FATIGUE BEHAVIOUR AND SIZE EFFECT PERPENDICULAR TO THE GRAIN
OF LAMINATED DOUGLAS FIR VENEER

by

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ABSTRACT

The rolling shear behaviour of laminated Douglas fir veneers was studied using specimens with 15 layers of 2.5 mm veneers. The 3 central layers were cross-ply with grain angle oriented perpendicular to the long axis of the specimen. The other layers were orthogonal to the central layers.

Based on three specimen sizes, the width effect and load configuration effect for rolling shear failures of laminated veneer panels subjected to “flatwise” three-point bending was experimentally and theoretically evaluated.

The rolling shear fatigue performance of the specimens was studied through cyclic testing until failure. Series of specimens were tested at three mean stress levels, 95%, 91%, and 83% of the of the mean static strength. A damage model which took the stress history into consideration was calibrated to the experimental data. Good agreement between model predictions and experimental results were obtained. Using a size adjustment factor, the model was verified by two cyclically tested series of specimens of different size. The size adjusted model predictions were found to agree well with the experimental data.

TABLE OF CONTENTS

	Page
ABSTRACT	ii
TABLE OF CONTENT	iii
LIST OF TABLES	v
LIST OF FIGURES	vi
1. INTRODUCTION	1
2. BACKGROUND	4
2.1 Laminated Veneer Lumber	4
2.2 Shear Strength of Laminated Veneer Products	5
2.3 Duration of Load and Damage Accumulation Models	7
2.4 Weakest Link Theory - Size Effects	13
3. Test Panels	17
3.1 Description of Test Panel	17
3.2 Materials and Methods	18
4. EXPERIMENTAL TEST PROGRAM	21
4.1 Static Test Program	21
4.1.1 Material and methods: Type A specimens	21
4.1.2 Material and methods: Type B specimens	23
4.1.3 Material and methods: Type C specimens	23
4.2 Cyclic Test Program	24
4.2.1 Material and methods: Type A specimens	24
4.2.2 Material and methods: Type B specimens	26
4.2.3 Material and methods: Type C specimens	27
4.3 Moisture Content Measurements	28
5. EXPERIMENTAL RESULTS FROM STATIC TESTS	29
5.1 Specimen Failure Modes and Load Capacity Evaluation	29
5.2 Rolling Shear Stress Evaluation	32
5.3 Test Statistics	35

	Page
5.4 Moisture Content of Static Series	39
6. WEIBULL WEAKEST LINK THEORY	41
6.1 Width Effect	41
6.2 Load Configuration Effect	43
7. EXPERIMENTAL RESULTS FROM CYCLIC TESTS	47
7.1 Specimen Failure Evaluation	47
7.2 Test Statistics	48
7.2.1 Fatigue behaviour of type A specimens	49
7.2.2 Fatigue behaviour of type B and C specimens	53
7.3 Moisture Content of Cyclic Series	59
8. DAMAGE ACCUMULATION MODEL	60
8.1 Ramp Load Case	61
8.2 Piecewise Linear Representation of Stress History	63
8.2.1 Ramp Load: Segment 1	64
8.2.2 Constant Load: Segment 2	65
8.2.3 Negative Ramp Load: Segment 3	66
8.3 Model Calibration	67
8.3.1 Calibration results	70
8.4 Verification of Damage Model	76
8.4.1 B-series	77
8.4.2 C-series	81
8.5 Final Remarks on Model Calibration and Verification	83
9. SUMMARY AND CONCLUSIONS	85
REFERENCES	87

LIST OF TABLES

	Page
Table 4.1 Description of load cycles.	25
Table 5.1 Input parameters for finite element program.	33
Table 5.2 Short term test results for various specimen types.	36
Table 5.3 Analysis of variance results on Type A specimens in load control mode.	37
Table 5.4 Weibull location, shape, and scale parameters for static test series.	39
Table 5.5 Moisture content statistics of static test series.	40
Table 6.1 Result from two-parameter Weibull fit.	42
Table 7.1 Cyclic Bending Test Results.	48
Table 7.2 Regression equations of individual cyclic test series.	51
Table 7.3 Moisture content statistics of cyclic test series.	59
Table 8.1 The mean and standard deviation of the model parameters for calibration based on the A91% and A83% series.	72
Table 8.2 Model predicted fatigue performance.	74

LIST OF FIGURES

	Page
Figure 2.1 Orientation of lathe checks.	4
Figure 2.2 Longitudinal shear failure planes.	5
Figure 2.3 Rolling shear plane.	6
Figure 3.1 Lay-up of test panel.	17
Figure 3.2 Selection of test specimens.	19
Figure 4.1 Specimen tested in 3-point bending for static shear strength.	22
Figure 4.2 Shape of the different load cycles applied to specimens of type A.	25
Figure 4.3 Shape of load cycles applied to type B specimens.	27
Figure 4.4 Shape of load cycles applied to type C specimens.	28
Figure 5.1 Deterioration of stiffness, load-deflection ratio versus time plot.	31
Figure 5.2 Deterioration of stiffness, deformation rate versus time plot.	32
Figure 5.3 Finite element mesh for type A and B specimens.	34
Figure 5.4 Example of calculated shear stress distribution over panel depth.	35
Figure 5.5 Load controlled static strength cumulative distributions.	38
Figure 7.1 Evaluation of number of cycles to failure for cyclic specimens.	47
Figure 7.2 Fatigue behaviour of type A specimens.	49
Figure 7.3 Fatigue behaviour of type A specimens based on equal rank assumptions.	50
Figure 7.4 Regression lines based on the different cyclic A-series	52
Figure 7.5 Fatigue behaviour of type B specimens based on equal rank assumptions.	54
Figure 7.6 Fatigue behaviour of type B specimens based on equal rank assumptions.	55
Figure 7.7 Schematic stress - strain relationship for specimen failure.	56
Figure 7.8 Fatigue behaviour of type A, B, and C specimens.	58
Figure 8.1 Lognormal fit of static strength distributions.	68

	Page
Figure 8.2 Cumulative distributions of number of cycles to failure for type A specimens.	70
Figure 8.3 Cumulative distribution of N_f -values based on the A91% calibration results.	72
Figure 8.4 Cumulative distribution of N_f -values based on the A83% calibration results.	73
Figure 8.5 Fatigue behaviour of A-specimens based on calibration of the A91%-series.	75
Figure 8.6 Fatigue behaviour of A-specimens based on calibration of the A83%-series.	75
Figure 8.7 Experimental and simulated N_f -values based on Weibull weakest link estimated based on estimated static strength distribution for B-specimens.	77
Figure 8.8 Simulated and actual fatigue behaviour of type B specimens.	79
Figure 8.9 N_f -values for B-specimens based on experimental static strength distribution.	80
Figure 8.10 Fatigue behaviour of type B specimens based on experimental static strength distribution.	80
Figure 8.11 N_f -values for C-specimens based on experimental static strength distribution.	82
Figure 8.12 Fatigue behaviour of type C specimens based on experimental static strength distribution.	83

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1. INTRODUCTION

Laminated veneer products can be divided into two categories, *parallel laminated veneer lumber* or LVL, and *plywood*. Applications utilizing LVL include beam flanges, headers, columns and truss chord members, whereas plywood is mainly used as structural sheathing material for floors, roofs and walls. Plywood is also used as web members in wooden I-beams, in form work, and for a wide range of non structural applications. One of the first attempts to utilize LVL material was made in 1944, during the production of high strength wooden aircraft parts. The attempt proved to be successful and parts of laminated Sitka spruce veneer were used as structural members in the mosquito airplanes. A part of the pioneer work was carried out by Luxford (1945) who showed that laminated material possessed strength properties equal to those of solid wood. At the time, higher manufacturing costs were justified by the dependable performance of the LVL. In recent years the dwindling supplies of high quality saw logs together with the possibility to improve timber yields have made wood composite products attractive as reliable value added products.

The basic mechanical behavior of wood and wood products under all conditions of short- and long-term loading is of great importance if new wood products are to be implemented in structural applications with the same confidence as other construction materials. The phenomenon of creep-rupture under long-term loads, usually known as duration of load, has attracted much attention from scientists over the years. Experimental studies within this field usually generate test data in form of time-to-failure data for constant load or ramp load conditions. This type of test gives a good general

understanding of the behavior of the materials under static loads, but does not provide information on the material behavior for the more complex stress histories encountered in real applications. To address this problem scientists have developed duration of load models, calibrated by test data, that can predict the material behavior for any given stress history. Some of these models have been used to establish reliability based DOL adjustment factors used in Canadian engineering design codes.

Another important phenomenon when dealing with wood is size effect. This size effect is determined by the relationship between structural load carrying capacity and the size, shape and stress distribution within the member. It is generally accepted that size effects are caused by the natural variation in strength properties within wood members so that under the same stress configuration a large volume of material has lower strength than a small volume.

Laminated veneer panels have been developed for various end uses in the past partly by trial and error techniques. By investigating how the individual veneer layer in a panel behaves when subjected to different stress conditions, it is possible to create engineered panel configurations for a specific end use. This is a major undertaking, however, since the study would include investigations over different species, load conditions, veneer thicknesses and other different grain orientations *etc.* As a step stone in this direction, the overall objective of this report is to evaluate the shear behavior of 2.5 mm ($\frac{1}{10}$ in) Douglas-fir veneers perpendicular to the grain in laminated veneer products under "plank-wise" applications for static and fatigue load conditions. The overall objective was divided into the following sub-objectives:

- 1) to experimentally study the shear behavior perpendicular to the grain of laminated Douglas-fir veneer specimens in static and cyclic loading conditions
- 2) to experimentally study the width- and load configuration effect of laminated Douglas-fir specimens subjected to shear forces perpendicular to the grain.
- 3) to use the Weibull weakest link approach to predict the relationship between rolling shear strength and specimen width.
- 4) to use the Weibull weakest link approach to predict the load configuration effect in rolling shear mode.
- 5) to use a damage accumulation model to predict the relationship between load level and time to failure for cyclically loaded beam specimens.

2. BACKGROUND

2.1 Laminated Veneer Lumber

To produce LVL, veneer is rotary peeled, dried, spread with adhesive, laminated together with the grain of all plies oriented longitudinally to the length of the billet and pressed in either a conventional batch-mode hot press or on a continuous or step-wise basis. During the rotary peeling of the veneer from the raw log, lathe checks appear on the veneer face as shown in Figure 2.1. Lathe checks occur perpendicular to the longitudinal / tangential (LT) plane of the veneers and their depths and frequencies are one of the major variables to take into consideration when predicting some of the mechanical properties of a laminated veneer product. Other factors are the size and distribution of the natural defects, the quality of the bonds and the strength of the clear wood.

Figure 2.1. Orientation of lathe checks.

In the 1950s Preston (1953)¹ observed increases in strength with decreasing veneer thickness. Preston found that part of the increase in strength could be attributed to the repair of lathe checks by flowing adhesive. It has been suggested that sufficiently thin laminations could result in strength characteristics close or equal to those of clear wood.

Phenol Formaldehyde (PF) resins are the most common type of resins used by the industry for manufacture of LVL. When laminating veneer, the desired result is to create a glue line that is stronger than the adjacent wood. Several factors have an impact on the quality of the glue bond such as uniform veneer thickness, surface quality of veneers, veneer moisture content and the pressing variables.

2.2 Shear Strength of Laminated Veneer Products

In laminated rotary peeled veneer composites, three shear planes can be identified. Shown in Figure 2.2 are the two shear planes in the longitudinal direction (parallel to the fiber direction).

Figure 2.2. Longitudinal shear failure planes.

¹ As referenced by Kunesh (1978)

Shear failure of this kind is called longitudinal shear and occurs when fibers start to slide upon each other. The remaining shear plane is shown in figure 2.3. In this case failure is called a rolling shear failure since the fibers tends to roll on top of one another when shear stresses close to the material strength are applied. The rolling shear failure plane is shown in Figure 2.3. The longitudinal shear failure tend to be more brittle whereas the rolling shear failure is more ductile.

Figure 2.3. Rolling shear plane.

