**Coordinates and Boundary Conditions
for the
General Relativistic Initial Data Problem**

by

Jonathan Thornburg
B.Sc. Honors 1st class
Simon Fraser University, 1983

A Thesis Submitted in Partial Fulfillment of
the Requirements for the Degree of
MASTER OF SCIENCE

in

The Faculty of Graduate Studies
Department of Geophysics and Astronomy

We accept this thesis as conforming
to the required standard

The University of British Columbia

November 1985

© Jonathan Thornburg, 1985

In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the head of my department or by his or her representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of Geophysics and Astronomy

The University of British Columbia
1956 Main Mall
Vancouver, Canada
V6T 1Y3

Date 11 November 1985

Abstract

Techniques for numerically constructing initial data in the $3+1$ formalism of general relativity (GR) are studied, using the theoretical framework described in Bowen and York (1980), *Physical Review D* 21(8), 2047–2056. The two main assumptions made are maximal slicing and 3-conformal flatness of the generated spaces. For ease of numerical solution, axisymmetry is also assumed, but all the results should extend without difficulty to the non-axisymmetric case.

The numerical code described in this thesis may be used to construct vacuum spaces containing arbitrary numbers of black holes, each with freely specifiable (subject to the axisymmetry assumption) position, mass, linear momentum, and angular momentum. It should be emphasised that the time evolution of these spaces has not yet been attempted.

There are two significant innovations in this work: the use of a new boundary condition for the surfaces of the black holes, and the use of multiple coordinate patches in the numerical solution.

The new boundary condition studied herein requires the inner boundary of the numerical grid to be a marginally trapped surface. This is in contrast to the approach used in much previous work on this problem area, which requires the constructed spaces to be conformally isometric under a “reflection mapping” which interchanges the interior of a specified black hole with the remainder of the space. The new boundary condition is found to be easy to implement, even for multiple black holes. It may also prove useful in time evolution problems.

The coordinate choice scheme introduced in this thesis uses multiple coordinate

patches in the numerical solution, each with a coordinate system suited to the local physical symmetries of the region of space it covers. Because each patch need only cover part of the space, the metrics on the individual patches can be kept simple, while the overall patch system still covers a complicated topology. The patches are linked together by interpolation across the interpatch boundaries. Bilinear interpolation suffices to give accuracy comparable with that of common second order difference schemes used in numerical GR.

This use of multiple coordinate patches is found to work very well in both one and two black hole models, and should generalise to a wide variety of other numerical GR problems. Patches are also found to be a useful (if somewhat over-general) way of introducing spatially varying grid sizes into the numerical code. However, problems may arise when trying to use multiple patches in time evolution problems, in that the interpatch boundaries must not become spurious generators or reflectors of gravitational radiation, due to the interpolation errors. These problems have not yet been studied.

The code described in this thesis is tested against Schwarzschild models and against previously published work using the Bowen and York formalism, reproducing the latter within the limits of error of the codes involved. A number of new spaces containing one and two black holes with linear or angular momentum are also constructed to demonstrate the code, although little analysis of these spaces has yet been done.

Table of Contents

Abstract	ii
Table of Contents	iv
List of Tables	v
List of Figures	vii
Acknowledgements	x
Dedication	xi
Introduction	1
Theoretical Background	4
Boundary Conditions	15
Choice of coordinates	22
Finite Differencing the Equations	26
Coordinate Patches – I	36
Coordinate Patches – II	42
The Extrinsic Curvature	44
Choice of Conformal Function	48
Solving the Algebraic Equations	52
Measuring the Masses of the Models	56
Testing the Code	59
Demonstrating the Code	67
Conclusions and Directions for Further Research	77
References	81
Appendix 1	84
Appendix 2	255

List of Tables

1	Comparison of Different Grid Sizes	60
2	Comparison of Different Patch Types	61
3	Comparison of Different Conformal Functions	61
4	Comparison of $\delta\Psi$ with Choptuik's Results	65
5	Model index for the Schwarzschild test series	85
6	Model index for the Choptuik comparison test series	86
7	Model index for the J test series	87
8	Model index for the RSP test series	87
9	Model index for the RSN test series	88
10	Model index for the NTO test series	88
11	Model index for the PP test series	89
12	Schwarzschild test series, 10plo100 selected Ψ values	93
13	Schwarzschild test series, 20plo100 selected Ψ values	97
14	Schwarzschild test series, 40plo100 selected Ψ values	101
15	Schwarzschild test series, c2 selected Ψ values	105
16	Schwarzschild test series, c01 selected Ψ values	109
17	Schwarzschild test series, c21 selected Ψ values	113
18	Schwarzschild test series, poo selected Ψ values	117
19	Schwarzschild test series, opo selected Ψ values	121
20	Schwarzschild test series, oop selected Ψ values	125
21	J test series, j0 selected Ψ values	171
22	RSP test series, p0 selected Ψ values	186
23	RSN test series, p0 selected Ψ values	201
24	NTO test series, p0 selected Ψ values	216

25	PP test series, m100 selected Ψ values	231
26	PP test series, m10 selected Ψ values	233
27	PP test series, m1 selected Ψ values	235
28	PP test series, mx1 selected Ψ values	237
29	PP test series, mx01 selected Ψ values	239
30	PP test series, 0 selected Ψ values	241
31	PP test series, px01 selected Ψ values	243
32	PP test series, px1 selected Ψ values	245
33	PP test series, p1 selected Ψ values	247
34	PP test series, p10 selected Ψ values	249
35	PP test series, p100 selected Ψ values	251
36	Directories where the code resides	256
37	File index for the main directory	257
38	File index for the fortran directory	258
39	File index for the patchutil directory	259
40	File index for the util directory	260

List of Figures

1	"Square" vs. "Triangle" Interpolation	39
2	Bilinear Interpolation Formulae	40
3	Baseline patch system for table 2	62
4	Augmented patch system for table 2	63
5	J Test Series Solutions	69
6	RSP Test Series Solutions	70
7	RSM Test Series Solutions	71
8	NT0 Test Series Solutions	72
9	Patch System for the PP Test Series	74
10	PP Test Series Solutions ($P \geq 0$)	75
11	PP Test Series Solutions ($P \leq 0$)	76
12	Choptuik comparison test series, ppx001 quantitative results	127
13	Choptuik comparison test series, ppx01 quantitative results	129
14	Choptuik comparison test series, ppx1 quantitative results	131
15	Choptuik comparison test series, pp1 quantitative results	133
16	Choptuik comparison test series, pp2x5 quantitative results	135
17	Choptuik comparison test series, pp7x5 quantitative results	137
18	Choptuik comparison test series, pp12x5 quantitative results	139
19	Choptuik comparison test series, mpx001 quantitative results	141
20	Choptuik comparison test series, mpx01 quantitative results	143
21	Choptuik comparison test series, mpx1 quantitative results	145
22	Choptuik comparison test series, mp1 quantitative results	147
23	Choptuik comparison test series, mp2x5 quantitative results	149
24	Choptuik comparison test series, mp7x5 quantitative results	151

25	Choptuik comparison test series, mp12x5 quantitative results	153
26	Choptuik comparison test series, jx001 quantitative results	155
27	Choptuik comparison test series, jx01 quantitative results	157
28	Choptuik comparison test series, jx1 quantitative results	159
29	Choptuik comparison test series, j1 quantitative results	161
30	Choptuik comparison test series, j10 quantitative results	163
31	Choptuik comparison test series, j100 quantitative results	165
32	Choptuik comparison test series, j1000 quantitative results	167
33	J test series, jx01 quantitative results	173
34	J test series, jx1 quantitative results	175
35	J test series, j1 quantitative results	177
36	J test series, j10 quantitative results	179
37	J test series, j100 quantitative results	181
38	RSP test series, px01 quantitative results	188
39	RSP test series, px1 quantitative results	190
40	RSP test series, p1 quantitative results	192
41	RSP test series, p10 quantitative results	194
42	RSP test series, p100 quantitative results	196
43	RSN test series, px01 quantitative results	203
44	RSN test series, px1 quantitative results	205
45	RSN test series, p1 quantitative results	207
46	RSN test series, p10 quantitative results	209
47	RSN test series, p100 quantitative results	211
48	NT0 test series, px01 quantitative results	218
49	NT0 test series, px1 quantitative results	220

50	NT0 test series, p1 quantitative results	222
51	NT0 test series, p10 quantitative results	224
52	NT0 test series, p100 quantitative results	226

Acknowledgements

First and foremost, I must acknowledge the many contributions of W. G. Unruh to this thesis. In addition to many stimulating conversations, he provided one of the two key ideas of this thesis (the marginally trapped surface boundary condition), as well as many other helpful suggestions. The exposition of this thesis has also benefited greatly from his comments.

F. Chen gave invaluable help with the $\text{\TeX}$ typesetting system used to prepare this thesis. M. Choptuik kindly reexamined the results from his own GR code and did a new perturbation analysis of them to provide me with comparison values to test out my code.

My choice of topic for this thesis was helped by early discussions with M. Choptuik, T. Piran and J. York. U. Ascher, M. Choptuik, and M. Yedlin provided valuable “physics” suggestions, and L. Brewin, G. Grieve, and T. Nicol helped me with a variety of computer problems.

I thank The University of British Columbia for financial support. The computer facility used to do the calculations of this thesis was funded by the Natural Sciences and Engineering Research Council.

Dedication

This thesis is dedicated to all of my professors, who together have shaped my mind and inspired me to learn. In particular, Hans Thom first interested me in physics, Ken Lyall nurtured that interest, Leigh Hunt Palmer first interested me in general relativity by giving me a copy of Price (1982), and Michael Ovenden gently guided me in the direction of Bill Unruh.

*There are certain common Privileges of a Writer,
the Benefit whereof, I hope, there will be no Reason to doubt;
Particularly, that where I am not understood, it shall be concluded,
that something very useful and profound is coucht underneath.*

— Jonathan Swift, “Tale of a Tub”, Preface (1704)

Introduction

Due to their complexity, the Einstein equations of general relativity (hereinafter “GR”) are very hard to solve analytically. Indeed, such solutions have been found only for systems with either a high degree of symmetry, or for systems in which the non-linearity of the equations can be taken to be “small”. Such restrictions exclude many interesting systems, for example the collision and subsequent merger of two black holes.

One way of examining systems not meeting these restrictions, is to *numerically* solve the Einstein equations, using the methods developed for other nonlinear PDE problems such as the Navier-Stokes equations. This approach, known as “numerical GR”, has seen substantial use in the past few decades. Most commonly, the “finite difference” technique is used to discretise the equations prior to solving them on a digital computer, although the alternative “finite elements” method has seen some limited use.

This thesis studies the problem of constructing spaces containing multiple moving and/or spinning black holes, and involves the construction of a numerical GR code to do this. This problem was selected for three main reasons. First, the time evolution of such spaces (which has not yet been performed) would be of great interest. Second, previous work has established a general theoretical framework for the solution of this problem. Third, and perhaps most important, it seems to provide a good “test-bed” problem for trying out various coordinates and boundary conditions.

The main results of this thesis are the introduction of a new and more general scheme for choosing coordinate systems, and of a new and physically well-motivated

boundary condition for black hole surfaces. Together, these should allow the above-mentioned theoretical framework to be more easily applied to a wider variety of problems, including the two black hole collision problem mentioned above.

In general, the emphasis in this thesis is more on the techniques used than on the results obtained. However, as well as replicating earlier work involving single black holes, some new spaces are constructed. In particular, the new boundary condition is used in the construction of a series of spaces containing moving single black holes. A series of spaces containing single spinning black holes are also constructed, although here the new boundary condition turns out to be equivalent to that used in previous work. Finally, both the new boundary condition and the new coordinate choice scheme are used in the construction of a series of spaces containing two black holes with equal and opposite momenta. The latter have not been numerically studied before.

The remainder of this thesis will now be outlined. Successive chapters discuss the theoretical background to the work, the various boundary conditions (both old and new) involved, and the coordinate choice problem. The general technique of finite differencing is then discussed, and applied to the problem of this thesis. The following chapters give more details of the code used in this thesis, including the internal representation of patch systems, the “human-readable” patch description language used to specify them, the detailed means by which the contents (for example “2 black holes with such-and-such positions, masses, momenta, and spins”) of the spaces are specified, and some final details of just how the equations are set up and solved. The various “mass functionals” used to measure the mass-energy of the black holes and spaces constructed are then discussed, followed by the means used to test the code. The new spaces used to demonstrate the code are described, followed by some conclusions and directions for

further research. Two appendices give the detailed input parameters to and quantitative graphs of the results of each model described in the thesis, and the source listings for the code itself.

For the most part, the chapters are organized so that they may be read independently. To this end, all notational conventions are defined near the beginning of the "Theoretical Background" chapter. Also, for the benefit of anyone wishing to replicate the results of this thesis, the main equations used in the code are all (with one exception described in the text) given in the body of the thesis as well, even if their derivations are merely tedious exercises in algebra. This should spare the reader the task of trying to "unprogram" the equations from the code listings.

Theoretical Background

As usually written, the Einstein equations refer to spacetime as a whole, without reference to any one coordinate being singled out as a “global time”. In contrast, many other physical theories (for example classical electromagnetism as described by the Maxwell equations) can be phrased in terms of the *time evolution* of various quantities. By analogy, we suspect that such a formulation of GR (as a *dynamical* theory) would be useful. Such a formulation does indeed exist, under suitable technical assumptions to guarantee the “non-pathological” nature of spacetime. However, the details of how this is set up are beyond the scope of this thesis, and will not be discussed here. The interested reader is referred to Choptuik (1982), Misner, Thorne, and Wheeler (1973 (hereinafter cited as “MTW”), chapter 21), Wald (1984, chapter 10 and appendix E), York (1979), York (1983), and York and Piran (1982) for further discussions of this topic. In particular, the cited works by York *et al* contain excellent discussions of how GR can be formulated as a dynamical theory in which the notion of “time evolution” is well defined. These works all build on the basic theoretical framework of the classic paper of Arnowitt, Deser, and Misner (1962).

We now discuss the theoretical foundations of this approach to GR, as well as the particular theoretical framework used in this thesis. Our discussion gives only a brief overview of the results used in this thesis, and will be kept to the least technical level possible. Tensor calculus will be used, but only in its coordinate form, i.e. the modern “coordinate-free” language and techniques of differential geometry will not be used. More advanced and comprehensive discussions can be found in the cited references.

The basic idea of the “ADM” framework is to view 4-dimensional spacetime as being composed of a 1-dimensionally infinite “stack” of 3-dimensional spacelike hyper-

surfaces (hereinafter “spaces” or “slices”), each labeled with a “time” coordinate. This decomposition of spacetime will often be referred to as “slicing” it. To give a dynamical interpretation to GR, we choose “initial data” defined on a single slice, and then try to “time evolve” it with the Einstein equations. This “3+1” approach to the dynamics of GR is often referred to as (solving) the “Cauchy problem”. However, there are two interrelated difficulties which arise when we try to do this.

First, not all the initial data can be *freely specified* on the initial slice. This is because of the Bianchi identities, which reduce by four the number of independent metric components. A useful analogy here is the specification in electromagnetism of E and B at a single moment in time—Gauss’s law and its magnetic analog prevent these from being completely freely specifiable. Such initial data as *can* be freely specified on the initial slice, presumably represent the true degrees of freedom of our problem. Recent work by York and others on the problem of defining freely specifiable GR initial data has yielded a general theoretical framework for dealing with this problem, which we adopt for this thesis.

Second, how do we remove the coordinate freedom inherent in the Einstein equations? The problem here is that a “solution” to a GR dynamical problem will presumably consist of a specification of all components of the metric over some time interval. This (together with the ranges of the coordinates) implicitly specifies the coordinate system in use throughout that interval, which clearly is *not* a true dynamical feature of the spacetime. Thus there can not and *must not* be enough Einstein equations to determine the evolution of all the metric components—some additional information must be supplied (thus defining the choice of coordinates). This must be done *prior* to obtaining the spacetime, so there is clearly a risk of choosing coordinates which are

inappropriate for the symmetries of the spacetime. Worse still, the chosen coordinates may fail to cover the whole spacetime, or they may cover undesired areas of it, for example the immediate neighborhoods of singularities. This problem still lacks a fully satisfactory solution, although considerable progress has been made. This thesis deals only with the construction of initial data, so this difficulty will not be further discussed.

Corresponding to the slicing of spacetime, the $3 + 1$ approach to GR splits the 10 Einstein equations into 4 “constraint” equations for the initial data, and 6 “evolution” equations which specify how this data evolves with time. This thesis deals solely with the former, i.e. with the construction of spaces which satisfy the constraint equations. The theoretical background for this construction will now be discussed, beginning with the $3 + 1$ slicing process itself.

Our notation generally follows that of MTW, with units chosen such that $G = c = 1$ and the metric having signature $(-, +, +, +)$. We shall sometimes use “ t ” as a synonym for the coordinate x^0 . Indices $a, b, c, \dots$ range from 0 to 3, while indices $i, j, k, \dots$ have the restricted range of 1 to 3. The usual summation convention is assumed for both types of indices. Greek letters will not be used for indices. ∂_a denotes the partial derivative operator with respect to the coordinate x^a , while ∇_a denotes the covariant derivative operator with respect to the same coordinate. Where ambiguity exists, quantities defined with respect to the full 4-dimensional spacetime are prefixed with a (4) , as in $(4)R$, while quantities defined with respect to a 3-dimensional space are prefixed with a (3) , as in $(3)R$.

Non-italic bold letters denote mathematical objects which do not transform as tensors, such as vectors and matrices (for which the index range conventions given above do not apply). Typewriter font identifies objects (for example names of models

or individual functions) known to or defined in the computer code itself.

We shall adopt one other important notational convention—the Penrose “abstract index” notation. This involves (almost) always writing tensors with the number of indices appropriate to their rank, even if the tensor as a whole is being referred to. Thus, we refer to the 4-metric (viewed as a single object) by the symbol $^{(4)}g_{ab}$, rather than $^{(4)}g$ as is sometimes done. The latter symbol is used to denote a (presumably related) *scalar* (rank zero) object. Another way to look at this notation is to just view all tensor equations as component equations with an implied “for all free (not summed over) indices” appended.

We begin the slicing process by considering the full 4-dimensional metric $^{(4)}g_{ab}$. We view $^{(4)}g_{ab}$ as the partitioned matrix

$$^{(4)}g_{ab} = \left[\begin{array}{c|c} ^{(4)}g_{00} & ^{(4)}g_{0j} \\ \hline ^{(4)}g_{i0} & ^{(4)}g_{ij} \end{array} \right] \quad (1)$$

and define the (covariant) “3-metric” $^{(3)}g_{ij}$ to be the spatial components of the 4-metric

$$^{(3)}g_{ij} = ^{(4)}g_{ij}, \quad (2)$$

so it denotes the lower right submatrix of (1). It can be shown that the $^{(3)}g_{ij}$ so defined is indeed the (covariant) metric induced on the hypersurface $t = \text{constant}$ by the 4-metric $^{(4)}g_{ab}$. Thus $^{(3)}g_{ij}$ is (as the notation implies) a (symmetric) 3-tensor. Given $^{(3)}g_{ij}$, we define the contravariant 3-metric $^{(3)}g^{ij}$ in the usual way as the matrix inverse of $^{(3)}g_{ij}$.

We now define a scalar α and a 3-vector β_i , known as the “lapse” and “shift” functions respectively, by

$$\left. \begin{array}{l} ^{(4)}g_{00} = -(\alpha^2 - \beta_i \beta^i) \\ ^{(4)}g_{0i} = ^{(4)}g_{i0} = \beta_i \end{array} \right\} \quad (3)$$

where the index on β^i is raised with the 3-metric. The timelike nature of t can be shown to guarantee that α comes out real and positive.

The lapse and shift functions defined above are the means by which the time evolution of our coordinate system itself is defined—they must be given as inputs to the Einstein evolution equations. They can be given an elegant geometric interpretation as the time and space components of a “time step” 4-vector (see for example, York (1979), p. 89). However, this is beyond the scope of this thesis; their only use herein is as a vehicle in the definition of the variables of our problem.

Finally, we define the “extrinsic curvature” K_{ij} by

$$\frac{\partial {}^{(3)}g_{ij}}{\partial t} = -2\alpha K_{ij} + {}^{(3)}\nabla_i \beta_j + {}^{(3)}\nabla_j \beta_i. \quad (4)$$

So defined, K_{ij} can be shown to transform as a (symmetric) 3-tensor under 3-coordinate transformations, and should perhaps be written ${}^{(3)}K_{ij}$, but no confusion will arise from the simpler form K_{ij} .

As defined, K_{ij} is essentially a “generalized velocity” conjugate to the “generalized position” ${}^{(3)}g_{ij}$. Indeed, the natural choice

$$\left. \begin{array}{l} \alpha = 1 \\ \beta^i = 0, \end{array} \right\} \quad (5)$$

(frequently referred to as “Gaussian Normal” or “harmonic” coordinates) leads to

$$K_{ij} = -\frac{1}{2} \frac{\partial {}^{(3)}g_{ij}}{\partial t}. \quad (6)$$

K_{ij} can also be interpreted as specifying just how the hypersurface $t = \text{constant}$ is embedded into spacetime—this is why it’s called the “extrinsic curvature” tensor. However, both of these interpretations are beyond the scope of this thesis, which uses K_{ij} only in specifying the contents of the spaces.

Having defined the 3-metric and the (3-)extrinsic curvature, we now consider the “initial data” problem of constructing suitable initial data to be time evolved. (This problem is sometimes referred to as the “initial value” problem, but the author feels that this usage invites confusion with the latter term’s well established meaning in a number of other fields, notably the study of numerical solutions of both ODEs and PDEs.) As described in the cited references, this amounts to constructing suitable 3-metrics and extrinsic curvatures at a single moment in time. This is the overall problem addressed by this thesis. By “suitable”, we mean both consistent with the Einstein constraint equations, and containing the desired physical objects. In this thesis, the latter are multiple moving and/or spinning black holes. The theoretical framework used in this thesis is that of Bowen and York (1980), which we will now discuss.

We begin by writing the Einstein constraint equations for the initial data as

$$\nabla^j \left(K_{ij} - {}^{(3)}g_{ij} \text{Tr}(K) \right) = 8\pi j_i \quad (7)$$

and

$${}^{(3)}R - {}^{(3)}g^{im} {}^{(3)}g^{jn} K_{ij} K_{mn} + (\text{Tr}(K))^2 = 16\pi\rho, \quad (8)$$

where ρ and j_i are the time and space components of the energy-momentum 4-vector of sources, ${}^{(3)}R$ is the Reimann curvature scalar of the 3-metric, $\text{Tr}(K)$ is the trace of K_{ij} defined by

$$\text{Tr}(K) = {}^{(3)}g^{ij} K_{ij}, \quad (9)$$

and indices are raised and lowered with the 3-metric.

We now make two simplifying assumptions. The first is that our initial slice satisfies

$$\text{Tr}(K) = 0. \quad (10)$$

This condition, which is widely used in numerical GR, can be interpreted (see, for example, York and Piran (1982), pp. 157–158) as requiring our space to be of maximal volume as embedded into spacetime, and is thus often referred to as the “maximal slicing” condition. It is known that many “interesting” spacetimes admit slicings satisfying this condition. York and Piran (1982) give as examples the Schwarzschild, Kerr, and Minkowski (flat) spacetimes as well as those “sufficiently close” (in a suitable norm) to them. They also state that “every stationary, axisymmetric, and asymptotically flat spacetime that is topologically $\mathbf{R}^4$ has maximal spacelike slices.” Ó Murchadha and Seifert (1982) have shown that the requirement of axisymmetry may be removed in this statement. Nonetheless, this condition probably excludes some physically interesting spacetimes. It should be noted that much of the present work relies only on $\text{Tr}(K)$ being a constant, although York and Piran suggest non-zero values as being appropriate for asymptotically null spaces, rather than the asymptotically strictly spacelike ones studied in this thesis.

The second simplifying assumption is that the space is 3-conformally flat,

$$^{(3)}g_{ij} = \Psi^4 f_{ij}, \tag{11}$$

where f_{ij} is a flat 3-metric and Ψ is an unknown (positive) “conformal factor”. The use of the fourth power here is for later convenience, and is of no fundamental significance. This assumption is perhaps the most serious limitation of this approach to the initial data problem. An arbitrary spacetime, even one known to possess maximal slices, in general does *not* possess *any* slicing satisfying this condition. However, the Schwarzschild solution does satisfy this assumption in any coordinate system which uses the usual Schwarzschild time coordinate, so we know that the existence of black holes isn’t ruled out.

The conformal factor can be interpreted as an isotropic “local stretching” of space, in that the transformation

$$^{(3)}g_{ij} \rightarrow \Psi^4 {}^{(3)}g_{ij} \quad (12)$$

leaves angles locally invariant, but isotropically multiplies local distances by a factor of Ψ^2 . Wald (1984, appendix D) discusses conformal transformations in more detail.

We now subject the other variables in the Einstein constraint equations (7) and (8) to the transformation

$$\left. \begin{aligned} \rho &= \Psi^{-8} \hat{\rho} \\ j_i &= \Psi^{-6} \hat{j}_i \\ K_{ij} &= \Psi^{-2} \hat{K}_{ij}, \end{aligned} \right\} \quad (13)$$

where “hatted” quantities refer to the “hat-means-flat” transformed (flat) space with 3-metric f_{ij} . This results in (the Einstein constraint equations) (7) and (8) simplifying to

$$\hat{\nabla}^j \hat{K}_{ij} = 8\pi \hat{j}_i \quad (14)$$

and

$$\nabla^2 \Psi + \frac{1}{8} \hat{K}_{ij} \hat{K}^{ij} \Psi^{-7} = 2\pi \hat{\rho} \Psi^{-3}, \quad (15)$$

where ∇^2 is the standard flat-space Laplacian operator. (Strictly speaking, it should have a “hat”, but no confusion should arise from the omission.)

We restrict our attention to vacuum spaces (outside black holes), so we take $\hat{j}_i = \hat{\rho} = 0$. All of the following discussion takes place in the transformed flat space; “hats” and $^{(3)}$ -prefixes are often omitted in the interests of typographical clarity. Indices are raised and lowered with the flat 3-metric f_{ij} .

We observe that (14) does not involve Ψ and hence is effectively decoupled from (15). In fact, this would still happen even if the maximal slicing assumption (10) were

replaced by any other slicing satisfying

$$\partial_i \text{Tr}(K) = 0. \quad (16)$$

Failure to satisfy this latter condition would result in our having to solve (14) and (15) simultaneously, rather than sequentially as we do in this thesis.

Bowen and York show that (14) may be solved analytically through the use of a vector potential. Their general solution is not needed herein; we shall use only two of their specific solutions, namely those representing single black holes with either specified (linear) momentum or specified spin (angular momentum).

For a single black hole with linear momentum P^i (and no spin), Bowen and York give the extrinsic curvature as (the following solution to (14))

$$\begin{aligned} \hat{K}_{ij} = & \frac{3}{2r^2} [P_i n_j + P_j n_i - (f_{ij} - n_i n_j) P^k n_k] \\ & - \kappa \frac{3a^2}{2r^4} [P_i n_j + P_j n_i + (f_{ij} - 5n_i n_j) P^k n_k], \end{aligned} \quad (17)$$

where n^i is the outward pointing unit normal to the 2-sphere of radius r centered on the black hole, a is an arbitrary parameter (later to be interpreted as one half the mass of the black hole), and $\kappa = \pm 1$. The significance of κ , as well as possible reasons for taking it equal to zero instead of ± 1 (thus dropping the second term of (17) entirely), will be discussed in the next chapter.

For a single black hole with spin J^i (and no momentum), they give the solution

$$\hat{K}_{ij} = \frac{3}{r^3} J^l n^k (\epsilon_{kil} n_j + \epsilon_{kjl} n_i), \quad (18)$$

where n^i and r are as before and

$$\epsilon_{ijk} = \begin{cases} +1 & \text{if } (ijk) \text{ is an even permutation of } (123) \\ -1 & \text{if } (ijk) \text{ is an odd permutation of } (123) \\ 0 & \text{otherwise} \end{cases} \quad (19)$$

is the usual completely antisymmetric unit alternating Levi-Civita symbol.

The only remaining point to be made here about the solution of (14) is that, since it's a linear equation, we may add solutions together to obtain $\hat{K}_{ij}$ (and hence K_{ij} by (13), once (15) has been solved) for spaces containing an arbitrary number of black holes, each with arbitrary and freely specifiable mass, position (obtained by translating the origin of the solutions given above), momentum, and spin.

Given a suitable solution to (14), we define a "source function" S by

$$S = \frac{1}{8} \hat{K}_{ij} \hat{K}^{ij}. \quad (20)$$

If we view (15), which we rewrite as

$$\nabla^2 \Psi + S \Psi^{-7} = 0, \quad (21)$$

as a nonlinear Poisson equation, $S \Psi^{-7}$ plays the role (modulo a factor of 4π) of a charge density. This analogy with classical electrostatics will prove helpful in our discussion of the solution of (21). It is the (numerical) solution of this equation which is the main topic of this thesis.

We now make one additional simplifying assumption, namely that our spaces are axisymmetric. This assumption is in no way implicit in the Bowen and York formalism, but is made purely to simplify our numerical solution of (21). It does this by allowing us to neglect one space dimension (coresponding to rotation about the symmetry axis) in the numerical solution, so we need only consider 2 space dimensions. Obviously, this greatly reduces the computational difficulty of numerically solving (21).

There is, however, another advantage to considering only 2 space dimensions. This is that we can obtain a global "bird's eye" view of a function (for example S or Ψ)

defined on our space, with a single plot of the function vs. (2-dimensional) position, viewed in perspective. No comparable technique is available for spaces of 3 or more dimensions—one must resort to the piecemeal examination of “slices” of the space. This lack of an easy means of viewing a function over the whole space is perhaps a more serious impediment to the treatment of fully 3-dimensional spaces than the greater computational difficulties, since the latter are diminishing with time as available computers increase in speed and memory capacity.

Boundary Conditions

We now consider what boundary conditions should be imposed on Ψ to specify a unique and physically meaningful solution to (21), and hence a space containing the desired objects. The following discussion takes place in the “hat-means-flat” space with (3-)metric ${}^{(3)}g_{ij} = f_{ij}$. As mentioned above, we take this space to be axisymmetric, with the axis of symmetry chosen as the z axis and ϕ the angle of rotation about it. We introduce the usual polar spherical coordinates (r, θ, ϕ) , with the line element $ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$. We also introduce cylindrical coordinates (ρ, z, ϕ) , with the line element $ds^2 = d\rho^2 + dz^2 + \rho^2 d\phi^2$.

We first consider the “outer” boundary. Since the physical “unhatted” space is asymptotically flat, we have

$$\lim_{r \rightarrow \infty} \Psi = 1. \quad (22)$$

However, our numerical “grid” will terminate at some finite radius r , so we must choose a boundary condition there which will best approximate the true boundary condition (22).

We could of course just take $\Psi = 1$ on our outer boundary, but this would not give very accurate results. Instead, York and Piran (1982) suggest exploiting more of the physics of the problem by observing that if E is the total energy of the space, the asymptotic behavior of Ψ as $r \rightarrow \infty$ will be

$$\Psi = 1 + \frac{E/2}{r} + O\left(\frac{1}{r^2}\right). \quad (23)$$

Thus

$$\frac{\partial \Psi}{\partial r} + \frac{\Psi - 1}{r} = O\left(\frac{1}{r^3}\right). \quad (24)$$

We now neglect the right hand side of this latter equation to obtain a fairly accurate “Robin” outer boundary condition,

$$\frac{\partial \Psi}{\partial r} + \frac{\Psi - 1}{r} = 0. \quad (25)$$

We next consider the problem of choosing an “inner” boundary condition for the “surfaces” of the black holes, assuming that we don’t want to extend our numerical grid inside the black holes. We first review the approach taken by Bowen and York (1980), which begins by considering the case of a single black hole only.

They suggest requiring that the space be conformally isometric under the “reflection” mapping

$$r \rightarrow \frac{a^2}{r}, \quad (26)$$

where both a and r are as defined in their choice (17) for the extrinsic curvature (which does indeed satisfy this requirement, as does their choice (18)). They then interpret the resulting space as the union of two distinct asymptotically flat spaces, joined at $r = a$ by an Einstein-Rosen bridge which they take as a model for the black hole. They observe that it suffices to solve (21) on only one of these spaces, the “outside” one defined by $r \geq a$, provided that an inner boundary condition enforcing the conformal isometry of the two spaces under (26) is chosen. They derive

$$\frac{\partial \Psi}{\partial r} + \frac{\Psi}{2r} = 0 \quad (\text{at } r = a) \quad (27)$$

as the desired “reflection symmetric” boundary condition for enforcing this.

We now digress to explain the significance of the parameter κ in (17), as promised in the previous chapter: the momentum P^i enters (21) quadratically (through S), so the “inner” space may possess either equal (if $\kappa = +1$ in (17)) or equal and opposite

(if $\kappa = -1$) momentum to the outer one. Bowen and York point out that the only reason for the second term in (17) is to satisfy their reflection symmetric boundary condition—taking $\kappa = 0$ in (17) (thus dropping the second term entirely) still yields a valid solution to (14). Thus if (as is done later in this thesis) Bowen and York’s inner boundary condition is abandoned, there seems no particular reason not to choose the simplest form of (17), namely that with $\kappa = 0$.

We now return to our discussion of the Bowen and York reflection symmetric inner boundary condition (27). To this author, it seems to suffer from several flaws. It may place the numerical boundary inside the actual black hole event horizon, which seems rather a waste of grid points (as was pointed out to the author by W. G. Unruh) if we are concerned only with the outside world. More serious difficulties may arise in time evolution problems: the grid points inside the horizon may encounter a singularity, and some care is needed to ensure that the numerical scheme remains well-behaved across the apparent horizon despite the change in the causal structure of spacetime which occurs there.

However, these latter effects should be fairly minor, because the reflection symmetric boundary condition seems (York and Piran (1982)) to result in the apparent horizon being only a small distance outside the inner grid boundary. When applied to the Schwarzschild solution, the two are found to coincide.

A more serious difficulty with the reflection symmetric inner boundary condition is that applying it to spaces containing multiple black holes is rather messy, involving (see, for example, Kulkarni, Shepley, and York (1983), Kulkarni (1984a), York (1984), Bowen, Rauber, and York (1984), and Kulkarni (1984b)) the summation of infinite series of “image charges” needed to enforce the isometry condition on reflection through

any of the a_i s.

This author also finds somewhat unnatural the modelling of a black hole by a vacuum region of spacetime, possessing a non-Euclidian topology which is a “mirror image” of the rest of the universe. In particular, why should the way a given black hole is modeled depend on the contents of the outside world?

As an alternative inner boundary condition, W. G. Unruh has suggested requiring that the inner boundary be a marginally trapped surface. This appears to avoid most of the difficulties mentioned above. In particular, the grid is kept outside the (apparent) event horizon and the generalization to multiple black holes is straightforward. The author also finds the physical significance of this boundary condition much clearer. It allows us to exclude the interiors of the trapped surfaces from our space, so we need not deal with such difficulties as how to model the actual (non-vacuum) singularities therein and how to treat them numerically.

It should be noted however, that there is no guarantee that the marginally trapped surface in question will be an *outer* trapped surface (apparent horizon). In particular, as Čadež (1974) showed, if we have two black holes close enough together, a third (outer) marginally trapped surface forms, enclosing them both. This latter surface is the desired inner grid boundary. One solution to this difficulty is to use the marginally trapped boundary condition on the “inner” grid boundary surrounding each black hole, then search the resulting space for additional marginally trapped surfaces. However, this has not yet been done for the spaces constructed in the present thesis.

The actual boundary condition on Ψ which results from the marginally trapped surface boundary condition will now be derived. Bowen and York (1980, equation

(44)) find that the equation for a marginally trapped surface (in the “hat-means-flat” space) is

$$\nabla_i s^i + 4s^i \nabla_i (\ln \Psi) + \frac{\hat{K}_{ij} s^i s^j}{\Psi^4} = 0, \quad (28)$$

where s^i is the outward pointing unit normal to the surface. The first term, being just the 3-divergence of the vector field s^i , can be rewritten (see for example, MTW page 222, equation (8.51c)) as

$$\nabla_i s^i = \frac{1}{\sqrt{{}^{(3)}g}} \partial_i \left(\sqrt{{}^{(3)}g} s^i \right), \quad (29)$$

where ${}^{(3)}g$ denotes the determinant of the matrix ${}^{(3)}g_{ij}$. The second term can be rewritten as

$$\begin{aligned} 4s^i \nabla_i (\ln \Psi) &= 4s^i \partial_i (\ln \Psi) \quad (\text{since } \Psi \text{ is a scalar}) \\ &= \frac{4s^i \partial_i \Psi}{\Psi}. \end{aligned} \quad (30)$$

Hence (28) becomes

$$\frac{1}{\sqrt{{}^{(3)}g}} \partial_i \left(\sqrt{{}^{(3)}g} s^i \right) + \frac{4s^i \partial_i \Psi}{\Psi} + \frac{\hat{K}_{ij} s^i s^j}{\Psi^4} = 0. \quad (31)$$

We must now find the coordinate version of (31) in whatever coordinate system will be used immediately surrounding black holes. Taking this to be polar spherical coordinates centered on the black hole in question and taking the marginally trapped surface to be the 2-sphere $r = \text{constant}$, we have $\sqrt{{}^{(3)}g} = r^2 \sin \theta$ and $s^i = (1, 0, 0)$. (31) then becomes

$$\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial r} (r^2 \sin \theta) + \frac{4}{\Psi} \frac{\partial \Psi}{\partial r} + \frac{\hat{K}_{rr}}{\Psi^4} = 0, \quad (32)$$

or

$$\frac{\partial \Psi}{\partial r} + \frac{\Psi}{2r} + \frac{1}{4} \frac{\hat{K}_{rr}}{\Psi^3} = 0. \quad (33)$$

This is the marginally trapped surface boundary condition used in the present work.

It is desirable to use the same computer code to handle as many as possible of the boundary conditions, so we merge (25), (27) and (33) into a single “generic” boundary condition

$$\frac{\partial \Psi}{\partial r} + b_1 \Psi + b_2 + \frac{b_3}{\Psi^3} = 0, \quad (34)$$

where the coefficients b_1 , b_2 , and b_3 are defined by

$$\left. \begin{array}{lll} \text{Robin} & b_1 & b_2 & b_3 \\ & +\frac{1}{r} & -\frac{1}{r} & 0 \\ \text{Reflection Symmetric} & +\frac{1}{2r} & 0 & 0 \\ \text{Marginally Trapped} & +\frac{1}{2r} & 0 & \frac{1}{4}\hat{K}_{rr} \end{array} \right\} \quad (35)$$

with both r and $\hat{K}_{rr}$ interpreted in a polar spherical coordinate system centered on either a “global origin” (for the “Robin” entries) or on the black hole in question (for the “Reflection Symmetric” and “Marginally Trapped” entries). Note that if multiple black holes are present, some care is needed to compute the correct $\hat{K}_{rr}$ values for the latter case; this will be discussed in detail later.

We shall frequently find that our spaces have a mirror image symmetry about the $z = 0$ plane (hereinafter the “equator”) which can be exploited by solving our problem only on (say) the $z \geq 0$ half-space and imposing a suitable boundary condition on the equator. To derive this boundary condition, we observe that mirror image symmetry about the equator implies that Ψ must be an even function across it. Hence

$$\frac{\partial \Psi}{\partial z} = 0 \quad (\text{on the equator}) \quad (36)$$

for cylindrical coordinates, and

$$\frac{\partial \Psi}{\partial \theta} = 0 \quad (\text{on the equator}) \quad (37)$$

for polar spherical coordinates.

We have now discussed boundary conditions for all the boundaries of our spaces. However, both polar spherical and cylindrical coordinates have the z axis as a boundary of their coordinate systems. We thus require a boundary condition there, which we now derive.

We assume that Ψ itself remains smooth across the z axis, in particular that all Ψ 's directional derivatives are continuous there. But if ℓ denotes a vector perpendicular to this axis, then the directional derivative operator in the ℓ direction switches from being (in cylindrical coordinates) a $-\rho$ derivative to a $+\rho$ derivative as we cross the axis in the ℓ direction, so we must have

$$\frac{\partial \Psi}{\partial \rho} = 0 \quad (\text{on the } z \text{ axis}) . \quad (38)$$

For polar spherical coordinates, the same argument yields

$$\frac{\partial \Psi}{\partial \theta} = 0 \quad (\text{on the } z \text{ axis}) . \quad (39)$$

Choice of coordinates

We now discuss the problem of choosing a suitable set of coordinates for spaces containing multiple (say two) black holes.

The numerical treatment of boundary conditions is much simpler if they occur only at constant coordinate surfaces, so we desire each black hole to have its surface be a constant coordinate surface of our coordinate system. Indeed, we would ideally like our coordinates to approach polar spherical (centered on the black hole's "center") as we approach each black hole, to reflect its approximate spherical symmetry.

As mentioned earlier, our space is asymptotically flat, so we again desire our coordinates to reflect the asymptotic spherical symmetry this implies by approaching polar spherical coordinates at large distances from the strong field region containing the black holes. Of course our actual numerical grid will only be of finite size, but it is still reasonable to require that its outer boundary resemble a sphere as much as possible. In particular, this boundary should be a constant coordinate surface (for the same reasons as the black hole surfaces), and its outward directed normal should be fairly close to a radial one, to minimise errors in computing surface integrals over this outer boundary. Because Ψ depends only weakly on θ near the outer boundary, this condition helps keep its radial and angular dependence decoupled. This allows anisotropic coordinate grids with (for example) very low angular resolution yet fairly high radial resolution.

Finally, it is certainly a reasonable requirement that our coordinate system be free of singularities, at least exterior to the black holes. This is important for numerical reasons, since a coordinate singularity would probably cause severe rounding errors, or even floating point over/underflow problems.

Unfortunately, the three conditions given above cannot be satisfied by any single coordinate system. This is because there are by assumption *two* black hole surfaces, each topologically a sphere, but only *one* outer grid boundary, also topologically a sphere. Clearly no coordinate system can make a *continuous* transition from having two spheres as constant coordinate surfaces, to only one. Thus one or more of the conditions given must be sacrificed.

Past work on this and similar problems, notably that of Smarr (1975), Eppley (1975), and Čadež (1971), (who studied the head-on collision of two black holes by time evolving analytically obtained initial data) has usually chosen to sacrifice the requirement that the coordinate system be free of singularities. Coordinates related to the equipotentials of electric charges placed at the black hole positions have often been used; these have saddle point singularities in between the black holes.

In a prototype of the code described in this thesis, bispherical coordinates were used, which placed a coordinate singularity at spatial infinity. One result of this was that the outer grid boundary was no longer a single constant coordinate surface, but was instead the union of three such surfaces. This proved inconvenient in a number of ways, including difficulty in varying the grid spacings in a local manner and problems with joining the two outer boundary surfaces. Physical insight into the numerical solutions was made more difficult by the same coordinate playing the dual role of “radius” near a black hole, and “angle” near the outer grid boundary. Also, these coordinates fail badly to approach polar spherical at large distances from the black holes.

Most such “global analytic” solutions to the coordinate choice problem yield coordinates which give rise to algebraically “messy” and complicated 3-metrics $^{(3)}g_{ij}$, even for the flat case f_{ij} . For non-flat 3-metrics (as would probably be used in a time

evolution calculation) this difficulty would be much more serious.

This thesis describes an alternative solution to the problem, namely the use of multiple coordinate patches in the numerical solution. Thus each black hole can be surrounded with a polar spherical patch centered on it. A polar spherical patch centered on some convenient origin can be used for the outer region of the numerical grid, and the volume in between can be filled with (for example) a rectangular coordinate patch.

Given that the boundaries between the patches can be dealt with, this scheme offers several advantages: It is not specialized to a fixed number of black holes, so the same computer code can deal with a wider variety of physical systems. Because each patch may have its own coordinate system, there is a great deal of flexibility in locally adapting the coordinates to the physics of the system, while still retaining a simple 3-metric in each patch.

In this thesis, the scheme mentioned above of using polar spherical coordinates both for the neighborhoods of black holes and for the outer regions of the numerical grid is used. For single black hole models, no other coordinate types are needed. For multiple (2 in this thesis) black hole models, cylindrical coordinates are used to fill the volume in between the neighborhoods of the black holes and the outer part of the grid. (These are preferred over the rectangular coordinates mentioned above because of the axisymmetry of our spaces.) Other types of coordinate systems have thus far been neither needed nor tried, but it would be easy to incorporate them into the computer code described herein.

Another advantage offered by the use of patches is that any remaining coordinate singularities (for example the one at the origin of polar spherical coordinates) can be

easily avoided by placing a nonsingular coordinate patch at the offending location. One place in this work where this might have been needed is the z axis, but the singularities there are sufficiently mild that it proved easy to deal with them analytically. None of the polar spherical coordinate patches used herein contain their origins, so the singularities occurring there are not a problem.

Another important issue in coordinate system selection is the need for varying and/or anisotropic resolution in the numerical grid. In the present work, this is dealt with by introducing new patches, while keeping the grid size constant (although possibly anisotropic) within each patch. The full generality of the patch structure used in this thesis is not really needed for this function, but the overhead of introducing new patches is small.

A detailed discussion of just how coordinate patches are used in the actual solution of our problem will be delayed until after the manner of discretizing the equations is discussed in the next chapter.

Finite Differencing the Equations

The previous chapters have shown that we must solve the nonlinear elliptic PDE

$$\nabla^2 \Psi + S \Psi^{-7} = 0 \quad (40)$$

on the region of space exterior to the black holes, using both polar spherical and cylindrical coordinates. We must do so subject to the boundary conditions

$$\frac{\partial \Psi}{\partial r} + b_1 \Psi + b_2 + \frac{b_3}{\Psi^3} = 0 \quad (41)$$

on both the inner boundary of the patch surrounding each black hole and the outer boundary of the outermost patch, and

$$\left. \begin{aligned} \frac{\partial \Psi}{\partial \rho} &= 0 && \text{(cylindrical coordinates, on the } z \text{ axis)} \\ \frac{\partial \Psi}{\partial z} &= 0 && \text{(cylindrical coordinates, on the equator)} \\ \frac{\partial \Psi}{\partial \theta} &= 0 && \text{(polar spherical coordinates, on the } z \text{ axis)} \\ \frac{\partial \Psi}{\partial \theta} &= 0 && \text{(polar spherical coordinates, on the equator)} \end{aligned} \right\} \quad (42)$$

on the given boundaries. The techniques by which these equations are converted into a set of coupled nonlinear algebraic equations will now be discussed. Only those aspects of finite differencing used in this thesis will be discussed; see Mitchell and Griffiths (1980) for a more detailed discussion and further references on the topic.

The basic idea is to consider the functions (eg. Ψ) to be defined on a finite “grid” of points (typically uniformly spaced), and to approximate the differential operators by suitable differences of grid-function values (thus the name “finite differences” for this technique). The resulting finite difference operators are then substituted for the differential operators to yield an algebraic equation, which must hold at each grid point. The boundary conditions are treated in a similar manner, as will be discussed later.

To illustrate the technique, consider first a single function f of a single variable x , and suppose that x has been discretized to a uniform grid $x_i = x_0 + i \Delta x$. (Within this chapter we will use subscripts mainly to denote evaluation at the corresponding grid point.) Taylor expanding $f(x)$ about x_i , we find that

$$f'_i \approx \frac{f_{i+1} - f_{i-1}}{2 \Delta x} \quad (43)$$

and that

$$f''_i \approx \frac{f_{i+1} - 2f_i + f_{i-1}}{(\Delta x)^2}, \quad (44)$$

where we have made the approximation of neglecting $O((\Delta x)^2)$ and higher terms in the Taylor series.

These two “centered second order” finite difference formulas are very widely used. The letter h is often used as a “generic” grid spacing Δx ; these formulas are then referred to as being “ $O(h^2)$ accurate”. Note that this “accuracy” (more properly known as “local truncation error”) refers only to the degree to which the difference equations *locally* approximate the differential equation. Whether or not the *solution* of the difference equations *globally* approximates the *solution* of the differential equation (assuming both to exist) to the same extent is in general a rather difficult question to answer. Fortunately, this latter property turns out to hold for all difference schemes discussed in this thesis, so this problem will not be further considered.

Continuing with our example, suppose that f satisfies the differential equation

$$f''(x) + x = 0. \quad (45)$$

We finite difference this by substituting the approximation (44) for f'' at a typical grid point x_i , to give the *algebraic* equation

$$\frac{f_{i+1} - 2f_i + f_{i-1}}{\Delta x^2} + x_i = 0, \quad (46)$$

which we require to be satisfied at each grid point, i.e. for each value of i .

We now consider boundary conditions for our example problem (45). Suppose we have that

$$f(0) = 0 \tag{47}$$

and

$$f'(0) - 1 = 0. \tag{48}$$

Suppose also that we have a grid starting at $x_0 = 0$ (it is usual in finite differencing to choose the grid so that all boundaries occur at grid points, as we have done here) and extending in the positive x direction (we ignore the right hand boundary of the grid for this example).

The boundary condition (47) is easily dealt with—just set $f_0 = 0$. However, boundary condition (48) requires that we have a finite difference approximation to $f'(0)$. Using the difference scheme (43) given above, this requires a value f_{-1} for $f(x)$ at the nonexistent grid point x_{-1} , as does the finite difference form (46) of the differential equation when evaluated at the grid point x_0 .

One solution to this difficulty lies in using the boundary condition to *extrapolate* a value for f_{-1} , which can then be substituted in the finite difference form (46) of the differential equation for the grid point x_0 . This substitution can be done either analytically (resulting in a modified finite difference form of the differential equation at the boundary point, which implicitly incorporates the boundary conditions), or numerically, by actually including the extrapolated function value as an extra variable in the algebraic equations. The latter technique is used in this thesis, so our example

boundary condition (48) becomes

$$\frac{f_1 - f_{-1}}{2 \Delta x} - 1 = 0. \quad (49)$$

This completes the finite differencing of our example; we now have one algebraic equation for each unknown function value.

We now return to our main problem, namely the finite differencing of (40) subject to the boundary conditions (41) and (42). We will need to do this for each coordinate system type used, namely cylindrical (ρ, z, ϕ) and polar spherical (r, θ, ϕ) . Finite differencing in a general curvilinear coordinate system is not quite as trivial as the one-dimensional case considered above, so we now discuss the finite difference approximation to $\nabla^2 \Psi$ for use in (40) above. The ϕ terms in the ∇^2 operator can be omitted due to the axisymmetry of our problem.

In cylindrical coordinates,

$$\nabla^2 \Psi = \frac{1}{\rho} \left[\frac{\partial}{\partial \rho} \left(\rho \frac{\partial \Psi}{\partial \rho} \right) \right] + \frac{\partial^2 \Psi}{\partial z^2} \quad (50)$$

$$= \frac{\partial^2 \Psi}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \Psi}{\partial \rho} + \frac{\partial^2 \Psi}{\partial z^2}, \quad (51)$$

assuming for the moment that $\rho \neq 0$.

While the two forms (50) and (51) are certainly equal, the straightforward application of the approximations (43) and (44) to them will not necessarily give the same finite difference equations. This is symptomatic of a general problem in finite differencing—the finite difference form of a given (continuum) equation is not unique. The different forms may vary greatly in accuracy. We shall defer consideration of the form (50) until later in this chapter, and proceed now with the finite differencing of (51).

We do this (still assuming that $\rho \neq 0$) by substituting the standard $O(h^2)$ centered

approximations (43) and (44) for the partial derivatives. This gives

$$\begin{aligned}
 (\nabla^2 \Psi)_{i,j} &\approx \left[\frac{\Psi_{i-1,j} - 2\Psi_{i,j} + \Psi_{i+1,j}}{(\Delta\rho)^2} \right] + \frac{1}{\rho_{i,j}} \left[\frac{\Psi_{i+1,j} - \Psi_{i-1,j}}{2\Delta\rho} \right] \\
 &\quad + \left[\frac{\Psi_{i,j-1} - 2\Psi_{i,j} + \Psi_{i,j+1}}{(\Delta z)^2} \right] \\
 &= c_1 \Psi_{i-1,j} + c_2 \Psi_{i+1,j} + c_3 \Psi_{i,j-1} + c_4 \Psi_{i,j+1} + c_5 \Psi_{i,j},
 \end{aligned} \tag{52}$$

where we have defined the coefficients

$$\left. \begin{aligned} c_1 &= \frac{1}{(\Delta\rho)^2} - \frac{1}{2\rho\Delta\rho} \\ c_2 &= \frac{1}{(\Delta\rho)^2} + \frac{1}{2\rho\Delta\rho} \\ c_3 &= c_4 = \frac{1}{(\Delta z)^2} \\ c_5 &= -(c_1 + c_2 + c_3 + c_4) \end{aligned} \right\} \tag{53}$$

at the (i,j) th grid point. It is common to write (52) as the convolution

$$(\nabla^2 \Psi)_{i,j} = \begin{bmatrix} & c_4 & \\ c_1 & c_5 & c_2 \\ & c_3 & \end{bmatrix} \circ (\Psi_{i,j}) \tag{54}$$

of the "molecule" with coefficients $\{c_i\}$ with the matrix of Ψ values centered on the point (i,j) . In this notation, the molecule coefficients are always assumed to be evaluated at the central grid point (i,j) , so for example " ρ " denotes $\rho_{i,j}$.

This difference scheme must be modified on the z axis, since the second term of (51) becomes singular there. The Neumann boundary conditions (42) allow us to apply L'Hopital's rule in its "0/0" form to the offending term of (51), to obtain

$$\nabla^2 \Psi = 2 \frac{\partial^2 \Psi}{\partial \rho^2} + \frac{\partial^2 \Psi}{\partial z^2} \quad (\rho = 0). \tag{55}$$

We finite difference this in the same manner as before, to give the molecule with coefficients

$$\left. \begin{aligned} c_1 &= c_2 = \frac{2}{(\Delta\rho)^2} \\ c_3 &= c_4 = \frac{1}{(\Delta z)^2} \\ c_5 &= -(c_1 + c_2 + c_3 + c_4). \end{aligned} \right\} \tag{56}$$

We now consider the finite differencing of $\nabla^2\Psi$ in polar spherical coordinates. We have

$$\nabla^2\Psi = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \Psi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \Psi}{\partial \theta} \right) \quad (57)$$

$$= \frac{\partial^2 \Psi}{\partial r^2} + \frac{2}{r} \frac{\partial \Psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Psi}{\partial \theta^2} + \frac{\cos \theta}{r^2 \sin \theta} \frac{\partial \Psi}{\partial \theta}, \quad (58)$$

assuming that $\theta \neq 0$ or π . We again defer consideration of the first of these forms (57), and proceed to finite difference (58).

Using the same procedure as before, we obtain the molecule with coefficients

$$\left. \begin{aligned} c_1 &= \frac{1}{(\Delta r)^2} - \frac{1}{r \Delta r} \\ c_2 &= \frac{1}{(\Delta r)^2} + \frac{1}{r \Delta r} \\ c_3 &= \frac{1}{r^2 (\Delta \theta)^2} - \frac{\cos \theta}{2r^2 \sin \theta \Delta \theta} \\ c_4 &= \frac{1}{r^2 (\Delta \theta)^2} + \frac{\cos \theta}{2r^2 \sin \theta \Delta \theta} \\ c_5 &= -(c_1 + c_2 + c_3 + c_4), \end{aligned} \right\} \quad (59)$$

still assuming that $\theta \neq 0$ or π . If this latter assumption does not hold, we can again apply L'Hopital's rule to the offending term (the last one) in (58) to obtain

$$\nabla^2\Psi = \frac{\partial^2 \Psi}{\partial r^2} + \frac{2}{r} \frac{\partial \Psi}{\partial r} + \frac{2}{r^2} \frac{\partial^2 \Psi}{\partial \theta^2} \quad (\theta = 0 \text{ or } \pi). \quad (60)$$

Finite differencing this in the same manner as before gives the molecule with coefficients

$$\left. \begin{aligned} c_1 &= \frac{1}{(\Delta r)^2} - \frac{1}{r \Delta r} \\ c_2 &= \frac{1}{(\Delta r)^2} + \frac{1}{r \Delta r} \\ c_3 &= c_4 = \frac{2}{r^2 (\Delta \theta)^2} \\ c_5 &= -(c_1 + c_2 + c_3 + c_4). \end{aligned} \right\} \quad (61)$$

We now return to consider the finite differencing of (57). If we were to apply the approximation (43) to each of the partial derivatives in (57), we would obtain a molecule

with nonzero entries farther away from the central point than we have previously seen (2 grid points rather than 1). This would greatly complicate the treatment of boundary points. As an alternative to this use of (43), we introduce the concept of a "staggered" grid. We first consider this in the context of our example problem (45).

We define the staggered grid by $x_{i+\frac{1}{2}} = x_0 + (i + \frac{1}{2}) \Delta x$. We Taylor expand $f(x)$ about $x_{i+\frac{1}{2}}$ to obtain the approximation

$$f'_{i+\frac{1}{2}} \approx \frac{f_{i+1} - f_i}{\Delta x}, \quad (62)$$

where we have again dropped the second order and higher terms in the Taylor expansion. Having defined f' on the staggered grid, we can now use it to derive an approximation for f'' on the original grid by Taylor expanding the function $f'(x)$ about x_i and dropping the second and higher order terms. This gives

$$f''_i = (f')'_i \approx \frac{f'_{i+\frac{1}{2}} - f'_{i-\frac{1}{2}}}{\Delta x}. \quad (63)$$

For our example problem (45), this doesn't get us anywhere, since substituting (62) into (63) just gives the approximation (44) again. However, the technique used here is directly applicable to the finite differencing of (57), to which we now return.

We apply the staggered grid technique to the finite differencing of (57) by first applying the approximation (63) to the "outer" derivatives, then using the approximation (62) to evaluate the "inner" derivatives on the staggered grid points where they are

needed. We have

$$\begin{aligned}
& (\nabla^2 \Psi)_{i,j} \\
& \approx \frac{1}{r_{i,j}^2} \frac{\left[\left(r^2 \frac{\partial \Psi}{\partial r} \right)_{i+\frac{1}{2},j} - \left(r^2 \frac{\partial \Psi}{\partial r} \right)_{i-\frac{1}{2},j} \right]}{\Delta r} \\
& \quad + \frac{1}{r_{i,j}^2 \sin \theta_{i,j}} \frac{\left[\left(\sin \theta \frac{\partial \Psi}{\partial \theta} \right)_{i,j+\frac{1}{2}} - \left(\sin \theta \frac{\partial \Psi}{\partial \theta} \right)_{i,j-\frac{1}{2}} \right]}{\Delta \theta} \\
& \approx \frac{1}{r_{i,j}^2} \frac{1}{\Delta r} \left[r_{i+\frac{1}{2},j}^2 \left(\frac{\Psi_{i+1,j} - \Psi_{i,j}}{\Delta r} \right) - r_{i-\frac{1}{2},j}^2 \left(\frac{\Psi_{i,j} - \Psi_{i-1,j}}{\Delta r} \right) \right] \\
& \quad + \frac{1}{r_{i,j}^2 \sin \theta_{i,j}} \frac{1}{\Delta \theta} \left[\sin \theta_{i,j+\frac{1}{2}} \left(\frac{\Psi_{i,j+1} - \Psi_{i,j}}{\Delta \theta} \right) - \sin \theta_{i,j-\frac{1}{2}} \left(\frac{\Psi_{i,j} - \Psi_{i,j-1}}{\Delta \theta} \right) \right],
\end{aligned} \tag{64}$$

which gives the molecule with coefficients

$$\left. \begin{aligned}
c_1 &= \frac{1}{(\Delta r)^2} \left(1 - \frac{\Delta r}{2r} \right)^2 \\
c_2 &= \frac{1}{(\Delta r)^2} \left(1 + \frac{\Delta r}{2r} \right)^2 \\
c_3 &= \frac{1}{r^2 (\Delta \theta)^2} \frac{\sin \theta_{i,j-\frac{1}{2}}}{\sin \theta_{i,j}} \\
c_4 &= \frac{1}{r^2 (\Delta \theta)^2} \frac{\sin \theta_{i,j+\frac{1}{2}}}{\sin \theta_{i,j}} \\
c_5 &= -(c_1 + c_2 + c_3 + c_4).
\end{aligned} \right\} \tag{65}$$

Finally, if $\theta = 0$ or π , we can again apply L'Hopital's rule to the offending term in (57) to obtain

$$\nabla^2 \Psi = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \Psi}{\partial r} \right) + \frac{2}{r^2} \frac{\partial^2 \Psi}{\partial \theta^2}, \quad (\theta = 0 \text{ or } \pi) \tag{66}$$

which gives the molecule with coefficients

$$\left. \begin{aligned}
c_1 &= \frac{1}{(\Delta r)^2} \left(1 - \frac{\Delta r}{2r} \right)^2 \\
c_2 &= \frac{1}{(\Delta r)^2} \left(1 + \frac{\Delta r}{2r} \right)^2 \\
c_3 &= c_4 = \frac{2}{r^2 (\Delta \theta)^2} \\
c_5 &= -(c_1 + c_2 + c_3 + c_4).
\end{aligned} \right\} \tag{67}$$

The application of the staggered grid differencing technique to the finite differencing of the form (50) turns out to give the same molecule (53) as was obtained by finite differencing (51) in the usual way, so we shall not consider this case further.

It can be shown that finite differencing ∇^2 (using the staggered grid differencing technique) in the form (50) or (57), results in the solution to the (linear) PDE $\nabla^2 \Psi = 0$ obeying a local finite difference form of Gauss's law. Such finite difference schemes (as (65)) are thus known as "locally conservative". Their properties (in contrast with the "independent terms" difference schemes such as (53) or (59)) are beyond the scope of this thesis, but they are often more accurate. Eppley (1977, pp. 1610–1612) discusses them in more detail, and gives further examples.

We now consider the boundary conditions (41) and (42). These are treated in the same manner as the example (48) above, i.e. the first derivatives are approximated using the standard $O(h^2)$ form (43), which is used to extrapolate a value for Ψ one grid spacing beyond the boundary. This value is then available for use with the ∇^2 molecules derived above; its substitution in the appropriate molecule is done numerically. Thus, if the actual boundary occurs at the "end" grid point Ψ_e (we suppress the coordinate running parallel to the boundary line), we extrapolate an "exterior" value $\Psi_{e\pm 1}$ one grid spacing beyond the boundary (which may be in the direction either of increasing or decreasing grid coordinate) based on Ψ_e and the "interior" value $\Psi_{e\mp 1}$ defined one grid spacing inside the boundary.

For the boundary condition (41), we thus have

$$\Psi_{e\pm 1} \approx \Psi_{e\mp 1} + d_1 \Psi_e + d_2 + \frac{d_3}{\Psi_e^3}, \quad (68)$$

where

$$d_i = \begin{cases} +2 \Delta r b_i & \text{(for an "inner" boundary)} \\ -2 \Delta r b_i & \text{(for the "outer" boundary)} \end{cases} \quad (69)$$

and the b_i are defined by (35). The Neumann boundary conditions (42) are treated the same way, and reduce to the simple expression

$$\Psi_{e\pm 1} \approx \Psi_{e\mp 1}. \quad (70)$$

Coordinate Patches – I

Now that we have discussed the finite differencing of our problem, the requirements on a patch system (i.e. the questions that the patch data structures must be able to answer) can be defined, and the means chosen to implement patch systems can be discussed in more detail. The key problem here is how to handle the interpatch boundaries. We must also be able to describe the geometry of the patch system, i.e. how the patches fit together to cover our space. We now discuss the basic principles of the patch system used, and the means by which it addresses these problems.

Each patch uses either polar spherical or cylindrical coordinates. The latter are also used as a “global” coordinate system to refer to positions in our space without reference to any particular patch. The coordinate system of any single patch need not be centered on the global origin, although in practice only polar spherical patches will be centered elsewhere, and these must always be centered somewhere on the z axis to preserve axisymmetry.

In order to keep the discussion applicable to both patch types, we introduce generic (2-dimensional) patch coordinates (x, y) , corresponding to (ρ, z) and (r, θ) in cylindrical and polar spherical patches respectively. Given the axisymmetry of our spaces, we will suppress the ϕ coordinate in the following discussion.

Each patch is then most simply described by first looking at it in its own (x, y) coordinate system, i.e. we view cylindrical patches in (ρ, z) coordinates and polar spherical patches in (r, θ) coordinates. We take the grid spacings to be uniform within each patch, so the coordinate grid becomes a uniform rectangular “mesh” of straight (coordinate) lines overlaying the patch. We label these lines with integer coordinates (i, j) , with

unit grid spacing in both directions. For the present work, the origins of the (x, y) and the (i, j) systems are taken to coincide. This is in no way essential to the development, but the extra generality of allowing them to be distinct has not been needed.

We now discuss in more detail just how the “geometry” of the patch is described. It suffices to describe the intersection of each (coordinate grid) line with the patch. A patch is said to be “manhattan convex” if and only if each such intersection is a finite line segment. All patches are taken to satisfy this constraint, which has not in practice proved onerous. (At worst, a region of our space which is logically a single patch but is not manhattan convex, must be split into two or more manhattan convex patches. This occurs, for example, in the patch system shown in figure 9.) Given this constraint, it suffices to describe the two *ends* of each line. In particular, the *shape* of a patch is described by giving the value of i or j (as appropriate) at each line end. This information is stored as part of a “line end descriptor” for each separate line end.

There remains the problem of how to describe the way the different patches fit together to cover the whole space. Truly “global” information (eg. “patch #4 is surrounded by patches #2, #5, and #9”) is very hard to describe in a useful way, but fortunately is not really needed. What is needed, though, is a means to describe “locally” how the patches fit together.

In particular, as well as describing *where* each line in a patch ends, a description is needed of *how* it ends. There are two basic possibilities for this: it can end either by “running into” an actual boundary of the overall coordinate system (hereinafter a “true” boundary), or it can end by encountering another patch (hereinafter a “patch” boundary). A flag is kept (in the line end descriptor) for each line end, telling which of these cases holds. An indication of which patch was “run into” or what type of

boundary condition (Neumann, Robin, Marginally Trapped, etc.) is in effect at the boundary is also encoded into the same flag value.

If the line ends in a true boundary, the appropriate choice of boundary condition (70) or of (68), (69), and (35), can then be made. The necessary coefficients d_1 , d_2 , and d_3 for the latter case are also stored in the line end descriptor.

If, on the other hand, the line ends by encountering another patch, then we must somehow obtain a $\Psi_{e\pm 1}$ (in the notation of the previous chapter) value for the “exterior” point which lies one grid step beyond the end of this line. We require that the exterior point lie *within* the other patch. This amounts to requiring that any gaps between patches can’t exceed the minimum of the grid sizes of the patches in question, which turns out to be fairly easy to arrange. (It usually shows up when switching between polar spherical and rectangular patches, where the effect is to sometimes require an extra polar spherical patch whose only purpose is to match up the grid sizes before the switch is made. An example of this is patch #2 of the patch system shown in figure 4.)

Since its grid point lies *within* the other patch, we can derive $\Psi_{e\pm 1}$ by *interpolating* Ψ from its values in the other patch. Since we are only approximating $\nabla^2 \Psi$ to $O(h^2)$, it suffices to use bilinear interpolation, whose error term is also $O(h^2)$. As shown in figure 1, there are essentially two cases to this interpolation: either we are interpolating into a square (of points in the other patch), or we are interpolating into a triangle.

The coefficients for the interpolation must be derived in a different manner (from the position of the interpolation point) in each case, with the triangle case further broken down into four subcases depending on the triangle’s orientation. However, as shown in figure 2, the net result in all cases is that the interpolated $\Psi_{e\pm 1}$ value is a

Figure 1 – “Square” vs. “Triangle” Interpolation

This figure shows interpolation of values from a rectangular coordinate patch into a polar spherical coordinate patch. The dotted lines and the points at the ends of them show the exterior points of the polar spherical coordinate patch, where the interpolation is done. Those points labelled with “□” lie within a square of the rectangular coordinate patch’s grid points; those labelled with “Δ” lie within a triangle of such points, the “outer” side of which is shown with a dashed line.

linear combination

$$\Psi_{e\pm 1} = d_{00}\Psi_{00} + d_{01}\Psi_{01} + d_{10}\Psi_{10} + d_{11}\Psi_{11} \quad (71)$$

of the other patch’s Ψ values on the four corners of a square (of side one) in the other patch’s (i, j) coordinate system. (The triangle cases all have the “weight” of the “missing corner” set to zero.) The (i, j) coordinates of the lower left corner of that square, and the four interpolation coefficients, are all stored in the line end descriptor.

There is one more detail to be discussed before the description of the patch system is complete. This concerns adjacent patches which have the same coordinate system type. (These can arise either as the result of splitting a “logical” patch into two or

Figure 2 – Bilinear Interpolation Formulae

$$\left. \begin{aligned} d_{00} &= (1-x)(1-y) \\ d_{01} &= (1-x)y \\ d_{10} &= x(1-y) \\ d_{11} &= xy \end{aligned} \right\} \quad (72)$$

$$\left. \begin{aligned} d_{00} &= 1-x-y \\ d_{01} &= y \\ d_{10} &= x \\ d_{11} &= 0 \end{aligned} \right\} \quad (73)$$

$$\left. \begin{aligned} d_{00} &= 1-y \\ d_{01} &= y-x \\ d_{10} &= 0 \\ d_{11} &= x \end{aligned} \right\} \quad (74)$$

$$\left. \begin{aligned} d_{00} &= 1-x \\ d_{01} &= 0 \\ d_{10} &= x-y \\ d_{11} &= y \end{aligned} \right\} \quad (75)$$

$$\left. \begin{aligned} d_{00} &= 0 \\ d_{01} &= 1-x \\ d_{10} &= 1-y \\ d_{11} &= x+y-1 \end{aligned} \right\} \quad (76)$$

Interpolation coefficients for the different possible cases of bilinear interpolation. *For this figure only*, (x, y) is the position of the interpolation point in the interpolation region, the latter being taken to run from 0 through 1 in both x and y .

more manhattan convex patches, or they can be the result of using extra patches to allow local variation in the grid spacing.) It would be possible to "butt joint" such patches, since the interpatch interpolation would then reduce to a simple transfer of Ψ values. However, due to finite differencing errors, the Ψ values obtained at the points common to both patches would not exactly match. It is thus simpler to always leave a gap between such patches, of width equal to the minimum of the two grid spacings perpendicular to their common boundary. This has been done in all patch systems used in this thesis.

Coordinate Patches – II

The previous chapter has discussed the type of coordinate patch system (also known as “grid”) used in this thesis. This system, although well-suited for “internal” computer use, is far too low-level a representation to be used for human input of any non-trivial patch system. Thus, a higher-level “patch description language” is needed, to describe the overall geometry of the patch system. This can then be “compiled” into the appropriate arrays of patch descriptors and line end descriptors. This chapter gives a brief overview of this language. It’s detailed syntax and semantics are fully defined only by the programs (listed in appendix 2) which process it, but an overview should be of interest to anyone trying to use coordinate patches in the numerical solution of PDEs. Numerous examples of patch description language input files may be found in appendix 1.

