DRIFT VELOCITY OF HOT ELECTRONS IN n-TYPE GERMANIUM

by

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ABSTRACT

The drift velocity of electrons in n-type germanium has been measured as a function of applied electric field at lattice temperatures of 77°K and 295°K . Three directions of applied field were used, viz. (1, 0, 0), (1, 1, 0) and (1, 1, 1) crystal directions. The range of field strength was from 100 v/cm to 75 kv/cm. A longitudinal anisotropy was observed at 77°K but not at 295°K . At 77°K the drift velocity for a given field strength was greatest in the (1, 0, 0) direction and least in the (1, 1, 1) direction. All specimens showed saturation of the drift velocity at high fields. In no case did the drift velocity show a further increase with increasing electric field once it had saturated. At 77°K , all (1, 0, 0) specimens exhibited a breakdown effect, the cause of which is not known. Its characteristics could not be correlated with any properties of the specimens.

The results are analysed on the basis of an extension of Stratton's theory to the case of a many-valley semiconductor. It is assumed that intervalley scattering does not affect energy relaxation, but does contribute a shift in the relative populations of the conduction band valleys. This theory is successful at low fields. It predicts an anisotropy at 77°K while predicting none at 295°K . However, it predicts too much anisotropy at low fields at 77°K . It also predicts an anisotropy at high fields at 295°K . Suggestions are made as to possible reasons for these discrepancies.

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Abstract

The drift velocity of electrons in n-type germanium has been measured as a function of applied electric field at lattice temperatures of 77°K and 295°K . Three directions of applied field were used, viz. $(1,0,0)$, $(1,1,0)$, and $(1,1,1)$ crystal directions. The range of field strength was from 100 v/cm to 75 kv/cm. A longitudinal anisotropy was observed at 77°K but not at 295°K . At 77°K the drift velocity for a given field strength was greatest in the $(1,0,0)$ direction and least in the $(1,1,1)$ direction. All specimens showed saturation of the drift velocity at high fields. In no case did the drift velocity show a further increase with increasing electric field once it had saturated. At 77°K , all $(1,0,0)$ specimens exhibited a breakdown effect, the cause of which is not known. Its characteristics could not be correlated with any properties of the specimens.

The results are analysed on the basis of an extension of Stratton's theory to the case of a many-valley semiconductor. It is assumed that intervalley scattering does not effect energy relaxation, but does contribute a shift in the relative populations of the conduction band valleys. This theory is successful at low fields. It predicts an anisotropy at 77°K while predicting none at 295°K . However, it predicts too much anisotropy at low fields at 77°K . It also predicts an anisotropy at high fields at 295°K . Suggestions are made as to possible reasons for these discrepancies.

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CHAPTER 1 - INTRODUCTION

For n-type germanium the carrier drift velocity ceases to be proportional to the applied field $\underline{F}$ at large values of F . This has been explained in the following manner. For small values of applied field the electrical power being fed into the electrons in the crystal is small. This can easily be dissipated through collisions with the lattice, leaving the electrons in thermal equilibrium with the lattice. As the applied field is increased, however, a point is reached at which the electrons can no longer lose this energy gained from the field as rapidly as they receive it. Consequently the mean energy (or temperature, if such can be defined) of the electrons increases. Since in germanium the energy loss mechanisms become more efficient as the electron energy is increased, an energy balance is again established. However, since the energy loss mechanisms are more efficient, the electron mobility is lower, and consequently the sample resistance higher, than at lower fields. Further increases in applied field lead to higher electron "temperatures" and to greater deviations from Ohm's law.

Ryder (1953) measured the drift velocity in n-type germanium as a function of the applied field up to a field of 6 kv/cm at a lattice temperature of 77°K and up to a field of 7 kv/cm at lattice temperatures of 193°K and 298°K . Gunn (1956) has repeated this experiment at a lattice temperature of 300°K and up to those fields required to produce avalanche breakdown in his specimens (70 kv/cm).

Sasaki et al. (1958) have also shown that for large fields the current and field in n-type germanium are not, in general, parallel. They measured the transverse anisotropy by studying the non-zero field component transverse to the current. Measurements of a longitudinal anisotropy have been briefly reported by Nathan (1960). In this case special field directions ((100), (110), (111)) are chosen for which field

and current are parallel. The drift velocity at a given field strength is then different for these different field directions. The anisotropy has been ascribed to the unequal sharing of current among the different valleys of the conduction band. When the field is large enough to cause heating of the electrons, the equilibrium "temperatures" of different valleys are not necessarily equal. Consequently, the low field isotropy is lost.

In the data of Ryder and Gunn, no orientation was stated for the crystals and the possibility of anisotropy was ignored. The present thesis reports a study of the longitudinal anisotropy at lattice temperatures of 77°K and 295°K , up to a field strength of 75 kv/cm.

Several attempts have been made at an analysis of the problem for an isotropic model. Yamashita and Watanabe (1954) achieved an analytical solution of the Boltzmann equation by neglecting the effects of electron-electron collisions and inelastic (optical mode) lattice scattering. Dykman and Tomchuk (1960) considered inelastic scattering as well, but still did not take into account the effect of electron-electron collisions. Following the work of Fröhlich and Paranjape (1956), Stratton (1958) has assumed that electron-electron scattering will ensure a displaced Maxwellian distribution for the energy of the hot carriers. He evaluated this distribution function, taking into account the effect of both acoustic and optical mode lattice scattering on the momentum and energy relaxation.

The analysis used here to discuss the anisotropy is in two parts (i) the transition probabilities for intervalley scattering (Herring, 1955) are used to find the carrier populations in the individual valleys (ii) the theory of Stratton (which seemed to give good agreement with the experiments of Gunn) is extended to cover the many-valley ellipsoidal conduction band of germanium.

CHAPTER 2 - EXPERIMENTAL METHODS

2.1 THE BASIC EXPERIMENT

The circuit used to produce and display the short high voltage pulses required (Fig. 1) was basically that due to Gunn and discussed by Lewis and Wells (1959). The pulse originated from the discharge of a capacitor C into a coaxial system of reasonably constant impedance Z_0 . This discharge was initiated by the firing of a suitably triggered hydrogen thyatron. The pulse so produced was then fed through a series spark gap (which considerably shortened its risetime) into a shorted transmission line. The output of the generator was taken from a point at an appropriate distance d from the end of the shorted line. However, instead of using a strip transmission line over an effectively infinite grounded plane (as did Gunn) for feeding the pulse from the generator to the sample, standard coaxial cable was used throughout. This made it possible to mount the sample-holder in the dewar required for the low-temperature measurements.

One end of the sample was attached to the inner conductor of the coaxial cable. The other end was attached to another coaxial line running directly to the deflection plates of a cathode ray oscilloscope which was used for measurement as well as display. In this way the voltage across the sample increased in a manner characteristic of the spark gap as the capacitor discharge pulse passed the point at which the sample was attached. This voltage was reduced to zero again, with the fall time equal to the rise time, at the passage of the reflected pulse, which was 180° out of phase due to reflection at the shorted termination. Hence, a pulse of length

$$\tau = \frac{2d}{c} \quad (2.1)$$

was applied across the sample (where c was the velocity of propagation of a disturbance along the coaxial cable). The deflection observed on the oscilloscope was proportional to the

FIGURE 1

Block diagram of the high field measurement circuit.

current through the sample.

In order to determine the voltage of the applied pulse it was necessary to replace the sample with a "standard resistor" which, ideally, was ohmic over the range of voltages used. The pulse height was then deduced from the current observed to flow through the standard resistor.

The quantities actually measured were the deflection voltages appearing on the oscilloscope: V_I with the sample in the line, and V_V with the standard resistor of value R . These measurements were reduced to the voltage V across the sample and the current I through it, and ultimately to a value of the drift velocity v_{drift} corresponding to a field F across the sample.

The cable leading to the oscilloscope was effectively terminated in an open circuit. The only loading on it was due to the capacitance of the deflection plates. Since this was of the order of 1 pF, the time constant of the deflection circuit with the 50 ohm cable used was about .05 nsec. This was completely negligible since the shortest pulses used were 5 nsec long. Therefore a current pulse I at the input would, by virtue of its reflection at the open circuit termination, produce a deflection voltage of $2IZ_o$ on the oscilloscope. Hence,

$$I = \frac{V_I}{2Z_o} \quad (2.2)$$

Considering the impedance Z_o of the cable as being in series with both the sample, and in turn, the standard resistor

$$V = \frac{(R + Z_o)}{2Z_o} V_V - \frac{1}{2} V_I \quad (2.3)$$

for the case of a rectangular specimen, and

$$V = \frac{(R + Z_o)}{2Z_o} V_V - \frac{1}{2} V_I - IR_s \quad (2.4)$$

for a dumb-bell sample. R_s was the spreading resistance associated with the dumb-bell ends. These equations would only hold for the case when twice the delay time of the cable

connecting the sample to the oscilloscope was greater than τ , as we have ignored the effect any reflected pulse would have at the sample end of the cable.

Since the hole concentration was negligible, the current in a sample with one dimensional geometry was

$$I = nq v_{drift} A \quad (2.5)$$

and since

$$\sigma_o = nq \mu_o \quad (2.6)$$

this reduced, under the assumption of constant n , to

$$v_{drift} = \frac{\mu_o R_o}{s} I \quad (2.7)$$

and of course

$$F = \frac{V}{s} \quad (2.8)$$

where: n = carrier concentration
 q = electronic charge
 A = cross-sectional area of the sample
 σ_o = low field conductivity
 μ_o = low field mobility
 R_o = low field resistance of the sample
 s = length of the sample

2.2 DETAILS OF THE APPARATUS

2.2(a) TIMING AND TRIGGERING CIRCUITS

The pulse repetition rate was controlled by a Tektronix Type 162 waveform generator. This unit could be run recurrently at rates down to 0.1 cps, and had the added provision that it could be actuated manually for one-shot operation. The output of this unit was used to trigger a Hewlett Packard Model 212A pulse generator. This instrument was used to supply the waveform required to turn on the electron beam in the oscilloscope during a pulse. It was selected because it gave a short sharp pulse which could be delayed or advanced

with respect to an auxillary trigger pulse. The 0 - 50 volt output pulse had rise and fall times less than 20 nsec, and a pulse length which could be varied precisely and continuously in the range 0 - 10 μ sec. The advance or delay time could be varied from 0 - 100 μ sec. This pulse was fed directly on to the grid of the cathode ray oscilloscope tube.

The positive "sync out" pulse from the HP212A was used to fire the thyatron trigger generator. The need for such a unit arose because of the discharge characteristics of hydrogen thyatrons. Both the rise time and the jitter associated with the discharge are functions, not only of the amplitude of the triggering pulse, but also of its rate of rise (Glascoe and Lebacqz, 1948). In order to ensure a positive triggering action, the circuit of Fig. 2 was constructed. Two 2D21 miniature gas-filled tetrode thyatrons were employed in a conventional manner. The first one discharged a 100 pF capacitor into a 1000 ohm load providing a sharp pulse to fire the second one. This tube in turn discharged a .02 μ F capacitor through a length of coaxial cable into a 50 ohm resistor which was connected from the grid pin of the hydrogen thyatron to ground. In this way a pulse of greater than 500 volts amplitude, with a rise time of less than 20 nsec, was applied to the grid of the hydrogen thyatron.

2.2(b) PULSE GENERATOR

The hydrogen thyatron which was used as the basis of the pulse generator (Fig. 3) was a positive control device. As long as the grid remained biased negatively, conduction would not occur for any value of anode voltage. However, when the grid was made positive, grid current was drawn which caused ionization of the gas in the grid-cathode region. As soon as the electron density outside the grid baffles built up sufficiently, the anode field was able to produce ionization outside the grid baffles and complete breakdown occurred. While it is true that in this state the tube drop was a function of the current, as

FIGURE 2

Thyatron Trigger Generator

well as the quality of the oxide coating on the cathode, the plate resistance of the tube was of the order of a few ohms and had little effect on the circuit performance.

In operation, the application of the trigger pulse caused the tube to conduct, effectively grounding its plate. The capacitor C, which was originally charged positively with respect to ground, therefore had an excess of charge on its lower plate. This discharged to ground through the coaxial cable, sending a negative pulse down the line.

Besides being able to withstand a d.c. voltage of 20 kv, the capacitor had to have a very low self-inductance. This was required in order that the rise time of the discharge pulse would not be appreciably longer than the 20 - 70 nsec ionization time typical of the thyatron. A low loss plastic film pulse capacitor was originally selected, but was found to be completely unsuited for this application. At times as long as 100 nsec after the thyatron had fired there was still no appreciable discharge current. The unit finally used was a rectangular array of twenty 500 pF 30 kv ceramic capacitors (Centralab Type TV8-503) connected in parallel by two 1/16" brass plates. This proved quite satisfactory in practice, and the rise times obtained were of the same order as those expected for the thyatrons themselves.

The capacitor was charged from an external high voltage power supply (0 - 20 kv at 50 ma). Choke charging was used to ensure that immediately following the discharge pulse, the voltage on the capacitor was kept to a low enough value for a sufficient length of time to allow the thyatron to de-ionize.

With the triggering circuit used, the best discharge characteristics were obtained with a 4C35 as the pulse tube. This gave a rise time of about 50 nsec. Unfortunately it flashed over internally at voltages somewhat below those required for the highest fields. For this reason it was necessary to use a 5C22, which gave a rise time of about 70 nsec.

FIGURE 3

Pulse Generator

2.2(c) SPARK GAP

Since the rise time of the pulse obtained from the thyatron generator was not short enough to allow the use of pulses a few nanoseconds in width, a high pressure series spark gap was constructed (Fig. 4).

The spark gap was made of brass and lucite with o-ring pressure seals. The electrodes and dielectric were shaped to give a characteristic impedance of 50 ohms, which matched the coaxial cable used. Two lucite windows were provided for the inspection of the electrode tips. The tips were electrolytically sharpened tungsten wire, 0.01" in diameter. The high pressure atmosphere was supplied by a commercial gas cylinder through a 0 - 1200 psi regulator valve. Although high purity Certified Grade argon was tried, it was found that more precise firing of the gap was possible when the less pure Welders Grade was used. It could have been that small amounts of oxygen were responsible for this improved stability.

In order that the gap width could be adjusted for optimum performance, an arrangement involving differential threads was incorporated in one end of the gap. This was driven by an electric motor and gear train. However, at the high pressures used the brass threads tended to bind, and adjustment under operating conditions was at first impossible. This difficulty was overcome by the use of molybdenum disulphide as a lubricant. In practice both the gap pressure and the gap width were adjusted for optimum pulse shape.

It was not possible to measure the actual breakdown time of the gap. The rise time of the pulse observed on the oscilloscope was estimated to be about 0.25 nsec. However, this was just the manufacturers specification for the electron transit time through the deflection plates of the cathode ray tube.

Full size side view cutaway of the constant impedance variable width spark gap

2.2(d) PULSE DISPLAY AND MEASUREMENT

A special cathode ray tube was used for observation and measurement of the current pulses. The tube, Type No. K1543P12, was designed and constructed by the Allen B. DuMont Laboratories of Passiac, New Jersey. It was a flat face tube 5 inches in diameter, utilizing electrostatic focus and deflection. One of the vertical deflection plates was internally grounded to provide the best termination possible for the signal pulse. In order to obtain sufficient light output from the phosphor to give a visible trace at the very high writing rates required (up to 2 nsec/cm), extremely high accelerating voltages were used in a post deflection acceleration arrangement. The tube also had as a fiducial mark a small aperture in the centre of the phosphor screen. This could be illuminated from the inside to give a zero parallax reference point for measuring the pulse height. Two high voltage power supplies were used to set the potentials of the electrodes in accordance with the manufacturers specifications. A circuit diagram is given in Fig. 5.