The depth and frequency of the lathe checks have a huge impact on the rolling shear behavior of laminated veneer products. Bohlen (1975) conducted a study where the shear behavior of laminated veneer products. Bohlen (1975) conducted a study where the shear strength of coastal Douglas-fir laminated veneer lumber was investigated. Both commercially fabricated and laboratory made LVL panels consisting of six layers of 6.3 mm ($\frac{1}{4}$ inch) thick rotary peeled veneers were studied. The LVL was tested in block shear tests according to ASTM D805-72 and oriented so that the shear plane was perpendicular to grain and parallel to the glue line (Fig. 2.3). Bohlen found that the laboratory specimens had twice the average rolling shear strength and half the variability

of the commercial material. Bohlen contributed the difference in strength to the depth of the lathe checks. The commercial material had an average lathe check depth of 92 percent compared to 74 percent for the laboratory made specimens.

The reduction in rolling shear strength due to the lathe checks is caused by insufficient connections between the individual wood fibers. This effect can, according to Preston (1953)², be partially repaired if resin flows into the lathe checks during the hot pressing process.

2.3 Duration of Load and Damage Accumulation Models.

Fatigue is the deterioration of a material due to a continued repetition of stress. Wood and wood composites are sensitive to fatigue creep rupture. Over the years scientists have tried to evaluate and model the behavior of wood when subjected to these types of conditions. In the middle 1800s Haubt (1840)³ recognized that prolonged loading would effect the bending strength of wood. The same phenomenon was also noted by Thurston (1881)³. Thurston found that small wood beams broke after 8 to 15 months when subjected to a load of 60 percent of their short term strength. In the early 1900s Tieman (1908)³ concluded that the strength of wood increases with the rate of loading. Tieman (1909)³ further stated that the static strength and stiffness of wood are not affected if long-term stresses below the static elastic limit are removed before any failure occurs. Eventually a duration of load factor for wood in bending was developed to account for the sustained load effect. The factor, $9/16$ was recognized as the ratio of the

² As referenced by Kunesh (1978)

³ As referenced by Gerhards (1977)

elastic limit to the modulus of rupture for wood for standard strength tests. Thus in the early 1920s, $\frac{9}{16}$ was recommended as a dead load factor without time limitations, determining how strong a wood member had to be to have a working stress of approximately 11.0 MPa. Combs (1939) believed the $\frac{9}{16}$ factor to correspond to about 500 years of sustained loading.

In 1943, Kommers (1943, 1944) presented a report based on cyclic fatigue tests conducted on cantilever-plank specimens made of solid sitka spruce and Douglas-fir and on yellow birch and yellow poplar 5-ply plywood. The solid wood specimens had the dimensions 229 x 32 x 5 mm ($9 \times 1\frac{1}{4} \times \frac{3}{16}$ inch) and the plywood specimens 229 x 32 x 8 mm ($9 \times 1\frac{1}{4} \times \frac{5}{16}$ inch). The specimens were tested at a rate of 1,790 cycles per minute over a span of 152 mm (6 inch) in completely reversed stress cycles and in cycles of repeated stress (from zero to a maximum stress in one direction only) respectively. Kommers reported an endurance strength of 27 percent of the static modulus of rupture for 50 million reversed stress cycles, independent on species and material. Kommers further found the endurance strength to be 36 percent of the mean static modulus of rupture after 50 million repeated stress cycles. Kommers found no species effect nor any significant differences in the fatigue behavior between plywood and solid wood specimens. The research, however, indicated that the shape of the applied stress curve and/or the amount of time the specimen is subjected to a higher stress level has an influence on the fatigue performance.

In the late 1940s, Wood (1947, 1951) presented a curve that related the 'percentage of short-term standard test strength' to the duration of load. The study was undertaken at the Forest Products Laboratory in Madison, Wisconsin and was based on

small clear wood specimens. The curve, known as the Madison curve, developed by Wood intercept with 100 percent of the standard test strength after about 5 minutes duration and with 56 percent (equivalent to the $9/16$ factor) after approximately 27 years of duration. Wood later considered the trend of rapid loading data combined with the data on sustained loading and developed a hyperbolic function to represent the load duration of wood. With D as the duration to failure in seconds, the equation for the curve by Wood is given as:

$$SL = \frac{108.4}{D^{0.04635}} + 18.03 \quad (2.1)$$

As can be seen, Equation 2.1 allows a stress level of 18.03 percent to be sustained forever. For a stress level of 56.25 percent ($9/16$ equivalent) Equation 2.1 gives a load duration of 216 years, which is in close agreement with the results derived by Combs (1939).

The Madison curve is an example of an earlier load duration based on small clear wood specimens. This approach to determining duration of load factors for wood was questioned by Madsen in the early 1970s. Instead of using small clear samples, Madsen (1973) based his study on structural lumber (hemlock) representing the various grades of commercial production. The results suggested that low quality material had less DOL effect than higher grade material. In a similar manner, the results indicated that the Madison curve was conservative in its estimation of the DOL effect. These results were later confirmed by Madsen and Barrett (1976) in a study of matched sets of Douglas-fir specimens subjected to constant bending loads at different percentiles of the short term strength.

In recent years researchers have presented different damage accumulation laws to model the special case of static fatigue (duration of load) in dimensional lumber. These models have been used in lifetime reliability analyses of wood members and systems during the development of reliability based design codes. The models are empirical due to the complexity involved with the creep rupture failure mechanism and have been calibrated with data from extensive laboratory work. A state variable is used to express the progression of accumulated damage within wood. The state variable, $\alpha(t)$, takes on values from 0, at an undamaged state, to 1 at failure. Barrett and Foschi (1978) undertook a massive test program to establish the load duration effect for dimensional lumber due to bending, tension, and compression stresses. Specimens were subjected to long term loading at different stress levels, and the time to failure was recorded. The relationship between stress level and time to failure were established, where the stress level was defined as the ratio between the applied stress to the short term strength. Two damage accumulation models were developed and calibrated. Barrett and Foschi (1978) found that the test data suggested the existence of a threshold level, below which no damage accumulates. Barrett and Foschi (1978) further proposed that the rate of damage, $\frac{d\alpha}{dt}$, depends on the existing damage as well as on the stress level, σ according to the following relations:

Barrett and Foschi Model I (Barrett and Foschi 1978a):

$$\frac{d\alpha}{dt} = A(\sigma - \sigma_0)^B \alpha^C \quad (2.2)$$

Barrett and Foschi Model II (Barrett and Foschi 1978a):

$$\frac{d\alpha}{dt} = A(\sigma - \sigma_0)^B + C\alpha \quad (2.3)$$

The model parameters A and C can be chosen as constants or as random variables. B and the threshold level, σ_0 , are model constants calibrated by the load duration test data. The random variables, A and C, as well as the constant B, have no physical meaning, which has been one of the major criticisms against the use of damage accumulation models.

Foschi and Yao (1986a and b) further expanded the original model (Barrett and Foschi, 1978) and made the applied stress, $\tau(t)$, the damage controlling function.

Foschi and Yao Model (Foschi and Yao 1986a and b):

$$\frac{d\alpha}{dt} = A[\tau(t) - \sigma_0\tau_s]^B + C[\tau(t) - \sigma_0\tau_s]^n \alpha(t) \quad (2.4)$$

The threshold level and short term strength is expressed by σ_0 and τ_s respectively, whereas A, B, C and n are random model parameters. The model was satisfactory calibrated with load duration data obtained from tests on dimensional lumber in bending, tension and compression. The calibration procedure was presented by Foschi, Folz and Yao (1987). The model was later used by Lam (1991) in a study of the fatigue behavior of the transDeck™ panel, an engineered laminated veneer plate intended for use as flooring in flat bed trailers.

Lam (1991) developed a theoretical framework based on the finite strip method to evaluate the performance of the transDeck™ panel in prototype trailer decking systems. The fatigue behavior of the transDeck™ panel was modeled through small size specimen tests in bending mode combined with damage accumulation laws. Lam (1991) also tested full size transDeck™ panels, 1.2 by 2.4 m (4 by 8 ft) subjected to both static and cyclic

loads. The relationship between fatigue life and failure mode of the small specimen tests and the full size panels were established.

Other DOL studies have found no indication of the existence of a threshold level (Gerhards 1979, 1988; Gerhards and Link 1986). The Gerhard and Link model is expressed as:

Exponential Damage Rate Model (EDRM) (Gerhards and Link 1979):

$$\frac{d\alpha}{dt} = e^{\left[-A + B \frac{\tau(t)}{\tau_s} \right]} \quad (2.5)$$

where A and B are model parameters. $\tau(t)$ represents the applied stress whereas the short term strength is expressed by τ_s . This model is a function of the current stress only and was calibrated with load duration tests on dimension lumber. The model was later calibrated with load duration data obtained from tests on plywood and oriented strandboard by McNatt and Laufenberg (1991) with satisfactory results.

The use of damage accumulation laws to describe duration of load in wood has received criticism for the lack of physical meaning attached to the model parameters. A different approach was presented by Nielsen (1978, 1985). The theory is based on the concept of materials behaving as a damaged viscoelastic material and is therefore called the DVM-theory. Wood is represented in the model by a single crack with viscoelastic properties and the propagation of the crack when subjected to a load history is considered. The model was calibrated by load duration results from dimension lumber (Nielsen 1985). Although the DVM-theory gives the model parameters physical meaning, it is fairly complicated to use, since it requires parameters that describe the viscoelastic behavior of wood. These properties cannot easily be obtained through experiments and are, hence,

usually obtained in the same way as the above mentioned damage accumulation models, through model calibration.

Factors such as moisture and temperature variations can contribute to the damage accumulation in structural wood members. Moisture and temperature are known to have an impact on the short term strength and stiffness of wood based materials. Fridley *et al.* (1992 a and b) undertook an extensive research program to evaluate the impact of environmental conditions on damage accumulation. Friedley found that damage accumulation was not sensitive to temperature and moisture as long as the short term strength was accurately adjusted for the existing hygrothermal conditions of the material. The researchers found, however, that cyclic moisture conditions had a significant impact on the damage accumulation that may be related to mechano-sorptive creep strain. The mechano-sorptive creep strain is a nonlinear interaction between changing moisture content and applied stress that results in excess creep. Other factors that have an impact on the fatigue behavior are the species, type of loading (*i.e.* bending, compression, tension, *etc.*) and the direction of loading (*i.e.* parallel- or perpendicular to the grain).

2.4 Weakest Link Theory - Size Effects

It has long been recognized that large brittle members tend to display a lower strength than smaller members of the same material when subjected to the same load conditions. Researchers have tried to explain this phenomenon by the use of conventional brittle fracture theory developed on the basis of the weakest link concept. The weakest

link theory was first proposed by Pierce (1926)⁴ during a study of cotton yarns and later by Tucker (1927)⁴ during a study of concrete.

Major developments of the weakest link concept were presented in a classic paper by Weibull (1939). Weibull verified his results based on tests of various brittle materials, but never on wood. The weakest link model proposed by Weibull (1939) assumes that the natural variation of strength properties within a member is random and uncorrelated. Consequently, larger members are weaker since they exhibit a higher probability of including a major flaw or defect in a high stress region. Weibull showed how the strength of a weakest link system can be represented by a cumulative distribution of the exponential type. Weibull weakest link theory enables the prediction of the probability of failure, p , of a homogeneous isotropic material at a given volume, V , as:

$$p = 1 - e^{-\frac{1}{V_0} \int_V \left(\frac{\tau - \tau_{\min}}{m} \right)^k dV} \quad (2.6)$$

where m and k are the scale and shape parameters for the distribution, $\tau_{\min}$ is the minimum material strength (location parameter), τ is the material strength and V_0 is a reference volume. The scale, shape, and location parameters are material constants. In the two parameter case ($\tau_{\min} = 0$), at an equal probability of failure, the strength, τ_1 , of a volume, V_1 , of a material can be predicted given the strength, τ_2 , of a geometrically similar volume, V_2 , of a common material subjected to the same stress distribution as:

$$\frac{\tau_1}{\tau_2} = \left(\frac{V_2}{V_1} \right)^{1/k} \quad (2.7)$$

⁴ As referenced by Madsen *et al.* (1985)

Using a similar approach it is possible to study the effects of length, depth and width by keeping two of the dimensions constant and varying the dimension of interest. Bohannon (1966) applied the Weibull brittle fracture theory to clear wood. He studied geometrically similar clear wood beams and found that the beams exhibited both a depth and a length effect of equal value. The strength was proportional to the depth of the beams to the power of $1/9$. Bohannon could not, however, detect any evidence of the existence of a width effect.

