A MEASUREMENT OF THE STRONG COUPLING CONSTANT USING
TWO JET EVENT RATES AT 91 GEV

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Date **8 - APRIL - 1992**
Abstract

An analysis of hadronic Z\(^0\) decays observed in the OPAL detector at LEP is presented. The analysis involves the selection of hadronic Z\(^0\) decays from the set of OPAL data and the selection of well resolved charged tracks and electromagnetic energy clusters observed in each event. These are then used to determine the relative rates of events with 2-, 3-, 4- and 5-jets using a number of different jet definition schemes. These rates are then compared with theoretical predictions, computed perturbatively to second order in the coupling constant \(\alpha_s\) using the theory of Quantum Chromodynamics. By fitting the theoretical predictions to the data, the QCD parameters \(\Lambda\) and \(\mu\) are determined and a value of the strong coupling constant is obtained for each jet definition scheme.

In the analysis, the experimental uncertainties in the measured n-jet event rate distributions are estimated. A particular emphasis is placed on the description of the techniques used to correct the measured distributions for the limited acceptance and response of the detector and for the unknown process of hadronization. The applicability of the corrections for detector acceptance and response are justified and an examination of the uncertainties in the measured values of \(\alpha_s\) due to the unknown level of virtuality of the primary partons is presented. It is found that the measured values of \(\alpha_s\) are consisted with other measurements made at LEP energies and that the values determined in each jet definition scheme are consistent with each other.
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Chapter 1

Introduction

The LEP accelerator, which produces $Z^0$ particles at rest by colliding high energy electrons and positrons, is perhaps the best facility for studying and testing Quantum Chromodynamics (QCD), the theory of strongly interacting quarks. This is true for a number of reasons. First, due to the narrow width of the $Z^0$ resonance and its high cross section, large numbers of $e^+e^- \rightarrow Z^0 \rightarrow q\bar{q}$ events have been produced with well known centre of mass energies. Because of the high $Z^0$ mass, the velocities of the final state quarks are sufficient to boost them and their decay products to form well collimated jets of particles. Also, these quarks have sufficient energy to radiate hard gluons which also form collimated jets of particles. These are the properties of such events which allow one to examine theoretical predictions made about small numbers of observed jets rather than very large numbers of observed particles in the detector.

One of the fundamental parameters of QCD is the strong coupling constant $\alpha_s$ which is analogous to the fine structure constant $\alpha$ in QED. In QCD, observables are generally computed as a perturbation series in the coupling constant and hence, a measurement of an observable defined in this way allows one to experimentally determine the value of $\alpha_s$. Furthermore, $\alpha_s$ is believed to decrease with increasing energy and at $E_{CM} = M_{Z^0}$ it should be small enough that observables computed to small orders in $\alpha_s$ should be able to describe experimental measurements well. Finally, the presence of jets in hadronic $Z^0$ decays makes it possible to deduce, as directly as possible, the momentum distributions of the underlying partons, since their momenta are expected to lie in the directions of the well distinguished jets. This allows QCD predictions of the momentum distributions of partons to be studied through the observation of jets in the detector.
In this analysis the strong coupling constant was determined by fitting the observed rates of two jet events to the QCD predictions. Although a measurement such as this requires a precise prescription for the way in which two jet events are defined, an unambiguous definition does not exist. Instead, a number of algorithms, or schemes, for defining jets were examined and \( \alpha_s \) was measured using jets defined according to each. These various jet definition schemes, the selection of "good" events for the analysis, the analytical treatement of the measured n-jet event rate distributions and the determination of \( \alpha_s \) from these distributions are described in the following chapters.
Chapter 2

QCD and n-jet Event Rates

The theory of Quantum Chromodynamics (QCD) is believed to describe processes involving quarks. Within the context of this theory, quarks may be charged with one of three possible "colours" and interact by the exchange of coloured gluons. This is distinctly different from Quantum Electrodynamics (QED) which describes the interactions of electrons which have only one type of electric charge and exchange electrically neutral photons.

The Feynman rules for QCD differ from those for QED because of these extra features. The vertices and propagators carry extra factors to keep track of the possible colours of the quarks and gluons and there are additional self-coupling diagrams such as three- and four-gluon vertices. A particular representation of the Feynman rules for QCD may be found in [1] and will not be reproduced here. However, to introduce the strong coupling constant, it is useful to consider the vertex factor in the Feynman rules which describe the coupling of quarks to gluons as shown in figure 2.1.

\[
-i \sqrt{4\pi} \gamma_{\nu} T_{ij}^a, \quad a, \nu \quad (a)
\]

\[
-i \sqrt{4\pi} \gamma_{\nu}, \quad \nu \quad (b)
\]

Figure 2.1: Feynman rules for (a) the quark-gluon vertices in QCD and for comparison with QED, (b) the electron-photon vertex.

3
Here, the numerical factor $T_{ij}^a$ in the quark-gluon vertex introduces a weight for quarks with colour indices $i$ and $j$ coupling to a gluon with colour $a$. The important property of these diagrams is the presence of the coupling constants $\alpha_s$ and $\alpha$. These are fundamental quantities which characterize the strength of the couplings and are present in all matrix elements and hence, in all expressions for observables computed using the Feynman rules. Their values may be deduced by actually measuring an observable and solving for $\alpha$ or $\alpha_s$.

2.1 The Strong Coupling Constant

When Feynman diagrams beyond leading order are considered, processes may be described by an effective coupling constant which is in fact not constant, but depends on the momentum transfer, $Q^2$ involved. In QCD, $\alpha_s(Q^2)$ becomes infinite in the limit as $Q^2 \to 0$ and hence, measurements of $\alpha_s$ must be made at some non-zero momentum transfer, $\mu^2$. The expression for $\alpha_s(\mu^2)$ is given to second order in $\log(\mu^2/\Lambda^2)$ by[5]:

$$\alpha_s(\mu^2) = \frac{1}{b_0 \log(\mu^2/\Lambda^2)} \left[ 1 - \frac{b_1 \log(\log(\mu^2/\Lambda^2))}{b_0 \log(\mu^2/\Lambda^2)} \right]$$  \hspace{1cm} (2.1)

where

$$b_0 = \frac{33 - 2N_f}{12\pi}$$  \hspace{1cm} (2.2)

and

$$b_1 = \frac{153 - 19N_f}{24\pi^2}$$  \hspace{1cm} (2.3)

The QCD scale parameter $\Lambda$ is a constant to be determined from experiment and the factor $N_f$ is the number of active light quark flavours which, at $\sqrt{s} = 91$ GeV will be treated as $N_f = 5$. Hence, a measurement of $\alpha_s$ at a particular $Q^2 = \mu^2$ provides an experimental determination of the parameter $\Lambda$ which may be used to predict the values of $\alpha_s$ at different values of $Q^2$.

As mentioned previously, the coupling constant may be determined by measuring an observable which has been calculated to some order in $\alpha_s$ from perturbation theory. However, QCD matrix elements have so far only been computed to second order, and hence can describe processes with at most four final state partons. Thus, there is a vast difference in the observables which may be calculated using perturbation theory and those which can be measured, since hadronic $Z^0$ decays result in large numbers ($\gg 4$) of hadrons. The useful observables are
those which are found to be somewhat insensitive to the process by which the initial partons fragment into the observed hadrons. Examples of such observables are the rates of suitably defined n-jet events and it is these which are used in this analysis.

2.2 Hadron  

e+e− annihilate in the OPAL detector possibly with initial state radiation (a) to produce a Z° (b) which subsequently decays to a q̅q pair. These radiate off gluons to form a parton shower (c). The final state partons form hadrons by way of an incalculable process (d), some of which decay (e) before being detected.
produced continue to radiate or pair produce more partons. This is believed to continue until the invariant masses of the partons drops below some cutoff, \( Q_0 \) at which point the remaining partons somehow rearrange themselves to form colourless hadronic bound states. The details of this \textit{hadronization} process are not known, but they are modelled in a number of different ways by various Monte Carlo programs.