In order to eliminate jitter from the display, it was found necessary to generate the horizontal deflection voltage for the oscilloscope from the actual pulse being applied to the sample. This was accomplished by integrating part of the output of the spark gap with a simple RC circuit. Since the voltage levels involved were much higher than those required to deflect the beam (about 100 v/cm) a comparatively long time constant (0.5 μ sec) could be used in the integration to ensure a linear sweep for the duration of the pulse. A series resistor was inserted in the line between the integrating network and the horizontal deflection plates so that the pulse appeared an appropriate size on the face of the oscilloscope. This resistor was easily changed to maintain this situation as the pulse height, and hence the deflection voltage, was varied during a series of measurements. The

Circuit diagram for the cathode ray oscilloscope.

horizontal shift voltage was supplied by a 0 - 300 volt regulated power supply connected across the deflection plates.

In order to eliminate the retrace line left by the returning beam, and to prevent the motionless beam from burning a hole in the phosphor between pulses, it was necessary to bias the electron beam off with a d.c. voltage applied between the grid and cathode of the cathode ray tube. The beam was then turned on with a voltage pulse synchronized to the applied signal. As mentioned above, this voltage pulse was obtained from the HP212A pulse generator which was used to fire the thyatron trigger circuit. In actual operation the pulse position control on the HP212A was adjusted until the beam was turned on just as the deflection voltage reached the horizontal deflection plates. The pulse length control was then adjusted until the beam was turned off when the desired signal had been observed.

The height of the signal pulse was measured by using a d.c. shift voltage applied across the vertical deflection plates to "buck out" the pulse. The vertical shift voltage was adjusted until the base line of the pulse trace coincided with the fiducial mark on the oscilloscope screen. Then with the trace suitably shifted horizontally, the vertical deflection voltage was increased until the top of the pulse trace passed through the fiducial mark. The amount by which the vertical shift voltage was increased was the pulse height. This shift voltage was supplied by a Technical Measurements Corporation power supply model HV-4A. The output of this unit was variable from 500 v to 1600 v, and was accurate to .5%. To extend the lower limit of the shift voltage available, a calibrated voltage divider was constructed of precision resistors. It had division ratios from 5×10^{-4} to 1. These ratios were checked several times over the period the measurements were made, but no changes were observed.

2.2(e) SAMPLE HOLDERS AND TEMPERATURE BATHS

Two sample holders were constructed of brass and lucite (Fig. 6). One was used for the standard resistors and the other for the germanium samples under test. This was necessary as no way was found to attach contacts to the silicon standards which would not inject at 77°K . It was therefore necessary to keep the standards at room temperature while the low temperature measurements were being made on the germanium samples.

The three cables connected to the sample (one from the spark gap, one to the oscilloscope, and one shorted cable which determined the pulse length) were securely anchored to the frame of the holder to avoid stressing the contacts. The inner conductors of the cables were soldered to phosphor bronze strips which were in turn bolted to a lucite plate. The outer conductors were braided and soldered together. To insert a sample into the circuit, it was only necessary to clamp its gold wire leads between the phosphor bronze strips and the lucite plate. The gold wire leads were then carefully bent until the sample was freely suspended, and not in contact with the holder except through the leads. The holder was then placed in a Pyrex dewar flask. This was filled with an appropriate liquid in order to keep the sample, as nearly as possible, in a constant thermal environment.

All of the liquids tested for the room temperature bath (water, glycerin, ethyl alcohol, methyl alcohol, transformer oil, DC 200 silicone fluid, and petroleum ether) caused the pulse to ring as observed on the oscilloscope. However, this effect was very small in the case of petroleum ether, and the oscillations had disappeared after 5 nsec. Since this did not affect the accuracy of the measurements, petroleum ether was used for the room temperature bath. The temperature of the bath was monitored with a thermometer immersed in the petroleum ether to within a few mm of the sample. While the measurements were being made the temperature was always within 1° of 22°C .

FIGURE 6

Sketch of the side and end views of the holder used to mount the germanium samples.

No deterioration of the pulse shape was noticed as the germanium samples were immersed in liquid nitrogen, which was used as the low temperature bath. As it was not possible to accurately measure the temperature of the nitrogen bath, care was taken to avoid contamination by atmospheric oxygen. Only freshly prepared liquid nitrogen was used, and the dewar flask was kept covered at all times. The temperature of the bath was taken to be 77°K . This would be accurate to within 1° over a range of 0 - 10 molecular percent of oxygen (Dodge and Dunbar, 1927).

During a series of measurements the display circuitry and the pulse generator were switched alternately to the germanium sample and to the standard resistor. This was accomplished with two single pole double throw coaxial relays. In order to prevent arcing from the reed to the case, it was necessary to oil fill the body of the relay being used to switch the high voltage pulse. Once this had been done, these relays had no noticeable effect on the pulse shape.

2.2(f) CABLES AND CONNECTORS

Coaxial cable type RG-58AU was used for the leads interconnecting the various pieces of equipment, and for the shorted line used to set the pulse length.

Care was taken to get rid of any reflections of the main pulse resulting from impedance mismatches. The length of the cables used to connect points where such reflections could occur was always greater than twice the length of the shorted line used to form the pulse. In this way no reflected pulse could be re-reflected while the main pulse was "on", thereby modifying the shape of the main pulse.

The constant impedance Improved Series BNC connectors were used to attach the leads carrying the main pulse and display signals. For all other applications, standard BNC fittings were adequate. Both types of connector gave considerable trouble due to voltage breakdown across the surface of the dielectric inside the body of the connector. The usual

anti-tracking compounds were not effective. Conduction paths would eventually develop even with pulse voltages as low as 1 kv. This problem was solved by packing the connectors with Dow Corning room temperature vulcanizing silicone rubber Type RTV 731. The connectors were first degreased in an ultrasonic cleaner. Then the inside of the connector and all fittings were primed with a suitable silicone varnish. The connector body was filled with silicone rubber and attached to the cable in the usual manner. The excess rubber was removed and the assembly left to cure for 72 hrs. Whenever possible mating connectors were also sealed together in a similar manner after preliminary adjustments to the circuit had been completed. After this treatment, no further difficulty was experienced, although the equipment was run for extended periods of time with pulses of up to 10 kv.

2.2(g) Lower Field Measurements

Since the high voltage pulse generator would not function reliably at low voltages, it was necessary to use a more conventional circuit for measurements below 750 v/cm. (Fig. 7). The pulse width and repetition rate were determined by a Tektronix Type 162 waveform generator. The output of this unit was used to trigger a pulse amplifier designed by Mr. J. B. Gunn. The pulse height was set by a separate variable (0 - 1 kv) power supply which provided the energy required for the output of the pulse amplifier.

As the sample heating was very much reduced at these lower values of applied field, longer pulses and higher repetition rates could be used. The measurements were made with a pulse width of 5 μ sec and a repetition rate of 50 pps. As before these were varied over a wide range to ensure that there was no noticeable effect due to sample heating.

The silicon standard and the germanium sample were connected in series across the output of the pulse amplifier. The output pulse from the amplifier, and the corresponding

FIGURE 7

Block diagram of the low field measuring circuit.

pulse appearing across the silicon standard were measured with the aid of a Tektronix Type 545 oscilloscope. Measurements were made over a range of field strength which overlapped that covered with the high voltage unit.

2.3 SAMPLE PREPARATION

2.3(a) CRYSTAL GROWTH AND ORIENTATION

The germanium samples used were prepared from a single crystal antimony doped boule grown by the vertical crystal pulling (Czochralski) technique. An antimony donor concentration of $3 \times 10^{14} \text{ cm}^{-3}$ ($\rho = 5 \text{ ohm cm}$) was chosen for the centre of the boule. With this value the contribution of holes to the total current would still be less than 2% at a field strength of 10^5 v/cm and a lattice temperature of 295°K (Prior, 1959). On the other hand, 99.8% of the Antimony atoms would be ionized at 77°K . Hence, the number of carriers would remain substantially constant over the range of temperature encountered in the measurements.

The boule was grown from a (1,1,0) seed, in order that several (1,0,0), (1,1,0), and (1,1,1) filaments could be obtained from a single slice cut perpendicular to the axis of growth. In this way the samples oriented along the different principal directions were as nearly identical in all other respects as possible. The choice of the (1,1,0) direction as the axis of growth was also fortunate in that it avoided large cross-sectional variations of impurity concentration discussed by Dikhoff (1960). During growth the crystal was pulled at an approximate rate of 1 inch/hr, while being rotated at about 60 rpm.

The boule was etched in a boiling aqueous solution of 30% H_2O_2 until the entire surface was roughened. As this was a preferential etch, any twinning or other imperfections of a polycrystalline nature could have easily been seen as

discontinuities in the reflectivity of the surface. No such effects were observed on this crystal.

From the growth facets on the side of the boule, the orientation of the crystallographic axes was tentatively established. This was confirmed by Laue photographs taken in the X-ray laboratory of the U.S.B.C. Metallurgy Department. The growth axis was confirmed to be a (1,1,0) direction, and patterns were obtained corresponding to the (1,0,0), (1,1,0), and (1,1,1) directions with the incident beam perpendicular to the growth axis. In this manner the orientation of the crystal axes was determined to within 3° .

2.3(b) FABRICATION OF FILAMENTS AND DUMB-BELLS

In order to prepare the samples, the boule was waxed to a glass slide which was in turn waxed to a metal support. Low melting point wax was used to avoid contamination of the crystal. This assembly was then mounted in the machinist's vise associated with a reciprocating wire saw (Eastman, 1960). With this arrangement a tungsten wire of about 0.003" diameter was drawn back and forth across the crystal at a velocity of some 30"/sec. With the cutting area liberally bathed in a smooth paste of 600 mesh carborundum dust and engine oil, cutting speeds of up to 1"/hr could be realized. However, it was found that a smoother cut could be obtained at the expense of some cutting speed by lowering the tension in the wire. Several slices were cut perpendicular to the growth direction from the top half of the boule where the longitudinal impurity gradient was least. These were lapped to a uniform thickness and etched in boiling H_2O_2 solution. Once again close inspection revealed no sign of polycrystalline nature.

The slices were then carefully examined for cross-sectional resistivity variations using the four-point probe technique. The equipment employed consisted of a Baird-Atomic probe assembly Type J-2 and a Rubicon Type 702 potentiometer, along with a specially constructed constant current source.

This latter device was set to give a current of 1 ma, as recommended by Valdez (1954), and was capable of regulating the current to better than 0.1%. Although the resolution was somewhat limited by the point spacing (0.15 cm) the equipment was very stable, and comparative measurements could be made over the area of a slice. At no time was there a variation in resistivity of over 10% noted in a single slice, and in the slice from which the majority of the samples were prepared the variation measured was of the order of 5%. Low current d.c. measurements which were made on prepared samples, confirmed a value of 5 ohm cm for the low field resistivity.

The selected slice was subsequently lapped until it was 0.5 mm thick, and was then rewaxed onto a support. It was finally cut into filaments 0.5 mm wide and oriented along the three previously determined crystalline directions. An adjacent slice 1mm thick was cut into filaments of assorted shapes in order to check for size effects.

During the course of the measurements it became apparent that because of contact failure these filaments were not suitable for measurements above 10 kv/cm. For that range specimens of the "dumb-bell" shape shown in Fig. 8 were prepared. The slice immediately adjacent to that from which the filaments were made was cemented to a glass slide with Eastman 910 adhesive. Using a stainless steel tool cone and a slurry of 600 mesh boron carbide three samples were cut (one along each principal direction) with a Raytheon Model 2-334 ultrasonic impact grinder.

2.3(c) CONTACTS

Contact was made to the germanium samples through the L - H (Low - High) type of alloy junctions. These were made by alloying to the germanium with gold wire leads very heavily doped (0.5%) with antimony.

The sample blanks were first thoroughly degreased. Several rinses in trichlorethylene were followed by a final

FIGURE 8

A sketch of the plan and elevation of the dumb-bell samples used for the high field measurements.

rinse in a clean volatile solvent e.g. reagent grade acetone. This phase was completed by boiling the samples in a 10% H_2O_2 solution. This served the dual purpose of actually etching the germanium, as well as oxidizing any remaining organic impurities. After being rinsed in deionized water and dried on filter papers, the blanks were customarily stored in this condition until it was actually desired to alloy the leads in place.

Immediately prior to alloying, the sample blanks were etched and polished in medium CP4. It is virtually impossible to over-emphasize the cleanliness which was required from this stage onwards. All the utensils were cleaned with chromic acid until they were water-break free, and rinsed five times in deionized water. Because of the hydrofluoric acid in the CP4, polyethylene dishes were used for the etching. The etch itself was prepared from reagent grade chemicals (semiconductor grade if possible) and stored in polyethylene bottles. After etching for about 30 sec, or until a shiny polished surface appeared, the blanks were thoroughly rinsed by allowing deionized water to run over them for several minutes. They were then left in a dish of deionized water until required.

The alloying itself was carried out in a small furnace of stainless steel equipped with a glass roof (Eastman). The blank, gripped in a pair of self-locking tweezers, was placed inside a coil of nichrome wire. The end to which it was desired to affix the wire was positioned at the top of the coil. An atmosphere of Certified Grade nitrogen was introduced near the bottom of the furnace and allowed to escape through the hole in the roof. A glass container, which served both to dispense the gold wire and to hold it in position, was gripped in a commercial micromanipulator. During the alloying operation the junction area was observed through a 12X binocular microscope. Again it was very important that all parts of the alloy furnace, as well as the tweezers used to handle the sample and gold wire, were scrupulously clean.

The variable transformer which powered the heater was carefully turned up until the sample was judged to be at the proper temperature. This depended largely upon the skill of the operator who had to use the colour of the nichrome heater to estimate the temperature. The gold wire was then lowered through a hole in the roof until it touched the end of the sample. If the surface of the germanium were sufficiently clean, a pool of alloy would form immediately and begin to flow across the surface. By carefully moving the gold wire, the alloy could be induced to flow over the entire area of the end of the sample without dripping over the side. The power to the heater was then slowly reduced over a period of 10 - 15 sec until the alloy was observed to solidify.

After the alloying was completed the sample was dipped into medium CP4 for about 1 sec. This was sufficient to clean the surface of the germanium, and did not appear to physically weaken the junction. An alloy contact of this type, properly prepared, was very strong. In fact, if an attempt were made to pull the leads off, the sample would fail just behind the junction, exposing germanium on both sides of the break.

2.3(d) TESTING OF PREPARED SAMPLES

A very quick and simple test was used to select samples for high field measurements.

The sample was placed in a bath of petroleum ether and connected to a precision Wheatstone bridge (a Grey Instruments Queen Bridge) through a reversing switch. This switch was specially prepared with wiper action rhodium contacts, and had a contact resistance of about 1 milliohm. The resistance was then measured with a current of a few microamps flowing through the sample. Using a sensitive galvanometer (Rubicon Type 702) as a null detector, it was possible to resolve differences in resistance of better than 2×10^{-5} as the reversing switch was thrown from one position to the other. If any difference in resistance was detected when the direction of current flow

was reversed, the sample was rejected. Any such failures (which were rare) were attributed to dirt on the germanium surface while alloying.

It is of course true that such a test was necessary, but not sufficient, to exclude the possibility of carrier injection at higher fields or lower temperatures. However, it is interesting to note that no sample which passed this simple low field d.c. test ever gave any indication of carrier injection. On the other hand, all samples which failed gave peculiar current pulse shapes which could be explained by carrier injection.

In the case of the dumb-bell samples, the spreading resistance was determined by subtracting the resistance calculated for the bridge of the sample from the observed low current d.c. resistance. Although this was a substantial correction at low fields, it was field independent and therefore became less important at high fields.

2.4 STANDARD RESISTORS

P-type silicon was considered to be the most suitable material for the standard resistors. Prior has shown that the higher the resistivity of p-type silicon, the less it will depart from Ohm's law as the electric field strength is increased. After failing to find a satisfactory method of attaching ohmic contacts to either 100 ohm cm or 50 ohm cm material, 11 ohm cm gallium doped silicon was selected. Even with this the yield of non-injecting contacts was relatively low. However, satisfactory standards were eventually prepared using the following techniques.

