A MICRO APPROACH TO MATHEMATICAL ARMS RACE ANALYSIS

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THESIS ABSTRACT

Even with the end of the Cold War, the question, Were the United States and the Soviet Union engaged in an action-reaction arms race? remains important and controversial. The bulk of empirical mathematical arms race research suggests that the US and USSR were not so engaged. Indeed, most such research into the matter suggests that US arms acquisitions were driven overwhelmingly by internal or domestic forces, as were Soviet arms acquisitions. Given the longstanding political, economic and military rivalry, between the US and USSR, the finding that they were not engaged in an arms race is perplexing. This is particularly so with respect to nuclear weapons acquisitions. Orthodox nuclear deterrence theory clearly posits that the attempt by each side to maintain a balance of nuclear forces with the other and hence deter the other from launching a first-strike should result in an action-reaction nuclear arms race. Why, then, does the overwhelming mass of quantitative research suggest that the opposite was true, in practice, in the US-Soviet case?

The problem, in part, has been that researchers have been using underspecified mathematical models of action-reaction arms race interaction. The most famous of these models is Richardson's 1960 action-reaction model. Researchers have long been aware that Richardson's model is underspecified and as such that it may not be capable of revealing the true nature of US-Soviet military interaction. Since the late 1960s, arms race researchers have attempted to move beyond Richardson's simple arms race specification. Several new approaches to arms race analysis have subsequently emerged: the game theoretic approach, the economic (stock adjustment) approach, and the expectations (adaptive, extrapolative, and rational) approach. Taken individually, neither of these approaches has, however, yielded much fruit.

In this dissertation, the game, stock adjustment, and rational expectations approaches were combined for the first time into a single, more comprehensive, analytical approach and a new action-reaction arms race model was derived, which we have named the GSR Model. In addition, it was argued that a new approach was needed for testing arms race models. Arms races are generally seen as competitions of total armed versus total armed might. Arms race models have, accordingly, been tested against data on states' annual military expenditures. We argued instead that an arms race is made of several subraces, the object of each subrace being a specific weapons system and a specific counter weapons system, deployed by an opponent and designed to thwart the former's political and military effect. Models should, then, be tested for each subrace in a given arms race, that is, against data on weapons system-counter weapons system deployment levels. Time frames for the analysis of a given weapons system-counter weapons system competition should be set to accord with the period in which those systems were dominant in the military calculations of the competing states.

In effect, we have specified an alternative approach to mathematical arms race analysis, the micro approach to mathematical arms race analysis. The GSR Model was tested against data on annual US and Soviet strategic nuclear warhead deployment levels, specifically, those onboard ICBMs (1960-71) and submarines (1972-87). The GSR model was also tested against annual US-Soviet aggregate strategic nuclear warhead deployment data (ICBM, SLBM and bomber based totals), 1967-84. Estimates of the GSR model suggest that the US and USSR were in fact engaged in an action-reaction arms race over submarine launched nuclear warheads. Regression analysis also indicates that the US and USSR strongly interacted, asymmetrically, over ICBM based nuclear warheads. There appears to have been no interaction over aggregate warhead deployments. Finally, the implications of these findings for the maintenance of a stable nuclear deterrent were discussed.

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PREFACE

The care of human life and happiness, and not their destruction, is the first and only legitimate object of good government.

Thomas Jefferson, US, 1809

For many years, politicians, journalists, and scholars alike have debated the merits of arms racing. Some argue the ancient Roman dictum "if you want peace, then prepare for war." By stockpiling armaments a nation can avoid war, because, knowing of that stockpile, its adversaries will fear it, and hence, avoid war with it. Others contend that arms races, on the contrary, lead to, or culminate in, war. Historically, proponents of this view argue, most states which have engaged each other in an arms race have also gone to war with each other. Neither side in the debate, however, has provided a completely convincing systematic analysis of cause and effect.

Yet in today's world, war, or peace, are not the only potential by-products of arms racing. Even a cursory look at available social and military expenditure statistics shows that throughout the world military armament programs are draining limited economic and industrial resources to the detriment of human development programs (Sivard, 1985). To cite only two examples, in 1985, developed nations expended 5.4 percent (on average) of their GNPs on military purposes. The same countries, in 1985, allocated only 0.3 percent (on average) of their GNPs to development aid to Third World countries. In 1983, an average of

US$45 per capita was spent by the governments of the world on military research while only US$11 was spent per capita on medical research.

Because arms races may lead to war, because arms races may help us to avoid war, because arms races drain away resources from other, potentially more useful, pursuits, efforts must be made to study them and to understand how they work. That is the purpose of my dissertation.

INTRODUCTION

In her Presidential Address to the International Studies Association, Zinnes (1980) issued a challenge to members of the international relations research community to develop a mathematical model, supported by data, of action-reaction arms race interaction among rival states in world politics. In this dissertation, I will take up that challenge. Indeed, in this dissertation, I will specify and apply an alternative approach to mathematical arms race analysis.

Mathematical arms race modelling goes back to 1960 when Richardson (1960a) presented his now famous linear differential arms race equations in his seminal work Arms and Insecurity. The assumptions behind his model were simple and intuitively appealing. Set in difference equation form, Richardson's model is as follows:

$$\mathbf{X}_t = k\mathbf{Y}_{t-1} + (1 - a)\mathbf{X}_{t-1} + g \qquad \text{(R28)}$$

$$\mathbf{Y}_t = l\mathbf{X}_{t-1} + (1 - b)\mathbf{Y}_{t-1} + h \qquad \text{(R29)}$$

State X's armaments at time t, denoted $\mathbf{X}_t$, depend positively upon the level of arms possessed by State X's rival, State Y, in time t-1, denoted $\mathbf{Y}_{t-1}$, times State X's defence coefficient, k, negatively upon the level of armaments possessed by State X at time t-1, denoted X_{t-1}, times State X's economic fatigue term (1-a) and on some constant amount, g, which reflects the degree of grievance State X has against State Y.

Richardson's work was indeed seminal. Until today, virtually every mathematical arms race study begins with some reference to Richardson and his model, the present study being no exception. Since Richardson presented his model, several other authors (e.g., Caspary, 1967; Majeski, 1983a, 1985; Ostrom, 1978b; Ward, 1984a; Gillespie et al, 1977c) have advanced mathematical models of action-reaction arms race interaction. Most of these models have been presented as alternatives to or as modifications of Richardson's model. Why, then, do we need another mathematical arms race model?

1. BACKGROUND: THE MACRO APPROACH TO ARMS RACE ANALYSIS

No attempt at mathematically modelling the arms race phenomenon has yet met with widespread approval in the international relations community, Richardson's model being no exception (Wallace, 1980a). Developing a model is one thing. Models must, however, be tested against empirical data.

International relations researchers have, for example, attempted to estimate the parameters of Richardson's model for a number of different arms races including the Arab-Iraeli race and the Iran-Iraq race. The most important application of Richardson's model, however, was to the US-Soviet arms race. There are a number of compelling reasons to believe that the US and Soviet Union are engaged in an action reaction arms race, the most compelling being the need for each side to maintain a balance of power with the other. Indeed both sides often justify their arms acquisitions on that basis.

Most researchers take the view that the US and Soviets wage their arms competition on the basis of total armed might versus total armed might. A state's annual aggregate military expenditures, it is felt, reflects its total military strength. Thus most researchers test Richardson's model for the US-Soviet arms race against annual US-Soviet aggregate military expenditure data. I will term this approach to mathematical arms race analysis the macro approach to emphasize its aggregate orientation.

Time and time again, in such instances, Richardson's model was disconfirmed by empirical data: estimates of Richardson's arms race model for the US-Soviet race suggest that each side's military expenditures are internally driven. There is no action-reaction interaction. In this study, I will term this result the non-interaction outcome. Still other research suggests that current US military expenditures depend upon previous period expenditures in the USSR, whereas Soviet military expenditures are internally driven (Majeski, 1985). I will term this result the asymmetric outcome.

2. NEW DIRECTIONS IN ARMS RACE RESEARCH METHODOLOGY: A CRITIQUE OF THE MACRO APPROACH

These counterintuitive findings, the non-interaction and asymmetric outcomes, have generated a great deal of debate within the mathematical arms race research community (Gillespie and Zinnes, 1982), culminating in Zinnes' challenge. Were US-Soviet arms acquisitions, as suggested by estimates of Richardson's and other's models, really independent of one another? Or have we yet to develop a methodology--a model, and model testing procedure--which can uncover the interactive component of that competition?

Those who have answered the latter question in the affirmative cite a number of deficiencies in conventional arms race research methodology which could account for the finding of non-interaction in quantitative studies of the US-Soviet arms race. These critiques can be grouped as those which focus on deficiencies in existing mathematical arms race formulations and those which focus on deficiencies in the way those models have been tested.

Some who argue that US-Soviet arms acquisitions were not independent of one another suggest that Richardson's model is somehow flawed, oversimplified, and as such that it could not be used to uncover the true nature of the US-Soviet arms race. Numerous factors, other than those suggested by Richardson, impact on a state's armament calculus. It was necessary, then, to expand if not totally rework Richardson's conceptualization of action-reaction arms interaction. The most important ideas presented in this rethink include the idea that state's base their armament calculations as much, if not more, on what they expect a rival will possess in the future as on what he held in the past (e.g., Majeski, 1985), a factor which Richardson's model does not take account of. Still others argue that states engaged in an arms race do so with a particular objective in mind such as maintaining a balance of power with a rival. Arms racing states, moreover, design and implement strategies to achieve their arming goals. Such strategies, if they are to be successful, must take into account the arming goals and strategies of rival states. Richardson's model does not contain any such decision calculus (Gillespie et al, 1977c). Others point out that those armament goals must be subject to economic, political and institutional constraints. Indeed there may be a disjuncture, in the end, between the level a state may wish to arm to and the level it can arm to (Caspary, 1967). Calculations such as these would need to be factored into any model of action-reaction arms race interaction. Indeed work was well underway on reconceptualizing action-reaction arms race interaction long before Zinnes issued her formal challenge to the international relations research community.

Still, there are those who have rejected, outright, the notion that US-Soviet military acquisitions are action-reaction driven. Some, taking their lead from developments in organizational theory (Davis, Dempster and Wildavsky, 1966; Kanter, 1973), have suggested that US-Soviet arms acquisitions could, in large measure, be independently driven. As Wallace (1979) notes, the organizational approach, as applied to the US-Soviet arms race, is two pronged. Some (Ostrom, 1977a, 1978b, Majeski, 1983a) see arms accumulation as the end product of a complex internal bureaucratic process. Still others (e.g. Nincic and Cusack, 1979) see arms accumulation as a product of internal political and economic forces. Under this perspective, military production is used to maintain or increase internal economic activity. Thus far, the organizational approach has met with little success in explaining the variance in US arms acquisitions (the focus of this approach, with few exceptions, notably Johnson and Wells, 1986, has been on the US).

An attempt, incidentally, was made to determine empirically which approach, the action-reaction or the organizational process, accounts for more variation in US-Soviet arms acquisitions. Ostrom (1977a) conducted a systematic comparison of a Richardson type action reaction model with a Davis, Dempster and Wildavsky type organizational process model and found that forecasts generated by these models for the US were indistinguishable from one another.

One of the most sophisticated attempts made thus far to uncover the basis of US-Soviet arms acquisitions is Williams and McGinnis (1988). They specified a set of conditions under which one could expect to find interaction in US-Soviet arms acquisitions and a set of conditions under which one could expect to find no interaction in US-Soviet arms acquisitions. They argued that two basic factors would determine a state's military expenditures for time t. The first is what it expected at time t its opponent would expend on armaments at time t. Under the Rational Expectations hypothesis, the expectation made by State K at time t, the current time period, regarding its opponent's, State J's, expenditures at time t is equal to State K's expectation at time t-1 of its opponent's arms expenditures at time t-1 and a prediction error showing, at time t, how far State K's current expectation was off. The second factor impacting a state's military expenditure decision at time t is its expectation at time t of its own economic condition at time t. This expectation would, likewise, be influenced by a state's previous period expectations and a prediction error.

Williams and McGinnis (1988) argue that in these circumstances, State J would, at time t, react not to the absolute level of expenditures which were observed to be made by its opponent, State K, at time t, but rather, State J would react at time t to the value of State K's prediction errors for time t. If, in the space leading up to time period t, State K were to find that the economic cost of a preferred level of military expenditure was too high and it subsequently reduced its expenditure level for time t, a negative error would then obtain in State K's time period t military expenditure calculus. Williams and McGinnis call such shocks innovations. If State J were vigilant in collecting information on innovations in State K's arming behaviour, as it would be under the Rational Expectations hypothesis, then State J should also lower its time period t military expenditures, seeing State K's reduction in its military expenditures as a reduction in threat. This would produce a negative error in State J's military expenditure calculus at time t.

States, then, react to errors in each other's military expenditure forecasts. If each side is efficient in its collection and use of information regarding innovations in the other's expenditure behaviour, then forecast errors should drive an arms race. Past expenditure levels effected by each side are important in determining current expenditures only if neither side is very efficient in its collection and use of information on innovations in the other's arming behaviour.

Where such inefficiencies do exist, and they likely would in any empirical context, it would then be important to have a model describing the sort of connection or interaction which could occur in the armament processes of rival states.

Other critiques of conventional arms race research methodology focus on the question, What exactly do the US and Soviets race over, if they race at all? The assumption that arms racing states compete on the basis of total armed might versus total armed might is widely accepted in the arms race research community. Indeed, most studies aimed at estimating the parameters of Richardson's model for the US-Soviet arms race are based on that very assumption. A new line of thought suggests that states engaged in an arms race, in particular the US and Soviet Union, do not compete on the basis of total armed might versus total armed might (McCubbins, 1983). The alternative view, the micro view, is that states, instead, compete thusly: when one side deploys a particular weapons systems, the other responds by deploying a system designed to counter the latter's military and political effect. An arms race between two rivals, then, is made of several subraces, the object of each particular subrace being a particular weapons systems and a particular counter weapons system. Theoretically, rival states could engage one another in several different subraces each with its own particular properties. Models, then, would have to be tested against each of these subcompetitions.

If states, in particular the US and USSR, did not compete on the basis of total armed might versus total armed might, then it should be no surprise to find that US and Soviet aggregate military expenditure series, which reflect total military capabilities, are independent of one another. In a given rivalry, some subraces could heat up while others cool down. In aggregate military expenditure data, peaks and valleys in individual weapons system acquisitions could cancel each other out. Indeed, evidence of action-reaction interaction in the arms acquisitions of two rivals might be totally distorted by aggregate military data. McCubbins (1983) application of the subrace approach to the US-Soviet arms race has shown a great deal of promise in demonstrating the existence of action-reaction interaction in their arms acquisitions.

Still others have focused on the question of time frames for the analysis of the US-Soviet arms competition. Lucier (1979) argues that arms race parameters do not remain constant in value over time. Parameters may, over time, increase or decrease in value depending upon factors such as changes in leadership in the competing countries. Most studies which have shown no interaction in US-Soviet arms acquisitions spanned the period

1950-60 to the present. Did the parameters of the US-Soviet competition remain constant in value over that time. If not, then an attempt to estimate the parameters of Richardson's, or any other, model from 1950 to the present might yield misleading results.

3. A NEW APPROACH TO ARMS RACE ANALYSIS: A MICRO APPROACH

It is my contention that the findings of non-interaction and asymmetric reaction in studies of the US-Soviet arms race are due more to deficiencies in conventional arms race methodology stemming from an insufficient understanding of what drives an arms race than to the genuine absence of interaction in US-Soviet arms acquisitions. The various modifications to conventional arms race research methodology suggested by Majeski (1985), Gillespie et al (1977c), McCubbins (1983) and Lucier (1979) and others, however, do represent a growth in the understanding of the factors which drive an arms race.

Unfortunately, never before have all of the modifications to conventional arms race methodology discussed above been employed all at once in a single study. Majeski (1985), for example, made the important point that a state engaged in an arms race focuses its attention more on what it expects its rival will possess in terms of arms in the future than on what its rival possessed in the past. After having derived a mathematical model which expressed this reality, he then estimated it against US and Soviet aggregate military expenditure data running 1949-81. The estimate of his model showed an asymmetric arms race. The US reacted to Soviet expenditures, but Soviet expenditures were internally driven. It is my position that, at a minimum, a study of the US-Soviet arms race, one which could reveal the so far illusive two-way interactive component of that competition, would require the amalgamation of all of the suggested modifications to conventional arms race methodology specified above into a single approach. That is what I propose to do in this dissertation.

More specifically, in this dissertation, I will specify and apply a micro approach, or disaggregate approach, to arms race analysis. There are two aspects to this approach: a new action-reaction arms race model and a specific approach to testing it.

The model I will derive in this thesis is an optimal behaviour model. It will give the conditions under which it is optimal for actors to engage each other in an action-reaction arms race. It will also give the conditions under which it is optimal for actors to engage each other in an asymmetric or non-interactive competition. Under my formulation of the arms race phenomenon, actors can both have non-zero defence coefficients and still engage each other in asymmetric or non-interactive competitions.

My model will be based on three key assumptions drawn from the rethink in arms race research which followed the unsuccessful testing of Richardson's model: (1) each state is assumed to arm in accordance with some goal, in particular, the goal of maintaining a balance of power with its rival (I will assume that weapons acquisitions are determined on a weapons systems versus cross-purpose weapons system basis), (2) that economic, political, and institutional constraints limit the extent to which each particular state is capable of realizing its armament goal, and (3) that each state forms and uses expectations of the other's future arming behaviour in calculating its own future armament requirements. I will formalize these assumptions by drawing on the methods of game theory, economic theory and Rational Expectations theory.

Secondly, I will test this model, not against military expenditure data, but against data on individual weapons systems with cross-purposes. More specifically, I will apply it in an analysis of the US-Soviet arms race. I will, of course, not examine every subrace in the larger US-Soviet military competition. I will focus my efforts on subraces within the US-Soviet strategic nuclear competition. More specifically, I will conduct three tests of my model: against data on US-Soviet SLBM warhead deployments, against data on US-Soviet ICBM warhead deployments, and against data on total US-Soviet strategic nuclear warhead deployments.

I will set the time frame for each test to reflect a period in which each system-counter system was dominant in US-Soviet strategic calculations. More specifically, I will set the time span of the US-Soviet SLBM warhead deployment analysis to run from 1972 to 1987 and the time span of the US-Soviet ICBM warhead deployment analysis to run from

1960 to 1971.

Previous attempts to uncover evidence of action-reaction interaction in US-Soviet nuclear weapons acquisitions, using the macro approach, have been largely unsuccessful (e.g., Kugler et al, 1980; McGuire, 1977). My micro approach to mathematical arms race analysis will yield strong evidence that the US and Soviets were engaged in an action-reaction strategic nuclear arms race. More specifically, I will conclude that the US and Soviets were engaged in an action-reaction competition over SLBM warhead deployments. This finding is important for it suggests that the logical conditions for the maintenance of strategic deterrence were being met. Maintaining a nuclear balance of power, and hence, maintaining nuclear deterrence, requires competing powers to engage one another in an action-reaction arms race. Indeed, it is reasonable to find that the US and Soviets did race over SLBM warhead deployments especially since both the US and the Soviets have viewed their SLBM based warheads as reserve second strike forces.

Secondly, from my analysis, we will be able to gain a deeper understanding of how arms races work. In particular, I will provide evidence which suggests that both the US and Soviets formed and used expectations of their own future arming requirements and expectations of each other's future arming requirements in accordance with the Rational Expectations hypothesis. The implication of this finding is that US and Soviet military calculations were intimately linked. My work will reveal several dimensions of that linkage.

4. ORGANIZATION OF THE DISSERTATION

I will organize this thesis in accordance with the five sequential steps of the mathematical modelling process. In that process, one must first abstract the phenomenon one wishes to model down to its main essential characteristics or elements. Second, one must formalize the substance and interrelationships among these elements using mathematical symbols. In this step, the model takes on its structure and its substance. One must then derive testable hypotheses concerning the sort of behaviour, mathematical properties, that the model obtained in the previous step predicts. As fourth step in the modelling process, one must design and implement tests of those predictions against observed reality. Finally, based on the degree to which one's model's predictions are confirmed by empirical observation, one must return to the first step of the modelling process and amend what one first thought constituted the basic elements making up the phenomenon in question. Therewith, steps two, three, four and five would be repeated.

I will begin my study with a review of the arms race literature. The arms race literature can be divided into two main categories, those which look at arms building models and those which look at arms using models (Moll and Lubbert (1980)). Arms building models describe the forces which determine the arms expenditures of states. Arms using models, as the name suggests, describe the rate at which a state's forces may be consumed in battle. The arms building category can be further divided into Richardson and non-Richardson type models. This latter category includes organizational process models. In Chapter 1, I will give a detailed review of Richardson's model. I will present a full derivation of that model and study its various properties. This will correspond to steps one, two and three of the modelling process where the arms race phenomenon will be abstracted down to its main essential elements and mathematized. In Chapter 2, I will report on the outcome of various attempts to estimate Richardson's model for the US-Soviet arms race. I will focus here on the types of data against which Richardson's and other's models have been tested for the US-Soviet arms race. Chapter two, then, corresponds to step four of the modelling process. In Chapter 3, I will look at the efforts that have been made to move beyond Richardson's simple conceptual framework. I will look at how game-control theory, economic theory and expectations theory have been recently employed in an effort to reconceptualize the arms race phenomenon. Chapter 3, then, corresponds to step five of the modelling process. In Chapter 4, I will return to steps one and two of the modelling process where I will present the derivation of my own arms race model. In deriving my model, I will draw on the concepts presented in the previous chapter, melding them into a single framework. Chapter 5 will be a chapter on old problems, new solutions. Here, I will use my model to derive testable propositions concerning the nature of the action-reaction, non-interaction and asymmetric outcomes. Chapter 5, then, corresponds to step three of the modelling process. In Chapter 6, I will test my model against data on the US-Soviet strategic nuclear arms race. I will, on the one hand, show that the US and Soviet Union do not race when it comes to each side's aggregate strategic warhead counts (the sum of ICBM, SLBM and bomber based nuclear warheads). In that same chapter, I disaggregate my US-Soviet nuclear warhead data into its component parts: SLBM, bomber and ICBM warhead deployments. I will present and discuss the results of an analysis made of the SLBM and ICBM warhead data sets. In the case of US and Soviet SLBM warhead deployments (1972-1987), I find clear cut evidence of action-reaction interaction between the US and USSR. Over the period 1960-71, I find that the US and Soviet Union were engaged in an asymmetric arms race over ICBM based nuclear warhead deployments. Both the SLBM warhead race and the ICBM based warhead are, moreover, stable. Finally, in Chapter 7, which corresponds to the final step in the modelling process, I will consider the degree to which my study has advanced the search for a model, confirmed by data, of action-reaction arms race interaction among rival states in world politics.

CHAPTER I: A REVIEW OF RICHARDSON'S MODEL

Mathematical arms race modelling and analysis began with Richardson's Arms and Insecurity (1960a) where he presented his now famous linear differential action-reaction arms race equations. In the subsequent literature on quantitative arms race analysis, one can discern, since Richardson's original work, a steady and logical progression in the methodology--model type, model operationalization, and model testing techniques--of the field. My own work is simply the next stepping stone in that progression. Accordingly, it is important to understand, in some detail, that progression if the significance of my work is to be understood. I will therefore begin my study with a detailed review of Richardson's model.

This review of Richardson's model will take us through the first three steps of the modelling process. More specifically, in this chapter, I will ask, How did Richardson come to his specific action-reaction arms race formulation? What properties does Richardson's model have? How have Richardson's assumptions been altered by others? What other types of models did Richardson construct besides his basic linear model? In the next chapter, I will ask, How well does Richardson's linear model account for the US-Soviet arms race? In short, the answer to this question is that Richardson's model is not well supported in the US-Soviet case. Why? This is one of the principal questions in mathematical arms race research today. In the third chapter, I will look at how arms race modelling has progressed since Arms and

Insecurity.

1. RICHARDSON'S DERIVATION

The insurance theory of armament stockpiling lies at the heart of most debates about arms racing. A nation can avoid war by stockpiling arms because, knowing of that stockpile, its potential adversaries will fear its strength, and hence, avoid war with it. Stockpiling can also benefit a nation should it become engaged in war. Richardson doubted the validity of the theory, but nevertheless based his model upon it. Regardless of its actual validity, Richardson thought, statesmen tend to believe the insurance theory and indeed act in accordance with it. How, then, did Richardson come to his specific action-reaction arms race formulation?

Richardson (1960a), in formulating his model, asked us to consider the arguments of a number of imaginary and well known real life public figures concerning the issues involved in national defence. A physicist by training, Richardson then converted these arguments into mathematical statements. First, he quoted a speech made by the Minister of Defence of Jedesland, a mythical country. The Minister stated:

> The intentions of our country are entirely pacific. We have given ample evidence of this by the treaties which we have recently concluded with our neighbors. Yet, when we consider the state of unrest in the world at large and the menaces by which we are surrounded, we should be failing in our duty as a government if we did not take adequate steps to increase the defence of our beloved land [p. 14].

There are a number of ways in which the Minister's statement could be converted into a mathematical statement. Richardson, limiting himself to an analysis of the rivalry between two states, X and Y, translated the Minister's words thusly.

$$dX/dt = k\mathbf{Y} \tag{R1}$$

$$dY/dt = l\mathbf{X} \tag{R2}$$

Here, **X** represents State X's armaments and **Y**, State Y's. k represents the degree of threat or menace which State X perceives State Y's armaments pose to it. l, similarly, represents the degree of threat or menace which State Y perceives State X's armaments pose to it. k and l have come to be known, respectively, as State X and Y's defence coefficients. The change, then, in State X's armaments per unit change in time (d**X**/dt) depends upon the level of arms possessed by State Y (**Y**) weighted by the threat which State X perceives those armaments to pose to it (k). State Y's armament level is similarly determined. The problem with this formulation is that if States X and Y each possess some positive level of armaments and if States X and Y each perceive the other's armaments to be some positive threat to it then each state will, as per the specification of d**X**/dt and d**Y**/dt, be spurred on to ever increasing arms expenditures. But, for economic reasons, no state could ever effect ever increasing arms expenditures. Richardson was well aware of this fact.

He thus quoted Churchill's address to the British cabinet in 1923 concerning Germany's naval armament program:

> Believing that there are practically no checks upon German naval expansion except those imposed by the increasing difficulties of getting money, I have had the enclosed report prepared with a view to showing how far those limitations are becoming effective. It is clear that they are becoming terribly effective [p. 15].

Richardson thus amended his preliminary specification as follows.

$$d\mathbf{X}/dt = k\mathbf{Y} - a\mathbf{X} \tag{R3}$$

$$d\mathbf{Y}/dt = l\mathbf{X} - b\mathbf{Y} \tag{R4}$$

The parameter a shows the economic burden to State X of its existing armament stockpile, **X**. The parameter b, similarly, shows the economic burden to State Y of its existing armament stockpile, **Y**. Richardson assumed both a and b to be positive. The products a**X** and b**Y**, Richardson thought, should be subtracted from d**X**/dt and d**Y**/dt respectively. Consider what this means for State X. While State Y's armaments act to stimulate growth in State X's armaments over time, State X's existing stockpile tends to dampen that growth by a factor of a. b**Y** has the same sort of effect in the case of State Y.

Richardson was still not satisfied with his model. He considered that British Foreign Secretary Grey wrote in 1925 that

> The increase of armaments that is intended in each nation to produce consciousness of strength, and a sense of security, does not produce these effects. On the contrary, it produces a consciousness of the strength of other nations and a sense of fear .... The enormous growth of armaments in Europe, the sense of insecurity and fear caused by them--it was these that made war inevitable .... This is the real and final account of the origin of the Great War [p. 15].

Richardson has already accounted for the sense of fear that Grey said nations experience from the armaments of their neighbors in his model via the parameters k and l, State X and Y's defence coefficients. But consider now British Member of Parliament Amery's challenge of Grey's view, in 1936, in an address to the House:

> With all respect to the memory of [the] eminent statesman, [Secretary Grey] I believe [his] statement to be entirely mistaken. The armaments were only the symptoms of the conflict of ambitions and ideals, of those nationalist forces, which created the war. The War was brought about because Serbia, Italy, Rumania passionately desired the incorporation in their States of territories which at the time belonged to the Austrian Empire and which the Austrian Government were not prepared to abandon without a struggle. France was prepared if the opportunity ever came to make an effort to recover Alsace-Lorraine. It was in those facts, in those insoluble conflicts of ambitions and not in the armaments themselves that the cause of the War lay [p. 15-16].

Richardson took account of Amery's retort not by eliminating State X and Y's defence coefficients from his model, but by working in new terms.

$$dX/dt = kY - aX + g \tag{R5}$$

$$dY/dt = lX - bY + h \tag{R6}$$

The parameters g and h are, respectively, State X and State Y's grievance terms. Richardson assumed g and h to be positive. g thus could positively impact the rate of change in State X's armaments per unit change in time independently of the level of armaments possessed by State X or the level possessed by State Y. g's impact on the rate of change in State X's armament level over time reflects the magnitude of State X's grievances against State Y. The parameter h has a similar interpretation with respect to State Y. This, then, is Richardson basic model.

2. PROPERTIES OF RICHARDSON'S MODEL: EQUILIBRIUM CONDITIONS

Richardson was also interested in the sorts of consequences he could deduce from his model (Caspary, 1967). His model is dynamic: State X and State Y's arms competition is specified in terms of changes in each respective state's armaments per unit change in time. Richardson thus asked, under what conditions would an arms race, as he had formulated it, stop? Mathematically, under what conditions will

$$dX/dt = 0 = dY/dt \tag{R7}$$

If we set

$$0 = k\mathbf{Y} - a\mathbf{X} + g \qquad \text{(R8)}$$

$$0 = l\mathbf{X} - b\mathbf{Y} + h \qquad \text{(R9)}$$

the answer becomes readily apparent. $d\mathbf{X}/dt = 0 = d\mathbf{Y}/dt$, equilibrium, occurs in Richardson's model when

$$\mathbf{X}^* = (kh + bg)/(ab - kl) \qquad \text{(R10)}$$

and

$$\mathbf{Y}^* = (lg + ah)/(ab - kl) \qquad \text{(R11)}$$

Thus ($\mathbf{X}^*$, $\mathbf{Y}^*$) is a general equilibrium solution for Richardson's arms race model. Most importantly, the point ($\mathbf{X}^*$, $\mathbf{Y}^*$) depends upon State X and Y's defence, economic fatigue and grievance terms. If, however,

$$(ab - kl) = 0 \qquad \text{(R12)}$$

then no equilibrium solution will exist for Richardson's arms race system. Yet even if the equilibrium point ($\mathbf{X}^*$, $\mathbf{Y}^*$) exists, it may be unstable. Richardson's arms race system is stable if, after reaching the point ($\mathbf{X}^*$, $\mathbf{Y}^*$), States X and Y return to it if some exogenous disturbance impacts on the system which causes them to move to some other point. Otherwise, the point ($\mathbf{X}^*$, $\mathbf{Y}^*$) is unstable. For Richardson, the distinction between unstable and stable equilibrium points was no small matter. Richardson believed that States X and Y's arming competition, if unstable, could culminate in war. We must, therefore, distinguish between stable and unstable equilibrium points.

To determine the stability conditions for Richardson's model, we first plot the locus of points in ($\mathbf{X}$, $\mathbf{Y}$) space where, for State X, $d\mathbf{X}/dt = 0$ and where, for State Y, $d\mathbf{Y}/dt = 0$. The point of intersection between these lines will give the equilibrium solution ($\mathbf{X}^*$, $\mathbf{Y}^*$). The relative slopes of the $d\mathbf{X}/dt = 0$ and $d\mathbf{Y}/dt = 0$ curves will, on the other hand, show whether or not the point ($\mathbf{X}^*$, $\mathbf{Y}^*$) is stable. The $d\mathbf{X}/dt = 0$ line and the $d\mathbf{Y}/dt = 0$ line in Graphs 1.1 and 1.2 were plotted as follows. From

$$0 = k\mathbf{Y} - a\mathbf{X} + g \tag{R8}$$

we obtain

$$\mathbf{Y} = (a/k)\mathbf{X} - (g/k) \tag{R13}$$

and from

$$0 = l\mathbf{X} - b\mathbf{Y} + h \tag{R9}$$

we obtain

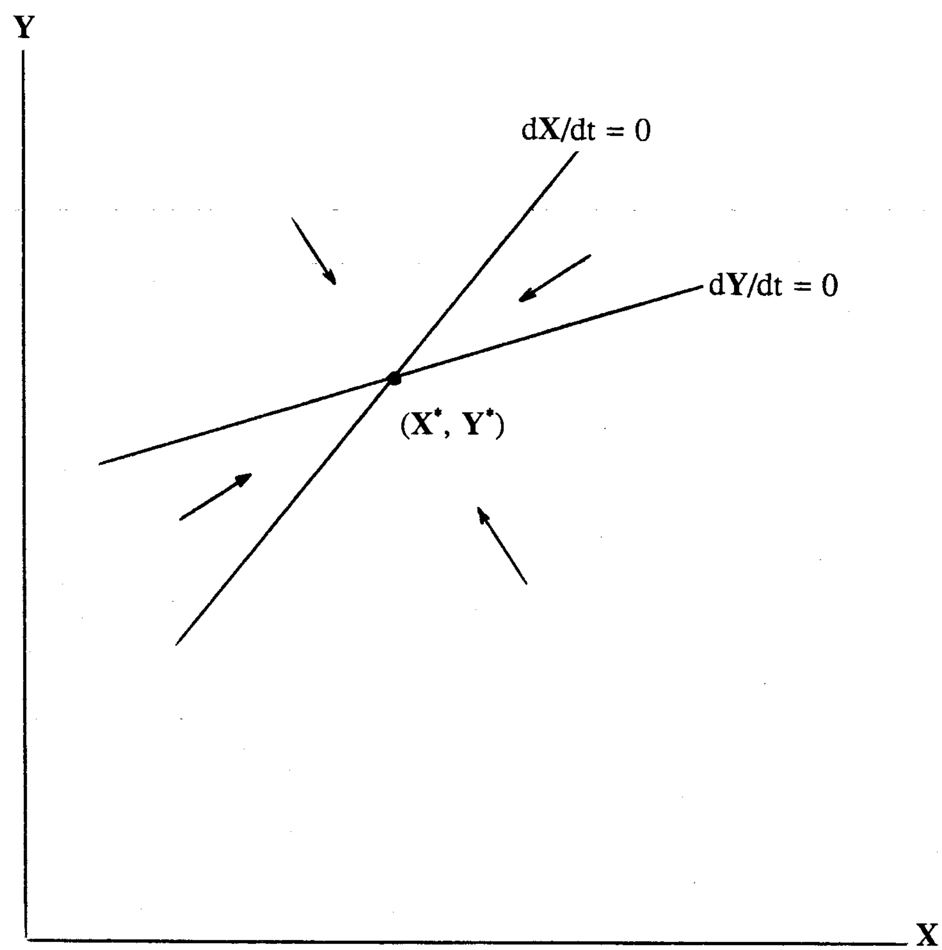
$$\mathbf{Y} = (l/b)\mathbf{X} - (h/b) \tag{R14}$$

Graphs 1.1 and 1.2 illustrate the dynamics of Richardson's model.

