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Date 27 July 92
ABSTRACT

Governments spend large sums of monies on various services provided to both firms and households. However, most open economy studies do not take government spending on industries into account. The present study deals exclusively with government spending on industries. This spending is incorporated into neoclassical production functions in terms of a public input. The purpose of this thesis is three fold: (i) to investigate the impact of terms-of-trade changes in a small public input economy; (ii) to explore the international transmission of government spending on public inputs; and (iii) to examine the relationship between government spending on public inputs and the pattern of international trade.

The thesis consists of three essays. In a three-period setting, the first essay examines the impact of terms-of-trade changes on the allocation of resources in a small open economy. The private sector of the economy produces two final goods by means of private inputs and a public input. The public input is produced by the public sector. The allocation of resources between the private and public sectors is endogenous and the public input is supplied with a lag of one period. The essay demonstrates that the timing of terms-of-trade changes is critical. The impact of terms-of-trade changes in the presence of labour unemployment is also considered.
The second essay develops a two-country, one-good, and two-factor general equilibrium model with a pure public input and international factor mobility. International transmission of government spending on a pure public input and the implications of potential international coordination are investigated in the short-run and the long-run. The essay also considers the international transmission of government spending on a pure public input in the context of a three-country model where two countries have formed an economic union.

The third essay develops a two-country, two-good, and two-factor general equilibrium model with a congestible public input. The model is used to investigate the relationship between government spending on a congestible public input and the pattern of international trade.
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CHAPTER 1

INTRODUCTION

Government spending constitutes a significant proportion of national incomes worldwide.\footnote{12\% and 14\% respectively was the share of government sector in the gross domestic product of the USA in 1950 and 1980. On the other hand, 20\% and 7\% respectively was the government’s share of Japanese gross domestic product in 1950 and 1980. The corresponding figures for the U.K. were 25\% and 23\%. See Summers and Heston (1984).} Such spending can be divided into two broad categories: spending on households, and spending on industries. In all real economies, large proportions of government budgets are directed towards services provided to industries. However, most open economy studies do not take government spending on industries into account and consider the private sector exclusively. These studies appear to be consistent with the assumption that the allocation of resources between the private and the public sectors is exogenous. In fact, despite separate management, the private and the public sectors are highly interdependent in most real economies. Accordingly, domestic and foreign shocks are likely to influence the output of both private and public sectors. Examples of these shocks include (exogenous) technological progress in the domestic private sector; an increase...
in the price of imported raw materials; and an improvement in the terms-of-trade.

Although government spending on industries has not received much attention in the theoretical literature, its importance has long been recognised. Pigou (1932) used Sidgwick's famous lighthouse example in this regard: the services of a lighthouse are an input into shipping companies' production functions for sea transport. Other examples include transportation facilities (i.e., roads, bridges, canals and harbours) and government financed scientific research whereby information on new production techniques is made available to all firms simultaneously.

The present study exclusively considers government spending on industries. This spending is incorporated in open economy models in terms of a public input. The private sector of an economy takes the supply of public inputs as given. However, for the society as a whole the supply of public inputs is endogenous. The supply of public inputs in an economy is determined by the government as the result of an optimisation process, such as the national welfare maximisation. In the presence of a public input, the domestic and foreign shocks influence the output of the private sector through the following channels: (1) an increase in the supply of public input influences the productivity of the private sector directly; (2) an increase in the production of public input leaves fewer resources for the private sector to work with; (3) induced factor
mobility within the private sector due to domestic and/or foreign shocks. If the interdependence of the public and the private sectors is assumed away, then domestic and foreign shocks affect the output of the private sector only through induced factor mobility.

Most available open economy studies do not take into account the lag between the production and supply of public inputs. For example, government financed scientific research conducted in the present is expected to benefit future users; production infrastructure utilised by firms in the present was built in the past. Due to the lag between the production and supply of public inputs, the timing of domestic and foreign shocks is critical.

Economic policies of the government also influence the provision of public inputs in an economy. For example, if the government uses tariffs to promote domestic import-competitive industries, it indirectly raises the demand for the relevant public inputs. Similar arguments can be made regarding the formation of Trade Development Zones and the Export Promotion Zones established by governments in some developing and developed countries. It is therefore desirable to investigate the impact of domestic and foreign shocks on the government and non-government sectors. This task can be accomplished only in the context of a multi-period model where the allocation of resources between the private and public sectors is endogenous.
The presence of public input within production functions allows one to consider the international transmission of government spending on industries.\(^2\) Government spending on industries is particularly important for economies engaged in tough competition in the international market. Abe (1990), Manning and McMillan (1979), and McMillan (1978) have shown that government spending on public inputs can influence the comparative advantage of an economy. Barro (1990) has shown that a positive relationship exists between the rate of economic growth and government spending on public inputs.

The statistical estimates obtained in a cross country study by Ram (1986) support the view that government size has a positive effect on economic performance and growth. The size of the government is measured by the output of the government sector. The private sector uses the output of the government sector as an input. Yamamura (1986) in his discussion of Japanese industrial policy indicates that from 1966 to 1980, the ministry of international trade and industry (MITI) provided services worth 663.1 million U.S. dollars to semiconductors and the computer industry alone. In general, government spending on the provision of public inputs affects the marginal productivity of private inputs.

\(^2\) Frenkel and Razin (1987), Devereux (1988), and Durlauf and Staiger (1990), among others, have examined the international transmission of government spending on households. Government spending on households is included in the utility functions in terms of a public good.
and thus influences the pattern of international trade and factor mobility.

The purpose of this thesis is three fold: (i) to examine the implications of terms-of-trade changes in a small open economy, where a public input is supplied by the government with a lag of one period; (ii) to explore the international transmission of government spending on public inputs and evaluate the implications of potential international economic policy coordination; and (iii) to investigate the relationship between government spending on public inputs and the pattern of international trade.

Significant labour unemployment exists in most real economies. The present study also examines the extent to which the results derived in this thesis are sensitive to a relaxation of the full-employment assumption.

The thesis consists of three essays, each of which is concerned with a specific issue outlined above. The first essay examines the impact of terms-of-trade changes on the allocation of resources in a small open economy. The private sector of the economy under consideration produces two final goods by means of labour, public input and some fixed factors, whereas the public sector produces a public input by means of labour which is fully mobile between the two sectors. The allocation of resources between the private and public sectors is therefore endogenously
determined. The public input, which is produced by the public sector, is made available to the private sector free of charge. In order to capture the effects of lags in the production and supply of public inputs, a three-period model is utilised.

In period one, the private sector utilises the pre-existing stock of the public input, whereas the public sector produces a public input. In period two, the private sector uses the public input produced in period one, whereas the public sector produces a public input which is made available to the private sector in period three. There is no public production in period three. In order to bring the role of lags in the supply of a public input into sharp focus, private investment is assumed away.

It is shown that an improvement in the terms-of-trade in period one decreases (increases) the production of public input in period one (two). Whereas, an anticipated improvement in the terms-of-trade in period two increases (decreases) the production of public input in period one (two). Finally, an anticipated improvement in the terms-of-trade in period three decreases (increases) the output of public input in period one (two). Due to intertemporal links, terms-of-trade changes in one period also influence the production of final goods in another period. It is shown that an improvement in the terms-of-trade in period one decreases (increases) the output of both final goods in period two (three). Whereas, an anticipated improvement in the terms-of-trade
in period two decreases the output of both final goods in period one and three. Finally, an anticipated improvement in the terms-of-trade in period three increases (decreases) the output of both final goods in period one (two).

The implications of labour unemployment in period one are also considered: it is shown that the output of public input in either period is not influenced by the terms-of-trade changes in period one. Consequently, terms-of-trade changes in period one have no impact on the output of either final good in period two and three. In addition, anticipated terms-of-trade changes in period two and three have no influence on the output of both final goods in period one. However, an increase in the minimum wage rate in period one influences both private and public sectors in each period. The essay explicitly considers a pure public input. However, after some modification, the results derived can also be extended to include an impure public input.

The focus of the second essay is the international transmission of government spending on public inputs and potential international economic policy coordination. The analysis is conducted by means of a two-country, one-good general equilibrium model with international factor mobility.\textsuperscript{3} Both countries produce an identical final good by means of capital, labour, and a pure

\textsuperscript{3} For analytical simplicity, the lags between the production and supply of public inputs are ignored in the rest of the thesis.
In the short-run, capital is fully mobile across international boundaries and labour is fully utilised in one country only. Whereas, in the long-run, both labour and capital are fully mobile across international boundaries and all resources are fully utilised in both countries. By means of a comparative statics exercise, the international transmission of government spending on a pure public input is considered.

It is shown that in the short-run equilibrium, the production of public input in a country where labour is fully utilised has no impact on the equilibrium rate of return on the internationally mobile factor. On the other hand, the production of public input in a country where labour is not fully utilised influences the equilibrium rate of return on the internationally mobile factor. In addition, an increase in the supply of public input in the country which fully utilises its resources can decrease labour employment in its trading partner. Furthermore, from the point of view of the country which fully utilises its resources in the short-run non-cooperative solution, the underemployed country spends too much (little) on the public input if it exports (imports) capital. In the short-run coordinated solution, the underemployed country spends too little (much) on the public input from the point of view of its residents if it exports (imports) capital.
In the long-run, the production of public input in both countries affects equilibrium factor prices. The results depend on relative capital intensity of the two countries. It is shown that in the long-run, non-cooperative solution both countries spend too much on public input from the point of view of each other; if the capital intensive country exports capital. On the other hand, in the long-run coordinated solution, both countries spend too little on the public input from the point of view of their residents; if the capital intensive country exports capital. The model is further extended to include a third country called the rest of the world, which is linked with the other two countries through international capital mobility. It is shown that although capital is fully mobile across international boundaries and resources are fully utilised everywhere, the supply of public input in the rest of the world has no influence on the equilibrium rate of return on capital. Economic policy coordination within the economic union is therefore desirable in order to exploit the rest of the world.

The third essay explores the link between government spending on public inputs and the pattern of trade in the context of a two-country, two-good general equilibrium model. Both goods are produced by means of capital, labour and a public input.

Abe (1990) has considered the relationship between the supply of a pure public input and the pattern of international trade. The purpose of the third essay is to extend Abe’s work in two
directions: (1) to consider the relationship between the supply of an impure public input and the pattern of trade, and (2) to consider the pattern of trade between underemployed economies.

Pure public inputs are non-congestible. On the other hand, impure public inputs are congestible within industries and among firms across industries. Congestion among firms across industries is inter-industry congestion. Due to their congestibility, differences in the production of impure public inputs can also influence the pattern of international trade. The pattern of trade between two economies where all resources are fully employed is considered first. It is shown that the country that produces more impure public input exports (imports) the output of the industry which causes more (less) congestion in the other industry, if the public input is equally congestible within each industry.

Furthermore, if the congestion created by each industry in the other is symmetric, then the country which produces more public input exports (imports) the output of the industry in which the public input is relatively less (more) congestible.

In other words, Abe’s result can be extended to include an impure public input only if (a) congestion caused by each industry is symmetric, and (b) public input is equally congestible across industries.
Finally, the relationship between government spending on an impure public input and the pattern of trade between underemployed economies is considered. It is shown that even if both industries derive equal benefits from an impure public input which is equally congestible across industries, and the congestion caused by each industry is symmetric, the pattern of international trade can still be influenced by its supply. Specifically, if two underemployed countries have identical preferences, production technology, primary factor supplies, the public input is equally congestible across industries and the congestion caused by each industry is symmetric; then the country that produces more public input exports (imports) the output of the industry that is relatively labour (capital) intensive.

Each of the following three chapters constitutes a self contained essay. The last chapter is the final summary of all the essays.
CHAPTER 2

TERMS-OF-TRADE CHANGES IN A PUBLIC INPUT ECONOMY

2.1 Introduction

Most open economy studies [for example Bhagwati and Srinivasan (1983), Dixit and Norman (1980), and Woodland (1982)] do not take government spending into account and consider the private economy exclusively. These studies provide an excellent survey of the alternative theories of international trade but do not explicitly consider either public goods or public inputs. These studies appear to be consistent with the assumption that the resource allocation problems of the private and public sectors are independent. In fact, despite separate management, the private and public sectors are highly interdependent in all mixed economies.

The assumed independence of resource allocation problems of the private and public sectors implies that neither domestic nor foreign shocks, which affect the private sector directly, influence the supply of public goods or public inputs in an open economy.
Examples of these shocks include (exogenous) technological progress in the domestic private sector; an increase in the price of imported raw materials; and an improvement in the terms-of-trade.

Governments spend large sums of monies on various services provided to firms and households in all mixed economies. Nevertheless, theoretical studies which take government spending into account often assume that such spending enters into household utility functions but not into production functions [see for example; Devereux (1988), Durlauf and Staiger (1990), Frenkel and Razin (1986a, 1986b, 1987), and Svensson (1987)]. These studies also assume that the utility functions are strongly separable in the public and private goods.\(^1\) The present study considers government spending on public inputs exclusively. Examples of these inputs include government financed scientific research whereby information on new production techniques is simultaneously made available to all firms, and production infrastructure.

Most open economy studies which include government spending on public inputs do not take into account the lag between the production and supply of these inputs. For example, government financed scientific research conducted in the present is expected to benefit future users; production infrastructure utilised by firms in the present was built in the past. Very often significant

\(^1\) An interesting example in this regard is Devereux (1988) where optimal government spending is zero.
repairs of the existing infrastructure involve a long period of time. Therefore, lags between the production and supply of public goods cannot be ignored. These lags can only be taken into account in a multi-period setting.

Existing theoretical studies which include government spending on industries, where the allocation of resources between the private and public sectors is endogenously determined, are almost entirely static in nature. In addition, these studies do not examine the impact of either tariffs or terms-of-trade changes on the provision of public inputs. It is well-known that changes in the terms-of-trade directly affect the private sector. However, in a mixed economy, terms-of-trade changes also affect the provision of public inputs which affects the private sector indirectly.

The purpose of this essay is to develop a simple three-period perfect foresight model of "productive" government, in which the allocation of resources between the public and private sectors is endogenously determined in a system that includes behavioural hypotheses about the agents exercising the power of the government to tax and spend. The small open economy under consideration produces two final goods by means of private inputs and a public input. The public input is produced by means of private inputs. The producers of final goods take the supply of public input as given, but in the full equilibrium the supply of public input is endogenous. The allocation of resources between the private and
public sectors in the small open economy is therefore endogenous. The model also takes into account lags between the production and supply (or the availability to firms) of the public input.

Through a comparative static exercise, the present study examines the impact of terms-of-trade changes on the provision of the public input and hence the production of final goods. There are intertemporal production links due to a lag between the production and supply of public input. These links provide a mechanism whereby terms-of-trade changes in either period are transmitted to the other periods. It is shown that an improvement in the terms-of-trade in period one decreases the supply of public input in the second period, but the supply in period three increases. An anticipated improvement in the terms-of-trade in period two increases the supply of public input in the second period but the supply in the third period decreases. Finally, an anticipated improvement in the terms-of-trade in period three decreases the supply of public input in the second period, but the supply in the third period increases.

Changes in the terms-of-trade influence the production of final goods both directly, and indirectly; through their impact on the supply of public input. An improvement in the terms-of-trade in period one decreases (increases) the production of both final goods in the second (third) period. An anticipated improvement in the terms-of-trade in the second period decreases the production of
both final goods in the first and third period. Finally, an anticipated improvement in the terms-of-trade in the third period increases (decreases) the production of both final goods in the first (second) period. The timing of terms-of-trade changes is therefore critical. An increase in the cost efficiency of producing the final goods is also considered.

Significant unemployment exists in most real economies. The model is therefore extended in section five to include unemployment in period one. The comparative static response of the private and public sectors is shown to be significantly influenced by the presence of unemployment.

The essay is organised as follows. Some related studies are briefly reviewed in section two. Section three develops a simple model of a small open economy. The effect of terms-of-trade changes on the private and public sectors is examined in the fourth section. The model developed in section three is extended in section five to include labour unemployment due to economy-wide rigid wages in period one. Section six contains a summary and concluding remarks.
2.2 Review of Related Literature

A significant proportion of government spending is directed towards the provision of public inputs in all mixed economies. Public inputs are intermediate goods and services which are non-rival in use. Due to their collective nature, the private provision is subject to market failure. However, the problem is less severe compared to the provision of public goods.

Meade (1952) distinguished two types of public inputs. He refers to these as "creation of atmosphere", and "unpaid factors of production" respectively. The "creation of atmosphere" is in fact the production analogue of the pure public (consumption) good as defined by Samuelson (1954). The use of the public input by one firm does not reduce the amount available for the other firms to use. Constant returns to scale has a very different meaning in such a case. Following Meade (1952), many authors (for example, Negishi (1973), Manning and McMillan (1982)) have indicated that the appropriate definition of constant returns to scale is a production technology linearly homogeneous in the private factors of production alone. Manning et al. (1985) have shown that in the presence of a pure public input, constant returns to scale in the private factors alone means that placing user charges on firms, e.g., Lindahl pricing, is not feasible.
A pure public input gives rise to increasing returns to scale which can result in a non-convex production set. In the relevant literature, government financed information on new production techniques has been widely cited as an example of pure public input. Feehan (1989) refers to the pure public input as the factor augmenting public input.

Unlike the pure public inputs which are non-congestible both across industries and among firms within each industry, Meade’s second type of public inputs, the unpaid factors of production, are congestible. These are therefore the production analogue of the impure public (consumption) goods. In the presence of an impure public input, doubling the amount of each private factor of production used in the industry leaves each unit of private factor with less public input to work with than before. An unpaid factors type public input is clearly a limiting input. Constant returns to scale in such a case means a production function linearly homogeneous with respect to all inputs, including the public input. Such public inputs include production infrastructure, for example, roads, bridges, canals, and harbours.

Kaizuka (1965) and Sandmo (1972) derived rules for the efficient provision of public inputs in a single period setting. Laffont (1975) and Pestieau (1976) extended these optimality rules to encompass technological uncertainty and distortionary taxes, respectively. Thompson (1968) and Negishi (1973) demonstrated that
competitive markets can achieve the efficient outcome.

Most theoretical studies in the literature on international trade consider the private economy exclusively. These studies appear to be consistent with the assumption that the allocation of resources between the production of private and public goods is exogenous. Some studies where the allocation of resources between private and public sectors is endogenous are reviewed in the remainder of this section.

