1/F COMPONENTS

in

SHORT TERM HEART RATE VARIABILITY SIGNALS

by

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We accept this thesis as conforming
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Abstract

When analyzing short term heart rate variability (HRV) signals using the FFT technique, the linear trend appears to be perfect 1/f signals. The non-linear trend in short term HRV signals produces the regression 1/f components. De-trending the data using a moving average is an effective technique for removing the 1/f components. However, though removing the trend may sometimes produce clearer spectral pictures of respiratory sinus arrhythmia or the breathing frequency, it has little impact otherwise. The linear trend is fractal, but not chaotic. The 1/f components in the signal does not imply the signal fractal; nor does a fractal signal implies it chaotic. No evidence in the present research suggests that the non-linear trend is fractal, nor is there evidence to suggest that the HRV signal is fractal, or chaotic.

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Acknowledgment

I thank Dr. David R. Jones for his guidance and support of the research presented in this thesis. It was his expectations and encouragement, his patience and trust, that made this thesis possible. Also, I thank my committee members, Dr. John M. Gosline and Dr. James F. Carolan for their valuable comments pertaining to this research.

I would also like to thank Dr. David R. Jones and Russell Andrews for serving as subjects by allowing me to record their heart rate variability signals, and also Dr. Pierre Signore for setting up the equipment and assisting in data acquisition. I would like to thank Dr. Doug Syme for his careful review of the thesis and his very helpful comments, and, finally, all my colleagues and friends who have given their care, encouragement, support and help to me during the course of my studies.

I thank especially my wife, Cynthia, for her interest and enthusiasm in my studies; I thank her for her help in performing the analytical Fourier transform of the linear trend using MATHEMATICA; I thank her for her technical support with respect to the computer at home and in my office; I thank her for her inspirational discussions with me; I thank her for her genuine love and unshakable faith in me that made all the hard work and struggles worthwhile.

INTRODUCTION

Using spectral analysis to investigate the frequency characteristics of heart rate variability (HRV) signals dates back to 1973, when Sayers published his paper 'Analysis of Heart Rate Variability'(1). In this paper spectral analysis was used to determine blood pressure fluctuations as well as those of respiration. In the early 1980s, spectral analysis coupled with pharmacological blockade (2,3,4) and direct nervous stimulation (5) revealed that peaks of short-term HRV spectra >0.15 Hz were mediated almost entirely by the vagus, whereas those <0.15 Hz could be mediated by both vagal and cardiac sympathetic nervous inputs to the sinoatrial node (4). These results give a rationale for investigating short-term HRV spectra in cardiovascular research as well as in clinical situations (6).

Many investigators using spectral analysis of HRV signals came across a phenomenon which they named the 1/f components of the HRV signals. In 1982, Kobayashi and Musha reported that the HRV signal had a 1/f power spectrum in the frequency range slower than 0.02 Hz, when the heartbeat period was analyzed over time scales longer than 50s (7). Earlier, Mandelbrot and Ness (1968) had proposed the concept of fractional noise to describe the origin of the 1/f components(14). Later Saul et. al. (1988) used 5-minute data segments and calculated that at frequencies between 0.00003 and 0.1 Hz, the power spectrum of the HRV signal was fundamentally 1/f (8). Finally, Yamamoto and Hughson (1991) proposed a Coarse-Graining Spectral Analysis (CGSA) to selectively remove the 1/f or fractal components(6) to improve the quality of the power spectral diagrams.

The 1/f spectrum was first reported in 1925 for an electric current passing through a vacuum tube(9). Later, the 1/f spectrum was reported in the cellular membrane potential(10), frequency fluctuation of the alpha brain wave(11), concentration modulation of action potential impulses propagating in the giant axon of a squid(12), as well as highway traffic current fluctuations(13).

There are a lot of terms to describe this phenomenon, such as 1/f fluctuation, 1/f noise, 1/f spectrum, 1/f-like power spectrum, ... etc. They describe a spectrum in which the power is inversely proportional to its frequency. In mathematical form: $P(f) = \frac{\alpha}{f^{\beta}}$, where P is power, f is frequency, α and β are positive real constants. Two cases illustrating the 1/f spectrum follow:

Case 1: Fig.1-1 shows $P(f) = \frac{\alpha}{f^{\beta}}$ when α=1 and β=2. Fig.1-2 shows $P(f) = \frac{\alpha}{f^{\beta}}$ when it is plotted in double log form $\log(P) = \log(\alpha) - \beta\log(f)$. It is a straight line with a negative slope.

Fig. 1-1

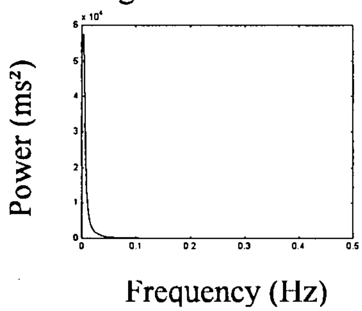
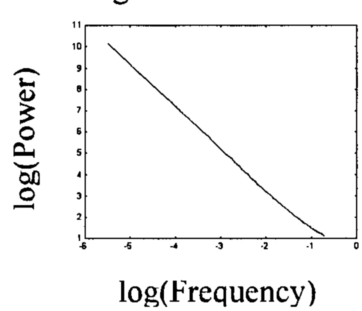


Fig. 1-2



Case 2: Usually, a perfect 1/f spectrum like those of Case 1 will not be seen. Instead, the more usual cases are those illustrated in Fig.1-3 and Fig.1-4.

Fig.1-3

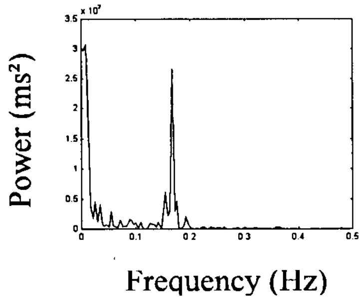
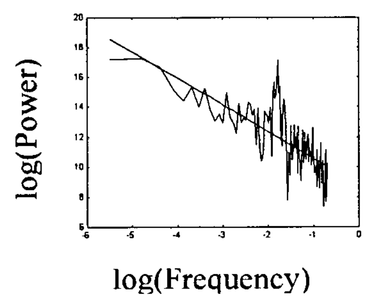


Fig.1-4



The spectrum is said to have 1/f components because there is a 1/f-like spectrum in the lower frequency range (0+ to 0.1+ Hz) of Fig.1-3 and the regression line of the double log spectrum (Fig.1-4) has a negative slope.

A perfect 1/f signal which produces the power spectra illustrated by Case-1 has never been observed before. This study identified its source as the linear trend of HRV. The origin of the regression 1/f spectra has been proposed (16), yet has not been identified before. This study shows it to be the non-linear trend of HRV. The CGSA technique (6) of removing the 1/f components is ineffective and mathematically incorrect. This study shows that de-trending by the method of moving average is quite effective for removing the lower frequency range 1/f components. An effective method for removing all 1/f components is also proposed.

What is the significance of knowing that the origin of the 1/f components is the trend? It was thought that the 1/f phenomenon had major implications with respect to the nature of HRV signals, in that the 1/f spectra indicated fractal noise. Further, these fractal components have been assumed to indicate chaos. This study clarifies that it is not the case: the 1/f component is nothing but the trend. This study also indicates that although the linear trend is fractal, it is by no means chaotic; clear evidence that being fractal does not imply chaos. No evidence in our research suggests that the non-linear trend is fractal, or the HRV signal is fractal, or the HRV signal is chaotic.

What is the significance of removing the 1/f components? The work of this thesis suggests that it is of little or no significance. Removal of the 1/f components may sometimes produce a clearer spectral picture of the breathing frequency, otherwise it is not necessary to remove the 1/f components as some used to believe (6).

METHODS

Data collecting: Electrocardiograph (EKG) signals were recorded from 3 subjects (DRJ, age 55; RA, age 32; YY, age 34) resting in an armchair using three electrical leads attached to the subjects' lower chest, upper chest and wrist. The leads were connected to an isolation unit which was connected to a Gould universal amplifier and filter. The EKG signals were sampled at 200 Hz on a computer (Lab Tech Notebook) and were stored as beat-to-beat or RR-intervals, also called heart rate variability (HRV) signals.

Data pre-processing: RR-intervals were linearly interpolated at 4 Hz to ensure equidistant sampling. This procedure inevitably introduced some higher frequency components into the signal. Therefore, these signals were filtered by a low pass filtering procedure at a cut-off frequency of 0.5 Hz. Then the signals were re-sampled at 1 Hz. This procedure reduces the data set size for calculation economy, yet keeps the information of interest intact. Since the highest frequency in the signals is 0.5 Hz, and the sampling frequency is 1 Hz, which is twice as much as the signal's highest frequency, and according the Nyquist sampling theorem (21), no aliasing should occur. The linear interpolations were done using the AWK program of the MKS TOOLKIT (Mortice Kern Systems Inc.). The filtering and re-sampling were done using MATLAB (Math Works, Inc. Cochituate Place, 24 Prime Park Way, Natick, Mass. 01760, USA).

