# TRANSIENT ANALYSIS OF SIX-PHASE SYNCHRONOUS MACHINES

by

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# ABSTRACT

Six-phase synchronous machines have been used in the past for high power generation, and presently are being used in high power electric drives and in ac and/or dc power supplies. This thesis presents a mathematical model for investigating the transient performance of a six-phase synchronous machine, and develops a model for the simulation of such machines with the EMTP. The effect of the mutual leakage coupling between the two sets of three-phase stator windings are included in the development of a d-q equivalent circuit. The six-phase machine is constructed from a three-phase machine by splitting the stator windings into two equal three-phase sets. The six-phase machine reactances and resistances are derived by relating their values to the known three-phase machine values. The transient performance of the machine is tested by solving the machine equations for a case of a six-phase short-circuit at the terminals of the machine. The EMTP model presented is based on representing the six-phase machine as two three-phase machines in parallel. The EMTP model verification is achieved by comparing the EMTP results against an independent computer program, for the case of a six-phase short-circuit test at the terminals of the machine.

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## DEDICATION

I would like to dedicate this work to my father Mohammad Hossein Mozaffari.

# Chapter 1

# INTRODUCTION

## 1.1 History of Six-Phase Synchronous Machines

In the late 1920's, the drive toward building larger generating units was hampered by limitations in circuit breaker interrupting capacity and by the large size of reactors needed to limit the fault currents. To overcome this problem, six-phase synchronous machines were built with two sets of three-phase windings [1], in which each set of three-phase stator windings was connected to its own station bus. The voltages of each set were equal and in phase. Such generators, rated up to 175 MVA, were in use until the fault current problem was solved by connecting each generator to its own step-up transformer, with switching accomplished on the high voltage side where the currents are lower.

During the last few years, six-phase synchronous motors have been used in high power electric drives. In these inverter-fed synchronous motors, each of the three-phase stator winding sets is connected to its own inverter. Figure 1.1 shows the basic circuit diagram of such a twelve-pulse inverter-fed synchronous motor with two three-phase windings. This circuit configuration is very beneficial for the reduction of harmonic losses and torque pulsations if the two sets of windings are displaced by 30° [2]. On the supply side, which consists of two bridge rectifiers fed by a three-winding transformer with $Y\Delta$ connection, giving a line shift of 30°, the 5th and 7th harmonic are canceled out. The two machine-side inverters each feed a set of three-phase windings of the machine. The two three-phase sets are displaced by 30° and cancel out the field caused by the 5th and 7th harmonics of the stator winding currents.

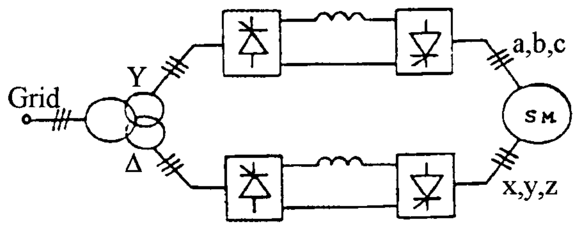



Figure 1.1: Basic Circuit of High-Power Drive with Six-Phase Synchronous Motor.

A second application of six-phase synchronous machines is for generators where one three-phase set feeds an ac load, while the other set supplies power through a three-phase rectifier to a dc load. This type of AC-DC generator has lower cost and lighter weight than a comparable three-phase generator transformer-rectifier configuration [3]. Currently, these types of generators are being used as power supplies on aircrafts and ships. If the rectifier is replaced with an inverter in this configuration, the six-phase machine acts as a DC to AC conversion system. The weight of such a system is less than that of a conventional DC to AC motor-generator set of the same capacity. For example, in a practical system with a 140-kVA 200-V rating on the ac side and 150-kW 1500-V rating on the DC side, the weight of the six-phase system is about 70% of a conventional motor-generator set [3]. This kind of dc-to-ac conversion system is being used in electric railway trains as ac power sources for the air-conditioning equipment.

### 1.2 Objective of the Thesis

The main objective of this thesis is to derive a mathematical model for investigating the transient performance of a six-phase synchronous machine with 30° displacement between the two sets of three-phase windings. With the help of this mathematical model, an equivalent model is developed for the simulation of six-phase synchronous machines with the EMTP(Electromagnetic Transients Program). In order to verify this EMTP model, a comparison will be made between the EMTP results and results from an independent computer program, for the case of a six-phase short-circuit test at the terminals of the machine.

In Chapter 2, the differential equations of a six-phase synchronous machine are first derived in phase variables. Park's transformation [4] will then be used to transform the machine equations to the d-q-0 rotor reference frame. This model includes the stator mutual leakage reactances [5], which have been ignored in earlier works [6,7].

Chapter 3 describes the calculation of the six-phase self and mutual reactances and resistances, which are needed for the simulation of transient phenomena. The six-phase machine parameters will be related to the better known three-phase machine parameters, based on turn ratios and winding factors. The transient performance of such a machine in case of a six-phase short-circuit at the terminals is presented in Chapter 4.

In Chapter 5, an EMTP model for six-phase synchronous machines is presented. In order to verify the validity of this model, the EMTP simulation results for a six-phase short circuit test are compared with the results obtained in the previous chapter. In Chapter 6, general conclusions are drawn, and recommendations are made for future research.

# Chapter 2

## EQUIVALENT CIRCUIT OF A SIX-PHASE SYNCHRONOUS MACHINE

### 2.1 Introduction

In this chapter, the differential equations for a six-phase synchronous machine and its equivalent circuit model are derived. The synchronous machine is assumed to have six identical windings in the stator, in which they are divided into two symmetrical three-phase sets, called a-b-c and x-y-z. The three-phase system x-y-z is displaced with respect to the system a-b-c by 30°. The three rotor windings are the field winding f, that produces flux in the direct axis, and two equivalent damper windings D and Q in the d- and q-axes. These nine windings are magnetically coupled, and the magnetic coupling between the windings is a function of the rotor position. Figure 2.1 shows a cross section of the synchronous machine with its six stator and three rotor windings.

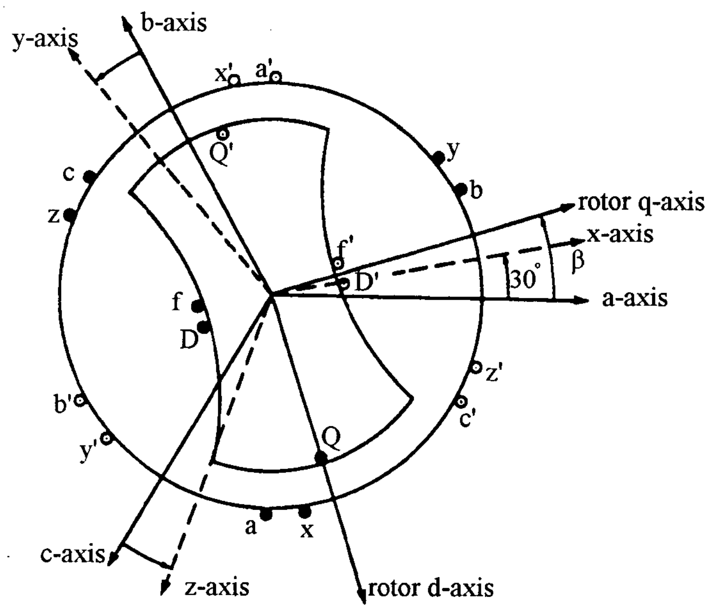



Figure 2.1: Cross Section of Synchronous Machine

**2.2 Differential Equations**

The voltage-current-flux relationship of the 9-coil machine in Figure 2.1 is described by the following equations:

$$[v]=[R][i]+\frac{d}{dt}[\lambda]; \tag{2.1}$$

$$[\lambda]=[L][i]; \tag{2.2}$$

$$[i]=\left[i_a, i_b, i_c, i_x, i_y, i_z, i_f, i_D, i_Q\right]^T; \tag{2.3}$$

$$[v]=\left[v_a, v_b, v_c, v_x, v_y, v_z, v_f, 0, 0\right]^T; \tag{2.4}$$

$$[\lambda]=\left[\lambda_a, \lambda_b, \lambda_c, \lambda_x, \lambda_y, \lambda_z, \lambda_f, \lambda_D, \lambda_Q\right]^T \text{;and} \tag{2.5}$$

$$[R]=diag\left[r_a, r_a, r_a, r_a, r_a, r_a, r_f, r_D, r_Q\right]. \tag{2.6}$$

The elements of the matrix [L] describing the linkage between the fluxes [λ] and the currents [i] depend on the rotor position β:

$$[L]=\left[L_{ij}(\beta)\right]. \tag{2.7}$$

With all the coils having the same number of turns, or with rotor quantities referred to the stator side, the elements $L_{ij}$ of the matrix in equation 2.7 are given bellow [5] (the lower case "$\ell$" represents the leakage inductances).

**- Stator self inductances:**

$$L_{aa}=\ell_s+L_1-L_2\cos 2\beta; \tag{2.8.1}$$

$$L_{bb}=\ell_s+L_1-L_2\cos 2\left(\beta-\frac{2\pi}{3}\right); \tag{2.8.2}$$

$$L_{cc}=\ell_s+L_1-L_2\cos 2\left(\beta+\frac{2\pi}{3}\right); \tag{2.8.3}$$

$$L_{xx}=\ell_s+L_1-L_2\cos 2\left(\beta-30^\circ\right); \tag{2.8.4}$$

$$L_{yy}=\ell_s+L_1-L_2\cos 2\left(\beta-30^\circ-\frac{2\pi}{3}\right); \tag{2.8.5}$$

$$L_{zz} = \ell_s + L_1 - L_2 \cos 2\left(\beta - 30^\circ + \frac{2\pi}{3}\right); \tag{2.8.6}$$

where

$\ell_s$ is the leakage inductance of a stator winding;

$L_1$ is the constant component of the magnetizing inductance of a stator winding; and

$L_2$ is the amplitude of the second harmonic component of the magnetizing inductance.

**- Stator mutual inductances:**

$$L_{ab} = -\frac{1}{2}L_1 - L_2 \cos\left(2\beta - \frac{2\pi}{3}\right) = L_{ba}; \tag{2.9.1}$$

$$L_{ac} = -\frac{1}{2}L_1 - L_2 \cos\left(2\beta + \frac{2\pi}{3}\right) = L_{ca}; \tag{2.9.2}$$

$$L_{ax} = \ell_{ax} + L_1 \cos\frac{\pi}{6} - L_2 \cos\left(2\beta - \frac{\pi}{6} - \frac{2\pi}{3}\right); \tag{2.9.3}$$

$$L_{ay} = \ell_{ay} + L_1 \cos\left(\frac{\pi}{6} + \frac{2\pi}{3}\right) - L_2 \cos\left(2\beta - \frac{\pi}{6} - \frac{2\pi}{3}\right); \tag{2.9.4}$$

$$L_{az} = \ell_{az} + L_1 \cos\left(\frac{\pi}{6} - \frac{2\pi}{3}\right) - L_2 \cos\left(2\beta - \frac{\pi}{6} + \frac{2\pi}{3}\right); \tag{2.9.5}$$

$$L_{bc} = -\frac{1}{2}L_1 - L_2 \cos 2\beta; \tag{2.9.6}$$

$$L_{bx} = \ell_{bx} + L_1 \cos\left(\frac{\pi}{6} - \frac{2\pi}{3}\right) - L_2 \cos\left(2\beta - \frac{\pi}{6} - \frac{2\pi}{3}\right); \tag{2.9.7}$$

$$L_{by} = \ell_{by} + L_1 \cos\frac{\pi}{6} - L_2 \cos\left(2\beta - \frac{\pi}{6} + \frac{2\pi}{3}\right); \tag{2.9.8}$$

$$L_{bz} = \ell_{bz} + L_1 \cos\left(\frac{\pi}{6} + \frac{2\pi}{3}\right) - L_2 \cos\left(2\beta - \frac{\pi}{6}\right); \tag{2.9.9}$$

$$L_{cx} = \ell_{cx} + L_1 \cos\left(\frac{\pi}{6} + \frac{2\pi}{3}\right) - L_2 \cos\left(2\beta - \frac{\pi}{6} + \frac{2\pi}{3}\right); \tag{2.9.10}$$

$$L_{cy} = \ell_{cy} + L_1 \cos\left(\frac{\pi}{6} - \frac{2\pi}{3}\right) - L_2 \cos\left(2\beta - \frac{\pi}{6}\right); \tag{2.9.11}$$

$$L_{cz} = \ell_{cz} + L_1 \cos\left(\frac{\pi}{6}\right) - L_2 \cos\left(2\beta - \frac{\pi}{6} - \frac{2\pi}{3}\right); \tag{2.9.12}$$

where

$\ell_{ax}, \ell_{ay}, \ell_{az}$ are the mutual leakage inductances between phase a and phases x, y and z;

$\ell_{bx}, \ell_{by}, \ell_{bz}$ are the mutual leakage inductances between phase b and phases x, y and z; and

$\ell_{cx}, \ell_{cy}, \ell_{cz}$ are the mutual leakage inductances between phase c and phases x, y and z.

**- Rotor self inductances:**

$$L_{ff} = \ell_f + (L_1 + L_2); \tag{2.10.1}$$

$$L_{DD} = \ell_D + (L_1 + L_2); \tag{2.10.2}$$

$$L_{QQ} = \ell_Q + (L_1 - L_2); \tag{2.10.3}$$

where

$\ell_f$ is the leakage inductance of the field winding referred to the stator;

$\ell_D$ is the leakage inductance of the d-axis damper winding referred to the stator; and

$\ell_Q$ is the leakage inductance of the q-axis damper winding referred to the stator.

**- Rotor mutual inductances:**

$$L_{fD} = L_{Df} = L_1 + L_2; \tag{2.11.1}$$

$$L_{fQ} = L_{Qf} = L_{DQ} = L_{QD} = 0.0; \text{ and} \tag{2.11.2}$$

**- Stator-to-rotor mutual inductances:**

$$L_{af} = (L_1 + L_2)\sin\beta; \tag{2.12.1}$$

$$L_{bf} = (L_1 + L_2)\sin\left(\beta - \frac{2\pi}{3}\right); \tag{2.12.2}$$

$$L_{cf} = (L_1 + L_2)\sin\left(\beta + \frac{2\pi}{3}\right); \tag{2.12.3}$$

$$L_{xf} = (L_1 + L_2)\sin\left(\beta - \frac{\pi}{6}\right); \tag{2.12.4}$$

$$L_{yf} = (L_1 + L_2)\sin\left(\beta - \frac{\pi}{6} - \frac{2\pi}{3}\right); \tag{2.12.5}$$

$$L_{zf} = (L_1 + L_2)\sin\left(\beta - \frac{\pi}{6} + \frac{2\pi}{3}\right); \tag{2.12.6}$$

$$L_{aD} = (L_1 + L_2)\sin\beta; \tag{2.12.7}$$

$$L_{bD} = (L_1 + L_2)\sin\left(\beta - \frac{2\pi}{3}\right); \tag{2.12.8}$$

$$L_{cD} = (L_1 + L_2)\sin\left(\beta + \frac{2\pi}{3}\right); \tag{2.12.9}$$

$$L_{xD} = (L_1 + L_2)\sin\left(\beta - \frac{\pi}{6}\right); \tag{2.12.10}$$

$$L_{yD} = (L_1 + L_2)\sin\left(\beta - \frac{\pi}{6} - \frac{2\pi}{3}\right); \tag{2.12.11}$$

$$L_{zD} = (L_1 + L_2)\sin\left(\beta - \frac{\pi}{6} + \frac{2\pi}{3}\right); \tag{2.12.12}$$

$$L_{aQ} = (L_1 - L_2)\cos\beta; \tag{2.12.13}$$

$$L_{bQ} = (L_1 - L_2)\cos\left(\beta - \frac{2\pi}{3}\right); \tag{2.12.14}$$

$$L_{cQ} = (L_1 - L_2)\cos\left(\beta + \frac{2\pi}{3}\right); \tag{2.12.15}$$

$$L_{xQ} = (L_1 - L_2)\cos\left(\beta - \frac{\pi}{6}\right); \tag{2.12.16}$$

$$L_{yQ} = (L_1 - L_2)\cos\left(\beta - \frac{\pi}{6} - \frac{2\pi}{3}\right); \tag{2.12.17}$$

$$L_{zQ} = (L_1 - L_2)\cos\left(\beta - \frac{\pi}{6} + \frac{2\pi}{3}\right). \qquad (2.12.18)$$