Since 4-momentum is conserved at each parton splitting, the momentum of a primary parton is carried away by the partons into which it splits and hence, by the hadrons into which they fragment. Since the available \( Q^2 \) at each splitting decreases, the transverse momenta of the radiated partons with respect to their parent decreases at each splitting, resulting in collimated \textit{jets} of particles whose total momenta reduces to those of the initial partons. Experimentally, this loose description of a jet is unsatisfactory. What is required is a consistent method for partitioning the observed hadrons into distinct classes which have the desired \textit{"jet-like"} properties. Since the sum of the momenta of the particles in the jets is expected to represent the momenta of the primary partons, such a definition allows one to compare experimental data with observables computed from second order QCD matrix elements in cases where fewer than five jets are observed.

2.3 The JADE-Type Algorithms

This section describes the algorithms used to define the jets observed in the detector. The original algorithm was first used by the JADE collaboration\cite{2, 3} in their studies of jet multiplicities. The algorithms partition an arbitrary list of 4-vectors \( p_i = (E_i, \vec{p}_i) \) into distinct jets by systematically merging pairs of 4-vectors until the scaled invariant mass of any pair becomes larger than some cutoff parameter, \( y_{cut} \).
The details of the algorithms are as follows:

i. Compute the "scaled invariant mass" $M_{ij}^2$ for each pair of vectors, $i, j$ in the list. $M_{ij}^2$ may be, for example, defined by $M_{ij}^2 = (p_i + p_j)^2 / E_{CM}^2$.

ii. If $\min_{i,j} M_{ij}^2$ is greater than the cutoff, $y_{\text{cut}}$, then terminate the algorithm.

iii. Otherwise, "merge" the two vectors $p_i$ and $p_j$ for which $M_{ij}^2$ is minimal and replace their entries in the list by a new vector $p_k$, thus reducing the number of elements in the list by one. $p_k$ may be defined, for example, simply by $p_k = p_i + p_j$ but other definitions are used.

iv. Return to step (i).

The scaled invariant mass and the formula for merging two vectors are not uniquely prescribed. Instead, there are a number of schemes each with their different prescriptions. The schemes for defining jets which were examined in this analysis are listed in table 2.1. The definitions of $M_{ij}^2$ which have the centre of mass energy squared, $s$, in the denominator are slightly modified when the algorithms are applied in experimental situations. In such cases, $s$ is replaced by $E_{\text{vis}}^2$, the visible energy in the event.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>$M_{ij}^2$</th>
<th>Recombination</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>$(p_i + p_j)^2 / s$</td>
<td>$p_k = p_i + p_j$</td>
<td>Lorentz invariant</td>
</tr>
<tr>
<td>JADE</td>
<td>$2E_i E_j (1 - \cos \theta_{ij}) / s$</td>
<td>$p_k = p_i + p_j$</td>
<td>Conserves $\sum p$</td>
</tr>
<tr>
<td>P</td>
<td>$(p_i + p_j)^2 / s$</td>
<td>$p_k = p_i + p_j$</td>
<td>Conserves $\sum E$ but not $\sum \vec{p}$</td>
</tr>
<tr>
<td>Durham</td>
<td>$2 \cdot \min(E_i^2, E_j^2) \cdot (1 - \cos \theta_{ij}) / s$</td>
<td>$p_k = p_i + p_j$</td>
<td>Conserves $\sum p$ and avoids experimental problems</td>
</tr>
<tr>
<td>Geneva</td>
<td>$8 \cdot \min(E_i^2, E_j^2) \cdot (1 - \cos \theta_{ij}) / (E_i + E_j)^2$</td>
<td>$p_k = p_i + p_j$</td>
<td>Conserves $\sum p$ and avoids experimental problems</td>
</tr>
</tbody>
</table>

Table 2.1: The various $M_{ij}^2$ and recombination definitions. In experimental applications, $s$ is replaced by $E_{\text{vis}}^2$, the visible energy in the event.
replaced by the visible energy squared $E_{\text{vis}}^2 = (\sum_i E_i)^2$, computed from the sum of the observed energies in the list of 4-vectors.

Clearly, the number of jets found by the algorithm depends on the quantity $y_{\text{cut}}$. For $y_{\text{cut}} \ll 1$, the algorithm will be terminated earlier, leaving a large number of unmerged 4-vectors and hence, a high jet multiplicity. For values of $y_{\text{cut}} \approx 1$, all vectors will be merged since any two tracks from a $Z^0$ decay must have a squared invariant mass of less than $s = M_{Z^0}^2$. These algorithms allow one to measure the rates of n-jet events as a function of the parameter $y_{\text{cut}}$ by counting the number of unmerged vectors which remain at a given value of $y_{\text{cut}}$.

To compare these results with the predictions of perturbative QCD, the algorithms may be applied to the primary partons with momentum distributions computed from the $O(\alpha_s)$ QCD matrix elements. This has been done analytically in the $\overline{\text{MS}}$ renormalization scheme and in this scheme, the rates of 3 and 4 jet events at a particular scale, $\mu^2$ may be expressed as a function of $y \equiv y_{\text{cut}}$ as follows:

$$R_3(y) = \frac{\sigma_3}{\sigma_{\text{tot}}} = \left( \frac{\alpha_s(\mu^2)}{2\pi} \right) A(y) + \left( \frac{\alpha_s(\mu^2)}{2\pi} \right)^2 \left( A(y) \cdot 2\pi b_0 \log(\mu^2/s) + B(y) \right) \quad (2.4)$$

$$R_4(y) = \frac{\sigma_4}{\sigma_{\text{tot}}} = \left( \frac{\alpha_s(\mu^2)}{2\pi} \right)^2 C(y) \quad (2.5)$$

where $b_0$ was defined in equation 2.2 and the functions $A(y)$, $B(y)$ and $C(y)$ depend on the particular jet definition used. The two jet event rate is then computed using the unitarity condition $R_2 + R_3 + R_4 = 1$ which should be valid only for those values of $y_{\text{cut}}$ where the fraction of events with five or more jets is negligible. Thus,

$$R_2(y) = \frac{\sigma_2}{\sigma_{\text{tot}}} = 1 - \left( \frac{\alpha_s(\mu^2)}{2\pi} \right) A(y) - \left( \frac{\alpha_s(\mu^2)}{2\pi} \right)^2 \left( A(y) \cdot 2\pi b_0 \log(\mu^2/s) + B(y) + C(y) \right) \quad (2.6)$$

The functions $A(y)$, $B(y)$ and $C(y)$ were tabulated for E, JADE and P schemes in [5] and were parameterized as fourth order polynomials in $\log(1/y)$ with coefficients listed in [6] for the Geneva and Durham schemes. Thus, by measuring $R_2$ as a function of $y_{\text{cut}}$, the strong coupling constant, $\alpha_s(\mu^2)$ may be obtained as a function of $\mu^2$ and $\Lambda_{\overline{\text{MS}}}$ by fitting the measured distributions to those predicted by equation 2.6. The subscript on $\Lambda$ indicates the fact that $\alpha_s(\mu^2)$
was determined from the rates calculated in the $\overline{\text{MS}}$ scheme and may be different from those computed using different renormalization techniques.
The LEP accelerator produces beams of electrons and positrons and accelerates them to approximately 45 GeV. These beams circulate in the LEP storage ring and are caused to collide at four points around the ring where detectors are located. The OPAL detector [4] is one of these and is of a conventional design, intended to serve as a multipurpose apparatus for observing a wide range of processes. The primary components of the OPAL detector are shown in figures 3.1 and 3.2. The coordinate system used is the polar coordinate system typical of such detectors with $\theta$ and $\phi$ shown in figure 3.1. Positrons enter the detector travelling in the positive z direction, while electrons travel in the negative z direction.

The important components for this analysis are the central tracking chambers and the electromagnetic calorimeter. These will be described briefly in the following sections. The other components shown in figure 3.1 play no major role in this analysis and will not be described here.