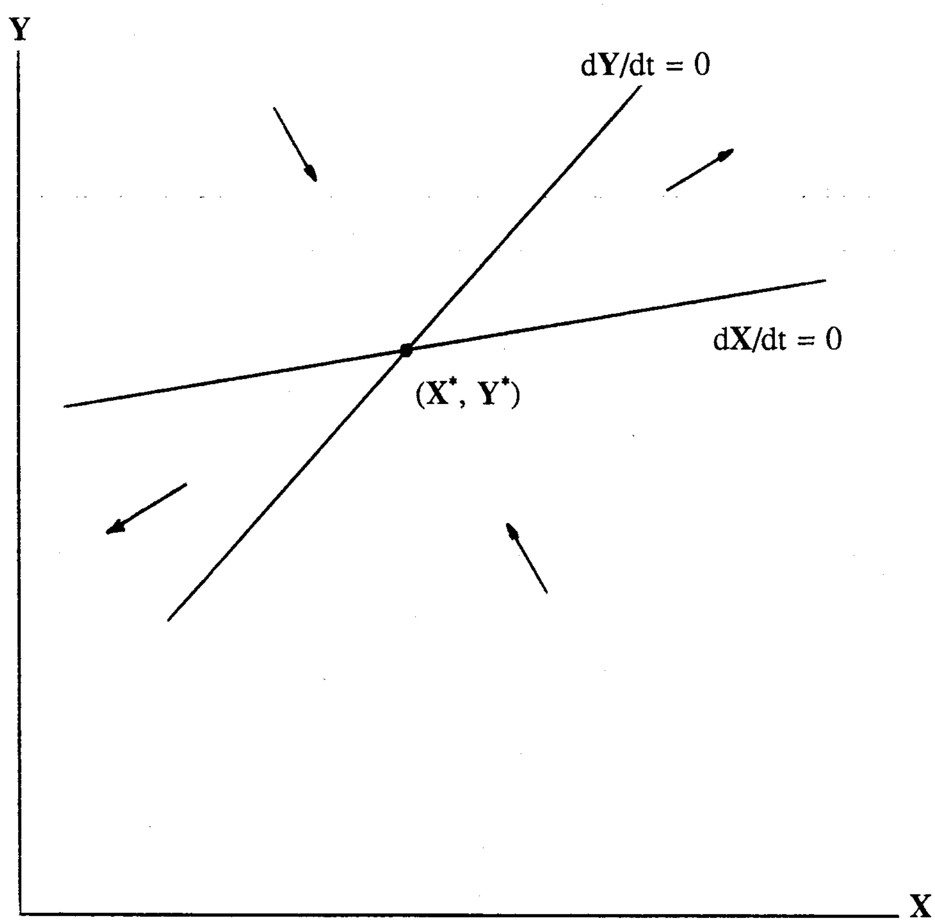
Graph 1.1 shows a stable arms race. The d**X**/dt = 0 line and the d**Y**/dt = 0 line were drawn under the assumption that k, l, a, b, g and h are all positive. In Graph 1.1, there exists an equilibrium point, the point of intersection between the d**X**/dt = 0 line and the d**Y**/dt = 0 line, and States X and Y's armaments are tending toward it. Graph 1.2 shows an unstable arms race. In Graph 1.2, the d**X**/dt = 0 line and the d**Y**/dt = 0 lines were also drawn under the assumption that k, l, a, b, g and h are all positive. Here there is also an equilibrium armament point. However, State X and Y's armaments are tending away from it.

Whether the slope arrangement given by Graph 1.1 or by Graph 1.2 prevails for any given arms race depends upon the relative values of k, l, a, b, g and h. Comparing the slopes of the d**X**/dt = 0 line and the d**Y**/dt = 0 in Graphs 1.1 and 1.2, we see that an arms race, as

GRAPH 1.1: Stability in Richardson's arms race system when all parameters are positive



GRAPH 1.2: Instability in Richardson's arms race system when all parameters are positive



Richardson has specified it, is stable if

$$(a/k) > (l/b) \tag{R15}$$

or

$$ab > kl \tag{R16}$$

Stability, then, in Richardson's model, depends on the relative values of State X and Y's economic fatigue terms and the values of their defence coefficients. For an arms race to be stable, by Richardson's formulation, the product of the racing states' domestic economic fatigue must exceed the product of their fear or defence needs.

3. VARYING RICHARDSON'S ASSUMPTIONS

Richardson had assumed that the parameters a, b, k, l, g and h all take on positive values. This assumption is not unreasonable, but it does limit the applicability of his model in the real world. Zinnes et al (1976a) argue that a, b, k, l, g and h need not and, in certain empirical contexts might not, all be positive.

For example, a state's arming program need not always constitute a drain on its economy. When countries' productive capacities are already heavily geared toward weapons production, one of or both of the economic fatigue terms, a and b, for a pair of rivals, may be positive. Secondly, negative values for the grievance terms g and h might indicate cooperation between two rivals. One question immediately arises, however. Can some of a, b, k, l, g and h be positive while others are negative?

In short, the answer is yes. Because the economic fatigue coefficients, a and b, refer to internal considerations in the rival states, their values, positive or negative, can stand independently of the values of k and l or g and h. k and l, the defence coefficients and g and h, the grievance terms, are relational parameters which take on their values in accordance with the nature of the rivalry between the states in question. Yet the relationship between the elements of the pair k and g and the elements of the pair l and h is not a simple one. Both elements of given pair, either k and g or l and h, need not be positive. For example, a nation could fear the armaments of a newly emerging rival, thus making its defence coefficient k positive, but have no history of conflict with that new rival, thus making its grievance term g negative.

It is important to note that if, in a given empirical context, a, b, k, l, g and h are not all positive, then the equilibrium conditions specified above may not hold. Indeed, the equilibrium dynamics of State X and Y's competition will vary as the signs of a, b, k, l, g and h vary.

If we know the values, positive or negative, of k, l, a, b, g and h for any pair of States X and Y, we can then plot the course of their competition as they move toward or away from equilibrium. Here, I will focus on the dynamics of State X and Y's arming process as they move toward equilibrium. Zinnes et al (1976a) show that the key to uncovering these patterns is to determine the sign of the discriminant D which, for Richardson's model, is given by

$$D = (a - b)^2 + 4kl \tag{R17}$$

The discriminant D can take on any one of three values: $D > 0$, $D < 0$, or $D = 0$. In addition to calculating D, we should also calculate the eigenvalues E_1 and E_2 for Richardson's model.

$$E_1 = [-(a+b)+[(a+b)^2-4(ab-lk)]^{1/2}]/2 \tag{R18}$$

$$E_2 = [-(a+b)-[(a+b)^2-4(ab-lk)]^{1/2}]/2 \tag{R19}$$

E_1 and E_2 can either be

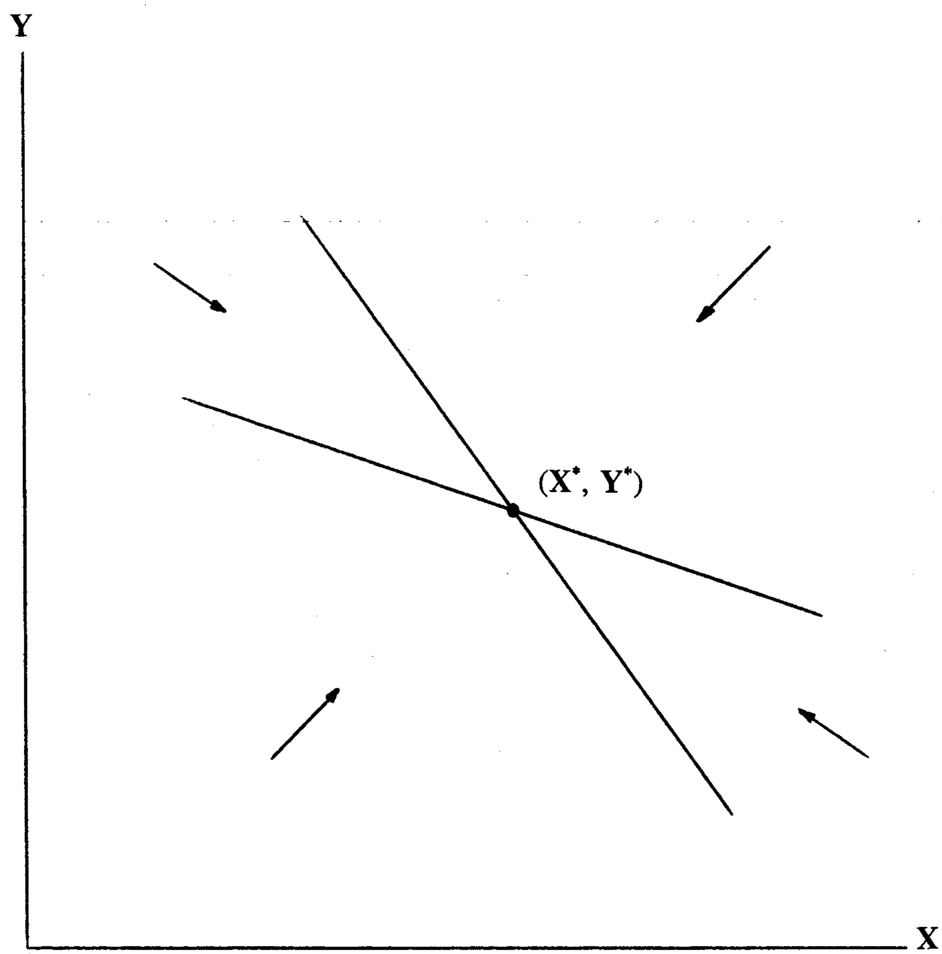
(i) distinct real numbers: $E_1 \neq E_2$

(ii) distinct complex numbers: $E_1 = u + iv$ and $E_2 = u - iv$

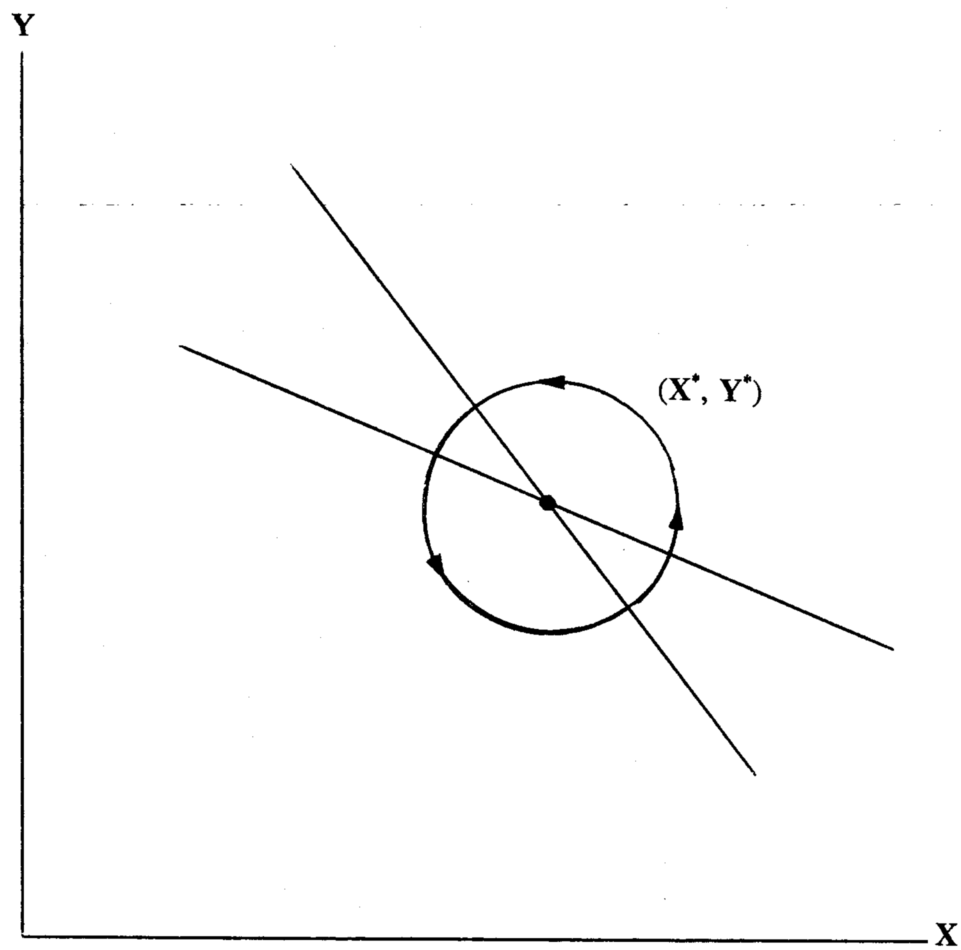
(iii) equal real numbers: $E_1 = E_2$

If $D > 0$, then E_1 and E_2 will be distinct real numbers. In this case, the arming pattern between States X and Y around equilibrium is given by Graph 1.3 (Note: Graphs 1.3 to 1.5 taken from Zinnes et al, 1976a). Graph 1.3 shows that when $D > 0$, State X and Y arm in a way that leads them nearly directly to their race's equilibrium point. If $D < 0$, however, the matter is less clear cut. Two arming patterns are possible when $D < 0$. When $D < 0$, E_1 and E_2 are distinct complex numbers. The arming dynamic between States X and Y around their race's equilibrium will depend upon the value of u. If $u = 0$, States X and Y will arm to levels that continually revolve around their race's equilibrium point, but they never approach it. This process is shown in Graph 1.4. If, on the other hand, $u < 0$, States X and Y will arm in a way which spirals in toward their race's equilibrium value. This result is shown in Graph 1.5. In the final case, $D = 0$. If, in addition to $D = 0$, State X and Y's defence coefficients, k and l, both equal zero, then, States X and Y will arm in pattern which tends quite strongly toward their race's equilibrium in a nearly straight line fashion. The dynamics of this outcome are similar to those portrayed in Graph 1.1. If instead l equals zero but k does not, then States X and Y will still arm in a pattern which leads them toward equilibrium in a straight line a fashion. The arming pattern in this case is similar to that shown in Graph 1.1. Finally, if k equals zero but l does not, then States X and Y will arm at levels that approach their race's equilibrium point in, again, nearly straight line fashion as shown in Graph 1.1. Zinnes and her colleagues have thus shed an important new light on arms race dynamics with their

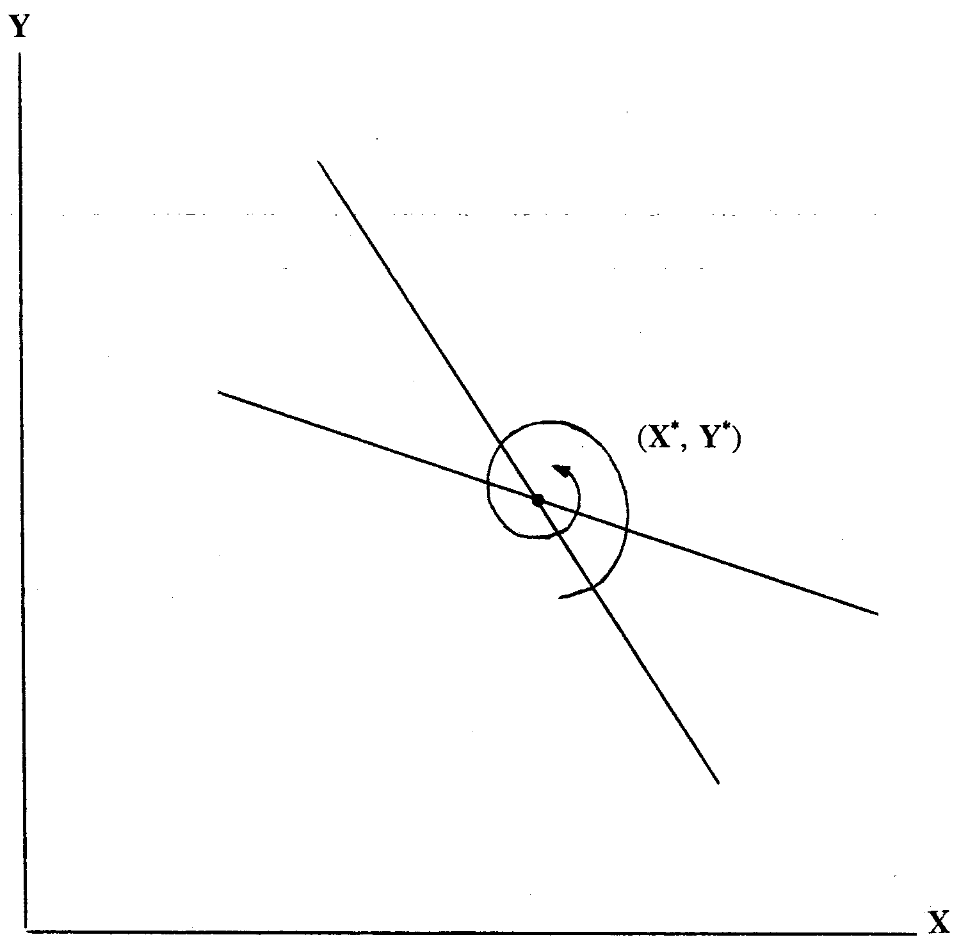
GRAPH 1.3: Stability in Richardson's arms race system when all parameters are not positive, Case I



GRAPH 1.4: Stability in Richardson's arms race system when all parameters are not positive, Case II



GRAPH 1.5: Stability in Richardson's arms race system when all parameters are not positive, Case III



analysis of Richardson's model.

4. OTHER MODELS: RICHARDSON'S RIVALRY MODEL

Richardson proposed still other arms race models. Two of the more interesting ones are the rivalry and the submissiveness models. The rivalry model is a simple extension of the his basic linear model. He suggests that a state will be concerned by a rival's armaments only if that rival possesses more armaments than it does. It is, then, the discrepancy between one state's arms levels and those of its adversary that drives an arms race. Richardson thus amends his basic model as follows.

$$d\mathbf{X}/dt = k(\mathbf{Y} - \mathbf{X}) - a\mathbf{X} + g \tag{R20}$$

$$d\mathbf{Y}/dt = l(\mathbf{X} - \mathbf{Y}) - b\mathbf{Y} + h \tag{R21}$$

The parameters k, l, a, b, g and h continue to be interpreted as before. Here, too, Richardson calculated stability conditions. Richardson found that

$$k + a + l + b > 0 \tag{R22}$$

and

$$kb + la + ab > 0 \quad \text{(R23)}$$

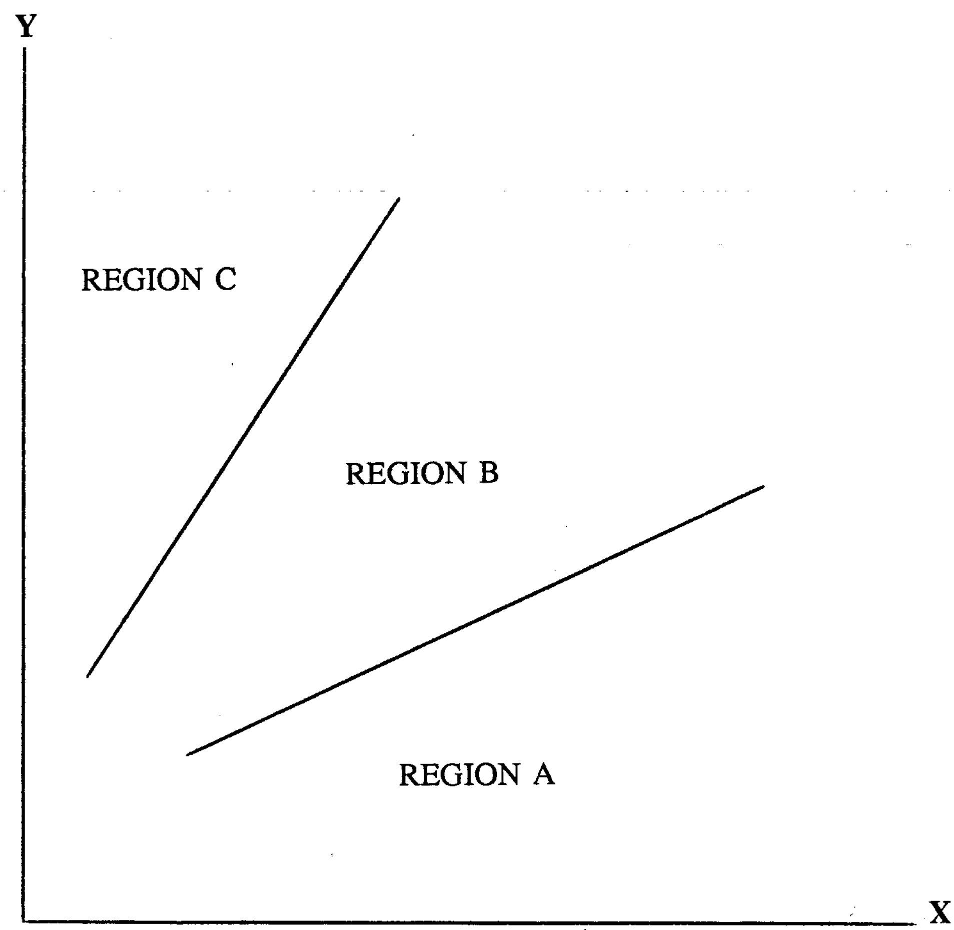
together constitute the stability conditions for his rivalry model. But k, l, a, b, g and h should, Richardson thought, all be positive. This, then, would mean that the rivalry model would always predict a given arms race to end up being stable. Because he was unwilling to vary his assumptions regarding the values of k, l, a, b, g and h and because he knew that, empirically, not all arms races are stable, Richardson abandoned the rivalry model.

5. THE SUBMISSIVENESS MODEL

With the submissiveness model, Richardson drops his assumption that arms races behave in a linear fashion. In region B of Graph 1.6, Richardson argued that the threat posed to State X by State Y's armaments would be such as to impel State X to increase the rate of growth of its armaments: $d\mathbf{X}/dt > 1$ (Zinnes, 1976). In region A, the threat posed to State X by State Y's armaments, Richardson thought, would be so small as to impel State X to reduce the rate of growth in its armaments and, in region C, to be so great as to overwhelm State X such that it could not hope to counter State Y and thus gives in by reducing the rate of growth in its armaments. In regions A and C, $d\mathbf{X}/dt < 1$. Richardson could have chosen to formalize this analysis in any one of a number of different ways. He chose

$$d\mathbf{X}/dt = k\mathbf{Y}(1 - s(\mathbf{Y} - \mathbf{X})) - a\mathbf{X} + g \quad \text{(R24)}$$

GRAPH 1.6: The dynamics of Richardson's submissiveness model



$$dY/dt = lX(1 - p(X - Y)) - bY + h \quad \text{(R25)}$$

In this model, k, l, a, b, g and h continue to hold the same meaning as before. The parameters s and p are new. Their meaning is not clear. Indeed the model itself is not easy to understand. But Zinnes (1976) shows that if States X and Y begin their race from an initial point of parity, the submissiveness model breaks down to Richardson's original formulation. If State Y initially possesses substantially more armaments than State X, then State X in effect becomes submissive and begins to disarm. If State X initially possesses substantially more armaments than State Y, then State X will continue to arm but at a decreasing rate as time progresses. Richardson did not, however, fully work out the stability conditions for the submissiveness model. The task would have been an extremely difficult one because the model is so complicated.

It is, finally, interesting to note that Hollist (1977b) tested Richardson's rivalry and submissiveness models against US and Soviet annual military expenditure data running 1948-70. Hollist's analysis supported neither model.

This, then, concludes the derivation of Richardson's basic linear action-reaction arms race model, and the discussion of its various properties. The model takes the form

$$dX/dt = kY - aX + g \quad \text{(R5)}$$

$$d\mathbf{Y}/dt = l\mathbf{X} - b\mathbf{Y} + h \tag{R6}$$

with equilibrium occurring at the point

$$\mathbf{X}^* = (kh + bg)/(ab - kl) \tag{R10}$$

and

$$\mathbf{Y}^* = (lg + ah)/(ab - kl) \tag{R11}$$

In the next chapter, I will turn to step four in the modelling process, where I will analyze the various attempts that have been made to test Richardson's and other's arms race models for the US-Soviet arms race. I will make several recommendations, based on that analysis, regarding testing procedures for action-reaction arms race models.

CHAPTER II: EMPIRICAL TESTS OF RICHARDSON'S AND OTHER A-R ARMS RACE MODELS

Richardson's basic linear model has been tested in hundreds of quantitative arms race studies. Attempts have been made to estimate it for many races, including the Arab-Israeli race, the India-Pakistan race and, most importantly, the US-Soviet race. Despite its basically sound conceptual framework, however, Zinnes (1980) was able to cite instance after instance where Richardson's linear action-reaction model was disconfirmed by empirical data.

The most perplexing of such examples given by Zinnes (1980) concerns the US-Soviet arms race. Most quantitative studies aimed at estimating Richardson's model for the US-Soviet military competition have concluded that the US and Soviet Union have not been engaged in an action-reaction arms race. In this chapter, I will look at the various attempts that have been made to estimate the parameters of Richardson's and others' models for the US-Soviet arms race with a special emphasis on the types of data that they have been tested against. Clear and logical advances have been made in this particular dimension of arms race research methodology. Indeed, much has changed since the earliest quantitative studies of the US-Soviet arms race.

In this chapter, I will begin with some introductory notes on the nature of the US-Soviet military competition. Secondly, I will discuss the general procedure for estimating Richardson's model. Third, I will look at the initial attempts that were made to estimate Richardson's model for the US-USSR. Fourth, I will discuss what researchers had learned from this initial round and where they went with their work. Finally, I will discuss the current studies of the US-Soviet arms race.

1. THE US-SOVIET MILITARY COMPETITION

Many professional observers of military affairs contend that the US and Soviet Union have been engaged in an action-reaction arms race. Arms racing is one way to maintain a balance of power between rivals. Indeed, a US Defence Department document entitled Soviet Military Power, dated 1981, states, in fact, that the aim behind America's armament program is to counter the threat posed by the build-up of armaments in the Soviet Union. A similar document, issued by the Soviet Ministry of Defence, entitled, From whence the threat to peace, dated 1982, states that the aim behind Soviet Union's armament program is to counter the threat posed by the build-up of armaments in the United States. Sivard's (1985) data, shown in Table 2.1, on weapons system development in the US and Soviet Union shows just how closely a weapons development in one country is matched by the other. Scholars have, accordingly, tried to estimate the parameters of Richardson's action-reaction model for the US-Soviet arms race. What did they find?

TABLE 2.1: Action-reaction in weapons system development in the US and USSR.

Action		System	Reaction	
USA	1945	atomic bomb	1949	USSR
USA	1948	intercontinental bomber	1955	USSR
USA	1952	thermonuclear bomb	1953	USSR
USSR	1957	ICBMs	1958	USA
USSR	1957	man-made satellite	1958	USA
USA	1960	SLBMs	1968	USSR
USA	1966	multiple warhead	1968	USSR
USSR	1968	anti-ballistic missile	1972	USA
USA	1970	MIRV system	1975	USSR
USA	1982	long-range cruise missile	1984	USSR
USA	1983	neutron bomb	198?	USSR

Source: Savard (1985)

2. ESTIMATING RICHARDSON'S MODEL FOR THE US-USSR: SOME INTRODUCTORY NOTES

First, a few methodological notes are in order. Before estimating Richardson's model, for any arms race, it must converted from differential equation form to difference equation form. Data on the US-Soviet race compiled by various research institutes is specified in discrete rather than continuous form. From

$$d\mathbf{X}/dt = k\mathbf{Y} - a\mathbf{X} + g \qquad \text{(R5)}$$

$$d\mathbf{Y}/dt = l\mathbf{X} - b\mathbf{Y} + h \qquad \text{(R6)}$$

we write the following equivalent statement:

$$\mathbf{X}_t - \mathbf{X}_{t-1} = k\mathbf{Y}_{t-1} - a\mathbf{X}_{t-1} + g \qquad \text{(R26)}$$

$$\mathbf{Y}_t - \mathbf{Y}_{t-1} = l\mathbf{X}_{t-1} - b\mathbf{Y}_{t-1} + h \qquad \text{(R27)}$$

We can simplify these statements as follows:

$$\mathbf{X}_t = k\mathbf{Y}_{t-1} + (1 - a)\mathbf{X}_{t-1} + g \tag{R28}$$

$$\mathbf{Y}_t = l\mathbf{X}_{t-1} + (1 - b)\mathbf{Y}_{t-1} + h \tag{R29}$$

Secondly, Richardson had assumed that hostile states compete with each other on the basis of each side's total armed might. He thought that an individual state's annual aggregate military expenditures would accurately reflect its total armed might. A state's annual aggregate military expenditures would include all monies budgeted to its armed services in a given year. I have term this perspective the macro perspective.

3. THE EARLY ATTEMPTS AT ESTIMATING RICHARDSON'S MODEL FOR THE US-USSR

Most studies which have attempted to estimate Richardson's model for the US-Soviet arms race test it against US and Soviet aggregate annual military expenditures. These macro studies (Abolfathi (1978), Chatterji (1969), Gillespie et al (1977a), Gregory (1974), Hamblin et al (1977), Hollist (1977a,b), Hollist and Guetzkow (1978), Lambelet (1973), Lambelet et al (1979), Luterbacher (1974, 1975), Majeski and Jones (1981), Milstein (1972), Rattinger (1976b), Saris and Middendorp (1980), Shisko (1977), Strauss (1972, 1978), Taagepera et al (1975), Wagner et al (1973), Wallace (1979, 1980a), and Zinnes and Gillespie (1973); see also Schrodt (1978b) for a discussion of the problems one faces when attempting to estimate

Richardson's model) generally show that the US and Soviet are not engaged in an action-reaction arms race. American military expenditures at time period t depend only upon American military expenditures at time t-1, the previous time period. Soviet previous period military expenditures do not seem to enter into the American expenditure calculus. Soviet military expenditures at time period t, likewise, seem to be self-driven.

More technically, many have found that the US defence coefficient, k, is zero and that the Soviet defence coefficient, l, is zero. The US, by custom, has been labeled as nation X, and the Soviet Union as nation Y. Substantively, neither the US nor the Soviet Union, Richardson's model accordingly suggests, feels threatened by the military expenditures of the other. This result will be termed, throughout this study, as the non-interaction outcome.

A smaller number of researchers (e.g., Majeski, 1985; Majeski and Jones, 1981; Ashley, 1980), on the other hand, have reported finding statistical evidence that suggests that k, the US defence coefficient, is in fact non-zero, but, curiously, that l, the Soviet Union's defence coefficient, is zero. This suggests, according to Richardson's model, that the US does feel threatened by the military expenditures of the Soviet Union, but that the Soviet Union does not feel threatened by the military expenditures of the US. This result will be termed the asymmetric outcome. Were US and Soviet military expenditures really independent of each other? Or were the non-interaction and asymmetric outcomes the result of some methodological deficiency.

4. THE US-SOVIET ARMS RACE REOPERATIONALIZED: THE USE OF WEAPONS STOCKS.

Serious doubts were raised about the propriety of testing Richardson's model for the US-Soviet arms race against each side's annual aggregate military expenditures. The assumption underlying the use of aggregate military expenditures in the US-Soviet case has been that increases in aggregate military expenditures reflect increases in armament levels (Hollist (1977b)). In fact, increases in military budgets may simply reflect increased costs of production without any increase in capability. Those who do use aggregate military expenditure data in their work must, moreover, assume that military cost-effectiveness remains constant not only over time, but over space as well (Wallace (1979)). Yet this is not likely to be true. Thus some researchers (for example, Lambelet (1973), Lambelet, Luterbacher, and Allan (1979), Allan and Luterbacher (1981), McGuire (1977), Desai and Blake (1981), and Squires (1982)) began to consider the prospect of testing Richardson's model against total US and Soviet weapons stockpile data rather than military budget data. Weapons stocks might more accurately reflect a nation's military strength than would its military expenditures.

While the unit of analysis for these researchers has changed from US and Soviet aggregate military expenditures to US and Soviet total weapons stocks, their outlook remains macro oriented. Lambelet (1973), for example, created an index of US total conventional weapons capability and an index for total Soviet conventional weapons capability. Each index was structured by multiplying manpower by firepower by mobility of forces for each state. McGuire (1977), conversely, tested Richardson's model against data on total US and Soviet strategic warhead deployment levels, total US and Soviet megatonnage levels, and total US and Soviet nuclear missile and bomber deployment levels running 1960-73. He found little evidence to suggest that the US and Soviet Union were engaged in an action-reaction arms race. Indeed, for the most part, such studies have not provided conclusive empirical evidence for the existence of an action-reaction arms race between the US and Soviet Union.

5. THE US-SOVIET ARMS RACE REOPERATIONALIZED: THE USE OF WEAPONS STOCKS AND MILITARY BUDGETS.

Another macro perspective developed in light of the finding of non-interaction in US-Soviet military expenditures was that evidence of action-reaction interaction in the US-Soviet arms race could be uncovered if both stocks of weapons and military budgets were analyzed together. Taagepera (1979-1980) first raised this question. This idea is just beginning to receive attention in the arms race literature. To date only Organski and Kugler (1980), Luterbacher, Allan and Imhoff (1979) and Ward (1984a) have conducted systematic analyses of the weapons stocks-military budget hypothesis. In his study, Ward developed and tested his own weapons stock-budget model. From his tests, he concluded (1984a: 309):

> The United States and the USSR do appear to be reactive to one another, yet not through budgets alone. Rather they each try to achieve or maintain a lead over the other in terms of the stocks of weapons, both strategic and conventional, for which the budget is spent.

Despite Ward's tentative successes, there are sound theoretical and methodological reasons to question the validity of his findings. In the next section, I will question the validity of the macro approach to US-Soviet arms race analysis.

6. THE US-SOVIET ARMS RACE REOPERATIONALIZED: A LOOK AT INDIVIDUAL WEAPONS SYSTEMS WITH CROSS PURPOSES.

McCubbins (1983) took arms race research methodology into a new direction. He argues that rival nations do not, as Richardson suggested, engage each other in a single race over "total armed might." The US and Soviets do not, as McCubbins argues, compete with each other, in an action-reaction mode, when it comes to total armed might because neither side in the US-Soviet case has the economic capability or institutional where-with-all to react to the full range of the other's military capability, as a single package, with a single response even though each side may fear the other's total armed might.

McCubbins (1983) instead argues for a disaggregate approach to US-Soviet arms race analysis. He argues that the US and Soviet Union compete with each other, in an action-reaction mode, when it comes to specific, individual weapons systems with cross-purposes.

For example, if one side steps up its deployments of heavy bombers, the other would respond by stepping up its deployments of jet interceptors. Conceivably, he furthermore argues, the US and Soviet Union could engage each other in several individual races, the object of any one race being a particular weapons systems and a particular counter weapons systems. At any given time, some sub-races, as we may call them, may be heating up while others are cooling down. Accordingly, testing an arms race model against aggregate military expenditures (the sum of all monies expended by each side on all weapons systems and general upkeep) could lead to misleading parameter estimates. A model would have to be tested against data on individual subraces.

McCubbins developed and then tested a model which set State X's deployment of a particular weapons system to be a function of State Y's deployment of a counter-purpose weapons system and State X's own GNP. He found, for example, that US deployment levels of strategic bombers, as he had expected, depend upon Soviet deployments of strategic interceptors. He, nevertheless, also looked for evidence of a system to system competition. For example he tested to see if US deployments of strategic bombers depended on Soviet deployments of strategic bombers. He found very little evidence of system to system competition.

To summarize, McCubbins' study is important in that it specifies a sound theoretical approach to the US-Soviet arms race analysis and in that it does show that the US and

Soviets are engaged in an action-reaction arms race. Rather than competing over aggregate capability levels, the US and Soviet Union compete when it comes to individual weapons systems with cross purposes.

7. SYNTHESIS

Beyond the question of what sort of data Richardson's or others' arms race models should be tested against in the US-Soviet case, there is the question of what constitutes an appropriate time frame for the analysis of the US-Soviet arms race. Lucier (1979) has argued that arms race parameters, for example k, a, g, l, b, and h from Richardson's model, do not remain constant over time. Changes in leadership, changes in standard operating procedures and even the signing by two adversaries of an arms control agreement can cause arms race parameters to change. If a data set spans a number of changes in the value of a particular arms race parameter, some complementary, some running in the opposite direction, then results obtained from statistical tests aimed at estimating its value might be misleading. To test his hypothesis, Lucier used a variant of Richardson's model, the organizational process model:

$$\mathbf{X}_t = q\mathbf{X}_{t-1}$$

He found, using data on the naval expenditures of Britain, Japan and the US during the interwar period, that indeed the value of q changed over time.

Few, if any, arms race researchers studying the US-Soviet race had considered the possibility that the parameters of that race might not be constant over time. Hollist (1977b), for example, did not consider this possibility. Hollist's analysis of the US-Soviet arms race was based on a military expenditure data set which spanned twenty-two years running 1948 to 1970. He tested Richardson's model against this data set and found no interaction in US-Soviet military expenditures. Surely things happened over this twenty-two year period which might have caused the values of the parameters of Richardson's model to change. For example the sub-period 1962-1970 must have seen a change in the value of the Soviet Union's defence coefficient. This was a period of unprecedented nuclear build-up for the Soviet Union, a build-up which historians tell us was incited by the Soviet Union's weakness in the face of America during the Cuban Missile Crisis. Since Hollist's data set spanned this likely parameter change we must be careful how much weight we attribute to his findings.

