VELOCITY MICROSTRUCTURE MEASUREMENTS IN THE WESTERN AND CENTRAL EQUATORIAL PACIFIC

by

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B.A.Sc., University Of Toronto, 1978
M.A.Sc., University Of Toronto, 1979

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

in

THE FACULTY OF GRADUATE STUDIES
Physics And Oceanography Departments

We accept this thesis as conforming
to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA
May 1984

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Date **July 9, 1984**
Abstract

Measurements of velocity microstructure were made in two quite different oceanic regimes using the free-falling profiler, Camel III. In conjunction with the Pacific Equatorial Ocean Dynamics (PEQUOD) expedition, profiles were made at or near the equator between 138°W and 153°W. Estimates of the rate of dissipation of turbulent kinetic energy, \( \epsilon \), made from the velocity microstructure measurements are surprisingly small in magnitude. Averaged values at 70, 90 and 110 meters (in the region of large mean shear just above the core of the equatorial undercurrent) are more than ten times smaller than those previously reported. The dissipation integrated from the level of no zonal velocity (\( \approx 70 \) meters) to the undercurrent core is less than 10% of an estimate made of the work done by the zonal pressure gradient. It is possible that the proposed balance between the work done by the zonal pressure gradient and the turbulent friction does not hold at all places at all times for the equatorial undercurrent.

A second set of measurements was made along 152°E between 27°N and 42°N, south of the Kuroshio Extension current. A strong main thermocline between 500 and 800 meters depth and south of 34°N manifested itself as a secondary subsurface maximum in buoyancy frequency, \( N \), which concurred with a subsurface maximum in averaged dissipation. A plot of \( \epsilon \) vs \( N \) indicates that \( \epsilon \) scales with \( N \) rather than depth. A simple model was developed to explain the relatively greater occurrence of turbulent patches in the main thermocline which assumes that
the turbulence is generated by internal waves. The prediction of the probability of occurrence of small Richardson number is \(-1/N\) proportional to \(e\) which predicts the shape of the distribution of the turbulence relatively successfully.
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Acknowledgement

I would like to express my gratitude to Professor T.R.Osborn for his guidance and for his support of my project. As well, I would like to thank Professor P.H.LeBlond for his freely-given time and thoughts. I have benefitted from time to time by being able to talk with Professors S.Pond, L.A.Mysak and R.W.Burling. I am also indebted to Dr.R.G.Lueck for many valuable discussions and much-appreciated technical advice and to Dr.W.R.Crawford for his input to the analysis of the equatorial data.

I would like to thank Dr.J.G.Richman for allowing me to participate in the PEQUOD expedition. Drs.P.P.Niiler and W.J.Schmitz permitted me to take measurements during the WESPAC cruise. Dr.J.R.Luyten supplied the White Horse data and Dr.P.Niiler the CTD data from WESPAC. The captain and crew of the R/V Thomas G.Thompson provided expert help in deploying and successfully recovering Camel III. Various members of the Woods Hole Buoy Group lent considerable time and expertise to the seagoing operation during both cruises. R.Noel assisted with the operation on the PEQUOD cruise and R.M.Ninnis played the role of technical assistant and devil's advocate on the WESPAC trip.

S.Milaire made the shear probes and, with B.Anderson, provided substantial technical assistance during the development stage of Camel III. H.Heckl did most of the machining.

Finally, I would like to thank the many graduate students and postdoctoral fellows with whom I have been fortunate to associate with and learn from during my tenure as a graduate student at UBC.

The Natural Sciences and Engineering Research Council of Canada supported me with a postgraduate scholarship during my studies.
I. INTRODUCTION

The study of three-dimensional turbulence in the ocean is essentially a study of the small scale mechanisms which are responsible for the mixing, dispersing, diffusing of heat, salt, momentum and other quantities. To understand the larger scale motions in the sea it is necessary to understand what is happening at the small scales. Due to the highly nonlinear nature of these small scale processes, the extent of the analytical treatment has been limited. Instead, the study of turbulence involves experimentation and empirical parameterization of the flow parameters in order to describe how they affect the large scale dynamics.

The first studies of oceanic turbulence by Grant, Stewart and Moilliet (1962) at the Canadian Defence Research Establishment, Pacific (DREP) were motivated by a fundamental interest in the nature of the turbulence itself. From experiments conducted under conditions of sufficiently high Reynolds number (10^8 in Seymour Narrows), they were able to provide the first solid evidence of the existence of the inertial subrange of turbulence proposed by Kolmogoroff. For a number of years, these were the only workers measuring ocean turbulence until, in the late 1960's a group headed by C.S. Cox at Scripps Institute of Oceanography developed instrumentation capable of resolving the small scales of the temperature field. Around this time, the term microstructure came into use to describe the scales ranging from ≈ 1 meter down to the smallest scales which exist in the ocean (a couple of centimeters for
velocity, one centimeter for temperature and a few millimeters for salt.)

The technique for measuring velocity microstructure in the ocean was developed by T.R.Osborn at the University of British Columbia and described in Osborn(1974). The measurement was made using an airfoil (or shear) probe capable of resolving the smallest scales of the cross-stream velocity fluctuations (except in regions of very intense turbulence). The probe was mounted on a vertical profiler and a measure was obtained of the vertical shear of the horizontal velocity fluctuations from which an estimate was made of the turbulent kinetic energy dissipation. The probe proved useful in obtaining a description of the vertical variability of velocity microstructure in the ocean. New rapid profiling techniques are providing valuable information on the temporal and horizontal variation.

The significance of the measurements made with the shear probes was demonstrated by Crawford and Osborn (1979b) who were able to balance the work done by the zonal pressure gradient (in the equatorial Atlantic) which drives the equatorial undercurrent with the turbulent kinetic energy dissipation, thereby linking the turbulence with the large scale dynamics. Added confidence in the estimate made of the dissipation with the shear probe measurements was recently given by Oakey(1982) who showed the agreement of dissipation estimates using two different techniques. From the high wavenumber cutoff of the temperature gradient spectrum, the dissipation was estimated to agree within a factor of two to the shear probe estimates when
the temperature gradient spectrum was well-resolved. Although considered to be a test of the method of computing dissipation from temperature data, an independent estimate consistent with that made by the shear probe was provided.

The sources of turbulence are mostly concentrated near the boundaries of the ocean. Surface processes (wind, wave, differential heating and cooling) often create a relatively well-mixed upper layer. Strong currents capable of producing large mean shears which may generate turbulence (as in equatorial regions) are generally confined to the upper layers. Other boundary layer effects at the ocean bottom and sides have been less well studied but appear to show greater levels of turbulence. In the interior of the ocean, the primary source for the turbulence is thought to be the loss of energy from the internal wave field, although the effects of double diffusive mixing may also play a strong role in some places.

It is difficult to assess the role of double diffusive processes on the measured dissipations discussed here, due to the lack of simultaneous temperature and salinity measurements. It is not generally possible to detect salt fingers with a vertical profiler since the signature of salt fingers is essentially horizontal (Schmitt and Evans, 1979). However, if the fingers are tilted from the horizontal in some way they may be detected. Recently, Larson and Gregg (1983) were able to estimate the relative importance of turbulence produced by the buoyancy flux of double diffusion using their measurements of temperature and salinity finestructure coincident with velocity
microstructure measurements. Regions above local T-s maxima were found which exhibited the distinctive steps in temperature and salinity characteristic of the diffusive regime of double diffusion. Here, the buoyancy flux was sufficiently large to be responsible for the measured dissipation. Where the T-s relation showed the necessary form for salt fingering, the measured dissipations were greater by a factor of 5 to 10 than in the diffusive regime case and were much greater than the estimated buoyancy flux, as well. It was suggested that the production of turbulent energy by the mean shear working against the Reynolds stress was at least partly responsible.

Studies of internal waves in the last 20 years have shown the spectrum of internal wave energy in the deep ocean to be remarkably constant (except in some quite special cases) in shape and level throughout the world's oceans. Garrett and Munk (1979) discuss a concept which may explain the universality of the internal wave spectrum. The spatial constancy may be a result of long-lasting waves, which, if they do not dissipate quickly, will propagate long distances thereby diffusing energy from a local source over a large region. The temporal constancy may be due to locally enhanced spectral levels saturating the internal wave spectrum, resulting in locally increased dissipation of internal wave energy into the turbulence. Since, as we shall see, the energy in the internal wave field scales as the local value of the buoyancy frequency, one suspects a similar relation for the turbulent kinetic energy dissipation.

In this thesis, I will describe a set of measurements of
velocity microstructure (made using the shear probes already discussed) which I conducted in two regions of the Pacific Ocean and I will attempt to put these measurements into their proper context. In February of 1982 a set of velocity microstructure profiles were made in the central equatorial Pacific. For this region there already exists a relatively large data bank of microstructure measurements and it is interesting to compare these. In May and June of 1982, measurements were made in the Western Pacific Ocean and a significant amount of data was collected at depths greater than 1000 meters (representing virtually all of the velocity microstructure data anywhere at these depths).

In Chapter 2 the equations which govern the turbulence are developed and the appropriate oceanic balance discussed. In Chapter 3 there is a discussion of seagoing experimental procedures, the on-deck signal processing, the two cruise tracks and other measurements made during the cruises. Chapter 3 is supplemented by the Appendices in which more detailed descriptions of the instrumentation, lab calibrations and data handling are found. As well, all of the vertical profiles of turbulent kinetic energy dissipation, CTD data and horizontal velocities are presented in Appendices K and L. The results from the western Pacific (WESPAC) cruise are discussed in Chapter 4. The notable result arising from the analysis of this data set is the strong relation between the turbulent kinetic energy dissipation and the buoyancy frequency in a region with a well-defined subsurface maximum in buoyancy frequency. An
attempt to explain this with a model based on internal wave shear instability is developed in Chapter 5. In Chapter 6 the results from the equatorial Pacific (PEQUOD) are presented. The PEQUOD data set is enhanced by the vertical profiles of mean horizontal velocity as well as by CTD data made with independent instrumentation, and nearly synoptic estimates of shear and Richardson number were obtained. Various estimates of turbulent eddy coefficients are discussed and compared in Chapter 7. A comparison of mean values of turbulent kinetic energy dissipation, lognormal statistics of dissipation and estimates of turbulent patch properties and distributions are given in Chapter 8. It is hoped that this may provide a format for comparing other data sets. Chapter 9 contains a short discussion and the conclusions.
II. BACKGROUND THEORY

2.1 Mean Kinetic Energy Equation

In the following discussion of the governing equations, Cartesian tensor notation is used. Spatial coordinates \( x_1 = x, x_2 = y \) are horizontal, and \( x_3 = z \) is positive upwards. The respective velocities are \( U_1 = U, U_2 = V \) and \( U_3 = W \). The density of the fluid is \( \rho \), \( \mu \) is the coefficient of viscosity, \( \nu = \mu/\rho \) the kinematic viscosity, \( g \) the acceleration due to gravity, \( \kappa \) the thermal diffusivity and \( \delta \) the Kronecker delta.

The conservation of momentum is expressed by the Navier-Stokes equations for an incompressible, non-rotating, viscous, stratified fluid (Phillips, 1977)

\[
\rho \left( \frac{\partial U_i}{\partial t} + U_i \frac{\partial U_i}{\partial x} \right) = - \frac{\partial p}{\partial x} - \rho g \delta + \mu \frac{\partial^2 U_i}{\partial x_j \partial x_j} \quad (2.1)
\]

The conservation of mass is expressed by

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho U_i)}{\partial x_i} = \kappa \frac{\partial^2 \rho}{\partial x_i \partial x_i}
\]

which, if the fluid is incompressible and the diffusion terms are small, reduces to

\[
\frac{\partial \rho}{\partial t} + U_i \frac{\partial \rho}{\partial x_i} = 0 \quad (2.2)
\]

and

\[
\frac{\partial U_i}{\partial x_i} = 0
\]
It is usually common to separate the flow field into mean plus fluctuating components (referred to as Reynolds' expansion after Osborne Reynolds who first suggested the separation). The measurements made using the shear probe are sensitive to scales of about 1-50 cm and it is believed that at these scales the motion is random and is responsible for mixing (i.e., the motion is turbulent). At scales greater than 50 cm (or at least greater than 1 meter) in the ocean, the spectrum of velocity shear does not scale with turbulence parameters (Gargett et al.(1981), Gargett et al.(1984)) and other processes dominate the flow field. For the purposes of this thesis, the scales of the fluctuating components will be taken as the scales of the turbulence (those scales measured by the shear probe) and all larger scales will be referred to as the mean motion. This implies that the mean motion includes currents, eddies and all scales of waves down to the smallest internal waves. Reynolds' expansion of the variables into mean plus fluctuating components is expressed by \( U = \bar{u} + u', \ p = \bar{p} + p', \ \rho = \bar{\rho} + \rho' \). Equations describing the mean and fluctuating components of the flow are separately derived. The overbar denotes an average and will henceforth be dropped from the mean quantities \( u, \ p \) and \( \rho \). The ergodic hypothesis is assumed, whereby spatial averages = time averages = ensemble averages. Averages of individual fluctuating components are equal to zero. The Boussinesq approximation is used so that the ratio \( \rho'/\rho \) is only of importance to the gravity term in (2.1).
To obtain the equation describing the kinetic energy of the mean field, (2.1) is multiplied by \(u\) and averaged so that

\[
\frac{1}{2} \rho \left\{ \frac{\partial (u u)}{\partial t} + u \frac{\partial (u u)}{\partial x} \right\} = - u \frac{\partial p}{\partial x} - \rho g u \delta
\]

\[
+ \mu \frac{\partial^2 u}{\partial x \partial x} - \rho \frac{\partial (u u' u')}{\partial x} + \frac{\rho u'u' \partial u}{\partial x} \tag{2.3}
\]

The mean kinetic energy is \(1 (u u)\) and, from (2.3), depends on the work done by the horizontal pressure gradient since the vertical hydrostatic pressure gradient accounts for the gravity term (under the Boussinesq approximation), viscous diffusion (which is generally considered to be small for the mean motion), the divergence of the mean advection of the Reynolds stresses \(u'u'\), and the Reynolds stresses working against the mean shear. This last term provides the means by which the energy is transferred from the mean to the turbulent field.

