PRINCIPAL-AGENT MODELS IN MULTI-DIMENSIONAL SETTINGS

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Abstract

The thesis applies the Principal-Agent models to the following two settings:

1. The agent is employed to work on a multi-stage project;

2. The agent is responsible for multiple tasks.

The method of analysis is an analytical one.

Part I studies the multi-stage problem in which periodic applications of effort by the agent are required. The agent also obtains private information as the project evolves and he decides if the project should be abandoned or continued. We show that the agent's decision to continue is not always aligned with the principal's desire. The result provides an economic rationale for the sunk cost phenomenon. There also exist conditions under which the agent chooses to prematurely abandon the contract.

Part II studies the effort allocation problem and provides insight with respect to the job design problem. When the agent is responsible for more than one task, the principal simultaneously studies the incentive problem for all the tasks and decides on the task grouping and assignment. The relative precision of the performance measures of the agent's effort in each task affects the cost to the principal of extracting high effort levels from each of the task. The principal should not settle for costlessly available but highly noisy information. Rather, the management accounting system of each firm should be designed to be consistent with the technology of the firm, its product strategy, and its organization structure. This allows the principal to more efficiently induce desired levels of effort.

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Chapter 1

Introduction and Overview

In the principal-agent relationship, the principal delegates to the agent the responsibility of managing part of the firm's operations. A major assumption of agency models is that individuals are motivated solely by self-interest. The agent's aversion to effort and the agent's private information result in tension in the relationship between the principal and the agent. Thus, the interests of the principal and the agent are unlikely to be aligned. The agent wishes to expend as little effort as possible in order to maximize his personal utility, thus his choice of effort level is unlikely to lead to a maximization of the principal's profit. Research in this area focuses on the optimal contractual relationships between the principal and the agent. It examines the relationship between the firm's information system and its employment contracts. The principal-agent model provides a coherent and useful framework for analyzing and understanding managerial accounting procedures. The purpose of the thesis is to extend the agency model to consider multidimensional aspects of the principal-agent problem. We specifically look at the following two problems in an agency setting:

1. The agent is employed to work on a multi-stage project;

2. The agent is responsible for multiple tasks.

The thesis consists of seven chapters. This chapter (Chapter 1) provides an introduction to the thesis. The remaining six chapters are grouped into two parts. Part I studies the multi-stage problem while part II examines the multiple task problem.

Part I consists of Chapters 2, 3 and 4. In the multi-stage problem, periodic applications of effort by the agent are required and the agent also obtains private information as the project evolves. A key feature of the model is that the project is subject to abandonment after the initial stage. In particular, we derive the optimal incentive contracts for a setting in which the agent is employed to undertake a two-stage project which may be subject to abandonment after the first stage. Effort is required in both stages, but there is only one outcome at the end of the project. After the first stage, the agent receives private information and decides if the project should be continued or abandoned. We show that under certain conditions, the agent chooses to continue although under first-best conditions, the project is abandoned. The agent's selection of the cutoff point is ex ante efficient but ex post inefficient. This result provides an economic rationale for the sunk cost phenomenon. Conversely, we show that there exist conditions under which the agent chooses to prematurely abandon the project.

Chapter 2 is an introduction to Part I. It discusses the incentive problems when agents are employed in risky multi-stage projects. In Chapter 3, we set up the general model and examine two special cases. The results provide us with useful insights when we analyze the general model in Chapter 4. We also examine a benchmark case in which information is publicly observable but not contractible.

Part II consists of Chapters 5, 6 and 7. It deals with the effort allocation problem and provides insights with respect to the job design problem. The agent is responsible for two tasks and his attitude towards performing the two tasks determines his personal cost of effort. We examine how changes in the agent's attitude affects the optimal effort levels. We vary the precision of the performance measure in the second task and explore how this affects task assignment and optimal effort levels. In the extreme case, we consider what happens if there is no costlessly available performance measure on the second task. The principal explores the option of investing in a costly monitoring technology to extract a signal that can be used in the compensation contract. We determine the factors that affect the optimal monitoring level.

The analysis indicates that when an agent is responsible for more than one task, incentive issues should not be addressed task by task. The principal should simultaneously study the incentive problem for all the tasks. While a good incentive plan is critical for motivating performance, the issue of effective job design should not be ignored. A good job design and a well-designed incentive plan are both necessary to motivate the agent to exert the optimal effort levels at the minimum cost. Also, since the precision of the performance measures affects the cost to the principal of extracting high effort levels, the principal should not just settle for costlessly available but highly noisy information. Rather, as Johnson and Kaplan (1987) advocate, the management accounting system of each firm should be designed to be consistent with the technology of the firm, its product strategy, and its organization structure. The provision of such an information system may be costly but it allows the principal to more efficiently induce desired levels of effort. When the benefits outweigh the cost of information collection, the principal invests in an accounting system that provides more congruent and less noisy performance measures.

Chapter 5 is an introduction to Part II. In Chapter 6, we examine the principal's monitoring decision in a single task model. This provides us with useful insight when we examine the two-task model in Chapter 7. With two tasks, task grouping and effort allocation are critical and we analyze the principal's monitoring decision in such a setting.

Chapter 2

Incentive Contracts with Continuation Decisions

2.1 Introduction

2.1.1 The Incentive Problem

This section of the dissertation examines the incentives of risk- and work-averse agents to work on projects with the following characteristics:

- risky – probability of failure is high, but if successful, the returns can be extraordinary;
- long-term and multi-stage; and
- subject to abandonment.

Motivating agents to take up such projects is different from motivating them in traditional type of work, like sales and manufacturing. Firms not only seek to provide incentives to induce the agents to take up such risky investments and work hard at them, but also seek to provide incentives for them to abandon the project if the profit prospect is low.[1]

The market value of the agents' human capital may depend on their past performance. By undertaking a risky investment, agents put their human capital at risk. A good outcome may help to increase the reputation value of the manager. On the other hand, a bad outcome, including abandonment, does not reflect well on the manager's talent and his value on the market may be adversely affected. Kanter (1989, p. 310) states that professional careers (i.e., careers defined by skill) are produced by projects,

[1] In our analysis, we do not examine the incentive problem of motivating the agents to select from among alternative projects.

with reputation as the key variable in success. Each project adds to the value of a reputation as it is successfully completed. On the other hand, project abandonment or failure blights the agents' career chances, destroys their merit increases and limits their scope to take risks again (Twiss and Goodridge, 1989, p. 51). Peters (1989) suggests that in innovative and highly risky work, managers should support failure instead of penalizing the agent. Otherwise, the agents will become afraid to take risk, or they will be reluctant to terminate a project they began even though the profitability prospect is low.

Such long-term projects usually involve the acquisition of firm/project-specific human capital. Milgrom and Roberts (1992, p.363) define firm-specific human capital as "knowledge, skills, and interpersonal relationships that increase workers' productivity in their current employment, but are useless if the workers leave to join other firms". If the project is abandoned and the agent's employment with the firm is terminated, then the market value for the agent's services upon reentering the job market is not higher than before he joined the firm and undertook the project. From the firm's perspective, these skills are also difficult and expensive to replace. Hence, it is in the interest of both parties that the employment relationship lasts at least for the duration of the project. In R & D work, skills may also be project-specific for legal reasons. If the skill is tied to the trade secret of the firm, the agent is not free to leave the firm and continue with the project on his own or with any other firm.

Williamson (1985, p. 259) argues that agents accepting employment of a firm-specific kind will recognize the risks of "one labor power and one job" and insist upon surrounding such jobs with protective governance structures. In the absence of such governance structures, Williamson (1985, p. 272) predicts that agents must be paid a wage premium to accept such employment.

We suggest that one type of protective governance structure in employment which involves firm- and project-specific skills is the reliance on long-term contracts which commit the firm to future compensation, and severance pay should the project be abandoned because newly-received information indicates that it is no longer profitable. The difficulty arises when the agent has built up a substantial amount of firm-specific and project-specific skill and knowledge which has little or no value for other firms, should the agent seek employment elsewhere. Therefore, an agent, who could choose between jobs which build up general skills and jobs which build up firm- and project-specific skills, will be reluctant to choose the latter types unless he is compensated should the project be abandoned. The termination payment provided for in an efficient contract acts as a contractual safeguard for the agent and it would encourage the continued investment in firm- and project-specific skill. Williamson (1985, pp. 33–34) states that, in general, transactions which require special purpose technology and which do not enjoy any protective safeguards are unstable contractually. They are either replaced by general purpose technology or some kind of contractual safeguards will be introduced to encourage the continued use of the special purpose technology.

2.1.2 Severance Pay

If the project is abandoned, the agent's firm-specific skill is no longer productive and the market does not value these skills. Therefore, it will be optimal to offer the agent a contract which has a severance pay. Project abandonment should not be treated like a failure as this would result in the agent becoming too risk averse in taking up investments, and at the same time, if the project is started, it will result in the agent being very unwilling to stop it even if profitability is low.

The use of severance pay when projects are abandoned is similar to the use of golden parachutes in takeovers. Just as golden parachutes help deter executives from resisting takeover attempts that are personally costly for them but beneficial to the shareholders, providing severance pay to executives in the event of project abandonment would help deter executives from resisting dropping projects that are no longer profitable. Also, providing for severance pay helps provide motivation for the agent to undertake risky projects which may be subject to abandonment.

2.2 The Abandonment Problem

The agent is usually most informed about the value of the project as he is directly involved in it. At the same time, some projects are very technical and may be beyond the understanding of the principal. Therefore, in most instances, the principal must rely on the agent to make any decisions about the project. At each stage of the project, the agent assesses the development of the project and decides if it is profitable to continue with it. Twiss (1992) lists a number of factors which he claims, cloud the issue of project abandonment. They include the following:

1. A sunk cost mentality - stick with the existing project because of the investment already made.

2. New project euphoria - abandon the old in favor of the new.

Our analysis shows that these two responses may be rational in the second-best world when the agent's effort is not observable and the agent makes the abandon/continue decision. In comparing the agent's abandon/continue decision with the principal's decision when effort is observable, we obtain the following two possibilities:

1. The agent continues the project even when information indicates it is no longer profitable to do so. We term this overcontinuation.

2. He prematurely abandons the project, which we term overabandonment.

What is commonly perceived as the 'sunk cost mentality' or escalation behavior may be explained by our result on overcontinuation, and what is perceived as 'new project euphoria' may be explained by our result on overabandonment. In escalation behavior, the agent adheres to and increases his earlier commitment even when new information indicates that continuing the earlier commitment will result in worse consequences. Such behavior has been generally termed as irrational and as evidence that decision-makers do not ignore sunk costs. Kanodia, Bushman and Dickhaut (1989) provide an explanation for such behavior based on reputation. They show that when the agent has private information about his human capital, a desire for reputation-building may lead the agent to demonstrate escalation behavior. My model provides an alternative explanation based on induced moral hazard, a term introduced by Demski and Sappington (1989). Unlike the overcontinuation problem, very little has been said about the overabandonment problem. With the benefit of hindsight, it is easy to spot the overcontinuation problem and conclude that an unsuccessful project should have been abandoned earlier. This is not possible with the overabandonment problem, because it results in missed opportunities which are much more difficult, if not impossible, to spot.

2.3 R & D as an Example

One example of a multi-stage project that is subject to abandonment is R & D type work. Holmstrom (1989) remarks that "the agency costs associated with innovation are likely to be high".[2] Features of R & D projects which cause contracting to be particularly demanding are:

- risky – probability of failure is high, but if successful, the returns can be extraordinary;
- labor intensive – substantial human effort is required at each stage;
- idiosyncratic – R & D projects are not easily compared with other projects; and
- long-term and multi-stage – projects are subject to termination notwithstanding efforts previously expended. Sometimes, the project is abandoned after many years of work, due for example, to information that competitors are far ahead in the research or that the research has little value. Of particular importance is that companies have often terminated R & D programs "for reasons that have nothing to do with the research quality." (Schneiderman, 1991)

[2] This suggests that finding ways to reduce these costs is a worthwhile endeavor.

Gibson (1981, pp.320–326) discusses three major phases in an R & D project. He states that as these phases are executed, "there should be a steadily decreasing risk caused by increased knowledge". At the initial selection point, risk is the highest. He also states that the cost of an R & D project is minimal at the beginning and rises with the state of certainty regarding the success of the project.

The first phase is called the intuitive/heuristic phase. The R & D idea is submitted and a feasibility check is performed. A tentative budget is set. If the initial indicators are positive, then the project proceeds into the critical phase. As the project progresses, new information on the project is received and the research scientist revises the probability of success of the project. At the same time, the expenditure on the project is increasing, and this is the point at which the scientist must decide if the project should be continued or abandoned. Gibson states that a decision to stop the project is hard to make since the scientist has an investment in the success of the project. The scientist tends to be overly optimistic and rarely objective. We propose that compensation packages which include a provision for severance pay (or a similar measure) helps motivate the agent to abandon the project if it is no longer sufficiently profitable. In the final phase, the project enters the commercial marketing phase.

Gibson's description of the major phases of the R & D project fits the characteristics of the projects examined (as discussed in Section 2.1) in this part of the dissertation. Since R & D projects are often critical to the earnings growth of a company, it is important that the research personnel be properly motivated to undertake such activities. Kanter (1987) observes that a controversy has been brewing in recent years over how to best and most fairly compensate those from whose efforts originate new products or technology. Proper compensation packages must be designed to attract, retain, and motivate the scientists and research engineers who undertake such projects. This part of the dissertation helps provide some insight into the nature of efficient compensation packages for this group of personnel.

2.4 Agency Literature Review

Twiss (1992) states that project selection and project abandonment are two critical and difficult decision areas in technology management. Project abandonment is important because of the high proportion of projects that are discontinued before their development is completed. Yet, hardly any work on multi-stage projects with project abandonment has been done, although there has been some work in the area of project selection. We review the work in this area as it is closely related to the idea of project abandonment. Lambert (1986) examines the incentives of an agent to invest in a risky single-stage project. His alternative is to invest in a safe project. The agent works to acquire private information about the risky project. Lambert derives conditions under which underinvestment or overinvestment in the risky project occurs. He concludes that underinvestment occurs when the risky project is *a priori* more profitable than the safe project. Balakrishnan (1991) examines a similar model with the additional feature that the agent has precontract private information on the agent's skill. This information is relevant because the ex ante probability of success in the risky project is strictly increasing in the agent's skill. By looking at the default project, i.e., the project that would have been chosen if the agent does not work to acquire information on the risky project and instead uses his precontract information alone, Balakrishnan shows that for the set of agents whose default project is the risk free project, overinvestment in the risky project occurs.

Banker, Datar and Gopi (1989) consider the project selection strategy of the agent, given that he has private post-contract information. A necessary condition for underinvestment and overinvestment is that the agent is risk-averse, and underinvestment and overinvestment occur as a result of a trade-off between risk premium cost and suboptimal project selection cost.

Another relevant area of literature is that on the value of communication of predecision information. Penno (1984) examines if there is strict value to communication of the agent's private pre-decision information, denoted by ξ in a one-period moral hazard setting. There are two crucial assumptions in his model:

1. There exists private information which informs the agent that effort is ineffectual; and

2. In response to this information, the agent has the option of reducing effort to a level of zero.

Penno shows that there exists a cutoff ξ_0 such that effort level equals zero for $\xi \leq \xi_0$ and effort level is strictly positive for $\xi > \xi_0$. He demonstrates that a strict improvement can always be generated by choosing communication, where the message space is either $\xi \leq \xi_0$ or $\xi > \xi_0$. If $\xi \leq \xi_0$, the agent receives a constant wage level. The gain from communication is achieved by allowing the risk-neutral principal to absorb risk from the risk-averse agent. Our model is, in some sense, similar to Penno's model. The termination/continuation decision is equivalent to a form of communication, and Penno's analysis shows that allowing for this decision is strictly valuable. Our analysis takes the model further by examining if both the principal and the agent agree on the same cutoff (which is denoted by ξ_0 in Penno's model).

Melumad (1989) examines a one-period model in which the agent acquires private post-contract pre-decision information and he is allowed to breach the contract by paying the principal predetermined damages. If the agent continues with the contract, he selects his effort level which is subject to the moral hazard problem. Melumad concludes that it is never optimal to include a severance payment in the compensation contract. This result is driven by the fact that the agent's market value after the breach of contract is the same as that before he joins the firm. Melumad does not examine whether the agent's choice of breach of contract is in line with the interest of the principal.

Demski and Sappington (1987) model an agent who is responsible for two activities, planning and implementation, and the latter entails no disutility to the agent. This means that in isolation, there is no moral hazard concerning the implementation activity. They show that it is sometimes optimal to create motivation concerns (termed as induced moral hazard) in the implementation activity in order to be more efficient in motivating the agent in the planning activity. We see a similar result in our model. Although the abandonment/continuation decision entails no disutility to the agent, we obtain overabandonment and overcontinuation as optimal outcomes in the second-best world.

2.5 Conclusion

In this part of the dissertation, we examine the incentives of risk- and work-averse agents to work on multi-stage projects which are subject to abandonment. We obtain overcontinuation and overabandonment as the possible outcomes. The circumstances leading to each situation are determined. With the benefit of hindsight, the overcontinuation behavior has generally been called irrational. Our results show that far from being irrational, they are optimal choices under the particular set of circumstances.

In the next chapter, we set up the general model and examine two special cases of the general model. These provide us with useful insights when we analyze the general model. Chapter 4 looks at the general model and a benchmark case in which information is publicly observable but not contractible.

Chapter 3

The Model and Two Single-Stage Settings

3.1 Introduction

The principal in our model faces a two-stage project. He seeks to attract an agent to join the firm and undertake the project. Working on the project will build up project-specific skills. A key feature of the model is that the agent receives private information after the first stage, from which he decides whether the project is to be abandoned or continued. The principal offers a long-term contract to the agent, which includes a provision for severance pay should the project be abandoned. The level of the severance pay plays a critical role in ensuring that the project will be terminated if it is no longer sufficiently profitable. Since the probability of success of the project is higher with higher effort level, the principal wants to motivate the agent to work hard. At the same time, the principal seeks to motivate the agent to make the abandon/continue decision in the principal's favor. A moral hazard problem exists in the effort level choice. The agent experiences no disutility from the abandon/continue decision, which is observable by the principal. Thus, viewed in isolation, there is no motivation concern in the abandon/continue decision, and the agent's incentive is aligned with the principal's.

Our results show that the moral hazard problem in the effort level choice leads to a motivation concern in the abandon/continue decision. Such concern is termed induced moral hazard by Demski and Sappington (1987). As a result, the principal may prefer to motivate the agent to choose to continue the project even when it does not appear profitable to do so. This explains why some firms appear reluctant to terminate their projects even when information received indicates that the probability of success is low. Such escalation behavior has been generally termed as irrational, and as evidence that decision-makers do not ignore sunk costs. Kanodia, Bushman and Dickhaut (1989) provide an explanation for such behavior based on reputation. They show that when the agent has private information about his human capital, a desire for reputation-building may lead the agent to demonstrate escalation behavior. Our model provides an alternative explanation based on induced moral hazard.

While escalation is one possibility, our results also indicate that when the return from a successful project is relatively high, the induced moral hazard problem may lead the principal to prefer to motivate a higher cutoff point. Thus, at times, firms may appear too hasty in terminating their project.

We set up the general model in section 3.2. In sections 3.3 and 3.4, we discuss two special cases which will provide us with useful insights when we analyze the general model. We examine a numerical example in section 3.5. All proofs are provided in the appendix. In the next chapter, we analyze the general case.

3.2 The Model

3.2.1 General Characteristics

We consider a three-date economy, $i = 0,\ 1$ and 2. The principal has a project, and he employs an agent to undertake the work. The project has two stages. The completion of the first stage coincides with date $i = 1$. The returns of the project are realized at the end of the second stage, which occurs at date $i = 2$. The agent takes t_{1m} at stage 1 and t_{2n} at stage 2 (if there is no abandonment), where $m, n \in \{h, l\}$, with h and l corresponding to high effort and low effort respectively. If the project is carried to completion, there are two possible cash flows: x_H represents a favorable outcome and x_L represents an unfavorable outcome. We assume that if the agent takes either t_{1l} or t_{2l}, the project crashes with probability one, i.e., $Pr(x_L|t_{1m}, t_{2n}) = 1$, if either m or n equals l.

After the agent has implemented his first-stage effort, he privately observes information signal y.

This signal allows him to update his probability assessment of a high cash flow, given the project has not crashed. In particular, let $Pr(x_H|t_{1h}, t_{2h}, y) = y$. We assume that the signal y is generated from the uniform distribution over the interval zero to one. Also, we assume that the agent is unable to communicate the information signal to the principal, because of the excessive cost of communication.

To utilize this information, we assume that the principal provides for the possibility of project abandonment after the first stage. Since the information is privately observed by the agent, the abandonment/continuation decision must be delegated to him.[1] After the agent has observed y and updated the probability of obtaining x_H, he decides if the project should be continued or abandoned. The agent experiences no direct disutility from making this decision. If the project is abandoned, the agent's employment is terminated and he enters the job market.

We assume the principal and the agent enter into a two-stage contract. The principal wants to motivate the agent to choose t_{1h}, and if the project is continued, the principal wants to motivate the agent to choose t_{2h}. The principal can commit to hire the agent for both stages, unless the project is abandoned. We assume it is in the principal's interest to commit to the contract for the duration of the project, since the project-specific skill of the agent is difficult and expensive to replace. The agent, on the other hand, cannot commit to remain with the firm for both stages. In particular, since the contract provides for project abandonment and subsequent termination of the agent's employment, the agent is free to leave the firm after the first stage concomitant with deciding that the project should be abandoned.

In this setting, the compensation package serves both to motivate the agent to choose t_{1h} and t_{2h}, and to induce an abandon/continue decision that is in the principal's interest. The compensation package consists of two components:

[1] Project abandonment may make the principal better off. For example, the project may require the principal to invest \$4 million in the first stage. If the project is continued, an additional investment of \$10 million is required. By providing for abandonment, if a very bad signal is received, the principal can avoid the further investment by allowing the agent to abandon the project.

1. If the project is carried on to completion, the agent earns a fixed wage, w_1, paid at the end of stage 1, and an amount $w_2(x_j)$ contingent on the cash flow x_j, $j = H, L$ realized, at the end of stage 2.

2. If the project is abandoned at the end of stage 1, the agent's employment is terminated and he is given severance pay, w_s. The agent enters the job market, earns a net wage of w_k in return for effort level t_{2l}.

Thus, the long-term compensation contract $\{[w_1, w_2(x_j)], w_s\}$ includes the following provision. At $i = 1$, if the agent decides that the project is to be abandoned, then his employment will be terminated and he receives severance pay w_s. Otherwise, if the project is to be continued, the agent will be paid w_1 in the first stage and $w_2(x_j)$ in the second stage.

The principal is risk-neutral while the agent is risk-averse. The agent also experiences a pecuniary private cost with effort supply, i.e., we assume that the direct impact of the agent's effort on his utility is represented as a "financial" cost to him. This cost might represent an opportunity cost of the time spent on the project. We assume that the agent's utility function exhibits constant absolute risk aversion, r, and is represented as follows:

$$H(\omega_1, \omega_2, t_{1m}, t_{2n}) = -\exp[-r(\omega_1 + \omega_2 - t_{1m} - t_{2n})],$$

where ω_1 is the first-stage aggregate income and ω_2 is the second-stage aggregate income. Hence,

$$\begin{aligned} H(\omega_1, \omega_2, t_{1m}, t_{2n}) &= U(\omega)v(t_{1m})v(t_{2n}), \\ \text{where } U(\omega) &= -\exp(-r\omega), \\ v(t_{im}) &= \exp(rt_{im}), \\ \text{and } \omega &= \omega_1 + \omega_2. \end{aligned}$$

In effect, the agent is only concerned with aggregate consumption.[2] Thus, we need only solve for the

[2]Thus, the analysis is not influenced by a lack of banking. The model is equivalent to a single-consumption date model, but there are multiple sequential acts.

total compensation $\omega = \omega_1 + \omega_2$. If the project is abandoned after the first stage, $\omega_1 = w_s$ and $\omega_2 = w_k$, which is determined by the job market.

The utility value is non-positive everywhere. We assume $v(t_{il}) = 1,\ i = 1,2$, and $v(t_{1h}) = v(t_{2h}) = v(t_h)$. The agent experiences a higher pecuniary cost with higher effort, so that $v(t_h) > v(t_l) = 1$. His reservation utility level is K, $K < 0$ and the equivalent wage level is $\bar{w}$, i.e., $U(\bar{w}) = K$.

The time-line of the game is as follows:

- At $i = 0$, the principal and the agent enter into a two-stage compensation contract.
- The agent chooses either low or high first-stage effort level $t_{1m}, m \in \{l, h\}$.
- At $i = 1$, a signal, y on the viability of continuing the project is privately observed by the agent. If the agent has chosen t_{1h}, he revises the probability of high cash flow, and accordingly decides if the project is to be continued or abandoned.
- If the project is abandoned, the agent's employment is terminated, he is paid his severance pay w_s, and he enters the job market and earns a wage of w_k. The principal faces alternative investment with return B.[3]
- If the project is continued, the agent is paid his first-stage wage, w_1. He chooses the second-stage effort level, incurring additional disutility $v(t_{2m})$.
- At $i = 2$, the outcome $x_j,\ j = L, H$ is publicly observed.
- The agent is paid his second-stage wage, $w_2(x_j)$.

Refer to Figure 3.1 in which we use a decision tree to depict the agent's sequential decision problem.

[3]For example, some of the capital equipment purchased for the project could be sold and the money reinvested at the assumed interest rate of zero. The principal considers abandonment only if $x_L < B + w_k + t_h$.

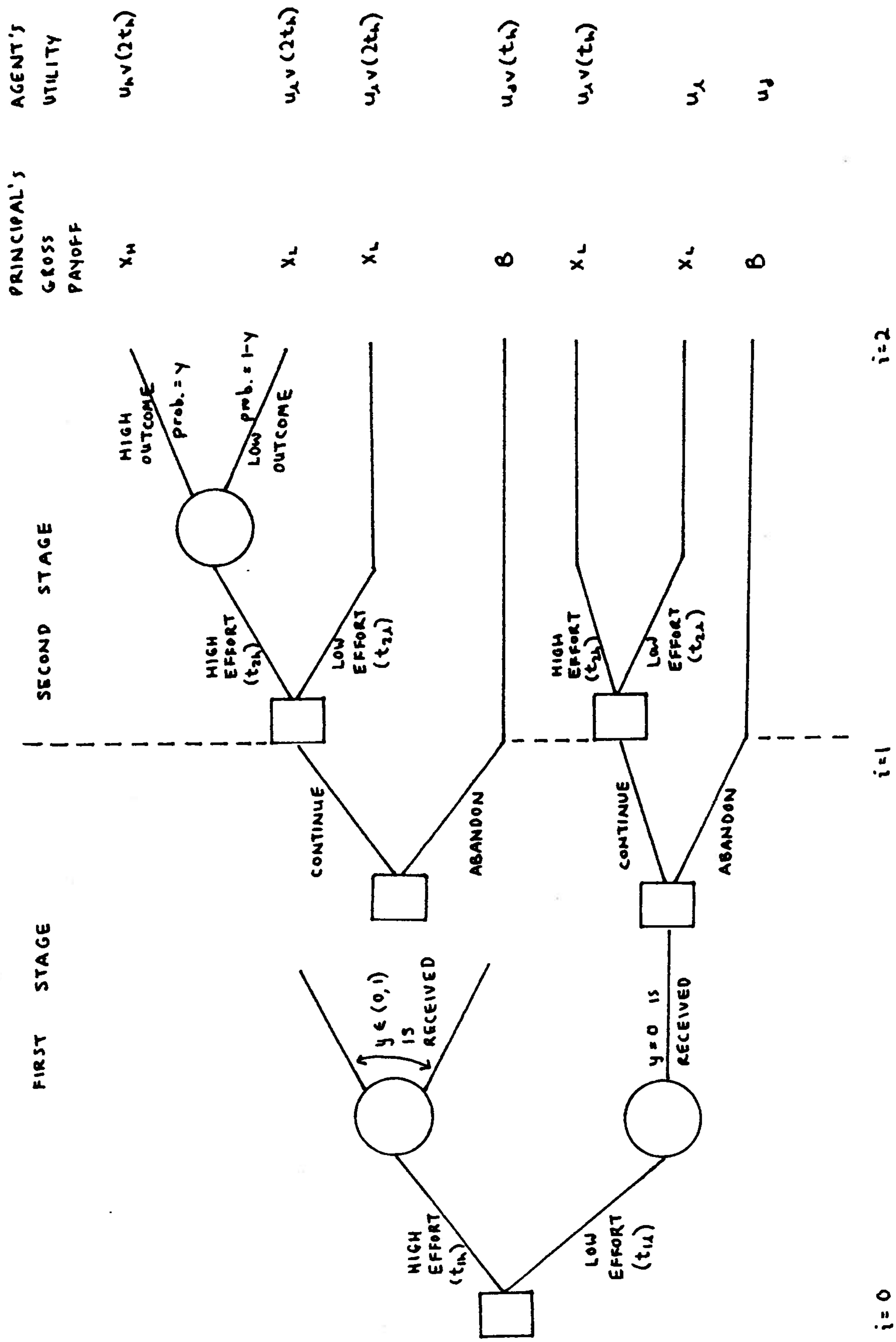


Figure 3.1: Decision Tree Depicting the Agent's Sequential Decision Problem

The principal seeks to motivate the agent to take t_{1h} and choose some cutoff point $\hat{y}$, such that if he observes $y < \hat{y}$, the agent abandons the project, while if he observes $y \geq \hat{y}$, he continues with the project. The agent's abandon/continue decision constitutes a level of communication to the principal of the former's private information. An abandonment (continuation) implies that $y < (\geq)\hat{y}$. The agent's effort is not observable, though his decision of whether to continue is observable. However, the principal cannot determine why the abandonment/continuation decision was made. For example, when the principal observes an abandonment decision, he cannot determine whether the agent has shirked in the first stage or if the information signal received indicates unfavourable conditions.

If the agent takes t_{1l}, the possible cash flows are B if the project is abandoned, and x_L if the project is continued. On the other hand, if the agent takes $\{t_{1h}, t_{2h}|\text{if continue}\}$, there are a total of three possible gross cash flows, namely, B, x_H and x_L. The probability of obtaining each cash flow depends on the cutoff point $\hat{y}$. Let p_{Bh}, p_{Hh} and p_{Lh} denote the probability of obtaining cash flow B, x_H and x_L, respectively, given the agent picks $\{t_{1h}, t_{2h}|\text{if continue}\}$ and $\hat{y}$.

$$\begin{aligned}
p_{Bh} &= \hat{y}, \\
p_{Hh} &= \int_{\hat{y}}^{1} P(x_H|t_{1h}, t_{2h}, y)f(y)dy \\
&= \frac{y^2}{2}\Big|_{\hat{y}}^{1} \\
&= \frac{1}{2}(1-\hat{y}^2), \\
p_{Lh} &= \int_{\hat{y}}^{1} [1 - P(x_H|t_{1h}, t_{2h}, y)]f(y)dy \\
&= (y - \frac{y^2}{2})\Big|_{\hat{y}}^{1} \\
&= \frac{1}{2}(1-\hat{y})^2. \qquad (3.1)
\end{aligned}$$

3.2.2 The Problem

The principal's problem is to choose a compensation contract and cutoff point that induces the agent to choose high effort and the desired investment. Following Grossman and Hart (1983), we decompose the principal's problem into two parts: (i) the **Contract Choice Problem** in which the principal identifies the optimal compensation contract for inducing high effort for each feasible cutoff $\hat{y} \in (0,1)$; (ii) the **Cutoff Point Selection Problem** in which the principal identifies the cutoff point $\hat{y}^*$ that maximizes his expected net profits. Assume that $\hat{y} \in (0,1)$.[4] Let

$$\begin{aligned} u_l &= U(w_1 + w_2(x_L)) = U(w_l), \\ u_h &= U(w_1 + w_2(x_H)) = U(w_h), \\ u_d &= U(w_s + w_k) = U(w_d). \end{aligned}$$

Observe that the contract can be represented by either (w_l, w_h, w_s) or (u_l, u_h, u_d) with $w_p = U^{-1}(u_p) = h(u_p)$, $p = l, h, d$.

Before we analyze the general problem, we consider the following two special cases:

1. No first-stage moral hazard.

2. No second-stage moral hazard.

3.3 Second-stage Moral Hazard, No First-stage Moral Hazard

3.3.1 The Model

We consider a simple scenario in which the agent chooses effort level only once, after he has observed the signal and decided if the project should be continued or abandoned. The time-line of the game is as follows:

[4] The cases for $\hat{y} = 0$ and $\hat{y} = 1$ are not interesting. If $\hat{y} = 0$, no abandonment is provided for in the contract. If $\hat{y} = 1$, the principal will never take up the project in the first place, since he will definitely be abandoning it after the first period.

1. At $i = 0$, the principal and the agent enter into a compensation contract.

2. A signal, y on the viability of continuing the project is privately observed by the agent. He revises the probability of high cash flow, and accordingly decides if the project is to be continued or abandoned.

3. If the project is abandoned, the agent's employment is terminated and he is paid w_s.[5] The agent enters the job market and earns a wage of w_k. The principal faces alternative investment with return B.

4. If the project is continued, the agent chooses the effort level, incurring disutility $v(t_m)$.

5. At $i = 1$, the outcome x_j, $j = L, H$ is publicly observed.

6. The agent is paid his wage $w(x_j)$.

The principal seeks to motivate the agent to choose some cutoff point $\hat{y}$, such that if he observes $y < \hat{y}$, he abandons the project, while if he observes $y \geq \hat{y}$, he continues with the project. If the project is continued, the principal seeks to motivate the agent to choose the high effort level.

This case is related to the literature dealing with post-contract information. In this strand of literature, it is usually assumed that the agent is committed to the firm. Even when the information received is not favorable, there is no provision in the contract to allow the agent to leave the firm. Melumad (1989) permits the agent to quit the contract if the information is unfavorable, upon the payment of damages and shows that allowing for a breach results in a Pareto improvement. However, he does not examine whether the agent's incentive to quit is aligned with the principal's incentive.

[5]Note that w_s may either be positive, i.e., the agent receives severance pay; or negative, i.e., the agent pays a penalty to withdraw from the contract.

3.3.2 First-best Solution

First, we consider a setting where the agent's effort is observable; thus, there is no incentive problem. The principal offers a compensation package to the agent such that the agent is indifferent between continuing or abandoning the project. We assume that when the agent is indifferent, he decides in the principal's best interest. If the project is abandoned, the transfer payment is w_s. If the project is continued, the agent receives compensation w_f. Thus, the agent is indifferent between abandoning or continuing the project when $U(w_s + w_k) = U(w_f)v(t_h)$. The participation constraint requires

$$U(w_s + w_k)\hat{y} + U(w_f)v(t_h)(1 - \hat{y}) = U(\bar{w}).$$

Consequently, the first-best compensation package is:

$$\begin{aligned} w_s &= \bar{w} - w_k, \\ w_f &= \bar{w} + t_h. \end{aligned}$$

Whether w_s is positive or negative depends on the level of w_k.

The principal's expected payoff, for a given cutoff $\hat{y}$, is

$$\hat{y}(B - w_s) + \frac{1}{2}(1 - \hat{y}^2)x_H + \frac{1}{2}(1 - \hat{y})^2 x_L - (1 - \hat{y})w_f.$$

Substituting for w_s and w_f, the first-order condition with respect to $\hat{y}$ is:

$$B - \bar{w} + w_k - \hat{y}x_H - (1 - \hat{y})x_L + \bar{w} + t_h = 0 \tag{3.2}$$

Note that the second-order condition on $\hat{y}$ is $-(x_H - x_L)$, which is negative. This implies that the principal's objective function is strictly concave, thus the first-order condition is necessary and sufficient to obtain the optimal cutoff. The first-best cutoff is

$$y^* = \begin{cases} 0 & \text{if } B + w_k + t_h \leq x_L \\ \frac{B+w_k+t_h-x_L}{x_H-x_L} & \text{if } x_L < B + w_k + t_h < x_H \\ 1 & \text{if } B + w_k + t_h \geq x_H. \end{cases}$$

To provide an intuitive explanation for y^*, we rewrite the first-order condition evaluated at y^* in the following manner:

$$B - [\bar{w} - w_k] = y^* x_H + (1 - y^*) x_L - [\bar{w} + t_h].$$

The left-hand side of the equality is the return to the principal if the project is abandoned, while the right-hand side of the equality gives the expected profitability if the project is continued. Thus, for $y \geq y^*$ ($y < y^*$), it is optimal to continue with (abandon) the project.

3.3.3 Second-best Solution

We now consider the setting in which the agent's effort is not observable. The principal wants to motivate the agent to choose some cutoff $\hat{y}$ and to take t_h if the project is continued. If the agent is offered the contract (u_l, u_h, u_d), and he takes effort t_h, then his expected utility with cutoff $\hat{y}$ is:

$$EH_h = \hat{y} u_d + \frac{1}{2}(1 - \hat{y}^2) u_h v(t_h) + \frac{1}{2}(1 - \hat{y})^2 u_l v(t_h).$$

The agent's first-order condition on $\hat{y}$ is:

$$u_d - \hat{y} u_h v(t_h) - (1 - \hat{y}) u_l v(t_h) = 0.$$

Hence, the agent will choose

$$\hat{y} = \begin{cases} 0 & \text{if } u_d \leq u_l v(t_h) \\ \frac{u_d - u_l v(t_h)}{v(t_h)[u_h - u_l]} & \text{if } u_l v(t_h) < u_d < u_h v(t_h) \\ 1 & \text{if } u_d \geq u_h v(t_h). \end{cases}$$

We note that for an interior solution, the agent's second-order condition on $\hat{y}$ is also satisfied.

The agent's first-order condition with respect to $\hat{y}$ implies the following:

- For $y < \hat{y}$, the agent is better off abandoning the project than continuing it at effort level t_h.
- For $y \geq \hat{y}$, the agent is better off continuing the project at effort level t_h than to abandon it.

However, the principal must ensure that for $y < \hat{y}$, the agent will prefer to abandon the project rather than continue it with effort level t_l. Also, for $y \geq \hat{y}$, the principal must ensure that the agent prefers to continue the project with effort level t_h than with effort level t_l. This is achieved by imposing the following two constraints:

$$\begin{aligned} u_d &\geq u_l, \\ \hat{y}u_h v(t_h) + (1-\hat{y})u_l v(t_h) &\geq u_l. \end{aligned}$$

Since the agent's first-order condition on $\hat{y}$ implies that $u_d = \hat{y}u_h v(t_h) + (1-\hat{y})u_l v(t_h)$, one of these two constraints is redundant. For subsequent analysis, we use the constraint $u_d \geq u_l$.

The Contract Choice Problem

The contract choice problem for each feasible cutoff $\hat{y} \in (0,1)$ is given as follows:

$$\begin{array}{lll} [P3.1] & \min_{\{u_d,u_h,u_l\}} & p_B(h(u_d) - w_k) + p_H h(u_h) + p_L h(u_l) \\ & \text{s.t.} & p_B u_d + p_H u_h v(t_h) + p_L u_l v(t_h) \geq K, \\ & & u_d \geq u_l, \\ & \text{and} & u_d - \hat{y}u_h v(t_h) - (1-\hat{y})u_l v(t_h) = 0. \end{array}$$

The first constraint is the participation constraint and ensures that the agent's expected utility from joining the firm is at least as high as his reservation utility level. The second constraint ensures that the agent will weakly prefer to abandon the project rather than continue it with effort level t_l if he observes that $y < \hat{y}$. The last constraint is the agent's first-order condition on $\hat{y}$, and it requires that for each value of the information signal y, the specified abandonment/continuation decision is optimal for the agent. The second and third constraints together also ensure that the agent weakly prefers to continue the project with effort level t_h than to continue with effort level t_l if he observes that $y \geq \hat{y}$.

Let λ_1, μ_1 and η_1 be the Lagrange multipliers of the first, second and third constraints respectively. Using first-order conditions, we obtain the following characterizations of the optimal compensation package.

$$\begin{aligned} h'(\hat{u}_d) &= \lambda_1 + \frac{\mu_1}{\hat{p}_B} + \frac{\eta_1}{\hat{p}_B}. \\ h'(\hat{u}_h) &= \lambda_1 v(t_h) - \frac{\eta_1 \hat{y} v(t_h)}{\hat{p}_H}. \\ h'(\hat{u}_l) &= \lambda_1 v(t_h) - \frac{\mu_1}{\hat{p}_L} - \frac{\eta_1(1-\hat{y})v(t_h)}{\hat{p}_L}. \end{aligned}$$

<u>Lemma 3.1:</u> At the optimal solution, all three constraints are binding, with $\lambda_1 > 0$, $\mu_1 > 0$ and $\eta_1 < 0$.

The optimal expressions for u_d, u_l and u_h are given as follows:[6]

$$\begin{aligned} \hat{u}_d = \hat{u}_l &= \frac{2K\hat{y}}{2\hat{y}v(t_h) - (v(t_h)-1)(1+\hat{y}^2)}, \\ \text{and} \qquad \hat{u}_h &= \frac{2K[\hat{y}v(t_h) - v(t_h) + 1]}{v(t_h)[2\hat{y}v(t_h) - (v(t_h)-1)(1+\hat{y}^2)]}. \end{aligned} \tag{3.3}$$

Note that $\hat{u}_d < K = U(\bar{w})$. Recall that $u_d = U(w_s + w_k)$. Thus, $w_s < \bar{w} - w_k$. w_k is the market's employment alternative if the agent abandons the project after he receives the information on the project. w_s may be positive (i.e., the principal pays the agent a severance pay for termination of the contract) or negative (i.e., the agent pays the principal a penalty to withdraw from the contract), and this partially depends on the level of w_k. If $w_k = \bar{w}$, then at the optimal solution, w_s is negative. This is consistent with Melumad (1989). He assumes that $w_k = \bar{w}$, i.e., the agent's employement alternative before and after he obtains the private information on the project is unchanged. He proves that it is never optimal for the principal to pay the agent a severance pay for termination of the contract.

Also, as $\hat{y}$ increases, $\hat{u}_d = \hat{u}_l$ increases, while $\hat{u}_h$ decreases, i.e., the spread between $\hat{u}_h$ and $\hat{u}_l$ decreases.[7] To provide incentive for the agent to choose high effort if the project is to be continued, the

[6] See appendix 3A for details.
[7] See appendix 3A for details.

principal imposes risk on the agent, who is then compensated for bearing this risk in the form of a risk premium. A high cutoff implies that the project is to be continued and effort is required to be exerted only when very good information is received. Thus, the higher is $\hat{y}$, the lesser is the amount of risk that is needed to be imposed on the agent and the smaller is the spread between $\hat{u}_h$ and $\hat{u}_l$.

On the other hand, a low cutoff implies that effort is required to be exerted even when information is not too favorable. A greater amount of risk is needed to be imposed on the agent. Lemma 3.2 gives the lower bound of the cutoff for a solution to exist.

<u>Lemma 3.2:</u> A necessary condition for a solution to exist is that $\hat{y} > (1 - \frac{1}{v(t_h)})$.

If $\hat{y} \leq (1 - \frac{1}{v(t_h)})$, the last constraint cannot be satisfied. The cost of motivating the agent becomes infinitely high and no feasible wage contract exists.

The Cutoff Point Selection Problem

To consider the implication of $\eta_1 < 0$, we examine the full principal's problem.

$$\begin{aligned}
\max_{\{u_d, u_h, u_l, \hat{y}\}} \quad & p_B[B - h(u_d) + w_k] + p_H[x_H - h(u_h)] + p_L[x_L - h(u_l)] \\
\text{s.t.} \quad & p_B u_d + p_H u_h v(t_h) + p_L u_l v(t_h) \geq K, \\
& u_d \geq u_l, \\
\text{and} \quad & u_d - \hat{y} u_h v(t_h) - (1 - \hat{y}) u_l v(t_h) = 0.
\end{aligned}$$

Taking the principal's first-order condition with respect to $\hat{y}$ and substituting in the agent's first-order condition for $\hat{y}$, we obtain

$$B - h(u_d) + w_k - \hat{y}^*[x_H - h(u_h)] - (1 - \hat{y}^*)[x_L - h(u_l)] - \eta_1 v(t_h)[u_h - u_l] = 0. \tag{3.4}$$

Since $u_h > u_l$, then

$$\text{Sign }(\eta_1) = \text{Sign }\{B - h(u_d) + w_k - \hat{y}^*[x_H - h(u_h)] - (1-\hat{y}^*)[x_L - h(u_l)]\}.$$

Lemma 3.1 states that $\eta_1 < 0$. This implies that at the optimal cutoff point $\hat{y}^*$,

$$B - h(u_d) + w_k < \hat{y}^*[x_H - h(u_h)] + (1-\hat{y}^*)[x_L - h(u_l)].$$

The left-hand side of the inequality is the return to the principal if the project is abandoned, while the right-hand side gives the expected profitability conditioned on $\hat{y}^*$ if the project is continued. Thus, at $\hat{y}^*$, the principal is not indifferent between abandoning and continuing the project. He strictly prefers that the project be continued at $\hat{y}^*$ and is indifferent at a cutoff point lower than $\hat{y}^*$. From the point of view of the principal, the second-best contract motivates the agent to overabandon the project. It is important to note that this conflict of interest refers to the abandonment/continuation decision after the agent observes the value of y.

The agent does not put in any effort before the abandon/continue decision and the principal needs to motivate the agent to choose high effort only if the project is continued. If $y > \hat{y}$ is observed and the project is continued, the probability of obtaining the high wage payment associated with the good outcome is y if the agent works hard. Therefore, as $\hat{y}$ increases, this probability increases and the cost of motivating the agent to work hard decreases. While the principal desires a lower cutoff, to keep the expected compensation cost low, he settles for a higher cutoff.

Next, we compare the second-best optimal cutoff point with the first-best cutoff point.

Proposition 3.1: The first-best cutoff point is lower than the second-best optimal cutoff point, i.e., $y^* < \hat{y}^*$.

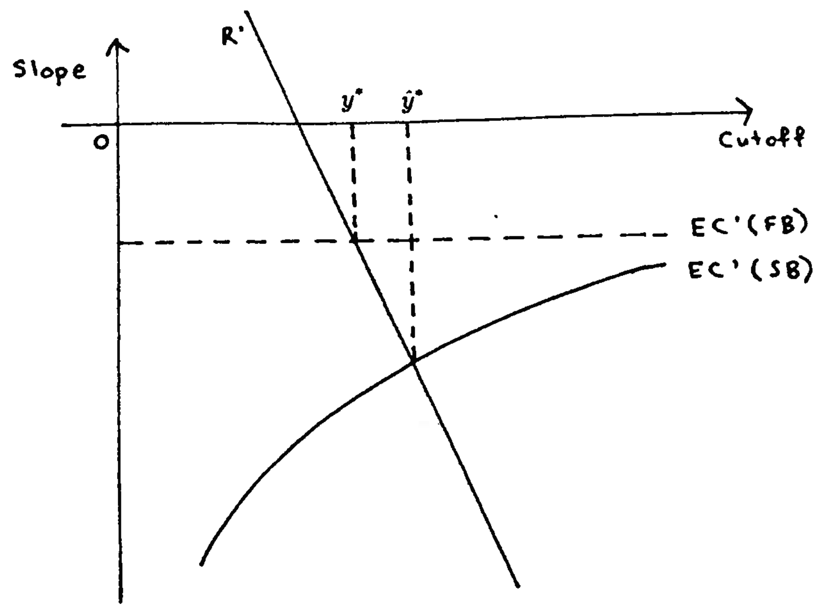


Figure 3.2: Comparison of $\hat{y}^*$ with y^* (no first-stage moral hazard)

R: Revenue
EC(FB): Expected Compensation Cost (first-best)
EC(SB): Expected Compensation Cost (second-best)

In the first-best case, the expected compensation cost is :

$$EC(FB) = \hat{y}(\bar{w} - w_k) + (1 - \hat{y})(\bar{w} + t_h).$$

Thus, the slope of the expected compensation cost is given by $-(w_k + t_h)$. At the second-best optimal cutoff point, the slope of the expected compensation cost is strictly more negative than that for first-best.[8] As Figure 3.2 shows, this implies that $y^* < \hat{y}^*$.