## 2.3 Stator Mutual Leakage Inductances

The mutual leakage fluxes between the two sets of stator windings a-b-c and x-y-z do not cross the machine air gap and couple the three phases of the two sets of stators. The inductance matrix related to these fluxes between the two sets of three-phase windings, can be written as:

$$[\ell_{12}] = \begin{bmatrix} \ell_{ax} & \ell_{ay} & \ell_{az} \\ \ell_{bx} & \ell_{by} & \ell_{bz} \\ \ell_{cx} & \ell_{cy} & \ell_{cz} \end{bmatrix}. \qquad (2.13)$$

The flux-current relationship equations for a two-layer stator winding with 30° difference in winding for a six-phase machine can be formulated as:

$$\lambda_{\ell a} = \ell_{ax} i_x + \ell_{ay} i_y + \ell_{az} i_z; \qquad (2.14.1)$$

$$\lambda_{\ell b} = \ell_{bx} i_x + \ell_{by} i_y + \ell_{bz} i_z; \text{ and} \qquad (2.14.2)$$

$$\lambda_{\ell c} = \ell_{cx} i_x + \ell_{cy} i_y + \ell_{cz} i_z. \qquad (2.14.3)$$

The two sets of stator windings a-b-c and x-y-z are uniformly distributed around the stator and therefore, the leakage inductance matrix $[\ell_{12}]$ is cyclic. This can be formulated as:

$$\ell_{ax} = \ell_{by} = \ell_{cz};$$

$$\ell_{ay} = \ell_{bz} = \ell_{cx}; \text{ and}$$

$$\ell_{az} = \ell_{bx} = \ell_{cy}.$$

## 2.4 Flux Equations

The flux equations of the six-phase synchronous machine, written in matrix form, are shown in equation 2.15:

$$[\lambda_{abc}] = [L_{s1}][I_{abc}] + [L_{12} + \ell_{12}][I_{xyz}] + [L_{r1}][I_{fDQ}]; \qquad (2.15.1)$$

$$[\lambda_{xyz}] = [L_{s2}][I_{xyz}] + [L_{12} + \ell_{12}]^t[I_{abc}] + [L_{r2}][I_{fDQ}]; \qquad (2.15.2)$$

$$\left[\lambda_{fDQ}\right]=\left[L_{r1}\right]^{t}\left[I_{abc}\right]+\left[L_{r2}\right]^{t}\left[I_{xyz}\right]+\left[L_{rr}\right]\left[I_{fDQ}\right]; \tag{2.15.3}$$

where

$$\left[L_{s1}\right]=\begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix}; \tag{2.16.1}$$

$$\left[L_{s2}\right]=\begin{bmatrix} L_{xx} & L_{xy} & L_{xz} \\ L_{yx} & L_{yy} & L_{yz} \\ L_{zx} & L_{zy} & L_{zz} \end{bmatrix}; \tag{2.16.2}$$

$$\left[L_{12}\right]=\begin{bmatrix} L_{ax} & L_{ay} & L_{az} \\ L_{bx} & L_{by} & L_{bz} \\ L_{cx} & L_{cy} & L_{cz} \end{bmatrix}; \tag{2.16.3}$$

$$\left[L_{r1}\right]=\begin{bmatrix} L_{af} & L_{aD} & L_{aQ} \\ L_{bf} & L_{bD} & L_{bQ} \\ L_{cf} & L_{cD} & L_{cQ} \end{bmatrix}; \tag{2.16.4}$$

$$\left[L_{r2}\right]=\begin{bmatrix} L_{xf} & L_{xD} & L_{xQ} \\ L_{yf} & L_{yD} & L_{yQ} \\ L_{zf} & L_{zD} & L_{zQ} \end{bmatrix}; \tag{2.16.5}$$

$$\left[L_{rr}\right]=\begin{bmatrix} L_{ff} & L_{fD} & L_{fQ} \\ L_{Df} & L_{DD} & L_{DQ} \\ L_{Qf} & L_{QD} & L_{QQ} \end{bmatrix}; \text{ and} \tag{2.16.6}$$

$$\left[\lambda_{abc}\right]=\begin{bmatrix} \lambda_{a} \\ \lambda_{b} \\ \lambda_{c} \end{bmatrix}. \tag{2.16.7}$$

## 2.5 Transformation of the Machine Equations

The inductance elements in equations 2.9 through 2.13 are all functions of the rotor position and this makes the solution of the equations difficult. However, these equations can be simplified by applying Park's coordinate transformation. This transformation projects the rotating fluxes onto the field axis, where they appear as stationary during steady-state operation. This transformation is identical for fluxes, voltages, and currents, and converts phase quantities a-b-c and x-y-z into d1, q1, 01 and d2, q2, 02 quantities. This transformation does not change the quantities on the field structure. The transformation matrix $[T]$ can be written as:

$$[T]=\begin{bmatrix} [T_1] & 0 & 0 \\ 0 & [T_2] & 0 \\ 0 & 0 & [I_{(3x3)}] \end{bmatrix}; \tag{2.17}$$

with

$$[I] = \text{identiy matrix}$$

$$[T_1]=\frac{\sqrt{2}}{\sqrt{3}}\begin{bmatrix} \cos\beta & \cos\left(\beta-\frac{2\pi}{3}\right) & \cos\left(\beta+\frac{2\pi}{3}\right) \\ \sin\beta & \sin\left(\beta-\frac{2\pi}{3}\right) & \sin\left(\beta+\frac{2\pi}{3}\right) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}; \text{ and} \tag{2.18}$$

$$[T_2]=\frac{\sqrt{2}}{\sqrt{3}}\begin{bmatrix} \cos\left(\beta-\frac{\pi}{6}\right) & \cos\left(\beta-\frac{\pi}{6}-\frac{2\pi}{3}\right) & \cos\left(\beta+\frac{\pi}{6}+\frac{2\pi}{3}\right) \\ \sin\left(\beta-\frac{\pi}{6}\right) & \sin\left(\beta-\frac{\pi}{6}-\frac{2\pi}{3}\right) & \sin\left(\beta+\frac{\pi}{6}+\frac{2\pi}{3}\right) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{2.19}$$

in which

$$[T_1]^{-1} = [T_1]^T. \tag{2.20}$$

Applying the transformation matrix to equations 2.1 (voltage equations) will result in:

$$[V_{dq01}] = [R_a][i_{dq01}] + [\omega_r][\lambda_{qd01}] + \frac{d}{dt}[\lambda_{dq01}]; \tag{2.21.1}$$

$$[V_{dq02}] = [R_a][i_{dq02}] + [\omega_r][\lambda_{qd02}] + \frac{d}{dt}[\lambda_{dq02}]; \tag{2.21.2}$$

$$[V_{fDQ}] = [R_r][i_{fDQ}] + \frac{d}{dt}[\lambda_{fDQ}]; \tag{2.21.3}$$

where

$$[\omega_r] = [T_1]\frac{d}{dt}[T_1^{-1}] = [T_2]\frac{d}{dt}[T_2^{-1}] = \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \tag{2.21.4}$$

$$\omega = \frac{d}{dt}\beta; \tag{2.21.5}$$

$$[R_a] = \begin{bmatrix} r_a & 0 & 0 \\ 0 & r_a & 0 \\ 0 & 0 & r_a \end{bmatrix}; \text{ and} \tag{2.21.6}$$

$$[R_r] = \begin{bmatrix} r_f & 0 & 0 \\ 0 & r_D & 0 \\ 0 & 0 & r_Q \end{bmatrix}. \tag{2.21.7}$$

Applying the transformation matrix to equations 2.15 (flux equations) will result in:

$$[\lambda_{dq01}] = [T_1][L_{s1}][T_1]^{-1}[i_{dq01}] + [T_1][L_{12} + \ell_{12}][T_2]^{-1}[i_{dq02}] + [T_1][L_{r1}][i_{fDQ}]; \tag{2.22.1}$$

$$[\lambda_{dq02}] = [T_2][L_{s2}][T_2]^{-1}[i_{dq02}] + [T_2][L_{12} + \ell_{12}]^T[T_1]^{-1}[i_{dq01}] + [T_2][L_{r2}][i_{fDQ}]; \tag{2.22.2}$$

and

$$[\lambda_{fDQ}] = [L_{r1}]^T[T_1]^{-1}[i_{dq01}] + [L_{r2}]^T[T_2]^{-1}[i_{dq02}] + [L_{rr}][i_{fDQ}]. \tag{2.22.3}$$

The products involving inductance matrices in the above equations are calculated as:

$$[T_1][L_{s1}][T_1]^{-1} = \begin{bmatrix} \ell_s + \frac{3}{2}(L_1 - L_2) & 0 & 0 \\ 0 & \ell_s + \frac{3}{2}(L_1 + L_2) & 0 \\ 0 & 0 & \ell_s \end{bmatrix} \quad (2.23.1)$$

$$[T_1][L_{12}][T_1]^{-1} = \begin{bmatrix} \frac{\sqrt{3}}{\sqrt{2}}(L_1 - L_2) & 0 & 0 \\ 0 & \frac{\sqrt{3}}{\sqrt{2}}(L_1 + L_2) & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad (2.23.2)$$

$$[T_1][\ell_{12}][T_2]^{-1} = \begin{bmatrix} A_{11} & -A_{12} & 0 \\ A_{12} & A_{11} & 0 \\ 0 & 0 & \ell_{ax} + \ell_{ay} + \ell_{az} \end{bmatrix}; \quad (2.23.3)$$

where

$$A_{11} = \ell_{ax}\cos(\frac{\pi}{6}) + \ell_{ay}\cos(\frac{\pi}{6} + \frac{2\pi}{3}) + \ell_{az}\cos(\frac{\pi}{6} - \frac{2\pi}{3});$$

$$A_{12} = \ell_{ax}\sin(\frac{\pi}{6}) + \ell_{ay}\sin(\frac{\pi}{6} + \frac{2\pi}{3}) + \ell_{az}\sin(\frac{\pi}{6} - \frac{2\pi}{3});$$

$$[T_1][L_{r1}] = \begin{bmatrix} \frac{\sqrt{3}}{\sqrt{2}}(L_1 - L_2) & 0 & 0 \\ 0 & \frac{\sqrt{3}}{\sqrt{2}}(L_1 + L_2) & \frac{\sqrt{3}}{\sqrt{2}}(L_1 + L_2) \\ 0 & 0 & 0 \end{bmatrix}; \quad (2.23.4)$$

$$[T_2][L_{s2}][T_2]^{-1} = [T_1][L_{s1}][T_1]^{-1}; \quad (2.23.5)$$

$$[T_2][L_{12}]^T[T_1]^{-1} = [T_1][L_{12}][T_2]^{-1}; \tag{2.23.6}$$

$$[T_2][\ell_{12}]^T[T_1]^{-1} = \left\{[T_1][\ell_{12}][T_2]^{-1}\right\}^T; \tag{2.23.7}$$

$$[T_2][L_{r2}] = [T_1][L_{r1}]; \tag{2.23.8}$$

$$[L_{r1}]^T[T_1]^{-1} = \left\{[T_1][L_{r1}]\right\}^T; \text{ and} \tag{2.23.9}$$

$$[L_{r2}]^T[T_2]^{-1} = \left\{[T_2][L_{r2}]\right\}^T. \tag{2.23.10}$$

## 2.6 Equivalent Circuit of the System

The voltage equations of the six-phase synchronous machine, once transformed into d, q and 0 quantities, will be as follows:

$$V_{d1} = r_a i_{d1} - \omega\left[(\ell_s + \frac{3}{2}(L_1 - L_2))i_{q1} + \frac{\sqrt{3}}{2}\left(\ell_{ax} - \ell_{ay}\right)(i_{q1} + i_{q2}) - \left(\frac{1}{2}\left(\ell_{ax} + \ell_{ay}\right) - \ell_{az}\right)i_{d2} + \frac{\sqrt{3}}{\sqrt{2}}(L_1 - L_2)\left(i_{q2} + i_Q\right)\right] + \frac{d}{dt}\left[(\ell_s + \frac{3}{2}(L_1 + L_2))i_{d1} + \frac{\sqrt{3}}{2}\left(\ell_{ax} - \ell_{ay}\right)(i_{d1} + i_{d2}) + \left(\frac{1}{2}\left(\ell_{ax} + \ell_{ay}\right) - \ell_{az}\right)i_{q2} + \frac{\sqrt{3}}{\sqrt{2}}(L_1 + L_2)\left(i_{d2} + i_D + i_f\right)\right] \tag{2.24.1}$$

$$V_{q1} = r_a i_{q1} + \omega\left[(\ell_s + \frac{3}{2}(L_1 + L_2))i_{d1} + \frac{\sqrt{3}}{2}\left(\ell_{ax} - \ell_{ay}\right)(i_{d1} + i_{d2}) + \left(\frac{1}{2}\left(\ell_{ax} + \ell_{ay}\right) - \ell_{az}\right)i_{q2} + \frac{\sqrt{3}}{\sqrt{2}}(L_1 + L_2)\left(i_{d2} + i_D + i_f\right) + \right] + \frac{d}{dt}\left[(\ell_s + \frac{3}{2}(L_1 - L_2))i_{q1} + \frac{\sqrt{3}}{2}\left(\ell_{ax} - \ell_{ay}\right)(i_{q1} + i_{q2}) - \left(\frac{1}{2}\left(\ell_{ax} + \ell_{ay}\right) - \ell_{az}\right)i_{d2} + \frac{\sqrt{3}}{\sqrt{2}}(L_1 - L_2)\left(i_{q2} + i_Q\right)\right] \tag{2.24.2}$$

$$V_{d2} = r_a i_{d2} - \omega\left[(\ell_s + \frac{3}{2}(L_1 - L_2))i_{q2} + \frac{\sqrt{3}}{2}\left(\ell_{ax} - \ell_{ay}\right)(i_{q1} + i_{q2}) + \left(\frac{1}{2}\left(\ell_{ax} + \ell_{ay}\right) - \ell_{az}\right)i_{d1} + \frac{\sqrt{3}}{\sqrt{2}}(L_1 - L_2)\left(i_{q2} + i_Q\right)\right] + \frac{d}{dt}\left[(\ell_s + \frac{3}{2}(L_1 + L_2))i_{d2} + \frac{\sqrt{3}}{2}\left(\ell_{ax} - \ell_{ay}\right)(i_{d1} + i_{d2}) - \left(\frac{1}{2}\left(\ell_{ax} + \ell_{ay}\right) - \ell_{az}\right)i_{q1} + \frac{\sqrt{3}}{\sqrt{2}}(L_1 + L_2)\left(i_{d2} + i_D + i_f\right)\right] \tag{2.24.3}$$

$$V_{q2} = r_a i_{q2} + \omega\left[(\ell_s + \frac{3}{2}(L_1 + L_2))i_{d2} + \frac{\sqrt{3}}{2}\left(\ell_{ax} - \ell_{ay}\right)(i_{d1} + i_{d2}) - \left(\frac{1}{2}\left(\ell_{ax} + \ell_{ay}\right) - \ell_{az}\right)i_{q1} + \frac{\sqrt{3}}{2}\left(L_1 + L_2\right)\left(i_{d2} + i_D + i_f\right)\right] + \frac{d}{dt}\left[(\ell_s + \frac{3}{2}(L_1 - L_2))i_{q2} + \frac{\sqrt{3}}{2}\left(\ell_{ax} - \ell_{ay}\right)(i_{q1} + i_{q2}) - \left(\frac{1}{2}\left(\ell_{ax} + \ell_{ay}\right) - \ell_{az}\right)i_{d1} + \frac{\sqrt{3}}{2}\left(L_1 - L_2\right)\left(i_{q2} + i_Q\right) + \right] \tag{2.24.4}$$

$$V_{01} = r_a i_{01} + \frac{d}{dt}\left[\ell_s i_{01} + \left(\ell_{ax} + \ell_{ay} - \ell_{az}\right) i_{02}\right] \tag{2.24.5}$$

$$V_{02} = r_a i_{02} + \frac{d}{dt}\left[\ell_s i_{02} + \left(\ell_{ax} + \ell_{ay} - \ell_{az}\right) i_{01}\right] \tag{2.24.6}$$

$$V_f = r_f i_f + \frac{d}{dt}\left[\frac{\sqrt{3}}{\sqrt{2}}(L_1 + L_2)\left(i_{d1} + i_{d2} + i_D + i_f +\right) + \ell_f i_f\right] \tag{2.24.7}$$

$$V_D = r_D i_D + \frac{d}{dt}\left[\frac{\sqrt{3}}{\sqrt{2}}(L_1 + L_2)\left(i_{d1} + i_{d2} + i_f + i_D\right) + \ell_D i_D\right] \tag{2.24.8}$$

$$V_Q = r_Q i_Q + \frac{d}{dt}\left[\frac{\sqrt{3}}{\sqrt{2}}(L_1 - L_2)\left(i_{q1} + i_{q2} + i_Q\right) + \ell_Q i_Q\right]. \tag{2.24.9}$$

The equivalent circuits of the six-phase synchronous machine corresponding to the equations 2.24 are shown in figure 2.2.