Manning and McMillan (1979), Tawada (1980), Tawada and Okamoto (1982), Manning and McMillan (1982), Tawada and Abe (1984), and Altenburgh (1987) mainly address the shape of the production possibility curve in the presence of a public input. In the context of a two-good, two-private and one public input model, it is shown that in the presence of an impure public input, the production possibility curve can be concave to the origin. However, in the presence of a pure public input, the production possibility curve can be concave to the origin only if the elasticity of both goods with respect to the public input is identical.

economy with a public input. Pugel (1982) considers technology transfer in the context of a Ricardian model incorporating Meade’s first type of public input. He uses a two-country model where each country completely specialises in the production of a private good. The home country produces a pure public input (e.g., an improved production management technique). The home country cannot exclude the foreign country from using the new technique. However, the foreign country (through some sort of binding international agreement) can be made to pay a royalty. Pugel derives and compares three different royalties for the foreign country: (1) a royalty which maximises the utility of the home country only, (2) a royalty which maximises the utility of the foreign country only, and (3) a royalty which maximises the aggregate utility of the two countries.

The framework of the studies mentioned above is static; public inputs are produced by means of private factors and these factors are fully mobile between the public and private sectors. In addition, all resources are fully utilised. What follows is a brief review of the relevant dynamic studies.

McMillan (1978) deals with the optimal supply of public input in a small open economy. The economy produces two private goods by means of labour and a public input. The public input is produced by means of labour which is fully mobile within national boundaries. There is no private investment in the economy. Private good
production functions are linear in labor, and consequently the 
underlying production possibility curve is strictly convex to the 
origin. McMillan (1978) derives the efficiency conditions for the 
supply of public input in this economy. The properties of the 
optimal trajectory are discussed and it is shown that despite one 
private factor of production, the comparative advantage of the 
small open economy is endogenously determined.

Barro (1990) introduces a public sector into a simple 
constant-returns model of economic growth. He considers a closed 
economy, where the private sector produces a final good by means of 
private and public inputs. The production of public input is 
financed by a proportional tax on domestic income: government 
converts its tax receipts into public input without additional 
cost. The rate of depreciation of the public input is 100%. Using 
a specific functional form for the production and utility 
functions, the steady state growth and saving rates are shown to 
depend on the proportional tax rate.

Devereux and Mansoorian (1989) develop a two-country, two-good 
model with a public good which is used by both consumers and 
producers. Each country is assumed to specialise in the production 
of one good in an infinite-horizon setting. Their model is similar 
to Barro (1990). They are concerned with the implications of

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In other words, McMillan (1978) considers Meade's first type 
of public intermediate good.
international fiscal cooperation for the growth rates of the two countries. Using Cobb-Douglas production and logarithmic utility functions, Devereux and Mansoorian (1989) argue that "the gains from international fiscal coordination will in general entail higher growth rates for each country".

None of the studies mentioned above explores the link between terms-of-trade changes and the provision of public inputs. Tawada and Okamoto (1983), and Okamoto (1985) re-investigate the validity of the Stolper-Samuelson theorem in the presence of a public input. However, they do not explicitly consider the impact of a tariff on the supply of public inputs. In addition, the studies mentioned so far do not take the lags in the production and supply of public inputs into account. These lags can be best considered in a multi-period framework.

Government economic policies which are designed to influence the production of private goods also affect the demand for and hence the supply of public inputs indirectly. The present study explicitly considers the impact of terms-of-trade changes on the public sector which indirectly influences the private sector.

In Anwar (1991), I developed a two-period, two-good, and two-country model with a durable public input. The public input which is used in the production of both final goods is produced by means of labour in period one. The two final goods produced are
industrial and primary goods. The industrial good is produced by means of capital and public input while the primary good is produced by means of labour and public input. Private investment therefore takes place in the industrial sector only. In period one, labour is mobile within each country but international labour mobility is restricted. The two countries are linked through perfect international labour mobility in period two. I have considered the implications of the introduction of a tariff by either country, in either period, on private and public investment. Due to anticipated perfect labour mobility in period two, the introduction of a tariff by either country affects private and public investment in both countries. The timing of the introduction of a tariff is shown to be critical. An increase in the cost efficiency of producing the public input is also considered. However, I have not taken into account the lag between the production and provision of a public input. In addition, I have not explicitly considered the impact of a tariff on the production of final goods.

During the last few decades trade among nations has increased significantly. Economic policy changes in any country in a quickly integrating world economy are therefore likely to influence both public and private sectors of all countries involved. The present study develops a simple model, where the allocation of resources between the private and public sectors is endogenous and lags in the production and supply of public inputs are taken into account.
The model is used to study the effects of temporary changes in the terms-of-trade on both private and public sectors.
2.3 A Three-Period Model

The purpose of this section is to develop a simple framework where the allocation of resources between the private and public sectors is endogenous and which allows an investigation of the effects of terms-of-trade changes on the two sectors.

The present study explicitly deals with government spending on a pure public input. However, after some minor modifications, the results presented in this study can also be extended to include an impure public input.

There are three periods, indexed $t = 1, 2$ and $3$, which can be interpreted as the past, the present, and the future respectively. In each period, the private sector produces two final goods by means of labour, a public input, and other specific factors. The government provides the public input, produced by means of labour, free of charge to the private sector. The pure public input under consideration is utilised by the producers of both final goods. Examples of such a public input, which is non-congestible both within an industry and across industries, include information on

---

3 The results presented in the present study can be generalised to any finite number of periods. However, a three-period setting captures the important role played by the lag in the production and eventual provision of public input without much mathematical complexity.
improved production management techniques.

In the first period, private producers use the pre-existing stock of public input, whereas the government produces a public input which is made available to the private sector in the second period. In the second period, the government produces a public input which is made available to the private sector in the third period. The public input can be used for only one period. In other words, the public input is durable but its rate of depreciation is 100%. This implies that the private sector utilises only the most recent information on production management techniques.⁴

There is no private investment. The purpose of this assumption is to bring the role of lags in the supply of public input into sharp focus. The supply of labour in each period is fixed and there is perfect labour mobility between the private and public sectors.

In each period, the two final goods (X and Y) are traded at

⁴ In the case of an impure public input, the assumption regarding the depreciation ensures that the profit in the last period is not unlimited. 100% depreciation, however, simplifies the algebra of the comparative statics considerably. A similar assumption is widely used in the related literature; see for example, Devereux (1988), Devereux and Mansoorian (1989), Durlauf and Staiger (1990), Frenkel and Razin (1986a, 1986b, 1987), and Barro (1990).
relative prices set by the rest of the world. The open economy under consideration can also borrow and lend from the rest of the world at a fixed rate of interest. In other words, the economy under consideration is small in both goods and credit markets. Demand conditions therefore have no role to play in the present study. Good X is the numéraire and the public input is also measured in its units. The economy under consideration exports good Y.

As indicated earlier, this study explicitly deals with a pure public input, the final good production functions are therefore assumed to exhibit constant returns to scale for a given level of public input. The private and public good production functions for the economy under consideration are given below, where the specific factors are not explicitly included.\(^{5}\)

\[
X_1 = \gamma_{1x} G_{0}^{\alpha_1} F_1(L_{1x}, T_{1x}) ; \quad 1 > \alpha > 0
\]

\[
Y_1 = \gamma_{1y} G_{0}^{\beta_1} H_1(L_{1y}, T_{1y}) ; \quad 1 > \beta > 0
\]

\[
G_1 = \gamma_{1g} L_{1g}
\]

\[
X_2 = \gamma_{2x} G_{1}^{\alpha_2} F_2(L_{2x}, T_{2x})
\]

\(^{5}\) The subscripts 1, 2, and 3 refer to the first, the second, and the third period respectively.
where

$X_t$: production of importable good in period $t$. ($t = 1, 2, 3$)

$Y_t$: production of exportable good in period $t$. ($t = 1, 2, 3$)

$G_t$: public input produced in period $t$. ($t = 1, 2, 3$)

$L_{tx}$: labour employed in the production of $X_t$.

$L_{ty}$: labour employed in the production of $Y_t$.

$L_{tg}$: labour employed in the production of $G_t$.

$T$: fixed factors.

$\alpha_t$ and $\beta_t$ are the elasticity of $X$ and $Y$ with respect to $G$ in period $t$ respectively.

$\gamma_{tx}$, $\gamma_{ty}$, $\gamma_{tg}$ capture the effect of exogenous technological progress.

The functional form of the above production functions implies that the public input is cooperative with private inputs in the production of both final goods.\(^6\) The labour-market clearing

\(^6\) The private production functions are separable in the public and the private inputs. The results of this and the next essay do not depend on this assumption.
conditions are given below where \( N_t \) refers to the supply of labour in period \( t \):

\[
L_{1g} + L_{1x} + L_{1y} = N_1 \tag{1}
\]

\[
L_{2g} + L_{2x} + L_{2y} = N_2 \tag{2}
\]

\[
L_{3x} + L_{3y} = N_3 \tag{3}
\]

Equations (1) to (3) indicate that the entire labour force is fully utilised in each period. The conditions for the optimal allocation of resources are derived in the following section.

2.3.1 Optimal Allocation of Resources

Efficiency conditions for the small open economy under consideration can be derived by maximising the present value of the final goods produced by the private sector. For the sake of simplicity, the relevant intertemporal discount factors are assumed to be unity. In other words, the rate of interest on foreign borrowing and lending is assumed to be zero. In addition, for simplicity \( \gamma_{tx}, \gamma_{ty}, \) and \( \gamma_{tq} \) are initially assumed to be unity.\(^7\) The

\[^7\text{An increase in these parameters from unity can be interpreted as technological progress.}\]
optimisation problem of the central planner is the following:

\[
\text{Max } \{G_0^{\alpha_1}F_1(L_{1x}, T_{1x}) + P_1 G_1^{\beta_1}H_1(L_{1y}, T_{1y})\} + \\
\{G_1^{\alpha_2}F_2(L_{2x}, T_{2x}) + P_2 G_1^{\beta_2}H_2(L_{2y}, T_{2y})\} + \\
\{G_2^{\alpha_3}F_3(L_{3x}, T_{3x}) + P_3 G_2^{\beta_3}H_3(L_{3y}, T_{3y})\}
\]

subject to full employment conditions (1) to (3).

\[G_1, G_2, L_{1y}, L_{1x}, L_{2y}, L_{2x}, L_{3y}, \text{ and } L_{3x}\] are the choice variables and \(P_c\) is the price of good \(Y\) (determined by the rest of the world) in terms of \(X\) in period \(t\). \(\{t = 1, 2, 3\}\)

The above constrained optimisation problem, by proper substitution, can be reduced to the following unconstrained problem:

\[
\text{Max } \{G_0^{\alpha_1}F_1(N_{1} - G_1 - L_{1y}, T_{1x}) + P_1 G_0^{\beta_1}H_1(L_{1y}, T_{1y})\} + \\
\{G_1^{\alpha_2}F_2(N_{2} - G_2 - L_{2y}, T_{2x}) + P_2 G_1^{\beta_2}H_2(L_{2y}, T_{2y})\} + \\
\{G_2^{\alpha_3}F_3(N_{3} - L_{3x}, T_{3x}) + P_3 G_2^{\beta_3}H_3(L_{3y}, T_{3y})\}
\]

with respect to \(G_1, G_2, L_{1y}, L_{2y}, \text{ and } L_{3y}\).

The first order conditions of the above optimisation problem are given below where for simplicity \(G_0\) is assumed to be unity;

\[
F_1L(N_{1} - L_{1y} - G_1, T_{1x}) = \alpha_2 G_1^{\alpha_2-1}F_2(N_{2} - L_{2y} - G_2, T_{2x}) + \\
P_2 \beta_2 G_2^{\beta_2-1}H_2(L_{2y}, T_{2y})
\]
\[ G_t^2 F_{2t}(N_t - L_{2y} - G_1, T_{2x}) = \alpha_2 G_t^2 F_3(N_3 - L_{3y}, T_{3x}) + \beta G_t^2 H_2(L_{3y}, T_{3y}) \]

\[ F_{1t}(N_1 - L_{1y} - G_1, T_{1x}) = \beta_1 H_1(L_{1y}, T_{1y}) \]

\[ G_t^{\alpha_2-\beta_2} F_{2t}(N_2 - L_{2y} - G_2, T_{2x}) = \beta_2 H_2(L_{2y}, T_{2y}) \]

\[ G_t^{\alpha_3-\beta_3} F_{3t}(N_3 - L_{3y}, T_{3x}) = \beta_3 H_3(L_{3y}, T_{3y}) \]

where

- \( H_{xt}(.) \): marginal product of labour in the production of \( Y \) in period \( t \).
- \( F_{xt}(.) \): marginal product of labour in the production of \( X \) in period \( t \).
- \( \alpha_t G_t^{\alpha-1} F_t(.) \): marginal product of pure public input in the production of \( X \) in period \( t \).
- \( \beta_t G_t^{\beta-1} H_t(.) \): marginal product of pure public input in the production of \( Y \) in period \( t \).

The economy described by equations (4) to (8) is a public input economy.\(^8\) There are five efficiency conditions [equations (4) to (8)] in five endogenous variables: \( G_1, G_2, L_{1y}, L_{2y}, \) and \( L_{3y} \). Equations (4) and (5) are the conditions for the optimal provision of public input in period two, and three respectively. The right-hand side of these equations is the present value of marginal

\(^8\) The term "public input economy" has been used by Abe (1990).
benefits to the final good producers from an additional unit of public input, whereas the left-hand side is its marginal cost.\(^9\) Equations (6) to (8) indicate the implications of perfect labour mobility within the private sector in each period. Equations (4) and (5) in conjunction with (6) and (7) also demonstrate the implications of perfect labour mobility between the private and public sectors in the first and second period: the wage rate in both sectors is identical.

If the firms behave competitively and the government supplies the optimal level of public input at market wages, the social planning optimum, described by equations (4) to (8) above, can be decentralised. The equilibrium can be interpreted as a perfect foresight equilibrium over time.

The present study explicitly assumes that the producers of final goods do not pay for the use of the pure public input. The reward of public input is captured by the private factors. The government uses a flat rate income tax to recover the cost of the public input. Since all agents and the government have access to the world credit market, the timing of these taxes does not matter. The government can also use a per-unit output tax to finance the public production but the tax rate must be identical across industries and time.

\(^9\) These conditions are similar to those derived by Kaizuka (1968), and Sandmo (1972) in a single period setting.
In the case of an impure public input, the reward of public input accrues to the owner of the firm. The government can therefore use a Lindahl pricing scheme to finance the cost of public production. Under this scheme the price paid by each producer, in each period, equals the marginal product of public input. The Lindahl pricing scheme is plausible because by observing the profits of the private producers in each period, the government can determine the benefits of public input to each industry.\textsuperscript{10}

In the next section comparative static properties of the model are explored. The results are derived by differentiating the equilibrium conditions (4) to (8) with respect to various exogenous variables.

\textsuperscript{10} The true marginal productivities of public input can also be determined by using a mechanism suggested by Groves and Loeb (1975).
2.4 Comparative Statics

The purpose of this section is to investigate the impact of temporary terms-of-trade changes on the private and public sectors. Svensson and Razin (1983) and Marion and Svensson (1984) define a temporary improvement in the (temporal) terms-of-trade as \( dP_t > 0 \) for only one \( t = \{1, 2, 3\} \). The impact of technological improvement in the production of final goods is also considered.

The public input and labour are cooperative in the production of both final goods. In the present study, the private and public sectors are linked through unrestricted labour mobility in period one and two. The comparative statics results presented in the following are derived under the assumption that \( \alpha_t = \beta_t \), which implies that the direct benefits of the public input to both industries are identical. This allows one to focus on the role of indirect benefits of a public input. The impact of terms-of-trade changes in period one is discussed in the following. The results are derived by using equations (4) to (8), where for algebraic simplicity \( G_o \) is initially assumed to be unity.

2.4.1 Terms-of-Trade Changes in Period one

The following equations describe the impact of a temporary
change in the terms-of-trade (in period one) on the optimal \( G_1, G_2, L_{1y}, L_{2y}, \) and \( L_{3y} : \)

\[
\frac{\partial G_1}{\partial P_1} = H_{1L}(.) F_{1LL}(.) [P_2 H_{2LL}(.) F_{2LL}(.) + \]
\[(\beta_2 - 1) G_2^{-1} G_1 \alpha_2 F_{2L}(.) (P_2 H_{2LL}(.) + F_{2LL}(.))] / H < 0 \tag{9}
\]

\[
\frac{\partial G_2}{\partial P_1} = \alpha_2 G_1^{\alpha_2 - 1} [F_{2L}(.) F_{1LL}(.)] [P_2 H_{2LL}(.) + F_{2LL}(.)] / H > 0 \tag{10}
\]

\[
\frac{\partial L_{1y}}{\partial P_1} = - [H_{1L}(.) + F_{1LL}(.) \partial G_1 / \partial P_1] /[F_{1LL}(.) + P_1 H_{1LL}(.)] > 0 \tag{11}
\]

\[
\frac{\partial L_{2y}}{\partial P_1} = - [F_{2LL}(.) /[F_{2LL}(.) + P_2 H_{2LL}(.)] [\partial G_2 / \partial P_1] < 0 \tag{12}
\]

\[
\frac{\partial L_{3y}}{\partial P_1} = 0 \tag{13}
\]

\( H = b_1 b_2 - b_3 b_4 > 0^{11} \)

\( b_1 = P_1 H_{1LL}(.) F_{1LL}(.) + (\alpha_1 - 1) G^{-1}_1 F_{1L}(.) [P_1 H_{1LL}(.) + F_{1LL}(.)] > 0 \)

\( b_2 = P_2 H_{2LL}(.) F_{2LL}(.) + (\alpha_2 - 1) G^{-1}_2 \alpha_2 F_{2L}(.) [P_2 H_{2LL}(.) + F_{2LL}(.)] > 0 \)

\( b_3 = - \alpha_1 G^{\alpha_1 - 1} F_{2L}(.) [P_1 H_{1LL}(.) + F_{1LL}(.)] > 0 \)

\( b_4 = - \alpha_2 G^{\alpha_2 - 1} F_{2L}(.) [P_2 H_{2LL}(.) + F_{2LL}(.)] > 0 \)

Equation (9) shows that an improvement in the terms-of-trade in period one leads to a decrease in the production of public input. This result can be explained by using efficiency conditions (4) and (6). According to equation (6), an improvement in the

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\(^{11} H\) is positive because the production functions are assumed to be strictly concave.
terms-of-trade in period one increases the wage rate in the private sector. Due to perfect labour mobility between the private and public sectors, an improvement in the terms-of-trade leads to labour outflow from the public sector which increases the marginal cost of the public input above its marginal benefits in the second period. According to the efficiency condition (4), the production of public input in period one must fall.