Thus, there exists a range of information signals $y \in (y^*, \hat{y}^*)$ in which the agent abandons the project even though the principal would have chosen to continue if he could observe the agent's effort. The moral hazard problem with the effort level choice leads the principal to prefer to motivate a higher cutoff point.

[8] See appendix 3A for proof

3.4 First-stage Moral Hazard, No Second-stage Moral Hazard

3.4.1 The Model

We consider another simple scenario in which the agent chooses his effort level only once, but in this case, the choice is made before he observes the signal and decides if the project should be continued or abandoned. The time-line of the game is as follows:

1. At $i = 0$, the principal and the agent enter into a compensation contract.
2. The agent chooses effort, incurring disutility $v(t_m)$.
3. A signal, y on the viability of continuing the project is privately observed by the agent. He revises the probability of high cash flow, and accordingly decides if the project is to be continued or abandoned.
4. If the project is abandoned, the agent's employment is terminated and he is paid w_s.[9] The agent enters the job market and earns a wage of w_k. The principal faces alternative investment with return B.
5. If the project is continued, at $i = 1$, the outcome x_j, $j = L, H$ is publicly observed.
6. The agent is paid his wage $w(x_j)$.

The principal seeks to motivate the agent to choose the high effort level and some cutoff point $\hat{y}$, such that if he observes $y < \hat{y}$, he abandons the project, while if he observes $y \geq \hat{y}$, he continues with the project.

Dye (1983) analyzes the value of communication when the agent receives private information after he has chosen his effort level. In his setting, the information is not used for any decision-making purposes.

[9]Note that w_s may either be positive (i.e., the agent receives severance pay,) or negative (i.e., the agent pays a penalty to withdraw from the contract).

Any value of communication derives from improved risk sharing in the compensation contract, and the agent's communication constitutes a choice from among a menu of compensation contracts that are contingent on the outcome. In my model, the agent's continuation decision is a form of communication with a coarse message space, i.e., abandonment implies that $y < \hat{y}$ while continuation implies that $y \geq \hat{y}$. My model is similar to Dye (1983) in that information is received and communication takes place after the agent has chosen his effort level. However, unlike Dye, the information in my model has decision-making value and the expected gross returns to the principal is different depending on the message. In Dye's model, the private signal received by the agent is correlated with the output but the message from the agent has no impact at all on the output.

3.4.2 First-best Solution

When the agent's effort is observable, there is no incentive problem. The principal offers a compensation package to the agent such that the agent is indifferent between abandoning or continuing the project. If the project is abandoned, the transfer payment is w_s. If the project is continued, the agent receives compensation w_f. Thus, the agent is indifferent between abandoning or continuing the project when $U(w_s + w_k) = U(w_f)$. The participation constraint requires

$$U(w_s + w_k)v(t_h)\hat{y} + U(w_f)v(t_h)(1 - \hat{y}) = U(\bar{w}).$$

Consequently, the first-best compensation package is:

$$\begin{aligned} w_s &= \bar{w} + t_h - w_k, \\ w_f &= \bar{w} + t_h. \end{aligned}$$

The first-best cutoff point in the present single-stage moral hazard problem is lower than that for the no-first-stage moral hazard problem. In the present problem, compensation for the agent's effort is a sunk cost at the time of the continuation decision, thus it is not relevant in the determination of the cutoff point. On the other hand, in the no-first-stage moral hazard problem, at the time of the continuation decision, compensation for effort is a relevant cost in the determination of the cutoff point. The first-best cutoff point in the present problem is given as follows:

$$y^* = \begin{cases} 0 & \text{if } B + w_k \leq x_L \\ \frac{B+w_k-x_L}{x_H-x_L} & \text{if } x_L < B + w_k < x_H \\ 1 & \text{if } B + w_k \geq x_H. \end{cases}$$

At the first-best cutoff,

$$B - [\bar{w} + t_h - w_k] = y^* x_H + (1 - y^*)x_L - [\bar{w} + t_h]. \tag{3.5}$$

The left-hand side of the equality is the return to the principal if the project is abandoned, while the right-hand side of the equality gives the expected profitability if the project is continued. Thus, at $y = y^*$, the principal is indifferent between abandoning and continuing the project, and for $y \geq y^*(y < y^*)$, he prefers to continue with (abandon) the project.

3.4.3 Second-best Solution

The Contract Choice Problem

The contract choice problem for each feasible cutoff $\hat{y}$ is given as follows:

$$\begin{array}{lll} [P3.2] & \min_{\{u_d,u_h,u_l\}} & p_B(h(u_d) - w_k) + p_H h(u_h) + p_L h(u_l) \\ & \text{s.t.} & [p_B u_d + p_H u_h + p_L u_l]v(t_h) \geq K, \\ & & [p_B u_d + p_H u_h + p_L u_l]v(t_h) \geq u_d, \\ & & [p_B u_d + p_H u_h + p_L u_l]v(t_h) \geq u_l, \\ & \text{and} & u_d - \hat{y}u_h - (1 - \hat{y})u_l = 0. \end{array}$$

The first constraint is the participation constraint and ensures that the agent's expected utility from joining the firm is at least as high as his reservation utility level. The second constraint ensures that the agent will weakly prefer to choose the high effort level and cutoff $\hat{y}$ rather than choose effort level t_l and abandon the project always. The third constraint ensures that the agent will weakly prefer to choose the high effort level and cutoff $\hat{y}$ rather than choose effort level t_l and continue the project always. The last constraint is the agent's first-order condition on $\hat{y}$, and it requires that for each value of the information signal y, the specified abandonment/continuation decision is optimal for the agent. To motivate the agent to work and choose $\hat{y}$, u_h must be greater than u_d, otherwise the agent will never work but will choose to abandon the project always. Also, u_l must be less than u_d, otherwise the agent will never abandon the project. This implies that the third constraint is never binding, thus the Lagrange multiplier for the constraint is zero. For subsequent analysis, we ignore the third constraint.

Let λ_2, μ_2 and η_2 be the Lagrange multipliers of the first, second and fourth constraints respectively. Using first-order conditions, we obtain the following characterizations of the optimal compensation package.

$$\begin{aligned} h'(\hat{u}_d) &= v(t_h)[\lambda_2 + \mu_2] - \frac{\mu_2}{\hat{p}_B} + \frac{\eta_2}{\hat{p}_B}. \\ h'(\hat{u}_h) &= v(t_h)[\lambda_2 + \mu_2] - \frac{\eta_2 \hat{y}}{\hat{p}_H}. \\ h'(\hat{u}_l) &= v(t_h)[\lambda_2 + \mu_2] - \frac{\eta_2(1-\hat{y})}{\hat{p}_L}. \end{aligned}$$

Lemma 3.3: At the optimal solution, $\lambda_2 > 0$, $\mu_2 > 0$ and $\eta_2 > 0$.

The optimal expressions for u_d, u_l and u_h are given as follows:

$$\begin{aligned} \hat{u}_d &= K, \\ \hat{u}_h &= \frac{K[2 - v(t_h)(1+\hat{y})]}{(1-\hat{y})v(t_h)}, \\ \text{and} \quad \hat{u}_l &= \frac{K[v(t_h)(1+\hat{y}^2) - 2\hat{y}]}{(1-\hat{y})^2 v(t_h)}. \end{aligned} \tag{3.6}$$

As $\hat{y}$ increases, $\hat{u}_h$ increases at an increasing rate, while $\hat{u}_l$ decreases at an increasing rate, i.e, the spread between $\hat{u}_h$ and $\hat{u}_l$ increases at an increasing rate.[10] To motivate the agent to exert effort, the principal needs to impose risk on the agent and then compensate him in the form of a risk premium. Here, effort is exerted before information is received. At the point of effort selection, the probability of obtaining a favorable outcome is $\frac{1}{2}(1-\hat{y}^2)$, which decreases as $\hat{y}$ increases. Thus, the higher is $\hat{y}$, the lower is the probability that outcome is informative of the agent's effort and a compensation contract with a bigger spread is necessary to motivate effort. Lemma 3.4 gives the upper bound on $\hat{y}$ for a solution to exist.

Lemma 3.4: Necessary conditions for a solution to exist are:

1. $\hat{y} < [\frac{2-v(t_h)}{v(t_h)}]$, and

2. $v(t_h) < 2$.

If $\hat{y} \geq [\frac{2-v(t_h)}{v(t_h)}]$, $\hat{u}_h$ becomes positive and no feasible wage contract exists. Here, the agent needs to exert effort before the abandon/continue decision. If the probability of abandoning the project is very high, the cost of motivating the agent to work hard in the first stage is too excessive.

The Cutoff Point Selection Problem

To consider the implication of $\eta_2 > 0$, we examine the full principal's problem.

$$\begin{aligned}
\max_{\{u_d,u_h,u_l,\hat{y}\}} \quad & p_B[B - h(u_d) + w_k] + p_H[x_H - h(u_h)] + p_L[x_L - h(u_l)] \\
\text{s.t.} \quad & [p_B u_d + p_H u_h + p_L u_l]v(t_h) \geq K, \\
& [p_B u_d + p_H u_h + p_L u_l]v(t_h) \geq u_d, \\
\text{and} \quad & u_d - \hat{y}u_h - (1-\hat{y})u_l = 0.
\end{aligned}$$

[10] See appendix 3A for details.

Taking the principal's first-order condition with respect to $\hat{y}$ and substituting in the agent's first-order condition for $\hat{y}$, we obtain

$$B - h(u_d) + w_k - \hat{y}^*[x_H - h(u_h)] - (1-\hat{y}^*)[x_L - h(u_l)] - \eta_2[u_h - u_l] = 0. \tag{3.7}$$

Since $u_h > u_l$, then

$$\text{Sign }(\eta_2) = \text{Sign }\{B - h(u_d) + w_k - \hat{y}^*[x_H - h(u_h)] - (1-\hat{y}^*)[x_L - h(u_l)]\}.$$

Lemma 3.3 establishes that $\eta_2 > 0$. This implies that at the optimal cutoff point $\hat{y}^*$,

$$B - h(u_d) + w_k > \hat{y}^*[x_H - h(u_h)] + (1-\hat{y}^*)[x_L - h(u_l)].$$

The left-hand side of the inequality is the return to the principal if the project is abandoned, while the right-hand side gives the expected profitability if the project is continued. Thus, at $\hat{y}^*$, the principal is not indifferent between abandoning and continuing the project. He strictly prefers that the project be abandoned at $\hat{y}^*$ and is indifferent at a cutoff point greater than $\hat{y}^*$. From the point of view of the principal, the second-best contract motivates the agent to overcontinue the project. It is important to note that this conflict of interest refers to the abandonment/continuation decision after the agent observes the value of y.

In this setting, the agent puts in the effort before the abandon/continue decision. The principal needs to motivate the agent to work hard and then choose cutoff $\hat{y}$. At the point of choosing effort input, the probability of obtaining the high wage payment is $\frac{1}{2}(1-\hat{y}^2)$. As $\hat{y}$ increases, this probability decreases and the cost of motivating the agent to work hard increases. While the principal desires a higher cutoff, he settles for a lower cutoff to keep the expected compensation cost low.

Unless the agent has invested high effort level in the project, he would not choose to continue the project. Overcontinuation in the project increases the probability that the project outcome is informative about the agent's effort level. Thus, a compensation contract with a smaller spread is enough to motivate effort, and the savings inherent in a smaller risk premium to the agent offset the cost of overcontinuation.

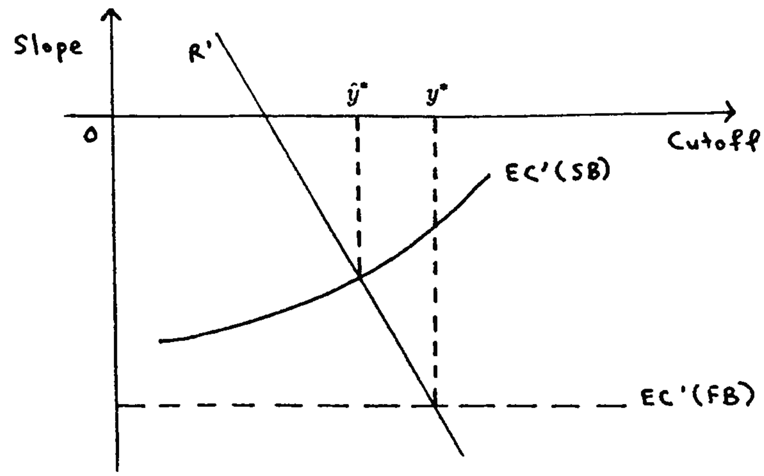


Figure 3.3: Comparison of $\hat{y}^*$ with y^* (no second-stage moral hazard)

R: Revenue
EC(FB): Expected Compensation Cost (first-best)
EC(SB): Expected Compensation Cost (second-best)

Next, we compare the second-best optimal cutoff point with the first-best cutoff point.

Proposition 3.2: The first-best cutoff point is higher than the second-best optimal cutoff point, i.e., $y^* > \hat{y}^*$.

The slope of the expected compensation cost for the first-best case is $-w_k$, and for the second-best case, the slope at the optimal cutoff point is strictly greater than $-w_k$.[11] Thus, as Figure 3.3 indicates, this implies that $y^* > \hat{y}^*$.

Thus, there exists a range of information signals $y \in (\hat{y}^*, y^*)$ in which the agent continues the project even though the principal would have chosen to abandon it if he could observe the agent's effort. This can be related to the sunk cost phenomenon, in which firms appear reluctant to abandon their projects even when the information received indicates that the probability of a good outcome is low. The moral hazard problem with the effort level choice results in an induced moral hazard problem in the abandon/continue

[11] See appendix 3A for details.

decision which leads the principal to prefer to motivate a lower cutoff point.

This result is consistent with Balakrishnan (1991) who examines a single-stage (first-stage) moral hazard problem with precontract information asymmetry on the agent's types. For a set of agent's type, the agent chooses a risk free project if he does not work to acquire information. This is termed the default project, i.e., the project that would have been chosen with the precontract information alone. He demonstrates that for the set of agent types whose default project is the risk free project, overinvestment in the risky project occurs. In our model without precontract information, the default option for the agent if he does not work hard is to terminate the project (the risk free option) and we show that overcontinuation of the project (the equivalence of overinvestment in the risky project) occurs.

3.5 Example

Using the following numerical values, we show how the expected compensation costs and the optimal wage levels vary as the cutoff point varies:

$$\begin{aligned} r &= 1 \\ w_k &= 0.36 \\ \bar{w} &= 0.693 \Rightarrow U(\bar{w}) = -0.5 \\ t_h &= 0.2 \Rightarrow v(t_h) = 1.2214 \end{aligned}$$

Figures 3.4, 3.5 and 3.6 relate to the no-first-stage moral hazard problem, while figures 3.7, 3.8 and 3.9 relate to the no-second-stage moral hazard problem.

3.5.1 No First-stage Moral Hazard Problem

Figure 3.4 shows that the expected compensation cost decreases as the cutoff point increases for both the first-best and second-best cases. While the rate of decrease is constant for the first-best case, the rate of decrease for the second-best case is decreasing in $\hat{y}$. Figure 3.5 shows the optimal wage levels for the second-best case as the cutoff point varies. Observe that w_h is decreasing in $\hat{y}$, while both w_l and w_s are increasing in $\hat{y}$. We also note that when the cutoff point is very low, w_s is negative, i.e., the agent pays the principal a penalty to withdraw from the contract.

We expand the example by varying the levels of x_H, while keeping the values of B and x_L constant.

$$\begin{aligned} B &= -5 \\ x_L &= -30. \end{aligned}$$

Figure 3.6 shows how the optimal cutoff points vary as x_H varies. We observe that $y^* < \hat{y}^*$. Also, the deviation from the first-best cutoff is larger at higher levels of x_H, when the optimal cutoff is lower. We recall that the rate of decrease of the second-best expected compensation cost decreases as $\hat{y}$ increases, while the rate of decrease of the expected compensation cost in the first-best case is constant. This implies that when the principal's desired cutoff is low, greater savings in expected cost result from moving to a higher cutoff than when the principal's desired cutoff is high. Thus, we observe greater deviation from the first-best cutoff when the optimal cutoff is lower.

3.5.2 No Second-stage Moral Hazard Problem

Figure 3.7 shows the expected compensation costs as the cutoff point varies. In the first-best case, the cost decreases at a constant rate as the cutoff point increases, while in the second-best case, the cost is convex. It decreases and subsequently increases at an increasing rate in $\hat{y}$. Figure 3.8 shows the optimal wage levels for the second-best case as the cutoff point varies. Observe that w_h is increasing in $\hat{y}$, while w_l is decreasing in $\hat{y}$.

Figure 3.9 shows how the optimal cutoff points vary as x_H varies. We observe that $y^* > \hat{y}^*$. Also, the deviation from the first-best cutoff is larger at lower levels of x_H, when the optimal cutoff is higher. We recall that in the first-best case, the expected compensation cost decreases at a constant rate, while in the second-best case, the expected cost is convex, and at higher $\hat{y}$, the expected cost increases at an increasing rate in $\hat{y}$. This implies that when the principal's desired cutoff is high, greater savings in expected cost result from moving to a lower cutoff than when the desired cutoff is low. Thus, we observe greater deviation from the first-best cutoff when the optimal cutoff is higher.

No First-Stage Moral Hazard Case

Figure 3.4

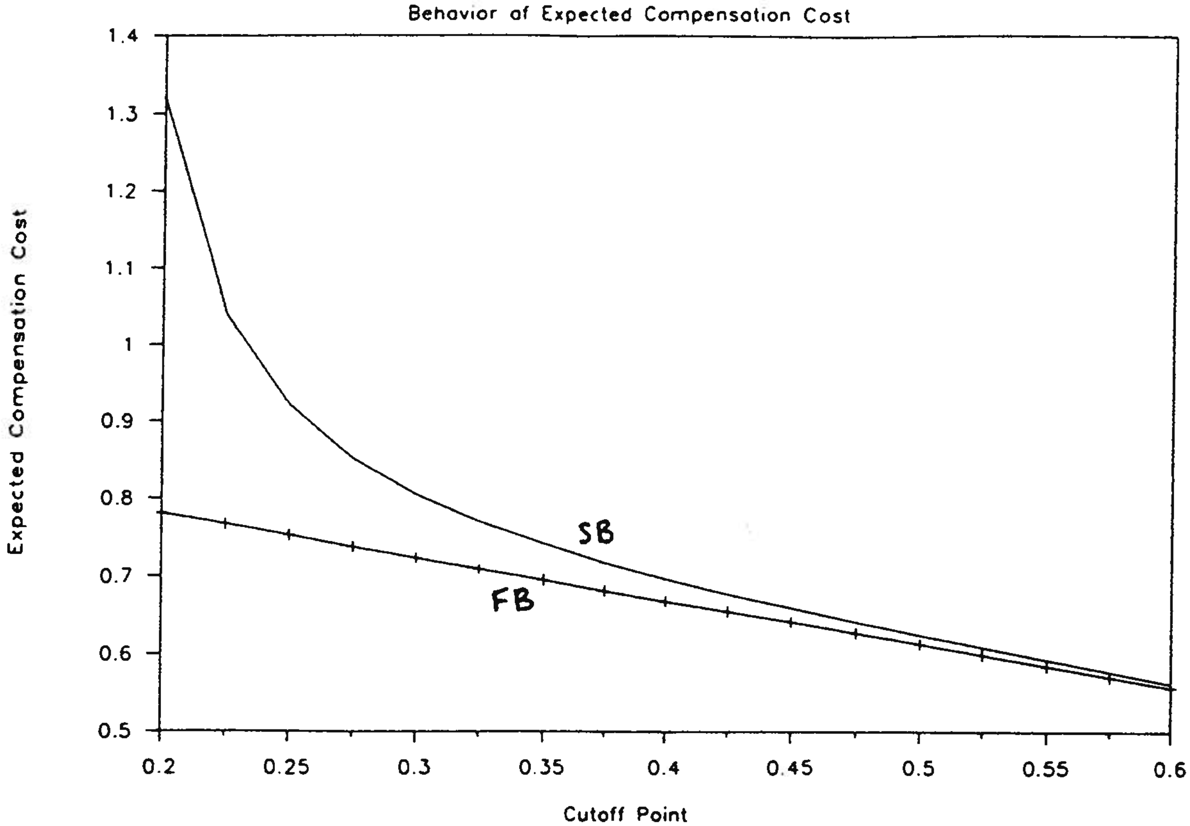


Figure 3.5

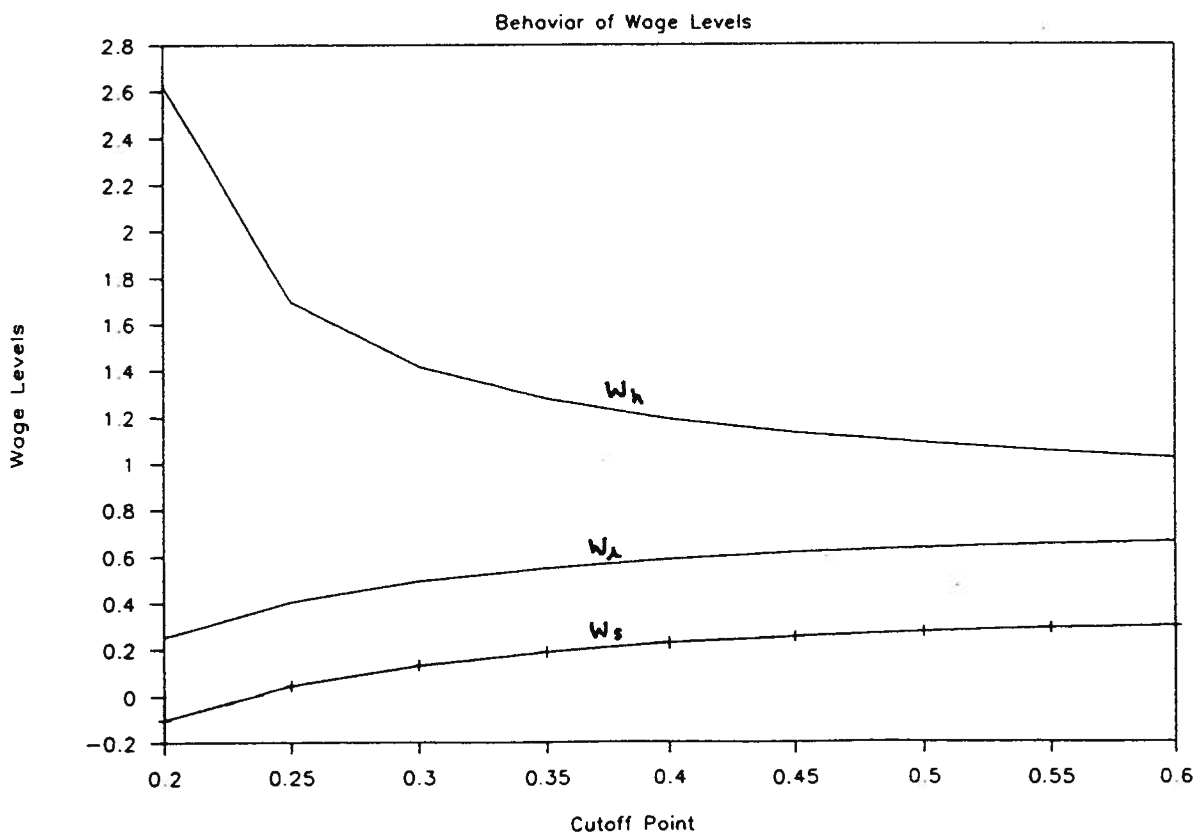
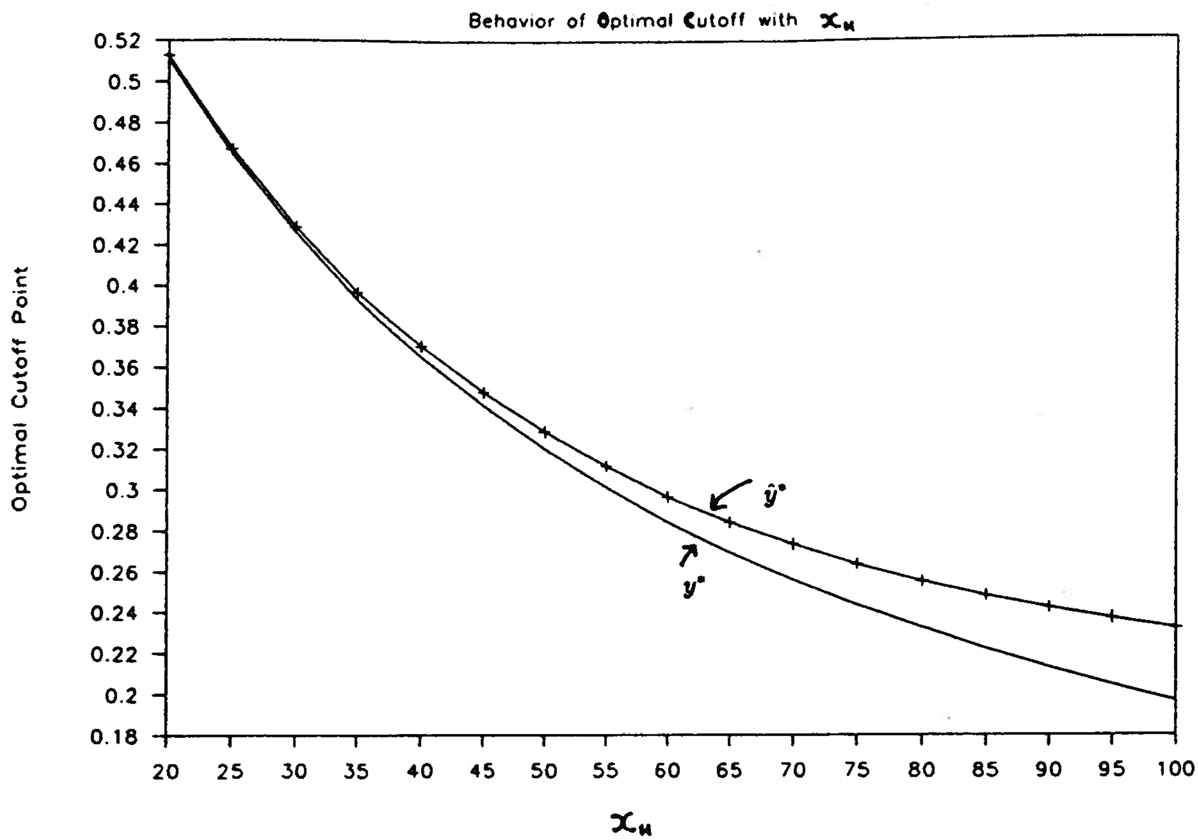


Figure 3.6


Behavior of Optimal Cutoff with x_H
Optimal Cutoff Point
0.52
0.5
0.48
0.46
0.44
0.42
0.4
0.38
0.36
0.34
0.32
0.3
0.28
0.26
0.24
0.22
0.2
0.18
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
$\hat{y}^*$
y^*
x_H

No Second-Stage Moral Hazard Case

Figure 3.7

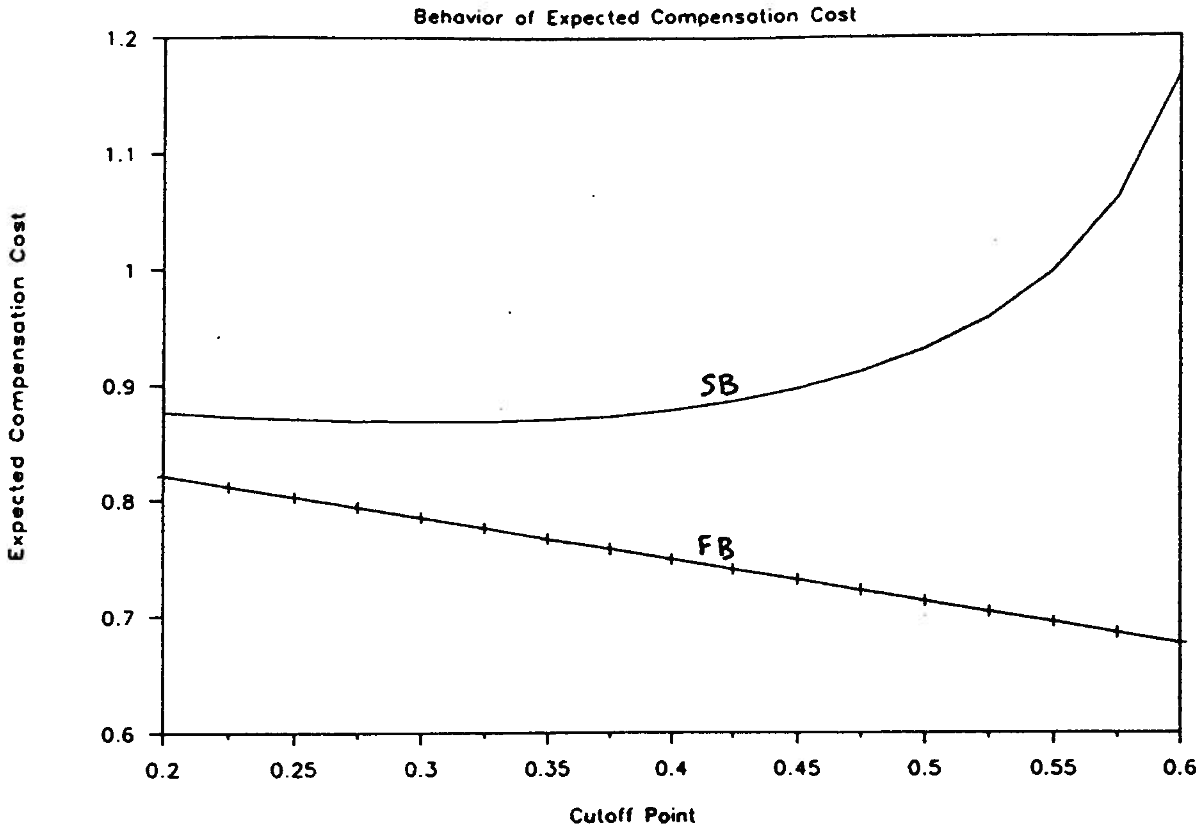


Figure 3.8

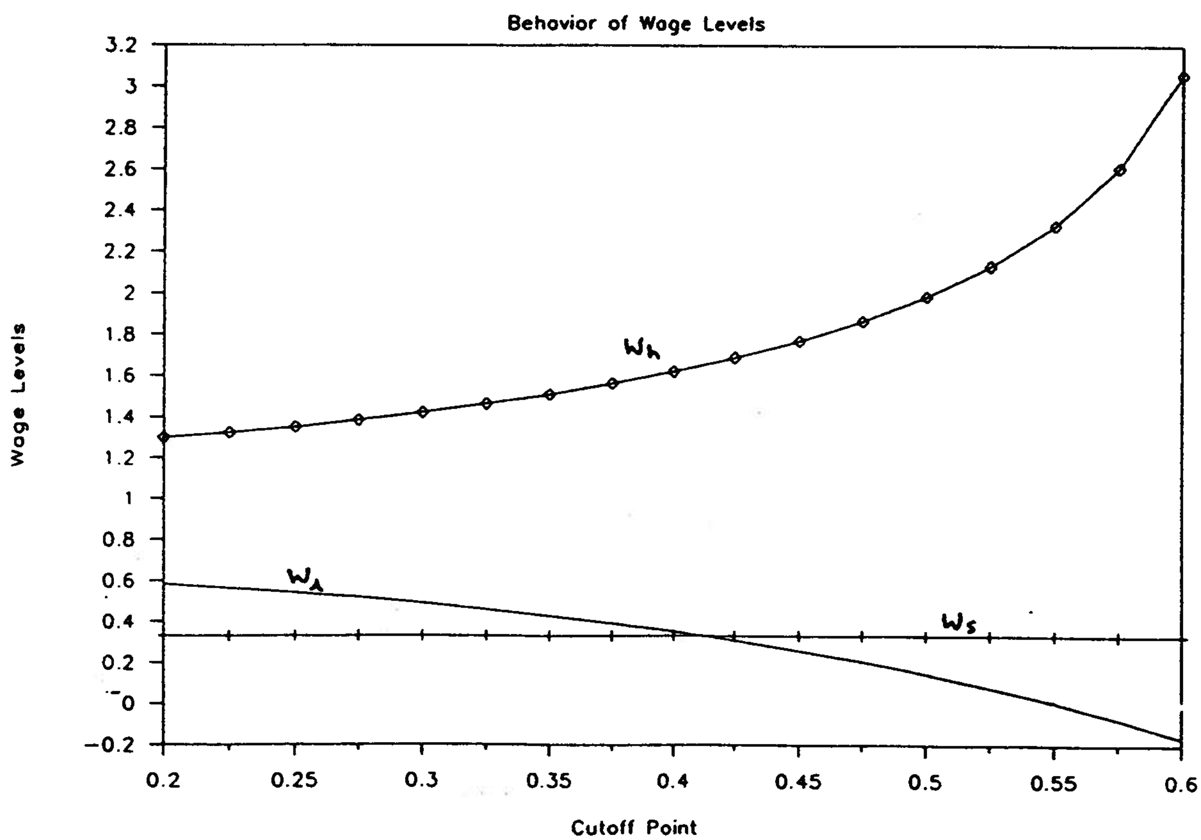
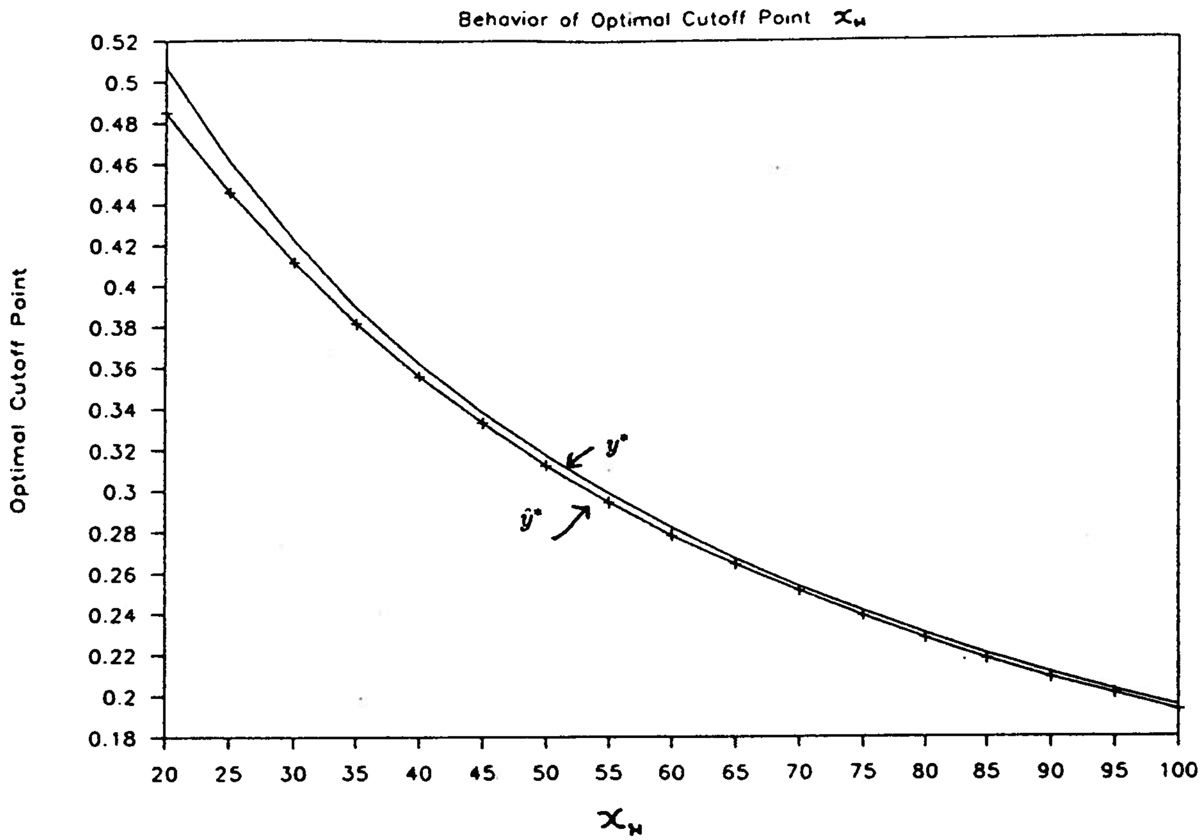


Figure 3.9


Behavior of Optimal Cutoff Point x_H
Optimal Cutoff Point
0.52
0.5
0.48
0.46
0.44
0.42
0.4
0.38
0.36
0.34
0.32
0.3
0.28
0.26
0.24
0.22
0.2
0.18
20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
y^*
$\hat{y}^*$
x_H

Appendix 3A

(I) No First-Stage Moral Hazard

(1) Proof of Lemma 3.1:

From Proposition 2 of Grossman & Hart (1983), we know that the participation constraint is binding. It is obvious that at least one of the second and third constraints of problem [P3.1] is binding.

- Second constraint is binding: The proof is by contradiction. Suppose not. Then $\mu_1 = 0$ and

$$\begin{aligned} h'(u_d) &= \lambda_1 + \frac{\eta_1}{p_B}, \\ h'(u_h) &= \lambda_1 v(t_h) - \frac{\eta_1 \hat{y} v(t_h)}{p_H}, \\ h'(u_l) &= \lambda_1 v(t_h) - \frac{\eta_1 (1-\hat{y}) v(t_h)}{p_L}. \end{aligned}$$

  If $\eta_1 > 0$, the agent prefers to abandon the project always.

  If $\eta_1 = 0$, a fixed wage contract results and the agent has no incentive to choose the high effort.

  If $\eta_1 < 0$, the second constraint is violated.

  Therefore, at the optimal solution, the second constraint is binding and $\mu_1 > 0$.

- Third constraint is binding and $\eta_1 < 0$: The proof is by contradiction. Suppose that $\eta_1 \geq 0$. Then

$$\begin{aligned} h'(u_d) &> \lambda_1, \\ h'(u_h) &\leq \lambda_1 v(t_h), \\ h'(u_l) &< \lambda_1 v(t_h). \end{aligned}$$

  The agent will strictly prefer to abandon the project always. Therefore, at the optimal solution, $\eta_1 < 0$.

(2) Derivation of $\hat{u}_d$, $\hat{u}_h$ and $\hat{u}_l$

Since the second constraint is binding (Lemma 3.1), $\hat{u}_d = \hat{u}_l$. Using the agent's first-order condition on

$\hat{y}$, $\hat{u}_d = \hat{u}_l = \frac{\hat{y}v(t_h)\hat{u}_h}{\hat{y}v(t_h)-v(t_h)+1}$. The agent's expected utility if he takes t_h is as follows:

$$\begin{aligned} EH &= \hat{y}\hat{u}_d + \frac{1}{2}(1-\hat{y}^2)\hat{u}_h v(t_h) + \frac{1}{2}(1-\hat{y})^2\hat{u}_l v(t_h) \\ &= \frac{\hat{u}_h v(t_h)}{\hat{y}v(t_h)-v(t_h)+1}[\hat{y}v(t_h) - \frac{1}{2}[v(t_h)-1](1+\hat{y}^2)]. \end{aligned}$$

Since $EH = K$,

$$\begin{aligned} \hat{u}_h &= \frac{K[\hat{y}v(t_h)-v(t_h)+1]}{v(t_h)[\hat{y}v(t_h)-\frac{1}{2}[v(t_h)-1](1+\hat{y}^2)]} \\ &= \frac{2K[\hat{y}v(t_h)-v(t_h)+1]}{v(t_h)[2\hat{y}v(t_h)-[v(t_h)-1](1+\hat{y}^2)]}. \\ \frac{d\hat{u}_h}{d\hat{y}} &= \frac{2K(v(t_h)-1)[v(t_h)(1+\hat{y}^2)-2\hat{y}(v(t_h)-1)]}{v(t_h)[2\hat{y}v(t_h)-[v(t_h)-1](1+\hat{y}^2)]^2} \\ &< 0. \end{aligned}$$

By substitution, we obtain the expressions for $\hat{u}_d$ and $\hat{u}_l$ for a given $\hat{y}$.

$$\begin{aligned} \hat{u}_d = \hat{u}_l &= \frac{2K\hat{y}}{[2\hat{y}v(t_h)-[v(t_h)-1](1+\hat{y}^2)]}. \\ \frac{d\hat{u}_l}{d\hat{y}} &= \frac{-2K(v(t_h)-1)(1-\hat{y}^2)}{[2\hat{y}v(t_h)-[v(t_h)-1](1+\hat{y}^2)]^2} \\ &> 0. \end{aligned}$$

(3) To show that $\hat{u}_d < K$:

Let $\hat{u}_d = \phi K$, where $\phi = \frac{2\hat{y}}{[2\hat{y}v(t_h)-(v(t_h)-1)(1+\hat{y}^2)]}$. We prove that $\phi > 1$. Suppose not. Then

$$\begin{aligned} 2\hat{y}v(t_h) - (v(t_h)-1)(1+\hat{y}^2) &\geq 2\hat{y} \\ (v(t_h)-1)[1+\hat{y}^2-2\hat{y}] &\leq 0 \\ (v(t_h)-1)[1-\hat{y}]^2 &\leq 0, \end{aligned}$$

which cannot hold. Therefore, $\phi > 1$ and $\hat{u}_d < K$.

(4) **Proof of Lemma 3.2**

Since $\hat{u}_d = \hat{u}_l$ (Lemma 3.1), the third constraint of [P3.1] can be expressed as follows:

$$\hat{y}v(t_h)u_h = u_d[\hat{y}v(t_h) - v(t_h) + 1].$$

For the constraint to hold, $\hat{y} > (1 - \frac{1}{v(t_h)})$.

(5) **Proof of Proposition 3.1:**

At the optimal cutoff point (see equation (3.4)),

$$(B - h(u_d) + w_k) - \hat{y}^*(x_H - h(u_h)) - (1 - \hat{y}^*)(x_L - h(u_l)) - \eta_1[u_h v(t_h) - u_l v(t_h)] = 0$$

For purpose of this proof, we redefine some variables:

$$\begin{aligned} U(w_h)v(t_h) &= \bar{u}_h \\ \Rightarrow w_h &= h(\bar{u}_h) + t_h. \\ U(w_l)v(t_h) &= \bar{u}_l \\ \Rightarrow w_l &= h(\bar{u}_l) + t_h. \end{aligned}$$

Then the agent's first-order condition on $\hat{y}$ is:

$$u_d = \hat{y}\bar{u}_h + (1 - \hat{y})\bar{u}_l.$$

Rewriting the principal's first-order condition on $\hat{y}$, we have:

$$(B - h(u_d) + w_k) - \hat{y}^*(x_H - h(\bar{u}_h) - t_h) - (1 - \hat{y}^*)(x_L - h(\bar{u}_l) - t_h) - \eta_1[\bar{u}_h - \bar{u}_l] = 0.$$

It can be rewritten as follows:

$$B + w_k + t_h - \hat{y}^* x_H - (1 - \hat{y}^*)x_L = h(u_d) - \hat{y}^* h(\bar{u}_h) - (1 - \hat{y}^*)h(\bar{u}_l) + \eta_1[\bar{u}_h - \bar{u}_l].$$

Since $u_d = \hat{y}\bar{u}_h + (1-\hat{y})\bar{u}_l$, and the agent is risk averse, therefore, $h(u_d) < \hat{y}^* h(\bar{u}_h) + (1-\hat{y}^*)h(\bar{u}_l)$. Lemma 3.1 states that $\eta_1 < 0$ and for $\bar{u}_h > \bar{u}_l$, this implies that

$$B + w_k + t_h - \hat{y}^* x_H - (1-\hat{y}^*)x_L < 0. \tag{3.8}$$

Recall that the principal's first-order condition with respect to the first-best cutoff point y^* is given by (equation (3.2)):

$$B + w_k + t_h - y^* x_H - (1-y^*)x_L = 0.$$

From section 3.3.1, the principal's objective function under first-best is strictly concave and is given by the following expression:

$$EP = \hat{y}[B + w_k + t_h] + \frac{1}{2}(1-\hat{y})x_H + \frac{1}{2}(1-\hat{y})^2 x_L - \bar{w} - t_h. \tag{3.9}$$

The derivative of EP with respect to $\hat{y}$ is given by:

$$EP' = B + w_k + t_h - \hat{y}x_H - (1-\hat{y})x_L. \tag{3.10}$$

We note that $EP' = 0$ at y^* and $EP' < 0$ at $\hat{y}^*$. This implies that $\hat{y}^* > y^*$.

(6) **Slope of Expected Compensation Cost**

In the first-best case, the expected compensation cost is:

$$\begin{aligned} EC(FB) &= \hat{y}(\bar{w} - w_k) + (1-\hat{y})(\bar{w} + t_h). \\ \text{Thus,} \quad EC'(FB) &= \bar{w} - w_k - \bar{w} - t_h \\ &= -(w_k + t_h). \end{aligned}$$

Let $R(\hat{y})$ denote the expected gross return at cutoff $\hat{y}$. Then $R' = B - \hat{y}x_H - (1-\hat{y})x_L$. In the second-best case, at the optimal cutoff $R' = EC'(SB)$. The proof of Proposition 3.1 (see (3.8)) indicates that $R' < -(w_k + t_h)$. This implies that at the second-best optimal cutoff, $EC'(SB) < -(w_k + t_h)$. Thus, $EC'(SB) < EC'(FB)$.

(II) No Second-Stage Moral Hazard

(1) Proof of Lemma 3.3:

- Second constraint of problem [P3.2] is binding: The proof is by contradiction. Suppose not. Then $\mu_2 = 0$ and

$$\begin{aligned} h'(u_d) &= \lambda_2 v(t_h) + \frac{\eta_2}{p_B}, \\ h'(u_h) &= \lambda_2 v(t_h) - \frac{\eta_2 \hat{y}}{p_H}, \\ h'(u_l) &= \lambda_2 v(t_h) - \frac{\eta_2 (1-\hat{y})}{p_L}. \end{aligned}$$

  If $\eta_2 > 0$, the agent will not work and prefers to abandon the project always.

  If $\eta_2 = 0$, a fixed wage contract results and the agent has no incentive to choose the high effort.

  If $\eta_2 < 0$, the agent will never abandon the project.

  Therefore, at the optimal solution, the second constraint is binding and $\mu_2 > 0$.

- Fourth constraint is binding and $\eta_2 > 0$: The proof is by contradiction. Suppose that $\eta_2 \leq 0$. Then

$$\begin{aligned} h'(u_d) &\leq v(t_h)[\lambda_2 + \mu_2], \\ h'(u_h) &\geq v(t_h)[\lambda_2 + \mu_2], \\ h'(u_l) &\geq v(t_h)[\lambda_2 + \mu_2]. \end{aligned}$$

  The agent will strictly prefer to continue the project always. Therefore, at the optimal solution, $\eta_2 > 0$.

(2) Derivation of $\hat{u}_d$, $\hat{u}_h$ and $\hat{u}_l$:

From the agent's first-order condition on $\hat{y}$, $\hat{u}_l = \frac{u_d - \hat{y}\hat{u}_h}{(1-\hat{y})}$. The agent's expected utility if he chooses t_h

is as follows:

$$\begin{aligned}
EH_h &= [\hat{y}\hat{u}_d + \frac{1}{2}(1-\hat{y}^2)\hat{u}_h + \frac{1}{2}(1-\hat{y})^2\hat{u}_l]v(t_h) \\
&= v(t_h)\{\hat{u}_d[\hat{y} + \frac{1}{2}(1-\hat{y})] + \hat{u}_h[\frac{1}{2}(1-\hat{y}^2) - \frac{1}{2}\hat{y}(1-\hat{y}))]\} \\
&= \frac{1}{2}v(t_h)[(1+\hat{y})\hat{u}_d + (1-\hat{y})\hat{u}_h].
\end{aligned}$$

Since $EH = \hat{u}_d = K$ (Lemma 3.3),

$$\begin{aligned}
(1-\hat{y})\hat{u}_h &= \frac{2K}{v(t_h)} - (1+\hat{y})K \\
\Rightarrow\ \hat{u}_h &= \frac{K[2 - v(t_h)(1+\hat{y})]}{(1-\hat{y})v(t_h)}. \\
\frac{d\hat{u}_h}{d\hat{y}} &= \frac{-2K[v(t_h)-1]}{(1-\hat{y})^2 v(t_h)} \\
&> 0. \\
\frac{d^2\hat{u}_h}{d\hat{y}^2} &> 0.
\end{aligned}$$

By substitution, we obtain the expression for $\hat{u}_l$ for a given $\hat{y}$.

$$\begin{aligned}
\hat{u}_l &= \frac{K}{(1-\hat{y})} - \frac{\hat{y}}{1-\hat{y})}\hat{u}_h \\
&= \frac{K[v(t_h)(1+\hat{y}^2) - 2\hat{y}]}{(1-\hat{y})^2 v(t_h)}. \\
\frac{d\hat{u}_l}{d\hat{y}} &= \frac{2K[v(t_h)-1](1+\hat{y})}{(1-\hat{y})^3 v(t_h)} \\
&< 0. \\
\frac{d^2\hat{u}_l}{d\hat{y}^2} &< 0.
\end{aligned}$$

(3) **Proof of Lemma 3.4:**

$$\hat{u}_h = \frac{K[2 - v(t_h)(1+\hat{y})]}{(1-\hat{y})v(t_h)}.$$

Since $\hat{u}_h$ is negative, thus

$$\begin{aligned} v(t_h)(1+\hat{y}) &< 2 \\ \Rightarrow \hat{y} &< \frac{2-v(t_h)}{v(t_h)}. \end{aligned}$$

For $\hat{y}$ to exist, $v(t_h) < 2$.

