A MAGNETOHYDRODYNAMICS STUDY USING
AN ELECTROMAGNETIC SHOCK TUBE

by

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ABSTRACT

This thesis is concerned with a theoretical and experimental investigation of Alfvén waves in an ionized medium, and magnetic interaction effects between a moving plasma and a magnetic coil external to the plasma.

Methods for generating Alfvén disturbances for varying conditions of gas density and magnetic fields are considered and various means for measuring any effects that may be produced. It will be seen that for propagation of m.h.d. waves, extremely strong coupling between the plasma and field is necessary with consequent necessary high fields and Alfvén speeds. The effect of an axial magnetic field modifying the shock speed in the plasma is investigated and also the effect of the field on incident and reflected shock speeds by placing a plain obstruction in the shock tube which blocks the plasma flow.

A further study of magnetic interaction effects between a moving plasma and a localized radial field was undertaken with the desire of correlating mechanical momentum transfer with varying conditions of applied field and gas pressures in the plasma (hence conductivity, density, and shock speed variations). Mechanical and electrical measurements of momentum transfer are compared with theory, and it will be seen that the mechanical method offers a fairly reliable means of measurement.
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SYMBOLS USED

\[ \begin{align*}
    \mathbf{u} &= \text{shock speed} \\
    \mathbf{p}_0 &= \text{initial gas pressure} \\
    \mathbf{p} &= \text{gas density} \\
    \mathbf{B} &= \text{magnetic induction} \\
    \mathbf{H} &= \text{magnetic field} \\
    \mathbf{E} &= \text{electric field} \\
    \mathbf{V} &= \text{particle velocity} \\
    \mathbf{J} &= \text{current density} \\
    \mathbf{a} &= \text{Alfvén speed} \\
    \mathbf{I} &= \text{current} \\
    \mathbf{CV} &= \text{condenser voltage of magnetic field system} \\
    \mathbf{L} &= \text{characteristic length} \\
    \mathbf{a}_s &= \text{speed of sound in Argon at room temperature} \\
    \mathbf{F} &= \text{force} \\
    \mathbf{L,M} &= \text{self, mutual inductance} \\
    \mathbf{C} &= \text{capacitance} \\
    \mathbf{V} &= \text{voltage} \\
    \mathbf{\Phi} &= \text{flux} \\
    \mathbf{\tau} &= \text{period of field discharge} \\
    \mathbf{r,e,h,s,d,l} &= \text{characteristic parameters of pendulum} \\
    \mathbf{P} &= \text{momentum transfer} \\
    \mathbf{b} &= \text{amplitude of induced voltage cosine curve} \\
    \mathbf{n} &= \text{number of turns per unit length} \\
    \mathbf{x} &= \text{linear variable} \\
    \mathbf{T} &= \text{temperature} \\
    \mathbf{\Omega} &= \text{ratio of magnetic energy to flow energy}
\end{align*} \]
\( \mu_0 = \) permeability
\( \sigma = \) gas conductivity
\( R_m = \) magnetic Reynolds number
\( q = \) charge
\( r, \theta, z = \) cylindrical coordinate system
\( \omega = \) angular frequency
\( k = \) wave number
\( \kappa = \) magnetic diffusivity, compression ratio
\( M = \) Mach number
\( \gamma = \) ratio of pressures across shock front
\( A = \) area
\( V = \) volume
\( t = \) time
\( g = \) acceleration due to gravity
\( \delta = \) skin depth
\( \gamma = \) ratio of specific heats
I. INTRODUCTION

An investigation of Alfvén waves in shock ionized Argon, and momentum transfer from a moving body of plasma to a magnetic coil has been conducted with some success.

The hydromagnetic disturbance which gives rise to Alfvén waves may be thought of as an elastic stretching of magnetic lines of force, or the propagation of a perturbation to the magnetic field. The experimental problem was to induce Alfvén waves and then to detect their propagation in ionized gas. To this end an electromagnetic shock tube was used to provide a body of Argon plasma moving with a velocity of the order of $u = 8 \times 10^5$ cm/sec down the tube, through which an axial magnetic field $\mathbf{B}_0/\mu_0$ (i.e. in the direction of $u$) was applied (Fig. 5). With high enough gas conductivity the plasma flow would be constrained to move along the magnetic lines of force, and so any interruption in flow would produce a radial component of magnetic field. Downstream, various geometrical bodies were used to disturb the fluid, and in so doing Alfvén waves should be propagated. In the instance where the Alfvén speed is greater than the shock speed, these waves should be detectable upstream from the disturbance.

Two methods were used to detect such propagation: photographs of the flow angle past a probe in the plasma, and search coil measurements of change in magnetic field. Smear camera photographs of incident and reflected shocks on a plain obstruction were used to try and correlate incident and reflected shock speeds with the applied magnetic field.

The problem of momentum transfer from a moving body of magnetized plasma to a magnetic coil is theoretically analyzed as a current inter-
action resulting from the mutual inductance between a copper ring around the shock tube and induced currents in the plasma.

![Diagram](image)

The copper ring is effectively diamagnetic and so a current $i_r$ is induced in such a manner that its field $\mathbf{B}_r/\mu_0$ opposes the applied field $\mathbf{B}_0/\mu_0$. The current $i_r$ gives a radial field $\mathbf{B}_r/\mu_0$ which interacts with the moving plasma to give azimuthal currents of density $\mathbf{j} = \sigma (\mathbf{u} \times \mathbf{B}_r)$. These currents in turn interact with the azimuthal current induced in the copper ring. The resulting repulsive force was measured experimentally by two methods: mechanical and electrical.

The copper coil was suspended as a pendulum (see Fig. 11) and the resulting $\mathbf{L}_r \mathbf{B}_r$ force imparted momentum to the coil whose amplitude of swing was measured for various applied fields. A galvanometer mirror system was used to measure the angular deflection. The mechanical measurement too, offered direct visual proof of the interaction.

Electrical measurement of the radial field $\mathbf{B}_r/\mu_0$ by a search coil technique, coupled with a current calculation also yielded a value of the force exerted on the coil (which is equal and opposite to the drag force on the plasma). Theory gives a value for the force approximately four times that measured electrically, and the mechanical measurement gives a value approximately the same as theory.
The method of presenting the current coil to the moving plasma, i.e. by using its diamagnetic properties, is preferable to pulsing an external current through the coil itself to provide a radial field. The coil can be so positioned that the magnetic field by itself offers no net force on the coil; and it is only on pulsing both plasma and field that a measurable effect takes place. The current torques acting on the coil can be compensated for by the use of trimmers (which modify local fields inductively) and by careful positioning of the copper ring in the field solenoid.

The plasma is generated by an electromagnetic shock tube (co-planar driver), and the axial magnetic field by a solenoid wound along the shock tube and connected to a 10⁴ joule capacitor bank. The LC discharge gives a sinusoidal field and the experiments are conducted when the field is maximum. Shock speeds of the order of 8 x 10⁵ cm/sec, conductivity 7 x 10⁻³ mhos/m, gas pressures from 0.1 to 1.0 mm Hg, and applied fields from 2000 gauss to 12,000 gauss were the conditions in the experiments. These fields induced currents in the copper coil of approximately 7400 amps at 1KV to 37,000 amps at 5 KV. The length of the interaction region is of the order of 8 cm.

A condition necessary for both the Alfvén wave and momentum transfer studies is that the magnetic Reynolds number be high; i.e. the plasma and field be "frozen". Conditions for a strong interaction in the momentum transfer experiment are present since, with the above parameters, \( R_m \approx 6 \). However for the Alfvén study it will be seen that a much higher \( R_m \) is necessary.
II. THEORY

1. The Magnetohydrodynamic Equations.

To determine the nature of Alfvén waves and their phase velocity, consider Maxwell's equations for a continuous ionized medium. Together with the Euler equation, the adiabatic gas law, and the continuity law, they constitute the mathematical basis for M.H.D.

\begin{align*}
\text{Ampere's Circuital Law} & \quad \nabla \times \vec{B} = \mu \mu_0 \vec{J} \\
\text{Faraday's Induction Law} & \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\
\text{Ohm's Law} & \quad \vec{J} = \sigma (\vec{E} + \nabla \times \vec{B}) \\
\text{Euler's Equation} & \quad \rho \frac{d\vec{v}}{dt} = \vec{J} \times \vec{B} - \nabla p
\end{align*}

(1) (2) (3) (4) (5)

neglecting viscous and gravitational forces.

\[
\frac{d\vec{v}}{dt} = -\frac{\nabla \times \vec{B}}{\rho} + (\nabla \cdot \vec{v}) \vec{v}
\]

Normally the \((\nabla \cdot \vec{v}) \vec{v}\) term is small and we shall neglect it.

2. Derivation of the Alfvén Velocity.

Looking at the physical processes, consider the following:

Fig. 2
Consider a long plasma subjected to uniform $B_0$ in the $Z$ direction. If segment ABCD is given velocity $\vec{V}$ parallel to the $r$-axis, then the force exerted on charge $q = q(\vec{V} \times \vec{B})$ tends to separate (+ve) and (-ve) charges as shown. Because plasma can form current loops, the plasma external to ABCD forms a closed electrical circuit. The induced current gives restoring force density $\vec{j} \times \vec{B}$ inside ABCD; whereas it accelerates the plasma outside. The process is repetitious on successive segments of plasma in the $z$ direction (since field and fluid are constrained to move together) and so there is propagation of the disturbance in the $rz$ direction.