3.1 Central Tracking Chamber

The central tracking system consists of a high resolution vertex detector, a large volume jet chamber and an array of z-chambers. The vertex detector is located between the beam pipe and the jet chamber while the z-chambers are located between the jet chamber and the coil. All three drift chambers are located within a pressure vessel filled with an argon-methane-isobutane mixture at a pressure of 4 bars.

The central vertex (CV) chamber is one metre long, 47 cm in diameter and has two layers of drift cells. The inner layer has 36 cells with axial wires and the outer layer has 36 stereo cells.
Figure 3.2: Cross sections of the OPAL detector a) perpendicular and b) parallel to the beam axis.
with wires inclined at 4° with respect to the beam axis. Each axial and stereo cell has 12 and 6 anode wires, respectively. The anode wires are read at both ends and the z-coordinate of a hit can be determined to an accuracy of ~ 4 cm from the difference in the arrival times of these signals. In addition, the stereo wires can determine the z-coordinate of a hit with a resolution of 700 μm and both sets of wires determine the coordinate \( \phi \) with a resolution of 55 μm.

The central jet (CJ) chamber has a cylindrical active volume about four metres long with inner and outer diameters of 50 and 370 cm, respectively. The chamber consists of 24 sectors, each consisting of a plane of 159 sense wires spaced 10 mm apart between the radii of 255 mm and 1835 mm. The position of a hit in the \( r-\phi \) plane is obtained from the wire position and the drift time to obtain an average resolution of \( \sigma_{r\phi} = 135 \mu m \) each sense wire is read out at both ends and the ratio of the integrated charge at each end provides an estimate of the z-coordinate of a hit with an average resolution of \( \sigma_z = 6 \text{ cm} \). For tracks with polar angles between 43° and 137°, all 159 hits are recorded, provided the track’s transverse momentum is sufficient to reach the outermost layers. In addition, the jet chamber provides \( dE/dx \) information, however this is not used for this analysis.

Beyond the jet chamber lie the Z-chambers which provide more precise measurement of the z-coordinates of tracks which in turn provides a more accurate measurement of their polar angles. They consist of 24 identical drift chambers, 4 metres long and 50 cm wide. Each chamber is divided into 8 cells in the z-direction with 6 anode wires each, oriented in the \( \phi \) direction so that the drift direction is along the z-axis. The z-coordinates of hits are determined with resolutions between 100 μm and 200 μm, depending on the drift distance, while the coordinate in the \( r-\phi \) plane is determined by charge division with a resolution of about 1.5 cm. The polar acceptance of the z-chambers is \( 44° < \theta < 136° \) and tracks with \( \theta \) in this range will have information from all three tracking chambers.
3.2 Electromagnetic Calorimeter

The endcap and barrel regions are instrumented with separate, overlapping assemblies of lead-glass blocks fitted with photomultiplier tubes. The barrel has an angular acceptance of $35^\circ < \theta < 145^\circ$ while the endcaps extend this range down to $11^\circ$ from the beam axis.

The barrel assembly is located outside the coil, directly behind the presamplers at a radius of 245.5 cm. This calorimeter consists of 2455 blocks arranged in an array of 59 blocks in $\theta$ by 160 blocks in $\phi$ and give an intrinsic spatial resolution of approximately 11 mm. These blocks have an effective depth of 24.6 radiation lengths ($X_0$) and point to positions slightly displaced from the interaction point to eliminate cracks through which neutral particles from an event could escape without being detected. The presence of the coil and the pressure vessel contributes to $\sim 2 X_0$ of material in front of the calorimeter which degrades energy and spatial resolution of electromagnetic showers.

Each endcap assembly consists of 1132 blocks mounted parallel to the beam axis. The blocks have 52 cm$^2$ cross sections and effective radiation lengths of typically 22 $X_0$. Their intrinsic spatial resolution is similar to that of the blocks in the barrel and likewise, there are $\sim 2 X_0$ of material in front of these blocks due to the pressure vessel.

3.3 Event Selection Criteria

The multihadronic events recorded by OPAL are often not ideally suited for analysis. For example, jets which lie too far forward in $\theta$ may have tracks with far less than the maximum 159 hits recorded in the jet chamber which would suffer from poorly measured momentum. Or, a jet might intersect the boundary between the endcap and the barrel — a region where the energy resolution is severely degraded. Another possibility is that the multihadronic event observed did not have the well defined initial state $Z^0 \rightarrow q\bar{q}$ but instead took place by way of a two photon process, $e^+e^- \rightarrow e^+e^- + q\bar{q}$ which has the Feynman diagram shown in figure 3.3. Each of these possibilities may be removed from the data sample by applying quality cuts on
Chapter 3. The OPAL Detector and Event Selection

the charged tracks in the drift chamber and on the energy clusters in the calorimeter and finally by performing cuts on the overall quality of the events.

3.3.1 Charged Track Cuts

The cuts used to select good charged tracks are based on the number of hits recorded in the jet chamber, a minimum momentum transverse to the beam axis and their transverse and longitudinal impact parameters measured with respect to the event vertex. The intent of these cuts is to leave a cleaner set of tracks in the event which retain the overall event structure. For the analysis performed here, the actual cuts used may not significantly change the shapes of the final distributions because of the correction procedures to be described in chapter 4. However, there are obvious cuts which can be made to remove tracks which are clearly not associated with a hadronic Z⁰ decay, or which are poorly resolved to the point where their analysis would give little useful information.

Figure 3.3: The two photon process for production of $q\bar{q}$ states.

The impact parameters $d'_0$ and $z'_0$ are defined as the distances from the event vertex to the point of closest approach on the helix fitted to the track's hits in the drift chambers. The sign of $d'_0$ is such that $d'_0 < 0$ when the track's circle in the $r - \phi$ plane contains the event vertex.¹

Hence, large, positive values of $d'_0$ are associated with secondary decays in the drift chamber.

¹These definitions differ those of $d_0$ and $z_0$ found in the OPAL DST banks.
and not with the primary $Z^0$ decay products. This is clearly shown by the long positive tail of the $d_0'$ distribution shown in figure 3.4.

The cuts used in this analysis are similar to those used in other studies of OPAL data. They are as follows:

i. At least 20 CJ hits.

ii. $p_T > 150$ MeV/c Minimum transverse momentum.

iii. $|d_0'| < 5$ cm Maximum transverse impact parameter.

iv. $|z_0'| < 40$ cm Maximum longitudinal impact parameter.

v. $20^\circ < \theta < 160^\circ$ Polar angular acceptance.

Tracks which are poorly reconstructed or which are only partially contained in the jet chamber will have inaccurate momentum measurements. Such tracks are typically associated with a low number of hits in the jet chamber and are removed by applying the cut (i) on the number of CJ hits. Because the number of drift chamber cells intersected by a track depends on the angle $\theta$, this cut also restricts the polar acceptance of the tracks. The a more definite polar acceptance cut is provided by (v).
The cut (ii) on transverse momentum effectively rejects tracks which curve in the magnetic field to the point where they do not enter the calorimeter. That is, tracks with transverse momenta less than 150 MeV/c have radii of curvature less than 115 cm and will either spiral in the drift chamber, or stop in the coil.\footnote{The relation \( p_T = (0.2998 \text{ MeV/c})B\rho \) is useful here, where \( \rho \) is measured in centimeters and \( B \) is in kiloGauss.}

Cuts (iii) and (iv) are intended to restrict all tracks retained for further analysis to originate from either a \( Z^0 \) decay or from prompt secondary decays. Hence, the cut values must not be chosen too small lest too many tracks from the decays of particles which \textit{did} originate from the event vertex be excluded. The distributions are shown in figures 3.5 and 3.6 with the cut values indicated. These show the impact parameter distributions for both real and Monte Carlo data and the approximate agreement between these samples indicates that no significant bias has been introduced by the placement of the cuts.
Figure 3.5: Transverse impact parameters for real tracks (solid line) and for Monte Carlo tracks (dashed line). Arrows indicate the placement of cuts of this parameter.