US-Soviet arms race researchers could take a lesson from Rattinger's (1976: 502) study of the Arab-Israeli arms race. Rattinger wrote, for

> most nations in the area, military spending is a rather meaningless indicator of capability because of the complex interplay of military aid, regular arms procurements, gifts and nonmaterial forms of payment by political allegiance to arms donors, and the like. This fact--together with the high degree of hostility and comparatively good information on hardware levels--makes defense spending less important as a perceptual variable in the Middle East

context.

Rattinger thus created indexes of strategic capability for individual weapons systems, e.g. for tanks, for each side in the Middle East conflict which included information on numbers and quality. He wanted to see, for example, if the Arabs and Israels were competing when it come to individual weapons systems such as tanks, or missile boats. On top of that, Rattinger broke down his data sets into two periods, one running 1956 to 1967, the other running 1967 to 1973. The reasoning behind this break-down involves a series of key military and political changes which beset the Middle East after the 1967 war. So Rattinger used, to summarize, desegregated data. In addition, he recognized that the parameters in the model that he was testing likely did not remain constant over time. His analysis, accordingly, showed evidence of a Richardsonian-type action-reaction race between the Arabs and Israelis, particularly in the period 1956-1967.

From this discussion, two important methodological points have been made. The process of estimating an action-reaction model, be it Richardson's or any other model, for the US-Soviet arms race, must be preceded by a careful consideration of the factors which drive that race. There is strong theoretical and empirical evidence to suggest that the US-Soviet arms race is made up of a number of subraces, the object of each being some weapons system, deployed by one side, and a weapons system with a cross-purpose deployed by the other. It would be necessary, then, to conduct several tests of a model, each test corresponding to a particular race, or subrace, over a particular weapons systems and a particular counter weapons system. Models should be tested against deployment levels of these systems. A picture of the overall US-Soviet military competition could then be sketched on the basis of those analyses. There is a need also to consider analytical time frames. Any particular subrace will have its own start and end point and these points may lie well within the start and end points of the overall military competition between two rivals. The start and end points of a subrace correspond with the period in which the weapons systems which were the object of the subrace were dominant in the arms race participants' strategic or tactical calculations.

I will term this approach to arms race model testing, the micro approach to mathematical arms race analysis.

While the micro approach may be theoretically sound, it is not without its own problems. In particular, sample sizes, under the micro approach, could end up being very small (less than 30 points). On the one hand, one must be concerned with choosing a valid approach to US-Soviet arms race analysis. Still, one must be concerned with the technical requirements of statistical estimation methods. Small sample sizes, for example, pose a number of technical problems when it comes to model estimation. In Chapter 5, I will consider, in detail, the technical problems associated with the micro approach to arms race analysis and suggest a way to put it into practise.

In the next chapter, I will examine an alternative approach to explaining the non-interaction and asymmetric outcomes. A number of attempts have been made to reconceptualize the arms race phenomenon since Richardson's original work. The motivating force behind these efforts has been the idea that Richardson's model has oversimplified important elements of the arms race phenomenon and as such that it could not capture, empirically, the true dynamics of real world races such as the US-Soviet race. What was needed, then, was a new action-reaction arms race model. Several new approaches to arms race modelling were subsequently to emerge, each with its own set of behaviorial assumptions and analytical techniques. In essence, arms race research returned to step one of the modelling process.

CHAPTER III: RECONCEPTUALIZING THE ARMS RACE PHENOMENON

By the mid 1970s serious questions were being raised about the validity and utility of Richardson's model. Richardson's model, many were to argue, oversimplified the arms race phenomenon, and as such could not be properly applied in the effort to determine the "true" nature of the US-Soviet military competition. A number of new mathematical conceptualizations of the arms race phenomenon were subsequently offered as alternatives to Richardson's model. Several excellent comprehensive reviews of this material have been offered including Moll and Luebbert, 1980; Isard and Anderton, 1988; Busch, 1970; Zinnes, 1976; Intrilligator and Brito, 1976. In this chapter, I, too, will offer a review of post-Richardson arms race modelling efforts. This review, however, will be selective, featuring those developments which are directly relevant to my own arms race modelling effort. More specifically, I will focus my review on the key works which have drawn on game theory, control theory (dynamic game theory), expectations theory and economic theory in modelling the arms race phenomenon. In what follows, I will outline the basic assumptions of each of these approaches and I will show the models which were derived from them. I will also consider the extent to which each of these models does capture the action-reaction element of the US-Soviet arms competition. We have returned, then, to step one of a second loop in the mathematical modelling process.

1. THE GAME-CONTROL THEORY APPROACH

Game theory is based on two key behaviorial assumptions (see for example, Zinnes et al 1978b; Lichbach, 1989, 1990; Shubik, 1982, 1984; Simman and Cruz, 1973, 1975a; Smale, 1980; Harasanyi, 1965; Brams and Wittman, 1981; Brito and Intrilligator, 1980; Gillespie et al, 1975, 1977a; Brams, 1975; Brams and Kilgour, 1988). First, players or states act in accordance with some goal. Second, a goal cannot, for the most part, be realized unless it is pursued in accordance with some sort of strategy. For that strategy to be successful, it must, to one degree or another, take into account the goals strategies of opposing players or states. The concepts of goal and strategy apply to a wide range of human activity and certainly to the activity of arms racing. The main criticism against the application of game theory to international relations research in general and arms race analysis in particular is that it assumes that the play in question is a one shot affair with costs and payoffs fixed over the life of the game. In reality, however, the game of international relations is continuous with costs and payoffs for the players involved constantly changing. It is on this score that arms race research turned to optimal control theory. Optimal control theory has provided one way for researchers to model state goals and strategies in an arms race situation taking into account the dynamic, continuous nature of international competition.

How, specifically, has game-optimal control theory been applied to arms race analysis. Like game theory, control theory views arms racing as a game where each player (or state) sets itself a goal and a strategy for obtaining that goal. An arms race can be conceptualized as a game to the extent that each participant takes into account the goal/strategy of its opponent when formulating its own goal oriented strategy. The concepts involved in game theory can be formalized using optimal control theory mathematics. Under optimal control theory, goals can be defined by a control variable, e.g., $u(t)$, and the environment by a state variable, e.g., $x(t)$. State, control and time variables together define a complete and closed system:

$$dx/dt = F(x(t), u(t), t) \tag{CT1}$$

Equation CT1 is known as the plant equation. Control theory assumes that at any point in time, a system is in some state. Control theory mathematics allows a nation to make the system respond in different ways at different time periods by manipulating the value of its control variable. Indeed a set of alternative "future histories" for a system can be plotted by a state by considering different control variable values. The desired yield from a system at any point in time can be mathematically expressed a function of the state and control variables and time:

$$J(x, u, t) \tag{CT2}$$

Equation CT2 is known as the objective function. In particular the optimization problem will take the general form

$$J = \int_{t_0}^{T} L[x(t), u(t), t]dt \tag{CT3}$$

where $u^*(t)$ is selected to optimize J subject to

$$dx/dt = F(x(t), u(t), t) \tag{CT1}$$

The optimal value for the control variable, $u^*(t)$, can be found by first defining a function known as the Hamiltonian, H, and then solving for $u^*(t)$. The Hamiltonian combines the object and plant equations through the use of auxiliary variables, namely, p(t). The auxiliary variables are called the adjoint vector.

$$H(x,u,p,t) = L(x,u,t) + p^T F(x,u,t) \tag{CT4}$$

In Equation CT4, p^T is the transpose of the adjoint vector and it satisfies the condition

$$-\dot{p} = dH/dX \tag{CT5}$$

$u^*(t)$, then, is calculated by finding

$$dH/du = 0 \tag{CT6}$$

Once $u^*(t)$, the optimal strategy, is calculated, then it can be substituted into the plant equation CT1 to obtain what is referred to as an optimal trajectory. An optimal trajectory is simply an optimal pattern of behavior given one's goals and one's operating environment.

Brito (1972), Brito and Intrilligator (1973) and Simaan and Cruz (1973) were the first to apply control theory mathematics to the study of arms races. Brito (1972), for example, conceptualized an arms race as a problem in welfare maximization. Each state, in an arms race dyad, seeks to find some optimal balance between consumption on the one hand, and defence on the other hand. State A's defence index, D_A, depends upon the stock of weapons currently in its possession, M_A, and the stock of weapons currently possessed by its opponent, State B, M_B. Mathematically,

$$D_A = D_A(M_A, M_B) \tag{B1}$$

D_A, State A's defence index, measures State A's well being in terms of security at any point in time. D_A increases with M_A and decreases with M_B. State A's utility, U_A, depends upon its defence index, D_A, and its level of consumption, C_A.

$$U_A = U_A[C_A, D_A(M_A, M_B)] \tag{B2}$$

State A will seek to maximize its welfare over all time periods given the present value of all future utility levels discounted over time. Mathematically, Brito expressed this assumption in the following way.

$$W_A = \int_0^\infty e^{-rt}\, U_A[C_A,\, D_A(M_A,\, M_B)]dt \tag{B3}$$

Brito then imposes constraints on the values of the defence and utility functions. He solves the welfare maximization problem by first specifying a parallel set of equations for State B and then applying control theory mathematics. The result is as given by the set of equations

$$dM_A/dt = F_A(M_A,\, M_B) \tag{B4}$$

$$dM_B/dt = F_B(M_A,\, M_B) \tag{B5}$$

In the special case where F_A and F_B are linear, the optimal solution to Brito's consumption-defence model is given by Richardson's arms race model. Brito was, thus, able to show one set of conditions under which Richardson's model might constitute an optimal trajectory for two nations engaged in an arms race. This was an important accomplishment because, as Gillespie et al (1977c: 226) write, "While Richardson's equations can be interpreted as describing how nations will react to one another in an arms race, they do not explain how the nations arrived at such a strategy." The principal shortcoming of Brito's work, however, is that he chose to discuss national goals and environmental conditions only in the most general of terms. Thus one could only draw general conclusions from his model.

Gillespie et al's (1977c) control theory study was the first to attempt a specific statement of national goals and environmental contexts for actors involved in an arms race. Nation U was posited to have the dual objective of, one, maintaining a balance of power with its rival, nation X, and, two, minimizing the total number of armaments possessed by it and nation X. The first goal was mathematized by Gillespie et al as follows

$$J_1 = \int_t^T [(u(t) - ax(t))^2]dt \tag{G1}$$

The second goal took the form

$$J_2 = \int_t^T [c(u(t) + x(t))]dt \tag{G2}$$

In Equations G1 and G2, u(t) represents State U's armaments and x(t), State X's armaments. Equations G1 and G2 were combined to give a comprehensive statement of State U's goals

$$J_2 = \int_t^T [(u(t) - ax(t))^2 + c(u(t) + x(t))]dt \tag{G4}$$

The operating environment for nation U is defined in terms of State X's arming behavior. Gillespie et al defined State X's arming behavior by Richardson's model where

$$dx/dt = lu(t) - bx(t) + h \tag{G5}$$

From Equations G1 to G5, Gillespie et al were able to calculate an optimal trajectory for States U and X and to calculate arms race stability conditions for State U and State X. They applied this entire framework to the US-Soviet arms race. Gillespie et al's study, based on annual aggregate military expenditure data, showed that the US-USSR arms race is unstable. Both sides in this race, they argue, would likely continue to arm to ever increasing levels.

2. THE EXPECTATIONS APPROACH

Over the years, international relations research has borrowed insights from a number of related academic disciplines. In the 1970s and 1980s arms race researchers began turning to economics for help in their efforts to develop more realistic models of arms races. In particular, arms race researchers had begun to consider that states calculate their arms requirements as much on the basis of what they expect an opponent's future armament level will be as they do on what an opponent's current and past arrament levels are or were.

Fortunately, economists had long recognized the role that people's expectations of the future values of different economic variables play in the functioning of an economic system. Accordingly, many conceptually deep, but mathematically simple ways were developed to model people's expectations, including the extrapolative expectations approach, the adaptive expectations approach and the Rational Expectations hypothesis. These approaches are general and can be applied to a wide range of economic and social problems, including the study of arms races.

Majeski (1985), for example, argued that certain aspects of a state's military expenditure behavior could be modelled by reference to the theories of extrapolative and adaptive expectations. Majeski began by assuming that State X's military expenditures at time period t should, first, depend upon internal bureaucratic momentum within that state. Majeski modelled the impact of bureaucratic processes in State X on its armament expenditure calculus as the sum of its past period expenditures as shown in Equation M1.

$$\mathbf{X}_t = \Sigma\, a_i \mathbf{X}_{t-i} \qquad \text{(M1)}$$

$$\mathbf{Y}_t = \Sigma\, d_i \mathbf{Y}_{t-i} \qquad \text{(M2)}$$

Equation M2 shows the impact of bureaucratic processes in State Y on its armament expenditure calculus. Next, when formulating its armament requirements for time period t, State X would consider the question, at what level would State Y's armaments at time period t be? State X must, in this case, formulate an expectation regarding State Y's future armament level. Extrapolative expectations theory provides that State X could formulate that expectation simply by taking a weighted average of State Y's past period deployment levels. Majeski denoted State X's expectation by $\mathbf{YP}_t$ where

$$\mathbf{YP}_t = \Sigma\, b_i \mathbf{Y}_{t-i} \tag{M3}$$

Thus Equation M1 is rewritten as follows

$$\mathbf{X}_t = \Sigma\, a_i \mathbf{X}_{t-i} + b\mathbf{YP}_t \tag{M4}$$

A parallel set of equations can also be specified for State Y

$$\mathbf{XP}_t = \Sigma\, c_i \mathbf{X}_{t-i} \tag{M5}$$

Thus Equation M2 can be rewritten as follows

$$\mathbf{Y}_t = \Sigma\, d_i \mathbf{Y}_{t-i} + c\mathbf{XP}_t \tag{M6}$$

Finally, Majeski drew on the adaptive expectations hypothesis. He posited that State X would also wish to take account of any errors it had made in the past when calculating its expectations of State Y's future armament levels and accordingly take compensatory action. Adaptive expectations theory provides a simple solution to this problem. Majeski simply incorporated an error term into his model which represented the difference between State Y's actual deployment level at time period t and State X's expectation of State Y's time period t deployment level. Majeski denoted State X's prediction error by $\mathbf{YE}_t$ where

$$\mathbf{YE}_t = \mathbf{Y}_t - \mathbf{YP}_t \tag{M7}$$

State Y's prediction error is, similarly, given by

$$\mathbf{XE}_t = \mathbf{X}_t - \mathbf{XP}_t \tag{M8}$$

Thus Equations M4 and M6 can be amended as follows

$$\mathbf{X}_t = \sum a_i\mathbf{X}_{t-i} + b\mathbf{YP}_t + \sum g_i\mathbf{YE}_{t-i} \tag{M9}$$

$$\mathbf{Y}_t = \sum d_i\mathbf{Y}_{t-i} + c\mathbf{XP}_t + \sum h_i\mathbf{XE}_{t-i} \tag{M10}$$

Equations M9 and M10 constitute the final form of Majeski's expectations model.

Majeski applied his expectations model to the US-Soviet arms expenditure race. He found evidence that the US and Soviet Union are engaged in an asymmetric reaction arms race. US military expenditures at time period t, he found, depend upon, one, previous period US military expenditures and, two, previous period Soviet military expenditures. Soviet military expenditures at time period t, in contrast, depend only upon Soviet previous period military expenditures. Majeski interpreted this result to have come about as a consequence of the disparity between US and Soviet economic-industrial capacity. The US, having a larger economic-industrial base than the Soviet Union, could more easily afford the luxury of matching Soviet expenditures while the Soviets, with their weaker economic-industrial base, could not do likewise.

3. ECONOMIC CONSTRAINT THEORY

Caspary (1967), dissatisfied with the way in which Richardson had formalized State X and Y's economic constraints, offered a new economic constraint formulation. Richardson, he felt, had oversimplified the way in which economic constraints could impact each nation's armament program. Caspary argued that the constraint should be non-linear rather than linear. He began by rewriting Richardson's equations. From

$$dX/dt = kY - aX + g \tag{R5}$$

$$dY/dt = lX - bY + h \tag{R6}$$

He wrote

$$D = dX/dt = a(kY/a - X + g/a) \tag{C1}$$

$$D' = dY/dt = b(lX/b - Y + h/b) \tag{C2}$$

This formulation is similar to Richardson's rivalry formulation in that it states that a nation will increase its military strength per unit time in response to a difference between some fraction of its existing force level and some fraction of its opponent existing force level. D, then, shows State A's desired arms increase per unit time. D' shows the same for State Y. Caspary made the point, however, that neither State A nor State B could arm without due consideration to its available resources. More specifically, he argues that there are diminishing returns to military expenditures. For example, additional expenditures by a state for military capability are likely to yield less security when expenditures are already high fraction of its total available resources for military items than if they are low. Caspary formalized these assumptions with the following equations.

$$p(d\mathbf{X}/dt) = a(C - M\mathbf{X})(1 - e^{-ND/C}) \tag{C3}$$

$$p'(d\mathbf{Y}/dt) = a(C' - M'\mathbf{X})(1 - e^{-N'D'/C'}) \tag{C4}$$

In Equations C3 and C4,

C = the total resources that State X has available for military items

D = as defined above

N = the per unit cost of new armaments in State X

M = the cost to State X of maintaining one unit of old military equipment

p = a dimensional constant used by Caspary to convert from armament levels/time to dollars/time in State X.

What Equation C3 shows is that if ND/C is large, which means that State X's desired military expenditures are set at a point which is a large fraction of its total available resources for military expenditures, then, in practice, the actual increase in State X's military expenditures will be less than its desired increase in military expenditures. Caspary never did operationalize his model. He was content to show how resource constraints could theoretically affect the levels of arms accumulation in states engaged in an arms race. That in itself was an important contribution.

3.1 STOCK ADJUSTMENT THEORY

A much simpler way to formalize the idea that each state must face economic and institutional constraints on its armament programs is suggested by the work of Nerlove (1958). Intrilligator (1975) suggested the potential of Nerlove's (1958) stock adjustment hypothesis in arms race research and so far, it has seen a fair amount of use (see Brito (1972), Gillespie and Zinnes (1975), Gillespie et all (1977c), Majeski (1983a, b) and Ward (1984)). There are different variations of the hypothesis, but essentially it states that the change that actually occurs in some agent's stock level from time t-1 to time t will be some fraction of what that agent wished to effect in his stock levels over that same time period.

4. SUMMARY: WHERE TO FROM HERE?

Arms race research has made important conceptual and analytical advances since Richardson's original work. Here, then, is where we stand today. Researchers now assume that a state engaged in an arms race with a rival power arms in accordance with some goal, such as maintaining a balance of power with that rival. Moreover, some assume that a state will design and implement a strategy to achieve its arming goal. For that strategy to be effective, a state must, to one degree or another, take into account the goals and strategies of its rival. In an effort to formalize the concepts of goal and strategy, researchers have employed the methodologies of game-control theory. Expectations theory has been employed to formalize the assumption that states calculate their arms requirements as much on the basis of what they expect an opponent's future armament levels will be as they do on what an opponent's current and past armament levels are or were. The idea that no state can arm without due regard to its available resources is important. The stock adjustment hypothesis shows a great deal of promise in facilitating the formalization of the economic constraint assumption.

Despite these conceptual advances in mathematical arms race modelling, however, researchers still have not been able to uncover any strong evidence of action-reaction in the US-Soviet arms race. Why? Each approach, game theory, expectations theory, stock adjustment theory, in practise, has been treated as a way to examine arms races in and of itself. Some studies focus only on the gaming aspects of arms racing. Some focus only on the expectations aspects of arms racing. Still others focus only the economic aspects of arms racing. But arms races have gaming dimensions, expectations dimensions and economic dimensions all at the same time. Focusing on one approach, then, amounts to an oversimplification of the arms race phenomenon. This could explain why researchers have still not been able to uncover any strong evidence of action-reaction interaction in the US-

Soviet military competition. An intuitively appealing strategy, then, would be to combine the game-control, expectations and economic approaches into a single, more comprehensive approach to arms race analysis. Indeed, three major attempts have already been made in this regard, namely, Ostrom (1978b) Majeski (1983a), and Ostrom and Marra (1986).

Each, first, argued that an arms expenditure process, particularly the US arms expenditure process, must be desegregated before it can be analyzed and modelled. Instead of developing a single model describing US military expenditures at time t, Ostrom (1978b), Majeski (1983a) and Ostrom and Marra (1986) developed four separate but interconnected models of the US arms expenditure process, each describing the behavior of a different policy making group in the process. The four models describe the expenditure decision rules of the Defence Services Agency, the President, the Congress, and the Department of Defence, each of which plays a crucial role in the determination of the US defence budget. In modelling the behavior of these groups, Ostrom (1978b), Majeski (1983a), and Ostrom and Marra (1986) drew on the theory of adaptive expectations, a theory which describes how actors might formulate forecasts of future conditions which have a bearing on the policy problems of concern to them. They also drew on game theory insofar as they argued that each of the policy making groups in question has a specific goal in mind with respect to the defence budget. They, finally, also drew on Nerlove's stock adjustment hypothesis as a way to formalize the real world fact that there will be a disjunction between what each policy group would like to have as a defence budget and what political and economic conditions will in the end allow as the budget. Majeski's (1983a) study was successful in many ways, but he was unable to find any statistical evidence that external threat, as measured by Soviet military expenditures, has any impact on the US military expenditure decision process over the period 1953-79. He concluded that this was further evidence for the growing belief that US military expenditures are driven principally, if not entirely, by domestic, rather than external, forces. Ostrom (1978b) and Ostrom and Marra (1986) uncovered evidence to the contrary.

My work, like Ostrom's (1978b), Majeski's (1983a), and Ostrom and Marra (1986) is an attempt to meld together some of the main assumptions concerning the arming behavior of states which have arisen since Richardson's original work. I have, however, approached this task from a much different angle. Unlike Ostrom's (1978), Majeski's (1983), and Ostrom and Marra (1986), I will approach my study of the arms race behavior of states from an aggregate perspective, as though states behaved as unitary rational actors. I do not mean to say here that a single person makes the arming decisions in any given state. Indeed the armament decision process in each country is competitive, involving many persons and many groups. Nevertheless, a single arming decision ultimately emerges from this competition. I will assume that the aggregate outcome of an armament decision process can be modelled as though it were made by a unitary rational actor (Williams and McGinnis, 1988). Secondly, I will draw on the Rational Expectations hypothesis rather than the adaptive expectations in modelling state forecasts of future conditions. The adaptive expectations hypothesis which Majeski (1983) used differs in critically important ways from the Rational Expectations hypothesis which Williams and McGinnis (1988) and McGinnis and Williams (1989a) and I have used. Under the Rational Expectations hypothesis, actors, be they states or individuals, can form unbiased expectations of future variables. Actors can and do make mistaken estimates from time to time, but their mistakes are not systematic. Under the adaptive expectations hypothesis which Majeski used in his work, actors, in contrast, can make systematic errors in their expectations. It seems reasonable, however, to assume that actors do pay attention to how well they have done in predicting future conditions. If the expectation rule that an actor happens to be using systematically has yielded over or under estimates, then it is reasonable to assume that actor will modify his expectation rule such that his expectation errors become less systematic. Hence my use of the Rational Expectations hypothesis.

More specifically, I will derive an action-reaction arms race model from the following set of assumptions, drawn from game-control theory, Rational Expectations theory and stock adjustment theory:

ASSUMPTION 1: The behavior of each of two states engaged in arms race is goal oriented. In my model, each state seeks to maintain some balance of forces between itself and its opponent in each successive time period throughout the life span of the race.

ASSUMPTION 2: Each state must face economic and institutional constraints on its armament program. The impact of political, economic, and institutional constraints on a state's arming program are such that the change that actually occurs in each state's force level from time t-1 to time t will be some fraction (between zero and one inclusive) of the change that each state wanted (desired) to effect in its force level over that one time period. This is essentially Nerlove's (1958) stock adjustment hypothesis.

ASSUMPTION 3: Each state takes account of its rival's future armament program when formulating its own armament program. But no state knows exactly what its rival's arming strategy will be. Each state is assumed, however, to be able to act in accordance with the Rational Expectations hypothesis. That is, each state can form an informed, unbiased estimate of its adversary's armament plans, and hence, each state can form an expectation of what its adversary's future armament level will be.

In essence, in the next chapter, I will return to step one of the modelling process. From my assumptions, I will be able to derive a new action-reaction arms race model. In a later chapter, I will demonstrate the utility of my model by testing it for the US-Soviet arms race. That test will reveal the illusive action-reaction component of the US-Soviet military competition.

CHAPTER IV: A GSR ACTION REACTION ARMS RACE MODEL

Arms race researchers have long been engaged in a quest to model the arms race phenomenon mathematically. Today, the quest continues. Many approaches have been applied along the way, notably, game-control theory, expectations theory and the economic-stock adjustment approach since Richardson's original work. Taken individually, none of these new approaches has, however, faired much better than Richardson's: no model has yet been able to adequately reveal, empirically, the dynamics of real world races such as the US-Soviet race, as scholars intuitively understand those dynamics. The problem is that arms races, like most social phenomena, are multi-dimensional. They have gaming dimensions, economic dimensions, and expectational dimensions all at once. I propose, therefore, to combine the assumptions and insights of the game theoretic, expectations, and economic approaches into a more comprehensive framework, and then derive there from my own action reaction arms race model. In effect, I will, in this chapter, carry out steps one and two of the modelling process.

I will begin by, (1), defining the phenomenon to be modeled, namely, an arms race, then, (2), consider various questions concerning the level of analysis at which my study will be conducted, (3), specify the unit of analysis, (4), discuss the behavioral assumptions behind my model, (5), formalize assumption 1: state goal formation, (6), formalize assumption 2: economic constraints on goals, (7), formalize assumption 3: forming a strategy, (8), present my new arms race model, then, (9), ask, does my model conform to my definition of an arms race?, and finally, (10), discuss the equilibrium properties of my model.

1. DEFINING THE PHENOMENON TO BE MODELED

In this chapter, I will derive an action-reaction arms race model. An arms race has two key defining features. First, two rivals can be said, at a minimum, to be engaged in an arms race to the extent that each side takes into account its rival's armaments, past, current, or future when deciding on its own future armament level (Gray, 1971, 1976). Further, for an arms race to be an arms race, it must be driven by the pressure of the military rivalry between the states involved and not by forces exogenous to that rivalry, such as domestic pressures in either or both of the states involved to promote economic growth through military development programs (Wallace, 1979). What specific decisions and processes could generate behavior consistent with this arms race definition?

2. LEVEL OF ANALYSIS

Most mathematical arms race research is conducted at a high level of aggregation. That is, analysis has mostly centered on the state. Conducting arms race research at the level of the state as actor is reasonable, for, despite arguments to the contrary, principally, those put forward by the Globalist school of international relations, the state is and remains the prime mover in international politics (for the Globalist perspective, see Mendlovitz (1975) and for the state centric perspective, see Holsti (1985)). States compete. States arm. States fight. That is not to take away from the fact that what a state does, to a degree, is the consequence of internal competition and bargaining between groups and people (Allison, 1971).

In the military expenditure field, two major works, Ostrom (1978b) and Majeski (1983a), analyze the military expenditure process in the US in terms of the competing individual groups which most directly influence it. Initial research by Stromberg (1970) and Ostrom (1978b) suggests that the military expenditure process in the US involves, sequentially, four groups: the Defense Services Agency which makes a budget request on behalf of the armed services, the President who submits a formal budget to Congress, the Congress who appropriates monies for defence, and finally the Department of Defense which expends those monies. Each of these groups has its own interests and its own concerns. But it remains that the competition between them ends in a single budgetary decision. Somehow, all of the various interests within the US (and elsewhere) aggregate and a single arming decision or policy emerges. In this study, I will focus on that aggregate decision, on how states act, on how states arm. This immediately raises the question of how to go about conceptualizing state behavior.

In this thesis, I will draw on the Rational Expectations hypothesis in my effort to model the outcome of states' military allocation processes. Originally developed by economists to model macro-economic phenomena (Muth, 1961; Begg, 1982), the Rational Expections hypothesis "refers to the aggregate result of private economic actors utilizing relevant information in forming unbiased expectations of the future behavior of the economy as a whole [Williams and McGinnis, 1988: 973]." The theory of Rational Expectations has its detractors. Some of its assumptions are indeed controversial. But as a theory, it is concise and internally consistent.

Although the Rational Expectations hypothesis was originally developed to model macro economic phenomena, Williams and McGinnis (1988) have argued its utility in modelling aggregate political processes and outcomes. They argue that its principal virtue, in contrast to alternative theories of expectations formation, is its emphasis on a sophisticated use of information by actors. The basic tennents of the Rational Expectations hypothesis as applied to political processes and outcomes generally and to military allocation processes and outcomes specifically are as follows.

1. Many actors are involved in the formation of public policy, be it defence policy or otherwise. The process of formulating public policy is competitive even in such countries as the USSR. Ministries, as they must in all countries, compete for a share of their nation's limited resources. The aggregate outcome of the competition between bureaucrats over alternative policy choices can be modeled as though it were set by a unitary rational actor (see Williams, 1988).

2. Bureaucratic actors will draw on a wide range of information in developing and defending their policy positions. Actors are assumed to know the relevant variables which pertain to their policy concerns and they are also assumed to know the connections between those variables. In effect, actors have some model in mind.

3. Actors all have access to the same quality of information at the same cost.

4. The process of gathering and processing information is highly efficient as is the process of transforming new information into changed behavior.

5. Defence policy today, by necessity, must be based on the political, strategic, economic and

technological conditions which are expected to prevail tomorrow. Under the Rational Expectations Hypothesis, actors may make mistaken forecasts regarding future conditions, but they must be able to correct for their mistakes when realized so that successive forecasts will not be systematically wrong. More specifically, the aggregate expectation's of these actors are assumed to be unbiased.

Williams and McGinnis (1988) note that both the US and the Soviet Union do put forth a great deal of effort to learn as much as possible about each other's positions and intentions. Because each is the other's principal rival, and thus because both are so interdependent, such information is crucial to defence policy formulation processes in each country. A good deal of US defence policy is based on information provided by its intelligence services, namely, the CIA. The CIA has, however, provided erroneous information to US policy makers in the past. In the 1960s, for example, the bomber and missile gaps were grossly overestimated. In contrast, Soviet ICBM deployment levels were underestimated by the CIA during the SALT negotiations. But, note Williams and McGinnis (1988), the pattern of overestimation-underestimation in CIA estimates is neither consistent nor systematic. The works of Freedman (1986) and Prados (1986) support this conclusion.

6. Following Begg (1982: 29), the hypothesis of Rational Expectations asserts that the unobservable subjective expectations of actors or states would be exactly the true

mathematical conditional expectations implied by the models in question.

3. UNIT OF ANALYSIS

The unit of analysis issue, put in the form of a question, is this: what, specifically, do arms race participants compete over? Traditionally, mathematical arms race analysis has been based on a macro approach. The unit of analysis in mathematical arms race studies has, traditionally, been the annual aggregate military expenditures of competing states, that is, all of the monies expended by competing states on maintaining previous and building new forces in each year. In this thesis, I will break with that tradition. The methodological basis, as I have argued previously, for the macro-approach to arms race analysis is weak. Following Ward (1984) and particularly McCubbins (1983), I will assume that arms race participants compete with each over individual weapons systems with cross-purposes. For example, if one side in a race increased its heavy bomber deployments, the other would increase its jet interceptor deployments. And my focus will not be on how much each side spends on these systems, but rather on the amount of system deployed per year. McCubbins (1983) presents a strong argument for this perspective which I have set out elsewhere in this thesis. Thus in developing my model, I will speak of states countering each other's deployments of particular weapons systems rather than of states countering each other's expenditures.

4. BEHAVIORAL ASSUMPTIONS

What specific assumptions, then, do I make regarding the behavior of states in the context of an arming competition? My model is based on assumptions woven together from game theory, stock adjustment theory and the Rational Expectations hypothesis. They are as follows:

ASSUMPTION 1: The behavior of each of two states engaged in an arms race is goal oriented. In my model, each state seeks to maintain a balance of forces between itself and its opponent in each successive time period throughout the life span of the race. That is not to say that each side seeks to maintain a one-to-one numerical correspondence between its own forces and those of its adversary. What any given state considers a balance to be will depend, inter alia, upon technological, geographic, and economic factors.

ASSUMPTION 2: Each state must face political, economic and institutional constraints on its armament program. The impact of political, economic and institutional constraints on a state's arming program are such that the change that actually occurs in a state's force level from time t-1 to time t will be some fraction (between zero and one inclusive) of the change that state wanted (desired) to effect in its force level over that one time period.

ASSUMPTION 3: Each state takes account of its rival's future armament program when formulating its own future armament program. But no state knows exactly what its rival's future armament plans will be. Each state is assumed, however, to be able to act in accordance with the Rational Expectations hypothesis. That is, each state can form an informed, unbiased estimate of its adversary's armament plans, and hence, each state can form an expectation of what its adversary's future armament level will be.

In what follows, I will formalize each of these assumptions into simple mathematical statements. I will then combine these statements in order to obtain my model.

5. FORMALIZING ASSUMPTION 1: STATE GOAL FORMATION

In my model, States K and X are military rivals. One purpose for arms racing, Caspary (1967) notes, is to be militarily prepared vis-a-vis an opponent should war breakout. In order to be so prepared, a state must seek to maintain some sort of balance or proportion between its own forces and those of its adversary. The question as to what constitutes a balance in any given situation must be left open. As already stated, balance need not mean a one-to-one numerical correspondence between opposing forces. Factors other than simple numbers are considered by policy makers when determining balance points between their own military forces and those of an adversary. For example, Dougherty and Pfaltzgraff (1981: 404) write:

> Rough parity [under SALT I] was arrived at by political intuition, rather than by the computation of strict mathematical equality of the superpowers' missile arsenals. The Soviets were assigned a 40 percent margin of superiority in the number of land-based ICBMs and about a one-third margin the number of ocean-based SLBMs. The agreement was widely criticized in the United States for conceding to the Soviets a substantial advantage in missile payload, but ratification was justified by the Nixon Administration on the grounds that the United States possessed several compensating advantages (such as overseas air and submarine bases, the total number of warheads deployed, the number of long-range bombers, and qualitative superiority in a variety of important technological dimensions) ....

As a first approximation, State K, then, is simply assumed to set its desired deployment level for time period t, denoted $\mathbf{K}^*_t$, to be a linear function of its expectation at time t-1 of State X's actual deployment level at time period t, denoted $(\mathbf{E}^k_{t-1}\mathbf{X}_t)$, times some balance or defence coefficient, B_1. $\mathbf{K}^*_t$ is assumed to also depend upon a grievance term B_0, and a prediction error $\mathbf{U}_t$, as per Equation 1. State K is assumed to form its expectations of State X's upcoming armament level on the basis of information contained in its defence information set I_{t-1}.

$$\mathbf{K}^*_t = B_0 + B_1(E^k_{t-1}\mathbf{X}_t) + \mathbf{U}_t, \quad B_0, B_1 \in R \tag{1}$$

$$\mathbf{X}^*_t = B'_0 + B'_1(E^x_{t-1}\mathbf{K}_t) + \mathbf{V}_t, \quad B'_0, B'_1 \in R \tag{2}$$

Equation 1 is, then, State K's goal equation and Equation 2 is State X's goal equation.

Equation 1 and Equation 2 are set on the basis of the principle that arms acquisition dynamics in today's world are, necessarily, anticipatory. According to Majeski (1985: 220), the arms race participant

> recognizes that expenditure decisions to obtain security ... are directly affected by the choices and behavior of his opponent. In fact, what his opponent does or will do is probably more important and relevant to the arms race participant in terms of meeting his objectives than his opponent's prior behavior. Expectations of current and future behavior play a crucial role in this context.