2.2 Turbulent Kinetic Energy Equation

The turbulent kinetic energy equation is derived by multiplying (2.1) by \(u'\), averaging and dividing by \(\rho\) to give

\[
\frac{1}{2} \left\{ \frac{\partial (u'u')}{\partial t} + u \frac{\partial (u'u')}{\partial x} \right\} = - \frac{1}{\rho} \frac{\partial \rho}{\partial x} \left\{ u'(p' + u'u'/2) \right\}
\]

\[
+ \frac{\rho'}{\rho} w' - \frac{u'u' \partial u}{\partial x} + \nu \frac{\partial (u'u' + 3u'u')}{\partial x} \tag{I} \tag{II}
\]

\[
- \rho' w' - \frac{u'u' \partial u}{\partial x} + \nu \frac{\partial (u'u' + 3u'u')}{\partial x} \tag{III} \tag{IV} \tag{V}
\]
I represents the total derivative of the turbulent kinetic energy \( \frac{1}{2} (\overline{u'u'}) \). II is the work done by the total dynamic pressure of the turbulence. Alternatively, this is the divergence of the transport of the pressure-velocity correlation plus that of the transport of turbulent energy by the turbulence. The \( p-u \) correlation is essential in redistributing the turbulent energy among the flow components. III is the work done by the buoyancy flux. IV is the work done by the Reynolds stresses working against the mean shear. This term is of opposite sign to the identical term in (2.3) and is responsible for energy exchange between the two fields. V is the work done by the viscous shear stresses of the turbulent motion. VI is the rate of dissipation of turbulent kinetic energy, \( \epsilon \).

Terms II and V represent a local spatial redistribution of the turbulence energy and do not change the total energy. If the turbulence is considered to be homogeneous, then spatial derivatives of mean turbulence quantities are equal to zero and the advective part of term I is zero. Further, if the flow is steady, then the time rates of change are zero, as well. This results in a balance between the mechanical production of turbulence by the Reynolds stresses and the mean shear, the production by the buoyancy flux and the dissipation of turbulent kinetic energy by viscosity, or

\[
- \frac{\rho \overline{w'w'}}{\rho} - \overline{u'u'} \frac{\partial u_i}{\partial x_j} - \epsilon = 0
\]  

(2.5)
Osborn (1980) discusses the ratio of the first two terms of (2.5) in light of laboratory measurements, pointing out that the buoyancy flux is the least significant of the terms in (2.5), representing at most 20% of each of the other two terms. In stratified turbulent flows with this expected ratio, it is usually convenient to retain this term in the form of the flux Richardson number,

\[
R = \frac{g \rho' w'}{(u'u' \partial u / \partial x)}. \]

2.3 The Estimate Of \( \varepsilon \)

With the shear probe sensors used to detect the velocity microstructure, an estimate is obtained of \( \varepsilon \), provided a major assumption is made pertaining to the form of the turbulence. In full component form, \( \varepsilon \) is written as

\[
\varepsilon = \frac{1}{2} \nu ( \partial u' / \partial x + \partial u' / \partial x )^2
\]

\[
= \nu [ 2(\partial u' / \partial x)^2 + 2(\partial v' / \partial y)^2 + 2(\partial w' / \partial z)^2
\]

\[
+ (\partial u' / \partial y + \partial v' / \partial x)^2 + (\partial u' / \partial z + \partial w' / \partial x)^2
\]

\[
+ (\partial v' / \partial z + \partial w' / \partial y)^2 ] .
\]

This can be simplified considerably if we make the assumption that the turbulence is isotropic. This may be a good assumption according to Gargett et al. (1984). From a submersible platform mounted with shear probes to detect the cross-stream turbulent fluctuations and a hot film probe to measure the streamwise fluctuations, measurements made over a wide range of \( \varepsilon \) indicate that isotropy at dissipation scales may be a relatively safe assumption to make down to a lower limit of \( \varepsilon = (75)^{3/4} \nu N^2 \), where
N is the local buoyancy frequency. In the deep ocean where N is \( \approx 0.001 \) rad/sec and \( \nu \approx 0.01 \text{ cm}^2/\text{sec} \), this results in \( \epsilon \approx 3 \times 10^{-7} \text{ W/m}^3 \), which is precisely the level which has been defined as the instrumental noise level (see Moum and Lueck(1984)). In the western Pacific thermocline (discussed in Chapter 4) where N is \( \approx 0.005 \) rad/sec and \( \epsilon \approx 8 \times 10^{-6} \text{ W/m}^3 \) this may degrade the reliability of the small estimates of \( \epsilon \) but should not seriously affect the averages which are dominated by individual estimates of \( \epsilon \) near or greater than \( \epsilon \).

Hinze(1975, p219) gives the following isotropic relations:

\[
(\partial u'/\partial x)^2 = (\partial v'/\partial y)^2 = (\partial w'/\partial z)^2
\]

\[
(\partial u'/\partial y)^2 = (\partial u'/\partial z)^2 = (\partial v'/\partial x)^2 = 2(\partial u'/\partial x)^2 = \ldots
\]

\[
(\partial u'/\partial y)(\partial v'/\partial x) = (\partial u'/\partial z)(\partial w'/\partial x) = -\frac{1}{2}(\partial u'/\partial x)^2 = \ldots
\]

The shear probes measure the cross-stream components of the turbulent velocity field. Vertical profiles yield vertical derivatives so that (as discussed in Appendix E) an estimate is obtained of the vertical derivatives, \( \partial u'/\partial z \) and \( \partial v'/\partial z \) (henceforth the primes will be dropped). Rewriting \( \epsilon \) in terms of these two components yields

\[
\epsilon = \frac{15\nu(\partial u'^2+\partial v'^2)}{4\partial z \partial z} \quad (2.6)
\]

This is, of course, the kinematic form for \( \epsilon \) with units of \([\text{L}^2/\text{T}^3]\). The units used in this thesis are \( \text{W/m}^3 \) requiring \( \epsilon \) to be written as

\[
\epsilon = \frac{15\rho\nu(\partial u'^2+\partial v'^2)}{4\partial z \partial z}
\]
or

\[ 
\epsilon = \frac{15\mu}{4} \left( \frac{\partial u^2 + \partial v^2}{\partial z} \right). 
\]

2.4 Stratified Flow Parameters

The Brunt-Vaisala or buoyancy frequency is defined by \( N^2 = -\frac{g \partial \rho}{\partial z} - \left( \frac{g}{c} \right)^2 \) where \( \rho \) is the in situ density and is a measure of the local static stability in the fluid. Alternatively, \( N \) may be thought of as the natural frequency of vertical oscillation of a parcel of fluid which has been disturbed from its equilibrium position. Frequencies greater than \( N \) are rapidly attenuated or do not propagate as waves while frequencies less than \( N \) travel as waves and are generated by a wide range of sources in the ocean. The parameter \( N \) represents the upper frequency limit to the internal wave spectrum of the ocean. On the other hand, \( N \) is associated with the largest scales of the turbulence. The length scale \( L = \left( \frac{\epsilon}{N^3} \right)^{\frac{1}{2}} \) which is discussed in Chapter 8 represents the scale at which buoyancy effects are of the same order as the nonlinear effects and the turbulence at larger scales is suppressed by the buoyancy.

Various Richardson numbers which roughly describe in one way or another the relative effects of the local static and dynamic stability are discussed in the text. The flux Richardson number, defined above, is the ratio of the gain in potential energy by raising mass (the buoyancy flux) to the kinetic energy required to accomplish this (the shear production
term). The gradient Richardson number is $R = N^2/(\partial u)^2$. From

the PEQUOD White Horse data estimates of $N$ and $S = \Delta u/\Delta z$ are
made over 25 meter depth intervals and a difference Richardson
number, $R_i = N^2/S^2 = g\Delta \rho \Delta z/(\Delta u)^2$ is calculated as an estimate of

$R$. Unfortunately, this estimate is only as good as the
g
difference estimates of $\rho$ and $u$ and conceals the information at
smaller scales.
III. EXPERIMENTAL CONSIDERATIONS

Detailed discussions of the instrumentation aboard Camel III are included in the appendices to this thesis and will only be referred to here. In this chapter, shipboard procedures for deployment and recovery of the instrument and the signal processing are discussed. As well, I will briefly discuss cruise tracks and other measurements made during the PEQUOD and WESPAC cruises.

A general point to be made concerns the 'design' of experiments such as this. A certain lack of control over experimental conditions distinguishes this type of experiment from a laboratory setup. In the lab, one has the luxury of being able to adjust parameters and repeat experiments over small time scales until satisfied with the results. Certainly, oceanic measurements may be repeated, but often due to time and expense, an extended period may lapse before doing so and oceanic conditions may be radically changed in the meantime. With this in mind, it would be ideal to make coherent sets of measurements of various parameters which are synoptic in space and time and to ensure that the scales of the measurements are such as to provide a basis for comparison. Especially in measuring turbulent quantities, which have small spatial scales and short time scales, this problem can be quite grave and may cause the experiment to resemble a fact finding mission rather than a study which concentrates on relating processes. In this thesis, I hope that I shall make proper reference to this limitation in discussing the results of the experiments.
Figure 1 - Schematic of Camel III as it was configured for the WESPAC and PEQUOD cruises.
3.1 A Brief Description Of Camel III

A schematic view of Camel III is shown in Figure 1. Two shear probes plus a thermistor are mounted on a streamlined nose piece which contains the pressure transducer and the circuitry for preamplifying the shear probe signal. Underwater multiconductor cables link the preamplified signals to the electronics in the main pressure housing. Lead ballast is affixed to the preamplifier - main body transition cone by wire links. These links are released by pressure activated ballast release mechanisms. The schematic also shows the launcher and recovery aids (flasher, radio, and underwater pinger). The ring around the top of Camel III is used to attach a snap hook and line for recovery.

The method used for deploying Camel III depends to a large extent on the equipment available. A new cherry picker crane was fitted on the R/V Thomas G. Thompson and this was used on the Pacific trips for lifting the Camel out of its cradle and over the side into the water. Once the weight of the instrument has been taken up by the water the launcher is activated to release the instrument. At a fall speed of about 80 cm/sec and a rising speed only slightly faster, the return trip to 1000 meters is about 45 minutes and a little over 90 minutes to 2000 meters since Camel III slows with depth. The surfacing time was chosen to coincide with the retrieval of the CTD on the WESPAC trip while we generally attempted to squeeze a Camel drop inside of a White Horse drop (which lasted approximately 2-3 hours) on the PEQUOD trip.
Surfacing of the instrument was indicated by the radio signal picked up by the shipboard OAR receiver unit which was pretuned to the frequency of the Camel III transmitter prior to launch. At night, sighting of the instrument was aided substantially by two strobe flashers mounted on the recovery end of the instrument. During daylight hours, however, surface glitter could make it quite difficult to spot the surfaced instrument. The longest time required to spot the instrument was just under two hours, with a dozen pairs of eyes straining and the radio direction finder zeroed in to an arc of about 45°. We experienced no failures of OAR flashers or transmitters on any of the drops.

Once the surfaced instrument had been located, the ship was manoeuvred alongside and downwind of the instrument, so that the snap hook could be attached from the hydrographic platform. With the snap hook in place, the instrument was allowed to drift towards the stern of the ship, where it was recovered using the capstan line through a snatch block in the ship's A-frame. The instrument was then secured to the cradle and moved inside the ship's lab for disassembly in order to replace the gel-cell batteries and cassette tapes.

3.2 Signal Processing

The instrumentation and electronics are discussed in detail in the appendices to this thesis. In this section I will describe the basic processing of the FM signal with reference to the block diagram Figure 2.
Figure 2 - Schematic of signal processing
In all, ten signals are input to voltage-controlled oscillators (VCOs) which convert the signal voltages to frequency modulated (FM) signals. These are summed to produce the FM multiplexed signal. This signal is recorded on cassette tape inside Camel III. To view the measurements in real time, an XBT wire link is used to transmit the signal to the ship, where it is discriminated (the frequency is converted back to a voltage) and viewed on a chart recorder. A big advantage of real time viewing is the peace of mind which goes with a successfully operating system. However, due to impedance losses when the wire is extended and the finite length of the wire, there is a practical limit of about 900 meters in depth when using the XBT wire link. Since the majority of the drops in this project were deeper, the cassette recorder system was the primary tool for signal recording.

The cassette was immediately copied once aboard the ship and viewed on a chart recorder. Back in the lab, the signals were digitized using an LSI-11 computer and recorded on magnetic tape. The high frequency shear signals are subject to several forms of glitches (bad data points) which are discussed in Lueck, Crawford and Osborn(1983). To eradicate these (which may be 1-100 data points long) an automatic deglitching routine was used which provides an objective criterion for the determination of bad data and agrees with 'eyeballed' data better than 90 percent of the time. Once the data were suitably 'clean' they were processed as in Appendix F.
3.3 The Study Areas

Figure 3 is a map of the Pacific Ocean with the PEQUOD and WESPAC cruise regions enclosed by rectangular boxes. The PEQUOD region was occupied in February, 1982 where nineteen Camel III profiles were made. Thirteen Camel III profiles were made in the WESPAC region in May and June of 1982.

