SPECTROSCOPIC DIAGNOSTIC TECHNIQUES
FOR SHOCK HEATED PLASMAS

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ABSTRACT

Photographic and Photoelectric measurements were made on the shock excited spectra of Argon and Helium. The plasma temperature and electron density in the region behind the shock wave were calculated from the spectroscopic measurements. These quantities were compared with the values obtained from the Rankine Hugoniot shock theory including the effect of ionization. Considerable disagreement was found between experimental results and theoretical predictions.
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I INTRODUCTION

In the greatly expanding field of high temperature or plasma physics much effort is going into the development of diagnostic techniques. The development of plasma diagnostics is rendered difficult by the transient nature of the laboratory plasmas produced, lifetimes of the order of microseconds being typical. Broadly speaking, plasma diagnostic techniques can be divided into two categories, the first being those techniques which perturb the plasma and then measure the effect of the perturbation, the second being those which do not perturb the plasma. Perturbing techniques as a rule introduce some uncertainty in that the perturbation may change the effect being measured.

Spectroscopic techniques, which do not involve plasma perturbations, are a useful and powerful means of investigation. Study of the emission spectra from a plasma can lead to identification of the elements present and yield a quantitative estimate of the electron density, electron temperature, and degree of ionization. In addition to the above, the ion kinetic temperature can be obtained for plasmas at very high temperatures (T $\sim 10^6$ °K).

The approach employed in this experiment was to drive a strong shock wave down a quartz tube and observe the emission spectra from the heated region behind the shock. The values of
electron density, electron temperature, and the degree of ionization obtained from spectroscopic observations were compared with those expected from theory for the shock velocities observed.

From the above comparison some inference may be made of conditions in the plasma being studied. The most important question is whether or not the plasma is in thermal equilibrium. Here equilibrium implies the existence of:

1. a Maxwellian velocity distribution for electrons, i.e., a unique electron temperature, and
2. a distribution of atoms and/or ions in various energy states, \( E_n \), the population of each describable by a term containing a Boltzmann factor, \( \exp(-E_n/kT) \), in which the temperature, \( T \), equals the electron temperature (\( k \) is the Boltzmann constant).

Spectroscopy is extremely well-suited to investigations of energy level populations.

The spectroscopic measurements to be made are of line broadening, line shifting, and of the relative intensities of lines of various adjacent atomic and ionic spectra*. It will be seen that in this experiment the temperature and densities are such that measurable line broadening and shifting are due only to the electric microfields within the plasma, so that measurement of these observables permits determination of electron and ion densities. The measurement of relative line intensities

*Adjacent spectra are the spectra of an atom and an ion (or of two ions) each possessing the same nucleus but whose complements of electrons differ by one.
from adjacent spectra, in an equilibrium plasma of known electron density, yields the plasma temperature.

Other workers (McLean et al, 1960) have done the above type of experiment with a helium plasma produced by a magnetically driven shock wave. In the present experiment both helium and argon plasmas have been investigated. The work on helium allows some comparison with that of McLean as the conditions are roughly similar though the analysis differs somewhat. Nothing was found in the literature on studies of argon spectra excited by magnetically driven shock waves.
II THEORY

Spectroscopic Theory

(a) Stark Broadening of Hydrogen Lines

In an assembly of emitting atoms and ions the principal spectral line broadening mechanisms are:

1. perturbation of one or both of the associated energy levels of the atom or ion* by the electric microfields due to surrounding ions and electrons (Stark effect),

2. Doppler effect on emission frequencies due to random thermal motion of the radiating atom or ion, and

3. perturbation of energy levels by van der Waals forces between atoms and/or ions (Pressure effect).

The above mechanisms have been discussed in many articles, the most useful of which was found to be a review article by R.G. Breene Jr. (1957). It can be shown, that at the temperatures and densities attained in this experiment, the Doppler and pressure broadening are less than a few tenths of an angstrom. This leaves only the Stark effect as significant. It should be pointed out here that the Stark effect has two manifestations, line broadening and line shifting. Line broadening without shifting results from the energy level perturbations changing sign with the fluctuating microfield (linear Stark effect).

*The associated energy levels, $E_m^1$ and $E_n^1$, are related to the spectral frequency, $\nu_{mn}$, by the well-known relation: $h\nu = E_m^1 - E_n^1$ where $h$ is Planck's constant ($6.6 x 10^{-27}$ erg-sec.).
The most pronounced linear Stark effect is seen in the spectrum of hydrogen which is present as an impurity in almost all plasmas. In this experiment broadening measurements were made only on hydrogen lines.

A theory for the linear Stark broadening of hydrogen lines was first developed by Holtsmark (1919). He assumed that the perturbations of the energy levels were due entirely to the quasi-stationary fields of the ions, the frequency of perturbation by the electron fields being too high to cause the energy levels to respond. Holtsmark calculated the probability for a given field at an emitting atom due to surrounding, stationary, singly charged ions of number density, $N_I$. If the field was expressed in units of $F_0$, where $F_0$ is given by:

$$F_0 = 2.61eN_I^{2/3}$$

(where $e$ is the electronic charge, $4.8 \times 10^{-10}$ coulombs), the probability distributions coincided for all $N_I$. $F_0$ is usually referred to as the Holtsmark normal field. Similarly, the profiles of a hydrogen line for different $N_I$ can be represented on a single curve by plotting the spectral intensity versus the parameter $\lambda = \frac{\Delta \lambda}{F_0}$ (where $\Delta \lambda$ is the displacement in angstroms from the line centre).

From work by Chandrasekhar (1943) the Holtsmark theory can be extended to multiply charged ions simply by replacing $N_I$ in Equation (1) by an "effective" density given by:

$$N_{eff} = N_1 + 2^{3/2}N_2 + 3^{3/2}N_3 + \ldots = \sum_{i=1}^{\infty} i^{3/2}N_i$$
where \( N_i \) is the number density of \( i \)-th stage ions. For a moderately ionized gas, \( N_e \), the electron density, is given by \( N_{\text{eff}} \) to a first approximation. The accurate expression for the electron density is:

\[
N_e = N_1 + 2N_2 + 3N_3 + \ldots = \sum_{i=1}^\infty iN_i
\]

Recent work was done by Griem, Kolb, and Shen (1959) on hydrogen line broadening due to electronic and ionic fields in a plasma of atoms, electrons, and singly charged ions. An elaborate numerical analysis was made and the theoretical line profiles obtained have shown better agreement with experiment than the previous Holtsmark theory.

The theoretical profiles of Griem, Kolb, and Shen (where \( F_0 = 2.61eN_e^{2/3} \)) are moderate corrections to the Holtsmark theory. Therefore, it will be assumed that for the purposes of this experiment the theory of Griem et al may be extended to include multiply charged ions by substituting \( N_{\text{eff}} \) (given by Equation (2)) for \( N_e \).

In this work, values of \( F_0 \) were determined by fitting the experimental line profiles with the theoretical profiles for temperature and density nearest that estimated to occur. The constant by which \( \mathcal{L} \) is multiplied to get the best fit is \( F_0 \), which yields \( N_{\text{eff}} \).