Let the field $\vec{B} = \vec{B}_0 + \vec{B}'$, where $\vec{B}'$ is the perturbed field. Let us consider for a derivation of the Alfvén speed the simplest wave motion characterized by $\nu_r, E_\theta, j_\theta, B_r$.

From (1) $\nabla \times \vec{E}_\theta = \mu_0 \vec{J}_\theta$ (6)

From (5) $\rho \frac{\partial \nu_r}{\partial t} = j_\theta B_0$ (7a)

$\nabla \times \vec{J}_\theta = 0 = -j_\theta B_r - \frac{\partial \rho}{\partial z}$ (7b)

$\frac{\partial \nu_r}{\partial t} = \frac{j_\theta B_0}{\rho} = \frac{B_0}{\mu_0 \rho} \frac{\partial B_r'}{\partial z}$ (8)

$\frac{\partial \rho}{\partial z} = -\frac{1}{\mu_0} \frac{\partial B_r'}{\partial z}$ (9)

From (14) $E_\theta = \frac{1}{\sigma} j_\theta + \nu_r B_0$ and using (6)

$E_\theta = \frac{1}{\mu_0 \sigma} \frac{\partial B_r'}{\partial z} + \nu_r B_0$ (10)

$\frac{\partial \nu_r}{\partial t} = \frac{1}{B_0} \frac{\partial E_\theta}{\partial t} - \frac{1}{\mu_0 \sigma B_0} \frac{\partial B_r'}{\partial z}$ differentiating (10)

but $\frac{\partial \nu_r}{\partial t} = \frac{B_0}{\mu_0 \rho} \frac{\partial B_r'}{\partial z}$ (8)
\[ \frac{B_0}{\mu_0 \rho} \frac{\partial B_y'}{\partial z} = - \frac{1}{B_0} \frac{\partial E_y}{\partial t} - \frac{1}{\mu_0 \sigma B_0} \frac{\partial^2 B_y'}{\partial t \partial z} \]  

(11)

Differentiating w.r.t. \( z \) and noting from (2) that

\[ \frac{\partial E_y}{\partial z} = \frac{\partial B_y'}{\partial t} \]

then

\[ \frac{B_0}{\mu_0 \rho} \frac{\partial^2 B_y'}{\partial z^2} = \frac{1}{B_0} \frac{\partial^2 B_y'}{\partial t^2} - \frac{1}{\mu_0 \sigma B_0} \frac{\partial^3 B_y'}{\partial t \partial z^2} \]

or

\[ \frac{\partial^3 B_y'}{\partial t \partial z^2} = \frac{B_0}{\mu_0 \rho} \frac{\partial B_y'}{\partial z^2} + \frac{1}{\mu_0 \sigma} \frac{\partial^3 B_y'}{\partial t \partial z^2} \]  

(12)

Trying a solution of the form

\[ B_y' = B' e^{i(kz-\omega t)} \]  

(13a)

and substituting in (12) yields for a dispersion relation,

\[ - \omega^2 = - \alpha_k^2 + i \omega \kappa \]

or \( \omega^2 = \kappa (\alpha^2 - i \omega \kappa) \) where \( \alpha^2 = \frac{B_0}{\mu_0 \rho} \) and \( \kappa = \frac{\omega}{\mu_0 \sigma} \). Assuming \( \omega \kappa \ll \alpha^2 \) which is justifiable since \( \alpha^2 \propto 10^3 \cos \) and \( \kappa = 10^{3} \cos \) and hence \( \omega \kappa \) would have to be \( \propto 10^4 \) to be comparable; then \( \frac{\omega}{\alpha} = \frac{\omega}{\alpha} (1 + i \omega \kappa) \). Substituting this value for \( k \) into (13a) gives

\[ B_y' = B' e^{i \frac{k}{\alpha^2 \kappa^2} e^{i(kz-\omega t)} \]  

(13b)

where \( z_0 = \frac{2 \alpha^2}{\omega \kappa} = \) distance in which the plane wave is damped to \( \frac{1}{e} \) of its initial amplitude, and \( k = \frac{\omega}{\alpha} \). The Alfvén wave then is a damped propagation of the perturbation to the magnetic field in the \( z \) direction. The distance \( z_0 \) for an initial gas pressure \( p_0 = 0.5 \text{mm Hg} \), \( B_0 = 6000 \text{ gauss} \) is 51.3 metres which is \( \gg \) any characteristic length in the experiment and so there should be negligible attenuation of the wave in our experiment.

Thus very nearly for large \( \sigma \)

\[ \frac{\partial^3 B_y'}{\partial t \partial z^2} = \alpha^2 \frac{\partial^3 B_y'}{\partial z^2} \]  

(13c)

where \( \alpha = \left( \frac{B_0}{\mu_0 \rho} \right)^{1/2} \) (14) is the phase velocity of the m.h.d. wave and is called the Alfvén velocity.

In order to achieve a strong interaction between plasma and field it is necessary that the field and plasma be effectively "frozen". Consider Maxwell's equations: (1), (2) and (4)

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{E} = \nabla \times \left\{ \frac{1}{\sigma} \mathbf{J} - \nabla \times \mathbf{B} \right\} \] (15)

but

\[ \mathbf{J} = \frac{1}{\mu_0} \nabla \times \mathbf{B} \] (1)

hence

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\nabla \times \mathbf{B}) - \frac{1}{\mu_0 \sigma} \nabla \times (\nabla \times \mathbf{B}) \] (16)

Now

\[ \nabla \times (\nabla \times \mathbf{B}) = \nabla (\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} \] vector identity

and

\[ \nabla \cdot \mathbf{B} = 0 \]

\[ \therefore \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\nabla \times \mathbf{B}) + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B} \] (17)

Let \( \kappa = \frac{1}{\mu_0 \sigma} \) and if \( \nabla \times (\nabla \times \mathbf{B}) = 0 \)

then \( \frac{\partial \mathbf{B}}{\partial t} = \kappa \nabla^2 \mathbf{B} \) which is the form of a diffusion equation, where \( \kappa \) is the magnetic diffusivity. On the other hand if \( \kappa \to \infty \) then

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\nabla \times \mathbf{B}) \]

which, analogous to vorticity in fluid flow, indicates the field lines are constrained to move with the fluid. The ratio of these two effects is an indication of the coupling of fluid motion and field. The dimensionless parameter \( R_M \) (magnetic Reynolds number) is a measure of degree of constraint. Taking \( L \) to be a characteristic length of the system i.e., a distance in which field variables change significantly, then define

\[ R_M = \frac{u \mathbf{B} / L}{\kappa \mathbf{B} / L^2} = \frac{\mu \sigma u L}{\kappa} \]

An \( R_M \gg 1 \) indicates strong coupling between the particle motion and magnetic field, i.e. the plasma and field are "frozen". For our conditions \( u = 8 \times 10^5 \) cm/sec, \( L = 5 \) cm, \( \sigma = 6 \times 10^3 \) mhos/m, and hence \( R_M \approx 3 \). It will be seen this \( R_M \) is high enough for magnetic interaction effects but is not
sufficiently high for detectable Alfvén disturbances.

4. Strong Shock Relations.

Properties of a strong shock wave give the following results:

\[ M = \frac{u}{a_0} \]  \hspace{1cm} (18)

\( u \) = shock speed

\( a_0 \) = local sound speed in Argon

\[ a_0 = \sqrt{\gamma RT} = 3.2 \times 10^4 \text{ cm/sec at room temperature} \]

Compression ratio \( \kappa = \frac{\rho_f}{\rho_i} = \frac{\gamma+1}{\gamma-1} \)  \hspace{1cm} (19)

\( \kappa \) for Argon \( \kappa = 4 \)

Pressure ratio across the shock front \( \frac{p_f}{p_i} = \frac{2\gamma M^2}{\gamma+1} \)  \hspace{1cm} (20)

\( \gamma \) for Argon \( \gamma = 1.67 \)

\( \kappa \) for Argon \( \kappa = 4 \)

at gas \( p_i = 0.5 \text{ mm Hg} \), \( B_0 = 5040 \text{ gauss} \).

Hence \( p_f = 380 \text{ mm Hg} \).


For the theoretical background of the momentum transfer between moving plasma and magnetized field coil system consider the following (see Fig. 1). A body of plasma travelling with velocity \( U \) enters the radial field region. The axial field \( B_0/\mu_0 \) is generated by a capacitor bank discharging through a solenoid (an LC discharge, with the experiment conducted when the field is at maximum). The copper ring acts as a dipole and a current \( i_1 \) is induced, tending to provide an equal and opposite field \( B_1/\mu_0 \) to that applied, \( B_0/\mu_0 \). The copper ring has an L/R time approximately 20 times that of the LC discharge so that the radial field \( B_1/\mu_0 \) does not decay appreciably during the experimental time (\( < 20 \mu\text{sec} \)). The current \( i_1 \) causes a radial field \( B_1/\mu_0 \) which interacts with the moving plasma.
inducing an azimuthal \( \vec{E} \) field given by
\[
\vec{E} = \vec{u} \times \vec{B}_r
\]  
(21)

The azimuthal current density \( \vec{J} \) is thus given by
\[
\vec{J} = \sigma \vec{E} = \sigma (\vec{u} \times \vec{B}_r)
\]  
(22)

and hence the induced current \( i_2 \) is related to \( \vec{u} \) and \( \vec{B}_r \) by
\[
i_2 = \sigma (\vec{u} \times \vec{B}_r) dA\]

or
\[
i_2 = \sigma u \vec{B}_r dA\]  
(23)

The differential coefficient of mutual inductance between current rings \( i_1 \) and \( i_2 \) gives an incremental repulsive force
\[
F' = i_1 i_2 \frac{dM}{dz}
\]  
(24)

The position of the copper ring is taken as the \( z = 0 \) origin.

