1.7, it can be shown that for a range of possible N values using mixed cash/in–kind subsidies is highly profitable. This is illustrated by the next proposition. It is difficult to show such a preference for mixed cash/in–kind subsidies when firms flow in both directions, in the circumstances described by propositions 1.8 and 1.9. Some intuition as to why this is the case is offered below.

Proposition 1.15 *Assume that the condition underlying proposition 1.7 holds; then, using mixed cash/in–kind subsidies constitutes a Subgame Perfect Nash Equilibrium if the following inequality is satisfied:*

$$\frac{[2t_2(1-t_1)+(t_2-t_1)]NL+t_1(2t_1+N)}{3Nt_2}$$
$$+\left[\frac{(t_2-1)NL+N-t_1}{3}\right]\left[\frac{(t_2-1)NL+N+2t_1}{3Nt_2}\right]\geq\frac{(N+2)^2}{9N}.$$

Proof: see appendix A.13. □

Figure 1.14 illustrates this proposition. The curve *b-d* represents the upperbound of the set of values for (t_1, t_2) such that using cash–only subsidies is preferable. The curve *a-c* in this figure represents the same boundary condition found in figure 1.6. From the set *a-b-c* of parameter values (t_1, t_2) that support cash/in–kind equilibria (with a one–way flow of firms) the subgame perfect requirement deletes the subset of values on or below the *b-d* curve. As N increases, the curve *b-d* moves up and gradually becomes quasi–horizontal, while the curve *a-c* moves continuously toward the left. For very large values of N, when the two industries are of drastically different sizes, it is only when t_2 tends toward one that in–kind subsidies remain preferable to cash subsidies.

As mentioned, it is difficult to show that mixed cash/in–kind subsidies are preferred in a subgame perfect sense when firms flow in both directions. No formal proof of this

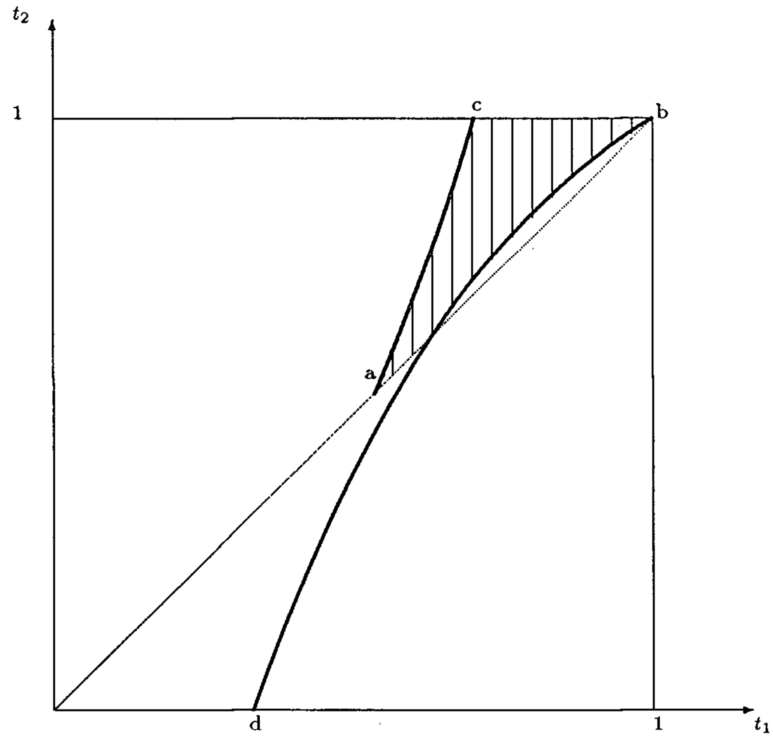

Figure 1.14: A Subgame Perfect Nash Equilibrium exists for the subset a-b-c of (t_1, t_2) values, as in Figure 1.6, *less* the subset on or below the line d-b. Values of (t_1, t_2) below the curve b-d are not admissible anymore, for in this case paying cash–only subsidies is preferred in a subgame perfect sense.

is offered. Extensive numerical experimentation shows that, for values of N and L that easily satisfy proposition 1.15, cash–only is preferred to mixed cash/in–kind subsidies in equilibria where there is a cross–flow of firms. The cross flow solutions described in section 1.5.3 are characterized by a much higher value of equilibrium in–kind bids, compared to the cash bids found in the cash–only solutions. This can be explained by the fact that regions are fighting over two margins simultaneously in the cross–flow solutions. Thus, region B benefits on two fronts from marginal increases in its cash subsidy offer: it allows it both to keep a larger fraction of its own type 2 firms, and to steal a higher fraction of the type 1 firms. Thus, fighting over two margins makes for more aggressive bidding. This fact is somewhat obscured in section 1.6.1. There, mixed cash/in–kind or cash–only subsidies were compared for the mixed cash/in–kind equilibrium value of b_B. Implicitly then, the comparison made was mostly in terms of the overall savings obtained from discriminating subsidy offers to firms, when using in–kind subsidies, versus the benefits of keeping all type 1 firms, in the cash subsidy solution. In a subgame perfect comparison, however, a clear added benefit of cash subsidies is that it leads to less aggressive bidding.

1.6.3 Best Reply for Region B

Throughout the preceding sections it has been assumed that region B's government used only cash subsidies. Quite obviously, this is not an assumption that can be maintained without cost. However, solving the firm allocation problem across regions becomes very complex without it. Establishing the superiority of using mixed cash/in–kind subsidies over cash subsidies for either government, in a simple Nash or a subgame perfect Nash sense, would also be difficult.

However, there is something to be said when firms flow one way, as in the case described by proposition 1.7. In that case, region A uses in–kind subsidies and is able to keep all of its own type 1 firms, while taking away some type 2 firms. The government of region B might contemplate using a strategy similar to government A's, using cash to pay off its own type 2 firms, and using γ–type subsidies targeted at type 1 firms. In as much as the following inequalities are respected, $b_A = t_1\theta_A > g_1\gamma_B$, and $g_2\gamma_B > b_B$, nothing would change and having region B offering a menu of subsidies is of no consequence.

Thus, to preserve qualitatively the results of proposition 1.7, and the associated proposition 1.10 and proposition 1.15, all that is required is to replace the constraint $t_1\theta_A > b_B$ by a stronger version, $b_A = t_1\theta_A > (g_1/g_2)b_B > b_B$, restricting somewhat the sets of subsets of t_1, t_2, N and L for which these propositions apply. In the cross–flow of firms cases, the problem has to be solved starting from first principles, which cannot be done with any degree of generality.

1.7 Discrete Differences in Bids and Equilibrium

Sections 1.4 and 1.5 described how regional governments compete in the case where overbidding the other region by an epsilon is sufficient to ensure that the very first firm from the other industry transfers to one's own region. In general, this is unlikely to be true. It is more likely the case that a minimum discrete differential is needed before the very first firm from one industry will change location. In the model, this possibility is represented by parameters δ_i, $i = 1, 2$. The impact of introducing these parameters is presented in this section. These δ_i parameters are a simple way to represent a common element of the preference displayed by firms of one industry for one region. One possible interpretation is to say that they represent the quality of the industry–specific infrastructure in place within the region. If this interpretation is used, it allows assessment of the impact of other types of industrial policy on the "firm stealing" industrial policy studied in this essay. However, in what follows these δ_i parameters are taken as strictly exogenous.

This added complication has important implications. In particular, introducing this type of friction means that the Nash equilibrium in pure strategies found in some of the cases discussed above no longer exist.

First, the problem is studied in the following section in some detail for the case when regions compete using only cash subsidies. Then, a sketch of what may happen with positive δ_i is presented for the restricted case when region A uses a mixture of cash and in–kind subsidies and region B is constrained to using cash.

1.7.1 Cash Subsidy Competition

The interest here is twofold. First, it shows that some results obtained in section 1.4 are fragile. In this way, it may help explain the apparent instability of regional governments' industrial policy. If PSNE do not exist, and yet the industrial policy must be introduced through legislation, it is no wonder that it changes often and the results are found to be not too satisfying by politicians. Also, as has been mentioned before, the cash only subsidy game bears a formal resemblance to Kanbur and Keen's [24] problem. Thus, the propositions presented here generalise what is found in their paper, and show that the problem is more complex than they consider[22].

Before presenting these propositions, the reaction functions for the two regions must be constructed again. For the cash subsidy game, the government of region A has the same objective function to maximize as before. The only difference is that the parameters $\hat{\alpha}$ and $\hat{\beta}$ have to be redefined as follows:

$$\hat{\alpha} \equiv b_B - b_A - \delta_1 \geq 0, \qquad \hat{\beta} \equiv b_A - b_B - \delta_2 \geq 0, \tag{1.7.1}$$

In terms of actual bidding strategy, it means that three things are possible: a government can overbid its opponent by a sufficient amount to attract firms; it can underbid its opponent by an amount sufficient to lose some of its own firms; or it can bid an amount sufficiently close to the opponent's bid such that no firms change location. It means that one of the three following situations is possible:

$$b_A \geq b_B + \delta_2, \quad b_A \leq b_B - \delta_1, \quad b_B + \delta_2 \geq b_A \geq b_B - \delta_1.$$

Consider the first case, $\hat{\beta} \geq 0, \hat{\alpha} = 0$; region A chooses to overbid or match region B,

[22] In the context of their commodity tax–competition, positive δ parameters, necessarily equal to each other, can be thought as the equivalent of "thick borders": while there is essentially no border between France and Luxembourg, there is an important one between France and the United Kingdom because one must cross the Channel to go to the other country.

$b_A \geq b_B + \delta_2$. Thus, the objective function to maximize is

$$V_A(b_A, b_B, \delta_1, \delta_2, N, L) = \max_{b_A} \left[1 + NH(\hat{\beta})\right](L - b_A), \tag{1.7.2}$$

and has first–order condition:

$$-1 - NH(\hat{\beta}) + Nh(\hat{\beta})(L - b_A) = 0, \tag{1.7.3}$$

which gives a reaction function

$$b_A = \frac{L - 1/N}{2} + \frac{b_B + \delta_2}{2}. \tag{1.7.4}$$

This function is consistent with inequality $b_A \geq b_B + \delta_2$ for any value $b_B \leq L - 1/N - \delta_2$. For $b_B > L - 1/N - \delta_2$, region A chooses $b_A = b_B + \delta_2$, to ensure that $\hat{\alpha} = 0$.

Substituting the value of b_A found above, equation 1.7.4, in the objective function 1.7.2, one obtains:

$$V_A = \frac{[NL + 1 - N(b_B + \delta_2)]^2}{4N}.$$

Thus, for $b_B \leq L - 1/N - \delta_2$, region A chooses 1.7.4, with payoff V_A. For $b_B > L - 1/N - \delta_2$, it chooses $b_A = b_B + \delta_2$ and has payoff $V_A = L - (b_B + \delta_2)$.

When region A responds to B's bid by inducing the outcome $\hat{\beta} = 0, \hat{\alpha} = 0$, then no firm moves. The problem to maximize becomes $\max_{b_A}(L - b_A)$, which is achieved by giving the lowest possible payment, $b_A = b_B - \delta_1$, for $b_B \geq \delta_1$ and $b_A = 0$, otherwise.

Finally, to induce $\hat{\alpha} \geq 0, \hat{\beta} = 0$, Region A maximizes the objective function $(1 - F(\hat{\alpha}))\,(L - b_A)$, which gives a reaction function

$$b_A = \begin{cases} \frac{L-1}{2} + \frac{b_B - \delta_1}{2} & \text{for } b_B \geq L - 1 + \delta_1 \\ b_B - \delta_1 & \text{for } b_B \leq L - 1 + \delta_1 \\ 0 & \text{for } b_B \leq \delta_1. \end{cases}$$

The value of the objective function may be found by substitution.

The introduction of the δ_i parameters makes the problem somewhat less transparent. While conceptually simple, it is no longer the case that one is able to obtain a clear reaction function for a region as was possible before. That is, it is no longer possible to do what is done in appendix A.1 to obtain expressions similar to 1.4.5 and 1.4.6. Instead, determining which segment of the "reaction function" is relevant has to be done on a case by case basis, given specific parameter values.

Collecting all the segments together gives the actual value of b_A, the first column, over certain ranges of region B's bid (in the second column), and according to which regime is followed (in the third column)

$$b_A = \begin{cases} \frac{L-1/N}{2} + \frac{b_B+\delta_2}{2} & \text{for } b_B \leq L - 1/N - \delta_2 \quad \text{and } b_A \geq b_B + \delta_2 \\ b_B + \delta_2 & \text{for } b_B \geq L - 1/N - \delta_2 \quad \text{and } b_A \geq b_B + \delta_2 \\ 0 & \text{for } b_B \leq \delta_1 \quad \text{and } b_B + \delta_2 \geq b_A \geq b_B - \delta_1 \\ b_B - \delta_1 & \text{for } b_B \geq \delta_1 \quad \text{and } b_B + \delta_2 \geq b_A \geq b_B - \delta_1 \\ b_B - \delta_1 & \text{for } b_B \leq L - 1 + \delta_1 \quad \text{and } b_A \leq b_B - \delta_1 \\ \frac{L-1}{2} + \frac{b_B-\delta_1}{2} & \text{for } b_B \geq L - 1 + \delta_1 \quad \text{and } b_A \leq b_B - \delta_1 \end{cases} \tag{1.7.5}$$

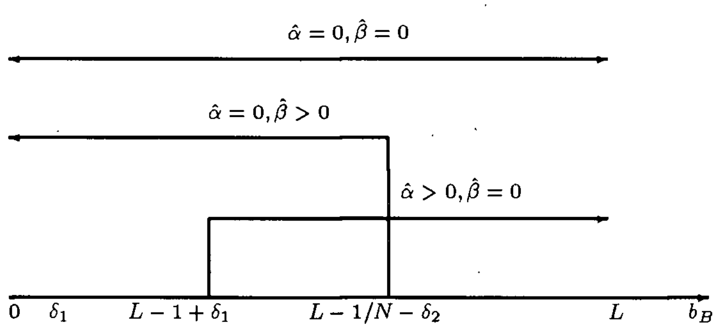

Figure 1.15: Choice of regimes as a function of b_B.

In general, these segments overlap. To illustrate a potential outcome, consider the case for which $L-1+\delta_1 < L-1/N-\delta_2$, which occurs provided that $(N-1)/N > \delta_1+\delta_2$. If one considers the "strong version" of the three regimes (with strict inequalities), then figure 1.15 illustrates what may obtain. For b_B between zero and $L - 1 + \delta_1$, region A may choose between regime ($\hat{\alpha} = 0, \hat{\beta} = 0$) and regime ($\hat{\alpha} = 0, \hat{\beta} > 0$), which are the two uppermost lines on figure 1.15. For b_B between $L - 1 + \delta_1$, and $L - 1/N - \delta_2$, the choice is between the three regimes. For b_B between $L - 1/N - \delta_2$ and L, the choice is between regimes ($\hat{\alpha} = 0, \hat{\beta} = 0$) and ($\hat{\alpha} > 0, \hat{\beta} = 0$), which are represented by the uppermost and lowermost lines on figure 1.15.

In other words, A always has the option of matching region B's bid. For relatively low values of b_B, it may be profitable to steal away some type 2 firms, while for large values of b_B, letting go some type 1 firms might be the preferred option. In the middle, anything is possible.

Which segments are actually relevant is determined by examining the value of the objective function, $V_A(b_A, b_A, \delta_1, \delta_2, N, L)$, for the different ranges of b_B and under the three regimes

$$V_A = \begin{cases} \frac{[NL+1-N(b_B+\delta_2)]^2}{4N} & \text{for } b_B \leq L - 1/N - \delta_2 & \text{and } b_A \geq b_B + \delta_2 \\ L - b_B - \delta_2 & \text{for } b_B \geq L - 1/N - \delta_2 & \text{and } b_A \geq b_B + \delta_2 \\ L - 0 & \text{for } b_B \leq \delta_1 & \text{and } b_B + \delta_2 \geq b_A \geq b_B - \delta_1 \\ L - b_B + \delta_1 & \text{for } b_B \geq \delta_1 & \text{and } b_B + \delta_2 \geq b_A \geq b_B - \delta_1 \\ L - b_B + \delta_1 & \text{for } b_B \leq L - 1 + \delta_1 & \text{and } b_A \leq b_B - \delta_1 \\ \left[\frac{L+1}{2} - \frac{b_B - \delta_1}{2}\right]^2 & \text{for } b_B \geq L - 1 + \delta_1 & \text{and } b_A \leq b_B - \delta_1. \end{cases} \tag{1.7.6}$$

For region B, the problem is similar, and so are the results:

$$b_B = \begin{cases} b_A - \delta_2 & \text{for } b_A \leq L - 1 + \delta_2 & \text{and } b_B \leq b_A - \delta_2 \\ \frac{L-1}{2} + \frac{b_A - \delta_2}{2} & \text{for } b_A \geq L - 1 + \delta_2 & \text{and } b_B \leq b_A - \delta_2 \\ 0 & \text{for } b_A \leq \delta_2 & \text{and } b_A + \delta_1 \geq b_B \geq b_A - \delta_2 \\ b_A - \delta_2 & \text{for } b_A \geq \delta_2 & \text{and } b_A + \delta_1 \geq b_B \geq b_A - \delta_2 \\ \frac{L-N}{2} + \frac{b_A + \delta_1}{2} & \text{for } b_A \leq L - N - \delta_1 & \text{and } b_B \geq b_A + \delta_1 \\ b_A + \delta_1 & \text{for } b_A \geq L - N - \delta_1 & \text{and } b_B \geq b_A + \delta_1; \end{cases} \tag{1.7.7}$$

$$V_B = \begin{cases} N\left[L - (b_A - \delta_2)\right] & \text{for } b_A \leq L - 1 + \delta_2 & \text{and } b_B \leq b_A - \delta_2 \\ N\left[\frac{L+1}{2} - \frac{(b_A + \delta_1)}{2}\right]^2 & \text{for } b_A \geq L - 1 + \delta_2 & \text{and } b_B \leq b_A - \delta_2 \\ N\left[L - 0\right] & \text{for } b_A \leq \delta_2 & \text{and } b_A + \delta_1 \geq b_B \geq b_A - \delta_2 \\ N\left[L - b_A + \delta_2\right] & \text{for } b_A \geq \delta_2 & \text{and } b_A + \delta_1 \geq b_B \geq b_A - \delta_2 \\ \left[\frac{L+N}{2} - \frac{(b_A + \delta_1)}{2}\right]^2 & \text{for } b_A \leq L - N - \delta_1 & \text{and } b_B \geq b_A + \delta_1 \\ N\left[L - b_A - \delta_1\right] & \text{for } b_A \geq L - N - \delta_1 & \text{and } b_B \geq b_A + \delta_1. \end{cases} \tag{1.7.8}$$

Now, using the "reaction functions" constructed above, it is possible to present some propositions.

First, consider the case for which industries are of the same size, $N = 1$, but governments now have to pay a discrete amount before getting the very first firm to move from one region to another. This changes the nature of the solution in a drastic way. The next proposition shows that, in general, no Nash Equilibrium exists in pure strategies.

Proposition 1.16 *For industries of the same size, and with at least one positive δ_i–parameter, in general no Nash equilibrium exist in pure strategies.*

Proof: See appendix A.14 . □

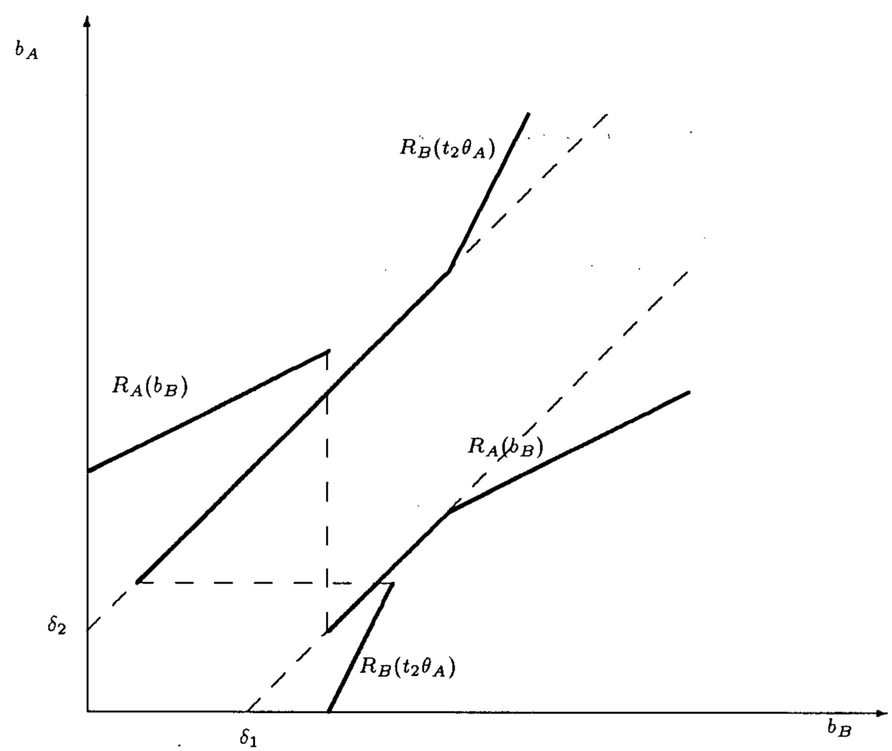

Figure 1.16: Positive common factors of attachment prevent a Nash equilibrium in pure strategies from existing by creating discontinuities in the reaction functions.

Figure 1.16 illustrates proposition 1.16. The intuition for the result (why the reaction functions do not cross) is clear. The only potential solutions involve having no firms moving from one region to another, but one region paying more than the other, the difference being made by the discrete amount needed to get the very first firm to relocate. However, if a region is unable to outbid its opponent by enough (by δ_i) to get any firms to transfer, minimizing the cost of matching the opponent's offer becomes the only consideration. Thus, for the regime $\hat{\alpha} = 0, \hat{\beta} = 0$, paying $b_A = b_B - \delta_1$ is strictly preferred to paying $b_B + \delta_2$, if $\delta_1 + \delta_2 > 0$. This result also implies that the reaction function for region A jumps down for some value of $b_B < L - 1 - \delta_2$, because at $b_B = L-1-\delta_2$ — which corresponds to the potential intersection point between the upper part of the reaction function and the upper diagonal, on figure 1.16 — region A is strictly better off on the lower branch of the reaction function. Region B's reaction function jumps from right to left before the level at which $b_B = b_A - \delta_1$.

