THREE ESSAYS IN REAL ESTATE ECONOMICS

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Abstract

This dissertation consists of three separate essays. The first two essays focus on real estate brokerage; one studies the conditions for efficient employment in the real estate brokerage industry under fixed commission rates and the other examines the role of real estate agents in buyer-seller bargaining. The third essay presents an integrated analysis of housing investment and consumption choices that takes into account both the uncertainty in investment returns and liquidity constraints.

Essay one presents a model of real estate trading with brokerage that integrates sequential search, two-sided matching, and the competitive entry and effort choice of real estate agents. The equilibrium employment pattern of the model helps to explain the observation that the number of agents is more sensitive to the expected transaction price than to the transaction volume. The condition for efficient employment requires the commission to be proportional to the opportunity cost of search time and the expected trading gain, with the proportion determined by the productivity of brokerage employment. Efficient employment also requires regulating the entry so as to achieve the productivity balance between the number of agents and individual effort.

Essay two examines asymmetric information and bargaining within the model of real estate trading developed in essay one. The equilibrium outcomes of bargaining with and without information asymmetry are characterized with the help of mechanism design methodology, and the associated welfare levels are compared. The analysis is applied to evaluating the role of real estate agents in the bargaining. Agents seek compromises between the buyer and seller by providing credible information to both parties. Such a role is welfare improving when the scale economy of brokerage with respect to the stock of buyers and sellers is not strong and brokerage employment is sufficient.
In essay three, Pratt's certainty-equivalent approximation is applied to the Henderson-Ioannides (1983) housing tenure choice model. The key trade-offs for housing investment and consumption choices induced by the uncertainty and liquidity constraints are clearly illustrated and the implications for tenure choice examined against the existing empirical evidence.
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Chapter 1

Introduction

The studies in this thesis concern two aspects of real estate markets. One is the trading process in the market, particularly the trading facilitated by real estate brokerage. The other aspect is the housing demand by homeowners, incorporating both investment and consumption considerations. In this chapter we discuss the purposes of these studies in a broad context.

1.1 Real Estate Brokerage

Real estate brokerage is an important market institution in many developed economies where real estate represents a significant portion of national wealth. Real estate brokerage affects the allocation of real estate resources in the society and employs considerable resources itself. However, real estate brokerage has received scanty attention in economics literature. The economic nature of real estate brokerage and the welfare performance of the industry are still largely open questions at the present.

In both Canada and the United States, there are continuing debates on two important issues pertaining to the real estate brokerage industry. One issue is whether the prevalent level of brokerage commission rates is too high so as to result in inefficient rent dissipation by excessive brokerage employment. The other is whether the conventional model of real estate agency, by which real estate agents owe their fiduciary duty solely to sellers, is too restrictive and whether alternative representation models, such as buyer agent or dual agent, would be more productive. These issues have much to do with the economic function and behavior of the real estate brokerage industry, and they would not be satisfactorily resolved unless we have a good understanding of the economics.
There are many interesting economic questions regarding the real estate brokerage industry. What determines the usefulness of real estate brokerage relative to other market institutions such as real estate auction? What advantages or disadvantages exist for the bundling of real estate brokerage services and the commission-based pricing, compared with alternative methods such as fixed fees for separated services? How does real estate brokerage contribute to the efficiency of resource allocation in real estate markets? And what determines the productivity of brokerage employment? In this thesis we attempt to examine the last two questions, which are central for the understanding of the allocation efficiency of the real estate brokerage industry. We will also discuss the implications of our results for the evaluation of commission rates and different real estate agency models.

Our discussion will be centred around the notion of information. We will argue that the key to understanding the nature and social value of real estate brokerage is in the information it provides. Information affects resource allocation in markets. And the nature of information affects the provision of brokerage services, the organization of the real estate brokerage industry, and the productivity of brokerage employment. The role of information in resource allocation and the public-good nature of market information has not received proper consideration in the existing studies of real estate brokerage and, hence, deserve some emphasis here.

Real estate agents (or salespeople) provide a variety of services to real estate buyers and sellers, including showing properties and handling paper works. But as far as the allocation efficiency is concerned, the most important ingredient of real estate brokerage is market information. In a world of imperfect information, as Arrow (1963) points out, information becomes a commodity, or a resource that can be used to create value. Arrow (1963) goes on further: “Like other commodities, it has a cost of production and a cost of transmission, and so it is naturally not spread out over the entire population but concentrated among those who can profit most from it.” The real estate brokerage industry produces and transmits information that is useful to real estate buyers and sellers in search for good trade opportunities. To understand the potential social value of the information, we need to understand how information affects
resource allocation in general and the resource allocation in real estate markets in particular.

Generally speaking, information affects resource allocation in two ways. First, it imposes technical constraints. Efficient resource allocation, whether through central planning or decentralized trading, requires information. Since the production and transmission of information are costly, the efficiency of resource allocation is constrained by the cost of information. Second, it imposes incentive constraints. Since private information is not negotiable or contractually enforceable, feasible resource allocation is limited to those outcomes that are incentive-compatible.

In real estate markets, efficient resource allocation requires, perhaps, more information than in the markets for most other commodities and assets. This is because, as is well recognized, each piece of real estate is unique and its value depends on its highest and best use. Ratcliff (1949, p365) summarizes the market allocation of real estate: "The result of this process, which is a competitive process, is an arraying of all the sites in a hierarchy of land values, each value for each site predicted on a given use." To determine the highest and best use of a piece of real estate, given many competing real estate assets and users, requires a lot of information transmission. The highest and best use of a property depends on the highest and best use for every other property in the market and thus depends on the attributes of all the properties and the preferences of all the potential users. The decentralized nature of real estate markets contributes further to the cost of information.

When information is perfect, the resource allocation generated by the competitive process in real estate markets would be Pareto efficient. This is demonstrated by the bid-rent model of Alonso (1964) and the hedonic price model of Rosen (1974). These models characterize the competitive outcomes in real estate markets under complete information, where every property gets the highest value and every buyer maximizes his utility at market clearing prices and no further gainful trade opportunity exists. The competitive prices that clears the market represent the value of the highest and best use of different properties.

The competitive outcomes in real estate markets under imperfect information will unlikely
to remain Pareto efficient as information becomes less incomplete. Because the competitive outcomes based on incomplete information fail to take into account all the potential trade opportunities, gainful trade opportunities may emerge when more information is available. Therefore, in markets where trade information is costly, there is a trade-off between the efficiency of resource allocation and the cost of information. Making more market information available to more buyers and sellers can improve the value of trading outcomes but also requires more resources.

Real estate brokerage provides two types of useful information that can facilitate real estate trading and the realization of the highest and best use for each property. The first type is competitive price information based on recent market transactions and current state of demand and supply in particular market segments. As we discussed above, the competitive price information tells a lot about the highest and best use of each property. The second type is the information of the buyers and sellers of various properties at various locations. The easy access to such information by buyers and sellers helps them to meet more potential trade opportunities and to find better matches given the competitive price information; thus more of the potential value of properties' highest and best use can be realized with less delay.

To characterize the social value of real estate brokerage we need a model of real estate trading under imperfect information. Sequential search and matching models, largely developed in labor economics literature [see Pissarides (1990) for a review of the literature], are now commonly used in modelling trading with imperfect information. Wheaton (1990) represents a recent application of this type of model in housing market context. In this type of model the expected gains of trading depend on both search productivity and matching quality. If the information provided by the real estate brokerage industry facilitates both search and matching, the social value of real estate brokerage should be reflected in the expected gains of trading.

Real estate trading is also subject to information asymmetry. Sellers may have experiential knowledge of the qualities of the building structure and the neighbourhood. Of course, serious buyers are not prevented from acquiring sufficient information about these qualities by
careful observation, employing an expert inspector, talking to neighbours, and visiting relevant municipal departments. Sellers' inability to verify individual buyers' preferences and buyers' inability to verify sellers' true eagerness to sell are, perhaps, more important sources of information asymmetry that affects price negotiation in real estate trading. Since information asymmetry tends to reduce feasible trade opportunities, real estate brokerage could make an additional contribution to the allocation efficiency in real estate trading if it helps to mitigate the incentive problems associated with asymmetric information.

The nature of real estate assets and the nature of market information are important factors determining the usefulness of real estate brokerage and the provision of brokerage services, as well as the productivity of brokerage employment. The spatial dispersion of real estate assets and their location specific attributes demand decentralized trading and decentralized brokerage operations. The decentralized nature of real estate markets also gives central importance to multiple listing services that facilitate sharing of information among real estate agents. Multiple listing services enhance the productivity of real estate agents because market information has a public-good nature: it can benefit many users with little additional cost. The dispersion of real estate assets and buyers and their heterogeneous characteristics, as well as the organization of the brokerage industry, affect the cost of information gathering and transmission and thus the productivity of brokerage employment.

Real estate brokerage may offer a number of advantages over the provision of information by buyers and sellers themselves. It provides the economy of scale in exploiting the public-good value of information. It provides the benefit of specialization, since there is a fixed cost in acquiring the skills for information gathering and for handling real estate transactions. It can be a useful institution to overcome the free-rider problem associated with the public-good nature of market information and the moral hazard problem associated with the intangible-good nature of market information. These factors are, however, less important for the marginal productivity of brokerage employment. A useful model for analyzing the efficiency of brokerage employment needs to allow the variation in the marginal productivity of employment due to
the differences in the nature of the real estate market, such as the density and heterogeneity
of buyers and sellers, in the organization of the industry, and in the information transmission
technologies.

Our model of brokerage employment productivity is based on the aggregate output of the
industry. A two-sided matching model of real estate trading [see Roth and Sotomayor (1990) for
an excellent treatment of two-sided matching models] suggests that the number of matches above
a given level of match quality increases with the number of buyers and sellers in the market. To
the extent that the information and other services provided by the real estate brokerage industry
help to determine the highest and best use of properties and facilitate the meeting between
buyers and sellers, the number of matches as well as the average quality of the matches increase
with brokerage employment. These relations can be represented with an aggregate matching
function that describes the number of matches above various match quality levels generated in
a given period of time at different market size and brokerage employment. The differences in
the nature of the market, in the organization of the industry, and in information technologies
can be reflected in the shape of the function. The function also allows scale economies (or
diseconomies) in brokerage employment and in market size as well as the effect of market size
on the productivity of brokerage employment.

The impact of information on allocation outcomes in real estate markets and the implications
of the public-good nature of market information for the productivity of brokerage employment
are largely ignored in existing models of real estate brokerage. The models that focus on
the aggregate performance of the real estate brokerage industry are based on Stigler's (1961)
notion of price- search under a fixed price distribution [Yinger (1981), Wu and Colwell (1986)].
In this context, search has no social value since it is a zero-sum game — one party's loss is
another's gain — and the search cost is a dead weight loss to the society. These models are
inadequate for analyzing the efficiency of the real estate brokerage industry. Other models focus
on the contractual relationship between the seller and the real estate agent [Anglin and Arnott
Chapter 1. Introduction

examine the search effort of an agent in the context of a single selling contract and ignore the potential impact of other agents and other selling opportunities on the productivity and the incentives of the particular agent. This principal-agent-type models fail to take into account the fact that much of an agent’s effort in terms of gathering market transaction information, advertising listings, and contacting buyers and sellers, contributes to the sale of not only a single listing but many listings. Individual agents may also benefit from the information provided by other agents, and individual selling effort may be complemented or offset by other agents’ selling effort. Thus, the productivity as well as the incentives of individual agents are properly specified in this type models.

With our characterization of the social value of real estate brokerage and the productivity of brokerage employment, it is possible for us to characterize the socially optimal level of brokerage employment. Such a model helps us to understand how different factors affect the marginal social value of brokerage employment and what information is needed to assess the marginal value. Furthermore, with a model of brokerage employment behavior under fixed commission rates, it is also possible for us to examine the employment efficiency of the industry at different levels of commission rates.

In practice, real estate agents are commonly involved in price negotiations between buyers and sellers; yet their role and the welfare consequence of their involvement are not formally examined. Understanding how information asymmetry affects real estate trading and what influence real estate brokerage may have on the bargaining process would have important ramifications for the evaluation of different representation models of real estate agency.

The essay in chapter 2 examines the real estate brokerage industry from the point of view of the technical constraints in real estate trading and addresses the issue of socially efficient commission rates. The essay in chapter 3 examines real estate brokerage from the point of view of the incentive constraints in real estate trading and addresses the evaluation of different representation models of real estate agency.
1.2 Housing Demand of Homeowners

A major part of housing policies in both Canada and the United States is focus on the promotion of homeownership. Not surprisingly a large body of housing studies have focused on housing demand of homeowners and the decision to rent or to own their home. During the inflationary periods of the 70's and 80's, major concern was expressed about the impact of liquidity constraints, and uncertain returns on housing investment, on homeownership rates and housing demand. Empirical studies have provided evidence of the impact of liquidity constraints and investment risk and return on homeownership rates and housing demand. Furthermore, empirical researchers have recognized the importance of estimating the tenure choice equation and housing demand equation simultaneously to take into account the potential correlation between the choices.

Theoretical models of housing tenure choice have provided "partial analysis" of the tenure choice problem. Tenure choice models that focus on liquidity constraints treat housing demand as well as the benefit of homeownership as exogenous [Artle and Varaiya (1978) and Brueckner (1986)]. Those that focus on the return and risk of housing investment fail to incorporate liquidity constraints [Henderson and Ioannides (1983)].

The essay in chapter 4 makes two contributions. First, we provide an integrated analysis of housing tenure choices and housing demand taking into account both investment uncertainty and liquidity constraints. The analysis helps to interpret many empirical findings concerning the interactions between tenure choice and housing demand and the impact of liquidity constraints and uncertainty on housing decisions. Second, with the help of Pratt's certainty-equivalent approximation, we are able to illustrate diagrammatically the trade-offs caused by liquidity constraints and uncertainty for housing investment and consumption choices in Henderson and Ioannides (1983) two-period setting.
Chapter 2

Employment Efficiency in Real Estate Brokerage Under Fixed Commission Rates

2.1 Introduction

One remarkable feature of the real estate brokerage industry is the lack of competition in commission rates. Although real estate prices and transaction activities vary widely over time and locations, commission rates vary little.\(^1\) High and stable commission rates have raised concern about the efficiency of the real estate brokerage industry\(^2\) [see Zumpano and Hooks (1988) for a review of the issues and literature concerning the efficiency of the real estate brokerage industry].

Given the competitive nature of the real estate brokerage industry, one important consequence of the lack of competition in commission rates could be excessive employment in the industry, resulting in inefficient production of brokerage services [see Crockett (1982)]. Brokerage employment has been found to be highly correlated with the dollar volume of real estate transactions [see Mittelbach and Case (1986)]. Clearly, brokerage employment is responsive to the level of commission income available in the market and excessive employment can result if commission rates are too high.\(^3\)

If non-price competition tends to prevail in the market for real estate brokerage services, as it does in many other markets where consumers are not experts [Arrow (1963) and Scitovsky

\(^1\)The commission rates charged by full service agents for a typical residential transaction has always been between 6% and 7%. See Wachter (1987) and Carney (1982) for discussions about the practice of commission charges.

\(^2\)In Canada, for example, public concern led to an investigation by Consumer and Corporate Affairs Canada on the practices of local real estate boards, which resulted in a Federal Court ruling in December 1988 prohibiting price fixing in the real estate brokerage industry. Commission rates, however, have changed little since then.

\(^3\)There is reason to believe that the demand for real estate brokerage is price inelastic, at least at the current prevailing level of commission rates; otherwise, price competition in the real estate brokerage industry would be common.
(1990)], then it is important for us to know what fixed commission rates would be consistent with efficient brokerage employment. Empirical evidence regarding employment efficiency in real estate brokerage is vague and inadequate, in part, due to the lack of a theoretical framework [see Zumpano and Hooks (1988) for a review of existing empirical evidences and some theoretical issues]. In view of both the importance of real estate brokerage industry and the public concern about its efficiency, a theoretical model capable of analyzing employment efficiency under fixed commission rates is desirable.

Three analytical studies in the existing literature address the efficiency of brokerage employment: Yinger (1981), Crockett (1982), and Wu and Colwell (1986). Yinger (1981) applies the Stigler (1961) price-search model in the context of real estate brokerage and examines brokers' employment decisions and the impact of multiple listing services on brokerage employment. Wu and Colwell (1986) use a similar model to investigate search effort incorporating interactions between broker search, owner search, and transaction prices. Crockett (1982) uses a simple model of non-price competition to examine the employment choices of brokerage firms. These studies are incomplete in their analysis of employment efficiency because they fail to incorporate a measure of the social benefit produced by brokerage employment.

This paper takes a new approach to analyzing brokerage employment and its efficiency under fixed commission rates. The approach builds upon dynamic search models [see Mortensen (1982), Pissarides (1990), and Hosios (1990)] and two-sided matching models [Roth and Sotomayor (1990)] recently developed in the economics literature. The dynamic search model characterizes the social value of real estate trading and the two-sided matching model links brokerage employment to the trading gain. The model presented in this paper accomplishes three major objectives. It describes the marginal value of brokerage employment; it describes the equilibrium behaviour of brokerage employment under a fixed commission rate; and it identifies the conditions for a commission rate and a licensing charge to generate efficient brokerage employment. The model is useful for analyzing the implications of government regulations in

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4At 7% commission rate, the commission income from multiple listing sales in 1990 amounts to $2.8 billion (Source: The Canadian Real Estate Association, Annual Statistical Survey 1990).
the real estate brokerage industry for brokerage employment and social welfare.

Three key elements in our model are the social value of real estate brokerage, the link between the social value and brokerage employment, and the link between brokerage employment and the level of commission rate. The social value of brokerage derives from the social value of real estate transactions. A transaction is socially valuable because it allows the property to be transferred from its lower-value user (the seller) to its higher-value user (the buyer) and hence increases the productivity of the property. The transaction, however, is a costly process due to both the uncertainty about competitive prices and the spatial dispersion of buyers and sellers. Therefore, the information about the competitive prices that would allocate properties to their highest and best uses as well as the information about the location of buyers and sellers are crucial for improving the efficiency of real estate market. The ability for organized real estate brokerage to internalize the social benefit of information enables real estate agents to specialize in collecting and disseminating the information about competitive prices as well as about the location of buyers and sellers. Thus the most important link between brokerage employment and the social value of brokerage lies in the impact of the improved availability and quality of information on the quality of matches between properties and their buyers and on the search time for buyer and sellers. The link between brokerage employment and the commission rate is based on the observation that the real estate brokerage industry is characterized by competitive entry of real estate agents and individual choice of search effort.

The relationship between brokerage employment and the quality of matches produced in a period is described by a matching function that characterizes the aggregate property of a two-sided matching market. We overlook the contractual relationship between individual agents and their customer principal that has been studied by many authors [see Anglin and Arnott (1991), Geltner et al (1991), Carroll (1989), and Zorn and Larsen (1986) for examples]. Our aggregate matching approach, however, does allow us to take into account the joint-production nature of real estate brokerage and the competitive or cooperative interactions among individual agents, which the principal-agent models of individual contracting problems tend to ignore. Indeed,
most of the effort spent by real estate agents cannot be linked exclusively to a particular
transaction. An open house, for example, is useful for selling other listings that the agent has
access to as well as for selling the house being shown. The productivity of individual agents also
depends on the aggregate employment; obviously, agents working in the organized real estate
brokerage industry today are far more productive than they would be without the organized
brokerage industry.

Our analysis proceeds as follows. In section 2.2 we present a search model of real estate
trading, where the gains from trading are determined to provide the basis for measuring the
marginal social value of brokerage employment. In section 2.3 we describe the matching tech-
nology with brokerage by means of the matching function. In section 2.4 we derive the marginal
value of brokerage employment. The competitive mechanism for brokerage employment is spec-
ified and its implications are examined in section 2.5. The conditions for the commission rate
and licensing charge to generate efficient brokerage employment are identified in section 2.6.
Section 2.7 discusses an assumption about sunk commission cost used in the analysis, and
section 2.8 concludes.

2.2 Search and Expected Trading Gain

2.2.1 The model and steady state

Real estate trading can be viewed as a search and matching process — buyers and sellers search
over time for desired trade opportunities or matches, and the trading probability in a given pe-
riod of time depends on the matching technology. We consider an open local real estate market
where trading takes place exclusively through real estate brokerage.\footnote{Federal Trade Commissio

n (1984) reports over 80% of all the homes sold in the United States involved real
estate agents, with approximately 90% of these homes listed with a multiple listing service [FTC (1984), p. 17].} Real estate brokerage
affects the matching technology; but in this section we will take the matching technology as
given and characterize expected trading gains for buyers and seller conditional upon a given
trading probability. In the next section we will describe the matching technology with real
It needs to be pointed out here that brokerage commission has two effects on the trading process. First, brokerage commission fees directly affect brokerage employment. Second, when the commission is paid upon transaction, it also affects the trading strategy of buyers and sellers. To incorporate both effects would unnecessarily complicate our analysis. Thus we will begin our analysis with these two effects separated by assuming the commission is paid in advance according to a fixed commission rate and the expected transaction price. As a result, the commission cost is sunk for buyers and sellers after entering the market. We leave the discussion of the impact of a contingent commission payment on the trading process and our results to the end of the chapter.

Buyers and sellers value properties differently. We denote the subjective value of a property to a buyer as $Y^b$, and that to a seller as $Y^s$. $Y^b$ is a random variable whose value depends on the match between the buyer and the property. To make our analysis tractable, we assume both the preferences of buyers and the attributes of properties are idiosyncratic, so that the distribution of $Y^b$ does not depend on the identity of the buyer or the property. Consequently, buyers and properties respectively are ex ante identical. We assume $Y^s$ is the same for all sellers. Variation in $Y^s$ affects the distribution of transaction gains between buyers and sellers but will not affect the social value of real estate brokerage. We define a transaction gain, denoted by $V$, to be $V = Y^b - Y^s$. Let $F(\cdot)$ be the probability distribution function for $Y^b$ with a support $[Y^b_L, Y^b_R]$. We assume $1 - F(Y^s) > 0$ so that there exist profitable trade opportunities.