Now, from \( F = I(\vec{I} \times \vec{B}) \), the force exerted on a current carrier by an external magnetic field;
\[
F' = -2\pi r \; i_2 \; B_r
\]  
(25)

where \( B_r \) is the radial field at the current filament \( i_2 \).

But
\[
F' = i_1 i_2 \frac{dM}{dz}
\]  
(24)

hence
\[
B_r = -\frac{i_1}{2\pi r} \left( \frac{dM}{dz} \right)
\]  
(26)

Substituting for \( B_r \) in (23) from (26) and then for \( i_2 \) in (24) from (23) yields
\[
\text{incremental } F' = \frac{\sigma u i_1^2}{2\pi r^2} \left( \frac{dM}{dz} \right)^2 dA
\]  
(27)
where \( dA = dr_2 \, dz \). Hence the total force repulsing the plasma from the copper ring is

\[
F = \frac{\sigma u_i}{2\pi} \int_A \frac{1}{r_2} (\frac{dM}{dz})^2 \, dA
\]

(28)

assuming \( \sigma, u \) and \( l \) are constant during the time of experiment. These conditions are satisfied with the experimental arrangement employed. Variables \( \sigma \) and \( u \) are determined by the shock tube and \( l \) is kept constant by having an \( L/R \) time for the copper ring approximately 20 times the \( L \) discharge time. The above derivation is dependent on the assumption of total current density \( j \) being given by \( j = \sigma \cdot u \cdot B \) entirely, i.e., neglecting any "back field" effects due to plasma currents. This is also justifiable since any magnitude correction resulting from self-inductance is of second order and therefore small.

An alternative derivation of (28) results from the force per unit volume being \( |j \times B| = \sigma \cdot u \cdot B \) and hence total \( F = \int V \sigma \cdot u \cdot B \, dV \)

where \( dV = 2\pi r_2 \, dr_2 \, dz \), and since \( B \cdot r_2 = -\frac{i}{2\pi r_2} (\frac{dM}{dz}) \)

\[
F = \frac{\sigma u_i}{2\pi} \int_A \frac{1}{r_2} (\frac{dM}{dz})^2 \, dA \, dz
\]

(28)

To obtain a value of \( F \) it is necessary to numerically integrate expression (28), for \( F \) is given in terms of an elliptic integral. However \( (\frac{dM}{dz}) \) has been extensively tabulated (reference 4) in terms of current filaments and so an accurate value for \( F \) can be obtained by numerically integrating over \( z \) and \( r_2 \).

From Grover, \( (\frac{dM}{dz}) \) is obtainable from a dimensionless variable \( k^2 \) relating the geometry and separation of the current loops. A value for \( F \) was arrived at by considering squares for \( dA \) and reducing these to equivalent current filaments (see Grover). The dimensions of the geometry are \( r_1 = 3.65 \, \text{cm} \), \( 2.54 \, \text{cm} \). Taking squares for \( \delta A \) i.e.,
dz = 0.5 cm, \( dr_2 = 0.508 \) cm, with \( r_2 \), and \( z \) taken at the centres of these squares and using equivalent filaments for \( \frac{dM}{dz} \) of radius \( r_2' = r_2 \left(1 + \left(\frac{dr_2}{2r_2}\right)^2\right) \) one arrives at a value for \( F \) from the simplification

\[
F = \frac{\sigma u_i^2}{2\pi} \left\{ \sum_{r_2} \frac{z}{r_2^2} \left( \frac{dM}{dz} \right)^2 \right\} \cdot 8A
\]

where the summations are over all \( r_2 \) and \( z \). The data is given in Appendix I. The result is

\[
F = 2.06 \times 10^{-15} \sigma u_i^2 \text{ newtons}
\]

where \( \sigma, u_i, i \) are in M.K.S. units.

A plot of \( \int \frac{1}{r_2} \left( \frac{dM}{dz} \right)^2 dr_2 \) versus \( z \) is given in Fig. 4.


The shape and ordinates of the \( \int \frac{1}{r_2} \left( \frac{dM}{dz} \right)^2 dr_2 \) versus \( z \) plot should compare with the voltage traces of the time differential of radial field as observed on the C.R.O., for:

\[
F = \sigma u_i i B_{r_2}
\]

from (25)

hence

\[
B_{r_2} = \sigma u_i i \int \frac{1}{r_2} \left( \frac{dM}{dz} \right)^2 dr_2 dz
\]

and since

\[
\frac{d}{dt} = \frac{d}{dz} \frac{dz}{dt} = u \frac{dz}{dz}
\]

then

\[
V' = -A \frac{dB_r}{dt} = -\frac{\sigma u_i A i}{4\pi^2 r_1} \int \frac{1}{r_2} \left( \frac{dM}{dz} \right)^2 dr_2
\]

where \( A \) is the area of the search coil, and so the voltage trace

\[
V' = -\frac{df}{dt}
\]

is proportional to \( \int \frac{1}{r_2} \left( \frac{dM}{dz} \right)^2 dr_2 \).
III. EXPERIMENTAL ARRANGEMENT

1. The Shock Tube.

An electromagnetically driven shock tube (co-planar driver) as shown in Fig. 5 with the solenoid in position gives a body of plasma 20 cm long travelling with a speed of the order of $8 \times 10^5$ cm/sec, depending on the voltage of the driver capacitor bank. This bank is rated at 20 KV and 50 $\mu$F although practically all experimentation was done at 16 KV. The resulting plasma is approximately 80% ionized and travels at Mach 2½ down a pyrex tube of 2 inch diameter around which the magnetic field solenoid is wound. The electromagnetic energy from the driver bank is dumped into a cylindrical slug of atmospheric air which is ejected through a mylar diaphragm and travels down the tube ionizing the Argon therein as it moves (Fig. 5). Depending on the initial downstream pressure of Argon, the conductivity of the gas is otherwise determined by the energy of the discharge. If the loss of energy in the discharge is neglected and all energy is imparted to the Argon (an incorrect assumption since the mass of air retains a portion); then, since the energy $\propto V^2$ of the bank goes to the thermal energy of the molecules $\propto kT$, the conductivity (which varies as $T^{\alpha/2}$) is therefore approximately proportional to $V^3$. The conductivity also increases with pressure since a greater number of charge carriers are then present.

2. The Magnetic Field System.

The magnetic field system (Fig. 5) is composed of three, 224 $\mu$F, 5 KV low inductance capacitors in parallel. Large copper strips are used
shock tube and solenoid

magnetic field system

luminous plasma

spark gap and muffler

Fig. 5
as leads to keep the inductance to a minimum and an air spark gap which is triggered externally is used as the series switch to initiate discharge. A portable D.C. charging unit was used to charge the bank through a series resistance of 27 $\Omega$ mounted in the portable magnetic field unit. A parallel connection of relay and 10,000 watt resistors gave a dumping bypass for the charged capacitor bank. The spark gap construction is shown in Fig. 5 with the muffler behind. This muffler was constructed of perspex and inch-thick insulating foam to deaden the noise from the discharge.

The copper strips used as leads to the solenoid were insulated with polythene and clamped together to reduce inductance and prevent flapping from current forces exerted when the capacitors were being discharged. Insulation was used liberally in the interests of safety and the whole system was enclosed with wire in a dexion cage. In addition, 3/4 inch plywood on one side, end, and top was added as a precaution, in the event of any loose material being thrown around by a charged bank.

The solenoid was wound of 1/4 inch copper tubing with a total of 21 turns of separation 1 1/2 inches. Braced wood spacers arranged equilaterally around the circumference of the solenoid provided rigidity for the assembly.

The field discharge is a damped LC sinusoid.

![Kirchhoff's Law Diagram](image)

Kirchhoff's Law
\[ V = iR + L \frac{di}{dt} = \frac{Q}{C} \]
Differentiate w.r.t. \( t \), then
\[
\frac{d^2i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = 0
\]
or
\[
\frac{d^2i}{dt^2} + 2\pi f \omega \frac{di}{dt} + \omega^2 i = 0
\]
where \( \omega = \frac{1}{\sqrt{LC}} \) (36) and \( 2\pi f = \frac{R}{L} \), whence
\[
\omega = \frac{2\sqrt{\pi f}}{\sqrt{L}} < 1.
\]

At \( t = 0 \), \( V = V_0 \) and \( i = 0 \); hence the current is given by
\[
i = i_0 e^{-\rho t} \sin \omega \sqrt{1-\rho^2} t
\]
where \( i_0 = V_0 \sqrt{\frac{L}{C}} \) assuming small resistance. The period of discharge (neglecting \( \frac{1}{\rho^2} \) correction) is
\[
T = 2\pi \sqrt{LC}
\]
(38)

For small \( \rho \) then, (neglecting \( \frac{1}{\rho^2} \) and higher order terms)
\[
i = i_0 e^{-\rho t} \sin \omega t
\]
(39) which is a damped LC sinusoid.

