LONGITUDINAL IMPEDANCE OF
A PROTOTYPE KICKER MAGNET SYSTEM

by

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Abstract

The longitudinal impedance of a prototype kicker magnet system for the proposed KAON Factory was measured from 0.3 to 200 MHz using the coaxial wire method. The method consists of transforming the aperture of the kicker magnet under test into a coaxial structure by inserting a central conductor so that the transmission coefficient can be measured. The longitudinal impedance was calculated from the transmission coefficient. The measurement was performed in two frequency ranges. From 0.3 to 50 MHz the magnet was transformed into a 50 Ω coaxial line, and from 45 to 200 MHz into a 180 Ω coaxial line. TSD calibration of the measurement assembly was performed in the higher frequency range while HP calibration of the test cables was performed in the lower frequency range.

Resonances in the longitudinal impedance are present below the cut-off frequency (30 MHz) of the magnet. The effects on the impedance of a speed-up network and a saturating inductor, installed on the input to the kicker magnet to improve its kick performance, were determined. It was found that a speed-up network damped only the high-frequency resonances whereas a saturating inductor reduced their number. Above the cut-off frequency, other components of the magnet system, such as the speed-up network, the saturating inductor and the cables, were found to have negligible influence on the impedance.

The maximum real longitudinal impedance of the magnet was measured to be 32 Ω. The total contribution to the Booster ring is 425 Ω and to the Driver ring is 1184 Ω, respectively. These contributions are small compared with that of rf cavities which have typical values of 10 kΩ. Hence the contribution of kicker magnets is negligible and furthermore any longitudinal instabilities due to kicker magnets can be damped with existing damping systems for the rf cavities except, perhaps, for short bunch (1 ns) operations.
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Introduction

Major goals of accelerator physics today are to increase beam intensity and energy. With the proposed KAON Factory, a high beam intensity is needed to improve the production of secondary beams, since cross-sections decrease as beam energy is increased. Beam intensity in a high energy accelerator such as the proposed KAON Factory is limited by beam stability. For example, space-charge tune spread at injection restricts the intensity of the beam because some particles are carried near unstable betatron resonances. A beam also interacts electromagnetically with its environment and coherent instabilities in the motion of the beam in the longitudinal and transverse directions can be induced by the interaction. If these motions are undamped, they can grow quickly as they are being driven repeatedly. In an extreme case, when the amplitude of the driven motions is sufficiently large, control of the beam could be lost, resulting in beam blow-up or even loss.

The interaction of the beam with its environment is characterized by a wake potential. It describes the electromagnetic field, usually called the wakefield, produced by the beam and its image currents in surrounding materials. Wake potentials are classified into two categories according to the type of motion they induce on a beam: longitudinal and transverse. The wake potentials are time dependent as well as source dependent. A more convenient concept to characterize the same interaction is impedance since it is a property of the accelerator component but not of the beam. The longitudinal and transverse impedances are the Fourier transforms of the respective wakefields of a point charge.

In the proposed KAON Factory, a beam of 100 μA is accelerated to 30 GeV by two synchrotron rings in series with three storage rings. Kicker magnets are used to extract and inject the beam from ring to ring. A large number of magnet modules is needed to achieve a sufficient “kick” because the momentum of the beam is high, especially in the Driver ring (30 GeV). The wakefields of the kicker magnets could greatly affect the motion of the beam. Of particular interest here are the longitudinal wakefields. Instability thresholds and energy loss can be calculated from the measured longitudinal
impedance. Comparing the measured longitudinal impedance of the magnets with that of other accelerator components in a ring, such as the rf cavities, one can determine whether the magnets play a major role in causing coherent instabilities.

This thesis describes the measurement of the longitudinal impedance of a prototype kicker magnet for the proposed KAON Factory. Chapter 1 deals with the definition of a wakefield and the concept of longitudinal impedance. It also describes the rôle of the kicker magnets in the KAON Factory and the configurations in which they would operate. The principles and the method of measurement are outlined in Chapter 2. The chapter ends with a discussion on transmission-coefficient measurements. Chapter 3 begins with an overview of the experimental set-up and the measurement. The remainder of the chapter describes the set-up, the calibration of the measurement assembly, and the measurement procedures. The measured longitudinal impedances along with discussions of the results are presented in Chapter 4. The thesis ends with some conclusions.
1 Impedance and Kicker Magnets

1.1 Impedance and Beam Instability

The electromagnetic field induced by a point charge in an accelerator component is usually called the wakefield. Subsequent charges following the point charge will see the wakefield and interact with it. The interaction can be described by a wake potential which is given in terms of a Green's function. In order to calculate the wake potential in a given accelerator component due to the passage of a bunch, firstly, the wakefield excited by a point charge must be found by solution of Maxwell's equations. Then, the bunch wake potential is found by the convolution of the point wake potential with the bunch charge distribution. The same interaction can be described in the frequency domain by the Fourier transforms of the wakefields. Wakefields are divided into two types depending on their effects on the beam motion: longitudinal and transverse. The transformed wakefields are the longitudinal and transverse impedances, respectively.

The impedance is a property of an accelerator component but not of the beam itself. This is the main advantage of the impedance concept. The real part of the longitudinal impedance is responsible for the energy loss or gain of the bunch. The imaginary part of the longitudinal impedance is associated with the incoherent tune shift and bunch lengthening of the beam. Instabilities, such as the microwave longitudinal instability, depend approximately on the absolute value of the longitudinal impedance as given by the Keil-Schnell criterion [20]. In general, single-bunch instabilities are due to the high-frequency broad-band impedance, while multi-bunch instabilities depend on the low-frequency narrow-band impedance. Narrow-band impedance is a term used to describe the narrow resonances in the impedance spectrum at low frequencies. Each resonance is produced by a local slow-decaying wakefield whose frequency is below or not much above the wave-guide cut-off frequency of the accelerator component. In the high frequency region well above the cut-off frequency, the resonances overlap, producing a smooth frequency dependence in the impedance.

The high-frequency narrow-band impedance describes the interaction of the beam
with abrupt changes of the beam pipe cross-sectional area as well as the high-frequency resonant modes of accelerator components such as rf cavities, bellows, and vacuum ports. For bunch lengths larger than the beam pipe radius, the detailed behavior of the high frequency impedance is not a major concern. It is approximated by a single broad-band impedance. For short bunches, the high-frequency impedance plays a major role in determining the single-particle motion.

1.2 Wake Potential and Longitudinal Impedance

In the frequency domain, the counterpart to the wakefield is the impedance. Suppose the beam current $I$ has a component of single frequency $\omega$ travelling in the longitudinal direction $z$, described by the real part of

$$I(z,t) = \hat{I}e^{i(kz - \omega t)}, \quad (1)$$

where $\hat{I}$ is the complex beam current amplitude and $k$ the wave number. The longitudinal component of the wakefield $E_z$ due to the beam can be obtained by adding up the contributions from all the charges $q$ preceding the field point in the form of a wakefield [17]

$$E_z(z,t) = -\frac{1}{v} \int_{-\infty}^{0} ds \ I(z, t - \frac{s}{v}) W'(s), \quad (2)$$

where the wakefield $W'(s)$ is the first derivative of the wake potential produced by a unit point charge with respect to $s$, its distance behind, and $v$ is the group velocity of the beam. Substituting Equation 1, the above equation can be written as

$$E_z(z,t) = -I(z,t) \int_{-\infty}^{0} e^{i\omega s/v} \ W'(s) \frac{ds}{v}. \quad (3)$$

A bunch traversing some length $L$ of the wake potential will then experience an energy loss due to the voltage drop $E_z L$. Borrowing the impedance concept from circuit theory, the voltage drop $V(z,t)$ can be expressed as a product of the beam current and an impedance $Z_{\parallel}(\omega)$ as

$$V(z,t) = -I(z,t)Z_{\parallel}(\omega), \quad (4)$$
where $V(z, t) = \dot{V} e^{(k z - i \omega t)}$ and the impedance is in Ohms ($\Omega$). Finally the impedance can be written as

$$\frac{Z_{ll}(\omega)}{L} = \frac{1}{v} \int_{-\infty}^{0} e^{i \omega s/v} W'(s) \, ds.$$  

The longitudinal impedance is just the one-sided Fourier transform of the wakefield $W'(s)$.

### 1.3 Wakefield of a Beam Pipe

Let us consider the wakefield of an component with a simple geometry. The simplest accelerator component is the beam pipe through which the beam circulates under
vacuum. Its cross-section is usually circular or rectangular and it is made of metal to
provide electromagnetic shielding and a path for image currents. The particle has a
charge \( e \) and is travelling with velocity \( v \) along the axis of a cylindrical pipe of radius
\( b \) as shown in Figure 1. First, assuming that the walls of the beam pipe have zero
resistance, then the wakefield at the walls will only have a radial component. The
resultant wakefield is plotted in Figure 1. The solution is worked out in detail in
Reference [11]. The impedance per unit length \( L \) is

\[
\frac{Z_{\parallel}(\omega)}{L} = -j \frac{Z_0 \omega g}{4\pi c \gamma^2 \beta^2},
\]

where \( \beta \) and \( \gamma \) are the relativistic parameters of the bunch, \( Z_0 \equiv 377 \, \Omega \) the free space
impedance and \( g \) a geometric factor defined by the beam radius \( a \) and beam pipe radius
\( b \) as \( g = 2 \ln(b/a) + 1 \). As expected, the real part of the impedance is zero since no
energy loss can occur in walls of zero resistance. If the walls of the beam pipe have
some resistance, then the axial field \( \vec{E}_z \) will not be zero. The wakefield of a beam pipe
with non-zero wall resistance is plotted in Figure 2 for the case where \( v \) approaches
the velocity \( c \) of light. The longitudinal impedance then contains a real term which is
responsible for the energy loss of the beam \([11]\]

\[
\frac{Z_{\parallel}(\omega)}{L} = -j \frac{Z_0 \omega g}{4\pi c \gamma^2 \beta^2} + (1 + j) \frac{1}{\sigma \delta r},
\]

where \( \sigma \) is the conductivity of the walls and \( \delta = (2/\omega \mu \sigma)^{1/2} \) is the skin depth.