Our real estate market operates over discrete time periods, where in each period new buyers and sellers arrive and some existing buyers and sellers trade and depart. Each buyer seeks one property and each seller has one for sale, and they never leave the market without trading. 

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6We may imagine our real estate brokerage operates like a club. The commission is like an admission fee and is paid to the club manager (the local real estate board) who will fully distribute the revenue to individual agents according to the number of transactions they produce in a period.

7This is a reasonable representation of many residential real estate markets where a substantial number of buyers are from other locations or are first-time home buyers and a substantial number of sellers either leave for other locations or switch to rental accommodation. Open market models are employed in other studies of real estate transactions [see Quan and Quigley (1991) for example].
Let \( N^b \) and \( N^s \) respectively denote the stock of buyers and sellers in the market at a given moment, and \( \lambda^b \) and \( \lambda^s \), the trading probabilities in a given period for buyers and sellers. Since transactions take place pairwise between buyers and sellers, we have \( \lambda^b N^b = \lambda^s N^s \equiv m \), where \( m \) is the number of transactions in a period and is also the number of pairs departing the market. We will focus only on the steady state, in which \( N^b \) and \( N^s \) are constant. Therefore, buyers and sellers must arrive in pairs and the number of pairs arriving each period, denoted by \( n \), must equal the number of pairs departing, \( m \). Transaction prices (or the distribution of transaction gains between buyers and sellers) will adjust until the steady-state conditions are satisfied. To further simplify our model, we assume that \( N^b = N^s \equiv N \). In this case, \( \lambda^b = \lambda^s = \lambda \) and, in steady state, 
\[
m \equiv \lambda N = n. \tag{2.1}
\]

### 2.2.2 The expected gains from trading and trading strategy

Given the steady-state trading process and the distribution of transaction gains defined above, we can determine the expected gains from trading for buyers and sellers following the models of Mortensen (1982) and Pissarides (1990). Let \( \bar{P} \) be the expected transaction price. Let \( \bar{Y}^b \) be the expected value of \( Y^b \) for a trade. The expected transaction gain of a trade, denoted by \( \bar{V} \), is given by \( \bar{Y}^b - Y^s \). Note that the distribution of \( V \) depends on the matching technology and is different from the distribution of \( Y^b \), since matching is a selective process.

Define the expected gains from trading, denoted by \( U^b \) and \( U^s \) respectively for buyers and sellers, to be the expected present value of transaction gains (at the beginning of a search period). Then, with the trading probability \( \lambda \) and a discount rate \( r \), \( U^b \) and \( U^s \) in steady state are determined by the following equations:

\[
U^b = \frac{\lambda(\bar{Y}^b - \bar{P} - U^b) + U^b}{1 + r}, \tag{2.2}
\]
\[
U^s = \frac{\lambda(\bar{P} - Y^s - U^s) + U^s}{1 + r}. \tag{2.3}
\]

These two equations state that in steady-state the expected gain at the beginning of a period
must equal the discounted value of the same expected gain at the end of the period plus an expected value of trading surplus. In steady state the expected trading gains are also the opportunity costs of transaction. Equations (2.2) and (2.3) can be reduced to

\[ rU^b = \lambda (\bar{Y}^b - \bar{P} - U^b), \]
\[ rU^s = \lambda (\bar{P} - Y^s - U^s), \]

which reveal that the returns on holding the option of continuing search must equal the expected "capital gains". Combining equations (2.4) and (2.5), we obtain the joint expected trading gain for a pair of buyer and seller, denoted by \( U \):

\[ U \equiv U^b + U^s = \frac{\lambda}{r + \lambda} (\bar{Y}^b - Y^s) = \frac{\lambda}{r + \lambda} \bar{V}. \]  

Equation (2.6) states that the joint expected trading gain equals the expected transaction gain, \( \bar{V} \), discounted for the expected search time \( 1/\lambda \) at the rate \( r \).

To define the trading strategy for buyers and sellers, we assume buyers and sellers have complete information with regard to \( U^b, U^s, V, \lambda \) and \( r \) and follow the Nash (1950) bargaining game. Therefore, buyers and sellers will trade as soon as the match provides a transaction gain no less than the joint expected trading gain \( U \). In other words, the trading strategy is defined by a reservation match quality \( V^* \) that equals \( U \). In equilibrium \( V^* \) will be determined by the following equation:

\[ V^* = U = \frac{\lambda}{r + \lambda} \bar{V}, \]

where \( \bar{V} = \text{Exp}\{V|V \geq V^*\} \) is an increasing function of \( V^* \) and, as will be explained in the next section, \( \lambda \) is a decreasing function of \( V^* \). For the equilibrium defined by equation (2.7) to exist, the contraction mapping theorem — see Kolmogorov and Fomin (1970, pp. 66-78) — requires the following sufficient condition to be satisfied:

\[ |U_{V^*}| = \left| \frac{r\lambda V^*}{(r + \lambda)^2} \bar{V} + \frac{\lambda}{r + \lambda} \bar{V}_{V^*} \right| < 1. \]  

For a discussion on the influence of brokerage on bargaining mechanism see Chapter 3.

Such a strategy maximizes \( U \) provided that \( N \) is taken as given.
where, as well for the rest of the paper, subscripted variable denotes the derivative of the variable with respect to the subscript. As will be explained later, the derivatives $\lambda_{V^*}$ and $\bar{V}_{V^*}$ depend on the matching function that characterizes brokerage production; therefore, condition (2.8) limits the admissible functional forms.

Moreover, the transaction price determined by the Nash bargaining game splits the surplus (or the bargaining gain) $V - U$ between the buyer and seller so that

\[ Y^b - P - U^b = P - Y^s - U^s = \frac{V - U}{2}. \]  

(2.9)

2.3 Matching Technology with Brokerage

Now we have determined, conditional upon the matching technology, the expected trading gain $U$ and the trading strategy defined by a reservation match quality $V^* = U$. The matching technology affects both the trading probability $\lambda$ and the distribution of $V$. In this section we characterize the matching technology with brokerage and relate brokerage employment to $\lambda$ and $\bar{V}$.

2.3.1 The matching technology and brokerage employment

To characterize the matching technology we first need a conceptual model for real estate brokerage. Our underlying brokerage model is a two-sided matching model [Roth and Sotomayor (1990)]. The real estate market is a two-sided matching market where many buyers and many sellers compete for the best available trade opportunities. The classic bid-rent model of Alonso (1964) and the hedonic model of Rosen (1974) both characterize the competitive market outcome as a matching between buyers and properties supported by a profile of competitive prices. In a world with perfect information, a competitive matching is Pareto efficient, in the sense that no alternative matching would give everyone as much value and someone more value. The essential function of real estate brokerage, the function that produces social value, is to facilitate the matching in an imperfect world by providing competitive price information and market exposure to buyers and sellers. Greater market exposure means greater opportunity for
Pareto improvement. The competitive price information eliminates the matches that are not competitive and hence increases the effective market exposure for buyers and sellers. Therefore, we conclude that brokerage employment produces market exposure for buyers and sellers and, as a result, improves the quality of the matches formed in a given period in the sense that the distribution of transaction gain $V$ among the matches skews further toward the upper bound $V_+ \equiv Y^b_+ - Y^s$. In Appendix 2.a we provide an informal proof that greater exposure improves the quality of competitive matches in a two-sided matching market.

We use two variables to represent brokerage employment: $K$, the number of real estate agents, and $e$, the search effort of individual agents. The search effort reflects the time and energy an agent spends and the amount of personal services (such as office manager and clerks) and capital services (such as office space, computers, and advertisement) the agent employs each period. We assume real estate agents are \textit{ex ante} identical, though they may specialize in terms of the preferences of buyers or the attributes of properties. Thus agents will choose an identical search effort in equilibrium so that we need only consider the search effort of a representative agent. The determination of $K$ and $e$ will be discussed in later sections. The productivity of $K$ and $e$ depends on market characteristics (like the heterogeneity embodied in the distribution function $F(Y)$), information technology, as well as the organization of the industry. Organizational innovations such as pooling information and cooperative sales through a multiple listing board significantly raise the productivity of the real estate brokerage industry.

The matching technology and the impact of brokerage employment on the matching outcome can be summarized by a matching function that describes the aggregate property of a two-sided matching process. Define $M(N, K, e, v)$ to be the matching function that describes the number of matches formed in a given period with transaction gain $V \geq v$, given market size $N$ and brokerage employment $K$ and $e$. $M$ is like a production function where $N$, $K$, and $e$ are factor inputs and $v$ is a threshold value. Based on our underlying brokerage model $M$ increases with $K$ and $e$ given $N$ and $v$, since brokerage employment improves the quality distribution of the matches. $M$ increases with $N$ given $K$, $e$, and $v$, simply because more trading opportunities are
in the market to be matched. By definition, $M$ decreases with $v$. Figures 2.1 and 2.2 illustrate the property of $M$.

Now the number of transactions in a period can be defined by the equation

$$m \equiv M(N, K, e, V^*)$$

where $V^*$ is the reservation match quality. The productivity of the factor inputs and the effect of $V^*$ can be represented by the elasticities of $m$ with respect to these variables. Let $\epsilon_{yx}$ denote the elasticity of $y$ with respect to $x$, that is, $\epsilon_{yx} \equiv (\partial y/y)/(\partial x/x)$. Then we have

$$\epsilon_{mN} > 0, \ \epsilon_{mK} > 0, \ \epsilon_{me} > 0, \ \epsilon_{mV^*} < 0.$$  

(2.11)

The magnitude and behavior of these elasticities embodies the technological and organizational factors that affect the productivity of brokerage employment.

The matching function $M$ should also display diminishing returns to individual factors of production. Thus we assume $\epsilon_{mx}$ is non-increasing in $x$, for $x \equiv N, K, e$. We can further assume that $K$ and $e$ are substitutes for each other, though not perfect, in the sense that $\epsilon_{mK}$ in non-increasing in $e$ and $\epsilon_{me}$ is non-increasing in $K$. On the other hand, $K$ and $e$ are complements to $N$, in the sense that $\epsilon_{mK}$ and $\epsilon_{me}$ are non-decreasing in $N$ and $\epsilon_{mN}$ is non-decreasing in $K$ and $e$ (see figure 2.2). We further assume that $v$ affects $\epsilon_{mK}$ and $\epsilon_{me}$ symmetrically such that $\epsilon_{mK}/\epsilon_{me}$ does not change with $v$.10

### 2.3.2 The behavior of the trading probability

In steady state $n = m \equiv \lambda N$. By equation (2.10) we have

$$n = M\left(\frac{n}{\lambda}, K, e, V^*\right),$$

(2.12)

which implicitly defines $\lambda$ as a function of $n$, $K$, $e$, and $V^*$. We treat $n$ as an exogenous variable. Thus, the productivity of factor inputs is reflected in the trading probability $\lambda$. It

---

10 This could be justified by the fact that, in the linear programming model in Appendix 2.a, the reservation values $U^v$ and $U^e$ do not affect the market exposure for individual buyers and sellers.
can be derived from equation (2.12) that
\[ \epsilon_{mN} \frac{d\lambda}{\lambda} = (\epsilon_{mN} - 1) \frac{dn}{n} + \epsilon_{mK} \frac{dK}{K} + \epsilon_{me} \frac{de}{e} + \epsilon_{mV*} \frac{dV*}{V*}. \]  
(2.13)

Based on (2.11), equation (2.13) indicates that \( \lambda \) increases with \( K \) and \( e \) and decreases with \( V^* \). An increase in the number of pairs of buyer and seller entering the market each period, \( n \), however, can have two offsetting effects on \( \lambda \). On the one hand, it can have a scale-economy effect: it increases market size and thickness, making closer matches easier to form. This increases \( \lambda \). One the other hand, \( n \) can have a crowding effect: it reduces brokerage employment per pair of buyer and seller and thus the individual market exposure. This decreases \( \lambda \). The net effect is determined by \( \epsilon_{mN} \). When \( \epsilon_{mN} > 1 \) the scale economy dominates and \( \lambda \) increases with \( n \). When \( \epsilon_{mN} < 1 \) the crowding effect dominates.

2.3.3 The behaviour of expected transaction gains

The quality of the matches formed in a given period not only affects \( \lambda \) but also \( \bar{V} \). The probability distribution of \( V \) can be expressed in terms of the frequency distribution of the matches with different transaction gains. That is, \( \Pr\{V < v|V \geq V^*\} = [M(N, K, e, V^*) - M(N, K, e, v)]/M(N, K, e, V^*) \). The expected transaction gain \( \bar{V} \), therefore, is determined by the following equation:
\[ \bar{V} = \operatorname{Exp}\{V|V \geq V^*\} = \int_{V^*}^{V_+} v d \frac{M(N, K, e, V^*) - M(N, K, e, v)}{M(N, K, e, V^*)} = V^* + \int_{V^*}^{V_+} \frac{M(N, K, e, v)}{m} dv. \]  
(2.14)

Differentiating equation (2.14) with respect to \( V^* \) we have
\[ \bar{V}_{V^*} = -\frac{mV^*}{m} \int_{V^*}^{V_+} \frac{M(N, K, e, v)}{M^*} dv = -\frac{\epsilon_{mV^*}}{V^*}(\bar{V} - V^*). \]  
(2.15)

Differentiating equation (2.14) with respect to factor inputs \( x \equiv N, K, e \) and applying the generalized integral mean value theorem, we have
\[ \bar{V}_x = \frac{1}{x} \int_{V^*}^{V_+} [\epsilon_{mX}(v) - \epsilon_{mx}] \frac{M(N, K, e, v)}{m} dv = \frac{1}{x} \left[ \epsilon_{\bar{m}X} - \epsilon_{mx} \right] (\bar{V} - V^*), \]  
(2.16)
where \( \epsilon_{Mx}(v) \equiv M_x(N, K, e, V)x/M(N, K, e, V) \), \( \epsilon_{\hat{m}x} \equiv \epsilon_{Mx}(\hat{\nu}(x)) \) with \( \hat{\nu}(x) \in [V^*, Y_b^k - Y^*] \), and \( \epsilon_{mx} \equiv \epsilon_{Mx}(V^*) \).

We make three assumptions about the distribution of \( V \) embodied in the matching function \( M \). First, we assume the hazard rate \( -M_v/M \) increases with \( v \); given that a match has a quality no less than a threshold \( v \), the match is less likely to be of a higher quality than the threshold as the threshold increases. This is true as long as the density of the distribution decreases less than exponentially. This is a reasonable assumption since, as the quality threshold increases, it becomes harder to find better matches. Second, we assume the hazard rate \( -M_v/M \) decreases with \( K, e, \) or \( N \). This means the quality distribution of the matches improves with \( K, e, \) or \( N \), so that it becomes more likely to find better matches at any quality threshold when brokerage employment increases or when the market becomes denser. Third, we assume the hazard rate decreases less with \( K, e, \) or \( N \) when the threshold \( v \) is higher. This means the improvement in quality distribution is less significant at higher levels of quality threshold.

\( \bar{V}_{V^*} > 0 \) follows from \( -M_v > 0 \). Moreover, it follows from the assumption of increasing hazard rate in \( v \) that \( \bar{V}_{V^*} < 1 \). This can be seen from equation (2.15), where \( \bar{V}_{V^*} \) equals the expected value of the ratio \(( -m_{V^*}/m)(-M_v/M) \). Since by the assumption this ratio is always less than one for \( v > V^* \), its expected value is less than one. Furthermore, from the assumption of decreasing hazard rate in \( x \) it follows that \( \bar{V}_x > 0 \). This is evident in equation (2.16) because the assumption that \( -M_v/M \) decreases in \( x \) is equivalent to that \( M_v/M \) increases in \( v \). Finally, it follows from the last assumption that \( \bar{V}_{V^*} \) decreases with \( x \) or, equivalently, \( \bar{V}_x \) decreases with \( V^* \), for \( x \equiv K, e, \) or \( N \). This also can be seen from equation (2.15), where two effects happen when \( x \) increases: the ratio of the hazard rates decreases for any \( v > V^* \) according to the last assumption and the probability weights shift towards the upper bound of the distribution according to the assumption of decreasing hazard rate with \( x \). Since the hazard rate increases with \( v \), the ratio of the hazard rates decreases with \( v \). Therefore, both of the two effects reduces \( \bar{V}_{V^*} \).

\( ^{11} \)For example, if \( V \) is exponentially distributed with parameter \( \beta \), \( \epsilon_{V_{V^*}} = V^*/(V^* + 1/\beta) < 1 \), if uniformly distributed with upper bound \( V_+ \), \( \epsilon_{V_{V^*}} = V^*/(V^* + V_+) < 0.5 \).
Chapter 2. Employment Efficiency

In equilibrium (2.7) holds and equations (2.15) and (2.16) can be rewritten as:

\[
\begin{align*}
\epsilon_{VV^*} & = - \frac{r}{r + \lambda} \epsilon_{e V^*}, \\
\epsilon_{VK} & = \frac{r}{r + \lambda} (\epsilon_{mK} - \epsilon_{mK}), \\
\epsilon_{Ve} & = \frac{r}{r + \lambda} (\epsilon_{me} - \epsilon_{me}), \\
\epsilon_{VN} & = \frac{r}{r + \lambda} (\epsilon_{mN} - \epsilon_{mN}).
\end{align*}
\]

(2.17)  
(2.18)  
(2.19)  
(2.20)

Thus, we have represented the matching technology with a matching function \(M(N, K, e, v)\). And with the matching function we have defined the trading probability \(\lambda\) and the expected transaction gain \(\bar{V}\) as functions of market size, brokerage employment, and reservation match quality.

2.4 Marginal Social Value of Brokerage Employment

To examine the impact of commission rates on the efficiency of brokerage employment, we need to determine the marginal social value of brokerage employment. The social value of brokerage arises from its contribution to the expected trading gain \(U\) determined by equation (2.7). With the matching technology described above, brokerage employment contributes to \(U\) in two ways: it increases \(\lambda\) according to equation (2.13) and increases \(\bar{V}\) according to equation (2.16). Since in equilibrium \(U = V^*\), and since \(V^*\) and \(\lambda\) affect each other, the impacts of \(K\) and \(e\) on \(U\) need to be determined simultaneously through equations (2.7) and (2.12). The following lemma summarizes the elasticities of \(\lambda\) and \(V^*\) with respect to \(K\) and \(e\) respectively.

**Lemma 1**

**Defined**

\[
\Theta \equiv \frac{\epsilon_{V V^*} (1 - \epsilon_{mN} + \epsilon_{mN})}{(1 - \epsilon_{V V^*}) \epsilon_{mN}}.
\]

At the steady-state equilibrium defined by equations (2.7) and (2.12), the elasticities of \(\lambda\) and \(V^*\) with respect to \(K\) and \(e\) are given by:

\[
\epsilon_{\lambda K} = \frac{\epsilon_{mK} - \epsilon_{mK} \epsilon_{V V^*} (1 - \epsilon_{V V^*}) \epsilon_{mN}}{1 + \Theta},
\]

\[
\epsilon_{\lambda e} = \frac{\epsilon_{me} - \epsilon_{me} \epsilon_{V V^*} (1 - \epsilon_{V V^*}) \epsilon_{mN}}{1 + \Theta}.
\]
Furthermore, the equilibrium is stable given the level of $K$ and $e$ if and only if $-1 < \Theta < 1$.

**Proof:** See Appendix 2.b.

$\Theta$ represents the feedback effect between $\lambda$ and $V^*$: $\Theta = - (\partial \lambda / \partial V^*) (\partial V^* / \partial \lambda)$. The equilibrium is stable when the magnitude of the feedback is not too large, which requires that $\epsilon_{V^*}$ is not too large and $\epsilon_{mN}$ is not too small. The term $\epsilon_{mN} - \epsilon_{mN}$ would be small relative to 1, because the scale-economy effect of an increase in $N$ tends to be offset by the crowding effect of it so that the net effect of a change in $N$ on $\bar{V}$ would be small. By the complementarity between brokerage employment and $N$, $\epsilon_{\lambda N}$ and $\epsilon_{V N}$, or equivalently $\epsilon_{mN}$ and $\epsilon_{mN} - \epsilon_{mN}$, increase with the employment. Moreover, an increase in brokerage employment reduces $\hat{V}_V^*$ due to the improvement in the quality distribution. Therefore, the stability can be maintained at reasonable levels of brokerage employment. It should be noted that $|\Theta| < 1$ guarantees the contraction mapping condition (2.8).

We will be concerned only with the stable equilibrium, where $1 + \Theta > 0$. Since the interpretation of $\epsilon_{\lambda e}$ and $\epsilon_{V^*e}$ is parallel to that of $\epsilon_{\lambda K}$ and $\epsilon_{V^*K}$, we only need to discuss equations (2.21) and (2.22). As equation (2.21) shows, the net effect of $K$ on $\lambda$ depends on how the direct effect, $\epsilon_{mK}/\epsilon_{mN}$, is offset by the indirect effect, $(\epsilon_{mK} - \epsilon_{mK})\epsilon_{V^*}/(1 - \epsilon_{V^*})\epsilon_{mN}$, due to the change in $V^*$. The net effect is positive if $\epsilon_{mK}/\epsilon_{mK} > \epsilon_{V^*}$. At low level of brokerage employment $\epsilon_{mK}/\epsilon_{mK}$ would be small and $\epsilon_{V^*}$ would be large, so that $\lambda$ can decrease with $K$; the opposite is true at high level of brokerage employment.
Equation (2.22) shows that when $\Theta > 0$ the marginal impact of $K$ on $U$ is always positive. Given $V^*$, $K$ increases both $\lambda$ and $\bar{V}$. The increase in $\lambda$ reduces $N$ when $n$ is constant; the reduction in $N$ can adversely affect the quality distribution of the matches and thus decrease $\bar{V}$. When $\epsilon_{mN} - \epsilon_{mN} < 1$, which implies $\Theta > 0$, the net effect of an increase in $\lambda$ on $U$ is always positive.