Equation (10) shows that an actual improvement in the period one terms-of-trade results in an increase in the output of public input in period two. This result follows from efficiency condition (5). An improvement in the terms-of-trade in period two decreases the output of the public input in period one and therefore decreases its marginal cost in period two [i.e., $G^a_t F_{2t}(\cdot)$] below its marginal benefits. Therefore, the optimal output of the public input in the second period must increase.

Equation (11) shows that the presence of a public input in the model strengthens the expected result. Equation (12) shows that an improvement in the terms-of-trade in period one lowers employment in the production of $Y_t$. This follows from the fact that an improvement in the terms-of-trade increases the output of public input, which is produced by means of labour. Consequently, fewer

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12 If the allocation of resources between the private and public sectors were exogenous, $\partial G_t/\partial P_t$, and $\partial G_z/\partial P_t$ would be zero for all $t = \{1, 2, 3\}$. 36
workers will be available for employment in the private sector. Equation (13) can be explained in the following way. Public input is not produced in period three, therefore all workers are available for employment in the private sector. A change in the terms-of-trade therefore does not affect employment within the private sector.

The impact of terms-of-trade changes in period one on the output of the private sector is discussed in the following:

\[ \frac{\partial x_1}{\partial P_1} = F_{1L}(.) [H_{1L}(.) - P_1 H_{1LL}(.) \frac{\partial G_1}{\partial P_1}] / [P_1 H_{1LL}(.) + F_{1LL}(.)] \]  \[ \left(14\right) \]

\[ \frac{\partial Y_1}{\partial P_1} = H_{1L}(.) \left[ \frac{\partial L_{1y}}{\partial P_1} \right] > 0 \]  \[ \left(15\right) \]

\[ \frac{\partial x_2}{\partial P_1} = \alpha_2 G_1^{2-1} F_2(\cdot) \frac{\partial G_1}{\partial P_1} - G_1^{2} \left[ P_2 H_{2LL}(\cdot) F_{2L}(\cdot) / \left[ P_2 H_{2LL}(\cdot) + F_{2LL}(\cdot) \right] \right] \frac{\partial G_2}{\partial P_1} < 0 \]  \[ \left(16\right) \]

\[ \frac{\partial Y_2}{\partial P_1} = \beta_2 G_1^{2-1} H_2(\cdot) \left[ \frac{\partial G_1}{\partial P_1} \right] + G_1^{2} H_2(\cdot) \left[ \frac{\partial L_{2y}}{\partial P_1} \right] < 0 \]  \[ \left(17\right) \]

\[ \frac{\partial x_3}{\partial P_1} = \alpha_3 G_2^{3-1} F_3(\cdot) \left[ \frac{\partial G_2}{\partial P_1} \right] > 0 \]  \[ \left(18\right) \]

\[ \frac{\partial Y_3}{\partial P_1} = \beta_3 G_2^{3-1} H_3(\cdot) \left[ \frac{\partial G_2}{\partial P_1} \right] > 0 \]  \[ \left(19\right) \]

Equation (15) shows that the incorporation of public spending in the production function strengthens the expected result: an improvement in the terms-of-trade in period one leads to an
increase in the production of \( Y_1 \). However, its effect on the production of \( X_1 \) is ambiguous because the sign of \( \partial L_{ix}/\partial P_1 \) is ambiguous. If the public input were absent from the model or if it were produced by sector-specific labour, then the sign of equation (14) would be unambiguously negative. But in the present case the production of public input decreases in response to an improvement in the terms-of-trade. Labour is released from the production of public input, but it is not clear if all of this labour finds employment in the production of \( Y_1 \). If in the new equilibrium all the labour released by the public sector is absorbed in the production of \( Y_1 \), then there will be no change in the production of \( X_1 \).

Equations (16) to (19) show that the impact of an improvement in the terms-of-trade in period one is transmitted to periods two and three through its effect on the supply of the public input. According to equations (16) and (17), its effect on production in the private sector in period two is negative whereas equations (18) and (19) indicate that the private sector in period three benefits. It is noticeable that if the public input was not included in the model then an improvement in the terms-of-trade in period one will not affect the output of the private sector in periods two and three.

An improvement in the terms-of-trade in period one leads to a decrease in the output of the private sector in period two because
it results in a decrease in the production of public input in period one (i.e., \( G_1 \)) which is used by the private sector in period two. However, it leads to an increase in the production of public input in period two (i.e., \( G_2 \)) which is used by the private sector in period three. This implies that less labour will be available to the private sector in period two, which reinforces the effect of a decrease in the supply of public input in period two.

The effect of an improvement in the terms-of-trade in period one on the output of the private sector in period three is positive because it results in an increase in the supply of public input without affecting the supply of labour to the private sector.

### 2.4.2 Terms-of-Trade Changes in Period Two

The model developed in the previous section is a perfect foresight model. The following equations describe the impact of a change in the terms-of-trade, in period two, on the optimal \( G_1, G_2, L_1, L_2, \) and \( L_3 \). This change was anticipated in the beginning of period one:

\[
\begin{align*}
\frac{\partial G_1}{\partial P_2} &= -\alpha_2 G_1^{\alpha_2 - 1} [F_{1,LL}(.) + P_1 H_{1,LL}(.)] [P_2 H_{2,LL}(.) F_{2,LL}(.) H_2 + \\
&\quad (\alpha_2 - 1) G_2^{-1} G_1^{\alpha_2} F_{2,LL}(.) (P_2 H_{2,LL}(.) + F_{2,LL}(.) - \\
&\quad G_1^{\alpha_2} F_{2,LL}(.) H_2 (.) F_{2,LL}(.) ) / H > 0
\end{align*}
\]
\[ \frac{\partial G_2}{\partial P_2} = G_1^2 F_{2LL}(.) H_2(.) \left[ F_{1LL}(.) P_1 H_{1LL}(.) + G_1^{\alpha-1}(F_{1LL}(.) + P_1 H_{1LL}(.)) \left[ (\alpha-1) F_{1LL}(.) - \alpha G_1^{-1} F_{2L}(.) H_2(.) \left( P_2 H_{2LL}(.) + F_{2LL}(.) \right) \right] \right] / H < 0 \] (21)

\[ \frac{\partial L_{1y}}{\partial P_2} = - \left( F_{1LL}(.) + P_1 H_{1LL}(.) \right) \left\{ \frac{\partial G_1}{\partial P_2} \right\} < 0 \] (22)

\[ \frac{\partial L_{2y}}{\partial P_2} = - \left( H_2(.) + F_{2LL}(.) \partial G_2 / \partial P_2 \right) / \left( F_{2LL}(.) + P_2 H_{2LL}(.) \right) > 0 \] (23)

\[ \frac{\partial L_{3y}}{\partial P_2} = 0 \] (24)

Equations (20) and (21) indicate that an anticipated improvement in the terms-of-trade in period two increases the production of public input in period one, but it results in a decrease in the production of public input in period two. An anticipated improvement in the terms-of-trade in period two increases the anticipated marginal cost of the production of public input in period two (i.e., \( G_2 \)) above the present value of its marginal benefits in period three. The optimal production of public input in period two therefore decreases. This however increases the marginal benefits of the public input supplied in period two (i.e., \( G_1 \)) above its marginal in period one. An anticipated improvement in the terms-of-trade in period two therefore leads to an increase in the production of public input in period one.

An anticipated improvement in the period two terms-of-trade also affects the demand for labour in the production of final goods.
Equation (22) shows that the demand for labour in the production of $Y_1$ decreases, whereas the demand for labour in the production of $Y_2$ increases (see equation (23)). Equation (24) shows that the demand for labour in the production of $X_3$ and $Y_3$ is unaffected by anticipated changes in the terms-of-trade in period two.

The following discussion pertains to the impact of an anticipated improvement in the terms-of-trade in period two on production by the private sector:

$$\frac{\partial X_1}{\partial P_2} = \frac{-[F_{1L}(.) P_{1H_{1L}}(.)]}{[P_{1H_{1L}}(.) + F_{1L}(.)]} [\frac{\partial G_1}{\partial P_2}] < 0 \quad (25)$$

$$\frac{\partial Y_1}{\partial P_2} = H_{1L}(.) \left[ \frac{\partial L_{1y}}{\partial P_2} \right] < 0 \quad (26)$$

$$\frac{\partial X_2}{\partial P_2} = G_2^{a_2} F_{2L}(.) \left\{ \left[ H_{2L}(.) - P_{2H_{2L}}(.) \frac{\partial G_2}{\partial P_2} \right] \right\}$$

$$+ \left[ a_2 G_2^{a_2-1} F_{2L}(.) \right] \left[ \frac{\partial G_1}{\partial P_2} \right] ? \quad (27)$$

$$\frac{\partial Y_2}{\partial P_2} = G_1^{b_2} H_{2L}(.) \left[ \frac{\partial L_{2y}}{\partial P_2} \right] + \left[ \beta_2 G_1^{b_2-1} H_2(.) \right] [\frac{\partial G_1}{\partial P_2}] > 0 \quad (28)$$

$$\frac{\partial X_3}{\partial P_2} = a_3 G_2^{a_3-1} F_{3(.)} \left[ \frac{\partial G_2}{\partial P_2} \right] < 0 \quad (29)$$

$$\frac{\partial Y_3}{\partial P_2} = \beta_3 G_2^{b_3-1} H_3(.) \left[ \frac{\partial G_2}{\partial P_2} \right] < 0 \quad (30)$$

According to equations (25), (26), (29), and (30), an anticipated improvement in the terms-of-trade in period two leads
to a decrease in the output of both final goods in period one and period three. The effect on the output of $X_2$ is ambiguous whereas the output of $Y_2$ increases.

### 2.4.3 Terms-of-Trade Changes in Period Three

The following equations describe the impact of an improvement in the terms-of-trade, in period three, on the optimal $G_1$, $G_2$, $L_{1y}$, $L_{2y}$, and $L_{3y}$. This improvement was anticipated in the beginning of period one:

\[
a^G_1/\partial P_3 = \left[ -\alpha_2 \alpha_3 G_2^{q_3-1} G_1^{q_2-1} F_{2L}(.) H_3(.) \right] \left[ P_2 H_{2LL}(.) + F_{2LL}(.) \right] / H < 0 \tag{31}
\]

\[
a^G_2/\partial P_3 = \left[ -\alpha_3 G_2^{q_3-1} H_3(.) \right] \left[ P_2 H_{2LL}(.) + F_{2LL}(.) \right] / H > 0 \tag{32}
\]

\[
a^{L_{1y}}/\partial P_3 = - \left( F_{1LL}(.) / [F_{1LL}(.) + P_1 H_{1LL}(.)] \right) \left\{ a^G_1/\partial P_3 \right\} > 0 \tag{33}
\]

\[
a^{L_{2y}}/\partial P_3 = - \left( F_{2LL}(.) / [F_{2LL}(.) + P_2 H_{2LL}(.)] \right) \left\{ a^G_2/\partial P_3 \right\} < 0 \tag{34}
\]

\[
a^{L_{3y}}/\partial P_3 = - \left( H_{3L}(.) / [F_{3LL}(.) + P_3 H_{3LL}(.)] \right) > 0 \tag{35}
\]

Equation (37) shows that an anticipated improvement in the terms-of-trade, in period three, leads to an increase in the public
input produced in period two. This result can be explained by using equation (5) which shows that an anticipated improvement in the terms-of-trade in period three directly increases the marginal benefits of public input above its marginal cost. Consequently, the output of public input in period two (i.e., $G_2$) increases. An anticipated increase in the production of public input in period two leads to a decrease in the marginal benefits of the public input produced in period one below its marginal cost [see the right-hand side of equation (4)]. The output of the public input in period one (i.e., $G_1$) therefore decreases.

Equation (33) shows that an anticipated improvement in the terms-of-trade in period one, due to its negative effect on the optimal $G_1$, leads to an increase in labour employed in the production of $Y_1$. On the other hand, the employment of labour in the production of $Y_2$ decreases, as indicated by equation (34). Equation (35) is independent of the supply of public input; it shows that the demand for labour in the production of $Y_3$ increases, which is not surprising.

The impact on private sector production of an anticipated improvement in the terms-of-trade in period three is discussed in the following:

$$\partial X_1/\partial P_3 = -(F_{1L}(.)P_{1H_{1LL}}(.))/(P_{1H_{1LL}}(.)+F_{1LL}(.))\{\partial G_1/\partial P_3\} > 0 \quad (36)$$
\[ \frac{\partial Y_1}{\partial P_3} = \frac{H_{1L}(.)}{} \left[ \frac{\partial L_{1y}}{\partial P_3} \right] > 0 \]  

(37)

\[ \frac{\partial X_2}{\partial P_3} = -[G_1^{a2}F_2(.) \{F_{2L}(.)/\{P_{2H}H_{2LL}(.) + F_{2LL}(.)\}\} \{\partial G_2/\partial P_3\} + \left[\alpha_1 G_1^{a2-1}F_2(.)\right] [\partial G_1/\partial P_3] < 0 \]  

(38)

\[ \frac{\partial Y_2}{\partial P_3} = \left[ G_1^{a2}H_{2L}(. \right] \left[ \partial L_{2y}/\partial P_3 \right] + \left[ \beta_2 G_2^{a2-1}H_{2L}(. \right] [\partial G_1/\partial P_3] < 0 \]  

(39)

\[ \frac{\partial X_3}{\partial P_3} = \left[ \alpha_3 G_2^{a2-1}F_3(.) \right] [\partial G_2/\partial P_3] - \left[ G_2^{a3}F_{3L}(.) \right] [\partial L_{3y}/\partial P_3] < 0 \]  

(40)

\[ \frac{\partial Y_3}{\partial P_3} = \left[ \beta_3 G_2^{a3-1}H_{3L}(.) \right] [\partial G_2/\partial P_3] + \left[ G_2^{a3}H_{3L}(.) \right] [\partial L_{3y}/\partial P_3] > 0 \]  

(41)

Equations (36) and (37) indicate that an anticipated improvement in the (temporal) terms-of-trade in period three results in an increase in the output of both final goods produced in period one. On the other hand, equations (38) and (39) indicate that the output of both final goods in the second period decreases. The above results depend entirely on the response of the public sector, i.e., the sign of \( \partial G_e/\partial P_3 \). Equation (40) shows that the presence of a public input in the model strengthens the expected result, whereas the impact on the output of \( X_3 \) is ambiguous.

The results presented in this section clearly indicate the importance of the timing of terms-of-trade changes. These results also demonstrate the important role played by lags in the production and supply of public inputs. The public and private sectors are linked through unrestricted labour mobility. In other
words, the allocation of resources between the two sectors is endogenous. Accordingly, terms-of-trade changes influence the private sector directly, as well as indirectly through their impact on the supply of public input.

The purpose of using a three period model is to spell out the transmission of terms-of-trade changes from one period to another. The pattern which emerges is the following.

In an n-period setting, it can be shown that the sign of $\frac{\partial G_t}{\partial P_1}$ will be negative for $t = 1, 3, 5, 7, \ldots$ and positive for other values of $t$. The effect on the output of $X_t$ will be ambiguous whereas the output of $Y_t$ will increase. The output of both final goods will decrease (increase) for $t = 2, 4, 6, \ldots \ldots \ldots \ (t = 3, 5, 7, \ldots )$.

As indicated earlier, the framework of the present study also allows one to examine the impact of domestic shocks on the output of private and public sectors. An example of such shocks is exogenous technological progress in the private sector. The impact of exogenous technological progress on the output of public sector is discussed in the following section. The results derived in the next section indicate that technological progress in the production of either good in either period has implications for the supply of the public input.
In the case of an impure public input, the reward of the public input is captured by the owner of the firm. The timing of technological progress therefore has implications for the profits of firms.

2.4.4 Technological Progress in the Production of Final Goods

The impact of exogenous technological progress (in the production of final goods) on the production of public input is discussed below, where $k = x$ and $y$

\[
\partial G_1 / \partial \gamma_{1k} > 0 \quad (42)
\]

\[
\partial G_2 / \partial \gamma_{1k} < 0 \quad (43)
\]

\[
\partial G_1 / \partial \gamma_{2k} < 0 \quad (44)
\]

\[
\partial G_2 / \partial \gamma_{2k} > 0 \quad (45)
\]

\[
\partial G_3 / \partial \gamma_{3k} > 0 \quad (46)
\]

\[
\partial G_2 / \partial \gamma_{3k} < 0 \quad (47)
\]
Equation (42) and (43) respectively indicate that, due to perfect labour mobility within the private sector, exogenous technological progress in the production of either $X_1$ or $Y_1$ (or both) increases the production of public input in period one but, decreases the production of public input in period two. The explanation for this result is simple: exogenous technological progress in the private sector reduces the marginal cost of public input below its marginal benefits in the second period, so it is appropriate to produce more public input in period one. Similar reasoning applies to other results presented in this section.

Equations (44) and (45) indicate that anticipated technological progress in period two decreases (increases) the output of public input in period one (two). Whereas equations (46) and (47) demonstrate that anticipated technological progress in period three increases (decreases) the output of public input in period one (two).

Due to a lag in the production and supply of public input, the effect of technological improvement in either period is transmitted to the other periods. It can be shown that technological improvement in the production of either or both final goods in period one increases (decreases) the output of both final goods in period two (three). Also, anticipated technological improvement in the production of either or both final goods in period two increases the output of both final goods in periods one and three.
Finally, anticipated technological improvement in the production of either or both final goods in period three decreases (increases) the output of both final goods in period one (two).

The results derived so far depend on the assumption that all resources are fully utilised in the small open economy under consideration. However, significant labour unemployment exists in most real economies. In the next section, the full employment assumption is relaxed.
2.5 Terms-of-Trade Changes in an Underemployed Economy

The model developed in section 2.3 is based on the assumption that labour is fully utilised in each period. However, significant unemployment is often present in real economies. The purpose of this section is to re-examine the results presented in section 2.4 when labour is not fully utilised.

