WRITING TO LEARN MATHEMATICS: STUDENT JOURNALS AND STUDENT-CONSTRUCTED QUESTIONS

by

RAMAKRISHNAN MENON
B.A., University of Malaya, 1973
B.Sc., University of London, 1974
M.A., University of Northern Iowa, 1983

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Department of **MATHS & SCIENCE E.D. (CURRICULUM & INSTRUCTION)**

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ABSTRACT

The purpose of the study was to investigate the mathematical learning indicated by using student journals (SJ) and student-constructed word problems (SCQ) in a mathematics class where the teacher initiated the procedures and conducted the lessons. The class was a multi-level Grade 5 and Grade 6 class but only the Grade 6 students were the focus of the study.

Three times a week, towards the end of their mathematics lesson students wrote in their SJ in response to teacher prompts. Once a week, the students also prepared SCQ, in groups and individually. These SCQ were collected, edited, typed and redistributed to the class by the teacher, and used as class exercises. Six students were interviewed (video-taped) three times each over the 15-week period of study. Records of these interviews, classroom observations and a teacher interview were all used to complement the analysis of SJ and the individually-prepared SCQ.

Although the SJ did give insights about students' mathematical knowledge, students' oral explanations indicated that they understood more than what was written in their SJ. Hence, the lack of ability to communicate through written words in the SJ was not an indicator of student's mathematical understanding.

In contrast, the SCQ indicated students' knowledge of fractions better, both (a) implicitly, through the type of question asked, information given, and the accompanying solution and (b) explicitly, through the type of fraction relationships, the amount of detail, and the number of steps and operations involved. The SCQ also revealed that students tended to
base their word problems on (a) their own experiences and interests, (b) an assumption of shared knowledge between the reader and writer of the word problem, (c) numbers which made computation easy and (d) the discrete model of fractions rather than the region model.

The results of this study indicate that the SCQ assisted mathematical learning in a classroom context but that the value of the SJ needs to be reconsidered.
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CHAPTER 1. INTRODUCTION

Background

Just as the fashion world sees ever changing styles and fabrics, mathematics curricula seem to pass through many fads. For example, the "modern mathematics" of the late 1950s was followed by the "back to the basics" mastery learning movement, which was itself soon replaced by a "problem solving" focus in school mathematics in the 1980s (Clements, 1989). Currently, "constructivism" (for example, Cobb, 1986) and, to a lesser extent, "writing to learn mathematics" (Connolly & Vilardi, 1989) seem to be in vogue.

This current interest in writing to learn mathematics was sparked by the "writing across the curriculum" movement (Fulwiler, 1987) which owed much to the earlier established theoretical links between written language and learning (Emig, 1983; Luria & Yudovich, 1971; Smith, 1982; Vygotsky, 1962). Prior to this burgeoning interest in writing to learn mathematics, many scholars and educators perceived language and mathematics as having little in common. For example, Snow (1965) reflected the feeling of the times when he differentiated scientific culture (as embodied in mathematics and natural sciences) from humanistic culture (as embodied in literature and the arts). He commented that members of the scientific community rarely communicated with members of the literary community, or vice versa and each were unaware and uncaring of the other's achievements. He lamented the lack of communication between these two cultures saying:

I believe the intellectual life of the whole of western society is increasingly being split into two polar groups... Literary intellectuals at one pole--at the other scientists, and as the most
representative, the physical scientists. Between the two a gulf of mutual incomprehension. (pp. 3-4)

However, it is now becoming more common not only to talk about the similarities and differences in language and mathematics learning (Zepp, 1989), but also to explore how writing can assist both language development (Emig, 1983; Smith, 1982) and mathematical development (Ellerton, 1989; Ellerton & Clements, 1991). Writing to learn mathematics, then, seems a step in the right direction to bridge the long-perceived dichotomy between language and mathematics, and hence between the two cultures Snow identified.

In addition to the links between mathematics and language development, many scholars perceive learning as recursive and not simply the accretion of discrete pieces of knowledge (Woditsch & Schmittroth, 1991). Such a view emphasizes that the process of learning is as important as the product of learning. As instances of this, we have philosophers (Dewey, 1933; Rorty, 1969); linguists and language educators (Atwell, 1990; Austin, 1975; Berthoff, 1981; Britton, Burgess, Martin, McLeod, & Rosen, 1975; Elbow, 1981; Graves, 1983; Kirby & Liner, 1988; Macrorie, 1988; Moffett, 1981; Pimm, 1987; Searle, 1969; Shaughnessy, 1977); social historians (Havelock, 1963; Ong, 1982); psychologists (Bruner, 1986); anthropologists (Geertz, 1973); and scientists (Kuhn, 1962).

Connolly (1989) sums up the importance of process in learning, by drawing from social constructivist (Ernest, 1991) and socio-psycholinguistic (Gawned, 1990) perspectives of the learning process, saying:

First... knowledge is socially constructed within a community, not discovered raw in nature by individual intellects; and second, that
the agents of construction are the symbol systems through which people "make meaning"—musical, mathematical, graphic, kinetic, but most important, verbal. (p.4)

Applied to the idea of writing to learn, the views above imply that one learns to develop and clarify concepts through the process of writing (both simultaneously with, and antecedent to conceptualization) rather than illustrating clearly-learned concepts through writing as a product (subsequent to conceptualization). Hence, approaches to learning mathematics through writing seem to be promising, especially as they are related to the well-established writing-learning link as well as the interest in process-learning.

Rationale

Rose (1989) attests that "despite the apparent disparity between the fields of writing and mathematics, a review of recent literature reveals a growing interest in the relationship of writing to teaching mathematics." (p. 17). Indeed, many studies on the theme "writing to learn mathematics" have been undertaken over the past decade and various suggestions have been made about the types of writing which might assist mathematics learning.

These suggestions include summaries, questions, explanations, definitions, reports and word problems (King, 1982); freewriting (Burton, 1985; Countryman, 1987; McMillen, 1986; Mett, 1987; Sachs, 1987); letters (King, 1982; Kennedy, 1985; Schmidt, 1985); admit slips (Schmidt, 1985); journals (Clarke, Waywood, & Stephens, 1992; McMillen, 1986; Nahrgang & Petersen, 1986; Powell, 1986; Rose, 1989; Schubert, 1987; Shaw, 1983; Vukovich, 1985; Waywood, 1991); writing problems (Ellerton, 1988) and story maths (Ellerton & Clements, 1992).
Most of the suggestions about the types of writing which might assist mathematics learning have come from researchers who have conducted studies at secondary or tertiary institutions in North America, where the researchers themselves instructed the students involved in the study. So it is puzzling why Waywood (1991) states that most reported research on writing to learn mathematics has focused on the elementary school level. It is not clear whether he was referring to Australian studies which may not have been reported in North American journals. Although he states that there were only two exceptions to the focus of such research on elementary schools, my readings indicate otherwise.

Whether or not results from studies on writing to learn mathematics in other countries such as Australia apply to Canada, there is a need for Canadian data because far too often mathematics educators have tried to implement the latest “trend” without considering its suitability for their own situation (Clements, 1989). In spite of the paucity of Canadian school-based research on writing to learn mathematics, especially in elementary schools, there are a few studies in this area. For example, Wason-Ellam (1987) reports on a Canadian grade one class using journals to learn mathematics but her analysis was predominantly on the linguistic, not mathematical, features of the journal entries. She reports, for instance, on compositional features of beginners' writing such as “beginnings, patterns of organization, and closures” (p. 11).

Regardless of the type of writing or level of research, very often past research in schools was “someone else's agenda.” There is a concern that the approach succeeded because of the luxury of time and resources available to the researcher, who has access to commodities that are normally in short supply to classroom teachers. Thus, there is a
need for such research to be complemented by research where only "normal" classroom resources are employed.

In this study, I investigate two types of writing to learn mathematics, namely student journals (SJ) and student-constructed questions (SCQ). By SJ, I mean students' in-class, individually written responses to the teacher's prompts, which mainly ask for explanations and clarifications. My use of the word "journal" is similar to that of Schubert's (1987), where she talks about mathematics journals for fourth graders as regular in-class written responses to teacher prompts. Similarly, Kirby and Liner (1988), use the term "class journal" to include in-class written responses to teacher prompts in language arts classes. I use the term SJ to focus on what students write about mathematics, not around mathematics. Most of the previous studies on journal writing have given prominence to the affective aspect of learning through exploratory and expressive (Britton et al., 1975) writing. I am not denying the importance of affect on mathematics learning, but affect is not the focus of my study.

By SCQ, I mean word problem(s) constructed and written out by the students either in groups or individually. I am using the term SCQ instead of the more common term "problem-posing," because I want to emphasize the centrality of students' direct involvement in the activity, as well as to emphasize the use of written words in these problems. Moreover, SCQ connotes a student constructing mathematics, rather than writing about mathematics.

I chose to study both SJ and SCQ in order to tap different aspects of mathematics learning: the SJ, to indicate how students communicate their mathematics understanding to the reader through predominantly transactional writing (Britton et al., 1975), and the SCQ to reveal the
depths of their mathematics knowledge through expressive writing (Del Campo & Clements, 1987). Britton uses the word “transactional” to mean writing such as that used to inform or explain (writing that is “directed towards external objectives” according to Bereiter & Scardamalia, 1987, p. 184). For Del Campo and Clements (1987), the word “expressive” indicates writing that is a result of the writer’s own constructions, based on personal experiences and referents and can be either to explore one’s feelings or to explain or both. For example, explaining in the SJ why $0.8 \neq 3/5$ would entail Britton’s transactional writing as the emphasis is on informing or explaining something. Writing a word problem about common and decimal fractions using the writer’s experiences and personal referents would be an instance of Del Campo and Clements’ expressive writing.

The two aspects of mathematics learning alluded to (communication through written words and knowledge of mathematics) are similar to, but not identical with, what Ellerton and Clarkson (1992) refer to as children writing about mathematics and children writing mathematics. While journal writing and problem posing in mathematics classes have been studied separately before, they have not been studied together in an elementary mathematics class. For example, Burton (1985) studied affect through the use of journals in mathematics methods courses at the college level and Ellerton (1989) studied mathematics word problems prepared by secondary school students.

My choice of both SJ and SCQ as appropriate vehicles for writing to learn mathematics is also in line with the National Council of Teachers of Mathematics (NCTM) Curriculum and Evaluation Standards (1989) where communication of mathematics as well as problem posing and
problem solving are stressed. In addition, the writing engendered through the SJ and SCQ might encourage students to “fish in the river” of their minds (Kirby & Liner, 1988, p.58) to reflect on and communicate the mathematics they have learned.

Moreover, in studying both, I hope to better understand mathematics learning in specific topics such as common and decimal fractions. In short, by choosing both SJ and SCQ, I hope to explore how students use written language (mainly words) to communicate mathematical understanding and ideas (through SJ) and how creating their own questions (through SCQ) might indicate their understanding of certain mathematical topics.

There are several reasons for selecting student journals as one of the two innovations to be implemented in this study. First, many reports of journals in mathematics have emphasized the affective domain (like students' feelings), and have neglected the cognitive aspect (like reports on specific mathematical gains). Second, where the journals did not emphasize feelings and attitudes, they were likely to be influenced by the process writing approach (for example, Graves, 1983), resulting in either the narrative genre overshadowing mathematics content or the genre being used peripherally in the mathematics class, “after the 'real' mathematics has been done” (Marks & Mousley, 1990, p.121). For example, research has shown that students sometimes write stories with numbers substituted for names of persons but without relating the mathematical associations among the numbers (Del Campo & Clements, 1987, p. 19).

Other than the research issue discussed above, the use of SJ relates to a pedagogical issue: the implementation of an innovation in
the classroom. Since most students and teachers have already used journals in subjects other than mathematics, they might not feel uncomfortable using them in mathematics, too.

I now turn to why I chose SCQ. First, my previous experience with SCQ (where SCQ were used intermittently in a Grade 6 class) was sufficiently positive to lead me to believe that the potential of SCQ needed to be explored. Moreover, SCQ, in providing opportunities for students to create and write their own problems, are providing opportunities to construct and record personal mathematical meanings (Del Campo & Clements, 1987). In addition, I feel that SCQ might be able to indicate what mathematics has been learned, as it would be almost impossible to construct and solve questions without understanding mathematical content.

**Purpose of the Study**

The main purpose of the study was to investigate the mathematical learning revealed through the use of SJ and SCQ in a mathematics class where the teacher herself initiated the procedures and conducted the lessons. In addition, I wanted to investigate the perceptions of the students and the teacher about the usefulness of writing in a mathematics class and also to investigate whether there were any patterns discernible in the word problems constructed by the students.

**Research Questions**

Specifically, the questions addressed in this study were:

1. What mathematical knowledge do students reveal through their writing?
2. What are the salient and recurrent features (themes) of the SCQ, if any?

3. What are the roles of SJ and SCQ in mathematics learning, according to (a) students? and (b) the teacher?

Overview of Study

This study focused on the mathematical learning of Grade 6 Canadian children in a multigrade (Grade 5 and 6) class where the teacher initiated two types of writing tasks (SJ and SCQ) as part of the mathematics class. The teacher was willing to participate in the study mainly because she wanted to try some different approaches to a topic (fractions) she has found difficult to teach.

Data for the study were obtained through what was written in the SJ and SCQ, together with individual student interviews, an interview with the teacher and my notes on classroom observations. Such data, together with the thick description (Geertz, 1973) provided in chapters three and four, encapsulating both the context and details of the study enable conceptual (Yin, 1984) or naturalistic generalisations (Stake, 1978) rather than statistical generalisations to be made.
CHAPTER 2. REVIEW OF LITERATURE

In this chapter I first review the literature on the use of writing as a tool for learning in general. Then I discuss current literature in various fields which, directly or indirectly, seem to support the use of writing to learn mathematics. Finally, I discuss previous studies on the use of journals and student-constructed questions in mathematics lessons.

Writing and learning

For many years, writing was thought to be a result of carefully-planned, well-conceptualized ideas. According to Emig (1983), these beliefs were based on three major assumptions: (a) there is a dichotomy between planning and writing, (b) writing never precedes planning and (c) writing is the recording of the already formulated conception. She points out the falsity of these assumptions by providing evidence from writers who agree that “planning is a tentative or sketchy affair and writing itself is the major act of discovery of how they think and feel” (p. 16).

The philosopher Ryle (1949), too, agrees that it is fallacious to believe that for an operation to be intelligent, it has to be guided by a previous intellectual operation. That is, Ryle denies that we always plan everything that we subsequently do successfully.

Emig (1983) says writing “connects the three major tenses of our experiences to make meaning” by “shuttling among past, present, and future” using the “processes of analysis and synthesis” (p. 129). She argues that writing is a unique mode of learning because it is
multirepresentational, in that it uses Bruner’s enactive, iconic and symbolic modes of representation concurrently or at least contiguously. Moreover, when writing, the hand, eye and brain are all working in concert.

Emig (1983, p. 129) draws some useful correspondences between learning and writing, saying that both give self-provided feedback, generate and connect concepts and are multirepresentational, integrative, active, personal and self-rhythmed. Others who echo similar beliefs about writing and learning are Elbow (1981), Luria and Yudovich (1971), Smith (1982), Vygotsky (1962). More recently, support for such beliefs come from the educationists Boyer (1987) and Fulwiler (1987), the futurists Naisbitt and Aburdene (1985) and writers such as Zinsser (1988). All those cited above emphasize that the very act of writing engenders thinking.

Writing has gone from an emphasis on a finished, polished product to writing as a process-product, where exploratory or tentative concept-building is encouraged, especially in informal writing. This building up of concepts through language and writing is emphasized by both Barnes (1976) and Smith (1982). Smith, in discussing how language and writing help learning, says that writing helps us find out what we know and think because “language creates as well as communicates” (p. 67).

The “whole language” movement (e.g. Shanahan, 1991) exhibits some of these process-oriented approaches. In this approach, children are encouraged to find their own voice by learning language (however “unpolished”) through actively using it, rather than learning just the rules about how to use grammatically correct language. The advocates of this
approach believe that the very process of using language (with feedback and guidance) brings about facility in using it correctly and appropriately.

The supporters of writing to learn also point out that because writing forces one to focus on certain aspects of the deluge of experience, one has to examine these aspects in greater detail, rather than leave them in the nebulous wash of unattended perceptions. As Pimm (1991) puts it, “externalizing thought through spoken or written language can provide greater access to one's own (as well as for others) thoughts, thus aiding the crucial process of reflection, without which learning rarely takes place” (p. 23). Similarly, Keith (1990) says, “Frequently we may think we understand something when we only recognize it; we confuse familiarity with understanding. This becomes obvious when we have to explain it in writing” (p. 6). In other words, writing helps in the clarification and generation of concepts and therefore is critical to the process of learning.

In pointing out the relationship between writing and learning, Mayher, Lester, and Pradl (1983) say:

Writing’s capacity to place the learner at the center of her own learning can and should make writing an important facilitator of learning anything that involves language. Writing that involves language choice requires each writer to find her own words to express whatever is being learned. Such a process may initially serve to reveal more gaps than mastery of a particular subject, but even that can be of immense diagnostic value for teacher and learner alike. And as the process is repeated, real and lasting mastery of the subject and its technical vocabulary is achieved. (p. 79)

In short, the proponents of writing to learn believe that writing helps thinking in two ways: one, by the process of writing, one’s thinking becomes clearer; and two, because it is a more permanent record,
writing allows one to look more closely at what is written and revise what has been written.

In spite of the benefits of writing to learn, it seems fitting to take note of some of the disadvantages which have been pointed out. For example, Pimm (1987, p.117-118) cautions that (a) expression through the written medium can be both time-consuming and arduous, (b) there is reluctance to write when there is no clear reason why one should write, (c) the problem of writing legibly and clearly makes one lose sight of what one was trying to express, (d) writing can act as a brake when one is caught up in a rush of ideas and (e) it is difficult to write from the standpoint of an uninformed reader or to make explicit assumptions which are self-evident to the writer.

Moreover, not all types of writing function to clarify and generate concepts. For instance, Mayher, Lester, and Pradl (1983) claim that "writing that involves minimal language choices, such as filling-in-blanks exercises or answering questions with someone else's language--the textbook's or the teacher's--are of limited value in promoting writing or learning" (p.78).

While the cautions against an uncritical acceptance of writing to learn are timely, there seems to be sufficient support for using writing as a learning tool.

**Writing to learn mathematics**

Initially, it might look odd to contemplate writing as a tool for the learning of mathematics especially the use of everyday language in a subject usually associated with precise and abstract mathematical symbols. This view is not surprising given that "most children are very
good at learning and using language—they make remarkable achievements in this domain before they commence schooling and in the absence of formal instruction—while very few children take so readily to mathematics" (Durkin, 1991, p. 4). Such a contrast in language and mathematical facility could be because everyday language can be comprehended even without knowing the meaning of every word. In contrast, mathematics is more difficult to comprehend because it uses words which are non-redundant and have densely-packed meaning.

Mathematics educators like Connolly (1989), Rose (1989) and Ellerton and Clarkson (1992) have documented the rapid and increased interest in writing to learn mathematics in recent years. Indeed, more and more educators perceive the similarities and differences in language and mathematics learning and believe that writing can help develop both language and mathematics learning.

While it would be useful to have a comprehensive model linking writing to mathematics learning, such a model has not been articulated yet. However, a model of learning that seems to link writing to mathematical problem-solving, is the Adaptive Control of Thought (ACT) model resulting from the application of computer science, information processing theory and linguistics (Anderson, 1983). In this model, learning is associated with two types of memory: declarative memory, where knowledge, such as propositions, spatial images and temporal strings are stored; and production memory, where skills based on the construction and addition of procedures originating from other procedures and propositions are stored (Kenyon, 1989, p. 75).

According to Kenyon, there seems to be a relationship among the ACT learning model, the three (recursive) phases of writing—namely the
prewriting or planning phase, the composition stage and the rewriting phase--and the mathematical problem solving approach. When faced with a problem, a search is initially executed to access propositions from declarative memory and procedures from production memory. So, when one writes at the prewriting or planning phase, one is attempting to understand what is being asked and what are the attendant conditions of the problem. A memory search (from both types of memories) for strategies and similar problems take place at this phase of writing. Then, during this exploratory, prewriting phase, possible strategies are planned and some strategy is selected from both types of memories.

In the second phase, composition, the writing is more organized and cohesive in order to execute the strategy selected. This phase is analogous to the actual solving of the problem when a certain approach is implemented, for example, by using some known procedures. If the strategy selected does not lead to the desired result, phase one is repeated, similar to the rejection of a procedure that did not lead to a solution and a search for another procedure. In the third phase, rewriting, the writing resembles that of the transactional mode as there is more clarity and focus because of an expected audience. This phase is similar to checking the reasonableness of the solution of a problem after which it may be either rewritten in order to communicate the solution to someone else or an attempt at a more elegant solution is made. Hence, the ACT model of learning seems to link writing, learning and mathematics learning, albeit more specifically to mathematical problem solving.

Mildren (1992) points out that just as learning language through expository essay writing involves “topic choice, planning and structuring..."
a text, organising information, drafting, revising, and editing" (p. 34), learning mathematics through problem-solving involves “defining the unknown, determining what information one already knows, designing a strategy or plan for solving the problem, reaching a conclusion and then checking the results” (Bell & Bell, 1985, p. 212). Hence, he too supports a link between writing and mathematical problem-solving.

According to Botstein (1989), a technologically-advanced society needs citizenry who are mathematically literate to maintain and improve itself. Because the mathematics on which technological devices are based is seldom derivable from daily experience, “the bridge between the technical and specialized worlds of modern mathematics and science and daily life and experience must be constructed out of ordinary language” (Botstein, 1989, p. xv). One way to build this bridge seems to be by expressive, exploratory writing in mathematics.

But just as there were cautions about the use of writing to learn, so also there are cautions about writing to learn mathematics. Most of the concerns about writing to learn mathematics have been about the trivial type of mathematics that seem to be reflected in students' writing in mathematics (Caughey & Stephens, 1987; Ormell, 1992; Pengelly, 1990). However, there is sufficient empirical and theoretical evidence contradicting these concerns. In the following sections, I elucidate some of these theoretical and empirical bases for supporting writing as a tool for learning mathematics.

Mathematics Learning and Language Learning

According to Ernest (1991), mathematical knowledge begins with linguistic knowledge. He says, “Natural language includes the basis of
mathematics through its register of elementary mathematical terms, through everyday knowledge of the uses and interconnections of these terms" (p. 75). As well, van Doren (1959, cited by Talman, 1990, p. 107) indicates that "language and mathematics are the mother tongues of our rational selves." Laborde (1990), too, states that "the functions of language in the context of mathematics classrooms are those that have been recognized for a long time in the development of thought: Language serves both as a means of representation and as a means of communication" (p. 53). Research from various parts of the world (Hughes, 1986; Clarke et al., 1992; Boero, 1989; Laborde, 1990; Stigler & Baranes, 1988; Rose, 1989) attest to language factors affecting mathematics learning.

While language in the mathematics classroom can represent and communicate mathematical ideas, and mathematics as language might be a useful notion, "mathematics is not a natural language in the sense that English and Japanese are" (Pimm, 1987, p.207) and language teaching methods may not be directly transferable to the teaching of mathematics (Ellerton and Clements, 1991, p. 125). Nevertheless, I will argue in this section that there are some parallels between the learning of a language and that of mathematics and that everyday language both helps and hinders mathematical learning. I would like to emphasize that, unless otherwise stated, I use the word “mathematics” to mean “school mathematics.”

While both Layzer (1989) and Pimm (1987) view mathematics as a foreign language, I think it might be more useful to view it as a second language. There have been many different views about what constitutes a foreign language and what constitutes a second language (e.g.
Richards, 1978; Stern, 1983) but my use of these two terms is clarified below.

To me, a foreign language can be construed as one that is seldom, if ever, used in everyday discourse, while a second language is one that has to be used, at least some of the time, to function more effectively in the society in which one lives. The second language, moreover, can be thought of as learned second, after the learning of the first language. For example, English is a second language while French is a foreign language to many recent Canadians of Asian origin.

Mathematics, specifically school mathematics, is somewhat like a second language in that it has to be learned to function more effectively in society. For example, the functionally numerate person has to make sense of percentage discounts in stores, simple statistics and graphs in newspapers. While children generally acquire (Krashen, 1981) their mother tongue or first language "naturally" without explicit instruction, mathematics as a second language has to be approached slightly differently and has to be learned. While Krashen's use of the term "learning" is restricted to the learning of procedures, I am using the term in a broader sense, to encompass whatever knowledge is gained from explicit instruction.