3.3.1 PEQUOD

The location of current meter moorings, White Horse nets and Camel III profiles for the PEQUOD trip are shown in Figure 4. Two transects of the equator were made along 138°W and 145°W. Five additional stations were occupied along the equator. J.Richman and C.Eriksen maintained the moorings while J.Luyten was responsible for the White Horse profiles. Since I have not worked with the current meter data, I will not discuss these any further.

The White Horse is a freely-falling, acoustically self-positioning dropsonde which is used to determine vertical profiles of horizontal ocean currents. It is positioned by a set of three transponders moored to the bottom which interrogate the instrument as it falls at about 1 m/sec. The bottom transponders are referred to as a transponder net. The White Horse also has a Neil Brown micro-profiling CTD mounted on it. Velocities are computed at 25 meter intervals and temperature and salinity at 2 meter intervals. A detailed description of the White Horse is given by Luyten, Needell and Thomson (1982).
Figure 3 - Map of Pacific Ocean showing the WESPAC and PEQUOD study regions
Figure 4 - PEQUOD cruise track (February, 1982). The solid dots with accompanying letters represent White Horse nets. Open circles are at locations of current meter moorings. Open triangles and associated Camel III drop numbers are followed by the depth of the drop in parentheses. The large concentration of drops near 0°, 145°W necessitated the lower left hand blowup.
For the purpose of this study, the White Horse data was used to determine the stability of a stratified fluid in terms of two parameters. The buoyancy frequency, $N = \left(-\frac{g\Delta \rho}{\rho \Delta z} - \left(\frac{g}{c}\right)^2\right)^{\frac{1}{2}}$ (where $c$ is the speed of sound in seawater), is a measure of the static stability of the fluid while $S = \frac{\Delta U}{\Delta z}$ is a measure of the dynamic stability. Small values of $N$ and large values of $S$ tend to destabilize the water column. A discussion of the treatment of the White Horse velocity and CTD data is given in Appendix J.

A total of nineteen Camel III profiles were made during the PEQUOD cruise. Sixteen of these yielded useful data, totalling 12,335 meters of data. Due to a pressure leak in the preamplifier case, drops were limited to 900 meters after drop 3. Eleven drops were made which were nearly synoptic with White Horse profiles. Prior to drop 13 Camel III was not deployed until the White Horse had been brought back safely aboard ship. The maximum time spacing between Camel III and White Horse profiles was about four hours. From drop 13 on, the Camel was deployed immediately after the White Horse and was brought back on board before the White Horse surfaced, resulting in a time lag of only minutes.

A drop log for PEQUOD, Camel III dissipation profiles, White Horse velocity, temperature, salinity and buoyancy frequency profiles are included in Appendix K.
3.3.2 WESPAC

The locations of ten current moorings and twenty-three CTD stations occupied along 152°E from 27°N to 41°N are marked in Figure 5. W.Schmitz deployed the moorings while P.Niiler commissioned the CTD data. Thirteen Camel III profiles were made, eleven of which yielded 13,070 meters of good data. Having solved the leaking preamplifier problem, it was possible to make drops to greater depths than were made along the equator. Consequently, 5805 meters of data below the depth of 1000 meters were obtained and three drops were made to nearly 2300 meters. Figure 6 shows the depth distribution of data from both the PEQUOD and WESPAC trips. Nearly equivalent total amounts of data were obtained but the PEQUOD data are concentrated above 900 meters while WESPAC data ranges to 2300 meters. For the WESPAC trip, Camel III was deployed so that it broke surface shortly after the CTD was brought aboard ship.

Appendix L includes a drop log for WESPAC, Camel III dissipation profiles and temperature, salinity and buoyancy frequency profiles from the CTD data (a discussion of the treatment of CTD data is given in Appendix J).
Figure 5 - WESPAC cruise track (May/June, 1982). Solid triangles and numbers to left of 152° meridian represent CTD casts, open circles are locations of current meter moorings and open triangles are Camel III profiles. The drop number accompanies the Camel III position and the drop depth (dbar) is in parentheses.
Figure 6 - Total lengths of data record from the specified 100 dbar depth intervals for the total PEQUOD and total WESPAC data sets.
IV. RESULTS FROM WESPAC

Camel III profiles were made in May and June of 1982 in the Western North Pacific Ocean along 152°E from 28°N to 42°N (and a single profile at 23°N, 148°W). These were made in conjunction with the final recovery of the WP1 array of moored instruments which were originally deployed in mid-summer of 1980. Preliminary results are presented in Schmitz et al. (1982). The region roughly bounds the most energetic segment of the Kuroshio Extension, a site which has been relatively sparsely sampled when compared to the analogous region of the Western North Atlantic (bounding the Gulf Stream). The WP1 array was planned as an exploratory array to characterize the geographical variability of the basic time averages and frequency distribution of low frequency currents and temperatures which have periods of two days or longer. Schmitz et al. (1982) refer to these low frequency fluctuations as eddies and point out that the array was not intended to resolve the spatial scales.

The CTD measurements made along 152°E (kindly supplied by P. Niiler) give a spatial snapshot of the temperature and salinity fields at the time of the microstructure measurements. Because of their importance in this study, I have included the T, S and N profiles in Appendix L along with the dissipation profiles from WESPAC.

There have been no previously reported measurements of microstructure in this region. As well, there have been no measurements of velocity microstructure at depths greater than 1000 meters anywhere. In these respects alone, the ensuing
pages describe completely new results.

A large scale description of the structure of the water column along 152°E from 28°N to 42°N in May/June, 1982 is given in terms of Figures 7-9 which are, respectively, sections of temperature, salinity and $e$ profiles.

The temperature section (Figure 7) shows a number of interesting large scale features at the time of the 1982 WESPAC cruise. A large cold core ring-like feature was centred at about 34°N (I will henceforth refer to this as a ring although with only a single section one cannot be sure that this is so). This is an important feature in separating the Camel III profiles into different regions. Drops 1-8 were made south of the ring while drop 9 was made directly in the middle of the ring and drops 10-13 were north of the ring (but due to instrument problems these latter drops contain only a small fraction of good data above 500 meters). The axis of the Kuroshio Extension is defined by Schmitz et al. (1982) as the location of the 15°C isotherm at 200 meters depth, and this occurs at about 37.5°N in early summer of 1982, which represents a northward shift of approximately 2° from its position in July, 1980 during the original deployment of WP1. The signatures of the ring and of the Kuroshio Extension front are both quite deep, apparent at least as deep as 1800 meters. The upward sloping isotherms north of the Kuroshio Extension front indicate the front of the Oyashio, which flows southward from the Bering Sea and Sea of Okhotsk. As well, north of the ring, there is considerable interleaving of water masses as seen by the closed
Figure 7 - Temperature section from CTD measurements taken along 152°E in May/June 1982. Isotherms are plotted every 1°C. A cold core ring is centred at 34°N, the Kuroshio Extension front at 38°N and the Oyashio front at 42°N.
contours in both the T and S sections.

South of the ring, the main thermocline lies between 350 and 800 meters depth and the temperature decreases from 15°C to 6°C across this range. The stratification is offset somewhat by the salinity which decreases to a minimum between 600 and 800 meters, thereafter increasing with depth (Figure 8). The isotherms of the main thermocline are nearly parallel south of the ring and tilt upwards to the south. The buoyancy frequency in the main thermocline has maximum values of 0.006-0.008 rad/sec and the maximum shifts downwards to the north.

Figure 8 shows the salinity section along 152°E from early summer, 1982. The ring and frontal features are readily apparent in the salinity section as are the deep signatures of each of these. A coherent salinity minimum bounded by the 34.1 ppt isohalines stretches across the section, generally sloping downward south of the Oyashio front, contorted by the ring and then slightly upward south of the ring corresponding to the upward tilt of the isotherms south of the ring. In all of the profiles south of the ring the maximum in N lies about 200 meters above the minimum in S.

Individual profiles of $\epsilon$ will be discussed in detail in pages to follow. Here I will refer to Figure 9 which represents averages of $\epsilon$ over 100 meter vertical intervals (or approximately 50 independent estimates of $\epsilon$). The profiles are numbered on Figure 9 for comparison with the detailed profiles in Appendix L. The scale of the $\epsilon$ bar graphs is linear in order to emphasize differences in adjacent values which may be reduced
Figure 8 - Salinity section from CTD measurements taken along 152°E in May/June 1982. Isohalines are plotted every 0.1 ppt. Note the shaded salinity minimum bounded by the 34.1 ppt isohalines.
Figure 9 - Turbulent kinetic energy dissipation averaged vertically over 100 meter intervals for 10 drops made along 152°E in May/June, 1982. The histogram scale is linear and proportional to the size indicated in the lower left hand corner.
by the heavy averaging. Blank sections of individual profiles represent bad data.

Except for drop 8, the near surface averaged dissipations are local maxima. Oakey and Elliott (1982) showed that a constant fraction of the energy input by the wind is dissipated by turbulent mixing in the surface mixed layer. The small near-surface value in drop 8 is likely due to reduced energy input at the surface. Unfortunately, I do not have a complete meteorological record but I did record wind speeds at the time of each Camel III drop, when the ship was stationary. These show that the wind speed relative to the ship dropped to nil two days prior to drop 8, rising slowly to 4 knots at the time of the drop. These were the smallest wind speeds recorded on the WESPAC cruise.

Drop 2 is notable due to the high turbulence levels throughout the entire drop, even when averaged over 100 meters. All of the profiles south of the ring except drop 4 have subsurface secondary maxima in $\epsilon$. These are quite distinct in drops 2, 6 and 8. although less apparent in drop 5. The tendency is for the maximum in $\epsilon$ to increase in depth northwards towards the ring. Recall that the depth range of the isotherms in the main thermocline encompasses the $\epsilon$ maxima and that these slope down towards the north between 27.5°N and 32.5°N, (Figure 7).

Drop 9 made inside the cold core ring is relatively quiet except for a small maximum in $\epsilon$ near 400 meters. The drops north of the ring are interesting in that they represent the
first measurements of velocity microstructure at these depths. They show signal levels greater than $10^{-6}$W/m$^3$ occurring over 12% of the data from depths > 1000 meters. Most of the patches are not thick (< 5 meters) with a notable exception being the 30 meter thick patch at 2050 meters from drop 11. This has a patch-averaged dissipation of $3 \times 10^{-6}$W/m$^3$.

4.1 The Drops South Of The Ring

Drops 1, 2, 4, 5, 6 and 8 shown in Appendix L were made south of the ring which is evident in the T and S sections along 152°E. Drop 1 was a test drop made well south of the ring and the CTD data were not available.

CTD profiles coinciding with these drops south of the ring indicate a shallow seasonal thermocline and a strong salinity minimum at the bottom of the main thermocline. The main thermocline is distinguished by a peak in N which is evident in Figure 10. Figure 10 represents averages of N over 25 meter depths over the five drops south of the ring that have associated CTD data. The maximum in Figure 10 at 500 meters is more than twice the value of the minimum at 300 meters. The minimum in N is approximately equal to N at 900 meters, below which N gradually decreases.

Individual profiles exhibit turbulent patches ranging in size from < 2 meters to nearly 50 meters thick and these extend over the depth range sampled (the deepest drop south of the ring was drop 2 which was to 1450 meters). Drop 2 was the most energetic of the profiles at depth, from both PEQUOD and WESPAC
Figure 10 - Vertical profile of averages of 25 meter estimates of buoyancy frequency over WESPAC drops 2, 4, 5, 6, 8.
trips, bearing little resemblance in either magnitude or 'patchiness' to any of the other profiles from WESPAC. In fact it most closely resembles drop 13 from the Fine and Microstructure Experiment (FAME) reported by Gargett and Osborn (1981) and which was made within the 2000 meter bathymetric contour about the island of Bermuda. Using the objective method described in Chapter 8 to determine patch thickness, 38% of the water column is turbulent in drop 2 (i.e., $> 10^{-6} \text{W/m}^3$) compared to an average value of 19% over all of the WESPAC drops.

There are two distinct features of the drops made south of the ring. Firstly, in the depth region bounded roughly by 200 and 450 meters, there are consistent and extensive (50-200 meters) intervals which are at or near the instrument noise level of $3 \times 10^{-7} \text{W/m}^3$. These regions concur with the subsurface minima in the profiles of buoyancy frequency, $N$. Secondly, the subsurface maxima of $N$ contain both more frequently occurring turbulent patches and higher levels of $\varepsilon$.

Figure 11 is a plot of 25 meter vertical averages of $\varepsilon$ which were then averaged over drops 2, 4, 5, 6, 8 south of the ring. The minimum in turbulent activity between 200 and 400 meters stands out distinctly. In fact, seven adjacent points in the range 200-400 meters are the smallest values over the entire depth range of the averages to 1350 meters (averaging was not done deeper than 1350 meters since only drop 2 was deeper while at least three drops were averaged into the remainder of the points). The maximum value of $\varepsilon$ is at 500 meters and it is
Figure 11 - Vertical profile of averages of 25 meter estimates of turbulent kinetic energy dissipation over WESPAC drops 2, 4, 5, 6, 8.
larger than the averaged value nearest to the surface. Below 500 meters there is considerable variability of $\varepsilon$, adjacent 25 meter averages differing by up to a factor of six. This variability is in contrast to the relatively smoothly varying averaged $\varepsilon$ profile over 200-450 meters.