This result is quite general. From 1.7.5, regime $(\hat{\beta} > 0, \hat{\alpha} = 0)$ is not desirable for region A for any value of b_B if $\delta_2 > L - 1$. Similarly for region B, from 1.7.7, for $\delta_1 > L - 1$, the regime $(\hat{\beta} = 0, \hat{\alpha} > 0)$ is never attractive. Thus, a Nash equilibrium in pure strategies may exist only for the trivial case in which neither region may wants to overbid the other, in any circumstances.

The absence of a Nash equilibrium in pure strategies does not preclude the existence of a mixed strategy Nash equilibrium. Given the institutional setting of the problem, it is hard to imagine such a mixed strategy; instead, it would lead to governments being constantly unsatisfied with their industrial policy and changing them often. However, the question is not pursued here.

To complete the description of the main properties of the cash subsidy game between region A and region B, consider the next proposition:

Proposition 1.17 *For regions of asymmetric size, and with at least one positive δ_i parameter, region A has firms of both types in equilibrium, if either of the following sets of conditions is met:*

1. $(N-1)/N > 2\delta_2$; $(N-1)/N > \delta_2 + 3\delta_1$,

$$and \quad \frac{[2+N(1-\delta_2)]^2}{9N} \geq \left[\frac{5N+1}{6N} + \frac{5\delta_1}{6} - \frac{\delta_2}{6}\right].$$

2. $(N-1)/N > 2\delta_2$; $(N-1)/N < \delta_2 + 3\delta_1$

$$and \quad \frac{[2+N(1-\delta_2)]^2}{9N} \geq \left[\frac{2N+1}{3N} + \delta_1 + \delta_2\right].$$

Proof: See appendix A.15 . □

This proposition illustrates that a common preference element for one location by all firms of an industry does not necessarily mean that Nash equilibrium in pure strategies never exist. For some set of (δ_1, δ_2) values, and for sufficient asymmetry between industries, for N large enough, region A will get both types of firms in equilibrium. On the other hand, if it is the case that $(N-1)/N < 2\delta_2$ (for perfect symmetry, $N = 1$, for example), then it could be shown that no Nash equilibrium exists in pure strategies. The proposition also shows how quickly the conditions under which an equilibrium may exist become hard to interpret and verified, in spite of the conceptual simplicity of the problem.

1.7.2 In–kind Subsidy Competition

Solving the problem of the optimal use of both cash and in–kind subsidies in the presence of positive factors of attachment δ_i is a daunting task. The objective of this section is more modest: here, it is shown only that the use of mixed cash/in–kind subsidies in fact may allow PSNE to be found in at least some circumstances.

When firms from at least one industry have a positive common factor of attachment to a region, it has been shown above that there no Nash equilibrium exist in pure strategies in the case of industries of same size. The next proposition shows the use of in–kind subsidies makes it possible to have a Nash Equilibrium in pure strategies in this case, if the following assumption is met.

Assumption 1.6 *The benefits for the two industries from θ–subsidies is sufficiently different such that* $(1-t_1)/(1-t_2) > L$.

This assumption is stronger than assumption 1.2, which merely imposes that $t_2 > t_1$, from this $(1-t_1)/(1-t_2) > 1$. By assumption 1.5, $L > 1$.

Proposition 1.18 *Assume that assumption 1.6 is met. For industries of the same size, there exist some positive values for the common factors of attachment for which a Nash equilibrium in pure strategies exists in the mixed cash/in–kind subsidy game.*

Proof: see appendix A.16. □

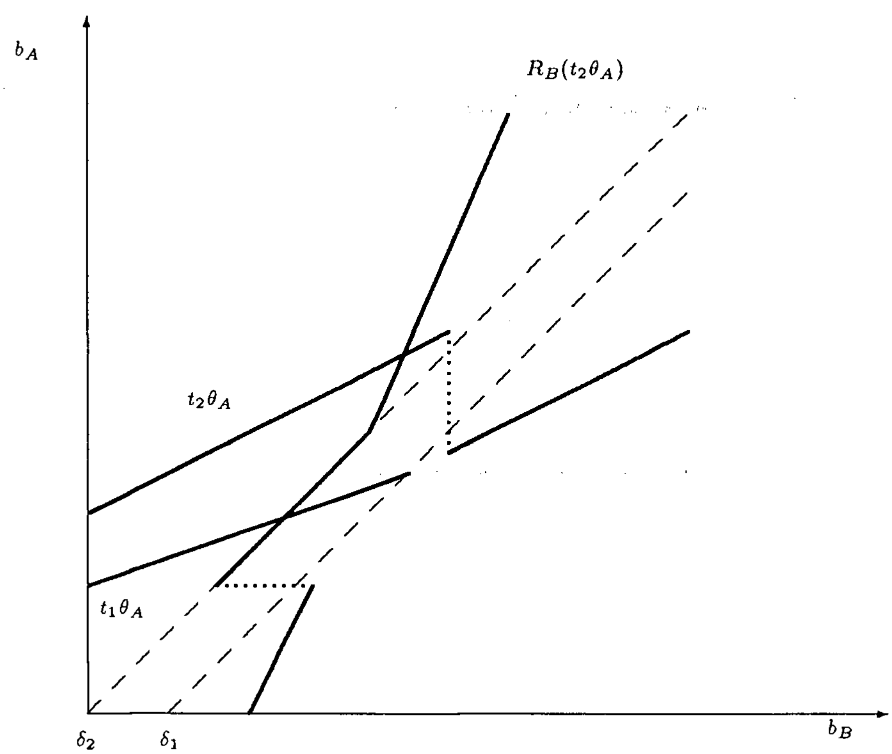

Figure 1.17: Nash equilibrium can arise in the symmetric case in spite of the positive common factors of attachement when region A uses in–kind subsidies.

Figure 1.17 illustrates the result. Note however, that the use of in–kind subsidies may allow a Nash equilibrium in pure strategies over a fairly restricted set of values for

δ_1 and δ_2, when $N = 1$. From region A's point of view, it is fairly straightforward to show that the key factor is the size of δ_2, which determines whether reaction functions will intersect off the diagonals; the larger δ_2 is, the less likely this is to happen. The value taken by δ_1 is quite important, too. It represents the effective protection region A has from region B's attempts at industrial policy. If δ_1 is large, region A need never worry about losing firms to region B, just as region A cannot to take away firms from region B when δ_2 is too large. But it could be that region A prefers to act in a purely defensive manner, setting $b_A = b_B - \delta_1$, because of the cost of the incentive–compatible transfer otherwise needed for type 1 firms.

1.8 Degenerate Distributions

This section describes how the state's strategy of offering a menu of mixed cash/in–kind subsidy programmes breaks down whenever all firms within an industry have the same degree of attachment to a region. That is, even though one may claim this scenario is similar to having each industry being akin to a large single plant firm, the type of equilibrium found usually is not sustainable.

The reason for this is quite clear: the fight over firms takes on an all–or–nothing nature. Say region A goes on the offensive and tries to get type 2 firms. Region B, in which these firms are best located will match region A's bid dollar for dollar. The outcome may appear predictable and simple, but it is not. Say that region B offers a penny more than region A's maximum bid, and gets to keep all the type 2 firms. This outcome, resembling a second price auction solution, is not an equilibrium for a very simple reason. Whenever region A offers subsidies to type 2 firms, it must offer something to the indistinguishable type 1 firms. Even if it does not succeed in attracting any type 2 firms, it must pay type 1 firms whenever it sets out to bid. Thus, it is better for region A not to bid, which means that region B does not need to offer a defensive bid. However, if region B offers nothing, then region A wants to offer just a little. In other words, no pure strategy Nash equilibrium exists.

No pure strategy Nash equilibrium exists whether regions offer cash or in–kind subsidies. It differs from the outcome obtained in the papers discussed in section 1.2 because of the presence of a second industry and the fact that governments try to attract very small firms. Attracting a large plant is a very different business, one for which discretion instead of rules is a better line of conduct.

1.9 Conclusion

The objectives of this essay were threefold: to explain why it can be rational to use seemingly wasteful forms of income transfers as part of an industrial policy; to determine which region is more likely to adopt an aggressive industrial policy; finally, to complement the existing literature, which has concentrated on the single large plant problem, by considering an industrial policy targeting entire industries.

Thus, some stylized facts about industrial policy can be explained. First, in a cash–only environment, proposition 1.2 provides a natural explanation for the commonly accepted idea that "have-not" regions pursue more aggressive industrial policy, an explanation that does not depend on regions giving different values to the same jobs. It shows that poorer regions can be more aggressive because fewer "natural clients" need to be bribed whenever special favours are extended to other firms. It shows quite clearly also that such an industrial policy leads to a spatial misallocation of firms, as compared to the *laisser–faire* situation.

The propositions contained in section 1.5 describe what kind of solutions are possible when in–kind subsidies are used, and whether or not using them is to be preferred to using only cash subsidies in the usual Nash or subgame perfect Nash sense. The most interesting case is surely the one illustrated by proposition 1.7 and its auxiliary propositions 1.10 and 1.15. In that case, region A gets to keep all the firms it would have got in the laisser–faire solution, and steals some type 2 firms away from region B.

Proposition 1.6 further indicates that using many subsidy programmes, even if they partly consist of wasteful in–kind subsidies, is likely to be rational if regions are of different size and are sufficiently different in the valuations they put on those in–kind subsidies. For this to occur, in–kind subsidies must be both sufficiently effective and have some discriminating power.

It has also been seen that as the asymmetry between industries increases, the set of (t_1, t_2) values such that in–kind subsidies are preferred to cash subsidies shrinks. This comes from the fact that an even larger fraction of subsidies are given to type 2 firms in an ineffective manner and that the savings made from discriminating against type 1 firms can only be of a limited size. This point has been made in different fashion by propositions 1.6, 1.10, and 1.15.

Some of the propositions contained in section 1.5 also make clear that it is possible to have no Nash equilibrium in pure strategies. This type of solutions exists also when some friction is introduced in the model, as in section 1.7. The common term δ_i in the degree of attachment firms have for a region may be used to think about the classical dilemma of industrial policy: should one tried to attract specific firms or should one improve the working condition for the overall industry? The δ_i factors, while exogenous in the model, could represent the benefits obtained by firms of one's own industry from the infrastructure of a region. Thus, investment in infrastructure could potentially shelter a region from raiding neighbours, as shown in proposition 1.16 and in general would certainly limit the extent of firm transfers. However, proposition 1.18 shows that using in–kind subsidies could be enough to re–establish the existence of a Pure Strategy Nash Equilibrium.

If one does not shy away from stretching the reach of this model, the logic at work in this model can be used to explain a good deal more. The assumptions of having only two types of industries and two possible types of subsidies have been made only for convenience. The model could be interpreted as indicating that the actual industries targeted by an industrial policy and the means of transfers selected are endogeneous[23]. That is, it may be of interest to pursue very peculiar types of industries, that are in themselves not so important economically, if they have specialized requirements. Similarly, a relatively ineffective form of transfer may be adopted to attract firms from a given industry if firms of other industries care even less about this form of transfer, and can be bribed cheaply in some other way. In other words, the choice of target and of forms of transfers, in designing the industrial policy, depend on the trade–offs that exist between all the potential t_j and (t_j, t_k) terms.

The main shortcoming of this essay comes from the fact that throughout the discussion on the use of mixed cash/in–kind subsidies, it has been maintained that region B would use only cash subsidies. As has been said, and should be obvious now, the problem becomes unrewardingly complex without this assumption.

A key extension to this model would be to allow for more general interaction between the private information parameters of the firms' profit function, as described in footnote 11. The set of strategies used by the government would then need to be expanded. It might want in this case to offer a continuum of cash and in–kind subsidy combinations to discriminate further between firms of the same industry. While the notion of "infra–marginal firms" would become different — it would not be synonymous anymore with infra–marginal industry — the qualitative result is unlikely to be affected: using in–kind subsidies constitutes a sensible strategy when one's own industry is effectively sheltered from competition, either by natural advantages — a large

[23]Industries can be here interpreted strictly, using some form of industrial classification, or as in the model in terms of input requirements.

δ_i — or by having industries of radically different size.

Another possible extension is to allow the benefits obtained from firms to depend on the subsidies given. For example labour training subsidies may promote the use of more labour intensive techniques, thus creating more jobs. This, and the possibility of having regions valuing jobs differently, would create other motives for the use of subsidies and change somewhat the efficiency properties of the equilibria achieved. Again, the main message of this essay would remain the same.

Chapter 2

Who Should Conduct a Technology–Oriented Industrial Policy in a Federation?

2.1 Introduction

Much has been written by economists on the competition between governments to affect the location of the production plant of an already established firm[1]. The main result in this literature has been that governments are unable to change the location of firms from what it would have been in a *laisser–faire* solution. Thus, while location efficiency is maintained, transfers from governments to firms are to be considered wasteful whenever the marginal cost of public funds is greater than one. This leads to the simple policy conclusion that in a federal country this type of industrial policy should be the prerogative of the central government, which would do best by doing nothing.

Much less has been written on an industrial policy that seeks to increase the number of new technologically–based firms (NTBFs) in the economy[2]. Governments want to

[1]See for example the papers by Biglaiser & Mezzetti [2], Black & Hoyt [3], King, McAfee & Welling[30], King & Welling [31], and Taylor[51], which have already been discussed in the first essay.

[2]Policy–oriented documents abound, but nothing has been found in terms of simple model–based analysis.

see such firms set up plants on their territory to benefit from the externalities they are thought to generate. Policy analysts and proponents of this type of intervention have sought to differentiate it from the former type of industrial policy by arguing that, while it requires government intervention to sustain newly-created firms, it involves none of the direct wasteful competition between governments[3] . The main insight of this essay is that in fact the two types of industrial policy are pretty much the same in that they imply ultimately that governments compete to get production plants built on their territory[4]. What differs, however, is that the government intervention may still overall be beneficial, and that a "do-nothing" policy prescription is unlikely to be the best possible solution.

The validity of using such an industrial policy rests on some basic premises, which are certainly open to debate. However, if one accepts them, determining which level of governments must conduct the industrial policy becomes of interest. This essay strictly tries to address that latter question.

Conceptually, the validity of such an industrial policy rests on two pillars. First, that such externalities exist; second, that they have a strongly local nature. The first aspect is less contentious. In fact, government intervention is generally justified by the following considerations, which are more or less supported by available evidence. First, high-technology firms, and Research and Development (R&D) in general, are perceived as important engines of growth, mostly in terms of productivity gains; the social return to R&D is higher than the private one because technological progress has spillover effects (See Henderson [21]). Much of the recent literature on endogenous growth rests on such a premise. This suggests that governments must try to promote

[3]See Lambright and Rahm [33], Lambright, Teich and O'Gorman [34], Piore [47], and Thornburgh [52].

[4]This aspect has not escaped everyone. See Smith [50] and the OECD report for Switzerland [43]. The papers by Moore [38] and Moore and Garnsey [39] discuss instead the crowding out effect that government programmes may have on private efforts.

further investment in R&D (See for example Grossman [18] and Ford & Suyker [17]). Second, an ever present objective of policy makers is to favour the creation of "good jobs", and in today's context, any job creation at all. While it is hotly debated among economists, it has been often said since the early eighties that small firms generate more new jobs than large firms do[5]. Third, and perhaps more importantly, small firms may not be as apt at separating the R&D and production functions as large firms are, especially multinational ones. In the latter case, their R&D function is footloose in the sense of beign divorcable from production. In contrast, successful discoveries from small firms would then mean both continuing employment of R&D staff and job creation at the production plant. Thus, a policy seeking to encourage the development of technologically–oriented small firms may go some distance towards meeting the twin objectives of getting productivity gains and achieving job creation.

The second aspect of the question is the extent to which these externalities are local, and what is their exact nature: are they internal to a narrowly defined industry or not ? To put it differently, are these externalities embedded into a product or associated to a production plant and its employees ? While mainstream economists have displayed heightened interest in spatial economics in recent years, see for example Paul Krugman's *Geography and Trade* [32], relatively little is known about the real importance of externalities for location decisions. Nevertheless, some empirical studies show that past location of manufacturing industries clearly helps explain their location today[6]. A region is then unlikely to be able to reinvent itself, but appears to be better off strenghtening the industries that constitute its economic base. Former Pennsylvania Governor Thornburgh defended his state's industrial policy by the necessity to sustain and improve the state's industrial base[7]. Walter Grünsteidl, an advisor to Philips, the

[5]The idea is associated to the work of David Birch. See Davis *et al.* [13] for an interesting analysis of the question.

[6]See the papers by Vernon Henderson [21] and Henderson *et al.* [22].

[7]"Technology — particularly process technology — is critical to helping our older, traditional

electronic products company, underlines the linkages between firms of broadly defined industries, from suppliers to skilled labour and training institutions, and how the loss of a narrowly defined industry may weaken many others[8]. Finally, and in the same spirit, Michael Wasylenko [53, page 14], in his review of the empirical literature, writes that "most firms do not relocate, and expansion on site and, to a lesser extent, branching are the most frequent methods of increasing production capacity." While all the facts enumerated above are far from being perfectly conclusive, it is possible to give some credence to the idea that attracting to and keeping NTBFs in a region may play an important role in its economic development if the firms and projects are well chosen.

Economists usually favour indirect intervention instruments to provide support for R&D, mostly through the taxation system for example. A more direct form of intervention is to have a government agency investing directly in firms, but it is considered unlikely to outperform the private sector in selecting promising technological avenues — see Gene Grossman [18]. However, if it is the intention of the government to favour small firms and discriminate against large firms — or at the very least to offer supplementary support to small firms — it may be preferable to use some other method than the tax system. NTBFs are often said to have difficulties getting the capital investment they require early on in their life–cycle. The term "capital gap" is often used to describe this phenomenon[9]. It is thus thought by many that direct support targeted at small NTBFs makes for sound economic policy. Moreover, it may be a superior way to achieve additionality, that is to achieve through public expenditures some sizable increases of total R&D expenditures. In other words, the policy could be designed in such a way as to have an impact at the margin[10].

industries regain their competitive edge by modernizing and increasing their productivity." [52, page 43]

[8]See [19]. The same factors are enumerated in Krugman's second lecture[32].

[9]See Schmandt and Wilson [48] on this topic and for descriptions of some programmes in existence in the United States to fill this gap.

[10]In Canada, firms obtain a 50% tax credit for all allowable R&D expenditures. In the U.S.A.,

In this paper, this diagnostic and the solutions offered are taken at face value. This paper seeks to determine which government level in a federation ought to control that type of policy, and what are the respective strengths and weaknesses of the central and regional governments in this domain. The literature on fiscal federalism is silent on this precise matter[11]. What crucially differs between the two levels of government, however, is that the central government is likely to acknowledge that it can potentially act on two margins: it can try to generate more entrepreneurship in the economy, an option that is not explored in this essay, or it can try to help more entrepreneurs discover the true value of their projects, by providing some financial support through the first stage of product development. A regional government has access to a third margin, because it need not take as given the total number of ideas. It can try to steal successful firms from a neighboring region. Working on this third margin may be detrimental to the overall performance of the economy if it does not improve the sorting properties of the system or leads to less attention being paid on the other margin. Thus, this essay tries to explore the implications of assigning this policy function to regional governments, which opens the third margin, as compared to the relative gains in flexibility inherent in a decentralized system. In other words, the question is to determine how badly must the federal system work with respect to local knowledge and flexibility for a decentralized solution to be preferred, even though it allows this costly third margin. Results obtained are presented either as observations or propositions. Observations present basic results derived from comparisons between the social planner's solution and given institutional settings. Propositions stem from comparative statics exercises that attempt to compare institutional settings. The next section presents in more

during the eighties, firms received a 20% tax credit on incremental R&D spending over a three-year moving average. Thus, the American system was better designed to work at the margin. See Katz and Ordover [29] on the American system.

[11] At best, it may be speculated that such an activity being of an allocative nature, in Musgrave's taxonomy, it would traditionally be recommended to assign this function to regional governments.

details the assumptions used and the basic logic of the problem. Section 2.3 formally presents the model and presents some observations on the structure of the problem. Section 2.4 contains the comparative statics exercises and the propositions. Section 2.5 concludes.

2.2 Model Description

To solve the policy–function assignment problem at hand, a simple model is developed, with some built–in biases in favour of the regional governments.

The industrial policy problem solved by any government can be decomposed into two stages. In the first stage, firms try to develop a new idea, at an exogenously fixed intensity level. They must finance their R&D programme using bank loans. Firms have a negative cash–flow during this stage and invest mostly in acquiring knowledge which has very little collateral value. Because they cannot judge the value of such science–oriented firms, banks may recall loans in this stage with some given probability[12]. If the loan is recalled, the firm is assumed to fold. This behavioural assumption on the part of lenders constitutes a way to implement the capital gap idea.

Only one decision has to be taken by the firms in this first stage. Firms may receive an offer of assistance from the government. Firms have then to decide whether to reject or accept the offer. It is seen below that they always accept the government's offer.

At the beginning of the second stage, having discovered the value of their project, firms decide whether or not to start production, and if so, where to locate their single plant for production.

Firms can be classified in two ways. First, it can be said that there are some good firms and some bad ones. In the absence of any financial constraint, the good ones

[12] This could be justified by saying that the debt/collateral ratio has become too high. Then, the problem has to be presented somewhat differently, as firms need to assess what is the critical value of this ratio. Firms having too little collateral to start with would never enter the game.

succeed in developing a product in the first stage. The bad ones fail in doing so. There is no information asymmetry with this respect; nobody knows beforehand which firms would be successful and which would not.

Second, firms may also differ in the benefits they would bring to a region if they are successful. Somewhat loosely, it is said that successful firms generate externalities which depend on the quality of the match between a firm and a region.

The total number of firms is considered exogeneous here; moreover, given the information firms have, the expected net payoff is sufficiently high that all want to start their R&D programme at time zero.

In the second stage, only some fraction of all firms are still in operation. The lack of financial means has forced some good and bad firms to fold. This provides grounds for government intervention. A government wants to intervene if it can partially solve the financial problems of the firms and allow a larger fraction of them to survive. In this way, it would achieve some degree of additionality.

This calls for a specific form of industrial policy. The policy in question may be conducted by setting up firm–incubators and inviting some firms to move into these incubators. The policy is attractive to firms if it offers workspace at subsidized rates and access to experienced managers. It is also attractive to governments in the sense that investing in the development of new firms in this manner has some good incentive–compatibility properties; transfers in kind attract chiefly firms which could benefit from it.