1.4 Wakefields of Kicker Magnets

For kicker magnets, analytic solutions of the wakefields excited by a beam are very
difficult due to their complicated geometry and electrical properties. Estimates of
impedance can be obtained for kicker magnet systems with simple electrical configu-
trations from equivalent electrical circuit analysis. The excitation of the wakefield is
modelled as mutual coupling between the kicker magnet and the beam. This method
of impedance calculation was pioneered by G. Nassibian \([12]\). It is limited to a few
ideal special cases where the input and output cables which connect the magnet to the
rest of the system and the magnet itself have zero attenuation and are non-dispersive. Such strict and unrealistic conditions would mean that calculated values are very far from actual values.

In the KAON Factory, kicker magnets will be used to extract and inject the beam. The wakefields of the magnets can greatly affect the beam if the magnitudes of the wakefields are large. Of particular interest here are the longitudinal wakefields of the kicker magnet systems, which can give rise to beam instabilities. Such concerns necessitate an accurate method of measuring the longitudinal impedance of kicker magnet.

1.5 Kicker Magnets for KAON

In the proposed KAON Factory a high intensity beam of 100 μA is accelerated to 30 GeV step by step in a series of five rings. The five rings are: the Accumulator ring, which accumulates beam from the cyclotron; the Booster ring, which initially accelerates the beam to 3 GeV; the Collector ring, which collects beam from 5 consecutive Booster cycles; the Driver ring, which accelerates the beam to 30 GeV, and the Extender ring, which stores the beam for slow extraction.

Kicker magnets will be used to inject and extract the beam from the accelerator rings. A 1 MHz beam chopper installed in the transport line from the TRIUMF cyclotron will be used to create gaps. The duration of a gap is approximately 108 ns. During the gap, the field in the kicker magnet must rise or fall from 1% to 99% in order to kick the next segment of the beam with minimum beam loss. The required rise (fall) -time of the magnetic field in each kicker magnet varies from ring to ring. The fastest rise (fall) -time will be 82 ns.

The design of the kicker magnets for the KAON Factory is based on those of the CERN PS Division, which are of the transmission line type. Each kicker magnet consists of usually ten LC cells connected together in series to form approximately a transmission line. Figure 3 shows an equivalent circuit diagram of an ideal kicker magnet with four cells. Each cell consists of a ferrite C-core sandwiched between high-
Figure 3: An equivalent circuit diagram for an ideal kicker magnet.

voltage capacitance plates. The ferrite C-core provides the inductance and shapes the magnetic field which kicks a beam. A long kicker magnet is divided into several smaller modules to improve its kick performance. A typical cross-section of a kicker magnet is shown in Figure 4 and a photo of the prototype kicker magnet with ten cells with capacitance plates visible is shown in Figure 5.

The characteristic impedance of a kicker magnet can be defined in terms of the inductance L and capacitance C of each of its LC cells as \( \sqrt{L/C} \). The injection and extraction kicker magnets in the KAON Factory will have a design characteristic impedance of 25 Ω and will be terminated with matched resistors. However, in the Booster ring, the magnets will be short-circuited due to space limitations. These kicker magnets will have an input impedance of 16.7 Ω. Short-circuiting the magnets has the effect of doubling the kick strength for a given driving voltage. To ensure maximum power transmission, the characteristic impedance of transmission cables which deliver power is matched to that of the kicker magnets.
Figure 4: Cross-section of a kicker magnet.
Figure 5: The prototype kicker magnet.
1.6 Performance of Kicker Magnet Systems

A kicker magnet system consists of a pulse-forming network, power-transmission cables, a kicker magnet perhaps with a speed-up network and a saturating inductor. A Speed-up network and a saturating inductor are components which improve the performance of a kicker magnet system. Pulse-forming networks supply current pulses which energize the kicker magnets. To avoid reflections in a system, the characteristic impedances of all the components are matched as closely as possible. The input cable of a magnet (T_x in Figure 6) is connected to the main-switch thyratron of a pulse-forming network. Thus when the main-switch thyratron is in the off state, this end of the cable is effectively an open-circuit (see Figure 6). The output cable (T_s in Figure 6) is usually connected to a resistive load. For some applications, the output of the magnet may be short-circuited and no output cables are needed. For example, the magnets in the Booster ring will be short-circuited magnets.

The performance of a kicker magnet is characterized by the rise-time of the magnetic field which kicks the beam and the uniformity of the field before and after the initial build-up. To a first approximation, the kick rise-time is the pulse rise-time (PT) plus the transit time (TT) through the magnet. The pulse rise-time is determined by the pulse-forming network. The transit time is given approximately by the propagation time of the leading edge of the pulse through the magnet. The propagation time in terms of the total inductance L and capacitance C of the magnet is given by $\sqrt{LC}$. In order to get a satisfactory kick rise-time, a long magnet is split into several identical modules, each driven by its own pulse-forming network. In this way the propagation time is reduced since the length of each module is shorter than the combined length.

A kicker magnet does not behave exactly like a transmission line because each LC cell consists of lumped elements instead of distributed ones. This difference makes the magnet behave more like a low pass network which attenuates the high frequency components of an applied pulse as it propagates through a magnet. As a result, the rise-time of the pulse is increased and hence the kick rise-time of the magnet is increased as well.
External circuits, called speed-up networks, are designed to control the overshoot (or undershoot) of a driving current pulse (Figure 7). These networks, which consists of series capacitance and resistance, are connected adjacent to a magnet (Figure 6). In Figure 7 an improvement in the rise-time due to a speed-up network which compensates the undershoot of a pulse is illustrated. Another effect to be considered is the network cut-off frequency of a magnet, which can be attributed to its low-pass behavior (Appendix B). High frequency components above the cut-off frequency are strongly attenuated.

Three-gap thyratrons are used as switches in pulse-forming networks to energize kicker magnets [6]. A three-gap thyratron turns on in three stages. Because of parasitic capacitance across the gaps, a flow of displacement currents occurs before a main current. A thyratron switch becomes conducting when the dielectric of the gap is ionized to form a conducting plasma. When this happens, the main current can then flow through the plasma to a magnet. In Figure 7, displacement currents are present before a main current pulse. These displacement currents in turn flow through the
magnet, exciting its prematurely and thus effectively increasing the rise-time of a pulse. An increase in the rise-time of a pulse will result in an increase in the kick rise-time of the magnet system.

One proposed method of eliminating displacement currents involves the use of a ferrite saturating inductor at the input of a magnet. Because of its non-linear response to current amplitudes, it has a high impedance for small currents and a low impedance for a large current pulse. Such an inductor will allow a main current pulse to pass through but stop small displacement currents. The effectiveness of a saturating inductor in eliminating displacement current is shown in Figure 7.

1.7 Longitudinal Impedance of a Prototype Kicker Magnet

In order to achieve high beam intensity and energy, collective instabilities driven by longitudinal impedance must be minimized or carefully controlled. Thus all accelerator
components which can give a large contribution to the total longitudinal impedance have to be carefully designed and measured before installation. Due to the large number of kicker magnet modules required in the KAON Factory, their contribution to the total longitudinal impedance could be significant and must be determined. How other components of a kicker magnet system affect the longitudinal impedance is described below.

The longitudinal impedance of a kicker magnet system can be characterized in terms of an equivalent circuit as the mutual coupling between a beam and a kicker magnet system. The beam would be the primary winding and the magnet the secondary winding. The speed-up network, saturating inductor, and transmission cables of the magnet system, all of which are connected to the magnet, are also coupled to the beam. Thus the mutual coupling is determined by the whole magnet system consisting of the magnet and other components. Consequently, the longitudinal impedance of the magnet system depends on all the components of the system not just on the magnet.

As part of the KAON Factory Project Definition Study, a prototype 30 Ω magnet had been designed and built at TRIUMF based on those of the CERN PS Division. The magnet has 10 LC cells. PSpice modelling [4, 5] was used to determine the optimal values of circuit elements for a speed-up network and a saturating inductor to improve the performance of the prototype magnet system. The system was set up for the two configurations [3] in which it would operate in the KAON Factory to determine its longitudinal impedance. The effects of transmission cable, speed-up network and saturating inductor on the longitudinal impedance was determined.
2 Principles of the Coaxial Wire Method

2.1 Principles of the Method

The set-up of the coaxial wire method to measure the longitudinal impedance in the frequency domain or the effects of a wakefield in the time domain is different from the actual environment in which a bunch and an accelerator component interact with each other. We reproduce the arguments put forward by M. Sands and J. Rees [8] to show that a central conductor can reproduce the wakefield in an accelerator component induced by a passing bunch. Thus, a measurement of the wakefield in terms of energy loss can be made with this method in the time domain. Although the arguments are presented for the time domain measurement, they are equally valid for the frequency domain measurement, since the set-ups are identical in both cases.

