ESTIMATES OF WASTEFUL COMMUTING IN A SAMPLE OF CANADIAN CITIES: A TEST OF THE MONOCENTRIC MODEL

by

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ABSTRACT

This thesis is based on an article by Bruce Hamilton published in the Journal of Political Economy 90(5) in 1982, titled "Wasteful Commuting". The analysis compares "optimal" and "observed" commuting behaviour in 23 Canadian cities in order to provide a test of the monocentric model's ability to predict commuting behaviour. Monocentric models are widely used in urban economics due to their simple structure but any model purporting to explain residential and job choice location should be able to explain observed commuting behaviour. In order to operationalize the model it was necessary to estimate employment and population density gradients for 23 Canadian cities using 1981 census data. Density gradients were estimated using a two-point estimation technique pioneered by Edwin Mills. The employment gradient estimates presented in Chapter 4 are the only existing Canadian estimates for a large set of cities. The density gradient estimates were used to calculate the minimum average commuting distance in each city. The minimum average commute was compared with observed commuting behaviour. Data on observed commuting was obtained from a 1977 household survey. The results indicate that observed commuting is, on average, eight times the minimum necessitated by the separation of homes and jobs. Randomly assigning residents to homes and jobs explains observed commuting better than the monocentric model. Like Hamilton's results, the results of this thesis draw into question the validity of the basic monocentric model.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

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My old man worked 50 years in a steel mill so his children could have the opportunity to do what would make them happy. My old man is also one of the most naturally curious people I have ever known. Thanks Dad - for instilling in me your natural curiosity and at least a fragment of your perseverance. I would be remiss if I did not thank Mike Goldberg who has been part mentor and part friend during my time at the University of British Columbia. Although I know it is arrogant to hope for, when I grow up I want to be just like Michael. I would also like to thank my thesis advisor Bob Helsley, a naturally gifted educator, brilliant economist and another who helped to feed my curiosity. Anyone with a compilation of Bob Dylan's complete lyrics on their desk must truly be a kindred spirit. Finally I would like to express my appreciation to the members of my committee for their interest in this project.

CHAPTER ONE

INTRODUCTION

The monocentric urban model is virtually synonymous with the urban economics sub-discipline and particularly with the paradigm of inquiry referred to as the "new urban economics". Since the work of Muth [45] and Mills [43] in the 1960s monocentric models have been widely employed in urban economic analysis.

The monocentric model yields a number of testable predictions about the distribution of phenomena, particularly housing and population, in urban areas. Many of the model's predictions have been empirically tested for numerous sets of U.S. cities. There has been a dearth of similar testing of the model in countries other than the United States. One area where the monocentric model has failed is in predicting observed commuting behaviour. Because this is a fundamental shortcoming in a spatial model that purports to explain the location of households and employment based on the trade-off between housing prices and commuting costs it is considered an area worthy of further analysis.

The purpose of this thesis is to test a "strong form" of the monocentric model. The test focuses on the monocentric model's ability to predict observed commuting behaviour. The test was originally developed and employed by Hamilton [26] for a sample of 14 U.S. cities. Specifically, this thesis attempts to answer two questions:

1. Given the existing distribution of homes and jobs in a

sample of Canadian cities is the monocentric model able to accurately predict observed commuting behaviour?

2. Is the model's performance significantly different in Canada than in the United States? Or put differently, in the aggregate, do Canadian and American commuters behave differently after controlling for the existing urban structure?

In order to operationalize the model used to estimate expected commuting it is first necessary to estimate population and employment density gradients for each city included in the sample of Canadian cities. Although this is not specifically listed among the goals of the thesis, the population and employment gradient estimates should be of interest to urban researchers. The employment gradient estimates are the first for any set of Canadian cities and the population gradient estimates are for the largest sample of Canadian cities covered by any research to date.

Chapter 2 reviews the importance of the monocentric model within the urban economics sub-discipline. A simple mathematical version of the model based on Hamilton and Mills [42] is presented in order to highlight the predictions of the monocentric model and the key role played by commuting. The spatial location equilibrium condition, one of the fundamental results in urban economics, is also presented. Finally criticisms of the monocentric model are reviewed.

Chapter 3 reviews a model that can be used to estimate aggregate commuting behaviour in monocentric cities. Originally, the model was developed and employed by Hamilton [26] for a sample of cities in the United States. The logic of the model is straightforward.

The average distance of homes and the average distance of jobs, from the CBD is estimated. The difference between the two is referred to as the optimum average commute. The optimum average commuting distance is then compared to observed commuting behaviour in a sample of cities.

Chapter 4 presents estimates of population and employment density gradients for a sample of 23 Canadian cities. The parameter estimates from the density gradients are necessary to estimate optimal commuting distance in each city. Considerable attention is given to the estimation technique and discussion of the gradient estimates.

Chapter 5 employs the model developed in Chapter 3 and the gradients estimated in Chapter 4, to test the ability of the monocentric model to predict observed commuting behaviour in 23 Canadian cities. The results for Canada are compared with Hamilton's results [26] for his sample of U.S. cities.

Chapter 6 concludes the study and provides suggestions for further research into the determinants of commuting behaviour.

CHAPTER TWO

A REVIEW OF THE MONOCENTRIC MODEL

This chapter reviews the monocentric city model. The importance of the monocentric model in urban economics and the evolution of the monocentric model is discussed. Monocentric models have been widely employed in urban economics due largely to the simple structure of the model and the associated mathematical tractability [13]. There are at least three essential but unrealistic assumptions common to most versions of the monocentric model:

1. There is a predetermined centre to which all households commute and to which all products are shipped;
2. Individuals are homogeneous in preferences and the urban area is homogeneous in land and neighbourhood characteristics; and
3. Cities are instantaneously developed and infinitely malleable (sometimes referred to as the putty-putty assumption).

The first two assumptions imply that distance to the central business district (CBD) fully characterizes the desirability of any location within the city: direction is irrelevant. Households are indifferent to a particular home or job except insofar as the transportation (commuting) costs associated with particular sites differ.[1] The third assumption implies that the spatial economy is characterized by a series of long run equilibria with capital

[1] The assumption of indifference among jobs is particularly unrealistic in monocentric models that attempt to incorporate multiple income groups. In reality an important factor in determining income differentials is probably employment income. The monocentric model assumes that all jobs are equally desirable so income differentials must arise due to differences in initial endowments or differences in non-employment income.

investment adjusted every period.

This chapter is divided into six sections. Section 2.1 discusses the central role played by transportation costs in urban economic analysis. Section 2.2 presents a simple version of the monocentric model. The model presentation relies heavily on Mills and Hamilton [42], (particularly Mills and Hamilton's Appendix A). Section 2.3 briefly touches upon the issue of functional form and the appropriateness of the negative exponential population density function. Section 2.4 examines employment location theory in the context of the simple urban model. The next section discusses some general criticisms of the monocentric model, including a critical test of the model developed by Hamilton [26]. Hamilton's methodology is subsequently built upon in the remaining chapters of this thesis. Finally section 2.6 concludes the chapter.

2.1 Transportation Costs, Land Values & Urban Economics

Transportation costs have long been recognized as a crucial determinant of both the formation of cities and the distribution of economic activity within cities.[2] In the absence of transportation costs geographic proximity is not necessary in order to preserve economic linkages. Once transportation is costly all interaction involves the cost of overcoming distance.

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[2] Transportation costs and scale economies in production (and to a lesser degree in consumption), are sufficient conditions for the existence of cities.

Traditional neoclassical economic theory is presented in a spaceless economy which is, quite clearly, artificial because spatial relationships and land use patterns affect every economic activity. In a spatial economy the demand for land arises from the consumption and production activity of individuals. Individual activities are brought together by spatial agglomeration economies which arise from interaction and it is this interaction that involves the cost of overcoming distance.

Urban economics developed as an explicit attempt to incorporate space into consumer and producer theory and this basic concern for the geographic distribution of phenomena (particularly population) in cities still defines urban economics [57]. The major difficulty inherent to incorporating space into neoclassical theory lies in the indivisibility of the land-location consumption decision. Locational fixity suggests that dwelling units differ greatly in their accessibility to production and consumption activities making it difficult to isolate the consumption of a good such as housing from the consumption of "accessibility" [49].

Many important insights into the operation of housing markets were derived during the 1960s from the realization that employment accessibility and housing are jointly purchased. However interest in the nexus between transportation costs and the value of land stretches back into the 19th century and the work of Johan von Thünen. Von Thünen postulated an agricultural land market where the price of land was determined by productivity (fertility) and the distance of land to the nearest market town.[3] William Alonso [3] built on the work of von Thünen and another early twentieth century economist, Robert Haig. Alonso's work made explicit the central role played by transportation costs within cities through the development of bid-rent functions. In equilibrium, sites differ by the transportation costs associated with distance from the CBD. The sum of land rent plus transportation costs must be a "constant" throughout the city for any particular land use, such as housing, in order to establish long run equilibrium.

From the work of Alonso urban economic analysis quickly evolved into spatial models focusing on equilibria and social optima. Of course, the purpose of such models is to abstract from reality using a few basic theoretical concepts in order to explain a large number of observed phenomena. The most important of the spatial models, the monocentric model, has become synonymous with urban economics. But as Wheaton cautioned, "If the monocentric models contain a lesson it is that spatial relationships substantially complicate microeconomics' simple analysis" [57].

2.2 The Basic Monocentric Model

According to Wheaton [57] the family of monocentric models represents a distinct branch of microeconomics. While Alonso [3] is generally credited with incorporating bid-rent functions into

[3] Throughout the 19th century and for the early part of the 20th century it was geographers rather than economists who were in the vanguard of locational and spatial research.

urban analysis, Mills [43] and Muth [45] are often cited among the first to have formalized mathematical versions of the monocentric model. In the Alonso model households have a direct preference for land which determines residential density. In the model developed by Muth and Mills consumers do not have a direct preference for the good land. Instead, residential density is determined on the supply side by the ability to substitute capital for land in the production of "housing services".

Monocentric models differ in sophistication and can incorporate a number of important complications that are either internal or external to the city. Internal complications include:

- Incorporating dispersed employment as well as dispersed housing;[4]
- The addition of public goods;
- Allowing for transportation congestion;
- Including externalities such as race, pollution or crime; and
- Incorporating multiple (usually two) income groups.

There are basically two variations in the outer structure of the spatial economy in which a city operates that have been examined within the framework of the monocentric model:

- Open versus closed cities - open cities allow migration into and out of the city, which implies that the level of utility is exogenously determined while closed cities do not allow migration implying that utility is determined within the city; and

[4] Employment decentralization is discussed in Section 2.4 of this chapter and again, briefly, in Chapter 3.

- Distribution of land rents - landlords can be assumed to reside in the city or it can be assumed that all rents are paid to absentee landlords. The former results in multiple income groups.

It is beyond the scope of this chapter to review each incarnation of the monocentric model as listed above. Only a simple version of the model is presented below, focusing on residential location theory. The model presented is, however, sufficient to highlight the central role that commuting plays in the model. And it is the inability of the monocentric model to predict commuting behaviour that was central both to Hamilton's [26] criticism of the monocentric model and to this thesis.

2.2.1 Derivation of the Locational Equilibrium Condition

Begin by assuming that all individual households are utility maximizers and that there are only two normal goods: housing (h) and a composite non-housing good (g). Household maximize:

$$U = u(h, g) \tag{2.1}$$

subject to the budget constraint:

$$P_g g(x) + P_h(x)\, h(x) + tx = y \tag{2.2}$$

where:

- y ≡ household income;
- t ≡ is the cost of a round trip kilometre;
- x ≡ distance from the CBD;
- P_h ≡ the price of one unit of the housing good;

- $P_g \equiv$ the price of one unit of the composite good.

In equilibrium, each household will maximize utility at a point where the slope of the indifference curve is tangent to the slope of the budget constraint:[5]

$$\frac{\Delta h(x)}{\Delta g(x)} = -\frac{P_g}{P_h(x)} \tag{2.3}$$

Now, in order to derive an important result, imagine the impact of a small change in location (Δx) on the budget constraint given in equation 2.2. To maintain equilibrium the following must obtain:

$$P_g \Delta g(x) + \Delta P_h(x)\, h(x) + P_h(x)\, \Delta h(x) + t\Delta x = 0. \tag{2.4}$$

Rearranging equation 2.3 yields:

$$P_g \Delta g(x) + P_h(x)\, \Delta h(x) = 0. \tag{2.5}$$

Subtracting equation 2.5 from both sides of 2.4 and rearranging gives:

$$\frac{\Delta P_h(x)\, h(x)}{\Delta x} = -t. \tag{2.6}$$

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[5] An indifference curve maps out all the possible combinations of (h) and (g) that will yield a given level of utility for a household. Households are indifferent among various combinations of (h) and (g) that yield the same level of satisfaction.

Equation 2.6 is an important result, often referred to as the locational equilibrium condition. The locational equilibrium condition implies that as distance to the CBD increases, the reduction in housing expenditure (the numerator on the LHS of equation 2.6) is exactly offset by an increase in commuting costs, -t (the RHS of equation 2.6). The importance of commuting behaviour in the model is illustrated by equation 2.6.

Rearranging equation 2.6 yields an expression for the slope of the housing price function:

$$\frac{\Delta P_h(x)}{\Delta x} = -\frac{t}{h(x)} \qquad (2.7)$$

Interesting implications arising from equation 2.7 include:

1. The minus sign on the RHS implies that the housing price function has a negative slope - i.e. prices fall with distance from the CBD;

2. The presence of h(x) in the denominator of the RHS implies that when housing consumption is small the house price function is steeper than when housing consumption is not small - i.e. P_h is convex with substitution in consumption; and

3. If we assume that non-land input prices do not vary with distance from the CBD, housing prices can be steep only where the land rent function is steep, near the centre of the city.

Two realistic implications of the locational equilibrium are:

1. Suburbanites consume more housing than central city residents in order to maintain spatial equilibrium; and

2. Suburban houses have lower capital land ratios because in the suburbs land is cheaper relative to other inputs in the production of housing.

Both implications entail lower population density in suburban locations.[6]

2.2.2 The Supply of Housing Services

To understand how the urban economy works is to understand how markets combine land with other inputs in varying proportions at different places. The previous derivation of the locational equilibrium condition ignored any formal inclusion of the supply of housing. This section will more formally examine the conditions which must obtain for equilibrium in the residential sector.

Muth/Mills models are able to account for the substitution between labour, land and capital in housing markets. In addition to the three assumptions cited in the introduction to this chapter the current exposition assumes:

- A Cobb-Douglas housing production function;
- The rental rate on capital (r) is not related to intra-urban location (x); and
- Housing input and output markets are competitive.

Using Cobb-Douglas notation assume the following housing production function:

[6] It is possible to construct a model with fixed lot sizes in which suburban residents do not consume more housing. With fixed lot sizes suburban housing is cheaper than in more central locations. This enables suburban households to achieve the equilibrium level of utility by increasing non-housing consumption by an amount exactly equal to the cost of commuting to the suburbs.

$$H_s(x) = AL(x)^{\alpha} K(x)^{1-\alpha} \tag{2.8}$$

where:

- $H_s(x)$ ≡ housing supply;
- A, α ≡ constants with $0 < \alpha < 1$;
- $L(x)$ ≡ land used in housing production; and
- $K(x)$ ≡ capital used in housing production.

Differentiating equation 2.8 yields the marginal product for land:

$$MP_{L(x)} = \frac{\alpha H_s(x)}{L(x)} \tag{2.9}$$

and the marginal product for capital:

$$MP_{K(x)} = \frac{(1 - \alpha) H_s(x)}{K(x)}. \tag{2.10}$$

Multiplying the marginal products of land and capital by the price of housing ($P_h(x)$) yields the respective value of marginal product expression for land and capital:[7]

$$R(x) = \frac{\alpha P_h(x) H_s(x)}{L(x)} \tag{2.11}$$

[7] R(x) is the rental rate on land at distance x from the CBD and r is the spatially invariant rental rate on capital. Given the assumption that input markets are competitive, in equilibrium the value of the marginal product of each factor of production must equal its rental rate.

$$r = \frac{(1 - \alpha) P_h(x)\ H_s(x)}{K(x)}. \tag{2.12}$$

2.2.3 The Demand for Housing Services

Assume that all workers in the city have the same income (y) (determined exogenously), the same preferences and the same individual demand for housing services:[8]

$$h(x) = By^{\theta_1} P_h(x)^{\theta_2} \tag{2.13}$$

where:

- $\theta_1 \equiv$ the income elasticity of demand for housing;
- $\theta_2 \equiv$ the price elasticity of demand for housing; and
- B $\equiv$ is a constant.

Because it was assumed that housing is a non-inferior good we know that $\theta_1 > 0$. Therefore, a downward sloping housing demand function implies that θ_2 must be less than zero. Total housing demand is the product of individual housing demand times the number of individuals, N(x):

$$H_D(x) = h(x)\, N(x) \tag{2.14}$$

[8] According to Mills and Hamilton [42], equation 2.13 is a demand function that has been widely employed in many demand studies. It assumes that the income and price elasticities are constant.

2.2.4 Solution to the Model

To complete the model and solve it assume that ϕ radians of a circle are available for development at every distance from the predetermined city centre with $\phi \leq 2\pi$.[9] Thus $2\pi - \phi$ radians are unavailable for development.

The model has five equilibrium conditions which must be satisfied:

$$H_d(x) = H_s(x) \tag{2.15}$$

$$P_h'(x)\, h(x) + t = 0 \tag{2.16}$$

$$L(x) = \phi x \tag{2.17}$$

$$R(\overline{x}) = \overline{R} \tag{2.18}$$

$$\int_0^{\overline{x}} N(x)\, dx = N. \tag{2.19}$$

Equation 2.15 simply says that in equilibrium total housing supply must equal total housing demand. Equation 2.16 is the locational equilibrium expression (derived in Section 2.2.1), equation 2.17 implies that, in equilibrium, the amount of land used for housing cannot exceed the total land available and no land can be left vacant. Equation 2.18 equates land rent at the urban boundary to rent in non-urban uses and 2.19 specifies that the total number of workers in the urban area, N, is equivalent to the number of workers at any distance, N(x), for all x.

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[9] ϕ cannot exceed 2π because a complete circle has 2π radians.

In order to solve the model the VMP equations 2.11 and 2.12 are rearranged isolating L(x) and K(x). L(x) and K(x) are then substituted into the housing supply expression, equation 2.8 yielding:

$$P_h(x) = \frac{r^{1-\alpha} R(x)^{\alpha}}{A\alpha^{\alpha}(1-\alpha)^{1-\alpha}}. \tag{2.20}$$

Equation 2.20 indicates that the price of housing is proportionate to land rent, R(x), raised to the power of α. Because we know that $0 < \alpha < 1$, we know that house prices are high when land rents are high but house prices rise less than land prices due to factor substitution in the housing production function. Taking the derivative of equation 2.20 with respect to x yields:

$$P_h'(x) = \frac{1}{A}\left(\frac{\alpha r}{(1-\alpha)}\right)^{1-\alpha} R(x)^{\alpha-1} R'(x) \tag{2.21}$$

where R'(x) is the slope of the rent function R(x). Equation 2.21 is a differential equation for housing prices but, because there is no initial condition for house prices, it is necessary to solve the model for R(x) rather than house prices. Equation 2.18 provides the initial condition necessary to solve the differential equation expressed in terms of land rents.

Substituting equation 2.13 into the locational equilibrium condition, equation 2.16, for h(x), equation 2.20 into 2.16 for $P_h(x)$ and equation 2.21 into 2.16 for $P_h'(x)$ yields:

$$E^{-1}R(x)^{\beta - 1}R'(x) + t = 0 \qquad \textbf{(2.22)}$$

where E and ß are collections of constants:

$$E^{-1} = \alpha\beta y^{\theta_1}[A\alpha^{\alpha}(1 - \alpha)^{1 - \alpha}]^{-(1 + \theta_2)} r^{(1 - \alpha)(1 + \theta_2)}$$

$$\beta = \alpha(1 + \theta_2).$$

Utilizing the initial condition provided by equation 2.18 results in a solution for R(x) from the differential equation, 2.22:

$$R(x) = [\overline{R}^{\beta} + \beta tE(\overline{x} - x)]^{\frac{1}{\beta}}. \qquad \textbf{(2.23)}$$

Equation 2.23 is a general expression for land rent, R(x). If ß equals zero land rent can be expressed as a negative exponential function of x:[10]

$$R(x) = \overline{R}e^{-tE(\overline{x} - x)}. \qquad \textbf{(2.24)}$$

From the definition of ß given above it can be seen that a zero value for ß implies that $\theta_2 = -1$: i.e. the price elasticity of demand for housing is -1.

The final step in this simple derivation of the model involves linking the rent gradient to the population density gradient.

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[10] e is the base of the natural logarithm.

Using the equilibrium condition given by equation 2.15 the expression for total housing demand, equation 2.14, can be rewritten:

$$N(x) = \frac{H_s(x)}{h(x)} . \tag{2.25}$$

Taking the ratio of the value of the marginal product of land and the value of the marginal product of capital (equation 2.11 and 2.12) results in the following expression for K(x):

$$K(x) = \frac{1 - \alpha}{\alpha r} R(x) L(x) . \tag{2.26}$$

Substituting equation 2.26 into the Cobb-Douglas production function, equation 2.8, yields:

$$H_s(x) = A\left[\frac{1 - \alpha}{\alpha r}\right]^{1 - \alpha} R(x)^{1 - \alpha} L(x) . \tag{2.27}$$

Substituting the expression obtained for $P_h(x)$ in equation 2.20, into the individual housing demand expression given by equation 2.13 and then substituting the entire expression into 2.25 for h(x) as well as substituting equation 2.27 into 2.25 for $H_s(x)$ yields:

$$\frac{N(x)}{L(x)} = ER(x)^{1 - \beta}. \tag{2.28}$$

Equation 2.28 indicates that resident workers per unit area (population density) is proportionate to land rent raised to the power (1 - ß). If ß is once again set to zero (i.e. assume that the price elasticity of demand is -1) population density is strictly proportionate to land rent and, therefore, population density declines exponentially with distance from the CBD, like land rent in equation 2.24.

2.3 The Negative Exponential Population Density Gradient

The empirical regularity of a negative exponential population density gradient was noted by Colin Clark almost 20 years before Muth and Mills (independently) constructed a formal theory to explain the empirical regularity. Because the negative exponential population density gradient is generated by a strong form of the monocentric model, it has been subject a great deal of scrutiny and criticism.

There have been numerous empirical tests of the negative exponential population density function using the Box-Cox method which can statistically test for functional form.[11] Employing Box-Cox tests Kau and Lee [32] found that only 50 percent of the U.S. cities they tested were well characterized by a negative exponential population density gradient. Anderson's [6] results were even less favourable with 22 of 30 cities poorly characterized by a negative exponential population density gradient. Kau Lee and Chen [31] found the negative exponential gradient did not apply in 50 percent of the cities they tested. McDonald and Bowman [40]

[11] The Box-Cox method is described in Johnston [30] and Meyer [46].

were able to identify several alternative functional forms that performed as well as the negative exponential density function in some cases, including generalized normal, gamma and standard normal.

Despite frequent criticism that the price elasticity of housing demand is not equal to -1 and the empirical evidence cited above, the negative exponential population density function has been used in numerous applied studies (e.g. [1], [4], [5], [14], [17], [18], [26], [38], [43], [45], [61] and [63]) since Mills formalized the monocentric model. This is largely because there are very few more mathematically complex models that are tractable [13].

It is important to understand the issues surrounding the correct functional specification because the test of the monocentric model developed by Hamilton [26] simply assumes that the negative exponential population (and employment function) obtains.[12] Although some would accuse Hamilton of constructing a "strawman" by using the negative exponential form, the accusation rings hollow considering the amount of academic research that has been based on the strong version of the monocentric model.

2.4 A Digression on Employment Location

Hamilton's [26] assumption of a negative exponential employment

[12] Hamilton's [26] model is described in detail in Chapter 3 of this thesis.

gradient raises employment location as an important issue.[13] In fact, one of the strongest criticisms of Hamilton's test of the monocentric model [62] focused on Hamilton's assumption that when employment decentralizes from the CBD it does not cluster but disperses in a uniform fashion.

In the monocentric model described in section 2.2 of this chapter it was assumed that all employment was located in the CBD. This is clearly an unrealistic assumption but it can be easily relaxed without disturbing the locational equilibrium condition. If a firm moves out of the CBD to a point 5 kilometres distant, any workers who choose to work at the firm would be better off than workers who work in the CBD, if both locations paid the same wage rate. Thus, a profit maximizing firm would offer a wage in the suburbs that is lower than the CBD wage by the amount of the potential commuting savings (in this case 5t). There is, therefore, a wage gradient with slope -t which leaves the residential location equilibrium condition, equation 2.6, undisturbed.[14] The somewhat counterintuitive result is that R(x) is unaltered by employment decentralization: workers still commute up the rent gradient, some all the way to the CBD and some only as far as the suburban

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[13] For the specifics regarding Hamilton's use of the negative exponential employment gradient see Chapter 3 of this thesis.

[14] This is true only if the number of workers located on the suburban side of a ray passing from the CBD and through the suburban firm location exceeds the demand for workers at the suburban employment location. If it does not then the firm would have to offer a higher wage in order to induce circumferential or backward commuting. See Chapter 3 of this thesis or White [62] for details.

employer.

Madden [39] provided one of the few studies to empirically test for a negatively sloped wage gradient. To test for a negative wage gradient Madden developed a regression model but had to assume that the negative exponential population gradient was a correct specification. Madden's results were consistent with a negatively sloped wage gradient for a sample of U.S. cities, although no conclusion regarding functional form was possible. Similarly, Leigh [36] found evidence of a wage gradient in several U.S. cities, but only for caucasians.

The assumption of negative exponential population and employment gradients requires an identical job/housing pattern along every ray emanating from the CBD [62]. This leads to the key question of whether firms are likely to cluster when they suburbanize or whether they are likely to disperse and what the decisions are based upon.

In general, employment location is poorly understood and much less researched than population location. Much of the employment location research has focused on the location of manufacturing (e.g. [10],[27] [51]), which is a very small portion of total employment in most North American cities. Work examining the location of non-industrial employment frequently examines the notion of agglomeration economies as a determinant of firm location (e.g. [11], [56]). Although the dictionary would treat agglomeration economies as a virtual synonym for clustering, the term carries a heavier, if poorly defined, meaning in economic writing on the location of industries.

It is important to note that even though firms can be allowed to decentralize in the monocentric model access to the CBD is still valuable or a firm would not choose to locate anywhere on the urban rent surface. Mills and Hamilton [42] suggest that firms with agglomeration economies that decline rapidly with distance from the CBD will have a steeper bid-rent function than firms with agglomeration economies that are less sensitive to distance.

There are two interesting implications of the Mills-Hamilton line of reasoning. First, improvements in communication technology might be expected to lessen the decline in agglomeration economies for a firm located at any distance from the CBD, with the result that more and more firms will choose suburban locations.[15] Second, by clustering in suburban locations firms may be able to generate some positive externalities (i.e. agglomeration economies) in the suburban location. The degree to which this is possible depends upon he micro-foundations of the agglomeration economies.[16] Of

[15] This argument is often used to explain the fairly recent suburbanization of office occupations which were typically thought to locate in the central city due to the need for face to face communications.

[16] The micro-foundations of agglomeration economies are poorly understood. Possible explanations include: the city performs a role as a warehouse, incubator effects such that new firms experience lower information costs due to proximity to other firms in the same industry, higher salvage values for capital assets, lower search costs for skilled labour, qualitative differences in

course, when suburbanized firms cluster they also offset some of the benefits of suburbanization by increasing both land rent and wages relative to a dispersed suburbanization scenario. Ultimately the choice to cluster or suburbanize will be dependant upon a trade-off between agglomeration benefits, land rent and wage costs.

Unfortunately, there is no way to extend the monocentric model to cover the formation of sub-centres because the monocentric model contains only one descriptor of location (distance to the CBD). Clustered suburban employment necessitates circumferential and/or backwards commuting making both distance and direction from the CBD important.[17] And two dimensional models are extremely complex and yield few analytic results.

2.5 Criticisms of the Monocentric Model

Criticisms of the monocentric model fall into two broad classes:

1. Criticism of the model's central assumptions (e.g. [49] [57]); and
2. Criticism of the predictive powers of the monocentric model (e.g. [7], [15], [26], [32], [36], [39]).

2.5.1 Assumptions of the Monocentric Model

All economic modelling involves abstraction which usually takes the

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the types of information exchanged etc.

[17] Dubin and Sung examined the impact of direction on the density gradient for Baltimore [16].

form of simplifying assumptions in the model and the monocentric model is no exception. The monocentric model has received criticism for a lack of realism in a number of areas including:

- The assumption that all housing is putty-putty;
- The lack of specificity regarding the reasons for the existence of the CBD;
- The assumption of homogeneous households;
- The assumption that there exists an everywhere dense network of transportation facilities; and
- The assumption that employment does not cluster when it decentralizes.

The putty-putty assumption is one of the least satisfactory aspects of the monocentric model. The model ignores the spatial fixity and durability of housing capital assuming instead that the city is restructured each period in order to achieve long-run equilibrium [49], [57]. Harrison and Kain [28] were able to overcome the putty-putty assumption by constructing a different logical edifice on the empirical regularity first observed by Colin Clark. Harrison and Kain argued that current spatial structure is logically viewed as an aggregation of historical patterns of development under the assumption of durable housing capital. Present density is a function of the weighted density of all past development in a particular city. The last decade has seen some important theoretical work with dynamic models but the specifications are often unwieldy and there has been only limited empirical testing of the models.

The lack of specificity regarding the reasons for the existence of a CBD has also been the subject of criticism. Wheaton argued that simple models of centrality are inadequate and what is required are models that explain the attraction of economic activity to cities [47], [57]. A limited number of researchers have responded to this criticism by developing non-monocentric models where the location of employment is endogenously determined (e.g. [47]). Agglomeration economies are fundamental to non-monocentric models but mathematical intractability is often severe with great difficulty in solving for equilibrium.

Multiple household types have been successfully incorporated into the monocentric model by a number of researchers including Wheaton [58] and White [64]. In an interesting paper, Steen examined the implications of relaxing the assumption of ubiquitous transportation networks [53]. Steen concluded that discreet transportation networks yield a more complex density pattern with households valuing access both to the transportation route and access to the CBD.

The central location of employment is the most widely criticized aspect of the monocentric model according to Wheaton [57]. Thus, employment decentralization was accorded a separate section, 2.4, above.

2.5.2 Predictions of the Monocentric Model

Blackley and Follain [7] noted that the monocentric model has been subjected to a great deal of testing and criticism.[18] Much of the testing has focused on the reduced form of the model by testing the monocentric model's ability to predict important spatial phenomena. While such tests may not refute the model's locational equilibrium condition they may weaken confidence in the monocentric model if the model fails repeatedly to predict important spatial patterns.

The monocentric model yields at least five important predictions:

1. There exists a negatively sloped housing price gradient;
2. There exists a negatively sloped land price gradient;
3. There exists a negatively sloped population density gradient;
4. There exists a negatively sloped wage gradient; and
5. People minimize commuting by travelling inwards on a ray between their home and job.

Each of these predictions has been empirically investigated, most with mixed results. The majority of studies have found evidence to support a negatively sloped house price and land price gradient although there are serious questions about some of the hedonic models used to test for a house price gradient.

While section 2.3 of this chapter highlighted concern surrounding the specific functional form of the population density gradient, most of the studies supported the conclusion that population density declines with distance to the city centre. There is,

[18] This has certainly been the case in the United States. There has been considerably less testing of the model in Canada.

however no similar support for a negatively sloped employment density gradient. Neither Kemper and Schmenner [33] or Schmenner [51] were able to find direct evidence of a rent gradient (and therefore an employment density gradient) for manufacturing using regression analyses similar to Muth's [45] model of residential location.

It has also proven extremely difficult to obtain data that allows for an empirical test of the existence of a negatively sloped wage gradient. Studies by Leigh [36] and Madden [39] provided weak evidence in support of the wage gradient.

Perhaps the greatest empirical failure of the monocentric model has been its inability to predict observed commuting behaviour. Hamilton [26] provided the essential test of the monocentric model in this regard. Hamilton's results indicated that observed commuting in 14 U.S. cities was eight times that predicted by the monocentric model. This is not surprising considering that the vast majority of urban travel is non-commuting trips. In a sense the monocentric model is too restrictive to adequately explain the location decisions of home owners who base their decisions upon much more than commuting cost.

2.6 Summary

The purpose of this chapter was to provide a brief overview of the monocentric model. The monocentric model is synonymous with urban economics. The importance of the monocentric model as its evolution was discussed. Monocentric models have been widely employed in urban economics due largely to the simple structure of the model and the associated mathematical tractability [13].

Sections 2.1 and 2.2 examined the logic behind the model and mathematically derived a simple version of the monocentric model based on Mills and Hamilton [42]. Sections 2.3 to 2.5 reviewed some of the work of those who challenged the assumptions and predictions of the monocentric model.

Section 2.3 discussed the negative exponential functional form. Most of the empirical work concluded that the negative exponential population density gradient is inappropriate, thereby implying that the price elasticity of the demand for housing is not -1. Section 2.3 is not an indictment of the model itself but an indictment of the use of equation 2.24 rather than 2.23. Section 2.4 described an important modification to the model by allowing employment decentralization. However, the inability to include clustered suburban employment remains an important weakness in the model. Finally Section 2.5 provided a brief overview of more general criticisms of the monocentric model. Hamilton's critique [26], comparing observed commuting behaviour with commuting estimated by the monocentric model forms the basis for the remainder of this thesis.

An appropriate epilogue to this chapter is provided by Wheaton [57]. Wheaton argues that while the utility of monocentric models in forecasting urban growth is limited they still serve an important educational function. Even if the assumptions and outcome of the models are seen as unrealistic the clarity of their simple structure has created a new awareness of spatial equilibrium and the role of transportation.

CHAPTER 3

HAMILTON'S MODEL FOR ESTIMATING WASTEFUL COMMUTING

This chapter describes and explains the methodology used to compare commuting behaviour in 23 Canadian cities in 1981. The basic model was derived by Hamilton [26]. Employing the same methodology as Hamilton's U.S. study allows direct comparison of commuting behaviour in Canada and the United States.

There are four sections in this chapter. Section 3.1 describes Hamilton's method for measuring wasteful commuting in cities. The exposition parallels that given by Hamilton [26] but attempts have been made to clarify areas where Hamilton's description was somewhat opaque. Section 3.2 reviews three methods for calculating two-point estimates of population and employment density gradients. Section 3.3 describes the data used to operationalize the model. The final section concludes the chapter.

3.1 Hamilton's Model For Estimating Wasteful Commuting

Hamilton realized that it is not sufficient to compare the average commuting distance (or time) for a sample of cities in order to judge the relative efficiency of commuting behaviour in each city. Simple comparison ignores extant structural differences among cities. The distribution of jobs and homes influences commuting behaviour. Hamilton's model is able to control for each city's internal distribution of people and jobs when assessing the ability of the monocentric model to predict observed commuting.