The key idea of the language is the means used to describe the geometry of each patch. This is done by describing all the different ways the patch’s grid lines can end, for each of the four “sides” of the patch—minimum x , maximum x , minimum y , and maximum y . Each such description takes the form of a series of “boundary definition terms”, each of which either says that the grid lines end at a (given) constant value of x or y (as appropriate), or says that they end by encountering such a boundary of some other patch. The actual shape of a patch is determined by the union of all its terms, i.e. a grid line ends as soon as it “hits” any term.

The compilation algorithm has two main phases: the determination of the shapes of the patches, and the determination of the precise boundary conditions in effect at each line end. The first phase is trivial for “regular” patches, defined as those whose definitions contain only “constant” terms. For “irregular” patches (all others),

we require the specification of rectangular (in the patch's own coordinate systems) "bounding boxes" completely enclosing the patches. These provide starting points for finding patch boundaries by sweeping along grid lines until a term that "hits" is found.

Once the patch shapes have all been determined, each line end of each patch is checked to see which of the available terms it hit. This allows the nature of the boundary conditions in effect at the line end to be determined, so the various flags and coefficients described in previous chapters can be calculated and stored in the appropriate descriptors.

This patch description language has proven adequate for the uses made of it in this thesis, but further enhancements might well be in order. For example, the current language lacks any facilities for parameterizing patch definitions. Defining non-trivial patch systems (such as those used for the two-black hole models of this thesis) is in practice a rather tedious and error-prone task, typically requiring several attempts before the "compilation" is error-free and produces the desired result. Part of this problem may be due to the fairly crude nature of the compilation algorithm and its present implementation. The basic problem is that almost all "computational geometry" is conceptually simple but complicated in practice. This certainly applies to the compilation code, which runs to about 2000 lines of code, and was very hard to debug.

The Extrinsic Curvature

The previous chapters have shown how our problem is discretized to obtain a set of coupled nonlinear algebraic equations. These depend on the particular black hole configuration chosen through the conformally scaled extrinsic curvature tensor $\hat{K}_{ij}$. We now discuss just how this latter object is obtained, starting with the Bowen and York solutions to (14) given by (17) and (18).

These solutions are most simply evaluated in polar spherical coordinates centered on a black hole. Choptuik (1982, pp. 56–57) gives the only non-zero components of $\hat{K}_{ij}$ in these coordinates as

$$\left. \begin{aligned} \hat{K}_{rr} &= \frac{3P \cos \theta}{r^2} \left(1 + \kappa \frac{a^2}{r^2} \right) \\ \hat{K}_{r\theta} &= -\frac{3P \sin \theta}{2r} \left(1 - \kappa \frac{a^2}{r^2} \right) \\ \hat{K}_{\theta\theta} &= -\frac{3P \sin \theta}{2} \left(1 + \kappa \frac{a^2}{r^2} \right) \\ \hat{K}_{\phi\phi} &= -\frac{3P \cos \theta \sin^2 \theta}{2} \left(1 + \kappa \frac{a^2}{r^2} \right) \end{aligned} \right\} \quad (77)$$

for a black hole with momentum P in the positive z direction, and (correcting 2 typographical errors in the cited source)

$$\hat{K}_{r\phi} = -\frac{3J \sin^2 \theta}{r^2} \quad (78)$$

for a black hole with spin vector J in the positive z direction.

For our problem, we in general have multiple black holes, each at a different position, and each possibly having both momentum and spin. Thus we need to transform (77) and (78) for each black hole into a common coordinate system, then sum over all black holes. We take cylindrical coordinates for this common system. The necessary partial

derivatives of $\{r, \theta\}$ with respect to $\{\rho, z\}$ are

$$\left. \begin{aligned} \frac{\partial r}{\partial \rho} &= \frac{\rho - \rho_{\text{BH}}}{r} \\ \frac{\partial r}{\partial z} &= \frac{z - z_{\text{BH}}}{r} \\ \frac{\partial \theta}{\partial \rho} &= \frac{z - z_{\text{BH}}}{r^2} \\ \frac{\partial \theta}{\partial z} &= -\frac{\rho - \rho_{\text{BH}}}{r^2} \end{aligned} \right\} \quad (79)$$

where we take the black hole coordinates to be $(\rho_{\text{BH}}, z_{\text{BH}})$. In practice, we must always have $\rho_{\text{BH}} = 0$ due to the assumed axisymmetry of our space.

Observing that a black hole with both momentum and spin will in general have all its polar spherical coordinate $\hat{K}_{ij}$ components non-zero except for $\hat{K}_{\theta\phi}$ and recalling the symmetry of $\hat{K}_{ij}$ in *any* coordinate system, we find the cylindrical coordinate $\hat{K}_{ij}$ components to be

$$\left. \begin{aligned} \hat{K}_{\rho\rho} &= \hat{K}_{rr} \left(\frac{\partial r}{\partial \rho} \right)^2 + 2\hat{K}_{r\theta} \frac{\partial r}{\partial \rho} \frac{\partial \theta}{\partial \rho} + \hat{K}_{\theta\theta} \left(\frac{\partial \theta}{\partial \rho} \right)^2 \\ \hat{K}_{\rho z} &= \hat{K}_{rr} \frac{\partial r}{\partial \rho} \frac{\partial r}{\partial z} + \hat{K}_{r\theta} \left[\frac{\partial r}{\partial \rho} \frac{\partial \theta}{\partial z} + \frac{\partial \theta}{\partial \rho} \frac{\partial r}{\partial z} \right] + \hat{K}_{\theta\theta} \frac{\partial \theta}{\partial \rho} \frac{\partial \theta}{\partial z} \\ \hat{K}_{\rho\phi} &= \hat{K}_{r\phi} \frac{\partial r}{\partial \rho} \\ \hat{K}_{zz} &= \hat{K}_{rr} \left(\frac{\partial r}{\partial z} \right)^2 + 2\hat{K}_{r\theta} \frac{\partial r}{\partial z} \frac{\partial \theta}{\partial z} + \hat{K}_{\theta\theta} \left(\frac{\partial \theta}{\partial z} \right)^2 \\ \hat{K}_{z\phi} &= \hat{K}_{r\phi} \frac{\partial r}{\partial z} \\ \hat{K}_{\phi\phi} &= \hat{K}_{\phi\phi} \end{aligned} \right\} \quad (80)$$

where the right hand sides are found (in polar spherical coordinates) from (77) and (78).

Having obtained $\hat{K}_{ij}$ in the global cylindrical coordinate system, there are two ways in which we shall use it. First, we must compute the source function

$$S = \frac{1}{8} \hat{K}_{ij} \hat{K}^{ij}. \quad (81)$$

Since this is a scalar (rank zero) object defined by a tensor equation, it is invariant under coordinate transformation. We thus may choose a coordinate system to compute it in solely on the basis of convenience. Since we have all $\hat{K}_{ij}$ components available in the global cylindrical coordinate system, we choose to compute S also in this system. Since

$$^{(3)}g^{ij} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & \rho^{-2} \end{bmatrix} \quad (82)$$

in cylindrical coordinates, we have (again recalling the symmetry of both $\hat{K}_{ij}$ and $\hat{K}^{ij}$)

$$S = \frac{1}{8} \left[\left(\hat{K}_{\rho\rho} \right)^2 + 2 \left(\hat{K}_{\rho z} \right)^2 + \frac{2}{\rho^2} \left(\hat{K}_{\rho\phi} \right)^2 + \left(\hat{K}_{zz} \right)^2 + \frac{2}{\rho^2} \left(\hat{K}_{z\phi} \right)^2 + \frac{1}{\rho^4} \left(\hat{K}_{\phi\phi} \right)^2 \right]. \quad (83)$$

This latter expression can be evaluated as it stands, except on the z axis, where $\rho = 0$. However, this difficulty is only apparent, since the $\hat{K}_{i\phi}$ components involved all contain cancelling factors of ρ . Since $\rho = r \sin \theta$ for polar spherical coordinates centered on (a black hole on) the z axis, the solution is to divide out a factor of $\sin^2 \theta$ from $\hat{K}_{\phi\phi}$ in (77) and a factor of $\sin \theta$ from $\hat{K}_{r\phi}$ in (78), then divide out a factor of r from $\hat{K}_{\rho\phi}$ and $\hat{K}_{z\phi}$ and a factor of r^2 from $\hat{K}_{\phi\phi}$ in (80). We then compute

$$S = \frac{1}{8} \left[\left(\hat{K}_{\rho\rho} \right)^2 + 2 \left(\hat{K}_{\rho z} \right)^2 + 2 \left(\frac{\hat{K}_{\rho\phi}}{\rho} \right)^2 + \left(\hat{K}_{zz} \right)^2 + 2 \left(\frac{\hat{K}_{z\phi}}{\rho} \right)^2 + \left(\frac{\hat{K}_{\phi\phi}}{\rho^2} \right)^2 \right]. \quad (84)$$

The second way the global cylindrical coordinate components of $\hat{K}_{ij}$ are used, is in computing $\hat{K}_{rr}$ for use in the marginally trapped surface boundary condition (33). We must thus transform $\hat{K}_{ij}$ from the global cylindrical coordinates back to the local polar spherical coordinates of each black hole that uses the boundary condition (33).

We find that

$$\hat{K}_{rr} = \hat{K}_{\rho\rho} \left(\frac{\partial \rho}{\partial r} \right)^2 + 2 \hat{K}_{\rho z} \frac{\partial \rho}{\partial r} \frac{\partial z}{\partial r} + \hat{K}_{zz} \left(\frac{\partial z}{\partial r} \right)^2, \quad (85)$$

where the necessary partial derivatives are given by

$$\left. \begin{aligned} \frac{\partial \rho}{\partial r} &= \sin \theta \\ \frac{\partial z}{\partial r} &= \cos \theta. \end{aligned} \right\} \quad (86)$$

Choice of Conformal Function

As mentioned earlier, our basic equation (21) can be viewed as a nonlinear Poisson equation, with the source function S determining the magnitude of the nonlinearity. To the extent that we can ignore this nonlinearity, our solutions to (21) should thus be superpositions of Schwarzschild solutions. In isotropic coordinates (required by our assumption of 3-conformal flatness), the Schwarzschild solution is

$$\Psi_{\text{SCHW}} = 1 + \frac{m/2}{r}, \quad (87)$$

where m is the black hole mass and r is the usual polar spherical radial coordinate in the “hat-means-flat” space. Thus we expect the “monopole sum”

$$\Psi = 1 + \sum_i \frac{\lambda_i}{r_i} \quad (88)$$

where the sum is over all black holes, $\{\lambda_i\}$ is a set of “monopole strengths” which we set to one half the black hole masses, and r_i is the distance from the i th black hole, to be an approximate solution to (21).

Because the boundary conditions play such a crucial role in determining the solution of (21), it is desirable that they be finite differenced as accurately as possible. In particular, this applies to our “generic” boundary condition (34) and its finite difference form (68). When this is applied at an inner (black hole surface) boundary, a single term in the sum of (88) will usually dominate. Likewise, when this is applied at the outer boundary, the asymptotic form (23) is again of the form of (88) with only a single term in the sum.

Thus any mechanism by which we can improve the accuracy of our finite differencing of the boundary condition (34) for solutions of the general form

$$\Psi = 1 + \frac{\lambda}{r}, \quad (89)$$

where λ is an unknown constant, should also improve our accuracy in more general problems. In particular, we want a good finite difference approximation to $\frac{\partial \Psi}{\partial r}$ for such solutions.

This leads to the idea of working with some function $\Omega(\Psi)$ which linearizes this radial derivative. We adopt the function

$$\Omega(\Psi) = \frac{1}{\Psi - 1} \quad (90)$$

for this purpose. This gives Ω as a linear function of r for the solution (89), so our finite difference approximation (68) to the boundary condition (34) is *exact* for this case.

An alternative way of improving the accuracy to which the boundary conditions (both inner and outer) are approximated was tried by Choptuik (1982), who replaced the r coordinate of polar spherical coordinates with the new coordinate

$$s = 1 - \frac{a}{r}. \quad (91)$$

This has the advantage of not putting any constraints on the range of Ψ , whereas (90) requires that Ψ be everywhere greater than unity (or everywhere less than it). On a few occasions when very poor initial guesses are used, the author's code produces extrapolated Ψ values which violate this requirement, with generally disastrous results.

Another advantage to Choptuik's s coordinate is that grid points evenly spaced in s are bunched up near the black hole. This is useful since closely spaced grid points are needed to resolve the rapid variation of Ψ in this region. (The author's code uses multiple patches to perform this function.) However, the definition (91) is closely tied to the use of polar spherical coordinates, and does not easily generalise to the case of multiple coordinate patches. A possible solution would be to adopt (91) on polar spherical patches around black holes and in the neighborhood of infinity, while retaining

more usual coordinates elsewhere. Used in the neighborhood of infinity, (91) allows the direct use of (22) in the form

$$\Psi(s=1) = 1 \quad (92)$$

for an outer boundary condition. Choptuik (1985c) has suggested that this will lead to more reliable values for the total mass-energy of the computed spaces than the Robin outer boundary conditions used in this thesis, but this has not been tried.

We now turn to a more detailed discussion of the use of (90) in the author's code. It was decided to continue to always store and finite difference Ψ (to obtain $\nabla^2\Psi$), but to allow interpatch interpolation and boundary extrapolation to use either Ψ or Ω , the choice being made independently for each patch. To obtain a value $\Psi_{e\pm 1}$ at an interpatch boundary, we use bilinear interpolation in either Ψ or Ω based on the *other* patch's choice of conformal function, since that's the patch $\Psi_{e\pm 1}$ is in. To obtain $\Psi_{e\pm 1}$ at a true boundary of a patch, we extrapolate either Ψ or Ω based on that patch's choice, since there is no other patch involved in this case.

For the interpolation case, (71) is merely replaced by a bilinear interpolation in Ω ,

$$\Omega_{e\pm 1} = \mathbf{d}_{00}\Omega_{00} + \mathbf{d}_{01}\Omega_{01} + \mathbf{d}_{10}\Omega_{10} + \mathbf{d}_{11}\Omega_{11}, \quad (93)$$

where the interpolation coefficients are found as before (figure 2). However, the extrapolation process at true boundaries isn't quite as simple. If the Neumann boundary conditions (42) apply, the extrapolation formula (70) is easily seen to still hold, so we need not consider this case further. However, the generalised boundary condition (41) requires a new treatment, which we now discuss.

From the defining expression (90), we have

$$\Psi = 1 + \frac{1}{\Omega}, \quad (94)$$

so that

$$\frac{\partial \Psi}{\partial r} = -\frac{1}{\Omega^2} \frac{\partial \Omega}{\partial r}. \quad (95)$$

Our boundary condition (41) thus becomes

$$-\frac{1}{\Omega^2} \frac{\partial \Omega}{\partial r} + b_1 \Psi + b_2 + \frac{b_3}{\Psi^3} = 0, \quad (96)$$

which we finite difference in the same manner as before to obtain

$$\Omega_{e\pm 1} = \Omega_{e\mp 1} - (\Omega_e)^2 \left(d_1 \Psi_e + d_2 + \frac{d_3}{\Psi_e^3} \right), \quad (97)$$

with the d_i being defined by (69) and (35) as before.

Solving the Algebraic Equations

The result of finite differencing the PDE under consideration is, as mentioned above, a large set of nonlinear algebraic equations, one for each grid point. In the interior of a patch, each point is “linked” to its four nearest neighbors only; points on interpatch boundaries are linked to the interpolation points in adjacent patches. In either case, the number of points a given point is linked to is far smaller than the total number of grid points in the patch system, i.e. the system of equations is “sparse”. We now discuss the methods used herein to solve these equations.

Most past work in numerical GR has used iterative methods to solve such systems of equations, typically without ever explicitly constructing the systems at once. These methods involve iteration even if the equations are linear. This was tried in early versions of the program described in this thesis, but the convergence of these methods (notably the standard “SOR” one) was found to be at best slow, and sometimes nonexistent.

To avoid this, we adopt a global Newton method, where the entire linearized system of equations is solved simultaneously, iteration occurring only due to the nonlinearity. This gives reliable convergence without “tuning parameters”, at the expense of very slow execution. Choptuik (1982) found the “multigrid” algorithm of Brandt *et al* to be very fast in execution. However, he reported that the programming effort to implement it is substantial—at least several person-months of programming over and above that needed to implement SOR. He also found that some tuning of the parameters of the multigrid algorithm was needed to ensure reliable convergence.

We will now discuss the global Newton method used in this thesis. We begin by

imposing a linear ordering on the grid points, say from 1 to n . (For the present work, the natural ordering of the 3-tuples (p, i, j) is used, where p denotes the patch number and (i, j) the integer patch coordinates of the grid point.) The system of equations can then be written as the single vector equation $\mathbf{F}(\mathbf{x}) = 0$, where $\mathbf{x}$ denotes the vector of n Ψ values, and $\mathbf{F}$ is a suitable vector-valued function.

We now linearize $\mathbf{F}$ about our current solution estimate $\mathbf{x}$,

$$\mathbf{F}(\mathbf{x} + \delta\mathbf{x}) \approx \mathbf{F}(\mathbf{x}) + \mathbf{J} \cdot \delta\mathbf{x}, \quad (98)$$

where $\mathbf{J}$ is the n by n “Jacobian” matrix (in general dependent on $\mathbf{x}$) defined by

$$\mathbf{J}_{kl}(\mathbf{x}) = \frac{\partial \mathbf{F}_k(\mathbf{x})}{\partial x_l}, \quad (99)$$

to be matrix multiplied by the column vector $\delta\mathbf{x}$. Finally, we solve for the “step” $\delta\mathbf{x}$ which would make $\mathbf{F}(\mathbf{x} + \delta\mathbf{x})$ zero if the linear approximation (98) were exact, finding that we must have

$$\mathbf{J} \cdot \delta\mathbf{x} = -\mathbf{F}(\mathbf{x}). \quad (100)$$

In this case, we interpret $\mathbf{F}$ as the vector of residuals, so $\mathbf{J}_{kl}$ represents the partial derivative of the k -th residual with respect to the l -th Ψ value. Note that this must take into account the indirect influence of values in other patches through interpatch interpolation at line ends, as well as the effects of boundary conditions. Given that either or both of these may be done using either Ψ or Ω , the actual calculation of the components of the Jacobian is a rather messy task, although conceptually straightforward. The detailed equations used may be found in the listing of the function `jacobian.c` in appendix 2; they have too many subcases to justify writing them out here.

Like the usual single-variable Newton iteration which it generalizes, this iteration may fail to converge if the initial guess is sufficiently bad, but when it does converge

it does so quadratically, roughly squaring the relative error (doubling the number of correct digits) each iteration. (This latter property provides a useful check that all components of the Jacobian have been calculated correctly.) Because of these two properties, there is a premium placed on having a fairly good initial guess.

The strategy adopted to provide this is twofold. First, we use the monopole sum (88), which corresponds to a linear superposition of Schwarzschild solutions. As discussed in the previous chapter, this provides a good initial guess when the source function S is small. Second, when computing a series of models differing only in black hole momenta or spins (i.e. in S), the Ψ values from previously computed models are used as initial guesses for new ones. This latter technique is known as using a “continuation method”, and was suggested by M. Yedlin.

Our problem is now reduced to that of solving a large sparse system of linear equations. This is a standard problem in numerical linear algebra, and many algorithms are available for solving it. For the present work, the “ILUCG” algorithm of Kernshaw (1978) was used, as embodied in a subroutine package written by P. Madderom and supplied by T. Nicol, both of the UBC Computing Center. This was used “as is” except for a minor modification in its internal convergence criteria, as described in the code listing in appendix 2.

However, one further problem arises in the numerical solution. The Jacobian entries are (ignoring boundary conditions) just the “molecule” components defined earlier, and these all depend on the grid spacing as $1/h^2$. Thus, when using a patch system in which different patches have very different grid spacings, the Jacobian will have entries which differ greatly in magnitude. This usually results in the matrix being numerically ill-conditioned. To try to avoid this, we scale the Jacobian so that the entries on each row

are approximately the same in size. We also scale the right hand side vector $-\mathbf{F}(\mathbf{x})$ by the same amounts, so the solution $\delta\mathbf{x}$ to the system (100) remains unchanged.

Measuring the Masses of the Models

It will prove useful to be able to measure the total energy of the spaces we construct. Also, to the extent that any proper subspace can be said to have a mass associated with it, we would like to measure the masses of the individual black holes which populate our spaces. We now discuss the means by which these measurements are made.

The first useful “mass functional” we shall consider is that associated with the area of the event horizon of a black hole. This quantity (summed over all black holes) always increases (second law of black hole dynamics) in any process involving black holes. Unfortunately, the event horizon of a black hole is a global property of spacetime as a whole, and can’t be found until the complete time evolution of the spacetime is known. However, every (outer) marginally trapped surface must lie *within* an event horizon, so we can place a lower bound on the “horizon area mass” of a black hole by calculating the mass associated with a marginally trapped surface surrounding it. We find the latter simply by specifying marginally trapped surface inner boundary conditions, so our inner grid boundary becomes the desired surface.

The mass M_h associated with any given horizon area A_h is

$$M_h = \sqrt{\frac{A_h}{16\pi}}. \quad (101)$$

This area is found by integrating over the marginally trapped surface,

$$\begin{aligned} A_{\text{MTS}} &= \int_0^{2\pi} \int_0^\pi \Psi^4 r^2 \sin \theta \, d\theta \, d\phi \\ &= 2\pi \int_0^\pi \Psi^4 r^2 \sin \theta \, d\theta, \end{aligned} \quad (102)$$

where we have exploited the axisymmetry to do the ϕ integral analytically.

We now consider means of computing estimates of the total energy of a model. Defining the total energy of a space is a somewhat difficult matter in GR, and is in any

case beyond the scope of this thesis. Wald (1984, section 11.2) discusses this topic in more detail. For our purposes it will suffice to consider the mass functional (see Eppley (1977, p. 1610) for a variety of different formulations)

$$E_{\infty} = -\frac{1}{2\pi} \oint_{\mathbf{S}} \nabla \Psi d\mathbf{S}, \quad (103)$$

where the integral is taken over the 2-sphere of infinite radius $\mathbf{S}$. We approximate this by taking $\mathbf{S}$ as the outer grid boundary. Since we are integrating over all angles, only the radial derivative of Ψ contributes towards the integral, and the Robin outer boundary condition (25) gives this “for free” as

$$\frac{\partial \Psi}{\partial r} \approx -\frac{\Psi - 1}{r}. \quad (104)$$

Hence (again using the axisymmetry to do the ϕ integral analytically)

$$E_{\infty} \approx \int_0^{\pi} \frac{\Psi - 1}{r} r^2 \sin \theta d\theta. \quad (105)$$

Unfortunately, most previous studies have found that such “surface integral” energy functionals are numerically rather inaccurate, due to finite differencing errors. By applying Gauss’s theorem to the volume occupied by the model, the bulk of the surface integral (103) can be replaced by a volume integral of $\nabla^2 \Psi$, which is known analytically by virtue of our basic equation (21). The remaining surface terms are also available “for free” through our inner boundary condition (34). This procedure, although it would almost certainly lead to more accurate energy values, is not used in the present thesis. This is due to the difficulty of doing a volume integral over the model in the presence of multiple coordinate patches with (small) gaps between them. This difficulty can probably be overcome by using information on the global geometry of the patch system, but this has not yet been tried.

Both (102) and (105) involve integrating a function which typically varies little with θ , weighted by $r^2 \sin \theta$. Although this can be done directly, numerical accuracy is considerably improved if a “representative” value of the function is subtracted off the numerical integral and it’s (weighted) integral is done analytically and added back. This is particularly useful for (105), since the patch systems used in this thesis tend to have low resolution in θ near the outer grid boundary.

The actual numerical integrations involved in (102) and (105) are done by fitting a cubic spline (using the Neumann boundary conditions (37) and (39) for the endpoints) and integrating this analytically (via the spline coefficients). We choose this method over the usual choice of Simpson’s rule because it does not constrain the number of integration points (angular grid points) to be an odd number.

Testing the Code

We use two main types of models to test out the code described in this thesis: Schwarzschild models with a variety of different patch systems, and both moving and spinning single black holes with parameters chosen to match those of Choptuik (1982, 1985a). Recall that detailed information about all models mentioned in this thesis is given in appendix 1.

We need to assess the accuracy of various Schwarzschild models. We will do this by comparing three quantities. The first of these is the mass M_{MTS} associated via (101) with the area of the marginally trapped surface at our inner grid boundary, as calculated by (102). The second is the total energy “at infinity” E_∞ , as calculated by (105). The third (and most reliable) is the maximum deviation $\max|\Psi - \Psi_{\text{SCHW}}|$ between the calculated Ψ and the exact Schwarzschild solution (87).

The first main result of the Schwarzschild tests is that the independent terms finite difference scheme given by (59) and (61) turns out to be *exact* (ignoring rounding errors) for the approximate solution (89). In other words, when (89) is substituted into the algebraic equations resulting from this difference scheme, they are satisfied to within the rounding errors, rather than just to $O(h^2)$. Because of this, the independent terms form is used in most of the models run in this thesis. The locally conservative finite difference scheme given by (65) and (67), in contrast, exhibits the expected $O(h^2)$ dependence of errors on grid size. Table 1 gives the results of a test series of models run to verify this dependence; note that each time the grid resolution is doubled, the errors are (approximately) divided by four.

The second main result of the Schwarzschild tests is that good results are (still)

Table 1 – Comparison of Different Grid Sizes

Model	Resolution	M_{MTS}	E_{∞}	$\max \Psi - \Psi_{\text{SCHW}} $
10plo100	10	1.00613	1.01808	6.125×10^{-3}
20plo100	20	1.00147	1.00429	1.474×10^{-3}
40plo100	40	1.00036	1.00105	3.621×10^{-4}

Results of running unit mass Schwarzschild models with different grids. “Resolution” is the value of $\Delta r/r$ on the inner boundary of each patch. This is also the approximate number of angular grid points used in the inner patches. Observe that the errors go down by approximately a factor of four each time the resolution is doubled.

obtained when polar spherical and cylindrical patches are intermixed. Table 2 gives the results of two test series of models run to verify this, one each for locally conservative and independent terms finite differencing schemes. Each test series consists of a baseline model using only polar spherical patches, followed by a model in which one of the patches has been augmented with a cylindrical coordinate patch. The same patch systems were used in both test series; the relevant portions of those patch systems are shown in figures 3 and 4. The table shows that inserting a cylindrical coordinate patch into the (otherwise spherically symmetric) model does not greatly increase the errors. We conclude that the accuracy of our solutions is probably not greatly degraded by the interpatch interpolation process.

The third main result of the Schwarzschild tests is that the use of Ω as a conformal function does indeed linearize the boundary condition (34), at both the inner and outer boundaries. It also helps to linearize the interpatch interpolation done across patch boundaries. Table 3 gives the results of a test series of models run to demonstrate the effects of using Ω compared to using Ψ as a conformal function. The series consists of a baseline case using Ω everywhere, followed by 3 models which successively substitute Ψ on the innermost patch, all the “middle” patches, and the the outermost patch. All

Table 2 – Comparison of Different Patch Types

Locally Conservative Finite Difference Schemes				
Model	M_{MTS}	E_{∞}	$\max \Psi - \Psi_{SCHW} $	Notes
20plo100	1.00147	1.00429	1.474×10^{-3}	(1)
c2	0.99884	0.99985	1.169×10^{-3}	(2)
Independent Terms Finite Difference Schemes				
Model	M_{MTS}	E_{∞}	$\max \Psi - \Psi_{SCHW} $	Notes
c01	1.00000	1.00000	0.0	(1)
c21	0.99767	0.99611	2.344×10^{-3}	(2)

Notes

- (1): Baseline patch system, as shown in figure 3. All patches are polar spherical.
- (2): Baseline patch system with patch #2 augmented with a cylindrical coordinate patch, as shown in figure 4.

Results of running unit mass Schwarzschild models with different grids. Observe that the model results change only slightly when cylindrical coordinate patches are introduced into otherwise spherically symmetric grids. This holds true for both types of finite difference schemes.

of these show larger errors than the baseline model, with the worst being the “inner” case where Ψ is being used to approximate the inner boundary condition. The overall conclusion to be drawn from this test series is that the best accuracy is obtained by using Ω everywhere.

Table 3 – Comparison of Different Conformal Functions

Model	M_{MTS}	E_{∞}	$\max \Psi - \Psi_{SCHW} $	Description
20plo100	1.00147	1.00429	1.474×10^{-3}	Ω used for all patches
poo	0.99648	0.99929	3.519×10^{-3}	Ψ used for innermost patch
opo	1.00196	1.00899	1.963×10^{-3}	Ψ used for “middle” patches
oop	1.00148	1.00493	1.480×10^{-3}	Ψ used for outermost patch

Results of running unit mass Schwarzschild models with different choices of conformal function. Observe that the smallest errors occur when Ω is used for all patches. The “middle” patches are those which are neither innermost nor outermost.

Figure 3 – Baseline patch system for table 2

Only the inner section of the patch system is shown (patches #1, #2, and #3, numbering from the inside out). The ρ axis runs horizontally at the bottom, and the z axis runs vertically at the left. The range of r shown is from 0.5 (at the inner boundary of patch #1) to 4.9 (at the outer boundary of patch #3).

Figure 4 – Augmented patch system for table 2

This should be compared to the patch system shown in figure 3 (previous page). The axes and coordinate ranges given there also apply here. The previous patch #3 is now #4, and a new cylindrical coordinate patch #3 has been inserted. Patch #2 (the "thin" polar spherical one) is used to change to the larger grid size before the polar spherical—cylindrical patch transition.

The other main way we test the code is by comparing its calculated results with those of Choptuik (1982, 1985a), who studied moving and spinning single black holes using the same Bowen and York formalism as this thesis. It should be noted that Choptuik's results are independent of those of this thesis, in that he used both a different coordinate system and two completely different techniques for solving (21). Choptuik (1982) did a full numerical solution of (21) using a different algorithm to solve the finite differenced equations, while Choptuik (1985a) numerically integrated a perturbation solution to (21) valid for small nonlinearities (S values). Since he in both cases used the Bowen and York reflection symmetric inner boundary condition (27), we also use it in constructing models meant for comparison with his.

We now discuss the results of these comparisons. There are three main series of models used, with results given in table 4. These series are moving black hole models with a range of momentum (i.e. P in (77)) values and $\kappa = +1$, moving black holes with the same range of momentum values but $\kappa = -1$, and spinning black holes with a range of angular momentum (i.e. J in (78)) values.

For these models, the differences

$$\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator}) \quad (106)$$

between the calculated Ψ at the "north pole" ($\theta = 0$) and on the equator ($\theta = \pi/2$) are compared with Choptuik's values. (A direct check of Ψ itself with his plots was also made, with satisfactory results.) These comparisons are made at the black hole surfaces ($r = a = 1$), where he gave his results. As can be seen from the table, the results agree with his (1985a) perturbation analysis whenever the nonlinearities are small, and generally agree with his (1982) full numerical solution in the nonlinear regime. The moving black hole models with $P = 7.5$ and $P = 12.5$ for $\kappa = -1$, however,

Table 4 – Comparison of $\delta\Psi$ with Choptuik's Results

Model	P	κ	J	$\delta\Psi(\text{Model})$	$\delta\Psi(\text{Choptuik})$	Notes
ppx001	0.001	+1	0	$+7.985 \times 10^{-9}$	$+8.0 \times 10^{-9}$	(1)
ppx01	0.01	+1	0	$+7.985 \times 10^{-7}$	$+8.0 \times 10^{-7}$	(1)
ppx1	0.1	+1	0	$+7.966 \times 10^{-5}$	$+8.0 \times 10^{-5}$	(1)
pp1	1.0	+1	0	$+6.550 \times 10^{-3}$	$+8.0 \times 10^{-3}$	(1) (2)
pp2x5	2.5	+1	0	$+2.335 \times 10^{-2}$	—	(3)
pp7x5	7.5	+1	0	$+5.586 \times 10^{-2}$	—	(3)
pp12x5	12.5	+1	0	$+7.209 \times 10^{-2}$	—	(3)
mpx001	0.001	-1	0	-1.375×10^{-9}	-1.4×10^{-9}	(1)
mpx01	0.01	-1	0	-1.375×10^{-7}	-1.4×10^{-7}	(1)
mpx1	0.1	-1	0	-1.372×10^{-5}	-1.4×10^{-5}	(1)
mp1	1.0	-1	0	-1.152×10^{-3}	-1.4×10^{-3}	(1) (2)
mp2x5	2.5	-1	0	-4.240×10^{-3}	-4.2×10^{-3}	(4) (5)
mp7x5	7.5	-1	0	-1.030×10^{-2}	-8.0×10^{-3}	(4) (5) (6)
mp12x5	12.5	-1	0	-1.328×10^{-2}	-1.1×10^{-2}	(4) (5) (6)
jx001	0	0	0.001	-2.340×10^{-9}	-2.3×10^{-9}	(1) (7)
jx01	0	0	0.01	-2.340×10^{-7}	-2.3×10^{-7}	(1) (7)
jx1	0	0	0.1	-2.338×10^{-5}	-2.3×10^{-5}	(1) (7)
j1	0	0	1.0	-2.210×10^{-3}	-2.3×10^{-3}	(1) (2) (7)
j10	0	0	10.0	-0.05451	-0.05	(7) (8)
j100	0	0	100.0	-0.1799	-0.18	(7) (8)
j1000	0	0	1000.0	-0.3622	-0.36	(7) (8)

Notes

- (1): Comparison made with Choptuik (1985a) perturbation analysis.
- (2): Discrepancy presumably due to nonlinearities invalidating assumptions of Choptuik (1985a) perturbation analysis.
- (3): No data available in Choptuik (1982) for this case.
- (4): Comparison made with values read graphically from Choptuik (1982, figure 12, page 113) full numerical solution.
- (5): Choptuik (1985d) confirms that $\kappa = -1$ is correct here, rather than $\kappa = +1$ as stated in Choptuik (1982).
- (6): See text for discussion of discrepancy.
- (7): Since $P = 0$, the value of κ is irrelevant here.
- (8): Comparison made with values read graphically from Choptuik (1982, figure 16, page 118) full numerical solution.

show disagreement at the 20% level. This corresponds to errors at the 10^{-3} level in Ψ itself. Choptuik (1985b) has said that he does not consider such disagreements to be significant, given the uncertainties (mostly truncation errors) of both his (1982) full numerical solution and the present one. Thus the overall conclusion of these test series is that the results of this thesis agree with those of Choptuik (1982, 1985a).

Demonstrating the Code

Although the main results of this thesis are the techniques developed, a number of test series of models were run to demonstrate the numerical GR code's capabilities. We now discuss these, beginning with the four single black hole test series. All of the black holes involved have unit nominal mass, in comparison to the unit nominal radius (hence nominal mass of two) used by York and Piran (1982) and Choptuik (1982, 1985a) in similar studies.

The first test series (hereinafter known as the "J" series) consists of models with zero linear momentum P (as defined by (17)) and varying (zero, then from 0.01 to 100 in decade steps) angular momentum J (as defined by (18)). The sign convention is that the angular momentum 3-vector of the black holes is directed along the positive z axis. Since for a single spinning black hole (78) shows $\hat{K}_{rr} = 0$, the marginally trapped surface inner boundary condition (33) used in this test series is identical to the Bowen and York reflection symmetric boundary condition (27) used in the previous studies cited above.

The remaining three test series all consist of models with zero angular momentum J and varying (zero, then from 0.01 to 100 in decade steps) linear momentum P . The sign convention is that the momentum 3-vector of the black holes is directed along the positive z axis. These test series differ in their choice of inner boundary condition, with one (hereinafter the "RSP" series) using the Bowen and York reflection symmetric condition (27) with $\kappa = +1$, one (hereinafter the "RSM" series) with $\kappa = -1$, and one (hereinafter the "NT0" series) using the marginally trapped surface condition (33) with $\kappa = 0$.

The J, RSP, and RSM models all have a mirror image symmetry about the equator, so Ψ is only numerically solved for on the half space $z \geq 0$ for these models. The MT0 models lack this symmetry due to the nonlinear $\hat{K}_{rr}$ term in (33), so they require that Ψ be numerically solved for on the whole space. All of the single black hole models used the same patch system, with all polar spherical patches. The solutions (Ψ shown as a function of position in the space) are shown in figures 5, 6, 7, and 8 for the J, RSP, RSM, and MT0 test series respectively. All of these figures show orthographic (perspective as seen from an “infinite” distance) views of the inner two patches (r from 0.5 to 1.95) of the numerical solution.

The first thing one notices on viewing these plots is the near spherical symmetry of the J, RSP, and RSM test series solutions. Although a detailed multipole analysis of the solutions has not been done, the perturbation analysis of Choptuik (1985a) found the dipole asymmetry to be dominant for small S (i.e. for small J or P).

We now consider the extent to which the different boundary conditions used in the three varying momentum test series just described, effect the solutions obtained. It is clear from examining figure 8 that the marginally trapped surface boundary condition gives models which differ noticeably from the two reflection symmetric cases. This is perhaps to be expected, since the former lack the mirror image symmetry of the latter, and the extra nonlinear $\hat{K}_{rr}$ term in (33) changes sign across the equator (see (77)). This seems to be the cause of the “tilt” which shows up in the high P models of this test series.

In contrast to the substantial difference the marginally trapped surface boundary condition makes, the two cases $\kappa = \pm 1$ of the reflection symmetric boundary condition produce very similar results, as can be seen by comparing figures 6 and 7. However,

Figure 5 - J Test Series Solutions

Ψ is plotted vertically from 0 to 6, ρ forward from 0 to 1.95, z to the right from 0 to 1.95. The lower set of (ρ, z) axes are at $\Psi = 0$, the upper set at $\Psi = 1$. The $J = 0.01$ and $J = 0.1$ models are visually indistinguishable from the $J = 0$ one at this scale, and are hence not plotted.

Figure 6 - RSP Test Series Solutions

Ψ is plotted vertically from 0 to 6, ρ forward from 0 to 1.95, z to the right from 0 to 1.95. The lower set of (ρ, z) axes are at $\Psi = 0$, the upper set at $\Psi = 1$. The $P = 0.01$ and $P = 0.1$ models are visually indistinguishable from the $P = 0$ one at this scale, and are hence not plotted.

Figure 7 - RSN Test Series Solutions

Ψ is plotted vertically from 0 to 6, ρ forward from 0 to 1.95, z to the right from 0 to 1.95. The lower set of (ρ, z) axes are at $\Psi = 0$, the upper set at $\Psi = 1$. The $P = 0.01$ and $P = 0.1$ models are visually indistinguishable from the $P = 0$ one at this scale, and are hence not plotted.

Figure 8 - NTO Test Series Solutions

Ψ is plotted vertically from 0 to 6, ρ forward from 0 to 1.95, z to the right from -1.95 to $+1.95$. The lower set of (ρ, z) axes are at $\Psi = 0$, the upper set at $\Psi = 1$. The black holes are all moving in the positive z direction. The $P = 0.01$ and $P = 0.1$ models are visually indistinguishable from the $P = 0$ one at this scale, and are hence not plotted.

they do produce rather different radial variations of the (small) nonspherical component of Ψ , as shown by the quantitative graphs in appendix 1.

We now consider models with two black holes. Recall that, due to our assumption of axisymmetry, the black holes must both lie on the z axis, as must their linear and angular momentum vectors. Due to computer memory limitations, only one series of two black hole models has been run so far. This "PP" series has two unit mass black holes with equal and opposite momenta (and no spin). These momenta vary from -100 to -0.01 in decade steps, then zero, then from $+0.01$ to $+100$ in decade steps. The sign convention is that positive momenta indicate the black holes are approaching each other. The two black holes lie at positions $z = \pm 1.5$. The patch system used for this test series is shown in figure 9.

The solutions for this test series are shown in figures 10 and 11, plotted in the same way as the other solutions (although at a different scale, as given in the figure caption). These models again have a mirror image symmetry about the equator, which is treated in the same manner as before. However, note that the contributions of both black holes must be taken into account in determining $\hat{K}_{rr}$ for the marginally trapped surface boundary condition (33) as well as in determining S itself.

Figure 9 – Patch System for the PP Test Series

Only the inner 5 patches are shown. The ρ axis runs horizontally at the bottom, and the z axis runs vertically at the left. Both axes range from 0 to 4.9. Note the splitting of the cylindrical coordinate region into two patches so that each patch will be manhattan convex, and the use of a "thin" polar spherical patch to change to a larger grid size before the polar spherical—cylindrical patch transition.

Figure 10 – PP Test Series Solutions ($P \geq 0$)

P=0

P=+1

P=+10

P=+100

Ψ is plotted vertically from 0 to 6, ρ forward from 0 to 4.9, z to the right from 0 to 4.9. The lower set of (ρ, z) axes are at $\Psi = 0$, the upper set at $\Psi = 1$. The black holes are approaching one another. The $P = +0.01$ and $P = +0.1$ models are visually indistinguishable from the $P = 0$ one at this scale, and are hence not plotted.

Figure 11 – PP Test Series Solutions ($P \leq 0$)

Ψ is plotted vertically from 0 to 6, ρ forward from 0 to 4.9, z to the right from 0 to 4.9. The lower set of (ρ, z) axes are at $\Psi = 0$, the upper set at $\Psi = 1$. The black holes are receding from one another. The $P = -0.01$ and $P = -0.1$ models are visually indistinguishable from the $P = 0$ one at this scale, and are hence not plotted.

Conclusions and Directions for Further Research

The two main innovations of this thesis (multiple coordinate patches and the marginally trapped surface boundary condition) both proved highly successful. Together, they should allow the Bowen and York formalism for the initial data problem to be used in a wide range of physical situations.

The difficulty of implementing the marginally trapped surface boundary condition seems to be no greater than that of the reflection symmetric one. The former generalizes easily to spaces containing multiple black holes, without the infinite series of “image charges” the latter seems to require. The marginally trapped surface condition also possesses a straightforward physical interpretation. It does, however, place a restriction on one’s freedom to choose spatial coordinates, both on the initial slice and on future slices in a time evolution problem.

There is another possible problem with using the marginally trapped surface condition in a time evolution GR code. In order to realise the advantages mentioned above, each marginally trapped surface used as an inner grid boundary should be the outermost such one surrounding its black hole. If the spatial geometry of the slices changes smoothly, this isn’t difficult to ensure. However, if a black hole suddenly gains some mass, or (say) two black holes merge, the outer marginally trapped surface will *discontinuously* move outwards, and the code may well “lose track” of it. A possible solution to this problem is to search the space for further marginally trapped surfaces each time step. This has not yet been done, but should pose no major difficulty. However, the cost of doing this search at each time step may prove prohibitive. Further investigation of this problem would be useful.

Multiple coordinate patches seem to work very well in providing spatial coordinates for spaces containing multiple black holes. At least for the initial data problem, the required interpolation across the interpatch boundaries is easy to implement and does not significantly degrade the accuracy of the numerical solution. These conclusions should extend to other elliptic PDE problems involving topologies which do not admit simple “analytic” coordinate systems.

However, if multiple coordinate patches are to be used for parabolic or hyperbolic PDE problems (such as time evolving GR initial data), further investigation is needed. The potential problems arise from the errors induced by the interpatch interpolation—they could introduce either instabilities at the interpatch boundaries, or spurious waves which will then propagate forward in time. Since the interpolation errors can be (and are, in this thesis) made of the same order as those incurred in finite differencing the PDE itself, these difficulties should be soluble, but the means needed have yet to be studied.

This thesis also used multiple coordinate patches as a means of spatially varying the grid resolution. This should probably be done automatically by an adaptive mechanism. Choptuik (1982) found such a scheme to be very effective in placing extra grid points just where they were needed, but his code only did adaptive gridding in one space dimension—the grid size in the other dimension was fixed. In multiple black hole systems, a full n -dimensional adaptive gridding system is probably needed.

The use of Ω to linearize the boundary conditions proved rather awkward. Given the desire to do some linearizing transformation, it seems that doing so with the domain (via Choptuik (1982)’s s coordinate or some similar device) is preferable to linearizing in the range (with Ω). The former can, if properly chosen, linearize a number of

different functions at once (useful for vector problems), and places no restrictions on the values of the functions being linearized.

The use of a global Newton method for solving the finite difference equations seems to be useful for “testbed” type codes such as the one constructed in this thesis. It offers reliable convergence, but the cost in run time and memory consumption probably precludes its use in a “production” code.

Like previous studies, this thesis found surface integral total energy formulas somewhat inaccurate. Volume integrals should be better, but means to do volume integrals in the presence of multiple coordinate patches (with gaps in between them) have yet to be developed.

Any means of relaxing either of the two main assumptions (maximal slicing and 3-conformal flatness) of this thesis would be of considerable significance. If $\partial_i \text{Tr}(K) \neq 0$, the Bowen and York formalism requires the solution of a vector elliptic equation, rather than the scalar one solved herein. An investigation of the difficulty of this would be very useful. Given that the vector equation is soluble, the 3-conformal flatness assumption could probably also be dropped; further investigation of this possibility is certainly warranted.

The new coordinates and boundary conditions could also be applied to the time evolution of these spaces, as well as to a wide variety of other problems. In particular, the work of Smarr (1975), Eppley (1975), and Čadež (1971) on the head-on collision of two black holes could be replicated and extended. Studies of the collision of spinning black holes would be of special interest, since they should show the effects of the GR spin-spin interaction. It would also be of interest to see if the amount of radiation

emitted in such collisions could be increased by “throwing” the two black holes together with initial linear momentum.

In the more distant future, the decay and subsequent merger of black hole binary systems would be of great interest, as would be a study of black hole-black hole scattering in general. Such studies (by numerical GR means) are probably some years away, since they require time evolution of 3 (space) dimensional data. Even assuming the required computational capability to be available (which it almost is today, and soon will be), the techniques involved in constructing suitable numerical GR codes are not yet fully understood.

References

- Arnowitt, R., Deser, S., and Misner, C. W. (1962): "The Dynamics of General Relativity", pp. 227–265 in Witten (1962).
- Bowen, J. M., Rauber, J., and York, J. W. (1984): "Two Black Holes with Axisymmetric Parallel Spins: Initial Data", *Classical and Quantum Gravity* 1, 591–610.
- Bowen, J. M., and York, J. W. (1980): "Time-Asymmetric Initial Data for Black Holes and Black Hole Collisions", *Physical Review D* 21(8), 2047–2056.
- Čadež, A (1971): "Colliding Black Holes", University of North Carolina at Chapel Hill Ph.D. Thesis.
- Čadež, A (1974): "Apparent Horizons in the Two-Black-Hole Problem", *Annals of Physics* 83, 449–457.
- Choptuik, M. (1982): "A Study of Numerical Techniques for the Initial Value Problem of General Relativity", University of British Columbia M.Sc. Thesis.
- Choptuik, M. (1985a): Personal communication.
- Choptuik, M. (1985b): Personal communication.
- Choptuik, M. (1985c): Personal communication.
- Choptuik, M. (1985d): Personal communication.
- Deruelle, N., and Piran, T., eds. (1983): "Rayonnement Gravitationnel", North-Holland, Amsterdam.
- Eppley, K. R. (1975): "The Numerical Evolution of the Collision of Two Black Holes", Princeton University Ph.D. Thesis.
- Eppley, K. R. (1977): "Evolution of Time-Symmetric Gravitational Waves: Initial Data and Apparent Horizons", *Physical Review D* 16(6), 1609–1614.
- Kernighan, B. W., and Ritchie, D. M. (1978): "The C Programming Language", Prentice-Hall, Englewood Cliffs.

- Kernshaw, D. S. (1978): "The Incomplete Cholesky—Conjugate Gradient Method for the Iterative Solution of Systems of Linear Equations", *Journal of Computational Physics* 26, 43–65.
- Kulkarni, A. D. (1984a): "Time-Asymmetric Initial Data for the N Black Hole problem in General Relativity", *Journal of Mathematical Physics* 25(4), 1028–1034.
- Kulkarni, A. D. (1984b): "Extrinsic Curvature for the Two-Black-Hole Problem", Preprint.
- Kulkarni, A. D., Shepley, L. C., and York, J. W. (1983): "Initial Data for N Black Holes", *Physics Letters* 96A(5), 228–230.
- Matzner, R. A., and Shepley, L. C. (1982): "Spacetime and Geometry: The Alfred Schild Lectures", University of Texas Press, Austin.
- Misner, C. W., Thorne, K. S., and Wheeler, J. A. (1973): "Gravitation", W. H. Freeman and Company, San Francisco.
- Mitchell, A. R., and Griffiths, D. F. (1980): "The Finite Difference Method In Partial Differential Equations", Wiley, Chichester.
- MTW: Alternate citation for Misner, Thorne, and Wheeler (1973).
- Ó Murchadha, N., and Seifert, H. J. (1982): personal communication.
- Price, R. H. (1982): "General Relativity Primer", *American Journal of Physics* 50(4), 300–329; see also Tryon (1984) for errata.
- Smarr, L. L. (1975): "The Structure of General Relativity With A Numerical Illustration: The Collision of Two Black Holes", University of Texas at Austin Ph.D. Thesis.
- Smarr, L. L., ed. (1979): "Sources of Gravitational Radiation", Cambridge University Press, Cambridge.
- Tryon, E. P. (1984): "Comment on 'General Relativity Primer'", *American Journal of Physics* 52(4), 366–367.
- Wald, R. M. (1984): "General Relativity", University of Chicago Press, Chicago.

- Witten, L., ed. (1962): "Gravitation—An Introduction to Current Research", Wiley, New York.
- York, J. W. (1979): "Kinematics and Dynamics of General Relativity", pp. 83–126 in Smarr (1979).
- York, J. W. (1983): "The Initial Value Problem and Dynamics", pp. 175–201 in Deruelle and Piran (1983).
- York, J. W. (1984): "Initial Data for N Black Holes", *Physica* 124A, 629–638.
- York, J. W., and Piran, T. (1982): "The Initial Value Problem and Beyond", pp. 147–176 in Matzner and Shepley (1982).

Appendix 1

This appendix contains listings of the input parameter files for each model discussed in this thesis. As described in the header file `patchutil:patch.h` (listed in appendix 2), the input parameters to a model `modelname` reside in two files, a `modelname.par` file describing the physical situation being modeled, and a `gridname.pat` file describing the patch system being used. The latter is typically shared between a number of models.

In addition, this appendix also contains quantitative graphs of both Ψ and $\delta\Psi$ as functions of s for all the non-Schwarzschild single black hole models, and tables of selected values of Ψ for the Schwarzschild and double black hole models. These should prove useful to the reader desiring specific numeric values to check against some other GR code's results.

The `modelname.par` files typically end with a sequence of comment lines (anything after a semicolon is a comment), which log all programs that modified the `modelname.gfn` data file.

The next 5 pages give indices to the listings and graphs/tables, giving the page numbers on which each model's information starts. The listings and graphs/tables themselves follow. When all the models of a test series share a common `.pat` file (as they do for all but the Schwarzschild models), this is listed after all the other information for that series.

Table 5 – Model index for the Schwarzschild test series

10plo100	90
20plo100	94
40plo100	98
c2	102
c01	106
c2i	110
poo	114
opo	118
oop	122

Table 6 – Model index for the Choptuik comparison test series

ppx001	126
ppx01	128
ppx1	130
pp1	132
pp2x5	134
pp7x5	136
pp12x5	138
mpx001	140
mpx01	142
mpx1	144
mp1	146
mp2x5	148
mp7x5	150
mp12x5	152
jx001	154
jx01	156
jx1	158
j1	160
j10	162
j100	164
j1000	166
matt.pat	168

Table 7 – Model index for the J test series

j0	170
jx01	172
jx1	174
j1	176
j10	178
j100	180
mt20.pat	182

Table 8 – Model index for the RSP test series

p0	185
px01	187
px1	189
p1	191
p10	193
p100	195
rs20.pat	197

Table 9 – Model index for the RSN test series

p0		200
px01		202
px1		204
p1		206
p10		208
p100		210
rs20.pat		212

Table 10 – Model index for the NT0 test series

p0		215
px01		217
px1		219
p1		221
p10		223
p100		225
mt20big.pat		227

Table 11 – Model index for the PP test series

m100	230
m10	232
m1	234
mx1	236
mx01	238
0	240
px01	242
px1	244
p1	246
p10	248
p100	250
m1.pat	252

Schwarzschild test series

10PLO100.PAR;2

18-OCT-1985 06:37

Page 1

title="Schwarzschild Solution"

N_black_hole=1 grid="10plo100"

;

black_hole=1

rho=0 z=0

mass=1 surrounding_patch_flag=1

momentum=0 reflection_term_sign=0 reflection_radius=0.5

spin=0

;

; mkgfn 10plo100

; directcg 10plo100 1.0e-10 (224.78 cpu seconds)

Schwarzschild test series

10PLO100.PAT;3

18-OCT-1985 06:37

Page 1

```

; Each patch approximately 10 x 10
; All patches polar spherical
; All patches using locally conservative differencing
; All patches using omega for conformal function
; Outer boundary at r = 100
;
N_patch=7
;
; Now describe each coordinate patch:
;
patch-1
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.05      delta_theta=10
r_min="constant=0.50"      bndry_cond="Reflection Symmetric"
r_max="constant=0.95"      bndry_cond="patch-2"
theta_min="constant=0"      bndry_cond="Neumann"
theta_max="constant=90"      bndry_cond="Neumann"
;
patch-2
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.1      delta_theta=10
r_min="constant=1.0"      bndry_cond="patch-1"
r_max="constant=1.9"      bndry_cond="patch-3"
theta_min="constant=0"      bndry_cond="Neumann"
theta_max="constant=90"      bndry_cond="Neumann"
;
patch-3
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.2      delta_theta=10
r_min="constant=2.0"      bndry_cond="patch-2"
r_max="constant=4.8"      bndry_cond="patch-4"
theta_min="constant=0"      bndry_cond="Neumann"
theta_max="constant=90"      bndry_cond="Neumann"
;
patch-4
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.5      delta_theta=10
r_min="constant=5.0"      bndry_cond="patch-3"
r_max="constant=9.5"      bndry_cond="patch-5"
theta_min="constant=0"      bndry_cond="Neumann"
theta_max="constant=90"      bndry_cond="Neumann"
;
patch-5
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=1      delta_theta=15

```

Schwarzschild test series

10PLO100.PAT;3

18-OCT-1985 06:37

Page 2

```

r_min="constant-10"      bndry_cond="patch-4"
r_max="constant-19"      bndry_cond="patch-6"
theta_min="constant-0"   bndry_cond="Neumann"
theta_max="constant-90"  bndry_cond="Neumann"
;
patch-6
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=2      delta_theta=15
r_min="constant-20"   bndry_cond="patch-5"
r_max="constant-48"   bndry_cond="patch-7"
theta_min="constant-0" bndry_cond="Neumann"
theta_max="constant-90" bndry_cond="Neumann"
;
patch-7
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=5      delta_theta=15
r_min="constant-50"   bndry_cond="patch-6"
r_max="constant-100"  bndry_cond="Robin"
theta_min="constant-0" bndry_cond="Neumann"
theta_max="constant-90" bndry_cond="Neumann"

```

Table 12 - Schwarzschild test series, 10plo100 selected Ψ values

Patch Number	i	j	Psi	rho	z	r	theta
1	10	0	2.006125			0.5000	0.0
1	10	3	2.006125			0.5000	30.0
1	10	6	2.006125			0.5000	60.0
1	10	9	2.006125			0.5000	90.0
1	19	0	1.530515			0.9500	0.0
1	19	3	1.530515			0.9500	30.0
1	19	6	1.530515			0.9500	60.0
1	19	9	1.530515			0.9500	90.0
2	10	0	1.504070			1.0000	0.0
2	10	3	1.504070			1.0000	30.0
2	10	6	1.504070			1.0000	60.0
2	10	9	1.504070			1.0000	90.0
2	19	0	1.265771			1.9000	0.0
2	19	3	1.265771			1.9000	30.0
2	19	6	1.265771			1.9000	60.0
2	19	9	1.265771			1.9000	90.0
3	10	0	1.252521			2.0000	0.0
3	10	3	1.252521			2.0000	30.0
3	10	6	1.252521			2.0000	60.0
3	10	9	1.252521			2.0000	90.0
3	24	0	1.105459			4.8000	0.0
3	24	3	1.105459			4.8000	30.0
3	24	6	1.105459			4.8000	60.0
3	24	9	1.105459			4.8000	90.0
4	10	0	1.101253			5.0000	0.0
4	10	3	1.101253			5.0000	30.0
4	10	6	1.101253			5.0000	60.0
4	10	9	1.101253			5.0000	90.0
4	19	0	1.053382			9.5000	0.0
4	19	3	1.053382			9.5000	30.0
4	19	6	1.053382			9.5000	60.0
4	19	9	1.053382			9.5000	90.0
5	10	0	1.050721			10.0000	0.0
5	10	2	1.050721			10.0000	30.0
5	10	4	1.050721			10.0000	60.0
5	10	6	1.050721			10.0000	90.0
5	19	0	1.026736			19.0000	0.0
5	19	2	1.026736			19.0000	30.0
5	19	4	1.026736			19.0000	60.0
5	19	6	1.026736			19.0000	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

Schwarzschild test series

20PLO100.PAR;4

12-NOV-1985 21:25

Page 1

title="Schwarzschild Solution"

N_black_hole=1 grid="20plo100"

;

black_hole=1

rho=0 z=0

mass=1 surrounding_patch_flag=1

momentum=0 reflection_term_sign=0 reflection_radius=0.5

spin=0

;

; mkgfn 20plo100

; directcg 20plo100 1.0e-10

(2480.11 cpu seconds)

Schwarzschild test series

20PLO100.PAT;3

18-OCT-1985 06:37

Page 1

```

; Each patch approximately 20 x 20
; All patches polar spherical
; All patches using locally conservative differencing
; All patches using omega for conformal function
; Outer boundary at r = 100
;
N_patch=7
;
; Now describe each coordinate patch:
;
patch-1
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.025  delta_theta=5
r_min="constant=0.500"      bndry_cond="Reflection Symmetric"
r_max="constant=0.975"      bndry_cond="patch-2"
theta_min="constant=0"      bndry_cond="Neumann"
theta_max="constant=90"     bndry_cond="Neumann"
;
patch-2
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.05    delta_theta=5
r_min="constant=1.00"      bndry_cond="patch-1"
r_max="constant=1.95"      bndry_cond="patch-3"
theta_min="constant=0"      bndry_cond="Neumann"
theta_max="constant=90"     bndry_cond="Neumann"
;
patch-3
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.1     delta_theta=5
r_min="constant=2.0"      bndry_cond="patch-2"
r_max="constant=4.9"      bndry_cond="patch-4"
theta_min="constant=0"      bndry_cond="Neumann"
theta_max="constant=90"     bndry_cond="Neumann"
;
patch-4
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.25    delta_theta=5
r_min="constant=5.00"      bndry_cond="patch-3"
r_max="constant=9.75"      bndry_cond="patch-5"
theta_min="constant=0"      bndry_cond="Neumann"
theta_max="constant=90"     bndry_cond="Neumann"
;
patch-5
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.5     delta_theta=7.5

```

Schwarzschild test series

20PLO100.PAT;3

18-OCT-1985 06:37

Page 2

```

r_min="constant-10.0"      bndry_cond="patch-4"
r_max="constant-19.5"      bndry_cond="patch-6"
theta_min="constant-0"     bndry_cond="Neumann"
theta_max="constant-90"    bndry_cond="Neumann"
;
patch-6
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=1      delta_theta=7.5
r_min="constant-20"      bndry_cond="patch-5"
r_max="constant-49"      bndry_cond="patch-7"
theta_min="constant-0"    bndry_cond="Neumann"
theta_max="constant-90"    bndry_cond="Neumann"
;
patch-7
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=2.5      delta_theta=7.5
r_min="constant-50"      bndry_cond="patch-6"
r_max="constant-100"     bndry_cond="Robin"
theta_min="constant-0"    bndry_cond="Neumann"
theta_max="constant-90"    bndry_cond="Neumann"

```


Table 13 - Schwarzschild test series, 20plo100 selected Ψ values

Patch Number	i	j	Ψ_i	ρ	z	r	θ
1	20	0	2.001474			0.5000	0.0
1	20	6	2.001474			0.5000	30.0
1	20	12	2.001474			0.5000	60.0
1	20	18	2.001474			0.5000	90.0
1	39	0	1.513811			0.9750	0.0
1	39	6	1.513811			0.9750	30.0
1	39	12	1.513811			0.9750	60.0
1	39	18	1.513811			0.9750	90.0
2	20	0	1.500975			1.0000	0.0
2	20	6	1.500975			1.0000	30.0
2	20	12	1.500975			1.0000	60.0
2	20	18	1.500975			1.0000	90.0
2	39	0	1.257023			1.9500	0.0
2	39	6	1.257023			1.9500	30.0
2	39	12	1.257023			1.9500	60.0
2	39	18	1.257023			1.9500	90.0
3	20	0	1.250602			2.0000	0.0
3	20	6	1.250602			2.0000	30.0
3	20	12	1.250602			2.0000	60.0
3	20	18	1.250602			2.0000	90.0
3	49	0	1.102344			4.9000	0.0
3	49	6	1.102344			4.9000	30.0
3	49	12	1.102344			4.9000	60.0
3	49	18	1.102344			4.9000	90.0
4	20	0	1.100298			5.0000	0.0
4	20	6	1.100298			5.0000	30.0
4	20	12	1.100298			5.0000	60.0
4	20	18	1.100298			5.0000	90.0
4	39	0	1.051457			9.7500	0.0
4	39	6	1.051457			9.7500	30.0
4	39	12	1.051457			9.7500	60.0
4	39	18	1.051457			9.7500	90.0
5	20	0	1.050171			10.0000	0.0
5	20	4	1.050171			10.0000	30.0
5	20	8	1.050171			10.0000	60.0
5	20	12	1.050171			10.0000	90.0
5	39	0	1.025739			19.5000	0.0
5	39	4	1.025739			19.5000	30.0
5	39	8	1.025739			19.5000	60.0
5	39	12	1.025739			19.5000	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

Schwarzschild test series

40PLO100.PAR;2

18-OCT-1985 06:37

Page 1

```
title="Schwarzschild Solution"
N_black_hole=1      grid="40plo100"
;
black_hole=1
rho=0              z=0
mass=1             surrounding_patch_flag=1
momentum=0         reflection_term_sign=0      reflection_radius=0.5
spin=0
;
; mkgfn 40plo100
; directcg 40plo100 1.0e-10      (60372.99 cpu seconds)
```

Schwarzschild test series

40PLO100.PAT;6

18-OCT-1985 06:37

Page 1

```

; Each patch approximately 40 x 40
; All patches polar spherical
; All patches using locally conservative differencing
; All patches using omega for conformal function
; Outer boundary at r = 100
;
N_patch=7
;
; Now describe each coordinate patch:
;
patch-1
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.0125 delta_theta=2.5
r_min="constant-0.5000"      bndry_cond="Reflection Symmetric"
r_max="constant-0.9875"      bndry_cond="patch-2"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-2
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.025 delta_theta=2.5
r_min="constant-1.000"      bndry_cond="patch-1"
r_max="constant-1.975"      bndry_cond="patch-3"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-3
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.05 delta_theta=2.5
r_min="constant-2.00"      bndry_cond="patch-2"
r_max="constant-4.95"      bndry_cond="patch-4"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-4
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.125 delta_theta=2.5
r_min="constant-5.000"      bndry_cond="patch-3"
r_max="constant-9.875"      bndry_cond="patch-5"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-5
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.25 delta_theta=3.75

```

Schwarzschild test series

40PLO100.PAT;6

18-OCT-1985 06:37

Page 2

```

r_min="constant-10.00"      bndry_cond="patch-4"
r_max="constant-19.75"      bndry_cond="patch-6"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"     bndry_cond="Neumann"
;
patch-6
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0    z_center=0    delta_r=0.5    delta_theta=3.75
r_min="constant-20.0"      bndry_cond="patch-5"
r_max="constant-49.5"      bndry_cond="patch-7"
theta_min="constant-0"     bndry_cond="Neumann"
theta_max="constant-90"    bndry_cond="Neumann"
;
patch-7
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0    z_center=0    delta_r=1.25    delta_theta=3.75
r_min="constant-50"        bndry_cond="patch-6"
r_max="constant-100"       bndry_cond="Robin"
theta_min="constant-0"     bndry_cond="Neumann"
theta_max="constant-90"    bndry_cond="Neumann"

```

Table 14 - Schwarzschild test series, 40plo100 selected Ψ values

Patch Number	i	j	Psi	rho	z	r	theta
1	40	0	2.000362			0.5000	0.0
1	40	12	2.000362			0.5000	30.0
1	40	24	2.000362			0.5000	60.0
1	40	36	2.000362			0.5000	90.0
1	79	0	1.506570			0.9875	0.0
1	79	12	1.506570			0.9875	30.0
1	79	24	1.506570			0.9875	60.0
1	79	36	1.506570			0.9875	90.0
2	40	0	1.500239			1.0000	0.0
2	40	12	1.500239			1.0000	30.0
2	40	24	1.500239			1.0000	60.0
2	40	36	1.500239			1.0000	90.0
2	79	0	1.253313			1.9750	0.0
2	79	12	1.253313			1.9750	30.0
2	79	24	1.253313			1.9750	60.0
2	79	36	1.253313			1.9750	90.0
3	40	0	1.250147			2.0000	0.0
3	40	12	1.250147			2.0000	30.0
3	40	24	1.250147			2.0000	60.0
3	40	36	1.250147			2.0000	90.0
3	99	0	1.101084			4.9500	0.0
3	99	12	1.101084			4.9500	30.0
3	99	24	1.101084			4.9500	60.0
3	99	36	1.101084			4.9500	90.0
4	40	0	1.100073			5.0000	0.0
4	40	12	1.100073			5.0000	30.0
4	40	24	1.100073			5.0000	60.0
4	40	36	1.100073			5.0000	90.0
4	79	0	1.050675			9.8750	0.0
4	79	12	1.050675			9.8750	30.0
4	79	24	1.050675			9.8750	60.0
4	79	36	1.050675			9.8750	90.0
5	40	0	1.050042			10.0000	0.0
5	40	4	1.050042			10.0000	15.0
5	40	8	1.050042			10.0000	30.0
5	40	24	1.050042			10.0000	90.0
5	79	0	1.025340			19.7500	0.0
5	79	4	1.025340			19.7500	15.0
5	79	8	1.025340			19.7500	30.0
5	79	24	1.025340			19.7500	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

Schwarzschild test series

C2.PAR;3

18-OCT-1985 06:40

Page 1

```
title="Schwarzschild Solution"
N_black_hole=1      grid="c2"
;
black_hole=1
rho=0              z=0
mass=1            surrounding_patch_flag=1
momentum=0        reflection_term_sign=0      reflection_radius=0.5
spin=0
;
; mkgfn c2
; directcg c2 1.0e-10      (4989.19 cpu seconds)
```

Schwarzschild test series

C2.PAT;14

18-OCT-1988 08:40

Page 1

```

; Each patch approximately 20 x 20
; All patches polar spherical, except (original) patch #2 is augmented by
;   a cylindrical one
; All patches using locally conservative differencing
; All patches using omega for conformal function
; Outer boundary at r = 100
;
N_patch=8
;
; Now describe each coordinate patch:
;
patch-1
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.025  delta_theta=5
r_min="constant-0.500"  bndry_cond="Reflection Symmetric"
r_max="constant-0.975"  bndry_cond="patch-2"
theta_min="constant-0"  bndry_cond="Neumann"
theta_max="constant-90" bndry_cond="Neumann"
;
patch-2
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.05   delta_theta=5
r_min="constant-1.00"  bndry_cond="patch-1"
r_max="constant-1.05"  bndry_cond="patch-3"
theta_min="constant-0"  bndry_cond="Neumann"
theta_max="constant-90" bndry_cond="Neumann"
;
patch-3
coord_type="cylindrical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_rho=0.05  delta_z=0.05
rho_min_bound=0    rho_max_bound=2
z_min_bound=0      z_max_bound=2
rho_min="patch-2 bndry-r_max"
rho_min="constant-0"  bndry_cond="Neumann"
rho_max="patch-4 bndry-r_min"
z_min="patch-2 bndry-r_max"
z_min="constant-0"  bndry_cond="Neumann"
z_max="patch-4 bndry-r_min"
;
patch-4
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.1    delta_theta=5
r_min="constant-2.0"  bndry_cond="patch-3"
r_max="constant-4.9"  bndry_cond="patch-5"
theta_min="constant-0"  bndry_cond="Neumann"
theta_max="constant-90" bndry_cond="Neumann"
;

```

Schwarzschild test series

C2.PAT;14

18-OCT-1985 06:40

Page 2

```

patch-5
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.25      delta_theta=5
r_min="constant-5.00"      bndry_cond="patch-4"
r_max="constant-9.75"      bndry_cond="patch-6"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-6
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.5      delta_theta=7.5
r_min="constant-10.0"      bndry_cond="patch-5"
r_max="constant-19.5"      bndry_cond="patch-7"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-7
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=1      delta_theta=7.5
r_min="constant-20"      bndry_cond="patch-6"
r_max="constant-49"      bndry_cond="patch-8"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-8
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=2.5      delta_theta=7.5
r_min="constant-50"      bndry_cond="patch-7"
r_max="constant-100"      bndry_cond="Robin"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"

```


Table 15 – Schwarzschild test series, c2 selected Ψ values

Patch Number	i	j	Ps1	rho	z	r	theta
1	20	0	1.998832			0.5000	0.0
1	20	6	1.998831			0.5000	30.0
1	20	12	1.998838			0.5000	60.0
1	20	18	1.998850			0.5000	90.0
1	39	0	1.511860			0.9750	0.0
1	39	6	1.511819			0.9750	30.0
1	39	12	1.511813			0.9750	60.0
1	39	18	1.511882			0.9750	90.0
2	20	0	1.499052			1.0000	0.0
2	20	9	1.498959			1.0000	45.0
2	20	18	1.499073			1.0000	90.0
2	21	0	1.475277			1.0500	0.0
2	21	9	1.475134			1.0500	45.0
2	21	18	1.475290			1.0500	90.0
3	30	0	1.332687	1.50	0.00		
3	20	10	1.446284	1.00	0.50		
3	30	10	1.315592	1.50	0.50		
3	10	20	1.446303	0.50	1.00		
3	20	20	1.352806	1.00	1.00		
3	30	20	1.276777	1.50	1.00		
3	0	30	1.332711	0.00	1.50		
3	10	30	1.315612	0.50	1.50		
3	20	30	1.276784	1.00	1.50		
4	20	0	1.249510			2.0000	0.0
4	20	6	1.249498			2.0000	30.0
4	20	12	1.249493			2.0000	60.0
4	20	18	1.249502			2.0000	90.0
4	49	0	1.101891			4.9000	0.0
4	49	6	1.101891			4.9000	30.0
4	49	12	1.101890			4.9000	60.0
4	49	18	1.101891			4.9000	90.0
5	20	0	1.099854			5.0000	0.0
5	20	6	1.099854			5.0000	30.0
5	20	12	1.099854			5.0000	60.0
5	20	18	1.099854			5.0000	90.0
5	39	0	1.051229			9.7500	0.0
5	39	6	1.051229			9.7500	30.0
5	39	12	1.051229			9.7500	60.0
5	39	18	1.051229			9.7500	90.0
6	20	0	1.049949			10.0000	0.0
6	20	6	1.049949			10.0000	45.0
6	20	12	1.049949			10.0000	90.0
6	39	0	1.025625			19.5000	0.0
6	39	6	1.025625			19.5000	45.0
6	39	12	1.025625			19.5000	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