We summarize the impact of brokerage employment on the expected trading gain $U$ and trading probability $\lambda$ in the following proposition.

**Proposition 1** The marginal social value of brokerage employment.

1. When the impact of $N$ on the quality distribution of the matches is small, the expected trading gain $U$ always increases with brokerage employment.

2. The trading probability $\lambda$ increases with brokerage employment if the expected transaction gain $\bar{V}$ is not very sensitive to the marginal change in either the reservation match quality $V^*$ or the employment.

3. The marginal value of brokerage employment $x$, for $x \equiv K$ or $e$,

   (a) increases with the opportunity cost of search time ($r/\lambda$);

   (b) increases with $\epsilon_{mX}$ and $\epsilon_{\tilde{m}X} - \epsilon_{mX}$ respectively; and

   (c) decreases with $\epsilon_{mN}$ and $\epsilon_{\tilde{m}N} - \epsilon_{mN}$ respectively if the trading probability $\lambda$ increases with brokerage employment.

*Proof:* Results 1, 2, 3a, and 3b follow directly from Lemma 1. To show the last result, we only need to find the derivative of $\epsilon_{V_K}$ with respect to $\Theta$:

$$\frac{\partial \epsilon_{V^*_K}}{\partial \Theta} = \frac{r - \epsilon_{mN}}{r + \lambda \epsilon_{V^*}} \frac{\epsilon_{\lambda K}}{1 + \Theta},$$

which is positive whenever $\epsilon_{\lambda K}$ is positive.

Q.E.D.
2.5 Brokerage Employment Under Fixed Commission Rates

We have derived the marginal social value of brokerage employment. To examine the impact of commission rates on the efficiency of brokerage employment, we still need a model of brokerage employment under a fixed commission rate. In this section we specify the behaviour of $K$ and $e$ and examine the implications of the resulting employment model.

2.5.1 The competitive entry and choice of effort

Our model of brokerage employment is based on two observations. First, it is relatively easy for individual agents to enter and exit brokerage business. Many individuals hold a real estate agent license and can become active or inactive according the conditions of the market. Moreover, to become a licensee requires some education and experience and a small licensing fee, which many people can fulfill in a relatively short period of time. Indeed Mittelbach and Case (1986) finds a high correlation between the number of real estate agents and the volume of real estate transactions in the United States.\footnote{They found that the number of transactions and the real transaction price index explained 90% of the variation in the number of real estate licensees in California during the period from 1959 to 1984. They also found that the existing home sales and the median home value of owner-occupied homes explained 85% of the variance in the number of employees in real estate brokerage establishments across 50 states in 1983.} Available evidence also shows that commission earnings by real estate agents on average are quite moderate and significantly less variable than transaction volumes [see Hamilton and Fu (1991)].\footnote{The average real commission earnings (in 1986 Canadian dollars) for the agents surveyed in British Columbia in 1987, 1988, and 1989 are, respectively, $44,001, $48,072, and $57,380. The real dollar volume of multiple listing sales changed by 37% between 1987 and 1988 and by 49% between 1988 and 1989, whereas the average real commission earnings changed by 9% and 19% respectively.}

Second, it is relatively easy for individual agents to choose a search effort. Individual agents are able to adjust their working time and energy. Brokerage office spending varies with commission income. Agents can also adjust the amount of personal and capital services they employ in their brokerage offices by adjusting their commission split or desk fees or by choosing different offices. Resources in brokerage offices can be shared with and shifted to and from non-brokerage businesses such as property management and leasing services. Thus, the search
effort of individual agents can vary according to market conditions.

Based on the first observation, we assume the employment of $K$ is determined by competitive entry or exit so that the normal earnings of an agent, denoted by $\pi^o$, is fixed and reflects the opportunity cost of employment in real estate brokerage. The earnings is the difference between the commission income and search cost. Let $c(e)$ denote the cost of individual effort $e$ including the opportunity of cost of additional personal effort and the cost of the personal and capital services purchased from brokerage offices through desk fees or commission sharing. Let $\sigma$ be the fixed commission rate. Then we have $\pi^o = \sigma \bar{P}n/K - c(e)$ or, equivalently,

$$K = \frac{\sigma \bar{P}n}{c(e) + \pi^o}. \tag{2.25}$$

Based on the second observation, we assume individual agents choose their own effort to maximize their expected net earnings per period, taking as given the market size $N$, the number of agents $K$, the effort of other agents $e$, and trading strategy $V^*$. Given the fixed commission rate $\sigma$ and the expected transaction price $\bar{P}$, an agent’s effort only affects his market share and operating cost. Let $\pi$ denote the net earnings per period, $e_k$ the effort of agent $k$, and $m_k$ the sales per period for agent $k$ (a cooperative sale can be counted as a fraction of a sale). Then the agent’s problem can be described as to maximize

$$\pi(e^k) = \sigma \bar{P}m^k - c(e^k), \tag{2.26}$$

where $m^k$ is determined by an individual output function $M^k(e^k, N, K, e, V^*)$. We assume $M^k$ is increasing and concave in $e^k$ and $c$ is increasing and convex. An increase in $e^k$ not only improves the quality of the matches for agent $k$’s clients, but also increases the agent’s share of clients. $M^k$ also increases with $N$ and decreases with $V^*$. The marginal impact of $K$ and $e$ is more ambiguous. More employment in the brokerage industry can generate more cooperative opportunities and more public good in terms of market information; but at the same time it generates more competition. The first-order condition is given by

$$\frac{d\pi(e^k)}{de^k} = \frac{\sigma \bar{P}m^k}{e^k} \epsilon_{m^k e^k} - c'(e^k) = 0. \tag{2.27}$$
Since agents are \emph{ex ante} identical, the output function and cost function are the same for all agents and in equilibrium they choose the same effort $e^k = e$. Moreover, $m^k = n/K$ and, by competitive entry and exit, $\pi(e) = \pi^0$; therefore, $\sigma \bar{P} n/K = \pi^0 + c(e)$. Let us define $\eta$ to be the elasticity of individual output with respect to individual own effort, that is $\eta \equiv \epsilon_{m^k e^k}$. In equilibrium the first-order condition (2.27) reduces to

$$c'(e) = \eta \frac{\pi^0 + c(e)}{e}.$$  

Equation (2.28) indicates that the equilibrium search effort is determined by the intersection of the marginal cost curve with the average cost curve multiplied by the elasticity $\eta$. Given the upward sloping marginal cost curve, $e$ increases with $\eta$ because an increase in $\eta$ shifts the adjusted average cost curve upward to intersect the upward sloping marginal cost curve further to the right. If $\eta$ is constant, $e$ is determined by $\pi^0$ and the cost function $c(e)$ only.

\subsection*{2.5.2 The implications of the employment model}

We would like to discuss two implications of the above specification of brokerage employment behaviour. The first is the implication of the individual output function for the aggregate matching function. Since $M = K M^k$ in equilibrium, we have $\epsilon_{m e} = \eta + \epsilon_{m^k e}$ and $\epsilon_{m K} = 1 + K \epsilon_{m^k K}$. The interactive terms $\epsilon_{m^k e}$ and $\epsilon_{m^k K}$ may be positive or negative depending on whether the cooperative and agglomerative or the competitive effects dominate. It is plausible to assume that $\epsilon_{m^k e}$ and $\epsilon_{m^k K}$ decrease with brokerage employment as the cooperative and agglomerative effects diminish and competitive effects intensify. This is consistent with the earlier assumption about the diminishing returns to individual factors.

The second implication regards the responses of $K$ and $e$ to changes in $n$ and $\bar{P}$, two most important indicators of market activities. Particularly, we want to examine whether the prediction of our model can explain the "puzzling" observations presented in Mittelbach and Case (1986). Their time-series model for California real estate licensees revealed that movements in the home-price index were a much better predictor of the number of licensees in the state than movements in transaction numbers. Their cross-state model revealed that the incidence
of real estate agents was positively and significantly correlated with median home price but negatively correlated with existing home sales per 1000 people. What explains the difference in the influences of average price and transaction activities on the number of agents in the market?

Equation (2.25) tells us that the number of agents $K$ would be equally sensitive to the changes in $P$ and $n$ if effort $e$ is constant. Thus $K$ would respond to $\bar{P}$ and $n$ differently only if $e$ responds to $\bar{P}$ and $n$ differently. The behaviour of $e$ depends critically on the behaviour of the elasticity of individual output to individual effort, $\eta \equiv \epsilon_{m}^{k,e}$. We assume $\eta$ changes in the same direction as $\epsilon_{me}$; thus

$$\epsilon_{\eta e} \leq 0, \quad \epsilon_{\eta K} \leq 0, \quad \epsilon_{\eta N} \geq 0, \quad \epsilon_{\eta V^*} \geq 0.$$  \hfill (2.29)

The first two properties reflect the diminishing return to brokerage employment, the third property reflects the complementary effect of $N$ on the productivity of brokerage employment, and the last property follows from the assumption of decreasing hazard rate with brokerage employment. With $\eta$ influenced by $N$, $K$, $e$, and $V^*$, the responses of $K$ and $e$ to changes in $n$ and $\bar{P}$ need to be determined simultaneously by trading strategy (2.7), matching technology (2.12), and employment behaviour (2.25) and (2.28). Before we present the results, we define $\delta \equiv c''/c' + 1 - \eta - \epsilon_{\eta e}$, which represents the difference between the elasticities on the two sides of equation (2.28) with respect to $e$. $\delta$ is positive because the marginal cost curve intersects the adjusted average cost curve from below. The elasticities of $K$ and $e$ with respect to $n$ and $\bar{P}$ are presented in the following lemma.

**Lemma 2**

*Given the equilibrium defined by trading strategy (2.7), matching technology (2.12), and employment behaviour (2.25) and (2.28), the response of $e$ and $K$ to changes in $\bar{P}$ and $n$ are given by*

$$\left( \frac{dK/K}{de/e} \right) = \frac{1}{\Delta} \left( \begin{array}{c} 1 \\ \epsilon_{eK} \end{array} \right) \left( \frac{d\bar{P}}{\bar{P}} + \frac{dn}{n} \right) + \frac{1}{\Delta} \left( \begin{array}{c} -\eta e_{n|K} \\ e_{n|K} \end{array} \right) \frac{dn}{n}, \hfill (2.30)$$
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where

\[ \epsilon_{eK} \equiv \frac{\epsilon_{\eta K} - \epsilon_{\eta N} \epsilon_{\lambda K} + \epsilon_{\eta V^*} \epsilon_{V^* K}}{\delta + \epsilon_{\eta N} \epsilon_{\lambda e} - \epsilon_{\eta V^*} \epsilon_{V^* e}}, \]

\[ \epsilon_{en|K} \equiv \frac{\epsilon_{\eta N} + (\epsilon_{m N} - 1) \epsilon_{\eta V^*} \epsilon_{V^*}/(r + \lambda)}{(\epsilon_{V^*}(1 - \epsilon_{m N}) + \epsilon_{m N})(\delta + \epsilon_{\eta N} \epsilon_{\lambda e} - \epsilon_{\eta V^*} \epsilon_{V^* e})}, \]

\[ \Delta \equiv 1 + \eta \epsilon_{eK}. \]

The response is convergent if and only if \(|\eta \epsilon_{eK}| < 1\). Moreover, the convergence of \(e\) with respect to a change in \(K\), with \(\lambda\) and \(V^*\) held constant, requires that \(-1 < -\eta \epsilon_{\eta K}/\delta < 1\); and that with respect to changes in \(\lambda\) and \(V^*\), with \(K\) held constant, requires that \(-1 < (\epsilon_{\eta N} \epsilon_{\lambda e} + \epsilon_{\eta V^*} \epsilon_{V^* e})/\delta < 1\).

**Proof:** See Appendix 2.b.

In Lemma 2, \(\epsilon_{eK}\) and \(\epsilon_{en|K}\) respectively represent the elasticities of \(e\) with respect to \(K\) and to \(n\) given \(K\), with the responses in \(\lambda\) and \(V^*\) being taken into account. \(K\) affects \(e\) in three ways: it affects \(\eta\) directly, it affects \(\lambda\) which in turn affects \(N\), and it affects \(V^*\). \(n\) affects \(e\) in two ways holding \(K\) constant: it affects \(N\) and \(V^*\). Note that the denominators of \(\epsilon_{eK}\) and \(\epsilon_{en|K}\) are positive if the requirement for \(e\) to be stable under constant \(K\) is satisfied. Thus \(\epsilon_{eK} < 0\) if \(-\epsilon_{\eta K} + \epsilon_{\eta N} \epsilon_{\lambda K} > \epsilon_{\eta V^*} \epsilon_{V^* K}\); and \(\epsilon_{en|K} > 0\) if \(\epsilon_{\eta N} + (\epsilon_{m N} - 1) \epsilon_{\eta V^*} \epsilon_{V^*}/(r + \lambda) > 0\). Given \(\epsilon_{\eta N} > 0\) and \(\epsilon_{\eta V^*} > 0\) according to (2.29), \(\epsilon_{en|K} > 0\) if \(\epsilon_{m N}\) is not too small, which is likely to be satisfied in equilibrium since the stability of the equilibrium requires \(\epsilon_{m N}\) not to be too small.

\(\Delta\) reflects the interaction between \(e\) and \(K\), with the responses in \(\lambda\) and \(V^*\) being taken into account. \(\Delta > 0\) follows from the convergence of \(K\) and \(e\) with respect to changes in \(n\) and \(\bar{P}\). If \(\epsilon_{eK} < 0\), \(\Delta < 1\) so that the interaction between \(e\) and \(K\) magnify the individual responses of \(e\) and \(K\) to exogenous shocks.

Lemma 2 can be used to explain the findings in Mittelbach and Case (1986). We present our case in Proposition 2.

**Proposition 2** The influences of \(n\) and \(\bar{P}\) on brokerage employment.
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1. In steady-state equilibrium, with the number of transactions \( n \) given, the number of real estate agents \( K \) always increases with the expected transaction price \( \bar{P} \). The response of \( K \) is greater if \( \epsilon_{eK} \) is smaller or more negative.

2. If \( \epsilon_{en|K} > 0 \), then for the same amount of increase in the dollar volume of transactions \((nP)\), the increase in \( \bar{P} \) has a greater positive impact on the number of agents \( K \) and a less positive impact on individual effort \( e \) than the increase in \( n \) does.

Proof:

1. From Lemma 2 we have \( \epsilon_{K\bar{P}} = 1/\Delta \), which is positive and increases as \( \epsilon_{eK} \) decreases.

2. From Lemma 2 we have \( \epsilon_{K\bar{P}} - \epsilon_{Kn} = \eta \epsilon_{en|K}/\Delta > 0 \) and \( \epsilon_{e\bar{P}} - \epsilon_{en} = -\epsilon_{en|K}/\Delta < 0 \).

Q.E.D.

Intuitively, when \( \epsilon_{en|K} > 0 \) an increase in \( n \) causes \( e \) to increase. The greater cost required by the higher level of \( e \) discourages more agents from entering. An increase in \( \bar{P} \), on the other hand, does not directly affect the choice of \( e \). Mittelbach and Case (1986) attributes the “puzzling” observations to the risk-seeking behaviour of real estate agents, as a higher average price with the same dollar volume of transactions implies higher but less frequent rewards to individual agents. Based on our model, however, the puzzling phenomena may well be the evidence for \( \epsilon_{en|K} > 0 \). Especially if excessive brokerage employment is present, \( \epsilon_{\bar{m}N} \) would be large and \( \epsilon_{en|K} \) would be large. Thus, our model of brokerage employment and production can explain the empirical observation of brokerage employment behavior.

2.6 Efficient Commission Rate

With Lemma 1 and 2 describing the marginal value and the behaviour of brokerage employment, we can characterize the efficient commission rate that would balance the marginal value and the marginal cost of brokerage employment. To relate the benefit and the cost of brokerage, we recall that \( n \) pairs of buyer and seller enter the real estate market each period seeking
brokerage services, each receiving an expected trading gain $U$ and paying a commission $\sigma \bar{P}$. The real estate brokerage industry employs $K$ and $e$ each period, totally financed by the expected commission revenue $n\sigma \bar{P}$. Thus for an exogenous $n$, brokerage employment is efficient when the net expected trading gain defined by $B \equiv U - \sigma \bar{P}$ is maximized. In this section, we examine the efficient commission rate in three steps. First, we let the number of agents $K$ be competitively determined but hold the search effort $e$ fixed. Second, we let both $K$ and $e$ be competitively determined. And third, we let licensing fee regulate the combination of $K$ and $e$.

2.6.1 The efficient commission rate with fixed effort

With $e$ fixed (or $\eta$ constant), the employment of $K$ is simply proportional to $\sigma$, $\bar{P}$, and $n$; that is,

$$\frac{dK}{K} = \frac{d\sigma}{\sigma} + \frac{d\bar{P}}{\bar{P}} + \frac{dn}{n}. \quad (2.31)$$

Thus a higher $\sigma$ leads to a larger $K$. The marginal cost of $K$ is fixed and equal to $c(e) + \pi^o$. Since $V$ is bounded above, it is natural to assume the marginal value of $K$, $V_K^*$, is diminishing with $K$. Therefore, there exists a commission rate $\sigma^*$, such that the resulting employment of agents $K^*$ generates a marginal value equal to the marginal cost. Proposition 3 summarizes this result.

Proposition 3 The efficient $\sigma$ under competitive $K$ and fixed $e$.

With $e$ constant and $|\Theta| < 1$, the commission rate that maximizes $B \equiv U - \sigma \bar{P}$, denoted by $\sigma^*$, exists and satisfies the following equation:

$$\sigma^* = \frac{U}{\bar{P} \epsilon V^*K} = \frac{r}{r + \lambda \bar{P}} \frac{U \epsilon_{mK} + \epsilon_m \epsilon_{\text{inhk}} - \epsilon_{\text{inhK}} + \epsilon_{mV} (1 - \epsilon_{\text{inhV}}) + \epsilon_{mV}}{\epsilon V_K^*}; \quad (2.32)$$

Furthermore, brokerage employment is excessive if $\sigma > H(\sigma)$, and is inadequate if $\sigma < H(\sigma)$.

Proof: By the first-order condition and equation (2.31) (with $dn = 0$), we have

$$\frac{\partial B}{\partial \sigma} = \frac{U}{\sigma \epsilon V^*K} \epsilon_{K\sigma} - \left( \bar{P} + \sigma \frac{\partial \bar{P}}{\partial \sigma} \right) = \left( \frac{U}{\sigma \epsilon V^*K} - \bar{P} \right) (1 + \epsilon_{\bar{P}\sigma}) = 0. \quad (2.33)$$
When $K$ is affected by $\sigma$, $1 + \epsilon P_{\sigma} \neq 0$ and equation (2.33) together with Lemma 1 gives the desired expression. Moreover by equation (2.25), equation (2.32) is equivalent to

\[ nV_{K}^* = \frac{n\sigma^* \bar{P}}{K} = c(e) + \pi^\circ. \tag{2.34} \]

Equation (2.34) states that the sum of the individual marginal benefit of $K$ equals the marginal cost of $K$, which is the familiar Samuelson condition for the optimal choice of a public good. That $V_{K}^*$ decreases with $K$ ensures that equation (2.34) is a welfare-maximizing condition for $K$ and $\sigma$. Let $K^*$ be the efficient employment associated with $\sigma^*$. Equation (2.34) tells us that, for $\sigma > \sigma^*$ (and correspondingly $K > K^*$),

\[ nV_{K}^* < \frac{n\sigma^* \bar{P}}{K} = \frac{n\sigma^* \bar{P}}{K^*} = c(e) + \pi^\circ, \]

or equivalently,

\[ \sigma > H(\sigma). \]

Similarly we can show that $\sigma < H(\sigma)$ for $\sigma < \sigma^*$ (and correspondingly $K < K^*$).

Q.E.D.

According to condition (2.32), when the commission rate is efficient it should equal the product of the ratio of the expected trading gain over the expected transaction price and the marginal value of employment $K$. Proposition 3 establishes the existence of an efficient commission rate within our model and the conditions under which the commission rate is efficient, too high, or too low. It provides an expression for the efficient commission rate in terms of the social value of trading and the productivity measures of brokerage employment. Equation (2.32) does not directly depend on the cost parameters of the real estate brokerage industry, although the behaviour of $H(\sigma)$ does since the response of $K$ to $\sigma$ depends on the cost function $c(e)$.

Although determining the parameters of the matching function is difficult, equation (2.32) allows us to make sensible judgments about the difference between the $\sigma$ we observe and the probable value for $H(\sigma)$ based on careful observations of trading activities. Some benchmark
values for $H(\sigma)$ can be provided based on reasonable assumptions. For example, suppose that $\epsilon_{\tilde{n}N} = \epsilon_{mN} = 1$, $\epsilon_{\tilde{n}K} = 1$, $r = 5\%$ per year, and $1/\lambda$ is $1/5$ year. Then $H(\sigma) = rU/(r + \lambda)\bar{P} = 0.99\%(U/\bar{P})$. Increasing $r$ to $8\%$ per year and $1/\lambda$ to $1/4$ year would give us $H(\sigma) = 1.96\%(U/\bar{P})$. Proposition 1 provides some guidance about how $H(\sigma)$ would change with the parameters of the matching function.

Whether or not the efficient commission rate is sensitive to the expected transaction price $\bar{P}$ and the transaction activity $n$ depends on how the function $H(\sigma)$ responds to variations in $\bar{P}$ and $n$. The impact of an increase in $\bar{P}$, for example, depends on how $V^*\epsilon_{V^*K}$ would change relative to $\bar{P}$ as $K$ increase with $\bar{P}$. The influence of $n$ depends on how $V^*\epsilon_{V^*K}$ changes with $K$ given that $K/n$ is constant.