A simple method of making the above fit is to replot the profiles on log-log scale with \( \mathcal{L} \) for the theoretical profile...
on the same scale as $\Delta \lambda$ for the experimental profile ($\lambda, \Delta \lambda$ abscissae). The horizontal shift required to align the profiles is $F_o$ (vertical shift is unimportant).

(b) **Spectral Line Shifting**

Considering again the Stark effect, line shifting as well as broadening occurs when the energy level perturbation is a function only of the magnitude of the fluctuating microfield (quadratic Stark effect). The functional relationship between field strength and shift of the spectral line wavelength or frequency is known from experiments in which a gaseous discharge is maintained in a steady, uniform electric field.

In the spectra of argon the stark shift is readily measured. The shifts of many argon lines have been investigated by Minnhagen (1948) and Maissel (1958) and been found more or less proportional to the square of the field strength. In helium the Stark effect is more complicated than in argon and as a result it is difficult to obtain useful information from helium line shifting. In this experiment line shift data will be used only in the work on argon.

It remains to relate the line shift observed in the shock spectra to the electron (ion) density in the plasma. The frequency shift, $\Delta \nu$, of a given line will be assumed to be related to the field strength $F$ by:

$$\Delta \nu = CF^2$$

(4)
where \( C \) is a constant known from Stark effect experiments. The line intensity distribution function \( I(\Delta \nu) \) is related to the normalized Holtsmark\(^*\) probability distribution \( W(F/F_0) \) by:

\[
(5) \quad I(\Delta \nu) d(\Delta \nu) = \frac{I_0}{F_0} W(F/F_0) dF
\]

where \( I_0 \) is the total spectral intensity of the line under discussion,

\[
(6) \quad I_0 = \int_{-\infty}^{+\infty} I(\Delta \nu) d(\Delta \nu)
\]

The Holtsmark function \( W(\beta) \) is obtained from Chandrasekhar (1943). \( W(\beta) \) vanishes at \( \beta = 0, \infty \), peaks at \( \beta = 1.6 \) and has a half width of \( \Delta \beta \approx 2.25 \). From Equations (4) and (5):

\[
(6) \quad I(\Delta \nu) = \frac{I_0}{2CF_0^2} \frac{W(F/F_0)}{F/F_0}
\]

Thus the intensity distribution function \( I(\Delta \nu) \) peaks as \( W(\beta) \). When plotted from the curve for \( W(\beta) \) the peak of \( W(\beta) \) is found to be at \( \beta \approx 1.05 \). For a given line, the frequency shift of the intensity distribution maximum is measured spectroscopically. The corresponding field strength, \( F_{\text{max}} \), calculated from Equation (4), and dividing \( F_{\text{max}} \) by 1.05 yields \( F_0 \). \( \text{Neff} \) can then be found from Equation (1) (with \( \text{Neff} \) replacing \( N_1 \)).

(c) **Temperature Determination from Spectral Intensities**

Assuming equilibrium between electron and ionic temp-

\( *W(\beta) \) is normalized by the requirement that the probability for \( 0 < \beta < \infty \) is unity ( \( \int_{0}^{\infty} W(\beta) \, d\beta = 1 \) ).
eratures, the temperature of a plasma can be determined from intensities of lines within a given spectrum or from lines of adjacent spectra. However, because of the large difference between energies of ionization for successive ion species compared with the small differences between energy states within a species, the temperature is most sensitively determined from comparison of measurements from adjacent spectra.

The absolute intensity of a spectral line resulting from a transition between the energy levels $E^i_m$ and $E^i_n$ of an i-th stage ion is given by (see for example Condon and Shortley):

$$I^i_{m,n} = \frac{N^i(m)}{\sigma^i_{m}} \frac{\hbar \pi^{i+} c}{3 \lambda^i_{m}(m,n)} S^i(m,n)$$

where $N^i(m)$ is the density of i-th stage ions of energy $E^i_m$, $\sigma^i_{m}$ is the degeneracy of the energy state $E^i_m$, $\lambda^i_{m}(m,n)$ is the wavelength of the line, $c$ is the speed of light, and $S^i(m,n)$ is the theoretical line strength of the transition $E^i_m - E^i_n$ ($E^i_m$ will always be taken as the upper level). Following Condon and Shortley the term line strength is taken to be the sum of the squares of the electric dipole matrix elements. The line strengths for the helium and argon lines studied in this experiment are tabulated in Appendix I.

Now in thermal equilibrium, at temperature $T$, the $N^i(m)$ can be expressed as the product of the statistical weight, $g^i_{m}$, for the energy level $E^i_m$ and the Boltzmann factor, $\exp(-E^i_m/kT)$ ($k$ is the Boltzmann constant). Therefore
(8) \[ N^i = \sum_m N^i_m = \sum_m g^i_m \exp\left(-\frac{E^i_m}{kT}\right) = \frac{N^i(0)}{g^i_o} \frac{\lambda^i(m, n)}{m^i} \exp\left(-\frac{E^i}{kT}\right) = \frac{N^i(0)}{g^i_o} Z^i \]

where \( Z^i \) is the partition function for \( i \)-th stage ions, and 
\( N^i(0) \) and \( g^i \) are the density and statistical weight respectively, 
for \( i \)-th stage ions in the ground state (\( E^i = 0 \)). From Equation (8)

\[
(9) \quad \frac{N^i(m)}{g^i_m} = \frac{N^i(0)}{g^i_o} \exp\left(-\frac{E^i}{kT}\right) = \frac{N^i}{Z^i} \exp\left(-\frac{E^i}{kT}\right)
\]

and Equation (7) becomes on substituting \( N^i(m)/g^i_m \):

\[
(10) \quad \frac{I^i_{m,n}}{g^i_m} = \frac{6^i \lambda^i(m, n) N^i}{3 \lambda^i(m, n) Z^i} \exp\left(-\frac{E^i}{kT}\right)
\]

A second relation involving the \( N^i \) is obtained by 
introducing Saha's equation which gives the ratio of the numbers 
of ions in the various stages of ionization:

\[
(11) \quad \frac{N^i Z^{i+1}}{N^{i+1} Z^i} = \frac{2(2m^i e^i/kT)^{3/2}}{N_e \hbar^2} \exp\left(-\frac{E^i}{kT}\right)
\]

where \( M_e \) is the electron mass and \( E_1^i \) is the ionization energy of 
the \( i \)-th stage ion (the energy required to remove the \((i+1)\)-th 
electron). Combining Equation (11) with Equation (10) taken 
for \( i \)-th and \((i+1)\)-th stage ions:

\[
(12) \quad \frac{I^i_{m,n}}{I^{i+1}_{m^1,n^1}} = \frac{\lambda^i_{1+1}(m^1, n^1)}{\lambda^i_1(m, n)} \frac{N_e \left(2m^i e^i/kT\right)^{3/2}}{\hbar^2} \frac{S^i_1(m, n)}{S_1^i(m, n)} \frac{\exp\left(-\frac{E^i}{kT}\right)}{\exp\left(-\frac{E^{i+1}}{kT}\right)}
\]
Taking $\log_{10}$ of Equation (12):

$$
(13) \quad kT = \frac{\left( \frac{1}{2} \cdot 3.03 \right) (E^i + E_{m+1} - E_{m}^i)}{3/2 \log_{10} kT + 3/2 \log_{10} \left( \frac{2 \pi m e}{\hbar^2} \right) + \log_{10} \left( \frac{2i^e \lambda_{i+1} S_{i+1}}{N e_i^i + 1 \lambda_{i+1} S_i} \right)}
$$

where $S^i(m,n)$ and $\lambda_i(m,n)$ have been rewritten $S^i$ and $\lambda_i$.