Schwarzschild test series

COI.PAR;2

18-OCT-1985 06:40

Page 1

title="Schwarzschild Solution"

N_black_hole=1 grid="c01"

;

black_hole=1

rho=0 z=0

mass=1 surrounding_patch_flag=1

momentum=0 reflection_term_sign=0 reflection_radius=0.5

spin=0

;

; mkgfn c01

; directcg c01 1.0e-10

Schwarzschild test series

COI.PAT;2

18-OCT-1985 06:40

Page 1

```

; Each patch approximately 20 x 20
; All patches polar spherical
; All patches using independent terms differencing
; All patches using omega for conformal function
; Outer boundary at r = 100
;
N_patch-7
;
; Now describe each coordinate patch:
;
patch-1
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.025  delta_theta=5
r_min="constant-0.500"      bndry_cond="Reflection Symmetric"
r_max="constant-0.975"      bndry_cond="patch-2"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"     bndry_cond="Neumann"
;
patch-2
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.05    delta_theta=5
r_min="constant-1.00"      bndry_cond="patch-1"
r_max="constant-1.95"      bndry_cond="patch-3"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"     bndry_cond="Neumann"
;
patch-3
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.1     delta_theta=5
r_min="constant-2.0"      bndry_cond="patch-2"
r_max="constant-4.9"      bndry_cond="patch-4"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"     bndry_cond="Neumann"
;
patch-4
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.25    delta_theta=5
r_min="constant-8.00"      bndry_cond="patch-3"
r_max="constant-9.75"      bndry_cond="patch-5"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"     bndry_cond="Neumann"
;
patch-5
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.5     delta_theta=7.5

```

Schwarzschild test series

COI.PAT;2

18-OCT-1985 06:40

Page 2

```

r_min="constant-10.0"      bndry_cond="patch-4"
r_max="constant-19.5"      bndry_cond="patch-6"
theta_min="constant-0"     bndry_cond="Neumann"
theta_max="constant-90"    bndry_cond="Neumann"
;
patch=6
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=1      delta_theta=7.5
r_min="constant-20"      bndry_cond="patch-5"
r_max="constant-49"      bndry_cond="patch-7"
theta_min="constant-0"   bndry_cond="Neumann"
theta_max="constant-90"  bndry_cond="Neumann"
;
patch=7
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=2.5      delta_theta=7.5
r_min="constant-50"      bndry_cond="patch-6"
r_max="constant-100"     bndry_cond="Robin"
theta_min="constant-0"   bndry_cond="Neumann"
theta_max="constant-90"  bndry_cond="Neumann"

```

Table 16 – Schwarzschild test series, c01 selected Ψ values

Patch Number	i	j	Ψ_i	ρ	z	r	θ
1	20	0	2.000000			0.5000	0.0
1	20	6	2.000000			0.5000	30.0
1	20	12	2.000000			0.5000	60.0
1	20	18	2.000000			0.5000	90.0
1	39	0	1.512821			0.9750	0.0
1	39	6	1.512821			0.9750	30.0
1	39	12	1.512821			0.9750	60.0
1	39	18	1.512821			0.9750	90.0
2	20	0	1.500000			1.0000	0.0
2	20	6	1.500000			1.0000	30.0
2	20	12	1.500000			1.0000	60.0
2	20	18	1.500000			1.0000	90.0
2	39	0	1.256410			1.9500	0.0
2	39	6	1.256410			1.9500	30.0
2	39	12	1.256410			1.9500	60.0
2	39	18	1.256410			1.9500	90.0
3	20	0	1.250000			2.0000	0.0
3	20	6	1.250000			2.0000	30.0
3	20	12	1.250000			2.0000	60.0
3	20	18	1.250000			2.0000	90.0
3	49	0	1.102041			4.9000	0.0
3	49	6	1.102041			4.9000	30.0
3	49	12	1.102041			4.9000	60.0
3	49	18	1.102041			4.9000	90.0
4	20	0	1.100000			5.0000	0.0
4	20	6	1.100000			5.0000	30.0
4	20	12	1.100000			5.0000	60.0
4	20	18	1.100000			5.0000	90.0
4	39	0	1.051282			9.7500	0.0
4	39	6	1.051282			9.7500	30.0
4	39	12	1.051282			9.7500	60.0
4	39	18	1.051282			9.7500	90.0
5	20	0	1.050000			10.0000	0.0
5	20	4	1.050000			10.0000	30.0
5	20	8	1.050000			10.0000	60.0
5	20	12	1.050000			10.0000	90.0
5	39	0	1.025641			19.5000	0.0
5	39	4	1.025641			19.5000	30.0
5	39	8	1.025641			19.5000	60.0
5	39	12	1.025641			19.5000	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

Schwarzschild test series

C2I.PAR;2

18-OCT-1985 06:40

Page 1

title="Schwarzschild Solution"

N_black_hole-1 grid="c2i"

;

black_hole-1

rho-0 z-0

mass-1 surrounding_patch_flag-1

momentum-0 reflection_term_sign=0 reflection_radius=0.5

spin-0

;

; mkgfn c2i

; directcg c2i 1.0e-10 (4946.00 cpu seconds)

Schwarzschild test series

C2I.PAT;2

18-OCT-1985 06:40

Page 1

```

; Each patch approximately 20 x 20
; All patches polar spherical, except (original) patch #2 is augmented by
;   a cylindrical one
; All patches using independent terms differencing
; All patches using omega for conformal function
; Outer boundary at r = 100
;
N_patch=8
;
; Now describe each coordinate patch:
;
patch-1
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.025  delta_theta=5
r_min="constant=0.500"  bndry_cond="Reflection Symmetric"
r_max="constant=0.975"  bndry_cond="patch-2"
theta_min="constant=0"  bndry_cond="Neumann"
theta_max="constant=90" bndry_cond="Neumann"
;
patch-2
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.05    delta_theta=5
r_min="constant=1.00"  bndry_cond="patch-1"
r_max="constant=1.05"  bndry_cond="patch-3"
theta_min="constant=0"  bndry_cond="Neumann"
theta_max="constant=90" bndry_cond="Neumann"
;
patch-3
coord_type="cylindrical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_rho=0.05  delta_z=0.05
rho_min_bound=0    rho_max_bound=2
z_min_bound=0      z_max_bound=2
rho_min="patch-2 bndry-r_max"
rho_min="constant=0"  bndry_cond="Neumann"
rho_max="patch-4 bndry-r_min"
z_min="patch-2 bndry-r_max"
z_min="constant=0"    bndry_cond="Neumann"
z_max="patch-4 bndry-r_min"
;
patch-4
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.1      delta_theta=5
r_min="constant=2.0"  bndry_cond="patch-3"
r_max="constant=4.9"  bndry_cond="patch-5"
theta_min="constant=0"  bndry_cond="Neumann"
theta_max="constant=90" bndry_cond="Neumann"
;

```

Schwarzschild test series

C2I.PAT;2

18-OCT-1985 06:40

Page 2

```

patch-5
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.25      delta_theta=5
r_min="constant-5.00"      bndry_cond="patch-4"
r_max="constant-9.75"      bndry_cond="patch-6"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-6
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.5      delta_theta=7.5
r_min="constant-10.0"      bndry_cond="patch-5"
r_max="constant-19.5"      bndry_cond="patch-7"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-7
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=1      delta_theta=7.5
r_min="constant-20"      bndry_cond="patch-6"
r_max="constant-49"      bndry_cond="patch-8"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-8
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=2.5      delta_theta=7.5
r_min="constant-50"      bndry_cond="patch-7"
r_max="constant-100"      bndry_cond="Robin"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"

```


Table 17 – Schwarzschild test series, c21 selected Ψ values

Patch Number	i	j	Psi	rho	z	r	theta
1	20	0	1.997657			0.5000	0.0
1	20	6	1.997656			0.5000	30.0
1	20	12	1.997663			0.5000	60.0
1	20	18	1.997675			0.5000	90.0
1	39	0	1.511095			0.9750	0.0
1	39	6	1.511054			0.9750	30.0
1	39	12	1.511048			0.9750	60.0
1	39	18	1.511117			0.9750	90.0
2	20	0	1.498301			1.0000	0.0
2	20	9	1.498208			1.0000	45.0
2	20	18	1.498322			1.0000	90.0
2	21	0	1.474552			1.0500	0.0
2	21	9	1.474409			1.0500	45.0
2	21	18	1.474566			1.0500	90.0
3	30	0	1.332126	1.50	0.00		
3	20	10	1.445593	1.00	0.50		
3	30	10	1.315052	1.50	0.50		
3	10	20	1.445612	0.50	1.00		
3	20	20	1.352222	1.00	1.00		
3	30	20	1.276281	1.50	1.00		
3	0	30	1.332151	0.00	1.50		
3	10	30	1.315072	0.50	1.50		
3	20	30	1.276288	1.00	1.50		
4	20	0	1.249046			2.0000	0.0
4	20	6	1.249034			2.0000	30.0
4	20	12	1.249028			2.0000	60.0
4	20	18	1.249037			2.0000	90.0
4	49	0	1.101644			4.9000	0.0
4	49	6	1.101644			4.9000	30.0
4	49	12	1.101644			4.9000	60.0
4	49	18	1.101644			4.9000	90.0
5	20	0	1.099611			5.0000	0.0
5	20	6	1.099611			5.0000	30.0
5	20	12	1.099611			5.0000	60.0
5	20	18	1.099611			5.0000	90.0
5	39	0	1.051083			9.7500	0.0
5	39	6	1.051083			9.7500	30.0
5	39	12	1.051083			9.7500	60.0
5	39	18	1.051083			9.7500	90.0
6	20	0	1.049806			10.0000	0.0
6	20	6	1.049806			10.0000	45.0
6	20	12	1.049806			10.0000	90.0
6	39	0	1.025541			19.5000	0.0
6	39	6	1.025541			19.5000	45.0
6	39	12	1.025541			19.5000	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

Schwarzschild test series

POO.PAR;2

18-OCT-1985 06:41

Page 1

title="Schwarzschild Solution"

N_black_hole-1 grid="poo"

;

black_hole-1

rho-0 z-0

mass-1 surrounding_patch_flag-1

momentum-0 reflection_term_sign-0 reflection_radius=0.5

spin-0

;

; mkgfn poo

; directcg poo 1.0e-10 (5911.45 opu seconds)

Schwarzschild test series

POO.PAT;3

18-OCT-1985 06:41

Page 1

```

; Each patch approximately 20 x 20
; All patches polar spherical
; All patches using locally conservative differencing
; Outer boundary at r = 100
;
; Conformal function psi / omega / omega
;
N_patch-7
;
; Now describe each coordinate patch:
;
patch-1
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="psi"
rho_center-0      z_center-0      delta_r=0.025  delta_theta=5
r_min="constant=0.500"      bndry_cond="Reflection Symmetric"
r_max="constant=0.975"      bndry_cond="patch-2"
theta_min="constant=0"      bndry_cond="Neumann"
theta_max="constant=90"     bndry_cond="Neumann"
;
patch-2
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center-0      z_center-0      delta_r=0.05    delta_theta=5
r_min="constant=1.00"      bndry_cond="patch-1"
r_max="constant=1.95"      bndry_cond="patch-3"
theta_min="constant=0"      bndry_cond="Neumann"
theta_max="constant=90"     bndry_cond="Neumann"
;
patch-3
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center-0      z_center-0      delta_r=0.1      delta_theta=5
r_min="constant=2.0"      bndry_cond="patch-2"
r_max="constant=4.9"      bndry_cond="patch-4"
theta_min="constant=0"      bndry_cond="Neumann"
theta_max="constant=90"     bndry_cond="Neumann"
;
patch-4
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center-0      z_center-0      delta_r=0.25    delta_theta=5
r_min="constant=5.00"      bndry_cond="patch-3"
r_max="constant=9.75"      bndry_cond="patch-5"
theta_min="constant=0"      bndry_cond="Neumann"
theta_max="constant=90"     bndry_cond="Neumann"
;
patch-5
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"

```

Schwarzschild test series

POO.PAT;3

18-OCT-1985 06:41

Page 2

```

rho_center=0      z_center=0      delta_r=0.5      delta_theta=7.5
r_min="constant-10.0"      bndry_cond="patch-4"
r_max="constant-19.5"      bndry_cond="patch-6"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-6
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=1      delta_theta=7.5
r_min="constant-20"      bndry_cond="patch-5"
r_max="constant-49"      bndry_cond="patch-7"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-7
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=2.5      delta_theta=7.5
r_min="constant-50"      bndry_cond="patch-6"
r_max="constant-100"      bndry_cond="Robin"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"

```

Table 18 - Schwarzschild test series, poo selected Ψ values

Patch Number	i	j	Ψ_i	ρ	z	r	θ
1	20	0	1.996481			0.5000	0.0
1	20	6	1.996481			0.5000	30.0
1	20	12	1.996481			0.5000	60.0
1	20	18	1.996481			0.5000	90.0
1	39	0	1.511249			0.9750	0.0
1	39	6	1.511249			0.9750	30.0
1	39	12	1.511249			0.9750	60.0
1	39	18	1.511249			0.9750	90.0
2	20	0	1.498477			1.0000	0.0
2	20	6	1.498477			1.0000	30.0
2	20	12	1.498477			1.0000	60.0
2	20	18	1.498477			1.0000	90.0
2	39	0	1.255742			1.9500	0.0
2	39	6	1.255742			1.9500	30.0
2	39	12	1.255742			1.9500	60.0
2	39	18	1.255742			1.9500	90.0
3	20	0	1.249353			2.0000	0.0
3	20	6	1.249353			2.0000	30.0
3	20	12	1.249353			2.0000	60.0
3	20	18	1.249353			2.0000	90.0
3	49	0	1.101834			4.9000	0.0
3	49	6	1.101834			4.9000	30.0
3	49	12	1.101834			4.9000	60.0
3	49	18	1.101834			4.9000	90.0
4	20	0	1.099798			5.0000	0.0
4	20	6	1.099798			5.0000	30.0
4	20	12	1.099798			5.0000	60.0
4	20	18	1.099798			5.0000	90.0
4	39	0	1.051200			9.7500	0.0
4	39	6	1.051200			9.7500	30.0
4	39	12	1.051200			9.7500	60.0
4	39	18	1.051200			9.7500	90.0
5	20	0	1.049921			10.0000	0.0
5	20	4	1.049921			10.0000	30.0
5	20	8	1.049921			10.0000	60.0
5	20	12	1.049921			10.0000	90.0
5	39	0	1.025610			19.5000	0.0
5	39	4	1.025610			19.5000	30.0
5	39	8	1.025610			19.5000	60.0
5	39	12	1.025610			19.5000	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

Schwarzschild test series

OPO.PAR;2

18-OCT-1985 06:41

Page 1

title="Schwarzschild Solution"

N_black_hole=1 grid="opo"

;

black_hole=1

rho=0 z=0

mass=1 surrounding_patch_flag=1

momentum=0 reflection_term_sign=0 reflection_radius=0.5

spin=0

;

; mkgfn opo

; directcg opo 1.0e-10 (2737.25 cpu seconds)

Schwarzschild test series

OPO.PAT;2

18-OCT-1985 06:41

Page 1

```

; Each patch approximately 20 x 20
; All patches polar spherical
; All patches using locally conservative differencing
; Outer boundary at r = 100
;
; Conformal function omega / psi / omega
;
N_patch=7
;
; Now describe each coordinate patch:
;
patch-1
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.025  delta_theta=5
r_min="constant=0.500"      bndry_cond="Reflection Symmetric"
r_max="constant=0.975"      bndry_cond="patch-2"
theta_min="constant=0"      bndry_cond="Neumann"
theta_max="constant=90"     bndry_cond="Neumann"
;
patch-2
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="psi"
rho_center=0      z_center=0      delta_r=0.05      delta_theta=5
r_min="constant=1.00"      bndry_cond="patch-1"
r_max="constant=1.95"      bndry_cond="patch-3"
theta_min="constant=0"      bndry_cond="Neumann"
theta_max="constant=90"     bndry_cond="Neumann"
;
patch-3
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="psi"
rho_center=0      z_center=0      delta_r=0.1      delta_theta=5
r_min="constant=2.0"      bndry_cond="patch-2"
r_max="constant=4.9"      bndry_cond="patch-4"
theta_min="constant=0"      bndry_cond="Neumann"
theta_max="constant=90"     bndry_cond="Neumann"
;
patch-4
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="psi"
rho_center=0      z_center=0      delta_r=0.25      delta_theta=5
r_min="constant=5.00"      bndry_cond="patch-3"
r_max="constant=9.75"      bndry_cond="patch-5"
theta_min="constant=0"      bndry_cond="Neumann"
theta_max="constant=90"     bndry_cond="Neumann"
;
patch-5
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="psi"

```

Schwarzschild test series

OPO.PAT;2

18-OCT-1985 06:41

Page 2

```

rho_center=0      z_center=0      delta_r=0.5      delta_theta=7.5
r_min="constant-10.0"      bndry_cond="patch-4"
r_max="constant-19.5"      bndry_cond="patch-6"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch=6
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="psi"
rho_center=0      z_center=0      delta_r=1      delta_theta=7.5
r_min="constant-20"      bndry_cond="patch-5"
r_max="constant-49"      bndry_cond="patch-7"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch=7
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=2.5      delta_theta=7.5
r_min="constant-50"      bndry_cond="patch-6"
r_max="constant-100"      bndry_cond="Robin"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"

```


Table 19 - Schwarzschild test series, opo selected Ψ values

Patch Number	i	j	Psi	rho	z	r	theta
1	20	0	2.001963			0.5000	0.0
1	20	6	2.001963			0.5000	30.0
1	20	12	2.001963			0.5000	60.0
1	20	18	2.001963			0.5000	90.0
1	39	0	1.514181			0.9750	0.0
1	39	6	1.514181			0.9750	30.0
1	39	12	1.514181			0.9750	60.0
1	39	18	1.514181			0.9750	90.0
2	20	0	1.501342			1.0000	0.0
2	20	6	1.501342			1.0000	30.0
2	20	12	1.501342			1.0000	60.0
2	20	18	1.501342			1.0000	90.0
2	39	0	1.257330			1.9500	0.0
2	39	6	1.257330			1.9500	30.0
2	39	12	1.257330			1.9500	60.0
2	39	18	1.257330			1.9500	90.0
3	20	0	1.250908			2.0000	0.0
3	20	6	1.250908			2.0000	30.0
3	20	12	1.250908			2.0000	60.0
3	20	18	1.250908			2.0000	90.0
3	49	0	1.102613			4.9000	0.0
3	49	6	1.102613			4.9000	30.0
3	49	12	1.102613			4.9000	60.0
3	49	18	1.102613			4.9000	90.0
4	20	0	1.100567			5.0000	0.0
4	20	6	1.100567			5.0000	30.0
4	20	12	1.100567			5.0000	60.0
4	20	18	1.100567			5.0000	90.0
4	39	0	1.051606			9.7500	0.0
4	39	6	1.051606			9.7500	30.0
4	39	12	1.051606			9.7500	60.0
4	39	18	1.051606			9.7500	90.0
5	20	0	1.050317			10.0000	0.0
5	20	4	1.050317			10.0000	30.0
5	20	8	1.050317			10.0000	60.0
5	20	12	1.050317			10.0000	90.0
5	39	0	1.025824			19.5000	0.0
5	39	4	1.025824			19.5000	30.0
5	39	8	1.025824			19.5000	60.0
5	39	12	1.025824			19.5000	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

Schwarzschild test series

OOP.PAR;2

18-OCT-1985 06:41

Page 1

title="Schwarzschild Solution"

N_black_hole=1 grid="oop"

;

black_hole=1

rho=0 z=0

mass=1 surrounding_patch_flag=1

momentum=0 reflection_term_sign=0 reflection_radius=0.5

spin=0

;

; mkgfn oop

; directcg oop 1.0e-10 (2685.24 cpu seconds)

Schwarzschild test series

OOP.PAT;2

18-OCT-1985 06:41

Page 1

```

; Each patch approximately 20 x 20
; All patches polar spherical
; All patches using locally conservative differencing
; Outer boundary at r = 100
;
; Conformal function omega / omega / psi
;
N_patch=7
;
; Now describe each coordinate patch:
;
patch-1
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.025  delta_theta=5
r_min="constant-0.500"  bndry_cond="Reflection Symmetric"
r_max="constant-0.975"  bndry_cond="patch-2"
theta_min="constant-0"  bndry_cond="Neumann"
theta_max="constant-90" bndry_cond="Neumann"
;
patch-2
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.05    delta_theta=5
r_min="constant-1.00"  bndry_cond="patch-1"
r_max="constant-1.95"  bndry_cond="patch-3"
theta_min="constant-0" bndry_cond="Neumann"
theta_max="constant-90" bndry_cond="Neumann"
;
patch-3
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.1    delta_theta=5
r_min="constant-2.0"  bndry_cond="patch-2"
r_max="constant-4.9"  bndry_cond="patch-4"
theta_min="constant-0" bndry_cond="Neumann"
theta_max="constant-90" bndry_cond="Neumann"
;
patch-4
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.25   delta_theta=5
r_min="constant-5.00"  bndry_cond="patch-3"
r_max="constant-9.75"  bndry_cond="patch-5"
theta_min="constant-0" bndry_cond="Neumann"
theta_max="constant-90" bndry_cond="Neumann"
;
patch-5
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"

```

Schwarzschild test series

OOP.PAT;2

18-OCT-1985 06:41

Page 2

```

rho_center=0      z_center=0      delta_r=0.5      delta_theta=7.5
r_min="constant-10.0"      bndry_cond="patch-4"
r_max="constant-19.5"      bndry_cond="patch-6"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-6
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=1      delta_theta=7.5
r_min="constant-20"      bndry_cond="patch-5"
r_max="constant-49"      bndry_cond="patch-7"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-7
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="psi"
rho_center=0      z_center=0      delta_r=2.5      delta_theta=7.5
r_min="constant-50"      bndry_cond="patch-6"
r_max="constant-100"      bndry_cond="Robin"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"

```

Table 20 - Schwarzschild test series, oop selected Ψ values

Patch Number	i	j	Psi	rho	z	r	theta
1	20	0	2.001480			0.5000	0.0
1	20	6	2.001480			0.5000	30.0
1	20	12	2.001480			0.5000	60.0
1	20	18	2.001480			0.5000	90.0
1	39	0	1.513816			0.9750	0.0
1	39	6	1.513816			0.9750	30.0
1	39	12	1.513816			0.9750	60.0
1	39	18	1.513816			0.9750	90.0
2	20	0	1.500980			1.0000	0.0
2	20	6	1.500980			1.0000	30.0
2	20	12	1.500980			1.0000	60.0
2	20	18	1.500980			1.0000	90.0
2	39	0	1.257027			1.9500	0.0
2	39	6	1.257027			1.9500	30.0
2	39	12	1.257027			1.9500	60.0
2	39	18	1.257027			1.9500	90.0
3	20	0	1.250606			2.0000	0.0
3	20	6	1.250606			2.0000	30.0
3	20	12	1.250606			2.0000	60.0
3	20	18	1.250606			2.0000	90.0
3	49	0	1.102347			4.9000	0.0
3	49	6	1.102347			4.9000	30.0
3	49	12	1.102347			4.9000	60.0
3	49	18	1.102347			4.9000	90.0
4	20	0	1.100302			5.0000	0.0
4	20	6	1.100302			5.0000	30.0
4	20	12	1.100302			5.0000	60.0
4	20	18	1.100302			5.0000	90.0
4	39	0	1.051460			9.7500	0.0
4	39	6	1.051460			9.7500	30.0
4	39	12	1.051460			9.7500	60.0
4	39	18	1.051460			9.7500	90.0
5	20	0	1.050175			10.0000	0.0
5	20	4	1.050175			10.0000	30.0
5	20	8	1.050175			10.0000	60.0
5	20	12	1.050175			10.0000	90.0
5	39	0	1.025742			19.5000	0.0
5	39	4	1.025742			19.5000	30.0
5	39	8	1.025742			19.5000	60.0
5	39	12	1.025742			19.5000	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

Choptuik comparison test series

PPX001.PAR;3

18-OCT-1985 06:47

Page 1

title="Replicate Matt's Results (P=0.001)"

N_black_hole=1 grid="matt"

;

black_hole=1

rho=0

z=0

mass=2

surrounding_patch_flag=1

momentum=0.001

reflection_term_sign=+1

reflection_radius=1

spin=0

;

; mkgfn ppx001

; directcg ppx001 1.0e-10

(1606.50 cpu seconds)

Figure 12 – Choptuik comparison test series, ppx001 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

Choptuik comparison test series

PPX01.PAR;4

18-OCT-1985 06:47

Page 1

title="Replicate Matt's Results (P=0.01)"

N_black_hole=1 grid="matt"

;

black_hole=1

rho=0 z=0

mass=2 surrounding_patch_flag=1

momentum=0.01 reflection_term_sign=+1 reflection_radius=1

spin=0

;

; mkgfn ppx01

; directcg ppx01 1.0e-10

(1763.90 cpu seconds)

Figure 13 – Choptuik comparison test series, ppx01 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

Choptuik comparison test series

PPX1.PAR;2

18-OCT-1985 06:47

Page 1

title="Replicate Matt's Results (P=0.1)"

N_black_hole=1 grid="matt"

;

black_hole=1

rho=0 z=0

mass=2 surrounding_patch_flag=1

momentum=0.1 reflection_term_sign=+1 reflection_radius=1

spin=0

;

; mkgfn ppx1

; cpgfn ppx01 psi ppx1 psi

; directcg ppx1 1.0e-10 (1820.39 cpu seconds)

Figure 14 – Choptuik comparison test series, ppx1 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

Choptuik comparison test series

PPl.PAR;2

18-OCT-1985 06:47

Page 1

title="Replicate Matt's Results (P-1)"

N_black_hole=1 grid="matt"

;

black_hole=1

rho=0 z=0

mass=2 surrounding_patch_flag=1

momentum=1 reflection_term_sign=+1 reflection_radius=1

spin=0

;

; mkgfn ppl

; cpgfn ppx1 psi ppl psi

; directcg ppl 1.0e-10 (2620.18 cpu seconds)

Figure 15 - Choptuik comparison test series, pp1 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

Choptuik comparison test series

PP2X5.PAR;2

18-OCT-1985 06:47

Page 1

title="Replicate Matt's Results (P=2.5)"
N_black_hole=1 grid="matt"

;
black_hole=1
rho=0 z=0
mass=2 surrounding_patch_flag=1
momentum=2.5 reflection_term_sign=+1 reflection_radius=1
spin=0
;
; mkgfn pp2x5
; cpgfn ppl psi pp2x5 psi
; directog pp2x5 1.0e-10 (3245.29 cpu seconds)

Figure 16 – Choptuik comparison test series, pp2x5 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

Choptuik comparison test series

PP7X5.PAR;2

18-OCT-1985 06:47

Page 1

title="Replicate Matt's Results (P=7.5)"

N_black_hole=1 grid="matt"

;

black_hole=1

rho=0 z=0

mass=2 surrounding_patch_flag=1

momentum=7.5 reflection_term_sign=+1 reflection_radius=1

spin=0

;

; mkgfn pp7x5

; cpgfn pp2x5 psi pp7x5 psi

; directcg pp7x5 1.0e-10

(4273.96 cpu seconds)

Figure 17 – Choptuik comparison test series, pp7x5 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

Choptuik comparison test series

PP12X5.PAR;2

18-OCT-1985 06:47

Page 1

title="Replicate Matt's Results (P=12.5)"

N_black_hole=1 grid="matt"

;

black_hole=1

rho=0 z=0

mass=2 surrounding_patch_flag=1

momentum=12.5 reflection_term_sign=+1 reflection_radius=1

spin=0

;

; mkgfn pp12x5

; cpgfn pp7x5 psi pp12x5 psi

; directcg pp12x5 1.0e-10 (4141.61 cpu seconds)

Figure 18 – Choptuik comparison test series, pp12x5 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

Choptuik comparison test series

MPX001.PAR;2

18-OCT-1985 06:47

Page 1

title="Replicate Matt's Results (P=0.001)"

N_black_hole=1 grid="matt"

;

black_hole=1

rho=0

z=0

mass=2 surrounding_patch_flag=1

momentum=0.001 reflection_term_sign=-1 reflection_radius=1

spin=0

;

; mkgfn mpx001

; directcg mpx001 1.0e-10

(1566.28 cpu seconds)

Figure 19 – Choptuik comparison test series, mpx001 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

Choptuik comparison test series

MPX01.PAR;2

18-OCT-1985 06:47

Page 1

title="Replicate Matt's Results (P=0.01)"

N_black_hole-1 grid="matt"

;

black_hole-1

rho-0 z-0

mass-2 surrounding_patch_flag-1

momentum-0.01 reflection_term_sign=-1 reflection_radius-1

spin-0

;

; mkgfn mpx01

; directcg mpx01 1.0e-10

(1612.97 cpu seconds)

Figure 20 – Choptuik comparison test series, mpx01 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

Choptuik comparison test series

MPX1.PAR;2

18-OCT-1985 06:47

Page 1

title="Replicate Matt's Results (P=0.1)"

N_black_hole=1 grid="matt"

;

black_hole=1

rho=0 z=0

mass=2 surrounding_patch_flag=1

momentum=0.1 reflection_term_sign=-1 reflection_radius=1

spin=0

;

; mkgfn mpx1

; cpgfn mpx01 psi mpx1 psi

; directcg mpx1 1.0e-10 (1766.72 cpu seconds)

Figure 21 – Choptuik comparison test series, mpx1 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

Choptuik comparison test series

MP1.PAR;2

18-OCT-1985 06:46

Page 1

title="Replicate Matt's Results (P-1)"

N_black_hole-1 grid="matt"

;

black_hole-1

rho=0 z=0

mass=2 surrounding_patch_flag=1

momentum-1 reflection_term_sign=-1 reflection_radius=1

spin=0

;

; mkgfn mpl

; cpgfn mpx1 psi mpl psi

; directcg mpl 1.0e-10 (2552.99 cpu seconds)

Figure 22 - Choptuik comparison test series, mp1 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

Choptuik comparison test series

MP2X5.PAR;3

18-OCT-1985 06:54

Page 1

title="Replicate Matt's Results (P-2.5)"

N_black_hole-1 grid="matt"

;

black_hole-1

rho-0 z-0

mass-2 surrounding_patch_flag-1

momentum-2.5 reflection_term_sign--1 reflection_radius-1

spin-0

;

; mkgfn mp2x5

; cpgfn mpl psi mp2x5 psi

; directcg mp2x5 1.0e-10

(2490.73 cpu seconds)

Figure 23 – Choptuik comparison test series, mp2x5 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

Choptuik comparison test series

MP7X5.PAR;2

18-OCT-1985 06:47

Page 1

title="Replicate Matt's Results (P=7.5)"

N_black_hole-1 grid="matt"

;

black_hole-1

rho-0 z=0

mass-2 surrounding_patch_flag-1

momentum-7.5 reflection_term_sign--1 reflection_radius-1

spin-0

;

; mkgfn mp7x5

; cpgfn mp2x5 ps1 mp7x5 ps1

; directcg mp7x5 1.0e-10 (4260.21 cpu seconds)

Figure 24 – Choptuik comparison test series, mp7x5 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

Choptuik comparison test series

MP12X5.PAR;2

18-OCT-1985 06:46

Page 1

title="Replicate Matt's Results (P-12.5)"

N_black_hole-1 grid="matt"

;

black_hole-1

rho-0 z-0

mass-2 surrounding_patch_flag-1

momentum-12.5 reflection_term_sign--1 reflection_radius-1

spin-0

;

; mkgfn mpl2x5

; cpgfn mp7x5 psi mpl2x5 psi

; directcg mpl2x5 1.0e-10

(4427.27 cpu seconds)

Figure 25 - Choptuik comparison test series, mp12x5 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

Choptuik comparison test series

JX001.PAR;3

18-OCT-1985 06:46

Page 1

title="Replicate Matt's Results (J=0.001)"

N_black_hole=1 grid="matt"

;

black_hole=1

rho=0 z=0

mass=2 surrounding_patch_flag=1

momentum=0 reflection_term_sign=0 reflection_radius=1

spin=0.001

;

; mkgfn jx001

; directcg jx001 1.0e-10

(1617.24 cpu seconds)

Figure 26 – Choptuik comparison test series, jx001 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

Choptuik comparison test series

JX01.PAR;2

18-OCT-1985 06:46

Page 1

title="Replicate Matt's Results (J=0.01)"

N_black_hole=1 grid="matt"

;

black_hole=1

rho=0

z=0

mass=2

surrounding_patch_flag=1

momentum=0

reflection_term_sign=0

reflection_radius=1

spin=0.01

;

; mkgfn jx01

; directcg jx01 1.0e-10

(1765.37 cpu seconds)

Figure 27 - Choptuik comparison test series, jx01 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

Choptuik comparison test series

JX1.PAR;3

18-OCT-1985 06:46

Page 1

title="Replicate Matt's Results (J=0.1)"

N_black_hole=1 grid="matt"

;

black_hole=1

rho=0 z=0

mass=2 surrounding_patch_flag=1

momentum=0 reflection_term_sign=0 reflection_radius=1

spin=0.1

;

; mkgfn jx1

; cpgfn jx01 psi jx1 psi

; directcg jx1 1.0e-10 (1772.80 cpu seconds)

Figure 28 - Choptuik comparison test series, jx1 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

Choptuik comparison test series

J1.PAR;2

18-OCT-1985 06:46

Page 1

title="Replicate Matt's Results (J-1)"

N_black_hole=1 grid="matt"

;

black_hole=1

rho=0 z=0

mass=2 surrounding_patch_flag=1

momentum=0 reflection_term_sign=0 reflection_radius=1

spin=1

;

; mkgfn j1

; cpgfn jx1 psi j1 psi

; directcg j1 1.0e-10 (1640.19 cpu seconds)

Figure 29 - Choptuik comparison test series, j1 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

Choptuik comparison test series

J10.PAR;2

18-OCT-1985 06:46

Page 1

```
title="Replicate Matt's Results (J-10)"
N_black_hole-1      grid="matt"
;
black_hole-1
rho-0               z-0
mass-2              surrounding_patch_flag-1
momentum-0          reflection_term_sign-0      reflection_radius-1
spin-10
;
; mkgfn j10
; opgfn j1 psi j10 psi
; directog j10 1.0e-10      (4633.32 cpu seconds)
```

Figure 30 - Choptuik comparison test series, j10 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

Choptuik comparison test series

J100.PAR;2

18-OCT-1985 06:46

Page 1

title="Replicate Matt's Results (J=100)"

N_black_hole=1 grid="matt"

;

black_hole=1

rho=0 z=0

mass=2 surrounding_patch_flag=1

momentum=0 reflection_term_sign=0 reflection_radius=1

spin=100

;

; mkgfn j100

; cpgfn j10 psi j100 psi

; directcg j100 1.0e-10 (6219.71 cpu seconds)

Figure 31 – Choptuik comparison test series, j100 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

Choptuik comparison test series

J1000.PAR;2

18-OCT-1985 06:46

Page 1

title="Replicate Matt's Results (J=1000)"

N_black_hole=1 grid="matt"

;

black_hole=1

rho=0 z=0

mass=2 surrounding_patch_flag=1

momentum=0 reflection_term_sign=0 reflection_radius=1

spin=1000

;

; mkgfn j1000

; cpgfn j100 psi j1000 psi

; directcg j1000 1.0e-10

(6920.95 cpu seconds)

Figure 32 - Choptuik comparison test series, j1000 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

Choptuik comparison test series

MATT.PAT;6

18-OCT-1985 06:45

Page 1

```

; Patch system to reproduce Matt's results
;
N_patch=6
;
; Now describe each coordinate patch:
;
patch-1
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.05      delta_theta=5
r_min="constant-1.00"      bndry_cond="Reflection Symmetric"
r_max="constant-1.95"      bndry_cond="patch-2"
theta_min="constant=0"      bndry_cond="Neumann"
theta_max="constant=90"      bndry_cond="Neumann"
;
patch-2
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.1      delta_theta=5
r_min="constant-2.0"      bndry_cond="patch-1"
r_max="constant-4.9"      bndry_cond="patch-3"
theta_min="constant=0"      bndry_cond="Neumann"
theta_max="constant=90"      bndry_cond="Neumann"
;
patch-3
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.2      delta_theta=5
r_min="constant-5.0"      bndry_cond="patch-2"
r_max="constant-9.8"      bndry_cond="patch-4"
theta_min="constant=0"      bndry_cond="Neumann"
theta_max="constant=90"      bndry_cond="Neumann"
;
patch-4
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.5      delta_theta=10
r_min="constant-10.0"      bndry_cond="patch-3"
r_max="constant-19.5"      bndry_cond="patch-5"
theta_min="constant=0"      bndry_cond="Neumann"
theta_max="constant=90"      bndry_cond="Neumann"
;
patch-5
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=1      delta_theta=10
r_min="constant-20"      bndry_cond="patch-4"
r_max="constant-49"      bndry_cond="patch-6"
theta_min="constant=0"      bndry_cond="Neumann"
theta_max="constant=90"      bndry_cond="Neumann"

```


Choptuik comparison test series

MATT.PAT;6

18-OCT-1985 06:45

Page 2

```
;
patch=6
coord_type="polar spherical"
difference_scheme="locally conservative"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=2      delta_theta=10
r_min="constant-50"      bndry_cond="patch-5"
r_max="constant-100"      bndry_cond="Robin"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
```

J test series

JO.PAR;2

18-OCT-1985 06:58

Page 1

```
title="Single Black Hole (J=0)"
N_black_hole=1      grid="mt20"
;
black_hole=1
rho=0               z=0
mass=1              surrounding_patch_flag=1
momentum=0          reflection_term_sign=+1      reflection_radius=0.5
spin=0
;
; mkgfn j0
; directcg j0 1.0e-10
```

Table 21 - J test series, j0 selected Ψ values

Patch Number	i	j	Psi	rho	z	r	theta
1	25	0	2.000000			0.5000	0.0
1	25	6	2.000000			0.5000	30.0
1	25	12	2.000000			0.5000	60.0
1	25	18	2.000000			0.5000	90.0
1	49	0	1.510204			0.9800	0.0
1	49	6	1.510204			0.9800	30.0
1	49	12	1.510204			0.9800	60.0
1	49	18	1.510204			0.9800	90.0
2	20	0	1.500000			1.0000	0.0
2	20	6	1.500000			1.0000	30.0
2	20	12	1.500000			1.0000	60.0
2	20	18	1.500000			1.0000	90.0
2	39	0	1.256410			1.9500	0.0
2	39	6	1.256410			1.9500	30.0
2	39	12	1.256410			1.9500	60.0
2	39	18	1.256410			1.9500	90.0
3	20	0	1.250000			2.0000	0.0
3	20	6	1.250000			2.0000	30.0
3	20	12	1.250000			2.0000	60.0
3	20	18	1.250000			2.0000	90.0
3	49	0	1.102041			4.9000	0.0
3	49	6	1.102041			4.9000	30.0
3	49	12	1.102041			4.9000	60.0
3	49	18	1.102041			4.9000	90.0
4	25	0	1.100000			5.0000	0.0
4	25	6	1.100000			5.0000	30.0
4	25	12	1.100000			5.0000	60.0
4	25	18	1.100000			5.0000	90.0
4	49	0	1.051020			9.8000	0.0
4	49	6	1.051020			9.8000	30.0
4	49	12	1.051020			9.8000	60.0
4	49	18	1.051020			9.8000	90.0
5	20	0	1.050000			10.0000	0.0
5	20	4	1.050000			10.0000	30.0
5	20	8	1.050000			10.0000	60.0
5	20	12	1.050000			10.0000	90.0
5	39	0	1.025641			19.5000	0.0
5	39	4	1.025641			19.5000	30.0
5	39	8	1.025641			19.5000	60.0
5	39	12	1.025641			19.5000	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

J test series

JX01.PAR;3

18-OCT-1985 06:58

Page 1

```
title="Single Black Hole (J=0.01)"
N_black_hole=1      grid="mt20"
;
black_hole=1
rho=0               z=0
mass=1              surrounding_patch_flag=1
momentum=0          reflection_term_sign=+1      reflection_radius=0.5
spin=0.01
;
; mkgfn jx01
; directcg jx01 1.0e-10      (4082.52 cpu seconds)
```

Figure 33 - J test series, jx01 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

J test series

JX1.PAR;3

18-OCT-1985 06:58

Page 1

```
title="Single Black Hole (J=0.1)"
N_black_hole=1      grid="mt20"
;
black_hole=1
rho=0              z=0
mass=1            surrounding_patch_flag=1
momentum=0        reflection_term_sign=+1      reflection_radius=0.5
spin=0.1
;
; mkgfn jx1
; cpgfn jx01 psi jx1 psi
; directcg jx1 1.0e-10      (6278.99 cpu seconds)
```

Figure 34 - J test series, jx1 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

J test series

J1.PAR;3

18-OCT-1985 06:58

Page 1

title="Single Black Hole (J-1)"

N_black_hole=1 grid="mt20"

;

black_hole=1

rho=0 z=0

mass=1 surrounding_patch_flag=1

momentum=0 reflection_term_sign=+1 reflection_radius=0.5

spin=1

;

; mkgfn j1

; cpgfn jx1 psi j1 psi

; directog j1 1.0e-10 (8750.83 cpu seconds)

Figure 35 - J test series, j1 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

J test series

J10.PAR;4

18-OCT-1985 06:58

Page 1

title="Single Black Hole (J-10)"

N_black_hole=1 grid="mt20"

;

black_hole=1

rho=0 z=0

mass=1 surrounding_patch_flag=1

momentum=0 reflection_term_sign=+1 reflection_radius=0.5

spin=10

;

; mkgfn j10

; cpgfn j1 psi j10 psi

; directcg j10 1.0e-10

(11688.20 cpu seconds)

Figure 36 - J test series, j10 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

J test series

J100.PAR;4

18-OCT-1985 06:58

Page 1

```
title="Single Black Hole (J-100)"
N_black_hole=1      grid="mt20"
;
black_hole=1
rho=0              z=0
mass=1            surrounding_patch_flag=1
momentum=0        reflection_term_sign=+1      reflection_radius=0.5
spin=100
;
; mkgfn j100
; cpgfn j10 psi j100 psi
; directog j100 1.0e-10      (15128.02 cpu seconds)
```

Figure 37 - J test series, j100 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

J test series

MT20.PAT;6

18-OCT-1985 06:57

Page 1

```

; One black hole patch system with:
; - Marginally Trapped inner boundary conditions
; - Delta r / r approximately 20-25 at inner boundary of each patch
; - All patches polar spherical
; - All patches using independent terms differencing
; - All patches using omega for conformal function
; - Outer boundary at r = 1000
;
N_patch=10
;
; Now describe each coordinate patch:
;
patch-1
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.02      delta_theta=5
r_min="constant-0.50"      bndry_cond="Marginally Trapped"
r_max="constant-0.98"      bndry_cond="patch-2"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-2
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.05      delta_theta=5
r_min="constant-1.00"      bndry_cond="patch-1"
r_max="constant-1.95"      bndry_cond="patch-3"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-3
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.1      delta_theta=5
r_min="constant-2.0"      bndry_cond="patch-2"
r_max="constant-4.9"      bndry_cond="patch-4"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-4
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.2      delta_theta=5
r_min="constant-5.0"      bndry_cond="patch-3"
r_max="constant-9.8"      bndry_cond="patch-5"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-5
coord_type="polar spherical"
difference_scheme="independent terms"

```

J test series

MT20.PAT;6

18-OCT-1985 06:57

Page 2

```

conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.5      delta_theta=7.5
r_min="constant-10.0"      bndry_cond="patch-4"
r_max="constant-19.5"      bndry_cond="patch-6"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-6
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=1      delta_theta=7.5
r_min="constant-20"      bndry_cond="patch-5"
r_max="constant-49"      bndry_cond="patch-7"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-7
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=2      delta_theta=7.5
r_min="constant-50"      bndry_cond="patch-6"
r_max="constant-98"      bndry_cond="patch-8"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-8
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=5      delta_theta=10
r_min="constant-100"      bndry_cond="patch-7"
r_max="constant-195"      bndry_cond="patch-9"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-9
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=10      delta_theta=10
r_min="constant-200"      bndry_cond="patch-8"
r_max="constant-490"      bndry_cond="patch-10"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-10
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=20      delta_theta=10
r_min="constant-500"      bndry_cond="patch-9"
r_max="constant-1000"      bndry_cond="Robin"
theta_min="constant-0"      bndry_cond="Neumann"

```

J test series

MT20.PAT;6

18-OCT-1985 06:57

Page 3

theta_max="constant-90"

bndry_cond="Neumann"

RSP test series

PO.PAR;12

10-NOV-1985 22:46

Page 1

title="Single Black Hole (P-0)"

N_black_hole=1 grid="rs20"

;

black_hole=1

rho=0 z=0

mass=1 surrounding_patch_flag=1

momentum=0 reflection_term_sign=+1 reflection_radius=0.5

spin=0

;

; mkgfn p0

; directcg p0 1.0e-10

Table 22 – RSP test series, p0 selected Ψ values

Patch Number	i	j	Ψ_i	ρ	z	r	θ
1	25	0	2.000000			0.5000	0.0
1	25	6	2.000000			0.5000	30.0
1	25	12	2.000000			0.5000	60.0
1	25	18	2.000000			0.5000	90.0
1	49	0	1.510204			0.9800	0.0
1	49	6	1.510204			0.9800	30.0
1	49	12	1.510204			0.9800	60.0
1	49	18	1.510204			0.9800	90.0
2	20	0	1.500000			1.0000	0.0
2	20	6	1.500000			1.0000	30.0
2	20	12	1.500000			1.0000	60.0
2	20	18	1.500000			1.0000	90.0
2	39	0	1.256410			1.9500	0.0
2	39	6	1.256410			1.9500	30.0
2	39	12	1.256410			1.9500	60.0
2	39	18	1.256410			1.9500	90.0
3	20	0	1.250000			2.0000	0.0
3	20	6	1.250000			2.0000	30.0
3	20	12	1.250000			2.0000	60.0
3	20	18	1.250000			2.0000	90.0
3	49	0	1.102041			4.9000	0.0
3	49	6	1.102041			4.9000	30.0
3	49	12	1.102041			4.9000	60.0
3	49	18	1.102041			4.9000	90.0
4	25	0	1.100000			5.0000	0.0
4	25	6	1.100000			5.0000	30.0
4	25	12	1.100000			5.0000	60.0
4	25	18	1.100000			5.0000	90.0
4	49	0	1.051020			9.8000	0.0
4	49	6	1.051020			9.8000	30.0
4	49	12	1.051020			9.8000	60.0
4	49	18	1.051020			9.8000	90.0
5	20	0	1.050000			10.0000	0.0
5	20	4	1.050000			10.0000	30.0
5	20	8	1.050000			10.0000	60.0
5	20	12	1.050000			10.0000	90.0
5	39	0	1.025641			19.5000	0.0
5	39	4	1.025641			19.5000	30.0
5	39	8	1.025641			19.5000	60.0
5	39	12	1.025641			19.5000	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

RSP test series

PX01.PAR;8

10-NOV-1985 22:46

Page 1

title="Single Black Hole (P=0.01)"
N_black_hole=1 grid="rs20"

;
black_hole=1

rho=0 z=0

mass=1 surrounding_patch_flag=1

momentum=0.01 reflection_term_sign=+1 reflection_radius=0.5

spin=0

;

; mkgfn px01

; directcg px01 1.0e-10 (4253.86 cpu seconds)

Figure 38 – RSP test series, px01 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

RSP test series

PX1.PAR;6

10-NOV-1985 22:46

Page 1

```
title="Single Black Hole (P-0.1)"
N_black_hole=1      grid="rs20"
;
black_hole=1
rho=0      z=0
mass=1      surrounding_patch_flag=1
momentum=0.1  reflection_term_sign=+1      reflection_radius=0.5
spin=0
;
; mkgfn px1
; cpgfn px01 psi px1 psi
; directcg px1 1.0e-10      (4198.79 cpu seconds)
```

Figure 39 – RSP test series, px1 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

RSP test series

P1.PAR;5

10-NOV-1985 22:46

Page 1

title="Single Black Hole (P-1)"

N_black_hole=1 grid="rs20"

;

black_hole=1

rho=0 z=0

mass=1 surrounding_patch_flag=1

momentum=1 reflection_term_sign=+1 reflection_radius=0.5

spin=0

;

; mkgfn p1

; cpgfn px1 psi p1 psi

; directcg p1 1.0e-10

(8308.23 cpu seconds)

Figure 40 – RSP test series, p1 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

RSP test series

P10.PAR;5

10-NOV-1985 22:46

Page 1

title="Single Black Hole (P-10)"

N_black_hole=1 grid="rs20"

;

black_hole=1

rho=0 z=0

mass=1 surrounding_patch_flag=1

momentum=10 reflection_term_sign=+1 reflection_radius=0.5

spin=0

;

; mkgfn p10

; cpgfn p1 psi p10 psi

; directcg p10 1.0e-10 (11212.95 cpu seconds)

Figure 41 – RSP test series, p10 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

RSP test series

P100.PAR;5

10-NOV-1985 22:46

Page 1

title="Single Black Hole (P=100)"

N_black_hole=1 grid="rs20"

;

black_hole=1

rho=0 z=0

mass=1 surrounding_patch_flag=1

momentum=100 reflection_term_sign=+1 reflection_radius=0.5

spin=0

;

; mkgfn p100

; cpgfn p10 psi p100 psi

; directcg p100 1.0e-10 (10257.00 cpu seconds)

Figure 42 - RSP test series, p100 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

RSP test series

RS20.PAT;5

10-NOV-1985 22:46

Page 1

```

; One black hole patch system with:
; - Reflection Symmetric inner boundary conditions
; - Delta r / r approximately 20-25 at inner boundary of each patch
; - All patches polar spherical
; - All patches using independent terms differencing
; - All patches using omega for conformal function
; - Outer boundary at r = 1000
;
N_patch=10
;
; Now describe each coordinate patch:
;
patch-1
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.02      delta_theta=5
r_min="constant-0.50"      bndry_cond="Reflection Symmetric"
r_max="constant-0.98"      bndry_cond="patch-2"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-2
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.05      delta_theta=5
r_min="constant-1.00"      bndry_cond="patch-1"
r_max="constant-1.95"      bndry_cond="patch-3"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-3
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.1      delta_theta=5
r_min="constant-2.0"      bndry_cond="patch-2"
r_max="constant-4.9"      bndry_cond="patch-4"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-4
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.2      delta_theta=5
r_min="constant-5.0"      bndry_cond="patch-3"
r_max="constant-9.8"      bndry_cond="patch-5"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-5
coord_type="polar spherical"
difference_scheme="independent terms"

```

RSP test series

RS20.PAT;5

10-NOV-1985 22:46

Page 2

```

conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.5      delta_theta=7.5
r_min="constant-10.0"      bndry_cond="patch-4"
r_max="constant-19.5"      bndry_cond="patch-6"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-6
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=1      delta_theta=7.5
r_min="constant-20"      bndry_cond="patch-5"
r_max="constant-49"      bndry_cond="patch-7"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-7
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=2      delta_theta=7.5
r_min="constant-50"      bndry_cond="patch-6"
r_max="constant-98"      bndry_cond="patch-8"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-8
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=5      delta_theta=10
r_min="constant-100"      bndry_cond="patch-7"
r_max="constant-195"      bndry_cond="patch-9"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-9
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=10      delta_theta=10
r_min="constant-200"      bndry_cond="patch-8"
r_max="constant-490"      bndry_cond="patch-10"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-10
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=20      delta_theta=10
r_min="constant-500"      bndry_cond="patch-9"
r_max="constant-1000"      bndry_cond="Robin"
theta_min="constant-0"      bndry_cond="Neumann"

```

RSP test series

RS20.PAT;5

10-NOV-1985 22:46

Page 3

theta_max-"constant-90"

bndry_cond-"Neumann"

RSN test series

PO.PAR;13

10-NOV-1985 22:42

Page 1

title-"Single Black Hole (P-0)"

N_black_hole=1 grid="rs20"

;

black_hole=1

rho=0 z=0

mass=1 surrounding_patch_flag=1

momentum=0 reflection_term_sign=-1 reflection_radius=0.5

spin=0

;

; mkgfn p0

; directog p0 1.0e-10

Table 23 -- RSM test series, p0 selected Ψ values

Patch Number	i	j	Psi	rho	z	r	theta
1	25	0	2.000000			0.5000	0.0
1	25	6	2.000000			0.5000	30.0
1	25	12	2.000000			0.5000	60.0
1	25	18	2.000000			0.5000	90.0
1	49	0	1.510204			0.9800	0.0
1	49	6	1.510204			0.9800	30.0
1	49	12	1.510204			0.9800	60.0
1	49	18	1.510204			0.9800	90.0
2	20	0	1.500000			1.0000	0.0
2	20	6	1.500000			1.0000	30.0
2	20	12	1.500000			1.0000	60.0
2	20	18	1.500000			1.0000	90.0
2	39	0	1.256410			1.9500	0.0
2	39	6	1.256410			1.9500	30.0
2	39	12	1.256410			1.9500	60.0
2	39	18	1.256410			1.9500	90.0
3	20	0	1.250000			2.0000	0.0
3	20	6	1.250000			2.0000	30.0
3	20	12	1.250000			2.0000	60.0
3	20	18	1.250000			2.0000	90.0
3	49	0	1.102041			4.9000	0.0
3	49	6	1.102041			4.9000	30.0
3	49	12	1.102041			4.9000	60.0
3	49	18	1.102041			4.9000	90.0
4	25	0	1.100000			5.0000	0.0
4	25	6	1.100000			5.0000	30.0
4	25	12	1.100000			5.0000	60.0
4	25	18	1.100000			5.0000	90.0
4	49	0	1.051020			9.8000	0.0
4	49	6	1.051020			9.8000	30.0
4	49	12	1.051020			9.8000	60.0
4	49	18	1.051020			9.8000	90.0
5	20	0	1.050000			10.0000	0.0
5	20	4	1.050000			10.0000	30.0
5	20	8	1.050000			10.0000	60.0
5	20	12	1.050000			10.0000	90.0
5	39	0	1.025641			19.5000	0.0
5	39	4	1.025641			19.5000	30.0
5	39	8	1.025641			19.5000	60.0
5	39	12	1.025641			19.5000	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

RSM test series

PX01.PAR;9

10-NOV-1985 22:42

Page 1

```
title="Single Black Hole (P=0.01)"
N_black_hole=1      grid="rs20"
;
black_hole=1
rho=0      z=0
mass=1      surrounding_patch_flag=1
momentum=0.01  reflection_term_sign=-1      reflection_radius=0.5
spin=0
;
; mkgfn px01
; directog px01 1.0e-10      (4310.60 cpu seconds)
```

Figure 43 - RSM test series, px01 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

RSN test series

PX1.PAR;6

10-NOV-1985 22:42

Page 1

```
title="Single Black Hole (P=0.1)"
N_black_hole=1      grid="rs20"
;
black_hole=1
rho=0              z=0
mass=1             surrounding_patch_flag=1
momentum=0.1       reflection_term_sign=-1      reflection_radius=0.5
spin=0
;
; mkgfn px1
; cpgfn px01 psi px1 psi
; directcg px1 1.0e-10      (4164.68 cpu seconds)
```

Figure 44 – RSN test series, px1 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

RSN test series

P1.PAR;5

10-NOV-1985 22:42

Page 1

title="Single Black Hole (P-1)"

N_black_hole-1 grid="rs20"

;

black_hole-1

rho-0 z-0

mass-1 surrounding_patch_flag-1

momentum-1 reflection_term_sign--1 reflection_radius-0.5

spin-0

;

; mkgfn pl

; cpgfn px1 psi pl psi

; directog pl 1.0e-10 (7422.40 cpu seconds)

Figure 45 – RSN test series, p1 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

RSM test series

P10.PAR;5

10-NOV-1985 22:42

Page 1

title="Single Black Hole (P-10)"

N_black_hole=1 grid="rs20"

;

black_hole=1

rho=0 z=0

mass=1 surrounding_patch_flag=1

momentum=10 reflection_term_sign=-1 reflection_radius=0.5

spin=0

;

; mkgfn p10

; cpgfn p1 psi p10 psi

; directog p10 1.0e-10 (11970.08 cpu seconds)

Figure 46 - RSM test series, p10 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

RSN test series

P100.PAR;5

10-NOV-1985 22:42

Page 1

```
title="Single Black Hole (P-100)"
N_black_hole=1      grid="rs20"
;
black_hole=1
rho=0              z=0
mass=1             surrounding_patch_flag=1
momentum=100       reflection_term_sign=-1      reflection_radius=0.5
spin=0
;
; mkgfn p100
; cpgfn p10 psi p100 psi
; directcg p100 1.0e-10      (10439.14 cpu seconds)
```

Figure 47 - RSN test series, p100 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

RSN test series

RS20.PAT;5

10-NOV-1985 22:42

Page 1

```

; One black hole patch system with:
; - Reflection Symmetric inner boundary conditions
; - Delta r / r approximately 20-25 at inner boundary of each patch
; - All patches polar spherical
; - All patches using independent terms differencing
; - All patches using omega for conformal function
; - Outer boundary at r = 1000
;
N_patch=10
;
; Now describe each coordinate patch:
;
patch-1
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.02      delta_theta=5
r_min="constant-0.50"      bndry_cond="Reflection Symmetric"
r_max="constant-0.98"      bndry_cond="patch-2"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-2
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.05      delta_theta=5
r_min="constant-1.00"      bndry_cond="patch-1"
r_max="constant-1.95"      bndry_cond="patch-3"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-3
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.1      delta_theta=5
r_min="constant-2.0"      bndry_cond="patch-2"
r_max="constant-4.9"      bndry_cond="patch-4"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-4
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.2      delta_theta=5
r_min="constant-5.0"      bndry_cond="patch-3"
r_max="constant-9.8"      bndry_cond="patch-5"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch-5
coord_type="polar spherical"
difference_scheme="independent terms"

```

RSN test series

RS20.PAT;5

10-NOV-1985 22:42

Page 2

```

conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.5      delta_theta=7.5
r_min="constant-10.0"      bndry_cond="patch-4"
r_max="constant-19.8"      bndry_cond="patch-6"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch=6
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=1      delta_theta=7.5
r_min="constant-20"      bndry_cond="patch-5"
r_max="constant-49"      bndry_cond="patch-7"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch=7
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=2      delta_theta=7.5
r_min="constant-50"      bndry_cond="patch-6"
r_max="constant-98"      bndry_cond="patch-8"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch=8
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=5      delta_theta=10
r_min="constant-100"      bndry_cond="patch-7"
r_max="constant-195"      bndry_cond="patch-9"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch=9
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=10      delta_theta=10
r_min="constant-200"      bndry_cond="patch-8"
r_max="constant-490"      bndry_cond="patch-10"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-90"      bndry_cond="Neumann"
;
patch=10
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=20      delta_theta=10
r_min="constant-500"      bndry_cond="patch-9"
r_max="constant-1000"      bndry_cond="Robin"
theta_min="constant-0"      bndry_cond="Neumann"

```

RSN test series

RS20.PAT;5

10-NOV-1985 22:42

Page 3

theta_max="constant-90"

bndry_cond="Neumann"

MTO test series

PO.PAR;9

10-NOV-1985 22:35

Page 1

title="Single Black Hole (P=0)"

N_black_hole=1 grid="mt20big"

;

black_hole=1

rho=0 z=0

mass=1 surrounding_patch_flag=1

momentum=0 reflection_term_sign=0 reflection_radius=0.5

spin=0

;

; mkgfn p0

; directcg p0 1.0e-10

Table 24 - NTO test series, p0 selected Ψ values

Patch Number	i	j	Psi	rho	z	r	theta
1	25	0	2.000000			0.5000	0.0
1	25	9	2.000000			0.5000	45.0
1	25	18	2.000000			0.5000	90.0
1	25	27	2.000000			0.5000	135.0
1	25	36	2.000000			0.5000	180.0
1	49	0	1.510204			0.9800	0.0
1	49	9	1.510204			0.9800	45.0
1	49	18	1.510204			0.9800	90.0
1	49	27	1.510204			0.9800	135.0
1	49	36	1.510204			0.9800	180.0
2	20	0	1.500000			1.0000	0.0
2	20	9	1.500000			1.0000	45.0
2	20	18	1.500000			1.0000	90.0
2	20	27	1.500000			1.0000	135.0
2	20	36	1.500000			1.0000	180.0
2	39	0	1.256410			1.9500	0.0
2	39	9	1.256410			1.9500	45.0
2	39	18	1.256410			1.9500	90.0
2	39	27	1.256410			1.9500	135.0
2	39	36	1.256410			1.9500	180.0
3	20	0	1.250000			2.0000	0.0
3	20	9	1.250000			2.0000	45.0
3	20	18	1.250000			2.0000	90.0
3	20	27	1.250000			2.0000	135.0
3	20	36	1.250000			2.0000	180.0
3	49	0	1.102041			4.9000	0.0
3	49	9	1.102041			4.9000	45.0
3	49	18	1.102041			4.9000	90.0
3	49	27	1.102041			4.9000	135.0
3	49	36	1.102041			4.9000	180.0
4	25	0	1.100000			5.0000	0.0
4	25	9	1.100000			5.0000	45.0
4	25	18	1.100000			5.0000	90.0
4	25	27	1.100000			5.0000	135.0
4	25	36	1.100000			5.0000	180.0
4	49	0	1.051020			9.8000	0.0
4	49	9	1.051020			9.8000	45.0
4	49	18	1.051020			9.8000	90.0
4	49	27	1.051020			9.8000	135.0
4	49	36	1.051020			9.8000	180.0
5	20	0	1.050000			10.0000	0.0
5	20	6	1.050000			10.0000	45.0
5	20	12	1.050000			10.0000	90.0
5	20	18	1.050000			10.0000	135.0
5	20	24	1.050000			10.0000	180.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

MTO test series

PX01.PAR;6

10-NOV-1985 22:35

Page 1

title="Single Black Hole (P=0.01)"

N_black_hole=1 grid="mt20big"

;

black_hole=1

rho=0 z=0

mass=1 surrounding_patch_flag=1

momentum=0.01 reflection_term_sign=0 reflection_radius=0.5

spin=0

;

; mkgfn px01

; directog px01 1.0e-10 (28784.57 cpu seconds)

Figure 48 - NTO test series, px01 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

NT0 test series

PX1.PAR;4

10-NOV-1985 22:35

Page 1

```
title="Single Black Hole (P=0.1)"
N_black_hole=1      grid="mt20big"
;
black_hole=1
rho=0      z=0
mass=1      surrounding_patch_flag=1
momentum=0.1      reflection_term_sign=0      reflection_radius=0.5
spin=0
;
; mkgfn px1
; cpgfn px01 psi px1 psi
; directog px1 1.0e-10      (36840.61 cpu seconds)
```

Figure 49 - MT0 test series, px1 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

NT0 test series

P1.PAR;3

10-NOV-1985 22:35

Page 1

title="Single Black Hole (P-1)"

N_black_hole=1 grid="mt20big"

;

black_hole=1

rho=0 z=0

mass=1 surrounding_patch_flag=1

momentum=1 reflection_term_sign=0 reflection_radius=0.5

spin=0

;

; mkgfn p1

; cpgfn px1 psi p1 psi

; directog p1 1.0e-10 (41402.16 cpu seconds)

Figure 50 - NT0 test series, p1 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

NT0 test series

P10.PAR;3

10-NOV-1985 22:35

Page 1

title="Single Black Hole (P-10)"

N_black_hole-1 grid="mt20big"

;

black_hole-1

rho=0 z=0

mass-1 surrounding_patch_flag-1

momentum-10 reflection_term_sign=0 reflection_radius=0.5

spin=0

;

; mkgfn p10

; cpgfn p1 psi p10 psi

; directcg p10 1.0e-10 (47072.84 cpu seconds)

Figure 51 - NT0 test series, p10 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

MTO test series

P100.PAR;3

10-NOV-1985 22:35

Page 1

title="Single Black Hole (P-100)"

N_black_hole=1 grid="mt20big"

;

black_hole=1

rho=0 z=0

mass=1 surrounding_patch_flag=1

momentum=100 reflection_term_sign=0 reflection_radius=0.5

spin=0

;

; mkgfn p100

; cpgfn p10 psi p100 psi

; directcg p100 1.0e-10 (46894.12 cpu seconds)

Figure 52 – NT0 test series, p100 quantitative results

This figure shows $\Psi(\text{equator})$ and $\delta\Psi = \Psi(\text{north pole}) - \Psi(\text{equator})$, both as functions of $s = 1 - a/r$. The former shows the general radial trend of Ψ , while the latter is a measure of the deviation of Ψ from spherical symmetry.