2.4(a) PREPARATION OF STANDARD RESISTORS

The resistor blanks were cut and degreased in the same fashion as were the germanium filaments. Again they were rectangular bars. In this case they had cross-sections from 0.5 mm x 0.5 mm to 2 mm x 2 mm, and lengths from 2 to 20 mm.

All attempts to duplicate the alloying technique so successful on the germanium filaments failed. In many cases no amount of rubbing of the gold wire across the surface would result in the formation of an alloy. In those cases where a small drop of alloy did form it did not wet the surface. On cooling, such a contact was very fragile and exhibited erratic, non-ohmic behavior. Apparently, all of the etches used, although resulting in highly polished clean surfaces, were leaving a tough oxide layer on the silicon which inhibited formation of the alloy. This problem was overcome by laying down a thick adherent gold plate on a silicon surface which was being etched as it entered the plating bath.

The gold plating bath was prepared from reagent grade chemicals and very pure gold wire (99.999%) according to the following formula.

gold wire	2.1 gm/litre
KCN	15 gm/litre
$\text{Na HPO}_4 \cdot 12\text{H}_2\text{O}$	4 gm/litre

A pure gold anode was used to prevent contamination and depletion of the bath.

The bath was set up on a hot-plate and its temperature maintained at 70°C . The external circuit was adjusted to give a plating current density of 15 ma/cm^2 . A 10% KOH etching solution was placed adjacent to the plating bath and maintained at a temperature just below its boiling point. Immediately prior to plating, a degreased silicon filament was held in tweezers and ground on all sides on a glass plate using a thin slurry of 600 mesh carborundum. The filament was then rinsed in deionized water and attached to the plating electrode. The end to be plated was dipped into the hot KOH solution for 30 - 60 sec, or until violent foaming action occurred. The filament was then pulled from the etch, dipped immediately into the already prepared plating bath, and held there for some 60 sec. This usually produced a rather dull adherent plate.

After several rinsings in deionized water the sample

was placed in the alloy furnace in the same manner as before. If the plating were successful, the appearance of the end of the sample would suddenly change, as the heater power was slowly increased, indicating the formation of an alloy. A pure gold lead was then alloyed to the surface.

If the alloy did not form properly (as was the case in two out of every three attempts), the end of the sample in question was reground and the process repeated. When both end contacts had been successfully applied, the specimen was carefully ground to remove any gold or alloy on the sides, and momentarily dipped in a silicon etch. For this purpose 50% HF, 70% HNO₃ and glacial acetic acid were used in the ratio of 1:3:6.

2.4(b) CONSISTENCY TESTS

A careful measurement of the dependence of the d.c. resistance on the direction of current flow, as described above for the germanium samples, proved to be equally useful for the silicon standards. When any irregularity appeared in such a measurement, the contributing contact(s) could be found by preferentially warming one, and then the other contact. A large or erratic change of resistance served to indicate the offending junction(s). With one general exception none of the standards whose resistance was independent of the direction of current flow at low fields gave any cause for concern during later measurements. Unfortunately, however, all of the contacts became injecting when lowered to liquid nitrogen temperature. No way was found to eliminate this, and it necessitated building a separate sample holder so that the silicon standard could be kept at room temperature at all times.

The following procedure was adopted to estimate the field dependence of the resistivity of the silicon used. Two samples were selected; one about ten times the length of the other. Under the assumption that the resistance of the longer bar was independent of field, the pulse equipment described

FIGURE 9

Normalized current density vs. electric field for the 11 ohm cm p-type silicon used as the standard resistors. The straight line is an ohmic characteristic given for comparison.

above was used to obtain measurements relating the resistance of the shorter bar to the applied field strength. The same results were obtained when the experiment was repeated with the physical positions of the two bars interchanged in the apparatus, and when measurements were carried out on several sets of standards. Since all of the bars were cut in a parallel direction from two adjacent slices of a zone levelled single crystal, it was assumed that these results (Fig. 9) were valid for all of the standards.

It can be seen that there is no appreciable change in resistivity up to a field strength of 2 kv/cm, and that it has increased by less than 10% at 8 kv/cm. During the actual measurements on germanium the field applied to the silicon standard was kept below 2 kv/cm whenever possible. If greater field strengths were required, a first order correction to the resistance of the standard was applied. In no case were measurements made with a field across the standard greater than 8 kv/cm.

A further check involving the consistency of the standards was made by measuring the ratio of their resistances on the pulse equipment, and ascertaining that these ratios were equal to the d.c. values.

2.5 EXPERIMENTAL CONSIDERATIONS

In evaluating the experimental results, it was felt that it was necessary to examine the following effects, to ensure that they did not influence the observed quantities.

2.5(a) SAMPLE HEATING

The thermal time constant of the samples under the conditions of the experiment was about 1 sec. This was determined by using an oscilloscope to observe the decay of the sample resistance when a current, sufficient to raise the sample temperature a few degrees, was switched off.

An estimate of the time constant appropriate to the decay of a temperature distribution inside the sample was obtained in the following way. For the long thin samples used, this internal time constant would be a maximum if heat were lost through the gold wire leads only. This was also a reasonable approximation, since the fluid surrounding the sample was a rather poor heat conductor. Under this assumption of no radial heat losses, the resulting temperature distribution of a sample initially at temperature T_0 , with ends maintained at an ambient temperature T_a , would not decay away exponentially (Williamson and Adams, 1919). However, the temperature at the centre of the sample would be closely approximated by an exponential function of the form.

$$T = T_a + (T_0 - T_a) \exp\left(\frac{-\delta \alpha t}{s^2}\right) \quad (2.9)$$

where;

s = the length of the sample
 α = the thermal diffusivity
 $= 2 \text{ cm}^2/\text{sec}$ for germanium

Most of the experimental measurements were made on samples of length 1 cm or less. Using this value, this internal thermal "time constant" would be 0.25 sec or less. Since this is nearly an exponential process, it was concluded that the internal temperature distribution would be essentially uniform while the sample was losing heat to the surroundings.

Since the applied pulse was very much shorter than both this internal time constant and the overall time constant, it was concluded that the temperature distribution within the sample would be uniform at all times.

Once the sample could be characterized by a single temperature T , and a single thermal relaxation time η , it was possible to calculate the temperature rise due to pulse heating. If a pulse of power W watts were applied to such a sample of heat capacity $H \text{ cal/}^\circ\text{C}$ for γ sec, with a pulse separation of δ sec, then assuming Newtonian cooling, in

steady state the accumulated temperature rise at the start of a pulse would be

$$T_i = T_a + \frac{\gamma S}{e^{\delta/\eta} - 1} \quad \begin{matrix} S = \frac{W}{JH} \\ \frac{\delta}{\eta} \ll 1 \end{matrix} \quad (2.10)$$

and the final temperature at the end of a pulse would be

$$T_f = T_i + \gamma S' = T_a + \gamma S' \frac{e^{\delta/\eta}}{e^{\delta/\eta} - 1} \quad (2.11)$$

During the experimental measurements it was always possible to keep the temperature at the end of a pulse within 2°C of the ambient temperature. It was also determined that no change in results could be observed when the pulse repetition rate was varied from one-shot pulsing up to a rate of 200 sec⁻¹. This is indicative that pulse heating was not an important factor.

2.5(b) CARRIER INJECTION

In order to suppress minority carrier injection as much as possible, alloy contacts of the L - H junction type were used. Sensitive d.c. tests were made, and any samples which showed a dependence of resistance upon direction of current flow were rejected.

However, any significant injection which did occur could be observed on the oscilloscope display. As the pulse length used was of the same order as the transit time of the carriers through the samples, any injection would show up as a sloping top on the current pulse. This was in fact observed with rejected samples which displayed non-ohmic d.c. behavior. No such effect was observed on any of the samples used for measurements, even though a 1% ramp could have been easily detected. To eliminate the possibility of accumulation of injected carriers, the pulse repetition rate was many times longer than the recombination time. As a final check on this point, measurements were carried out with one-shot pulsing.

Unfortunately, the contacts broke down irreversibly when field strengths of the order of 10 kv/cm were applied

to the rectangular bar samples. This necessitated the use of dumb-bell type samples for higher field measurements. In these cases, the contacts were not exposed to high fields, and any carriers which might have been injected could not have reached the main part of the samples during a pulse. In any event, no sloping tops were observed on the current pulses for any of the dumb-bell samples (not even those which were rejected on account of non-ohmic d.c. behavior).

2.5(c) SKIN EFFECT

A theory of the skin effect in semiconductors has been derived by Frei and Strutt (1960). They have shown that for a semiconductor of conductivity σ , dielectric constant ϵ , and magnetic permeability μ , the skin depth at angular frequency ω is given by

$$d = d_0 \left\{ (1 + \gamma^2)^{1/2} - \gamma \right\}^{-1/2} \quad (2.12)$$

where

$$d_0 = \left\{ \frac{10^9}{2\pi\sigma\omega\mu} \right\}^{1/2}$$

$$\gamma = \frac{1}{36\pi \cdot 10^{11}} \frac{\omega\epsilon}{\sigma}$$

For the germanium samples used

$$\sigma = 0.2 \text{ } \Omega^{-1} \text{ cm}^{-1}$$

$$\epsilon = 16$$

$$\mu = 1$$

Using a frequency $\omega = 2\pi \times 10^8$ rad/sec, corresponding to the fundamental of the shortest pulse employed (5 nsec),

$$\gamma = .004$$

$$d \approx d_0 = 1.1 \text{ cm}$$

Since the cross sections of the samples were of the order of 0.5 mm x 0.5 mm, this effect was of no importance.

2.5(d) VARIATION OF CARRIER CONCENTRATION WITH APPLIED ELECTRIC FIELD

The fact that the current pulse was observed to be flat would seem to rule out the possibility of the carrier concentration changing during a pulse. However, at the high values of electric field applied in this experiment, several effects could conceivably alter the carrier concentration during the rise time of the pulse. Upon close examination the magnitude of these effects is seen to be very small.

Stark Effect

At fields of the order of 10^5 v/cm, the donor states (characterized by a Bohr radius of some 45 Å) will almost certainly be ionized. However, the impurity concentration was chosen so that essentially all of the donor states would be ionized at a lattice temperature of 77°K. Thus this high-field ionization can do no more than increase the free carrier concentration by about 0.2%.

Franz-Keldysh Effect

Keldysh (1960) has calculated that the change in the band gap due to a strong applied field is

$$\Delta E = \frac{1}{2} \left\{ (eE)^2 \frac{\hbar^2}{m^*} \right\}^{1/3} \quad (2.13)$$

At the highest fields reached in this experiment this effect will decrease the band gap by about 0.02 ev. For the impurity concentration and temperature range under consideration

$$p = \frac{n_i^2}{N_D} \propto \exp \left(- \frac{E_{gap}}{kT} \right) \quad (2.14)$$

This change in the band gap would be most significant for the room temperature case when it could increase the hole concentration by a factor of 1.5. However, since the equilibrium hole concentration was only about 1% of the electron concentration, this is not important.

Zener Effect

The probability of an electron tunneling from the valence band across the forbidden energy gap to the conduction

band can become quite large at high fields. The probability is a very rapidly varying exponential function of field. Because of this it is very difficult to estimate the exact value of the field at which it would become appreciable. However, simple calculations indicate that the increase in carrier concentration would reach a value of about $4 \times 10^{12} \text{ cm}^{-3}$ at a field of $4 \times 10^5 \text{ v/cm}$, and would be completely negligible at the fields attained in this experiment.

Impact Ionization

Again, it is difficult to calculate at exactly what field strength the charge carriers would gain enough energy between collisions to knock a bound electron free.

However, the onset of both Zener tunneling and impact ionization would be so sudden, and the incremental resistance so low, that there would have been no difficulty observing them had one or the other occurred. Since no sudden or discontinuous increases in current were observed which could be interpreted as being due to either of these phenomena, it is not thought that they were of any importance over the range in which measurements were made.

Effect of Traps on the Carrier Concentration

If there were trapping levels in the forbidden gap at an energy near that of the antimony donor states, these could conceivably alter the carrier concentration while the voltage pulse was being applied. This could occur if the probability of a carrier being trapped by such a level were strongly dependent on carrier velocity. However, these levels would have to be present with a concentration comparable to 10^{14} cm^{-3} . No such levels are known to exist in such numbers.

CHAPTER 3 - EXPERIMENTAL RESULTS

3.1 CONTACT BREAKDOWN AT 10 KV/CM

As mentioned above it was not possible to make all of the measurements on one type of sample. The rectangular bars which were originally prepared gave reproducible measurements up to fields of the order of 10 kv/cm. However, when any of these samples was pulsed with fields of that order, either at room temperature or 77°K, an irreversible change took place. The top of the previously flat, steady, pulse began to slope upwards. This slope increased with increasing field and was unstable, often continuing to rise even after the field had stopped rising. When the field was increased indefinitely, the sample began to vibrate with the applied pulse and eventually exploded. The fragments showed signs of local melting down the centre of the sample. However, when the field was reduced after the breakdown was first observed, the sloping top remained and was clearly visible at fields as low as 1 kv/cm. Subsequent testing showed that the low current d.c. resistance had decreased and was no longer independent of the direction of current flow. In view of the fact that Gunn reported a surface breakdown effect at similar values of field strength, the sample was carefully examined under a microscope. No evidence of any surface tracking was found. The anomalous behavior persisted even after the samples were re-etched. It was therefore concluded that the gold alloy contact employed suffered some sort of irreversible breakdown at electric field strengths of the order of 10 kv/cm. The interpretation was strengthened when the dumb-bell specimens were prepared. Although they were pulsed at fields almost an order of magnitude greater, no evidence of any such effect was observed.

FIGURE 10

A sketch of several consecutive current pulses, typical of the breakdown which occurred in the (100) direction at 77°K. Note the characteristic delay time before the onset of the breakdown, and the envelope which contained all of the pulses.

3.2 BREAKDOWN IN (1,0,0) AT 77°K

A phenomenon was observed when measurements were made on the (1,0,0) samples at an ambient temperature of 77°K. As the field strength was increased a disturbance appeared at the falling edge of the current pulse. Further increases in field caused the disturbance to occur at earlier times during the pulse. A typical waveform is given in Fig. 10.

The disturbance could best be described as an apparently random, but monotonic, increase of the previously constant current. The interval of time after the onset of the pulse before the disturbance started depended on the value of the field strength, decreasing with increasing field. This delay was reproducible for all (1,0,0) samples. However, the actual current vs. time characteristic varied from pulse to pulse. No correlation could be observed between pulses for any pulse repetition rate, excepting for the fact that all the pulses were confined within an envelope. The phenomenon had no irreversible effect on the samples, and did not damage them in any way.

This effect was observed with all (1,0,0) samples tested, and with none of the samples cut in the other directions. However, the field at which it set in and the way in which the delay changed with field strength varied from sample to sample. They could not be correlated with the length, cross-section, or surface treatment of the samples. Typically the onset was around 4 kv/cm for a 50 nsec pulse, and 10 kv/cm for a 5 nsec one.

3.3 FINAL RESULTS

The results for different samples did not always agree exactly. However, the variation in measured drift velocity at a given field was usually less than 5% and never more than 10%. These discrepancies were attributed to variation in impurity concentration across the slice of the crystal from which the samples were cut.

FIGURE 11

Drift velocity vs. electric field for $T_L = 295^\circ\text{K}$.

- — — — — theoretical curves with neglect of energy relaxation due to acoustic modes.
- - - - - theoretical curve for (100) field direction with inclusion of this process.

FIGURE 12

Drift velocity vs. electric field for $T_L = 77^\circ\text{K}$.

- — — theoretical curves with neglect of energy relaxation due to acoustic modes.
- - - - - theoretical curve for (100) field direction with inclusion of this process.

The results obtained from the dumb-bell samples agreed with those obtained from the rectangular bars in the 2.5 - 10 kv/cm region where the measurements overlapped.

The curves given in Figs. 11 and 12 represent the smoothed data for 12 samples. The (1,0,0) curve for 77°K is terminated at 10 kv/cm because the breakdown mentioned in Section 3.2 above was distorting the shortest pulses attainable.