Barrett (1974) used the weakest link concept in a study of the relationship between specimen volume and load carrying capacity of Douglas-fir specimens loaded in uniform tension perpendicular to the grain. Barrett established that the tensile strength of Douglas-fir perpendicular to the grain is strongly effected by the volume and stress distribution within a specimen. He further found that the weakest link concept based on a two-parameter Weibull cumulative distribution function was in good agreement with the test data and data retrieved from the literature. Lastly Barrett concluded that the experimental data suggested that the magnitude of the size effect is strongly dependent on the quality of the investigated material.

To explain the concept of the weakest link theory, Madsen (1985) considered a member that conceptually consists of a large number of small elements, where the strength of each individual element varies according to a strength distribution. The member is subjected to a gradually increasing uniform stress until failure occurs in the weakest of the elements. At this point, the stresses will be redistributed within the member. If the beam is of a brittle material, the failed element loses all its strength and the adjacent elements receive an instantaneous increase of stress. It is also likely that stress concentrations will

develop in the proximity of the failed element. With these stress increases in mind, it is highly probable that the strength of the adjacent elements will be exceeded and thus, a fracture will propagate suddenly throughout the member and cause immediate failure. A material that behaves in such a fashion is called a *perfectly brittle* material, and its strength is governed by its weakest link. If a ductile material were to be studied in the same fashion, the weakest element would start to yield but still be able to carry a part of the load. Thus, no sudden stress increases will occur in the adjacent elements. Wood as a material exhibits a partially ductile behavior in compression but is close to perfectly brittle in tension.

3. TEST PANELS

3.1 Description of Test Panel

This chapter describes the manufacturing process of test specimens. Since the research objective for this report was to investigate the shear performance of veneer perpendicular to grain for panels used in a flooring application, it was necessary that a rolling shear failure mode was achieved when specimens were subjected to loads in a bending configuration. To accomplish this, a panel type was developed exclusively for this study. The test panel consisted of 15 plies of 2.54 mm ($1/10$ inch) thick C-grade Douglas-fir veneers, of which the three central plies (cross plies) were oriented so that their grain direction had an angle of 90 degrees to the length of the specimen (see Figure 3.1). The remaining 12 plies, 6 on each side of the cross plies were oriented with their grain along the length of the specimen and at a 90 degrees angle to the cross plies.

Figure 3.1. Lay-up of test panel.

The test panel functioned more or less as an I-beam when subjected to bending stresses. The outer plies functioned as flanges taking up the tension / compression stresses, while the weaker cross plies functioned as the web taking up the shear stresses. The test panel was designed in such a way that failure occurred in rolling shear mode in the cross plies before it occurred in bending in the parallel plies.

3.2 Materials and Methods

The Douglas-fir veneers used to form the panels were of grade C and had a thickness of 2.5 mm ($\frac{1}{10}$ inch). Veneers, 1200 x 1200 mm (4.0 x 4.0 ft) in size had been dried to a moisture content of 6 to 8 percent upon delivery. Before resin was applied, they were cut into 600 x 600 mm (24 x 24 inch) sheets by use of a skill saw. A standard phenol formaldehyde resin commonly used in plywood and concrete forming panels was used as a binding agent. Resin was coated on one face in a semi automatic glue spreader that applied the resin evenly onto the veneer sheets at a quantity of 193.7 g per m² (39.7 lb per ft). After resin coating, the veneers were stacked and oriented into the predetermined panel lay-up. The veneers were randomly selected, all but for the two face plies on each panel. These two sheets were visually selected from the original population to be knot free and to have a smooth and even surface.

The panels were pressed in a hydraulic hot press with a capacity of 760 x 760 mm (30 x 30 inch). Aluminum cauls were placed on top and bottom of the stacked veneers to prevent flowing resin to damage the press platen. The panels were pressed for 19 minutes at a platen temperature of 149° C (305° F) and at a pressure of 1.38 MPa (200 psi). The length of the press time depended on the thickness of the panel. For the innermost glue

line to cure, the resin had to reach a temperature of approximately 95° C (203° F) for a period of one minute. To prevent warping and cupping, the panels were hot-stacked directly after they were removed from the hot press and put under pressure by weights. This allowed a slow cooling procedure. A total of 40 panels with dimensions 610 x 610 x 38 mm (24 x 24 x 1½ inch) were pressed. When the panels had cooled down, the weights were removed and the panels were stored in room climate conditions (20° C and 50% RH) until they reached equilibrium.

The test panels were randomly divided into two groups of 20 panels. Three different sizes of specimens, type A, B, and C, were used in the experimental study. The specimens were cut by use of a table saw from the test panels and the specimens were given a code indicating panel number and original location in the panel. From the first group of panels (No's 1-20) each panel was cut into 5 specimens with the dimensions 406.4 x 101.6 x 38.1 mm (16.0 x 4.0 x 1.5 inch) as Figure 3.2 indicates.

Figure 3.2. Selection of test specimens.

The specimens were divided into five matched sets of 20 specimens. Since the location in the original panel can have an impact on the quality of the glue bond, specimens from one

panel were randomly divided into the five groups. These specimens will be called type A. From the second group of test panels (No's 21-40) five pieces were cut into the same dimensions as type A specimens (406.4 x 101.6 x 38.1 mm) from each panel and randomly divided into five matched sets of 20 pieces. The first two sets were cut into the dimensions 406.4 x 25.4 x 38.1 mm (16.0 x 1.0 x 1.5 inch) and will henceforth be referred to as specimens of type B. The next two sets were cut into the dimensions 304.8 x 50.8 x 38.1 mm (12.0 x 2.0 x 1.5 inch) and will be called type C. The last set of twenty specimens remained unchanged (type A) and were used as a control group between the two groups of tests panels.

4. EXPERIMENTAL TEST PROGRAM

The three specimen types were used in this study as follows: Specimens of type A were used to evaluate fatigue performance and to calibrate a damage accumulation model. Specimens of type B were used in comparison to type A specimens to study the width effect but also to perform as a control group to verify the damage accumulation model. The last type of specimens, C, was used to evaluate the load configuration effect between type A- and C-specimens, and function as a control group for verification of the damage accumulation model.

The different tests conducted in this report can generally be divided into two groups, *static*- and *cyclic*-tests. The static tests were used to determine the short term strength and to evaluate the size effect and load configuration effect. On the basis of the cyclic tests, the fatigue behavior was determined.

4.1 Static Test Program

4.1.1 Material and methods: Type A specimens

The specimens were tested as simply supported beams with a test span of 304.8 mm (12.0 inch) and loaded under a center point load. The specimens were 406.4 mm (16.0 inch) long which resulted in an overhang on both sides of 50.8 mm (2.0 inch). The widths of the specimens were 101.6 mm (4 inch). Figure 4.1 shows a specimen being tested in bending. Two test series, one under deflection control and the other under load control, were performed on matched sets of 20 specimens. The results from the deflection control series were used to determine the ramp rate used in the load control series so that

failure occurred approximately after 15 seconds. The same ramp rate was later used in the cyclic tests where one full load cycle lasted 30 seconds. The specimens were cut to length immediately before testing and a piece from each specimen was taken for moisture content measurements. The specimen width and thickness at mid-span and at two points near each edge were measured, averaged and recorded with a pair of digital calipers.

Figure 4.1. Specimen tested in 3-point bending for static shear strength.

A MTS model 810 hydraulic close loop universal testing machine with a capacity of 222.4 kN (50000 lb) applied the load in both deflection control and load control mode.

An uniform rate of cross head motion of $1.37 \frac{\text{mm}}{\text{min.}}$ ($0.054 \frac{\text{inch}}{\text{min.}}$) was used in the

deflection control series, which resulted in specimen failure between 5 to 7 minutes of

loading. The load control series was tested under a uniform ramp rate of $58.74 \frac{\text{kN}}{\text{min}}$ ($13200 \frac{\text{lb}}{\text{min}}$) which resulted in specimen failure after approximately 15 seconds of loading. A load cell with a 50 kN (11240 lb) capacity was used to monitor the loads and a computer based data acquisition software was used to acquire the load, deformation and time data.

4.1.2 Materials and methods: Type B specimens

Moisture content samples and specimen dimensions for specimens of type B were taken and measured in the same manner as described in the previous Section 4.1.1. The specimens of type B were tested with the same type of equipment in a simply supported bending configuration over a test span of 304.8 mm (12.0 inch). The specimens had a length of 406.4 mm (16.0 inch) and width of 25.4 mm (1.0 inch). The overhangs were, as before, equal to 50.8 mm (2.0 inch) on each edge. The same test equipment was used as in the static tests for the type A specimens. Only tests in load control mode were conducted and the ramp rate was set to $14.68 \frac{\text{kN}}{\text{min}}$ ($3300 \frac{\text{lb}}{\text{min}}$), a quarter of the value used for the specimens of type A so that an equivalent rate of stress was applied to the type B specimens.

4.1.3 Materials and methods: Type C specimens

Moisture content samples and specimen dimensions for type C-specimens were taken and measured in the same manner as described in Chapter 4.1.1. The specimens

were simply supported over a span of 20.3 cm (8.0 in) and tested in load control mode under center-point loading until failure. The load rate was set to $29.36 \frac{\text{kN}}{\text{min.}} (6600 \frac{\text{lb}}{\text{min.}})$, so that the stress rate was equal that of specimen type A and B. Since the length of the specimens was equal to 30.5 cm (12.0 in), the tests were performed with the same length of the overhangs, 50.8 mm (2.0 inch) on each end, as for previous specimen types.

4.2 Cyclic Test Program

4.2.1 Material and methods: Type A specimens

Three series of matched specimens were tested flat-wise in center-point cyclic bending at three different mean stress levels. The levels were chosen such that the peak load within each cycle corresponded to approximately 95%, 91%, and 83% of the static strength obtained in the load controlled static tests. The test levels were chosen in such way that the number of cycles to failure in the lowest mean stress level series (83%) were not too high such that specimen failure occurred within reasonable time. A trapezoidal type of stress cycle was used in this study. The ramp rate for all test level series was set to equal that of the static load controlled series, $58.74 \frac{\text{kN}}{\text{min.}} (13200 \frac{\text{lb}}{\text{min.}})$ both during up- and down loading. The duration of a load cycle for the different load level series was kept constant at 30 seconds. This was achieved by letting the load be constant at peak value as shown in Figure 4.2. The number of specimens tested in the different series and specifics for each mean stress level are given in Table 4.1.

Figure 4.2. Shape of the different load cycles applied to the specimens of type A.

Table 4.1. Description of load cycles.

Cyclic Mean Stress Level	Number of Specimens	Peak Load kN (lbs)
95 %	15	13.74 (3087.5)
91 %	20	13.16 (2957.5)
83 %	15	12.00 (2697.5)

The test specimens were cut to length immediately before testing and a moisture content sample was taken from each specimen from the excess material. Specimen width and thickness were measured at mid-point and at points close to each edge. The specimens were tested in the same test configuration as the corresponding static tests for

specimen type A: Simply supported at two ends over a test span of 304.8 mm (12.0 inch) and loaded under a center point load. The specimens were 406.4 mm (16.0 inch) long which resulted in an overhang on both sides of 50.8 mm (2.0 inch). The width of the specimen was 101.6 mm (4 inch). To stabilize the specimen and to prevent it from siding, a minimum load of 0.22 kN (50 lb) was kept on the specimens at all times during the load cycles.

A MTS model 810 hydraulic control close loop universal testing machine with a capacity of 222.4 kN (50000 lb) was used in load control mode. The machine was programmed to repeat the predetermined load cycles until failure occurred. The load history of each specimen was recorded using a sampling rate of 9 Hz. The number of cycles to failure was obtained from the output data.

4.2.2 Material and methods: Type B specimens

The cyclic test of type B specimens were conducted at one stress level, at 91% of the static strength, to verify the damage accumulation model calibrated by the cyclic test data of type A specimens. The same type of trapezoidal load cycles were used, but with the same ramp rate as was used in the static load control mode test of type B specimens, $14.68 \frac{kN}{min.} (3300 \frac{lb}{min.})$. The duration of one test cycle was as in the previous tests set to 30 seconds. The test setup was identical to the previous setup for cyclic test performed on type A specimens. The shape of the load cycles can be viewed in Figure 4.3.

Figure 4.3. Shape of load cycles applied to type B specimens.