I say "slightly differently" because of some similarities between first language and second language learning. For example, just as a child moves from holophrases to complete sentences through an interlanguage (Corder, 1981) in first language acquisition, so too a child moves through the phases of using natural (but possibly mathematically imprecise) language to the more sophisticated register of the mathematician.
Currently, the approach to second language learning is based more on communicative competence (Hymes, 1972; Widdowson, 1990) than on the memorisation of rules of syntax. That is, how to use language appropriate to the context is more important than being able to parse a sentence into clauses. Similarly, communicating mathematical ideas is considered an important component of mathematical competence (NCTM Standards, 1989).

In comparing second language instruction with mathematics instruction, Borasi and Agor (1990) argue that many of the approaches to second language instruction (for example, the “Delayed Oral Production” of Postovsky, 1977; “Silent Way” of Gattegno, 1976; and “Counseling Learning/Community Language Learning” of Curran, 1976) might be usefully modified and applied to mathematics instruction.

For example, by delaying the actual plotting and drawing of graphs by the students and instead asking students to interpret a given graph (p. 13), the teacher is directing students to use mathematics meaningfully rather than concentrating on the mechanics of plotting accurately--similar to using language in context, rather than learning, say, the rules of grammar. Another example is the problem-posing approach of Brown and Walter (1990) who argue that even with limited factual mathematical knowledge and partial understanding, one can engage in creative learning in mathematics. For instance, in the case of the Pythagorean theorem, a student-generated question on what would happen if the theorem did not deal with a right triangle (Brown & Walter, 1990, p. 45) would open up many possibilities for mathematical investigation through student initiative and responsibility and allow for informative discussion. Such an approach, according to Borasi and Agor
(1990, pp. 15-18), is similar to the Silent Way and Counseling Learning/Community Language Learning approaches to second language instruction.

Mathematics is usually taught in a context-reduced manner, while second language teaching emphasizes context-embeddedness (Cummins, 1981). As Spanos, Rhodes, Dale, and Crandall, (1988) say, “the pedagogical advantages of incorporating real situations in an interactive framework, a common practice in most English as a second language (ESL) curricula, seem to be absent in the traditional mathematics curriculum” (p. 232).

When children write in mathematics using their own words and experiences they are embedding their learning in a context and are communicating their mathematical ideas, both to themselves and to others. Such writing therefore helps move the learner from embedded thought (embedded in the context) to “disembedded thought” (Donaldson, 1984)--thought which abstracts the concept from the context. Hence, the route to abstract mathematical symbolisms via formal mathematical language has to be paved by everyday language.

However, linguistic ambiguities, especially lexical ambiguities, abound when language from an everyday context is used in a mathematics context (Pimm, 1987; Durkin & Shire, 1991). Accordingly, writing that promotes discussion of ambiguous words might resolve these conflicting roles of everyday language (of helping and hindering mathematical learning) by fostering an awareness of how words used in everyday language can have different meanings in mathematics. Such writing may also act as a window to the mathematical thinking of the child.
There are factors other than lexical ambiguity associated with the transfer from everyday language to the mathematical register. One such factor is the linguistic structure of natural language. According to Laborde (1990), “linguistic features of natural language can affect the transition of a situation from natural language into an algebraic statement” (p. 61).

This linguistic factor is one of the reasons many tertiary students cannot translate relationships expressed in everyday language into corresponding mathematical expressions (Clement, Lochhead & Monk, 1981; Mestre & Lochhead, 1983; Rosnick & Clement, 1980). The example most cited by these researchers is the following problem: There are six times as many students as professors. If S represents students and P represents professors, write a mathematical relationship connecting S and P. Most students write 6S = P rather than S = 6P as a solution to this problem. One explanation for this confusion is that the linguistic structure of the problem statement, where the expression “six times as many students” precedes the word “professors,” could have led to the left-to-right translation of the problem.

Another factor associated with the transfer from everyday language to the mathematical register is the difference in levels of language required as one progresses from the colloquial to the mathematical (Freudenthal, 1978, pp. 233-242). New concepts are exemplified by exemplary or demonstrative language first. Later refinement leads to relative language and finally to functional language. Examples of these levels of language follow:

1. demonstrative (pointing out instances, without explanations): half is like this part here.
2. relative (using words to indicate relationship or procedures): when something is cut into two equal parts, each part is called a half.

3. functional (generalisations or relationship between relationships): The common fraction 1/2 is the same as the decimal fraction 0.5 because one out of two equal parts is equivalent to five out of ten equal parts.

The levels suggested by Freudenthal seem to form a continuum ranging from personal referents to more abstract ones. Therefore, if children are encouraged to write in mathematics, their initial exploratory writing using their own words might lead them from demonstrative language to relative and functional languages later.

Even though facility in sophisticated mathematical language is the final objective, initial exploratory writing in mathematics, using personal, everyday language can help the learner construct meaning. Having something written down allows a re-vision, which in turn helps reconstruct meaning. From the point of view of language learning, each construction may be perceived as an interlanguage (rather than an error or misconception) leading to the appropriate mathematical register.

What I have discussed so far suggests that certain aspects of language learning might be usefully linked to mathematics learning and that everyday language might assist in learning mathematics. In short, just as written language and learning seem to be linked, writing and the learning of mathematics might also be linked.
Mathematics Learning and Pedagogy

In this section, I discuss research that supports the notion that writing in mathematics, especially informal exploratory writing, is pedagogically sound.

Many students seem to view mathematics with anxiety. To allay such anxiety, Tobias (1989), suggests that "getting students to write about their feelings and misconceptions would relieve their anxiety and unlearn models and techniques that were no longer useful to them" (p. 50). Even though she does caution that her techniques might have worked because her students were already "predisposed to verbal expression" (p. 50), I believe her point about relieving student anxiety is well taken, and writing might very well act as a catharsis.

LeGere (1991), too, concurs that writing does seem to diminish stress and anxiety about getting the "correct" answer, and allows risk taking. Morrow and Schifter (1988) have this to say about the "anxiety-reducing" role of writing in mathematics:

Turning to a more familiar and often more comfortable mode, such as writing, can provide a sense of security to a math-anxious student. Alternating among various modes of discourse (writing, talking, drawing, and symbolic representation) builds bridges between formal mathematics knowledge embedded in students' everyday experiences; insistence that mathematically valid thought is restricted to rule-governed manipulation of strings of symbols keeps the mathematics insulated from their personal knowledge. (p. 380)

Mathematics educators have recently started emphasizing student ownership of their learning (e.g. Ellerton & Clements, 1991). Because writing to learn mathematics entails students using their own language and ideas to express mathematical concepts and principles, some ownership of learning seems to be engendered.
According to Nahrgang and Petersen (1986) and Wilde (1991), writing in mathematics promotes the learning of mathematics. Wilde (1991) has shown that, even for ESL students, writing about mathematical ideas initially in their first language, and then translating these ideas (also in written form) using their limited knowledge of English, does benefit their mathematical learning.

Associated with ownership of mathematics learning is the idea of empowering the learner. Buerk (1990) states that many students view mathematics as truths external to themselves, derived from the indisputable authority of the textbook, teacher or mathematician. They believe that “mathematics is something they can have no ideas about” (p. 79) and that they have a sense of “powerlessness in its presence” (p. 78).

Students seldom realise that mathematicians struggle to understand, clarify and verify mathematical ideas just as students themselves do. Moreover, mathematics is often presented as objective truths, unsullied by subjectivity or human struggle. Brookes puts this picturesquely “... the most neglected existence theorem in mathematics is the existence of people” (1970, p.vii, cited in Pimm, 1987, p. xvii). By encouraging exploratory writing in mathematics which itself entails uncertainty, students are empowered and given dignity as learners (Hoffman & Powell, 1989; Worsley, 1989).

Writing to learn mathematics finds support in the current transactional-interactive model of teaching. In this model, teachers are facilitators and their role is to provide situations that give rise to discussion and learning. As Charbonneau and John-Steiner (1988) say, “the teacher will be more a catalyst to learning rather than a presenter of a body of facts and figures” (p. 98).
Such models are actually not new. For example, Piaget's notion of cognitive dissonance, Vygotsky's zone of proximal development, and Bruner's discovery learning all implicitly encapsulate the interactive model. More recently, we have the 4 MAT System (McCarthy, 1986) and other suggestions that emphasize learning through reflection and interaction. Since writing in mathematics is closely associated with reflection on what is written and interaction with both what is written and the expected reader, writing seems to be an appropriate vehicle for interactive learning and teaching.

For instance, Keith (1990) says, when talking about calculus courses, that "to keep mathematics alive as a subject in college, we must be prepared to create a more vigorous, interactive classroom environment" (p. 6). One way to achieve this, according to her, is to give writing assignments in the calculus course. Powell and Lopez (1989) agree that such writing in mathematics courses allows crystallisation and generation of mathematical concepts through constant reflection and revision.

Another facet of reflective learning can be viewed through the idea of active involvement in mathematics through doing mathematics. But, as Pimm (1987) suggests, it is not just doing, but thinking about doing, that counts. Thus, this reflection, which is aided by writing, allows for greater active involvement in mathematics learning (Azzolino, 1990; Rose, 1989; LeGere, 1991).

Another pedagogical benefit from writing in mathematics is that it gives feedback on the learning-teaching enterprise, by helping assess and monitor both learning and teaching (Powell, 1986; Powell and Lopez, 1989; Sachs, 1987). Such feedback, according to Countryman
(1987), also allows for diagnosis and remediation, whether teacher- or pupil-initiated. Certainly, writing mathematics, using mainly mathematical symbols with minimal explanatory words (as in finding the maximum or minimum points using differentiation), can also give such feedback. For example, the student might have written all the appropriate steps to show that a maximum point exists, but may not have any idea why the “tests” seem to work and under what conditions these tests might have to be modified or extended. Or in the case of fractions, students may be able to add fractions using common denominators, but may not be aware why they need to do so. However, such writing (predominantly mathematical symbols) cannot identify students who may only be using procedural knowledge (Hiebert, 1986) by manipulating symbols, apparently logically and sequentially, to arrive at a correct solution without substantive understanding of the mathematical concepts or principles involved.

Even if they get an incorrect answer, and subsequently exhibit error patterns in their work—which might be a basis for remediation—such error patterns can only be detected by careful construction of a diagnostic test. With the use of writing in mathematics, not only does the teacher not have to prepare and administer the diagnostic test but also idiosyncratic misunderstandings can be attended to, as long as students can articulate their thoughts in writing.

With the emphasis on conceptual rather than procedural learning (Hiebert, 1986), elementary school students seem to have a poor recall of mathematical facts and procedures. While learning concepts are important, it would seem that students should benefit if certain mathematical facts and procedures had become automatised. This
The supposed benefit of automaticity might seem at odds with writing to learn, but I will show why such automaticity is actually helpful to students writing to learn mathematics and conversely, how such writing will help automaticity.

Because writing has a communicative function (even to oneself), it results in further clarity and better retention of mathematical concepts (Stempien & Borasi, 1985). Such retention of concepts leads to being able to recall and use them in further writing, without having to re-think the concepts, thereby allowing the short term memory (from the information processing point of view) to work more efficiently to enhance problem solving processes.

Another pedagogical concern is the problem of attending to individual needs of the students. Because writing, unlike speaking, can be done simultaneously by the whole class, every individual is actively involved. Each student can write about his or her own perceptions, difficulties, solutions and feelings. While speaking allows for articulating one's difficulties, not every student would risk making mistakes in front of their peers. Writing is more private and allows for a confidential dialogue between the teacher and the student. Responses from the teacher make it a personal learning experience as well, because students know that the teacher hears and cares (Thompson, 1990; Watson, 1980). In short, writing in mathematics is like individualising instruction in a group setting (Bemiller, 1987).

Another concern of mathematics teachers is to help students become better problem solvers. One major difficulty is that students do not know where, or how, to start. Bemiller (1987), and King (1982) found
that when students were stuck with a problem, writing out their thoughts often helped them resolve the problem by themselves.

Additional support for writing as a means of assisting students become better problem solvers is given by Bell and Bell (1985) who compared ninth graders with and without a writing component during the teaching of mathematics problem-solving. The experimental group—who used writing to analyze information needed to solve the problem and describe the process used to solve the problem—outperformed the control group in a posttest on problem solving.

Helping weaker students to develop and learn mathematics is also a concern of mathematics teachers. According to Evans (1984), writing in mathematics does help the weaker mathematics students. She compared two groups of fifth graders on units of geometry and multiplication, using three types of writing for the experimental group: explanations on how to do something, definitions, and explanation of errors on homework and tests. Even though the control group, unlike the experimental group, comprised some gifted students and also scored better in the pretests, the experimental group performed better on both units in the posttest.

Pallmann's (1982) study also included an experimental group using writing (in their remedial college mathematics course) to explain in their own words how, for example, they worked out problems and interpreted graphs and algebraic laws. Results of her study indicate that though there was a mean gain for the experimental group on the posttest, this was not statistically significant. However, the fact that this group had more of the weaker students made it educationally significant.
In summary, pedagogical theory and practice suggest the following benefits for writing in mathematics classrooms: more student ownership of learning, less student anxiety, individualised learning in a group setting, monitoring and diagnosis of learning, and enhanced achievement in tests on problem solving.

**Mathematics as a Social Enterprise**

Views about the nature of knowledge have inspired many philosophical debates and driven many practices. For example, those viewing knowledge essentially as dualistic (Perry, 1970), would rely heavily on authority (be it the text book, teacher, mathematician or logic itself) and believe in answers to mathematical questions as either absolutely right or wrong. Such views would emphasize correct procedures and exclude conscious guessing (Lakatos, 1976) and tentative procedures, like exploratory writing in mathematics.

In contrast, those who hold a relativistic outlook (Perry, 1970), acknowledge multiple approaches and solutions to mathematical problems relative to evaluative criteria in a given framework. Many of those who hold such a view of mathematics would say that mathematics is a social enterprise. For example, Wittgenstein (1956) states that mathematics is a human practice, depending crucially on the consensus of a community and is very much a socialisation process. Similar views were expressed by Restivo (1981) and by Lakatos (1976). Lakatos, for instance, points to the fallibility of mathematics and views mathematics as a human enterprise, relying on conjectures, criticisms and corrections.

Radical constructivists like von Glasersfeld (1987), who believe that knowledge is constructed by the individual, would be considered
Relativistic. So would social constructivists such as Cobb, Yackel and Wood (1992), Ernest (1991) and Steffe (1989) who believe that mathematical knowledge is a result of the individual interacting in a social setting. (For an elaboration of the differences between radical constructivism and social constructivism, see Ernest, 1991, p. 71). A social constructivist perspective supports the notion of mathematics as a social enterprise because social constructivists believe that there is no reality outside the learners’ own experiential interpretations and that mathematical knowledge is actively constructed by the learners and consensually validated by the community (Bloor, 1984; Ernest, 1991).

When children write mathematics using their own experiential referents and discuss their writing with peers or the teacher (this discussion could be oral or in the form of written responses), they will come to realize that mathematics is also subject to consensual validation and shared meaning (Lampert, 1988; Pirie, 1991). This is in keeping with the view that mathematics “is an ever-changing field that exists not as an abstraction but as a piece of linguistic and cultural fabric” (Scholnick, 1988, p. 86).

In Lampert’s study (1988) on how to make mathematics classrooms reflect what mathematicians actually do, she emphasizes the role of negotiation of meaning in the social context of the class. She claims that her fifth grade students “learned to participate as active members of a community of discourse about mathematics” (p. 465), citing both Bauersfeld (1979) and Steiner (1987). She argues that “mathematics learning is a process of social as well as individual construction” (p. 473). The increasing interest in ethnomathematics and a corresponding decrease in eurocentrism (“Western mathematics is
best") and findings from studies of mathematics in different cultures (Bishop, 1985; Cocking & Mestre, 1988; Harris, 1989) also indicate that mathematics is socially and culturally bound.

Mellin-Olsen (1987) also views mathematics as a social enterprise, arguing that activity is a means of survival in society, where the individual acts on society and in turn is acted upon by society. Language and mathematics can then be perceived as thinking and communicative tools for changing the individual and society, especially by resolving problems in society by the use of mathematics and language.

Mellin-Olsen gives examples of how students identified and solved problems affecting their neighbourhood by using both mathematics and language (for example, by verbalising their ideas and then by writing letters to people in authority, giving reasons based on mathematics and statistical data) to implement change. By allowing children to use whatever mathematics they felt suitable to support their case, teachers helped children to see the relevance of mathematics. And by accomplishing something, largely by themselves, they also felt a sense of ownership and autonomy.

Earlier, I discussed the relationship between learning and language. Hence, using language by writing and subsequently sharing one's writing with others, should help with the renegotiation of knowledge in a social context. Because writing to learn mathematics encourages reflection, interaction and renegotiation of meaning, such writing should be able to help reinforce socially constructed mathematical knowledge.
In summary, I argue that the view of mathematics as socially constructed supports the approach of writing mathematics to learn mathematics, because (a) language itself is a social enterprise and (b) use of everyday language in writing to learn mathematics, will, after helping construct and reconstruct idiosyncratic mathematical concepts, be subjected to a critical audience (peers or the teacher), thereby promoting communication, validation and renegotiation of socially constructed mathematical meaning.

Journals

In this section I first discuss the use of journals as a learning tool in language arts and then discuss in more detail the use of journals as a learning tool in mathematics specifically. For purposes of this review, I am using the word journal to include a range of writings, from “diary-like entries to focused (i.e. assigned) entries such as summaries of lectures and discussions of problems” (Sipka, 1990, p. 12).

Journals in Language Arts

According to Fulwiler (1987), the increased attention to informal writing in the late 1960s made journal writing more accepted in education. He lists some cognitive activities associated with such writing. Among such activities are speculations, revisions, questions, observations and synthesis of ideas. Other advocates of journal writing in language arts include Atwell (1990); Kirby and Liner (1988); Macrorie (1988); and Martin et al. (1976). Kirby and Liner (1988), for instance, found journals an effective tool for improving fluency in written English.
They felt that a journal is effective because "it's a private, protected place, . . . to explore" (p. 58). Also, according to Carswell (1988),

"The journal writing activity was certainly a success in both my and my students' estimation. The journal clearly enhanced communication between the instructor and the students, appeared to provide some therapy, stimulated conceptualization and reconceptualization, and generated more than occasional enjoyment." (p.112)

However, there are some problems associated with the use of journals as a pedagogical tool. Autry (1991) points out that the journal "is the product of two contradictory genres--the commonplace book and the diary" (p. 74) where the former may be a rehearsal for an intended audience and the latter is for private consumption. Accordingly, journal entries may not reveal spontaneous and sincere struggles to learn because of the intended audience, unless a risk-taking and trusting atmosphere has been established.

Anderson (1992), too, while acknowledging the role of journal writing to enhance learning, warns of some problems associated with their use, such as overuse of the genre, ethical issues, aversion to writing, problem of grading, lack of analysis, lack of synthesis, lack of growth in writing ability, and writing for the teacher.

**Mathematics Journals**

In this section, I review literature on the use of mathematics journals. Journal writing at all levels involved some or all of the following: (a) writing explanations of mathematical definitions, procedures or solutions, (b) writing students' feelings about, and attitudes towards, mathematics and (c) writing summaries. Examples of how these types of writing were used at different age levels and what benefits were
derived are given next. Note that in most studies, more than one type of writing was used, but in the examples below, I am highlighting only one type of writing.

**Writing explanations of mathematical definitions, procedures or solutions**

Watson (1980) asked her second-year algebra classes to write explanations in their journals about specific topics that had been just taught and at other times to write without her specifying the topics. She reports that journal writing encouraged student reflection and also served as a two-way communication between herself and her students. Vukovich (1985) reports similar benefits about the use of weekly journals in a basic mathematics program for college students but in addition she reports that the journals enabled the teacher to identify students who needed help.

Selfe, Petersen and Nahrgang (1986) asked their college students to explain in their journals concepts studied in their analytic geometry and calculus courses. Although their study used an experimental design and multiple measures, quantitative measures did not give a clear indication of whether journal writing affected mathematical learning. However, analysis of the qualitative data (open ended questions and journal entries) enabled them to conclude that students had demonstrated an understanding of mathematics concepts. In addition, they suggest that using students' own words enabled students to view mathematical ideas less abstractly.

Bemiller (1987) used what he called a mathematics workbook for his college students. He used both expressive and transactional writing (Britton et al., 1975). For the three types of transactional writing Bemiller
assigned, students did directed writing during class time, conceptual writing outside of class and mini-reports on assigned problems. He reports that writing in journals enabled students to clarify and concretise their mathematics as well as allowed for individualised learning. He states that the journals allow the instructor to monitor both the “academic progress and personal growth” (p. 366) of the student.

Mett (1987) used daily journals to help her students learn business calculus. She asked her students to explain, as if to a precalculus student, concepts such as average and instantaneous velocity. She reports that writing such explanations prepared students for, and helped students to understand, more difficult but related concepts such as differentiation. She states that student-teacher communication was improved through these journals.

Schubert (1987) worked with Grade 4 students and used journals for a unit on fractions. Students wrote in their journals an explanation of specific mathematical concepts, principles or operations in their own words. Not only did her students find word problems less intimidating, but their use of appropriate mathematical terms improved considerably. When she compared the mathematics test scores of students who had used journals in their mathematics class with those who had not, even after a year, she found that the mean score of the former were better than those of the latter. She found the student journal entries and her responses “a way for me to keep up with the life of each of these fourth grade children in a sympathetic, understanding way” (p. 356).
Writing students' feelings about, and attitudes towards, mathematics

Brandau (1990) worked with elementary school teachers and student teachers. She used journals to explore students' feelings about mathematics, as well as students' feelings and emotions about solving mathematical problems. This she did by asking them to solve problems of their choice and write down their feelings as they attempted to solve these problems. A number of problems had no solution given, either by her or by the textbook from which the problems were chosen. She reports that students' confidence in doing mathematics problems increased and that the instructor received feedback about student difficulties.

Powell and Lopez (1989) used journals for college-level students enrolled in the developmental mathematics course (designed for mathematically-underprepared college students). Students wrote what they felt about their learning of mathematics in general or on the mathematics course itself. They also wrote about discoveries they had made about mathematical patterns, relationships and procedures. They report that journals provided an effective two-way communication between the instructor and the students, that students developed more confidence in their own ideas, and that students moved from expressive to transactional writing.

Rose (1989, 1990) used journal writing for college students taking her calculus course. Her students were required to write at least as many one-page entries as there were calculus classes attended. She reports that students wrote more about their feelings about mathematics and the course and less about mathematics itself. Yet such writing encouraged students' reflective thought, provided a diagnostic tool as well as
feedback on the course for the teacher, and allowed a more personalized approach to the teaching-learning interaction.

Tobias (1989), in studying reasons for mathematics anxiety, used journals for peers (from fields outside mathematics) and students (from secondary and tertiary levels) to explore negative feelings about mathematics by using “divided-page” exercises (pp. 51-52). The left-hand page was for the feelings (for example, struggles and false starts) as they worked through problems and the right-hand page was for the actual working of the correct solution. She reports that such writing allowed (a) students to move from unfocused “emotional detritus” (p. 52) to ways of identifying and overcoming their misconceptions and difficulties in mathematics and (b) instructors to get valuable feedback.

Burton (1985) refers to students in preservice and in-service mathematics education courses and recommends recording personal reactions to memorable lessons (either particularly difficult or interesting lessons) in students' and instructors' journals as soon as possible after the lesson, preferably towards the end of a lesson. According to her, such immediate recording of reactions by students and instructors can be used for reflection later on, as “the expressed emotions of the experience are relived and the cognitive aspects recalled” (p. 41).

Stempien and Borasi (1985) required their preservice teachers to keep weekly journals and share their writings with their peers and the instructors. The student teachers found that keeping journals about their feelings about mathematics, the mathematics education classes and their experiences in the practicum (student teaching) and sharing their writings encouraged communication and made them aware that mathematics was not controversy-free. The instructors found the journals
provided valuable feedback on their teaching and also helped establish better rapport with the students.

Shaw (1983) suggests using daily logs for junior high school students to record their feelings about the mathematics lessons and to write a review of the daily events in these logs. She reports that such logs allow the teacher to get to know the students better and to receive feedback on the teaching. In addition, she states that students become more aware that what they write has to be understandable to everyone, as they are required to take turns reading their entries to the class.

**Writing summaries**

Clarke, Waywood, and Stephens (1992) conducted a 4-year longitudinal study involving about 500 junior secondary school girls (all the students from Grades 7 through 11 from an all-girls school in Australia). The students wrote in their journals after every mathematics lesson as a major part of their homework. They were asked to write a summary of the main ideas of the day's lesson, together with some comments and questions. Clarke et al. report that the journal entries could be classified under the categories (a) narrative, (b) summary and (c) dialogue. The narrative category refers to students recounting, without evaluative or thoughtful questions, what transpired in the lesson. The summary refers to a synthesis of knowledge useful for the learner in terms of examination preparation or content mastery by recognizing and sequencing important ideas, and the dialogue refers to creating and recreating knowledge through posing and attempting to answer questions leading to more sophisticated knowledge. They report that the
summary category was most prevalent, implying a utilitarian view towards the learning of mathematics by most of the students.