(4) **Proof of Proposition 3.2:**

The expected compensation cost at $\hat{y}$ is:

$$EC(\hat{y}) = \hat{y}[h(u_d) - w_k] + \frac{1}{2}(1-\hat{y}^2)h(u_h) + \frac{1}{2}(1-\hat{y})^2 h(u_l).$$

Totally differentiating $EC(\hat{y})$ with respect to $\hat{y}$, we obtain the following:

$$\begin{aligned} EC' &= h(u_d) - \hat{y}h(u_h) - (1-\hat{y})h(u_l) - w_k \\ &+ \hat{y}h'(u_d) + \frac{1}{2}(1-\hat{y}^2)h'(u_h) + \frac{1}{2}(1-\hat{y})^2 h'(u_l). \end{aligned}$$

At the optimal cutoff, $R' = EC'$. Thus,

$$\begin{aligned} B + w_k - \hat{y}^* x_H - (1-\hat{y}^*)x_L &= h(u_d) - \hat{y}^* h(u_h) - (1-\hat{y}^*)h(u_l) \\ &+ \hat{y}^* h'(u_d) + \frac{1}{2}(1-\hat{y}^{*2})h'(u_h) + \frac{1}{2}(1-\hat{y}^*)^2 h'(u_l). \end{aligned}$$

The optimal compensation contract is (equation (3.6)):

$$\begin{aligned} \hat{u}_d &= K, \\ \hat{u}_h &= \frac{K[2-v(t_h)(1+\hat{y})]}{(1-\hat{y})v(t_h)}, \\ \text{and} \quad \hat{u}_l &= \frac{K[v(t_h)(1+\hat{y}^2) - 2\hat{y}]}{(1-\hat{y})^2 v(t_h)}. \end{aligned}$$

Also, $h(u_p) = -\frac{1}{r}\ln(-u_p)$. Through substituition, differentiation and rearrangement of the terms, we obtain the following:

$$h(u_d) - \hat{y}^* h(u_h) - (1-\hat{y}^*)h(u_l) + \frac{1}{2}(1-\hat{y}^{*2})h'(u_h) + \frac{1}{2}(1-\hat{y}^*)^2 h'(u_l)$$

$$\begin{aligned}
&= \frac{1}{r} \;\{\frac{2(v(t_h)-1)^2(1+\hat{y}^*)}{[v(t_h)(1+\hat{y}^{*2})-2\hat{y}^*][2-v(t_h)(1+\hat{y}^*)]} - \ln\frac{(1-\hat{y}^*)^2 v(t_h)}{v(t_h)(1+\hat{y}^{*2})-2\hat{y}^*} \\
&- \;\hat{y}^* \ln\frac{v(t_h)(1+\hat{y}^{*2})-2\hat{y}^*}{[2-v(t_h)(1+\hat{y}^*)](1-\hat{y}^*)}\}. \qquad (3.11)
\end{aligned}$$

Let

$$\begin{aligned}
P &= \frac{2(v(t_h)-1)^2(1+\hat{y})}{[v(t_h)(1+\hat{y}^2)-2\hat{y}][2-v(t_h)(1+\hat{y})]} - \ln\frac{(1-\hat{y})^2 v(t_h)}{v(t_h)(1+\hat{y}^2)-2\hat{y}} \\
&\quad -\hat{y}\ln\frac{v(t_h)(1+\hat{y}^2)-2\hat{y}}{[2-v(t_h)(1+\hat{y})](1-\hat{y})}. \qquad (3.12)
\end{aligned}$$

Using the following steps, we prove that for $1 < v(t_h) < 2$ and $0 < \hat{y} < \frac{2-v(t_h)}{v(t_h)}$, $P > 0$.

1. When $v(t_h) = 1$,

$$\begin{aligned}
P &= 0 - [\ln\frac{(1-\hat{y})^2}{(1-\hat{y})^2} + \hat{y}\ln\frac{(1-\hat{y})^2}{(1-\hat{y})^2}] \\
&= 0.
\end{aligned}$$

2. For each feasible cutoff $\hat{y}$, let P_v denote the derivatives of P with respect to $v(t_h)$.

$$\begin{aligned}
P_v &= \frac{4(v(t_h)-1)[v(t_h)\hat{y}^3(v(t_h)-2)+\hat{y}^2(2v(2t_h)-3v(t_h)+4)+v(t_h)\hat{y}(v(t_h)-4)+v(t_h)]}{v(t_h)[v(t_h)\hat{y}^2-2\hat{y}+v(t_h)]^2[v(t_h)\hat{y}+v(t_h)-2]^2} \\
&= M * [v(t_h)\hat{y}^3(v(t_h)-2)+\hat{y}^2(2v(2t_h)-3v(t_h)+4)+v(t_h)\hat{y}(v(t_h)-4)+v(t_h)],
\end{aligned}$$

where $M = \frac{4(v(t_h)-1)}{v(t_h)[v(t_h)\hat{y}^2-2\hat{y}+v(t_h)]^2[v(t_h)\hat{y}+v(t_h)-2]^2}$ which is strictly greater than zero.

3. Let

$$\begin{aligned}
Q &= v(t_h)\hat{y}^3(v(t_h)-2)+\hat{y}^2(2v(2t_h)-3v(t_h)+4)+v(t_h)\hat{y}(v(t_h)-4)+v(t_h) \\
&= v(2t_h)\hat{y}[\hat{y}^2+2\hat{y}+1] - v(t_h)[2\hat{y}^3+3\hat{y}^2+4\hat{y}-1]+4\hat{y}^2.
\end{aligned}$$

Q is convex in $v(t_h)$ and reaches a minimum at a value of $v(t_h) < 1$. This implies that for $v(t_h) \geq 1$, Q is monotonically increasing.

4. When $v(t_h) = 1$, $Q = (1-\hat{y})^3 > 0$. This implies that for $v(t_h) \geq 1$, $Q > 0$.

5. Recall that $P_v = M * Q$. We conclude that for $v(t_h) \geq 1$, $P_v > 0$.

6. If P is monotonically increasing for $v(t_h) \geq 1$ and $P = 0$ for $v(t_h) = 1$, then $P > 0$ for $v(t_h) > 1$.

Therefore, at the optimal cutoff $\hat{y}^*$,

$$h(u_d) - \hat{y}^* h(u_h) - (1-\hat{y}^*)h(u_l) + \frac{1}{2}(1-\hat{y}^{*2})h'(u_h) + \frac{1}{2}(1-\hat{y}^*)^2 h'(u_l) > 0.$$

This implies that

$$B + w_k - \hat{y}^* x_H - (1-\hat{y}^*)x_L > 0.$$

Recall from equation (3.5) that at the first-best cutoff,

$$B + w_k - y^* x_H - (1-y^*)x_L = 0.$$

The principal's objective function under first-best is given by the following expression:

$$EP = \hat{y}[B + w_k] + \frac{1}{2}(1-\hat{y})x_H + \frac{1}{2}(1-\hat{y})^2 x_L - \bar{w} - t_h. \tag{3.13}$$

The derivative of EP with respect to $\hat{y}$ is given by:

$$EP' = B + w_k - \hat{y}x_H - (1-\hat{y})x_L. \tag{3.14}$$

The second-order condition on $\hat{y}$ is negative, thus EP is strictly concave in $\hat{y}$. We note that $EP' = 0$ at y^* and $EP' > 0$ at $\hat{y}^*$. This implies that $\hat{y}^* < y^*$.

(5) Slope of Expected Compensation Cost

In the first-best case, the expected compensation cost is:

$$\begin{aligned} EC(FB) &= \hat{y}(\bar{w} + t_h - w_k) + (1-\hat{y})(\bar{w} + t_h). \\ \text{Thus,} \quad EC'(FB) &= \bar{w} + t_h - w_k - \bar{w} - t_h \\ &= -w_k. \end{aligned}$$

In the second-best case, as the proof for Proposition 3.2 indicates, at the optimal cutoff point,

$$EC'(SB) > -w_k.$$

Therefore, $EC'(SB) > EC'(FB)$.

Chapter 4

Analysis of the Multi-Stage Setting

We analyze the general model where there are two stages of moral hazard. In section 4.1, we introduce a benchmark case in which information is publicly observable but not contractible. In section 4.2, both the first-best and the second-best cases for the general model are analyzed. We examine if there is any value to communication of the specific value of the information in section 4.3. We provide a numerical example in Section 4.4. All proofs are provided in the appendix.

4.1 Benchmark Case

4.1.1 The Model

Consider as a benchmark case, the cutoff that would be employed if the agent's information is publicly observable but not contractible. Thus, the principal makes the abandon/continue decision based on y, but y cannot be used as an argument in the agent's compensation. We assume that if the agent takes t_{1l}, the information signal $y = 0$ is observed, while if the agent takes t_{1h}, the information signal $y \in (0, 1)$ is observed. Since y is not contractible, the agent will not be penalized if $y = 0$ is observed and the principal abandons the project.

4.1.2 Analysis of the Problem

The Contract Choice Problem

For a given cutoff $\hat{y}$, the principal selects the compensation contract to solve the following:

$$
\begin{array}{lll}
[P4.1] & \min_{\{u_d,u_h,u_l\}} & p_{Bh}(h(u_d)-w_k)+p_{Hh}h(u_h)+p_{Lh}h(u_l) \\
& \text{s.t.} & p_{Bh}u_d v(t_h)+p_{Hh}u_h v(2t_h)+p_{Lh}u_l v(2t_h) \geq K, \\
& & p_{Bh}u_d v(t_h)+p_{Hh}u_h v(2t_h)+p_{Lh}u_l v(2t_h) \geq u_d, \\
& \text{and} & \hat{y}u_h v(t_h)+(1-\hat{y})u_l v(t_h) \geq u_l,
\end{array}
$$

where we recall from Chapter 3, equation (3.1) that $p_{Bh}=\hat{y}$, $p_{Hh}=\frac{1}{2}(1-\hat{y}^2)$, and $p_{Lh}=\frac{1}{2}(1-\hat{y})^2$.

The first constraint is the participation constraint and ensures that the agent's expected utility from joining the firm is at least as high as his reservation utility level. The second constraint is the incentive constraint; it requires that the agent's expected utility from working hard in both periods weakly exceeds the utility level he could obtain by taking t_{1l}. Recall that if the agent takes t_{1l}, information signal $y=0$ is observed and the principal abandons the project. The third constraint ensures that if the project is continued, the agent will weakly prefer to take t_{2h} than t_{2l}.

For a given $\hat{y}$, there are three constraints and three unknowns. All three constraints are binding, thus for a given $\hat{y}$, the optimal contract $(\tilde{u}_d,\tilde{u}_h,\tilde{u}_l)$ is uniquely defined as follows:

$$
\begin{aligned}
\tilde{u}_d &= K, \\
\tilde{u}_h &= \frac{2K[1-(1-\hat{y})v(t_h)][1-\hat{y}v(t_h)]}{v(2t_h)(1-\hat{y})[1+\hat{y}-v(t_h)(1-\hat{y})]}, \\
\text{and } \tilde{u}_l &= \frac{2K\hat{y}[1-\hat{y}v(t_h)]}{v(t_h)(1-\hat{y})[1+\hat{y}-v(t_h)(1-\hat{y})]}.
\end{aligned} \tag{4.1}
$$

As $\hat{y}$ increases, $\tilde{u}_l$ increases and the spread between $\tilde{u}_h$ and $\tilde{u}_l$ decreases.[1] Intuitively, the abandon/continue decision made by the principal will determine if the agent will face a riskless or risky compensation package. If the project is abandoned, the agent obtains a riskless payment, $\tilde{u}_d$. If the project is continued, the agent receives a risky compensation package. If the probability that the agent receives a risky pay package is low, the agent will require a lower risk premium. Thus, if the principal chooses a high cutoff point, then the probability that the agent receives a risky pay package is low, and the agent will require a lower risk premium as incentive to choose $\{t_{1h}, t_{2h}|\text{if continue}\}$. Thus, as $\hat{y}$ increases, the spread between $\tilde{u}_h$ and $\tilde{u}_l$ decreases. Also, the probability that the agent must be compensated for his second-stage effort decreases. Therefore, the expected compensation cost is monotonically decreasing in $\hat{y}$.

Lemma 4.1 gives the upper and lower bounds of $\hat{y}$ for a feasible solution to exist. From the two single-stage moral hazard problems in chapter 3, we know that the second-stage moral hazard problem results in the lower bound (Lemma 3.2) while the first-stage moral hazard problem determines the upper bound (Lemma 3.4). When $\hat{y}$ is too low, motivating the agent to work hard given poor information (second-stage moral hazard problem) is very costly. On the other hand, when $\hat{y}$ is too high, motivating the agent to work hard when the chances of abandonment is very high (first-stage moral hazard problem) is too excessive.

Lemma 4.1: Necessary conditions for a solution to exist are:

1. $(1 - \frac{1}{v(t_h)}) < \hat{y} < \frac{1}{v(t_h)}$, and

2. $v(t_h) < 2$.

If $\hat{y} \geq \frac{1}{v(t_h)}$, the second constraint cannot be satisfied, while if $\hat{y} \leq (1 - \frac{1}{v(t_h)})$, the third constraint

[1] See appendix 4A for details.

cannot be satisfied. Since $\frac{1}{v(t_h)} > (1 - \frac{1}{v(t_h)})$ for $\hat{y}$ to exist, $v(t_h) < 2$.

The Cutoff Point Selection Problem

Using the solutions from the Contract Choice Problem, the principal chooses the cutoff point that maximizes

$$\hat{p}_{Bh}[B - \tilde{w}_d(\hat{y}) + w_k] + \hat{p}_{Hh}[x_H - \tilde{w}_h(\hat{y})] + \hat{p}_{Lh}[x_L - \tilde{w}_l(\hat{y})].$$

The relative levels of B, x_H and x_L determine what cutoff point the principal seeks to implement. If x_H is relatively high, the principal prefers a lower cutoff point, while a relatively low x_H implies that the agent prefers a higher cutoff point. Let $\tilde{y}^*$ denote the optimal cutoff in the benchmark case.

4.2 The General Model

We now analyze the general model in which information is privately observed by the agent. Here, the compensation contract seeks to motivate the agent to do two things:

1. Choose t_{1h}, and t_{2h} if the project is to be continued.

2. After choosing t_{1h} and observing the information signal y privately, make the abandon/continue decision which is in the principal's best interest.

In the benchmark case, the compensation contract only needs to motivate the agent to do the former.

4.2.1 First-best Solution

First, we consider a setting where the agent's effort is observable, and thus there is no incentive problem. The principal offers a compensation package to the agent such that the agent is indifferent between continuing or abandoning the project. We assume that when the agent is indifferent, he decides in the principal's best interest. The agent receives severance pay w_s if the project is abandoned, and a total pay package of w_f if the project is continued. Let $u_d = U(w_s + w_k)$ and $u_f = U(w_f)$. Thus, the agent's utility if he abandons the project is given by $u_d v(t_h)$, while his utility if he continues the project is $u_f v(2t_h)$. The agent is indifferent between continuing or abandoning the project if $u_d v(t_h) = u_f v(2t_h)$. The first-best cutoff point is denoted y^*. The participation constraint requires

$$u_d v(t_h)\hat{y} + u_f v(2t_h)(1-\hat{y}) = U(\bar{w}).$$

Consequently, the first-best compensation package is:

$$\begin{aligned} w_s &= \bar{w} - w_k + t_h, \\ w_f &= \bar{w} + 2t_h. \end{aligned}$$

The principal's expected payoff, for a given cutoff $\hat{y}$, is

$$\hat{y}(B - w_s) + \frac{1}{2}(1-\hat{y}^2)x_H + \frac{1}{2}(1-\hat{y})^2 x_L - (1-\hat{y})w_f.$$

Substituting for w_s and w_f, the first-order condition with respect to $\hat{y}$ is:

$$\begin{aligned} B - \bar{w} + w_k - t_h - \hat{y}x_H - (1-\hat{y})x_L + \bar{w} + 2t_h &= 0 \\ \Rightarrow \quad B + w_k + t_h - \hat{y}x_H - (1-\hat{y})x_L &= 0. \end{aligned} \tag{4.2}$$

Note that the second-order condition on $\hat{y}$ is $-(x_H - x_L)$, which is negative. This implies that the principal's objective function is strictly concave, thus the first-order condition is necessary and sufficient to obtain the optimal cutoff. The first-best cutoff is

$$y^* = \begin{cases} 0 & \text{if } B + w_k + t_h \leq x_L \\ \frac{B+w_k+t_h-x_L}{x_H-x_L} & \text{if } x_L < B + w_k + t_h < x_H \\ 1 & \text{if } B + w_k + t_h \geq x_H. \end{cases}$$

The first-best cutoff for the general model is identical to that for problem [P3.1], the no-first-stage moral hazard case, and is higher than that for problem [P3.2], the no-second-stage moral hazard problem.

We note that past effort does not determine the cutoff level. Only future effort is relevant. To provide an intuitive explanation for y^*, we rewrite the first-order condition evaluated at y^* in the following manner:

$$B - [\bar{w} - w_k + t_h] = y^* x_H + (1 - y^*) x_L - [\bar{w} + 2t_h].$$

The left-hand side of the equality is the return to the principal if the project is abandoned, while the right-hand side of the equality gives the expected profitability if the project is continued. Thus, for $y \geq y^*$ $(y < y^*)$, it is optimal to continue with (abandon) the project.[2]

4.2.2 Analysis of the Second-best Problem

We now consider the setting in which the agent's effort is not observable. The principal wants to motivate the agent to take t_{1h} and choose some cutoff point $\hat{y}$. Assume that $\hat{y} \in (0, 1)$. If the agent is offered the contract (u_l, u_h, u_d), and he takes effort $\{t_{1h}, t_{2h} | \text{if continue}\}$, then his expected utility with cutoff $\hat{y}$ is:

$$EH_h = \hat{y} u_d v(t_h) + \frac{1}{2}(1 - \hat{y}^2) u_h v(2t_h) + \frac{1}{2}(1 - \hat{y})^2 u_l v(2t_h).$$

The agent's first-order condition on $\hat{y}$ is:

$$u_d v(t_h) - \hat{y} u_h v(2t_h) - (1 - \hat{y}) u_l v(2t_h) = 0. \tag{4.3}$$

Hence, the agent will choose

$$\hat{y} = \begin{cases} 0 & \text{if } u_d \leq u_l v(t_h) \\ \frac{u_d - u_l v(t_h)}{v(t_h)[u_h - u_l]} & \text{if } u_l v(t_h) < u_d < u_h v(t_h) \\ 1 & \text{if } u_d \geq u_h v(t_h). \end{cases}$$

We note that for interior solutions, the agent's second-order condition on $\hat{y}$ is also satisfied.

The agent's first-order condition with respect to $\hat{y}$ implies the following:

[2] We are unable to establish formally the relationship between the benchmark cutoff $\bar{y}^*$ and the first-best cutoff y^*. The example in section 4.4 indicates the following relations:

- When x_H is relatively high, and y^* is low, then $\bar{y}^* > y^*$.
- When x_H is relatively low, and y^* is high, then $\bar{y}^* < y^*$.

- For $y < \hat{y}$, the agent is better off abandoning the project than continuing it at effort level t_{2h}.
- For $y \geq \hat{y}$, the agent is better off continuing the project at effort level t_{2h} than to abandon it.

However, the principal must ensure that for $y < \hat{y}$, the agent will prefer to abandon the project rather than continue it with effort level t_{2l}. Also, for $y \geq \hat{y}$, the principal must ensure that the agent prefers to continue the project with effort level t_{2h} than with effort level t_{2l}. This is achieved by imposing the following two constraints:

$$\begin{aligned} u_d &\geq u_l, \\ \hat{y}u_h v(t_h) + (1-\hat{y})u_l v(t_h) &\geq u_l. \end{aligned}$$

Since the agent's first-order condition on $\hat{y}$ implies that $u_d = \hat{y}u_h v(t_h) + (1-\hat{y})u_l v(t_h)$, one of these two constraints is redundant. For subsequent analysis, we use the constraint $u_d \geq u_l$. This constraint also implies that if the agent takes t_{1l}, he is better off abandoning the project at $i = 1$ than to continue with it.

The Contract Choice Problem

For a given cutoff $\hat{y}$, the principal selects a compensation contract which solves the following:

$$\begin{aligned} [P4.2] \qquad & \min_{\{u_d,u_h,u_l\}} \quad p_{Bh}(h(u_d) - w_k) + p_{Hh}h(u_h) + p_{Lh}h(u_l) \\ & \text{s.t.} \quad p_{Bh}u_d v(t_h) + p_{Hh}u_h v(2t_h) + p_{Lh}u_l v(2t_h) \geq K, \\ & \qquad p_{Bh}u_d v(t_h) + p_{Hh}u_h v(2t_h) + p_{Lh}u_l v(2t_h) \geq u_d, \\ & \qquad u_d \geq u_l, \\ & \text{and} \quad u_d - \hat{y}u_h v(t_h) - (1-\hat{y})u_l v(t_h) = 0. \end{aligned}$$

The first constraint is the participation constraint and ensures that the agent's expected utility from joining the firm is at least as high as his reservation utility level. The second constraint is the incentive constraint and requires that the agent's expected utility from working hard in both stages, and choosing $\hat{y}$ as the cutoff point, weakly exceeds the utility level he could obtain by taking the lower effort level and abandoning the project after the first stage. The third constraint ensures that the agent will weakly prefer to abandon the project rather than continue it with effort level t_{2l} if he observes that $y < \hat{y}$. The last constraint is the agent's first-order condition on $\hat{y}$, and it requires that for each value of the information signal y, the specified abandonment/continuation decision is optimal for the agent.

Let λ, μ_1, μ_2 and η be the Lagrange multipliers of the four constraints. The principal's Lagrangian formulation is as follows:

$$\begin{aligned} L &= -p_{Bh}(h(u_d) - w_k) - p_{Hh}h(u_h) - p_{Lh}h(u_l) \\ &+ \lambda[p_{Bh}u_d v(t_h) + p_{Hh}u_h v(2t_h) + p_{Lh}u_l v(2t_h) - K] \\ &+ \mu_1[u_d\{p_{Bh}v(t_h) - 1\} + u_h p_{Hh}v(2t_h) + u_l p_{Lh}v(2t_h)] \\ &+ \mu_2[u_d - u_l] \\ &+ \eta[u_d - \hat{y}u_h v(t_h) - (1-\hat{y})u_l v(t_h)]. \end{aligned}$$

Using first-order conditions, we obtain the following characterizations of the optimal compensation package, denoted $(\hat{u}_d, \hat{u}_h, \hat{u}_l)$,

$$\begin{aligned} h'(\hat{u}_d) &= v(t_h)(\lambda+\mu_1) - \frac{\mu_1}{\hat{p}_{Bh}} + \frac{\mu_2}{\hat{p}_{Bh}} + \frac{\eta}{\hat{p}_{Bh}}. \\ h'(\hat{u}_h) &= v(2t_h)(\lambda+\mu_1) - \frac{\eta\hat{y}v(t_h)}{\hat{p}_{Hh}}. \\ h'(\hat{u}_l) &= v(2t_h)(\lambda+\mu_1) - \frac{\mu_2}{\hat{p}_{Lh}} - \frac{\eta(1-\hat{y})v(t_h)}{\hat{p}_{Lh}}. \end{aligned}$$

Lemma 4.2 establishes the signs of the Lagrange multipliers for the two incentive constraints.

<u>Lemma 4.2:</u> At the optimal solution, $\mu_1 \geq 0$ and $\mu_2 \geq 0$ with at least one of μ_1 and μ_2 strictly greater than zero.

The sign of η, the Lagrange multiplier on the first-order condition on the agent's choice of cutoff is determined later, and as we shall see, it depends on the sign of μ_1 and μ_2. We note that in the principal's problem, for a given $\hat{y}$, there are four constraints and three unknowns. At the optimal solution, one of the four constraints is redundant. The participation constraint is always binding. We consider the following three cases:

- Case A: $\mu_1 > 0$ and $\mu_2 > 0$.
- Case B: $\mu_1 > 0$ and $\mu_2 = 0$.
- Case C: $\mu_1 = 0$ and $\mu_2 > 0$.

Subsequent analysis shows that the value of the optimal cutoff determines which one of the last three constraints is redundant, thus which one of the above three cases applies. Lemma 4.3 gives us the value of $\hat{y}$ that results in case A.

Lemma 4.3: The first three constraints are binding if, and only if, $\hat{y} = \hat{y}_A$, where

$$\hat{y}_A = \frac{v(t_h) + 1 - \sqrt{(2v(t_h)+1)}}{v(t_h)}.$$

Furthermore, in this case, $\hat{u}_d = \hat{u}_d(A)$, $\hat{u}_l = \hat{u}_l(A)$ and $\hat{u}_h = \hat{u}_h(A)$, where

$$\begin{aligned} \hat{u}_d(A) &= \hat{u}_l(A) = K, \\ \text{and} \quad \hat{u}_h(A) &= \frac{K[2 - \sqrt{(2v(t_h)+1)}]}{v(t_h) + 1 - \sqrt{(2v(t_h)+1)}}. \end{aligned} \tag{4.4}$$

The fourth constraint is redundant, since given the above contract, the agent will find it optimal to select $\hat{y}_A$ as the cutoff point.

The cutoff point $\hat{y}_A$ is solely determined by $v(t_h)$. As $v(t_h)$ increases, both $\hat{y}_A$ and $\hat{u}_h(A)$ increase.

In case B, the binding incentive constraint is that the contract must motivate the agent to choose $\{t_{1h}, t_{2h}|\text{if continue}\}$ compared to choosing t_{1l} and always abandoning the project. The level of $\hat{u}_d$ is independent of $\hat{y}$ and is given by $\hat{u}_d = K$. We derive the following expressions for $\hat{u}_h$ and $\hat{u}_l$ for a given $\hat{y}$.

$$\begin{aligned} \hat{u}_h &= \frac{K[2 - v(t_h)(1+\hat{y})]}{(1-\hat{y})v(2t_h)}, \\ \text{and} \quad \hat{u}_l &= \frac{K[v(t_h)(1+\hat{y}^2) - 2\hat{y}]}{(1-\hat{y})^2 v(2t_h)}. \end{aligned} \tag{4.5}$$

We observe that the optimal expressions for u_d, u_l and u_h are similar to those for problem [P3.2], the no-second-stage moral hazard case. (See Chapter 3, equation (3.6).) The moral hazard problem in the first stage drives the results of case B.

As $\hat{y}$ increases, $\hat{u}_h$ increases at an increasing rate, while $\hat{u}_l$ decreases at an increasing rate. Thus, the spread $(\hat{u}_h - \hat{u}_l)$ increases at an increasing rate as $\hat{y}$ increases. To motivate the agent to exert effort, the principal needs to impose risk on the agent and then compensate him in the form of a risk premium. The binding incentive constraint is that which motivates him to work hard in both stages, and at that point of effort selection in the first stage, the probability of obtaining a favorable outcome is $\frac{1}{2}(1-\hat{y}^2)$ which decreases as $\hat{y}$ increases. The higher is $\hat{y}$, the lower is the probability that outcome is informative of the agent's effort and a compensation contract with a bigger spread is necessary to motivate effort.

In case C, the binding incentive constraint is that if the project is continued, the contract must motivate the agent to choose t_{2h} versus choosing t_{2l}. For a given $\hat{y}$,

$$\begin{aligned} \hat{u}_d = \hat{u}_l &= \frac{2K\hat{y}}{v(t_h)[2\hat{y}v(t_h) - (v(t_h)-1)(1+\hat{y}^2)]}, \\ \text{and} \quad \hat{u}_h &= \frac{2K[\hat{y}v(t_h) - v(t_h) + 1]}{v(2t_h)[2\hat{y}v(t_h) - (v(t_h)-1)(1+\hat{y}^2)]}. \end{aligned} \tag{4.6}$$

We observe that the optimal expressions for u_d, u_l and u_h are similar to those for problem [P3.1], the no-first-stage moral hazard case. (See Chapter 3, equation (3.3).) The moral hazard problem in the second stage drives the results of case C.

As $\hat{y}$ increases, $\hat{u}_h$ decreases and $\hat{u}_l = \hat{u}_d$ increases, i.e., the spread between $\hat{u}_h$ and $\hat{u}_l$ decreases. In this case, the binding incentive constraint is that which motivates the agent to work hard in the second stage. A high $\hat{y}$ implies that the project is to be continued and effort is required to be exerted only when very good information is received. Thus, the higher is $\hat{y}$, the lesser is the amount of risk that is needed to be imposed on the agent to motivate him to work hard and the smaller is the spread between $\hat{u}_h$ and $\hat{u}_l$.

Proposition 4.1 demonstrates that to motivate a cutoff point other than $\hat{y}_A$, either case B or case C applies.

Proposition 4.1:

1. When the principal wants to motivate a cutoff point $\hat{y} = \hat{y}_A$, case A applies and $\mu_1 > 0$, $\mu_2 > 0$ and $\eta = 0$.

2. When the principal wants to motivate a cutoff point $\hat{y} > \hat{y}_A$, case B applies and $\mu_1 > 0$, $\mu_2 = 0$ and $\eta > 0$.

3. When the principal wants to motivate a cutoff point $\hat{y} < \hat{y}_A$, case C applies and $\mu_1 = 0$, $\mu_2 > 0$ and $\eta < 0$.

The relative levels of x_H, x_L and B determine the cutoff point the principal would want the agent to select. If x_H is relatively high, the principal will prefer a relatively low cutoff point, whereas if x_H is relatively low, the principal will prefer a relatively high cutoff point.

As in the single-stage moral hazard problems, for a feasible solution to exist, there are upper and lower bounds to $\hat{y}$. From chapter 3, Lemma 3.2, the second-stage moral hazard problem results in the lower bound. When $\hat{y}$ is too low, motivating the agent to work hard given poor information (second-stage moral hazard problem) is very costly. From Lemma 3.4, the first-stage moral hazard problem determines the upper bound. When $\hat{y}$ is too high, motivating the agent to work hard when the chances of abandonment are very high (first-stage moral hazard problem) is too expensive. Lemma 4.4 gives the upper and lower bounds of $\hat{y}$ for a feasible solution to exist in this two-stage moral hazard problem.

Lemma 4.4: Necessary conditions for a solution to exist are:[3]

1. $\frac{v(t_h)-1}{v(t_h)} < \hat{y} < \frac{2-v(t_h)}{v(t_h)}$, and

2. $v(t_h) < \frac{3}{2}$.

Outside these bounds, $\hat{u}_h$ becomes infinitely high and no feasible wage contract exists.

Next, we examine the behavior of the required risk premium if the project is continued, as $\hat{y}$ varies. If the project is continued, the principal motivates the agent to choose t_{2h}. Also, the principal seeks to motivate the agent to choose t_{1h} in stage 1, otherwise there is no gain from continuing the project. The principal provides the necessary incentive for the agent to choose high effort by imposing some risk on him, and the agent is then compensated for bearing this risk in the form of a risk premium. In Equation (4.7), $\pi(\hat{y})$ represents the risk premium and the compensation for effort.

$$U[\hat{y}h(u_h) + (1-\hat{y})h(u_l) - \pi(\hat{y})] = [\hat{y}u_h + (1-\hat{y})u_l]v(t_h). \tag{4.7}$$

In the subsequent discussion, for simplicity, we call $\pi(\hat{y})$ the required risk premium if the project is continued. Besides, it is the element of risk premium that is the critical factor in our results. Let $\hat{\pi}(\hat{y})$ ($\bar{\pi}(\hat{y})$) denote the required risk premium in the second-best (benchmark) case if the project is continued.

[3] In appendix 4C, we show that if abandonment of the project is not allowed, the principal will not employ the agent to undertake the project if $v(t_h) \geq \sqrt{2}$.

Lemma 4.5:

1. In the benchmark case, the required risk premium if the project is continued is decreasing in $\hat{y}$, i.e., $\tilde{\pi}'(\hat{y}) < 0$.

2. In the second-best case:

   - If $\hat{y} < \hat{y}_A$, the required risk premium if the project is continued is decreasing in $\hat{y}$, i.e., $\hat{\pi}'(\hat{y}) < 0$.

   - If $\hat{y} > \hat{y}_A$, the required risk premium if the project is continued is increasing in $\hat{y}$, i.e., $\hat{\pi}'(\hat{y}) > 0$. Also, the rate of increase of the required risk premium is increasing in $\hat{y}$.

In the benchmark case, the principal makes the abandon/continue decision. When the project is continued and if the agent is motivated to work hard when he observes $\hat{y}$, then he is motivated to work hard when he observes $y > \hat{y}$. Hence, $\hat{y}$ is a key determinant of the incentive contract and increasing $\hat{y}$ reduces the risk premium imposed on the agent.

In the second-best case, if $\hat{y} < \hat{y}_A$, the severance pay w_s is low and the agent is strictly better off choosing $\{t_{1h}, t_{2h}|\text{if continue}\}$ compared to choosing t_{1l} and abandon always. The binding incentive constraint is that if the project is continued, the contract must motivate the agent to choose t_{2h} versus choosing t_{2l}. At $i = 1$, as $\hat{y}$ increases, the probability of obtaining the high wage payment associated with the good outcome increases, the cost of motivating the agent to work hard decreases, and the required risk premium decreases.

If $\hat{y} > \hat{y}_A$, the binding incentive constraint is that the contract must motivate the agent to choose $\{t_{1h}, t_{2h}|\text{if continue}\}$ compared to choosing t_{1l} and always abandoning the project. At $i = 0$, the probability of obtaining the high wage payment is $\frac{1}{2}(1 - \hat{y}^2)$ and it decreases as $\hat{y}$ increases. The cost of motivating the agent to work hard increases and the required risk premium increases as $\hat{y}$ increases.

The next proposition compares the second-best optimal compensation contract with the benchmark optimal contract, for a given $\hat{y}$.

Proposition 4.2: To implement a given $\hat{y}$, the following relations between the benchmark and the second-best compensation contracts hold.

$$\begin{array}{ccc} \underline{\hat{y} = \hat{y}_A} & \underline{\hat{y} > \hat{y}_A} & \underline{\hat{y} < \hat{y}_A} \\ \\ \hat{u}_d = \tilde{u}_d = K & \hat{u}_d = \tilde{u}_d = K & \hat{u}_d < \tilde{u}_d = K \\ \hat{u}_h = \tilde{u}_h & \hat{u}_h > \tilde{u}_h & \hat{u}_h > \tilde{u}_h \\ \hat{u}_l = \tilde{u}_l & \hat{u}_l < \tilde{u}_l & \hat{u}_l > \tilde{u}_l \end{array} \tag{4.8}$$

For a given $\hat{y}$, the expected compensation cost in the second-best case is weakly higher than that in the benchmark case, and they are equal at $\hat{y} = \hat{y}_A$.

If $\hat{y} > \hat{y}_A$, the severance pay for the second-best case equals that for the benchmark case, and the level is the same for all $\hat{y}$. The spread between u_h and u_l is greater in the second-best case. As $\hat{y}$ increases, the spread in the second-best case increases at an increasing rate, while the spread in the benchmark case decreases.

If $\hat{y} < \hat{y}_A$, the severance pay in the second-best case is lower than that in the benchmark case, which remains constant at K. The former decreases as $\hat{y}$ decreases. The spread between u_h and u_l in the second-best case is less than that in the benchmark case. As $\hat{y}$ increases, the spread in both the second-best and the benchmark cases decreases.

Recall that when information is privately observed by the agent, the compensation contract must motivate the agent to do two things:

1. Choose t_{1h}, and t_{2h} if the project is to be continued.

2. After choosing t_{1h} and observing the information signal y privately, make the abandon/continue decision which is in the principal's best interest.

On the other hand, when information is publicly observable and the principal chooses the cutoff point, the compensation contract only needs to motivate the agent to choose t_{1h}, and t_{2h} if the project is to be continued. Therefore, for a given $\hat{y}$, the expected compensation cost in the second-best case is weakly higher than that in the benchmark case, and they are equal at $\hat{y} = \hat{y}_A$. Given the optimal contract at $\hat{y}_A$, it is in the agent's interest to select $\hat{y}_A$ as the cutoff. The higher compensation cost in the second-best case reflects the cost arising from the induced moral hazard problem with the abandon/continue decision because information is not publicly observable.

The Cutoff Point Selection Problem

Using the solutions from the Contract Choice Problem, the principal chooses the cutoff point that maximizes

$$\hat{p}_{Bh}[B - w_d(\hat{y}) + w_k] + \hat{p}_{Hh}[x_H - w_h(\hat{y})] + \hat{p}_{Lh}[x_L - w_l(\hat{y})].$$

Let $\hat{y}^*$ represent the optimal cutoff point in the second-best case. The following proposition compares the second-best cutoff with the benchmark cutoff.

Proposition 4.3:

- If $\tilde{y}^* = \hat{y}_A$, then $\hat{y}^* = \tilde{y}^* = \hat{y}_A$.
- If $\tilde{y}^* < \hat{y}_A$, then $\tilde{y}^* < \hat{y}^* < \hat{y}_A$ and $\eta < 0$.
- If $\tilde{y}^* > \hat{y}_A$, then $\hat{y}_A < \hat{y}^* < \tilde{y}^*$ and $\eta > 0$.

The sign of η for $\hat{y}^*$ tells us if $\hat{y}^* > \tilde{y}^*$ or $\hat{y}^* < \tilde{y}^*$ and its relation to $\hat{y}_A$.

We examine the behavior of the expected compensation cost to help us understand why the proposition holds. As $\hat{y}$ increases, two factors simultaneously determine the behavior of the expected compensation cost:

1. The probability that the agent's employment will be terminated after the first stage increases, thus the probability that the principal incurs second-stage compensation cost decreases. This causes the expected compensation cost to decrease.

2. The required risk premium if the project is continued may either increase or decrease, causing the expected compensation cost to either increase or decrease respectively.

If the required risk premium is decreasing in $\hat{y}$, then the expected compensation cost will be decreasing in $\hat{y}$. If the required risk premium is increasing, then the net effect on the expected cost can either be decreasing or increasing, depending on which is the dominant factor. In the benchmark case, from Lemma 4.5, the required risk premium is decreasing in $\hat{y}$; therefore, the expected compensation cost is monotonically decreasing in $\hat{y}$.

We consider the second-best case. When $\hat{y} < \hat{y}_A$, Lemma 4.5 tells us that the required risk premium is decreasing in $\hat{y}$. Therefore, the expected compensation cost is monotononically decreasing in $\hat{y}$. For any $\hat{y}$, the expected compensation cost in the benchmark case is lower than that in the second-best case, and they are equal at $\hat{y} = \hat{y}_A$. The proof for Proposition 4.3 indicates that for any $\hat{y}$, the slope of the expected compensation cost in the second-best case is steeper than that in the benchmark case. Since the expected gross return is concave in $\hat{y}$, this implies that $\tilde{y}^* < \hat{y}^* < \hat{y}_A$.

When $\hat{y} > \hat{y}_A$, Lemma 4.5 tells us that the required risk premium is increasing in $\hat{y}$. If the first factor dominates, the expected compensation cost decreases as $\hat{y}$ increases. However, if the second factor dominates, the expected cost increases as $\hat{y}$ increases. The proof for Proposition 4.3 indicates that for any $\hat{y}$, the slope of the expected compensation cost in the second-best case is strictly greater than that in the benchmark case. Since the expected gross return is concave in $\hat{y}$, this implies that $\tilde{y}^* > \hat{y}^* > \hat{y}_A$.

If $\hat{y}^* > \tilde{y}^*$, the principal would prefer a lower cutoff point than $\hat{y}^*$ if he could observe y and make the abandon/continue decision. There exists a range of information signals $y \in (\tilde{y}^*, \hat{y}^*)$ in which the agent abandons the project even though it appears optimal to continue with it. In fact, in the second-best case, the moral hazard problem with the effort level choice results in an induced moral hazard problem in the abandon/continue decision, so that the principal prefers to motivate a higher cutoff point. This result indicates that there can arise cases when firms may appear too hasty in abandoning their projects.

On the other hand, when $\hat{y}^* < \tilde{y}^*$, the principal would prefer a higher cutoff point than $\hat{y}^*$ if he could observe y and make the abandon/continue decision. Therefore, there exists a range of information signals $y \in (\hat{y}^*, \tilde{y}^*)$ in which the agent continues the project even though it is not optimal to do so. In fact, in the second-best case, the induced moral hazard problem in the abandon/continue decision leads the principal to prefer a lower cutoff point. In the abovementioned range of information signals, the principal is better off allowing the agent to continue with the project.

Twiss (1992) studies the causes of successes and failures in technological innovation. An examination of projects which fail leads him to the conclusion that these projects should never have been initiated or should never have been allowed to proceed thus far in the first place. This is because when he looks back at the information available at the time of project selection or evaluation, he concludes that proper use of this information would have avoided these failures. However, if such information was private to the agent at the time of project evaluation and the agent makes the abandon/continue decision, then our results indicate that this phenomenon may not be avoidable in a principal-agent relationship with a first-stage moral hazard problem. The induced moral hazard problem in the abandon/continue decision leads the principal to prefer to motivate a lower cutoff point.

Next, we compare the second-best cutoff point with the first-best cutoff.

Proposition 4.4:

- If $\hat{y}^* \leq \hat{y}_A$, then $y^* < \hat{y}^* \leq \hat{y}_A$.
- If $\hat{y}^* > \hat{y}_A$, then $\hat{y}_A < \hat{y}^* < y^*$.

The moral hazard problem in the effort level choice leads to a different optimal cutoff point. Compared to the first-best cutoff point, the agent tends to overabandon the project when x_H is relatively high and $\hat{y}^* \leq \hat{y}_A$, and tends to overcontinue the project when x_H is relatively low and $\hat{y}^* > \hat{y}_A$. This relation is similar to that between the optimal benchmark cutoff $\tilde{y}^*$ and the second-best cutoff.

The results in the two specific cases in Chapter 3, the no-first-stage and no-second-stage moral hazard problems, provide us with insights into this problem. Compared to the first-best cutoff, in the no-first-stage moral hazard case, overabandonment results, while in the no-second-stage moral hazard case, overcontinuation results. In the general model where there are two stages of moral hazard, we conclude that:

- when $\hat{y}^* < \hat{y}_A$, the second-stage moral hazard problem dominates the first-stage moral hazard problem, resulting in overabandonment; and
- when $\hat{y}^* > \hat{y}_A$, the first-stage moral hazard problem dominates the second-stage moral hazard problem, resulting in overcontinuation.

From the numerical example in section 3.5, we observe that in the no-first-stage moral hazard problem, the deviation of the second-best cutoff from the first-best cutoff is greater when the first-best cutoff is low. On the other hand, in the no-second-stage moral hazard problem, the deviation is greater when the first-best cutoff is high. It is thus not surprising that in this two-stage moral hazard problem, we observe that when y^* is low, the second-stage moral hazard problem dominates, and when y^* is high, the first-stage moral hazard problem dominates.

4.3 Communication

Next, we examine if there is value to communication of the agent's signal to the principal. In the analysis above, although there is no communication of the signal itself, the abandon/continue decision serves as one level of communication. If the agent chooses abandonment, he is in effect communicating to the principal that the signal is below the cutoff. On the other hand, the decision to continue implies that a signal above the cutoff has been received. Our results indicate that the first-best cutoff cannot be achieved in this setting. If x_H is relatively high and the first-best cutoff is low, then the second-best cutoff is higher than the first-best. There is an overabandonment of the project. On the other hand, if x_H is relatively low and the first-best cutoff is high, then the second-best cutoff is lower than the first-best. There is overcontinuation of the project.

There are two potential benefits if the agent is able to communicate the specific value of y to the principal:

1. Planning function value: a smaller distortion of the cutoff from first-best, which implies a less severe overabandonment or overcontinuation problem; and

2. Control function value: a lower cost of motivating the agent to work hard (this derives from a lower risk premium that needs to be imposed on the agent to induce him to work hard).

We assume that the principal announces and precommits to a menu of contracts, $(u_d, \{u_h(y), u_l(y)\})$. After the agent chooses first-stage effort and observes the signal y, he decides whether to continue or not and which contract to select. If the project is to be continued, communication of the specific value of y has no planning value, and Proposition 4.5 shows that communication of the specific value of y has no control function value too.

Proposition 4.5: Communication of the specific value of y has no value.

Our proof indicates that to satisfy the truth-telling constraints, the compensation contract cannot be made contingent on communication. Thus, the least cost contract is the original contract under no communication, i.e., $(\hat{u}_d, \{\hat{u}_h, \hat{u}_l\})$. Since the compensation contract cannot be made contingent on communication, this implies that communication of the specific value of y cannot be used to solve the overabandonment and overcontinuation problem. We see similar results in Lambert (1986).

Lambert (1986) examines a single-stage moral hazard problem. The agent chooses between a risky project or a risk-free project. Before the project selection, the principal seeks to motivate him to work to acquire private information on the risky project. Lambert derives conditions under which underinvestment or overinvestment in the risky project occurs. When communication is introduced, he demonstrates that the underinvestment problem disappears. Balakrishnan (1991) examines a similar model to Lambert's with the additional consideration that the agent has precontract information. He also demonstrates that strict value to communication arises only if, absent communication, underinvestment results. However, both Lambert's and Balakrishnan's results depend critically on the assumption that the risky project does not require any more effort than does the risk-free project. Thus, we see that in our model, the presence of the second-stage moral hazard problem implies that the overabandonment problem does not disappear when communication is introduced. Lambert (1986) states that it is unclear if the overinvestment problem disappears when communication is introduced.

4.4 Example

4.4.1 Expected Compensation Cost, Wage Levels and Cutoff Points

Using the following numerical values, we show how the expected compensation costs vary as the cutoff point varies:

$$
\begin{aligned}
r &= 1 \\
w_k &= 0.36 \\
\bar{w} &= 0.693 \Rightarrow U(\bar{w}) = -0.5 \\
t_h &= 0.2 \Rightarrow v(t_h) = 1.2214
\end{aligned}
$$

Figure 4.10 shows how the expected compensation cost varies as the cutoff point varies for the three cases, benchmark, first-best, and second-best. In both the benchmark and the first-best cases, expected compensation is monotonically decreasing as the cutoff point increases. The probability of compensating the agent for t_{2h} is decreasing as the cutoff point increases. Also, recall that in the benchmark case, the required risk premium is also decreasing in $\hat{y}$. In the second-best case, the expected compensation cost is convex in $\hat{y}$ and reaches a minimum at $\hat{y} > \hat{y}_A$. We denote the cutoff point where the expected cost is minimum as $\hat{y}_m$. Note that at the cutoff point $\hat{y}_A$, the expected compensation for the benchmark case is equal to that in the second-best case. This is consistent with our analysis. In the second-best case, if the principal seeks to implement $\hat{y} = \hat{y}_A$, he does not need to motivate the agent to select $\hat{y}_A$. Given the optimal compensation contract to motivate the agent to choose $\{t_{1h}, t_{2h}|\text{if continue}\}$, it is in the agent's interest to select $\hat{y}_A$. For subsequent discussion, we partition the range of cutoff points into three regions (see Figure 4.10):

- Region 1: $\hat{y} < \hat{y}_A$,
- Region 2: $\hat{y}_A \leq \hat{y} \leq \hat{y}_m$, and

- Region 3: $\hat{y} > \hat{y}_m$.