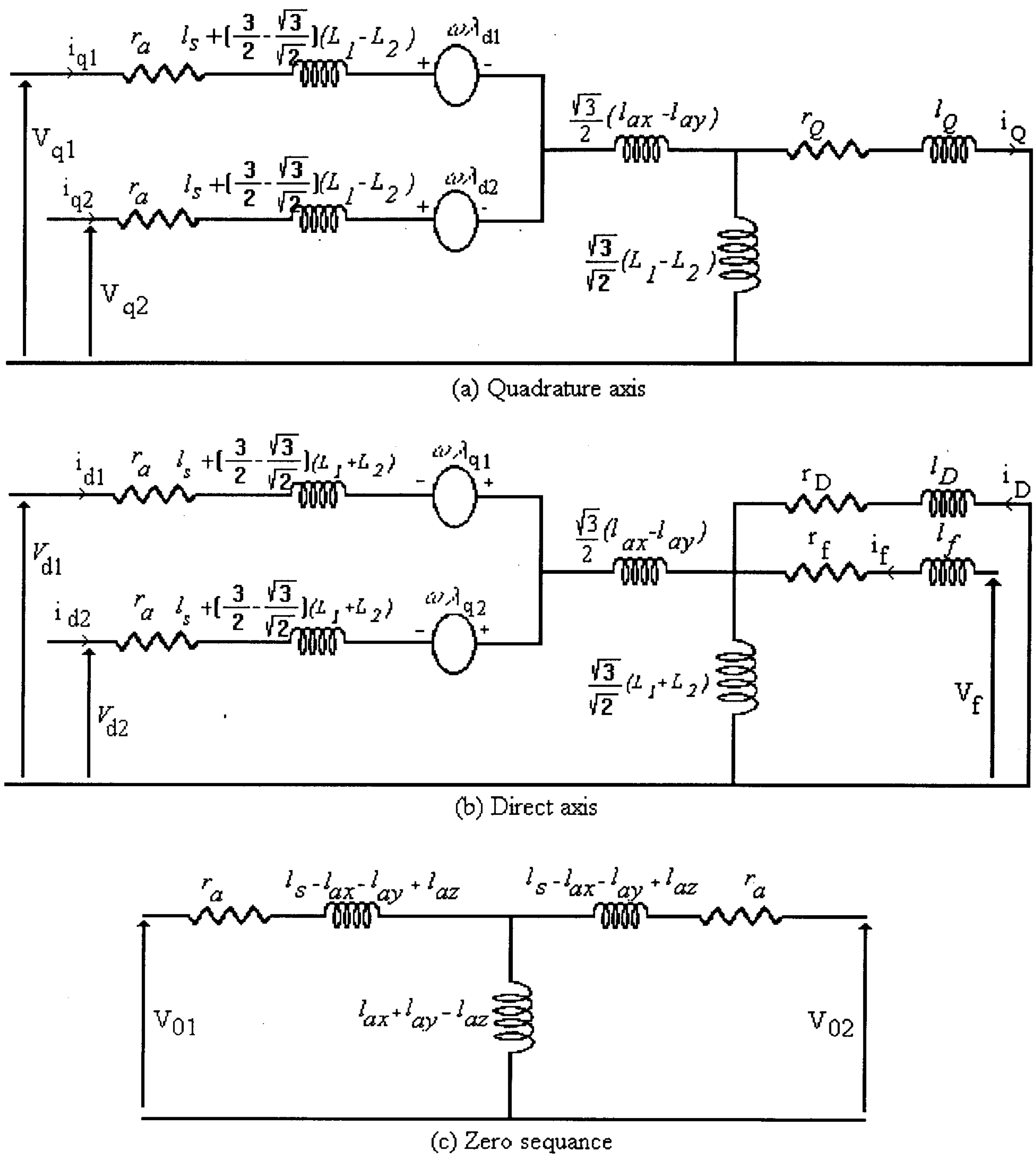



Figure 2.2: Equivalent Circuit of the System, without the Mutual Leakage Coupling between q1,d2 and q2,d1

# Chapter 3

## SIX-PHASE MACHINE PARAMETERS

### 3.1 Introduction

In this chapter, the six-phase synchronous machine reactances and resistances will be derived, assuming that the machine is constructed by splitting each 60° phase belt of a three-phase machine into two equal 30° phase belts, one for each three-phase winding set [8]. The six-phase machine parameters will be related to those of the three-phase machine, based on the turns ratios and winding factors.

### 3.2 Synchronous Self-Reactances of a Three-Phase Machine

For a three-phase salient pole synchronous machine with $N_3$ equivalent stator turns per phase, the reactances in the rotor reference frame can be written as [9]:

$$X_{d3} = \omega \frac{3}{2} N_3^2 P_d \text{: and} \tag{3.1.1}$$

$$X_{q3} = \omega \frac{3}{2} N_3^2 P_q; \tag{3.1.2}$$

where

$P_d$ is the d-axis equivalent permeance dependent on machine dimensions;

$P_q$ is the q-axis equivalent permeance dependent on machine dimensions;

$\omega$ is the base frequency; and

$N_3$ is the turns per coil of a three-phase machine.

The subscript 3 is used to denote three-phase machine values.

### 3.3 Stator Leakage Reactances of a Three-Phase Machine

The stator leakage reactances will be derived assuming that the stator winding has two layers with sixty degree phase belts, and that the winding phase belt has either unity or 5/6 pitch. In both cases, the leakage reactances can be divided into two distinct components:

(a) slot dependent leakage reactance;

(b) all non-slot related leakage reactance including zigzag-leakage reactance, the belt-leakage reactance, and the coil end leakage reactance.

From the above definitions, the leakage reactances of a three-phase machine can be written as:

$$X_{\ell s} = X_{\ell} + X_{\ell slot}, \tag{3.2}$$

where

$X_{\ell \mathrm{slot}}$ is the slot dependent leakage reactance; and

$X_{\ell}$ is the sum of all non-slot related leakage reactances.

**- Slot leakage reactances**

The slot leakage reactances are made up of three parts: the self reactance of the coil sides in the top; the mutual reactances between top and bottom layer coil sides in the stator slots and the self-reactances of the coil sides in the bottom.

$$X_{\ell slot} = X_{\ell T} + K_S X_{\ell TB} + X_{\ell B}, \tag{3.3}$$

where

$X_{\ell T}$ is the slot leakage reactance due to coil sides in the top;

$X_{\ell B}$ is the slot leakage reactance due to coil sides in the bottom;

$X_{\ell TB}$ is the slot leakage reactance between top and bottom layer coil; and

$K_s$ is the coupling factor that is dependent on the winding pitch.

The slot leakage reactance components in equation 3.3 are each defined as:

$$X_{\ell T} = \omega N_3^{\,2} L \frac{P_T}{S_3}, \tag{3.4.1}$$

$$X_{\ell B} = \omega N_3^{\,2} L \frac{P_B}{S_3}, \tag{3.4.2}$$

$$X_{\ell TB} = \omega N_3^{\,2} L \frac{P_{TB}}{S_3}, \tag{3.4.3}$$

where

$P_T, P_B$ are the permeances in top half and bottom half of the slot;

$P_{TB}$ is the permeance of mutual coupling between coils in the top and bottom half;

$L$ is the slot length;

$N_3$ is the turns per coil of a three-phase machine; and

$S_3$ is the number of stator slots per phase of the three-phase machine.

For a three-phase winding, the effective value of the coupling factor $K_s$ will vary linearly with pitch, so that for pitch values $p$ between 2/3 and 1 [9]

$$K_s = 3p - 1. \qquad (3.5)$$

Substituting equation 3.5 into 3.3 results in the following slot leakage reactances:

for a unity pitch ------- $X_{\ell slot} = X_{\ell T} + 2X_{\ell TB} + X_{\ell B}$; and (3.6.1)

for a 5/6 pitch--------- $X_{\ell slot} = X_{\ell T} + 1.5X_{\ell TB} + X_{\ell B}$. (3.6.2)

**- Non-slot leakage reactances**

The non-slot leakage reactances are approximately proportional to the square of the equivalent stator turns and base frequency,

$$X_\ell = \omega K_\ell N_3^{\,2}, \qquad (3.7)$$

where

$K_\ell$ is the proportionality factor dependent upon the end winding and slot dimensions and the number of turns per pole.

### 3.4 Relating Six-Phase Machine Parameters to a Three-Phase Machine

A six-phase machine can be constructed by splitting the 60 degrees phase belt of a three-phase machine into two equal portions each spanning 30 degrees. This technique is illustrated in figure 3.1 for a machine having six slots per pole.

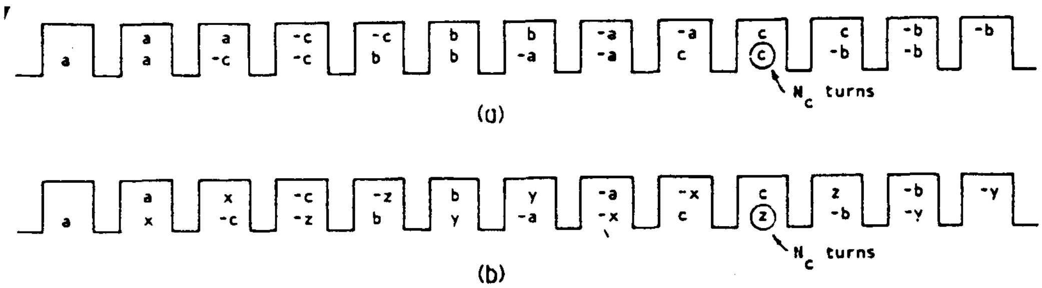



Figure 3.1: Winding Distributions for a 5/6 Pitch Machine a) Three-Phase Machine b) Six-Phase Machine With Split Phase Belts

If the turns and winding factors of the two machines are the same, then the following relationships between the six- and the three-phase machine can be written as follows:

$$N_6 = \frac{N_3}{2} K_{pd}, \tag{3.8.1}$$

$$K_{pd} = \frac{K_{p6} K_{d6}}{K_{p3} K_{d3}}, \tag{3.8.2}$$

$$S_6 = \frac{S_3}{2}, \tag{3.8.3}$$

where

$N_6, N_3$ are the equivalent turns per phase of the six-phase and the original three-phase windings, respectively;

$K_{p6}, K_{p3}$ are the pitch factors of the six-phase and three-phase machine, respectively;

$K_{d6}, K_{d3}$ are the winding distribution factors of the six-phase and three-phase machine, respectively; and

$S_6$ is number of stator slots per phase of the six-phase machine.

The magnetizing reactances $X_q$ and $X_d$ of a six-phase machine are defined as:

$$X_{q6} = \omega \frac{3}{2} N_6^{\,2} P_q; \text{ and} \tag{3.9.1}$$

$$X_{d6} = \omega \frac{3}{2} N_6^{\,2} P_d . \tag{3.9.2}$$

From equations 3.8 and the impedance relationships of equations 3.1 and 3.9 , the following equations relating six-phase parameters to their three-phase counterparts can be derived:

$$X_{q6} = X_{q3} \frac{1}{4} \left( K_{pd} \right)^2 ; \tag{3.10.1}$$

$$X_{d6} = X_{d3} \frac{1}{4} \left( K_{pd} \right)^2 ; \tag{3.10.2}$$

$$X_{\ell Q6} = X_{\ell Q3} \frac{1}{4} \left( K_{pd} \right)^2 ; \tag{3.10.3}$$

$$X_{\ell D6} = X_{\ell D3} \frac{1}{4} \left( K_{pd} \right)^2 ; \tag{3.10.4}$$

$$X_{\ell f6} = X_{\ell f3} \frac{1}{4} \left( K_{pd} \right)^2 ; \tag{3.10.5}$$

$$r_{Q6} = r_{Q3} \frac{1}{4} \left( K_{pd} \right)^2 ; \tag{3.10.6}$$

$$r_{D6} = r_{D3} \frac{1}{4} \left( K_{pd} \right)^2 ; \text{ and} \tag{3.10.7}$$

$$r_{f6} = r_{f3} \frac{1}{4} \left( K_{pd} \right)^2 . \tag{3.10.8}$$

The stator resistances of a six-phase machine are directly proportional to the number of stator turns, as in the case of a three-phase machine. Therefore, for an even stator split, the stator resistance will be

$$r_{a6} = \frac{1}{2} r_{a3} . \tag{3.11}$$

### 3.5 Stator Leakage Reactance of a Six-Phase Machine

As in the case of a three-phase machine the stator leakage reactances of a six-phase machine can be broken into two parts:

$$X_{\ell s} = X_{\ell 6} + X_{\ell slot6}; \tag{3.12}$$

where

$X_{\ell slot6}$ is the slot dependent leakage reactance of a six-phase machine; and

$X_{\ell 6}$ is the sum of all non-slot related leakage reactances of a six-phase machine.

The slot leakage reactance can be broken into three components so that

$$X_{\ell slot6} = X_{\ell T6} + K_{s6} X_{\ell TB6} + X_{\ell B6}. \tag{3.13}$$

For the same number of turns per pole of the three-phase machine, each component of equation 3.13 can be defined as:

$$X_{\ell T6} = \omega N_6^2 L \frac{P_T}{S_6}, \tag{3.14.1}$$

$$X_{\ell B6} = \omega N_6^2 L \frac{P_B}{S_6}, \tag{3.14.2}$$

$$X_{\ell TB6} = \omega N_6^2 L \frac{P_{TB}}{S_6}, \tag{3.14.3}$$

where

$K_{s6}$ is the coupling factor that is dependent on the winding pitch.