Figure 3.6: Longitudinal impact parameters for real tracks (solid line) and for Monte Carlo tracks (dashed line). Arrows indicate the placement of cuts of this parameter.
3.3.2 Electromagnetic Cluster Cuts

The selection of the significant electromagnetic energy clusters is performed with the same intent as the selection of charged tracks. That is, an attempt is made to remove clusters which are not likely to have been caused by the primary $Z^0$ decay products. This is performed by the following cuts which depend on whether the cluster is in the barrel or the endcap calorimeter.

i. $E_{\text{raw}} > 300 \text{ MeV}$ for endcap clusters.
   
   $E_{\text{raw}} > 100 \text{ MeV}$ for barrel clusters.

ii. Energy in at least 2 blocks for endcap clusters.

iii. When both tracks and clusters are analysed, clusters are required to have no associated drift chamber track.

The cuts (i) on the raw energy measured in the cluster simply remove clusters caused by low energy particles or noisy calorimeter channels. The cut (ii) on the number of blocks spanned by the cluster is not applied to the barrel because of the pointing geometry of the lead-glass blocks in that region. However, there is no pointing geometry in the endcaps and thus, a track from the origin would create a cluster spanning more than one block, especially for larger angles $\theta$.

Cut (iii) suppresses the effects of counting particles twice when they are detected both in the tracking chambers and in the calorimeter. The track-cluster association is performed offline after both the charged tracks, presampler clusters and calorimeter clusters have been reconstructed. Tracks are extrapolated from their last measured point in the drift chamber into the outer detectors by simulating their propagation using GEANT. At each outer detector, the extrapolated tracks are associated with a cluster if they lie within the angular range spanned by the cluster.
3.3.3 Event Selection Cuts

Due to the long running periods and the nature of the apparatus, not all subdetectors in OPAL remain active all of the time. The redundancy in the OPAL trigger allows runs and the recording of data to continue even if one of the subdetectors is not active. Before the cuts on event quality are made, the status of each of the subdetectors involved in the analysis is determined and events are rejected if the essential subdetector elements were not functioning as required. Specifically, events were rejected from the analysis of charged tracks if any of the vertex, jet or Z-chambers, were inactive and events from electromagnetic cluster analyses were rejected if information from the calorimeters or presamplers was not present. With the assurance that the required components were running, events were selected based on the quality of the data obtained from them.

Given that all the required detectors are active the following initial cuts, based on the numbers of “good” tracks and clusters, are applied:

i. At least 5 charged tracks.

ii. At least 3 electromagnetic clusters.

Cut (i) is applied when the analysis involves charged tracks and (ii) is applied when electromagnetic clusters are to be analysed. For events which pass these cuts and depending on the type of analysis to be performed, a list of 4-vectors is constructed from the tracks, clusters or a combination of the two, which have passed their respective quality cuts.

Since only momentum is determined from the jet chamber and only energy is measured in the calorimeter, charged tracks are assumed to be pions and clusters are assumed to be photons. Hence, the 4-vectors $(E_i, \vec{p}_i)$ are constructed as follows:

\[
E_i = \sqrt{|\vec{p}_i|^2 + m^2} \quad \text{for charged tracks}, \tag{3.1}
\]

\[
\vec{p}_j = E_j \hat{n}_j \quad \text{for electromagnetic clusters}, \tag{3.2}
\]

where $\hat{n}_j$ is a unit vector from the event vertex to the centre of cluster number $j$. Once this list
has been formed, no further distinction is made between electromagnetic clusters and charged tracks. The remaining cuts are made only on quantities derived from this list of 4-vectors.

For electromagnetic clusters, the energy $E_i$ is the corrected energy which is obtained from an empirical function of the polar angle of the cluster, the energy measured in the associated subdetectors, such as the presampler, and the raw energy in the lead-glass. The parameters in the empirical formula have been determined from Monte Carlo simulations of the detector's response to electrons in a range of energies and angles. Thus, an estimate of the actual energy of an electromagnetic particle may be obtained even when some energy is not observed since showering may start in the coil or the pressure vessel.

While the corrected energy may not be reliable for values below 1 GeV, this should not introduce any systematic-bias for two reasons. First, the significance of clusters in this analysis is effectively weighted by the cluster energy. Secondly, as will be described in the following chapter, the measured distributions are corrected for any limited detector response. These corrected distributions should then be free from bias, provided the energy response of the calorimeter is adequately modelled by the Monte Carlo used in the correction procedure.

Next, the following quantities are computed from the contents of the list of 4-vectors:

Visible energy: $E_{\text{vis}} = \sum_i E_i$

Momentum balance: $P_{\text{bal}} = |\sum_i \vec{p}_i|/|\sum_i |\vec{p}_i|$

Thrust vector: $\hat{t}$

Invariant hemisphere mass: $M_{+(-)}$

The thrust vector is defined as the vector $\hat{t}$ which maximizes the quantity:

$$T = \frac{\sum_i |\vec{p}_i \cdot \hat{t}|}{\sum_i |\vec{p}_i|}.$$  \hspace{1cm} (3.3)

For well collimated two jet events, $\hat{t}$ points roughly along the jet axis but for events with higher jet multiplicities its direction is less well defined. Even so, it assigns an axis which would approximate the direction of the most well defined jet-like features of the event. Once the thrust vector has been obtained, the tracks in the list may be grouped according to the side of a plane perpendicular to $\hat{t}$ on which they lie. The invariant mass in each hemisphere is then
Chapter 3. The OPAL Detector and Event Selection

calculated as follows:

\[ M^2_{+(-)} = \left( \sum_{+(-)} E_i \right)^2 - \left| \sum_{+(-)} \vec{p}_i \right|^2 \]  

(3.4)

where \(+(-)\) denotes the subset of all tracks indices \(i\) for which \(\vec{p}_i \cdot \hat{t} > 0(< 0)\). The following cuts are then applied to these quantities:

iii. \(E_{vis} > 20\) GeV for electromagnetic cluster analysis,

\(E_{vis} > 20\) GeV for charged track analysis,

\(E_{vis} > 40\) GeV for both CT and EM cluster analysis.

iv. \(P_{bal} < 0.4\) Maximum momentum balance.

v. \(|t_3| < \cos(43°) = 0.731\) Minimum polar angle of thrust axis.

vi. \(\min(M_+, M_-) > 2\) GeV Minimum hemisphere invariant mass.

All of these cuts reduce the background from two-photon events which are characterized by low numbers of tracks or clusters, a thrust axis which lies close to the beam axis and a large missing energy, since the final state electrons are generally scattered at small angles and escape down the beam-pipe. Figure 3.7 shows the visible energy versus the number of electromagnetic clusters. The large cluster of points in the centre of the plot is due to hadronic \(Z^0\) decays while the cluster at low energies and low multiplicities corresponds to two-photon events. A similar result is observed when the visible energy is plotted against the number of charged tracks. These motivate the requirements that events pass cuts (i) and (ii).

The correlation between the momentum balance and the direction of the thrust axis are shown in figure 3.8. This shows that events with a large momentum imbalance are generally associated events lying close to the beam axis. Such events are generally caused by events which are not well contained in the detector, or which have a portion of their tracks removed by the track quality cuts. This acceptance of partially contained events is prevented by cutting on \(t_3\) as shown in the figure. The choice of the \(t_3\) cut value is such that events passing the cut will lie in the barrel region of the calorimeter, where there remains approximately 28° between the
Figure 3.7: Visible energy versus electromagnetic cluster multiplicity.
Figure 3.8: Momentum balance versus $t_3$. 

Charged tracks + EM clusters
effective cut on the polar angles of the tracks. This is sufficiently large that entire jets will be retained for further analysis.