Labour unemployment in the present study is assumed to be due to economy wide rigid wages in period one. Several alternative reasons can be found for downward real wage rigidity: the real wage may be indexed institutionally; the efficiency wage theory, as expounded by Shapiro and Stiglitz (1984), Wiess (1980) and Yellen (1984) provides a mechanism whereby the real wage becomes downward rigid. This assumption is often adopted in the literature on international trade, see Bhagwati and Srinivasan (1983), Itoh and Negishi (1989) for an elegant survey. One can also appeal to the idea of "Surplus Labour" developed by Lewis (1954) to explain unemployment in period one. The present study does not attempt to explain why wages are rigid. The focus of the present study is on the outcomes when unemployment is present.

Due to the rigidity of the real wage rate in period one,
labour cannot be fully utilised and therefore the allocation of resources cannot be optimal. A competitive equilibrium can be characterised by the following conditions:

\[ \hat{w}_1 = \alpha_2 G_1^{a_2-1} F_2 (N_2 - L_{2y} - G_2, T_{2x}) + P_2 \beta_2 G_2^{\beta_2-1} H_2 (L_{2y}, T_{2y}) \quad (48) \]

\[ G_1^{a_2} F_2 (N_2 - L_{2y} - G_1, T_{2x}) = \alpha_3 G_2^{a_3-1} F_3 (N_3 - L_{3y}, T_{3x}) + P_3 \beta_3 G_3^{\beta_3-1} H_2 (L_{3y}, T_{3y}) \quad (49) \]

\[ a_1 F_{1L} (L_{1e} - L_{1y} - G_1, T_{1x}) = \hat{w}_1 \quad (50) \]

\[ P_1^{\beta_1} H_{1L} (L_{1y}, T_{1y}) = \hat{w}_1 \quad (51) \]

\[ G_1^{\alpha_2-\beta_2} F_2 (N_2 - L_{2y} - G_2, T_{2x}) = P_2 H_{2L} (L_{2y}, T_{2y}) \quad (52) \]

\[ G_2^{\alpha_3-\beta_3} F_3 (N_3 - L_{3y}, T_{3x}) = P_3 H_{3L} (L_{3y}, T_{3y}) \quad (53) \]

Where

\[ \hat{w}_1: \text{institutionally fixed minimum wage rate in period one.} \]

\[ L_{1e} = L_{1y} + L_{1y} + L_{1x}: \text{labour employed in period one.} \]

Equations (48) to (53) are six equations in six endogenous variables; \( G_1, G_2, L_{1e}, L_{1y}, L_{2y}, \) and \( L_{3y}. \) Equation (48), in conjunction with other conditions, determines the equilibrium \( G_1 \) in the presence of unemployment in period one. The left-hand side of this equation, \( \hat{w}_1, \) is the marginal cost of public input in period.
one. Inefficiency in the present formulation arises from the fact that the wage rate in period one, $\hat{\omega}_1$, is fixed above its market clearing value.

Equilibrium conditions (48) to (53) can be used to derive the impact of terms-of-trade changes on the public and private sectors of an underemployed economy:

\[
\frac{\partial G_t}{\partial P_t} = 0 \quad \text{for } t = \{2, 3\} \\
\frac{\partial X_t}{\partial P_t} = 0 \quad \text{for } t = \{1, 2, 3\} \\
\frac{\partial Y_t}{\partial P_t} = 0 \quad \text{for } t = \{2, 3\} \\
\frac{\partial X_i}{\partial P_t} = 0 \quad \text{for } t = \{2, 3\}
\]

Equation (54) shows that an improvement in the terms-of-trade in period one does not influence the production of the public input, $G_1$, in period one. This implies that terms-of-trade changes

---

13 Some comparative static results presented in section four are not affected by the presence of unemployment in period one. These results are not included in this section.
in period one will not be transmitted to periods two and three, see equations (55) to (57). An improvement in the terms of trade leads to an increase in demand for labour in the production of $Y_i$. However, the wage rate in period one is rigid; there is no outflow of labour from the public sector. The public sector is therefore insulated from terms-of-trade changes in period one. Additional workers are hired from the existing pool of unemployed workers. The output of $X_i$ also does not depend on $P_i$, which is a standard result under real wage rigidity.

Equations (58) and (59) also indicate that anticipated terms-of-trade changes in the second and the third periods have no influence on the output of the private sector in period one. An anticipated improvement in the terms-of-trade in period two leads to an increase in the production of public input in period one. However, additional workers required in the public sector can be hired from the existing pool of unemployed. Consequently, the output of the private sector is not affected. Similar reasoning applies to the result given by equation (59).

In most real economies, unions often demand an increase in the real wage. The implications of such a policy change are discussed in the following:

$$\frac{\partial G_i}{\partial \hat{w}_i} < 0$$ (60)

52
Equations (60) and (61) respectively indicate that an increase in the minimum wage, in period one, decreases the production of public input in period one, but increases the production of public input in period two. An increase in the minimum wage also affects the output of both final goods in each period: the output of final goods in period two (three) decreases (increases). An increase in the minimum wage rate increases the marginal cost of the public input produced in period one, which explains the results presented in this section.
2.6 Concluding Remarks

The present study develops a three-period perfect foresight model of a small open economy. The model is used to demonstrate that terms-of-trade changes in either period affect both private and public sectors of an economy, in that period, and lags in the production and supply of a public input transmit these effects to the other periods.

The economy under consideration produces two final goods by means of a public input, labour and other fixed factors. The public input is produced by means of labour which is fully mobile between the private and public sectors. The allocation of resources between the private and public sectors is therefore endogenous. The public input produced in period \( t \) is made available to the final good producers in period \( t+1 \). In other words, lags in the production and supply of public inputs are explicitly taken into account.

The model is used to investigate the impact of terms-of-trade changes, in either period, on the private and public sectors. The results presented in section four demonstrate the important role played by the timing of a change in the terms-of-trade. It has been shown that an improvement in the terms-of-trade in period one leads to a decrease (increase) in the production of public input in period one (two). On the other hand, an anticipated improvement in
the terms-of-trade in period two leads to an increase (decrease) in the production of public input in period one (two). Finally, an anticipated improvement in the terms-of-trade in period three leads to a decrease (increase) in the production of public input in period one (two).

Due to a lag in the production and supply of public input, the effect of terms-of-trade changes in either period is transmitted to the other periods. An improvement in the terms-of-trade in period one results in a decrease (increase) in the production of both final goods in period two (three). On the other hand, an anticipated improvement in the terms-of-trade in period two results in a decrease in the production of both final goods in period one and three. Finally, an anticipated improvement in the terms-of-trade in period three results in an increase (decrease) in the production of both final goods in period one (two).

The presence of labour unemployment in period one, due to economy wide rigid wages, breaks the intertemporal production links. A change in the terms-of-trade in period one does not change the production of public input in either period. It therefore does not influence the production of final goods in the second and third period. In addition, anticipated changes in the terms-of-trade in period two and period three do not influence the production of final goods in period one. The impact of a change in the minimum wage rate is also considered: it has been shown that a small
decrease in the minimum wage leads to an increase (decrease) in the production of public input in period one (two). Consequently, the production of both final goods in period two (three) increases (decreases).

The model can be extended further to include uncertainty about the future provision of the public input and the future terms of trade.
CHAPTER 3

GOVERNMENT SPENDING ON INDUSTRIES, INTERNATIONAL FACTOR MOBILITY, AND POLICY COORDINATION

3.1 Introduction

In all real economies, governments spend large sums on services provided to households and industries in all real economies. A change in government spending in a large open economy, like the United States or Japan, affects not only the domestic economy but also the economies of trading partners. Concerns of a similar nature led to the formation of the so called G-7 (group of seven), an important objective of which is to coordinate monetary and fiscal policies. Frenkel and Razin (1986a, 1986b, 1987), Svensson (1987), Devereux (1988), and Durlauf and Staiger (1990), among others, have discussed the international (indirect) spillovers of government spending in an economy. However, these and
most other relevant studies do not consider government spending on industries.¹ In addition, these studies have not investigated the implications of potential international economic policy coordination.

The idea of international fiscal coordination has not received much attention in the theoretical literature. Kehoe (1987, 1989) is concerned with government spending on households only. Using a Cobb-Douglas specification for the utility function, Kehoe (1987) argues that in a world economy in which all countries are small, equilibrium under international fiscal cooperation may not coincide with the non-cooperative equilibrium. Kehoe (1989) presents a counter-example to the belief that fiscal cooperation among benevolent governments is desirable. His result is partly driven by the time inconsistency problem. Using a logarithmic specification of the utility and Cobb-Douglas production functions, Devereux and Mansoorian (1989) argue that the gains from international fiscal coordination may entail higher growth rates for the countries involved.

In Anwar (1992), I have considered government spending on what Meade (1952) described as "atmosphere externalities". This spending

¹ The importance of this is examined by Abe (1990), Manning and McMillan (1979), and McMillan (1978), who suggest that government spending on industries can influence the direction of international trade.
is incorporated into a one-good, two-country, and two-factor general equilibrium model in the form of a pure public input. The public input is available to all firms in the industry. Under competitive conditions, each country produces an identical consumption good by means of labour, capital, and a non-traded pure public input. The two countries are linked through international factor mobility: in the short-run, only one private factor is fully mobile across international boundaries; whereas in the long-run, both private factors are fully mobile. In addition, both factors are fully utilised in the short-run and the long-run. International transmission of government spending on a pure public input and policy coordination are examined in the short-run and the long-run.

In the short-run, an increase in the supply of public input in either country increases the reward of the fully mobile factor. On the other hand, in the long-run, the impact of an increase in the supply of public input in either country on the equilibrium wage rate and the rate of return on capital depends on the relative capital intensity.

In the short-run non-cooperative solution, a country which exports the mobile factor spends too much on its industries from the point of view of the its trading partner. Whereas, in the long-run non-cooperative solution, both countries spend too much (little) on their industries from the point of view of each other, if the capital (labour) intensive country exports capital. On the
other hand, in the short-run cooperative solution, a country which exports the mobile factor spends too little on its industries from the point of view of its residents. Whereas, in the long-run cooperative solution, both countries spend too little (much) on their industries from the point of view of their residents, if the capital (labour) intensive country exports capital.

This study extends my earlier work in the following directions: (1) international transmission of government spending on industries in the presence of labour unemployment is considered; (2) a three-country model is developed at the end of the essay which allows an investigation of international transmission of government spending when two countries have formed an economic union. The present study extends and generalises the results derived in my previous study [Anwar (1992)].

The present study incorporates a pure public input into a two-country, one-good, two-factor general equilibrium model. Under competitive conditions, each country produces an identical consumption good by means of labour, capital, and a pure public input. The public input in each country is provided free of charge by the government. Both governments use an income tax to finance the cost of public production. The two countries are linked through international factor mobility.

This study focuses on the implications of public inputs across
international boundaries. The public inputs do not spillover directly, but because of their implications for relative prices, there are impacts on countries which have integrated economies. These impacts are examined under the following alternative assumptions regarding factor mobility and labour employment: (1) capital is fully mobile across international boundaries but labour mobility is restricted, while labour is not fully employed in one country due to rigid wages; (2) both private factors, i.e., labour and capital, are fully mobile across international boundaries and all factors are fully employed. These two cases can be interpreted as the short-run and the long-run respectively. The model developed in the next section is also used to compare the coordinated policy outcome to the uncoordinated one.

In the short-run, the supply of the public input in a country where wages are fully flexible does not influence the equilibrium rate of return on capital, which is fully mobile across international boundaries. On the other hand, the supply of public input in a country where wages are rigid can influence the equilibrium rate of return on capital. The supply of public input in a country which does not fully utilise labour can also influence the equilibrium wage rate in the country which fully utilises its labour. Consequently, in the absence of international policy

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2 The long-run model is identical to the one presented in Anwar (1992). However, the present version explicitly considers the existence and stability of the interior solution.
coordination, a country which exports (imports) capital and where wages are rigid spends too much (little) on its industries from the point of view of the other country where wages are fully flexible. On the other hand, in the absence of international policy coordination, a country which does not fully utilise labour spends too much on its industries from the point of view of the country which fully utilises its labour irrespective of the direction of international capital mobility.

In the long-run, the supply of public input in both countries determines all factor prices. A relatively capital (labour) intensive country can increase the equilibrium rate of return on capital (labour) by increasing its supply of public input. In the absence of international policy coordination, each country spends too much on its industries from the point of view of the other country, if the production technologies are such that the capital (labour) intensive country exports capital (labour). On the other hand, under international policy coordination both countries spend too little on their industries from the point of view of their residents, if the capital (labour) intensive country exports capital (labour).

The model is further extended to include a third country called the rest of the world. A three-country model allows an investigation of international transmission of government spending on industries when two countries have formed an economic union.
Capital is fully mobile across international boundaries, including the rest of the world. On the other hand, international labour mobility is restricted: labour is fully mobile between the other two countries only. The other two countries can therefore be considered as members of an economic union, such as the European Economic Community. All resources are fully utilised in each country.

It is shown that despite free international capital mobility and full utilisation of resources, the supply of the public input in the rest of the world does not influence the equilibrium rate of return on capital. In addition, the supply of public input in the rest of the world does not influence the equilibrium wage rate in the economic union. The rest of the world therefore cannot influence the consumption of the members of the economic union through its supply of public input. On the other hand, the supply of public inputs in the economic union can influence the equilibrium rate of return on capital and the wage rate in the rest of the world. Policy coordination within the economic union is therefore desirable in order to exploit the rest of the world.

The essay is organised as follows. The next section develops a simple two-country, two-sector general equilibrium model with a pure public input and international capital mobility. Due to economy wide rigid wages, labour is not fully utilised in one country. International transmission of economic policy and the
implications of potential international policy coordination in the short-run are explored in section three. A long-run model is developed in section four, where both private factors are fully mobile across international boundaries and there is no unemployment. In the fifth section, international transmission of economic policy and the ramifications of international economic policy coordination are examined in the long-run. A three-country model is developed in section six, where two countries have formed an economic union. The model is used to re-investigate the international transmission of government spending on industries. The last section contains concluding remarks.
3.2 A Short-Run Model

The purpose of this section is to develop a simple two-country model which allows an investigation of international transmission of government spending on public inputs. There are no international spillovers of public inputs directly, but because of their implications for factor prices, a change in the supply of a public input in either country can influence the consumption of both countries. The two countries under consideration are home and the foreign countries. Each country produces a final good by means of a pure public input (G), capital, and labour. All inputs are essential in the production of the relevant goods. The pure public input is provided free of charge by the government of each country.

A widely cited (see for instance, Laffont (1975), Negishi (1973), and Sandmo (1972)) example of such pure public input is government financed applied scientific research whereby information on new production techniques is made available to all firms simultaneously.

Each country has a fixed endowment of private factors, i.e., capital and labour. Capital is fully mobile across international boundaries. On the other hand, there are barriers to free

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3 Throughout the paper, the foreign variables will be distinguished by an asterisk (*).
international labour mobility. Significant labour unemployment exists in many real economies. In addition, international rates of labour unemployment differ considerably. Unemployment in most open economy studies is assumed to be due to rigid wages, Bhagwati (1883) and Batra and Beladi (1990) for example. The countries under consideration (in the short-run) are therefore assumed to be asymmetric: wages are fully flexible in the foreign country, whereas wages are rigid in the home country. Consequently, home labour is not fully utilised. In other words, the home country is a labour surplus economy. Batra and Beladi (1990) have considered the pattern of trade between two asymmetric economies.

The final good is the numéraire and public input is also measured in its units. Since the public input is measured in the units of the final good, the marginal rate of transformation between the two is constant. Public production is financed by means of a flat rate tax on domestic income. The government receives taxes in terms of the numéraire good which are converted into a pure public input without additional cost: these are assumptions widely used in the existing literature, see Barro (1990) and Devereux and Mansoorian (1989) for instance.4

In the case of a pure public input, the possibility of

4 This is essentially a simplifying assumption. The results presented in this essay continue to hold even if the public input is produced by means of (fully mobile) primary factors.
congestion in use does not arise. Constant returns to scale therefore has a very different meaning. Following Meade (1952), many authors (for example, Negishi (1973), Manning and McMillan (1982)) have indicated that the appropriate definition of constant returns to scale is a production technology linearly homogeneous in the private factors of production alone. Consequently, the final good production function exhibits increasing returns to scale as a whole.

If for a given level of pure public input, the final good technology is homogeneous of degree one in private inputs then there is no economic profit under competitive conditions because the entire output is exhausted by payments to the private factors. In other words, the reward for the pure public input (which is supplied free of charge by the government) is appropriated by the private factors of production.

The production functions for the final good for the home and the foreign countries are given below:

\[ Y = G^a F(K_y, L_y); \ 1 > \alpha > 0 \]

\[ \tilde{Y} = \tilde{G}^\beta \tilde{F}(\tilde{K}_y, \tilde{L}_y); \ 1 > \beta > 0 \]

where \( \alpha \) and \( \beta \) are constants.

\( G \): supply of pure public input in the home country.
\( K_y \): capital used in the production of the final good in the home
country.

$L_y$: labour used in the production of the final good in the home country.

The functional form of the above production technologies implies that the public input is cooperative with private inputs in the production of $Y$ and $R$ and there are diminishing returns with respect to the public input.

Competitive firms in both countries take the supply of public input as given. $F(.)$ and $F(.)$ are linearly homogeneous with respect to the relevant inputs. The production functions for the home and the foreign country described above are therefore the industry production functions. There are economies of scale in the present case but these economies are external to both the firm and the industry. Due to the Marshallian nature of economies of scale the market structure is assumed to be competitive.

An increase in the supply of the public input in the present study can also be interpreted as an improvement in the respective production techniques. However, such a technological improvement can be achieved only by an increase in the relevant tax rate. In other words, technological progress is endogenous.

The relevant cost functions for the home and the foreign countries are derived below:
\[ Y \ C(r, w)/G^a = \text{Min} \ [wL_y + rK_y : Y = G^a F(K_y, L_y)] \]

with respect to \( L_y \) and \( K_y \).

\[ \hat{Y} \ C(r, \hat{w})/G^\beta = \text{Min} \ [\hat{w}L_y + rK_y : \hat{Y} = G^\beta \ F(\hat{K}_y, \hat{L}_y)] \]

with respect to \( \hat{L}_y \) and \( \hat{K}_y \).

where

- \( w_0 \): minimum wage rate in the home country.
- \( r \): rate of return on capital in the home and foreign country.
- \( C(r, w_0)/G^a \): home country's unit cost function for the final good.