As with the three-phase case, the coupling factors $K_{s6}$ vary linearly with the pitch factor $p$

$$K_{s6} = 12p - 10 \qquad \text{for} \quad \frac{5}{6} \le p \le 1. \tag{3.15}$$

From equations 3.13 and 3.15, the slot leakage reactance for a unity pitch can be written as:

$$X_{\ell slot6} = X_{\ell T6} + 2X_{\ell TB6} + X_{\ell B6}; \tag{3.16}$$

and for a 5/6 pitch factor

$$X_{\ell slot6} = X_{\ell T6} + X_{\ell B6}. \tag{3.17}$$

The mutual leakage coupling between the two sets of windings is only caused by stator winding from each three phase set occupying the same stator slot. Therefore, all mutual leakage reactances can be related to $X_{\ell TB6}$ by coupling factors:

$$X_{\ell ax} = K_x X_{\ell TB6}; \tag{3.18.1}$$

$$X_{\ell ay} = K_y X_{\ell TB6}; \text{ and} \tag{3.18.2}$$

$$X_{\ell az} = K_z X_{\ell TB6}. \tag{3.18.3}$$

For a 5/6 pitch winding, the coupling factors can be determined by inspection of figure 3.1 as follows:

$$K_x = 1; \qquad K_y = -1; \text{ and } \quad K_z = 0, \qquad \text{for } p = \frac{5}{6}. \tag{3.19}$$

From equations 3.4 , 3.7 , 3.8 and 3.14 , the following relationships between the six-phase and three-phase leakage reactances can be derived:

$$X_{\ell 6} = X_{\ell 3}\frac{1}{4}\left(K_{pd}\right)^2; \tag{3.20.1}$$

$$X_{\ell T6} + X_{\ell B6} = \frac{1}{2}\left(X_{\ell T3} + X_{\ell B3}\right); \text{ and} \tag{3.20.2}$$

$$X_{\ell TB6} = \frac{1}{2} X_{\ell TB3}. \tag{3.20.3}$$

### 3.6 Machine Parameters

The six-phase synchronous machine parameters will be derived from a three-phase, four pole, salient pole, 125 kVA, 480 V synchronous machine whose parameters are given in table 3.1. For a machine of this type, the slot component of the leakage reactance is 35% of the total leakage reactance [9] and therefore

$$X_{\ell solt3} = \frac{35}{100}(0.147) = 0.05145\ \Omega \text{ and} \tag{3.22.1}$$

$$X_{\ell 3} = \frac{65}{100}(0.147) = 0.0956\ \Omega. \tag{3.22.2}$$

| | |
|---|---|
| $X_d = 4.0735\ \Omega$ | $X_{\ell D} = 0.1842\ \Omega$ |
| $X_q = 1.9612\ \Omega$ | $X_{\ell f} = 0.168\ \Omega$ |
| $X_d^{'} = 0.3041\ \Omega$ | $r_s = 0.0332\ \Omega$ |
| $X_d^{''} = 0.236\ \Omega$ | $r_D = 0.00826\ \Omega$ |
| $X_q^{''} = 0.3557\ \Omega$ | $r_f = 0.00558\ \Omega$ |
| $X_{\ell s} = 0.147\ \Omega$ | $r_Q = 0.00872\ \Omega$ |
| $X_{\ell Q} = 0.2354\ \Omega$ | |

Table 3.1: Three-Phase Machine Parameters

From equation 3.6.2, for a 5/6 pitch, the slot leakage reactance was calculated as:

$$X_{\ell slot3} = X_{\ell T3} + 1.5X_{\ell TB3} + X_{\ell B3}. \tag{3.23}$$

Assuming a straight stator slot and neglecting tooth top fringing, the slot top and bottom mutual leakage reactances are related by:

$$X_{\ell TB3} = 0.3(X_{\ell T3} + X_{\ell B3}). \tag{3.24}$$

By substituting equations 3.22 and 3.24 into equation 3.23, the following values of leakage reactances can be determined:

$$X_{\ell T3} + X_{\ell B3} = 0.0355\ \Omega \text{ and} \tag{3.25.1}$$

$$X_{\ell TB3} = 0.0106\ \Omega. \tag{3.25.2}$$

The pitch and distribution factors for the sixty-degree phase belt are given as [9] :

$$K_{p3} = \sin\frac{p\pi}{2} \text{ and} \tag{3.26.1}$$

$$K_{d3} = \frac{\sin(30^\circ)}{\pi/6}. \tag{3.26.2}$$

For a 5/6 pitch, the above factors are :

$$K_{p3} = 0.966 \text{ and} \tag{3.27.1}$$

$$K_{d3} = 0.955. \tag{3.27.2}$$

From all the above parameters for a three-phase machine, their six-phase counterparts can now be calculated.

**- Six-phase synchronous machine parameters**

The value of the pitch factor $K_p$ is the same as in the case of a three-phase machine. However, the value of the distribution factor will depend on the number of phase belts per pole and the number of slots per phase belt:

$$K_{d6} = \frac{\sin\left(\frac{\pi}{2q}\right)}{n\sin\left(\frac{\pi}{2nq}\right)}, \tag{3.28}$$

where

q is the number of phase belts per pole; and

n is the number of slots per phase belt.

For a 5/6 pitch the distribution factor is:

$$K_{d6} = 0.989. \tag{3.29}$$

Substituting the values of $K_{p3}, K_{d3}, K_{p6}$ and $K_{d6}$ into equation 3.8.2 will result in

$$K_{pd} = 1.0356. \tag{3.30}$$

Substituting the three-phase impedance values along with equation 3.30 into equation 3.10 will result in the six-phase machine parameters that are shown in table 3.2. From equations 3.18, 3.19 and 3.20, the leakage reactances and the mutual leakage coupling are calculated and their values are shown in table 3.3.

| | |
|---|---|
| $X_{d6} = 1.0921\ \Omega$ | $X_{\ell D6} = 0.0493\ \Omega$ |
| $X_{q6} = 0.5258\ \Omega$ | $X_{\ell f6} = 0.0437\ \Omega$ |
| $X_{d6}^{'} = 0.0815\ \Omega$ | $r_{s6} = 0.0166\ \Omega$ |
| $X_{d6}^{''} = 0.0633\ \Omega$ | $r_{f6} = 0.00149\ \Omega$ |
| $X_{q6}^{''} = 0.0953\ \Omega$ | $r_{D} = 0.0022\ \Omega$ |
| $X_{\ell Q6} = 0.0631\ \Omega$ | $r_{Q} = 0.0023\ \Omega$ |

Table 3.2: Six-Phase Machine Parameters

| | |
|---|---|
| $X_{\ell 6} = 0.0256\ \Omega$ | $X_{\ell az} = 0.0\ \Omega$ |
| $X_{\ell TB6} = 0.0053\ \Omega$ | $X_{\ell slot6} = 0.01775\ \Omega$ |
| $X_{\ell ay} = -0.0053\ \Omega$ | $X_{\ell s6} = 0.0433\ \Omega$ |
| $X_{\ell ax} = 0.0053\ \Omega$ | $X_{\ell T6} + X_{\ell B6} = 0.01775\ \Omega$ |

Table 3.3: Leakage Reactances of the Six-Phase Machine

## Chapter 4

## TRANSIENT ANALYSIS

### 4.1 Introduction

The transient performance of synchronous machines is as important as its steady-state behaviour. This is because the synchronous machine is part of the power system whose overall behaviour and stability is greatly influenced by the dynamic characteristics of individual devices, and by their dynamic interaction. The sudden short-circuit test, in which all terminals of an unloaded synchronous machine are short-circuited simultaneously, is a well-established method of checking the transient characteristics of the machine. In this chapter, the differential equations of the six-phase synchronous machine, developed in chapter 2, will be solved for the case of a six-phase short-circuit, under the condition of constant field voltage and speed.

### 4.2 Symmetrical Short-Circuit of an Unloaded Six-Phase Synchronous Machine

The differential equations for a synchronous machine under condition of constant speed can be written in the following matrix format in the d-q reference frame:

$$[v]=[R][i]-\omega[L_1][i]+\frac{d}{dt}[L_2][i]. \tag{4.1}$$

The elements of the matrix equation 4.1 are then defined as:

$$[v]=\left[v_{d1},v_{q1},v_{d2},v_{q2},v_f,v_D,v_Q\right]^T; \tag{4.2.1}$$

$$[i]=\left[i_{d1},i_{q1},i_{d2},i_{q2},i_f,i_D,i_Q\right]^T; \tag{4.2.2}$$

$$[R]=diag\left[r_a,r_a,r_a,r_a,r_f,r_D,r_Q\right]; \tag{4.2.3}$$

$$[L_1]=\begin{bmatrix} 0 & \ell_s+\frac{3}{2}(L_1-L_2) & -\frac{1}{2}(\ell_{ax}+\ell_{ay})+\ell_{az} & \frac{\sqrt{3}}{2}(\ell_{ax}-\ell_{ay})+\frac{3}{2}(L_1-L_2) & 0 & 0 & L_1-L_2 \\ -\ell_s+\frac{3}{2}(L_1+L_2) & 0 & -\frac{\sqrt{3}}{2}(\ell_{ax}-\ell_{ay})-\frac{3}{2}(L_1+L_2) & -\frac{1}{2}(\ell_{ax}+\ell_{ay})+\ell_{az} & -L_1-L_2 & -L_1-L_2 & 0 \\ \frac{1}{2}(\ell_{ax}+\ell_{ay})-\ell_{az} & \frac{\sqrt{3}}{2}(\ell_{ax}-\ell_{ay})+\frac{3}{2}(L_1-L_2) & 0 & \ell_s+\frac{3}{2}(L_1-L_2) & 0 & 0 & L_1-L_2 \\ -\frac{\sqrt{3}}{2}(\ell_{ax}-\ell_{ay})-\frac{3}{2}(L_1+L_2) & \frac{1}{2}(\ell_{ax}+\ell_{ay})-\ell_{az} & -\ell_s-\frac{3}{2}(L_1+L_2) & 0 & -L_1-L_2 & -L_1-L_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \tag{4.3}$$

and

$$[L_2]=\begin{bmatrix} \ell_s+\frac{3}{2}(L_1+L_2) & 0 & \frac{\sqrt{3}}{2}(\ell_{ax}-\ell_{ay})+\frac{3}{2}(L_1+L_2) & \frac{1}{2}(\ell_{ax}+\ell_{ay})-\ell_{az} & L_1+L_2 & L_1+L_2 & 0 \\ 0 & \ell_s+\frac{3}{2}(L_1-L_2) & \frac{1}{2}(\ell_{ax}+\ell_{ay})+\ell_{az} & \frac{\sqrt{3}}{2}(\ell_{ax}-\ell_{ay})+\frac{3}{2}(L_1-L_2) & 0 & 0 & L_1-L_2 \\ \frac{\sqrt{3}}{2}(\ell_{ax}-\ell_{ay})+\frac{3}{2}(L_1+L_2) & -\frac{1}{2}(\ell_{ax}+\ell_{ay})+\ell_{az} & \ell_s+\frac{3}{2}(L_1+L_2) & 0 & L_1+L_2 & L_1+L_2 & 0 \\ 0 & \frac{\sqrt{3}}{2}(\ell_{ax}-\ell_{ay})+\frac{3}{2}(L_1-L_2) & 0 & \ell_s+\frac{3}{2}(L_1-L_2) & 0 & 0 & L_1-L_2 \\ \frac{3}{2}(L_1+L_2) & 0 & \frac{3}{2}(L_1+L_2) & 0 & \ell_f+(L_1+L_2) & L_1+L_2 & 0 \\ \frac{3}{2}(L_1+L_2) & 0 & \frac{3}{2}(L_1+L_2) & 0 & L_1+L_2 & \ell_D+(L_1+L_2) & 0 \\ 0 & \frac{3}{2}(L_1-L_2) & 0 & \frac{3}{2}(L_1-L_2) & 0 & 0 & \ell_Q+(L_1-L_2) \end{bmatrix}. \tag{4.4}$$

The elements of the inductance matrices $[L_1]$ and $[L_2]$ with constant coefficients have already been defined in chapter 2. However, for each three-phase set, the addition of currents in all three phases will equal to zero. Therefore by knowing that $i_x+i_y+i_z=0$ or $i_z=-(i_x+i_y)$, the stator mutual leakage inductances matrix of equation 2.13 can be simplified as :

$$[\ell_{12}]=\begin{bmatrix} \ell_{ax}-\ell_{az} & \ell_{ay}-\ell_{az} & 0 \\ 0 & \ell_{ax}-\ell_{az} & \ell_{ay}-\ell_{az} \\ \ell_{ay}-\ell_{az} & 0 & \ell_{ax}-\ell_{az} \end{bmatrix}.$$

To solve equation 2.1 for the currents it must be rearranged as follow:

$$[L_2]\frac{d}{dt}[i]=-[R][i]+\omega[L_1][i]+[v], \tag{4.5}$$

$$\frac{d}{dt}[i]=-[L_2]^{-1}[R][i]+\omega[L_2]^{-1}[L_1][i]+[L_2]^{-1}[v], \tag{4.6}$$

$$\frac{d}{dt}[i]=\left\{-[L_2]^{-1}[R]+\omega[L_2]^{-1}[L_1]\right\}[i]+[L_2]^{-1}[v]. \tag{4.7}$$

Since all six phases are short circuited, the voltages are:

$$v_{d1}=v_{d2}=v_{q1}=v_{q2}=v_D=v_Q=0. \tag{4.8}$$

Substituting equation 4.8 into 4.7 results in the following expression for the short circuit currents:

$$\frac{d}{dt}[i]=\left\{-[L_2]^{-1}[R]+\omega[L_2]^{-1}[L_1]\right\}[i]+[L_2]^{-1}[v_f]. \tag{4.9}$$

The initial value of the field current is calculated with $i_d = i_q = 0$ as

$$i_{f(0)}=\frac{V_{q(0)}}{2\pi\,(f)\frac{3}{2}(L_1+L_2)} \quad \text{with} \tag{4.10}$$

$$V_{q(0)}=V_{\ell-\ell\ rated(RMS)}. \tag{4.11}$$

The value of $i_{f(0)}$ is used to initialize $v_f$,

$$v_{f0}=R_f i_{f(0)}. \tag{4.12}$$

### 4.3 Solution of the Differential Equation

Equation 4.9 is a set of first order differential equations that has the following general form:

$$\frac{di_n(t)}{dt}=f_n'(t,i,\ldots\ldots i_N) \quad \text{n= 1,.........., N,} \tag{4.13}$$

with known initial conditions $i_n(0)$, n=1,.....N ("initial value problem"). With the parameters from chapter 3, these differential equations become:

$$\frac{d}{dt}i_{d1}=-118.6i_{d1}+1795.9i_{q1}+66.2i_{d2}+1415.9i_{q2}+2.4i_f+3.2i_D+1015.4i_Q-1101.3, \tag{4.14.1}$$

$$\frac{d}{dt}i_{q1}=-2110.8i_{d1}-106.6i_{q1}-1736.8i_{d2}+76.9i_{q2}-1251.4i_f-1251.4i_D+3.5i_Q+0.0, \tag{4.14.2}$$

$$\frac{d}{dt}i_{d2} = 66.2i_{d1} + 1415.9i_{q1} - 118.6i_{d2} + 1795.9i_{q2} + 2.4i_f + 3.2i_D + 1015.4i_Q - 1101.30, \quad (4.14.3)$$

$$\frac{d}{dt}i_{q2} = -1736.8i_{d1} + 76.9i_{q1} - 2110.8i_{d2} - 106.6i_{q2} - 1251.4i_f - 1251.4i_D + 3.5i_Q + 0.0, \quad (4.14.4)$$

$$\frac{d}{dt}i_f = 40.3i_{d1} - 2471.5i_{q1} + 40.3i_{d2} - 2471.5i_{q2} - 10.0i_f + 3.8i_D - 1562.8i_Q + 4552.1, \quad (4.14.5)$$

$$\frac{d}{dt}i_D = 35.8i_{d1} - 2191.7i_{q1} + 35.8i_{d2} - 2191.7i_{q2} + 2.6i_f - 13.5i_D - 1385.8i_Q - 1166.1, \text{ and} \quad (4.14.6)$$

$$\frac{d}{dt}i_Q = 4825.2i_{d1} + 37.3i_{q1} + 4825.2i_{d2} + 37.3i_{q2} + 3138.8i_f + 3138.8i_D - 11.0i_Q + 0.0. \quad (4.14.7)$$

Even though it is known that the zero sequence currents are zero for a six-phase short circuit , its equations were included for the completeness, as:

$$\frac{d}{dt}i_{01} = -8055.55i_{01}, \text{ and} \quad (4.14.8)$$

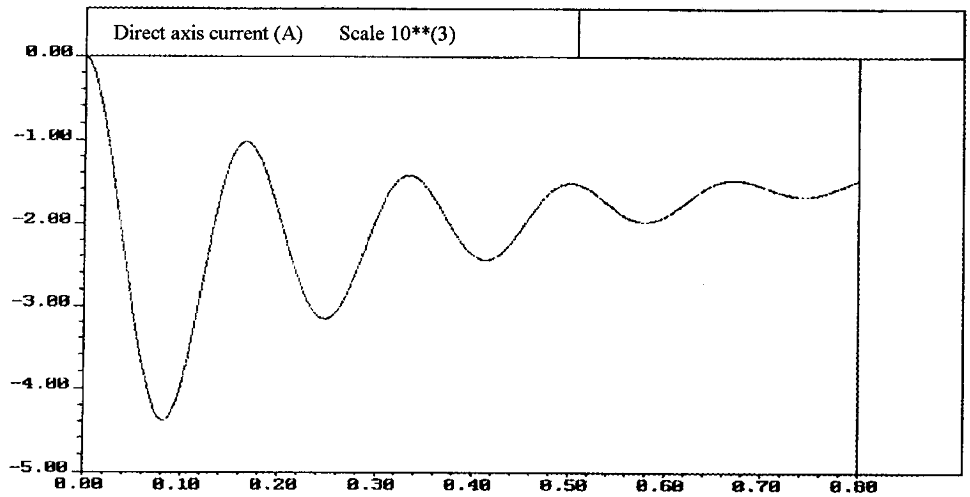
$$\frac{d}{dt}i_{02} = -8055.55i_{02}. \quad (4.14.9)$$