3.4 DISCUSSION

The following points are considered worth mentioning.

(a) There was no observable longitudinal anisotropy at room temperature. The results obtained from the different crystallographic directions agreed within the experimental scatter.

(b) Once the current in a sample "saturated" no further increases were observed for higher values of field strength.

This is in disagreement with the results of Gunn. At 9 kv/cm, a field greater than that required to saturate the sample, he found a region where $v_{\text{drift}} \propto F^{0.134}$ which ultimately ended in a breakdown at 63 kv/cm. The value of 9 kv/cm corresponds closely with the field strength at which Gunn had trouble with surface tracking, and to the field strength at which we found contact breakdown. It is therefore felt that the current increase, beyond the saturation region, as measured by Gunn, could be attributed to a spurious effect: either surface tracking or contact breakdown.

(c) The two most serious problems associated with this type of experiment are sample heating and carrier injection. It is felt that in the present work sample heating effects (discussed in Chapter II, Section 4) were avoided. Even though the measurements were made at a rate of 10 pulses per second,

they were independent of pulse repetition rate over a range from one-shot pulsing to over 200 pulses per second. Since the pulses were very much shorter than any of the thermal time constants involved, there would be no non-uniformity in the temperature of the sample during a single pulse.

The current wave forms produced by injecting contacts could be, and in fact were, observed on the oscilloscope. Since no such effects were observed with any of the samples used to obtain the final results, even though the pulse was flat to better than 1% over most of its range, it was felt that injected carriers were of no significance.

(d) Low field d.c. measurements showed no evidence of anisotropy, even at 77°K. It was therefore concluded that the anisotropy observed was induced by the applied electric field, and not by oriented dislocations (Read, 1954).

CHAPTER 4 - ANALYSIS OF RESULTS

4.1 ANALYSIS

Reik and Risken (1961) have published a detailed analysis of the hot electron problem in many-valley semi-conductors. They have also published (1962) the results of the application of their general theory to the case of n-type germanium. We have chosen to analyse our data on the basis of a simpler theory; we have extended the work of Stratton (1958) to the many-valley case of n-type germanium and have used this in the analysis of our results. Stratton assumed simple spherical constant energy surfaces in the wave-number space of the electrons and assumed also that the equilibrium distribution function in the presence of the applied field had the form of a displaced Maxwellian distribution. In the extension of Stratton's theory it is assumed that internal scattering is sufficient to establish a displaced Maxwellian distribution in each valley with different valleys having in general different electron temperatures, but is insufficient to effect the energy relaxation. We can then treat the individual valleys separately and sum up their individual contributions to the total current density. When the corresponding approximation is made in the general theory of Reik and Risken, the two theories give similar results.

For a single valley with the electron energy ϵ given as a function of the electron wave-number vector $\underline{k}$ by the relation

$$\epsilon(\underline{k}) = (\hbar^2/2m_0) \left\{ \left(\frac{m_0}{m_t} \right) k_x^2 + \left(\frac{m_0}{m_t} \right) k_y^2 + \left(\frac{m_0}{m_l} \right) k_z^2 \right\} \quad (4.1)$$

and with a displaced Maxwellian distribution function

$$f(\underline{k}) \propto \exp \left\{ -\epsilon(\underline{k} - \underline{k}_0) / k T_e \right\} \quad (4.2)$$

the four parameters $\underline{k}_0$, T_e are to be determined from the four equations

$$\int d\underline{k} \epsilon(\underline{k}) \left\{ \left(\frac{\partial f}{\partial t} \right)_F + \left(\frac{\partial f}{\partial t} \right)_a + \left(\frac{\partial f}{\partial t} \right)_o \right\} = 0 \quad (4.3)$$

$$\int d\underline{k} \underline{k} \left\{ \left(\frac{\partial f}{\partial t} \right)_F + \left(\frac{\partial f}{\partial t} \right)_a + \left(\frac{\partial f}{\partial t} \right)_o \right\} = 0 \quad (4.4)$$

In these, the subscripts F, a and o denote that the changes in the distribution function are produced respectively by the applied field, acoustic and optical mode scattering. The equations (4.3) and (4.4) basically state the conservation of energy and momentum respectively.

We make the following change of variables:

$$\underline{a}^* = \left\{ \left(\frac{m_o}{m_e} \right)^{1/2} a_x, \left(\frac{m_o}{m_e} \right)^{1/2} a_y, \left(\frac{m_o}{m_e} \right)^{1/2} a_z \right\} \text{ with } \underline{a} \text{ equal}$$

in turn to $\underline{k}$, $\underline{k}_0$, $\underline{q}$ and $\underline{F}$; $\underline{V}^{**} = (\hbar/m_o) \underline{k}_0^*$; $\underline{q}$ is here a phonon wave-number vector.

The transition probability for both acoustic and optical mode scattering is, for absorption

$$\rho = \frac{2\pi}{\hbar} B(\underline{q}) N_{\underline{q}} \delta \{ \epsilon_f - \epsilon_i + \hbar \omega(\underline{q}) \}$$

with a corresponding term for emission.

For acoustic modes:

$$B(\underline{q}) = \frac{2C^2 \hbar q}{9 M \mu V' N}$$

$$\text{with } \omega(\underline{q}) = u q$$

for optical modes

$$B(\underline{q}) = B_o$$

$$\hbar \omega(\underline{q}) = \hbar \theta_o$$

In these, M is the atomic mass, V the crystal volume, N the number of atoms per unit volume, C an interaction constant, u is an averaged velocity of sound in the crystal (taken to be 5.4×10^5 cm/sec) and θ_o is the temperature associated with the optical modes (taken to be 390°K). We have therefore ignored in the calculation the anisotropy in q-space, taking this to be considerably less significant than

the anisotropy in $\mathbf{k}$ -space. To be consistent, the scattering probabilities discussed by Herring and Vogt (1956) should have been used; this has been done in the theory of Reik and Risken. For the acoustic modes it was assumed that $\hbar\omega \ll kT_L$

The equations (4.3) and (4.4) then reduce to (see

Appendix I)

$$\underline{F}^* \underline{v}^{**} = \frac{32}{9\pi} \left(\frac{m_e^2 m_l}{m_0^3} \right)^{1/2} \frac{3}{4e} (2\pi m_0 kT_L)^{1/2} \quad (4.5)$$

$$\left\{ \frac{4m_0^2 kT_L}{3\pi \hbar^4} \frac{C^2}{MN} \left(\frac{\gamma_0}{\gamma} \right)^{3/2} \left(1 - \frac{\gamma}{\gamma_0} \right) \frac{2m_e + m_l}{3m_0} \right. \\ \left. + \frac{3m_0 k\theta_0}{4\pi \hbar^4} B_0 V' \left(\frac{\gamma_0}{\gamma} \right)^{1/2} \{ (n_0 + 1) e^{-\gamma} - n_0 \} \frac{\gamma}{2} e^{\gamma/2} K_1 \left(\frac{\gamma}{2} \right) \right\} \\ \underline{F}^* = \underline{v}^{**} \frac{32}{9\pi} \left(\frac{m_e^2 m_l}{m_0^3} \right)^{1/2} \frac{3}{4e} (2\pi m_0 kT_L)^{1/2} \left(\frac{\gamma_0}{\gamma} \right)^{1/2} \quad (4.6) \\ \left[\frac{4m_0^2 kT_L}{9\pi \hbar^4 \mu^2} \frac{C^2}{MN} + \frac{m_0^2}{2\pi \hbar^4} B_0 V' \{ ([n_0 + 1] e^{-\gamma} + n_0) \frac{\gamma^2}{4} e^{\gamma/2} K_2 \left(\frac{\gamma}{2} \right) \right. \\ \left. + ([n_0 + 1] e^{-\gamma} - n_0) \frac{\gamma^2}{4} e^{\gamma/2} K_1 \left(\frac{\gamma}{2} \right) \} \right]$$

where

$$n_0 = \{ e^{\theta_0/T_L} - 1 \}^{-1} \quad \gamma = \theta_0/T_e \quad \gamma_0 = \theta_0/T_L$$

$$K_n(z) = \frac{\Gamma(\frac{1}{2}) (\frac{1}{2}z)^n}{\Gamma(n + \frac{1}{2})} \int_0^\infty e^{-zt} (t^2 - 1)^{n-1/2} dt$$

The procedure of analysis is then the following (assuming for the moment that C^2 and $B_0 V^1$ are known). Values of T_e are inserted in the equations (4.5) and (4.6) and the equations solved for $\underline{v}^{**}$ and F^* . Graphs are then drawn of $\underline{v}^{**}$ vs. F^* and T_e vs. F^* . For a given $\underline{F}$, F^* is found for each valley and the corresponding values of $\underline{v}^{**}$ and T_e for that valley are read off from the graphs. The contributions to the electron current density from the individual valleys are then summed. The total drift velocity is written as $\underline{v}$. The valley with its major axis in the (111) direction is referred to as the (111) valley; similarly for the others. For the three directions of $\underline{F}$ under discussion, the following is

obtained (see Appendix II).

$$(i) \quad \underline{F} = (F, 0, 0); \quad F^* = F \left\{ \frac{2m_0}{3m_t} + \frac{m_0}{3m_e} \right\}^{1/2}$$

for all valleys; all valleys have the same T_e ;

$$\underline{V} = (V, 0, 0) \quad \text{with} \quad V = v^{**} \left\{ \frac{2m_0}{3m_t} + \frac{m_0}{3m_e} \right\}^{1/2}$$

$$(ii) \quad \underline{F} = \left(\frac{F}{\sqrt{2}}, \frac{F}{\sqrt{2}}, 0 \right); \quad F_1^* = F \left(\frac{m_0}{m_t} \right)^{1/2}$$

for the $(\bar{1}\bar{1}1)$ and $(1\bar{1}\bar{1})$ valleys (the "hot" valleys);

$$F_2^* = F \left\{ (m_0/3m_t) + (2m_0/3m_e) \right\}^{1/2}$$

for the $(11\bar{1})$ and $(\bar{1}\bar{1}\bar{1})$ valleys (the "cold" valleys);

$$\underline{V} = \left(\frac{V}{\sqrt{2}}, \frac{V}{\sqrt{2}}, 0 \right) \quad \text{with} \quad V = \frac{n_1 V_1 + n_2 V_2}{n_1 + n_2};$$

$$V_1 = v_1^{**} \left(\frac{m_0}{m_t} \right)^{1/2} \quad V_2 = v_2^{**} \left\{ \frac{m_0}{3m_t} + \frac{2m_0}{3m_e} \right\}^{1/2};$$

n_1 and n_2 are the concentrations of electrons in a "hot" and "cold" valley respectively.

$$(iii) \quad \underline{F} = \left(\frac{F}{\sqrt{3}}, \frac{F}{\sqrt{3}}, \frac{F}{\sqrt{3}} \right) \quad F_2^* = F \left(\frac{m_0}{m_e} \right)^{1/2}$$

for the (111) valley (the "cold" valley);

$$F_1^* = F \left\{ (8m_0/9m_t) + (m_0/9m_e) \right\}^{1/2}$$

for the $(\bar{1}\bar{1}1)$, $(1\bar{1}\bar{1})$, and $(\bar{1}\bar{1}\bar{1})$ valleys (the "hot" valleys);

$$\underline{V} = \left(\frac{V}{\sqrt{3}}, \frac{V}{\sqrt{3}}, \frac{V}{\sqrt{3}} \right) \quad \text{with} \quad V = \frac{n_2 V_2 + 3n_1 V_1}{n_2 + 3n_1}$$

$$V_2 = v_2^{**} \left(\frac{m_0}{m_e} \right)^{1/2} \quad \text{and} \quad V_1 = v_1^{**} \left\{ (8m_0/9m_t) + (m_0/9m_e) \right\}^{1/2}$$

n_1 and n_2 are the concentrations of electrons in a "hot" and "cold" valley respectively.

For cases (ii) and (iii) the electron concentrations n_1 and n_2 were found using the relation $n_i \langle \frac{1}{\tau_{iv}} \rangle = \text{constant}$, i labelling the valleys, and τ_{iv} being the intervalley scattering rate introduced by Herring (1955) (cf. Paige, 1960) (see Appendix III).

Before the above procedure can be applied C^2 and $B_0 V^1$ need to be known. These could be determined from a detailed

analysis of the low-field mobility data where Ohm's Law is obeyed. This would not however be very accurate. Instead the results for the (100) specimens were used in finding values to ascribe to these two parameters. This was done in the following manner.

If the first term on the right hand side of equation (4.5), i.e. the contribution of the acoustic modes to the energy relaxation, is neglected the equations predict a saturation drift velocity given by

$$v_{sat}^{**2} = \frac{3kT_0}{4m_0} \left\{ (2n_0 + 1) + \frac{4kT_L}{9mN\mu_2} \frac{C^2}{B_0 V^1} \right\}^{-1}$$

From this a saturation drift velocity can be found for a (100) field direction:

$$v_{sat} = v_{sat}^{**} \left\{ \frac{2m_0}{3m_L} + \frac{m_0}{3m_L} \right\}^{1/2}$$

The experimental value of the saturation drift velocity for this direction of field then gives a value for $(C^2/B_0 V^1)$.

Still neglecting the first term on the right hand side of (4.5), values of C^2 and $B_0 V^1$ individually were found by best fitting the onset of saturation.

This procedure was carried out at both lattice temperatures. At $T_L = 77^\circ K$, $C^2 = 280 \text{ (ev)}^2$ and $B_0 V^1 = 3.8 \times 10^{-72} \text{ (ev)}^{-2} \text{ cm}^3$; at $T_L = 295^\circ K$, $C^2 = 360 \text{ (ev)}^2$ and $B_0 V^1 = 3.1 \times 10^{-72} \text{ (ev)}^{-2} \text{ cm}^3$. Using these values in a rough calculation of the low-field mobility gives a value of $2400 \text{ cm}^2 \text{ v}^{-1} \text{ sec}^{-1}$ compared with an experimental value of $3900 \text{ cm}^2 \text{ v}^{-1} \text{ sec}^{-1}$.

The theoretical curves were then found for the other two directions of field, still neglecting this term in (4.5). The results are shown in Fig. 11 and Fig. 12.

With these values of C^2 and $B_0 V^1$ the first term on the right hand side of (4.5) was then included and the results obtained for the (100) direction of field are shown in Fig. 11 and Fig. 12; the inclusion of this term modifies the curves

for the other field directions in a similar manner; for the sake of clarity these are not included. As can be seen from the graphs this evaluation of C^2 and $B_0 V^1$ would be reliable only if theory were incorrect in that it made the first term on the right hand side of (4.5) significant at too low a field. However the values quoted for these parameters give reasonable agreement below the onset of saturation and do not disagree violently with low-field mobility data. These values were therefore retained.

Figures 13 and 14 show T_e as a function of applied field for the different valleys and directions of field. Figure 15 shows the intervalley transition probability as a function of electron temperature. Figures 16 and 17 show n_1/n_2 as a function of applied field for the different field directions.

4.2 DISCUSSION

The above theory cannot be expected to hold when the electron temperature in any valley is greater than 1500°K . The relationship between electron energy and wave-number is no longer that given by equation (4.1); the effective masses in the valleys under consideration will have changed and higher valleys in the conduction band will have started to be populated. Our discussion should therefore be limited to fields up to that giving the onset of saturation.

There are two important points of disagreement between theory and experiment in this range. First, the theoretical anisotropy is too large at low fields at 77°K . The theory is not expected to be entirely satisfactory here. The electron temperatures are less than the energy of the optical modes and the assumption of a displaced Maxwellian distribution is not likely to be satisfactory (cf. Reik and Risken). Secondly, theory predicts an anisotropy at high fields at 295°K . The theory would have to be modified in such a way that this anisotropy would be removed without the corresponding

anisotropy at 77°K being removed. This could possibly be accomplished by the inclusion of the effect of intervalley scattering on the energy relaxation.