4.2 $\varepsilon$ and Eddy Kinetic Energy

In the ensuing pages and in the next chapter, I will try to show that the turbulence in the drops south of the ring is related to the internal wave field, and provide some reasons to support this contention. As this set of measurements was made in a region of the ocean for which a strong energy source exists in the form of the western boundary current whose energy contribution to the surroundings may be estimated by determining the local eddy kinetic energy, $K$, results from Schmitz et al. (1982) are presented to attempt to convince the reader of the lack of correlation between $\varepsilon$ and $K$. $K$ is calculated using the mean values of the variances of the horizontal velocity components measured by moored current meters which were deployed in mid-summer, 1980 and recovered in early summer, 1981. Averaging is done over about eleven months of data which, hopefully, describes the climate of $K$ in this part of the ocean. Schmitz et al. (1982) discuss the data handling in more detail.

Schmitz et al. (1982) describe selected results from the first setting of the WP1 mooring array from mid-summer 1980 to
mid-summer 1981. A CTD section from July 1980 (Figure 2 from Schmitz et al. (1982)) indicates a cold core ring at 33.5°N and the front of the Kuroshio Extension at 35.5°N, which represents a southward shift of 2° for the front and about 1° for the ring when compared to the 1982 T and S sections of Figures 7 and 8 (this is likely an entirely different ring from 1982). With this in mind consider Figure 12a which is derived from Figures 5 and 8 of Schmitz et al. (1982). These data are eddy kinetic energies estimated along 152°E for the initial WP1 deployment, at three depth intervals which correspond to depth intervals covered by the Camel drops. In Figure 12b are the dissipations from the Camel drops averaged over the corresponding depth intervals. The two plots have been shifted 1° with respect to each other to align the positions of the rings and Kuroshio Extension fronts from 1980 and 1982. A severe limitation to this comparison is the relative averaging of the two data sets. The current meter data are heavily averaged over time while the dissipation profiles represent spatial snapshots through the field of turbulence, which does itself vary considerably.

There are two distinct trends to the latitudinal section of $K$. First of all, for all three depth intervals, $K$ increases northward from 28°N by about a factor of ten, with peak values at (for the deeper meters) or just north of (for the shallow meters) the position of the ring. Secondly, at each mooring position, vertically adjacent and deeper meters record substantially lower values of $K$. Vertically adjacent values
Figure 12 - a) Eddy kinetic energy estimated by Schmitz et al. (1982) for current meters located at depth intervals 250-300m (dots), 500-700m (triangles) and 1000-1500m (squares) along 152°E. Longitude is marked above the plots. b) Turbulent kinetic energy dissipation averaged over depth intervals corresponding to a) one year after the recovery of the moorings by Schmitz et al. (1982). The plot is shifted 1° to the south for reasons discussed in the text. Note that (in contrast to Figures 7-9) south is to the right.
differ by up to a factor of ten. These two traits are in contrast to the $\epsilon$ section. Indeed, the 500-700 meter (mid-depth) averaged values of $\epsilon$ are the greatest for every drop south of the ring and these reach a peak which is located about $2^\circ$ south of the ring where the peak in $K$ occurs. There is no apparent peak in the average value of $\epsilon$ in either of the 250-300 meter or 1000-1500 meter segments. Drop 9, which was made inside the ring, actually has some of the smallest values of averaged $\epsilon$. From these albeit rough and qualitative results there is no apparent reason to suggest a relation between turbulence levels and kinetic energy in the eddy field.

4.3 $\epsilon$ And $N$

The other available information to which the turbulence may be compared is the CTD data. As discussed in Appendix L, values of $N$ were calculated from the $T$, $S$ data over 25 meter depth intervals. For each of the data sets, independent values of $\epsilon$ were averaged over corresponding 25 meter depth intervals to produce the set $(\epsilon, N)_{25}$. Subsets of these data were chosen so that the upper (20-300m) and lower (300m-bottom) portions of the water column could be considered separately (this grouping is more crucial for the PEQUOD data where the effects of the current fields are generally confined to the upper 300 meters).

From the scatter plots of Figure 13 it is difficult to distinguish any trends. However, it is noted that the highest values of $N$ are associated with the highest values of $\epsilon$ while the lowest values of $N$ coincide with lower values of $\epsilon$, except
Figure 13 - Scatter plots of buoyancy frequency and turbulent kinetic energy dissipation, each estimated for 25 meter depth intervals. Averages were made over 1/3 decade intervals in N (solid dots).
for the low N, near surface values in 13a which are strongly
turbulent due to wind mixing. The large black dots represent
averages of $e$ made over all the values lying within half decade
intervals of $N$. A paucity of values in the upper $N$ bin of 13b
precluded meaningful averaging. In each of the three plots,
there is a strong positive correlation of the averaged values
between $e$ and $N$ (because of wind mixing, the averaged value in
the lowest $N$ bin in 13a is not representative). The line drawn
in 13c has a slope of 1.

A distinctly different form of averaging is presented in
Figures 14 and 15. These averages of $\bar{e}$ and $\bar{N}$ were made by
averaging vertically over 100 meters and then over all of the
drops. The resultant vertical profiles are 14a,b. Even though
the drops made in and north of the cold core ring have now been
included in the averaging, the maxima and minima in and above
the thermocline are still apparent (cf. Figures 10 and 11).
The high averaged $e$ value at 2100 meters is dominated by the
event in drop 11 due to the small amount of data available at
this depth.

The $\bar{e}$ and $\bar{N}$ profiles of 14a,b are combined in Figure 15 to
show a relatively strong correlation of $e$ with $N$ over one full
decade in each parameter. Bars were drawn about the 850 meter
data point in order to provide an indication of the spread of
the data about the mean. For each data point, three consecutive
25 meter values of $e$ were averaged for each drop and then
averaged over all of the drops. For the 850 meter data point,
Figure 14 - a) 100 meter vertical averages of buoyancy frequency averaged over all of the WESPAC drops. b) 100 meter vertical averages of turbulent kinetic energy dissipation averaged over all of the WESPAC drops.
Figure 15 - Log-log plots of $\bar{\varepsilon}$ vs $\bar{N}$ from Figures 14a,b.
there were 20 points in the depth range 825≤z≤875 meters. The average value was 1.0×10⁻⁶ W/m³ and the standard deviation about the mean was 1.8×10⁻⁶ W/m³. 90% of the points had values of ε < 1.7×10⁻⁶ W/m³ and 90% had values > 2×10⁻⁷ W/m³. The middle 80% of the data, then, is bounded by 2×10⁻⁷ < ε < 1.7×10⁻⁶ W/m³. These uncertainty or data spread bars are drawn in Figure 15.

A point to note concerns the differences in averaging. Although the averaging done in Figure 15 is over depth intervals as compared to N-intervals in Figure 13, both exhibit a strong positive slope which is not much different from 1.

As noted by Lueck, Crawford and Osborn (1983), the observation that e/N may be a constant for midocean measurements at any location is not well established. However, there exist arguments based on internal wave dynamics which suggest that the rate of energy lost by the internal wave field to the turbulence scales with N where γ is between 1 and 2. WKB scaling indicates that the total energy in the internal wave field (TE) is linearly proportional to N (Munk (1981)) and, if τ is the decay time scale of the internal wave field, then ε = TE/τ a N⁻γ. As well, a recent discussion by Gargett and Holloway (1984) suggests that γ is 1 or 1.5 depending on the appropriate scaling for the vertical velocity variance of the internal wave field.

Various accounts, then, suggest that, when e has a strong positive dependence on N, the source of the turbulent energy is the internal wave field. This will be pursued further in the following chapter.
To complete the presentation of the $\bar{e} - \bar{N}$ statistics, a comparison is made with other data sets. Lueck, Crawford and Osborn (1983) made a series of measurements over the continental slope of Vancouver Island in May, 1980. The data were cluster-averaged over the depth intervals 100-200m, 200-500m, and 500m-bottom. These cluster averages are plotted as solid triangles in Figure 16. Gargett and Osborn (1981) averaged $\varepsilon$ for $N \approx 2$ cph and these are the open circles in Figure 16. The upper and lower circles are joined by a bar indicating the range within which lies the true value of $\bar{e}$. Solid circles are WESPAC values reproduced from Figure 15 and solid squares are from PEQUOD (only the data below 300m is shown from PEQUOD). A line has been drawn with a slope of 1 and the same intercept as Figure 15.

A comparison of the data sets is best accomplished by considering the constant of proportionality of each estimated from $\bar{e} = a_0 \bar{N}$. This was done in a relatively crude manner by 'eyeballing' a best fit line of slope = 1 through each data set (Lueck, Crawford and Osborn (1983) and Gargett and Osborn (1981) have already done this for their respective data sets). These result in the following values of $a_0$ (in units of $m^2/sec^3$-sec):

- WESPAC - $2.2 \times 10^{-7}$
- PEQUOD - $1.4 \times 10^{-7}$
- Lueck, Crawford and Osborn (1983) - $1.8 \times 10^{-7}$
- Gargett and Osborn (1981) - $4.0 \times 10^{-7}$. 
Figure 16 - Log-log plots of $\bar{\varepsilon}$ vs $\bar{N}$ for the four data sets in the legend and further discussed in the text. The WESPAC and Sargasso Sea values are greater than those from PEQUOD and the Vancouver Island slope.
The Gargett and Osborn (1981) intercept was calculated using an additional data point lying outside the range of the present plot. Although the uncertainty in the estimate of $a_0$ is quite large, it is certainly indicative of the relative levels of $\epsilon$ from each region. From Figure 16, it is quite apparent that the WESPAC levels are greater than both of the PEQUOD and Vancouver Island slope levels and that the PEQUOD data set includes the smallest averages of this present plot. In the spirit of Lueck, Crawford and Osborn (1983), then, Figure 16 is offered as an appended guideline with which to compare other values of $\epsilon/N$ in the ocean.

Although it is clearly impossible to determine an indisputable parameterization of the dependence of $\epsilon$ and $N$ with the available data, it is nonetheless encouraging and perhaps surprising that the data behave so well. An omnipresent problem in relating the rapidly changing turbulence parameters to the 'mean' state involves the averaging process required to get a stable estimate of $\epsilon$. With a few spatial snapshots of the vertical field of the turbulence at numerous locations, this criterion is met only to a minimal degree. However, there do exist distinct trends and these should not be ignored. With the relationship inferred from Figures 13, 15 and 16, and the apparent agreement with the internal wave scaling, one is encouraged to examine further the dependence of the turbulence on the internal wave field.
V. A MODEL OF TURBULENCE IN AN INTERNAL WAVE FIELD

In this chapter, an attempt is made to model the distribution of shear instability in a field of internal waves. An oft-quoted condition for stability in a stratified shear flow requires that the Richardson number, \( \text{Ri} = \frac{N^2}{(u')^2} \), > 1/4 everywhere in the flow (Turner(1973) discusses experimental results and Miles(1961) makes the initial derivation for a variably stratified water column which experiences a mean shear). For the purposes of this chapter, \( u' \) refers to the vertical derivative of the mean flow (that is, the mean shear). Locally the Richardson number may be reduced to a value less than 1/4 due to the superposition of internal wave shears and one may infer that \( \text{Ri} < 1/4 \) is an indicator of instability in the internal wave field (breaking). Indeed, provocative evidence of a Richardson number cutoff at 1/4 in an internal wave field is provided by Eriksen(1978 ,Table 1). Eriksen describes an almost flat distribution of the frequency of occurrence of Richardson number versus \( \arctan(\text{Ri}) \) down to \( \text{Ri} = 1/4 \), below which there is an abrupt cutoff. Eriksen makes the following remarks: 'The measurements may be interpreted as evidence that breaking does occur. The remarkable experimental result is that a cutoff at a Richardson number of 1/4 exists over vertical scales of the order of a few meters'.

To simulate the effect of a field of randomly superposed internal waves, an estimate for the shear is required to evaluate the Richardson number from a given distribution of \( N \) (the distribution used is the profile of Figure 10). With the
aid of the Garrett and Munk expressions for the internal wave dynamics, the shear, $u'$, is calculated from the mean square of the predicted shear spectrum. A Rayleigh distribution is used to estimate the likelihood of occurrence of a shear of sufficient magnitude to create a Richardson number small enough for instability. This is then compared to the data.

5.1 Internal Wave Energy Profiles

On the basis of linear theory and the available observations, Garrett and Munk (1972, 1975, 1979) have synthesized a model of the complete frequency-wavenumber spectrum of internal waves. This model has withstood the test of vigorous experimental and theoretical investigation since its conception and now provides a standard for the comparison of new results. This study will begin with an estimate of the energy in the internal wave field calculated from the measured buoyancy frequency, $N$, and the GM spectral levels.

Munk (1981) gives a recent discussion of internal waves and small scale processes. The most recent forms of the GM spectra are presented as follows, with $F$ the spectrum of vertical displacement, $F_{\xi}$ that of horizontal velocities and $F_e$ that of the total wave energy:

\[
F(\omega,j) = \frac{b^2 N_0 (\omega^2 - f^2) E(\omega,j)}{\omega^2} \quad \text{(5.1)}
\]

\[
F_{\xi}(\omega,j) = b^2 N_0 N (\omega^2 + f^2) E(\omega,j) \quad \text{(5.2)}
\]

\[
F_e(\omega,j) = \frac{1}{2} (F_e + N^2 F_{\xi}) = b^2 N_0 NE(\omega,j) \quad \text{(5.3)}
\]
where \( j \) is the vertical mode number, \( b \) is the e-folding scale of \( N(z) \) and is taken to be 1300 meters, \( N_0 = 5.2 \times 10^{-3} \) sec\(^{-1} \) (3 cph) is the surface-extrapolated buoyancy frequency used by GM, and \( f = 7.3 \times 10^{-5} \) sec\(^{-1} \) is the Coriolis frequency at 30\(^{\circ} \) latitude. \( E(\omega,j) \) is the dimensionless energy density defined by

\[
E(\omega,j) = B(\omega)H(j)E,
\]

\[
B(\omega) = \frac{2f}{(\omega^2 - f^2)\frac{1}{2}},
\]

\[
N \int \frac{B(\omega)d\omega}{f} = 1,
\]

\[
H(j) = (j^2 + j_0^2)^{-1}/(\Sigma(j^2 + j_0^2)^{-1}),
\]

\[
\Sigma H(j) = 1,
\]

where \( j_0 = 3 \) is a mode scale number. \( E \) is the dimensionless internal wave 'energy parameter' and is set at \( 6.3 \times 10^{-5} \).