Inserting numerical values in Equation (13) gives:

$$
(14) \quad kT = \frac{\left( \frac{1}{2} \cdot 3.03 \right) (E^i + E_{m+1} - E_{m}^i)}{3/2 \log_{10} kT + 21.8 + \log_{10} \left( \frac{2i^e \lambda_{i+1} S_{i+1}}{N e_i^i + 1 \lambda_{i+1} S_i} \right)}
$$

where $kT$ is in electron volts.

It is seen that Equation (14) has the form:

$$
(15) \quad kT = \frac{A}{3/2 \log_{10} kT + B}
$$

and thus lends itself easily to graphical solution. This is best accomplished by introducing a scaling parameter $p$, such that $pA$ is constant. For convenience $pA$ will be set equal to 10, and Equation (15) then becomes:

$$
(16) \quad \frac{10}{pkT} - 3/2 \log_{10} pkT = (B + 3/2 \log_{10} \frac{A}{10}) = L
$$

Figure 1 is a graph of $L$ versus $pkT$ from which $kT$ can be obtained for given $B$ and $A$. 


It should be added here that an exact value of $N_e$ for insertion in Equation (14) will not always be available from line broadening and shifting measurements. If these measurements give only $N_{\text{eff}}$ as defined by Equation (2), a first approximation to $kT$ can be obtained by inserting $N_{\text{eff}}$ for $N_e$ in Equation (14). $N_1, N_2$ etc. are found from Saha's equation using these approximations to $kT$ and $N_e$. Equation (3) then yields a better value for $N_e$ which in turn will improve the approximation for $kT$.

**Shock Theory**

We will consider here a strong, one dimensional shock
wave propagating with velocity $v_s$ into a gas at rest. Following the notation on Figure 2, the subscript $_0$ will denote quantities before the shock. The symbols $p$, $T$, $U$, $N$, $v$, denote respectively pressure, temperature, internal energy per particle, number density of ions and/or atoms combined, and flow velocity.

Figure 2 - One Dimensional Shock Wave

As it is assumed that the shock is strong: $p < p_0$ and $U_o < U$ and the equations for conservation of mass, linear momentum, and energy are:

(a) $N_0 v_s = N(v_s - v)$

(b) $p = mN_0 v_s v$

(c) $p v = N_0 v_s (U + \frac{1}{2}mv^2)$

where $m$ is the mass of an atom or ion (neglecting electron mass).

From Equation (17) (b) and (c):

(18) $U = \frac{1}{2}mv^2$

To proceed further with this development the processes occurring behind the shock must be considered. The principal
processes in this region are collisional ionization and excitation. From the overall charge neutrality of the plasma we may write:

\[ \sum \alpha_i N_i = N \]

where the symbol \( \alpha_i \) denotes the fraction of \( i \) times ionized atoms \( \alpha_i = N_i/N \).

Assuming thermal equilibrium between ions and electrons, (i.e. \( T_{ion} = T_e \)) Equations (17) (a), (b) and (18) can be supplemented with the equation of state and the equation for the internal energy of an ideal gas:

(a) \[ p = (N+N_e)kT = (1+\sum \alpha_i)NkT \]

(b) \[ U = \frac{3}{2} (1+\sum \alpha_i)kT + U_{ie} \]

where \( U_{ie} \) is the ionization and the excitation energy per ion.

Solving Equations (17) (a), (b), (18) and (20) for \( N_o \) and \( v_s^2 \), and introducing numerical values:

(a) \[ N_o = \frac{N_e(1+\sum \alpha_i)/\sum \alpha_i}{(4(1+\sum \alpha_i) + 2U_{ie}/kT} \]

(b) \[ v_s^2 = 1.92/M \frac{(2kT(1+\sum \alpha_i) + U_{ie})^2}{3/2 kT(1+\sum \alpha_i) + U_{ie}} \text{ (cm./microsec.)}^2 \]

where \( kT \) and \( U_{ie} \) are in electron volts and \( M \) is the atomic weight of the rest gas.

For comparison with values of \( kT \) and \( N_e \) obtained from spectroscopic measurements it is necessary to express \( kT \) and \( N_e \)
in terms of the observables, \( v_s \) and \( N_0 \). It should be noted that \( \mathcal{L}_i \) and \( U_{ie} \) are themselves functions of \( kT \) and \( N_e \). In this experiment the temperature is of the order of a few electron volts and so the excitation energy of an ion, given by:

\[
E = \sum_{n} E_n g_n \exp\left(-\frac{E_n}{kT}\right)
- \sum_{n} g_n E_n \exp\left(-\frac{E_n}{kT}\right)
\]

is small compared with the ionization energy. Thus, neglecting the excitation energies, \( U_{ie} \) can be expressed in terms of the \( \mathcal{L}_i \):

\[
(22) \quad U_{ie} = \mathcal{L}_1 E^0 + \mathcal{L}_2 (E^0 + E^1) + \ldots + \mathcal{L}_r (E^0 + \ldots + E^{r-1}) + \ldots
\]

Assuming thermal equilibrium, the \( \mathcal{L}_i \) are given by the Saha equations:

\[
(23) \quad \frac{\mathcal{L}_{r+1}}{\mathcal{L}_r} = \frac{N_{r+1}}{N_r} = \frac{2^r N_e}{Z^r} \left(\frac{2m_e kT}{\hbar^2}\right)^{3/2} \exp\left(-\frac{E_r}{kT}\right)
\]

and by:

\[
(24') \quad \sum \mathcal{L}_i = 1
\]

To solve Equations (21) (a), (b), (22), (23) and (24) for \( kT \), \( N_e \) and the \( \mathcal{L}_i \) a method of successive approximations was adopted. We will find it convenient to rewrite Equation (21) in form:

\[
(25a) \quad kT = \frac{b^+ \sqrt{\frac{Mv_s^2}{I} + 2c^+}}{8(1+\mathcal{L})}
\]

\[
(25b) \quad N_e = \frac{N_0 \mathcal{L}}{(1+\mathcal{L})} \left(1 + \mathcal{L} + \frac{2U_{ie}}{kT}\right)
\]
where \( b \) and \( c \) are given by:

\[
b = (4U_{ie} - \frac{3}{2}Mv^2_s/1.92)
\]

\[
c = (4U_{ie} + 9/4Mv^2_s/1.92)
\]

Inserting estimated values for the \( \mathcal{L}_1 \) in Equations (22) and (25) gives a first approximation to \( kT \) and \( N_e \). Substitution of these first approximations in Saha's equation yields better values for the \( \mathcal{L}_1 \). This process is continued until self consistent values of \( \mathcal{L}_1 \) are obtained.