In the cutoff point selection problem, the principal chooses the optimal cutoff point to maximize his expected gross return less the expected compensation costs. $R(\hat{y})$ is the expected gross return given cutoff point $\hat{y}$.

$$R(\hat{y}) = \hat{y}B + \frac{1}{2}(1 - \hat{y}^2)x_H + \frac{1}{2}(1 - \hat{y})^2 x_L.$$

The expected gross return given cutoff point $\hat{y}$ is concave in $\hat{y}$. At the optimal cutoff point, marginal expected gross return equals marginal expected compensation cost.

In both regions 1 and 2, the expected compensation cost in the second-best case is decreasing in $\hat{y}$. In region 1, the rate of decrease in the second-best case is greater than that for the benchmark case, thus $\hat{y}^* > \tilde{y}^*$. In region 2, the rate of decrease in the second-best case is smaller than that in the benchmark case, therefore $\hat{y}^* < \tilde{y}^*$. In region 3, expected compensation cost for the second-best case is increasing, while that for the benchmark case is decreasing. Therefore, $\hat{y}^* < \tilde{y}^*$. This is consistent with Proposition 4.3 in section 4.2.2. Figure 4.11 shows the optimal wage levels as the cutoff point varies for the two cases, benchmark and second-best.

We expand the example by varying the levels of x_H, while keeping the the values of B and x_L constant.

$$\begin{aligned} B &= -5 \\ x_L &= -30. \end{aligned}$$

Figure 4.12 shows how the optimal cutoff points vary as x_H varies for the three cases, benchmark, first-best and second-best. We observe that when $\tilde{y}^* > \hat{y}_A$, then $\hat{y}_A < \hat{y}^* < \tilde{y}^*$. When $\tilde{y}^* < \hat{y}_A$, then $\hat{y}_A > \hat{y}^* > \tilde{y}^*$.

Also, Figure 4.12 indicates that the optimal cutoff point decreases as x_H increases. Recall that the expected gross return is

$$R(\hat{y}) = \hat{y}B + \frac{1}{2}(1-\hat{y}^2)x_H + \frac{1}{2}(1-\hat{y})^2 x_L.$$

Thus,

$$R'(\hat{y}) = B - \hat{y}x_H - (1-\hat{y})x_L.$$

As x_H increases, holding B and x_L constant, $R'(\hat{y})$ decreases for a given $\hat{y}$. In the benchmark case, the expected compensation cost is decreasing in $\hat{y}$ at a decreasing rate. At the optimal cutoff point, marginal expected gross return equals marginal expected compensation cost. Thus, as x_H increases, the optimal cutoff point decreases. In the second-best case, the expected compensation cost is convex in $\hat{y}$ with a minimum at $\hat{y}_m$. Thus, the optimal cutoff point also decreases as x_H increases.

Figure 4.10

Behavior of Expected Compensation Cost

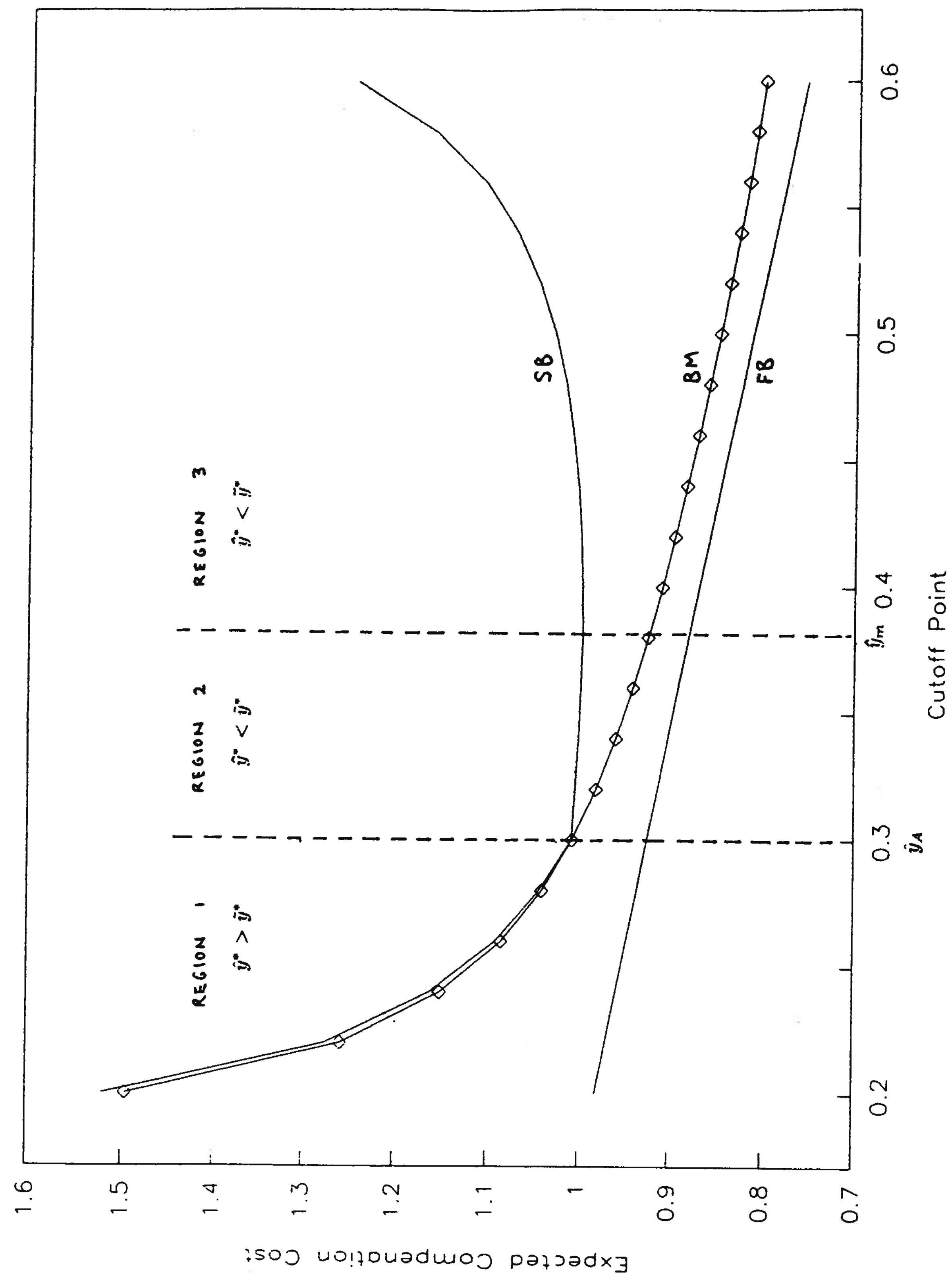


Figure 4.11

Behavior of Wage Levels

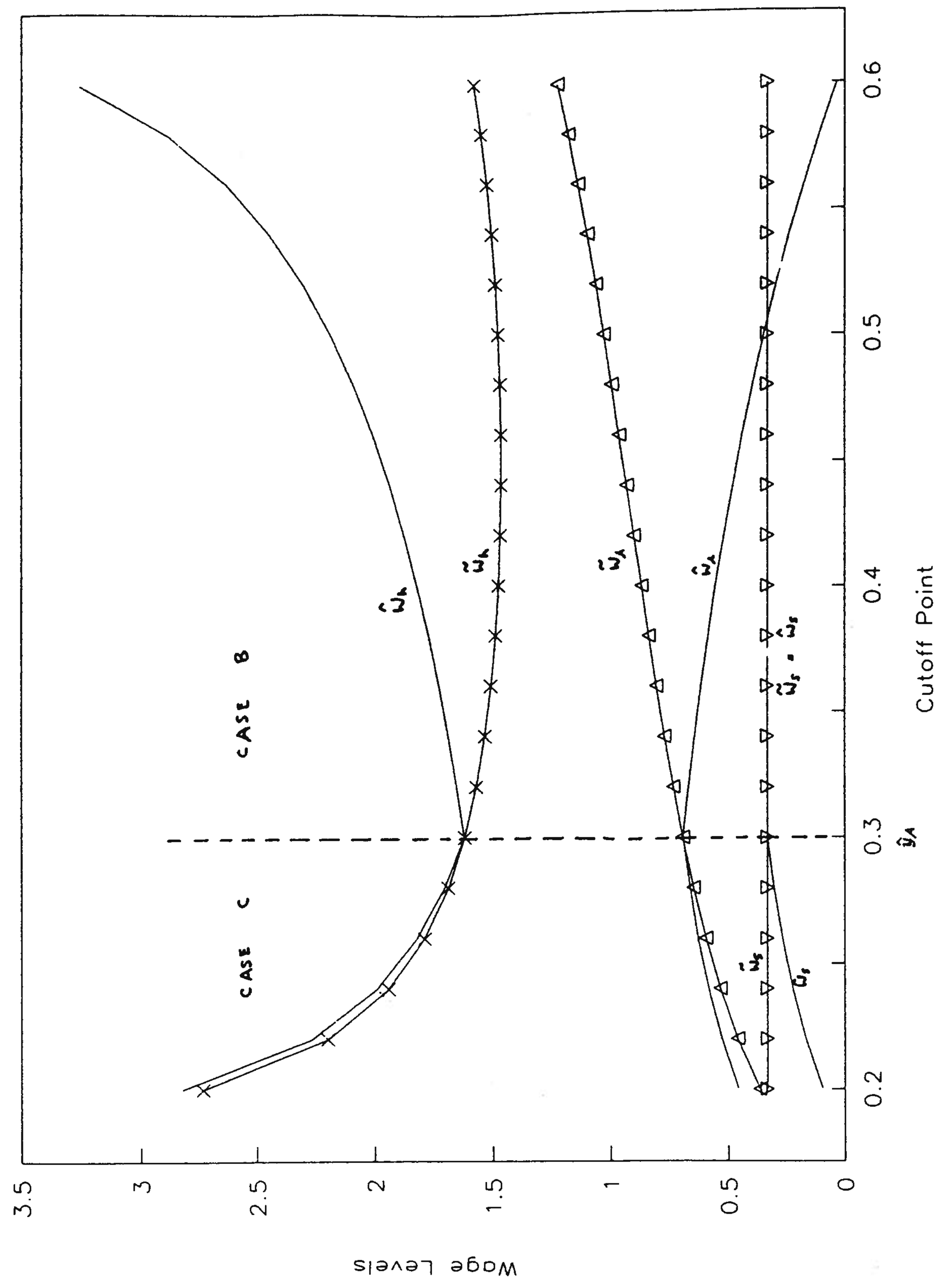
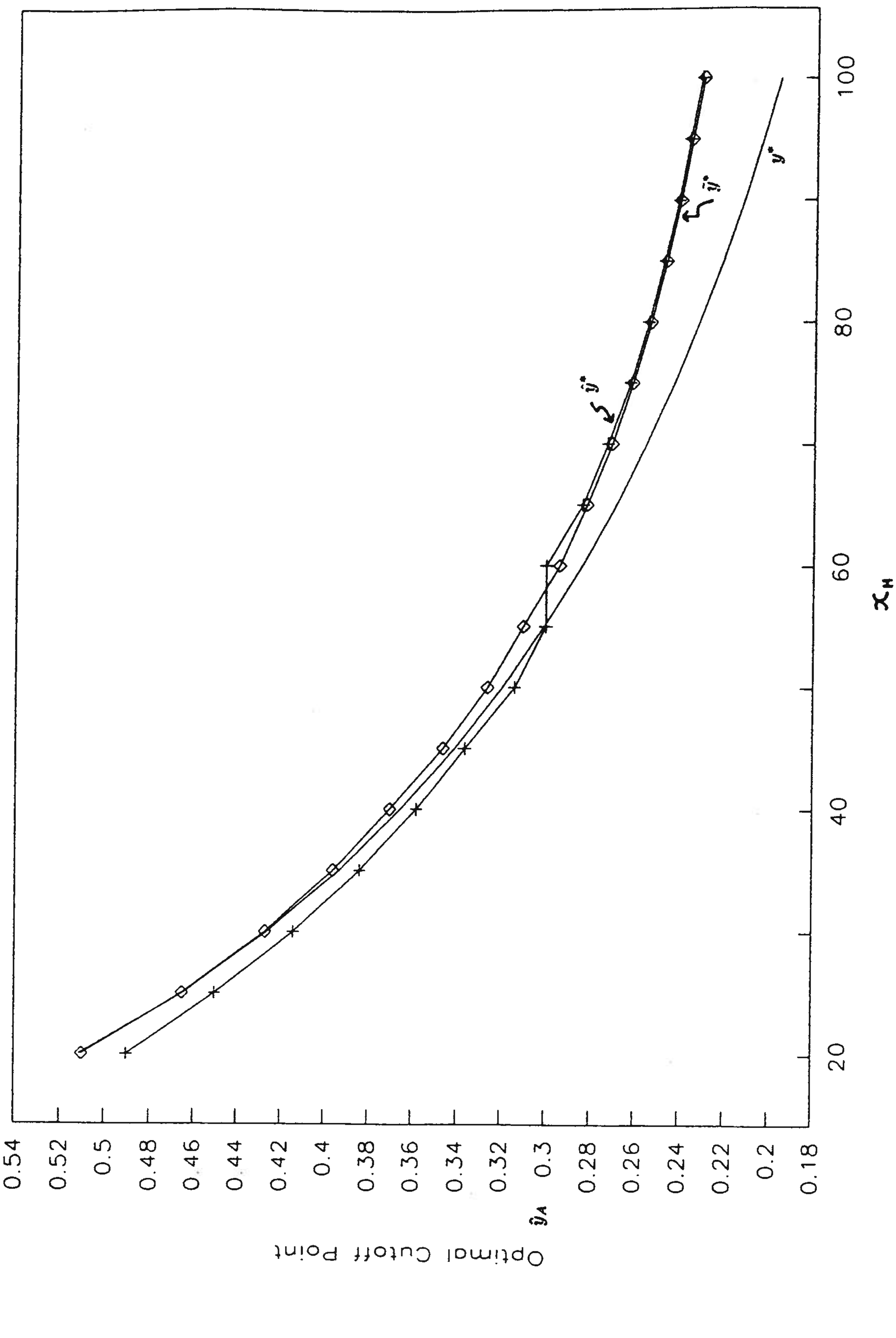


Figure 4.12

Behavior of Optimal Cutoff with x_H



4.4.2 Review of Results

In the second-best case, the desired cutoff $\hat{y}$ is ex ante efficient from the principal's viewpoint and ex ante incentive compatible from the agent's viewpoint. However, it appears inefficient from the principal's viewpoint ex post. We continue with the above example and examine two cases for specific values of x_H.

Overcontinuation

Let $x_H = 40$, then $y^* = .365$ and $\hat{y}^* = .356$. To implement $\hat{y}^*$ as the cutoff, the optimal compensation contract is $\hat{w}_h = 1.7208$, $\hat{w}_l = 0.6222$ and $\hat{w}_s = 0.333$. The expected compensation cost is 0.9989 and the expected returns (after deducting the compensation cost) to the principal is 8.4652. On the other hand, if the principal were to implement y^* as the cutoff, the optimal compensation contract is $\hat{w}_h = 1.73936$, $\hat{w}_l = 0.6094$ and $\hat{w}_s = 0.333$. The expected compensation cost is 0.9982 and the expected return (after deducting the compensation cost) to the principal is 8.4641. Thus, the principal will not choose to implement y^* but is better off implementing $\hat{y}^*$.

Next, we consider what happens if at $i = 1$, $y = 0.36$ is privately observed by the agent. Given the compensation contract, the agent is strictly better off continuing with the project and he chooses to do so. However, the principal's expected return (after deducting the expected compensation cost) is higher if the project is abandoned as shown below:

- If the project is abandoned, the net gain to the principal is:

$$\begin{aligned} \text{Net gain} &= B - \hat{w}_s \\ &= -5 - .333 \\ &= -5.333. \end{aligned}$$

- If the project is continued, the net gain to the principal is:

$$\text{Net gain} = y * (x_H - \hat{w}_h) + (1 - y) * (x_L - \hat{w}_l)$$

$$= .36 * (40 - 1.7208) + (.64) * (-30 - .6222)$$
$$= -5.8177.$$

Therefore, at $i = 1$, given the second-best optimal compensation contract and that $y = 0.36$ is observed, the principal is better off if the project is abandoned. From the point of view of an external observer who subsequently sees the realization of $y = 0.36$, it may appear that the agency has been unwilling to forego projects due to the investments made in the first stage. This is the sunk cost phenomenon and is frequently hailed as irrational.

The principal will not additionally compensate the agent to abandon the project. Otherwise, it will provide the agent with incentive to always report an observed y between $\hat{y}^*$ and y^* if he observes a lower value of y. The principal would then be strictly worse off relative to implementing the second-best optimal cutoff.

Overabandonment

In this second case, we let $x_H = 80$, then $y^* = 0.23236$ and $\hat{y}^* = .255$. To implement $\hat{y}^*$ as the cut-off, the optimal compensation contract is $\hat{w}_h = 1.8582$, $\hat{w}_l = 0.6174$ and $\hat{w}_s = 0.2574$. The expected compensation cost is 1.1057 and the expected returns (after deducting the compensation cost) to the principal is 26.69. On the other hand, if the principal were to implement y^* as the cutoff, the optimal compensation contract is $\hat{w}_h = 2.0784$, $\hat{w}_l = 0.5636$ and $\hat{w}_s = 0.2036$. The expected compensation cost is 1.1964 and the expected returns (after deducting the compensation cost) to the principal is 26.64. Thus, the principal will not choose to implement y^* but is better off implementing $\hat{y}^*$.

Next, we consider what happens if at $i = 1$, $y = 0.24$ is privately observed by the agent. Given the compensation contract, the agent is strictly better off abandoning the project and he chooses to do so. However, the principal's expected return (after deducting the expected compensation cost) is higher if the project is continued as shown below:

- If the project is abandoned, the net gain to the principal is -5.257.
- If the project is continued, the net gain to the principal is:

$$\begin{aligned}
\text{Net gain} &= y * (x_H - \hat{w}_h) + (1 - y) * (x_L - \hat{w}_l) \\
&= .24 * (80 - 1.8582) + (.76) * (-30 - .6174) \\
&= -4.5152.
\end{aligned}$$

Therefore, at $i = 1$, given the second-best optimal compensation contract and that $y = 0.24$ is observed, the principal is better off if the project is continued. From the point of view of an external observer who subsequently sees the realization of $y = 0.24$, it may appear that the agency is not rational and is too fast in dropping projects.

The principal will not additionally compensate the agent to continue the project. Otherwise, it will provide the agent with incentive to always report an observed y between y^* and $\hat{y}^*$ if he observes a higher value of y. The principal would then be strictly worse off relative to implementing the second-best optimal cutoff.

4.5 Literature Review

Hardly any work on multi-stage projects with project abandonment decision has been done. However, we often hear that decision-makers are reluctant to terminate projects when new information received indicates that the probability of success is low. Such escalation behavior has been generally termed as irrational. Kanodia, Bushman and Dickhaut (1989) provide an economic explanation for such behavior based on reputation. They show that when the agent has private information about his human capital, a desire for reputation-building may lead the agent to demonstrate escalation (overcontinuation) behavior. In their model, there is no possibility of overabandonment behavior. Our model provides an alternative explanation for escalation behavior based on induced moral hazard. We show that such behavior is ex ante efficient but ex post inefficient from the principal's perspective. Our model also shows that overabandonment behavior may occur when the return from a successful project is relatively high.

A closely related area of literature is project selection. The idea of overcontinuation (overabandonment) is similar to Lambert's (1986) overinvestment (underinvestment). Lambert's model is a first-stage moral hazard problem. If the agent does not work, the probability of success in the risky project is 0.5. Thus, in Lambert's model, both underinvestment and overinvestment can occur depending on whether the first-best cutoff is less than or greater than 0.5. In contrast, in our model, if the agent does not work, the probability of success in the project is nil. If there is only a first-stage moral hazard problem, only overcontinuation can occur.

Banker, Datar and Gopi (1989) also examine the project selection strategy of the agent. They compare the strategy of the agent when information is private to him and he makes the project selection with the case in which the principal himself receives the information and makes the selection. The latter case is similar to our benchmark case. When information is private to the agent, the principal's objective is to motivate the agent to choose the appropriate cutoff. If the project is to be undertaken, there is no uncertainty in the outcome and no moral hazard concern as a fixed outcome is expected. Thus, the compensation of the agent depends on only the invest/do not invest decision. In contrast, the principal in our model seeks to motivate the agent to work hard and choose the appropriate cutoff. If the project is to be continued, there is uncertainty in the outcome and a moral hazard concern exists. In both Banker, Datar and Gopi (1989) and our model, underinvestment (overabandonment) and overinvestment (overcontinuation) occur as a result of trade-off between the risk premium cost and suboptimal project selection (continuation decision) cost. However, in Banker, Datar and Gopi (1989), underinvestment prevails when project returns are relatively low and overinvestment prevails when project returns are high. The reverse occurs in our model where overabandonment prevails when returns are high and overcontinuation prevails when returns are low.

4.6 Implications and Conclusions

The results from our model show that the problem of overabandonment and overcontinuation is likely to be less severe in small firms in which the principal plays a key role in the project. He is well informed and can closely monitor the work done. The first-best cutoff may be attainable. Malidique and Hayes (1987, p. 157) state that the ease of innovation in small firms has inspired both puzzlement and jealousy in larger firms. Also, many successful large technological firms recreate the climate of the small firm by divisionalization, with each division manager given much autonomy in running the division. The literature gives excellent communication and freedom from bureaucracy as the main factors for success in small firms. Our model provides a different explanation for the advantage small firms have over large firms in the management of risky, multi-stage projects. In small firms, the principal is able to monitor the project closely and be kept well-informed of the progress of the project. Thus, he is in the position to make the abandon/continue decision that is in his best interest. On the other hand, in a large firm where the principal is very detached from the project, the abandonment/continuation decision has to be delegated to the agent. Some welfare loss is suffered since the agent does not share the same objective as the principal.

The current literature on project management focuses only on the overcontinuation (or sunk cost) problem. Our model shows clearly that both overcontinuation and overabandonment problem can actually happen. For projects with relatively high returns if they are successful, the overabandonment problem can occur. On the other hand, for projects with relatively low returns if successful, the overcontinuation problem can occur. The lack of focus on the overabandonment problem in the literature may be because, in practice, it is very hard to pinpoint overabandonment. The problem of overabandonment results in missed opportunities, while the overcontinuation problem may result in failure. While a failure is glaringly obvious, missed opportunities are not so clearly seen. Our model shows that overabandonment is a problem too and should not be ignored. A firm may lose its competitive advantage due to such missed opportunities. As discussed in the previous paragraph, we expect the problem to be less severe in a small firm in which the principal is very much involved in the project. This may help explain the finding of Scherer (1984), who indicates that small firms have been responsible for a disproportionate share of innovations.

Appendix 4A

Benchmark Case

(1) To show that both the second and third constraints are binding:

It is obvious that at least one of the second and third constraints of problem [P4.1] must be binding.

- Second constraint is binding: Proof is by contradiction. We drop the second constraint and assume that the optimal solution induces $\{t_{1h}, t_{2h} | \text{if continue}\}$. The principal only needs to impose risk on the agent to motivate him to work hard in stage 2. If the project is abandoned, the agent's utility is $u_d v(t_h)$. If the project is continued, we let the agent's expected utility be $\bar{u}_c v(2t_h)$, where $\bar{u}_c$ is defined using the participation constraint.

$$\begin{aligned} \hat{y}u_d v(t_h) &+ (1-\hat{y})\bar{u}_c v(2t_h) = K \\ \Rightarrow (1-\hat{y})\bar{u}_c &= \frac{1}{2}(1-\hat{y}^2)u_h + \frac{1}{2}(1-\hat{y})^2 u_l \\ \Rightarrow \bar{u}_c &= \frac{1}{2}[(1+\hat{y})u_h + (1-\hat{y})u_l], \end{aligned}$$

  where the ratio of u_h to u_l satisfies the third constraint. To minimize the risk imposed on the agent, the principal sets $u_d v(t_h) = \bar{u}_c v(2t_h)$. To satisfy the participation constraint, this implies that $u_d = \frac{K}{v(t_h)} > K$ and $\bar{u}_c = \frac{K}{v(2t_h)}$. Since the participation constraint is binding, $u_d > K$ implies that the second constraint is violated. Thus, at the optimal solution, the second constraint is binding.

- The third constraint is binding: Proof is by contradiction. If the third constraint is dropped, the principal could set $u_l = u_h = \bar{u}$. To satisfy the second constraint, $\bar{u} = \frac{K[1-\hat{y}v(t_h)]}{(1-\hat{y})v(2t_h)}$. But this implies that the agent will prefer to take t_{2l} which violates the third constraint.

(2) Derivation of $\tilde{u}_h$ and $\tilde{u}_l$:

Since both the first and second constraints are binding, $\tilde{u}_d = K$ and from the third constraint,

$$\tilde{u}_h = \frac{\tilde{u}_l[\hat{y}v(t_h) - v(t_h) + 1]}{\hat{y}v(t_h)}.$$

Substituting for u_d and u_h in the participation constraint, we obtain $\tilde{u}_l$:

$$\tilde{u}_l = \frac{2K\hat{y}[1 - \hat{y}v(t_h)]}{v(t_h)(1-\hat{y})[1 + \hat{y} - v(t_h)(1-\hat{y})]},$$

and

$$\frac{d\tilde{u}_l}{d\hat{y}} > 0.$$

By substitution, we obtain $\tilde{u}_h$ as follows:

$$\tilde{u}_h = \frac{2K[1 - v(t_h)(1-\hat{y})][1 - \hat{y}v(t_h)]}{v(2t_h)(1-\hat{y})[1 + \hat{y} - v(t_h)(1-\hat{y})]}.$$

To prove that the spread between $\tilde{u}_h$ and $\tilde{u}_l$ decreases as $\hat{y}$ increases:

We note that the ratio of $\tilde{u}_h$ to $\tilde{u}_l$ is

$$\frac{\tilde{u}_h}{\tilde{u}_l} = \frac{\hat{y}v(t_h) - [v(t_h) - 1]}{\hat{y}v(t_h)}.$$

This ratio increases as $\hat{y}$ increases and, since $\tilde{u}_h$ and $\tilde{u}_l$ are negative values and $\tilde{u}_l$ is increasing in $\hat{y}$, this implies that the spread between $\tilde{u}_h$ and $\tilde{u}_l$ decreases.

(3) Proof of Lemma 4.1:

The second constraint can be rewritten as:

$$u_d\{\hat{y}v(t_h) - 1\} + \frac{1}{2}(1-\hat{y}^2)u_h v(2t_h) + \frac{1}{2}(1-\hat{y})^2 u_l v(2t_h) \geq 0.$$

Since u_d, u_h and u_l are negative values, for the constraint to hold,

$$\hat{y} < \frac{1}{v(t_h)}.$$

The third constraint can be rewritten as:

$$\hat{y}u_h v(t_h) \geq u_l\{1 - v(t_h) + \hat{y}v(t_h)\}.$$

For the constraint to hold,

$$1 - v(t_h) + \hat{y}v(t_h) > 0.$$

This implies that

$$\hat{y} > 1 - \frac{1}{v(t_h)}.$$

For $\hat{y}$ to exist, $1 - \frac{1}{v(t_h)} < \frac{1}{v(t_h)}$, which implies that $v(t_h) < 2$.

Appendix 4B

The General Model

(1) **Derivation of the optimal compensation package:**

Since the principal's objective function in [P4.2] is convex in u_p, while the constraints are linear in u_p, first-order conditions are sufficient to ensure optimality. The first-order conditions for the optimal compensation contract are as follows:

$$
\begin{array}{lrcl}
(1) & -\hat{p}_{Bh}h'(u_d) + \lambda\hat{p}_{Bh}v(t_h) + \mu_1\hat{p}_{Bh}v(t_h) - \mu_1 + \mu_2 + \eta & = & 0 \\
& \Rightarrow \quad h'(u_d) = v(t_h)(\lambda + \mu_1) - \dfrac{\mu_1}{p_{Bh}} + \dfrac{\mu_2}{p_{Bh}} + \dfrac{\eta}{p_{Bh}}. & & \\
(2) & -\hat{p}_{Hh}h'(u_h) + \lambda\hat{p}_{Hh}v(2t_h) + \mu_1\hat{p}_{Hh}v(2t_h) - \eta\hat{y}v(t_h) & = & 0 \\
& \Rightarrow \quad h'(u_h) = v(2t_h)(\lambda + \mu_1) - \dfrac{\eta\hat{y}v(t_h)}{p_{Hh}}. & & \\
(3) & -\hat{p}_{Lh}h'(u_l) + \lambda\hat{p}_{Lh}v(2t_h) + \mu_1\hat{p}_{Lh}v(2t_h) - \mu_2 - \eta(1-\hat{y})v(t_h) & = & 0 \\
& \Rightarrow \quad h'(u_l) = v(2t_h)(\lambda + \mu_1) - \dfrac{\mu_2}{p_{Lh}} - \dfrac{\eta(1-\hat{y})v(t_h)}{p_{Lh}}. & &
\end{array}
$$

(2) **Proof of Lemma 4.2:**

For a given cutoff point $\hat{y}$, the principal needs to choose an optimal contract to motivate the agent to work hard. Given $\hat{y}$, the probabilities p_{jm}, $j = B,\ H,\ L$ and $m = l,\ h$ are fixed. Thus the principal's problem is a standard nonlinear programming problem of minimizing a convex function subject to a finite number of linear constraints. Thus, for any value of $\hat{y}$, the Lagrange multipliers on the inequality constraints are nonnegative.

Suppose both μ_1 and μ_2 equal zero. The optimal contract is as follows:

$$
\begin{aligned}
h'(u_d) &= \lambda v(t_h) + \frac{\eta}{\hat{p}_{Bh}}. \\
h'(u_h) &= \lambda v(2t_h) - \frac{\eta\hat{y}v(t_h)}{\hat{p}_{Hh}}.
\end{aligned}
$$

$$h'(u_l) = \lambda v(2t_h) - \frac{\eta(1-\hat{y})v(t_h)}{\hat{p}_{Lh}}.$$

If $\eta > 0$, the agent prefers to abandon the contract whatever the value of observed y.

If $\eta = 0$, a fixed wage contract results and the agent will prefer to take t_{1l}.

If $\eta < 0$, the third constraint $u_d \geq u_l$ is violated.

Therefore, at the optimal solution, at least one of μ_1 and μ_2 must be strictly greater than zero.

(3) **Proof of Lemma 4.3:**

If the first three constraints are binding, then $\hat{u}_d = \hat{u}_l = K$. Thus,

$$\begin{aligned} \hat{p}_{Bh}Kv(t_h) &+ \hat{p}_{Hh}\hat{u}_h v(2t_h) + \hat{p}_{Lh}Kv(2t_h) = K, \\ \hat{u}_h &= \frac{K[1-\hat{p}_{Bh}v(t_h) - \hat{p}_{Lh}v(2t_h)]}{\hat{p}_{Hh}v(2t_h)}. \end{aligned}$$

Substituting for $\hat{p}_{Bh}$, $\hat{p}_{Hh}$ and $\hat{p}_{Lh}$, we obtain:

$$\hat{u}_h = \frac{K\{2[1-\hat{y}v(t_h)] - (1-\hat{y})^2 v(2t_h)\}}{(1-\hat{y}^2)v(2t_h)}. \tag{4.9}$$

The agent's first-order condition for $\hat{y}$ (equation (4.3)) implies that

$$\hat{u}_h = \frac{K[1-v(t_h)+\hat{y}v(t_h)]}{\hat{y}v(t_h)}.$$

Equating the above two expressions for $\hat{u}_h$, and by substitution and rearrangement, we obtain the following expression for $\hat{y}$:

$$\hat{y} = \frac{v(t_h)+1 \pm \sqrt{(2v(t_h)+1)}}{v(t_h)}.$$

Since $\hat{y}$ must be less than one, thus:

$$\hat{y} = \frac{v(t_h)+1 - \sqrt{(2v(t_h)+1)}}{v(t_h)}.$$

By substitution, we obtain the following expression for $\hat{u}_h$:

$$\hat{u}_h = \frac{K[1-v(t_h)+\hat{y}v(t_h)]}{\hat{y}v(t_h)}$$

$$= \frac{K[2-\sqrt{(2v(t_h)+1)}]}{v(t_h)+1-\sqrt{(2v(t_h)+1)}}.$$

(4) **Differentiation of $\hat{y}_A$ and $u_h(A)$:**

$$\begin{aligned}
\hat{y}_A &= \frac{v(t_h)+1-\sqrt{(2v(t_h)+1)}}{v(t_h)}.\\
\frac{d\hat{y}_A}{dv(t_h)} &= \frac{v(t_h)[1-(2v(t_h)+1)^{-1/2}]-[v(t_h)+1-\sqrt{(2v(t_h)+1)}]}{v(2t_h)}\\
&= \frac{v(t_h)+1-\sqrt{(2v(t_h)+1)}}{v(2t_h)\sqrt{(2v(t_h)+1)}}.
\end{aligned}$$

Since $v(t_h)+1>\sqrt{(2v(t_h)+1)}$, this implies that $\frac{d\hat{y}_A}{dv(t_h)}>0$.

$$\begin{aligned}
u_h(A) &= \frac{K[2-\sqrt{(2v(t_h)+1)}]}{v(t_h)+1-\sqrt{(2v(t_h)+1)}}.\\
\frac{du_h(A)}{dv(t_h)} &= \frac{-K\{2\sqrt{(2v(t_h)+1)}-v(t_h)-2\}}{[v(t_h)+1-\sqrt{(2v(t_h)+1)}]^2\sqrt{(2v(t_h)+1)}}.
\end{aligned}$$

Since $2\sqrt{(2v(t_h)+1)}>v(t_h)+2$, this implies that $\frac{du_h(A)}{dv(t_h)}>0$.

(5) **Case B: Derivation of $\hat{u}_h$, $\hat{u}_l$ and $\hat{u}_d$:**

In case B, $EH_h=\hat{u}_d$ $(\mu_1>0)$ and $\hat{u}_d>\hat{u}_l$ $(\mu_2=0)$. Thus, $\hat{u}_d=K$ while $\hat{u}_l=K-\epsilon$, $\epsilon>0$. We derive expressions for $\hat{u}_h$ and $\hat{u}_l$ for a given $\hat{y}$. From the agent's first-order condition on $\hat{y}$, $\hat{u}_l=\frac{\hat{u}_d-\hat{y}v(t_h)\hat{u}_h}{(1-\hat{y})v(t_h)}$. The agent's expected utility is as follows:

$$\begin{aligned}
EH_h &= \hat{y}\hat{u}_d v(t_h)+\frac{1}{2}(1-\hat{y}^2)\hat{u}_h v(2t_h)+\frac{1}{2}(1-\hat{y})^2\hat{u}_l v(2t_h)\\
&= \hat{u}_d[\hat{y}v(t_h)+\frac{1}{2}(1-\hat{y})v(t_h)]+\hat{u}_h[\frac{1}{2}(1-\hat{y}^2)v(2t_h)-\frac{1}{2}\hat{y}(1-\hat{y})v(2t_h)]\\
&= \frac{1}{2}v(t_h)[(1+\hat{y})\hat{u}_d+(1-\hat{y})\hat{u}_h v(t_h)]. \qquad (4.10)
\end{aligned}$$

Since $EH=\hat{u}_d=K$,

$$(1-\hat{y})\hat{u}_h v(t_h) = \frac{2K}{v(t_h)}-(1+\hat{y})K$$

$$\begin{aligned}
\Rightarrow \hat{u}_h &= \frac{K[2-v(t_h)(1+\hat{y})]}{(1-\hat{y})v(2t_h)}. \\
\frac{d\hat{u}_h}{d\hat{y}} &= \frac{-2K[v(t_h)-1]}{(1-\hat{y})^2 v(2t_h)} \\
&> 0. \\
\frac{d^2\hat{u}_h}{d\hat{y}^2} &> 0.
\end{aligned}$$

By substitution, we obtain the expression for $\hat{u}_l$ for a given $\hat{y}$.

$$\begin{aligned}
\hat{u}_l &= \frac{K}{(1-\hat{y})v(t_h)} - \frac{\hat{y}}{(1-\hat{y})}\hat{u}_h \\
&= \frac{K[v(t_h)(1+\hat{y}^2)-2\hat{y}]}{(1-\hat{y})^2 v(2t_h)}. \\
\frac{d\hat{u}_l}{d\hat{y}} &= \frac{2K[v(t_h)-1](1+\hat{y})}{(1-\hat{y})^3 v(2t_h)} \\
&< 0. \\
\frac{d^2\hat{u}_l}{d\hat{y}^2} &< 0.
\end{aligned}$$

(6) **Case C: Derivation of $\hat{u}_h$, $\hat{u}_l$ and $\hat{u}_d$:**

In case C, $EH_h > \hat{u}_d$ ($\mu_1 = 0$) and $\hat{u}_d = \hat{u}_l$ ($\mu_2 > 0$). Thus, $\hat{u}_d = \hat{u}_l = K - \delta$, $\delta > 0$. We derive expressions for $\hat{u}_h$ and $\hat{u}_l$ for a given $\hat{y}$. $\hat{u}_d = \hat{u}_l$ and from the agent's first-order condition on $\hat{y}$, $\hat{u}_d = \hat{u}_l = \frac{\hat{y}v(t_h)\hat{u}_h}{\hat{y}v(t_h)-v(t_h)+1}$. The agent's expected utility is as follows:

$$\begin{aligned}
EH_h &= \hat{y}\hat{u}_d v(t_h) + \frac{1}{2}(1-\hat{y}^2)\hat{u}_h v(2t_h) + \frac{1}{2}(1-\hat{y})^2\hat{u}_l v(2t_h) \\
&= \frac{\hat{u}_h v(2t_h)}{\hat{y}v(t_h)-v(t_h)+1}[\hat{y}v(t_h) - \frac{1}{2}[v(t_h)-1](1+\hat{y}^2)]. \qquad (4.11)
\end{aligned}$$

Let Υ represents a positive expression. It is generally the square of the denominator. Since $EH_h = K$,

$$\begin{aligned}
\hat{u}_h &= \frac{K[\hat{y}v(t_h)-v(t_h)+1]}{v(2t_h)[\hat{y}v(t_h)-\frac{1}{2}[v(t_h)-1](1+\hat{y}^2)]} \\
&= \frac{2K[\hat{y}v(t_h)-v(t_h)+1]}{v(2t_h)[2\hat{y}v(t_h)-[v(t_h)-1](1+\hat{y}^2)]}. \\
\frac{d\hat{u}_h}{d\hat{y}} &= \frac{2K[v(t_h)-1][v(t_h)(1+\hat{y}^2)-2\hat{y}(v(t_h)-1)]}{\Upsilon}
\end{aligned}$$

$$< \ 0.$$

By substitution, we obtain the expression for $\hat{u}_l$ for a given $\hat{y}$.

$$\begin{aligned}
\hat{u}_l &= \frac{2K\hat{y}}{v(t_h)[2\hat{y}v(t_h) - [v(t_h) - 1](1 + \hat{y}^2)]}. \\
\frac{d\hat{u}_l}{d\hat{y}} &= \frac{-2K[v(t_h) - 1](1 - \hat{y}^2)}{\Upsilon} \\
&> 0.
\end{aligned}$$

(7) **Proof of Proposition 4.1:**

(a)Proof that $\eta = 0$ in case A:

In case A, $\mu_1 > 0$ and $\mu_2 > 0$. From the first three binding constraints, we obtain the following expression for $\hat{u}_h$:

$$\hat{u}_h = \frac{K\{2[1 - \hat{y}v(t_h)] - (1 - \hat{y})^2 v(2t_h)\}}{(1 - \hat{y}^2)v(2t_h)}.$$

If the principal wants to motivate the agent to select the cutoff point $\hat{y}_A$, then substituting the value of $\hat{y}_A$ into u_h, we obtain the following:

$$u_h(A) = \frac{K[2 - \sqrt{(2v(t_h) + 1)}]}{v(t_h) + 1 - \sqrt{(2v(t_h) + 1)}}.$$

Given that $\hat{u}_d = \hat{u}_l = K$ and $\hat{u}_h = u_h(A)$, the agent will find it optimal to select $\hat{y}_A$ as the cutoff point. This implies that the last constraint is not binding, thus $\eta = 0$.

(b) Case B applies when the principal wants to motivate a cutoff point $\hat{y} > \hat{y}_A$:

In case B,

$$\hat{u}_l = \frac{K[v(t_h)(1 + \hat{y}^2) - 2\hat{y}]}{(1 - \hat{y})^2 v(2t_h)}.$$

Since $\hat{u}_d = K$ and $\hat{u}_d > \hat{u}_l$, this implies that

$$
\begin{aligned}
\frac{[v(t_h)(1+\hat{y}^2)-2\hat{y}]}{(1-\hat{y})^2 v(2t_h)} &> 1 \\
\Rightarrow \quad \hat{y}^2 v(t_h)[v(t_h)-1] - 2\hat{y}[v(2t_h)-1] + v(t_h)[v(t_h)-1] &< 0 \\
\Rightarrow \quad \hat{y} > \frac{v(t_h)+1-\sqrt{(2v(t_h)+1)}}{v(t_h)} &= \hat{y}_A.
\end{aligned}
$$

Next, we prove that $\eta > 0$ in case B:

In case B, $\mu_1 > 0$ and $\mu_2 = 0$. Suppose that $\eta \leq 0$. Then

$$
\begin{aligned}
h'(u_d) &< v(t_h)(\lambda + \mu_1), \\
h'(u_h) &> v(2t_h)(\lambda + \mu_1), \\
h'(u_l) &> v(2t_h)(\lambda + \mu_1).
\end{aligned}
$$

Then the agent will strictly prefer to continue the contract no matter what value of y is observed. Thus, in case B, $\eta > 0$.

(c) Case C applies when the principal wants to motivate the agent to select a cutoff point $\hat{y} < \hat{y}_A$:

In case C,

$$
\hat{u}_d = \hat{u}_l = \frac{2K\hat{y}}{v(t_h)[2\hat{y}v(t_h) - (v(t_h)-1)(1+\hat{y}^2)]}.
$$

Since $EH = K$ and $EH > \hat{u}_d$, thus,

$$
\begin{aligned}
\frac{2\hat{y}}{v(t_h)[2\hat{y}v(t_h) - (v(t_h)-1)(1+\hat{y}^2)]} &> 1 \\
\Rightarrow \quad \hat{y}^2 v(t_h)[v(t_h)-1] - 2\hat{y}[v(2t_h)-1] + v(t_h)[v(t_h)-1] &> 0 \\
\Rightarrow \quad \hat{y} < \frac{v(t_h)+1-\sqrt{(2v(t_h)+1)}}{v(t_h)} &= \hat{y}_A.
\end{aligned}
$$

Next, we prove that $\eta < 0$ in case C:

In case C, $\mu_1 = 0$ and $\mu_2 > 0$. Suppose that $\eta \geq 0$. Then

$$\begin{aligned} h'(u_d) &> \lambda v(t_h), \\ h'(u_h) &< \lambda v(2t_h), \\ h'(u_l) &< \lambda v(2t_h). \end{aligned}$$

Then the agent will strictly prefer to abandon the project no matter what value of y is observed. Thus, in case C, $\eta < 0$.

(8) **Proof of Lemma 4.4:**

In case A, from (4.4),

$$\hat{u}_h = \frac{K[2 - \sqrt{(2v(t_h) + 1)}]}{v(t_h) + 1 - \sqrt{(2v(t_h) + 1)}}.$$

Since the denominator is clearly positive, then $\hat{u}_h < 0$ and $K < 0$ imply that

$$2 - \sqrt{(2v(t_h) + 1)} > 0.$$

Simplifying the equation, we obtain the upper bound on $v(t_h)$, i.e., $v(t_h) < \frac{3}{2}$.

In case B, from (4.5),

$$\hat{u}_h = \frac{K[2 - v(t_h)(1 + \hat{y})]}{(1 - \hat{y})v(2t_h)}.$$

Since the denominator is clearly positive, then $\hat{u}_h < 0$ and $K < 0$ imply that

$$2 - v(t_h)(1 + \hat{y}) > 0.$$

Simplifying the equation, we obtain the upper bound of $\hat{y}$, i.e., $\hat{y} < \frac{2-v(t_h)}{v(t_h)}$.

In case C, from (4.6),

$$\begin{aligned} \hat{u}_h &= \frac{2K[\hat{y}v(t_h) - v(t_h) + 1]}{v(2t_h)[2\hat{y}v(t_h) - [v(t_h) - 1](1 + \hat{y}^2)]} \\ \hat{u}_l &= \frac{2K\hat{y}}{v(t_h)[2\hat{y}v(t_h) - [v(t_h) - 1](1 + \hat{y}^2)]}. \end{aligned}$$

$\hat{u}_l < 0$ and $K < 0$ imply that $[2\hat{y}v(t_h) - [v(t_h) - 1](1 + \hat{y}^2)] > 0$. Therefore, the denominator of $\hat{u}_h$ is positive, and $\hat{u}_h < 0$ and $K < 0$ imply that

$$\hat{y}v(t_h) - v(t_h) + 1 > 0.$$

Simplifying the equation, we obtain the lower bound of $\hat{y}$, i.e., $\hat{y} > \frac{v(t_h)-1}{v(t_h)}$.

(9) **Proof of Lemma 4.5:**

From (4.7),

$$U[\hat{y}h(u_h) + (1 - \hat{y})h(u_l) - \pi(\hat{y})] = [\hat{y}u_h + (1 - \hat{y})u_l]v(t_h).$$

1. In both the benchmark case (binding third constraint) and the second-best case for $\hat{y} < \hat{y}_A$ (agent's first-order condition on $\hat{y}$ and that $\hat{u}_l = \hat{u}_d$):

$$[\hat{y}u_h + (1 - \hat{y})u_l]v(t_h) = u_l.$$

Therefore,

$$\begin{aligned} \pi &= \hat{y}h(u_h) + (1 - \hat{y})h(u_l) - h(u_l) \\ &= \hat{y}[h(u_h) - h(u_l)]. \end{aligned}$$

Next, the ratio of u_h to u_l in both the benchmark case and the general case are equal and is given by:

$$\begin{aligned} \frac{u_h}{u_l} &= \frac{\hat{y}v(t_h) - v(t_h) + 1}{\hat{y}v(t_h)} \\ \frac{-\exp[-rh(u_h)]}{-\exp[-rh(u_l)]} &= \frac{\hat{y}v(t_h) - v(t_h) + 1}{\hat{y}v(t_h)} \\ h(u_h) - h(u_l) &= -\frac{1}{r}\ln[\frac{\hat{y}v(t_h) - v(t_h) + 1}{\hat{y}v(t_h)}]. \end{aligned}$$

Thus,

$$\begin{aligned}
\pi &= -\frac{1}{r}\{\hat{y}\ln[\frac{\hat{y}v(t_h)-v(t_h)+1}{\hat{y}v(t_h)}]\}. \\
\pi' &= -\frac{1}{r}\{\ln[\frac{\hat{y}v(t_h)-v(t_h)+1}{\hat{y}v(t_h)}]+\frac{v(t_h)-1}{\hat{y}v(t_h)-v(t_h)+1}\}. \\
&= -\frac{1}{r}\{\frac{v(t_h)-1}{\hat{y}v(t_h)-v(t_h)+1}-\ln[\frac{\hat{y}v(t_h)}{\hat{y}v(t_h)-v(t_h)+1}]\} \\
&= -\frac{1}{r}\{\frac{v(t_h)-1}{\hat{y}v(t_h)-v(t_h)+1}-\ln[\frac{v(t_h)-1}{\hat{y}v(t_h)-v(t_h)+1}+1]\}.
\end{aligned}$$

Let

$$\begin{aligned}
f &= a-\ln(a+1), a \geq 0. \\
\Rightarrow f' &= 1-\frac{1}{a+1}.
\end{aligned}$$

$f=0$ at $a=0$ and $f'>0$ for $a>0$. Thus, $a>\ln(a+1)$ for $a>0$. Since $\frac{v(t_h)-1}{\hat{y}v(t_h)-v(t_h)+1}>0$,

$$\frac{v(t_h)-1}{\hat{y}v(t_h)-v(t_h)+1}-\ln[\frac{v(t_h)-1}{\hat{y}v(t_h)-v(t_h)+1}+1]>0.$$

Thus, $\pi'<0$.