3.1.1 The Theoretical Basis of Hamilton's Model

Hamilton begins with the standard monocentric model (described in the previous chapter). Recall from Chapter 2 that individuals are assumed to maximize a utility function, (U), defined over a housing good, (h), and a composite non-housing good, (g):

$$U = u(h, g) \tag{3.1}$$

Utility is maximized subject to a budget constraint in which commuting is costly:

$$y - t\hat{x} = P_h(x)\, h(x) + P_g g(x) \tag{3.2}$$

where:

- $y \equiv$ household income;
- $t \equiv$ the cost of a round trip kilometre;
- $x \equiv$ distance from home to the CBD;
- $\hat{x} \equiv$ the distance from home to work;
- $P_h(x) \equiv$ the price of one unit of the housing good;
- $P_g \equiv$ the unit price of the composite good.

Notice that the model incorporates a spatial component into standard consumption theory by specifying P_g as spatially invariant while P_h is specified as a function of distance to the CBD, (x). Equilibrium requires that P_h decline as x increases in order to compensate individuals for the higher transportation costs (i.e. commuting costs) associated with residences more distant from the CBD.

In the case of complete employment centralization, (i.e. $\hat{x} = x$ for all individuals), the possibility of wasteful commuting does not exist because aggregate commuting is fixed. There is no possible reassignment of homes or jobs among existing residents that will reduce aggregate commuting. The average commute is simply aggregate commuting divided by total employment in the city [pg. 1037, 26].

When employment decentralizes, but doesn't cluster, households maximize utility by trading off accessibility to $\hat{x}$, (rather than x_o), with the price of housing $P_h(x)$.[1] Land rent, however, varies only with distance to the CBD (x) as in the case of complete employment centralization. Hamilton and Mills [42], [43] show that in the case of non-clustered decentralized employment, there exists a wage gradient, with slope -t, that leaves the rent gradient and the locational equilibrium unchanged from the case of complete employment centralization. Lower commuting costs associated with a suburban job result in a lower wage rate at the suburban job location [pg. 114, 42].

As long as the population located on the ray $\hat{x} \rightarrow F$ in Figure 3.1 exceeds the total demand for workers of a firm at $\hat{x}$, wages at $\hat{x}$ will be $(w^* - \hat{x}t)$, which is $\hat{x}t$ less than wages at the CBD (w^*). The discount amount, $(\hat{x}t)$, is exactly equal to the savings in commuting

[1] The importance of the assumption that suburban employment does not cluster is discussed in Chapter 2 of this thesis. See also White [62].

costs for an employee who lives at x_1 and works at $\hat{x}$ instead of working in the CBD. Individuals who work at $\hat{x}$ will always choose to live on the portion of the ray between $\hat{x} \rightarrow F$ in order to minimize both $P_h(x)$ which declines with distance from the CBD and transportation costs.[2]

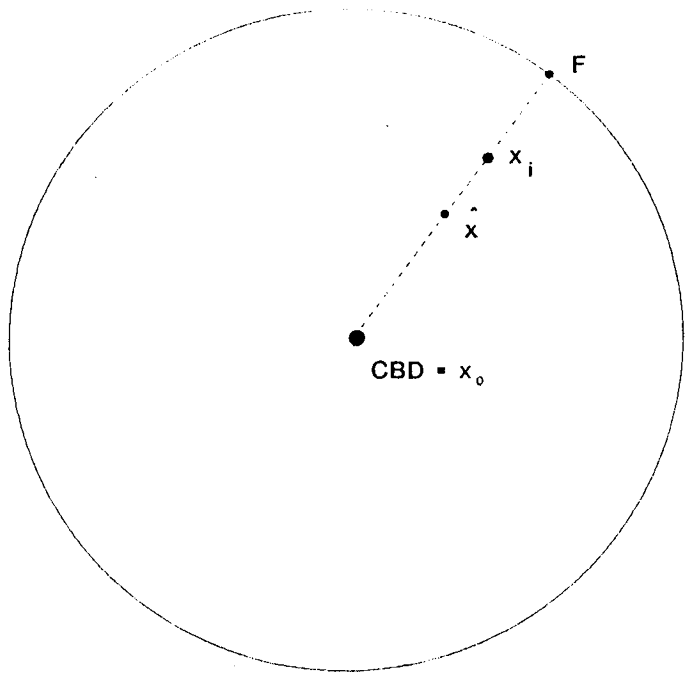
The market solution minimizes aggregate commuting within a city if every household chooses to live on the suburban side of the ray $x_o \rightarrow \hat{x} \rightarrow F$ for all possible $\hat{x}$. Any violation of this criterion makes possible a house or job swap that reduces total commuting resulting in a pareto improvement. The solution that minimizes aggregate commuting is not unique, however. Any two workers on the ray $\hat{x} \rightarrow F$ can swap houses (or jobs) as long as both remain on the suburban side of their respective jobs [pg. 1038, 26].

The introduction of a negatively sloped wage gradient in the presence of decentralized employment preserves the original locational equilibrium which is based solely on the trade-off of commuting costs and housing costs. However, employment decentralization raises the possibility that aggregate commuting can exceed the minimum necessary. Wasteful commuting, impossible in the case of complete employment centralization, becomes possible when the model allows for decentralized employment. The matching of households and jobs is no longer irrelevant, but can effect the

---

[2] If a firm at $\hat{x}$ demands more workers than are living on $\hat{x} \rightarrow F$ the suburban wage rate will be greater than $(w^* - \hat{x}t)$ but less than w^*. This must be the case. For the firm at $\hat{x}$ to attract workers it must compensate them for either backward or circumferential commuting. See Chapter 2, Section 2.4 for more details as well as White [62].

FIGURE 3.1
CITY WITH DISPERSED EMPLOYMENT



- CBD = Central Business District (x_0).
- x_i = household location of individual i.
- F = the urban boundary.
- $\hat{x}$ = suburban employment location.
- w^* = wage rate at the CBD.
- $x_i t$ = commuting cost for person at x_i working at x_0.
- $(x_i - \hat{x})$ = commuting cost for person at x_i working at $\hat{x}$.
- $\hat{x}t$ = commuting savings from working at $\hat{x}$.
- $(w^* - \hat{x}t)$ = wage rate at $\hat{x}$.

amount of aggregate commuting in a city.

3.1.2 Calculating the Optimal Average Commute

If all households are assumed to behave as individual utility maximizers, it is relatively easy to calculate the minimum required average commute for any city. The minimum required commute equals the average distance between homes and jobs throughout the metropolitan area.

When all jobs are located in the CBD the average commute can be estimated as the aggregate distance of all people from the CBD divided by the total population:[3]

$$A = \frac{1}{P} \int_0^{\bar{x}} x P(x)\, dx \qquad (3.3)$$

where:

- $A \equiv$ the average distance of individuals from the CBD;
- $P(x) \equiv$ the population at any distance, (x) from the CBD;
- $P \equiv$ the total metropolitan population;
- $\bar{x} \equiv$ the urban boundary;
- $x \equiv$ distance from the CBD.

[3] Hamilton's model assumes that the participation rate does not vary with distance from the CBD. This assumption is necessary so that the average distance of the population from the CBD can be interpreted as the average distance of workers from the CBD. If the participation rate increases with distance from the CBD then "A" will likely underestimate the average distance of workers from the CBD; if the participation rate declines with distance from the CBD "A" will overestimate the average distance of workers from the CBD.

The logic behind equation 3.3 is straightforward. P(x)dx can be interpreted as the number of people in a ring of width dx located x kilometres from the CBD. In a circular city this can be expressed as:

$$P(x)\,dx = D(x)\,2\pi\,x\,dx \tag{3.4}$$

where:

- $D(x) \equiv$ population density x kilometres from the CBD;
- $2\pi \equiv$ the circumference of a circle in radians (2π = 360°).[4]

The right hand side of equation 3.4 can be substituted into equation 3.3 to replace the term P(x)dx yielding:

$$\begin{aligned} A &= \frac{1}{P}\int_0^{\bar{x}} xD(x)\,2\pi x\,dx \\ &= \frac{2\pi}{P}\int_0^{\bar{x}} x^2 D(x)\,dx \end{aligned} \tag{3.5}$$

The total population of the city can be obtained by integrating equation 3.4 from $0 \rightarrow \bar{x}$:

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[4] The circumference can vary from 0 to 360 degrees to account for variations in the land available for development among different cities. Typically $2\pi - \phi \equiv$ the number of radians unavailable for urban development. When $\phi = 2\pi$, 360° are available for development; $\phi = \pi$ implies only 180° are available for development.

$$
\begin{aligned}
P &= \int_0^{\tilde{x}} P(x)\, dx \\
&= \int_0^{\tilde{x}} D(x)\, 2\pi\, x\, dx \\
&= 2\pi \int_0^{\tilde{x}} D(x)\, x\, dx
\end{aligned} \tag{3.6}
$$

All that remains unspecified in equation 3.5 is the functional form of the density function, D(x), and the diameter of the city, $\tilde{x}$.

The majority of urban economic research has assumed a negative exponential population density function (e.g. [17] [18], [28], [38], [42], [43], [45], [58], [59], [61]):[5]

$$D(x) = D_0 e^{-\gamma x} \tag{3.7}$$

where:

- D_o ≡ the population density at the city centre;
- γ ≡ the slope of the population density gradient;
- e ≡ the natural logarithm; and
- x ≡ distance from the CBD.

Substituting equation 3.7 into 3.6 for D(x) yields:

[5] Because Hamilton wanted to test the efficacy of the monocentric model he correctly maintained the simplest assumptions. Most researchers begin with the simple model although they usually modify the basic model somewhat. White [62] and several other researchers subsequently accused Hamilton of creating a "straw man" which Hamilton then proceeds to knock down in the paper estimating wasteful commuting [25], [26].

$$A = \frac{2\pi}{P}\int_0^{\bar{x}} x^2 D_0 e^{-\gamma x}\,dx \\ = \frac{2\pi D_0}{P}\int_0^{\bar{x}} x^2 e^{-\gamma x}\,dx \tag{3.8}$$

Integrating equation 3.8 by parts yields:[6]

$$A = \frac{2}{\gamma} - \frac{2\pi D_0}{\gamma P}\bar{x}^2 e^{-\gamma \bar{x}} \tag{3.9}$$

In order to estimate A, values for D_o, γ and $\bar{x}$ are required. Values for $\bar{x}$ are estimated using equation 3.7 once γ and D_o are known.[7] Estimation of D_o and γ is discussed in section 3.2 of this chapter.

Hamilton interpreted employment decentralization as the potential savings in aggregate commuting arising from jobs moving closer to residences over time. To measure the potential commute savings for each city Hamilton calculated the average distance of jobs from the CBD. This was completely analogous to the calculation of the average distance of people from the CBD described in equations 3.3 to 3.9. The average distance of jobs from the CBD can be written:

---

[6] Appendix 1 provides the detailed integration.

[7] See the section titled **Choosing the Urban Boundary** in Chapter 5 of this thesis for estimates of $\bar{x}$ using different density gradient parameters.

$$B = \frac{1}{J}\int_0^{\bar{x}} xJ(x)\,dx \tag{3.10}$$

where:

- $B \equiv$ the average distance of jobs from the CBD;
- $J(x) \equiv$ employment at any distance from the CBD;
- $J \equiv$ total metropolitan employment;
- $\bar{x} \equiv$ the urban boundary; and
- $x \equiv$ distance from the CBD.

As in the case of population, $J(x)dx$ can be interpreted as the number of jobs in a ring of width dx located at distance x from the CBD. In a circular city this can be expressed as:

$$J(x)\,dx = E(x)\,2\pi\,x\,dx \tag{3.11}$$

where:

- $E(x) \equiv$ employment density x kilometres from the CBD; and
- $2\pi \equiv$ the circumference of a circle in radians ($2\pi = 360^{\circ}$).

The right hand side of equation 3.11 can be substituted into equation 3.10 to replace the term $J(x)dx$ yielding:

$$\begin{aligned} B &= \frac{1}{J}\int_0^{\bar{x}} xE(x)\,2\pi x\,dx \\ &= \frac{2\pi}{J}\int_0^{\bar{x}} x^2 E(x)\,dx \end{aligned} \tag{3.12}$$

Total employment in the city can be obtained by integrating equation 3.11 from $0 \rightarrow \bar{x}$:

$$\begin{aligned} J &= \int_0^{\bar{x}} J(x)\, dx \\ &= \int_0^{\bar{x}} E(x)\, 2\pi x\, dx \\ &= 2\pi \int_0^{\bar{x}} E(x)\, x\, dx \end{aligned} \tag{3.13}$$

Like the population gradient, the employment density gradient, E(x), is assumed to be a negative exponential function:[8]

$$E(x) = E_0 e^{-\delta x} \tag{3.14}$$

where:

- E_0 ≡ the employment density at the city centre;
- δ ≡ the slope of the employment density gradient;
- e ≡ the natural logarithm; and
- x ≡ distance from the CBD.

Substituting equation 3.14 into 3.12 for E(x) yields:

$$\begin{aligned} B &= \frac{2\pi}{J} \int_0^{\bar{x}} x^2 E_0 e^{-\delta x}\, dx \\ &= \frac{2\pi E_0}{J} \int_0^{\bar{x}} x^2 e^{-\delta x}\, dx \end{aligned} \tag{3.15}$$

---

[8] Hamilton's assumption of a negative exponential employment gradient is problematic. While there is both theoretical and some empirical support for a negative exponential population density gradient, there is neither for a negative exponential employment gradient. This issue is discussed in more detail in Chapter 2. Important references include: [10], [11], [19], [21], [27], [33], [41], [50], [51], [55], [56].

Integrating equation 3.15 by parts yields:

$$B = \frac{2}{\delta} - \frac{2\pi E_0}{\delta J}\tilde{x}^2 e^{-\delta\tilde{x}} \tag{3.16}$$

Hamilton defined the difference between A and B is as the minimum possible average commute, given the underlying urban structure as summarized by equations 3.7 and 3.14. Thus optimal commute, (C), is defined:

$$C = A - B \tag{3.17}$$

The parameters D_o, E_o, γ and δ control for differences in the existing distribution of people and jobs within individual cities. Thus the model can be used to answer the question:

> "Given the current distribution of homes and jobs, is observed commuting behaviour consistent with the predictions of the monocentric model?".

In order to answer this question C must be compared with observed commuting behaviour in a sample of cities.

3.1.3 Calculating the Average Random Commute

Once the failure of the monocentric model to predict observed commuting behaviour was confirmed, Hamilton introduced the concept of random commuting in order to illustrate the degree to which the monocentric model failed. Calculating the average random commute, (E), for a city with a finite radius, $\bar{x}$, turns out to be mathematically tedious. The estimate is much less tedious if the mean random commute is calculated for a city without limits. It can be shown that allowing the city boundary to approach ∞ imparts only a small upward bias in the estimate of random commuting, (E). Both procedures are described in an appendix to Hamilton's paper [26]. Only the infinite boundary estimate is described here and later utilized in Chapter 5.

To begin, suppose that households and jobs are distributed throughout the city according to equations 3.7 and 3.14. The city is assumed to be an everywhere dense network or radial roads and beltways such that every household is located at an intersection. Commuters take the shortest route to work. The city is circular with $2\pi - \phi$ radians unavailable for development. Homes and jobs are distributed according to the population and employment density functions given in equations 3.7 and 3.14. By integrating twice over homes and jobs the average one way commute is defined as:

$$E = \frac{2(\gamma + \delta)}{\gamma\delta} + 2M\left[\frac{\gamma^2 + 3\gamma\delta}{(\gamma + \delta)^2}\right] \qquad \textbf{(3.18)}$$

where:

E ≡ the average one way random commute for a city without limits; and

M ≡ a function of ϕ such that:

$$\begin{aligned} \phi \le 2 \quad & M = \left(\frac{\phi}{3}\right) - 2 \\ 2 \le \phi < 2\pi \quad & M = \left[\frac{\frac{8}{3}}{\phi^2}\right] - \left(\frac{4}{\phi}\right) \\ \phi = 2\pi \quad & M = \frac{-2}{\pi} \end{aligned} \tag{3.19}$$

Basically, the procedure specifies a distance function and integrates that function over house and job locations then divides by the total number of commuters. The assumption of an everywhere dense network of roads imparts a small downward bias into estimates of E.

3.2 Two-Point Estimates of Employment and Population Density Gradients

In section 3.1 Hamilton's method for measuring the minimum possible average commute in any city was described. Recall that the minimum possible average commute, (C), was defined as the difference between the average distance of homes from the CBD, (A), and the average distance of jobs from the CBD, (B). In order to solve equations 3.9 and 3.16 (for A and B) Hamilton required estimates of D_o, γ, E_o and δ.

There are two different methods available for estimating the density gradient parameters in equations 3.7 and 3.14.[9] Both methods assume density, D(x), declines exponentially with distance,

[9] Each broad method has a number of variants but, essentially, all are subsets of the two methods described below.

x, from a predetermined city centre.[10]

Muth [45] suggested taking the natural log of both sides of equation 3.7 or 3.14 and estimating γ or δ using OLS:

$$\ln D(x) = \ln D_0 - \gamma x \tag{3.20}$$

For each city estimated using the Muth method, data for D(x) and x must be collected at the census tract level.

Rather than estimate equation 3.20, Hamilton used values for γ, D_o, E_o and δ calculated by Macauley [38]. Macauley in turn used a modified version of Mills' two-point estimation technique.

In his pioneering work, Studies in the Structure of the Urban Economy, [43], Mills developed the two-point estimation technique. Two-point estimation is actually a misnomer since the technique involves the use of two large integrals.

---

[10] See Chapter 2 for the implicit assumptions embedded in a negative exponential gradient. It is important to disentangle two separate but related issues. Questions regarding the appropriateness of the two-point technique as an estimator of γ and D_o are distinct from questions about the use of the negative exponential function as a descriptor of population or employment density. The latter is a fundamental debate in urban economics while the former issue is a methodological wrangle. Both Mills' two-point method and Muth's regression-based alternative proceed under the assumption $D(x)=D_oe^{-\gamma x}$ is an accurate descriptor of urban density. Papers contributing to the debate surrounding the appropriateness of the negative exponential density function include Brueckner [8], Griffith [25], Harrison and Kain [28], Kau, Lee and Chen [31], Kau and Lee [32], Kim and McDonald [34] and Hamilton and Mills [42]. Kau and Lee, for example, found that 6 of the same 14 SMSAs used by Hamilton [26] were not well characterized by a negative exponential density function. Kau and Lee obtained their result using a Box-Cox test for functional form.

The logic behind Mills method is straightforward.[11] Assume that equation 3.7 is an accurate depiction of population density in a particular city and that the metropolitan area is circular with $2\pi - \phi$ radians unavailable for development.[12] [13] In such a CMA, the number of people, n(x), in a ring of width dx, located x miles from the city would be:

$$n(x)\,dx = D(x)\,\phi x\,dx \tag{3.21}$$

The total population, N(k), within k miles of the CBD is simply the integral of equation 3.21 from 0 → k:

$$N(k) = \int_0^k n(x)\,dx. \tag{3.22}$$

Substituting the right hand side of equation 3.21 into 3.22 gives:

$$\begin{aligned} N(k) &= \int_0^k D(x)\,\phi x\,dx \\ &= \int_0^k D_0 e^{-\gamma x}\,\phi x\,dx \\ &= D_0\phi \int_0^k e^{-\gamma x}\,x\,dx. \end{aligned} \tag{3.23}$$

Integration of equation (3.23) yields:

---

[11] For greater detail see Mills [42] (Chapter 3) or Edmonston [16], [17].

[12] In equation 3.7, D_0 represents density at the city centre and γ is a measure of the rate at which density declines with distance from the CBD. If γ is large density falls off rapidly; if γ is small density falls off slowly.

[13] The implicit circularity of this assumption was modified by Edmonston [17]. See section 3.2.3 of this thesis for details.

$$N(k) = \frac{\Phi D_0}{\gamma^2}\,[1-(1+\gamma k)\,e^{-\gamma k}]\,. \tag{3.24}$$

where:

- $N(k) \equiv$ total population from $0 \rightarrow k$;
- $\phi \equiv$ radians of land available for development;
- $k \equiv$ the radius of the metropolitan area;
- $e \equiv$ the natural logarithm; and
- D_o, $\gamma \equiv$ parameters to be estimated.

The basic insight provided by Mills was recognizing that equation 3.7 could be estimated with only two observations provided by the central city - suburb data. The estimation technique is much less onerous than alternative techniques which use large samples of census tract data. The following quotation from Mills summarizes the elegance of the Mills method:

> "The basic insight exploited in the present chapter is that if the negative exponential density function is an accurate representation, its estimation does not depend on where the central city boundary is drawn or on whether its location changes over time. Furthermore, since it is a two-parameter family of curves, it can be estimated with the two observations provided by the cental city - suburb data.
>
> It must be emphasized that the cental city - suburb data are not merely a sample of two observations. They provide two exhaustive and exclusive integrals of the density function and thus make use of the entire population of data. There is no reason to believe that they provide less accurate estimates than would a large sample of census tract observations." [pg. 35, 43].

In contrast to Hamilton's work, there were no recent estimates of the population and employment density gradient parameters for a substantial number Canadian CMAs that could be used for this thesis. Thus, it was necessary to solve equation 3.24 for both the population and employment density gradients in each Canadian city. As presented above equation 3.24 is not solvable because it is a non-linear equation with two unknown parameters (D_o and γ). Three separate methods for solving 3.24 are presented below.

3.2.1 The Mills Estimation Technique

To solve equation 3.24 Mills noted that letting $k \rightarrow \infty$ implies:

$$\frac{\phi D_0}{\gamma^2} = N \tag{3.25}$$

where $N \equiv$ total metropolitan population. Substituting N into equation 3.24 yields:

$$N(\hat{k}) = N[1-(1+\gamma\hat{k})\,e^{-\gamma x}]\ . \tag{3.26}$$

where:

- $N(\hat{k}) \equiv$ central city population;
- $N \equiv$ SMSA (CMA) population;
- $\hat{k} \equiv$ radius of the central city;
- $\gamma \equiv$ the parameter to be estimated.

Equation 3.26 is a non-linear equation in one unknown, γ. Non-linear equations are solved using numerical methods. Once γ is estimated it can be substituted into 3.25 to calculate D_o.

Data for N, $N(\hat{k})$ and $\hat{k}$ for each of 23 CMAs was obtained from the 1981 Census of Canada and equation 3.26 was solved iteratively for

γ.[14] Estimates of γ are then substituted into equation 3.25 to solve for D_o.

Rearranging 3.25 to isolate D_o yields:

$$D_0 = \frac{N\gamma^2}{\phi} \tag{3.27}$$

In order to solve for D_o an estimate of ϕ is required. Mills solved for ϕ by assuming that each city was (semi)circular and then equated the known area of the city to the appropriate value of ϕ.[15] If we define $2\pi - \phi$ as the radians excluded from development in each city, when $2\pi = \phi$ the entire circle is available for development. The area of a circle, (A_c), is defined as:

$$A_c = \pi r^2 \tag{3.28}$$

Rearranging the expression $2\pi = \phi$ to isolate π implies that $\pi = \phi/2$. Substituting into equation 3.28 for π and isolating ϕ yields:

$$\phi = 2\left[\frac{A_c}{r^2}\right] \tag{3.29}$$

Data for the area of the city, (A_c), and the radius were obtained from the 1981 Census of Canada. ϕ was estimated from equation 3.29

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[14] The algorithm employed is described below in Section 3.3.3.

[15] Recall that the circumference of a circle is equal to 2π (360 degrees).

and then substituted, with γ, into 3.27 in order to solve for D_o.

Finally, it is necessary to calculate $\hat{k}$, the distance at which population density declines to 100 people per square mile (or 36.61 per square kilometre).[16] γ and D_o were substituted into equation 3.7 and D(x) was set equal to 36.61 persons per square kilometre. The non-linear equation was then solved for $\hat{k}$ using numerical methods.

The procedure used to estimate the employment density gradient parameters, E_o and δ, was similar to the procedure described for population density gradient parameter estimates. Employment was substituted for population in equations 3.24 to 3.29.

3.2.2 The Macauley Estimation Technique

Macauley modified Mills' technique in order to remove a slight upward bias in Mills' estimate of γ and δ. Mills' assumption that the urban boundary was infinite was a simplification which allowed him to solve equation 3.24. Mills recognized that allowing $k \rightarrow \infty$ would impart a slight upward bias to estimates of γ and D_o but he conjectured that the bias would be small.[17]

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[16] The population density figure of 100 people per square mile was chosen arbitrarily by Hamilton, as the density dividing rural from urban.

[17] The upward bias in the Mills method implies that his estimates of γ and δ underestimate the suburbanization of homes and jobs, respectively. This was shown to be the case for the 1981 Canadian estimates included in Chapter 4 of this thesis. Estimates using Mills' technique yielded values for γ and δ that were slightly greater than values obtained with Macauley's method. The

The method used by Macauley is identical to Mills' method up to equation 3.24. Rather than solve equation 3.24 by letting $k \rightarrow \infty$ Macauley, [38], avoided the Mills bias by estimating two separate equations that are similar in form to 3.24: one for the central city and one for the metropolitan area.

The first step in Macauley's method is to isolate D_0 in one of the two equations:

$$D_0 = \frac{N(k_2)\,\gamma^2}{\phi\left[1 - (1 + \gamma k_2)\, e^{-\gamma k_2}\right]} \tag{3.30}$$

D_0 is then substituted back into the second equation. The resulting expression is the ratio of metropolitan area to central city population:

$$\frac{N(k_1)}{N(k_2)} = \frac{[1 - (1 + \gamma k_1)\, e^{-\gamma k_1}]}{[1 - (1 + \gamma k_2)\, e^{-\gamma k_2}]} \tag{3.31}$$

where:

$N(k_1) \equiv$ metropolitan area population;

$N(k_2) \equiv$ central city population;

$k_1 \equiv$ metropolitan radius; and

$k_2 \equiv$ central city radius.

---

difference between Mills and Macauley estimates was not large nor was it statistically significant.

Macauley [38] also confirmed Mills' conjecture. Macauley's estimates using both Mills' original and corrected techniques indicated that the original estimates of γ and δ were only slightly larger than corrected estimates.

To estimate 3.31 data for $N(k_1)$, $N(k_2)$, k_1 and k_2 were obtained from the 1981 Census of Canada. Equation 3.31 is a non-linear equation in one unknown (γ) and was solved using numerical methods. Estimates of γ were substituted back into 3.30 to solve for D_o. ϕ was estimated as in the Mills technique.

3.2.3 The Edmonston Estimation Technique

Both Mills' and Macauley's estimates of γ, D_o, δ and E_o implicitly assume that all cities are (semi) circular. Irregularly shaped cities were excluded by Mills [43] and Macauley [38] in an ad hoc fashion. Because of the limited number of cities in the Canadian urban system, removal of irregularly shaped cities was deemed impractical. Furthermore neither Mills nor Macauley provide criteria for evaluating which cities are irregularly shaped and should, thus, be excluded from the sample.

Edmonston [17] developed a technique that attempts to overcome this the need to exclude irregularly shaped cities from the Mills model.[18] Edmonston incorporated a method that measures the average distance to the urban boundary from the CBD, for a variety of different city shapes. Like Macauley, Edmonston solved for γ in equation 3.31. However, in order to implement Macauley's model k_1

[18] The Mills estimation procedure is used to describe both his original and corrected technique. Mills developed the technique so it seems appropriate to refer to it as the Mills technique. However, Macauley provided the corrected estimates that Hamilton used in his estimates of wasteful commuting.

and k_2 were measured from census maps using a large compass.[19]

Rather than measure k_1 and k_2 directly Edmonston suggested coding each metropolitan area's shape based on its current political (and therefore, statistical) boundaries. Possible shapes included:

1. Circle
2. Square
3. Hexagon
4. Rectangles of various length to width ratios:

   1 x 2

   1 x 4

   1 x 6

   1 x 8

   1 x 10.

Given the metropolitan (central city) land area, the coded shape and the assumption that the CBD is centred within the region, k_1 (k_2) can be estimated as the average distance to the boundary. For example, if a city is circular the area is given by equation 3.28. Rearranging 3.28 to isolate r yields:[20]

$$\begin{aligned} r &= \sqrt{\frac{A_c}{\pi}} \\ &= (0.5642)\sqrt{A_c} \end{aligned} \tag{3.32}$$

---

[19] Edmonston described this method of measuring the central city and metropolitan area radius as "time consuming and not always as accurate as desired." [17].

[20] The symbol, r is being used to denote the radius or average distance to the urban boundary. Using the notation of equation 3.31, $r \equiv k_1$ or k_2 for the CMA and central city, respectively.

The average distance to the boundary for other shapes was calculated in a similar manner, based on the formula for the area of each shape. Table 3.1 summarizes the derivation of r (i.e. k_1 or k_2) for eight stylized city shapes.

Equation 3.31 requires a value for both k_1 and k_2 so both the CMA and central city shape had to be coded. In the majority of cases the shape was similar for both the central city and the CMA. The complete data set is discussed in Section 3.3. With Edmonston's method it was still necessary to estimate ϕ in order to solve 3.30 for D_o. Edmonston suggested using a protractor and census maps to determine the radians available for urban development.

Table 3.1: Radius Calculation Based on Eight City Shapes

SHAPE		Calculated k_1 and k_2
Circle		$r = (0.5642)\sqrt{A_c}$
Square		$r = (0.5611)\sqrt{A_c}$
Hexagon		$r = (0.5629)\sqrt{A_c}$
Rectangle		
	1 x 2	$r = (0.5416)\sqrt{A_c}$
	1 x 4	$r = (0.4909)\sqrt{A_c}$
	1 x 6	$r = (0.4523)\sqrt{A_c}$
	1 x 8	$r = (0.4247)\sqrt{A_c}$
	1 x 10	$r = (0.4023)\sqrt{A_c}$

Source: Edmonston [17].

It is obvious from Table 3.1 that the Mills' assumption of circularity is not likely to substantially impact gradient estimates for cities coded as square or hexagonal. Rectangular cities, particularly those with length to width ratio exceeding 1:2, are more likely to be affected by the assumption of circularity. Using the 1981 census boundaries, cities with width to length ratios exceeding 1:2 included Kitchener, Saint John, St. John's, Thunder Bay, Victoria and Winnipeg.

3.2.4 Two-Point Versus OLS Density Gradient Estimation

Muth's regression method (equation 3.20) has been widely applied (e.g. Alperovich [4], [5] and Muth [45]), to estimate population density gradients. The paucity of employment data at sub-metropolitan geographic levels has precluded OLS estimation of employment gradients using Muth's method. Hamilton's [26] remark regarding Muth's method illustrates the tacit belief present in much of the urban economics literature:

> "There is a widespread view that the Mills' two-point estimates of density gradients are inherently inferior to the Muth technique of regressing log density on distance" [26].

White [60] examined carefully the legitimacy of the prevailing attitude toward two-point estimation techniques. White argued that the bias in favour of OLS arises, in part, because the statistical properties of the OLS estimator are well understood, in contrast to the two-point estimate's unknown distributional properties.

There are a number of concerns regarding OLS estimates of γ and mounting empirical evidence to suggest that "researchers using two-point estimates need not apologize profusely" [60]. While it is often noted that the two-point method ignores potential information, Hamilton argued the converse is also true: OLS ignores prior information by not constraining the integral of the estimated density function to "add up" to the actual metropolitan area population. Indeed the "thrust of Mills' method is to note that there is only one parameter vector which satisfies the prior information for an exponential density gradient" [pg. 1045, 26].[21] McDonald and Bowman [40] found constrained OLS yielded estimates of γ that were steeper than ordinary OLS estimates.[22] McDonald and Bowman's result is consistent with other empirical work suggesting that Mills two-point estimates are, on average, steeper than Muth's regression estimates [26], [28], [42], [43].

A second difficulty with OLS estimates of γ is a potentially severe upward bias generated by a systematic relationship between census tract geographic areas and population density at any given distance from the city centre [6], [20]. The relationship arises because sparsely populated city districts are consolidated to a greater degree than densely populated districts, in order to form census tracts of approximately uniform population. Thus, sparsely populated districts tend to be under-represented in a random sample of census tracts [20].[23] Frankena suggested using weighted least

[21] See Hamilton's [26] footnote #12 for greater detail.

[22] Constrained estimation forces the integral to "add up" to total population. See McDonald and Bowman for more detail on the constrained estimation procedure employed.

[23] The relevant point is that the data required to estimate γ are available by census tract, not city district.

squares (WLS) to correct the bias, and Anderson [6] employed a Box-Cox transformation of the dependant variable, D(x), in order to determine the proper weights for WLS estimation.

White [60] noted a third, often overlooked, limitation of OLS estimates of population density gradients. While OLS estimates of γ are BLUE (Best Linear Unbiased Estimates), when the standard assumptions regarding the error structure of equation 3.7 are made, estimates of D_o are biased. OLS estimates of ln D_o are BLUE but, under the natural log transformation, estimates of D_o are not. This limitation is usually overlooked because most researchers have been more interested in the gradient parameter, γ, than the intercept parameter, D_o.

White's [60] paper provides perhaps the strongest endorsement of two-point estimation. Given the impossibility of determining the distributional properties of two-point estimators White instead conducted a Monte Carlo simulation experiment. An artificial city was created, in which both the density gradient and the random error term applied to the population density of each metropolitan district was known.[24] White then compared the ability of OLS and two-point estimators to predict the "true" γ.

White had no prior information regarding the correct error structure for equation 3.7 Assuming a multiplicative error structure ensured that OLS estimates were BLUE and thus superior to

[24] See White [60], (pg. 298-299), for greater methodological detail.

two-point estimates. However, the magnitude of the difference between OLS and two-point estimates of γ was less than 5 percent. When White assumed an additive error structure, OLS estimates were inferior to two-point estimates. White concluded that, overall, two-point estimates performed as well as OLS estimates. Two-point estimates of γ appeared to be biased downward no more than 4-5 percent, while the bias in D_0 depended on the assumed error structure (multiplicative versus additive).[25]

For this thesis two-point estimates were believed appropriate estimators for γ, D_0, δ and E_0 for four reasons:

1. Hamilton used two-point estimates and one of the primary goals was to compare my results with Hamilton's results;
2. The evidence reviewed above suggested two-point estimates are at least as good as OLS estimates;
3. The data requirements for two-point estimates were far less daunting than for regression estimates; and
4. The data necessary for OLS estimates of δ and E_0 are not available.

3.3 Data Sources

Data for this study came from a variety of sources. Employment and population density gradient parameters were estimated for twenty-two Census Metropolitan Areas (CMAs) and for one Census

[25] Another of White's results is particularly encouraging. White estimated γ using a variety of central city-suburban population splits. He found γ was not sensitive to the particular split except for extreme cases. When the central city had ≤ 5 percent or ≥ 80 percent of the total metropolitan area population the variability of two-point estimates increased dramatically (pg. 303), [60].

Agglomeration (CA). Figure 3.2 ranks the CMAs included in this study according to their 1981 population. Figure 3.3 lists the abbreviations for each CMA used throughout the remainder of this thesis.

In order to estimate wasteful commuting the following data were required:

- Total CMA population;
- Central city population;[26]
- Total CMA employment;
- Central city employment;
- Central city land area;
- Land available for urban development (ϕ);[27]
- Radius of the CMA, (k_1);
- Radius of the central city, (k_2);
- Shape of the CMA and cental city; and
- Observed commuting, (D).