2.6.2 The efficient commission rate with competitive effort

When both the number of agents and individual search effort are competitively determined, the efficiency of brokerage employment involves both the level and the combination of the factors. $K$ and $e$ are substitutes, but not perfect: the quality of brokerage services requires certain amount of individual effort. Moreover, the marginal cost of additional effort can differ from the marginal cost of an additional agent, which reflects the average cost of effort. The combination of $K$ and $e$ is efficient if the ratio of their marginal output equals the ratio of their cost, that is,

$$\frac{M_e}{M_K} = \frac{Kc'(e)}{c(e) + \pi^o},$$

which, by equation (2.28) in equilibrium, is equivalent to

$$\frac{\epsilon_{me}}{\epsilon_{mK}} = \eta. \tag{2.35}$$

Moreover, since $\epsilon_{\lambda_x}$ and $\epsilon_{V^*x}$ are linear combinations of $\epsilon_{m_x}$ and $\epsilon_{\tilde{m}_x}$, for $x \equiv e$ or $K$, and since $\epsilon_{\tilde{m}e}/\epsilon_{\tilde{m}K} = \epsilon_{me}/\epsilon_{mK}$ by an earlier assumption, condition (2.35) also implies that $\epsilon_{\lambda e}/\epsilon_{\lambda K} = \epsilon_{V^*e}/\epsilon_{V^*K} = \eta$. When $\epsilon_{me} > \eta\epsilon_{mK}$, factor $e$ is under-employed relative to factor $K$.

When the combination of $K$ and $e$ is inefficient, adjusting the commission rate alone can only achieve a second-best brokerage employment. Proposition 4 presents the second-best
commission rate.

**Proposition 4** The second-best \( \sigma \) under competitive \( K \) and \( e \).

Given that \( |\Theta| < 1 \) and \( |\eta e e K| < 1 \), the commission rate that maximizes \( B \equiv U - \sigma \tilde{P} \), denoted by \( \sigma^{**} \), exists and satisfies the following equation:

\[
\sigma^{**} = \frac{U}{\tilde{P}} \left[ \epsilon_{V-K} + (\epsilon_{V-e} - \eta e e K) \frac{e e K}{1 + \eta e e K} \right] \equiv \tilde{H}(\sigma^{**}).
\] (2.36)

Furthermore, \( \sigma > (<) \sigma^{**} \) if and only if \( \sigma > (<) \tilde{H}(\sigma) \).

**Proof**: By the first-order condition and equation (2.30) in Lemma 2, where \( d\tilde{P}/\tilde{P} \) is replaced by \( d\tilde{P}/\tilde{P} + d\sigma/\sigma \) and \( d\eta = 0 \), we have

\[
\frac{\partial B}{\partial \sigma} = \frac{U}{\sigma} (\epsilon_{V-K} e K \epsilon + \epsilon_{V-e} e e \sigma) - \left( \tilde{P} + \sigma \frac{\partial \tilde{P}}{\partial \sigma} \right) = \left( \frac{U \epsilon_{V-K} + \epsilon_{V-e} e e K}{\sigma} - \tilde{P} \right) \frac{1 + \epsilon_K K}{\Delta} = 0.
\] (2.37)

When \( K \) is affected by \( \sigma \), \( 1 + \epsilon_K K \neq 0 \) and equation (2.37) gives our desired result (2.36).

Moreover by equation (2.25) and (2.28), equation (2.36) is equivalent to

\[
nV_K^* + nV_e^* \frac{de}{dK} = \frac{n \sigma^{**} \tilde{P}}{K} (1 + \eta e K) = c(e) + \pi^o + K \epsilon'(e) \frac{de}{dK},
\]

or,

\[
nV_K^* - (\pi^o + c(e)) + (nV_e^* - K \epsilon'(e)) \frac{de}{dK} = 0.
\] (2.38)

Equation (2.38) states that the optimal level of \( K \) and \( e \) should be such that the net benefit of an additional \( K \) is just offset by the net benefit of the resulting change in \( e \). With \( V_K^* \) and \( V_e^* \) non-increasing in \( K \) and \( e \) and \( \epsilon'(e) \) increasing in \( e \), it is easy to verify that the left-hand side of equation (2.38) becomes negative (positive) and, hence, \( \sigma > (<) \tilde{H}(\sigma) \) whenever \( \sigma > (<) \sigma^{**} \).

Q.E.D.

It is clear from equation (2.36) that when the combination of \( K \) and \( e \) is efficient, that is \( \epsilon_{V-K} - \eta e e K = 0 \), \( \tilde{H}(\sigma) = H(\sigma) \) and, therefore, Proposition 4 reduces to Proposition 3.
\[ \epsilon_{V,e} > \eta_{e,K} \], or effort \( e \) is under-employed relative to \( K \), however, the optimal commission rate \( \sigma^{**} \) is smaller than \( \sigma^* \) if \( \epsilon_{e,K} \) is negative. This is so because reducing \( \sigma \) reduces \( K \), which would in turn encourage more effort \( e \) and thus improve the efficiency of factor combination.

2.6.3 The efficient commission rate with optimal licensing charges

The competitive levels of \( K \) and \( e \) depend on entry costs to the brokerage business or the opportunity cost of employment — equations (2.25) and (2.28) indicate that a larger \( \pi^0 \) leads to greater \( e \) and smaller \( K \). Thus, real estate license fees, education requirements, and the restriction on secondary employment have the effect of regulating the composition of brokerage employment.

The proponents of real estate licensing regulations argue that they improve the quality of brokerage services; the critics point out that the restricted entry reduces competition and supports high commission rates [see Johnson and Loucks (1986) for example]. Our model suggests that lowering both the opportunity cost of employment in real estate brokerage and the commission rate does not necessarily improve consumer welfare. The equilibrium brokerage employment with both low \( \sigma \) and low \( \pi^0 \) may be associated with substantial under-employment of \( e \), resulting in poor quality of brokerage services. Our model also suggests that regulating both the commission rate and the opportunity cost of employment can achieve first-best brokerage employment.

We use \( l \) to denote a charge or a subsidy per period imposed on real estate agents by regulations, so that the opportunity cost of employment becomes \( \pi^0 + l \). Let us call \( l \) a licensing charge. Proposition 5 presents the first-best commission rate and licensing charge.

**Proposition 5** The first-best commission rate and licensing charge.

Given that \(|\Theta| < 1 \) and \(|\eta_{e,K}| < 1 \), the commission rate and licensing charge, denoted by \( \sigma^{***} \) and \( l^* \) respectively, that jointly maximize \( B \equiv U - K(\pi^0 + c(e))/n = U - \sigma^* + Kl/n \) exist.
and satisfy the following equations:

\[ \sigma^{***} = \frac{U}{\bar{P} \eta} \]  
\[ l^* = \left( \pi^* + l^* + c(e) \right) \left( 1 - \frac{\eta \epsilon V^* K}{\epsilon V^* e} \right) \]

**Proof:** See Appendix 2.b.

Note that the first-best commission rate is proportional to the marginal value of \( e \). Moreover, the first-best licensing charge is proportional to the opportunity cost of employment and the proportion is positive if and only if \( \epsilon V^* e - \eta \epsilon V^* K > 0 \) or, equivalently, \( \epsilon_{mc} - \eta \epsilon_{mK} > 0 \). A positive \( \epsilon_{mc} - \eta \epsilon_{mK} \) means \( e \) is relatively under-employed and, therefore, the opportunity cost of brokerage employment should be increased to discourage \( K \) and encourage more \( e \). Note also that \( \sigma^{***} \) and \( l^* \) changes with \( \epsilon V^* e / \eta \) in the same direction. Thus, a higher marginal value of \( e \), or a lower elasticity of individual output with respect to own effort, which would lead to a lower \( e \), requires the efficient commission rate to be higher and at the same time the efficient licensing charge to be larger to restrict the number of agents.

In the case where the employment composition is efficient, that is \( \epsilon V^* e = \eta \epsilon V^* K \), \( l^* = 0 \) and condition (2.39) is equivalent to condition (2.32) in Proposition 3. It can be verified that equation (2.39) is equivalent to \( nV^* e = n \sigma \bar{P} \eta / e = Kc'(e) \), which states that total marginal benefit of \( e \) equals total marginal cost of \( e \). Moreover, equation (2.40) is equivalent to \( (\pi^* + c(e))/V^*_K - Kc'(e)/V^*_e = 0 \), which states that the ratio of the marginal cost of \( K \) to its marginal value equals the ratio of the marginal cost of \( e \) to its marginal value.

### 2.7 The Implication of Contingent Commission Payment

In the above analysis we assume that the brokerage commission is paid in advance so that it does not affect bargaining outcomes. When the commission is paid by the seller upon transaction, as it is in practice, two things will happen. First, the transaction price \( \bar{P} \) increases with the commission rate. In other words, a part of the commission will be capitalized into the transaction price. To show the impact of contingent commission on price, we provide the
equations for $U^b$, $U^s$, and $\bar{P}$ below:

$$rU^b = \lambda(\bar{Y}^b - \bar{P} - U^b) = \lambda\frac{\bar{V} - \sigma\bar{P} - U}{2},$$

$$rU^s = \lambda((1 - \sigma)\bar{P} - Y^s - U^s) = \lambda\frac{\bar{V} - \sigma\bar{P} - U}{2},$$

which result in

$$U^b = \frac{U}{2},$$

$$U = \frac{\lambda}{r + \lambda}(\bar{V} - \sigma\bar{P}),$$

$$\bar{P} = \frac{1}{1 - 0.5\sigma\sigma}[\bar{V} + Y^s].$$

Since the commission is delayed for the expected duration of search time, the expected present value of the commission, which equals $\sigma\bar{P}/(1 + r/\lambda)$, may change only slightly. With $r = 8\%$ per year, $1/\lambda = 1/4$ year, and $\sigma = 6\%$, for example, the expected present value of the commission increases by only 1%. Thus, the impact of this change in the expected present value of the commission is likely to be minimal.

Second, the reservation match quality $V^*$ increases with $\sigma$, as the transaction gain is now shared among the buyer, the seller, and the agent. To show this effect, we have

$$V^* - \sigma\bar{P} = U,$$

which by equation (2.45) gives,

$$V^* = \frac{\lambda}{r + \lambda}\bar{V} + \frac{r}{r + \lambda}\sigma\bar{P}.$$  

A higher $V^*$ reduces $\lambda$ and increases $\bar{V}$, the net benefit of which depends on $\epsilon_{mN}$ and the efficiency of employment composition. A larger $\epsilon_{mN}$ makes $\lambda$ less sensitive to $V^*$ [see equation (2.13)], so that the net change in $U$ is likely to be positive. Furthermore, the increase in $V^*$ and the corresponding increase in $N$ due to a smaller $\lambda$ would raise $\eta$ and thus $\epsilon$. If $\epsilon$ is relatively under employed, the increase in $\epsilon$ would improve $U$. Thus, with the contingent commission
payment, the efficient commission rate could be higher if $e_{mN}$ is large and $e$ is relatively under employed.

2.8 Conclusions

In this paper we have developed a model that integrates the gains from real estate trading, the matching technology with real estate brokerage, and the behaviour of brokerage employment. Our model can explain the observed behavior of brokerage employment. More importantly, our model can characterize the marginal social value of brokerage employment and the level of fixed commission rates and licensing charges that maximize the net social value of brokerage employment. The characterization helps us to understand the factors that determine the productivity and the marginal social value of brokerage employment and the information we need in order to evaluate the efficiency of commission rates. The model provides a useful framework for analyzing the employment and welfare implications of government regulations in the real estate brokerage industry.

The social value of real estate brokerage may go beyond the matching between buyers and sellers. The information generated and disseminated by the industry may improve the decisions of real estate investors — buyers and sellers — whether or not to enter the market at a particular time. Brokerage services may also reduce transaction costs faced by buyers and sellers in real estate conveyancing.

It is hoped that our model also provides a useful framework for further empirical studies of the real estate brokerage industry. Particularly, we need to know more about the matching function, which is very important for us to understand the efficiency of the industry.
2.a The Effect of Exposure on the Value of Competitive Matches in a Two-sided Matching Market

Consider a two-sided matching market where many buyers and many sellers are seeking best available trade opportunities, or matches. If a buyer, identified by \( i \), is matched with a seller, identified by \( j \), they obtain a potential transaction gain of \( V^{ij} = V^i + V^j = Y^i - Y^j \), where the partition of the gain, \( V^i \) and \( V^j \), will be determined in a competitive equilibrium. Assume buyer \( i \) is exposed to a set of sellers (properties), denoted by \( S^i \), and a seller \( j \), to a set of buyers, denoted by \( B^j \). Buyers and sellers have an option of continuing search which gives them an expected present value of \( U^b \) and \( U^s \) respectively. Buyers and sellers seek matches to maximize \( V^i \) and \( V^j \) respectively. The competitive outcome of the market is a solution to the following linear programming problem [see Roth and Sotomayor (1990)]:

\[
\begin{align*}
\max_{\{x_{ij}\}} & \quad \sum_i \sum_j x_{ij} V^{ij} + \sum_i \tau_i U^b + \sum_j \tau_j U^s \\
\text{st} & \quad \sum_{i \in B^j} x_{ij} + \tau_j = 1, \text{ for all } j, \\
& \quad \sum_{j \in S^i} x_{ij} + \tau_i = 1, \text{ for all } i, \\
& \quad x_{ij}, \tau_i, \tau_j \geq 0,
\end{align*}
\]

where \( x_{ij} = 1 \) indicates buyer \( i \) is matched with seller \( j \), and \( \tau_i = 1 \) or \( \tau_j = 1 \) indicates buyer \( i \) or seller \( j \) is matched with him/herself and thus has the value of continuing search. The dual problem can be written:

\[
\begin{align*}
\min_{\{V^i, V^j\}} & \quad \sum_i V^i + \sum_j V^j \\
\text{st} & \quad V^i + V^j \geq V^{ij} \forall j \in S^i \text{ for all } i, \\
& \quad V^i + V^j \geq V^{ij} \forall i \in B^j \text{ for all } j, \\
& \quad V^i \geq U^b,
\end{align*}
\]
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\[ V^j \geq U^s. \]

By *Le Chatelier Principle*, the optimal value of the dual problem is non-decreasing in the size of \( S^i \)'s and \( B^j \)'s, which measure the market exposure for buyers and sellers. A greater value of the objective function means either that more of the \( x_{ij} \)'s equal 1 (thus a larger \( m \)), or that some of the \( V^ij \)'s are larger (thus a larger \( V \)), or both.

2.b Proof of Lemmas and Propositions

**Proof of Lemma 1**

Since \( K \) and \( e \) are parallel, we only need to derive \( \epsilon_{\lambda K} \) and \( \epsilon_{V^* K} \). Differentiating equations (2.7) and (2.12) we have

\[
\begin{pmatrix}
\epsilon_{mN} & -\epsilon_{mV^*} \\
-r/(r+\lambda) + \epsilon_{\psi N} & 1 - \epsilon_{\psi V^*}
\end{pmatrix}
\begin{pmatrix}
d\lambda/\lambda \\
dV^*/V^*
\end{pmatrix}
= \begin{pmatrix}
\epsilon_{mK} \\
\epsilon_{\psi K}
\end{pmatrix}
\frac{dK}{K}.
\] (2.1b)

The stability of the equilibrium requires \(-1 < \Theta \equiv -(\partial\lambda/\partial V^*)(\partial V^*/\partial \lambda) < 1\). By equations (2.17) and (2.20) we have

\[
\Theta \equiv -\frac{\epsilon_{mV^*} r (1 - \epsilon_{\psi N} + \epsilon_{mN})}{\epsilon_{mN} (r+\lambda)(1 - \epsilon_{\psi V^*})} = \frac{\epsilon_{\psi V^*} (1 - \epsilon_{\psi N} + \epsilon_{mN})}{(1 - \epsilon_{\psi V^*}) \epsilon_{mN}}.
\] (2.2b)

Given that \(|\Theta| < 1\), equation (2.1b) can be solved, with \( \epsilon_{\psi K} \) substituted by equation (2.18), to give the desired result:

\[
\begin{pmatrix}
d\lambda/\lambda \\
dV^*/V^*
\end{pmatrix}
= \frac{1}{1+\Theta}
\begin{pmatrix}
\frac{\epsilon_{mK} - \epsilon_{mK} \epsilon_{\psi V^*}}{r (1 - \epsilon_{\psi V^*}) \epsilon_{mN}} \\
\frac{\epsilon_{mK} (1 - \epsilon_{\psi N} + \epsilon_{mN})}{(1 - \epsilon_{\psi V^*}) \epsilon_{mN}} - \frac{\epsilon_{mK} - \epsilon_{mK} \epsilon_{\psi V^*} \rho - \epsilon_{mK}}{r+\lambda}
\end{pmatrix}
\frac{dK}{K}.
\]

**Proof of Lemma 2**

Let \( f^K, f^e, f^\lambda, \) and \( f^{V^*} \) denote the system of equations:

\[
\begin{align*}
f^K & \equiv (\pi^0 + c(e)) - \frac{\sigma \bar{P} n}{K} = 0, \\
f^e & \equiv c'(e) - \eta \frac{\pi^0 + c(e)}{e} = 0, \\
f^\lambda & \equiv M \bar{N} \lambda, K, e; V^* \equiv n = 0, \\
f^{V^*} & \equiv \frac{\lambda}{r + \lambda} \bar{V} - V^* = 0.
\end{align*}
\]
Differentiating $f^K$, $f^e$, $f^\lambda$, and $f^{V^*}$ we have

\[
\begin{align*}
A \left( \frac{dK/K}{de/e} \right) + B \left( \frac{d\lambda/\lambda}{dV^*/V^*} \right) &= \left( \frac{1}{0} \right) \left( \frac{d\bar{P}/\bar{P} + dn}{n} \right) + \left( 0 \right) \left( \frac{dn}{n} \right), \\
C \left( \frac{dK/K}{de/e} \right) + D \left( \frac{d\lambda/\lambda}{dV^*/V^*} \right) &= \left( \frac{1 - \epsilon_{mN}}{-\epsilon_{V^*N}} \right) \left( \frac{dn}{n} \right),
\end{align*}
\]

where

\[
\begin{align*}
A &= \begin{pmatrix} (\pi^o + c(e))^{-1} & 0 \\ 0 & c'(e)^{-1} \end{pmatrix} \left( \begin{array}{cc} \frac{\partial (f^K, f^e)^T}{\partial (K, e)} & K \\ \frac{\partial (f^K, f^e)^T}{\partial (\lambda, V^*)} & 0 \end{array} \right) \begin{pmatrix} K \\ 0 \end{pmatrix}, \\
B &= \begin{pmatrix} (\pi^o + c(e))^{-1} & 0 \\ 0 & c'(e)^{-1} \end{pmatrix} \left( \begin{array}{cc} \frac{\partial (f^K, f^e)^T}{\partial (K, e)} & \lambda \\ \frac{\partial (f^K, f^e)^T}{\partial (\lambda, V^*)} & 0 \end{array} \right) \begin{pmatrix} 0 \\ V^* \end{pmatrix}, \\
C &= \begin{pmatrix} n^{-1} & 0 \\ 0 & (V^*)^{-1} \end{pmatrix} \left( \begin{array}{cc} \frac{\partial (f^\lambda, f^{V^*})}{\partial (K, e)} & K \\ \frac{\partial (f^\lambda, f^{V^*})}{\partial (\lambda, V^*)} & 0 \end{array} \right) \begin{pmatrix} K \\ 0 \end{pmatrix}, \\
D &= \begin{pmatrix} n^{-1} & 0 \\ 0 & (V^*)^{-1} \end{pmatrix} \left( \begin{array}{cc} \frac{\partial (f^\lambda, f^{V^*})}{\partial (K, e)} & \lambda \\ \frac{\partial (f^\lambda, f^{V^*})}{\partial (\lambda, V^*)} & 0 \end{array} \right) \begin{pmatrix} 0 \\ V^* \end{pmatrix}.
\end{align*}
\]

We define

\[
\begin{pmatrix} \epsilon_{\lambda n|K_e} \\ \epsilon_{V^* n|K_e} \end{pmatrix} = D^{-1} \begin{pmatrix} 1 - \epsilon_{mN} \\ -\epsilon_{V^*N} \end{pmatrix} = \begin{pmatrix} \frac{\epsilon_{V^* n|K_e} (1 - \epsilon_{mN}) (1 - \epsilon_{V^* n|K_e})}{\epsilon_{V^* n|K_e} (1 - \epsilon_{mN}) + \epsilon_{mN}} \\ \frac{\epsilon_{V^* n|K_e} (1 - \epsilon_{mN}) (1 - \epsilon_{V^* n|K_e})}{\epsilon_{V^* n|K_e} (1 - \epsilon_{mN}) + \epsilon_{mN}} \end{pmatrix}.
\]

From (2.b4) we have

\[
\begin{pmatrix} d\lambda/\lambda \\ dV^*/V^* \end{pmatrix} = D^{-1} \begin{pmatrix} 1 - \epsilon_{mN} \\ -\epsilon_{V^*N} \end{pmatrix} \left( \frac{dn}{n} \right) - D^{-1} C \left( \begin{array}{c} dK/K \\ de/e \end{array} \right) = \left( \begin{array}{c} \epsilon_{\lambda n|K_e} \\ \epsilon_{V^* n|K_e} \end{array} \right) \left( \frac{dn}{n} \right) + \left( \begin{array}{cc} \epsilon_{\lambda K} & \epsilon_{\lambda e} \\ \epsilon_{V^* K} & \epsilon_{V^* e} \end{array} \right) \left( \begin{array}{c} dK/K \\ de/e \end{array} \right).
\]

The convergence of $d\lambda$ and $dV^*$ in equation (2.b5) requires $|\Theta| < 1$. Substituting (2.b5) into (2.b3) we obtain

\[
\begin{pmatrix} 1 \\ -\epsilon_{\eta K} + \epsilon_{\eta V^*} \epsilon_{\lambda K} - \epsilon_{\eta V^*} \epsilon_{V^* K} \end{pmatrix} \begin{pmatrix} dK/K \\ de/e \end{pmatrix} = \begin{pmatrix} \eta \\ \delta + \epsilon_{\eta N} \epsilon_{\lambda e} - \epsilon_{\eta V^*} \epsilon_{V^* e} \end{pmatrix} \begin{pmatrix} dK/K \\ de/e \end{pmatrix}.
\]
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\[
\begin{pmatrix}
1 & 0 \\
0 & -\varepsilon_{\varepsilon K}
\end{pmatrix}
\begin{pmatrix}
\frac{dP}{P} + \frac{dn}{n} \\
\frac{dP}{P} + \frac{dn}{n}
\end{pmatrix}
+ \begin{pmatrix}
0 \\
\varepsilon_{\eta N} - \varepsilon_{\eta \varepsilon \lambda n[K]e} + \varepsilon_{\eta V^* \varepsilon V^* n[K]e}
\end{pmatrix}
\frac{dn}{n},
\]

or equivalently,

\[
\begin{pmatrix}
1 & \eta \\
-\varepsilon_{\varepsilon K} & 1
\end{pmatrix}
\begin{pmatrix}
dK/K \\
de/e
\end{pmatrix}
= \begin{pmatrix}
1 \\
0
\end{pmatrix}
\begin{pmatrix}
\frac{dP}{P} + \frac{dn}{n} \\
\frac{dP}{P} + \frac{dn}{n}
\end{pmatrix}
+ \begin{pmatrix}
0 \\
\varepsilon_{\varepsilon n[K]e}
\end{pmatrix}
\frac{dn}{n},
\] (2.6)

which leads to the desired expression in Lemma 2. The convergence of \(dK\) and \(de\) in equation (2.6) requires that \(-1 < \eta \varepsilon_{\varepsilon K} < 1\).