Suppose we wish to measure the longitudinal wakefield in a section of beam pipe. Inserting a central conductor along the axis transforms the beam pipe into a coaxial structure with the pipe as the ground conductor and the wire as the inner positive conductor. A short pulse can now be sent down this coaxial structure from one end to the other. To emulate a charged particle bunch passing through the beam pipe, the current pulse should be made to have the same shape as that of the bunch. The current pulse will travel along the central conductor with approximately the same speed as that of the bunch, namely close to the speed of light, and will retain an instantaneous charge density that corresponds closely to that of the travelling bunch.

Figure 2 shows a wakefield in the beam pipe due to a point-like bunch. The longitudinal wakefield at the bunch subtracts energy from it but does not significantly alter the charge distribution. The energy lost by the bunch due to the wakefield in this way is the integral of the field along its path. Under the same circumstances the energy lost by the current pulse due to its image currents is the same as that of the bunch. If we can measure the energy lost by the current pulse, then we will have determined the energy lost by the bunch as it traverses the same beam pipe. In general, the energy lost by the bunch due to the longitudinal wakefield of any accelerator component can
be determined in this way by transforming the component into a coaxial structure with the insertion of a central conductor.

The most striking difference between the coaxial structure and the beam pipe is the presence of the central conductor in the coaxial structure. In the absence of the central conductor, wakefield energy is stored in a large number of normal-mode oscillations of the beam pipe and is dissipated as heat energy in the resistive walls. With the central conductor, the characteristics of the normal-mode oscillations are changed, and the oscillations, to a certain extent, are coupled to the central wire and travel along it to be dissipated in the terminations at the two ends. On the surface it may appear that the difference between the two cases is so great as to make them irreconcilable. However, if the central conductor is sufficiently thin, as is usually the case, then the normal-mode oscillations are only slightly modified. In addition, if the transit time of the current pulse or bunch is short compared to the decay times of the oscillations, then the energy removed from the pulse and left behind in the oscillations will be similar to the energy removed from the bunch.

In Appendix A, the error due to a central conductor is roughly estimated. We used two conductor sizes of 3 mm and 41 mm for the measurements. The error due to the large conductor (50 Ω line) is estimated to be comparable with the measurement uncertainty of the data, which is at most 5%. However, the error due to the smaller conductor (180 Ω line) is estimated to be negligible. Also in the appendix, the transit time is roughly estimated to be 1.5 ns which is small compared with the decay times of the normal-mode oscillations with a typical time of 15 ns.

2.2 Energy Loss by a Beam Bunch

We will start with a description of a coaxial wire set-up for sending and recording a current pulse. In the time domain, the energy loss can be measured by the coaxial wire method. Equations of energy loss will be derived in the time domain for use in the following section where they are Fourier transformed into their frequency domain counterparts to identify the longitudinal impedance. It is much simpler to derive these
A schematic diagram of an experimental set-up for sending and recording a current pulse is shown in Figure 8. The beam pipe segment or accelerator component is transformed into a coaxial structure by inserting a wire along the axis. Tapered sections, matching the coaxial structure to the test cables, are attached to both ends and are in turn connected to a pulse generator and an oscilloscope by test cables. It is necessary that the entire coaxial line is matched to avoid reflections. The pulse generator must produce a current pulse with the same time shape as that of a bunch because time domain measurement is bunch-shape dependent.

To determine the energy difference or loss, it is necessary to record the passage of a current pulse through a reference pipe. The length and cross-sectional dimensions of the reference pipe must be the same as that of the accelerator component to be measured because the same matching sections are used. It should be made of a good conducting material. A current pulse passing through the metallic reference pipe is only slightly attenuated and hence it should retain its original time shape. An over-
all check on the performance of the pulse generator, tapered matching sections, and terminations can be obtained by comparing the time shape of the current pulse before and after its passage through the reference pipe.

The basic measurement of consists of recording the time shape of the current pulse \( I_r(t) \) after its passage through the reference pipe and \( I_m(t) \) then again when the reference pipe is replaced by the accelerator component. The raw data will be the two time functions recorded by the oscilloscope. The secondary current \( \Delta I(t) \) is defined as the difference between the reference pulse and the modified one as

\[
\Delta I(t) = I_r(t) - I_m(t).
\]  

The energy \( U_r \) contained in the reference pulse is

\[
U_r = Z_0 \int I_r^2 \, dt,
\]  

where \( Z_0 \) is the impedance of the matched termination. Similarly, the energy contained in the modified pulse is

\[
U_m = Z_0 \int I_m^2 \, dt.
\]  

The energy loss \( \Delta U \) by the pulse as it passes through the accelerator component is given by

\[
\Delta U = U_r - U_m,
\]

\[
= 2Z_0 \int I_m \Delta I \, dt + Z_0 \int \Delta I^2 \, dt.
\]  

The last equation is obtained by rewriting it in terms of \( \Delta I(t) \). This energy loss is attributed to the work done against the electric wakefield \( E_w \) as the pulse passes through it. The energy loss in terms of the electric wakefield \( E_w \) is

\[
\Delta U = q \int E_w \cdot d\vec{x},
\]

where \( q \) is the total charge of the pulse.

After the reference pulse \( I_r \) and modified pulse \( I_m \) have been recorded by the oscilloscope, the above equations can be used to calculate the energy loss by the pulse.
as it traverses the accelerator component. The energy loss by a bunch depends on its length and shape. Hence many measurements must be made for the different time shapes and lengths that might be encountered. This dependence makes the direct measurement of the energy loss in the time domain inefficient. If the response of the whole coaxial structure is linear at all the relevant frequencies and amplitudes, then an equivalent measurement can be made in the frequency domain. The energy loss by a particular bunch can then be calculated using Fourier analysis. The frequency domain measurement is also known as the coaxial wire method and is explained in the following sections.

2.3 Method of Transmission Measurement

Time domain measurements of the type described in the last section, are limited to measuring the energy loss and phase difference of a bunch, whereas frequency domain measurements can completely characterize the longitudinal interaction of a bunch with an accelerator component in the measured frequency range. Frequency domain measurements are relatively easy compared to time domain measurements and can provide much more useful information. We will start with a description of the experimental set-up. Fourier transformation of the energy loss (Equation 12) into the frequency domain will be used to identify the longitudinal impedance.

Figure 9 shows a schematic experimental set-up for measuring the transmission coefficient \( S_{21} \) of a coaxial structure. In the frequency domain, the transmission coefficient of a coaxial structure is measured instead of the energy loss. The transmission coefficient is the ratio of the amplitudes of the transmitted voltage over the incident voltage. In transmission measurements, a coaxial structure is known as a two-port device with input and output ports. The definition of the transmission coefficient of a two port device is given in Appendix C.

The set-up in Figure 9 is made up of the transformed coaxial structure of a reference pipe or an accelerator component, tapered matching sections, transmission lines and a network analyzer. The method consists of measuring the transmission coefficients of
the reference pipe and the accelerator component in some frequency range. The raw data will be two transmission coefficients at each frequency. One is the transmission coefficient $S_{21}^{ref}$ of the reference pipe and the other is the transmission coefficient $S_{21}$ of the accelerator component in place of the reference pipe.

The energy loss $\Delta U$ by a current pulse is given (Equation 12) by

$$\Delta U \approx 2Z_0 \int I_m \Delta I \, dt,$$

if the second order term $\Delta I^2$ is neglected. The magnitude of $\Delta I$ is small compared with $I_m$ so neglecting the second order term is a good approximation. The same energy loss can also be written in terms of the work done against the wake potential $W_\parallel(t)$ as

$$\Delta U = q \int W_\parallel I_m \, dt,$$

where $q$ is the total charge. Equating Equations 14 and 15, we obtain

$$qW_\parallel(t) = 2Z_0 \Delta I(t).$$
Fourier transforming the above equation into the frequency domain yields

\[ Z_{\parallel}(\omega)\bar{I}(\omega) = -2Z_0[\bar{I}_r(\omega) - \bar{I}(\omega)], \]  

(17)

where \( Z_0 \) is frequency independent. Using the definition of transmission coefficient, the above equation can be written as

\[ Z_{\parallel} = 2Z_0 \frac{S_{\tau\tau}^{\text{ij}} - S_{21}}{S_{21}}. \]  

(18)

Equation 18 gives the longitudinal impedance directly in terms of the complex transmission coefficients involved.

### 2.4 Longitudinal Impedance and Transmission Coefficient

The naive analysis leading to Equation 18 glosses over the question of whether \( Z_{\parallel} \) is a localized impedance, i.e. almost zero length, or an extended impedance, such that

\[ Z_{\parallel} = \int_0^L Z(s) \, ds, \]  

(19)

where the integration is performed over the length \( L \). The analysis of Hahn and Pedersen [9] makes this distinction more explicit and indicates that Equation 18 is appropriate to a localized impedance. They derive a new expression for an extended impedance based on repeated application of Equation 18 to each differential element. This new expression is appropriate to kicker magnets whose impedance is distributed. In this section, we summarize their derivation of the expression that relates the transmission coefficient to the longitudinal impedance of an extended and uniform source.

For accelerator components which exhibit uniform structure in the axial direction, the longitudinal impedance can be assumed to be uniformly distributed. Beam pipes, bellows, and kicker magnets are examples of accelerator components that have approximately uniform axial structure and so for these components, a longitudinal impedance per unit length can be defined. In Section 3.2, we derived an expression (Equation 18) which relates a localized longitudinal impedance between the ports to the transmission coefficients. For an extended and uniform component for which an impedance per unit
length can be defined, a new expression can be derived by the repeated application of Equation 18 to each differential element of the extended impedance.