To solve the ordinary differential equations 4.14, the fourth-order Runge-Kutta method is chosen [10]. This method advances the solution over an interval by combining the information from several forward Euler-type steps, and by using this information to match a fourth-order Taylor series expansion to it. In the fourth-order Runge-Kutta method, the derivative is evaluated four times: once at the initial point, twice at trial midpoints, and once at a trial endpoint. From these derivatives, the final function value is calculated. The FORTRAN program "solve 1" in Appendix A was written to solve equations 4.14 for the case of a six-phase short circuit at the terminals of the machine. Figure 4.1 shows the computed wave forms for the direct and quadrature axis currents. From the results of this run it can be concluded that:

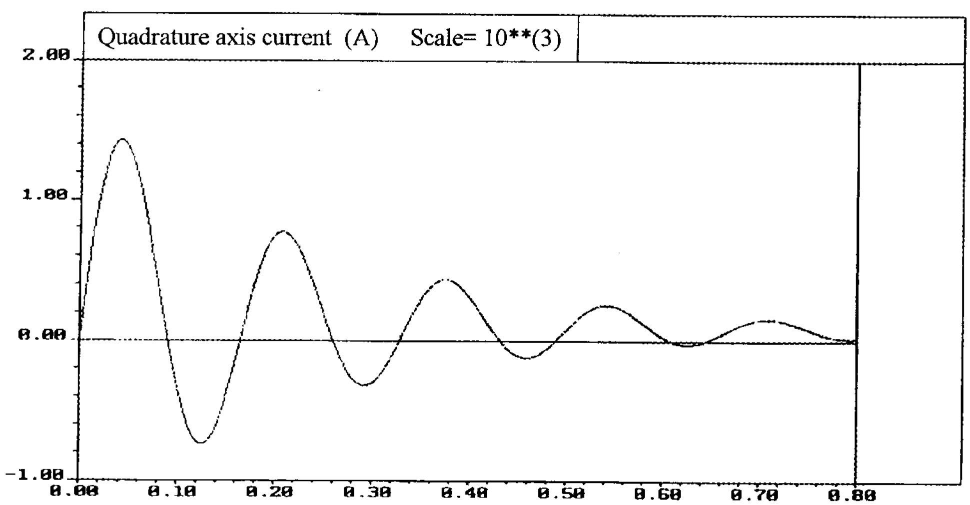
$$i_{d1} = i_{d2}; \text{ and} \quad (4.15.1)$$

$$i_{q1} = i_{q2}. \quad (4.15.2)$$

However, in the transformation from d,q,0- quantities to phase quantities, the transformation matrices $[T_1]$ and $[T_2]$ are different. This causes the two sets of currents for windings a,b,c and x,y,z to be different. The two FORTRAN programs "phase1" and "phase2" in Appendix A transform the d1,q1-quantities into a,b,c and d2,q2-quantities into x,y and z. Figure 4.2 and 4.3 show the curves for the stator currents after the six-phase short circuit at the terminals of the machine.




Time (s) (a) Scale 10**(-1)




Time (s) Scale 10**(-1)

(b)

Figure 4.1: Computed Short Circuit Current: a) direct axis; b)quadrature axis.

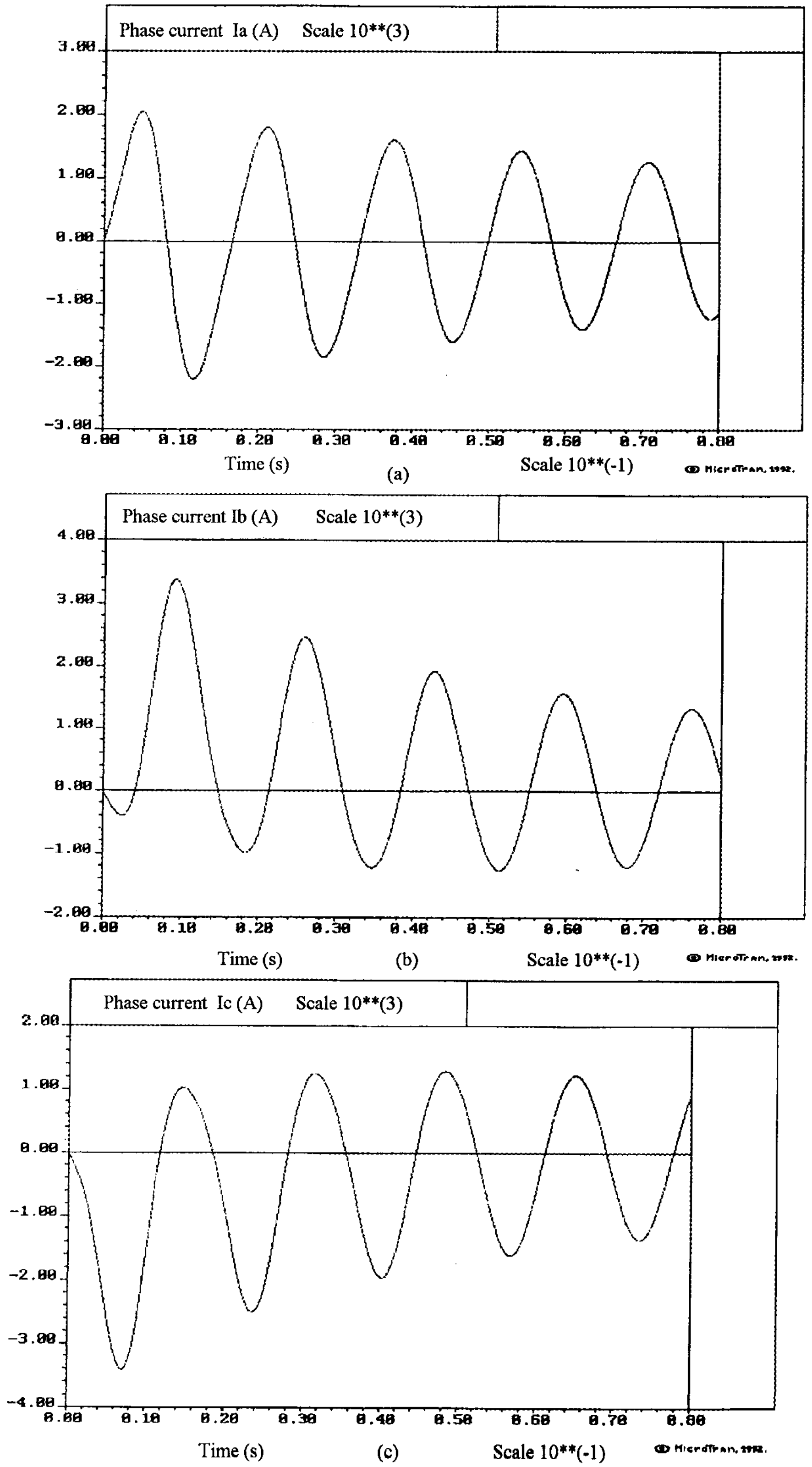



Figure 4.2: Short Circuit Current in Windings a,b,c: a) Phase a; b) Phase b; c) Phase c.

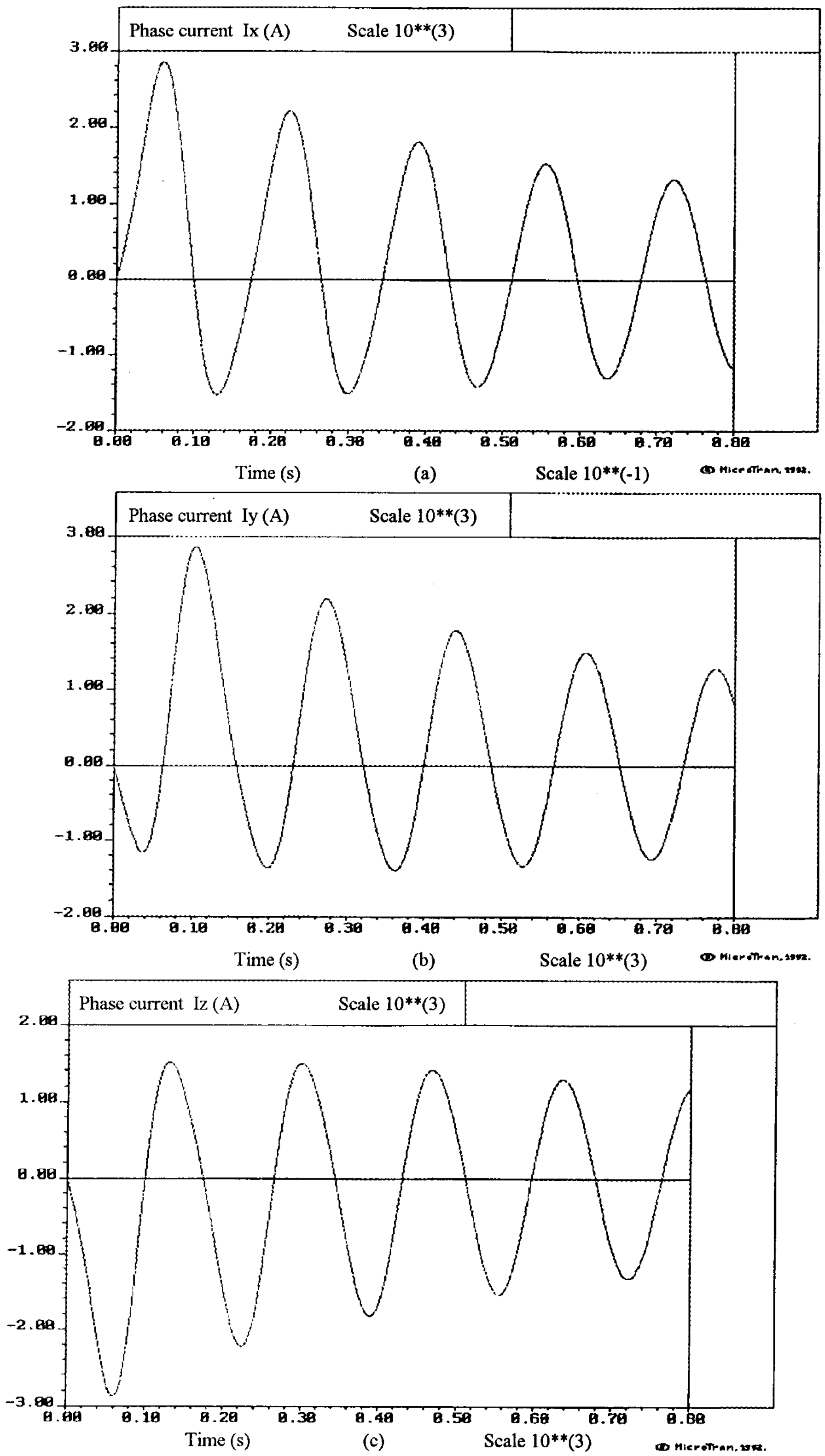



Figure 4.3: Short Circuit Current in Windings x,y,z: a)Phase x; b)Phase y; c)Phase z.

The short-circuit armature currents in figures 4.2 and 4.3 consist of a exponentially decaying unidirectional current and a superimposed sinusoidal current with decaying amplitude. The dc components of the armature currents all decrease to zero exponentially with the same time constant $\tau$. The initial values of these components depend upon the point in the cycle at which the short-circuit occurs. The ac component can be separated into, steady, transient, and subtransient components. The transient component with a long time constant $T_d'$ and the subtransient component with a very short time constant $T_d''$. Figure 4.4 shows the field current, which like the armature current consists of dc and ac components. The dc component can be divided into steady, transient, and subtransient components. The transient and subtransient dc components decrease with the time constant $T_d'$ and $T_d''$. The ac component of field current decay with time constant $\tau$, which is the same as the dc component of the armature currents.

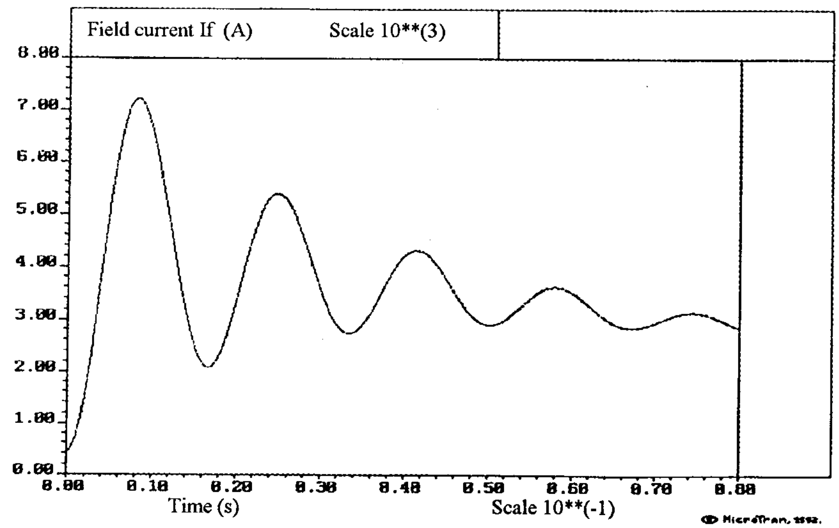



Figure 4.4: Field Current After a Short-Circuit

## Chapter 5

## EMTP MODEL FOR THE SIX-PHASE SYNCHRONOUS MACHINE

### 5.1 Introduction

The Electromagnetic Transients Program is a general purpose computer program whose principal application is the simulation of transient effects in electric power systems. It was developed by H.W. Dommel in the late 1960's at the Bonneville Power Administration [11]. The U.B.C. MicroTran version of EMTP has a model for a three-phase synchronous machine, in which the electrical part of the machine is modelled as an equivalent two-pole machine with seven windings [12]. These windings are: three armature windings; one field winding which produces flux in the direct axis; one winding in the quadrature axis to represent fluxes produced by eddy currents; one winding in the direct axis and one winding in the quadrature axis to represent damper bar effects. In this chapter, an EMTP model for a six-phase, salient pole synchronous machine, that includes the stator mutual leakage inductances, will be developed. This is achieved by paralleling two identical three-phase synchronous machines in such a way that the magnetic coupling between the two machines is included. The magnetic axes of the two three-phase sets are displaced by an angle of 30°. The windings of each set are uniformly distributed and have axes displaced 120° apart. In the first part of this chapter, the equations and the physical aspects of the model will be discussed. In the second part, the EMTP model will be tested by simulating a six-phase short circuit test at the terminals of the machine. The results of this simulation are then compared against the results obtained from chapter 4.

### 5.2 Model of the Machine

The first proposal for including a multiple coil synchronous machine in the EMTP was based on the fact that the magnetic coupling between the two three-phase sets can be approximated with equivalent voltages sources, which the program user can specify as TACS (Transients Analysis of Control System) variables, connected in series with the machine terminals [13]. This model was verified for several short-circuit tests. However, its implementation was numerically unstable. L. Bompa [13] suggested another EMTP model for the six-phase synchronous machine in 1988, in which the magnetic coupling between the two three-phase sets was accounted for by injecting currents from one machine into the other and vice versa. In this model, the extra leakage inductance coupling effects between the two three-phase machines were ignored. In addition, both magnetic axes of the two machines were not displaced with respect to each other.