NT0 test series

MT2OBIG.PAT;4

10-NOV-1985 22:35

Page 1

```

; One black hole patch system with:
; - Marginally Trapped inner boundary conditions
; - Delta r / r approximately 20-25 at inner boundary of each patch
; - All patches polar spherical
; - All patches using independent terms differencing
; - All patches using omega for conformal function
; - Outer boundary at r = 1000
;
N_patch=10
;
; Now describe each coordinate patch:
;
patch-1
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.02      delta_theta=5
r_min="constant-0.50"      bndry_cond="Marginally Trapped"
r_max="constant-0.98"      bndry_cond="patch-2"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-180"      bndry_cond="Neumann"
;
patch-2
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.05      delta_theta=5
r_min="constant-1.00"      bndry_cond="patch-1"
r_max="constant-1.95"      bndry_cond="patch-3"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-180"      bndry_cond="Neumann"
;
patch-3
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.1      delta_theta=5
r_min="constant-2.0"      bndry_cond="patch-2"
r_max="constant-4.9"      bndry_cond="patch-4"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-180"      bndry_cond="Neumann"
;
patch-4
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.2      delta_theta=5
r_min="constant-5.0"      bndry_cond="patch-3"
r_max="constant-9.8"      bndry_cond="patch-5"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-180"      bndry_cond="Neumann"
;
patch-5
coord_type="polar spherical"
difference_scheme="independent terms"

```

NTO test series

MT20BIG.PAT;4

10-NOV-1985 22:35

Page 2

```

conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.5      delta_theta=7.5
r_min="constant-10.0"      bndry_cond="patch-4"
r_max="constant-19.5"      bndry_cond="patch-6"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-180"      bndry_cond="Neumann"
;
patch=6
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=1      delta_theta=7.5
r_min="constant-20"      bndry_cond="patch-5"
r_max="constant-49"      bndry_cond="patch-7"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-180"      bndry_cond="Neumann"
;
patch=7
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=2      delta_theta=7.5
r_min="constant-50"      bndry_cond="patch-6"
r_max="constant-98"      bndry_cond="patch-8"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-180"      bndry_cond="Neumann"
;
patch=8
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=5      delta_theta=10
r_min="constant-100"      bndry_cond="patch-7"
r_max="constant-195"      bndry_cond="patch-9"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-180"      bndry_cond="Neumann"
;
patch=9
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=10      delta_theta=10
r_min="constant-200"      bndry_cond="patch-8"
r_max="constant-490"      bndry_cond="patch-10"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-180"      bndry_cond="Neumann"
;
patch=10
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=20      delta_theta=10
r_min="constant-500"      bndry_cond="patch-9"
r_max="constant-1000"      bndry_cond="Robin"
theta_min="constant-0"      bndry_cond="Neumann"

```

MTO test series

MT20BIG.PAT;4

10-NOV-1985 22:35

Page 3

theta_max="constant-180"

bndry_cond="Neumann"

PP test series

M100.PAR;1

18-OCT-1985 07:22

Page 1

title="Two Receding Black Holes (P-100)"

N_black_hole=2 grid="m1"

;

black_hole-1

rho=0 z=+1.5

mass=1 surrounding_patch_flag=+1

momentum=+100 reflection_term_sign=0 reflection_radius=0.5

spin=0

;

black_hole-2

rho=0 z=-1.5

mass=1 surrounding_patch_flag=-1

momentum=-100 reflection_term_sign=0 reflection_radius=0.5

spin=0

;

; mkgfn m100

; cpgfn m10 psi m100 psi

; directcg m100 1.0e-10 (63406.59 cpu seconds)

Table 25 - PP test series, m100 selected Ψ values

Patch Number	i	j	Ps1	rho	z	r	theta
1	25	0	6.006155			0.5000	0.0
1	25	9	5.889163			0.5000	45.0
1	25	18	5.630431			0.5000	90.0
1	25	27	5.426208			0.5000	135.0
1	25	36	5.366942			0.5000	180.0
1	49	0	4.240542			0.9800	0.0
1	49	9	4.202186			0.9800	45.0
1	49	18	4.184557			0.9800	90.0
1	49	27	4.325007			0.9800	135.0
1	49	36	4.472483			0.9800	180.0
2	20	0	4.203141			1.0000	0.0
2	20	18	4.151684			1.0000	90.0
2	20	36	4.454380			1.0000	180.0
2	21	0	4.115249			1.0500	0.0
2	21	18	4.074012			1.0500	90.0
2	21	36	4.413143			1.0500	180.0
3	0	0	4.251956	0.00	0.00		
3	20	0	3.857759	1.00	0.00		
3	40	0	3.264515	2.00	0.00		
3	60	0	2.830528	3.00	0.00		
3	20	20	4.065191	1.00	1.00		
3	40	20	3.253724	2.00	1.00		
3	60	20	2.807309	3.00	1.00		
4	20	40	3.960098	1.00	2.00		
4	40	40	3.143068	2.00	2.00		
4	0	60	3.559160	0.00	3.00		
4	20	60	3.300082	1.00	3.00		
5	35	0	3.190235			3.5000	0.0
5	35	10	2.990383			3.5000	30.0
5	35	20	2.745623			3.5000	60.0
5	35	30	2.667309			3.5000	90.0
5	49	0	2.623991			4.9000	0.0
5	49	10	2.525446			4.9000	30.0
5	49	20	2.382282			4.9000	60.0
5	49	30	2.332927			4.9000	90.0
6	25	0	2.596496			5.0000	0.0
6	25	9	2.423789			5.0000	45.0
6	25	18	2.314035			5.0000	90.0
6	49	0	1.897579			9.8000	0.0
6	49	9	1.829939			9.8000	45.0
6	49	18	1.783860			9.8000	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

PP test series

M10.PAR;3

18-OCT-1985 07:22

Page 1

title="Two Receding Black Holes (P-10)"

N_black_hole=2 grid="m1"

;

black_hole=1

rho=0 z=+1.5

mass=1 surrounding_patch_flag=+1

momentum=+10 reflection_term_sign=0 reflection_radius=0.5

spin=0

;

black_hole=2

rho=0 z=-1.5

mass=1 surrounding_patch_flag=-1

momentum=-10 reflection_term_sign=0 reflection_radius=0.5

spin=0

;

; mkgfn m10

; cpgfn m1 psi m10 psi

; directcg m10 1.0e-10 (53590.01 cpu seconds)

Table 26 – PP test series, m10 selected Ψ values

Patch Number	i	j	Psi	rho	z	r	theta
1	25	0	3.511156			0.5000	0.0
1	25	9	3.456947			0.5000	45.0
1	25	18	3.338895			0.5000	90.0
1	25	27	3.249710			0.5000	135.0
1	25	36	3.226276			0.5000	180.0
1	49	0	2.503289			0.9800	0.0
1	49	9	2.490086			0.9800	45.0
1	49	18	2.491959			0.9800	90.0
1	49	27	2.568467			0.9800	135.0
1	49	36	2.646260			0.9800	180.0
2	20	0	2.482042			1.0000	0.0
2	20	18	2.473143			1.0000	90.0
2	20	36	2.635094			1.0000	180.0
2	21	0	2.432203			1.0500	0.0
2	21	18	2.428829			1.0500	90.0
2	21	36	2.609735			1.0500	180.0
3	0	0	2.511518	0.00	0.00		
3	20	0	2.296484	1.00	0.00		
3	40	0	1.981559	2.00	0.00		
3	60	0	1.760077	3.00	0.00		
3	20	20	2.422029	1.00	1.00		
3	40	20	1.977030	2.00	1.00		
3	60	20	1.747980	3.00	1.00		
4	20	40	2.359027	1.00	2.00		
4	40	40	1.916064	2.00	2.00		
4	0	60	2.120753	0.00	3.00		
4	20	60	1.985254	1.00	3.00		
5	35	0	1.920234			3.5000	0.0
5	35	10	1.827226			3.5000	30.0
5	35	20	1.715477			3.5000	60.0
5	35	30	1.679407			3.5000	90.0
5	49	0	1.629380			4.9000	0.0
5	49	10	1.592270			4.9000	30.0
5	49	20	1.538970			4.9000	60.0
5	49	30	1.519926			4.9000	90.0
6	25	0	1.615920			5.0000	0.0
6	25	9	1.552232			5.0000	45.0
6	25	18	1.511187			5.0000	90.0
6	49	0	1.304326			9.8000	0.0
6	49	9	1.290242			9.8000	45.0
6	49	18	1.280085			9.8000	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

PP test series

M1.PAR;2

18-OCT-1985 07:22

Page 1

```

title="Two Receding Black Holes (P-1)"
N_black_hole=2      grid="m1"
;
black_hole=1
rho=0               z=+1.5
mass=1              surrounding_patch_flag=+1
momentum=+1         reflection_term_sign=0      reflection_radius=0.5
spin=0
;
black_hole=2
rho=0               z=-1.5
mass=1              surrounding_patch_flag=-1
momentum=-1         reflection_term_sign=0      reflection_radius=0.5
spin=0
;
; mkgfn m1
; cpgfn mx1 psi m1 psi
; directcg m1 1.0e-10      (47018.38 cpu seconds)

```

Table 27 — PP test series, m1 selected Ψ values

Patch Number	i	j	Ψ	ρ	z	r	θ
1	25	0	2.476853			0.5000	0.0
1	25	9	2.470882			0.5000	45.0
1	25	18	2.464894			0.5000	90.0
1	25	27	2.477059			0.5000	135.0
1	25	36	2.490642			0.5000	180.0
1	49	0	1.821982			0.9800	0.0
1	49	9	1.827481			0.9800	45.0
1	49	18	1.852638			0.9800	90.0
1	49	27	1.911104			0.9800	135.0
1	49	36	1.961148			0.9800	180.0
2	20	0	1.808130			1.0000	0.0
2	20	18	1.839552			1.0000	90.0
2	20	36	1.951830			1.0000	180.0
2	21	0	1.775717			1.0500	0.0
2	21	18	1.808897			1.0500	90.0
2	21	36	1.930789			1.0500	180.0
3	0	0	1.850764	0.00	0.00		
3	20	0	1.710494	1.00	0.00		
3	40	0	1.515366	2.00	0.00		
3	60	0	1.385855	3.00	0.00		
3	20	20	1.806741	1.00	1.00		
3	40	20	1.513728	2.00	1.00		
3	60	20	1.378632	3.00	1.00		
4	20	40	1.750647	1.00	2.00		
4	40	40	1.473670	2.00	2.00		
4	0	60	1.577118	0.00	3.00		
4	20	60	1.500688	1.00	3.00		
5	35	0	1.455685			3.5000	0.0
5	35	10	1.412833			3.5000	30.0
5	35	20	1.358943			3.5000	60.0
5	35	30	1.340448			3.5000	90.0
5	49	0	1.294193			4.9000	0.0
5	49	10	1.281592			4.9000	30.0
5	49	20	1.261733			4.9000	60.0
5	49	30	1.253732			4.9000	90.0
6	25	0	1.287189			5.0000	0.0
6	25	9	1.265370			5.0000	45.0
6	25	18	1.249105			5.0000	90.0
6	49	0	1.136699			9.8000	0.0
6	49	9	1.133951			9.8000	45.0
6	49	18	1.131489			9.8000	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

PP test series

MX1.PAR;2

18-OCT-1985 07:22

Page 1

title="Two Receding Black Holes (P=0.1)"

N_black_hole=2 grid="m1"

;

black_hole-1

rho=0 z=+1.5

mass=1 surrounding_patch_flag=+1

momentum=+0.1 reflection_term_sign=0 reflection_radius=0.5

spin=0

;

black_hole-2

rho=0 z=-1.5

mass=1 surrounding_patch_flag=-1

momentum=-0.1 reflection_term_sign=0 reflection_radius=0.5

spin=0

;

; mkgfn mx1

; cpgfn mx01 psi mx1 psi

; directcg mx1 1.0e-10 (68193.69 cpu seconds)

Table 28 - PP test series, mx1 selected Ψ values

Patch Number	i	j	Psi	rho	z	r	theta
1	25	0	2.356239			0.5000	0.0
1	25	9	2.367343			0.5000	45.0
1	25	18	2.401762			0.5000	90.0
1	25	27	2.452931			0.5000	135.0
1	25	36	2.482240			0.5000	180.0
1	49	0	1.761109			0.9800	0.0
1	49	9	1.771717			0.9800	45.0
1	49	18	1.807236			0.9800	90.0
1	49	27	1.872953			0.9800	135.0
1	49	36	1.924833			0.9800	180.0
2	20	0	1.748296			1.0000	0.0
2	20	18	1.794592			1.0000	90.0
2	20	36	1.915251			1.0000	180.0
2	21	0	1.718290			1.0500	0.0
2	21	18	1.764988			1.0500	90.0
2	21	36	1.893645			1.0500	180.0
3	0	0	1.811867	0.00	0.00		
3	20	0	1.673609	1.00	0.00		
3	40	0	1.484595	2.00	0.00		
3	60	0	1.360992	3.00	0.00		
3	20	20	1.767419	1.00	1.00		
3	40	20	1.482569	2.00	1.00		
3	60	20	1.353941	3.00	1.00		
4	20	40	1.703457	1.00	2.00		
4	40	40	1.442725	2.00	2.00		
4	0	60	1.533857	0.00	3.00		
4	20	60	1.464282	1.00	3.00		
5	35	0	1.420980			3.5000	0.0
5	35	10	1.383382			3.5000	30.0
5	35	20	1.334922			3.5000	60.0
5	35	30	1.317976			3.5000	90.0
5	49	0	1.271404			4.9000	0.0
5	49	10	1.260719			4.9000	30.0
5	49	20	1.243413			4.9000	60.0
5	49	30	1.236270			4.9000	90.0
6	25	0	1.264936			5.0000	0.0
6	25	9	1.246307			5.0000	45.0
6	25	18	1.231928			5.0000	90.0
6	49	0	1.126269			9.8000	0.0
6	49	9	1.124065			9.8000	45.0
6	49	18	1.122008			9.8000	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

PP test series

MX01.PAR;1

18-OCT-1985 07:22

Page 1

```

title-"Two Receding Black Holes (P=0.01)"
N_black_hole-2      grid="m1"
;
black_hole-1
rho-0              z=+1.5
mass-1            surrounding_patch_flag=+1
momentum=+0.01    reflection_term_sign=0      reflection_radius=0.5
spin=0
;
black_hole-2
rho-0              z=-1.5
mass-1            surrounding_patch_flag=-1
momentum=-0.01    reflection_term_sign=0      reflection_radius=0.5
spin=0
;
; mkgfn mx01
; cpgfn 0 psi mx01 psi
; directog mx01 1.0e-10      (35585.18 cpu seconds)

```

Table 29 - PP test series, mx01 selected Ψ values

Patch Number	i	j	Psi	rho	z	r	theta
1	25	0	2.348612			0.5000	0.0
1	25	9	2.361687			0.5000	45.0
1	25	18	2.400759			0.5000	90.0
1	25	27	2.456440			0.5000	135.0
1	25	36	2.487596			0.5000	180.0
1	49	0	1.758715			0.9800	0.0
1	49	9	1.769840			0.9800	45.0
1	49	18	1.806580			0.9800	90.0
1	49	27	1.873518			0.9800	135.0
1	49	36	1.925954			0.9800	180.0
2	20	0	1.745978			1.0000	0.0
2	20	18	1.793944			1.0000	90.0
2	20	36	1.916320			1.0000	180.0
2	21	0	1.716144			1.0500	0.0
2	21	18	1.764359			1.0500	90.0
2	21	36	1.894600			1.0500	180.0
3	0	0	1.812425	0.00	0.00		
3	20	0	1.673693	1.00	0.00		
3	40	0	1.484351	2.00	0.00		
3	60	0	1.360696	3.00	0.00		
3	20	20	1.767408	1.00	1.00		
3	40	20	1.482236	2.00	1.00		
3	60	20	1.353621	3.00	1.00		
4	20	40	1.702260	1.00	2.00		
4	40	40	1.442211	2.00	2.00		
4	0	60	1.532598	0.00	3.00		
4	20	60	1.463397	1.00	3.00		
5	35	0	1.420130			3.5000	0.0
5	35	10	1.382791			3.5000	30.0
5	35	20	1.334570			3.5000	60.0
5	35	30	1.317686			3.5000	90.0
5	49	0	1.270959			4.9000	0.0
5	49	10	1.260343			4.9000	30.0
5	49	20	1.243130			4.9000	60.0
5	49	30	1.236019			4.9000	90.0
6	25	0	1.264505			5.0000	0.0
6	25	9	1.245990			5.0000	45.0
6	25	18	1.231680			5.0000	90.0
6	49	0	1.126096			9.8000	0.0
6	49	9	1.123906			9.8000	45.0
6	49	18	1.121861			9.8000	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

PP test series

O.PAR;13

18-OCT-1985 07:22

Page 1

title="Two Stationary Black Holes"

N_black_hole=2 grid="m1"

;

black_hole=1

rho=0 z=+1.5

mass=1 surrounding_patch_flag=+1

momentum=0 reflection_term_sign=0 reflection_radius=0.5

spin=0

;

black_hole=2

rho=0 z=-1.5

mass=1 surrounding_patch_flag=-1

momentum=0 reflection_term_sign=0 reflection_radius=0.5

spin=0

;

; mkgfn 0

; directog 0 1.0e-10

(44591.88 cpu seconds)

Table 30 - PP test series, 0 selected Ψ values

Patch Number	i	j	Psi	rho	z	r	theta
1	25	0	2.347831			0.5000	0.0
1	25	9	2.361127			0.5000	45.0
1	25	18	2.400718			0.5000	90.0
1	25	27	2.456905			0.5000	135.0
1	25	36	2.488268			0.5000	180.0
1	49	0	1.758501			0.9800	0.0
1	49	9	1.769682			0.9800	45.0
1	49	18	1.806559			0.9800	90.0
1	49	27	1.873638			0.9800	135.0
1	49	36	1.926140			0.9800	180.0
2	20	0	1.745772			1.0000	0.0
2	20	18	1.793923			1.0000	90.0
2	20	36	1.916500			1.0000	180.0
2	21	0	1.715956			1.0500	0.0
2	21	18	1.764340			1.0500	90.0
2	21	36	1.894766			1.0500	180.0
3	0	0	1.812545	0.00	0.00		
3	20	0	1.673752	1.00	0.00		
3	40	0	1.484362	2.00	0.00		
3	60	0	1.360692	3.00	0.00		
3	20	20	1.767458	1.00	1.00		
3	40	20	1.482237	2.00	1.00		
3	60	20	1.353614	3.00	1.00		
4	20	40	1.702174	1.00	2.00		
4	40	40	1.442188	2.00	2.00		
4	0	60	1.532500	0.00	3.00		
4	20	60	1.463336	1.00	3.00		
5	35	0	1.420071			3.5000	0.0
5	35	10	1.382757			3.5000	30.0
5	35	20	1.334559			3.5000	60.0
5	35	30	1.317680			3.5000	90.0
5	49	0	1.270934			4.9000	0.0
5	49	10	1.260324			4.9000	30.0
5	49	20	1.243120			4.9000	60.0
5	49	30	1.236012			4.9000	90.0
6	25	0	1.264481			5.0000	0.0
6	25	9	1.245976			5.0000	45.0
6	25	18	1.231672			5.0000	90.0
6	49	0	1.126088			9.8000	0.0
6	49	9	1.123900			9.8000	45.0
6	49	18	1.121856			9.8000	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

PP test series

PX01.PAR;4

18-OCT-1985 07:28

Page 1

```
title="Two Approaching Black Holes (P=0.01)"
N_black_hole=2      grid="m1"
;
black_hole=1
rho=0               z=+1.5
mass=1              surrounding_patch_flag=+1
momentum=-0.01      reflection_term_sign=0      reflection_radius=0.5
spin=0
;
black_hole=2
rho=0               z=-1.5
mass=1              surrounding_patch_flag=-1
momentum=+0.01      reflection_term_sign=0      reflection_radius=0.5
spin=0
;
; mkgfn px01
; cpgfn 0 psi px01 psi
; directcg px01 1.0e-10      (35406.68 cpu seconds)
```

Table 31 - PP test series, px01 selected Ψ values

Patch Number	i	j	Psi	rho	z	r	theta
1	25	0	2.347064			0.5000	0.0
1	25	9	2.360580			0.5000	45.0
1	25	18	2.400692			0.5000	90.0
1	25	27	2.457385			0.5000	135.0
1	25	36	2.488955			0.5000	180.0
1	49	0	1.758297			0.9800	0.0
1	49	9	1.769535			0.9800	45.0
1	49	18	1.806549			0.9800	90.0
1	49	27	1.873769			0.9800	135.0
1	49	36	1.926339			0.9800	180.0
2	20	0	1.745576			1.0000	0.0
2	20	18	1.793913			1.0000	90.0
2	20	36	1.916693			1.0000	180.0
2	21	0	1.715777			1.0500	0.0
2	21	18	1.764330			1.0500	90.0
2	21	36	1.894945			1.0500	180.0
3	0	0	1.812677	0.00	0.00		
3	20	0	1.673821	1.00	0.00		
3	40	0	1.484380	2.00	0.00		
3	60	0	1.360694	3.00	0.00		
3	20	20	1.767519	1.00	1.00		
3	40	20	1.482244	2.00	1.00		
3	60	20	1.353613	3.00	1.00		
4	20	40	1.702099	1.00	2.00		
4	40	40	1.442173	2.00	2.00		
4	0	60	1.532411	0.00	3.00		
4	20	60	1.463283	1.00	3.00		
5	35	0	1.420020			3.5000	0.0
5	35	10	1.382730			3.5000	30.0
5	35	20	1.334552			3.5000	60.0
5	35	30	1.317679			3.5000	90.0
5	49	0	1.270914			4.9000	0.0
5	49	10	1.260310			4.9000	30.0
5	49	20	1.243113			4.9000	60.0
5	49	30	1.236008			4.9000	90.0
6	25	0	1.264462			5.0000	0.0
6	25	9	1.245966			5.0000	45.0
6	25	18	1.231668			5.0000	90.0
6	49	0	1.126083			9.8000	0.0
6	49	9	1.123895			9.8000	45.0
6	49	18	1.121852			9.8000	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

PP test series

PX1.PAR;3

18-OCT-1985 07:28

Page 1

title="Two Approaching Black Holes (P=0.1)"

N_black_hole=2 grid="m1"

;

black_hole=1

rho=0 z=+1.5

mass=1 surrounding_patch_flag=+1

momentum=-0.1 reflection_term_sign=0 reflection_radius=0.5

spin=0

;

black_hole=2

rho=0 z=-1.5

mass=1 surrounding_patch_flag=-1

momentum=+0.1 reflection_term_sign=0 reflection_radius=0.5

spin=0

;

; mkgfn px1

; cpgfn px01 psi px1 psi

; directog px1 1.0e-10 (48170.14 cpu seconds)

Table 32 - PP test series, px1 selected Ψ values

Patch Number	i	j	Psi	rho	z	r	theta
1	25	0	2.340766			0.5000	0.0
1	25	9	2.356280			0.5000	45.0
1	25	18	2.401102			0.5000	90.0
1	25	27	2.462366			0.5000	135.0
1	25	36	2.495811			0.5000	180.0
1	49	0	1.756930			0.9800	0.0
1	49	9	1.768673			0.9800	45.0
1	49	18	1.806924			0.9800	90.0
1	49	27	1.875466			0.9800	135.0
1	49	36	1.928674			0.9800	180.0
2	20	0	1.744276			1.0000	0.0
2	20	18	1.794287			1.0000	90.0
2	20	36	1.918971			1.0000	180.0
2	21	0	1.714628			1.0500	0.0
2	21	18	1.764700			1.0500	90.0
2	21	36	1.897096			1.0500	180.0
3	0	0	1.814382	0.00	0.00		
3	20	0	1.674888	1.00	0.00		
3	40	0	1.484881	2.00	0.00		
3	60	0	1.360975	3.00	0.00		
3	20	20	1.768532	1.00	1.00		
3	40	20	1.482647	2.00	1.00		
3	60	20	1.353860	3.00	1.00		
4	20	40	1.701852	1.00	2.00		
4	40	40	1.442344	2.00	2.00		
4	0	60	1.531987	0.00	3.00		
4	20	60	1.463140	1.00	3.00		
5	35	0	1.419875			3.5000	0.0
5	35	10	1.382767			3.5000	30.0
5	35	20	1.334743			3.5000	60.0
5	35	30	1.317905			3.5000	90.0
5	49	0	1.270956			4.9000	0.0
5	49	10	1.260388			4.9000	30.0
5	49	20	1.243240			4.9000	60.0
5	49	30	1.236151			4.9000	90.0
6	25	0	1.264508			5.0000	0.0
6	25	9	1.246070			5.0000	45.0
6	25	18	1.231808			5.0000	90.0
6	49	0	1.126139			9.8000	0.0
6	49	9	1.123956			9.8000	45.0
6	49	18	1.121917			9.8000	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

PP test series

P1.PAR;3

18-OCT-1985 07:22

Page 1

```

title="Two Approaching Black Holes (P-1)"
N_black_hole=2      grid="m1"
;
black_hole=1
rho=0               z=+1.8
mass=1              surrounding_patch_flag=+1
momentum=-1         reflection_term_sign=0      reflection_radius=0.5
spin=0
;
black_hole=2
rho=0               z=-1.8
mass=1              surrounding_patch_flag=-1
momentum=+1         reflection_term_sign=0      reflection_radius=0.5
spin=0
;
; mkgfn p1
; cpgfn px1 psi p1 psi
; directcg p1 1.0e-10

```

(38466.47 cpu seconds)

Table 33 - PP test series, p1 selected Ψ values

Patch Number	i	j	Ps1	rho	z	r	theta
1	25	0	2.334259			0.5000	0.0
1	25	9	2.369034			0.5000	45.0
1	25	18	2.459010			0.5000	90.0
1	25	27	2.564154			0.5000	135.0
1	25	36	2.615783			0.5000	180.0
1	49	0	1.784179			0.9800	0.0
1	49	9	1.799919			0.9800	45.0
1	49	18	1.849937			0.9800	90.0
1	49	27	1.934115			0.9800	135.0
1	49	36	1.996160			0.9800	180.0
2	20	0	1.771787			1.0000	0.0
2	20	18	1.836919			1.0000	90.0
2	20	36	1.985722			1.0000	180.0
2	21	0	1.742668			1.0500	0.0
2	21	18	1.806419			1.0500	90.0
2	21	36	1.962210			1.0500	180.0
3	0	0	1.873554	0.00	0.00		
3	20	0	1.722127	1.00	0.00		
3	40	0	1.518051	2.00	0.00		
3	60	0	1.385800	3.00	0.00		
3	20	20	1.816994	1.00	1.00		
3	40	20	1.514541	2.00	1.00		
3	60	20	1.377993	3.00	1.00		
4	20	40	1.736197	1.00	2.00		
4	40	40	1.470350	2.00	2.00		
4	0	60	1.560439	0.00	3.00		
4	20	60	1.490559	1.00	3.00		
5	35	0	1.445917			3.5000	0.0
5	35	10	1.407431			3.5000	30.0
5	35	20	1.357425			3.5000	60.0
5	35	30	1.339889			3.5000	90.0
5	49	0	1.290289			4.9000	0.0
5	49	10	1.278718			4.9000	30.0
5	49	20	1.260251			4.9000	60.0
5	49	30	1.252731			4.9000	90.0
6	25	0	1.283461			5.0000	0.0
6	25	9	1.263329			5.0000	45.0
6	25	18	1.248098			5.0000	90.0
6	49	0	1.135581			9.8000	0.0
6	49	9	1.133012			9.8000	45.0
6	49	18	1.130708			9.8000	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

PP test series

P10.PAR;3

18-OCT-1985 07:22

Page 1

title="Two Approaching Black Holes (P-10)"

N_black_hole=2 grid="m1"

;

black_hole=1

rho=0 z=+1.5

mass=1 surrounding_patch_flag=+1

momentum=-10 reflection_term_sign=0 reflection_radius=0.5

spin=0

;

black_hole=2

rho=0 z=-1.5

mass=1 surrounding_patch_flag=-1

momentum=+10 reflection_term_sign=0 reflection_radius=0.5

spin=0

;

; mkgfn p10

; cpgfn p1 psi p10 psi

; directcg p10 1.0e-10 (67574.98 cpu seconds)

Table 34 - PP test series, p10 selected Ψ values

Patch Number	i	j	Psi	rho	z	r	theta
1	25	0	2.965542			0.5000	0.0
1	25	9	3.069733			0.5000	45.0
1	25	18	3.321834			0.5000	90.0
1	25	27	3.587860			0.5000	135.0
1	25	36	3.708586			0.5000	180.0
1	49	0	2.377784			0.9800	0.0
1	49	9	2.398004			0.9800	45.0
1	49	18	2.486178			0.9800	90.0
1	49	27	2.652415			0.9800	135.0
1	49	36	2.770211			0.9800	180.0
2	20	0	2.362016			1.0000	0.0
2	20	18	2.467595			1.0000	90.0
2	20	36	2.754718			1.0000	180.0
2	21	0	2.324464			1.0500	0.0
2	21	18	2.423815			1.0500	90.0
2	21	36	2.719835			1.0500	180.0
3	0	0	2.588565	0.00	0.00		
3	20	0	2.336737	1.00	0.00		
3	40	0	1.992679	2.00	0.00		
3	60	0	1.762283	3.00	0.00		
3	20	20	2.460842	1.00	1.00		
3	40	20	1.982404	2.00	1.00		
3	60	20	1.748400	3.00	1.00		
4	20	40	2.312972	1.00	2.00		
4	40	40	1.908432	2.00	2.00		
4	0	60	2.071845	0.00	3.00		
4	20	60	1.957137	1.00	3.00		
5	35	0	1.894204			3.5000	0.0
5	35	10	1.813647			3.5000	30.0
5	35	20	1.713200			3.5000	60.0
5	35	30	1.679913			3.5000	90.0
5	49	0	1.620723			4.9000	0.0
5	49	10	1.586126			4.9000	30.0
5	49	20	1.536458			4.9000	60.0
5	49	30	1.518728			4.9000	90.0
6	25	0	1.607733			5.0000	0.0
6	25	9	1.548220			5.0000	45.0
6	25	18	1.509946			5.0000	90.0
6	49	0	1.302333			9.8000	0.0
6	49	9	1.288613			9.8000	45.0
6	49	18	1.278820			9.8000	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

PP test series

```
P100.PAR;1                                18-OCT-1985 07:22                Page 1

title="Two Approaching Black Holes (P-100)"
N_black_hole=2                          grid="m1"
;
black_hole=1
rho=0                                z=+1.5
mass=1                              surrounding_patch_flag=+1
momentum=-100                      reflection_term_sign=0          reflection_radius=0.5
spin=0
;
black_hole=2
rho=0                                z=-1.5
mass=1                              surrounding_patch_flag=-1
momentum=+100                      reflection_term_sign=0          reflection_radius=0.5
spin=0
;
; mkgfn p100
; cpgfn p10 psi p100 psi
; directog p100 1.0e-10                (66746.36 cpu seconds)
```

Table 35 - PP test series, p100 selected Ψ values

Patch Number	i	j	Psi	rho	z	r	theta
1	25	0	4.901730			0.5000	0.0
1	25	9	5.106945			0.5000	45.0
1	25	18	5.600149			0.5000	90.0
1	25	27	6.115628			0.5000	135.0
1	25	36	6.347410			0.5000	180.0
1	49	0	4.004863			0.9800	0.0
1	49	9	4.028501			0.9800	45.0
1	49	18	4.176307			0.9800	90.0
1	49	27	4.489573			0.9800	135.0
1	49	36	4.712551			0.9800	180.0
2	20	0	3.978409			1.0000	0.0
2	20	18	4.143881			1.0000	90.0
2	20	36	4.685657			1.0000	180.0
2	21	0	3.915004			1.0500	0.0
2	21	18	4.067229			1.0500	90.0
2	21	36	4.625083			1.0500	180.0
3	0	0	4.397018	0.00	0.00		
3	20	0	3.933659	1.00	0.00		
3	40	0	3.286599	2.00	0.00		
3	60	0	2.836516	3.00	0.00		
3	20	20	4.142018	1.00	1.00		
3	40	20	3.265734	2.00	1.00		
3	60	20	2.810220	3.00	1.00		
4	20	40	3.875294	1.00	2.00		
4	40	40	3.131921	2.00	2.00		
4	0	60	3.474133	0.00	3.00		
4	20	60	3.253143	1.00	3.00		
5	35	0	3.148182			3.5000	0.0
5	35	10	2.969486			3.5000	30.0
5	35	20	2.743901			3.5000	60.0
5	35	30	2.670249			3.5000	90.0
5	49	0	2.612543			4.9000	0.0
5	49	10	2.517660			4.9000	30.0
5	49	20	2.380019			4.9000	60.0
5	49	30	2.332729			4.9000	90.0
6	25	0	2.585816			5.0000	0.0
6	25	9	2.419215			5.0000	45.0
6	25	18	2.313751			5.0000	90.0
6	49	0	1.895961			9.8000	0.0
6	49	9	1.828647			9.8000	45.0
6	49	18	1.782991			9.8000	90.0

This table gives values of Ψ at selected points in the space. These points are identified by triples (patch number, i, j), where (i, j) are the generic integer patch coordinates defined in the text. Depending on the patch type, either (ρ, z) or (r, θ) are also shown.

PP test series

M1.PAT;4

18-OCT-1985 07:29

Page 1

```

; Two black hole patch system with:
; - Mirror image symmetry about equator
; - Marginally Trapped inner boundary conditions
; - Delta r / r approximately 20-25 at inner boundary
; - All patches using independent terms differencing
; - All patches using omega for conformal function
; - Outer boundary at r = 1000
;
N_patch-12
;
; Now describe each coordinate patch:
;
patch-1
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=1.5      delta_r=0.02      delta_theta=5
r_min="constant-0.50"      bndry_cond="Marginally Trapped"
r_max="constant-0.98"      bndry_cond="patch-2"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-180"      bndry_cond="Neumann"
;
patch-2
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=1.5      delta_r=0.05      delta_theta=5
r_min="constant-1.00"      bndry_cond="patch-1"
r_max="constant-1.05"      bndry_cond="patch-3"
r_max="constant-1.05"      bndry_cond="patch-4"
theta_min="constant-0"      bndry_cond="Neumann"
theta_max="constant-180"      bndry_cond="Neumann"
;
patch-3
coord_type="cylindrical"      ; lower one
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_rho=0.05      delta_z=0.05
rho_min_bound=0.00      rho_max_bound=3.50
z_min_bound=0.00      z_max_bound=1.45
rho_min="patch-2 bndry-r_max"
rho_min="constant-0.00"      bndry_cond="Neumann"
rho_max="patch-5 bndry-r_min"
z_min="constant-0.00"      bndry_cond="Neumann"
z_max="patch-2 bndry-r_max"
z_max="patch-5 bndry-r_min"      bndry_cond="patch-4"
z_max="constant-1.45"
;
patch-4
coord_type="cylindrical"      ; upper one
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_rho=0.05      delta_z=0.05
rho_min_bound=0.00      rho_max_bound=3.50
z_min_bound=1.50      z_max_bound=3.50

```

PP test series

M1.PAT;4

18-OCT-1985 07:29

Page 2

```

rho_min="patch-2 bndry-r_max"
rho_min="constant-0.00"
rho_max="patch-5 bndry-r_min"
z_min="patch-2 bndry-r_max"
z_min="constant-1.50"
z_max="patch-5 bndry-r_min"
;
patch-5
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.1      delta_theta=3
r_min="constant-3.5"      bndry_cond="patch-3"
r_max="constant-3.5"      bndry_cond="patch-4"
theta_min="constant-4.9"  bndry_cond="patch-6"
theta_max="constant-0"    bndry_cond="Neumann"
theta_max="constant-90"   bndry_cond="Neumann"
;
patch-6
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.2      delta_theta=5
r_min="constant-5.0"      bndry_cond="patch-5"
r_max="constant-9.8"      bndry_cond="patch-7"
theta_min="constant-0"    bndry_cond="Neumann"
theta_max="constant-90"   bndry_cond="Neumann"
;
patch-7
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=0.5      delta_theta=5
r_min="constant-10.0"     bndry_cond="patch-6"
r_max="constant-19.5"     bndry_cond="patch-8"
theta_min="constant-0"    bndry_cond="Neumann"
theta_max="constant-90"   bndry_cond="Neumann"
;
patch-8
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=1      delta_theta=7.5
r_min="constant-20"      bndry_cond="patch-7"
r_max="constant-49"      bndry_cond="patch-9"
theta_min="constant-0"    bndry_cond="Neumann"
theta_max="constant-90"   bndry_cond="Neumann"
;
patch-9
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=2      delta_theta=7.5
r_min="constant-50"      bndry_cond="patch-8"
r_max="constant-98"      bndry_cond="patch-10"

```

PP test series

M1.PAT;4

18-OCT-1985 07:29

Page 3

```

theta_min="constant-0"
theta_max="constant-90"
;
patch-10
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=5      delta_theta=7.5
r_min="constant-100"      bndry_cond="patch-9"
r_max="constant-195"      bndry_cond="patch-11"
theta_min="constant-0"    bndry_cond="Neumann"
theta_max="constant-90"   bndry_cond="Neumann"
;
patch-11
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=10      delta_theta=10
r_min="constant-200"      bndry_cond="patch-10"
r_max="constant-490"      bndry_cond="patch-12"
theta_min="constant-0"    bndry_cond="Neumann"
theta_max="constant-90"   bndry_cond="Neumann"
;
patch-12
coord_type="polar spherical"
difference_scheme="independent terms"
conformal_fn_type="omega(psi)"
rho_center=0      z_center=0      delta_r=20      delta_theta=10
r_min="constant-500"      bndry_cond="patch-11"
r_max="constant-1000"     bndry_cond="Robin"
theta_min="constant-0"    bndry_cond="Neumann"
theta_max="constant-90"   bndry_cond="Neumann"

```

Appendix 2

This appendix contains the source listings of the numerical GR code described in this thesis. The bulk of the code is written in the C programming language (Kernighan and Ritchie (1978)). The "ILUCG" sparse matrix subroutine package used to solve the linearized algebraic equations is written in FORTRAN. There is one short assembler routine, used to support the passing of variable numbers of arguments to C functions.

The code is written for (and runs on) a VAX/750 with floating point accelerator, running the VMS operating system, version 3.2. Double precision arithmetic is used throughout, giving about 17 decimal digits (56 bits) of precision in floating point quantities. The time for a floating point add/multiply pair is about 20 microseconds.

The C dialect used extends the Kernighan and Ritchie standard in only one significant way—enumerated data types are allowed, with semantics similar to those of (for example) PASCAL. The author uses a number of private "language extensions", via the header file "util:stdc.h". The most significant of these are the definition of "new" data types `void` and `boolean`, and the definition of a `then` keyword for use in the C `if-else` statement.

Most of the source files begin with a list of the external objects (functions and variables) they contain. Those items preceded by a single asterisk are "public" (visible outside the file), while those preceded by a double asterisk are declared as `static` and hence have scope limited to the file.

The code resides in 4 distinct directories, which have been given "logical names" so files in them may be easily referred to from other subdirectories. The syntax of such a reference is that the logical name precedes the filename, separated by a colon. The

Table 36 – Directories where the code resides

Logical Name	Contents
<code>main</code>	Routines to set up and solve equation (21)
<code>fortran</code>	"ILUCG" Sparse Matrix package
<code>patchutil</code>	Patch utility routines
<code>util</code>	General purpose utility routines

directories and their contents are listed in table 36.

The next 4 pages give indices to the listings for each of these directories, giving the page numbers on which each file's listing starts. The listings themselves follow. Within each directory, header files are listed first, then "normal" source code files. Within these groupings, the files are ordered alphabetically. In general, the name of an executable program is the same as that of the file holding its main function. The sole exception to this is the executable program `directcg`, derived from the main function in the file `direct.c`, both in the `main` directory (the name comes from "DIRECT solver via Conjugate Gradient method").

The reader wishing a general understanding of how the code works is advised to start by reading the header files in the `patchutil` directory, especially `patch.h`. After this, the `mkgfn.c` and `direct.c` files, together with the functions they call, are probably the next place to look. The `directcg` executable program derived from `direct.c` was used to obtain all the model solutions in this thesis.

Table 37 – File index for the main directory

solve.h		261
conformal.c		262
cpgfn.c		265
curvature.c		267
direct.c		273
dopsor.c		277
dumpgrd.c		279
extrap.c		280
info.c		283
jacobian.c		286
mkgfn.c		292
mkgrd.c		297
monopole.c		298
physics.c		302
psor.c		308
setextrap.c		311
solve.c		314
solvecg.c		319

Table 38 – File index for the fortran directory

mydilucg.for		329
mysilucg.for		339

Table 39 – File index for the patchutil directory

blackhole.h	349
cvt.h	350
inout.h	351
patch.h	352
tempbndry.h	356
allocgfna.c	357
binaryio.c	358
coord.c	362
cvt.c	365
dump.c	371
inout.c	377
inputpar.c	379
inputpat.c	382
ipartest.c	384
ipattest.c	385
ngridpts.c	386
parlog.c	387
readpat.c	388
setbndry.c	398
setpatch.c	405

Table 40 – File index for the util directory

bandmat.h	419
keyword.h	420
stdc.h	421
arrayio.c	422
bgecl.c	423
calltrail.mar	425
dumparray.c	427
errorexit.c	430
fixname.c	431
fuzz.c	432
keyword.c	435
miscfp.c	440
round.c	441
spline.c	442
time.c	447

main directory

SOLVE.H;15

21-SEP-1985 19:16

Page 1

```
/*
 * This defines the mapping from (p,i,j) into equation number (note that the
 * range of the mapping is from 0 to neqn-1):
 */
#define index(p_,i_,j_)      (pij_mapping[p_][i_] + (j_))
#define INDEX(p_,i_,j_)      (index(p_,i_,j_) + 1) /* for FORTRAN */

extern int **pij_mapping;
extern int neqn;
extern float *f_rhs, *f_solution;
extern double *d_rhs, *d_solution;

extern enum precision { FLOAT, DOUBLE } mode;
```

main directory

CONFORMAL.C;9

21-SEP-1985 19:18

Page 1

```

/* This file contains conformal function conversion routines. */
/*
 * The following functions are available:
 * cvt_psi_to_cfn - Converts psi to conformal function of a given patch
 * cvt_cfn_to_psi - Converts conformal function of a given patch back to psi
 * cvt_psi_to_omega - (obvious!)
 * cvt_omega_to_psi - (obvious!)
 */

#include "util:stdio.h"
#include "patchutil:patch.h"

/*****/

/*
 * This function converts 'psi', the 'true' conformal factor, to the conformal
 * function type used in a given patch.
 *
 * Arguments:
 * ptr --> The coordinate patch descriptor for the conformal function to be
 *         converted.
 * psi - The psi value to be converted.
 *
 * The resulting conformal function value is returned as the function value.
 */
double cvt_psi_to_cfn(ptr, psi)
register struct coord_patch_dscr *ptr;
double psi;
{
    void error_exit();
    double cvt_psi_to_omega();

#define WHATSIT ptr->conformal_fn_type
    switch (WHATSIT)
    {
    case PSI:
        return(psi);

    case OMEGA_OF_PSI:
        return(cvt_psi_to_omega(psi));

    default:
        error_exit(OK_EXIT,
            "***** cvt_psi_to_cfn: unknown conformal fn type code %d\n",
            WHATSIT); /* DOES NOT RETURN */
    }
}

/*****/

/*
 * This function converts the conformal function type used in a given patch,
 * back to 'psi', the 'true' conformal factor.
 *
 * Arguments:

```

main directory

CONFORMAL.C:9

21-SEP-1985 19:18

Page 2

```

* ptr --> The coordinate patch descriptor for the conformal function to be
*         converted.
* cfn - The conformal function value.
*
* The resulting psi value is returned as the function value.
*/
double cvt_cfn_to_psi(ptr, cfn)
register struct coord_patch_dscr *ptr;
double cfn;
{
void error_exit();
double cvt_omega_to_psi();

#define WHATSIT ptr->conformal_fn_type
switch (WHATSIT)
{
case PSI:
return(cfn);

case OMEGA_OF_PSI:
return(cvt_omega_to_psi(cfn));

default:
error_exit(OK_EXIT,
          "***** cvt_cfn_to_psi: unknown conformal fn type code %d\n",
          WHATSIT); /* DOES NOT RETURN */
}
}

/*****/

/*
* This function converts psi to omega.
*
* Arguments:
* psi - The psi value to be converted.
*
* The resulting omega value is returned as the function value.
*/
double cvt_psi_to_omega(psi)
double psi;
{
return(1.0 / (psi - 1.0));
}

/*****/

/*
* This function converts omega to psi.
*
* Arguments:
* omega - The omega value to be converted.
*
* The resulting psi value is returned as the function value.
*/

```

main directory

CONFORMAL.C;9

21-SEP-1985 19:18

Page 3

```
double cvt_omega_to_psi(omega)
double omega;
{
return(1.0 + 1.0/omega);
}
```


main directory

CPGFN.C;15

21-SEP-1985 19:18

Page 1

```

#include "util:stdc.h"
#include "patchutil:patch.h"
#include "patchutil:blackhole.h"
#include "patchutil:cvt.h"

main(argc,argv) /* cpgfn - copy grid functions from one model to another */
int argc;
char *argv[];
{
    static void copy_gfn();

    void error_exit();
    char *fixname();
    void input_pat(), input_output_grd(), input_output_gfn(), par_log();
    int cvt_grid_fn_name_to_offset();

    struct grid_dscr src_gd, dst_gd;
    struct bhs_dscr src_bhsd, dst_bhsd;
    char *src_title, *dst_title;
    char *src_grid, *dst_grid;
    int src_offset, dst_offset;

    /* decode argument list: */
    if (argc != 5)
        then error_exit(OK_EXIT,
            "usage: %s src_model_name src_grid_fn_name dst_model_name dst_grid_fn_name\n",
                fixname(argv[0]));
    #define SRC_NAME      argv[1]
    #define SRC_GFN_NAME  argv[2]
    #define DST_NAME      argv[3]
    #define DST_GFN_NAME  argv[4]
    src_offset = cvt_grid_fn_name_to_offset(SRC_GFN_NAME);
    if (src_offset < 0)
        then error_exit(OK_EXIT,
            "***** unknown or illegal grid function name '%s'\n",
                SRC_GFN_NAME); /* DOES NOT RETURN */
    dst_offset = cvt_grid_fn_name_to_offset(DST_GFN_NAME);
    if (dst_offset < 0)
        then error_exit(OK_EXIT,
            "***** unknown or illegal grid function name '%s'\n",
                DST_GFN_NAME); /* DOES NOT RETURN */

    /* read both source and destination arrays into memory: */
    input_par(SRC_NAME, &src_title, &src_grid, &src_bhsd);
    input_output_grd(src_grid, "read", &src_gd);
    input_output_gfn(SRC_NAME, "read", &src_gd);
    input_par(DST_NAME, &dst_title, &dst_grid, &dst_bhsd);
    input_output_grd(dst_grid, "read", &dst_gd);
    input_output_gfn(DST_NAME, "read", &dst_gd);

    /* update the in-memory copy of the destination array */
    if (strcmp(src_grid, dst_grid) != 0)
        then error_exit(OK_EXIT,
            "***** copying different grids not yet supported -- exiting *****\n"
                ); /* DOES NOT RETURN */

```

main directory

CPGFN.C;15

21-SEP-1985 19:18

Page 2

```

copy_gfn(&src_gd, src_offset, &dst_gd, dst_offset);    /* copy data over */

/* write updated destination array back out */
input_output_gfn(DST_NAME, "write", &dst_gd);
par_log(DST_NAME, argo, argv, 0.0);
}

/...../

/*
 * This function copies a grid function from one grid to another.  The grids
 * must have the same structure (EXACTLY!).
 *
 * Arguments:
 * (src_pgd,dst_pgd) --> The source and destination grid descriptors.
 * (src_offset,dst_offset) - The offsets, in 'double's, of the source and
 *                           destination grid functions in a 'struct grid_fn'.
 */
static void copy_gfn(src_pgd, src_offset, dst_pgd, dst_offset)
register struct grid_dsor *src_pgd, *dst_pgd;
int src_offset, dst_offset;
{
    register int p, i, j;
    double *src_ptr, *dst_ptr;

    for (p = 1 ; p <= dst_pgd->N_patch ; ++p)
    {
        for (i = dst_pgd->patch[p].i_min ;
             i <= dst_pgd->patch[p].i_max ;
             ++i)
        {
            for (j = dst_pgd->patch[p].line_end[J_MIN][i].end_ij ;
                 j <= dst_pgd->patch[p].line_end[J_MAX][i].end_ij ;
                 ++j)
            {
                src_ptr = (double *) GRID_FNS(&src_pgd->patch[p],i,j);
                dst_ptr = (double *) GRID_FNS(&dst_pgd->patch[p],i,j);
                dst_ptr[dst_offset] = src_ptr[src_offset];
            }
        }
    }
}

```

main directory

CURVATURE.C:41

21-SEP-1985 19:19

Page 1

```

/* This file contains functions to compute the extrinsic curvature tensor: */
/*
 * The following routines are contained in this file:
 ** momentum_K_hat_sub_ij - computes K-hat-sub-ij for a black hole with momentum
 ** spin_K_hat_sub_ij - computes K-hat-sub-ij for a black hole with spin
 ** K_hat_sub_ij - computes K-hat-sub-ij (total) for all black holes
 ** K_hat_sub_rr - computes K-hat-sub-r-r (total) in polar spherical coords
 */

#define DEBUG 0

#include "util:stdio.h"

#include "patchutil:blackhole.h"
#include "patchutil:patch.h"

/*****

/*
 * This function computes all non-vanishing extrinsic curvature components for
 * a black hole with a given momentum (but no spin), in a polar spherical
 * coordinate system centered on that black hole.
 *
 * Arguments:
 * momentum - The linear momentum of the black hole, assumed to be in the
 *             (positive) z-direction.
 * reflection_term_sign - The sign (+1, 0, or -1) for the 2nd ("reflection
 *                       symmetry") term in the Kij's.
 * reflection_radius - The radius 'a' (usually 1/2 the black hole mass) for
 *                    the 2nd ("reflection symmetry") term in the Kij's. This
 *                    is ignored if 'reflection_term_sign' (above) is zero.
 * (r,theta) - The coordinates of the point to compute the Kij's at.
 * pK_r_r --> Where K-hat-sub-r-r will be stored, or NULL to skip this.
 * pK_r_theta --> Where K-hat-sub-r-theta (same as K-hat-sub-theta-r) will be
 *               stored, or NULL to skip this.
 * pK_theta_theta --> Where K-hat-sub-theta-theta will be stored, or NULL to
 *                  skip this.
 * pK_phi_phi --> Where (K-hat-sub-phi-phi / sin-squared-theta) will be stored,
 *                or NULL to skip this.
 *
 * Note 1:      The formulas are taken verbatim from Matt Choptuik's M.Sc.
 *               thesis, p. 58.
 *
 * Note 2:      The reason for the missing sin-squared-theta factor is to keep
 *               everything finite and non-zero on the z-axis.
 */
static void momentum_K_hat_sub_ij
(
    momentum, reflection_term_sign, reflection_radius,
    r, theta,
    pK_r_r, pK_r_theta, pK_theta_theta, pK_phi_phi
)
double momentum;
int reflection_term_sign;
double reflection_radius;

```

main directory

CURVATURE.C;41

21-SEP-1985 19:19

Page 2

```

double r, theta;
double *pK_r_r, *pK_r_theta, *pK_theta_theta, *pK_phi_phi;
{
double sin(), cos();
double square();

double temp, factor_p, factor_m;

temp = reflection_term_sign * square(reflection_radius / r);
factor_p = 1.0 + temp;
factor_m = 1.0 - temp;

temp = 3.0 * momentum * cos(theta);

if (pK_r_r != NULL)
    then *pK_r_r = factor_p * (temp) / square(r);
if (pK_r_theta != NULL)
    then *pK_r_theta = factor_m * (-3.0 * momentum * sin(theta)) / (2.0 * r);
if (pK_theta_theta != NULL)
    then *pK_theta_theta = factor_p * (-temp) / 2.0;
if (pK_phi_phi != NULL)
    then *pK_phi_phi = factor_p * (-temp) / 2.0; /*
                                                * Note missing factor of
                                                * sin-squared-theta!!!
                                                */
}

/*****/

/*
 * This function computes the only non-vanishing extrinsic curvature component
 * for a black hole with a given spin (but no momentum), in a polar spherical
 * coordinate system centered on that black hole.
 *
 * Arguments:
 * spin - The angular momentum of the black hole, assumed to be in the positive
 *        z-direction.
 * (r,theta) - The coordinates of the point to compute at.
 * pK_r_phi --> Where (K-hat-sub-r-phi / sin-theta) will be stored, or NULL to
 *              skip this.
 *
 * Note 1:      The formulas are taken verbatim from Matt Choptuik's M.Sc.
 *              thesis, p. 57. (Note there are 2 misprints in the top equation
 *              on this page, as can be verified by checking against Matt's
 *              actual code on p. 237. This is also what is needed to make the
 *              next equation, (3.27) come out correct -- it's the one which
 *              Matt's code actually uses.)
 *
 * Note 2:      The reason for the missing sin-theta factor is to keep things
 *              finite and non-zero on the z-axis.
 */
static void spin_K_hat_sub_ij(spin, r, theta, pK_r_phi)
double spin, r, theta;
double *pK_r_phi;
{

```

main directory

CURVATURE.C;41

21-SEP-1985 19:19

Page 3

```

double sin();
double square();

if (pK_rho_phi != NULL) then *pK_rho_phi = -3.0 * spin * sin(theta) / square(r);
}

/*****/

/*
 * This function computes all components of the extrinsic curvature tensor
 * 'K-hat-sub-ij' (summed over all black holes) in the global reference
 * coordinate system (cylindrical).
 *
 * Arguments:
 * pbhsd --> The black hole system descriptor.
 * (rho,z) = Where to evaluate the tensor at.
 * pK_rho_rho --> Where K-hat-sub-rho-rho is to be stored, or NULL to skip
 *               this.
 * pK_rho_z --> Where K-hat-sub-rho-z is to be stored, or NULL to skip this.
 * pK_rho_phi --> Where (K-hat-sub-rho-phi / rho) is to be stored, or NULL to
 *               skip this.
 * pK_z_z --> Where K-hat-sub-z-z is to be stored, or NULL to skip this.
 * pK_z_phi --> Where (K-hat-sub-z-phi / rho) is to be stored, or NULL to skip
 *               this.
 * pK_phi_phi --> Where (K-hat-sub-phi-phi / (rho-squared)) will be stored, or
 *               NULL to skip this.
 *
 * Note:      The reason for the extra factors of rho is to keep everything
 *             finite and non-zero on the z-axis.
 */
void K_hat_sub_ij(pbhsd, rho, z, pK_rho_rho, pK_rho_z, pK_rho_phi,
                  pK_z_z, pK_z_phi, pK_phi_phi)

register struct bhs_dscr *pbhsd;
double rho, z;
double *pK_rho_rho, *pK_rho_z, *pK_rho_phi, *pK_z_z, *pK_z_phi, *pK_phi_phi;
{
double square(), hypot(), atan2();      /* use latter as 'atan2(y, x)' */

static void momentum_K_hat_sub_ij();
static void spin_K_hat_sub_ij();

int bh;
double delta_rho, delta_z, r, theta;
double temp_r_r, temp_r_theta, temp_r_phi, temp_theta_theta, temp_phi_phi;
double K_rho_rho, K_rho_z, K_rho_phi, K_z_z, K_z_phi, K_phi_phi;
double d_r_d_rho, d_r_d_z, d_theta_d_rho, d_theta_d_z; /* partial deriv's */

/* make sure we have someplace to accumulate our results: */
if (pK_rho_rho == NULL) then pK_rho_rho = &K_rho_rho;
if (pK_rho_z == NULL) then pK_rho_z = &K_rho_z;
if (pK_rho_phi == NULL) then pK_rho_phi = &K_rho_phi;
if (pK_z_z == NULL) then pK_z_z = &K_z_z;
if (pK_z_phi == NULL) then pK_z_phi = &K_z_phi;
if (pK_phi_phi == NULL) then pK_phi_phi = &K_phi_phi;

```

main directory

CURVATURE.C;41

21-SEP-1985 19:19

Page 4

```

/* initialize sums: */
*pK_rho_rho = *pK_rho_z = *pK_rho_phi =
*pK_z_z = *pK_z_phi = *pK_phi_phi = 0.0;

/* accumulate results for each black hole: */
for (bh = 1 ; bh <= pbhsd->N_black_hole ; ++bh)
{
    /* how far from the black hole are we: */
    delta_rho = rho - pbhsd->black_hole[bh].rho;
    delta_z = z - pbhsd->black_hole[bh].z;
    r = hypot(delta_rho, delta_z);          /* black hole's local coords */
    theta = atan2(delta_rho, delta_z);

    /* find curvature components in black hole's local coordinates: */
    momentum_K_hat_sub_ij(pbhsd->black_hole[bh].momentum,
        pbhsd->black_hole[bh].reflection_term_sign,
        pbhsd->black_hole[bh].reflection_radius,
        r, theta,
        &temp_r_r, &temp_r_theta, &temp_theta_theta,
        &temp_phi_phi);
    spin_K_hat_sub_ij(pbhsd->black_hole[bh].spin, r, theta,
        &temp_r_phi);

    /* evaluate partial derivatives: (my notes, p. 143) */
    d_r_d_rho = delta_rho / r;
    d_r_d_z = delta_z / r;
    d_theta_d_rho = delta_z / square(r);
    d_theta_d_z = - delta_rho / square(r);

    /*
     * Note: Recall that, in order to keep things finite and non-
     * zero on the z-axis, we are returning the values of the
     * Kij's divided by one 'rho' for each 'phi' appearing in
     * indices (i,j). But rho = r * sin(theta), and only the
     * sin(theta) has been divided out by the 'momentum' and
     * 'spin' Kij routines. Thus we must divide by an extra
     * factor of 'r' for each 'phi' in the indices.
     */

    /* sum curvature components in global coordinates: (p. 144-146) */
    *pK_rho_rho += temp_r_r * square(d_r_d_rho)
        + temp_r_theta * 2.0 * d_r_d_rho * d_theta_d_rho
        + temp_theta_theta * square(d_theta_d_rho);

    *pK_rho_z += temp_r_r * d_r_d_rho * d_r_d_z
        + temp_r_theta * ( d_r_d_rho * d_theta_d_z
            + d_theta_d_rho * d_r_d_z )
        + temp_theta_theta * d_theta_d_rho * d_theta_d_z;

    *pK_rho_phi += (temp_r_phi / r) * d_r_d_rho;    /* o.f. Note above */

    *pK_z_z += temp_r_r * square(d_r_d_z)
        + temp_r_theta * 2.0 * d_r_d_z * d_theta_d_z
        + temp_theta_theta * square(d_theta_d_z);

```

main directory

CURVATURE.C;41

21-SEP-1985 19:19

Page 5

```

        *pK_z_phi += (temp_r_phi / r) * d_r_d_z;          /* c.f. Note above */
        *pK_phi_phi += (temp_phi_phi / square(r));        /* c.f. Note above */
    }
}
/*****/

/*
 * This function computes the K-hat-sub-r-r curvature component, summed over
 * all black holes, in the polar spherical coordinate system of a selected
 * patch.
 *
 * Arguments:
 * pgd --> The grid descriptor describing the coordinate patch system.
 * pbhsd --> The black hole descriptor describing the black hole system.
 * ptr --> The coordinate patch descriptor. ptr->type must be POLAR_SPHERICAL.
 * (r,theta) = Polar spherical coordinates to compute the curvature component
 * at.
 *
 * The curvature component is returned as the function value.
 */
double K_hat_sub_rr(pgd, pbhsd, ptr, r, theta)
register struct grid_dscr *pgd;
register struct bhs_dscr *pbhsd;
register struct coord_patch_dscr *ptr;
double r, theta;
{
    double sin(), cos();
    double square();
    void cvt_xy_to_xy();
    void K_hat_sub_ij(); /* in (rho,z) global cylindrical coordinates */

    double rho, z;
    double K_rho_rho, K_rho_z, K_z_z; /* all 'phi' terms are irrelevant!!! */
    double d_rho_d_r, d_z_d_r;
    double sum;

    /* compute global coordinates: */
    cvt_xy_to_xy(ptr, r, theta, &pgd->patch[GLOBAL_REF], &rho, &z);

    /* find all relevant K-hat-sub-ij in global coordinates: */
    K_hat_sub_ij(pbhsd, rho, z, &K_rho_rho, &K_rho_z, NULL,
                &K_z_z, NULL,
                NULL);

    /* transform to local coordinates: */
    d_rho_d_r = sin(theta);
    d_z_d_r = cos(theta);

    sum =
        + K_rho_rho * square(d_rho_d_r)
        + 2.0 * K_rho_z * d_rho_d_r * d_z_d_r
        + K_z_z * square(d_z_d_r);

```

main directory

CURVATURE.C;41

21-SEP-1985 19:19

Page 6

```
return(sum);  
}
```


main directory

DIRECT.C:82

21-SEP-1985 19:20

Page 1

```

#include "util:stdio.h"
#include "util:keyword.h"

#include "patchutil:patch.h"
#include "patchutil:blackhole.h"

/*****

main(argc, argv)      /* direct - driver for direct solver */
int argc;
char *argv[];
{
double compute_residual();
void setup_direct_solve();
double direct_solve();

int keyword_scan();
double square();
void input_par(), input_output_grd(), input_output_gfn();
void set_extrapolation_coeff();
void par_log();
double cpu_time();
void error_exit();
char *fixname();

struct grid_dscr gd;
struct bhs_dscr bhsd;
char *model_title;
char *grid_name;
double time;

enum { ITERATIONS, RESIDUAL } termination_test_type;
double eps, omega;
int max_iterations;
#define BUFSIZE 100
char iteration_limit[BUFSIZE];
double max_residual;
int iteration_count;
double residual;

/*
 * Keyword scanner variables and table:
 */
static boolean solve_exactly_each_iteration = FALSE;
static char *char_omega = "1.0";
static char *dump_file_name = NULL;      /*
 * If non-NULL, this is file name to
 * dump Jacobian/rhs to each iteration
 */

static struct var_dscr cmd_table[] =
{
{ "-exact",      &solve_exactly_each_iteration,  "%d" }, /* really 'boolean' */
{ "omega",      &char_omega,                    "%s" },
{ "dump",      &dump_file_name,                  "%s" } /* var is "extern" */
};

```

main directory

```

DIRECT.C:82                                21-SEP-1985 19:20                                Page 2

#define CMD_TABLE_SIZE ( sizeof(cmd_table) / sizeof(struct var_dscr) )

/* check for proper usage: */
if ((argc >= 2) && (strcmp(argv[1], "-help") == 0))
    then error_exit(OK_EXIT, "\n\n
----- Options: -----\n\n
-exact          Forces each Newton step to be calculated to full (double\n\
precision) accuracy. Default is only to calculate to 0.1%\n\
of the expected residual after the step.\n\n
omega-value     Causes 'SOR-style' multiplication of calculated Newton step\n\
vector by specified value. Default is to use full step.\n\n
omega-auto      Similar, but a value for omega is 'automatically' chosen (by\n\
trial and error) that minimises the resulting residual.\n\n
dump-filename   Dumps Jacobian, rhs vectors out to specified filename each\n\
Newton iteration, just before trying direct solution.\n\n
"); /* DOES NOT RETURN */

if (argc < 3)
    then error_exit(OK_EXIT, "\n\n
usage: %s model_name #it_count/max_residual [ options ... ]\n\n
('s -help' gives more info)\n\n
        fixname(argv[0]),
        fixname(argv[0])); /* DOES NOT RETURN */

/* decode 'fixed' arguments: */
#define MODEL_NAME      argv[1]
if (argv[2][0] == '*')
    then {
        termination_test_type = ITERATIONS;
        sscanf(argv[2], "%d", &max_iterations);
        sprintf(iteration_limit, "%s", &argv[2][1]);
    }
    else {
        termination_test_type = RESIDUAL;
        sscanf(argv[2], "%lf", &max_residual);
        sprintf(iteration_limit, "(max residual %s)", argv[2]);
    }

/* decode 'options' arguments: */
{
    int i, status;
    for (i = 3; i < argc; ++i)
    {
        status = keyword_scan(CMD_TABLE_SIZE, cmd_table, argv[i]);
        if (status < 0)
            then error_exit(OK_EXIT,
                "***** bad option '%s' -- exiting *****\n",
                argv[i]); /* DOES NOT RETURN */
    }
}

if (strcmp(char_omega, "auto") == 0)
    then error_exit(OK_EXIT,
        "***** 'omega-auto' option not yet supported -- exiting *****\n"
        ); /* DOES NOT RETURN */
sscanf(char_omega, "%lf", &omega);

```

main directory

DIRECT.C;82

21-SEP-1985 19:20

Page 3

```

/* get input data: */
input_par(MODEL_NAME, %model_title, %grid_name, %bhsd);
input_output_grd(grid_name, "read", %gd);
set_extrapolation_coeff(%gd, %bhsd); /* update for our black hole system */
input_output_gfn(MODEL_NAME, "read", %gd);

printf("%s\n", model_title);

iteration_count = 0;
time = 0.0;
setup_direct_solve(%gd); /* sets up Jacobian sparsity structure */
while (TRUE)
{
  if (iteration_count == 0) then residual = compute_residual(%gd);

  switch (termination_test_type)
  {
  case ITERATIONS:
    if (iteration_count >= max_iterations)
      then goto quit;
    break;
  case RESIDUAL:
    if (residual < max_residual)
      then goto quit;
    break;
  }

  /*
  * Choose allowable relative error in Newton step:
  *
  * The strategy used is to allow a fairly large (0.1%) relative error
  * in the steps when we are 'not near' the solution, but cut this down
  * when the Newton iteration starts quadratically 'homing in' on the
  * solution. In this latter domain, the residual should be roughly
  * squared each iteration. However, we can never go below the roundoff
  * floor, and there's no need to go far below the desired residual.
  *
  * In addition, the "-exact" command line argument forces an "exact"
  * solution each iteration. (Actually, this is done by allowing the
  * last 3 decimal digits of the solution to be in error, but that's
  * close enough...)
  */
#define OK_RELEERR 0.001
  eps = OK_RELEERR;
  if (residual < 1.0)
    then eps := square(residual);
  if (eps < OK_RELEERR * max_residual)
    then eps = OK_RELEERR * max_residual;
  if (solve_exactly_each_iteration) then eps = -1000; /* "exact" */

  time += cpu_time();
  residual = direct_solve(%gd, eps, omega, dump_file_name);
  time += cpu_time();
  ++iteration_count;

```

main directory

DIRECT.C:82

21-SEP-1985 19:20

Page 4

```
    printf("Iteration %d of %s: Maximum residual = %.21e\n",
           iteration_count, iteration_limit, residual);

    printf("<<<<disk update ");
    input_output_gfn(MODEL_NAME, "write", &gd);
    printf("done>>>>\n");
}

quit:

printf("%d iterations / %.2f cpu seconds", iteration_count, time);
if (iteration_count != 0)
    then printf(" --> %.2f cpu sec/iteration", time/iteration_count);
printf("\n");

/* final disk update */
input_output_gfn(MODEL_NAME, "write", &gd);

/* log to '.par' file: */
par_log(MODEL_NAME, argc, argv, time);
}
```

main directory

DOPSOR.C;16

21-SEP-1985 19:21

Page 1

```

#include "util:stdio.h"
#include "patchutil:patch.h"

/*****

double compute_residual(pgd)
struct grid_dscr *pgd;
{
double do_point_sor();
return(do_point_sor(pgd, 0.0));
}

*****/

/*
 * This function does a single point SOR iteration, and returns the infinity-
 * norm of the (updated) dynamic residual.
 *
 * Arguments:
 * pgd --> The grid descriptor describing the coordinate grid.
 * omega - The overrelaxation parameter.
 */
double do_point_sor(pgd, omega)
register struct grid_dscr *pgd;
double omega;
{
double extrapolate();

register int p, i, j;
register struct coord_patch_dscr *ptr;
register struct grid_fn *pij;
double residual, derivative;
double max_residual, temp;
register struct line_end_dscr *pled_min, *pled_max;
int min, max;

max_residual = 0.0;

    for (p = 1 ; p <= pgd->N_patch ; ++p)
    {
        ptr = &pgd->patch[p];
#define PSI(I_,J_)      GRID_FNS(ptr,(I_),(J_))->psi_

        /* extrapolate the boundary values that will not be line endpoints: */
        for (j = ptr->j_min ; j <= ptr->j_max ; ++j)
        {
            pled_min = &ptr->line_end[I_MIN][j];
            pled_max = &ptr->line_end[I_MAX][j];
            min = pled_min->end_ij;
            max = pled_max->end_ij;
            PSI(min-1, j) = extrapolate(pgd, p, pled_min, min,j, min+1,j);
            PSI(max+1, j) = extrapolate(pgd, p, pled_max, max,j, max-1,j);
        }

        /* do the point SOR iteration: */

```

main directory

DOPSOR.C;16

21-SEP-1985 19:21

Page 2

```

for (i = ptr->i_min ; i <= ptr->i_max ; ++i)
{
/* set up to do this line (of constant 'i'): */
pled_min = & ptr->line_end[J_MIN][i];
pled_max = & ptr->line_end[J_MAX][i];
min = pled_min->end_ij;
max = pled_max->end_ij;
PSI(i, min-1) = extrapolate(pgd, p, pled_min, i,min, i,min+1);
PSI(i, max+1) = extrapolate(pgd, p, pled_max, i,max, i,max+1);

/* relax the line: */
for (j = min ; j <= max ; ++j)
{
pij = GRID_FNS(ptr, i,j);
temp = pij->psi_ * pij->psi_; /* psi ** 2 */
temp = temp * temp; /* psi ** 4 */
temp = temp * temp; /* psi ** 8 */
temp = pij->S_ / temp; /* S * (psi ** -8) */
residual = pij->c1_ * PSI(i-1,j)
+ pij->c2_ * PSI(i+1,j)
+ pij->c3_ * PSI(i,j-1)
+ pij->c4_ * PSI(i,j+1)
+ pij->psi_ * (pij->c5_ + temp);
derivative = pij->c5_ - 7.0 * temp;
pij->R_ = residual;
pij->psi_ -= omega * residual / derivative;

if (residual < 0.0) then residual = - residual;
if (residual > max_residual)
then max_residual = residual;
}
/* end of this line (of constant 'i') */
}

/* end of this patch */
}

return(max_residual);
}

```

main directory

DUMPGRD.C;6

21-SEP-1985 19:21

Page 1

```
#include "util:stdio.h"
#include "patchutil:patch.h"

/*****

main(argc, argv)      /* dumpgrd - dumps out a '.grd' file */
int argc;
char *argv[];
{
void error_exit();
char *fixname();
void input_output_grd();
void dump_grid();

struct grid_dscr gd;

if (argc != 2)
    then error_exit(OK_EXIT,
                    "usage: %s gridname\n",
                    fixname(argv[0])); /* DOES NOT RETURN */

input_output_grd(argv[1], "read", &gd);
dump_grid(&gd);
}
```

main directory

EXTRAP.C:32

21-SEP-1985 19:21

Page 1

```

#include "util:stdio.h"
#include "patchutil:patch.h"

/*
 * This function extrapolates psi values at line ends, taking into account all
 * boundary conditions etc. Note that it must be supplied with some redundant
 * arguments -- this is simpler than deriving them all from a minimal set, and
 * the information is easily available to the caller, anyway..... (To be
 * precise, the last 2 arguments are redundant.)
 *
 * Arguments:
 * pgd --> The grid descriptor for the coordinate patch system.
 * p = The patch number.
 * pled --> The line end descriptor for the line end.
 * (i_end,j_end) = The coordinates of the 'end_ij' point.
 * (i_int,j_int) = The coordinates of the point adjacent to the above point, in
 *                  the 'interior' direction, i.e. away from the boundary.
 *
 * The extrapolated psi value is returned as the function value.
 */
double extrapolate(pgd, p, pled, i_end,j_end, i_int,j_int)
register struct grid_dscr *pgd;
register int p;
register struct line_end_dscr *pled;
int i_end,j_end, i_int,j_int;
{
double cvt_psi_to_ofn(), cvt_ofn_to_psi();
double cvt_psi_to_omega(), cvt_omega_to_psi();
double square(), cube();

register struct coord_patch_dscr *ptr;
double sum;

ptr = & pgd->patch[p];
if (pled->flag > 0)
    then {
        /*
         * 'pled->flag' gives the patch number to interpolate into.
         */
        register struct coord_patch_dscr *other_ptr;

        other_ptr = & pgd->patch[pled->flag];

#define ROOT    pled->boundary_info.interpolate

#define PSI_AT_OFFSET(delta_i_,delta_j_)          \
    GRID_FNS(other_ptr,                          \
              ROOT.i_low + delta_i_,              \
              ROOT.j_low + delta_j_->psi_         \
              /* end of macro defn */

        /*
         * Note we do the (bilinear) interpolation in the conformal function of
         * the other patch, then convert the result back to psi.
         */

```


main directory

EXTRAP.C;32

21-SEP-1985 19:21

Page 2

```

sum = 0.0;
sum += ROOT.d00 * cvt_psi_to_cfn(other_ptr, PSI_AT_OFFSET(0,0));
sum += ROOT.d01 * cvt_psi_to_cfn(other_ptr, PSI_AT_OFFSET(0,1));
sum += ROOT.d10 * cvt_psi_to_cfn(other_ptr, PSI_AT_OFFSET(1,0));
sum += ROOT.d11 * cvt_psi_to_cfn(other_ptr, PSI_AT_OFFSET(1,1));
sum = cvt_cfn_to_psi(other_ptr, sum);

return(sum);
}
else {
/*
 * 'pled->flag' is a code value describing the type of boundary
 * condition; we must extrapolate.
 */
double psi_end, psi_int;
double omega_end, omega_int;

psi_end = GRID_FNS(ptr, i_end, j_end)->psi_;
psi_int = GRID_FNS(ptr, i_int, j_int)->psi_;

switch (pled->flag)
{
case DIRICHLET:
return(1.0);

case NEUMANN:
return(psi_int);

case ROBIN:
case REFLECTION_SYMMETRIC:
case marginally_trapped:

#define ROOT      pled->boundary_info.extrapolate
sum = ROOT.d1*psi_end + ROOT.d2 + ROOT.d3/cube(psi_end);

#define WHATSIT    ptr->conformal_fn_type
switch (WHATSIT)
{
case PSI:
/* extrapolate in 'PSI' variable: */
sum = psi_int + sum;
break;
case OMEGA_OF_PSI:
/* extrapolate in 'OMEGA' variable: */
omega_end = cvt_psi_to_omega(psi_end);
omega_int = cvt_psi_to_omega(psi_int);
sum = omega_int - square(omega_end) * sum;
sum = cvt_omega_to_psi(sum);
break;
default:
error_exit(OK_EXIT,
***** extrapolate: unknown conformal function type code=%d in patch %d\n",
WHATSIT,
p); /* DOES NOT RETURN */
}

```

main directory

EXTRAP.C;32

21-SEP-1985 19:21

Page 3

```
        return(sum);

    default:
        error_exit(OK_EXIT,
***** extrapolate: bad pled->flag=%d in patch %d at (%d,%d)\n",
                pled->flag,
                p,
                i_end, j_end);          /* DOES NOT RETURN */

        /* end of 'switch' on type of boundary condition */
    }

    /* end of pled->flag <- 0 logic */
}
```

main directory

INFO.C;26

4-NOV-1985 00:14

Page 1

```

/* This file contains routines for the 'info' program. */
/*
 * The following functions live here:
 * main - main routine for 'info'
 ** maximum_abs_psi - finds maximum absolute value of psi.
 */

#include "util:stdio.h"

#include "patchutil:patch.h"
#include "patchutil:blackhole.h"

/*****

main(argc, argv)          /* info - print out information about models */
int argc;
char *argv[];
{
void error_exit();
char *fixname();
void input_par(), input_output_grd(), input_output_gfn();
void set_extrapolation_coeff();

int N_grid_points();
double compute_residual();
double mass_of_mts_bndry();
double surface_total_energy();
void set_mass_monopoles(), set_psi_minus_monopoles();
static double maximum_abs_psi();

struct grid_dscr gd;
struct bhs_dscr bhsc;
char *model_title;
char *grid_name;
int size, total;
int i, p, bh;
struct black_hole_dscr *pbhd;

/* check for proper usage: */
if ((argc < 2) || (argc > 3))
    then error_exit(OK_EXIT,
                    "usage: %s model_name [ -grid ]\n",
                    fixname(argv[0])); /* DOES NOT RETURN */
#define MODEL_NAME      argv[1]

/* get model data: */
input_par(MODEL_NAME, &model_title, &grid_name, &bhsc);
input_output_grd(grid_name, "read", &gd);
set_extrapolation_coeff(&gd, &bhsc); /* update for our black hole system */
input_output_gfn(MODEL_NAME, "read", &gd);

/* general stuff on model: */
printf("Model = \"%s\"\n", MODEL_NAME);
printf("Title = \"%s\"\n", model_title);
printf("Maximum residual = %g\n", compute_residual(&gd));