Linear trends were calculated by the method of least squares. Non-linear trends were calculated by moving averages of 24 second periods in the case of 2 minute HRV signals, and 38 second periods in the case of 4 minute HRV signals.

All spectral analyses were done using MATLAB on PC.

Analytical Fourier Transform of the linear trend was done using MATHEMATICA. Courtesy of UBC TeleCom.

RESULTS

1) The 1/f Components due to the Linear Trend: Fig.3-1 is a one minute HRV signal segment. Its power spectrum (Fig.3-2) and its double-log plot (Fig.3-3) show that it has 1/f components. It has a linear trend (Fig.3-4). The power spectrum of the linear trend is a perfect 1/f spectrum (Fig.3-5). This is also shown in its double-log plot (Fig.3-6). When the trend is removed from the signal (Fig.3-7), the power spectrum and its double-log plot show that no 1/f components remain (Fig.3-8 and Fig. 3-9).

Note: 1. The tangent of the regression line of Fig.3-3 is -1.4242; whereas that of Fig.3-9 is 0.0471;

2. Changing the frequency range of interest will change the tangent of the regression line;

3. The peak at 0.1667 Hz in Fig.3-2 and Fig.3-8 correspond to the breathing frequency of the subject.

Fig.3-10 is another one minute HRV signal segment. Its power spectrum (Fig.3-11) and its double-log plot (Fig.3-12) show that it has 1/f components. It has a linear trend (Fig.3-13). The power spectrum of the linear trend is a perfect 1/f spectrum (Fig.3-14). This is also shown in its double-log plot (Fig.3-15). When the trend is removed from the signal (Fig.3-16), the power spectrum and its double-log plot show no 1/f components remain (Fig.3-17 and Fig. 3-18).

Note: 1. The tangent of the regression line of Fig.3-12 is -1.4115; whereas that of Fig.3-18 is 0.0579;

2. Changing the frequency range of interest will change the tangent of the regression line;

3. The peak at 0.1667 Hz in Fig.3-11 and Fig.3-17 correspond to the breathing frequency of the subject.

Signal Power Double-log

Original signal

Fig. 3-1

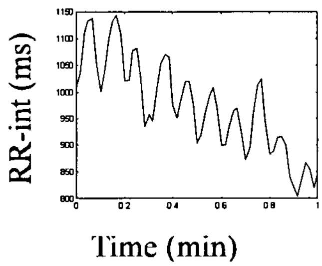


Fig. 3-2

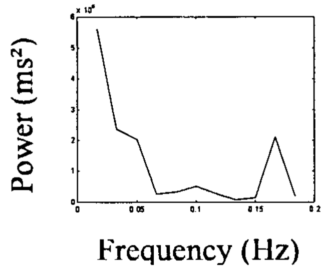
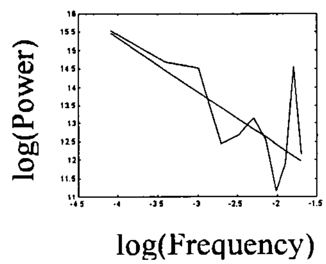


Fig. 3-3



Trend

Fig. 3-4

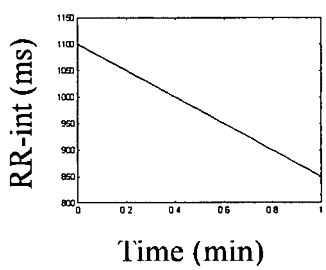


Fig. 3-5

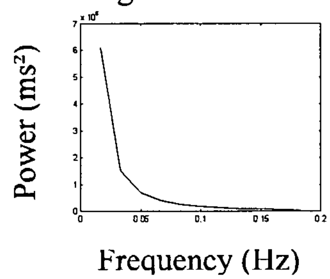
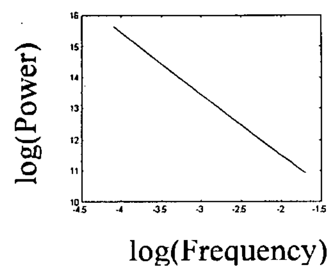


Fig. 3-6



Trend-free signal

Fig. 3-7

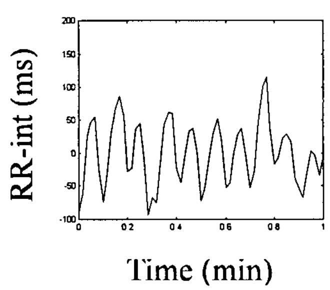


Fig. 3-8

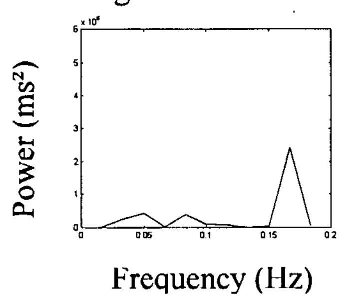
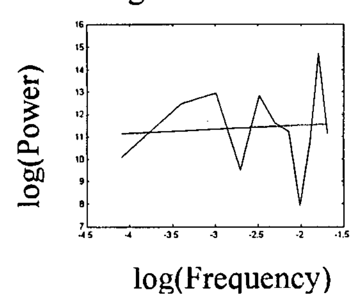


Fig. 3-9



Signal Power Double-log

Original signal

Fig. 3-10

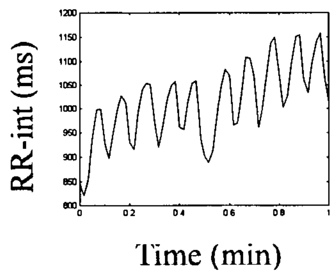


Fig. 3-11

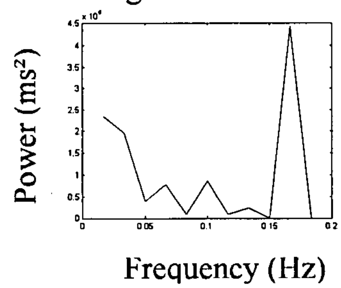
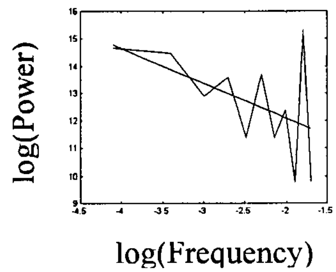


Fig. 3-12



Trend

Fig. 3-13

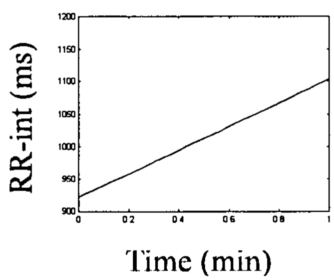


Fig. 3-14

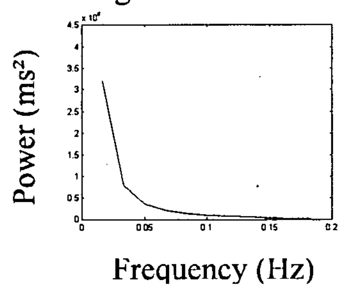
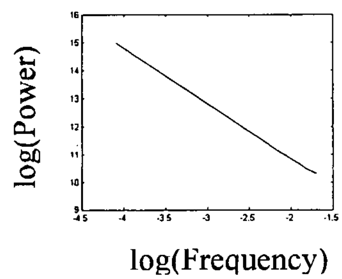


Fig. 3-15



Trend-free signal

Fig. 3-16

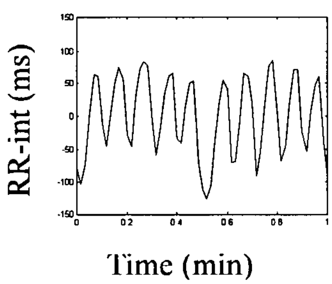


Fig. 3-17

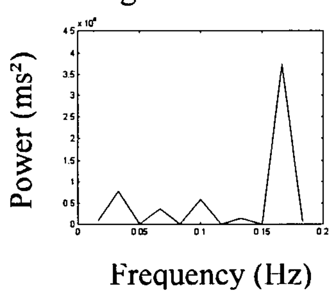
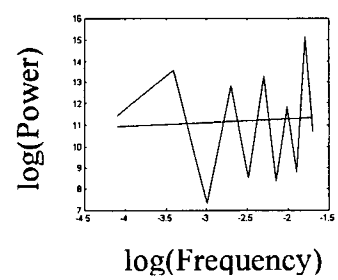


Fig. 3-18



2) The 1/f Components due to the Non-linear Trend: Fig.3-19 is a two minute HRV signal segment. Its power spectrum (Fig.3-20) and its double-log plot (Fig.3-21) show that it has 1/f like components. It has a non-linear trend (Fig.3-22). The power spectrum of the non-linear trend is not a perfect 1/f spectrum (Fig.3-23). This is also shown in its double-log plot (Fig.3-24). However, the regression line of the double-log plot is 1/f (Fig.3-24). When the trend is removed from the signal (Fig.3-25), power spectrum and its double-log plot show little remaining 1/f component (Fig.3-26 and Fig. 3-27).