We will start with a combination of two localized impedances and then generalize to a distributed source of impedance. Figure 10 shows a schematic diagram of two localized impedances \( Z^a \) and \( Z^b \) along the beam pipe of characteristic impedance \( Z_0 \) separated by a distance \( l \). The transmission coefficients \( S_{21}^a \) and \( S_{21}^b \) of the two localized impedances can be obtained from Equation 18

\[
S_{21}^a = 1 - \frac{Z^a}{Z^a + 2Z_0}, \quad (20)
\]

\[
S_{21}^b = 1 - \frac{Z^b}{Z^b + 2Z_0}. \quad (21)
\]

The combined transmission coefficient \( S_{21}^{ab} \) of the two cascaded impedances can be shown [9] to be

\[
\frac{S_{21}^{ab}}{S_{21}^{ref}} = \frac{S_{21}^a S_{21}^b}{1 - S_{11}^a S_{11}^b}, \quad (22)
\]

where \( S_{11}^a \) and \( S_{11}^b \) are the reflection coefficients of the two impedances, respectively. Substituting expressions for \( S_{21}^a \) and \( S_{21}^b \) (Equation 18) into the above equation yields

\[
\frac{S_{21}^{ab}}{S_{21}^{ref}} \approx 1 - \frac{Z^a + Z^b}{2Z_0}, \quad (23)
\]
after simplification, since $|Z_{||}| \ll Z_0$ and $|Z_{\perp}| \ll Z_0$ as required by Equation 18. Thus the two contributions simply add up independent of their location provided that each contribution is small compared to $Z_0$.

This result can easily be extended to a distributed impedance $[9] Z_{||}$ in Ohms ($\Omega$)

$$\frac{S_{21}}{S^\|_{21}} \cong \left(1 - \frac{R_{||}}{2Z_0}\right) \exp\left(-jX_{||}/2Z_0\right),$$

where the total impedance $Z_{||} = R_{||} + jX_{||}$ is separated into the resistive part $R_{||}$ and the reactive part $X_{||}$. One can see that the real part affects the magnitude while the imaginary part affects the phase of a transmitted signal. This equation will be used to extract the longitudinal impedance of the prototype kicker magnet from the measured transmission coefficients.
3 Experimental Set-up and Experiment

3.1 Overview

In order to determine the longitudinal impedance of the prototype kicker magnet system, the magnet aperture was transformed into a coaxial structure by inserting a central wire. The magnet was enclosed in an aluminum tank to simulate a vacuum tank coupled to a section of beam pipe. To minimize unwanted reflections in the coaxial line, tapered matching sections were used. The transmission coefficient of the coaxial line was measured with an HP network analyzer. The longitudinal impedance of the magnet was calculated from the transmission coefficient. A reference coaxial line was built which has the same length and cross-sectional dimensions as the magnet aperture. This was necessary to fix the phase of the transmission coefficient. It also provided a check on the overall accuracy of the measurements.

The longitudinal impedance of our prototype kicker system was determined in the frequency range of 0.3 to 200 MHz. The transmission coefficient from 0.3 to 200 MHz was measured in two steps: from 0.3 to 50 MHz we transformed the magnet aperture into a 50 Ω coaxial line and from 45 to 200 MHz into a 180 coaxial Ω line. A 50 Ω line was chosen to match the impedance of an HP network analyzer and standard test cables. This also permitted the use of existing matching sections and cable adaptors from another experiment [21]. A line with a larger impedance was necessary in the higher frequency range to improve the accuracy of the approximate formula used to calculate impedance. One of the conditions of the formula is that longitudinal impedance must be smaller than line impedance. A 180 Ω was chosen to utilize existing 180 Ω apparatus from another experiment [21].

For both lines, calibration was necessary to eliminate systematic errors of the HP network analyzer and test cables. Semi-rigid 50 Ω test cables were used to connect the coaxial line to the HP network analyzer. For the 50 Ω line, HP calibration procedures and HP standard terminations were used to eliminate the systematic errors of the test cables and the HP network analyzer. However, for the 180 Ω lines, we used the
TSD (Through, Short, Delay) calibration method, which calibrated the measurement assembly from the network analyzer up to the transformed coaxial line. With the TSD calibration method, the impedance of the line did not need to be matched to the 50 Ω test cables. The raw data of a calibration and measurement were processed by an HP computer. Error-correction factors were calculated from the calibration and then used to extract corrected transmission coefficients.

3.2 Experimental Set-up

3.2.1 Magnet Tank

The prototype kicker magnet was placed in an aluminum tank which has two openings aligned with the aperture of the magnet. The opening dimensions of the tank are the same as the cross-section of the aperture. Figure 11 shows the magnet in the tank. The dimensions of the tank are rectangular with the bottom 27.5 cm of the tank 4 cm wider in the axial direction of the magnet to accommodate the magnet stand. The tank is approximately 90 cm tall and 55 cm by 42 cm in cross-section. The aluminum tank simulated the condition of a vacuum tank in a beam line and ensured the presence of a ground conductor around the central conductor of the line. Two gaps between the matching sections and the magnet were bridged by rf finger stocks (Figure 11). The rf finger stocks simulated the beam pipe connection to the magnet. The ground connection from the input matching section to the output matching section was continuous through the ground conductor of the magnet when rf finger stocks were installed.

3.2.2 Reference Pipe

The reference pipe for the prototype kicker magnet was made of brass for mechanical stiffness. It has the same rectangular cross-section as the magnet aperture. The dimensions are 15.5 cm by 7.8 cm and 45.1 cm long. Its length is the same as that of the tank. It ends are welded to flanges for mating with other sections as shown in Figures 12 and 13. Brass is a very good conductor in the frequency range of 0.3
Figure 11: The prototype kicker magnet in the ground tank.
Figure 12: The 50 Ω coaxial line.

to 200 MHz and so the brass reference pipe has a very low longitudinal impedance ($\leq 1.6\Omega/m$).

3.2.3 Matching Sections

The purpose of the matching sections is to match segments of the coaxial line with different cross-sectional dimensions. Two matching sections, one at each end of the magnet tank or reference pipe, were used to connect the tank or pipe to the end sections which were in turn connected to test cables of the HP network analyzer. See Figures 12 and 13 for more details. The cross-sectional dimensions of the matching sections are 15.5 cm by 7.8 cm at one end and taper to 10.3 cm by 7.8 cm. They are 18.4 cm in length and also made of brass. For the 50 Ω line, the central conductor was also tapered at the end sections, but for the 180 Ω line, the central conductor did not need to be tapered.

3.2.4 Central Conductors

Central conductors were required to transform the reference pipe and magnet aperture into coaxial lines. For the 50 Ω transformed line, the central conductor was a circular
Figure 13: The 180 Ω coaxial line.

copper pipe with a diameter of 41.35 mm. The pipe was held in the center by plastic spacers in the end sections. Because the pipe was rigid, no additional spacers were required in the middle section. In the 180 Ω transformed line, a copper wire with a diameter of 3.175 mm was strung between the end sections. It was held in place under tension to reduce sagging at the middle.

### 3.3 Calibrations

#### 3.3.1 Full Two-Port Calibration

In any measurement there are errors associated with the measurement system that contribute to the uncertainty of the results. Over the years, transmission measurement errors have been studied and methods developed to correct them. The standard method of correcting transmission measurements is to use an error model which characterizes the errors of the measurement ports. Both the HP calibration and TSD calibration are based on the principles of a full two-port calibration, the principles and procedures of which are outlined in this section.

Our measurement system consisted of an HP 8753B network analyzer and 50 Ω semi-rigid test cables. Sources of systematic errors that can be modelled are slight impedance
mismatch and leakage in the test cables, isolation between the reference and test signal paths, and system frequency response. All these errors introduce uncertainty in the magnitude and phase of a measured transmission coefficient. The two-port calibration measures the reflection and transmission coefficients of test cables, a standard short-circuit termination and a matched resistive load at each of the two calibration planes. Error parameters of the sources being modelled are calculated from the calibration using equations from Speciale and Franzen [12]. The correction factor of the two-port transmission measurement can then be calculated from the error parameters of the calibration. When the correction factor is applied to a measured transmission coefficient the sources of error being modelled are effectively eliminated.

A check on the overall accuracy of a calibration was provided by measuring the longitudinal impedance of the reference pipe and comparing the measured values to the expected values. If an excessively large impedance of the reference pipe was measured then the calibration was faulty and was repeated.

3.3.2 HP Calibration for the 50 Ω Line

In this section we will describe the method of HP calibration and steps taken to assure the accuracy of the calibration.

For the 50 Ω line, the calibration planes were at the ends of the semi-rigid 50 Ω test cables coming from the two ports of the network analyzer. See Figure 14 for details. Full two-port calibration was performed following the menu and using the standard terminations of the HP 8753B network analyzer. The frequency range was from 0.3 to 45 MHz. The HP standard terminations, which consist of a short-circuit termination and a resistive 50 Ω load, were connected one by one to the ends of the test cables. The transmission and reflection coefficients of the terminations could then be measured. A measurement of the transmission and reflection coefficients of the test cables was also included in the calibration procedures. The test cables were connected together for this measurement.