The above method of solution simplifies in practice for values of \( kT \) such that:

\[
kT \leq \text{minimum} \left( \frac{E_{r+1} - E_{r}}{3} \right)
\]

At such temperatures only the \( \mathcal{L}_1 \) for which

\[
\frac{\mathcal{L}_{i+1}}{\mathcal{L}_1} \sim 1
\]

need be considered. This is at once apparent from Saha's equation. Thus, in most cases only two of the \( \mathcal{L}_1 \) need be used in the approximating procedure. However, if \( \mathcal{L}_{i-1} \) or \( \mathcal{L}_{i+2} \) are significant as calculated from Saha's equation using the approximate values of \( kT \) and \( N_e \), then the approximating procedure must be carried one step further. Inserting in Equation (24) whichever of \( \mathcal{L}_{i-1} \) or \( \mathcal{L}_{i+2} \) is significant, the above procedure is repeated giving better values for \( \mathcal{L}_1 \) and \( \mathcal{L}_{i+1} \). This whole process is continued until a consistent value of \( \mathcal{L}_{i-1} \) or \( \mathcal{L}_{i+2} \) is found.
III APPARATUS

Shock Tube

The shock tube used for this work consisted of a quartz tube of 2.5 cm inside diameter, approximately 100 cm long, and fitted with an electromagnetic driver. The general details of the driver, energy storage circuit and velocity measuring apparatus are shown in figure 3 (not to scale).

Figure 3 - Shock Tube

The spark switch, when triggered, feeds current to the driver from the capacitor bank which is rated at four mfd. at 15 KV. It can be seen from the driver geometry that the arc current is tightly coupled to the current in the backstrap and is thus given a strong magnetic repulsion which in addition
to sudden expansion by heating causes a shock wave to be propagated down the tube. The shock velocity is calculated from the time interval between the photomultiplier responses to the light from the luminous front following the shock.

The design, construction, and operation of this shock tube are fully described by Cormack (1960).

Spectroscopic Equipment

Time integrated spectra were obtained using a Hilger El spectrograph. This spectrograph could be fitted with a seven step neutral density filter for determination of the emulsion density versus light intensity relation for any plate required.

To study time variation of spectral intensities a Hilger constant deviation spectrograph was modified by the addition of an adjustable slit in the focus plane, followed by a photomultiplier. The resulting spectrophotometer could only be used in the visible spectral region. This spectrograph was later replaced by a Bausch and Lomb grating monochromator when one became available.

For time integrated studies, the wavelength range 2000-7200 Å could be covered by the Hilger El spectrograph. By using an RCA IP 28 photomultiplier in the spectrophotometer, time resolved measurements could be made in the range 2500-6000 Å.

The dispersion of the El spectrograph ranges from approximately 1.2Å/mm at 2000Å to 4.8Å/mm at 6600Å while
that of the grating spectrophotometer is $16\text{A}^0/\text{mm}$ throughout the spectrum. The slit width used on the El instrument and the average grain size of plate emulsion allowed resolution of lines on the plate separated by approximately .1 mm. The grating spectrophotometer with the slit widths used (entrance slit $\sim .04 \text{ mm}$, exit slit $\sim .09 \text{ mm}$) was capable of resolving two lines of half width $1.5\text{A}^0$ separated by 4 to $5\text{A}^0$.

Electronic Ancilliary Equipment

The standard electronics used consisted of a plate voltage supply, a variable, calibrated 0 - 1.5 KV. supply for the photomultipliers and a Tektronix type 551 dual beam oscilloscope fitted with a single input preamplifier, a difference preamplifier, and a Dumont trace recording camera. The photomultiplier circuits were made up in the laboratory.

The output of the velocity measuring photomultipliers was fed through shielded cable to the difference amplifier on the oscilloscope. The output of the photomultiplier on the monochromator was fed into a cathode follower of standard design. The output of the cathode follower was fed through shielded cable to the single input preamplifier. The rise time of the photomultiplier, cathode follower and preamplifier circuit was of the order of .1 microsecond. The oscilloscope was triggered by a pick-up coil coupled to the current in the shock tube discharge.

The variable high voltage power supply to the photomultipliers allowed adjustment of spectrophotometer sensitivity
to accommodate a great range of spectral intensities without changing the spectrophotometer entrance slit.

Arrangement of the Equipment and Miscellaneous Details

The apparatus was arranged as in figure 4 (not to scale).

Figure 4 - Arrangement of Apparatus

The iron arc and electrodeless discharge tube yield comparison spectra which can be superimposed by use of a Hartmann
diaphragm on the time integrated shock spectra obtained by the Hilger El instrument. The electrodeless discharge tube was connected to the shock tube vacuum system so that the pressure of the gas being excited could be regulated by the same valves as controlled the pressure in the shock tube. The circuit diagram for the electrodeless discharge is given in figure 5.

**Figure 5 - Electrodeless Discharge Circuit**

![Circuit Diagram](image)
Preliminary Investigation

Several time integrated shock spectra were taken on Ilford HP3 plates of the illumination of a station 10 cm from the driving electrodes. Each spectra was taken at a different shock velocity. The shock velocity could be changed readily by adjusting the initial voltage on the capacitor bank. After fifteen to twenty firings the inside of the shock tube became blackened and it was necessary to remove the driver and clean the tube (see figure 3). Fifteen firings of the tube were found ample for a good exposure. The photographic plate so obtained was analyzed using the iron arc spectrum as a reference. Relative intensities of the lines were estimated from the emulsion densities. From the spectral lines identified on the plate some were chosen for further study. The criteria for the selection were freedom from impurity interference, proximity to other lines of the same and adjacent spectra and availability in the literature of Stark shift coefficients. The second criterion was desirable as no ready means was available for checking the manufacturers' spectral sensitivity versus wavelength relation for the spectrophotometer. It may be added that spectral lines which were expected and not found on the plates were looked for with the spectrophotometer. This was tried because in the region 2500 - 5000Å the spectrophotometer gave a strong response to spectral lines which were very weak on the plate.
As the Stark broadening of the $H_{\alpha}$ and $H_{\beta}$ lines were to be used for the determination of electron density it was desirable to know their time histories. Their arrival times at the 10 cm station and pulse shape were observed using the spectrophotometer and compared with histories of the line intensities of the rest gas being used.

After the exploratory work above had been completed the spectrophotometer was adjusted and calibrated for the lines to be studied. The photomultiplier supply voltage was set to 1200 volts and the entrance slit width was set for "on scale" responses from the weakest line to be investigated. Then the exit slit was adjusted by setting the spectrophotometer on the widest line and narrowing the exit slit by small increments until the response showed a sudden decrease. Next, the relation between spectrophotometer sensitivity and photomultiplier supply voltage was determined by setting the instrument on a spectral line whose response was eleven to thirteen volts and then decreasing the supply voltage by one hundred volts and taking the response again. In this manner intensities taken at 1200, 1100, 1000 volts etcetera could be related. It was assumed that the photomultiplier-cathode follower circuit gave linear response with spectral intensity below the saturation point (output $\sim$16 volts).

Finally, the electrodeless discharge was adjusted to give the best possible reference spectrum for the gas being studied, i.e. narrow unshifted spectral lines of the same spectra.
as were observed in the shock tube. The best conditions were chosen by comparing several exposures taken with various capacitances and spark gaps in the discharge circuit (figure 5) and with the gas pressure at a low value \((p \sim 20 \text{ microns})\).