Furthermore, from equation (2.13) and (2.15), the convergence of \(de\) with respect to \(d\lambda\) and \(dV^*\) holding \(K\) constant requires that \(-1 < (-\varepsilon_{\eta N} \varepsilon_{\lambda e} + \varepsilon_{\eta V^* \varepsilon V^* e})/\delta < 1\); the convergence of \(de\) with respect to \(dK\) holding \(\lambda\) and \(V^*\) constant requires that \(-1 < \eta \varepsilon_{\eta K}/\delta < 1\).

**Proof of Proposition 5**

The system of equations for brokerage employment, matching technology, and trading strategy are as follows:

\[
f^K \equiv (\pi^o + l + c(e)) - \frac{\sigma \hat{P} n}{K} = 0,
\]

\[
f^\varepsilon \equiv d(e) - \eta \frac{\pi^o + l + c(e)}{e} = 0,
\]

\[
f^\lambda \equiv M\left(\frac{n}{\lambda}, K, e; V^*\right) = 0,
\]

\[
f^{V^*} \equiv \frac{\lambda}{r + \lambda} \hat{V} - V^* = 0.
\]

The first-order conditions for \(\sigma^{***}\) and \(l^{*}\) are

\[
\frac{\partial B}{\partial \sigma} = \frac{\partial V^*}{\partial \sigma} - \left(\hat{P} + \sigma \frac{\partial \hat{P}}{\partial \sigma}\right) + \left(\frac{l}{n} \frac{\partial K}{\partial n}\right) = 0, \quad (2.7)
\]

\[
\frac{\partial B}{\partial l} = \frac{\partial V^*}{\partial l} - \left(\frac{\sigma}{n} \right) \frac{\partial K}{\partial n} + \left(\frac{l}{n} \frac{\partial K}{\partial l}\right) = 0.
\]

(2.8)

Differentiating \(f^K\) and \(f^\varepsilon\) we get

\[
A \begin{pmatrix}
dK/K \\
de/e
\end{pmatrix} + B \begin{pmatrix}
d\lambda/\lambda \\
dV^*/V^*
\end{pmatrix} = \]
Define the elasticity of \( e \) with respect to \( l \) under a constant \( K \):

\[
\epsilon_{el|K} \equiv \frac{Kl}{\sigma Pn} \delta + \epsilon_{\eta N} \epsilon_{\lambda e} - \epsilon_{\eta V} \epsilon_{V^* e}.
\]

Following the proof of Lemma 2, we have

\[
\left( \frac{dK}{K} \right)_{de/e} = \left( A - BD^{-1}C \right)^{-1} \left[ \left( \frac{d\bar{P}}{\bar{P}} + \frac{d\sigma}{\bar{P} \sigma} - \frac{K dl}{\bar{P} \sigma Pn} \right) + \frac{0}{1} \right] \left( \frac{K dl}{\sigma Pn} \right),
\]

and

\[
dV*/V^* = \frac{\epsilon_{V^* k} + \epsilon_{V^* e} \epsilon_{K} K}{\Delta} \left( \frac{d\bar{P}}{\bar{P}} + \frac{d\sigma}{\bar{P} \sigma} - \frac{K dl}{\bar{P} \sigma Pn} \right) + \frac{\epsilon_{V^* e} \epsilon_{K} K}{\Delta} \frac{dl}{l}.
\]

By (2.b10) and (2.b11), equation (2.b7) reduces to

\[
\frac{U}{\sigma} \left( \epsilon_{V^* K} \epsilon_{K} + \epsilon_{V^* e} \epsilon_{e} \sigma \right) - \left( 1 + \epsilon_{\bar{P} n} \right) \bar{P} n + \frac{Kl}{\sigma n} \epsilon_{K} = \left( \frac{U}{\sigma} \epsilon_{V^* K} + \epsilon_{V^* e} \epsilon_{e} \sigma \right) \bar{P} + \frac{Kl}{\sigma n} \delta \left( 1 + \epsilon_{\bar{P} n} \right) = 0.
\]

By (2.b10), (2.b11), and (2.b12), equation (2.b8) reduces to

\[
\frac{U}{\sigma} \left( \epsilon_{V^* K} \epsilon_{K} + \epsilon_{V^* e} \epsilon_{e} \right) - \left( \epsilon_{\bar{P} l} - \frac{Kl}{\sigma Pn} \right) \bar{P} + \frac{Kl}{\sigma n} \epsilon_{K} = \left( \frac{U}{\sigma} \epsilon_{V^* e} \epsilon_{K} + \frac{Kl}{\sigma n} \right) \epsilon_{e} \epsilon_{e} + \left( \frac{U}{\sigma} \epsilon_{V^* K} + \epsilon_{V^* e} \epsilon_{e} \sigma \right) \bar{P} + \frac{Kl}{\sigma n} \delta = 0.
\]

Substituting (2.b13) into (2.b12) we get

\[
\frac{V^*}{\sigma} \left( \epsilon_{V^* K} + \epsilon_{V^* e} \epsilon_{e} K + \frac{\epsilon_{V^* e} \epsilon_{V^* K}}{\eta} \right) \frac{1}{\Delta} - \bar{P} = 0,
\]

which gives the desired equation (2.39). According to equations (2.39) and (2.25), equation (2.b13) gives the desired equation (2.40).
Figure 2.1: Matching Function: the Impact of Brokerage Employment and Market Size on Quality Distribution
Figure 2.2: Matching Function: the Complementarity between Market Sizes and Brokerage Employment
Chapter 3

Information, Bargaining, and Real Estate Brokerage

3.1 Introduction

Real estate transactions take place in a thin market with imperfect information. In thin market buyers and sellers have a small chance to find multiple trade opportunities at the same time or place. As a result, both sides of a transaction have some bargaining power and the incentive to trade as well as the transaction price depend on the bargaining mechanism. Moreover, the bargaining mechanism is affected by asymmetric information. Particularly, sellers in the real estate market typically cannot directly observe or verify individual buyers' valuation for the property. The purpose of this chapter is to examine bargaining in the context of real estate brokerage, the welfare implications of alternative bargaining mechanisms, and the impact of real estate brokerage on the bargaining mechanism.

Real estate agents not only assist buyers and sellers in their search for trade opportunities, but also act as a medium for price negotiations between buyer and seller. We want to show how real estate brokerage can potentially overcome the information asymmetry problem and why agents have incentive to do so. It is well known that information asymmetry can reduce the set of feasible trades [see Myerson (1979), Myerson and Satterthwaite (1981), and Kennan and Wilson (1993)]. Furthermore, in a search market individual trades can affect the productivity of search of the remaining buyers and sellers [Hosios (1990)]. Therefore, agents' influence on bargaining can affect the welfare of buyers and sellers. The welfare implications need to be considered in the context of an equilibrium model of real estate trading with brokerage.

The analysis has important policy implications. Traditionally, a real estate agent is bound by law to be an exclusive agent of the seller. Such an agency relationship prevents agents from
serving the interests of both the buyer and seller at the same time and, particularly, transmitting private information between the buyer and seller. Moreover, buyers would be unable to trust agents to reveal their true valuation for different properties. However, surveys by Federal Trade Commission (1983) and Ball and Nourse (1988) revealed that buyers and sellers as well as agents themselves thought agents represented buyers also, and commonly participated in the negotiations by conveying private information between the buyer and seller and advising both sides. The tension between the traditional agency relationship and the reality is evident in the on-going legal contention about the proper role of real estate agents [see Currier (1981), Marsh and Zumpano (1988), and Black (1993)]. Alternative models for the fiduciary responsibilities of real estate agents have been proposed. Since fiduciary responsibilities affect agents' role in bargaining, a better understanding of the welfare implications of the agents' role and their incentives to influence bargaining certainly helps to solve the controversy.

Existing economic models of real estate brokerage mostly focus on the search effort of agents or the impact of brokerage on prices [see, for example, Yinger (1981), Jud (1983), Miceli (1991), and Geltner et al (1991)]. Brokerage's impact on bargaining has not received due attention. Bargaining in real estate transactions is examined by Quan and Quigley (1991), focusing on a learning process and the implications for real estate appraisal. Incentive constraints in bargaining and their welfare consequences for real estate trading have not been systematically examined.¹

Our analysis is based on the model of real estate trading with brokerage developed in chapter 2 and the mechanism design methodology [see Myerson (1979) and Myerson and Satterthwaite (1981)]. We find that the presence of information asymmetry in bargaining raises the reservation match quality, which results in a lower expected net trading gain for buyers and sellers when the scale economy of real estate brokerage with respect to the stock of buyers and sellers is not large and the commission rate is high. In such a case the role of real estate agents in overcoming information asymmetry in bargaining benefits society.

¹Rubinstein and Wolinsky (1987) examine the impact of middlemen on the distribution of bargaining power but not on incentive constraints.
Our analysis proceeds as follows. Section 3.2 summarizes the model of chapter 2, which provides the context for evaluating bargaining mechanisms. Section 3.3 examines bargaining under both asymmetric and symmetric information. The welfare implications of alternative bargaining mechanisms are examined in section 3.4. Section 3.5 considers more sources of asymmetric information in bargaining. The potential and the incentives for real estate agents to overcome the information asymmetry problem are analyzed and some policy implications of such potential and incentives are discussed in section 3.6. Section 3.7 concludes.

3.2 The Model of Real Estate Trading With Brokerage

The model of chapter 2 can be summarized in four parts: the market and trading process, the expected trading gain, the matching technology, and brokerage employment.

3.2.1 Market and trading process

We consider a steady-state local real estate market, where buyers and sellers arrive over time, search through real estate brokerage, and leave only after trade. Each buyer seeks one property and each seller has one for sale. A transaction gain is defined as \( V = Y^b - Y^s \), where \( Y^b \) is a stochastic variable representing the buyer’s subjective value for a property and \( Y^s \) is a constant representing the seller’s subjective value. The heterogeneous preferences of buyers and attributes of properties are assumed to be idiosyncratic so that the probability distribution of transaction gain is independent of the identity of the buyer or the property. The trading can be represented as a dynamic search process over discrete time periods. In a given period, each buyer or seller faces a trade opportunity, which is a random draw from the competitive matches produced by the real estate brokerage industry. Each match has a quality measured in terms of the transaction gain \( V \), the distribution of which depends, among other things, on the size of the market and brokerage employment. The trading probability for each buyer or seller in a given period depends on the quality distribution of the matches as well as the bargaining game. Brokerage services are paid for with sales commissions. To focus on the informational influence...
of brokerage on bargaining, however, we assume that the commission is paid in advance by
buyers and sellers so that it is a sunk cost as far as trading is concerned.

Let \( N \) be the number of pairs of buyers and sellers in the market at any time, \( n \) be the
number of pairs of buyers and sellers arriving in the market each period, and \( \lambda \) be the trading
probability. In steady state,

\[
 n = \lambda N. \tag{3.1}
\]

### 3.2.2 Expected trading gain

Let \( \bar{P} \) be the expected transaction price and \( r \) be a discount rate. The expected gains from
trading for buyers and sellers, denoted by \( U^b \) and \( U^s \) respectively, are determined by the
following equations:

\[
 U^b = \frac{\lambda(\bar{Y}^b - \bar{P} - U^b) + U^b}{1 + r}, \tag{3.2}
\]

\[
 U^s = \frac{\lambda(\bar{P} - Y^s - U^s) + U^s}{1 + r}. \tag{3.3}
\]

These two equations state that in steady-state the expected gain at the beginning of a period
must equal the discounted value of the same expected gain at the end of the period plus an
expected value of trading surplus. Define the expected trading gain \( U \) to be the joint expected
trading gain of a pair of buyer and seller, and define \( \bar{V} \equiv \bar{Y}^b - Y^s \) to be the expected transaction
gain of a trade. Combining equations (3.2) and (3.3), we obtain

\[
 U \equiv U^b + U^s = \frac{\lambda}{r + \lambda}(\bar{Y}^b - Y^s) = \frac{\lambda}{r + \lambda}\bar{V}. \tag{3.4}
\]

Thus the expected trading gain equals the expected transaction gain discounted for the expected
search time \( 1/\lambda \) at the rate \( r \).

### 3.2.3 Matching technology

Employment in the real estate brokerage industry is represented by two variables: the number
of agents \( K \) and the search effort of an individual agent \( e \). The matching outcomes from real
estate brokerage can be characterized by an aggregate matching function \( M(N, K, e, v) \), which describes the number of matches with quality no less than \( v \) generated in a given period at factor input levels \( N, K, \) and \( e \). \( M \) increases with \( N, K, \) and \( e \) and decreases with \( v \). \( M/N \) describes the quality distribution of the matches. We assume the hazard rate \(-M_v/M\) increases with \( v \); given that a match has a quality no less than the threshold \( v \), the match is less likely to be of higher quality than the threshold as the threshold increases. This is true as long as the density of the distribution decreases less than exponentially. Furthermore, we assume the quality distribution of the matches improves with \( N, K, \) and \( e \), which means the hazard rate \(-M_v/M\) decreases with \( N, K, \) and \( e \) so that a match becomes more likely to be of a higher quality than \( v \) given its quality is no less than \( v \).

Let \( V^* \) be the reservation match quality, or the minimum value of \( V \) for a trade. The value of \( V^* \) depends on the bargaining game and will be examined in the next section. Then the number of transactions in a period can be defined as \( m \equiv \lambda N \equiv M(N, K, e, V^*) \). Thus, by steady-state condition (3.1), \( \lambda \) is determined by the following implicit function of \( n, K, e, \) and \( V^* \):

\[
n = M\left(\frac{n}{\lambda}, K, e, V^*\right). \quad (3.5)
\]

Define \( \epsilon_{xy} \equiv (\partial x/\partial y)(y/x) \) to be the elasticity of \( x \) with respect to \( y \). From equation (3.5) we have

\[
\epsilon_{mN} \frac{d\lambda}{\lambda} = (\epsilon_{mN} - 1) \frac{dn}{n} + \epsilon_{mK} \frac{dK}{K} + \epsilon_{me} \frac{de}{e} + \epsilon_{mV^*} \frac{dV^*}{V^*}. \quad (3.6)
\]

The expected transaction gain is determined by the following equation:

\[
\bar{V} = \text{Exp}\{V|V \geq V^*\} = V^* + \int_{V^*}^{V^+} \frac{M(N, K, e, V)}{m} dV, \quad (3.7)
\]

where \( V^+ \) is the upper bound of the support for the distribution of \( V \). The derivatives of \( \bar{V} \) are given by

\[
\bar{V}_{V^*} = -\frac{mV^*}{m} \int_{V^*}^{V^+} \frac{M(N, K, e, V)}{m} dV = -\frac{\epsilon_{mV^*}}{V^*} (\bar{V} - V^*), \quad (3.8)
\]

\[
\bar{V}_x = \frac{1}{x} \int_{V^*}^{V^+} \left[ \epsilon_{Mx}(v) - \epsilon_{mx} \right] \frac{M(N, K, e, v)}{m} dv \equiv \frac{1}{x}[\epsilon_{Mx} - \epsilon_{mx}] (\bar{V} - V^*), \quad (3.9)
\]
where \( x \equiv N, K, e \) and \( \epsilon_{Mx}(\hat{v}) \), with \( \hat{v} \in [V^*, Y^b - Y^s] \) according to the generalized integral mean value theorem. It follows from the assumption of increasing hazard rate in \( v \) that \( \tilde{V}_v < 1 \) and from the assumption of decreasing hazard rate in \( x \) that \( \tilde{V}_x > 0 \).

### 3.2.4 Brokerage employment

Brokerage employment is determined competitively based on the following individual earnings function

\[
\pi(e^k) = \sigma \bar{P} m^k - c(e^k),
\]

where \( \sigma \) is the commission rate, \( m^k \) is the output for individual agent \( k \), and \( e^k \) is the effort of agent \( k \). \( m^k \) is an increasing function of own effort \( e^k \) but also depends on \( N, K, e, \) and \( V^* \). In equilibrium, agents are identical so that \( e^k = e \) and \( m^k = m/K = n/K \), and competitive entry and exit result in \( \pi(e) = \pi^0 \). Thus,

\[
K = \frac{\sigma \bar{P} n}{c(e) + \pi^0}.
\]

Furthermore, individual agents choose \( e^k \) competitively to maximize \( \pi(e^k) \), resulting in the first-order condition for \( e \)

\[
c'(e) = \sigma \bar{P} \frac{\partial m^k}{\partial e^k} = \frac{\sigma \bar{P} n}{K e} \eta = \eta \frac{\pi^0 + c(e)}{e},
\]

where \( \eta \equiv \epsilon_{m^k e^k} \) is the elasticity of individual output with respect to individual effort.

### 3.3 Bargaining

An important variable in the trading process described in section 3.2 is the reservation match quality \( V^* \), which affects both the trading probability \( \lambda \) and the expected transaction gain of a trade \( \tilde{V} \) and, consequently, affects the expected trading gain \( U \). How \( V^* \) is determined, however, depends on the bargaining game, and the mechanism of the game depends on the information structure faced by the players in the game. There can be many information problems in a real estate transaction. In this section we restrict our attention to just one particular information problem — the verification of the buyer’s valuation for the property \( Y^b \). This is the most
important information asymmetry problem in our model of trading with brokerage. It would be very difficult for a seller to verify a particular buyer's \( Y^b \). Moreover, since buyers with a higher value of \( Y^b \) have no incentive to tell the truth to the seller, direct communication of \( Y^b \) from the buyer to the seller is not credible. Thus, without a third party providing verification of \( Y^b \), information in the bargaining game is asymmetric and the bargaining outcome is affected by the incentive constraint of the buyer [Myerson and Satterthwaite (1983)].

The objective of this section is to characterize the bargaining outcomes with and without the information asymmetry. The outcomes are evaluated in the next section in the context of trading with brokerage. In section 3.5 we consider other sources of asymmetric information.

### 3.3.1 Bargaining under asymmetric information

Consider the following bargaining problem. In a given period, a buyer meets a seller. The buyer observes his private valuation for the property \( Y^b \). Since the match is randomly drawn from the matches produced by the brokerage, the distribution of \( V = Y^b - Y^s \) is determined by the matching function \( M(N, K, e, v) \). \( Y^s \) is a constant and is assumed to be common knowledge. We further assume that \( M, U^b, \) and \( U^s \) are common knowledge for the buyer and seller are taken as given. This assumption may be justified on the ground that \( M, U^b, \) and \( U^s \) can be observed or inferred from the trading process by real estate agents, and can be learned by buyers and sellers through consultation with agents. \( U^b \) and \( U^s \) are not fixed, however; they adjust according to buyers’ and sellers’ strategies and converge to their equilibrium values. The bargaining problem is to determine whether the transaction takes place and what the transaction price is, given that \( V \) is the private knowledge of the buyer and \( M, U^b, \) and \( U^s \) are common knowledge.