4.2.3 Material and methods: Type C specimens

The cyclic tests were performed using the same test set-up as in the static tests of type C specimens (section 4.1.3). The load was applied in pre-programmed load cycles of trapezoidal shape until specimen failure. The load cycles had a ramp rate of $29.36 \frac{\text{kN}}{\text{min}}$ (6600 $\frac{\text{lb}}{\text{min}}$), a duration of 42.7 seconds, and reached a peak load at 85 % of the

static strength. The shape of the load cycle can be seen in Figure 4.4.

Figure 4.4. Shape of load cycles applied to type B specimens.

4.3 Moisture Content Measurements

The weight, w , of each piece taken from the test specimens was measured by an electronic scale with an accuracy of 0.01 grams. The pieces were oven-dried for 48 hours at a temperature of 101°C , whereupon the oven-dry weight, w_0 , was measured. The moisture content, MC, was calculated for each piece as:

$$\text{MC} = \frac{w - w_0}{w_0} \quad (4.1)$$

5. EXPERIMENTAL RESULTS FROM STATIC TESTS

5.1 Specimen Failure Modes and Load Capacity Evaluation

Three failure modes were observed at the time of the tests: 1) rolling shear failure in the cross-ply (RS), 2) tension failure of the bottom ply (B), and 3) both rolling shear failure at the cross-ply and tension failure of the bottom ply occurring at the same time (RS+B). For specimens of type A, the percentage of specimens with failure modes RS, B and RS+B were 80%, 10%, and 5%, respectively, for the displacement controlled tests and 82.5%, 7.5% and, 7.5%, respectively, for the load controlled tests. The type B specimen displayed 75% RS-, 15% B-, and 10% RS+B-failures. The type C specimens exhibited 65% RS-, 20% B-, and 15% RS+B-failures. No significant differences in failure mode distribution were found between displacement controlled and load controlled series of type A, and the type B series. The C-type specimens exhibited a lower percentage of RS-failures than the other test series. An explanation can be found in the fact that the C-type series had a shorter test span than the other series. The length of the overhangs were kept constant for all three types of specimens and since the C-type specimens exhibited a smaller deflection at mid span for a given shear force compared to the other series (due to their shorter test span), propagation of a rolling shear failure through the overhangs was more difficult to achieve.

Due to the specific veneer lay-up of the specimens, the exact failure loads in rolling shear mode were difficult to determine. Failures were in most cases initiated in rolling shear, with cracks at a 45° angle starting to appear in the high shear stress zone of the three cross-ply and propagated towards one or both ends of the specimen.

When the cracks had propagated through the overhangs, the cross-plyies had lost their capacity to carry any load and the specimen was practically divided into two parts, the top and bottom six layers of veneers with fibers along the length of the specimen. At this moment, the loads were transferred to these layers to be taken in bending-compression and -tension respectively. In most cases the increase in stress led instantly to a tension failure at the outer ply and progressed rapidly into total collapse of the specimen. However, in some cases the outer layers were strong enough to allow further increase in load before total collapse occurred. To more accurately determine the actual time of rolling shear failure and the corresponding load, the output data were studied in load-deflection ratio versus time and deflection rate versus time plots for each specimen. Examples of such plots are shown in Figures 5.1 and 5.2 respectively. The load-deflection ratio versus time plots were normalized against the maximum value for easier comparison between specimens. The load-deflection ratio can be viewed as a simplified measure of stiffness. Thus, Figure 5.1 shows the different phases in the deterioration of the stiffness of a specimen. The behavior was more or less identical for all specimens tested and three phases were identified as: 1) elastic behavior, load-deflection ratio ≈ 1.0 . 2) nonlinear region. 3) imminent collapse zone, large increases in deflection over small increases of load. Rolling shear failure was judged to occur at the end of phase 2, where the load-deflection ratio exhibits a nonlinear trend against time.

Figure 5.1. Deterioration of stiffness, load-deflection ratio versus time plot.

In Figure 5.2, similar behavior can be recognized, in this case three phases of changes in deformation rate were identified as: 1) constant deflection rate, 2) gradual increase in deflection rate, 3) rapid increase in deflection rate. Rolling shear failure was deemed to occur at the end of phase 2, which was generally in good agreement with the evaluation of the load-deflection ratio versus time plots. The rolling shear failure load was primarily determined on the basis of the load-deflection ratio versus time plots, the load rate versus time plots were primarily used for confirmation but in few cases where the primary method was inconclusive the latter method was used.

Figure 5.2. Deterioration of stiffness, deformation rate versus time plot.

5.2 Rolling Shear Stress Evaluation

Because of the complicated failure mechanism, a result of the special veneer lay-up of the test panels, the assumption of plane section remain plane was not valid and consequently elementary elastic beam theory could not be used to evaluate the specimens for rolling shear strength. Thus the finite element method (FEM) was used for calculation of specimen rolling shear stresses. A two dimensional finite element program⁵ using an eight node quadratic element with nine Gaussian points was used to evaluate the stress distribution over a specimen for a given failure load. The cross-ply and the parallel veneer plies were modeled as two different kinds of materials. The material input parameters are presented in Table 5.1.

Table 5.1. Input parameters for finite element model

Parameter	Material 1 (parallel plies)	Material 2 (cross-ply)
MOE parallel to x-axis	13,580 MPa (E_L)	630 MPa (E_T)
MOE perpendicular to x-axis	960 MPa (E_R)	960 MPa (E_R)
Shear modulus	830 MPa (G_{LR})	80 MPa (G_{TR})
Poisson ratio, strain in y-direction due to stress in x-direction	0.376 (ν_{RL})	0.425 (ν_{LR})
Poisson ratio, strain in x-direction due to stress in y-direction	0.021 (ν_{RT})	0.304 (ν_{TR})

*Characters within brackets denote the direction or plane of the material parameter.

Since the shear stress distribution of a 3-point load configuration is symmetric around the mid point, only one half of the specimen was modeled to keep the number of elements as low as possible. Here, all nodes along the symmetry line were restricted to move only in the vertical plane (y-direction). A mesh consisting of 117 elements and 398 nodes was used to model the specimens, of type A and B. Specimens of type C were modeled by a mesh consisting of 99 elements and 338 nodes. The mesh for type A and B specimens is shown in Figure 5.3. The width of the specimens were given as an input parameter. The program assumed the stress distribution to be uniform along the width (z-direction) for given (x,y)-position. Smaller elements were used where stress concentrations were expected (*ie.* close to the support and the load). The support was modeled as shown in Figure 5.3, by three nodes of which two were restrained to prevent movement in the vertical direction (y-direction) and one central node, prevented movement in both horizontal and vertical directions.

⁵ Finite element program developed by Ricardo Foschi, UBC Department of Civil Engineering.

Figure 5.3. Finite element mesh for type A and B specimens.

The load was modeled as uniformly distributed over one element, this was judged to be suitable since during the tests the load head transferred the loads to a small area, 5 mm long, after some crushing had taken place. The loads used as input were taken as the mean value of the static load controlled tests for each test series. An example of the FEM calculated shear stress distribution over a cross section of a specimen is shown in Figure 5.4. Noticeable is the irregular shape of shear stresses over the specimen depth caused by the lower values of MOE and shear modulus of the cross-plyies compared to those of the longitudinal plyies.

Figure 5.4. Example of calculated shear stress distribution over panel depth.

5.3 Test Statistics

Summary statistics from static test series of specimen types A, B, and C are presented in Table 5.2. The test series A1 and A2 represent series of specimens tested in load control from the two groups of test panels (1-20 and 21-40 respectively) and AD represents the series tested in displacement control.

The coefficients of variation (COV) of specimen strength are, over all, very low for all five test series, as low as 4.77 % for the A2-series. Low variation is natural for a laminated veneer product, since defects are distributed within and between the different layers of the panel. The differences in COV between the series are small.

Table 5.2. Short term tests results for the various specimen types.

Test Series	Test Mode	Number of Specimens	Rolling Shear Strength, MPa			
			Mean	Median	STDV	COV (%)
AD	Displacement Control	20	2.23	2.23	0.160	7.19
A1	Load Control	20	2.25	2.23	0.125	5.75
A2	Load Control	20	2.26	2.26	0.108	4.77
B	Load Control	20	2.32	2.33	0.171	7.40
C	Load Control	20	2.77	2.76	0.169	6.10

It is notable, however, that series A1 and A2 have a lower COV than the other test series. Since the specimens of type A are wider than those of type B and C, it is natural that the series of wider specimens exhibits lower variation of strength assuming that failure is governed by the weakest defect. Following this line of discussion, the AD-series should have a lower COV value, however, the evaluation of failure load in rolling shear mode proved to be extremely difficult for displacement controlled specimens and inconsistencies of the evaluation may account for a part of the higher variation. This is of no concern since the single purpose of the AD series was to establish an approximate failure load for specimens of type A, so that the ramp rate used in the load controlled test series could be determined.

The rolling shear strength values in Table 5.2 are based on the finite element calculations using the individual specimen dimensions and failure loads as input to the FEM-program. The three series of type A specimens have almost identical mean strength

values that range from 2.23 - 2.26 MPa. Due to width- and load configuration-effect, the B and C series have slightly higher rolling shear strength values than the three A series.

An analysis of variance was conducted on the A1 and A2 series to evaluate whether the test results from the two groups of specimens were significantly different. This evaluation was necessary since one group (A1) was used for the width- and load configuration-effect study and the other group (A2) was used for the calibration of the fatigue model. The results from the analysis of variance are shown in Table 5.3.

Table 5.3. Analysis of variance results of type A specimens in load control mode.

Source	Degree of Freedom	Sum of Squares	Mean Square	F value
Dependent Variable: Failure Load for Rolling Shear Failure				
Panel Series	1	4916	4916	0.15
Error	34	1086835	31965.72	
Total	35	1091752		

Since the F-value is less than the critical value ($f_{0.05}(1,34) = 4.14$), the results indicate that the difference in failure loads between the two A-type series tested in load control were not statistically significant at the 95% probability level.

Figure 5.5 shows the rolling shear strength cumulative probability distribution for the four static test series tested in load control mode. The displacement controlled series for type A specimens was not included in the graph. A three parameter Weibull distribution was fitted to each set of test data. Visual inspection of Figure 5.5 indicates that a three parameter Weibull distribution fit each set of test data reasonably well. The

data clearly shows that the two load controlled series A1, and A2, are almost identically distributed. The Weibull distribution fitted to the A1 data set have, however, a slightly wider spread than that (dotted line) fitted to the A2 data set. This is in good agreement with the test data since the A1 series had a slightly higher COV value, but the difference is not large enough to question whether the A1 and A2 specimens come from the same population.

Figure 5.5. Load controlled static strength cumulative distributions.

The distributions of the static B- and C-series have higher strength values than the two distributions of the A1- and A2 series. This is in agreement with the weakest link theory. It is worth noting, however, that the distribution of the C-series overlaps with the distributions of the A-series at the lower tail. Based on theoretical analysis (presented in Chapter 6) the static strength of the type C-specimens should be distributed at higher

strength values. Similarly, the static strength values of the C-series should be distributed at lower strength values, though, still higher than the two A-series. Based on Table 2 and Figure 5.5, it is clear that the B- and C-series have larger variation of strength than the A-series.

The Weibull location, shape, and scale parameters for each of the four static data sets are given in Table 5.4.

Table 5.4. Weibull location, shape, and scale parameters for static series.

Test Series	Location parameter	Scale Parameter	Shape Parameter
A1	1.90	0.401	2.89
A2	2.05	0.242	2.04
B	1.87	0.501	2.71
C	2.11	0.728	4.23

5.4 Moisture Content of Static Series

The statistics of the moisture content measurements of the specimens in the static test series are presented in Table 5.5. Very low variation within and between series was observed. The mean moisture content of the different series ranged from 8.14%-8.49%. Since the specimens of the different static series exhibited close to identical moisture content values, it can be concluded that differences in static strength between series can not be contributed to variations of moisture content.

Table 5.5. Moisture content statistics of static test series.

Test Series	Mean (%)	STDV (%)	COV (%)
AD	8.26	0.31	3.75
A1	8.14	0.29	3.53
A2	8.15	0.27	3.32
B	8.49	0.30	3.48
C	8.36	0.28	3.34

6. WEIBULL WEAKEST LINK THEORY

6.1 Width-Effect.

In this chapter, the width effect between the specimens of type B and A will be evaluated theoretically and compared to static test results. The two specimen types were identical in length and thickness but different in width. The A-specimens had an average width of 102.0 mm compared to 25.4 mm for the B-specimens. The two specimen types were tested under identical circumstances, same load configuration and stress rate. Thus the specimens of the two series were subjected to the same loading distribution.