McMillen (1986) studied college students and reported the use of double-entry journals, where students wrote summaries of their readings on one page, with their reaction to the readings on the facing page. She reports that such journals help students understand the mathematical content better as well as encourage them to be more critical. Indeed, students could even synthesize their notes to create their own textbooks.

King (1982), in describing transactional writing tasks for high school and college level students in mathematics, regards making summaries of mathematics lessons as helpful for students to identify, focus and synthesize their mathematical learning. Johnson (1983) states that even rewriting an unclear page or paragraph of an algebra or calculus textbook or a word problem can be perceived as writing a summary, as it entails similar skills to those suggested by King.

Talman (1990) required college students to keep weekly journals, containing three kinds of writing: summary of topics covered during the week, a report of students' own relevant activities during the week, and an analysis of the week's work. He also expected an analysis of the solution to at least one problem not solved in class, and in the week following an examination, part of the analysis was an analysis of the examination and their performance in it. He reports enhanced confidence in their own mathematical abilities and better performance in mathematics problems and tests.
In summary, using journals in mathematics classes seems to have the following benefits:


3. Ability to solve problems by themselves (King, 1982; Powell, 1986; Watson, 1980).


5. Enhanced student-student interaction and cooperative learning (Stempien & Borasi, 1985; Vukovich, 1985).


Some Cautions about Mathematics Journals

In spite of the optimistic reports on the benefits of mathematics journals, there are some dissenting voices. For example, Mildren (1992), investigated the use of journals among 120 Australian children from
Grades 4, 5 and 6 over a period of between six and ten months. He used the headings “What I did” (to obtain student reflection), “What I learnt” (to obtain knowledge about student knowledge and beliefs) and “Problems and difficulties” (to obtain knowledge of attitudes) in order to initiate the entries. Mildren reports that none of the entries included reasons for statements and very few attitude statements were made. He inferred that such results could be reflecting classroom practice where justification for statements were not asked and student attitudes towards mathematics were not considered important. In short, he says that the classroom mathematics program was text-centred rather than student-centred (p. 88).

Clarke, Waywood, and Stephens (1992) caution that self-reporting of the value of journals by students may not be indicative of their benefits. Waywood is presently working on ways to corroborate inferences about the nature of students' mathematical writing. I believe that one of the reasons for being unsure of the veracity of students' statements is a lack of rapport between the teacher or researcher and the students (for example, the students may be afraid of writing down negative feelings about something they perceive the teacher values). Moreover, when marks are given for journal entries, as Clarke et al. (1992) did, there could be a tendency to be more careful and less spontaneous.

Ormell (1992) suggests that journal writing might not reveal the extent of, or student involvement in mathematical learning. For example, he criticizes the examples of journal entries in Mildren's (1992) study, saying that the entries had "virtually no sense of internalised meaning" (p. 230).
Some researchers look to quantitative studies in journal writing to give evidence of mathematical learning through journal writing. Selfe et al. (1986) embarked on just such an enterprise, but they did not find any "hard" evidence, one way or the other. They warn that not only are quantitative measures about journals enhancing mathematics learning difficult to come by, but such measures have to be viewed with caution, because the questions answered by such quantitative measures may not be answers to the substantive questions raised in the first place (p. 202). According to them, they obtained more valuable answers from the qualitative analysis employed, and they suggest that a combination of quantitative and qualitative measures be employed in future research.

For teachers, one of the main drawbacks about the use of mathematics journals (or other journals, for that matter), is the amount of time required to respond effectively to such writing. Among the suggested solutions are: random collection of a limited number of journal entries every day (Schubert, 1987); using journals only for classes that are not too large (Talman, 1990); using peer responses and discussion (Kirby & Liner, 1988); and requiring only short, in-class writing (Powell, 1986).

**Student-constructed questions (SCQ)**

The literature on problem solving in general, and word problems in particular, covers a wide range of factors associated with the difficulties students and adults find in trying to solve these problems. For example, some variables commonly studied are the number of words in a problem, the presence of cues or key words, the semantic structure of the problem and order of presentation of the given numbers (Carpenter, 1985; De
Corte & Verschaffel, 1991; Krutetskii, 1976; Riley, Greeno, & Heller, 1983). In these studies, the emphasis has been on the solving of the problems, rather than on students constructing their own problems to use as a learning tool. Because of this emphasis, there has been very little research on the use of SCQ in mathematics or the effectiveness of SCQ as a learning tool in mathematics.

A notable exception to the problem-solving focus has been the problem-posing emphasis of Brown and Walter (1990), who organised a whole graduate course around problem-posing, considering problem-posing as “a worthy activity in its own right” (Pimm, 1987, p. 205). According to Brown and Walter (1990), “essentially no understanding can take place without some effort at problem generation” (p. 139). Also, generating a problem reduces mathematics anxiety “because posing of problems or asking of questions is potentially less threatening than answering them” (p. 140).

According to Willoughby (1990, p. 56), the process of formulating problems, even as a member in a group, helps students understand problems created by others. He also states that in industry and in real life situations, “The fact that the mathematician knows an answer is of absolutely no interest unless the mathematician can communicate the answer to someone who will use it. . . . Communication is, in many respects, the most important part of mathematics” (p. 55). Such views were echoed years ago by Einstein and Infield (1938) about science when they said

The formulation of a problem is often more essential than its solution, which may be merely a matter of mathematical or experimental skill. To raise new questions, new possibilities, to
regard old problems from a new angle, requires creative imagination and marks real advances in science. (p. 29)

The largest study on problem-posing (and subsequent solving) in mathematics has been the investigation by Ellerton (1989). She asked 10,500 secondary school students in Australia and New Zealand to make up a difficult mathematics problem, and then solve it. She found that they used problems similar to the ones encountered in class, and rarely referred to real world contexts.

King (1982) asked secondary and college level students to write word problems of their own to demonstrate their understanding of mathematical concepts. According to her, writing their own word problems tended to enhance communication as they had to write clear, specific and complete instructions. A prerequisite for such communication was a good understanding of underlying mathematical concepts.

Johnson (1983) asked students in his basic algebra class to rewrite story problems or construct their own problems. By rewriting story problems, students became aware of key words and relationships. By writing their own problems, students became aware of the contents of the question as well as how to solve the questions. Abel (1987) and Ackerman (1987) conducted studies with college students, similar to that of Johnson (1983) and found similar benefits (such as awareness of mathematical relationships).

In Kennedy's (1985) study, middle school students were sometimes supplied data on which they were to prepare questions. At other times, both data and questions were prepared by the students. Sometimes he also asked them to prepare problems with insufficient
information. When the questions were put forward for class discussion, clarifications were sought and given, thus promoting critical thinking and learning.

Stempie and Borasi (1985) report that when students were given some situations where they had to relate numerical data to mathematical operations or principles, they chose to share experientially-embedded contexts relating to real-life applications. Such practice seemed to increase confidence in tackling word problems from textbooks. Their findings about context-embedded questions contrast with Ellerton's (1989) study.

Hodgin (1987) reports that her elementary school students constructed their own problems based on a picture or cartoon. After writing the question, discussion and evaluation (in pairs) took place. Then, after appropriate revision if necessary by the students themselves, the questions were displayed on the class bulletin board. Students seemed to enjoy this activity. Moreover, they learned to apply mathematical concepts by constructing these problems.

Instead of SCQ for classroom exercises, some researchers have encouraged the use of student-constructed tests. For example, Clarke, Clarke, & Lovitt (1990) have used tests made up partly of student-constructed test items duly edited. Students seemed to find this a very effective revision strategy and they also felt a sense of participation in the assessment process.
Significance of the Study

Previous researchers have suggested a number of benefits for learners arising out of using writing in mathematics classrooms. Among the suggested benefits are: (a) encouraging students to express their own experiences and language allows for more autonomy and ownership of learning, (b) individualising learning in a group setting because every child is involved at his or her level of understanding and pace, (c) monitoring and diagnosing children's mathematical progress and (d) developing an interest in mathematics.

These suggested benefits seem to emphasize the affective domain. In contrast, I am investigating the possibility of specific mathematical (cognitive) benefits through writing to learn. The two types of writing tasks (the SJ and the SCQ), focus on specific mathematical knowledge and the extent to which such knowledge can be communicated. By focusing on these two writing tasks, I am also addressing criticisms about the trivial mathematics that seems to be reflected in student journals (e.g. Caughey & Stephens, 1987; Ormell, 1992; Pengelly, 1990). Because this study also emphasizes communication of mathematical ideas through written words, it should enhance knowledge about language factors affecting the learning of mathematics. Moreover, the SJ and SCQ (as used here) have never been used together in previous studies, and should thus be able to provide data to form a broader picture of how different writing tasks in the classroom might help the learning of mathematics.

There has been considerable research on journal writing at secondary and tertiary levels, but not many studies have been conducted at the elementary school level. Results of studies conducted at
secondary and tertiary levels might not be applicable to students at the elementary school level (for example, because of differences in writing experiences and levels of language proficiency) and so this study should augment existing knowledge about writing to learn mathematics at the elementary school level.

Previous studies were usually conducted and reported by the researcher as teacher (e.g. Rose, 1989; Selfe et al., 1986; Stempien & Borasi, 1985) at the tertiary level. In this study, the researcher (who does not teach the class) reports on an elementary classroom teacher who initiates and conducts the mathematics lessons using writing as a routine component of the mathematics lesson.

While Brown and Walter (1990) and Einstein and Infield (1938) talk about problem-posing in a more global manner (for example, by changing certain initial or boundary conditions), I confine myself in this study to word problems involving common and decimal fractions. Another difference from previous studies on problem posing is that the word problems constructed by the students are used as classroom exercises in this study. In contrast, Ellerton's (1989) study is about students constructing problems for the researcher to analyze and not for students to attempt as class exercises. Although Ellerton's sample is large, she studied only one question per child. This study uses a small sample but investigates a range of questions over a 15-week period. Moreover, this study involves Grade 6 children, in contrast to Ellerton's secondary school students. Even where the students' problems were used in the classroom (e.g. Abel, 1987; Kennedy, 1985; King, 1982), they were at secondary and tertiary levels and were used peripherally and
sporadically and not centrally, or as an ongoing component of the mathematics lesson over a 15-week period, as in this study.

Whereas King (1982) dealt with secondary and college level students and topics, this study focuses on elementary school students and topics. This study also has students generating their own word problems, but the students are much younger and do not have as much exposure to word problems as the students in the studies described by Johnson (1983), Abel (1987) and Ackerman (1987). In this study, too, the focus is not on the discussion generated by the SCQ as in Kennedy's (1985) study but is on the mathematical learning reflected through students' written questions. Unlike the study by Stempien and Borasi (1985) where numerical data are given by the teacher for students to use in their problems, in this study generally the numbers in the SCQ are chosen by the students themselves. While Hodgin's (1987) study reports on problems constructed by elementary school children, these problems were based on pictures and cartoons, in contrast to the SCQ in this study which are based on the teacher's verbal prompts such as "write a word problem involving common and decimal fractions."

Teachers may find this study significant for the following reasons:

1. The study documents an attempt to encourage students to take more responsibility for their own learning and to become active participants in the learning of mathematics by generating their own ideas and constructing their own meanings rather than treating mathematics as a spectator sport.

2. Valuable information on students' understanding of, and difficulties in, mathematics might be obtained by examining what students write in their SJ and SCQ.
3. Some ideas on implementing change in the mathematics class might be obtained by examining how SJ and SCQ were initiated in this study.

4. Teachers might become more aware of discrepancies in student- and teacher-perceptions on the usefulness of certain innovations and try to address these discrepancies.

Conclusion

In this chapter, I have tried to relate writing to learning, first by briefly surveying the evidence that writing supports learning in general and learning mathematics in particular. Then, I gave theoretical and empirical support to the notion of writing to learn mathematics based on the literature in various disciplines and from various perspectives. Finally, I reviewed the literature on how journals and student-constructed questions seem to help the learning of mathematics, and at the same time attempted to clarify how this study seeks to extend those previously done.
CHAPTER 3. METHOD

In this chapter, I discuss how I conducted the study, by describing the context of the study and the procedures for collection and analysis of data. In order to contextualize the study, I first describe my own background and the teacher’s and then explain my basis for selecting this particular group of students for study. Then I detail the procedures for collecting and analyzing the data.

Context of the study

My Background

On the basis of good scores on school and public examinations, I had been considered a good student in my school days. It was only when I began my undergraduate studies in mathematics that I realized that I owed much of my success in examinations to a good memory and procedural knowledge (Hiebert, 1986). So, after my undergraduate studies, I decided to pursue an M.A. in mathematics education rather than in mathematics per se as I became interested in why many children did not seem to be learning mathematics successfully, even though they seemed to succeed at other school subjects. As well, because language learning fascinated me, I pursued an advanced diploma in Teaching of English as a Second Language (TESL). In addition, I have had more than 12 years of experience in working with high school students and preservice elementary and junior high school teachers who have had a negative attitude towards mathematics but a positive attitude towards language. These interests in language and mathematics education have
led me to pursue doctoral studies, with a focus on how using written language might assist the learning of mathematics.

Next, I describe that part of a previous research project which involved the particular teacher who subsequently agreed to help me in the present study.

The Previous Project

The project, funded by the Social Sciences and Humanities Research Council of Canada (SSHRC, Grant No. 410-90-1369) and conducted by Dr. Douglas Owens of the University of British Columbia (UBC), lasted from January 23 to March 22, 1991.

The project supported a substitute teacher for the cooperating teacher, Linda, to be released periodically for half day planning sessions with us. We developed tasks in the form of worksheets for class use. Unlike the usual use of worksheets, where pupils tried to complete as many exercises as possible, these worksheets were to be taken as starting points for class discussion. For example, students would work on one problem, or a series of related problems, either individually or in groups and then Linda would lead a class discussion on their solutions or difficulties. Manipulatives such as fraction strips, flats (10 x 10 cm grid paper), longs (10 x 1 cm strips) and cubes (wooden 1 cm cubes) were used to introduce, develop and relate common and decimal fraction concepts and operations.

Students also wrote in their mathematics "learning logs" (Linda's form of journal), usually once a week, (but sometimes once every two weeks) about how they felt about the mathematics lessons and tried to answer the teacher's question about "how can you help me teach you?"
Linda responded to all the journal entries. Also, once a week students prepared their own questions (and solutions) in groups and Linda selected student-prepared questions for group and class discussion.

Lessons were video taped, as were four interviews with six children (who were recommended by Linda as representative of a range of mathematical ability). Linda and the researchers evaluated the project towards the end of July, 1992, after we had viewed the video tapes.

**The Teacher's Background**

Linda, the teacher, started teaching in 1966 but had no opportunities to attend in-service mathematics courses during her early teaching career. She reached a turning point in her teaching outlook when, in the summer of 1990, she attended a professional development course on whole language which had one session on mathematics. Even though the course made her more aware of her teaching philosophy, most of the mathematics part of it was “just filed away” without any connection to her teaching practices in mathematics.

The second turning point came after she attended a professional development course in the Fall of 1990 on the implementation of the Year 2000 document, a major curriculum change in British Columbia. At this point she felt that she should change her teaching to try to establish a classroom environment conducive to risk taking.

However, she was acutely aware that she needed more confidence in teaching mathematics, especially the topic of common and decimal fractions. So when she was approached to take part in the project on students’ understanding of common and decimal fractions, she agreed enthusiastically “because division with decimals was the most
difficult concept I tried to teach my Grade 6 class last year.” But she made it clear that she would need a lot of support from us (the project supervisor and me). In spite of her interest in division of decimal fractions, by the time the project was completed, it turned out that we included decimal multiplication, not decimal division.

**Linda's Growing Professional Development**

Initially, Linda viewed us as experts and herself as a novice. Also, her early lack of confidence made her very dependent on our ideas on what would be suitable as lessons and worksheets. However, she did suggest that she needed an overview of the lessons before getting into specific lessons. During the planning stages she wrote down which concept was to be the focus, which worksheet would go with which lesson and so on. We prepared all worksheets but before using any worksheet, Linda examined it thoroughly. Throughout the project she rarely used the textbook as most lessons were based on the worksheets. These worksheet exercises were meant to initiate discussion whereas textbook exercises were usually practice exercises.

During the lessons she consulted with us on anything about which she was unsure. She said that she was confident of her procedural knowledge but was unsure of some of the different ways of conceptualizing and relating common and decimal fractions. For example, she said that she had “never thought of 0.2 x 0.3 as 2/10 x 3/10” and just placed the decimal point in the answer 0.06 procedurally, by using the rule of counting the number of places after the decimal point.

As the lessons progressed, Linda became less and less dependent on us. For example, at one stage we asked her whether she
would like to look at the worksheet for the next day and she replied that she did not feel she needed to go over it with us as closely as she used to before. According to her, there were two crucial points when she started becoming more confident of herself: one, when she saw what children could do and learn while preparing their own questions (during the fifth lesson) and the other, when she saw the children's excitement and involvement when they tried to find their own rule for placing the decimal point in the product of two decimal fractions.

She found students highly motivated while preparing questions for their classmates to answer. They also asked and answered more difficult questions than they were accustomed to. In trying to find the rule for the placement of the decimal point, children were so involved that they lost track of time. Students were encouraged to use, as Linda put it, a “think/pair/share strategy” where they tried to solve the problem themselves and then compared and justified their procedure and solution with a partner.

She also had to respond to unexpected and unplanned situations. Instead of being dismayed by this, she began to enjoy “learning together” with the pupils. She became more and more comfortable with children setting the pace and direction of the lessons. In short, instead of addressing “your (the researcher’s) agenda,” she did not mind being “sidetracked” by her students, and began to assume more ownership of the project as the days passed. Linda had this to say about the lessons on multiplication of decimal fractions in her report on the project:

My teaching method underwent radical change during these lessons. In the past when I “taught” this concept, I would present and explain the algorithm, guide the practice and assign some independent practice. This time I did not tell the students how to
multiply decimal fractions. They were asked to struggle and mess around with the problems and try to make a little sense out of multiplying decimals. I was never sure what direction the lesson would take or what the outcome might be. One student reported in her log that this search for the rule made her frustrated and angry at first, but excited and proud when she was successful. The quiet murmur of OH!s was quite wonderful.

A key factor in Linda's growing confidence was her excellent rapport with the class and her willingness to cultivate a risk-taking environment, without fear of children or of herself making mistakes. Her de-emphasis on getting the right answer also helped pupils to feel comfortable about "messing around," without having to worry about the answer or about completing the problem. Often she celebrated different answers by saying "Aha, we have someone with a different answer, let's hear this!" or she even expressed disappointment when there was no disagreement!

Because of her participation in the project, she said:

1. She increased her confidence in teaching mathematics conceptually.
2. She would never separate the teaching of common fractions from decimal fractions: these topics would be taught together.
3. Student journals and student constructed questions had good potential as learning tools.
4. Her experience in the project encouraged her to participate in similar projects and share her experience with colleagues.
5. She could study problems in her teaching-learning situation more systematically and collaboratively, if not with university researchers, at least with other teachers.
6. Student interviews were informative, and viewing videotapes of such interviews would help plan future lessons. Alternatively, such
interviews should somehow be incorporated into the teaching-learning process.

7. Spending a longer time discussing rather than telling, helped students develop mathematics more meaningfully.

8. Manipulatives not only helped develop mathematical concepts, they became a "communicative tool, after learning the concepts" by their use to explain and clarify concepts.

Her professional development continued to grow, even after the project was over. Below are some activities indicating her continued professional growth and empowerment.

1. She taught division of decimal fractions herself--a topic she had no confidence in teaching prior to her involvement in the project--with minimal support from us.

2. She shared her project experiences at a workshop, because she felt like a "born again mathematics teacher."

3. She enrolled for a Master's program in Curriculum and Instruction, specifically to improve her ability to undertake classroom-based research that might help her become a better teacher.

4. One year later, she attended a mathematics education course and gathered data from her class as part of her project for the course.

5. She conducted regular, short, in-class interviews ("oral" journals) with her students during journal writing time.

6. She video taped some of her mathematics lessons to get feedback and plan for future lessons.

7. She agreed to participate in data collection for my doctoral study.
Linda's Commitment

It is important to recognize that this teacher had very good rapport with her class, and that could have contributed to the students' enhanced understanding of common and decimal fraction concepts studied in the research project. Moreover, she felt that she had developed professionally as a result of her involvement with the project. Indeed, she said she was positively “evangelical” about her experiences with the project, and wanted to get involved in other projects that might enhance her professional development. In other words, I had a very committed teacher who was willing to try some innovative approaches.

The Present Research Setting

While actively involved in the research project mentioned earlier as a research assistant investigating middle school children's concepts of common and decimal fractions, I piloted the use of student journals (SJ) and student-constructed questions (SCQ). The SJ used previously were designed to provide feedback to the teacher on student's difficulties and feelings about mathematics. Moreover, they were used as and when time permitted or when the teacher thought it appropriate to use them. I found that using the SJ to get feedback once a week or so was not beneficial and neither were the student entries allowing me to gauge their understanding or communication of their understanding of mathematics. Hence, I decided that for this study I would attempt a more systematic approach to the SJ (such as three times a week, with specific teacher prompts focused mostly on mathematics than on feelings about mathematics).
The SCQ in the previous study were prepared in groups and were used mainly to initiate discussion, but again they were not used routinely in the class. Students evidenced enthusiasm in preparing the SCQ, but sometimes they tended to give problems that they themselves were not sure how to solve. Moreover, certain students dominated the discussion and not every group member contributed to the preparation of the SCQ. So I decided that for this study I would attempt SCQ once a week, with both group and individually-constructed SCQ, but with only the latter to be used for data analysis. Hence, from the previous project experiences in a Grade 6 class for a period of about 2 months and then in a Grade 4 and 5 (combined) class, for another 2 months, I became more aware of how to refine and use SJ and SCQ and also gained more confidence in the viability and significance of my proposed study. The Grade 6 teacher, Linda, also offered to help me for this study.

In this study, Linda was teaching a Grade 5 and 6 combined class. At the beginning of the study she told me that she had yet to establish the rapport she had with her previous Grade 6 class. At the end of the study, she said that she still felt her rapport with her previous class was much better. She thought that one of the reasons for her lack of rapport had something to do with the different mix of students. For example, previously she had four or five "bright" students who acted as "sparks" and "catalysts" for initiating and leading discussions whereas in the present class there was only one such student. In addition, she had no ESL (English as a Second Language) students the previous year.

Linda had agreed to participate in the study because of her continued commitment to try to improve her students' mathematical learning while working within the usual constraints of time and topic...
coverage. Moreover, based on her experience in the previous project, she felt that the SJ and SCQ might be beneficial for the learning of mathematics.

It should be emphasized that the learning log used previously was mainly to give Linda feedback on her teaching whereas the SJ used here focused on how the students could communicate mathematical ideas through written explanations. The SCQ in the previous project were used to initiate discussion, and were used sporadically, whereas those in the present study were used as class exercises (with and without discussion) on a regular basis.

In the present study, Linda started with the same topics that she had taught in the previous project, namely common and decimal fractions. Linda and I agreed that because she had taught these topics in the previous project, she could use her prior experience to advantage, and perhaps be more systematic than before in the use of SJ and SCQ.

Linda was free to discuss with me (before and after classes) possible approaches to instruction and types of resources that might be made available for her. I did not teach the class, but observed the lessons and kept notes of salient aspects of the lessons and discussions.

**Student Participants**

Some students had exposure to journal writing in a mathematics class before, but their experience was so minimal (about three or four times a year) as to be negligible. Neither had any student ever used SCQ in previous classes. As well, none of the students had Linda as their teacher before.
All students in the combined Grade 5 (n = 15) and 6 (n = 12) class took part in the mathematics lessons, but I chose to focus on the Grade 6 children (6 girls, 6 boys) because, according to the teacher, most of her Grade 6 children have had sufficient experiences with language and writing for the purposes of this study. Moreover, four of the 15 fifth graders children were ESL (English as a Second Language) students who had to attend extra English lessons which were scheduled during some of the mathematics periods.