This effect is also shown clearly in figure 3.9 where the visible energy is plotted against $t_3$. This shows a significant drop in visible energy for events with large $t_3$ which again may be attributed to only partial acceptance of their tracks or clusters. In all, approximately 58,000 multihadronic events were analysed of which 40,000 were accepted for the charged track analysis, 34,000 for the electromagnetic cluster analysis and 37,000 for the combined analysis.
Chapter 4

Experimental n-jet Event Rates and Data Correction

The lists of good 4-vectors from events which passed the selection cuts were used as input to the jet finding algorithms based on the E, JADE, P, Geneva and Durham schemes. For the analysis involving electromagnetic clusters, the measured jet rate distributions as functions of $y_{\text{cut}}$ in each of these schemes are shown in figure 4.1. These are the raw distributions measured at the detector level and are biased by the limited detector acceptance and response. Also, they are unsuitable for a direct comparison with the theoretical expressions for $R_2$, $R_3$ and $R_4$ found in equations 2.5 and 2.6. This is because the theoretical rates were determined from the momentum distributions of the primary partons, whereas the distributions shown in figure 4.1 were measured using the detected particles. These particles are believed to have evolved from the initial partons through the incalculable process of hadronization and clearly, the rates at the detector and parton levels may not necessarily be directly compared.

This chapter describes the way in which both the detector and hadronization effects can be removed by correcting the measured n-jet event rate distributions using Monte Carlo simulations of the OPAL detector and a model for hadronization. The resulting distributions may be compared directly with the QCD predictions. In principle, the unfolding procedure used to correct this data may be applied to any measured distribution and its use and limitations are described in the following sections.

4.1 Bin-by-Bin Corrections

To illustrate the principle used to correct an arbitrary distribution for limited detector acceptance and response, it is useful to examine a simple situation. Consider a spectroscopy
Figure 4.1: Experimental n-jet event rate distributions as functions of $y_{cut}$. 
experiment where the energies of gamma rays from a source are measured using a detector and are histogrammed using some form of multichannel analyser. We assume that the energy spectrum is divided up into $N$ discrete bins and say that the energy $E^0$ of an incident gamma ray falls in bin $i$ when $E_{i-1} < E^0 < E_i$, $i = 1, \ldots, N$ where the $E_j$'s represent the boundaries of the bins in the histogram. With a perfect detector, every incident gamma ray which has energy in bin $i$ would contribute one count to bin $i$ in the measured distribution. However, for real detectors, there are several factors which may cause this gamma ray to contribute to the contents of some bin other than $i$. This effect is referred to as migration between bins.

First, the intrinsic energy resolution of the detector may cause an incident gamma ray with energy $E^0$ in bin $i$ to be detected in some other bin. The response of such a detector to a gamma ray with energy $E^0$ may be modelled, for example, by a Gaussian with a mean at $E^0$ and with some width $\sigma(E^0)$ representing the detector's intrinsic energy resolution for that particular energy. Hence, if this were the only factor limiting the measured energy distribution, monochromatic gamma rays with energy in bin $i$ would sometimes have a measured energy in bin $i$ but could also have energies in bins $j \neq i$. If, however, $\sigma$ were reasonably narrow, then one would expect the number of gamma rays with detected energy in bins $j < i$ and $j > i$ to be small.

Next, if the detector had some material in front of its active elements, some energy may be lost in this material and the detected energy would be less than the incident energy. Thus, for monochromatic gamma rays with energy in bin $i$, the mean of the detected energy distribution described above would lie in some bin $j$ with $j \leq i$.

A third factor which could affect the measured distribution is that of limited detector acceptance. If the detector did not trigger on every gamma ray, then the number of detected gamma rays would be less than the number which were incident. Furthermore, the acceptance may be different in different energy regions and a uniform energy distribution of incident gamma rays could result in a nonuniform distribution of measured energies.

All these detector characteristics may be modelled, in a statistical sense, by a transfer matrix
Chapter 4. Experimental n-jet Event Rates and Data Correction

$G_{ij}$, the elements of which represent the probability that incident gamma rays with energy in bin $j$ are detected in bin $i$. Thus, if the source emitted gamma rays with an energy distribution given by $E_j^0$, then one would detect a distribution given by

$$E_{i}^{\text{det}} = \sum_j G_{ij} E_j^0. \quad (4.1)$$

Clearly, if one knew the elements of the matrix $G$ exactly, then one could correct a measured energy distribution for the detector's limited acceptance and response to arrive at a distribution

$$E_{i}^{\text{corr}} = \sum_j (G^{-1})_{ij} E_{i}^{\text{det}}. \quad (4.2)$$

Then, one would expect that $E_{i}^{\text{corr}} \equiv E_i^0$ — the true energy spectrum of the source.

One way to determine $G^{-1}$ would be to simulate the response of the detector to gamma rays using a Monte Carlo program. Then, for a large number of simulated incident gamma ray with incident energy in bin $i$ one could count the number detected in bins $j$ for $j = 1, \ldots, N$ and obtain an estimate for the elements $G_{ij}$. With sufficient statistics, one could obtain an estimate for the entire matrix $G$ which could then be inverted to obtain an estimate of $G^{-1}$. However, the statistical uncertainties of the matrix elements are only reduced as $1/\sqrt{N}$ when the number of simulated events, $N$, becomes large. Due to the large number of arithmetic operations required to compute the inverse of a matrix, the statistical uncertainties in $G^{-1}$ would be reduced far less rapidly and only for very large Monte Carlo data samples would it be possible to estimate $G^{-1}$ reliably in this way.

If, however, the matrix $G$ were diagonal, that is, $G_{ij} = b_i \delta_{ij}$, then the inverse $G^{-1}$ would also be diagonal and could be written $(G^{-1})_{ij} = c_i \delta_{ij}$ where $c_i \equiv 1/b_i$. This matrix could be readily determined using the Monte Carlo technique outlined above without the need for enormous sets of simulated data. Furthermore, the bin widths for the energy distribution may be suitably chosen so that the situation where $G$ is diagonal is realized, at least approximately. The factors $c_i$ may then be determined simply as the ratio of the normalized initial distribution and final distributions:

$$c_i = \frac{(E_i^0 / \sum_i E_i^0)}{(E_{i}^{\text{det}} / \sum_i E_{i}^{\text{det}})}. \quad (4.3)$$
where $E_i^0$ and $E_i^{\text{det}}$ are the simulated incident and detected energy distributions. Once the $c_i$'s have been determined in this way, the energy spectrum of an arbitrary source may be corrected for the limited acceptance and response of the detector by

$$E_i^{\text{corr}} = c_i \cdot E_i^{\text{det}}$$

(4.4)

where $E_i^{\text{det}}$ now represents the number of real gamma rays detected having energy in bin $i$. This technique is referred to as bin-by-bin correction.

4.2 Potential Sources of Bias

The bin-by-bin correction technique may be reliably applied to distributions for which $G$ is diagonal. However, in some circumstances, the same technique can be applied to distributions where some of the off-diagonal elements of $G$ are non-zero. This situation would arise when the bin widths are chosen smaller than the intrinsic resolution of the detector. This does not necessarily make the technique outlined above inapplicable, however it does introduce the possibility of biasing the corrected distributions.

This potential for bias arises when the shapes of the simulated and detected distributions do not match. To examine this effect more closely let us continue to consider the gamma ray source and detector experiment. When the bin widths are small, there are contributions to the fraction of gamma rays with detected energy in bin $i$ not only from the incident gamma rays with energy in bin $i$, but also from those with energies in bins $j \neq i$ which migrate into bin $i$ at the detector level. If the fraction which remained in bin $j$ was the same in both the real experiment and in the simulated data but the fraction migrating into bin $i$ were significantly different, then the correction coefficients $c_i$, computed using equation 4.3 would be determined incorrectly and would bias the corrected distribution towards the simulated one. This effect generally takes place where there are sharp discontinuities occurring near different bins in the distributions of the simulated and real data. For this reason it is desirable to have the simulated data model the real data as closely and as smoothly as possible.
Chapter 4. Experimental n-jet Event Rates and Data Correction

Of course, the bin-by-bin correction procedure may be applied to any measurable distribution and it is used in this analysis of OPAL data. To apply bin-by-bin corrections to arbitrary distributions measured using the OPAL detector, it is necessary that the simulated properties of multihadronic events agree closely with the observed properties. To achieve this agreement, considerable attention has been given to the problem of tuning the parameters of JETSET and other Monte Carlo programs so that they reproduce the overall properties of the observed hadronic events [7]. Figure 4.2 shows several event shape distributions for both measured events and events simulated using JETSET 7.2 with suitably tuned parameters. Definitions and a discussion of these parameters may be found, for example, in [8]. In general, the agreement is good and this suggests that the limited detector acceptance and response may be corrected for without biasing results when histogram bins are smaller than the intrinsic resolution of the detector.