```

main directory

INFO.C;26

4-NOV-1985 00:14

Page 2

```

/* general stuff on grid: */
total = N_grid_points(&gd, 0);          /* total # of grid points */
printf("Grid = \"%s\" (%d patches, %d grid points)\n",
      grid_name, gd.N_patch, total);
if ((argc >= 3) && (strcmp(argv[2], "-grid") == 0))
  then {
    /* detailed grid size breakdown info wanted -- print it: */
    for (p = 1 ; p <= gd.N_patch ; ++p)
    {
      size = N_grid_points(&gd, p);
      printf("Patch %d has %d grid points, %4.11f%% of total\n",
            p, size, 100.0 * size/total);
    }
  }

/* 'physics' stuff on model: */
for (bh = 1 ; bh <= bhsd.N_black_hole ; ++bh)
{
  pbhd = &bhsd.black_hole[bh];  /* --> black hole descr for this b.h. */

  p = pbhd->surrounding_patch_flag;
  if (p == 0)
    then error_exit(OK_EXIT,
      "***** Black hole %d has no surrounding patch!\n",
      bh);
  /* DOES NOT RETURN */

  if (p < 0) then p = - p;

  printf("\
Black hole %d: (rho=%f, z=%f) P=%g(%o), J=%g\n\
      nominal mass = %g, 'horizon area' mass = %g\n\
      ",
    bh, pbhd->rho, pbhd->z,
    pbhd->momentum, "-O+"[1 + pbhd->reflection_term_sign],
    pbhd->spin,
    pbhd->mass, mass_of_mts_bndry(pbhd, &gd)
  );
}

printf("Total energy via surface integral = %g\n",
      surface_total_energy(&gd.patch[gd.N_patch]));

/* note following computation destroys psi!!! */
set_mass_monopoles(&bhsd);
set_psi_minus_monopoles(&gd, &bhsd);  /* zap!!! */
printf("Maximum absolute value deviation from nominal monopole sum = %g\n",
      maximum_abs_psi(&gd));
}

/*****

/*
* This function computes the maximum (over all grid points) of the absolute
* value of psi. It is most useful if a monopole sum has been subtracted off
* from psi before calling.
*

```

main directory

INFO.C;26

4-NOV-1985 00:14

Page 3

```

* Arguments:
* pgd --> The grid descriptor.
*
* The maximum is returned as the function result.
*/
static double maximum_abs_psi(pgd)
register struct grid_dscr *pgd;
{
register int p, i, j;
register struct coord_patch_dscr *ptr;
double psi, max;

max = 0.0;
for (p = 1 ; p <= pgd->N_patch ; ++p)
{
ptr = &pgd->patch[p];
for (i = ptr->i_min ; i <= ptr->i_max ; ++i)
{
for (j = ptr->line_end[J_MIN][i].end_ij ;
j <= ptr->line_end[J_MAX][i].end_ij ;
++j)
{
psi = GRID_FNS(ptr, i, j)->psi;
if (psi < 0) then psi = -psi;
if (psi > max) then max = psi;
}
}

/* end of this patch */
}

return(max);
}

```

main directory

JACOBIAN.C:67

21-SEP-1985 19:22

Page 1

```

/* This file contains routine that set up the Jacobian. */
/*
 * The following functions live in this file:
 * compute_Jacobian - calculates all non-zero Jacobian elements
 ** handle - helper function to 'compute_Jacobian'
 ** invoke - helper function to 'handle'
 */

#include "util:stdio.h"

#include "patchutil:patch.h"
#include "patchutil:inout.h"

/*****
/*
 * This function calculates all non-zero Jacobian elements, and sends them to a
 * specified function for recording. Typically, it will be invoked twice, once
 * to record the sparsity structure of the Jacobian, and once to actually set
 * the numerical values.
 *
 * Arguments:
 * pgd --> The grid descriptor for the system.
 * p - The patch number to compute the Jacobian for (with all other patches
 * 'frozen' and used as Dirichlet boundary conditions), or 0 if all patches
 * are to be allowed to vary at once in the Jacobian.
 * pfn --> A function which is to be called once for each non-zero Jacobian
 * element. The call will be '(*pfn)(p1,i1,j1, p2,i2,j2, J12)' to say
 * that 'J12' is the partial derivative of the residual at patch #p1,
 * point (i1,j1), with respect to psi at patch #p2, point (i2,j2). The
 * ordering of the calls will be such that (p1,i1,j1) are cycled in row
 * major order (i.e. p1 varies slowest, j1 varies fastest). All the
 * calls for a given value of (p1,i1,j1) will be done in a group, with
 * no other (p1,i1,j1) values interleaved within them. Within such a
 * (p1,i1,j1) group, any given (p2,i2,j2) value may only occur more
 * than once, with the partial derivative values passed to be added.
 * Such calls may have other (i2,j2) calls interleaved within them.
 *
 * Note that if the 'p' argument is non-zero, then the 'p1' and 'p2' arguments
 * to *pfn will both be equal to 'p'.
 */
void compute_Jacobian(pgd, p, pfn)
register struct grid_dscr *pgd;
int p;
void (*pfn)();
{
    static void handle();          /* helper function */
    double square();

    boolean full_J = (p == 0);    /* full Jacobian wanted??? */
    register struct coord_patch_dscr *ptr;
    int i,j;
    int p_low, p_hi;
    struct grid_fn *pgfn;
    double deriv;

```

main directory

JACOBIAN.C;67

21-SEP-1985 19:22

Page 2

```

if (full_J)
  then {
    p_low = 1;
    p_hi = pgd->N_patch;
  }
  else {
    p_low = p_hi = p;
  }

/* scan through all 'valid' patches: */
for (p = p_low ; p <= p_hi ; ++p)
{
  /* scan all points in this patch: */
  ptr = & pgd->patch[p];          /* --> coord_patch_dscr for patch #p */
  for (i = ptr->i_min ; i <= ptr->i_max ; ++i)
  {
    for (j = ptr->line_end[J_MIN][i].end_ij ;
         j <= ptr->line_end[J_MAX][i].end_ij ;
         ++j)
    {
      /*
       * This loop body will be executed once for each (p,i,j)
       * triple we want to calculate partial derivatives of the
       * residual at.
       */

      /* do the central point (diagonal element of Jacobian): */
      pgfn = GRID_FNS(ptr, i, j);
      deriv = pgfn->c5_ - 7.0 * pgfn->S_
                / square(square(square(pgfn->psi_)));
      (*pfn)(p,i,j, p,i,j, deriv);

      /* now do the outer points of the del-squared star: */
#define CONSTANT_PART handle(pfn, full_J, pgd, p,i,j
      CONSTANT_PART, pgfn->c1_, -1, 0, & ptr->line_end[I_MIN][j]);
      CONSTANT_PART, pgfn->c2_, 1, 0, & ptr->line_end[I_MAX][j]);
      CONSTANT_PART, pgfn->c3_, 0,-1, & ptr->line_end[J_MIN][i]);
      CONSTANT_PART, pgfn->c4_, 0, 1, & ptr->line_end[J_MAX][i]);

      /* end of this 'j' */
    }

    /* end of this 'i' */
  }

  /* end of this patch */
}

/* all done now!!! */
}

/*****

```

main directory

JACOBIAN.C;67

21-SEP-1985 19:22

Page 3

```

/*
 * This function handles the details of checking for patch boundaries for the
 * off-diagonal Jacobian entries.
 *
 * Arguments:
 * pfn --> The function to call if we find a non-zero Jacobian (see header
 *         comments to 'compute_Jacobian' above for details).
 * full_J - TRUE if the full (multi-patch) Jacobian is wanted, FALSE if only a
 *         single patch (pl) is wanted. In this latter case, the remaining
 *         patches are treated as being 'frozen in place' to provide Dirichlet
 *         boundary conditions along all patch boundaries.
 * pgd --> The grid descriptor structure.
 * (pl,il,jl) - The patch number (pl) and point (il,jl) where the residual is
 *              being evaluated.
 * weight - The weight of (i2,j2) in the del-squared 'star' centered on
 *           *pl(il,jl).
 * (di,dj) - The offset of the second (i2,j2) point from the first (il,jl).
 *           This must be one of (-1,0), (1,0), (0,-1), or (0,1), i.e. it must
 *           be in the del-squared 'star' centered on *pl(il,jl).
 * pled --> The line end descriptor for (i2,j2), if it's outside patch *pl. If
 *           (i2,j2) isn't outside patch *pl, this argument is ignored.
 *
 * The function computes the partial derivative of the residual at (pl,il,jl)
 * with respect to psi at (pl,il+di,jl+dj) (where the latter may be the result
 * of boundary extrapolation or inter-patch interpolation), and passes it to
 * (*pfn), using the call sequence defined in the header comments to
 * 'compute_Jacobian' above.
 */
static void handle(pfn, full_J, pgd, pl,il,jl, weight, di,dj, pled)
void (*pfn)();
boolean full_J;
register struct grid_dscr *pgd;
int pl,il,jl;
double weight;
int di,dj;
register struct line_end_dscr *pled;
{
double square(), cube(), fourth();
double extrapolate();
double cvt_psi_to_omega();
void error_exit();
enum relation inout_ij();

static void invoke();

register struct coord_patch_dscr *ptr1, *ptr2;
int p2;
int i2,j2, i2_r,j2_r;
double psi1, psi2, psi2_r;
double omega1, omega2, omega2_r, omega_other;
boolean S_A_flag;

ptr1 = &pgd->patch[pl];
i2 = il + di;          i2_r = il - di;

```


main directory

JACOBIAN.C;67

21-SEP-1985 19:22

Page 4

```

j2 = j1 + dj;          j2_r = j1 - dj;

/* easy case??? */
if (inout_1j(ptr1,12,j2) != OUTSIDE)
  then {
    (*pfn)(p1,11,j1, p1,12,j2, weight);
    return;
  }

/* we now know that (12,j2) is OUTSIDE patch #p1 --> hard case: */
psi1 = GRID_FNS(ptr1,11,j1)->psi_;
psi2 = extrapolate(pgd, p1, pled, 11,j1, 12_r,j2_r);

if (pled->flag > 0)
  then {
    /* 'patch' boundary */
    if (! full_J) then return;      /* skip other patches??? */

    p2 = pled->flag;      /* other patch # */
    ptr2 = & pgd->patch[p2];

    /*
     * We must include Jacobian entries for effects of psi in patch #p2 on
     * the residual at #p1(11,j1). Since we in general are interpolating
     * into a square, there are in general 4 points in patch #p2 to deal
     * with. However, we can filter out those with zero 'dij' coefficients
     * since they can't effect the result.
     *
     * The interpolation will have been done using the conformal function
     * of the patch #p2. The function 'invoke(...)' is where this is taken
     * into account.
     */

#define ROOT    pled->boundary_info.interpolate
#define INVOKE(di_, dj_, deriv_) \
    invoke(pfn, p1, 11, j1, p2, ROOT.i_low+di_, ROOT.j_low+dj_, \
           deriv_, ptr2, psi2)

    if (ROOT.d00 != 0.0) then INVOKE(0,0, weight * ROOT.d00);
    if (ROOT.d01 != 0.0) then INVOKE(0,1, weight * ROOT.d01);
    if (ROOT.d10 != 0.0) then INVOKE(1,0, weight * ROOT.d10);
    if (ROOT.d11 != 0.0) then INVOKE(1,1, weight * ROOT.d11);
  }
  else {
    /* 'true' boundary */
    switch (pled->flag)      /* code for boundary condition type: */
    {
      case DIRICHLET:
        break;

      case NEUMANN:
        (*pfn)(p1,11,j1, p1,12_r,j2_r, weight);
        break;
    }
  }

```

main directory

JACOBIAN.C:67

21-SEP-1985 19:22

Page 5

```

#define ROOT      pled->boundary_info.extrapolate
#define SUM       ( ROOT.d1*psi1 + ROOT.d2 + ROOT.d3 / cube(psi1) )
#define SUM_DER   ( ROOT.d1 - 3.0 * ROOT.d3 / fourth(psi1) )

      case ROBIN:
      case REFLECTION_SYMMETRIC:
      case marginally_trapped:
#define WHATSIT ptr1->conformal_fn_type
      switch (WHATSIT)
      {
      case PSI:
        (*pfn)(p1,i1,j1, p1,i2_r,j2_r, weight * 1.0);
        (*pfn)(p1,i1,j1, p1,i1,j1, weight * SUM_DER);
        break;

      case OMEGA_OF_PSI:
        psi2_r = GRID_FNS(ptr1, i2_r, j2_r)->psi_;
        omeg1 = cvt_psi_to_omega(psi1);
        omeg2 = cvt_psi_to_omega(psi2);
        omeg2_r = cvt_psi_to_omega(psi2_r);
        (*pfn)(p1,i1,j1, p1,i2_r,j2_r,
              weight * square(omeg2_r / omeg2));
        (*pfn)(p1,i1,j1, p1,i1,j1,
              weight * square(omeg1 / omeg2)
              * (SUM_DER - 2.0 * omeg1 * SUM));
        break;

      default:
        error_exit(OK_EXIT, "\
***** compute_Jacobian: unknown conformal function type code=%d in patch %d\n\
",
                  WHATSIT,
                  p1); /* DOES NOT RETURN */
      }
      break;

      default:
        /* trouble! */
        error_exit(OK_EXIT,
        "***** compute_Jacobian: bad 'pled->flag' value=%d at %d(%d,%d)\n",
          pled->flag,
          p1,
          i1, j1); /* DOES NOT RETURN */

        /* end of switch on boundary condition code */
      }

    /* end of 'true boundary' case */
  }

/*****
/*
* This function is used to compute the partial derivatives due to inter-patch
* interpolation. It is called ONLY from 'handle(...)' above.

```

main directory

JACOBIAN.C:67

21-SEP-1985 19:22

Page 6

```

*
* Arguments:
* (pfn, p1,i1,j1, p2,i2,j2) - As in description of 'pfn' argument to
*                             'compute_Jacobian(...)' above.
* deriv - Partial derivative if the conformal function used in patch #p2 is
*         'psi' itself.
* ptr2 --> Coordinate patch descriptor for patch #p2.
* psi2 - 'psi' returned by 'extrapolate(...)'.
*/
static void invoke(pfn, p1,i1,j1, p2,i2,j2, deriv, ptr2, psi2)
void (*pfn)();
int p1,i1,j1, p2,i2,j2;
double deriv;
register struct coord_patch_dscr *ptr2;
double psi2;
{
double cvt_psi_to_omega();
double square();

double factor;
double psi_ij, omega_ij, omega2;

#define WHATSIT ptr2->conformal_fn_type
switch (WHATSIT)
{
case PSI:
    factor = 1.0;
    break;

case OMEGA_OF_PSI:
    psi_ij = GRID_FNS(ptr2, i2, j2)->psi_;
    omega_ij = cvt_psi_to_omega(psi_ij);
    omega2 = cvt_psi_to_omega(psi2);
    factor = square(omega_ij / omega2);
    break;

default:
    error_exit(OK_EXIT, "\
***** compute_Jacobian: unknown conformal function type code-%d in patch %d\n\
", WHATSIT,
    p2); /* DOES NOT RETURN */
}

/* now tell 'user' about partial derivative!!! */
(*pfn)(p1,i1,j1, p2,i2,j2, deriv * factor);
}

```

main directory

MKGFN.C;64

21-SEP-1985 19:23

Page 1

```

/* This file includes the main routine for 'mkgfn' and her friends. */
/*
 * Routines in this file are:
 * main - 'mkgfn' - makes a '.gfn' file
 ** compute_ci - computes del-squared coefficients c1, c2, c3, c4, c5
 ** source_fn - computes 'source function' = 1/8 * extrinsic curvature squared
 */

#include "util:stdio.h"

#include "patchutil:blackhole.h"
#include "patchutil:patch.h"

/*****/

main(argc, argv)      /* mkgfn - makes a '.gfn' file */
int argc;
char *argv[];
{
void input_par();
void input_output_grd();
void input_output_gfn();
void error_exit();
void par_log();

void cvt_ij_to_xy();
void cvt_xy_to_xy();
int fuzzy_EQ();
char *fixname();

void set_mass_monopoles();
double sum_monopoles();
static void compute_ci();
static double source_fn();

char *title, *gridname;
struct bhs_dscr bhsd;
struct grid_dscr gd;
int p;
register struct coord_patch_dscr *ptr;
int i, j;
double x, y;
double rho, z;
boolean on_z_axis;
register struct grid_fn *pgfn;

if (argc != 2)
    then error_exit(OK_EXIT, "usage: %s model_name\n", fixname(argv[0]));
    /* DOES NOT RETURN */
#define MODEL_NAME          argv[1]

/* fetch input data: */
input_par(MODEL_NAME, &title, &gridname, &bhsd);
input_output_grd(gridname, "read", &gd);

```

main directory

MKGFN.C;64

21-SEP-1985 19:23

Page 2

```

/* set up '.gfn' file contents: */
set_mass_monopoles(&bhsd);
  for (p = 1 ; p <= gd.N_patch ; ++p)
  {
    ptr = &gd.patch[p];

    allocate_grid_fn_array(ptr);
    /* if not, storage already allocated (above!) */

    for (i = ptr->i_min ; i <= ptr->i_max ; ++i)
    {
      for (j = ptr->line_end[(int)J_MIN][i].end_ij ;
           j <= ptr->line_end[(int)J_MAX][i].end_ij ;
           ++j)
      {
        /* compute grid functions for this point: */
        pgfn = GRID_FNS(ptr, i, j);
        cvt_ij_to_xy(ptr, i, j, &x, &y);
        cvt_xy_to_xy(ptr, x, y,
                     &gd.patch[GLOBAL_REF], &rho, &z);

        on_z_axis = fuzzy_EQ(rho, 0.0);
        compute_ci(ptr, p,
                   x, y, on_z_axis,
                   (double *) &pgfn->cl_ - 1);

        pgfn->S_ = source_fn(&bhsd, rho, z);
        pgfn->psi_ = sum_monopoles(&bhsd, rho, z);
        pgfn->R_ = 0.0;
      }
    }

    /* end of this patch */
  }

/* create '.gfn' file: */
input_output_gfn(MODEL_NAME, "write", &gd);

par_log(MODEL_NAME, argc, argv, 0.0); /* log in '.par' file */

/*****
/*
* This function computes the del-squared coefficients.
*
* Arguments:
* ptr --> The coord_patch_dscr for the patch in which the coefficients are to
*         be used.
* p - The patch number (used only to label error messages).
* (x,y) - The 'x/y' coordinates of the point where the coefficients are to be
*         computed, in the system of patch in use.
* on_z_axis = TRUE if (x,y) is on the z-axis, FALSE otherwise.
* c --> (1 origin) A 5-element array in which to store the coefficients.
*/

```

main directory

MKGPN.C;64

21-SEP-1985 19:23

Page 3

```

static void compute_c1(ptr, p, x, y, on_z_axis, c)
register struct coord_patch_desc *ptr;
int p;
double x, y;
boolean on_z_axis;
double c[];
{
double sin(), cos();
double square();
void error_exit();

double term1, term2;

switch (ptr->coord_type)
{
case CYLINDRICAL:
/*
 * Formulas taken from p. 119 in my notes, or from p. 157/158 -- both
 * types of difference scheme give the same results.
 */
#define RHO          x
#define Z            y
#define DELTA_RHO    ptr->i_step
#define DELTA_Z       ptr->j_step

c[3] = c[4] = 1.0 / square(DELTA_Z);

if (on_z_axis)
then c[1] = c[2] = 2.0 / square(DELTA_RHO);
else {
term1 = 1.0 / square(DELTA_RHO);
term2 = 1.0 / (RHO * 2.0 * DELTA_RHO);
c[1] = term1 - term2;
c[2] = term1 + term2;
}

break;

case POLAR_SPHERICAL:
/*
 * Here, the different difference schemes DO make a difference!!! (Pun
 * not intended.)
 */

#define R          x
#define THETA      y
#define DELTA_R    ptr->i_step
#define DELTA_THETA ptr->j_step

#define WHATSIT ptr->difference_scheme
switch (WHATSIT)
{
case INDEPENDENT_TERMS:
/* formulas taken from p. 117-118 in my notes */

term1 = 1.0 / square(DELTA_R);

```

main directory

MKGPN.C;64

21-SEP-1985 19:23

Page 4

```

term2 = 1.0 / (R * DELTA_R);
c[1] = term1 - term2;
c[2] = term1 + term2;

if (on_z_axis)
then c[3] = c[4] = 2.0 / square(R * DELTA_THETA);
else {
    term1 = 1.0 / square(R * DELTA_THETA);
    term2 = cos(THETA)
            / (square(R) * sin(THETA) * 2.0 * DELTA_THETA);
    c[3] = term1 - term2;
    c[4] = term1 + term2;
}

break;

case LOCALLY_CONSERVATIVE:
/* formulas taken from p. 159-162 in my notes */

term1 = 1.0 / square(DELTA_R);
term2 = DELTA_R / (2.0 * R);
c[1] = term1 * square(1.0 - term2);
c[2] = term1 * square(1.0 + term2);

if (on_z_axis)
then c[3] = c[4] = 2.0 / square(R * DELTA_THETA);
else {
    term1 = 1.0 / (square(R * DELTA_THETA) * sin(THETA));
    term2 = 0.5 * DELTA_THETA;
    c[3] = term1 * sin(THETA - term2);
    c[4] = term1 * sin(THETA + term2);
}

break;

default:
    error_exit(OK_EXIT,
***** compute_ci: unknown difference scheme type code=%d for patch%d\n",
                WHATSIT,
                p); /* DOES NOT RETURN */
    }

break;

default:
    error_exit(OK_EXIT,
***** compute_ci: unknown coordinate type code=%d for patch %d\n",
                ptr->coord_type,
                p); /* DOES NOT RETURN */
    }

c[5] = - (c[1] + c[2] + c[3] + c[4]); /* always!!! */
}

/*****
/*
* This function computes the 'source function', given by

```

main directory

MKGPN.C;64

21-SEP-1985 19:23

Page 5

```

*
*      1   ^   ^1j
*      S = --- K   K
*      8   1j
*
* Arguments:
* pbhsd --> The black hole system descriptor.
* (rho,z) - The global cylindrical coordinates of the place where the source
*           function is desired.
*
* The resulting source function is returned as the function result.
*/
static double source_fn(pbhsd, rho, z)
register struct bhs_dscr *pbhsd;
double rho, z;
{
double square();
void K_hat_sub_ij();

double K_rho_rho, K_rho_z, K_rho_phi, K_z_z, K_z_phi, K_phi_phi;
double sum;

K_hat_sub_ij(pbhsd, rho, z, &K_rho_rho, &K_rho_z, &K_rho_phi,
                                &K_z_z, &K_z_phi,
                                &K_phi_phi);

/*
* Note:      Recall that, in order to keep things finite and non-zero on the
*            z-axis, the Kij's returned have already been divided by one
*            factor of 'rho' for each 'phi' occurring in the indices (i,j).
*            This is exactly what's needed to avoid having to divide by any
*            more factors of 'rho' in forming the scalar invariant K-squared
*            below.
*/

sum = 0;

sum += square(K_rho_rho);
sum += 2.0 * square(K_rho_z);
sum += 2.0 * square(K_rho_phi);           /* c.f. note above */
sum += square(K_z_z);
sum += 2.0 * square(K_z_phi);           /* c.f. note above */
sum += square(K_phi_phi);               /* c.f. note above */

sum /= 8.0;

return(sum);
}

```


main directory

MKGRD.C;6

21-SEP-1985 19:24

Page 1

```

#include "util:stdio.h"
#include "patchutil:patch.h"

/*****

main(argo,argv)      /* mkgrd -- makes a '.grd' file from a '.pat' file */
int argo;
char *argv[];
{
void input_pat();
void input_output_grd();
char *fixname();

struct grid_dscr gd;

if (argo != 2)
    then error_exit(OK_EXIT,
                    "usage: %s gridname\n",
                    fixname(argv[0])); /* DOES NOT RETURN */

input_pat(argv[1], &gd);
input_output_grd(argv[1], "write", &gd);
}

```

main directory

MONOPOLE.C;28

4-NOV-1985 00:14

Page 1

```

/* This file contains routines that deal with 'monopoles' in psi */
/*
 * Routines in this file are:
 * sum_monopoles - Sums current monopoles for specified position
 * set_mass_monopoles - Sets monopole strengths to one half b.h. masses
 * set_area_monopoles - Sets monopole strengths to one half horizon areas
 * normalise_monopoles - normalises monopoles so they sum to specified amount
 * set_psi_minus_monopoles - computes 'psi -- (monopole sum)'
 */

#include "util:stdio.h"

#include "patchutil:blackhole.h"
#include "patchutil:patch.h"

/*****
/*
 * This function computes the sum of all the monopoles (associated with the
 * black hole system), plus 1.0. Thus this value is a suitable 'psi' value to
 * associate with the current set of monopoles.
 *
 * Arguments:
 * pbhsd --> The black hole system descriptor.
 * (rho,z) - The global cylindrical coordinates of the place where the 'psi
 *           value' is desired.
 *
 * The 'psi value' is returned as the function value.
 */
double sum_monopoles(pbhsd, rho, z)
register struct bhs_dscr *pbhsd;
double rho, z;
{
double hypot();

register int bh;
register double sum, r;

sum = 1.0;

    for (bh = 1 ; bh <= pbhsd->N_black_hole ; ++bh)
    {
        r = hypot(rho - pbhsd->black_hole[bh].rho,
                  z - pbhsd->black_hole[bh].z);
        sum += pbhsd->black_hole[bh].lambda / r;
    }

return(sum);
}

/*****
/*
 * This function sets the monopole strength associated with each black hole to
 * one half that black hole's mass.

```

main directory

MONOPOLE.C;28

4-NOV-1985 00:14

Page 2

```

*
* Arguments:
* pbhsd --> The black hole system descriptor.
*/
void set_mass_monopoles(pbhsd)
register struct bhs_dscr *pbhsd;
{
    register int bh;

    for (bh = 1 ; bh <= pbhsd->N_black_hole ; ++bh)
    {
        pbhsd->black_hole[bh].lambda = pbhsd->black_hole[bh].mass / 2.0;
    }
}

/*****

/*
* This function sets the monopole strength associated with each black hole to
* one half the mass of that black hole's horizon.
*
* Arguments:
* pgd --> The grid descriptor for the coordinate patch system.
* pbhsd --> The black hole system descriptor.
*/
void set_area_monopoles(pgd, pbhsd)
register struct grid_dscr *pgd;
register struct bhs_dscr *pbhsd;
{
    double mass_of_mts_bndry();

    register int bh;
    double m;

    for (bh = 1 ; bh <= pbhsd->N_black_hole ; ++bh)
    {
        m = mass_of_mts_bndry( & pbhsd->black_hole[bh], pgd);
        pbhsd->black_hole[bh].lambda = m / 2.0;
    }
}

/*****

/*
* This function normalises the monopole strengths by multiplying them all by
* a constant chosen so they sum to a specified amount.
*
* Arguments:
* pbhsd --> The black hole system descriptor.
* desired_sum = The desired monopole sum.
*/
void normalise_monopoles(pbhsd, desired_sum)
register struct bhs_dscr *pbhsd;
double desired_sum;
{

```

main directory

MONOPOLE.C:28

4-NOV-1985 00:14

Page 3

```

int bh;
double actual_sum, ratio;

actual_sum = 0;
for (bh = 1 ; bh <= pbhsd->N_black_hole ; ++bh)
{
    actual_sum += pbhsd->black_hole[bh].lambda;
}

ratio = desired_sum / actual_sum;

for (bh = 1 ; bh <= pbhsd->N_black_hole ; ++bh)
{
    pbhsd->black_hole[bh].lambda *= ratio;
}
}

/*****
/*
* This function subtracts the monopole sum from psi.
*
* Arguments:
* pgd --> The grid descriptor.
* pbhsd --> The black hole descriptor, with all 'lambda' fields already filled
*         in.
*/
void set_psi_minus_monopoles(pgd, pbhsd)
register struct grid_dsor *pgd;
register struct bhs_dsor *pbhsd;
{
    double sum_monopoles();
    void cvt_ij_to_xy(), cvt_xy_to_xy();

    register int p, i, j;
    register struct coord_patch_dsor *ptr;
    double x, y, rho, z;

    for (p = 1 ; p <= pgd->N_patch ; ++p)
    {
        ptr = &pgd->patch[p];

        for (i = ptr->i_min ; i <= ptr->i_max ; ++i)
        {
            for (j = ptr->line_end[J_MIN][i].end_ij ;
                j <= ptr->line_end[J_MAX][i].end_ij ;
                ++j)
            {
                cvt_ij_to_xy(ptr, i, j, &x, &y);
                cvt_xy_to_xy(ptr, x, y,
                    &pgd->patch[GLOBAL_REF], &rho, &z);
                GRID_FNS(ptr, i, j)->psi_
                    -= sum_monopoles(pbhsd, rho, z);
            }
        }
    }
}

```

main directory

MONOPOLE.C;28

4-NOV-1985 00:14

Page 4

```
    /* end of this patch */  
  }  
}
```

main directory

PHYSICS.C;37

21-SEP-1985 19:27

Page 1

```

/* This file contains routines to calculate 'physics' functionals of models */
/*
 * The following functions live here:
 * mass_of_area - Finds mass associated with given horizon area
 * integrate_r_const - Integrates a specified fn on a 'r = constant' surface
 * area_r_const - Finds area of 'r = constant' surface
 * area_fn - Function to be integrate in finding above area
 * mass_of_mts_bndry - Finds mass of m.t.s. boundary surrounding given b.h.
 * surface_total_energy - Finds total energy of model via surface integral
 * surface_energy_fn - Function to be integrated for above
 */

#include "util:stdio.h"
#include "patchutil:patch.h"
#include "patchutil:blackhole.h"

/*****
/*
 * This function finds the mass associated with a given horizon area.
 *
 * Arguments:
 * area - The horizon area.
 *
 * The associated mass is returned as the function result.
 */
double mass_of_area(area)
double area;
{
double sqrt();

return(sqrt(area / (16.0*PI)));
}

/*****
/*
 * This function integrates a specified function (computed 'on the fly') over a
 * given 'r = constant' surface in a patch with coord_type="polar spherical".
 * The boundary conditions on theta must be 'Neumann'. The range of theta must
 * be either [0,PI/2], [PI/2,PI], or [0,PI]; in the former 2 cases, reflection
 * (mirror image) symmetry about theta-PI/2 is assumed. Optionally, the usual
 * "r-squared-sin-theta" weighting may be done internal to this function. If
 * the remainder of the integrand is almost constant, this option yeilds much
 * improved accuracy.
 *
 * Arguments:
 * pfn --> The function to be integrated. This is used in the form
 *         function = (*pfn)(ptr, i, j);
 *         to evaluate the function at (i,j) in the specified patch.
 * weight_flag - FALSE to integrate (function as given),
 *               - TRUE to integrate square(r)*sin(theta) * (function as given).
 * ptr --> The coordinate patch descriptor for the surface.
 * i - The 'i' integer radial coordinate of the surface. Note that this way of
 *     specifying the constant value of r implies that the surface must be a

```

main directory

PHYSICS.C;37

21-SEP-1985 19:27

Page 2

```

*      grid line (in 2 dimensions).
*
* The integral (over theta and phi), is returned as the function result.
*/
double integrate_r_const(pfn, weight_flag, ptr, i)
double (*pfn)();
boolean weight_flag;
register struct coord_patch_dsor *ptr;
int i;
{
double spline_int();
boolean fuzzy_EQ();
void cvt_ij_to_xy();
void error_exit();
char *calloc();
void cfree();
double square(), sin(), cos();

register struct line_end_dsor *pled_min, *pled_max;
int j, j_min, j_max, n_points;
double theta, theta_min, theta_max, r;
double integral;
double *work_ptr, *work;

/* make sure all our assumptions are valid */
if (ptr->coord_type != POLAR_SPHERICAL)
    then error_exit(OK_EXIT,
***** integrate_r_const: integrating requires a polar spherical patch\n"
    ); /* DOES NOT RETURN */
pled_min = & ptr->line_end[J_MIN][i];
pled_max = & ptr->line_end[J_MAX][i];
if (! ((pled_min->flag == NEUMANN) && (pled_max->flag == NEUMANN)))
    then error_exit(OK_EXIT, "\n
***** integrate_r_const: integrating requires Neumann bndry conds on theta\n"
    ); /* DOES NOT RETURN */

/* set up function in array: */
j_min = pled_min->end_ij;
j_max = pled_max->end_ij;
n_points = j_max - j_min + 1;
work_ptr = ALLOC(n_points, double);
if (work_ptr == NULL)
    then error_exit(OK_EXIT,
***** integrate_r_const: insufficient memory for %d grid points in theta\n",
n_points); /* DOES NOT RETURN */
work = work_ptr - j_min; /* adjust for non-zero index origin */
for (j = j_min; j <= j_max; ++j)
{
    work[j] = (*pfn)(ptr, i, j);
}

/* do weighting internally? */
cvt_ij_to_xy(ptr, i, j_min, &r, &theta_min);
cvt_ij_to_xy(ptr, i, j_max, &r, &theta_max);
if (weight_flag)

```

main directory

PHYSICS.C;37

21-SEP-1985 19:27

Page 3

```

    then {
        /*
         * Subtract off theta_min value of function, do that part of integral
         * analytically:
         */
        double temp;
        temp = work[j_min];
        for (j = j_min ; j <= j_max ; ++j)
        {
            work[j] -= temp;
            cvt_1j_to_xy(ptr, 1, j, &r, &theta);
            work[j] += square(r) * sin(theta);
        }
        integral = cos(theta_min) - cos(theta_max);
        integral += temp * square(r);
    }
    else integral = 0;

/* integrate: */
integral += spline_int(theta_min, theta_max, n_points-1, work,
                        1, 0.0, /* Neumann boundary condition at theta_min */
                        1, 0.0); /* (ditto at theta_max) */

/* find integral over theta range [0, PI] by taking symmetry into account: */
if (fuzzy_EQ(theta_min, 0.0) && fuzzy_EQ(theta_max, PI))
    then { } /* theta range is [0, PI] ==> do nothing */
else if (fuzzy_EQ(theta_min, 0.0) && fuzzy_EQ(theta_max, PI/2.0))
    then integral *= 2.0; /* theta range is [0, PI/2] ==> must double */
else if (fuzzy_EQ(theta_min, PI/2.0) && fuzzy_EQ(theta_max, PI))
    then integral *= 2.0; /* theta range is [PI/2, PI] ==> must double */
else error_exit(OK_EXIT,
***** integrate_r_const: specified theta interval [%3f,%3f] is invalid\n",
                theta_min, theta_max); /* DOES NOT RETURN */

/* return storage & quit: */
cfree(work_ptr); /* must supply original 'calloc(...)' ptr: NO OFFSET */
return(2.0 * PI * integral); /* 2.0*PI is for phi integration!!! */
}

/*****
/*
 * This function finds the area of a given 'r = constant' surface in a patch
 * with coord_type="polar spherical". The boundary conditions on theta must
 * be 'Neumann'. The range of theta must be either [0,PI/2], [PI/2,PI], or
 * [0,PI]; in the former 2 cases, reflection (mirror image) symmetry about
 * theta-PI/2 is assumed.
 *
 * Arguments:
 * ptr --> The coordinate patch descriptor for the surface.
 * i - The '1' integer radial coordinate of the surface. Note that this way of
 * specifying the constant value of r implies that the surface must be a
 * grid line (in 2 dimensions).
 *
 * The area is returned as the function result.
 */

```


main directory

PHYSICS.C;37

21-SEP-1985 19:27

Page 4

```

*/
double area_r_const(ptr, i)
register struct coord_patch_dscr *ptr;
int i;
{
double integrate_r_const();
static double area_fn();

return(integrate_r_const(&area_fn, TRUE, ptr, i));
}

/*****

/*
* This function is integrated in the computation of the area of 'r = constant'
* surfaces. In particular, it's integral over the sphere, is that area.
*
* Arguments:
* ptr --> Coordinate patch descriptor for patch in which the surface lies.
*         This patch is assumed to have coord_type="polar spherical".
* (i,j) - (Integer) coordinates (in the system of this patch) to evaluate the
*         function at.
*
* The returned result is obvious!
*/
static double area_fn(ptr, i, j)
register struct coord_patch_dscr *ptr;
register int i, j;
{
double fourth();

double psi;

psi = GRID_FNS(ptr, i, j)->psi_;

return( fourth(psi) );
}

/*****

/*
* This function computes the mass of the marginally trapped surface grid
* boundary which is assumed to surround a given black hole.
*
* Arguments:
* pbhd --> The black hole descriptor.
* pgd --> The grid descriptor.
*
* The mass is returned as the function result.
*/
double mass_of_mts_bndry(pbhd, pgd)
register struct black_hole_dscr *pbhd;
register struct grid_dscr *pgd;
{
void error_exit();

```

main directory

PHYSICS.C:37

21-SEP-1985 19:27

Page 5

```

double mass_of_area(), area_r_const();

int p;
register struct coord_patch_dscr *ptr;

p = pbhd->surrounding_patch_flag;

if (p == 0)
    then error_exit(OK_EXIT,
***** mass_of_mts_bndry: black hole has surrounding patch flag of zero\n"
    ); /* DOES NOT RETURN */
if (p < 0) then p = - p;

ptr = & pgd->patch[p];
return(mass_of_area(area_r_const(ptr, ptr->i_min)));
}

/*****
/*
* This function computes the total energy of a model, using a surface integral
* (not very accurate). Robin outer boundary conditions are assumed.
*
* Arguments:
* ptr --> The coordinate patch descriptor for the model's outermost patch.
*
* The total energy is returned as the function result.
*/
double surface_total_energy(ptr)
register struct coord_patch_dscr *ptr;
{
double integrate_r_const();
static double surface_energy_fn();

double temp;

temp = integrate_r_const(&surface_energy_fn, TRUE, ptr, ptr->i_max);

return(temp / (2.0 * PI));
}

/*****
/*
* This function is integrated in the computation of the total energy of a
* model by the surface integral.
*
* Arguments:
* ptr --> Coordinate patch descriptor for patch in which the surface lies.
* This patch is assumed to have coord_type="polar spherical".
* (i,j) = (Integer) coordinates (in the system of this patch) to evaluate the
* function at.
*
* The returned result is obvious!
*/

```

main directory

PHYSICS.C;37

21-SEP-1985 19:27

Page 6

```
static double surface_energy_fn(ptr, i, j)
register struct coord_patch_dscr *ptr;
register int i, j;
{
void cvt_ij_to_xy();
double r, theta, psi;

cvt_ij_to_xy(ptr, i, j, &r, &theta);
psi = GRID_FNS(ptr, i, j)->psi_;

return( (psi - 1.0) / r );
}
```

main directory

PSOR.C;38

21-SEP-1985 19:28

Page 1

```

#include "util:stdio.h"

#include "patchutil:patch.h"
#include "patchutil:blackhole.h"

/*****

main(argc, argv)      /* psor - driver for point SOR solver */
int argc;
char *argv[];
{
void error_exit();
double do_point_sor(); /*
                        * Does a single point SOR iteration, returns infinity-
                        * norm of new dynamic residual.
                        */

void input_par(), input_output_grd(), input_output_gfn(), par_log();
void set_extrapolation_coeff();
double cpu_time();
char *fixname();

struct grid_dscr gd;
struct bhs_dscr bhsd;
char *model_title;
char *grid_name;
double time;

enum { ITERATIONS, RESIDUAL } termination_test_type;
int max_iterations;
#define BUFSIZE 100
char iteration_limit[BUFSIZE];
double max_residual;

double omega;
int print_interval, disk_write_interval;

int iteration_count;
double old_residual, residual;

/* check for proper usage: */
if (! ((argc == 5) || (argc == 6)) )
    then error_exit(OK_EXIT,
"usage: %s model_name #it_cnt/max_residual omega print_int [disk_write_int]\n",
        fixname(argv[0])); /* DOES NOT RETURN */

/* decode arguments: */
#define MODEL_NAME      argv[1]
if (argv[2][0] == '#')
    then {
        termination_test_type = ITERATIONS;
        sscanf(argv[2], "%d", &max_iterations);
        sprintf(iteration_limit, "%s", &argv[2][1]);
    }
    else {
        termination_test_type = RESIDUAL;

```

main directory

PSOR.C;38

21-SEP-1985 19:28

Page 2

```

        sscanf(argv[2], "%lf", &max_residual);
        sprintf(iteration_limit, "(max residual %s)", argv[2]);
    }
    sscanf(argv[3], "%lf", &omega);
    sscanf(argv[4], "%d", &print_interval);
    if (argc == 6)
        then sscanf(argv[5], "%d", &disk_write_interval);
        else disk_write_interval = 0;          /* only at end */

/* get input data: */
input_par(MODEL_NAME, &model_title, &grid_name, &bhsd);
input_output_grd(grid_name, "read", &gd);
set_extrapolation_coeff(&gd, &bhsd);          /* update for our black hole system */
input_output_gfn(MODEL_NAME, "read", &gd);

/* iterate..... */
if (print_interval != 0)
    then printf("%s\n", model_title);

time = 0.0;
iteration_count = 0;
while (TRUE)
{
    switch (termination_test_type)
    {
        case ITERATIONS:
            if (! (iteration_count < max_iterations) )
                then goto quit;
            break;
        case RESIDUAL:
            if (iteration_count != 0)
                then if (residual < max_residual) then goto quit;
            break;
    }

    old_residual = (iteration_count == 0) ? 1.0 : residual;

    time -= cpu_time();
    residual = do_point_sor(&gd, omega);
    time += cpu_time();
    ++iteration_count;

    if ( (print_interval != 0)
        && (iteration_count % print_interval == 0) )
        then printf("Iteration %d of %s: Max residual = %.21e (old*%.5f)\n",
                    iteration_count, iteration_limit,
                    residual, residual/old_residual);
    if ( (disk_write_interval != 0)
        && (iteration_count % disk_write_interval == 0) )
        then {
            input_output_gfn(MODEL_NAME, "write", &gd);
            if (print_interval != 0) then printf("<disk update done>\n");
        }
}

quit:

```

main directory

PSOR.C;38

21-SEP-1985 19:28

Page 3

```
if ((print_interval != 0) && (iteration_count != 0))
    then printf("%d iterations/%.2f cpu seconds --> %.2f cpu msec/iteration\n",
               iteration_count, time, 1.0e3 * time/iteration_count);

/* final disk update */
input_output_gfn(MODEL_NAME, "write", &gd);

/* log to '.par' file: */
par_log(MODEL_NAME, argc, argv, time);
}
```

main directory

SETEXTRAP.C;27

21-SEP-1985 19:28

Page 1

```

#include "util:stdio.h"

#include "patchutil:patch.h"
#include "patchutil:blackhole.h"

/*****
/*
/* This function sets the extrapolation, or 'true', boundary coefficients, for
/* all boundaries of all patches.
/*
/* Arguments:
/* pgd --> The grid_dscr structure describing the coordinate grid system.
/* pbhsd --> The bhs_dscr structure describing the black hole system. This is
/*          needed since 'marginally trapped surface' boundary conditions
/*          involve the extrinsic curvature.
*/
void set_extrapolation_coeff(pgd, pbhsd)
register struct grid_dscr *pgd;
register struct bhs_dscr *pbhsd;
{
void cvt_ij_to_xy();
double K_hat_sub_rr();

int p;
register struct coord_patch_dscr *ptr;
enum bndry bndry;
register struct line_end_dscr *pled;

int i, j;
int *pij_scan, *pij_end, start, count;

double x, y;
double b1, b2, b3;
double sign;

    for (p = 1 ; p <= pgd->N_patch ; ++p)
    {
        ptr = &pgd->patch[p];

        if (ptr->coord_type != POLAR_SPHERICAL)
            then continue; /*
/* Only polar spherical patches have 'true'
/* boundary conditions of the types that need
/* coefficients (note NEUMANN doesn't).
*/

/*
/* get to here --> we must check each boundary point of the patch to
/* see what, if anything, needs doing:
*/
        for (bndry = I_MIN ; bndry <= J_MAX ; ++bndry)
        {
            /* first, decide just which variable to scan: */
            switch (bndry)

```

main directory

SETEXTRAP.C;27

21-SEP-1985 19:28

Page 2

```

{
case I_MIN:
case I_MAX:
    pij_scan = &j;
    start = ptr->j_min;
    count = ptr->j_max - start + 1;
    pij_end = &i;
    break;
case J_MIN:
case J_MAX:
    pij_scan = &i;
    start = ptr->i_min;
    count = ptr->i_max - start + 1;
    pij_end = &j;
    break;
}

/* now scan the points of this boundary: */
for (*pij_scan = start ; count-- > 0 ; ++*pij_scan)
{
    pled = &ptr->line_end[(int)bndry][*pij_scan];

    switch (pled->flag)
    {
    case ROBIN:
    case REFLECTION_SYMMETRIC:
    case marginally_trapped:
        break; /* work to do!!! */
    default:
        continue; /* with next boundary point */
    }

    /*
     * get to here --> extrapolation coefficients needed!!!
     */
    *pij_end = pled->end_ij;
    cvt_ij_to_xy(ptr, i, j, &x, &y);

#define R      x
#define THETA  y
#define DELTA_R ptr->i_step

    switch (pled->flag)
    {
    case ROBIN:
        b1 = 1.0 / R;
        b2 = - 1.0 / R;
        b3 = 0.0;
        break;
    case REFLECTION_SYMMETRIC:
        b1 = 1.0 / (2.0 * R);
        b2 = 0.0;
        b3 = 0.0;
        break;
    case marginally_trapped:

```


main directory

SETEXTRAP.C;27

21-SEP-1985 19:28

Page 3

```

b1 = 1.0 / (2.0 * R);
b2 = 0.0;
b3 = 0.25 * K_hat_sub_rr(pgd, pbhsd,
                          ptr,
                          R, THETA);

break;
}

/* set actual extrapolation coefficients: */
switch (bndry)
{
case I_MIN:
case J_MIN:
    sign = 1.0;
    break;
case I_MAX:
case J_MAX:
    sign = - 1.0;
    break;
}

#define ROOT      pled->boundary_info.extrapolate
ROOT.d1 = b1 * 2.0 * DELTA_R * sign;
ROOT.d2 = b2 * 2.0 * DELTA_R * sign;
ROOT.d3 = b3 * 2.0 * DELTA_R * sign;

/* end of this boundary point */
}

/* end of this boundary */
}

/* end of this patch */
}

```

main directory

SOLVE.C;96

21-SEP-1985 19:30

Page 1

```

/* Direct solution stuff which is independent of Ax-b solution method. */
/*
 * The following global variables and functions live in this file:
 * psi_mapping - Records mapping from (p,i,j) to equation number
 * neqn - Number of equations
 * mode - FLOAT or DOUBLE -- gives precision to which next step will be taken
 * (f_rhs,d_rhs) --> Right-hand-side vector
 * (f_solution,d_solution) --> Solution vector
 *
 * first_time - Helper function: have we been called before with same args???
 * setup_direct_solve - Sets up rhs, solution, Jacobian sparsity structure
 * setup_initial_ordering - Sets up initial ordering of grid points
 * setup_rhs_solution - Sets up the rhs/solution vectors for Newton iteration
 * update_psi - Takes Newton step.
 */

#define DEBUG          1
#define DEBUG_TRACE    0

#include "util:stdio.h"
#include "patchutil:patch.h"
#include "solve.h"

#define TROUBLE error_exit(OK_EXIT,
"\n***** Insufficient 'calloc(...)' memory -- exiting *****\n" \
) /* DOES NOT RETURN */

/*****/

/*
 * Note that 'pij_mapping' is only used via the 'index(...)' macro, defined in
 * the 'solve.h' header file!!!
 */
int **pij_mapping; /* records (p,i,j) --> equation # mapping */
int neqn; /* number of equations */
enum precision mode;
float *f_rhs, *f_solution;
double *d_rhs, *d_solution;

/*****/

/*
 * This function returns TRUE if either (a) this is the first call on it, or
 * (b) the previous call had any arguments different. Otherwise, the function
 * returns FALSE.
 *
 * Arguments:
 * (p,i,j) - Arguments to be checked.
 */
boolean first_time(p,i,j)
int p,i,j;
{
    static boolean first_flag = TRUE;
    static int old_p, old_i, old_j;

```

main directory

SOLVE.C;96

21-SEP-1985 19:30

Page 2

```

boolean return_value;

return_value = (first_flag || !((p == old_p) && (i == old_i) && (j == old_j)));

first_flag = FALSE;
old_p = p;
old_i = i;
old_j = j;

return(return_value);
}

/*****

/*
 * This function sets up the rhs and solution vectors, and the sparsity
 * structure for the Jacobian.
 *
 * Arguments:
 *   pgd -- The grid descriptor for the system.
 */
void setup_direct_solve(pgd)
struct grid_dscr *pgd;
{
    char *calloc();

    void compute_Jacobian();
    void setup_initial_ordering();
    void record_sparsity_structure();          /* called via 'compute_Jacobian' */
    void allocate_Jacobian_storage();

    setup_initial_ordering(pgd);

    #if      DEBUG
    printf("setup_direct_solve: neqn=%d\n", neqn);
    #endif

    /* allocate rhs[], solution[] storage: */
    f_rhs = ALLOC(neqn, float);                f_solution = ALLOC(neqn, float);
    d_rhs = ALLOC(neqn, double);                d_solution = ALLOC(neqn, double);
    if ( (f_rhs == NULL) || (f_solution == NULL)
        || (d_rhs == NULL) || (d_solution == NULL))
        then TROUBLE;

    compute_Jacobian(pgd, 0, &record_sparsity_structure);

    #if      DEBUG
    printf("setup_direct_solve: all sparsity structure recorded\n");
    #endif

    allocate_Jacobian_storage();
}

*****/

```

main directory

SOLVE.C;96

21-SEP-1985 19:30

Page 3

```

/*
 * This function sets up the initial ordering (i.e. (p,i,j) <--> pij mapping)
 * for the system. Actually, only the '-->' mapping is stored, since we never
 * need to map from pij back to (p,i,j). This mapping is stored in 'calloc()'
 * storage, pointed to by the static variable 'pij_mapping'.
 *
 * Arguments:
 * pgd --> The grid descriptor for the system.
 */
void setup_initial_ordering(pgd)
register struct grid_dscr *pgd;
{
    char *calloc();
    void error_exit();

    int posn;
    int p, i;
    register struct coord_patch_dscr *ptr;

    pij_mapping = ALLOC(pgd->N_patch, int *);
    if (pij_mapping == NULL) then TROUBLE;
    --pij_mapping; /* allow for origin-1 array indexing with patch # */

    posn = 0;
    for (p = 1 ; p <= pgd->N_patch ; ++p)
    {
        ptr = &pgd->patch[p];

#define SIZE (ptr->i_max - ptr->i_min + 1)
        pij_mapping[p] = ALLOC(SIZE, int);
        if (pij_mapping[p] == NULL) then TROUBLE;
        pij_mapping[p] -- ptr->i_min; /* allow for non-zero origin */

        for (i = ptr->i_min ; i <= ptr->i_max ; ++i)
        {
            pij_mapping[p][i] = posn - ptr->line_end[J_MIN][i].end_ij;

#ifdef DEBUG_TRACE >= 2
            printf("pij_mapping[p=%d][i=%d] = %d (posn=%d)\n",
                p, i, pij_mapping[p][i], posn);
#endif

            posn += ptr->line_end[J_MAX][i].end_ij
                  - ptr->line_end[J_MIN][i].end_ij
                  + 1;
        }

        /* end of this patch */
    }

    neqn = posn;
}

/*****

```

main directory

SOLVE.C:96

21-SEP-1985 19:30

Page 4

```

/*
 * This function sets up the rhs vector for a new Newton iteration. It assumes
 * that the Jacobian has been multiplied throughout by -1 to make it positive
 * definite, so the rhs vector can just be the residual itself. It also sets
 * up the solution vector to be the current value of psi, as an initial guess.
 *
 * Arguments: (explicit)
 * pgd --> Grid descriptor for system.
 *
 * Arguments: (implicit, as global variables)
 * pij_mapping = (p,i,j) --> pij mapping vector. Used via macro 'index(...)'.
 * mode = FLOAT or DOUBLE to specify the precision to use.
 * (f_rhs[],d_rhs[]) = RHS vector. (MODIFIED)
 * (f_solution[],d_solution[]) = Solution vector. (MODIFIED)
 */
void setup_rhs_solution(pgd)
register struct grid_dscr *pgd;
{
double compute_residual(); /* computes them in grid function array */

int p, i, j;
register struct coord_patch_dscr *ptr;
struct grid_fn *gfp;

compute_residual(pgd);
for (p = 1 ; p <= pgd->N_patch ; ++p)
{
ptr = &pgd->patch[p];

for (i = ptr->i_min ; i <= ptr->i_max ; ++i)
{
for (j = ptr->line_end[J_MIN][i].end_ij ;
j <= ptr->line_end[J_MAX][i].end_ij ;
++j)
{
gfp = GRID_FNS(ptr, i, j);
if (mode == FLOAT)
then {
f_rhs[index(p,i,j)] = gfp->R;
f_solution[index(p,i,j)] = gfp->psi;
}
else {
d_rhs[index(p,i,j)] = gfp->R;
d_solution[index(p,i,j)] = gfp->psi;
}
}
}

/* end of this patch */
}
}

/*****
/*

```

main directory

SOLVE.C;96

21-SEP-1985 19:30

Page 5

```

* This function uses the computed solution to the linear system (with negated
* Jacobian) to update psi in the grid function array by the Newton step size.
*
* Arguments (explicit):
* pgd --> The grid descriptor for the system.
* omega - Fraction of the computed step to take. Standard Newton iteration is
*         omega=1.0.
*
* Arguments (implicit, as global variables):
* mode - Either FLOAT or DOUBLE as appropriate.
* (f_solution[],d_solution[]) - The vector giving the Newton step amount.
* pij_mapping - (p,i,j) --> pij mapping vector. Used via macro 'index(...)'.
*/
void update_psi(pgd, omega)
register struct grid_dscr *pgd;
double omega;
{
register int p, i, j;
register struct coord_patch_dscr *ptr;
double calc_step;

    for (p = 1 ; p <= pgd->N_patch ; ++p)
    {
        ptr = &pgd->patch[p];

        for (i = ptr->i_min ; i <= ptr->i_max ; ++i)
        {
            for (j = ptr->line_end[J_MIN][i].end_ij ;
                 j <= ptr->line_end[J_MAX][i].end_ij ;
                 ++j)
            {
                calc_step = (mode == FLOAT)
                    ? f_solution[index(p,i,j)]
                    : d_solution[index(p,i,j)] ;
                GRID_FNS(&pgd->patch[p], i,j)->psi_
                    += omega * calc_step;
            }
        }
    }
}

```

main directory

SOLVECG.C;64

21-SEP-1985 19:31

Page 1

```

/* Direct solution stuff for Incomplete LU - Conjugate Gradient solver: */
/*
 * The following functions live in this file:
 * <global variables> - Storage for Jacobian.
 * record_sparsity_structure - Records where 'compute_Jacobian' has non-zeros
 * allocate_Jacobian_storage - Allocates storage for the Jacobian
 * direct_solve - Does a single Newton iteration on the system
** init_Jacobian_storage - Sets Jacobian to all zeros
** setup_Jacobian - Sets up sparsity structure and Jacobian numerical values
** scale_Jacobian_rhs - Rescales Jacobian/rhs to improve numerical conditioning
** dump_Jacobian_rhs_solution - Dumps out Jacobian/rhs/solution to a file
*/

#define DEBUG          1
#define DEBUG_TRACE    0

#include <stdio.h>
#include "util:stdio.h"
#include "patchutil:patch.h"
#include "solve.h"

/*****
/*
 * These variables hold the Jacobian, in the format defined by the 'SILUCG'
 * 'DILUCG' FORTRAN subroutines used to do the actual sparse matrix solution.
 *
 * Note that the 'storage' arrays are subscripted in origin 1 and the 'INDEX'
 * macro is used in preference to the 'index' one, since it too is set up for
 * origin 1 indexing. This is in deference to FORTRAN.
 */
static int non_zero_count = 0; /* name in 'SILUCG'/'DILUCG' documentation: */
static int *row_INDEX;        /* 'MAX' */
static int *col_INDEX;        /* 'IA' */
static float *f_JACOBIAN;     /* 'JA' */
static double *d_JACOBIAN;    /* 'A' for SILUCG */
static int *itemp;            /* 'A' for DILUCG */
static float *f_temp;         /* 'ITEMP' (SILUCG/DILUCG scratch storage) */
static double *d_temp;        /* 'RTEMP' (SILUCG scratch storage) */
                                /* 'RTEMP' (DILUCG scratch storage) */

/*****
/*
 * This function is called once for each non-zero element in the Jacobian. It
 * records information about the sparsity structure for later use in storage
 * allocation.
 *
 * Arguments (explicit):
 * (p1,i1,j1) - 'Source' grid point.
 * (p2,i2,j2) - 'Destination' grid point.
 * j12 - Partial derivative of residual at 'source' grid point w.r.t. psi at
 *       'destination' grid point. This argument is ignored.
 *
 * Arguments (implicit, as global variables):

```

main directory

SOLVECG.C;64

21-SEP-1985 19:31

Page 2

```

    * non_zero_count = (incremented as appropriate)
    */
void record_sparsity_structure(pl,i1,j1, p2,i2,j2, J12)
int pl,i1,j1, p2,i2,j2;
double J12;      /* IGNORED */
{
    boolean first_time();
    void error_exit();

    /*
     * To count the number of non-zeros in the Jacobian before storage is allocated
     * for it, we must buffer the indices of non-zero elements already seen in the
     * current row to avoid duplication if they are incremented again.
     */
    #define BUFSIZE 25
    static int buffer[BUFSIZE];
    static int bufptr;      /* index of highest valid entry in buffer */

    int i, INDEX2;

    if (first_time(pl,i1,j1))
        then {
            bufptr = -1;          /* initialize buffer */
        }

    INDEX2 = INDEX(p2,i2,j2);

    #if (DEBUG_TRACE >= 2)
    printf("record_sparsity_structure: %d(%d,%d) %d(%d,%d)-%d init bufptr=%d\n",
           pl,i1,j1, p2,i2,j2, INDEX2, bufptr);
    #endif

    /* search to see if #2 grid point is already in buffer for this row: */
    #if (DEBUG_TRACE >= 2)
    printf("searching");
    #endif
    for (i = 0 ; i <= bufptr ; ++i)
    {
        #if (DEBUG_TRACE >= 2)
        printf(" %d", buffer[i]);
        #endif
        if (buffer[i] == INDEX2)
            then {
                /* found it! */
                #if (DEBUG_TRACE >= 2)
                printf("\n");
                #endif
                return;          /* already present --> already recorded */
            }
    }

    #if (DEBUG_TRACE >= 2)
    printf(" (insert)\n");
    #endif

    /* not present --> record and buffer it: */

```


main directory

SOLVECG.C;64

21-SEP-1985 19:31

Page 3

```

if (bufptr >= BUFSIZE-1)
    then error_exit(OK_EXIT,"
**** record_sparsity_structure: row buffer overflow at src %d(%d,%d)=%d,\n
****                                dst %d(%d,%d)=%d -- exiting\n",
                                pl,il,jl, INDEX(pl,il,jl),
                                p2,i2,j2, INDEX2
                                ); /* DOES NOT RETURN */

buffer[++bufptr] = INDEX2;
++non_zero_count;
}

/*****

/*
* This routine allocates storage for the Jacobian, in the format required by
* the 'SILUCG'/'DILUCG' subroutines.
*
* Arguments: (implicit, as global variables:)
* neqn - Number of equations.
* non_zero_count - Number of non-zero entries in the Jacobian.
* (row_INDEX,col_INDEX,F_JACOBIAN,D_JACOBIAN,itemp,f_temp,d_temp)
*   - Set as appropriate.
*/
void allocate_Jacobian_storage()
{
    char *calloc();
    void error_exit();

    row_INDEX = ALLOC(neqn+1, int);
    col_INDEX = ALLOC(non_zero_count, int);
    itemp = ALLOC( 3*(neqn+non_zero_count)+2 , int );
    f_JACOBIAN = ALLOC(non_zero_count, float);
    f_temp = ALLOC( 4*neqn + non_zero_count , float );
    d_JACOBIAN = ALLOC(non_zero_count, double);
    d_temp = ALLOC( 4*neqn + non_zero_count , double );

    if ( (row_INDEX == NULL) || (col_INDEX == NULL) || (itemp == NULL)
        || (f_JACOBIAN == NULL) || (f_temp == NULL)
        || (d_JACOBIAN == NULL) || (d_temp == NULL)
        )
        then error_exit(OK_EXIT,
        "\n**** Insufficient 'calloc(...)' storage for Jacobian -- exiting ****\n"
        ); /* DOES NOT RETURN */

    /* use origin 1 indexing for FORTRAN-style arrays: */
    --row_INDEX;    --col_INDEX;    --f_JACOBIAN;    --d_JACOBIAN;

    #if    DEBUG
    printf("allocate_Jacobian_storage: ok with neqn=%d non_zero_count=%d\n",
        neqn, non_zero_count
        );
    #endif
}

```

main directory

SOLVECG.C;64

21-SEP-1985 19:31

Page 4

```

/*****
/*
* This function does a single 'direct solution' Newton iteration.
*
* Arguments: (explicit)
* pgd --> The grid descriptor for the system.
* eps = Permitted relative error in Newton step. This can be used to set a
*       convergence criteria if an iterative method is used in an inner
*       iteration to calculate the Newton step.
* omega = Fraction of calculated Newton step to take. Normal Newton iteration
*         is omega=1.0.
* dump_file_name = (Pointer to standard C character string) file name to dump
*                  Jacobian and rhs vector to, or NULL to skip this.
*
* The updated residual is returned as the function result.
*/
double direct_solve(pgd, eps, omega, dump_file_name)
register struct grid_dsor *pgd;
double eps, omega;
char *dump_file_name;
{
void compute_Jacobian();
void setup_rhs_solution();
unsigned SILUCG(), DILUCG(); /* FORTRAN codes to solve sparse A*x=b */
void update_psi();
double compute_residual();
static void init_Jacobian_storage();
static void setup_Jacobian(); /* called via 'compute_Jacobian(...)' */
static void scale_Jacobian_rhs();
static void dump_Jacobian_rhs_solution();

float f_eps = eps;
char *subname;
int iterations, max_iterations;
unsigned status;
boolean ILUCG_success;

if (eps < 0)
    then mode = DOUBLE; /* want high-accuracy */
    else { /* variable-accuracy --> possibly FLOAT? */
        float test;
        test = 1.0;
        test += eps/1000; /*
            * Allow 3 digits room between roundoff floor
            * and precision -- if it fits, FLOAT; if not,
            * DOUBLE.
            */
        mode = (test -- 1.0) ? DOUBLE : FLOAT;
    }
subname = (mode == FLOAT) ? "SILUCG" : "DILUCG";

setup_rhs_solution(pgd);
init_Jacobian_storage();
compute_Jacobian(pgd, 0, &setup_Jacobian);

```

main directory

SOLVECG.C:64

21-SEP-1985 19:31

Page 5

```

scale_Jacobian_rhs();

if (dump_file_name != NULL) then dump_Jacobian_rhs_solution(dump_file_name);

#if    DEBUG
printf("direct_solve: about to try '%s(...)' with eps=%g\n", subname, eps);
#endif

max_iterations = neqn; /* should converge in much less than this.... */
status = (mode == FLOAT)
    ? SILUCG(&neqn, &row_INDEX[1], &col_INDEX[1], &f_JACOBIAN[1],
            f_rhs, f_solution, itemp, f_temp,
            &f_eps, &max_iterations, &iterations)
    : DILUCG(&neqn, &row_INDEX[1], &col_INDEX[1], &d_JACOBIAN[1],
            d_rhs, d_solution, itemp, d_temp,
            &eps, &max_iterations, &iterations);
ILUCG_success = (status & 01) ? TRUE : FALSE; /*
                                           * FORTRAN only uses low bit to
                                           * store booleans.
                                           */

if (! ILUCG_success)
    then error_exit(OK_EXIT,
"direct_solve: '%s(...)' error return -- row %d in error -- exiting\n",
    subname, iterations
    ); /* DOES NOT RETURN */
if (iterations < 0)
    then printf(
"direct_solve: '%s(...)' convergence failure after %d iterations\n",
    subname, -iterations
    );

#if    DEBUG
printf("direct_solve: back from '%s(...)' after %d/%d iterations\n",
    subname, iterations, max_iterations);
#endif

update_psi(pgd, omega);
return(compute_residual(pgd));
}

/*****/

/*
 * This function sets the Jacobian to all zeros, and does other initialization
 * on the sparsity structure arrays.
 *
 * Arguments (implicit, as global variables):
 * mode - FLOAT or DOUBLE -- tells what precision to use.
 * non_zero_count - Gives size of following array arguments.
 * (f_JACOBIAN[], d_JACOBIAN[]) - Set as appropriate.
 * row_INDEX - Set as appropriate.
 */
static void init_Jacobian_storage()
{
int i;

```

main directory

SOLVECG.C;64

21-SEP-1985 19:31

Page 6

```

        for (i = 1 ; i <= non_zero_count ; ++i)
        {
            if (mode == FLOAT)
                then f_JACOBIAN[i] = 0.0;
            else d_JACOBIAN[i] = 0.0;
        }

row_INDEX[1] = 1;          /* where first row will start */
}

/*****
/*
* This function sets up the actual numeric values of the Jacobian and its
* associated pointer arrays.  It is called at least once for each non-zero
* Jacobian value, with the passed 'J12' values additive.
*
* Note that the negative of the passed values are actually stored, so that the
* diagonal will be all positive rather than all negative.
*
* Arguments (explicit):
* (p1,i1,j1) = 'Source' grid point.
* (p2,i2,j2) = 'Destination' grid point.
* J12 = Partial derivative of residual at 'source' grid point w.r.t. psi at
*       'destination' grid point.  This argument is ignored.
*
* Arguments (implicit, as global variables):
* (non_zero_count,neqn) = These give sizes to check for buffer overflow etc.
* mode = FLOAT or DOUBLE -- tells what precision to use.
* (row_INDEX,col_INDEX,f_JACOBIAN,d_JACOBIAN) = Set to appropriate values.
*/
static void setup_Jacobian(p1,i1,j1, p2,i2,j2, J12)
int p1,i1,j1, p2,i2,j2;
double J12;
{
    boolean first_time();
    void error_exit();

    static int EQN;
    int COL;
    int i;

    if (first_time(p1,i1,j1))
        then {
            EQN = INDEX(p1,i1,j1);
            row_INDEX[EQN+1] = row_INDEX[EQN];          /* INDEX of next free slot */
            #if (DEBUG_TRACE >= 2)
            printf("setup_Jacobian: starting new row at %d(%d,%d), INDEX=%d\n",
                p1,i1,j1, EQN);
            #endif
        }

    COL = INDEX(p2,i2,j2);

```

main directory

SOLVECG.C;64

21-SEP-1985 19:31

Page 7

```

#if (DEBUG_TRACE >= 3)
printf("... dst is %d(%d,%d), INDEX-%d: searching", p2,i2,j2, COL);
#endif

/* search to see if there already is an entry for this (p2,i2,j2) in row: */
for (i = row_INDEX[EQN] ; i < row_INDEX[EQN+1] ; ++i)
{
#if (DEBUG_TRACE >= 3)
printf(" %d", i);
#endif
    if (col_INDEX[i] == COL)
        then {
#if (DEBUG_TRACE >= 3)
printf("\n");
#endif
            goto store;
        }
}

#if (DEBUG_TRACE >= 3)
printf(" (not found)\n");
#endif

/* not found --> create new entry: */
i = row_INDEX[EQN+1] ++;
if (i > non_zero_count)
    then error_exit(OK_EXIT,
        "***** setup_Jacobian: array overflow at INDEX-%d/MAX-%d -- exiting *****\n",
            i, non_zero_count
        ); /* DOES NOT RETURN */

#if (DEBUG_TRACE >= 2)
printf("inserting into posn %d\n", i);
#endif

store:
col_INDEX[i] = COL;
if (mode == FLOAT)
    then f_JACOBIAN[i] += -1.0 * J12; /* store negated */
    else d_JACOBIAN[i] += -1.0 * J12; /* store negated */
}

/*****
/*
* This function does row scaling on the Jacobian and the rhs to improve the
* numerical conditioning of the system. In particular, each row is divided by
* its (original) diagonal element, so all the diagonal elements of the scaled
* Jacobian are +1.0. Note that this scaling does NOT effect the solution to
* the system (in the absence of rounding errors), since the rhs vector is also
* scaled as needed.
*
* Arguments: (implicit)
* mode - Tells which Jacobian/rhs pair to scale.
* neqn - Number of equations in system to scale.
* (row_INDEX,col_INDEX,f_JACOBIAN,d_JACOBIAN) - (modified) Give the Jacobian

```

main directory

SOLVECG.C;64

21-SEP-1985 19:31

Page 8

```

*
* (f_rhs,d_rhs) = (modified) Give the rhs vector, which must also be scaled by
* the same amounts the Jacobian is scaled by.
*/
static void scale_Jacobian_rhs()
{
  int EQN;
  int POSN;
  double diagonal;

  for (EQN = 1 ; EQN <= neqn ; ++EQN)
  {
    /* find diagonal: */
    for (POSN = row_INDEX[EQN] ; POSN < row_INDEX[EQN+1] ; ++POSN)
    {
      if (col_INDEX[POSN] == EQN) then goto found;
    }
    error_exit(OK_EXIT,
    "***** scale_Jacobian_rhs: no diagonal element found for EQN=%d\n",
    EQN); /* DOES NOT RETURN */

  found: diagonal = (mode == FLOAT) ? f_JACOBIAN[POSN] : d_JACOBIAN[POSN];

    /* rescale: */
    if (diagonal == 0.0)
      then error_exit(OK_EXIT,
    "***** scale_Jacobian_rhs: zero diagonal found for EQN=%d\n",
    EQN); /* DOES NOT RETURN */
    for (POSN = row_INDEX[EQN] ; POSN < row_INDEX[EQN+1] ; ++POSN)
    {
      if (mode == FLOAT)
        then f_JACOBIAN[POSN] /= diagonal;
      else d_JACOBIAN[POSN] /= diagonal;
    }
    if (mode == FLOAT)
      then f_rhs[EQN-1] /= diagonal;
      else d_rhs[EQN-1] /= diagonal;

    /* end of this equation */
  }
}

/*****/

/*
* This function dumps out the Jacobian/rhs/solution to a specified file. The
* format is a compromise between (a) not making the file too huge for big
* problems, (b) making it human-readable for ease of debugging, (c) making it
* an accurate dump so other programs can read the results back in & try their
* hands at matters, and (d) making it fixed-format for the sake of FORTRAN
* programs which are too stupid to read nice-format input.
*
* Arguments: (explicit)
* filename = (A pointer to) a standard C character string giving the filename
* to dump the Jacobian/rhs/solution out to.