I used the SJ and SCQ data from 11 Grade 6 children in the class, but selected six of these students (3 girls, 3 boys) for individual interviews. (One student returned to class after one and a half months, on March 23, three weeks before the end of the study, so I did not include him in the data analysis.) My sample selection for the interviews can be thought of as reputational-case sampling (Goetz & LeCompte, 1984). The interview data were meant to clarify and complement the SJ and SCQ data, as well as answer research question number three. The choice of the six Grade 6 children was dependent on a number of factors, such as the following:

1. Their writing in the SJ and SCQ. I wanted a range of SJ entries, from those by students who were not very fluent in expressing their mathematical ideas in written words to those who were fluent. For the SCQ, I wanted a range of word problems from brief word problems involving one or two operations to long, multi-step word problems with extraneous information.

2. A range of mathematical and writing abilities as identified by the teacher, based on her observations of, and interaction with her
students, such as students who were considered good at writing but weak in mathematics and vice versa.

From these criteria, I believed that the interview data drawn from the six students would be sufficient to analyze and interpret the unit of analysis. Note that this is a case study where the unit of analysis is a phenomenon, an approach (using SJ and SCQ in a mathematics classroom) and not a particular child and this unit of analysis is what I would like "to be able to say something about at the end of the study" (Patton, 1980, p. 100).

Teaching-Learning Environment

To give a better picture of Linda's approach and her rapport with the students, I describe the teaching-learning environment. Linda generally used heterogeneous groups of four to five students in her mathematics classes, and changed group members every month. She used a number of criteria in deciding the membership of any group, and tried getting a good mix of (a) assertive and non-assertive students, (b) girls and boys, (c) language ability and (d) mathematics ability.

She used mainly discussion and manipulatives to teach common and decimal fractions. When students were given exercises for discussion in groups, she insisted that "everyone in the group has to be able to explain." In order to emphasize the relationship between common and decimal fractions, she encouraged the reading of decimal fractions in common fraction notation (e.g. 0.3 was to be verbalised as "three tenths"), by asking all the other students to literally hold their hands out and point at anyone who read 0.3 as "zero point three," just as the "point" was read. The students, including the one who had read "point",
seemed to treat this action as a jocular reminder of how to read the
decimal fraction.

Linda used “good” questions (Clarke et al., 1990) effectively. Note
that for Clarke et al. (1990), a question does not necessarily end with a
question mark and a good question enables students to learn and the
teacher to know more about the student as well as allows for more than
one correct answer. For example, she asked “in your groups, find out as
many different names or equivalent fractions for \(1 \frac{7}{10}\).” Generally she
used such “questions” to encourage exploration and a variety of
answers.

Sometimes she got student attention by saying “it’s ‘can I trick you
now’ time” and giving them some challenging questions. She constantly
reminded students that she wanted to “see your thinking,” or “explain so
that it makes sense.” Some expressions she used in class were: I don’t
understand that; give me an example. Who are confused? Good, people
who are confused will learn something. You are stuck? Good, those
sitting next can ask some good questions to help those stuck. I’m
interested in how you think, not just what the answer is. Convince me,
justify your answer.

Most of the expressions she used were designed to promote a
non-threatening, risk-taking atmosphere in the classroom. She
emphasized to students that explaining their thinking, even if they had an
answer they were unsure about, was more important than just getting the
solution correct. That her encouragement was not in vain was evidenced
by students who wrote that they learned “by making mistakes,” and that
learning through mistakes was a priority in their list of “important things in
the learning of mathematics.” I did not see any evidence of a student
afraid to voice his or her tentative solutions, or being embarrassed at
giving a wrong answer. Just as in her previous class, Linda celebrated
errors and disagreements, saying “good, we have some disagreement
here; let’s see why.”

In spite of her slight dissatisfaction with class rapport, I saw signs
to the contrary. For example, on April Fool's Day, she told the students
that cutbacks in funding had forced the school to lay off some teachers.
As a consequence, their class was to be dissolved and the students were
to be distributed over a number of other classes. (All this was said with a
straight face!) The students reluctantly packed their bags and books to
go to their new classes, unhappy to leave their friends or Linda. Then
Linda said “I’ve got something important to say. I hope all of you are
paying attention. Are you ready for this? April Fool's Day!” The students
laughed at being fooled (but relief was written on their faces at not having
to leave their present class) while some first muttered good-naturedly,
and then directly to her “We'll get you for this!” Indeed, Linda agreed that
they could play an April Fool's Day trick on her, as long as it was not
dangerous or messy (in the sense of dirtying the classroom). All in all,
Linda’s trick was taken in good humour, and the class settled back to
their mathematics lesson without undue delay.

Another example showing good rapport was a question
constructed by the students. They were to make up a word problem
involving multiplication of the numbers 32 and 0.125. Two students
made up the following problem and brought it for Linda's comments:
“Mrs. Lomax eats 32 chocolate bars a year. If each chocolate bar makes
her 0.125 lb heavier, how much weight does she gain in a year?”
Neither the students nor Linda (Mrs. Lomax) were uncomfortable with the
problem, a sign of the rapport between her and the students, indicating that the children did not mind joking with, and about, their teacher.

That they liked her teaching approach was evidenced by student comments (during interviews) such as:

- Mrs. Lomax tells us to write down how we think, what goes on in our brain. Teacher last year just told us to write down the answer. I like it this way.

- She doesn't give us work out of the book as often; she makes it fun for me. I don't have as much pressure as I used to. I feel comfortable in her class.

- She lets us figure out a lot of things ourselves, and I like that.

- I think Mrs. Lomax is the best person I have had to teach mathematics because the previous teacher used only the textbook, and there were people crying because they couldn't do division.

In addition to good rapport between the students and the teacher, Linda was also firm in that she made sure that students did involve themselves actively in learning tasks. For example, she checked that students had prepared their SCQ. The SCQ were collected and edited by Linda before distributing them to the class as typewritten exercises (with names of those who prepared the question written next to the relevant problem). These SCQ, together with a few teacher-prepared problems, became the source of exercises for the students. A few of these questions were attempted every day, first individually, and then through group discussion. Sometimes students were given a choice of doing the questions individually or in groups, but either way, they had to explain how they obtained the solution. The students seemed very animated
during the preparation of the SCQ. Students commented on their enjoyment of the SCQ in their journals as well as during interviews.

The SJ were used on alternate days (three times a week) in the mathematics class. Students wrote in their SJ in response to specific teacher prompts such as "Explain why you agree or disagree that $0.8 = \frac{3}{5}$." Most of the time students wrote in their SJ towards the end of the mathematics lesson (the last five to nine minutes). All individually-constructed SCQ were also written in the SJ, but students were given more time to do this (ten to twelve minutes) than when explanations of mathematical concepts were required.

Very few textbook exercises were used for the duration of the study. One reason for this was that the teacher was attempting to use student-generated questions for discussion and as exercises. She also wanted to see how students would fare on standard textbook exercises, even though they followed an approach that minimised the role of the textbook. She found that the students thought that the textbook exercises were "no big deal"—that is, they had no difficulty with these exercises.

**Procedure**

The study covered a period of 15 weeks, from January 7, 1992, to April 14, 1992. I was in the class for three days a week, namely Mondays, Wednesdays and Fridays, except for the Spring break from March 16 to 20, 1992. I observed the mathematics lessons, but did not take part in teaching or other in-class activities such as group and class discussions. However, I kept notes of salient and significant events that took place. For example, I noted what happened in the mathematics lessons (e.g. discussion, the teaching-learning environment, use of
manipulatives, type of journal entry initiated, and interesting or insightful questions).

The students wrote in their SJ for an average of seven minutes a lesson (usually towards the end of the lesson) for the three days a week I was there. The teacher responded in writing to the entries once a week, with responses such as "How did you decide this?" and "Good thinking, Shaun." I did not respond to the students about their journal entries but I kept track of the entries in three ways: (a) by reading the entries as they wrote, while walking around the class; (b) by collecting and skimming through the SJ as soon as they had finished writing, and returning them to Linda within an hour; and (c) by collecting the journals and perusing them before the next journal entry was due. In all three instances, I kept notes about salient points I found in the SJ.

Although Linda and I discussed the journal entries informally and I summarized these discussions for my records, I did not offer, on a regular basis, any suggestions on what entries might be suitable. However, once or twice, when Linda asked for some suggestions on how to make the journal entries more interesting or mathematically relevant, I gave her some suggestions. For example, I suggested a possible way to get the students to write about what they thought mathematics was like. At times, Linda was worried that I was not getting the data that I wanted, but I assured her that I was quite satisfied, because what I wanted to know was what could be done to assist students' mathematical learning in a mathematics classroom with SJ and SCQ implemented by a practising teacher rather than a researcher from outside the school.

The students also wrote SCQ once a week, on Fridays, in groups of three or four and discussed their questions both in groups and in class.
I kept notes on these discussions, as well as copies of each group's questions (both edited and original versions). In addition, each person in the group was asked to write a question in their SJ, either similar to or different from the group question. I examined and analyzed these individually-constructed questions (which were in the SJ) of the Grade 6 students. I also looked through the SCQ prepared in groups but did not analyze them because these SCQ were prepared by both Grade 6 and Grade 5 students (and the latter were not the focus of my study).

I initially interviewed (and videotaped) nine of the Grade 6 children on February 5, 1992. I did not interview all 12 of the Grade 6 students because two were absent and one was hearing-impaired. I interviewed nine rather than my target of six because it was too early to use any of the criteria suggested earlier for the selection, and also because of possible attrition. Then, on March 4, 1992, together with the teacher I selected the six Grade 6 students based on the student writings in SJ and SCQ (using the criteria mentioned earlier) and interviewed them. Each student interview took about twenty minutes. There were three interviews per student, the first within four weeks into the study (with the nine students), the second within eight weeks into the study and the other towards the end of the study (April 14, 1992). Patton's (1987) suggestions about qualitative interviewing guided these semi-structured interviews which were designed to elaborate on SJ entries and SCQ, and to indicate answers to some of the research questions. Sample questions from all three interviews are in the appendix.

I interviewed Linda once (audio-recorded) on April 30, 1992, two weeks after the end of the study (because she was not free earlier), to find out her perceptions regarding the use of SJ and SCQ. But I also
kept notes of my informal discussions with her throughout the study and used these data to supplement data from the interview. Linda wanted to keep a journal of her own but could not because of her busy schedule. However, she did keep notes of topics taught and exercises given, including notes of lessons on the days I was not there. Her notes, together with mine, as well as the SJ, SCQ and the interview (students' and teacher's) transcripts served as a form of triangulation of data for the study.

Data analysis

Analysis of SJ

Analyses of the individually-constructed SCQ, students' written explanations in the SJ, and their verbal explanations in the interview, coupled with the notions of conceptual and procedural knowledge in mathematics (Hiebert, 1986) helped answer research question #1 (What mathematical knowledge do students reveal through their writing?)

In addition to prompts requesting explanations of mathematical concepts, some of the prompts for the SJ were about the role of SJ and SCQ in the mathematics class. Responses to such prompts, together with interview data, were analyzed using the constant comparative method (Glaser & Strauss, 1967), inductive analysis (Patton, 1987) and notions of convergence (Guba, 1978) as guides for the emergent categories, in order to answer research question #3 (What are the roles of SJ and SCQ in mathematics learning, according to (a) students? and (b) the teacher?)

If communication of mathematical knowledge is an indication of a mathematical competency (NCTM Curriculum and Evaluation Standards,
1989) and if communication plays a crucial part in the growth of mathematical knowledge (e.g. Ernest, 1991), then it would be appropriate to include students' use of language to communicate their knowledge of mathematics (fractions, in this instance) under research question #1 (What mathematical knowledge do students reveal through their writing?) Hence, I attempted to identify students' use of language to communicate their knowledge of fractions, using Freudenthal's (1978, pp. 233-242) levels of language. According to Freudenthal, new mathematical concepts are first articulated in words by using demonstrative language (just pointing at or giving examples), followed by relative language (showing an understanding of relationships), and finally by functional language (showing relationship between relationships, or making generalisations). I also analyzed the interview transcripts for evidence to support Freudenthal's classification. One of the reasons for choosing the SJ as a writing task in this study was to find out how students use words to communicate their mathematical knowledge to others, and so Freudenthal's levels of language provide an indication of this communicative ability (although there were no research questions per se on levels of language).

Analysis of SCQ

For an analysis of the SCQ, I looked for salient and recurrent features (themes) and knowledge of fractions evidenced by the individually-prepared SCQ. For example, were the SCQ becoming progressively more difficult in terms of the number of steps involved in their solution? These analyses helped answer research questions #1
and #2 (What are the salient and recurrent features (themes) of the SCQ, if any?)

Conclusion

In this chapter, in order to contextualize the study, I first described my background, the teacher's background, the reason for choosing the Grade 6 students for the study, and the teaching-learning environment. I also emphasized that the SJ and SCQ of all the 11 Grade 6 students were used for data analysis, but that the interview data from six of the Grade 6 students were used to complement data from the SJ and SCQ and to answer research question number three. Then I described the procedures for data collection and analysis.
CHAPTER 4. RESULTS AND DISCUSSION

In this chapter, I discuss results of this study in relation to the research questions. Before going any further, I wish to emphasize that the “case” in my study is the use of SJ and SCQ as part of a mathematics class. The work of the eleven Grade 6 students together with the interview data from the six Grade 6 students are just instances of the case “within which issues are indicated, discovered or studied so that a tolerably full understanding of the case is possible” (Adelman, Jenkins, & Kemmis, 1983, p. 3).

Below each research question, I interpret the Grade 6 students' written responses relating to that research question. I also include interview and other relevant data from my study, where necessary. Then I synthesize information from the eleven instances of the case, together with information from the interviews, and summarise the findings in relation to the research question. Note that in using students' responses, I attempt to present words they used, and avoid correcting spelling or other errors, unless clarity of meaning necessitates such corrections. To ensure anonymity, pseudonyms are used for the students and the teacher.

Research question #1 and results

Research question #1: What mathematical knowledge do students reveal through their writing?

By mathematical knowledge I mean knowledge about common and decimal fractions such as part-whole, equivalence and renaming. Even though the research question is on mathematical knowledge, I will
juxtapose my discussion of mathematical knowledge with the use of language to communicate mathematical understanding, as language is inextricably linked with thought (Vygotsky, 1962) and hence to understanding. Admittedly, mathematical knowledge and communication of mathematical knowledge can be seen as distinct from each other but the prevailing view of mathematics as socially-constructed knowledge (e.g. Bishop, 1985,1988; Ernest, 1991; Lampert, 1988) implies that communication is vital for the growth of such knowledge. Indeed, the NCTM Standards (1989) emphasizes that communication of mathematical ideas is an essential component of the mathematics curriculum. For discussing the communication of mathematical knowledge, I will use the levels of language suggested by Freudenthal (1978, pp. 233-242). According to him, such communication progresses through three levels of language: demonstrative (pointing out instances, without explanations), relative (using words to indicate relationship or procedures) and functional (generalisations or relationship between relationships).

**Part-whole (Common Fractions)**

To the prompt "What do you understand by fractions, for example, the fraction 3/8? Show 3/8 on the pink strip of paper," students typically wrote as in Figures 4.01 and 4.02.

All the pieces have to be equal. 3/8 of the parts you colour. It's a 8 strip and the three parts are called 3 8ths.

**Figure 4.01:** Jackie, January 7.
Fractions are mainly pieces of something. Take 3/8 for instance. It simply means 3 parts of 8 parts. So 5/8 would be 5 of 8 and 6/7 would be 6 of 7.

Figure 4.02: Mike, January 7.

Possibly because they had used such fraction strips in recent lessons, the students had no difficulty shading in 3 out of 8 equal parts on the pink strip of paper provided. The entries indicate that students had the notion that fractions had something to do with equal parts and parts of a whole and that there was some sort of comparison involved, namely a comparison of the part to the whole. This part-whole concept was illustrated by diagrams such as those in Figure 4.03 and Figure 4.04:

Figure 4.03: Sam, January 7.
The first entry illustrates a region model (geometric shapes divided into equal-sized parts) and this model was used predominantly by the students in explaining basic fraction concepts. The second entry illustrates a discrete model (where a number of elements in a subset is compared to the total number of elements in the whole set) shown by the circles and a region model shown by the rectangle. Indeed, the student who had the second entry was the only one who illustrated her explanation with a discrete model (as well as a region model). Even she, however, drew equal-sized circles, although the number of circles rather than the size of the circles determines the fraction represented by the discrete model (e.g. 3/8 of 8 students is 3 students, irrespective of the size of the students). Students' explanatory diagrams in the SJ indicate
that, for them, the part-whole concept is a comparison of *equal* parts to a whole, whether the parts are from a whole region or from a total number of objects. Since fraction concepts are usually taught using region models (where the parts are equal), it seems natural that most students use region models to represent fraction concepts and even if they use discrete models, equal-sized parts are used.

The part-whole concept also seems to be influenced by the image of a physical representation, as exemplified in Figures 4.05 and 4.06, in response to the prompt "Is half always equal to a half?"

---

No, because if you have a medium pizza, and a large pizza, then you cut out them in half, what you have is 2 larger pieces from the large than the medium.

---

**Figure 4.05:** Tommy, January 17.

---

No, because if one whole can be bigger than the other whole, so if the wholes are different, the 1/2 will be different too.

---

**Figure 4.06:** Shaun, January 17.

From Figure 4.05 and 4.06, students seem to know that the fraction "1/2" is indeed a relationship between a part and a whole. On the other hand, the part-whole relationship seems to be perceptually bound. For example, when they said that a half is not always equal to a half, they usually based their explanations on images such as two different-sized rectangles (or circles) indicating different sized wholes and comparing half of one rectangle with half of another rectangle. No student argued
that because the fraction 'half' was a relation of a part to a whole, in that sense, as long as that relationship held good, a half is always a half.

One student (Tanya), however, did seem to suggest that a half had different symbolic representations but retained its value (see Figure 4.07).

I think 1/2 is always equal to one half no matter what number you have you can still make it one half like 4/8, 3/6, 6/12.

Figure 4.07: Tanya, January 17.

But even she thought she had misunderstood the question and said that a half is not always equal to a half when Linda drew two different-sized rectangles and asked her to compare the halves in the two rectangles. It must be admitted that Linda did not point out the possibility of perceiving the symbol “1/2” as representing a number disembedded from concrete referents. Anyway, the crucial role of the whole seemed well-understood by all students.

In terms of levels of language for communicating the part-whole relationship, three students (whom the teacher had perceived as weak in mathematics), exemplified demonstrative language by drawing a diagram and writing that the diagram represented the fraction as in the responses shown in Figure 4.03 and Figure 4.04.

Five students used the relative level of language, for example, by relating the fraction to pieces of an object, as in Jackie's response (Figure 4.01). Three students used the functional level of language, for example, by giving some sort of generalisation, as shown by Mike's
response (Figure 4.02). Mike's explanation indicates a generalisation that fractions are a relationship between parts and wholes, without, however, explicitly stating that the parts have to be equal in size.

It must be noted that when I say that students used demonstrative language, I am saying that they used words not so much to explain but rather to point out instances or examples. For example, if a student were to explain what an even number is, the response “Two, four and six are even numbers” would be an example of the use of demonstrative language whereas the responses “a number that ends in 2, 4, 6, 8 or zero is an even number” and “an even number is any number which is divisible by 2” would be examples of the relative and functional levels of language, respectively. Furthermore, in saying that students used the demonstrative level of language, I am only indicating the level of language used to communicate the student's mathematical knowledge, and am not implying that use of demonstrative language indicates a lack of conceptual knowledge.

In summary, the SJ indicated that students generally used the region model to explain the part-whole relationship of fractions. Since most teaching of fraction concepts makes use of region models, it seems natural that such models dominate students' explanations of fractions. The equality of the parts was an important aspect of this explanation, even in the only instance when discrete models (which did not necessitate such equal-sized parts) were used. It seems to be the case that while students recognized the importance of the whole and the relationship of the part to the whole, they had difficulty disembedding the abstract concept of fractions from perceptual cues such as the size of the part (e.g. when they explained why a half is not always a half). It is
instructive that even though early (whole) number concepts are embedded in images—such as 2 apples, 2 books and 2 people—most students can abstract the ‘twoness’ of different sets of two objects by perceiving the number 2 as referring to the numerosity of a set rather than the type of objects in the set. Students find such disembeddedness more difficult in the case of the part-whole relationship in fractions, as evidenced by their perceptually bound explanations in the SJ. Such a difficulty could stem in part from the way fractions are taught (for example, by limiting the type of manipulatives used to region models). In terms of communicating the part-whole relationship through the SJ, the students seemed to indicate differences in communicative competence consistent with Freudenthal's (1978) levels of language.

**Equivalence and Renaming (Common Fractions)**

To probe students' understanding of equivalent fractions and renaming, they were asked to “explain how would you rename 1/2.” Three responses are given in Figures 4.08, 4.09 and 4.10. There seems to be a slight difference in the written explanations by Tommy and Jackie (the best in mathematics and the weakest in mathematics, respectively, according to Linda), see Figures 4.08 and 4.09. For example, though Tommy wrote 16 equivalent fractions, starting from 1/2 and successively multiplying both the numerator and denominator by two, and Jackie wrote fewer equivalent fractions, Tommy had two different ways of generating equivalent fractions (two “rules”) while Jackie had only one rule. Judging from the type of fractions generated, Tommy seems to be following an arithmetic algorithm whereas Jackie seems to be selecting specific fractions which are easily recognisable as equivalent to a half
(for example, 2/4, 3/6, 4/8 and 5/10) or are obtainable by following a pattern (for example, the fractions 3/6, 30/60 and 300/600 can be generated by affixing zeroes). Moreover, even though Jackie's use of the word "imagine" might indicate spatial partitioning, she drew only one diagram and interview data indicated that she could not partition a region, mentally or on paper, into a large number of equal parts. I would infer from these entries that both Tommy and she used the diagram for illustrating rather than for generating equivalent fractions. Another difference was in the type of language used. While Tommy used a functional level of language to express a generalisation, for example, by writing "I will choose an even number and divide it by two," Jackie used a relative level of language in trying to relate the equivalence to the size of the piece by writing "because if you imagine cutting it in half again it still is half." Their explanations seem to emphasize procedural rather than conceptual knowledge.

1/2, 2/4, ...32768/65536. I will choose an even number and divide by 2. I could also multiply both top and bottom by the same number.

---

Figure 4.08: Tommy, February 12.
1/2, half, 2/4, 3/6, 5/10, 4/8, 30/60, 300/600. 1/2 is = to 2/4 because if you imagine cutting it in half again it still is half, but 2/4.

Figure 4.09: Jackie, February 12.

1/2 = 2/4, 3/6, 6/12, 9/18, 12/24, 4/8; My procedure that happens in my head is you say start from the least number that is equal to 1/2. Then you think of a number that could be split in half like 2/4 s or 12/24.

Figure 4.10: Maureen, February 12.

Maureen's explanation (Figure 4.10) also shows the use of an arithmetic algorithm, but she seems to have made a generalisation in that she writes "start from the least number that is equal to 1/2." Her verbal explanation complemented by her use of two partitioned rectangles.
indicate that she is using both relative language and functional language. She seems to reflect conceptual and procedural knowledge.

Further examples of students' knowledge of equivalence and renaming are shown (see Figures 4.11, 4.12 and 4.13) when students were asked to justify whether 8/15 was to the left or to the right of 1/2 on a number line:

Today I learned that 8 would be more higher on the 8 15 because it would have say bigger pieces of pie.

Figure 4.11: Maureen, February 12.

Today I learned about 1/2. I learned that is half of 15 is 7 and a little bit. We were renaming. I also learned how number can be half.

Figure 4.12: Darlene, February 12
I learned today that \( \frac{8}{15} \) is more than \( \frac{1}{2} \). Because if you had \( \frac{1}{2} \) to share something with 15 people and you had to share some thing the same size with 16 people you would get more with 15.

![Example](image)

Figures 4.11 and 4.12 relate 8/15 to 1/2, with no explicit renaming in either one. The first compares the fractions with pieces of pie and also uses a number line (with the mark next to 1/2 unlabelled) with the whole renamed as 15/15 and the half to indicate 8/16 (as confirmed during class discussions). Maureen uses everyday language (such as “pieces of pie”) in her verbal explanation and then uses her explanation to answer the question which referred to a number line. Perhaps she is more comfortable using everyday language to explain the concept in words, before transferring her answer to the number line, rather than use the number line directly to answer the question. The second excerpt had no diagrams but I gathered from classroom discussions that Darlene perceived halving 15 as equivalent to dividing 15 by 2 and also that the expression “how number can be half” implied fractions equivalent to half. In the third excerpt, Sam does not explicitly state any relationship.

Figure 4.13: Sam, February 12.
between 8/15 and 1/2, but like Maureen, indicates sharing something, and uses circles, rectangles and a number line as illustrations. Sam uses relative language by making a meaningful comparison between the size of a fraction with the size of a person’s share and the number of persons getting a share. Indeed, all the three excerpts mentioned can be said to be using the relative level of language because they attempt to indicate meaningful relationships. Moreover, all indicate that they are aware that fractions could be written differently but nevertheless have the same value, thereby evidencing conceptual knowledge.