The value range of B_1, in Equation 1, is left open in recognition of the fact that states do not necessarily calculate balances in strict one-to-one terms. In a specific context, one could hypothesize a value for B_1, but otherwise, its value must be determined empirically. State K's desired deployment level at time t is also influenced by its grievance term B_0. One could also hypothesize a value for B_0 in a specific context, but otherwise, its value, like B_1, must be determined empirically. Finally, State K's desired deployment level at time t depends on the error term $\mathbf{U}_t$. In accordance with the Rational Expectations hypothesis, $\mathbf{U}_t$ is assumed to have a mean of zero: $E(\mathbf{U}_t) = 0$ and it is assumed to be uncorrelated with the expectation term $E^k_{t-1}\mathbf{X}_t$. $\mathbf{U}_t$, by definition, reflects any unanticipated developments in State X's arming behavior in State K's armament decision calculus. State X's desired deployment goal is assumed to be set in a similar fashion based on information contained in its defence information set I'_{t-1}. Finally, in accordance with the Rational Expectations hypothesis, $\mathrm{corr}(\mathbf{U}_t, \mathbf{V}_t)$ is assumed not equal to zero.

6. FORMALIZING ASSUMPTION 2: ECONOMIC CONSTRAINTS ON GOALS

Each state must weigh its armament objectives in light of political, economic, industrial and institutional realities. Ostrom (1978b) and Majeski (1983a) recognized this fact and made compensation by formalizing the potential impact of these factors, the political, the economic, the industrial, and the institutional, on a state's armament decision process by using Nerlove's (1958) stock adjustment hypothesis. Ostrom, for example, noted that defence expenditures cannot be made without regard to public opinion and that public opinion may have a dampening effect on the final amount expended.

The stock adjustment hypothesis systematically relates a state's desired deployment level at time t, its actual deployment level at time t and its actual deployment level at time t-1. More specifically, the stock adjustment hypothesis is this: the change that actually occurs in State K's force level from time t-1 to time t will be some fraction, namely S, of the change that State K wanted (desired) to effect in its force level over that one time period. The same hypothesis can be applied to State X. S and S' (for State X) are called stock adjustment parameters and they take values in the range $0 \leq S,S' \leq 1$. They show the extent to which each respective state is capable, politically, economically and institutionally, of achieving some predefined desired armament goal. Mathematically,

$$\mathbf{K}_t - \mathbf{K}_{t-1} = S(\mathbf{K}^*_t - \mathbf{K}_{t-1}), \quad 0 \leq S \leq 1 \tag{3}$$

$$\mathbf{X}_t - \mathbf{X}_{t-1} = S'(\mathbf{X}^*_t - \mathbf{X}_{t-1}), \ \ 0 \leq S' \leq 1 \tag{4}$$

Equations 3 and 4 are general and can accommodate any number of definitions for $\mathbf{K}^*_t$ and $\mathbf{X}^*_t$ respectively. In general, $S = 1$, in Equation 3, implies that State K's actual deployment level will fully adjust to some predefined desired deployment level in each t. $0 < S < 1$ implies State K's actual deployment level will only partially adjust to some predefined desired deployment level in each t. $S = 0$ implies that there will be no adjustment, that is, change, in State K's actual deployment level over time t.

Having specified for State K its desired deployment goal equation and its stock adjustment equation, I then substitute the former into the latter. I do the same for State X. These substitutions yield a preliminary description of each state's arming behavior. I term these equations, State K and X's constrained goal equations. Mathematically,

$$\mathbf{K}_t - \mathbf{K}_{t-1} = S(B_0 + B_1(E^k_{t-1}\mathbf{X}_t) + \mathbf{U}_t - \mathbf{K}_{t-1}) \tag{5}$$

By multiplying out Equation 5 and setting the result in terms of $\mathbf{K}_t$, we obtain,

$$\mathbf{K}_t = SB_0 + SB_1(E^k_{t-1}\mathbf{X}_t) + (1 - S)\mathbf{K}_{t-1} + S\mathbf{U}_t \tag{6}$$

Equation 6 gives State K's actual deployment level at time period t, $\mathbf{K}_t$, which is a function of its expectation at time t-1 of State X's actual deployment level at time period t, $E^k_{t-1}\mathbf{X}_t$, State K's own deployment level at time t-1, $\mathbf{K}_{t-1}$, and State K's prediction error term $\mathbf{U}_t$. S, State K's stock adjustment coefficient, now refers specifically to State K's political, economic and institutional capability to effect a desired balance, as defined by Equation 1, between its forces and those it expects its rival, State X, to hold in each successive time period over the life span their race. S = 1 would indicate that State K had the economic and institutional capacity to fully adjust its actual deployment level for time t to a level where a desired balance between its own forces and those it expected State X was going to deploy at time t would obtain. If $0 < S < 1$, then that adjustment would only be partial and if S = 0 no such adjustment would occur over time t.

When S = 1, Equation 6 reduces to Equation 1. That is, State K's actual deployment level at time t will equal its desired deployment level for time t. When $0 < S < 1$, the term $(1 - S)\mathbf{K}_{t-1}$, in Equation 6, comes into play. $(1 - S)\mathbf{K}_{t-1}$ is State K's carry-over term. To reiterate, when $0 < S < 1$, State K can only partially adjust its actual deployment level at time t to a level where a balance between its own forces and those it expected State X would deploy at time t would obtain. $(1 - S)\mathbf{K}_{t-1}$ shows the extent to which State K, accordingly, underdeployed forces in time t-1. State K would, thus, need to compensate at each successive time t by factoring into its armament balancing calculus, as given by Equation 6, the amount $(1 - S)\mathbf{K}_{t-1}$.

A solution can, similarly, be found for $\mathbf{X}_t$, State K's actual deployment level. By substitution,

$$\mathbf{X}_t - \mathbf{X}_{t-1} = S'(B'_0 + B'_1(E^x_{t-1}\mathbf{K}_t) + \mathbf{V}_t - \mathbf{X}_{t-1}) \tag{7}$$

Therefore, State B's actual deployment level in time t is given by,

$$\mathbf{X}_t = S'B'_0 + S'B'_1(E^x_{t-1}\mathbf{K}_t) + (1 - S')\mathbf{X}_{t-1} + S'\mathbf{V}_t \tag{8}$$

Equation 8 can be read as Equation 6 was read.

7. FORMALIZING ASSUMPTION 3: FORMING A STRATEGY

Thus State K and X are assumed to each have a particular arming goal, balancing the other's future military capabilities, subject to political, economic and institutional constraints. When developing an arming policy for itself, a state would realize that policy, if it is to be successful, must, to one degree or another, take into account the expected arming goals and strategies of its opponent. The Rational Expectations hypothesis provides a concise and systematic way to model such expectations.

Under the Rational Expectations hypothesis, States K and X may form their expectations of each other's arming strategies thusly. We can assume that State K's defence information set I_{t-1} contains Equations 1 to 8. Similarly let us assume that State X's defence information set I'_{t-1}, also contains Equations 1 to 8. Placing Equations 1 through 8 into each states information set is completely consistent with the Rational Expectations hypothesis. As Willams and McGinnis (1988: 977) note there should be a

> considerable overlap in the two states' information sets. This restriction is justified, indeed required, by the presumption that these two states are locked into a competitive relationship in which each devotes considerable effort to determining the likely behavior of the other. Thus, each should obtain access to essentially the same information to predict each other's military expenditures. For if one state finds a certain piece of information useful in forming expectations of some factor relevant to their joint security competition, even if it relates to domestic constraints, then the other state must necessarily find this same piece of information useful.

Thus State K would find all of the equations in its information set which pertain to State X's behavior useful. State X would, similarly, find the equations which pertain to State K's arming behavior useful. Let us assume further that State K routinely takes measurements, expectations, of those equations pertaining to State X, contained in its defence information set, at time period t-1. Let us, likewise, assume the same with respect to State X. Let E^k_{t-1} denote an expectation made by State K at time period t-1 and let E^x_{t-1} denote an expectation made by State X at time period t-1.

The problem now for State K is that its constrained goal equation contains an unknown variable, $E^k_{t-1}\mathbf{X}_t$, its expectation at time t-1 of State X's actual deployment level at time period t. In order to determine a value for $E^k_{t-1}\mathbf{X}_t$, State K would need to be able to form an expectation as to what State X's arming strategy might be.

$$\mathbf{K}_t = SB_0 + SB_1(E^k_{t-1}\mathbf{X}_t) + (1 - S)\mathbf{K}_{t-1} + S\mathbf{U}_t \qquad (6)$$

State X must, similarly, try to solve for $E^x_{t-1}\mathbf{K}_t$, its expectation at time t-1 of State K's actual deployment level at time period t from Equation 8, its constrained goal equation.

$$\mathbf{X}_t = S'B'_0 + S'B'_1(E^x_{t-1}\mathbf{K}_t) + (1 - S')\mathbf{X}_{t-1} + S'\mathbf{V}_t \qquad (8)$$

Under the Rational expectations hypothesis, State K can obtain a value for $E^k_{t-1}\mathbf{X}_t$ by taking the expectation at time t-1 of Equation 8, State X's constrained goal equation, which is contained in its information set. This course would be an efficient use of information in that Equation 8 contains a great deal of information regarding State X's arming behavior. Accordingly, the Rational Expectations hypothesis would assert that State K's expectation of Equation 8 would be given by

$$E^k_{t-1}\mathbf{X}_t = S'B'_0 + S'B'_1(E^k_{t-1}(E^x_{t-1}\mathbf{K}_t)) + (1 - S')\mathbf{X}_{t-1} +$$

$$S'(E^{k}_{t-1}\mathbf{V}_t) \tag{9}$$

Equation 9 gives State K's counter strategy equation. It gives State K's expectation at time t-1 of State X's actual deployment level at time period t, $E^{k}_{t-1}\mathbf{X}_t$. $E^{k}_{t-1}\mathbf{X}_t$ depends upon $E^{k}_{t-1}(E^{x}_{t-1}\mathbf{K}_t)$, which is State K's expectation at time t-1 of State X's expectation at time t-1 of State K's actual deployment level at time t. $E^{k}_{t-1}\mathbf{X}_t$ also depends upon $\mathbf{X}_{t-1}$, State X's actual deployment level at time t-1 and $E^{k}_{t-1}\mathbf{V}_t$, State K's expectation at time t-1 of State X's prediction error at time t. State X can, similarly, solve for $E^{x}_{t-1}\mathbf{K}_t$ by taking the expectation at time t-1 of Equation 6, State K's constrained goal equation, which is contained in its information set. In this instance, Equation 10 would obtain.

$$E^{x}_{t-1}\mathbf{K}_t = SB_0 + SB_1(E^{x}_{t-1}(E^{k}_{t-1}\mathbf{X}_t)) + (1 - S)\mathbf{K}_{t-1} +$$

$$S(E^{x}_{t-1}\mathbf{U}_t) \tag{10}$$

Equation 10 gives State X's counter strategy equation. Thus State K can directly substitute Equation 9 into 6 and State X can directly substitute Equation 10 into Equation 8. The result of these substitutions is given, respectively, by Equations 11 and 12.

$$\mathbf{K}_t = SB_0 + SB_1S'B'_0 + SB_1S'B'_1(E^{k}_{t-1}(E^{x}_{t-1}\mathbf{K}_t)) +$$

$$SB_1(1 - S')\mathbf{X}_{t-1} + SB_1S'(E^k_{t-1}\mathbf{V}_t) + (1 - S)\mathbf{K}_{t-1} + S\mathbf{U}_t \tag{11}$$

$$\mathbf{X}_t = S'B'_0 + S'B'_1SB_0 + S'B'_1SB_1(E^x_{t-1}(E^k_{t-1}\mathbf{X}_t)) +$$

$$S'B'_1(1 - S)\mathbf{K}_{t-1} + S'B'_1S(E^x_{t-1}\mathbf{U}_t) + (1 - S')\mathbf{X}_{t-1} + S'\mathbf{V}_t \tag{12}$$

Equations 11 and 12 give a preliminary description of State K and X's arming behavior. But they each still contain unknown expectational variables. Equations 11 and 12 can, however, be substantially simplified by referring back to the basic assumptions which underlie the Rational Expectations hypothesis.

First, State K must solve for the term $E^x_{t-1}\mathbf{K}_t$, State X's expectation at time t-1 of State K's actual deployment level for time t, in Equation 11. State K could figure out the structure of State X's counter-strategy equation and use it to solve for $E^x_{t-1}\mathbf{K}_t$. But State X's counter strategy equation contains information which State K has already used since State X's counter strategy equation is simply State X's expectation at time t-1 of State K's constrained goal equation. State K would do better to refer to the Rational Expectations assumption that any expectation will equal the real value in question plus some error term. State K can thus assume that

$$E^{x}_{t-1}\mathbf{K}_t = \mathbf{K}_t + \mathbf{M}_t, \text{ where } E(\mathbf{M}_t) = 0 \tag{13}$$

Equation 13's value to State K is that it is concise and that it does contain new information, namely, the error term $\mathbf{M}_t$. The variable $E^{k}_{t-1}\mathbf{X}_t$, State K's expectation at time t-1 of State X's actual deployment level for time t, from Equation 12 can, similarly, be simplified. State X can assume that

$$E^{k}_{t-1}\mathbf{X}_t = \mathbf{X}_t + \mathbf{N}_t, \text{ where } E(\mathbf{N}_t) = 0 \tag{14}$$

The relationships specified in Equations 13 and 14 can be used to simplify Equations 11 and 12. By substituting Equation 13 into Equation 11 and Equation 14 into Equation 12 we obtain.

$$\mathbf{K}_t = SB_0 + SB_1S'B'_0 + SB_1S'B'_1(E^{k}_{t-1}(\mathbf{K}_t)) +$$

$$SB_1S'B'_1(E^{k}_{t-1}(\mathbf{M}_t)) + SB_1(1 - S')\mathbf{X}_{t-1} +$$

$$SB_1S'(E^{k}_{t-1}\mathbf{V}_t) +$$

$$(1 - S)\mathbf{K}_{t-1} + S\mathbf{U}_t \tag{15}$$

$$\mathbf{X}_t = S'B'_0 + S'B'_1SB_0 + S'B'_1SB_1(E^x_{t-1}(\mathbf{X}_t)) +$$

$$S'B'_1SB_1(E^x_{t-1}(\mathbf{N}_t)) + S'B'_1(1 - S)\mathbf{K}_{t-1} +$$

$$S'B'_1S(E^x_{t-1}\mathbf{U}_t) +$$

$$(1 - S')\mathbf{X}_{t-1} + S'\mathbf{V}_t \tag{16}$$

Equations 15 and 16 can be further simplified. Again, we note that under the Rational Expectations Hypothesis, the term $E^k_{t-1}(\mathbf{K}_t)$, State K's expectation at time t-1 of its own deployment level for time t, from Equation 15, will be that value plus some error term. The same holds true for $E^x_{t-1}(\mathbf{X}_t)$, State X's expectation at time t-1 of its own deployment level for time t, from Equation 16.

$$E^k_{t-1}\mathbf{K}_t = \mathbf{K}_t + \mathbf{P}_t, \text{ where } E(\mathbf{P}_t) = 0 \tag{17}$$

$$E^x_{t-1}\mathbf{X}_t = \mathbf{X}_t + \mathbf{Q}_t, \text{ where } E(\mathbf{Q}_t) = 0 \tag{18}$$

With Equations 17 and 18, Equations 15 and 16 reduce as follows

$$\mathbf{K}_t = SB_0 + SB_1S'B'_0 + SB_1S'B'_1\mathbf{K}_t + SB_1S'B'_1\mathbf{P}_t +$$

$$SB_1S'B'_1(E^k_{t-1}\mathbf{M}_t) + SB_1(1 - S')\mathbf{X}_{t-1} +$$

$$SB_1S'(E^k_{t-1}\mathbf{V}_t) + (1 - S)\mathbf{K}_{t-1} + S\mathbf{U}_t \qquad (19)$$

$$\mathbf{X}_t = S'B'_0 + S'B'_1SB_0 + S'B'_1SB_1\mathbf{X}_t + S'B'_1SB_1\mathbf{Q}_t +$$

$$S'B'_1SB_1(E^x_{t-1}\mathbf{N}_t) + S'B'_1(1 - S)\mathbf{K}_{t-1} +$$

$$S'B'_1S(E^x_{t-1}\mathbf{U}_t) + (1 - S')\mathbf{X}_{t-1} + S'\mathbf{V}_t \qquad (20)$$

Finally, if States K and X set Equations 19 and 20 in terms of $\mathbf{K}_t$ and $\mathbf{X}_t$, respectively, then Equations 21 and 22 would result. Equations 21 and 22 give, finally, my GSR action reaction arms race model (GSR is an acronym for Game, Stock Adjustment, Rational Expectations).

8. A NEW ARMS RACE MODEL

My formalization of Assumptions 1, 2 and 3 ultimately yields the arms race Equations 21 and 22. What is particularly interesting about Equations 21 and 22 is that they are specified solely in terms of observed, measurable variables. State K's actual military armament requirement for time period t becomes a function of its own armaments at time period t-1, the previous period, the armaments of State X at time period t-1 and some error $\mathbf{Y}_t$. State X's actual military armament requirements for time period t are similarly driven. Equations 21 and 22, given immediately below, constitute the final form of my action-reaction arms race model.

$$\mathbf{K}_t = \frac{(SB_0 + SB_1S'B'_0)}{1 - SB_1S'B'_1} + \frac{SB_1(1 - S')}{1 - SB_1S'B'_1}\mathbf{X}_{t-1} + \frac{(1 - S)}{1 - SB_1S'B'_1}\mathbf{K}_{t-1} + \mathbf{Y}_t \tag{21}$$

$$\mathbf{X}_t = \frac{(S'B'_0 + S'B'_1SB_0)}{1 - SB_1S'B'_1} + \frac{S'B'_1(1 - S)}{1 - SB_1S'B'_1}\mathbf{K}_{t-1} + \frac{(1 - S')}{1 - SB_1S'B'_1}\mathbf{X}_{t-1} + \mathbf{Z}_t \tag{22}$$

where $\mathbf{Y}_t$ and $\mathbf{Z}_t$ are error terms which take the form

$$\mathbf{Y}_t = \frac{SB_1S'E^k_{t-1}\mathbf{V}_t + S\mathbf{U}_t + SB_1S'B'_1(\mathbf{P}_t + E^k_{t-1}\mathbf{M}_t)}{1 - SB_1S'B'_1} \qquad (21a)$$

$$\mathbf{Z}_t = \frac{S'B'_1SE^x_{t-1}\mathbf{U}_t + S'\mathbf{V}_t + S'B'_1SB_1(\mathbf{Q}_t + E^x_{t-1}\mathbf{N}_t)}{1 - SB_1S'B'_1} \qquad (22a)$$

The parameters B_0 and B_1 are, respectively, State K's Richardsonian grievance and defence coefficients. S is State K's stock adjustment coefficient. The parameters B'_0, B'_1 and S' are, respectively, State X's Richardsonian grievance, defence and stock adjustment coefficients. In what follows, I will refer to the parameter arrangements $[(SB_0 + SB_1S'B'_0)/1 - SB_1S'B'_1]$ and $[(S'B'_0 + S'B'_1SB_0)/1 - SB_1S'B'_1]$ from Equations 21 and 22 as State K and X's GSR grievance terms. I will refer to the arrangements $[SB_1(1 - S')/1 - SB_1S'B'_1]$ and $[S'B'_1(1 - S)/1 - SB_1S'B'_1]$ as State K and X's GSR defence coefficients. Finally, I will refer to the arrangements $[(1 - S)/1 - SB_1S'B'_1]$ and $[(1 - S')/1 - SB_1S'B'_1]$ as State K and X's GSR stock adjustment terms. (GSR is an acronym for game, stock adjustment, Rational Expectations.)

9. DOES MY GSR MODEL CONFORM TO MY DEFINITION OF AN ARMS RACE?

The structure of my arms race model is determined by how one chooses to specify each state's desired deployment goal. By specifying each state's goal as one of maintaining some balance of forces with its rival in each successive time period, as given by Equations 1 and 2,

$$\mathbf{K}^*_t = B_0 + B_1(E^k_{t-1}\mathbf{X}_t) + \mathbf{U}_t,\ \ B_0, B_1 \in R \tag{1}$$

$$\mathbf{X}^*_t = B'_0 + B'_1(E^x_{t-1}\mathbf{K}_t) + \mathbf{V}_t,\ \ B'_0, B'_1 \in R \tag{2}$$

the final step in my arms race derivation will yield a model which consists of two difference equations which take the form: $\mathbf{K}_t = f(\mathbf{X}_{t-1}, \mathbf{K}_{t-1}, \mathbf{Y}_t)$ and $\mathbf{X}_t = g(\mathbf{K}_{t-1}, \mathbf{X}_{t-1}, \mathbf{Z}_t)$. The structural specification $\mathbf{K}_t = f(\mathbf{X}_{t-1}, \mathbf{K}_{t-1}, \mathbf{Y}_t)$ and $\mathbf{X}_t= g(\mathbf{K}_{t-1}, \mathbf{X}_{t-1}, \mathbf{Z}_t)$ is important insofar as it constitutes an irreducible minimum specification for an arms race as I have defined it.

First, I defined an arms race to be a situation where, at a minimum, each side takes into account its rival's armaments, past, current, or future, when deciding on its own current armament level. In the final step of the derivation of my model, each state takes into account its adversary's past period armament level when calculating its own current armament requirements. Secondly, I stated that for an arms race to be an arms race, it must be driven by the pressure of the military rivalry between the states involved and not by forces exogenous to that rivalry. Operationally, this means that each state's arming behavior must be described solely on the basis of perceived external threat, and not domestic imperatives such as employment. Why then does each state, as posited by my model, take account of is own past period armament level when calculating its future armament requirements? In Richardson's model, the fact that a nation considers its own previous armament level in determining its future armament level has less to do with pressures emanating from its rivalry than with pressures emanating from within itself, namely, its own economy and its own institutions. Richardson believed that a state's own previous armament expenditures would have a dampening effect on its future expenditure levels insofar as those previous armaments had drained resources from its economy. In fact, however, the opposite can also be true. Previous expenditures can spur on future expenditures for purely domestic reasons such as the need to maintain employment levels. Neither of these instances would, however, mesh with my definition of what an arms race is. What I have done is to alter, radically, the nature of the impact that a state's own previous armament levels has on its future armament calculus through my use of the stock adjustment hypothesis. Through the stock adjustment hypothesis, each state takes account of its own previous period deployment level when calculating its future armament requirements only to the extent that its previous period armament level was below the level necessary to achieve its goal of maintaining a desired balance of forces between itself and its rival in that period. Thus the structure of my model is consistent with my definition of an arms race.

10. EQUILIBRIUM PROPERTIES

Loosely speaking, the term arms race equilibrium refers to a point, if it exists, of mutually agreeable arms balance between two rivals engaged in an arms race, though it need not necessarily refer to the occurrence of one-to-one balance. Equilibrium points can be stable or unstable. If, after having reached equiiibrium, two adversaries return to it whenever some exogenous shock causes them to move to a different armament level, then that equilibrium point can be said to be stable. Otherwise, such a point would be said to be unstable. In this section, I will set out the equilibrium and stability-instability conditions for my GSR model.

Equilibrium analysis has long held an important place in mathematical arms race analysis. Richardson (1951, 1960a) contended, though never systematically demonstrated, that arms races, particularly unstable ones, lead to war. A great deal of energy has been expended in determining the validity of that position (Caspary, 1967; Saaty, 1964; Gray, 1971, 1973, 1974, 1975, 1976; Smoker, 1965; Wohlstetter, 1974; Lambelet, 1971, 1975; Wallace, 1979, 1980a, 1980b, 1980c, 1982; Weede, 1980; Smith, 1980; Siverson and Diehl, 1990; Diehl, 1983; Altfeld, 1983; Morrow, 1989). Still, the question has not yet been settled. Leidy and Staiger (1985) suggest three other practical reasons why arms race equilibrium analysis is important. First, the existence of a stable equilibrium point for a given arms race suggests that the race will have a finite duration. This in turn suggests that there will be a bound on the drain of resources expended in executing the race. Second, the atmosphere attending a race which has a stable equilibrium point might be less tense, making it easier for the participants to negotiate their way out of the race. Thirdly, the character of an arms race can be altered by changing its parameters. One could, for example, suggest through equilibrium analysis how to turn an unstable race into a stable race.

Under what conditions, then, will State K and X's arms race behavior, as described by Equations 21 and 22, tend toward equilibrium? Under what conditions will that equilibrium be stable or unstable? I will begin by rewriting Equations 21 and 22 in a simplified form. This form will make easier the task of calculating equilibrium and stability conditions.

$$\mathbf{K}_t = L_0 + L_1\mathbf{X}_{t-1} + L_2\mathbf{K}_{t-1} + \mathbf{Y}_t \tag{23}$$

$$\mathbf{X}_t = L'_0 + L'_1\mathbf{K}_{t-1} + L'_2\mathbf{X}_{t-1} + \mathbf{Z}_t \tag{24}$$

Equation 23 corresponds to Equation 21 and Equation 24 corresponds to Equation 22. L_0 and L'_0 represent, respectively, State K and X's GSR grievance terms, L_1 and L'_1 State K and X's GSR defence terms and L_2 and L'_2 represent State K and X's GSR stock adjustment terms. In my model, equilibrium occurs when the difference between State K's actual deployment level at time t and State K's actual deployment level at time t-1 and State X's actual deployment level at time t and State X's actual deployment level at time t-1 is, simultaneously, zero. Mathematically

$$\mathbf{K}_t - \mathbf{K}_{t-1} = 0 \tag{25}$$

$$\mathbf{X}_t - \mathbf{X}_{t-1} = 0 \tag{26}$$

Thus, if we disregard the error terms $\mathbf{Y}_t$ and $\mathbf{Z}_t$, we can rewrite Equations 23 and 24 as follows:

$$\mathbf{K}_t - \mathbf{K}_{t-1} = L_0 + L_1\mathbf{X}_{t-1} + L_2\mathbf{K}_{t-1} - \mathbf{K}_{t-1} = 0 \tag{27}$$

$$\mathbf{X}_t - \mathbf{X}_{t-1} = L'_0 + L'_1\mathbf{K}_{t-1} + L'_2\mathbf{X}_{t-1} - \mathbf{X}_{t-1} = 0 \tag{28}$$

Equilibrium in my arms race model, then, occurs at the point

$$\mathbf{K}^e_{t-1} = \frac{L_0(1 - L'_2) + L_1L'_0}{(1 - L_2)(1 - L'_2) - L'_1L_1} \tag{29}$$

$$\mathbf{X}^e_{t-1} = \frac{L'_0(1 - L_2) + L'_1L_0}{(1 - L_2)(1 - L'_2) - L'_1L_1} \tag{30}$$

Under what conditions will the arms race equilibrium point ($\mathbf{K}^e_{t-1}$, $\mathbf{X}^e_{t-1}$) be stable? Two methods can be applied here, the graphical method and the characteristic root method. First, from Equations 27 and 28, we can obtain the following demarcation curves. Equation 31 represents the locus of points in the ($\mathbf{K}_{t-1}$, $\mathbf{X}_{t-1}$) space where the condition $\mathbf{K}_t - \mathbf{K}_{t-1} = 0$ is satisfied. Equation 32, similarly, represents the locus of points in the ($\mathbf{K}_{t-1}$, $\mathbf{X}_{t-1}$) space where the condition $\mathbf{X}_t - \mathbf{X}_{t-1} = 0$ is satisfied.

$$\mathbf{X}_{t-1} = -\frac{L_0}{L_1} - \frac{(L_2 - 1)}{L_1}\mathbf{K}_{t-1} \tag{31}$$

$$\mathbf{X}_{t-1} = \frac{L'_0}{(1 - L'_2)} + \frac{L'_1}{(1 - L'_2)}\mathbf{K}_{t-1} \tag{32}$$

What, then, are the arms race stability conditions for my model as implied by Equations 31 and 32? It should, first of all, be noted that the parameters L_0, L_1, L_2, L'_0, L'_1 and L'_2 can take on a wide range of values, from negative to positive. This means that there is a large number of possible stability conditions. The equilibrium point ($\mathbf{K}^e_{t-1}$, $\mathbf{X}^e_{t-1}$) can be graphically determined to be stable or unstable by plotting Equations 31 and 32 and determining their phase trajectories as was done in Chapter 1 using Richardson's model.

Basic examples of a stable and an unstable race are given below in Graphs 4.1 and 4.2. In Graph 4.1, stability obtains: $(L_2 - 1)/L_1 < 0$ and $L'_1/(1 - L'_2) > 0$ where $|(L_2 - 1)/L_1| > |L'_1/(1 - L'_2)|$. Here, the dynamics of State K and X's arms race are such that both side's armaments tend nearly directly toward their race's equilibrium point. Graph 4.2 shows an unstable arms race: $(L_2 - 1)/L_1 < 0$ and $L'_1/(1- L'_2) > 0$ where $|(L_2 - 1)/L_1| < |L'_1/(1 - L'_2)|$. In this instance, States K and X arm in a way which always leads them away from their race's equilibrium point.

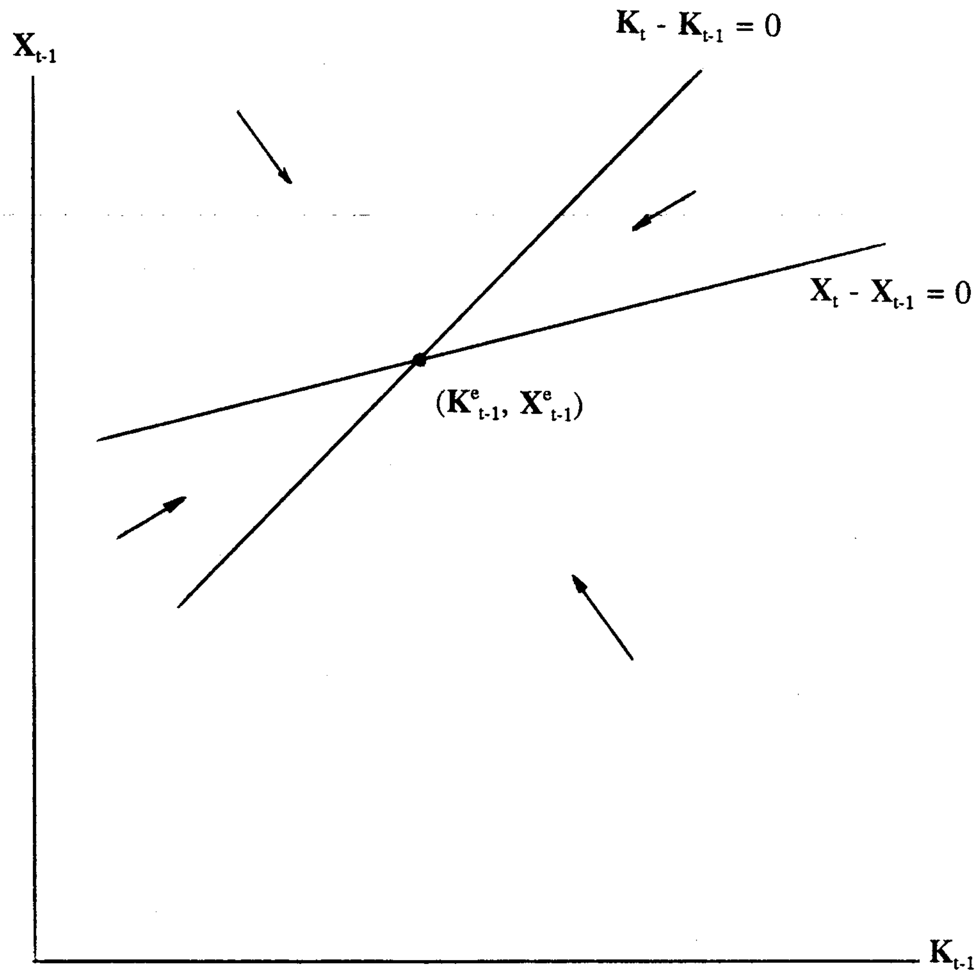
Alternatively, one could determine the equilibrium characteristics of an arms race by calculating the Eigen values or characteristic roots, A_1 and A_2, of the characteristic equation for Equations 23 and 24. A_1 and A_2 are as follows.

$$A_1 = \frac{(L_1 + L'_1) + [(L_1)^2 - 2L_1L'_1 + 4L_2L'_2 + (L'_1)^2]^{1/2}}{2} \tag{33}$$

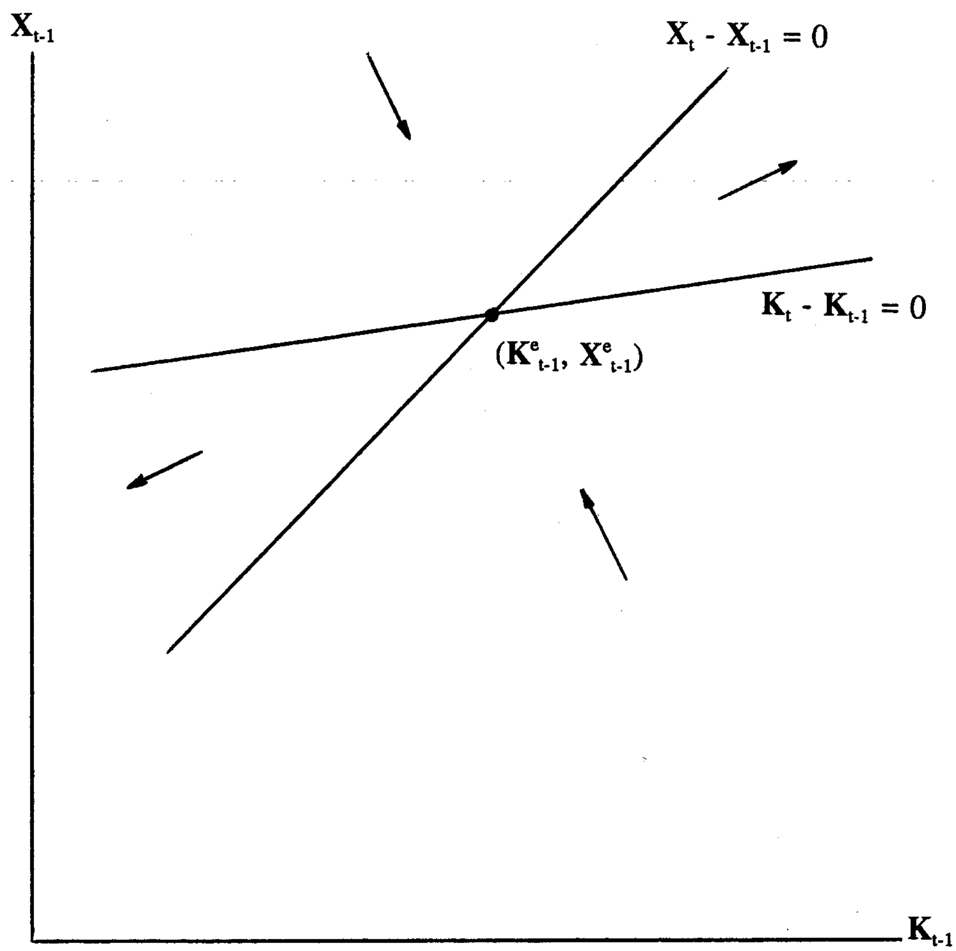
$$A_2 = \frac{(L_1 + L'_1) - [(L_1)^2 - 2L_1L'_1 + 4L_2L'_2 + (L'_1)^2]^{1/2}}{2} \tag{34}$$

In the case where estimates of L_1, L_2, L'_1 and L'_2 yield values for A_1 and A_2 which are both less than unity in absolute value, the arms race in question whose parameters are represented by L_1, L_2, L'_1 and L'_2 can be said to be stable. If either A_1 or A_2 or both are

GRAPH 4.1: Stable equilibrium in the GSR model



GRAPH 4.2: Unstable equilibrium in the GSR model



greater than unity in absolute value, then the race in question has an unstable equilibrium point.