Extensive measurements have proven \( E \) to be remarkably universal (to within a factor of two). The contribution to \( F \) of the vertical velocity spectrum is considered to be negligible by Munk(1981) compared to those of \( F \) and \( F \) (compare Figures 7 and 14 of Eriksen(1978)).

The mean-square quantities are, then, (for \( f \ll N \))

\[
\langle \xi^2 \rangle = \int d\omega \Sigma F(\omega,j) = \frac{1}{2}b^2EN_0 = 53N_0 \text{ [m}^2\text{]}
\]

\[
\langle u^2 \rangle = \int d\omega \Sigma F(\omega,j) = \frac{3}{2}b^2EN_0N = 44 \times 10^{-4}N \text{ [m}^2\text{/s}^2\text{]}
\]

from which the potential(PE) and kinetic(KE) energies are calculated as a function of depth, \( z \);
Figure 17 - Plots of internal wave potential energy (PE), kinetic energy (KE) and total energy (TE) calculated from the equations in section 5.1 of the text and the buoyancy frequency profile of Figure 10.
\[ PE = \frac{1}{2} N^2(z) \langle \xi^2(z) \rangle \]
\[ KE = \frac{1}{2} \langle u^2(z) \rangle \]
\[ TE = PE+KE = \left\{ \frac{53N_0}{2+22\times10^{-4}/N_0} \right\}N \text{ [m}^2/\text{sec}^2] \]

Using the \( N \) profile in Figure 10, which represents the WESPAC drops south of the ring, and the calculation of \( \langle u^2 \rangle \) and \( \langle \xi^2 \rangle \) given above, PE, KE and TE are plotted in Figure 17 and will be referred to later.

5.2 Internal Wave Shears

The mean-square value of the vertical shear due to the internal wave field is:
\[ \langle (u')^2 \rangle = \langle (\partial u)^2 \rangle + \langle (\partial v)^2 \rangle = m^2 \langle u^2 \rangle \]

The dispersion relation for linear internal waves is (Olbers, 1983):
\[ m^2 = k^2 \frac{(N^2 - \omega^2)}{\omega^2 - f^2}, \quad (5.5) \]

where \( m \) is the vertical and \( k \) the horizontal wavenumber. The vertical velocities may be expressed as \( w = \text{asin}(mz) \). For a constant value of \( N \) the requirement that \( w = 0 \) at the ocean bottom, \( H \), yields \( mH = j\pi, \ j = 1,2,... \)

When \( N \) is a function of \( z \), a WKB solution is required and the condition \( mH = j\pi \) becomes (Munk, 1981)
\[ m = \frac{j\pi \sqrt{(N^2 - \omega^2)}}{b \sqrt{N_0^2 - \omega^2}}, \quad (5.6) \]

This form of the boundary condition assures attenuation outside the waveguide. Because of the relative intractability of this form for \( m \), the limiting case of \( \omega << N \) is taken so that
\[ m = \frac{j \pi N}{b N_0} \tag{5.7} \]

This simplification is made with some concern about the contribution of high frequency waves trapped in the thermocline. However, there is evidence to indicate that although the shear spectrum is relatively flat in the vertical wavenumber domain (Gargett et al. (1981)), it slopes downward from low to high frequencies (McComas and Muller (1981)) implying that the internal wave shear is concentrated in the low frequency modes, and that the simplification suggested here may be justified.

The expression for the mean-square shear becomes
\[ \langle (u')^2 \rangle = \int d\omega \sum_m^N F (\omega, j) \]

which, with (5.2) and (5.7) is
\[ \langle (u')^2 \rangle = 2\pi f E N^3 \sum_j^N j^2 H(j) f \frac{\omega^3}{N_0} \frac{(\omega^2 + f^2)}{\sqrt{\omega^2 - f^2}} d\omega. \]

The integral is \( \frac{3\pi}{4} f \) and the summation \( \sum j^2 H(j) = j J \), where \( J = 2.13 \) and it is required that \( j_* \), the upper limit to the finite estimate of \( \sum j^2 H(j) \), be \( >> j_0 = 3 \). Thus,
\[ \langle (u')^2 \rangle = \frac{3\pi^2 N^3 E j_* J}{2 N_0} \tag{5.8} \]

The upper limit to the vertical mode number, \( j_* \), is a critical parameter which is related to the vertical wavelength by
\[ m_* = 2\pi/\lambda_* = j_* \pi N/b N_0 \]

A parameter study showing the dependence of \( \lambda_* \) on \( j_* \) for a range of realistic \( N \) values is shown in Figure 18. The choice of \( j_* \) must concur with the upper wavenumber cutoff of the internal
Figure 18 - Plot of the dependence of the upper wavelength cutoff, $\lambda_*$, on the upper limit, $j_*$, of the vertical mode number for four different values of buoyancy frequency, $N$. A horizontal line at $\lambda_* = 10$ meters is drawn for comparison to the $\approx 10$ meter break in the vertical shear spectrum found by Gargett et al(1981).
wave spectrum. A composite spectrum of vertical shear in the ocean measured nearly simultaneously by three separate velocity profilers with differing spatial bandwidths was compiled by Gargett et al. (1981). For the spectra presented, there is a consistent break in slope at about 10 meters separating the spectral range which scales with internal wave parameters from that which scales with buoyancy parameters. Gregg (1977) found that the break in slope of the temperature gradient spectra for the internal wave bandwidth was also near 10 meters. A horizontal reference line at 10 meters is drawn in Figure 18. For \( j_+ < 100 \), the wavelengths become many times greater than 10 meters. For \( j_+ > 600 \), the wavelengths for \( N = 2-4 \) cph converge at < 5 meters. Choosing \( 200 \leq j_+ \leq 400 \) allows vertical wavelength cutoffs at 4 cph of 10 (\( j_+ = 200 \)) and 5 meters (\( j_+ = 400 \)) and at 2 cph of 20 (\( j_+ = 200 \)) and 10 meters (\( j_+ = 400 \)). Further calculations are restricted to this range of \( j_+ \).

5.3 The Distribution Of Shear And Richardson Number

In order to calculate the likelihood of small Richardson number in an ocean of known \( N \), the internal waves must be superposed in a realistic fashion in order to estimate the total shear in the field. Desaubies and Smith (1982) use Gaussian statistics for shear and stratification to develop a distribution function for \( R_i \). Indeed, Evans (1982) suggests that shear measurements from the main thermocline in the North Atlantic show no departure from a normal distribution based on a \( \chi^2 \) test. However, the data were not tested for any other distribution and there is no reason, a priori, to reject the
hypothesis that the data follow another distribution (for example, the Rayleigh distribution). A practical consideration is that, although an estimate for the mean-square value of the shear is easily obtained (as we shall see), there is no obvious way to get an estimate for the mean value of the shear from the available data. Both of these values are required for the normal distribution but only the mean-square value for the Rayleigh distribution. There are also some solid grounds for assuming the latter distribution as a descriptor of the statistics of a large number of independent shear vectors. The useful property of the Rayleigh distribution (which was used by Longuet-Higgins (1952) to describe the superposition of a field of surface waves) is that it describes the distribution of amplitudes of a large number of independent two-dimensional vectors having random phase (Maisel (1971)). The form of the probability density function is

\[ p(x) = \frac{x}{\sqrt{2\pi}} e^{-x^2/2}, \quad \text{for } x \geq 0 \]

\[ = 0, \quad \text{for } x < 0 \]

where \( a^2 = \frac{x^2}{2} \) and \( x^2 \) is the mean square value. The cumulative density function is \( \int_{-\infty}^{x} p(x) \, dx \), or

\[ p'(x) = 1 - e^{-x^2/2}. \]

In terms of internal wave shear,

\[ p'(u') = 1 - e^{-(u')^2/\langle (u')^2 \rangle} \]

describes the probability that the shear is less than \( u' \). The upper limit to the corresponding Richardson number, \( \text{Ri} = \)
\( \frac{N^2}{(u')^2} \) requires \( u' \) to have a maximum value described by \( 1 - p'(u') \) so that

\[
\Pr(Ri) = 1-p'(u') = \exp[-(u')^2/(\langle u' \rangle^2)]
\]
describes the probability that the Richardson number is less than some value, \( Ri \). With (5.8),

\[
\Pr(Ri) = \exp\left[ -\frac{2N_o(u')^2}{3\pi^2 N^3 E^J} \right] \quad (5.9)
\]

The squared shear corresponding to an arbitrary constant \( Ri = c_0^{-1} = \frac{N^2}{(u')^2} \) is \( (u')^2 = c_0 N^2 \) so that

\[
\Pr(Ri < c_0^{-1}) = \exp\left[ -\frac{2N_o c_0}{3\pi^2 N E^J} \right]^{-1/N} \quad (5.10)
\]

The expression (5.10) gives an \( e^{-1/N} \) dependence which is an intuitively acceptable choice. This implies that the likelihood of local shear instability in the internal wave regime, formulated in terms of small Richardson number, is greater in regions of large \( N \), such as the thermocline, than in regions of small \( N \). Munk(1981) takes a different approach to determine that 'layers of largest gravitational stability (largest \( N \)) are also layers of largest shear instability (largest \( u'/N \))'. Carried one step further, if the shear instability results in wave breaking and the breaking causes turbulence, then one expects to find the type of \( \epsilon-N \) dependence found in the previous chapter.
5.4 Comparison With The Data

In Figure 19 vertical profiles of $Pr(R_i<1)$ and $Pr(R_i<1/4)$ calculated from (5.10) have been plotted for $j_+ = 200, 300$ and 400. The values of $N$ used are from Figure 10 to simulate the WESPAC region south of the cold core ring. The critical dependence on $j_+$ unfortunately makes it impossible to actually determine a number for $Pr(R_i)$ at any position. On this scale, the curves of $Pr(R_i<1/4)$ are almost indistinguishable from the right hand axis except for high $N$ and $j_+$. The shape of the curves, however, indicate the strong $N$-dependence.

To compare with the probability distribution of small $R_i$, an estimate was made of the fraction of the water column that was turbulent over 50 meter depth intervals (call this PCT for percent turbulent) for the profiles 2,4,5,6,8 south of the ring. The criterion used for turbulence detection was a critical threshold level of dissipation, $\epsilon^c$. The thickness of patches with successive independent estimates of $\epsilon$ which were $> \epsilon^c$ were added to calculate the thickness of individual patches. Over the 50 meter interval the turbulent fraction is $\Sigma$ (patch thicknesses)/50. The scale used in Figure 20 for PCT is the same as in Figure 19. The choice of $\epsilon^c$ is subjective and is, at the low end, limited by the noise level of the instrumentation. Although it is not clear how to interpret the magnitudes of Figure 20 (as well as Figure 19), the shapes of the vertical profiles appear not to be affected by the choice of
Figure 19 - Vertical profiles of Pr(Ri<1) and Pr(Ri<1/4) calculated from equation (5.10) and the buoyancy frequency profile of Figure 10 for j. = 200, 300, 400.
Figure 20 - Vertical profiles of fraction of turbulent water column (PCT) estimated for two different threshold levels of turbulent kinetic energy dissipation. The horizontal scale is identical to that in Figure 19.
ε. Two values of ε are used in Figure 20. 10^-6 \ W/m^3 is 3\times(\text{instrumental noise level} = 3\times10^{-7} \ W/m^3) while 3\times10^{-6} \ W/m^3 is 10\times(\text{noise level}). The difference in PCT using these two threshold levels varies by factors of 2-6. But both curves indicate that the water column is significantly less turbulent (by a factor of 4-5) in the low N region above the thermocline than in the thermocline itself and this decreases below the thermocline. Presumably, other choices of ε would give similar shapes (until such a low value were chosen that it would be noise-saturated or such a high value chosen that it would not exist in the set of measured values).

Figure 20 may be compared to Figure 11 (the averaged ε over 25 meter depth intervals for the drops south of the ring). In general, low PCT concurs with low ε and vice-versa. Anomalies to this trend are indicative of either relative patch thickness or relative turbulence magnitude. Regions where the ratio \overline{ε}/PCT is greater than normal levels are interpreted as having relatively thin and/or strongly turbulent patches and where the ratio \overline{ε}/PCT is smaller, the patches are thicker and/or more weakly turbulent.

A rough test was devised to indicate the degree to which the shape of the model (5.10) agrees with the measured data plotted in Figure 20. Since it has been acknowledged that an averaging problem exists with the limited available data set, it was decided that it was acceptable to smooth the profiles in
Figure 20 with a three-point running average. In order to compare these directly with each other and with the profiles of Pr(Ri<1/4), each profile was normalized using the peak values nearest to the thermocline. The scaling factors and the titles given to each data set are given in Table 1. With the scaling factor, one may recover the original values of Pr(Ri<1/4) and PCT.

<table>
<thead>
<tr>
<th>Data</th>
<th>Set Title</th>
<th>Scaling Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon = 3 \times 10^{-6}$</td>
<td>PCT1</td>
<td>.38 @ 700m</td>
</tr>
<tr>
<td>c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\epsilon = 10^{-6}$</td>
<td>PCT2</td>
<td>.087 @ 500m</td>
</tr>
<tr>
<td>c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pr(Ri&lt;1/4) from (5.10)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$j_0 = 200$</td>
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<td>$6.9 \times 10^{-7}$ @ 500m</td>
</tr>
<tr>
<td>$j_0 = 300$</td>
<td>PR300</td>
<td>$1.7 \times 10^{-5}$ @ 500m</td>
</tr>
<tr>
<td>$j_0 = 400$</td>
<td>PR400</td>
<td>$8.3 \times 10^{-5}$ @ 500m</td>
</tr>
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</table>

Table 1 - Scaling factors for normalization and titles given to data sets.