Population, employment and land area for each CMA and central city was obtained from the 1981 Census of Canada (Table 3.2). The 1981 Census represents the first time that Statistics Canada tabulated employment by place of work, as well as place of residence. Thus, 1981 represents the first opportunity to estimate employment

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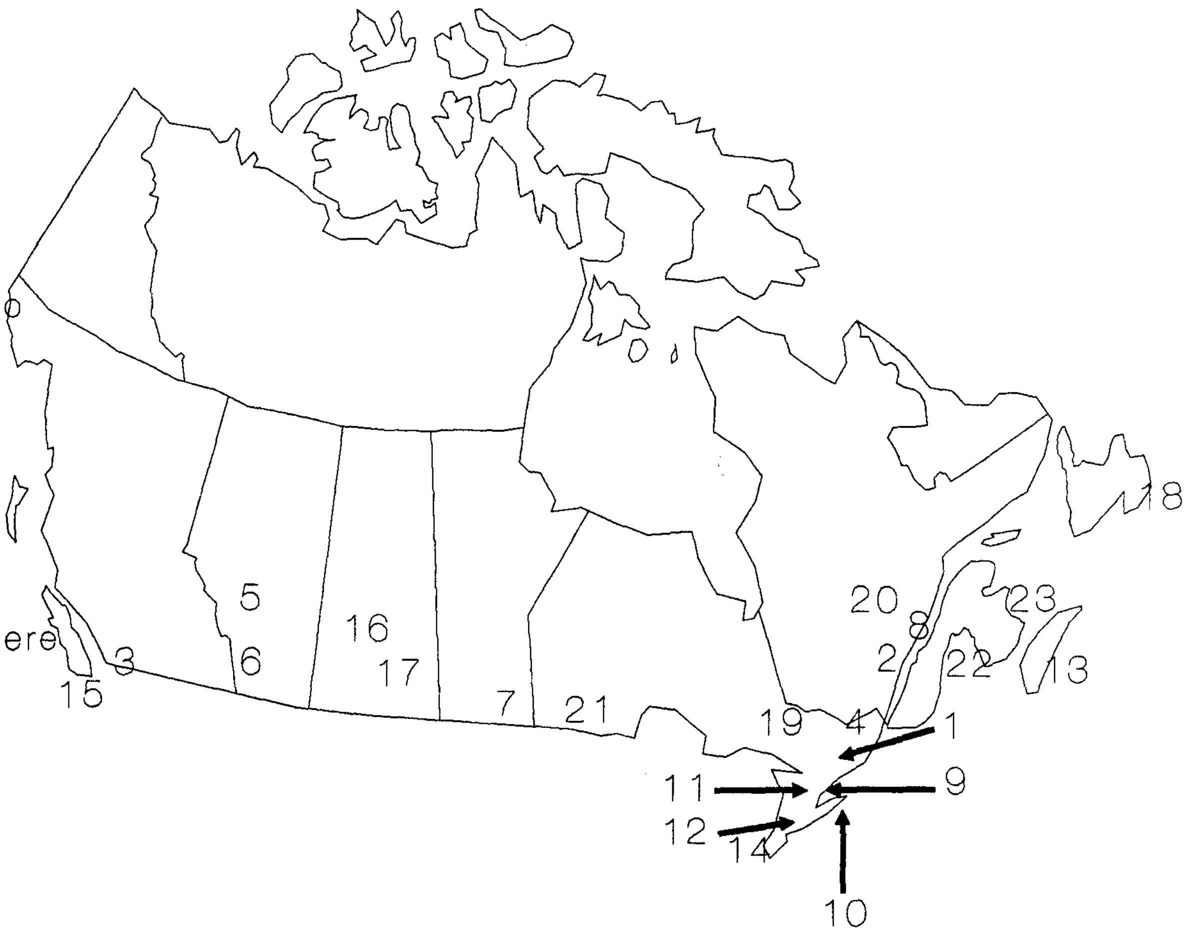
[26] Taking the case of the Vancouver CMA as an example implies the central city would include data for the City of Vancouver only, while the metropolitan area includes Surrey, Richmond, Burnaby etc.

[27] $(2\pi - \phi)$ radians are unavailable.

FIGURE 3.2
LOCATION OF CMAs RANKED BY 1981 POPULATION

1	Toronto
2	Montreal
3	Vancouver
4	Ottawa-Hull
5	Edmonton
6	Calgary
7	Winnipeg
8	Quebec City
9	Hamilton
10	St Catherines
11	Kitchener-Waterloo
12	London
13	Halifax
14	Windsor
15	Victoria
16	Saskatoon
17	Regina
18	Saint John's
19	Sudbury
20	Chicoutimi-Jonquiere
21	Thunder Bay
22	Saint John
23	Charlottetown



Source: 1981 Census of Canada

FIGURE 3.3
CMA Abbreviations

Abbreviation	CMA
Tor	*Toronto*
Mtl	*Montreal*
Van	*Vancouver*
Ott	*Ottawa*
Edm	*Edmonton*
Cgy	*Calgary*
Wpg	*Winnipeg*
Que	*Quebec City*
Ham	*Hamilton*
SC	*Saint Catherines-Niagara*
KW	*Kitchener-Waterloo*
Lon	*London*
Hal	*Halifax*
Wsr	*Windsor*
Vic	*Victoria*
Sas	*Saskatoon*
Reg	*Regina*
SJs	*Saint John's*
Sud	*Sudbury*
Chi	*Chicoutimi-Jonquiere*
TB	*Thunder Bay*
SJ	*Saint John*
Cha	*Charlottetown*

Table 3.2: Population and Employment Data Used to Estimate Density Gradients

CMA	Population			Employment			Area	
	CMA	CC	CC Share	CMA	CC	CC Share	CC	CMA
Calgary	625,966	592,743	94.7%¶	325,205	300,310	92.3%¶	504.96	5055.96
Charlottetown	44,999	15,282	34.0%	18,380	13,460	73.2%	6.99	550.73
Chicoutimi	135,172	55,465	41.0%	44,715	40,845	91.3%¶	147.42	1132.54
Edmonton	657,057	532,246	81.0%	339,075	289,035	85.2%	321.65	4142.51
Halifax	277,727	176,871	63.7%	127,660	107,180	84.0%	120.92	2508.10
Hamilton	542,095	306,434	56.5%	228,435	153,545	67.2%	122.82	1358.50
Kitchener	287,081	189,162	65.9%	132,825	87,460	65.8%	199.79	823.64
London	283,668	254,280	89.6%¶	131,955	116,675	88.4%¶	162.28	1601.91
Montreal	2,828,349	980,354	34.7%	1,265,055	615,180	48.6%	158.31	2814.43
Ottawa	717,978	351,388	48.9%	347,975	259,675	74.6%	139.44	3998.03
Quebec	576,075	166,474	28.9%	237,260	103,485	43.6%	89.05	2817.96
Regina	173,226	162,613	93.9%¶	79,565	74,605	93.8%¶	109.83	3421.58
Saint John	114,048	73,389	64.3%	45,865	40,795	88.9%¶	322.71	1476.08
Saskatoon	175,058	154,210	88.1%¶	71,730	67,395	94.0%¶	122.04	4749.30
St. Catherines	304,353	124,018	40.7%	126,110	78,975	62.6%	94.43	1068.07
St John's	154,820	83,770	54.1%	60,460	51,370	85.0%	35.10	1127.47
Sudbury	149,923	89,773	59.9%	58,180	45,630	78.4%	262.73	2379.84
Thunder Bay	121,379	109,365	90.1%¶	54,000	50,785	94.0%	323.46	2032.38
Toronto	2,998,947	599,217	20.0%	1,571,455	545,160	34.7%	97.15	3742.94
Vancouver	1,268,183	414,281	32.7%	646,435	276,215	42.7%	113.13	2786.22
Victoria	233,481	64,379	27.6%	105,130	60,820	57.9%	18.78	488.52
Windsor	246,110	192,083	78.0%	96,080	84,975	88.4%¶	119.76	768.87
Winnipeg	584,842	562,059	96.1%¶	284,785	268,275	94.2%¶	571.60	2310.03
Mean	586,980	271,733	60.2%	278,188	162,254	75.2%	181.06	2311.11

Source: 1981 Census of Canada.

¶ ≡ a CMA with an extreme population or employment split. This is likely to make estimates of the density gradient parameters less reliable according to Edmonston, Goldberg and Mercer [18] and White [60].

density gradients for a large sample of Canadian cities.[28]

Section 3.2 described differences between the population and employment gradient estimation techniques of Mills and Edmonston. The differences revolved around measurement of ϕ and k_1 and k_2. The Mills estimates of ϕ, described in equation 3.29, are given in Table 3.3.[29] Of course, in order to estimate ϕ using equation 3.29, an estimate of the CMA radius was required. The "measured radius" in Table 3.3 was obtained from 1981 Census maps using a large compass.

In contrast to the Mills method, Edmonston devised a system to calculate the radius of each CMA based on the shape of the CMA. Table 3.3 presents the coded shape of each city and the implied "calculated radius" for each CMA.[30] On average, both the measured and calculated radii are substantially greater than the values of F (where population density is estimated to decline to 100 people per square mile) presented in Chapters 4 and 5.

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[28] Unfortunately the published census data does not permit estimation of employment gradients by industry or occupation. Disaggregated gradients would be of interest to determine which jobs are most suburbanized.

[29] The figures in Table 3.3 have been multiplied through by π. Thus a figure of 6.28319 (i.e. 2π) represents a complete circle of 360 degrees available for development. Remember $2\pi - \phi$ was defined as the area of land **NOT** available for development.

[30] Table 3.3 also presents the "circle radius" for each CMA. This was calculated based on land area published in the census and assuming that the city was a complete circle of 360 degrees. Cities where this figure is substantially different from the other radius measures are likely cities that are not well characterized as a circle.

Table 3.3: Estimation of CMA Radius and the Parameter ϕ

CMA	Shape		ϕ		Radius		
	CMA	CC	Mills	Edmonston	Calculated	Measured	Circle
Calgary	1 x 2	1 x 2	6.25722	6.28319	40.2	38.5	40.1
Charlottetown	Square	1 x 2	4.30258	5.06145	16.0	12.7	13.2
Chicoutimi	1 x 2	1 x 4	5.03978	6.28319	21.2	16.5	19.0
Edmonton	1 x 2	Square	5.79842	6.28319	37.8	36.1	36.3
Halifax	1 x 2	1 x 2	4.21441	3.66519	34.5	27.1	28.3
Hamilton	Hexagon	Square	5.04793	4.88692	23.2	20.7	20.8
Kitchener	1 x 3	Hexagon	5.83645	6.28319	16.8	15.5	16.2
London	1 x 2	Hexagon	5.80139	6.28319	23.5	21.7	22.6
Montreal	Hexagon	1 x 2	5.49693	6.28319	32.0	29.9	29.9
Ottawa	Square	Hexagon	5.84080	6.28319	37.0	31.0	35.7
Quebec	Square	1 x 2	5.57328	6.28319	31.8	28.8	29.9
Regina	1 x 2	Square	5.91969	6.28319	34.0	31.7	33.0
Saint John	1 x 3	1 x 2	3.90368	3.75246	27.5	20.8	21.7
Saskatoon	Square	Square	5.99644	6.28319	39.8	38.7	38.9
St Catherines	Square	Square	3.86807	2.35619	23.5	17.7	18.4
St John's	1 x 4	1 x 2	3.46780	2.61799	25.5	16.5	18.9
Sudbury	Square	1 x 2	5.65955	6.28319	29.0	26.4	27.5
Thunder Bay	1 x 3	1 x 2	4.51640	4.01426	30.0	22.1	25.4
Toronto	1 x 2	1 x 2	4.14443	3.49066	42.5	34.3	34.5
Vancouver	1 x 2	1 x 2	3.66367	2.87979	39.0	29.6	29.8
Victoria	1 x 4	Square	3.19033	3.05433	17.5	12.0	12.5
Windsor	1 x 2	1 x 2	3.84435	2.96706	20.0	13.6	15.6
Winnipeg	1 x 3	Hexagon	5.68798	6.28319	28.5	23.4	27.1
Mean			4.91616	4.96281	29.2	24.6	25.9

Two sources of data provided information on observed commuting behaviour for a sample of Canadian cities:

- The Urban Concerns Survey (1978);
- The Vehicle Survey Data (1975).

3.3.1 The Urban Concerns Survey (UCS)

In 1978 Canada Mortgage and Housing (CMHC), and the Ministry of State for Urban Affairs (MSUA) conducted a survey of 11,061 households located in urban areas with population greater than 100,000. The survey covered all 23 cities listed in Figure 3.2 and included a question on the distance to work and the time it took to travel the given distance. The survey used a stratified random sample to ensure broad geographic coverage within metropolitan areas. The sample was stratified in each CMA on the following location criteria:

- Central city;
- Mature suburbs;
- New Suburbs;
- Exurban;

The strata were of different size for each CMA based upon the population distribution recorded by the 1976 Census. Nationally, the sample distribution was: central city 28 percent, mature suburbs 29 percent, new suburbs 30 percent and exurban 10 percent. Not all strata types were present in every city. Some CMAs had no exurban observations, for example, while Charlottetown was not

Table 3.4: Summary of Observed Commuting Data

	One Way Commute				Ave Speed		Sample Size	
	UCS	VS	UCS	VS	UCS	VS	UCS	VS
	kilometres		minutes		km per hour		#	#
Calgary	10.64	12.28	23.9	20.1	26.7	36.7	152	528
Charlottetown	4.22		10.5		24.1		230	
Chicoutimi	8.30		14.0		35.6		150	
Edmonton	10.49	12.36	25.4	19.0	24.8	39.0	207	503
Halifax	7.97		19.0		25.2		203	
Hamilton	15.63		22.5		41.7		175	
Kitchener	12.09		17.6		41.2		184	
London	8.42		18.0		28.1		146	
Montreal	12.23	13.32	24.6	22.9	29.8	34.9	300	1,021
Ottawa	10.40	10.89	21.6	18.5	28.9	35.3	269	464
Quebec	11.28	11.49	21.2	16.1	31.9	42.8	227	74
Regina	8.53		17.4		29.4		181	
St. John	11.10		17.1		39.0		160	
Saskatoon	7.95		16.0		29.8		222	
St. Catherine	10.59		15.9		40.0		122	
St. John's	8.19		20.3		24.2		246	
Sudbury	13.61		18.0		45.4		132	
Thunder Bay	10.12		17.1		35.5		116	
Toronto	12.25	15.27	25.2	24.2	29.2	37.9	318	952
Vancouver	13.57	14.11	24.0	21.0	33.9	40.3	206	846
Victoria	10.64		20.6		31.0		205	
Windsor	10.17		16.1		37.9		167	
Winnipeg	11.09		24.1		27.6		217	
Observations							4,535	4,388
Mean	10.41		19.6		32.2		197	
Sub-sample	11.55	12.82	23.7	20.3	29.3	38.1	240	627

Notes: UCS ≡ Urban Concerns Survey; VS ≡ Vehicle Survey; Sub-sample ≡ the mean values for the seven city sub-sample of the UCS that allows for comparison of the UCS and VS.

stratified at all. Most CMAs had some of each strata type although proportions were often different from the national share.

A summary of observed commuting data is given in Table 3.4. The total number of responses that included a useable answer for the commuting distance question was 4,535. The average number of responses for each CMA was only 197. From this data the average commute in 1978 was estimated to be 10.4 kilometres. The average commuting speed was 32 kilometres per hour. This includes information for all modes of transit.

3.3.2 Vehicle Survey (VS)

A second source of observed commuting data was obtained from a vehicle survey conducted by a private consulting company for Environment Canada in 1975. The survey consisted of 7,838 observations. In this case each vehicle represented an observation so the number of separate households was approximately 5,800. The survey only included seven cities: Calgary, Edmonton, Montreal, Ottawa, Quebec City, Toronto and Vancouver.

A summary of observed commuting data from the vehicle survey is also included in Table 3.4. The average observed commute was 12.8 kilometres or nearly 1.3 kilometres more than the average obtained from the urban concerns survey for the same seven CMAs (sub-sample mean in Table 3.4). Velocity was more than 9 kilometres per hour greater in the vehicle survey. The differences are not surprising considering that the urban concerns survey includes all types of transit while the vehicle survey included only the primary automobile in each household. The vehicle survey probably over sampled inner city locations because it was not geographically stratified.

Given these considerations the results of the vehicle survey were considered as confirmation of the veracity of observed commuting data from the urban concerns survey. To the extent that the urban concerns survey may be an under-estimate of observed commuting distance the estimates of wasteful commuting in Chapter 5 will be biased downward and the Hamilton model will be biased in favour of finding the monocentric model acceptable.

3.3.3 Estimating the Zeros of Non-Linear Equations

Mills [43] employed a Newton-Raphson algorithm to solve for γ in equation 3.26.[31] This thesis employed two alternative algorithms to solve for the zeros of the non-linear equations:

- DRZFUN based on Muller's method; and
- ZERO1 based on Bus and Dekker's method.

Two methods were used in order to check for consistency. The first solution method was based on an algorithm developed by Muller [44]. It is a double precision analogue and had the advantage of not requiring initial estimates of the roots. Using this method single roots are generally accurate to within five significant figures and

[31] Some researchers have had problems with convergence using the Newton-Raphson algorithm.

converge within 20 or 30 iterations.

The second method proceeds iteratively by a method combining linear interpolation, rational interpolation and bisection based on the work of Bus and Dekker. The method will find one root of the specified function but the root must lie within a user specified interval. Both methods produced estimates of γ and δ that were identical when reasonable starting intervals were used for the Bus and Dekker algorithm.

3.4 Summary

This chapter reviewed a method for estimating wasteful commuting developed by Hamilton [25], [26]. The model estimates the average distance of people and jobs from the CBD using the parameters from negative exponential employment and population density gradients. The difference between the average distance of homes and jobs from the CBD is interpreted as the optimum average commute. This is then compared with observed commuting for a sample of cities.

Hamilton employed density gradient parameters estimated by Macauley [38] to operationalize his model. There were no readily available gradient estimates for a large sample of Canadian CMAs so gradients had to be estimated for this thesis. Three variations of the two-point technique developed by Mills', were discussed in Section 3.2. Section 3.3 described the data required to estimate the model including two sources of observed commuting data.

CHAPTER 4

DENSITY GRADIENT ESTIMATES

This chapter presents estimates of population and employment density gradients for 22 Canadian Census Metropolitan Areas (CMA) and one Census Agglomeration (CA).[1] The key inputs into the model developed by Hamilton [26] are the parameters from each city's employment and population density gradient. Hamilton [26] used estimates of D_o, E_o, γ and δ provided by Macauley [38] who, in turn, employed a modified version of Mills' [43] two-point technique to estimate the four gradient parameters.

There are four sections in this chapter. The first section reviews previous two-point estimates for several different sets of cities. The second section presents estimates of population density gradients for 23 Canadian cities in 1981. Section 4.3 summarizes 1981 employment gradient estimates for the same 23 cities. The final section summarizes the chapter.

4.1 Previous Two-Point Density Gradient Estimates

In his pioneering work Studies in the Structure of the Urban Economy [43] Mills estimated 360 density functions for 18 metropolitan areas in the United States.[2] Cities with irregularly

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[1] Charlottetown was included despite its status as a CA for two reasons: (i) observed commuting data were available for Charlottetown; and (ii) to ensure complete provincial coverage.

[2] Estimates were made for 4 years (1948,1954,1958,1963), and 5 sectors (population, manufacturing, retailing, services and wholesaling). Thus, 18 x 5 x 4 = 360.

shaped boundaries (e.g. San Francisco) or cities considered to be polycentric (e.g. New York-Newark) were excluded from Mills' sample. Mills' average gradients for each sector and year are presented in Table 4.1.

Table 4.1 : Average Density Gradients, Mills and Macauley

γ / YEAR	1948	1954	1958	1963	1970/72	1977/80
Population	0.58	0.47	0.42	0.38	0.29	0.24
Manufacturing	0.68	0.55	0.48	0.42	0.34	0.32
Retailing	0.88	0.75	0.59	0.44	0.35	0.30
Services	0.97	0.81	0.66	0.53	0.41	0.38
Wholesaling	1.00	0.86	0.70	0.56	0.43	0.37

Mills' important conclusions included:

- Density gradients in all sectors tended to flatten over time.
- Gradient estimates varied much less than central density estimates. Gradients ranged from 0.20 to 1.00 while central population and employment densities ranged from 6,000 to 60,000 persons per square mile.
- γ was inversely related to metropolitan population and D_0 was directly related to metropolitan population. Central density estimates appeared to be more sensitive to differences in total population than gradient estimates.

It is important to recognize Mills' criteria for inclusion of cities because Hamilton [26] employed a sub-sample of Mills' 18 cities in his own work. Additionally, the small size of the Canadian urban system made strict application of Mills' arbitrary criteria inappropriate for this thesis. Irregularly shaped Canadian CMAs would likely include Vancouver, Montreal, Halifax and

St. John's using Mills' criteria.[3] If density gradients cannot be efficiently estimated for irregularly shaped CMAs using Mills' (Macauley's) methodology, then including such cities reduces the comparability between the Canadian results presented here and Hamilton's results.

Macauley [38] updated Mills' estimates in 1985.[4] The figures for 1970 and 1980 given in Table 4.1 (above) are Macauley's estimates. While Macauley's mean γ, based on the same 18 SMSAs used in 1972 by Mills, was 0.24 Hamilton [26] employed a sub-sample of Macauley's estimates with an average population gradient of only 0.22. The conclusions of Macauley regarding her gradient estimates were:

- Density gradients in all sectors continued to flatten over time but at a decreasing rate; and
- Employment and population gradients appeared to be converging. Macauley's result combined with Mills' results seems to indicate that, initially, population suburbanized and then employment followed.

Research has shown that Mills' two point gradient estimates are biased upwards, (i.e. are too steep), because of Mills' simplifying assumption that population density declines to zero at the urban boundary [17], [38], [42]. Mills believed the bias would be small

[3] Mills gave no specifics regarding what constitutes an irregular city shape, so it is not clear which Canadian CMAs would be excluded by Mills but the four just listed would be prime candidates. Making any generalizations about Canadian urban structure while ignoring Montreal and Vancouver is clearly nonsensical. For this reason, it was important to include density gradients estimated using Edmonston's method. Edmonston's method attempts to control for city shape.

[4] Data for Macauley's employment gradient estimates were available for 1972 and 1977 rather than 1970 and 1980.

for typical values of γ and k (equation 4.8). Macauley [38] verified empirically Mills' conjecture. In all cases Mills' estimates of γ were greater than or equal to her own but the difference between the estimates was small and not significantly different from zero.[5]

There was one important caveat to Macauley's conclusion that the Mills method does not seriously bias estimates of γ and D_0. Macauley's results held only when SMSA level data were employed. Macauley found Mills' estimates 68 percent larger than her own estimates if Urbanized Area (UA) data were employed.[6] Edmonston Goldberg and Mercer, (pg. 213) [18], argued that the Statistics Canada definition of CMA is more analogous to the U.S. definition of UA than SMSA. In the light of Macauley's results it is important to test the Edmonston, Goldberg, Mercer claim that CMAs approximate UAs, because the statistical unit employed may have the potential to radically alter density gradient parameter estimates.

The Mills (Macauley) technique requires that the urban boundary be

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[5] Macauley (pg. 259) showed the bias of Mills' estimates is:

$$\frac{N(\infty)-N(k)}{N(\infty)} = (1+\gamma k)\, e^{-\gamma k}.$$

[6] Urbanized Area (UA) is a more rigid concept of "urban". SMSA uses counties as the basic unit of aggregation, while UA only includes areas meeting specific density criteria. The average UA in Macauley's sample was 462 square miles with radius, (k), 13.2 miles. For the same cities the average SMSA area was 3539 square miles with radius (k) 33.9 miles (pg. 254) [37], [38].

regularly shaped.[7] Cities not meeting this requirement were simply excluded by Mills and Macauley. Edmonston [17] attempted to overcome this limitation and adapted the two-point method to include irregularly shaped cities.[8] Edmonston found a simple correlation of 0.92 between estimates of γ using his modified method and those obtained using Mills' method. The correlation was 0.98 for estimates of D_0. Such close agreement between the two techniques for cities with regular boundaries indicates that Edmonston's method should not incorporate any new bias in attempting to include irregularly shaped cities.

Edmonston, Goldberg and Mercer [18] applied Edmonston's modified gradient estimation method to a sample of 20 Canadian CMAs using data for the period 1950-1976. Prior to 1981 data required to estimate employment gradients was not available. Edmonston et.al. compared Canadian estimates with U.S. estimates for the same period. Mean values for γ and D_0 are presented in Table 4.2.

Several patterns are evident from Table 4.2:

- Canadian cities had higher central densities than U.S. cities in all periods but Canadian and U.S. gradients converged since 1970;
- Like their U.S. counterparts, Canadian urban populations have suburbanized (i.e. density gradients flattened) at a decreasing rate since the 1950's; and

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[7] White (pg. 299) [60] showed that an irregularly shaped boundary imparts a small upward bias to γ. However the upward bias may be partially or totally offset by a downward bias that results when the central city is not centred within the CMA. The magnitude and direction of the net bias depends upon the degree of irregularity and eccentricity respectively.

[8] See Chapter 3 of this thesis for specific methodological details.

- Estimates of γ and D_o for Canadian cities exhibited the same relationship with total metropolitan population as previous U.S. estimates: i.e. gradients tended to be flatter in larger cites while central density tended be higher in larger centres.[9]

Table 4.2: Average Values For γ and D_o Edmonston Goldberg and Mercer [18]

	γ		D_o	
YEAR	**CDN**	**US**	**CDN**	**US**
1950/51	0.93	0.76	50,000	24,000
1960/61	0.67	0.60	33,000	17,000
1970/71	0.45	0.50	22,000	13,000
1975/76	0.42	0.45	20,000	11,000

4.2 Population Gradients: New Canadian Estimates

Population gradients for 23 Canadian cities are presented in table 4.3. Estimates using Mills' technique as well as estimates using methods incorporating two slight modifications to Mills' technique (Macauley and Edmonston) are included.[10] Macauley estimates are considered the base case estimates throughout this thesis. Mills

[9] Consider the following results for 1976:

CMA Population	γ	D_o
$\leq$ 250,000	0.42	15,000
250-499,000	0.47	17,000
$\geq$ 500,000	0.39	26,000

Curiously, middle sized cities had γ exceeding the smallest group of CMAs but the sample size in each group was very small.

[10] Chapter 3 outlines the methodological differences among the Mills, Macauley and Edmonston estimation techniques.

and Edmonston estimates are included for two reasons:

1. To check the accuracy of the estimates obtained with the Macauley technique; and
2. To gauge the sensitivity of Hamilton's wasteful commuting model to the particular density gradient parameters employed.

It turns out that estimates obtained with the Mills and Macauley methods are virtually indistinguishable. Edmonston gradients are steeper with higher central densities than Macauley estimates, on average.[11]

4.2.1 Macauley Estimates

Hamilton [26] used Macauley's estimates of γ, D_o, δ and E_o to calculate his measure of wasteful commuting for 14 U.S. metropolitan areas. The same method employed by Macauley is used to estimate γ and D_o, for the 23 Canadian CMAs listed in Table 4.3. The population gradient estimates presented in Table 4.3 and Figure 4.1 seem reasonable.[12] The average γ for 1981 is 0.3290 with a median of 0.2835. Values range from a low of 0.1306 in Toronto to a maximum of 0.7179 in Regina. The figures for γ are consistent with the results obtained by Mills [43] and Macauley [38] presented

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[11] The implications for estimates of wasteful commuting are discussed in the next chapter.

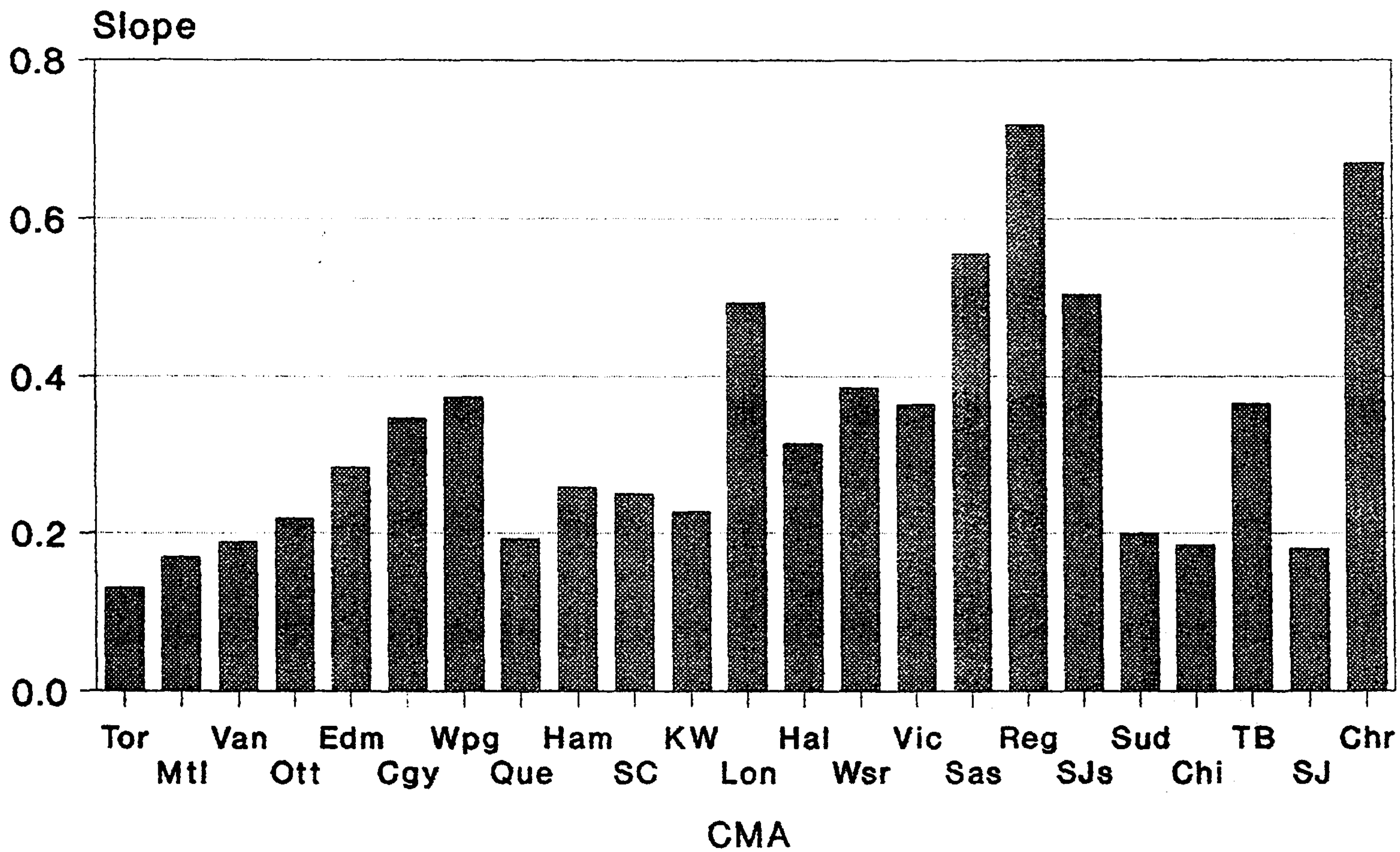
[12] The observations in Figure 4.1 and all subsequent figures are arranged in descending order according to the total CMA population (i.e. from Toronto to Charlottetown).

Table 4.3: 1981 Population Density Gradient Parameter Estimates.

CMA	Mills		Macauley		Edmonston	
	γ	D_0	γ	D_0	γ	D_0
Calgary	0.3460377	11978.88	0.3460159	11977.53	0.3828995	14606.40
Charlottetown	0.6701036	4696.32	0.6699682	4695.64	0.8614193	6589.55
Chicoutimi	0.2008953	1082.47	0.1844091	1011.68	0.1657379	780.07
Edmonton	0.2835978	9113.79	0.2834614	9107.39	0.3031348	9611.40
Halifax	0.3137889	6488.68	0.3137013	6486.59	0.3488784	9230.50
Hamilton	0.2634134	7451.39	0.2586705	7312.23	0.3014035	10221.54
Kitchener	0.2585306	3295.85	0.2271979	2847.02	0.2629334	3465.41
London	0.4929754	11883.06	0.4928113	11876.55	0.5338795	12869.68
Montreal	0.1726244	15332.64	0.1687077	15081.19	0.1756098	14352.38
Ottawa	0.2193046	5911.99	0.2187339	5897.66	0.2482884	7072.34
Quebec	0.1940538	3892.36	0.1917721	3862.96	0.2064706	3980.87
Regina	0.7179666	15084.21	0.7179664	15121.40	0.7594731	15915.71
Saint John	0.1889714	1043.29	0.1796146	984.39	0.2077067	1411.02
Saskatoon	0.5558713	9020.63	0.5558713	9020.63	0.5917340	9755.63
St. Catherines	0.2541840	5083.70	0.2500605	5016.86	0.2377767	7916.03
St. John's	0.5036675	11325.58	0.5036489	11325.15	0.5661072	18969.25
Sudbury	0.2038195	1100.47	0.1991318	1073.00	0.2163606	1142.33
Thunder Bay	0.3647303	3575.15	0.3645032	3571.46	0.4006837	4861.30
Toronto	0.1328807	12776.96	0.1306086	12666.03	0.1519010	20519.44
Vancouver	0.1887848	12336.72	0.1879939	12300.64	0.1988397	17750.81
Victoria	0.3683036	9927.21	0.3643718	9861.18	0.4171162	13857.68
Windsor	0.3877557	9625.49	0.3852291	9537.88	0.4772608	19110.81
Winnipeg	0.3736645	14356.36	0.3730519	14313.34	0.3700665	12768.70
Sample Mean	0.3328660	8103.62	0.3290220	8041.24	0.3645950	10293.70

NOTES: and D_0 are parameters from the negative exponential density gradient: $D(x) = D_0 e^{- x}$. D(x) represents the population density at any distance x from the central city; D_0 represents the population density (persons per square kilometre) at the city centre; represents the rate at which density declines as we move away from the city centre.

FIGURE 4.1
1981 POPULATION GRADIENT ESTIMATES
MACAULEY ESTIMATION TECHNIQUE



in Table 4.1.[13] A median value to the left of the mean indicates a distribution with a heavy righthand tail (i.e. there are more extremely large gradients than extremely small gradients).

The Macauley type estimates of γ presented in Table 4.3 are inversely related to CMA population.[14] Regression of γ on CMA population yields:[15]

$$\gamma = 0.38555 - \underset{(4.01 \times 10^{-8})}{9.63 \times 10^{-8}}\, POP \qquad R^2 = .2157$$

Based on the above estimate, a population increase of 100,000 (i.e. 17 percent of the mean 1981 CMA population) would flatten γ by only .00963 (i.e. 2.0 percent of the mean γ).