Sources of error during calibration and measurement were insecure connections and
any movement of the test cables that could loosen connections and cause slight variations in the lengths of the test cables. Small movements of the test cables were unavoidable during calibration procedures when standards were being mounted and dismounted. However, large movements could be avoided by securing the test cables at various points along their length. For best results, the calibration was done as close as possible to the transformed line being measured. This eliminated large movements of the test cables after calibration when they were reconnected to the transformed line for test measurements.

The success of the calibration was confirmed by measuring the transmission coefficient of the reference line to determine its longitudinal impedance. It was expected that the brass reference pipe should have a distributed longitudinal impedance less than 1 Ω/m [7] in the 0.3 to 45 MHz frequency range. For our reference pipe of about 0.5 m, the measured longitudinal impedance was 0.3 Ω, satisfying the expectation.

3.3.3 TSD Calibration for the 180 Ω line

The TSD calibration is based on the same principles as those of the HP calibration. A full two-port calibration requires the use of a resistive load as one of its standard terminations. Because of the difficulty of building accurate 180 Ω resistive load for pipe-size dimensions, a delay pipe, of the same dimensions as the reference pipe, was used to simulate a resistive load. In this section we will describe the the method of TSD calibration.

For the 180 Ω line, the TSD (Through, Short, Delay) method of calibration was used to calibrate the measurement assembly. The calibration planes were at the end flanges of the end caps and are shown in Figure 14. The frequency range was from 45 to 200 MHz. The TSD standard terminations consisted of a short-circuit plate, and a 180 Ω load in the form of a pipe segment. The pipe segment, used to simulate a matched load, was a short section of pipe less than one-quarter wavelength at the highest frequency. For a pipe segment of known length, the input impedance can be calculated for use as a matched load. The reference pipe was a segment less than one-half wavelength at the
The 50 Ω Line

Test cable
Connector

Mating Flanges
50 Ω Test cable adaptor

Reference pipe
or Magnet tank

Matching section

Calibration plane

The 180 Ω Coaxial Line

Test cable
Connector

Mating Flanges
End cap

Reference pipe
or Magnet tank

Matching section

Calibration plane

Figure 14: Calibration planes.
highest frequency. The terminations were mounted on the end caps and held in place by nuts and bolts to ensure good electrical contact. The transmission and reflection coefficients of the terminations were measured. The reference pipe was also used in the calibration to measure the transmission of the matching sections, end sections, and test cables. The measured transmission and reflection coefficients were used to calculate the error parameters of the two-port error model of the measurement assembly.

The calibration procedures were adopted from Walling [7]. A computer was needed to control the calibration procedures. For best results, movements of the test cable were kept to a minimum and the same precautions outlined in Section 4.3.2 were followed. Unlike the small standard HP terminations, which were mounted at the ends of the test cables using standard cable connectors, TSD terminations were mounted at the end flanges of the end caps using nuts and bolts. These terminations were large and heavy and required a great deal of care to ensure good electrical contact. A flash light was used each time to search for air gaps between the interfaces. If any gaps were found, the bolts were further tightened until the gaps closed. The presence of an air gap would mismatch different sections of the transformed coaxial line leading to faulty error parameters.

The success of the calibration was confirmed by the measured longitudinal impedance of the reference pipe which was 1Ω for a length of 0.451 m. Comparing this to a measured value of 1 Ω/m for a similar pipe [7] indicated the calibration was not faulty.

3.4 Digital Noise Reduction

The 8753B HP network analyzer is a highly sophisticated instrument which offers many features enhancing accuracy. For instance, greater accuracy in the test data can be obtained with little effort by utilizing its digital-data processing capability. An average sampling size of 8 was used for both calibrations and test measurements. Averaging reduces the noise level of the signal. The value of each data point was based on a weighted average [19] of 8 consecutive samplings. An averaging rate of 8 was moderate and hence did not slow down the update time significantly. Another noise
reduction feature IF (Input Frequency) Bandwidth Reduction was also used to lower the noise floor. This reduction was accomplished by digitally reducing the receiver input bandwidth. It is more reliable than sample averaging in filtering out unwanted responses such as spurs, high-frequency spectral noise, and line-related noise. An IF bandwidth of 10 Hz was used, the smallest bandwidth available.

3.5 Computer Calibration Program

For the 180 Ω line, which used the TSD method of calibration, a computer program, originally written by Walling [7], was adapted to take the measurements and perform the calibration. First the calibration was performed following the calibration menu of the program and then the error correction factor of the measurement assembly was calculated. To check the accuracy of the calibration, the transmission coefficient of the reference pipe was measured and the corrected transmission coefficient calculated using the error correction factor. The longitudinal impedance of the reference pipe was calculated from the corrected transmission coefficient and compared with that of a pipe of similar dimensions. If the measured value was within an acceptable range of other measured values, the calibration was used for subsequent measurements of the kicker magnet.

For the 50 Ω line, which used the HP method of calibration, a computer was not needed to take the measurements or perform the calibration. A calibration menu for the 50 Ω line was available on the HP network analyzer. The measurement steps were the same as those for the 180 Ω line.

3.6 Longitudinal Impedance Calculations

The basic transmission data consisted of the transmission coefficients $S_{21}^{ef}$ and $S_{21}$ of the reference pipe and the kicker magnet, respectively. These transmission coefficients were measured only after a successful calibration of the test instrument had been accomplished. The longitudinal impedance was then calculated from the transmission coefficients using Equation 24, from which the resistance $R_{||}$ and the reactance $X_{||}$ can
be extracted separately as follows

\[
\begin{align*}
\text{phase} \left[ \frac{S_{21}}{S_{21}^{\text{ref}}} \right] & \approx -\frac{X_{||}}{2Z_0}, \\
\text{magnitude} \left[ \frac{S_{21}}{S_{21}^{\text{ref}}} \right] & \approx 1 - \frac{R_{||}}{2Z_0},
\end{align*}
\]

where \( Z_0 \) is the characteristic impedance of the reference line.

A slightly different method of determining longitudinal impedance was required for the reference pipe. In order to determine the longitudinal impedance of the reference pipe, for use as an overall check on the accuracy of the measurement assembly, its transmission coefficient must be compared to that of another pipe as required by Equations 25 and 26 to fix the phase and background of the test signal. A transmission-coefficient equation for an ideal lossless reference pipe was substituted into Equations 25 and 26,

\[ S_{21} = \exp(-jkl), \]

where \( k \) is the wave number and \( l \) is the total length of the line between the calibration planes. The magnitude of \( S_{21} \) is unity because an ideal line has no losses. The measured longitudinal impedance of the reference pipe would then include any errors in the line and the calibration.

The length of a line is the distance between the calibration planes. The length of the 180 \( \Omega \) ideal line was given by the length of the reference pipe whose ends were at the calibration planes. The pipe was measured with calipers to be 0.820 m. For the 50 \( \Omega \) ideal line, the calibration planes were at the ends of the test cables, which were further back from the ends of the reference pipe. Its length could not be precisely measured with calipers. A different method, consisted of fitting the measured \( S_{21} \) of the reference pipe to Equation 27, was used to extract the length from the equation. Its length was found to be 1.62 m.
4 Measurement Results and Discussions

4.1 Longitudinal Impedance of the Reference pipe

In order to extract the impedance of the magnet using Equations 25 and 26, one must obtain a transmission measurement of a perfectly matched reference coaxial line. Such a reference line must include all the transition sections of the magnet measurement line, but with a reference pipe in place of the magnet (Figure 11). In practice, a coaxial line with different segments cannot be perfectly matched at all frequencies. A well matched line can be used instead, provided any loss and mismatch are negligible. The reference line for the magnet was formed with a brass pipe.

In a transmission measurement of the magnet, the mismatch between different segments of the line causes reflections which reduce the transmission. If this reduction in transmission is attributed to attenuation rather than reflection, then the measured impedance is not the true impedance but also includes a contribution from the mismatch. We shall call this contribution the mismatch impedance. The mismatch is approximately the same for the reference line because the components are the same for both lines except for the interchange of the magnet with the (brass) reference pipe.

Under the assumption that the measured impedance can be separated into the true impedance and a mismatch impedance, and that the mismatch impedance is additive, we may write

\[
Z_{\text{ meas}}^{\text{mag}} = Z_{\text{ true}}^{\text{mag}} + Z_{\text{mismatch}}, \tag{28}
\]

\[
Z_{\text{ meas}}^{\text{ref}} = Z_{\text{ true}}^{\text{ref}} + Z_{\text{mismatch}}. \tag{29}
\]

If we now form the quotient of transmission parameters we find

\[
\frac{\sigma_{\text{meas}}^{\text{mag}}}{\sigma_{\text{meas}}^{\text{ref}}} = \exp[Z_{\text{true}}^{\text{mag}} - Z_{\text{true}}^{\text{ref}}]. \tag{30}
\]

In principle, the systematic error \(Z_{\text{true}}^{\text{ref}}\) could be removed if the impedance of the reference line was precisely known. However, no greater accuracy could be obtained if the measured reference pipe impedance were used, because of the presence of a mismatch.
impedance which could be of comparable size. A direct measurement of the mismatch impedance requires an ideal lossless line, which is not available. Consequently, no attempt was made to remove the systematic error, $Z_{\text{true}}^{\text{ref}}$. However, its upper limit can be determined from the inequality

$$|Z_{\text{true}}^{\text{ref}}| \leq |Z_{\text{meas}}^{\text{ref}}|.$$  

Consequently, the systematic error will be small if the measured reference pipe impedance is small compared with the measured magnet impedance.