Other instances of SJ entries indicating the extent of students’ understanding of fractions through renaming was shown in answers to the prompt “In your journal, I want you to order these: 1/2, 3/4, 3/8, 5/8, 1/4. Then write how you made the decisions. You can explain with diagrams. I want to know how you think.” Three excerpts are shown in Figures 4.14, 4.15 and 4.16:

\[
\frac{1}{2} = \frac{2}{4} = \frac{4}{8}, \quad \frac{3}{4} = \frac{6}{8}, \quad \frac{5}{8}, \quad \frac{1}{4} = \frac{2}{8}.
\]

\( \checkmark \) Thinks \( \frac{1}{2} \) is the lowest fraction, because \( \checkmark \) changed it into weights like \( \frac{2}{2} \). To \( \checkmark \), had the same amount of groups the fraction was then changed to \( \frac{4}{4} \). To \( \checkmark \), was clearly lower. \( \checkmark \) then changed the rest to weights. The worst was super hard now. The 2nd lowest was \( \frac{3}{4} \). Next was \( \frac{1}{4} \). Highest was \( \frac{5}{8} \), then \( \frac{5}{8} \). Now they were in order. All fractions had weight groups to which ever number had the highest top number when changed was the largest. The order was \( \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8} \), and \( \frac{3}{4} \).

\[\text{Figure 4.14: Tommy, February 17.}\]
Figure 4.15: Jackie, February 17.

1. I think that $\frac{2}{3}$ is the smallest one, because the pieces are the smallest and it has only 2 of them.

2. I think that $\frac{1}{2}$ is the second one, because it can't be $\frac{2}{3}$ because that's bigger than $\frac{1}{2}$.

3. I think that $\frac{1}{4}$ will be the next one, because it's more than $\frac{1}{2}$ and less than $\frac{2}{3}$.

4. I think that $\frac{5}{8}$ is the next one, because $\frac{5}{8}$ is 1 right more than $\frac{1}{2}$ and less than $\frac{3}{4}$.

5. I think that $\frac{3}{4}$ is the biggest one, because it's bigger than $\frac{2}{3}$.

Figure 4.16: Shaun, February 17.
Tommy (Figure 4.14) seems to have understood equivalent fractions to the extent that he can order fractions by renaming and comparing the resulting equivalent fractions. For example, he renamed 1/4 to its equivalent 2/8, and 1/2 to its equivalent 4/8 and compared the resulting fractions 2/8 and 4/8. He did not use diagrams to explain his reasoning. As well, the “rule” for renaming fractions was not taught by the teacher. Rather, many “rules” were attempted by students, and Tommy was one of the few who could justify his rule verbally and pictorially. Hence, I could conclude that when he wrote “All fractions had eight groups so whichever number had the highest top number when changed was the largest,” Tommy had abstracted and generalised the concept that as long as the denominators are equal, the numerators indicate the magnitude of the fractions. Although he did not use any specific mathematical terminology, I would say that he was operating at the functional language level, as he could make generalisations.

In the second excerpt, Jackie drew same-sized units/wholes for 1/2, 3/4, and 1/4 to show 3/4 > 1/2 > 1/4 but she had two bigger rectangles to represent the wholes for 3/8 and 5/8 (her rectangles for the eightths were double the size of her rectangles for the fourths, see Figure 4.15). So, even though she showed 5/8 > 3/8, she had changed the whole (for the fourths and eighths), and it was difficult to see how to order the fractions. From her statement “witch one has the biggest pecies” and the diagram, I infer that she considered 5/8 the largest and 1/4 the least. But her diagram gives no indication of whether she considered 3/8 equal to, less than or larger than 3/4.

She seemed very dependent on her perception of diagrams. The concept of a common “whole” on which to base comparisons of fractions
was not evident. Though she articulated relationships through diagrams (not entirely accurately), thereby indicating some conceptual knowledge, the fact that she did not use words to elaborate on these relationships indicated that she was using the demonstrative level of language.

Just as in Figure 4.14, in the third excerpt (Figure 4.16), too, no diagrams or number lines were used for explanations. Even without diagrams, I gathered that it was clear to Shaun that 5/8 was more than a half, and that 5/8 was 1/8 bigger than a half. Hence he evidenced conceptual knowledge. The interview data also indicated that he used visual imagery rather than overt diagrams to “see” fraction relationships such as the ones described in the excerpt above and that he had no difficulty drawing the appropriate diagrams when asked to do so. From his statement (see Figure 4.16) “5/8 is 1 eight more than 1/2 and less than 3/4,” I concluded that he was using the relative level of language, as he could relate the fractions 5/8, 1/8, 1/2 and 3/4.

In summary, students explained equivalent (common) fractions by using region models. In addition, renaming was used throughout to generate equivalent fractions and to order common fractions. As well, the majority of students explained correct symbolic procedures for renaming common fractions and used relative language to communicate their understanding of renaming and equivalent fractions. Their conceptual knowledge seemed evident when they related everyday experiences like sharing pieces of pie to explain the order of magnitude of fractions. Their awareness that fractions could be written differently but nevertheless have the same value is also indicative of their conceptual knowledge.
This difference between the form (e.g. the actual symbols used to represent half) and substance (e.g. the equivalence of the fractions) in mathematics has a parallel in language where the surface and deep structure of an utterance are differentiated, with the same form used to convey different meanings or different forms used to convey the same meaning. For example, while the English expressions "How are you today?" is just one of many equivalent forms of greeting or opening gambits in conversation, the mathematical symbols 2/4, 5/10, and 31/62 are all equivalent to the fraction 1/2. Where communicative competence in language is indicated by being able to use different expressions to convey the same meaning, conceptual knowledge of fractions is indicated by the use of renaming.

Decimal Fractions and Inter-relationship between Common and Decimal Fractions

Some SJ excerpts demonstrating student understanding of decimal fractions and the inter-relationships between common and decimal fractions are shown (Figures 4.17, 4.18 and 4.19) in response to the prompt "What does 3.1 mean? Explain with words and a diagram."

3.1 is another way of saying 31/10 or 3 1/10. 3.1 means 3 whole pieces and 1/10 added on.

---

Figure 4.17: Tommy, January 21.
3.1 means 3 whole units and 1 tenths of a whole unit.

Figure 4.18: Darlene, January 21.

3.1 is another way of saying $\frac{31}{10}$ or $\frac{31}{10}$. Day you had four sixth. You divide each cake up into 10 pieces. 3 cakes have candles in all the ten pieces. On the cake that is left, there is 1 candle out of the ten.

Figure 4.19: Tanya, January 21.

The first excerpt (Figure 4.17) shows that Tommy can represent the decimal fraction 3.1 as the common fraction $\frac{31}{10}$ and the mixed number $3\frac{1}{10}$. In relating his explanation to his diagram, he has indicated that 3 wholes is equivalent to $\frac{30}{30}$. His flexibility in representing the decimal fraction using various symbols and showing their inter-relationship
evidences the use of a relative level of language and also demonstrates his understanding of common and decimal fractions.

In Figure 4.18, Darlene (supposedly a weak mathematics student, according to the teacher) demonstrates her understanding of the decimal fraction 3.1 by using relative language and relating it to a figure (once again a region model) which is very similar to that given by Tommy. Hence, like Tommy, she too evidences conceptual knowledge and utilizes relative language to communicate in words her knowledge about the inter-relationship between common and decimal fractions. But unlike Tommy, she does not indicate different common fraction notations for 3.1, possibly because she felt she had answered the question adequately, as she was asked to explain the meaning of 3.1 “with words and a diagram.” Hence, though she was supposed to be weak at mathematics, she seemed to be at the same level of conceptual knowledge as Tommy, at least for this topic.

In the last excerpt (Figure 4.19) the student explains her understanding of 3.1 through the use of circles (representing cakes) with candles drawn in each of the sectors, instead of shading the sectors as students usually do for the region model. In a sense she was using a discrete model (31 candles) but her explanation that “you divide each cake up into 10 pieces,” resembles the language used with the region model. Unlike Tommy and Darlene who use a mathematical context (rectangles) to illustrate 3.1, Tanya relates 3.1 to an everyday context (cakes and candles), but like them, she, too, uses relative language. Both the words and diagrams demonstrate students understood the different ways of writing 3.1, as either a decimal or common fraction and
also that the students could move from the decimal fraction to the common fraction notation.

Further SJ entries (Figures 4.20 and 4.21) indicating students' knowledge of decimal fractions and an awareness of the relationship between common and decimal fractions, are given in response to the prompt

Find out who received the gold medal for the downhill skiing championships, if the top three won in the following times: 7:37.59, 7:31.57 and 7:39.80. write how you thought it out. Also work out how much faster was the gold medal winner compared to the one who got second. (Feb. 19)

Table: Tommy, February 19/92

<table>
<thead>
<tr>
<th>All times</th>
<th>7:37.59</th>
<th>7:31.57</th>
<th>7:39.80</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Second</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Third</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>The fastest</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7:31.57</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7:37.59</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7:39.80</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

0:06.02 seconds faster.

Figure 4.20: Tommy, February 19.

In the excerpt in Figure 4.20, Tommy subtracted and got the answer "0:06.02 seconds faster." He looked at the minutes column first, saw it was the same, then examined the other digits systematically. He also used his everyday experience to decide that the polarised comparative (Lean, Clements, & Del Campo, 1990) lower/higher was not compatible with slower/faster, and that lower was related to faster rather than slower. He used an efficient strategy for ordering these decimal fractions, and
communicated well his knowledge of decimal fractions through his writing.

37.59 - Big second #2
31.57 - Small second #1
39.80 - Big second #3
31.57 because it has the smallest numbers
39.80 because it has biggest numbers
37.59 it is in the middle

Figure 4.21: Jackie, February 19.

Jackie (figure 4.21) seemed to have a grasp of decimal fractions. Admittedly, her explanation is very terse, resembling telegraphic speech. However, she too used a procedure similar to Tommy's to order the winners correctly. When she wrote "Big second," she meant the number of seconds was large, referring to the time taken to complete the race. She had ignored the minutes (because all are the same, that is 7 minutes), just as Tommy had done. Where Tommy has made his explanation more explicit, her explanation has assumed, implicitly, certain shared knowledge between herself and the teacher.

To me, her understanding of decimal fractions in this context was no less than Tommy's understanding. The only difference seemed to be that Jackie was writing in a "think aloud" way, to solve the word problem, and not trying to explicitly explain her solution to someone else. That is, Tommy seemed aware that he was writing for an audience but Jackie was not. Given that communicating clearly involves awareness of
audience and that the teacher had repeatedly reminded the students to explain "as if you are explaining to someone else," Jackie's seeming lack of awareness of audience is indicative of a lower level of communicative competence compared to Tommy. According to Vygotsky (1962), verbal explanation is about 7 years ahead of written explanation. On the other hand, even though Jackie's explanation lacks the coherence expected in everyday prose, it could be argued that, by using such a parsimonious explanation, she is communicating as a mathematician would—that is, by concentrating on the bare essentials and disregarding superfluous words. Hence, her lack of fluency in using everyday written prose to communicate should not be taken as a lack of mathematical understanding.

Further evidence of the students' facility to relate common and decimal fractions by renaming is shown in Figures 4.22 and 4.23, when they were asked to explain which was larger, 4 3/10 or 4.03.

4 and 3/10 is bigger than 4.03 because 4 3/10 is also 4.30, and that is more than 4.03. Just like 4.03 can also be changed to 4 3/100 which is smaller than 4 3/10.

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**Figure 4.22:** Mike, February 5.

---

I think 4 3/10 is bigger than 4.03 because 4 3/10 means 4 whole units and 3 of a tenth. 4.03 means 4 whole units and 3 one hundredths. What I means 3/10 is bigger.

---

**Figure 4.23:** Darlene, February 5.
The first excerpt (Figure 4.22) indicates that Mike can relate common and decimal fractions. He has given two explanations, one by comparing decimal fractions (4.30 and 4.03) and the other by comparing common fractions (4 3/100 and 4 3/10). Although he did not use diagrams in this instance, he is able to use words to describe different representations of the same mathematical concept.

In the second excerpt (Figure 4.23), Darlene's explanation of what 4 3/10 means and what 4.03 means showed a clear understanding of these symbols. Although she did not explain clearly that 3/10 is bigger than 3 hundredths, her understanding of it is implicit in her writing "I means 3/10 is bigger." Whereas Mike has used more symbols, Darlene has used less, but both explanations reflect similar understanding of the relationship between common and decimal fractions.

More evidence of students' ability to relate common and decimal fractions is given in Figures 4.24, 4.25 and 4.26, in response to the prompt "Jason thinks 0.8 = 3/5. Do you agree or disagree? Explain, prove, convince me. Whoever reads this should understand your thinking."
Figure 4.24: Tommy, February 26.

Say you divided up 2 granola bars.

You divided one into 10 and one into 5. For the 10 shade in 5, in the 5 shade in 3. As you can see it is bigger. If you wanted it equal you would need \( \frac{4}{5} = \frac{4}{5} \).

Figure 4.25: Tanya, February 26
All the three excerpts indicate students can relate common and decimal fractions through renaming. For example, 0.8 is renamed as the common fraction 8/10, the common fraction 3/5 is renamed 6/10 and the resulting common fractions 8/10 and 6/10 are compared, implying a knowledge of inter-relationships between decimal and common fractions.

The explanation in the first excerpt (Figure 4.24) is more detailed than in the other two, but is still not very explicit (for example, "5ths would be 2 by 2"). It also demonstrates a combination of the demonstrative and relative levels of language: demonstrative, because of statements like "you can see 8/10 is larger," and "this is 10ths," and relative, because of an attempt to relate fractions to everyday experience such as the size and number of pieces. However, on discussing further with Tommy, I found that he could make explicit what was implicit in his written explanation (such as "5ths would be 2 by 2" meant that 2 of the spaces marked in tenths was equivalent to a fifth). Hence, I infer that he has an understanding of the inter-relationships between common and decimal fractions. The second excerpt (Figure 4.25) relates fractions to everyday examples (such as granola bars) and could be said to demonstrate the
relative level of language. The third excerpt (Figure 4.26) indicates the
demonstrative level of language, as it points to instances without much
explanation using words. Although Darlene seemed to have difficulty in
using written words to explain what she meant, from interview data I
found that she could explain orally (and with the help of diagrams) why
$0.8 \neq \frac{3}{5}$. She was helped in her oral explanation by the fact that she
could point at parts of the diagram during her explanation, demonstrating
that she understood the relationship between common and decimal
fractions, although she was much less articulate in using written words.

In summary, students demonstrated conceptual understanding of
the relationship between common and decimal fractions, and used
relative language to communicate such understanding. Where students
used demonstrative language, further discussions and interviews
revealed that they, too, understood common and decimal fractions
relationships, once again indicating that demonstrative language cannot
be equated with lack of mathematical understanding.

Addition of Common Fractions

Students also used renaming to add fractions with different, but
related denominators. Figures 4.27, 4.28 and 4.29 show responses to
the prompt “Explain to Adam, who was absent, how to do this $\frac{1}{4} + \frac{3}{8}$.”
First I would probably draw a diagram.

\[
\begin{array}{c}
\text{This is } \frac{3}{8} \\
\text{This is } \frac{1}{4}
\end{array}
\]

1/4 could also be cut into eighths. So 1/4 could also be 2/8. Take the 2 top numbers and add them up. So 2 + 3 = 5. Put the answer of the 2 top numbers on top of the 8 to make 5/8.

Figure 4.27: Tommy, March 5.

Well first what you could do is you could draw fraction strips:

So the answer is 5/8 because if you put those in 8's then it would be 5/8.

Figure 4.28: Maureen, March 5.

To make it easier to add we will rename 1/4 to 2/8, now its easier to add because the bottom # is the same. So 2 + 3 = 5 and the bottom # stays the same = 5/8.

Figure 4.29: Shaun, March 5.
Tommy (Figure 4.27) explains using relative language and also indicates conceptual and procedural knowledge by using two separate fractions (3/8 and 1/4) and an algorithm to get the sum of 5/8. Maureen's (Figure 4.28) referent for "those" in the expression "if you put those in 8's" is not very clear. She seems to expect the reader to grasp her meaning from the diagram that 1/4 can be renamed as 2/8. In other words, she resorts to demonstrative language, but her diagrams indicate her conceptual knowledge of addition of these fractions. In contrast to the other two, Shaun (Figure 4.29) uses words and symbols but no diagrams in his explanation, and it seems that he uses procedural knowledge.

Even though the addition was related through diagrams or words to equivalent fractions, from subsequent interviews I found that students (other than Tommy) could not add other fractions where one denominator was not a multiple of the other. Indeed, for the addition of 1/3 and 1/2, five of the six interviewees did not try renaming the fractions. Rather, they had the two fractions represented by shaded parts in one rectangle (even Shaun, who usually did not resort to diagrams) and estimated the sum to be either 2/3 or 3/4. So, even though they could add by first renaming the fractions when one denominator was a multiple of the other (as in 1/4 and 3/8), they could not extend such renaming to fractions without common multiples (as in 1/3 and 1/2), thus demonstrating they needed perceptual cues to support and illustrate renaming and equivalent fractions. Even Shaun, who had said "now its easier to add because the bottom # is the same" for his procedural addition of 1/4 and 3/8 (Figure 4.29), resorted to a diagram for 1/3 + 1/2. Hence I infer that the relative level of language was used by most students in communicating their knowledge of addition of common fractions.
Tommy, on the other hand, used diagrams, words and renaming (to obtain fractions with common denominators) to add fractions. For example, during the third interview, he stated that 1/2 and 1/4 could not be added without renaming as “they are different items.” He went on to say that “if you have someone who doesn't understand, they might give the answer as 2/6,” and he explained how 2/6 was obtained as well as why 2/6 was wrong. He also could add 1/3 and 1/5, both by renaming using symbols and by drawing diagrams. Although not warranted from the one entry used (Figure 4.27), I was able to conclude from the interview and from listening to earlier discussions in class that he was operating at the functional level of language in communicating about his knowledge of addition of fractions, as he could explain the need to use common denominators. In general, however, he evidenced conceptual and procedural knowledge of addition of common fractions.

In summary, students can add fractions where one denominator is a multiple of the other, by using renaming with or without diagrams for illustrative purposes. From my observations, interview data and SJ, I found that no Grade 6 student added numerator to numerator and denominator to denominator, perhaps reflecting Linda's de-emphasis on symbol manipulation and emphasis on understanding mathematical concepts and relationships. Linda used word problems to initiate discussion on addition of fractions. She encouraged students to justify their solutions (whatever the solution) and to use manipulatives and diagrams. She did not teach a rule for finding common denominators nor did she give any exercises involving only numbers, such as 1/2 + 3/8. Other than Tommy, no student had articulated the notion of common denominators for adding fractions or used the functional level of
language to communicate their knowledge of addition of fractions (most students used relative language).

**Word Problems (Decimal Fractions)**

Written explanations of how to solve word problems involving decimal fractions are shown in Figures 4.30 and 4.31. They were in response to the teacher’s question “Mr Campbell drove about 95 km an hour for 2.5 h and then 60 km an hour for 1.25 h. How far did he drive?”

\[ 95 \text{ km each hour, so } 95 \times 2 \text{ hours is } \frac{190}{\text{km}} \text{ plus } \frac{1}{2} \text{ an hour, so divide } 95 \text{ in half } 2 \frac{1}{2} \text{ in half an hour. Mr Campbell went } 47.5 \text{ km. Add them up } 190. \text{ Now he changed speed, } 60 \text{ km an hour. So } 47.5 \times 0.25 \text{ is left. } \frac{25}{100} \text{ is equal to } \frac{1}{4} \text{ of an hour is } 15. \text{ Now Mr Campbell actually drove a km each minute, so } 60 \text{ km } + 15 \text{ km } = 75. \text{ 237.5} \]

\[ \text{Mr Campbell drove } 312.5 \text{ km.} \]

---

**Figure 4.30**: Tommy, March 13.
All the students (except Tommy) operated with mathematical symbols, without words of explanation, as typified by Figure 4.31. Even so, they are able to indicate conceptual knowledge, as they use relationships among speed, time and distance as well as that between common and decimal fractions. Indeed, they were answering the question asked, just as many mathematicians would do, without superfluous words. However, in the sense that they are computing without an accompanying explanation in words, I would infer that they were using demonstrative language. That they used demonstrative language is not surprising because they were more interested in arriving at a solution to the problem (as that is what was asked) than in trying to communicate to the teacher by using words. Moreover, the assumption seemed to be that the steps that they showed, though symbolic, would be self-explanatory to someone like the teacher.

Figure 4.31: Mike, March 13.
Tommy (Figure 4.30) was the only one to explain in words what he did in order to arrive at his solution. Just like the other students, Tommy, too, inter-relates different aspects of the problem, indicating his conceptual knowledge. In addition, he seems to be putting his thoughts to paper—a kind of “thinking aloud”—and because he makes these “relationships between relationships” (for example 60 km an hour is equivalent to “a km each minute” and “0.25 is left . . . 1/4 of an hour is 15/60”) explicit by using words, I infer that he was using the functional level of language.

It is instructive to note that this question was part of a class test to be done in the SJ, and so it should not be surprising that getting a solution was more important for the student than giving an elaborate explanation to the teacher. However, according to the teacher (and I observed this happening too), even in solving non-test word problems in class, students generally tended to concentrate on the computations needed to arrive at the solution, as in the second excerpt (Figure 4.31). Only when they were explicitly asked to explain their reasoning (and reminded repeatedly to do so) did they make attempts to use words in addition to symbols. On the other hand, when they were asked to present their solutions in front of the class, the students tended to explain and justify more articulately “in response to an explicit request” or in an attempt to “communicate aspects of their mathematical thinking that they think were not readily apparent to others” (Cobb, Wood, Yackel, & McNeal, 1992, p. 577). It is not surprising that such classroom discourse involves negotiations of meaning, and is in fact expected by classroom routines. But (a) the lack of practice in writing for peers, (b) the lack of an awareness that writing in a mathematics class for an audience other than
the teacher is legitimate, and (c) the way students have been generally taught mathematics all these years all seem to explain in part why the functional level of language is seldom used in solving word problems.

In summary, students evidenced conceptual knowledge in solving word problems involving decimal fractions. However, they seldom used words to explain their solutions. Rather, they tended to show only the computational steps, unless reminded explicitly to do so. It looks as if students have a difficult time reconciling old habits of getting an answer as quickly as possible with the present teacher's emphasis on written explanations of their solutions.

Rule for Placement of Decimal Point in Decimal Multiplication

Just as in the addition of common fractions, no rule was given to the students for multiplication of decimals. Instead, they were to find out a suitable rule by themselves first without, then with, calculators to help them, by attempting some word problems given by the teacher. When asked the rule for placing the decimal point when multiplying decimals, some of them wrote as shown in Figures 4.32, 4.33, 4.34 and 4.35.

I think the rule is if you are multiplying 2 numbers with so and so tenths times so and so tenths, the answer will be so and so hundredths. If your multiplying 2 numbers that both have so and so hundredths, the answer would be so and so ten thousandths. Example 2.56 x 2.56 = 6.5536 [He showed the vertical form of multiplication]. If you have one number with hundredths and one with tenths the answer would be thousandths. [Gave the examples 4.2 x 3.8 = 15.96, 0.67 x 1.38 = 0.9246 and 0.02 x 0.3 = 0.006].

Figure 4.32: Tommy, March 25.
When you multiply with numbers on both sides of the decimal, the answer can be a whole number, but when it is only the right side, it can't be a whole. [He elaborated by computing the following: \(4.2 \times 3.8 = 15.96\) (correct), \(0.67 \times 1.38 = 0.9246\) (incorrect), and \(0.02 \times 0.30 = 0.60\) (incorrect)].

**Figure 4.33:** Mike, March 25.

My rule is that all you have to do is just answer the whole question instead of putting it wherever you want. [She exemplified her rule by multiplying 0.2 by 0.7 to get 0.14, ignoring decimal points in the partial products]. If that doesn't work then be logical. Or count digits to the right of the decimal. [Then she used her "counting" rule on the following: \(4.2 \times 3.8 = 15.96\), \(0.67 \times 1.38 = 0.9246\) (incorrect) and \(0.02 \times 0.3 = 0.006\)].