Setting up the model such that the first stage is truly a continuous time period and having an explicit incubator policy is unnecessarily complex. It is much simpler to imagine that governments are simply investing in new technology, and neglect formally the mechanism by which they do so. Intuitively, it may still be helpful to think in the incubator/temporal framework[13].

Looking first at the social planner solution allows establishment of a benchmark case. Assume that the social planner has no more information than anyone else. That is, it cannot distinguish a good firm from a bad one before the second stage. Moreover, the social planner cannot change the banks' lending policy[14]. Only a fraction of all surviving firms turn out to be successful. The social planner's optimal policy is to invest just enough so that the marginal cost of helping all these firms is just equal to the benefits obtained from the externalities generated by the successful firms that would have folded.

In a multiregional context, there are two ways to implement such a policy. The industrial policy may be conducted at the regional level or it may be centralized.

Consider first what happens if regional governments have jurisdiction over industrial policy. In the second stage, full knowledge is obtained about the value of a firm. To benefit from the externalities generated by a good firm, a region may entice it to move with a benefit package. That is, it may well be unnecessary to invest in the first stage in firms if they can be bought in the second stage. However, if adopting such a policy makes sense for one region, it typically makes sense for all of them. The amount any region may rationally bid for a firm is constrained by the benefits it receives from acquiring such a firm. At this stage, a Nash equilibrium calls for a second–price auction result; the best region wins the firm, and pays the maximum bid of the second region.

If that is the outcome of the bidding war in the second stage of the game, then two things could be observed about the optimal government investment in the first stage;

[13]Another potential aspect of the problem is lost when one proceeds this way. It is often said that decentralization allows for continual experimentation on ways to solve problems. That is, a successful public policy might be easier to find if a large number of state governments experiment on their own, allowing them later to imitate the most successful one. This diversity in mechanism choice is lost in the present framework. See David Osborne's book [44] for such an argument.

[14]The social planner here is more like a government facing real constraints. It is not the usual omnipotent social planner.

first, in comparison with the social planner's solution, there is going to be *under–investment* in new technology, as the gains to a region from a good firm are partly dissipated in the bidding process. This represents the costs of having opened the competitive margin. Second, governments invest only in firms well–suited to their region. This is so because any other firm, if successful, cannot be retained in the second–stage bidding war. This equilibrium then involves some form of under–investment, but has good sorting properties.

Now, consider how such a policy could be conducted at the centralized level. In the second stage, when all information is revealed about surviving firms, an all-knowing central government can give a small prize to firms conditional on their moving to the desired region. This implies that in the first stage, this central solution entails the same amount of investment as in the social planner's solution, but for the cost of this small prize. If the policy were to invite firms to very specifically located incubators in the first place, then even this small prize is not needed, as firms themselves are assumed to have no preferences over location.

However, it is unlikely to be true that the central government can do as well as the social planner. Here, two types of constraints on the behaviour of the federal government are considered in turn. The first results from supposing that uniform rules have to be followed by the central government in all regions. The second results from assuming that the central government has less information about firms' optimal location than the regional ones. Moreover, it is assumed that the private benefits obtained by firms are uniform across regions. If it were assumed instead that private benefits and the value of externalities generated are correlated across regions, the uninformed central government could rely on private firms to generate the information it needs, and the absence of direct information would not be a real constraint. If it can be shown that despite these constraints the federal government does as well as the regional ones, it will certainly be true in a less restrictive environment.

These constraints may be justified in the following manner: it is quite possible that the central government has the choice of implementing such an industrial policy in two different ways. First, it could set a unique central bureau in charge of the policy; second, it could have regional offices conducting the policy on its behalf. A decentralized solution such as the latter could lead to competition between offices, either caused by "yardstick–competition" evaluation by the central bureau, or by having the offices captured by local constituencies. Thus, if the centre chooses to use regional offices, it may want to impose uniform investment rules for regional offices as a way to limit harmful competition between them. If not, regional offices could engage in "pre–emptive" investment, to capture firms before other regional offices do so, and dissipate all rents in this manner. This constitutes one way to justify the constrained central government assumption. If the central government centralizes all its operation to prevent inter–offices competition, then the uninformed central government assumption becomes easier to understand.

If this central government has less information than the regional governments about firms' optimal location, or if it were forced upon it to adopt a uniform policy, in terms of not giving an advantage to one region over another, the problem would be quite different. Consider first the impact of not so perfect information. In the second stage, the central government knows which firms are still active, but does not know their ideal location. One potential policy for the government would be to move all firms to the region for which the expected value of the externalities is the highest. It thus resembles the solution to the so–called one–arm bandit problem in the sense that the government picks the lottery with the highest average payoff. Since net returns are lower, it also implies that in the first stage of the game investment is not optimal. Again, the comparison point is the social planner's solution. Thus, there might be too many or too few firms surviving the first stage. It may easily be seen that in some circumstances this policy could perform worse than the one followed by competing regional governments. It is obvious here that the policy followed by the central government is deficient in both stages.

If the central government were constrained to adopt some uniform policy across regions, then surviving firms' allocation across regions is optimal. However, in the first stage, its investment is constrained to be similar across regions; i.e. either it cannot invite firms to move from one region to another, or it is forced to offer the same assistance everywhere. Again, the consequence is that investment is not optimal, as compared to the social planner's solution. In this case, inefficiency arises only in the first stage.

2.3 Formal Model

2.3.1 The Firms' Problem

In this section, what has been presented in the preceding section is done in a more formal way.

Firms decide to explore the value of a project. They know that their project can have two possible outcomes in the second stage. Either the project is a success and give future profits $F_g > 0$, or is a failure and has payoff $F_b = 0$. Firms do not learn the value of their project before the second stage. Thus, in the second stage of the game, the decisions to be made by the firms are quite simple. If the payoff for a firm l turns out to be low, $F_l = F_b$, no further work is done on the project. If it is a high payoff, $F_l = F_g$, the firm decides where to set up its unique plant for production. As of now, it is assumed that firms are indifferent between the several possible locations in the absence of government intervention, as their private return to production is identical across regions. If governments offer location–specific incentive, the successful firms move to the region offering the highest bid.

Now turn to the first stage of the game. Overall, there is a proportion λ of good projects at time zero. In the first stage, there is a probability $1 - s(v)$ of seeing the bank loan recalled. v represents the amount of outside investment made into the firm by governments. The function $s(v)$ has the following properties:

$$s(0) > 0, \qquad s'(v) > 0, \qquad s''(v) < 0. \tag{2.3.1}$$

The private cost of conducting R&D in the first stage is c. This implies that the expected net payoff at time zero is

$$F_g \lambda s(0) + F_b(1 - \lambda)s(0) - c, \tag{2.3.2}$$

λ represents the proportion of total firms that are going to find an innovation. This proportion is assumed to be known. It is further assumed that the expected payoff is high enough to start the project, even without government support. A successful project has two kinds of return. The first is the private payoff to the firm, as described above. The second is a social return not captured by the firm, some externality β_k^l which is region–dependent. The latter can be thought as the result of some externalities generated by the firms, which depend on the quality of the match between the firm and the region in which it sets up its plant. All the investment made up to this point in R&D can be considered a sunk cost.

2.3.2 The Social Planner's Problem

What matters to the social planner is to see that some projects survive to generate the social return. A social planner may be concerned by all the firms that fail in the first stage of this game. It may also want to see firms making the right choice when time comes to set up a production plant. This latter problem is easy to solve.

For a value of externality β^l_j, where j represents the region and l represents the firm, the planner simply dictates a location k for each firm l according to

$$\beta^l_k = \text{argmax}\left[\beta^l_1, \beta^l_2\right]. \tag{2.3.3}$$

In the first stage of the game, the social planner intervenes by investing in firms an amount v such that the marginal benefit of doing so, in terms of total payoff of firms rescued, is equal to the marginal extra cost it imposes in terms of subsidizing all firms more intensively.

That is, the social planner maximizes the following objective function

$$\max_v \lambda s(v)\beta^l_k - v. \tag{2.3.4}$$

The first-order condition is

$$\lambda \frac{\partial s(v)}{\partial v} \beta^l_k - 1 = 0. \tag{2.3.5}$$

Its interpretation is straightforward. Equation 2.3.5 defines implicitly how much the social planner wants to invest in new technologies, assuming that an interior solution obtains. This is written as the function:

$$v = s\prime^{-1}\left(\frac{1}{\lambda\beta^l_k}\right) = v(\beta^l_k), \tag{2.3.6}$$

the term λ can suppressed because it remains constant throughout the analysis. The shape of the function $v(\beta)$ is of interest for later reference. From equation 2.3.5, one may observe that

$$\frac{\mathrm{d}\, v}{\mathrm{d}\, \beta^l_k} = \frac{-1}{\lambda(\beta^l_k)^2 \frac{\partial^2 s(v)}{\partial v^2}} > 0, \tag{2.3.7}$$

when $\frac{\partial^2 s(v)}{\partial v^2} < 0$. In other words, the level of investment increases as the value of $\lambda\beta^l_k$ increases.

Furthermore, this function grows at a decreasing rate if some condition is met on the shape of the survival function, $s(v)$.

$$\frac{\mathrm{d}^2 v}{\mathrm{d}\,\beta_k^{l\ 2}} = \frac{1}{\lambda(\beta_k^l)^2 \frac{\partial^2 s(v)}{\partial v^{\ 2}}} \left[\frac{2}{\beta_k^l} + \frac{\frac{\mathrm{d}\,v}{\mathrm{d}\,\beta_k^l}}{\frac{\partial^3 s(v)}{\partial v^{\ 3}}} \right] < 0. \tag{2.3.8}$$

The expression in square bracket in the equation 2.3.8 is positive if

$$\frac{\partial^3 s(v)}{\partial v^{\ 3}} > \frac{-\beta_k^l \frac{\mathrm{d}\,v}{\mathrm{d}\,\beta_k^l}}{2}. \tag{2.3.9}$$

This condition is met if the third derivative of $s(v)$ is positive. To complete the description of the social planner's solution, the following assumptions are made. First, it is assumed that all firms going to one region generate the same level of externalities. This level may differ across regions. It eliminates the need to carry some separate notation with respect to the firms' identity and simply to designate them by their optimal location. Second, there is a fraction ρ_j of all firms present at the beginning that have their highest externalities in region j.

The social planner has two independent problems to solve, similar to equation 2.3.4 above, one for each region. Total investments made by the social planner can be written as

$$\sum_{j=1}^{2} \rho_j \lambda v(\beta_j). \tag{2.3.10}$$

Similarly, the total net benefits generated in the social planner's solution is

$$V^{sp}(\beta_1, \beta_2) = \sum_{j=1}^{2} \rho_j \left[\lambda s(v(\beta_j))\beta_j - v(\beta_j)\right], \tag{2.3.11}$$

where *sp* stands for "social planner".

2.3.3 The Regional Governments' Problem

Turn now to the problem that regional governments have to solve. While it is true that overall the number of innovative firms is fixed, from the point of view of a region there are two ways to get such firms: the first is to nurture them by investing early, the second is to pay enough to take them away from some other region when those firms are observed as being successful. Thus the need to think of a region's problem as a two–stage one.

Second Stage

In the second stage of the game, one knows exactly the value of all surviving firms. Each government needs to attract a firm to its region to benefit from it. In order to do that, it just needs to offer a small production subsidy to a firm. Recall that firms are indifferent with respect to their plant location, as far as private returns are concerned.

With several regions competing to attract the firms, the Nash equilibrium resembles an ascending–bid auction. The maximum bid a region is ready to make is equal to its benefits from having the firm.

A region j wants to maximize the objective function:

$$\max_{b_j^l} \left[\beta_j^l - b_j^l, 0\right]. \tag{2.3.12}$$

The maximum bid for a firm l by region j is equal to β_j^l. The winning region k is the one for which[15]:

$$\beta_k^l = \operatorname{argmax} \left[\beta_{kj}^l, \beta_{kj}^l\right]. \tag{2.3.13}$$

The bid it has to pay is equal to the value of the firm's externalities in the other region[16].

The net benefit for this region is thus

$$\beta_k^l - \beta_j^l, \qquad \text{for } j \neq k, \tag{2.3.14}$$

[15]The notation $\beta_{k,j}$ should be read like this: k stands for the optimal location of a firm; j represents where it actually ends up. In the social planner problem, the two are always the same. In this case, the notation β_k is used instead.

[16]It is assumed that a tie is broken in favour of the region k. It is one way to take care of the problem created by the use of real numbers. If the use of some currency units is imposed, this problem disappears quite naturally.

and it is equal to zero for the other region.

This allows one to make the following observation:

Observation 2.1 *Firms having survived to the second stage of this game are going to be allocated across regions optimally.*

This follows immediately from maximizing 2.3.12.

First Stage

Turning now to the first stage of the game, the following observation can be made. A government trying to keep alive some of these firms must always take into account what will happen in the next stage. It is never interested in investing in a firm that ultimately is going to set up its production plant in the other region. It might want to do that if it could force the firm to commit to staying in the region in the second stage. Short of taking over the firm, there is no obvious mechanism the government could use to obtain such a commitment[17]. Thus,

Observation 2.2 *In the first stage, governments invest only in firms that are best matched to their region. In other words, each firm is going to receive at most one subsidy offer.*

Once this is established, it must be determined how much the government wants to invest. Define $\alpha^l_{k,sb}$ as

$$\alpha^l_{k,sb} = \beta^l_k - \beta^l_j, \qquad \text{for } j \neq k. \tag{2.3.15}$$

Here, *sb* represents simply the "second–best" alternative. It represents the difference between externalities generated in the best and second–best locations. In the context

[17] Even if governments' assistance to firms take the form of loans, conditional on the firm's success, what could prevent firms to pay back their loans in the second stage, and then move to some other region?

of a bidding war, it also represents the net benefits for the best region, after transfers. Regional government k solves the following problem

$$\max_{v} \lambda \; s(v)\alpha^{l}_{k,sb} - v, \tag{2.3.16}$$

for the fraction ρ_k of firms that are best suited to its region. The level of investment can be written as

$$v(\alpha^{l}_{k,sb}). \tag{2.3.17}$$

There are two independent problems like this one solved by the two governments. Comparing with equation 2.3.4, the social planner's problem, it can be seen that equation 2.3.16 differs only in the net payoff received per firm in the second stage of the game. Whenever the second–highest bid is greater than zero, a regional government invests less than a social planner does. This follows immediately from the second–order–condition. Thus, a third observation can be made:

Observation 2.3 *Competition among regional governments for new firms leads these governments to* under–*invest in these firms, as compared to the social planner's solution.*

The total level of investment is

$$\sum_{j=1}^{2} \rho_j v(\alpha_{j,sb}) \tag{2.3.18}$$

The total net benefits obtained are

$$\begin{aligned} V^{dc}(\beta_1, \beta_2) &= \sum_{j=1}^{2} \rho_j V^{dc}_j(\alpha_{j,sb}) \\ &= \sum_{j=1}^{2} \rho_j \left[\lambda s(v(\alpha_{j,sb}))\beta_j - v(\alpha_{j,sb})\right], \end{aligned} \tag{2.3.19}$$

where dc stands for "decentralized competition".

Note for future use that in this solution neither the second stage total payoff per firm, nor the investment level depends directly on the ρ_j parameters. Investment level obviously does depend on the second–best location for the firms.

2.3.4 Central Government's Problem

Now turn to the problem a central government has to solve. In the absence of any constraint on its behaviour, and assuming it knows exactly as much as regional governments do, it seems obvious that the central government is able to do as well as the social planner, both in terms of allocating firms to the appropriate region in the second stage, and in terms of choosing the correct level of investment in the first stage of the game.

However, it is known that central governments are often constrained in their behaviour by some notion of horizontal equity. That is, they must either offer the same subsidy programmes in most regions, or they cannot really promote firms' mobility across regions, as they could then be perceived as robbing some regions of their chance at improving their lot[18]. Either way, it forces only one policy to be adopted across the regions.

Consider first this latter case. The government knows about the average externalities obtained from firms in each region. However, in the absence of information about the best location of individual firms, the government solves instead the following problem in the second stage of the game. It must choose to send all firms to a region m, for m defined by

$$\overline{\beta}_m = \text{argmax} \left[\sum_{i=1}^{2} \rho_i \beta_{i,1}, \sum_{i=1}^{2} \rho_i \beta_{i,2} \right], \qquad j = 1, 2. \tag{2.3.20}$$

Without loss of generality, it is assumed that region 1 has higher average benefits.

[18]This latter case is not pursued here. Dealing with it would entail making assumptions about the *ex ante* distribution of innovative firms.

This problem is sometimes called the one–arm bandit problem. One has to choose between two random strategies, and adopts the one with the best average payoff. If both regions and firms *are* different, it is bound to be the case that no one region always constitutes the best location.

In the first stage, the central government maximizes this objective function

$$\max_{v} \lambda s(v)\overline{\beta}_1 - v. \tag{2.3.21}$$

The investment level, for the region 1, is found implicitly from the first–order condition of this problem. Solving the first–order–condition, it gives the total net benefits obtained as

$$V^{ug}(\beta_1, \beta_2) = V^{ug}(\overline{\beta}_1) = \lambda s(v(\overline{\beta}_1))\overline{\beta}_1 - v(\overline{\beta}_1). \tag{2.3.22}$$

The notation ug stands for "uninformed government" solution. It should be noted that both the investment level and the average benefit depend on the firms' optimal distribution, the ρ_j's. It also depends on the value of firms' alternative location.

Return now to the other case mentioned above. If the central government has perfect information about firms' optimal location but is constrained to invest equally across regions, its problem is closer to the social planner's.

In the second stage of the game, the government is allowed to allocate firms as it wishes. It then acts optimally and obtains an average payoff

$$\hat{\beta} = \sum_{j=1}^{2} \rho_j \beta_j \geq \overline{\beta}_1. \tag{2.3.23}$$

This inequality is strict whenever not all firms should be sent to one region, i.e. whenever $\rho_j < 1, \forall j$.

In the first stage of the game, the government is again constrained to have only one investment level. Before, this was so because of its ignorance. Now, it is an outcome of some institutional constraint, of some imposed notion of horizontal equity. The prize to be obtained being larger in the second stage, it is quite easy to determine that the outcome to this problem is superior to the one found above.

The problem the government maximizes is

$$\max_{v} \lambda s(v)\hat{\beta} - v. \tag{2.3.24}$$

From the optimal solution of the problem and the net total benefits obtained:

$$V^{cb}(\beta_1, \beta_2) = \sum_{j=1}^{2} \rho_j V^{cb}(\hat{\beta}) = \sum_{j=1}^{2} \rho_j \lambda s(v(\hat{\beta}))\beta_j - v(\hat{\beta}). \tag{2.3.25}$$

where cb stands for "constrained behaviour" solution.

What can be noted here is that the constrained government gets a higher surplus per surviving firms than the uninformed one, because it is able to locate them properly in the second stage. Also, it would be possible for the constrained government to choose the same investment level, per firm, in the first stage, if it chose to do so. These two facts together lead to a fourth observation about the model:

Observation 2.4 *The lack of information of the central government is much more costly than some uniform policy requirement could be.*

The next two observations provide a more detailed comparison of the central government's solutions with the social planner's solution.

Observation 2.5 *A central government constrained to provide the same level of support to firms in all regions is going to* over–*invest, as compared to the social planner's solution.*

This observation rests on two assumptions. The first is that inequality 2.3.8 is satisfied, while the second is that the benefits of optimally located firms differ across regions. If the inequality is satisfied, then the investment function, $v(\cdot)$, is concave. The investment decision has to be based on the average of the externality values. Thus,

$$\hat{\beta} = \rho\beta_1 + (1-\rho)\beta_2, \tag{2.3.26}$$

and it implies,

$$v(\hat{\beta}) > \rho v(\beta_1) + (1-\rho)v(\beta_2). \tag{2.3.27}$$

Similarly, the following observation can be made about the uninformed central government.

Observation 2.6 *An uninformed central government may either* under–*invest or* over–*invest in firms' support, as compared to the social planner's solution.*

Without loss of generality, assume that all firms are moved and supported in region 1, by the decision rule 2.3.20. If firms best located in region 2 yield lower benefits there, in region 1, than firms best located in region 1, $\beta_{21} < \beta_{11}$, then it is the case that

$$\rho\beta_{11} + (1-\rho)\beta_{21} < \beta_1, \tag{2.3.28}$$

which implies

$$v(\overline{\beta}_1) < v(\beta_1), \tag{2.3.29}$$

and it guarantees under–investment.

Conversely, for $\beta_{21} > \beta_{11}$, one obtains that

$$v(\overline{\beta}_1) > v(\beta_1), \tag{2.3.30}$$

and it becomes then possible to observe

$$v(\overline{\beta}_1) > \rho v(\beta_1) + (1-\rho)v(\beta_2), \tag{2.3.31}$$

for β_{21} sufficiently close to β_{22}. Again, this latter result depends on the concavity of the investment function, $v(\cdot)$.

2.4 Comparative Statics

Having determined how the problem is solved by the social planner and the two levels of government, consider next in which circumstances one institutional setting is likely to outperform the other. It is also of interest to see if there exist non–trivial cases for which one of these two settings could approach the optimal solution of a social planner.

The four possible solutions are:

- social planner's solution.

$$V^{sp}(\beta_1, \beta_2) = \sum_{j=1}^{2} \rho_j \left[\lambda_g s(v(\beta_j))\beta_j - v(\beta_j)\right]. \tag{2.4.1}$$

- competing governments' solution.

$$\begin{aligned} V^{dc}(\beta_1, \beta_2) &= \sum_{j=1}^{2} \rho_j V_j^{dc}(\alpha_{j,sb}) \\ &= \sum_{j=1}^{2} \rho_j \left[\lambda_g s(v(\alpha_{j,sb}))\beta_j - v(\alpha_{j,sb})\right] \end{aligned} \tag{2.4.2}$$

- constrained behaviour central government's solution

$$V^{cb}(\beta_1, \beta_2) = \sum_{j=1}^{2} \rho_j V^{cb}(\hat{\beta}) = \sum_{j=1}^{2} \rho_j \lambda_g s(v(\hat{\beta}))\beta_j - v(\hat{\beta}). \tag{2.4.3}$$

- uninformed central government's solution

$$V^{ug}(\beta_1, \beta_2) = V^{ug}(\overline{\beta}_1) = \lambda_g s(v(\overline{\beta}_1))\overline{\beta}_1 - v(\overline{\beta}_1). \tag{2.4.4}$$

Remember first that in three of these four cases the externalities obtained per firm in the second stage are identical. It is only in the case of an uninformed government that surviving firms are poorly allocated across regions.