According to the Weibull weakest link theory, the probability of failure, p , of a homogeneous isotropic material of a given volume, V , can be predicted as:

$$p = 1 - e^{-\frac{1}{V_0} \int_V \left(\frac{\tau - \tau_{\min}}{m} \right)^k dV} \quad (6.1)$$

where m , k , and $\tau_{\min}$ are the scale, shape, and location parameters for the distribution. The material strength is expressed by τ , and V_0 is a reference volume. The theory assumes that the scale, shape, and location parameters are material constants. In the two parameter case ($\tau_{\min} = 0$) at an equal probability of failure, the static strength ratio between the A-

and B- specimen, $\frac{\tau_B}{\tau_A}$, at an equal probability of failure as:

$$\frac{\tau_B}{\tau_A} = \left(\frac{V_A}{V_B} \right)^{1/k} \quad (6.2)$$

where V_A and V_B are the volume of the A- and B-specimens respectively. τ_A and τ_B are the mean short term strengths of type A- and B-specimens respectively. Since the two

specimen types have identical stress distribution, length, and thickness, Equation 6.2 can be rewritten as:

$$\frac{\tau_B}{\tau_A} = \left(\frac{b_A}{b_B} \right)^{1/k} \quad (6.3)$$

where b_A and b_B are the width of the A- and B-specimens respectively. To determine the shape parameter, k , the recorded specimen strength values of the static A1-, B-, and C-series were each fitted to a two-parameter Weibull distribution. The shape parameters are presented in Table 6.1.

Table 6.1. Results from two-parameter Weibull fit.

Static Test Series	Weibull Shape Parameter, k
A1	20.5
B	16.2
C	18.2

The Weibull shape parameter is related to the coefficient of variation of a data set with the approximate relationship:

$$k \cong \text{COV}^{-1.087} \quad (6.4)$$

where COV is the coefficient of variation of the data set. Based on the COV of the static test series, the shape parameter values can also be calculated using Equation 6.4 which are in the same range as the results in Table 6.1. Entering a range of the shape parameter,

$16.2 \leq k \leq 20.5$, and the average specimen thickness for the A1- and B-series into

Equation 6.3 yields: $1.070 \leq \frac{\tau_B}{\tau_A} \leq 1.089$.

Hence, according to the Weibull weakest link theory, a width effect in the range of 7.0%-8.9% should exist between the strength's of type A- and B-specimens. Based on the static test results presented in Table 5.2, the stress ratio, $\frac{\tau_B}{\tau_A}$, was found to be equal to 1.045. Consequently, the experimental results yielded a lower value of the width-effect than the theoretical computed value. The difference between theoretical and experimental value is small and roughly in the same range as the COV of the two series. Further, such differences are within the sampling and experimental errors expected from a limited sample size. Unfortunately, it is not possible to conclusively establish a value of the width-effect between the static A1- and B-series. Based on the test results, however, it is likely that a width-effect between the two specimen types exists at a lower level than predicted by theory.

To evaluate the quantity of a width effect for rolling shear failures of laminated veneer products, a larger sample size and greater difference of volume under stress between the specimen types is necessary, preferably in the range larger than 1:8

6.2 Load Configuration Effect

In the following section, the load configuration effect between specimens of type A and C will be evaluated and discussed. Due to the shorter test span of the C-configuration, the specimens of the A1- and C-series were not geometrically similar and

consequently, the two specimen types were not exposed to equal stress distributions. Thus, the methodology used in the section 6.1 cannot be applied to this case. Based on Equation 6.1, the two specimen types can be evaluated at an equal probability of failure level as:

$$p = 1 - e^{-\frac{1}{V_0} \int_{V_A} \left(\frac{\tau_A(x,y,z) - \tau_{\min}}{m} \right)^k dV_A} = 1 - e^{-\frac{1}{V_0} \int_{V_C} \left(\frac{\tau_C(x,y,z) - \tau_{\min}}{m} \right)^k dV_C} \quad (6.5)$$

where V_A and V_C are the volume of the cross-ply for a A- and C-specimen respectively. $\tau_A(x,y,z)$ and $\tau_C(x,y,z)$ are the distributions of shear stresses in the cross-ply of type A- and C-specimens. Assuming the results of the static test series can be described by a two-parameter Weibull distribution, the location parameter, $\tau_{\min}$, is equal to zero and consequently Expression 6.5 can be simplified to:

$$\int_{V_A} \tau_A(x,y,z)^k dV_A = \int_{V_C} \tau_C(x,y,z)^k dV_C \quad (6.7)$$

Since the panel lay-up prohibited use of elastic beam theory, the integration of the shear stresses in the cross-ply was done numerically by the same finite element program used to evaluate the shear strength of the static test series. For a given range of the Weibull shape parameter ($18.204 \leq k \leq 20.503$), load level and average specimen width, the program numerically computed the volume integral over the shear stress in the cross-ply using the same material parameters and meshes as in chapter 5.2. It was decided to use the A-configuration as control. The integral, $\int_{V_A} \tau_A(x,y,z)^k dV_A$ was computed using the mean failure load of the A1-series as input. The corresponding rolling shear strength to this load was, as mentioned in Table 5.2, equal to 2.25 MPa. An iteration procedure

was set up to estimate the mean failure load for the C-configuration such that the integral,

$\int_{V_C} \tau_C(x, y, z)^k dV_C$, equalled that for the A-configuration (Expression 6.7).

The resulting range of calculated mean rolling shear strength values for the C-configuration based on the range of the shape parameter was $2.36 \leq \tau_C \leq 2.38$ MPa. Thus, the theoretically calculated ratio between the static rolling shear strength of the C- and A-

configuration was found to be within the range: $1.048 \leq \frac{\tau_C}{\tau_A} \leq 1.058$. Compared to the

experimental value, $\frac{\tau_C}{\tau_A} = \frac{2.77}{2.25} = 1.23$, there is a difference of close to 17%. The large

difference between experimental- and theoretical value indicates that the specimens of the C-configuration exhibited a different failure mechanism than those of the A-configuration.

Upon closer examination of the test set up for the C-configuration, it is clear that the propagation of the rolling shear failures is likely to be restrained by the overhangs due to the shorter test span. Thus, complete failure was prolonged until the cross plies of the overhangs had been thoroughly penetrated. This behaviour resulted in an exaggeration of the failure load of the C-configuration compared to the A-configuration.

In conclusion, the Weibull weakest link theory proved not to be suitable in evaluating the load configuration effect between type A- and C-specimens. The reasons are:

- 1) Type A- and C-specimens did not experience similar shear stress distributions.
- 2) The overhangs had a larger impact on the rolling shear strength of type C-specimens than on type A-specimens.

3) Type C-specimens exhibited a somewhat ductile failure behaviour, which is not in agreement with the assumptions of Weibull weakest link theory.

7. EXPERIMENTAL RESULTS FROM CYCLIC TESTS

7.1 Specimen Failure Evaluation

The cyclically tested specimens exhibited a similar failure behaviour as the static specimens. Failures were induced in the cross-plyies in rolling shear mode, but in some cases the parallel to grain layers at the bottom of the specimen prolonged the final collapse of the specimen. However, the evaluation procedure to establish the number of cycles to failure in rolling shear mode proved to be easier than in the case of static strength. For each stress cycle, the total amount of deflection from minimum load until peak load was determined. These deflection values were plotted against the number of cycles of survival as shown in Figure 7.1. Failure was defined when a drastic change of slope was encountered.

Figure 7.1. Evaluation of number of cycles to failure for cyclic specimens.

7.2 Test statistics

Results from the cyclic test program are summarized in Table 7.1. The variability in the number of cycles to failure in each group was found to be large. This is consistent with previous research on the fatigue performance of laminated veneer products (Lam, 1991 and Kommers, 1943 and 1944).

Table 7.1. Cyclic Bending Test Results.

Specimen Type	Number of Specimens	Peak Load (kN)	Mean Stress Level S_R	Number of Cycles to Failure	
				Mean	STDV
A	15	14.0	0.946	93	131
A	20	13.4	0.907	331	552
A	15	12.2	0.828	1964	3033
B	20	3.38	0.906	308	640
C	20	9.64	0.854	537	664

The peak loads were normalized with the corresponding mean strength values obtained from the static test in load control, to compute the mean stress level (S_R) for each group. Thus, the three series of type A were tested at approximately 95%, 91%, and 83% mean stress level and the B- and C-series were tested at approximately 91% and 85% respectively. The specimens failed predominately in rolling shear mode. For specimens of type A, a total of 2 out of 50 specimens failed in a bending mode. In the cyclic series of B- and C-specimens, two bending failures were encountered in each case, the remaining specimens failed in a rolling shear mode.

7.2.1 Fatigue behaviour of type A specimens

The rolling shear fatigue behaviour is presented in Figures 7.2 and 7.3. In Figure 7.2, the stress level is based on the assumption that all specimens have a static strength equal to the mean static strength. Thus, the stress level of each specimen within a group is equal. If a regression analysis was to be made, it would mainly be based on only the three average values. The variation within each group is extremely high, consequently, it is difficult to determine whether the average values at each stress level is an accurate estimation.

Figure 7.2. Fatigue behaviour of type A-specimens.

Since the static strength is distributed around the mean strength, it can be assumed that specimens that survived more than the average number of cycles to failure, in reality

had a static strength exceeding that of the mean value. In Figure 7.3, the static strength of each specimen is predicted based on equal rank assumptions. First, the distribution of static strength of both short term- and fatigue testing-series are assumed to follow a three parameter Weibull distribution with identical location, scale, and shape parameters. Now, static strength and fatigue resistance are assumed positively correlated. Therefore, if the rank (percentile) of a specimen in a cyclic fatigue group is determined, an estimation of its static strength can be made by selecting a value at the equivalent rank in the distribution of static strength.

Figure 7.3. Fatigue behaviour of type A-specimens based on equal rank assumptions.

From Figure 7.3, it is clear that the latter method gives a clearer graphical illustration of the fatigue resistance than the first method. A least square regression was made based on the equal rank assumption data for all three cyclic groups, the regression equation is equal to:

$$S = 0.9923 - 0.0239 \ln(N_f) \quad (7.1)$$

N_f is the number of cycles to failure and S is the stress level defined as the ratio between applied stress and predicted static rolling shear strength for specimens of type A. The coefficient of determination, r^2 , equals 0.976. The largest number of stress cycles survived by a specimen was close to 10^4 . According to Equation 7.1, this correspond to a stress level of 77%. Visual inspection of the trend of the data suggests that an endurance limit for rolling shear failures of laminated veneers is well below a stress level of 77%.

If the data points of the three test series in Figure 7.3 are visually compared, it can be seen that the trend of the specimens tested at a mean stress level of 83% differs in terms of slope from the trends of those specimens tested at mean stress levels of 95% and 91%. It was also observed that the trend of the A-83% series is not in linear relation to the $\ln$ (number of cycles). Least squares regressions were made based on each individual series, the regression results are presented in Table 7.2 and Figure 7.4.

Table 7.2. Regression equations of individual cyclic test series.

Cyclic Test Series	Regression Equation	Coefficient of Determination r^2
A-95%	$S = 0.9943 - 0.0196 \ln(N_f)$ (7.2)	0.953
A-91%	$S = 0.9654 - 0.0172 \ln(N_f)$ (7.3)	0.971
A-83%	$S = 1.0177 - 0.0298 \ln(N_f)$ (7.4)	0.914

Comparing the various values of the coefficient of determination, it is evident that the data of the A-83% series does not fit its regression line as well as the data in the other two test series which confirms the visual observations of Figure 7.3. The values of the y-intercept of the three series are in good agreement with each other. Though, there is a difference of approximately 3.5% between the y-intercept of the A-91% and A-83% series, this is well within the natural variation of strength of wood based products.

Figure 7.4. Regression lines based on the different cyclic A-series.

There are very small differences in terms of slope between the A-95% and A-91% series. However, these two series have less steep slopes than the A-83% series. Since the regression line did not fit the A-83% series well, it is of more interest to investigate the

reason for the non-linear behaviour of the data points than compare it to the other two regression lines. From a visual standpoint, it is clear that the three weakest specimens of the A-83% series, within a stress level range of 90% to 84%, should have failed after a lower number of cycles if a linear relationship between stress level and $\text{Ln}(N_f)$ existed. A plausible explanation may be that the irregularity of linear behaviour is within the experimental error. If this is the case, the data points, with the exemption of the three weakest, have a trend parallel to that of the other two series. The explanation might, however, also result from the fact that the A-83% series was conducted at a considerably lower mean stress level than the other two series.