Due to unrestricted international capital mobility, the rate of return on capital in the two countries is identical. The zero profit conditions for the home and the foreign country respectively are given below:

\[ C(r, w_0)/G^a = 1 \] \hspace{1cm} (1)

\[ \hat{C}(r, \hat{w})/G^\beta = 1 \] \hspace{1cm} (2)

For a given supply of the pure public input, the above zero profit conditions determine the equilibrium factor prices independent of factor market clearing conditions. However, in a full equilibrium, only those values of \( G \) and \( \hat{G} \) are considered for which the economy's minimum wage constraint is binding. The factor market clearing conditions given below actually determine the
output of the final good in the two countries and labour employment in the home country:

\[ K_0 + \dot{K}_0 = Y \left\{ C_r(r, w_0)/G^a \right\} + \dot{Y} \left\{ \dot{C}_r(r, \dot{w})/G^b \right\} \tag{3} \]

\[ L = Y \left\{ C_w(r, w_0)/G^a \right\} \tag{4} \]

\[ \dot{N}_e = \dot{Y} \left\{ \dot{C}_w(r, \dot{w})/G^b \right\} \tag{5} \]

where

- \( K_0 \): capital endowment of the home country.
- \( L \): labour employed in the home country.
- \( \dot{N}_e \): labour supply in the foreign country.
- \( C_r(.) / G^a \): capital requirement per unit of \( Y \) in the home country.
- \( C_w(.) / G^a \): labour requirement per unit of \( Y \) in the home country.

Equation (3) is the international capital market clearing condition. Equation (4) in conjunction with other equations determines the labour employment in the home country. Whereas, equation (5) determines the labour market clearing wage rate in the foreign country.

(1) and (2) are equations in two endogenous variables, \( r \) and \( \dot{w} \). Once the optimal \( r \) and \( \dot{w} \) are determined the output of final good in the two countries and labour employment in the home country can be determined from equations (3) to (5). Clearly, the optimal \( r \) and
are influenced by the supply of the public input in both countries. This completes the discussion of the production side of the model.

Because both countries produce an identical final good (which is also the numéraire), supply determines demand in the present model.\(^5\) The net consumption of the home and the foreign country (\(c\) and \(c^*\) respectively) is given below, where gross national product is used as the tax base:

\[
c = [rK_e + w_oL] - G
\]

\[
c^* = [rK_e^* + wN_e^*] - G
\]

The government in each country determines the level of public input. In other words, each government acts as a monopolist in its provision of public input within national boundaries. The provision of public input by the two governments can be modelled as a standard two person game where the following solutions can be adopted: (1) a non-cooperative solution where each government passively observes the other and takes its supply of public input as given, then determines its own supply; (2) a cooperative solution which involves international economic policy coordination.

\(^5\) In other words, Walras' law ensures that the market for consumption goods clears in both countries.
The supply of public input in the home and the foreign countries (i.e., $G$ and $\hat{G}$ respectively) enter as parameters in the reduced form solutions of $r$ and $\hat{w}$. Consequently, a change in the supply of public input in either country influences the relevant variables and hence the net consumption of both countries. In the present study, the effects of a change in the supply of public input in one country are transmitted to another through international factor mobility. These effects and the possibility of international economic policy coordination are examined in the next section.

The public input in the present study can also be considered as a local public (intermediate) good. In such a case, the model laid out in this section describes two regions in a closed economy where capital is fully mobile between the two regions but labour mobility is restricted. Also, the earnings of the mobile factors are repatriated. Each region is managed by a separate government which provides a pure public input for use within its own jurisdiction.\(^6\) The results presented in this essay can easily be reinterpreted.

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\(^6\) See Mera (1973) for an empirical investigation of the role of infrastructure investment in raising the income level of various regions in Japan.
3.3 Transmission of Economic Policy

and International Coordination

in the Short-Run

The purpose of this section is to demonstrate that due to its implications for factor prices, a very small change in the provision of public input in either country can significantly influence the welfare level of its trading partner. Accordingly, there is a need for international economic policy coordination. In the rest of this section, the implications of such coordination are analysed by means of a comparative static exercise. The countries considered in this section are asymmetric: wages are rigid in the home country, whereas wages are fully flexible in the foreign country.

The impact of a small change in the provision of public input in either country on the optimal \( r \) and \( \hat{w} \) is considered below. These results are derived by differentiating equations (1) and (2):

\[
\frac{\partial r}{\partial G} = \alpha G^{a-1}/C_r(.) > 0 \quad (8)
\]

\[
\frac{\partial r}{\partial \hat{G}} = 0 \quad (9)
\]

\[
\frac{\partial \hat{w}}{\partial G} = - \left[ \frac{\hat{C}_r(.)}{\hat{C}_w(.)} \right] \left[ \frac{\partial r}{\partial G} \right] < 0 \quad (10)
\]
Equation (8) shows that a small increase in the supply of public input in the home country increases the equilibrium rate of return on the internationally mobile factor, capital. Whereas, a small increase in the supply of public input in the foreign country does not influence the equilibrium rate of return on capital, see equation (9). These results can be explained by means of equations (1) and (2): for a given G, wage rigidity implies that the equilibrium rate of return on capital does not depend on the supply of public input in the foreign country. In other words, the equilibrium international rate of return on capital coincides with the home country’s pre-capital mobility rate.

Equation (10) shows that a small increase in the supply of public input in the home country, through its impact on capital mobility, decreases the equilibrium wage rate in the foreign country. This result follows from equation (8): an increase in the supply of public input in the home country increases the equilibrium rate of return on capital in the home country. Capital moves from the foreign country to the home country to take advantage of the higher rate of return. This leads to a situation where there are too many workers per-unit of capital in the foreign country. The equilibrium wage rate in the foreign country therefore falls. On the other hand, equation (11) shows that a small increase
in the supply of public input in the foreign country increases its equilibrium wage rate because such an increase directly increases the marginal productivity of foreign labour.

Equations (4) and (5) can be used to eliminate $Y$ and $\dot{Y}$ from equation (3). The resulting equation is given below:

$$K_o + \dot{K}_e = L \left\{ C_r(r, w_o)/C_w(r, w_o) \right\} + N_o \left\{ C_r(r, \dot{w})/C_w(r, \dot{w}) \right\}$$

In the following, the above equation is used to determine the impact of a change in the supply of public input on labour employment in the home country.

$$\frac{\partial L}{\partial G} > 0 \quad (12)$$

$$\frac{\partial L}{\partial G} = - \left[ N_o/\dot{C}_w(.) \right] \left[ C_w(.)/C_r(.) \right] \left[ \dot{C}_r(.) \dot{C}_w(.) \right]
- \dot{C}_w(.) \dot{C}_w(.) \left[ \frac{\partial \dot{w}}{\partial G} \right] < 0 \quad (13)$$

Equation (12) shows that the supply of public input in the home country is positively related to labour employment. This result follows from the implicit assumption that labour and the public input are cooperative in the production of final good. On the other hand, equation (13) shows that an increase in the supply of public input in the foreign country, where labour is fully employed, decreases labour employment in the home country due to its negative impact on the equilibrium wage rate.
In a Cournot-Nash equilibrium (i.e., in the absence of international economic policy coordination), each country takes the supply of public input in the other country as given and determines its own supply such that the net consumption of its residents is maximised. The following equations, which are derived by using equations (6), (7), and (9), determine the optimal supply of public input in the home country and the foreign country respectively:

\[ K_e \left( \frac{\partial r}{\partial G} \right) + w_e \left( \frac{\partial L}{\partial G} \right) = 1 \]

\[ N_e \left( \frac{\partial w}{\partial G^*} \right) = 1 \]

The right hand side of the above equations is the marginal cost of the provision of public input to tax payers whereas the left hand side is the net marginal benefit to the owners of the private factors. Clearly, the above rules ignore the indirect spillovers of public input across international boundaries due to international capital mobility.

The impact of a small change in the provision of public input on the net consumption of the two countries is given below where the initial supply of public input in each country is determined by using the above optimality conditions. These results are derived using equations (6) and (7):

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7 In the context of a closed economy model, Grossman and Lucas (1975) have derived a similar condition.
\[ \frac{\partial c^*}{\partial G} = K_e \left\{ \frac{\partial r}{\partial G} \right\} + N_e \left\{ \frac{\partial \hat{c}^*}{\partial G} \right\} \]
\[ = [K_e - N_e \frac{C_r(\cdot)}{C_w(\cdot)}] \left\{ \frac{\partial r}{\partial G} \right\} \]
\[ = [K_e - \hat{K}] \left\{ \frac{\partial r}{\partial G} \right\} \] (14)

\[ \frac{\partial c}{\partial G} = K_e \left\{ \frac{\partial r^*}{\partial G} \right\} + w_o \left\{ \frac{\partial L}{\partial G} \right\} \]
\[ = w_o \left\{ \frac{\partial L}{\partial G} \right\} < 0 \] (15)

A small increase in G has no first order effect on home consumption because it is optimally chosen. However, such an increase in G can affect foreign consumption as indicated by equation (14) above. The sign of \( \frac{\partial c^*}{\partial G} \) depends on the direction of international capital mobility. If in the initial Cournot-Nash equilibrium the foreign country exports capital, then \( \frac{\partial c^*}{\partial G} \) is positive. In such a case, it can be claimed that the home country spends too little on its industries from the point of the foreign country.

This follows from the fact that a small increase in G does not change the net consumption of the home country, but increases the net consumption of the foreign country. Therefore, from the point of view of foreign country, the home country spends too little on its industries.\(^8\) On the other hand, if the foreign country imports

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\(^8\) Negish (1973) and Grossman and Lucas (1975) have used a similar argument to determine the excess of public spending in an economy.
capital then the sign of $\partial \hat{c}/\partial G$ is negative which implies that from the point of view of the foreign country, the home country spends too much on its industries.

Equation (15) shows that the impact of an increase in the foreign country's supply of public input on the net consumption of the home country is negative, irrespective of the direction of international capital mobility. An increase in the supply of public input in the foreign country, where labour is fully employed decreases the net consumption of the home country through its adverse impact on the employment of labour. In other words from the point of view of the home country where labour is not fully utilised, the foreign country spends too much on its industries irrespective of the direction of capital mobility.

Since each country can influence the net consumption and therefore the welfare level of its trading partner by its choice of spending on industries, there is a need for international policy coordination. Most available studies on international economic policy coordination derive optimality rules by maximising the aggregate welfare of the countries involved; the present study follows the existing literature in this respect.\(^9\) From a theoretical point of view, the joint welfare maximisation can also

be justified if lump-sum transfers between the two countries are allowed. However, the actual level of public spending would probably be the result of international bargaining. The rest of this section deals with the implications of international economic policy coordination.

In the present framework, the welfare of each country is measured by its net consumption of the final good. Consequently, the objective of international economic policy coordination is to maximise aggregate net consumption of the two-country world. This problem is formally stated in the following:

\[
\text{Max } [r(K_e + K_i) + w_o L + w N_e - G - G] \quad \text{with respect to } G \text{ and } G. 
\]

The first-order conditions for a maximum are the following, where equation (9) has been used in deriving condition (17):

\[
(K_e + K_i) \frac{\partial r}{\partial G} + w_o \frac{\partial L}{\partial G} + N_e \frac{\partial w}{\partial G} = 1 \quad (16)
\]

\[
w_o \frac{\partial L}{\partial G} + N_e \frac{\partial w}{\partial G} = 1 \quad (17)
\]

Equations (16) and (17) state that the aggregate consumption of the two-country world will be maximised when each government supplies a level of public input such that its net marginal benefit to the owners of private factors in both countries equal its
marginal cost to taxpayers. The right hand side of the above equations is marginal cost of the provision of public input to taxpayers, whereas the left hand side is the net marginal benefit to owners of private factors in the two countries.

It can easily be shown that international economic policy coordination in the above model is not optimal either for the home or for the foreign country in isolation. The following derivatives describe the impact on net consumption of the home and the foreign country when the respective governments increase supply of public input by a small amount. These are derived by differentiating equation (6) with respect to $G$, and equation (7) with respect to $\hat{G}$. Equation (9) has also been used in deriving $\partial c^*/\partial \hat{G}$:

$$\frac{\partial c}{\partial G} = K_e \left( \frac{\partial r}{\partial G} \right) + w_o \left( \frac{\partial L}{\partial G} \right) - 1$$

$$\frac{\partial c^*}{\partial \hat{G}} = \hat{N}_e \left( \frac{\partial \hat{w}}{\partial \hat{G}} \right) - 1$$

The above equations, after making use of equation (16) and (17) and further substitution, can be written as the following:

$$\frac{\partial c}{\partial G} = - K_e \left( \frac{\partial r}{\partial G} \right) - \hat{N}_e \left( \frac{\partial \hat{w}}{\partial \hat{G}} \right)$$

$$= - \left[ K_e - \hat{N}_e \frac{\hat{C}_r(\cdot)}{\check{C}_w(\cdot)} \right] \left[ \frac{\partial r}{\partial G} \right]$$

$$= - \left[ K_e - \hat{K} \right] \left[ \frac{\partial r}{\partial G} \right] \quad (18)$$

80
\[ \frac{\partial c^*}{\partial G^*} = - w_o \left\{ \frac{\partial L^*}{\partial G^*} \right\} > 0 \] (19)

Since \( \frac{\partial c}{\partial G} \) and \( \frac{\partial c^*}{\partial G^*} \) are not zero, international economic policy coordination does not maximise the net consumption of either country in isolation. If in the initial equilibrium the home country exports capital, then from the point of view of residents of the home country, the home government spends too little on industries. On the other hand, if the home country imports capital, then from the point of view of the home residents their government spends too much on domestic industries.

On the other hand, equation (19) shows that from the point of view of the residents of the foreign country, its government spends too little on industries. This result does not depend on the direction of international capital mobility.

The results presented in this section can be extended to include more than two (private) factors of production. However, these results are sensitive to the assumption that only capital is fully mobile across international boundaries. The analysis conducted thus far can therefore be interpreted as the short-run analysis, where only one factor is internationally mobile and labour is not fully utilised in one country. In the next section, the international transmission of economic policy and the implications of potential international policy coordination are re-
investigated in a long-run context where all private factors are free to move across international boundaries.
3.4 A Long-Run Model

The purpose of this section is to extend the model developed in the previous section by allowing unrestricted international mobility of both private factors. The modified framework allows an investigation of the international transmission of government spending on industries and potential international policy coordination in the long-run.

The countries under consideration are symmetrical. Each country produces a final good (Y). The final good is produced by means of a pure public input (G), capital, and labour. All resources are fully utilised in both countries. Both private factors (i.e., capital and labour) are fully mobile across international boundaries.

In the present framework, due to the non-convexity associated with the overall allocation of capital and labour, corner solutions are also possible with both private factors moving to one country or another depending on the production technologies and the level of public input provision. However, in a two-country setting, the interior solution is both interesting and relevant. Corner solutions are possible if one country is uniformly more productive than the other country in the sense that \( F(z, v) = k \hat{F}(z, v) \) for all \( z, v \); where \( k \) is some constant greater than unity. The
implications of corner solutions are briefly examined towards the end of next section. The interior solution is considered in the following.

The relevant zero profit conditions for the home and the foreign country are given below, where $w$ is the wage rate:

$$\frac{C(r, w)}{G^a} = 1$$  \hspace{1cm} (20)

$$\frac{\hat{C}(r, w)}{G^\beta} = 1$$  \hspace{1cm} (21)

Due to unrestricted international factor mobility, the wage rate and the rate of return on capital in the two countries are identical. For a given supply of pure public input, the above zero profit conditions determine the factor prices independent of factor market clearing conditions. The factor market clearing conditions given below determine the equilibrium output of the final goods in the two countries:

$$K_e + \hat{K}_e = Y \left\{ \frac{C_v(r, w)}{G^a} \right\} + Y \left\{ \frac{\hat{C}_v(r, w)}{G^\beta} \right\}$$  \hspace{1cm} (22)

$$N_e + \hat{N}_e = Y \left\{ \frac{C_u(r, w)}{G^a} \right\} + Y \left\{ \frac{\hat{C}_u(r, w)}{G^\beta} \right\}$$  \hspace{1cm} (23)

where

$K_e$: capital endowment of the home country.

$N_e$: labour supply of the home country.
Equations (22) and (23) are the international market clearing conditions for capital and labour respectively. These equations indicate that both primary factors of production are fully utilised in the present two-country world. The existence of an interior solution is considered in the following.

For a given supply of pure public input in the home and the foreign country, equations (22) and (23) can be solved for $Y$ and $\check{Y}$, if and only if the determinant of relevant Jacobian matrix is non-zero. The relevant determinant is non-zero, if the following condition holds:\textsuperscript{10}

$$G^{a\gamma\beta} [C_w(\cdot)\check{C}_r(\cdot) - C_r(\cdot)\check{C}_w(\cdot)] \neq 0$$

In other words, a unique interior solution in which both $Y$ and $\check{Y}$ are positive exists as long as $(\check{K}_y/\check{L}_y)$ is not equal to $(K_y/L_y)$. If $(\check{K}_y/\check{L}_y)$ is greater (less) than $(K_y/L_y)$ then the foreign country is relatively capital (labour) intensive. The results derived in the next section depend on the relative capital intensity of the home and the foreign country; therefore the foreign country is assumed

\textsuperscript{10} For certain values of $K_e$, $N_e$, $\check{K}_e$, and $\check{N}_e$, the output of the final good in the two countries may not be positive. The present study assumes away such values.
to be relatively capital intensive. The stability of the interior solution is examined in the appendix.

Equations (20) and (21) are two equations in two endogenous variables; r w. Once the optimal r and w are determined, the output of the final goods in the two countries can be determined by equations (22) and (23). Clearly, the optimal r and w are influenced by the supply of the public input in both countries. This completes the discussion of production side of the model.

The net consumption of the home and the foreign country (c and c* respectively) are given below:

\[ c = [rK_e + wN_e] - G \]  \hspace{1cm} (24)

\[ c^* = [r^*K_e + w^*N_e] - ^*G \]  \hspace{1cm} (25)

The supply of public input in the home and the foreign country (i.e., G and G* respectively) enter as parameters in the reduced form solutions of r, w. Consequently, a change in the supply of public input in either country influences the relevant variables and hence the net consumption of both countries. In the present study, the effects of a change in the supply of public input in one country are transmitted to another through international factor mobility. These effects and the possibility of international economic policy coordination are examined in the next section.
3.5 Transmission of Economic Policy and International Coordination in the Long-Run

The purpose of this section is to re-investigate the international transmission of government spending on a pure public input in the long-run. The results derived in this section are compared with those derived in the short-run. The implications of potential international policy coordination are also examined by means of a comparative static exercise.