It is well known that any game with incomplete information can be recast as a direct revelation game subject to incentive-compatibility and individual rationality constraints [see Myerson (1979) and Harris and Townsend (1985)]. In a bargaining situation, the direct revelation game is a direct bargaining mechanism, in which the players simultaneously report their private values and the bargaining outcomes are determined based on the reported values. Following Myerson
and Satterthwaite (1983) and Samuelson (1984), the bargaining mechanism for our problem can be characterized by two functions: $s(V)$, denoting the probability of transaction given the buyer's report of $V$, and $t(V) \equiv s(V)P(V)$, denoting the expected transfer payment. The expected bargaining gain for the buyer is given by

$$w^b(V) = s(V)(V + Y^* - U^b) - t(V). \quad (3.13)$$

The individual rationality requires

$$w^b(V) \geq 0, \quad (3.14)$$

and the incentive compatibility requires

$$w^b(V) \geq s(V')(V + Y^* - U^b) - t(V'), \quad (3.15)$$

for all $V' \neq V$, so that the truth telling is the dominant strategy for the buyer. Applying condition (3.15) when the buyer observes $V$ but considers reporting $V'$ and vice versa, we obtain the following inequality:

$$s(V')(V - V') \leq w^b(V) - w^b(V') \leq s(V)(V - V'),$$

which implies, first,

$$\frac{ds(V)}{dV} \geq 0 \quad (3.16)$$

and, second,

$$\frac{dw^b(V)}{dV} = s(V). \quad (3.17)$$

Since there are $N$ pairs of buyers and sellers in the market, the probability for the buyer to observe a transaction gain of at least $V$ is $M(V)/N$ (for notational simplicity we will suppress the arguments $N$, $K$, and $e$ in function $M$ when they are held constant). Taking into account the constraint (3.17) and that $M(V_-) \equiv N$, where $V_-$ is the lower bound of the support for the distribution of $V$, we obtain the expected value of $w^b(V)$:

$$w^b \equiv \int_{V_-}^{V_+} w^b(V)d \left[ -\frac{M(V)}{N} \right] = \int_{V_-}^{V_+} \frac{M(V)}{N} s(V)dV + w^b(V_-). \quad (3.18)$$
The expected bargaining gain, or trade surplus, for the pair of buyer and seller is given by

\[ w^s + w^b \equiv \int_{V_-}^{V_+} (V - U) s(V) d \left[ -\frac{M_v(V)}{N} \right]. \tag{3.19} \]

Since the seller cannot verify \( V \), the best the seller can do is to maximize \( w^s \) subject to the constraints (3.14), (3.16) and (3.17), where \( w^s \) is given by

\[ w^s = \int_{V_-}^{V_+} (V - U) s(V) d \left[ -\frac{M(V)}{N} \right] - w^b = \int_{V_-}^{V_+} \left( V - U + \frac{M(V)}{M_v(V)} \right) s(V) d \left[ -\frac{M_v(V)}{N} \right] - w^b(V_-). \tag{3.20} \]

Define \( H(V) \equiv V - U + \frac{M(V)}{M_v(V)} \), which is increasing in \( V \) given \( U \) by the assumption of the increasing hazard rate in \( v \). Equation (3.20) reveals that the integral, and hence \( w^s \), is maximized if \( s(V) \) is a single step function defined by\(^2\)

\[ s(V) = \begin{cases} 1 & \text{if } V \geq V^*_a \\ 0 & \text{otherwise} \end{cases} \tag{3.21} \]

where \( V^*_a \) is determined by the following equation:

\[ H(V^*_a) = V^*_a - U + \frac{M(V^*_a)}{M_v(V^*_a)} = 0. \tag{3.22} \]

Clearly, the function defined by (3.21) satisfies the constraint (3.16). Moreover, differentiating equation (3.13) and taking into account equation (3.17), we have \( s'(Y)(Y - U) - t'(Y) = 0 \). Since \( s(Y) = 1 \) for \( V \geq V^*_a \), \( t(V) \) must be constant for \( V \geq V^*_a \). To maximize \( w^s \) the sellers should choose \( t(V) \) to minimize \( w^b \). Since \( w^b(V) \) is increasing when \( s(V) > 0 \) according to equation (3.17), \( w^b \) is minimized when the constraint (3.14) is binding at \( V = V^*_a \). Therefore, letting \( w^b(V^*_a) = 0 \) and using equation (3.22) we have

\[ \bar{P} = t(V^*_a) = V^*_a + Y^s - U^b = U^s + Y^s - \frac{M(V^*_a)}{M_v(V^*_a)}. \tag{3.23} \]

Thus we have shown that in the case where the seller cannot verify the valuation of the buyer, the bargaining mechanism that results from the sellers’ strategic behavior is a take-it-or-leave-it mechanism. In other words, the sellers would commit to a price \( \bar{P} = t(V^*_a) \) and trade

\(^2\)A formal proof can be found in Samuelson (1984).
with any buyer willing to take the price. \( V^*_a \) is the reservation match quality resulting from bargaining with asymmetric information.

The take-it-or-leave-it mechanism can be implemented as the Nash equilibrium of a two-stage game in the search context. In the first stage, individual sellers set the price, taking the prices of the other sellers and the buyers' opportunity cost of transaction as given. In the second stage, individual buyers choose whether or not to take the price in a given match. We start with the second stage.

Suppose every seller in the market has chosen a price \( \tilde{P} \). A buyer's expected trading gain (or the opportunity cost of transaction) \( U^b \) is determined by equation (3.2), or equivalently by

\[
rU^b = \lambda(\tilde{V} + Y^s - \tilde{P} - U^b),
\]

where \( \lambda = M(V^*)/N \). The buyer will choose \( V^* \) to maximize \( U^b \) given \( Y^s, \tilde{P}, N, K, \) and \( e \). This results in

\[
V^* = \tilde{P} + U^b - Y^s. \tag{3.25}
\]

Equation (3.24) and (3.25) together determine \( V^* \) and \( U^b \) given \( \tilde{P} \). A buyer will agree to trade at price \( \tilde{P} \) if the buyer values the property at least as much as \( Y^b = V^* + Y^s = \tilde{P} + U^b \).

Now we consider the first stage. Suppose a seller is to set a price \( P \) to maximize his expected trading gain \( U^s(P) \), taking the price set by the other sellers \( \tilde{P} \) and the buyers' expected trading gain \( U^b \) as given. The trading probability for the seller is \( \lambda = M(V(P))/N \), where \( V(P) \equiv P + U^b - Y^s. U^s(P) \) is determined by equation (3.3), or equivalently by

\[
rU^s(P) = \frac{M(V(P))}{N}(P - Y^s - U^s(P)). \tag{3.26}
\]

Maximizing \( U^s(P) \) with respect to \( P \), we get \( (P - Y^s - U^s(P))M + M(V(P)) = 0 \), or equivalently,

\[
P = Y^s + U^s + \frac{M(V(P))}{M(V(P))}. \tag{3.27}
\]

\(^3\)The first-order condition is: \( (\tilde{V} + Y^s - \tilde{P} - U^b)M + M\tilde{V} = 0 \), where \( \tilde{V} = -(\tilde{V} - V^*)M/\tilde{M}\) according to equation (3.7).
Equation (3.26) and (3.27) together determine $U^s$ and $P$, given $U^b$. In equilibrium $P = \bar{P}$ and $V(\bar{P}) = V^*$. Therefore, we obtain

$$V^* = U - \frac{M(V^*)}{M(N(V^*)}. \tag{3.28}$$

Equations (3.27) and (3.28) are the same as equations (3.23) and (3.22) respectively.

3.3.2 Bargaining under symmetric information

Now we consider the bargaining problem without asymmetric information. In this case, the expected bargaining gain for the buyer and seller given by equation (3.19) can be maximized with the mechanism that requires

$$s(V) = \begin{cases} 1 & \text{if } V \geq U \\ 0 & \text{otherwise} \end{cases} \tag{3.29}$$

and

$$t(V) = \begin{cases} Y^s + U^s + (V - U)/2 & \text{if } V \geq U \\ 0 & \text{otherwise} \end{cases} \tag{3.30}$$

There is no incentive-compatibility constraint associated with the revelation of $V$. Such a mechanism can be implemented as a Nash (1950) bargaining game or a Rubinstein (1982) bargaining game.

Therefore, the reservation match equality resulting from bargaining with symmetric information is given by

$$V^*_e = U, \tag{3.31}$$

and the expected transaction price is given by

$$\bar{P} = Y^s + U^s + \frac{\bar{V} - U}{2}. \tag{3.32}$$

Indeed, the trading strategy $V^*_e = U$ maximizes the expected trading gain $U$ given $N$ and brokerage employment.
3.4 Welfare Implications of Alternative Bargaining Mechanisms

Different bargaining mechanisms result in different reservation match qualities. As $M(v)/M_\nu(v) < 0$, the reservation match quality resulting from bargaining with asymmetric information is greater than that from bargaining with symmetric information. The impact of the bargaining mechanism on the welfare of buyers and sellers depends on how it affects the expected trading gain $U$ and brokerage employment. We analyze the two effects separately: first consider the effect of $V^*$ on $U$ holding brokerage employment fixed and then consider the variation in expected transaction price and brokerage employment.

3.4.1 The effect on the expected trading gain

We examine the effect of bargaining mechanism on the expected trading gain $U$ by comparing $V_a^*$ and $V_s^*$ with the reservation match quality $V_o^*$ that maximizes $U$. Differentiating $U$ with respect to $V^*$ in equation (3.4), we get

$$\frac{dU}{dV^*} = \frac{rV}{(r+\lambda)^2} + \frac{\lambda}{r+\lambda} \left( \bar{V}V^* - \bar{V}N \frac{N}{\lambda} \right).$$

(3.33)

An increase in $V^*$ has two effects on $U$: it reduces $\lambda$ and increases $\bar{V}$. By equations (3.6), (3.8), and (3.9) and with $K$ and $e$ constant, equation (3.33) becomes

$$\frac{V^*}{U} \frac{dU}{dV^*} = \frac{e_{mV^*}e_{mN}}{V_{mN}} \left( V^* - U + \frac{r\bar{V}}{r+\lambda} \frac{1 - \epsilon_{mN}}{\epsilon_{mN}} \right).$$

(3.34)

Note that $e_{mV^*} < 0$. Thus equation (3.34) reveals that $U$ is maximized when

$$V_o^* = U + \frac{r\bar{V}}{r+\lambda} \frac{\epsilon_{mN} - 1}{\epsilon_{mN}}.$$  

(3.35)

Condition (3.35) defines the optimal reservation match quality.

The behavior of $U$, $Y_o^*$, $Y_a^*$, and $Y_s^*$, are illustrated in figures 3.3 through 3.5. A number of results follow from the comparisons. First, $V_o^*$ is greater, equal, or less than $U$ if $\epsilon_{mN}$ is greater, equal, or less than 1. When $\epsilon_{mN} > 1$, the matching technology exhibits increasing returns to scale with respect to $N$; individual trades bestow a positive externality on the search
productivity of other buyers and sellers. Thus, the optimal reservation match quality, which takes into account the social cost of individual trades, is greater than $U$, which represents only the private opportunity cost of trade. Since $V_s^* = U$, it is optimal only if $\epsilon_{\tilde{m},N} = 1$. Furthermore, $V_s^*$ is less or greater than $V_o^*$ if $\epsilon_{\tilde{m},N}$ is greater or less than one. This first result is consistent with the general result concerning the efficiency of exit choices in search models [see Hosios (1990)].

Second, if $\epsilon_{\tilde{m},N} \leq 1$ then $V_s^*$ always results in a greater $U$ than $V_o^*$ does. In other words, without increasing returns to scale with respect to $N$ in the matching technology, the expected trading gain is always greater when there is less information asymmetry in bargaining.

Third, by equation (3.4) and (3.8), we can rewrite equation (3.22) as

$$V_o^* = U + \frac{r V}{r + \lambda \frac{1}{1 + V_{V^*}}}.$$  

Comparing equations (3.36) and (3.35) indicates that $V_o^*$ can be closer to $V_o^*$ than $V_s^*$ is if $\epsilon_{\tilde{m},N}$ is close to $1 + 1/V_{V^*} \geq 2$. However, $V_{V^*}$ can be small when the density of the quality distribution to the right of $V^*$ is flat or increasing. With uniform distribution, for example, $V_{V^*} = 1/2$. Thus, $V_s^*$ can still be more efficient relative to $V_o^*$ even if $\epsilon_{\tilde{m},N} > 1$.

### 3.4.2 The effect on brokerage employment

Reservation match quality affects brokerage employment in two ways: it affects expected transaction $\bar{P}$ and the stock of buyers and sellers $N = n/\lambda$, which affect the number of agents $K$, and individual effort $e$, according to equations (3.11) and (3.12). The trade-off between the cost and the benefit of the variation in brokerage employment can be evaluated in terms of the net expected trading gain $B \equiv U - \sigma \bar{P}$.

Let $\bar{P}_o$ and $\bar{P}_s$, respectively, be the expected transaction prices associated with bargaining with asymmetric and symmetric information. Substituting for $U^*$ with equation (3.3) and for $V_o^*$ with equation (3.36), we can rewrite equations (3.27) and (3.32) as follows:

$$\bar{P}_o = Y^* + \frac{\bar{V}}{1 + V_{V^*}}.$$  

$$\bar{P}_s = Y^* + \frac{\bar{V}}{1 + V_{V^*}}.$$
\[ \bar{P}_{s} = Y + \frac{\bar{V}}{2}. \] (3.38)

Clearly, \( \bar{P}_{a} > \bar{P}_{s} \) because \( 1 + V_{t} < 2 \) and \( \bar{V} \) increases with \( V^{*} \). A higher expected transaction price results in greater brokerage employment and higher commission cost. Whether the net gain \( B \) is higher with \( \bar{P}_{a} \) or \( \bar{P}_{s} \) depends on \( \sigma \). If the commission rate \( \sigma \) leads to efficient or excessive employment at the expected price \( \bar{P}_{s} \), that is \( \partial B / \partial (\sigma \bar{P}) \leq 0 \) at \( \bar{P}_{s} \), then an increase from \( \bar{P}_{s} \) to \( \bar{P}_{a} \) reduces \( B \); otherwise, \( B \) can be higher with \( \bar{P}_{a} \). Moreover, greater brokerage employment reduces the hazard rate \( -m_{V} \omega / m \), as the quality distribution of the matches improves. Consequently, \( V_{a}^{*} - U = -m / m_{V}^{*} \) increases, which further increases \( \bar{P} \).

Thus, taking into account the change in brokerage employment reinforces the difference between \( V_{a}^{*} \) and \( V_{s}^{*} \) and that between the associated expected trading gains. In summary, if the scale economy of brokerage with respect to \( N \) is not large and the commission rate is high, the bargaining mechanism with asymmetric information results in a lower net expected trading gain \( B \) than the bargaining mechanism without asymmetric information.

The composition of brokerage employment can vary with reservation match quality as \( N \) and \( \bar{P} \) are affected. As chapter 2 shows, a higher \( \bar{P} \) encourages greater \( K \) and discourages \( e \), whereas a higher \( V^{*} \) and \( N \) increase \( \eta \) which encourages \( e \) and discourages \( K \). The difference in the composition of brokerage employment associated with \( V_{a}^{*} \) and \( V_{s}^{*} \) cannot be determined here without more detailed assumptions about the cost function \( c(e) \) and the individual output function \( m^{e} \). When the composition differs, the change in the composition enhances the productivity of brokerage employment if the combination of \( K \) and \( e \) becomes more efficient in the sense that the marginal value of \( K \) over the marginal cost of \( K \) becomes closer to the marginal value of \( e \) over the marginal cost of \( e \).

### 3.5 Other Sources of Asymmetric Information

Buyers may also face asymmetric information when, for example, the subjective value \( Y^{s} \) of sellers is uncertain. The variation in \( Y^{s} \), however, has only small influence on bargaining. This
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is so because the opportunity cost of trade for buyers, $U^b$, is determined by the expected value $\bar{Y}^s$ and the $Y^s$ of a particular seller can affect the price by only the opportunity cost of $Y^s - \bar{Y}^s$. Consequently, the variation in sellers’ threat point $U^s(Y^s) + Y^s$ would be much smaller than that in $Y^s$.

A more important source of uncertainty other than that associated with transaction gain $V$ is the uncertainty in discount rates. Each buyer or seller may have different opportunity costs of search time, resulting in different expected trading gains or opportunity costs of trade. As far as bargaining is concerned, the uncertainty in $U^b$ and $U^s$ has the same effect as the uncertainty in $Y^b$ and $Y^s$, resulting in an uncertain bargaining gain $w = \bar{Y}^b - \bar{Y}^s$, where $\bar{Y}^b \equiv Y^b - U^b$ and $\bar{Y}^s \equiv Y^s + U^s$.

Bargaining with two-sided asymmetric information is studied by Myerson and Satterthwaite (1981). In general, the bargaining mechanism that maximizes the expected bargaining gain and satisfies individual rationality and incentive compatibility when both $\bar{Y}^b$ and $\bar{Y}^s$ are private knowledge has the trading probability function $s(\bar{Y}^b, \bar{Y}^s)$ defined by

$$s(\bar{Y}^b, \bar{Y}^s) = \begin{cases} 1 & \text{if } \frac{\bar{Y}^b - \alpha}{1+\alpha} \cdot \frac{1-F(\bar{Y}^b)}{f(\bar{Y}^b)} \geq \bar{Y}^s + \frac{\alpha}{1+\alpha} \cdot \frac{G(\bar{Y}^s)}{g(\bar{Y}^s)} \\ 0 & \text{otherwise} \end{cases}$$

for $\bar{Y}^b \in [\bar{Y}^b_-, \bar{Y}^b_+]$ and $\bar{Y}^s \in [\bar{Y}^s_-, \bar{Y}^s_+]$, where $F$ and $G$ are respectively the distribution functions of $\bar{Y}^b$ and $\bar{Y}^s$, $f$ and $g$ are the density functions, and $\bar{Y}^b_-, \bar{Y}^b_+, \bar{Y}^s_-$, and $\bar{Y}^s_+$ are the boundaries of the supports. $\alpha$ is a positive parameter whose value reflects the sensitivity of the expected value of bargaining gain $w$ to an easing in incentive constraints, that is a decrease in $1 - F(\bar{Y}^b)/f(\bar{Y}^b)$ or $G(\bar{Y}^s)/g(\bar{Y}^s)$. Equation (3.21) is a special case where $U^b$, $U^s$, and $Y^s$ are constants and known.

The comparison between equations (3.21) and (3.39) reveals that the uncertainty in $\bar{Y}^s$ has the equivalent effect of an increase in $V^*$ in that it further reduces the set of feasible trades. Thus analysis in the previous section applies when more uncertainty and information asymmetry

---

4 It can be shown that with symmetric Nash bargaining the expected transaction price for a seller with subjective value $Y^s$ is $\bar{P}(Y^s) = (\bar{Y}^s + \bar{Y}^*)/2 + r(Y^s - \bar{Y}^*)/(2r + \lambda)$ and the expected trading gain for the seller is $U^s(Y^s) = \lambda(\bar{Y}^s - \bar{Y}^s)/(2r + \lambda) - \lambda(Y^s - \bar{Y}^s)/(2r + \lambda)$.
associated with $\hat{Y}^b$ and $\hat{Y}^s$ are considered. The presence of more uncertainty and asymmetric information only strengthens our results about the impact of information asymmetry on bargaining outcomes and the net expected trading gain.

3.6 The Impact of Real Estate Brokerage on Bargaining

Real estate agents not only match buyers and sellers so as to facilitate their search, but also act as a medium for price negotiations between buyers and sellers so as to facilitate transactions. The FTC (1984) survey revealed that over 80% of both buyers and sellers agreed that the agents involved in their transactions played a major role in the negotiation. Real estate agents commonly seek compromises from both buyers and sellers to bring about trades. The agents surveyed by Ball and Nourse (1988) perceived that their role is best reflected by the following statement:

Having the best knowledge of what the property is worth, the selling agent promotes a sale at what the agent considers to be fair in price and terms so that both the buyer and seller end up with a good deal.

In this section, we examine the potential and incentives for agents to influence bargaining. We want to show that (i) real estate brokerage changes the content of the revelation game and thus the incentives for buyers and sellers to reveal their private information, and (ii) agents have incentives to promote the Nash bargaining outcome by seeking compromises from both the buyer and the seller. We will also discuss the policy implications of our view of the impact of real estate brokerage on bargaining.

3.6.1 Revelation of private information with brokerage

With real estate brokerage, buyers and sellers play a different revelation game. The payoffs of the game are no longer the expected gains from a single bargaining event, but are the expected gains from the entire search event. The incentives to reveal private information, such as the
preferences of a buyer and the opportunity cost of delaying for a seller, depend on both the effect of the information on bargaining outcomes and the effect on matching quality.

Consider the case where the buyers' valuation $Y^b$ is the only private information. The revelation game with respect to agent-assisted search involves buyers' report of their preference set. Let $\mathcal{Y}^{bi}$ be the preference set of buyer $i$, $\hat{\mathcal{Y}}^{bi}$ be the reported preference set, and $\rho \equiv ||\mathcal{Y}^{bi} - \hat{\mathcal{Y}}^{bi}||$ be a measure of the distance between the two sets (such as the average distance between the corresponding elements in the two sets). We assume that either the trading probability $\lambda$ or the expected match quality $\tilde{V}$ or both for the buyer is decreasing in $\rho$, as the distorted preferences reduce the effectiveness of agents' matching effort. The expected trading gain for the buyer, given the reporting bias $\rho$, is

$$U^b(\rho) = \frac{\lambda(\rho)}{r + \lambda(\rho)} (\tilde{V}(\rho) + Y^* - \hat{P}(\rho)),$$

(3.40)

where $\hat{P}(\rho)$ depends on the bargaining mechanism. With the take-it-or-leave-it bargaining mechanism, $\hat{P}$ is not affected by $\rho$; therefore, $U^b(\rho)$ is decreasing in $\rho$. In this case, truthful reporting of the preference set is the dominant strategy. With the Nash bargaining mechanism, under-reporting of the valuations may reduce $\hat{P}$ for the buyer. The sensitivity of $\hat{P}$ to the under-valuation depends on how the reported bargaining gain is divided between the seller and the buyer. Thus, if $\lambda$ and $\tilde{V}$ are relatively more sensitive to the under-valuation than $\hat{P}$ is, truth revealing is still the dominant strategy for the buyer. To the extent that agents can influence the division of bargaining gain and are trusted by buyers, agents can induce the truthful revelation from buyers.

The same thing can be said when there is private information on both the buyers' side and the sellers' side. With a proper division of bargaining gain, agents can potentially induce the truthful revelation from both buyers and sellers.

### 3.6.2 The incentives of agents to influence bargaining

We have seen that if agents either commit to keeping the private information, so that the reports of buyers and sellers do not affect the transaction price, or commit to seeking compromises from
both sides, so that the under-valuation by buyers and the over-valuation by sellers become less valuable in bargaining, truthful revelation can be induced. And agents want to seek truthful revelation since misinformation reduces the effectiveness of their matching effort. Then, which commitment would agents choose to make?