Profiles of normalized PCT and Pr(Ri<1/4) are shown in Figure 21. Above 600 meters, a depth which is still in the main thermocline, the agreement with the data is very good. However, below about 700 meters the agreement is poor. Sample correlation coefficients were calculated as

$$ r = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{\sqrt{\Sigma(x - \bar{x})^2\Sigma(y - \bar{y})^2}} $$

and these are listed in Table 2. Over the entire depth interval 100-1250 meters, the correlations are poor. But over the interval 100-600 meters the calculated correlation coefficient
<table>
<thead>
<tr>
<th>SETS</th>
<th>DEPTH RANGES</th>
<th>r</th>
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<tbody>
<tr>
<td>PCT1, PR200</td>
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<td>100-600</td>
<td>.89</td>
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<td>PCT2, PR200</td>
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<td>PCT2, PR400</td>
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<td>.94</td>
</tr>
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</table>

Table 2 - Correlation coefficient, r, calculated for the pairs of data sets defined in Table 1 and over the depth ranges specified.
Vertical profiles of normalized PCT from Figure 20 and normalized $Pr(Ri<1/4)$ from Figure 19. The profiles were normalized by the maximum values nearest the thermocline. The normalization factors are listed in Table 1.
is better than .89 and is .94 between the set PCT2 \((\epsilon = 3 \times 10^{-6}\) \(W/m^3\)) and each of the family PR200, PR300 and PR400.

5.5 Discussion

A number of assumptions which were made in order to derive (5.10) should be emphasized. First of all, the GM model upon which this model relies was developed with the use of WKB theory, requiring a slowly varying N(z). The degree to which N(z) in the thermocline is slowly varying is open to question. Secondly, it was assumed that the high frequency internal wave components \((\omega = N)\) do not contribute significantly to the shear variance. Thirdly, a major problem is the high sensitivity of the model to \(j\). and the inability to make a clear choice for this parameter. The other choice to be made is that of \(\epsilon\) which, although it does not affect the model, does affect the data to which it is compared. Finally, it was attempted to correlate two quite different parameters. PCT is an actual measure of the fraction of the water column which has \(\epsilon\) levels greater than a detector threshold, \(\epsilon\). This is compared to the probability that the water column at any point meets the condition for shear instability \((Pr(R_i<1/4))\) due to the local superposition of internal waves. The implicit assumption made requires the instability to manifest itself as turbulence at scales which may be measured by Camel III.

The model is unable to predict the magnitude of PCT. In fact, the uncertainty in the choice of \(j\) alone results in a
scaling factor difference of >100 between \( j_+ = 200 \) and 400. As well, below about 700 meters, neither shape nor magnitudes of PCT or \( Pr(Ri<1/4) \) agree. As has been mentioned, though, this may be in part due to the unrepresentative intense \( \epsilon \) levels exhibited in drop 2 which is discussed in the previous chapter. (Perhaps a greater number of drops in the region would have reduced the influence of drop 2 on the averaging. Conversely, since drop 2 appears to be quite different in character from drops 4,5,6,8 it may be argued that it could be dropped from the averaging scheme in order to improve the behaviour of the averages. This subjective tampering could not be accepted on ethical grounds, however, and it was decided to live with the shortcomings of the limited data set, with the understanding that the averaging is not likely 'correct').

With due regard to the inadequacies stated above, a number of pertinent comments may be made in light of the limited success of the model. Apparently the \( \epsilon^{1/N} \) behaviour of the model is followed by PCT in at least part of the water column. This positive dependence of PCT on \( N \) agrees qualitatively with the relation \( \epsilon \propto N' \) suggested in the previous chapter.

The fact that the magnitudes of PCT and \( Pr(Ri<1/4) \) differ so greatly is in part due to the choices of \( j_+ \) and \( \epsilon \). However, it seems clear that the shear instabilities occur only rarely (the scaling factor for \( Pr400 \) is 1000 times smaller than that of PCT2). To attempt to match PCT and \( Pr(Ri<1/4) \), one might infer either a larger \( j_+ \) (for which we have already determined would
give an unrealistically high wavenumber cutoff in light of Gargett et al. (1981) or a detector threshold level, $\epsilon_c$, which is greater by some factors of 10. This leads to the question of the actual amount of energy which is released by a single breaking event and the mechanism by which the energy is converted to turbulence. For this, no answers are given. However, it is noted that, from the entire WESPAC data set, which constitutes 13,070 meters of data, representing approximately 7,000 independent estimates of $\epsilon$, there is not a single occurrence of $\epsilon > 10^{-4}$ W/m$^3$ and, below 100 meters, < 1% of the data indicates $\epsilon > 10^{-5}$ W/m$^3$ (which translates to a lot of zeroes in a vertical profile such as Figure 20). Hence, there is no evidence of the dissipation levels required to satisfy the above argument. In short, it appears that the probability of a shear instability leading to a breaking event estimated from (5.10) is much smaller than the measured levels of PCT in the ocean.

An interesting sideline involves the question 'how long does an internal wave last'. If the waves are long-lasting, then energy may propagate long distances, thereby effectively dispersing energy in the internal wave field far from a local source and, as suggested by Garrett and Munk (1979), may help to explain why the oceans are 'filled to almost the same equilibrium spectral level everywhere'. The estimate made for the internal wave energy (5.4) and the relation $\epsilon = a_0 N$ may be used to estimate the time scale representing the decay of the internal wave spectrum, provided that $\epsilon$, which is a parameter of
the turbulence, is representative of the energy lost from the internal wave field. Defining \( \tau = \frac{TE}{\epsilon} \),

\[
\tau = \left\{ \frac{53 N_0/2 + 22 \times 10^{-4}}{N_0} \right\} / a_0.
\]

Lueck, Crawford and Osborn (1983) used a value of .0045 rad/sec for \( N_0 \). Figure 14a indicates that a value of \( N_0 \) extrapolated from > 900 meters from WESPAC (to avoid the thermocline maximum) is \( \approx .0055 \) rad/sec. A suitable value from PEQUOD extrapolated from > 300 meters is .005 rad/sec (Figure 25). Gargett and Osborn (1981) show profiles of \( N \) which indicate that \( N_0 \) is \( \approx .0055 \) rad/sec. Together with the appropriate values of \( a_0 \), these yield \( \tau \approx 46 \) days for PEQUOD, 36 days for the Vancouver Island slope data, 29 days for the WESPAC data and 16 days for the Sargasso Sea data. The factor of two variability in the energy parameter, \( E \), via equation (5.4) limits these estimates to within a factor of two uncertainty. If the large scale, low frequency, energy-containing internal waves travel at 10 cm/sec (McComas and Muller (1981)) then they will propagate 200-500 km, which is considerably smaller than the scale of an ocean basin. Garrett and Munk (1979) use a propagation speed of 20 cm/sec, but this only increases the range to 400-1000 km, which is still less than basin scale.
VI. RESULTS FROM PEQUOD

6.1 Currents And Hydrography

The large scale structure of the currents and hydrographic parameters in equatorial regions is now relatively well known. Many standard oceanographic texts (Pond and Pickard(1983), Gill(1982)) contain discussions of equatorial observations and dynamics. Preliminary theoretical and experimental work is regularly reported in the Tropical Ocean-Atmosphere Newsletter (D.Halpern, JISAO, University of Washington, editor). A recent review is given by Leetmaa, McCreary and Moore(1981).

The equatorial current regime is basically comprised of a series of surface and subsurface zonal jets which are of the order $10^6$ meters long by $10^5$ meters wide and $10^2$ meters deep. Prevailing easterly winds in the tropics drive the westward flowing South Equatorial Current (SEC) at latitudes of $10^\circ$S to 4$^\circ$N, and the North Equatorial Current (NEC) between 10$^\circ$N and 20$^\circ$N. In the region of the doldrums from 4$^\circ$N to 10$^\circ$N, the winds are considerably weaker (although still easterly) and the resulting meridional wind shear results in an eastward flowing North Equatorial Countercurrent (NECC).

More remarkable and more pertinent to this study is the existence of the Equatorial Undercurrent (EUC). Due to the predominantly westward transport near the equator, a buildup occurs at the western boundary of the ocean resulting in a west-east pressure gradient. Near the surface, the wind stress is strong enough to dominate, resulting in the westward flowing
SEC, but with increasing depth the effect of the winds is diminished and, in the thermocline below the mixed layer, the pressure gradient dominates. The result is the eastward flowing EUC.

Included in Appendix K with the Camel III profiles from PEQUOD are vertical profiles of horizontal current, temperature, salinity and the calculated buoyancy frequency. These were measured with the White Horse. The treatment of the White Horse data is discussed in Appendix J.

The basic features of the currents and hydrographic parameters are seen in the profiles of net D which is associated with the Camel III drop 3 at 1/2°N, 138°W (see Appendix K). At net D the surface SEC flows westward at about 75 cm/sec while the core of the EUC is situated at about 120 meters and has a maximum eastward velocity of about 75 cm/sec (unfortunately, it is quite possible that the real maximum of the EUC core is missed due to the 25 meter depth resolution of the White Horse velocities). The resulting mean shear is 150 cm/sec in 120 meters or about .01 sec⁻¹. Beneath the EUC core the velocity decreases to near zero. Between 600 and 700 meters there is structure in the velocity field which can be detected above the 4 cm/sec resolution (see Appendix J) of the White Horse measurements. This structure is evident in some of the other profiles (notably net K on 20/02/82 and net Q) but should not be considered typical.

The temperature data (Appendix K) generally indicate a shallow wind mixed layer (the winds in mid-winter are typically
slacker than average). A low gradient in T underlies this and the very strong thermocline is situated at about 100-140 meters depth. The near-surface salinity shows the effects of wind mixing. The typical strong salinity maximum located in the main thermocline is not evident in all of the profiles (but can be seen in nets L and F in Appendix K, for example). All of the salinity profiles exhibit strong finestructure features in the regions near and in the thermocline. The maximum value of N in the thermocline is about .016-.024 rad/sec. A second maximum of about .005 rad/sec persists from profile to profile near 350 meters depth, which is just below the depth of the thermostad (a region of constant temperature of about 12°C which has been consistently found in equatorial CTD profiles).

A unique and interesting feature is the local minimum in many of the N profiles above the thermocline in the large mean shear region of the SEC-EUC interface. Evident in nets D and E especially and to a lesser extent in other profiles, the minimum appears to be due to a combination of locally constant temperature and local weakening of the salinity gradient above the thermocline. M. McPhaden (personal communication) has found a corresponding minimum in the temporal variations of potential temperature and the vertical density gradient. It will be interesting to compare the microstructure data in this region.
6.2 Previous Equatorial Microstructure Measurements

The previous notable equatorial measurements of microstructure are reported in Gregg (1976), Crawford and Osborn (1979a, 1979b, 1981), Osborn and Bilodeau (1980) and Crawford (1982).

Gregg (1976) made six profiles of temperature microstructure at 155°W on the equator in July, 1972. The drops were limited to the upper 500 meters and the characteristics emphasized were those of the large mean shear region of the SEC-EUC interface, the core of the EUC and the thermostad. The intensity of the turbulence was found to be exceptionally high in the SEC-EUC interface, quite weak in the EUC core, moderate in the thermostad between 300 and 400 meters and very weak below the thermostad. Osborn and Bilodeau (1980) collected temperature microstructure data in the equatorial Atlantic between 24°W and 33°W in June and July, 1974. Nearly coincident velocity microstructure data allowed them to make the following points; temperature microstructure is generally found concurrently with velocity turbulence fluctuations unless the temperature gradient microstructure is consistently of one sign; turbulent patches extend horizontally over at least some tens of meters; the temperature microstructure and finestructure are more intense in the Osborn and Bilodeau Atlantic measurements than in the Gregg Pacific measurements.

Velocity microstructure measurements in the equatorial Atlantic in June and July, 1974 were made by Crawford and Osborn (1979a, b) and in the equatorial Pacific in January and
February, 1979 (Crawford and Osborn(1981a), Crawford(1982)). Again, the measurements were confined to the upper 500 meters. These results indicate large turbulent intensities in the SEC-EUC interface and low intensities in the EUC core. In the Atlantic, high turbulent intensities were also found below the EUC core but were not found below the EUC core in the Pacific. The remarkable result of these studies was the connection made between the turbulence measurements and the large scale dynamics of equatorial currents. The Atlantic data indicated that the rate of dissipation of turbulent kinetic energy in the EUC above the core is comparable in magnitude to the rate at which the EUC gains energy from the zonal pressure gradient, thereby defining the role of turbulent friction as a sink of kinetic energy in equatorial currents.

6.3 PEQUOD Microstructure

Profiles of turbulent kinetic energy dissipation, $\epsilon$, calculated from the velocity microstructure data obtained from the central equatorial Pacific in February, 1982 are contained in Appendix K. Thirteen profiles were made at or within $1^\circ$ of the equator (these will be referred to as on-equator) and the other three were made within $2^\circ$ of the equator (and will be referred to as off-equator).

Without exception, the on-equator profiles in Appendix K display high levels of dissipation in the large mean shear region of the SEC-EUC interface. The estimated $\epsilon$ is usually much smaller in the velocity core of the EUC where the mean
shear is a minimum. Below the core, the mean shear is high but only occasionally is the $\varepsilon$ very large in this region (as found by Crawford and Osborn (1981b) for the Pacific data but which is contrary to the results from the Atlantic). For example, although drop 3 has the second largest value of $\varepsilon$ averaged over 20 to 140 meters depth, in the large mean shear region at the base of the EUC, the individual estimates of $\varepsilon$ are quite small. However, the maximum $\varepsilon$ values from drop 5 are at the base of the EUC.