The impact of a small change in the provision of public input in either country on optimal $r$ and $w$ is considered below. These results are derived by differentiating equations (20) and (21):

\[
\frac{\partial r}{\partial G} = - \alpha G^{\alpha-1} C_r(\cdot) / H < 0 \tag{26}
\]

\[
\frac{\partial r}{\partial G^*} = \beta G^* \beta^{-1} C_r(\cdot) / H > 0 \tag{27}
\]

\[
\frac{\partial w}{\partial G} = \alpha G^{\alpha-1} C_w(\cdot) / H > 0 \tag{28}
\]

\[
\frac{\partial w}{\partial G^*} = - \beta G^* \beta^{-1} C_w(\cdot) / H < 0 \tag{29}
\]

where
$H = [C_w(.)^\ast C_r(.) - C_r(.)^\ast C_w(.)] > 0$

Clearly, the sign of the above comparative static results depends on the sign of $H$. Since the foreign country is assumed to be relatively capital intensive, $H$ is positive. In other words, $(K_y/L_y)^\ast$ is greater than $(K_y/L_y)$.

Equations (26) and (28) respectively indicate that a very small increase in the supply of public input in the home country decreases the equilibrium rate of return on capital and increases the equilibrium wage rate. This result follows from the assumption that the foreign country is relatively capital intensive compared to the home country. For a given rate of return on capital, an increase in $G$ increases the wage rate in the home country above its initial equilibrium value. Consequently, labour moves from the foreign country to the home country to take advantage of higher wages. However, the foreign country is less labour intensive compared to the home country. Consequently, not enough labour is released from the foreign country. Accordingly, the equilibrium wage rate stays at a higher level. Also, due to outflow of labour, the foreign country ends up with too much capital per unit of labour which puts downwards pressure on the equilibrium rate of return on capital.

In other words, during the adjustment period, both capital and
labour move from the foreign country to the home country, but too much capital and too little labour are released from the foreign industries. Therefore the equilibrium rate of return on capital decreases and the wage rate increases.

Equations (27) and (29) indicate that a small increase in the supply of public input in the foreign country results in a higher rate of return on capital and lower wage rate. This is because the home country is relatively labour intensive compared to the foreign country. An increase in the foreign supply of public input results in too much labour but too little capital inflow.

The above results indicate that in the present framework, a capital (labour) intensive country can increase (decrease) the equilibrium rate of return on capital, and decrease (increase) the equilibrium wage rate, by increasing its spending on industries.

These results sharply differ from those derived in section three where only capital is fully mobile across international boundaries: when only capital is fully mobile and wages are rigid in the home country, the foreign country cannot influence the equilibrium rate of return on the mobile factor.

In a Cournot-Nash equilibrium (i.e., in the absence of international economic policy coordination), each country takes the supply of public input in the other country as given and determines
its own supply such that the net consumption of its residents is maximised. The following equations determine the optimal supply of public input in the home and foreign country respectively:

\[ K_e \left( \frac{\partial r}{\partial G} \right) + N_e \left( \frac{\partial w}{\partial G} \right) = 1 \]

\[ \tilde{K}_e \left( \frac{\partial \tilde{r}}{\partial \tilde{G}} \right) + \tilde{N}_e \left( \frac{\partial \tilde{w}}{\partial \tilde{G}} \right) = 1 \]

The right hand side of the above equations is the marginal cost to tax payers of the provision of public input, whereas the left hand side is the net marginal benefits to owners of internationally mobile factors. Clearly, the above rules ignore indirect spillovers of the public input across international boundaries due to international factor mobility.

The impact of a small change in the provision of public input on the net consumption of the two countries is given below, where the initial supply of public input in each country is determined by using the above optimality conditions. These results are derived by using equations (24) and (25):

\[ \frac{\partial \tilde{c}}{\partial G} = \tilde{K}_e \left( \frac{\partial r}{\partial G} \right) + \tilde{N}_e \left( \frac{\partial w}{\partial G} \right) \]

\[ = \tilde{N}_e \left( \frac{\tilde{K}_e}{\tilde{N}_e} - \frac{\tilde{K}_y}{\tilde{L}_y} \right) \frac{\partial r}{\partial G} / H < 0 \quad (30) \]

Since the home country is relatively labour intensive, the owners of domestic capital will prefer a lower level of public input.
\[
\frac{\partial c}{\partial \hat{G}} = K_e \left( \frac{\partial r}{\partial \hat{G}} \right) + N_e \left( \frac{\partial w}{\partial \hat{G}} \right) \\
= N_e \left[ (K_e/N_e) - (K_f/L_f) \right] \left( \frac{\partial r}{\partial \hat{G}} \right)/H < 0
\]

A small change in the supply of public input in the home country does not change its consumption since the initial level of G is optimally chosen. Such an increase however has implications for foreign consumption as indicated by equation (30). If the foreign country exports capital and imports labour in the initial equilibrium, then both \( \frac{\partial c}{\partial G} \) and \( \frac{\partial c}{\partial \hat{G}} \) are negative. Therefore, following Negishi (1973), it can be argued that each country spends too much on its industries from the point of view of its trading partner.\(^{12}\)

This follows from the fact that a small increase in G does not change the net consumption of the home country, but decreases the net consumption of the foreign country. Therefore, from the point of view of the foreign country, the home country spends too much on its industries. Consider the foreign country: a small increase in its public spending does not change its own net consumption since \( \hat{G} \) is optimally chosen, but equation (31) shows that it decreases the net consumption of the home country due to the specified direction of factor mobility. Accordingly, from the point of view

\(^{12}\) It can easily be established that if the foreign country exports capital and imports labour, and H is positive, then the foreign country must be capital abundant, i.e., \( \frac{K_e}{N_e} > \frac{K_f}{N_f} \).
of the home country, the foreign country spends too much on its industries.

Since each country can influence the net consumption and therefore the welfare level of its trading partner by its choice of spending on industries, there is a need for international economic policy coordination. The rest of this section deals with the implications of international economic policy coordination.

The objective of international economic policy coordination is to maximise the aggregate net consumption of the two-country world. This problem is formally stated in the following:

$$\text{Max } [r(K_e + K_o) + w(N_e + N_o) - G - \bar{G}] \text{ with respect to } G \text{ and } \bar{G}.$$  

In other words, each government selects its spending such that aggregate net consumption is maximised. The first-order conditions for a maximum are the following:

$$\begin{align*}
(K_e + K_o) \left( \frac{\partial r}{\partial G} \right) + (N_e + N_o) \left( \frac{\partial w}{\partial G} \right) &= 1 \\
(K_o + K_o) \left( \frac{\partial r}{\partial \bar{G}} \right) + (N_o + N_o) \left( \frac{\partial w}{\partial \bar{G}} \right) &= 1
\end{align*}$$  

Equations (32) and (33) state that the aggregate consumption of the two-country world will be maximum when each government supplies a level of public input such that its net marginal benefit
to the owners of private factors in both countries equals its marginal cost to tax payers.

It can easily be shown that international economic policy coordination in the above model is not optimal either for the home or for the foreign country in isolation. By differentiating equation (24) with respect to $G$, and equation (25) with respect to $G^*$

$$
\frac{dc}{dg} = K_e \left( \frac{dr}{dg} \right) + N_e \left( \frac{dw}{dg} \right) - 1
$$

$$
\frac{dc^*}{dg^*} = K_e \left( \frac{dr^*}{dg^*} \right) + N_e \left( \frac{dw^*}{dg^*} \right) - 1
$$

The above equations, after making use of equation (32) and (33) and further substitution, can be written as follows:

$$
\frac{dc}{dg} = - K_e \left( \frac{dr}{dg} \right) - N_e \left( \frac{dw}{dg} \right) = - N_e \left[ \frac{K_e}{N_e} \right] \left[ \frac{K_y}{L_y} \right] \frac{r}{H} > 0 \quad (34)
$$

$$
\frac{dc^*}{dg^*} = K_e \left( \frac{dr^*}{dg^*} \right) + N_e \left( \frac{dw^*}{dg^*} \right) = - N_e \left[ \frac{K_e}{N_e} \right] \left[ \frac{K_y}{L_y} \right] \frac{r^*}{H} > 0 \quad (35)
$$

Since $\frac{dc}{dg}$ and $\frac{dc^*}{dg^*}$ are not zero, international economic policy coordination does not maximise net consumption of either country in isolation. If the foreign country exports capital and imports labour in the initial equilibrium, then both $\frac{dc}{dg}$ and
\frac{\partial c}{\partial \delta} are positive, which implies that by increasing their supply of public input, both governments can increase the net consumption of their respective countries. In other words, under international economic policy coordination both governments spend too little on their industries. This result differs sharply from the one derived in the short-run: it has been shown that when labour is not fully mobile across international boundaries and wages are rigid in one country, international policy coordination leads to a situation where the country which fully utilises labour spends too little on its industries, irrespective of the direction of international capital mobility.

The analysis conducted so far exclusively considers the interior solution. As indicated earlier, corner solutions are possible when one country is uniformly more productive than the other country. If the foreign country is uniformly more productive compared to the home country, then both capital and labour will move to the foreign country. Consequently, the optimal supply of public input in the home country will be zero. In such a case, the net consumption of the two countries will be the following:

\[ c = K_e r(0, \delta_e) + N_e w(0, \delta_e) \]

\[ c^* = K_e r(0, \delta_e^*) + N_e w(0, \delta_e^*) - \delta_e^* \]
where \( w(.) \) and \( r(.) \) are respectively the equilibrium wage rate and the rate of return on capital when both private factors move to the foreign country.

In a non-cooperative solution, \( \hat{G} \) will be chosen such that the net consumption of the foreign country (i.e., \( \hat{c} \)) is maximised. Whereas, in a cooperative solution, \( \hat{G} \) will be chosen such that the aggregate consumption of the two-country world (i.e., \( c + \hat{c} \)) is maximised.

The analysis conducted thus far considers two countries only, where both private factors (i.e., capital and labour) are fully mobile across international boundaries. The above two-country framework is therefore interpreted as the long-run. In the next section, the above model is extended to include a third country: the rest of the world. The rest of the world is linked with the home and the foreign country through unrestricted capital mobility; labour is fully mobile between the home and the foreign country only. The three-country framework can be interpreted as the short-run, where the home and the foreign country are members of an economic union such as the European Economic Community (EEC). The modified framework allows an investigation of international transmission of government spending on industries, when two countries have formed an economic union.
3.6 International Transmission of Economic Policy in the Presence of an Economic Union

The purpose of this section is to develop a simple three-country model, which allows an investigation of international transmission of economic policy in the presence of an economic union. The home and the foreign country have formed an economic union, which is linked with the rest of the world through unrestricted international capital mobility. Capital is fully mobile within each country and across international boundaries. Labour is fully mobile within each country but its mobility across international boundaries is restricted: labour is fully mobile within the economic union but there is no labour mobility between either the rest of the world and the home country, or the rest of the world and the foreign country. The rest of the model is similar to the long-run model.

The competitive producers in the rest of the world take the supply of public input as given. The production functions for the rest of the world are given below:  

\[ \text{(13)} \]

13 Variables pertaining to the rest of the world will be distinguished by (\textsubscript{A}).
\[ \hat{Y} = G^\delta F(\hat{K}_y, \hat{L}_y); \ 1 > \delta > 0 \]

where

\( \hat{Y} \): production of final good in the rest of the world.
\( \hat{G} \): pure public input in the rest of the world.
\( \hat{K}_y \): capital used in the production of final good in the rest of the world.
\( \hat{L}_y \): labour used in the production of final good in the rest of the world.

The relevant cost functions for the rest of the world are derived below:

\[ \hat{Y} \cdot C(r, \hat{w})/G^\delta = \text{Min} [\hat{w}L_y + r\hat{K}_y : \hat{Y} = G^\delta F(L_y, K_y)] \]

with respect to \( \hat{L}_y \) and \( \hat{K}_y \).

where

\( \hat{w} \): wage rate in the rest of the world.

For a given supply of public input, the corresponding zero profit condition is the following:

\[ \hat{C}(r, \hat{w})/G^\delta = 1 \] (36)

For a given supply of public input in each country, equations (20), (21), and (36) are three equations in three endogenous
variables: \( w, r, \) and \( \hat{w} \). These equations can determine the equilibrium factor prices independent of the relevant market clearing conditions. The results presented in this section depend on the relative capital intensity of the home and the foreign country only. Consequently, the foreign country is assumed to be relatively capital intensive as compared to the home country.

The present three-country model is recursive: equations (20) and (21) determine the equilibrium \( w \) and \( r \) as a function of \( G \) and \( \hat{G} \) only. Equation (36) can then be used to determine the equilibrium \( \hat{W} \). Consequently, the equilibrium wage rate in the rest of the world depends on the supply of public input in all three countries.

The recursive nature of the above model suggests that a change in the supply of public input in the rest of the world (which is linked with the economic union through capital mobility) cannot influence the equilibrium \( w \) and \( r \). However, by changing their supply of public input both home and foreign country can influence the equilibrium \( \hat{w} \). The following factor market clearing conditions determine the equilibrium output of final goods in the three countries:

\[
\begin{align*}
K_e + \hat{K}_e + \hat{K}_e &= \hat{Y} \left\{ C_r(r, w)/\hat{G}^a \right\} + \hat{Y} \left\{ \hat{C}_r(r, \hat{w})/\hat{G}^b \right\} \\
&+ \hat{Y} \left\{ C_r(r, \hat{w})/\hat{G}^b \right\} 
\end{align*}
\] (37)
Equation (37) shows that capital is fully mobile both within and across international boundaries. Whereas, equations (38) and (39) indicate that labour is fully mobile within each country but international labour mobility is restricted: labour is internationally mobile only between the home and the foreign country.

The net consumption of the rest of the world (c) is given below:

\[ \hat{c} = \hat{r} \hat{K}_e + \hat{w} \hat{N}_e - \hat{G} \]

The impact of a change in the supply of public input on equilibrium factor prices is discussed in the following, where the foreign country is assumed to be relatively capital intensive as compared to the home country:

\[ \partial r / \partial G < 0 \]

\[ \partial r / \partial \hat{G} > 0 \]
\[
\frac{\partial r}{\partial G} = 0 \quad (40)
\]
\[
\frac{\partial w}{\partial G} > 0
\]
\[
\frac{\partial w}{\partial \hat{G}} < 0
\]
\[
\frac{\partial w}{\partial \hat{G}} = 0 \quad (41)
\]
\[
\frac{\partial \hat{w}}{\partial G} > 0 \quad (42)
\]
\[
\frac{\partial \hat{w}}{\partial \hat{G}} < 0 \quad (43)
\]
\[
\frac{\partial \hat{w}}{\partial \hat{G}} > 0 \quad (44)
\]

Equations (40) and (41) indicate that although the rest of the world is linked with the home and the foreign country through perfect capital mobility and the wage rate is fully flexible, its supply of public input does not influence the equilibrium \(r\) and \(w\). Equation (42) shows that an increase in the supply of public input in the home country increases the equilibrium wage rate in the rest of the world. On the other hand, equation (43) shows that an increase in the supply of public input in the foreign country decreases the equilibrium wage rate in the rest of the world. An increase in the supply of public input in the rest of the world increases its equilibrium wage rate, see equation (44).
The above results indicate that the supply of public input in the rest of the world cannot influence the consumption of either the home or the foreign country. This implies that in a Cournot-Nash equilibrium, from the point of view of these countries, spending on industries in the rest of the world is optimal. However, the supply of public input in the home and the foreign country influences the consumption of the rest of the world. The following derivatives can be used to determine the impact of an increase in the supply of public input on net consumption of the rest of the world:

\[
\frac{dc}{dG} = \frac{K_e}{N_e} \frac{dr}{dG} + \frac{\hat{N}_e}{\hat{N}_e} \frac{d\hat{w}}{dG} = -\frac{[\hat{N}_e]}{[\hat{K}_e/\hat{N}_e] - [\hat{K}_y/\hat{L}_y]} \frac{[dr/dG]}{H} \tag{45}
\]

\[
\frac{dc}{d\hat{G}} = \frac{K_e}{N_e} \frac{dr}{d\hat{G}} + \frac{\hat{N}_e}{\hat{N}_e} \frac{d\hat{w}}{d\hat{G}} = \frac{[\hat{N}_e]}{[\hat{K}_e/\hat{N}_e] - [\hat{K}_y/\hat{L}_y]} \frac{[dr/d\hat{G}]}{H} \tag{46}
\]

Equation (45) indicates that an increase in the supply of public input in the home country decreases the consumption of the rest of the world, if the rest of the world exports capital in the initial equilibrium. On the other hand, equation (46) shows that an increase in the supply of public input in the foreign country increases the consumption of the rest of the world, if the rest of the world exports capital in the initial equilibrium. In other words, in a Cournot-Nash equilibrium, the home country spends too much on its industries, whereas the foreign country spends too
little from the point of view of the rest of the world.

Clearly, members of the economic union can exploit the rest of the world through policy coordination within the union. It is in the best interests of the rest of the world to join the economic union. However, the members of economic union are likely to oppose this since only the rest of the world may be gain from this move.
3.7 Concluding Remarks

This essay investigates the international transmission of economic policy in a static framework. The implications of potential international policy coordination are also examined. The purpose of international economic policy coordination in the present study is to maximise the aggregate consumption of a two-country world, where each country produces a final good by means of a pure public input, capital, and labour. The two countries are linked through international factor mobility.

International transmission of economic policy is considered under the following alternative assumptions: (1) capital is fully mobile across international boundaries but international labour mobility is restricted, and due to economy wide rigid wages in one country, labour is not fully employed; (2) both capital and labour are fully mobile across international boundaries. These two cases can be interpreted as the short-run and the long-run respectively.

In the short-run, the supply of public input in a country where wages are fully flexible does not influence the equilibrium rate of return on the internationally mobile factor, i.e., capital. On the other hand, the supply of public input in a country where wages are rigid can influence the equilibrium rate of return on capital and the equilibrium wage rate in a country where wages are
flexible. Consequently, in the absence of international economic policy coordination, the country which exports (imports) capital and where wages are rigid spends too much (little) on its industries from the point of view of the other country, where wages are fully flexible. Whereas, a country where wages are fully flexible spends too much on its industries from the point of view of the country where wages are rigid irrespective of the direction of international capital mobility.