**Measurements**

The observations were all made at the 10 cm station and at three values of shock velocity corresponding to driving voltages of 10 KV, 11.25 KV and 12.5 KV. As the seven step neutral density filter was not available until the work was almost completed, only one exposure was made through the filter, at a driving voltage of 12.5 KV with argon. Some of the time integrated spectra were exposed in juxtaposition with electrode-less discharge spectra in order to measure Stark shifts.

Whenever possible, time integrated and time resolved data were obtained from the same firings. Quadruple Polaroid exposures were taken of the oscilloscope tracings to average out random fluctuations in intensity and velocity. In this manner the average shock velocity was recorded for each time integrated spectrum and any abnormal deviation for a single shot could easily be seen. A typical oscilloscope trace photo is pictured in Figure 6. Referring to Figure 6, the upper traces are the responses of the spectrophotometer to radiation in the vicinity of the line. After four exposures had been made of the spectrophotometer response to a given spectral line the background continua was recorded. The spectro-
photometer was turned to a nearby wavelength region free from spectral lines and four more exposures were taken (with the lower trace removed). The lower trace is the response of the two velocity measuring photomultipliers taken through the difference preamplifier. The shock velocity is obtained from the lower trace, the separation of the discontinuities in the trace marking the time for shock passage through 5 cm.

Figure 6 - Trace Photograph

Where possible time resolved profiles were obtained of the $H_\alpha$ and $H_\beta$ lines. This was done by recording average time histories at 5° intervals ranging from the line centre to a point where only the background signal was observed.

Data Reduction of Time Integrated Spectra

A Jarrel-Ash microphotometer coupled with a Bristol pen recorder was used to scan the lines of interest on the time integrated spectra for spectral intensity and the broadening of the $H_\alpha$ and $H_\beta$ lines. These pen recorder traces were made for the exposure transmitted through each segment of the seven step filter.
The maximum emulsion densities read from the above traces (excepting those for hydrogen lines) were plotted for each spectral line against the logarithm of the intensity (taking the unfiltered intensity as unity and the logarithms of filtered intensities as the negative of the filter densities). All the curves so obtained were fitted to a single curve by shifting parallel to the log intensity axis. From this curve the relative intensity versus emulsion density was read.

The procedure for determination of hydrogen line broadening differed from the above. In this case the emulsion density, $\rho$, for each filter step was plotted versus wavelength (on the same scale) as sketched in Figure 7. Next, plots of wavelength versus log intensity were made for several values of constant $\rho$; the filter density difference and hence $\Delta \log \text{intensity}$ being known between each two steps. As the plots of $I$ versus $\lambda$ were symmetrical, only a half profile was used as is seen in Figure 8. The profile of log intensity versus $\lambda$ is obtained by vertically shifting the curves of Figure 8 to a best fit single curve.
**Data Reduction of Time Resolved Spectral Intensities**

The spectrophotometer traces were plotted on a larger scale with their respective backgrounds. The backgrounds were subtracted leaving the net intensity of each spectral line. The intensity scale was arbitrarily chosen so that the responses in volts for the highest photomultiplier supply voltage could be plotted without change. Traces for which the supply voltage was reduced to avoid saturation of the electronics were corrected using the sensitivity versus supply voltage relation previously determined.

Corrections for variation in spectrophotometer sensitivity with wavelength were required for comparison of two lines separated by more than 100-200°A. A spectrophotometer sensitivity curve (Figure 9) was drawn from the grating efficiency curve for the Bausch and Lomb monochromator combined with the
spectral sensitivity curve for the IP 28 photomultiplier taken from RCA tube data.

**Figure 9 - Spectrophotometer Sensitivity**

Time resolved hydrogen line profiles were obtained directly from the Polaroid trace photographs by plotting the response at a given time from each averaged trace against the wavelength at which the trace was taken (after subtracting the background signal for the given time from each response).
V RESULTS

This chapter will be divided into two parts (A) Argon and (B) Helium.

(A) Argon

Preliminary

The preliminary plates of the argon shock spectra contained a generous number of strong AII and AIII lines. Impurity lines present were H\textsc{\large L}, H\textsc{\large F}, many CII, SiIII, CuI lines, the stronger CaII lines and only the strongest CIII lines, 4650.16 and 4651.35\AA (unresolvable). The lines chosen for intensity measurement were:

<table>
<thead>
<tr>
<th>AII</th>
<th></th>
<th>AIII</th>
</tr>
</thead>
<tbody>
<tr>
<td>3293.95\AA</td>
<td>3285.85\AA</td>
<td></td>
</tr>
<tr>
<td>3307.24\AA</td>
<td>3301.88\AA</td>
<td></td>
</tr>
<tr>
<td>3350.94\AA</td>
<td>3311.25\AA</td>
<td></td>
</tr>
<tr>
<td>3376.46\AA</td>
<td>3336.13\AA</td>
<td></td>
</tr>
<tr>
<td>3388.17\AA</td>
<td>3344.72\AA</td>
<td></td>
</tr>
</tbody>
</table>

while those for measurement of Stark shift were:

<table>
<thead>
<tr>
<th>AII</th>
</tr>
</thead>
<tbody>
<tr>
<td>3559.53\AA</td>
</tr>
<tr>
<td>3561.04\AA</td>
</tr>
<tr>
<td>3576.62\AA</td>
</tr>
<tr>
<td>3588.44\AA</td>
</tr>
<tr>
<td>4474.76\AA</td>
</tr>
</tbody>
</table>

The HP3 plate cut off in sensitivity at about 6570\AA so that measurement of the H\textsc{\large L} profile necessitated an exposure with a Kodak type F plate which has relatively constant sensitivity in this region.
Time resolved analysis could not be made of the Hβ line because many AII lines overlap its profile and the Hδ line lay outside the sensitive region of the spectrophotometer.

The electrodeless discharge tube was found to yield strong AII and AIII lines with $C = 0.0075$ mfd. and a spark gap of one inch.

**Time Integrated Measurements**

(a) Ne Determination from Hα Broadening and Line Shifting

The Hα profile as obtained by the procedure outlined in Chapter IV (Figures 7 and 8) was fitted best in the wings by the theoretical profile for $T=20,000^\circ K$ and $Ne=10^{17}$ cm$^{-3}$. The ratio $\frac{\Delta\lambda}{\lambda} = F_o$ for best fit was $(212 \pm 24)$ stat volts/cm which yielded, from $F_o=2.61eN_{eff}^{2/3}$, $Ne \approx N_{eff} = (0.7 \pm 1)10^{17}$ cm$^{-3}$.

The AII line shifting gave $F$ values tabulated below:

**Table I**

<table>
<thead>
<tr>
<th>Line (Å)</th>
<th>$F$ (stat volts/cm)</th>
<th>Weighted* Average $F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4474.8</td>
<td>747</td>
<td>833stv/cm</td>
</tr>
<tr>
<td>3588.4</td>
<td>1001</td>
<td></td>
</tr>
<tr>
<td>3576.6</td>
<td>1147</td>
<td></td>
</tr>
<tr>
<td>3561.0</td>
<td>1900</td>
<td></td>
</tr>
<tr>
<td>3559.5</td>
<td>977</td>
<td></td>
</tr>
</tbody>
</table>

Taking average $F=833$ stat v/cm, $F_o = \frac{833}{1.05} = 793$ stat v/cm and $Ne \approx N_{eff}=(5\pm1)10^{17}$ cm$^{-3}$.