```

main directory

SOLVECG.C;64

21-SEP-1985 19:31

Page 9

```

*
* Arguments: (implicit)
* mode - Tells which Jacobian/rhs/solution pair to dump.
* neqn - Number of equations in system to dump.
* non_zero_count - Number of non-zeros in Jacobian to dump.
* (row_INDEX,col_INDEX,f_JACOBIAN,d_JACOBIAN) - Gives the Jacobian to be
*                                                dumped.
* (f_rhs,d_rhs) - Gives the rhs vector to be dumped.
* (f_solution,d_solution) - Gives the solution vector (initial guess) to be
*                           dumped.
*/
static void dump_Jacobian_rhs_solution(filename)
char *filename;
{
FILE *fopen();
void fprintf(), fclose();
void error_exit();
void dump_int_array(), dump_float_array(), dump_double_array();

FILE *fp;
int I, EQN;
char cmode = (mode == FLOAT) ? 'f' : 'd';

fp = fopen(filename, "w");
if (fp == NULL)
    then error_exit(OK_EXIT,
        "***** dump_Jacobian: failed to open '%s' for writeing\n",
        filename); /* DOES NOT RETURN */

#define IDUMP(N,A) \
    dump_int_array(fp, (A),1,(N), 10, 1,"%6d:", "%6d", 0,"\n")
#define FDUMP(N,A) \
    dump_float_array(fp, (A),1,(N), 5, 1,"%6d:", "%25.17e", 0,"\n")
#define DDUMP(N,A) \
    dump_double_array(fp, (A),1,(N), 5, 1,"%6d:", "%25.17e", 0,"\n")

fprintf(fp, "neqn=%6d\n", neqn);
fprintf(fp, "non_zero_count=%6d\n", non_zero_count);
fprintf(fp, "mode=%s\n", (mode == FLOAT) ? "FLOAT" : "DOUBLE");

fprintf(fp, "\n");
fprintf(fp, "----- Dump of 'row_INDEX[1...%d]' follows ----- \n", neqn+1);
IDUMP(neqn+1, row_INDEX);

fprintf(fp, "\n");
fprintf(fp, "----- Dump of 'col_INDEX[1...%d]' follows ----- \n",
    non_zero_count);
IDUMP(non_zero_count, col_INDEX);

fprintf(fp, "\n");
fprintf(fp, "----- Dump of '%c_JACOBIAN[1...%d]' follows ----- \n",
    cmode,
    non_zero_count);
if (mode == FLOAT)
    then FDUMP(non_zero_count, f_JACOBIAN);

```

main directory

SOLVECG.C:64

21-SEP-1985 19:31

Page 10

```
    else DDUMP(non_zero_count, d_JACOBIAN);

    fprintf(fp, "\n");
    fprintf(fp, "----- Dump of '%c_RHS[1...%d]' follows ----- \n",
            cmode,
            neqn);
    if (mode == FLOAT)
        then FDUMP(neqn, f_rhs-1);
        else DDUMP(neqn, d_rhs-1);

    fprintf(fp, "\n");
    fprintf(fp, "----- Dump of '%c_SOLUTION[1...%d]' follows ----- \n",
            cmode,
            neqn);
    if (mode == FLOAT)
        then FDUMP(neqn, f_solution-1);
        else DDUMP(neqn, d_solution-1);

    fclose(fp);
}
```


fortran directory

MYDILUCG.FOR;13

21-SEP-1985 19:47

Page 1

LOGICAL FUNCTION DILUCG(N,IA,JA,A,B,X,ITEMP,RTEMP,EPS,ITER,IE)
 IMPLICIT REAL*8(A-H,O-Z)

```

C
C   INCOMPLETE LU DECOMPOSITION-CONJUGATE GRADIENT
C   -
C WHERE:
C   |N| IS THE NUMBER OF EQUATIONS. IF N < 0, ITEMP AND
C   RTEMP CONTAIN THE ILU FROM A PREVIOUS CALL AND
C   B AND X ARE THE NEW RHS AND INITIAL GUESS.
C   IA IS AN INTEGER ARRAY DIMENSIONED |N|+1. IA(1) IS THE
C   INDEX INTO ARRAYS JA AND A OF THE FIRST NON-ZERO
C   ELEMENT IN ROW I. LET MAX=IA(|N|+1)-IA(1).
C   JA IS AN INTEGER ARRAY DIMENSIONED MAX. JA(K) GIVES
C   THE COLUMN NUMBER OF A(K).
C   A IS A REAL*8 ARRAY DIMENSIONED MAX. IT CONTAINS THE
C   NONZERO ELEMENTS OF THE MATRIX STORED BY ROW.
C   B CONTAINS THE RHS VECTOR.
C   X IS A REAL*8 ARRAY DIMENSIONED |N|. ON ENTRY, IT CONTAINS
C   AN INITIAL ESTIMATE; ON EXIT, THE SOLUTION.
C   ITEMP IS AN INTEGER SCRATCH ARRAY DIMENSIONED 3*(|N|+MAX)+2.
C   RTEMP IS A REAL*8 SCRATCH ARRAY DIMENSIONED 4*|N|+MAX.
C   EPS IS THE CONVERGENCE CRITERIA. IT SPECIFIES THE RELATIVE
C   ERROR ALLOWED IN THE SOLUTION. TO BE PRECISE, CONVERGENCE
C   IS DEEMED TO HAVE OCCURED WHEN THE INFINITY-NORM OF THE
C   CHANGE IN THE SOLUTION IN ONE ITERATION IS .LT. EPS * THE
C   INFINITY-NORM OF THE CURRENT SOLUTION. HOWEVER, IF EPS
C   .LT. 0.000, IT IS INTERNALLY SCALED BY THE MACHINE PRECISION,
C   SO THAT, FOR EXAMPLE, EPS = -256.0 WILL ALLOW THE LAST TWO
C   HEXADECIMAL DIGITS OF THE SOLUTION TO BE IN ERROR.
C   ITER GIVES THE REQUESTED NUMBER OF ITERATIONS.
C   IE IS AN INTEGER VARIABLE. FOR NORMAL RETURN, IT GIVES
C   THE NUMBER OF ITERATIONS DONE (NEGATIVE IS NO CONVERGENCE).
C   ON ERROR RETURN, THE ROW IN ERROR.
C
C (MODIFIED TO RETURN LOGICAL VALUE, J. THORNBURG, 13/MAY/85.)
C (MODIFIED TO ADD CONVERGENCE CRITERIA, J. THORNBURG, 17/MAY/85.)
C THE FUNCTION RETURNS .TRUE. ON A NORMAL RETURN, .FALSE. ON AN ERROR RETURN.
C
C REFERENCE:
C   D.S.KERSHAW, "THE INCOMPLETE CHOLESKY-CONJUGATE GRADIENT
C   METHOD FOR INTERATIVE SOLUTION OF LINEAR EQUATIONS",
C   J.COMPUT.PHYS. JAN 1978 PP 43-65
C
C   DIMENSION IA(1),JA(1),A(1),B(1),X(1),ITEMP(1),RTEMP(1)
C   NP=IABS(N)
C   IE=0
C   IF (NP.EQ.0) GO TO 20
C CALCULATE INDICES FOR BREAKING UP TEMPORARY ARRAYS.
C   N1=NP+1
C   MAX=IA(N1)-IA(1)
C   ILU=1
C   JLU=ILU+N1
C   ID=JLU+MAX
C   IC=ID+NP
C   JC=IC+N1

```

fortran directory

MYDILUCG.FOR;13

21-SEP-1985 19:47

Page 2

```

      JCI=JC+MAX
      IR=1
      IP=IR+NP
      IS1=IP+NP
      IS2=IS1+NP
      IALU=IS2+NP
      IF (N.LT.0) GO TO 10
C DO INCOMPLETE LU DECOMPOSITION
      CALL DLU_(NP,IA,JA,A,ITEMP(IC),ITEMP(JC),ITEMP(JCI),RTEMP(IALU),
      *   ITEMP(ILU),ITEMP(JLU),ITEMP(ID),RTEMP(IR),IE,#20)
C AND DO CONJUGATE GRADIENT ITERATIONS
C CALL MODIFIED TO ADD ADJUSTABLE CONVERGENCE CRITERIA EPS
C - J. THORNBURG, 17/MAY/85.
10   CALL DNCG_(NP,IA,JA,A,B,X,ITEMP(ILU),ITEMP(JLU),ITEMP(ID),
      *   RTEMP(IALU),RTEMP(IR),RTEMP(IP),RTEMP(IS1),RTEMP(IS2),
      *   EPS,ITER,IE)
      DILUCG = .TRUE.
      RETURN
20   DILUCG = .FALSE.
      RETURN
      END
      SUBROUTINE DLU_(N,IA,JA,A,IC,JC,JCI,ALU,ILU,JLU,ID,V,IE,*)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION IA(1),JA(1),A(1),IC(1),JC(1),JCI(1),
      *   ALU(1),ILU(1),JLU(1),ID(N),V(N)
      LOGICAL NODIAG
      COMMON /ICBD___/ ICBAD
C   INCOMPLETE LU DECOMPOSITION
C WHERE:
C   N,IA,JA, AND A ARE DESCRIBED IN SUBROUTINE ILUCG
C   IC IS AN INTEGER ARRAY DIMENSIONED N+1, IC(J) GIVES THE
C   INDEX OF THE FIRST NONZERO ELEMENT IN COLUMN J IN
C   ARRAY JC.
C   JC IS AN INTEGER ARRAY WITH THE SAME DIMENSION AS A.
C   JC(K) GIVES THE ROW NUMBER OF THE K'TH ELEMENT IN
C   THE COLUMN STRUCTURE.
C   JCI IS AN INTEGER ARRAY WITH THE SAME DIMENSION AS A.
C   JCI(K) GIVES THE INDEX INTO ARRAY A OF THE K'TH ELEMENT
C   OF THE COLUMN STRUCTURE.
C   ALU HAS THE SAME DIMENSION AS A. ON EXIT, IT WILL
C   CONTAIN THE INCOMPLETE LU DECOMPOSITION OF A WITH THE
C   RECIPROCAL OF THE DIAGONAL ELEMENTS OF U.
C   ILU AND JLU CORRESPONDS TO IA AND JA BUT FOR ALU.
C   ID IS AN INTEGER ARRAY DIMENSIONED N. IT CONTAINS
C   INDICES TO THE DIAGONAL ELEMENTS OF U.
C   V IS A REAL SCRATCH VECTOR OF LENGTH N.
C   IE GIVES THE ROW NUMBER IN ERROR.
C NOTE: DLU_ SETS ARGUMENTS IC THROUGH V.
C
      ICBAD=0
C ZERO COUNT OF ZERO DIAGONAL ELEMENTS IN U.
C
C FIRST CHECK STRUCTURE OF A AND BUILD COLUMN STRUCTURE
      DO 10 I=1,N
        IC(I)=0

```

fortran directory

MYDILUCG.FOR;13

21-SEP-1985 19:47

Page 3

```

10  CONTINUE
    DO 30 I=1,N
        KS=IA(I)
        KE=IA(I+1)-1
        NODIAG=.TRUE.
        DO 20 K=KS,KE
            J=JA(K)
            IF (J.LT.1.OR.J.GT.N) GO TO 210
            IC(J)=IC(J)+1
            IF (J.EQ.I) NODIAG=.FALSE.
20  CONTINUE
    IF (NODIAG) GO TO 210
30  CONTINUE
C MAKE IC INTO INDICES
    KOLD=IC(1)
    IC(1)=1
    DO 40 I=1,N
        KNEW=IC(I+1)
        IF (KOLD.EQ.0) GO TO 210
        IC(I+1)=IC(I)+KOLD
        KOLD=KNEW
40  CONTINUE
C SET JC AND JCI FOR COLUMN STRUCTURE
    DO 60 I=1,N
        KS=IA(I)
        KE=IA(I+1)-1
        DO 50 K=KS,KE
            J=JA(K)
            L=IC(J)
            IC(J)=L+1
            JC(L)=I
            JCI(L)=K
50  CONTINUE
60  CONTINUE
C FIX UP IC
    KOLD=IC(1)
    IC(1)=1
    DO 70 I=1,N
        KNEW=IC(I+1)
        IC(I+1)=KOLD
        KOLD=KNEW
70  CONTINUE
C FIND SORTED ROW STRUCTURE FROM SORTED COLUMN STRUCTURE
    NP=N+1
    DO 80 I=1,NP
        ILU(I)=IA(I)
80  CONTINUE
C MOVE ELEMENTS, SET JLU AND ID
    DO 100 J=1,N
        KS=IC(J)
        KE=IC(J+1)-1
        DO 90 K=KS,KE
            I=JC(K)
            L=ILU(I)
            ILU(I)=L+1

```

fortran directory

MYDILUCG.FOR;13

21-SEP-1985 19:47

Page 4

```

          JLU(L)=J
          KK-JCI(K)
          ALU(L)=A(KK)
          IF (I.EQ.J) ID(J)=L
90      CONTINUE
100     CONTINUE
C RESET ILU (COULD JUST USE IA)
      DO 110 I=1,NP
          ILU(I)=IA(I)
110     CONTINUE
C FINISHED WITH SORTED COLUMN AND ROW STRUCTURE
C
C DO LU DECOMPOSITION USING GAUSSIAN ELIMINATION
      DO 120 I=1,N
          V(I)=0.0D0
120     CONTINUE
      DO 200 IROW=1,N
          I-ID(IROW)
          PIVOT=ALU(I)
          IF (PIVOT.NE.0.0D0) GO TO 140
C THIS CASE MAKES THE ILU LESS ACCURATE
          ICBAD=ICBAD+1
          KS=ILU(IROW)
          KE=ILU(IROW+1)-1
          DO 130 K=KS,KE
              PIVOT=PIVOT+DABS(ALU(K))
130         CONTINUE
          IF (PIVOT.EQ.0.0D0) GO TO 220
140         PIVOT=1.0D0/PIVOT
          ALU(I)=PIVOT
          KKS=I+1
          KKE=ILU(IROW+1)-1
          IF (KKS.GT.KKE) GO TO 160
          DO 150 K=KKS,KKE
              J=JLU(K)
              V(J)=ALU(K)
150         CONTINUE
C FIX L IN COLUMN IROW AND DO PARTIAL LU IN SUBMATRIX
160         KS=IC(IROW)
          KE=IC(IROW+1)-1
          DO 190 K=KS,KE
              I=JC(K)
              IF (I.LE.IROW) GO TO 190
              LS=ILU(I)
              LE=ILU(I+1)-1
              DO 180 L=LS,LE
                  J=JLU(L)
                  IF (J.LT.IROW) GO TO 180
                  IF (J.GT.IROW) GO TO 170
                  AMULT=ALU(L)*PIVOT
                  ALU(L)=AMULT
                  IF (AMULT.EQ.0.0) GO TO 190
                  GO TO 180
170             IF (V(J).EQ.0.0D0) GO TO 180
                  ALU(L)=ALU(L)-AMULT*V(J)

```

fortran directory

MYDILUCG.FOR:13

21-SEP-1985 19:47

Page 5

```

180      CONTINUE
190      CONTINUE
C RESET V
      IF (KKS.GT.KKE) GO TO 200
      DO 195 K=KKS,KKE
        J=JLU(K)
        V(J)=0.0DO
195      CONTINUE
200      CONTINUE
      RETURN
210      IE=I
      RETURN 1
220      IE=IROW
      RETURN 1
      END
      SUBROUTINE DNCG_(N,IA,JA,A,B,X,ILU,JLU,ID,ALU,R,P,S1,S2,
* EPS,ITER,IE)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION IA(1),JA(1),A(1),B(N),X(N),ILU(1),JLU(1),ALU(1),ID(N),
* R(N),P(N),S1(N),S2(N)
C      NONSYMMETRIC CONJUGATE GRADIENT
C WHERE:
C      N,IA,JA,A,B, AND X ARE DESCRIBED IN SUBROUTINE DILUCG.
C      ILU GIVES INDEX OF FIRST NONZERO ELEMENT IN ROW OF LU.
C      JLU GIVES COLUMN NUMBER.
C      ID GIVES INDEX OF DIAGONAL ELEMENT OF U.
C      ALU HAS NONZERO ELEMENTS OF LU MATRIX STORED BY ROW
C      WITH RECIPROCAL OF DIAGONAL ELEMENTS OF U.
C      R,P,S1, AND S2 ARE VECTORS OF LENGTH N USED IN THE
C      ITERATIONS.
C FOLLOWING PARAMETER ADDED BY J. THORNBURG, 17/MAY/85.
C      EPS IS CONVERGENCE CRITERIA. (DESCRIBED IN SUBROUTINE
C      DILUCG).
C      ITER IS MAX NUMBER OF ITERATIONS.
C      IE GIVES ACTUAL NUMBER OF ITERATIONS, NEGATIVE IF
C      NO CONVERGENCE.
C
C RO=B-A*XO
      CALL DMUL1_(N,IA,JA,A,X,R)
      DO 10 I=1,N
        R(I)=B(I)-R(I)
10      CONTINUE
C PO=(UT*U)(-1)*AT*(L*LT)(-1)*RO
C FIRST SOLVE L*LT*S1=RO
      CALL DSUBL_(N,ILU,JLU,ID,ALU,R,S1)
C TIMES TRANSPOSE OF A
      CALL DMUL2_(N,IA,JA,A,S1,S2)
C THEN SOLVE UT*U*P=S2
      CALL DSUBU_(N,ILU,JLU,ID,ALU,S2,P)
      IE=0
C INPROD IS DOT PRODUCT ROUTINE IN *LIBRARY (UBC MATRIX P 28)
C ALL CALLS ON IT COMMENTED OUT AND REPLACED WITH CALLS TO NEW
C ROUTINE DGVV(...); SOURCE CODE FOR LATTER ADDED TO END OF
C THIS FILE. - J. THORNBURG, 10/MAY/85.
C      CALL INPROD(R,S1,EDOT,RDOT,N)

```

fortran directory

MYDILUCG.FOR;13

21-SEP-1985 19:47

Page 6

```

      RDOT = DGVV(R,S1,N)
C LOOP BEGINS HERE
20  CALL DMUL3_(N,ILU,JLU,ID,ALU,P,S2)
C    CALL INPROD(P,S2,EDOT,PDOT,N)
      PDOT = DGVV(P,S2,N)
C
      IF (PDOT.EQ.0.0D0) RETURN
C
      ALPHA=RDOT/PDOT
C EQUATION 9PA                                ALPHA=(R,LINV*R)/(P,UT*U*P)
      XMAX=1.0D0
      XDIF=0.0D0
      DO 30 I=1,N
        AP=ALPHA*P(I)
        X(I)=X(I)+AP
C EQUATION 9PB                                X=X+ALPHA*P
        AP=DABS(AP)
        XX=DABS(X(I))
        IF (AP.GT.XDIF) XDIF=AP
        IF (XX.GT.XMAX) XMAX=XX
30  CONTINUE
      IE=IE+1
C
C CONVERGENCE TEST (CHANGED BY J. THORNBURG, 17/MAY/85)
C
      IF ((EPS .GT. 0.0D0) .AND. (XDIF .LT. EPS * XMAX)) RETURN
      IF ((EPS .LT. 0.0D0) .AND. (XMAX + XDIF/DABS(EPS) .EQ. XMAX))
        * RETURN
C
      IF (IE.GE.ITER) GO TO 60
      CALL DMUL1_(N,IA,JA,A,P,S2)
      DO 40 I=1,N
        R(I)=R(I)-ALPHA*S2(I)
C EQUATION 9PC                                R=R-ALPHA*A*P
40  CONTINUE
      CALL DSUBL_(N,ILU,JLU,ID,ALU,R,S1)
C    CALL INPROD(R,S1,EDOT,RRDOT,N)
      RRDOT = DGVV(R,S1,N)
      BETA=RRDOT/RDOT
C EQUATION 9PD                                BETA=(R+,LINV*R+)/(R,LINV*R)
      RDOT=RRDOT
      CALL DMUL2_(N,IA,JA,A,S1,S2)
      CALL DSUBU_(N,ILU,JLU,ID,ALU,S2,S1)
      DO 50 I=1,N
        P(I)=S1(I)+BETA*P(I)
C EQUATION 9PE                                P=(UT*U)(-1)*AT*(L*LT)(-1)*R+BETA*P
50  CONTINUE
      GO TO 20
60  IE=--IE
      RETURN
      END
      SUBROUTINE DMUL1_(N,IA,JA,A,B,X)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION IA(1),JA(1),A(1),B(N),X(N)
C MULTIPLY A TIMES B TO GET X

```

fortran directory

MYDILUCG.FOR;13

21-SEP-1985 19:47

Page 7

```

C WHERE:
C   N IS THE ORDER OF THE MATRIX
C   IA GIVES INDEX OF FIRST NONZERO ELEMENT IN ROW
C   JA GIVES COLUMN NUMBER
C   A CONTAINS THE NONZERO ELEMENTS OF THE NONSYMMETRIC
C   MATRIX STORED BY ROW
C   B IS THE VECTOR
C   X IS THE PRODUCT (MUST BE DIFFERENT FROM B)
C
      DO 20 I=1,N
        KS=IA(I)
        KE=IA(I+1)-1
        SUM=0.0D0
        DO 10 K=KS,KE
          J=JA(K)
          SUM=SUM+A(K)*B(J)
10      CONTINUE
        X(I)=SUM
20      CONTINUE
      RETURN
      END
      SUBROUTINE DMUL2_(N,IA,JA,A,B,X)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION IA(1),JA(1),A(1),B(N),X(N)
C MULTIPLY TRANSPOSE OF A TIMES B TO GET X
C WHERE:
C   N IS THE ORDER OF THE MATRIX
C   IA GIVES INDEX OF FIRST NONZERO ELEMENT IN ROW
C   JA GIVES COLUMN NUMBER
C   A CONTAINS THE NONZERO ELEMENTS OF THE NONSYMMETRIC
C   MATRIX STORED BY ROW
C   B IS THE VECTOR
C   X IS THE PRODUCT (MUST BE DIFFERENT FROM B)
C
      DO 10 I=1,N
        X(I)=0.0D0
10      CONTINUE
      DO 30 I=1,N
        KS=IA(I)
        KE=IA(I+1)-1
        BB=B(I)
        DO 20 K=KS,KE
          J=JA(K)
          X(J)=X(J)+A(K)*BB
20      CONTINUE
30      CONTINUE
      RETURN
      END
      SUBROUTINE DMUL3_(N,ILU,JLU,ID,ALU,B,X)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION ILU(1),JLU(1),ID(1),ALU(1),B(N),X(N)
C MULTIPLY TRANSPOSE OF U TIMES U TIMES B TO GET X
C WHERE:
C   N IS THE ORDER OF THE MATRIX
C   ILU GIVES INDEX OF FIRST NONZERO ELEMENT IN ROW OF LU

```

fortran directory

MYDILUCG.FOR;13

21-SEP-1985 19:47

Page 8

```

C      JLU GIVES COLUMN NUMBER
C      ID GIVES INDEX OF DIAGONAL ELEMENT OF U
C      ALU HAS NONZERO ELEMENTS OF LU MATRIX STORED BY ROW
C      WITH RECIPROCAL OF DIAGONAL ELEMENTS
C      B IS THE VECTOR
C      X IS THE PRODUCT  $U^T \cdot U \cdot B$  (X MUST BE DIFFERENT FROM B)
C
      DO 10 I=1,N
        X(I)=0.0D0
10     CONTINUE
      DO 50 I=1,N
        KS=ID(I)+1
        KE=ILU(I+1)-1
        DIAG=1.0D0/ALU(KS-1)
        XX=DIAG*B(I)
        IF (KS.GT.KE) GO TO 30
        DO 20 K=KS,KE
          J=JLU(K)
          XX=XX+ALU(K)*B(J)
20      CONTINUE
30     X(I)=X(I)+DIAG*XX
        IF (KS.GT.KE) GO TO 50
        DO 40 K=KS,KE
          J=JLU(K)
          X(J)=X(J)+ALU(K)*XX
40      CONTINUE
50     CONTINUE
      RETURN
      END
      SUBROUTINE DSUBU_(N,ILU,JLU,ID,ALU,B,X)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION ILU(1),JLU(1),ID(1),ALU(1),B(N),X(N)
C DO FORWARD AND BACK SUBSTITUTION TO SOLVE  $U^T \cdot U \cdot X = B$ 
C WHERE:
C      N IS THE ORDER OF THE MATRIX
C      ILU GIVES INDEX OF FIRST NONZERO ELEMENT IN ROW OF LU
C      JLU GIVES COLUMN NUMBER
C      ID GIVES INDEX OF DIAGONAL ELEMENT OF U
C      ALU HAS NONZERO ELEMENTS OF LU MATRIX STORED BY ROW
C      WITH RECIPROCAL OF DIAGONAL ELEMENTS OF U
C      B IS THE RHS VECTOR
C      X IS THE SOLUTION VECTOR
C
      NP=N+1
      DO 10 I=1,N
        X(I)=B(I)
10     CONTINUE
C FORWARD SUBSTITUTION
      DO 30 I=1,N
        KS=ID(I)+1
        KE=ILU(I+1)-1
        XX=X(I)*ALU(KS-1)
        X(I)=XX
        IF (KS.GT.KE) GO TO 30
        DO 20 K=KS,KE

```


fortran directory

MYDILUCG.FOR;13

21-SEP-1985 19:47

Page 9

```

      J=JLU(K)
      X(J)=X(J)-ALU(K)*XX
20    CONTINUE
30    CONTINUE
C BACK SUBSTITUTION
      DO 60 II=1,N
        I=NP-II
        KS=ID(I)+1
        KE=ILU(I+1)-1
        SUM=0.0DO
        IF (KS.GT.KE) GO TO 50
        DO 40 K=KS,KE
          J=JLU(K)
          SUM=SUM+ALU(K)*X(J)
40      CONTINUE
50      X(I)=(X(I)-SUM)*ALU(KS-1)
60    CONTINUE
      RETURN
      END
      SUBROUTINE DSUBL_(N,ILU,JLU,ID,ALU,B,X)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION ILU(1),JLU(1),ID(1),ALU(1),B(N),X(N)
C DO FORWARD AND BACK SUBSTITUTION TO SOLVE L*LT*X=B
C WHERE:
C   N IS THE ORDER OF THE MATRIX
C   ILU GIVES INDEX OF FIRST NONZERO ELEMENT IN ROW LU
C   JLU GIVES THE COLUMN NUMBER
C   ID GIVES INDEX OF DIAGONAL ELEMENT OF U
C   ALU HAS NONZERO ELEMENTS OF LU MATRIX STORED BY ROW
C   DIAGONAL ELEMENTS OF L ARE 1.0 AND NOT STORED
C   B IS THE RHS VECTOR
C   X IS THE SOLUTION VECTOR
C
      NP=N+1
      DO 10 I=1,N
        X(I)=B(I)
10    CONTINUE
C FORWARD SUBSTITUTION
      DO 30 I=1,N
        KS=ILU(I)
        KE=ID(I)-1
        IF (KS.GT.KE) GO TO 30
        SUM=0.0DO
        DO 20 K=KS,KE
          J=JLU(K)
          SUM=SUM+ALU(K)*X(J)
20      CONTINUE
        X(I)=X(I)-SUM
30    CONTINUE
C BACK SUBSTITUTION
      DO 50 II=1,N
        I=NP-II
        KS=ILU(I)
        KE=ID(I)-1
        IF (KS.GT.KE) GO TO 50

```

fortran directory

MYDILUCG.FOR;13

21-SEP-1985 19:47

Page 10

```
      XX=X(I)
      IF (XX.EQ.0.0) GO TO 50
      DO 40 K=KS,KE
        J=JLU(K)
        X(J)=X(J)-ALU(K)*XX
40     CONTINUE
50     CONTINUE
      RETURN
      END
      DOUBLE PRECISION FUNCTION DGVV(V,W,N)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION V(N),W(N)
C     SUBROUTINE TO COMPUTE DOUBLE PRECISION VECTOR DOT PRODUCT.
C
      SUM = 0.0D0
      DO 10 I = 1,N
        SUM = SUM + V(I)*W(I)
10     CONTINUE
      DGVV = SUM
      RETURN
      END
```

fortran directory

MYSILUCG.FOR;14

21-SEP-1985 19:48

Page 1

LOGICAL FUNCTION SILUCG(N,IA,JA,A,B,X,ITEMP,RTEMP,EPS,ITER,IE)
 IMPLICIT REAL*4(A-H,O-Z)

```

C
C   INCOMPLETE LU DECOMPOSITION-CONJUGATE GRADIENT
C   --
C WHERE:
C   N IS THE NUMBER OF EQUATIONS. IF N < 0, ITEMP AND
C   RTEMP CONTAIN THE ILU FROM A PREVIOUS CALL AND
C   B AND X ARE THE NEW RHS AND INITIAL GUESS.
C   IA IS AN INTEGER ARRAY DIMENSIONED N+1. IA(I) IS THE
C   INDEX INTO ARRAYS JA AND A OF THE FIRST NON-ZERO
C   ELEMENT IN ROW I. LET MAX=IA(N+1)-IA(1).
C   JA IS AN INTEGER ARRAY DIMENSIONED MAX. JA(K) GIVES
C   THE COLUMN NUMBER OF A(K).
C   A IS A REAL*4 ARRAY DIMENSIONED MAX. IT CONTAINS THE
C   NONZERO ELEMENTS OF THE MATRIX STORED BY ROW.
C   B CONTAINS THE RHS VECTOR.
C   X IS A REAL*4 ARRAY DIMENSIONED N. ON ENTRY, IT CONTAINS
C   AN INITIAL ESTIMATE; ON EXIT, THE SOLUTION.
C   ITEMP IS AN INTEGER SCRATCH ARRAY DIMENSIONED 3*(N+MAX)+2.
C   RTEMP IS AN REAL*4 SCRATCH ARRAY DIMENSIONED 4*(N+MAX).
C   EPS IS THE CONVERGENCE CRITERIA. IT SPECIFIES THE RELATIVE
C   ERROR ALLOWED IN THE SOLUTION. TO BE PRECISE, CONVERGENCE
C   IS DEEMED TO HAVE OCCURED WHEN THE INFINITY-NORM OF THE
C   CHANGE IN THE SOLUTION IN ONE ITERATION IS .LT. EPS * THE
C   INFINITY-NORM OF THE CURRENT SOLUTION. HOWEVER, IF EPS
C   .LT. 0.0, IT IS INTERNALLY SCALED BY THE MACHINE PRECISION,
C   SO THAT, FOR EXAMPLE, EPS = -256.0 WILL ALLOW THE LAST TWO
C   HEXADECIMAL DIGITS OF THE SOLUTION TO BE IN ERROR.
C   ITER GIVES THE REQUESTED NUMBER OF ITERATIONS.
C   IE IS AN INTEGER VARIABLE. FOR NORMAL RETURN, IT GIVES
C   THE NUMBER OF ITERATIONS DONE (NEGATIVE IS NO CONVERGENCE).
C   ON ERROR RETURN, THE ROW IN ERROR.
C
C (MODIFIED TO RETURN LOGICAL VALUE, J. THORNBURG, 13/MAY/85.)
C (MODIFIED TO ADD CONVERGENCE CRITERIA, J. THORNBURG, 17/MAY/85.)
C THE FUNCTION RETURNS .TRUE. ON A NORMAL RETURN, .FALSE. ON AN ERROR RETURN.
C
C REFERENCE:
C   D.S.KERSHAW, "THE INCOMPLETE CHOLESKY-CONJUGATE GRADIENT
C   METHOD FOR INTERATIVE SOLUTION OF LINEAR EQUATIONS",
C   J.COMPUT.PHYS. JAN 1978 PP 43-65
C
C   DIMENSION IA(1),JA(1),A(1),B(1),X(1),ITEMP(1),RTEMP(1)
C   NP=IABS(N)
C   IE=0
C   IF (NP.EQ.0) GO TO 20
C CALCULATE INDICES FOR BREAKING UP TEMPORARY ARRAYS.
C   N1=NP+1
C   MAX=IA(N1)-IA(1)
C   ILU=1
C   JLU=ILU+N1
C   ID=JLU+MAX
C   IC=ID+NP
C   JC=IC+N1

```

fortran directory

MYSILUCG.FOR;14

21-SEP-1986 19:48

Page 2

```

      JCI=JC+MAX
      IR=1
      IP=IR+NP
      IS1=IP+NP
      IS2=IS1+NP
      IALU=IS2+NP
      IF (N.LT.0) GO TO 10
C DO INCOMPLETE LU DECOMPOSITION
      CALL SLU_(NP,IA,JA,A,ITEMP(IC),ITEMP(JC),ITEMP(JCI),RTEMP(IALU),
      * ITEMP(ILU),ITEMP(JLU),ITEMP(ID),RTEMP(IR),IE,&20)
C AND DO CONJUGATE GRADIENT ITERATIONS
C CALL MODIFIED TO ADD ADJUSTABLE CONVERGENCE CRITERIA EPS
C - J. THORNBURG, 17/MAY/85.
10   CALL SNCG_(NP,IA,JA,A,B,X,ITEMP(ILU),ITEMP(JLU),ITEMP(ID),
      * RTEMP(IALU),RTEMP(IR),RTEMP(IP),RTEMP(IS1),RTEMP(IS2),
      * EPS,ITER,IE)
      SILUCG = .TRUE.
      RETURN
20   SILUCG = .FALSE.
      RETURN
      END
      SUBROUTINE SLU_(N,IA,JA,A,IC,JC,JCI,ALU,ILU,JLU,ID,V,IE,*)
      IMPLICIT REAL*4(A-H,O-Z)
      DIMENSION IA(1),JA(1),A(1),IC(1),JC(1),JCI(1),
      * ALU(1),ILU(1),JLU(1),ID(N),V(N)
      LOGICAL NODIAG
      COMMON /ICBD_/ ICBAD
C INCOMPLETE LU DECOMPOSITION
C WHERE:
C   N,IA,JA, AND A ARE DESCRIBED IN FUNCTION SILUCG
C   IC IS AN INTEGER ARRAY DIMENSIONED N+1. IC(J) GIVES THE
C   INDEX OF THE FIRST NONZERO ELEMENT IN COLUMN J IN
C   ARRAY JC.
C   JC IS AN INTEGER ARRAY WITH THE SAME DIMENSION AS A.
C   JC(K) GIVES THE ROW NUMBER OF THE K'TH ELEMENT IN
C   THE COLUMN STRUCTURE.
C   JCI IS AN INTEGER ARRAY WITH THE SAME DIMENSION AS A.
C   JCI(K) GIVES THE INDEX INTO ARRAY A OF THE K'TH ELEMENT
C   OF THE COLUMN STRUCTURE.
C   ALU HAS THE SAME DIMENSION AS A. ON EXIT, IT WILL
C   CONTAIN THE INCOMPLETE LU DECOMPOSITION OF A WITH THE
C   RECIPROCAL OF THE DIAGONAL ELEMENTS OF U.
C   ILU AND JLU CORRESPONDS TO IA AND JA BUT FOR ALU.
C   ID IS AN INTEGER ARRAY DIMENSIONED N. IT CONTAINS
C   INDICES TO THE DIAGONAL ELEMENTS OF U.
C   V IS A REAL SCRATCH VECTOR OF LENGTH N.
C   IE GIVES THE ROW NUMBER IN ERROR.
C NOTE: SLU_ SETS ARGUMENTS IC THROUGH V.
C
      ICBAD=0
C ZERO COUNT OF ZERO DIAGONAL ELEMENTS IN U.
C
C FIRST CHECK STRUCTURE OF A AND BUILD COLUMN STRUCTURE
      DO 10 I=1,N
        IC(I)=0

```

fortran directory

MYSILUCG.FOR;14

21-SEP-1985 19:48

Page 3

```

10    CONTINUE
      DO 30 I=1,N
        KS=IA(I)
        KE=IA(I+1)-1
        NODIAG=.TRUE.
        DO 20 K=KS,KE
          J=JA(K)
          IF (J.LT.1.OR.J.GT.N) GO TO 210
          IC(J)=IC(J)+1
          IF (J.EQ.I) NODIAG=.FALSE.
20    CONTINUE
      IF (NODIAG) GO TO 210
30    CONTINUE
C MAKE IC INTO INDICES
      KOLD=IC(1)
      IC(1)=1
      DO 40 I=1,N
        KNEW=IC(I+1)
        IF (KOLD.EQ.0) GO TO 210
        IC(I+1)=IC(I)+KOLD
        KOLD=KNEW
40    CONTINUE
C SET JC AND JCI FOR COLUMN STRUCTURE
      DO 60 I=1,N
        KS=IA(I)
        KE=IA(I+1)-1
        DO 50 K=KS,KE
          J=JA(K)
          L=IC(J)
          IC(J)=L+1
          JC(L)=I
          JCI(L)=K
50    CONTINUE
60    CONTINUE
C FIX UP IC
      KOLD=IC(1)
      IC(1)=1
      DO 70 I=1,N
        KNEW=IC(I+1)
        IC(I+1)=KOLD
        KOLD=KNEW
70    CONTINUE
C FIND SORTED ROW STRUCTURE FROM SORTED COLUMN STRUCTURE
      NP=N+1
      DO 80 I=1,NP
        ILU(I)=IA(I)
80    CONTINUE
C MOVE ELEMENTS, SET JLU AND ID
      DO 100 J=1,N
        KS=IC(J)
        KE=IC(J+1)-1
        DO 90 K=KS,KE
          I=JC(K)
          L=ILU(I)
          ILU(I)=L+1

```

fortran directory

MYSILUCG.FOR;14

21-SEP-1985 19:48

Page 5

```

180      CONTINUE
190      CONTINUE
C RESET V
      IF (KKS.GT.KKE) GO TO 200
      DO 195 K=KKS,KKE
        J=JLU(K)
        V(J)=0.0
195      CONTINUE
200      CONTINUE
      RETURN
210      IE=I
      RETURN 1
220      IE=IROW
      RETURN 1
      END
      SUBROUTINE SNCG_(N,IA,JA,A,B,X,ILU,JLU,ID,ALU,R,P,S1,S2,
* EPS,ITER,IE)
      IMPLICIT REAL*4(A-H,O-Z)
      DIMENSION IA(1),JA(1),A(1),B(N),X(N),ILU(1),JLU(1),ALU(1),ID(N),
* R(N),P(N),S1(N),S2(N)
C NONSYMMETRIC CONJUGATE GRADIENT
C WHERE:
C N,IA,JA,A,B, AND X ARE DESCRIBED IN SUBROUTINE SILUCG.
C ILU GIVES INDEX OF FIRST NONZERO ELEMENT IN ROW OF LU.
C JLU GIVES COLUMN NUMBER.
C ID GIVES INDEX OF DIAGONAL ELEMENT OF U.
C ALU HAS NONZERO ELEMENTS OF LU MATRIX STORED BY ROW
C WITH RECIPROCAL OF DIAGONAL ELEMENTS OF U.
C R,P,S1, AND S2 ARE VECTORS OF LENGTH N USED IN THE
C ITERATIONS.
C FOLLOWING PARAMETER ADDED BY J. THORNBURG, 17/MAY/85.
C EPS IS CONVERGENCE CRITERIA. (DESCRIBED IN SUBROUTINE
C SILUCG).
C ITER IS MAX NUMBER OF ITERATIONS.
C IE GIVES ACTUAL NUMBER OF ITERATIONS, NEGATIVE IF
C NO CONVERGENCE.
C
C RO=B-A*X0
      CALL SMUL1_(N,IA,JA,A,X,R)
      DO 10 I=1,N
        R(I)=B(I)-R(I)
10      CONTINUE
C PO=(UT*U)(-1)*AT*(L*LT)(-1)*RO
C FIRST SOLVE L*LT*S1=RO
      CALL SSUBL_(N,ILU,JLU,ID,ALU,R,S1)
C TIMES TRANSPOSE OF A
      CALL SMUL2_(N,IA,JA,A,S1,S2)
C THEN SOLVE UT*U*P=S2
      CALL SSUBU_(N,ILU,JLU,ID,ALU,S2,P)
      IE=0
      RDOT = SGVV(R,S1,N)
C LOOP BEGINS HERE
20      CALL SMUL3_(N,ILU,JLU,ID,ALU,P,S2)
      PDOT = SGVV(P,S2,N)
C

```

fortran directory

MYSILUCG.FOR;14

21-SEP-1985 19:48

Page 4

```

          JLU(L)=J
          KK-JCI(K)
          ALU(L)=A(KK)
          IF (I.EQ.J) ID(J)=L
90      CONTINUE
100     CONTINUE
C RESET ILU (COULD JUST USE IA)
      DO 110 I=1,NP
          ILU(I)=IA(I)
110     CONTINUE
C FINISHED WITH SORTED COLUMN AND ROW STRUCTURE
C
C DO LU DECOMPOSITION USING GAUSSIAN ELIMINATION
      DO 120 I=1,N
          V(I)=0.0
120     CONTINUE
      DO 200 IROW=1,N
          I=ID(IROW)
          PIVOT=ALU(I)
          IF (PIVOT.NE.0.0) GO TO 140
C THIS CASE MAKES THE ILU LESS ACCURATE
          ICBAD=ICBAD+1
          KS=ILU(IROW)
          KE=ILU(IROW+1)-1
          DO 130 K=KS,KE
              PIVOT=PIVOT+ABS(ALU(K))
130         CONTINUE
          IF (PIVOT.EQ.0.0) GO TO 220
140         PIVOT=1.0/PIVOT
          ALU(I)=PIVOT
          KKS=I+1
          KKE=ILU(IROW+1)-1
          IF (KKS.GT.KKE) GO TO 160
          DO 150 K=KKS,KKE
              J=JLU(K)
              V(J)=ALU(K)
150         CONTINUE
C FIX L IN COLUMN IROW AND DO PARTIAL LU IN SUBMATRIX
160         KS=IC(IROW)
          KE=IC(IROW+1)-1
          DO 190 K=KS,KE
              I=JC(K)
              IF (I.LE.IROW) GO TO 190
              LS=ILU(I)
              LE=ILU(I+1)-1
              DO 180 L=LS,LE
                  J=JLU(L)
                  IF (J.LT.IROW) GO TO 180
                  IF (J.GT.IROW) GO TO 170
                  AMULT=ALU(L)*PIVOT
                  ALU(L)=AMULT
                  IF (AMULT.EQ.0.0) GO TO 190
                  GO TO 180
170                 IF (V(J).EQ.0.0) GO TO 180
                  ALU(L)=ALU(L)-AMULT*V(J)

```

fortran directory

MYSILUCG.FOR;14

21-SEP-1985 19:48

Page 6

```

      IF (PDOT.EQ.0.0) RETURN
C
      ALPHA=RDOT/PDOT
C EQUATION 9PA                                ALPHA=(R,LINV*R)/(P,UT*U*P)
      XMAX=1.0
      XDIF=0.0
      DO 30 I=1,N
        AP=ALPHA*P(I)
        X(I)=X(I)+AP
C EQUATION 9PB                                X=X+ALPHA*P
      AP=ABS(AP)
      XX=ABS(X(I))
      IF (AP.GT.XDIF) XDIF=AP
      IF (XX.GT.XMAX) XMAX=XX
30    CONTINUE
      IE=IE+1
C
C CONVERGENCE TEST (CHANGED BY J. THORNBURG, 17/MAY/85)
C
      IF ((EPS .GT. 0.0) .AND. (XDIF .LT. EPS * XMAX)) RETURN
      IF ((EPS .LT. 0.0) .AND. (XMAX + XDIF/ABS(EPS) .EQ. XMAX))
        * RETURN
C
      IF (IE.GE.ITER) GO TO 60
      CALL SMUL1_(N,IA,JA,A,P,S2)
      DO 40 I=1,N
        R(I)=R(I)-ALPHA*S2(I)
C EQUATION 9PC                                R=R-ALPHA*A*P
40    CONTINUE
      CALL SSUBL_(N,ILU,JLU,ID,ALU,R,S1)
C      CALL INPROD(R,S1,EDOT,RRDOT,N)
      RRDOT = SGVV(R,S1,N)
      BETA=RRDOT/RDOT
C EQUATION 9PD                                BETA=(R+,LINV*R+)/(R,LINV*R)
      RDOT=RRDOT
      CALL SMUL2_(N,IA,JA,A,S1,S2)
      CALL SSUBU_(N,ILU,JLU,ID,ALU,S2,S1)
      DO 50 I=1,N
        P(I)=S1(I)+BETA*P(I)
C EQUATION 9PE                                P=(UT*U)(-1)*AT*(L*LT)(-1)*R+BETA*P
50    CONTINUE
      GO TO 20
60    IE=IE-IE
      RETURN
      END
      SUBROUTINE SMUL1_(N,IA,JA,A,B,X)
      IMPLICIT REAL*4(A-H,O-Z)
      DIMENSION IA(1),JA(1),A(1),B(N),X(N)
C MULTIPLY A TIMES B TO GET X
C WHERE:
C   N IS THE ORDER OF THE MATRIX
C   IA GIVES INDEX OF FIRST NONZERO ELEMENT IN ROW
C   JA GIVES COLUMN NUMBER
C   A CONTAINS THE NONZERO ELEMENTS OF THE NONSYMMETRIC
C   MATRIX STORED BY ROW

```


fortran directory

MYSILUCG.FOR;14

21-SEP-1985 19:48

Page 7

```

C      B IS THE VECTOR
C      X IS THE PRODUCT (MUST BE DIFFERENT FROM B)
C
      DO 20 I=1,N
        KS=IA(I)
        KE=IA(I+1)-1
        SUM=0.0
        DO 10 K=KS,KE
          J=JA(K)
          SUM=SUM+A(K)*B(J)
10      CONTINUE
        X(I)=SUM
20    CONTINUE
      RETURN
      END
      SUBROUTINE SMUL2_(N,IA,JA,A,B,X)
      IMPLICIT REAL*4(A-H,O-Z)
      DIMENSION IA(1),JA(1),A(1),B(N),X(N)
C      MULTIPLY TRANSPOSE OF A TIMES B TO GET X
C      WHERE:
C      N IS THE ORDER OF THE MATRIX
C      IA GIVES INDEX OF FIRST NONZERO ELEMENT IN ROW
C      JA GIVES COLUMN NUMBER
C      A CONTAINS THE NONZERO ELEMENTS OF THE NONSYMMETRIC
C      MATRIX STORED BY ROW
C      B IS THE VECTOR
C      X IS THE PRODUCT (MUST BE DIFFERENT FROM B)
C
      DO 10 I=1,N
        X(I)=0.0
10    CONTINUE
      DO 30 I=1,N
        KS=IA(I)
        KE=IA(I+1)-1
        BB=B(I)
        DO 20 K=KS,KE
          J=JA(K)
          X(J)=X(J)+A(K)*BB
20      CONTINUE
30    CONTINUE
      RETURN
      END
      SUBROUTINE SMUL3_(N,ILU,JLU,ID,ALU,B,X)
      IMPLICIT REAL*4(A-H,O-Z)
      DIMENSION ILU(1),JLU(1),ID(1),ALU(1),B(N),X(N)
C      MULTIPLY TRANSPOSE OF U TIMES U TIMES B TO GET X
C      WHERE:
C      N IS THE ORDER OF THE MATRIX
C      ILU GIVES INDEX OF FIRST NONZERO ELEMENT IN ROW OF LU
C      JLU GIVES COLUMN NUMBER
C      ID GIVES INDEX OF DIAGONAL ELEMENT OF U
C      ALU HAS NONZERO ELEMENTS OF LU MATRIX STORED BY ROW
C      WITH RECIPROCAL OF DIAGONAL ELEMENTS
C      B IS THE VECTOR
C      X IS THE PRODUCT UT*U*B (X MUST BE DIFFERENT FROM B)

```

fortran directory

MYSILUCG.FOR;14

21-SEP-1985 19:48

Page 8

```

C
DO 10 I=1,N
  X(I)=0.0
10 CONTINUE
DO 50 I=1,N
  KS-ID(I)+1
  KE-ILU(I+1)-1
  DIAG=1.0/ALU(KS-1)
  XX-DIAG*B(I)
  IF (KS.GT.KE) GO TO 30
  DO 20 K=KS,KE
    J-JLU(K)
    XX-XX+ALU(K)*B(J)
20 CONTINUE
30 X(I)-X(I)+DIAG*XX
  IF (KS.GT.KE) GO TO 50
  DO 40 K=KS,KE
    J-JLU(K)
    X(J)-X(J)+ALU(K)*XX
40 CONTINUE
50 CONTINUE
RETURN
END
SUBROUTINE SSUBU_(N,ILU,JLU,ID,ALU,B,X)
  IMPLICIT REAL*4(A-H,O-Z)
  DIMENSION ILU(1),JLU(1),ID(1),ALU(1),B(N),X(N)
C DO FORWARD AND BACK SUBSTITUTION TO SOLVE UT*U*X=B
C WHERE:
C   N IS THE ORDER OF THE MATRIX
C   ILU GIVES INDEX OF FIRST NONZERO ELEMENT IN ROW OF LU
C   JLU GIVES COLUMN NUMBER
C   ID GIVES INDEX OF DIAGONAL ELEMENT OF U
C   ALU HAS NONZERO ELEMENTS OF LU MATRIX STORED BY ROW
C   WITH RECIPROCAL OF DIAGONAL ELEMENTS OF U
C   B IS THE RHS VECTOR
C   X IS THE SOLUTION VECTOR
C
  NP=N+1
  DO 10 I=1,N
    X(I)-B(I)
10 CONTINUE
C FORWARD SUBSTITUTION
DO 30 I=1,N
  KS-ID(I)+1
  KE-ILU(I+1)-1
  XX-X(I)*ALU(KS-1)
  X(I)-XX
  IF (KS.GT.KE) GO TO 30
  DO 20 K=KS,KE
    J-JLU(K)
    X(J)-X(J)-ALU(K)*XX
20 CONTINUE
30 CONTINUE
C BACK SUBSTITUTION
DO 60 II=1,N

```

fortran directory

MYSILUCG.FOR;14

21-SEP-1985 19:48

Page 9

```

      I-NP-II
      KS-ID(I)+1
      KE-ILU(I+1)-1
      SUM=0.0
      IF (KS.GT.KE) GO TO 50
      DO 40 K=KS,KE
        J-JLU(K)
        SUM=SUM+ALU(K)*X(J)
40      CONTINUE
50      X(I)=(X(I)-SUM)*ALU(KS-1)
60      CONTINUE
      RETURN
      END
      SUBROUTINE SSUBL(N,ILU,JLU,ID,ALU,B,X)
      IMPLICIT REAL*4(A-H,O-Z)
      DIMENSION ILU(1),JLU(1),ID(1),ALU(1),B(N),X(N)
C DO FORWARD AND BACK SUBSTITUTION TO SOLVE L*LT*X=B
C WHERE:
C   N IS THE ORDER OF THE MATRIX
C   ILU GIVES INDEX OF FIRST NONZERO ELEMENT IN ROW LU
C   JLU GIVES THE COLUMN NUMBER
C   ID GIVES INDEX OF DIAGONAL ELEMENT OF U
C   ALU HAS NONZERO ELEMENTS OF LU MATRIX STORED BY ROW
C   DIAGONAL ELEMENTS OF L ARE 1.0 AND NOT STORED
C   B IS THE RHS VECTOR
C   X IS THE SOLUTION VECTOR
C
      NP=N+1
      DO 10 I=1,N
        X(I)=B(I)
10      CONTINUE
C FORWARD SUBSTITUTION
      DO 30 I=1,N
        KS=ILU(I)
        KE=ID(I)-1
        IF (KS.GT.KE) GO TO 30
        SUM=0.0
        DO 20 K=KS,KE
          J=JLU(K)
          SUM=SUM+ALU(K)*X(J)
20      CONTINUE
        X(I)=X(I)-SUM
30      CONTINUE
C BACK SUBSTITUTION
      DO 50 II=1,N
        I=NP-II
        KS=ILU(I)
        KE=ID(I)-1
        IF (KS.GT.KE) GO TO 50
        XX=X(I)
        IF (XX.EQ.0.0) GO TO 50
        DO 40 K=KS,KE
          J=JLU(K)
          X(J)=X(J)-ALU(K)*XX
40      CONTINUE

```

fortran directory

MYSILUCG.FOR;14

21-SEP-1985 19:48

Page 10

```
50    CONTINUE
      RETURN
      END
      REAL FUNCTION SGVV(V,W,N)
      IMPLICIT REAL (A-H,O-Z)
      DIMENSION V(N),W(N)
C     SUBROUTINE TO COMPUTE REAL VECTOR DOT PRODUCT.
C
      SUM = 0.0
      DO 10 I = 1,N
        SUM = SUM + V(I)*W(I)
10    CONTINUE
      SGVV = SUM
      RETURN
      END
```

patchutil directory

BLACKHOLE.H;27

21-SEP-1985 18:55

Page 1

/* This file describes how black holes are represented: */

```

struct black_hole_dscr
{
    double rho, z;                /* position of black hole */
    /* Note rho must be zero to retain
    * axisymmetry (2 space dimensions).
    */
    double mass;                  /* obvious!!! */
    double lambda;                /*
    * monopole coefficient for approximate
    * solution -- normally one half times
    * some estimate of mass of this b.h.
    */
    int surrounding_patch_flag;    /*
    * > 0 --> patch # of closest
    * surrounding patch
    * = 0 --> illegal
    * < 0 --> negative of patch # of
    * mirror image symmetric
    * b.h.'s closest surrounding
    * patch
    */
    double momentum;              /* in positive z-direction */
    int reflection_term_sign;      /*
    * +1 --> use "+" sign for Kij 2nd term
    * 0 --> ignore Kij 2nd term
    * -1 --> use "-" sign for Kij 2nd term
    */
    double reflection_radius;      /*
    * radius 'a' for Kij 2nd term; should
    * normally be one half the mass
    */
    double spin;                  /* in positive z-direction */
};

/*
* Each black hole will, if within the volume covered by the patch system, be
* immediately surrounded by a 'coord_type="polar spherical"' patch, as given
* by the 'surrounding_patch_flag' item above. If the black hole is outside
* the patch system's volume, then presumably some symmetry (mirror image about
* theta = 90 degrees for now) is being exploited, and this item in the black
* hole descriptor will give (the negative of) the 'symmetric' black hole's
* immediately surrounding patch.
*/

struct bhs_dscr                  /* black hole system descriptor */
{
    int N_black_hole;              /* how many b.h.s are there? */
    struct black_hole_dscr *black_hole; /*
    * --> array of b.h. dscr's
    * (indexed in origin 1)
    */
};

```

patchutil directory

CVT.H;18

21-SEP-1985 18:50

Page 1

```
/* Integer codes for 'cvt_' conversion functions: */
#define CVT_ERROR -1
#define CVT_BNDRY_COND_TYPE_ERROR 0

#define OFFSET_C1 0
#define OFFSET_C2 1
#define OFFSET_C3 2
#define OFFSET_C4 3
#define OFFSET_C5 4
#define OFFSET_S 5
#define OFFSET_PSI 6
#define OFFSET_R 7

#define PSI_MINUS_MASS_MONOPOLES -10
#define PSI_MINUS_AREA_MONOPOLES -11
#define PSI_MINUS_TOT_E_PER_M_MONOPOLES -12
#define PSI_MINUS_TOT_E_PER_A_MONOPOLES -13
#define PSI_MINUS_SPHERICALLY_SYM_MEAN -14
#define PSI_MINUS_SPHERICALLY_SYM_MIN -15
#define PSI_MINUS_Z_AXIS -16
#define PSI_MINUS_EQUATOR -17

#define TRUE_RESIDUAL -25
#define K_HAT_RR -26
```

patchutil directory

INOUT.H;2

21-SEP-1985 18:50

Page 1

```
/*  
 * The following 'enum' is used in classifying points w.r.t. patch boundaries  
 * and/or patch boundary terms:  
 */  
enum relation { INSIDE, ON_BNDRY, OUTSIDE };
```

patchutil directory

PATCH.H;79

21-SEP-1985 18:48

Page 1

```

/* This file describes how patches, grids, and functions are represented: */

/*
 * The basic idea is that each patch is a 'Manhattan convex' subset of a
 * rectangle in the i-j plane, where 'i' and 'j' are the 2 integer indices of
 * it's coordinate system. This subset is defined by the minimum and maximum
 * values of i on each line of constant j, and of j on each line of constant i.
 * (The 'Manhattan convexity' condition gaurantees that these will suffice,
 * i.e. that any constant i or constant j line will intersect the patch in a
 * finite closed interval.)
 *
 * To transform from i-j coordinates back to 'physical' rho-z coordinates, the
 * following scheme is used:
 *
 *      x = i * i_step,          y = j * y_step
 *      if (polar spherical) then { r = x, theta = y, transform to rho, z }
 *      if (cylindrical) then { rho = x, z = y }
 *      rho += rho_center,      z += z_center
 */

/* 'flag' code values for boundary condition types: */
#define DIRICHLET -1
#define NEUMANN -2
#define ROBIN -3
#define REFLECTION_SYMMETRIC -4
#define MARGINALLY_TRAPPED -5

struct line_end_dsor /* describes the end of a coordinate line */
{
    int end_ij; /* i or j value for last point of line */
    int flag; /*
        * < 0 --> 'true' boundary (see above for codes)
        * = 0 --> invalid
        * > 0 --> patch number to interpolate into
        */
};

/*
 * For Dirichlet or Neumann boundary conditions, no coefficients of any kind
 * are needed.
 */
union {
/*
 * If we have Robin, Reflection Symmetric, or Marginally Trapped boundary
 * conditions here, we need these coefficients to extrapolate the 'end_ij'
 * value from 'inner' ones.
 */
    struct {
        double d1, d2, d3;
    } extrapolate;
/*
 * If we are interpolating into another coordinate patch (as given by 'flag'
 * above), we need this information to do it.
 */
    struct {
        int i_low, j_low;
        /*
         * Coordinates of the interpolation 'square'

```


patchutil directory

PATCH.H;79

21-SEP-1985 18:48

Page 2

```

        * are (i_low,j_low), (i_low,j_low+1),
        * (i_low+1,j_low), (i_low+1,j_low+1), in the
        * system of the 'other' patch.
        */
        double d00, d01, d10, d11;
        /*
        * The interpolated value is found by taking a
        * 'dot product' of these coefficients with the
        * function values at the 4 coordinates given
        * above.
        */
        } interpolate;
    } boundary_info;
};

/*
 * Grid functions are stored as arrays of 'grid_fn' structures, each of which
 * holds all the functions defined at that grid point. This is convenient to
 * use, and also improves reference locality for the program.
 */
struct grid_fn
{
    double c1_, c2_, c3_, c4_, c5_; /* del-squared coefficients */
    double S_; /* source function */
    double psi_; /* conformal factor itself */
    double R_; /* residual */
};

enum bndry { I_MIN, I_MAX, J_MIN, J_MAX };
enum coord_type { CYLINDRICAL, POLAR_SPHERICAL };
enum difference_scheme { INDEPENDENT_TERMS, LOCALLY_CONSERVATIVE };
enum conformal_fn_type { PSI, OMEGA_OF_PSI };

struct coord_patch_dscr
{
    enum coord_type coord_type;
    enum difference_scheme difference_scheme;
    enum conformal_fn_type conformal_fn_type;
    double rho_center, z_center; /* position for i=0, j=0 */
    double i_step, j_step;
    int i_min, i_max; /* referred to as 'size info' */
    int j_min, j_max; /* (ditto) */
    struct line_end_dscr * line_end[4];
    /*
    * This 4-element array is indexed by (an 'int' type-
    * case of) an 'enum bndry'; each element points to an
    * array of line_end_dscr's to describe that boundary.
    *
    * Note these arrays are indexed in origin j_min,
    * j_min, i_min, and i_min respectively, so the
    * pointers are offset as appropriate. The 1 line ends
    * are indexed by j and vice versa (!!!).
    *
    * Note also that the 'size info' may be 'downsized'
    * after these arrays are allocated --> don't try to

```

patchutil directory

PATCH.H;79

21-SEP-1985 18:48

Page 3

```

        * deallocate them via 'cfree(...)'.
        */
    int fn_i_step, fn_j_step;
    /*
     * These define the storage layout of the grid function
     * arrays in memory -- they are in units of 'struct
     * grid_fn's.
     */
    struct grid_fn *fn_00_ptr;      /* --> i=0, j=0 in storage array */
};

struct grid_dscr
{
    int N_patch;      /* 'real' patches are numbered #1 to #N_patch */
    struct coord_patch_dscr *patch;      /* --> array of patch dscrs */
};
#define GLOBAL_REF      0      /* patch # to use as global reference coord */
/*
 * By convention, the highest numbered patch is of coord_type="polar spherical"
 * and has the model's outer boundary as it's J_MAX boundary.
 */

/*
 * The actual accessing of grid functions is done as follows:
 * (Note '+ 3' below allows a 'border' space for boundary extrapolation!)
 * (Note 'ptr_' is a 'struct coord_patch_dscr *', typically of the form
 * '&pgd->patch[p]' etc.)
 */
#define GRID_FN_SIZE      ( sizeof(struct grid_fn) / sizeof(double) )
#define GRID_FN_I_SIZE(ptr_)      ( (ptr_)->i_max - (ptr_)->i_min + 3 )
#define GRID_FN_J_SIZE(ptr_)      ( (ptr_)->j_max - (ptr_)->j_min + 3 )
#define GRID_FNS(ptr_,i_,j_)      \
    ( (ptr_)->fn_00_ptr + (i_)*(ptr_)->fn_i_step + (j_)*(ptr_)->fn_j_step )

/*
 * On disk, the data is represented in the following files:
 * '.pat' file - Describes a coordinate patch system, in human readable form.
 * '.grd' file - Same thing, but in a 'binary image' form. This is independent
 *               of the black hole system and hence does NOT include the 'true'
 *               boundary condition coefficients.
 * '.par' file - Describes all parameters for a given black hole and coordinate
 *               system, in human readable form; coordinate system is described
 *               by naming a '.pat'/''.grd' pair.
 * '.gfn' file - Stores all grid functions, in 'binary image' form.
 *
 *      gridname.pat
 *      |
 *      V
 *      +-----+
 *      | mkgrd |
 *      +-----+
 *      |
 *      V

```


patchutil directory

TEMPBNDRY.H;7

21-SEP-1985 18:51

Page 1

/* This file describes the temporary patch boundary data structure */

```

struct bndry_defn_term
{
    struct bndry_defn_term *next; /*
                                   * The terms are in a one-way linked
                                   * list, since we have no knowledge in
                                   * advance of how many there will be.
                                   */

    /* first define how the boundary location is chosen: */
    enum { CONSTANT_BNDRY_TYPE, PATCH_BNDRY_TYPE } defn_type;
    union {
        double value; /* if CONSTANT type */
        struct { /* if PATCH type */
            int patch; /* the other patch number */
            enum bndry bndry; /* which bndry of it do we hit??? */
        } patch_info;
    } defn_value;

    /* now decide how we will get the boundary values: */
    enum { TRUE_BNDRY_COND, PATCH_BNDRY_COND } bndry_cond_type;
    union {
        char *bndry_cond; /* if TRUE_BNDRY type */
        int patch; /* if PATCH type */
    } bndry_cond_value;
};

struct patch_bndry
{
    double x_min_bound, x_max_bound, y_min_bound, y_max_bound;
    /* bounding box of patch */
    struct bndry_defn_term * bdt_list[4];
    /*
     * This (4-element) array is indexed by
     * (an 'int' type-cast of) an 'enum
     * bndry'; each element points to a
     * list of boundary definition terms
     * describing that boundary.
     */
};

/*
 * The overall temporary data structure will be an array (1 origin) of
 * patch_bndry structures, indexed by patch number.
 */

```

patchutil directory

ALLOCGFNA.C;7

21-SEP-1985 18:59

Page 1

```

#include "util:stdio.h"
#include "patchutil:patch.h"

/*
 * This function makes all policy decisions on the storage layout of grid
 * functions -- i.e. are they stored by column (FORTRAN) or by row (the rest of
 * the world), and sets the fn_i_step, fn_j_step, and fn_00_ptr fields of a
 * coord_patch_dscr appropriately. The latter is set up pointing to a suitable
 * offset from a suitable-size block of storage, so the access macros defined
 * in 'patch.h' can be used.
 *
 * Arguments:
 * ptr --> The coordinate patch descriptor to do the allocation for.
 */
void allocate_grid_fn_array(ptr)
register struct coord_patch_dscr *ptr;
{
    char *calloc();
    void error_exit();

    /* make storage layout policy -- currently stored by rows: */
    ptr->fn_i_step = GRID_FN_J_SIZE(ptr);
    ptr->fn_j_step = 1;

    /* allocate storage: */
    ptr->fn_00_ptr = ALLOC(GRID_FN_I_SIZE(ptr) * GRID_FN_J_SIZE(ptr),
                          struct grid_fn);
    if (ptr->fn_00_ptr == NULL)
        then error_exit(OK_EXIT,
            "allocate_grid_fn_array(...): insufficient memory -- exiting\n"
        ); /* DOES NOT RETURN */

    /* adjust for proper origin in indexing: */
    ptr->fn_00_ptr -= ptr->fn_i_step * (ptr->i_min - 1);
    ptr->fn_00_ptr -= ptr->fn_j_step * (ptr->j_min - 1);
}

```

patchutil directory

BINARYIO.C:53

21-SEP-1985 19:00

Page 1

```

/* This file contains routines to do i/o on 'machine readable' files. */
/*
 * Routines in this file are:
 * input_output_grd - Inputs or Outputs a '.grd' files from/to disk.
 * input_output_gfn - Inputs or Outputs a '.gfn' file from/to disk.
 */

#include <stdio.h>
#include "util:stdio.h"

#include "patchutil:patch.h"

/*****
/*
 * This function does input/output of '.grd' grid files from/to disk. It's the
 * only function which knows the grid data structure's disk representation.
 *
 * Arguments:
 * gridname - The name of the grid (i.e. the filename without the '.grd').
 * dir --> 'w' for writing, 'r' for reading. Note that 'dir' may actually be a
 *         'normal' C string, so long as it's 1st character is appropriate.
 * pgd --> The 'grid_dscr' structure. (For reading, this is filled in!)
 *
 * The function handles all errors 'internally', so if it returns, the caller
 * may presume that all is well.
 *
 * BUGS:      Error checking isn't very strict!!!
 */
void input_output_grd(gridname, dir, pgd)
char gridname[];
char *dir;
register struct grid_dscr *pgd;
{
void set_global_ref();
void error_exit();

char *calloc();
FILE *fopen();
void fread(), fwrite();          /* non-standard, VMS specific */
void fclose();

register FILE *fp;
#define BUFSIZE 100
char filename[BUFSIZE];

void (*frwp)();                  /* 'fread/write pointer' */
int p;
enum bndry bndry;
int start, count;

sprintf(filename, "%s.grd", gridname);
fp = fopen(filename, (*dir == 'w') ? "w" : "r");
if (fp == NULL)
    then error_exit(OK_EXIT,

```

patchutil directory

BINARYIO.C;53

21-SEP-1985 19:00

Page 2

```

        "input_output_grd: unable to open file '%s' -- exiting",
        filename); /* DOES NOT RETURN */

#define NOMEM
error_exit(OK_EXIT,
        "input_output_grd: insufficient memory for grid '%s' -- exiting",
        gridname) /* DOES NOT RETURN */
/* END OF MACRO DEFN */

frwp = (*dir == 'w') ? fwrite : fread;

/* do the grid_dscr structure itself: */
(*frwp)(% pgd->N_patch, sizeof(int), 1, fp);
if (*dir != 'w')
    then {
        /* read --> allocate coord_patch_dscr's and set up global ref. patch */
        pgd->patch = ALLOC(pgd->N_patch + 1, struct coord_patch_dscr);
        if (pgd->patch == NULL) then NOMEM;
        set_global_ref(% pgd->patch[GLOBAL_REF]);
    }

/* do the coord_patch_dscr's themselves: */
(*frwp)(% pgd->patch[1],
        sizeof(struct coord_patch_dscr),
        pgd->N_patch,
        fp);

/* finally, do the line_end_dscr arrays: */
for (p = 1 ; p <= pgd->N_patch ; ++p)
{
    for (bndry = I_MIN ; bndry <= J_MAX ; ++bndry)
    {
        /* process this boundary's line_end_dscr array: */
        switch (bndry)
        {
            case I_MIN:
            case I_MAX:
                start = pgd->patch[p].j_min;
                count = pgd->patch[p].j_max - start + 1;
                break;
            case J_MIN:
            case J_MAX:
                start = pgd->patch[p].i_min;
                count = pgd->patch[p].i_max - start + 1;
                break;
        }
    }

#define PTR    pgd->patch[p].line_end[(int)bndry]

    if (*dir != 'w')
        then {
            /* read --> allocate line_end_dscr array: */
            PTR = ALLOC(count, struct line_end_dscr);
            if (PTR == NULL) then NOMEM;
            PTR -- start; /* adjust index origin */
        }

```

patchutil directory

BINARYIO.C;53

21-SEP-1985 19:00

Page 3

```

    }

    (*frwp)(& PTR[start],
            sizeof(struct line_end_dscr),
            count,
            fp);
}

/* end of this patch */
}

fclose(fp);
}

/*****
/*
* This function does input/output of '.gfn' grid files from/to disk. It's the
* only function which knows the grid functions' disk representation.
*
* Arguments:
* grdname - The filename of the grid function data file (without '.gfn' on the
*           end).
* dir --> 'w' for writing, 'r' for reading. Note that 'dir' may actually be a
*         'normal' C string, so long as it's 1st character is appropriate.
* pgd --> The 'grid_dscr' structure describing the grid to be used in the data
*         transfer. For reading, it is assumed that all the non-grid-function
*         fields have been filled in, by (for example) 'input_pat' and
*         'allocate_grid_fn_arrays', or by the "read" option of
*         'input_output_grd'.
*
* The function handles all errors 'internally', so if it returns, the caller
* may presume that all is well.
*
* BUGS:      Error checking isn't very strict!!!
*/
void input_output_gfn(grdname, dir, pgd)
char grdname[];
char *dir;
register struct grid_dscr *pgd;
{
void allocate_grid_fn_array();
void error_exit();

char *calloc();
FILE *fopen();
void fread(), fwrite();      /* non-standard, VMS specific */
void fclose();

#define BUFSIZE 100
char filename[BUFSIZE];
void (*frwp)();              /* 'fread/write pointer' */
register FILE *fp;
register int p, i, j;
register struct coord_patch_dscr *ptr;