On the other hand, under international policy coordination, the country where wages are rigid and which exports (imports) capital spends too little (much) on its industries from the point of view of its residents. Whereas, the country where wages are fully flexible spends too little on its industries from the point of view of its residents, irrespective of the direction of international capital mobility.

In the long-run, the supply of public input in both countries determines all factor prices. A relatively capital (labour) intensive country can increase the equilibrium rate of return on capital (labour) by increasing its supply of public input. Consequently, in the absence of international coordination, each country spends too much (little) on its industries from the point of view of the other country, if the capital (labour) intensive country exports capital. On the other hand, under international policy coordination both countries spend too little (much) on their
industries from the point of view of their residents, if the capital (labour) intensive country exports capital.

The model has been further extended to include a third country: the rest of the world. The rest of the world is linked with the home and the foreign country through unrestricted capital mobility, whereas labour is fully mobile between the home and the foreign country only. The home and the foreign country can therefore be considered as members of an economic union. A three-country model allows an investigation of international transmission of government spending on industries when two countries have formed an economic union.

It is shown that the supply of public input in the rest of the world has no influence on the equilibrium rate of return on capital, which is fully mobile across international boundaries. On the other hand, the supply of public input in the home and the foreign country has implications for the equilibrium rate of return on capital in the rest of the world. Clearly, policy coordination within the economic union is desirable in order to exploit the rest of the world.

This essay exclusively deals with the international transmission of government spending on a pure public input. In the case of an impure public input it can be shown that both in the short-run and the long-run, an increase in the supply of public
input in either country increases the relative price of all inputs in the home and the foreign country. In the short-run, an increase in the supply of public input in either country increases employment in the underemployed country. In addition, in the short-run uncoordinated equilibrium, the underemployed country spends too much (little) on the impure public input from the point of view of the fully employed country, if it exports (imports) capital. On the other hand, in the short-run coordinated equilibrium, the underemployed country spends too much on the impure public input. However, in the long-run interior solution, where one country exports capital and the other exports labour, unambiguous results cannot be derived.
Appendix 3.1: Stability of the Interior Solution

Equations (20) to (23) can also be used to show that the interior solution satisfies Routh-Hurwitz stability conditions. The postulated dynamic adjustment process is described by means of the following equations, where the left hand side is the time derivative of the relevant variable:

\[
\frac{dY}{dt} = a_y [1 - G^\alpha C(w, r)]
\]

\[
\frac{\dot{Y}}{dt} = b_y [1 - G^\beta C(r, \dot{w})]
\]

\[
\frac{dw}{dt} = a_w [Y \left\{ C_v(r, w)/G^\alpha \right\} + \dot{Y} \left\{ C_v(r, w)/G^\beta \right\} - N_o - \dot{N}_o]
\]

\[
\frac{dr}{dt} = a_r [Y \left\{ C_r(r, w)/G^\alpha \right\} + \dot{Y} \left\{ C_r(r, w)/G^\beta \right\} - K_o - \dot{K}_o]
\]

where the relevant speeds of adjustment \((a_y, b_y, a_w, a_r)\) are assumed to be positive constants.

The economic meanings of the above equations are obvious, therefore the interpretation is not included in this essay. The relevant Jacobian matrix, denoted by \(J\) is the following:
\[
J = \begin{bmatrix}
0 & 0 & a_{13} & a_{14} \\
0 & 0 & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} & a_{34} \\
a_{41} & a_{42} & a_{43} & a_{44}
\end{bmatrix}
\]

where
\[
\begin{align*}
a_{13} &= -C_w(.)G^{-\alpha} \\
a_{14} &= -C_r(.)G^{-\alpha} \\
a_{23} &= -\dot{C}_w(.)G^{-\beta} \\
a_{24} &= -\dot{C}_r(.)G^{-\beta} \\
a_{31} &= C_w(.)G^{-\alpha} \\
a_{32} &= \dot{C}_w(.)G^{-\beta} \\
a_{33} &= [YC_{ww}(.)G^{-\alpha} + \dot{Y}C_{ww}(.)G^{-\beta}] \\
a_{34} &= [YC_{wr}(.)G^{-\alpha} + \dot{Y}C_{wr}(.)G^{-\beta}] \\
a_{41} &= C_r(.)G^{-\alpha} \\
a_{42} &= \dot{C}_r(.)G^{-\beta} \\
a_{43} &= [YC_{wr}(.)G^{-\alpha} + \dot{Y}C_{wr}(.)G^{-\beta}] \\
a_{44} &= [YC_{rr}(.)G^{-\alpha} + \dot{Y}C_{rr}(.)G^{-\beta}]
\end{align*}
\]

One of the Routh-Hurwitz stability condition requires that

\((-1)^4|J| > 0\)
where \(|\ |\) stands for the determinant. The determinant of the above Jacobian matrix is the following:

\[
|J| = G^{-2\alpha} \hat{G}^{-2\beta} \left\{ C_w(.) \hat{C}_v(.) - C_r(.) \hat{C}_u(.) \right\}^2 > 0
\]

Clearly, the determinant condition is satisfied for all values of \(G\) and \(\hat{G}\). Also, the trace of the above Jacobian matrix is negative. Hence, the interior solution is stable.
4.1 Introduction

One of the main objectives of international trade theory is to explain the pattern of international trade. Explanations based on differences in production technologies led to the Ricardian Theorem, whereas explanations based on differences in factor endowments resulted in the Heckscher-Ohlin-Samuelson Theorem. More recent explanations consider the role of increasing returns and imperfect competition in the world economy. However, the role of government spending on industries in determining the trade pattern has not received much attention.

Most open economy studies which explicitly include government spending on industries are mainly concerned with the shape of the

In the context of a one-factor and two-good model, Manning and McMillan (1979) have shown that the comparative advantage of an economy depends on the level of government spending on a pure input. In a multi-period setting, Barro (1990) and Devereux and Mansoorian (1989) have shown that the growth rate of an economy depends on the level of government spending on public inputs. The empirical investigation by Ram (1986) also appears to support this view. However, Abe (1990) is the only available study where the relationship between government spending on industries and the pattern of international trade is explicitly considered.

Abe (1990) has incorporated government spending on a pure public input into a two-country general equilibrium model. Each country produces two final goods by means of capital, labour, and a pure public input. The pure public input which is supplied free

of charge by the government is produced by means of capital and labour. The cost of the public input is financed by means of an income tax. Both countries fully utilise all resources. By means of a comparative statics exercise, Abe has shown that when two countries have identical homothetic preferences, production technology, factor supplies, and the factor intensity of public sector is the same as that of private sectors, then the country that produces more public input exports (imports) the output of the industry which derives more (less) benefits from its supply. This implies that when both industries derive equal benefits from the supply of a pure public input and the other conditions stated above also hold, then differences in the production (and hence the supply) of a pure public input cannot influence the pattern of international trade.

A significant proportion of government budget is directed towards the provision of impure public (or semi-public) inputs in most real economies. Examples of such public inputs include roads, canals, bridges and harbours. However, Abe (1990) has not considered the relationship between government spending on impure public inputs and the pattern of international trade.

An important difference between the pure and impure public inputs is their availability to firms. The entire supply of a pure public input can be utilised by all firms simultaneously. Consequently, a pure public input is non-congestible both across
industries and among firms within each industry. On the other hand, the entire supply of an impure public good cannot be utilised by all firms simultaneously. In other words, an impure public input is congestible within industries and among firms across industries. Due to this asymmetry, the supply of pure and impure public inputs influences the pattern of trade differently.

For a given supply of impure public input, the entry of an extra firm not only increases congestion within the relevant industry, but also increases the level of congestion in the other industries. Congestion affects the size of benefits derived by each industry from the use of an impure public input. An impure public input may not be equally congestible across industries. In addition, each industry is likely to contribute in different ways to the degree of congestion. The pattern of international trade is therefore also influenced by (a) congestion within the industries, and (2) inter-industry congestion.

The above discussion pertains to those economies which fully utilise all resources. However, significant labour unemployment exits in most real economies. It is therefore desirable to investigate the validity of Abe's result in the presence of labour unemployment. In a recent study, Batra and Beladi (1990) have examined patterns of trade between underemployed economies, but they do not consider the role of public inputs in determining the comparative advantage. The underemployment in Batra and Beladi
(1990) refers to labour unemployment due to rigid wages.

The purpose of this essay is to extend Abe (1990) in two directions: (1) to consider the relationship between government spending on impure public inputs and the pattern of trade between economies which fully utilise all resources; and (2) to consider the relationship between government spending on pure and impure public inputs and the pattern of trade between underemployed economies.

The essay develops a two-country, two-good, and two-factor general equilibrium model with government spending on a public input. The private sector produces two final goods by means of capital, labour, and the public input. The public input, which is provided to firms free of charge, is produced by the public sector. The cost of public production is financed by means of a proportional income tax.

The essay is organised as follows. In section two, the relationship between government spending on an impure public input and the pattern of trade is considered; the economies under consideration fully utilise all resources. The public input is congestible both across industries and among firms within industries. In other words, both congestion across industries and inter-industry congestion are explicitly taken into account. It is shown that Abe's result can be extended to include an impure public input.
input only if (i) the public input is equally congestible across industries, and (ii) congestion created by one industry in the other is symmetric.

Section three deals with the relationship between government spending on an impure public input and the pattern of trade between underemployed economies. It is shown that even if both industries derive equal benefits from an impure public input which is equally congestible across industries and the congestion caused by each industry is symmetric, the pattern of international trade can still be influenced by its supply: the country that produces more public input exports (imports) the output of the industry which is relatively labour (capital) intensive. The last section contains concluding remarks.
4.2 Government Spending on an Impure Public Input and the Trade Pattern between Fully Employed Economies

The purpose of this section is to investigate the relationship between government spending on an impure public input and the pattern of international trade between economies which fully utilise all resources. A simple two-country, two-good framework is utilised in which the respective governments supply an impure public input for use within national boundaries. Examples of impure public inputs include production infrastructure: roads, bridges, canals, dams, and harbours. The public input is cooperative with the private inputs in the production of both goods. The economies under consideration fully utilise their resources. Since this essay attempts to explore the relationship between government spending on industries and the pattern of trade in goods, international factor mobility is not considered.

Consider a self sufficient economy that produces two final goods (X and Y) by means of two primary inputs and an impure public input. The primary inputs are labour and capital, which are fully utilised. The public input is produced by means of capital and labour. The supply of labour and capital in the economy is fixed. The public input is provided free of charge by the government. The cost of public production is financed by a proportional income tax.
Distortionary taxes are not considered in this essay: a distortionary tax (for example, a per-unit commodity tax) in itself influences the trade pattern, see Melvin (1970). The final good X is the numéraire. All markets are competitive and private producers take the supply of public input as given.

The present study considers an impure public input which is congestible across industries and among firms within each industry. The firms in each industry are assumed to be identical. Consequently, congestion caused by each firm within an industry is identical. However, the public input may not be equally congestible across industries. In addition, each industry is likely to contribute in different ways to the degree of congestion. In other words, congestion caused by different industries is unlikely to be symmetric. Congestion is assumed to be positively related to the output of each industry. Accordingly, when an industry expands, the resulting congestion decreases the usefulness of the public input to all industries. In the present study, congestion is modelled as a negative externality. Each industry consists of a large number of identical firms. The production functions of the ith firm in industry X and Y are given below:

\[ X_i = S_x(G, X, Y) \cdot F(K_{ix}, L_{ix}); \quad X = n_x X_i \quad (i = 1, 2, \ldots n_x) \]

\[ Y_i = S_y(G, X, Y) \cdot H(K_{iy}, L_{iy}); \quad Y = n_y Y_i \quad (i = 1, 2, \ldots n_y) \]
where

$X_i$: output of firm $i$ in industry $X$.

$n_x$: number of firms in industry $X$.

$G$: supply of impure public input.

$K_{ix}$: capital used in the production of $X_i$.

$L_{ix}$: labour used in the production of $X_i$.

Each firm takes $G$, $X$, and $Y$ as given. $F(.)$ and $H(.)$ are linearly homogeneous. $S_x(.)$ and $S_y(.)$ measure the contribution of the public input to production. For a given $X$ and $Y$, an increase in $G$ reduces the congestion. In other words, $S_x(.)$ and $S_y(.)$ are positively related to the supply of congestible public input. However, for a given supply of public input, an increase in the output of either or both industries increases congestion. In other words both $S_x(.)$ and $S_y(.)$ are negatively related to $X$ and $Y$. However, $S_x(.)$ and $S_y(.)$ are concave. The above production functions are separable in the private and the public inputs. Consequently, an increase in the supply of impure public input affects the output of the two industries in a Hicks neutral fashion.

For the sake of algebraic simplicity the following functional forms for $S_x(.)$ and $S_y(.)$ are used

$$S_x(.) = G^{\alpha X^{-\gamma} Y^{-\lambda}}; \quad 1 > \alpha > 0, \gamma > 0, \lambda > 0$$

$$S_y(.) = G^{\beta Y^{-\zeta} X^{-\mu}}; \quad 1 > \beta > 0, \zeta > 0, \mu > 0$$
\[ \alpha = \frac{\partial X}{\partial G} \frac{G}{X} : \text{elasticity of } X \text{ with respect to } G. \]

\[ \beta = \frac{\partial Y}{\partial G} \frac{G}{Y} : \text{elasticity of } Y \text{ with respect to } G. \]

\[ \gamma: \text{congestion rate in industry } X. \]

\[ \zeta: \text{congestion rate in industry } Y. \]

\[ \lambda: \text{Y industry's contribution to congestion in industry } X. \]

\[ \mu: \text{X industry's contribution to congestion in industry } Y. \]

For a given \( G, X, \) and \( Y, \) the cost functions for the \( i^{th} \) firm in industry \( X \) and \( Y \) can be derived as follows:

\[ C^x(r, w)[X_i/S_x(.)] = \min \{ wL_x + rK_x : X_i/S_x(.) = F(K_x, L_x) \} \]

with respect to \( L_x \) and \( K_x. \)

\[ C^y(r, w)[Y_i/S_y(.)] = \min \{ wL_y + rK_y : Y_i/S_y(.) = H(K_y, L_y) \} \]

with respect to \( L_y \) and \( K_y. \)

where

\( w: \text{wage rate.} \)

\( r: \text{rate of return on capital.} \)

Since all firms in both industries are identical, the industry cost functions can be derived by aggregating the cost functions of the firms as follows:

\[ C^x(r, w)[X/S_x(.)] \]
The technology for public production is the following, where \( g(.) \) is linearly homogeneous:

\[
G = g(K_g, L_g)
\]

The corresponding cost function is derived in the following:

\[
GC^y(r, w) = \min \{ wL_g + rK_g : G = g(K_g, L_g) \}
\]

with respect to \( L_g \) and \( K_g \).

By using the properties of cost functions, the factor market clearing conditions can be written as follows:

\[
L_e = \frac{X}{S_x(.)} c^x_v(.) + \frac{Y}{S_y(.)} c^y_w(.) + GC^v(.) \tag{1}
\]

\[
K_e = \frac{X}{S_x(.)} c^x_r(.) + \frac{Y}{S_y(.)} c^y_r(.) + GC^r(.) \tag{2}
\]

where

\[
\frac{X}{S_x(.)} c^x_v(.) : \text{labour used in the production of } X.
\]

\[
\frac{X}{S_x(.)} c^x_r(.) : \text{capital used in the production of } X.
\]

\[
GC^v(.) : \text{labour used in the production of } G.
\]

\[
GC^r(.) : \text{capital used in the production of } G.
\]

\( K_e \): supply of capital.

\( L_e \): supply of labour.
Due to unrestricted factor mobility within the economy, the wage rate and the rate of return on capital in the private and the public sectors is identical. The zero profit conditions for industry X and Y are the following:

$$C^*(r, w)/[S_x(.)] = 1 \quad (3)$$

$$C'(r, w)/[S_y(.)] = p \quad (4)$$

For a given supply of pure public input (G) and the autarky relative goods price ratio (p), equations (1) to (4) determine the equilibrium w, r, X and Y. The autarky price ratio in the economy is determined by the interaction of supply and demand. On the demand side, preferences are assumed to be homothetic. The relative demand for the two goods in the economy therefore depends on the relative price ratio alone. Market equilibrium for the final goods can be written as

$$Y/X = D(p) \quad (5)$$

where

D(p): relative demand of the two final goods.

$$\partial D(p)/\partial p$$ is negative.

For a given supply of pure public input, equilibrium of the private sector can be characterised by equations (1) to (5). In five equations there are five endogenous variables; w, r, p, X, and
These variables are determined as the function of the supply of public input. As indicated earlier, the cost of public production is financed by means of income tax. The budget constraint of the government is given in the following, where $t$ is the income tax rate:

$$t(wL_w + rKe) = Gc^g(w, r)$$

(6)

Right hand side of the above equation is the total cost of public input; whereas the left hand side is government tax receipts. For the purposes of the present study, the supply of public input does not have to be optimal. The full equilibrium of the closed economy under consideration is given by equations (1) to (6). These are six equations in six endogenous variables: $w$, $r$, $p$, $X$, $Y$, and $t$. This completes the description of a representative closed economy.

The relationship between the supply of a pure public input and the pattern of trade is examined in the next section.

4.2.1 The Pattern of Trade

For a given supply of impure public input, equations (1) to (5) describe an autarky equilibrium in the economy. Consider two

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2 The budget constraint of the representative consumer is the following: $(1 - t)(wL_w + rKe) = X + pY$
such economies. If the two countries are identical in every respect, then there is no basis for international trade, since the autarky price ratio in the two countries will be identical. However, differences in the supply of public input can lead to differences in the autarky price ratio.

The supply of an impure public input influences the autarky goods price ratio (p) through the following channels:

(i) for a given wage and the rate of return on capital, an increase in G influences p through a reduction in the cost of production of both industries. This is the direct effect of an increase in the supply of impure public input.

(ii) an increase in G also increases the cost of both industries through its positive impact on wages and the rate of return on capital. This is the indirect effect of an increase in the supply of an impure public input.