4.3 Corrected Jet Rate Distributions

The jet rate distributions shown in figure 4.1 were corrected for the limited acceptance and response of the OPAL detector in the manner just described. A first set of hadronic \( Z^0 \) decays was generated using the JETSET 7.2 Monte Carlo program [9] and the n-jet hadronic event rates \( R_{n}^{\text{had}}(y_i) \) were measured for each jet definition scheme shown in table 2.1. In a second set, JETSET was used to simulate hadronic \( Z^0 \) decays including initial state radiation followed by a simulation of the OPAL detector using the COPAL detector simulation program. The n-jet event rates of these simulated events, \( R_{n}^{\text{sim}}(y_i) \), were then measured in each jet definition scheme using good 4-vectors from these events which passed the same cuts applied to the real data described in chapter 3.

From these pairs of distributions, the bin-by-bin correction coefficients for the n-jet event rates were computed using equation 4.5:

\[
\epsilon_i^{(n)} = \frac{(R_{n}^{\text{had}}(y_i)/N_{\text{had}})}{(R_{n}^{\text{sim}}(y_i)/N_{\text{sim}})}
\] (4.5)
Figure 4.2: Event shape distributions for both real events (solid curves) and events simulated using the JETSET Monte Carlo followed by a simulation of the OPAL detector (broken curves). (EM+CT analysis)
where $N_{\text{sim}}$ and $N_{\text{had}}$ were the total number of events in the first and second Monte Carlo data sets, respectively. The corrected rates $R_n^{\text{corr}}(y_i)$ were then computed from these coefficients and the n-jet rates $R_n(y_i)$:

$$R_n^{\text{corr}}(y_i) = c_i \cdot R_n(y_i)$$

(4.6)

The n-jet event rates measured using electromagnetic clusters, corrected in this way for detector acceptance and response, are shown in figure 4.3. Also, the correction coefficients for the two and three jet events are shown in figure 4.4.

### 4.4 Hadronization Corrections

The distributions which have been corrected for the acceptance and response of the detector are still unsuitable for comparison with equations 2.5 and 2.6 since they have not been corrected for the hadronization process. To achieve this, the bin-by-bin correction procedure was applied to the detector level data with the correction coefficients $c_i$ computed as follows.

The JETSET Monte Carlo was used to simulate the parton shower only, and the 4-vectors of the partons were used as input to the various jet finding schemes, thus determining the n-jet event rates at the parton level, $R_n^{\text{part}}(y_i)$. In cases where only two partons were generated in the parton shower the event was classified as a two jet event for all values of $y_i > 0$. The same set of Monte Carlo data which included initial state radiation, hadronization and detector simulation was used to simulate the detected distributions and the correction coefficients were computed in the usual way:

$$c_i^{(n)} = \frac{(R_n^{\text{part}}(y_i)/N_{\text{part}})}{(R_n^{\text{sim}}(y_i)/N_{\text{sim}})}.$$  (4.7)

Then, the measured n-jet event rate distributions, corrected for the limited detector acceptance and response as well as for hadronization process were computed simply using equation 4.6.

This step in the correction procedure requires still more justification. Since the real distributions at the parton level and the process of hadronization are not known there is the possibility of biasing the corrected distributions using the phenomenological JETSET model or any other Monte Carlo generator. In general, if it can be shown that the results obtained are insensitive
Figure 4.3: N-jet event rate distributions corrected for detector acceptance and response as functions of $y_{cut}$. Dashed lines show the uncorrected distributions. (EM cluster analysis)
Figure 4.4: Bin coefficients for correcting 2- and 3-jet event rate distributions for detector acceptance and response. (EM cluster analysis)
to the particular Monte Carlo model, or to the parameter values used to simulate the events, then one can assume that modelling the unknown parton distributions in this way will not bias the corrected measurements. Any variation in the parameters deduced from the corrected distributions when different Monte Carlo models were used, or when a range of Monte Carlo parameters were used may be quoted as a systematic uncertainty associated with the correction procedure.

The n-jet event rates measured using electromagnetic clusters, corrected in this way are shown in figure 4.5 and the correction coefficients $c_i^{(2)}$ and $c_i^{(3)}$ are shown in figure 4.6.
Figure 4.5: N-jet event rate distributions corrected for detector acceptance and response and for hadronization as functions of $y_{\text{cut}}$. Dashed lines show the uncorrected detector level distributions. (EM cluster analysis)
Figure 4.6: Bin coefficients for correcting 2- and 3-jet event rate distributions for detector acceptance and response and for hadronization. (EM cluster analysis)
Experimental Determination of $\alpha_s$

The distributions shown in figure 4.5 were corrected for both detector acceptance and response and for hadronization. Now the functions given in equations 2.5 and 2.6 may be fit to these curves to obtain $\alpha_s$ and $\mu^2$. In practice, it is more meaningful to fit the differential two jet rate distribution defined by

$$D_2(y) = \frac{R_2(y) - R_2(y - \Delta y)}{\Delta y}.$$  \hspace{1cm} (5.1)

This, and the details of the procedure used to it the data are described in this chapter. Also, a description of the treatment of the uncertainties in the data and the fitted parameters is presented here.

5.1 Statistical Correlation in $R_2$

The n-jet event rate distributions $R_n(y_{cut})$ were obtained by counting the number of events with $n$ jets at each value of $y_{cut}$. If $y_{3-jet}$ and $y_{2-jet}$ were the smallest values of $y_{cut}$ for which an event had two and three jets, respectively, then this event would be counted as a three jet event for all $y$ such that $y_{3-jet} < y < y_{2-jet}$ and a two jet event for all $y > y_{2-jet}$. Hence, a single event is counted in the $R_n$ distributions many times, which introduces correlations between the contents of neighboring bins in $y$. For this reason, the value of $\chi^2$ computed from a fit to these distributions is not well defined in a statistical sense.

The correlations betwixt adjacent bins in the 2-jet distribution may be removed by considering the differential 2-jet rates defined in equation 5.1. This new distribution represents the number of events which were counted as 3-jet events for $y_{cut} < y - \Delta y$ and as 2-jet events for $y_{cut} > y$ and hence, each event is counted in this distribution only once. Thus, the statistical
errors in $D_2(y)$ are uncorrelated and a meaningful value of $\chi^2$ may be obtained from a fit to this distribution.

5.2 Estimation of Uncertainties in $D_2(y)$

The $D_2(y)$ distributions defined above were measured in each of the jet definition schemes using three sets of 4-vectors obtained from the selection of charged tracks, electromagnetic clusters and the combination of these, as described in chapter 3. These distributions were corrected for detector acceptance and response as well as for hadronization by the techniques described in chapter 4 using approximately 36,000 Monte Carlo $Z^0$ decays with full detector simulation and 100,000 events simulated at the parton level.

Table 5.1 shows the resulting differential 2-jet event rates obtained for each of these schemes. In each jet definition scheme, the values of $D_2(y)$ were obtained by the weighted mean of the rates $D_2^{(CT)}(y)$, $D_2^{(EM)}(y)$ and $D_2^{(EM+CT)}(y)$:

$$<D_2(y)> = \frac{\sum_i D_2^{(i)}(y)/(\sigma^{(i)})^2}{\sum_i 1/(\sigma^{(i)})^2}$$

(5.2)

where the index $i$ runs over the three sets of data, $i = EM, CT, EM + CT$. The statistical uncertainties of $<D_2(y)>$ were obtained by adding those of the $D_2^{(CT)}$, $D_2^{(EM)}$ and $D_2^{(EM+CT)}$ distributions in quadrature and the experimental uncertainty was computed using

$$\sigma_{exp}(y) = \sqrt{\frac{<D_2(y)^2> - <D_2(y)>^2}{2}}.$$  

(5.3)

The final error quoted for $<D_2(y)>$ in table 5.1 is the result of these statistical and experimental errors, added in quadrature.