*Average is weighted in favour of AII 4474.8Å which showed best agreement with $\Delta\nu = CF^2$ and against AII 3561Å which showed worst agreement.
It is seen that the two estimates of $N_e$ differ by a factor of seven, though both will be lowered after obtaining an approximation for $kT$ and the $L_i$ and then solving equations (2) and (3) for a second approximation to $N_e$.

(a) **Observed Line Intensities**

The AII and AIII time integrated total line intensities were taken as the product of the intensity corresponding to the peak of the line emulsion density profile as taken from the pen recorder tracing and the width of the profile taken at a density reading corresponding to one-half peak intensity. These intensities observed at $v_s=1.92$ cm/microsecond, are tabulated below with upper energy levels, $E^{\text{m}}_i$, and ionization energys, $E^i$, as taken from Moore (1949).

<table>
<thead>
<tr>
<th>Multiplet Line ($A^0$)</th>
<th>AII</th>
<th>$E^2=40.7\text{ev}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$83$ 3293.9</td>
<td>44</td>
<td>23.53</td>
</tr>
<tr>
<td>$83$ 3307.2</td>
<td>52.5</td>
<td>23.45</td>
</tr>
<tr>
<td>$109$ 3350.9</td>
<td>40</td>
<td>24.72</td>
</tr>
<tr>
<td>$109$ 3376.5</td>
<td>45</td>
<td>24.71</td>
</tr>
<tr>
<td>$96$ 3388.1</td>
<td>50</td>
<td>23.53</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiplet Line ($A^0$)</td>
<td>AIII</td>
<td>$E^2=40.7\text{ev}$</td>
</tr>
<tr>
<td>$1$ 3285.8</td>
<td>40</td>
<td>25.28</td>
</tr>
<tr>
<td>$1$ 3301.9</td>
<td>40</td>
<td>25.26</td>
</tr>
<tr>
<td>$1$ 3311.2</td>
<td>23</td>
<td>25.25</td>
</tr>
<tr>
<td>$3$ 3336.1</td>
<td>25</td>
<td>27.98</td>
</tr>
<tr>
<td>$3$ 3344.7</td>
<td>19</td>
<td>27.96</td>
</tr>
<tr>
<td>$3$ 3358.4</td>
<td>12</td>
<td>27.94</td>
</tr>
</tbody>
</table>
(c) Temperatures from Observed Line Intensities

Solutions to equation (14) using line strengths, $S_i$, from Appendix I, energy levels, ionization energies, and pairs of line intensities from (b) above and for each value of $N_e$ in (a) above are tabulated below. The lower numbers are values of $kT$ for $N_e = 5 \times 10^{17} \text{cm}^{-3}$, the upper numbers values for $N_e = 0.7 \times 10^{17} \text{cm}^{-3}$.

**Table III - Plasma Temperature (ev)**

<table>
<thead>
<tr>
<th>AIII LINES</th>
<th>3285.8</th>
<th>3301.9</th>
<th>3311.2</th>
<th>3336.1</th>
<th>3344.7</th>
<th>3358.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>AII (Å)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3293.9</td>
<td>2.06</td>
<td>2.10</td>
<td>2.10</td>
<td>2.11</td>
<td>2.13</td>
<td>2.13</td>
</tr>
<tr>
<td></td>
<td>2.36</td>
<td>2.42</td>
<td>2.41</td>
<td>2.39</td>
<td>2.42</td>
<td>2.41</td>
</tr>
<tr>
<td>3307.2</td>
<td>1.93</td>
<td>1.97</td>
<td>1.97</td>
<td>1.97</td>
<td>1.99</td>
<td>1.97</td>
</tr>
<tr>
<td></td>
<td>2.19</td>
<td>2.24</td>
<td>2.24</td>
<td>2.22</td>
<td>2.25</td>
<td>2.24</td>
</tr>
<tr>
<td>3350.9</td>
<td>1.39</td>
<td></td>
<td></td>
<td>1.34</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.61</td>
<td></td>
<td></td>
<td>1.58</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3356.5</td>
<td>1.4</td>
<td></td>
<td></td>
<td>1.34</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.62</td>
<td></td>
<td></td>
<td>1.61</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3388.1</td>
<td>2.11</td>
<td>2.16</td>
<td>2.15</td>
<td>2.15</td>
<td>2.18</td>
<td>2.17</td>
</tr>
<tr>
<td></td>
<td>2.41</td>
<td>2.49</td>
<td>2.48</td>
<td>2.45</td>
<td>2.46</td>
<td>2.46</td>
</tr>
</tbody>
</table>

Only a few temperature calculations were made using AII 3350.9Å and 3376.5Å as the line strength theory seemed to break down for these lines and give inconsistent results (see Appendix I). A better approximation to the higher estimate of $N_e$ was made using the average value of $kT$ from Table III (discounting calculations involving AII 3350.9 and 3376.5Å).
solving for $L_1, L_2$, (assuming $L_1 + L_2 = 1$) and then substituting $L_1, L_2$ in equations (2) and (3). This calculation yielded $N_e = 3.9 \times 10^{17} \text{cm}^{-3}$ which in turn caused the second approximation to $kT$ to be reduced...04 ev (a 2% reduction).

**Time Resolved Measurements**

In the work with argon time resolved histories could not be obtained from the $\text{H}_\alpha$ and $\text{H}_\beta$ lines. The AII and AIII intensity histories could, however, be used to check the total intensity values obtained from the time integrated spectra.

Typical averages of AII and AIII intensity histories are replotted in Figures 10 and 11.

*Figure 10* - $I(t)$ for AII 3294\,Å

*Figure 11* - $I(t)$ for AIII 3285\,Å

The second peak, which is quite apparent in AII 3294, is caused by a second shock which occurs due to ringing in the discharge circuit (see Cormack, 1960). For comparison with time integrated
intensities the integral

\[ \int I(t) dt \]

was calculated for each intensity versus time curve. The integrated intensities are tabulated below for the three driving voltages (i.e. for shock velocities \( v_s \)).

**Table IV - Line intensities for various shock velocities \( v_s \) (in cm/microsecond)**

<table>
<thead>
<tr>
<th>Line (( A^0 ))</th>
<th>10KV ( v_s=1.5 )</th>
<th>11.25KV ( v_s=1.72 )</th>
<th>12.5KV ( v_s=1.92 )</th>
<th>Line (( A^0 ))</th>
<th>10KV ( v_s=1.5 )</th>
<th>11.25KV ( v_s=1.72 )</th>
<th>12.5KV ( v_s=1.92 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3294</td>
<td>24.1</td>
<td>36.5</td>
<td>45.0</td>
<td>3286</td>
<td>4.6</td>
<td>25.6</td>
<td>42.5</td>
</tr>
<tr>
<td>3307</td>
<td>20.1</td>
<td>29.9</td>
<td>33.6</td>
<td>3302</td>
<td>5.9</td>
<td>11.9</td>
<td>22.3</td>
</tr>
<tr>
<td>3351</td>
<td>21</td>
<td>22.7</td>
<td>42.7</td>
<td>3336</td>
<td>8.7</td>
<td>12.1</td>
<td>8.6</td>
</tr>
<tr>
<td>3388</td>
<td>no readings taken</td>
<td>41.2</td>
<td></td>
<td>3345</td>
<td>8.2</td>
<td></td>
<td>8.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3358</td>
<td></td>
<td></td>
<td>8.7</td>
</tr>
</tbody>
</table>

From Tables II and IV it can be seen that the two methods of measuring total line intensity show only rough agreement. However, when the temperature calculation is made using the intensities of Table IV the values of \( kT \) obtained cover the same range as those in Table III.