The conditions for the existence of a stable equilibrium point can be further refined. Smith (1980) argued that if an arms race did have a stable equilibrium point, the practical impact on the outcome of a race (e.g., war-no war) may be no more different than if the race had been unstable. Specifically, she drew a distinction between interesting and uninterestingly stable arms race equilibria. She defined an interestingly stable arms race as one where its solution, 90 percent of its equilibrium value, is realized within the lifespan of the race. For example, she estimated that the Russo-Japanese race which lasted for 18 years would not have approached equilibrium, given its dynamics, for about 90 years after its start. In this context, the fact that the Russo-Japanese race had a stable equilibrium point has little meaning. An uninterestingly stable race will likely occur when equilibrium occurs at some extraordinarily high armament level (as compared to the levels which obtained at the start of a race). The time it takes for a race to approach 90 percent of its equilibrium solution, Smith states, can be calculated using the equation

$$t = \log_{10}(0.1)/\log_{10}(A)$$

where A is the largest of the characteristic roots A_1 or A_2.

In this chapter, I have derived a new action-reaction arms race model based on assumptions taken from game theory, Rational Expectations theory and stock adjustment theory. This model was, further, derived under assumption that states compete over individual weapons systems with cross-purposes. It is, therefore, a micro arms race model and as such constitutes a key additional component of the micro approach to mathematical arms race analysis which I have been developing in this thesis. Having derived my GSR model and having demonstrated its equilibrium properties, I will, in the next chapter, move on to discuss four testable propositions concerning the arms race behavior of states which derive from my model. In the chapter which follows that, I will apply my micro approach to mathematical arms race analysis to the US-Soviet arms race where I have been able to uncover strong evidence of action-reaction interaction in the US-Soviet strategic nuclear warhead competition.

CHAPTER V: DERIVING TESTABLE PROPOSITIONS FROM THE GSR MODEL

In the last chapter, I derived my GSR action-reaction arms race model. My model specifies optimal arming behaviors to be followed by States K and X, two arms race participants, given that each state has the goal of maintaining a balance of forces with the other over each successive time period over the lifespan of their race, given that each state's arming goal must be subject to economic constraints, and given that each state takes account of the arming strategy of the other. State K's actual deployment level for time period t becomes a function of its own deployment level at time period t-1, the previous period, State X's deployment level at time period t-1, and an error $\mathbf{Y}_t$. State X's actual deployment level for time period t is similarly driven. My model is, accordingly, structurally identical to Richardson's action-reaction arms race model. Given immediately below is the final form of my action-reaction arms race model

$$\mathbf{K}_t = \frac{(SB_0 + SB_1S'B'_0)}{1 - SB_1S'B'_1} + \frac{SB_1(1 - S')}{1 - SB_1S'B'_1}\mathbf{X}_{t-1} + \frac{(1 - S)}{1 - SB_1S'B'_1}\mathbf{K}_{t-1} + \mathbf{Y}_t \qquad (21)$$

$$\mathbf{X}_t = \frac{(S'B'_0 + S'B'_1SB_0)}{1 - SB_1S'B'_1} + \frac{S'B'_1(1 - S)}{1 - SB_1S'B'_1}\mathbf{K}_{t-1} + \frac{(1 - S')}{1 - SB_1S'B'_1}\mathbf{X}_{t-1} + \mathbf{Z}_t \tag{22}$$

where $\mathbf{Y}_t$ and $\mathbf{Z}_t$ are error terms

$$\mathbf{Y}_t = \frac{SB_1S'E^k_{t-1}\mathbf{V}_t + S\mathbf{U}_t + SB_1S'B'_1(\mathbf{P}_t + E^k_{t-1}\mathbf{M}_t)}{1 - SB_1S'B'_1} \tag{21a}$$

$$\mathbf{Z}_t = \frac{S'B'_1SE^x_{t-1}\mathbf{U}_t + S'\mathbf{V}_t + S'B'_1SB_1(\mathbf{Q}_t + E^x_{t-1}\mathbf{N}_t)}{1 - SB_1S'B'_1} \tag{22a}$$

The parameters B_0 and B_1 are, respectively, State K's Richardsonian grievance and defence coefficients. S is State K's stock adjustment coefficient. The parameters B'Corline Scott B_0, B'_1 are, respectively, State X's Richardsonian grievance and defence coefficients. S' is State X's stock adjustment coefficient. $(SB_0 + SB_1S'B'_0)/1 - SB_1S'B'_1$ from Equation 21 is State K's GSR grievance term. Moreover, from Equation 21, $SB_1(1 - S')/1 - SB_1S'B'_1$ is State K's GSR defence term and $(1 - S)/1 - SB_1S'B'_1$ is State K's GSR economic term. From Equation 22, $(S'B'_0 + S'B'_1SB_0)/1 - SB_1S'B'_1$ is State X's GSR grievance term, $S'B'_1(1 - S)/1 - SB_1S'B'_1$ is State X's GSR defence term and $(1 - S')/1 - SB_1S'B'_1$ is State X's GSR economic term.

In this chapter, I will continue the modelling process sequence by discussing four main propositions concerning the arming behavior of states which derive from that model. Firstly, I will derive testable propositions which follow from my application of the Rational Expectations hypothesis to arms race modelling. These propositions concern the extent to which arms race actors, in given empirical context, do form their expectations of their armament requirements in accordance with the dictates of the Rational Expectations hypothesis. I will then determine the conditions under which State K and X's arms competition will manifest itself in the form of an action-reaction competition and under what conditions it will manifest asymmetric reaction or non-interaction. Finally, I will discuss the procedure to be followed in estimating my GSR model.

1. DERIVING TESTABLE PROPOSITIONS: RATIONAL EXPECTATIONS AND ARMS RACE MODELLING.

A principal assumption underlying my GSR arms race model is that states form expectations of each other's arming goals and strategies and use those expectations in determining their own force requirements. Most importantly, my model is based on the assumption that states form these expectations in accordance with the Rational Expectations hypothesis. How valid is that assumption? Can its validity be empirically established in a specific arms race context? In their premiere work with the Rational Expectations hypothesis and the US-Soviet military competition, Williams and McGinnis (1988) suggested that if the

US and Soviet Union do calculate their military expenditure requirements in accordance with the Rational Expectations hypothesis, then certain implications or propositions can be derived regarding the dynamics of those expenditures over time. The validity of using Rational Expectations in arms race modelling could be assessed by analyzing US and Soviet military expenditure series and determining if those propositions hold.

Williams and McGinnis' (1988) case has already been set out in an early chapter, but essentially, they argued that each side in a dyad would base its military expenditure calculations on what it expected the other would be expending, on its expected domestic economic fatigue, and on some prediction error. These expectations would be based on information which both held in common (a basic assumption of the Rational Expectations hypothesis). Suppose, then, argue Williams and McGinnis, that some exogenous shock caused State K to deploy armaments at a level which fell below the level it calculated that it would deploy for time t. This would translate into a negative error in State K's actual deployment level at time t. If State J were vigilant in collecting and analyzing data on State K's arming behavior, as the Rational Expectations hypothesis would assert, then State J would have become aware of this upcoming deviation between State K's planned deployment level for time t and State K's actual deployment at time t. State J should take this as a decrease, albeit temporary, in the security threat posed to it by State K. Accordingly, State J should lower its deployment level for time t below what it had planned. This would lead to a negative error in State J's actual deployment level at time t. The reverse of this case would also be true if

States K and J did form the expectations in accordance with the Rational Expectations hypothesis. If State J were highly efficient in transforming newly obtained information on exogenous shocks in State K's arming behavior into changes in its own behavior and if State K were likewise efficient, then one should expect to find a strong contemporaneous cross correlation in State K and State J's prediction errors. This is a key proposition in Williams and McGinnis and it did hold using US-Soviet military expenditure data over the years 1954-1987 (using a simple distributed lag model).

My GSR model, similarly, suggests that if States K and X are highly efficient in altering their arming behavior on the basis of new information on exogenous shocks in the other's arming behavior, then one should find a high contemporaneous cross correlation in their prediction errors $\mathbf{Y}_t$ and $\mathbf{Z}_t$. In an empirical context, it would be necessary then to determine if the following proposition holds or not in order for one to be able to conclude that States K and X do indeed form their expectations of each other's arming behavior in accordance with the Rational Expectations Hypothesis.

PROPOSITION 1.0: If States K and X do in fact form their expectations of each other's arming behavior in accordance with the Rational Expectations hypothesis, then the errors in State K and X's arming behavior, as modelled by Equations 21a and 22a, should be strongly correlated. Mathematically $\text{corr}(\mathbf{Y}_t, \mathbf{Z}_t) \neq 0$. This result, it must be noted, is conditional upon the information contained in State K and X's defence information sets.

Proposition 1.0, to reiterate, is identical to the one advanced by Williams and McGinnis (1988) in their study of Rational Expectations and reaction in the US-Soviet arms race. But a high contemporaneous cross correlation in the prediction errors of rival states, they correctly note, may also result if rivals form their expectations adaptively. An adaptive expectations process could generate such a correlation if expectations formed under that process equaled expectations formed under the Rational Expectations process. Thus, the result corr($\mathbf{Y}_t$, $\mathbf{Z}_t$) does not equal zero is only a causally necessary, but not sufficient, condition for Rational Expectations formation in the armament processes of rival states.

Secondly, Williams and McGinnis argued that prediction errors drive an arms race and not absolute expenditure levels. Williams and McGinnis (1988: 980) predicted that "In an arms race under Rational Expectations, neither military expenditure series will Granger cause the other." That is, a knowledge of State K's military expenditure series will not help one to predict State J's expenditure series and vice-versa. Both this prediction and the prediction concerning State K and J's error terms were borne out in Williams and McGinnis. They note, however, that

> systematic delay in observing or acting upon previously available information [regarding an opponent's error term] may have the consequence that one series does indeed help to improve predictions of the other series (p. 981).

More specifically, they argue, a Rational Expectations arms race model would allow

> for the possibility that one state may successfully keep some information secret and that the time needed to develop new weapons or to overcome bureaucratic inertia may delay any response to a change in expectations. All of these factors may be interpreted as 'inefficiencies' in the transformation of new information into changed behavior .... If one state manages to keep some relevant information secret, the rival's reaction to this specific event may be delayed, but the overall correlation [in prediction errors] would be eliminated only in the presence of a continuing pattern of successful deception or concealment. ... If bureaucratic or technical inefficiencies delay each state's response to new information approximately the same length of time, then their joint reaction would be distributed across adjacent time periods, reducing the size of any particular year's [error] correlation.... Only if there is a pronounced asymmetry in the two state's adjustment times would contemporaneous residuals no longer be correlated, whereas lagged ones would be (p. 981).

Accordingly, a companion proposition to Proposition could be specified as follows

PROPOSITION 1.1: If States K and X do in fact form their expectations of each other's arming behavior in accordance with the Rational Expectations hypothesis, but if there are substantial inefficiencies in the transmission and transformation of information between States K and X, then the errors in State K and X's arming behavior, as modelled by Equations 21a and 22a, should show a weak contemporaneous correlation or a strong non-contemporaneous correlation. Mathematically $\mathrm{corr}(\mathbf{Y}_t, \mathbf{Z}_t) = 0$ or $\mathrm{corr}(\mathbf{Y}_{t-i}, \mathbf{Z}_{t-j}) \neq 0$, where i and $j > 0$ and $i = j$ or $i \neq j$. This result is conditional upon the information contained in State K and X's defence information sets.

Proposition 1.1, like Proposition 1.0, must be qualified. A very weak contemporaneous, or a strong non-contemporaneous cross correlation in State K and X's errors is a causally necessary, but not sufficient, condition for Rational Expecations. Such correlations would also be consistent with an adaptive expectations formation process. Inefficiency in the transformation of new information into new or changed behavior is consistent with both Rational Expectations formation and adaptive expectations formation. But this fact would not eliminate the key theoretical distinction between Rational and adaptive expections formation processes, namely, that an actor's prediction errors would be unbiased under Rational Expectations and biased under adaptive expectations.

In the instance where States K and X do experience inefficiencies in the transmission and transformation of information about the other, then, as Williams and McGinnis suggest, a knowledge of one state's military deployment series should help to predict the other's deployment series. In the next sections, I will spell out, in detail, the predictions my model makes with respect to interaction between State K and X's deployment series.

2. DERIVING TESTABLE PROPOSITIONS: THE CAUSE OF THE ASYMMETRIC ARMS RACE

Ashley (1980), Freeman (1983), Majeski (1985), Majeski and Jones (1981), and Williams and McGinnis (1988) have studied the asymmetric outcome. Ashley (1980), Majeski (1985), and Majeski and Jones' (1981) studies of the USA-USSR military expenditure competition indicate that the US and Soviets are engaged in an asymmetric arms race. American military expenditures, at time period t, show up as a function of Soviet military expenditures at time t-1 and US military expenditures at time t-1, just as the classical Richardson model would predict. The Soviets, on the other hand, do not seem to be responding to American military expenditures. Soviet military expenditures at time period t have been found to be a function of Soviet military expenditures at time t-1 only. The US-Soviet arms competition is defined here, then, by a zero defence coefficient for the Soviet Union and a non-zero defence coefficient for the US.

When Majeski (1985) found statistical evidence that the US and USSR were engaged in an asymmetric arms race, he adopted Ashley's (1980) explanation. Ashley had himself found a similar result. Ashley writes: the asymmetry is

> the result of an 'ordered relationship.' The United States is a more rapidly growing society with greater resources and latitude to invest in military capability to respond to the activities of others than the Soviet Union. ... [T]he Soviet Union has sought to extract the maximum resources from its economy to build its military instruments to overcome obstacles and threats posed not so much by U.S. arms but by 'sustained American lateral pressure.' Having already committed their resources to the fullest to meet the external threat, the Soviet Union has little latitude to react to changes in U.S. capabilities [Majeski, 1985: 239].

Ashley's reasoning can, in fact, be formalized through my GSR model.

If, accordingly, we take the United States to be State K we could in fact assign its coefficient of adjustment (S) the maximum value, namely, one. The coefficient of adjustment, to reiterate, is a measure of institutional or technical rigidities, or cost of change. America seems little constrained. But what about the Soviet Union, State X? Its coefficient of adjustment (S') should, accordingly, be assigned a value of less than one, but more than zero. The Soviets seem to be facing, at a minimum, institutional rigidities which prevent it from mirroring US deployments. High opportunity costs would seem, too, to be a factor pushing the Soviet Union's coefficient of adjustment downward. In short, these value assignments, S = 1 and 0 < S' < 1, if substituted into Equations 21 and 22, will yield an asymmetric arms race outcome. Given, now, is the resultant, Equations 33 and 34, an asymmetric arms race outcome.

$$\mathbf{K}_t = \frac{(B_0 + B_1S'B'_0)}{1-B_1S'B'_1} + \frac{B_1(1 - S')}{1-B_1S'B'_1}\mathbf{X}_{t-1} + \mathbf{YY}_t \tag{33}$$

$$\mathbf{X}_t = \frac{(S'B'_0 + S'B'_1B_0)}{1-B_1S'B'_1} + \frac{(1 - S')}{1-B_1S'B'_1}\mathbf{X}_{t-1} + \mathbf{ZZ}_t \tag{34}$$

where $\mathbf{YY}_t$ and $\mathbf{ZZ}_t$ are error terms.

$$\mathbf{YY}_t = \frac{B_1S'E^k_{t-1}\mathbf{V}_t + \mathbf{U}_t + B_1S'B'_1(\mathbf{P}_t + E^k_{t-1}\mathbf{M}_t)}{1-B_1S'B'_1} \tag{33a}$$

$$\mathbf{ZZ}_t = \frac{S'B'_1E^x_{t-1}\mathbf{U}_t + S'\mathbf{V}_t + S'B'_1B_1(\mathbf{Q}_t + E^x_{t-1}\mathbf{N}_t)}{1-B_1S'B'_1} \tag{34a}$$

Clearly this result conforms with Ashley (1980). A deeper cause for asymmetric reaction is, however, suggested by the Rational Expectations hypothesis.

Consider that my GSR model, Equations 21 and 22, was derived on the basis that each of the states in question, States K and X, each formed an informed expectation of the other's arming goal and arming strategy and used that expectation in coming to a final determination as to what its time period t armament requirements should be, as given by Equation 21 for State K and Equation 22 State X. Under the Rational Expectations hypothesis, State K would know what the values of State X's defence policy parameters, B'_0, B'_1 and S' are. Similarly, State X should know the values of State K's defence policy parameters, B_0, B_1 and S are (this information should be contained in their defence information sets). If State X knows that S, State K's stock adjustment coefficient is equal to one, and that its own lies between zero and one, then State X would know that State K should have no carry over term in its GSR

Equation 21. That is, the term $(1 - S)/1 - SB_1S'B'_1$ times State K's own previous period deployment level, $\mathbf{K}_{t-1}$, would disappear from Equation 21. State K would, under $S = 1$, $0 < S' < 1$, only take account of State X's previous period deployment level when determining its current deployment needs. Indeed State X should know that if $S = 1$ and $0 < S' < 1$, then State K's GSR Equation 21 should break down to Equation 33.

Since State X's GSR Equation 22 was derived on the basis that it had formed and used an accurate expectation of State K's arming goal, and arming strategy, combined with a knowledge of the values of State K's defence policy parameters, it follows then that State X's GSR Equation 22 should break down to Equation 34. If State K is not taking account of its own previous period armament level when determining its current period requirement, then why should State X If State K is only taking account of State X's previous period armament level when determining its current period requirement, then State X must use that information, and attempt to balance State K's current period armament level on that basis.

This accounting of asymmetric reaction, I will show, directly contradicts the work done by Williams and McGinnis (1988: 990). They consider one-way or asymmetric arms race formulations to be of rather dubious validity.

> fundamental deficienc[y] in any undirectional causation formulation. Although an international dispute may indeed have been originally caused by one state's aggressive intentions or behavior, once the target state has learned of the other's continued animosity and has decided to resist its advances, then both

> should form and act upon their expectations of each other's future behavior. Any attribution of most of the contemporaneous correlation to one side or the other must be justified by reference to a persistent asymmetry in their respective capabilities to form and act upon expectations of the other's behavior, such as a Soviet inability to predict the outcomes of the complex U.S. domestic policy process or the inability of the U.S. analysts to penetrate the veil of secrecy surrounding Soviet decision making.

Ironically, what I have shown with my Rational Expectations arms race model is that one-way causation or asymmetric reaction can indeed come about as a consequence of both sides forming and acting "upon their expectations of each other's future behavior." But most importantly, I have shown that, under the Rational Expectations hypothesis, asymmetric reaction can occur even when their is no asymmetry in "their respective capabilities to form and act upon expectations of the other's behavior ...." Indeed asymmetric reaction can occur just because each side in an arms race is forming efficient expectations of the other's arming behavior.

Can an asymmetric arms race really be called an arms race? Wallace (1979: 5) writes:

> at a minimum, we can only speak of an 'arms race' between nations whose foreign and defense policies are heavily interdependent; the behavior and capabilities of each nation must be highly salient to the other nations.

In Equations 33 and 34, we see the possibility that asymmetric arms races can be highly interactive affairs. Consider, accordingly, that Equation 34, which gives States X's actual deployment level at time t, contains B_1, State K's defence coefficient and B_0, State K's grievance term. This suggests that State X's own deployment behavior is affected by State K's defence policy parameters. Equation 33 which gives State's K's actual deployment level at time t, similarly, depends upon State X's defence policy parameters. More specifically, it depends upon State X's grievance term B'_0 and State X's defence coefficient B'_1.

This is important to note because the term asymmetric reaction suggests that one side is reacting and the other is not. Indeed, Equation 34 shows that $\mathbf{X}_t$, State X's actual deployment level at time t, can be a function of B'_1, State X's own defence coefficient even though it is not taking explicit account of State K's past period armament level when calculating its deployment level for time t. And, as one might expect, as the value of B'_1 increases, the value of $\mathbf{X}_t$ increases. This is a significantly new way to look at asymmetric arms races. The traditional Richardsonian explanation of why asymmetric arms races occur involved reasoning that State X's defence coefficient was zero. This was never really very convincing especially when two very antagonistic states were the object of the analysis. From this analysis, the following proposition emerges.

PROPOSITION 2.0: If, for any given arms race dyad, estimates of Equations 21 and 22 show an asymmetric arms race, that outcome will be due the conditions summarized by either $S = 1$, $0 < S' < 1$ and B_1, B'_1 do not equal zero or $S' = 1$, $0 < S < 1$ and B_1, B'_1 do not equal zero.

It should also be noted that my GSR model will show an asymmetric arms race obtains if, for any pair of states, one state's Richardsonian defence coefficient is zero, the other's is non-zero and both have stock adjustment coefficients which lie between zero and one. If, for example, $B_1 = 0$, B'_1 does not equal zero and $0 < S,S' < 1$, then my GSR model breaks down as follows

$$\mathbf{K}_t = SB_0 + (1 - S)\mathbf{K}_{t-1} + e_t \tag{21b}$$

$$\mathbf{X}_t = (S'B'_1SB_0) + S'B'_1(1 - S)\mathbf{K}_{t-1} + (1 - S')\mathbf{X}_{t-1} + \ell_t \tag{22b}$$

Equation 21b shows that State K's reaction to State X is effected only through its Richardsonian grievance term B_0. That is, State K's reaction to State X is constant despite variations in State X's armament level. State X's reaction to State K's, in contrast, is variable. State X's armament decision calculus at time t, as given by Equation 22b, depends, inter alia, on State K's previous period armament deployment level. A companion proposition to Proposition 2.0, then, can be put as follows.

PROPOSITION 2.1: If, for any given arms race dyad, estimates of Equations 21 and 22 show an asymmetric arms race, that outcome will be due the conditions summarized by 0 < S, S'

< 1 and either $B_1 = 0$ and B'_1 does not equal zero, or B_1 does not equal zero and $B'_1 = 0$.

3. DERIVING TESTABLE PROPOSITIONS: CONDITIONS FOR AN ACTION-REACTION ARMS RACE

My GSR model specifies that States K and X will engage each other in an action-reaction arms race only under certain conditions.

PROPOSITION 3.0 If, for any given arms race dyad, estimates of Equations 21 and 22 show an action-reaction arms race, that outcome will be due the conditions summarized by $0 < S, S' < 1$ and B_1, B'_1 do not equal zero.

That is, each side must have a non-zero Richardsonian defence coefficient and each side must have a stock adjustment coefficient which is greater than zero but less than one in order for an action-reaction arms race to occur between States K and X. Why? Under the Rational Expectations hypothesis, State K would know that State X's Richardsonian defence coefficient is non-zero and that its stock adjustment coefficient lies between zero and one. State X would, similarly, know that State K's Richardsonian defence coefficient is non-zero and that its stock adjustment coefficient lies between zero and one. This information would be contained in their defence information sets.

State K thus could draw two basic conclusions regarding State X's arming behavior. First, State K would have to conclude that in an effort to realize its arming goal of maintaining a balance between its own forces at time t and those it expects that State K will deploy at time t, State X will take State K's previous period deployment level into account when calculating its own time t period deployment level (since $S'B'_1(1 - S)/1 - SB_1S'B'_1$ times $\mathbf{K}_{t-1}$). But, because State X's stock adjustment term is less than one (but greater than zero), State K would also have to conclude that State X would not be able to adjust fully its actual deployment level at time t to a level where balance between its own forces at time t and those it expected State K's was going to deploy at time t obtained. State X would have to make up for the shortfall in each successive time period t through its carry over term in the amount of $[(1 - S')/1 - SB_1S'B'_1]$ times its previous deployment level $\mathbf{X}_{t-1}$. That is, in addition to taking into account State K's previous period deployment level when calculating its own current period deployment needs, State X would also take into account its own previous period period deployment level. State X would come to the same sorts of conclusions with respect to State K. Because each state would take into account the other's arming calculus when determining its own armament requirements under Rational Expectations, the net effect will be an action-reaction arms race.

4. DERIVING TESTABLE PROPOSITIONS: THE CAUSE OF THE NON-INTERACTION OUTCOME.

A special case of arming behavior occurs when $S = S' = 0$, that is, when both State K and State X have zero stock adjustment coefficients. It happens that when $S = S' = 0$, the form of interaction posited by my GSR model between States K and X will break-down from action-reaction interaction, as given by Equations 21 and 22, to non-interaction, as given by Equations 37 and 38 below.

$$\mathbf{K}_t = (1)\mathbf{K}_{t-1} \tag{37}$$

$$\mathbf{X}_t = (1)\mathbf{X}_{t-1} \tag{38}$$

The significance of Equations 37 and 38 becomes more apparent when they are rewritten as follows

$$\mathbf{K}_t - \mathbf{K}_{t-1} = 0 \tag{37a}$$

$$\mathbf{X}_t - \mathbf{X}_{t-1} = 0 \tag{38a}$$

Equations 37a and 38a indicate that when both states have zero stock adjustment coefficients, no change occurs from time t-1 to time t in either side's deployment level. In effect, there is no arms race. However, it is interesting to note that Equations 21 and 22 will break down to Equations 37 and 38 when S = S' = 0 irrespective of the values of State K's defence and grievance terms and State X's defence and grievance terms. That is, the form of arms race interaction in my GSR model can break down from action-reaction to non-interaction even if States K and X have non-zero defence coefficients, that is, even States K and X do fear each other's military deployments.

This property of my GSR model can be used to shed some light on results obtained in previous research on the US-Soviet arms race. Mathematical arms race analysis began with Richardson's (1960a) linear action reaction arms race model. His model is given immediately below, specified in difference equation form.

$$\mathbf{X}_t = k\mathbf{Y}_{t-1} + (1 - a)\mathbf{X}_{t-1} + g \tag{R28}$$

$$\mathbf{Y}_t = l\mathbf{Y}_{t-1} + (1 - b)\mathbf{Y}_{t-1} + h \tag{R29}$$

In Richardson's model, States X and Y will engage each other in an action-reaction arms race if k and l, respectively, State X and Y's defence coefficients, are both non-zero. If, for any given arms race, k and l take on values of zero, then Richardson's model breaks down to an organizational process model, as shown immediately below.

$$\mathbf{X}_t = (1 - a)\mathbf{X}_{t-1} + g \tag{R30}$$

$$\mathbf{Y}_t = (1 - b)\mathbf{Y}_{t-1} + h \tag{R31}$$

In this instance, State X and Y's military expenditures are internally driven. It is interesting to note that most studies aimed at estimating the parameters of Richardson's model for the US-Soviet arms race (see citations in Chapter 2) suggest that both the US and Soviet defence coefficients are zero, indeed, that their military expenditures are internally driven. Given the longstanding political, economic and military rivalry between the US and Soviet Union, this conclusion is, to the least, counter-intuitive, for it suggests, according to Richardson's model, that neither side really fears, nor even pays attention to, the military expenditures of the other.

As discussed elsewhere in this dissertation in more detail, Richardson had assumed that hostile states engaged in an arms race would compete with each other on the basis of each side's total armed might. He suggested that a state's total armed might could be represented by its annual aggregate military expenditure figures. Richardson's assumption implies, rather dubiously, that each state is capable of reacting to the other's military capability as a single package with a single response. A more reasonable assumption, however, is that while states may fear each other's total military capability, they react to each other on a weapons-system by weapons-system basis (McCubbins, 1983). In particular, they compete over individual weapons systems with cross-purposes. This suggests that many sub-races can occur within the context of a larger military competition. With some sub-races heating up while others cool down, one could well find no-interaction between the military expenditure series of two rivals even thought each side may really fear the other's total armed might. In effect, evidence of action-reaction interaction between two rivals might be masked by measures of their aggregate military strength. This could well explain why most mathematical arms race research shows no interaction in the US-Soviet aggregate military expenditure race.

One should, then, expect to find no interaction in the aggregate military expenditure series, or indeed in any aggregate measure of military capability, of rival states. Institutions in State K and X simply may not be capable of reacting to each other's total armed might as a single package with a single response, even though each might fear the total might of the other. In this context, State K would know that State X's stock adjustment would be zero and State X would know that State K's stock adjustment coefficient is zero. That information would be contained in their defence information sets. With that information, the form of arms race interaction formalized by my GSR model will break down from action-reaction to non-interaction if $S = S' = 0$ irrespective of the values of B_1 and B'_1. Substantively, B_1 and B'_1 both equal zero simply means that neither of the rivals in question fears the armaments of the other. But in the case of a true rivalry, B_1 and B'_1 should not, then, both equal zero. This analysis suggests the following proposition

PROPOSITION 4.0: If, for any given arms race dyad, estimates of Equations 21 and 22 using data showing the aggregate military strength of each of the dyad members shows a non-interaction outcome, that outcome will be due the conditions summarized by $S = S' = 0$ and B_1 and B'_1 do not equal zero.

Put in terms of my GSR formulation, non-interaction in the US-Soviet aggregate expenditure competition is more palatable. It should, finally, be noted that interaction will break from action-reaction to non-interaction in my GSR model if State K and X have zero defence coefficients (i.e., $B_1 = B'_1 = 0$), irrespective of the values of their stock adjustment coefficients. But where two truly competitive states are the object of the analysis, B_1 and B'_1 should be non-zero.

PROPOSITION 4.1: If, for any given arms race dyad, estimates of Equations 21 and 22 using data showing the aggregate military strength of each of the dyad members shows a non-interaction outcome, that outcome will be due the conditions summarized by $B_1 = B'_1 = 0$ and $0 \leq S\ S' \leq 1$.

Alternatively, if for any pair of rival states, both have zero stock adjustment coefficients, then my GSR model will break down to a non-interaction model even if one of the states in question has a zero Richardsonian defence term.

PROPOSITION 4.2: If, for any given arms race dyad, estimates of Equations 21 and 22 using data showing the aggregate military strength of each of the dyad members shows a non-interaction outcome, that outcome will be due the conditions summarized by $S = S' = 0$ and either $B_1 = 0$ and B'_1 does not equal zero, or B_1 does not equal zero and $B'_1 = 0$.

5. MODEL AND PROPOSITION TESTING PROCEDURES

Equations 21 and 22 are specified in a form which can be easily and readily estimated. Estimates of these equations would give an indication, through the values obtained for each GSR parameter, what sort of arms race States K and X happen to be engaged in: action-reaction, asymmetric, non-interactive. But in order to determine which of Propositions 1-4 hold in a given case, estimates would have to be obtained for the Richardsonian parameters B_1, S, B'_1 and S'. Given the complexity of each GSR parameter, however, how could we tell, in a specific case study, what the values of B_0, B_1, S, B'_0, B'_1 and S' were? The values of these parameters could, with a little work, be obtained by also estimating Equations 6 and 8, State K and X's constrained goal equations.

$$\mathbf{K}_t = SB_0 + SB_1(E^k_{t-1}\mathbf{X}_t) + (1 - S)\mathbf{K}_{t-1} + SU_t \tag{6}$$

$$\mathbf{X}_t = S'B'_0 + S'B'_1(E^x_{t-1}\mathbf{K}_t) + (1 - S')\mathbf{X}_{t-1} + S'\mathbf{V}_t \tag{8}$$

First, Equations 6 and 8 could not be estimated as they stand. Each contains an unobserved variable. Equation 6 contains $E^k_{t-1}\mathbf{X}_t$, State K's expectation at time t-1 of State X's actual deployment level for time t. Equation 8 contains $E^x_{t-1}\mathbf{K}_t$, State X's expectation at time t-1 of State K's actual deployment level for time t. But by referring back to the Rational Expectations hypothesis, we could assume that

$$E^k_{t-1}\mathbf{X}_t = \mathbf{X}_t + \mathbf{R}_t, \text{ where } E(\mathbf{R}_t) = 0 \tag{37}$$

$$E^x_{t-1}\mathbf{K}_t = \mathbf{K}_t + \mathbf{S}_t, \text{ where } E(\mathbf{S}_t) = 0 \tag{38}$$

Thus, Equations 6 and 8 could be rewritten as

$$\mathbf{K}_t = SB_0 + SB_1\mathbf{X}_t + (1 - S)\mathbf{K}_{t-1} + S(\mathbf{U}_t + B_1\mathbf{R}_t) \tag{6a}$$

$$\mathbf{X}_t = S'B'_0 + S'B'_1\mathbf{K}_t + (1 - S')\mathbf{X}_{t-1} + S'(\mathbf{V}_t + B'_1\mathbf{S}_t) \tag{8a}$$

Then, from Equation 6a, we could obtain an estimate for the parameter (1 - S) and from that estimate, calculate the value of S. With a value for S in hand, we could then the divide the estimate obtained for the parameter SB_1 by S to obtain a value for B_1. Similarly, we could divide the value obtained for SB_0 by S to obtain B_0. The same procedure could be followed with Equation 8a to obtain values for S', B'_0 and B'_1.

Note, however, that if a value of zero is obtained for S from Equation 6a and if either one or both of the parameters SB_1 and SB_0 estimates is zero, then it may be impossible to tell from Equation 6a whether or not B_1 and B_0 have non-zero values. One would know if B_1, B_0, B'_1, and B'_0 were non-zero by estimating Equations 1 and 2. Estimating Equations 1 and 2 is, however, a complicated task. In addition to containing the variables $E^k_{t-1}\mathbf{X}_t$ and $E^x_{t-1}\mathbf{K}_t$, Equations 1 and 2 also contain the variables $\mathbf{K}^*_t$ and $\mathbf{X}^*_t$. $\mathbf{K}^*_t$ and $\mathbf{X}^*_t$ are, as you will recall from my earlier specifications, theoretical, unobserved variables. $\mathbf{K}^*_t$ refers to State K's desired deployment level at time period t and $\mathbf{X}^*_t$ refers to State X's desired deployment level at time period t. $\mathbf{K}^*_t$ and $\mathbf{X}^*_t$ can, nevertheless, be calculated. Equations 1 and 2 are reproduced immediately below.

$$\mathbf{K}^*_t = B_0 + B_1(E^k_{t-1}\mathbf{X}_t) + \mathbf{U}_t, \quad B_0, B_1 \in R \tag{1}$$

$$\mathbf{X}^*_t = B'_0 + B'_1(E^x_{t-1}\mathbf{K}_t) + \mathbf{V}_t, \quad B'_0, B'_1 \in R \tag{2}$$

First, Equation 37 can be substituted into Equation 1 and Equation 38 can be substituted into

Equation 2 in order to eliminate the unknowns $\mathbf{E}^{k}_{t-1}\mathbf{X}_t$ and $E^{x}_{t-1}\mathbf{K}_t$ respectively. The specified substitutions will give

$$\mathbf{K}^{*}_{t} = B_0 + B_1\mathbf{X}_t + (B_1\mathbf{R}_t + \mathbf{U}_t) \tag{1a}$$

$$\mathbf{X}^{*}_{t} = B'_0 + B'_1\mathbf{K}_t + (B_1\mathbf{S}_t + \mathbf{V}_t) \tag{2a}$$

How can the variables $\mathbf{K}^{*}_{t}$ and $\mathbf{X}^{*}_{t}$ be calculated?

$\mathbf{K}^{*}_{t}$ and $\mathbf{X}^{*}_{t}$ are theoretical, unobserved variables. How then can their values be obtained? The key to estimating Equations 1a and 2a is to first estimate Equations 6a and 8a and extract from them, respectively, the values S and S'. Next, rewrite Equations 3 and 4, displayed immediately below, setting them in terms, respectively, of $\mathbf{K}^{*}_{t}$ and $\mathbf{X}^{*}_{t}$. From

$$\mathbf{K}_t - \mathbf{K}_{t-1} = S(\mathbf{K}^{*}_{t} - \mathbf{K}_{t-1}) \tag{3}$$

$$\mathbf{X}_t - \mathbf{X}_{t-1} = S'(\mathbf{X}^{*}_{t} - \mathbf{X}_{t-1}) \tag{4}$$

we obtain,

$$\mathbf{K}^*_t = \frac{\mathbf{K}_t - (1 - S)\mathbf{K}_{t-1}}{S} \tag{39}$$

$$\mathbf{X}^*_t = \frac{\mathbf{X}_t - (1 - S')\mathbf{X}_{t-1}}{S'} \tag{40}$$

As set, Equations 39 and 40 can yield, on the basis of known values, the theoretical values, respectively, $\mathbf{K}^*_t$ and $\mathbf{X}^*_t$. With these values, Equations 1a and 2a can readily be estimated.