The impedance of the brass reference line was determined as outlined in Section 3.6. Its value indicates the upper limit on the systematic error of the measured impedance of the magnet. The longitudinal impedance of the brass reference pipe was measured from 0.3 to 50 MHz by transforming it into a 50 $\Omega$ coaxial line. The results of the measurements are shown in Figure 15. The absolute maximum resistance is less than 0.3 $\Omega$ for a pipe length of 0.451 m and the absolute maximum reactance is less than $j0.1 \Omega$.

From 45 to 200 MHz, the reference pipe was transformed into a 180 $\Omega$ coaxial line, and the longitudinal impedance was measured. The absolute maximum resistance is 0.8 $\Omega$ and the absolute maximum reactance is $j3.8 \Omega$. The results of the measurements are shown in Figure 16. Note in Figure 16 there is a rising trend in the longitudinal reactance. This was likely due to the phase instability of the test cables caused by cable movements during the calibration procedures. Hence a large longitudinal reactance was measured.

With the exception of the longitudinal reactance in the 45–200 MHz range, the values obtained are approximately the same as those measured by Walling [7] for similar pipe dimensions and length. Direct comparison of the measured values with the measurements of Reference [7] is not possible because those values are for slightly different pipe dimensions. However, comparison between pipes of similar dimensions can provide a strong indication of the accuracy of the measurements.
Figure 15: Longitudinal Impedance of the Reference Pipe from 0.3–50 MHz.
Figure 16: Longitudinal Impedance of the Reference Pipe from 45–200 MHz.
4.2 Electrical Resonant Modes

The magnet system behaves like a transmission line up to its network cut-off frequency (30 MHz). In this frequency range, the resonances in the impedance of the magnet system can be explained in terms of the propagation of voltage waves in a transmission line of the same length. Using this model, resonant frequencies are determined by those of the standing waves. These frequencies can easily be calculated from the termination and length of the line. The widths and heights of the resonances in the longitudinal impedance, however, cannot be obtained from this simplistic model. The agreement between the calculated and measured resonant frequencies was confirmed by the data. Above the cut-off frequency, there is strong attenuation of travelling waves and hence difficulty in obtaining standing waves. This was also confirmed by the lack of resonances in the longitudinal impedance from the data. Consequently, below 30 MHz, we expect the transmission line model to work well. We show data that support this model.

In an equivalent circuit diagram (Figure 17), the prototype kicker magnet is modelled as a section of transmission line. The one-way delay time of the magnet is 30 ns and its characteristic impedance is 30 Ω. The ends of the transmission line correspond to the input and the output of the magnet. They can be short-circuited, open-circuited, or connected to input or output 30 Ω cables. The magnet and the cables form an extended transmission line which can support standing waves.
Figure 17: Resonant structures of the prototype kicker magnet.
When the magnet is excited by a sinusoidal current travelling along the central conductor, image currents are set up in the magnet and propagate as travelling waves towards the ends of the magnet. These travelling waves will be either absorbed or reflected at the ends depending on the terminations. If an end is terminated resistively with an impedance matching the characteristic impedance of the magnet, no waves will be reflected. For voltage waves reflected from a short-circuit termination, there is a 180 degree phase change. For an open-circuit termination, however, there is no phase change in the reflected waves. If the line is not terminated resistively, then at certain frequencies the right conditions exist for standing waves to resonate along the line. With the assumption that the wakefield excited by a current pulse on the central conductor is the same as the wakefield excited by a charged particle beam, these standing waves are identical to the local modes of the oscillating wakefield, which in turn interacts with the beam. In a longitudinal impedance graph the local modes show up as resonances.
Table 1: Resonant frequencies of the 30 ns quarter-wavelength resonator and the 30 ns half-wavelength resonator.

In Figure 18, the measured longitudinal impedance of the prototype kicker magnet is shown. 30 Ω feed cables were not connected to either the input or the output of the magnet. The solid line is the longitudinal impedance of the magnet with the input (or output) open-circuited and the output (or input) short-circuited. Note that with no cables connected, the input and the output of the magnet are interchangeable. The resonances appearing in the solid line correspond to those of a quarter-wavelength resonator: a resonance appears for the first three \((2n + 1)\lambda/4\) modes where \(n = 1, 2, 3\). The dashed line in Figure 18 is the longitudinal impedance of the magnet with both the input and the output open-circuited. The resonances in the dashed line correspond to those of a half-wavelength resonator: a resonance appears for the first two \(n\lambda/2\) modes where \(n = 1, 2\).

Table 1 lists the frequencies of the first three resonances of the magnet from measurements and calculations. The measured resonant frequencies are very close to the calculated frequencies. Above the network cut-off frequency (30 MHz), the magnet ceases to behave like a transmission line and no more standing waves can exist: hence, the lack of resonances.
Figure 18: Longitudinal impedance of the prototype kicker magnet
Figure 19: Resonant structure of the kicker magnet system with a feed cable.
Figure 20 shows the longitudinal impedances of the magnet with a feed cable, its output being short-circuited or open-circuited (Figure 19). The one-way delay of the cable is 30 ns and it has a characteristic impedance of 30 Ω, matching that of the magnet and thus forming a combined line of 60 ns. The resonance distribution is similar to that of Figure 18 but corresponds to that of resonators that are twice as long but with lower resonant frequencies. Table 2 shows the calculated and measured resonant frequencies of the two resonators. The agreement is very close, within 4%.

The longitudinal impedance shown in Figure 21 is that measured for the magnet with a longer feed cable connected to its input. The one-way delay of the long feed cable is 193 ns and it has a characteristic impedance (30 Ω) matching those of the magnet. The combined length of the cable and the magnet is 223 ns. The resonance distributions correspond to those of quarter-wavelength and half-wavelength resonators. The resonant frequencies can be calculated assuming an ideal transmission line as in the last two cases. An envelope on the overall magnitude of the longitudinal impedance can be observed.
Figure 20: Longitudinal impedance of the prototype kicker magnet with a 30 ns feed cable.
Figure 21: Longitudinal impedance of the prototype kicker magnet with a 193 ns feed cable.
<table>
<thead>
<tr>
<th>Resonant mode (n)</th>
<th>quarter-wavelength resonator</th>
<th>half-wavelength resonator</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculated</td>
<td>Measured</td>
</tr>
<tr>
<td>1</td>
<td>4.1 MHz</td>
<td>5.3 MHz</td>
</tr>
<tr>
<td>2</td>
<td>12.5</td>
<td>12.1</td>
</tr>
<tr>
<td>3</td>
<td>20.8</td>
<td>21.0</td>
</tr>
</tbody>
</table>

Table 2: Resonant frequencies of the 60 ns quarter-wavelength resonator and the 60 ns half-wavelength resonator.

The heights and the widths of resonances do not exhibit any simple dependence on the parameters of the magnet and the cable that can easily be deduced. In general, the attenuation of travelling waves by the magnet is responsible for broadening the resonance widths. Above the cut-off frequency, the travelling waves become evanescent and hence standing waves cannot be maintained. Consequently, the conditions for resonances do not exist. The cut-off frequency of the prototype kicker magnet was measured to be 30 MHz (Appendix B). Losses in the C-core ferrite largely determine the degree of attenuation and hence the cut-off frequency.

Figure 23 shows the effect of terminating the kicker magnet system with a 30 Ω resistor. The 30 Ω resistor was connected at the end of the output cable. The equivalent circuit diagram of the system is shown in Figure 22. For a magnet system terminated with a matched resistor, standing waves cannot exist because reflections do not occur from a matched resistor. Without standing waves, resonances do not occur.

Nevertheless, some reflections are possible as indicated by the figure. The reflections occurred since the magnet could not be really terminated at all frequencies with a 30 Ω resistor. This is due to the fact that the input impedance of the magnet slightly varies from 30 Ω below the cut-off frequency (Appendix D). Above the cut-off frequency (30 MHz), the input impedance varies considerably and can no longer be considered constant nor be terminated with a resistor.
Figure 22: Equivalent diagram of the prototype kicker magnet with a 30 ns output cable.
Figure 23: Longitudinal impedance of the prototype kicker magnet with a 30 ns output cable resistively terminated.
4.3 Magnet Configurations

In the KAON Factory, kicker magnets will be operated in two electrical configurations. In the short-circuit configuration the magnet output is short-circuited, and in the matched-termination configuration the magnet output is connected to a cable which is resistively terminated (Figures 25 and 24, respectively). As shown in Section 4.2, the longitudinal impedance of a magnet system depends not only on the magnet itself but also on other components of the system. The longitudinal impedance of the prototype magnet for both configurations is described so that it can be used for comparison in later sections.

Figure 26 and Figure 27 show the longitudinal impedance of the two configurations from 0.3–50 MHz and 45–200 MHz, respectively. For the short-circuit configuration, the longitudinal impedance contains a resonance distribution that is typical of a quarter-wavelength resonator. Resonances are not present in the matched-termination configuration, as discussed in Section 4.2.
Figure 25: Schematic diagram of short-circuit configuration.