To develop a complete model, it must first be shown how the magnetic coupling and extra leakage inductances can be included. To do so, the voltage equations of 2.27 are partitioned as follow:

$$\begin{bmatrix} V_{d1} \\ V_{q1} \\ V_{01} \\ V_{d2} \\ V_{q2} \\ V_{02} \end{bmatrix} = \begin{bmatrix} [A_{11}] & [A_{12}] \\ [A_{21}] & [A_{22}] \end{bmatrix} \begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{01} \\ i_{d2} \\ i_{q2} \\ i_{02} \end{bmatrix}, \tag{5.1}$$

where

$$[A_{11}] = [A_{22}] = \begin{bmatrix} r_a + \frac{d}{dt}\left(\ell_s + \frac{3}{2}(L_1 + L_2)\right) & -\omega\left(\ell_s + \frac{3}{2}(L_1 - L_2)\right) & 0 \\ \omega\left(\ell_s + \frac{3}{2}(L_1 + L_2)\right) & r_a + \frac{d}{dt}\left(\ell_s + \frac{3}{2}(L_1 - L_2)\right) & 0 \\ 0 & 0 & r_a + \frac{d}{dt}\ell_s \end{bmatrix}. \tag{5.2}$$

The matrix of equation 5.2 represents a model of a three-phase synchronous machine. The fact that the two matrixes $[A_{11}]$ and $[A_{22}]$ are equal supports the idea of representing the six-phase machine as two three-phase machines in parallel. Since the stator winding position is fixed in the EMTP machine model, the two sets of stator windings of the two machine in parallel can not be set 30° apart. However, a second approach is to keep the two sets of stator windings in the same position and rotate the rotor position of one machine with respect to another by 30°. The off-diagonal matrices $[A_{12}]$ and $[A_{21}]$ represent the extra magnetic couplings and leakage inductances that must be taken into account in paralleling the two three phase machines. The matrixes $[A_{12}]$ and $[A_{21}]$ are found to be:

$$[A_{12}]=\begin{bmatrix} \omega\left[\frac{1}{2}(\ell_{ax}+\ell_{ay})+\ell_{az}\right]+\frac{d}{dt}\left[\frac{3}{2}(L_1+L_2)+\frac{\sqrt{3}}{2}(\ell_{ax}-\ell_{ay})\right] & -\omega\left[\frac{3}{2}(L_1-L_2)+\frac{\sqrt{3}}{2}(\ell_{ax}-\ell_{ay})\right]+\frac{d}{dt}\left[\frac{1}{2}(\ell_{ax}+\ell_{ay})-\ell_{az}\right] & 0 \\ \omega\left[\frac{3}{2}(L_1+L_2)+\frac{\sqrt{3}}{2}(\ell_{ax}-\ell_{ay})\right]-\frac{d}{dt}\left[\frac{1}{2}(\ell_{ax}+\ell_{ay})+\ell_{az}\right] & \omega\left[\frac{1}{2}(\ell_{ax}+\ell_{ay})-\ell_{az}\right]+\frac{d}{dt}\left[\frac{3}{2}(L_1-L_2)+\frac{\sqrt{3}}{2}(\ell_{ax}-\ell_{ay})\right] & 0 \\ 0 & 0 & \frac{d}{dt}(\ell_{ax}+\ell_{ay}-2\ell_{az}) \end{bmatrix}; \tag{5.3}$$

and

$$[A_{21}]=\begin{bmatrix} -\omega\left[\frac{1}{2}(\ell_{ax}+\ell_{ay})-\ell_{az}\right]+\frac{d}{dt}\left[\frac{3}{2}(L_1-L_2)+\frac{\sqrt{3}}{2}(\ell_{ax}-\ell_{ay})\right] & -\omega\left[\frac{3}{2}(L_1-L_2)+\frac{\sqrt{3}}{2}(\ell_{ax}-\ell_{ay})\right]+\frac{d}{dt}\left[\frac{1}{2}(\ell_{ax}+\ell_{ay})-\ell_{az}\right] & 0 \\ \omega\left[\frac{3}{2}(L_1+L_2)+\frac{\sqrt{3}}{2}(\ell_{ax}-\ell_{ay})\right]+\frac{d}{dt}\left[\frac{1}{2}(\ell_{ax}+\ell_{ay})-\ell_{az}\right] & -\omega\left[\frac{1}{2}(\ell_{ax}+\ell_{ay})+\ell_{az}\right]+\frac{d}{dt}\left[\frac{3}{2}(L_1-L_2)+\frac{\sqrt{3}}{2}(\ell_{ax}-\ell_{ay})\right] & 0 \\ 0 & 0 & \frac{d}{dt}(\ell_{ax}+\ell_{ay}-2\ell_{az}) \end{bmatrix}. \tag{5.4}$$

Based on the theoretical analysis of chapter 3, the above matrices can be simplified for a case of 5/6 pitch windings. Using equations 3.18 and 3.19, the mutual leakage inductances between the two machines can be calculated as:

$$\ell_{ax} = -\ell_{ay} \text{ and} \tag{5.5.1}$$

$$\ell_{az} = 0. \tag{5.5.2}$$

Substituting equation 5.5 into the matrix equations 5.3 and 5.4 will result in the following simplified off-diagonal matrix $[A_{12}]'$ :

$$[A_{12}]' = [A_{21}]' = \begin{bmatrix} \frac{d}{dt}\left[\frac{3}{2}(L_1+L_2)+\frac{\sqrt{3}}{2}(\ell_{ax}-\ell_{ay})\right] & -\omega\left[\frac{3}{2}(L_1-L_2)+\frac{\sqrt{3}}{2}(\ell_{ax}-\ell_{ay})\right] & 0 \\ \omega\left[\frac{3}{2}(L_1+L_2)+\frac{\sqrt{3}}{2}(\ell_{ax}-\ell_{ay})\right] & \frac{d}{dt}\left[\frac{3}{2}(L_1-L_2)+\frac{\sqrt{3}}{2}(\ell_{ax}-\ell_{ay})\right] & 0 \\ 0 & 0 & \end{bmatrix}. \quad (5.6)$$

The rewriting of the voltage equations 2.27 with the above simplified matrices and the rearranging of them, will show a common flux $\phi_m$ which is created by the addition of the two currents $i_{d1}$ , $i_{d2}$ in the direct axis and $i_{q1}$ , $i_{q2}$ in the quadrature axis. This common flux $\phi_m$ in the direct and quadrature axis are defined as:

$$\phi_{md} = \frac{3}{2}(L_1 + L_2)(i_{d1} + i_{d2}) \text{ and} \quad (5.7.1)$$

$$\phi_{mq} = \frac{3}{2}(L_1 - L_2)(i_{q1} + i_{q2}). \quad (5.7.2)$$

The flux $\phi_m$ which is created by currents coming from both machines represents the magnetic coupling effect between the two three-phase sets. This relationship is important because it suggests the representation of this magnetic coupling between the two machines by injecting to the terminals of each machine the current coming from the other machine. This is the basis of the EMTP model shown in figure 5.1.

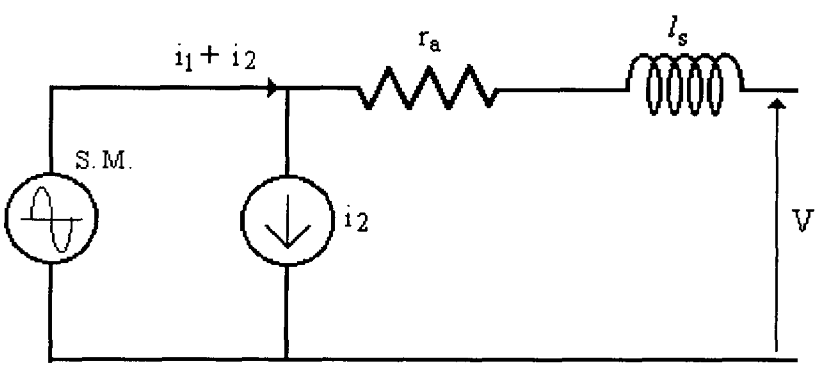



Figure 5.1: The EMTP Model Diagram

According to equation 5.7 and diagram 5.1, the armature resistances and self leakage inductances do not participate in the magnetic coupling phenomena between the two machines. Therefore, they are placed outside the machine. At the same time, the armature resistance $R_a$ and the armature leakage reactance $X_l$, must be set to zero in the machine input data (card six of the EMTP synchronous machine data deck). Along with the armature resistance and self leakage inductance, the effect of the mutual leakage inductances between the two machine was calculated and connected in series to the outside of the two machines. Also, it is assumed that the synchronous machine maintains its synchronous speed, and therefore the mechanical behaviour of the machine need not be modelled.

### 5.3 Method of Current Injection

To inject the current from one machine to the other, the FORTRAN subroutine "source" in Appendix B was written and interfaced with the EMTP. This subroutine uses as input the values of the currents at the terminals of the machines at each time step. These values are put into a vector "X" of length "LX" by the EMTP. The subroutine "source" will then use the values in vector X, along with constant parameters appended to vector X, to generate six user-defined current sources to be injected back into the two machines at each time step. The constant parameters which modifies the values of currents in vector X are necessary because the two machines are rotated with respect to each other by 30°. The EMTP solves the machine equations in d,q,0 quantities and only transforms them into phase quantities to connect it to the outside network. This is shown graphically in Figure 5.2. Therefore, when injecting the current $i_{abc}$ as an extra load on machine 2, it must be rotated by $-30^\circ$ and then injected as $i_{dq02}$ . For the same reason the current $i_{xyz}$ must be rotated by $+30^\circ$ and then injected as $i_{dq01}$ .

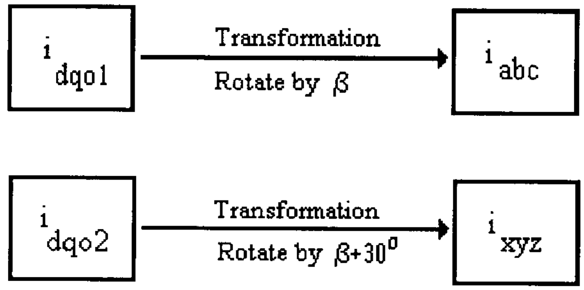



Figure 5.2: Transformation to Phase Quantities

To calculate the constant parameters matrix $[D_1]$, the transformation matrix $[T]^{-1}$ is used along with the modified transformation matrix $[C_1]$, according to equations 5.11. The transformation matrix $[T]^{-1}$ as defined in the EMTP is:

$$\begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{01} \end{bmatrix} = [T_1]^{-1} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}, \tag{5.8}$$

where

$$[T_1]^{-1} = \frac{\sqrt{2}}{\sqrt{3}} \begin{bmatrix} \cos\beta & \cos(\beta - 120^\circ) & \cos(\beta + 120^\circ) \\ \sin\beta & \sin(\beta - 120^\circ) & \sin(\beta + 120^\circ) \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}. \tag{5.9}$$

Rotating the transformation equation of 5.9 by another $30^\circ$, as required here, will result in the new transformation matrix

$$[C_1] = \frac{\sqrt{2}}{\sqrt{3}} \begin{bmatrix} \cos(\beta - 30^\circ) & \cos(\beta - 120^\circ - 30^\circ) & \cos(\beta + 120^\circ - 30^\circ) \\ \sin(\beta - 30^\circ) & \sin(\beta - 120^\circ - 30^\circ) & \sin(\beta + 120^\circ - 30^\circ) \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}. \tag{5.10}$$

Matrix $[C_1]$ can be written as the product of two matrices. One matrix is the original transformation matrix $[T_1]^{-1}$ and the other is a constant matrix $[D_1]$ :

$$[C_1] = [T_1]^{-1}[D_1]; \tag{5.11.1}$$

$$[D_1] = [T_1][C_1]; \text{ and} \tag{5.11.2}$$

$$[T_1] = \left\{[T_1]^{-1}\right\}^t. \tag{5.11.3}$$

The constant matrix $[D_1]$ from equation 5.11 becomes:

$$[D_1] = \frac{2}{3}\begin{bmatrix} \cos\left(\frac{\pi}{6}\right)+\frac{1}{2} & \cos\left(\frac{5\pi}{6}\right)+\frac{1}{2} & \cos\left(\frac{\pi}{2}\right)+\frac{1}{2} \\ \cos\left(\frac{\pi}{2}\right)+\frac{1}{2} & \cos\left(\frac{\pi}{6}\right)+\frac{1}{2} & \cos\left(\frac{5\pi}{6}\right)+\frac{1}{2} \\ \cos\left(\frac{5\pi}{6}\right)+\frac{1}{2} & \cos\left(\frac{\pi}{2}\right)+\frac{1}{2} & \cos\left(\frac{\pi}{6}\right)+\frac{1}{2} \end{bmatrix}. \tag{5.12}$$

In an analogous way, the constant matrix $[D_2]$ for machine 2 becomes:

$$[D_2] = \frac{2}{3}\begin{bmatrix} \cos\left(\frac{\pi}{6}\right)+\frac{1}{2} & \cos\left(\frac{\pi}{2}\right)+\frac{1}{2} & \cos\left(\frac{5\pi}{6}\right)+\frac{1}{2} \\ \cos\left(\frac{5\pi}{6}\right)+\frac{1}{2} & \cos\left(\frac{\pi}{6}\right)+\frac{1}{2} & \cos\left(\frac{\pi}{2}\right)+\frac{1}{2} \\ \cos\left(\frac{\pi}{2}\right)+\frac{1}{2} & \cos\left(\frac{5\pi}{6}\right)+\frac{1}{2} & \cos\left(\frac{\pi}{6}\right)+\frac{1}{2} \end{bmatrix}. \tag{5.13}$$

The modification of the currents from one machine before injection into the other machine can then be formulated as follow:

$$\begin{bmatrix} i_{a\text{modified}} \\ i_{b\text{modified}} \\ i_{c\text{modified}} \end{bmatrix} = [D_1]\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}; \text{ and} \tag{5.14}$$

$$\begin{bmatrix} i_{x\text{modified}} \\ i_{y\text{modified}} \\ i_{z\text{modified}} \end{bmatrix} = [D_2]\begin{bmatrix} i_x \\ i_y \\ i_z \end{bmatrix}. \tag{5.15}$$

This modification is the essential part of subroutine source.

## 5.4 Numerical Instability Caused by Time Delays

The EMTP model of the six-phase synchronous machine developed in the previous section can cause numerical oscillations due to the time delay introduced by the interaction between the subroutine Source and the main program EMTP. The delay is caused when the subroutine calculates the current sources for the time step at time t from the results of the preceding time step at (t-Δt). Therefore, the supplemental source equations (subroutine Source) and the power system equations (EMTP) are not solved simultaneously, and a time delay of Δt is introduced. To show how this time delay may cause numerical instability, consider a simple example of a "constant resistance" simulation using a one time-step lagging current generator [14]. The equivalent circuit of such a system and its z-transform equivalent network is shown in figure 5.3. The current source $i_k$ is represented as rz, where z represents the time delay introduced by the interaction between the source routine and the EMTP.

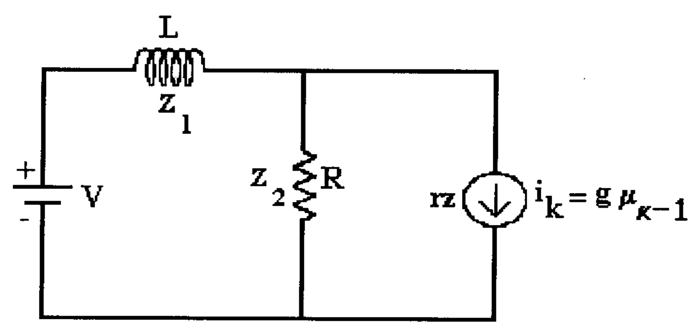



Figure 5.3: Equivalent z-Transform Network

The z-transform current response of the circuit in figure 5.3 is:

$$I = \frac{V/Z_1}{1 + rz\dfrac{Z_1 + Z_2}{Z_1 Z_2}} = \frac{C(z)}{P(z)}. \tag{5.16}$$

The stability of the system defined by equation 5.16 can be determined from the location of the closed-loop poles in the z-plane, or the roots of the characteristic equation

$$P(z) = 1 + rz\frac{Z_1 + Z_2}{Z_1 Z_2} = 0. \qquad (5.17)$$

For the system to be stable, the closed-loop poles or the roots of the characteristic equation must lie within the unit circle in the z-plane. Using the trapezoidal rule of integration, $Z_1$ and $Z_2$ are defined as:

$$Z_1 = \frac{2L}{\Delta t}\frac{z-1}{z+1} \text{ and} \qquad (5.18.1)$$

$$Z_2 = R. \qquad (5.18.2)$$

By substituting equation 5.18 into 5.17, the roots of the characteristic equation are found to be:

$$z = -\frac{r\left(R - \frac{2L}{\Delta t}\right) + \frac{2LR}{\Delta t}}{2r\left(\frac{2L}{\Delta t} + R\right)} \pm \sqrt{\left[\frac{r\left(R - \frac{2LR}{\Delta t}\right)}{2r\left(\frac{2L}{\Delta t} + R\right)}\right]^2 + \frac{\frac{2LR}{\Delta t}}{r\left(\frac{2L}{\Delta t} + R\right)}}. \qquad (5.19)$$

By letting $\Delta t \to 0.0$, the root of P(z) was calculated as:

$$Z_a = -\frac{R-r}{2r} + \frac{R+r}{2r} = 1 \text{ and} \qquad (5.20.1)$$

$$Z_b = -\frac{R-r}{2r} - \frac{R+r}{2r} = -\frac{R}{r}. \qquad (5.20.2)$$

For cases where R>r, the roots of equation 5.17 will be outside the unit circle, which means instability with a force free component of growing amplitude. To overcome this problem, the time delay between the solution of the current generator and the EMTP should be eliminated. Therefore the power system equations should be solved simultaneously with the other equations. Research on this topic is beyond the objective of this thesis. It is the research topic of a Ph.D. thesis project at the University of British Columbia which addresses the problem of interfacing control system equations with the power system equations in the EMTP [15].