Note: 1. The tangent of the regression line of Fig.3-12 is -1.2651; whereas that of Fig.3-18 is -0.0032;

2. Changing the frequency range of interest or the length of the moving average period will change the tangent of the regression line;

3. The peak at 0.1667 Hz in Fig.3-20 and Fig.3-26 corresponds to the breathing frequency of the subject.

Fig.3-28 is a four minute HRV signal segment. Its power spectrum (Fig.3-29) and its double-log plot (Fig.3-30) show that it has 1/f like components. It has a non-linear trend (Fig.3-31). The power spectrum of the non-linear trend is not a perfect 1/f spectrum (Fig.3-32). This is also shown in its double-log plot (Fig.3-33). However, the regression line of the double-log plot is 1/f (Fig.3-33). When the trend is removed from the signal (Fig.3-34), the power spectrum and its double-log plot show no 1/f components remain (Fig.3-35 and Fig. 3-36).

Note: 1. The tangent of the regression line of Fig.3-30 is -0.8936; whereas that of Fig.3-36 is 0.0295;

2. Changing the frequency range of interest or the length of the moving average period will change the tangent of the regression line;

3. The peak at 0.1667 Hz in Fig.3-29 and Fig.3-35 corresponds to the breathing frequency of the subject.

Signal | Power | Double-log

Original signal

Fig. 3-19

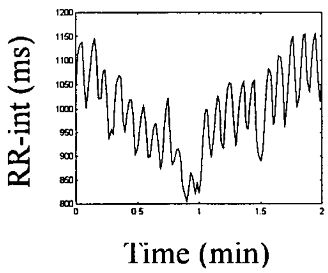


Fig. 3-20

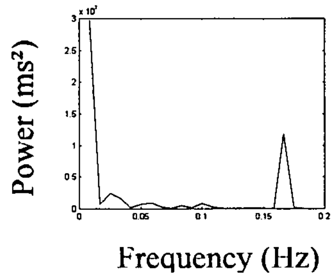
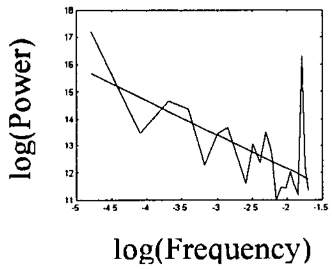


Fig. 3-21



Trend

Fig. 3-22

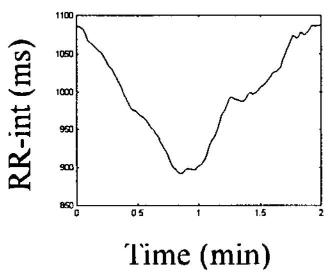


Fig. 3-23

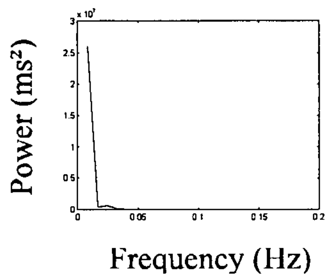
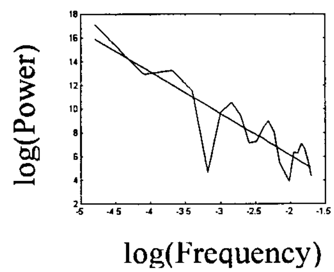


Fig. 3-24



Trend-free signal

Fig. 3-25

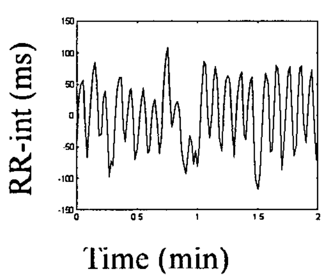


Fig. 3-26

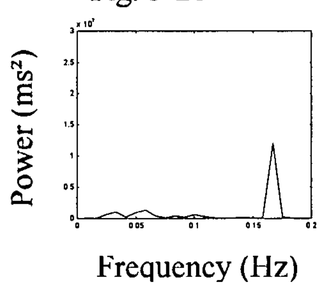
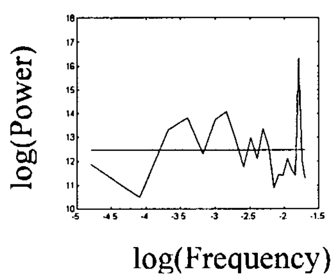


Fig. 3-27



Signal | Power | Double-log

Original signal

Fig. 3-28

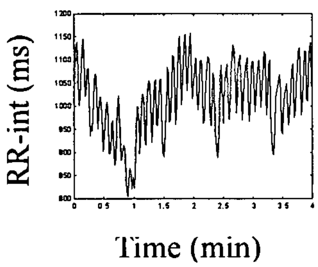


Fig. 3-29

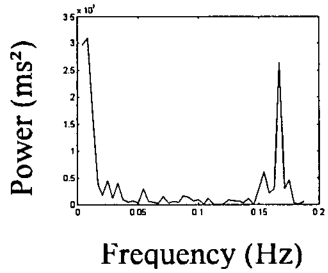
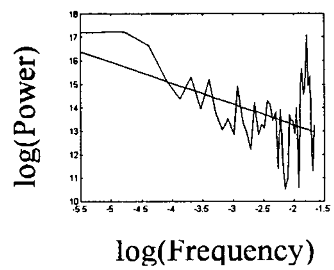


Fig. 3-30



Trend

Fig. 3-31

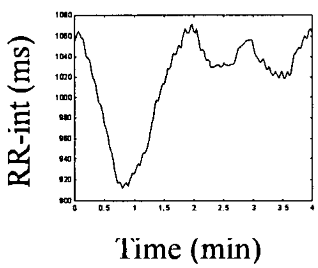


Fig. 3-32

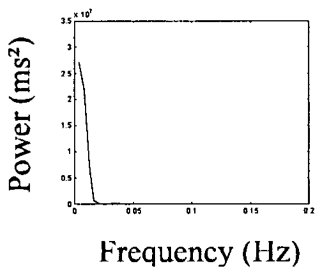
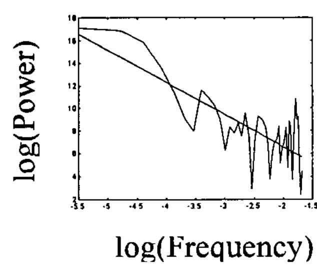


Fig. 3-33



Trend-free signal

Fig. 3-34

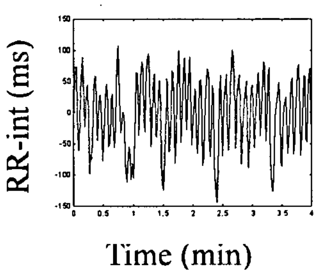


Fig. 3-35

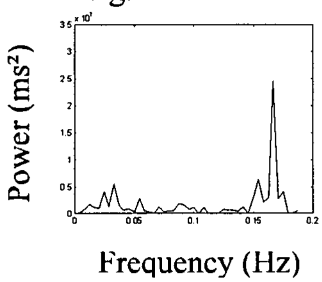
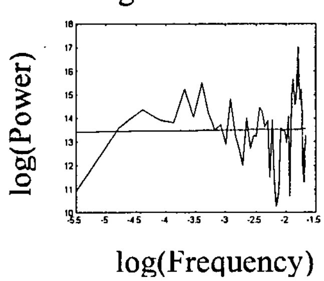


Fig. 3-36



DISCUSSION

1) The Linear Trend *Why does the linear trend appear to be a perfect 1/f signal?*

Every linear trend can be expressed by the equation:

$$y(t) = a \cdot t + b \qquad (0 \le t \le T) \qquad (1)$$

Where y is the dependent variable, t is time the independent variable, T is the signal length, a and b are real constants.

Its Fourier transform is

$$Y(\omega) = \frac{1}{T}\int_0^T y(t) * e^{-i\omega t} dt$$

And its result is

$$Y(\omega) = \begin{cases} a\dfrac{T}{2} + b & ,\omega = 0 \\ \dfrac{1}{T}\left[-\dfrac{a+ib\omega}{\omega^2} + \dfrac{e^{-iT\omega}(a+ib\omega+iaT\omega)}{\omega^2}\right] & ,\begin{bmatrix} 0 < \omega < +\infty \\ -\infty < \omega < 0 \end{bmatrix} \end{cases} \qquad (2)$$

where $\omega = 2\pi f$.