Secondly, note also that in the first two solutions, there are two investment levels to be picked. In the two possible scenarios for the central government only one investment level is chosen, based on some average externality value. Thus, there is some interdependence observed between regions. Finally, note that there is always some relationship between the investment levels of competing governments (or of an uninformed central government) and the size of externalities when the firms are not optimally located.

There seems to be two different types of comparative statics that can be done on this model. The first is to examine at the impact of changes in the optimal distribution of firms, that is at the value of the ρ parameter. The second consists in an examination of the importance taken by the relative advantage of one region versus another in terms of the externalities generated.

2.4.1 Optimal Firm Distribution

The symbol ρ is used to designate the proportion of firms that should be allocated in region one. The following result clearly obtains:

Proposition 2.1 *For ρ equal either to zero or one, both the uninformed central government and the constrained central government can replicate the social planner's solution, for arbitrary values of the potential externalities, β_{ij}.*

This result is fairly trivial. If all firms should be located in one region, there is no meaning to the concept of a lack of information. Also, the constraint on the behaviour of the government could never be binding.

Any change in the value of ρ has an impact on the optimal policy only for the central government. For the uninformed central government, its impact is felt in the second stage by changing the value of $\overline{\beta}_1$. For the constrained central government, its impact is felt through the non–linearity of the relationship between some β_j and the ideal investment level, $v_j(\lambda_g \beta_j)$, as in the social planner's solution.

The solution obtained by competing regional governments is not qualitatively affected by changes in ρ.

2.4.2 Relative Size of Externalities

There are two ways to examine how the relative performance of the different institutional settings depends on the relative size of the externalities. First, one may examine the relative importance of choosing the right location for a firm. For example, if the firm is developing a new process, it might be that this process could be adopted to the different needs of the industrial base of the region, and that this would be done simply through the close contacts developed between the innovator and the firms already in place. If the firm is developing a new product, it may be the case that its actual location does not really matter, in the sense that the inputs it needs are of a common type, and its output is mostly or solely exported. Benefits are then more in terms of increased employment. Secondly, one may examine to what extent the relative benefits of optimally located firms matter. In this case, different firms, developing their own products or processes, are going to bring more or less the same level of benefits to their optimal locations.

The statement that the location of a firm becomes relatively less important is equivalent to saying that $\beta_{ij} \rightarrow \beta_{ii}, i \neq j$. If β_{ii} is kept constant, the solution of the social planner's problem stays unchanged, and the benchmark is maintained. The behaviour of the constrained central government remains the same, as it depends only on $\hat{\beta} = \sum_i^2 \rho_i \beta_{ii}$. However, the two other institutional settings are affected.

For the uninformed central government, $\beta_{ij} \rightarrow \beta_{ii}, i \neq j$, means that the cost of ignorance decreases. It also means that in the limit the notion of having different regions, at least for this aspect of industrial policy, is somewhat meaningless. In the limiting case of $\beta_{ij} = \beta_{ii}, i \neq j$, it is found that in any region the average externality is equal to the average of the best location externalities, i.e.

$$\hat{\beta} = \sum_{k=1}^{2} \rho_k \beta_k = \overline{\beta}_1 = \sum_{i=1}^{2} \rho_i \beta_{(i,1)} \geq \sum_{i=1}^{2} \rho_i \beta_{(i,2)}. \tag{2.4.5}$$

It thus follows that

Proposition 2.2 *If firms bring equal benefits in their first and second best locations, an uninformed central government does as well as a constrained central government.*

The problem becomes more interesting for the competing regional governments.

When the optimal location of the firms becomes less important, the surplus a government can get after having paid its second stage location subsidy diminishes. The exact value of the surplus was found to be

$$\alpha_{k,sb} = \beta_k - \beta_j, \qquad \text{for } j \neq k. \tag{2.4.6}$$

The level of investment made by the government depends on this surplus. Because $\beta_{ij} \rightarrow \beta_{ii} \Rightarrow \alpha_{k,sb} \rightarrow 0, i \neq j$, each regional government has then less incentive to invest in firms in the first stage of the game because the surplus it can keep in the second stage has decreased. Thus

Proposition 2.3 *As the respective benefits obtained from firms in their first and second best locations approach each other in value, the regional governments invest less in the first stage. At the limit, for equal benefits in both regions, no investment is made in the first stage of the game by competing regional governments.*

Conversely, as the value of externalities generated in the second best location approaches zero, competing governments get closer to the social planner's solution. In the limit, when $\beta_{ij} = 0, i \neq j$, the optimal solution is attained.

Within this framework, it can also be shown that even if competing governments all invest less per firm than the uninformed central government in the first stage, a better solution can be reached in the former case.

Proposition 2.4 *There exist cases for which the competing government allocation is better than the uninformed central government allocation even if the latter invests more per firm than competing governments.*

Proof: A continuity argument is used. There exist some values for the externality parameters $\beta^b_{11}, \beta^b_{12}, \beta^b_{21}, \beta^b_{22}$ and ρ such that

$$\alpha^b_{1,2} \equiv \beta_{11} - \beta_{12} = \alpha^b_{2,1} \equiv \beta_{22} - \beta_{21} = \overline{\beta}^b_1 \equiv \rho\beta_{11} + (1-\rho)\beta_{21}. \tag{2.4.7}$$

This last equality implies that the investment level in the first stage is the same for the three governments. The surpluses in the second stage are such that

$$\sum_{j=1}^{2} \rho_j V(\beta^b_{j,sb}) - V(\overline{\beta}^b_1) = s(v(\overline{\beta}^b_1))(\beta^b_{22} - \beta^b_{21})(1-\rho) > 0, \tag{2.4.8}$$

which shows that the decentralised solution is superior to the uninformed central government solution.

Keeping all other parameters the same, increase the value of β_{21} to β^a_{21} (thus increasing investment by the uninformed government and decreasing it for region 2's government in the competing solution) as to obtain the same overall surplus in the second stage:

$$V(\overline{\beta}^a_1) = \sum_{j=1}^{2} \rho_j V(\beta^a_{j,sb}). \tag{2.4.9}$$

Any intermediate value, $\beta^b_{21} < \beta_{21} < \beta^a_{21}$, gives the desired result. □

Return now to the second type of comparative statics relating to the size of externalities. If it is the case that optimally located firms do bring the same level of benefits to their region, then only the constrained central government problem is affected directly.

If $\beta_{ii} = \beta_{jj}, i \neq j$, the uninformed central government has exactly the same problem to solve as described previously; the only difference is that its choice of a region m now depends only on the relative performance of firms when they are poorly allocated.

However, for the constrained central government, having $\beta_{ii} = \beta_{jj}$ implies that the constraint it faces in offering the same level of investment everywhere is not strictly binding; this in fact has become its preferred option, thus:

Proposition 2.5 *Whenever best–located firms bring the same benefits to both regions, the constrained government is able to attain the first best solution, as defined by the social planner's problem.*

2.5 Conclusion

This essay develops a model to analyse the investment decision of governments in start–up firms in a multi–regional context. No attempt is made to justify such investment in the first place. The main insight of this essay is that regional governments have a tendency to compete away the rents that can be earned from such an industrial policy. In that respect, it resembles a "location" type of industrial policy. In the latter case, to do nothing is the best possible policy, and this outcome can be achieved simply by taking away this policy function from regional government.

Taking as given that there exists a capital gap and that externalities are local in nature, a technology oriented industrial policy can be valuable. However, the main objective of this essay has been to compare the relative performance of different ways to conduct this type of industrial policy. This has been done in the context of a simple model.

The industrial policy can either be in the hands of a central authority or in the hands of regional governments. If a central government has access to all the instruments and information regional governments have, assigning it the task of performing this industrial policy clearly dominates. Thus, two types of limitations have been imposed on the central government. The first is to limit the information it has relative to the regional governments. The second has been to limit the possibility of it using differentiated rules across regions. These two constraints do not seem to be too implausible.

Observations 2.1, 2.2 and 2.3 relate to the solution obtained in the competing region framework. It has been seen that firms are going to be allocated in the correct manner in this framework, that they invest only in firms they can retain, and that governments would *under*–invest in their development.

Observations 2.4, 2.5 and 2.6 are for the two possible policies followed by the central government. It is first observed that a lack of information is much more costly than a behavioral constraint, as it leads to some firms' being misallocated across regions. Moreover, it has been seen that the constrained government would systematically *over*–invest in firms' development, as compared to the social planner's solution, while the misinformed government may over– or under–invest.

The propositions found in section 2.4 throw some lights on the circumstances in which one setting would be preferable to another one. This has been done by varying the different parameters, ρ and the β_{ij}'s.

Proposition 2.1 shows that in the extreme case when all firms generate higher externalities in the same region, the central governments can achieve the first–best in spite of the constraints. Similarly, proposition 2.2 shows that whenever firms do have the same impact in any regions the uninformed government does as well as the constrained one.

Proposition 2.3 shows that in the limit case where firms generate positive externalities in one region and none in the other, the competing governments' solution is equivalent to the social planner's one. In other words, it occurs when regions have drastically different industrial bases. Conversely, if the externalities generated by one firm in the two regions tend to be the same, there is a drastic level of under–investment made by the governments, so much so that in the limit no investment at all is being made. This is relevant for a country where all regions share a common industrial base.

Proposition 2.5 shows that the more equal are the benefits across regions to correctly allocated firms, the closer the constrained central government tend to get to the first–best solution. This is relevant if innovations generate externalities of the same magnitude in all industrial sectors.

Finally, proposition 2.4 shows that it is possible to observe less investment per firm being made by the competing governments than by an uninformed central government and yet reach a better solution. This is true because the competing regional government solution has good sorting properties.

No direct comparison of the solutions achieved by the competing governments and the constrained government has been done here. It is known that in the latter case there is a tendency to over–invest, while the tendency is to under–invest in the former case. The comparison has to be done indirectly, using the propositions developed in section 2.4, and by inferring which situation is more likely to arise.

All this suggest that a way to replicate the sorting properties of the competing government setting, without its associated rent dissipation, is for the central government to channel funding to firms through specialized incubators. If incubators are structured by industrial branches, and if their location is chosen by industrial associations, then an appropriate allocation of firms across regions can be achieved.

When more than one incubator is created by industrial branches, or when these latter are too finely defined, the central government must limit competition between incubators by imposing uniform rules. If not, then rent dissipation can occur by investing too much in developing firms. If it were to happen, the outcome may be worse than when the industrial policy is assigned to competing regional governments.

Chapter 3

Income Redistribution in a Federation when Regional Price Levels Differ

3.1 Introduction

Intergovernmental grants play an important role in federal countries. They are used to provide appropriate incentives in the presence of many kinds of either real or fiscal externalities, they serve to equalize the fiscal burden across regions, and play many other roles[1]. In this paper, it is argued that intergovernmental grants can serve one more purpose, which appears not to have been noticed before: they can be used by the federal government to achieve its income redistribution objectives across agents, irrespective of their location.

In Richard Musgrave's familiar taxonomy, there are three functions to be fulfilled by the government: to achieve economic stabilization, to reach economic efficiency, and to redistribute income across agents. Economists usually prescribe assigning the income redistribution function to the highest government level in a federation because it is thought that the mobility of the tax base would prevent regional governments

[1]See Robin Boadway and David Wildasin's texbook,[8], for a complete list of the use made of intergovernmental grants and further discussions.

from redistributing income efficiently. To quote Musgrave [40, page 12]:

> "Member jurisdictions, when forming a federation, therefore do not have a simple option of choosing between federation–wide and regional redistribution. Rather, the choice may be between central or no redistribution. At best, decentralized redistribution policy would carry a high efficiency cost. The issue between fiscal centralization and decentralization thus easily becomes one of realizing or voiding redistributive objectives. In practice, most federations seem to view redistribution policy as primarily a federation–wide issue, with lower level tax systems substantially less progressive than those of central government. Allowing for deductibility from the base of a progressive federal tax, the latter indeed tends to become regressive."

Hans Werner Sinn [49] tends to be more pessimistic as to the ability of regional governments to redistribute income[2]. Most recent contributions that have sought to determine if Musgrave is right have used the following approach[3]: they seek to determine if population mobility imposes high efficiency costs when regional governments try to redistribute income among people living on their territory. It is only in papers by Johnson [27] and by Boadway, Marchand and Vigneault [7] that the authors consider the implications of having both federal and states' governments involved in redistributing income. However, Johnson neglects potentially important strategic interactions between levels of government. Moreover, the problem is presented from a partial equilibrium perspective. In Boadway and al., unlike here, all regions are perfectly identical.

In this paper, two layers of governments are present, and Musgrave's prescription

[2]This pessimism may not be warranted, given that some income redistribution is done at the provincial level in Canada, and that welfare programmes like AFDC are administered by state governments in the U.S.A.

[3]See for example the papers by John Burbidge and Gordon Myers [11] and by Wildasin [54].

is obeyed: the income redistribution function is assigned exclusively to the federal government; furthermore, it is assumed that regional governments behave myopically with respect to population mobility, and that the relative distribution of agent's types within regions stay constant and identical in all circumstances. Admittedly, these are restrictive assumptions. However, imposing them appears necessary to obtain sharp and easily interpretable results[4].

Considering together three well accepted facts makes it easy to understand why income redistribution by the federal government necessitates the use of intergovernmental grants. These three facts are the following. First, governments restrict themselves to using a single nominal tax schedule for a very large and diverse economy, mainly, it has to be assumed, for political economy reasons[5]. Second, in large federal countries like Canada or the United States, nominal prices vary systematically across regions, even though one suspects that population mobility would guarantee that utility levels are the same in the long run for given types of workers. Non–tradeable prices are among the ones that vary the most across regions, chiefly wages and housing[6]. McMahon [37] presents some figures to the effect that costs of living may differ by up to 40% between American states. Thus, identical nominal incomes translate into widely different real incomes across regions. Third, the federal government knows that what really matters is real income net of all taxes, federal and regional. Thus, if it wants to achieve its income redistribution objective, it may want to complement its own tax schedule with grants to regional governments as a way to obtain the correct net–of–

[4]Note that Boadway and Michael Keen [6] obtain some very interesting results that do not depend on whether population is mobile or immobile. This model used here shares many similarities with Boadway and Keen's, which may make the myopic behaviour assumption more palatable.

[5]This is best thought of in terms of a voting model; see Geoffrey Brennan and James Buchanan [9]. Nevertheless, there are exceptions to such a rule, most notably the Great North in Canada and the center of Australia. Quite obviously, any limits put on the number of instruments available to a government is of great consequence for the type of results obtained.

[6]Létourneau [35] computes provincial consumer price indices which include adjustments for housing costs, for Canada.

all–tax real income distribution. Grants that vary systematically with the price level in the regions allow regional taxes to be reduced in some regions, thereby eliminating, or at least alleviating, the horizontal inequity created by the federal nominal income tax schedule.

While this problem seems obvious enough, very little has been written on it. Louis Kaplow [28] recently suggested different ways to correct for these nominal price differences, from an income redistribution perspective, but at the cost of violating the well founded political economy principle of preserving equal fiscal treatment of equals. Moreover, while discussing implicitly the American case, Kaplow never considers explicitly the federal structure of the economy, or even the presence of state governments. Oded Hochman and David Pines [25] consider the problem of financing a nation wide public good in the context of a unitary state, and with (homogeneous) population living in cities of different sizes. Nominal wages are a function of city size, but so are some prices. The use of a nominal income tax system is shown to distort some location decision, as well as the provision of local public goods.

While being close to the main concern of this paper, neither of these two papers considers the federal structure of the country, nor the potential use of intergovernmental grants. Moreover, their prescriptions to solve the problem created by differences between nominal and real incomes appear unrealistic. In both papers, it is suggested that wages earned by workers in similar occupations across regions be compared to "benchmark" local federal income tax schedules. This appears to be difficult to implement, as it could lead to extensive rent seeking, occupation "renaming" and bargaining problems. Here, it is shown that intergovernmental grants can lead to the same solution. Moreover, it could be argued that it would be much easier to implement from a political economy point of view.

To study the extent to which nominal income differences may impede the central government capacity to redistribute income across ability level, and how intergovernmental grants may be used to alleviate this problem, it is best to try and develop a model that eliminates most, if not all, other possible rationales for intergovernmental grants. It appears crucial also to model explicitly the existence of the two levels of governments, which is quite surprisingly almost never done. In this respect, this paper borrows extensively from the recent paper by Boadway and Keen [6]. A key aspect of their model is that the federal government is taken to be a leader in a non–cooperative Nash game: it sets its tax and transfer parameters first, knowing how regional governments are going to react to them.

Several other issues are crucial for the model to be sensible, and have an important impact on the nature of the results obtained. First, given that an income redistribution problem is at the heart of the question, it is essential to have a tax base that is sensitive to the tax rate in place. If not, the simple and easy solution would be to tax away all work income and redistribute it arbitrarily. Second, the overlap between the federal and regional tax bases matters. Here the problem is examined when the regions use a commodity tax base, which is more comprehensive than the federal tax base. A payroll tax base could have been chosen instead; then, some results presented here could not be obtained. Third, how changes in one government's tax rate affect the tax base of the other government is quite important, too. When a region uses a payroll tax, an increase in the payroll tax rate decreases the workforce participation rate and affects the tax revenues of the federal government. If regions use a commodity tax, which applies uniformly to everyone, it is potentially the case that changes in regional tax rates will not have any impact on the federal tax base. It is argued below that such a tax field assignment is quite sensible in the present context. This could be argued as representing one good reason why both direct and indirect tax bases are used at the same time.

All these issues must be tackled in turn. Section 3.2 presents the basic model and the unitary government's optimum. Section 3.3 presents the problems solved by the different regional and federal governments, while section 3.4 concludes.

3.2 The Basic Model

3.2.1 Preliminaries

Achieving a good balance between simplicity and realism is the main concern dictating the form taken by the model used in this paper. A few key elements are required if this model is to be able to answer the questions asked of it: there must exist some motive for income redistribution present in this economy; both government levels must have some minimal financial requirements, some reasons to tax agents; the tax base has to be sensitive to tax rates; and, finally, the distinction between nominal and real prices must be present. Ideally, the model must be such as to eliminate most other potential motives for intergovernmental grants and concentrate on the one of interest here, that is grants that allow the federal government to get closer to its desired income distribution.

There are many potential reasons why nominal price levels might differ markedly across regions even though the relative price between labour and a widely defined basket of goods is the same in all regions. For example, one may think of an economy made of a hierarchy of cities, in the fashion of urban economists and new regional economists[7]. Some cities may be involved in the production of high level services, advertising and banking for example, which benefit from being in a very large city, some others are mid–sized differentiated manufacturing cities, while others might be resource–dependent, one–industry cities in isolated or undesirable locations. In such a

[7]That is in fact what Hochman and Pines [25] do; one also needs to add that each region contains only one major city that dictates the price level for the entire region.

hierarchy of cities prices would depend on whether a good is tradeable or not. Non–tradeable goods, especially housing, would have higher prices in the more densely populated regions, while for some tradeable goods prices would be uniform across the country. While it is true that amenities may be more plentiful in the densely populated regions, which may offset partly the higher costs, nominal wages nevertheless need to be higher in those areas to compensate, and attract or keep workers at the margin. Thus, it is quite conceivable that workers of identical ability get markedly different nominal incomes from city to city and yet achieve identical utility levels. Introducing a nominal income tax schedule that does not take into account the cost of living differences would distort the location choice made by agents.

While building a model that incorporates such a hierarchy of city–states with tradeable and non–tradeable goods, and superposing to it a second layer of government may be deemed a good description of reality, it would quickly become unwieldy.

A simple way to obtain similar qualitative results while retaining tractability is obtained from the following fable: in all regions, workers are engaged in the extraction of some natural resources, which can be exported on the world market at a fixed price. The production technology for this natural resource sector is concave. There exists a unique consumption good, foodstuff, which is imported from the rest of the world, at an internationally given price, and must transit through all regions 1 through region $i-1$ to reach region i. This is assumed to be done following an "iceberg" transportation technology, i.e. the stock transported decays with distance travelled. Thus, its cost in region i is some multiple P of the price in region $i-1$. When workers are mobile, they will move from one region to another until an equal utility level is achieved in all regions.

In some respects, this model resembles the one used by Boadway and Keen, [6], which is one of the few models that deals with fiscal federalism and explicitly includes both levels of government. It differs from their model in several important ways. First, agents are not strictly identical in this model, as they have different level of distaste for work. This introduces a motive for "utility equalization" which is implemented through an income redistribution scheme. Second, here taxes affect labour supply at the extensive margin; with agents choosing between working full time or not at all; in their model the impact is felt at the intensive margin with workers deciding on the number of hours worked. Finally, this model differs from theirs in introducing some asymmetry between regions. More precisely, regions potentially differ in their nominal price levels, even though key within–region relative prices are identical across regions.

Pronounced differences exist also between this model and Johnson's [27] and Boadway et al.'s [7] models. First, Johnson does not consider strategic interactions between levels of government in any details, while Boadway and al. consider in great detail strategic interactions, but only for perfectly symmetric regions. Both use linear production technologies, and both present the work/leisure choice at the intensive margin. Finally, neither allows for intergovernmental grants.

The next subsection presents the basic characteristics of this economy, in terms of the production function, relative prices and the roles assigned to each level of government. Then, the following two subsections present in turn the agents' problems, and the unconstrained unitary government's problem.

3.2.2 The Macro Structure of the Economy

In what follows, the following notation is adopted: N^i is the population of region i, with $\bar{N}$ being the total population[8]. Each region has an identical production function $F(L^i)$, with $L^i = N^i\delta^{i*}$, where δ^{i*} is the fraction of the local population that works[9].

[8]Table 3.2.2 provides a summary of variables used in this essay.

[9]How δ^{i*} is determined is explained in the next section.