There are, however, some notable exceptions to the underlying

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[13] The mean γ is reasonably close to Edmonston, Goldberg and Mercer's estimate for 1976 (Table 4.2). Remember, however, that EGM used a slightly different methodology as well as a smaller sample of CMAs (20). The implied reduction in γ from 0.42 to 0.32 in the 5 years between Edmonston's estimates and this work is more rapid than expected, a priori. The difference is due, partly, to methodological differences. The mean γ obtained using Edmonston's method with 1981 Canadian data (0.3646) was somewhat higher than the Macauley estimate (0.3290), (Table 4.3).

[14] The estimated equation is obviously simplistic. While Mills and Macauley found γ tended to be smaller in cities with higher populations, Alperovich [5] found the opposite once he controlled for the land area of the metropolitan area:

> "Our results show that holding land supply constant, cities which are more populous tend to be less suburbanized. Suburbanization is primarily associated with high supply of land and not with increased population per se" (pg. 293) [5].

[15] Standard error in parentheses.

inverse relationship between γ and CMA population. Saint John, Chicoutimi and Sudbury are ranked 22nd, 20th, and 19th in terms of CMA population but had the third, fourth, and seventh flattest population gradients (Figure 4.2). Further, Regina's gradient appears excessively steep ($\gamma = 0.7179$) compared with Saskatoon ($\gamma = 0.5559$) given that the 1981 population in the two CMAs is almost identical (Regina = 173,226, Saskatoon = 175,058).

Two of the apparently anomalous cities, Chicoutimi and Saint John, violate the Mills' assumption of circularity. Both city's boundary resemble an elongated rectangle, rather than a circle. The result may be a poor estimate for γ in each city.[16] The discrepancy between Regina and Saskatoon may result from the small proportion of the total CMA population outside the central city in Regina (i.e. what White [60] termed an extreme split). As a rule of thumb Edmonston, Goldberg and Mercer [18] suggested that a reliable estimate of γ cannot be obtained using the two-point method for any metropolitan area with a suburban population of less than 10,000. In 1981, Regina had a suburban population of 10,613 while Saskatoon's suburban population was 20,848.[17] No obvious explanation for Sudbury's anomalous gradient is apparent.

Using the Macauley estimation method, the average population

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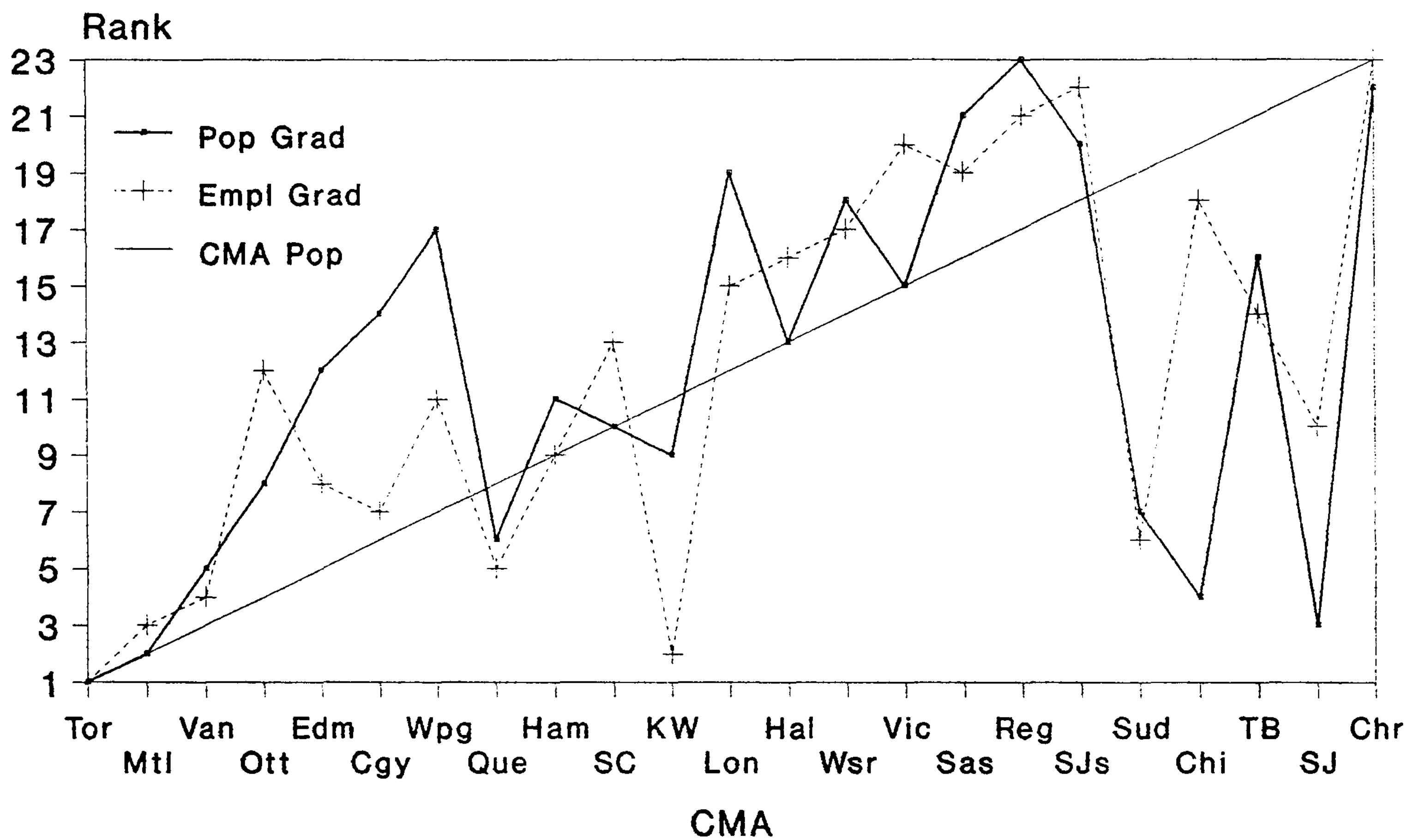
[16] Recall White's [60] reminder that irregular boundaries and eccentricity will bias estimates of γ in opposite directions.

[17] The term suburban refers to population within the CMA but outside of the central city. For example, a resident of North York would be a suburban resident of the Toronto CMA or a resident of Burnaby would be a suburban resident in the Vancouver CMA.

FIGURE 4.2
GRADIENT SLOPE RANK
1981, MACAULEY ESTIMATION TECHNIQUE



density at the centre of Canadian CMAs is 8,041 people per square kilometre (20,827 per square mile). Individual estimates range from a low of 984 per square kilometre (2,550 per mile) in Saint John, to 15,121 per square kilometre (39,164 per mile) in Regina (Figure 4.3). The median value, 9,021 persons per square kilometre, was greater than the mean value, indicating a distribution with a heavy lefthand tail (i.e. there are more extremely small values than extremely large values in the distribution).[18] These central density estimates are not inconsistent with those provided by Mills [42], Macauley [38], and Edmonston, Goldberg and Mercer [18]. The mean central density reported in Table 4.3 is almost identical to that reported by Edmonston, Goldberg and Mercer for 1976 (Table 4.2).

As expected, central population density is positively related to total CMA population. Regressing D_o on CMA population yields:

$$D_0 = 6350.9 + \underset{(1.11 \times 10^{-3})}{2.88 \times 10^{-3}} POP \qquad R^2 = .2411$$

Based on this estimated equation, an increase in CMA population of 100,000 would increase central density by 288 people per square kilometre (i.e. a 17 percent increase in population would yield a 3.6 percent increase in D_o, at the mean).[19] D_o appears slightly

[18] This is the opposite result compared to the estimates of γ which were skewed toward larger values.

[19] The estimate of 2.88×10^{-3} is remarkably close to Alperovich's figure of 2.46×10^{-3} (pg 292) [5]. However, the equation estimated by Alperovich controlled for other variables,

more sensitive than γ to changes in total CMA population.[20] Mills also found that D_0 was more sensitive than γ to changes in population. (Chapter 3) [43].

As with the estimates of γ, there are some obvious exceptions to the overall positive relationship between D_0 and CMA population. Figure 4.4 illustrates that the population central density relationship breaks down, particularly in the middle of the urban hierarchy.[21] The cluster of CMAs including Quebec City (Que), Hamilton (Ham), St. Catherines (SC), and Kitchener-Waterloo (KW) ranked far too low, while Windsor (Wsr), Victoria (Vic), Saskatoon (Sas), Regina (Reg), and St. John's (SJs) rank too high relative to their population ranking. The largest and smallest CMAs behave closer to expectations, with population and central density rankings much more equal.

Most researchers who have employed the two-point method to estimate population density functions have been more interested in estimates of γ than D_0. Even the technique's pioneer, Mills [43] did not discuss his estimates of central density in detail. While at first it seems perverse that the Canadian estimates presented above rank

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such as metropolitan land area. In an earlier footnote it was pointed out that Alperovich found controlling for land area reversed the effect of total metropolitan population on γ. This does not seem to be the case for estimates of D_0.

[20] A 17 percent increase in population reduced γ by only 2.0 percent.

[21] Observations below the diagonal line in Figure 4.4 have a central population density that is higher than expected given the CMA population while observations above the diagonal line have a central population density that is lower than expected.

FIGURE 4.3
CENTRAL POPULATION DENSITY ESTIMATE 1981, MACAULEY ESTIMATION TECHNIQUE

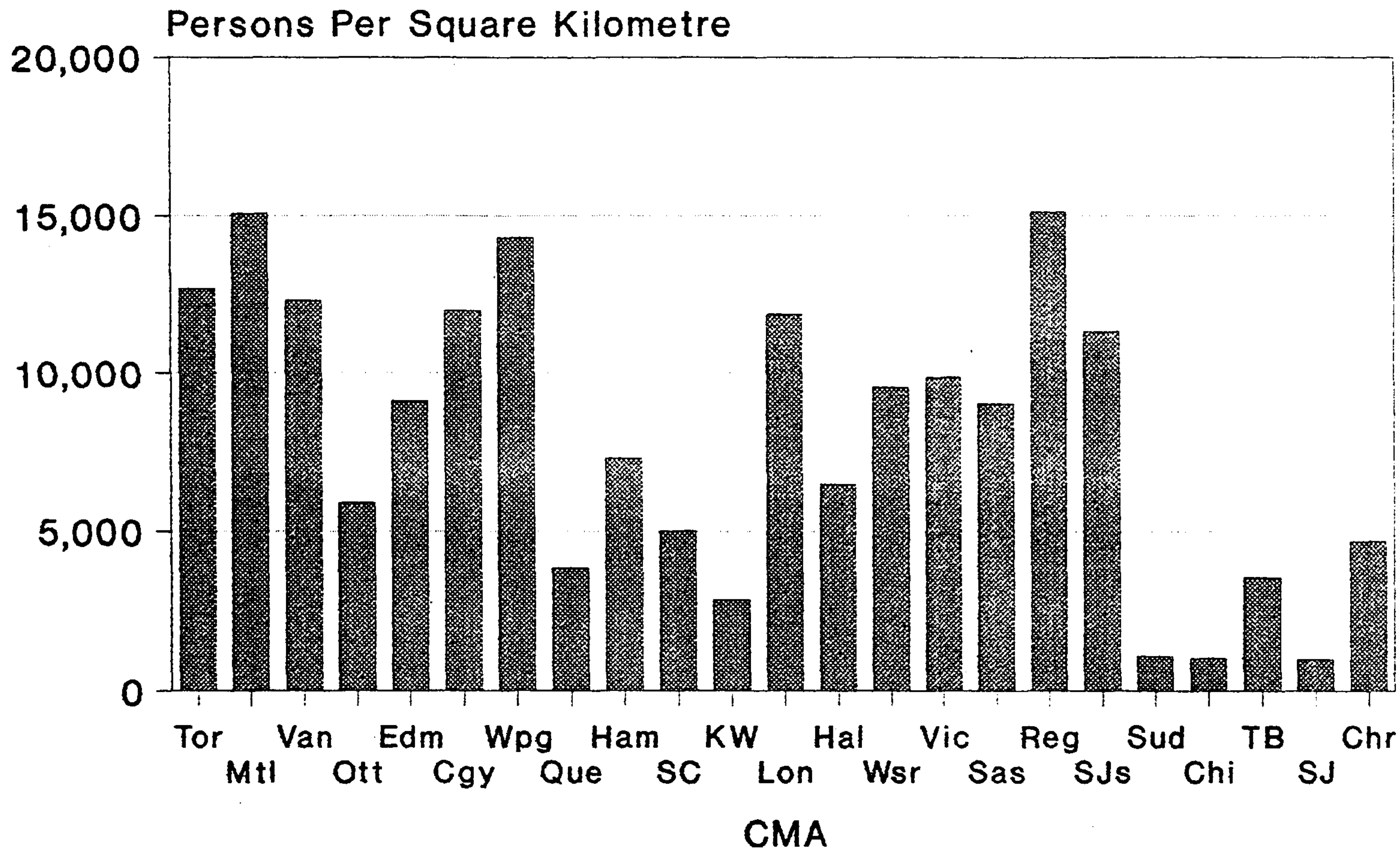
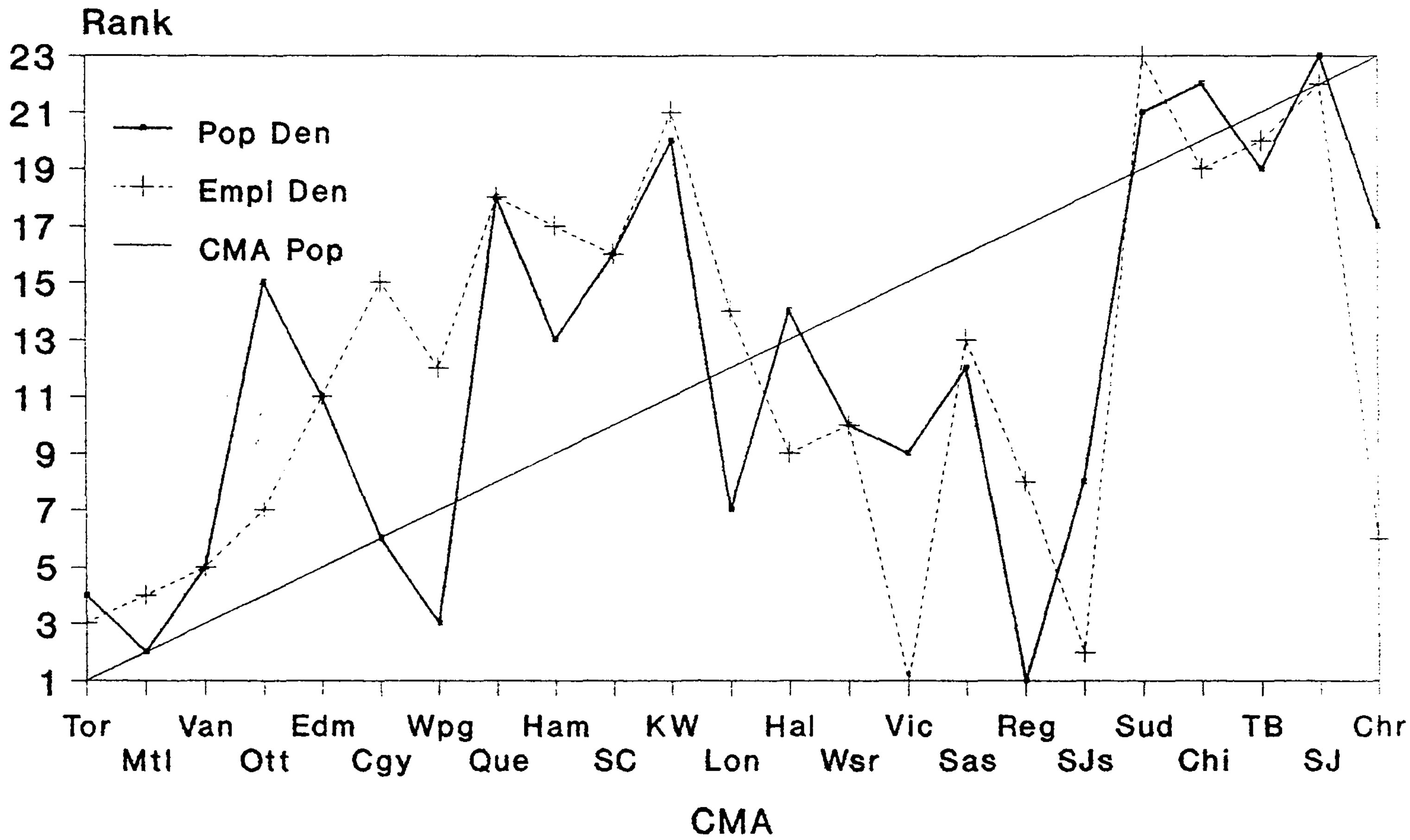


FIGURE 4.4
CENTRAL DENSITY RANK
1981, MACAULEY ESTIMATION TECHNIQUE



the central population density in smaller CMAs (e.g. Regina and St. John's) among the highest in Canada, this is not entirely inconsistent with Mills' results (pg. 40, Table 11) [43]. Mills' results showed smaller cities such as Columbus and Toledo had among the highest central population densities, ranking ahead of larger cities including Boston and Pittsburg.

There are at least three potential explanations for the divergence of D_0 from the rank-size relationship:

1. In order to estimate D_0, the radians of land available for development, ϕ, must be known.[22] Most procedures used to measure ϕ are ad hoc, contributing an element of uncertainty to estimates of D_0. In contrast ϕ is not required to estimate γ. In the results presented above in many of the CMAs in which D_0 seemed anomalous, (Figures 4.3 and 4.4), ϕ is not equal to 2π (e.g. Victoria, Windsor, Hamilton and St. Catherines).[23]

2. Estimates of D_0 for CMAs with "extreme population splits" tend to exceed the expected central density based on the rank-size rule. Consider Table 4.4 below.[24]

3. Three of five cities with unexpectedly low values for D_0 are part of a dual (polycentric) CMA: Ottawa-Hull, St. Catherines-Niagara Falls and Kitchener-Waterloo. The other two CMAs both contain a large secondary city: Hamilton (Burlington) and Quebec (Levi).

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[22] Estimation methods are described in detail in Chapter 3 of this thesis. For the present it is sufficient to recall the following equation:

$$D_0 = \frac{N(k_2)\gamma^2}{\phi[1-(1+\gamma k_2)e^{-\gamma k_2}]}$$

One of the differences between the Edmonston and Macauley estimates was the measurement of ϕ.

[23] Recall that 2π is the maximum number of radians available: i.e. $2\pi = 360^\circ$. Thus, if a CMA is circular, $\phi = 2\pi$ and ϕ does not have to be estimated in order to calculate D_0.

[24] White suggested that $\geq$ 80 percent of the population in the central city constitutes an "extreme split" [60].

Table 4.4 : Cities With Small Suburban Population and Large Values For D_0

CMA	Suburban #	Pop. % of CMA
London	29,338	10.3
Regina	10,613	6.1
Saskatoon	20,848	11.9
Winnipeg	22,783	3.9

One test of the reasonableness of the estimates for γ and D_0 is to examine the predicted distance at which population density declines to 100 people per square mile. Both Macauley [38] and Hamilton [26] used this criterion and both argued that it was reasonable to expect the predicted distance (denoted F) to be less than or equal to the political (statistical) boundary used in the census. While the cutoff value of 100 people per square mile is arbitrary, it is also reasonable: densities below 100 people per square mile are not normally thought of as urban. Figure 4.5 compares the predicted distance, (F), with the measured radius, (G), taken from Statistics Canada maps, for each CMA.[25]

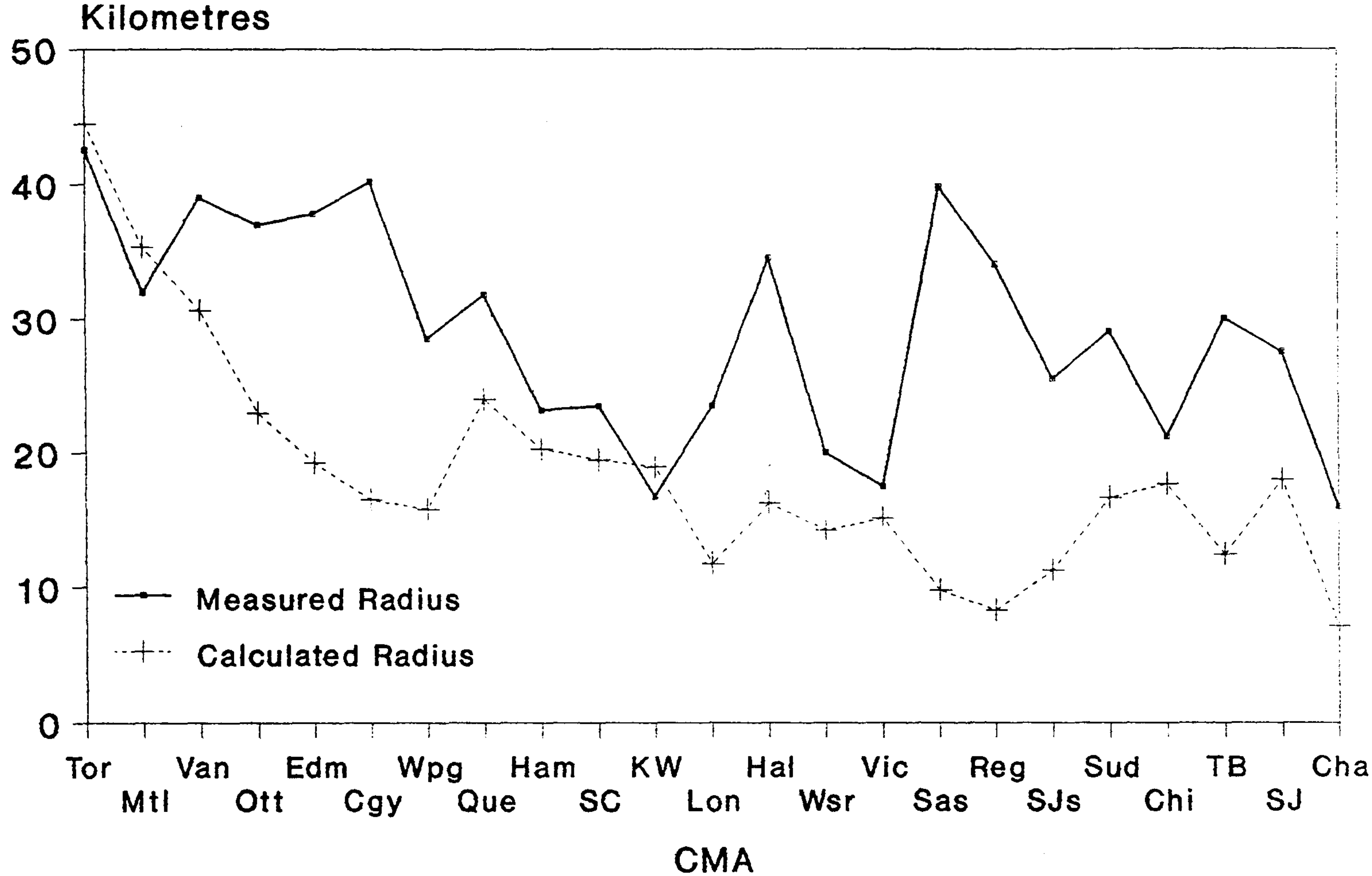
The average predicted radius (F) for the Canadian sample of cities is 18.6 kilometres while the average measured radius, (G), was 29.2 kilometres. In only three cases was F beyond G: Kitchener (by

[25] If γ and D_0 are known and D(x) is assumed to be equal to 100 persons per square mile (or 38.6 persons per square kilometre) F can be calculated by substituting into:

$$D(x) = D_0 e^{-\gamma x}$$

and solving for x=F.

FIGURE 4.5
MEASURED VERSUS CALCULATED* CMA RADIUS



* Calculated is where population density = 100 people per square mile.

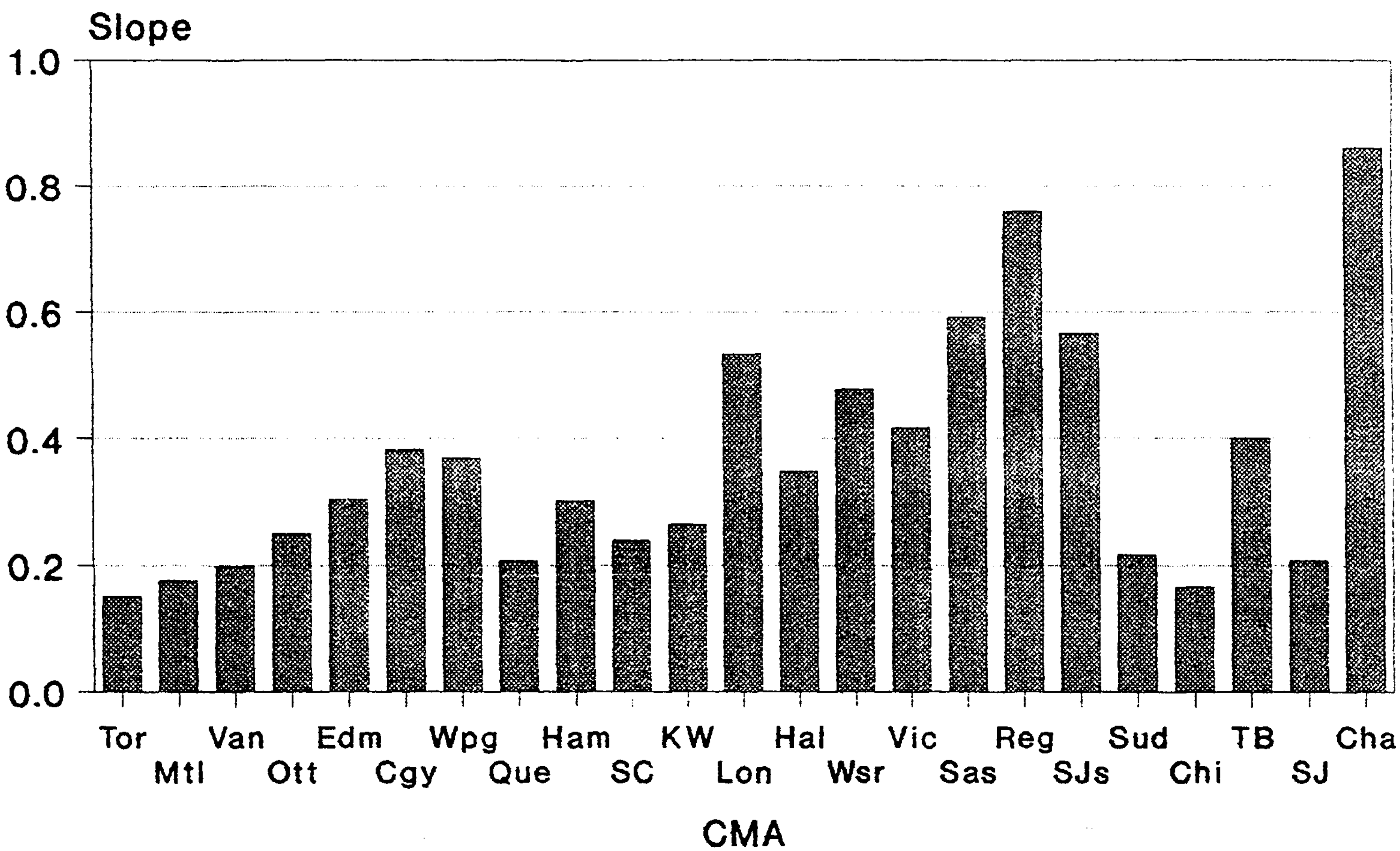
2.1 km), Montreal (by 3.4 km), and Toronto (by 2.0 km). A priori, it was expected that F would more closely approximate the political boundary, G, in larger cities. Figure 4.5 partially confirms this expectation. However, for the Canadian sample of cities, a regional pattern seems to dominate the size relationship. In more densely populated central Canada, F and G converge, while the greatest absolute differences occur in the prairies.[26] The latter result is not surprising considering that the 1981 legal (political) civic areas of Calgary, Edmonton and Saskatoon are larger than Montreal, Toronto or Vancouver.

4.2.2 Edmonston Estimates

Population gradients obtained using Edmonston's method are presented in Table 4.3 and Figure 4.6. On average, estimates using Edmonston's method are steeper than the Macauley estimates, and exhibit higher central population density. The mean γ is 0.3646 and the median is 0.3031, indicating a distribution skewed toward larger values of γ (i.e. skewed right). Estimates range from 0.1519 for Toronto to 0.8614 for Charlottetown. The mean estimated γ is smaller than the 0.42 reported by Edmonston, Goldberg and Mercer in 1976 using the same methodology. While continued suburbanization is expected to flatten the density gradient over time the implied rate between 1976 and 1981 appears excessively

[26] The five smallest absolute differences were: Toronto (1.96 km), Kitchener (2.13 km), Victoria (2.29 km), Hamilton (2.93 km), and Montreal (3.37 km). The five largest absolute differences were: Saskatoon (29.99 km), Regina (25.68 km), Calgary (23.62 km), Edmonton (18.53 km), and Halifax (18.17 km).

FIGURE 4.6
POPULATION GRADIENT ESTIMATES
1981, EDMONSTON ESTIMATION TECHNIQUE



rapid. Two factors most likely account for this:

- The 1981 estimates presented in Table 4.3 include 3 CMAs not included in the EGM study; and

- Several boundary changes occurred between 1976 and 1981 reducing the comparability of the samples still further.[27]

The mean difference between the Macauley and Edmonston estimates of γ is 0.03557, equivalent to 9.8 percent of the mean Macauley gradient. The difference is significantly different from zero, at the $\alpha = .01$ level ($t = 4.10$).[28] In only three instances are Macauley estimates steeper than Edmonston estimates: Chicoutimi, St. Catherines and Winnipeg. The greatest absolute differences occur in Windsor (24.9 percent), Charlottetown (22.0 percent), Hamilton (14.2 percent) and Victoria (12.6 percent).

Differences in the rank ordering of the CMAs are more likely to influence estimates of optimum commute (using Hamilton's method) than absolute differences between Macauley and Edmonston estimates.

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[27] One of the supposed advantages of the two-point method is that estimates are not affected by changes in political boundaries over time. The claim of insensitivity to political change is founded upon three restrictive assumptions, however:

- The city is circular;

- The negative exponential function provides an exact fit; and

- All municipal annexations are circular (pg. 60) [17].

In practice, violation of any of these assumptions will reduce inter-temporal comparability.

[28] The mean absolute difference was 0.03852 or 10.6 percent of the mean Macauley gradient. This difference was significant at the $\alpha = .01$ level ($t = 4.77$).

Systematic differences between the Macauley and Edmonston methods should not affect estimates of wasteful commuting if the differences apply to both the population and employment density gradients.[29] Using Hamilton's notation, differences in A and B should cancel out in the estimate of waste, (C).[30] Figure 4.7 confirms that Edmonston estimates of γ are essentially an order-preserving transformation of the Macauley estimates.

Like the Macauley estimates, Edmonston estimates of γ are inversely related to total CMA population. A simple regression yields:

$$\gamma = 0.4295 - \underset{(4.74 \times 10^{-8})}{1.12 \times 10^{-7}} POP \qquad R^2 = .2055$$

The estimated regression coefficient indicates that a population increase of 100,000 (17.0 percent of the mean 1981 CMA population) would flatten the mean gradient by .0112 (3.1 percent of the mean γ). Thus, Edmonston gradients are slightly more sensitive to changes in population than Macauley gradients. Recall that a 17 percent increase in CMA population flattened the mean Macauley gradient by only 2.0 percent.

The Edmonston estimate of mean central density is 10,293 (Table 4.3

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[29] Of course both the gradient and central density rankings should be preserved between methods.

[30] See Hamilton's footnote #2 (pg. 1038) [25], [26] for a similar argument regarding the measure of ϕ used to calculate A, B and C.

people per square kilometre, (26,827 per mile). Values range from 780 per square kilometre, (2,020 per mile) in Chicoutimi, to 20,519 per square kilometre, (53,145 per mile) in Toronto (Figure 4.8). The median value is 9,757 per square kilometre. In contrast to the Macauley estimates, D_0 is skewed toward large values (i.e. skewed right). The mean value is 34 percent larger than the mean value reported by Edmonston, Goldberg and Mercer [18] for 1976. Like the Macauley estimates presented above, D_0 exhibited a direct relationship to total metropolitan population. Simple regression yields:

$$D_0 = 8229.2 + 3.52 \times 10^{-3} POP \qquad R^2 = .2085$$
$$(1.50 \times 10\text{-}3)$$

This implies that a population increase of 100,000 (17 percent) would raise mean central population density by 352 people per square kilometre (3.4 percent).[31]

The mean difference between the Macauley and Edmonston estimates of D_0 is 2,252 people per square kilometre. This is 22 percent of the mean Macauley estimate of D_0.[32] The difference is statistically significant at the α = .01 level (t=3.73). As with

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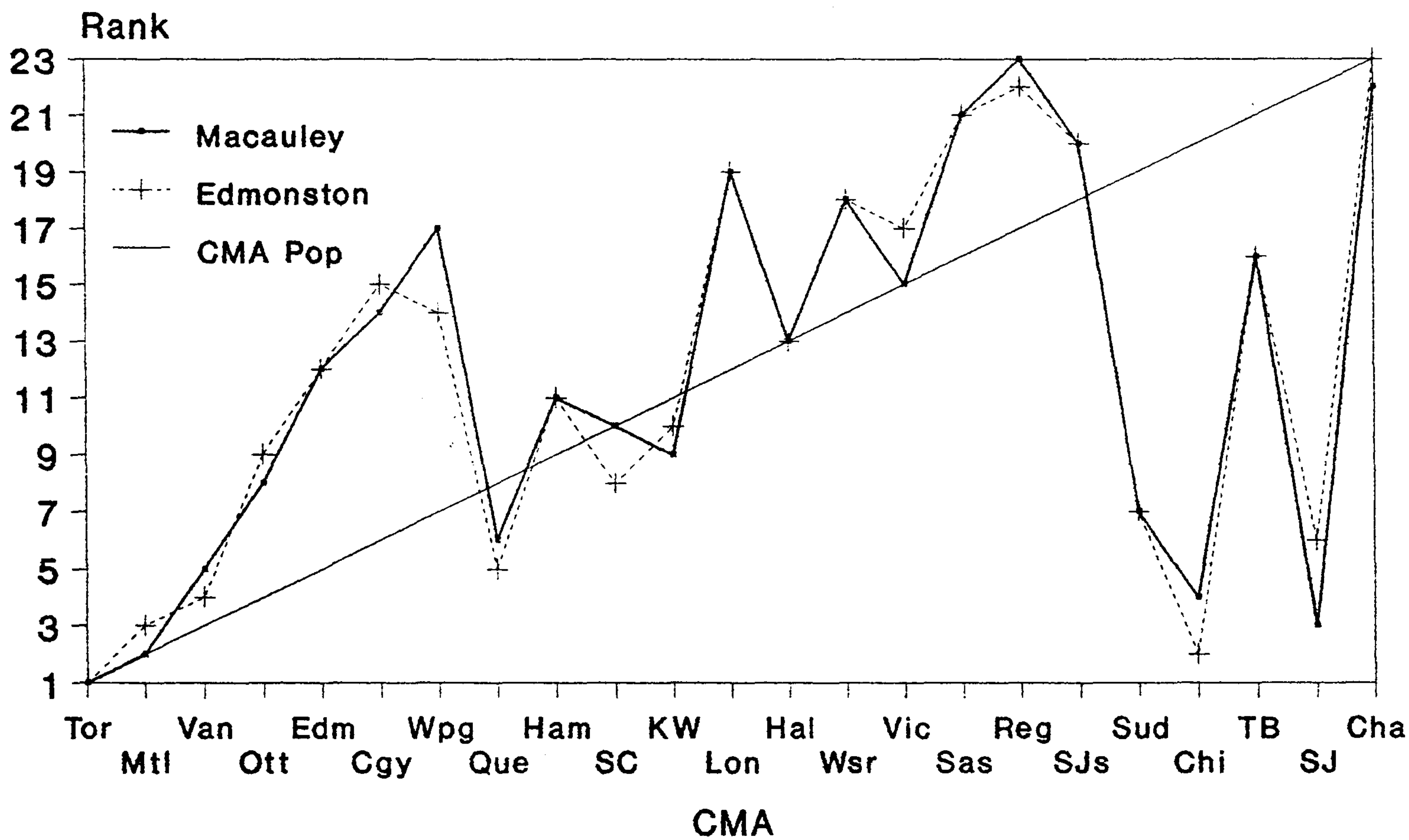
[31] This is almost identical to the 3.6 percent for Macauley-type estimates.