7.2.2 Fatigue behaviour of type B and C specimens

The rolling shear fatigue behaviour of type B specimens is presented in Figure 7.5 using the equal rank assumptions. The B-series of cyclic specimens was tested at approximately 91% mean stress level. For comparison reasons, the test data of the A-91% series are included in the figure. The line through the data points of Figure 7.5 is determined by least squares regression over the B-91% series, and the regression equation is equal to:

$$S = 0.9997 - 0.0256 \text{Ln}(N_f) \quad (7.2)$$

The stress level, S , is in this case defined as the ratio between applied stress and the predicted rolling shear strength for specimens of type B. The coefficient of determination, r^2 , is equal to 0.9708. Visual inspection of Figure 7.5 suggests that the fatigue resistance of the B-91% series is in good agreement with that of the A-91% series. Comparison between the two regression equations, 7.3 and 7.5, finds that the two series are almost

identical in terms of slope and y-intersect, which means that the fatigue resistance of type A and B specimens are similar.

Figure 7.5. Fatigue behaviour of type B-specimens based on equal rank assumptions.

The fatigue resistance of the C-series is presented in Figure 7.6. The mean stress level of the specimens of type C tested in cyclic fatigue was approximately 85%. Thus the data of the A-83% series are included in the graph for comparisons between specimen types. The stress level of the individual specimens of the C-85% series was determined through equal rank assumptions as the ratio between applied stress and predicted static rolling shear strength of type C.

Figure 7.6. Fatigue behaviour of type C-specimens based on equal rank assumptions.

Visual inspection of Figure 7.6, suggests that the trend of the data is not completely linear and appears to have a “bump” with a steeper slope between the stress levels 90%-84%. If the data points above a stress level of 90% are excluded, the remaining data are in very good agreement with the A-83% series. It is notable, that the data points of both test series exhibit similar behaviour within the stress levels of 90%-84% after which they have a similar slope as the other cyclic series. The data points above 90% stress levels follow the trend of the B-91% series and the two A-series tested at 95% and 91% mean stress level. Thus, it is not possible to attribute the irregularities in linearity of the C-85% and A-83% series to experimental error. Moreover, it confirms the

indication that the non-linear behaviour between stress level and $\ln(N_f)$ is caused by the stress level of the individual specimens at which the tests were conducted.

A plausible explanation of the non-linear behaviour of series A-83% and C-85% may be found by studying a stress-strain relationship of a specimen tested for static strength. In Figure 7.7, a typical relationship between stress and strain is shown for a specimen tested for static strength.

Figure 7.7. Schematic stress - strain relationship for specimen failure.

The curve has been divided into three zones depending on the behaviour of the stress-strain relationship; plastic zone, transition zone between elastic and plastic behaviour, and elastic zone. In Figure 7.6, three different trends of fatigue resistance of the C-85% series can be identified: 1) For the 91%-100% stress level, these specimens

have relatively low strengths considering their rank therefore, it is likely that they were cyclically tested at a stress level within the plastic zone of Figure 7.7;. 2) For the 84%-90% stress level, following the same line of thought, the specimens within this region can be assumed to have been tested at stresses within the transition zone; 3) For the 78%-83% stress level, these specimens are likely to have been tested at stresses within the elastic zone. Now, assume that a specimen tested in cyclic fatigue fails when the strain accumulates to a critical value, ϵ_c . The specimens belonging to the first trend were all tested at stresses that instantly caused plastic deformations, whereas the specimens belonging to the second trend group were tested at stresses that instantly or after a few stress cycles caused plastic deformations. Then, the difference in slope of trend 1 and 2, can be explained by differences in the ratio $\frac{\Delta\sigma_i}{\Delta\epsilon_i}$. According to Figure 7.7, the variation of strain is higher for specimens tested in the plastic region than for specimens tested in the transition zone for a given range of stress, $\Delta\sigma$ (*i.e.* $\frac{\Delta\sigma_1}{\Delta\epsilon_1} \ll \frac{\Delta\sigma_2}{\Delta\epsilon_2}$). This means that the initial strain / stress ratio have more impact on the number of cycles to failure for specimens tested in the plastic zone than those tested in the transition zone. It is thus, likely that the variation of numbers of cycles to failure is higher for the former group of specimens than in the latter and consequently, the slope of trend 1 is less steep than that of trend 2.

The specimens of the third trend group were tested at stresses within the elastic region and each specimen was probably subjected to a substantial number of cycles until plastic deformations started to occur. It is also likely that the number of cycles until

plastic deformations occurred is positively correlated to the stress at which the cyclic test was conducted. Hence, the specimens of the third group exhibited a larger variation of cycles to failure than the specimens tested within the transition zone.

Looking back at Figure 7.3, it is possible to detect a similar, if not as pronounced, behaviour in the A-91% series at approximately a stress level of 89%-91% and in the A-95% series at a stress level around 88%-90%. In these two cases it is possible, however, to discard the behaviour as experimental error.

The fatigue resistance of all five cyclic series are presented in Figure 7.8.

Figure 7.8. Fatigue behaviour of type A, B, and C specimens.

The line through the data points is based on least squares regression of the five series.

The regression equation is given as:

$$S = 0.9975 - 0.0250 \ln(N_f) \quad (7.3)$$

The coefficient of determination is equal to 0.9283. In conclusion, based on visual inspection of Figure 7.8, it is evident that, in general, the data fits the regression line well and the fatigue resistance is documented by the five cyclic series in a satisfactory way.

7.3 Moisture Content of Cyclic Series.

The results from the moisture content evaluation of specimens tested for cyclic fatigue is presented in Table 7.3. Very low variation within test series was observed, the COV of the A83%-series was as low as 2.81%. The variation between series was also low, the mean moisture content of the individual series ranged from 8.18%-8.37%. The results compare well with the moisture content evaluation of the static test series presented in Table 5.5.

Table 7.3. Moisture content statistics of cyclic test series.

Test Series	Mean (%)	STDV (%)	COV (%)
A95%	8.25	0.26	3.16
A-1%	8.31	0.30	3.61
A83%	8.18	0.23	2.81
B91%	8.37	0.31	3.70
C85%	8.31	0.26	3.12

8. DAMAGE ACCUMULATION MODEL

The fatigue behavior of type A specimens has been experimentally examined in the previous chapter. It is desirable to identify and calibrate a damage accumulation model so that the relationship between stress level and cycles to failure in rolling shear can be predicted for similar types of specimen. A similar study has previously been conducted by Lam (1991) and the following chapter in parts follows closely the model development made by him. Based on the literature survey, it was decided to use an existing damage accumulation model developed by Foschi and Yao (1986a and b). The model takes the form:

$$\frac{d\alpha}{dt} = a[\tau(t) - \sigma_0 \tau_s]^b + c[\tau(t) - \sigma_0 \tau_s]^n \alpha(t) \quad (8.1)$$

where a , b , c , n and σ_0 are random model parameters, that are constant for a given member but vary between members. The accumulated damage is expressed by a state variable, α . At an undamaged state $\alpha = 0$, while failure of the member occurs when $\alpha \geq 1$. $\tau(t)$ is the stress history experienced by the specimen and τ_s is the median value of the short term strength of the specimens obtained from the load controlled test. σ_0 denotes the threshold ratio, accumulation of damage occurs only for stresses larger than $\sigma_0 \tau_s$.

If a width effect parameter, Z , is introduced to the model so that $Z = 1.0$ for specimen of type A with a width equal to 101.6 mm, Equation 8.1 can be rewritten as:

$$\frac{d\alpha}{dt} = a[\tau(t) - \sigma_0 \tau_s Z]^b + c[\tau(t) - \sigma_0 \tau_s Z]^n \alpha(t) \quad (8.2)$$

Z will take on values lower than 1.0 for specimens wider than type A specimens and values higher than 1.0 for specimens with lesser width.

Let $f_1 = [\tau(t) - \sigma_0 \tau_s Z]^b$ and $f_2 = [\tau(t) - \sigma_0 \tau_s Z]^n$, through multiplication of the expression $e^{(-\int c f_2 dt)}$ to both sides of Equation 8.2, the equation can be rewritten as:

$$\frac{d\alpha}{dt} e^{(-\int c f_2 dt)} = [a f_1 + c f_2 \alpha(t)] e^{(-\int c f_2 dt)} \quad (8.3)$$

$$\frac{d}{dt} \left[\alpha e^{(-\int c f_2 dt)} \right] = a f_1 e^{(-\int c f_2 dt)} \quad (8.4)$$

Integration of equation 8.4 over a time period, T, gives:

$$\alpha e^{(-\int c f_2 dt)} \Big|_0^T = \int_0^T a f_1 e^{(-\int c f_2 dt)} dt \quad (8.5)$$

If the stress history, $\tau(t)$, the width effect, and the model parameters are known, Equation 8.5 can be evaluated by performing the integration over the intervals where $\tau(t) > \sigma_0 \tau_s Z$ to estimate the current accumulated damage in the specimen.

8.1 Ramp Load Case

The static tests performed in a load control mode were subjected to ramp load with an increase of load at a constant rate, K_s . Assuming that the stress also increases at the same rate, the stress history for a ramp load case can be expressed by $\tau(t) = K_s t$. Since damage only occurs if $\tau(t) > \sigma_0 \tau_s Z$, a time t_0 can be defined at which $\tau(t_0) = \sigma_0 \tau_s Z$. Substituting the stress history of the ramp load case into Equation 8.5 and integrating from $t = t_0$ to $t = T$ yields:

$$\alpha e^{\left\{ \frac{-c}{K_s(n+1)} [K_s t - \sigma_0 \tau_s Z]^{(n+1)} \right\}} \bigg|_{t_0}^T = \int_{t_0}^T a [K_s t - \sigma_0 \tau_s Z]^b e^{\left\{ \frac{-c}{K_s(n+1)} [K_s t - \sigma_0 \tau_s Z]^{(n+1)} \right\}} dt \quad (8.6)$$

If the time to failure of a specimen in a short term ramp test is defined as T_s , then at $t = T_s$, $\alpha = 1$ and $K_s T_s = \tau_s Z$. At $t = t_0$, $\alpha = 0$ and $K_s t_0 = \sigma_0 \tau_s Z$; hence, simplification of Equation 8.6 yields:

$$e^{\left\{ \frac{-c}{K_s(n+1)} [\tau_s Z - \sigma_0 \tau_s Z]^{(n+1)} \right\}} = \int_{t_0}^{T_s} a [K_s t - \sigma_0 \tau_s Z]^b e^{\left\{ \frac{-c}{K_s(n+1)} [K_s t - \sigma_0 \tau_s Z]^{(n+1)} \right\}} dt \quad (8.7)$$

Foschi and Yao (1986a and b) showed that K_s is as a rule large compared to the model parameter c and consequently, Equation 8.7 can be rewritten as:

$$1 \cong \int_{t_0}^{T_s} a [K_s t - \sigma_0 \tau_s Z]^b dt = \frac{a}{K_s(b+1)} [\tau_s Z - \sigma_0 \tau_s Z]^{(b+1)} \quad (8.8)$$

The model parameter, a , can now be expressed in an approximate relationship by the ramp rate, short term strength, size effect and the model parameters b and σ_0 as:

$$a \cong \frac{K_s(b+1)}{(\tau_s Z - \sigma_0 \tau_s Z)^{(b+1)}} \quad (8.9)$$

The relationship dictates that the model parameter cannot be chosen independently from the other model parameters b and σ_0 , the ramp rate K_s , the short term strength τ_s and the width effect term Z .

8.2 Piecewise Linear Representation of Stress History

Let the damage after the I^{th} cycle be expressed in the form of a recurrence relationship as

$$\alpha_I = \alpha_{I-1} K_0(I) + K_1(I) \quad (8.10)$$

Since the stress cycles used in this study have identical shape, $K_0(I)$ and $K_1(I)$ are equal to K_0 and K_1 , respectively, for any given cycle I . Assuming initial damage, $\alpha_0 = 0$, the two unknown coefficients K_0 and K_1 can be determined by evaluating the accumulated damage during the first two stress cycles as:

$$K_1 = \alpha_1 \text{ and } K_0 = \left(\frac{\alpha_2}{K_1} - 1 \right) \quad (8.11)$$

where α_1 and α_2 are the accumulated damage after the first and second cycle respectively.