### 5.5 Testing the EMTP Model

In order to test the EMTP six-phase synchronous machine model developed in the previous section , a six-phase short circuit at the terminals of the machine is considered. The results from this run are then compared against the solution obtained from solving the differential equations with the fourth order Runge-Kutta method (RK4), as shown in chapter 4. The synchronous machine model of the EMTP accepts as data input either the transient and subtransient reactances and time constants, which are then converted to self and mutual impedances, or directly the self and mutual impedances of the windings. In this thesis, input was in the form of self and mutual impedances, to insure that the data is identical with that of the Runge-Kutta solution. The complete EMTP input data listing for modelling the six-phase synchronous machine and the six-phase short circuit is shown in Appendix C. The numerical simulation with the EMTP has been carried out from t=0s to t=0.12s, with a step size of $50\mu s$. The generator was open circuited before the fault was applied at time zero. Figure 5.4 shows the comparisons between the EMTP model and the corresponding solution of the differential equations with the Runge-Kutta method, for the armature current for the first few cycles, in phase "a" and "x". The EMTP model gives, practically, the same results than those obtained from chapter 4. The fault current takes a long time to reach steady state because of the small value of the resistances. Figure 5.5 shows the comparison of the steady-state short circuit current in phase "a", for the two solution methods.

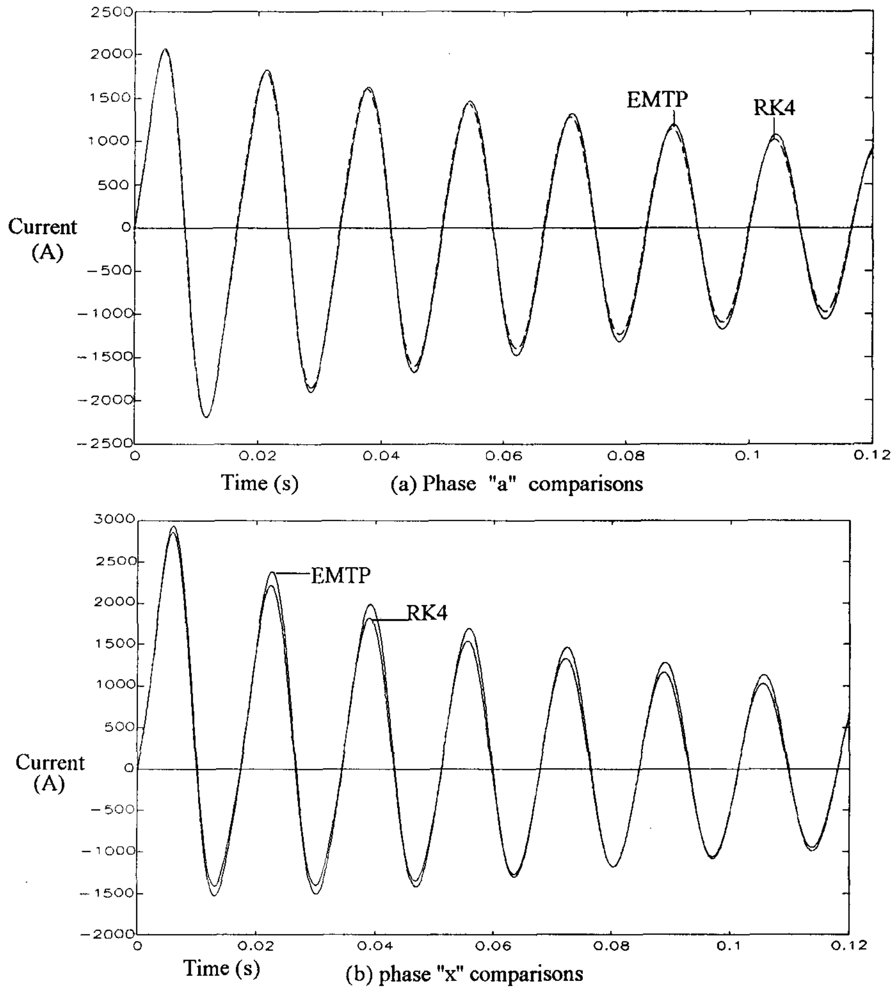



Figure 5.4: Short-Circuit Armature Current: a)Phase a; b)Phase x.

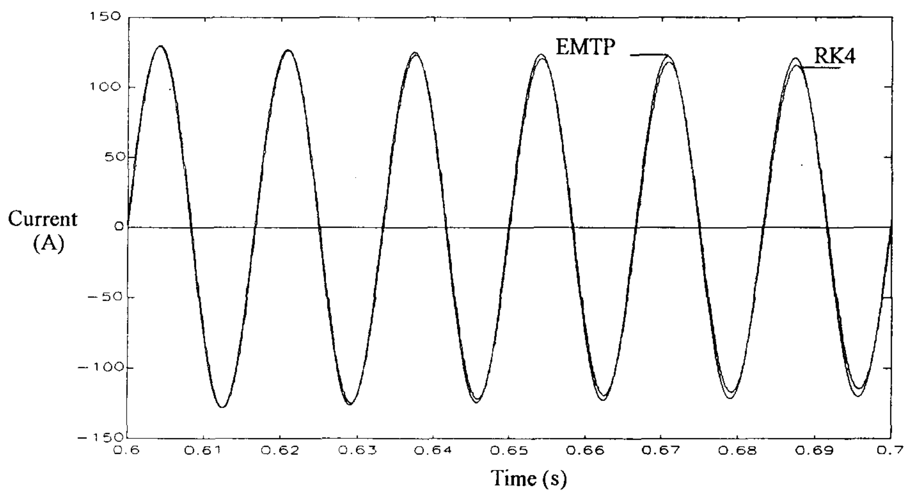



Figure 5.5: Steady-State Short-Circuit Current in Phase "a".

The problem of numerical instability caused by the time delay between the EMTP and the subroutine Source, as discussed in section 5.4 , was a main concern in testing the EMTP model. As shown in figure 5.6, the solution is numerically stable in the beginning, but numerical oscillations eventually start building up. It was hoped that the numerical oscillations could be avoided if the currents in subroutine "source" from the preceding time step were replaced by predicted values, with linear extrapolation. However, numerical oscillations still appeared with this approach.

The final cure for the numerical oscillations was an "occasional" switch to the critical damping adjustment scheme [16], which replaces the trapezoidal rule of integration with step size $\Delta t$ by two integration steps of step size $\Delta t/2$ with the backward Euler method. This switch was done after every 100 time steps, and produces the stable solutions shown in figures 5.4 and 5.5.

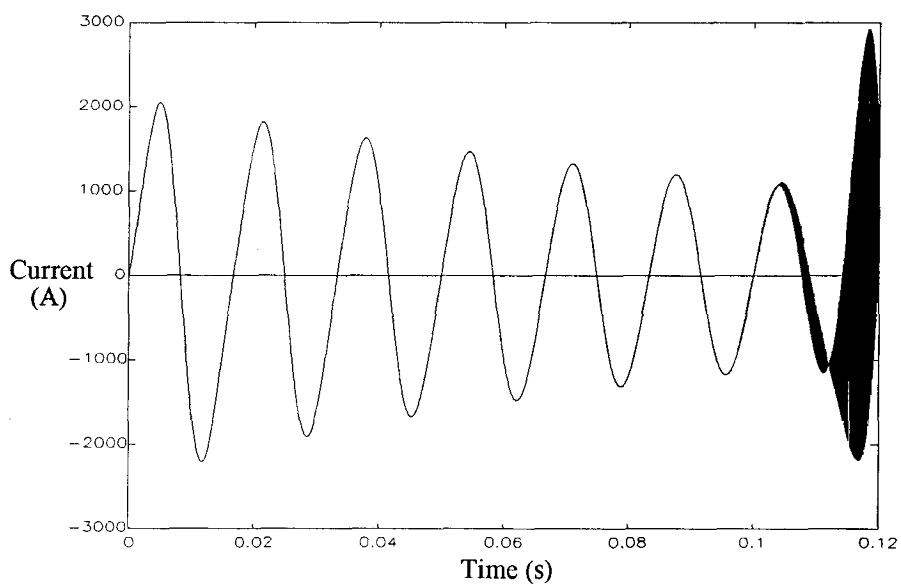



Figure 5.6: Numerical Oscillations

# Chapter 6

# CONCLUSIONS

## 6.1 Results in Thesis

Six-phase synchronous machines are increasingly being used as part of 12-pulse converter-fed high-power drives. This kind of arrangement, used in power plant auxiliary systems, is very beneficial for the reduction of harmonic losses and torque pulsations. The aim of this thesis was to derive a mathematical model for investigating the transient performance of a six-phase synchronous machine and to develop a model for the simulation of such machines with the EMTP.

The first part of this thesis was devoted to the derivation of the equivalent circuit of a six-phase synchronous machine with 30° displacements between the two sets of three-phase windings. This equivalent circuit includes the effects of stator winding mutual leakage coupling between the two three-phase winding sets. Since the machine inductances are all functions of the rotor position, Park's coordinate transformation was used to project the rotating fluxes onto the field axis, where they appear as stationary during steady-state operation. The six-phase machine reactances and resistances were then derived by relating their values to known three-phase machine values. These parameters were derived assuming the six-phase machine was constructed from a three-phase machine by splitting the stator windings into two equal three-phase sets while retaining the same number of turns per pole.

The transient performance of the six-phase synchronous machine was simulated by solving the differential equations of the machine, for the case of a six-phase short-circuit at the terminals of the machine. To solve the ordinary differential equations, the fourth-order Runge-Kutta method was used. The obtained curves for both, the short-circuit armature currents and the field, current were consistent with the behaviour of short-circuit currents for three-phase machines [17].

The EMTP model for a six-phase synchronous machine presented in this thesis was based on the idea of representing the six-phase machine as two three-phase machines in parallel. The magnetic coupling effect between the two three-phase sets, which was created by currents coming from both machines, was represented by injecting to the terminals of each machine the current coming from the other machine. This method of current injection was achieved by adding a FORTRAN subroutine inside the EMTP source code. This subroutine uses the values of the currents at the terminals of the machines, and after a phase rotation, generates six user-defined current sources to be injected back into the two machines at each time step. The test validation of the proposed EMTP model was carried out by simulating a six-phase sudden short circuit at the terminals of the machine. The results from this simulation were then compared against the solution obtained from solving the differential equations with a fourth-order Runge-Kutta method. The EMTP model gave, practically, the same results as those obtained from solving the differential equations with the Runge-Kutta method. The results of this comparison confirm the accuracy of the proposed EMTP model. The problem of numerical oscillations was solved by using the critical damping adjustment scheme.

### 6.2 Suggestions for Future Work

In a paper presented at the European EMTP User Group Meeting in October 1992 [18], H. Knudsen describes an ongoing Ph.D. project, involving the Danish utility company ELKRAFT and the Technical University of Denmark, on modelling of six-phase synchronous machines with 2 three-phase windings spatially displaced by 30°. In this work the six-phase machine equations were solved using the "MODELS" input option for control system simulations. With this approach the results appear in the network modelled by the EMTP in the next time step, as controlled current sources. Knudsen felt that CDA is needed in the synchronous machine equations of MODELS to suppress numerical oscillations at time of discontinuities (switch closing and opening). However, the program version ATP which he uses does not have CDA implementation. There is also the problem of introducing a time delay into the interaction between the synchronous machine equations in MODELS and the network equations in the EMTP, which may cause numerical instabilities.

In order to overcome the above limitations, the six-phase synchronous machine equations should be solved simultaneously with the network. This is possible in UBC's EMTP version MicroTran with subroutine CONNEC [12], which represents the network as a Thevenin equivalent circuit, seen from the machine terminals, to the machine equations inside the subroutine. MicroTran also uses the CDA scheme for the suppression of numerical oscillations, which is not available in the EMTP version ATP. Implementation of the six-phase machine equations with the trapezoidal rule of integration, and with the backward Euler method for the CDA scheme, is a major effort, which was beyond the scope of this thesis.

## REFERENCES

[1]. Barton T.F., "The Double Winding Generator," General Electric Rev., June 1929, pp. 302-308.

[2]. Liyu, C., L. Fahai, Y. Bingshou and G. Jingde, "A Mathematical Model for Converter-Fed Synchronous Machines With Dual Three-Phase Windings," International Conference on Electrical Machines, Zurich, Switzerland, August 29, 1991, pp. 347-351.

[3]. Kataoka, Teruo and E.H. Watanabe, "Steady-State Characteristic of a Current-Source Inverter/Double-Wound Synchronous Machine System for Ac Power Supply," IEEE Transactions on Industry Applications, Vol. 16, No. 2, 1980, pp. 262-270.

[4]. Park, R.H., "Two-Reaction Theory of Synchronous Machines: Generalized Method of Analysis-Part 1," Transactions AIEE, Vol. 29, No. 33, 1929, pp. 716-730.

[5]. Schiferl, R.F., "Six Phase Synchronous Machine With AC and DC Stator Connection," IEEE Transactions on Power Apparatus and Systems, Vol. PAS-102, No.8, August 1983, pp. 2685-2700.

[6]. Bunzel, E. and G. Muller, "General Analysis of a 6-Phase Synchronous Machine," International Conference on Electrical Machines, Zurich, Switzerland, August 29, 1991, pp. 333-340.

[7]. Fuchs, E.F. and L.T. Rosenberg, "Analysis of Alternator With Two Displaced Stator Windings," IEEE Transactions on Power Apparatus and System, Vol. 93, No. 6, 1974, pp. 1776-1786.

[8]. Moustafa, E., J.P. Chassande and M. Poloujadoff, "A d-q Modelling of A Six-Phase Armature Synchronous Machine," Paper B6.3, 16th University Power Engineering Conference, Sheffield, England, April 1981.

[9]. Alger, P.L., *Induction Machines*, 2nd ed., Gordon and Breach Publishers, New York, 1970.

[10]. Press, W.H., et. al., *Numerical Recipes*, Cambridge University Press, New York, 1986.

[11]. Dommel, H.W., "Digital Computer Solution of Electromagnetic Transients in Single and Multiphase Networks," IEEE Trans., Vol. PAS-88, April 1969, pp. 388-399.

[12]. Dommel, H.W., *Electromagnetic Transients Program Reference Manual* (EMTP Theory Book), Bonneville Power Administration, Portland, U.S.A., August 1986.

[13]. Bompa, L., "EMTP Model for Multiple Coil Synchronous Machines," 14th EMTP European User Group Meeting, Trondheim, Norway, May 1988.

[14]. Lima, J.A., "Numerical Instability Due to EMTP_TACS Inter-relation," EMTP Newsletter, Vol. 5, No. 1, Jan. 1985, pp. 21-33.

[15]. Araujo, A.E.A., H.W. Dommel and J.R. Marti, "Numerical Instabilities in Power System Transient Simulations," Proceedings of the IASTED International Conference, Vancouver, Canada, August 7, 1992, pp. 176-180.

[16]. Marti, J.R. and Lin-Jiming, "Suppression of Numerical Oscillations in the EMTP," IEEE Trans. Power Systems, Vol. 4, May 1989, pp. 739-747.

[17]. Adkins, B. and Harley, R.G., *The General Theory of Alternating Current Machines*, John Wiley & Sons, Inc., New York, 1975.

[18]. Knudsen, H., "Implementation of machine model of a 6-phase synchronous machine with 2 three-phase windings spatially displaced by 30° using EMTP MODELS," EMTP Autumn Meeting, Heverlee, Belgium, November 9, 1992.