There is agreement in that the theory does predict, at low fields, an anisotropy (although too large) at 77°K while predicting no anisotropy at 295°K. This is reflected in the graph of (n_1/n_2) vs. F and is due to the increasing importance, with increasing T_L of intervalley scattering due to the absorption of phonons. At high fields the electron mean energy is sufficiently high compared with the lattice temperature that the role played by the absorption of phonons, as compared with that played by emission, is less important and the anisotropy is then almost independent of lattice temperature.

For fields above 2 Kv/cm no satisfactory theory at present exists. Such a theory has to explain (a) the saturation of drift velocity and (b) the absence of anisotropy at 77°K at very high fields. Scattering to the higher valley with minimum at the centre of the Brillouin zone could accomplish both of these.

Electron Temperatures as functions of applied field for $T_L = 295^\circ\text{K}$.

(110)H, for instance, denotes the "hot" valley when the field is in the (110) direction.

FIGURE 14

Electron Temperatures as functions of applied field for $T_L = 77^\circ\text{K}$.
 (110)H, for instance, denotes the "hot" valley when the field is in the (110) direction.

FIGURE 15

Intervalley transition probability $\langle \frac{1}{\omega \tau} \rangle$ vs. electron temperature

FIGURE 16

(n_1/n_2) as a function of electric field for $T_L = 295^\circ\text{K}$.
The curves are labelled by the field direction.

FIGURE 17

(n_1/n_2) as a function of electric field for $T_L = 77^\circ\text{K}$.
The curves are labelled by the field direction.

APPENDIX I

EXTENSION OF STRATTON'S THEORY

Having assumed a displaced Maxwellian distribution function for electrons in a conduction band with ellipsoidal constant energy surfaces near the minima, we have

$$f(\underline{k}) = a \exp \left[-\frac{\hbar^2}{2m_0 k T_e} \left\{ \frac{m_0}{m_t} (k_x - k_{x0})^2 + \frac{m_0}{m_c} (k_y - k_{y0})^2 + \frac{m_0}{m_e} (k_z - k_{z0})^2 \right\} \right] \quad (\text{A1.1})$$

and

$$\epsilon(\underline{k}) = \frac{\hbar^2}{2m_0} \left(\frac{m_0}{m_t} k_x^2 + \frac{m_0}{m_c} k_y^2 + \frac{m_0}{m_e} k_z^2 \right) \quad (\text{A1.2})$$

where

$$a = \frac{n \hbar^3}{(2\pi m_0 k T_e)^{3/2}} \left(\frac{m_0^3}{m_t^2 m_e} \right)^{1/2}$$

so that

$$\int f(\underline{k}) \frac{d\underline{k}}{2\pi^3} = n. \quad (\text{A1.3})$$

The conservation laws

$$\int d\underline{k} \underline{k} \left\{ \left(\frac{\partial f}{\partial t} \right)_F + \left(\frac{\partial f}{\partial t} \right)_a + \left(\frac{\partial f}{\partial t} \right)_o \right\} = 0 \quad (\text{A1.4})$$

$$\int d\underline{k} \epsilon(\underline{k}) \left\{ \left(\frac{\partial f}{\partial t} \right)_F + \left(\frac{\partial f}{\partial t} \right)_a + \left(\frac{\partial f}{\partial t} \right)_o \right\} = 0 \quad (\text{A1.5})$$

which follow from the Boltzmann eqn. allow us to solve for the four parameters $\underline{k}_0$ and T_e which appear in the distribution function. Here

$$\left(\frac{\partial f}{\partial t} \right)_F, \left(\frac{\partial f}{\partial t} \right)_a, \left(\frac{\partial f}{\partial t} \right)_o$$

represent the respective contributions to the time rate of change of the distribution function due to the applied electric field, and due to scattering by the acoustic and optical modes of lattice vibration.

Field Dependent Terms

$$\int d\underline{k} \epsilon(\underline{k}) \left(\frac{\partial f}{\partial t} \right)_F = \int d\underline{k} \epsilon(\underline{k}) \left\{ -\frac{e}{\hbar} \underline{F} \cdot \underline{\nabla}_k f \right\} \quad (\text{A1.6})$$

introducing the transformation

$$\underline{a}^* = \left(\sqrt{\frac{m_0}{m_t}} a_x, \sqrt{\frac{m_0}{m_c}} a_y, \sqrt{\frac{m_0}{m_e}} a_z \right)$$

this becomes

$$\int d\underline{k} \epsilon(\underline{k}) \left(\frac{\partial f}{\partial t} \right)_F = -\frac{e}{\hbar} \left(\frac{m_t^2 m_e}{m_0^3} \right)^{1/2} \int d\underline{k}^* \epsilon(\underline{k}^*) \left\{ \underline{F}^* \cdot \underline{\nabla}_{\underline{k}^*} f(\underline{k}^*) \right\}$$

$$= \frac{\pi \hbar^3}{(2\pi m_0 k T_e)^{3/2}} \frac{e \hbar}{m_0} \underline{F}^* \cdot \int d\underline{k}^* \underline{k}^* \exp \left[-\frac{\hbar^2}{2m_0 k T_e} (\underline{k}^{*2} - 2\underline{k}^* \cdot \underline{k}_0 + k_0^2) \right]$$

If we assume the displacement of the distribution is small we may ignore the term k_0^2 . This is equivalent to assuming that the drift velocity of the electrons under the influence of the applied electric field can be neglected with respect to their mean thermal velocities. So

$$\int d\underline{k} \epsilon(\underline{k}) \left(\frac{\partial f}{\partial t} \right)_F = \frac{\pi \hbar^3}{(2\pi m_0 k T_e)^{3/2}} \frac{e \hbar}{m_0} \underline{F}^* \cdot \int d\underline{k}^* \underline{k}^* e^{-\frac{\hbar^2 \underline{k}^{*2}}{2m_0 k T_e}} \left\{ 1 + \frac{\hbar^2}{m_0 k T_e} \underline{k}^* \cdot \underline{k}_0 \right\}$$

and finally

$$\int \frac{d\underline{k}}{(2\pi)^3} \epsilon(\underline{k}) \left(\frac{\partial f}{\partial t} \right)_F = ne \left(\underline{F}^* \cdot \frac{\hbar \underline{k}_0}{m_0} \right) \quad (\text{A1.7})$$

In a similar manner the other integral can be evaluated to give

$$\int \frac{d\underline{k}}{(2\pi)^3} \underline{k} \left(\frac{\partial f}{\partial t} \right)_F = \frac{ne}{\hbar} \underline{F}^* \sqrt{\frac{m_t, m_e}{m_0}} \quad (\text{A1.8})$$

where the (m_t, m_1) notation implies that m_t is to be used with the x and y components and m_1 with the z component.

We can express both of these equations in terms of the drift velocity $\underline{v}$ of a single valley defined by

$$\underline{j} = ne \underline{v} \quad (\text{A1.9})$$

where

$$\underline{j} = -e \int \frac{d\underline{k}}{(2\pi)^3} \underline{v}(\underline{k}) f(\underline{k}) \quad (\text{A1.10})$$

and

$$\underline{v}(\underline{k}) = \frac{\hbar \underline{k}}{(m_t, m_e)} \quad (\text{A1.11})$$

Evaluating we find that

$$\underline{v} = \left(\frac{\hbar k_{0x}}{\sqrt{m_0 m_t}}, \frac{\hbar k_{0y}}{\sqrt{m_0 m_t}}, \frac{\hbar k_{0z}}{\sqrt{m_0 m_e}} \right)$$

Making the transformation

$$\underline{v}^{**} = \left(\sqrt{\frac{m_e}{m_0}} v_x, \sqrt{\frac{m_t}{m_0}} v_y, \sqrt{\frac{m_e}{m_0}} v_z \right)$$

this becomes

$$\underline{v}^{**} = \frac{\hbar \underline{k}_0^*}{m_0}$$

so that the field dependent terms may be written

$$\int \frac{d\underline{k}}{(2\pi)^3} \epsilon(\underline{k}) \left(\frac{\partial f}{\partial t} \right)_F = ne (\underline{F}^* \cdot \underline{v}^{**}) \quad (\text{A1.12})$$

$$\int \frac{d\underline{k}}{(2\pi)^3} \underline{k} \left(\frac{\partial f}{\partial t} \right)_F = \frac{ne}{\hbar} \sqrt{\frac{m_e m_e}{m_0}} \underline{F}^* \quad (\text{A1.13})$$

Acoustic Mode Terms

Under the assumption of classical statistics, the time rate of change of the distribution function for an electron of wave number $\underline{k}$ by scattering by an acoustic mode of lattice vibration is

$$\left(\frac{\partial f(\underline{k})}{\partial t} \right)_a = - \sum_{\underline{q}} \left\{ f(\underline{k}) \rho_a(\underline{k}, \underline{k}+\underline{q}) - f(\underline{k}+\underline{q}) \rho_e(\underline{k}+\underline{q}, \underline{k}) \right. \quad (\text{A1.14}) \\ \left. + f(\underline{k}) \rho_e(\underline{k}, \underline{k}-\underline{q}) - f(\underline{k}-\underline{q}) \rho_a(\underline{k}-\underline{q}, \underline{k}) \right\}$$

where $\underline{q}$ is the acoustic phonon wave number vector and

$$\rho_a(\underline{k}, \underline{k}+\underline{q}) = \frac{2\pi}{\hbar} B(\underline{q}) n_{\underline{q}} \delta \left\{ \epsilon(\underline{k}+\underline{q}) - \epsilon(\underline{k}) - \hbar \omega_{\underline{q}} \right\}$$

is the probability per unit time of an electron scattering from a state with wave number $\underline{k}$ and energy $\epsilon(\underline{k})$ to a state with wave number $(\underline{k} + \underline{q})$ and energy $\epsilon(\underline{k} + \underline{q})$ by absorbing a phonon of wave number $\underline{q}$ and energy $\hbar \omega_{\underline{q}}$. Similarly the probabilities for the other absorption and emission processes are

$$\rho_a(\underline{k}-\underline{q}, \underline{k}) = \frac{2\pi}{\hbar} B(\underline{q}) n_{\underline{q}} \delta \left\{ \epsilon(\underline{k}) - \epsilon(\underline{k}-\underline{q}) - \hbar \omega_{\underline{q}} \right\} \quad (\text{A1.15})$$

$$\rho_e(\underline{k}, \underline{k}-\underline{q}) = \frac{2\pi}{\hbar} B(\underline{q}) (n_{\underline{q}}+1) \delta \left\{ \epsilon(\underline{k}-\underline{q}) - \epsilon(\underline{k}) + \hbar \omega_{\underline{q}} \right\}$$

$$\rho_e(\underline{k}+\underline{q}, \underline{k}) = \frac{2\pi}{\hbar} B(\underline{q}) (n_{\underline{q}}+1) \delta \left\{ \epsilon(\underline{k}) - \epsilon(\underline{k}+\underline{q}) + \hbar \omega_{\underline{q}} \right\}$$

where $n_{\underline{q}}$ is the density of phonons with wave number $\underline{q}$ and

$$B(\underline{q}) n_{\underline{q}} = \frac{2}{9} \frac{c^2 \hbar q}{m u \sqrt{N}} \frac{1}{e^{\frac{\hbar \omega_{\underline{q}}}{kT}} - 1}$$

where:

C = an interaction constant

u = an averaged velocity of sound in the crystal

M = the atomic mass

V = the crystal volume

N = the number of atoms per unit volume

T_L = the crystal lattice temperature

On replacing the summation over all q values by an integration over q space we obtain,

$$\left(\frac{\partial f(\underline{k})}{\partial t}\right)_a = \frac{-V}{(2\pi)^2} \left(\frac{m_e^2 m_l}{m_0^3}\right)^{1/2} \frac{C^2}{9 m_l u V N} \frac{m_0}{\hbar^2 k^*} \int d\mathbf{q}^* g^* \int d(\cos\beta) \int d\psi$$

$$e^{\frac{q}{\hbar \omega_q / k T_L}} \left[\left\{ f(\underline{k}^*) - e^{\frac{\hbar \omega_q}{k T_L}} f(\underline{k}^* + \underline{q}^*) \delta \left\{ \cos\beta + \frac{q^*}{2k^*} - \frac{m_0 \omega_q}{\hbar k^* q^*} \right\} \right\} \right.$$

$$\left. + \left\{ f(\underline{k}^*) e^{\frac{\hbar \omega_q}{k T_L}} - f(\underline{k}^* - \underline{q}^*) \delta \left\{ \cos\beta - \frac{q^*}{2k^*} - \frac{m_0 \omega_q}{\hbar k^* q^*} \right\} \right\} \right]$$

where β is the angle between $\underline{k}^*$ and $\underline{q}^*$, and ψ is the angle between the $\underline{q}^* \underline{k}^*$ plane and the $z \underline{k}^*$ plane.

In the above $\omega_q = u q$. In transforming from $\underline{k}$ to $\underline{k}^*$, it was convenient for most purposes to carry out the same transformation on $\underline{q}$ to $\underline{q}^*$. This transformation has changed a isotropic dependence of ω on $\underline{q}$, to an anisotropic dependence of ω on $\underline{q}^*$. What we need now is ω as a function of q^* . The appropriate expression is

$$\omega = u q = u q^* \sqrt{\frac{m_l}{m_0} - \frac{(m_l - m_e)}{m_0} \{ \cos\gamma \cos\beta + \sin\gamma \sin\beta \cos\psi \}^2}$$

where γ is the angle between $\underline{k}^*$ and the z axis. The chief problem in extending Stratton's work to the case of an ellipsoidal constant electron energy surface is the complicated dependence of ω on q^* . This complicates the finding of the singularity of the δ function. We require the root of

$$\cos\beta + \frac{q^*}{2k^*} - \frac{m_0 u}{\hbar k^*} \sqrt{\frac{m_l}{m_0} - \frac{(m_l - m_e)}{m_0} \{ \cos\gamma \cos\beta + \sin\gamma \sin\beta \cos\psi \}^2}$$

This is $\cos \beta_n = -\frac{q^*}{2k^*} + \frac{m_0 u}{\hbar k^*} X_1 - \frac{m_0 u}{\hbar k^*} Y_1$

where:

$$X_1 = \sqrt{\frac{m_L}{m_0} - \frac{(m_L - m_E)}{m_0}} \left\{ -\frac{q^*}{2k^*} \cos \gamma + \sqrt{1 - \frac{q^{*2}}{4k^{*2}}} \sin \gamma \cos \psi \right\}^2$$

$$Y_1 = \left(\frac{m_L - m_E}{m_0} \right) \left(-\frac{q^*}{2k^*} \cos \gamma + \sqrt{1 - \frac{q^{*2}}{4k^{*2}}} \sin \gamma \cos \psi \right) \left(\cos \gamma + \frac{\sin \gamma \cos \psi}{\sqrt{1 - \frac{q^{*2}}{4k^{*2}}}} \frac{q^*}{2k^*} \right)$$

We also need the root of

$$\cos \beta - \frac{q^*}{2k^*} - \frac{m_0 u}{\hbar k^*} \sqrt{\frac{m_L}{m_0} - \frac{(m_L - m_E)}{m_0}} \left\{ \cos \gamma \cos \beta + \sin \gamma \sin \beta \cos \psi \right\}^2$$

which is

$$\cos \beta_n = \frac{q^*}{2k^*} + \frac{m_0 u}{\hbar k^*} X_2 - \frac{m_0 u}{\hbar k^*} Y_2$$

$$\text{where: } X_2 = \sqrt{\frac{m_L}{m_0} - \frac{(m_L - m_E)}{m_0}} \left\{ \frac{q^*}{2k^*} \cos \gamma + \sqrt{1 - \frac{q^{*2}}{4k^{*2}}} \sin \gamma \cos \psi \right\}^2$$

$$Y_2 = \left(\frac{m_L - m_E}{m_0} \right) \left(\frac{q^*}{2k^*} \cos \gamma + \sqrt{1 - \frac{q^{*2}}{4k^{*2}}} \sin \gamma \cos \psi \right) \left(\cos \gamma - \frac{\sin \gamma \cos \psi}{\sqrt{1 - \frac{q^{*2}}{4k^{*2}}}} \frac{q^*}{2k^*} \right)$$