2. In the second-best case for $\hat{y}>\hat{y}_A$ (agent's first-order condition on $\hat{y}$ and that $\hat{u}_d=K$):

$$[\hat{y}u_h+(1-\hat{y})u_l]v(t_h)=K.$$

Therefore,

$$\begin{aligned}
\pi &= \hat{y}h(u_h)+(1-\hat{y})h(u_l)-h(K) \\
&= \hat{y}[h(u_h)-h(u_l)]+h(u_l)-h(K).
\end{aligned}$$

Next,

$$\begin{aligned}
\frac{u_h}{u_l} &= \frac{[2-v(t_h)(1+\hat{y})](1-\hat{y})}{v(t_h)(1+\hat{y}^2)-2\hat{y}}. \\
\Rightarrow h(u_h)-h(u_l) &= -\frac{1}{r}\ln[\frac{[2-v(t_h)(1+\hat{y})](1-\hat{y})}{v(t_h)(1+\hat{y}^2)-2\hat{y}}].
\end{aligned}$$

Thus,

$$\begin{aligned}
\pi &= -\frac{1}{r}\{\hat{y}\ln[\frac{[2-v(t_h)(1+\hat{y})](1-\hat{y})}{v(t_h)(1+\hat{y}^2)-2\hat{y}}] + \ln[-u_l] - \ln[-K]\}.\\
\pi' &= -\frac{1}{r}\{\ln[\frac{[2-v(t_h)(1+\hat{y})](1-\hat{y})}{v(t_h)(1+\hat{y}^2)-2\hat{y}}] + \frac{2[v(t_h)-1][\hat{y}^2v(t_h)-2\hat{y}v(t_h)-v(t_h)+2]}{[\hat{y}^2v(t_h)-2\hat{y}-v(t_h)+2][\hat{y}^2v(t_h)-2\hat{y}+v(t_h)]}\}.\\
&= -\frac{1}{r}\{\frac{2[v(t_h)-1][\hat{y}^2v(t_h)-2\hat{y}v(t_h)-v(t_h)+2]}{[\hat{y}^2v(t_h)-2\hat{y}-v(t_h)+2][\hat{y}^2v(t_h)-2\hat{y}+v(t_h)]} - \ln[\frac{\hat{y}^2v-2\hat{y}+v}{[2-v(t_h)(1+\hat{y})](1-\hat{y})}]\}\\
&= \frac{1}{r}\{\ln[\frac{2(v(t_h)-1)}{\hat{y}^2v(t_h)-2\hat{y}-v(t_h)+2}+1] - \frac{2[v(t_h)-1][\hat{y}^2v(t_h)-2\hat{y}v(t_h)-v(t_h)+2]}{[\hat{y}^2v(t_h)-2\hat{y}-v(t_h)+2][\hat{y}^2v(t_h)-2\hat{y}+v(t_h)]}\}
\end{aligned}$$

Note that $\frac{2(v(t_h)-1)}{\hat{y}^2v(t_h)-2\hat{y}-v(t_h)+2} > 0$ and $\frac{[\hat{y}^2v(t_h)-2\hat{y}v(t_h)-v(t_h)+2]}{[\hat{y}^2v(t_h)-2\hat{y}+v(t_h)]} < 1$.

Let

$$\begin{aligned}
g &= \ln(b+1) - \phi b.\\
g' &= \frac{1}{b+1} - \phi.
\end{aligned}$$

$g = 0$ at $b = 0$. If $\frac{1}{b+1} > \phi$ for $b > 0$, then $g > 0$ for $b > 0$.

If $b = \frac{2(v(t_h)-1)}{\hat{y}^2v(t_h)-2\hat{y}-v(t_h)+2} > 0$, then,

$$\frac{1}{b+1} = \frac{\hat{y}^2v(t_h)-2\hat{y}-v(t_h)+2}{\hat{y}^2v(t_h)-2\hat{y}+v(t_h)}.$$

$\phi = \frac{[\hat{y}^2v(t_h)-2\hat{y}v(t_h)-v(t_h)+2]}{[\hat{y}^2v(t_h)-2\hat{y}+v(t_h)]} < 1$, and $\frac{1}{b+1} > \phi$. Thus, $[\ln(b+1)-\phi b] > 0$ and $\pi' > 0$.

$\pi'' = N * \{3\hat{y}^4v(2t_h) + 2\hat{y}^3v(t_h)(v(t_h)-4) - 4\hat{y}^2(v(t_h)-1) - 2\hat{y}[v(t_h)[v(t_h)-2]-2] + (v(2t_h)-4)\}$,

where N denotes a negative expression.

Let

$$\begin{aligned}
h &= 3\hat{y}^4v(2t_h) + 2\hat{y}^3v(t_h)(v(t_h)-4) - 4\hat{y}^2(v(t_h)-1) - 2\hat{y}[v(t_h)[v(t_h)-2]-2] + (v(2t_h)-4).\\
h_{\hat{y}} &= 12\hat{y}^3v(2t_h) + 6\hat{y}^2v(t_h)(v(t_h)-4) - 8\hat{y}(v(t_h)-1) - 2[v(t_h)[v(t_h)-2]-2].
\end{aligned}$$

For $0 < \hat{y} < \frac{2-v(t_h)}{v(t_h)}$, $h_{\hat{y}} > 0$, i.e., h is an increasing function in $\hat{y}$ for $0 < \hat{y} < \frac{2-v(t_h)}{v(t_h)}$. At $\hat{y} = 0$, $h = v(2t_h) - 4 < 0$. At $\hat{y} = \frac{2-v(t_h)}{v(t_h)}$, $h = \frac{4(v(t_h)-2)(v(t_h)-1)^2}{v(t_h)} < 0$. Therefore, since h is continuous and increasing, $h < 0$ for $0 < \hat{y} < \frac{2-v(t_h)}{v(t_h)}$. Thus, $\pi'' > 0$.

(10) **Proof of Proposition 4.2:**

Recall that in the benchmark case (equation (4.1)),

$$\begin{aligned} \tilde{u}_h &= \frac{2K[1-(1-\hat{y})v(t_h)][1-\hat{y}v(t_h)]}{v(2t_h)(1-\hat{y})[1+\hat{y}-v(t_h)(1-\hat{y})]}, \\ \text{and } \tilde{u}_l &= \frac{2K\hat{y}[1-\hat{y}v(t_h)]}{v(t_h)(1-\hat{y})[1+\hat{y}-v(t_h)(1-\hat{y})]}. \end{aligned}$$

(a) At $\hat{y} = \hat{y}_A$:

By substituting $\hat{y} = \hat{y}_A = \frac{v(t_h)+1-\sqrt{(2v(t_h)+1)}}{v(t_h)}$ into $\tilde{u}_h$ and $\tilde{u}_l$, we obtain the following:

$$\begin{aligned} \tilde{u}_h &= \hat{u}_h(A), \\ \tilde{u}_l &= \hat{u}_l(A) = K. \end{aligned}$$

(b) If $\hat{y} > \hat{y}_A$:

$$\hat{u}_h - \tilde{u}_h = \frac{K[v(t_h)-1][\hat{y}^2v(t_h) - 2\hat{y}v(t_h) - 2\hat{y} + v(t_h)]}{v(2t_h)(1-\hat{y})[\hat{y}v(t_h)+\hat{y}-v(t_h)+1]}.$$

Within the feasible range of $\hat{y}$, $[\hat{y}v(t_h)+\hat{y}-v(t_h)+1] > 0$. For $\hat{y} > \hat{y}_A$, $[\hat{y}^2v(t_h)-2\hat{y}v(t_h)-2\hat{y}+v(t_h)] < 0$. Thus, $\hat{u}_h > \tilde{u}_h$.

The participation constraints of both problems [P4.1] (benchmark case) and [P4.2] (general case) are binding and since $\tilde{u}_d = \hat{u}_d$, we obtain the following:

$$\frac{1}{2}(1-\hat{y}^2)\hat{u}_h v(2t_h) + \frac{1}{2}(1-\hat{y})^2\hat{u}_l v(2t_h) = \frac{1}{2}(1-\hat{y}^2)\tilde{u}_h v(2t_h) + \frac{1}{2}(1-\hat{y})^2\tilde{u}_l v(2t_h).$$

Therefore,

$$(1+\hat{y})[\hat{u}_h - \tilde{u}_h] = (1-\hat{y})[\tilde{u}_l - \hat{u}_l].$$

Since $\hat{u}_h > \tilde{u}_h$, the equality implies that $\tilde{u}_l > \hat{u}_l$. Thus, if $\hat{y} > \hat{y}_A$, for a given cutoff,

$$\begin{aligned}\hat{u}_d &= \tilde{u}_d = K,\\ \hat{u}_h &> \tilde{u}_h,\\ \text{and}\quad \hat{u}_l &< \tilde{u}_l.\end{aligned}$$

Next, let $\tilde{s}(\hat{s})$ denote the spread between u_h and u_l in the benchmark (second-best) case.

$$\begin{aligned}\tilde{s} = \tilde{u}_h - \tilde{u}_l &= -\frac{2K(v(t_h)-1)(1-\hat{y}v(t_h))}{v(2t_h)(1-\hat{y})[\hat{y}v(t_h)+\hat{y}-v(t_h)+1]}.\\ \hat{s} = \hat{u}_h - \hat{u}_l &= -\frac{2K(v(t_h)-1)}{v(2t_h)(1-\hat{y})^2}.\\ \hat{s}' &= -\frac{4K(v(t_h)-1)}{v(2t_h)(1-\hat{y})^3} > 0.\\ \hat{s}'' &> 0.\end{aligned}$$

Then,

$$\begin{aligned}\hat{s} - \tilde{s} &= \frac{2K(v(t_h)-1)[\hat{y}^2v(t_h)-2\hat{y}v(t_h)-2\hat{y}+v(t_h)]}{v(2t_h)(1-\hat{y})^2[\hat{y}v(t_h)+\hat{y}-v(t_h)+1]}\\ &> 0.\end{aligned}$$

(c) If $\hat{y} < \hat{y}_A$:

$$\hat{u}_h - \tilde{u}_h = \frac{2K\hat{y}(v(t_h)-1)(\hat{y}v(t_h)-v(t_h)+1)[\hat{y}^2v(t_h)-2\hat{y}v(t_h)-2\hat{y}+v(t_h)]}{v(2t_h)[\hat{y}^2v(t_h)-\hat{y}^2-2\hat{y}v(t_h)+v(t_h)-1](1-\hat{y})[\hat{y}v(t_h)+\hat{y}-v(t_h)+1]}.$$

The expression $[\hat{y}^2v(t_h)-2\hat{y}v(t_h)-2\hat{y}+v(t_h)] = 0$ at $\hat{y} = \hat{y}_A$ and $[\hat{y}^2v(t_h)-2\hat{y}v(t_h)-2\hat{y}+v(t_h)] > 0$ for $\hat{y} < \hat{y}_A$. Also, for $\hat{y} < \hat{y}_A$, from (4.6),

$$\hat{u}_l = \frac{2K\hat{y}}{v(t_h)[2\hat{y}v(t_h)-[v(t_h)-1](1+\hat{y}^2)]}.$$

$\hat{u}_l < 0$ and $K < 0$ imply that $[2\hat{y}v(t_h)-[v(t_h)-1](1+\hat{y}^2)] > 0$ or $[\hat{y}^2v(t_h)-\hat{y}^2-2\hat{y}v(t_h)+v(t_h)-1] < 0$.

Therefore, $\hat{u}_h > \tilde{u}_h$.

We compare the ratio of u_h to u_l at a given $\hat{y}$ for the second-best case with the benchmark case. In the second-best case,

$$\frac{\hat{u}_h}{\hat{u}_l} = \frac{\hat{y}v(t_h) - v(t_h) + 1}{\hat{y}v(t_h)}.$$

Similarly, in the benchmark case,

$$\frac{\tilde{u}_h}{\tilde{u}_l} = \frac{\hat{y}v(t_h) - v(t_h) + 1}{\hat{y}v(t_h)}.$$

Since $\hat{u}_h > \tilde{u}_h$, $\frac{\hat{u}_h}{\hat{u}_l} = \frac{\tilde{u}_h}{\tilde{u}_l}$ implies that $\hat{u}_l > \tilde{u}_l$. Therefore, if $\hat{y} < \hat{y}_A$, for a given cutoff,

$$\begin{aligned}
\hat{u}_d &< \tilde{u}_d = K, \\
\hat{u}_h &> \tilde{u}_h, \\
\hat{u}_l &> \tilde{u}_l.
\end{aligned}$$

$$\begin{aligned}
\hat{s} &= -\frac{2K(v(t_h) - 1)}{v(2t_h)[\hat{y}^2 - \hat{y}^2 v(t_h) + 2\hat{y}v(t_h) - v(t_h) + 1]}. \\
\hat{s} - \tilde{s} &= \frac{2K\hat{y}(v(t_h) - 1)^2[\hat{y}^2 v(t_h) - 2\hat{y}v(t_h) - 2\hat{y} + v(t_h)]}{v(2t_h)[\hat{y}^2 - \hat{y}^2 v(t_h) + 2\hat{y}v(t_h) - v(t_h) + 1](1 - \hat{y})[\hat{y}v(t_h) + \hat{y} - v(t_h) + 1]}. \\
&< 0.
\end{aligned}$$

(11) **Proof of Proposition 4.3:**

1. If $\tilde{y}^* = \hat{y}_A$:

   From Proposition 4.2, at $\hat{y} = \hat{y}_A$, $\hat{u}_p = \tilde{u}_p$, $p = d, h, l$. If the optimal benchmark cutoff is $\hat{y}_A$ and in the general case, if the principal wishes the agent to select cutoff $\hat{y}_A$, Lemma 4.4 states that given the compensation contract in (4.4), the agent will find it optimal to select $\hat{y}_A$ as the cutoff. Therefore, if $\tilde{y} = \hat{y}_A$, then $\hat{y}^* = \tilde{y}^*$.

2. If $\tilde{y}^* < \hat{y}_A$:

   Let $\hat{EC}(\hat{y})[\tilde{EC}(\hat{y})]$ denote the expected compensation cost at $\hat{y}$ for the second-best [benchmark] case. Totally differentiating the expected compensation cost with respect to $\hat{y}$, we obtain the following:

$$\begin{aligned}
\hat{EC}' &= h(\hat{u}_d) - w_k + \hat{y}h'(\hat{u}_d) - h(\hat{u}_l) - \hat{y}[h(\hat{u}_h) - h(\hat{u}_l)] \\
&+ \frac{1}{2}(1-\hat{y}^2)h'(\hat{u}_h) + \frac{1}{2}(1-\hat{y})^2h'(\hat{u}_l). \\
\tilde{EC}' &= h(\tilde{u}_d) - w_k + \hat{y}h'(\tilde{u}_d) - h(\tilde{u}_l) - \hat{y}[h(\tilde{u}_h) - h(\tilde{u}_l)] \\
&+ \frac{1}{2}(1-\hat{y}^2)h'(\tilde{u}_h) + \frac{1}{2}(1-\hat{y})^2h'(\tilde{u}_l).
\end{aligned}$$

We prove that $\hat{EC}' < \tilde{EC}'$ for $0 < \hat{y} < \hat{y}_A$.

$$\begin{aligned}
\hat{EC}' - \tilde{EC}' &= h(\tilde{u}_l) - h(\tilde{u}_d) + \hat{y}h'(\hat{u}_d) + (1-\hat{y})[h'(\hat{u}_h) - h'(\tilde{u}_h)] \\
&= -\frac{1}{r}[A], \\
\text{where} \quad A &= \ln \frac{2\hat{y}(1-\hat{y}v(t_h)}{v(t_h)(1-\hat{y})(1-v(t_h)+\hat{y}v(t_h)+\hat{y})} \\
&- \frac{(v(t_h)-1)[\hat{y}^3(3v(2t_h)-1) - \hat{y}^2(7v(2t_h)+1) + \hat{y}(5v(2t_h)-3) - (v(2t_h)-1)]}{(1-v(t_h)+2\hat{y}v(t_h)-\hat{y}^2v(t_h)+\hat{y}^2)(1-v(t_h)+\hat{y}v(t_h)+\hat{y})(1-\hat{y}v(t_h))}.
\end{aligned}$$

For any $0 < \hat{y} < \hat{y}_A$, A reaches its minimum at $v(t_h) = 1$, with a value of zero. Thus, for $v(t_h) > 1$, $A > 0$. Therefore, we conclude that for $0 < \hat{y} < \hat{y}_A$, $\hat{EC}' < \tilde{EC}'$. At the optimal cutoff, $R' = EC'$. Since R' is negatively sloping, this implies that $\tilde{y}^* < \hat{y}^* < \hat{y}_A$. Proposition 4.1 establishes that $\eta < 0$ for $\hat{y} < \hat{y}_A$.

3. If $\tilde{y}^* > \hat{y}_A$:

   Proposition 4.2 states that

$$\begin{aligned}
\hat{u}_d &= \tilde{u}_d, \\
\hat{u}_h &> \tilde{u}_h, \\
\text{and } \hat{u}_l &< \tilde{u}_l.
\end{aligned}$$

There is more compensation risk in the second-best case and $\hat{EC} > \tilde{EC}$. From section 4.1.1, in the benchmark case, the spread between $\tilde{u}_h$ and $\tilde{u}_l$ decreases as $\hat{y}$ increases. From section 4.2.2, in the second-best case, the spread $(\hat{u}_h - \hat{u}_l)$ increases at an increasing rate as $\hat{y}$ increases. This implies that as $\hat{y}$ increases, the difference $(\hat{EC} - \tilde{EC})$ increases, thus $\hat{EC}' > \tilde{EC}'$. At the optimal cutoff, $R' = EC'$. Since R' is negatively sloping, this implies that $\tilde{y}^* > \hat{y}^* > \hat{y}_A$. Proposition 4.1 establishes that $\eta > 0$ for $\hat{y} > \hat{y}_A$.

(12) **Proof of Proposition 4.4:**

We consider the full principal's problem. Using a Lagrangian formulation, we write the problem as follows:

$$\begin{aligned}
L &= \hat{p}_{Bh}[B - h(u_d) + w_k] + \hat{p}_{Hh}[x_H - h(u_h)] + \hat{p}_{Lh}[x_L - h(u_l)] \\
&+ \lambda[\hat{p}_{Bh}u_d v(t_h) + \hat{p}_{Hh}u_h v(2t_h) + \hat{p}_{Lh}u_l v(2t_h) - K] \\
&+ \mu_1[u_d\{\hat{p}_{Bh}v(t_h) - 1\} + u_h\hat{p}_{Hh}v(2t_h) + u_l\hat{p}_{Lh}v(2t_h)] \\
&+ \mu_2[u_d - u_l] \\
&+ \eta[u_d - \hat{y}u_h v(t_h) - (1-\hat{y})u_l v(t_h)].
\end{aligned}$$

Differentiating the problem with respect to $\hat{y}$ and using the agent's first-order condition with respect to $\hat{y}$ (equation (4.3)), we obtain

$$B - h(u_d) + w_k - \hat{y}^*[x_H - h(u_h)] - (1-\hat{y}^*)[x_L - h(u_l)] - \eta v(t_h)[u_h - u_l] = 0.$$

If $u_h > u_l$, then

$$\text{Sign }(\eta) = \text{Sign }\{B - h(u_d) + w_k - \hat{y}^*[x_H - h(u_h)] - (1-\hat{y}^*)[x_L - h(u_l)]\}$$

For purpose of this proof, we redefine some variables:

$$\begin{aligned}
U(w_h)v(t_h) &= \bar{u}_h \\
\Rightarrow \quad w_h &= h(\bar{u}_h) + t_h.
\end{aligned}$$

$$\begin{aligned} U(w_l)v(t_h) &= \bar{u}_l \\ \Rightarrow \; w_l &= h(\bar{u}_l) + t_h. \end{aligned}$$

Then the agent's first-order condition on $\hat{y}$ is:

$$u_d = \hat{y}\bar{u}_h + (1-\hat{y})\bar{u}_l$$

1. If $\hat{y}^* \leq \hat{y}_A$:

   Rewriting the principal's first-order condition on $\hat{y}$, we have

   $$B - h(u_d) + w_k - \hat{y}^*[x_H - h(\bar{u}_h) - t_h] - (1-\hat{y}^*)[x_L - h(\bar{u}_l) - t_h] - \eta[\bar{u}_h - \bar{u}_l] = 0.$$

   It can be rewritten as follows:

   $$B + w_k + t_h - \hat{y}^* x_H - (1-\hat{y}^*)x_L = h(u_d) - \hat{y}^* h(\bar{u}_h) - (1-\hat{y}^*)h(\bar{u}_l) + \eta[\bar{u}_h - \bar{u}_l].$$

   Since $u_d = \hat{y}\bar{u}_h + (1-\hat{y})\bar{u}_l$, and the agent is risk averse, therefore, $h(u_d) < \hat{y}h(\bar{u}_h) + (1-\hat{y})h(\bar{u}_l)$. If $\hat{y}^* \leq \hat{y}_A$, we know from Proposition 4.1 that $\eta \leq 0$. With $\bar{u}_h > \bar{u}_l$, this implies that

   $$B + w_k + t_h - \hat{y}^* x_H - (1-\hat{y}^*)x_L < 0.$$

   Recall that in the first-best case, the principal's first-order condition with respect to the cutoff (equation (4.2)) is given by:

   $$B + w_k + t_h - y^* x_H - (1-y^*)x_L = 0.$$

   From section 4.2.1, the principal's objective function under first-best is strictly concave in $\hat{y}$ and is given by the following:

   $$\begin{aligned} EP &= \hat{y}[B - w_s] + \frac{1}{2}(1-\hat{y}^2)x_H + \frac{1}{2}(1-\hat{y})^2 x_L - (1-\hat{y})w_f. \\ &= \hat{y}[B - \bar{w} + w_k - t_h] + \frac{1}{2}(1-\hat{y}^2)x_H + \frac{1}{2}(1-\hat{y})^2 x_L - (1-\hat{y})(\bar{w} + 2t_h). \quad (4.12) \end{aligned}$$

The derivative of EP with respect to $\hat{y}$ is given by:

$$\begin{aligned} EP' &= B - \bar{w} + w_k - t_h - \hat{y}x_H - (1-\hat{y})x_L + \bar{w} + 2t_h \\ &= B + w_k + t_h - \hat{y}x_H - (1-\hat{y})x_L. \end{aligned} \tag{4.13}$$

We note that $EP' = 0$ at y^* and $EP' < 0$ at $\hat{y}^*$. This implies that for $\hat{y}^* \leq \hat{y}_A$, $\hat{y}^* > y^*$.

2. If $\hat{y}^* > \hat{y}_A$:

The expected compensation cost at $\hat{y}$ is:

$$\begin{aligned} EC(\hat{y}) &= \hat{y}[h(u_d) - w_k] + \frac{1}{2}(1-\hat{y}^2)[h(\bar{u}_h) + t_h] + \frac{1}{2}(1-\hat{y})^2[h(\bar{u}_l) + t_h] \\ &= \hat{y}[h(u_d) - w_k] + \frac{1}{2}(1-\hat{y}^2)h(\bar{u}_h) + \frac{1}{2}(1-\hat{y})^2 h(\bar{u}_l) + (1-\hat{y})t_h. \end{aligned}$$

Totally differentiating $EC(\hat{y})$ with respect to $\hat{y}$, we obtain the following:

$$\begin{aligned} EC' &= h(u_d) - \hat{y}h(\bar{u}_h) - (1-\hat{y})h(\bar{u}_l) - w_k - t_h \\ &+ \hat{y}h'(u_d) + \frac{1}{2}(1-\hat{y}^2)h'(\bar{u}_h) + \frac{1}{2}(1-\hat{y})^2 h'(\bar{u}_l). \end{aligned}$$

At the optimal cutoff, $R' = EC'$. Thus,

$$\begin{aligned} B + w_k + t_h - \hat{y}^* x_H - (1-\hat{y}^*)x_L &= h(u_d) - \hat{y}^* h(\bar{u}_h) - (1-\hat{y}^*)h(\bar{u}_l) \\ &+ \hat{y}^* h'(u_d) + \frac{1}{2}(1-\hat{y}^{*2})h'(\bar{u}_h) + \frac{1}{2}(1-\hat{y}^*)^2 h'(\bar{u}_l). \end{aligned} \tag{4.14}$$

For $\hat{y}^* > \hat{y}_A$, the optimal compensation contract (equation (4.5)) is:

$$\begin{aligned} u_d &= K, \\ \bar{u}_h &= \frac{K[2 - v(t_h)(1+\hat{y})]}{(1-\hat{y})v(t_h)}, \\ \text{and} \quad \bar{u}_l &= \frac{K[v(t_h)(1+\hat{y}^2) - 2\hat{y}]}{(1-\hat{y})^2 v(t_h)}. \end{aligned}$$

Also, $h(u_p) = -\frac{1}{r}\ln(-u_p)$.

Through substituition, differentiation and rearrangement of the terms, we obtain the following:

$$
\begin{aligned}
& h(u_d) - \hat{y}^* h(\bar{u}_h) - (1-\hat{y}^*)h(\bar{u}_l) + \frac{1}{2}(1-\hat{y}^{*2})h'(\bar{u}_h) + \frac{1}{2}(1-\hat{y}^*)^2 h'(\bar{u}_l) \\
= & \ \tfrac{1}{r} \ \{\frac{2(v(t_h)-1)^2(1+\hat{y}^*)}{[v(t_h)(1+\hat{y}^{*2})-2\hat{y}^*][2-v(t_h)(1+\hat{y}^*)]} - \ln\frac{(1-\hat{y}^*)^2 v(t_h)}{v(t_h)(1+\hat{y}^{*2})-2\hat{y}^*} \\
- & \ \hat{y}^* \ln\frac{v(t_h)(1+\hat{y}^{*2})-2\hat{y}^*}{[2-v(t_h)(1+\hat{y}^*)](1-\hat{y}^*)}\}. \qquad (4.15)
\end{aligned}
$$

The right-hand side (RHS) of equation (4.15) is exactly the same as that of equation (3.11) of Chapter 3. In appendix 3A(II)(4), we prove that for $1 < v(t_h) < 2$ and $0 < \hat{y} < \frac{2-v(t_h)}{v(t_h)}$, the RHS of (4.15) is strictly greater than zero. This implies that at the second-best optimal cutoff (see equation (4.14)),

$$B + w_k + t_h - \hat{y}^* x_H - (1-\hat{y}^*)x_L > 0.$$

From equation (4.2), at the first-best cutoff,

$$B + w_k + t_h - y^* x_H - (1-y^*)x_L = 0.$$

We note that $EP' = 0$ at y^* and $EP' > 0$ at $\hat{y}^*$. The principal's objective function is a strictly concave function of $\hat{y}$, so this implies that for $\hat{y}^* > \hat{y}_A$, $\hat{y}^* < y^*$.

(13) **Proof of Proposition 4.5:**

If a project is to be abandoned, there is no value to communication of the specific value of y. The severance payment cannot be made contingent on the agent's message.

Next, we prove that communication of the specific value of y given continuation has no value. Consider a menu of contracts $(u_d, \{u_h(y), u_l(y)\})$. At the cutoff $\hat{y}$, $(u_d, \{u_h(\hat{y}), u_l(\hat{y})\}) = (\hat{u}_d, \{\hat{u}_h, \hat{u}_l\})$. Note that no risky contract should strictly dominate another. This implies that if $(u_h(s), u_l(s))$ and $(u_h(t), u_l(t))$ are the risky contracts selected when $y = s$ and $y = t$ respectively, then $u_h(s) > u_h(t)$ if and only if $u_l(s) < u_l(t)$. Also, $u_h(y)$ is a weakly increasing function of y and $u_l(y)$ is a weakly decreasing function of y, i.e., as y increases, the relevant contract becomes more risky.

For a given $y > \hat{y}$, the principal seeks to find a contract $[u_h(y), u_l(y)]$ that minimizes the expected compensation cost. The problem can be represented as follows:

$$\begin{aligned}
\min \quad & yh[u_h(y)] + (1-y)h[u_l(y)] \\
\text{s.t.} \quad & \hat{y}\hat{u}_h + (1-\hat{y})\hat{u}_l \geq \hat{y}u_h(y) + (1-\hat{y})u_l(y), \\
& yu_h(y) + (1-y)u_l(y) \geq y\hat{u}_h + (1-y)\hat{u}_l.
\end{aligned}$$

We prove that the cost-minimizing contract is $u_h(y) = \hat{u}_h$ and $u_l(y) = \hat{u}_l$.

Suppose not. Then $u_h(y) > \hat{u}_h$ and $u_l(y) < \hat{u}_l$ and $yh[u_h(y)]+(1-y)h[u_l(y)] < yh[\hat{u}_h]+(1-y)h[\hat{u}_l]$. Define $\tau(y)$ and $\tau(y, \hat{y})$ as follows:

$$\begin{aligned}
U[yh[u_h(y)] + (1-y)h[u_l(y)] - \tau(y)] &= yu_h(y) + (1-y)u_l(y) \\
U[yh[\hat{u}_h] + (1-y)h[\hat{u}_l] - \tau(y,\hat{y})] &= y\hat{u}_h + (1-y)\hat{u}_l.
\end{aligned}$$

$\tau(y, \hat{y})$ represents the required risk premium when the agent observes $y > \hat{y}$ but chooses a less risky contract $\{\hat{u}_h, \hat{u}_l\}$. Therefore, $\tau(y) > \tau(y, \hat{y})$.

The expected compensation cost can be rewritten as follows:

$$\begin{aligned}
yh[u_h(y)] + (1-y)h[u_l(y)] &= h[yu_h(y) + (1-y)u_l(y)] + \tau(y) \\
yh[\hat{u}_h] + (1-y)h[\hat{u}_l] &= h[y\hat{u}_h + (1-y)\hat{u}_l] + \tau(y,\hat{y}).
\end{aligned}$$

$$\begin{aligned}
\text{If} \quad & yh[u_h(y)] + (1-y)h[u_l(y)] < yh[\hat{u}_h] + (1-y)h[\hat{u}_l] \\
\text{then} \quad & h[yu_h(y) + (1-y)u_l(y)] - h[y\hat{u}_h + (1-y)\hat{u}_l] + \tau(y) - \tau(y,\hat{y}) < 0.
\end{aligned}$$

$\tau(y) > \tau(y, \hat{y})$ implies that to satisfy the inequality,

$$\begin{aligned}
h[yu_h(y) + (1-y)u_l(y)] &< h[y\hat{u}_h + (1-y)\hat{u}_l] \\
\Rightarrow \; yu_h(y) + (1-y)u_l(y) &< y\hat{u}_h + (1-y)\hat{u}_l.
\end{aligned}$$

This violates the second truth-telling constraint. Therefore, we conclude that the prinicpal's cost minimizing contract for a given $y > \hat{y}$ is given by $u_h(y) = \hat{u}_h$ and $u_l(y) = \hat{u}_l$. The principal does not condition the contract on the message.

Appendix 4C

No Abandonment Provided

We consider the case when the principal does not provide for abandonment in the contract. In this instance, the receipt of private information by the agent at date $i = 1$ is not utilised. The principal wants to motivate the agent to take t_h in both periods.

$$\begin{aligned} Pr(x_H|t_{1h}, t_{2h}) &= \int_0^1 yf(y)dy \\ &= \frac{1}{2}. \\ Pr(x_L|t_{1h}, t_{2h}) &= \frac{1}{2}. \end{aligned}$$

If the agent takes t_l in either period, the project crashes with probability 1.

The principal's problem is given as follows:

$$\begin{array}{ll} \max_{u_h, u_l} & \frac{1}{2}[x_H - h(u_h)] + \frac{1}{2}[x_L - h(u_l)] \\ \text{s.t.} & \frac{1}{2}u_h v(2t_h) + \frac{1}{2}u_l v(2t_h) \geq K \\ & \frac{1}{2}u_h v(2t_h) + \frac{1}{2}u_l v(2t_h) \geq u_l. \end{array}$$

The first constraint is the principal's participation constraint. The second constraint is the incentive constraint and it ensures that the agent is weakly better off choosing $\{t_{1h}, t_{2h}\}$ than choosing t_l in either of the two periods. The second constraint can be rewritten as follows:

$$\frac{1}{2}u_h v(2t_h) = u_l[1 - \frac{1}{2}v(2t_h)].$$

Both u_h and u_l must be strictly negative, as the agent's utility function is negative exponential. This implies that for the above equality to hold, $[1 - \frac{1}{2}v(2t_h)] > 0 \Rightarrow v(t_h) < \sqrt{2}$. Thus, the principal will not take up the project if $v(t_h) \geq \sqrt{2}$, since the cost of motivating the agent to choose t_h is now extremely high.

Since both constraints are binding, we can solve explicitly for u_l and u_h.

$$\begin{aligned} u_l &= K, \\ u_h &= \frac{K[2 - v(2t_h)]}{v(2t_h)}. \end{aligned}$$

We note that $u_h > u_l$ as $\frac{2-v(2t_h)}{v(2t_h)} < 1$.

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Chapter 5

Effort Allocation and Job Design

5.1 Introduction

There are many tasks that have to be carried out in the running of an organization. These tasks normally include basic production activities, service activities, supervisory/training activities, and product and technology development activities. When the principal employs agents to help him manage the organization, it is usual that each agent is made responsible for more than one task. In a single task situation, the optimal contract is designed to motivate the agent to work hard. However, in a multitask situation, the optimal incentive contract is designed not only to motivate the agent to work hard, but also serves to direct the agent to devote an optimal amount of effort to each of these activities. Current research, in dealing mainly with single task situations, treats the problem of motivating current productive effort separately from that of motivating research/investment effort, for example. However, if the agent's disutility from the two effort types are not additively separable (i.e., interactions between the two effort types exist in the agent's disutility function), it is important that the two tasks be considered together in determining the optimal incentive contract to properly motivate the agent.[1] Holmstrom and Milgrom (1991) deal with a multitask situation. They comment that "when there are multiple tasks, incentive pay serves not only to allocate risks and to motivate hard work, it also serves to direct the allocation of the agents' attention among their various duties" (p.25).

However, the cost of motivating an agent to achieve a particular combination of effort levels in the

[1] Even if the agent's disutility from the two effort types are additively separable, the two must be considered jointly since the compensation is evaluated jointly.

multiple tasks for which he is responsible is determined, to a large extent, by the availability and precision of the performance measures of the agent's effort in each task. Holmstrom and Milgrom (1991) suggest that if one activity is impossible to observe and its outcome impossible to measure, and an agent controls multiple activities, then using incentive contracts based on the output of the measurable activities leads the agent to spend little or no time on the former activity. Thus, flat wage contracts are used to ensure that the agent works on the various activities. However, flat wage contracts imply that the agent puts in minimal effort in the activities. This does not sound appealing in a highly competitive environment in which product improvement and development are very important. The principal should instead consider the possibility of investing in a costly post-decision monitoring system to obtain performance measures of the agent's effort. If the benefits from higher effort levels outweigh the cost of obtaining and using the information, then the investment in monitoring is worthwhile. This part of the dissertation addresses the effort allocation issue in a multitask setting and examines how changes in the precision of the performance measures of the agent's effort in each task affects the optimal effort levels in the multiple tasks.

In a multitask setting, the agent's attitude towards performing a given set of tasks appears to influence the optimal effort levels given the precision of the performance measures of the agent's effort in each task. Since a different combination of tasks may have a different impact on the agent, one of the problems with which the principal needs to deal is how to efficiently group these tasks into individual jobs. In some cases, task grouping is straightforward due to skill requirements. In other situations, the principal has much more flexibility in the task assignment and the grouping of tasks. We ignore the skill requirement issue in subsequent discussion. Assuming that the prinicpal has perfect freedom in the grouping of the tasks, he seeks an optimal grouping and assignment of tasks to the agents. Behavioral scientists have often asserted the value of job design (job enlargement and job enrichment) in motivating employees. However, the agency theory literature has given only limited attention to job design. In fact, in most of the models examined, the agent's action space is single dimensional. In this part of the dissertation, while we do not directly model the job design problem, we believe the analysis of the effort allocation issue provides useful insights with respect to that problem.

5.2 Tasks with Long-term and Short-term Impact

In this section, we examine a setting in which the agent is responsible for both current operations and innovation activities, which differ in their impact on the firm's profit position. Incentive pay serves not only to motivate hard work, it also serves to direct the allocation of the agent's effort among these tasks. If the incentive plan uses current year's profit as the sole criterion for evaluation, this may result in the agent concentrating on tasks with a short-term impact, and foregoing projects that bring long-term benefits but hurt the short-term results. Rappaport (1982) suggests that one reason for the low research and capital spending in the United States leading to a slowdown in the long-term growth of the economy is that firms have been preoccupied with short-term results, and this is in part due to the poor design of the management incentive compensation plans. These plans compel the managers "to concentrate on short-run results and adopt policies that may discourage growth and acceptance of reasonable risk" (p. 370). Stock options and long-term contracts have been used to help correct the myopic tendency in agents. However, as Rich and Larson (1987) point out, since these plans pay out at the end of a four- to five-year period, while annual bonuses offer opportunities for substantial rewards in the near future, the agents are still motivated to direct more attention to annual performance goals as opposed to long-term goals. They prefer to take the cash and let the credit go.

Rappaport suggests three possible approaches that firms could take to better integrate management incentives and strategic planning. One of these approaches is termed a strategic factors approach. "This involves identification of the strategic factors governing future profitability, periodic measurement of the progress achieved in accomplishing each goal, and incorporation of the results in incentive packages" (p. 372). Some examples of such strategic factors are:

- target market share,
- productivity levels,
- product quality measures,
- product development measures, and
- personnel development measures.

A similar suggestion was made by Ira Kay (1991) in the article "Beyond Stock Options: Emerging Practices in Executive Incentive Programs". He suggests that shorter-term strategic circumstances or achievements are important since they serve as critical performance markers. Thus, they should be designed into the annual incentive plans. Kay gives the following examples of strategic mileposts:

- Progress or achievements in the research and development of new products;
- The development of proprietary/unique production methods;
- Improved employee productivity not attained through capital substitution;
- An improvement in the capability and potential of employees, particularly managers and middle-level people;
- Improved marketing methods resulting in greater market share; and
- The successful development of a plan, such as a strategic business plan.

An example of a firm that integrates management incentives with strategic planning is McDonald's. McDonald's identifies the following six areas as key success factors that affect long-run profits:

- Product quality
- Service

- Cleanliness
- Sales volume
- Personnel training
- Cost control

Accordingly, McDonald's assesses its store managers based on their performance in these areas. "Focusing on these key success factors, rather than short-run profits, identifies these factors as the key influences on long-run profitability" (Kaplan and Atkinson, 1989). The effectiveness of these incentive plans may be a contributing factor to the immense success that McDonald's enjoys worldwide. Similarly, General Electric uses multiple measures of divisional performance like market position, product leadership and personnel development. To be able to use these success factors as performance measures requires that the management accountants design and maintain appropriate information systems to reflect the necessary information. As Kim and Suh (1991) point out, different information systems may result in different optimal incentive plans and different optimal effort levels. They analyze the effect of different information systems on the corresponding expected minimum compensation costs in inducing a given effort level when the agent is responsible for only one task. They provide a ranking of the information system in inducing a given effort level when the agent has a square root utility function.

5.3 Objectives of the Model

In this part of the dissertation, we consider a two task situation, in a one-period setting. Both tasks should be undertaken for the well-being of the firm. The agent's attitude towards performing the set of tasks is captured in the agent's personal cost of effort function. We call this attitude the interactive effect on the agent's personal cost of effort function. It can either be negative, zero or positive. A negative value implies that the two effort levels are complementary in the agent's cost function, i.e., the marginal disutility of achieving one task decreases as the effort level in the other task increases. A zero value implies that the two effort levels are independent in the agent's cost function. A positive value implies that the two effort levels are substitutable in the agent's cost function, i.e., the marginal disutility of achieving one task increases as the effort level in the other task increases. The grouping of tasks determines the value of the interactive effect, and we examine how this effect, together with the incentive contracts, affect the cost to the principal of extracting high effort levels from the agent. We vary the precision of the performance measure in the second task and explore how this affects the task assignment and optimal effort levels. In the extreme case, we consider what happens if there is no costlessly available performance measure on this second task. The principal explores the option of investing in a costly monitoring technology to extract a signal that can be used in the compensation contract. The more precise the information to be extracted, the higher the monitoring cost, and we explore what factors determine the optimal monitoring level.

The analysis yields the following results.

1. The analysis emphasizes that when an agent is responsible for more than one task, incentive issues should not be addressed task by task. It is necessary that the principal studies the incentive problems for all the tasks together. Since the grouping of tasks affects the agent's personal cost of effort function, the principal chooses the grouping that affects the agent most favorably. The principal is then able to more efficiently motivate higher effort levels and achieve higher profitability.

2. A good job design and a well-designed incentive plan are both necessary to motivate the agent to exert the optimal effort levels at the minimum cost. A good incentive plan with poor job design either limits the ability of the principal to extract optimal effort levels from the agent or if the effort levels are achieveable, raises the compensation cost substantially. On the other hand, a good job design with a poorly-designed compensation plan result in suboptimal effort allocation.

3. The relative precision of the performance measures determines the cost to the principal of extracting

high effort levels from the multiple tasks. If the agent is responsible for both task 1 and task 2, the performance measure of task 2 is relatively very noisy compared to that of task 1, and the interactive effect on the agent's personal cost function is positive, there can arise cases when the principal is better off removing task 2 from the agent so that he concentrates on only task 1.

4. The principal should not just settle for costlessly available but highly noisy information since the use of such measures increases the cost to the principal of extracting high effort levels. Rather, he should investigate the potential benefits from investing in a costly monitoring technology to obtain more precise information before deciding on the basis for the incentive contracts.

5.4 Agency Literature Review

The following three classes of literature are relevant for this part of the dissertation:

- Multitask setting;
- Job design; and
- Investment in Monitoring Technology.

5.4.1 Multitask Literature

There are a number of articles in the literature examining different aspects of the multitask setting. Our approach is similar to the multitask model examined by Holmstrom and Milgrom (1991). They focus on the case where the agent's personal costs depend only on the total effort the agent devotes to all his tasks, i.e., all activities are equal substitutes in the agent's cost function. They conclude that the desirability of providing incentives for any one activity decreases with the difficulty of measuring performance in any other activities that make competing demands on the agent's attention. Incentives for a task can be rewarded in two ways: either the task itself is rewarded or the marginal opportunity cost for the task can be lowered by removing or reducing the incentives on competing tasks. Our analysis differs from theirs in that we allow for a range of interactive effects on the agent's cost of effort, i.e., the tasks may not be equal substitutes in the agent's cost function and they could even be complementary in the agent's cost function. This has an important effect on how jobs should be designed.

Feltham and Xie (1994) explore the economic impact of variations in performance measure congruence and the use of multiple measures to deal with both problems of goal congruence and the impact of uncontrollable events on performance measures. They assume that the effort levels are independent in the agent's cost function, i.e., the interactive effect on the agent's personal cost of effort function is zero. Their analysis shows that a contract based on a noncongruent measure induces suboptimal effort allocation across tasks, whereas noise in a performance measure results in suboptimal effort intensity. Our study differs from theirs in that we assume congruent performance measures and we focus on the impact of changes in the relative precision of the performance measures on the effort levels. We also examine the impact on the optimal effort levels of variations in the interactive effect on the agent's cost of effort function.

Bushman and Indjejikian (1993) study the use of both accounting earnings and stock price in compensation contracts for executives involved in two tasks. Their analysis focuses on the role of accounting earnings as the information content of earnings varies. In Paul (1991), the agent performs two tasks which he interprets as pertaining to short and long run cash flows. The agent's contract is a function of the stock price. The analysis shows that depending upon which type of information has the more pronounced effect on price, overemphasis on either short run or long run actions can occur. In our analysis, if the agent is responsible for two tasks pertaining to short and long run cash flows, we suggest that for the task affecting long run cash flow, a strategic milepost instead of stock price is used to motivate the agent to allocate effort to long-term projects. The strategic milepost may be costly to obtain and we examine the principal's decision to invest in obtaining the information.

5.4.2 Job Design

Itoh (1991) examines the factors that lead a principal to choose to induce workers to work separately on their tasks rather than to induce them to spend some effort helping others. The two determining factors are strategic interaction between agents and their attitudes towards performing multiple tasks. The latter factor is similar to our consideration of the interactive effect on the agent's personal cost of effort function. Itoh (1991) only allows for the interactive effect to be positive or zero. He obtains the result that the principal wants either an unambiguous division of labour or substantial teamwork.

Holmstrom and Milgrom (1991) also examine the issue of job design. They obtain the result that each task should be made the responsibility of just one agent, i.e., an unambiguous division of labour. This is because they assume that the agent's effort types are perfect substitutes in the agent's cost function so that the interactive effect is positive. In our analysis, we allow for the interactive effect to range from negative to positive and we examine the impact on job design and optimal effort levels.

While behavioral scientists assert that job enlargement and enrichment can motivate workers to work hard, our analysis attempts to indicate how the benefit is achieved. Job enlargement and enrichment may lead to a decrease in the value of the interactive effect on the agent's cost of effort function. This makes it less costly for the principal to motivate higher effort levels.

5.4.3 Investment in Monitoring Technology

Most of the studies dealing with the choice of monitoring systems assume a costless monitoring technology with an exogenously specified quality in a single-task setting. Shavell (1979) and Holmstrom (1979) show that an information system which reports both the output and an imperfect monitor of the agent's effort is more valuable than one which reports only the outcome if the monitor conveys information about effort not already conveyed by the output. This arises because a contract with improved motivational effects is achieved. In the studies that examine the choice of acquiring costly signals, most assume that the principal has in place a costless information system that reveals the production output. The principal's problem is then when to acquire the costly additional signal. Singh (1985) derives the amount of monitoring of the agent's effort endogenously in such a setting, and shows that if the marginal costs of gathering information are always positive, there is a minimal optimal level of monitoring by the principal.

Baiman and Rajan (1994) study the design of costly post-decision monitoring systems when there is no alternative costlessly available signal. The system reports either success or failure. The principal's monitoring decision is to choose the probability that the desired action generates the 'failure' signal. Our model assumes a very different monitoring technology. The principal's monitoring decision is to select the precision level of the signal, thus the signal is not dichotomous but is continuous. Also, we examine the principal's decision to invest in the monitoring technology in a multitask setting when effort allocation becomes an issue.

5.5 Conclusion

This part of the dissertation focusses on a two-task setting. We examine the role of an incentive plan for effort allocation, and consider the implications of variations in the interactive effect on the agent's personal cost of effort function. Our study also explores how changes in the relative precision of the performance measures affect the optimal combination of effort levels in the multiple tasks. Finally, we examine the principal's decision to invest in a costly monitoring technology when performance measures are not costlessly available or they are relatively very noisy.

In the next chapter, we look at a single-task principal-agent model. Costly monitoring is engaged and we examine the principal's selection of the optimal level of monitoring. This provides us with valuable insights when we examine the two-task model in chapter 7. When there is more than one task, effort allocation and job design become critical and we analyze the principal's monitoring decision in such a setting.

Chapter 6

Single-Task Principal-Agent Model with Costly Monitoring Technology

6.1 Introduction

In this chapter, we consider a single period principal-agent model, in which the agent is responsible for one task. The outcome of the task is not observable when the agent is paid for his effort. This situation arises, for example, if the agent is responsible for research and development projects or long-term investment projects, in which the outcome of these activities are not realized till some periods later. Thus, the principal and the agent cannot contract on the outcome of the task, since it is not observable when the agent is to be paid. To motivate an effort level higher than the minimal level, the principal employs a costly monitoring technology which provides information on the agent's effort. We examine the principal's monitoring decision. This provides us with useful insights when, in the next chapter, we analyze a two-task situation in which effort allocation becomes critical.

The next section presents the general model. In section 6.3, we use specific linear profit and quadratic cost functions to facilitate the derivation of closed form expressions for the incentive rate, the activity level and the monitoring level. We also examine how these optimal levels vary as the exogenous variables change. Proofs of the lemma and the propositions are provided in appendix 6A.

6.2 The Model

Let t be a measure of the activity or accomplishment level that the agent can choose with certainty for the task. The agent supplies t at a personal cost of $V(t)$, which we assume to be convex and increasing with t. The incremental profit (before any wage payment to the agent) is given by $\Pi(t)$, which accrues directly to the principal. $\Pi(t)$ is assumed to be weakly concave. No discounting is considered in the model.