# APPENDIX A

## Fourth-order Runge-Kutta Procedure

This Appendix shows the FORTRAN listing of the simulation program and its associated subroutines for the solution of the six-phase synchronous machine current equations for the case of a six-phase short circuit at the terminals of the machine, as presented in chapter 4. The FORTRAN program "Solve.for" solves the short-circuit current equations with the fourth-order Runge-Kutta method, and gives the results as $i_{d1}$, $i_{q1}$ and $i_{01}$ or $i_{d2}$, $i_{q2}$ and $i_{02}$ . The program "Phase1.for" transforms the d, q and 0 quantities of windings a,b,c into phase quantities. The program "phase2.for" transforms $i_{d2}$, $i_{q2}$ and $i_{02}$ into the phase quantities x, y and z.

```
      PROGRAM  SOLVE.FOR
C     This program solves the short-circuit current of the six-phase machine and
C     gives the result in d, q and 0 quantities for winding a,b,c and x,y,z.
C     The method used is fourth-order Runge-Kutta.
C
      PARAMETER(NVAR=9)
      DIMENSION ISTART(NVAR)
      EXTERNAL DERIVS
      REAL*8 XBEGIN,XEND,ISTART,PI,DEITAT
      OPEN(10,FILE='solve1.OUT',STATUS='UNKNOWN')
      XBEGIN=0.0D0
      DELTAT=5.0D-5
      NSTEP=1600
      XEND=NSTEP*DELTAT
C     ISTART(1) is the initial value of I(1)--x(0)
C     ISTART(2) is the initial value of I(2)--xdot(0)
      ISTART(1)=0.0
      ISTART(2)=0.0
      ISTART(3)=0.0
```

```
      ISTART(4)=0.0
      ISTART(5)=456.35
      ISTART(6)=0.0
      ISTART(7)=0.0
      ISTART(8)=0.0
      ISTART(9)=0.0
      CALL RKDUMB(ISTART,NVAR,XBEGIN,XEND,NSTEP,DERIVS)
      END
      SUBROUTINE DERIVS(T,I,DIDT)
      DIMENSION I(*),DIDT(*)
      REAL*8 I,DIDT,T
      DIDT(1)=-118.6*I(1)+1795.9*I(2)+66.2*I(3)+1415.9*I(4)+2.4*I(5)+
     *      3.2*I(6)+1015.4*I(7)-1101.3
      DIDT(2)=-2110.8*I(1)-106.6*I(2)-1736.8*I(3)+76.9*I(4)-1251.4*I(5)
     *      -1251.4*I(6)+3.5*I(7)+0.0
      DIDT(3)=66.2*I(1)+1415.9*I(2)-118.6*I(3)+1795.9*I(4)+2.4*I(5)+
     *      3.2*I(6)+1015.4*I(7)-1101.3
      DIDT(4)=-1736.8*I(1)+76.9*I(2)-2110.8*I(3)-106.6*I(4)-1251.4*I(5)
     *      -1251.4*I(6)+3.5*I(7)+0.0
      DIDT(5)=40.3*I(1)-2471.5*I(2)+40.3*I(3)-2471.5*I(4)-10.0*I(5)+
     *      3.8*I(6)-1562.8*I(7)+4552.1
      DIDT(6)=35.8*I(1)-2191.7*I(2)+35.8*I(3)-2191.7*I(4)+2.6*I(5)-
     *      13.5*I(6)-1385.8*I(7)-1166.1
      DIDT(7)=4825.2*I(1)+37.3*I(2)+4825.2*I(3)+37.3*I(4)+3138.8*I(5)+
     *      3138.8*I(6)-11.0*I(7)+0.0
      DIDT(8)=-8055.55*I(8)
      DIDT(9)=-8055.55*I(9)
      RETURN
      END

      SUBROUTINE RKDUMB(ISTART,NVAR,X1,X2,NSTEP,DERIVS)
      PARAMETER (NMAX=10)
      INTEGER CONT
      REAL*8 ISTART,V,DV,X1,X2,H,X
      DIMENSION ISTART(NVAR),V(NMAX),DV(NMAX)
      DO 11 I=1,NVAR
       V(I)=ISTART(I)
11    CONTINUE
      X=X1
      H=(X2-X1)/NSTEP
      CONT=0
      WRITE(10,100) X1,V(1),V(2),V(8)
C     WRITE(10,100) X1,V(3),V(4),V(9)
```

```
      DO 13 K=1,NSTEP
       CALL DERIVS(X,V,DV)
       CALL RK4(V,DV,NVAR,X,H,V,DERIVS)
      WRITE(10,100) X,V(1),V(2),V(8)
C     WRITE(10,100)X,V(3),V(4),V(9)
       IF(X+H.EQ.X)PAUSE 'Stepsize not significant in RKDUMB.'
       X=X+H
13     CONTINUE
100    FORMAT(F16.6,2X,F10.4,2X,F10.4,2X,F10.4)
      RETURN
      END

      SUBROUTINE RK4(Y,DYDX,N,X,H,YOUT,DERIVS)
      PARAMETER (NMAX=10)
      REAL*8 Y,DYDX,YOUT,YT,DYT,DYM,X,H,H6,HH,XH
      DIMENSION
     Y(N),DYDX(N),YOUT(N),YT(NMAX),DYT(NMAX),DYM(NMAX)
      HH=H*0.5
      H6=H/6.
      XH=X+HH
      DO 11 I=1,N
       YT(I)=Y(I)+HH*DYDX(I)
11     CONTINUE
      CALL DERIVS(XH,YT,DYT)
      DO 12 I=1,N
       YT(I)=Y(I)+HH*DYT(I)
12     CONTINUE
      CALL DERIVS(XH,YT,DYM)
      DO 13 I=1,N
       YT(I)=Y(I)+H*DYM(I)
       DYM(I)=DYT(I)+DYM(I)
13     CONTINUE
      CALL DERIVS(X+H,YT,DYT)
      DO 14 I=1,N
       YOUT(I)=Y(I)+H6*(DYDX(I)+DYT(I)+2.*DYM(I))
14     CONTINUE
      RETURN
      END
```

---

```
      PROGRAM PHASE1.FOR
C     This program transforms the d, q and 0 (1) quantities into
C     phase quantities a, b and c.
      DIMENSION TINV(3,3),X(3),Y(3)
```

```
      REAL*8 BETA,LAMDA,TIME,TINV,OMEGA,X,PI,Y
      OPEN(12,FILE='PHASE1.OUT',STATUS='UNKNOWN')
      OPEN(10,FILE='SOLVE1.OUT',STATUS='OLD')
      PI=DACOS(-1.0D0)
      WRITE(*,*) 'OMEGA,LAMDA'
      READ(*,*) OMEGA,LAMDA
 300  READ(10,*,END=200) TIME,X(1),X(2),X(3)
      BETA=OMEGA*TIME+LAMDA
      TINV(1,1)=DSQRT(2.D0)/DSQRT(3.D0)*DCOS(BETA)
      TINV(1,2)=DSQRT(2.D0)/DSQRT(3.D0)*DSIN(BETA)
      TINV(1,3)=1/DSQRT(3.D0)
      TINV(2,1)=DSQRT(2.D0)/DSQRT(3.D0)*DCOS(BETA-2.D0*PI/3.D0)
      TINV(2,2)=DSQRT(2.D0)/DSQRT(3.D0)*DSIN(BETA-2.D0*PI/3.D0)
      TINV(2,3)=1/DSQRT(3.D0)
      TINV(3,1)=DSQRT(2.D0)/DSQRT(3.D0)*DCOS(BETA+2.D0*PI/3.D0)
      TINV(3,2)=DSQRT(2.D0)/DSQRT(3.D0)*DSIN(BETA+2.D0*PI/3.D0)
      TINV(3,3)=1/DSQRT(3.D0)
      CALL MULTVECTOR(TINV,X,Y)
      WRITE(12,100) TIME,Y(1),Y(2),Y(3)
 100  FORMAT(F16.6,2X,F16.4,2X,F16.4,2X,F16.4)
      GO TO 300
 200  CONTINUE
      END

      SUBROUTINE MULTVECTOR(A,X,Y)
      DIMENSION A(3,3),X(3),Y(3)
      REAL*8 A,X,SUM,Y
      DO 1 I=1,3
      SUM=0.D0
      DO 2 K=1,3
      SUM=SUM+A(I,K)*X(K)
 2    CONTINUE
      Y(I)=SUM
 1    CONTINUE
      RETURN
      END
-----------------------------------------------------------------
      PROGRAM PHASE2.FOR
C     This program transforms the d, q, and 0 (2) quantities into
C     the phase quantities x, y and z.
      DIMENSION TINV(3,3),X(3),Y(3)
      REAL*8 BETA,LAMDA,TIME,TINV,OMEGA,X,PI,Y
      OPEN(12,FILE='phase2.OUT',STATUS='UNKNOWN')
```

```
      OPEN(10,FILE='solve2.out',STATUS='OLD')
      PI=DACOS(-1.0D0)
      WRITE(*,*) 'OMEGA,LAMDA'
      READ(*,*) OMEGA,LAMDA
300   READ(10,*,END=200) TIME,X(1),X(2),X(3)
      BETA=OMEGA*TIME+LAMDA
      TINV(1,1)=DSQRT(2.D0)/DSQRT(3.D0)*DCOS(BETA-PI/6.D0)
      TINV(1,2)=DSQRT(2.D0)/DSQRT(3.D0)*DSIN(BETA-PI/6.D0)
      TINV(1,3)=1/DSQRT(3.D0)
      TINV(2,1)=DSQRT(2.D0)/DSQRT(3.D0)*DCOS(BETA-5.D0*PI/6.D0)
      TINV(2,2)=DSQRT(2.D0)/DSQRT(3.D0)*DSIN(BETA-5.D0*PI/6.D0)
      TINV(2,3)=1/DSQRT(3.D0)
      TINV(3,1)=DSQRT(2.D0)/DSQRT(3.D0)*DCOS(BETA+5.D0*PI/6.D0)
      TINV(3,2)=DSQRT(2.D0)/DSQRT(3.D0)*DSIN(BETA+5.D0*PI/6.D0)
      TINV(3,3)=1/DSQRT(3.D0)
      CALL MULTVECTOR(TINV,X,Y)
      WRITE(12,100) TIME,Y(1),Y(2),Y(3)
100   FORMAT(F16.6,2X,F16.4,2X,F16.4,2X,F16.4)
      GO TO 300
200   CONTINUE
      END

      SUBROUTINE MULTVECTOR(A,X,Y)
      DIMENSION A(3,3),X(3),Y(3)
      REAL*8 A,X,SUM,Y
      DO 1 I=1,3
      SUM=0.D0
      DO 2 K=1,3
      SUM=SUM+A(I,K)*X(K)
2     CONTINUE
      Y(I)=SUM
1     CONTINUE
      RETURN
      END
```

# APPENDIX B

## Subroutine Source

This Appendix shows the FORTRAN listing of the subroutine Source. This subroutine uses as input the values of the currents at the terminals of the two machines at each time step. The current values are generated by EMTP and stored in x(1) to x(6). The subroutine then uses these values, along with the constant parameters x(7) to x(9) needed for the definition of $[D_1]$ and $[D_2]$, to generate six user-defined current sources to be injected back into the network at each time step. This subroutine also uses a linear extrapolation, to reduced the severity of the time delay between the EMTP and the subroutine Source.

```
      PROGRAM  SOURCE.FOR
      SUBROUTINE SOURCE(X,LX,SOURC,T,DELTAT,IHALF,KREAD)
      DIMENSION X(*),SOURC(*),AUX1(10)
      IMPLICIT REAL*8 (A-H,O-Z),INTEGER*4 (I-N)
      SAVE AUX1
      KREAD=0
      SOURC(1)=X(1)*X(7)+X(2)*X(8)+X(3)*X(9)
      SOURC(2)=X(1)*X(9)+X(2)*X(7)+X(3)*X(8)
      SOURC(3)=X(1)*X(8)+X(2)*X(9)+X(3)*X(7)
      SOURC(4)=X(4)*X(7)+X(5)*X(9)+X(6)*X(8)
      SOURC(5)=X(4)*X(8)+X(5)*X(7)+X(6)*X(9)
      SOURC(6)=X(4)*X(9)+X(5)*X(8)+X(6)*X(7)
C     Linear Extrapolation for improving the time delay.
      IF(X(10).EQ.0.0D0) RETURN
      IF(T.EQ.0.0D0) THEN
       DO 15 I=1,6
        AUX1(I)=SOURC(I)
   15   CONTINUE
      END IF
```

```
      DO 20 I=1,6
       AUX2=2.D0*SOURC(I)-AUX1(I)
       AUX1(I)=SOURC(I)
       SOURC(I)=AUX2
20     CONTINUE
      END
```

# APPENDIX C
## EMTP Input File

This Appendix shows the EMTP input listing for the simulation of a six-phase short circuit at the terminals of a six-phase synchronous machine. "Cards" 4, 5 and 6 define the machine self and mutual impedances. The six switches at the terminals of the machine represent the six-phase short circuit.

```
   EMTP INPUT FILE
*  . . . . . . . . Case identification card
  SIX-PHASE  SYNCHRONOUS MACHINE MODEL, SHORT CIRCUIT TEST
        TO THE TERMINAL OF THE MACHINES
*  . . . . . . . . Time card
  50.E-6 0.12                                1
*
*  . . . . . . . . Lumped RLC branch
  GM1-aNODE1a          .03320 .5200                    1
  GM1-bNODE1b          .03320 .5200                    1
  GM1-cNODE1c          .03320 .5200                    1
  GM2-aNODE2a          .03320 .5200                    1
  GM2-bNODE2b          .03320 .5200                    1
  GM2-cNODE2c          .03320 .5200                    1
$ == End of level 1: Linear and nonlinear elements = = = = = = = = = = = =
*
*  . . . . . . . . Time-controlled switch
 NODE1a    .000000     100.0
 NODE1b    .000000     100.0
 NODE1c    .000000     100.0
 NODE2a    .000000     100.0
 NODE2b    .000000     100.0
 NODE2c    .000000     100.0
*
$ = = =  End of level 2: Switches and piecewise linear elements = = = = = = = =
```

```
*
* . . . . . . . . S.M. Node names for armature windings (Card 1)
50 GM1-a        60.0   391.920      0.0
*
* . . . . . . . . S.M. Node names for armature windings (Card 2)
  GM1-b        .1250    .480                        1
*
* . . . . . . . . S.M. Node names for armature windings (Card 3)
  GM1-c
*
* . . . . . . . . S.M. Impedances and time constants (Card 4)
3.9265   3.9265   3.9265   3.9265   4.0896   4.1107   0.00558   0.00826
*
* . . . . . . . . S.M. Impedances and time constants (Card 5)
1.8142   0.0     1.8142   0.0     1.0     2.0496   0.0     0.00872
*
0.0      0.000     0.16515    0.0
01010100000
*
* . . . . . . . . S.M. Design parameters (Card 7)

* . . . . . . . . S.M. Node names for armature windings (Card 1)
*
50 GM2-a        60.0   391.920      30.0
*
* . . . . . . . . S.M. Node names for armature windings (Card 2)
  GM2-b        .1250    .480                        1
*
* . . . . . . . . S.M. Node names for armature windings (Card 3)
  GM2-c
*
* . . . . . . . . S.M. Impedances and time constants (Card 4)
3.9265   3.9265   3.9265   3.9265   4.0896   4.1107   0.00558   0.00826
*
* . . . . . . . . S.M. Impedances and time constants (Card 5)
1.8142   0.0     1.8142   0.0     1.0     2.0496   0.0     0.00872
0.0      0.000    0.16515   0.0
01010100000
*
* . . . . . . . . S.M. Design parameters (Card 7)
 1 GM2-a-1
 2 GM2-b-1
 3 GM2-c-1
```

```
 4 GM1-a-1
 5 GM1-b-1
 6 GM1-c-1
$ = = =  End of level 3: Sources  = = = = = = = = = = = = = = = = = = = = = = = = =
  GM1-a
$ = = =  End of level 4: User-defined voltage output  = = = = = = = = = = = = =
.USER
 I GM1-aNODE1a
 I GM1-bNODE1b
 I GM1-cNODE1c
 I GM2-aNODE2a
 I GM2-bNODE2b
 I GM2-cNODE2c
P          -0.9106836
P           0.2440169
P          -0.3333333
P           0.00000
.END
```