Calculations were made of average temperatures, \( kT \), at \( v_s=1.5 \) and \( 1.72 \) cm/microsecond using intensities of AII 3294, 3307\( A^0 \) and AIII 3286, 3302, 3336, and 3344\( A^0 \) from Table IV. These values are seen in Table V. The average electron densities used in the above calculations were estimated from shock theory,
as time did not permit Stark shift measurements at these shock velocities. At \( v_s = 1.92 \text{ cm/microsecond} \) the electron densities as calculated from the shock theory and from the Stark shifts differed significantly, and therefore the electron densities calculated from shock theory were scaled down in proportion for the lower shock speeds. In Table V the upper entries are \( kT \) values for \( v_s = 1.72 \text{ cm/microsecond} \), \( N_e = 3.4 \times 10^{17} \text{ cm}^{-3} \) while the lower ones are for \( v_s = 1.5 \text{ cm/microsecond} \), \( N_e = 2.95 \times 10^{17} \text{ cm}^{-3} \).

**Table V - Plasma Temperature kT(ev)**

<table>
<thead>
<tr>
<th>AIII Lines (Å)</th>
<th>3285.8</th>
<th>3301.9</th>
<th>3336.1</th>
<th>3344.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>AII 3293.9</td>
<td>2.25</td>
<td>2.19</td>
<td>2.21</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>2.05</td>
<td>2.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3307.2</td>
<td>2.15</td>
<td>2.09</td>
<td>2.11</td>
<td>2.15</td>
</tr>
<tr>
<td></td>
<td>1.96</td>
<td>2.04</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Theoretical Temperatures and Densities**

The values of the plasma temperature, \( kT \), and electron density, \( N_e \), calculated from the shock theory for \( v_s = 1.5, 1.72, \) and \( 1.92 \text{ cm/microsecond} \) and initial rest gas density, \( N_0 = 2.12 \times 10^{16} \text{ cm}^{-3} \) (from \( p_0 = .6 \text{ mm Hg at 68°F} \)) are displayed in Table VI. Also shown in Table VI are the average values of spectroscopically determined temperatures and densities.
Table VI

<table>
<thead>
<tr>
<th>$v_s$ (cm/microseconds)</th>
<th>Spectroscopic Values</th>
<th>Shock Theory Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$kT$(ev)</td>
<td>$N_e(10^{17}\text{cm}^{-3})$</td>
</tr>
<tr>
<td>1.92</td>
<td>2.32</td>
<td>3.9(±.8)</td>
</tr>
<tr>
<td>1.72</td>
<td>2.18</td>
<td>-</td>
</tr>
<tr>
<td>1.50</td>
<td>2.04</td>
<td>-</td>
</tr>
</tbody>
</table>

(B) Helium

Preliminary

On the plates taken of the helium shock spectra only two He lines were obviously present; these were HeI 5876 and HeII 4686. A very faint line was seen at 3888Å which may have been HeI 3888 (though there is a weak CII line at 3889Å). This line was clearly observed with the spectrophotometer as was the line HeII 3203. No other He lines could be found.

Impurity lines present in addition to Hα and Hβ were many CII, CIII, OII, SiII lines and the stronger CuI, CaII, and SiIII lines. The identification of these lines was verified by noting that the intensities of lines of the third spectra showed a more rapid increase with shock velocity than in the case of the second spectra.

HeI 5876, HeII 4686 and 3203 were far enough from impurity lines (5Å) to be resolved by the Bausch and Lomb monochromator. An estimate of the intensity of HeI 3888Å
could be made by correcting for the response at 3888A° due to CII 3889. The intensity of CII 3889 was in turn estimated from the response of other CII lines of strength.

Both the Hære and Hø lines were free from interference by other lines.

Time Integrated Measurements

No quantitative measurements were made on helium spectra as the neutral density filter was not available while this work was in progress.

Time Resolved Measurements

Usable measurements on helium were made only at a driving voltage of 12.5KV. Much exploratory work in the visible spectral region was done with the modified Hilger constant deviation spectrograph but no specific results came from this work as the spectral sensitivity of the instrument could not readily be determined. After the arrival of the Bausch and Lomb monochromator only a limited amount of work was done on helium.

(a) Hydrogen and Helium Line Intensities

Time resolved observations of spectral intensity in the vicinity of the Hβ line yielded Hβ profiles for different times. Typical Hβ intensity histories are shown in Figures 12 and 13 for wavelengths near the line centre (4861A°) and in the wings.
It is readily seen from Figures 12 and 13 that if a time integrated intensity is measured for the $H_\beta$ line (i.e. from a photographic exposure) such a measurement will be heavily weighted by the second pulse (caused by the second current surge in the ringing of the discharge). The time integrated measurement would give a much narrower profile than is actually the case immediately behind the shock.

Typical intensity histories of HeI and HeII lines are shown in Figures 14 and 15. From these curves it can be seen that the ionic lines peak and decay sooner than do the atomic lines.
(b) $N_e$ and $kT$ Behind Shock

Approximations to $N_e$ (i.e., $N_{\text{eff}}$) were determined from the H profiles at various times after the passing of the shock; these densities are plotted in Figure 16. Having time resolved values of electron densities, the plasma temperature could be calculated using the HeI and HeII line intensity histories. The temperatures so obtained are also plotted in Figure 16. The origin of the time axis in Figure 16 is taken to be the point where the luminosity first begins to rise as seen in Figures 12 - 15. The temperatures and densities shown are such that $\gamma_2 \ll 1$ and thus $N_{\text{eff}} = N_e$ to the accuracy of this experiment.

Figure 16 - $kT$ and $N_e$ versus $t$
Theoretical Temperatures and Densities

The plasma temperature and electron density calculated from the shock theory for $v_s = 4.8$ cm/microsecond and $N_o = 1.17 \times 10^{16}$ cm$^{-3}$ (from $p_o = 0.33$ mm Hg at 68°F) are shown in Table VII with the spectroscopic values for comparison.

Table VII

<table>
<thead>
<tr>
<th>Spectroscopic Values</th>
<th>Shock Theory Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$kT$(ev)</td>
<td>$N_e(10^{17}$cm$^{-3}$)</td>
</tr>
<tr>
<td>3.73</td>
<td>5.3($\pm$1)</td>
</tr>
</tbody>
</table>
VI CONCLUSIONS

For the gases used in this experiment, the temperature measured spectroscopically was in all cases equal to or less than that expected from the shock theory. However, the electron densities determined from spectroscopic measurements were greater for helium and less for argon than the shock theory predicted.

The above results are at variance with those observed by McLean et al (1960), their spectroscopic temperatures being higher than expected. A possible factor involved here is the distance from the discharge to the station at which the observations were made. McLean's observations were made at 6 cm, those here at 10 cm.