The principal is risk neutral. The agent is risk averse, and has exponential utility $u_m(z) = -\exp(-rz)$, where r is the agent's coefficient of absolute risk aversion and z is the agent's net income, which consists of his wage ω minus the personal cost of effort $V(t)$, i.e. $z = \omega - V(t)$. Thus, we assume that $V(t)$ is expressed in dollar terms.

In the first-best setting in which the effort of the agent is observable and the agent could be severely penalized if he does not provide the required level of t, the principal uses a flat wage contract. We assume that the certainty equivalent of the agent's reservation net income level is zero. The interior solution is characterized by the following:

$$\begin{aligned} \Pi'(t) &= V'(t) \\ w &= V(t). \end{aligned}$$

In the second-best setting, without any signal about the agent's effort, if the principal uses a flat wage contract, the agent puts in the minimal level of effort, which we assume to be zero. The expected profit would also be zero.

Now, we assume that a costly monitoring technology is available and it provides a noisy signal of the agent's effort. This signal could then be used for compensation purposes. Our model is a special case of the model used by Huddart (1993). We work with a risk neutral principal who holds all the shares of the firm. We assume that the signal obtained from the monitoring technology is related to the agent's effort and the level of monitoring in the following manner:

$$y = t + \frac{1}{\sqrt{h}}\theta, \;\; \theta \sim N(0,1).$$

The cost of the signal depends on h, $C(h)$, where h equals the precision of the monitoring technology.

Assume that $C(\cdot)$ is continuous and monotonically increasing, and that

$$\lim_{h\to\infty} C(h) = \infty \text{ and } \lim_{h\to 0} C(h) = 0.$$

The higher the level of h, the more precise the signal and the more costly the monitoring. The cost of a perfect signal is infinite, while there is no cost if the principal decides not to use the costly monitor. We also assume that the level of h is observable and verifiable, thus it is contractible. For example, h may be related to the number of auditor or computer hours devoted to retrieving the data.

With the signal from the monitor, the principal can now base the wage contract on the signal. We restrict our analysis to the use of linear wage contracts, which take the following form, $w(y) = ay + b$. The manager's problem is

$$\max_t \; Eu_m[w(y) - V(t)].$$

Since his utility function is exponential and all random variables are normally distributed, maximizing the expected utility is equivalent to maximizing the certainty equivalent, which is given by:

$$CE_m(a, b, t) = at + b - V(t) - \frac{1}{2} r \frac{a^2}{h}.$$

That is, the agent's certainty equivalent consists of the expected wage less the personal cost of effort and less a risk premium.

The principal's objective is to maximize his expected profit, subject to the agent's participation and incentive compatibility constraints.

$$\begin{array}{cc} \max\limits_{w(\cdot),h,t} & \Pi(t) - C(h) - E[w(y)] \\ \text{s.t.} & CE_m(a,b,t) \geq 0 \\ \text{and} & t \in \arg\max_{t'} CE_m(a,b,t'). \end{array}$$

Under the linear wage scheme, the certainty equivalent of the principal is given by:

$$CE_p(a, b, d, t) = \Pi(t) - C(h) - (at + b).$$

The total certainty equivalent of the principal and the agent is then given by:

$$CE_p + CE_m = \Pi(t) - C(h) - V(t) - \frac{1}{2}r\frac{a^2}{h}.$$

Note that the total certainty equivalent is independent of the intercept component of the wage contract, b. It serves only to allocate the total certainty equivalent between the two parties, such that the agent's reservation utility level is reached. As explained in Holmstrom and Milgrom (1991), this implies that the incentive-efficient linear contracts are those that maximize the total certainty equivalent subject to the incentive compatibility constraints. Therefore the problem reduces to the following:

$$\begin{aligned} \max_{a,h,t} \quad & \Pi(t) - C(h) - V(t) - \tfrac{1}{2}r\tfrac{a^2}{h} \\ \text{s.t.} \quad & t \in \arg\max_{t'} \; at' - V(t'). \end{aligned}$$

From the constraint, we know that $a = V'(t)$, which can be substituted into the objective function to obtain the following:

$$\max_{h,t} \; \Pi(t) - C(h) - V(t) - \frac{1}{2}r\frac{[V'(t)]^2}{h}.$$

Using the first-order conditions on h and t, we characterize the optimal level of effort t and the optimal level of monitoring h at the optimal t.

$$V'(t) = \frac{\Pi'(t)h}{h + rV''(t)} \tag{6.1}$$

$$C'(h) = \frac{r}{2}[\frac{\Pi'(t)}{h + rV''(t)}]^2. \tag{6.2}$$

6.3 Quadratic Cost Setting

In this section, we analyze in greater depth settings in which the cost function is quadratic and the profit function is linear. Let $\Pi(t) = t$, $V(t) = \frac{1}{2}\delta t^2$ and $C(h) = ch$, $c > 0$. Then $\Pi'(t) = 1$, $V'(t) = \delta t$, $V''(t) = \delta$ and $C'(h) = c$.

6.3.1 First-best Setting

In the first-best setting, the optimal effort level and the optimal wage contract are as follows:

$$\begin{aligned} t^* &= \frac{1}{\delta} \\ w^* &= \frac{1}{2\delta}. \end{aligned}$$

The total certainty equivalent is given by:

$$\begin{aligned} CE(FB) &= \Pi(t^*) - V(t^*) \\ &= \frac{1}{2\delta}. \end{aligned}$$

6.3.2 Use of Monitoring Technology

Using (6.1) and (6.2), we characterize the optimal levels of effort t and monitoring h as follows:

$$V'(t) = \delta t = \frac{h}{h + r\delta} \tag{6.3}$$

$$C'(h) = c = \frac{r}{2(h + r\delta)^2}. \tag{6.4}$$

The principal monitors the agent only when the benefit exceeds the cost of monitoring. Lemma 6.1 establishes when monitoring is worthwhile for the principal.

<u>Lemma 6.1:</u> Necessary and sufficient condition for the principal to engage in monitoring is that $c < \frac{1}{2r\delta^2}$.

As the risk aversion of the agent increases, a higher risk premium is required when imperfect information is used. As the agent's private cost of effort increases, the agent requires more compensation for taking effort. Thus, the bound on the cost of monitoring tightens as r increases or as δ increases.

Proposition 6.1: When $c < \frac{1}{2r\delta^2}$, the principal engages in monitoring and the optimal level of monitoring, $\hat{h}$, the optimal commision rate, $\hat{a}$ and the optimal effort level, $\hat{t}$ are:

$$\begin{aligned}
\hat{h} &= \sqrt{(\frac{r}{2c})} - r\delta \\
&= \sqrt{(\frac{r}{2c})}[1 - \delta\sqrt{(2rc)}], && (6.5) \\
\hat{a} &= 1 - \delta\sqrt{(2rc)}, && (6.6) \\
\hat{t} &= \frac{1}{\delta} - \sqrt{(2rc)}. && (6.7)
\end{aligned}$$

The deviation from first-best effort level is $\sqrt{(2rc)}$. First-best effort intensity is achieved if the agent is risk neutral or the monitoring technology is costless, i.e., if $rc = 0$. Costless monitoring technology implies that the intensity of monitoring is infinite, so that perfect information is obtained.

6.3.3 Comparative Statics

Next, we examine how the the optimal monitoring level, incentive rate and the effort level vary with the various parameters.

Proposition 6.2:

1. An increase in the cost of monitoring (c) results in reduced monitoring ($\hat{h}$), reduced incentive rate ($\hat{a}$), and reduced effort level ($\hat{t}$).
2. (a) An increase in the agent's risk aversion (r) results in reduced incentive rate ($\hat{a}$), and reduced effort level ($\hat{t}$).

   (b) Monitoring level is concave in r and is most intense at $r = \frac{1}{8c\delta^2}$.

To provide incentive for the agent to work at a level higher than the minimal level, the principal imposes risk on the agent who is then compensated for bearing this risk in the form of a risk premium. When r is low, the risk premium required by the agent for a given effort level is not high, the principal settles for noisy information and he uses a less intense level of monitoring. The expression for $\hat{h}$ shows that as r approaches zero, the level of $\hat{h}$ approaches zero. For high levels of r, the principal uses weak incentives to reduce the risk imposed on the agent, he settles for a low effort level, and we expect the intensity of monitoring to be reduced. Huddart (1993) states that "monitoring is valuable only when coupled with an incentive scheme responsive to the signals generated". For very risk averse agents, the principal settles for low monitoring, low incentives, and low output.

Appendix 6A

(1) Derivation of (6.1) and (6.2)

Substituting $a = V'(t)$ into the objective function, we obtain the following:

$$\max_{h,t} \Pi(t) - C(h) - V(t) - \frac{r}{2h}[V'(t)]^2$$

We have the following first-order conditions on t and h:

$$\begin{aligned} \Pi'(t) - V'(t) - \frac{r}{h}V'(t)V''(t) &= 0 \\ -C'(h) + \frac{r}{2h^2}[V'(t)]^2 &= 0 \end{aligned}$$

Thus, the optimal level of t is characterized by:

$$V'(t) = \frac{\Pi'(t)h}{h + rV''(t)}.$$

The optimal level of h is characterized by:

$$C'(h) = \frac{r}{2h^2}[V'(t)]^2$$

At the optimal level of t, we obtain the following characterization of the optimal h:

$$C'(h) = \frac{r}{2}[\frac{\Pi'(t)}{h + rV''(t)}]^2.$$

(2) Proof of Lemma 6.1

When no monitoring is undertaken, $h = 0$, $a = 0$ and $t = 0$ and the total certainty equivalent $= 0$. With positive level of monitoring, the total certainty equivalent is:

$$\begin{aligned} CE &= \Pi(t) - V(t) - \frac{1}{2}r\frac{[V'(t)]^2}{h} - C(h) \\ &= t - \frac{1}{2}\delta t^2 - \frac{r}{2h}[\frac{h}{h + r\delta}]^2 - ch. \end{aligned}$$

From (6.3), $t = \frac{h}{\delta(h+r\delta)}$. By substituting for t and simplifying, we obtain the following:

$$CE = \frac{h}{2\delta(r\delta + h)} - ch.$$

Monitoring is strictly worthwhile if, and only if, $CE > 0$ for some $h \in (0, \infty)$, which implies that

$$\begin{aligned} \frac{h}{2\delta(r\delta + h)} - ch &> 0 \\ \Rightarrow h[\frac{1}{2\delta(r\delta + h)} - c] &> 0. \end{aligned}$$

Hence, if $c < \frac{1}{2r\delta^2 + 2\delta h}$, the certainty equivalent is srictly greater than 0 if monitoring is undertaken. Therefore, $h > 0$ and $c < \frac{1}{2r\delta^2 + 2\delta h}$ imply that $c < \frac{1}{2r\delta^2}$. From (6.5), the optimal level of h is $h = \surd(\frac{r}{2c})[1 - \delta\surd(2rc)]$. Thus, $h > 0$ implies that $c < \frac{1}{2r\delta^2}$.

(3) **Proof of Proposition 6.1**

To obtain the optimal level of h, we use (6.4).

$$\begin{aligned} c &= \frac{r}{2(h + r\delta)^2}. \\ \text{Hence, we obtain } h &= \surd(\frac{r}{2c}) - r\delta. \end{aligned}$$

To obtain the optimal level of t, we use (6.3).

$$t = \frac{h}{\delta(h + r\delta)}.$$

We substitute for the optimal level of h derived above as follows:

$$\begin{aligned} t &= \frac{\surd(\frac{r}{2c}) - r\delta}{\delta(\surd(\frac{r}{2c}) - r\delta + r\delta)} \\ &= \frac{1}{\delta} - \surd(2rc). \end{aligned}$$

The optimal level of a is characterized by $a = V'(t)$. Hence, we obtain:

$$\begin{aligned} a &= \delta t \\ &= \delta[\frac{1}{\delta} - \surd(2rc)] \\ &= 1 - \delta\surd(2rc). \end{aligned}$$

(4) **Proof of Proposition 6.2**

By differentiating (6.5), (6.6) and (6.7) of Proposition 6.1, the results in parts (1), (2) and (3a) are obvious.

To prove (3b):

$$\begin{aligned} h &= \sqrt{(\frac{r}{2c})} - r\delta. \\ \frac{dh}{dr} &= \frac{1}{2\sqrt{(2cr)}} - \delta. \\ \frac{d^2h}{dr^2} &< 0. \end{aligned}$$

Thus, h is at its maximum when

$$\begin{aligned} \frac{1}{2\sqrt{(2cr)}} &= \delta \\ \Rightarrow \quad r &= \frac{1}{8c\delta^2}. \end{aligned}$$

Chapter 7

Multitask Principal-Agent Model with Costly Monitoring Technology

7.1 Introduction

In this chapter, we consider a single period principal-agent model. The principal owns the firm and thus owns the outcome of all tasks undertaken for the firm. There are two tasks which the agent is employed to perform. In such a setting, the principal is not only concerned with motivating hard work, he is also concerned with directing the agent's attention between the two tasks. We assume that the post-action value of the firm is not observable prior to the termination of the agent's contract. Incentive contracts are based on imperfect performance measures associated with each task undertaken by the agent. For one of the tasks, we let the precision of the performance measure vary. At the extreme, we assume that the performance measure is not costlessly available and the principal considers investing in a costly monitoring technology. We examine the principal's monitoring decision in such a setting when effort allocation is critical, and compare the results with those in a one-task setting (which we analyze in Chapter 6) when effort allocation is not an issue.

In the next two sections, we present and analyze the general model. In section 7.4, we use specific linear profit and quadratic cost functions to facilitate the derivation of closed form expressions for the incentive rates, the activity levels, and monitoring level. We focus on the use and the value of the costly monitoring technology when the performance measure of one of the tasks is not costlessly available. We also examine how the optimal incentive rates, activity and monitoring levels vary as the exogenous variables change. In section 7.5, we consider some implications of the results for job design and organization structure. Proofs of the lemmas and the propositions are provided in appendix 7A.

7.2 The Model

7.2.1 General Characteristics

Let t_i, $i = 1, 2$ be a measure of the activity or accomplishment level that the agent can choose with certainty for task i.[1] The agent makes a one-time choice of a vector of activity levels $t = (t_1, t_2)$ at a personal cost of $V(t)$, which we assume to be increasing with t_i and convex, that is, $V_i(t) > 0$, $V_{ii}(t) > 0$ and $V_{11}V_{22} - V_{12}^2 > 0$. The subscripts on $V(t)$ refer to the first and second partial derivatives with respect to t_i. For both tasks, a public signal on the agent's activity level y_i is observed at the end of the period, and is related to the agent's activity level in the following manner:

$$y_i = t_i + \theta_i, \quad \theta_i \sim N(0, h_i), \; i = 1, 2, \tag{7.1}$$

where h_i is the precision (i.e., the inverse of variance).[2] We assume that the agent's activity level t_i does not influence the precision of the signal, h_i, and that θ_1 is independent of θ_2. The principal could contract with the agent based on y_1 and y_2, since they are observable and contractible.[3]

The principal is risk neutral. The agent is risk averse, and has exponential utility $u_m(z) = -\exp(-rz)$, where r is the agent's coefficient of absolute risk aversion and z is the agent's final income, which consists of the realized wage ω minus the personal cost of effort incurred to achieve the activity level, $V(t)$, i.e. $z = \omega - V(t)$. Thus, we assume that $V(t)$ can be expressed in dollar terms. Assume that the principal uses a compensation contract linear in y_i, $i = 1, 2$.

The timing of the game is as follows:

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[1] This activity level may be a transformed measure. For example, we may transform the number of hours worked by the agent into t_i which may represent the degree of completion of the task or the expected outcome. Thus, t_i may be interpreted as an output measure.

[2] See appendix 7B for the model's application to more general expressions of signals.

[3] y_i may be a financial measure which provides noisy information about the agent's choice of activity level. For example, y_1 may represent reported accounting income. As a result of uncontrollable events influencing sales demand and prices as well as input prices, accounting income provides a noisy measure of the activity level of the agent. Also, accounting adjustments and provisions may cause reported income to be a noisy measure of the agent's effort.

1. The principal offers the agent a wage contract;
2. The agent selects his activity levels, $t = (t_1, t_2)$;
3. y_1 and y_2 are observed publicly;
4. The agent is paid.

For subsequent analyses in this section, we consider two different cases:

- Case 1: Interior solution with $(t_1, t_2) >> 0$.
- Case 2: Corner solution with either t_1 or $t_2 = 0$.

7.2.2 Interdependency in either $V(t)$ or $\Pi(t)$

We assume that there is some form of interdependency of the two activity levels in either $V(t)$ or $\Pi(t)$. This is essential, otherwise the problem reduces to two single-task principal-agent problems. The interdependency in $V(t)$ is characterized by V_{ij}, $i, j = 1, 2$, and $i \neq j$. V_{ij} could be either negative, positive or 0. A negative V_{ij} means that the two activity levels are complementary in the agent's private cost function, i.e. the marginal disutility of achieving task i decreases as the activity level in task j increases. This means that achieving a higher level on one task makes achieving a higher level on the other task less costly (painful). Perhaps this could be due to "learning" or due to "variety" in the sense that "a break is as good as a rest". If V_{ij} is positive, the two activity levels are substitutable in the agent's cost function, i.e. the marginal disutility of achieving task i increases as the activity level in task j increases. Here, the tasks are rather similar and they make competing demands on the agent, thus, the agent is only concerned about the total activity level. When $V_{ij} = 0$, the two activity levels are independent and the marginal disutility of achieving task i is not affected by the activity levels in task j. In this case, the tasks may be of very different types and the agent has task specific disutility for each task.

To illustrate, we consider a two-product firm. For each product, both the marketing and the after-sales/customer service aspects are critical for success. The firm employs two agents and considers the following options:

- The firm may be organized by product line and each agent is put in charge of a product. He is responsible for both the marketing and the after-sales service of the product.
- The firm could be organized by functions, i.e., each agent is put in charge of a particular function.

In the first option, the agent is responsible for both the marketing and the after-sales service of a product to a particular set of customers. Such organization is generally termed divisionalization. There is task variety which may add to the job satisfaction of the agent so that his effort levels are complementary in the agent's cost of effort function, i.e., V_{ij} is negative. In the second option, each agent is responsible for either the marketing or the after-sales service of both products. This option is generally termed a functional structure. Since the tasks are similar, they make competing demands on each of the agent. This is likely to result in each agent's effort level being substitutes in the agent's cost of effort function and V_{ij} is positive.

Interdependency in Π is characterized by Π_{ij}, $i, j = 1, 2$ and $i \neq j$. Π_{ij} could also be negative, positive or zero. A negative Π_{ij} implies that the marginal profit from task i decreases as the activity level in task j increases.[4] A positive Π_{ij} implies that the marginal profit from task i increases as the activity level in task j increases. This could be due to the presence of an indivisible input which is shared between the two activities.[5] A zero Π_{ij} means that the marginal profit of task i does not depend on the activity level of task j. Here, the production technology for the two activities and the markets for the resulting products may be very different, thus the two activities are independent in their impact on the expected profit.

[4] If so, there is no reason to produce the two products/services together, unless it is legally required.

[5] For example, $\Pi = R_1 + R_2 - I$, where I is the cost of the input. If $I_{ij} < 0$, then $\Pi_{ij} > 0$.

7.2.3 First-Best Setting

We first characterize the first-best situation: the activity levels of the agent in the two tasks are observable. We assume that the agent could be severely penalized if he does not provide the required level of t_1 and t_2. The wage contract is a constant, w, since there is no incentive problem. The principal's objective is to maximize his profit subject to the agent's participation constraint. We assume that the certainty equivalent of the agent's reservation net income level is zero. The first-best problem can be stated as follows:

$$\begin{array}{ll} \max\limits_{w,t} & \Pi(t) - w \\ \text{s.t.} & u_m[w - V(t)] \geq u_m(0). \end{array}$$

<u>Case 1</u>

Assuming an interior solution, it is characterized by the following:

$$\begin{aligned} w &= V(t) \\ \Pi_i &= V_i(t), \quad i = 1, 2, \end{aligned}$$

where the subscripts on Π and V denote the partial derivatives with respect to t_i. Notice that any complementarities in the principal's profit function and the agent's cost function do not enter directly into the determination of the optimal effort level.[6]

<u>Case 2</u>

When a corner solution applies, the principal sets the appropriate t_i to be zero, and solves the problem for a single-task situation.

[6] The second order conditions for a maximum require that the following conditions are satisfied.

1.
$$\Pi_{ii} - V_{ii} \leq 0, \quad i = 1, 2.$$

2.
$$(\Pi_{ii} - V_{ii})(\Pi_{jj} - V_{jj}) - (\Pi_{ij} - V_{ij})^2 \geq 0, \quad i, j = 1, 2.$$

7.3 Analysis of the Second-Best Setting

7.3.1 General Solution for Costless Performance Measures

In the second-best situation, the agent's activity levels are noncontractible. The principal utilizes two costless signals, which are observable in this current period, to provide the necessary incentives for the agent. The wage contract is assumed to be linear in y_1 and y_2 and is given by $w(y_1, y_2) = a_1y_1 + a_2y_2 + b$.

The manager's problem is

$$\max_{t} Eu_m[w(y_1, y_2) - V(t)].$$

Since his utility function is exponential and all random variables are normally distributed, maximizing the expected utility is equivalent to maximizing the certainty equivalent, which is given by:

$$CE_m(a_1, a_2, b, t) = a_1t_1 + a_2t_2 + b - V(t) - \frac{1}{2}r[\frac{a_1^2}{h_1} + \frac{a_2^2}{h_2}].$$

That is, the agent's certainty equivalent consists of the expected wage less the personal cost of effort and less a risk premium.

The principal's objective is to maximize his expected profit, subject to the agent's participation and incentive compatibility constraints.

$$\begin{array}{ll} \max\limits_{w(\cdot,\cdot),t} & \Pi(t) - E[w(y_1, y_2)] \\ \text{s.t.} & CE_m(a_1, a_2, b, t) \geq 0 \\ \text{and} & t \in \arg\max_{t'} CE_m(a_1, a_2, b, t'). \end{array}$$

Under the linear wage scheme, the certainty equivalent of the principal is given by:

$$CE_p(a_1, a_2, b, t) = \Pi(t) - (a_1t_1 + a_2t_2 + b).$$

The total certainty equivalent of the principal and the agent is then given by:

$$CE_p + CE_m = \Pi(t) - V(t) - \frac{1}{2}r[\frac{a_1^2}{h_1} + \frac{a_2^2}{h_2}].$$

Note that the total certainty equivalent is independent of the intercept component of the wage contract, b. It serves only to allocate the total certainty equivalent between the two parties, such that the agent's reservation utility level is reached. As explained in Holmstrom and Milgrom (1991), this implies that the incentive-efficient linear contracts are those that maximize the total certainty equivalent subject to the incentive compatibility constraints. Therefore the problem reduces to the following:

$$[\text{P7.1}] \quad \max_{a_1,a_2,t} \quad \Pi(t) - V(t) - \tfrac{1}{2}r[\tfrac{a_1^2}{h_1} + \tfrac{a_2^2}{h_2}]$$
$$\text{s.t.} \quad t \in \arg\max_{t'} \; a_1 t_1' + a_2 t_2' - V(t').$$

<u>Case 1</u>

If both t_1 and t_2 are strictly positive, the incentive compatibility constraint is:

$$a_i = V_i(t), \quad i = 1,2. \tag{7.2}$$

The solutions of a_1 and a_2 are then given by:[7]

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \left[I_2 + r \begin{bmatrix} V_{11} & V_{21} \\ V_{12} & V_{22} \end{bmatrix} \begin{bmatrix} \frac{1}{h_1} & 0 \\ 0 & \frac{1}{h_2} \end{bmatrix} \right]^{-1} \begin{bmatrix} \Pi_1 \\ \Pi_2 \end{bmatrix}. \tag{7.3}$$

Note that Π_i and V_{ij} are functions of t, but to simplify the notation, (t) is dropped from $\Pi_i(t)$ and $V_{ij}(t)$. Expression (7.3) can be rewritten as follows:

$$a_i = \frac{h_i[\Pi_i(rV_{jj} + h_j) - \Pi_j r V_{ij}]}{(rV_{ii} + h_i)(rV_{jj} + h_j) - r^2 V_{ij}^2}, \quad i,j = 1,2 \text{ and } i \neq j. \tag{7.4}$$

Notice that if the activities are technologically independent, i.e. $V_{ij} = 0$, $i,j = 1,2$ and $i \neq j$, then $a_i = \Pi_i h_i [rV_{ii} + h_i]^{-1}$, $i = 1,2$. In the model, the error terms have been assumed to be stochastically independent, i.e. θ_1 is assumed to be independent of θ_2. The rates a_1 and a_2 are set independently of each other, except for the fact that Π_i may depend on the optimal level of the other task. Observe that the smaller is h_i, i.e., the greater the noise in the performance measure, the smaller is the rate a_i and

[7] See appendix 7A for derivation of (7.3).

the lower is the effort exerted on task i.

Case 2

If the optimal solution entails that either t_1 or t_2 is set to zero, then the principal sets the appropriate t_i to be zero, and solves the problem for a single-task situation.

7.3.2 Costless Performance Measure Available for Only Task 1

We now consider the situation where the precision of the signal on the agent's activity level for the second task, h_2, is assumed to be zero. This implies that a costless, independent, noisy performance measure is available for only the first task. If an incentive contract based on the signal for the agent's activity level in the first task is used, the agent may be motivated to work only on the first task and neglect the second task. If a flat wage contract is used, the agent chooses the minimal activity levels for both tasks. Without loss of generality, we assume that this minimal activity level is zero. The incremental expected profit from the two tasks (before any wage payment to the agent) is given by $\Pi(t_1, t_2) = \Pi(t)$, which accrues directly to the principal. $\Pi(t)$ is assumed to be weakly concave. No discounting is considered in this model. We assume that the principal and the agent cannot contract based on $\Pi(t)$ because it is not observable and verifiable prior to the termination of the agent's contract.

Case 1

Assuming an interior solution, we apply (7.4) which gives the formula for the incentive rates when noisy performance measures are available for both tasks. By setting $h_2 = 0$ and simplifying, we obtain:[8]

$$\begin{aligned} a_1 &= \frac{h_1(\Pi_1 - \Pi_2 \frac{V_{12}}{V_{22}})}{h_1 + r(V_{11} - \frac{V_{12}^2}{V_{22}})} \\ a_2 &= 0. \end{aligned} \tag{7.5}$$

The agent's choice of the optimal effort level is characterized by $a_i = V_i(t),\ i = 1, 2$. Notice that the denominator of a_1 is strictly greater than zero.[9] To ensure that $a_1 > 0$, some restrictions apply to

[8] See appendix 7A for the details.

[9] Convexity of the cost of effort function implies that $V_{11}V_{22} - V_{12}^2 > 0$.

the ratio of the marginal profits, namely, $\frac{\Pi_1}{\Pi_2} > \frac{V_{12}}{V_{22}}$. It is then logical to ask why the agent should be motivated to work on task 2, when he is not explicitly rewarded for his effort.

If the two activity levels are complementary in the agent's private cost function, that is, $V_{12} < 0$, then a_1 is strictly greater than 0, and the more negative is V_{12}, the higher is a_1. A negative V_{12} implies that the marginal disutility of achieving task 1 decreases as the activity level in task j increases. Although the agent is paid on only the outcome of task 1, he can decrease the marginal disutility of achieving task 1 by working on task 2. Thus, there is incentive for the agent to work on task 2, so as to incur less "cost" on task 1.

Case 2

If the two activity levels are substitutes in the agent's cost function, i.e. $V_{12} > 0$, there is no incentive for the agent to work on task 2 when the compensation contract takes the form of $w(y_1) = a_1 y_1 + b$. Given such a contract, and assuming an interior solution, the agent's reduced incentive constraints become $V_1(t) = a_1$ and $V_2(t) = 0$. However, for $a_1 > 0$, this implies that t_1 and t_2 must have opposite signs to satisfy the two reduced incentive constraints, i.e. if $t_1 > 0$, then $t_2 < 0$, and vice versa. This solution is not feasible and the principal should reconsider his problem by setting $t_2 = 0$ and solving the problem as in a single-task situation.

$$[\text{P7.2}] \quad \max_{a_1, t_1} \; \Pi(t_1, 0) - V(t_1, 0) - \frac{1}{2} r [\frac{a_1^2}{h_1}]$$

$$\text{s.t. } a_1 = V_1(t_1, 0).$$

The solution of a_1 is given by

$$a_1 = \frac{\Pi_1 h_1}{h_1 + r V_{11}}. \tag{7.6}$$

In section 7.4, we analyze in more depth a quadratic cost setting which further clarifies the explanations given above.

7.3.3 A Costly Monitoring Technology

In this section, we investigate the principal's decision to invest in a costly monitoring technology, given that no costless signal on the agent's activity level in the second task is available. The monitor provides a noisy signal of the agent's activity level for the second task. This signal could then be used for compensation purposes. We could interpret this costly signal to include nonfinancial measures, for example, on-time delivery performance, response time to customers' requests, and defect rates detected on shipped products and during manufacturing. These measures are not readily available in the accounting records, thus extra cost must be incurred to extract this information. The signal may also include a consultant's report. The principal chooses the monitoring intensity to obtain the desired precision on this costly signal. The higher the desired precision, the greater the monitoring intensity and the higher the cost. Assume that the costly signal obtained is related to the agent's activity level and the monitoring intensity in the following way:

$$y_2 = t_2 + \frac{1}{\sqrt{h_2}}\theta_2, \;\; \theta_2 \sim N(0,1).$$

Assume that θ_1 and θ_2 are independent of one another, that is, the error terms are stochastically independent. The cost of the signal depends on the level of h_2 and we denote the cost by $C(h_2)$. Assume that $C(\cdot)$ is continuous and monotonically increasing, and that

$$\lim_{h_2 \to \infty} C(h_2) = \infty \text{ and } \lim_{h_2 \to 0} C(h_2) = 0.$$

The higher the level of h_2, the more precise the signal and the more costly the monitoring. The cost of a perfect signal is infinite, while there is no cost if the principal decides not to use the costly monitor. We also assume that the level of h_2 is observable and verifiable, thus it is contractible. For example, h_2 may be related to the number of auditor or computer hours devoted to retrieving the data.

We assume an interior solution for the optimal level of monitoring. With the signal from the monitor, the principal can now base the wage contract on the signal. The wage contract takes the following form,

$w(y_1, y_2) = a_1 y_1 + a_2 y_2 + b$. The total certainty equivalent of the principal and the agent is now given by:

$$CE_p + CE_m = \Pi(t) - C(h_2) - V(t) - \frac{1}{2} r [\frac{a_1^2}{h_1} + \frac{a_2^2}{h_2}].$$

The principal determines the optimal wage contract, monitoring and effort levels by solving the following reduced problem:

$$\begin{array}{lll} \text{[P7.3]} & \max\limits_{a_1, a_2, h_2, t} & \Pi(t) - C(h_2) - V(t) - \frac{1}{2} r [\frac{a_1^2}{h_1} + \frac{a_2^2}{h_2}] \\ & \text{s.t.} & t \in \arg\max_{t'} a_1 t'_1 + a_2 t'_2 - V(t'). \end{array}$$

<u>Case 1</u>

Assuming an interior solution, the solutions of a_1 and a_2 are given as follows:[10]

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} I_2 + r \begin{bmatrix} V_{11} & V_{21} \\ V_{12} & V_{22} \end{bmatrix} \begin{bmatrix} \frac{1}{h_1} & 0 \\ 0 & \frac{1}{h_2} \end{bmatrix} \end{bmatrix}^{-1} \begin{bmatrix} \Pi_1 \\ \Pi_2 \end{bmatrix}. \tag{7.7}$$

Simplified further, we obtain the following expressions:

$$\begin{aligned} a_1 &= \frac{h_1[\Pi_1(rV_{22} + h_2) - \Pi_2 r V_{12}]}{(rV_{11} + h_1)(rV_{22} + h_2) - r^2 V_{12}^2}, \\ a_2 &= \frac{h_2[\Pi_2(rV_{11} + h_1) - \Pi_1 r V_{12}]}{(rV_{11} + h_1)(rV_{22} + h_2) - r^2 V_{12}^2}. \end{aligned} \tag{7.8}$$

The expressions for a_i, $i = 1, 2$ are similar to those obtained in the second-best setting and both outcomes are observable. The difference is that in the present situation, the precision of the signal on the agent's effort in task 2, h_2, is endogenously determined. The principal chooses the optimal level of monitoring h_2, determined by the following first-order necessary conditions on h_2, given by

$$C'(h_2) = \frac{r a_2^2}{2 h_2^2}. \tag{7.9}$$

The level of h_2 is determined by substituting for a_2.[11] The commission rates a_1 and a_2 are then set

[10] See Appendix 7A for the derivation.

[11] Note that the second order conditions are also satisfied. $-C''(h_2) - \frac{r a_2^2}{h_2^3} < 0$.

accordingly, and the agent chooses his optimal action, given by

$$V_i(t) = a_i, \; i = 1, 2.$$

Case 2

If a corner solution in which $t_2 = 0$ applies, then the principal does not undertake any monitoring and he sets $a_2 = h_2 = 0$ and solves the problem for a single-task situation. Such a corner solution occurs if the cost of obtaining and using the signal on the agent's activity level in task 2, y_2, is too high.

7.4 Quadratic Cost Setting

7.4.1 Introduction

In this section, we analyze in more depth a setting in which the agent's personal cost function is quadratic and the expected payoff function is linear. We assume that the profit function, $\Pi(t)$ is additively separable in the two effort types. By appropriately re-expressing the problem, this assumption allows us, without loss of generality, to concentrate on a linear profit function. A concave profit function can be transformed into a linear function. This transformation changes the activity level measure and the agent's personal cost function becomes more convex. Appendix 7C gives an example of the tranformation. Therefore, for subsequent analysis, we use a linear profit function, denoted by $\Pi(t) = t_1 + t_2$. We consider a general symmetric quadratic cost of effort function, given by $V(t) = \delta(\frac{1}{2}t_1^2 + \frac{1}{2}t_2^2 + \nu t_1 t_2)$. Then $\Pi_i = 1$, $V_i = \delta(t_i + \nu t_j)$, $V_{ii} = \delta$ and $V_{ij} = \nu\delta$ for $i, j = 1, 2$ and $i \neq j$. These functions allow us to derive closed form expressions for the optimal activity levels and the incentive components. δ influences the overall cost of attaining the two profit levels and ν influences the complementarity of these costs. To satisfy the convexity requirements, $\nu^2 < 1$, i.e., $-1 < \nu < 1$. If $0 < \nu < 1$, the two effort types are substitutable in the agent's cost function, and if $-1 < \nu < 0$, the two effort types are complementary in the agent's cost function. $\nu = 0$ implies that the two activities are technologically independent.[12] Figure 7.13 illustrates

[12] Symmetry is a key element of this example.

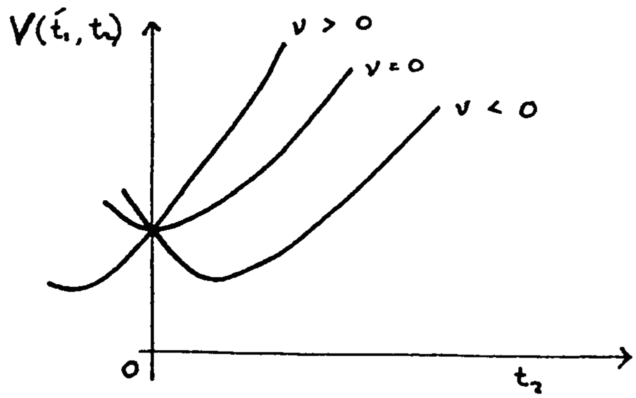

Figure 7.13: Behavior of $V(\acute{t}_1, t_2)$ with changes in t_2

the behavior of the cost of effort function for a fixed level of effort in task 1, $\acute{t}_1$.

Note that at $t_2 = 0$,

$$V_2(t) \begin{cases} < 0, & \text{if } \nu < 0 \\ = 0, & \text{if } \nu = 0 \\ > 0, & \text{if } \nu > 0. \end{cases}$$

From (7.2), the agent's incentive constraint is given by

$$V_i(t) = a_i, \quad i = 1, 2. \tag{7.10}$$

Differentiating (7.10), we obtain the following:

$$\begin{bmatrix} \frac{\partial a_1}{\partial t_1} & \frac{\partial a_1}{\partial t_2} \\ \frac{\partial a_2}{\partial t_1} & \frac{\partial a_2}{\partial t_2} \end{bmatrix} = \begin{bmatrix} \delta & \nu\delta \\ \nu\delta & \delta \end{bmatrix}.$$

By the inverse function theorem, we obtain the following:

$$\begin{bmatrix} \frac{\partial t_1}{\partial a_1} & \frac{\partial t_1}{\partial a_2} \\ \frac{\partial t_2}{\partial a_1} & \frac{\partial t_2}{\partial a_2} \end{bmatrix} = \begin{bmatrix} \frac{1}{\delta(1-\nu^2)} & \frac{-\nu}{\delta(1-\nu^2)} \\ \frac{-\nu}{\delta(1-\nu^2)} & \frac{1}{\delta(1-\nu^2)} \end{bmatrix}. \tag{7.11}$$

Equation (7.11) characterises how changes in the incentive rates a_i affect the activity level that will be supplied.

When the two effort types are complementary in the agent's personal cost function, an increase in a_i affects the activity levels in both tasks positively. The principal uses both a_1 and a_2 to motivate the agent's effort in both tasks. On the other hand, when the effort types are substitutable in the agent's personal cost function, an increase in a_i results in an increase in activity level t_i and a decrease in t_j, $i \neq j$. The principal can only use incentive component a_1 to motivate effort in task 1 and a_2 to motivate effort in task 2. In this latter case, relatively high a_1 implies that the opportunity cost of working in task 2 is high, thus the agent's attention is partially directed from task 2. Similarly, relatively high a_2 discourages effort in task 1. We observe that the higher is $|\nu|$, the more responsive is the activity level t_i to a change in a_j. From (7.11), we note that δ also determines how responsive the agent is to incentives. High δ lowers the responsiveness to incentives.

7.4.2 First-best Solution

In the first-best situation, a flat wage contract is paid. Optimal effort levels are $t_1^* = t_2^* = \frac{1}{\delta(1+\nu)}$.[13] The total certainty equivalent is given by:

$$\begin{aligned} CE_T^* &= \Pi(t^*) - V(t^*) \\ &= \frac{1}{\delta(1+\nu)}. \end{aligned} \tag{7.12}$$

As ν increases, t_1^*, t_2^* and thus, CE_T^* decrease. This implies that the optimal effort levels are lower if the two effort types are substitutes in the agent's cost function, as compared to when they are complements. Similarly, as δ increases, t_1^*, t_2^* and thus, CE_T^* decrease. A high δ indicates that it is more costly to employ the agent, thus the optimal effort levels are lower.

[13] See appendix 7A for details of the derivation.

7.4.3 Second-best Solution - Costless Noisy Performance Measures

In the second-best situation, if independent, costless, noisy performance measures are available for both tasks, then substituting $\Pi_i = 1$, $V_{ii} = \delta$ and $V_{ij} = \nu\delta$, $i \neq j$ into (7.4), we obtain the following:

$$\bar{a}_i = \frac{h_i[h_j + r\delta(1-\nu)]}{(r\delta + h_i)(r\delta + h_j) - \nu^2 r^2 \delta^2}. \tag{7.13}$$

The agent's choice of activity level is characterized by $V_i(t) = a_i$, $i = 1, 2$. Since $V_i = \delta(t_i + \nu t_j)$, $i = 1, 2$, we have two equations for the two tasks which we solve simultaneously to obtain the following:[14]

$$\bar{t}_i = \frac{\bar{a}_i - \nu \bar{a}_j}{\delta(1-\nu^2)}, \quad i, j = 1, 2, \text{and } i \neq j. \tag{7.14}$$

Substituting (7.13) for $\bar{a}_i$ and $\bar{a}_j$ gives

$$\bar{t}_i = \frac{h_i h_j + r\delta(h_i - \nu h_j)}{\delta(1+\nu)[(r\delta + h_i)(r\delta + h_j) - \nu^2 r^2 \delta^2]}, \quad i, j = 1, 2, \text{and } i \neq j. \tag{7.15}$$

The total certainty equivalent if both tasks are undertaken is:

$$\begin{aligned} \bar{CE}_T &= \Pi(\bar{t}) - V(\bar{t}) - \frac{1}{2} r [\frac{\bar{a}_1^2}{h_1} + \frac{\bar{a}_2^2}{h_2}] \\ &= \frac{r\delta(1-\nu)(h_1 + h_2) + 2h_1 h_2}{2\delta(1+\nu)[(r\delta + h_1)(r\delta + h_2) - \nu^2 r^2 \delta^2]}. \end{aligned} \tag{7.16}$$

We make the following observations on the behavior of the total certainty equivalent:[15]

- $\bar{CE}_T$ is increasing and concave in h_1 and h_2. The principal is better off with more precise performance measures.

- The levels of $\bar{t}_i$, $i = 1, 2$ and $\bar{CE}_T$ are higher when ν is negative than when ν is positive, holding all other parameters constant. In fact, $\frac{\partial \bar{CE}_T}{\partial \nu} < 0$, i.e., the total certainty equivalent decreases as ν increases. Recall that a negative ν implies that the marginal disutility of achieving task i decreases as the activity level in task j increases, thus it is not surprising that the principal is better off the

[14] See appendix 7A for details.
[15] See appendix 7A for details.

lower the value of ν. This implies that if the principal has perfect freedom in the grouping and assignment of tasks to the agents, he is better off if such grouping and assignment achieve as low a value of ν as possible. The principal is able to motivate higher effort levels more efficiently and earn higher profit levels with a more negative value of ν.

- We denote the loss to the principal from being unable to observe the agent's effort by L. Then $L = CE_T^* - \bar{CE}_T$, where CE_T^* is the total certainty equivalent in the first-best setting. While CE_T^* is independent of the risk aversion of the agent r, $\bar{CE}_T$ is decreasing in r, thus, the level of L increases as r increases. Also, observe that $\lim_{r \to 0} L(r) = 0$, i.e., when the agent is nearly risk neutral, the loss to the principal approaches zero. This result is analogous to Grossman and Hart (1983) (Propositions 15 and 16) in a single-task pure moral hazard setting with binary outcomes. Arya, Fellingham and Young (1993) obtain a similar result in a setting in which the agent has private productive information. They express the efficiency loss to the principal as the sum of expected lost production and a risk premium, and they show that the loss in expected production increases as the agent becomes more risk averse.

Next, we examine if the principal is always better off using both performance measures in the incentive contract and motivating the agent to work in both tasks. First, consider the case when $\nu < 0$. If the principal chooses to use only the costless performance measure for task 1, then the total certainty equivalent is as follows (see equation (7.21)):

$$\bar{CE}_{T1} = \frac{h_1(1-\nu)}{2\delta(1+\nu)[h_1 + r\delta(1-\nu^2)]}.$$

Figure 7.14 shows that the principal always achieves a higher certainty equivalent using both pieces of information, regardless of how noisy the performance measure of task 2 is.

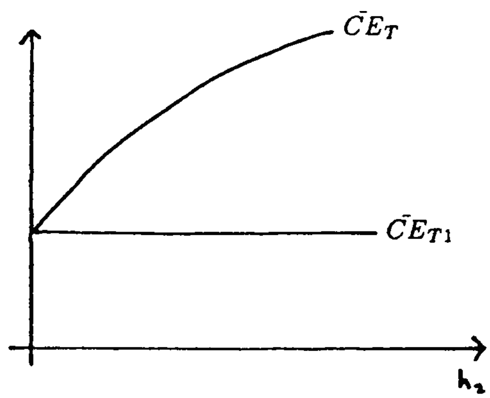

Figure 7.14: Behavior of total certainty equivalent with h_2 ($\nu < 0$)

On the other hand, if $\nu > 0$, depending on the relative levels of h_1 and h_2, there can arise situations where the principal is better off motivating the agent in just one of the tasks. Assume that $h_1 > h_2$. The total certainty equivalent if only task 1 is undertaken is given by:

$$\bar{CE}_{T1} = \frac{h_1}{2\delta(h_1 + r\delta)}. \tag{7.17}$$

Then the principal motivates the agent to work on both tasks if, and only if,[16]

$$h_2 > \bar{h}_2 = \frac{\nu r\delta h_1[r\delta(1-\nu)(2+\nu) + 2h_1]}{(1-\nu)[h_1 + r\delta]^2}. \tag{7.18}$$

Note that $\bar{h}_2$ increases as h_1 increases. This implies that the higher is h_1, the higher h_2 must be before task 2 is undertaken. The intuition for this is very straightforward. The higher is h_1, the more attractive is task 1 and a higher incentive rate for task 1 results. This implies that to motivate task 2, a relatively high incentive rate is required. However, this is costly for a low h_2, and the principal does not motivate the agent to work on task 2 if $h_2 \leq \bar{h}_2$.

[16] Equation (7.18) holds if, and only if, the total certainty equivalent when both tasks are undertaken exceeds that when only task 1 is underaken. See appendix 7A for derivation of $\bar{h}_2$.

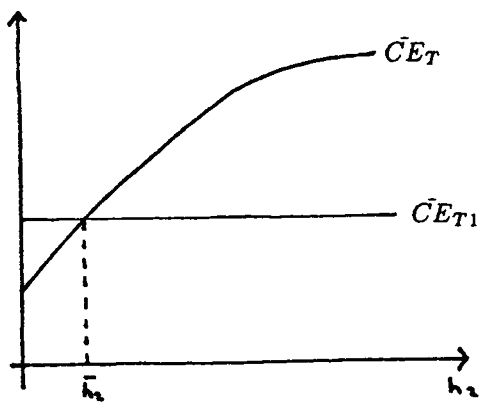

Figure 7.15: Behavior of total certainty equivalent with h_2 ($\nu > 0$)

There are two cost elements in motivating the agent to work on task 2, namely, the required risk premium as a result of incentive rate a_2, and the increased cost of motivating the agent in task 1. The latter cost element arises because with $a_2 > 0$, the opportunity cost of working on task 1 increases, so that it is now more expensive to motivate any particular level of effort in task 1. Figure 7.15 depicts the relationship between the respective total certainty equivalent and the level of h_2.

If h_2 is less than $\bar{h}_2$, the information is too noisy to be of any value to the principal.[17] The optimal activity level in task 2 is low and the principal is better off motivating the agent to work on task 1 alone. This is consistent with Itoh (1991) in a multi-agent setting. He investigates whether it is optimal for the principal to induce teamwork or unambigious division of labour. He concludes that in a situation similar to $\nu > 0$, the principal wants either a specialized structure or substantial teamwork. A low level of help is suboptimal.[18]

[17]The cost of using the noisy information outweighs the benefit.

[18]Itoh (1991) uses a binary outcome structure in his analysis, i.e., outcome is denoted by either success or failure.

Lemma 7.1 applies for both positive and negative values of ν. We assume that when $\nu > 0$, it is optimal for the principal to motivate the agent to work on both tasks.

<u>Lemma 7.1:</u>

- If $h_1 = h_2$, then $\bar{a}_1 = \bar{a}_2$ and $\bar{t}_1 = \bar{t}_2$.
- If $h_1 > h_2$, then $\bar{a}_1 > \bar{a}_2$ and $\bar{t}_1 > \bar{t}_2$.

Lemma 7.1 states that if there is relatively more precise information available on the activity level of one task, then the principal pays a higher incentive rate and induces a higher activity level in that task. Lal and Srinivasan (1993) demonstrate a similar result in a multiproduct salesforce setting for the case when the agent's effort types are perfect substitutes in the agent's cost function. Products with lower uncertainty should be given higher commission rates to generate maximum profits to the firm.

7.4.4 Comparative Statics - Costless Noisy Performance Measures

We examine how the optimal incentive components and the optimal effort levels vary as h_2, ν and r vary. Proposition 7.1 examines the effect of changes in h_2, the precision of the performance measure in task 2. For $\nu > 0$, we assume that as the various parameters vary, it is still optimal for the principal to motivate the agent to undertake both tasks.