The strong thermocline appears to be an effective barrier to the exchange of turbulent energy from the surface to the deeper waters. In fact, a recurrent feature in many of the profiles is the very high level of turbulence with varying degrees of intermittency in the upper 150 meters or so, below which there is a sharp cutoff and individual estimates of $\varepsilon$ are at or near the noise level. At greater depths in the equatorial profiles the occurrence of turbulent patches is much less frequent and the levels of turbulence are much lower.

Another interesting feature, especially in light of McPhaden's estimate of low finestructure activity in the region of minimum static stability above the EUC core and which was briefly mentioned in the previous section, is strong evidence in at least three drops of low microstructure activity in the low $N$ region above the EUC core. Drop 2 (100 meters), drop 3 (65 meters) and drop 6 (80 meters) all show reduced $\varepsilon$ in the minimum $N$ region; at least one and up to two decades smaller compared to thick patches of turbulence immediately above and below. In
other drops the minimum in $N$ is either very weak or nonexistent and there is no obvious minimum $\epsilon$ region.

Of eight drops made within $1/2^\circ$ of the equator at $145^\circ W$, four ($12, 14, 15, 17$) show strikingly similar turbulent patches at 500 meters depth. Drop 10 shows no sign of a 500 meter turbulent patch, drops 7 and 8 have thinner patches at 500 meters and drop 13 has three nearly equidistant 5 meter thick patches separated by $\approx 25$ meters and centred at 500 meters. These are summarized in Table 3.

Figure 22 compares on-equator and off-equator profiles. Independent 2 meter estimates of $\epsilon$ were averaged vertically over 50 meters and then over all of the drops within the respective latitude range. The two upper values (50, 100 meters) indicate averaged $\epsilon$ greater near the equator by a factor of 4-5 over the off-equator drops. Averaged over the upper 200 meters, the dissipation is $1.6 \times 10^{-5}$ W/m$^3$ for the on-equator data and $3.3 \times 10^{-6}$ W/m$^3$ for off-equator drops. At 150 meters, corresponding to a depth just below the EUC core, the difference is only a factor of two. Below 200 meters, only three (of fourteen) separate on-equator averages of $\epsilon$ are smaller than off-equator averages. When averaged over the depth range 200-900 meters $\epsilon$ is $5.6 \times 10^{-7}$ W/m$^3$ for on-equator drops and $3.3 \times 10^{-7}$ W/m$^3$ for off-equator drops.

As a comparison with the measurements of Crawford and Osborn (1979a,b) and Crawford (1982), dissipations were averaged over the depth range 20-140 meters and plotted on Figure 3 of Crawford (1982) which is shown here as Figure 23. The two single
<table>
<thead>
<tr>
<th>DROP</th>
<th>PATCH CENTRE</th>
<th>THICKNESS</th>
<th>$\varepsilon (W/m^3 \times 10^7)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td></td>
<td></td>
<td>weak</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td>weak</td>
</tr>
<tr>
<td>10</td>
<td>no sign</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>483 meters</td>
<td>35 meters</td>
<td>50</td>
</tr>
<tr>
<td>13</td>
<td>3 patches each 5 meters</td>
<td></td>
<td>10-20</td>
</tr>
<tr>
<td>14</td>
<td>510</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>540</td>
<td>15</td>
<td>30</td>
</tr>
<tr>
<td>15</td>
<td>492</td>
<td>10</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>518</td>
<td>27</td>
<td>20</td>
</tr>
<tr>
<td>17</td>
<td>492</td>
<td>35</td>
<td>40</td>
</tr>
</tbody>
</table>

Table 3 - Depth, thickness and patch-averaged dissipations for the patches located near 500 meters depth for the drops within 1/2° of the equator at 145°W.
Figure 22 - Vertical profiles of 50 meter vertically averaged values of \( \log e \) averaged over all of the Camel III profiles taken within 1/2° of the equator (dots) and over the profiles outside of 1° of the equator (triangles).
Figure 23 - Averaged dissipations observed during the Parizeau cruise in 1979, the Atlantis II cruise in 1974 and the Thomas G. Thompson cruise in 1982. The 1982 data were added to Figure 3 from Crawford (1982).
largest values from 1982 are within 1/2° of the equator and the three off-equator values are all less than $7 \times 10^{-6}$ W/m$^3$, in general agreement with Crawford (1982). However, of the thirteen on-equator drops, only the two drops mentioned are distinguishable in magnitude from the off-equator drops. The large and recurrent peak at 0° found by Crawford (1982) for both the Atlantic, 1974 and Pacific, 1979 equatorial data is clearly not so dominant for the 1982 data, which were taken, it should be noted, well prior to the onset of the 1982 El Nino event.

6.4 The On-equator Profiles

The longitude of on-equator stations occupied where joint Camel-White Horse profiles were made are listed in Table 4 and shown in Figure 26. Three of the profiles are from 138°W, four from 145°W and one from 153°W. The other five equatorial profiles were made at or near 145°W but were not synoptic with White Horse profiles. Crawford made nineteen profiles within 1/2° of the equator in early 1979, all at 150°W, and nine of these were accompanied by over-the-side current meter profiles from which the shear over the depth range 20 to 140 meters was estimated.

White Horse velocity profiles were smoothed using a 3-point running mean filter and then first-differenced to estimate the magnitude of the shear over the 25 meter range. The Brunt-Vaisala frequency was calculated over approximately 25 meters to correspond to the calculated shear. The eight individual profiles are shown in Figures 24 and 25, where the thick line represents the average value at each depth of the eight
profiles. Figure 26 includes the symbol key for these diagrams.

The mean shear profile shows a minimum near the surface in the South Equatorial Current (SEC) which increases to a maximum value of about .014 sec\(^{-1}\) in the region of the SEC-Equatorial Undercurrent (EUC) interface. Values span almost a factor of 3 above the EUC core from .008 to .022 sec\(^{-1}\), with the larger shears generally found in the eastern profiles. As well as can be determined, given the 25 meter depth resolution of the White Horse velocity measurements, the EUC core was about 130 meters deep at 138°W, only slightly deeper at 145°W and about 150 meters deep at 153°W. The core is indicated by the minimum in the shear profile. Below the EUC core is a second shear maximum of about half the magnitude of the upper one.

The Brunt-Vaisala profile shows a subsurface maximum of average value .018 rad/sec located in the EUC core. This maximum deepens westward. A second maximum located at approximately 350 to 420 meters is considerably smaller but historically persistent. A comparison of shear and Brunt-Vaisala profiles indicate bulk Richardson numbers (Ri) less than one only above the core, where in fact there are many individual occurrences of Ri<1/4 when calculated over 25 meter intervals.

Vertical profiles of \(\epsilon\) are shown in Figure 26. Individual estimates of \(\epsilon\) were averaged over 25 meters and plotted. Individual 25 meter averages range over 4 decades from 2x10\(^{-4}\) W/m\(^3\) above the EUC core on drop 3 to 3x10\(^{-8}\) W/m\(^3\) at depth. The thick line represents the average over the eight Camel profiles which were associated with White Horse profiles and shows a near
Figure 24 - Eight vertical profiles of vertical shear as estimated from White Horse horizontal velocities taken within 1/2° of the equator in February, 1982. Longitudes of individual profiles are listed in the key to Figure 26. The thick line is the average of the eight profiles.
Figure 25 - Eight vertical profiles of Brunt-Vaisala frequency measured simultaneously as the shears of Figure 24. The symbols are keyed in Figure 26. The thick line is the average of the eight profiles.
Figure 26 - Vertical profiles of turbulent kinetic energy dissipation averaged over 25 meters depth and which are nearly synoptic with the data of Figures 24 & 25. The thick line is the average of the eight profiles. Large solid dots are 20 meter averages over nineteen profiles taken at 150°W and within 1/2° of the equator by Wm. Crawford in January/February, 1979. The large diamonds are 20 meter averages over thirteen equatorial profiles from February, 1982.
surface maximum of $4 \times 10^{-5}$ W/m$^3$ and a general decrease of $\epsilon$ with depth, with the exception of a peak near 500 meters which indicates averaged dissipations three times greater than vertically adjacent values. The 500 meter peak in $\epsilon$ lies directly beneath the peak in Brunt-Vaisala frequency. With the inclusion of the five Camel profiles at 145°W which were made independently of White Horse profiles (these are the large diamonds and were averaged over 20 meters to compare to Crawford(1982)), the near surface and 500 meter peaks are enhanced while the qualitative description remains unchanged.

In marked contrast to the 1982 data are the 1979 profiles of Crawford. The 20 meter averages of nineteen profiles at 150°W are denoted by large solid dots. These differ somewhat from Figure 5 of Crawford(1982) due to a slightly different averaging scheme used. Crawford averaged over individual days 'to reduce the bias caused by the tendency of multiple profiles to be on days of high turbulent intensity'. For the present purpose, we have averaged over individual profiles for direct comparison with our data. The data are from Crawford and Osborn(1981a). The 1979 profiles were much shallower (to 300 meters) and show quantitative agreement below 160 meters with the 1982 data (this gives us confidence that the two sets of measurements are comparable). However, in the large mean shear region above the EUC core the 1982 profiles show 20 meter averaged values which are smaller by more than a factor of ten at 70, 90 and 110 meters! In fact, the 13 profiles from February, 1982, depth-averaged from 20 to 140 meters give $\bar{\epsilon}$ of
0.28x10^-4 W/m^3 while averaging over the 19 equatorial profiles from 1979 gives 1.2x10^-4 W/m^3, more than a factor of four difference.

Unfortunately, neither the 1979 profiles nor the 1982 profiles have completely synoptic shear measurements. Nine of the 1979 dissipation profiles were associated with current meter profiles from which the shear was estimated while eight of the 1982 profiles were synoptic with White Horse profiles. Averaged dissipation over 20 to 140 meters for these data indicate absolute values which are smaller by about 50 percent from those mentioned above, but \( \bar{\varepsilon} \) estimated from the 1979 equatorial drops remains about four times greater. This is shown in Table 4 along with the shear and Ri from 20 to 140 meters estimated by differencing between those depths. Dissipations averaged over the profiles were four times greater in January/February 1979 than in February 1982 while the averaged shear was considerably less and Ri considerably greater. Crawford's 1979 data indicate that the larger values of \( \varepsilon \) are accompanied by smaller values of Ri and larger values of shear, as one might reasonably expect. This trend does not appear to be so clear for the 1982 data set, either on a drop by drop or an averaged basis. However, considering only the average of the four equatorial drops made in February 1979, three of which are 33, 36 and 38 in Table 4, \( \bar{\varepsilon} \) from 20 to 140 meters is 0.29x10^-4 W/m^3 which agrees surprisingly well with the figure noted in the preceding paragraph.
### Table 4 - Comparison of $\bar{\epsilon}$ averaged over 20-140 meters from 1982 and 1979 drops. The velocity and depth are differenced over 20-140 meters by Crawford (1982) for the 1979 data and over 12.5-137.5 meters for the 1982 data. At the bottom are averages over the listed drops.

<table>
<thead>
<tr>
<th>Drop</th>
<th>$\epsilon (W/m^2 \times 10^4)$</th>
<th>$\Delta U/\Delta z$</th>
<th>$Ri = N^2/(\Delta U/\Delta z)^2$</th>
<th>Drop</th>
<th>$\epsilon (W/m^2 \times 10^4)$</th>
<th>$\Delta U/\Delta z$</th>
<th>$Ri = N^2/(\Delta U/\Delta z)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3(138°W)</td>
<td>.66</td>
<td>.011</td>
<td>1.3</td>
<td>13-14</td>
<td>1.5</td>
<td>.0094</td>
<td>.53</td>
</tr>
<tr>
<td>4(138°W)</td>
<td>.16</td>
<td>.0090</td>
<td>2.2</td>
<td>21</td>
<td>.66</td>
<td>.0096</td>
<td>1.3</td>
</tr>
<tr>
<td>5(138°W)</td>
<td>.071</td>
<td>.015</td>
<td>.81</td>
<td>22-24</td>
<td>.71</td>
<td>.0083</td>
<td>2.0</td>
</tr>
<tr>
<td>10(145°W)</td>
<td>-</td>
<td>.011</td>
<td>1.5</td>
<td>33</td>
<td>.42</td>
<td>.0051</td>
<td>1.2</td>
</tr>
<tr>
<td>13(145°W)</td>
<td>.17</td>
<td>.0070</td>
<td>2.0</td>
<td>36</td>
<td>.58</td>
<td>.0047</td>
<td>4.3</td>
</tr>
<tr>
<td>14(145°W)</td>
<td>.082</td>
<td>.0048</td>
<td>3.9</td>
<td>38</td>
<td>.12</td>
<td>.0035</td>
<td>3.3</td>
</tr>
<tr>
<td>17(145°W)</td>
<td>.16</td>
<td>.0089</td>
<td>1.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19(153°W)</td>
<td>.036</td>
<td>.0089</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\bar{\epsilon} = 7.7 \times 10^{-5} W/m^3$

$\Delta U/\Delta z = 0.0074 \text{ sec}^{-1}$

$Ri = 2.5$

$\Delta U/\Delta z = 0.0093 \text{ sec}^{-1}$

$Ri = 1.8$
The effect of the winds on the estimated $\epsilon$ values is shown in Figure 27. Averaged over 20 to 140 meters, the dissipation is plotted against time. Each day, six measurements of wind speed and direction were made. The direction was relatively steady and easterly. Wind speeds were averaged over the six daily measurements and the cube of the wind speed plotted at 1200 hours of the particular day in Figure 27. For comparison, the data of Crawford (1982) are also plotted. The abscissa of Figure 27 is calendar days and the distinction is emphasized that Crawford's values are from 1979 and the others from 1982. The winds are comparable in strength between the two data sets with somewhat less variability for the 1982 data aside from the large three day drop in wind speeds from February 20 to 22. The two smallest values of averaged $\epsilon$ occur during and just after the drop in wind speed. However, there is no other indication to give one confidence of any relation between wind speed and $\epsilon$, in agreement with the finding of Crawford (1982).