Digital Fourier Transform (DFT) calculates only part of the results in (2):

$$Y(\omega_n) = \begin{cases} a\dfrac{T}{2} + b & ,n = 1 \\ \dfrac{1}{T}\left[-\dfrac{a+ib\omega_n}{\omega_n^2} + \dfrac{e^{-iT\omega_n}(a+ib\omega_n+iaT\omega_n)}{\omega_n^2}\right] & ,n = 2,3,4\ldots N/2 \\ conj[Y(\omega_{N-n+1})] & ,n = (N/2+1),(N/2+2),\ldots,N \end{cases}$$

This is because in the analytical form, the period $F = \lim_{dt \to 0} \dfrac{1}{dt} = \infty$;

whereas in digital form $F = \frac{1}{\Delta t}$.

When in the FFT format where $\Delta f = \frac{1}{T}$

$$N = \frac{F}{\Delta f} = \frac{T}{\Delta t}.$$

$$\begin{aligned}\omega_n &= 2\pi f_n \\ &= 2\pi \cdot (n-1)\Delta f \\ &= 2\pi \cdot (n-1) \cdot \frac{1}{T}\end{aligned}$$

$$\begin{aligned}T\omega_n &= T \cdot 2\pi \cdot (n-1) \cdot \frac{1}{T} \\ &= 2\pi(n-1)\end{aligned}$$

$$\begin{aligned}e^{-iT\omega_n} &= \cos(T\omega_n) - i \cdot \sin(T\omega_n) \\ &= \cos[2\pi(n-1)] - i \cdot \sin[2\pi(n-1)] \\ &= 1\end{aligned}$$

$$\begin{aligned}Y(\omega_n) &= \frac{1}{T}[-\frac{a + ib\omega_n}{\omega_n^2} + \frac{e^{-iT\omega}(a + ib\omega_n + iaT\omega_n)}{\omega_n^2}] \\ &= \frac{1}{T}[-\frac{a + ib\omega_n}{\omega_n^2} + \frac{1 \cdot (a + ib\omega_n + iaT\omega_n)}{\omega_n^2}] \\ &= i\frac{a}{\omega}\end{aligned}$$

$$\begin{aligned}A(\omega_n) &= |Y(\omega_n)| \\ &= \left|\frac{a}{\omega_n}\right|\end{aligned}$$

$$\begin{aligned}P(\omega_n) &= A^2(\omega_n) \\ &= \left(\frac{a}{\omega_n}\right)^2\end{aligned}$$

Substituting $2\pi f_n$ for ω_n, we obtain:

$$P(f_n) = \left(\frac{a}{2\pi f_n}\right)^2$$
$$= \frac{(a/2\pi)^2}{f_n^{\,2}}$$

Let $\alpha = (a/2\pi)^2$, and $\beta = 2$,

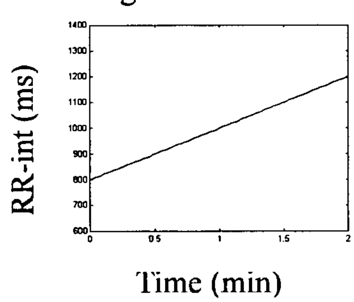
$$P(f_n) = \frac{\alpha}{f_n^{\,\beta}} \qquad , \; n = 2,3,4 \ldots N/2$$

Therefore the linear trend is a perfect 1/f signal when expressed in the FFT format.

The graphs shown below are of the trend equation $y(t) = \frac{10}{3}t + 800 \quad (0 \le t \le 120s)$ expressed in the FFT format:

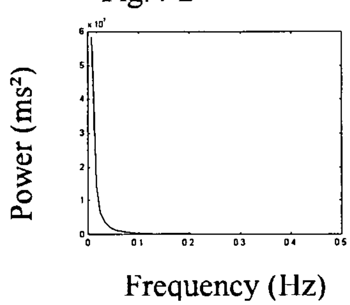
Signal

Fig.4-1



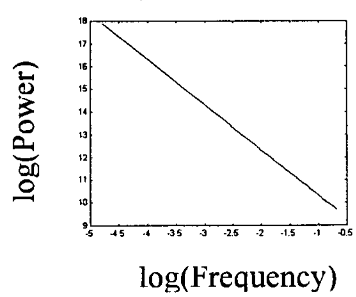
Power

Fig.4-2



Double-log

Fig.4-3



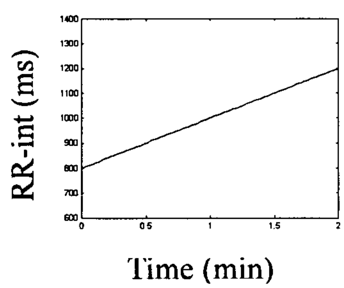
When in the DFT format of $\Delta f = \dfrac{1}{10*T}$

Then $N = \dfrac{F}{\Delta f} = \dfrac{10*T}{\Delta t}$.

and $Y(\omega_n) = \dfrac{1}{T}[-\dfrac{a+ib\omega_n}{\omega_n^2} + \dfrac{e^{-iT\omega_n}(a+ib\omega_n+iaT\omega_n)}{\omega_n^2}]$ $\quad ,n = 2,3,4 \ldots N/2$

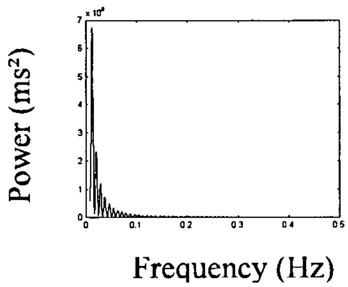
Signal

Fig.4-4



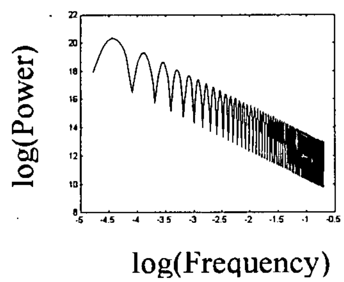
Power

Fig.4-5



Double-log

Fig.4-6

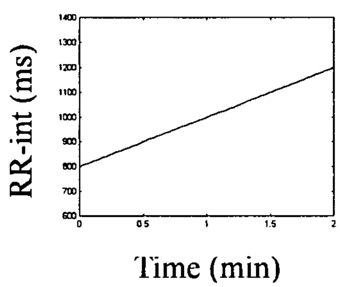


Here, it does not appear to be a perfect 1/f signal anymore. However, the base line of the power and the double-log form is what we have seen earlier in the FFT format (Fig.4-2 and Fig.4-3). So strictly speaking, the linear trend is not a perfect 1/f signal; it only appears to be one when its power spectrum is expressed in the FFT format where $\Delta f = \dfrac{1}{T}$. This is usually the case in spectral analysis.

Aliasing in the FFT of the linear trend. When using FFT at the sampling rate of 1 Hz to calculate the power and then the double-log of the power spectrum, we obtain Fig.4-8 and Fig.4-9 which are very similar to those of Fig.4-2 and Fig.4-3. However, there are some differences. The major differences being that the end of the line is tipped upward (Fig.4-9). It is due to aliasing. From formula (2) we know that the linear trend has infinite frequency components. The FFT's sampling rate here is set at 1 Hz. According to the Nyquist sampling theorem (21), the sampling rate should be at least twice that of the signal's highest frequency, otherwise, aliasing occurs. In the case of the linear trend, however, there is no way that one can have a sampling frequency twice that of its highest frequency, since that highest frequency is infinite. Therefore, aliasing is bound to occur. Fortunately, the power of the higher frequencies is usually very small compared with those of the lower frequency ones, so aliasing is negligible.

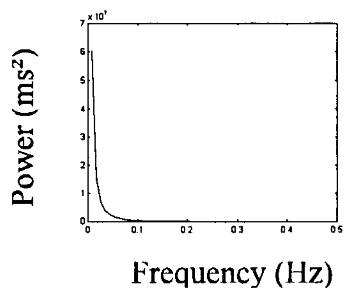
Signal

Fig.4-7



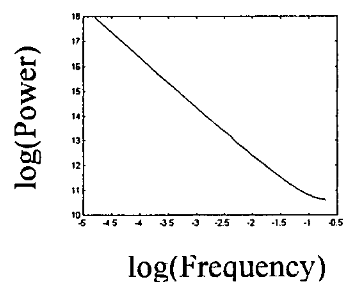
Power

Fig.4-8



Double-log

Fig.4-9



2) The Non-linear Trend *The frequency response of the de-trending procedure by moving average.* Fig.4-10 and Fig.4-11 show the frequency response of the transfer function which converts the signal of Fig.3-19 to the signal of Fig.3-25. Fig.4-12 and Fig.4-13 show the frequency response of the transfer function which converts the signal of Fig.3-28 to the signal of Fig.3-34. From Fig.4-10 and Fig.4-12 it can be seen that the de-trending procedure attenuates the amplitude of the lower frequency range of the signal, having little effect on the higher frequency range. From Fig.4-11 and Fig.4-13 it is can be seen that it also has little effect on the phase of the signals over all of the frequency range. De-trending by moving average only removes the lower frequency range of the 1/f components. Removal of the 1/f components over the entire frequency range needs a special technique which is discussed in the following section.