[32] The percent differential for central density is almost twice that for γ. This result is not surprising. One of the main differences between the Macauley and Edmonston methods was the measurement of ϕ (see chapter 3 of this thesis for complete methodological details). Recall that ϕ is needed to estimate D_0 but not to estimate γ (compare equations 4.9 and 4.10) above).

FIGURE 4.7
COMPARISON OF EDMONSTON AND MACAULEY POPULATION GRADIENT ESTIMATE RANKINGS



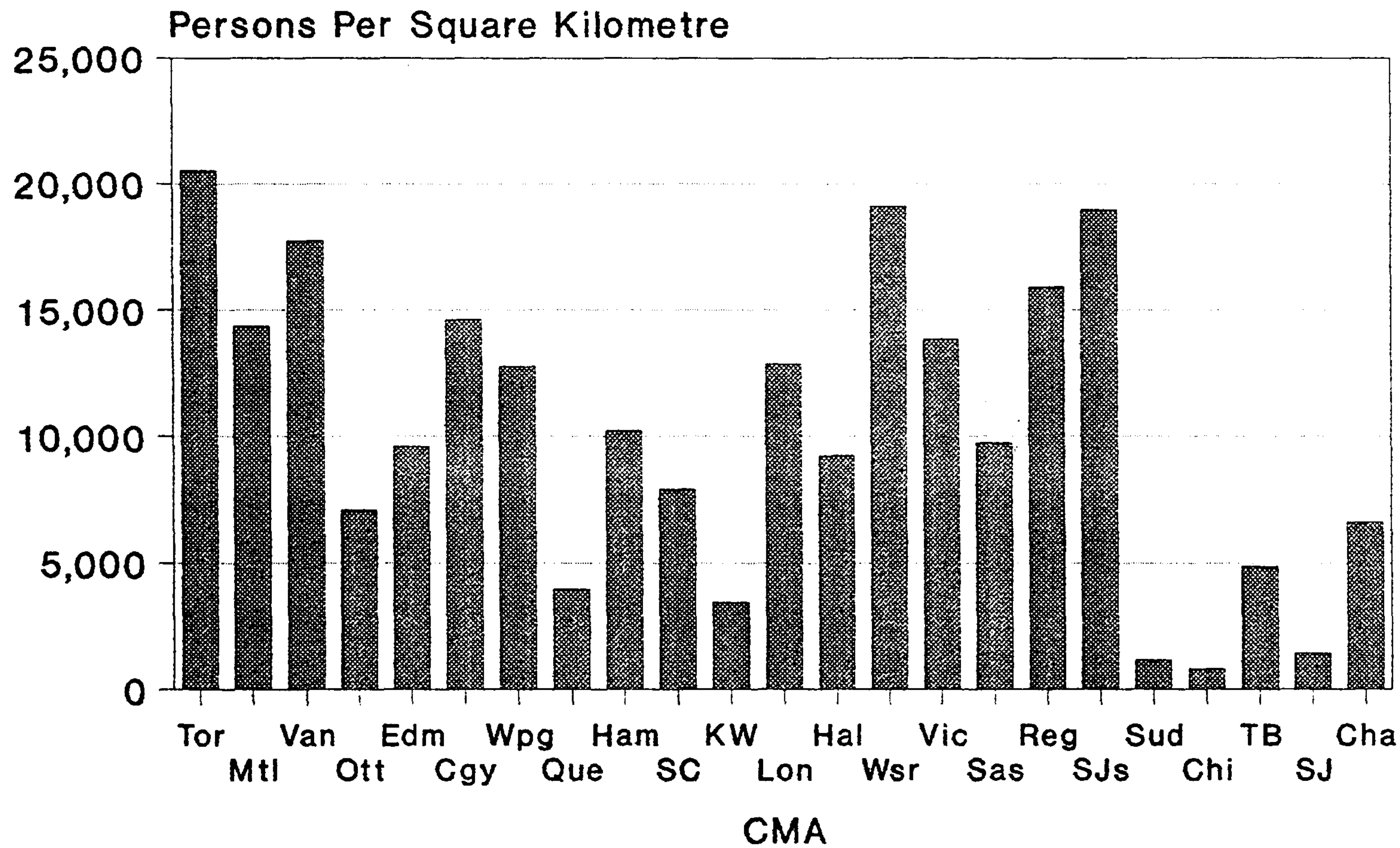
γ, in only three instances are Macauley estimates greater than Edmonston estimates: Chicoutimi, St. Catherines, Winnipeg. The greatest absolute differences are in Windsor 50.1 percent, St. John's 40.3 percent, Toronto 38.3 percent, Vancouver 30.7 percent and Victoria 28.8 percent. In each of these cities $\phi \neq 2\pi$. Figure 4.9 illustrates that while the CMA rankings of estimates of D_0 obtained using the Macauley and Edmonston are similar, they are less so than the rankings for estimates of γ. Significant D_0 rank changes occurred for Windsor (9th Macauley, 2nd Edmonston), Winnipeg (3rd, 10th), Montreal (2nd, 7th), Toronto (4th, 1st) and Regina (1st, 4th).

Overall it appears that the Macauley and Edmonston methods rank the CMA gradients similarly. There are greater discrepancies in the ranking of central population density. However, Edmonston gradients are steeper and have higher central density in all but three CMAs. If these results extend to the respective employment gradients, differences are expected to cancel out when Hamilton's method is applied in to estimate wasteful commuting, (C).

4.2.3 Mills Estimates

The results of this thesis employing Canadian data confirm Mills' conjecture that the bias in estimating density gradients resulting from the simplifying assumption that population density is zero at the city periphery is small. Macauley [38] found the bias was small when SMSA data were used, but the bias was much larger if

FIGURE 4.8
CENTRAL POPULATION DENSITY ESTIMATE
1981, EDMONSTON ESTIMATION TECHNIQUE



more compact Urbanized Area (UA) data were used. This discrepancy is of interest because Edmonston, Goldberg and Mercer [18] argued that the Statistics Canada definition of CMA more closely approximates the U.S. Census Bureau definition of UA than SMSA. Results presented above are not consistent with the Edmonston, Goldberg and Mercer claim.

In each CMA Mills estimates of γ are greater than or equal to the Macauley estimates presented in Table 4.3. The mean difference is .00384 or 1.2 percent of the mean Macauley estimate of γ. In only two CMAs is the difference large: Kitchener, 12.1 percent and Chicoutimi, 8.2 percent. Results for estimates of D_0 are similar. The mean difference is 62.4 people per square kilometre, or 0.77 percent of the mean Macauley estimate of D_0. The only large differential is Kitchener, at 13.6 percent.

Mills and Macauley estimates of γ and D_0 are essentially the same. The ranking of CMAs is preserved in all cases and absolute differences are small. It can also be concluded that Canadian CMAs more closely approximate the U.S. definition of SMSA than UA.

4.3 Employment Gradients: Canadian Estimates

Employment gradient estimates for a sample of Canadian cities in 1981 are presented in Table 4.5. Prior to the 1981 census even the modest data requirements of two-point estimation could not be satisfied for employment gradients. Thus, these are the only Canadian employment gradient estimates of which I am aware.

Previous U.S. employment gradient estimates (e.g. Edmonston [17], Macauley [38], Mills [42], [43]), were disaggregated by employment sector and are not directly comparable to these results.

4.3.1 Macauley Estimates

Employment gradient estimates obtained using Macauley's method are presented in Table 4.5 and Figure 4.10. It was expected that the absolute value of δ would exceed γ in every CMA. If the opposite were true, it would mean that population was more centralized than employment, violating the central assumption of the monocentric model (i.e. the assumption that there exists a trade-off between accessibility, and housing consumption). For three CMAs (Calgary, London and Winnipeg) the estimate of δ is less than the estimate of γ (i.e. the employment gradient is flatter than the population gradient).

Despite the individual anomalies, on average, the results seem plausible. The mean δ in 1981 is estimated to be 0.4666 with a median of 0.3563. This is 42 percent steeper than the mean population gradient estimate. Values for δ range from 0.1974 in Toronto to 1.4436 in Charlottetown. A median value to the left of the mean indicates a distribution with a heavy righthand tail (i.e. there are more extremely large gradients than extremely small). This was also the case for γ.

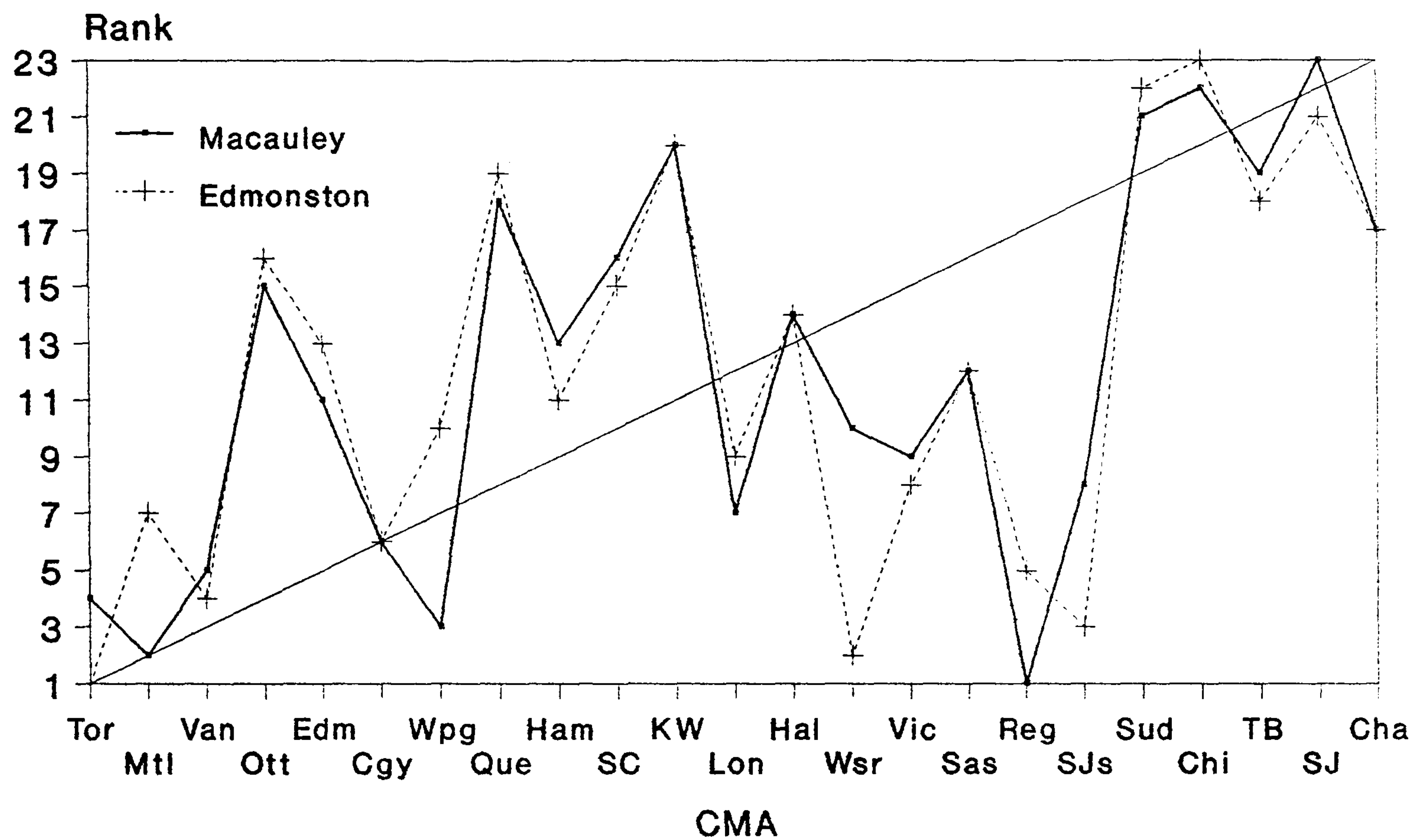
Like population gradients, employment gradients are inversely related to CMA population (Figure 4.10). Regression of δ on CMA

Table 4.5 : 1981 Employment Density Gradient Parameter Estimates.

CMA	Mills		Macauley		Edmonston	
	δ	E_0	δ	E_0	δ	E_0
Calgary	0.3128020	15085.28	0.3127501	5083.83	0.3461048	6200.16
Charlottetown	1.4435890	8902.33	1.4435890	8902.34	1.8560400	12509.68
Chicoutimi	0.5814989	3000.12	0.5813880	2999.16	0.5975073	2542.18
Edmonton	0.3142159	5773.54	0.3141539	5771.78	0.3359407	6090.77
Halifax	0.4760949	6866.02	0.4760936	6865.99	0.5298379	9777.95
Hamilton	0.3212670	4670.69	0.3191913	4634.22	0.3711974	6466.61
Kitchener	0.2591098	1527.91	0.2279318	1320.93	0.2637403	1607.66
London	0.4749254	5130.32	0.4747097	5126.55	0.5142696	5555.24
Montreal	0.2302758	12203.53	0.2291056	12146.07	0.2389976	11574.80
Ottawa	0.3562899	7562.80	0.3562756	7562.39	0.4048457	9077.54
Quebec	0.2694734	3091.33	0.2690403	3087.08	0.2900491	3183.80
Regina	0.7218454	7003.44	0.7218454	7003.44	0.7585494	7286.35
Saint John	0.3244227	1236.60	0.3232313	1229.21	0.3848479	1815.73
Saskatoon	0.6838427	5593.97	0.6838425	5593.96	0.7279616	6049.76
St. Catherines	0.3849621	4831.61	0.3844290	4835.03	0.3809130	7855.38
St. John's	0.9359563	15273.01	0.9359562	15273.01	1.0529500	25604.43
Sudbury	0.2922224	877.85	0.2912226	873.62	0.3167404	931.00
Thunder Bay	0.4235179	2144.59	0.4234428	2143.92	0.4664317	2927.71
Toronto	0.1978160	14837.47	0.1974853	14819.42	0.2308162	24062.52
Vancouver	0.2348140	9728.74	0.2345953}	9721.10	0.2498919	14089.96
Victoria	0.6941540	15878.23	0.6941023	15876.97	0.8092844	22557.74
Windsor	0.5009179	6271.09	0.5002666	6257.90	0.6251126	12678.34
Winnipeg	0.3380818	5722.72	0.3370394	5691.55	0.3328864	5040.95
Sample Mean	**0.4683520**	**6661.44**	**0.4665950**	**6644.33**	**0.5254310**	**8933.41**

NOTES: δ and E_0 are parameters from the negative exponential density gradient: $E(x)=E_0\, e^{-\delta x}$. E(x) represents the employment density at any distance, x, from the city centre; E_0 represents the employment density (jobs per square kilometre) at the city centre; δ represents the rate at which density declines as we move away from the city centre.

FIGURE 4.9
COMPARISON OF EDMONSTON AND MACAULEY CENTRAL POPULATION DENSITY RANKINGS



population yields:[33]

$$\delta = 0.5607 - \underset{(7.10 \times 10^{-8})}{1.60 \times 10^{-7}}\, POP \qquad R^2 = .1952$$

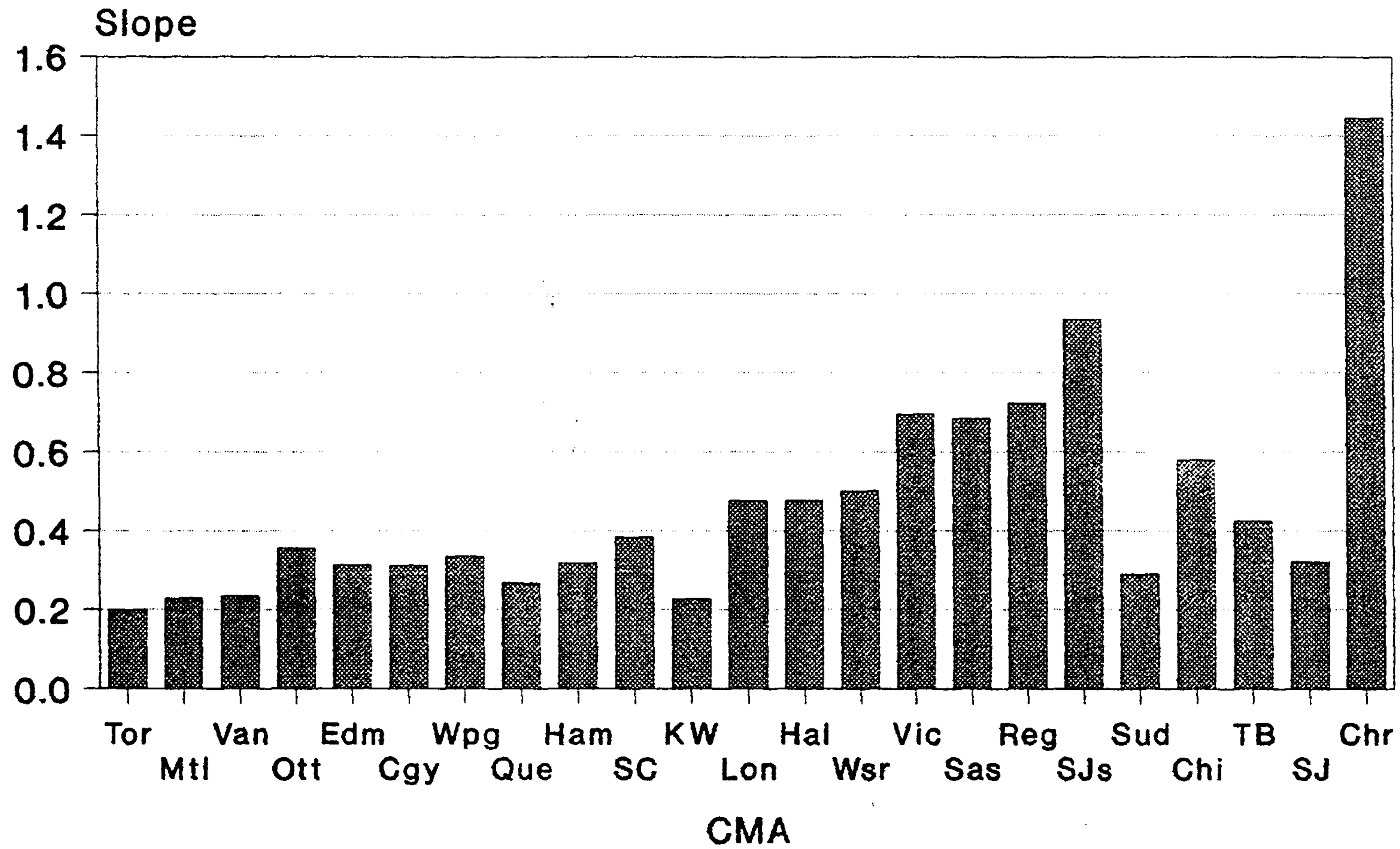
The estimated equation indicates that an increase in population of 100,000 (i.e. 17 percent of mean CMA population) would flatten the mean employment gradient by .0160 (i.e. 3.4 percent of the mean estimated δ). Thus, as population grows, δ appears to flatten slightly more quickly than γ. This is consistent with the notion that jobs follow people to suburban locations and is also consistent with Macauley's contention that population and employment gradients converge over time [38].

While Figure 4.10 confirms that employment gradient estimates flatten with CMA size, as population gradients estimates did, the relationship appears weaker for the employment gradient estimates. Once again several small cities, Sudbury, Chicoutimi, Thunder Bay and Saint John, have inexplicably flat gradients.

Using the Macauley estimation method, average employment density at the centre of Canadian CMAs in 1981 is 6,644 employees per square kilometre (17,209 per square mile). E_0 ranges from a low of 874 per square kilometre (2,263 per mile) in Sudbury, to 15,877 per square kilometre (41,121 per mile) in Victoria (Figure 4.11). The median value, 5,692, indicates a distribution with a heavy

33 Standard error is in parentheses.

FIGURE 4.10
EMPLOYMENT GRADIENT ESTIMATES
1981, MACAULEY ESTIMATION TECHNIQUE



righthand tail (i.e. there are more extremely large central densities than extremely small).

Like central population density, central employment density is positively related to total CMA population. Regressing E_0 on CMA population yields:

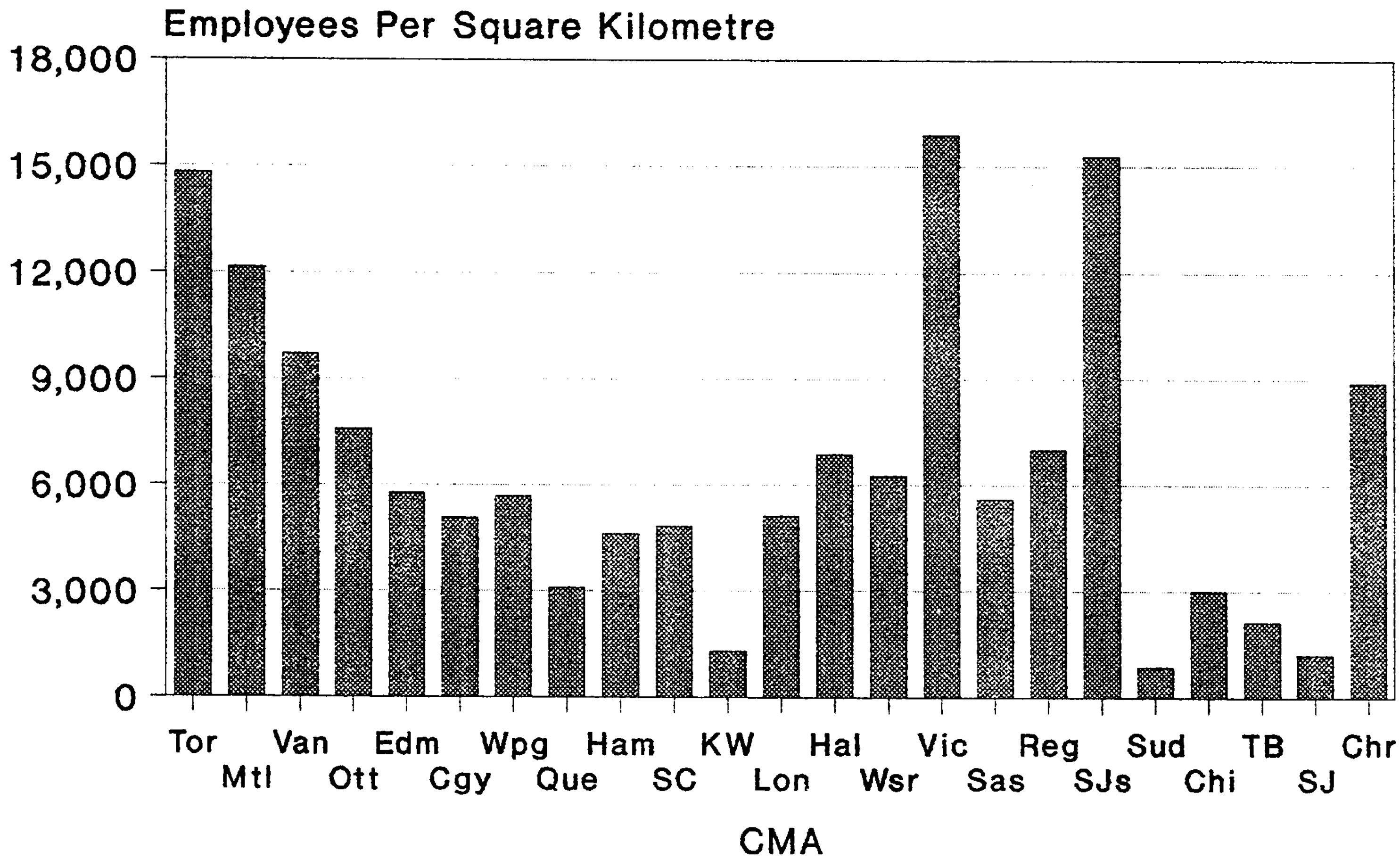
$$E_0 = 5011.3 + 2.78 \; x \; 10^{-3} \; POP \qquad R^2 = .2477$$
$$(1.06 \; x10^{-3})$$

The estimated equation indicates that an increase in CMA population of 100,000 would increase the mean central employment density by 278 people per square kilometre (i.e. a 17 percent increase in population yields a 4.2 percent increase in E_0, at the mean). Thus, E_0 appears to be more sensitive than δ to changes in total population.[34]

The relationship between E_0 and total CMA population appears strongest at the upper end of the urban hierarchy. In medium sized cities, (e.g. Winnipeg, Quebec City, Hamilton, St. Catherines and Kitchener), E_0 is lower than expected based on the simple rank-size relationship. Some of the smallest cities have much higher than expected central employment density (e.g. Victoria, Regina, St. John's and Charlottetown).

[34] A 17 percent increase in population reduced δ by only 3.4 percent.

FIGURE 4.11
CENTRAL EMPLOYMENT DENSITY ESTIMATE
1981, MACAULEY ESTIMATION TECHNIQUE



4.3.2 Edmonston Estimates

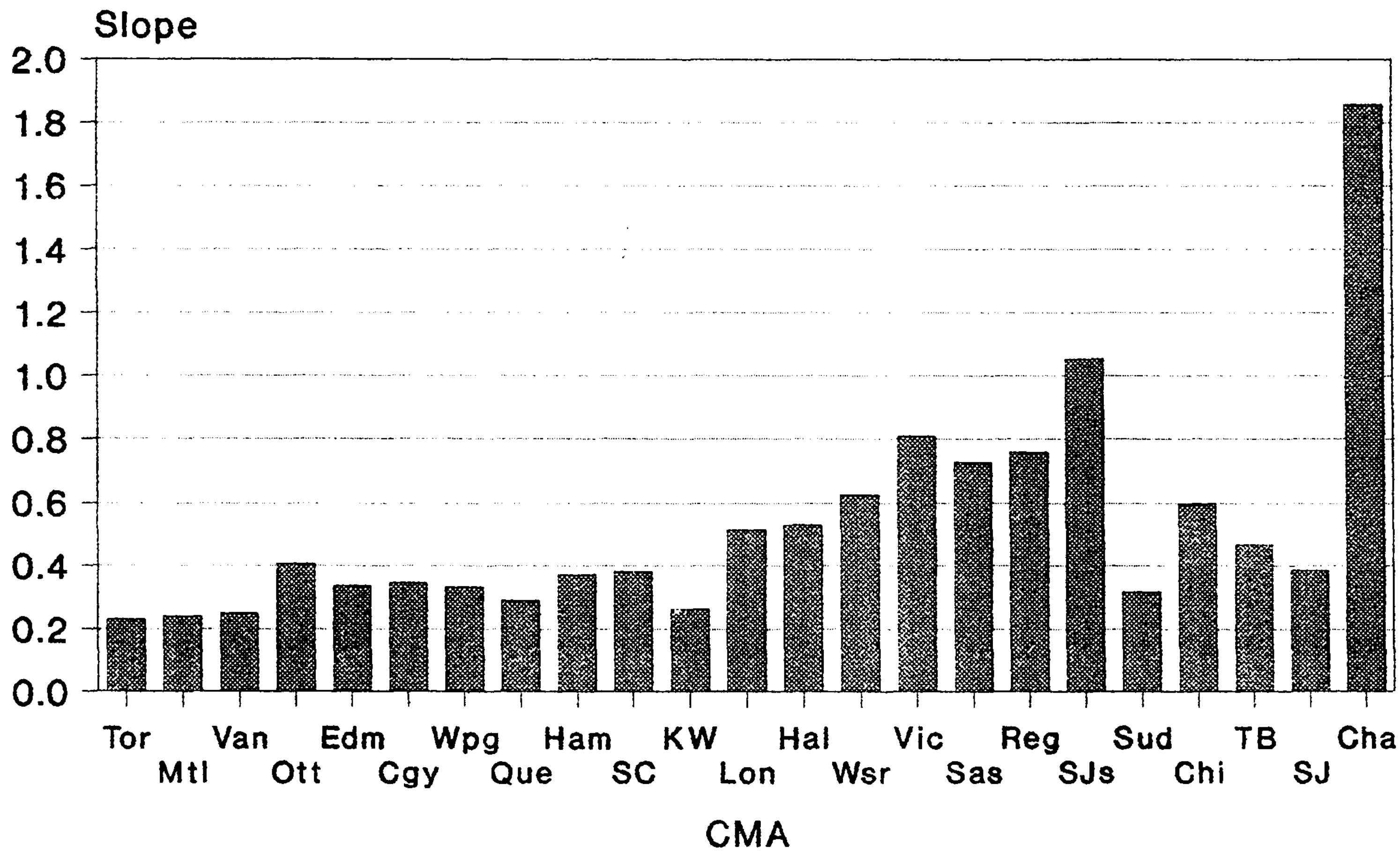
Edmonston employment gradient estimates are presented in Table 4.5 and Figure 4.12. Four CMAs have estimated employment gradients that are flatter than their population gradients (i.e. $\delta < \gamma$). In addition to Calgary, London and Winnipeg (the same three as in the Macauley estimates) Regina has an estimated δ < estimated γ.[35] The mean estimated δ for 1981 is 0.52543. This is 44 percent steeper than the mean population gradient (γ) and 13 percent larger than the mean Macauley estimate of δ. Individual estimates of δ range from 0.2308 in Toronto to 1.8560 in Charlottetown. The median value, 0.3848, is to the left of the mean by a large amount, indicative of a strongly right-skewed distribution. Regression of δ on total CMA population yields:

$$\boldsymbol{\delta} = 0.6359 + \underset{(9.09 \times 10^{-8})}{1.88 \times 10^{-7}}\, POP \qquad R^2 = .1693$$

The estimated equation indicates that an increase in population of 100,000 (i.e. 17 percent of the mean CMA population) would flatten the mean gradient by .0188 (i.e. 3.6 percent of the mean δ). This is almost identical to the result for the Macauley estimates. Thus, using Edmonston or Macauley estimates, δ appears to flatten more quickly than γ as CMA population increases. Smaller cities

[35] Essentially the population and employment gradients were equal in Regina. The difference occurs in the third decimal place. For the other three cities the population gradient estimate was 5-10 percent larger than the employment gradient estimate, regardless of which estimation technique was used.

FIGURE 4.12
EMPLOYMENT GRADIENT ESTIMATES
1981, EDMONSTON ESTIMATION TECHNIQUE



such as Sudbury, Chicoutimi, Thunder Bay and Saint John have inexplicably flat gradients based on the rank-size relationship between δ and CMA population, regardless of which gradient estimation technique is employed.

As was the case for estimates of γ, the Edmonston estimates of δ appear to be an order-preserving transformation of the Macauley estimates. Figure 4.13 demonstrates the close agreement between the rankings of δ for the two estimation methods. The similarity in rankings implies that Hamilton's wasteful commuting methodology should not be unduly sensitive to the technique used to estimate the population and employment gradients.

The average estimated employment density at the centre of Canadian CMAs is 8,933 employees per square kilometre (23,137 per mile) using Edmonston's method. This is 34 percent greater than the Macauley estimate. In percentage terms, the difference in the estimates average central density is three times greater than the difference in the estimate of average δ. In addition to the significant absolute difference in E_o, Edmonston estimates do not appear to be a strictly order preserving transformation of Macauley estimates of E_o. This is illustrated by Figure 4.14. The relationship between the ranking of estimates of D_o (Figure 4.4) appears to be much closer than the relationship for the estimates of E_o. The greatest discrepancies occur near the centre of the urban hierarchy (e.g. Hamilton, St. Catherines, Windsor and Halifax).