The production function is concave and of the power class[10], $F' > 0, F'' \leq 0$. Workers are paid their marginal productivity, $W^i = F'(L^i)$. Nominal per capita rents in the region equal $R^i \equiv F(L^i)/N^i - \delta^i F'(L^i) \geq 0$. Real wages and real per capita rents are equal to $w^i = F'(L^i)/P^i$ and $r^i = R^i/P^i$ respectively, for a regional price index P^i. The production function assumed here has the property that having equal real wages across regions also means that real rents are equal across regions.

variables	definitions
N	total population of the country
N_i	population in region i
L_i	workforce in region i
δ_i	fraction of the population working in region i
P_i	nominal price level in region i
$F(L_i)$	common production function
$W_i = F'(L_i)$	nominal wages in region i
w_i	real wages in region i
$R_i = F(L_i) - L_i W_i$	nominal rents in region i
$r_i = R_i/P_i$	real rents in region i
θ^F	share of rents owned by federal government
θ^R	share of rents owned by regional governments
G	nation–wide public good
g_i	publicly–provided private good in region i
T	federal income tax rate
E	transfers made to non–working agents
S_i	nominal intergovernmental grant to region i
z_i	commodity tax rate of regional government i
$\bar{c}_i$	consumption when working in region i
$\underline{c}_i$	consumption when not working in region i
τ_i	unitary government income tax rate in region i
x_i	unitary government commodity tax rate in region i
A_i	unitary government transfer to non–working agents in region i when using commodity tax

Table 3.1: definition of variables used in this model.

[10]This assumption allows both a model similar to Boadway and Keen's, and a symmetric equilibrium to the unitary government's problem. Instead, it could have been assumed that each agent owns an equal fraction of rents in each region. This latter assumption is quite commonly used in this literature. In that case, any concave production function could be used in the model.

Rents' ownership is divided into two parts: the share captured by the regional government, θ^R, and the share that goes to the federal government, θ^F, $\theta^R + \theta^F = 1$. Changes in these shares may allow some potential motives for intergovernmental grants to be isolated.

There are two levels of government in this economy, the federal and regional levels. The federal government is assigned the following functions: first, it provides a pure public good to everyone, G, and second, it seeks to redistribute income across agents. The income redistribution function is assigned to the federal government for the reasons usually mentioned in the fiscal federalism literature. The federal government has access to a single two–parameter tax function, which must be applied uniformly across the country. T is the tax rate imposed on nominal income while E is a nominal transfer made to agents who have no work income. While the federal is constrained to behave in the exact same manner in its *direct* dealings with agents, in the sense of the services it provides and the taxes it imposes — this would correspond to some notion of "procedural" horizontal equity — it faces no such constraint in its relationship with regional governments. Thus, it is assumed to have the ability to give differentiated intergovernmental grants, S^i, in that they need not be the same in per capita or aggregate terms.

Regional governments publicly provide a private good, g^i, and have access to a consumption tax base, using a tax rate z^i. The fact that regional governments provide such a private good instead of a pure regional public good matters in the sense that it eliminates one other potential motive for intergovernmental grants from this model[11]. Having instead a pure regional public good would introduce potentially some economy of scale motive for intergovernmental grants whenever the regions' population size differ

[11] This is not objectionable if regional governments' main purpose is to provide health and education services.

in equilibrium[12].

The choice of the tax base allocated to the regional governments is not without consequences. It is quite conceivable that governments have a choice between using a consumption tax base and a payroll tax base. The two tax bases may differ noticeably in their impact on some marginal choices made by agents, especially in a model where income redistribution and work/leisure consumption choice are central concerns. Here, the consumption tax base has been chosen because it is slightly more general, as is made clear in subsection 3.2.3. Regional expenditures can also be financed with the intergovernmental grants received from the federal government. Both levels of government use their share of rents to finance their expenditures.

3.2.3 Workers

In each region, there is a continuum of agents who differ in their distaste for work. The value of this distaste parameter is uniformally distributed between zero and one[13]. Agents decide whether or not to work by comparing their net–of–tax wages with the utility level obtained when not working and receiving the minimum guaranteed nominal income paid by the federal government[14]. When the federal government imposes a tax T on wage income, and the regional governments uses a sales tax z, the integrated tax rate is $T + z(1-T)$; $\bar{c} = (1-T)(1-z)w$ is the net–of–total–tax real income of an agent who works, which allows the agent to buy some composite good c; an agent who chooses not to work gets a real net income $\underline{c} = (1-z)e$, e being the real value of a welfare transfer paid by the federal government. δ is the psychological cost of working, which varies for each agent; g and G are respectively the provincially provided private

[12]However, there remains yet another motive for intergovernmental grants, in terms of differences in what Buchanan calls fiscal residua, which would create inefficient population relocation. Later on, this difficulty is going to be assumed away, quite inelegantly.

[13]This distribution assumption is of no consequence. It simply lightens the notation.

[14]Quite clearly, the discrete nature of the work/leisure decision matters for the type of results obtained with this model.

good and the federally provided public good[15].

The common utility function is assumed to be additively separable into four components: goods consumption, distaste for work, and the two publicly provided goods:

$$V(\bar{c}, \delta, g, G) = U(\bar{c}) - \delta + b(g) + B(G), \tag{3.2.1}$$

if working, and

$$V(\underline{c}, \delta, g, G) = U(\underline{c}) + b(g) + B(G), \tag{3.2.2}$$

when the agent does not work.

For given values of the parameters, an agent characterized by a distaste for work δ^* is just indifferent between working and not working:

$$\delta^* \equiv U(\bar{c}) - U(\underline{c}). \tag{3.2.3}$$

Because of the additively separable nature of the utility function, this value of δ^* does not depend directly on the publicly provided goods, but strictly on the two possible consumption levels, $\bar{c}$ and $\underline{c}$, and thus indirectly on the parameters of the income tax schedule, T and e, the sales tax rate, z, and the wage paid, w: $\delta^* = \delta^*(T, e, z, w)$.

$$\frac{\mathrm{d}\,\delta^*}{\mathrm{d}\,T} = \frac{-w(1-z)U'(\bar{c})}{1 - U'(\bar{c})(1-T)(1-z)\frac{\mathrm{d}\,w}{\mathrm{d}\delta^*}} < 0; \quad \frac{\mathrm{d}\,\delta^*}{\mathrm{d}\,e} = \frac{-(1-z)eU'(\underline{c})}{1 - U'(\bar{c})(1-T)(1-z)\frac{\mathrm{d}\,w}{\mathrm{d}\delta^*}} < 0.$$

$$\frac{\mathrm{d}\,\delta^*}{\mathrm{d}\,z} = \frac{\underline{c}\,U'(\underline{c}) - \bar{c}\,U'(\bar{c})}{(1-z)\left[1 - U'(\bar{c})(1-T)(1-z)\frac{\mathrm{d}\,w}{\mathrm{d}\delta^*}\right]}.$$

Note that $\frac{\mathrm{d}\delta^*}{\mathrm{d}E} = \frac{1}{P}\frac{\mathrm{d}\delta^*}{\mathrm{d}e}$. If the utility function displays relative risk aversion equal to unity, then $\frac{\mathrm{d}\delta^*}{\mathrm{d}z} = 0$: changes in the commodity tax rate have no impact on the work/leisure decision. In the present context, it is more natural to think in terms of

[15] One may think of the former as education or health care, while the latter would be a pure public good like justice or defense.

an aggregate labour supply elasticity with respect to commodity taxes, $\frac{z}{\delta^*}\frac{\mathrm{d}\delta^*}{\mathrm{d}_z}$; changes in z would lead to a higher or lower labour force participation rate, depending on the case. This can be contrasted with the case of regional governments using a payroll tax base; then, the sum of regional governments' tax bases would overlap perfectly with the federal tax base. In that case, the impact of changes in federal and regional tax rates would be qualitatively identical.

Quite naturally, only the real values of the parameters matter for the agents. However, in a federal context some transfers may have uniform nominal values across regions. More precisely in this case, a transfer E received in region i translates into a real transfer $e^i = E/P^i$.

The specification detailed above represents a simple form of a two–parameter tax function: E is the basic guaranteed income, while T is the tax rate applied on earned income above level E.

3.2.4 Unitary Government's Solution

The solution obtained under an unconstrained unitary government constitutes the benchmark solution. The unconstrained unitary government's problem differs from the one to be solved by the divided governments in two ways. First, all decision making is centralized, save for the work/leisure choice made by workers, and second, the unitary government is not required to follow any notion of "procedural" horizontal equity. To the contrary, it is free to impose region–specific tax rates and transfers.

The unitary government is assumed to have a utilitarian social welfare function. For convenience, the country–wide public good G is assumed to be produced solely in region 1 which has the lowest nominal price level[16].

[16]It is easy to think of other conventions that could have been adopted instead: one would be for the federal government to provide region–specific public goods, G^i's. However, it would violate the convention adopted here, for realism purposes, that the federal government is essentially constrained to behave exactly in the same way in its direct dealing with all citizens, irrespective of their location,

The choice of a tax base for the unitary government is to some extent irrelevant. To allow comparisons to be made between the unitary government's solution and that of the federal government, it is useful to write the problem as if the unitary government had access to the same income tax base, and made the same choices as the federal government, except for the fact that it is allowed to discriminate across regions. This is done first. Next, to allow comparisons to be made directly between its taxation decision and the regional governments' taxation decisions, the problem must be presented as if the unitary government's had access to a commodity tax base. This is done later.

For the case when it has access to an income tax base, its objective function is:

$$\sum_{i=1}^{I}\left[N^i\int_0^{\delta^{i*}}(U(\bar{c}^i)-\delta^i+b(g^i))\mathrm{d}\,\delta^i+\int_{\delta^{i*}}^{1}(U(\underline{c}^i)+b(g^i))\mathrm{d}\,\delta^i\right]+\bar{N}B(G),$$

where $\bar{c}^i = (1-\tau^i)w^i$ and $\underline{c}^i = e^i$. τ^i is the income tax. Then, δ^{i*} is a function of τ^i, w^i, e^i: $\delta^{i*}(\tau^i, w^i, e^i)$. Overall budget balance for the government requires:

$$\sum_{i=1}^{I} N^i P^i\left[\tau^i\delta^{i*}w^i + r^i - g^i - (1-\delta^{i*})e^i\right] = G. \tag{3.2.4}$$

The unitary government has access to the sum of rents taxable by the regional and federal governments, $\theta^R + \theta^F = 1$.

The concept of horizontal equity is implemented in the following way: the unitary government's budget constraint is broken down into I regional budget constraints. Each region contributes a fraction γ^i of the public good cost, G, the fraction γ^i being chosen in such a way that the real resource cost per capita in all regions is identical. This would leave the same level of real per capita resources in each region to finance consumption $\bar{c}^i, \underline{c}^i$ and public spending g^i, in per capita terms.

The unitary government also knows that population will move across regions until utility levels are equal across regions. It thus chooses a tax and transfer scheme to

both on the tax and expenditure side. A second one would be to assume that the public good G is produced using labour coming from regions in proportions equal to their population weight. This alternative specification is not pursued here.

accommodate this behaviour which in any case does not conflict with its own objective. The location decision of the agents is assumed not to change the distribution of agents' type within regions: all regions contain the same proportion of "lazy" and "hard–working" agents, as determined by their individual δ coefficient[17].

Thus, a unitary government solves the following problem: choose variables τ^i, e^i, G, and g^i to solve:

$$\max_{\tau^i,e^i,G,g^i} \sum_{i=1}^{I} N^i \left[\int_0^{\delta^{i*}} (U(\bar{c}^i) - \delta^i + b(g^i)) \mathrm{d}\,\delta^i + \int_{\delta^{i*}}^{1} (U(\underline{c}^i) + b(g^i)) \mathrm{d}\,\delta^i \right] + \bar{N}B(G) \tag{3.2.5}$$

subject to the constraints:

$$(\lambda^i) \qquad \tau^i \delta^{i*} w^i + r^i - g^i - (1-\delta^{i*})e^i \geq \frac{\gamma^i G}{N^i P^i}; \tag{3.2.6}$$

$$(\lambda^N) \qquad \sum_{i=1}^{I} N^i - \bar{N} = 0. \tag{3.2.7}$$

The first–order conditions for this problem are[18]:

$$\begin{aligned} &N^i \delta^{i*} U'(\bar{c}^i) \left[w^i - (1-\tau^i) \tfrac{\partial w^i}{\partial \delta^{i*}} \tfrac{\mathrm{d}\,\delta^{i*}}{\mathrm{d}\,\tau^i} \right] \\ &= \lambda^i \left[\delta^{i*} w^i + \left[\tau^i \left(w^i + \delta^{i*} \tfrac{\partial w^i}{\partial \delta^{i*}} \right) + e^i + \tfrac{\partial r^i}{\partial \delta^{i*}} \right] \tfrac{\mathrm{d}\,\delta^{i*}}{\mathrm{d}\,\tau^i} \right]; \end{aligned} \tag{3.2.8}$$

$$\begin{aligned} &N^i \delta^{i*} U'(\bar{c}^i)(1-\tau^i) \tfrac{\partial w^i}{\partial \delta^{i*}} \tfrac{\mathrm{d}\,\delta^{i*}}{\mathrm{d}\,e^i} + N^i(1-\delta^{i*})U'(\underline{c}^i) \\ &= \lambda^i \left[(1-\delta^{i*}) - \left[\tau^i \left(w^i + \delta^{i*} \tfrac{\partial w^i}{\partial \delta^{i*}} \right) + e^i + \tfrac{\partial r^i}{\partial \delta^{i*}} \right] \tfrac{\mathrm{d}\,\delta^{i*}}{\mathrm{d}\,e^i} \right]; \end{aligned} \tag{3.2.9}$$

$$\bar{N}B'(G) - \sum_{i=1}^{I} \frac{\lambda^i \gamma^i}{N^i P^i} = 0; \tag{3.2.10}$$

$$N^i b'(g^i) - \lambda^i = 0; \tag{3.2.11}$$

[17]As mentioned in the introduction, it is not an objective of the present paper to contribute to the literature on income redistribution within regions. Without this fixed within–region distribution assumption, all the margins would have to adjust, and the argument would not be as clear as it could be.

[18]Simplified by repeatedly using condition 3.2.3 which characterizes the work/leisure choice made by agents.

Now, if the unitary government uses instead a commodity tax base, with tax rate x^i, and makes a transfer A^i to non–working agents, it has region–specific constraints:

$$(\lambda^{ci}) \qquad x^i\left[\delta^{i*}w^i + (1-\delta^{i*})A^i\right] + r^i - g^i - (1-\delta^{i*})A^i \geq \frac{\gamma^i G}{N^i P^i}. \qquad (3.2.12)$$

Given this new tax base, consumption levels by agents become $\bar{c}^i = (1-x^i)w^i$ and $\underline{c}^i = (1-x^i)A^i$. The first–order condition for the choice of the tax rate is:

$$\begin{aligned}&\delta^{i*}\bar{c}^i\left[U'(\bar{c}^i) - b'(g^i)\right] + (1-\delta^{i*})\underline{c}^i\left[U'(\underline{c}^i) - b'(g^i)\right] = U'(\bar{c}^i)(1-x^i)^2\tfrac{\mathrm{d}\delta^{i*}}{\mathrm{d}x^i}\\ &+b'(g^i)\left[\delta^{i*}\tfrac{\partial w^i}{\partial\delta^{i*}} + x^i\left(w^i + \delta^{i*}\tfrac{\partial w^i}{\partial\delta^{i*}}\right) + (1-x^i)A^i + \tfrac{\partial r^i}{\partial\delta^{i*}}\right](1-x^i)\tfrac{\mathrm{d}\delta^{i*}}{\mathrm{d}x^i}.\end{aligned} \qquad (3.2.13)$$

It is quite clear that by choosing $x^i = \tau^i, A^i = e^i/(1-x^i)$ and setting all the other parameters to the same value, whatever solution is achieved with an income tax base can be replicated with a commodity tax base.

Return now to the income tax base case. A symmetric optimum can be obtained from these $3I+1$ first–order conditions[19] in the following manner: assign to region i a cost–share $\gamma^i = P^i/\sum^i P^i$; let the agents locate across regions in inverse relation to the regional price level, such that real wages *and* per–capita rents are equal, which is possible with a power production function; then from conditions 3.2.10 and 3.2.11 one obtains a version of the Samuelson rule:

$$\bar{N}B'(\hat{G}) = \frac{1}{\sum_i P^i}\left[\sum_i b(\hat{g}^i)\right] = \frac{Ib'(\hat{g})}{\sum_i P^i} = \frac{b'(\hat{g})}{\bar{P}}, \qquad (3.2.14)$$

where $\bar{P}$ is the average nominal price level across regions. For the case where the population is so distributed across the regions, it is possible to obtain that $\delta^{i*} = \delta^*, \forall\, i$ by setting both $e^i = \hat{e}$ and $\tau^i = \hat{\tau}, \forall\, i$ and thus obtain a perfectly symmetric solution in terms of real wages, real per–capita rents and labour force participation rate. This involves making real direct transfers $E^i = \hat{e}P^i$ to region i's non–working agents.

[19]Clearly, there are only $2I-1$ independent parameters.

From condition 3.2.8, it is possible to determine implicitly the optimal value $\hat{\tau}$:

$$\frac{U'(\hat{c})}{b'(\hat{g})} = \frac{\left[\delta^* w + \left[\hat{\tau}\left(w + \delta^* \frac{\partial w}{\partial \delta^*}\right) + \hat{e} + \frac{\partial r^i}{\partial \delta^*}\right] \frac{\mathrm{d}\,\delta^{i*}}{\mathrm{d}\,\hat{\tau}}\right]}{\delta^* \left[w - (1 - \hat{\tau}) \frac{\partial w}{\partial \delta^*} \frac{\mathrm{d}\,\delta^*}{\mathrm{d}\,\hat{\tau}}\right]}. \tag{3.2.15}$$

From condition 3.2.9 one obtains:

$$\begin{aligned} &\frac{\delta^*(1-\hat{\tau}) \frac{\partial w}{\partial \delta^*} \frac{\mathrm{d}\,\delta^*}{\mathrm{d}\,\hat{e}} U'(\hat{c}) + (1-\delta^*) U'(\underline{c})}{b'(\hat{g})} \\ &= (1 - \delta^*) - \left[\hat{\tau}\left(w + \delta^* \frac{\partial w}{\partial \delta^*}\right) + \hat{e} + \frac{\partial r^i}{\partial \delta^*}\right] \frac{\mathrm{d}\,\delta^*}{\mathrm{d}\,\hat{e}}. \end{aligned} \tag{3.2.16}$$

These conditions are not exactly transparent. They both correspond to some idea of marginal cost of public expenditures. That is, there are essentially two types of public expenditures in this model: on welfare, which has an impact on the labour force participation decision, and on goods g and G, which has no impact on participation rates by the separability assumptions imposed on the utility function. Changes in the tax rate τ or non–work benefits e have an impact on the labour force participation rate in the region, which in turn changes the wage rate and the rents obtained by the unitary government. In the simple case of a common linear production technology[20], 3.2.15 and 3.2.16 may be rewritten as:

$$\frac{U'(\hat{c})}{b'(\hat{g})} = \left[1 + \frac{\hat{\tau} w + \hat{e}}{\delta^* w} \frac{\mathrm{d}\,\delta^*}{\mathrm{d}\,\hat{\tau}}\right]; \tag{3.2.17}$$

and

$$\frac{U'(\hat{\underline{c}})}{b'(\hat{g})} = \left[1 - \frac{\hat{\tau} w + \hat{e}}{1 - \delta^*} \frac{\mathrm{d}\,\delta^*}{\mathrm{d}\,\hat{e}}\right]. \tag{3.2.18}$$

These two conditions are considerably simpler to explain. Condition 3.2.17 simply says that, at the optimum, expenditures on the publicly provided private good g must be such that its marginal benefits is inferior to the marginal benefits of extra consumption by working agents, the gains from higher tax being equal to $\delta^* w$ but at the cost of

[20]Note that in such a case the optimum solution would also entail locating all workers in the region with the lowest nominal price level P.

reduced labour force participation, which has a cost at the margin of $\hat{\tau}w + \hat{e}$ in terms of lost tax income and increased welfare expenditures.

Similarly, from condition 3.2.18, it can be seen that at the optimum the marginal utility from welfare expenditures is lower than the marginal utility of the good g, increased spending on welfare bringing benefits to $1 - \delta^*$ non–working agents at the cost of increasing the welfare load and reducing tax income by $\hat{\tau}w + \hat{e}$. Finally, it can be said that with a strongly concave production technology, the interpretation of the those marginal costs of public expenditures remains essentially the same, but for the fact that changes in wage rates and per capita rents complicate their interpretation. From the unitary government's solution the following proposition may be obtained:

Proposition 3.1 *When horizontal equity is a paramount concern of the unitary government, an optimal solution to the income redistribution problem in a federation entails having only one common tax function* in real terms *imposed on all regions of the country.*

This means that a common tax rate $\hat{\tau}$ is used everywhere, and that nominal transfers $E^i = \hat{e}P^i$ are adjusted to take into account differences in nominal prices.

Kaplow [28] comes to the same conclusion in his paper[21]. Moreover, both he and Hochman and Pines [25] suggest that a practical way to implement this optimal scheme would be to compare the nominal wages of workers in similar occupations across regions to compute relative cost of living indices and determine the size of the adjustments made locally to the basic deduction E^i of the tax schedule. This prescription appears unrealistic for political economy reasons, as it would lead to too complex a negotiation process involving too many parties. Moreover, it appears to be too close to a "pie–division" problem to be conducted efficiently in a democracy. In the next section, it is shown that an equivalent solution can be achieved in some circumstances in

[21]The same lesson could also be derived indirectly from the paper by Hochman and Pines [25].

a decentralized setting by using intergovernmental grants. Such grants have as clear advantages that, first, they allow the federal government to adhere to the principle of equal treatment of equals in its direct dealing with citizens, and, second, that negotiation over the grants would involve a much smaller number of parties.

To complete this discussion, it can be noted that the unitary government's solution described above constitutes an optimal solution in a restricted sense only. It is quite clear that the government would do better by asking all non–working agents —those for whom $\delta^i \geq \delta^{i*}$— to move to the region that has the lowest nominal price level. In this fashion, the cost of supporting them would be kept to a minimum. However, this possibility has been ruled out with the assumption that all regions have the same distribution of agents' type.

3.3 Governments' Problems

Following the lead of Boadway and Keen [6], it appears sensible to model the federal government as being a leader in a game played with regional governments. This Stackelberg assumption means that the federal government sets its policy parameters first, knowing how regional governments will react. It can then use its different instruments, tax rates and lump–sum transfers, to manipulate regional governments' choices of tax and spending parameters. If regions differ, the principle of horizontal equity may prevent the federal government from achieving the unitary government's solution.