**Figure 4.34:** Maureen, March 25.

I think the rule is connected to the estimation. What I mean is that the digits you get in your estimation that's how many digits you will have before the decimal. Here's an example:
question 3.64 x 8.8, estimation 4 x 9 = 36, so answer is 32.032

**Figure 4.35:** Darlene, March 25.

Figure 4.32 shows that Tommy was using the functional level of language as he made a generalisation about the placement of the decimal point involving decimal multiplication. He seemed to have no difficulties in placing the decimal point correctly and his procedure was based on a pattern he had found. But when asked why he obtained a number (0.06) less than the numbers he started with (0.3 x 0.2), he
replied “Because not multiplying by whole numbers.” So, even though he used the functional level of language in his SJ to communicate his knowledge of decimal multiplication, and he seemed to have a correct procedure for placing the decimal point, it seems that he (just like the other students I talked to) treats decimal numbers as obeying rules of multiplication different from that of whole number multiplication. Hence, his conceptual base for multiplication of decimals does not seem very strong, even though he has discovered some patterns in decimal multiplication.

In spite of his facility in moving from common to decimal fraction notation and vice versa, he did not use renaming and common fractions to justify his generalisation that multiplying tenths results in hundredths. The reason for not comparing decimal fraction multiplication with common fraction multiplication is not surprising, given that he had not come across such multiplication before in class, even though he could answer verbal questions such as “What is a third of six tenths?” and “What is a third of zero point six?” To me, Tommy seemed to have a quicker grasp of decimal multiplication, even though he seemed to have started with conceptual knowledge not very different from his classmates. I say this because during interview three, when I asked him to think about the symbol “x” as representing “of” as in “1/2 of 0.4” being equivalent to “1/2 x 0.4” he said that he could now see why the product could become less, unlike in whole number multiplication. In contrast, the other students did not seem to understand why the product of decimal fractions could sometimes become less than the numbers they started with, even though I used the same analogy.
The examples in Figure 4.33 indicate that Mike felt that if one of
the multiplicands were less than one, then the product would be less
than one. His rule depended on the type of numbers that were being
multiplied: If the numbers were greater than one (e.g. 4.2 x 3.8), then one
set of rules applied; if the numbers were less than one (e.g. 0.02 x 0.30),
then another set of rules applied. While it is true that he conjectures--and
conjecturing is mathematical--his rules are number specific and not
generalised: he sees no contradictions in changing rules to fit particular
situations rather than perceiving rules as a generalisation with a wide
applicability. So he, too, has a conceptual base for decimal
multiplication, albeit not a very strong one.

The language used in Figure 4.33 seems closer to the relative
level of language than the functional because he attempts to relate his
rules to specific sets of numbers rather than to numbers in general.
However, it has lexical ambiguities (Durkin & Shire, 1991). For example,
when Mike used the word “numbers” in “multiply with numbers,” he
meant non-zero digits, but when he used “number” in “whole number,” he
meant that the digits to the left of the decimal point represented whole
numbers, not that the product itself was a whole number. Such ambiguity
in the use of the word “number” could hinder the abstraction of the
mathematical concept of number, as “number” seems to mean only
whole numbers to Mike. It is possible that such an abstraction is made
even more difficult because teachers seldom explicitly point out that
fractions are numbers, too (just as Linda did not).

In Figure 4.34, Maureen shows two rules for the placement of the
decimal point, and two levels of language: (a) by counting the total
number of digits to the right of the decimal point in the multiplier and
multiplicand, she is using the demonstrative level of language as she is just stating a procedure without any justification; and (b) by using an estimation to get reasonable answers (which, according to her verbal explanation later, was what she meant by the word “logical,” and her first sentence, too), she is using the relative level of language, as she is relating the answer to her everyday experience and “common sense.” In other words, Maureen perceived the placement of the decimal point in decimal multiplication as dependent on the type of decimal fractions involved. However, judging by the placement of estimation before her “counting” rule in her explanation (Figure 4.34) her counting rule seemed to be an alternative only when estimation or logic did not “work” (agree with the calculator-computed answer).

In Figure 4.35, Darlene, too, uses the relative level of language by stating a meaningful approach (estimation) to locating the decimal point in the product of decimal fraction multiplication. Both Maureen's and Darlene's strategies had their advantages. For instance, Darlene's strategy has the advantage that it applies to all instances of decimal fractions as long as estimation skills are good. But she did have difficulties with numbers close to zero, such as 0.1 x 0.2 and 0.02 x 0.3, when her initial estimations resulted in zero for both the products. On the other hand, Maureen had another strategy to fall back on if one failed. Moreover, her rule was also applicable to all cases, except that she had to be careful about numbers like 0.5 x 0.2 which would result in 0.10 but with the ever-present danger of discarding the zero in the hundredths place because of rules like “trailing zeroes can be ignored,” resulting in confusion as to the placement of the decimal point. But because
Maureen was prepared to resort to an arbitrary (but possibly pattern-engendered) rule, she might do so in other topics, too.

In summary, students obtained rules for placement of the decimal point in decimal multiplication through conjectures, estimation and following patterns, based on word problems. For example, in order to find the amount of tax to be paid for a pair of jeans costing $30, if the tax were 7%, students realised that $210 and $21 would be unreasonable, and so they obtained $2.10 and attempted to find a pattern for other situations involving decimal fraction multiplication. Although most of them used estimation and obtained reasonable solutions, they had difficulties in estimating answers for multiplying numbers like 0.02 x 0.3. None of them could satisfactorily explain why the product of decimal numbers could sometimes be less than the numbers with which they started. Overall, they had some conceptual understanding of decimal multiplication but had better procedural knowledge. As well, most of them used either demonstrative or relative language in their writing to communicate their knowledge of decimal multiplication.

SCQ and Mathematical Knowledge

So far, I have only discussed journal entries. I now examine some of the SCQ briefly but postpone more detailed discussion to a later section (e.g. when discussing results pertaining to research questions #2 and #3). Following are some examples of SCQ indicating students' knowledge of common and decimal fractions.
Someone ate 3/8 of a pizza. You and your friend are going to share the rest. How much of the pizza will you get?

Figure 4.36: Darlene, January 7.

Caroline went to the mall. She brot $100.00. She went to "Off The Wall" and spent $32.00 and 1/4 of the money went to the government. How much does the store keep? Answer: The store keeps $24.00.

Figure 4.37: Jackie, February 10.

Each person got a mark out of 10. But someone renamed some of them! Who was first? Who was third? Can you rename them again a different way?

Answer: \[
\frac{2}{2} \times \frac{4 - 20}{10 - 50} \times \frac{60}{100} = \frac{2}{10} \times \frac{4}{10} = \frac{3}{10} \times \frac{1}{10}
\]

Figure 4.38: Mike, February 19.

The question shown in Figure 4.36 was constructed after Linda had asked them to write about where fractions are used in the students' daily
life. Many of the students had given the pizza as an example of fractions in daily life. Moreover, the region model had been exemplified by granola bars and pizzas in previous classes (before Grade 6, with teachers other than Linda) and by fraction strips in the present class. So, it was not surprising that Darlene—supposedly a weak mathematics student, according to the teacher—constructed a “pizza problem.”

Although Figure 4.37 has superfluous information like $100.00 and the name of the store, such information draws on the student’s own meaningful experiences and situates the word problem in a realistic context. The mathematical operations involved in the SCQ are multiplication and subtraction, with the former as yet untaught. Even though the student was supposedly weak at mathematics (according to the teacher) and had no knowledge of the multiplication algorithm for common fractions, she had no difficulty in explaining (verbally) how she solved her SCQ, indicating her awareness that a fourth of 32 is the same as dividing 32 by 4.

In Figure 4.38, Mike has chosen all except one fraction with multiples of 10 in the denominator. Even so, he did not seem to have any difficulty in converting 2/2 to 10/10, or 20/50 and 60/200 to fractions with 10 in the denominator. He admitted that while preparing the question, he tried a few fractions before choosing numbers which he could rename in tenths. A week before this he had explained both algorithmically and through illustrative diagrams in his SJ how to rename fractions so as to obtain equivalent fractions, indicating that he had both procedural and conceptual knowledge of renaming and equivalent fractions. This excerpt (Figure 4.38) seems to give further support that he understands equivalent fractions and how they relate to ordering fractions. His
solution has no verbal comments, once again indicating what I said earlier (p. 106) about students' habit of not writing elaborate verbal explanations of solutions to (decimal fraction) word problems.

While the previous examples of SCQ give an indication of the students' knowledge of common and decimal fractions, the following examples of the SCQ give an indication of the growing competence of the majority of students to write questions that not only made sense to them, but were mathematically (but not necessarily computationally) more sophisticated.

Consider one student's SCQ written about two weeks apart (Figures 4.39 and 4.40):

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Figure 4.39: Tanya, January 24.
Stepheny wants to watch her favorite show at 5:30 p.m. It's a 1/2 hour show. In the morning she gets up at 10 a.m. She is at the ice arena an hour later. She gets off the ice at 2/3 from regular skating time which is 3 hours. She eats lunch for 1/3 of an hour. Then she goes shopping for twice as long as she went skating. It takes her 1/2 an hour to walk home. How much of her show will she get to watch?

Figure 4.40: Tanya, February 6.

Tanya had gone from a fraction question (Figure 4.39) involving a region model together with the operations of addition, subtraction and whole number multiplication to one that involved no explicit region models but dealt with intervals of time, fractions (including equivalent fractions) and multiplication of fractions and whole numbers (e.g. 2/3 of 3 hours).

Moreover, the second SCQ (Figure 4.40) had a wealth of realistic details as well as a temporal sequence that had to be followed in order to solve the problem.

Another student wrote these SCQ (Figures 4.41 and 4.42) about six weeks apart:

| Winning Spirit had 76 baseball bats and 38/76 were wood and 13/76 were metal. How many baseball bats are left over. |

Figure 4.41: Anne, February 10.
Jack made $0.25 an hour and he worked 6 hr a day. He worked for 7 days. How much did he make? [To which the teacher responded "Jack was underpaid!"]

**Figure 4.42**: Anne, March 23.

Anne's SCQ progressed from a word problem (Figure 4.41) that looked superficially like a fraction problem with addition and subtraction of whole numbers, to a word problem (Figure 4.42) involving multiplication of decimal fractions (for example, in her solution accompanying the question, she wrote "0.25 x 6 h = $1.50, $1.50 x 7 = $10.50"). Admittedly, she uses only one operation, and she uses a problem involving money, which might not have needed a knowledge of decimal fractions for a solution. But given that she was weak at mathematics and was asked to write a word problem "involving multiplication of two numbers with at least one of the numbers a decimal fraction less than one," she did prepare a question satisfying the conditions.

Figures 4.43 and 4.44 show Tommy's SCQ over two weeks.

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You buy a box of Upper Deck Hockey cards. Every box you receive 36 packs. Each pack you get 12 cards. You are missing 1/6 of cards in each pack. How many cards are you missing, and how many cards will you have now?

**Figure 4.43**: Tommy, January 27.
There was a pie eating contest that was held at Stanley Park. There were 4 contestants in the contest. Tom ate 4 pies, and 1/3 of another. It took Tom 1:54.03 to pass out. Exactly :24.04 later Roxanne passed out eating 1 pie more than Tom. 1.54.44 later Jake passed out in 1.00.00 later eating 4 more pies than Roxanne. This declared Jeremy the winner. Jeremy didn’t stop, he was trying to beat the record. The record was 13 pies eaten in 6.04.00. Jeremy ate 12 pies and 2/3 of another in 5.45.35. How much pie was eaten, in how much time? Counting their times together.

Figure 4.44: Tommy, February 10.

Figure 4.43 shows a word problem involving multiplication and subtraction of whole numbers with the fraction 1/6 which could be treated as dividing by 6. Figure 4.44 is not only a longer question but involves addition of common and decimal fractions. Tommy has incorporated both common and decimal fractions in the second SCQ, possibly because he has learned decimal fractions in addition to common fractions. On the other hand he has shown an increase in his knowledge of fractions by including addition of fractions, whereas his first SCQ could have been solved using just operations on whole numbers.

Another instance of the improvement in the SCQ is shown in Figures 4.45 and 4.46, which took place over a five week period.

There were 160 parking spaces at the mall. The parking lot was 1/4 full. How many parking spaces were left?

Figure 4.45: James, January 24.
I had to walk back and forth to work every day for one week and the walk was 63.12 metres each way. I also had to walk to the grocery store twice that week. The grocery store was 32.15 m each way. I also walked to the doctor’s office which was 67.94 m. How far did I walk that week? For every metre that I walked I gained 0.05 kg. How much weight did I gain that week?

Figure 4.46: James, March 4.

Figure 4.45 indicates an SCQ with two operations (division and subtraction) whereas Figure 4.46 shows an SCQ that is multi-step, with addition and multiplication of decimal fractions. While it is unrealistic to expect someone to gain weight by walking, other details in the question seem realistic enough to indicate everyday experience. Moreover, the second SCQ (Figure 4.46) has more mathematical information in it as well, compared to the first SCQ. So I infer that there is an improvement in the SCQ, both in terms of mathematical knowledge and background details.

As the study progressed, even the “weaker” students wrote SCQ that were rich in realistic, everyday details. For example, the weakest student, Jackie, went from the question in Figures 4.47 to that in Figure 4.48 in just over two weeks.

We had 200 salmon eggs. 1/4 died, how many were still alive?

Figure 4.47: Jackie, January 24.
Caroline went to the mall. She brought $100. She went to "Off The Wall" and spent $32.00 and 1/4 of the money went to the government. How much does the store keep?

Figure 4.48: Jackie, February 10.

As can be seen, the later question (Figure 4.48) has a richer background of details (for example, use of "mall," "brought $100," the name of the store "Off The Wall," and "money went to the government") than the earlier question (Figure 4.46), which seemed rather bare. While it could be argued that the richer background has nothing to do with the mathematics involved, it must be remembered that such background situates mathematics in a meaningful context and could make problems more amenable to solution (e.g. Ellerton & Clements, 1991; Laborde, 1990; Mason & Davis, 1991; Spanos et al., 1988). As well, though the operations involved are the same (division and subtraction), in the second question, the fact that the $100 was not needed in the computation could point to not only everyday experience, but also could be thought of as providing extraneous information. Such extraneous information has to be recognized as unnecessary for the solution and that too speaks of a level of mathematical sophistication.

In summary, initially, students wrote word problems without many words and the solutions needed few steps, possibly indicating both their unfamiliarity with SCQ and the level of mathematical understanding about common and decimal fractions as well as the number of topics taught. Later on, students embellished their word problems with background (and sometimes even superfluous) information and required
multiple steps for their solution, demonstrating familiarity with the SCQ as well as more understanding of common and decimal fractions, especially when they included both common and decimal fractions in their word problems. Students themselves embellished their questions with background and Linda neither encouraged nor discouraged their embellishment. Because each SCQ had to be handed up with the solution, students' understanding of the problem was demonstrated by the accompanying explanation or solution as well. Where there was only the numerical solution with no accompanying explanation to the solution, students' understanding was demonstrated through their verbal explanation of the solution to the group, class or researcher. Overall, the SCQ showed a slight, but noticeable improvement in students' mathematical knowledge about common and decimal fractions, with all the students making longer questions, but with the weaker students giving more non-mathematical background and the better students including more topics, operations and steps.

Summary of results pertaining to research question #1

Initially, students tended to communicate their mathematical knowledge orally by using demonstrative language when grappling with common and decimal fraction concepts and progress to the relative and functional levels of language as they developed better understanding of these concepts. The findings of this study about oral communication seem to confirm Freudenthal's (1978) view that communication in mathematics progresses through three levels of language (demonstrative, relative and functional). In writing, however, while all students moved from the demonstrative to the relative level of language
for a limited number of concepts (such as the part-whole concept), the majority of the students used relative language when writing about the other common and decimal fraction concepts, and sometimes used both demonstrative and relative levels of language. A minority also sometimes used the functional level of language. In short, there was no noticeable progression in the levels of written language, although students seemed to understand fraction concepts better. Indeed, there were instances of students using demonstrative language in the SJ but showing a good level of understanding through their pictorial and oral explanations that seem to indicate that the levels of language used for communicating mathematical knowledge (a) vary according to the mode of communication (say, oral versus written), (b) the fraction concepts being learned and (c) are not unambiguous indicators of a student's level of mathematical understanding or knowledge. Hence, the findings of this study indicate that Freudenthal's levels of language about communication in mathematics need to be re-examined where written communication is concerned.

Most of the students used diagrams to complement their written, verbal explanations. Where their writing did not give satisfactory explanations, follow-up interviews revealed that they understood much more mathematics than initially indicated by their writing. As Linda, the teacher, said “sometimes they do understand, they just don't know how to say it in words”--which agrees with Freudenthal's (1978) statement that “most of us understand more language than we can speak” (p. 234).

Only one student in this study could articulate fraction concepts and relationships clearly through written words. My observations of class discussions and the interview data where students satisfactorily
explained fraction concepts confirm that the majority of students in this study were more articulate speaking than writing about fraction concepts and relationships. This finding contradicts Loban's (1976) study which indicates that language proficiency in writing catches up with that in speech by the age of twelve. A possible reason for the difference in oral and written proficiency is in the mode of communication itself: where oral discourse lends itself to immediate feedback from the audience, writing does not (Bereiter & Scardamalia, 1987). For example, when students try to explain verbally why $8/15$ is larger than $1/2$, any lack of clarity in the explanation is immediately brought to the attention of the speaker who can then modify the explanation accordingly. In contrast, in writing about the relative magnitudes of $8/15$ and $1/2$, the writer may be making assumptions about shared knowledge between the writer and the reader which might make the import of the writing unclear to the reader.

Interestingly, those students perceived as poor writers by the teacher (in other subject areas such as Social Studies and Language Arts) seemed to be also poor at writing in mathematics. So it would seem that the difficulty in writing was not a function of the content or topic (fractions, in this instance) but of something more generic such as written communication. Thus transactional writing (Britton et al., 1975), as used in the SJ for this group of students, was not as effective as verbal and pictorial communication about mathematics. That is not to say that the SJ did not reflect students' mathematical knowledge at all. On the contrary, the SJ did give insights about students' mathematical knowledge. It is only that verbal explanations demonstrated students' mathematical knowledge more clearly, possibly because of an audience who gave feedback so as to allow for immediate clarification. In contrast, the SCQ,
which required the writer to give solutions, showed the students’ mathematical knowledge implicitly (through the type of question asked, information given, and the accompanying solution) and also their improvement in their fraction knowledge explicitly (through the type of fraction relationships, the amount of detail, and the number of steps and operations involved).

Although most students were not fluent in expressing themselves in written language, they did demonstrate conceptual knowledge (Hiebert, 1986) and relational understanding (Skemp, 1976) of common and decimal fractions by using relative language (Freudenthal, 1978), that is, “where objects are described by their relations to other objects” (p. 237) either in their SJ or orally. For example, Shaun used mathematical symbols frequently in his SJ to explain his method of solution rather than diagrams or everyday language and so he might be construed (as the teacher did) as using procedural knowledge (Hiebert, 1986). Most of his SJ entries indicated that he used such abbreviated explanations correctly (e.g. he used “rules” such as “multiply top and bottom by 2” for equivalent fractions) but he could explain his reasoning when asked to do so verbally and by referring to diagrams, thereby demonstrating conceptual knowledge of common and decimal fractions and the use of relative language as well. Hence a preponderance of symbols in writing about fractions is not indicative of a lack of conceptual understanding. Neither is a lack of everyday prose as evidenced by demonstrative language (for example, both Tommy and Darlene used demonstrative language in justifying that $0.8 \neq \frac{3}{5}$, see comments on Figures 4.24 and 4.26) an indicator of lack of conceptual understanding.
In contrast to Shaun, the writing of a student like Mike indicated that he was not very familiar with mathematical terminology and sometimes obtained wrong answers (like misplacing the decimal point). However, he used everyday language in his journal to relate mathematical concepts and principles (such as the interchangeability of common and decimal fractions). In addition, his self-constructed questions, though computationally not very difficult for students of grade 6, did demonstrate a grasp of relationships between common and decimal fractions, thereby evidencing conceptual knowledge (Hiebert, 1986). Overall, I can say that this group of students did reveal conceptual and procedural knowledge of mathematics as well as growth in such knowledge through their writing.

**Research question #2 and results**

*Research question # 2: What are the salient and recurrent features (themes) of the SCQ, if any?*

Four themes emerged from the student-constructed questions (SCQ). They were:

1. The use of student experience and interest as the context of the SCQ.
2. The assumption of shared knowledge between the reader and writer of the SCQ.
3. The use of numbers which made computation easy.
4. The use of questions reflecting the discrete model of fractions rather than the region model.
Theme 1: use of student experience and interest

The SCQ reflected students’ daily, out-of-school experiences or in-school experiences such as other subject areas currently being studied. For example, some students used their shopping experience in their SCQ, as in Figure 4.49.

A regular Bulls parka costs $175 and the regular Bulls jacket costs $75. There was a half off sale. Brian bought 1 parka and one jacket. Jason bought 2 parkas. How much money did Brian and Jason each pay? How much did the store collect?

Figure 4.49: Anne, March 2.

Others drew upon topics currently being learned in other subject areas. For example, the students were learning about whales in Social Studies, and they wrote SCQ related to whales (Figure 4.50).

Vancouver Aquarium has a weekly budget of $100, of which 0.5 will go for the killer whales, 0.44 for belugas, and the leftover money for the dolphin. 1. Find out if 0.5, 0.44 in $. 2. Find out how much goes for the dolphin.

Figure 4.50: Shaun, February 19.

Others wrote on recent or ongoing experiences and activities in which they had been involved. For example, the students were responsible for a salmon tank in the classroom. They took turns to feed the salmon and measure the temperature of the water but some of the salmon died. An SCQ indicating their recent involvement in salmon rearing is given in Figure 4.51.
We had 200 salmon eggs. 1/4 died, how many were still alive?

Figure 4.51: Jackie, March 3.

Some of the students wrote SCQ reflecting their interest in games and sports, such as ice hockey and gymnastics. Two examples of SCQ showing these interests are given in Figures 4.52 and 4.53.

The Winnipeg Jets were on a 4 game road trip. Through all games, the Jets scored 24 goals. 1/3 were against the Vancouver Canucks, 1/2 were on the Sharks 1/8 were on the Nordiques. How many did they score against the Canadians?

Figure 4.52: Tommy, February 7.

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<thead>
<tr>
<th>Girls</th>
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<td>Maryon</td>
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How much more did Jackie get in all events?

Figure 4.53: Jackie, March 2.

Some students wrote SCQ based on specialised interests. For example, only two students--the father of one was a pilot--were very interested in aircraft and an SCQ related to aircraft is shown in Figure 4.54.
An airplane was flying at 50.2 m at 12.00 a.m. It flew for 4 hours and gained 1.4 m of altitude each hour. What time would it be and what altitude would the plane be at?

Figure 4.54: Sam, March 4.

The SCQ in Figures 4.49 to 4.54 illustrate that students seem to draw upon their interests or ongoing daily in-school and out-of-school experiences in which to situate their SCQ. Moreover, recent or ongoing activities (e.g. salmon rearing) and what is being discussed in other subject areas (e.g. whales in Social Studies)—which generally did not involve any fractions—seem to play a part in the type of SCQ being constructed. It looks as if students can, given the opportunity, write SCQ which are meaningful to them, basing them on their experiences, rather than limiting the SCQ to what is learned in the mathematics class. Such experientially-based SCQ are not surprising, given that how an individual constructs meaning is dependent “on the context and on the mathematical content underlying the formulations” (Laborde, 1990, p. 62).

Another factor that might have encouraged students to look to other sources of ideas for the SCQ was the fact that frequent textbook exercises were not part of the practice in this class. Instead, group discussions formed the basis for the SCQ initially, after which both group and individual SCQ were collected, edited and distributed by the teacher as class exercises. (I am not including examples of SCQ resulting from group discussion, as the groups comprised both Grade 5 and Grade 6 students, whereas my focus is on the Grade 6 students.) I would
speculate that these group discussions helped students clarify ideas for the preparation of individual SCQ.

**Theme 2: assumption of shared knowledge**

Time and again the SCQ indicated that the writer of the question assumed that the reader was privy to all the information the writer possessed, even though such information was not explicitly stated in the question. Furthermore, such unstated information was often crucial to the determination of the answer. Examples of such SCQ follow.

In band there are 60 people that play different instruments. 29/60 are clarinets, 10/60 are flutes and 10/60 are saxaphones. How many trumpets are there? (The answer is in sixtieths) Answer: the amount of the trumpets is 11/60.

**Figure 4.55:** Shaun, January 27.

The first assumption in the SCQ (Figure 4.55) is that the reader knows that there are only four different kinds of musical instruments in this band. The second, non-crucial assumption is that the reader is aware that the symbols expected by the writer in the answer ("in sixtieths") can be reconciled with the reader's expectation of a whole number as the answer. Indeed, even though students found the computation easy, there was disagreement and discussion on this particular issue ("How many trumpets are there?" as being indicative of a whole number answer rather than a fraction) when solutions were attempted.