FIGURE 4.13
COMPARISON OF EDMONSTON AND MACAULEY EMPLOYMENT GRADIENT ESTIMATE RANKINGS

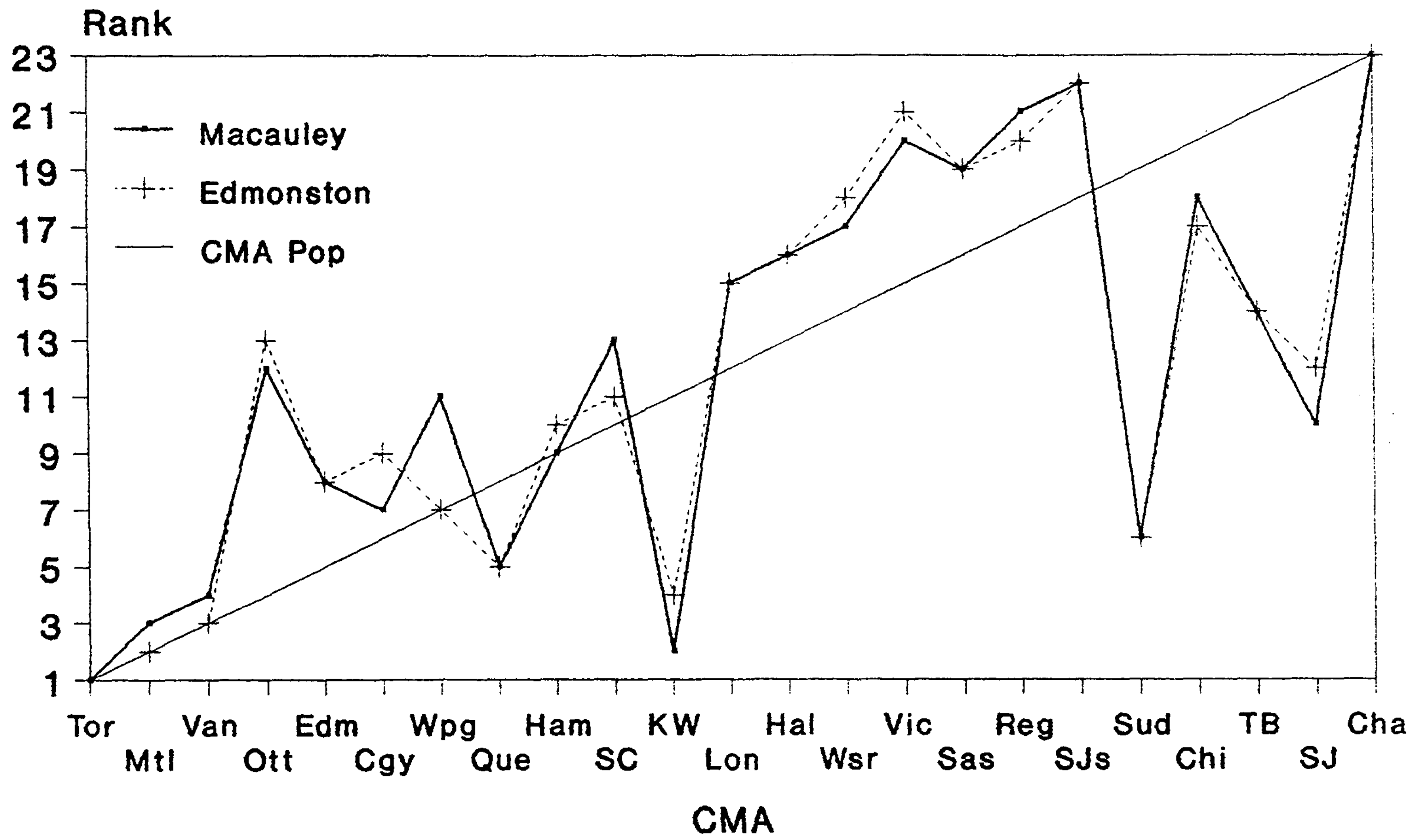
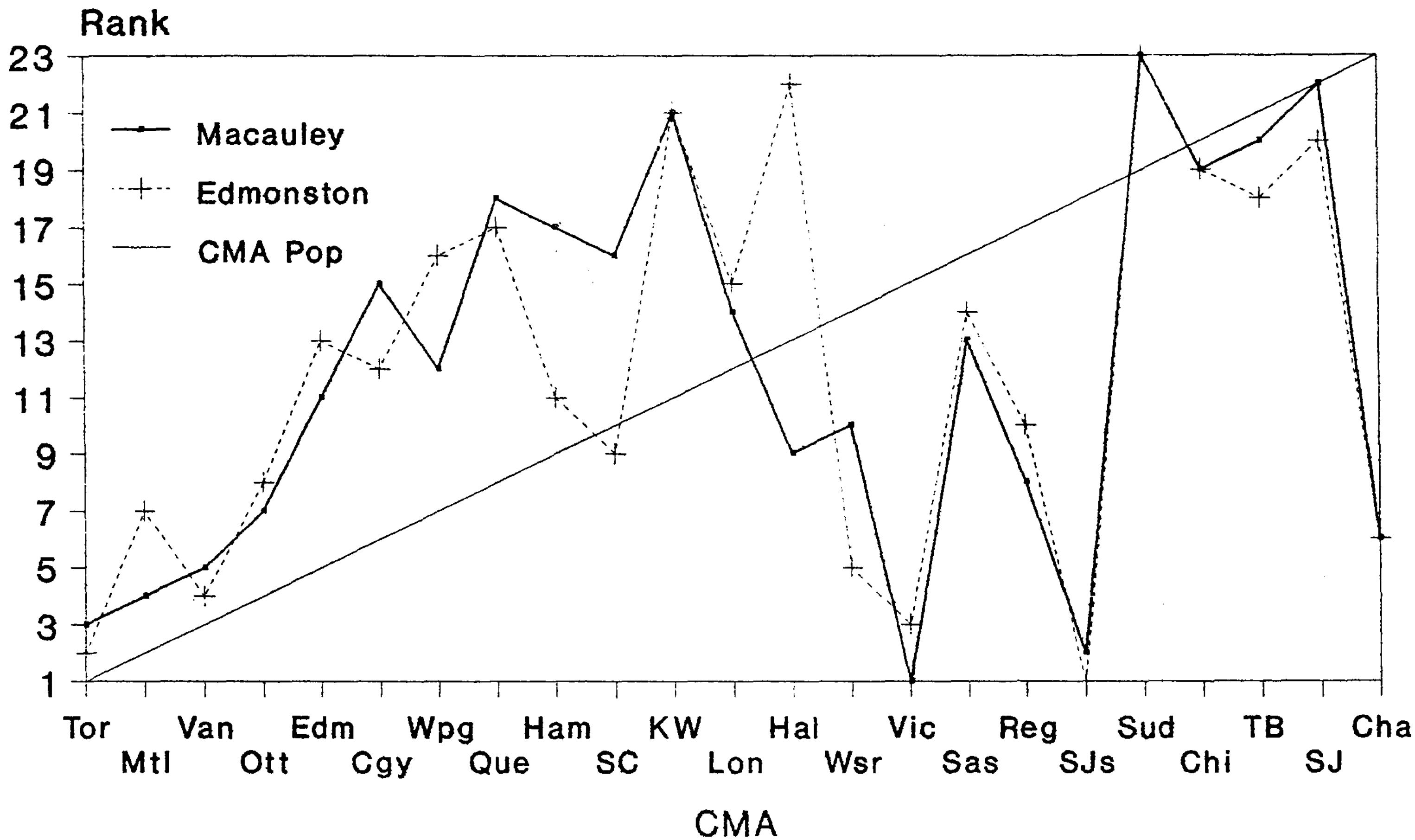


FIGURE 4.14
COMPARISON OF EDMONSTON AND MACAULEY CENTRAL EMPLOYMENT DENSITY RANKINGS



Once again, central employment density is positively related to total CMA population. Regressing E_0 on CMA population yields:[36]

$$E_0 = 6348.9 + 3.75 \times 10^{-3}\, POP \qquad R^2 = .1622$$
$$(1.83 \times 10^{-3})$$

The estimated equation indicates that an increase in CMA population of 100,000 would increase mean central employment density by 375 employees per square kilometre (i.e. a 17 percent increase in population yields a 4.2 percent increase in E_0, at the mean). Thus, E_0 is slightly more sensitive than δ to changes in total CMA population.[37]

4.3.3 Mills Estimates

Mills and Macauley estimates of employment gradients are almost identical. In all cases Mills estimates of δ are greater than or equal to Macauley estimates (Table 4.5). The mean difference is only .00175 or 0.4 percent of the mean estimated Macauley δ. Again, the only large differential is in Kitchener (13.6 percent). The result for estimates of E_0 is similar. The mean difference is only 17.1 employees per square kilometre, or 0.25 percent of the mean Macauley estimate of E_0.

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[36] Standard error in parentheses.

[37] The Edmonston and Macauley estimates of E_0 appear equally sensitive to changes in CMA population.

4.4 Summary

The purpose of this chapter was to present estimates of population and employment density gradients for 23 Canadian cities using data from the 1981 Census of Canada. The estimated parameters are key inputs for the model used to determine optimal and wasteful commuting behaviour. It was argued that two-point estimates of population density are at least as good as estimates obtained using alternative techniques. Furthermore, two-point estimation is the only technique available to estimate employment gradients, due to a lack of geographically disaggregated employment data.

Density gradients were estimated using three variations of the two-point method. The results are summarized in Table 4.6. The average estimated γ ranged from 0.32-0.36, while the mean estimate for δ ranged from 0.46-0.52. The average central population density ranged from 8,000 to 10,000 people per square kilometre and the average central employment density ranged from 6,600 to 9,000 employees per square kilometre.

Mills and Macauley estimates were virtually indistinguishable. Edmonston and Macauley estimates exhibited some significant differences. In all cases γ and δ were inversely related to CMA population, while D_o and E_o were directly related to CMA population. All parameter estimates were inelastic with respect to CMA population: sensitivities ranged from 0.15-0.20.

Table 4.6: Summary of Macauley and Edmonston Gradient Estimates

	Mean	Median	Elasticity With Respect to Population	Distribution Skew
		Macauley	Summary	
γ	0.3290	0.2835	-0.12	Right
D_o	8,041	9,021	+0.21	Left
δ	0.4666	0.3563	-0.20	Right
E_o	6,644	5,692	+0.25	Right
		Edmonston	Summary	
γ	0.3646	0.3031	-0.18	Right
D_o	10,294	9,756	+0.20	Right
δ	0.5254	0.3848	-0.21	Right
E_o	8,933	6,466	+0.25	Right

Results presented in this chapter were consistent with previous U.S. and Canadian estimates. However, several CMAs had estimated 1981 population gradients that were steeper than the estimated employment gradients. This is perverse and violates one of the key assumptions of the monocentric model.

CHAPTER 5

ESTIMATES OF WASTEFUL COMMUTING IN 23 CANADIAN CMAs

The purpose of this chapter is to compare optimal and actual commuting behaviour for 23 Canadian cities using a model developed by Hamilton [26].[1] When Hamilton applied his model to a sample of U.S. cities, he found aggregate commuting to be almost eight times the amount predicted by the simple monocentric model. On the strength of this result Hamilton questioned the validity of the monocentric model as a description of location behaviour in cities.

This chapter uses Hamilton's model to answer two basic questions:

- Is commuting behaviour in Canadian cities consistent with the predictions of the monocentric model?; and
- Do Canadian commuters behave in a manner significantly different from their U.S. counterparts after controlling for differences in the structure of urban areas?.[2]

Chapter 2 highlighted the importance of commuting behaviour in the monocentric model, or in any model that purports to explain urban residential and job choice location (pg. 1097) [61]. Monocentric models have been widely used in urban economics due to their simple

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[1] Hamilton's methodology was reviewed in detail in Chapter 3 of this thesis.

[2] Goldberg and Mercer [23] meticulously documented differences between cities in Canada and the United States. They noted U.S. cities tend to be more dispersed with lower central population density. A host of reasons are given for the differences. Even accepting that structural differences exist, does not necessarily imply that Hamilton's measure of waste, (D-C), should be greater for U.S. cities. Underlying structural differences would, perhaps, be reflected in larger estimates for A in the U.S. cities, but not necessarily in more "waste" if both jobs and homes are more suburbanized in the United States.

structure and perceived explanatory power. If the monocentric model fails to predict fundamental economic behaviour, such as commuting, the model must be deemed to have crossed the line between simplification and over-simplification. Use of a model cannot be justified by simplicity alone.[3]

Comparing the performance of the monocentric model in several different countries is also an important task. International comparison provides an excellent crucible in which to test the robustness of basic economic theories.[4] Comparing the monocentric model's performance in Canada and the United States can be interpreted as a weak test of theoretical robustness, because Canada and the United States are socially, culturally and politically similar.

The remainder of this chapter is comprised of five sections. Section 5.1 is presented as the base case. The Macauley population and employment gradients from Chapter 4 are used to calculate the

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[3] In describing his minimum requirements for a scientific model Stephen Hawking takes what he calls a simple minded view of what a model must do:

> "It must accurately describe a large class of observations on the basis of a model that contains only a few arbitrary elements, and it must make definite predictions about the results of future observations." Stephen Hawking, A Brief History of Time.

[4] Marc Bloch, an eminent European historian argued that:

> "Correctly understood the primary interest of the comparative method is...the observation of differences, whether they are original or the result of divergent developments from a common origin." Marc Bloch, Toward a Comparative History of European Society, in Enterprise and Secular Change.

optimum commute (C) for 23 Canadian cities. Estimates of optimum commute are compared with observed commuting behaviour. Section 5.2 adjusts the estimates of optimum commute to recognize the fact that most cities have a central area devoted almost exclusively to non-residential land uses. The third section of this chapter examines the sensitivity of estimates of optimum commute to the choice of urban boundary (F).[5] In Section 5.4, estimates of A, B and C, using Edmonston population and employment gradients, are presented. This is important in order to gauge the Hamilton model's sensitivity to the particular method used to estimate density gradient parameters. Section 5.5 concludes the chapter.

5.1 Measuring Waste Using Macauley Gradient Estimates: Base Case

Table 5.1 is strictly analogous to Hamilton's Table 1 (pg. 1041 [26]). The notation is identical to Hamilton's and was described in Chapter 3 of this thesis. All distances are in kilometres.

The average distance of each household from the CBD is given by A. This can also be interpreted as the average commute, if all employment is located in the CBD. The mean value for average distance of households from the CBD in the 23 Canadian cities is

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[5] In order to calculate A and B equations 3.9 and 3.15 were integrated to a finite distance, $\hat{x}$. Hamilton chose $\hat{x}$ to equal the distance at which population density, D(x), declined to 100 people per square mile. Given the arbitrary nature of Hamilton's choice, it was prudent to evaluate the impact different values of $\hat{x}$ had on estimates of A, B and C.

Table 5.1: Optimal and Actual Commute Characteristics Using 1981 Macauley Gradient Estimates

CMA	A	B	C	D	E	F	G
Calgary	5.466	5.896	-0.430	10.64	11.01	16.58	40.2
Charlottetown	2.689	1.381	1.306	4.21	3.19	7.17	16.0
Chicoutimi	8.209	3.434	4.775	8.30	13.45	17.71	21.2
Edmonton	6.596	6.088	0.509	10.49	12.23	19.27	37.8
Halifax	5.858	4.147	1.711	7.97	9.18	16.33	34.5
Hamilton	7.151	6.060	1.091	15.63	12.70	20.27	23.2
Kitchener	7.614	7.599	0.015	12.09	16.36	18.93	16.8
London	3.849	3.962	-0.113	8.42	7.04	11.79	23.5
Montreal	11.304	8.643	2.662	12.24	19.42	35.37	32.0
Ottawa	8.356	5.561	2.794	10.39	13.72	22.99	37.0
Quebec	9.258	7.189	2.070	11.29	16.72	24.02	31.8
Regina	2.657	2.645	0.011	8.54	4.36	8.32	34.0
Saint John	8.388	5.872	2.516	11.11	15.94	18.03	27.5
Saskatoon	3.362	2.844	0.519	7.96	5.40	9.81	39.8
St. Catherines	7.234	5.120	2.114	10.59	11.71	19.46	23.5
St. John's	3.747	2.134	1.614	8.19	4.61	11.28	25.5
Sudbury	7.679	6.211	1.469	13.62	15.80	16.70	29.0
Thunder Bay	4.840	4.372	0.468	10.13	8.76	12.42	30.0
Toronto	14.520	10.067	4.453	12.25	24.02	44.46	42.5
Vancouver	10.072	8.358	1.714	13.56	17.49	30.66	39.0
Victoria	5.150	2.877	2.273	10.65	6.79	15.21	17.5
Windsor	4.864	3.917	0.947	10.18	7.60	14.30	20.0
Winnipeg	5.104	5.517	-0.414	11.09	10.02	15.86	28.5
Sample Mean	6.69	5.21	1.48	10.41	11.63	18.56	29.2
Adj. Mean	6.98	5.23	1.75	10.47	11.97	19.14	28.9
Hamilton Mean	13.55	11.76	1.79	14.06	19.46	36.18	n.a.

NOTES: All distances in kilometres. A ≡ necessary commute if complete centralization of employment is assumed; B ≡ potential commute savings resulting from employment decentralization; C ≡ optimum commute (i.e. A - B); D ≡ observed mean commute from the 1977 urban concerns survey; E ≡ average commute randomly assigning jobs to houses; F ≡ radius at which population density declines to 38.61 people per square kilometre (i.e. 100 per square mile); G ≡ actual radius of the CMA; n.a. ≡ not available; Adj mean excludes Calgary, London, and Winnipeg because these CMA's had values for C less than zero.

6.69 kilometres. Excluding CMAs for which $B \geq A$ yields a slightly greater mean distance - 6.98 kilometres.[6] The mean for Hamilton's sample of U.S. cities was 13.55 kilometres. By itself, it is not particularly significant that the estimate of average distance of households from the CBD in Hamilton's sample was more than twice that for the Canadian cities. The cities included in Hamilton's sample were larger, on average, than the Canadian cities and it is reasonable to expect the average distance of households to increase with total CMA (SMSA) population. Regressing the estimate of average household distance from the CBD on CMA population confirms the expected relationship with total CMA population:[7]

$$A = 5.01 + \underset{(5.07 \times 10^{-7})}{2.87 \times 10^{-6}}\, POP \qquad R^2 = .6033$$

The estimated equation implies that an increase in CMA population of 100,000 (17 percent of the mean CMA population) would result in an increase in A of 0.287 kilometres (only 4 percent of the adjusted mean A).

In Table 5.1, B represents the average distance of jobs from the CBD. Hamilton referred to this distance as the potential average commute savings attributable to employment decentralization within the metropolitan area. The adjusted mean estimate for B for the 20 Canadian CMAs is 5.23 kilometres. Hamilton's mean estimate for the

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[6] When Calgary, London and Winnipeg are excluded it is referred to as the adjusted mean in Table 5.1.

[7] Standard error in parentheses.

average distance of jobs from the CBD was 11.76 kilometres. Like the average distance of households from the CBD (A), the average distance of jobs from the CBD (B) is positively related to total CMA population. Regressing the estimate of the average job distance from the CBD on total CMA population yields:

$$B = 3.97 + \underset{(4.08 \times 10^{-7})}{2.12 \times 10^{-6}}\, POP \qquad R^2 = .5624$$

The estimated equation implies that an increase in CMA population of 100,000 (17 percent of the mean CMA population) would result in an increase in the average distance of jobs from the CBD of 0.212 kilometres (4 percent of the adjusted mean B).

Figure 5.1 illustrates that cities most likely to diverge from the rank-size relationship (for either A or B) are second order centres (e.g. Edmonton, Calgary, Winnipeg), or very small cities (e.g. Sudbury, Chicoutimi, Thunder Bay).[8] The POP coefficients in the two regression equations estimated above are consistent with the notion that in Canadian cities population suburbanizes more rapidly than employment, as city size increases.[9]

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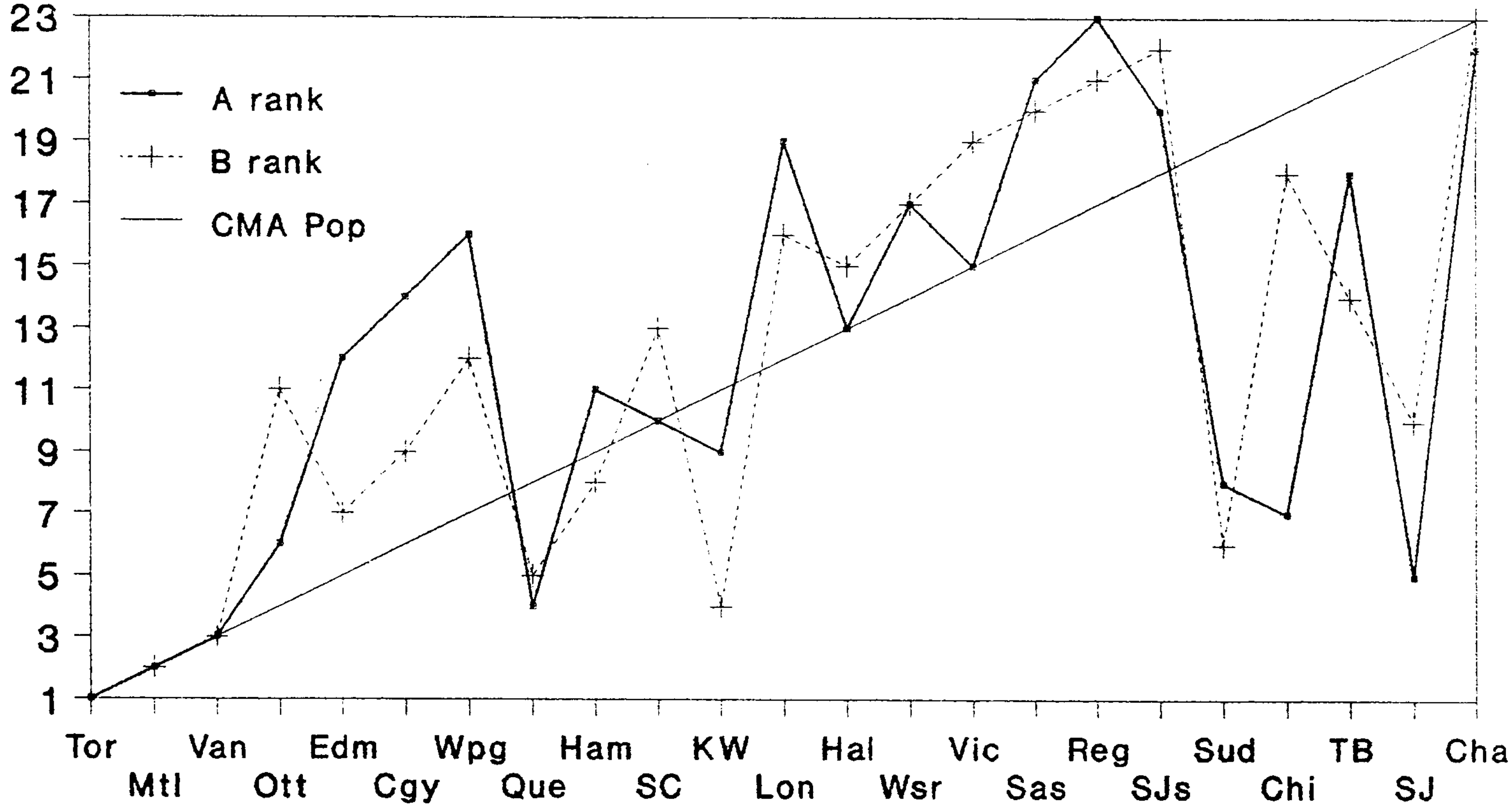
[8] The cities included in Figure 5.1, and all subsequent Figures throughout this chapter, are arranged from largest to smallest in terms of CMA population. The population rank is thus an upward sloping straight line, dividing the figure into two equal halves (triangles). A and B were also ranked from largest to smallest. A rank of 1 is assigned to the largest value of A or B, 23 to the smallest value.

[9] The POP coefficient in the first equation is 35 percent larger than in the second equation. This result arises because γ was more sensitive than δ to changes in CMA population. See Chapter 4 for details regarding this result.

FIGURE 5.1
RANKING OF AVERAGE HOUSEHOLD AND JOB DISTANCE FROM THE CBD BASED ON 1981 MACAULEY DENSITY GRADIENT ESTIMATES



In Table 5.1 C represents the difference between A and B, or the minimum possible average commute in each CMA. This distance (C) can be thought of as the average distance between homes and jobs in each city. Hamilton referred to it as the optimum commute and, despite the normative overtones of his nomenclature, the terminology is maintained in this thesis. The estimated (adjusted) mean optimum commute is 1.75 kilometres for the Canadian cities and 1.79 for Hamilton's sample of 14 U.S. cities. In contrast to the average distance of households and jobs from the CBD, C is not strongly related to total CMA population.

Consider the following regression of optimum commute on CMA POP:[10]

$$C = 1.04 + 7.46 \times 10^{-7} POP \qquad R^2 = .1765$$
$$(3.52 \times 10^{-7})$$

The longest estimated optimum commute occurred in Chicoutimi (4.77 kilometres) and the shortest in Regina (0.011 kilometre). Figure 5.2 illustrates the weak relationship between CMA population and C.

It would seem reasonable to expect people in larger cities to commute greater distances than people in smaller cities, on average. The mean observed commute for each Canadian city in 1977,

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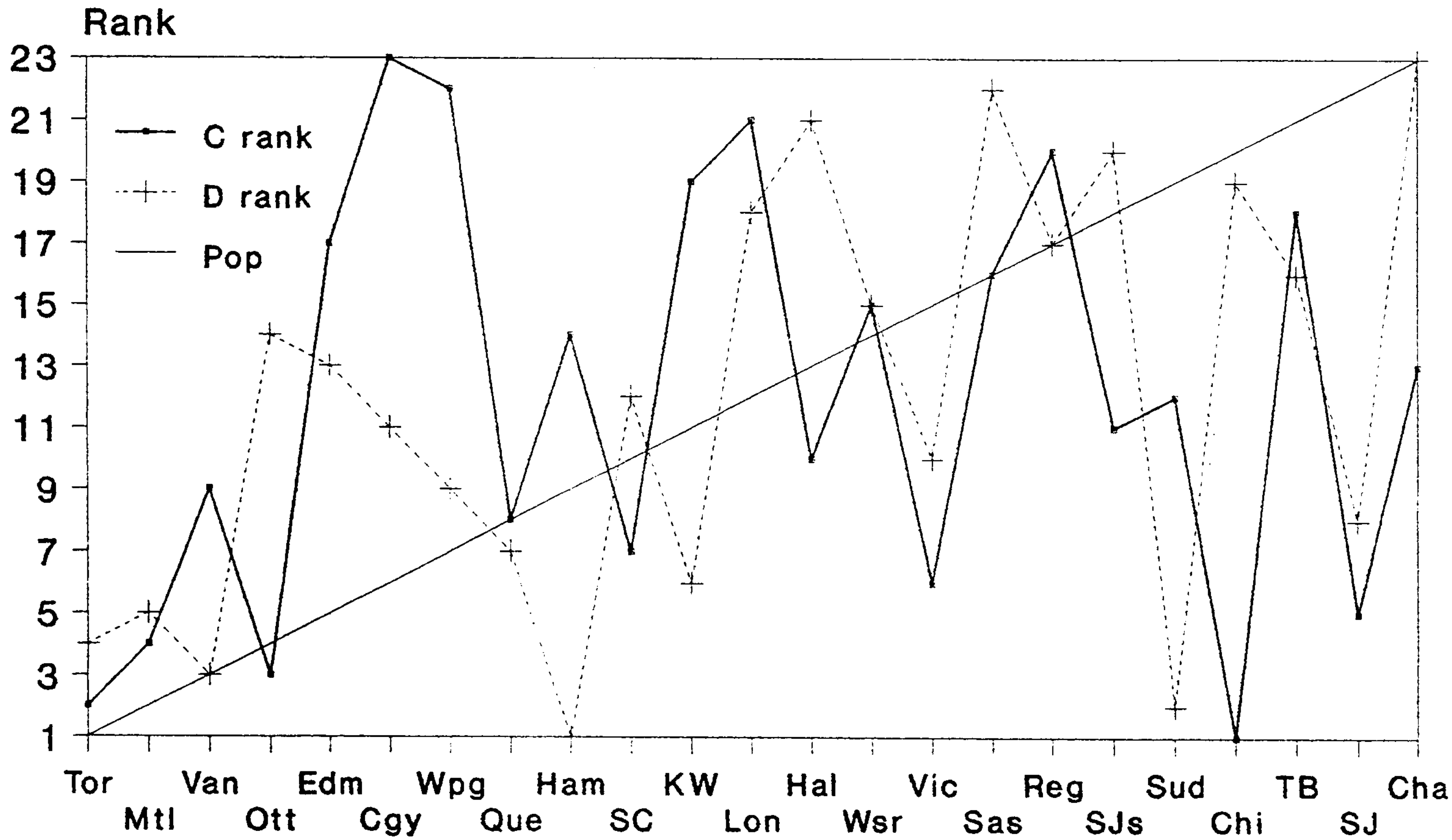
[10] Excluding Calgary, London and Winnipeg yielded a marginally stronger relationship:

$$C = 1.32 + 7.15 \times 10^{-7} POP \qquad R^2 = .2185$$
$$(3.19 \times 10^{-7})$$

Log-linear and log-log estimation yielded poorer results than the simple linear estimation.

FIGURE 5.2
RANKING OF OPTIMAL AND OBSERVED COMMUTE
1981 MACAULEY DENSITY GRADIENT ESTIMATES



(D), is given in Table 5.1. Contrary to expectation no obvious relationship between observed commuting distance and city population is discernable in Figure 5.2. Regressing observed commuting distance on population yields:

$$D = 9.69 + 1.22 \times 10^{-6} \, POP \qquad R^2 = .1626$$
$$(6.09 \times 10^{-7})$$

Unlike optimal commuting, log-linear and log-log regressions of observed and ln(D) on the natural logarithm of total CMA population results in a much better fit. Consider the following equations:

$$D = -7.42 + 1.40 \ln POP \qquad R^2 = .3555$$
$$(0.412)$$

$$\ln D = 0.27 + 0.16 \ln POP \qquad R^2 = .3825$$
$$(0.045)$$

Figure 5.3 illustrates the variability in both observed and optimal commuting distances independent of CMA population. The Hamilton CMA has the longest observed commute (15.63 kilometres) and Charlottetown the shortest observed commute (4.21 kilometres). Kitchener and Regina are two small CMAs with optimal commutes estimated near 0, and long observed average commutes (more than 8 kilometres).

5.1.1 Assessing the Monocentric Model

Comparing estimates of optimal and observed commuting distances provides an indirect test of the predictive power of the monocentric model. Blackley and Follain [7] viewed Hamilton's work as part of a growing body of indirect tests of the monocentric model. Such indirect tests focus on the monocentric model's ability (or inability) to predict important spatial patterns. The tests are considered indirect because they focus upon the monocentric model's reduced form.

In this study, the monocentric model did a poor job of predicting commuting behaviour for the 23 Canadian CMAs, as illustrated by Figure 5.3. Table 5.2 summarizes five different measures of the monocentric model's performance. The five measures are defined as:

- (D - C) ≡ gross one way difference between the average observed and average optimal commute. Larger values indicate more waste.
- (D/C) ≡ the ratio of average observed to average optimal commute. For example, Vancouver's observed commute was 7.9 times the optimal commute.
- (C/D) ≡ the ratio of average optimal to average observed commute expressed in percentage terms. While this measure is simply the inverse of D/C it has two advantages: first, it avoids extreme values associated with small C's (e.g. Kitchener, Regina) and second, it has an easily understood interpretation: it is the percentage of actual commuting that can be accounted for by the separation of home and work. Smaller values indicate more waste.
- (A/D) ≡ the ratio of optimal to actual commute under the assumption of completely centralization of employment.
- (D/E) ≡ the ratio of average observed commute to average random commute. Random commute was determined by assigning households to jobs randomly.[11]

By each measure the monocentric model did an almost unbelievably poor job of predicting observed commuting behaviour in 23 Canadian CMAs in 1981. The average wasteful commute (one way), (D - C), is

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[11] See Hamilton (pg. 1042) [25], [26] for details on the computation of random commute.

FIGURE 5.3
OPTIMUM VERSUS OBSERVED COMMUTE
1981 MACAULEY DENSITY GRADIENT ESTIMATES

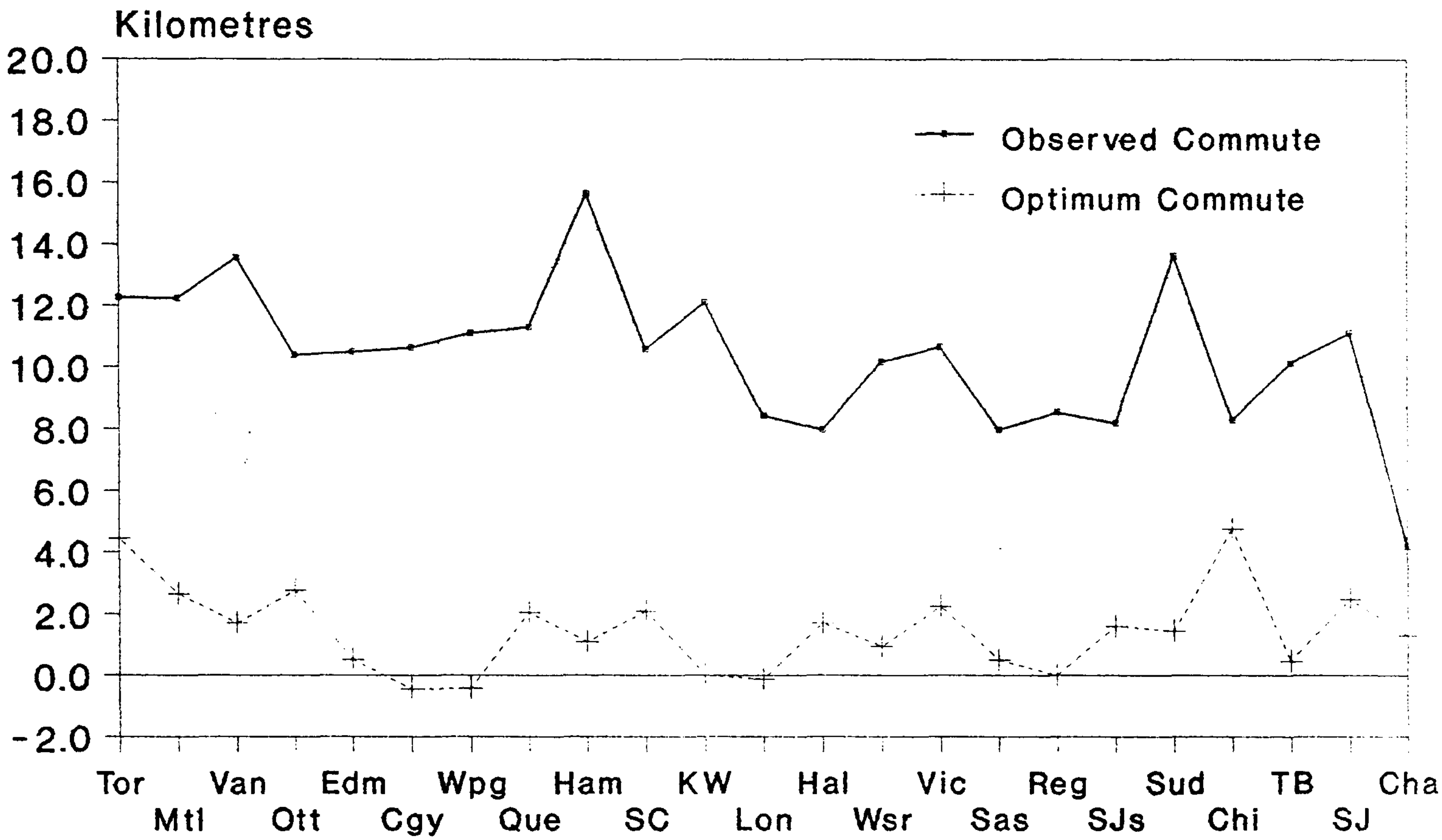


Table 5.2: Alternative Measures of Wasteful Commuting Derived From Estimates in Table 5.1

CMA	D-C	D/C	C/D	A/D	D/E	F/G	Total
	kms	#	%	%	%	#	Population
Calgary	11.07	24.8	4.0	51.7	96.7	0.41	625,966
Charlottetown	2.91	3.2	31.0	63.8	132.0	0.44	44,999
Chicoutimi	3.53	1.7	57.5	98.9	61.7	0.84	135,172
Edmonton	9.98	20.6	4.9	62.9	85.7	0.51	657,057
Halifax	6.26	4.7	21.5	73.5	86.8	0.47	277,727
Hamilton	14.54	14.3	7.0	45.7	123.1	0.87	542,095
Kitchener	12.07	822.0	0.1	63.0	73.9	1.13	287,801
London	8.53	74.3	1.3	45.7	119.6	0.50	283,668
Montreal	9.57	4.6	21.8	92.4	63.0	1.10	2,828,349
Ottawa	7.60	3.7	26.9	80.4	75.7	0.62	717,978
Quebec	9.22	5.5	18.3	82.0	67.5	0.76	576,075
Regina	8.52	742.8	0.1	31.1	195.8	0.24	173,226
Saint John	8.60	4.4	22.6	75.5	69.7	0.66	114,048
Saskatoon	7.44	15.3	6.5	42.2	147.3	0.25	175,058
St. Catherines	8.48	5.0	20.0	68.3	90.5	0.83	304,353
St. John's	6.58	5.1	19.7	45.8	177.7	0.44	154,820
Sudbury	12.15	9.3	10.8	56.4	86.2	0.58	149,923
Thunder Bay	9.66	21.6	4.6	47.8	115.6	0.41	121,379
Toronto	7.79	2.8	36.4	118.6	51.0	1.05	2,998,947
Vancouver	11.85	7.9	12.6	74.3	77.6	0.79	1,268,183
Victoria	8.37	4.7	21.3	48.4	156.7	0.87	233,481
Windsor	9.23	10.8	9.3	47.8	134.0	0.72	246,110
Winnipeg	11.50	26.8	3.7	46.0	110.7	0.56	584,842
Sample Mean	8.93	68.9	14.9	63.6	104.3	0.64	n.a
Mean w/o -C's	8.72	85.5	17.6	65.9	103.6	0.68	n.a.
Adjusted Mean	n.a.	8.07	19.6	n.a.	n.a.	n.a.	n.a.
Hamilton Mean	12.28	12.14	12.6	95.9	78.3	n.a.	n.a.