It is important to note that the procedure just specified is valid, methodologically, only if the error terms $S(\mathbf{U}_t + B_1\mathbf{R}_t)$ from Equation 6a and $S'(\mathbf{V}_t + B'_1\mathbf{S}_t)$ from Equation 8a are not each autocorrelated. Why? Specifically, if $S(\mathbf{U}_t + B_1\mathbf{R}_t)$ and $S'(\mathbf{V}_t + B'_1\mathbf{S}_t)$ are each autocorrelated, the standard errors of the estimates obtained for (1 - S) and (1 - S') would likely be underestimated. This could lead to misleading conclusions about the statistical significance of the estimates obtained for (1 - S) and (1 - S'). In that eventuality, the subsequent calculation of $\mathbf{K}^*_t$ and $\mathbf{X}^*_t$ and hence the estimation of B_0, B_1, B'_0, and B'_1 would be correspondingly misleading. This leads, finally, to two questions: which regression technique should be used to estimate Equations 1a, 2a, 6a, 8a, 21 and 22 and what method should be used to test the regression results for autocorrelation?

One of the three pillars of the micro approach to mathematical arms race analysis which I have developed in this thesis is that an arms race between two rivals is made up of a series of subraces. Each subrace has its own focus: each involves a competition wherein one side in the dyad in question deploys a particular weapons system and the other responds by deploying a weapons systems designed to counter the former's effectiveness or purpose. Secondly, these subraces have a start and an end point which may lie well within the start and end points of the overall military rivalry. This is the second pillar of the micro approach. When preparing to study a given arms race, one must, therefore, make a decision as to what weapons system-counter weapons system subcompetition to study and determine its start and end points. More specifically, a sample used to estimate my GSR model and its component equations should be set, timewise, to correspond with a historical, economic, and political period in which parameter values for that subrace are constant (see Lucier (1979)). This approach to parameter estimation could lead to very small sample sizes (less than 30 points).

Basic statistical theory is, though, large sample based. However, small sample (n = 10) experimental studies by Kmenta (1971) suggest that parameter estimates obtained for $\mathbf{Y}_t = a + b\mathbf{X}_t + \mathbf{U}_t$ would still be unbiased. Standard error estimates would, though, increase as the sample size decreased under both OLS and GLS. This simply means that it would be more difficult to obtain significant estimates for the parameters in my GSR component Equations 1a and 2a.

Equations 1a and 2a could first be estimated by OLS. The standard Durbin Watson

DW test could then be applied to the results to test for autocorrelation. If autocorrelation is found, then Equations 1a and 2a could be reestimated using GLS.

Small sample experimental studies by Malinvaud (1970) suggest that parameter estimates for autoregressive equations, such as my GSR component Equations 6a and 8a and my GSR Equations 21 and 22, will be biased in a downward direction (about 15%) under OLS. Standard error estimates would be unbiased. This, as in the former case, means that it would be more difficult to obtain statistically significant estimates for the parameters in Equations 6a, 8a, 21 and 22. Thus OLS could be effectively used to estimate Equations 6a, 8a, 21 and 22.

Malinvaud's conclusions regarding small sample properties of estimates of autoregressive models are, however, contingent upon the existence of non-autocorrelated error terms. OLS would not be appropriate if the error terms in Equations 6a, 8a, 21 and 22 were autocorrelated. What test could be applied in order to determine if the errors in Equations 6a, 8a, 21 and 22 were autocorrelated?

When a lagged independent variable is used as a dependent variable, the standard Durbin-Watson test (using Ordinary Least Squares) would give a misleading indication of the extent of that autocorrelation. It could even show no autocorrelation exists when in fact it does exist. Normally, equations which do contain a lagged independent variable as a dependent variable could be estimated by OLS and the extent of autocorrelation could be determined through a relatively new test, Durbin's h. But Durbin's h is a large sample test. Its properties have not yet been determined for small samples (Ostrom, 1978c).

The autocorrelation test best suited to the conditions under which we must operate under the micro approach to mathematical arms race analysis is the Run Test. The Run Test can be applied against both autoregressive and non-autoregressive models and its conclusions are valid under sample sizes as small as nine. The Run Test for autocorrelation is applied as follows (Gujurati (1978). First arrange the signs (+ or -) of the estimated errors for the model in question in a consecutive sequence. It must be that $n = N_1 + N_2$ where n is the sample size, N_1 is the total number of positive signs and N_2 is the total number of negative signs. Define a run as an uninterrupted sequence of one symbol (+ or -) from the consecutive sequence of all plus or minus signs. The test for randomness of errors is done by asking if the observed number of runs for a given n is too high, too low, or about right as compared to the number of runs expected in a strictly random sequence of n observations. Too many runs suggests negative autocorrelation and too few suggests positive auto correlation. If N_1 or N_2 is smaller than ten, special tables (see Gujurati (1978: 440-441)) give critical values of the runs expected in a strictly random sequence of n observations. If N_1 or N_2 are large, then the number of runs is distributed normally, and therefore, the Z-test can be used to test if the number of runs observed is statistically significant.

How likely is it that estimates of Equations 6a and 8a and hence Equations 21 and 22 would exhibit autocorrelation? The errors in autoregressive models such as the adaptive expectations model or the Koyck model can be theoretically shown to have a high likelihood of being autocorrelated (Gujurati (1978: 266-276). It can also be demonstrated that the errors in the partial adjustment model, which forms the GSR Equations 6a, 8a, 21 and 22, do in fact theoretically satisfy the OLS assumption of non-autocorrelation. Hibbs (1974) shows that OLS estimates obtained for a partial adjustment model will be consistent, but biased in small samples. This is consistent with Malinvaud (1970).

What happens if, nevertheless, it turns out that the errors in any of Equations 6a, 8a, 21 and 22 are autocorrelated? Normally, that is when n is large, when the error term in an autoregressive Equation is found to be autocorrelated, one must reestimate the equation using a technique such as IV-pseudo GLS. This procedure would give more efficient estimates than OLS (Hibbs (1974)). But the small sample properties of the IV-pseudo GLS method have not yet been determined (Smith (1980). Here, then, is where the greatest weakness of the micro approach to mathematical arms race analysis lies. If the errors in any of Equations 6a, 8a, 21 and 22 are found to be autocorrelated, and if one then reestimated those equations using IV-pseudo GLS, the resulting estimates would have unknown properties. The utility of such estimates would be quite limited.

Thus, by following the procedures set out in this section, my GSR model and its component equations can be estimated. With those estimates in hand, one could, in a given arms race context, determine which of propositions 1-4 hold.

In this chapter, I have laid out the conditions under which the form of arms interaction between two military rivals can materialize as action-reaction interaction, and with the rivals being no less hostile toward each other, when it can materialize as a non-interaction outcome or an asymmetric outcome. I have also set out propositions concerning the validity of the Rational Expectations formation in arms competitions. More formally, there are four main propositions concerning the arms race behavior of states which derive from my model. They are

PROPOSITION 1.0: If States K and X do in fact form their expectations of each other's arming behavior in accordance with the Rational Expectations hypothesis, then the errors in State K and X's arming behavior, as modelled by Equations 21a and 22a, should be strongly correlated. Mathematically $\mathrm{corr}(\mathbf{Y}_t, \mathbf{Z}_t) \neq 0$.

PROPOSITION 2.0: If, for any given arms race dyad, estimates of Equations 21 and 22 show an asymmetric arms race, that outcome will be due the conditions summarized by either $S = 1$, $0 < S' < 1$ and B_1, B'_1 do not equal zero or $S' = 1$, $0 < S < 1$ and B_1, B'_1 do not equal zero.

PROPOSITION 3.0: If, for any given arms race dyad, estimates of Equations 21 and 22 show an action-reaction arms race, that outcome will be due the conditions summarized by $0 < S, S' < 1$ and B_1, B'_1 do not equal zero.

PROPOSITION 4.0: If, for any given arms race dyad, estimates of Equations 21 and 22 using data showing the aggregate military strength of each of the dyad members show a non-interaction outcome, that outcome will be due the conditions summarized by $S = S' = 0$ and B_1 and B'_1 do not equal zero.

In the next chapter, I will estimate the parameters of my GSR model and its component equations for the US-Soviet nuclear warhead race. More specifically, I will test my model against data on SLBM and the ICBM based warhead deployment levels, and against data on US and on Soviet aggregate strategic nuclear warhead deployment levels. With the estimates of my GSR model in hand, I will then make a determination as to which of Propositions 1-4 hold in which cases.

CHAPTER VI: AN EMPIRICAL TEST OF THE GSR MODEL: THE US-SOVIET NUCLEAR ARMS RACE

Were the United States and the Soviet Union engaged in an arms race? A great deal of time and energy has been expended over the years in a search for quantitative evidence suggesting that the US and Soviets are (or now were) so engaged. The effort, thus far, has been to little avail. Most quantitative analyses of the US-Soviet arms competition show a complete absence of interaction. Was it really the case that the US and Soviet Union acquired arms without regard to what the other was acquiring? Or is it that we have yet to develop a methodology which can uncover the interactive component of US-Soviet arms acquisitions?

How one approaches a study of this competition determines, in large measure, what one will find. One can either take a macro approach or a micro approach. The former approach, most common among arms race researchers, takes the US-Soviet arms competition as a competition of total armed might versus total armed, in the tradition of Richardson. The unit of analysis under the macro perspective is most commonly aggregate military expenditures. The micro approach, developed in this dissertation, sees the larger US-Soviet military competition as made up of a series of subraces, the object of each subrace being a particular weapons system, deployed by one side, and individual weapons system with a cross purpose, deployed by the other, and designed to counter the former's political and military effect. Under this perspective, two rivals can engage each other in several different races, or subraces, the object of each being a particular weapons system and a corresponding weapons system with a cross purpose. A picture of an overall military competition can then sketched from an analysis of each of those subraces.

There are sound theoretical and methodological reasons, which I have set out elsewhere in this dissertation, for approaching the US-Soviet military competition via the micro approach, as opposed to the macro approach. In my study of the US-Soviet arms race, I will adopt the micro approach. The specific micro-level questions I will attempt to answer are, Were the US and Soviets engaged in individual action-reaction arms races over SLBM warhead deployments and ICBM warhead deployments? Were they engaged in an arms race over total strategic nuclear warhead deployments? In effect, mine will be a study of the US-Soviet nuclear arms race. The micro approach will yield the intuitively appealing conclusion that the US and Soviets were indeed engaged in an action-reaction nuclear arms race.

1. A GENERAL OVERVIEW OF TRENDS IN THE US-SOVIET NUCLEAR ARMS RACE

The US-Soviet nuclear arms race began, for all intents and purposes, on 6 June 1945 when the US had tested the world's first atomic bomb. In August of that same year, the US dropped a 15-20 kiloton bomb on the Japanese city of Hiroshima. In 1949, the Soviet Union detonated its first atomic bomb. By this time, the US and Soviets were bitter political, economic and military rivals. Each came to see the other as a threat to its very existence. In this section, I will provide an overview of the basic trends and developments of what became the potentially most dangerous arms race in history, the US-Soviet nuclear arms race (Freedman, 1983, 1986; Holloway, 1985; Gray, 1976; MccGwire, 1987a, 1987b, 1991; Koenig, 1982; SIPRI, 1971-90; Military Balance, 1969-1990; Sagan, 1989; Trachtenberg, 1988-89; Garthoff, 1990; Catudal, 1988).

In the early 1950s, the only means that the US and USSR possessed for delivering nuclear payloads was the longrange bomber. As arms races go, the US-Soviet bomber race was really not a race at all. In the very early 1950s, the Soviets were able to field two different intercontinental bombers, the M4 Bison, with a range of 7000 miles and the Tu95 Bear, with a range of 7800. The Bison proved unequal to its appointed task and was eventually relegated to the role of in-flight tanker-refueler (Koenig, 1982). The Bear proved a much better aircraft and by 1980 some 113 Bears remained in service in the Soviet Airforce as longrange bombers. Neither plane was, however, produced in any real quantity and thus the Soviet bomber threat was considered small in the US. The US, on the other hand, had built a large force of B52s. The Soviets, in contrast to the Americans, were thus obliged to construct a formidable air defence system. Soviet air defence rested on a missile defence network and on some 2600 interceptor aircraft (Koenig, 1982).

By 1960, ICBM technology had improved to the point where both sides had begun to deploy large numbers of longrange nuclear tipped missiles. The ICBM quickly began to supplant the long range bomber as the principal means for delivering nuclear payloads (Koenig, 1983). ICBMs were more accurate than the longrange bomber and more difficult to intercept. The first generation US and Soviet ICBMs were primitive, by today's standards. They were inaccurate and each could only carry one warhead. The USSR first deployed the hugh SS-6 and the US deployed the Atlas.

It was not long, however, before the US had developed MIRV technology which allowed it to place more than one warhead inside each missile. Guidance systems also improved making ICBMs more accurate. Eventually, the Soviets also developed improved guidance technology and MIRV technology. In consequence, both American and Soviet ICBMs became vulnerable to a first strike by the late 1960s. Both sides, thus, made an effort to develop anti-ballistic missile defence systems. Both the US and Soviets eventually put operational systems into place, but today, only the Soviet system remains operational.

Another way both sides secured their second strike forces was to step up SLBM deployments. Indeed, the US had begun to place a strong emphasis on SLBM basing in the early 1960s. The SLBM basing mode meshed well with US nuclear doctrine. By the mid 1960s, US doctrine called for nuclear strikes against the USSR only in retaliation for a Soviet strike on the US. SLBMs were secure from Soviet preemptive attack and thus could be used either to deter an initial first strike by the Soviets or to retaliate for a first strike by the Soviets. The Soviets, on the other hand, did not begin to emphasize SLBM deployments in their strategic calculations until the early 1970s. MIRV and improved guidance technology was making their ICBM force vulnerable to a US first strike. This was no small concern for the Soviets since the bulk of their nuclear forces were ICBM based. Also, Soviet military doctrine had, in the mid-late 1960s, changed. Initial Soviet plans called for a first-strike against US nuclear forces if nuclear war looked imminent. The Soviets now began to see the virtue in shifting to a second strike posture. This would require the development of large SLBM force and, accordingly, Soviet SLBM deployments began to increase substantially throughout the 1970s and 1980s.

Originally, launchers (ICBMs, SLBMs, and bombers) were the focus of the US-Soviet nuclear competition. In 1972, SALT I came into effect. It was significant because it placed limits on the number of launchers each side could deploy. It was also significant for what it did not limit. It placed no controls on the number of warheads each side could deploy. From 1972 onward, the US-Soviet race thus shifted from a race over launchers to a race over warheads (Keating, 1985). Soon, START will cut back the number of strategic nuclear warheads from 11,000 to 7,000 on the Soviet side and from 12,000 to 9,000 on the US side.

2. THEORETICAL BASIS FOR A US-SOVIET NUCLEAR ARMS RACE

Were the US and the Soviets engaged in an action-reaction arms competition? Key US defence officials have stated that the US-Soviet military rivalry must, logically, be conducted in an action-reaction mode, particularly when it comes to nuclear weapons. Enthoven and Smith (1971:176-177) who were two of McNamara's key assistants during his time as US Secretary of Defense wrote

> It is important to understand [... the] interaction of opposing strategic forces and its relation to the strategic force planning process. If the overriding objective of our strategic nuclear forces is to deter a first-strike against us, the United States must have a second-strike capability .... This capability to destroy him after absorbing his surprise attack must be a virtual certainty, and clearly evident to the enemy. This is the foundation of the U.S. deterrent strategy. Consequently, as long as deterrence remains the priority objective, the United States must be prepared to offset any Soviet effort to reduce the effectiveness of our assured destruction capability below the level we consider necessary.
>
> At the same time, however, if deterrence is also the Soviets' objective (as the available evidence has consistently and strongly suggested), we would expect them to react in much the same way to any effort on our part to reduce the effectiveness of their deterrent (or assured destruction) capability against us. And we would also expect them, in their planning, to view our strategic offensive forces as a potential first-strike threat (just as we do theirs) and provide for second-strike capability. In other words, any attempt on our part to reduce damage to our society would put pressure on the Soviets to strive for an offsetting improvement in their assured destruction forces, and vice versa. Each step by either side, however sensible or precautionary, would elicit a precautionary response from the other side. This 'action-reaction' phenomenon is central to all strategic force planning issues as well as to any theory of an arms race.

The logic that obliges the US and Soviet Union to engage each other in an action-reaction nuclear arms race is compelling. Action-reaction arms accumulation is an essential means to maintaining the balance of terror between the US and USSR. What empirical evidence has been adduced to verify the existence of a nuclear arms race between the US and Soviet Union?

3. PAST QUANTITATIVE STUDIES OF THE US-SOVIET NUCLEAR ARMS RACE

Little quantitative empirical evidence of a nuclear arms race between the US and Soviet has been found. Kugler, Organski and Fox (1980), for example, argued that if the US and Soviet Union were set upon maintaining a balance of terror between them, then each side should increase its nuclear capabilities in response and in proportion to increases in the nuclear capabilities of the other in order to maintain that balance. In other words, the US and Soviet Union should be engaged in an action-reaction nuclear arms race. The purpose of their study was to find empirical evidence to support this proposition. They found none. They wrote:

> According to our data, then, the presence of nuclear arms race, far from constituting a given of international politics, proves to be a chimera. We have tried again and again to test for the presence of arms competition or arms racing and we have failed to find anything each time. It is obvious that the US and USSR are building arms, but are not doing so, as they allege, because they are racing or competing with one another [p. 128].

McGuire (1976) has, similarly, found little evidence to support the view that the US and Soviet Union are engaged in a nuclear arms race. Why could neither Kugler et al (1980) nor McGuire (1976) find any evidence indicating the existence of a US-Soviet nuclear arms race?

Kugler et al's and McGuire's failure to find any empirical evidence to support the view that the US and USSR are engaged in an action reaction nuclear arms race is due more to their particular approach to the problem than it is to the genuine absence of such a competition. In their study, Kugler et al estimated the parameters of Richardson's model using data showing annual US and Soviet expenditures on all strategic nuclear forces, including, ICBM, submarine and bomber forces (given in US dollars). They considered the race, then, to be one of total strategic nuclear capability versus total strategic nuclear capability. McGuire took a slightly different approach. Instead of using expenditure data, he used nuclear weapons inventory levels to indicate US and Soviet strategic capability. For example, he tested Richardson's model against data on the total annual megatonage in the US and Soviet strategic arsenals. He also tested Richardson's model against data on the total strategic warhead counts of the US and Soviet Union. Why did these studies not reveal evidence that the US and Soviet Union are engaged in a nuclear arms race? Indeed, if the US and Soviets are engaged in a nuclear arms race, over what, specifically, are they racing?

4. A NEW APPROACH TO THE STUDY OF THE US-SOVIET NUCLEAR ARMS RACE

As discussed in more detail elsewhere in this thesis, McCubbins (1983) argues that rival nations do not, as Richardson (1960a) suggested, engage each other in a single race over "total armed might." McCubbins argues that the US and Soviet Union compete with each other, in an action-reaction mode, when it comes to specific, individual weapons systems with cross-purposes, e.g., one side deploys long range bombers, the other side will counter with jet interceptors. Theoretically, two rivals could engage each other in a number of different races over different sets of weapons and corresponding counter-weapons systems. At any given time, some of these sub-races, as we may call them, may be heating up while others are cooling down. As argued elsewhere in this thesis, looking at the US-Soviet rivalry as a competition of total armed might versus total armed might, then, can result in any evidence of action-reaction interaction being masked.

McCubbins' empirical analysis, indeed, showed that the US-Soviet conventional weapons competition occurs over individual weapons systems with cross-purposes and that the competition is action-reaction in nature. Yet McCubbins' study was limited in its scope. His analysis of the US-Soviet arms race was confined to the competition over conventional weapons. He might have done well to consider applying his analysis to the US-Soviet nuclear arms race as well.

Both the US and Soviet Union do try to counter the nuclear force deployments of the other and thereby maintain a balance of terror. For example, each side has attempted to counter the other's strategic nuclear missile deployments with anti-ballistic missile systems (Rathjens, 1969). This is an example of a race over individual weapons systems with cross-purposes. A less obvious example would be the attempt by the superpowers in the early 1960s to out deploy each other in ICBMs. An ICBM-ICBM race might seem like a race over individual weapons systems with identical purposes. But, if, in this case, the policy goal of each side was to prevent the other from gaining a first strike capability, and hence avoid nuclear war, then one way to achieve that goal would have been for each side to prevent the other from gaining numerical superiority in ICBMs.

A further point about that ICBM race is that it did not last long. By the 1970s, both the US and Soviets had begun to focus more resources into SLBM basing. Accordingly, it begins to become clear why Kugler et al and McGuire were unable to discern any real evidence that the US and Soviet Union are engaged in a nuclear arms race. The total weapons counts for the US and Soviet Union that McGuire used contain information on three different weapons systems: ICBMs, SLBMs and long range bombers. Clearly, while the ICBM race was heating up, the bomber race was dying down. When taken together, the peaks and valleys in the ICBM, the SLBM and the bomber race cancel each other out. This may also explain the outcome of Kugler et al's study. Kugler et al used US and Soviet data which reflected total expenditures on ICBM, submarine and bomber based nuclear forces by each side.

From this discussion, it is clear that the conventional approach to the quantitative analysis of the US-Soviet nuclear arms race must be amended. In my study of the US-Soviet nuclear arms race, I will follow the McCubbins' data approach. In particular, as Snow (1981) argues, the US and USSR consider each side's strategic nuclear warhead count to be a good indicator of each side's strategic capability. Missile counts can be misleading since some missiles contain more than one warhead and it is the warhead which does the actual damage in a war. The unit of analysis in my study, then, will be annual US-Soviet strategic nuclear warhead deployments. More specifically, following the historical trend of the US-Soviet nuclear arms race, laid out above, I will conduct three separate tests of my GSR model. Specifically, I will test my GSR model against data on annual US and Soviet deployments of SLBM warheads from 1972 to 1987 and against data on US-Soviet annual ICBM warhead deployments from 1960 to 1971 for evidence of action-reaction interaction. Setting the start and end points of each of these races to match the historical record should yield valid results (see Lucier, 1979). Finally, I will test my GSR model against data on total annual US-Soviet strategic nuclear warhead counts from 1967 to 1984. The reason for this test is that it will establish whether or not the US and Soviets did react to each other's total strategic capability. The data set time span for this study is arbitrarily set. If the US and Soviet Union did not so react, then any sample drawn between 1949 and 1991 should show this to be the case. From the estimates obtained in each of these tests, I will then make a determination as to which of the arms race Propositions 1-4, derived in the last chapter, hold, and in which cases they do hold.

6. THE SLBM WARHEAD RACE

Of all the strategic nuclear warheads distributed across the US and Soviet triads, none are more invulnerable to a surprise first-strike than those based onboard the submarines of the US and Soviet navies. Neither side has yet been able to develop a device, such as a satellite, which is capable of scanning vast areas beneath the surface of the seas and locating enemy submarines. Each side does have systems to help narrow down the location of enemy submarines, but these systems are effective only when the general location of a submarine is already known (Handler, 1987). Because the seas are so vast and because the SLBMs which the submarines carry have a long range, a submarine can hide virtually anywhere and remain undetected while awaiting launch instructions.

The US was the first to develop SLBM technology. By 1960, it had some 32 single warhead SLBMs, the Polaris A1, deployed at sea (Military Balance, 1969-70: 55). It was not until 1963 that the Soviets were able to field their first SLBM. From the beginning, the US had placed a high priority on developing and deploying SLBM systems. The US was committed to a doctrine of engaging in nuclear war only in response to a first-strike by the Soviet Union. SLBM basing mode meshed well with the US strategic doctrine of second-strike because submarines are invulnerable to surprise attack and destruction. Thus by basing warheads onboard submarines, the US would always have a force which, in the event of a Soviet first-strike, could be used to retaliate against Soviet cities. The bulk (50% as of 1984) of the US strategic warhead inventory is currently based on SLBMS. The Soviets, on the other hand, had from the beginning of the nuclear race, placed a high priority on the development of landbased ICBM systems. For one thing, it lagged behind the US in sophisticated SLBM and SLBM basing technologies. In emphasizing landbased ICBM development, the Soviets were drawing on the greatest strength, the enormous size of the Soviet Union. Much of the Soviet landmass is uninhabited. Large numbers of ICBMs could, then, be safely deployed in these areas, away from population centers. The US, then, would have to commit large numbers of missiles to target those systems (Snow, 1980: 147).

By the 1970s, several factors had come together to channel Soviet efforts into increasing its SLBM warhead deployments: the development of MIRV technology, the SALT I arms control agreement, changes in Soviet military doctrine. One of the most significant technological developments to occur in the US-Soviet nuclear arms race was the development, first by the US and then by the Soviets, of MIRV technology. MIRV technology allowed each side to place more than one nuclear warhead inside each of its ballistic missiles. Each warhead could be individually and independently targeted. This development was to put at risk the Soviet Union's landbased ICBM force as it made it theoretically easier for the U.S. to destroy those ICBMs while still in their silos. This was no small concern for the Soviets since the bulk of their strategic nuclear capability was in their landbased ICBMs. Something had to be done. As Scoville (1972: 37) writes:

> By the late 1960s it must have been obvious to military planners in the U.S.S.R. that their land-based ICBM's would become increasingly vulnerable to the U.S. MIRV's, which were then under development and which had been publicly justified as providing an improved counterforce capability. The Russian deterrent needed shoring up with a more effective SLBM force, whose value had been demonstrated by the U.S.

Indeed, from 1972-87, Soviet SLBM warhead deployments rose from 458 to 3408 (Nuclear Notebook, May, 1988).

Second, in 1972, the Strategic Arms Limitation Treaty (SALT) I came into effect. SALT I placed ceilings on the number of strategic delivery vehicles that the US and Soviet Union could possess. The term delivery vehicle applies to longrange bombers, and ICBM and SLBMs, excluding the warheads they each carry. Each side was assigned strict maximum launcher deployment levels in each category. The US took the opportunity afforded it during the SALT negotiations to influence Soviet thinking and Soviet action. The US, as already mentioned, placed a great deal of emphasis on SLBM development and deployment. Because SLBMs were invulnerable to preemptive attack, the US saw them as a stabilizing element in the overall East-West nuclear game. The US thus wanted the Soviets to also emphasize SLBM basing in their strategic calculations. The Americans were able to get the Soviets to agree to a "one-way mixing clause" in SALT I. Each side would be permitted to dismantle ICBMs and replace them with SLBMs, but not vice-versa, while keeping total launcher ceilings constant (Snow, 1980: 98).

Thirdly, Soviet military planners began in the late 1960s to rethink Soviet military doctrine (MccGwire, 1987, 1991). Originally, Soviet military planners had expected that war with the United States would ultimately turn nuclear and would become a decisive showdown between the capitalist and the communist systems. The Soviet Union intended to emerge the victor in this contest. In order to limit the damage that the USSR would have to sustain in such a war, Soviet planners adopted a strategy of striking American nuclear (particularly ICBM) forces first in order to blunt the weight of an American nuclear attack. But by the late 1960s, Soviet planners began to think that war with the United States might not necessarily involve large scale strategic nuclear strikes and counter strikes. Indeed a conventional war with the West need not escalate beyond the conventional level. Strategic nuclear war, and hence the devastation of the Soviet Union, could be avoided. Certainly, in the event of war, the US would have to be ejected from Europe, but this could be done by conventional means. In order to deter the US from launching nuclear strikes against the Soviet Union itself when faced with the defeat of its forces in Europe, the Soviet Union had to have a secure second strike capability. Because of MIRV technology, however, its main nuclear forces, landbased ICBMs, were becoming vulnerable to an American first strike. Soviet efforts to develop a mobile land based ICBM were not bearing fruit and so it was decided that SLBM deployments should be stepped up in order to secure the Soviet Union's second strike capability. The Soviet Union thus began expending vast resources on its navy throughout the 1970s and into the 1980s. By the 1980s, the Soviets began to reduce this emphasis on SLBM development and deployment. Relations with the West were easing, and significant new strides were being made in mobile landbased ICBM technology. Mobile ICBMs are much more difficult to hit than fixed based ICBMs. Currently, the Soviets have deployed the railbased SS-24 and the roadmobile SS-25.

These factors, the development of MIRV technology, SALT I, and a new Soviet military doctrine were critical in setting in motion a US-Soviet SLBM warhead deployment competition spanning the early 1970s to the late 1980s. It was these factors, principally, critically, which determined the course and dynamics of that competition. Because it is the destructive capacity of the second strike forces of each side which deter the other from launching a first strike, it would be reasonable to expect that each side would be trying to maintain a second strike force comparable to that of the other. Parameter estimates for my GSR model suggest that the US and Soviet Union have, in fact, been engaged in an action-reaction competition over SLBM warhead deployments over the years 1972-1987. I began by estimating the simplified versions of Equations 21 and 22, Equations 23 and 24.

$$\mathbf{K}_t = L_0 + L_1\mathbf{X}_{t-1} + L_2\mathbf{K}_{t-1} + \mathbf{Y}_t \quad \text{(USA)} \tag{23}$$

$$= 2023.0 + 0.158\mathbf{X}_{t-1} + 0.550\mathbf{K}_{t-1}$$

$$\underset{**}{(432.6)} \quad \underset{*}{(0.11)} \qquad \underset{**}{(0.130)}$$

$R^2 = 0.9132$, n = 15, ** = sig. at 0.05 and * = sig. at 0.10. (Note Equation 23/21 was estimated using OLS. Run Test results indicated no autocorrelation at a 0.05 level of significance: see Table 6.1 for test results).

TABLE 6.1: Autocorrelation test results (Run Test). See page 141 for an explanation of this test. Figures contained in this table show the observed number of runs, the number of positive errors, and the number of negative errors, in that order, for each indicated equation and for each indicated race.

	SLBM Warheads 1972-87	ICBM Warheads 1960-71	TOTAL Warheads 1967-84
Equation 1a	6,6,9	8,4,7	7,8,9
Equation 2a	7,6,9	6,4,7	7,8,9
Equation 6a	6,6,9	8,4,7	7,8,9
Equation 8a	7,6,9	6,4,7	7,8,9
Equation 23	7,7,8	8,4,7	7,8,9
Equation 24	9,7,8	8,6,5	7,8,9

$$\mathbf{X}_t = L'_0 + L'_1\mathbf{K}_{t-1} + L'_2\mathbf{X}_{t-1} + \mathbf{Z}_t \qquad \text{(USSR)} \tag{24}$$

$$= -399.9 + 0.20\mathbf{K}_{t-1} + 0.80\mathbf{X}_{t-1}$$

$$\underset{}{(323.4)} \quad \underset{**}{(0.097)} \quad \underset{**}{(0.087)}$$

$R^2 = 0.9775$, n = 15 (Note Equation 22/24 was estimated using OLS. Run Test results indicated no autocorrelation at a 0.05 level of significance: see Table 6.1 for test results).

My estimates of the GSR coefficients in Equations 23/21 and 24/22 show that over the period 1972 to 1987, the US and Soviet Union have been engaged in an action-reaction arms race over SLBM warhead deployments. The US GSR grievance, defence and stock adjustment terms are all significant at 0.05 or 0.10. Of those parameter estimates which are significant, only the US GSR defence term is significant at 0.10. Less weight must, accordingly, be attributed to this estimate compared to the others. Specifically, my estimates show that US SLBM warhead deployments at time t depend upon 0.158 times Soviet SLBM warhead deployments at time t-1 and 0.55 times US SLBM warhead deployments at time t-1 plus a GSR grievance term of 2023. The Soviet GSR defence and stock adjustment terms are significant at 0.05. Soviet SLBM warhead deployments at time t, similarly, depend upon 0.20 times US SLBM warhead deployments at time t-1 and 0.80 times Soviet SLBM warhead deployments at time t-1. The Soviet GSR grievance term is not statistically significant at 0.10.

The finding that the US and USSR were engaged in an action-reaction race over

SLBM warhead deployments (1972-87) is not surprising. Indeed it is important. It suggests that the logical conditions for the maintenance of strategic deterrence were being met.

The mechanism behind the action-reaction competition of the US and Soviet Union with respect to SLBM warhead deployments can be explained thusly. Consider first, that both the US and the Soviet Union see their sea based nuclear forces as second-strike, or deterrent forces. The theoretical link between action-reaction arms racing and deterrence, as set out by Kugler et al (1980: 108) is as follows:

> It should be noted that in the context of deterrence theory it is not necessary to assume that both sides in nuclear arms races must make the same amount of effort or have the same level of capabilities. One must assume, however, that each of the contestants will allocate substantial portions of the resources scheduled to be used in the improvement of nuclear capabilities in direct response to the other's allocations. Hence one must compete and even race with one's opponent. And the race continues even after both contestants reach a second strike capability. One must always keep in mind that the invulnerability of the defendant's deterrent depends on the power of the aggressor's initial attack.

Two points must, accordingly, be made. The first is that both the US and Soviets have, with their SLBM forces, each achieved a second strike capability. However, each side has made a point, and this is particularly true of the Americans, of developing the means and the methods for destroying the other's SLBM forces in the event of war. The US Navy has long had a policy of pursuing and destroying, in the event of war, Soviet nuclear missile carrying submarines (Handler, 1987). Never was that policy made more explicit than in 1983 by US

Navy Secretary Lehman. In this context it would, then, be prudent for each side to try to maintain an SLBM force proportional to that of the other at all times. What that proportion would be would depend on how capable each side thought the other was of destroying its SLBM forces and on how each side evaluated the technological span between its own SLBMs and those of its adversary. In this regard, much can be learned by referring to the estimates of Equations 1a, 2a, 6a and 8a.

What are the values of the parameters, B_0, B_1 and S, the US Richardsonian grievance, defence and stock adjustment parameters and B'_0, B'_1 and S', the Soviet Richardsonian grievance, defence and stock adjustment parameters? Equations 6a and 8a, reproduced immediately below.

$$\mathbf{K}_t = SB_0 + SB_1\mathbf{X}_t + (1 - S)\mathbf{K}_{t-1} \qquad \text{(USA)} \qquad \text{(6a)}$$

$$= \underset{\substack{(439.9)\\ **}}{2177.0} + \underset{\substack{(0.131)\\ *}}{0.226}\mathbf{X}_t + \underset{\substack{(0.143)\\ **}}{0.479}\mathbf{K}_{t-1}$$

$R^2 = 0.9200$, n = 15. (Note: Equation 6a was estimated using OLS. Run Test results indicated no autocorrelation at a 0.05 level of significance: see Table 6.1 for test results).

$$\mathbf{X}_t = S'B'_0 + S'B'_1\mathbf{K}_t + (1 - S')\mathbf{X}_{t-1} \qquad \text{(USSR)} \tag{8a}$$

$$= -958.2 + 0.322\mathbf{K}_t + 0.77\mathbf{X}_{t-1}$$

$$\underset{**}{(501.8)} \quad \underset{**}{(0.13)} \quad \underset{**}{(0.08)}$$

$R^2 = 0.9794$, $n = 15$. (Note: Equation 8a was estimated using OLS. Run Test results indicated no autocorrelation at a 0.05 level of significance: see Table 6.1 for test results).

The estimate of Equations 6a reveals that the US stock adjustment coefficent is $S = 1 - 0.479 = 0.521$ and the estimate of Equation 8a reveals that the Soviet stock adjustment coefficient $S' = 1 - 0.77 = 0.23$. Within the context of the SLBM warhead race, then, the US has more than double the political, economic and institutional capacity, compared to the USSR, to adjust its actual deployment level at time t to a level where a desired balance between its own SLBM warhead levels and those it expected the Soviet Union was going to deploy at time t would obtain. This finding is not unreasonable given the US's stronger economic-industrial base and the longstanding US doctrinal preference for the SLBM basing model. But since $0 < S < 1$, US could not fully adjust its actual SLBM warhead levels at time t to their desired deployment levels at any t, nor could the Soviet Union. From Equations 6a and 8a, we can also calculate, respectively, the US Richardsonian grievance and defence parameters, B_0 and B_1 and the Soviet Richardsonian grievance and defence parameters B'_0 and B'_1. From Equation 6a

$$B_0 = SB_0/S = 2177.0/0.521 = 4178.5$$

$B_1 = SB_1/S = 0.226/0.521 = 0.43$

From Equation 8a

$B'_0 = S'B'_0/S' = -958.2/0.23 = -4166.08$

$B'_1 = S'B'_1/S' = 0.322/0.23 = 1.4$

Before I will attempt to analyze the meaning of the values obtained for the parameters B_0 and B_1, B'_0 and B'_1 from Equations 6a and 8a, I will verify their accuracy by estimating Equations 1a and 2a.

$$\mathbf{K}^*_t = B_0 + B_1\mathbf{X}_t \qquad \text{(USA)} \qquad (1a)$$

$$= \underset{\substack{(237.7)\\ **}}{4183.9} + \underset{\substack{(0.102)\\ **}}{0.43}\mathbf{X}_t$$

$R^2 = 0.5830$, n = 15. (Note: Equation 1a was estimated using OLS. Run Test results indicated no autocorrelation at a 0.05 level of significance: see Table 6.1 for test results).

$$\mathbf{X}^*_t = B'_0 + B'_1\mathbf{K}_t \qquad \text{(USSR)} \qquad (2a)$$

$$= -4285 + 1.43\mathbf{K}_t$$

$$\underset{**}{(1257)} \quad \underset{**}{(0.25)}$$

$R^2 = 0.7115$, n = 15. (Note: Equation 2a was estimated using OLS. Run Test results indicated no autocorrelation at a 0.05 level of significance: see Table 6.1 for test results).