A self-inductance calculation for the solenoid (from formula and tables in the "Rubber Handbook of Physics and Chemistry") gives
\[
L = 6.5 \mu H
\]
for a geometry of:

- length of solenoid = 76.2 cm
- mean radius of solenoid = 5.5 cm
- diameter of copper wire used in solenoid = 0.64 cm
- number of turns = 21

Allowing for one large turn from the leads of the magnetic system, an inductance of approximately \( 2 \mu H \) is present. Hence total inductance
\[
\approx 8.5 \mu H , \quad \text{capacitance} = \cap (2\pi \mu f) = 672 \mu F
\]
\[
\therefore \quad T = 0.474 \text{ millisecond}
\]
From the measured period of the discharge (C.R.O. trace), \( T = 0.48 \text{ millisecond} \) which agrees very well with the calculated value.
We wish now to calculate the maximum field available from the system.

Energy of the bank = \( \frac{1}{2} CV^2 = \frac{1}{2} \left( 672 \times 10^{-6} \right) \left( 5 \times 10^3 \right)^2 \)

= 8400 joules.

Maximum current \( I_m \) neglecting resistance, is given by

\[
\frac{1}{2} CV^2 = \frac{1}{2} L I_m^2
\]

\[
I_m = \sqrt{\frac{16800}{3.5 \times 10^{-6}}}
\]

\[
I_m = 4.45 \times 10^4 \text{ amps.}
\]

hence

\[
B_m = \mu_0 n I_m
\]

\[
= 4\pi \times 10^{-7} \left( \frac{2}{0.762} \right) \left( 4.45 \times 10^4 \right) \text{ w/m}^2
\]

\[
B_m = 15400 \text{ gauss.}
\]

Alternatively we can consider the energy of the condenser bank going into magnetic energy of the solenoid.

\[
\frac{1}{2} CV^2 = \int_{V} \frac{B^2}{2\mu_0} dV \equiv \frac{B^2}{2\mu_0} V
\]

(41)

where the volume is that of the solenoid, \( V = Al \), which yields a \( B_m = 17,000 \) gauss. Correcting for the inductance of the solenoid gives \( B_m = 15,000 \) gauss.

From a preliminary calibration of the field using 69 volts across the gap, a maximum field of 11,600 gauss is obtainable on scaling up to a supply of 5 KV. In actual fact the maximum field attained in final calibration was 11,450 gauss. The magnetic field generated in the solenoid varies linearly with the initial voltage of the condenser bank and all experiments were conducted at a time near the maximum of the field, i.e., at 0.12 millisecond after triggering. In fact the applied axial field slows the plasma somewhat (from \( M = 24 \) to \( M = 22 \) at an applied field of 5040 gauss.
with gas $p_0 = 0.5 \text{ mm Hg}$ due to some radial field in the solenoid, and synchronizing plasma and field meant conducting the experiment at a distance $s = u t$ down the shock tube from the driver. With a shock speed of $7.8 \times 10^5 \text{ cm/sec}$ the test section was located 93 cm from the driver to have field and plasma synchronized.


The magnetic field calibration was accomplished by mounting a single turn search coil around the shock tube and pulsing the solenoid discharge as shown in Fig. 7.

![Diagram of solenoid and shock tube](image)

Fig. 7

The LC sinusoidal discharge gives a sinusoidal field pick-up of the flux through the search coil. Since the voltage pick-up $V'$ is given by

$$V' = -\frac{d \Phi}{dt}$$

the time rate of change of flux through the coil, then since $\Phi = AB$ where $A$ is the area enclosed and $B \mu_0$ is the instantaneous field, hence:

$$V' = -A \frac{dB}{dt}$$

or

$$B = -\frac{1}{A} \int_0^t V' dt$$

Therefore integrating the voltage time curve gives the magnetic induction value.
At $\tau/4$, a quarter period of the discharge, $\frac{dI}{dt} = 0$ and the field is maximum in the LC discharge. The shaded area then, from $t = 0$ to $t = \tau/4$ gives the field at time $t = \tau/4$.

4. Production and Detection of Alfvén Waves.

(i) Probes and Camera.

The first means attempted to detect an Alfvén propagation utilized a lucite probe in the plasma flow as shown in Fig. 9. The gas flow would be interrupted by the probe, sending out a hydromagnetic disturbance. The resulting velocity distribution should show that the glancing angle of the plasma off the point of the probe is gradually shifted towards a plane structure at the tip of the probe as the applied magnetic field is increased; since the disturbance is being propagated sideways faster as the Alfvén speed increases. No definite effect is observable from the photographs of the plasma flow. Another probe technique used a conical brass plug on the end of the lucite rod (Fig. 9). No effect was again distinguished.

The photographs were obtained by holding open the shutter of a polaroid camera (film speed 3000 ASA) in a darkened room for the duration of plasma flow. The camera was mounted about two and one half feet above the shock tube.

Yet another probe arrangement used a pointed brass piece placed
in the lucite rod and photos were again taken of the flow angle past the probe for situations of no field and varied applied fields. Because the magnetic field cannot penetrate the brass (the brass is essentially diamagnetic at the frequency of the field), any disturbance in the field would be diverted radially outwards. This last method gives a result which might be interpreted as supporting the change of flow angle due to Alfvén wave propagation, but it seems improbable since the magnetic Reynolds number is low.

(ii) Search Coil Measurements.

To achieve more accurate results, a search coil technique was next attempted with a brass plug effectively sealing off the tube to axial magnetic field lines. A doubly wound search coil with opposing turns was placed a few centimeters upstream of the brass plug with the view in mind of measuring any change in radial magnetic field due to Alfvén propagation upstream from the plug. Conditions in the shock tube are such that the Alfvén speed $a > \omega$, the shock speed. The plasma, synchronized to the field discharge, would strike the plug which would initiate a large radial magnetic disturbance and hopefully be detectable upstream as the plasma is reflected from the face of the brass plug. With this arrangement, a succession of measurements (with a camera mounted on the C.R.O. to photograph the traces) to determine the effect of field, search coil-reflector distance, pressure, and material used for reflector, on the field pattern was completed. The effect of the brass reflector on the applied field was also investigated. To obtain sufficient time resolution of the search coil pick-up, a photomultiplier placed just upstream from the search coil was used to trigger the C.R.O. The luminous plasma streaming past the photomultiplier
lucite probe in plasma

brass cone in plasma

photo-multiplier and shock tube

Fig. 9
gave enough light to initiate the triggering.

(iii) Shock Speed Measurements.

This photomultiplier (Fig. 9) also gave a means of measuring shock velocities. An external loop was used to trigger the C.R.O. on pickup of an electromagnetic signal from the beginning of the plasma discharge. The negative output signal from the photomultiplier was fed into the cathode beam plates of the C.R.O. and used as a cutoff voltage. Hence as the luminous plasma passed the photomultiplier the trace on the C.R.O. was cut off and gave a time measurement. The distance of the photo tube from the driver diaphragm was measured, and so the shock speed was determined.

(iv) Smear Camera.

An experimental investigation to determine the dependence of incident and reflected shock speeds on the local applied field involved the use of a smear camera mounted perpendicular to the shock tube. The luminosity of the moving plasma is reflected from a mirror mounted above the shock tube and is smeared out in time by a revolving mirror system and focused on an exposed film. Thus the slope of the luminous front of the film gives a measurement of shock speed. The linear speed is determined by the geometry of the camera arrangement and the angular speed of the rotating mirror system. This camera was used to photograph the incident and reflected shock speeds for varying applied fields to see if any correlation between reflected shock speed and applied field existed. Both lucite and brass reflectors were used. To obtain synchronization, an electrical signal from the camera when the rotating mirror is aligned in the correct direction is provided, and this is used to trigger the experiment via an
electronic delay (reference 7).

The resulting photos (Fig. 20) were inconclusive but indicate < 5% variation of shock speed with different applied axial fields.

5. Momentum Transfer.

From the current interaction, the copper coil with current \( i \) (Fig. 1) experiences a repulsive force equal and opposite to that acting on the body of plasma. Hence we can determine the drag force on the plasma by measuring the effects on the copper coil. The repulsive force \( F \), acts over a time \( \Delta t \) and therefore the impulse exerted is \( F \Delta t \).

(i) Mechanical Measurement - The Pendulum.

Consider the coil suspended as a pendulum, with an effective radius of swing \( r \) determined by the centre of gravity of the pendulum, and a mass \( m \).

![Diagram of pendulum](attachment:pendulum.png)

The copper ring was attached by aluminum supports (3/16" dia.) to form a rigid pendulum swinging freely on ball bearings mounted in lucite, which in turn are attached to a dexion support structure (Fig. 11).

The advantages of the above pendulum are: the inherent strength of the coil, the highly divergent field produced over the region of the shock tube adjacent to the coil and the uniformity of field generated since
there are no current leads.

Let the momentum transfer from plasma to copper coil as a result of impulse $F\Delta t$ be $p$. Then from energy conservation (Fig. 10)

$$\frac{p^2}{2m} = mgh \quad (44)$$

For our arrangement $r = 20.3 \text{ cm}$, $l = 74 \text{ cm}$, $m = 110 \text{ gm}$. A typical swing with field applied gave a measurement $d = 0.1 \text{ cm}$. Thus since $\frac{2\epsilon}{l} = \frac{0.05 \text{ (cm)}}{2(74) \text{ (cm)}} < 1^\circ$ we can make a simplifying approximation that $\sin \theta \approx \theta$, $\cos \theta = 1$.