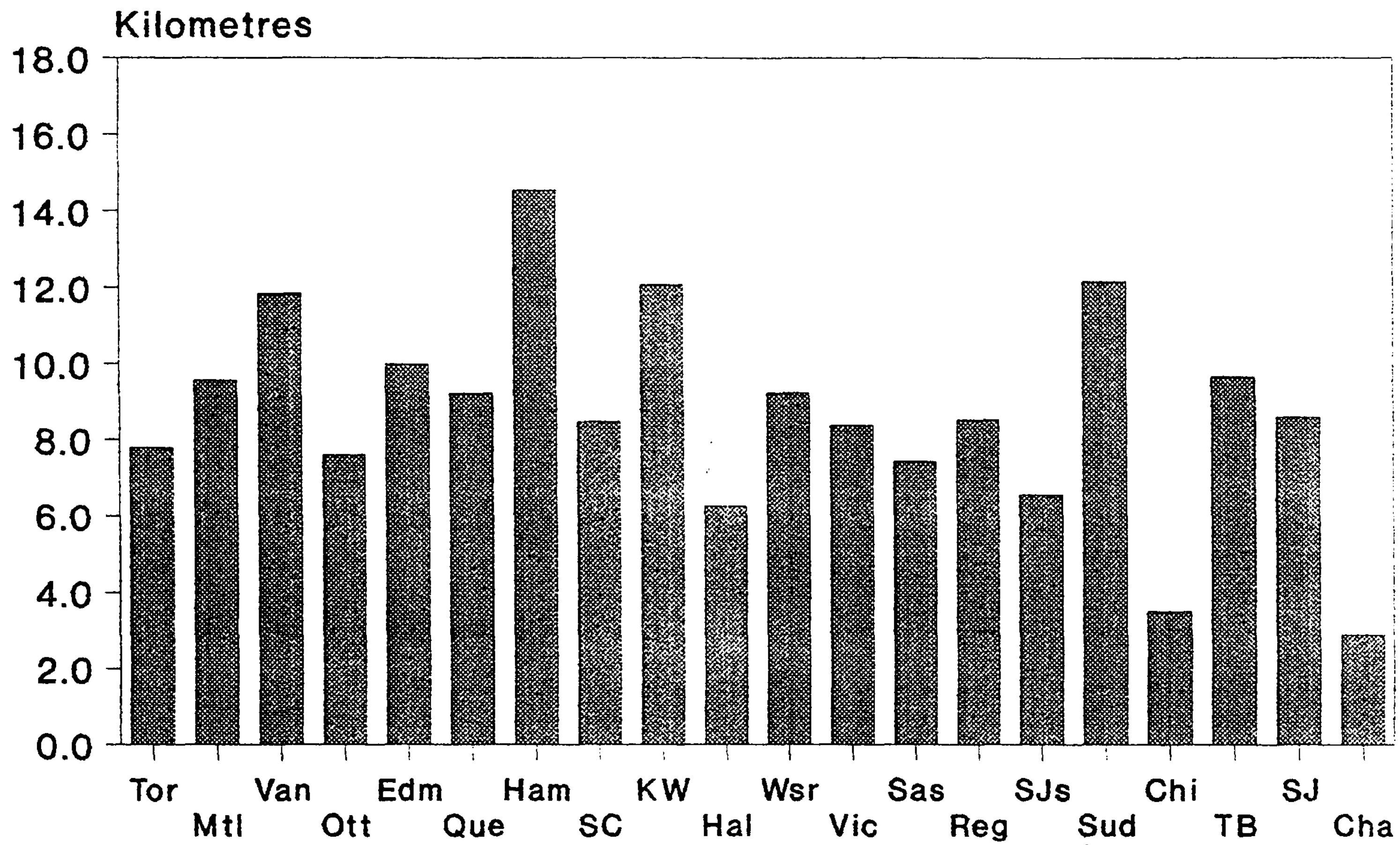
NOTES: The mean w/o CMAs with negative estimated C values excludes Calgary, London and Winnipeg; the adjusted mean excludes Kitchener and Regina in addition to the three CMA's excluded from mean w/o -C's; n.a. ≡ not applicable.

8.72 kilometres, (Table 5.2). The largest values for (D - C) are in Hamilton (14.54 kilometres), Kitchener (12.07 kilometres.), Sudbury (12.15 kilometres) and Vancouver (11.85 kilometres.), (Figure 5.4).

Expressed another way, observed commuting in the Canadian cities is an estimated 68.9 times greater than that predicted by the monocentric model. This result is heavily influenced by Kitchener and Regina where the optimal commuting distance ≈ 0. Even after excluding Kitchener, Regina and the three CMAs with optimal commuting distances less than zero, (D/C) is 8.07. Put differently, the monocentric model is able to account for only 19.6 percent of observed commuting, on average, (C/D, Table 5.2). The greatest share of observed commuting that the monocentric model explains in any Canadian city is 58 percent in Chicoutimi (Figure 5.5). In Toronto, approximately 36 percent of observed commuting is accounted for by the separation of homes and jobs.

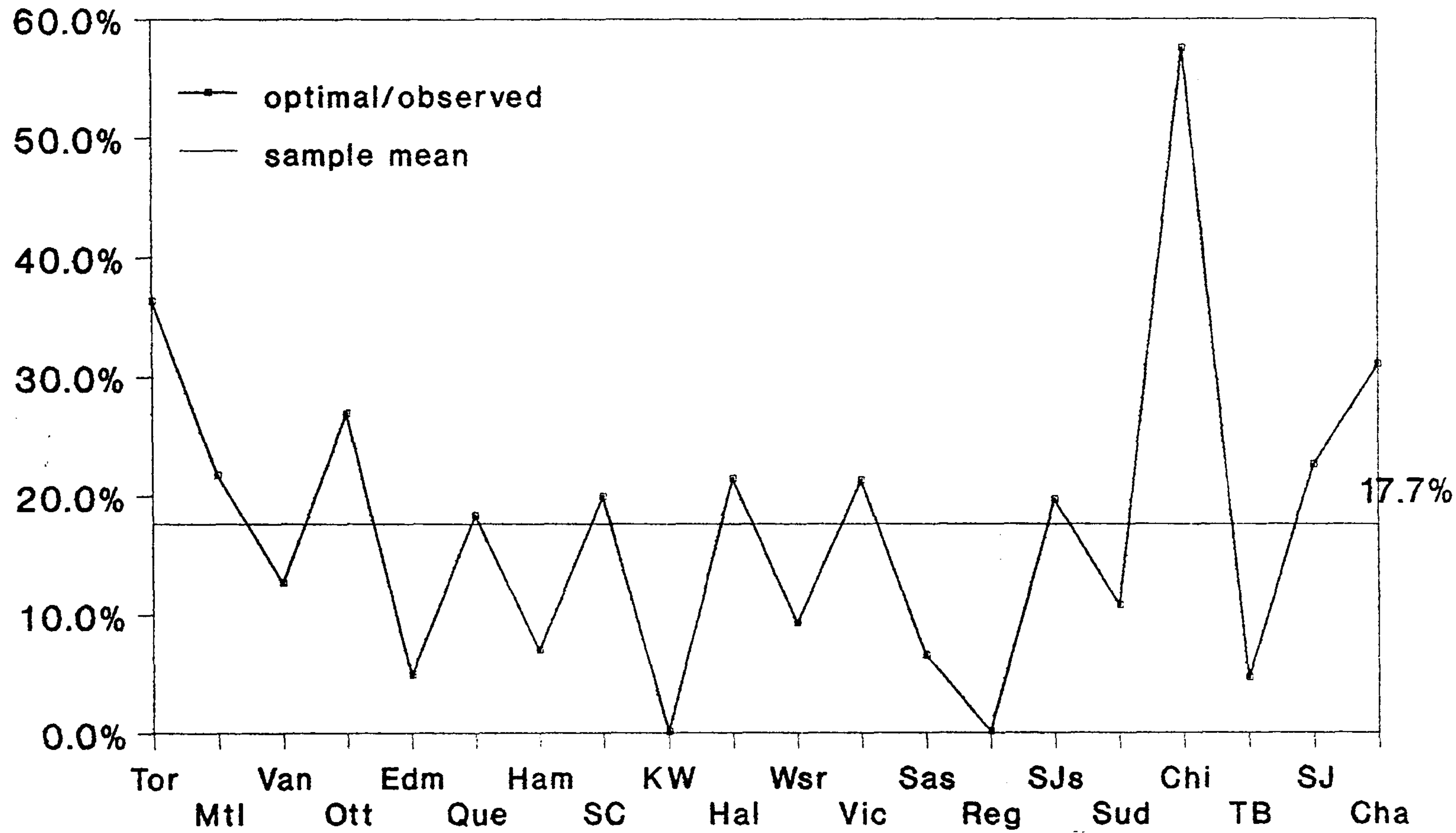
The abject failure of the monocentric model is further illustrated by comparing the average distance of households from the CBD and observed commuting distance (Table 5.2). (A/D) represents the ratio of the optimal to observed average commute under the assumption that all 1981 metropolitan employment is located in the in the CBD. Appendix 2 indicates that this is clearly an oversimplification. On average, 25 percent total 1981 CMA employment was outside the central city, (CC) and central city is a much broader geographic concept than the CBD. Even under the clearly extreme assumption of total employment centralization, the

FIGURE 5.4
AVERAGE ONE WAY WASTEFUL COMMUTE USING 1981 MACAULEY DENSITY GRADIENT ESTIMATES *



* Calgary, Winnipeg and London are excluded because C < 0.

FIGURE 5.5
OPTIMAL AS A SHARE OF OBSERVED COMMUTE
1981 MACAULEY DENSITY GRADIENT ESTIMATES *



* Calgary, Winnipeg and London are excluded because C < 0.

monocentric model only accounted for two-thirds of observed commuting (65.9 percent, Table 5.2).

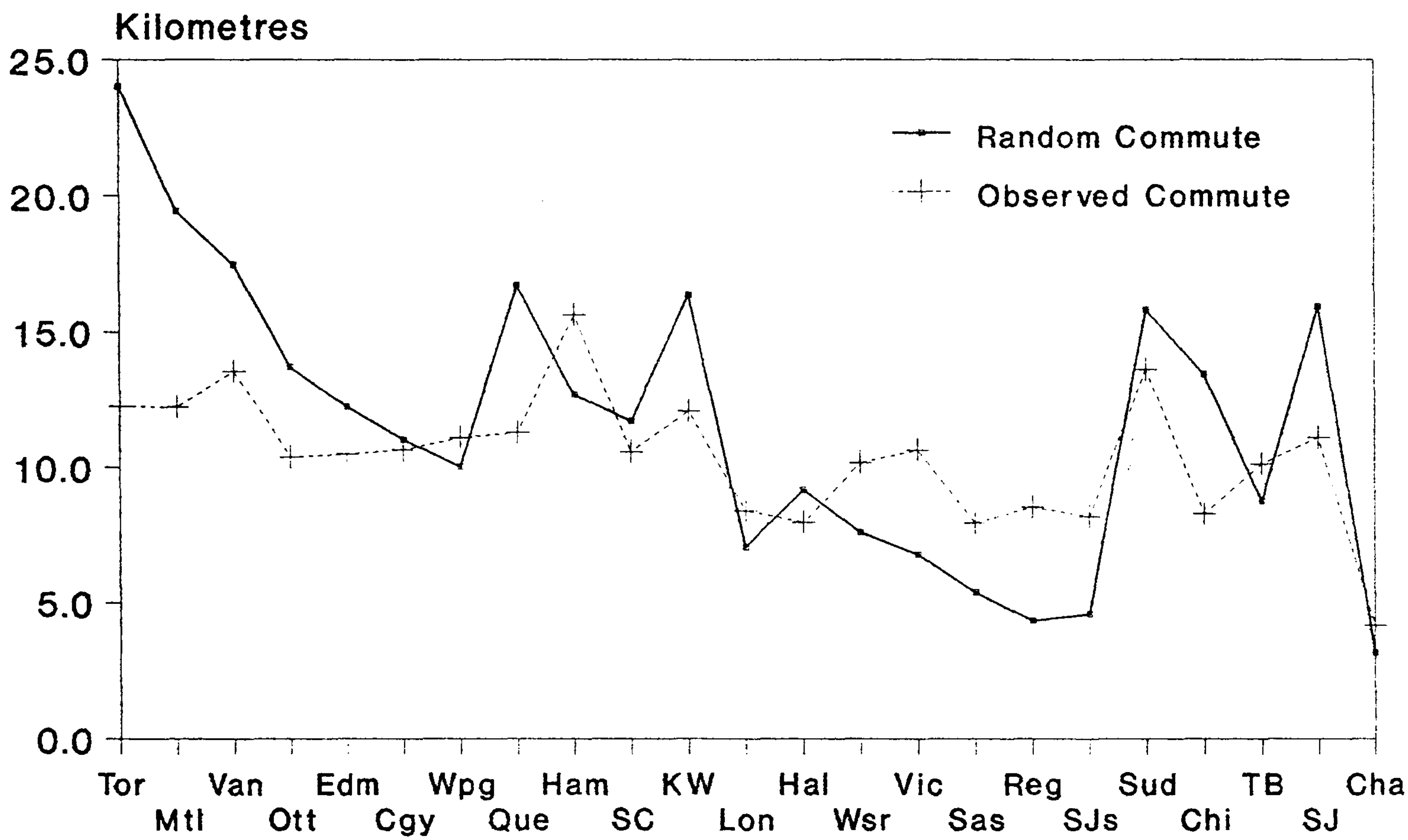
The final indication of the monocentric model's poor performance is provided by comparing the average observed and the average random commute. The average observed commuting distance in the Canadian CMAs was far more accurately predicted by the assumption that commuting is a random behaviour, than by the monocentric model.[12] Assuming that households choose their home and job locations randomly resulted in an average random commute, (E), of 12 kilometres (Table 5.1). (D/E) in Table 5.2 indicates the average observed commute was 103.6 percent of the average random commute. Despite the close mean values for observed and random commuting distance the (D/E) variable exhibits wide variation among cities. Observed average commute exceeds random commute in seven of 23 Canadian CMAs (Figure 5.6). In some CMAs observed commuting is 1½-2 times greater than the random commute (e.g. D/E: Regina 195.8, St. John's 177.7, Victoria 156.7, Saskatoon 147.3 and Charlottetown 132.0). Other cities had values of D only ½ that of E (e.g. D/E: Toronto 51.0, Chicoutimi 61.7 and Montreal 63.0).

5.1.2 Comparing Canadian and American Cities

One implicit thesis of Goldberg and Mercer's The Myth of the North American City was Canadian cities are more efficient than U.S. cities, because cities in Canada have a more compact internal

[12] See Chapter 3 for details regarding the random commuting model and the derivation of E.

FIGURE 5.6
OBSERVED VERSUS RANDOM COMMUTE
1981 MACAULEY DENSITY GRADIENT ESTIMATES



structure [23]. Comparison of the estimates of wasteful commuting presented in Table 5.2 with Hamilton's results [26] provides an excellent opportunity to test Goldberg and Mercer's hypothesis that Canadian cities are more efficient.[13]

The estimated mean distance of households from the CBD, (A), for the U.S. sample of cities is almost twice that for the Canadian CMAs (Table 5.3). The difference is statistically significant at the α = .01 level, (t=5.49). By itself, a difference in the average distance of households from the CBD is not very illuminating. Recall that previously it was demonstrated that the average distance of households from the CBD exhibits a strong positive relationship with total metropolitan population. Results for the average distance of jobs from the CBD are similar. Given that the U.S. cities were larger, on average, the average distance of households and jobs from the CBD was expected to be larger, as well. In contrast, the mean estimated optimal commuting distance for the Canadian CMAs is 97 percent of the U.S. estimate of optimal commuting distance, despite the fact that Canadian estimates of the distance of households and jobs from the CBD are only 50 percent and 43 percent of the respective U.S. values. The average estimated optimum commute is 1.79 kilometres for Hamilton's U.S. cities and 1.75 kilometres for the 20 Canadian CMAs. The difference in C is not statistically significant (t=0.112). This is an interesting result. According to the monocentric model, even

[13] All subsequent comparisons are based upon Hamilton's sample of 14 cities and a Canadian sample of 20 cities (i.e. Calgary, London and Winnipeg are excluded) for the purposes of comparison.

Table 5.3: Summary Statistics Comparing Canadian and American Estimates of Urban Structure and Commuting Behaviour

	Hamilton Sample			Canadian Sample			Diff	t-statistic	Significance	
	Mean	σ^2	σ	Mean	σ^2	σ	of Means	H_0: Diff=0	.95	.99
A	13.55	12.70	3.56	6.98	8.56	2.93	6.57	5.49	X	X
B	11.76	9.98	3.16	5.23	5.32	2.31	6.53	6.37	X	X
C	1.79	0.56	0.75	1.75	1.58	1.26	0.04	0.11		
D	14.07	1.43	1.20	10.47	6.09	2.47	3.60	5.48	X	X
E	19.46	22.26	4.72	11.97	30.29	5.50	7.49	4.12	X	X
D-C	12.28	1.53	1.24	8.72	7.29	2.70	3.56	5.02	X	X
C/D	12.65	43.66	6.61	17.65	180.58	13.44	5.00	1.39		
A/D	95.85	809.27	28.44	65.93	445.70	21.11	29.92	3.23	X	X
D/E	78.25	478.77	21.88	103.59	1651.78	40.64	25.34	2.27	X	

Notes: σ^2 ≡ variance; σ ≡ standard deviation; t-statistics calculated under the null hypothesis that the difference between the two samples was zero using a two tailed difference of means test; degrees of freedom = 32.

though the U.S. cities are more dispersed than Canadian cities, (higher values for A), U.S. cities do not necessitate increased commuting, because jobs appear to have followed people to suburban locations, (higher values for B).[14]

Both Hamilton's results, and the results for Canadian CMAs, indicate that, in 1981, the average observed commute exceeded the average optimum commute by a large amount. The mean observed commute for the American sample of cities is 3.6 kilometres greater than for the Canadian CMAs. This difference is statistically significant at the α = .01 level (t=5.48). In order to assess wasteful commuting in the two countries, values for (C/D) are compared. Approximately 12.7 percent of observed commuting in Hamilton's sample is necessitated by the separation of home and work. The corresponding figure for the Canadian sample is 17.7 percent. The difference between the estimates of (C/D) is not statistically significant even at the α = .10 level (t=1.39). Employing a one-sided t-test, the estimate of (C/D) for the Canadian CMAs is significantly greater (at the α = .10 confidence level), than Hamilton's estimate of (C/D).[15] Thus, commuters in Hamilton's U.S. cities may be marginally more profligate than commuters in the Canadian CMAs, but the results presented in Table 5.3 are not conclusive.

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[14] The result implies that the U.S. cities in Hamilton's sample are not innately less efficient due to their internal structure. This is counter to some of the arguments contained in Goldberg and Mercer's [23] analysis.

[15] For a one-sided test the critical t-value, at the 90 percent confidence level with 32 degrees of freedom, is 1.31.

One reason for the inconclusive result is probably the large variation in city size in the Canadian sample. The estimates of waste (D - C) and (C/D) for the Canadian CMAs have very large standard deviations (Table 5.3). This is likely due to greater variation in the size of cities included in the Canadian sample. CMA population ranges from Toronto, near 3,000,000, to Charlottetown at less than 100,000. The large variance in C and (C/D) may account for the lack of a statistically significant difference between Canadian and U.S. estimates of commuting waste.

5.2 Adjusting for the Areal Extent of the CBD

The CBD in the monocentric model is like a black hole; it is a single point in space without internal dimension, into which all economic activity is drawn. Not only is this theoretically unsatisfying, it also introduces a potential source of bias into estimates of the population density gradient. Gradient estimates obtained using the negative exponential function yield maximum population density at the dimensionless CBD. In reality, the central area of most cities is devoted, almost exclusively, to non-residential land uses. Hamilton [pg. 1044, 26] referred to this phenomena as a crater in the density function.

It is possible to choose an alternative functional form for the population gradient that allows for a density profile with a central crater. Rather than complicate his model, Hamilton simply assumed the CBD in each CMA was one mile in diameter and devoid of residences. This has the effect of increasing the average household's distance to the CBD (i.e. A), by one half mile. Optimum commute also increases by one half mile, because the employment gradient and the average distance of jobs from the CBD are unaffected by the crater in the population gradient.

Although Hamilton's adjustment was simple it was clearly ad hoc. Since economic theory does not strongly support any particular functional form for the population gradient, Hamilton could have chosen an alternative function. McDonald and Bowman [40] suggested several alternative equations, such as standardized normal or quadratic, that allow for the possibility of a central density crater.[16] However, given that many applications of the monocentric model employ a negative exponential function, Hamilton was correct to employ the same function for his critique of the usefulness of the monocentric model.

Perhaps the greatest weakness in choosing an arbitrary value for the diameter of the CBD is that it assumes the same size central area for each city. While a CBD diameter of 1.0 kilometre may be reasonable for larger cities in the Canadian sample, it is probably too large for some of the smaller cities. However, the same criticism, (i.e. assuming one size fits all), must be extended to a great body of urban literature. It should be stressed that the

[16] Mills and Hamilton [42] and Chapter 2 of this thesis showed that the negative exponential density gradient requires the rather unrealistic assumptions of Cobb-Douglas housing production functions and a price elasticity of demand for housing equal to negative 1.

Table 5.4: Alternative Measures of Wasteful Commuting Assuming a CBD Radius of 1.0 Kilometres

	VALUE			RANK			
	D/C	C/D	A/D	D/C	C/D	A/D	POP
	#	%	%	#	#	#	#
Calgary	18.66	5.36	60.77	2	22	14	6
Charlottetown	1.83	54.74	87.57	22	2	6	23
Chicoutimi	1.44	69.54	110.89	23	1	2	20
Edmonton	6.95	14.38	72.42	7	17	11	5
Halifax	2.94	34.02	86.08	19	5	7	13
Hamilton	7.48	13.37	52.14	6	18	22	9
Kitchener	11.91	8.39	71.26	3	21	12	11
London	9.50	10.53	57.58	4	20	19	12
Montreal	3.34	29.92	100.56	15	9	3	2
Ottawa	2.74	36.51	90.03	20	4	5	4
Quebec	3.68	27.20	90.89	13	11	4	8
Regina	8.44	11.85	42.84	5	19	23	17
Saint John	3.16	31.64	84.47	17	7	8	22
Saskatoon	5.24	19.08	54.81	10	14	21	16
St. Catherines	3.40	29.40	77.73	14	10	10	10
St. John's	3.13	31.91	57.96	18	6	15	18
Sudbury	5.52	18.13	63.73	9	15	13	19
Thunder Bay	6.90	14.50	57.67	8	16	17	21
Toronto	2.25	44.53	126.73	21	3	1	1
Vancouver	5.00	20.01	81.64	12	12	9	3
Victoria	3.25	30.74	57.77	16	8	16	15
Windsor	5.23	19.13	57.62	11	13	18	14
Winnipeg	18.92	5.29	55.01	1	23	20	7
Sample Mean	6.13	25.22	73.83	na	na	na	na
Hamilton Mean	5.69	19.83	103.04	na	na	na	na

base case assumes the same size CBD in all cities: zero. One advantage of assuming a one kilometre radius is that it eliminates the negative values for optimal commuting distance in three Canadian CMAs.

In order to simulate a non-residential central area, 1.0 kilometre is added to each CMA's estimated average distance of households from the CBD. For example, in Vancouver the average increases from 10.07 kilometres to 11.07 kilometres; the average distance of jobs from the CBD is unchanged and the optimum commute increases from 1.71 to 2.71 kilometres. The increase in the optimum commute makes cities appear less wasteful. New values for D/C, C/D and A/D are presented in Table 5.4. The average observed commute for all cities declines from 8.7 times the average optimal commute to only 6.1 times optimal, (D/C). The monocentric model now explains 25 percent of observed commuting, (C/D). Under the strong assumption of completely centralized employment, the monocentric model is able to explain 73.8 percent of 1981 observed commuting in the 23 Canadian CMAs.

Table 5.4 ranks commuting waste in the 23 Canadian cities. Winnipeg, Calgary and Kitchener rank as the most wasteful cities with observed commuting at least 11 times the optimal commute.[17] Chicoutimi, Charlottetown and Toronto rank as the most efficient Canadian cities with 69, 54 and 44 percent of observed commuting

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[17] D/C and C/D are opposite sides of the same coin. **Higher** values for D/C indicate greater wasteful commuting, while **lower** values for C/D indicate greater waste. Thus, Winnipeg ranked 1st for D/C and 23rd for C/D.

explained by the separation of houses and jobs.

Two patterns in the rankings warranted further investigation. Figure 5.7 illustrates that there is no strong linear relationship between CMA population and the CMA's wasteful commuting rank. In fact, comparison Figures 5.7 and 5.3 vindicates Hamilton's methodology demonstrateing the importance of controlling for urban structure when assessing commuting behaviour. While the largest CMAs have some of the longest observed commutes, the largest CMAs are not among the most wasteful. The four largest CMAs, (Toronto, Montreal Vancouver and Ottawa), and three of the smallest CMAs (Charlottetown, Saint John and Chicoutimi), are among the most efficient cities.

In order to explore the relationship between metropolitan population and wasteful commuting in more detail the possibility of a non-linear relationship was investigated. Table 5.5 summarizes the results of regressing several various measures of commuting waste on CMA population and the natural log of CMA population. Linear estimations yield weak results. Log-linear and log-log estimates with (C/D) and (D/C) as the dependant variables do not yield better results. Both log-linear and log-log regressions of (D - C) on CMA population yield improved results, compared with other alternative specifications. This latter result is not unexpected because it was shown above that the average observed commute is related to total CMA population in a non-linear fashion.

FIGURE 5.7
RANKING OF WASTEFUL COMMUTING (C/D) VERSUS CMA POPULATION

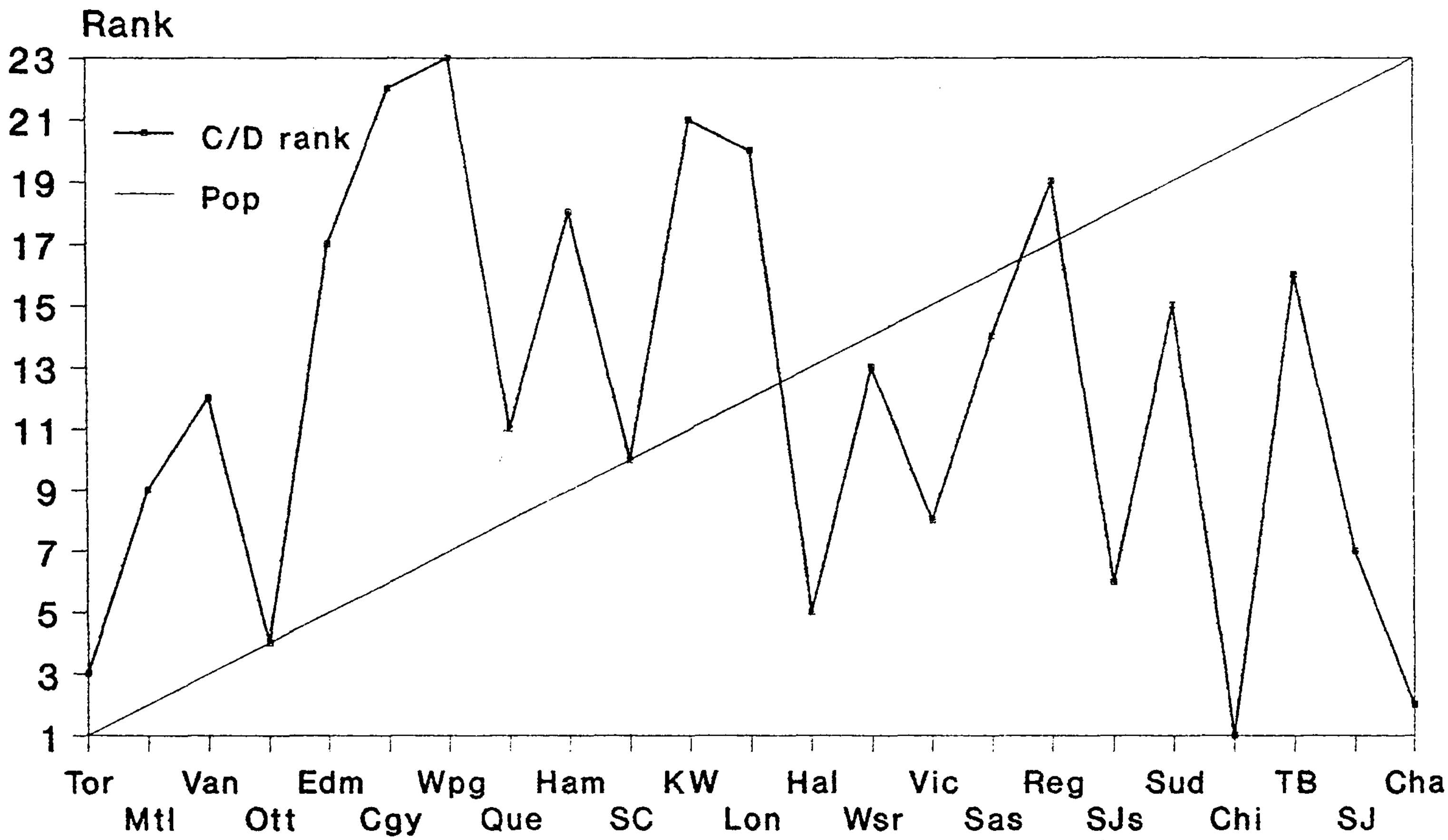

Table 5.5: Regression Estimates of the Relationship Between CMA Population and Wasteful Commuting

Dependant Variable	Intercept Term	Coefficient Estimate	Independant Variable	Standard Error	R-squared
D-C	8.65	4.84×10^{-7}	POP	7.37×10^{-7}	$R^2 = .020$
C/D	13.43	3.93×10^{-6}	POP	3.73×10^{-6}	$R^2 = .050$
D/C	106.34	-4.39×10^{-5}	POP	6.17×10^{-5}	$R^2 = .024$
D-C	- 5.22	1.11	ln POP	0.52	$R^2 = .179$
C/D	19.13	-0.27	ln POP	2.95	$R^2 = .000$
D/C	435.27	-27.87	ln POP	47.5	$R^2 = .016$
ln(D-C)	-0.07	0.17	ln POP	0.07	$R^2 = .228$
ln(C/D)	1.62	0.14	ln POP	0.34	$R^2 = .008$
ln(D/C)	4.32	-0.14	ln POP	0.34	$R^2 = .008$

The implication in the context of Hamilton's model appears to be that the monocentric model does not perform well over a certain range of city sizes. There may be other variables that better explain variations in wasteful commuting and thus that account for the poor performance of the monocentric model.

5.3 Sensitivity to the Choice of Urban Boundary

In order to calculate the average distance of households and jobs from the CBD (i.e. A and B) the integrals in equations 3.9 and 3.15 had to be evaluated over a definite range. Hamilton [26] chose the urban boundary, $\hat{x}$, to equal the distance at which population density declined to 100 people per square mile, (denoted as F by Hamilton).[18] Thus, the boundary for each city is unique and depends upon the population gradient parameters estimated in (see Chapter 4). Although Hamilton's choice of F was arbitrary, it seems reasonable. A density of 100 people per square mile is roughly equivalent to 10 acre lot sizes. Furthermore, as is shown below, the influence of the boundary choice on estimates of optimum commute tends to wash out if the impact of $\hat{x}$ upon both A and B is similar.

Column F in Table 5.1 represents the estimated distance at which population density declines to 100 people per square mile in each Canadian CMA, based on the density gradients estimated in Chapter 4. The boundary (F) is calculated using the density gradient

[18] This is equivalent to 38.61 people per square kilometre.

parameters presented in Table 4.3, and then substituted for $\hat{x}$ in equations 3.10 and 3.16. For comparison, column G in Table 5.1 gives the CMA radius as measured from Statistics Canada maps. Figure 5.8 reveals some considerable differences between the boundary measures (F and G). The calculated boundary is greater than the measured (political) boundary (G) in only three CMAs: Toronto, Montreal and Kitchener.

Hamilton suggested the impact of a particular boundary choice on estimates of wasteful commuting would be minor.[19] However, when Hamilton applied his model to a sample of Japanese cities he found his assumption of 100 people per square mile as the dividing line between urban and rural was inappropriate and yielded cities with infinite radius. Further, Macauley [38] found density gradient estimates were sensitive to the particular data set employed. Macauley found that using UA data rather than SMSA data caused major differences in density gradient estimates for a given set of cities. Thus it is worthwhile to check the model's sensitivity to the choice of urban boundary.