The longitudinal impedance in the 30–200 MHz range for both configurations is dominated by losses in the C-core ferrite of the LC cells. The increase in loss as a function of frequency is characteristic of hysteresis loss in the ferrite. In general, the longitudinal impedance from 30–200 MHz does not depend on the termination of the magnet nor on other components of the magnet system. The maximum real impedance of both configurations is approximately 32 Ω.
Figure 26: Longitudinal impedance of short-circuit and matched-termination configurations (0.3–50 MHz).
Figure 27: Longitudinal impedance of short-circuit and matched-termination configurations (45-200 MHz).
Losses due to the ferrite can be reduced by shielding them with a conducting material. Aluminum sheets 1.5 mm thick were placed over the walls of the magnet aperture. Figure 28 shows the effectiveness of this shielding in reducing the longitudinal impedance in the 50 to 200 MHz range. It appears there is some degradation in the longitudinal impedance below about 100 MHz, attributable to undamped resonances of some unidentified modes. The reactance is reduced almost at all frequencies and the resistance is increased at frequencies below 100 MHz.

In actual operation, such a thickness is not practical due to its adverse effect on the rise—time of the magnetic field. The thickness of the shielding material must be less than the skin depth of the high—frequency components of the main current pulse that drives the magnet, in order not to increase the rise—time of the magnet drastically. Due to the cut—off frequency of the magnet, the high—frequency components of the current pulse are restricted to below 30 MHz. For aluminum, the skin depth (15 μm) at 30 MHz is much less than the 1.5 mm aluminum sheets used. The improvement in the longitudinal impedance above about 100 MHz shown in Figure 28 is therefore an upper limit. A feasible solution would be to insert a ceramic chamber in the magnet aperture, with its inner surface lined with a metallic film, thin relative to the skin depth (15 μm) [13].
Figure 28: Effect of shielding the ferrite of the prototype kicker magnet.
4.4 Effect of a Speed-up Network and a Saturating Inductor

It has been proposed [5] that a saturating inductor be connected in series between the input of the magnet and the feed cable (Figure 29) to absorb small displacement currents before the main power pulse so as to improve the rise-time of the magnetic field. In addition, travelling waves would be reflected by such an inductor because it has a high impedance for small current magnitudes. As a result, the length of the magnet system is reduced and so there are fewer resonances.

Figure 30 shows the longitudinal impedance of the prototype magnet in the short-circuit configuration with and without the ferrite saturating inductor. An equivalent circuit diagram for the magnet system is shown (Figure 29). The saturating inductor has a very high impedance for small currents below saturation. As a result, it effectively terminated the feed cable at the magnet input as an open-circuit. Consequently the effective length of the magnet system did not include the length of the feed cable when the saturating inductor was connected. For a shorter line, the number of resonances in a given frequency interval is reduced. Therefore, the saturating inductor has the effect of reducing the number of resonances.
Figure 30: Effect of saturating inductor.
A speed-up network was connected at the input of the prototype magnet to improve the performance of the magnet system [4, 5]. It consisted of a capacitor and resistor in series. Figure 33 shows the effect of the speed-up network, with different values of capacitance and resistance, on the longitudinal impedance of the prototype magnet, in the matched-termination configuration (Figure 31). Below the cut-off frequency, the resistor of the network slightly modifies the impedance. Most importantly, the speed-up network did not give rise to any resonances if the resistor of the network was present. Hence connecting the speed-up network did not adversely affect the longitudinal impedance.

Figure 34 shows the damping effect of a 30 $\Omega$ network resistor on the resonances of the prototype magnet in the short-circuit configuration (Figure 32). The network resistor damped the last two resonances but did not influence the first (5 MHz) resonance. At low frequencies, the impedance of the capacitor is high, thus preventing the resistor from absorbing energy and thereby providing some additional damping for the first resonance.
Matched termination Configuration

```plaintext
matched-termination configuration
```

![Matched termination Configuration diagram](image)

Figure 31: Schematic diagram of magnet system with speed-up network (matched-termination configuration).

Short-Circuit Configuration

```plaintext
short-circuit configuration
```

![Short-Circuit Configuration diagram](image)

Figure 32: Schematic diagram of magnet system with speed-up network (Short-circuit configuration).
Figure 33: Effect of speed-up network (matched-termination configuration).
Figure 34: Effect of speed-up network (short-circuit configuration).
4.5 Gaps Between the Beam Pipe and Kicker Magnets

In the proposed KAON Factory, the design of the kicker magnet system follows that of CERN where there is a gap between the kicker magnet, which is in a vacuum tank, and the beam pipe which is connected to the vacuum tank. This is the easiest design choice because there is no direct coupling between the beam pipe and the kicker magnet. Such a gap can contribute additional strong resonances compared to the longitudinal impedance of the magnet system.

See Figure 11 for the location of the gaps. The gaps, approximately 1 cm wide, were at both ends of the magnet aperture facing the beam pipe. The gaps were bridged by rf finger stock when the longitudinal impedance of the kicker magnet was measured. Finger stock is usually used for electrical connections in rf devices and consists of spring loaded wires for making pressure contact.

The resonances due to the gap were measured when the rf finger stock was removed from the gaps. Figures 35 and 36 show the gap resonances for the case of the matched-termination configuration. The longitudinal impedances of the gaps are of the localized type because the gaps are discrete sources. The measured peak values of the resonances were approximately 100–200 Ω, which are large compared to the longitudinal impedance of the magnet system. In general, these gaps should be avoided because they give rise to strong resonances.
Figure 35: Resonances due to gaps (0.3–50 MHz).
Figure 36: Resonances due to gaps (45-200 MHz).
4.6 Kicker Magnet Contribution to the Total Longitudinal Impedance

The collective instabilities of a beam in a synchrotron ring are determined by the total longitudinal and transverse impedance of all the components which make up the ring. The total impedance can be broken down into major contributions from rf cavities, resistive beam pipe, beam instrumentation devices, and kicker magnets. In all rings of the KAON Factory, the largest contribution to the total impedance is from the rf cavities. The longitudinal impedance of a cavity consists of many sharp peaks at its resonant frequencies. An upper limit of 1 kΩ has been set for each peak[14].

For the purpose of instability calculations in the *KAON Accelerator Design Report* [14], it was assumed that a kicker magnet module has maximum real longitudinal impedance equal to half its characteristic impedance. Thus a maximum real longitudinal impedance of 12.5 Ω was assumed for each 25 Ω magnet module.

The total impedance of the kicker magnets in a ring is the sum of the impedances of the magnet modules that make up the kicker magnets. Each ring has a few kicker magnets and several magnet modules make up a kicker magnet. The number of magnet modules required in a ring depends on the total kick angle and the beam momentum. Because the number of cells that make up a magnet module varies from ring to ring, the total magnetic length of the magnet modules in a ring is used instead as the measure of kick strength. The magnetic length of the prototype kicker magnet is 34.5 cm. Because the maximum longitudinal impedance of the prototype kicker magnet does not depend on other components that make up the magnet system, the total maximum longitudinal impedance of the kicker magnets in a ring can be scaled by the total magnetic length.

The maximum real longitudinal impedance of the prototype kicker magnet was measured to be approximately 32 Ω up to 200 MHz in both the short-circuit and the matched-termination configurations (Section 4.3). The total magnetic length in the Booster ring is 4.58 m and so the total longitudinal impedance is 425 Ω, by magnetic length scaling. In the Driver ring, the total magnetic length is 12.77 m and so the total longitudinal impedance is 1184 Ω. The total kicker magnet impedance in each ring is
small compared with the rf cavity impedance, 12 kΩ in the Booster ring and 18 kΩ in
the Driver ring, respectively. Due to their small contribution to the total longitudinal
impedance of each ring, the kicker magnets are not expected to be a significant source
of instabilities.

For short bunch (1 ns) operations of the rings, it is required that the total impedance
$Z_{\parallel}$ of a ring divided by the mode number $n$ must be smaller than 2 Ω. The impedance
of the magnet does not satisfy such a low $|Z_{\parallel}/n|$ nor do other components of the rings
without major efforts. Nothing less than a redesign of the magnet system can such a
low impedance be obtained.
5 Conclusions

The longitudinal impedance of a prototype kicker magnet system for the KAON Factory has been determined in both the short-circuit and the matched-termination configurations. The effects on the longitudinal impedance of a saturating inductor and a speed-up network, which were installed to improve the kick performance of the system, have also been assessed. When a saturating inductor was installed, the number of resonances in the impedance was reduced. The resistor of a speed-up network, which was connected to the input of the magnet, damped the high-frequency resonances. Most importantly, the network did not give rise to any additional resonances provided the network resistor was present. In the frequency range from 30 to 200 MHz, the longitudinal impedance did not depend much on components external to the magnet (such as the saturating inductor, the speed-up network, and the feed cables), because of the onset of strong attenuation of travelling waves by the magnet. Gaps between the kicker magnet and the beam pipe contributed very strong resonances to the longitudinal impedance of the magnet system. Therefore gaps must be eliminated.

The maximum real longitudinal impedance of the prototype kicker magnet was measured to be approximately 32 Ω up to 200 MHz in both the short-circuit and the matched-termination configurations. The total real longitudinal impedance of the kicker magnets in the Booster ring would then be 425 Ω and in the Driver ring 1184 Ω, respectively. The total kicker magnet impedance in each ring is small compared with the total rf cavity impedance with typical values of 10 kΩ. Hence the contribution of kicker magnets to the total longitudinal impedance of each ring is negligible and any longitudinal instabilities due to kicker magnets can be damped with existing damping systems for parasitic resonances of rf cavities. However, the magnet impedance does not satisfy the strigent requirements for short bunch (1 ns) operations of the rings.
References


Appendix A

Effects of a Central Conductor and Transit Time

A central conductor perturbs the wakefield of an accelerator component. The larger the radius of the conductor, the more it modifies the wakefield. However, if the central conductor is sufficiently thin, then the wakefield is only slightly modified and more accurate measurements can be made. For mechanical support and impedance matching, the radius cannot be as small as one wishes. A chosen radius size is a compromise between practicality and measurement accuracy. In this appendix, we make an estimate of the error caused by using a central conductor [8]. We also show by an example that the transit time of a bunch is small compared with the decay time of normal-mode oscillations.