Now

$$h = r(1 - \cos \theta)$$

$$h(1 + \cos \theta) = r(1 - \cos^2 \theta) = rsin^2 \theta$$

$$\therefore \quad h = \frac{r \theta^2}{2} \quad (45)$$

and so from (44)

$$p^2 = m^2 qr \theta^2$$

or

$$p = m\theta \sqrt{qr} \quad (46)$$

Hence, measuring the angular swing of the pendulum gives a measure of the momentum transfer $= F\Delta t$ and by obtaining a measurement of $\Delta t$ in a separate experiment we can obtain an average value of the force $F$ exerted.

To check for the effect of momentum transfer the coil was positioned so that pulsing the field alone gave no effect. Then simultaneous application of plasma and field resulted in a definite measurable effect. The angle of swing was measured for various applied fields at a gas pressure of $p_0 = 0.5 \text{ mm Hg}$.

(ii) Electrical Measurement.

The second method of measurement was electrical. This measurement is dependent on accurately measuring the radial field $Br/\mu_0$ at the coil, for since the drag force on the plasma is equal to the impulsive
force on the coil, we can calculate the force which is

\[ F = N l_i B_r = 2\pi r_i B_r. \]  \hfill (47)

One method of obtaining the current \( l_i \) is to measure the residual field in the coil and equate flux due to field and self-inductance. The flux \( \phi \) through the copper loop is given by

\[ \phi = L i_1, \]

but

\[ \phi = -A B_i \]  \hfill (48)

where \( B_i/\mu_0 \) is the induced opposing field in the central area \( A \) of the coil due to current \( l_i \). The self-inductance \( L \) was again calculated from Grover and the residual flux measured by taping a single turn search coil to the copper ring and pulsing the field (Fig. 12).

![Copper ring and search coil](image)

Fig. 12

A further calculation of the mutual inductance of the system allows for a correction to the induced field.

![Mutual and self-inductance](image)

Fig. 13

\( B_0/\mu_0 = \) applied field

\( B_i/\mu_0 = \) induced opposing field due to current

\( B_r/\mu_0 = \) residual field measured
Hence $B_i = B_0 - \frac{L}{M} B_r$ (49) since the search coil measures only the field proportional to the mutual inductance of the system.

However, since the coil is to all purposes nearly diamagnetic, a more accurate value of $B_i$ is obtained by putting $B_i \approx -B_0$. The $(L/R)$ time of the coil supports this argument, and since the mutual inductance $M$ is very sensitive to the separation of loops there could be considerable error in $M$. This $L/M$ correction to $B_r$ assumes currents at the centres of the copper coil and search coil, whereas in fact the skin effect at our frequencies would probably prove to show most current on the surface of the coil.

From equating flux

$$i_i = -\frac{A B_i}{L}$$

where $A = \pi r_1^2$ and $B_i = -B_0$, and hence

$$F = i_i B_r = 2\pi r_1 \left( \frac{\pi r_1^2}{L} \right) B_0 B_r$$

$$F = 2\pi^2 r_1^2 \frac{L}{B_0} B_r$$

The method of measuring $B_r$ was to use a doubly wound search coil with the windings opposed and spaced equally on each side of the central copper ring. A lucite frame was constructed to hold the ring and search coils rigidly in place around the shock tube.

The search coil has two leads to a Tektronix 551 dual beam C.R.O. (Fig. 14); one to look at the single turn search coil, i.e., the axial flux, and the other to look at the net pickup of the two search coils, i.e., the radial flux. Integrating the area under the curve of the double coil pickup yields the radial field.
IV. RESULTS AND CALCULATIONS

1. Field Calibration.

The pickup voltage by the search coil is a damped cosine curve as is verified from the following:

\[ i = i_0 e^{-\frac{\xi}{\tau} \sin \omega t} \]  

where \( \omega = \frac{1}{\sqrt{LC}} \) \quad \( \tau = \frac{2\pi}{\omega} \) \quad \( i_0 = \frac{V_0}{\sqrt{L}} \)

since \( V = V_0 \) and \( i = 0 \) at \( t = 0 \)

in the LC discharge of the field coil system. Now the axial field in the solenoid is given by

\[ B = \mu_0 n i \]  

where \( n = \) number of turns/metre.

hence

\[ B = \mu_0 n i_0 e^{-\frac{\xi}{\tau} \sin \omega t} \]

Since the pickup voltage \( V' \) of the search coil is given by

\[ V' = -A \frac{dB}{dt} = \frac{A}{\mu_0} \frac{d}{dt} B \]

where \( A = \) area of search coil

then

\[ V' = -\mu_0 n A \omega i_0 \left( \cos \omega t - \frac{\xi}{\tau} \sin \omega t \right) e^{-\frac{\xi}{\tau} \sin \omega t} \]

or

\[ V' = -\mu_0 n A \frac{V_0}{L} \left( \cos \omega t - \frac{\xi}{\tau} \sin \omega t \right) e^{-\frac{\xi}{\tau} \sin \omega t} \]

(53)

At \( \omega t = \pi/2 \), the induced voltage \( V' \) is given by

\[ V' = b e^{-\frac{\xi}{\tau} \sin \omega t} \left( \cos \omega t - \frac{\xi}{\tau} \sin \omega t \right) \bigg|_{\omega t = \pi/2} \]

From the amplitude damping a calculation gives \( \frac{\xi}{\tau} = 0.06 \), and therefore

\[ V' = b e^{-\frac{\xi}{\tau} \sin \omega t} \approx \frac{1}{2} b \]

To within 5\% we are justified in neglecting the \( \frac{\xi}{\tau} \sin \omega t \) term of \( V' \).

Integrating the expression \( \int_0^{\pi/4} e^{-\xi \omega t} \cos \omega t \, dt \) and expanding the exponential (neglecting terms involving \( \frac{\xi}{\tau} \) and higher order) the
result is

$$B_{\text{max}} \approx \frac{1}{A} \left( \frac{b}{\omega} \right)$$

The initial calibration (June 8, 1962) of axial field versus condenser voltage is given in Fig. 15. These values apply to the Alfvén wave experiment. A re-calibration on August 10, 1962 shows a slight increase in field (most probably because of the change in geometry of the solenoid after many discharges) and these values are applicable to the momentum transfer study.

The effect of many discharges on the coil at high currents can be observed by comparing Fig. 5 and 9. The end turns have been bent inwards considerably because of current forces acting on them. The turns through the middle of the solenoid have no net force, and so it is only the end turns which suffer a deformation.

As a comparison of the amplitude of the induced voltage $V^1$

$$b = \mu_0 n A V_0$$

$$n = \frac{21}{0.762} \text{ turns/m} \quad A = 3.96 \times 10^{-3} \text{ m}^2 \quad L = 8.5 \mu \text{H}$$

For the case of condenser voltage (CV) = 1 KV

$$b = 16.1 \text{ volts from (55)}$$

versus a measured value from the voltage trace (Fig. 16) of $b = 12.0 \text{ volts}$. This is a difference of 25% between values; the error being attributable to the neglected $\frac{c}{b}$ term, and considerable error because of the voltage drop across the neglected resistance.

2. Alfvén Waves.

(i) Probes and Camera.

The first photos with a lucite probe (Fig. 9) do not show any
Magnetic Field Calibration

Calibration
Aug. 10/62

Calibration
June 8/62

Condenser Voltage (CV) of field bank (KV)

Fig. 15
effect, although this is definitely in part due to lack of synchronization between plasma and maximum field at the probe. These preliminary runs were done with the shock tube driver at 12 KV. From the field discharge, the $\tau/4$ time is 0.12 millisec as compared to the gas time (as seen by photomultiplier cut-off when the luminosity reaches the probe) of 0.4 millisec for the next probe type used (Fig. 16). The lack of synchronization is apparent. The cone is of brass. The driver gap was adjusted so that 18 KV could be used to obtain higher shock velocities. With placement of the probe further down the shock tube synchronization of field and plasma is consistently attained with the field applied. However from the resulting photos it would appear that the inference of an m.h.d. propagation cannot be made.


With the driver at 18 KV, measured shock speeds with and without field, and at different positions along the shock tube, give the following results at a gas pressure of 0.5 mm Hg.

With no field applied:

at $s = 67$ cm $t = 0.078$ millisec

hence $u = 8.59 \times 10^5$ cm/sec and $M = 26.8$

at $s = 93$ cm $t = 0.12$ millisec

hence $u = 7.75 \times 10^5$ cm/sec and $M = 24.2$

The sound speed in Argon at room temperature is,

$$Q_0 = \sqrt{\gamma RT} = \sqrt{1.67 \times 8.31 \times 15 \times 298} \text{ cm/sec}$$

$$Q_0 = 3.2 \times 10^4 \text{ cm/sec}.$$  

Thus there is an attenuation of shock speed from $M = 27$ to $M = 24$ over 26 cm of tube length. This is accountable for by a simple momentum
a. $\frac{d\phi}{dt}$ trace 50 $\mu$sec/cm

b. cut-off of C.R.O. trace

c. brass probe in plasma ($B = 0$)

d. brass probe in plasma ($B = 10^4 \text{g}$)
conservation argument.