With the limited amount of data available from the work done to date no specific conclusions can be drawn as to quantitative departure from the shock theory. Much more investigation is required of shock behavior at different speeds and stations in order to determine if the observed discrepancies are functions of the apparatus or are due to invalidity of the theory. Also, the effect of impurities, which has not been considered in this work, requires some study.

For reasons outlined in the results, time integrated or average values of $N_e$ (or $N_{eff}$) can best be determined from shift or broadening measurements made on the spectra of the
Determination of electron densities from observation of impurity spectra such as that of hydrogen must be made from time resolved spectra. The use of the spectrophotometer in obtaining hydrogen line profiles and hence electron densities appears to be a very promising technique.

While the techniques of measuring line intensities are adequate for rough (± 10%) determination of temperatures they require much refinement in order to check the equilibrium assumption on which the spectroscopic theory is based. To make such a check, intensities of far separated lines of many different multiplets must be accurately determined and then compared. Such a comparison would require calibration of the spectroscopic apparatus for absolute intensity versus wavelength. Having absolute intensities for the spectral lines, Equations (3), (10) and (11) can be solved for $kT$ and $N_e$. A separate determination of $N_e$ which is independent of the equilibrium assumption can be made from measurements such as hydrogen line broadening. Agreement between the two values of $N_e$ would substantiate the original assumption of equilibrium.

Future investigations could be directed towards plasma properties and behaviors which are sensitive to the existence or non-existence of equilibrium. Attempts could be made to determine the time required to establish equilibrium conditions.

In summary, the conclusions to be drawn from this experiment are that while spectroscopy is a very useful tool
for aiding in the determination of conditions within a plasma, it requires refinement in both theory and experimental technique. However, to further develop these techniques requires a more thorough understanding of the processes within a plasma.
APPENDIX I

THEORETICAL LINE STRENGTHS

The theoretical line strengths to be used in calculations for argon and helium will be taken directly from published values where available. As no values are available for the strengths of argon lines used, these must be calculated.

Argon Line Strengths

Following Condon and Shortley the line strength, S, can be expressed as follows:

\[ S = M L \epsilon^2 \]

where \( M \) is a factor depending on the multiplet*, \( L \) is a factor depending on the particular line in the multiplet, and where \( \epsilon^2 \) is related to the initial electron angular momentum quantum number, \( l \), and to the radial wave functions \( R_i \) and \( R_f \) of the initial and final states by:

\[ \epsilon^2 = \frac{1}{4l^2_{l-1}} \int_{0}^{\infty} R_i R_f r \, dr. \]

The products \( ML \) are easily found from the electron configurations of the associated energy levels (Moore, 1949 and 1959) and from formulae and tables in Chapter 9, Condon and Shortley.

*A multiplet is a group of lines emitted by transitions from upper energy states of common principal and orbital angular momentum quantum numbers, \( (n, l) \), but differing total angular momentum quantum numbers, \( (j) \), to lower energy states having the same relationship to one another.
Bates and Damgaard (1950) have integrated (2) using wave functions which are solutions in the Coulomb approximation** to the Schröedinger equation. Bates and Damgaard use the expression:

\[
\sigma = \frac{1}{\pi} Z(n^*_l, l) f(n^*_{l-1}, n^*_l, l)
\]

where \( C \) is the excess electronic charge on the nucleus (one for neutral atom, two for singly ionized atom, etc.), and \( Z, f \) are:

\[
Z = \frac{3/2}{l} \frac{C/n^*_l}{(n^*_l^2 - \ell^2)^{1/2}}
\]

\[
f = \frac{2/3}{n^*_l^2 - l^2} \left[ \int_0^\infty R(n^*_{l-1}, l-1, C) R(n^*_l, l, C) r \, dr \right]^{1/2}
\]

The quantity \( n^*_l \) is the effective principal quantum number corresponding to the state with electron orbital angular momentum \( l \). The \( n^*_l \) and \( n^*_{l-1} \) are given by the simple relation:

\[
n^*_l = \frac{C}{(E_l)^{1/2}}, \quad n^*_{l-1} = \frac{C}{(E_{l-1})^{1/2}}
\]

where \( E \) is the energy of a level below the ionization limit for the particular atom or ion. In the above work the \( Z(n^*_l, l) \) and \( f(n^*_{l-1}, n^*_l, l) \) are tabulated for given \( n^*_l, n^*_{l-1}, \) and \( l \).

The calculated line strengths for the argon lines of interest are shown in the table below. Values of \( E_l \) were

**In the Coulomb approximation the electron is assumed to move in a Coulomb potential (Potential energy = \( C \) where \( C \) is excess electronic charges on nucleus).
taken from Moore (1949).

<table>
<thead>
<tr>
<th>AII</th>
<th></th>
<th>S</th>
<th>AIII</th>
<th></th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line (Å)</td>
<td></td>
<td></td>
<td>Line (Å)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3293.6</td>
<td></td>
<td>5.26</td>
<td>3285.8</td>
<td></td>
<td>25.2</td>
</tr>
<tr>
<td>3307.2</td>
<td></td>
<td>2.10</td>
<td>3301.9</td>
<td></td>
<td>18.0</td>
</tr>
<tr>
<td>3350.9</td>
<td></td>
<td>0.18</td>
<td>3311.2</td>
<td></td>
<td>10.8</td>
</tr>
<tr>
<td>3376.5</td>
<td></td>
<td>0.25</td>
<td>3336.1</td>
<td></td>
<td>42.9</td>
</tr>
<tr>
<td>3388.5</td>
<td></td>
<td>9.2</td>
<td>3344.7</td>
<td></td>
<td>29.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3358.5</td>
<td></td>
<td>20.0</td>
</tr>
</tbody>
</table>

The line strengths for AII 3350.9 and 3376.5 are abnormally low; the Coulomb approximation seems to break down for these lines.

Helium Line Strengths

The line strengths of helium were obtained from the absorption oscillator strengths. The absorption oscillator strength, \( f \), is related to the line strength, \( S \), by

\[
f = K \frac{1}{\lambda} \frac{S}{g_n}
\]

where \( \lambda \) is the wavelength of the line, \( g_n \) is the degeneracy of the lower energy level and \( K \) is a constant which will be taken as unity here as only ratios of \( f \) or \( S \) are used in calculations.

The oscillator strengths of neutral helium were obtained from the work of Treffetz et al (1957) while those of ionized helium are simply those of hydrogen (see for example
The values of $f$, $g_n$, and $S$ for the helium lines to be studied in this experiment are tabulated below.

<table>
<thead>
<tr>
<th>$f$</th>
<th>$\lambda$ (cm x 10$^5$)</th>
<th>$g_n$</th>
<th>$S = \lambda g_n f$</th>
<th>$f$</th>
<th>$\lambda$ (cm x 10$^5$)</th>
<th>$g_n$</th>
<th>$S = \lambda g_n f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.057</td>
<td>3.888</td>
<td>3</td>
<td>.665</td>
<td>.151</td>
<td>3.203</td>
<td>50</td>
<td>24.2</td>
</tr>
<tr>
<td>.623</td>
<td>5.876</td>
<td>9</td>
<td>32.9</td>
<td>.842</td>
<td>4.686</td>
<td>32</td>
<td>126</td>
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</tbody>
</table>
BIBLIOGRAPHY