The energy loss factor $K$ can be obtained from the reference pulse $I_r(t)$ and the modified pulse $I_m(t)$ of the test component by the equation

$$K = \frac{2Z_0}{q^2} \int I_d I_r(t) \, dt + \delta K,$$

where $I_d = I_m - I_r$, $Z_0$ is the characteristic impedance of the line, and $q$ the total charge in the pulse. The correction term $\delta K$ acknowledges the effect of the central conductor. A first-order-of-magnitude estimate for the size of $\delta K$ is given by

$$\frac{\delta K}{K} \approx \frac{I_d}{I_r},$$

where $I_d$ and $I_r$ are the maximum magnitudes of the currents. For the case of zero energy loss $\delta K \to 0$ since $I_d \to 0$. Let us assume that $\delta K$ is small compared to $K$ for a case where the conductor is sufficiently thin, and that it is approximately equal to the measurement uncertainty $\Delta K$, that is,

$$\frac{\delta K}{K} \approx \frac{\Delta K}{K}.$$
For this conductor, where the effect of size is minimal, it follows from Equation 33 that

\[
\frac{I_d}{I_r} \approx \frac{\Delta I_d}{I_r}, \quad (35)
\]

\[
< \frac{\Delta I_d}{I_d}, \quad (36)
\]

where \( I_a < I_r \) and \( \Delta I_a \) is the measurement uncertainty. The above inequality shows that, for a sufficiently thin conductor, the effect of the conductor size is not as important as the measurement uncertainty.

For short pulses (10 ns), the integral of Equation 32 can be approximately evaluated using rectangular pulses. The reference current \( I_r(t) \) can be approximated by a rectangular pulse of magnitude \( I_r \) and duration \( \tau \) and, likewise, \( I_a(t) \) by \( I_a \) and \( \tau \). With this approximation the integral is roughly equal to \( I_r I_a \tau \), and so Equation 32 can be rearranged as

\[
\frac{I_d}{I_r} \approx \frac{\tau K}{2Z_0}, \quad (37)
\]

where \( q = I_r \tau \). \( K \) and \( \tau \) are constant, so the only free parameter is \( Z_0 \), which is determined by the size of the central conductor. This equation justifies the statement, made in Section 2.1, that the effect of the central conductor can be made small by choosing a central conductor of a sufficiently small radius.

Now, we have all the equations to estimate the measurement error due to a central conductor. For a typical bunch length in the KAON Factory, we take \( \tau = 10 \) ns. We estimate the loss factor \( K \) of the prototype kicker magnet to be 100 times less than that of a typical cavity. This estimate is very conservative since cavities are known to have very large loss factors compared with those of kicker magnets. We take a cavity [20] with a loss factor of \( 4.8 \times 10^{10} \) V/C and so the loss factor \( K \) of the kicker magnet is \( = 4.8 \times 10^8 \) V/C. We used two conductor sizes (41 mm and 3 mm). The small conductor size gives the coaxial line a characteristic impedance \( Z_0 \) of 180 \( \Omega \) and the bigger one 50 \( \Omega \).
Equation 37 gives

\[
\frac{I_d}{I_r} \approx \frac{4.8 \, \Omega}{2 \times 50 \, \Omega} \approx 0.05, \tag{38}
\]

\[
\frac{I_d}{I_r} \approx \frac{4.8 \, \Omega}{2 \times 180 \, \Omega} \approx 0.01. \tag{39}
\]

We assume that the uncertainty in the measured data is at most 5%. According to the arguments that lead to Equation 36, the error due to the presence of the central conductor of the 50 Ω line is comparable to the measurement uncertainty. For the 180 Ω line, this kind of error is negligible compared with the measurement uncertainty.

To estimate the effect of transit time, we can roughly calculate the time it takes a pulse to pass through the prototype kicker magnet and the decay time constant of the wakefield. For the prototype kicker magnet, the transit time is approximately the speed of light divided by its length, which is about 1.5 ns. The decay time constant is given by \(2Q/\omega_0\), where \(Q\) is the quality factor and \(\omega_0\) the resonant frequency. For the kicker magnet, there are no resonances above 30 MHz. We assume a poor quality factor of 1. The decay time constant is about 11 ns, which is much larger than the transit time.
Appendix B

The Cut-off Frequency of the Prototype Kicker Magnet

The low-pass behavior of a kicker magnet is characterized by the cut-off frequency $f_c$. The cut-off frequency of a low-pass network is the frequency at the half power point on the roll-off curve of frequency versus power transmitted. Because of the mutual inductance $L_m$ between adjacent cells, the cut-off frequency of the prototype kicker magnet is further reduced. The cut-off frequency is given approximately by [16, 17]

$$f_c = \frac{1}{\pi \sqrt{(L + 4L_m)C}}$$  \hspace{1cm} (40)

where $L$ and $C$ are the inductance and capacitance of each cell respectively. For the prototype kicker magnet, $L$ is 139 nH for an end cell, $L_m$ is 119 nH, and $C$ is 74 pF. The cut-off frequency given by Equation 40 is approximately 47 MHz. The measured
cut-off frequency of the prototype magnet at the 3 dB point is 30 MHz (Figure 38).
The discrepancy between the calculated and measured values is due to the fact that
Equation 40 does not take into account the shunt capacitance between adjacent cells
or resistive losses in the cells.
Figure 38: Magnitude of transmission coefficient of the prototype kicker magnet as a low pass filter.
Appendix C

The Scattering Matrix Definitions

The scattering matrix is a convenient and standard way to describe the transmission of a two-port device in a transmission line environment [15]. The incident and reflected wave amplitudes $a$ and $b$ are defined by

$$ a_1 = \frac{1}{2} \left( \frac{V_1}{\sqrt{R_{01}}} + \sqrt{R_{01}} I_1 \right), \quad (41) $$

$$ a_2 = \frac{1}{2} \left( \frac{V_2}{\sqrt{R_{02}}} + \sqrt{R_{02}} I_2 \right), \quad (42) $$

$$ b_1 = \frac{1}{2} \left( \frac{V_1}{\sqrt{R_{01}}} - \sqrt{R_{01}} I_1 \right), \quad (43) $$

$$ b_2 = \frac{1}{2} \left( \frac{V_2}{\sqrt{R_{02}}} - \sqrt{R_{02}} I_2 \right), \quad (44) $$
where $R_{01}$ and $R_{02}$ are arbitrary, positive, real reference impedances. The scattering parameters $S_{ij}$ are defined by
\[
\begin{bmatrix}
  b_1 \\
  b_2
\end{bmatrix} =
\begin{bmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
  a_1 \\
  a_2
\end{bmatrix},
\]
(45)

The input reflection coefficient $S_{11}$ is defined by
\[
S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} = \left. \frac{Z_1 - R_{01}}{Z_1 + R_{01}} \right|_{R_2=R_{02}},
\]
(46)

where $Z_1$ is the input impedance with the output port terminated in the reference impedance $R_{02}$.

The forward transmission coefficient $S_{21}$ is given by
\[
S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0} = \left. \frac{2V_2}{V_{g1}} \sqrt{\frac{R_{01}}{R_{02}}} \right|_{R_1=R_{01}, R_2=R_{02}},
\]
(47)

where $R_1 = R_{02}$ gives a simple relationship between $V_{g1}$ and $a_1$, and $R_2 = R_{02}$ implies $a_2 = 0$.

By reversing input and output ports we get the output reflection coefficient
\[
S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0} = \left. \frac{Z_2 - R_{02}}{Z_2 + R_{02}} \right|_{R_1=R_{01}},
\]
(48)

and the reverse transmission coefficient
\[
S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0} = \left. \frac{2V_1}{V_{g2}} \sqrt{\frac{R_{02}}{R_{01}}} \right|_{R_1=R_{01}, R_2=R_{02}}.
\]
(49)

If the two–port is a lossless transmission line it is convenient to choose the reference impedance equal to the characteristic impedance: $R_{01} = R_{02} = Z_0$. The wave parameters can then be related to incident and reflected voltage. The scattering matrix is then
\[
[S] = \begin{bmatrix}
  0 & e^{-jkl} \\
  e^{-jkl} & 0
\end{bmatrix},
\]
(50)

where $k$ is the propagation constant and $l$ the line length.
Appendix D

Input Impedance of the Prototype Kicker Magnet

Figure 40 shows the input impedance of the prototype kicker magnet when the output of the magnet is short-circuited. Even though the characteristic impedance of the prototype kicker magnet is 30 Ω, the input impedance of the short-circuited magnet depends on the frequency in a predictable way until the cut-off frequency (30 MHz).

Figure 41 shows the input impedance of the kicker magnet with the output terminated with a resistor whose impedance matches the characteristic impedance of the magnet (30 Ω). The input impedance of the terminated magnet would be a constant 30 Ω if the magnet behaved like an ideal transmission line. In fact the magnet behaves like a low-pass filter with a cut-off frequency of 30 MHz. As shown in Figure 41, the input impedance is approximately 30 Ω below the cut-off frequency and drops considerably above the cut-off frequency.
Figure 40: Magnitude of input impedance of the short-circuited prototype kicker magnet.
Figure 41: Magnitude of input impedance of the terminated prototype kicker magnet