Another example of a question which assumed shared knowledge is in Figure 4.56.
In Vancouver Aquarium there are 10 belugas. 4/10 are young belugas. 0.5 of the whole herd are adults. How many baby belugas are there? Answer: There is 1 baby beluga because 4 + 5 = 9, 10 - 9 = 1.

**Figure 4.56:** Shaun, February 10.

In Figure 4.56, Shaun assumes that the reader will know that "young" belugas are not equivalent to "baby" belugas. Without such an assumption, the reader might work out the problem by assuming that "young" and "baby" mean the same, and get a solution different from the one intended by Shaun.

Yet another example of an SCQ assuming shared knowledge is the one in Figure 4.57.

**Figure 4.57:** Maureen, March 31.

A boy went to a roller ring. People borrowed 5/20, then gave back 4/20, then borrowed 1/20, then how much do they have left?

Evidently, Maureen assumed that the reader was aware that she was writing about skates, and that she had given the number of pairs of skates (20) to start with. Without assuming such shared knowledge, the reader would have been unable to solve the above SCQ.

In the examples of the SCQ given in Figures 4.55 to 4.57, students, irrespective of whether they were considered weak, average or good at mathematics by the teacher, seemed to be unable to view the SCQ from the reader's perspective. A partial reason for providing
insufficient information in the SCQ could be that the development of
decentration from egocentrism (e.g. Barnes, 1976; Piaget & Inhelder,
1969) takes time and the grade 6 students here are still too egocentric to
be aware of the need to provide information which seems obvious to
them. It may also be the case that students feel they know their readers
(her classmates) well enough to take such shared knowledge for
granted. Another reason, which I have alluded to earlier when
discussing students' written and oral explanations in the SJ, is that the
lack of an audience militates against self-monitoring for clarity because
immediate feedback in the form of verbal or non-verbal cues is absent.

Yet another way to interpret the assumption of shared knowledge
in the SCQ is to base the reasons for such assumptions on Bereiter and
Scardamalia's (1987) knowledge telling and knowledge transforming
models of writing. Knowledge telling here refers to a model of writing
which makes use of "readily available material from memory" (p. 29),
whereas knowledge transforming refers to a writing model where "re-
processing of knowledge" (p. 7-8) takes place through metacognition.
While admitting that writing down something from memory might itself
have a "knowledge-transforming effect" (p. 29), they maintain that writers
working from a knowledge telling model present content that "is salient in
the mind of the writer but not necessarily sufficient or relevant for the
reader" (p. 345).

Even though they caution that the two models they propose are
models that "refer to mental processes by which texts are composed, not
to texts themselves" (p. 13), I would infer that these SCQ reflect the
mental processes characteristic of the knowledge telling model of writing,
as they evidence assumption of shared knowledge. The SJ indicate that
the students in the study did not explain mathematics concepts clearly in writing, thereby revealing themselves as "novice" writers.

According to Laborde (1990), "the necessity of communicating a message to someone requires an awareness that what is obvious or known to the speaker is neither automatically clear nor necessarily known to the listener or reader" (p. 54). An absence of such awareness in the SCQ would seem to indicate that the students had provided insufficient or vague information and therefore had failed to communicative effectively, as was evident a number of times when groups discussed how to solve the SCQ prepared by others.

Theme 3: The use of numbers which made computation easy

Most of the SCQ, even if they were multistep problems, contained numbers that were easy to compute. Some examples of SCQ with numbers that allowed uncomplicated computing follow.

#1 movie is 108.55 min. long. #2 movie is 90.2 min. long. #3 movie is 98.551 min. long. Use addition and predict which two, put together, would take the longest, in a 2 for 1 movie special? Which two would be shortest?

Figure 4.58: Mike, March 4.
Stepheny wants to watch her favorite show at 5.30 p.m. It’s a 1/2 hour show. In the morning she gets up at 10 a.m. She is at the ice arena an hour later. She gets off the ice at 2/3 from regular skating time which is 3 hours. She eats lunch for 1/3 of an hour. Then she goes shopping for twice as long as she went skating. It takes her 1/2 an hour to walk home. How much of her show will she get to watch?

Figure 4.59: Tanya, February 6.

A new couple just moved in there new house. Their new living room is exactly (square) 6.82 m^2. (for each wall). The couple decided to put their 1.75 m piano on one side. On another side they put their 1.52 m sofa. Across they put their 0.84 m television set. Beside their sofa they put their 1.02 m loveseat. Attached to (Part of one wall in) the living room was a 2.04 m (log) fire place. How much (wallspace) was left of the living room along the sides?

Figure 4.60: Tommy, March 4.

There was a pie eating contest that was held at Stanley Park. There were 4 contestants in the contest. Tom ate 4 pies, and 1/3 of another. It took Tom 1:54.03 to pass out. Exactly :24.04 later Roxanne passed out eating 1 pie more than Tom. 1.54.44 later Jake passed out in 1.00.00 later eating 4 more pies than Roxanne. This declared Jeremy the winner. Jeremy didn’t stop, he was trying to beat the record. The record was 13 pies eaten in 6.04.00. Jeremy ate 12 pies and 2/3 of another in 5.45.35. How much pie was eaten, in how much time? Counting their times together.

Figure 4.61: Tommy, February 10.

In Figure 4.58, though the decimal fractions look complicated, the computation involves addition and subtraction, with no regrouping across
units of time such as seconds and minutes (Mike's solution showed
108.550 + 98.551 = 207.101 min). Note that the expression “2 for 1
movie special” is familiar to students, and such realistic contexts for the
SCQ (as pointed out earlier), seem to aid students in its solution.

The second question (Figure 4.59) is remarkable because it is
seemingly convoluted, but on closer inspection proves to be a
reasonable reflection of some students' routine for a weekend. Although
the context is realistic enough, because of the sequence of actions, one
has to be careful about computing each step. However, the computation
itself is not difficult, as fractions such as 1/2, 1/3 and 2/3 are easy
numbers for the students to deal with (both according to the students and
the teacher).

In Figure 4.60, the words in parentheses were added on by the
teacher to make the problem easier to read and comprehend. All the
students attempted this problem with the aid of diagrams. But once the
diagram was in place, the computations were not difficult. Although it
was a long question, once a mathematical model--in this instance a
diagram representing the data--was used, the actual computation posed
no difficulties for the students.

The fourth SCQ (on pie eating, Figure 4.61) had both common and
decimal fractions based on a rather involved “story” and used rather
unconventional and inconsistent notation for the time periods, with no
indication whether hours, minutes or seconds were involved. (From the
accompanying solution, it seemed that 1:54.03 referred to 1 minute and
54.03 seconds.) In spite of the common fractions in the SCQ, the
computation involved only adding decimal fractions and converting
seconds to minutes.
A glance at the SCQ presented so far under all the themes, as well as some SCQ I will discuss under the next theme, will reveal the extent to which students chose numbers which were easy to compute with. How, then, did students come to choose such numbers?

From observations of group discussions and talks with the students and teacher, I gathered that there were basically four ways by which they came to choose such computationally easy fractions: (a) by sheer coincidence or "luck", (b) by first choosing the fractions and then deciding the whole numbers which were divisible by the denominator of the chosen fraction, (c) by first choosing the whole numbers and then deciding the fraction whose denominator was a factor of the whole number, and (d) from personal interest and experience.

Although (d) above did feature in a number of questions (e.g. Jackie said because she was interested in Rhythmic Gymnastics, she used that as a basis of one of her questions), by and large, (b) and (c) above were the most frequent ways of choosing such easy-to-compute numbers. For example, Mike admitted that while preparing the SCQ shown in Figure 4.62, he tried a few fractions before choosing numbers which he could rename in tenths, indicating that such choices were mainly a result of trial and error.

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Each person got a mark out of 10. But someone renamed some of them! Who was first? Who was third? Can you rename them again in a different way? Given: 4/10, 20/50, 10/100, 2/2, 60/200. Answer: 2/2, 4/10 = 20/50, 60/200, 10/100; 10/10, 4/10 = 4/10, 3/10, 1/10

Figure 4.62: Mike, February 19.
In summary, students prepared SCQ which had numbers that were easy to compute with, even though the questions were long and required a number of steps for their solution. Generally, students tended to use trial and error to get these “easy to compute” numbers, though there were some instances of students using personal experience (as in scoring of events they had taken part in, like Rhythmic gymnastics and skating). Where the SCQ were long, a diagram was sufficient to model the situation and assist in arriving at the solution. In other words, the SCQ seem to provide the students an opportunity to mathematise (Mason & Davis, 1991), for example, by taking into account all the information given, understanding what the question was about, and translating their mental imagery of the problem into mathematical models such as diagrams and other forms of representation before actually carrying out the computation.

**Theme 4: use of discrete rather than region models**

Most mathematics teachers would agree that the region model (such as a rectangular or circular region divided up into a number of parts) is the one very frequently used in the teaching-learning of fractions, almost to the total exclusion of other models, such as the set or discrete model (like a fraction of a number of people). But the SCQ here show a surprising amount of the seldom-emphasized discrete model. Before speculating on some reasons for this high frequency of discrete models in the SCQ, let me give two examples of SCQ which used the discrete model of fractions (Figures 4.63 and 4.64):
There are 5000 computer discs. The night before last, someone stole 2/3 of the discs. How many are left?

Figure 4.63: Jackie, March 31.

Tiffany had 200 cookies. 3/4 had chocolate chips and 1/4 were plain. Tiffany ate 2/50 of the chocolate chip cookies and 1/50 of the plain cookies. How many cookies did Tiffany eat? How many are left?

Figure 4.64: Anne, February 6.

Figure 4.64 was typical of many questions reflecting a discrete model SCQ on fractions, where a countable set of objects was given and a part of the number of objects had to be computed. The first SCQ (Figure 4.63) was one of the few examples where the numbers did not result in whole number solutions. Even so, the computer discs are countable, and hence represent a discrete set. While there was nothing to prevent students drawing a region like a rectangle and letting it represent the whole (5000), the student who constructed the question indicated that she expected a solution using a discrete model.

There were many other SCQ using the discrete model for fractions (such as those referring to a fraction of a number of flowers, trucks, cards, instruments, skates, balloons, basketballs and so on). Instead of giving further examples of SCQ using the discrete model for fractions, suffice it to say that slightly over 85% of the SCQ, especially those involving common fractions, used the discrete model.
This finding does not seem consistent with Ellerton’s (1989) where problems constructed by students tended to be the type of questions provided by teachers and textbooks. (An exception did surface during initial SCQ involving decimal fractions when students tended to use unrealistic situations such as “15.27 people were blond” until the teacher gave examples of decimal fraction word problems in sports.) Even though the teacher in the present study did provide some exercises from the textbook and from teacher-generated questions, most were questions arising out of group discussion in class. The non-threatening atmosphere, the de-emphasis on correct answers and encouragement to justify their “thinking” could have contributed to students’ going beyond the usual classroom model of regions to using the discrete model which is more closely related to students’ daily (non-mathematics classroom) experiences.

Summary of results pertaining to research question #2

Overall, the SCQ revealed that students seemed to construct experience- and interest-based questions which assumed shared knowledge and were computationally simple, but were nevertheless multi-step problems. Almost all the questions were rich in details reflecting everyday experience--details considered extraneous to the mathematical information needed for solutions to these questions, but nevertheless details that served to situate the questions in contexts meaningful to the student. The SCQ were also generally more complex, in terms of the steps or number of operations required to arrive at solutions, compared to standard textbook word problems on the same topics.
Research question #3 and results

Research question #3: What are the roles of SJ and SCQ in mathematics learning, according to (a) students? and (b) the teacher?

Students' View of the Role of SJ in Mathematics Learning

I start with Tommy, the only Grade 6 student who articulated both orally and in writing that he did not find any benefit in writing journals in a mathematics class. Right at the outset, however, I have to say that I have given him more “voice” because (a) he was the only one who had a negative attitude towards the SJ, (b) he was the most articulate, both orally and in writing, and (c) he was present throughout the study and kept his SJ entries current. If some of the students have not been given much voice, it is because they were either not very articulate or had a number of absences or both.

In an interview, when asked whether SJ in a mathematics class were useful, Tommy said “All you’re doing is what’s in your head, it doesn’t help in any way” although “writing down is much easier because there is no one there to stop and say they don’t understand” (March 4). He wrote the following in response to what he thinks about writing in mathematics:

During Grade 6 I have changed my mind constantly, right now I think writing in my math journal is a waste of time kind of. It's just like when you (Mrs. Lomax) don't like it wasting your time. We can just tell you in person what we think. With the time we use writing you can give a slow understanding lesson. When this is over, you only have a quick 10 minute review. I don't know what this is for, all I know is it is for Mr Rama. (Tommy, April 10)

To which the teacher responded “Tommy, I wish all the students felt confident enough to say in person what they think.” Tommy was referring
to journal writing and not writing questions in the mathematics classroom in the above excerpt since in another journal entry he indicated that he liked to write his own questions, stating “Yes, I like to make my own questions. I like to because I put in every thing I have learned into the question.” (March 13)

In answer to my question on whether writing should be done in mathematics classes, Tommy replied (during an interview) that “it sounded odd to do writing in mathematics classes when we’re only supposed to usually write numbers and explain how you got the answer” and that it was a waste of time because

> when we speak, let’s say I write something down and I can talk it out when I write it, and someone times me and I see when I’m done and say, I memorize it, I can speak out before writing. (Tommy, April 14)

From these excerpts, it is clear that Tommy has maintained that he saw no advantages to using SJ in a mathematics class. From the interview data, it seems that Tommy feels that writing takes a longer time than speaking, and therefore less time is given for the teacher to teach. He also did not see any purpose for this activity. His comments seem to echo Pimm (1987) who remarked “Many adults and children alike are reluctant to write things down, particularly in situations where there is no clear reason why they should” (p. 118). But in spite of his negative attitude towards SJ, he “obeyed” the teacher’s injunction to write—an indication of the asymmetrical power relationship existing in the classroom, however well-intentioned or “democratic” the teacher might be. In spite of his stated aversion to writing in the mathematics class, he wrote the most, perhaps because of his fluency in writing. Moreover, most of his writing indicated that he was aware that he was supposed to
explain to an audience. As well, he sometimes wrote as if he were thinking aloud or having a conversation with an absentee audience.

When I spoke to the teacher about Tommy's attitude towards SJ, she had this to say:

You see, the thing with Tommy is that he doesn't really need this to solidify his understanding, because he has such good understanding, so maybe in his case it's not as useful, because he is doing it for himself in his mind. (April 30)

From the teacher's comments and from my own interaction with Tommy over the period of study, I feel that he is one of those who did not benefit from the use of SJ in this study. He was very articulate, and did not seem to feel the need to write something down and look over it, to clarify ideas. He seems to typify students capable of reflecting on and monitoring their learning without having to put everything down "on paper" as seems to be the assumption underlying writing as a metacognitive tool. The relative ineffectiveness of SJ for Tommy could be because most of the writing done in SJ in this study was not the exploratory type. Rather, it was a transactional type of writing, and since he could explain orally just as well as he could in writing, the type of SJ used here did not seem to have any particular advantage for him. Another reason could be that he perceived mathematics as something to do with numbers (recall his statement that he felt it "odd" to do writing in mathematics classes when "we're only supposed to usually write numbers . . . ") and he preferred to solve problems that made him "think" (according to what he told me about what he likes about mathematics questions). So, for him, while thinking about and writing down (and even orally explaining) a solution to a word problem would constitute a valid mathematical task, writing about it in words would be considered not a worthwhile or valid
mathematical task. Exploratory writing (like “what would happen if ?”), on the other hand, might have satisfied his liking for challenging questions and led to a more positive attitude towards writing in mathematics.

What the other students felt about the role of SJ for mathematical learning can be categorised under the headings (a) clarification of ideas, and (b) feedback. Some excerpts exemplifying each of these categories are given next.

**Clarification of ideas**

I think you learn more when you write. Like, say, I don't know, say Mrs. Lomax asks us a question and we actually write it down in our books and solve it and everything, like it just helps, like you can explain how you do it and everything. (Jackie, April 14)

If she talked to you, you might not fully understand, but if you wrote it yourself, you’d understand and probably the others would understand. (Mike, April 14)

In spite of her stated support for the SJ in mathematics classes (as shown in her excerpt), Jackie's written explanations were rather terse and seemed to depend on diagrams, with very few words of explanation (e.g. she just drew diagrams when asked to order fractions). Jackie typified the students who stated that SJ assisted in mathematical learning through clarification and explanation of ideas but provided little overt evidence of such a role for SJ in their writing, possibly because they were writing for themselves. On the other hand, their oral explanations showed more elaboration of their mathematical understanding, as did their SCQ.

From the second excerpt (Mike's)--and after requesting for clarification from Mike--I gathered that “she” referred to the teacher and “others” referred to his classmates, implying that an advantage of writing...
student explanations in the SJ would be that such explanations would be more understandable than those of a teacher to other students in the class. In other words, he seemed to be saying that writing in the SJ serves two purposes: (a) writing for oneself helps one understand, and (b) other students who read the SJ can also understand, possibly because of shared experiences, difficulties and language. His SJ did provide evidence of his understanding (e.g. when he explained what a fraction meant), but there was no evidence of other students’ understanding his explanation, as the SJ were meant to be read by the teacher. One reason for his support of the SJ as a learning tool for mathematics could be that he generally liked to write (according to the teacher and also from interview data).

These two excerpts indicate that the students view writing in the SJ as a means to clarify ideas and to promote understanding of mathematics. Such a view is in keeping with that of proponents of writing to learn mathematics, that the very act of writing forces one to grapple with, and clarify, mathematical ideas (e.g. Kenyon, 1989; Keith, 1990; Pimm, 1991). However, while the SJ did reflect students’ mathematical learning, most of the written explanations were not as clear as their oral explanations. So, while it may be true that they learned through writing in the SJ, they seemed to be learning just as well, if not better, through oral discussion and explanations. Hence, though SJ might be a useful learning tool for clarifying ideas in mathematics, not all students who say they benefit in such a way from the SJ seem to be able to show it in their writing (for example, their written explanations towards the end of the study did not seem to be very different from those at the beginning of the study).
Feedback

For instance, one of our math classes, she said, try to put down your answer and explain how you thought, and even to me it showed me how I think about, in words, how I think and how I progress, even myself, not just my teacher. (Mike, Feb. 5)

Helps one understand things, because when I think something, I write something in there and Mrs. Lomax corrects it and I figure out what I did wrong. If you think something the wrong way the teacher could fix it. (Shaun, March 4)

Well, that’s good. Well, sometimes she can, if you write something and she asks you a question, you can talk to her later, and even if you are confused or something, she can help you. Because if you are just thinking about it, you might not be able to do it in your head, because you have, maybe, to write it down or draw a picture or something. (Maureen, March 4)

I sort of like it because it’s sort of different, get your ideas out and what you think about it. I guess it’s sort of easier because you can draw what you are thinking and write down what you’re thinking. (Jackie, March 4)

The excerpts under the heading “Feedback” seem to indicate that students viewed the role of SJ as providing feedback from the teacher about their mathematical learning. The type of feedback referred to seems to differ. For example, there is feedback from the text to the writer, as exemplified by Mike, who was the only one who said that writing, even to himself as reader, helped him clarify ideas. The other type of feedback implied is the feedback from the teacher to the writer, as indicated by the second and (first part of the) third excerpts.

When I pointed out during an interview that the teacher could help even if the student were to talk rather than write about their mathematics, Shaun said

Because, if you just spoke to the teacher, she wouldn’t remember what you said. (Shaun, April 14)
When asked to elaborate, he said that it would be impossible for the teacher to remember what everyone in the class said to her, but she could remember if they had put everything in writing. I infer that he was emphasizing two points here: one, that SJ provided feedback from the teacher to the student; and two, the relative permanence of written expression as opposed to oral expression allowed for more concerted attention.

This last point seems to be what Jackie, and to some extent, Maureen, too, are making when they talk about writing it down or drawing a picture to "see" their thoughts. In the sense of easier accessibility to their thoughts, such writing seems to encourage interactive writer-text feedback. Mike, too, seemed to agree that writing was a more permanent record allowing for easier accessibility when he said "thoughts, just thinking in your head, you can't etch it in stone in your brain" (April 14). But where Jackie and Maureen implied that writing allows their thoughts to be "seen" on paper, he said that writing helped him "make pictures in my mind" (March 4).

Students' View of the Role of SCQ in Mathematics Learning

As to the role of SCQ in mathematics learning, there seemed to the following categories: (a) to get a better understanding, (b) practice in solving problems, and (c) to provide questions at an appropriate level. Excerpts of SCQ follow under each category.
To get a better understanding

Makes you think more. When you write, you have to have a solution, so you are writing for everyone else, but you are also writing for yourself and answering. So you have to do two things, not one thing. (Tommy, March 4 Interview)

Writing my own problems help me because when you're questioning yourself for the answer or trying to think up a good problem. (Maureen, March 13)

I think when I write my own questions it helps me understand how to make problems and get my own answers. (Darlene, March 13)

The excerpts imply that preparing SCQ cannot be done without an attempt at understanding the mathematics involved, when a solution has to be worked out, too. Hence, preparing SCQ made them understand mathematics better because in doing so, they had to think about both the question and the solution. Although the excerpt by Tommy indicates the role SCQ play in understanding mathematics quite clearly, I have included the other two excerpts to show how the less articulate students have stated the same view.

Practice in solving problems

In the future, you may get a similar question and it will be a lot easier. (Tommy, Feb. 5)

It will help you get a review of it, practice stuff. (Maureen, March 13 Journal Entry)

It helps because you know how you do it in your own question, so you can solve somebody else's question, too. (Shaun, Feb. 5)

From the excerpts, it is clear that the students perceive that the SCQ can serve as review and practice material. In trying to solve their own question, and by the practice obtained in doing so, they also found it easier to solve other students' word problems.
To provide questions at an appropriate level.

I know how people think when they make up questions. I know how other 11 year olds think. (Mike, March 13)

I learn more solving other students problems because they are about something that we know and understand. (Shaun, March 13)

I think solving students problems are better because, one student may put all their knowledge in one question and that would make you think real hard on finishing it, while a teacher puts an easy one, and gradually make it harder. So in 100 questions the teacher makes, it would be the same as 1 of the students hardest problem they can make. (Tommy, March 13)

The excerpts indicate that, according to the students, the SCQ are better able to provide questions at a level appropriate to the students than questions by the teacher. The reason for the appropriateness is that the SCQ are prepared by peers.

In summary, except for Tommy, all the other students feel that SJ had a number of roles to play in the mathematics classroom. Among the ones the students mentioned were: to clarify thoughts, to help express oneself, to understand better, to help teachers assist students having difficulties, and to help remember and review. All these roles I encapsulated under the two categories of “clarification of ideas" and “feedback.” And some of the roles for SCQ, according to the students, were: to get a better understanding (as preparing and solving questions needed clearer understanding of concepts), to give practice in solving problems (as every question had to be accompanied by an answer), and to provide questions at an appropriate level (as the questions were prepared by their peers).
Teacher's View of the Role of SCQ in Mathematics Learning

The teacher's views recorded below were mostly obtained through an interview on April 30, 1992 (some were obtained through discussion with her and also through what she had written in her assignment for her night class). According to the teacher (Linda), she initially felt positive about SJ and SCQ, mostly because of her experience with them in the previous project. She changed her mind about the SJ approximately half-way into the study, but not about the usefulness of the SCQ. She said:

I think that the SCQ definitely are motivational, the kids really liked writing them and they liked solving each other's questions and I think that they have potential for showing the teacher what the kids' conceptual understanding is like. Think you can use them as sort of an evaluation of the understanding.

When asked to comment specifically on whether SCQ helped the mathematical learning of the students, this is what she said:

I think that I got more from the SCQ in terms of their mathematical learning. I could know from what they have written, who had a pretty good grasp of this and who didn't. And it was easier to see with the questions when there had been a change in their mathematical learning. Like Chris is a good example of that. Because at first he wrote problems that had no fractions in them and then he whoops, in one week, wrote a really good interesting problem about fractions that shows that he had gained a whole lot of understanding about what these really meant.

And in terms of equivalence, that was really obvious, too. So that when the Shaun's band problem, when he had 12 sixtieth of the band played the clarinet, some sixtieth played this and that, he had a good understanding of what the fractions meant, but it was only if he had the fractions with the same denominator as the number in the whole.

But it was interesting to see which students were ready to look at that, they all thought that it was an okay problem to solve, but that it was easy. And it was interesting to work on that to see which kids could see how to make that more difficult and they could see that the factors of 60 and they could be rewritten and all of those
things. So I really like the potential of what that reveals, and also that you can work with that in the terms of the context, to see which kids you can shove along a little bit further in their understanding.