To understand the problem to be solved here, it may be useful to make an analogy with the classical industrial organisation problem of the monopolist dealing with resellers having exclusive territories. The monopolist must prevent harmful double marginalization by choosing carefully its own wholesale price and side–payments such as to obtain the profit–maximising retail price. Here, the federal government chooses its own tax rate and intergovernmental grants to obtain the correct marginal conditions.

To complete the analogy, nominal price differences across regions create difficulty when the monopolist is forced to charge the same wholesale price to all its resellers.

3.3.1 Regional Governments

The I regional governments have a simple objective function: they maximize the utility level of their residents by choosing a commodity tax rate, z, and the level of per capita government expenditure, g, taking as given the behaviour of the federal government, and neglecting the potential mobility of the workers. Given that relative price within one region are constant, it is possible to examine this problem directly in real terms. Because all regions have the same real characteristics, one may consider the case of one region as representative of the problem solved by all.

Remember that as far as the regional government is concerned, only federal transfers to it, $s^i = S^i/P^i$, and to non–working agents, $e^i = E/P^i$, have to be deflated; otherwise, all other variables can be considered in real terms. Regional government i solves:

$$\max_{z^i} \int_0^{\delta^{i*}} \left[U(\bar{c}^i) - \delta^i + b(g^i)\right] \mathrm{d}\delta^i + \int_{\delta^{i*}}^1 \left[U(\underline{c}^i) + b(g^i)\right] \mathrm{d}\delta^i, \tag{3.3.1}$$

subject to the budget constraint:

$$g^i = z^i \left[\delta^{i*}(1-T)w^i + (1-\delta^{i*})e^i\right] + \frac{s^i}{N^i} + \theta^R r^i \tag{3.3.2}$$

The first–order condition becomes:

$$\begin{aligned}&\delta^{i*}\bar{c}^i \left[U'(\bar{c}^i) - b'(g^i)\right] + (1-\delta^{i*})\underline{c}^i \left[U'(\underline{c}^i) - b'(g^i)\right] = U'(\bar{c}^i)(1-T)(1-z^i)^2 \tfrac{\mathrm{d}\delta^{i*}}{\mathrm{d}z^i} \\ &+b'(g^i)\left[z^i(1-T)\left(w^i + \delta^{i*}\tfrac{\partial w^i}{\partial \delta^{i*}}\right) - z^i e^i + \theta^R \tfrac{\partial r^i}{\partial \delta^{i*}}\right](1-z^i)\tfrac{\mathrm{d}\delta^{i*}}{\mathrm{d}z^i}.\end{aligned} \tag{3.3.3}$$

This expression is markedly different from the one found for the unitary government's solution using a commodity tax, 3.2.13. First, the regional government cares only about its own share of the rents, θ^R, and not about the share owned by the federal government. Moreover, it cares only about its own tax rate and tax revenues, not those of the federal government. Also, it does not take into account the cost of a higher welfare load when tax rate z is increased. Note that the sign of the right–hand side expression depends on the impact that changes in commodity tax have on labour force participation. In the case where they have no impact, for $\frac{d\delta^{i*}}{dz^i} = 0$, then at the margin none of the considerations just mentioned has an impact on the regional government's decision. It merely chooses its level of spending on g^i such that its marginal benefits are equal on average to private consumption's marginal benefits. To conclude, it is generally the case that 3.3.3 and 3.2.13 are different, which leads to the proposition:

Proposition 3.2 *The marginal cost of public funds for the regional government diverges from the optimal level found in the unitary government problem.*

This proposition means that the federal government must attempt to manipulate the regions' choice if it wants to be able to replicate the unitary government's solution. Given that there is only one federal government and several regional governments, that the former is often much more important in terms of expenditures than any regional government might be, the federal government is assumed to have the benefit of being a "first–mover". That is, it is assumed to be a Stackelberg leader, which makes it possible to write the region's choice variables as functions of the federal government's choice, $g^i(G, T, E, S^i)$ and $z^i(G, T, E, S^i)$, using 3.3.2 and 3.3.3 together.

3.3.2 Federal Government

Consider now the problem the federal government has to solve for a I–region country. It must use its two tax rate parameters T and E, uniform across regions, its ability to make differential grants to regional governments, S^i, and its spending power for the federal public good such as to redistribute income, or increase some agents' utility level, for people with different distaste for work. When real prices are different across regions, the federal government's inability to make different nominal transfers to agents may prevent it from replicating the unitary government's solution.

The problem to be solved is similar to the first one considered for the unitary government. The federal government has a different set of instruments at its disposition; in fact, it has $3+I$ instruments to choose directly, the tax rate T, the expenditure level E and public good G, plus the I transfers S^i. Control over the $2I$ instruments z^i and g^i is in the hands of the regional governments. Thus, the federal government must find a way to manipulate the regional governments' choice of these instruments to replicate the unitary government's solution. Agents' choice over their location will dictate *ex–post* that utility levels are the same across locations. Given that the federal government seeks to achieve an horizontally equitable solution, that is such that all agents of a given type achieve the same level of utility in all regions, the *ex–post* solution and the desired solution can be the same. Thus, the government maximizes the objective function:

$$\max_{T,E,G,S^i} \sum_{i=1}^{I} N^i \left[\int_0^{\delta^{i*}} \left[U(\bar{c}^i) - \delta^i + b(g^i) \right] \mathrm{d}\,\delta^i + \int_{\delta^{i*}}^{1} \left[U(\underline{c}^i) + b(g^i) \right] \mathrm{d}\,\delta^i \right] + \bar{N} B(G), \tag{3.3.4}$$

subject to the constraints that all revenues raised in one region is spent in that region, save for the share γ^i of public good G that each region contributes:

$$(\lambda^i) \qquad T\delta^{i*} w^i P^i + \theta^F P^i r^i - S^i - (1-\delta^{i*})E \geq \gamma^i G. \tag{3.3.5}$$

The first–order conditions[22] become:

$$\begin{aligned} &\textstyle\sum_{i=1}^{I} N^i \left[\delta^{i*} U'(\bar{c}^i) \left[(1-z^i) w^i - (1-T)(1-z^i) \frac{\partial w^i}{\partial \delta^{i*}} \frac{\mathrm{d}\,\delta^{i*}}{\mathrm{d}\,T} \right] - b'(g^i) g^i_T \right] \\ &= \textstyle\sum_{i=1}^{I} \lambda^i P^i \left\{ \delta^{i*} w^i + \left[T\left(w^i + \delta^{i*} \frac{\partial w^i}{\partial \delta^{i*}} \right) + \frac{E}{P^i} + \theta^F \frac{\partial r^i}{\partial \delta^{i*}} \right] \left(\frac{\mathrm{d}\,\delta^{i*}}{\mathrm{d}\,T} + \frac{\mathrm{d}\,\delta^{i*}}{\mathrm{d}\,z^i} z^i_T \right) \right\}; \end{aligned} \tag{3.3.6}$$

$$\begin{aligned} &\textstyle\sum_{i=1}^{I} N^i \left[\delta^{i*} U'(\bar{c}^i)(1-T)(1-z^i) \frac{\partial w^i}{\partial \delta^{i*}} \frac{\mathrm{d}\,\delta^{i*}}{\mathrm{d}\,E} + (1-\delta^{i*})(1-z^i) U'(\underline{c}^i) + b'(g^i) g^i_E \right] \\ &= \textstyle\sum_{i=1}^{I} \lambda^i P^i \left\{ \frac{(1-\delta^{i*})}{P^i} - \left[T\left(w^i + \delta^{i*} \frac{\partial w^i}{\partial \delta^{i*}} \right) + \frac{E}{P^i} + \theta^F \frac{\partial r^i}{\partial \delta^{i*}} \right] \left(\frac{\mathrm{d}\,\delta^{i*}}{\mathrm{d}\,E} + \frac{\mathrm{d}\,\delta^{i*}}{\mathrm{d}\,z^i} z^i_E \right) \right\}; \end{aligned} \tag{3.3.7}$$

[22]Simplified by using repeatedly both the solutions to the regional government problem, 3.3.3, and the agents' problems, 3.2.3

$$\bar{N}B'(G) = \sum_{i=1}^{I} \lambda^i \gamma^i; \tag{3.3.8}$$

$$N^i b'(g^i) g^i_{S^i} = \lambda^i + \lambda^i P^i \left[T \left(w^i + \delta^{i*} \frac{\partial w^i}{\partial \delta^{i*}} \right) + \frac{E}{P^i} + \theta^F \frac{\partial r^i}{\partial \delta^{i*}} \right] \frac{\mathrm{d}\, \delta^{i*}}{\mathrm{d}\, z^i} z^i_{S^i}. \tag{3.3.9}$$

These conditions 3.3.6–3.3.9 differ in a few ways from the unitary government's ones, 3.2.8–3.2.11. First, they display a dependence on P^i that is quite different. Second, there is an interdependence between budget constraints, as the presence of terms like g^i_T, g^i_E and $g^i_{S^i}$ show. Third, the term $\frac{\mathrm{d}\, \delta^{i*}}{\mathrm{d}\, z^i}$ appears repeatedly; this third difference is the crucial one, really. It shows how changes in regional tax parameters may affect the federal tax base. In what follows, isolating its tax base from actions taken by regional governments turns out to be a major intermediary objective of the federal government.

Before tackling the complete problem when nominal prices differ across regions, consider first the simpler case where all price levels are the same in the economy and, furthermore, normalized to be equal to one, $P = 1$. Then, it can be seen that the conditions 3.3.8 and 3.3.9 turn out to be equivalent to the optimal Samuelson condition 3.2.14 if the second term on the right–hand side of 3.3.9 is equal to zero.

Two ways to set that second term equal to zero are explored here. The first is to set the term in square brackets equal to zero, while the second involves the possibility that the aggregate labour supply elasticity is equal to zero, that is, $\frac{\mathrm{d}\, \delta^{i*}}{\mathrm{d}\, z^i} = 0$. The relevance of these two possible solutions differ. In the first case, the term in square brackets depends directly on the instruments T, E, and θ^F that are directly under federal government control. They are amenable to direct manipulation. In the other case, the aggregate labour supply elasticity depends on the preferences of the agents. It is a primitive of the economy, and as such not directly controllable by the government. In other words, any solution to the federal government problem obtained by the first method is more generally applicable than the one found using the second method. The former possibility is explored first; the other method is explored below.

Consider what happens if T is chosen such as to make the expression in square brackets equal to zero:

$$T = \frac{-(E + \theta^F \frac{\partial r^i}{\partial \delta^{i*}})}{w^i + \delta^{i*}\frac{\partial w^i}{\partial \delta^{i*}}}. \tag{3.3.10}$$

Then, from 3.3.8 and 3.3.9, and using the value of $g^i_{S^i}$,B.0.4, the optimal Samuleson condition obtains[23]:

$$\bar{N}B'(G) = \frac{Ib'(g^i)}{I} = b'(\hat{g}). \tag{3.3.11}$$

Substituting in 3.3.6 and 3.3.7 the value for g^i_T and g^i_E, B.0.2 and B.0.3, and adding and subtracting $T(w^i + \delta^{i*}\frac{\partial w^i}{\partial \delta^{i*}})$ to their right–hand side, the two expressions can be rewritten as:

$$\begin{aligned} &\textstyle\sum_{i=1}^{I} N^i \left[\delta^{i*}U'(\bar{c}^i)\left[w^i - (1-T)\frac{\partial w^i}{\partial \delta^{i*}}\frac{\mathrm{d}\,\delta^{i*}}{\mathrm{d}\,T}\right]\right] \\ &= \textstyle\sum_{i=1}^{I} \lambda^i \left[\delta^{i*}w^i + \left[\frac{(T+z^i(1-T))}{(1-z^i)}(w^i + \delta^{i*}\frac{\partial w^i}{\partial \delta^{i*}}) + \frac{c^i}{(1-z^i)} + \frac{\theta^R+\theta^F}{(1-z^i)}\frac{\partial r^i}{\partial \delta^{i*}}\right]\frac{\mathrm{d}\,\delta^{i*}}{\mathrm{d}\,T}\right]; \end{aligned} \tag{3.3.12}$$

and

$$\begin{aligned} &\textstyle\sum_{i=1}^{I} N^i \left[\delta^{i*}U'(\bar{c}^i)(1-T)\frac{\partial w^i}{\partial \delta^{i*}}\frac{\mathrm{d}\,\delta^{i*}}{\mathrm{d}\,E} + (1-\delta^{i*})U'(\underline{c}^i)\right] \\ &= \textstyle\sum_{i=1}^{i} \lambda^i \left[(1-\delta^{i*}) - \left[\frac{(T+z^i(1-T))}{(1-z^i)}(w^i + \delta^{i*}\frac{\partial w^i}{\partial \delta^{i*}}) + \frac{\underline{c}}{(1-z^i)} + \frac{\theta^R+\theta^F}{(1-z^i)}\frac{\partial r^i}{\partial \delta^{i*}}\right]\frac{\mathrm{d}\,\delta^{i*}}{\mathrm{d}\,T}\right]. \end{aligned} \tag{3.3.13}$$

Now, given that all regions are identical, that the sum of tax rates paid by an agent working in this decentralized setting are $T + z^i(1-T)$, that $\theta^R + \theta^F = 1$, and that

$$\frac{\mathrm{d}\,\delta^{i*}}{\mathrm{d}\,T} = (1-z^i)\frac{\mathrm{d}\,\delta^{i*}}{\mathrm{d}\,\tau}, \tag{3.3.14}$$

when $(1-T)(1-z) = (1-\tau)$, it can be seen that the two expressions above are equivalent to the sum over the I regions of the unitary government's optimal conditions 3.2.8 and 3.2.9. Similarly, using the same tax rate T, adding and subtracting $T\left(w^i + \delta^{i*}\frac{\partial w^i}{\partial \delta^{i*}}\right)$ from the second element on the right–hand side of the expression 3.3.3,

[23]Expressions B.0.2, B.0.3, and B.0.4 are derived in B.

the first–order condition describing the choice of expenditure level g^i by each regional government becomes identical to the unitary government's version of it, 3.2.13:

$$\begin{aligned}\delta^{i*}\bar{c}^i\left[U'(\bar{c}^i)-b'(g^i)\right]+(1-\delta^{i*})\underline{c}^i\left[U'(\underline{c}^i)-b'(g^i)\right] &= U'(\bar{c}^i)(1-T)(1-z^i)^2\frac{\mathrm{d}\delta^{i*}}{\mathrm{d}z^i} \\ +b'(g^i)\left[[T+z^i(1-T)]\left[w^i+\delta^{i*}\frac{\partial w^i}{\partial\delta^{i*}}\right]+\underline{c}^i+(\theta^R+\theta^F)\frac{\partial r^i}{\partial\delta^{i*}}\right](1-z^i)\frac{\mathrm{d}\delta^{i*}}{\mathrm{d}z^i}, & \end{aligned} \tag{3.3.15}$$

given that

$$\frac{\mathrm{d}\delta^{i*}}{\mathrm{d}z^i}=\frac{(1-x^i)}{(1-z^i)}\frac{\mathrm{d}\delta^{i*}}{\mathrm{d}x^i}, \tag{3.3.16}$$

when consumption levels and tax rates are equivalent to what they are in the unitary government's solution.

Thus, equations 3.3.12–3.3.16 together show that the federal government can replicate the unitary government's solution. This leads to the following proposition:

Proposition 3.3 *When the federal government behaves as a Stackelberg leader, and when nominal prices are identical across regions, there exists a tax and transfer scheme that allows the federal government to replicate the unitary government's second–best solution.*

The optimal policy may be implemented with the federal government giving a subsidy to working agents. The subsidy paid is larger than the transfer made to agents not working. Equal intergovernmental grants are received by regional governments.

Note that in the decentralized case, the transfer made to non–working agents has to be grossed up by the taxes paid locally. Thus, in the decentralized setting, $e/(1-z)=\hat{e}$.

The key to understanding this result is to see that the federal government has been able to choose its tax rate in such a way as to delegate all tax decisions at the margin to the regional governments. They have the incentive to set their tax rates at the required level because the value of keeping agents working has been increased by the federal subsidy T. This can be seen more clearly when the production function is linear. Then, the federal tax rate is $T=-e/w$, which is equivalent to giving an equal lump–sum subsidy to everyone; all taxation decisions at the margin are delegated to the regions, and they have the same incentive as the unitary government does to keep as many agents working as possible. The parallels with the problem of a monopolist dealing with exclusive–territory resellers are quite clear: all marginal decisions are left to resellers, and the profits are extracted using lump–sum payments.

Proposition 3.3 is similar to propositions 2 and 3 obtained by Boadway and Keen [6] in a slightly different model. However, note that, contrary to their result, the value of the subsidy in not just dependent on rents. For example, if all rents were to remain in the hands of the regional governments, there would remain a motive for subsidies, while it would disappear in Boadway and Keen's model. This is because regional governments, when choosing their tax rates, do not take into account the double impact it has on the federal tax base when it changes the labour force participation decision. First, it has an impact in terms of rents obtained, and regional governments underestimate this aspect because they are not the sole beneficiaries of rents; second, the impact is also felt in terms of lower tax revenues and higher welfare transfers, $Tw + e$. This second motive would not disappear even if all rents were kept by regional governments.

Similarly, it is also the case that in this model it is impossible to ascertain in which direction intergovernmental grants would flow. It is however obvious that it takes the same value for every region. It might be the case that the share of rents obtained by the federal government, θ^F, is sufficient to cover all expenditures, in terms of work subsidies, welfare transfers and public good expenditures. However, for the case when the federal government is deprived of all other sources of income, $\theta^F = 0$, as in Boadway and Keen, it is possible to determine that intergovernmental grants flow from the regions to the center.

Now, consider what happens when nominal price levels differ across regions. The scheme suggested above does not work anymore, for the obvious reason that it becomes impossible with a single parameter T to isolate the federal government tax base from the action of all regional governments simultaneously.

$$T = \frac{-(\frac{E}{P^i} + \theta^F \frac{\partial r^i}{\partial \delta^{i*}})}{w^i + \delta^{i*} \frac{\partial w^i}{\partial \delta^{i*}}} \Rightarrow T \neq \frac{-(\frac{E}{P^j} + \theta^F \frac{\partial r^j}{\partial \delta^{j*}})}{w^j + \delta^{j*} \frac{\partial w^j}{\partial \delta^{j*}}}. \tag{3.3.17}$$

The source of the required adjustment is unlikely to come from the other parameters in this expression, although it appears difficult to rule it out completely; different real wages and real rents across regions would surely lead to countervailing population movement, which in turn would surely recreate the dependence of the federal tax base on the taxation decisions of the regions.

Now, given that the more general method fails in some circumstances, it is worth exploring what happens when the labour participation rate is not sensitive to the commodity tax rate. Looking again at the first order conditions 3.3.8 and 3.3.9, reproduced below, it is clear that the desired Samuelson condition might be obtained in a different manner:

$$\bar{N}B'(G) = \sum_{i=1}^{I} \lambda^i \gamma^i;$$

$$N^i b'(g^i) g^i_{S^i} = \lambda^i + \lambda^i P^i \left[T \left(w^i + \delta^{i*} \frac{\partial w^i}{\partial \delta^{i*}} \right) + \frac{E}{P^i} + \theta^F \frac{\partial r^i}{\partial \delta^{i*}} \right] \frac{\mathrm{d}\, \delta^{i*}}{\mathrm{d}\, z^i} z^i_{S^i}.$$

If it is the case that $\frac{\mathrm{d}\,\delta^{i*}}{\mathrm{d}\,z^i} = 0$, then the federal tax base is independent of the taxation decisions made by regional governments. When nominal prices are equal everywhere, the optimal unitary government's solution can be obtained for a wider range of possible tax and transfers parameters. That is, the value of T is not determined anymore in the same fashion as before, and several combinations of z, T and E are consistent with an optimal solution, given that z can always be manipulated by the choice of S. Note for instance that having $z = 0$ is one potential solution. This in turns means that the direction of the intergovernmental grants is of no import in this admittedly quite special case.

When price levels are different, and $\frac{d\delta^{i*}}{dz^i} = 0$, then a 'third–best' solution can be obtained in the following manner. In that case, the federal government problem comes essentially from its inability to offer differentiated nominal welfare payments across regions. An indirect way to obtain the same result is by giving differentiated subsidies to regions to lower the commodity tax rate imposed in some regions such that the *after tax* real welfare payments are identical. That is, region i's overall tax schedule, made of the two parameters $(1-z^i)T$ and $(1-z^i)E$, can be shifted up and down by changing the tax rate z^i. This allows the federal government to obtain a 'third–best' solution such that

$$\bar{N}B'(\tilde{G}) = \frac{b'(\tilde{g})}{\bar{P}}, \tag{3.3.18}$$

and

$$\begin{aligned}&\textstyle\sum_{i=1}^{I} \tilde{N}^i \left[\tilde{\delta}^{i*}U'(\tilde{\bar{c}}^i)\left[\tilde{w}^i - (1-T)\frac{\partial\tilde{w}^i}{\partial\tilde{\delta}^{i*}}\frac{d\tilde{\delta}^{i*}}{dT}\right]\right] \\ &= \textstyle\sum_{i=1}^{I} \lambda^i \left[\tilde{\delta}^{i*}\tilde{w}^i + \left[\frac{(T+z^i(1-T))}{(1-z^i)}(\tilde{w}^i + \tilde{\delta}^{i*}\frac{\partial\tilde{w}^i}{\partial\tilde{\delta}^{i*}}) + \tilde{e}^i + \frac{1}{(1-z^i)}\frac{\partial\tilde{r}^i}{\partial\tilde{\delta}^{i*}}\right]\frac{d\tilde{\delta}^{i*}}{dT}\right];\end{aligned} \tag{3.3.19}$$

and

$$\begin{aligned}&\textstyle\sum_{i=1}^{I} \tilde{N}^i \left[\tilde{\delta}^{i*}U'(\tilde{\bar{c}}^i)(1-T)\frac{\partial\tilde{w}^i}{\partial\tilde{\delta}^{i*}}\frac{d\tilde{\delta}^{i*}}{dE} + (1-\tilde{\delta}^{i*})U'(\tilde{\underline{c}}^i)\right] \\ &= \textstyle\sum_{i=1}^{i} \lambda^i \left[(1-\tilde{\delta}^{i*}) - \left[\frac{(T+z^i(1-T))}{(1-z^i)}(\tilde{w}^i + \tilde{\delta}^{i*}\frac{\partial\tilde{w}^i}{\partial\tilde{\delta}^{i*}}) + \tilde{e}^i + \frac{1}{(1-z^i)}\frac{\partial\tilde{r}^i}{\partial\tilde{\delta}^{i*}}\right]\frac{d\tilde{\delta}^{i*}}{dT}\right].\end{aligned} \tag{3.3.20}$$

This constitutes a third-best because in the presence of nominal price differences the federal government can not impose perfect horizontal equity and maintain the correct allocation of workers simultaneously. To obtain that real after-tax welfare payments are identical everywhere can be achieved only by manipulating the tax rates z^i. In other words, one can get

$$\frac{(1-z^i)E}{P^i} = \frac{(1-z^j)E}{P^j} \tag{3.3.21}$$

only by manipulating regional tax rates z^k, by the choice of transfers S^k, in such a way that $\frac{(1-z^i)}{P^i} = \frac{(1-z^j)}{P^j}$. However, this can not be done without having at the same time identical nominal wage rates being paid to working agents, if they are to have the same after tax real wages

$$\frac{(1-z^i)(1-T)W^i}{P^i} = \frac{(1-z^j)(1-T)W^j}{P^j} \Rightarrow W^i = W^j, \tag{3.3.22}$$

when $\frac{(1-z^i)}{P^i} = \frac{(1-z^j)}{P^j}$. This is inefficient. An efficient solution calls for nominal gross wages to be proportional to the local price level. Thus, high price regions have too many workers. To complete the description of this solution, note that both the unitary government condition 3.2.13 and the regional government condition 3.3.3 call simply for setting equal average benefits from private and public spending within the region, which can be done simply by choosing the transfer level S^k:

$$\delta^{i*}\bar{c}^i\left[U'(\bar{c}^i) - b'(g^i)\right] = -(1-\delta^{i*})\underline{c}^i\left[U'(\underline{c}^i) - b'(g^i)\right]. \tag{3.3.23}$$

Thus, this leads to having a 'third–best'solution.