Bearing this in mind the calculation follows the standard procedures (Stratton, and Wilson, 1959). We finally obtain

$$\left(\frac{\partial f(\underline{k}^*)}{\partial t} \right)_a = \left(\frac{\partial f(\underline{k}^*)}{\partial t} \right)_a' + \left(\frac{\partial f(\underline{k}^*)}{\partial t} \right)_a''$$

where

$$\left(\frac{\partial f(\underline{k}^*)}{\partial t} \right)_a'$$

is independent of $(\underline{q}^* \cdot \underline{k}_0^*)$ and contributes only to the energy integral, while

$$\left(\frac{\partial f(\underline{k}^*)}{\partial t} \right)_a''$$

contains a multiplicative factor of $(\underline{q}^* \cdot \underline{k}_0^*)$ and contributes only to the momentum integral.

$$\begin{aligned} \left(\frac{\partial f(\underline{k}^*)}{\partial t} \right)_a' &= -\frac{V}{2\pi} \left(\frac{m_L^2 m_E}{m_0^3} \right)^{1/2} \frac{C^2 k_{Te}}{M \sqrt{N}} \frac{m_0}{\hbar^2} \frac{f_0(\underline{k}^*)}{k^*} \left(1 + \frac{\hbar^2 k^* k_{Te}^* \cos \theta}{m_0 k_{Te}} \right) \left(\frac{1}{k_{Te}} - \frac{1}{k_L} \right) \\ &\times \left[\frac{\hbar}{k_{Te}} 4k^{*4} \left\{ \frac{m_L}{m_0} - \frac{(m_L - m_E)}{m_0} \left(\frac{1}{6} + \frac{1}{2} \cos^2 \gamma \right) \right\} + \frac{m_0}{\hbar} \frac{1}{2} (1 - 3 \cos^2 \gamma) \left(\frac{m_L - m_E}{m_0} \right) 4k^{*2} \right] \end{aligned}$$

$$\left(\frac{\partial f(\underline{k}^*)}{\partial t} \right)_a'' = -\frac{V}{2\pi} \left(\frac{m_L^2 m_E}{m_0^3} \right)^{1/2} \frac{C^2}{M \sqrt{N} \hbar u^2} \frac{T_L}{T_E} f_0(\underline{k}^*) k_0^* k^{*2} \cos \theta$$

Finally

(A1.16)

$$\begin{aligned}
 - \int \frac{d\mathbf{k}}{(2\pi)^3} \epsilon(\mathbf{k}) \left(\frac{\partial f(\mathbf{k})}{\partial t} \right)_a &= \frac{32}{9\pi} \frac{3ne u^2}{\mu_a} \left(\frac{\gamma_0}{\gamma} \right)^{3/2} \left(1 - \frac{\gamma}{\gamma_0} \right) \left(\frac{2m_e + m_e}{m_0} \right)^{1/3} \left(\frac{2m_0}{m_e} + \frac{m_0}{m_e} \right) \\
 - \int \frac{d\mathbf{k}}{(2\pi)^3} \mathbf{k} \left(\frac{\partial f(\mathbf{k})}{\partial t} \right)_a &= \frac{32}{9\pi} \left(\frac{\gamma_0}{\gamma} \right)^{1/2} \frac{\eta e}{m_0 \mu_a} \frac{1}{3} \left(\frac{2m_0}{m_e} + \frac{m_0}{m_e} \right) \left\{ \begin{aligned} &\sqrt{\frac{m_e}{m_0}} \mathbf{k}_{ox}^* \\ &\sqrt{\frac{m_e}{m_0}} \mathbf{k}_{oy}^* \\ &\sqrt{\frac{m_e}{m_0}} \mathbf{k}_{oz}^* \end{aligned} \right\}
 \end{aligned}$$

where: $\gamma = \theta_0 / T_e$ $\gamma_0 = \theta_0 / T_L$ $n_0 = (e^{\gamma_0} - 1)^{-1}$

Optical Mode Terms

(A1.17)

In a similar manner to the above we may write

$$\begin{aligned}
 \left(\frac{\partial f(\mathbf{k})}{\partial t} \right)_o &= - \sum_{\mathbf{q}} \left\{ f(\mathbf{k}) \rho_a(\mathbf{k}, \mathbf{k} + \mathbf{q}) - f(\mathbf{k} + \mathbf{q}) \rho_e(\mathbf{k} + \mathbf{q}, \mathbf{k}) \right. \\
 &\quad \left. + f(\mathbf{k}) \rho_e(\mathbf{k}, \mathbf{k} - \mathbf{q}) - f(\mathbf{k} - \mathbf{q}) \rho_a(\mathbf{k} - \mathbf{q}, \mathbf{k}) \right\}
 \end{aligned}$$

(A1.18)

where in this case

$$\begin{aligned}
 \rho_a(\mathbf{k}, \mathbf{k} + \mathbf{q}) &= \frac{2\pi}{\hbar} B_0 n_0 \delta(\epsilon(\mathbf{k} + \mathbf{q}) - \epsilon(\mathbf{k}) - \hbar\omega_0) \\
 \rho_a(\mathbf{k} - \mathbf{q}, \mathbf{k}) &= \frac{2\pi}{\hbar} B_0 n_0 \delta(\epsilon(\mathbf{k}) - \epsilon(\mathbf{k} - \mathbf{q}) - \hbar\omega_0) \\
 \rho_e(\mathbf{k}, \mathbf{k} - \mathbf{q}) &= \frac{2\pi}{\hbar} B_0 (n_0 + 1) \delta(\epsilon(\mathbf{k} - \mathbf{q}) - \epsilon(\mathbf{k}) + \hbar\omega_0) \\
 \rho_e(\mathbf{k} + \mathbf{q}, \mathbf{k}) &= \frac{2\pi}{\hbar} B_0 (n_0 + 1) \delta(\epsilon(\mathbf{k}) - \epsilon(\mathbf{k} + \mathbf{q}) + \hbar\omega_0)
 \end{aligned}$$

(A1.19)

where we are assuming a monoenergetic optical mode spectrum of density $n_0 = (e^{\hbar\omega_0 / T_L} - 1)^{-1}$ and energy $\hbar\omega_0$ independent of $\mathbf{q}$ and therefore of $\mathbf{q}^*$.

The calculation follows as before except that

$$B(\mathbf{q}) = B_0$$

and

$$\hbar\omega_{\mathbf{q}} = \hbar\omega_0$$

which makes the evaluation of the position of the singularities of the δ functions straightforward.

Again we may write

$$\left(\frac{\partial f}{\partial t} \right)_o = \left(\frac{\partial f_0}{\partial t} \right)_o + \left(\frac{\partial f_1}{\partial t} \right)_o \cos \theta$$

where as before only the

$$\left(\frac{\partial f_0}{\partial t} \right)_o$$

contributes to the energy integral, and

$$\left(\frac{\partial f_1}{\partial t}\right)_0 \cos \theta$$

contributes to the momentum integral.

$$\left(\frac{\partial f_0(\underline{k}^*)}{\partial t}\right)_0 = -\frac{\sqrt{m_e^2 m_e}}{\pi \hbar} \left(\frac{m_e^2 m_e}{m_0^3}\right)^{1/2} \frac{m_0}{\hbar^2} f_0(\underline{k}^*) B_0 \underline{k}^* \left\{ (n_0+1) \bar{e}^{-\gamma} - n_0 \right\} \\ \times \left[\sqrt{1 + \frac{\hbar \omega_0}{\epsilon}} - \sqrt{1 - \frac{\hbar \omega_0}{\epsilon}} H\left(1 - \frac{\hbar \omega_0}{\epsilon}\right) e^{\gamma} \right]$$

$$\left(\frac{\partial f_1(\underline{k}^*)}{\partial t}\right)_0 = -\frac{\sqrt{m_e^2 m_e}}{\pi \hbar} \left(\frac{m_e^2 m_e}{m_0^3}\right)^{1/2} m_0 f_0(\underline{k}^*) B_0 \underline{k}_0^* \underline{k}^* \\ \times \left[n_0 \sqrt{1 + \frac{\hbar \omega_0}{\epsilon}} + (n_0+1) \sqrt{1 - \frac{\hbar \omega_0}{\epsilon}} H\left(1 - \frac{\hbar \omega_0}{\epsilon}\right) \right]$$

Finally,

$$-\int \frac{d\underline{k}}{(2\pi)^3} \epsilon(\underline{k}) \left(\frac{\partial f}{\partial t}\right)_0 = \frac{32}{9\pi} \frac{3\pi e}{4m_0 \mu_0} \frac{\{(n_0+1) \bar{e}^{-\gamma} - n_0\}}{(2n_0+1)} K_1\left(\frac{\gamma}{2}\right) \left(\frac{\gamma_0}{\gamma}\right)^{1/2} \underline{k}_0^* \left(\frac{2m_0 + m_0}{3m_e + 3m_e}\right)^{1/2} \quad (A1.20)$$

$$\text{and} \quad -\int \frac{d\underline{k}}{(2\pi)^3} \underline{k} \left(\frac{\partial f}{\partial t}\right)_0 = \frac{32}{9\pi} \frac{\hbar}{m_0} \frac{\pi e}{2(2n_0+1)\mu_0} \left(\frac{\gamma_0}{\gamma}\right)^{1/2} \left\{ \{(n_0+1) \bar{e}^{-\gamma} + n_0\} \frac{\gamma^2}{4} e^{\gamma/2} K_2\left(\frac{\gamma}{2}\right) \right. \\ \left. + \{(n_0+1) \bar{e}^{-\gamma} - n_0\} \frac{\gamma}{2} K_1\left(\frac{\gamma}{2}\right) \left(\frac{2m_0 + m_0}{3m_e + 3m_e}\right) \sqrt{\frac{m_e + m_e}{m_0}} \underline{k}_0^* \right\} \quad (A1.21)$$

Substituting for the various terms in the conservation laws

we finally have

$$\underline{F}^* = \underline{v}^{**} = \frac{32}{9\pi} \left(\frac{m_e^2 m_e}{m_0^3}\right)^{1/2} \frac{3}{4e} (2\pi m_0 k T_L)^{1/2} \quad (A1.22) \\ \left\{ \frac{4 m_0^2 k T_L}{3\pi \hbar^4} \frac{c^2}{MN} \left(\frac{\gamma_0}{\gamma}\right)^{3/2} (1 - \frac{\gamma}{\gamma_0}) \left(\frac{2m_e + m_e}{3m_0}\right) \right. \\ \left. + \frac{3m_0 k T_0}{4\pi \hbar^4} B_0 \sqrt{\left(\frac{\gamma_0}{\gamma}\right)^{1/2}} \{(n_0+1) \bar{e}^{-\gamma} - n_0\} \frac{\gamma}{2} e^{\gamma/2} K_1\left(\frac{\gamma}{2}\right) \right\}$$

$$\underline{F}^* = \underline{v}^{**} = \frac{32}{9\pi} \left(\frac{m_e^2 m_e}{m_0^3}\right)^{1/2} \frac{3}{4e} (2\pi m_0 k T_L)^{1/2} \left(\frac{\gamma_0}{\gamma}\right)^{1/2} \quad (A1.23) \\ \left[\frac{4 m_0^2 k T_L}{9\pi \hbar^4 \mu^2} \frac{c^2}{MN} + \frac{m_0^2}{2\pi \hbar^4} B_0 \sqrt{\{(n_0+1) \bar{e}^{-\gamma} + n_0\} \frac{\gamma^2}{4} e^{\gamma/2} K_2\left(\frac{\gamma}{2}\right)} \right. \\ \left. + \{(n_0+1) \bar{e}^{-\gamma} - n_0\} \frac{\gamma^2}{4} e^{\gamma/2} K_1\left(\frac{\gamma}{2}\right) \right]$$

$$\text{where } K_n(\gamma) = \frac{\Gamma(\frac{1}{2}) \Gamma(\frac{1}{2} \gamma)}{\Gamma(n + \frac{1}{2})} \int_0^\infty e^{-\gamma t} (t^2 - 1)^{n-1/2} dt$$

APPENDIX II

COMBINING THE CURRENT CONTRIBUTIONS FROM DIFFERENT VALLEYS

In Appendix I, we worked in a representation in which the minimum energy occurred at $\underline{k} = 0$, and the k_z direction was that of the major axis of the ellipsoid of revolution. In germanium the conduction band structure has four valleys. In order to combine the effects of all four we shall have to transform, for each valley, to the axes (k_x, k_y, k_z) , the usual principal directions in the Brillouin zone of germanium (Fig. 18). For the aspect in which we are interested here, we can think of each of the four valleys as having its centre at the centre of the Brillouin zone.

Any vector $\underline{a}$ can be transformed from one co-ordinate system to the other by means of the usual transformation of the form

$$\begin{aligned} a_x &= \begin{pmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{pmatrix} \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} = T \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \end{aligned}$$

and similarly

$$\begin{aligned} a_x &= \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix} \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} = T^{-1} \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \end{aligned}$$

For the four conduction band valleys of germanium these matrices have the form

(111) valley $T = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \end{pmatrix}$

FIGURE 18

Orientation of one of the conduction band valleys of germanium with respect to the principal directions of the Brillouin zone.

$$(\bar{1}11) \text{ valley} \quad T = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{6} & -1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \end{pmatrix}$$

$$(\bar{1}\bar{1}1) \text{ valley} \quad T = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & -1/\sqrt{3} \\ 0 & 2/\sqrt{6} & 1/\sqrt{3} \end{pmatrix}$$

$$(\bar{1}\bar{1}\bar{1}) \text{ valley} \quad T = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{6} & -1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & -1/\sqrt{3} \\ 0 & 2/\sqrt{6} & 1/\sqrt{3} \end{pmatrix}$$

So, in particular, for the electric field F we have

$$\begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix} \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix}$$

and

$$\begin{pmatrix} F_x^* \\ F_y^* \\ F_z^* \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{m_0}{m_c}} & 0 & 0 \\ 0 & \sqrt{\frac{m_0}{m_c}} & 0 \\ 0 & 0 & \sqrt{\frac{m_0}{m_2}} \end{pmatrix} \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix}$$

If we evaluate each of these for each valley for each direction of the applied field we find the following.