With field applied (at CV = 3 KV)

at s = 93 cm \[ t = 0.13 \text{ millisec} \]

hence \[ u = 7.15 \times 10^5 \text{ cm/sec and } M = 22.3 \]

and thus an applied field of 6000 gauss slows the shock wave from \( M = 24 \) to \( M = 22 \). This slowing down of the plasma by the applied magnetic field is due to the retarding force/unit volume = \( \sigma u B_r^2 \) (a nozzle effect since the solenoid gives some radial field).

(iii) Tabulated Alfven Speeds.

With the brass tipped probe (Fig. 16) synchronization is good, and comparing Fig. 16c and 16d there might be interpreted an effect of propagation upstream, though it is unlikely since \( R_M \) is \( \approx 3 \) and the magnetic and mechanical coupling is therefore not strong enough. The Alfven speed is given by

\[ a = \frac{B}{(\mu_0 \rho)^{1/2}} \]  

Now for Argon the atomic weight is 39.94. For a gas, one gram mole occupies 22.4 litres and hence at room temperature and gas pressure \( p_o = 0.5 \) mm Hg, the density \( \rho \) of Argon is:

\[ \rho = \frac{39.94}{22450} \times \frac{273}{298} \times \frac{0.5}{760} \text{ g/m}^3 \]

\[ \rho = 1.07 \times 10^{-6} \text{ g/m}^3 \]

Now the density increase across the shock for Argon is \( \kappa = 4 \) (from formula 19). Hence the Alfven speed for an applied field of 5910 gauss (CV = 3 KV) is \( a = 8.06 \times 10^5 \text{ cm/sec} \) which is only slightly greater than the shock speed \( u = 7.2 \times 10^5 \text{ cm/sec} \) at those conditions. Tabulated Alfven speeds for fields corresponding to condenser voltages from 1 KV to 5 KV and gas pressures from 0.1 mm Hg to 1.0 mm Hg are given in Fig. 17. Comparing
CALCULATED ALFVEN SPEEDS FOR VARYING MAGNETIC FIELDS AND GAS PRESSURES

\[ a = \frac{B}{\sqrt{\mu_0 \rho}} \, \text{m/sec} \]

<table>
<thead>
<tr>
<th>Po (mm Hg)</th>
<th>Bo (gauss)</th>
<th>a (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>2000 (1 KV)</td>
<td>6.02 x 10^3</td>
</tr>
<tr>
<td></td>
<td>5910 (3 KV)</td>
<td>1.80 x 10^4</td>
</tr>
<tr>
<td></td>
<td>7875 (4 KV)</td>
<td>2.40 x 10^4</td>
</tr>
<tr>
<td></td>
<td>9840 (5 KV)</td>
<td>3.00 x 10^4</td>
</tr>
<tr>
<td>0.2</td>
<td>2000</td>
<td>4.25 x 10^3</td>
</tr>
<tr>
<td></td>
<td>5910</td>
<td>1.27 x 10^4</td>
</tr>
<tr>
<td></td>
<td>7875</td>
<td>1.70 x 10^4</td>
</tr>
<tr>
<td></td>
<td>9840</td>
<td>2.12 x 10^4</td>
</tr>
<tr>
<td>0.5</td>
<td>2000</td>
<td>2.69 x 10^3</td>
</tr>
<tr>
<td></td>
<td>5910</td>
<td>8.06 x 10^3</td>
</tr>
<tr>
<td></td>
<td>7875</td>
<td>1.07 x 10^4</td>
</tr>
<tr>
<td></td>
<td>9840</td>
<td>1.34 x 10^4</td>
</tr>
<tr>
<td>1.0</td>
<td>2000</td>
<td>1.90 x 10^3</td>
</tr>
<tr>
<td></td>
<td>5910</td>
<td>5.70 x 10^3</td>
</tr>
<tr>
<td></td>
<td>7875</td>
<td>7.60 x 10^3</td>
</tr>
<tr>
<td></td>
<td>9840</td>
<td>9.40 x 10^3</td>
</tr>
</tbody>
</table>

Fig. 17
Fig. 16c and 16d again, the conditions were gas \( p_0 = 1.0 \text{ mm Hg} \) and field 9840 gauss which gives \( a = 9.4 \times 10^3 \text{ m/sec} > u = 6.6 \times 10^3 \text{ m/sec} \) (but not much greater). Nonetheless the luminosity shows up considerably better for this pressure and so is more easily interpreted. With sufficiently high pressures \((\approx 1 \text{ mm Hg})\), magnetic fields (probably \( \approx 15 \text{ Kgauss} \)) and magnetic Reynolds number \((\approx 10)\) it might be possible to detect m.h.d. waves in shock ionized Argon.

(iv) Search Coil Measurements.

In the investigation of Alfvén propagation by measuring the change in the radial magnetic field, the following arrangement was adopted.

\[
\begin{align*}
\text{search coil} & \quad \text{dia.} \\
\text{photomultiplier} & \quad \text{to trigger} \\
& \quad \text{C.R.O.}
\end{align*}
\]

![Diagram of search coil arrangement](image)

An attempt was made to correlate the resulting C.R.O. traces of \( \frac{dB_r}{dt} \) with applied magnetic field, initial gas pressure, search coil to reflector distance, and nature of the reflector in order to observe linear or non-linear effects. With a brass reflector, a comparison of Fig. 19c and 19d shows an observable effect. The figures a, b, c, d respectively, show normal field pattern with no plasma, field pattern with plasma and reflector, field pattern with plasma but no reflector, and the same results using greater time resolution (c and d). The doubly-wound search coil
measures radial field and so the reflected shock is resulting in a change in magnetic field, though it is presumptuous to try and ascribe a velocity to the propagation upstream.

It can be observed that there is a flat in the voltage for a period of $\sim 3\mu\text{sec}$. For this duration the radial field $B_r/\mu_0$ is nearly constant, which would be accountable for by the reflected shock carrying field lines back with it. Synchronization of field and plasma is good to within $5\mu\text{sec}$.

Measurements to obtain the accuracy of the search coil and the effect of the brass reflector on the applied field were undertaken. In the absence of the brass reflector the opposing-turns search coil gave a $B \approx 103$ gauss at a field applied of $B_o = 5910$ gauss. This is a 1.75% residual field and so the coil gives a fairly accurate measurement showing that the turns almost exactly oppose. A measurement with the brass reflector in position gave a value of residual $B = 62$ gauss which shows further attenuation, but also the trace had the appearance of a sine curve rather than cosine, indicating possibly a delay in pick-up. This effect was completely reproducible.

The C.R.O. traces (Fig. 19) are reproducible in general waveform for varying conditions of $p_o, B_o$, and $x$ (the distance from reflector to search coil) though the peak amplitudes vary tremendously; and so no useful quantitative measurements can be made from these results. Varying the search coil to reflector distance, applied field, and gas pressure, again gave inconsistent results until it was discovered a ring of brass was being coated on the wall of the shock tube close to, and upstream of, the brass reflector. This would explain the inconsistent results since as the brass
is deposited in a ring, current rings would be formed modifying the local field; and also the skin depth of the deposit would seriously affect any measurement of field. The skin depth $\delta$, of a conducting medium such as brass is given by

$$\delta = \left( \frac{2}{\mu_0 \sigma \omega} \right)^{1/2}$$  \hspace{1cm} (56)

For $\sigma = 4 \times 10^{-6} \text{ mhos/cm}$, $\mu = 4\pi \times 10^{-7} \text{ H/m}$, $\omega = \frac{2\pi}{\tau} = 1.31 \times 10^4 \text{ cps}$ we obtain a value for $\delta$ of 2.7 mm.

After many discharges enough brass to give a thickness of approximately $\delta/4$ was deposited on the shock tube walls and so the results are invalidated.

On substituting a lucite reflector the traces show a reproducibility of waveform but they are consistent at conditions such that the Alfvén speed, $\alpha < \mathbf{u}$, the shock speed. This would indicate the traces are attributable only to reflected plasma and not Alfvén propagation. The traces obtained for both lucite and brass reflectors were at conditions $\alpha \approx \mathbf{u}$ and so it is hard to interpret any significance from them. At conditions of lower pressures and higher fields (i.e. at sufficiently high magnetic Reynolds numbers), synchronization became a problem and so no useful information could be obtained at Alfvén speeds at least double the shock speed.

(v) Smear Camera Results.

The results from using a smear camera add little to the above information. There is no readily measurable effect of increase in reflected shock velocity as conditions for higher Alfvén speeds are experimented with. The smear camera gives a velocity measurement directly by smearing out the luminosity of the incident and reflected shocks in time (Fig. 20). The photos do point out the advantage of the smear camera for
accurate velocity measurement. The reflected shock speeds do not appear to increase for applied fields (up to 9000 gauss) as Fowler and Turner found.

3. **Momentum Transfer.**

   (i) **Mechanical Measurement.**

   In the momentum transfer study the coil was first suspended as a pendulum by two strings. The net current torque acting on the copper ring (as a result of the non-homogeneity of the solenoidal field) caused it to swing violently about a vertical axis. This torquing was eliminated by using aluminum tubing screwed to both the ring and the upper end of the assembly to form a rigid pendulum, and by the use of trimmers. A lucite join prevented a current path from being formed in the suspension. The bearings used for support gave a freely swinging pendulum that took 27 complete vibrations to damp out an initial amplitude of around 1 cm to half amplitude. Measurements were taken on the third swing.