Table 5.6 presents revised estimates of waste, (D - C), and the share of commuting that is necessary, (C/D), allowing $\hat{x}$ to vary from $F \rightarrow G \rightarrow \infty$.[20] The average estimated wasteful commute declines by 0.56 kilometres when $\hat{x}$ is increased from F to ∞. The difference was is statistically different from zero (t=0.671). Comparing the

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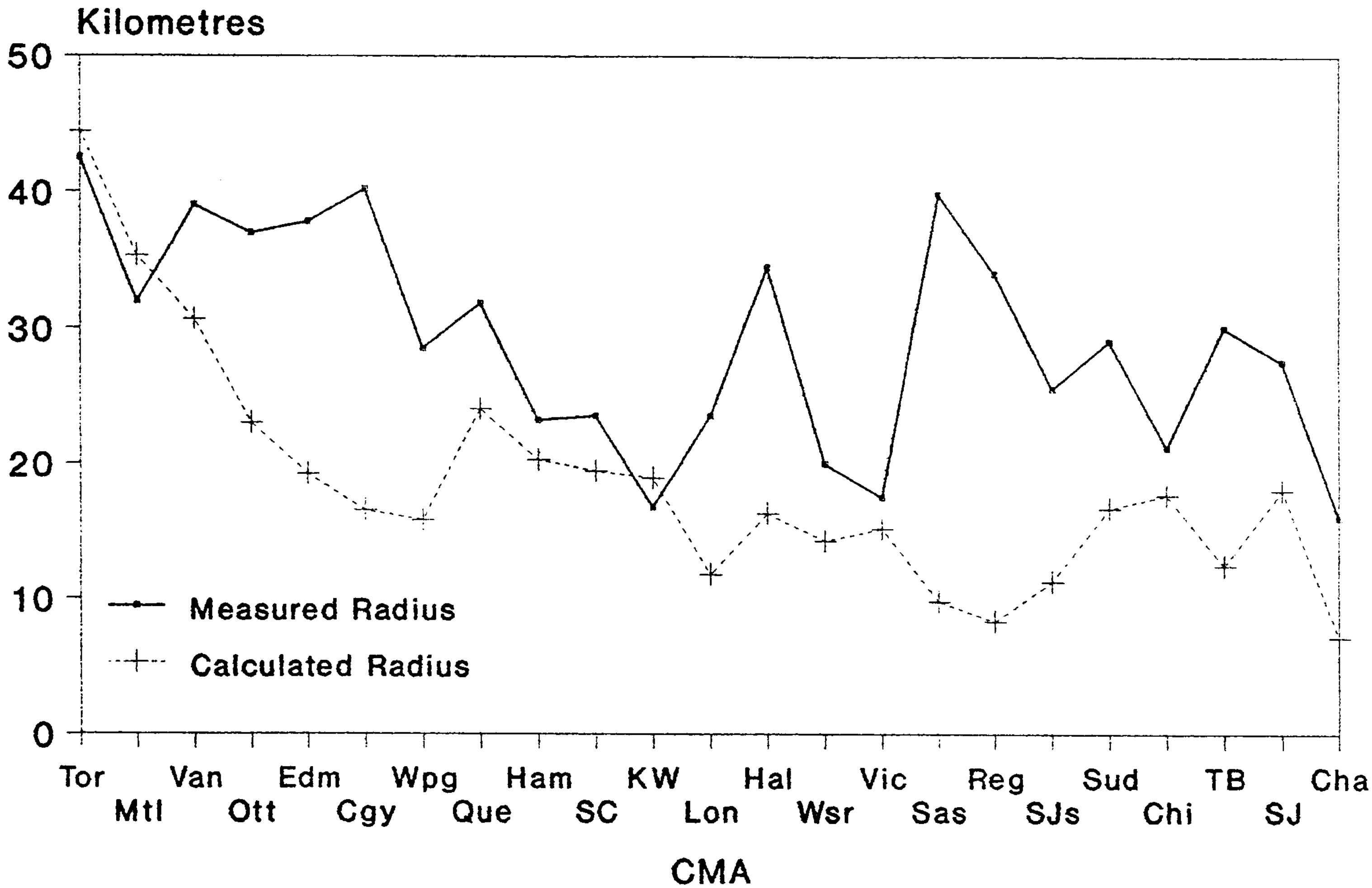
[19] See Hamilton (pg. 1040) [25], [26] footnote #6.

[20] The figures generated using F are identical to those in Table 5.4. They are repeated here to make comparison easier.

FIGURE 5.8
MEASURED VERSUS CALCULATED* CMA RADIUS



* Calculated is where population density = 100 people per square mile.

Table 5.6: Alternative Measures of Wasteful Commuting Assuming Different CBD Boundaries

CMA	D - C			C/D		
	F	G	∞	F	G	∝
Calgary	10.07	10.25	10.26	5.36	3.63	3.62
Charlottetown	1.91	1.61	1.61	54.74	61.63	61.72
Chicoutimi	2.53	1.74	-0.10	69.54	79.04	101.22
Edmonton	8.98	8.80	8.80	14.38	16.05	16.11
Halifax	5.26	4.80	4.79	34.02	39.75	39.85
Hamilton	13.54	13.41	13.17	13.37	14.20	15.77
Kitchener	11.07	11.08	11.06	8.39	8.37	8.51
London	7.53	7.58	7.58	10.53	10.05	10.04
Montreal	8.57	8.76	8.11	29.92	28.40	33.71
Ottawa	6.59	5.95	5.86	36.51	42.72	43.59
Quebec	8.21	7.68	7.29	27.20	31.94	35.40
Regina	7.52	7.52	7.52	11.85	11.89	11.89
Saint John	7.60	6.15	5.16	31.64	44.67	53.51
Saskatoon	6.44	6.29	6.29	19.08	21.02	21.02
St Catherines	7.48	7.17	6.80	29.39	32.34	35.83
St John's	5.58	5.36	5.36	31.91	34.59	34.60
Sudbury	11.15	9.92	9.44	18.12	27.15	30.66
Thunder Bay	8.65	8.37	8.36	14.50	17.36	17.42
Toronto	6.79	6.92	6.06	44.53	43.50	50.51
Vancouver	10.85	10.60	10.45	20.01	21.85	22.96
Victoria	7.37	7.23	7.04	30.74	32.09	33.89
Windsor	8.23	8.04	7.98	19.13	20.96	21.56
Winnipeg	10.50	10.66	10.67	5.29	3.95	3.85
Sample Mean	7.93	7.65	7.37	25.22	28.14	30.75

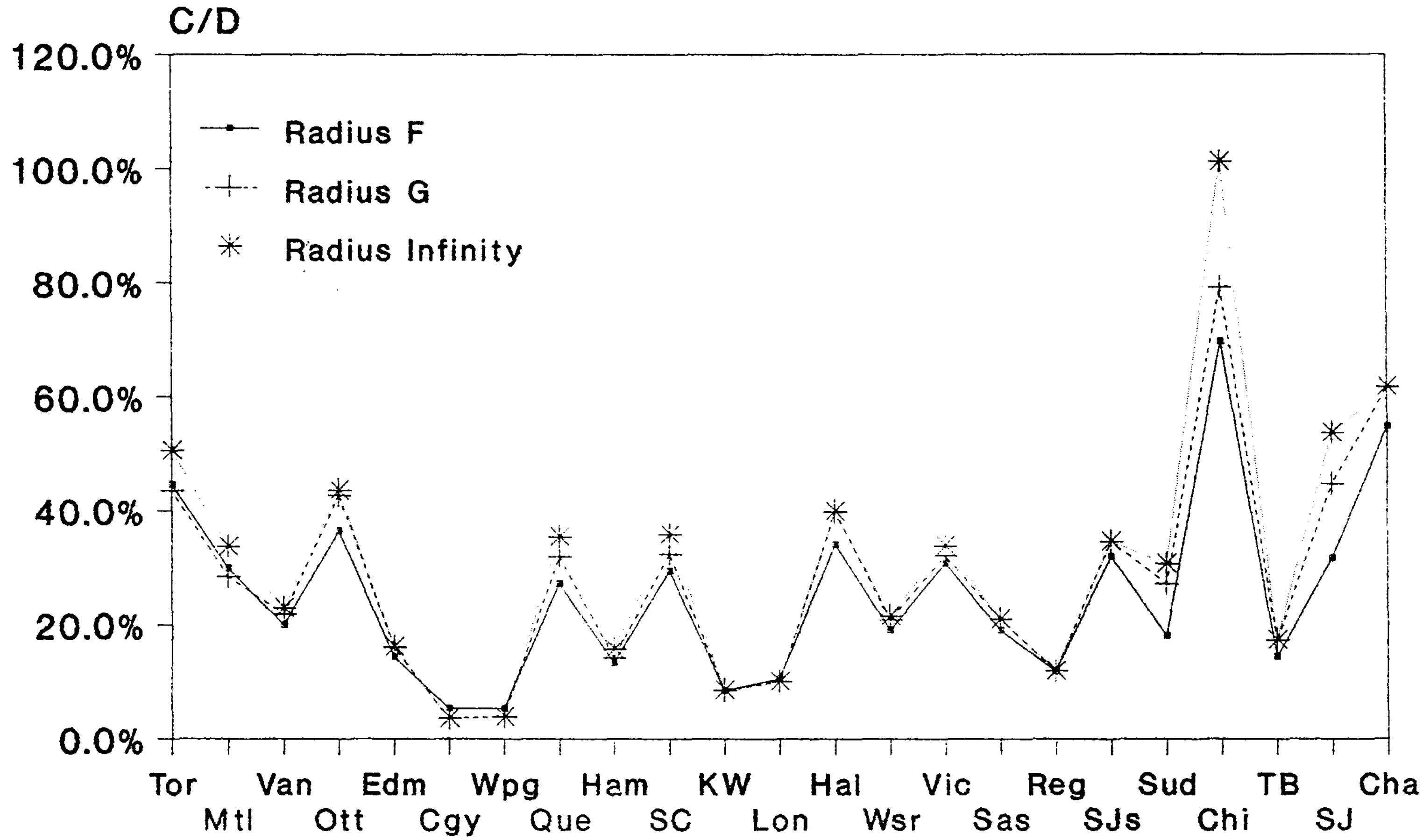
mean values for (C/D) resultsin the same conclusion. The average share of observed commuting deemed necessary increases from 25 to 31 percent. Again the difference is not statistically different from zero (t=0.974).

Figure 5.9 graphically illustrates the insensitivity of Hamilton's technique to the choice of urban boundary for individual Canadian CMAs. The result holds for cities throughout the urban hierarchy. The greatest changes in (C/D) occurr in Chicoutimi and Saint John. Recall that in Chapter 4 it was argued both these cities were expected to behave erratically, due to their irregularly shaped boundaries. Interestingly, in Chicoutimi allowing the boundary to approach ∞ increases (C/D) to the point where there is no wasteful commuting.

5.4 Using Edmonston Density Gradients

Chapter 4 highlighted some significant differences between density gradients estimated using Edmonston's versus Macauley's estimation technique. It was argued that, if the differences between gradient estimates were systematic for both the employment and population gradients, the Hamilton technique for calculating the optimum commute would not be adversely effected by the choice of gradient estimation technique. Evidence in Chapter 4 suggested the two techniques ranked the density gradient parameters α and δ similarly across the 23 CMAs. However, differences between the D_o and E^o parameters appeared less systematic. Although Edmonston population and employment gradient estimates are steeper than Macauley

FIGURE 5.9
OPTIMAL AS A SHARE OF OBSERVED COMMUTE ALLOWING THE URBAN BOUNDARY TO VARY



estimates, the effect on estimates of wasteful commuting should be negligible, if estimates of both the average distance of households and jobs (A and B) are equally effected because wasteful commuting is defined as (A - B).

Table 5.7 summarizes the difference in estimates of several key measures of waste using Edmonston rather than Macauley gradient estimates. Values presented in Table 5.7 are for the estimates which added 1.0 kilometre to the average distance of households from the CBD to represent a non-residential core in each CMA. This was described in Section 5.2.

The mean difference between the estimated average distance of households from the CBD is 0.47 kilometres and the difference between the estimated average distance of jobs from the CBD is 0.45 kilometres (Table 5.7). Both differences are significantly different from zero at the α = .01 level (t=4.74 and t=6.34). Notice that the difference between estimates of A and B is almost equal. Thus, it is not surprising that the difference between the average estimated wasteful commute is not significantly different from zero (t=0.04). The mean estimated waste, (D - C), is not statistically different between Edmonston and Macauley density gradient estimates.

To understand how this is possible, despite large differences in the estimates of D_o and E_o, recall equations 3.10 and 3.16 from Chapter 3:

Table 5.7: Summary Statistics Comparing Urban Structure and Commuting Behaviour Using Edmonston Versus Macauley Density Gradient Estimates

	Macauley		Edmonston		Difference	t-statistic	Significance	
	Mean	σ^2	Mean	σ^2	of Means	H_0: Diff=0	.95	.99
A	7.69	8.40	7.22	7.39	0.473	4.74	X	X
B	5.21	4.93	4.77	4.17	0.448	6.34	X	X
C	2.48	1.95	2.46	2.02	0.026	0.61		
D-C	7.93	7.19	7.96	7.30	-0.030	0.04		
C/D	25.22	253.09	24.90	260.65	0.650	0.14		
A/D	73.83	440.60	69.27	414.03	4.560	0.73		

Notes: $\sigma^2 \equiv$ variance; t-statistics calculated under the null hypothesis that the difference between the two samples was zero using a two tailed difference of means test; degrees of freedom = 44.

$$A = \frac{2}{\gamma} - \frac{2\pi D_0}{\gamma P}\,\overline{x}^2 e^{-\gamma \overline{x}}$$

$$B = \frac{2}{\delta} - \frac{2\pi E_0}{\delta J}\,\overline{x}^2 e^{-\delta \overline{x}}$$

Hamilton [26] showed that the second term in each equation is small relative to the first term. Indeed, when distance (x) approaches ∞ 3.10 reduces to $2/\alpha$ and 3.16 reduces to $2/\delta$. Since D_o and E_o appear only in the second term in each equation, Hamilton's method is robust with respect to the gradient estimation technique, despite large variance in estimates of D_o and E_o, as long as α and δ are ranked consistently. This conclusion holds for individual city comparisons as well as for the average estimate of wasteful commuting. Appendix 3 presents tables that are analogous to Tables 5.1 and 5.2 using Edmonston gradients rather than Macauley estimates. There are no noticeably large differences for any individual CMA.

5.5 Summary

The purpose of this chapter was to compare estimated optimal commuting distances with observed commuting distances in 23 Canadian metropolitan areas in 1981. The methodology used to calculate the optimal mean commute was developed by Hamilton [26]. The overall goals were to:

- Test the ability of the monocentric model to predict observed commuting behaviour in a sample of Canadian cities; and
- Compare the amount of wasteful commuting occurring in a sample of Canadian versus American cities.

Observed commuting in 23 Canadian CMAs was found to be six to eight times greater than the amount necessary due to the separation of homes and jobs. Put another way, the monocentric model was only able to explain an average 20-25 percent of observed commuting in 23 Canadian CMAs in 1981. The average wasteful commute (one way) is in excess of 8.7 kilometres because, on average, homes and jobs are only 1-2 kilometres apart but the average observed commute exceeds 10 kilometres. The total failure of the monocentric model in predicting commuting behaviour was clearly evident for the sample of Canadian cities, as it was for Hamilton's [26] sample of U.S. cities. In fact, randomly assigning workers to houses and jobs provided a much better explanation of commuting behaviour than the monocentric model which is based on an implicit trade off of accessibility and transportation costs.

Comparison of Canadian and U.S. estimates of wasteful commuting did not provide strong support for the hypothesis that Canadian commuting behaviour is less wasteful, due to more efficient urban form. Approximately 75-80 percent of observed commuting in the Canadian cities was "wasteful", while Hamilton found that 85-90 percent of observed commuting in his sample of U.S. cities was "wasteful". A two-tailed difference of means test indicated that the difference in means was not statistically different from zero, even at the 90 percent confidence level. A one-tailed difference of means test indicated that wasteful commuting in Canadian cities was significantly less than wasteful commuting in U.S. cities but only at the 90 percent level of confidence.

This chapter also demonstrated that Hamilton's method for estimating wasteful commuting is robust with respect to the technique used to estimate the density gradient parameters. Estimates of wasteful commuting were also shown to be robust with respect to the choice of urban boundary, within reasonable limits. Difference in estimates of wasteful commuting (i.e. the failure of the monocentric model to explain observed commuting) among cities does not appear to be related to the population of the metropolitan area. Differences are more likely explained by other variables.

CHAPTER 6

SUMMARY AND DIRECTIONS FOR FURTHER RESEARCH

The overall goals of this thesis were twofold:

1. To test the ability of the monocentric model to predict observed commuting behaviour in a sample of Canadian cities; and

2. To compare commuting behaviour in a sample of American and Canadian cities.

The technique used to test the ability of the monocentric model to predict observed commuting behaviour in Canadian cities was developed by Hamilton [26]. Hamilton's model was used to estimate the average distance of homes and the average distance of jobs from the CBD in each of 23 Canadian cities. The difference between the average distance of homes from the CBD and the average distance of jobs from the CBD is deemed to be the minimum possible average commuting distance (optimal commute). The optimal commute was compared with the average observed commute in each city.

In order to estimate the average optimal commute in each CMA the model developed by Hamilton requires estimates of population and employment density gradients for each CMA.

6.1 Density Gradient Estimates

The population and employment density gradient parameters for each city are the key inputs into the model used to estimate optimal and wasteful commuting.[1] When Hamilton [26] estimated the model for a sample of 14 U.S. cities he was able to use density gradient parameters previously estimated by Macauley [38]. Unfortunately, there was not a similar set of density gradient parameter estimates available for a lareg sample of Canadian cities. Therefore, Chapter 4 of this thesis presents 1981 estimates of population and employment density gradients for 23 Canadian urban areas.

Although it was not the goal of this thesis to estimate population and employment density gradients, the gradients presented in Chapter 4 represent a significant contribution to urban analysis for three reasons:

1. They are the only employment gradient estimates for a large sample of Canadian cities;
2. Both the population and employment gradient estimates cover a much larger sample of Canadian cities than any previous work; and
3. Three separate variants of the two-point estimation technique were compared for 23 Canadian cities.

Results presented in Chapter 4 are broadly consistent with previous Canadian and U.S. density gradient estimates. A detailed summary of the gradient estimates is provided by Table 4.6 in Chapter 4. Despite the overall reasonableness of the parameter estimates there were at least three cities in which the estimated employment gradient was flatter than the estimated population gradient: Calgary, London, Winnipeg (and Regina for one estimation technique but not the others). Such a result is perverse and violates one of

[1] The model is described in detail in Chapter 3.

the key assumptions of the monocentric model.

The Mills and Macauley gradient parameter estimates were virtually indistinguishable as was expected a priori. Edmonston gradient parameter estimates, which attempt to control for the shape of the CMA, were steeper with higher central densities for both the population and employment gradients. It was not possible to assess which estimates were superior, however. In all cases population and employment density gradient parameters were found to be inelastic with respect to total CMA population.

6.2 Wasteful Commuting - A Critique of the Monocentric Model

Commuting behaviour must play a fundamental role in any model that purports to explain urban residential and employment location. Monocentric models have been widely employed in urban economic analysis because of their simple structure and presumed explanatory power.

Hamilton's model [26] as employed in this thesis represents part of a growing body of literature aimed at providing tests of the monocentric model's ability to predict important spatial patterns. Recent research has also focused on criticism of the assumptions underlying the monocentric model.[2] If, as was shown by this thesis, the monocentric model fails to predict fundamental economic behaviour, such as commuting, the usefulness of the model is in

[2] See Wheaton [57] for a review of the literature critical of the monocentric model.

question. A model cannot be used just for its simplicity if the model fails to predict crucial aspects of economic behaviour.

How badly did the monocentric model tested in 23 Canadian cities fail? Observed commuting in the sample of Canadian cities was an average of 6.0 to 8.0 time greater than the commuting predicted by the monocentric model. Put another way, the version of the monocentric model tested in this thesis explained only 20-25 percent of observed commuting. In fact, randomly assigning workers to jobs and residences provided a much better explanation of observed commuting behaviour in a sample of Canadian cities.

6.3 International Comparison

By replicating the methodology developed by Hamilton [26] this thesis was able to directly assess the relative performance of the monocentric model in Canada versus the United States. International comparison is an important test of an economic model's robustness. In this case, the performance of the monocentric model was uniformly poor in both the United States and Canada.

Comparison of estimates for wasteful commuting were used to evaluate the relative efficiency of Canadian versus U.S. cities. If the optimal average commute, as calculated by the Hamilton model, represents the minimum possible average commute, then the greater the divergence between observed commuting and optimal commuting the more "wasteful" commuting that is occurring in a city. There was only weak evidence to suggest that, on average, observed commuting behaviour more closely resembles optimal commuting behaviour in Canadian cities. Based on the work of Goldberg and Mercer [23] it was expected that there would be significant differences between cities in the two countries.

6.4 Criticisms of Hamilton's Methodology

Hamilton's model for estimating wasteful commuting incorporates several important assumptions including:

- When employment decentralizes it does so in a disperse manner, without clustering[3];
- Both population and employment density are well characterized by a negative exponential density gradient;[4]
- Labour force participation rates are independent of intra-urban location; and
- All jobs and homes are equally desirable[5].

With the exception of the assumption regarding labour force participation rates, none of the key assumptions are specific to Hamilton's model but are instead implicit in the version of the monocentric model which Hamilton [26] chose to test. The assumption that labour force participation rates do not vary within cities was necessary to allow Hamilton to interpret the average

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[3] This issue was discussed in Chapter 2, Section 2.4.

[4] The functional form issue was discussed in some detail in Chapter 2, Section 2.3.

[5] See Wheaton [58] for a model that relaxes this assumption.

distance of population from the central city as the average distance of the "labour force" from the central city. It is improbable that the overall participation rate shows strong intraurban variation but it is possible that there are significant differences in female participation rates among intraurban locations.

The strongest criticism of Hamilton's model was put forward by White [62]. White focused on Hamilton's assumption of non-clustered decentralized employment. It is in some sense ironic that the most avuncular critic of Hamilton's method for testing the monocentric model focused on the employment assumption. White finds that there is minimal wasteful commuting when suburban employment is permitted to cluster [62]. On the surface White's critique of Hamilton seems to lead to the same conclusion as Hamilton's original work. As soon as White allows clustered suburban employment her model becomes polycentric and the land rent gradient is no longer characterized by a single location variable. Thus White's conclusion amounts to the statement that if the monocentric model is really polycentric then there is no wasteful commuting.[6]

6.5 Directions for Further Research

Although White's critique of Hamilton's work perhaps missed the

[6] Mills and Hamilton [42] convincingly argue that it is impossible to reconcile clustered suburban employment with the monocentric model. See Chapter 2 of this thesis, in particular Section 2.4 for a discussion of this issue.

point Hamilton was trying to make, White's work did point to one of the weaknesses in urban economics. While there is a large body of literature examining residential choice, there has been much less investigation into the determinants of employment location. Instead centrality has typically been assumed.

On the theoretical side, two types of models need to be developed further. First, are non-monocentric models which have no prespecified centre allowing the model to determine the location and number of employment clusters. While these models are intuitively appealing mathematical complexity is likely to limit advances made. Ogawa and Fujita developed a non-monocentric model that yielded some interesting results [47]. When commuting is modelled as very expensive relative to the transaction costs of doing business, the city has a completely mixed economy with no centre. If the reverse is modelled (commuting is cheap relative to transaction costs) the monocentric result is achieved. Reality is, no doubt, somewhere between these extreme results.

The second area of investigation must be in the area of polycentric models. In contrast to non-monocentric models, polycentric models have several prespecified employment nodes. Although this is less satisfying theoretically, it seems more likely to yield empirically verifiable results. Dubin and Sung [16], Griffith [25] and Wieand [65] have each developed simple polycentric models and tested the implications for population density and rent gradients.

There is also a great deal of empirical work that needs to be done examining the determinants of commuting behaviour. It is clear that the attempt by individuals to minimize commuting distance (i.e. the monocentric model) can explain only a small fraction of observed commuting behaviour. Other variables influencing commuting behaviour might include:

- The sex of the household head;
- The income of the household head;
- The variable used to measure commuting (distance versus time);
- The presence of a second wage earner in a household;
- The demand for other travel;
- Heterogeneity of jobs and houses and the need for "matching"; and
- Tenure.

The influence of some of these variable on commuting behaviour has been investigated to a limited degree. Frankena [19] and White [63], [64] examined differences in commuting behaviour among men and women for a limited number of U.S. cities. Adler [1] investigated the impact of tenure (rent versus own) on commuting behaviour. Coulson [12] examined the differential impacts of changes in time versus money costs of commuting on the decisions of individuals.

The data base employed in this thesis would permit a detailed analysis of individual commuting decisions in Canadian cities. Variables available for analysis include sex of the household head, tenure, whether there is a second wage earner, the number of non-commuting trips to downtown, the place of work for the household head and the second wage earner, the occupation of the household head, the transit mode of the household head and the length of tenure for each household. It is suggested that the next step in the analysis might logically be to empirically test the influence of each of these variables on commuting behaviour in Toronto, Montreal and Vancouver. Results obtained for the three largest cities could then be tested in smaller cities to see if there are important difference in commuting behaviour among individuals in different size cities.

In concluding, the words of Michelle White provide an admirable goal for urban economists:

> "It is hoped that in the future urban economists will not have to characterize any commuting behaviour as wasteful and instead will be able to explain it" [62].

Although this is clearly overly optimistic, detailed investigation into the determinants of commuting behaviour represents a promising extension to this work.

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APPENDIX 1

INTEGRATION OF EQUATIONS 3.8 AND 3.15

In Chapter three the following equation, 3.8 was presented to represent the average distance of homes from the CBD, (A), in a particular monocentric city:

$$A = \frac{2\pi D_0}{P} = \int_0^{\bar{x}} x^2 e^{-\gamma \bar{x}} dx \qquad \textbf{(A1.1)}$$

Integration by parts is based on the following identity:

$$\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx} \qquad \textbf{(A1.2)}$$

Now if we define the following:

- $u \equiv x^2$; and
- $dv/dx \equiv e^{-\gamma x}$

then:

- $du/dx = 2x$; and
- $v = -(e^{-\gamma x}/\gamma)$.

Applying the formula described by equation A1.2 to equation A1.1 yields:

$$= \frac{D_0 2\pi}{P}\left[-\frac{x^2 e^{-\gamma x}}{\gamma} - \int_0^{\tilde{x}} -\frac{2x e^{-\gamma x}}{\gamma}\, dx\right] \qquad \textbf{(A1.3)}$$

Multiplying through and simplifying yields:

$$= -\frac{D_0 2\pi x^2 e^{-\gamma x}}{\gamma P}\Bigg|_0^{\tilde{x}} + \frac{2D_0 2\pi}{\gamma P}\int_0^{\tilde{x}} x e^{-\gamma x}\, dx \qquad \textbf{(A1.4)}$$

Now recall from equation 3.6 in Chapter 3 that the total population of the city, (P) can be expressed as:

$$P = 2\pi \int_0^{\tilde{x}} D(x)\, x\, dx \qquad \textbf{(A1.5)}$$

Substituting in the negative exponential density function, 3.7, for D(x) in equation A1.5 yields:

$$P = 2\pi D_0 \int_0^{\tilde{x}} x e^{-\gamma x}\, dx \qquad \textbf{(A1.6)}$$

Substituting A1.6 into the second term in A1.4 reduces A1.4 to:

$$
\begin{aligned}
&= -\frac{D_0 2\pi \tilde{x}^2 e^{-\gamma\tilde{x}}}{\gamma P} + \frac{2}{\gamma P}P \\
&= \frac{2}{\gamma} - \frac{2\pi D_0}{\gamma P}\tilde{x}^2 e^{-\gamma\tilde{x}}
\end{aligned}
\qquad \textbf{(A1.7)}
$$

Equation A1.7 is equivalent to 3.9 as required.

One other point must be made regarding Hamilton's model. In estimating A and B Hamilton was unconcerned whether $\phi = 2\pi$ because, as long as 2π is used to calculate P, the 2π in the numerator and denominator of A1.7 cancel out. This implies, however, that for Canadian CMAs where 2π does not equal ϕ it is not correct to simply use the population given in the census to estimate A. Instead population had to be estimated using equation A1.5 as though the city were a complete circle. This also makes the use of Edmonston density gradient estimates based on shapes other than a circle somewhat inconsistent with the Hamilton wasteful commuting model.

APPENDIX 2

COMPARISON OF CMA EMPLOYMENT AND CENTRAL CITY EMPLOYMENT BASED ON DATA FROM THE 1981 CENSUS

Chapter 5 showed that when it is assumed all employment in each CMA is located in the CBD the monocentric model does a much better job of explaining commuting behaviour. The Table below clearly illustrates that a significant portion of CMA employment in 1981 was not located in the central city. On average, 25 percent of employment was located in suburban municipalities. Larger cities tended to have a smaller share of total employment located in the central city.

Central city is a much broader geographic concept than CBD. Using Vancouver as an example, the central city refers to the City of Vancouver (this would include downtown and the Broadway corridor) The CBD, as it is used in the monocentric model, refers to the downtown area only. Thus, it is reasonable to conclude that a significant proportion of total metropolitan employment was located outside the central business district in 1981.

CMA	CMA	Central City	
	Employment	Employment	Share
Calgary	325,205	300,310	0.9234
Charlottetown	18,380	13,460	0.7323
Chicoutimi	44,715	40,845	0.9135
Edmonton	339,075	289,035	0.8524
Halifax	127,660	107,180	0.8396
Hamilton	228,435	153,545	0.6722
Kitchener	132,825	87,460	0.6585
London	131,955	116,675	0.8842
Montreal	1,265,055	615,180	0.4863
Ottawa	347,975	259,675	0.7462
Quebec	237,260	103,485	0.4362
Regina	79,565	74,605	0.9377
Saint John	45,865	40,795	0.8895
Saskatoon	71,730	67,395	0.9396
St. Catherines	126,110	78,975	0.6262
St. John's	60,460	51,370	0.8497
Sudbury	58,180	45,630	0.7843
Thunder Bay	54,000	50,785	0.9405
Toronto	1,571,455	545,160	0.3469
Vancouver	646,435	276,215	0.4273
Victoria	105,130	60,820	0.5785
Windsor	96,080	84,975	0.8844
Winnipeg	284,785	268,275	0.9420
Mean	278,188	162,254	0.7518

APPENDIX 3

ESTIMATES OF WASTEFUL COMMUTING DERIVED USING EDMONSTON DENSITY GRADIENT ESTIMATES

Chapter 5 presented estimates of wasteful commuting based on density gradient parameters obtained using Macauley estimation techniques. This appendix presents estimates of wasteful commuting based on density gradient parameters obtained using Edmonston estimation techniques. The tables presented in this appendix are analagous to Tables 5.1 and 5.2 in Chapter 5.

Table A3.1: Optimal and Actual Commute Characteristics Using 1981 Edmonston Gradient Estimates

CMA	A	B	C	D	E	F	G
Calgary	4.977	5.381	-0.404	10.64	9.70	15.50	38.5
Charlottetown	2.140	1.077	1.064	4.21	2.37	5.97	12.7
Chicoutimi	8.640	3.343	5.297	8.30	14.71	18.14	16.5
Edmonton	6.187	5.705	0.482	10.49	11.31	18.20	36.1
Halifax	5.365	3.743	1.622	7.97	7.94	15.70	27.1
Hamilton	6.240	5.255	0.984	15.63	10.69	18.51	20.7
Kitchener	6.715	6.702	0.013	12.09	13.92	17.10	15.5
London	3.554	3.660	-0.105	8.42	6.35	10.88	21.7
Montreal	10.845	8.282	2.563	12.24	18.60	33.70	29.9
Ottawa	7.446	4.904	2.542	10.39	11.91	20.99	31.0
Quebec	8.635	6.677	1.959	11.29	15.43	22.45	28.8
Regina	2.516	2.519	-0.003	8.54	4.00	7.93	31.7
Saint John	7.716	5.049	2.667	11.11	13.41	17.33	20.8
Saskatoon	3.171	2.677	0.497	7.96	4.93	9.35	38.7
St. Catherines	7.792	5.213	2.579	10.59	11.57	23.39	17.7
St. John's	3.393	1.898	1.495	8.19	3.60	10.95	16.5
Sudbury	7.212	5.753	1.459	13.62	14.43	15.66	26.4
Thunder Bay	4.509	4.039	0.470	10.13	7.69	12.07	22.1
Toronto	12.674	8.636	4.038	12.25	20.21	41.31	34.3
Vancouver	9.640	7.896	1.744	13.56	16.06	30.83	29.6
Victoria	4.561	2.470	2.091	10.65	5.65	14.10	12.0
Windsor	4.026	3.163	0.858	10.18	5.46	13.00	13.6
Winnipeg	5.124	5.551	-0.427	11.09	10.11	15.68	23.4
Sample Mean	6.221	4.765	1.456	10.41	10.44	17.73	24.6
Adj Mean	6.679	4.868	1.812	10.57	11.05	18.83	23.7
Hamilton Mean	14.001	12.199	1.802	14.00	19.46	36.21	n.a.

NOTES: All distances in kilometers; A ≡ necessary commute if complete centralization of employment is assumed; B ≡ potential commute savings resulting from employment decentralization; C ≡ optimum commute (i.e. A - B); D ≡ observed mean commute from the 1977 urban concerns survey; E ≡ average commute randomly assigning jobs to houses; F ≡ radius at which population density declines to 38.61 people per square kilometer (i.e. 100 people per square mile); G ≡ actual radius of the CMA; n.a. ≡ not available; Adj mean excludes Calgary, London, Regina and Winnipeg because these CMA's had values for C less than zero.

Table A3.2: Alternative Measures of Wasteful Commuting Derived From Estimates in Table A3.1

CMA	D-C kms	D/C #	C/D %	A/D %	D/E %	F/G #	Total Population
Calgary	11.04	26.3	3.8	46.8	109.7	0.40	625,966
Charlottetown	3.15	4.0	25.2	50.8	177.7	0.47	44,999
Chicoutimi	3.01	1.6	63.8	104.0	56.5	1.10	135,172
Edmonton	10.01	21.8	4.6	59.0	92.7	0.50	657,057
Halifax	6.35	4.9	20.4	67.3	100.3	0.58	277,727
Hamilton	14.65	15.9	6.3	39.9	146.2	0.89	542,095
Kitchener	12.07	907.7	0.1	55.6	86.9	1.10	287,801
London	8.53	80.0	1.2	42.2	132.6	0.50	283,668
Montreal	9.97	4.8	20.9	88.6	65.8	1.13	2,828,349
Ottawa	7.85	4.1	24.5	71.7	87.2	0.68	717,978
Quebec	9.33	5.8	17.4	76.5	73.1	0.78	576,075
Regina	8.54	3443.9	29.5	213.6	0.25		173,226
Saint John	8.45	4.2	24.0	69.4	82.9	0.83	114,048
Saskatoon	7.47	16.1	6.2	39.8	161.6	0.24	175,058
St. Catherines	8.01	4.1	24.4	73.6	91.5	1.26	304,353
St. John's	6.70	5.5	18.3	41.4	227.5	0.66	154,820
Sudbury	12.16	9.3	10.7	53.0	94.4	10.59	149,923
Thunder Bay	9.66	21.6	4.6	44.5	131.8	0.55	121,379
Toronto	8.21	3.0	33.0	103.5	60.6	1.20	2,998,947
Vancouver	11.82	7.8	12.9	71.1	84.4	1.04	1,268,183
Victoria	8.55	5.1	19.6	42.8	188.4	1.18	233,481
Windsor	9.32	11.9	8.4	39.6	186.4	0.96	246,110
Winnipeg	11.52	26.0	3.8	46.2	109.8	0.67	584,842
Sample Mean	8.96	109.5	14.6	59.0	120.1	0.76	n.a.
Mean w/o -C's	8.76	55.7	18.2	62.7	115.6	0.83	n.a.
Adjusted Mean	n.a.	8.4	19.2	n.a.	n.a.	n.a.	n.a.
Hamilton Mean	12.29	12.2	12.7	95.9	78.3	n.a.	n.a.

NOTES: The mean w/o CMAs with negative estimated C values excludes Calgary, London, Regina and Winnipeg; the adjusted mean excludes Kitchener in addition to the four CMA's excluded from mean w/o -C's; n.a. ≡ not applicable.