Proposition 3.4 *When nominal prices differ across regions, and horizontal equity is an overriding concern of the federal government, there exists a tax and transfer scheme that gives agents the same real after tax wages and welfare payments in all regions, provided that the labour force participation rate is not sensitive to local commodity tax rates.*

It constitutes a 'third–best' solution in the sense that population is not allocated optimally across regions.

One lesson to be drawn from this proposition concerns the tax–field assignment in a federation. The broadest possible tax base should be given to the regional governments, because it would have a lesser impact on the work/leisure decision of the agents. Highly distortionary tax instruments are best kept at the central level. In terms of direct/indirect taxation, it means that there would be a reason to use both types of taxation in a federal setting, and that the indirect tax base should be assigned to the regional governments[24].

Turn now to the relative size of intergovernmental grants. If local commodity tax rates z^i must be manipulated in the manner described above to reach the third-best solution, is it possible to characterize the relative intergovernmental grants S^i, and describe how they may differ across regions?

It is quite easily seen that, first, the nominal tax bases of the regional governments are all equal, which translates into a lower real tax base for regions with higher price levels. Second, for $\frac{(1-z^i)}{P^i} = \frac{(1-z^j)}{P^j}$, it can be seen that $z^j < z^i$ if $P^j > P^i$; thus a lower real tax base is taxed at a lower tax rate in the high price level regions. Given that regional governments have the same per-capita spending, in terms of g, this leads clearly to the following conclusion:

Proposition 3.5 *To implement its "third-best" solution, the federal government needs to make larger intergovernmental grants to high nominal price regions.*

This result may have some surprising implications. It is often thought that higher nominal price regions are also wealthier. Here, the optimal strategy calls for the federal government to transfer larger sums to regions that are likely to be wealthier. Many would doubt that the observed pattern of relative grants conforms to the one suggested in proposition 3.5. As mentioned in the introduction, there exist several other motives for intergovernmental grants. When all these motives are taken into consideration simultaneously, it is hard to predict what the net grant structure would be like.

[24]Note that considering explicitly population mobility would strenghten this argument. In the context of an income redistribution problem, the key difficulty for regional governments is that taxation often affects different types of agents differently, attracting agents with positive fiscal residua, pushing away those for whom it is negative. Using as broad as possible a uniform tax base, like it is suggested here, would dampen the incentive differential.

3.4 Conclusion

This paper has shown how intergovernmental grants can be used to maintain efficient level of expenditures in a federation when the federal government seeks to redistribute income across types of agents, not regions. This paper differs from others found in the literature because the income redistribution function has not been assigned to regional governments; instead, the usual fiscal federalism prescription to assign this function to the central government has been obeyed.

It has been shown first that in the absence of local amenities, the unitary government's optimum tax schedule requires only light adjustments in the presence on nominal price differences. In effect, the same tax rate can be used everywhere; only the basic income deduction needs to be adjusted upwards to take into account the higher cost of living in a region.

Furthermore, it has been shown that some important results obtained by Boadway and Keen[6] carry through in the absence of nominal price differences. That is, to maintain efficient spending level, the federal government needs to isolate its own tax base from the decisions made by regional governments. This is done, in the absence of nominal price differences, by giving some subsidies to workers, i.e. by having a negative tax rate. However, this becomes impossible to do in most circumstances when nominal prices differ across regions. Then, the federal government is generally unable to isolate itself from the decisions taken locally, having too few instruments at its disposal. This means that it is unable to implement its horizontally equitable income redistribution scheme.

In the very special case when the regional tax rate does not affect the work/leisure consumption decision of the workers, then nominal price differences do not matter as much. Changes in local tax rates do not change the revenues obtained locally by the central government. A careful choice of intergovernmental grants S^k allows the central government to set equal real after-tax welfare benefits in all regions, and to implement its horizontal equity objective. However, this comes at the cost of having a misallocation of workers across regions. The optimal solution requires having fewer workers as the price level increases; the third–best solution obtained here would lead to having the same number of workers everywhere, in a free mobility, long–run equilibrium. This is clearly inefficient from a production point of view.

In reality, it appears unlikely that local tax rates have no effect at all on the local tax base. Here, it is possible in some circumstances because of the very stark nature of the work/leisure choice made by agents. Thus the importance of having chosen a commodity tax base for regional governments in this model. Nevertheless, it is likely to be indicative of the type of intergovernmental grants that are required to bring into being the desired income redistribution in a federation, both in terms of direction and in terms relative size. Moreover, it leads to the policy conclusion that indirect fields of taxation should be given to regional governments, while direct taxation should remain under federal jurisdiction.

To conclude, one may note that the task of implementing some income redistribution scheme in a large and diversified economy appears to be easier if the country adopts a federal structure. The possibility the federal government has of manipulating the taxation and expenditure decisions of regional governments by its use of intergovernmental grants compensates to some extent its inability to adopt region–specific income tax schedules. Thus, a federal structure may be beneficial to the economy.

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Appendix A

Proofs of Propositions Found in Chapter 1

A.1 Cash Reaction Functions

The problem to solve for region A, when $b_A > b_B$ is:

$$\max_{b_A}(1 + N(b_A - b_B))(L - b_A), \tag{A.1.1}$$

from the first-order condition,its reaction function is easily found to be

$$b_A = \frac{L - 1/N}{2} + \frac{b_B}{2}. \tag{A.1.2}$$

If region A underbids region B, it solves

$$\max_{b_A}(1 - (b_B - b_A))(L - b_A), \tag{A.1.3}$$

and it has a reaction function:

$$b_A = \frac{L - 1}{2} + \frac{b_B}{2}. \tag{A.1.4}$$

Inserting these reaction functions in the relevant objective functions, one obtains

$$V_A = \begin{cases} N\left[\frac{L+1/N-b_B}{2}\right]^2 & \text{for } b_A \geq b_B \\ \left[\frac{L+1-b_B}{2}\right]^2 & \text{for } b_A \leq b_B. \end{cases}$$

Comparing these two expressions, one obtains that they are equally preferable for $b_B = L - 1/\sqrt{N}$:

$$\begin{aligned} N\left[\frac{L+1/N-b_B}{2}\right]^2 &= \left[\frac{L+1-b_B}{2}\right]^2 \\ \Rightarrow (L-b_B)(\sqrt{N}-1) &= \frac{1}{\sqrt{N}}(\sqrt{N}-1) \\ \Rightarrow L - \frac{1}{\sqrt{N}} &= b_B. \end{aligned} \tag{A.1.5}$$

For lower values of b_B, overbidding is to be preferred. Finally, one verifies that given the reaction function when region A overbids region B, its bid is actually larger than region B's. This is the case provided that $N \geq 1$.

$$b_A = \begin{cases} \frac{L-1/N}{2} + \frac{b_B}{2} & \text{for } b_B \leq L - 1/\sqrt{N} \\ \frac{L-1}{2} + \frac{b_B}{2} & \text{for } b_B \geq L - 1/\sqrt{N}. \end{cases}$$

Similarly, region B's government maximizes the following objective function when it underbids region A's bid:

$$\max_{b_B} N(1-(b_A-b_B))(L-b_B), \tag{A.1.6}$$

which gives a reaction function:

$$b_B = \frac{L-1}{2} + \frac{b_A}{2}. \tag{A.1.7}$$

If it overbids region A, it maximizes:

$$\max_{b_B}(N+b_B-b_A)(L-b_B), \tag{A.1.8}$$

which gives the reaction function:

$$b_B = \frac{L-N}{2} + \frac{b_A}{2}. \tag{A.1.9}$$

Inserting into the proper objective function, one obtains:

$$V_B \begin{cases} \left[\frac{L+N-b_A}{2}\right]^2 & \text{for } b_B \geq b_A \\ N\left[\frac{L+1-b_A}{2}\right]^2 & \text{for } b_B \leq b_A. \end{cases}$$

The value of b_A such that region B's government is indifferent between overbidding or underbidding can be found in a similar manner to what has been done above for region A. It gives a value of $b_A = L - \sqrt{N}$. Next, it can be verified region B does not actually overbid region A, using the optimal reaction function A.1.9, whenever $L - N \leq b_A \leq L - \sqrt{N}$. This is true because

$$b_B = \frac{L-N}{2} + \frac{b_A}{2} \geq b_A \qquad \text{if} \quad b_A \leq L - N. \tag{A.1.10}$$

This gives a final reaction function:

$$b_B = \begin{cases} \frac{L-N}{2} + \frac{b_A}{2} & \text{for } b_A \leq L - N \\ b_A & \text{for } L - N \leq b_A \leq L - \sqrt{N} \\ \frac{L-1}{2} + \frac{b_A}{2} & \text{for } b_A \geq L - \sqrt{N}. \end{cases}$$

A.2 Proposition 1.2

From the statement of the proposition, it is known that $N > 1$. Inspection of expressions 1.4.5 and 1.4.6, shows that an equilibrium is possible only for:

$$b_A = \frac{L - 1/N + b_B}{2}, \qquad b_B = \frac{L - 1 + b_A}{2}. \tag{A.2.1}$$

One obtains:

$$b_A = L - \frac{2+N}{3N}, \qquad b_B = L - \frac{2N+1}{3N}. \tag{A.2.2}$$

Verifying that these values of b_A and b_B respect the boundary conditions is easy:

$$b_B = L - \frac{2N+1}{3N} \leq L - 1/N, \quad b_A = L - \frac{2+N}{3N} \geq L - \sqrt{N}, \tag{A.2.3}$$

these conditions being met for any $N > 1$. □

A.3 Proposition 1.4

To attract the desired $NH(\hat{\beta})$ type 2 firms the subsidy package to these firms, $(\theta_{2A}, \gamma_{2A}, b_{2A})$, must satisfy

$$\hat{\beta} = t_2\theta_{2A} + g_2\gamma_{2A} + b_{2A}$$

or,

$$\theta_{2A} = \frac{\hat{\beta}}{t_2} - \frac{g_2}{t_2}\gamma_{2A} - \frac{b_{2A}}{t_2}.$$

Substituting this subsidy package into firm 1's profit function gives the smallest level of profit that a subsidy package to firms of type 1 must achieve in order to satisfy incentive compatibility. Region A is interested in minimizing the value of this payout to type 1 firms. The profit expression is

$$\begin{aligned}\Pi_1(\theta_{2A}, \gamma_{2A}, b_{2A}) &= \frac{t_1}{t_2}\hat{\beta} - \frac{t_1 g_2}{t_2}\gamma_{2A} - \frac{t_1 b_{2A}}{t_2} + g_1\gamma_{2A} + b_{2A} + \alpha \\ &= \frac{t_1}{t_2}\hat{\beta} + \frac{\gamma_{2A}}{t_2}(t_2 g_1 - t_1 g_2) + \frac{t_2 - t_1}{t_2}b_{2A} + \alpha,\end{aligned}$$

by assumptions 1.2 and 1.3 this expression is minimized when $\gamma_{2A} = 0 = b_{2A}$. Thus region A prefers not to use cash to attract marginal firms. These assumptions together ensure that the θ–subsidy is the least attractive to type 1 firms of the three possible ways to subsidize type 2 firms. It is chosen to minimize the cash equivalent needed to maintain incentive–compatibility for type 1 firms. Incentive compatibility of the subsidy package $(\theta_{1A}, \gamma_{1A}, b_{1A})$ now requires that

$$t_1\theta_{1A} + g_1\gamma_{1A} + b_{1A} = t_1\theta_{2A} + g_1\gamma_{2A} + b_{2A}.$$

Substituting for the values $(\theta_{2A}, \gamma_{2A}, b_{2A}) = (\hat{\beta}/t_2, 0, 0)$, and for b_{1A} from the IC constraint, allows the cost–minimizing problem to be written

$$\begin{aligned}&\min_{\theta_{1A},\gamma_{1A},b_{1A}} \quad \theta_{1A} + \gamma_{1A} + b_{1A} + NH(\hat{\beta})(\theta_{2A} + \gamma_{2A} + b_{2A}) \\ &= \min_{\theta_{1A},\gamma_{1A}} \quad \theta_{1A} + \gamma_{1A} + \tfrac{t_1}{t_2}\hat{\beta} - t_1\theta_{1A} - g_1\gamma_{1A} + NH(\hat{\beta})\hat{\beta}/t_2.\end{aligned}$$

This programme is solved by $\theta_{1A} = 0 = \gamma_{1A}$, since $1 > t_1$ and $1 > g_1$. Thus, given $\hat{\beta}$ the government of region A chooses $\theta_{2A} = \hat{\beta}/t_2$, $b_{1A} = t_1\hat{\beta}/t_2$.□

A.4 Proposition 1.5

By contradiction, assume that region A prefers to use in–kind subsidies. For $t_1 = t_2$, the share of type 2 firms that region A obtains is:

$$\hat{\beta} = \frac{t(NL-1)}{2N}. \tag{A.4.1}$$

Inserting this value in the objective function, and computing the payments made to type 1 and type 2 firms, $b_A = \theta_A$ and $t\theta_A$, one obtains:

$$V_A = L - \frac{t(NL-1)}{2N} + N\frac{t(NL-1)}{2N}\left[L - \frac{NL-1}{2N}\right]. \tag{A.4.2}$$

The pure cash strategy payoff is

$$V_A^c = \frac{(NL+1)^2}{4N}. \tag{A.4.3}$$

The in–kind strategy is preferred if A.4.2 is larger than A.4.3; simple manipulations show that it is true for

$$(1-t)\left[\frac{3NL-1}{4N}\right] + \left[\frac{(t-1)NL}{2}\right]\left[\frac{NL+1}{2N}\right] + \frac{(NL+1)^2}{4N} > \frac{(NL+1)^2}{4N}. \tag{A.4.4}$$

$$\Rightarrow (1-t)\frac{3NL-1}{4N} + \frac{(t-1)NL}{2}\frac{NL+1}{2N} > 0$$

$$\Rightarrow (NL-1)^2 < 0,$$

which can never be true because $N \geq 1$ and $L > 1$. □

A.5 Proposition 1.6

The key of the proof is to move from the three basic conditions implicit in the proposition statement to the following two conditions:

$$t_2 \geq 1 - \frac{(N-1)}{1 + 2N - 5NL + NL(3NL-N)}; \tag{A.5.1}$$

$$t_1 = 1 - (1 - t_2)NL; \tag{A.5.2}$$

The three conditions that must be met in this proposition are the following.

1. The first is to have the same equilibrium point. If region A pays in–kind subsidies, it must be the case that

$$t_2 L - \frac{t_1}{N_2} = L - \frac{1}{N_2} \Rightarrow (1 - t_1)/(1 - t_2) = NL, \tag{A.5.3}$$

a variation on assumption 1.5, which implies that the two reaction functions are identical everywhere, in terms of actual value of payouts to type 2 firms.

2. Second, if this strategy is to work, it must be the case that $t_1 \theta_A \geq b_B$, at the equilibrium values, which is equivalent to saying:

$$t_1 \theta_A = \frac{t_1}{t_2}\left[L - \frac{(2+N)}{3N}\right] \geq b_B = \left[L - \frac{(2N+1)}{3N}\right],$$

or,

$$\frac{(2N+1)t_2 - (2+N)t_1}{3N(t_2 - t_1)} \geq L. \tag{A.5.4}$$

Given that $N \geq 1$, it is clear that $2N + 1 \geq 2 + N$, and that the left–hand side of this expression is positive.

3. Third, if the strategy is to generate as least as high benefits for region A, it has to be the case that

$$(1 - t_1)\bar{\theta}_A + \left[1 + NH(\tilde{b}_A)\right]\left[\tilde{b}_A - \bar{\theta}_A\right] + N\left[H(\bar{\theta}_A) - H(\tilde{b}_A)\right]\left[L - \bar{\theta}_A\right] \geq 0.$$

$\bar{\theta}_A$ is for the in-kind subsidy solution, while $\tilde{b}_A$ represents the cash solution. Given that the third term is equal to zero, and exploiting the fact that at any point $\tilde{b}_A = t_2 \bar{\theta}_A$, this is equivalent to

$$\frac{(t_2 - t_1)}{(1 - t_2)} \geq \frac{(N-1)}{3}. \tag{A.5.5}$$

The left–hand side of this expression may be interpreted as meaning that difference in valuation for the in–kind subsidy θ by the two industries must be sufficiently larger than its inherent inefficiency, $t_2 < 1$. Note that this expression does not depend on L.

If conditions A.5.3, A.5.4, and A.5.5 are satisfied, then region A obtains the same number of type 2 firms, keeps all its type 1 firms and benefits from having reduced the payments made to type 1 firms, in equilibrium.

It is possible to write equation A.5.3 as $t_1 = 1 - (1 - t_2)NL$, as in A.5.2; subtract this expression from t_2, to get that $t_2 - t_1 = (NL - 1)(1 - t_2)$; substitute $t_2 - t_1$ in A.5.5, to get

$$L \geq \frac{1}{3} + \frac{2}{3N},$$

which is always true, by assumptions 1.5, $L > 1$, and because a primitive of this model is that $N \geq 1$.

Using A.5.2, the inequality A.5.4 may be rewritten as the boundary condition A.5.1 above.

For a given value of L, from the equation A.5.1

$$N \to 1 \Rightarrow t_2 \to 1; \quad N \to \infty \Rightarrow t_2 \to 1;$$

while for intermediate values, t_2 is bounded away from 1.

From equation A.5.2, $t_1 < t_2$, for any finite N, L. □

A.6 Proposition 1.7

If region A is to keep all type 1 firms and steal some type 2 firms, it has to be the case that: $t_2\theta_A > b_B$; $b_A = t_1\theta_A > b_B$. The problem to be solved is

$$\max_{\theta_A} [L - t_1\theta_A] + N [t_2\theta_A - b_B] [L - \theta_A]; \tag{A.6.1}$$

from the first–order condition, one obtains

$$\theta_A = \frac{t_2 NL - t_1}{2Nt_2} + \frac{b_B}{2t_2}; \tag{A.6.2}$$

$$b_A = t_1\theta_A = \frac{t_1}{t_2}\left[\frac{t_2 NL - t_1}{2N} + \frac{b_B}{2}\right]. \tag{A.6.3}$$

The government of region B solves

$$\max_{b_B} N\left[1 - (t_2\theta_A - b_B)\right]\left[L - b_B\right]; \tag{A.6.4}$$

and from the first–order condition one obtains

$$b_B = \frac{L - 1 + t_2\theta_A}{2};. \tag{A.6.5}$$

Solving these three conditions together give

$$\theta_A = \frac{(2t_2+1)NL - (2t_1+N)}{3Nt_2}. \tag{A.6.6}$$

$$b_B = \frac{(2+t_2)NL - (2N+t_1)}{3N}. \tag{A.6.7}$$

To obtain that all type 1 firms are kept by region A, it is required that $t_1\theta_A - b_B \geq 0$. Using conditions A.6.6 and A.6.7 this means:

$$\begin{aligned} &\frac{\left[(1+t_2)(t_1-t_2) + t_2(t_1-1)\right]NL}{3Nt_2} \\ &+\frac{(t_2-t_1)(N+t_1)}{3Nt_2} + \frac{Nt_2 - t_1t_1}{3Nt_2} > 0. \end{aligned} \tag{A.6.8}$$

Inequality A.6.8 imposes a minimum value for t_1 as a function of the three other parameters t_2, N and L. The point of intersection for these two constraints is informative. At $t_1 = t_2$, inequality A.6.8 becomes: $t \geq N(L-1)/(NL-1)$. For $N = 1$, the in–kind subsidy game is not substainable as it would mean that $t \geq (L-1)/(L-1) = 1$, which is impossible. Thus, for $N = 1$, there exist no values for t_1 and t_2 that respect the assumptions of this model and meet the constraints of proposition 1.7

Using equations A.6.6 and A.6.7 together, one obtains the share of firms of type 2 firms obtained by region A equal to:

$$N\frac{(t_2-1)NL+N-t_1}{3N}=\frac{(t_2-1)NL+N-t_1}{3}. \qquad \text{(A.6.9)}$$

□

A.7 Propositions 1.8 and 1.9

In a two–way flow equilibrium as described in 1.5.3, the government of region A has to solve the following constrained problem:

$$\max_{b_A,\theta_A,\lambda}\,[1-(b_B-b_A)]\,[L-b_A]+N\,[t_2\theta_A-b_B]\,[L-\theta_A]\,, \qquad \text{(A.7.1)}$$

subject to

$$(\lambda) \qquad b_A \geq t_1\theta_A, \qquad \text{(A.7.2)}$$

where λ represents the Lagrange multiplier. The government of region B solves the following problem:

$$\max_{b_B}\,[b_B-b_A]\,[L-b_B]+N\,[1-(t_2\theta_A-b_B)]\,[L-b_B]\,. \qquad \text{(A.7.3)}$$