Amplitude Response | Phase Response

2-minutes

Fig.4-10

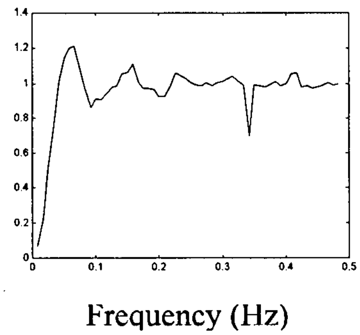
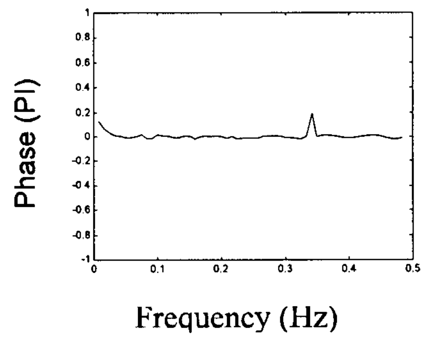


Fig.4-11



4-minutes

Fig.4-12

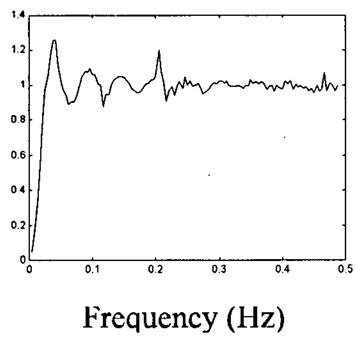
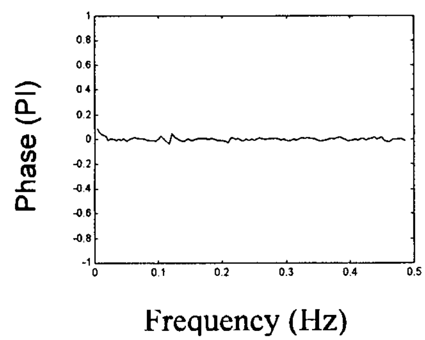


Fig.4-13



The procedure for removing 1/f components over the entire frequency range. A brief explanation of how this is accomplished is given here. Fig.4-14 is the same 4 minute signal as Fig. 3-28. Fig.4-15 is its power spectrum. Fig.4-16 is the power spectrum plotted in the double log form with its regression line (tangent =-1.7732), the 1/f component. The 1/f component free power, in the double log form with its regression line (tangent = 0) (Fig.4-19), is obtained by subtracting the components of the regression line from the components of the double log power spectrum then adding the results to the mean value of the components of the double log power spectrum. The anti-log is taken to obtain the 1/f component free power spectrum (Fig.4-18). The phases are not changed in the procedure. The 1/f component free signal (Fig.4-17) is obtained by inverse Fourier transform.

Fig.4-14

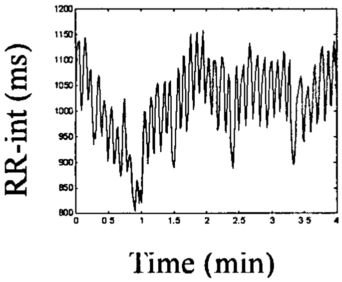


Fig.4-15

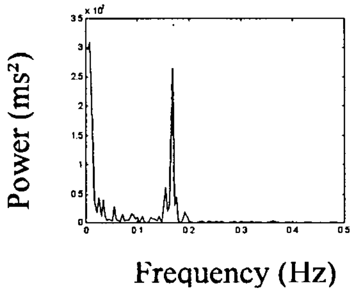


Fig.4-16

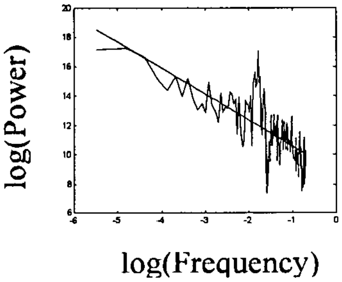


Fig.4-17

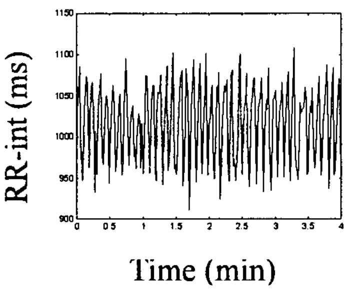


Fig.4-18

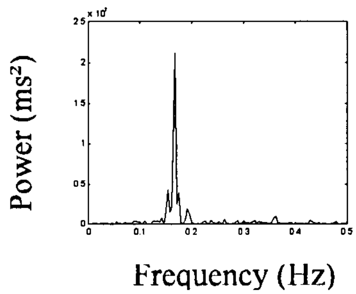
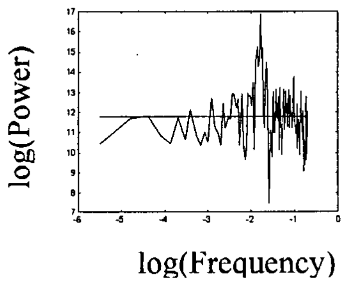


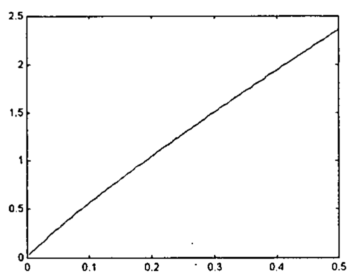
Fig.4-19



The *frequency response of the transfer function.* Figs.4-20 and 4-21 describe the frequency response of the transfer function which converts the signal of Fig.4-14 to that of Fig.4-17. From Fig.4-20 we can see that the procedure attenuates the amplitude of the lower frequency range of the signal, and amplifies that of the higher frequency range. From Fig.4-21 we can see that the procedure does not affect the phase of the signals. This procedure removes all the 1/f components over the entire frequency range.

Amplitude Response

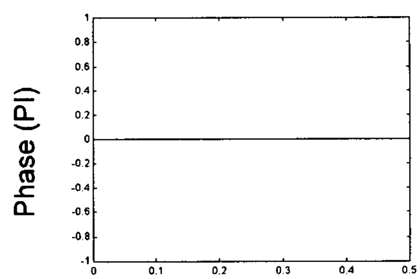
Fig.4-20



Frequency (Hz)

Phase Response

Fig.4-21



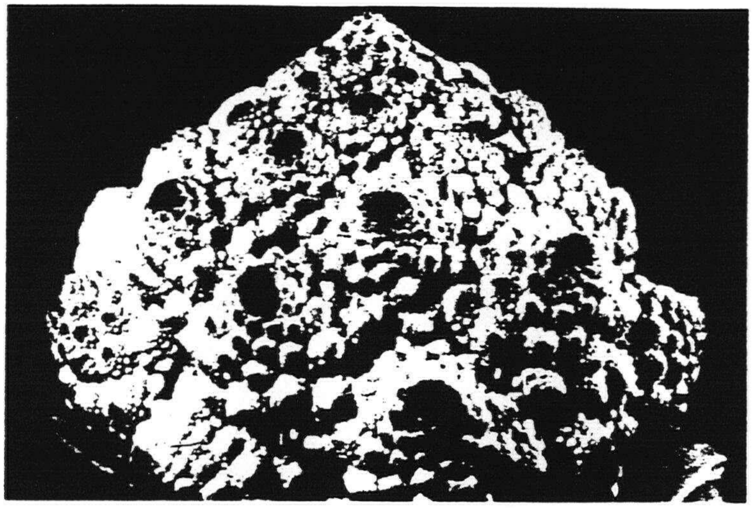
Frequency (Hz)

3) 1/f Components in HRV Signals, Fractals and Chaos

Fractal is a word coined by Mandelbrot in 1975 from the Latin *fractus*, which describes a broken stone. A fractal object looks the same when examined from far away or nearby - it is self-similar (20). Mandelbrot himself gave two examples of the fractal: the cauliflower (Fig.4-22) (20)and the Sierpinski gasket (Fig.4-23) (20). The fractal of the cauliflower is self-similar and irregular, whereas that of the Sierpinski gasket is self-similar and regular. So whether it is irregular or not, if the object is self-similar, then it is fractal.