5.3 Determination of $\Lambda_{\overline{MS}}$ and $\mu^2$ from the $D_2(y)$ Distributions

When $\mu^2$ is constrained by the relation $\mu^2 = M_{Z^0}^2$, the strong coupling constant may be parameterized only in terms of $\Lambda_{\overline{MS}}$ as given by equation 2.1. In this form, $\Lambda_{\overline{MS}}$ may be
Chapter 5. Experimental Determination of $\alpha_s$

<table>
<thead>
<tr>
<th>$y$</th>
<th>$\Delta y$</th>
<th>$&lt;D_2(y)&gt;_{(E)}$</th>
<th>$&lt;D_2(y)&gt;_{(JADE)}$</th>
<th>$&lt;D_2(y)&gt;_{(P)}$</th>
<th>$&lt;D_2(y)&gt;_{(Durham)}$</th>
<th>$&lt;D_2(y)&gt;_{(Geneva)}$</th>
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<td>0.010</td>
<td>0.005</td>
<td>15.46 ± 1.57</td>
<td>23.91 ± 1.08</td>
<td>26.74 ± 0.95</td>
<td>26.51 ± 0.72</td>
<td>17.59 ± 1.13</td>
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<td>0.015</td>
<td>0.005</td>
<td>16.16 ± 1.02</td>
<td>18.38 ± 0.55</td>
<td>19.94 ± 0.56</td>
<td>15.31 ± 0.57</td>
<td>17.41 ± 0.84</td>
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<tr>
<td>0.020</td>
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<td>14.29 ± 0.54</td>
<td>15.69 ± 0.62</td>
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<td>0.030</td>
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<td>11.01 ± 0.30</td>
<td>11.66 ± 0.36</td>
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<td>13.31 ± 0.48</td>
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<td>3.46 ± 0.13</td>
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<td>4.08 ± 0.15</td>
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<td>0.100</td>
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<td>2.68 ± 0.14</td>
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<td>0.140</td>
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<td>0.68 ± 0.05</td>
<td>0.49 ± 0.04</td>
<td>0.25 ± 0.03</td>
<td>0.98 ± 0.06</td>
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Table 5.1: Differential 2-jet event rates.

<table>
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<tr>
<th>$\Lambda_{MS}$ (MeV)</th>
<th>793 ± 31</th>
<th>303 ± 31</th>
<th>245 ± 27</th>
<th>341 ± 30</th>
<th>234 ± 38</th>
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</thead>
<tbody>
<tr>
<td>$\alpha_s(M^2_{Z^0})$</td>
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<td>0.123 ± 0.002</td>
<td>0.119 ± 0.002</td>
<td>0.126 ± 0.002</td>
<td>0.119 ± 0.003</td>
</tr>
<tr>
<td>$\chi^2/DOF$</td>
<td>4.13/3</td>
<td>3.68/5</td>
<td>5.96/6</td>
<td>2.26/8</td>
<td>0.13/2</td>
</tr>
</tbody>
</table>

Table 5.2: Results of the one parameter fit to $D_2(y)$ with $\mu^2 = M^2_{Z^0}$.

determined from a one-parameter fit of the data in table 5.1 to the $D_2(y)$ distribution, defined by equations 2.6 and 5.1.

In all schemes, the coefficients $A(y)$, $B(y)$ and $C(y)$ were computed using the parameterization found in [6]. Because the $O(\alpha_s^2)$ calculations appear to underestimate the production of 4-jet events when $\mu^2$ is constrained to $M^2_{Z^0}$[2] this fit was performed in the region where $R_4(y) < 1\%$. The resulting values of $\alpha_s$, $\Lambda_{MS}$ and the corresponding $\chi^2$ for each fit are presented in table 5.2. The uncertainties on $\Lambda_{MS}$ correspond to 68.3% confidence limits and the uncertainties quoted on $\alpha_s(M^2_{Z^0})$ were estimated by recording its maximum and minimum values found within the 68.3% confidence region of $\Lambda_{MS}$. 

Chapter 5. *Experimental Determination of $\alpha_s$*

One may also write $\mu^2 = fE_{CM}^2$ in equations 2.6 and 2.1 and treat $f$ as a free parameter. When $\mu^2$ is given this freedom the $\mathcal{O}(\alpha_s^2)$ calculations can adequately describe the 4-jet event rates in the region $y < 0.06$ [12]. However, the 5-jet event rates are not predicted by these calculations and for this reason the fits were performed in the region where $R_5(y) < 1\%$. The resulting values for $\Lambda_{\overline{\text{MS}}}$ and $f$ are shown in table 5.3*. Again, the quoted uncertainties of $\Lambda_{\overline{\text{MS}}}$ and $f$ correspond to 68.3% confidence intervals. From these fitted parameters, $\alpha_s$ is computed in two ways. $\alpha_s(\mu^2)$ is simply the value computed using equation 2.1 with both $\Lambda_{\overline{\text{MS}}}$ and $\mu^2 = fM_{Z0}^2$ determined from the fits. $\alpha_s(M_{Z0}^2)$ is computed using the value of $\Lambda_{\overline{\text{MS}}}$ determined from the two parameter fits, but with $\mu^2 \equiv M_{Z0}^2$. The uncertainties on $\alpha_s(\mu^2)$ and $\alpha_s(M_{Z0}^2)$ were determined from the maximum and minimum values obtained within the 68.3% confidence intervals of their parameters.

The measured differential 2-jet event rate distributions with the both fitted curves superimposed on the data are shown in figure 5.1.

### Table 5.3: Results of the two parameter fit to $D_2(y)$ with $\Lambda_{\overline{\text{MS}}}$ and $f \equiv \mu^2/E_{CM}^2$ as free parameters.

<table>
<thead>
<tr>
<th>$\Lambda_{\overline{\text{MS}}}$ (MeV)</th>
<th>E</th>
<th>JADE</th>
<th>P</th>
<th>Durham</th>
<th>Geneva</th>
</tr>
</thead>
<tbody>
<tr>
<td>119 ±9</td>
<td>121 ±9</td>
<td>159 ±18</td>
<td>212 ±78</td>
<td>145 ±26</td>
<td></td>
</tr>
<tr>
<td>$f = \mu^2/s$</td>
<td>0.000039 ±0.000001</td>
<td>0.00241 ±0.00048</td>
<td>0.034 ±0.014</td>
<td>0.0025 ±0.0011</td>
<td>0.031 ±0.030</td>
</tr>
<tr>
<td>$\alpha_s(\mu^2)$</td>
<td>0.398 ±0.028</td>
<td>0.186 ±0.019</td>
<td>0.148 ±0.019</td>
<td>0.216 ±0.065</td>
<td>0.147 ±0.022</td>
</tr>
<tr>
<td>$\alpha_s(M_{Z0}^2)$</td>
<td>0.108 ±0.002</td>
<td>0.108 ±0.002</td>
<td>0.112 ±0.002</td>
<td>0.117 ±0.008</td>
<td>0.111 ±0.002</td>
</tr>
<tr>
<td>$\chi^2/DOF$</td>
<td>2.32/6</td>
<td>3.43/8</td>
<td>10.40/9</td>
<td>1.98/9</td>
<td>3.48/5</td>
</tr>
</tbody>
</table>

*In practice, the parameter $\log(f)$ was used to fit the data.*

5.4 Parton Virtuality and Hadronization Effects

The way in which the measured $D_2(y)$ distributions are corrected for the limited acceptance and response of the detector was described in chapter 4. The procedure is justified on the grounds that the properties of the detector, although complicated, are well understood and
Chapter 5. Experimental Determination of $\alpha_s$

Figure 5.1: Differential 2-jet event rates compared with fits to $\mathcal{O}(\alpha_s^2)$ calculations. Broken lines show the one parameter ($\Lambda_{\text{MS}}$) fits and solid lines shown the two parameter ($\Lambda_{\text{MS}}, f$) fits. Arrows indicate the regions of $y$ used for the fits.
may be modelled in arbitrarily great detail. So, the response of the detector can be, and indeed has been properly simulated and thus, the bin-by-bin correction coefficients computed using the Monte Carlo followed by the detector simulation may be used to correct for the detector's limited resolution and response.