If K_0 and K_1 are known, the accumulated damage after $I + N$ cycles can be estimated through evaluation of Equation 8.10 recurrently as:

$$\alpha_{I+N} = \alpha_{I-1} K_0^N + K_1 \left(K_0^{N-1} + K_0^{N-2} + \dots + K_0 + 1 \right) \quad (8.12)$$

If $I = 1$ and $\alpha_0 = 0$; hence

$$\alpha_{1+N} = K_1 \left(K_0^{N-1} + K_0^{N-2} + \dots + K_0 + 1 \right) \quad (8.13)$$

Equation 8.13 can be rewritten in the form:

$$\alpha_{1+N} = K_1 \left(\frac{1 - K_0^N}{1 - K_0} \right) \quad (8.14)$$

If failure occurs after $N + 1$ cycles, then $\alpha_{1+N} = 1$. Thus, the number of cycles to failure can be determined through evaluation of Equation 8.14 as:

$$N_f = \frac{\log\left(\frac{K_1 + K_0 - 1}{K_1}\right)}{\log(K_0)} + 1 \quad (8.15)$$

The stress history of one load cycle, experienced by the cyclically tested specimens, can be described by three piecewise linear segments. For each segment, the accumulated damage can be evaluated by integration of Equation 8.5.

8.2.1 Ramp-load: segment 1

Assuming accumulation of damage only occurs when $\tau(t) - \sigma_0 \tau_s Z > 0$, let us define a time t_0 at which $\tau(t) = \sigma_0 \tau_s Z$ as starting point of the first segment. From $t = t_0$ to $t = t_1$, the stress increases at a constant rate, K_s . Thus, the stress history over the first segment can be expressed as $\tau(t) = K_s t$. Let us further define the peak stress in any given cycle as τ_c , thus, $\tau(t_1) = K_s t_1 = \tau_c$. The accumulated damage after the first segment can be estimated by integration of Equation 8.5 from $t = t_0$ to $t = t_1$ as:

$$\alpha e^{\left\{ \frac{-c}{K_s(n+1)} [K_s t - \sigma_0 \tau_s Z]^{(n+1)} \right\}} \bigg|_{t_0}^{t_1} = \int_{t_0}^{t_1} a [K_s t - \sigma_0 \tau_s Z]^b e^{\left\{ \frac{-c}{K_s(n+1)} [K_s t - \sigma_0 \tau_s Z]^{(n+1)} \right\}} dt \quad (8.16)$$

Assuming $c \ll K_s$ and $\alpha(t_0) = 0$, equation 8.16 can be rewritten as:

$$\alpha(t_1) \cong \int_{t_0}^{t_1} a (K_s t - \sigma_0 \tau_s Z)^b dt = \frac{a}{K_s(b+1)} [K_s t_1 - \sigma_0 \tau_s Z]^{(b+1)} \quad (8.17)$$

with $a \cong \frac{K_s(b+1)}{[\tau_s Z - \sigma_0 \tau_s Z]^{(b+1)}}$ and $\tau_c = K_s t_1$, the accumulated damage after the first

segment can be expressed by:

$$\alpha(t_1) \cong \left[\frac{\tau_c - \sigma_0 \tau_s Z}{\tau_s Z - \sigma_0 \tau_s Z} \right]^{(b+1)} \quad (8.18)$$

8.2.2 Constant-load: segment 2

During the second segment, the stress is at a constant level, τ_c , from the time, $t = t_1$ to $t = t_2$. Consequently, the stress history during the second segment is given as $\tau(t) = \tau_c$. The accumulated damage after the second segment can be evaluated through Equation 8.5 as:

$$\begin{aligned} \alpha e^{-c(\tau_c - \sigma_0 \tau_s Z)^n t} \Big|_{t_1}^{t_2} &= \int_{t_1}^{t_2} a [\tau_c - \sigma_0 \tau_s Z]^b e^{-c(\tau_c - \sigma_0 \tau_s Z)^n t} dt \\ \alpha(t_2) e^{-c t_2 (\tau_c - \sigma_0 \tau_s Z)^n} - \alpha(t_1) e^{-c t_1 (\tau_c - \sigma_0 \tau_s Z)^n} &= \\ &= \frac{-a}{c} [\tau_c - \sigma_0 \tau_s Z]^{(b-n)} e^{-c(\tau_c - \sigma_0 \tau_s Z)^n t} \Big|_{t_1}^{t_2} \end{aligned} \quad (8.19)$$

By dividing both sides by $e^{-c t_2 (\tau_c - \sigma_0 \tau_s Z)^n}$ Equation 8.19 yields:

$$\alpha(t_2) = \alpha(t_1) e^{c \Delta t (\tau_c - \sigma_0 \tau_s Z)^n} + \frac{a}{c} [\tau_c - \sigma_0 \tau_s Z]^{(b-n)} \left[c \Delta t (\tau_c - \sigma_0 \tau_s Z)^n - 1 \right] \quad (8.20)$$

where $\Delta t = (t_2 - t_1)$. Equation 8.20 can be modified by implementation of Equation 8.9:

$$\alpha(t_2) = \alpha(t_1) e^{c \Delta t (\tau_c - \sigma_0 \tau_s Z)^b} + \frac{K_s(b+1)}{c} \frac{[\tau_c - \sigma_0 \tau_s Z]^{(b-n)}}{[\tau_s Z - \sigma_0 \tau_s Z]^{(b+1)}} \left[c \Delta t (\tau_c - \sigma_0 \tau_s Z)^b - 1 \right] \quad (8.21)$$

8.2.3 Negative ramp-load: segment 3

The third segment stretches from the end of the constant stress state at which $t = t_2$ to the time, t_3 , where $\tau(t) = \sigma_0 \tau_s Z$. The stress decreases at a constant rate of $-K_s$. Hence, the stress history during the third segment is given by: $\tau(t) = \tau_c - K_s(t - t_2)$. Using the same methodology as in the previous two cases, the accumulated damage after the third can be derived from:

$$\alpha e^{-\int_{t_2}^{t_3} c [\tau_c - K_s(t-t_2) - \sigma_0 \tau_s Z]^n dt} = \int_{t_2}^{t_3} a [\tau_c - K_s(t-t_2) - \sigma_0 \tau_s Z]^b e^{-\int_{t_2}^t c [\tau_c - K_s(t-t_2) - \sigma_0 \tau_s Z]^n dt} dt \quad (8.22)$$

$$\alpha e^{\frac{c}{K_s(n+1)} [\tau_c - K_s(t-t_2) - \sigma_0 \tau_s Z]^{(n+1)}} \bigg|_{t_2}^{t_3} = \int_{t_2}^{t_3} a [\tau_c - K_s(t-t_2) - \sigma_0 \tau_s Z]^b e^{\frac{c}{K_s(n+1)} [\tau_c - K_s(t-t_2) - \sigma_0 \tau_s Z]^{(n+1)}} dt \quad (8.23)$$

Assuming, as before, that $c \ll K_s$, Equation 8.22 can be modified so that the accumulated damage after one complete load cycle is given by:

$$\begin{aligned}
\alpha(t_3) &= \alpha_1 = \alpha(t_2) + \int_{t_2}^{t_3} a \left[\tau_c - K_s(t - t_2) - \sigma_0 \tau_s Z \right]^b dt = \\
&= \alpha(t_2) + \frac{a}{K_s(b+1)} \left[\tau_c - \sigma_0 \tau_s Z \right]^{(b+1)}
\end{aligned} \tag{8.24}$$

Substituting the expression for model parameter a (Equation 8.9) together with Equations 8.18 and 8.21 into the above expression yields:

$$\begin{aligned}
\alpha_1 &= \left[\frac{\tau_c - \sigma_0 \tau_s Z}{\tau_s Z - \sigma_0 \tau_s Z} \right]^{(b+1)} \left[1 + e^{c \Delta t (\tau_c - \sigma_0 \tau_s Z)^n} \right] + \\
&\quad + \frac{K_s(b+1) \left[\tau_c - \sigma_0 \tau_s Z \right]^{(b-n)}}{c \left[\tau_s Z - \sigma_0 \tau_s Z \right]^{(b+1)}} \left[e^{c \Delta t (\tau_c - \sigma_0 \tau_s Z)^n} - 1 \right]
\end{aligned} \tag{8.25}$$

The same three calculation steps were made for the second stress cycle, hence, the accumulated damage after two complete stress cycles is given as:

$$\begin{aligned}
\alpha_2 &= \left[\frac{\tau_c - \sigma_0 \tau_s Z}{\tau_s Z - \sigma_0 \tau_s Z} \right]^{(b+1)} \left[1 + e^{c \Delta t (\tau_c - \sigma_0 \tau_s Z)^n} \right]^2 + \\
&\quad + \frac{K_s(b+1) \left[\tau_c - \sigma_0 \tau_s Z \right]^{(b-n)}}{c \left[\tau_s Z - \sigma_0 \tau_s Z \right]^{(b+1)}} \left[e^{2c \Delta t (\tau_c - \sigma_0 \tau_s Z)^n} - 1 \right]
\end{aligned} \tag{8.26}$$

If the model parameters are determined and knowing that $K_1 = \alpha_1$ and $K_0 = \left(\frac{\alpha_2}{K_1} - 1 \right)$, the number of cycles to failure of a specimen can be estimated from equation 8.15.

8.3 Model Calibration

The damage model was calibrated against the test data from the three type A test series following the procedures developed by Foschi *et al.* (1987). Foschi *et al.* assumed the static strength data, τ_s , to be lognormally distributed. In Figure 8.1, the static A-, B-, and C-series are fitted to lognormal distributions. Visual inspection of Figure 8.1 indicates

that the lognormal distributions fit the data well. Thus, it was assumed that the static strength of all three series could be represented with lognormal distributions. Each of the four independent model parameters (b , c , n , and σ_0) were also assumed to be lognormally distributed.

Figure 8.1. Lognormal fit of static strength distributions.

A Fortran program⁶ was used to estimate the mean and standard deviation of each individual model parameter through a non linear function minimization procedure using quasi-Newton method. Thus, a vector, X , with eight unknowns matching the mean and standard deviation of the four independent model parameters, was estimated through the minimization procedure. The procedure was summarized by Lam (1991) as follows:

⁶ Program written by Foschi, Folz, and Yao (1987).

- 1) Initial estimates of the lognormal distributions in the X -vector, was provided.
- 2) Using Equations 8.15, 8.25, and 8.26, a random sample of 1000 number of cycles to failure values ($N_f(i)$, $i = 1, 2, \dots, 1000$) was generated based on the initial estimate of the X -vector.
- 3) The cumulative probability distribution of the randomly generated N_f -values was obtained and compared to the experimental data by computing a objective function, Φ , as:

$$\Phi = \sum_{i=1}^L \left(1.0 - \frac{N_{f_i}^s}{N_{f_i}^a} \right)^2 \quad (8.27)$$

where L is the number of probability levels considered, $N_{f_i}^s$ is the simulated number of cycles to failure at the i^{th} probability level and $N_{f_i}^a$ is the actual number of cycles to failure obtained from the experimental data at the same probability level.

- 4) Equation 8.27 was minimized according to the quasi-Newton method. The initially chosen distribution parameters, X_i ($i = 1, \dots, 8$) were modified automatically by the program through an iteration procedure. In this study the increment ΔX_i was chosen as $0.001X_i$ and the convergence criterion, ϵ , for the objective function was set to 0.001. The program was considered to have found an optimal solution when changes in X_i of a magnitude, ΔX_i , did not reduce the objective function value (for $i = 1, \dots, 8$) between two consecutive iterations more than $\epsilon \Phi$.

8.3.1 Calibration results

Figure 8.2 shows the cumulative distributions of the number of cycles to failure, N_f , (in ln scale) of type A-specimens under bending at three mean stress levels.

Figure 8.2. Cumulative distribution of number of cycles to failure for type A-specimens.