Since the Nash bargaining mechanism minimizes the reservation match quality and thus maximizing the trading probability, the dominant choice for agents would be to seek compromises so as to promote the Nash bargaining mechanism. To see this, we can rewrite the expected profit of an agent $k$ as follows:

$$
\pi^k(V^*) = \sigma \bar{P}(V^*)N^k(U(V^*)) \frac{\lambda^k(V^*)}{\lambda + \lambda^k(V^*)} - c(e^k),
$$

(3.41)

where $V^*$ is the reservation match quality achieved by agent $k$, $\bar{P}$ increases with $V^*$, $\lambda^k$ is the trading probability due to agent $k$ and decreases with $V^*$, $\lambda$ is the trading probability due to other agents, and $N^k$ is the stock of buyers and sellers assisted by agent $k$. $N^k$ increases with expected trading gain $U$ and otherwise is determined by the available time and energy of the agent. Since the Nash bargaining mechanism maximizes the expected trading gain for individual buyers and sellers, given the bargaining mechanism of all other buyers and sellers, $U$ and hence $N^k$ decrease with $V^*$. Thus $\pi^k(V^*)$ decreases with $V^*$ as long as the elasticity of $\bar{P}$ with respect to $V^*$ is smaller than that of $N^k \lambda^k$.

Of course, the ability for an agent to promote the Nash bargaining mechanism depends on the agent’s ability to seek compromises from both the buyer and the seller. The competence and trustworthiness of the agent as well as the agent’s effort are important factors for the agent’s success in seeking compromises.

The existing evidence supports our view that real estate agents induce truthful revelation from buyers and sellers and commit to seeking compromises. The FTC (1983) survey reveals that the majority of buyers and sellers indicated they told the agent their reservation price and usually learned from the agent the confidential information of the other side. The survey reported in Ball and Nourse (1988) confirms the FTC finding. The agents’ self-reflection and the opinion of the buyers and sellers cited at the beginning of this section provide further
3.6.3 Policy implications

The above analysis indicates that with real estate brokerage the information asymmetry problem associated with bargaining can be overcome and the Nash bargaining mechanism can be achieved. Based on the analysis in section 3.4, when the increasing returns to scale with respect to \( N \) in the matching technology is not strong and the commission rate is high, the Nash bargaining mechanism leads to greater expected trading gains than does the bargaining mechanism subject to information asymmetry. In this sense, both the potential and the incentives for agents to overcome asymmetric information and seek compromises should be promoted.

This brings us to the controversy about the fiduciary duties of real estate agents. Conventionally, the agent is a representative of the seller. This creates the confusion about the role of real estate agents in real estate transactions among buyers, sellers, and agents. Alternative representation models such as buyer agency and dual agency have received much attention in the industry [see Currier (1981) and Black (1993)]. The fiduciary duties of real estate agents affect both the potential and the incentives for agents to seek compromises and promote the Nash bargaining mechanism. We submit that alternative representation models need to be evaluated in terms of their compatibility with welfare enhancement and with the profit incentives of agents. The seller-agent model obstructs both the potential and the incentives for agents to promote the Nash bargaining mechanism. The incompatibility of this representation model with the incentives of agents contributes to the confusion regarding the role of agents. To the extent that the increasing returns to scale with respect to the stock of buyers and sellers is not strong and brokerage employment is not inadequate, the seller-agent model also discourages a socially beneficial function of real estate brokerage in facilitating the bargaining.

\(^5\)In the sense that the marginal social value of brokerage employment is no less than its marginal social cost.
3.7 Conclusions

In this chapter the role of real estate agents in bargaining has been examined. Our analysis starts with identifying the potential information asymmetry problems related to bargaining in real estate trading. The impact of asymmetric information on the bargaining mechanism and on the welfare of buyers and sellers are analyzed in a model of real estate trading with brokerage. The presence of asymmetric information raises the reservation match quality. In the absence of strong increasing returns to scale in real estate brokerage with respect to the stock of buyers and sellers, and when the commission rate is sufficiently high, the higher reservation match quality results in lower expected net trading gains for buyers and sellers. We have shown that with a proper bargaining mechanism real estate agents can induce the truthful revelation from buyers and sellers, and thus are able to overcome the information asymmetry problem in the bargaining. Furthermore, the profit-maximization incentive motivates agents to seek compromises between buyers and sellers. Because the role played by real estate agents in bargaining affects the bargaining mechanism and, consequently, affects the welfare of buyers and sellers, evaluating alternative representation models of real estate brokerage should take into account the incentives and potential for real estate agents to facilitating bargaining.
Figure 3.3: Reservation Match Quality with $\epsilon_{n_1,n} > 1$
Figure 3.4: Reservation Match Quality with $\epsilon_{m,V} = 1$
Figure 3.5: Reservation Match Quality with $\epsilon_{m,V} < 1$
Chapter 4

Uncertainty, Housing Investment, and Housing Consumption — A Certainty-Equivalent Approach

4.1 Introduction

Economic analysis of housing tenure choice follows two basic approaches. The first one examines tenure choice in an intertemporal choice framework and emphasizes down payment and liquidity constraints [Artle and Varaiya (1978) and Brueckner (1986)]. This approach reveals that a liquidity-constrained household must sacrifice consumption in the early years of life in order to enjoy the benefit of homeownership in later years. The second approach examines the relative costs of owning versus renting using the concept of the user cost of capital [Mills (1990) and Rosen, Rosen, and Holtz-Eakin (1984)]. This approach highlights the impact of the capital gains and risks associated with housing investment on tenure choice. Both approaches are incomplete in that the liquidity constraint and portfolio considerations are not integrated in the analysis [Smith, Rosen, and Fallis (1988)]. Moreover, both treat housing consumption as exogenous.

Henderson and Ioannides (HI) (1983) incorporate risky housing investment into an intertemporal tenure choice model and treat housing consumption as endogenous; however, the model is very complicated to solve even in the absence of a liquidity constraint. Moreover, although the solution provided in Fu (1991) clarifies the results of the HI model, the impact of the liquidity constraint on the household’s choice behavior is not systematically studied.

The purpose of this chapter is to examine tenure choice under both uncertainty and the liquidity constraint. The HI model is extended by adding the liquidity constraint and a short
Chapter 4. Housing Investment and Consumption

selling restriction on housing investment. The model is solved using a certainty-equivalent approach, which substantially simplifies the analysis and allows a diagrammatic illustration of the important trade-offs induced by the uncertainty and liquidity constraint. The effects of the income path and wealth on housing investment and consumption demands are examined, as are the effects of mortgage loan availability, expected housing price appreciation, and future housing price uncertainty. The impact of income taxes on tenure choice is also examined.

Unlike other tenure choice models, which focus directly on the discrete choice between owning and renting, the HI model examines the demands for housing investment and consumption as separate but continuous choices. The model does not seek to determine whether a household is better off owning or renting, which depends on the factors that are beyond the specification of the model, such as the completeness of the rental housing market [Bossons (1978)] and the utility that the household may derive from owning its home per se. The model determines, instead, the amount of housing assets the household would be willing to own if the household were allowed to own any portion of the house it is going to occupy. The usefulness of this approach is two fold. First, it generates comparative statics that apply to housing tenure choice, in that an increase in the demand for housing investment relative to the demand for housing consumption, other things being equal, increases the likelihood of owning. Second, it allows us to examine housing demand along with tenure choice, which will also provide a theoretical basis for the simultaneous estimation of housing demand and tenure choice found in many of the recent empirical housing studies [Goodman (1988) and Horioka (1988)].

The chapter proceeds as follows. Section 4.2 describes the model, followed by the discussion of the certainty-equivalent approach in section 4.3. The behavior of housing investment and consumption and its implications for tenure choice are examined in section 4.4. Section 4.2a' considers the impact of income taxes and section 4.6 concludes.

1Recently Gagnon (1992) studied the HI model in the presence of the liquidity constraint. Unfortunately, some major results in that paper are incorrect and the author mainly focuses on the effects of the income path and on the case of risk neutrality. Also the restriction on short selling of housing assets is not considered.

2The potential conflict between the demand for housing as a consumption good and that as an investment is also discussed in Bossons (1978).
4.2 The Model

Consider two periods, period 1 and 2. The household chooses a consumption bundle \( \{x, h_c\} \) and an investment portfolio \( \{S, h_I\} \) in period 1, where \( x \) is the numeraire, \( h_c \) is housing consumption, \( S \) is savings, and \( h_I \) is housing investment. The return on housing investment in period 2 is uncertain but the expected rate of return is greater than that on savings, which bears a riskless interest rate \( r \). The household receives income \( y_1 \) and \( y_2 \) respectively in each period. The utility in period 1 is given by \( U_1(x, h_c) \) and that in period 2 is given by \( V_2(W_2) \), where \( W_2 \) is the wealth remaining after period 1. \( W_2 \) depends on the household’s choice of the consumption bundle and the investment portfolio. The utility functions \( U_1(x, h_c) \) and \( V_2(W_2) \) are both increasing and concave.

The market price of housing in period 1 is \( P \) per unit and the market rent for housing services in period 1 is \( R \) per unit. The housing price in period 2 will be \( P(1 + \theta) \), where the appreciation rate \( \theta \) is stochastic. The capital market imposes the constraints that \( S \geq 0 \) and \( h_I \geq 0 \). \( S \geq 0 \) is a liquidity constraint, which means that the household cannot borrow against its wealth in period 2; \( h_I \geq 0 \) is a short selling constraint. Housing investment is financed through a mortgage loan of \( L \) dollars per unit of \( h_I \). The required down payment is thus \( P - L \) dollars per unit of \( h_I \). The mortgage loan bears an interest rate \( r \) and calls for a balloon payment of principal and interest in period 2. The household also receives the market rent in period 1 for its housing investment.

We assume real capital gains on housing investment to be the only source of uncertainty. Other sources of uncertainty, such as the uncertainty in rental costs and in the returns on security investment, might usefully be considered [Bosch, Morris, and Wyatt (1986)], but empirically they are less important. Rosen, Rosen, and Holtz-Eakin (1984), based on their finding that the forecast error variances of the price of renting are very small compared to those of the price of owning, conclude that the risks associated with owning are most important to the

\(^3\)The error variances of the price of owning is estimated based on a model where real capital gains are the only source of uncertainty.
tenure decision. Feldstein and Feenberg (1983) find that 80% of those under the age of 56 held less than $5000 in financial assets in 1972.

Also, to make our results directly comparable to those in Henderson and Ioannides (1983) and Fu (1991), we assume a zero income tax for the moment but will consider the case of positive income taxes later in section 5.

The household’s problem, then, is to choose the values \( \{h_c^*, h_I^*, S^*\} \) to maximize the expected utility:

\[
\max_{h_c, h_I, S} U_1(x, h_c) + E\{V_2(W_2)\}
\]

subject to

\[
\begin{align*}
y_1 &= x + Rh_c + S + (P - L - R)h_I, \\
W_2 &= y_2 + S(1 + r) + (P(1 + \theta) - L(1 + r))h_I, \\
S &\geq 0, \\
h_I &\geq 0.
\end{align*}
\]

Since the optimal choice of housing consumption, \( h_c^* \), increases with \( W_1 \) (housing is a normal good), we can just focus on a reduced version of problem (4.1):

\[
\max_{h_I, S} V_1(W_1) + E\{V_2(W_2)\},
\]

subject to

\[
\begin{align*}
W_1 &= y_1 - S - (P - L - R)h_I, \\
W_2 &= y_2 + S(1 + r) + (P(1 + \theta) - L(1 + r))h_I, \\
S &\geq 0, \\
h_I &\geq 0,
\end{align*}
\]

where

\[
V_1(W_1) = \{\max_{h_c} U(x, h_c) : x + Rh_c = W_1\}.
\]
The first-order conditions are

\[-(P - L - R)V_1' + E\{[P(1 + \theta) - L(1 + r)]V_2'] + \mu = 0, \quad (4.2a)\]

\[-V_1' + E\{(1 + r)V_2'\} + \lambda = 0. \quad (4.2b)\]

where \(\lambda\) and \(\mu\) are the Lagrange multipliers associated with the constraints \(S \geq 0\) and \(h_I \geq 0\) respectively. The complementary slackness conditions are

\[\lambda > 0, \text{ if } S^* = 0, \text{ and } \lambda = 0, \text{ if } S^* > 0;\]

\[\mu > 0, \text{ if } h_f^* = 0, \text{ and } \mu = 0, \text{ if } h_f^* > 0.\]

4.3 The Certainty-Equivalent Approach

In order to simplify our analysis and also to illustrate the trade-offs diagrammatically, we propose the following certainty-equivalent transformation of the period 2 expected utility. Let

\[V_2(\bar{W}_2 - gh_I^2) \equiv E\{V_2(W_2)\}, \quad (4.3)\]

where, \(\bar{W}_2 \equiv E\{W_2\} = y_2 + S(1 + r) + (P(1 + \bar{\theta}) - L(1 + r))h_I\), with \(\bar{\theta} = E\{\theta\}\), is the expected wealth in period 2 and \(g\) is a risk premium per unit of housing investment. The idea is to determine a risk premium \(g\) such that the household is indifferent between an uncertain wealth \(W_2\) in period 2 and a certain wealth \(\bar{W}_2 \equiv \bar{W}_2 - g(h_I^*)^2\). To determine \(g\), we apply Taylor series expansion to both sides of equation (4.3). Expanding \(V_2(W_2)\) at \(\theta = \bar{\theta}\), we have\(^4\)

\[E\{V_2(W_2)\} \approx V_2(\bar{W}_2) + V'_2(\bar{W}_2)Ph_I E\{\theta - \bar{\theta}\} + \frac{1}{2} V''_2(\bar{W}_2)(Ph_I)^2E\{\theta - \bar{\theta}\}^2\]

\[= V_2(\bar{W}_2) + \frac{1}{2} V''_2(\bar{W}_2)(Ph_I)^2\text{var}(\theta). \quad (4.4)\]

And expanding \(V_2(\bar{W}_2 - gh_I^2)\) at \(g = 0\), we get

\[V_2(\bar{W}_2) \approx V_2(\bar{W}_2) - V'_2(\bar{W}_2)gh_I^2. \quad (4.5)\]

\(^4\)This quadratic approximation of the expected utility is accurate if the third and higher order moments of the distribution of \(\theta\) are negligible. Rosen, Rosen, and Holz-Eakin (1984) and Bosch, Morris, and Wyatt (1986) also employ the quadratic representation of expected utilities.
Let \((4.4) = (4.5)\) and we obtain
\[ g \approx -\frac{1}{2} \frac{V_2''(\bar{W}_2)}{V_2'(\bar{W}_2)} P^2 \text{var}(\theta). \quad (4.6) \]

Equation \((4.6)\) allows us to interpret \(g\) in terms of risk aversion and the uncertainty in housing investment. Our results will not depend on the precise functional form of \(g\). It is clear, however, that \(g\) is positive and increases with the Arrow-Pratt absolute risk aversion function \(-V_2''V_2'/V_2'^2\) and with the variance of \(P\theta\).

Substituting \((4.3)\) into the first-order conditions \((4.2a)\) and \((4.2b)\) we obtain
\[ -(P - L - R)V_1' + (P(1 + \bar{\theta}) - L(1 + r) - 2gh_I)V_2' + \mu = 0, \quad (4.2a') \]
\[ -V_1' + (1 + r)V_2' + \lambda = 0. \quad (4.2b') \]

Equation \((4.2a')\) says that when \(h_I \geq 0\) is not binding, the optimal housing investment \(h_I^*\) should be such that the marginal rate of substitution between period 1 consumption and period 2 consumption equals the risk-adjusted marginal gross rate of return on housing investment. The risk-adjusted marginal gross rate of return on housing investment is defined by \((P(1 + \bar{\theta}) - L(1 + r) - 2gh_I^*)/(P - L - R)\), which decreases with \(h_I^*\) and \(g\) since larger \(h_I^*\) and \(g\) respectively mean more risk and greater risk aversion. Similarly as equation \((4.2b')\) indicates, when the household is not liquidity constrained, the optimal savings \(S^*\) requires that the marginal rate of substitution equals the marginal gross rate of return on savings \((1 + r)\).

The difference in the marginal gross rates of return on housing investment and on savings is
\[ \frac{P(1 + \bar{\theta}) - L(1 + r) - 2gh_I^*}{P - L - R} - (1 + r) = \frac{P(\bar{\theta} - r) + R(1 + r) - 2gh_I^*}{P - L - R}, \quad (4.7) \]
where \(P(\bar{\theta} - r) + R(1 + r)\) is the expected excess return per unit of housing investment and is assumed to be positive. Obviously, the right hand side of equation \((4.7)\), the risk-adjusted rate of excess return at the margin, is positive when \(h_I\) is small. In the absence of the liquidity constraint, the optimal portfolio requires that the risk-adjusted marginal rate of return on housing investment be equal to the return on savings. Thus by setting the right hand side of
equation (4.7) equal to zero, we get the first-best housing investment $h_I^{**}$, which is
\[ h_I^{**} = \frac{P(\bar{\theta} - r) + R(1 + r)}{2g} \approx \frac{P(\bar{\theta} - r) + R(1 + r)}{-V''(W_2)P^2\text{var}(\bar{\theta})}. \] (4.8)

Equation (4.8) shows that $h_I^{**}$ increases with the expected excess return on housing investment and decreases with the Arrow-Pratt measure of absolute risk aversion and the variance of $P\theta$.

The household wants to invest in housing first, since the risk-adjusted marginal rate of return on housing investment exceeds that on savings when $h_I$ is small. After housing investment reaches the first-best level $h_I^{**}$, the household wants to make its further investment in savings, because the risk-adjusted marginal rate of return on housing investment beyond $h_I^{**}$ is less than that on savings.

Now we can describe the budget curves in a $\tilde{W}_2 - W_1$ diagram (see Figure 4.6) that represent feasible resource reallocations between period 1 and period 2 under various capital market conditions. In the absence of uncertain housing investment, the household achieves intertemporal resource reallocation through savings only; the frontier of the feasible set of intertemporal resource allocations (the budget set) is represented by the budget line $aa'$, which has a slope of $-(1 + r)$, the marginal gross rate of return on savings. This budget line is censored to the line $aa'$ if the household faces liquidity constraint $S \geq 0$. With the opportunity to invest in housing and in the absence of savings, the appropriate budget curve is the curve $ABC$, which represents the diminishing risk-adjusted marginal rate of return on housing investment. With both housing investment and savings, the budget set can be expanded to the frontier $ABb$. Since the household in this case can invest in savings whenever the risk-adjusted marginal rate of return on housing investment becomes less than that on savings, line $BB$ has a slope of $-(1 + r)$ and is tangent to the curve $ABC$ at point $B$, where the risk-adjusted marginal rate of return on housing investment equals $(1 + r)$. Finally, if the household can also borrow at the interest rate $r$, the budget set can be further expanded to the frontier $bB'$, which is an extension of the line $BB$. In fact $h_I^{**}$ maximizes the certainty-equivalent wealth $W_1 + \tilde{W}_2/(1 + r)$ and, accordingly, the budget line $bb'$ represents the largest budget set attainable with perfect capital markets.

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5Equation (4.8) is the same as equation (4) in Fu (1991).
4.4 The Behavior of $h^*_t$ and $h^*_f$ and the Implications for Tenure Choice

A number of factors may affect the household’s housing investment and consumption. We will examine the effects of the following factors in the presence of both perfect and imperfect capital markets: (1) variation in the income path, holding wealth constant, (2) variation in wealth, holding the income path constant, (3) variation in the expected rate of housing price increase $\tilde{\theta}$, (4) variation in the uncertainty of housing price increase, $\text{var}(\theta)$, and (5) variation in the availability of mortgage loan $L$.

We define $T \equiv y_1 - y_2/(1 + r)$ to measure the tilt in the income path toward period 1, and $W \equiv y_1 + y_2/(1 + r)$ to denote the household’s wealth. Equivalently, $y_1 = (T + W)/2$ and $y_2/(1 + r) = (W - T)/2$. For notational simplicity, when we take derivatives with respect to $T$, we always assume $dW = 0$, and vice versa.

We will also simply assume that the Arrow-Pratt measure of absolute risk aversion, and hence $g$, is decreasing with $W$ and, in the presence of the liquidity constraint, is non-increasing with $T$. The first part of the assumption is natural: since consumption in both periods are normal goods, both $W_1$ and $W_2$ will increase with $W$. To understand the second part of the assumption, we notice that the household cannot be worse off with an increase in $T$, since the household can invest only a portion of the income transferred from $y_2/(1 + r)$ to $y_1$ in housing to make $W_2$ unchanged. In the presence of perfect capital markets, $g$ is not affected by $T$ since any change in $T$ will be totally offset by a corresponding change in $S$ [see Fu (1991)].

4.4.1 With Perfect Capital Markets

With perfect capital markets individuals can lend or borrow freely at the market interest rate $r$. The absence of the liquidity constraint has two important implications for the household’s choices. First, the household would always choose $h^*_t = h^*_f$, the level of housing investment that maximizes the household’s certainty-equivalent wealth $W_1 + \hat{W}_2/(1 + r)$, independent of the marginal rate of substitution between current and future consumption and the income path. Second, the household’s choice of an intertemporal resource allocation $\{W^*_1, \hat{W}^*_2\}$ is constrained
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by the budget line $bb'$, which is determined by the maximum wealth attained through housing investment. Figure 4.7 provides an illustration. It is clear that the Fisherian separation of investment and consumption [Fisher (1930)] applies in this setting.

Fu (1991) shows analytically that the income path $\{y_1, y_2\}$ does not affect either the housing investment or the consumption with perfect capital markets. This is also apparent in Figure 4.7. A change in the income path, holding wealth $y_1 + y_2/(1 + r)$ constant, changes the position of point A along the line $aa'$ and the position of point $B$ along the line $bb'$, leaving $h_1^* = h_1^{**}$ and $\{W_1, W_2^*\}$ unchanged.