The estimates of the Equations 1a and 2a coincide with the values obtained for B_1 and B'_1 from Equations 6a and 8a. In Equation 1a, $B_1 = 0.43$ and from Equation 2a, $B'_1 = 1.43$. Clearly, the Soviet Richardsonian defence coefficient, the defence context being the US-Soviet SLBM warhead race, is much larger than the US Richardsonian defence coefficient. This result is consistent with the Soviet Union's reappraisal of its military doctrine in the late 1960s (MccGwire, 1987, 1991). It reflects the heavy emphasis placed by the Soviets on the development of a sea based deterrent in the 1970s, as discussed above, as a hedge against some American technological break through which might render their ICBM impotent. In contrast, the US has long recognized the great disparities between US and Soviet technology and as such has felt secure with a numerically larger Soviet SLBM force. Indeed, under SALT I, the US agreed to give the Soviets a 30% higher ceiling on its SLBM deployment levels than it took on for itself. Nevertheless, as of 1987, the US had 5632 SLBM warheads deployed while the Soviet had only 3408 deployed. The Soviets have always been behind in this race.

The fact that the Soviet Richardsonian grievance terms, in Equation 8a and Equation 2a, were negative, and that the Soviet Union's GSR grievance term in Equation 22/24 was zero is worthy of consideration. The negative values of the Soviet Richardsonian grievance terms, in Equations 8a and 2a, may well be indicative of some internal constraint on the Soviet Union's ability to match American armament (SLBM warhead) levels, a constraint which is not reflected in the value of its stock adjustment coefficient S'. It acts to reduce the "match." Whatever this constraint may be, however, its impact is neutralized when the Soviet Union explicitly takes into account the US's SLBM warhead deployment program as indicated by the zero value of the Soviet Union's GSR grievance term. When negative, the constraint may well reflect the impact of Soviet doves on armament policy in the Soviet Union. The fact that it is ultimately neutralized may reflect the impact of Soviet hawks.

One of the most interesting aspects of the US-Soviet SLBM warhead race is its equilibrium properties. Equations 29 and 30 show the equilibrium SLBM deployment levels for the US and USSR.

$$\mathbf{K}^{e}_{t-1} = \frac{L_0(1 - L'_2) + L_1L'_0}{(1 - L_2)(1 - L'_2) - L'_1L_1} \qquad \text{(USA)} \qquad (29)$$

$$\mathbf{K}^{e}_{t-1} = \frac{2023(1 - 0.8) + (0.158)(0)}{(1 - 0.55)(1 - 0.8) - (0.158)(0.20)}$$

$$= 6928.08$$

$$\mathbf{X}^e_{t-1} = \frac{L'_0(1 - L_2) + L'_1L_0}{(1 - L_2)(1 - L'_2) - L'_1L_1} \qquad \text{(USSR)} \tag{30}$$

$$\mathbf{X}^e_{t-1} = \frac{0(1 - 0.55) + 0.207(2032)}{(1 - 0.55)(1 - 0.80) - 0.207(0.158)}$$

$$= 6928.08$$

Equilibrium in the US-Soviet SLBM warhead race, ($\mathbf{K}^e_{t-1}$, $\mathbf{X}^e_{t-1}$), then, occurs at the point (6928.08, 6928.08). That is, US and Soviet SLBM warhead deployments should converge to 6928.08 SLBM warheads each. The equilibrium point (6928.08, 6928.08) in the US-Soviet SLBM warhead race is, moreover, stable as indicated by the slopes of Equations 31 and 32 which give the equilibrium demarcation curves for the US and Soviet Union, respectively.

$$\mathbf{X}_{t-1} = -\frac{L_0}{L_1} - \frac{(L_2 - 1)}{L_1}\mathbf{K}_{t-1} \qquad \text{(USA)} \tag{31}$$

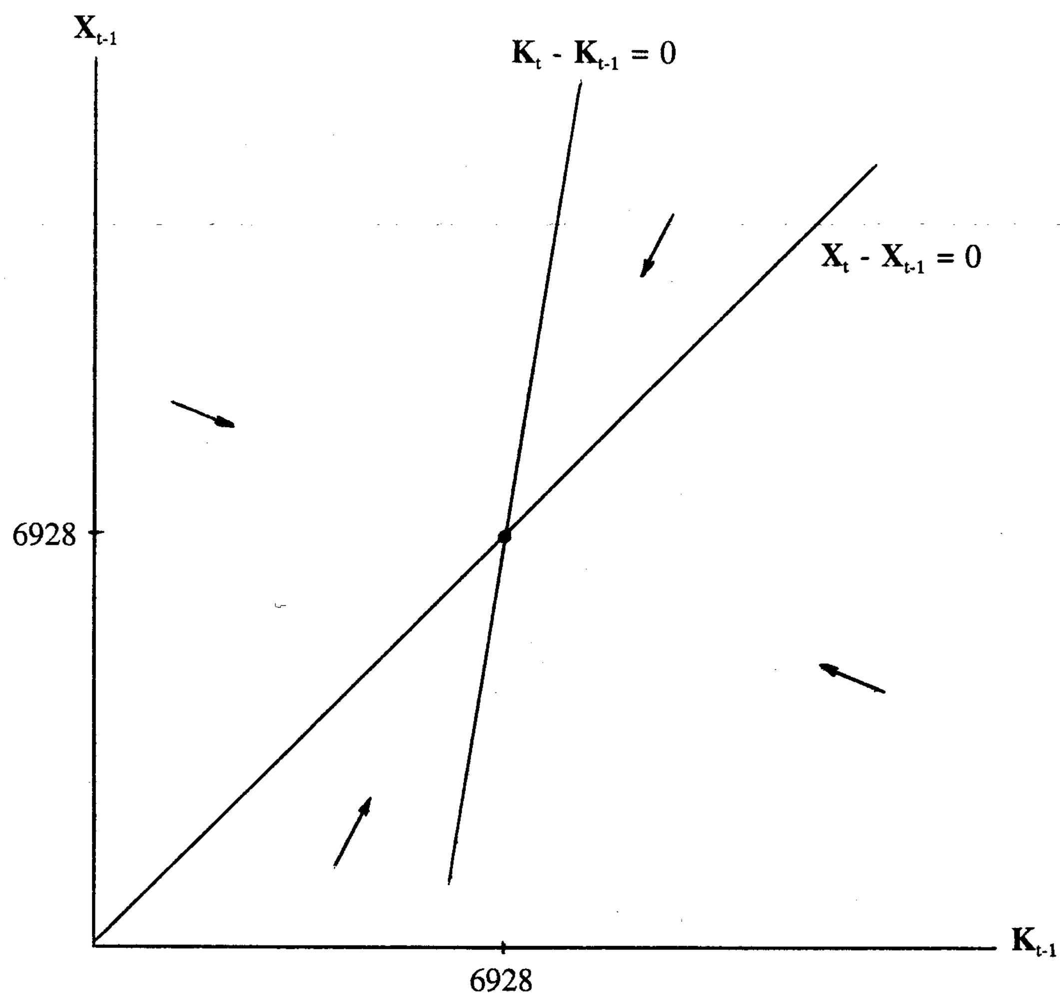
$$\mathbf{X}_{t-1} = -12803.7 + 2.8\mathbf{K}_{t-1}$$

$$\mathbf{X}_{t-1} = \frac{L'_0}{(1 - L'_2)} + \frac{L'_1}{(1 - L'_2)}\mathbf{K}_{t-1} \quad \text{(USSR)} \tag{32}$$

$$\mathbf{X}_{t-1} = 1.0\mathbf{K}_{t-1}$$

Graph 6.1 shows the plots of Equations 31 and 32. The plot suggests that the US-Soviet SLBM nuclear warhead race (1972-87) was stable. The robustness of this conclusion can be demonstrated by adding the estimate obtained for L_1, the US GSR defence term to its estimated standard error, the value obtained for L_2, the US GSR economic term, to its estimated standard error, L'_1, the Soviet GSR defence term to its estimated standard error and by adding L'_2, the Soviet GSR economic term to its estimated standard error and then recalculating Equations 31 and 32. If the SLBM race is still found to be stable, then the original conclusion that the race is stable is strengthened. An additional robustness test could be performed by subtracting, rather than adding, the estimated standard errors for the GSR parameters from the corresponding GSR parameter estimates and then recalculating Equations 31 and 32. Here, too, if the resulting calculations show that the US-Soviet SLBM warhead race is stable, then the original conclusion regarding the stable nature of the race is strengthened. In the latter instance, the data continue to show that the US-Soviet SLBM

GRAPH 6.1: Stable equilibrium in the US-Soviet SLBM warhead race, 1972-87



warhead race was stable. A stable node obtains. In the former instance, the recalculation of Equations 31 and 32 shows a saddle point. That is, the evidence, one way or another, is not conclusive. The original finding, then, that the US-Soviet SLBM warhead race (1972-87) was stable seems to be robust.

Substantively, it must be emphasized that Equations 31 and 32 were plotted on the basis of the values obtained for B_1 and S, the US defence and stock adjustment parameters, and B'_1 and S', the Soviet defence and stock adjustment parameters. The interplay of these values dictates that the Soviet Union would consider any point where it and the US have an equal number of SLBM warheads to be an equilibrium point. Because the Soviet Union's stock adjustment coefficient is so small, it could not surpass the US in SLBM warhead deployments as its high defence coefficient suggests it would like to. At best, Graph 6.1 indicates, the Soviets would be satisfied to match the US SLBM warhead for SLBM warhead. The US in contrast will consider any point where it has 0.35 SLBM warheads to every 1.0 Soviet SLBM warhead to be an equilibrium point.

6. THE ICBM WARHEAD RACE

The perfection of Intercontinental Ballistic Missile technology, such as it was in the late 1950s early 1960s, marked a turning point military history (Koenig, 1982). Tipped with a nuclear warhead, the ICBM provided its masters with the potential to do what had never been done before: obliterate an enemy's society without ever having to engage his military forces in the field.

In the beginning (1960) ICBM technology was relatively primitive. The first Soviet ICBM, the SS-6, was enormous. It had no less than 32 engines all of which had to be fired in concert if it was to get off the ground. The American Altas was not much smaller. The Atlas had to be lifted out of its silo and fuelled before it could be fired. Each carried a single, highly inaccurate warhead in the megaton range. The Atlas had a CEP of about 1800 meters and the SS-6 had a CEP of about 2500. By today's standards, these missiles were crude. Given the fact that they were inaccurate, their only real utility was city busting.

ICBM technology, however, advanced rapidly. Solid fuels were developed which was to reduce the time between the order to launch and the launch to minutes. Guidance technology improved making warheads more accurate. The second generation US ICBM, the Titan II, had a CEP of 1500 meters. The third generation Minutemen had CEPs of 350 meters. The newer Soviet SS-9s and SS-11s had a CEP of about 1300m. Secondly, a technique was developed which allowed first the US and then the Soviet Union to pack more than one warhead inside each side each missile. Later, the Americans developed a technique called MIRVing which allowed them to independently target multiple warheads. Why were these developments significant?

While both sides have encased their land based ICBMs in concrete and steel silos, they are not completely secure. In order to destroy a silo with a nuclear blast, the attacking warhead must land almost nearly on top of it in order to exert the maximum overhead pressure thereby shattering it and the missile within it. Warheads must, accordingly, be highly accurate and it may take several, detonated in concert, to actually destroy a silo. The effect of improved guidance technology and MIRV technology, then, was to make each side's ICBM forces became vulnerable to attack and destruction in a first-strike. At present, US silos are designed to withstand a maximum over head blast pressure of about 2000 psi. Concrete silos could not be made much stronger. The technical limit on concrete's ability to withstand overhead blast pressure is about 3000 psi (Davis and Schilling, 1973). In a nuclear war, ICBMs, because of their vulnerability, would be the first nuclear weapons launched by an aggressor and they would be the first of the victim's nuclear weapons to be destroyed.

From the Soviet point of view, these developments were at once both good and bad. Soviet military doctrine called for preemptive strikes against US nuclear forces (ICBMs) if nuclear war appeared imminent. However, since the US also possessed these same technologies (and in each case had them first), Soviet ICBM forces were also vulnerable to an American preemptive-strike. The Soviets had invested more heavily in their ICBM forces than they had in SLBM or long range bomber forces. Indeed the bulk of Soviet nuclear power is ICBM based. In order to reduce the actual and perceived probability that the Americans could successfully destroy their ICBM forces on the ground, the Soviets developed an anti-ballistic missile defence system, still employed to this day, and began to step up their ICBM deployments, basing them throughout their vast country, thereby complicating American war calculations by giving them more targets to hit. The Soviets ought reasonably to have tied their ICBM deployments to US ICBM deployments.

The Americans were faced with the same sorts of problems. Their ICBM forces might eventually also become vulnerable to a Soviet first strike. The Americans were to mount three responses. One was to defend their ICBM bases with anti-ballistic missile defence systems. An operational system was actually put into place, the Safeguard system, but was shut down in the 1970s. A second American response was to step up their ICBM deployments. A third American response to the growing vulnerability of their ICBM force was to begin basing more warheads at sea where they would be safe from a Soviet first strike. The US, in fact, had a preference for SLBM basing. Submarines are extremely difficult to detect and hence extremely difficult to destroy. Eventually, both sides would look to the sea as the safest basing mode for their nuclear forces. By the early 1970s, both the US and the Soviet Union were to step up the rate of SLBM deployment.

There is today a resurgence of interest in the ICBM. ICBM technology continued to progress through the deployment stasis of the 1970s and 80s. The US has recently deployed the 100 new MX missiles in refurbished (superhardened) Minute Man silos. The MX carries ten warheads and is said to have a CEP of 100m which would make it the most accurate

ICBM ever developed. The Soviets have finally developed mobile ICBM systems. They have recently deployed the railmobile SS-24 and the roadmobile SS-25. The SS-24 is MIRVed, carrying 7 to 10 warheads of 550kt each. Because these systems are mobile they will be extremely difficult for the Americans to target and destroy in the event of war.

To what extent did the US and Soviet Union tie their ICBM warhead deployment calculations in the 1960s to what each thought the other was going to deploy? The estimates of Equations 23/21 and 24/22 for US and Soviet ICBM warhead deployments over the years 1972-1987 are given immediately below.

$$\mathbf{K}_t = L_0 + L_1\mathbf{X}_{t-1} + L_2\mathbf{K}_{t-1} + \mathbf{Y}_t \quad \text{(USA)} \tag{23}$$

$$= \underset{\substack{(80.37)\\ **}}{206.18} + \underset{(0.120)}{0.0347}\mathbf{X}_{t-1} + \underset{\substack{(0.149)\\ **}}{0.843}\mathbf{K}_{t-1}$$

$R^2 = 0.9102$, $n = 12$ (Note Equation 23/21 was estimated using OLS. Run Test results indicate no autocorrelation at a 0.05 level of significance: see Table 6.1 for test results).

$$\mathbf{X}_t = L'_0 + L'_1\mathbf{K}_{t-1} + L'_2\mathbf{X}_{t-1} + \mathbf{Z}_t \quad \text{(USSR)} \tag{24}$$

$$= \underset{(72.61)}{-10.70} + \underset{\substack{(0.134)\\ **}}{0.254}\mathbf{K}_{t-1} + \underset{\substack{(0.109)\\ **}}{0.937}\mathbf{X}_{t-1}$$

$R^2 = 0.9676$, $n = 12$ (Note Equation 24/22 was estimated using OLS. Run Test results indicate no autocorrelation at a 0.05 level of significance: see Table 6.1 for test results).

What do the estimates of Equations 23/21 and 24/22 show?

The estimates of Equations 23/21 and 24/22, respectively, for US and Soviet ICBM warhead deployments over the years 1960-71 show an asymmetric arms race. The US GSR grievance and stock adjustment terms only are significant at 0.05. US ICBM warhead deployments at time t depend only upon 0.84 times US ICBM warhead deployments at time t-1 plus a grievance of 206.18. In contrast, the USSR GSR defence and stock adjustment terms are significant at 0.10 and 0.05 respectively. Soviet ICBM warhead deployments at time t depend upon 0.25 times US ICBM warhead deployments at time t-1 and 0.93 times Soviet ICBM warhead deployments at time t. In this case, the USSR is racing over ICBM warhead deployments with the US, while the US's ICBM warhead deployments are self-driven. Much can be learned about the nature of the US-Soviet ICBM competition from the estimates of Equations 6a and 8a.

$$\mathbf{K}_t = SB_0 + SB_1\mathbf{X}_t + (1 - S)\mathbf{K}_{t-1} \qquad \text{(USA)} \tag{6a}$$

$$= \underset{\substack{(80.6)\\ **}}{204.4} + \underset{(0.122)}{0.0240}\mathbf{X}_t + \underset{\substack{(0.169)\\ **}}{0.848}\mathbf{K}_{t-1}$$

$R^2 = 0.9097$, n = 12. (Note: Equation 6a was estimated using OLS. Run Test results indicate no autocorrelation at a 0.05 level of significance: see Table 6.1 for test results).

$$\mathbf{X}_t = S'B'_0 + S'B'_1\mathbf{K}_t + (1 - S')\mathbf{X}_{t-1} \qquad \text{(USSR)} \tag{8a}$$

$$= -31.805 + 0.225\mathbf{K}_t + 0.969\mathbf{X}_{t-1}$$

$$\underset{}{(97.487)} \quad \underset{*}{(0.152)} \quad \underset{**}{(0.113)}$$

$R^2 = 0.9633$, n = 12. (Note: Equation 8a was estimated using OLS. Run Test results indicate no autocorrelation at a 0.05 level of significance: see Table 6.1 for test results).

From the estimates of Equations 6a and 8a, we see that the US stock adjustment coefficient S = 1 - 0.84 = 0.16 and the Soviet stock adjustment coefficient S' = 1 - 0.96 = 0.04. Both the US and Soviet stock adjustment coefficients are very small. From the estimates of Equations 1a and 2a, we obtain values for the US and Soviet Richardsonian defence and grievance terms. The regression results were as follows.

$$\mathbf{K}^*_t = B_0 + B_1\mathbf{X}_t \qquad \text{(USA)} \tag{1a}$$

$$= 1295.7 + 0.1825\mathbf{X}_t$$

$$\underset{**}{(345.6)} \quad (0.416)$$

$R^2 = 0.0209$, n = 12 (Note: Equation 1a was estimated using OLS. Run Test results indicate no autocorrelation at a 0.05 level of significance: see Table 6.1 for test results).

$$\mathbf{X}^*_t = B'_0 + B'_1\mathbf{K}_t \quad \text{(USSR)} \tag{2a}$$

$$= -867.4 + 5.85\mathbf{K}_t$$

$$(2134) \quad \underset{**}{(2.40)}$$

$R^2 = 0.3977$, n = 12 (Note: Equation 2a was estimated using OLS. Run Test results indicate no autocorrelation at a 0.05 level of significance: see Table 6.1 for test results).

The estimates of Equations 1a and 2a show that the US Richardsonian defence coefficient, the defence context being the US-Soviet ICBM warhead race, is not significantly different from zero. The US Richardsonian grievance terms is, on the other hand, significant at 0.05 and has a value of 1295.7. In contrast, the Soviet Union's Richardsonian defence term is significant at 0.05 and has a value of 5.85. It Richardsonian grievance terms is not, however, significantly different from zero.

In order to get a more meaningful interpretation of these parameter estimates it would be useful to first summarize the fact that for the US B_0 is non-zero, B_1 is zero and S lies between zero and one. For the USSR, B'_0 is zero, B'_1 is non-zero and S' lies between zero and one. When these values are plugged into Equations 21 and 22, the GSR model, the following result obtains:

$$\mathbf{K}_t = SB_0 + (1 - S)\mathbf{K}_{t-1} + e_t \tag{21b}$$

$$X_t = (S'B'_1SB_0) + S'B'_1(1 - S)\mathbf{K}_{t-1} + (1 - S')\mathbf{X}_{t-1} + \ell_t \qquad (22b)$$

Equations 21b and 22b do show an asymmetric arms race. Here State X, in this case, the Soviet Union, does take into account State K's, the US's, defence policy parameters when calculating its ICBM warhead deployment requirements for time t. Equation 22b contains B_0 and S, the US's Richardsonian grievance and stock adjustment terms. Equation 21b shows, in contrast, that the US did not take account of any of the Soviet Union's defence policy parameters when it calculated its ICBM warhead deployment requirements for time t.

The underlying structure of this race, as suggested by Equations 21b and 22b, is not inconsistent with historical, technological and doctrinal developments in the US and USSR in the 1960s. By 1960, the US had not only developed and deployed ICBM systems it had developed and deployed SLBM systems. The Soviet Union, in contrast, was not able to begin deploying SLBMs until 1963 (Military Balance, 1969-70). ICBMs were the Soviet's main nuclear force and during the 1960s, Soviet military doctrine called for preemptive nuclear strikes against US landbased nuclear forces if it appeared nuclear war was imminent. This would mean that the Soviets would have to take account of the size of the US ICBM force when determining their own ICBM warhead deployment requirements, as Equation 22b suggests that they have. The US, on the other hand, operating under a second strike doctrine, had a preference for SLBMs. It would be less important for the US to maintain an ICBM force numerically large enough to survive a first strike by Soviet ICBM forces. A second strike mission could easily be carried out by the US's SLBM force. In this sense, the US could maintain its deterrent without tying their ICBM warhead deployment calculations to expected Soviet ICBM warhead deployments. This is clearly supported by the construction of Equation 21b. US ICBM warhead deployments were internally driven. That is not to say, however, that the US was oblivious to Soviet ICBM warhead deployments as the US did have a non-zero Richardsonian grievance term, B_0.

Equations 29 and 30 show the equilibrium ICBM deployment levels for the US and USSR.

$$\mathbf{K}^e_{t-1} = \frac{L_0(1 - L'_2) + L_1 L'_0}{(1 - L_2)(1 - L'_2) - L'_1 L_1} \qquad \text{(USA)} \tag{29}$$

$$\mathbf{K}^e_{t-1} = \frac{206.18(1-0.93) + 0(0)}{(1 - 0.84)(1 - 0.93) - (0.25)(0)}$$

$$= 1288.39$$

$$\mathbf{X}^e_{t-1} = \frac{L'_0(1 - L_2) + L'_1 L_0}{(1 - L_2)(1 - L'_2) - L'_1 L_1} \qquad \text{(USSR)} \tag{30}$$

$$\mathbf{X}^e_{t-1} = \frac{0(1-0.84) + 0.25(206.18)}{(1 - 0.84)(1 - 0.93) - 0.25(0)}$$

$$= 4602.23$$

Equilibrium in the US-Soviet ICBM warhead race, ($\mathbf{K}_{t-1}$, $\mathbf{X}_{t-1}$), then, occurs at the point (1288.39, 4602.23). The point (1288.39, 4602.23) is, moreover, stable. This can be shown with Equations 31 and 32 give the equilibrium demarcation curves for the US and Soviet Union, respectively.

$$\mathbf{X}_{t-1} = -\frac{L_0}{L_1} - \frac{(L_2 - 1)}{L_1}\mathbf{K}_{t-1} \qquad \text{(USA)} \tag{31}$$

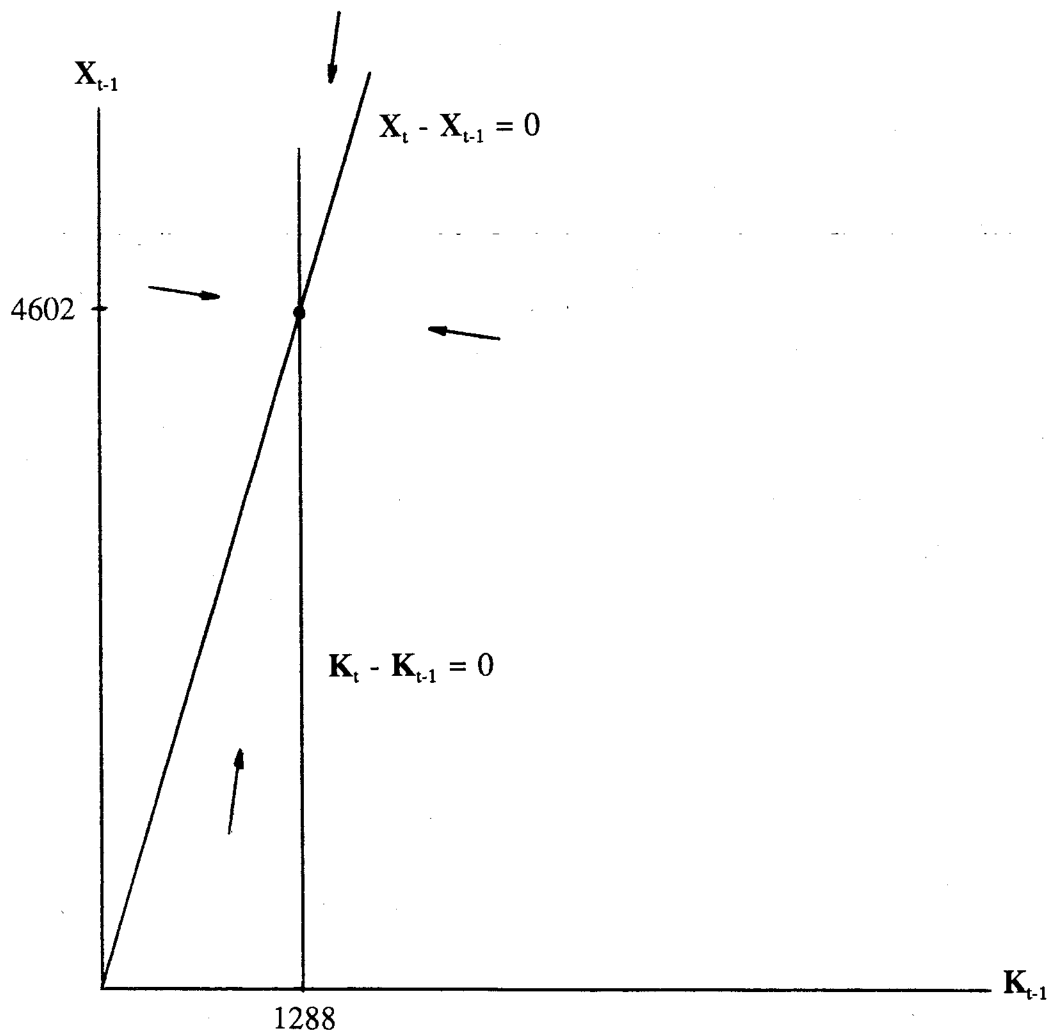
$$\mathbf{X}_{t-1} = \text{oo}\mathbf{K}_{t-1}$$

$$\mathbf{X}_{t-1} = \frac{L'_0}{(1 - L'_2)} + \frac{L'_1}{(1 - L'_2)}\mathbf{K}_{t-1} \qquad \text{(USSR)} \tag{32}$$

$$\mathbf{X}_{t-1} = 3.57\mathbf{K}_{t-1}$$

Graph 6.2 shows the plots of Equations 31 and 32. One can conclude from this plot that the

GRAPH 6.2: Stable equilibrium in the US-Soviet ICBM warhead race, 1960-71



US-Soviet ICBM warhead race (1960-71) was stable. That this conclusion is robust is suggested by the fact that if one were to add the estimated standard errors of the GSR parameter estimates to the corresponding GSR parameter estimates and then, using that data, recalculate Equations 31 and 32, the result would still indicate that the US-Soviet ICBM warhead race was stable. The same conclusion would still hold if one were to subtract the estimated standard errors of the GSR parameter estimates from the corresponding GSR parameter estimates.

Substantively, Graph 6.2 suggests is that the US would be satisfied with 1288 ICBM warheads deployed irrespective of the number deployed by the Soviet Union. The Soviets, in contrast, wanted numerical superiority, on the order of 3.57 to 1, as indicated by the slope of Equation 32. The ratio 3.57 to one may reflect Soviet calculations as to what they deemed they needed to have a first strike capability vis-a-vis the US ICBM force. The US position is not an unreasonable one given that it had a substantial warhead force kept onboard its submarines which the Soviets could not destroy and which could be used in retaliation for a Soviet strike on their ICBMs.

7. TOTAL STRATEGIC WARHEAD COUNTS

The findings recorded above that the US and Soviet Union were engaged in an asymmetric arms race over ICBM warhead deployments 1960-71 and an action reaction arms race over SLBM warhead deployments over the years 1972-87 lend a great deal of support for the validity of the micro approach to mathematical arms race analysis. States do, these findings suggest, compete over individual weapons systems with cross-purposes. It would, now, be useful to conduct a test of total US and Soviet strategic nuclear warhead levels (ICBM, SLBM and bomber based warhead counts) for evidence of interactive deployment. A finding of action-reaction interaction in US-Soviet aggregate strategic nuclear warhead deployments would undermine the validity of the micro approach to mathematical arms race analysis and a finding of non-interaction would add further support for the approach.

I tested my GSR model against a data set comprised of aggregate US and Soviet warhead deployments (the sums of ICBM, SLBM and Bomber warheads for each country) running from the period 1967 to 1984. The time span for this study, 1967-84, is arbitrary. If the US and Soviet Union do compete over aggregate strategic nuclear warhead deployments, then any sample drawn from period 1945-91 should show interaction.

The figures contained in this data set are official US estimates, compiled by SIPRI. It is important to note that over the years, the US has amended its published count of US and Soviet aggregate strategic nuclear warheads for the year 1976. In 1976, it listed 8900 warheads for the US and 3500 warheads for the Soviet Union. Later, in 1981, the US relisted the 1976 count as 8400 for the US and 3300 for the Soviet Union. Which set of values should be used? Ostrom (1978b) and Ostrom and Marra (1986) make the point that arms race models should be tested using the data that the actors in question would have had at their disposal at the time that they were making their armament decisions. This is an important point because, in general, my model suggests that the US would base a 1977 weapons deployment decision on what it and the Soviet Union had deployed in 1976. In the specific case at hand, in 1977, the US would have been working with the figures 8900 warheads for the US and 3500 warheads for the Soviet Union. The amended figures of 8400 for the US and 3300 for the Soviet Union were not available in 1977. Thus in testing my GSR model, I will use the figures of 8900 warheads for the US and 3500 warheads for the Soviet Union.

I estimated the simplified versions of Equations 21 and 22, Equations 23 and 24.

$$\mathbf{K}_t = L_0 + L_1\mathbf{X}_{t-1} + L_2\mathbf{K}_{t-1} + \mathbf{Y}_t \quad \text{(USA)} \tag{23}$$

$$= 577.91 - 0.018\mathbf{X}_{t-1} + 0.971\mathbf{K}_{t-1}$$

$$\underset{}{(499.2)} \quad (0.08) \quad \underset{**}{(0.097)}$$

$R^2 = 0.9629$, n = 17, ** = significant at 0.05, * = significant at 0.10. (Note: Equation 23/21 was estimated using OLS. Run Test results indicate no autocorrelation at a 0.05 level of significance: see Table 6.1 for test results).

$$\mathbf{X}_t = L'_0 + L'_1\mathbf{K}_{t-1} + L'_2\mathbf{X}_{t-1} + \mathbf{Z}_t \quad \text{(USSR)} \tag{24}$$

$$= -263.37 + 0.117\mathbf{K}_{t-1} + 0.967\mathbf{X}_{t-1}$$

$$(489.5) \quad (0.095) \quad \underset{**}{(0.0826)}$$

$R^2 = 0.9706$, n = 17 (Note: Equation 24/22 was estimated using OLS. Run Test results indicate no autocorrelation at a 0.05 level of significance: see Table 6.1 for test results).

Accordingly, the estimates of the GSR parameters in Equations 23/21 and 24/22 suggest a non-interaction outcome in the US-Soviet aggregate warhead race. In Equation 23, only the US GSR stock adjustment term is significant, at 0.05. US aggregate warhead deployments at time period t are a function of US aggregate warhead deployments at time t-1 only. Similarly, in Equation 24, only the US's GSR stock adjustment coefficient is significant, at 0.05. Soviet aggregate warhead deployments at time period t-1 depend only on Soviet aggregate warhead deployments at time t-1.

The estimates of Equations 6a and 8a tend to conform well with the estimates of Equations 23 and 24. Estimates of Equations 6a and 8a also show non-interaction.

$$\mathbf{K}_t = SB_0 + SB_1\mathbf{X}_t + (1 - S)\mathbf{K}_{t-1} \quad \text{(USA)} \qquad \text{(6a)}$$

$$= \underset{(503)}{548.47} - \underset{(0.082)}{0.02}\mathbf{X}_t + \underset{\substack{(0.101)\\ **}}{0.981}\mathbf{K}_{t-1}$$

$R^2 = 0.9641$, n = 17 (Note: Equation 6a was estimated using OLS. Run Test results indicate no autocorrelation at a 0.05 level of significance: see Table 6.1 for test results).

$$\mathbf{X}_t = S'B'_0 + S'B'_1\mathbf{K}_t + (1 - S')\mathbf{X}_{t-1} \quad \text{(USSR)} \tag{8a}$$

$$= -196.76 + 0.093\mathbf{K}_t + 0.987\mathbf{X}_{t-1}$$

$$(521.01) \quad (0.093) \quad \underset{**}{(0.079)}$$

$R^2 = 0.9696$, n = 17 (Note: Equation 8a was estimated using OLS. Run Test results indicate no autocorrelation at a 0.05 level of significance: see Table 6.1 for test results).

Estimates of Equations 6a and 8a suggest that both the US and Soviet Union have extremely small stock adjustment coefficients, the defence context being aggregate strategic nuclear warhead deployments. The US stock adjustment coefficient is S = 0.019 and the Soviet coefficient is S' = 0.013. As suggested in Chapter 5, a non-interaction outcome can occur if the rivals in question have extremely small (if not zero) stock adjustment terms or if the rivals both have zero Richardsonian defence coefficients or both. We can determine which is the case by estimating Equations 1a and 2a.

The estimates of Equations 1a and 2a are given immediately below.

$$\mathbf{K}_t = B_0 + B_1\mathbf{X}_t \quad \text{(USA)} \tag{1a}$$

$$= 29082 - 1.37\mathbf{X}_t$$

$$\underset{**}{(11867)} \quad (2.37)$$

$R^2 = 0.0218$, n= 17. Equation 1a was estimated using OLS. Run Test results indicate no autocorrelation at a 0.05 level of significance: see Table 6.1 for test results).

$$X_t = B'_0 + B'_1\mathbf{K}_t \qquad \text{(USSR)} \qquad (2a)$$

$$= -15350 + 7.2\mathbf{K}_t$$

$$(34066) \quad \underset{*}{(4.34)}$$

$R^2 = 0.1567$, n = 17. Equation 2a was estimated using OLS. Run Test results indicate no autocorrelation at a 0.05 level of significance: see Table 6.1 for test results).

The estimate of Equation 1a suggests that the US has a zero Richardsonian defence coefficient and a non-zero Richardsonian grievance term. The estimate of Equation 2a, on the other hand, suggests that the Soviet Union has a non-zero Richardsonian defence term and a zero grievance term. What these findings suggests is that the US does not set strategic nuclear policy on the basis of total capability. Total warhead counts do, in contrast, seem to enter into the Soviet strategic calculus although their extremely small adjustment coefficient prevents an interactive response on their part to total US strategic nuclear warhead deployments. In effect, the US seems to address the total strategic nuclear problem on a weapons systems by weapons systems basis as suggested by estimates of the US-Soviet SLBM and ICBM warhead races.