It can be observed that the two distributions based on results of the A95% and A91% series, are slightly skewed at the lower tail. The skewed behaviour can, to some degree, be explained by the high mean stress level at which the cyclic tests were conducted. Of the fifteen specimens of the A95%-series, four failed within two stress cycles. The specimens that failed during the upward segment of the stress cycle, were considered to have survived one complete stress cycle. It can further be observed that the two series tested at mean stress levels 95% and 91%, exhibited larger variation than the

series tested at 83%. Due to limited sample size and high variability in the number of cycles to failure of each distribution, model calibration based on all three cumulative distributions proved to be difficult. Regardless of the initial model parameters, the minimization procedure fitted the cumulative distributions with higher mean stress level rather well at the lower tail, but did not provide a good fit at higher levels of cumulative probability nor did it provide a good fit for the A83%-series. The objective function in Equation 8.27 evaluated the square of the relative difference between the actual and simulated N_F -values. This procedure favors the cumulative distributions with lower N_F -values to those with higher N_F -values.

It was decided to calibrate the model based on one cumulative distribution only. Thus, the model was evaluated based on calibration results of both the A91%- and the A83%-series. To account for the inconsistencies at the lower tail of the distribution for the A91%-series, the specimen with N_F -values below a cumulative probability level of 0.3 were not included in the calibration procedure. For each of the two sets of calibrated model parameters, simulated N_F -values were produced for the three mean stress levels and compared to the experimental data. Table 8.1 shows the mean- and standard deviation-values of the two sets of the calibrated model parameters.

Figures 8.3 and 8.4 show the cumulative distributions of the experimental and simulated N_F -values of the three cyclic A-series based on model calibration of the modified A91%- and the A83%-series respectively.

Table 8.1. The mean and standard deviation of the model parameters for calibration based on the A91%- and A83%-series.

Parameter	Mean	STDV
Calibration results based on A91%-series:		
b	40.6628	2.40414×10^{-1}
c	1.93638×10^{-6}	9.85818×10^{-7}
n	4.48950×10^{-1}	1.15399×10^{-1}
σ_0	7.59441×10^{-2}	1.21265×10^{-2}
Calibration results based on A83%-series:		
b	33.7724	3.23337×10^{-1}
c	1.31652×10^{-6}	9.87354×10^{-7}
n	1.14480	2.65217×10^{-2}
σ_0	2.43012×10^{-2}	8.59007×10^{-3}

Figure 8.3. Cumulative distributions of N_F -values based on the A91% calibration result.

From Figure 8.3, it is obvious that the matching between simulated and actual number of cycles to failure was inconsistent. The simulated values fit the distributions of the A91%- and A95%-series reasonably well, but clearly overestimate the number of cycles to failure for the A83%-distribution. Similar inconsistent behaviour can be observed from Figure 8.4. In this case the simulated data perfectly match the actual values of the A83%-series, but underestimate the number of cycles to failure of both the A91%- and A95%-series.

Figure 8.4. Cumulative distributions of N_f -values based on calibration of the A83%-series.

Table 8.2 shows the fatigue performance based on the two sets of calibrated model parameters. Notable is the large standard deviation (37,782) of the A83%-series based on

the set of model parameters calibrated by the A91%-series. This indicates that the model calibration based on the A91%-series is inconsistent at lower stress levels.

Tale 8.2. Model predicted fatigue performance.

Mean Stress Level S_r	Mean N_f	STDV
Calibration based on A91%-series:		
0.95	45	81
0.91	340	666
0.83	2073	37782
Calibration based on A83%-series:		
0.95	16	22
0.91	75	118
0.83	1840	2730

Figures 8.5 and 8.6 compare the model predicted and actual fatigue performance of the type A-specimens under cyclic loading. The simulated fatigue performance in Figures 8.5 and 8.6 is based on the model calibration of the A91%- and A83%-series respectively. The assumptions of equal rank have been applied to both the experimental and simulated values to calculate the stress levels. The simulated N_f -values were picked from the cumulative distributions at probability levels corresponding to those of the experimental values. From Figure 8.5 it is clear that the model calibrated by the A91%-series provides a relatively good fit at higher stress levels but vastly overestimates the number of cycles to failure at lower stress levels.

Figure 8.5. Fatigue behaviour of A-specimens based on calibration of the A91%-series.

Figure 8.6. Fatigue behaviour of A-specimens based on calibration of the A83%-series.

The model calibrated by the A83%-series generates a general trend of the fatigue resistance that is in better agreement with the experimental data. Although this model, in certain regions, underestimates the number of cycles to failure, it provides values very close to the experimental data at lower stress levels. The model calibrated by the A83%-series is conservative at higher stress levels and more accurately describes the fatigue performance at lower stress levels than the model calibrated by the A91%-series. Consequently, it was decided that the model calibrated by the A83%-series provided the “best” representation of the fatigue behaviour in rolling shear mode for the laminated veneer specimens.

A least square regression line was fitted to the simulated data in Figure 8.6. The regression model equation takes the form:

$$SL = 0.9867 - 0.0261 \ln(N_f) \quad (8.28)$$

where SL is the stress level and N_f is the number of cycles to failure. The coefficient of determination, r^2 , was equal to 0.9919.

8.4 Verification of Damage Model

The model was verified by comparing simulated fatigue behaviour of type B- and C-specimens to the experimental data of the B91%- and C85%-series respectively. A size adjustment parameter, Z , was used to estimate the static strength distributions of the type B- and C-specimens based on the results from the static A-series.

8.4.1 B-series

Based on the width effect calculations in Chapter 6.1, the theoretical calculated size adjustment parameter for type B-specimens was set to 1.07. Thus, the simulated static strength for specimens of type B is equal to $1.07\tau_A$. Using the set of model parameters calibrated by the A83%-series and the same stress cycles applied to the experimental B91%-series, a cumulative distribution of N_f -values was generated by the model.

The cumulative distributions (in Ln-scale) of both simulated and experimental N_f -values for the type B-specimens are presented in Figure 8.7.

Figure 8.7. Experimental and simulated N_f -values based on Weibull weakest link estimation static of strength distribution for type B-specimens.

It is clear that the experimental data exhibit a larger variation of number of cycles to failure than the simulated values. This can be explained by the fact that the simulated N_F -values are based on the static strength distribution of the A-series, which had less variation of strength than the static B-series. Consequently the simulated values overestimate the number of cycles to failure at lower probability levels and underestimates them at higher probability levels. The cumulative distribution of the simulated values are also to some degree shifted to the right in relation to the experimental distribution. This is caused by the use of the theoretical calculated size adjustment factor that was higher than the experimentally determined width-effect. Thus, the static strength of the B-specimens used by the model was overestimated by approximately 5%.

In Figure 8.8, the predicted fatigue performance of type B-specimens is compared to the experimental results. Using the assumptions of equal rank, the stress level of the simulated and experimental data corresponding to a rank, i , was calculated as:

$$SL_i = \frac{\tau_{appl}}{\tau_{A_i} Z} \quad (8.29)$$

where τ_{appl} is the applied stress, τ_{A_i} is the estimated static strength of a type A-specimen at the i^{th} percentile and Z is the size adjustment factor between A- and B-specimens. The line through the data is the model equation (Equation 5.28). From Figure 8.8, it is obvious that the simulated and the experimental values have different slopes. Although the model fits the experimental values rather well at lower stress levels, it overestimates the fatigue performance at higher stress levels. It should, however, be stated that predictions in the low cycle region are very difficult to make with accuracy since the behaviour is very close to static loading.

Figure 8.8. Model simulated and actual fatigue behaviour of type B-specimens.

The above described procedure was repeated using the experimentally determined static strength distribution for type B-specimens. The cumulative distributions of simulated and experimental N_f -values are presented in Figure 8.9. In this case the simulated distribution is in very close agreement with the experimental data. In Figure 8.10, the predicted fatigue performance of type B-specimens is compared to the experimental results using the assumptions of equal rank. Here, the stress level at a rank (percentile), i , is given by:

$$SL_i = \frac{\tau_{\text{appl}}}{\tau_{B_i}} \quad (8.30)$$

where τ_{B_i} is the estimated static strength of a type B-specimen at the i^{th} percentile.

Figure 8.9. N_f -values for B-specimens based on experimental static strength distribution.

Figure 8.10. Fatigue behaviour of type B-specimens based on experimental static strength distribution.

From Figure 8.10 it is evident that the simulated values and the experimental data are close to identical in terms of spread and slope.

8.4.2 C-series

The load configuration effect study conducted in Chapter 6.2 revealed large differences between the theoretical calculated and experimental values. According to Weibull weakest link theories, the load configuration adjustment parameter, Z , between type A and C specimens should be approximately equal to 1.05. The experimentally established value of Z equaled 1.24. Due to the large difference between theoretical and experimental value, it was decided not to try and verify the damage accumulation model using the theoretical value of the load configuration adjustment factor. Instead the model was verified based on the established static strength distribution of type C-specimens and compared to the series of C-specimens cyclically tested at a mean stress level of 85%.

Based on the set of model parameters calibrated by the A83%-series and input parameters of the stress cycles experienced by the C85%-series, a cumulative distribution of N_f -values was generated by the model. The cumulative distribution is presented in Figure 8.11 together with the corresponding distribution of the experimentally established values for the type C-specimens. Visual inspection of Figure 8.11 suggests that the model slightly underestimates the number of cycles to failure at lower stress levels. The simulated actual fatigue resistance of type C-specimens is shown together with the model equation line in Figure 8.12. The assumptions of equal rank have been used to generate the stress levels for the data points in Figure 8.12. The stress level corresponding to a rank (percentile), i of the static strength distribution of type C-specimens was calculated as:

$$SL_i = \frac{\tau_{\text{appl}}}{\tau_{C_i}} \quad (8.31)$$

where τ_{appl} is the applied stress and τ_{C_i} is the estimated static strength at the i^{th} percentile.

Clearly the model-predicted and actual fatigue performance of type C-specimens agree reasonably well. Although the simulated values deviate from the experimental data within the 20 - 300 stress cycle region, the model predictions are conservative. More importantly, the model predictions are accurate at higher numbers of stress cycles.

Figure 8.11. N_F -values for C-specimens based on experimental static strength distribution.

Figure 8.12. Fatigue behaviour of type C-specimens based on experimental static strength distribution.

8.5 Final Remarks on Model Calibration and Verification

Comparing the modeled and actual fatigue behaviour for type B- and C-specimens as shown in Figures 8.11 through 8.16, the following conclusions can be made:

- 1) A model calibrated by all three cumulative distributions of N_F -value, failed to generate values consistent with the experimental data.
- 2) The damage accumulation model provides a good estimation of the fatigue behaviour in rolling shear mode for laminated veneer products of the various shapes and sizes used in this study.

- 3) The model is more accurate in the region of higher number of stress cycles than at lower numbers of cycles.
- 4) Deviations in fatigue behaviour between experimental data and simulated values can largely be contributed to an incorrectly determined size adjustment parameter rather than inconsistencies of the model.
- 5) Even if a correct value of the size adjustment parameter is chosen, the model has to be corrected for a size-related change of the variation of the static strength distribution.

9. SUMMARY AND CONCLUSIONS

A panel construction to evaluate the rolling shear behaviour of laminated Douglas fir veneer subjected to "flat-wise" bending was developed. The panel consisted of 15 layers of 2.5 mm thick veneer sheets of which the 3 central layers (cross-ply) were oriented at an angle of 90° to the remaining layers.

Based on three specimen sizes, the width and load configuration-effect for rolling shear failures of laminated veneer were experimentally evaluated. The experimental results were compared to theoretical values based on Weibull weakest link theory. It was found that the increase in strength due to width effect was approximately equal to 4% and 8% for the experimental and theoretical evaluations respectively. The experimentally established value of the width effect was in a range well within the sampling and experimental errors expected from a limited sample size. Consequently, it is not possible to conclusively establish a value of the width-effect between the two specimen sizes. It is nevertheless likely that a width effect exists, but of a lower magnitude than predicted by the theory.

Based on Weibull weakest link theory the increase in static strength due to the load configuration effect was found to be approximately 5%, whereas the experimental results yielded an increase in strength as high as 24%. The large difference between theoretical and experimental values indicated that the specimen types exhibited different failure mechanisms. Consequently, the Weibull weakest link theory proved not to be suitable in evaluating the load configuration effect between the two specimens types.

The rolling shear fatigue performance of panels was studied through cyclic testing of specimens until failure. Specimens were tested at three different mean stress levels, namely 95%, 91% and 83% of the static strength.

A damage model which took the stress history into consideration was calibrated to the experimental data of the cyclically tested specimens. Good agreement between model predictions and experimental results were obtained. Using size adjustment factors, the model was verified by two series of specimens of different size. It was found that the size adjusted model predictions and the experimental data agreed well if the size adjustment factors were correctly determined.

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