An increase in wealth pushes line $aa'$ outward. At the same time, the Arrow-Pratt measure of absolute risk aversion is reduced, resulting in a smaller $g$ and a greater risk-adjusted marginal rate of return on housing investment. Accordingly, the optimal housing investment $h_1^* = h_1^{**}$ increases and so does the maximum certainty-equivalent wealth $\bar{W}_2$. As the consumption in both periods are normal goods, $W_1^*$ also increases as the budget line $bb'$ moves further from the origin, resulting in a higher demand for housing consumption $h_c^*$. The relative changes in $h_1^*$ and $h_c^*$, however, cannot be determined without knowing how much the risk aversion and the marginal rate of substitution between current and future consumption will change with wealth.\(^6\)

The risk-adjusted marginal rate of return on housing investment increases with the expected rate of housing price appreciation $\bar{\theta}$ and decreases with the uncertainty in $\bar{\theta}$ (see equation (4.7)); so do housing investment $h_1^{**}$ (see equation (4.8)) and housing consumption $h_c^*$. The mortgage loan $L$ plays no special role here, since the household is free to borrow against its future income with perfect capital markets.

4.4.2 With Imperfect Capital Markets

With imperfect capital markets, the household faces the constraints $S \geq 0$ and $h_1 \geq 0$. The appropriate budget curve in this case is $ABb$. The three possible situations where the constraints may affect the choice of $\{S^*, h_1^*, W_1^*, W_2^*\}$ are illustrated in Figure 4.8. In the first situation, the

\(^6\)The analytical expressions for $dh_1^*/d(y_1 + y_2/(1 + r))$ and $dh_c^*/d(y_1 + y_2/(1 + r))$ are provided in Henderson and Ioannides (1983) and Fu (1991).
household has a strong preference for future consumption relative to current consumption, so that its indifference curve \((I)\) is flat and is tangent to the segment \(Bb\) of the budget curve. The household in this situation wants to save enough income in period 1 to be able to make a first-best housing investment, \(h_1^* = h_1^{**}\). Since the liquidity constraint does not affect the household's choice in this situation, the following analysis will focus on the other two situations.

In the second situation the household has a steeper indifference curve \((I')\), which is tangent to the segment \(AB\) of the budget curve. The household in this situation still wants to save some of its income in period 1 but the amount to be saved is not sufficient to make the first-best housing investment. As a result, the household is liquidity constrained and \(h_1^* < h_1^{**}\).

In the third situation, the household has a very strong preference for current consumption relative to future consumption, so that its indifference curve \((I'')\) is very steep and passes point \(A\). The household in this situation does not want to save any of its income in period 1. Consequently, both constraints \(S \geq 0\) and \(h_1 \geq 0\) are binding.

Housing investment in the presence of the liquidity constraint has two implications: it enhances the household's certainty-equivalent wealth but also entails a resource reallocation between the two periods. Consequently, the Fisherian separation of investment and consumption no longer applies and the household's intertemporal consumption preferences have to enter into its investment decision. If the household has a strong preference for present consumption, it is rational for the household to choose a level of housing investment that is less than the first-best in order to accommodate its current consumption needs.

**The effects of the income path**

A tilt of income path toward period 1, such that the position of point \(A\) moves down along the line \(aa'\), will push the household from the third situation, where both constraints bind, toward the first situation, where neither constraint binds. The indifference curve that is tangent to the budget curve or passes point \(A\) will shift outward from the origin. Obviously, when the indifference curve passes point \(A\) but is not tangent to the budget curve, a small increase in \(T\)
will not affect housing investment but will increase consumption in period 1. For a household in the second situation its choice of \( W_1^* \) (and hence \( h_1^* \)) and \( h_1^* \) will both increase (as long as \( S \geq 0 \) is binding). This is so because an increase in \( T \) makes the liquidity constraint less binding and thus allows the household to improve both its certainty-equivalent wealth and its current consumption.

This can also be shown using the first-order condition (4.2a'). Let

\[
Q \equiv -(P - L - R)V_1^1 + [P(1 + \theta) - L(1 + r) - 2gh_1^1]V_2^1 = 0. \tag{4.9}
\]

Without \( h_I \geq 0 \) binding, then,

\[
\frac{dh_1^*}{dT} = -\frac{\partial Q}{\partial h_1^*}, \tag{4.10}
\]

where

\[
\frac{\partial Q}{\partial h_1^*} = (P - L - R)^2V_1'' + [P(1 + \theta) - L(1 + r) - 2gh_1^1]^2V_2'' - 2gV_2', \tag{4.11}
\]

\[
\frac{\partial Q}{\partial T} = \left(-\frac{1}{2} + \frac{\partial S}{\partial T}\right)\{(P - L - R)V_1'' + [P(1 + \theta) - L(1 + r) - 2gh_1^1](1 + r)V_2''\}
- 2\frac{\partial g}{\partial T}h_1^*V_2'. \tag{4.12}
\]

By the concavity assumption of the indirect utility functions it is clear that \( \partial Q/\partial h_1^* < 0 \).

Further, when \( S \geq 0 \) is binding, \( \partial S/\partial T = 0, h_1^* < h_1^{**} \) and \( P(1 + \theta) - L(1 + r) - 2gh_1^* > (P - L - R)(1 + r) \). Thus \( \partial Q/\partial T > 0 \) as long as \( S \geq 0 \) is binding and the net down payment \( (P - L - R) \) is positive. Consequently, \( dh_1^*/dT > 0 \). That is, the optimal choice of housing investment increases as the income path tilts toward period 1.

Moreover, by the fact that \( P(1 + \theta) - L(1 + r) - 2gh_1^* > (P - L - R)(1 + r) > 0 \), it can be shown easily that

\[
\frac{dh_1^*}{dT} < \frac{1}{2(P - L - R)},
\]

and by the definition of \( W_1^* \).

\[
\frac{dW_1^*}{dT} = \frac{1}{2} - (P - L - R)\frac{dh_1^*}{dT} > 0. \tag{4.13}
\]

That is, the period 1 consumption \( W_1^* \) also increases as the income path tilts toward period 1.
The effects of wealth

The impact of an increase in wealth $W$, holding the income path constant, on $h_1^*$ and $W_1^*$ is less obvious. An increase in $W$ pushes the budget curve $ABb$ outward. At the same time $g$ decreases, resulting in a greater risk-adjusted marginal rate of return on housing investment (or a steeper budget curve). For the household facing a binding constraint $h_I \geq 0$, there is no substitution between current and future consumption and $W_1^*$ increases with wealth. For the household that invests in housing, a greater risk-adjusted marginal rate of return on housing investment would induce more housing investment, or in other words, more substitution of period 2 consumption for period 1 consumption. An increase in wealth, holding risk aversion constant, however, can cause the investment to change in either direction.

Analytically,

$$\frac{\partial Q}{\partial W} = -\frac{P - L - R}{2}V''_1 + [P(1 + \theta) - L(1 + r) - 2gh_1^*][1 + r] - \frac{\partial g}{\partial W}(h_1^*)^2]V''_2 - 2\frac{\partial g}{\partial W}h_1^*V'_2. \tag{4.14}$$

The first two terms in equation (4.14) represents the change in the marginal rate of substitution between current and future consumption, with the risk-adjusted marginal rate of return on housing investment being constant. The third term represents the change in the marginal rate of return on housing investment caused by the change in risk aversion. While the third term is positive, the sign of the first two terms is indeterminate. However, if we assume that the marginal rate of substitution between current and future consumption does not change with wealth, the first two terms in equation (4.14) vanishes. As a result $\partial Q/\partial W > 0$ and housing investment increases with wealth.

The net effect on $W_1^*$ of an increase in wealth also depends on the magnitudes of the effects associated with the “pure” wealth change and the greater risk-adjusted marginal rate of return on housing investment; whereas the latter causes $W_1^*$ to decrease, the former causes it to increase. As equation (4.13) indicates, $W_1^*$ will increase with wealth if and only if $(P - L - R)\partial h_1^*/\partial W < 1/2.7$

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7In an empirical study of housing tenure choice, Ioannides and Rosenthal (1992) identify housing investment
The effects of the expected housing price increase

An increase in $\bar{\theta}$ increases the risk-adjusted marginal rate of return on housing investment, causing an increase in the substitution of future consumption for current consumption. Consequently, $h_I^*$ increases and $W_1^*$ decreases. When the constraint $h_I \geq 0$ is binding, however, a change in $\bar{\theta}$ has no effect on $W_1^*$.

The effects of future housing price uncertainty

An increase in $\text{var}(\theta)$ decreases the risk-adjusted marginal rate of return of housing investment, causing a decrease in the substitution of future consumption for current consumption. Consequently, $h_I^*$ decreases and $W_1^*$ increases. Again, when the constraint $h_I \geq 0$ is binding a change in $\text{var}(\theta)$ has no effect on $W_1^*$.

The effects of mortgage loan

With imperfect capital markets, the availability of mortgage loans eases the liquidity constraint. A larger $L$, and thus a smaller net down payment $P - L - R$, reduces the requirement by housing investment for the substitution of future consumption for current consumption. Furthermore, a greater leverage with $L$ increases the rate of return on housing investment, making the budget curve steeper. This is clear from the following equation:

$$\frac{\partial}{\partial L} \left( \frac{\partial W_1}{\partial h_I^*} \right) = \frac{\partial}{\partial L} \left( \frac{P(1 + \bar{\theta}) - L(1 + r) - 2gh_I}{P - L - R} \right) = \frac{P(\bar{\theta} - r) + R(1 + r) - 2gh_I}{(P - L - R)^2},$$

which is positive for $h_I < h_I^*$.

Both the reduction in the substitution requirement and the increase in the rate of return encourages housing investment. Under the extreme circumstance that $P - L - R = 0$, the household will always choose the wealth-maximizing investment $h_I^{**}$ (which is clear from the demand and housing consumption demand separately and find that both are positively correlated with household wealth and current income. Moreover, wealth has a significant impact on investment demand but an insignificant impact on consumption demand, whereas income has a significant impact on consumption demand but an insignificant impact on investment demand.
first-order condition (4.2a')). This is because housing investment no longer requires the substitution of future consumption for current consumption and, hence, intertemporal consumption preferences no longer enter into the investment decision (see Figure 4.9).

Analytically,

\[
\frac{\partial Q}{\partial L} = h_1^*\{-(P - L - R)V_1'' + [P(1 + \theta) - L(1 + r) - 2gh_1^*](1 + r)V_2'' \} + V_1' - (1 + r)V_2', \tag{4.15}
\]

which is positive when \( h_1^* < h_1^{**} \) and \( S = 0 \). Accordingly, \( \partial h_1^{**}/\partial L > 0 \).

The impact of a small change in \( L \) on \( W_1^* \) depends on the initial choice of \( h_1^* \). Using equations (4.11) and (4.15), we have

\[
\frac{dW_1^*}{dL} = h_1^* - (P - L - R)\frac{\partial h_1^*}{\partial L} = \left( \frac{\partial Q}{\partial h_1^*} \right)^{-1}\{[P(1 + \theta) - L(1 + r) - 2gh_1^*][P(\theta - r) + R(1 + r) - 2gh_1^*]V_2''\}
+ [P(\theta - r) + R(1 + r) - 4gh_1^*]V_2'. \tag{4.16}
\]

Equation (4.16) is positive if \( h_1^* \) is close to \( h_1^{**} \), as the saving from a reduction in the down payment requirement more than offsets the cost of an increase in \( h_1^* \). It is negative if \( h_1^* \) is close to zero and \( V_2'' \) is small in absolute value, as the saving from a reduction in down payment requirement less than offsets the cost of an increase in \( h_1^* \).

4.4.3 Summary of the Comparative Statics and Their Implications for Tenure Choice

We summarize the comparative static results in table 4.1. The results listed in the third and fourth columns of the table show that the presence of the liquidity constraint changes the behavior of \( h_1^* \) and \( h_1^{**} \) substantially. The third column recapitulates the results in Henderson and Ioannides (1983) and Fu (1991), where the household faces no liquidity constraint. There are three important implications of those results. First, the income path and mortgage credit does not affect the housing decision. Second, both housing investment and consumption are normal goods and increase with wealth. Third, the demand for housing consumption will
move in the same direction when the demand for housing investment changes, because housing investment affects the certainty-equivalent wealth.

The fourth column summarizes the choice behavior of a liquidity-constrained household. In this case, both the income path and mortgage credit affect housing investment and consumption: the more resources available in period 1, other things being equal, the more investment and consumption will be chosen. Wealth does not necessarily affect the demands for housing consumption and investment in the same direction. And finally, the demands for housing investment and consumption tend to move in the opposite direction; an increase in the expected housing price appreciation or a decrease in the uncertainty of future housing prices, for example, increases the demand for housing investment but decreases the demand for housing consumption. As housing investment becomes more productive, the household is willing to forego more of its current consumption in exchange for future consumption by investing more in housing assets.

Empirical studies find that transitory income or the forward tilting in the income path has a strong and positive impact on the housing demand of homeowners. The forward tilting also has a strong and positive impact on the demand for homeownership, whereas transitory income has a small impact [Goodman (1988), Henderson and Ioannides (1987), Dynarski and Sheffrin (1985)]. These empirical findings are consistent with the results in the fourth column but not with those in the third column, indicating the importance of the liquidity constraint. Furthermore, wealth and permanent income are found to have a major impact on tenure choice but a minor impact on the housing demand of homeowners. This is also consistent with the results in the fourth column, in which case an increase in housing investment demand partially offsets the positive impact of wealth on housing consumption demand.

The results can also provide some explanations for the coefficient of the Mill’s ratio in the housing demand equation, which represents the covariance between the errors in the estimation of the probability of owning and those in the estimation of the housing demand of homeowners. There are several possibilities. A negative coefficient [significant in Goodman (1988) and
Lee and Trost (1978), and insignificant in Horioka (1988), Dynarski and Sheffrin (1985), and Rosen (1979) may suggest that the omitted variables in the tenure choice equation represent idiosyncratic preferences for homeownership. In this case a smaller predicted probability of owning indicates that the household more likely needs to overcome a deficiency in its housing investment demand to be a homeowner and, when the household is liquidity constrained, it needs to suppress its current consumption demands. A positive coefficient [insignificant in Henderson and Ioannides (1987)],\(^8\) on the other hand, may arise if the omitted variables are correlated with risk aversion. In this case a smaller predicted probability of owning indicates that the household is less risk averse and, when the household is not liquidity constrained, a greater certainty-equivalent wealth results and will translate into greater current consumption. Furthermore, the more liquidity-constrained households in the sample, the more likely the coefficient of the Mill’s ratio will be negative.\(^9\)

Another interesting hypothesis implied by the results in the fourth column is that an increase in the expected real capital gains, or a decrease in the uncertainty in future housing prices, will increase the demand for homeownership but decrease the average housing demand of first-time home buyers (who are likely liquidity constrained). So far no empirical evidence has been presented to test such a joint hypothesis.

### 4.5 The Impact of Income Taxes

Suppose now the household pays income taxes at rate \(\tau\). The interest income is taxed, the imputed rents and capital gains are not. In the United States the interest expense on mortgage loans is tax deductible. Thus the budget constraints for the U.S. household becomes

\[
\begin{align*}
W_1 &= (1 - \tau)y_1 - S - (P - L - R)h_I, \\
W_2 &= (1 - \tau)y_2 + S(1 + r - \tau r) + (P(1 + \theta) - L(1 + r - \tau r))h_I,
\end{align*}
\]

\(^8\)Gillingham and Hagemann (1983) obtain significant and positive coefficients for the Mill’s ratio, but in their case transitory income is missing in both tenure choice and housing demand equations and the Mill’s ratio may well pick up the effect of transitory income.

\(^9\)Indeed the positive coefficient obtained in Henderson and Ioannides (1987) is much more significant for the subsample of older homeowners (age> 45) than that for the subsample of young homeowners (age< 35).
In the case of the household borrowing against its future income to finance its current consumption, the interest expense on consumer loans is not tax deductible and the second period budget becomes

\[ W_2 = (1 - \tau)y_2 + S(1 + r) + (P(1 + \theta) - L(1 + r - \tau r))h_I, \]

\[ S < 0. \]

The budget curve with income taxes is depicted in Figure 4.10. The slope of the segment \( bb \), representing the after-tax interest rate, decreases with income tax rate \( \tau \); on the other hand, the slope of the curve \( CBB'A \), given by

\[ \frac{P(1 + \bar{\theta}) - L(1 + r) + L\tau r - 2gh_I^*}{P - L - R}, \]

increases with \( \tau \) (we assume the increase in risk aversion, and hence in \( g \), due to an increase in income tax rate, has a smaller impact than does the decrease in after-tax mortgage interest expense). Based on the behavior of these slopes, the following conclusions can be made. First, for the household that holds positive savings (its indifference curve is tangent to the segment \( Bb \)) the optimal housing investment \( h_I^B \) is determined by the tangent condition at point \( B \) and is given by

\[ h_I^B = \frac{P(\bar{\theta} - r) + R(1 + r) + (P - R)\tau r}{2gh_I^*}. \]

\( h_I^B \) increases with \( \tau \). Second, for the household that holds positive consumer loans (its indifference curve is tangent to the line \( B'b' \)) the optimal housing investment \( h_I^{B'} \) is determined by the tangent condition at point \( B' \) and is given by

\[ h_I^{B'} = \frac{P(\bar{\theta} - r) + R(1 + r) + L\tau r}{2gh_I^*}. \]

\(^{10}\)This budget curve also applies to the situation where the household faces a higher interest rate on consumer loans than that it earns on savings.
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$h_f^P$ also increases with $\tau$ but is less than $h_f^R$ (as long as $P - L - R > 0$). Third, for the household that holds neither savings nor consumer loans, or is liquidity constrained (its indifference curve is tangent to the curve $BB'A$) the optimal housing investment is less than $h_f^R$ and increases with $\tau$.

The consumption demands in both periods will be adversely affected by income taxes, as an increase in $\tau$ moves point $A$ toward the origin and thus draws the budget curves inward. In the case of the household holding positive savings, income taxes have a relatively smaller impact on current consumption since the household will reduce its savings when the after-tax rate of return on savings decreases. When the household is liquidity constrained, income taxes have a relative larger impact on current consumption since the increase in housing investment due to a higher income tax rate compounds the reduction in current income.

The above analysis shows that an increase in the income tax rate increases the demand for homeownership, as the demand for housing investment increases and that for housing consumption decreases. Furthermore, with income taxes the income path affects the demands for housing investment and consumption even in the absence of the liquidity constraint. Specifically, the demand for housing investment increases and the demand for housing consumption decreases as the household switches from holding consumer loans to holding savings when its income path tilts forward. The demand for housing investment increases because the marginal rate of return decreases. The demand for housing consumption decreases because wealth decreases as the income path tilts forward. The decrease in wealth is due to a lower after-tax rate of return on savings relative to the discount rate $r$. Income taxes do not change the effects of the income path when the household is liquidity constrained and its risk-adjusted marginal rate of return on housing investment is greater than $r$.

With progressive income taxes housing investment becomes more sensitive to wealth, because the reduction in after-tax mortgage interest expense enhances the effect of the decrease in risk aversion as wealth increases. As a result, for the liquidity-constrained household wealth will

\[11\] If the income path $T$ is calculated according to after-tax interest rates, then an increase in $T$ will not decrease wealth.
have a larger impact on the demand for homeownership and a smaller impact on the demand for owner-occupied housing consumption; for the household holding positive savings, wealth will have a larger impact on homeownership rate but more or less the same impact on housing consumption.

In Canada, mortgage interest expense is not tax deductible. As a result the risk-adjusted marginal rate of return on housing investment decreases with income tax rate because risk aversion increases when income taxes increase. Therefore, the demands for housing investment and consumption of the liquidity-constrained household both decrease with income tax rate, and the impact of income taxes on tenure choice is indeterminate. Moreover, with progressive income taxes the demands for housing investment and consumption of the liquidity-constrained household both will be less sensitive to (before tax) wealth, although for the household holding positive savings wealth will have a larger impact on the demand for housing investment but more or less the same impact on the demand for housing consumption.

4.6 Conclusions

Using the certainty-equivalent approach we are able to characterize the demands for housing investment and housing consumption in the presence of both uncertainty and a liquidity constraint. The analysis has produced a number of useful comparative statics which explain several empirical facts associated with housing tenure choice and housing demand. The analysis has also shown the important effects of the uncertainty in future housing prices, the liquidity constraint, and income taxes on the housing decisions.

A useful extension of our model is to incorporate risky securities in the household's portfolio. A rigorous solution to such a model would be difficult; however, the impact of the presence of risky securities can be intuitively understood based on the analysis in this chapter. For those not liquidity constrained, diversification through security investment enhances the demand for housing investment as well as wealth. For those liquidity constrained, the opportunity cost of homeownership will also include the lost benefit of portfolio diversification.
Table 4.1: A Summary of Comparative Statics

<table>
<thead>
<tr>
<th>Response variables</th>
<th>Effects</th>
<th>With perfect capital markets $S \geq 0$ is not binding</th>
<th>With imperfect capital markets $h_j^* \geq 0$ is binding</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_j^*$</td>
<td>$T$ increases</td>
<td>No effect</td>
<td>Positive</td>
</tr>
<tr>
<td></td>
<td>$W$ increases</td>
<td>Positive</td>
<td>Indeterminate</td>
</tr>
<tr>
<td>$h_c^*$</td>
<td>$T$ increases</td>
<td>No effect</td>
<td>Positive</td>
</tr>
<tr>
<td></td>
<td>$W$ increases</td>
<td>Positive</td>
<td>Indeterminate</td>
</tr>
<tr>
<td>$\theta$ increases</td>
<td>$h_j^*$</td>
<td>Positive</td>
<td>Positive</td>
</tr>
<tr>
<td>var($\theta$) increases</td>
<td>$L$ increases</td>
<td>No effect</td>
<td>Positive</td>
</tr>
<tr>
<td>$h_c^*$</td>
<td>$\theta$ increases</td>
<td>Positive</td>
<td>Negative</td>
</tr>
<tr>
<td>var($\theta$) increases</td>
<td>$L$ increases</td>
<td>No effect</td>
<td>Positive when $h_j^* \gg 0$</td>
</tr>
</tbody>
</table>
Figure 4.6: The Budget Curves
Figure 4.7: The Case of Perfect Capital Markets
Figure 4.8: The Case of Imperfect Capital Markets
Figure 4.9: The Case of $P - L - R = 0$
Figure 4.10: The Budget Curves With Income Taxes
Bibliography