One reason for her viewing SCQ as useful was that she could see students' involvement and interest in the SCQ, as evidenced by their discussion and willingness to work on the SCQ without being aware of the time passing. For example, sometimes the students were so engrossed in their SCQ, they looked surprised when the mathematics period came to an end. At other times, students were eager to present their solutions to the class, which were usually done with transparencies and overhead projectors, something which was usually done by the teacher. In other words, other than the novelty of the SCQ and the presentation, they had an opportunity to take some responsibility for their mathematics learning. Moreover, they had some sense of ownership of the exercises as their SCQ were used for class exercises.

As a teacher, Linda realised the value of the feedback she obtained through what they wrote, as their SCQ mirrored their understanding of the topic being taught. The SCQ also revealed the improvement of the students' knowledge of fractions, as she pointed out above about Chris and Shaun. She was also amazed at the range of questions students seemed capable of, revealing the SCQ as a potential resource for meaningful exercises (this she told me during an informal talk with her during the course of the study).

When I asked her how she felt about the frequency of SCQ, and whether she would use them for other topics as well, she replied:

in terms of SCQ in math, I was really happy about writing once a week and then, so the next week, so that when that routine was happening, that worked really well. They would write, I would publish and then we would work on those. And one week that happened that there were a lot of really valuable problems to
solve and we could do one a day and on weeks where there were several that were of similar kind we could chunk them and say do one of this kind, and then we could talk and talk about problem solving more generally than just this particular problem. So once a week worked pretty well for that, I think that was okay, but I don't think we'd wanna do it for every topic in math.

Her reservation about using SCQ for other topics stemmed in part from her experience when students were asked to prepare SCQ on decimal fractions. According to her, initially they prepared unrealistic questions which included information such as “15.27 people were blond.” She realised that the students' difficulties arose from a lack of meaningful situations in their everyday experience where they had to use decimal fractions. She said “They know about 1/2 a chocolate bar and 1/4 of a pizza but they have no reason to know about 12.3 metres of anything!” So, after she gave examples of word problems from sports, students generated many interesting SCQ on decimal fractions, with many of them different from the type of examples Linda had given. Anyway, she said that she would only use SCQ for topics which she felt students could relate to their own experiences.

In addition to what she said in the interview about the role of SCQ, from my discussions with her and from her night class assignment, I gathered that the SCQ allowed her to give increased attention to problem solving, communication and reasoning. As well the SCQ helped her accommodate the diverse needs of the students, and was a source of information for evaluation. Overall, she felt that the SCQ was a valuable teaching-learning tool for mathematics.
Teacher's View of the Role SJ in Mathematics Learning

As to the SJ, she had this to say:

In terms of the journals, I've been thinking about why I'm not as enthusiastic about the journals and I think that's because the problem writing was my project and the journal writing was yours, and I, whether I realised that, viewed it that way, and the kids view it that way, so that they view writing in their journal as something we are doing for you, for your research. They don't view it as something they are doing for themselves or even something they are doing for me, so I think that it has to be clearer, the purpose of it to the kids: we are doing this because I believe that it will help you to understand this better, and I also think that it has to be clearer in my mind what the purpose is of the question I am asking them: I am asking them this because I want to know this from them, not just what are we going to write in our journal today, oh we'll write a question about how we feel about something, well there wasn't, it wasn't purposeful enough for my teaching and their learning. We shifted that purpose for "this is for Mr. Rama's research."

I think the crucial point she was making is brought out by her expression that "it wasn't purposeful enough for my teaching and their learning. We shifted that purpose for 'this is for Mr Rama's research'." Her view seems to parallel Stubbs' (1980) statement that "a general principle in teaching any kind of communicative competence, spoken or written, is that the speaking, listening, writing or reading should have some genuine communicative purpose" (p. 115). The SJ as used here did not seem to have this genuine communicative purpose.

A reason for her waning enthusiasm for the SJ was that she was involved in a number of time-consuming activities. For example, her time commitment to her ongoing project (an assignment for her night class which she was doing for credit at the university focused on "The use of student written problems to promote mathematical power") did not allow her to give as much time as she might have liked for the SJ. She also had a student-teacher to supervise and she was involved in a number of
professional development courses. In addition, she had her normal
teaching load.

She elaborated on the apparent lack of success of the SJ, saying

I think that as a teacher, I need to think more clearly about what is
it that I am trying to find out when I ask them to write in their
journal. Is it for them or for me? What are they getting out of it?
What is it that I am gonna look for, when I look at this? Ya, it
wasn't, the purpose wasn't clear enough in my mind or their mind.
That's basically why I don't think that was as successful as SCQ.

One of her reasons for assuming lack of success for the SJ was that
sometimes she had to think really hard to initiate appropriate entries for
the SJ, so much so that she felt it was rather artificial to push for
something when learning seemed to be going on reasonably
successfully in her mathematics classes. Some of the entries were very
short and unclear as to the intent of the writer. In other words, the
students were not fluent writers. She commented on her difficulty in
assessing student understanding of mathematics from entries in the SJ.
Was it “the language that is not clear or the math they are confused
about?” According to her, the SCQ gave her better information about
students' mathematical understanding. Moreover, she seemed to learn
much more about student understanding from classroom discussions
and presentations by the students than from the SJ.

She felt that in order for the SJ to be more successful, she had to
spend more time and effort on her responses to students entries, as well
as plan what she hoped to get out of asking students to write in their SJ.
And she just could not afford the time and effort involved, especially
when the students seemed to be learning pretty effectively with one of the
innovations, namely the SCQ and she had so many other competing
commitments.
When asked about the frequency of the SJ, she said

When I first started doing learning logs, it was done on Friday and I used to have the topics from the week, and some I wanted them to specifically to comment on and then there was choices what do you think of this or that and how is this or that going, but I think that that was, somewhat useful as a sort of a summary of the week and it was a good way to report to parents and it did go home and come back and all that stuff, but I think that is different than this kind of learning log where it is specific to the subject and today's lesson and it has to happen right now and it's part of the lesson. So, I am not sure about how frequently. Not too often is the big fear.

While she could see the benefits of writing summaries once a week (as in her previous project), she was doubtful about the benefits of SJ entries on specific topics done on alternate days, as was done here. Overall, she felt disappointed with the use of SJ in the mathematics classroom. She felt that the way she used SJ did not seem worth the time and effort, and for it to be more successful she needed to be really aware of specific purposes for using them. In other words, she felt she and the students did not assume ownership of the SJ, as they did of the SCQ. But she was convinced that the SCQ were really motivating, and helped the students improve their understanding of specific mathematics topics, and the teacher to get feedback on student understanding.

I shared both her misgivings about SJ and her enthusiasm for the SCQ. My misgivings were mainly due to the wide disparity in students' levels of discussion and verbal explanations, compared to their writings in the SJ. While students were articulate during the oral discussions, they were rather stilted in their writing, or seemed unable to express themselves clearly, except for a small minority.

However, perhaps we (Linda and I) were unduly pessimistic, given that the majority of the students felt that SJ did assist in their mathematics
learning. Even if the students were trying to be accommodating, there is some evidence from their SJ that they were useful, at least insofar as identifying misconceptions or partial understanding as well as focusing on certain concepts, principles and relationships in mathematics. Moreover, if "one use of written language is to externalize thought in a relatively stable and permanent form, so it may be reflected upon by the writer, as well as providing access to it for others" (Pimm, 1991, p. 20-21), the SJ did serve a purpose. Even so, the findings of this study about the benefits of SJ (as used here) are that these benefits do not seem to be obtainable only through the SJ or that these benefits are superior to those that could have just as easily been obtained through verbal explanations and discussions.

**Summary of Results Pertaining to Research Question #3**

Students were generally positive about the role of SJ and SCQ in the learning of mathematics. But while not all agreed about the positive role of SJ, the teacher and students were unanimous about the benefits of SCQ. In contrast, the teacher did not feel that the SJ were very successful.
CHAPTER 5. CONCLUSION

This chapter is divided into five sections: summary of the study, general discussion about the results of the study, limitations of the study, implications for practice, and possibilities for future research.

Summary of study

In this study, I investigated the mathematical learning revealed through the use of student journals (SJ) and student-constructed word problems (SCQ) in a mathematics class where the teacher initiated the procedures and conducted the lessons. The class was a multi-level Grade 5 and Grade 6 class but only the Grade 6 students were the focus of the study.

Three times a week, towards the end of their mathematics lesson students wrote in their SJ in response to teacher prompts. Once a week, the students also prepared SCQ, in groups and individually. These SCQ were collected, edited, typed and redistributed to the class by the teacher, and used as class exercises. Six Grade 6 students were interviewed (video-taped) three times each over the 15-week period of study. Records of these interviews, classroom observations and a teacher interview were all used to complement the analysis of SJ and the individually-prepared SCQ.

Although the SJ did give insights about students' mathematical knowledge, the SCQ was a better indicator of students' knowledge of fractions as they showed the students'

1. mathematical knowledge implicitly (through the type of question asked, information given, and the accompanying solution) and
2. improvement in their fraction knowledge explicitly (through the type of fraction relationships, the amount of detail, and the number of steps and operations involved).

The SCQ also revealed that students tended to base their word problems on

1. their own experiences and interests
2. an assumption of shared knowledge between the reader and writer of the word problem
3. numbers which made computation easy and
4. the discrete model of fractions rather than the region model.

The results of this study indicate that the SCQ assisted mathematical learning in a classroom context but that the value of the SJ needs to be reconsidered.

**General discussion**

In this study I set out to investigate the mathematical learning revealed through the use of SJ and SCQ in a "normal" mathematics class where the teacher herself initiated the procedures and conducted the lessons. The results of the study indicate that students demonstrate mathematical knowledge about common and decimal fractions through their SCQ and SJ, but that their ability to communicate such knowledge, especially through the SJ, is not necessarily an indicator of their mathematical knowledge. Indeed, as evidenced by discussions and oral explanations, some students seemed to have more knowledge than indicated by their writing in the SJ, so much so that it could be said that ability to communicate mathematical knowledge through written words
seems to be a sufficient but not a necessary condition for possession of such knowledge. Use of diagrams to complement explanation in words seemed to be a common feature of the SJ as well as of oral explanations.

Many benefits of the SJ were documented in this study (such as providing feedback for the students) but such benefits could have been obtained as well, if not better, through group and class discussions as well as oral explanations, according to my observations throughout this study. Indeed, the benefits derived through the SJ do not seem worth the amount of effort put in by the teacher (to read and respond) and the student (to write).

Most journal usage in previous studies had encouraged exploratory language and feelings about mathematics but here the SJ were used specifically to communicate the student's understanding of a mathematical concept or principle in written language. Perhaps this type of SJ, where written explanations of concepts and principles were the focus, is not suitable for this age level. I found that most of the students could express themselves better orally than in written form. This finding is consistent with what others have found about student writing in mathematics at higher levels. For example, Miller (1992), in discussing benefits of writing in some high school algebra classes, said that "students' written responses do not always accurately portray their understanding" because "at this stage the student was better able to communicate mathematical knowledge verbally than in writing" (p. 338).

While it is true that Linda's busy schedule (such as supervising a student teacher and attending night class) and gradual disenchantment with the SJ (such as lack of clear purpose for the SJ) might have contributed to what she perceived as lack of success of the SJ, perhaps it
was too much to expect her to try to implement two innovations (SJ and SCQ) concurrently for the duration of almost three months.

The students' and teacher's perceptions about the usefulness of the SJ were different. Whereas the former stated a number of benefits (such as being able to clarify ideas) for the SJ, the latter was doubtful if the SJ could reveal whether the student's difficulty was with language or with mathematics. Perhaps the very novelty of the SJ might have led students to accept the SJ as beneficial, just as the teacher herself had been interested in SJ earlier. In spite of the students' stated views of the SJ as contributing to their understanding, I did not find much evidence of this in the writing of the less articulate students, though it is certainly possible that such understanding is covert and may not be revealed through their writing.

The success of the SCQ might at least in part be attributed to Linda's extra motivation in the assignment she had chosen for her night class. The assignment was entitled “Classroom Inquiry” and she focussed on SCQ as a way to “promote mathematical power.” Linda's interest in and attention to the SCQ could have been “caught” by the students. Even if the students were not influenced by Linda's feelings about the SCQ, they seemed to enjoy the SCQ, perhaps for the novelty, sense of ownership, or the student-student interaction they engendered. The discussion that evolved during attempts to solve the SCQ served to inform the teacher about her students' current state of mathematical knowledge and gave an opportunity for the students to ask for clarification of unclear questions, unstated assumptions, or to evaluate unrealistic problems. Whatever the reasons, the students and teacher
were unanimous about the benefits of SCQ as a teaching-learning tool for mathematics.

Among the four themes identified in the SCQ, the fourth theme of the predominance of discrete models in their SCQ seemed at variance with the students' use of the region model to explain fraction concepts in an interview context. While more than 85% of the SCQ were based on the discrete model, almost all the explanations during interviews were based on the region model—possibly because basic fraction concepts such as equivalence and ordering were involved. As well, perhaps students associated interviews with the formal context of classrooms (where the region model was dominant), whereas the SCQ were more experience-oriented and student-owned.

Both SJ and SCQ did make use of class time—with the SJ generally taking less time—but the time expended for these two activities seemed to be viewed differently: the SJ seemed to be something that had to be done but the SCQ seemed to be something the students wanted to do. From my observations in class, I could witness the students' enthusiasm (e.g. animated discussion) during the SCQ, but such enthusiasm was not evident when they were asked to do their SJ. Even though students did not evidence enthusiasm for the SJ, they treated the SJ as part of the class routine.

The SJ required the teacher to think about appropriate prompts, the students to write (individually, with hardly any discussion), and the teacher to read and respond. In contrast, the SCQ (prepared both in groups and individually) did not need too much extra time or effort by the teacher because the teacher did not have to prepare questions catering to individual needs and interests. She only had to edit the SCQ and
group them in order to optimize learning during the discussion of their solutions by groups and individuals. For the students, their enthusiasm in preparing the SCQ seemed to make them overlook the effort needed to prepare the SCQ.

Overall, in the context of this study, the SCQ seem an effective teaching-learning tool that can be integrated into the normal routine of a mathematics class. The SCQ also seem to reflect the importance of student experience and ownership of learning. In contrast, the SJ, as used here, do not seem to have any decided advantage over verbal classroom discussion and explanation by the students and whatever benefits derived from the SJ seem disproportionate to the time and effort required.

**Limitations of the study**

One of the main limitations of the study was the difficulty in deciding the extent to which written language could reflect mathematical knowledge. While this limitation was less severe in the SCQ (because of the emphasis on asking rather than explaining), the nature of the tasks involved in the SJ--explaining mathematical concepts and principles using written language--made written proficiency imperative. Class discussions and follow-up interviews revealed students could explain things better orally than in writing. If it were not for these discussions and interviews, I might have concluded that lack of written proficiency might indicate lack of mathematical understanding. Now, however, I infer that written proficiency was a sufficient but not a necessary criterion for mathematical understanding.
Another limitation of the study was that while the teacher had made the SCQ part of her own agenda, she did not seem as committed to the SJ. She had admitted her lack of enthusiasm for the SJ about halfway into the study and later said she felt the SJ had not seemed purposeful enough for her. Perhaps her view that SJ was not very successful might have altered if she had been as enthusiastic about the SJ as she had been about the SCQ.

A further limitation of the study was that it did not take much account of the student-student interaction during the preparation of the SCQ. The language students used during such interactions could have provided data on the extent and type of mathematical discourse that resulted in the SCQ. Such information could have contributed to an awareness of conditions influencing the production and quality of SCQ. Even though I knew that classroom discourse was an important factor in a teaching-learning situation, I did not want to make that a focus of my study, just as I did not want to focus on affect, in spite of its importance. In retrospect, it looks as if I have erred on the side of caution, as by trying to be cautious about being drawn into factors such as classroom discourse—which have potential for complete studies in themselves—I have stayed too much at the fringes to inform my own study advantageously.

Implications for practice

It is a truism that unless the teacher sees a purpose for an innovation and is committed to it, it will not be worth the time or effort of the teacher. In this instance, it seems that the teacher's views about the usefulness of the SJ did not correspond to the students' view. Because the students indicated they did find benefits for the use of SJ, the teacher
might try using SJ for a period of time before deciding to continue or discontinue their use. Even if the teacher and students wanted to use the SJ, the students' written proficiency would have to be taken into account before attempting the type of SJ used here. Where student proficiency is in doubt, they should be encouraged to use diagrams and oral explanations to complement their written explanations. As the findings indicate, students seem to use diagrams to complement their explanation, even without being asked to do so. And, instead of the written journal being the culminating task, it could be used to initiate oral discussion. Perhaps some examples by the teacher--either her own or those from other students--might also prove helpful for students not used to journals or for those unsure of what is expected of the journal entries.

The evidence for the benefits of SCQ seems indubitable. From the first theme of student experience and interest, it would seem appropriate to encourage students to prepare SCQ as part of the routine of the mathematics lesson. Since teachers are always looking for ways to individualize and vary exercises for the students, I would suggest that SCQ seem suited for these purposes.

The assumption of shared knowledge, the second theme of the SCQ, could be used to good advantage in a classroom situation. For example, such SCQ could be used to initiate discussion of ambiguities in questions, other ways of looking at questions, the artificial or realistic aspect of a question, and justification of arguments in the solution process.

The third theme of the SCQ, the use of numbers which make computation easier, can make teachers aware that a word problem need not have complex computations to make it mathematically adequate. In
other words, teachers can use SCQ to identify student understanding of mathematics principles, even without including complex manipulation of numbers. It is well known that many students have procedural knowledge (Hiebert, 1986) and can manipulate numbers by using an algorithm without understanding the mathematical concepts involved. Hence, just as a child who can recite the number names need not necessarily understand cardinality and ordinality of numbers, so a student who can obtain correct answers for fraction questions need not understand fraction concepts. A move away from equating computational competence to mathematical competence (such as estimating, problem-posing and problem solving) through the SCQ might prove pedagogically more defensible and optimal, especially with the availability of calculators and computers.

The fourth theme, the use of discrete rather than region models, should make teachers pause and reflect on the type of models they usually use for fractions. In spite of the predominance of the region model in previous classroom exercises and manipulatives used during the present study, students chose the discrete model. Their choice showed that left to their own devices, students tend to revert to familiar experiences to situate their problems and make the problems meaningful to them. Perhaps such a choice is also indicative that discrete sets are more amenable to the operation of division (e.g. a fourth of 24 is viewed as 24 divided by 4)—which might be how students view basic fraction concepts and operations.

I would also speculate that if teachers believe students should start from what they know, and have some responsibility and choice for their learning, then SCQ and SJ might have a role to play in the
mathematics classroom. Furthermore, if one aspect of mathematical competence is communicative competence--knowing how to communicate effectively according to context-dependent cues--then writing in mathematics by using SJ and SCQ should be encouraged in the mathematics classroom.

**Possibilities for future research**

A number of issues raised by this study could be useful starting points for further research. For example, the SJ used here attempted to gather information about students' communication of their understanding of mathematical concepts and principles. The students wrote to and for the teacher, and they were dependent on the teacher to tell them how well they had succeeded in their task. There were two facets of this task. One was writing proficiency. The other was external accountability through written explanation to the teacher.

To compensate for the lack of written proficiency, students could be encouraged to explain orally and draw diagrams complementing their verbal explanation. An analysis of the student diagrams and oral explanation could provide data on students' understanding of mathematical concepts and principles. In addition, rather than requiring only explanations, they could be asked to question themselves or the teacher about things that were puzzling them. In questioning themselves or the teacher, students are attempting the use of metacognition to aid their learning. Thus a study comprising both these facets--explanations (drawn diagrams and verbal explanation) and questions puzzling students--could expand our knowledge of students' communicative
competence in mathematics as well as their control and internalization of mathematics learning.

In this study, the issue—whether lack of writing proficiency might hinder the expression of mathematical understanding through written communication—kept coming up. A study comparing mathematics journals of students in different grade levels on similar mathematics topics might indicate how language background and writing proficiency might influence communicative competence in mathematics.

It is instructive that one student—who was the best in mathematics in that class and who could also articulate well in his SJ—did not see any advantages in using SJ in a mathematics class. Perhaps there are students in other classes who might also find SJ not beneficial and who might, like Tommy, prefer oral to written explanations in mathematics. If so, how can teachers address such an issue, especially if they believe in the SJ and also in allowing students in general and such students in particular, more choice in, and responsibility for, their learning of mathematics? What are the wider implications to using writing as a tool for learning mathematics? While this study has raised these questions, I have not attempted to address them. Perhaps future research might address these concerns.

Another area of research could be an analysis of discourse in mathematics classrooms. For example, Pimm (1987) has studied teaching gambits as a factor in initiating verbal communication in the mathematics classroom. Because the student-teacher and student-student interactions influence mathematics learning in the mathematics classroom, the two aspects of discourse analysis—production of text and interpretation of text—should reveal information about how students are
helped to produce text and how students and teachers interpret text related to mathematical learning.

A related aspect is whether the SCQ prepared in groups would differ from those prepared individually. For example, were the SCQ similar in quality? What if students prepared SCQ individually first and then in groups (that is, the reverse of what was done in this study)? These, and other such questions, might prove a fruitful area for research about SCQ.

The SJ, as used here, focussed on how students attempted to communicate their understanding of fraction concepts and principles. Previous studies have shown that attitudes toward and difficulties in mathematics affect students' mathematical competence. So, for those interested in gaining a more comprehensive picture of student understanding of mathematics, they might want to use a modified form of the SJ used here together with Clarke's (1987) IMPACT procedure. Such a combination might be useful as the IMPACT procedure allows for information about attitudes and difficulties to be obtained without requiring a high level of written proficiency, and a modified form of the SJ allowing for lower language proficiency might be incorporated into an expanded version of the IMPACT procedure.

For example, if students seem to prepare SCQ based on recent experiences in school, teachers might wish to consider how such experiences could be incorporated into the mathematics classroom to complement or provide an alternative to teacher-generated mathematics exercises. Suppose the student changes the unit or whole when comparing the magnitude of fractions--for example, by drawing two different-sized rectangles to compare 2/3 and 3/4. Then the teacher can
infer that the student has some difficulty in understanding basic fraction concepts. As well, if there is evidence of cognitive benefits, then elementary school mathematics teachers might reflect on and try implementing some ideas from this study that seem appropriate to them. Even if the supposed benefits do not accrue, a close analysis of the study might suggest whether such an approach, suitably modified, might work in different circumstances or in a different class.
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APPENDIX
(Sample interview questions)

Interview 1  (Wed., Feb. 5, 1992)

1. When you hear the word “Christmas,” what picture comes to your mind?

2. When you hear the word “mathematics,” what picture comes to your mind? Explain.

3. Is there anything different about this year’s math lessons? What is different?

4. Do you think writing in the math journal/notebook helps you learn maths? How does it help?

5. Do you think writing your own questions helps you learn maths? How does it help?

6. Give me an equivalent fraction for 1 fourth. Explain how you got your answer.

7. Alex and Maureen share a chocolate bar. If Alex eats 1 half of the chocolate bar and Mercedes eats 1 third of it, how much of the chocolate bar has been eaten? Explain how you worked it out.
Interview 2. (Wednesday, March 4, 1992)

1. What is a fraction? How would you explain to your younger brother or sister (who does not know about fractions) what a fraction is?

2. You have been doing quite a lot of journal writing in mathematics. Earlier you told me it helps you learn mathematics. Do you still feel journal writing helps you learn mathematics? Explain.

3. What about writing your own questions? How do you feel about it now? Do you prefer to work alone or in groups when preparing questions? Do you think your questions are better now than before? Explain.

4. You seem to have got a rule for equivalent fractions. What is your rule? Does it always work? What helped you in getting the rule? (discussing, journal writing, preparing own questions)

5. How do you compare and order fractions?
Interview 3 (April 15, 1992)

1. Do you think that learning math is important? What would be 2 important things that you should learn in math?

2. How do you learn math best? How should math be taught?

3. In math classes, usually we do sums. But you have been doing quite a lot of writing in addition to doing sums. Do you think writing should be done in math classes? What advantages do you see for writing in math classes? What disadvantages? How do you think writing helps you learn math?

4. Earlier you had said “math is like ...” Explain. Complete “fractions are like ...”

5. How do you add common fractions, like 1/2 + 1/4; 1/2 + 1/3?

6. How would you multiply decimal fractions?

7. Why do you think multiplying decimal fractions sometimes makes the answer smaller?

8. Do you think my being present for your math classes affected the way Mrs. Lomax taught you? The way you learned math? Did you feel comfortable, uncomfortable, or whatever, because of my being in the classroom?