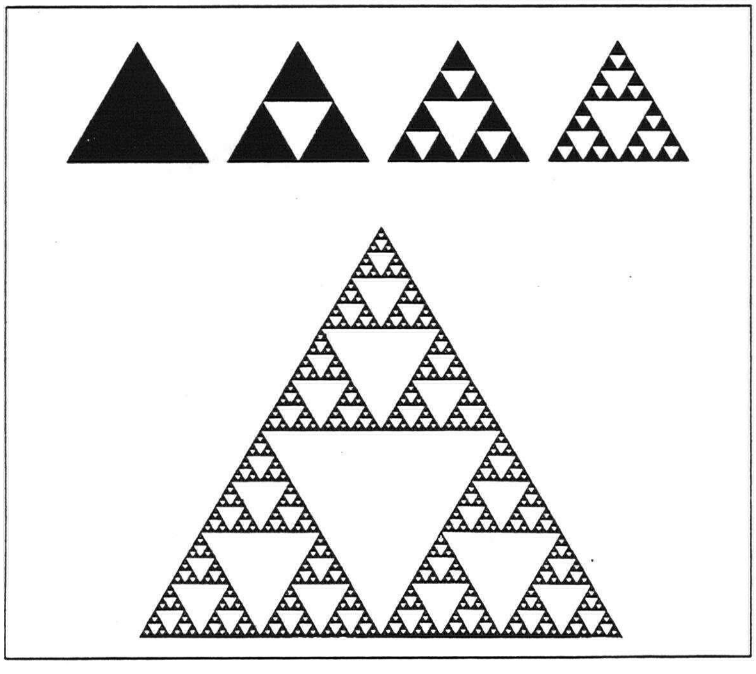
A chaotic system, as its name implies, is inherently unstable. It has two important properties: 1) it is sensitive to its initial point, i.e. small changes in input cause big changes in the output; 2) its behavior is unpredictable. A well-known example is a swinging pendulum with a iron bob that is attracted equally to two magnets positioned below it. When the bob moves near to a point midway between the magnets, it is affected almost equally by the force from each magnet. Its future motion becomes extremely sensitive to small changes in its present position and velocity, and therefore its motion is chaotic. If we assume the sensitivity is so great that the error in measuring the position of the bob increases by 10 times in one swing between the magnets, which is not at all exceptional, prediction of its position to within a centimeter after one swing entails measuring its position at any point in the swing to within a millimeter. For a prediction with the same degree of precision after four swings, its position would have to be

Fig.4-22



This cauliflower, a variety called *c. Romanesco*, is an example of a natural fractal.

Fig.4-23



The Sierpiński gasket – a simple fractal produced by breaking up a triangle into successively smaller ones.

measured to within the size of a bacterium, and after nine swings, to within less than the size of an atom. The pendulum obeys Newton's deterministic laws, but any attempt to predict its future behavior over long times is impossible (20).

Chaos and fractals are fascinating new ideas, which have impinged on many fields across the scientific spectrum. Of special interest to physiologists is the role played by fractals in the spectrum of heart rate variability (HRV) signals: a 1/f spectrum suggests a fractal signal, and hence, a chaotic process (Goldberger, 1991) (17). One of the objectives of this thesis is to show that this hypothesis is not true:

1) Fractal does not imply chaos: According to the rule set by Mandelbrot (Fig.4-23), the linear trend (Fig.4-24) is fractal, because any part of the signal is similar to each other and to the whole signal, yet it is obviously not chaotic because it is totally predictable and its power spectrum is not sensitive to the signal's initial point. Therefore, this is evidence that being fractal does not imply being chaotic;

2) 1/f does not imply fractal: Fig.4-24 shows a linear trend; Fig.4-25 shows its 1/f power spectrum; Fig.4-26 shows the phase spectrum (Note: It was calculated by FFT. The phases shift from 0.5 π to 1 π from 0 to 0.5 Hz is due to aliasing). However, the same 1/f power spectrum (Fig.4-28) with random phases (Fig.4-29) has a completely different time domain signal (Fig.4-27); Fig.4-28 is a signal which has the same 1/f power spectrum yet the phases of the 2-min HRV signal of Fig.3-19. The same 1/f power spectrum does not have a linear trend. The signals of Fig.4-27 , Fig.4-30 and any other signal with the same 1/f power spectrum yet different phases are not necessarily all fractal. A 1/f spectrum does not implies fractal components in the original signal.

Therefore, the Hypothesis that a 1/f power spectrum suggests a fractal signal, and hence, a chaotic process, is not valid.

In the case of the non-linear trends, its double-log regression power spectrum is 1/f. Is it chaotic? Is it fractal? As a special case, the double-log frequency domain regression power spectrum of the linear trend is certainly 1/f as well, but it is not chaotic. Therefore, the 1/f double-log regression power spectrum does not imply a chaotic signal. Is it fractal? The work of this thesis suggests that not every non-linear trend is self-similar, therefore, the 1/f double-log regression power spectrum does not imply a fractal signal.

As for the HRV signals, my experiments show that not every HRV signal is self-similar, therefore, not every HRV signal is fractal; not every HRV signal's power spectrum is sensitive to small changes of the initial point of the signal, therefore, not every HRV signal is chaotic.

Signal | 1/f Power Spectrum | Phase Spectrum

Linear Trend

Fig.4-24

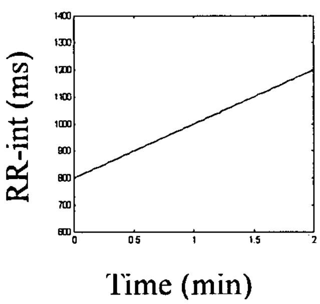


Fig.4-25

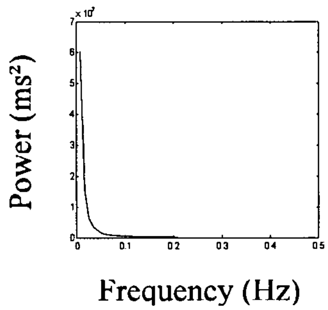
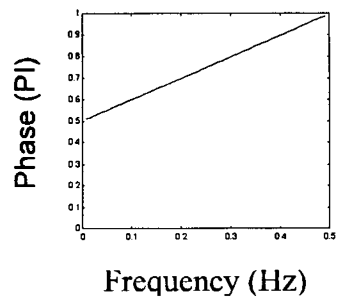


Fig.4-26



Signal with Random Phases

Fig.4-27

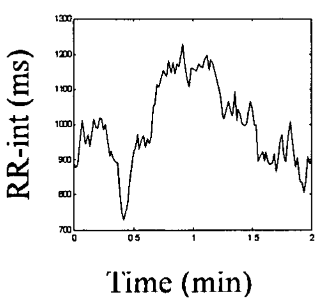


Fig.4-28

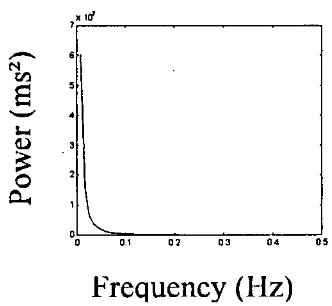
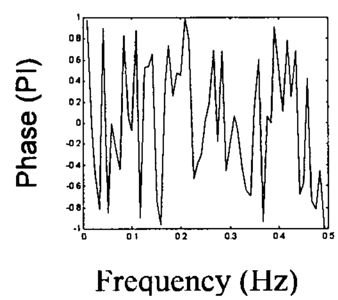


Fig.4-29



Signal with the Phases of the 2-min HRV

Fig.4-30

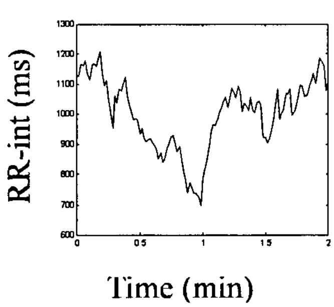


Fig.4-31

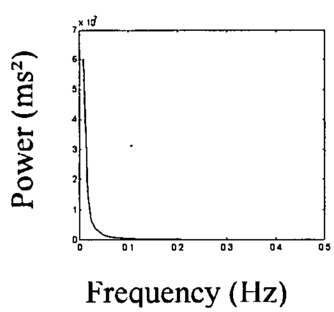
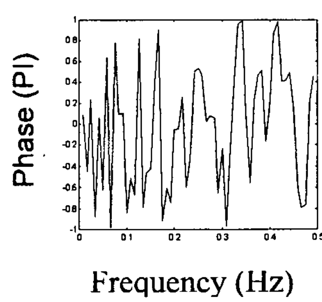