```


patchutil directory

BINARYIO.C;53

21-SEP-1985 19:00

Page 4

```

sprintf(filename, "%s.gfn", grdname);
fp = fopen(filename, (*dir == 'w') ? "w" : "r");
if (fp == NULL)
    then error_exit(OK_EXIT,
                    "input_output_gfn: unable to open file '%s' -- exiting",
                    filename); /* DOES NOT RETURN */

frwp = (*dir == 'w') ? fwrite : fread;

for (p = 1 ; p <= pgd->N_patch ; ++p)
{
    ptr = &pgd->patch[p];

    if (*dir != 'w') then allocate_grid_fn_array(ptr);

    /* process this patch's function array: */
    for (i = ptr->i_min ; i <= ptr->i_max ; ++i)
    {
        for (j = ptr->line_end[J_MIN][i].end_ij ;
             j <= ptr->line_end[J_MAX][i].end_ij ;
             ++j)
        {
            (*frwp)(GRID_FNS(ptr, i, j),
                    sizeof(struct grid_fn),
                    1,
                    fp);
        }
    }

    /* end of this patch */
}

fclose(fp);
}

```

patchutil directory

COORD.C;28

21-SEP-1985 19:00

Page 1

```

/* This file contains coordinate conversion routines */
/*
 * The following functions are available:
 * cvt_ij_to_xy - Converts (i,j) to (x,y) within a given patch
 * cvt_xy_to_ij - Converts (x,y) to (i,j) within a given patch
 * cvt_xy_to_xy - Converts (x,y) in one patch to (x,y) in another
 * cvt_rtheta_to_rhoz - Converts (r,theta) to (rho,z)
 * cvt_rhoz_to_rtheta - Converts (rho,z) to (r,theta)
 */
#include "util:stdio.h"
#include "patchutil:patch.h"

/*****

/*
 * This function converts integer 'i/j' to floating point 'x/y' coordinates, as
 * described in 'patch.h'. Note that the interpretation of the 'x/y' coord's
 * depends on the type of coordinate system the patch uses.
 *
 * Arguments:
 * ptr --> The coord_patch_dscr for the patch.
 * (i,j) - The integer coordinates to be converted.
 * (px,py) --> Where the (x,y) results are to be stored, or NULL to omit the
 *             storing.
 */
void cvt_ij_to_xy(ptr, i, j, px, py)
register struct coord_patch_dscr *ptr;
int i, j;
double *px, *py;
{
    if (px != NULL) then *px = ( (double) i ) * ptr->i_step;
    if (py != NULL) then *py = ( (double) j ) * ptr->j_step;
}

/*****

/*
 * This function converts floating point 'x/y' to integer 'i/j' coordinates, as
 * described in 'patch.h'. Note that the interpretation of the 'x/y' coord's
 * depends on the type of coordinate system the patch uses.
 *
 * Arguments:
 * ptr --> The coord_patch_dscr for the patch.
 * (x,y) - The floating point coordinates to be converted.
 * (pi,pj) --> Where the (i,j) results are to be stored, or NULL to omit the
 *             storing.
 */
void cvt_xy_to_ij(ptr, x, y, pi, pj)
register struct coord_patch_dscr *ptr;
double x, y;
int *pi, *pj;
{
    int round_double_to_int();

    if (pi != NULL) then *pi = round_double_to_int( x / ptr->i_step );

```

patchutil directory

COORD.C:26

21-SEP-1985 19:00

Page 2

```

if (pj != NULL) then *pj = round_double_to_int( y / ptr->j_step );
}

/*****
/*
* This function converts floating point 'x/y' coordinates in the system of one
* patch to floating point 'x/y' coordinates in the system of another patch, as
* described in 'patch.h'. Note that the interpretation of the 'x/y' coord's
* depends on the types of coordinate system the patches use.
*
* Arguments:
* ptr1 --> The coord_patch_dscr for the 'source' patch.
* (x1,y1) = The 'source' coordinates to be converted.
* ptr2 --> The coord_patch_dscr for the 'destination' patch.
* (px2,py2) --> Where the 'destination' coordinates are to be stored, or NULL
*               to omit the storing.
*/
void cvt_xy_to_xy(ptr1, x1, y1, ptr2, px2, py2)
register struct coord_patch_dscr *ptr1;
double x1, y1;
register struct coord_patch_dscr *ptr2;
double *px2, *py2;
{
void cvt_rtheta_to_rhoz();
void cvt_rhoz_to_rtheta();

double rho, z;

if ( (ptr1->coord_type == POLAR_SPHERICAL)
    && (ptr2->coord_type == POLAR_SPHERICAL)
    && (ptr1->rho_center == ptr2->rho_center)
    && (ptr1->z_center == ptr2->z_center)
    )
then {
/* special case: coincentric spherical coords: */
if (px2 != NULL) then *px2 = x1; /* 'r' is identical */
if (py2 != NULL) then *py2 = y1; /* so is 'theta'!!! */
}
else {
/* 'normal' case */
if (ptr1->coord_type == POLAR_SPHERICAL)
then cvt_rtheta_to_rhoz(x1, y1, &rho, &z);
else {
rho = x1;
z = y1;
}

rho += ptr1->rho_center;
z += ptr1->z_center;

rho -= ptr2->rho_center;
z -= ptr2->z_center;

if (ptr2->coord_type == POLAR_SPHERICAL)

```

patchutil directory

COORD.C;26

21-SEP-1985 19:00

Page 3

```

        then cvt_rhoz_to_rtheta(rho, z, px2, py2);
        else {
            if (px2 != NULL) then *px2 = rho;
            if (py2 != NULL) then *py2 = z;
        }

        /* end of 'normal' case */
    }

/*****

/*
 * This function converts polar spherical to cylindrical coordinates:
 *
 * Arguments:
 * (r,theta) - Polar spherical coordinates to be converted.
 * (prho,pz) --> Where cylindrical coordinate result is to be stored, or NULL
 *               to omit the storing.
 */
void cvt_rtheta_to_rhoz(r, theta, prho, pz)
double r, theta, *prho, *pz;
{
    double sin(), cos();

    if (prho != NULL) then *prho = r * sin(theta);
    if (pz != NULL) then *pz = r * cos(theta);
}

/*****

/*
 * This function converts cylindrical to polar spherical coordinates:
 *
 * Arguments:
 * (rho,z) - Cylindrical coordinates to be converted.
 * (pr,ptheta) --> Where polar spherical coordinate result is to be stored, or
 *                 NULL to omit the storing.
 *
 * Note that the origin is deemed to be at r=0, theta=0.
 */
void cvt_rhoz_to_rtheta(rho, z, pr, ptheta)
double rho, z, *pr, *ptheta;
{
    double hypot();          /* hypot(x,y) returns sqrt(x*x + y*y) */
    double atan2();          /* 4-quadrant arctangent (with backwards args) */

    double temp_r;

    temp_r = hypot(rho, z);

    if (pr != NULL) then *pr = temp_r;
    if (ptheta != NULL) then *ptheta = (temp_r == 0.0) ? 0.0 : atan2(z, rho);
}

```

patchutil directory

CVT.C:126

21-SEP-1985 19:00

Page 1

```

/* Conversions between numeric codes and character names */
/*
 * --- function name ---          --- convert from ---          --- to ---
 * cvt_bndry_to_char              enum bndry                  char *
 * cvt_coord_type_to_char        enum coord_type             char *
 * cvt_difference_scheme_to_char  enum difference_scheme      char *
 * cvt_conformal_fn_type_to_char  enum conformal_fn_type      char *
 *
 * cvt_bndry_to_code              char *                      int
 * cvt_coord_type_to_code         char *                      int
 * cvt_difference_scheme_to_code  char *                      int
 * cvt_conformal_fn_type_to_code  char *                      int
 *
 * cvt_bndry_cond_type_to_code    char *                      int
 * cvt_grid_fn_name_to_offset     char *                      int
 */

#include "util:stdio.h"

#include "patchutil:patch.h"
#include "patchutil:cvt.h"

/*****

static struct table_entry      { char *name; };
#define TABLE_ENTRY_SIZE      sizeof(struct table_entry)
static struct full_table_entry { char *name; int code; };
#define FULL_TABLE_ENTRY_SIZE  sizeof(struct full_table_entry)

static struct table_entry
    coord_type_table [] = { "cylindrical", "polar spherical" };

static struct table_entry
    difference_scheme_table [] = {
        "independent terms",
        "locally conservative"
    };

static struct table_entry
    conformal_fn_type_table [] = { "psi", "omega(psi)" };

*****/

/*
 * This function converts a boundary code to character form.
 *
 * Arguments:
 * bndry - The boundary code to convert.
 *
 * A pointer to the character form is returned as the function result. The
 * string pointed at is NOT to be altered!!!
 */
char *cvt_bndry_to_char(bndry)
enum bndry bndry;
{

```

patchutil directory

CVT.C;126

21-SEP-1985 19:00

Page 2

```

static struct table_entry
    bndry_table [] = { "I_MIN", "I_MAX", "J_MIN", "J_MAX" };

return(bndry_table[bndry].name);
}

/*****
/*
* This function converts a coordinate type code to character form.
*
* Arguments:
* coord_type - The coordinate type code to convert.
*
* A pointer to the character form is returned as the function result. The
* string pointed at is NOT to be altered!!!
*/
char *cvt_coord_type_to_char(coord_type)
enum coord_type coord_type;
{
return(coord_type_table[coord_type].name);
}

/*****
/*
* This function converts a difference scheme code to character form.
*
* Arguments:
* difference_scheme - The difference scheme code to convert.
*
* A pointer to the character form is returned as the function result. The
* string pointed at is NOT to be altered!!!
*/
char *cvt_difference_scheme_to_char(difference_scheme)
enum difference_scheme difference_scheme;
{
return(difference_scheme_table[difference_scheme].name);
}

/*****
/*
* This function converts a conformal function type code to character form.
*
* Arguments:
* conformal_fn_type - The conformal function type code to convert.
*
* A pointer to the character form is returned as the function result. The
* string pointed at is NOT to be altered!!!
*/
char *cvt_conformal_fn_type_to_char(conformal_fn_type)
enum conformal_fn_type conformal_fn_type;
{
return(conformal_fn_type_table[conformal_fn_type].name);
}

```

patchutil directory

CVT.C;126

21-SEP-1985 19:00

Page 3

```

}

/*****

/*
 * This function converts a character string representing a boundary code to
 * an integer which is either 'CVT_ERROR' (defined in 'cvt.h') or which can be
 * legally type cast into an 'enum bndry'.
 *
 * Arguments:
 * string - The string to convert.
 *
 * The integer described above is returned as the function result.
 *
 * Note that the mapping from string to integer code, is many to one, since the
 * same integer code has (in general) a different string form for each type of
 * coordinate system it occurs in.
 */
int cvt_bndry_to_code(string)
char string[];
{
    char *table_search();

    struct full_table_entry *p;

    static struct full_table_entry bndry_table [] =
    {
        { "r_min",      I_MIN },      { "rho_min",      I_MIN },
        { "r_max",      I_MAX },      { "rho_max",      I_MAX },
        { "theta_min",  J_MIN },      { "z_min",       J_MIN },
        { "theta_max",  J_MAX },      { "z_max",       J_MAX }
    };
    #define BNDRY_TABLE_SIZE          (sizeof(bndry_table) / FULL_TABLE_ENTRY_SIZE)

    p = (struct bndry_table_entry *) table_search(BNDRY_TABLE_SIZE,
                                                    FULL_TABLE_ENTRY_SIZE,
                                                    bndry_table,
                                                    string);

    return( (p == NULL) ? CVT_ERROR : p->code );
}

/*****

/*
 * This function converts a character string representing a coordinate system
 * type into an integer which is either 'CVT_ERROR' (defined in 'cvt.h') or
 * which can be legally type cast into an 'enum coord_type'.
 *
 * Arguments:
 * string - The string to convert.
 *
 * The integer described above is returned as the function result.
 */
int cvt_coord_type_to_code(string)
char string[];

```

patchutil directory

CVT.C;126

21-SEP-1985 19:00

Page 4

```

{
char *table_search();

#define TABLE_SIZE      (sizeof(coord_type_table) / TABLE_ENTRY_SIZE)
struct table_entry *p;

p = (struct table_entry *) table_search(TABLE_SIZE,
                                         TABLE_ENTRY_SIZE,
                                         coord_type_table,
                                         string);
return( (p == NULL) ? CVT_ERROR : (p - coord_type_table) );
}

/*****
/*
* This function converts a character string representing a difference scheme
* into an integer which is either 'CVT_ERROR' (defined in 'cvt.h') or which
* can be legally type cast into an 'enum difference_scheme'.
*
* Arguments:
* string - The string to convert.
*
* The integer described above is returned as the function result.
*/
int cvt_difference_scheme_to_code(string)
char string[];
{
char *table_search();

#define TABLE_SIZE      (sizeof(difference_scheme_table) / TABLE_ENTRY_SIZE)
struct table_entry *p;

p = (struct table_entry *) table_search(TABLE_SIZE,
                                         TABLE_ENTRY_SIZE,
                                         difference_scheme_table,
                                         string);
return( (p == NULL) ? CVT_ERROR : (p - difference_scheme_table) );
}

/*****
/*
* This function converts a character string representing a type of conformal
* function into an integer which is either 'CVT_ERROR' (defined in 'cvt.h') or
* which can be legally type cast into an 'enum conformal_fn_type'.
*
* Arguments:
* string - The string to convert.
*
* The integer described above is returned as the function result.
*/
int cvt_conformal_fn_type_to_code(string)
char string[];
{

```


patchutil directory

CVT.C;126

21-SEP-1985 19:00

Page 5

```

char *table_search();

#define TABLE_SIZE      (sizeof(conformal_fn_type_table) / TABLE_ENTRY_SIZE)
struct table_entry *p;

p = (struct table_entry *) table_search(TABLE_SIZE,
                                         TABLE_ENTRY_SIZE,
                                         conformal_fn_type_table,
                                         string);

return( (p == NULL) ? CVT_ERROR : (p - conformal_fn_type_table) );
}

/*****
/*
* This function converts a character string representing a boundary condition
* type into an integer which is either 'CVT_BNDRY_COND_TYPE_ERROR' (defined in
* 'cvt.h') or which is one of the codes defined in 'patch.h'.
*
* Arguments:
* string - The string to convert.
*
* The integer described above is returned as the function result.
*/
int cvt_bndry_cond_type_to_code(string)
char string[];
{
    char *table_search();

    struct full_table_entry *p;

    static struct full_table_entry bct_table [] =
    {
        { "Dirichlet",          DIRICHLET },
        { "Neumann",           NEUMANN },
        { "Robin",             ROBIN },
        { "Reflection Symmetric", REFLECTION_SYMMETRIC },
        { "Marginally Trapped", MARGINALLY_TRAPPED }
    };
    #define BCT_TABLE_SIZE (sizeof(bct_table) / FULL_TABLE_ENTRY_SIZE)

    p = (struct full_table_entry *) table_search(BCT_TABLE_SIZE,
                                                  FULL_TABLE_ENTRY_SIZE,
                                                  bct_table,
                                                  string);

    return( (p == NULL) ? CVT_BNDRY_COND_TYPE_ERROR : p->code );
}

/*****
/*
* This function converts a character string representing a grid function name
* into the offset in 'doubles' of that grid function in a 'struct grid_fn' (as
* defined in 'patch.h') or into one of several possible negative 'code' values
* (as defined in 'cvt.h', including CVT_ERROR for unknown grid function name).

```

patchutil directory

CVT.C;126

21-SEP-1985 19:00

Page 6

```

*
* Arguments:
* string - The string to convert.
*
* The integer described above is returned as the function result.
*/
int cvt_grid_fn_name_to_offset(string)
char string[];
{
char *table_search();

struct full_table_entry *p;

static struct full_table_entry grid_fn_name_table [] =
{
{ "c1",          OFFSET_C1 },
{ "c2",          OFFSET_C2 },
{ "c3",          OFFSET_C3 },
{ "c4",          OFFSET_C4 },
{ "c5",          OFFSET_C5 },
{ "s",           OFFSET_S },
{ "psi",         OFFSET_PSI },
{ "r",           OFFSET_R },
{ "psi-mm",      PSI_MINUS_MASS_MONOPOLES },
{ "psi-am",      PSI_MINUS_AREA_MONOPOLES },
{ "psi-tem",     PSI_MINUS_TOT_E_PER_M_MONOPOLES },
{ "psi-tea",     PSI_MINUS_TOT_E_PER_A_MONOPOLES },
{ "psi-ssmean",  PSI_MINUS_SPHERICALLY_SYM_MEAN },
{ "psi-ssmin",   PSI_MINUS_SPHERICALLY_SYM_MIN },
{ "psi-zaxis",   PSI_MINUS_Z_AXIS },
{ "psi-equator", PSI_MINUS_EQUATOR },
{ "r(true)",     TRUE_RESIDUAL },
{ "krr",         K_HAT_RR }
};
#define GFN_TABLE_SIZE (sizeof(grid_fn_name_table) / FULL_TABLE_ENTRY_SIZE)

p = (struct full_table_entry *) table_search(GFN_TABLE_SIZE,
                                             FULL_TABLE_ENTRY_SIZE,
                                             grid_fn_name_table,
                                             string);

return( (p == NULL) ? CVT_ERROR : p->code );
}

```

patchutil directory

DUMP.C;63

21-SEP-1985 19:02

Page 1

```

/* This file contains routines to dump out the various data structures */
/*
 * The following functions are contained in this file:
 * dump_bhs - Dumps 'bhs' data structure.
 * dump_grid - (ditto for 'grid')
 * dump_patch_bndry - (ditto for 'patch_bndry')
 */

```

```
#include "util:stdc.h"
```

```
#include "patchutil:blackhole.h"
```

```
#include "patchutil:patch.h"
```

```
#include "patchutil:tempbndry.h"
```

```

/*****

```

```

/*
 * This function dumps out the black-hole-system data structure.
 *

```

```

 * Arguments:
 * pbhsd --> The black-hole-system descriptor to be dumped.
 */

```

```
dump_bhs(pbhsd)
```

```
struct bhs_dscr *pbhsd;
```

```
{
```

```
int i;
```

```
printf("\n----- DUMP OF BLACK HOLE SYSTEM ----- \n");
```

```
if (pbhsd == NULL )
```

```
then {
```

```
    printf("----- PANIC!!! -- pbhsd=NULL ----- \n");
```

```
    return;
```

```
}
```

```
if (pbhsd->black_hole == NULL)
```

```
then {
```

```
    printf("----- PANIC!!! -- pbhsd->black_hole=NULL ----- \n");
```

```
    return;
```

```
}
```

```
for (i=1 ; i <= pbhsd->N_black_hole ; ++i)
```

```
{
```

```
    if (i != 1) then printf("\n");
```

```
    printf("Black Hole #d:\n", i);
```

```
    printf("rho=%lf z=%lf\n",
```

```
        pbhsd->black_hole[i].rho,
```

```
        pbhsd->black_hole[i].z);
```

```
    printf("mass=%lf lambda=%lf surrounding_patch_flag=%d\n",
```

```
        pbhsd->black_hole[i].mass,
```

```
        pbhsd->black_hole[i].lambda,
```

```
        pbhsd->black_hole[i].surrounding_patch_flag);
```

```
    printf("\n
```

```
momentum=%lf reflection_term_sign=%d reflection_radius=%lf\n",
```

```
        pbhsd->black_hole[i].momentum,
```

```
        pbhsd->black_hole[i].reflection_term_sign,
```

patchutil directory

DUMP.C:63

21-SEP-1985 19:02

Page 2

```

        pbhsd->black_hole[i].reflection_radius);
    printf("spin=%lf\n",
        pbhsd->black_hole[i].spin);
}

printf("----- END OF BLACK HOLE SYSTEM DUMP -----\n");
}

/*****

/*
 * This function dumps out the 'grid' data structure.
 *
 * Arguments:
 * pgd --> The grid data structure to be dumped.
 */
dump_grid(pgd)
struct grid_dscr *pgd;
{
    int p;
    enum bndry bndry;
    int ij;
    char *ij_string;
    int ij_min, ij_max;
    register struct coord_patch_dscr *ptr;
    register struct line_end_dscr *pled;

    int N_grid_points();
    char *cvt_coord_type_to_char();
    char *cvt_difference_scheme_to_char();
    char *cvt_conformal_fn_type_to_char();
    char *cvt_bndry_to_char();

    printf("\n----- DUMP OF GRID SYSTEM -----\n");

    if (pgd == NULL )
        then {
            printf("----- PANIC!!! -- pgd=NULL -----\n");
            return;
        }
    if (pgd->patch == NULL)
        then {
            printf("----- PANIC!!! -- pgd->patch=NULL -----\n");
            return;
        }

    printf("This Grid System has a total of %d patches, %d grid points\n",
        pgd->N_patch, N_grid_points(pgd, 0));

    for (p=1 ; p <= pgd->N_patch ; ++p)
    {
        printf("\n----- patch %d (has %d grid points) start:\n",
            p, N_grid_points(pgd, p));

        ptr = & pgd->patch[p];

```

patchutil directory

DUMP.C;63

21-SEP-1985 19:02

Page 3

```

printf("coord_type=%d (%s)\n",
      ptr->coord_type,
      cvt_coord_type_to_char(ptr->coord_type)
);
printf("difference_scheme=%d (%s)\n",
      ptr->difference_scheme,
      cvt_difference_scheme_to_char(ptr->difference_scheme)
);
printf("conformal_fn_type=%d (%s)\n",
      ptr->conformal_fn_type,
      cvt_conformal_fn_type_to_char(ptr->conformal_fn_type)
);
printf("rho_center=%lf  z_center=%lf\n",
      ptr->rho_center, ptr->z_center);
printf("i_step=%lf  j_step=%lf\n", ptr->i_step, ptr->j_step);
printf("i_min=%d  i_max=%d\n", ptr->i_min, ptr->i_max);
printf("j_min=%d  j_max=%d\n", ptr->j_min, ptr->j_max);
printf("fn_i_step=%d  fn_j_step=%d  fn_00_ptr=0x%08x\n",
      ptr->fn_i_step, ptr->fn_j_step,
      ptr->fn_00_ptr);

for(bndry = I_MIN ; bndry <= J_MAX ; ++bndry)
{
  if (ptr->line_end[bndry] == NULL)
  then {
    printf("%d line_end array not yet allocated\n",
          bndry);
    continue; /* with next boundary */
  }

  /* get to here --> line_end array exists --> print it: */
  switch (bndry)
  {
    case I_MIN:
    case I_MAX:
      ij_string = "j";
      ij_min = ptr->j_min;
      ij_max = ptr->j_max;
      break;
    case J_MIN:
    case J_MAX:
      ij_string = "i";
      ij_min = ptr->i_min;
      ij_max = ptr->i_max;
      break;
  }

  for (ij = ij_min ; ij <= ij_max ; ++ij)
  {
    pled = & ptr->line_end[bndry][ij];

    printf("line_end[%s][%s-%d] = { end_ij=%d flag=%d ",
          cvt_bndry_to_char(bndry),
          ij_string, ij,

```

patchutil directory

DUMP.C;63

21-SEP-1985 19:02

Page 4

```

        pled->end_ij,
        pled->flag
    );

    if (pled->flag > 0)
    then {
#define ROOT      pled->boundary_info.interpolate
#define HANDLE(x)  if      (x == 0.0) then printf(" 0.0 ");
                  else if (x == 1.0) then printf(" 1.0 ");
                  else      printf(" %.3lf", x);
                      HANDLE(ROOT.d00)
                      HANDLE(ROOT.d01)
                      HANDLE(ROOT.d10)
                      HANDLE(ROOT.d11)
                  }
        else switch (pled->flag)
        {
            case DIRICHLET:
            case NEUMANN:
                printf("==> (no coefficients needed)");
                break;
            case ROBIN:
            case REFLECTION_SYMMETRIC:
            case marginally_trapped:
                printf("==> %.3lg %.3lg %.3lg",
#define ROOT      pled->boundary_info.extrapolate
                                ROOT.d1, ROOT.d2, ROOT.d3);
                break;
            default:
                printf("==> *** (illegal value) ***");
                break;
        }

        printf(" }\n");

        /* end of this boundary */
    }

    printf("----- patch #d stop:\n", p);

printf("----- END OF GRID SYSTEM DUMP ----- \n");
}

/*****

/*
 * This function dumps out the temporary patch boundary data structure.
 *
 * Arguments:
 * N_patch - The number of coordinate patches.
 * patch_bndry --> (1 origin) The patch boundary data structure to dump.
 */

```

patchutil directory

DUMP.C;63

21-SEP-1985 19:02

Page 5

```

dump_patch_bndry(N_patch, patch_bndry)
int N_patch;
register struct patch_bndry *patch_bndry;
{
register struct bndry_defn_term *ptr;
int p;
enum bndry bndry;

printf("\n----- DUMP OF PATCH BOUNDARY TEMPORARY DATA STRUCTURE ----- \n");

if (patch_bndry == NULL)
    then {
        printf("patch_bndry=NULL\n");
        return;
    }

    for (p = 1 ; p <= N_patch ; ++p)
    {
        printf("\n----- patch %d start:\n", p);

        printf("x_min_bound=%lf    x_max_bound=%lf\n",
            patch_bndry[p].x_min_bound, patch_bndry[p].x_max_bound);
        printf("y_min_bound=%lf    y_max_bound=%lf\n",
            patch_bndry[p].y_min_bound, patch_bndry[p].y_max_bound);

        /* print out list, if any: */
        for (bndry = I_MIN ; bndry <= J_MAX ; ++bndry)
        {
            ptr = patch_bndry[p].bdt_list[bndry];
            if (ptr == NULL)
                then {
                    printf("\n%d list --> (NULL)\n", bndry);
                    continue; /* with next bndry */
                }

            printf("\n%d list:\n", bndry);
            for ( ; ptr != NULL ; ptr = ptr->next )
            {
                printf("--> defn_type=%d", ptr->defn_type);
                if (ptr->defn_type == CONSTANT_BNDRY_TYPE)
                    then printf("    defn_value.value=%lf\n",
                        ptr->defn_value.value);
                else printf(
                    "    defn_value.patch_info. ( patch=%d    bndry=%d )\n",
                        ptr->defn_value.patch_info.patch,
                        ptr->defn_value.patch_info.bndry);
                printf("    bndry_cond_type=%d", ptr->bndry_cond_type);
                if (ptr->bndry_cond_type == TRUE_BNDRY_COND)
                    then printf("    bndry_cond_value.bndry_cond='%s'",
                        ptr->bndry_cond_value.bndry_cond);
                else printf("    bndry_cond_value.patch=%d",
                        ptr->bndry_cond_value.patch);
            }
            printf("\n");
        }
    }
}

```

patchutil directory

DUMP.C;63

21-SEP-1985 19:02

Page 6

```
        /* end of this bndry */  
    }  
  
    printf("----- patch #%d stop:\n", p);  
}  
  
printf("----- END OF PATCH BOUNDARY TEMPORARY DATA STRUCTURE DUMP ----- \n");  
}
```


patchutil directory

INOUT.C;476

21-SEP-1985 19:03

Page 1

```

/* This file contains routines which classify points w.r.t. patches */
/*
 * The following functions are contained in this file:
 * inout_xy - Checks a point for in/on/out status w.r.t. a specified patch
 * inout_ij - ditto, for 'integral' points only
 */

#include "util:stdio.h"

#include "patchutil:patch.h"
#include "patchutil:inout.h"

/*****

/*
 * This function checks a specified point for in/on/out status w.r.t. (the
 * 'discrete', as defined by line_end_dscr arrays version of) a specified
 * patch.
 *
 * Arguments:
 * ptr --> The coordinate patch descriptor for the patch to interpolate into.
 * (x,y) - The coordinates of the point to check (in the system of patch 'p').
 */
enum relation inout_xy(ptr, x, y)
register struct coord_patch_dscr *ptr;
double x, y;
{
    boolean fuzzy_LT(), fuzzy_EQ(), fuzzy_GT();
    int fuzzy_floor();

    enum bndry bndry;
    double i_float, j_float;
    int i, j_i, j_ip1;
    double fract;
    double j_float_limit;

    /* first check against bounding box to see if we can get a quick 'OUTSIDE': */
    i_float = x / ptr->i_step;
    j_float = y / ptr->j_step;

    if (    fuzzy_LT(i_float, (double)ptr->i_min)
        || fuzzy_GT(i_float, (double)ptr->i_max)
        || fuzzy_LT(j_float, (double)ptr->j_min)
        || fuzzy_GT(j_float, (double)ptr->j_max)
        )
        then return(OUTSIDE);

    /* it's inside bounding box (or on edge) -- now check actual boundaries: */
    i = fuzzy_floor(i_float);
    fract = i_float - i;

    /* ..... both lower and upper: */
    for (bndry = J_MIN; bndry <= J_MAX; ++bndry)
    {
        /* this gets done once for J_MIN and once for J_MAX: */

```

patchutil directory

INOUT.C;476

21-SEP-1985 19:03

Page 2

```

    j_i = ptr->line_end[bndry][i].end_ij;
    if (i == ptr->i_max)
        then j_ipl = j_i;
        else j_ipl = ptr->line_end[bndry][i + 1].end_ij;
    if (j_ipl > j_i + 1) then j_ipl = j_i + 1; /* limit slope ... */
    if (j_ipl < j_i - 1) then j_ipl = j_i - 1; /* ... to +/- 1.0 */

    j_float_limit = j_i + ( fract * (j_ipl-j_i) );

    if (fuzzy_EQ(j_float, j_float_limit)) then return(ON_BNDRY);
    if ( ((bndry == J_MIN) && (j_float < j_float_limit))
        || ((bndry == J_MAX) && (j_float > j_float_limit))
        )
        then return(OUTSIDE);
    }
return(INSIDE);
}

/*****

/*
 * This function checks a specified 'integral' point for in/on/out status
 * w.r.t. a specified patch.
 *
 * Arguments:
 * ptr --> The coordinate patch descriptor for the patch to check.
 * (i,j) - The 'integral' coordinates to check.
 */
enum relation inout_ij(ptr, i, j)
register struct coord_patch_dscr *ptr;
int i, j;
{
enum relation inout_xy();
void cvt_ij_to_xy();

double x, y;
cvt_ij_to_xy(ptr, i, j, &x, &y);
return(inout_xy(ptr, x, y));
}

```

patchutil directory

INPUTPAR.C;362

21-SEP-1985 19:03

Page 1

```

#include <stdio.h>
#include "util:stdio.h"

#include "util:keyword.h"
#include "patchutil:blackhole.h"

/*****
/*
* This function inputs a '.par' grid file from disk, copying the information
* into the various descriptors (details described below).
*
* Arguments:
* modelname - The name of the file to read (before appending '.par').
* ptitle --> Where the title is to be stored.
* pgridname --> Where the name of the associated '.pat'/''.grd' file pair is to
*               be stored.
* pbhsd --> A black-hole-system descriptor to be updated. The associated
*            array of black_hole_dscr's is allocated and completely filled in,
*            as are all arrays in the original black-hole-system descriptor
*            itself.
*
* Result:
*/
void input_par(modelname, ptitle, pgridname, pbhsd)
char *modelname;
char *ptitle;           /* --> variable of type 'char *' */
char *pgridname;        /* --> variable of type 'char *' */
register struct bhs_dscr *pbhsd;
{
    int keyword_scan();
    void error_exit();

    char *calloc();

#define FILENAME_SIZE 100
    char filename[FILENAME_SIZE];
#define MAX_LINE_LEN 250
    char line_buffer[MAX_LINE_LEN];

    FILE *fp;
    int i, status, old_bh, bh;
    boolean allocated_yet;

    static struct var_dscr par_table [] =          /* for keyword scanner */
    {
        /* note 'NULL' pointers below will be filled in later..... */
        { "title",                NULL,    "%s" },
        { "N_black_hole",         NULL,    "%d" },
        { "grid",                 NULL,    "%s" },
        { "black_hole",           NULL,    "%d" },
        { "rho",                  NULL,    "%lf" },
        { "z",                    NULL,    "%lf" },
        { "mass",                 NULL,    "%lf" },
        { "surrounding_patch_flag", NULL,    "%d" },
    }

```

patchutil directory

INPUTPAR.C;362

21-SEP-1985 19:03

Page 2

```

{ "momentum",          NULL,    "%lf" },
{ "reflection_term_sign", NULL,    "%d" },
{ "reflection_radius",  NULL,    "%lf" },
{ "spin",              NULL,    "%lf" }
};
#define PAR_TABLE_SIZE \
    ( sizeof(par_table) / sizeof(struct var_dscr) )

/*
 * EXECUTION STARTS HERE:
 */

/* fill in keyword scanner table entries, as much as possible: */
i = 0; /* offset in par_table of 1st entry to be filled in */
par_table[i++].value = (char *) ptitle;
par_table[i++].value = (int *) &pbhsd->N_black_hole;
par_table[i++].value = (char *) pgridname;
par_table[i++].value = (int *) &bh;

/* open file: */
sprintf(filename, "%s.par", modelname); /* form 'full' file name */
fp = fopen(filename, "r");               /* open for reading only */
if (fp == NULL)
    then error_exit(OK_EXIT,
***** input_par: could not open file \"%s\",
                    filename); /* DOES NOT RETURN */

/* process file: */
bh = pbhsd->N_black_hole = 0;
allocated_yet = FALSE;
while (fgets(line_buffer, MAX_LINE_LEN, fp) != NULL)
{
    old_bh = bh;
    status = keyword_scan(PAR_TABLE_SIZE, par_table, line_buffer);
    if (status < 0)
        then error_exit(OK_EXIT,
***** input_par: syntax error (in next line) in model \"%s\" \"\n%s\",
                    modelname,
                    line_buffer); /* DOES NOT RETURN */

    if (bh != old_bh)
        then {
            /* fill in remaining keyword scanner table entries: */
            i = 4; /* offset in par_table of 1st entry to be filled in */
            par_table[i++].value
            & pbhsd->black_hole[bh]
#define LHS
#define RHS_ROOT
            LHS = (double *) RHS_ROOT.rho;
            LHS = (double *) RHS_ROOT.z;
            LHS = (double *) RHS_ROOT.mass;
            LHS = (int *) RHS_ROOT.surrounding_patch_flag;
            LHS = (double *) RHS_ROOT.momentum;
            LHS = (int *) RHS_ROOT.reflection_term_sign;
            LHS = (double *) RHS_ROOT.reflection_radius;
            LHS = (double *) RHS_ROOT.spin;
        }

```

patchutil directory

INPUTPAR.C;362

21-SEP-1985 19:03

Page 3

```

    if ( (!allocated_yet) && (pbhsd->N_black_hole != 0) )
        then {
            /* allocate array of black_hole_dscr's: */
            pbhsd->black_hole = ALLOC(pbhsd->N_black_hole,
                                     struct black_hole_dscr);
            if (pbhsd->black_hole == NULL)
                then error_exit(OK_EXIT,
            ***** input_par: insufficient memory for %d black holes in model \"%s\"\\n",
                                     pbhsd->N_black_hole,
                                     modelname); /* DOES NOT RETURN */
            --pbhsd->black_hole; /* adjust for 1 origin indexing */
            allocated_yet = TRUE;
        }

fclose(fp);
}

```

patchutil directory

INPUTPAT.C;24

21-SEP-1985 19:03

Page 1

```

/* This file contains 'input_pat(...)' and her friends. */
/*
 * The following functions live in this file:
 * input_pat - Inputs a '.pat' file from disk into grid data structure
 * dismantle - Dismantles temporary patch boundary data structure
 */

#include <stdio.h>
#include "util:stdio.h"

#include "patchutil:patch.h"
#include "patchutil:tempbndry.h"

/*****
/*
 * This function inputs a '.pat' file from disk into the grid data structure.
 * Note that this results in all fields of the grid data structure being filled
 * in, EXCEPT for (a) the 'true' boundary condition extrapolation coefficients
 * (which would involve 'physics' knowledge, specifically the black hole system
 * to be used, which isn't known here) and (b) the 'fn_i_step', 'fn_j_step',
 * and 'fn_00_ptr' fields of the coord_patch_dscr's, which are part of the grid
 * function structure, rather than the grid structure itself.
 *
 * Arguments:
 * gridname = The name of the file to input from (before appending '.pat').
 * pgd --> A grid descriptor to be filled in.
 */
void input_pat(gridname, pgd)
char *gridname;
struct grid_dscr *pgd;
{
    struct patch_bndry * read_pat();           /* read into temp data structure */
    void set_patch_shapes();                   /* set up patch shape info */
    void set_bndry_conds();                   /* set up boundary conditions */

    static void dismantle();                  /* dismantle temp data structure */

    struct patch_bndry *patch_bndry;

    patch_bndry = read_pat(gridname, pgd);
    set_patch_shapes(gridname, patch_bndry, pgd); /*
                                                * 'gridname' supplied only for
                                                * error messages.
                                                */
    set_bndry_conds(gridname, patch_bndry, pgd); /* (ditto) */
    dismantle(pgd->N_patch, patch_bndry);
}

/*****
/*
 * This function deallocates the temporary patch boundary data structure:
 */
static void dismantle(N_patch, patch_bndry)

```

patchutil directory

INPUTPAT.C;24

21-SEP-1985 19:03

Page 2

```

int N_patch;
register struct patch_bndry *patch_bndry;
{
void cfree();

int p;
enum bndry bndry;
register struct bndry_defn_term *ptr, *next_ptr;

    for (p = 1 ; p <= N_patch ; ++p)
    {
        for (bndry = I_MIN ; bndry <= J_MAX ; ++bndry)
        {
            /* free bndry'th chain: */
            ptr = patch_bndry[p].bdt_list[(int)bndry];
            while (ptr != NULL)
            {
                next_ptr = ptr->next;
                cfree(ptr);
                ptr = next_ptr;
            }
        }
    }
cfree(patch_bndry);
}

```

patchutil directory

IPARTEST.C:12

21-SEP-1985 19:04

Page 1

```

#include "util:stdio.h"

#include "patchutil:blackhole.h"
#include "patchutil:patch.h"

/*****

main(argc, argv)          /* iparttest - 'input_par' test */
int argc;
char *argv[];
{
    void error_exit();
    char *fixname();
    double cpu_time(), clock_time();
    void input_par();
    void dump_bhs();

    struct bhs_dscr bhsd;
    char *title, *gridname;

    double time1, time2;

    if (argc != 2)
        then error_exit(OK_EXIT,
                        "usage: %s modelname\n",
                        fixname(argv[0])); /* DOES NOT RETURN */

    time2 = - clock_time();
    time1 = - cpu_time();
    input_par(argv[1], &title, &gridname, &bhsd);
    time1 += cpu_time();
    time2 += clock_time();

    printf("cpu/clock time used for input_par(...) was %.2lf/%.2lf sec\n",
           time1, time2);

    printf("title='%s'\n", title);
    printf("gridname='%s'\n", gridname);
    dump_bhs(&bhsd);
}

```


patchutil directory

IPATTEST.C;34

4-NOV-1985 00:15

Page 1

```

#include "util:stdio.h"

#include "patchutil:patch.h"

/*****

main(argc, argv)      /* ipattest - 'input_pat' test */
int argc;
char *argv[];
{
double cpu_time(), clock_time();
void error_exit();
char *fixname();
void input_pat();

struct grid_dscr gd;
double time1, time2;

if (argc != 2)
    then error_exit(OK_EXIT,
                    "usage: %s grid-name\n",
                    fixname(argv[0])); /* DOES NOT RETURN */

time2 = - clock_time();
time1 = - cpu_time();
input_pat(argv[1], &gd);
time1 += cpu_time();
time2 += clock_time();

printf("cpu/clock time used for input_pat(...) was %.2lf/%.2lf sec\n",
       time1, time2);

dump_grid(&gd);
}

```

patchutil directory

NGRIDPTS.C;3

21-SEP-1985 19:04

Page 1

```

#include "util:stdio.h"
#include "patchutil:patch.h"

/*****
/*
* This function returns the number of grid points in a grid.
*
* Arguments:
* pgd --> The grid descriptor.
* p = The patch # for which the number of grid points is wanted, or 0 for the
*     whole grid to be counted.
*/
int N_grid_points(pgd, p)
register struct grid_dscr *pgd;
register int p;
{
register struct coord_patch_dscr *ptr;
register int count;
register int p_min, p_max, i;

if (p == 0)
    then {
        p_min = 1;
        p_max = pgd->N_patch;
    }
    else p_min = p_max = p;

count = 0;
for (p = p_min ; p <= p_max ; ++p)
{
    ptr = &pgd->patch[p];
    for (i = ptr->i_min ; i <= ptr->i_max ; ++i)
    {
        count += ptr->line_end[J_MAX][i].end_ij
                - ptr->line_end[J_MIN][i].end_ij
                + 1;
    }
}

return(count);
}

```

patchutil directory

PARLOG.C;12

21-SEP-1985 19:04

Page 1

```

#include <stdio.h>
#include "util:stdio.h"

/*****

/*
 * This function is used to log operations on '.gfn' files in their associated
 * '.par' files.
 *
 * Arguments:
 * modelname is a (pointer to a standard C null-terminated) character string
 *             giving the model name (i.e. '.par' and '.gfn' filename).
 * (argc,argv) are the standard C arguments to 'main' describing the command
 *             whose execution is to be logged.
 * time is the cpu time used. If this is non-zero, it is logged.
 */
void par_log(modelname, argc, argv, time)
char modelname[];
int argc;
char *argv[];
double time;
{
FILE *fopen();
void fclose();

char *fixname();

FILE *fp;
int i;
#define BUFSIZE 50
char name_buffer[BUFSIZE];

sprintf(name_buffer, "%s.par", modelname);
fp = fopen(name_buffer, "a"); /* opening in 'append' mode --> posn at eof */
if (fp == NULL) then return; /* ... forget it!!! */

fprintf(fp, ";");
for (i=0 ; i<argc ; ++i)
{
fprintf(fp, " %s", (i == 0) ? fixname(argv[0]) : argv[i]);
}
if (time != 0.0) then fprintf(fp, "\t\t(%.2f cpu seconds)", time);
fprintf(fp, "\n");

fclose(fp);
}

```

patchutil directory

READPAT.C;361

21-SEP-1985 19:05

Page 1

```

/* This file contains 'read_pat' and friends */
/*
 * The following functions are contained in this file:
 * read_pat - Reads a '.pat' grid file from disk.
 * set_global_ref - Sets up a patch (normally #0) as a global reference patch
 ** process_header_line - Processes '.pat' file line from 'header' section
 ** process_pat_line - Processes '.pat' file line from 'patch' section
 ** cvt_angles_to_radians - Converts all stored angles from degrees to radians
 */

#include <stdio.h>
#include "util:stdio.h"

#include "util:keyword.h"

#include "patchutil:patch.h"
#include "patchutil:tempbndry.h"
#include "patchutil:cvt.h"

/*****

/*
 * This function inputs a '.pat' grid file from disk.
 *
 * Arguments:
 * gridname - The name of the file to read (before appending '.pat').
 * pgd --> A grid descriptor to be updated. The associated array of
 *          coord_patch_dscr's is allocated and partly filled in; some fields in
 *          the original grid descriptor are filled in.
 *
 * Result:
 * The patch boundary information is stored in an array of patch_bndry
 * descriptors (with sublists of bndry_defn_term's), which is allocated
 * and filled in. A pointer (to be used in l origin indexing) to this
 * array is returned as the function result.
 */
struct patch_bndry * read_pat(gridname, pgd)
char *gridname;
register struct grid_dscr *pgd;
{
char *calloc();
void error_exit();

static boolean process_header_line();
static void process_pat_line();
static void cvt_angles_to_radians();

#define FILENAME_SIZE 100
char filename[FILENAME_SIZE];
#define MAX_LINE_LEN 250
char line_buffer[MAX_LINE_LEN];

FILE *fp;
struct patch_bndry *patch_bndry;          /* will become function result */

```

patchutil directory

READPAT.C;361

21-SEP-1985 19:05

Page 2

```

enum { IN_HEADER, IN_PATCHES } state;
int done_flag;

/* open file: */
sprintf(filename, "%s.pat", gridname); /* form 'full' file name */
fp = fopen(filename, "r"); /* open for reading only */
if (fp == NULL)
    then error_exit(OK_EXIT,
        "***** read_pat: unable to open file \"%s\"\\n",
        filename); /* DOES NOT RETURN */

/* process file: */
state = IN_HEADER;
while (fgets(line_buffer, MAX_LINE_LEN, fp) != NULL)
{
    switch (state)
    {
        case IN_HEADER:
            done_flag = process_header_line(gridname, line_buffer, pgd);
            if (done_flag)
            then {
                /* allocate space: */
                pgd->patch = ALLOC(pgd->N_patch + 1,
                    struct coord_patch_dscr);
                patch_bndry = ALLOC(pgd->N_patch, struct patch_bndry);
                if ((pgd->patch == NULL) || (patch_bndry == NULL))
                    then error_exit(OK_EXIT,
                        "***** read_pat: insufficient memory for %d patches in grid \"%s\"\\n",
                        pgd->N_patch,
                        gridname); /* DOES NOT RETURN */
                set_global_ref(&pgd->patch[0]);
                -- patch_bndry; /* shift to origin 1 */
                state = IN_PATCHES;
            }
            break;
        case IN_PATCHES:
            process_pat_line(gridname, line_buffer, pgd, patch_bndry);
            break;
    }
}

fclose(fp); /* we're done with the '.pat' file now! */
cvt_angles_to_radians(pgd, patch_bndry); /* fix stored angles */
/* (input in degrees) */
return(patch_bndry);
}

/*****
/*
* This functions sets up a patch (normally #0) to serve as a global reference
* coordinate system for all other patches.
*
* Arguments:
* ptr --> The coord_patch_dscr for the patch to be so set up.
*/

```

patchutil directory

READPAT.C;361

21-SEP-1985 19:05

Page 3

```

*/
void set_global_ref(ptr)
register struct coord_patch_dscr *ptr;
{
/* set up reference coordinate system: */
ptr->coord_type = CYLINDRICAL;
ptr->rho_center = ptr->z_center = 0.0;

/* set up 'null' values in remaining fields: */
ptr->difference_scheme = INDEPENDENT_TERMS; /* doesn't matter! */
ptr->conformal_fn_type = PSI; /* (ditto) */
ptr->i_step = ptr->j_step = 0.0;
ptr->i_min = 0; ptr->i_max = -1;
ptr->j_min = 0; ptr->j_max = -1;
}

/*****
/*
* This function processes a line from the '.pat' file (header section).
*
* Arguments:
* gridname - Grid name for error messages.
* line_buffer --> The text line to be processed.
* pgd --> A grid descriptor to be filled in. (Only the N_patch field.)
*
* The function returns TRUE if it has found and processed the end of the
* section, FALSE if not.
*/
static boolean process_header_line(gridname, line_buffer, pgd)
char gridname[];
char line_buffer[];
register struct grid_dscr *pgd; /* OUTPUT PARAMETER! */
{
void error_exit();
int keyword_scan();

int i, status;

static int first_time = TRUE;
static int count = 0;
static struct var_dscr header_table [] = /* for keyword scanner */
/*
* Note the 'commented' entry will be filled in -- it isn't known at link time,
* so the compiler can't do it for us.
*/
{
{ "N_patch", NULL /* &pgd->N_patch */ , "%d" }
};
#define HEADER_TABLE_SIZE \
( sizeof(header_table) / sizeof(struct var_dscr) )

if (first_time)
then {
/* fill in run-time keyword scanner table values: */

```

patchutil directory

READPAT.C;361

21-SEP-1985 19:05

Page 4

```

    i = 0;
    header_table[i++].value = (int *) &pgd->N_patch;
    first_time = FALSE;          /* no need to do it again..... */
}

status = keyword_scan(HEADER_TABLE_SIZE, header_table, line_buffer);

if (status < 0)
    then error_exit(OK_EXIT,
***** read_pat: syntax error (in next line) in grid \"%s\" \"n%s\n\",
                gridname,
                line_buffer);          /* DOES NOT RETURN */

count += status;

if (count == HEADER_TABLE_SIZE)
    then return(TRUE);
    else return(FALSE);
}

/*****
/*
* This function processes a line from the '.pat' file (patch description, or
* 'main', section)
*
* Arguments:
* gridname = Grid name for error messages.
* line_buffer --> The text line to be processed.
* pgd --> The grid descriptor.
* patch_bndry --> (1 origin) The patch boundary temporary data structure.
*/
static void process_pat_line(gridname, line_buffer, pgd, patch_bndry)
char gridname[];
char line_buffer[];
register struct grid_dsor *pgd;
register struct patch_bndry *patch_bndry;
{
    int cvt_coord_type_to_code();
    int cvt_bndry_to_code();
    void error_exit();
    void keyword_scan();

    char *calloc();

    int i, status, rescan_status;
    int old_p;          /* old patch number */
    char *min_max_defn;
    struct bndry_defn_term *pbdt;

/*
* The following variables are made static only so we can initialize the
* keyword scanner tables as much as possible at compile time -- their values
* not needed across calls.
*/
static int p;          /* patch number */

```

patchutil directory

READPAT.C;361

21-SEP-1985 19:05

Page 5

```

static char *coord_type, *difference_scheme, *conformal_fn_type;
static char *i_min_defn, *i_max_defn, *j_min_defn, *j_max_defn;
static *bndry_cond_defn;
static int patch_temp;
static char *bndry_temp;
static double constant_temp;

static struct var_dscr patch_table [] =          /* for keyword scanner */
{
  { "patch",          &p,          "%d" },
  { "coord_type",     &coord_type, "%s" },
  { "difference_scheme", &difference_scheme, "%s" },
  { "conformal_fn_type", &conformal_fn_type, "%s" },
  { "rho_center",     NULL,        "%lf" },
  { "z_center",       NULL,        "%lf" },
  /* polar spherical coordinate patch items: */
  { "delta_r",        NULL,        "%lf" },
  { "delta_theta",    NULL,        "%lf" },
  { "r_min_bound",    NULL,        "%lf" },
  { "r_max_bound",    NULL,        "%lf" },
  { "theta_min_bound", NULL,        "%lf" },
  { "theta_max_bound", NULL,        "%lf" },
  { "r_min",          &i_min_defn,  "%s" },          /* will be rescanned */
  { "r_max",          &i_max_defn,  "%s" },          /* will be rescanned */
  { "theta_min",      &j_min_defn,  "%s" },          /* will be rescanned */
  { "theta_max",      &j_max_defn,  "%s" },          /* will be rescanned */
  /* cylindrical coordinate patch items: */
  { "delta_rho",      NULL,        "%lf" },
  { "delta_z",        NULL,        "%lf" },
  { "rho_min_bound",  NULL,        "%lf" },
  { "rho_max_bound",  NULL,        "%lf" },
  { "z_min_bound",    NULL,        "%lf" },
  { "z_max_bound",    NULL,        "%lf" },
  { "rho_min",        &i_min_defn,  "%s" },          /* will be rescanned */
  { "rho_max",        &i_max_defn,  "%s" },          /* will be rescanned */
  { "z_min",          &j_min_defn,  "%s" },          /* will be rescanned */
  { "z_max",          &j_max_defn,  "%s" },          /* will be rescanned */
  { "bndry_cond",     &bndry_cond_defn, "%s" } /* will be rescanned */
};
#define PATCH_TABLE_SIZE \
  ( sizeof(patch_table) / sizeof(struct var_dscr) )

static struct var_dscr rescan_table [] =          /* for keyword scanner */
{
  { "patch",          &patch_temp,  "%d" },
  { "bndry",          &bndry_temp,  "%s" },
  { "constant",       &constant_temp, "%lf" }
};
#define RESCAN_SUBTABLE_SIZE 1 /* size to only look for 'patch' keyword */
#define RESCAN_TABLE_SIZE \
  ( sizeof(rescan_table) / sizeof(struct var_dscr) )

/*
 * EXECUTION BEGINS HERE:

```


patchutil directory

READPAT.C;361

21-SEP-1985 19:05

Page 6

```

*/
coord_type = difference_scheme = conformal_fn_type = NULL;
i_min_defn = i_max_defn = j_min_defn = j_max_defn = NULL;
bndry_cond_defn = NULL;

old_p = p;
status = keyword_scan(PATCH_TABLE_SIZE, patch_table, line_buffer);
if (p != old_p)
    then {
        /* start of new patch definition found: */

        /* fill in keyword scanner table to point to new patch: */

        i = 4;
        patch_table[i++].value = (double *) &pgd->patch[p].rho_center;
        patch_table[i++].value = (double *) &pgd->patch[p].z_center;

        /* polar spherical items: */
        patch_table[i++].value = (int *) &pgd->patch[p].i_step;
        patch_table[i++].value = (int *) &pgd->patch[p].j_step;
        patch_table[i++].value = (double *) &patch_bndry[p].x_min_bound;
        patch_table[i++].value = (double *) &patch_bndry[p].x_max_bound;
        patch_table[i++].value = (double *) &patch_bndry[p].y_min_bound;
        patch_table[i++].value = (double *) &patch_bndry[p].y_max_bound;
        i += 4; /* skip over 4 initialized values */

        /* cylindrical items: */
        patch_table[i++].value = (double *) &pgd->patch[p].i_step;
        patch_table[i++].value = (double *) &pgd->patch[p].j_step;
        patch_table[i++].value = (double *) &patch_bndry[p].x_min_bound;
        patch_table[i++].value = (double *) &patch_bndry[p].x_max_bound;
        patch_table[i++].value = (double *) &patch_bndry[p].y_min_bound;
        patch_table[i++].value = (double *) &patch_bndry[p].y_max_bound;
    }

switch (status)
{
case EOF:
case SYNTAX_ERROR:
case VALUE_ERROR:
case UNKNOWN_KEYWORD:
    error_exit(OK_EXIT,
    ***** read_pat: syntax error (in next line) in grid \"%s\"\\n%s\\n\",
                gridname,
                line_buffer); /* DOES NOT RETURN */

default:
    if (coord_type != NULL)
        then {
            int temp_int_coord_type;
            temp_int_coord_type = cvt_coord_type_to_code(coord_type);
            if (temp_int_coord_type == CVT_ERROR)
                then error_exit(OK_EXIT,
                ***** read_pat: unknown coordinate type \"%s\" in grid \"%s\"\\n\",
                            coord_type,

```

patchutil directory

READPAT.C;361

21-SEP-1985 19:05

Page 7

```

                                gridname); /* DOES NOT RETURN */
        else pgd->patch[p].coord_type = (enum coord_type)
                                temp_int_coord_type;
    }

    if (difference_scheme != NULL)
    then {
        int temp_int_difference_scheme;
        temp_int_difference_scheme
            = cvt_difference_scheme_to_code(difference_scheme);
        if (temp_int_difference_scheme == CVT_ERROR)
            then error_exit(OK_EXIT,
***** read_pat: unknown difference scheme \"%s\" in grid \"%s\"\\n\",
                                difference_scheme,
                                gridname); /* DOES NOT RETURN */
        else pgd->patch[p].difference_scheme
            = (enum difference_scheme)
                                temp_int_difference_scheme;
    }

    if (conformal_fn_type != NULL)
    then {
        int temp_int_conformal_fn_type;
        temp_int_conformal_fn_type
            = cvt_conformal_fn_type_to_code(conformal_fn_type);
        if (temp_int_conformal_fn_type == CVT_ERROR)
            then error_exit(OK_EXIT,
***** read_pat: unknown conformal fn type \"%s\" in grid \"%s\"\\n\",
                                conformal_fn_type,
                                gridname); /* DOES NOT RETURN */
        else pgd->patch[p].conformal_fn_type
            = (enum conformal_fn_type)
                                temp_int_conformal_fn_type;
    }

    if ( (i_min_defn != NULL) || (i_max_defn != NULL)
        || (j_min_defn != NULL) || (j_max_defn != NULL))
    then {
        /* keyword_scan has got a min/max defn in it */
        /* (we assume exactly 1) */
        pbdt = ALLOC(1, struct bndry_defn_term);
        if (pbdt == NULL)
            then error_exit(OK_EXIT,
***** read_pat: insufficient memory for grid \"%s\"\\n\",
                                gridname); /* DOES NOT RETURN */

#define COLLECT_AND_INSERT(end, end_defn)
        if (end_defn != NULL)
        then {
            min_max_defn = end_defn;
            /* insert (*pbdt) at head of list: */
            pbdt->next = patch_bndry[p].bdt_list[(int)end];
            patch_bndry[p].bdt_list[(int)end] = pbdt;
        } /* end of macro definition */
    }

```

patchutil directory

READPAT.C;361

21-SEP-1985 19:05

Page 8

```

COLLECT_AND_INSERT(I_MIN, i_min_defn)
COLLECT_AND_INSERT(I_MAX, i_max_defn)
COLLECT_AND_INSERT(J_MIN, j_min_defn)
COLLECT_AND_INSERT(J_MAX, j_max_defn)

patch_temp = 0;
keyword_scan(RESCAN_TABLE_SIZE,
             rescan_table,
             min_max_defn);

/*
 * We know that either the two keywords 'patch' and 'bndry', or
 * the keyword 'constant' have/has been assigned by the last
 * keyword scan. We find out which by checking to see if
 * 'patch_temp' has changed; patches are numbered in origin 1
 * so 0 is not a valid patch number.
 */
if (patch_temp == 0)
then {
    /* 'constant' keyword assignment made: */
    pbdt->defn_type = CONSTANT_BNDRY_TYPE;
    pbdt->defn_value.value = constant_temp;
}
else {
    /* 'patch' keyword assignment made: */
    int temp_int_bndry;
    temp_int_bndry = cvt_bndry_to_code(bndry_temp);
    if (temp_int_bndry == CVT_ERROR)
        then error_exit(OK_EXIT,
***** read_pat: unknown boundary type \"%s\" in grid \"%s\"\\n\",
                        bndry_temp,
                        gridname); /* DOES NOT RETURN */
    else {
        pbdt->defn_type = PATCH_BNDRY_TYPE;
#define ROOT pbdt->defn_value.patch_info
        ROOT.patch = patch_temp;
        ROOT.bndry = (enum bndry) temp_int_bndry;
    }
}

if (pbdt->defn_type == PATCH_BNDRY_TYPE)
then {
    /* no 'bndry_cond' keyword assignment needed */
    pbdt->bndry_cond_type = PATCH_BNDRY_COND;
    pbdt->bndry_cond_value.patch = patch_temp;
}
else {
    /* keyword 'bndry_cond' should have it: */
    /*
     * It will either be a keyword assignment to the
     * keyword 'patch', or a 'true' boundary condition --
     * we must rescan to find out which. Note we use only
     * the subset of the main rescan keyword table which
     * contains the entry for the single keyword 'patch'.
     */
    patch_temp = 0;
    rescan_status = keyword_scan(RESCAN_SUBTABLE_SIZE,

```

patchutil directory

READPAT.C;361

21-SEP-1985 19:05

Page 9

```

                                rescanner_table,
                                bndry_cond_defn);
if (rescan_status == UNKNOWN_KEYWORD)
then {
    pbdt->bndry_cond_type = TRUE_BNDRY_COND;
    pbdt->bndry_cond_value.bndry_cond
        = bndry_cond_defn;
}
else {
    pbdt->bndry_cond_type = PATCH_BNDRY_COND;
    pbdt->bndry_cond_value.patch
        = patch_temp;
}
}
}
}

/*****
/*
* This function converts all angular values stored in either the temporary
* patch boundary data structure or the grid descriptor data structure, from
* degrees (in which they were read in) to radians (in which they will be used
* from now on). Note the line_end_dscr arrays aren't looked at, since they
* won't exist when this function is supposed to be called.
*
* Arguments:
* pgd --> The grid descriptor data structure.
* patch_bndry --> (1 origin) The patch boundary temporary data structure.
*/
static void cvt_angles_to_radians(pgd, patch_bndry)
register struct grid_dscr *pgd;
register struct patch_bndry *patch_bndry;
{
#define DEGREES_TO_RADIAN (PI/180.0) /* multiply by this to cvt */

register int p;
register enum bndry bndry;
register struct bndry_defn_term *ptr;

/* do the temporary patch boundary data structure: */
for (p = 1; p <= pgd->N_patch; ++p)
{
    if (pgd->patch[p].coord_type != POLAR_SPHERICAL)
        then continue; /* with next patch */

#define FIX(x) x *= DEGREES_TO_RADIAN /* convert x in place */

    /* get to here --> patch is polar spherical --> angles stored to fix */
    FIX(patch_bndry[p].y_min_bound);
    FIX(patch_bndry[p].y_max_bound);

    /* now do the lists: */
    /* note we only do the 'J' (theta) lists!!! */

```

patchutil directory

READPAT.C;361

21-SEP-1985 19:05

Page 10

```

    for (bndry = J_MIN ; bndry <= J_MAX ; ++bndry)
    {
        for (ptr = patch_bndry[p].bdt_list[(int)bndry] ;
            ptr != NULL ;
            ptr = ptr->next)
        {
            /* fix up ptr->(list element): */
            if (ptr->defn_type == CONSTANT_BNDRY_TYPE)
                then FIX(ptr->defn_value.value);
        }
    }

/* do the grid descriptor data structure: */
for (p = 1 ; p <= pgd->N_patch ; ++p)
{
    if (pgd->patch[p].coord_type != POLAR_SPHERICAL)
        then continue; /* with next patch */

    /* get to here --> patch is polar spherical --> angles stored to fix */
    FIX(pgd->patch[p].j_step);
}

```

patchutil directory

SETBNDRY.C;491

21-SEP-1985 19:06

Page 1

```

/* This file contains 'set_bndry_conds' and subsidiary routines: */
/*
 * The following functions are contained in this file:
 * set_bndry_conds - Sets all 'non-physics' boundary condition information
 ** do_bndry_point - Does all boundary condition setup for a single bndry point
 ** set_interpolation - Sets up interpolation coefficients
 */

#include "util:stdc.h"

#include "patchutil:patch.h"
#include "patchutil:tempbndry.h"
#include "patchutil:inout.h"
#include "patchutil:cvt.h"

/*****
/*
 * This function sets up all boundary condition information in the grid_dscr
 * data structure, except for extrapolation coefficients for 'true' boundary
 * conditions. The latter require 'physics' knowledge, which this file doesn't
 * make use of.
 *
 * Arguments:
 * gridname - The grid name (before appending '.par'). This is used only to
 *             label error messages.
 * patch_bndry --> (1 origin) The temporary patch boundary data structure.
 * pgd --> The grid_dscr data structure.
 */
void set_bndry_conds(gridname, patch_bndry, pgd)
char gridname[];
struct patch_bndry *patch_bndry;
struct grid_dscr *pgd;
{
static void do_bndry_point();

int p;
enum bndry bndry;
struct line_end_dscr *pled;
int i, j, i_end, j_end;
int *pij_end, *pij_scan;
int extrap_dir;
int ij_scan_min, ij_scan_max;

    for (p = 1 ; p <= pgd->N_patch ; ++p)
    {
        for (bndry = I_MIN ; bndry <= J_MAX ; ++bndry)
        {
            /* figure out which variable is which & limits for scan: */
            switch (bndry)
            {
            case I_MIN:
            case I_MAX:
                pij_end = &i; /* 'end' values are of i */
                pij_scan = &j; /* scan j */

```

patchutil directory

SETBNDRY.C;491

21-SEP-1985 19:06

Page 2

```

        ij_scan_min = pgd->patch[p].j_min;
        ij_scan_max = pgd->patch[p].j_max;
        break;
case J_MIN:
case J_MAX:
        pij_end = &j; /* 'end' values are of j */
        pij_scan = &i; /* scan i */
        ij_scan_min = pgd->patch[p].i_min;
        ij_scan_max = pgd->patch[p].i_max;
        break;
    }
    switch (bndry)
    {
case I_MIN:
case J_MIN:
        extrap_dir = -1;
        break;
case I_MAX:
case J_MAX:
        extrap_dir = 1;
        break;
    }

    /* do each point on this boundary: */
    for (*pij_scan = ij_scan_min ;
        *pij_scan <= ij_scan_max ;
        ++*pij_scan)
    {
        pled = &pgd->patch[p].line_end[bndry][*pij_scan];
        *pij_end = pled->end_ij;
        i_end = 1; /* save boundary point */
        j_end = j;
        *pij_end += extrap_dir; /* extrapolate */
        do_bndry_point(gridname,
            patch_bndry, pgd, p, bndry,
            i_end, j_end, i, j,
            pled);
    }

    /* end of this boundary */
}

/* end of this patch */
}

/*****
/*
* This function sets up all the fields other than 'end_ij' in a line_end_dscr.
*
* Arguments:
* gridname - Grid name to use in error messages.
* patch_bndry --> (origin 1) The patch boundary temporary data structure.
* pgd --> The grid_dscr data structure.

```

patchutil directory

SETBNDRY.C;491

21-SEP-1985 19:06

Page 3

```

* p - The patch number.
* bndry - The boundary this point lies on in this patch.
* (i_end, j_end) - The integer coordinates of the 'end_ij' point.
* (i_extrap, j_extrap) - The integer coordinates of the extrapolated point.
* pled --> The line_end_dsor itself to be filled in.
*/
static void do_bndry_point(gridname,
                           patch_bndry, pgd, p, bndry,
                           i_end, j_end, i_extrap, j_extrap,
                           pled)

char gridname[];
register struct patch_bndry *patch_bndry;
register struct grid_dsor *pgd;
int p;
enum bndry bndry;
int i_end, j_end, i_extrap, j_extrap;
register struct line_end_dsor *pled;
{
void cvt_ij_to_xy();
void cvt_xy_to_xy();
enum relation temp_bndry_inout_term();
enum relation inout_xy();
int cvt_bndry_cond_type_to_code();
char *cvt_bndry_to_char();
void error_exit();
static void set_bndry_trouble();
static void set_interpolation();

register struct bndry_defn_term *ptr;
double x_end, y_end;
double x_extrap, y_extrap;
int other_patch;
double x_extrap_other, y_extrap_other;

/* Find applicable bndry_defn_term in patch boundary data structure: */
for (ptr = patch_bndry[p].bdt_list[bndry] ;
     ptr != NULL ;
     ptr = ptr->next)
{
if ( (ptr->defn_type == CONSTANT_BNDRY_TYPE)
    && (ptr->bndry_cond_type == TRUE_BNDRY_COND)
    )
then {
/* 'constant-... bndry_cond-...(true)...' : */
cvt_ij_to_xy(& pgd->patch[p], i_end, j_end, &x_end, &y_end);
if (temp_bndry_inout_term(patch_bndry, pgd, p,
                           x_end, y_end,
                           bndry,
                           ptr)

    != INSIDE)
then {
#define WHATSIT ptr->bndry_cond_value.bndry_cond
pled->flag = cvt_bndry_cond_type_to_code(WHATSIT);
if (pled->flag == CVT_BNDRY_COND_TYPE_ERROR)
/* Note non-standard error code!!! */

```


patchutil directory

SETBNDRY.C;491

21-SEP-1985 19:06

Page 4

```

        then error_exit(OK_EXIT, "\
***** set_bndry_conds: unknown boundary condition\n\
        \"s\" for patch %d in grid \"s\"\\n\",
        WHATSIT,
        P,
        gridname); /* DOES NOT RETURN */
        return;
    }
else {
    /*
    * Either 'constant=... bndry_cond="patch=..."
    * or 'patch=... bndry=...'. Either way, check to see if
    * the extrapolated point really resides in the other patch:
    */
    cvt_ij_to_xy(& pgd->patch[p], i_extrap, j_extrap,
        &x_extrap, &y_extrap);
    other_patch = ptr->bndry_cond_value.patch;
    cvt_xy_to_xy(& pgd->patch[p], x_extrap, y_extrap,
        & pgd->patch[other_patch],
        &x_extrap_other, &y_extrap_other);

    if (inout_xy(& pgd->patch[other_patch],
        x_extrap_other, y_extrap_other)
        != OUTSIDE)
    then {
        pled->flag = other_patch; /*
        * patch to interpolate
        * into
        */

        set_interpolation(gridname,
            other_patch,
            & pgd->patch[other_patch],
            x_extrap_other, y_extrap_other,
            pled); /* set coefficients */

        return;
    }

    /* end of trying this boundary definition term */
}

/* get to here --> no applicable bndry_defn_term in list --> trouble: */
error_exit(OK_EXIT, "\
***** set_bndry_conds: bad boundary condition specification found at\n\
        (%d,%d) on bndry \"s\" of patch %d of grid \"s\"\\n\",
        i_end, j_end,
        cvt_bndry_to_char(bndry),
        P,
        gridname); /* DOES NOT RETURN */
}

/*****
/*

```

patchutil directory

SETBNDRY.C:491

21-SEP-1985 19:06

Page 5

```

* This routine sets up the coefficients for bilinear interpolation.
* Formulas used are from my notes, pages 113-114 (interpolation into a square)
* and pages 122-123 (interpolation into a triangle). Note that coefficients
* which are 'fuzzily' equal to zero/one will be set to exact zeros/ones!
*
* Arguments:
* gridname - The grid name for error messages.
* p - The patch we are interpolating into, again for error messages (only).
* ptr --> The coordinate patch descriptor for the patch to interpolate into.
* (x,y) - The coordinates of the point to interpolate a value at.
* pled --> The line end descriptor to store the coefficients in.
*/
static void set_interpolation(gridname, p, ptr, x, y, pled)
char gridname[];
int p;
struct coord_patch_dscr *ptr;
double x, y;
struct line_end_dscr *pled;
{
    int fuzzy_floor();
    boolean fuzzy_EQ(), fuzzy_NE();
    enum relation inout_ij();
    void error_exit();

    double xx, yy;
    double i_float, j_float;
    int i_low, j_low;
    int ok00, ok01, ok10, ok11;
    double d00, d01, d10, d11;
    int sum_ok;

    i_float = x / ptr->i_step;
    j_float = y / ptr->j_step;

    i_low = fuzzy_floor(i_float);
    j_low = fuzzy_floor(j_float);

#define ROOT    pled->boundary_info.interpolate
    ROOT.i_low = i_low;
    ROOT.j_low = j_low;

    /*
     * Find out status (1 --> inside or on edge, 0 --> OUTSIDE) of all 4
     * 'square' points:
     */
#define IS_IT_OK(status)    ( ((status) == OUTSIDE) ? 0 : 1 )
    ok00 = IS_IT_OK(inout_ij(ptr, i_low, j_low ));
    ok01 = IS_IT_OK(inout_ij(ptr, i_low, j_low+1 ));
    ok10 = IS_IT_OK(inout_ij(ptr, i_low+1, j_low ));
    ok11 = IS_IT_OK(inout_ij(ptr, i_low+1, j_low+1 ));
    sum_ok = ok00 + ok01 + ok10 + ok11;

    if (sum_ok == 0)
        then error_exit(OK_EXIT, "\
***** set_bndry_conds: unable to interpolate (%lf,%lf) into\n\

```

patchutil directory

SETBNDY.C;491

21-SEP-1985 19:06

Page 6

```

        patch %d of grid \"%s\"\\n",
        x, y,
        p,
        gridname); /* DOES NOT RETURN */

xx = i_float - i_low;
yy = j_float - j_low;

if (sum_ok == 3)
then {
    /* bilinear interpolation into a triangle: */
    /*
     * There are 4 cases, depending on which point of the square is missing
     * (to form the triangle):
     */
    if (! ok00)
    then {
        d00 = 0.0;
        d01 = 1.0 - xx;
        d10 = 1.0 - yy;
        d11 = -1.0 + xx + yy;
    }
    if (! ok01)
    then {
        d00 = 1.0 - xx;
        d01 = 0.0;
        d10 = xx - yy;
        d11 = yy;
    }
    if (! ok10)
    then {
        d00 = 1.0 - yy;
        d01 = yy - xx;
        d10 = 0.0;
        d11 = xx;
    }
    if (! ok11)
    then {
        d00 = 1 - xx - yy;
        d01 = yy;
        d10 = xx;
        d11 = 0.0;
    }
}
else {
    /* bilinear interpolation into a square: */
    d00 = (ok00) ? ((1.0 - xx) * (1.0 - yy)) : 0.0;
    d01 = (ok01) ? ((1.0 - xx) * yy) : 0.0;
    d10 = (ok10) ? (xx * (1.0 - yy)) : 0.0;
    d11 = (ok11) ? (xx * yy) : 0.0;
}

/* Force coefficients fuzzily equal to zero/one, to be exact zeros/ones: */
#define FIX(dij,val)    if (fuzzy_EQ(dij, val)) then dij = val;
#define FULLFIX(dij)    FIX(dij, 0.0)  FIX(dij, 1.0)

```

patchutil directory

SETBNDRY.C;491

21-SEP-1985 19:06

Page 7

```

FULLFIX(d00)    FULLFIX(d01)    FULLFIX(d10)    FULLFIX(d11)

/* "Firewall" check for correct normalization: */
#define SUM      d00 + d01 + d10 + d11
if (fuzzy_NE(SUM, 1.0))
    then error_exit(OK_EXIT, "\
***** set_bndry_conds: d00+d01+d10+d11-%lf (should be 1.0) on\n\
                        attempt to interpolate (%lf,%lf) into\n\
                        patch %d of grid \"%s\"\n",
                        SUM,
                        x, y,
                        p,
                        gridname); /* DOES NOT RETURN */

/* all ok --> store results & quit: */
ROOT.d00 = d00;
ROOT.d01 = d01;
ROOT.d10 = d10;
ROOT.d11 = d11;
}

```

patchutil directory

SETPATCH.C;417

21-SEP-1985 19:07

Page 1

```

/* This file contains 'set_patch_shapes' and her friends */
/*
 * The following functions are contained in this file:
 * set_patch_shapes - Sets up all patch shape information
 ** set_up_const_bndrys - Allocates, fills in line_end_dscr arrays: some patches
 ** set_up_patch_bndrys - (ditto): remaining patches
 ** fix_up_patch_bndrys - Fixes up inconsistencies produced by above function
 * all_bndrys_constant - Checks if all bndrys of a patch are 'constant-...'
 ** set_constant_size_info - If above fn returns 'TRUE', sets size info
 ** temp_bndry_inout - Checks if a point is inside a patch
 * temp_bndry_inout_term - (ditto, for a single single boundary defn term)
 ** allocate_line_end_dscrs - Allocates array storage, given size info
 */

#include "util:stdio.h"

#include "patchutil:patch.h"
#include "patchutil:tempbndry.h"
#include "patchutil:inout.h"

/*****

/*
 * This function sets up the shapes of all patches. This includes allocating
 * storage space for the descriptor arrays etc.
 *
 * Arguments:
 * gridname - Grid name for error messages (only).
 * patch_bndry --> (1 origin) The temporary patch boundary data structure.
 * pgd --> A grid descriptor to be filled in.
 */
void set_patch_shapes(gridname, patch_bndry, pgd)
char gridname[];
register struct patch_bndry *patch_bndry;
register struct grid_dscr *pgd;
{
static void set_up_const_bndrys();
static void set_up_patch_bndrys();
static void fix_up_patch_bndrys();

set_up_const_bndrys(gridname, patch_bndry, pgd);
set_up_patch_bndrys(gridname, patch_bndry, pgd);
fix_up_patch_bndrys(gridname, patch_bndry, pgd);
}

/*****

/*
 * This function uses the temporary patch boundary data structure to compute
 * and fill in the coordinate patch size and line descriptor fields in the grid
 * data structure, for patches with all 'constant-...' type boundaries.
 *
 * Arguments:
 * gridname - The grid name for error messages.
 * patch_bndry --> (1 origin) The temporary patch boundary data structure.