Proposition 7.1:

An increase in the precision of the performance measure for task 2 (h_2) results in:

1. a decrease (increase) in the incentive rate for the performance measure of task 1 if the activities are complements (substitutes), i.e., $\frac{\partial \bar{a}_1}{\partial h_2} \begin{cases} < 0 & \text{for } -1 < \nu < 0 \\ > 0 & \text{for } 0 < \nu < 1; \end{cases}$
2. an increase in the incentive rate for the performance measure of task 2, i.e., $\frac{\partial \bar{a}_2}{\partial h_2} > 0$;
3. an increase (decrease) in the level of activity of task 1 if the activities are complements (substitutes), i.e., $\frac{\partial \bar{t}_1}{\partial h_2} \begin{cases} > 0 & \text{for } -1 < \nu < 0 \\ < 0 & \text{for } 0 < \nu < 1; \end{cases}$
4. an increase in the level of activity in task 2, i.e., $\frac{\partial \bar{t}_2}{\partial h_2} > 0$.

As the precision of the signal of agent's effort in task 2 increases, the risk imposed on the agent for using the signal to motivate a given effort level decreases. It is now less costly to motivate the agent to work on task 2, thus the principal increases $\bar{a}_2$ and $\bar{t}_2$. For $-1 < \nu < 0$, the principal reduces the incentive component $\bar{a}_1$ since the marginal cost of motivating the agent through $\bar{a}_2$ is now relatively lower. $\bar{t}_1$ also increases since the overall marginal cost of motivating the agent is lower. For $0 < \nu < 1$, as $\bar{a}_2$ increases, the opportunity cost of working in task 1 increases, thus, $\bar{a}_1$ increases to balance the agent's motivation to work on task 1. $\bar{t}_1$ decreases, since the relative cost of motivating effort in task 2 decreases and the principal partially redirects the agent's attention from task 1 to task 2. Note that a change in h_1 affects $\bar{a}_1$, $\bar{a}_2$, $\bar{t}_1$ and $\bar{t}_2$ the same way that a change in h_2 affects $\bar{a}_2$, $\bar{a}_1$, $\bar{t}_2$ and $\bar{t}_1$, respectively.

The next proposition examines how changes in ν and r affect the optimal incentive rates and activity levels. When $-1 < \nu < 0$, an increase in the value of ν implies that the complementary effect of the effort types in the agent's cost function decreases. When $0 < \nu < 1$, an increase in the value of ν implies that the substitution effect of the effort types in the agent's cost function increases.

Proposition 7.2

1. An increase in the value of the interactive effect of the two effort types in the agent's cost function (ν) results in:

   (a) a decrease in the incentive rate for both measures, i.e., $\frac{\partial \bar{a}_i}{\partial \nu} < 0, \;\; i = 1, 2$;

   (b) a decrease in the level of activity for both tasks if the activities are complements, while it is unclear what happens if the activities are substitutes,[19] i.e.,
   $$\frac{\partial \bar{t}_i}{\partial \nu} \begin{cases} < 0 & \text{for } -1 < \nu < 0 \\ \textit{ambiguous} & \text{for } 0 < \nu < 1, \;\; i = 1, 2. \end{cases}$$

2. An increase in the agent's coefficient of absolute risk aversion, (r) results in:

   (a) a decrease in the incentive rate for both measures if the activities are substitutes, while it is ambiguous if the activities are complements, i.e., $\frac{\partial \bar{a}_i}{\partial r} \begin{cases} \textit{ambiguous} & \text{for } -1 < \nu < 0 \\ < 0 & \text{for } 0 < \nu < 1, \;\; i = 1, 2; \end{cases}$

   (b) a decrease in the level of activity for both tasks, i.e., $\frac{\partial \bar{t}_i}{\partial r} < 0, \;\; i = 1, 2$.

When ν is negative, motivating a higher level of $\bar{t}_1$ has a double benefit, since the direct cost of motivating $\bar{t}_2$ becomes less costly. As ν becomes less negative, this benefit decreases. Generally, as ν increases, it becomes relatively more expensive to motivate any particular level of effort. We expect $\bar{a}_i$ and $\bar{t}_i$, $i = 1, 2$ to decrease. However, for $\nu > 0$, a decrease in $\bar{a}_i$ implies that the opportunity cost of working in task j, $i \neq j$, is decreased, and the indirect effect is an increase in t_j. Thus, we see that the effect of an increase in ν for $\nu > 0$ on $\bar{t}_i$, $i = 1, 2$ is ambigious.[20]

[19] While it is generally possible to characterize the sign of $\frac{\partial \bar{t}_i}{\partial \nu}$ for alternative sets of parameter values, we do not because the expressions are complicated and yield little economic insight. This note applies to subsequent cases of ambiguity as well. However, if the characterizations are useful, we place them in the appendix and provide the discussion on the economic insight in the section subsequent to the respective proposition in this main paper.

[20] The direct effect on t_j (due to a change in a_j) is relatively larger than the indirect effect (due to an equal change in a_i). However, the rate of change in a_i due to changes in ν may be different from that of a_j depending on the precision of the signal.

As r increases, the required risk premium to motivate a given effort level increases and the cost of motivating the agent increases. We expect $\bar{t}_i$, $i = 1,2$ to decrease. For $\nu > 0$, $\bar{a}_i$, $i = 1,2$ decreases as the principal settles for a lower activity level. For $\nu < 0$, the behavior of $\bar{a}_i$ appears to depend on the relative level of h_i to h_j. If h_i is comparable to h_j, both $\bar{a}_i$ and $\bar{a}_j$ decrease. If h_i is relatively very large compared to h_j, then it is possible for $\bar{a}_i$ to increase and $\bar{a}_j$ to decrease as r increases. There appears to be a substitution effect taking place. The principal now emphasizes the use of the signal of task 1 due to its relatively high precision and focuses less on the use of the signal of task 2.

7.4.5 Costless Performance Measure Available for Only Task 1

From Proposition 7.1, we learn that a change in the precision of the signal on task 2, h_2, affects the incentive rates and activity levels differently depending on whether the effort types are complementary or substitutable in the agent's cost function. In this section, we consider the extreme case in which h_2 equals zero, i.e., costless performance measure is available for only task 1.

<u>Case 1</u>

If the effort types are complementary in the agent's cost function, i.e. $\nu < 0$, then substituting $\Pi_i = 1$, $V_{ii} = \delta$ and $V_{ij} = \nu\delta$ into (7.5), we obtain the following:

$$
\begin{aligned}
\tilde{a}_1 &= \frac{h_1(1-\nu)}{h_1 + r\delta(1-\nu^2)} \\
\tilde{a}_2 &= 0.
\end{aligned} \tag{7.19}
$$

Without any information about the second task, the principal relies on information about the first task to motivate agent's effort in both task 1 and task 2. The combination of effort levels in the two tasks that can be induced are thus restricted. The agent's choice of activity level is characterized by $V_i(t) = a_i$, $i = 1,2$. Substituting $V_i = \delta(t_i + \nu t_j)$ into $V_i = a_i$, $i = 1,2$ and solving simultaneously, we obtain the following:[21]

$$\begin{aligned}
\tilde{t}_1 &= \frac{\tilde{a}_1}{\delta(1-\nu^2)} \\
&= \frac{h_1}{\delta(1+\nu)[h_1 + r\delta(1-\nu^2)]} \\
\tilde{t}_2 &= \frac{-\nu\tilde{a}_1}{\delta(1-\nu^2)} \\
&= \frac{-\nu h_1}{\delta(1+\nu)[h_1 + r\delta(1-\nu^2)]}.
\end{aligned} \tag{7.20}$$

Since $\nu < 0$, we obtain $\tilde{a}_1 > 0$, $\tilde{t}_1 > 0$ and $\tilde{t}_2 > 0$. Recall that the incentive component, $\tilde{a}_1$ is used to motivate both $\tilde{t}_1$ and $\tilde{t}_2$. Thus, for any changes in the exogenous parameters, $\tilde{a}_1$, $\tilde{t}_1$ and $\tilde{t}_2$ move in the same direction. As h_1 increases, the cost of using $\tilde{a}_1$ to motivate effort decreases, resulting in an increase in $\tilde{a}_1$, $\tilde{t}_1$ and $\tilde{t}_2$. As ν or r increases, the cost of motivating the agent increases, thus, $\tilde{a}_1$, $\tilde{t}_1$ and $\tilde{t}_2$ decrease. The total certainty equivalent is given by:

$$\begin{aligned}
\tilde{CE}_T(\nu < 0) &= \Pi(\tilde{t}) - V(\tilde{t}) - \frac{r\tilde{a}_1^2}{2h_1} \\
&= \frac{h_1(1-\nu)}{2\delta(1+\nu)[h_1 + r\delta(1-\nu^2)]}.
\end{aligned} \tag{7.21}$$

<u>Case 2</u>

If the effort types are substitutable in the agent's cost function, i.e. $\nu > 0$, then from (7.6),

$$\begin{aligned}
\tilde{a}_1 &= \frac{h_1}{h_1 + r\delta} \\
\tilde{a}_2 &= 0.
\end{aligned} \tag{7.22}$$

Without any information about the second task, it is impossible to induce effort in that task. The agent sets $t_2 = 0$ and his choice of activity level in t_1 is characterized by $V_1(t_1, 0) = a_1$. Since $V_1(t_1, 0) = \delta t_1$, we obtain

$$\tilde{t}_1 = \frac{1}{\delta}\tilde{a}_1$$

[21] See appendix 7A for details.

$$= \frac{h_1}{\delta(h_1 + r\delta)}$$

$$\tilde{t}_2 = 0. \tag{7.23}$$

Notice that the optimal values of $\tilde{a}_1$ and $\tilde{t}_1$ are independent of both the nature of task 2 and any interaction between the two tasks. The result is consistent with Holmstrom & Milgrom (1991). They show that when effort types are perfectly substitutable in the agent's cost function, and if costless performance measure is not available for say, task 2, then the use of an incentive contract on task 1 implies that the agent allocates no effort to task 2. Our analysis indicates that this result holds for any $0 < \nu < 1$, and not only when the effort types are perfectly substitutable in the agent's cost function.

Observe that $\tilde{a}_1$ and $\tilde{t}_1$ increase with h_1 and decrease with r. The total certainty equivalent is given by:

$$\tilde{CE}_T(\nu > 0) = \frac{h_1}{2\delta(h_1 + r\delta)}. \tag{7.24}$$

The moral hazard problem created by the unobservable nature of the agent's effort and the additional incentive problem caused by having an observable performance measure for one task only, result in an efficiency loss and the principal is worse off than in the first-best setting in which the agent's effort in both tasks are observable and verifiable.

7.4.6 Second Best Solution - Costly Monitoring Technology

In this subsection, we assume that there is no costlessly observable performance measure for the second task. Instead, the principal uses a costly monitoring technology. Assuming that the optimal monitoring level is strictly positive, then by substituting for $\Pi_i = 1$, $V_{ii} = \delta$ and $V_{ij} = \nu\delta$, $i, j = 1, 2$, $i \neq j$ into (7.8), we obtain the following:

$$\hat{a}_i = \frac{h_i[h_j + r\delta(1-\nu)]}{(r\delta + h_i)(r\delta + h_j) - \nu^2 r^2 \delta^2}, \quad i, j = 1, 2, \quad i \neq j. \tag{7.25}$$

The agent chooses the optimal activity levels such that $V_i(t) = \delta(t_i + \nu t_j) = a_i$. Therefore, by solving simultaneously, we obtain

$$\acute{t}_i = \frac{\hat{a}_i - \nu \hat{a}_j}{\delta(1-\nu^2)}, \quad i,j = 1,2, \quad i \neq j. \tag{7.26}$$

Notice that these expressions are identical with those in the second-best setting with both outcomes costlessly observable (equations (7.13) and (7.14)). In that situation, the precision of the signal on the agent's effort in the second task is exogenously specified, while in this case, it is endogenously determined. We assume that the cost of the monitoring technology is $C(h_2) = ch_2,\ c > 0$. The principal chooses the optimal level of monitoring $\hat{h}_2$ which is determined as follows:

$$\hat{h}_2 = \max\{0, \acute{h}_2\},$$

where $\acute{h}_2$ is defined below. From (7.9), the level of $\acute{h}_2$ is determined as follows:

$$\begin{aligned} c &= \frac{r\hat{a}_2^2}{2\acute{h}_2^2}. \\ \text{Hence, we obtain} \quad \acute{h}_2 &= \hat{a}_2\sqrt{(\frac{r}{2c})}. \end{aligned}$$

By substituting for $\hat{a}_2$ as given in (7.25) and simplifying, we obtain[22]

$$\acute{h}_2 = \frac{h_1 + r\delta(1-\nu) - \delta\sqrt{(2rc)}[h_1 + r\delta(1-\nu^2)]}{\sqrt{(\frac{2c}{r})}(h_1 + r\delta)}. \tag{7.27}$$

When $\hat{h}_2 = 0$, no monitoring is undertaken and the results in the previous section in which only one performance measure is available applies. Also, we observe from (7.27) that $\lim_{c\to 0} \hat{h}_2 = \infty$, implying that as the cost of obtaining information approaches zero, the principal purchases perfect information.[23]

We assume an interior solution for $\hat{h}_2$. Thus, $\hat{h}_2 = \acute{h}_2$. Using (7.27) for the optimal level of h_2 and substituting into (7.25) and (7.26), the expressions for a_i and t_i, $i = 1, 2$, we obtain the following:

$$\hat{a}_1 = \frac{h_1(1 - \nu\delta\sqrt{(2rc)})}{h_1 + r\delta}$$

[22] See appendix 7A for details of the derivation.

[23] From (7.27), we also observe that $\lim_{r\to 0} \hat{h}_2 = 0$, i.e., as the risk aversion of the agent approaches zero, the principal does not purchase any information. When the agent is risk neutral, the incentive rates are set at $a_i = 1$ and the agent pays a fixed fee to the principal.

$$\hat{a}_2 = \frac{h_1 + r\delta(1-\nu) - \delta\sqrt{(2rc)}[h_1 + r\delta(1-\nu^2)]}{h_1 + r\delta} \quad (7.28)$$

$$\hat{t}_1 = \frac{h_1 - \nu r\delta[1 - \delta\sqrt{(2rc)}(1+\nu)]}{\delta(h_1 + r\delta)(1+\nu)}$$

$$\hat{t}_2 = \frac{1 - \delta\sqrt{(2rc)}(1+\nu)}{\delta(1+\nu)}. \quad (7.29)$$

We make the following observations of the optimal level of effort in task 2 $\hat{t}_2$.[24]

- It is independent of h_1, which is the precision of the signal of the agent's effort in task 1. Here, the principal determines the precision of the signal of the agent's effort in task 2 through the monitoring intensity, $\hat{h}_2$. Since y_1 provides no information of the agent's effort in task 2, the principal offsets any variation in h_1 through his choice of $\hat{h}_2$ and maintains the level of $\hat{t}_2$. On the other hand, when the precision of the signal of the agent's effort in task 2 is exogenously specified, the agent's activity level in task 2 depends on h_1. For $-1 < \nu < 0$, t_2 increases with h_1 and for $0 < \nu < 1$, t_2 decreases with h_1. See Proposition 7.1.

- If $\nu = 0$, $\hat{t}_2 = \frac{1}{\delta} - \sqrt{(2rc)}$. This is also the optimal effort level if task 2 is the only task and costly monitoring technology is employed.

- Under the first-best setting, the optimal effort level in task 2 is given by $t_2^* = \frac{1}{\delta(1+\nu)}$. The deviation of $\hat{t}_2$ from first-best is thus given by $\sqrt{(2rc)}$. From (6.7), we note that if task 2 is the only task, which implies that effort allocation is not an issue, and costly monitoring technology is employed, then the deviation from first-best effort level is also $\sqrt{(2rc)}$. This implies that in the two-task setting, the deviation from the first-best effort intensity depends on only the risk aversion of the agent, and the cost of monitoring, and is not affected by the effort allocation issue. This result is consistent with Feltham and Xie (1994). They show that a contract based on a noncongruent measure induces suboptimal effort allocation across tasks, whereas performance measure noise results in suboptimal effort intensity. In our model, the performance measures used are congruent

[24] Note that $\hat{a}_2 > \hat{a}_1 \Leftrightarrow \hat{t}_2 > \hat{t}_1$. $\hat{t}_2 > \hat{t}_1$ holds when $\sqrt{(2rc)} < \frac{r}{h_1 + r\delta(1+\nu)}$. Thus, low c and low h_1 may result in $\hat{t}_2 > \hat{t}_1$.

with the principal's expected profit, thus suboptimal effort allocation is not an issue.

- First-best effort intensity in task 2 is achieved if the agent is risk neutral or the monitoring technology is costless (i.e., if $rc = 0$). A costless monitoring technology implies that perfect information is obtained and the performance measure is noiseless. As the cost of monitoring or the risk aversion of the agent increases, the deviation from first-best increases. This is a result observed in most models in the principal/agent literature.

Next, we derive the condition on c for monitoring to be worthwhile for $-1 < \nu < 0$ and $0 < \nu < 1$. From (7.16), for a given level of h_1 and h_2, the total certainty equivalent before deducting the cost of monitoring if both effort levels are strictly positive is

$$CE_T(h_2) = \frac{r\delta(1-\nu)(h_1+h_2)+2h_1h_2}{2\delta(1+\nu)[(r\delta+h_1)(r\delta+h_2)-\nu^2r^2\delta^2]}. \tag{7.30}$$

$CE_T(h_2)$ is increasing and concave in h_2 which implies that the principal experiences diminishing returns to monitoring.

Case 1

If $-1 < \nu < 0$ and no monitoring is undertaken, the agent still works on both tasks and the total certainty equivalent is given by (see (7.21)):

$$CE_T(h_2 = 0) = \frac{h_1(1-\nu)}{2\delta(1+\nu)[h_1+r\delta(1-\nu^2)]}.$$

When monitoring is costly, Lemma 7.2 provides a condition for monitoring to be worthwhile.

Lemma 7.2: For $-1 < \nu < 0$, a necessary and sufficient condition for the principal to engage in monitoring is

$$c < \frac{[h_1+r\delta(1-\nu)]^2}{2r\delta^2[h_1+r\delta(1-\nu^2)]^2}.$$

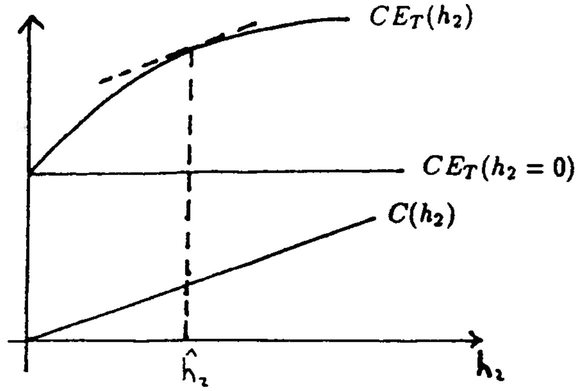

Figure 7.16: Determining the optimal level of monitoring ($\nu < 0$)

At the optimal level of monitoring, the marginal benefit of monitoring equals the marginal cost of monitoring c. The marginal benefit (MB) of monitoring is as follows:[25]

$$MB = \frac{r[h_1 + r\delta(1-\nu)]^2}{2[r^2\delta^2(1-\nu^2) + r\delta(h_1 + h_2) + h_1 h_2]^2}. \tag{7.31}$$

At $h_2 = 0$, $MB(h_2 = 0) = \frac{[h_1 + r\delta(1-\nu)]^2}{2r\delta^2[h_1 + r\delta(1-\nu^2)]^2}$. Thus, if $c \geq MB(h_2 = 0)$, the principal does not engage in monitoring and he relies on the costless signal of the agent's effort in task 1 to motivate the agent's effort in both tasks. He settles for a lower effort level in task 2 than if he engages in monitoring. As h_1 increases, the bound on the cost of monitoring tightens. Figure 7.16 shows that once the condition in Lemma 7.2 is met, monitoring is always worthwhile even when the optimal level of monitoring is very low.

Case 2

If $0 < \nu < 1$ and no monitoring is undertaken, it is impossible to induce effort in task 2 and the agent works on only task 1. The total certainty equivalent is given by (see (7.24)):

$$CE_T(h_2 = 0) = \frac{h_1}{2\delta(h_1 + r\delta)}.$$

[25] See appendix 7A for details.

A necessary condition for the principal to motivate the agent to undertake both tasks is that $CE_T(h_2 > 0) > \frac{h_1}{2\delta(h_1+r\delta)}$. This implies that (see (7.18))

$$\hat{h}_2 > \bar{h}_2 = \frac{\nu r\delta h_1[r\delta(1-\nu)(2+\nu)+2h_1]}{(1-\nu)[h_1+r\delta]^2}.$$

When the monitoring intensity is too low, the information obtained is too noisy to be of any value. Using the expression for optimal h_2, we determine the upper bound for the cost of monitoring, c.

<u>Lemma 7.3:</u> For $0 < \nu < 1$, necessary conditions for the principal to engage in monitoring and motivate the agent to work on both tasks are:

1. $c < \frac{(1-\nu)^2\{h_1^2+r\delta h_1(2-\nu)+r^2\delta^2(1-\nu)\}^2}{2r\delta^2(1+\nu)^2\{h_1^2+r\delta(1-\nu)[2h_1+r\delta(1-\nu)]\}^2}$, and

2. $h_1 > \nu r\delta[1-\delta\surd(2rc)(1+\nu)]$.

If the cost of monitoring is too high, the principal does not engage in monitoring. He foregoes any benefit from task 2 and concentrates on task 1 alone. Similarly, if h_1 is too small, i.e., the information on activity level in task 1 is very noisy, the principal can be better off concentrating on task 2 alone, even though the signal for activity level in task 1 is costlessly available. Note that the lower bound on h_1 depends on the level of c. As c increases, the lower bound on h_1 decreases. Figure 7.17 illustrates that the principal does not engage in a low level of monitoring.

Propositions 7.3 and 7.4 compare the optimal levels of t_1, t_2 and a_1 in the non-monitoring (denoted by $\tilde{}$) and monitoring (denoted by $\hat{}$) environments. In the former environment, $\tilde{a}_2 = 0$.

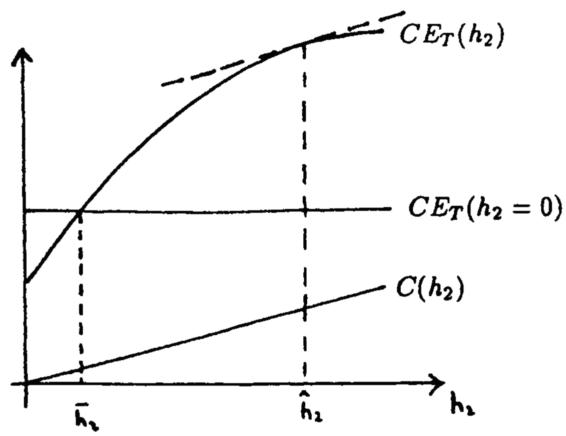

Figure 7.17: Determining the optimal level of monitoring ($\nu > 0$)

Proposition 7.3: If the activities are complements, i.e., $-1 < \nu < 0$, the following relations hold:

1. $\hat{t}_1 > \tilde{t}_1$,

2. $\hat{t}_2 > \tilde{t}_2$, and

3. $\hat{a}_1 < \tilde{a}_1$.

The principal chooses to extract higher effort levels, and can do so more efficiently, with the monitoring technology. With the information about the agent's effort in task 2, the principal uses both $\hat{a}_1$ and $\hat{a}_2$ to motivate the agent in the two tasks. Without the monitoring technology, no information is available about the agent's effort in task 2 and the principal uses only $\tilde{a}_1$ to motivate the agent.

Proposition 7.4: If the activities are substitutes, i.e., $0 < \nu < 1$, the following relations hold. Recall that $\tilde{t}_2 = 0$.

1. $\hat{t}_1 < \tilde{t}_1$,
2. $\hat{t}_2 > \tilde{t}_2$, and
3. $\hat{a}_1 < \tilde{a}_1$.

In this setting, incentive component a_1 motivates the agent to work on task 1 only. Without the monitoring technology, no information is available about the agent's effort in task 2, thus, the principal motivates the agent to concentrate totally on task 1. With the monitoring technology, the principal redirects the agent's attention partially to task 2.

7.4.7 Comparative Statics - Costly Monitoring Technology

We examine how the intensity of monitoring, the incentive components and the effort levels vary as the other variables vary. We assume interior solutions for both the monitoring level and the effort levels.

Proposition 7.5:

An increase in the cost of monitoring (c) results in:

1. reduced monitoring, i.e., $\frac{\partial \hat{h}_2}{\partial c} < 0$;
2. increased (decreased) incentive rate for task 1 if the tasks are complements (substitutes), i.e.,
$$\frac{\partial \hat{a}_1}{\partial c} \begin{cases} > & 0 \quad \text{for } -1 < \nu < 0 \\ < & 0 \quad \text{for } 0 < \nu < 1; \end{cases}$$
3. decreased incentive rate for task 2, i.e., $\frac{\partial \hat{a}_2}{\partial c} < 0$;
4. decreased (increased) effort level in task 1 if the tasks are complements (substitutes), i.e.,
$$\frac{\partial \hat{t}_1}{\partial c} \begin{cases} < & 0 \quad \text{for } -1 < \nu < 0 \\ > & 0 \quad \text{for } 0 < \nu < 1; \end{cases}$$
5. reduced effort level in task 2, i.e., $\frac{\partial \hat{t}_2}{\partial c} < 0$.

As the costliness of monitoring, c, increases, it becomes relatively more expensive to motivate the agent using incentive component $\hat{a}_2$. The intensity of monitoring, $\hat{h}_2$, and consequently, the incentive component on task 2, $\hat{a}_2$, decrease. For $-1 < \nu < 0$, since the effort types are complements in the agent's private cost function, the principal now uses a relatively cheaper means of motivation, and $\hat{a}_1$ increases. Both $\hat{t}_1$ and $\hat{t}_2$ also decrease since the cost of motivation has increased and the principal could not be better off than before the increase in c. For $0 < \nu < 1$, with the decrease in $\hat{a}_2$, the principal seeks to reduce the opportunity cost of working on task 2 to maintain a proper allocation of the agent's effort between the two tasks, and $\hat{a}_1$ decreases. The principal partially redirects the agent's effort from task 2 to task 1, since it is now relatively cheaper to motivate effort in task 1 as compared to task 2. Thus, $\hat{t}_1$ increases while $\hat{t}_2$ decreases.

Proposition 7.6: An increase in the precision of the performance measure for task 1 (h_1) results in

1. reduced (increased) monitoring if the tasks are complements (substitutes), i.e.,
$\frac{\partial \hat{h}_2}{\partial h_1} \begin{cases} < 0 & \text{for } -1 < \nu < 0 \\ > 0 & \text{for } 0 < \nu < 1; \end{cases}$
2. increased incentive rate for task 1, i.e., $\frac{\partial \hat{a}_1}{\partial h_1} > 0$;
3. decreased (increased) incentive rate for task 2 if the tasks are complements (substitutes), i.e.,
$\frac{\partial \hat{a}_2}{\partial h_1} \begin{cases} < 0 & \text{for } -1 < \nu < 0 \\ > 0 & \text{for } 0 < \nu < 1; \end{cases}$
4. increased level of activity for task 1, i.e., $\frac{\partial \hat{t}_1}{\partial h_1} > 0$;
5. no change in the activity level for task 2, i.e., $\frac{\partial \hat{t}_2}{\partial h_1} = 0$.

The optimal level of $\hat{t}_2$ does not depend on h_1, since the signal y_1 is not informative on the agent's effort in task 2. We compare this result with the case when the precision of the signal of the agent's effort in task 2 is exogenously specified. Proposition 7.1 tells us that as h_1 increases, the optimal level of t_2 is not maintained. Rather, as h_1 increases, t_2 increases for $-1 < \nu < 0$, while for $0 < \nu < 1$, t_2 decreases.

As h_1 increases, it is now less expensive to motivate effort in task 1 using incentive component a_1, so the principal increases $\hat{a}_1$ and $\hat{t}_1$. For $-1 < \nu < 0$, with the increased level of $\hat{a}_1$, the principal reduces the intensity of monitoring. $\hat{h}_2$ and $\hat{a}_2$ decrease, and the level of $\hat{t}_2$ is maintained. For $0 < \nu < 1$, since $\hat{a}_1$ increases, the opportunity cost of working on task 2 increases. The principal increases the intensity of monitoring, $\hat{h}_2$, and the incentive component on task 2, $\hat{a}_2$, otherwise the agent's effort in task 2 is partially redirected to task 1.

Proposition 7.7: An increase in the value of the interactive effect of the two effort types in the agent's cost function (ν) results in:

1. reduced monitoring, i.e., $\frac{\partial \hat{h}_2}{\partial \nu} < 0$;

2. reduced incentive rates for both measures, i.e., $\frac{\partial \hat{a}_i}{\partial \nu} < 0, \; i = 1, 2$;

3. reduced level of activity for task 1 if the two tasks are complements, while the relationship is unclear if the two tasks are substitutes, i.e., $\frac{\partial \hat{t}_1}{\partial \nu} \begin{cases} < 0 & \text{for } -1 < \nu < 0 \\ \textit{ambiguous} & \text{for } 0 < \nu < 1; \end{cases}$

4. reduced level of activity for task 2, i.e., $\frac{\partial \hat{t}_2}{\partial \nu} < 0$.

As ν varies, $\hat{a}_1$, $\hat{h}_2$ and $\hat{a}_2$ should move in the same direction since there is no gain to using one incentive component over another. As ν increases, it becomes relatively more expensive to motivate any particular level of effort, thus, $\hat{a}_1$, $\hat{h}_2$ and $\hat{a}_2$ decrease. Also, we expect both $\hat{t}_1$ and $\hat{t}_2$ to decrease as ν increases. However, we observe that for $\hat{t}_1$, this behavior does not always hold for $\nu > 0$. There can arise situations when $\hat{t}_1$ may increase as ν increases. This is because the rate of decrease in the incentive rate $\hat{a}_2$ may be much faster than that of $\hat{a}_1$ (due, for example, to a very low h_1). Thus, the attractiveness of working in task 2 drops by more than that for task 1, and the opportunity cost of working in task 1 decreases. The agent's attention is partially redirected from task 2 to task 1. These relationships are illustrated in Figure 7.18.

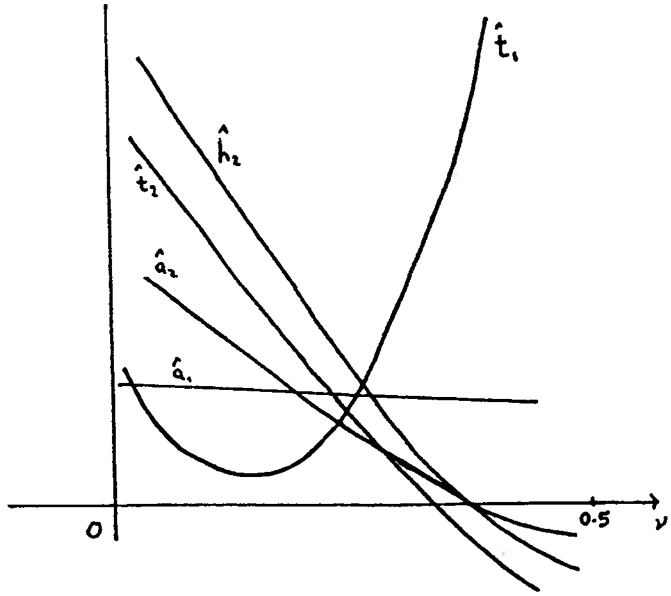

Figure 7.18: Behavior of $\hat{h}_2$, $\hat{t}_i$ and $\hat{a}_i$ as ν varies

Values of the parameters: $c = 0.2$, $\delta = 0.5$, $r = 5$ and $h_1 = 0.1$

Proposition 7.8: Increasing the agent's coefficient of absolute risk aversion (r) results in:

1. reduced incentive rate for task 1 if the tasks are substitutes, but the relationship is unclear if the tasks are complements, i.e., $\frac{\partial \hat{a}_1}{\partial r} \begin{cases} \textit{ambiguous} & \text{for } -1 < \nu < 0 \\ < 0 & \text{for } 0 < \nu < 1; \end{cases}$
2. reduced level of activity for task 1 if the tasks are complements, but the relationship is unclear if the tasks are substitutes, i.e., $\frac{\partial \hat{t}_1}{\partial r} \begin{cases} < 0 & \text{for } -1 < \nu < 0 \\ \textit{ambiguous} & \text{for } 0 < \nu < 1; \end{cases}$
3. reduced level of activity for task 2, i.e., $\frac{\partial \hat{t}_2}{\partial r} < 0$.

We were unable to establish the behavior of the level of monitoring and the incentive rate on task 2 with changes in r. However, numerical examples indicate that the level of monitoring is concave in r and is most intense at an intermediate level of risk aversion. This result is similar to that for a single-task model with costly monitoring. Proposition 6.2 states that monitoring is most intense at an intermediate level of risk aversion.

It is also interesting to contrast the results with Proposition 7.2 for the case when the precision of the signal of the agent's effort in task 2 is exogenously specified. As r increases, the required risk premium for a given effort level increases and the activity levels of the two tasks decrease. On the other hand, when the monitoring precision is endogenous, as r increases, the principal is able to partially control for the increased risk premium by choosing a higher precision. This benefit is offset by the direct cost of monitoring. Using cost-benefit analysis, the principal determines the optimal precision and activity levels. As Proposition 7.8 shows, as r increases, the behavior of the incentive rates and activity levels is not clear.

We first examine the case where $-1 < \nu < 0$. The incentive rate $\hat{a}_1$ is concave in r and reaches a maximum at a very low level of r. The incentive rate $\hat{a}_2$ may increase or decrease in r. It appears to increase when both h_1 and c are at low levels. As r increases, a higher risk premium is required for a given incentive rate. When h_1 and c are low, it becomes relatively more attractive to obtain more precise information on task 2 so that the principal can use a higher $\hat{a}_2$. At the same time, he reduces the use of $\hat{a}_1$ to avoid the high risk premium due to the low h_1. Recall from (7.11) that an increase in a_i affects the activity levels in both tasks positively. The diagrams below show some of these relationships.

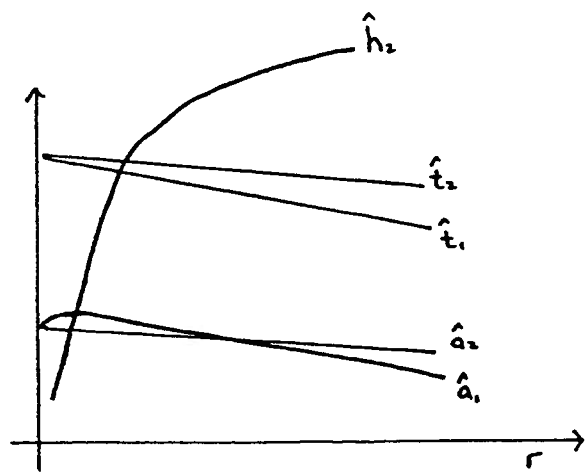

Figure 7.19: Behavior of $\hat{h}_2$, $\hat{t}_i$ and $\hat{a}_i$ as r varies

Values of the parameters: $c = 0.01$, $\delta = 0.5$, $\nu = -0.3$ and $h_1 = 5$

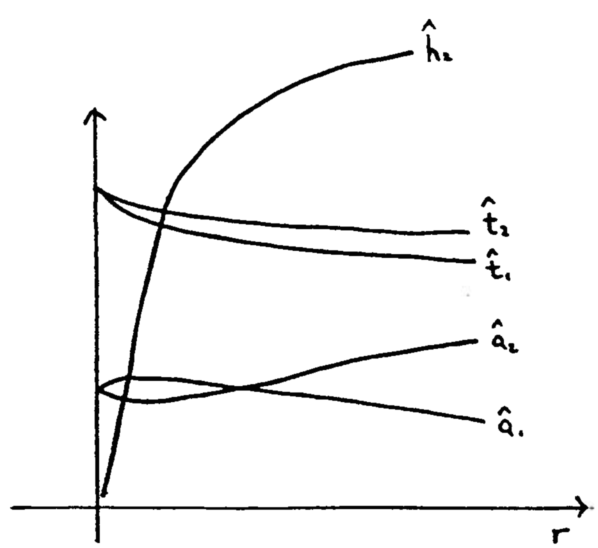

Figure 7.20: Behavior of $\hat{h}_2$, $\hat{t}_i$ and $\hat{a}_i$ as r varies

Values of the parameters: $c = 0.01$, $\delta = 0.5$, $\nu = -0.3$ and $h_1 = 1$

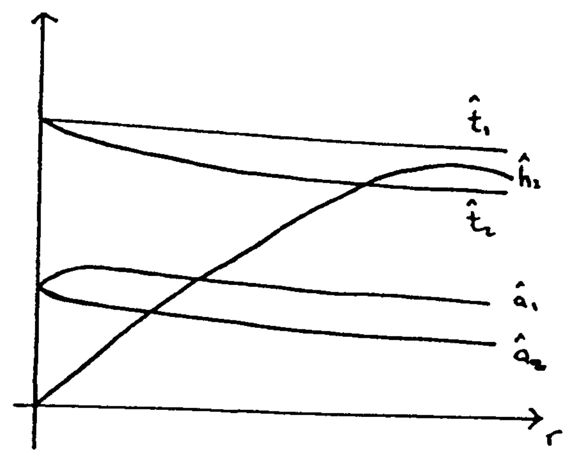

Figure 7.21: Behavior of $\hat{h}_2$, $\hat{t}_i$ and $\hat{a}_i$ as r varies

Values of the parameters: $c = 0.2$, $\delta = 0.5$, $\nu = -0.3$ and $h_1 = 5$

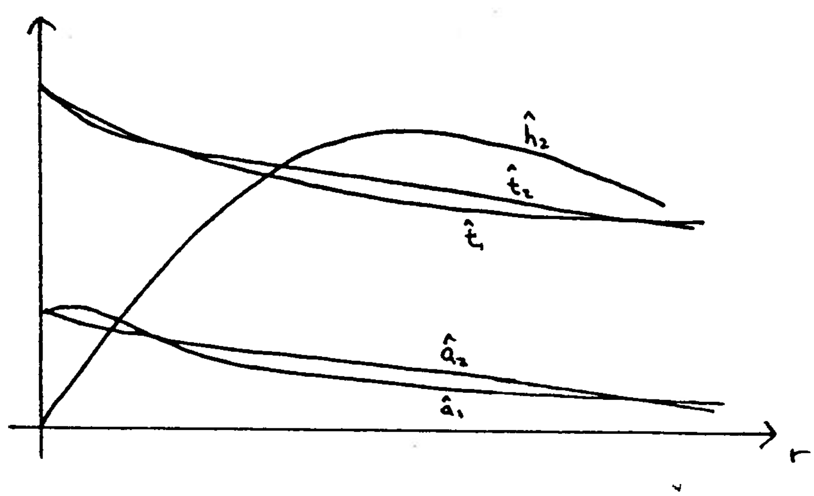

Figure 7.22: Behavior of $\hat{h}_2$, $\hat{t}_i$ and $\hat{a}_i$ as r varies

Values of the parameters: $c = 0.2$, $\delta = 0.5$, $\nu = -0.3$ and $h_1 = 1$

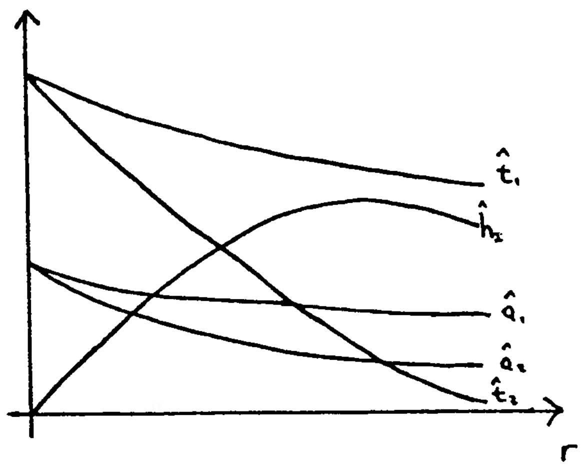

Figure 7.23: Behavior of $\hat{h}_2$, $\hat{t}_i$ and $\hat{a}_i$ as r varies

Values of the parameters: $c = 0.2$, $\delta = 0.5$, $\nu = 0.2$ and $h_1 = 5$

Next, we look at the case where $0 < \nu < 1$. The incentive rate $\hat{a}_2$ decreases in r for $\nu \leq \surd(\frac{2}{3})$, while the relationship is unclear for $\nu > \surd(\frac{2}{3})$. The relationship of the activity level of task 1 with r is not clear, but generally, it appears that $\hat{t}_1$ decreases as r increases. We illustrate some of these relationships in Figure 7.23.

7.4.8 The Value of Monitoring

The role of monitoring in this problem is performance evaluation rather than belief revision or information-verification. Without any information about task 2 and without monitoring, the combination of effort levels in the two tasks that can be induced are restricted or it may be impossible to induce effort in task 2. The availability of noisy performance measures on both tasks enables the principal to motivate higher effort levels and more satisfactory effort allocation. The issue then is the determination of the optimal effort level in each task given the acquired information. However, this information is imperfect. In fact, the principal chooses the level of precision of the information, but since the cost of perfect information is infinite, the principal never acquires perfect information. The use of imperfect information in the incentive contract increases uncertainty to the agent. If the principal uses only the imperfect information about the agent's effort in task 1, he just needs to pay a risk premium on that piece of information. However, with an imperfect monitor of task 2, the principal also chooses to pay a risk premium on the use of this second piece of information. The total risk premium is $\frac{1}{2}r[\frac{a_1^2}{h_1} + \frac{a_2^2}{h_2}]$. The cost of monitoring includes:

1. the direct cost of monitoring, $C(h_2) = ch_2$; and
2. the risk premium required on the second piece of imperfect information.

The availability of monitoring allows the principal to motivate higher effort levels or more satisfactory effort allocation. In fact, when $\nu > 0$, the second task is not undertaken if there is no monitoring. The benefits of monitoring are:

1. additional profit arising from higher effort levels or a more satisfactory effort allocation; and
2. the savings on the required risk premium on a_1, since the incentive component on task 1, a_1, is lower in the monitoring environment.

The principal has two main decisions to make about monitoring. First, he decides whether monitoring is to be undertaken. Generally, if the benefits of monitoring exceed the cost of monitoring, then monitoring is undertaken. This is equivalent to the total certainty equivalent in the monitoring environment being greater than that in the environment without monitoring. Next, the principal decides on the intensity of monitoring, represented by the precision of the signal obtained from monitoring, h_2. Our analysis does not distinguish between the two decisions. In fact, the first is not explicitly modelled.

Ignoring the direct cost of monitoring, the analysis above shows that:

1. For $-1 < \nu < 0$, information, no matter how noisy it is, is always valuable;

2. For $0 < \nu < 1$, information that is too noisy is not valuable. The principal is better off not using the information.

Here, we show that in a two-task setting, the value of a little bit of information depends on the degree of substitutability of the two effort types in the agent's cost function ν. It is positive for $\nu < 0$. However, when $\nu > 0$, a little bit of information is not valuable.

Next, we discuss how the intensity of monitoring varies with the agent's risk aversion coefficient, r. When r is very low, the risk premium required by the agent for a given effort level is not high, the principal settles for noisy information, and he uses a less intense level of monitoring. As the expression for $\hat{h}_2$ (see (7.27)) shows, as r approaches zero, the level of $\hat{h}_2$ approaches zero. For high levels of r, the principal chooses to use weak incentives to reduce the risk imposed on the agent, and we expect the intensity of monitoring to be reduced. Huddart (1993) states that "monitoring is valuable only when coupled with an incentive scheme responsive to the signals generated". For very risk averse agents, the principal settles for low monitoring, low incentives and low output. Numerical examples indicate that the intensity of monitoring is highest at intermediate levels of risk aversion. A similar behavior is observed in a one-task setting when a costly monitoring technology is employed.[26]

The principal may also find it worthwhile to invest in monitoring even when there is costlessly available information. This occurs when the costlessly available information is very noisy and the cost of monitoring is relatively low. With the advances in the information technology, we expect that the present cost of data collection and information analysis is very low. With such low cost monitoring, the principal should no longer just settle for freely available information which is collected for different purposes altogether. Using highly noisy information as a performance measure forces the principal

[26] See Proposition 6.2.

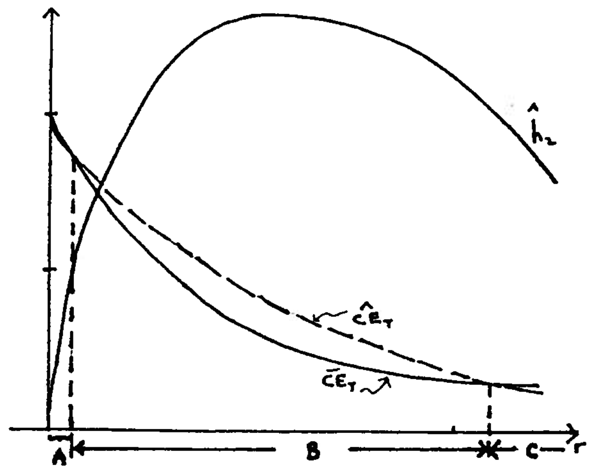

Figure 7.24: Behavior of total certainty equivalent with r

Values of parameters: $\delta = 0.5$, $h_1 = 5$, $h_2 = 0.5$, $\nu = -0.5$, and $c = 0.1$

to settle for low effort intensity and low output. Here, we are assuming that the costlessly available information is a noisy version of the monitoring information, thus it has zero value given the availability of monitoring information. We construct two numerical examples to illustrate that the principal may be better off investing in inexpensive costly monitoring technology than settling for highly noisy information. We assume that the principal seeks to motivate the agent to work on both tasks. Recall that $\bar{CE}_T$ is the total certainty equivalent when costless performance measures are available for both tasks and both tasks are undertaken. For the two examples, let $h_2 = 0.5$. When a costless performance measure is available for only task 1 and the principal invests in monitoring, we let the total certainty equivalent net of monitoring cost be denoted by $\hat{CE}_T$, and we let $c = 0.1$. The first example is for the case when the agent's effort types are complementary in his cost function. We let $\nu = -0.5$. Figure 7.24 depicts the behavior of the optimal monitoring level and the total certainty equivalent as r varies.

The second example is for the case when the agent's effort types are substitutable in his cost function, and we let $\nu = 0.5$. The behavior of the optimal monitoring level and the total certainty equivalent as

r varies is similar to that for $\nu = -0.5$ as depicted in Figure 7.24. Note, however, that when $\nu < 0$, the principal is able to achieve higher certainty equivalent levels.

From Figure 7.24, we observe three partitions to r. In region A, the agent is not very risk averse. Even when the information is very noisy, the required risk premium is low and the principal can still use strong incentives to motivate high output. The principal does not invest in monitoring. As the agent becomes more risk averse, a higher risk premium is required for the same incentive rate. We observe in region B that $\hat{CE}_T > \bar{CE}_T$, i.e., the principal is better off investing in monitoring to obtain more precise information to motivate higher effort levels. As the agent becomes even more risk averse, the principal settles for lower effort levels and lower incentive rates and we see in region C that the principal does not invest in monitoring but uses the costlessly available information. When information is noisy, using low incentive rates result in a low risk premium.

7.5 Some Implications

7.5.1 Job Design, Organization Structure and Incentive Plans

The above analysis shows clearly that when an agent is responsible for more than one task, incentive issues should not be addressed task by task. It is necessary that the principal studies the incentive problems for all the tasks together. This allows the principal to take advantage of any interdependency in the agent's cost of effort function through proper job design. The presumption is that any change in the task assignment only changes the interactive term of the agent's cost function. The principal is then able to efficiently motivate higher effort levels and achieve higher profitability.