Applied Field in the (100) Direction $\underline{F} = (F, 0, 0)$

(111) valley

$$F_x = \frac{1}{\sqrt{2}} F$$

$$F_y = \frac{1}{\sqrt{6}} F$$

$$F_z = \frac{1}{\sqrt{3}} F$$

($\bar{1}11$) valley

$$F_x = \frac{1}{\sqrt{2}} F$$

$$F_y = -\frac{1}{\sqrt{6}} F$$

$$F_z = -\frac{1}{\sqrt{3}} F$$

$$F_x^* = \sqrt{\frac{m_0}{m_t}} \frac{F}{\sqrt{2}}$$

$$F_y^* = \sqrt{\frac{m_0}{m_t}} \frac{F}{\sqrt{6}}$$

$$F_z^* = \sqrt{\frac{m_0}{m_t}} \frac{F}{\sqrt{3}}$$

$$F_x^* = \sqrt{\frac{m_0}{m_t}} \frac{F}{\sqrt{6}}$$

$$F_y^* = -\sqrt{\frac{m_0}{m_t}} \frac{F}{\sqrt{6}}$$

$$F_z^* = -\sqrt{\frac{m_0}{m_t}} \frac{F}{\sqrt{3}}$$

($\bar{1}\bar{1}1$) valley

$$F_x = \frac{1}{\sqrt{2}} F$$

$$F_y = -\frac{1}{\sqrt{6}} F$$

$$F_z = \frac{1}{\sqrt{3}} F$$

$$F_x^* = \sqrt{\frac{m_0}{m_t}} \frac{F}{\sqrt{2}}$$

$$F_y^* = -\sqrt{\frac{m_0}{m_t}} \frac{F}{\sqrt{6}}$$

$$F_z^* = \sqrt{\frac{m_0}{m_t}} \frac{F}{\sqrt{3}}$$

($\bar{1}1\bar{1}$) valley

$$F_x = \frac{1}{\sqrt{2}} F$$

$$F_y = \frac{1}{\sqrt{6}} F$$

$$F_z = -\frac{1}{\sqrt{3}} F$$

$$F_x^* = \sqrt{\frac{m_0}{m_t}} \frac{F}{\sqrt{2}}$$

$$F_y^* = \sqrt{\frac{m_0}{m_t}} \frac{F}{\sqrt{6}}$$

$$F_z^* = -\sqrt{\frac{m_0}{m_t}} \frac{F}{\sqrt{3}}$$

For all valleys

$$F^{*2} = F_x^{*2} + F_y^{*2} + F_z^{*2} = F^2 \left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l} \right)$$

therefore

$$F^* = F \left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l} \right)^{1/2}$$

Applied Field in the (110) Direction $\underline{F} = \left(\frac{F}{\sqrt{2}}, \frac{F}{\sqrt{2}}, 0 \right)$

(111) valley

$$F_x = 0$$

$$F_y = \frac{F}{\sqrt{3}}$$

$$F_z = \frac{2F}{\sqrt{6}}$$

$$F_x^* = 0$$

$$F_y^* = \sqrt{\frac{m_0}{m_t}} \frac{F}{\sqrt{3}}$$

$$F_z^* = \sqrt{\frac{m_0}{m_t}} \frac{2F}{\sqrt{6}}$$

(111) valley

$$F_x = F$$

$$F_y = 0$$

$$F_z = 0$$

$$F_x^* = \sqrt{\frac{m_0}{m_t}} F$$

$$F_y^* = 0$$

$$F_z^* = 0$$

(111) valley

$$F_x = F$$

$$F_y = 0$$

$$F_z = 0$$

$$F_x^* = \sqrt{\frac{m_0}{m_t}} F$$

$$F_y^* = 0$$

$$F_z^* = 0$$

(111) valley

$$F_x = 0$$

$$F_y = \frac{F}{\sqrt{3}}$$

$$F_z = -\frac{2F}{\sqrt{6}}$$

$$F_x^* = 0$$

$$F_y^* = \sqrt{\frac{m_0}{m_t}} \frac{F}{\sqrt{3}}$$

$$F_z^* = -\sqrt{\frac{m_0}{m_t}} \frac{2F}{\sqrt{6}}$$

For the (111) and (111) valleys

$$F^* = F \left(\frac{m_0}{3m_t} + \frac{2m_0}{3m_e} \right)^{1/2}$$

For the (111) and (111) valleys

$$F^* = F \sqrt{\frac{m_0}{m_t}}$$

Applied Field in the (111) Direction $\underline{F} = \left(\frac{F}{\sqrt{3}}, \frac{F}{\sqrt{3}}, \frac{F}{\sqrt{3}} \right)$ (111) valley

$$F_x = 0$$

$$F_y = 0$$

$$F_z = F$$

$$F_x^* = 0$$

$$F_y^* = 0$$

$$F_z^* = \sqrt{\frac{m_0}{m_t}} F$$

(111) valley

$$F_x = \sqrt{\frac{2}{3}} F$$

$$F_y = -\frac{\sqrt{2}}{3} F$$

$$F_z = \frac{1}{3} F$$

$$F_x^* = \sqrt{\frac{2m_0}{3m_t}} F$$

$$F_y^* = \sqrt{\frac{m_0}{m_t}} \frac{\sqrt{2} F}{3}$$

$$F_z^* = \sqrt{\frac{m_0}{m_t}} \frac{F}{3}$$

(111) valley

$$F_x = \sqrt{\frac{2}{3}} F$$

$$F_y = \frac{\sqrt{2}}{3} F$$

$$F_z = \frac{1}{3} F$$

$$F_x^* = \sqrt{\frac{2m_0}{3m_t}} F$$

$$F_y^* = \sqrt{\frac{m_0}{m_t}} \frac{\sqrt{2} F}{3}$$

$$F_z^* = \sqrt{\frac{m_0}{m_t}} \frac{F}{3}$$

(111) valley

$$F_x = 0$$

$$F_y = \frac{2\sqrt{2}}{3} F$$

$$F_z = -\frac{1}{3} F$$

$$F_x^* = 0$$

$$F_y^* = \sqrt{\frac{m_0}{m_e}} \frac{2\sqrt{2}}{3} F$$

$$F_z^* = -\sqrt{\frac{m_0}{m_e}} \frac{F}{3}$$

For the (111) valley

$$F^* = \sqrt{\frac{m_0}{m_e}} F$$

For the (111), (111), and (111) valleys

$$F^* = F \left(\frac{8}{9} \frac{m_0}{m_e} + \frac{1}{9} \frac{m_0}{m_e} \right)^{1/2}$$

Relationship Between $\underline{v}^{**}$ and the Drift Velocity $\underline{V}$

Since $\underline{F}^*$ and $\underline{v}^{**}$ are parallel we have

$$v_x^{**} = \frac{F_x^*}{F^*} v^{**}$$

$$v_y^{**} = \frac{F_y^*}{F^*} v^{**}$$

$$v_z^{**} = \frac{F_z^*}{F^*} v^{**}$$

and also

$$v = \sqrt{\frac{m_0}{m_e}} v^{**}$$

$$v = \sqrt{\frac{m_0}{m_e}} v^{**}$$

$$v = \sqrt{\frac{m_0}{m_e}} v^{**}$$

and finally

$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{pmatrix} \begin{pmatrix} v_x^{**} \\ v_y^{**} \\ v_z^{**} \end{pmatrix}$$

We can find the total drift velocity $\underline{V}$ by summing the contributions from each valley; each contribution being weighted by the population of the corresponding valley.

$$ne\underline{V} = \sum_{i=1}^4 n_i e \underline{v}_i$$

$$\underline{V} = \frac{\sum_i n_i \underline{v}_i}{\sum_i n_i}$$

(a) Field Applied in the (100) Direction

Since for this particular orientation of the field, F^* has the same value in each of the four valleys, v^{**} will also be the same in all of the valleys.

(111) valley

$$v_x^{**} = \frac{v^{**} \sqrt{\frac{m_0}{m_t}} \frac{1}{\sqrt{2}}}{\left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2}}$$

$$v_x = \frac{v^{**} \frac{m_0}{m_t} \frac{1}{\sqrt{2}}}{\left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2}}$$

$$v_y^{**} = \frac{v^{**} \sqrt{\frac{m_0}{m_t}} \frac{1}{\sqrt{6}}}{\left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2}}$$

$$v_y = \frac{v^{**} \frac{m_0}{m_t} \frac{1}{\sqrt{6}}}{\left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2}}$$

$$v_z^{**} = \frac{v^{**} \sqrt{\frac{m_0}{m_l}} \frac{1}{\sqrt{3}}}{\left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2}}$$

$$v_z = \frac{v^{**} \frac{m_0}{m_l} \frac{1}{\sqrt{3}}}{\left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2}}$$

$$v_x = \alpha_1 v_x + \beta_1 v_y + \gamma_1 v_z = \left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2} v^{**}$$

$$v_y = \alpha_2 v_x + \beta_2 v_y + \gamma_2 v_z = \left(-\frac{1}{3} \frac{m_0}{m_t} + \frac{1}{3} \frac{m_0}{m_l}\right) v^{**} \left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2}$$

$$v_z = \alpha_3 v_x + \beta_3 v_y + \gamma_3 v_z = \left(-\frac{1}{3} \frac{m_0}{m_t} + \frac{m_0}{3m_l}\right) v^{**} \left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2}$$

($\bar{1}11$) valley

$$v_x = \sqrt{\frac{m_0}{m_t}} v_x^{**} = \frac{\frac{m_0}{m_t} \frac{1}{\sqrt{2}}}{\left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2}} v^{**}$$

$$v_y = \sqrt{\frac{m_0}{m_t}} v_y^{**} = \frac{-\frac{m_0}{m_t} \frac{1}{\sqrt{6}}}{\left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2}} v^{**}$$

$$v_z = \sqrt{\frac{m_0}{m_t}} v_z^{**} = \frac{-\frac{m_0}{m_t} \frac{1}{\sqrt{3}} v^{**}}{\left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2}}$$

$$v_x = \left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2} v^{**}$$

$$v_y = \frac{\left(\frac{m_0}{3m_t} - \frac{m_0}{3m_l}\right) v^{**}}{\left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2}}$$

$$v_z = \frac{\left(\frac{m_0}{3m_t} - \frac{m_0}{3m_l}\right) v^{**}}{\left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2}}$$

(111) Valley

$$v_x = \sqrt{\frac{m_0}{m_t}} v_x^{**} = \frac{\frac{m_0}{m_t} \frac{1}{\sqrt{2}} v^{**}}{\left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2}}$$

$$v_y = \sqrt{\frac{m_0}{m_t}} v_y^{**} = \frac{-\frac{m_0}{m_t} \frac{1}{\sqrt{6}} v^{**}}{\left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2}}$$

$$v_z = \sqrt{\frac{m_0}{m_l}} v_z^{**} = \frac{\frac{m_0}{m_l} \frac{1}{\sqrt{3}} v^{**}}{\left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2}}$$

$$v_x = \left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2} v^{**}$$

$$v_y = \frac{\left(\frac{m_0}{3m_t} - \frac{m_0}{3m_l}\right) v^{**}}{\left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2}}$$

$$v_z = \frac{\left(-\frac{m_0}{3m_t} + \frac{m_0}{3m_l}\right)}{\left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2}} v^{**}$$

(111) Valley

$$v_x = \sqrt{\frac{m_0}{m_t}} v_x^{**} = \frac{\frac{m_0}{m_t} \frac{1}{\sqrt{2}}}{\left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2}} v^{**}$$

$$v_y = \sqrt{\frac{m_0}{m_t}} v_y^{**} = \frac{\frac{m_0}{m_t} \frac{1}{\sqrt{6}}}{\left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2}} v^{**}$$

$$v_z = \sqrt{\frac{m_0}{m_l}} v_z^{**} = \frac{-\frac{m_0}{m_l} \frac{1}{\sqrt{3}}}{\left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2}} v^{**}$$

$$v_x = \left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2} v^{**}$$

$$v_y = \frac{\left(-\frac{m_0}{3m_t} + \frac{m_0}{3m_l}\right)}{\left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)} v^{**}$$

$$v_z = \frac{\left(\frac{m_0}{3m_t} - \frac{m_0}{3m_l}\right)}{\left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)} v^{**}$$

Since for this orientation the populations of all the valleys will be equal, we have

$$V_x = \left(\frac{2m_0}{3m_t} + \frac{m_0}{3m_l}\right)^{1/2} v^{**}$$

$$V_y \equiv 0$$

$$V_z \equiv 0$$

(b) Field Applied in the (110) Direction

Since F^* has one value in one pair of valleys, and a different value in the other pair, the carrier drift velocities associated with these pairs of valleys will also be different.

$$F = \left(\frac{F}{\sqrt{2}}, \frac{F}{\sqrt{2}}, 0 \right)$$

(111) Valley

$$F^* = F \left(\frac{m_0}{3m_t} + \frac{2m_0}{3m_l} \right)^{1/2}$$

$$v_{2x} = \sqrt{\frac{m_0}{m_t}} v_{2x}^{**} = 0$$

$$v_{2y} = \sqrt{\frac{m_0}{m_t}} v_{2y}^{**} = \frac{\frac{m_0}{m_t} \frac{1}{\sqrt{3}} v_2^{**}}{\left(\frac{m_0}{3m_t} + \frac{2m_0}{3m_l} \right)^{1/2}}$$

$$v_{2z} = \sqrt{\frac{m_0}{m_l}} v_{2z}^{**} = \frac{\frac{m_0}{m_l} \frac{2}{\sqrt{6}} v_2^{**}}{\left(\frac{m_0}{3m_t} + \frac{2m_0}{3m_l} \right)^{1/2}}$$

$$v_{2x} = \frac{\frac{m_0}{\sqrt{18}m_t} + \frac{2m_0}{\sqrt{18}m_l}}{\left(\frac{m_0}{3m_t} + \frac{2m_0}{3m_l} \right)^{1/2}} v_2^{**}$$

$$v_{2y} = \frac{\frac{m_0}{\sqrt{18}m_t} + \frac{2m_0}{\sqrt{18}m_l}}{\left(\frac{m_0}{3m_t} + \frac{2m_0}{3m_l} \right)^{1/2}} v_2^{**}$$

$$v_{2z} = \frac{\frac{-2m_0}{\sqrt{18}m_t} + \frac{2m_0}{\sqrt{18}m_l}}{\left(\frac{m_0}{3m_t} + \frac{2m_0}{3m_l} \right)^{1/2}} v_2^{**}$$

($\bar{1}\bar{1}\bar{1}$) Valley

$$F^* = \sqrt{\frac{m_0}{m_t}} F$$

$$v_{1x} = \sqrt{\frac{m_0}{m_t}} v_{1x}^{**} = \sqrt{\frac{m_0}{m_t}} v_1^{**}$$

$$v_{2y} = 0$$

$$v_{2z} = 0$$

$$v_{1x} = \sqrt{\frac{m_0}{m_t}} \frac{v_1^{**}}{\sqrt{2}}$$

$$v_{1y} = \sqrt{\frac{m_0}{m_t}} \frac{v_1^{**}}{\sqrt{2}}$$

$$v_{1z} = 0$$

(111) Valley

$$F^* = \sqrt{\frac{m_0}{m_t}} F$$

$$v_{1x} = \sqrt{\frac{m_0}{m_t}} v_{1x}^{**} = \sqrt{\frac{m_0}{m_t}} v_1^{**}$$

$$v_{1y} = 0$$

$$v_{1z} = 0$$

$$v_{1x} = \sqrt{\frac{m_0}{m_t}} \frac{v_1^{**}}{\sqrt{2}}$$

$$v_{1y} = \sqrt{\frac{m_0}{m_t}} \frac{v_1^{**}}{\sqrt{2}}$$

$$v_{1z} = 0$$

(111) Valley

$$F^* = F \left(\frac{m_0}{3m_t} + \frac{2m_0}{3m_e} \right)^{1/2}$$

$$v_{2x} = 0$$

$$v_{2y} = \sqrt{\frac{m_0}{m_t}} v_{2y}^{**} = \frac{\frac{m_0}{m_t} \frac{1}{\sqrt{3}} v_2^{**}}{\left(\frac{m_0}{3m_t} + \frac{2m_0}{3m_e} \right)^{1/2}}$$

$$v_{1x} = \sqrt{\frac{m_0}{m_t}} v_{1x}^{**} = \sqrt{\frac{m_0}{m_t}} v_1^{**}$$

$$v_{2y} = 0$$

$$v_{2z} = 0$$

$$v_{1x} = \sqrt{\frac{m_0}{m_t}} \frac{v_1^{**}}{\sqrt{2}}$$

$$v_{1y} = \sqrt{\frac{m_0}{m_t}} \frac{v_1^{**}}{\sqrt{2}}$$

$$v_{1z} = 0$$

(111) Valley

$$F^* = \sqrt{\frac{m_0}{m_t}} F$$

$$v_{1x} = \sqrt{\frac{m_0}{m_t}} v_{1x}^{**} = \sqrt{\frac{m_0}{m_t}} v_1^{**}$$

$$v_{1y} = 0$$

$$v_{1z} = 0$$

$$v_{1x} = \sqrt{\frac{m_0}{m_t}} \frac{v_1^{**}}{\sqrt{2}}$$

$$v_{1y} = \sqrt{\frac{m_0}{m_t}} \frac{v_1^{**}}{\sqrt{2}}$$

$$v_{1z} = 0$$

(111) Valley

$$F^* = F \left(\frac{m_0}{3m_t} + \frac{2m_0}{3m_e} \right)^{1/2}$$

$$v_{2x} = 0$$

$$v_{2y} = \sqrt{\frac{m_0}{m_t}} v_{2y}^{**} = \frac{\frac{m_0}{m_t} \frac{1}{\sqrt{3}} v_2^{**}}{\left(\frac{m_0}{3m_t} + \frac{2m_0}{3m_e} \right)^{1/2}}$$

$$v_{2z} = \sqrt{\frac{m_0}{m_e}} v_{2z}^{**} = \frac{-\frac{m_0}{m_e} \frac{2}{\sqrt{6}}}{\left(\frac{m_0}{3m_e} + \frac{2m_0}{3m_e}\right)^{1/2}} v_2^{**}$$

$$v_{2x} = \frac{\left(\frac{m_0}{\sqrt{18}m_e} + \frac{2m_0}{\sqrt{18}m_e}\right)}{\left(\frac{m_0}{3m_e} + \frac{2m_0}{3m_e}\right)^{1/2}} v_2^{**}$$

$$v_{2y} = \frac{\left(\frac{m_0}{\sqrt{18}m_e} + \frac{2m_0}{\sqrt{18}m_e}\right)}{\left(\frac{m_0}{3m_e} + \frac{2m_0}{3m_e}\right)^{1/2}} v_2^{**}$$

$$v_{2z} = \frac{\left(\frac{2m_0}{\sqrt{18}m_e} - \frac{2m_0}{\sqrt{18}m_e}\right)}{\left(\frac{m_0}{3m_e} + \frac{2m_0}{3m_e}\right)^{1/2}} v_2^{**}$$