   With the pendulum, the amplitudes of swing at gas \( p_0 = 0.5 \) mm Hg were, for fields:

<table>
<thead>
<tr>
<th>( B_0 ) (gauss)</th>
<th>( d ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2290</td>
<td>0.5</td>
</tr>
<tr>
<td>4580</td>
<td>2</td>
</tr>
<tr>
<td>6870</td>
<td>5</td>
</tr>
</tbody>
</table>

   which shows a square law increase within 10%. Then from Fig. 11 and formula 46, since \( m = 110 \) gm; \( l = 74 \) cm; \( r = 20.3 \) cm and \( \theta = \frac{d}{2l} \), we obtain values of momentum transfer, \( p \), for applied fields of 1 KV, 2 KV, and 3 KV at a gas \( p_0 = 0.5 \) mm Hg. Values are:
<table>
<thead>
<tr>
<th>C.V. (KV)</th>
<th>( B_0 ) (gauss)</th>
<th>( p ) (gm-cm/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2290</td>
<td>5.25</td>
</tr>
<tr>
<td>2</td>
<td>4580</td>
<td>21.00</td>
</tr>
<tr>
<td>3</td>
<td>6870</td>
<td>52.50</td>
</tr>
</tbody>
</table>

obtained from \( p = m \theta \sqrt{qr} \).

(ii) Electrical Measurement.

With the arrangement for electrical measurement it was first necessary to determine the cancellation of the fluxes from the two oppositely wound turns. The ratio of the voltages \( V' \) for the single turn \( V_s' \) (which measures axial flux) and the double turn \( V_d' \) (which measures the residual axial flux and radial flux) at an applied field of 2000 gauss is

\[
\frac{V_d'}{V_s'} = \frac{0.2V}{9.0V} = 0.0443
\]

and so there is cancellation to within ±5% by the opposed windings. Conditions for the electrical measurements were: driver at 16 KV and a few trials at 12 KV, initial gas pressures of 0.01, 0.1, 0.5, and 1.0 mm Hg, and applied fields of 1, 2, 3, and 4 KV. Typical induced voltages \( V' = -\frac{d\phi}{dt} \) for the radial field pickup can be seen in Fig. 20b and Fig. 20c which gives greater time resolution by using the photomultiplier to trigger the C.R.O. Integrating the curves gives a magnitude of the induced radial field \( B_r/\mu_0 \) and a separate measurement gives the induced current from the flux linkage.

\[
\phi = L i, = -ABi
\]

where \( B_r/\mu_0 \) is the induced opposing axial field and is equal to the applied field \( B_s/\mu_0 \). The self-inductance \( L \) as calculated from Grover is 0.13 \( \mu \)H.

The integrated fields \( B_r/\mu_0 \) for varying gas pressures and applied fields \( B_s/\mu_0 \) are shown in Fig. 21. From the graph of \( B_r \) versus \( B_s \) at various pressures, the slope of the curve for the average \( B_r \) for all
C.R.O. traces of $\frac{dq}{dt}$

Fig. 20
$B_r$ versus $p_0$

Induced Radial Field $B_r/\mu_0$ (gauss)

Initial Pressure (mm Hg)

$B_r$ versus $B_0$

Induced Radial Field $B_r/\mu_0$ (gauss)

Applied Field $B_0/\mu_0$ (KV)

$\circ \ p_0 = 1$ mm Hg
$\triangle \ p_0 = 0.5$ mm Hg
$\times \ p_0 = 0.1$ mm Hg

driver at 16 KV

Fig. 21
pressures is \( \frac{B_r}{B_0} = \frac{14.7 \text{ gauss}}{4580 \text{ gauss}} = 3.21 \times 10^{-3} \). Hence the force, as measured electrically is \( F = 2\pi r_i^3 (3.21 \times 10^{-3}) B_0^2 \), which substituting in values gives for \( B_0 \) in gauss,

\[
F = 23.8 \times 10^{-8} B_0^2 \text{ dynes.} \tag{58}
\]

The force acts over an average time of 10 \( \mu \text{sec} \) as determined from the oscillograms (Fig. 20) and hence the momentum transfer \( p \), is given by

\[
P = 23.8 \times 10^{-8} B_0^2 \text{ gm-cm/sec,}
\]

which for fields corresponding to condenser voltage C.V. yield:

<table>
<thead>
<tr>
<th>C.V. (KV)</th>
<th>( B_0 ) (gauss)</th>
<th>( P ) (gm-cm/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2290</td>
<td>1.25</td>
</tr>
<tr>
<td>2</td>
<td>4580</td>
<td>5.00</td>
</tr>
<tr>
<td>3</td>
<td>6870</td>
<td>11.25</td>
</tr>
</tbody>
</table>

(iii) Theoretical Results.

Theoretically the maximum force attainable is given by

\[
F = 2.06 \times 10^{-15} \sigma u i^2 \text{ newtons} \tag{30}
\]

and the current \( i \), by (50), which for fields \( B_0/\mu_0 \) as given above yields:

at 1 KV \[
\dot{i} = \frac{\pi (3.45) \times 10^{-4} (2290)}{0.13 \times 10^{-6}} \text{ amps}
\]

\[
\dot{i} = 7380 \text{ amps}
\]

and which varies linearly with \( B_0 \). Taking \( \sigma = 6.7 \times 10^3 \text{ mhos/m, } u = 7.2 \times 10^3 \text{ m/sec, } \) a force acts on the coil of 5.42 newtons for an applied field of 1 KV. The momentum transferred is thus 5.42 dyne-sec and increases as the square of the applied field \( B_0/\mu_0 \). The values of \( p \) from the three methods are shown in Fig. 22 as a function of applied field for \( P_o = 0.5 \text{ mm Hg.} \)

4. Discussion of Results.

The mechanical values agree with theory to within probably 10%
$p$ versus $B_0^2$

(Applied Field)$^2$ in ($10^3$ gauss$^2$)

Fig. 22
considering errors in measurements, which is fairly good agreement. The electrical measurements fall far short of the expected result. Electrical values are $\approx \frac{1}{4}$ of those predicted by theory.

(i) Discrepancy in Electrical Measurements.

Some error in the electrical measurement is introduced as a result of lack of synchronization between $B_{\text{max}}$ and plasma; however this is slight. Generally the plasma arrives at the coil at a time $t = 0.12$ millisecond whereas $B_{\text{max}}$ occurs $20 \mu s$ later. Hence the actual field is

$$\frac{\sin\left(\frac{0.12}{0.14} \frac{\pi}{2}\right)}{\sin\left(\frac{\pi}{2}\right)} = 0.97$$

that of the maximum and so the force is $(0.97)^2$ of the maximum attainable. This correction is small. In fact it is believed that the principal errors involved are:

a) the conductivity $\sigma$ attained is not as high as theory predicts, probably because of fall-off behind the shock front where the gas is cooled appreciably.

b) the finite separation of the turns of the search coil gives an averaged flux that is significantly smaller than that which would be measured with turns very close to the copper coil.

Any induced fields in the plasma due to the circulating currents $i_2$ contribute very little flux linkage to the copper coil and so do not modify the current $i_1$ to any extent. A calculation subsequently justified this assumption. In any event the induced currents $i_2$ are in such a direction that they would produce a field $B/\mu_0$ aiding the applied field and so would result in a larger value of radial field than that measured. A graph of conductivity for varying initial pressures and shock Mach numbers is given in Fig. 23 (the values are obtained from reference 2). Add to any variation in $\sigma$, a considerable error in the search coil measurement and the low values from the electrical measurements are understandable. A discussion of the error in search coil measurement will follow.
Conductivity of Argon as a Function of Shock Speed

<table>
<thead>
<tr>
<th>$p_0$ (mmHg)</th>
<th>$U$ ($10^3$ W/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>8.3</td>
</tr>
<tr>
<td>0.5</td>
<td>7.2</td>
</tr>
<tr>
<td>1.0</td>
<td>6.4</td>
</tr>
</tbody>
</table>

Fig. 23
(ii) Invariance of $\sigma U$.

The induced radial field $B_r / \mu_0$ should be linear with applied field and the graph of $B_r$ versus $B_0$ supports this, neglecting experimental scatter of a few points. The $B_r$ versus $\rho_0$ plot shows an almost constant $B_r$ for any pressure but there is indication of a slight peaking around $\rho_0 = 0.5 \text{ mm Hg}$, especially at low applied fields. A slight variation of $B_r$ at differing pressures is understandable but the peaking is not so readily apparent. Since for our system:

$$\rho = \rho RT \quad \text{gas law (59)}$$

$$\rho u = \text{const.} \quad \text{continuity (60)}$$

$$P \left( \frac{1}{\rho} \right)^{\gamma} = \text{const.} \quad \text{adiabatic law (61)}$$

whence

$$\rho^{-\gamma} T = \text{const.}$$

or

$$\rho u^{-\gamma} T^{-1}$$

and since the conductivity,

$$\sigma \propto T^{3/2}$$

$$\sigma u \propto T^{3/2} T^{-\gamma} = T^{\gamma}$$

$$\sigma u \propto T^{3/2 - \gamma/2}$$

or

$$\sigma u \propto u^{3/2 - \gamma/2}$$

(62)

Now for Argon $Y = \frac{5}{3}$ and one might therefore expect $\sigma u$ to be invariant, however for ionized Argon $Y < \frac{5}{3}$ and so $\sigma u \propto u^n$ where $0 < n < 1$.