This benefit is derived by controlling the agent's personal cost of effort. The principal seeks to keep the value of the interactive effect of the agent's effort on his cost function as low as possible. The above analysis shows that the principal is clearly better off when the effort levels are complementary in the agent's cost function, i.e., the marginal disutility of achieving task i decreases as the effort level in task

j increases. Thus, if possible, jobs should be designed to achieve this. The lower cost of effort implies that it is now optimal for the principal to motivate the agent to attain a higher effort level, and a higher profit level can be attained.[27]

In section 7.2, we relate organization structure to the interactive effect of the agent's effort on his cost function. In divisionalized firms, it is likely that the agent's effort levels are complementary in his cost function. On the other hand, in functionally-structured firms, it is likely that the agent's effort levels are substitutes in the agent's cost function. It has been observed that the divisionalized forms have largely displaced the centralized functional forms as the dominant structure for such firms (Rumelt (1986), pp. 63–69). Armour and Teece's (1978) survey of the petroleum industry shows that the divisionalized firms outperformed the functionally-structured firms. Our analysis shows that a contributing factor to better performance in divisionalized-structured firms may be that the structure takes advantage of the negative interactive effect of the agent's effort on his cost function. Thus, the lower cost of effort makes it efficient for the principal to motivate a higher level of effort to achieve higher profit. However, empirical evidence on the performance of divisionalized firms suggests that expected efficiency gains in such firms may not always hold. In fact, a number of studies (Hill, 1985; Hoskisson and Hitt, 1988) have indicated that tight financial controls and incentives based on divisional performance in divisionalized organizations result in short-run profit maximization and risk-avoidance behavior in the divisional managers. Our analysis indicates that such a result is not unexpected if proper incentives are not provided to the divisional managers to allocate his effort between activities.

For example, using accounting earnings as the only basis for performance evaluation is certainly not going to encourage agents to actively undertake and oversee risky projects that are healthy in the long-run but hurt short-term profits. In fact, if the interactive effect of the agent's effort on his cost function is positive, the agent concentrates on only the short-term effort. Examples of such long-term projects

[27] In our analysis, we ignore the interactive effect of effort on profit, i.e., we assume that the marginal profit from each task is not affected by the activity level in the other task.

are innovation and R & D work. The problem is further confounded by the accounting standards for R & D, which is very conservative. Most of the R & D costs are to be charged off as expenses of the period in which they are incurred. Thus, if accounting earnings is used for performance evaluation, there is no incentive at all for the agent to work on R & D projects if he does not expect to be there to reap the future benefits. Allocating effort to such a task not only reduces the effort that can be allocated to current operations, but it also results in a lowering of current period's earnings. Therefore, using the divisionalized structure in large multiproduct firms is not sufficient for the advantages of the structure to follow. Proper performance measures should be used and proper incentives must be provided to motivate the agent to allocate his effort between the numerous tasks.

The analysis in this paper implies that organization structure, job design and incentive plans cannot be designed separately. The principal needs to be very clear what he wishes the agent to achieve. Proper job design and incentive plans can then be used to induce the agent to achieve these objectives.

7.5.2 Investment in Monitoring Technology

The literature indicates extensive use of stock prices in the compensation contracts with the objective of motivating the agent to take a long term focus. While the stock price can be a congruent measure (in the sense of Feltham and Xie (1994)), it is likely to lack precision. Similarly, some costlessly available information, produced to meet financial reporting needs, may be too noisy to be able to lead to high optimal effort levels. Our analysis indicates that the principal should consider investing in a costly monitoring technology to extract more precise and congruent performance measures. With the advances in information technology, the cost of information extraction is unlikely to be high. The principal should investigate the possible benefits from obtaining more precise and congruent measures and should compare these benefits with the cost of monitoring. Information on the firm's key success factors should not be omitted from the firm's information system simply because they are costly to obtain. Johnson and

Kaplan (1987) state that measuring and reporting nonfinancial indicators is important. These indicators should be based on the company's strategy and include key measures of manufacturing, marketing and R & D success. A company should not settle for information extracted from a system designed to satisfy external reporting and auditing requirements. Rather, a management accounting system should be designed to be consistent with the technology of the organization, its product strategy, and its organization structure. They also warn that poor management accounting systems can contribute to the decline of the organization. Our analysis indicates that this warning should indeed be taken seriously. When inappropriate or very noisy performance measures are used, the principal settles for low effort and low profit.

Appendix 7A

Proofs

(I) The Model

(1) Derivation of incentive rates (7.3)

The two constraints of problem [P7.1] are

$$\begin{aligned} a_1 - V_1(t) &= 0 \\ \text{and } a_2 - V_2(t) &= 0. \end{aligned}$$

Let λ_1 and λ_2 be the respective lagrange multipliers of the two constraints. Computing the first order necessary conditions, we obtain:

$$\begin{aligned} t_1 : \quad & \Pi_1 - V_1(t) - \lambda_1 V_{11}(t) - \lambda_2 V_{21}(t) = 0 \\ t_2 : \quad & \Pi_2 - V_2(t) - \lambda_1 V_{12}(t) - \lambda_2 V_{22}(t) = 0 \end{aligned} \tag{7.32}$$

$$\begin{aligned} a_1 : \quad & -\frac{ra_1}{h_1} + \lambda_1 = 0 \\ a_2 : \quad & -\frac{ra_2}{h_2} + \lambda_2 = 0 \end{aligned} \tag{7.33}$$

From (7.33), we obtain:

$$\begin{aligned} \lambda_1 &= \frac{ra_1}{h_1} \\ \text{and } \lambda_2 &= \frac{ra_2}{h_2}. \end{aligned} \tag{7.34}$$

From (7.2), we know that $V_i(t) = a_i$. Substituting a_i for $V_i(t)$ and (7.34) for λ_1 and λ_2 into (7.32), we obtain:

$$\begin{aligned} \Pi_1 - a_1 - \frac{ra_1}{h_1}V_{11} - \frac{ra_2}{h_2}V_{21} &= 0 \\ \Pi_2 - a_2 - \frac{ra_1}{h_1}V_{12} - \frac{ra_2}{h_2}V_{22} &= 0. \end{aligned} \tag{7.35}$$

Equations (7.35) can be written in matrix notation as follows:

$$\begin{bmatrix} \Pi_1 \\ \Pi_2 \end{bmatrix} - \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} - r \begin{bmatrix} V_{11} & V_{21} \\ V_{12} & V_{22} \end{bmatrix} \begin{bmatrix} \frac{a_1}{h_1} \\ \frac{a_2}{h_2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} \Pi_1 \\ \Pi_2 \end{bmatrix} - \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} - r \begin{bmatrix} V_{11} & V_{21} \\ V_{12} & V_{22} \end{bmatrix} \begin{bmatrix} \frac{1}{h_1} & 0 \\ 0 & \frac{1}{h_2} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} \Pi_1 \\ \Pi_2 \end{bmatrix} = \left[I_2 + r \begin{bmatrix} V_{11} & V_{21} \\ V_{12} & V_{22} \end{bmatrix} \begin{bmatrix} \frac{1}{h_1} & 0 \\ 0 & \frac{1}{h_2} \end{bmatrix} \right] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \left[I_2 + r \begin{bmatrix} V_{11} & V_{21} \\ V_{12} & V_{22} \end{bmatrix} \begin{bmatrix} \frac{1}{h_1} & 0 \\ 0 & \frac{1}{h_2} \end{bmatrix} \right]^{-1} \begin{bmatrix} \Pi_1 \\ \Pi_2 \end{bmatrix}.$$

(2) Costless performance measure available for only task 1:

Derivation of incentive rates [(7.5) and (7.6)]

- Case 1: Expression (7.5)

  Substituting $h_2 = 0$ into (7.4), we obtain the following:

$$\begin{aligned} a_1 &= \frac{h_1[\Pi_1 r V_{22} - \Pi_2 r V_{12}]}{(rV_{11} + h_1) r V_{22} - r^2 V_{12}^2} \\ &= \frac{h_1[\Pi_1 - \Pi_2 \frac{V_{12}}{V_{22}}]}{h_1 + r(V_{11} - \frac{V_{12}^2}{V_{22}})}. \\ a_2 &= 0. \end{aligned}$$

- Case 2: Expression (7.6)

  Let λ be the lagrange multiplier for the constraint of problem [P7.2]. The FOC are:

$$t_1: \quad \Pi_1 - V_1(t) - \lambda V_{11}(t) = 0 \tag{7.36}$$

$$a_1: \quad -\frac{r a_1}{h_1} + \lambda = 0 \tag{7.37}$$

From (7.37), we obtain

$$\lambda = \frac{ra_1}{h_1}.$$

Substituting a_1 for V_1 and $\frac{ra_1}{h_1}$ for λ into (7.36), we obtain the following:

$$\begin{aligned} a_1 &= \Pi_1 - \frac{ra_1}{h_1}V_{11} \\ &= \frac{\Pi_1 h_1}{h_1 + rV_{11}}. \end{aligned}$$

(3) Using a monitor: Derivation of incentive rates (7.7)

The two constraints of problem [P7.3] are

$$\begin{aligned} a_1 - V_1(t) &= 0 \\ \text{and } a_2 - V_2(t) &= 0. \end{aligned}$$

Let λ_1 and λ_2 be the respective lagrange multipliers of the constraints. Computing the respective first order necessary conditions, we obtain:

$$t_1: \quad \Pi_1 - V_1(t) - \lambda_1 V_{11}(t) - \lambda_2 V_{21}(t) = 0$$

$$t_2: \quad \Pi_2 - V_2(t) - \lambda_1 V_{12}(t) - \lambda_2 V_{22}(t) = 0 \tag{7.38}$$

$$a_1: \quad -\frac{ra_1}{h_1} + \lambda_1 = 0$$

$$a_2: \quad -\frac{ra_2}{h_2} + \lambda_2 = 0 \tag{7.39}$$

$$h_2: \quad -C'(h_2) + \frac{ra_2^2}{2h_2^2} = 0 \tag{7.40}$$

From (7.39), we obtain:

$$\lambda_1 = \frac{ra_1}{h_1}$$

$$\lambda_2 = \frac{ra_2}{h_2} \tag{7.41}$$

The agent chooses t_i so that $a_i = V_i(t)$. Substituting a_i for $V_i(t)$ and (7.41) for λ_1 and λ_2 into (7.38), we obtain:

$$\begin{aligned} \Pi_1 - a_1 - \frac{ra_1}{h_1}V_{11} - \frac{ra_2}{h_2}V_{21} &= 0 \\ \Pi_2 - a_2 - \frac{ra_1}{h_1}V_{12} - \frac{ra_2}{h_2}V_{22} &= 0. \end{aligned} \tag{7.42}$$

Equations (7.42) can be written in matrix notation as follows:

$$\begin{bmatrix} \Pi_1 \\ \Pi_2 \end{bmatrix} - \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} - r \begin{bmatrix} V_{11} & V_{21} \\ V_{12} & V_{22} \end{bmatrix} \begin{bmatrix} \frac{1}{h_1} & 0 \\ 0 & \frac{1}{h_2} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} \Pi_1 \\ \Pi_2 \end{bmatrix} = \left[I_2 + r \begin{bmatrix} V_{11} & V_{21} \\ V_{12} & V_{22} \end{bmatrix} \begin{bmatrix} \frac{1}{h_1} & 0 \\ 0 & \frac{1}{h_2} \end{bmatrix} \right] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \left[I_2 + r \begin{bmatrix} V_{11} & V_{21} \\ V_{12} & V_{22} \end{bmatrix} \begin{bmatrix} \frac{1}{h_1} & 0 \\ 0 & \frac{1}{h_2} \end{bmatrix} \right]^{-1} \begin{bmatrix} \Pi_1 \\ \Pi_2 \end{bmatrix}$$

(II) Quadratic Cost Setting

(1) First-best setting: Derivation of optimal effort levels

The principal's problem is to select t_i to maximize $\Pi - V(t)$. Therefore, the first order condition with respect to t_i is given by $\Pi_i = V_i(t)$. Thus, we obtain the following:

$$\begin{aligned} \Pi_1 = 1 &= \delta(t_1 + \nu t_2) \\ \Pi_2 = 1 &= \delta(t_2 + \nu t_1). \end{aligned}$$

Hence, we obtain

$$t_1^* = t_2^* = \frac{1}{\delta(1+\nu)}$$

The total CE is given by:

$$\begin{aligned} CE_T(FB) &= \Pi(t^*) - V(t^*) \\ &= \frac{1}{\delta(1+\nu)}. \end{aligned}$$

(2) Derivation of optimal effort levels: Equation (7.14)

From (7.2), the optimal effort level is characterized by $V_i(t) = a_i$, $i = 1, 2$. Since $V_i = \delta(t_i + \nu t_j)$, we obtain the following equations:

$$\begin{aligned} \delta(t_1 + \nu t_2) &= a_1 \\ \text{and } \delta(t_2 + \nu t_1) &= a_2. \end{aligned}$$

Solving the two equations simultaneously, we obtain the following:

$$\begin{aligned} t_1 &= \frac{a_1 - \nu a_2}{\delta(1-\nu^2)} \\ t_2 &= \frac{a_2 - \nu a_1}{\delta(1-\nu^2)}. \end{aligned}$$

(3) Behavior of total certainty equivalent

1. $\frac{\partial \bar{CE}_T}{\partial h_1} = \frac{r[r^2\delta^2(1-\nu)^2+2r\delta h_2(1-\nu)+h_2^2]}{2[r^2\delta^2(\nu^2-1)-r\delta(h_1+h_2)-h_1h_2]^2}$

   > 0, since all terms are positive.

2. $\frac{\partial^2 \bar{CE}_T}{\partial h_1^2} = -\{\frac{r(r\delta+h_2)[r^2\delta^2(1-\nu)^2+2r\delta h_2(1-\nu)+h_2^2]}{[r^2\delta^2(1-\nu^2)+r\delta(h_1+h_2)+h_1h_2]^3}\}$

   < 0, since all terms in the $\{\}$ bracket are positive.

3. $\frac{\partial \bar{CE}_T}{\partial \nu} = -\{\frac{r^3\delta^3(1+\nu)(1-\nu)^2(h_1+h_2)+r^2\delta^2[h_1^2+h_1h_2(3(1-\nu^2)-2\nu)+h_2^2]+2r\delta h_1h_2(h_1+h_2)+h_1^2h_2^2}{\delta[r^2\delta^2(\nu^2-1)-r\delta(h_1+h_2)-h_1h_2]^2(1+\nu)^2}\}$

   < 0, since all terms in the $\{\}$ bracket are positive, which we prove below.

   This is obvious for $\nu < 0$. For $0 < \nu < 1$, the first, third and fourth terms in the $\{\}$ bracket are positive. The second term is also positive as we show below.

   To prove that $[h_1^2 + h_1h_2(3(1-\nu^2) - 2\nu) + h_2^2] > 0$:

   As ν increases, $[3(1-\nu^2) - 2\nu]$ decreases. As $\nu \to 1$, $[3(1-\nu^2) - 2\nu] \to -2$. Thus, at $\nu = 1$,

$$
\begin{aligned}
[h_1^2 + h_1h_2(3(1-\nu^2) - 2\nu) + h_2^2] &= [h_1^2 - 2h_1h_2 + h_2^2] \\
&= (h_1 - h_2)^2 \\
&> 0.
\end{aligned}
$$

   Therefore, within the range of $0 < \nu < 1$, $[3(1-\nu^2)-2\nu] > -2$ and $[h_1^2+h_1h_2(3(1-\nu^2)-2\nu)+h_2^2] > 0$.

4. $\frac{\partial \bar{CE}_T}{\partial r} = -\{\frac{r^2\delta^2(1-\nu)^2(h_1+h_2)+4r\delta h_1h_2(1-\nu)+h_1h_2(h_1+h_2)}{2[r^2\delta^2(\nu^2-1)-r\delta(h_1+h_2)-h_1h_2]^2}\}$

   < 0, since all terms in the $\{\}$ bracket are positive.

(4) Derivation of $\bar{h}_2$ given $\nu > 0$: Equation (7.18)

The principal motivates the agent to work on both tasks if, and only if, the total certainty equivalent when both tasks are undertaken exceeds that when only task 1 is undertaken. Equation (7.16) gives the total certainty equivalent when both tasks are undertaken, while (7.17) gives the total certainty equivalent when only task 1 is undertaken. Thus, the principal motivates the agent to work on both tasks if, and only if,

$$\begin{aligned}
\bar{CE}_T &> \bar{CE}_{T1} \\
\text{i.e.,}\quad \frac{r\delta(1-\nu)(h_1+h_2)+2h_1h_2}{2\delta(1+\nu)[(r\delta+h_1)(r\delta+h_2)-\nu^2r^2\delta^2]} &> \frac{h_1}{2\delta(h_1+r\delta)} \\
\text{which implies that}\quad h_2 &> \frac{\nu r\delta h_1[r\delta(1-\nu)(2+\nu)+2h_1]}{(1-\nu)[h_1+r\delta]^2} = \bar{h}_2.
\end{aligned}$$

(5) Proof of Lemma 7.1:

1. We use (7.13) and (7.15). Set $h_1 = h_2$ and we obtain the following:

$$\begin{aligned}
\bar{a}_1 = \bar{a}_2 &= \frac{h_2[h_2+r\delta(1-\nu)]}{(r\delta+h_2)^2-\nu^2r^2\delta^2} \\
\bar{t}_1 = \bar{t}_2 &= \frac{h_2^2+r\delta h_2(1-\nu)}{\delta(1+\nu)[(r\delta+h_2)^2-\nu^2r^2\delta^2]}.
\end{aligned}$$

2. We set $h_1 = h_2+\epsilon$, $\epsilon > 0$. Then, by substitution into (7.13), we obtain the following:

$$\begin{aligned}
\bar{a}_1 &= \frac{(h_2+\epsilon)[h_2+r\delta(1-\nu)]}{(r\delta+h_2+\epsilon)(r\delta+h_2)-\nu^2r^2\delta^2} \\
\bar{a}_2 &= \frac{h_2[h_2+\epsilon+r\delta(1-\nu)]}{(r\delta+h_2+\epsilon)(r\delta+h_2)-\nu^2r^2\delta^2}.
\end{aligned}$$

It is clear that $\bar{a}_1 > \bar{a}_2$. Similarly, by substituting $h_1 = h_2+\epsilon$ into (7.15), we obtain the following:

$$\begin{aligned}
\bar{t}_1 &= \frac{(h_2+\epsilon)h_2+r\delta(h_2-\nu h_2+\epsilon)}{\delta(1+\nu)[(r\delta+h_2+\epsilon)(r\delta+h_2)-\nu^2r^2\delta^2]} \\
\bar{t}_2 &= \frac{(h_2+\epsilon)h_2+r\delta(h_2-\nu h_2-\nu\epsilon)}{\delta(1+\nu)[(r\delta+h_2+\epsilon)(r\delta+h_2)-\nu^2r^2\delta^2]}.
\end{aligned}$$

Since $-\nu\epsilon < \epsilon$, $\bar{t}_1 > \bar{t}_2$.

(6) Proof of Proposition 7.1:

From (7.13),

$$\bar{a}_i = \frac{h_i[h_j + r\delta(1-\nu)]}{(r\delta + h_i)(r\delta + h_j) - \nu^2 r^2 \delta^2}, \quad i = 1, 2.$$

From (7.15),

$$\bar{t}_i = \frac{h_i h_j + r\delta(h_i - \nu h_j)}{\delta(1+\nu)[(r\delta + h_i)(r\delta + h_j) - \nu^2 r^2 \delta^2]}, \quad i = 1, 2.$$

Let Υ denote positive expressions.[28]

1. $\frac{\partial \bar{a}_1}{\partial h_2} = \frac{\nu r \delta h_1[h_1 + r\delta(1-\nu)]}{\Upsilon}$.

   Hence, the sign is the sign of ν.

2. $\frac{\partial \bar{a}_2}{\partial h_2} = \frac{r\delta[h_1 + r\delta(1-\nu)][h_1 + r\delta(1-\nu^2)]}{\Upsilon}$

   > 0, since all components are positive.

3. $\frac{\partial \bar{t}_1}{\partial h_2} = -\frac{\nu r^2 \delta[h_1 + r\delta(1-\nu)]}{\Upsilon}$.

   Hence, the sign is opposite to the sign of ν.

4. $\frac{\partial \bar{t}_2}{\partial h_2} = \frac{r(r\delta + h_1)[h_1 + r\delta(1-\nu)]}{\Upsilon}$

   > 0, since all components are positive.

(7) Proof of Proposition 7.2:

Effect on a_i

From (7.13),

$$\bar{a}_i = \frac{h_i[h_j + r\delta(1-\nu)]}{(r\delta + h_i)(r\delta + h_j) - \nu^2 r^2 \delta^2}, \; i = 1, 2.$$

Let Υ denote positive expressions.

[28] It is generally the square of the denominator in the corresponding expression above.

1. $\frac{\partial a_i}{\partial \nu} = -\{\frac{r\delta h_i[r^2\delta^2(1-\nu)^2+h_i(r\delta+h_j)+r\delta h_j(1-2\nu)]}{\Upsilon}\}$

   < 0, since the term in the $\{\}$ bracket is positive as we prove below.

   To prove that $h_i(r\delta + h_j) + r\delta h_j(1-2\nu) > 0$:

   From (7.15), $t_i > 0$ only if $h_i > \frac{r\delta h_j \nu}{r\delta + h_j}$. For $-1 < \nu < 1$:

$$\begin{aligned}
\nu &> 2\nu - 1 \\
\Rightarrow \frac{r\delta h_j \nu}{r\delta + h_j} &> \frac{r\delta h_j(2\nu-1)}{r\delta + h_j} \\
\Rightarrow h_i &> \frac{r\delta h_j(2\nu-1)}{r\delta + h_j} \\
\Rightarrow h_i(r\delta + h_j) &> r\delta h_j(2\nu - 1) \\
\Rightarrow h_i(r\delta + h_j) + r\delta h_j(1-2\nu) &> 0.
\end{aligned}$$

2. $\frac{\partial a_i}{\partial r} = -\{\frac{\delta h_i[r^2\delta^2(1-\nu)(1-\nu^2)+2r\delta h_j(1-\nu^2)+h_j(\nu h_i+h_j)]}{\Upsilon}\}$

   For $\nu > 0$, every term in the $\{\}$ bracket is positive, thus the sign is negative. However, we were unable to sign it for $\nu < 0$. We observe that if h_i is less than or comparable to h_j, then the term in the $\{\}$ bracket is positive and $\frac{\partial a_i}{\partial r} < 0$. However, the term in the $\{\}$ bracket can be negative if h_i is relatively very large compared to h_j. Then $\frac{\partial a_i}{\partial r} > 0$. We show this below:

$$\begin{aligned}
[r^2\delta^2(1-\nu)(1-\nu^2) + 2r\delta h_j(1-\nu^2) + \nu h_i h_j + h_j^2)] &< 0 \\
\Rightarrow h_i &> \frac{r^2\delta^2(1-\nu)(1-\nu^2) + 2r\delta h_j(1-\nu^2) + h_j^2}{-\nu h_j} \\
\Rightarrow h_i &> \frac{r^2\delta^2(1-\nu)(1-\nu^2)}{-\nu h_j} + \frac{2r\delta(1-\nu^2)}{-\nu} + \frac{h_j}{-\nu}.
\end{aligned}$$

<u>Effect on t_1</u>

From (7.15),

$$\bar{t}_i = \frac{h_i h_j + r\delta(h_i - \nu h_j)}{\delta(1+\nu)[(r\delta + h_i)(r\delta + h_j) - \nu^2 r^2 \delta^2]}$$

1. $\frac{\partial t_i}{\partial \nu} = -\frac{1}{\Upsilon}\{r^3\delta^3(1+\nu)[h_i(1-3\nu) + h_j(2\nu^2 - \nu + 1)]$

   $+r^2\delta^2[h_i^2 + h_i h_j(3(1-\nu^2) - 2\nu) + h_j^2] + 2r\delta h_i h_j(h_i + h_j) + h_i^2 h_j^2\}.$

   For $\nu < 0$, every term in the {} bracket is positive, thus the expression is negative. However, we were unable to sign it for $\nu > 0$.

2. $\frac{\partial t_i}{\partial r} = -\{\frac{r\delta(1-\nu)[r\delta(h_i - \nu h_j) + 2h_i h_j] + h_i h_j^2}{\Upsilon}\}$

   < 0, since the term in the {} bracket is positive as we prove below.

   To prove that $[r\delta(h_i - \nu h_j) + 2h_i h_j] > 0$:

   From (7.15), $t_i > 0$ only if $[r\delta(h_i - \nu h_j) + h_i h_j] > 0$. If this holds, then $[r\delta(h_i - \nu h_j) + 2h_i h_j] > 0$.

<u>**Costless performance measure for only task 1**</u>

(8) Derivation of optimal effort levels, (7.20)

When $\nu < 0$, we obtain an interior solution for t_2. The agent chooses the optimal effort levels so that $V_i(t) = a_i$, $i = 1, 2$. Since $V_i = \delta(t_i + \nu t_j)$ and from (7.19), $a_2 = 0$, we obtain the following equations:

$$\begin{aligned} \delta(t_1 + \nu t_2) &= a_1 \\ \text{and } \delta(t_2 + \nu t_1) &= 0. \end{aligned}$$

Solving the two equations simutaneously, we obtain the following:

$$\begin{aligned} t_1 &= \frac{a_1}{\delta(1-\nu^2)} \\ t_2 &= \frac{-\nu a_1}{\delta(1-\nu^2)}. \end{aligned}$$

(9) Behavior of incentive rates and effort levels

This proposition applies for $-1 < \nu < 0$ only.

<u>Effect on a_1</u>

From (7.19),

$$\tilde{a}_1 = \frac{h_1(1-\nu)}{h_1 + r\delta(1-\nu^2)}.$$

Let Υ denote positive expressions.[29]

1. $\frac{\partial a_1}{\partial h_1} = \frac{r\delta(1-\nu)(1-\nu^2)}{\Upsilon}$

   > 0, since all components are positive.

2. $\frac{\partial a_1}{\partial \nu} = -\{\frac{h_1[r\delta(1-\nu)^2 + h_1]}{\Upsilon}\}$

   < 0, since all components in the $\{\}$ bracket are positive.

[29] It is generally the square of the denominator in the corresponding expression above.

3. $\frac{\partial a_1}{\partial r} = -\{\frac{\delta h_1(1-\nu)(1-\nu^2)}{\Upsilon}\}$

   < 0, since all components in the {} bracket are postive.

Effect on t_1

From (7.20),

$$\tilde{t_1} = \frac{h_1}{\delta(1+\nu)[h_1 + r\delta(1-\nu^2)]}$$

Let Υ denote positive expressions.[30]

1. $\frac{\partial t_1}{\partial h_1} = \frac{r(1-\nu)}{\Upsilon}$

   > 0, since all components are positive.

2. $\frac{\partial t_1}{\partial \nu} = \frac{h_1[r\delta(3\nu-1)(1+\nu)-h_1]}{\Upsilon}$

   < 0, since the term in the [ ] bracket is negative.

3. $\frac{\partial t_1}{\partial r} = -\{\frac{h_1(1-\nu)}{\Upsilon}\}$

   < 0, since all the terms in the {} bracket are positive.

Effect on t_2

From (7.20),

$$\tilde{t_2} = \frac{-\nu h_1}{\delta(1+\nu)[h_1 + r\delta(1-\nu^2)]}$$

Let Υ denote positive expressions.[31]

1. $\frac{\partial t_2}{\partial h_1} = -\frac{\nu r(1-\nu)}{\Upsilon}$

   > 0, since $-\nu$ is positive.

[30] It is generally the square of the denominator in the corresponding expression above.
[31] It is generally the square of the denominator in the corresponding expression above.

2. $\frac{\partial t_2}{\partial \nu} = -\{\frac{h_1[r\delta(\nu^2(1+2\nu)+1)+h_1]}{\Upsilon}\}$

   < 0, since all the terms in the $\{\}$ bracket are positive.

   To prove that $\nu^2(1+2\nu)+1 > 0$: For $-1 < \nu < 0$,

$$\begin{aligned} 1+2\nu &> -1 \\ \Rightarrow \nu^2(1+2\nu) &> -\nu^2 \\ \Rightarrow 1+\nu^2(1+2\nu) &> 1-\nu^2 > 0. \end{aligned}$$

3. $\frac{\partial t_2}{\partial r} = \frac{h_1\nu(1-\nu)}{\Upsilon}$

   < 0, since ν is negative.

<u>Use of monitoring technology</u>

(10) Derivation of $\acute{h}_2$, (7.27)

From (7.9), the optimal level of h_2 is determined as follows:

$$\begin{aligned} c &= \frac{ra_2^2}{2h_2^2} \\ h_2 &= a_2\surd(\frac{r}{2c}). \end{aligned}$$

By substituting for a_2 from (7.25), we obtain the following:

$$\begin{aligned} h_2 &= \frac{h_2[r\delta + h_1 - \nu r\delta]}{(r\delta+h_1)(r\delta+h_2) - \nu^2r^2\delta^2}\surd(\frac{r}{2c}) \\ r\delta + h_2 &= \frac{[r\delta + h_1 - \nu r\delta]\surd(\frac{r}{2c}) + \nu^2r^2\delta^2}{r\delta+h_1} \\ h_2 &= \frac{h_1 + r\delta(1-\nu) + \nu^2r^2\delta^2\surd(\frac{2c}{r}) - \surd(\frac{2c}{r})r\delta(r\delta+h_1)}{\surd(\frac{2c}{r})(r\delta+h_1)} \\ &= \frac{h_1 + r\delta(1-\nu) - \delta\surd(2rc)[h_1 + r\delta(1-\nu^2)]}{\surd(\frac{2c}{r})(h_1+r\delta)}. \end{aligned}$$

(11) Proof of Lemma 7.2:

From (7.27),

$$\acute{h}_2 = \frac{h_1 + r\delta(1-\nu) - \delta\sqrt{(2rc)}[h_1 + r\delta(1-\nu^2)]}{\sqrt{(\frac{2c}{r})}(h_1 + r\delta)}.$$

1. $\acute{h}_2 > 0$ when the following holds:

$$\begin{aligned} h_1 + r\delta(1-\nu) &> \delta\sqrt{(2rc)}[h_1 + r\delta(1-\nu^2)] \\ \Rightarrow c &< \frac{[h_1 + r\delta(1-\nu)]^2}{2r\delta^2[h_1 + r\delta(1-\nu^2)]^2}. \end{aligned}$$

2. From (7.31), the marginal benefit of monitoring (before deducting the cost of monitoring) is

$$MB = \frac{r[h_1 + r\delta(1-\nu)]^2}{2[r^2\delta^2(1-\nu^2) + r\delta(h_1 + h_2) + h_1 h_2]^2}.$$

At $h_2 = 0$,

$$MB = \frac{[h_1 + r\delta(1-\nu)]^2}{2r\delta^2[h_1 + r\delta(1-\nu^2)]^2}.$$

When $c < \frac{[h_1 + r\delta(1-\nu)]^2}{2r\delta^2[h_1 + r\delta(1-\nu^2)]^2}$, the marginal cost of monitoring is less than the marginal benefit of monitoring at $h_2 = 0$. At the optimal level of monitoring, the marginal cost of monitoring equals the marginal benefit of monitoring. Thus, when $c < \frac{[h_1 + r\delta(1-\nu)]^2}{2r\delta^2[h_1 + r\delta(1-\nu^2)]^2}$, the optimal level of monitoring is strictly greater than zero.

(12) Derivation of marginal benefit MB of monitoring, (7.31)

From (7.30), for any given level of h_2, the total certainty equivalent before deducting the cost of monitoring is

$$CE_T(h_2) = \frac{r\delta(1-\nu)(h_1 + h_2) + 2h_1 h_2}{2\delta(1+\nu)[(r\delta + h_1)(r\delta + h_2) - \nu^2 r^2 \delta^2]}$$

The MB of monitoring is given by the partial differentiation of $CE_T(h_2)$ with respect to h_2:

$$\frac{\partial CE_T}{\partial h_2} = \frac{r[h_1 + r\delta(1-\nu)]^2}{2[r^2\delta^2(1-\nu^2) + r\delta(h_1 + h_2) + h_1 h_2]^2}.$$

The bound on the cost of monitoring is given by $MB(h_2 = 0) = \frac{[h_1 + r\delta(1-\nu)]^2}{2r\delta^2[h_1 + r\delta(1-\nu^2)]^2}$. Then

$$\begin{aligned}\frac{\partial MB(h_2 = 0)}{\partial h_1} &= \frac{\nu(1-\nu)[h_1 + r\delta(1-\nu)]}{\delta[h_1 + r\delta(1-\nu^2)]^3} \\ &< 0, \quad \text{since } \nu < 0.\end{aligned}$$

(13) Proof of Lemma 7.3:

1. If the signal on the agent's effort in the second task is costlessly available and $h_1 > h_2$, the principal motivates the agent to work on both tasks if, and only if, (see (7.18))

$$h_2 > \bar{h}_2 = \frac{\nu r\delta h_1[r\delta(1-\nu)(2+\nu) + 2h_1]}{(1-\nu)[h_1 + r\delta]^2}.$$

If the signal on the agent's effort in the second task is costly, then a necessary condition for monitoring to be undertaken is that the level of monitoring (7.27) is greater than $\bar{h}_2$, i.e.,

$$\frac{h_1 + r\delta(1-\nu) - \delta\sqrt{(2rc)}[h_1 + r\delta(1-\nu^2)]}{\sqrt{(\frac{2c}{r})}(h_1 + r\delta)} > \frac{\nu r\delta h_1[r\delta(1-\nu)(2+\nu) + 2h_1]}{(1-\nu)[h_1 + r\delta]^2}.$$

For the expression to hold,

$$c < \frac{(1-\nu)^2\{h_1^2 + r\delta h_1(2-\nu) + r^2\delta^2(1-\nu)\}^2}{2r\delta^2(1+\nu)^2\{h_1^2 + r\delta(1-\nu)[2h_1 + r\delta(1-\nu)]\}^2}.$$

2. At the optimal level of monitoring intensity, from (7.29),

$$\hat{t_1} = \frac{h_1 - \nu r\delta[1 - \delta\sqrt{(2rc)}(1+\nu)]}{\delta(h_1 + r\delta)(1+\nu)}.$$

For $\hat{t_1} > 0$, $h_1 > \nu r\delta[1 - \delta\sqrt{(2rc)}(1+\nu)]$.

(14) Proof of Proposition 7.3:

In the monitoring environment, (7.28) and (7.29) give the optimal levels of $\hat{a}_i$ and $\hat{t}_i$. In the non-monitoring environment, for $\nu < 0$, (7.19) and (7.20) give the optimal levels of $\tilde{a}_i$ and $\tilde{t}_i$. Then,

$$\begin{aligned}
\hat{t}_1 - \tilde{t}_1 &= \frac{-\nu r[h_1 + r\delta(1-\nu) - \delta\sqrt(2rc)[h_1 + r\delta(1-\nu^2)]}{(h_1 + r\delta(1-\nu^2))(r\delta + h_1)} \\
&> 0. \\
\hat{t}_2 - \tilde{t}_2 &= \frac{h_1 + r\delta(1-\nu) - \delta\sqrt(2rc)[h_1 + r\delta(1-\nu^2)}{\delta(h_1 + r\delta(1-\nu^2))} \\
&> 0. \\
\hat{a}_1 - \tilde{a}_2 &= \frac{\nu h_1[h_1 + r\delta(1-\nu) - \delta\sqrt(2rc)[h_1 + r\delta(1-\nu^2)]}{(h_1 + r\delta(1-\nu^2))(r\delta + h_1)} \\
&< 0.
\end{aligned}$$

(15) Proof of Proposition 7.4:

In the monitoring environment, (7.28) and (7.29) give the optimal levels of $\hat{a}_i$ and $\hat{t}_i$. In the non-monitoring environment, for $\nu > 0$, (7.22) and (7.23) give the optimal levels of $\tilde{a}_i$ and $\tilde{t}_i$. Note that $\tilde{a}_2 = 0$ and $\tilde{t}_2 = 0$. Then,

$$\begin{aligned}
\hat{t}_1 - \tilde{t}_1 &= -\frac{\nu[h_1 + r\delta(1 - \delta\sqrt(2rc)(1+\nu))]}{\delta(r\delta + h_1)(1+\nu)} \\
&< 0. \\
\hat{a}_1 - \tilde{a}_1 &= -\frac{\nu\delta h_1\sqrt(2rc)}{r\delta + h_1} \\
&< 0.
\end{aligned}$$

(16) Proof of Proposition 7.5:

We assume interior solutions for optimal monitoring level and optimal effort levels. Therefore, $\hat{h}_2 = \acute{h}_2$.

From (7.27),

$$\acute{h}_2 = \frac{h_1 + r\delta(1-\nu) - \delta\sqrt{(2rc)}[h_1 + r\delta(1-\nu^2)]}{\sqrt{(\frac{2c}{r})}(h_1 + r\delta)}.$$

From (7.28) and (7.29),

$$\begin{aligned}
\hat{a}_1 &= \frac{h_1(1-\nu\delta\sqrt{(2rc)})}{h_1 + r\delta} \\
\hat{a}_2 &= \frac{h_1 + r\delta(1-\nu) - \delta\sqrt{(2rc)}[h_1 + r\delta(1-\nu^2)]}{h_1 + r\delta} \\
\hat{t}_1 &= \frac{h_1 - \nu r\delta[1 - \delta\sqrt{(2rc)}(1+\nu)]}{\delta(h_1 + r\delta)(1+\nu)} \\
\hat{t}_2 &= \frac{1 - \delta\sqrt{(2rc)}(1+\nu)}{\delta(1+\nu)}.
\end{aligned}$$

These are also used for propositions 7.6, 7.7 and 7.8.

1. $\frac{\partial h_2}{\partial c} = -\{\frac{\sqrt{(\frac{2r}{c})}(h_1+r\delta(1-\nu))}{4c(r\delta+h_1)}\}$

   < 0, since all terms in the {} bracket are positive.

2. $\frac{\partial a_1}{\partial c} = -\frac{\nu\delta h_1\sqrt{(\frac{2r}{c})}}{2(h_1+r\delta)}$.

   Hence the sign is opposite to the sign of ν.

3. $\frac{\partial a_2}{\partial c} = -\{\frac{\sqrt{(\frac{2r}{c})}\delta(h_1+r\delta(1-\nu^2))}{2(h_1+r\delta)}\}$

   < 0, since all terms in the {} bracket are positive.

4. $\frac{\partial t_1}{\partial c} = \frac{\nu r\delta\sqrt{(\frac{2r}{c})}}{2(h_1+r\delta)}$.

   Hence the sign is the sign of ν.

5. $\frac{\partial t_2}{\partial c} = -\sqrt{(\frac{r}{2c})}$

   < 0.

(17) Proof of Proposition 7.6:

1. $\frac{\partial h_2}{\partial h_1} = \frac{\sqrt{(2)}\nu r\delta[1-\nu\delta\sqrt{(2rc)}]}{2\sqrt{(\frac{c}{r})}(r\delta+h_1)^2}$.

   Hence the sign is the sign of ν. Recall from (7.28) that $[1-\nu\delta\sqrt{(2rc)}] > 0$ is necessary for $\hat{a}_1 > 0$.

2. $\frac{\partial a_1}{\partial h_1} = \frac{r\delta(1-\nu\delta\sqrt{(2rc)})}{(h_1+r\delta)^2}$

   > 0, since all terms are positive.

3. $\frac{\partial a_2}{\partial h_1} = \frac{\nu r\delta(1-\nu\delta\sqrt{(2rc)})}{(h_1+r\delta)^2}$.

   Hence the sign is the sign of ν.

4. $\frac{\partial t_1}{\partial h_1} = \frac{r(1-\nu\delta\sqrt{(2rc)})}{(h_1+r\delta)^2}$

   > 0, since all terms are positive.

5. $\frac{\partial t_2}{\partial h_1} = 0$.

(18) Proof of Proposition 7.7:

1. $\frac{\partial h_2}{\partial \nu} = -\{\frac{r\delta[1-2\nu\delta\sqrt{(2rc)}]}{\sqrt{(\frac{2c}{r})}(r\delta+h_1)}\}$

   < 0, since all terms in the $\{\}$ bracket are positive.

   To prove that $[1-2\nu\delta\sqrt{(2rc)}] > 0$:

   It is obvious if $\nu < 0$. If $\nu > 0$, we prove by contradiction:

$$\begin{aligned} \text{Suppose } 1-2\nu\delta\sqrt{(2rc)} &< 0. \\ \text{Then } \nu &> \frac{1}{2\delta\sqrt{(2rc)}}. \end{aligned}$$

   From (7.29), $\hat{t}_2 > 0$ only if $\nu < \frac{1}{\delta\sqrt{(2rc)}} - 1$. Thus, for an interior solution to exist:

$$\begin{aligned} \frac{1}{2\delta\sqrt{(2rc)}} &< \frac{1}{\delta\sqrt{(2rc)}} - 1 \\ \Rightarrow \frac{1}{2\delta\sqrt{(2rc)}} &> 1 \end{aligned}$$

$$\begin{aligned}
\Rightarrow \delta &< \frac{1}{2\sqrt{(2rc)}}. \\
\text{For } 0<\nu<1,\ \frac{1}{2\sqrt{(2rc)}} &< \frac{1}{2\nu\sqrt{(2rc)}}, \\
\Rightarrow \delta &< \frac{1}{2\nu\sqrt{(2rc)}}, \\
\Rightarrow \nu &< \frac{1}{2\delta\sqrt{(2rc)}} \quad \text{a contradiction.}
\end{aligned}$$

Therefore, $[1-2\nu\delta\sqrt{(2rc)}]>0$.

2. $\frac{\partial a_1}{\partial \nu} = -\{\frac{c\delta h_1\sqrt{(\frac{2r}{c})}}{h_1+r\delta}\}$

   <0, since all terms in the $\{\}$ bracket are positive.

3. $\frac{\partial a_2}{\partial \nu} = -\{\frac{r\delta[1-2\nu\delta\sqrt{(2rc)}]}{(r\delta+h_1)}\}$

   <0, since all terms in the $\{\}$ bracket are positive.

4. $\frac{\partial t_1}{\partial \nu} = -\{\frac{r\delta(1-\delta\sqrt{(2rc)}(1+\nu)^2)+h_1}{\delta(h_1+r\delta)(1+\nu)^2}\}$.

   The terms in the $\{\}$ bracket is positive for $\nu<0$, as we prove below. We prove that $(1-\delta\sqrt{(2rc)}(1+\nu)^2)>0$ for $-1<\nu<0$:

$$\begin{aligned}
(1+\nu)^2 &< (1+\nu) \\
\Rightarrow 1-\delta\sqrt{(2rc)}(1+\nu)^2 &> 1-\delta\sqrt{(2rc)}(1+\nu).
\end{aligned}$$

   From (7.29), $\hat{t}_2>0$ only if $[1-\delta\sqrt{(2rc)}(1+\nu)]>0$.

   Therefore, we conclude that $[1-\delta\sqrt{(2rc)}(1+\nu)^2]>0$ and $\frac{\partial t_1}{\partial \nu}<0$.

   For $\nu>0$, we were unable to determine the sign of $\frac{\partial t_1}{\partial \nu}$. We observe that for high ν and low h_1, it is possible for the term in the $\{\}$ bracket to be negative, thus the sign is positive.

5. $\frac{\partial t_2}{\partial \nu} = -\frac{1}{\delta(1+\nu)^2}$

   <0.

(19) Proof of Proposition 7.8:

1. $\frac{\partial h_2}{\partial r} = -\frac{\sqrt(2)}{4r(r\delta+h_1)^2}[(r\delta h_1(3\nu-2)-r^2\delta^2(1-\nu)-h_1^2)\sqrt(\frac{r}{c})+2\sqrt(2)r\delta(r^2\delta^2(1-\nu^2)+2r\delta h_1(1-\nu^2)+h_1^2)]$.

   We were unable to sign the expression.

2. $\frac{\partial a_1}{\partial r} = -\{\frac{\sqrt(2)\delta h_1[\sqrt(\frac{2r}{c})-\nu r\delta+\nu h_1)]}{2\sqrt(\frac{r}{c})(h_1+r\delta)^2}\}$.

   The term in the {} bracket is positive for $\nu > 0$ but we were unable to sign it for $\nu < 0$.

   To prove that for $\nu > 0$, $[\sqrt(\frac{2r}{c}) - \nu r\delta] > 0$:

   Suppose not. Then,

$$\begin{aligned}\sqrt(\frac{2r}{c}) &< \nu r\delta \\ \Rightarrow \nu &> \frac{2}{\delta\sqrt(2rc)}.\end{aligned}$$

   For $\hat{t}_2 > 0$, $\nu < \frac{1}{\delta\sqrt(2rc)} - 1$. However, since $\frac{2}{\delta\sqrt(2rc)} > \frac{1}{\delta\sqrt(2rc)} - 1$, therefore, $\nu > \frac{2}{\delta\sqrt(2rc)}$ cannot hold. Thus, we conclude that $[\sqrt(\frac{2r}{c}) - \nu r\delta] > 0$.

3. $\frac{\partial a_2}{\partial r} = -\frac{\delta}{\sqrt(\frac{2r}{c})(h_1+r\delta)^2}[h_1\nu\sqrt(\frac{2r}{c}) + r^2\delta^2(1-\nu^2) - h_1(r\delta(3\nu^2-2) - h_1)]$.

   We were unable to sign the expression.

4. $\frac{\partial t_1}{\partial r} = -\{\frac{\sqrt(2)h_1 - \nu\delta\sqrt(rc)(3h_1+r\delta)}{\sqrt(2)(h_1+r\delta)^2}\}$.

   The term in the {} bracket is positive for $\nu < 0$ but we were unable to sign it for $\nu > 0$.

5. $\frac{\partial t_2}{\partial r} = -\sqrt(\frac{c}{2r})$

   < 0.

Appendix 7B

Applicability to General Expressions of Signal

The model applies to general expressions of signals with some modifications. The signals on the agent's effort could be related to the agent's effort in the following manner:

$$y_1 = f_1(t_1) + \theta_1, \quad \theta_1 \sim N(0, h_1),$$

$$y_2 = f_2(t_2) + \theta_2, \quad \theta_2 \sim N(0, h_2),$$

where we assume that f_1 and f_2 are increasing and concave functions. Now, let

$$\mu_1 = f_1(t_1),$$

$$\mu_2 = f_2(t_2).$$

$$\text{Then,} \quad y_1 = \mu_1 + \theta_1,$$

$$y_2 = \mu_2 + \theta_2.$$

Since a choice of $t = (t_1, t_2)$ is equivalent to a choice of $\mu = (\mu_1, \mu_2)$, we could use μ as the action choice variable of the agent. Then the agent's personal cost function $V(t)$ and the expected profit function from the two tasks will need to be re-expressed in terms of μ.

$$V(f_1^{-1}(\mu_1), f_2^{-1}(\mu_2)),$$

$$\Pi(f_1^{-1}(\mu_1), f_2^{-1}(\mu_2)).$$

Since functions f_1 and f_2 are concave and increasing, their respective inverse functions are convex and increasing. As the agent's cost function $V(t)$ is convex with respect to t, this implies that it is also convex with respect to μ. The expected profit function $\Pi(t)$ is assumed to be concave. As such, there is no assurance that the transformed expected profit function will be concave with respect to μ. A sufficient condition to ensure that it will be concave is that $\Pi(t)$ is more concave than $f_1(t)$ and $f_2(t)$.

Appendix 7C

Applicability to General Profit Functions

Let $\Pi(\tau) = g_1(\tau_1) + g_2(\tau_2)$, where g_1 and g_2 are increasing and weakly concave. Define

$$t_1 = g_1(\tau_1) \Rightarrow \tau_1 = g_1^{-1}(t_1),$$

$$t_2 = g_2(\tau_2) \Rightarrow \tau_2 = g_2^{-1}(t_2).$$

We can re-express both the profit functions, $\Pi(\tau)$, and the agent's private cost function, $V(\tau)$, in terms of t_1 and t_2, given by:

$$\Pi = t_1 + t_2,$$

$$V(g_1^{-1}(t_1), g_2^{-1}(t_2)).$$

We use an example to show the transformation.

$$\begin{aligned} \Pi(\tau) &= \tau_1^{1/2} + \tau_2^{1/2}, \\ V(\tau) &= \tau_1^2 + \tau_2^2 + \nu\tau_1\tau_2. \end{aligned}$$

Define:

$$\begin{aligned} t_1 &= \tau_1^{1/2} \Rightarrow \tau_1 = t_1^2, \\ t_2 &= \tau_2^{1/2} \Rightarrow \tau_2 = t_2^2. \end{aligned}$$

We can then re-express the problem in terms of t_1 and t_2.

$$\begin{aligned} \Pi(t) &= t_1 + t_2, \\ V(t) &= t_1^4 + t_2^4 + \nu t_1^2 t_2^2. \end{aligned}$$

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