Estimates of Equations 1a, 2a, 6a, 8a, 23 and 24 suggest that the US and Soviet Union were not engaged in an action-reaction competition over total strategic warhead deployments. The extremely small stock adjustment coefficients obtained for each side may suggest an inability or disinclination on the part of the US and Soviet Union to react to each other's total strategic strength as a single package with a single response. Indeed, the historical record shows that the preference for one nuclear weapons basing mode over another in the US and Soviet Union rises and falls, in part, in concert with technological change. For example, by 1960, US bomber forces lost standing as ICBM technology developed. And ICBM forces lost standing as SLBM technology came to fruition. It may be that the total strategic count does not constitute a degree of weapons system individuation sufficient to constitute a basis for action-reaction interaction between the US and Soviet Union.

In the next section, I will, on the basis of estimates obtained for the US-Soviet SLBM, ICBM, and aggregate strategic nuclear warhead competitions, attempt to determine which of Propositions 1-4, derived in the last chapter, concerning the arming behavior of states, hold.

8. PROPOSITIONS 1-4.

Did the US form its expectations of its future SLBM and ICBM nuclear warhead deployment requirements and its expectations of the Soviet Union's future SLBM and ICBM warhead deployment behavior in accordance with the Rational Expectations hypothesis? Did Soviet behavior also accord with the Rational Expectations hypothesis? These questions can be answered by referring to the value corr($\mathbf{Y}_{t-i}$, $\mathbf{Z}_{t-j}$), the correlation between the US and the Soviet Union's GSR error terms, for each of the specified races, as per Propositions 1 and 1.1, derived in the previous chapter.

Corr($\mathbf{Y}_t$, $\mathbf{Z}_t$) for the US-Soviet SLBM race (1972-87) race is 0.3245. This correlation is not statistically significant at even 0.10. That is, the errors in the US-Soviet SLBM warhead deployment race are not contemporaneously correlated. However, a systematic evaluation of the correlation between all possible combinations of $\mathbf{Y}_t$ and $\mathbf{Z}_t$ and lagged values of $\mathbf{Y}_t$ and $\mathbf{Z}_t$ up to t-2 reveals that corr($\mathbf{Y}_t$, $\mathbf{Z}_{t-2}$) = -0.414. This correlation is significant at 0.10. Corr($\mathbf{Y}_t$, $\mathbf{Z}_{t-1}$), corr($\mathbf{Y}_{t-1}$, $\mathbf{Z}_t$), corr($\mathbf{Y}_{t-2}$, $\mathbf{Z}_t$), corr($\mathbf{Y}_{t-1}$, $\mathbf{Z}_{t-1}$) and corr($\mathbf{Y}_{t-2}$, $\mathbf{Z}_{t-2}$) were found to not be statistically significant.

Corr($\mathbf{Y}_t$, $\mathbf{Z}_t$) = 0.0733 in the US-Soviet ICBM warhead race (1960-71). This correlation is not significant at 0.10. But, again, a systematic evaluation of the correlation between all possible combinations $\mathbf{Y}_t$ and $\mathbf{Z}_t$ and lagged values of $\mathbf{Y}_t$ and $\mathbf{Z}_t$ up to t-2 does reveal a significant correlation: corr($\mathbf{Y}_{t-1}$, $\mathbf{Z}_t$) = -0.48 which is significant at 0.10. Corr($\mathbf{Y}_{t-2}$, $\mathbf{Z}_t$), corr($\mathbf{Y}_t$, $\mathbf{Z}_{t-1}$) corr($\mathbf{Y}_t$, $\mathbf{Z}_{t-2}$), corr($\mathbf{Y}_{t-1}$, $\mathbf{Z}_{t-1}$) and corr($\mathbf{Y}_{t-2}$, $\mathbf{Z}_{t-2}$) are not significant.

Substantively, these results suggest that inefficiencies exist in US and Soviet reactions to newly received information regarding innovations in the other's SLBM and ICBM warhead deployment behavior. The non-zero value of Corr($\mathbf{Y}_t$, $\mathbf{Z}_{t-2}$), from the SLBM warhead deployment race, suggests that the US needed two years to respond to an innovation in Soviet SLBM warhead deployment behavior. The Soviets, on the other hand, seemed to have needed more than two years to respond to an innovation in American SLBM warhead deployment behavior. The negative value of corr($\mathbf{Y}_t$, $\mathbf{Z}_{t-2}$), furthermore, suggests that when some shock caused the Soviet Union to lower its SLBM warhead deployments for a given year below a previously planned level, the US responded two years later by setting its SLBM warhead deployment level above what it otherwise had planned for that year. In short, whatever the Soviet the action, the US reaction was to do the opposite.

The non-zero value of corr($\mathbf{Y}_{t-1}$, $\mathbf{Z}_t$) for the US-Soviet ICBM warhead competition suggests that the Soviet Union required one year to respond to innovations in US ICBM deployment behavior. The US required more than two years to respond to innovations in Soviet ICBM warhead deployment behavior. The negative value of corr($\mathbf{Y}_{t-1}$, $\mathbf{Z}_t$) suggests that US and Soviet behavior was opposite.

In short, these results suggest that Proposition 1.1 holds. The US and Soviet Union did form their expectations of their own armament requirements and of the other's arming behavior in accordance with the Rational Expectations hypothesis, but bureaucratic and technological inefficiencies existed in each side's ability to transform newly received information regarding innovations in the other's arming behavior into changes in each side's own behavior.

It must be stated at this point that such inefficiencies in information transformation could also be indicative of an adaptive expectations formation process, as discussed in Chapter 5. The existence of a non-contemporaneous cross correlation in State K and X's prediction errors is only a necessary, but not sufficient, condition for Rational Expectations. What is more certain is that because these inefficiencies do exist, the US-Soviet nuclear arms race is not driven by prediction errors alone.

It was, accordingly, revealed that the US and Soviet Union were engaged in an action-reaction arms race over SLBM warhead deployments (1972-87). In this instance, the US and Soviet Richardsonian defence terms were both non-zero and that their stock adjustment terms both lay between zero and one. Accordingly, in the case of the US-Soviet SLBM warhead race, Proposition 3.0 holds.

The US-Soviet ICBM warhead race is an interesting case. Estimates of Equations 21 and 22 showed the US and Soviet Union to be engaged in an asymmetric arms race over ICBM warhead deployments. The USSR's Richardsonian defence term was non-zero and its stock adjustment term lay between zero and one. The US's Richardsonian defence term, on the other hand, was zero and its stock adjustment term fell between zero and one. Thus Proposition 2.1, as set out in the previous chapter, does hold in the case of the US-Soviet ICBM warhead race (1960-71).

Finally, an analysis of US-Soviet aggregate strategic nuclear warhead deployments (bombers, ICBMs and SLBMs) data (1967-84) showed no interaction. The US Richardsonian defence coefficient was zero and its stock adjustment coefficient was so small as to be nearly zero. The Soviet Richardsonian defence coefficient was non-zero, and its stock adjustment coefficient was even small than the US coefficient. These conditions are those set out in Proposition 4.2 for the occurrence of non-interaction.

Estimates of my GSR model for the US-Soviet SLBM, ICBM and aggregate strategic nuclear warhead competitions are summarized in Table 6.2.

9. SUMMARY

The findings in this chapter reverse much of the previous quantitative work on the US-Soviet nuclear arms race. For the most part, the existence of a US-Soviet nuclear arms race could not be confirmed through quantitative analysis. This result has always been puzzling given the theoretical connection between the maintenance of nuclear deterrence and nuclear arms racing. Tests of my GSR model, on the other hand, suggest that the US and Soviet Union were engaged in an action reaction arms race over SLBM warhead deployments over the years 1972-87, and an asymmetric arms race over ICBM warhead deployments from 1960-71. Tests of Equations 21 and 22 for US and Soviet aggregate warhead deployments, in contrast, showed no interaction. In the next chapter, I will attempt to assess why my study was more successful than previous attempts to uncover the interactive components of the US-Soviet nuclear competition.

TABLE 6.2: GSR and Richardsonian parameter estimates for the US-Soviet nuclear warhead competition. (Note: parameters significant at 0.05 are marked **. Those which are significant at 0.10 are marked *)

	SLBM Warheads 1972-87	ICBM Warheads 1960-71	Total Warheads 1967-84
GSR Grievance Term			
USA	2023 (432.68) **	201.6 (80.37) **	577.9 (499)
USSR	-399 (323.49)	-10.7 (72.61)	-263.37 (489)
GSR Defence Term			
USA	0.158 (0.1) *	0.03 (0.12)	-0.018 (0.0842)
USSR	0.20 (0.097) **	0.25 (0.134) **	0.1172 (0.09)
GSR Stock Adjustment Term			
USA	0.550 (0.130) **	0.843 (0.149) **	0.9717 (0.09) **
USSR	0.80 (0.087) **	0.937 (0.109) **	0.9677 (0.08) **

TABLE 6.2 continued

TABLE 6.2 (Continued): GSR and Richardsonian parameter estimates for the US-Soviet nuclear warhead competition. (Note: Parameters which are significant at 0.05 are marked **. Those which are significant at 0.10 are marked *.)

	SLBM Warheads 1972-87	ICBM Warheads 1960-71	Total Warheads 1967-84
Richardsonian Grievance Term			
USA	4183 (237.72) **	1295 (345.6) **	29082 (11867) **
USSR	-4285 (1257) **	-867 (2134.5)	-15350 (34066)
Richardsonian Defence Term			
USA	0.43 (0.102) **	0.18 (0.416)	-1.3 (2.37)
USSR	1.43 (0.25) **	5.85 (2.40) **	7.26 (4.34) *
Stock Adjustment Coefficient			
USA	0.521	0.16	0.019
USSR	0.23	0.04	0.013
Form of Interaction	Action Reaction	Asymmetric Reaction	No Reaction

My data, furthermore, suggest that the SLBM race was stable with equilibrium occurring at 6928 SLBM warheads for the US and 6928 SLBM warheads for the Soviet Union. The ICBM warhead competition, too, was found to have been stable with an equilibrium point of 1288 warheads for the US and 4602 for the Soviet Union.

Error analysis tentatively suggests, finally, that the US and Soviet Union formed expectations of each other's future nuclear weapons arming behavior in accordance with the Rational Expectations hypothesis. But each side's reactions were non-contemporaneous to stimulation from the other. Reaction time may have been slowed due to bureaucratic interia or to the time needed to develop new weapons systems. My error analysis, nevertheless, suggests a two-way linkage existed between nuclear weapons deployment policy in the US and nuclear weapons deployment policy in the USSR. One would expect such a linkage to have existed if mutual nuclear deterrence was to exist and be maintained.

In the next and final chapter, I will summarize the overall achievements of my research effort and raise a number of questions, suggested by my research, for future consideration.

CHAPTER VII: SUMMARY

In this, the final chapter of my dissertation, I will assess the overall contributions of my research effort to mathematical arms race research in general and to the quantitative study of the US-Soviet arms race in particular. In this dissertation, I rejected the traditional macro approach to arms race analysis. I specified and then applied a new approach, which I have termed the micro approach. Did I uncover anything new about the nature of the US-Soviet arms race with this approach? Could we have learned as much using the macro approach?

To summarize, the micro approach to arms race analysis revolves around the idea that states engaged in an arms race, in particular the US and Soviet Union, compete on a weapons system versus a cross-purpose weapons system basis. My GSR model, reproduced immediately below, was formulated on that basis and thus constitutes an appropriate analytical framework for an analysis of the US-Soviet arms race.

$$\mathbf{K}_t = \frac{(SB_0 + SB_1S'B'_0)}{1-SB_1S'B'_1} + \frac{SB_1(1 - S')}{1-SB_1S'B'_1}\mathbf{X}_{t-1} + \frac{(1 - S)}{1-SB_1S'B'_1}\mathbf{K}_{t-1} + \mathbf{Y}_t \qquad (21)$$

$$X_t = \frac{(S'B'_0 + S'B'_1SB_0)}{1-SB_1S'B'_1} + \frac{S'B'_1(1 - S)}{1-SB_1S'B'_1}K_{t-1} + \frac{(1 - S')}{1-SB_1S'B'_1}X_{t-1} + Z_t \qquad (22)$$

Other formulations, yet to be derived, may be as suitable. Finally, time frames for any weapons system, cross-purpose weapons system analysis must be set to accord with the historical period in which those systems were dominant in US-Soviet military calculations.

The macro approach, in contrast, is based on the view that states engaged in an arms race compete on the basis of total armed might versus total armed might. A state's annual aggregate military expenditures are, generally, thought to constitute a good indicator of its total armed might. Models, then, are most often tested against such data. Analytical time frames are generally set to cover as lengthy a period as possible. No reasons are normally given for so setting analytical time frames, but one can surmise that it is done in order to achieve statistical consistency in arms race parameter estimations.

1. COMPARING THE OLD AND THE NEW

As I stated at the outset of the previous chapter, how one approaches the study of the US-Soviet arms race determines, in large measure, what one will find.

My application of the micro approach to the US-Soviet nuclear arms has yielded strong evidence that the US and Soviets were engaged in an action-reaction arms race over SLBM warhead deployments over the years 1972-87. This finding stands in marked contrast to finding reported by Kugler et al (1980) in their detailed macro study of the US-Soviet nuclear arms race. They could find no quantitative evidence of action-reaction in US-Soviet aggregate expenditures on strategic nuclear forces over the years 1952-76. Kugler et al (1980: 128), accordingly, stated that

> The absence of competition or races between the two countries [the US and USSR] leads to the startling finding that the logical conditions for deterrence are absence, and by inference to the conclusion that, mutual deterrence is not taking place.

They further state

> In the absence of competition, furious nuclear arms stockpiling is not easy to evaluate. While it may not be appropriate to think of the process whereby the two countries acquire strategic weapons as 'neurotic', it is the term that nevertheless comes to mind. 'Neurotic', then, may have to do [p. 131].

My findings, in contrast to Kugler et al (1980), permit the intuitively appealing conclusion that US-Soviet nuclear arms acquisitions, at least US-Soviet SLBM warhead acquisitions, were systematically driven, specifically, action-reaction driven. The logical conditions for the maintenance of strategic deterrence, one may accordingly surmise, were in fact being met.

Who's findings should one accept: mine or Kugler et als? The answer to this question rests, in part, on the theoretical validity of the macro approach as compared to the micro approach to arms race analysis. Secondly, one must also weigh the findings generated by the micro approach against the findings generated by the macro approach. Which best accord with past experience, informed intuition and established theory?

The macro approach is based, to reiterate, on the assumption that states engaged in an arms race compete on the basis of total armed might versus total armed might. In fact, the historical record is replete with examples of the US and Soviets each justifying their acquisitions on a weapons system, cross-purpose weapons system basis. ICBM deployments, for example, were originally justified on this basis.

Secondly, it was reasonable to find that the US and Soviets were engaged in action-reaction arms race over SLBM warheads in the period 1972-87 and it was reasonable to find that they were engaged in an asymmetric race over ICBM warhead deployments over the period 1960-71. The period 1972-87 was one where both sides, at the same time, placed a strong emphasis on having second strike reserve forces. Given the vulnerability of longrange bombers to interception and given the vulnerability of the ICBM to preemptive attack, the

SLBM basing mode provided the best protection for each side's nuclear warheads.In the worst case, where each side's bomber and ICBM forces were decimated in a series of counter-force, counter-counter force strikes, then the side which had the most effective, if not the largest, SLBM warhead force would be in a position to dictate terms to the other. Action-reaction interaction, in this context, is reasonable. Similarly, one should expect to find that the USSR was racing against the US over ICBM warhead deployments in 1960-71, and the Americans racing themselves. Soviet doctrine, over that period, called for preemptive strikes against US ICBM forces in the event that nuclear war appeared imminent. In order to for the Soviet Union to be able to effect this mission, Soviet ICBM forces would have to be kept in some mathematical proportion to US ICBM forces. (For most of the period 1960-71, both US and Soviet ICBMs carried only one warhead each. Thus the missile count and the warhead count, for most of that period, were one and the same.) The US in the period 1960-71, in contrast, focused its efforts on developing a secure second-strike force. They did not think the ICBM basing mode to be particularly secure. Instead, the US invested heavily in SLBMs in the period 1960-71. It is, finally, interesting to note that tests of my GSR model against US-Soviet aggregate warhead deployments showed no interaction, that is, no race. Indeed, a macro analysis of the US-Soviet nuclear arms race masks any notion of an action-reaction SLBM warhead race (1972-87) and an asymmetric ICBM warhead race (1960-71).

My contribution to the field of mathematical arms race research, then, is a case for reorienting away from macro analysis to the micro approach to mathematical arms race analysis which I have specified in this dissertation.

2. QUESTIONS FOR FUTURE ARMS RACE RESEARCH

Two sets of questions emerge from my theoretical and empirical arms race analysis. The first set of questions concerns the role of Rational Expectations formation in the US-Soviet arms competition. In their studies of the impact of Rational Expectations formation on the US-Soviet arms competition, Williams and McGinnis (1988) and McGinnis and Williams (1989) concluded that the non-interaction and asymmetric reactions puzzles were not puzzles at all. Under Rational Expectations, they argued, the US and Soviet Union would not react in the current time period to each other's previous period military expenditures. Instead, the US and the Soviet Union would, in the current period, react only to innovations, deviations, from the other's planned level of military expenditure. Only if the US and Soviet Union were inefficient in their collection and use of information regarding innovations in the other's behavior would past period expenditures play a role in determining current period expenditures. Williams and McGinnis thought that such inefficiencies would be negligible, however.

On the basis of their findings, Williams and McGinnis (1988: 992) suggested that arms race research turn away from the attempt to

> verify empirically the mere existence of reaction [in the US-Soviet arms competition] to focus instead on a broader set of research questions concerning the consequences of the intimate linkage between domestic policy processes of rival states.

I concur with Williams and McGinnis that we should now focus attention on the consequences of the linkages between US and Soviet arms decision making processes. In particular, we should begin to focus on questions such as, What specific information did the US and USSR draw on when forming their expectations of each other's future arming goals and strategies. To what extent did each side factor information used by the other in its armament calculations into its own calculations? For example, public opinion, as Williams and McGinnis note, has an impact on US military expenditure levels. To what extent did the Soviets take that opinion into account when formulating their estimates of future US arming goals and strategies?

In contrast to Williams and McGinnis, however, I suggest, based on my research effort, that efforts to uncover the existence of action-reaction interaction in US-Soviet arms acquisitions be stepped. The search, then, should focus on action-reaction in the deployment of individual weapons systems with cross-purposes. McCubbins (1983) began this process with an analysis of the US-Soviet conventional arms competition. I carried on the search with an analysis of the US-Soviet strategic nuclear arms competition. Tests should now be conducted to determine if US-Soviet short range nuclear weapons acquisitions follow the weapons system-cross purpose weapons system deployment pattern established at the conventional and strategic nuclear levels. From such efforts, we would have an invaluable base from which to assess the overall nature and dynamics of the US-Soviet military competition, arguably, the most important military competition in human history. There are, however, certain issues which must be resolved before we could set about sketching a picture of the overall nature and dynamics of the US-Soviet military competition.

Under the macro perspective where arms races are seen as competitions of total armed might versus total armed might, the meaning of the term "arms race equilibrium" is straight forward. "Arms race equilibrium" refers to overall equilibrium in the military forces of the contending parties. But when arms races between rival states are viewed as occurring over individual weapons systems with cross purposes, and when, theoretically, any given pair of rivals could thus engage each other in several distinct system level arms races, the meaning of the term arms race equilibrium becomes less clear. Theoretically, each individual system level competition can have its own distinctive dynamics and its own distinctive equilibrium point. We have seen this to be the case with respect to the US-Soviet SLBM warhead and the US-Soviet ICBM warhead races. Can we, then, speak of overall arms race stability or instability if, for any given pair of rivals, we find that some individual system level races are stable while others are unstable. How do the dynamics of one individual system level competition between two rivals affect other individual level competitions between the same rivals? How do these questions impinge on the theory that unstable arms races lead competing states inevitably to war? These are extremely difficult questions, which must be answered before a sketch of the overall US-Soviet arms competition, based on system level analyses, can be drawn. Their importance merits them a position at the top of the arms race research agenda.

3. THE MICRO APPROACH TO MATHEMATICAL ARMS RACE ANALYSIS AND THE END OF THE COLD WAR.

With the end of the US-Soviet Cold-War, one might question the continued relevance of the micro approach to mathematical arms race analysis as a tool of international analysis. It is true that East-West relations have become much friendlier in recent years. It is true that Eastern Europe, set free by Moscow, is now in the process of democratizing itself. It is also true that East and West have recently signed a major European conventional reduction treaty and major strategic arms reduction agreement.

But the US-Soviet military, political and economic rivalry was not an isolated event. Rivalry, aggressive intentions and aggression itself have been the hallmarks of international interaction since the time of Alexander. The US-Soviet rivalry was a particularly important, and dangerous, historical example in a long line of cases.

Even though the Cold War is over, the US-Soviet rivalry will stand as an excellent example of a broader theoretical problem in international relations, the problem of rivalry, and arms racing. It will continue to be the focus of much scholarly debate and research. The micro approach to mathematical arms race analysis could serve as a useful analytical device in that research. And, unfortunately, so long as the world remains divided into nation-states, the potential for some new rivalry, and hence, a new arms race, to arise will not diminish.

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APPENDIX A: US-Soviet Nuclear Arms Control Agreements

HOT LINE AGREEMENT (1963): This agreement allowed for the establishment of direct radio and telegraph links between Washington and Moscow to ensure direct communication between heads of government in times of extreme crisis.

HOTLINE MODERNIZATION AGREEMENT (1971): This agreement allows for the establishment of a satellite link between Moscow and Washington to be used in times of extreme crisis.

ACCIDENTS MEASURES AGREEMENT (1971): Requires the signatories to take steps to improve safeguards against the accidental or unauthorized use of nuclear weapons.

ANTI-BALLISTIC MISSILE TREATY (1972): This treaty limits the deployment of anti-ballistic missile defence systems to two sites for each signatory. A 1974 Protocol further limited the deployment of ABM systems to one site for each signatory.

STRATEGIC ARMS LIMITATION TREATY (SALT) I INTERIM AGREEMENT (1972): This treaty imposes ceilings (maximum deployment levels) on each side's strategic nuclear forces. More specifically, it places ceilings on the numbers of strategic launchers (delivery vehicles) that each side can deploy across its triad. (Note: Under SALT I, the USSR was given higher launcher ceilings than the US. Richard Nixon allowed this arguing the US had a substantial technological lead over the Soviet Union and thus the Soviet Union should be allowed to have numerically larger nuclear forces. In 1972, the principal technological advantage that the US had over the Soviet Union was its MIRV technology (the ability to place more than one nuclear warhead inside a launch vehicle.)

PREVENTION OF NUCLEAR WAR AGREEMENT (1973): This agreement commits the signatories to consultation in the event that there is a danger of nuclear war.

STRATEGIC ARMS LIMITATION TREATY (SALT) II (1979): (Note: The treaty came about due mainly to US disaffection with SALT I. The USSR had quickly developed the technology to MIRV its missiles and by the late 1970s some Americans were arguing that the USSR had a substantial advantage over the US. The USSR had larger a larger delivery vehicle force than the US due to SALT I and that force was being MIRVed.) SALT II thus aimed at establishing an overall equivalence between US and Soviet nuclear forces. SALT II was never ratified by the US Senate.

APPENDIX A (CONT)

THRESHOLD TEST BAN TREATY (1974): This treaty prohibits the signatories from conducting underground nuclear test explosions where the yield exceeds 150 kilotons.

PEACEFUL NUCLEAR EXPLOSIONS TREATY (1974): This treaty bans "group explosions" of nuclear weapons when the aggregate yield would exceed 1500 kilotons. (Note: Such tests are useful for determining the force configuration and yield required to destroy ICBM silos. Both sides agree that attacking ICBM silos would only be done in a first-strike situation.) The PEACEFUL NUCLEAR EXPLOSIONS TREATY also provided for on-site inspection where nuclear test detonations would have a yield in excess of 150 kilotons.

INTERMEDIATE NUCLEAR FORCES (INF) TREATY (1987): This treaty committed the US and USSR to remove and destroy all of their intermediate range ballistic missiles from the European theatre, the US Tomahawk Cruise and Pershing II Missiles and the Soviet SS-20s. INF also provided for intrusive verification procedures to ensure compliance with the terms of the treaty.

STRATEGIC ARMS REDUCTION TREATY (START) (signed in Moscow on 31 July 1991): This treaty requires the US and Soviets to reduce their current strategic nuclear warhead deployments levels. The Soviets will reduce their strategic nuclear warhead inventory from 11,000 to 7,000 and the US will reduce from 12,000 to 9,000. (Note: While SALT I did place ceilings on the number of launchers each side could deploy, it did not place any sort of limits on warhead deployments. MIRV technology was to allow each side to substantially increase its warhead count while staying within the launcher ceiling imposed by SALT I.)

APPENDIX B: Multilateral Nuclear Arms Control Agreements

ANTARCTIC TREATY (1959): This treaty prohibits signatory states from using the Antarctic for military purposes. In particular, it forbids nuclear testing in the Antarctic. (30 states have signed this treaty.)

PARTIAL TEST BAN TREATY (1963): This treaty bans nuclear testing in the atmosphere, outerspace, and underwater. (111 states)

OUTER SPACE TREATY (1967): Under this treaty, signatories are prohibited from basing nuclear weapons in earth orbit or outerspace. (81 states)

LATIN AMERICAN NUCLEAR FREE ZONE TREATY (1967): This treaty prohibits signatory states from testing, possessing, or deploying nuclear weapons in Latin America. It further requires signatories to strict safeguards on any nuclear facilities they may possess. (25 states)

NON-PROLIFERATION TREATY (1968): This treaty bans the US, China, the USSR, France and Britain from transferring nuclear weapons and nuclear weapons technology to non-nuclear states. It also commits the five nuclear powers to seek to end the nuclear arms race. (129 states)

SEABED TREATY (1971): This treaty forbids the deployment of nuclear weapons on the seabed beyond a 12-mile costal limit. (74 states)

SOUTH PACIFIC NUCLEAR FREE ZONE TREATY (1985): In this case, signatories are prohibited from testing, manufacturing, acquiring, or stationing nuclear weapons in the South Pacific area. The SOUTH PACIFIC NUCLEAR FREE ZONE TREATY further requests that the five major nuclear states sign a protocol banning the use, the threat of use, and the testing of nuclear weapons. (8 states)

APPENDIX C: Annual US-Soviet SLBM
Nuclear Warhead Deployments

1972-1987

(Source: BULLETIN OF THE ATOMIC SCIENTISTS May, 1988)

YEAR	US	USSR
1972	2384	458
1973	3536	556
1974	3824	688
1975	3968	828
1976	4688	954
1977	4832	1503
1978	5120	1970
1979	5088	2105
1980	4896	2198
1981	4976	2714
1982	4992	2762
1983	5152	2750
1984	5536	2934
1985	5760	3160
1986	5632	3176
1987	5632	3408

APPENDIX D: Annual US-Soviet ICBM
Nuclear Warhead Deployments

1960-1971

(Source: Military Balance, 1969-70, SIPRI Yearbook, 1976, 1981)

YEAR	US	USSR
1960	18	35
1961	63	50
1962	294	75
1963	424	100
1964	834	200
1965	854	262
1966	934	338
1967	1054	722
1968	1054	902
1969	1054	1198
1970	1074	1498
1971	1274	1527

APPENDIX E: Annual Aggregate US-Soviet Strategic Nuclear Warhead Deployments

1967-1984

(Source: SIPRI Yearbook 1976, 1981, 1982, 1983, 1984)

YEAR	US	USSR
1967	4500	1000
1968	4200	1100
1969	4200	1350
1970	4000	1800
1971	4600	2100
1972	5700	2500
1973	6784	2200
1974	7650	2500
1975	8500	2500
1976	8900	3500
1977	8500	4000
1978	9000	4500
1979	9200	5000
1980	9200	6000
1981	9000	7000
1982	9540	8802
1983	9681	8781
1984	9665	8880

APPENDIX F: Alternative tests of the GSR model

In Chapter 6, the GSR model and its component equations were tested against data on US-Soviet strategic nuclear warhead deployments. Equations 21/23, 22/24, 6a, 8a, 1a, and 2a were estimated from data on US-Soviet SLBM warhead deployments, 1972-87, from data on US-Soviet ICBM warhead deployments, 1960-71 and from data on US-Soviet total strategic nuclear warhead deployments, 1967-84. For each competition, the GSR model and its component equations were estimated as single equation models. Arguably, however, each of the equation pairs 21/23 and 22/24, 8a and 6a, and 1a and 2a constitutes a simultaneous equation system and must be estimated as such. Because the errors in each pair of equations are theoretically cross correlated, estimation by the method of Seemingly Unrelated Regression would be appropriate.

In this appendix, I will present estimates of the GSR model and its component equations under the assumption that each of the equation pairs, Equations 6a and 8a, and Equations 1a and 2a, constitutes a simultaneous system. It will not be necessary to reestimate the system comprised of Equations 21/23 and 22/24. Because Equations 21/23 and 22/24 contain the same independent variables, the method of Seemingly Unrelated Regression would yield results identical to those obtained when Equations 21/23 and 22/24 are estimated separately.

The estimates obtained under the method of Seemingly Unrelated Regression for Equations 6a and 8a and Equations 1a and 2a for the US-Soviet SLBM warhead race, 1972-87, for the US-Soviet ICBM warhead competition 1960-71 and for the US-Soviet total strategic nuclear warhead competition were not much different from those presented in Chapter 6 when each equation was estimated as a single equation model. In fact, the results presented below are completely consistent with those presented in Chapter 6. Thus, no alteration is necessary in the substantive analysis presented in Chapter 6.

1. THE US-SOVIET SLBM WARHEAD RACE, 1972-87 (SEEMINGLY UNRELATED REGRESSION RESULTS)

$$\mathbf{K}_t = SB_0 + SB_1\mathbf{X}_t + (1 - S)\mathbf{K}_{t-1} \qquad \text{(USA)} \qquad (6a)$$

$$= 2292.5 + 0.27\mathbf{X}_t + 0.43\mathbf{K}_{t-1}$$

$$\underset{**}{(431)} \quad \underset{**}{(0.129)} \quad \underset{**}{(0.14)}$$

$R^2 = 0.9192$, n = 15. ** indicates significant at 0.05 and * indicates significant at 0.01. (Run Test results indicate no autocorrelation at a 0.05 level of significance: 6 Runs, 6 positive and 9 negative)

$$\mathbf{X}_t = S'B'_0 + S'B'\mathbf{K}_t + (1 - S')\mathbf{X}_{t-1} \qquad \text{(USSR)} \qquad (8a)$$

$$= -1287.8 + 0.41\mathbf{K}_t + 0.72\mathbf{X}_{t-1}$$

$$\underset{**}{(492)} \quad \underset{**}{(0.128)} \quad \underset{**}{(0.085)}$$

$R^2 = 0.9787$, n = 15 (Run Test results indicate no autocorrelation at a 0.05 level of significance: 7 Runs, 7 positive and 8 negative)

1. SLBM WARHEAD REGRESSIONS (CONT)

$$\mathbf{K}^*_t = B_0 + B_1\mathbf{X}_t \qquad \text{(USA)} \tag{1a}$$

$$= 4045.3 + 0.48\mathbf{X}_t$$

$$\underset{**}{(215)} \quad \underset{**}{(0.09)}$$

$R^2 = 0.6573$, n = 15 (Run Test results indicate no autocorrelation at a 0.05 level of significance: 6 Runs, 6 positive and 9 negative)

$$\mathbf{X}^*_t = B'_0 + B'_1\mathbf{K}_t \qquad \text{(USSR)} \tag{2a}$$

$$= -4746.8 + 1.5\mathbf{K}_t$$

$$\underset{**}{(1034)} \quad \underset{**}{(0.20)}$$

$R^2 = 0.7719$, n = 15 (Run Test results indicate no autocorrelation at a 0.05 level of significance: 7 Runs, 7 positive and 8 negative)

2. THE US-SOVIET ICBM WARHEAD RACE 1960-71 (SEEMINGLY UNRELATED REGRESSION RESULTS)

$$\mathbf{K}_t = SB_0 + SB_1\mathbf{X}_t + (1 - S)\mathbf{K}_{t-1} \quad \text{(USA)} \tag{6a}$$

$$= 209.1 + 0.06\mathbf{X}_t + 0.80\mathbf{K}_{t-1}$$

$$\underset{**}{(80.2)} \quad (0.12) \quad \underset{**}{(0.16)}$$

$R^2 = 0.9087$, n = 11 (Run Test results indicate no autocorrelation at a 0.05 level of significance: 8 Runs, 5 positive and 6 negative)

$$\mathbf{X}_t = S'B'_0 + S'B'\mathbf{K}_t + (1 - S)\mathbf{X}_{t-1} \quad \text{(USSR)} \tag{8a}$$

$$= -68.6 + 0.29\mathbf{K}_t + 0.92\mathbf{X}_{t-1}$$

$$(96.7) \quad \underset{**}{(0.14)} \quad \underset{**}{(0.11)}$$

$R^2 = 0.9623$, n = 11 (Run Test results indicate no autocorrelation at a 0.05 level of significance: 6 Runs, 5 positive and 6 negative)

2. ICBM WARHEAD REGRESSIONS (CONT)

$\mathbf{K}^*_t = B_0 + B_1\mathbf{X}_t$ (USA) (1a)

$= 1055.8 + 0.33\mathbf{X}_t$

(273) (0.32)
**

$R^2 = 0.0599$, n = 11, (Run Test results indicate no autocorrelation at a 0.05 level of significance: 8 Runs, 5 positive and 6 negative)

$\mathbf{X}^*_t = B'_0 + B'_1\mathbf{K}_t$ (USSR) (2a)

$= -940 + 3.8\mathbf{K}_t$

(1050) (1.17)
**

$R^2 = 0.4620$, n = 11 (Run Test results indicate no autocorrelation at a 0.05 level of significance: 6 Runs, 5 positive and 6 negative)

3. THE US-SOVIET AGGREGATE STRATEGIC NUCLEAR WARHEAD COMPETITION, 1967-84 (SEEMINGLY UNRELATED REGRESSION RESULTS)

$$\mathbf{K}_t = SB_0 + SB_1\mathbf{X}_t + (1 - S)\mathbf{K}_{t-1} \quad \text{(USA)} \tag{6a}$$

$$= 593.6 - 0.11\mathbf{X}_t + 0.96\mathbf{K}_{t-1}$$

(502) (0.08) (0.10)
**

$R^2 = 0.9518$, n = 17 (Run Test results indicate no autocorrelation at a 0.05 level of significance: 7 Runs, 8 positive and 9 negative)

$$\mathbf{X}_t = S'B'_0 + S'B'_1\mathbf{K}_t + (1 - S')\mathbf{X}_{t-1} \quad \text{(USSR)} \tag{8a}$$

$$= -304 + 0.115\mathbf{K}_t + 0.972\mathbf{X}_{t-1}$$

(502) (0.09) (0.07)
**

$R^2 = 0.9695$, n = 17 (Run Test results indicate no autocorrelation at a 0.05 level of significance: 7 Runs, 8 positive and 9 negative)

3. US-SOVIET AGGREGATE STRATEGIC WARHEAD REGRESSIONS (CONT)

$\mathbf{K}^*_t = B_0 + B_1\mathbf{X}_t$ (USA) (1a)

$= 17460 - 0.30\mathbf{K}_t$

(6605) (1.3)
**

$R^2 = 0.0072$, n = 17 (Run Test results indicate no autocorrelation at a 0.05 level of significance: 7 Runs, 8 positive and 9 negative)

$\mathbf{X}^*_t = B'_0 + B'_1\mathbf{K}_t$ (USSR) (2a)

$= -11255 + 4.18\mathbf{K}_t$

(15738) (2.0)
**

$R^2 = 0.1942$, n = 17 (Run Test results indicate no autocorrelation at a 0.05 level of significance: 7 Runs, 8 positive and 9 negative)