Over the pressure range 0.1 mm to 1.0 cm Hg the shock speed $u$ varies from $8.3 \times 10^3$ to $6.6 \times 10^3 \text{ m/sec}$ and so there is little change in the product $\sigma U$. Thus discounting any small change in field due to induced currents in the plasma, $B_r$ versus $B_0$ should be linear at any given pressure; and $B_r$ versus $\rho_0$ should experience only a small effect. At pressures 0.1, 0.5, 1.0 mm the change in $B_r$ can be determined; since $B_r \propto \sigma U$, then

$$\left( \frac{\sigma U}{\rho_0} \right)_{p = 1} = \frac{7.4 \times 10^4}{5.8 \times 10^3} = 1.27$$

$$\left( \frac{\sigma U}{\rho_0} \right)_{p = 0.1}$$
and \[
\frac{(\sigma u)_{p_0 = 0.5}}{(\sigma u)_{p_0 = 0.1}} = \frac{4.7 \times 7.2}{5.6 \times 6.3} = 1.00
\]

Now from Fig. 40, the averaged curve at \( p_0 = 0.1, 1.0, \) and 0.5 mm agrees with the above calculation (excepting the two points believed in error at 1.0 mm and at 0.1 mm).

Measurements obtained with the driver at 12 KV gave a considerably lower value of \( B_r \) than that at 16 KV. The measured radial field \( B_r/\mu_0 \) for an initial pressure of 0.1 mm, applied field of 3 KV, and driver at 12 KV was 5.54 gauss. The same conditions with driver at 16 KV (Fig. 23) gave \( B_r = 19.38 \) gauss. Thus \( \frac{(B_r)_{12KV}}{(B_r)_{16KV}} = 0.285 \). Since \( \omega \propto \sqrt{\text{voltage of driver}} \) and \( \sigma \propto \sqrt{V} \) then the ratio of
\[
\frac{(\sigma u)_{12KV}}{(\sigma u)_{16KV}} = \left(\frac{12}{16}\right)^3 = 0.315
\]
which very nearly agrees with the ratio \( \frac{B_r_{12}}{B_r_{16}} \). The difference of course is attributable to the fact that since all electromagnetic energy does not go to the gas, \( \sigma \) varies less than \( \sqrt{V} \).

(iii) Comparison of Theoretical and Observed Voltage Waveforms.

Looking closer at the observed voltage waveforms, we wish to compare amplitudes and durations of the pulses \( \frac{d\phi}{dt} \). The theoretical pulse is given by
\[
\frac{d\phi}{dt} = -\frac{\sigma u^2 i_1 A}{4\pi^2 n} \int \frac{1}{r_n} \left( \frac{dM}{dz} \right)^2 dr_n . \tag{33}
\]
These have been plotted for an applied field of \( CV = 1 \) KV for varying initial pressures, in Fig. 24. Along with these curves are those corresponding to the observed waveforms. The time scale \( t \) of the C.R.O. traces has been converted to an axial distance \( z \) by \( z = ut \). It will be noted that using the corresponding shock speeds for initial pressures, the peak amplitudes of the voltage traces are nearly axially coincident at all pressures. Any discrepancy would be due to a time delay in the C.R.O. pick-up, since
All at \( eV = 1 \text{keV} \)

---

Theory

Exp't.

---

\( p_0 = 0.5 \)

---

\( p_0 = 1 \)

---

\( p_0 = 0.1 \)

---

\( p_0 = 0.01 \)

---

Fig. 24
the search coil separation is finite.

The observed voltages $V'$ are in every case lower than the theoretical values. This is consistent with the electrical values of drag being less than theoretical values. This also supports the argument of the search coil winding separation resulting in a lower measured value of $B_r$. A comparison of the area under the theoretical curve with the area under the experimental curve was made by approximating the curves as triangles. This gave the ratio of areas as $(\frac{\text{observed}}{\text{theory}})^{1/3} = 1/3$ for $r = 0.5$ mm. The ratio is less at higher pressures and is greater at lower pressures. This correction would bring electrical measurements much closer in line with theory.

The difference in peak amplitudes, which is most noticeable at low pressures, can also be noted. The work done on the plasma by the magnetic field is given by $\int F dz$ where $F$ is the breaking force due to a radial field component, i.e. $F = \sigma u B_r^2$ per unit volume. Hence the work done per unit volume is $\sigma u B_r^2 L$ where $L$ is a characteristic length for the interaction. If the ratio of this energy to the flow energy, $\frac{1}{2} \rho u^2$ is significant, then the flow can be modified appreciably due to the slowing down of the plasma. The ratio of these energy densities, $\frac{1}{2} \rho u^2$ is given by

$$\frac{1}{2} \rho u^2 L = 2 \sigma B_r^2 L$$

where for comparison purposes (see reference 6) we are using the value of applied field. At a pressure of 0.1 mm, $B_0 = 2290$ gauss, $\sigma = 5 \times 10^3$ mhos/m, $u = 8.3 \times 10^3$ m/sec, and assuming a characteristic length of 8 cm (comparable to the interaction length); a value for $\frac{1}{2} \rho u^2$ is 2$\mu$ which is in a region for strong interaction. The mutual inductance coupling when the plasma has reached the copper coil is at a maximum and so the greatest induced field coupling exists which would increase the radial field as the
Complete Observed C.R.O. Trace

$$v' = -\frac{d\phi}{dt}$$ (volts)

$CV = 1 \text{ KV}$

$p_0 = 0.5 \text{ mm Hg}$

Fig. 25
plasma passes through the other side of the coil. This is a non-linear field effect and is consequently greater at lower pressures when $\Omega >> 1$. At a pressure of 0.01 mm the effect is even greater (Fig. 24) and is unimportant at pressures of 0.5 mm and higher when $\Omega \leq 5$ (where the energies are comparable). The effect of the profile distortion may also be due to non-uniform $\sigma$ since the conductivity behind the shock falls off.

Fig. 25 shows a complete $V' = -\frac{d\phi}{dt}$ trace which shows the cancellation of areas $\int V'dt$ as the plasma passes through and beyond the interaction region. Any difference between areas enclosed by the curve, above and below the z axis, results from inaccuracy in picking the neutral axis on the voltage traces.

V. CONCLUSIONS

The Alfvén wave study gave inconclusive results; chiefly because of the conditions for generating m.h.d. disturbances, the time scale for measurement, and the sensitivity of the C.R.O. to triggering at conditions for sufficiently high Alfvén speeds. With more elaborate care in generating and detecting these disturbances, it is felt that some quantitative results could be obtained on Alfvén waves in shock ionized Argon. Very much higher fields would be essential and therefore too, care in triggering.

The effect of the applied axial field on the incident and reflected shock speeds was investigated and indicated that up to 10,000 gauss the applied field does not modify the shock speeds appreciably. (The smear camera results show less than 5% variation of shock speed with field applied.) This result is contradictory to what Fowler and Turner (reference 3) found for various applied fields, where a significant increase in shock speeds
with fields $> 5000$ gauss was noted. The shock slowing that was detected is due to the presence of some radial field in a solenoid and so there is a magnetic retarding force on the plasma.

The search coil measurements of change in radial field in the Alfvén study would indicate that the reflected plasma carrying frozen magnetic field lines rather than m.h.d. wave propagation is responsible for any measured values of change in $B_r$.

The magnetic interaction study between a moving magnetized plasma and a localized radial field was less critically dependent upon a high magnetic Reynold's number and yielded some useful qualitative and quantitative results. Mechanical measurement of the drag force on the plasma (or alternatively the momentum transfer between plasma and magnetic coil) gave values which agreed very well with theory (less than 10% difference). The electrical measurement gave results which differed significantly from mechanical measurement and theory (approximately $1/4$ of the other values), however the greater part of the error could be accounted for. The chief error with the electrical measurement was the finite separation of the search coil windings which yielded an averaged flux over the cylindrical area of the search coil rather than a value right at the magnetic coil. Even with this evident error, the relatively small difference between values of momentum transfer from theory, mechanical, and electrical measurement lends credence to the data obtained.

The momentum transfer obeys a square law increase with applied magnetic field. A highly non-linear effect in the observed oscillograms at lower initial gas pressures is noticed and should be investigated further. From theory the waveforms should change only in amplitude for varying gas pressures and the observed non-linearity is thus attributable to a
very much different mechanism in the interaction. One possibility is the effect of non-uniform conductivity in the plasma which could distort the measured fields (since $B_r \propto \sigma$). Another possibility is the effect of induced currents in the plasma giving rise to larger measured values of $B_r$ as the plasma passes through and beyond the magnetic coil region. This would result from the greater mutual inductance coupling as the plasma approaches closer to and passes through the coil region together with the stronger magnetic interaction as the initial gas pressure is lowered ($\mathcal{N} \gg 1$).
# Appendix I: Values of Differential Coefficient of Mutual Inductance

\[
\begin{align*}
\beta_z &= \frac{4\pi R_z}{(R_z + r_0)^2 + z^2} \\
\beta_i &= \frac{14.6 R_z}{(3.65 + r_0)^2 + z^2}
\end{align*}
\]

\[
\delta = \frac{z}{R_i}, \quad \alpha = \frac{r_0}{R_i}
\]

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\[ z = \left(14 \frac{\phi}{\sigma^2} \right)^{1/4} \]
BIBLIOGRAPHY


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