Fig.4-32



3) The Mistakes of the CGSA Let's see how CGSA works. In their paper 'Coarse-graining spectral analysis: new method for studying heart rate variability', Yamamoto and Hughson wrote, 'Let us now consider a discrete stationary stochastic process [X(t) | t=1,2,...,hT], which consists of some harmonic components and ΔB_H (i.e., the 1/f noise). For simplicity, we set $h = 2^n$, where n is a positive integer. We can obtain the two subsets of X(t): $[x(t)|t = 1,2,\ldots,T]$ and $[x'(t)|t = h,2h,\ldots,hT]$. To make $x(t)$ and $x'(t)$ from one ensemble, one can take the first T samples from X(t) for $x(t)$ and take T samples every h samples for $x'(t)$. Therefore it can be said that $x'(t)$ is a coarse-grained process of X(t). ... Now we consider the autopower spectrum S_{xx} of $x(t)$ and the cross-power spectrum $S_{xx'}$ between $x(t)$ and $x'(t)$, which are the Fourier transform of $C_{xx}(\tau)$ (auto-correlation function of $x(t)$) and $C_{xx'}(\tau)$ (cross-correlation function between $x(t)$ and $x'(t)$), respectively. As the phase relationship between $x(t)$ and $x'(t)$ is apparently random, it is expected that $\langle \mathrm{Im}\, S_{xx'} \rangle \to 0$ (where Im is the imaginary part of the complex variable). Consequently, $|S_{xx'}|$ is thought to express only the power of ΔB_H components, and the quantities $S_{xx} - |S_{xx'}|$ can be used to evaluate ΔB_H -free spectrum of X(t)....,these procedures are called the coarse-graining spectral analysis (CGSA).'

These statements can be examined by an example:

Fig.4-33

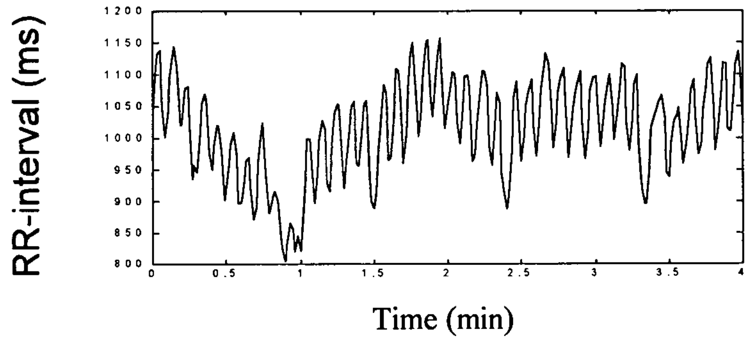


Fig.4-34

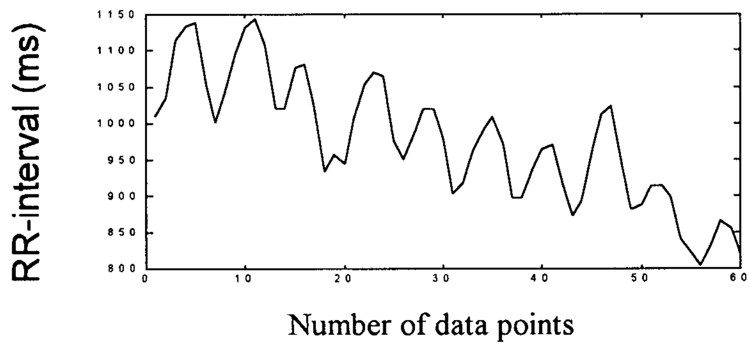


Fig.4-35

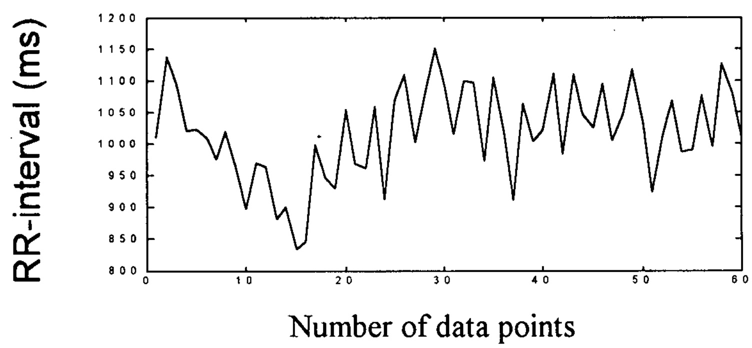


Fig.4-33. the signal of $X(t)$;

Fig.4-34. the signal of $x(t)$;

Fig.4-35. the signal of $x'(t)$.

Fig.4-36

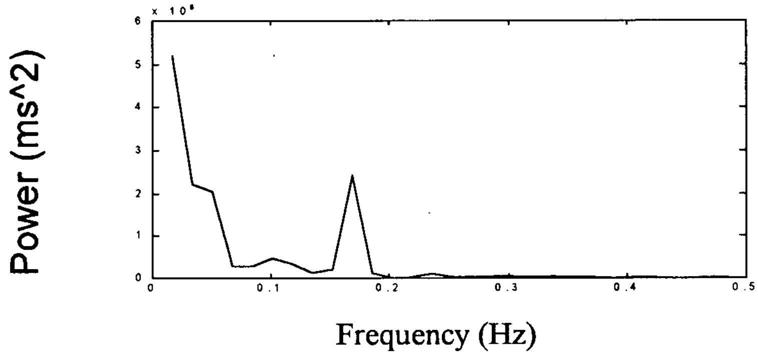


Fig.4-37

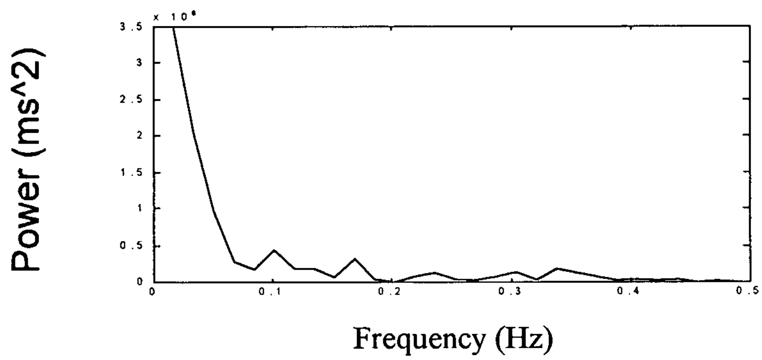


Fig.4-38

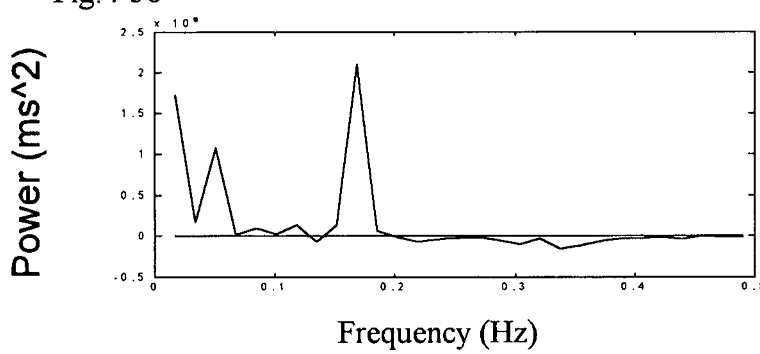


Fig.4-36. S_{xx} , the autopower spectrum of $x(t)$.

Fig.4-37. $|S_{xx'}|$, the cross-power spectrum between $x(t)$ and $x'(t)$.

Fig.4-38. $S_{xx}-|S_{xx'}|$, the supposedly ΔB_H -free spectrum of X(t).

From Fig.4-38 it is seen that 1) not all the 1/f-like components in the lower frequency range are removed by this procedure; 2) the $S_{xx}-|S_{xx'}|$ gave negative results, which is not possible, since the power is the amplitude squared and can never be negative. The reason for this may be that CGSA assumes that the harmonic power components, P_H , plus the 1/f components, P_F , equals the total power component, P, of the signal. Such a linear relationship does not exist for the power spectrum. We know that if the signal $x = x_H + x_F$, (where x is the original signal, x_H is the harmonic part of the signal, and x_F is the fractal part of the signal), and if $Y = FFT(x)$, $Y_H = FFT(x_H)$ and $Y_F = FFT(x_F)$, then $Y = Y_H + Y_F$, thus $P = |Y|^2$, $P_H = |Y_H|^2$ and $P_F = |Y_F|^2$. Since $|Y|^2 = |Y_H + Y_F|^2 \neq |Y_H|^2 + |Y_F|^2$, then $P \neq P_H + P_F$. Therefore, the assumption of CGSA that the harmonic power components, P_H , plus the 1/f components, P_F , equals the total power component, P, of the signal, is wrong. 3) CGSA was wrong when it written 'As the phase relationship between $x(t)$ and $x'(t)$ is apparently random, it is expected that $\langle \mathrm{Im}\, S_{xx'} \rangle \to 0$ (where Im is the imaginary part of the complex variable).' The data shown below are the second to eleventh data points of $S_{xx'}$ and it is clear that $\langle \mathrm{Im}\, S_{xx'} \rangle$ s do not approach 0.