If n_2 is the number of electrons in each of the (111) and $(\bar{1}\bar{1}1)$ valleys, and n_1 the number in each of the $(\bar{1}11)$ and $(1\bar{1}1)$ valleys then,

$$V_x = \frac{1}{\sqrt{2}(n_1+n_2)} \left\{ \left(\frac{m_0}{3m_e} + \frac{2m_0}{3m_e}\right)^{1/2} v_2^{**} n_2 + \left(\frac{m_0}{m_e}\right)^{1/2} v_1^{**} n_1 \right\}$$

$$V_y = \frac{1}{\sqrt{2}(n_1+n_2)} \left\{ \left(\frac{m_0}{3m_e} + \frac{2m_0}{3m_e}\right)^{1/2} v_2^{**} n_2 + \left(\frac{m_0}{m_e}\right)^{1/2} v_1^{**} n_1 \right\}$$

$$V_z = 0$$

Alternately, we may consider

$$V = \frac{n_1 V_1 + n_2 V_2}{n_1 + n_2}$$

$$V_1 = \left(\frac{m_0}{m_e}\right)^{1/2} v_1^{**} \quad V_2 = \left(\frac{m_0}{3m_e} + \frac{2m_0}{3m_e}\right)^{1/2} v_2^{**}$$

(c) Field Applied in the (111) Direction(111) Valley

$$\underline{F} = \left(\frac{F}{\sqrt{3}}, \frac{F}{\sqrt{3}}, \frac{F}{\sqrt{3}} \right)$$

$$F^* = F \left(\frac{m_0}{m_l} \right)^{1/2}$$

$$v_{2x} = \sqrt{\frac{m_0}{m_t}} v_{2x}^{**} = 0$$

$$v_{2y} = \sqrt{\frac{m_0}{m_t}} v_{2y}^{**} = 0$$

$$v_{2z} = \sqrt{\frac{m_0}{m_l}} v_{2z}^{**} = \sqrt{\frac{m_0}{m_l}} v_2^{**}$$

$$v_{2x} = v_{2y} = v_{2z} = \frac{1}{\sqrt{3}} \left(\frac{m_0}{m_l} \right)^{1/2} v_2^{**}$$

($\bar{1}\bar{1}\bar{1}$) Valley

$$F^* = F \left(\frac{8}{9} \frac{m_0}{m_t} + \frac{1}{9} \frac{m_0}{m_l} \right)^{1/2}$$

$$v_{1x} = \frac{\sqrt{\frac{2}{3}} \frac{m_0}{m_t}}{\left(\frac{8}{9} \frac{m_0}{m_t} + \frac{1}{9} \frac{m_0}{m_l} \right)^{1/2}} v_1^{**}$$

$$v_{1y} = \frac{-\frac{\sqrt{2}}{3} \frac{m_0}{m_t}}{\left(\frac{8}{9} \frac{m_0}{m_t} + \frac{1}{9} \frac{m_0}{m_l} \right)^{1/2}} v_1^{**}$$

$$v_{1z} = \frac{\frac{1}{3} \frac{m_0}{m_l}}{\left(\frac{8}{9} \frac{m_0}{m_t} + \frac{1}{9} \frac{m_0}{m_l} \right)^{1/2}} v_1^{**}$$

$$v_{1x} = \frac{\frac{4}{3\sqrt{3}} \frac{m_0}{m_t} - \frac{1}{3\sqrt{3}} \frac{m_0}{m_l}}{\left(\frac{8}{9} \frac{m_0}{m_t} + \frac{1}{9} \frac{m_0}{m_l} \right)^{1/2}} v_1^{**}$$

$$v_{1y} = \frac{\frac{2}{3\sqrt{3}} \frac{m_0}{m_t} + \frac{1}{3\sqrt{3}} \frac{m_0}{m_l}}{\left(\frac{8}{9} \frac{m_0}{m_t} + \frac{1}{9} \frac{m_0}{m_l}\right)^{1/2}} v_1^{**}$$

$$v_{1z} = \frac{\frac{2}{3\sqrt{3}} \frac{m_0}{m_t} + \frac{1}{3\sqrt{3}} \frac{m_0}{m_l}}{\left(\frac{8}{9} \frac{m_0}{m_t} + \frac{1}{9} \frac{m_0}{m_l}\right)^{1/2}} v_1^{**}$$

(111) Valley

$$F^* = F \left(\frac{8}{9} \frac{m_0}{m_t} + \frac{1}{9} \frac{m_0}{m_l}\right)^{1/2}$$

$$v_{1x} = \frac{\sqrt{\frac{2}{3}} \frac{m_0}{m_t}}{\left(\frac{8}{9} \frac{m_0}{m_t} + \frac{1}{9} \frac{m_0}{m_l}\right)^{1/2}} v_1^{**}$$

$$v_{1y} = \frac{\frac{\sqrt{2}}{3} \frac{m_0}{m_t}}{\left(\frac{8}{9} \frac{m_0}{m_t} + \frac{1}{9} \frac{m_0}{m_l}\right)^{1/2}}$$

$$v_{1z} = \frac{\frac{1}{3} \frac{m_0}{m_l}}{\left(\frac{8}{9} \frac{m_0}{m_t} + \frac{1}{9} \frac{m_0}{m_l}\right)^{1/2}} v_1^{**}$$

$$v_{1x} = \frac{\frac{2}{3\sqrt{3}} \frac{m_0}{m_t} + \frac{1}{3\sqrt{3}} \frac{m_0}{m_l}}{\left(\frac{8}{9} \frac{m_0}{m_t} + \frac{1}{9} \frac{m_0}{m_l}\right)^{1/2}} v_1^{**}$$

$$v_{1y} = \frac{\frac{4}{3\sqrt{3}} \frac{m_0}{m_t} - \frac{1}{3\sqrt{3}} \frac{m_0}{m_l}}{\left(\frac{8}{9} \frac{m_0}{m_t} + \frac{1}{9} \frac{m_0}{m_l}\right)^{1/2}} v_1^{**}$$

$$v_{1z} = \frac{\frac{2}{3\sqrt{3}} \frac{m_0}{m_t} + \frac{1}{3\sqrt{3}} \frac{m_0}{m_l}}{\left(\frac{8}{9} \frac{m_0}{m_t} + \frac{1}{9} \frac{m_0}{m_l}\right)} v_1^{**}$$

(111) Valley

$$F^* = F \left(\frac{8}{9} \frac{m_0}{m_t} + \frac{1}{9} \frac{m_0}{m_l} \right)^{1/2}$$

$$v_{1z} = 0$$

$$v_{1y} = \frac{\frac{2\sqrt{2}}{3} \frac{m_0}{m_t}}{\left(\frac{8}{9} \frac{m_0}{m_t} + \frac{1}{9} \frac{m_0}{m_l}\right)} v_1^{**}$$

$$v_{1z} = \frac{-1/3 \frac{m_0}{m_l}}{\left(\frac{8}{9} \frac{m_0}{m_t} + \frac{1}{9} \frac{m_0}{m_l}\right)} v_1^{**}$$

$$v_{1x} = \frac{\frac{2}{3\sqrt{3}} \frac{m_0}{m_t} + \frac{1}{3\sqrt{3}} \frac{m_0}{m_l}}{\left(\frac{8}{9} \frac{m_0}{m_t} + \frac{1}{9} \frac{m_0}{m_l}\right)} v_1^{**}$$

$$v_{1y} = \frac{\frac{2}{3\sqrt{3}} \frac{m_0}{m_t} + \frac{1}{3\sqrt{3}} \frac{m_0}{m_l}}{\left(\frac{8}{9} \frac{m_0}{m_t} + \frac{1}{9} \frac{m_0}{m_l}\right)^{1/2}} v_1^{**}$$

$$v_{1z} = \frac{\frac{4}{3\sqrt{3}} \frac{m_0}{m_t} - \frac{1}{3\sqrt{3}} \frac{m_0}{m_l}}{\left(\frac{8}{9} \frac{m_0}{m_t} + \frac{1}{9} \frac{m_0}{m_l}\right)^{1/2}} v_1^{**}$$

If there are n_2 electrons in the (111), and n_1 in each of the others, then

$$V_x = V_y = V_z = \frac{1}{\sqrt{3}} (n_2 V_2 + 3n_1 V_1) / (n_2 + 3n_1)$$

$$V_2 = v_2^{**} \left(\frac{m_0}{m_l} \right)^{1/2}$$

$$V_1 = v_1^{**} \left(\frac{8m_0}{9m_t} + \frac{m_0}{9m_l} \right)^{1/2}$$

APPENDIX III

INTERVALLEY SCATTERING PROBABILITY

The total probability of an electron with energy ϵ being scattered from some state in one conduction band valley to some other state in a different valley, by the emission or absorption of an optical mode phonon is given by Herring

$$\frac{1}{\tau} = W \left\{ \frac{\left(\frac{\epsilon}{\hbar\omega} + 1\right)^{1/2}}{\exp\left(\frac{\hbar\omega}{kT_L}\right) - 1} + \frac{\left(\frac{\epsilon}{\hbar\omega} - 1\right)^{1/2} H(\epsilon - \hbar\omega)}{1 - \exp\left(-\frac{\hbar\omega}{kT_L}\right)} \right\}$$

where the first term deals with the absorption process, and the second with the emission.

Since we have assumed that the distribution of the electrons in each of the valleys is Maxwellian, this probability could be averaged over the entire distribution in a particular valley according to the usual formula

$$\langle \frac{1}{\tau} \rangle = \frac{1}{N} \int_0^{\infty} \frac{1}{\tau} f(\epsilon) d\epsilon$$

where

$$f(\epsilon) = \frac{2N}{\sqrt{\pi}} (kT_e)^{-3/2} \epsilon^{1/2} e^{-\epsilon/kT_e}$$

This calculation involves the integral

$$I_1 = \int_0^{\infty} \left(\frac{\epsilon}{2c} + 1\right)^{1/2} \epsilon^{1/2} e^{-\epsilon/kT_e} d\epsilon \quad c = \frac{\hbar\omega}{2}$$

If we substitute $u = \epsilon + c$ and simplify the result

$$I_1 = \frac{e^{c/kT_e}}{\sqrt{2c}} \int_c^{\infty} (u^2 - c^2)^{1/2} e^{-u/kT_e} du$$

Substituting $u = c \cosh v$ we get

$$I_1 = 2 \left(\frac{c}{2}\right)^{3/2} e^{a} \int_0^{\infty} \sinh^2 v e^{-a \cosh v} dv$$

$$a = \frac{c}{kT_e} = \frac{\hbar\omega}{2kT_e}$$

An integration by parts gives

$$I_1 = 2 \left(\frac{C}{2} \right)^{3/2} e^a \left\{ \frac{1}{a} \int_0^\infty \cosh v e^{-a \cosh v} dv - \frac{1}{a} \sinh v \right\}$$

and so

$$I_1 = \frac{2}{a} \left(\frac{C}{2} \right)^{3/2} e^a K_1(a)$$

The other integral which arises

$$I_2 = \int_{2C}^\infty \left(\frac{\epsilon}{2C} - 1 \right)^{1/2} \epsilon^{1/2} e^{-\epsilon/kT_e} d\epsilon$$

can be solved in a similar manner if the initial substitution

$u = \epsilon - C$ is made. So,

$$I_2 = \frac{2}{a} \left(\frac{C}{2} \right)^{3/2} e^{-a} K_1(a)$$

Finally,

$$\left\langle \frac{1}{\tau_c} \right\rangle = \omega \sqrt{\frac{2}{\pi}} \left(\frac{\hbar \omega}{2kT_e} \right)^{1/2} K_1 \left(\frac{\hbar \omega}{2kT_e} \right) \left\{ \frac{\exp \left(\frac{\hbar \omega}{2kT_e} \right)}{\exp \left(\frac{\hbar \omega}{kT_L} \right) - 1} + \frac{\exp \left(-\frac{\hbar \omega}{2kT_e} \right)}{1 - \exp \left(-\frac{\hbar \omega}{kT_L} \right)} \right\}$$

BIBLIOGRAPHY

- Dikhoff, J.A.M. 1960. Solid State Electronics, 1, 202.
- Dodge, B.F. and Dunbar, A.K. 1927. J. Amer. Chem. Soc. 49, 591.
- Dykman, I.M. and Tomchuk, P.M. 1961. Sov. Physics-Solid State, 2, 1988.
- Eastman, P.C. 1960. Ph.D. Thesis, U.B.C.
- Frei, A.H. and Strutt, M.J.O. 1960. Proc. I.R.E. 48, 1272.
- Frohlich, H. and Paranjape, B.V. 1956. Proc. Phys. Soc. (London), B, 69, 21.
- Glascoe, G.N. and Lebacqz, J.V. 1948. Pulse Generators, M.I.T. Radiation Laboratory Series volume 5, (McGraw Hill Book Company).
- Gunn, J.B. 1956. J. Electronics, 2, 87.
- Herring, C. 1955. Bell System Tech. J. 34, 237.
- Herring C. and Vogt, E. 1956. Phys. Rev. 101, 944.
- Keldysh, L.V., Vavilov, V.S., Bricin, K.J. 1960. Proceedings of the International Conference on Semiconductor Physics, Prague, (Academic Press), p. 824.
- Lewis, I.A.D. and Wells, F.H. 1959. Millimicrosecond Pulse Techniques, 2nd ed., (Pergamon Press).
- Nathan, M.I. 1960. Bull. Am. Phys. Soc. 5, 194.
- Paige, E.G.S. 1960. Proc. Phys. Soc. (London), A, 75, 174.
- Prior, A.C. 1959. Phys and Chem. Solids, 12, 175.
- Read, W.T., Jr. 1954. Phil. Mag. 45, 775.
- Reik, H.G. and Risken, H. 1961. Phys. Rev. 124, 777.
- Reik, H.G. and Risken, H. 1962. Phys. Rev. 126, 1737.
- Ryder, E.J. 1953. Phys. Rev. 90, 766.
- Sasaki, W., Shibuya, M., and Mizuguchi, K. 1958. J. Phys. Soc. Japan, 13, 456.
- Stratton, R. 1958. Solid state physics in electronics and telecommunications: Proceedings of an international conference, Brussels, edited by M. Desirant and J.L. Michiels, p. 343.
- Valdes, L.B. 1954. Proc. I.R.E. 42, 420.
- Williamson, E.D. and Adams, L.H. 1919. Phys. Rev. 14, 99.
- Wilson, A.H. 1959. Theory of Metals, (Cambridge University Press).
- Yamashita, J. and Watanabe, M. 1954. Progr. Theor. Phys. 12, 443.