(iii) the public input is congestible within industries and among firms across industries. In general the public input is not likely to be equally congestible across industries. In addition, the congestion caused by each industry in the other is also not likely to be symmetric. Accordingly, for a given wage rental ratio, an increase in G directly affects the output of both industries and hence the autarky price ratio.

In the case of a pure (i.e., non-congestible) public input, Abe (1990) has shown that the pattern of international trade is not
influenced by differences in the production of public input if (i) both industries derive equal benefits from its supply, i.e., $\alpha$ is equal to $\beta$ in terms of the present study; and (ii) the factor intensity of the public sector is the same as that of the private sectors, i.e., $[(K_x + K_y)/(L_x + L_y)]$ is equal to $[K_g/L_g]$. The present study deals with an impure (i.e., congestible) public input. In order to facilitate comparison with the results derived by Abe, it is assumed that (1) $\alpha$ is equal to $\beta$, and (2) the factor intensity of the public and the private sectors is the same. In other words, the present study attempts to examine the role of congestion in determining the pattern of trade exclusively.

The relationship between the supply of an impure public input and the pattern of trade can be examined by means of the following derivative, which describes the impact on the autarky price ratio of a country when its supply of public input increases by a small amount

$$\frac{\partial p}{\partial G} = - \frac{p}{G} \left[ (\mu - \lambda) + (\zeta - \gamma) \right] \frac{H_1 H_2 + H_3 H_4}{H_d} \tag{7}$$

---

3 Unlike the present study, in Abe (1990) the elasticity of $X$ and $Y$ with respect to $G$ is variable.

4 This implies that $L_g \{L_x \{(K_x/L_x)-(K_g/L_g)\} + L_y \{(K_y/L_y)-(K_g/L_g)\}\}$ is zero. In other words, the factor intensity of the public sector is a weighted average of the factor intensity of the private sectors.

5 The properties of the cost functions are used in the derivation of equation (7). For an excellent survey of duality theory, see Diewert (1974).
where

\[ H_1 = (\alpha K_x + \beta K_y) > 0 \]
\[ H_2 = (L_x a_{xw} + L_y a_{yw} + L_g a_{gw}) < 0 \]
\[ H_3 = (\alpha K_x + \beta K_y) > 0 \]
\[ H_4 = (K_x a_{xr} + K_y a_{yr} + K_g a_{gr}) < 0 \]
\[ a_{yr} = (r/C_r(.)) [ (dC_r(.))/dr ] < 0 \]
\[ a_{xw} = (w/C_w(.)) [ (dC_w(.))/dw ] < 0 \]
\[ a_{xr} = (r/C_r(.)) [ (dC_r(.))/dr ] < 0 \]
\[ a_{yw} = (w/C_w(.)) [ (dC_w(.))/dw ] < 0 \]
\[ a_{gw} = (w/C_g(.)) [ (dC_g(.))/dw ] < 0 \]
\[ a_{qr} = (r/C_r(.)) [ (dC_r(.))/dr ] < 0 \]

Equation (7) is derived by using equations (1) to (5) and conditions (i) and (ii). The sign of the above derivative depends on: (1) the relative size of inter-industry congestion \((\mu - \lambda)\); (2) the relative size of congestion within each industry \((\gamma - \zeta)\); and (3) the sign of \(H_d\). \(H_d\) is positive provided the equilibrium is stable.

If the impure public input is equally congestible within each industry (i.e., \(\gamma = \zeta\)) and the congestion caused by each industry is symmetric (i.e., \(\lambda = \mu\)), then the above derivative is zero. This implies that both pure and impure public inputs influence the pattern of international trade symmetrically.

If the congestion caused by each industry in the other is
symmetric (i.e., \( \lambda = \mu \)), then the following proposition follows from equation (7).

PROPOSITION 1. When two countries have (i) identical homothetic preferences, production technology, factor supplies, (ii) both industries derive equal benefits from the impure public input, (iii) the factor intensity of the public sector is the same as that of the private sectors, and (iv) the congestion caused by each industry in the other is symmetric; then the country that produces more public input exports (imports) the output of that industry in which the public input is relatively less (more) congestible.

An increase in the supply of public input increases the output of both industries. Suppose that the public input is relatively more congestible in X industry as compared to Y. Hence if G increases, then the output of X industry rises relatively less than Y. Consequently, the output of Y industry where the public input is relatively less congestible is exported.

The above proposition also implies that the congestibility can actually reverse the pattern of international trade based only on the size of the benefits of an impure public input across industries. Suppose that in the initial equilibrium the public input is equally congestible in both industries, but the direct benefits of the public input across industries are not symmetric.
In such a case, a country which produces more public input exports the output of the industry which derives more benefits from its supply. However, if the industry which derives more benefits from the supply of an impure public input is also the industry where the public input is relatively more congestible, then it may be cheaper for the country in question to import the relevant good.

If the public input is equally congestible within each industry (i.e., $\gamma = \zeta$), then the following proposition follows from equation (7).

PROPOSITION 2. When two countries have (i) identical homothetic preferences, production technology, factor supplies, (ii) both industries derive equal benefits from the impure public input, (iii) the factor intensity of the public sector is the same as that of the private sectors, and (iv) the public input is equally congestible within each industry; then the country that produces more public input exports (imports) the output of that industry which causes more (less) congestion in the other industry.

The above result can be explained as follows. Suppose that $Y$ industry does not cause any congestion in $X$, but $X$ industry does cause congestion in $Y$. In other words, $\lambda = 0$ but $\mu > 0$. Consequently if $G$ increases, then the output of $X$ industry rises relatively more than $Y$ (since $X$ does not suffer from congestion). Hence, the output of industry $X$ which causes congestion in $Y$ is
exported.

The analysis conducted so far is based on the assumption that all resources are fully utilised in both countries. However, significant labour unemployment exists in most real economies. In the next section, the models used in the previous sections are extended to include labour unemployment.
4.3 Government Spending on an Impure Public Input and the Trade Pattern between Underemployed Economies

Although significant labour unemployment exits in most real economies, most open economy models do not take unemployment into account. Batra and Beladi (1990) have considered the trade pattern between underemployed economies, but they do not consider the role of public inputs in determining comparative advantage. The term underemployment refers to labour unemployment due to rigid wages. The purpose of this section is examine how the distortion created by institutionally fixed minimum wages influences the results derived in the previous section.

The self sufficient economy under consideration produces two final goods (X and Y) by means of two primary inputs and an impure public input. The primary factors are labour and capital, of which only capital is fully employed. Due to economy wide rigid wages, labour is not fully employed. All other assumptions of section three are maintained in this section.

The equilibrium conditions are the following, where \( L \) is labour employment:

\[
L = \left[ \frac{X}{S_x(\cdot)} \right] \nu(\cdot) + \left[ \frac{Y}{S_y(\cdot)} \right] \nu_w(\cdot) + Gc_m(\cdot)
\]  
(8)
For a given supply of impure public input, the above are five equations in five endogenous variables: $L, r, p, X$ and $Y$. These equations determine the equilibrium of a representative closed economy. By means of a comparative statics exercise, the pattern of international trade between two such economies is considered in the following section.

### 4.3.1 The Pattern of Trade

The following derivative can be derived by using equations (9) to (12). The two industries are assumed to derive equal benefits from the supply of an impure public input (i.e., $\alpha = \beta$), public input is equally congestible across industries (i.e., $\gamma = \zeta$) and the congestion caused by each industry in the other industry is identical (i.e., $\mu = \lambda$).

\[
\frac{\partial p}{\partial G} = \frac{(p/G)T[a_{xr} - a_{yr}]}{H_{sd}}
\]
Where
\[ T = [\alpha(K_x + K_y) + (\gamma + \mu)K_y] > 0 \]

The sign of the above derivative depends on: (1) the relative capital intensity \((K_x/L_x - K_y/L_y)\); and (2) the sign of \(H_{dd}\). \(H_{dd}\) is negative provided the equilibrium is stable.

The above derivative shows that even if (1) the congestion caused by the two industries is symmetric, (2) public input is equally congestible within each industry, and (3) both industries derive equal benefits from the supply of public input; the pattern of trade can still be influenced by differences in the supply of an impure public input. The wage rate is fixed. However an increase in \(G\) affects the equilibrium rate of return on capital which increases the cost of production of both industries. The sign of the above derivative is positive if \(Y\) industry is relatively more capital intensive.

The following proposition follows from equation (13).

**PROPOSITION 3.** When two countries have (i) identical preferences, production technology, factor supplies; (ii) a minimum wage rate; (iii) both industries derive equal benefits from an impure public input; (iv) the congestion caused by each industry is
symmetric; and (v) the public input is equally congestible within each industry; then the country that produces more public input exports (imports) the output of the industry which is relatively less (more) capital intensive.

The wage rate is fixed, but an increase in G increases r through an increase in the marginal productivity of capital. Suppose that industry Y is relatively more labour intensive compared to X. If G increases, then the per unit cost of industry X rises relatively more than Y (since Y is capital intensive). Consequently, the output of labour intensive industry Y is exported.

The above result differs sharply from the one derived in the previous section where it was shown that when (i) both industries derive equal benefits from the supply of an impure public input, (ii) the factor intensity of the public and private sectors is the same, (iii) the congestion across the two industries is symmetrical, and (iv) inter-industry congestion is symmetrical; then the trade pattern is not influenced by the differences in the production of a public input.

It can easily be shown that the above result also holds in the case of a pure public input.
4.4 Concluding Remarks

Abe (1990) has shown that the differences in the production of a pure public input alone can explain the pattern of trade between countries which fully utilise their resources. This essay has attempted to extend Abe's work in two directions: (1) the relationship between government spending on an impure public input and the pattern of trade between economies which fully utilise their resources has been considered; and (2) the relationship between government spending on a public input and the pattern of trade between underemployed economies has been considered.

In the context of a two-country and two-good general equilibrium model, the relationship between government spending on an impure public input and the trade pattern is examined in section two. All resources are fully utilised in both countries. An impure public input is congestible within industries and among firms across industries. Both types of congestion are considered. It is shown that when the public input is equally congestible within each industry but the congestion caused by each industry is not symmetric, then the differences in its production can determine the pattern of trade even if both industries equally benefit from its supply and the factor intensity of the public and private sectors are identical (or sufficiently close to each other): a country which produces more public input exports (imports) the output of that industry which creates more (less) congestion in the other
Furthermore, when the congestion caused in each industry is symmetric but the public input is not equally congestible within each industry, then the differences in its production can determine the pattern of trade even if both industries equally benefit from its supply and the factor intensity of the public and private sector are identical: a country which produces more public input exports (imports) the output of that industry in which the public input is relatively less (more) congestible.

On the other hand, if both industries benefit equally from the supply of the public input, the congestion caused by each industry is symmetric, public input is equally congestible within each industry, and conditions (i) and (ii) hold, then differences in its supply cannot influence the pattern of international trade. This implies that congestion can actually reverse the results derived by Abe (1990). If the public input is not congestible and the factor intensity of the public and private sectors are identical, then the country which produces more public input exports the output of the industry which derives more benefits from its supply. However, in the case of a congestible public input, the country which produces more public input may import the output of the industry which derives more benefits from its supply, if the public input is relatively more congestible in the relevant industry.
Finally, it is shown that the results derived in section three are significantly influenced by the presence of labour unemployment: if both industries benefit equally from the supply of a congestible public input, public input is equally congestible within each industry, and the congestion caused by each industry is symmetric, then the underemployed country which produces more public input exports the output of the relatively less capital intensive industry.
Appendix 4.1: Stability Conditions

Equations (5), and (8) to (11) can also be used to derive the Routh-Hurwitz stability conditions. The postulated dynamic adjustment process is described by means of the following equations, where the left hand side is the time derivative of the relevant variable:

\[
\frac{dw}{dt} = a_w \left[ \frac{X}{S_x(\cdot)} c_w(\cdot) + \frac{Y}{S_y(\cdot)} c'_w(\cdot) + Gc_w(\cdot) - L_w \right]
\]

\[
\frac{dr}{dt} = a_r \left[ \frac{X}{S_x(\cdot)} c_r(\cdot) + \frac{Y}{S_y(\cdot)} c'_r(\cdot) + Gc_r(\cdot) - K_r \right]
\]

\[
\frac{dX}{dt} = a_x \left[ 1 - C_x(r, w) / S_x(\cdot) \right]
\]

\[
\frac{dY}{dt} = a_y \left[ p - C_y(r, w) / S_y(\cdot) \right]
\]

\[
\frac{dp}{dt} = a_p \left[ D(p) - Y/X \right]
\]

where the relevant speeds of adjustment \((a_w, a_r, a_p, a_y, a_x)\) are assumed to be positive constants.

The economic meanings of the above equations are obvious, therefore the interpretation is not included in this essay. The relevant Jacobian matrix, denoted by \(J\) is the following:
$J = \begin{bmatrix}
    a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\
    a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\
    a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\
    a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\
    a_{51} & a_{52} & a_{53} & a_{54} & a_{55}
\end{bmatrix}$

where

\[ a_{11} = \left[ \{X/S_x(.)\}c_w^x(.) + \{Y/S_y(.)\}c_y^x(.) + Gc_g^x(.) \right] \]

\[ a_{12} = \left[ \{X/S_x(.)\}c_w^x(.) + \{Y/S_y(.)\}c_y^x(.) + Gc_g^x(.) \right] \]

\[ a_{13} = c_w^x\left[ XS_{xx}(.) - S_x(.) \right]/\left[ S^2_x(.) \right] - c_y^xYS_{yx}(.)/S_y(.) \]

\[ a_{14} = c_y^x\left[ YS_{yy}(.) - S_y(.) \right]/\left[ S^2_y(.) \right] - c_y^xYS_{xy}(.)/S_x(.) \]

\[ a_{15} = 0 \]

\[ a_{21} = \left[ \{X/S_x(.)\}c_w^x(.) + \{Y/S_y(.)\}c_y^x(.) + Gc_g^x(.) \right] \]

\[ a_{22} = \left[ \{X/S_x(.)\}c_w^x(.) + \{Y/S_y(.)\}c_y^x(.) + Gc_g^x(.) \right] \]

\[ a_{23} = c_w^x\left[ XS_{xx}(.) - S_x(.) \right]/\left[ S^2_x(.) \right] - c_y^xYS_{yx}(.)/S_y(.) \]

\[ a_{24} = c_y^x\left[ YS_{yy}(.) - S_y(.) \right]/\left[ S^2_y(.) \right] - c_y^xYS_{xy}(.)/S_x(.) \]

\[ a_{25} = 0 \]

\[ a_{31} = - c_w^x(.)/S_x(.) ; a_{32} = - c_w^x(.)/S_x(.) \]

\[ a_{33} = c_w^x\left[ XS_{xx}(.)\right]/\left[ S^2_x(.) \right] ; a_{34} = c_w^x\left[ XS_{xy}(.)\right]/\left[ S^2_x(.) \right] \]

\[ a_{35} = 0 \]

\[ a_{41} = - c_y^x(.)/S_y(.) ; a_{42} = - c_y^x(.)/S_y(.) \]

\[ a_{43} = c_y^x\left[ YS_{yy}(.)\right]/\left[ S^2_y(.) \right] ; a_{44} = c_y^x\left[ YS_{xy}(.)\right]/\left[ S^2_y(.) \right] \]

\[ a_{45} = 1 \]

\[ a_{51} = 0 ; a_{52} = 0 ; a_{53} = Y/X^2 ; a_{54} = - (1/X) ; a_{55} = D'(p) \]

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One of the Routh-Hurwitz stability condition requires that

\((-1)^5 |J| > 0\)

where "| |" stands for the determinant. The determinant of the above Jacobian matrix is the following:

\[|J| = -H_d\]

Clearly, the determinant condition is satisfied if \(H_d\) is positive.
CHAPTER 5

FINAL SUMMARY AND CONCLUDING REMARKS

This thesis consists of three essays. Each essay examines the role of government spending on industries in open economies. Government spending on industries is incorporated in production functions in terms of a public input. The public input is cooperative with the private inputs in the production of final goods. Additionally, the public input is provided to the firms by the government free of charge. Since each essay deals with a distinct issue regarding public inputs in open economies, different models are utilised in chapters 2 to 4.

The purpose of the first essay (chapter 2) is to investigate the impact of terms-of-trade changes in a small public input economy. Lags in the production and supply of public inputs are explicitly taken into account. These lags provide a mechanism whereby terms-of-trade changes in either period influence the output of both private and public sectors in other periods. Lags in the production and supply of public inputs can only be taken into account in the context of a multi-period model. A three period
model is therefore appropriate. The public input is produced by means of labour, whereas the private goods are produced by means of labour, public input, and some fixed factors. Labour is fully mobile between the public and the private sectors. The allocation of resources between the public and private sector is therefore endogenous. Because the focus of the essay is on the role of public inputs, private investment is assumed away. The model can further be extended to include uncertainty about the future terms-of-trade and the future provision of public inputs. Another possible extension is to allow distortionary taxes to finance the cost of public production.

The second essay (chapter 3) examines the international transmission of government spending on public inputs in the short-run and the long-run. The implications of potential international economic policy coordination are also considered. International indirect spillovers of government spending on public inputs are possible only if the countries under consideration are linked. International factor mobility provides one such link. When the countries are linked through international factor mobility, a single final good model can be used. Accordingly, the analysis is conducted by means of a two-country, one-good general equilibrium model with international factor mobility. The focus of the essay is on international transmission of government spending on public inputs. For simplicity, the public input is assumed to be produced out of the final good which is also the numéraire.
In the context of a three-country model, the essay also examines the international transmission of government spending on a public input when two countries have formed an economic union. The analysis is conducted by means of a one-period model. A possible direction for further research is to consider a multi-period model. Such a framework also allows one to consider the international transmission of fiscal deficits.

Abe (1990) considered the relationship between the supply of a pure public input and the pattern of trade. The third essay (chapter 4) examines the relationship between government spending on impure public inputs and the pattern of trade. An impure public input is incorporated into a standard two-country, two-good, two-primary-factor general equilibrium model. For simplicity, international factor mobility is assumed away.

Unlike pure public input, an impure public input is congestible. It is shown that results derived by Abe (1990) can be extended to include an impure public input only if (i) the public input is equally congestible across industries, and (ii) inter-industry congestion is symmetric. The relationship between the supply of an impure public input and the trade pattern among underemployed economies is also considered. One possible direction for further research is to consider the pattern of international trade in the presence of more than one public service, where some services are utilised by both households and firms.
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