This is not the case when the possible effects of hadronization are considered. Unlike the response of the detector, almost all details of the hadronization process are unknown and it is difficult to argue that the bin-by-bin coefficients which are be used to correct the data for hadronization process are correct. In this case, the application of bin-by-bin corrections is justified only when it is demonstrated that the results are insensitive to the details of the hadronization process. This may be achieved by comparing the results computed using either different hadronization models, or a wide range of model parameters. To compare the results of different hadronization models, one repeats the entire analysis with correction coefficients for detector acceptance and response as well as for the hadronization process computed using a different Monte Carlo, such as HERWIG [10]. This was not done in this analysis, but others have estimated the uncertainty in $\alpha_s$ by such methods and found it to be less than 3% [11].

In this study, $\alpha_s$ was determined by the methods described in the previous sections except with correction coefficients determined from several sets of JETSET Monte Carlo data generated with different values of the parton shower cutoff parameter, $Q_0$. This parameter represents a level of parton virtuality such that partons with invariant masses less than $Q_0$ will not split. Since the response of the OPAL detector was simulated only for JETSET events with $Q_0 = 1$ GeV, the corrected $D_2(y)$ distributions were computed as shown in equation 5.4

$$D_2^{corr}(y_i) = c_i^{had}(Q_0) \cdot c_i^{det} \cdot D_2(y_i) \quad (5.4)$$

where the coefficients $c_i^{det}$ correct only for the detector's acceptance and response for hadrons generated with $Q_0 = 1$ GeV while $c_i^{had}$ correct for the hadronization of partons simulated with various values of the shower cutoff, $Q_0$. The variation of $\alpha_s(M_Z^2)$ with $Q_0$ determined from both the one-parameter and the two-parameter fits is shown in figure 5.2.

It was found that the Geneva scheme depends only weakly on the parameter $f$ because
Figure 5.2: Variation of $\alpha_s(M^2_{Q_0})$ with the parton shower cutoff, $Q_0$, as determined from one- and two-parameter fits to the differential 2-jet distributions.
change in $f$ may be absorbed in $\alpha_s$ by a change in $\Lambda_{\overline{MS}}$ since the expression for $\alpha_s$ given in equation 2.1 depends only on the ratio $\mu^2/\Lambda_{\overline{MS}}^2$. The only explicit dependence on $f$ enters in the region where the function $B(y)$ in equation 2.6 contributes, which is generally the region of small $y$. Since the Geneva scheme produces a larger fraction of 4- and 5-jet events, as can be seen from figure 4.5, the region of $y$ used for the fit was typically $y > 0.1$ and hence the explicit dependence on $f$ is reduced. Thus, it was found that for some sets of Monte Carlo data, the weak dependence of $\alpha_s$ on $f$ in the Geneva scheme prevented a minimal $\chi^2$ from being found in a physical region of the parameters. For this reason, the dependence of $\alpha_s$ on $Q_0$ for the two parameter fit in the Geneva scheme is not presented here and no attempt has been made to estimate the systematic uncertainty due to parton virtuality of $\alpha_s$ determined in this scheme.

5.5 Summary of $\alpha_s$ Measurements

It has become customary to average the values of $\alpha_s(M^2_{Z^0})$ obtained for the one- and two-parameter fits to obtain a quantity $\bar{\alpha}_s(M^2_{Z^0})$ and quote half the difference of the two values as the uncertainty in this average due to the ambiguity of the renormalization scale $\mu^2$ at which $\alpha_s$ is evaluated. The mean values of $\alpha_s$ were determined in this way and are presented in table 5.4 along with the various estimates of the systematic errors.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>$\bar{\alpha}<em>s(M^2</em>{Z^0})$</th>
<th>$\Delta \alpha_s$(exp.)</th>
<th>$\Delta \alpha_s$(scale)</th>
<th>$\Delta \alpha_s(Q_0)$</th>
<th>$\Delta \alpha_s$(tot.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>0.127</td>
<td>0.004</td>
<td>0.019</td>
<td>0.004</td>
<td>0.020</td>
</tr>
<tr>
<td>JADE</td>
<td>0.116</td>
<td>0.003</td>
<td>0.008</td>
<td>0.001</td>
<td>0.009</td>
</tr>
<tr>
<td>P</td>
<td>0.116</td>
<td>0.003</td>
<td>0.004</td>
<td>0.003</td>
<td>0.006</td>
</tr>
<tr>
<td>Durham</td>
<td>0.122</td>
<td>0.007</td>
<td>0.005</td>
<td>0.002</td>
<td>0.009</td>
</tr>
<tr>
<td>Geneva</td>
<td>0.115</td>
<td>0.005</td>
<td>0.004</td>
<td>—</td>
<td>0.006</td>
</tr>
</tbody>
</table>

Table 5.4: Final results of $\bar{\alpha}_s(M^2_{Z^0})$ for different jet definition schemes.

The experimental uncertainties $\Delta \alpha_s$(exp.) consist of the mean errors on $\alpha_s(M^2_{Z^0})$ from tables 5.2 and 5.3 added in quadrature while $\Delta \alpha_s$(scale) was determined from half the difference
of these two values. $\Delta \alpha_s(Q_0)$ was computed using

$$\Delta \alpha_s(Q_0) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( \alpha_s(M^2_{Z^0}; Q_0 \equiv 1) - \alpha_s(M^2_{Z^0}; Q_0^{(i)}) \right)^2}$$  \hspace{1cm} (5.5)$$

where $\alpha_s(M^2_{Z^0}; Q_0^{(i)})$ are the $N$ values of $\alpha_s(M^2_{Z^0})$ from the two-parameter fit corrected with various values of the parton shower cutoff, $Q_0$. 
Chapter 6

Conclusion

It has been demonstrated that the data from the OPAL detector is ideal for the determination of the strong coupling constant $\alpha_s$ using $n$-jet event rates. This follows from the fact that the high rates and good jet resolution at LEP energies prevent the experimental measurements from being limited by statistics, as may be the case and energies off the $Z^0$ resonance. The limitations are only those of detector resolution and acceptance and theoretical uncertainties in the hadronization process and the QCD matrix elements. Some of these systematic uncertainties have been analysed in detail in this study and have been shown not to be a severe limitation of the precision with which $\alpha_s$ can be measured.

With the current $\mathcal{O}(\alpha_s^3)$ matrix elements, the rates of 2-, 3- and 4-jet events were predicted using a number of jet definition schemes and were compared with the rates measured in this analysis as functions of the jet resolution parameter, $y_{cut}$. The agreement among the values of $\alpha_s$ obtained in this way lead one to conclude that this parameter is not sensitive to the particular scheme used to define jets. Hence it would appear that one has the freedom to choose any scheme based on its particular merits for other analyses and in particular, one may quote the value of $\alpha_s$ which was most precisely measured using these schemes. Thus, one may conclude that the value of $\alpha_s$ measured from $n$-jet event rates using the techniques described here is

$$\alpha_s(M_{Z^0}^2) = 0.116 \pm 0.006.$$  \hspace{1cm} (6.1)

This value is in agreement with $0.120 \pm 0.006$ obtained by a combined analysis of OPAL studies of event shapes, jet rates and energy correlations [13]. In general, to improve such a measurement, one needs theoretical calculations to higher orders in $\alpha_s$. 

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Bibliography


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