```

patchutil directory

SETPATCH.C;417

21-SEP-1985 19:07

Page 2

```

* pgd --> The grid descriptor structure to be filled in.
*/
static void set_up_const_bndrys(gridname, patch_bndry, pgd)
char gridname[];
register struct patch_bndry *patch_bndry;
register struct grid_dscr *pgd;
{
boolean all_bndrys_constant();
static void set_constant_size_info();
static void allocate_line_end_dscls();

int p; /* current patch number */
enum bndry bndry;
int range_end;
int ij, ij_min, ij_max;

for (p = 1 ; p <= pgd->N_patch ; ++p)
{
/* process patch #p: */
if (! all_bndrys_constant(& patch_bndry[p]))
then continue; /* with next patch */

/* get to here --> patch is of 'constant=...' type: */
set_constant_size_info(patch_bndry, pgd, p);
allocate_line_end_dscls(gridname, p, & pgd->patch[p]);

/* fill in that storage: */
for (bndry = I_MIN ; bndry <= J_MAX ; ++bndry)
{
switch (bndry)
{
case I_MIN:
case I_MAX:
ij_min = pgd->patch[p].j_min;
ij_max = pgd->patch[p].j_max;
break;

case J_MIN:
case J_MAX:
ij_min = pgd->patch[p].i_min;
ij_max = pgd->patch[p].i_max;
break;
}

switch (bndry)
{
case I_MIN:
range_end = pgd->patch[p].i_min;
break;
case I_MAX:
range_end = pgd->patch[p].i_max;
break;
case J_MIN:
range_end = pgd->patch[p].j_min;
break;
case J_MAX:

```

patchutil directory

SETPATCH.C;417

21-SEP-1985 19:07

Page 3

```

        range_end = pgd->patch[p].j_max;
        break;
    }

    /* now actually fill in the array: */
    for (ij = ij_min ; ij <= ij_max ; ++ij)
    {
        pgd->patch[p].line_end[bndry][ij].end_ij
            = range_end;
    }

    /* end of this boundary */
}

/* end of this patch */
}

/*****
/*
* This function uses the temporary patch boundary data structure to compute
* and fill in the coordinate patch size and line descriptor fields in the grid
* data structure, for patches with one or more 'patch=... bndry=...' type
* boundaries.
*
* Arguments:
* gridname - The grid name for error messages.
* patch_bndry --, (1 origin) The temporary patch boundary data structure.
* pgd --> The grid descriptor structure to be filled in.
*/
static void set_up_patch_bndrys(gridname, patch_bndry, pgd)
char gridname[];
register struct patch_bndry *patch_bndry;
register struct grid_dscr *pgd;
{
    void cvt_ij_to_xy();
    void cvt_xy_to_ij();
    boolean all_bndrys_constant();
    void error_exit();
    static void allocate_line_end_dsdrs();
    static enum relation temp_bndry_inout();

#define FIX_UNRUE_GAP    0        /* # extra steps to advance at sweep end */

    int p;
    enum bndry bndry;
    int i,j;
    double x, y;
    int *pij_fixed, ij_fixed_start, ij_fixed_stop;
    int *pij_scan, ij_scan_start, ij_scan_dir, scan_count, N_scan;
    struct bndry_defn_term *hit_term;

    for (p = 1 ; p <= pgd->N_patch ; ++p)
    {

```

patchutil directory

SETPATCH.C;417

21-SEP-1985 19:07

Page 4

```

/* process patch *p: */

if (all_bndrys_constant(& patch_bndry[p]))
    then continue; /* with next patch */

/*
 * get to here --> patch has one or more 'patch-... bndry-...' type
 * boundaries
 */
/* convert bounding box (x,y) to (i,j) coordinates: */
cvt_xy_to_ij(& (pgd->patch[p]), patch_bndry[p].x_min_bound,
             patch_bndry[p].y_min_bound,
             &pgd->patch[p].i_min, &pgd->patch[p].j_min);
cvt_xy_to_ij(& (pgd->patch[p]), patch_bndry[p].x_max_bound,
             patch_bndry[p].y_max_bound,
             &pgd->patch[p].i_max, &pgd->patch[p].j_max);

/* allocate array storage: */
allocate_line_end_dscols(gridname, p, & pgd->patch[p]);

/*
 * The desired limits are those points which first give a 'not-inside'
 * status, in a scan 'outward' from the patch.
 */

/* Fill in array storage: */
for (bndry = I_MIN ; bndry <= J_MAX ; ++bndry)
{
    /* fill in array storage for this boundary: */

    /*
     * Set up pointers so we know what variables to use; also do
     * limits & counts so we know just how much to do:
     */
    switch (bndry)
    {
        case I_MIN:
        case I_MAX:
            pij_fixed = &j; /* keep j fixed */
            ij_fixed_start = pgd->patch[p].j_min;
            ij_fixed_stop = pgd->patch[p].j_max;
            pij_scan = &i; /* scan i */
            N_scan = pgd->patch[p].i_max - pgd->patch[p].i_min + 1;
            break;
        case J_MIN:
        case J_MAX:
            pij_fixed = &i; /* keep i fixed */
            ij_fixed_start = pgd->patch[p].i_min;
            ij_fixed_stop = pgd->patch[p].i_max;
            pij_scan = &j; /* scan j */
            N_scan = pgd->patch[p].j_max - pgd->patch[p].j_min + 1;
            break;
    }
    switch (bndry)
    {

```


patchutil directory

SETPATCH.C;417

21-SEP-1985 19:07

Page 5

```

case I_MIN:
    ij_scan_start = pgd->patch[p].i_max;
    ij_scan_dir = -1;
    break;
case I_MAX:
    ij_scan_start = pgd->patch[p].i_min;
    ij_scan_dir = 1;
    break;
case J_MIN:
    ij_scan_start = pgd->patch[p].j_max;
    ij_scan_dir = -1;
    break;
case J_MAX:
    ij_scan_start = pgd->patch[p].j_min;
    ij_scan_dir = 1;
    break;
}

/*
 * First step the 'fixed' variable, which specifies the grid
 * line being considered: (Note that the order in which the
 * different values of this variable are tried is unimportant)
 */
for (*pij_fixed = ij_fixed_start ;
    *pij_fixed <= ij_fixed_stop ;
    ++*pij_fixed)
{
    /*
     * Then step the 'scan' variable to find the first
     * 'non-inside' value, checking against the desired
     * boundary only. This is the desired limit value,
     * modulo a bit of fixing up for the Unruh gap et al.
     */
    for (*pij_scan = ij_scan_start , scan_count=0 ;
        scan_count < N_scan ;
        *pij_scan += ij_scan_dir, ++scan_count)
    {
        cvt_ij_to_xy( &(amp;pgd->patch[p]), i, j, &x, &y);

        if (temp_bndry_inout(patch_bndry, pgd, p,
                             x, y,
                             bndry,
                             &hit_term)

            != INSIDE)
            then goto found;
    }

    /*
     * get to here --> trouble -- we failed to find the
     * boundary!!!
     */
    error_exit(OK_EXIT, "\
***** set_patch_shapes: bounding box too small at (%d,%d)\n\
in patch %d of grid \"%s\"\n",
              i, j,
              p,

```

patchutil directory

SETPATCH.C;417

21-SEP-1985 19:07

Page 6

```

                                gridname);  /* DOES NOT RETURN */

/*
 * 'pij_scan' is now the desired 'end_ij' value, modulo some +/- 1 and/or
 * Unruh gap type fixups. In particular:
 *   - We want to sweep right to 'constant=...' boundaries; no fixups needed
 *     here.
 *   - We don't want overlap when interpolating into another patch; we have
 *     gone one step too far here.
 *   - In general, if we are interpolating into another patch, we should go
 *     FIX_UNRUH_GAP steps farther.
 */
found:

                                if (hit_term->defn_type == PATCH_BNDRY_TYPE)
                                    then {
                                        /* fixup: */
                                        *pij_scan -= ij_scan_dir;
                                        *pij_scan += ij_scan_dir * FIX_UNRUH_GAP;
                                    }
                                pgd->patch[p].line_end[bndry][*pij_fixed].end_ij
                                    = *pij_scan;

                                /* end of this 'fixed' point, i.e. line_end_dscr */
                                }

                                /* end of this boundary */
                                }

                                /* end of this patch */
                                }
}

/*****
/*
 * This function fixes up the inconsistencies in the patch boundaries produced
 * by 'set_up_patch_bndrys(...)'. In particular:
 *   - It adjusts the 'size info' downwards, if needed, to correspond to the
 *     actual patch sizes, which are in general smaller than the 'bounding
 *     box' sizes which were originally used.
 *   - It enforces consistency between endpoints of vertical and horizontal
 *     scans, requiring that they end on the same grid points.
 *
 * Note also that only 'non-constant boundary' patches need fixing!!!
 *
 * Arguments:
 *   gridname - The grid name for error messages. (Currently none exist!!!)
 *   patch_bndry --> (1 origin) The temporary patch boundary data structure.
 *   pgd --> The grid descriptor structure to be filled in.
 */
static void fix_up_patch_bndrys(gridname, patch_bndry, pgd)
char gridname[];
register struct patch_bndry *patch_bndry;
register struct grid_dscr *pgd;
{
    boolean all_bndrys_constant();

```

patchutil directory

SETPATCH.C;417

21-SEP-1985 19:07

Page 7

```

#define BIG      ( ((unsigned) -1) >> 1 )      /* largest positive integer */
int p;
int i, j;
register struct coord_patch_descr *ptr;
enum bndry bndry;
int changes;
int ij, ij_min, ij_max; /* case (bndry) of j j i i */
int temp, limit;        /* case (bndry) of i i j j */
enum bndry min_limit_bndry, max_limit_bndry;
int *ip;

    for (p = 1; p <= pgd->N_patch ; ++p)
    {
        if (all_bndrys_constant(& patch_bndry[p]))
            then continue; /* with next patch ('constant' --> no fixes) */

        ptr = & pgd->patch[p];

        /*
         * 'Downsize' 'size info' if needed:
         *
         * Since a given 'downsizing' will in general exclude some lines which
         * had a role in determining the limits, we must iterate until no
         * further changes occur. This iteration will always terminate, since
         * each iteration shrinks the patch size in at least one coordinate,
         * while the size is bounded below (by zero).
         */
        do
        {
            changes = FALSE;

            /* fix up min/max for i: */
            ij_min = BIG;
            ij_max = -BIG;
            for (j = ptr->j_min ; j <= ptr->j_max ; ++j)
            {
                temp = ptr->line_end[I_MIN][j].end_ij;
                if (temp < ij_min) then ij_min = temp;

                temp = ptr->line_end[I_MAX][j].end_ij;
                if (temp > ij_max) then ij_max = temp;
            }
            if (ij_min != ptr->i_min) then changes = TRUE;
            ptr->i_min = ij_min;
            if (ij_max != ptr->i_max) then changes = TRUE;
            ptr->i_max = ij_max;

            /* fix up min/max for j: */
            ij_min = BIG;
            ij_max = -BIG;
            for (i = ptr->i_min ; i <= ptr->i_max ; ++i)
            {
                temp = ptr->line_end[J_MIN][i].end_ij;
                if (temp < ij_min) then ij_min = temp;
            }
        } while (changes);
    }

```

patchutil directory

SETPATCH.C;417

21-SEP-1985 19:07

Page 8

```

        temp = ptr->line_end[J_MAX][i].end_ij;
        if (temp > ij_max) then ij_max = temp;
    }
    if (ij_min != ptr->j_min) then changes = TRUE;
    ptr->j_min = ij_min;
    if (ij_max != ptr->j_max) then changes = TRUE;
    ptr->j_max = ij_max;
}
while (changes);

/*
 * Enforce endpoint consistency between different sweep directions:
 */
for (bndry = I_MIN ; bndry <= J_MAX ; ++bndry)
{
    /* find loop limits: */
    switch (bndry)
    {
        case I_MIN:
        case I_MAX:
            ij_min = ptr->j_min;
            ij_max = ptr->j_max;
            min_limit_bndry = J_MIN;
            max_limit_bndry = J_MAX;
            break;
        case J_MIN:
        case J_MAX:
            ij_min = ptr->i_min;
            ij_max = ptr->i_max;
            min_limit_bndry = I_MIN;
            max_limit_bndry = I_MAX;
            break;
    }

    /* do enforcement for this boundary: */
    for (ij = ij_min ; ij <= ij_max ; ++ij)
    {
        limit = ptr->line_end[bndry][ij].end_ij;

        /* extend down??? */
        ip = & ptr->line_end[min_limit_bndry][limit].end_ij;
        if (ij < *ip) then *ip = ij;

        /* extend up??? */
        ip = & ptr->line_end[max_limit_bndry][limit].end_ij;
        if (ij > *ip) then *ip = ij;
    }

    /* end of this boundary */
}

/* end of this patch */
}

```

patchutil directory

SETPATCH.C;417

21-SEP-1985 19:07

Page 9

```

/*****
/*
 * This function finds out whether or not all the boundaries of a given patch
 * defined by 'constant-...' terms, using the temporary patch boundary data
 * structure. If so it returns TRUE, if not, FALSE.
 *
 * Arguments:
 * ppbs --> The patch_bndry structure for the patch.
 */
boolean all_bndrys_constant(ppbs)
register struct patch_bndry *ppbs;
{
    register struct bndry_defn_term *ptr;
    register enum bndry bndry;

    for (bndry = I_MIN ; bndry <= J_MAX ; ++bndry)
    {
        for (ptr = ppbs->bdt_list[bndry] ;
             ptr != NULL ;
             ptr = ptr->next)
        {
            if (ptr->defn_type != CONSTANT_BNDRY_TYPE) then return(FALSE);
        }
    }

    return(TRUE);
}

/*****
/*
 * Assuming that all boundaries of a patch are defined by 'constant-...' terms,
 * this function sets the size info fields in the patch descriptor, using the
 * temporary patch boundary data structure. Since only one 'constant-...' term
 * per patch boundary is meaningful for setting the boundary position (and
 * hence the patch size info), only the first entries in the boundary def'n
 * term lists are used -- any and all remaining entries are ignored.
 *
 * Arguments:
 * patch_bndry --> (1 origin) The temporary patch boundary data structure.
 * pgd --> the grid_dsor structure.
 * p is the patch number.
 */
static void set_constant_size_info(patch_bndry, pgd, p)
register struct patch_bndry *patch_bndry;
register struct grid_dsor *pgd;
register int p;
{
    int round_double_to_int();

    register struct coord_patch_dsor *ptr;
    double x_min, x_max, y_min, y_max;

```

patchutil directory

SETPATCH.C:417

21-SEP-1985 19:07

Page 10

```

/* get floating point coordinate limits: */
x_min = patch_bndry[p].bdt_list[I_MIN]->defn_value.value;
x_max = patch_bndry[p].bdt_list[I_MAX]->defn_value.value;
y_min = patch_bndry[p].bdt_list[J_MIN]->defn_value.value;
y_max = patch_bndry[p].bdt_list[J_MAX]->defn_value.value;

/* compute and store integer coordinate limits: */
ptr = &pgd->patch[p];
ptr->i_min = round_double_to_int( x_min / ptr->i_step );
ptr->i_max = round_double_to_int( x_max / ptr->i_step );
ptr->j_min = round_double_to_int( y_min / ptr->j_step );
ptr->j_max = round_double_to_int( y_max / ptr->j_step );
}

/*****
/*
* This function uses the temporary patch boundary data structure to find the
* in/on/out relationship of a given point and a given patch.
*
* Arguments:
* patch_bndry --> (1 origin) The temporary patch boundary data structure.
* pgd --> The grid descriptor structure.
* p - The patch number to check.
* (x,y) - The coordinates of the point to check (in the system of patch 'p').
* bndry - The boundary to check against.
* pptrm --> Where to store a pointer the 'hitting' term, or NULL to take no
*          action to save this information. The 'hitting' term is the one
*          with an OUTSIDE status if this function returns OUTSIDE, the last
*          encountered one with an ON_BNDRY status if this function returns
*          ON_BNDRY, and is undefined if this function returns INSIDE. In
*          this last case, a 'request to store' causes NULL to be stored.
*
* The relationship is returned as the function value.
*/

static enum relation temp_bndry_inout(patch_bndry, pgd, p, x, y, bndry, pptrm)
register struct patch_bndry *patch_bndry;
register struct grid_dscr *pgd;
register int p;
double x, y;
enum bndry bndry;
struct bndry_defn_term * *pptrm;          /* --> a (struct bndry_defn_term *) */
{
enum relation temp_bndry_inout_term();

register struct bndry_defn_term *ptr, *dummy;
enum relation r;
boolean on_flag;

if (pptrm == NULL) then pptrm = &dummy;
*pptrm = NULL;

/* check against each term on this boundary's list: */
on_flag = FALSE;

```

patchutil directory

SETPATCH.C:417

21-SEP-1985 19:07

Page 11

```

    for (ptr = patch_bndry[p].bdt_list[bndry] ;
        ptr != NULL ;
        ptr = ptr->next )
    {
        r = temp_bndry_inout_term(patch_bndry, pgd, p, x, y, bndry, ptr);
        if (r != INSIDE) then *ppterm = ptr;

        if (r == OUTSIDE) then return(r);
        if (r == ON_BNDRY) then on_flag = TRUE;
    }

/* get to here --> failed all 'outside or on this boundary?' tests: */
return(on_flag ? ON_BNDRY : INSIDE);
}

/*****
/*
* This function uses the temporary patch boundary data structure to find the
* in/on/out relationship of a given point w.r.t. a given boundary def'n term.
*
* Arguments:
* patch_bndry --> (1 origin) The temporary patch boundary data structure.
* pgd --> The grid descriptor structure.
* p - The patch number to check.
* (x,y) - The coordinates of the point to check (in the system of patch 'p').
* bndry - The boundary that the term is on -- this is needed to set the parity
*         of the test.
* ptr --> The boundary definition term to check for a hit.
*
* The returned function value is the desired relationship.
*/
enum relation temp_bndry_inout_term(patch_bndry, pgd, p, x, y, bndry, ptr)
register struct patch_bndry *patch_bndry;
register struct grid_dscr *pgd;
int p;
double x, y;
enum bndry bndry;
register struct bndry_defn_term *ptr;
{
    boolean fuzzy_EQ();          /* fuzzy (as in APL) equality comparison */
    void cvt_xy_to_xy();         /* converts (x,y) coord's between patches */

    double val;
    double x_other, y_other;
    int other_patch;
    enum bndry other_bndry;
    double other_val;

    if (ptr->defn_type == CONSTANT_BNDRY_TYPE)
        then {
            /* boundary defn term is of 'constant-...' type: */
            val = ptr->defn_value.value;

            /* does it fall fuzzily on the boundary? */

```

patchutil directory

SETPATCH.C;417

21-SEP-1985 19:07

Page 12

```

switch (bndry)
{
case I_MIN:
case I_MAX:
    if (fuzzy_EQ(x, val)) then return(ON_BNDRY);
    break;
case J_MIN:
case J_MAX:
    if (fuzzy_EQ(y, val)) then return(ON_BNDRY);
    break;
}

/* does it fall outside the boundary: */
switch (bndry)
{
case I_MIN:
    if (x < val) then return(OUTSIDE);
    break;
case I_MAX:
    if (x > val) then return(OUTSIDE);
    break;
case J_MIN:
    if (y < val) then return(OUTSIDE);
    break;
case J_MAX:
    if (y > val) then return(OUTSIDE);
    break;
}
}
else {
    /* bndry defn term is of 'patch-... bndry-...' type: */

    /* get coordinates of point in other patch's coordinate system: */
    other_patch = ptr->defn_value.patch_info.patch;
    other_bndry = ptr->defn_value.patch_info.bndry;
    other_val = patch_bndry[other_patch]
        .bdt_list[other_bndry]
        ->defn_value.value;
    cvt_xy_to_xy(&pgd->patch[p], x, y,
        &pgd->patch[other_patch], &x_other, &y_other);

    /* does it fall fuzzily on the (other) boundary? */
    switch (other_bndry)
    {
    case I_MIN:
    case I_MAX:
        if (fuzzy_EQ(x_other, other_val)) then return(ON_BNDRY);
        break;
    case J_MIN:
    case J_MAX:
        if (fuzzy_EQ(y_other, other_val)) then return(ON_BNDRY);
        break;
    }
}

/*

```


patchutil directory

SETPATCH.C:417

21-SEP-1985 19:07

Page 13

```

    * Note reversed sense of comparisons in code below!!!
    */
    /* does it fall outside our boundary, i.e. inside the other one: */
    switch (other_bndry) /* check them! */
    {
    case I_MIN:
        if (x_other > other_val) then return(OUTSIDE);
        break;
    case I_MAX:
        if (x_other < other_val) then return(OUTSIDE);
        break;
    case J_MIN:
        if (y_other > other_val) then return(OUTSIDE);
        break;
    case J_MAX:
        if (y_other < other_val) then return(OUTSIDE);
        break;
    }

    /* get to here --> failed all chances at a hit --> inside */
    return(INSIDE);
}

/*****

/*
 * Given that the size info fields have been filled in, this routine allocates
 * (and stores the pointers to) the 4 arrays of line_end_dscr structures used
 * in a coordinate patch descriptor.
 *
 * Arguments:
 * gridname - The grid name for error messages (only).
 * p - The patch number, also for error messages (only).
 * ptr --> the coordinate patch descriptor.
 */
static void allocate_line_end_dsors(gridname, p, ptr)
char gridname[];
int p;
register struct coord_patch_dscr *ptr;
{
    void error_exit();
    char *calloc();

    /* note '1' line end arrays sized by 'JSIZE' and vice versa!!! */
#define ISIZE ptr->i_max - ptr->i_min + 1
#define JSIZE ptr->j_max - ptr->j_min + 1
    ptr->line_end[I_MIN] = ALLOC(JSIZE, struct line_end_dscr);
    ptr->line_end[I_MAX] = ALLOC(JSIZE, struct line_end_dscr);
    ptr->line_end[J_MIN] = ALLOC(ISIZE, struct line_end_dscr);
    ptr->line_end[J_MAX] = ALLOC(ISIZE, struct line_end_dscr);

    if ( (ptr->line_end[I_MIN] == NULL)
        || (ptr->line_end[I_MAX] == NULL)
        || (ptr->line_end[J_MIN] == NULL)

```

patchutil directory

SETPATCH.C:417

21-SEP-1985 19:07

Page 14

```
    || (ptr->line_end[J_MAX] == NULL)
  )
  then error_exit(OK_EXIT, "\
**** set_patch_shapes: insufficient memory for ISIZE=%d, JSIZE=%d\n\
        line_end arrays in patch %d of grid \"%s\"\n",
        ISIZE, JSIZE,
        P,
        gridname); /* DOES NOT RETURN */

/* offset pointers to allow for non-zero-origin array indexing: */
ptr->line_end[I_MIN] == ptr->j_min;
ptr->line_end[I_MAX] == ptr->j_min;
ptr->line_end[J_MIN] == ptr->i_min;
ptr->line_end[J_MAX] == ptr->i_min;
}
```

util directory

BANDMAT.H:8

21-SEP-1985 20:02

Page 1

```
/* This file defines my C 'row-oriented' storage format for band matrices: */
#ifdef COMMENT
The format is perhaps best illustrated by the following example - this matrix:
```

a00	a01	a02	0	0	0	0	0	0	0
a10	a11	a12	a13	0	0	0	0	0	0
a20	a21	a22	a23	a24	0	0	0	0	0
0	a31	a32	a33	a34	a35	0	0	0	0
0	0	a42	a43	a44	a45	a46	0	0	0
0	0	0	a53	a54	a55	a56	a57	0	0
0	0	0	0	a64	a65	a66	a67	a68	0
0	0	0	0	0	a75	a76	a77	a78	a79
0	0	0	0	0	0	a86	a87	a88	a89
0	0	0	0	0	0	0	a97	a98	a99

will be stored like this:

0	0	a00	a01	a02
0	a10	a11	a12	a13
a20	a21	a22	a23	a24
a31	a32	a33	a34	a35
a42	a43	a44	a45	a46
a53	a54	a55	a56	a57
a64	a65	a66	a67	a68
a75	a76	a77	a78	a79
a86	a87	a88	a89	0
a97	a98	a99	0	0

Note that my conventions are such that this matrix has 'bandwidth' 5.
#endif

```
#define BAND_MATRIX_ELT(mat_, bandwidth_, i_, j_) \
( ((mat_) + ((bandwidth_) >> 1) + (i_)*(bandwidth_))[(j_)-(i_)] )
```

util directory

KEYWORD.H;11

21-SEP-1985 19:58

Page 1

```
/* Variable Descriptor (structure) definition: */
struct var_dscr
{
    char *name;           /* points to 'character string' name */
    char *value;          /* points to (value of) variable itself */
                          /* NULL --> skip assignment */
                          /* Note that this must be type-cast before */
                          /* it is either used or set. */
    char *in_format;      /* points to 'scanf' format string */
                          /* (including '%' character at the start) */
                          /* (may be NULL if 'value' also is) */
                          /* Note that '%s' here need not imply the */
                          /* absence of white space in the string!!! */
};

/* Return codes (other than EOF) from 'keyword_scan': */
#define SYNTAX_ERROR    -10
#define VALUE_ERROR     -11
#define UNKNOWN_KEYWORD -12
```

util directory

STDC.H;27

21-SEP-1985 19:58

Page 1

```

/* My "standard c" include file: */
#define then
#define void      int
#define boolean unsigned

#define ungetchar(o)      ungeto((o), stdin)
/* note that the next line overrides the VMS library 'gets' routine!!! */
#define gets(line, maxline)      fgets(line, maxline, stdin)

#define GOBBLE              { do { } while ((getchar()) != '\n'); }

#define ALLOC(count_, type_)      ( (type_ *) calloc(count_, sizeof(type_)) )

#define NULL      0
#define EOF      (-1)
#define TRUE      1
#define FALSE      0
#define OK_EXIT      1      /* 'exit()' code for ok (VMS-specific) */
#define TROUBLE_EXIT      0      /* 'exit()' code for trouble (VMS-specific) */

#define PI      3.14159265358979323846264338
#define E      2.718281828459054

#define BIG_INT      ((int) ( ((unsigned) -1) >> 1))
#define BIG_FLOAT      (1.0e38)

#define strmov(src,dst)      strcpy(dst,src)
#define strnmov(src,dst,max)      strncpy(dst,src,max)
#define arctan2(x_,y_)      atan2(y_,x_)

```

util directory

ARRAYIO.C;2

21-SEP-1985 20:03

Page 1

```

#include <stdio.h>
#include "util:stdio.h"

/*
 * This function reads or writes an array from a disk file. It is the only
 * function which knows about how arrays are stored in such files -- currently
 * in 'core image' ('binary') form.
 *
 * Arguments:
 * filename is (a pointer to) the character string name of the file.
 * type is (a pointer to) one of the character strings "float" or "double",
 *     telling what type of floating point numbers are to be read/written.
 * dir is (a pointer to) one of the character strings "read" or "write",
 *     telling which operation to perform. (Actually, only the 1st character
 *     is looked at -- the rest are ignored!)
 * count is the number of values to read/write.
 * array is a pointer to the data array area in memory.
 *
 * The function returns the number of items actually read. If an error occurs
 * in trying to open or read/write the file, EOF is returned.
 *
 * BUGS:
 * - I'm not quite sure what happens if end-of-file is encountered in a read.
 */
int array_io(filename, type, dir, count, array)
char *filename;
char *type, *dir;
int count;
char *array; /* actually it's either 'float *' or 'double *' */
{
    int fread(), fwrite(); /* VMS-specific??? */
    FILE *fopen();
    void fclose();

    int status, item_size;
    char sdir[2]; /* 'short dir' -- only 1st character wanted for 'fopen' */
    FILE *fp;

    sprintf(sdir, "%c", *dir);
    fp = fopen(filename, sdir);
    if (fp == NULL) then return(EOF); /* error flag */

    item_size = (strcmp(type, "float") == 0) ? sizeof(float) : sizeof(double);

    status = (*sdir == 'w')
        ? fwrite(array, item_size, count, fp)
        : fread(array, item_size, count, fp);
    if (status == 0) then status = EOF; /* error flag */

    fclose(fp);
    return(status);
}

```

util directory

BGEC1.C;36

21-SEP-1985 20:03

Page 1

```

#include <stdio.h>
#include "util:stdio.h"
#include "util:bandmat.h"

typedef double PRECISION;

/*
 * This routine solves a system of linear equations by the method of (banded)
 * Gaussian Elimination.
 *
 * Arguments:
 * neqn is a pointer to the number of equations.
 * bandwidth a pointer to the 'bandwidth' of the (coefficient) matrix, as
 *   defined in 'bandmat.h'.
 * matrix is a pointer to the matrix storage area. The matrix format is as
 *   defined in 'bandmat.h'. Note that this matrix is modified during the
 *   solution process.
 * rhs is a pointer to the right-hand-side vector. Note that this vector is
 *   modified during the solution process.
 * soln is a pointer to where the solution is to be stored.
 *
 * Notes:
 *   - No pivoting done --> all diagonal matrix elements MUST be
 *     non-zero (KLUGE!).
 *   - 'neqn' and 'bandwidth' are pointers so this is FORTRAN-
 *     compatible!!!
 */
void bge(neqn, bandwidth, matrix, rhs, soln)
int *neqn, *bandwidth;
PRECISION *matrix, *rhs, *soln;
{
    make_triangular(*neqn, *bandwidth, matrix, rhs);
    back_substitute(*neqn, *bandwidth, matrix, rhs, soln);
}

/*****/

#define mat(row, col)  BAND_MATRIX_ELT(matrix, bandwidth, row, col)
#define HALF_WIDTHTH  (bandwidth >> 1)

#define GETLIM
    limit = 1 + HALF_WIDTHTH;
    if (limit >= neqn) then limit = neqn-1;

/*****/

static void make_triangular(neqn, bandwidth, matrix, rhs)
register int neqn, bandwidth;
register PRECISION *matrix, *rhs;
{
    register int i, j, k, limit;
    register PRECISION factor;

    for (i=0 ; i<neqn ; ++i)
    {
        /*

```

util directory

BGEC1.C;36

21-SEP-1985 20:03

Page 2

```

    * Use i'th equation (row in matrix) to kill off non-zero elements in
    * all equations (typical one is 'number j') below this one:
    */
GETLIM /* compute 'limit' */
      for (j=i+1 ; j<=limit ; ++j)
      {
        factor = - mat(j,i) / mat(i,i);
        rhs[j] += factor * rhs[i];
        mat(j, i) = 0.0;
        for (k=i+1; k<=limit ; ++k)
        {
          mat(j,k) += factor * mat(i,k);
        }
      }
    }
  }

/*****/

static void back_substitute(neqn, bandwidth, matrix, rhs, soln)
register int neqn, bandwidth;
register PRECISION *matrix, *rhs, *soln;
{
  register int i, j, limit;
  register PRECISION sum;

  for (i=neqn-1 ; i>=0 ; --i)
  {
    /* solve for soln[i], using soln[i+1]...soln[neqn-1]: */
    GETLIM /* compute 'limit' */
    sum = 0.0;
    for (j=limit ; j>=i ; --j)
    {
      sum += mat(i,j) * soln[j];
    }
    soln[i] = (rhs[i] - sum) / mat(i,i);
  }
}

```


util directory

CALLTRAIL.MAR;13

4-NOV-1985 00:16

Page 1

```
.title call_trail
.sbtbl call_trail
```

Usage: (C)

```
call_trail(pfn, ptal, ...);
```

or:

```
result = call_trail(pfn, ptal, ...);
```

This function calls a specified function with the caller's 'trailing' arguments, and a specified (possibly empty) vector of 'leading' arguments.

Arguments:

pfn --> The function to be called.

ptal --> The first trailing argument of our caller.

Any remaining arguments are considered as 'leading' ones.

Example:

```
#include <stdio.h>
foo(file, format)
char file[], format[];
{
    FILE *fopen();
    void fprintf(), fclose();
    void call_trail();
    FILE *fileptr;
    fileptr = fopen(file, "a");
    call_trail(&fprintf, &format, fileptr);
    fclose(fileptr);
}
```

This C function, when invoked with

```
foo("file.ext", "i-%d j-%d\n", i, j);
```

logs a line giving the values of i and j to the file 'file.ext'.

Example argument lists:

```

+-----+-----+
|          | 3 | <----- our ap
+-----+-----+
<<<fprintf>>> <-----*
+-----+-----+
|          | * |
+-----+-----+
|      fileptr      |
+-----+-----+
+-----+-----+
|          | 4 | <----- foo's ap
+-----+-----+
"file.ext" <-----*
+-----+-----+
"i-%d j-%d\n" <-----*
+-----+-----+
|          | 1 |
+-----+-----+
```

util directory

CALLTRAIL.MAR;13

4-NOV-1985 00:16

Page 2

```

;
;                                     +-----+
;                                     |       j       |
;                                     +-----+
;
; What we want, is to set up to do the call
;     fprintf(fileptr, "i-%d j-%d\n", i, j);
; (on our stack).
;
;::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
; Offsets:
;
; pfn          = 4      ; our arguments, from our ap
; ptal         = 8
; first_trail  = 12
;
; saved_ap     = 8      ; our caller's saved ap, from our fp
;
;::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
;
;     .entry call_trail, ^m(r2,r3,r4,r5, r6)
;
; Copy our caller's trailing arguments (if any) to our stack:
;
; We already have a pointer to the 1st of these, so what we need to know is how
; many of them there are:
;
;     movl      @saved_ap(fp), r0      ; --> start of caller's arglist
;     movzbl    (r0), r1               ; # caller's args
;     movl      (r0)[r1], r1           ; --> caller's last argument
;     subl      ptal(ap), r1           ; 4 * ((* caller's trailing args) - 1)
;     addl      #4, r1                 ; 4 * (* caller's trailing args)
;                                     ; = length in bytes of caller's
;                                     ; trailing args
;     ashl      #-2, r1, r6            ; # caller's trailing args
;     subl      r1, sp                 ; allocate space on our stack
;     movc3     r1, @ptal(ap), (sp)    ; copy caller's trailing args
;
; Copy our own trailing arguments (if any) to our stack.
;
; non_trail_count = 2
;     movzbl    (ap), r0               ; # our args
;     subl      #non_trail_count, r0   ; # our trailing args
;     addl      r0, r6                 ; bump running count of final call
;                                     ; arguments
;     ashl      #2, r0, r0              ; length in bytes of our trailing args
;     subl      r0, sp                 ; allocate space on our stack
;     movc3     r0, first_trail(ap), (sp) ; copy our trailing args
;
; Do the call:
;
;     calls     r6, @pfn(ap)           ; ZAP!
;
;     ret
;     .end

```

util directory

DUMPARRAY.C;7

21-SEP-1985 20:04

Page 1

```

/* This file contains routines to dump out arrays in human-readable format */
/*
 * The following functions live here:
 * <description of common argument list for dump routines>
 * dump_int_array - For 'int' arrays.
 * dump_float_array - For 'float' arrays.
 * dump_double_array - For 'double' arrays.
 */

#include <stdio.h>
#include "util:stdio.h"

/*****
/*
 * Arguments: (common for all dump functions)
 * fp --> FILE (defined in 'stdio.h') to dump into (must already be open).
 * array - Points to [0] entry in array -- actual dump will be of segment
 *         'array[start...stop]'.
 * start - Starting index in array to dump.
 * stop - Stopping index in array to dump.
 * n_per_line - Number of array elements per 'line'. A 'line' is defined by
 *              the 'bol' and 'eol' arguments below.
 * bol_val - Defines beginning of line to be when the condition
 *            ((index % n_per_line) == bol_val)
 *            is TRUE.
 * bol_fmt - If non-NULL, 'fprintf(...)' format to dump index in at beginning
 *            of line.
 * array_fmt - 'fprintf(...)' format to dump each array element in.
 * eol_val - Defines end of line to be when the condition
 *           ( ((index % n_per_line) == eol_val) || (index == stop) )
 *           is TRUE.
 * eol_fmt - If non-NULL, will be 'fprintf(...)'ed at end of line. This is
 *            typically the string "\n".
 *
 * Example of usage to dump FORTRAN array A(1...N), from a C program:
 *         dump_float_array(fp, A,1,N, 10, 1,"%6d:", "%6d", 0,"\n");
 */
*****/

void dump_int_array(fp,
                   array, start, stop,
                   n_per_line,
                   bol_val, bol_fmt,
                   array_fmt,
                   eol_val, eol_fmt)

FILE *fp;
int array[];
int start, stop, n_per_line;
int bol_val;
char bol_fmt[];
char array_fmt[];
int eol_val;
char eol_fmt[];

```

util directory

DUMPARRAY.C;7

21-SEP-1985 20:04

Page 2

```

{
void fprintf();

int i;

    for (i = start ; i <= stop ; ++i)
    {
        if ((bol_fmt != NULL) && ((i % n_per_line) == bol_val))
            then fprintf(fp, bol_fmt, i);
        fprintf(fp, array_fmt, array[i]);
        if ( (eol_fmt != NULL)
            && ( ((i % n_per_line) == eol_val) || (i == stop) )
        )
            then fprintf(fp, eol_fmt);
    }
}

/*****/

void dump_float_array(fp,
                    array, start, stop,
                    n_per_line,
                    bol_val, bol_fmt,
                    array_fmt,
                    eol_val, eol_fmt)

FILE *fp;
float array[];
int start, stop, n_per_line;
int bol_val;
char bol_fmt[];
char array_fmt[];
int eol_val;
char eol_fmt[];
{
void fprintf();

int i;

    for (i = start ; i <= stop ; ++i)
    {
        if ((i % n_per_line) == bol_val) then fprintf(fp, bol_fmt, i);
        fprintf(fp, array_fmt, array[i]);
        if ( (eol_fmt != NULL)
            && ( ((i % n_per_line) == eol_val) || (i == stop) )
        )
            then fprintf(fp, eol_fmt);
    }
}

/*****/

void dump_double_array(fp,
                    array, start, stop,
                    n_per_line,
                    bol_val, bol_fmt,

```

util directory

DUMPARRAY.C;7

21-SEP-1985 20:04

Page 3

```

                                array_fmt,
                                eol_val, eol_fmt)
FILE *fp;
double array[];
int start, stop, n_per_line;
int bol_val;
char bol_fmt[];
char array_fmt[];
int eol_val;
char eol_fmt[];
{
void fprintf();

int i;

    for (i = start ; i <= stop ; ++i)
    {
        if ((i % n_per_line) == bol_val) then fprintf(fp, bol_fmt, i);
        fprintf(fp, array_fmt, array[i]);
        if ( (eol_fmt != NULL)
            && ( (i % n_per_line) == eol_val) || (i == stop) )
        )
            then fprintf(fp, eol_fmt);
    }
}

```

util directory

ERROREXIT.C;2

21-SEP-1985 20:05

Page 1

```
#include <stdio.h>
#include "util:stdio.h"

/*
 * This function optionally prints an error message to the standard error
 * output stream before exiting.  IT DOES NOT RETURN TO IT'S CALLER!!!
 *
 * Arguments:
 * code - Code to send to 'exit(...)', which does actual exiting.
 *
 * Note:      If any further arguments are supplied, 'fprintf(stderr, ...)'
 *            is called with them.
 */
void error_exit(code, format)
int code;
char format[];
{
void fprintf();
void exit();

void call_trail();
int getarg();

if (getarg(0) > 1)
    then call_trail(&fprintf, &format, stderr);

exit(code);
}
```

util directory

FIXNAME.C;13

21-SEP-1985 20:05

Page 1

#include "util:stdio.h"

```

/*
 * This function 'fixes up' a VMS C style pathname to produce just the program
 * name itself, as would be typed on UNIX or with my VMS 'install' utility.
 *
 * Arguments:
 *   pathname is the VMS C pathname. Typically this will be argv[0] from the C
 *   'main(argo,argv)' function.
 *
 * A pointer to the 'fixed' name is returned. This points to a private static
 * character array which is overwritten on successive calls.
 */
char *fixname(pathname)
char *pathname;
{
/*
 * What is wanted is the substring starting just after the '[' and running to
 * just before the next '.' in the VMS pathname.
 */
#define START_CHAR '['
#define STOP_CHAR '.'
#define SIZE 50 /* make sure its big enough!!! */
static char fixed_name[SIZE];
int j;
boolean copy_flag;

copy_flag = FALSE;
for (j = 0 ; *pathname != '\0' ; ++pathname)
{
    if (copy_flag && (*pathname == STOP_CHAR)) then break;
    if (copy_flag) then fixed_name[j++] = *pathname;
    if (*pathname == START_CHAR) then copy_flag = TRUE;
}
fixed_name[j] = '\0';

return(fixed_name);
}

```

util directory

FUZZ.C;23

4-NOV-1988 00:18

Page 1

```

/* This file contains 'fuzzy arithmetic' functions -- all are 'global' */
/*
 * Functions available:
 * fuzzy_LT - TRUE/FALSE x < y test
 * fuzzy_LE - TRUE/FALSE x <= y test
 * fuzzy_EQ - TRUE/FALSE x == y test (note all other fuzzy fns based on this)
 * fuzzy_NE - TRUE/FALSE x != y test
 * fuzzy_GE - TRUE/FALSE x >= y test
 * fuzzy_GT - TRUE/FALSE x > y test
 * fuzzy_floor - Largest integer fuzzily less than or equal to argument
 * fuzzy_ceil - Smallest integer fuzzily greater than or equal to argument
 */

/*
 * WARNING:
 * Although the design of these functions is based on those of APL, the
 * definition of 'fuzzy_EQ' is somewhat different from that of the ISO
 * (proposed) APL standard. However, the other fuzzy-relational-functions
 * are based on 'fuzzy_EQ' in the (ISO standard) APL manner. One of the
 * (desirable) consequences of this is that exactly one of the three fuzzy
 * comparisons 'fuzzy_LT(x,y)', 'fuzzy_EQ(x,y)', and 'fuzzy_GT(x,y)' will
 * be TRUE, for any pair of arguments x and y.
 */

#include "util:stdio.h"

double quad_CT = 1.0e-10; /* this can be changed if desired */

/*****
boolean fuzzy_LT(x, y) /* 'x < y' comparison with APL-type 'fuzz' */
double x, y;
{
    boolean fuzzy_EQ();

    if (fuzzy_EQ(x, y)) then return(FALSE);
    if (x < y) return(TRUE);
    return(FALSE);
}

/*****
boolean fuzzy_LE(x, y) /* 'x <= y' comparison with APL-type 'fuzz' */
double x, y;
{
    boolean fuzzy_EQ();

    if (fuzzy_EQ(x,y)) then return(TRUE);
    if (x < y) return(TRUE);
    return(FALSE);
}

/*****
boolean fuzzy_EQ(x, y) /* 'x == y' comparison with APL-type 'fuzz' */

```


util directory

FUZZ.C;23

4-NOV-1985 00:18

Page 2

```

/*
 * NOTE WARNING AT TOP OF FILE -- SEMANTICS DIFFER FROM
 * APL'S FOR ARGUMENTS NEAR ZERO.
 */

double x, y;
{
double fabs();

double abs_x, abs_y;
double big, epsilon;

abs_x = fabs(x);
abs_y = fabs(y);
big = (abs_x > abs_y) ? abs_x : abs_y; /* max(|x|, |y|) */

epsilon = quad_CT * big; /* width of interval to use */
if (epsilon < quad_CT) then epsilon = quad_CT; /* THIS DIFFERS FROM APL */

if (fabs(x-y) <= epsilon)
    then return(TRUE); /* fuzzily equal */
    else return(FALSE); /* not fuzzily equal */
}

/*****/

boolean fuzzy_NE(x, y) /* 'x != y' comparison with APL-type 'fuzz' */
double x, y;
{
boolean fuzzy_EQ();

if (fuzzy_EQ(x, y))
    then return(FALSE); /* not fuzzily not-equal */
    else return(TRUE); /* fuzzily not-equal */
}

/*****/

boolean fuzzy_GE(x, y) /* 'x >= y' comparison with APL-type 'fuzz' */
double x, y;
{
boolean fuzzy_EQ();

if (fuzzy_EQ(x,y)) then return(TRUE);
if (x > y) then return(TRUE);
return(FALSE);
}

/*****/

boolean fuzzy_GT(x, y) /* 'x > y' comparison with APL-type 'fuzz' */
double x, y;
{
boolean fuzzy_EQ();

if (fuzzy_EQ(x,y)) then return(FALSE);

```

util directory

FUZZ.C;23

4-NOV-1985 00:18

Page 3

```

if (x > y) then return(TRUE);
return(FALSE);
}

/*****

int fuzzy_floor(x)      /* largest integer which is fuzzy_LE x */
/*
/* note result returned as an 'int', unlike VMS (non-
/* fuzzed) library routine, which returns a 'double'.
*/

double x;
{
double floor();        /* VMS-specific library routine!!! */
int round_double_to_int();

/*
/* The algorithm is the one given on page 24 of the ISO draft APL standard.
*/
int o;
double cc;

o = round_double_to_int(x);
cc = o;                /* convert back to floating point */
if (cc < x) then return(o);
if (o == 0)
    then if ((-quad_CT <= x) && (x <= 0))
        then return(0);
        else return(-1);
    else if (fuzzy_EQ(x, cc))
        then return(o);
        else return(o - 1);
}

*****/

int fuzzy_ceiling(x)    /* smallest integer which is fuzzy_GE x */
double x;
{
int fuzzy_floor();

return(- fuzzy_floor(-x) );
}

```

util directory

KEYWORD.C;49

21-SEP-1985 20:08

Page 1

/* Keyword Scanning And Processing Routines */

/*

* This file contains the following functions:
 * keyword_scan - Scans for and processes keyword assignments to variables
 * table_search - Does a linear search of a table for a value
 */

```
#include <stdio.h>
#include <ctype.h>
#include "util:stdio.h"
#include "util:keyword.h"
```

/*****

/*

* This function processes keyword assignments (hereafter 'KAs') to variables.

* The format of a single KA is either 'keyword' or 'keyword-value'. No
 * embedded blanks are allowed except as part of a character string 'value'
 * enclosed in '"' double quotes. The normal '\\' escapes are NOT allowed.

* Arguments:

* n is the number of entries in 'table' (next argument).
 * table is (a pointer to) an array of variable descriptors, each of which is a
 * structure (defined in detail in the 'keyword.h' file) defining a
 * variable available for definition by KAs. The action of this function
 * is to modify the appropriate entries of this array to reflect the KAs.
 * KAs of the form 'keyword' are expanded into 'keyword-1'. KAs of the
 * form 'keyword-value' cause the variable with that name to be set to
 * the specified value. The actual assignment of 'value' to the variable
 * is via 'sscanf' using the variable's 'in_format' descriptor entry,
 * except for '%s' 'in_format' entries, in which case the value is copied
 * to a 'safe place' (obtained via 'calloc()') and the variable is set to
 * point to it. In any case, the variable itself is found by the 'value'
 * pointer in it's descriptor. (If this pointer is NULL, the assignment
 * to the variable is skipped.)
 * string is (a pointer to) a character array which is to be scanned for KAs.
 * It may contain multiple KAs, separated by white space (blanks/tabs/
 * newlines). Except as part of a '"' double quoted string, a ';'
 * semicolon (delimited from any KAs before it by white space) marks the
 * start of a comment -- all text from the semicolon to the next newline
 * inclusive is ignored.

* The function returns the number of KAs made, or one of the following error
 * signals (defined in 'stdio.h' or 'keyword.h':
 * EOF - Unexpected null byte found in 'string'.
 * SYNTAX_ERROR - KA found with incorrect syntax.
 * VALUE_ERROR - 'keyword-value' KA found, but 'sscanf' could not scan 'value'.
 * UNKNOWN_KEYWORD - KA found, but 'keyword' not in 'table'.
 * Note that a 'skipped' KA ('value' field in variable descriptor NULL) doesn't
 * count towards the returned 'number of KAs made'.

*/

```
int keyword_scan(n, table, string)
int n;
```

util directory

KEYWORD.C;49

21-SEP-1985 20:08

Page 2

```

struct var_dscr *table;
char *string;
{
char *calloc();
int strcmp(), strlen();
void strcpy();
char *table_search();

#define MAX_KEYWORD_LENGTH      50
#define MAX_VALUE_LENGTH       100
char keyword[MAX_KEYWORD_LENGTH];
char value[MAX_VALUE_LENGTH];

enum      {
START, QUIT, COMMENT, GET_KEYWORD, GET_VALUE, GET_STRING_VALUE
}
state;

char c, *p;
int i, count;
struct var_dscr *var_ptr;

count = 0;
state = START;
while (TRUE)
{
if (state == QUIT) then return(count);
c = * (string++);
switch (state)
{
case START:
if (c == '\0')      then state = QUIT;
else if (c == ';')  then state = COMMENT;
else if (isspace(c)) then { /* do nothing */ }
else
{
state = GET_KEYWORD;
i = 0;
keyword[i++] = c;
}

break;
case COMMENT:
if (c == '\0')      then state = QUIT;
else if (c == '\n')  then state = START;
else
{ /* do nothing */ }
break;
case GET_KEYWORD:
if (c == '\0')      then
{
state = QUIT;
keyword[i] = '\0';
goto KEYWORD_FOUND;
}
else if (c == '-')   then
{
state = GET_VALUE;
keyword[i] = '\0';
i = 0;
}
}
}

```

util directory

KEYWORD.C;49

21-SEP-1985 20:08

Page 3

```

    }
    else if (isspace(c)) then {
        state = START;
        keyword[i] = '\0';
        goto KEYWORD_FOUND;
    }
    else
        break;
case GET_VALUE:
    if (c == '\0') then {
        state = QUIT;
        value[i] = '\0';
        goto KEYWORD_VALUE_FOUND;
    }
    else if (c == '"') then {
        state = GET_STRING_VALUE;
        i = 0;
    }
    else if (isspace(c)) then {
        state = START;
        value[i] = '\0';
        goto KEYWORD_VALUE_FOUND;
    }
    else
        break;
case GET_STRING_VALUE:
    if (c == '\0') then return(SYNTAX_ERROR);
    else if (c == '\n') then return(SYNTAX_ERROR);
    else if (c == '"') then {
        state = START;
        value[i] = '\0';
        goto KEYWORD_VALUE_FOUND;
    }
    else
        break;
default:
    fprintf(stderr,
"keyword_scan: Internal error -- unknown state -- code is %d\n", state
    );
    exit(TROUBLE_EXIT);
}
continue; /* with scanning the next character */

KEYWORD_FOUND: /* A 'keyword' KA has been found: */
    strmov("l", value); /* default value */
    /* ... now fall through into common code: */

KEYWORD_VALUE_FOUND: /* A 'keyword-value' KA has been found: */
    var_ptr = (struct var_dscr *)
        table_search(n, sizeof(struct var_dscr), table, keyword);
    if (var_ptr == NULL) then return(UNKNOWN_KEYWORD);
    if (var_ptr->value == NULL) then continue; /* skip it??? */
    if (strcmp(var_ptr->in_format, "%s") == 0)
    then {
        /* string value --> copy it in to safe place, save ptr: */

```

util directory

KEYWORD.C;49

21-SEP-1985 20:08

Page 4

```

        p = calloc(strlen(value) + 1 /* for '\0' */ , sizeof(char));
        if (p == NULL)
            then {
                fprintf(stderr,
"keyword_scan: Insufficient memory to save string value\n"
                );
                exit(OK_EXIT);
            }
        strmov(value, p);
        *( (char **) (var_ptr->value) ) = p;
        ++count;
    }
else {
    /* non-string value --> let 'sscanf' store it: */
    i = sscanf(value, var_ptr->in_format, var_ptr->value);
    if (i != 1) then return(VALUE_ERROR);
    ++count;
}
}

/*****

/*
* This function does a linear search of a table for a specified value.
*
* Arguments:
* n is the number of entries in the table.
* size is the size of a table entry in bytes, as returned by 'sizeof()'.
* table is a pointer to the table itself. The format of an individual table
* entry is 'struct { char *item; ... }', where '...' represents fields
* ignored by this function.
* value is a pointer to a string which is matched against the strings pointed
* to by the 'item' fields (see above) of the table entries.
*
* The function returns a pointer to the 1st table entry found which gives a
* match, or NULL if no match is found.
*/
char *table_search(n, size, table, value)
int n, size;
char *table, *value;
{
    int strcmp();

    int i;
    char *table_ptr;
    char *item_ptr;

        for (i=0 ; i<n ; ++i)
        {
            table_ptr = table + i*size;                /* table_ptr -> table[i] */
            item_ptr = * (char **)table_ptr;             /* table[i].item */
            if (strcmp(item_ptr, value) == 0) then return(table_ptr);
        }

    return(NULL);

```

util directory

KEYWORD.C:49

21-SEP-1985 20:08

Page 5

}

util directory

MISCFP.C;2

21-SEP-1985 20:08

Page 1

```
double square(x)
double x;
{
return(x * x);
}
```

```
double cube(x)
double x;
{
return(x * x * x);
}
```

```
double fourth(x)
double x;
{
return(x * x * x * x);
}
```


util directory

ROUND.C;1

21-SEP-1985 20:08

Page 1

```
#include "util:stdio.h"
```

```
int round_double_to_int(d)
```

```
double d;
```

```
{
```

```
int i;
```

```
/* note C double-->int conversion truncates */
```

```
if (d > 0.0)
```

```
then i = d + 0.5;
```

```
else {
```

```
    d = -d;
```

```
    i = d + 0.5;
```

```
    i = -i;
```

```
    }
```

```
/* force positive -- negatives are system-dependent */
```

```
/* compensate!!! */
```

```
return(i);
```

```
}
```

util directory

SPLINE.C;9

21-SEP-1985 20:08

Page 1

```

/* This file contains routines for using one dimensional cubic splines. */
/*
 * The following functions live in this file:
 * <definition of spline functional form>
 * spline_calc - Calculates coefficients for a cubic spline given data points
 * spline_int - Integrates evenly spaced 1-dimensional data by spline fitting
 */

#include "util:stdio.h"
#include "util:bandmat.h"

/*****
/*
 * The splines dealt with in this file are all one dimensional cubic splines.
 *
 * Reference:  Richard L. Burden, J. Douglas Faires, and Albert C. Reynolds,
 *             "Numerical Analysis", 2nd edition (Prindle, Weber, & Schmidt,
 *             1981, Boston), p. 109-115.
 *
 * Given (distinct) nodes x[0], x[1], ..., x[n], and corresponding data values
 * y[0], y[1], ..., y[n], the spline has the functional form
 *    $S[k](x) = y[k] + a[k](x-x[k]) + b[k](x-x[k])**2 + c[k](x-x[k])**3$ 
 * on the (closed) interval from x[k] to x[k+1], for each k=0, 1, ..., n-1.
 * The nodes need not be equally spaced, but they should be ordered so that
 * x[0] < x[1] < ... < x[n].
 *
 * Clearly the resulting (composite) spline function S(x) will agree with the
 * data values at the given nodes. In addition, the coefficients a[i], b[i],
 * and c[i] can be chosen so that S(x) is continuous and has continuous first
 * and second derivatives everywhere. At each endpoint, either the first or
 * the second derivative may be freely chosen.
 */
*****/

/*
 * This function calculates coefficients for a one dimensional cubic spline,
 * given the data points and suitable end conditions.
 *
 * Arguments:
 * n - One less than number of data points.
 * x[0...n] - Array of x coordinates of data points.
 * y[0...n] - Array of y coordinates of data points.
 * flag_0 - Selector for 'deriv_0', below.
 * deriv_0 - Desired S'(x[0]) if flag_0 is 1, or S''(x[0]) if flag_0 is 2.
 * flag_n - Selector for 'deriv_n', below.
 * deriv_n - Desired S'(x[n]) if flag_n is 1, or S''(x[n]) if flag_n is 2.
 * a[0...n] - Array in which linear coefficients are to be stored (see note).
 * b[0...n] - Array in which quadratic coefficients are to be stored (see
 *           note).
 * c[0...n] - Array in which cubic coefficients are to be stored. (see note).
 *
 * Note:      The [n] elements of these 3 vectors are used internally, but
 *            are of no external significance. The values in them on return

```

util directory

SPLINE.C:9

21-SEP-1985 20:08

Page 2

```

*           from this function should be ignored.
*/
void spline_calc(n, x, y, flag_0, deriv_0, flag_n, deriv_n, a, b, c)
int n;
register double x[], y[];
int flag_0;
double deriv_0;
int flag_n;
double deriv_n;
register double a[], b[], c[];
{
    char *calloc();
    void cfree();
    void error_exit();
    void bge(); /* banded Gauss elimination A*x=b solver */

    int k;
    int neqn, bandwidth;
    double h_k, h_kml;
    double (*matrix)[3], *rhs;

    /*
     * To find the coefficients, we first solve for the quadratic coefficients b[k]
     * which satisfy a tridiagonal linear system of order n+1. We can then easilly
     * calculate the linear and cubic coefficients from the quadratic ones.
     */
    matrix = ALLOC(3 * (n+1), double);
    rhs = ALLOC(n+1, double);
    if ((matrix == NULL) || (rhs == NULL))
        then error_exit(OK_EXIT,
        ***** spline_calc: insufficient memory for a %d point spline\n",
            n+1); /* DOES NOT RETURN */

    /* fill in interior coefficient matrix and rhs vector: */
    for (k = 1 ; k <= n-1 ; ++k)
    {
        /* formulas from the reference, p. 111-112: */
        h_k = x[k+1] - x[k];
        h_kml = x[k] - x[k-1];
        matrix[k][0] = h_kml;
        matrix[k][1] = 2.0 * (h_k + h_kml);
        matrix[k][2] = h_k;
        rhs[k] = 3.0 * ( (y[k+1] - y[k])/h_k - (y[k] - y[k-1])/h_kml );
    }

    /* fill in lower end point values: */
    matrix[0][0] = 0.0; /* not used */
    if (flag_0 == 1)
        then {
            /* deriv_0 is desired first derivative at x[0]: */
            h_k = x[1] - x[0];
            matrix[0][1] = 2.0 * h_k;
            matrix[0][2] = h_k;
            rhs[0] = 3.0 * ( (y[1] - y[0])/h_k - deriv_0 );
        }
}

```

util directory

SPLINE.C:9

21-SEP-1985 20:08

Page 3

```

    else {
        /* deriv_0 is desired second derivative at x[0]: */
        matrix[0][1] = 1.0;
        matrix[0][2] = 0.0;
        rhs[0] = 0.5 * deriv_0;
    }

/* fill in upper end point values: */
matrix[n][2] = 0.0;      /* not used */
if (flag_n == 1)
    then {
        /* deriv_n is desired first derivative at x[n]: */
        h_kml = x[n] - x[n-1];
        matrix[n][0] = h_kml;
        matrix[n][1] = 2.0 * h_kml;
        rhs[n] = 3.0 * (deriv_n - (y[n] - y[n-1])/h_kml);
    }
    else {
        /* deriv_n is desired second derivative at x[n]: */
        matrix[n][0] = 0.0;
        matrix[n][1] = 1.0;
        rhs[n] = 0.5 * deriv_n;
    }

/* solve tridiagonal system for b[0...n] coefficient vector: */
neqn = n + 1;
bandwidth = 3;
bge(&neqn, &bandwidth, matrix, rhs, b);

/* calculate a[0...n] and c[0...n] coefficient vectors: */
for (k = n-1; k >= 0; --k)
{
    h_k = x[k+1] - x[k];
    a[k] = (y[k+1] - y[k])/h_k - (b[k+1] + 2.0*b[k])*(h_k/3.0);
    c[k] = (b[k+1] - b[k])/(3.0*h_k);
}

/*
 * free storage in reverse acquisition order --> stack discipline --> should
 * be most efficient!!!
 */
cfree(rhs);
cfree(matrix);
}

/*****/

/*
 * This function does (1-dimensional) numerical integration. The method used
 * is to fit a cubic spline to the data, then (analytically) integrate this.
 * This is good to  $O(h^4)$  error for smooth data, provided that either 1st or
 * 2nd derivatives are correctly given at each endpoint.
 *
 * Arguments:
 * low - Lower limit of integral.

```

util directory

SPLINE.C;9

21-SEP-1985 20:08

Page 4

```

* high - Upper limit of integral.
* n - Number of intervals.
* y[0...n] - Vector of data values to be integrated.
* flag_low - Selector for 'deriv_low', below.
* deriv_low - y'[0] if flag_low is 1, or y''[0] if flag_low is 2.
* flag_high - Selector for 'deriv_high', below.
* deriv_high - y'[n] if flag_high is 1, or y''[n] if flag_high is 2.
*
* The estimated integral is returned as the function value.
*/
double spline_int(low, high, n, y, flag_low, deriv_low, flag_high, deriv_high)
double low, high;
int n;
register double y[];
int flag_low;
double deriv_low;
int flag_high;
double deriv_high;
{
char *calloc();
void cfree();
void error_exit();
void spline_calc();

double h, h22, h33, h44;
register double *x, *a, *b, *c;
register double sum;
register int i;

/* allocate storage: */
x = ALLOC(n+1, double);
a = ALLOC(n+1, double);
b = ALLOC(n+1, double);
c = ALLOC(n+1, double);
if ((x == NULL) || (a == NULL) || (b == NULL) || (c == NULL))
    then error_exit(OK_EXIT,
        "***** integrate: insufficient memory for n=%d points\n",
        n); /* DOES NOT RETURN */

/* set up for and do spline fit: */
h = (high - low) / n;
for (i = 0 ; i <= n ; ++i)
{
    x[i] = low + i * h;
}
spline_calc(n, x, y, flag_low, deriv_low, flag_high, deriv_high, a, b, c);

/* (analytically) integrate fitted spline to get result: */
h22 = h * h / 2.0;
h33 = h * h * h / 3.0;
h44 = h * h * h * h / 4.0;
sum = 0;
for (i = 0 ; i < n ; ++i)
{
    /*

```

util directory

SPLINE.C:9

21-SEP-1985 20:08

Page 5

```
    * Note we do each interval, not each point ==> "i < n", not "i <= n"!!  
    */  
    sum += h*y[i] + h22*a[i] + h33*b[i] + h44*c[i];  
}  
  
cfree(c);  
cfree(b);  
cfree(a);  
cfree(x);  
return(sum);  
}
```

util directory

TIME.C:20

21-SEP-1985 20:11

Page 1

```

/* Timing routines live here */
/*
 * The following functions live in this file:
 * clock_time - returns clock time in seconds since 17-NOV-1858
 * cpu_time - returns total process cpu time in seconds
 * time_stamp - returns character time-stamp for labelling output
 */
#include "util:stdio.h"

/*****

double clock_time() /* returns clock time in seconds since 17-NOV-1858 */
/* NOTE VAX/VMS SPECIFIC */
/* Note "base" is Julian Date 2400000. Note also that
values in late 20'th century will be around 4.0e9,
so about 32 bits of precision will be lost when
subtracting "before" from "after" values (for a 1
second time interval). Thus double precision will
be needed on most machines, and will give adequate
precision to do the job. */
{
int LIB$STAT_TIMER();

int clock_code = 1;
short unsigned answer[4]; /* must be 16-bit integers here */
/* Note we have to use "short unsigned" here
because there is a compiler bug that causes
"unsigned" to be incorrectly converted to
"double", whereas the conversion from
"short unsigned" to "double" seems ok. */

int i;
double sum, weight;

/* returns "answer" in 100-nanosecond units as a 64-bit integer: */
LIB$STAT_TIMER(&clock_code, &answer[0]);

sum = 0.0;
weight = 1.0;
for (i=0 ; i<4 ; ++i)
{
sum += weight * (double)answer[i];
weight *= 65536.0; /* 2 to the 16 */
}

return (100.0e-9 * sum);
}

*****/

double cpu_time() /* returns total process cpu time in seconds */
/* NOTE VAX/VMS SPECIFIC */
{
int LIB$STAT_TIMER();

int cpu_code = 2;

```

util directory

TIME.C:20

21-SEP-1985 20:11

Page 2

```

unsigned answer;          /* must be 32-bit integer here */

/* returns "answer" in 10-millisecond units as a 32-bit integer:*/
LIB$STAT_TIMER(%cpu_code, %answer);

return (10.0e-3 * ((double) answer));
}

/*****

/*
 * Returns pointer to a 'time-stamp' string for labelling output.  The current
 * format is 'hh:mm wkd dd mmm yyyy\0'.  The next 2 lines label these fields:
 *      123456789 12345678901      (0 origin)
 *      123456789 123456789 1      (1 origin, printing only)
 * If the argument is non-null, it is assumed to point to a string buffer to
 * store the result in; if it's null, an internal buffer is used.  Successive
 * calls with the same argument will overwrite any previous time-stamp strings.
 */
char *time_stamp(p)
char p[];          /* must have room for 22 chars! */
{
long time();        /* returns time in seconds since 00:00 1 Jan 1970 */
char *ctime();      /* converts time() result to unix character style string */
/* format is 'wkd mmm dd hh:mm:ss yyyy\n\0', in positions */
/*      123456789 123456789 1234 5 */
char strncpy();     /* used via my 'strnmov' macro */

static char our_buffer[22];
long bintim;
char *unix_tsp;

bintim = time(NULL);          /* this is original source of timing data */
unix_tsp = ctime(&bintim);    /* set unix_tsp --> unix time-stamp string */

if (p == NULL) then p = our_buffer;      /* now p --> destination buffer */

strnmov(unix_tsp+11,    p+0,    5);      /* 'hh:mm' */
p[5] = '\0';
strnmov(unix_tsp+0,    p+6,    4);      /* 'wkd ' */
strnmov(unix_tsp+8,    p+10,   3);      /* 'dd ' */
strnmov(unix_tsp+4,    p+13,   4);      /* 'mmm ' */
strnmov(unix_tsp+20,   p+17,   4);      /* 'yyyy' */
p[21] = '\0';

return(p);
}

```


Publications:

Thornburg, Jonathan

Hobson, R. F., Hannon, P., and Thornburg, J. (1981): "High-Level Microprogramming with APL Syntax", Proc. 14th Annual Microprogramming Workshop, 131-139.

Hobson, R. F., Gudaitis, J., and Thornburg, J. (1984): "A New Machine Model for High-Level Language Interpretation", Simon Fraser University Computer Science Technical Report TR84-18.

Thornburg, J. (1985): "On Polygon Clipping", APL Quote Quad 15(3), 29-30; errata in press.

Thornburg, J. (1985): "An Upper Limit for the Solar Acceleration", Monthly Notices of the Royal Astronomical Society 213, 27p-28p.

Thornburg, J. (1985): "Further Notes on the Modulo Operator", Sigplan Notices 20(9), 94.

Thornburg, J. (1985): "Global Character String Search and Replace", APL Quote Quad, in press.