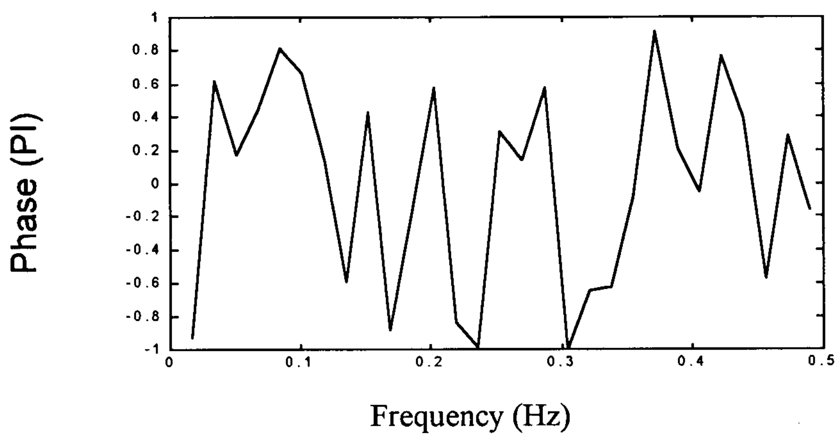
```
1.0e+006 *

-3.3873 + 0.7801i
-0.7412 - 1.9016i
 0.8255 - 0.5007i
 0.0472 - 0.2763i
-0.1443 - 0.0921i
-0.2184 - 0.3836i
 0.1650 - 0.0789i
-0.0516 + 0.1711i
 0.0141 - 0.0627i
-0.3026 + 0.1126i
```

On the other hand, if $\langle \mathrm{Im}\, S_{xx'} \rangle \to 0$, then the phase spectrum of $S_{xx'}$ should be zero all across the frequency range. The phase spectrum (Fig.4-39) shows clearly that it is not the case.

Phase Spectrum

Fig.4-39



The reason for that is because 'as the phase relationship between $x(t)$ and $x'(t)$ is apparently random', it is NOT to be expected that $\langle \mathrm{Im}\, S_{xx'} \rangle \to 0$.

The discussion above shows that CGSA is mathematically incorrect.

4) The Significance of the Current Study A perfect 1/f has never been observed before; this study it is identified as the linear trend. It is also a new observation that the origin of the regression 1/f spectra is due to the non-linear trend. The CGSA (6) technique of removing the 1/f components is ineffective and mathematically incorrect. This study showed that de-trending using a moving average is quite effective in removing the 1/f components of the lower frequency range of the signal. A method of removing 1/f components over entire frequencies was also discussed in this thesis. What is the significance of removing the 1/f components? It is of little significance. Removal of the 1/f components may sometimes produce a clearer spectral picture of the breathing frequency, otherwise it is not necessary to remove as some used to believe. It was thought that the 1/f phenomenon had major implications for the nature of the HRV signals, and that the 1/f spectra indicated fractal noise, and the fractal components indicated chaos. This study shows that this is not the case: the 1/f component is nothing but the trend, and the *major implications* are that it is no more than the trend. This study also indicated that although the linear trend is fractal, it is by no means chaotic. This is clear evidence that being fractal does not imply being chaotic. No evidence in the present research suggests that the non-linear trend is fractal, nor is there evidence to suggest that the HRV signal is fractal or chaotic.

Finally, although the results and discussions in this thesis are limited to short term HRV signals, it is conceivable that similar arguments may hold true in some other areas of 1/f signals (9, 10, 11, 12, 13).

CONCLUSIONS

1) Linear trend appears to be a perfect 1/f signal when FFT is used.

2) Non-linear trends are responsible for the regression 1/f components.

3) Removing the 1/f components by removing non-linear trend is more effective and accurate than CGSA.

4) The linear trend is fractal, but not chaotic; 1/f components, fractal and chaos are not *'cause and effect'* related. No evidence in my research suggests that the non-linear trend is fractal, nor is there evidence to suggest that the HRV signal fractal or chaotic.

5) Although the results and discussion in this thesis are limited to the short term HRV signals, it is conceivable that similar arguments may hold true in some other areas of 1/f signals (9, 10, 11, 12, 13).

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Appendix

The question has been raised that whether the data sets used in this thesis are too short to draw serious conclusions from their analysis. Since the title of this thesis is '1/F COMPONENTS IN **SHORT TERM** HEART RATE VARIABILITY SIGNALS', then my choice of short data sets are obvious but to dispel further arguments I now show that longer data sets do not invalidate the results and conclusions made earlier. It is true that when the HRV data set becomes longer, the linear trend becomes less prominent. However, that does not invalidate my conclusion that the linear trend appears to be a perfect 1/f signal when FFT is used. The fact of the matter is that however long the data set of the linear trend is, it appears to be a perfect 1/f signal (p14-16). On the other hand, no matter how long the data set gets, the non-linear trend never disappear. Fig.6-1 is a forty minute HRV signal segment. Its power spectrum (Fig.6-2) and its double-log plot (Fig.6-3) show that it has 1/f like components. It has a non-linear trend (Fig.6-4). The power spectrum of the non-linear trend is not a perfect 1/f spectrum (Fig.6-5). This is also shown by double-log plot (Fig.6-6). However, the regression line of the double-log plot is 1/f (Fig.6-6). When the trend is removed from the signal (Fig.6-7), the power spectrum and its double-log plot show that no 1/f components remain (Fig.6-8 and Fig.6-9).

Note: 1. The tangent of the regression line of Fig.6-3 is -0.5661; whereas that of Fig.6-9 is 0.0013;

2. Changing the frequency range of interest or the length of the moving average period will change the tangent of the regression line;

3. The peak around 0.1667 Hz in Fig.3-29 and Fig.3-35 corresponds to the breathing frequency of the subject.

Signal | Power | Double-log

Original signal

Fig. 6-1

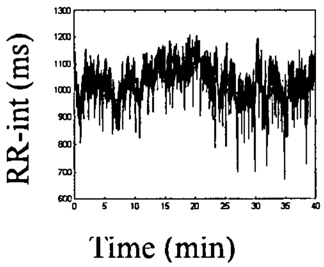


Fig. 6-2

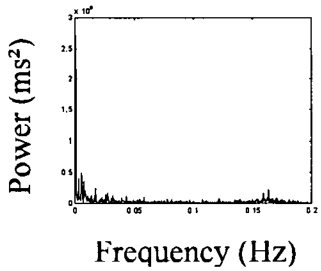
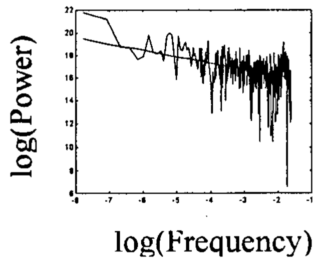


Fig. 6-3



Trend

Fig. 6-4

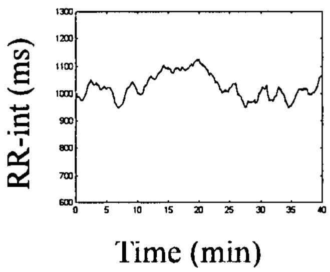


Fig. 6-5

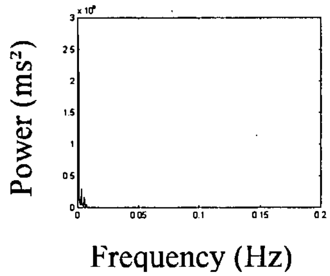
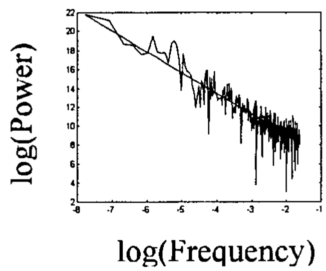


Fig. 6-6



Trend-free signal

Fig. 6-7

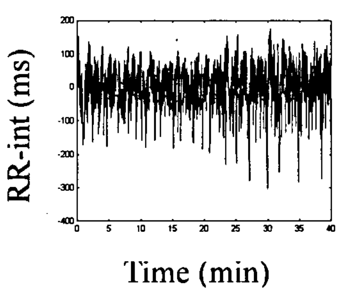


Fig. 6-8

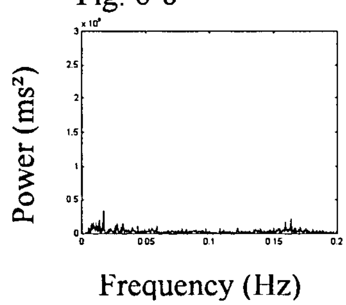


Fig. 6-9