The first–order conditions to region A's problem are

$$b_A=\frac{L-1+b_B+\lambda}{2}; \qquad \text{(A.7.4)}$$

$$\theta_A=\frac{N(t_2L+b_B)-t_1\lambda}{2Nt_2}; \qquad \text{(A.7.5)}$$

$$\lambda\,[b_A-t_1\theta_A]=0, \quad \lambda\geq 0, \quad b_A-t_1\theta_A\geq 0. \qquad \text{(A.7.6)}$$

Solving simultaneously for A.7.4, A.7.5 and $b_A-t_1\theta_A=0$, one gets:

$$b_A=\frac{t_1\,[(t_1+Nt_2)L-t_1+b_B(t_1+N)]}{2((t_1)^2+Nt_2)}; \qquad \text{(A.7.7)}$$

$$\theta_A = \frac{(t_1 + Nt_2)L - t_1 + b_B(t_1 + N)}{2((t_1)^2 + Nt_2)}; \tag{A.7.8}$$

$$\lambda = \frac{N\left[(1 - t_1)t_2L + t_2 + b_B(t_1 - t_2)\right]}{(t_1)^2 + Nt_2}. \tag{A.7.9}$$

The first–order condition to region B's problem, A.7.3, is:

$$b_B = \frac{(1 + N)L - N + b_A + Nt_2\theta_A}{2(1 + N)}. \tag{A.7.10}$$

Solving for equations A.7.7– A.7.10 simultaneously gives:

$$\theta_A = \frac{\left[N + 3t_1 + 2Nt_2\right](1 + N)L - \left[(N)^2 + t_1(2 + 3N)\right]}{(4N + 3)(t_1)^2 - Nt_1 + 4Nt_2 + 3(N)^2t_2 - Nt_1t_2}; \tag{A.7.11}$$

$$b_A = t_1\theta_A; \tag{A.7.12}$$

$$\begin{aligned} b_B = {} & \frac{\left(N\left[2t_2(1 + N) + 2t_1(t_1 + t_2) + N(t_2)^2\right] + 3(t_1)^2\right)L}{(4N + 3)(t_1)^2 - Nt_1 + 4Nt_2 + 3(N)^2t_2 - Nt_1t_2} \\ & - \frac{\left[t_1(t_1 + Nt_2) + 2N\left[(t_1)^2 + Nt_2\right]\right]}{(4N + 3)(t_1)^2 - Nt_1 + 4Nt_2 + 3(N)^2t_2 - Nt_1t_2}; \end{aligned} \tag{A.7.13}$$

$$\begin{aligned} \lambda = {} & \frac{\left[3t_1(1 + t_2) + 2Nt_1(1 + 2t_2) - t_2(6 + 5N + Nt_2)\right]NL}{(4N + 3)(t_1)^2 - Nt_1 + 4Nt_2 + 3(N)^2t_2 - Nt_1t_2} \\ & - \frac{N\left[t_1(1 + 2N) - t_2(4 + 5N)\right]}{(4N + 3)(t_1)^2 - Nt_1 + 4Nt_2 + 3(N)^2t_2 - Nt_1t_2}. \end{aligned} \tag{A.7.14}$$

Expression A.7.14 represents the value of the Lagrange multiplier for the problem; this is the expression 1.5.9 used in section 1.5.3. If it is positive, then the constraint is binding, and the two conditions $t_2\theta_A \geq b_B$, and $b_B \geq t_1\theta_A$ can be written, using expressions A.7.11– A.7.13, as in proposition 1.9:

$$\begin{aligned} & t_2\left\{\frac{\left[N + 3t_1 + 2Nt_2\right](1 + N)L - \left[(N)^2 + t_1(2 + 3N)\right]}{(4N + 3)(t_1)^2 - Nt_1 + 4Nt_2 + 3(N)^2t_2 - Nt_1t_2}\right\} \\ & - \left\{\frac{\left(N\left[2t_2(1 + N) + 2t_1(t_1 + t_2) + N(t_2)^2\right] + 3(t_1)^2\right)L}{(4N + 3)(t_1)^2 - Nt_1 + 4Nt_2 + 3(N)^2t_2 - Nt_1t_2}\right. \end{aligned} \tag{A.7.15}$$

$$-\frac{[t_1(t_1+Nt_2)+2N[(t_1)^2+Nt_2]]}{(4N+3)(t_1)^2-Nt_1+4Nt_2+3(N)^2t_2-Nt_1t_2}\Bigg\} \geq 0.$$

and
$$\frac{(N[2t_2(1+N)+2t_1(t_1+t_2)+N(t_2)^2]+3(t_1)^2)L}{(4N+3)(t_1)^2-Nt_1+4Nt_2+3(N)^2t_2-Nt_1t_2}$$
$$-\frac{[t_1(t_1+Nt_2)+2N[(t_1)^2+Nt_2]]}{(4N-3)(t_1)^2-Nt_1+4Nt_2+3(N)^2t_2-Nt_1t_2}$$
$$-t_1\left\{\frac{[N+3t_1+2Nt_2](1+N)L-[(N)^2+t_1(2+3N)]}{(4N-3)(t_1)^2-Nt_1+4Nt_2+3(N)^2t_2-Nt_1t_2}\right\} \geq 0.$$

□

If A.7.14 is non–positive, then region A's government prefers to offer more than the minimum incentive–compatible cash payment to type 1 firms, and the first–order conditions A.7.4 and A.7.5, together with region B's first–order condition A.7.10, have to be solved for $\lambda \equiv 0$:

$$b_A = \frac{[6+5N+Nt_2]L}{6(1+N)} - \frac{(4+5N)}{6(1+N)}; \tag{A.7.16}$$

$$b_B = \frac{[6+4N+2Nt_2]L}{6(1+N)} - \frac{(2+4N)}{6(1+N)}; \tag{A.7.17}$$

$$\theta_A = \frac{[3+3t_2+2N+4Nt_2]L}{6t_2(1+N)} - \frac{(1+2N)}{6t_2(1+N)}. \tag{A.7.18}$$

Verifying that $t_2\theta_A > b_B$, using equations A.7.17 and A.7.18, gives condition:

$$t_2 \geq 1 - \frac{(1+2N)}{(3+2N)L}. \tag{A.7.19}$$

Verifying that $b_B > b_A$, using equations A.7.16 and A.7.17, gives

$$t_2 \geq 1 - \frac{(2+N)}{NL}. \tag{A.7.20}$$

Verifying that $t_2\theta_A > b_A$, using equations A.7.16 and A.7.18, gives

$$t_2 \geq 1 - \frac{1}{L}. \tag{A.7.21}$$

It can be computed that constraint A.7.19 always takes on a larger value than constraints A.7.20 and A.7.21. This gives proposition 1.8. □

A.8 Proposition 1.10

Proposition 1.10 states that offering mixed cash/in–kind subsidies leads to a higher surplus for region A than offering cash–only subsidies, *given* region B's equilibrium cash bid, b_B, when region A offers mixed subsidies. This corresponds to

$$[L - t_1\theta_A] + N\,[t_2\theta_A - b_B]\,[L - \theta_A] > (L - b_A^c)(1 + N(b_A^c - b_B)), \qquad \text{(A.8.1)}$$

which becomes the inequality from the proposition, once θ_A and b_B are replaced by expressions A.6.6 and A.6.7, that is the values found in the appendix containing the proof of proposition 1.7, appendix A.6; b_A^c is replaced by expression 1.6.1. □

A.9 Proposition 1.11

The government of region A prefers mixed cash/in–kind subsidies to cash–only subsidies, even if it leads to a cross–flow of firms —when it pays strictly more than the minimum incentive–compatible cash payments to type 1 firms— if the following inequality is verified,

$$[1 - (b_B - b_A)]\,[L - b_A] + N\,[t_2\theta_A - b_B]\,[L - \theta_A] > (L - b_A^c)(1 + N(b_A^c - b_B)) \qquad \text{(A.9.1)}$$

for

$$b_A > t_1\theta_A. \qquad \text{(A.9.2)}$$

Replace b_A, b_B and θ_A by the values found when proposition 1.8 is valid, equations A.7.16, A.7.17, and A.7.18 from appendix A.7; for b_A^c, substitute 1.6.1. After these substitutions, one obtains the inequality contained in the proposition statement.□

A.10 Proposition 1.12

The government of region A prefers mixed cash/in–kind subsidies to cash–only subsidies, even if it leads to a cross–flow of firms —when it pays the minimum incentive–

compatible cash payments to type 1 firms— if the following inequality is verified,

$$[1-(b_B-b_A)]\,[L-b_A]+N\,[t_2\theta_A-b_B]\,[L-\theta_A] \qquad \text{(A.10.1)}$$
$$> (L-b^c_A)(1+N(b^c_A-b_B));$$

for

$$b_A = t_1\theta_A. \qquad \text{(A.10.2)}$$

Replace θ_A and b_B by the values found when proposition 1.9 holds, equations A.7.11 and A.7.13 from appendix A.7; replace b^c_A by 1.6.1. These substitutions give the inequality found in proposition 1.12. □

A.11 Proposition 1.13

To show this, proceed by contradiction. Δ is defined to be:

$$\Delta \equiv ((1-\bar{t}_2)\bar{N}\bar{L}+t_1-1)/2. \qquad \text{(A.11.1)}$$

Set t_1 high enough such that the next expression equals zero:

$$\left[(\bar{t}_2-1)\theta^*_A+\Delta/\bar{N}\right]. \qquad \text{(A.11.2)}$$

This can always be done because Δ is a positive function of t_1, while θ^*_A is a negative function of t_1, as seen with 1.7; moreover, for $\Delta = 0$, the expression A.11.2 is negative, while for $t_1 = \bar{t}_2$, it is positive. Thus, the second term of expression 1.6.4, reproduced here below, equals zero.

$$(1-t_1)\theta^*_A+\left[1+\bar{N}(\frac{\bar{N}-1+\Delta}{3\bar{N}})\right]\left[(\bar{t}_2-1)\theta^*_A+\frac{\Delta}{\bar{N}}\right]$$
$$-\bar{N}\frac{\Delta}{\bar{N}}\left[\bar{L}-\theta^*_A\right].$$

When $\Delta = -\bar{N}\left[(\bar{t}_2 - 1)\theta_A^*\right] > 0$, the two other terms can be rewritten as:

$$(1 - t_1)\theta_A^* + \bar{N}(\bar{t}_2 - 1)\theta_A^*\left[\bar{L} - \theta_A^*\right] \geq 0,$$

$$\Rightarrow \qquad (1 - t_1)\theta_A^* + \bar{N}\bar{L}(\bar{t}_2 - 1)\theta_A^* \geq \bar{N}(\bar{t}_2 - 1)\theta_A^*\theta_A^*,$$

Using the definition for Δ, A.11.1, it becomes

$$\left[(1 - t_1) + \bar{N}\bar{L}(\bar{t}_2 - 1)\right]\theta_A^* = -2\Delta\theta_A^* \geq \bar{N}(\bar{t}_2 - 1)\theta_A^*\theta_A^*$$

$$-2\Delta \geq \bar{N}(\bar{t}_2 - 1)\theta_A^* \;\Rightarrow\; 2 \leq 1,$$

by A.11.2, which shows that it is not possible to spend the same amount per type 2 firms (A.11.2), and have that 1.6.4 be positive, for a *given* value of b_B. □

A.12 Proposition 1.14

Consider the relative benefits of the mixed cash/in–kind subsidy and cash–only subsidy at the two potential equilibria: where region A's in–kind "reaction function" crosses region B's reaction function, and where A's cash "reaction function" crosses region B's. Using expression 1.6.4, these differences in benefits can be written as:

$$(1 - t_1)\theta_A^* + \left[1 + N\left(\frac{N - 1 + \Delta}{3N}\right)\right]\left[(t_2 - 1)\theta_A^* + \frac{\Delta}{N}\right] - N\frac{\Delta}{N}\left[L - \theta_A^*\right], \qquad \text{(A.12.1)}$$

at the mixed cash/in–kind strategy solution, and

$$(1 - t_1)\theta_A^{**} + \left[1 + N\left(\frac{N - 1}{3N}\right)\right]\left[(t_2 - 1)\theta_A^{**} + \frac{\Delta}{N}\right] - N\frac{\Delta}{N}\left[L - \theta_A^{**}\right], \qquad \text{(A.12.2)}$$

at the cash–only solution. One needs to show that there exists some combination of parameter values such that expression A.12.1 is negative while A.12.2 is negative.

Subtract expression A.12.1 from A.12.2 to get

$$(1-t_1)z(\Delta)+\Delta z(\Delta)+\left[1+N(\frac{N-1}{3N})\right][(t_2-1)z(\Delta)]$$
$$-\frac{\Delta}{3}\left[(t_2-1)\theta_A^*+\frac{\Delta}{N}\right], \qquad (A.12.3)$$

where $z(\Delta)=\theta_A^{**}-\theta_A^*$, a simple (positive) trigonometric function of Δ.

Now, consider the case for which expression A.12.1 is strictly equal to zero. This can occur only for $\left[(t_2-1)\theta_A^*+\frac{\Delta}{N}\right]<0$, as shown by proposition 1.13. Using A.12.3, one can determine the sign of expression A.12.2. In this case, A.12.3 can be rewritten (using the definition of Δ) as:

$$-\Delta z(\Delta)+z(\Delta)(1-t_2)N\left[L-(\frac{1}{3}+\frac{2}{3N})\right]-\frac{\Delta}{3}\left[(t_2-1)\theta_A^*+\frac{\Delta}{N}\right];$$

the last two terms of this expression are strictly positive. Using the definition of Δ, one gets

$$z(\Delta)\left[(1-t_2)\frac{N}{2}\left[L-2(\frac{1}{3}+\frac{2}{3N})\right]+\frac{1-t_1}{2}\right]-\frac{\Delta}{3}\left[(t_2-1)\theta_A^*+\frac{\Delta}{N}\right],$$

and it is sufficient to have $\left[L-2(\frac{1}{3}+\frac{2}{3N})\right]$ positive to guarantee that the overall expression is positive. By continuity, by making A.12.1 slightly negative, A.12.2 remains positive and the non–existence of a Pure Strategy Nash Equilibrium obtains. □

A.13 Proposition 1.15

Proposition 1.15 states that the government of region A prefers to use mixed cash/in–kind subsidies to cash–only subsidies in a subgame perfect sense when the following inequality is verified:

$$L-b_A^{ik}+N(t_2\theta_A^{ik}-b_B^{ik})(L-\theta_A^{ik}) > \qquad (A.13.1)$$
$$(1+N(b_A^c-b_B^c))(L-b_A^c),$$

where the superscript ik indicates the mixed cash/in–kind subsidy solution, while the superscript c indicates the cash–only solution. θ_A^{ik} and b_B^{ik}, on the left–hand side term of the inequality, can be replaced by equations A.6.6 and A.6.7 from appendix A.6 (proof of proposition 1.7); it is also true in these circumstances that region A offers the minimum incentive–compatible cash payment to type 1 firms: $b_A^{ik} = t_1\theta_A^{ik}$. Values of b_A^c and b_B^c on the right–hand side correspond to A.2.2 in appendix A.2 (proof of proposition 1.2).□

A.14 Proposition 1.16

By assumption either δ_1 or δ_2, or both, are different from zero. For $N = 1$, only three equilibrium configuration might exist: i) $b_A = b_B = 0$; ii) $b_A = b_B + \delta_2$, and iii) $b_A = b_B - \delta_1$. One can verify this first claim by looking for compatible intervals for which both regions may find the regime $(\hat{\beta} > 0, \hat{\alpha} = 0)$ optimal, that is they choose to have $b_A > b_B + \delta_2$ and $b_B < b_A - \delta_1$. Inspection of expressions 1.7.5 and 1.7.7 quickly reveals that there is no such interval[1]; thus, only solutions on the "diagonals" are possible.

Examining the three potential solutions in turn, one gets that for case i), when $b_B = 0$, the optimal bid for region A is $b_A = (L - 1 + \delta_2)/2$, which is larger than zero for any value $\delta_2 < L - 1$. Thus, $b_A = b_B = 0$ cannot be an equilibrium.

ii) $b_A = b_B + \delta_2$ cannot be a solution, because it is easily seen that

$$V_A(b_B - \delta_1, b_B) = L - (b_B - \delta_1) > V_A(b_B + \delta_2, b_B) = L - (b_B + \delta_2). \quad \text{(A.14.1)}$$

iii) $b_A = b_B - \delta_1$ is identical to the second case, but with respect to V_B. That is, region B prefers to pay $b_A - \delta_2$ to paying $b_A + \delta_1$, given that in both cases it only gets to keep firms from its own industry. □

[1]By symmetry, if that regime is impossible, it is also impossible for the regime $\hat{\beta} = 0, \hat{\alpha} > 0$.

A.15 Proposition 1.17

If $(N-1)/N > 2\delta_2$, then both regions may find the regime $(\hat{\beta} > 0, \hat{\alpha} = 0)$ optimal. For the range $b_B \leq L - 1/N - \delta_2$, $b_A \geq L - 1 + \delta_2$ (from 1.7.5 and 1.7.7), the two reaction functions may cross. Solving, one gets:

$$b_A = L - \frac{2+N}{3N} + \frac{\delta_2}{3}, \quad b_b = L - \frac{2N+1}{3N} - \frac{\delta_2}{3}; \tag{A.15.1}$$

If $(N-1)/N > \delta_2 + 3\delta_1$, then one compares the value of the objective function between the regimes $(\hat{\beta} > 0, \hat{\alpha} = 0)$ and $(\hat{\beta} = 0, \hat{\alpha} > 0)$, that is

$$\frac{[2 + N(1-\delta_2)]^2}{9N} \geq \left[\frac{5N+1}{6N} + \frac{5\delta_1}{6} - \frac{\delta_2}{6}\right]. \tag{A.15.2}$$

If $(N-1)/N < \delta_2 + 3\delta_1$, then one compares with the regime $(\hat{\beta} = 0, \hat{\alpha} = 0)$

$$\frac{[2 + N(1-\delta_2)]^2}{9N} \geq \left[\frac{2N+1}{3N} + \delta_1 + \delta_2\right]. \tag{A.15.3}$$

□

A.16 Proposition 1.18

The strategy followed consists in giving in–kind subsidies to marginal firms, and cash to infra–marginal ones, as described in section 1.5.1 above. If $t_2\theta_2 \geq b_B + \delta_2$ and $t_1\theta_2 = b_A \geq b_B - \delta_1$, then the programme that region A solves is

$$\max_{\theta_A} (L - t_1\theta_A) + NH(\hat{\beta})[L - \theta_A], \tag{A.16.1}$$

for $t_1\theta_A = b_A$, and $\hat{\beta} = t_2\theta_A - b_B - \delta_2 \geq 0$.

From the first order condition, one obtains a reaction function for region A:

$$t_2\theta_A = \frac{(t_2L - t_1)}{2} + \frac{(b_B + \delta_2)}{2}, \tag{A.16.2}$$

when

$$b_B \leq t_2L - t_1 - \delta_2, \quad \text{and } b_B \leq \frac{t_1(t_2L - t_1 + \delta_2)}{(2t_2 - t_1)} + \delta_1, \tag{A.16.3}$$

this second condition coming from the restriction that $t_1\theta_A = b_A \geq b_B - \delta_1$.

On the other hand, for $b_B \leq t_2\theta_A - \delta_2$ and for $b_A \geq L - 1 + \delta_2$, then the reaction function for region B is

$$b_B = \frac{L-1}{2} + \frac{(t_2\theta_A - \delta_2)}{2}. \tag{A.16.4}$$

These two reaction functions may cross each other if $t_2L - t_1 - \delta_2 > L - 1 + \delta_2$. If $\delta_2 = 0$ and assumption 1.6 is met, then $t_2L - t_1 > L - 1$. Solving for the two reaction functions in this range, one obtains:

$$t_2\tilde{\theta}_A = \frac{(2t_2+1)L}{3} - \frac{(2t_1+1)}{3}, \quad \tilde{b}_B = \frac{(2+t_2)L}{3} - \frac{(2+t_1)}{3}. \tag{A.16.5}$$

Now, comparing the welfare obtained in this case to what it would be if region A merely matched region B's bid, one gets:

$$(L - \tilde{b}_A) + (t_2\tilde{\theta}_A - \tilde{b}_B)\left[L - \tilde{\theta}_A\right] > \left[L - (\tilde{b}_B - \delta_1)\right]. \tag{A.16.6}$$

After manipulations, the expression becomes

$$\left[\frac{(t_2-1)L}{3} + \frac{(1-t_1)}{3}\right]\left[L - \tilde{\theta}_A\right] > \left[t_1\tilde{\theta}_A - (\tilde{b}_B - \delta_1)\right]. \tag{A.16.7}$$

The left–hand side of this expression is positive given that $\tilde{\theta}_A < L$, and assumption 1.6 is met, while the right–hand side is equal to zero if $\delta_1 = \tilde{b}_B - t_1\tilde{\theta}_A$. There exist other, larger, values of δ_1 such that the off–diagonal equilibrium is preferred by region A. Thus, it is established that using in–kind subsidies may generate a pure strategy Nash Equilibrium for some combinations of the parameters δ_1, δ_2. □

Appendix B

Extra Derivations Used in Chapter 3

For the regional budget constraint

$$g^i = z^i \left[\delta^{i*}(1-T)w^i + (1-\delta^{i*})e^i\right] + \frac{s^i}{N^i} + \theta^R r^i \tag{B.0.1}$$

it is easy to find the impact of changes in T, E and S^i on the expenditure level g^i.

$$g^i_T = -z^i\delta^{i*}w^i + z^i\left[(1-T)w^i - e^i\right] + \left[z^i\delta^{i*}(1-T)\frac{\partial w^i}{\partial\delta^{i*}} + \theta^R\frac{\partial r^i}{\partial\delta^{i*}}\right]\frac{\mathrm{d}\,\delta^{i*}}{\mathrm{d}\,T}; \tag{B.0.2}$$

$$g^i_E = z^i(1-\delta^{i*})e^i + z^i\left[(1-T)w^i - e^i\right] + \left[z^i\delta^{i*}(1-T)\frac{\partial w^i}{\partial\delta^{i*}} + \theta^R\frac{\partial r^i}{\partial\delta^{i*}}\right]\frac{\mathrm{d}\,\delta^{i*}}{\mathrm{d}\,E}; \tag{B.0.3}$$

$$g^i_{S^i} = \frac{1}{P^iN^i}. \tag{B.0.4}$$