6.5 $\epsilon$ And The Zonal Pressure Gradient

Crawford and Osborn (1979b) suggested that a reasonable balance of the turbulent kinetic energy equation (2.4) in the upper equatorial waters is between the production and the dissipation terms. If this holds true, an estimate may be made of the turbulence production term in the mean kinetic energy equation (2.3) from the dissipation measurements. With this basic premise, a balance of the mean kinetic energy was proposed. From the surface to the level of no zonal velocity,
Figure 27 - Time variations of turbulent kinetic energy dissipation averaged over 20 to 140 meters. Dots represent stations within $1/2^\circ$ of the equator and crosses are outside of $1^\circ$ of the equator. Small dots and crosses are from 1979 and large dots and crosses from 1982. The solid line is the cube of the average daily wind speed from 1979 and the dotted line is that from 1982 (revised from Figure 4 of Crawford (1982)).
it was found that the energy input by the wind stress at the surface was approximately balanced by the sum of the work done on the zonal pressure gradient, $udP/dx$, in piling up water towards the west plus the losses to turbulent friction, $\epsilon = u'w'du/dx$. Below the level of no zonal velocity, the work done by the zonal pressure gradient in driving the EUC was approximately balanced by losses to turbulent friction. At the same time, it was suggested that the balance of terms was better than could be expected from the accuracy of the observations and that other terms for which no estimate could be made may be significant.

Five White Horse CTD profiles on the equator are available for computing dynamic heights. These profiles were made at net E at 138°W on 02/09/82, net K at 145°W on 02/15/82, 02/20/82 and 02/21/82 and net Q at 153°W on 02/24/82. Dynamic heights computed from the CTD data at the surface relative to 1000 dbar (0/1000 dbar in the standard notation) are 15.7 m²/s² at net E, 17.8 m²/s² at net Q but range from 16.0 to 16.7 m²/s² at net K. The variation at net K is much greater than that suggested by Wyrtki (1983) of ±0.2 m²/s² due to internal waves and tides. A close look at the data indicates that a 20 to 30 meter depression of the thermocline occurred at net K on 02/20/82. The $\sigma = 24, 25$ and 26 surfaces were depressed by 20 to 30 meters and the position of the measured velocity maximum in the EUC deepened by 25 meters. Coincidentally, the nearest surface velocity measurement (at 12.5 meters) indicated a reversal from
strong westward (-53 cm/sec) to strong eastward (+57 cm/sec) flow between 02/15/82 and 02/20/82 and back to 10 cm/sec westward flow on 02/21/82. Also, on 02/21/82, the depressed $\sigma_t$ surfaces as well as the EUC core returned to within 1 meter of the original depths of 02/25/82. Due to this large variability, it was impossible to obtain a good estimate of the zonal pressure gradient from the PEQUOD data.

However, there exists historical data from which estimates may be made of the zonal pressure gradient. Knauss (1966) computed a gradient of $2.6 \times 10^{-7}$ m/s$^2$ at the depth of the EUC core between 140°W and 104°W in May, 1958. Lemasson and Piton (1968) show a dynamic height section from which Katz et al. (1977) computed a gradient of $4.8 \times 10^{-7}$ m/s$^2$ at 50/700 dbar and $3.3 \times 10^{-7}$ m/s$^2$ at 100/700 dbar between 160°E and 105°W. An AXBT section along the Pacific equator from 172°E to 110°W made in April and May, 1979 is reported in Halpern (1980). A number of coincident CTD profiles provided Halpern with the information to compute a zonal pressure gradient at 0/270 dbar of $5.4 \times 10^{-7}$ m/s$^2$ between 153°W and 133°W. From the AXBT section I have made an estimate of the dynamic heights at 100/270 dbar at 150°W and 130°W using inferred salinities. These indicate a zonal pressure gradient of $2.8 \times 10^{-7}$ m/s$^2$ at 100/270 dbar.

A reasonable estimate, then, of the zonal pressure gradient at the surface is $5 \times 10^{-7}$ m/s$^2$ and is $3 \times 10^{-7}$ m/s$^2$ near the depth of the EUC core. Following the method of Crawford and Osborn (1979b), the work done by the zonal pressure gradient and the loss to the turbulent dissipation is integrated over the two
regions. Between the surface and the level of no zonal velocity (≈ 70 meters) the average velocity is ≈ 30 cm/sec westward and the average pressure gradient is 4.5x10⁻⁷ m/s². The work done by the zonal pressure gradient, then, is (.3 m/s)(4.5x10⁻⁷ m/s²)(70 m)(1028 kg/m³) = 10x10⁻³ W/m². The dissipation averaged over the equatorial drops between 20 and 70 meters is 6x10⁻⁵ W/m³, resulting in a depth-integrated dissipation of 4x10⁻³ W/m². If the balance of Crawford and Osborn (1979) holds, this indicates a net input by the wind at the surface of 14x10⁻³ W/m², which is identical to the wind stress in the equatorial Atlantic quoted by Crawford and Osborn (1979b) at the time of their measurements. Unfortunately, I do not know of a reliable estimate of wind stress which is synoptic with the PEQUOD data.

Below the level of no zonal velocity and to the EUC core (which is ≈ 110 to 140 meters), the average zonal pressure gradient is 3.5x10⁻⁷ m/s², the average velocity is ≈ 45 cm/sec eastward and the average dissipation is 1x10⁻⁵ W/m³. The resulting work done by the zonal pressure gradient is 11x10⁻³ W/m² and the depth-integrated dissipation is about 1x10⁻³ W/m² or about 10% of the work done by the zonal pressure gradient. Recall that the 1982 dissipations at 70, 90 and 110 meters (Figure 26) were 10% of the 1979 values. Presumably, then, the 1979 data yields a closer balance to the estimate made of the work done by the historical zonal pressure gradient. Since there is no reason to doubt the reliability of the dissipation estimates, one may suspect that the zonal pressure gradient in February, 1982 was much smaller. Although the
estimates of dynamic height from PEQUOD are certainly not representative of the mean state, they do indicate a much larger zonal pressure gradient rather than a smaller one. Apparently, then, there is good reason to believe that there must be another sink for the energy which is input to the EUC by the zonal pressure gradient. Crawford and Osborn (1979b) have suggested that meridional divergence terms which could not be estimated from the Atlantic data set may play a role. As well, there is some indication from the PEQUOD data and other data sets that the strength of the undercurrent is not steady due to either meandering or pulsing and hence the time rate of change of the mean kinetic energy may play a significant role in the balance of terms in (2.3).

6.6 $\epsilon$ And N, S, Ri

As in Chapter 4, the data from PEQUOD are compared to the other available data from the cruise in order to determine the existence of trends. With the White Horse velocity data it is possible to estimate shear, $S$, and the difference Richardson number, $\text{Ri} = N^2/S^2$, as well as the buoyancy frequency. The errors involved in calculating $\text{Ri}$, $S$ and $N$ are discussed in Appendix J.

Figure 28 shows scatter plots of the three parameters $N$, $S$, and $\text{Ri}$ calculated over 25 meter depth intervals plotted against $\epsilon$ averaged over 25 meters. The large black dots represent averages of $\epsilon$ over respective $N$, $S$, and $\text{Ri}$ bins. The data span approximately one and one half decades in $N$, two decades in $S$ and three decades in $\text{Ri}$. In each of the three plots the scatter
Figure 28 - Scatter plots of log(turbulent kinetic energy dissipation averaged over 25 meter vertical intervals) vs log(buoyancy frequency(N)), log(vertical shear(S)) and log(difference Richardson number(Ri) calculated from N and S). The data represent the entire water column sampled synoptically by both Camel III and by White Horse. Large black dots are averages over 1/3 decade intervals in N, 1/2 decade intervals in S and over the ranges <1, 1-4, and 4-40 in Ri.
is considerable. Generally, however, the largest values of $\epsilon$ are associated with large $S$ and $N$ and small $Ri$. Similarly, the smallest values of $\epsilon$ are associated with small $S$ and small $N$. For $Ri > 2$ there is no trend although the smaller values of $\epsilon$ are associated with $Ri > 2$ but are indistinguishable from those for $Ri > 10$. Certainly, the small $\epsilon$ values are not associated with $Ri < 1$. The bin-averaged dissipations support these trends. A significant limitation in making this type of comparison is due to the 25 meter spatial resolution of $N$, $S$ and $Ri$ compared to the much finer resolution of the $\epsilon$ measurements.

A cursory glance at the data in Appendix K indicates that many of the patch sizes, especially below 200 meters, are much smaller than 25 meters. But these thin patches dominate the 25 meter average of $\epsilon$. On the other hand, the estimates of $S$ and $N$ from which $Ri$ is calculated are differenced over full 25 meter intervals, thereby substantially obscuring locally large gradients. Hence, the comparison is best made only for the heavily averaged parameters as is done in Figures 16, 29 and 30.

In Figure 16, the solid squares represent 100 meter vertical averages of $\epsilon$ and $N$ which have then been averaged over all of the drops. Only the data below 300 meters have been included in Figure 16. As discussed in Chapter 4, the arguments which support the type of relation $\epsilon \propto N^1$ are based on internal wave scaling. Since the large mean shear in the surface layers of the equatorial data would seem to dominate the turbulence generation, it was decided that plotting the upper 300 meter values would be inappropriate. The plotted data do not differ
substantially from those of the other data sets. A line of slope = 1 gives a reasonable description of the data. As previously noted, the PEQUOD data represent the smallest averaged data measured to date, indicating a dearth of turbulent activity in the deeper waters of the central equatorial Pacific.

Identical averaging as was done to produce Figures 15 and 16 was done to generate S and Ri vs \( \varepsilon \) plots. Neither of Figures 29 nor 30 offer strong evidence for trends at high levels of Ri or low shear, but for low Ri and high shear (in the upper 300 meters), the expected trends are quite apparent. The upper three shear values exhibit successively and substantially larger values of \( \varepsilon \), as do the lower three values of Ri. Of course, due to the dependence of Ri on S, it is not surprising that these are the same \( \varepsilon \) values.

In the upper 300 meters, there appears to be a strong dependence of \( \varepsilon \) on both S and Ri calculated over 25 meter depth intervals and averaged over 100 meters depth. The vertical scales of the shear are resolved by the White Horse measurement in the upper 300 meters. But at greater depths the situation is quite different, and we believe that the mechanism by which the turbulence is generated is distinct from that in the upper waters due in part to the isolation of the deep water by the strong thermocline. In fact, the scaling \( \varepsilon \propto N' \) has hinted that the turbulence is due to internal waves. If the turbulence is due to internal waves, there is little chance that the shear is resolved by the White Horse measurement. As was mentioned in
Figure 29 - Plot of log $\epsilon$ vs log S, where $\epsilon$ and S have been vertically averaged over 100 meter intervals and then over all of the PEQUOD drops with synoptic Camel III-White Horse data.
Figure 30 - Plot of log $\epsilon$ vs log $R_i$, where $\epsilon$ and $R_i$ have been vertically averaged over 100 meter intervals and then over all of the PEQUOD drops with synoptic Camel I-I-White Horse data.
Chapter 5, Gargett et al. (1981) provide evidence for an upper wavenumber limit to the internal wave horizontal velocity shear spectrum corresponding to ten meters while the measurements of Eriksen (1978) indicate breaking at vertical scales of several meters.

One may speculate a little on the relative dependence of $\epsilon$ on $S$ and $Ri$. If $\epsilon \propto N$, $\epsilon \propto S$ and $\epsilon \propto Ri$, then $Ri = N^2/S^2 \propto \epsilon^{2/\gamma} \propto 2/a$. This requires $1/\beta = 2/\gamma - 2/a$. But it has been proposed that $\gamma = 1$ and apparently, $\beta < 0$, thereby implying that $0 < a < 1$, or $\epsilon$ has a stronger dependence on $N$ than $S$.

Two factors tend to obscure any trends of both $S$ and $Ri$ at depths greater than 300 meters. One of these is the small dynamic range in both parameters. And the other is due to the huge error in estimating large values of $Ri$.

6.7 Statistics Of $Ri$ And $\epsilon$

In Figure 31 $N$ and $S$ calculated over 25 meter depth intervals are plotted against each other. The diagonals $Ri = 1/4$, 1, 4 have been plotted for reference. The data are grouped by depth intervals, 20 - 300 meters and 300 meters - bottom of drop. The character of each is quite distinctive. In the upper waters, there exist both higher values of $N$ and $S$ and also larger dynamic ranges of at least $S$ if not $N$. Successive vertical data points have been joined and these indicate (especially for $Ri > 1$) that, in the upper waters, $Ri$ is reduced more frequently by increased $S$ rather than reduced $N$ since the lines are more nearly horizontal than vertical. In the deep
Figure 31 - Scatter plots of log N vs log S estimated over 25 meter intervals for the depth ranges noted in the plots. Adjacent vertical points are joined.
waters, no pattern is obvious. The 20 - 300 meter plot indicates many values of $1/4 < Ri < 1$. In the 300 meter - bottom range most values of $Ri$ are greater than 4. These two data sets are plotted on the same plot labelled 20 meters - bottom for comparison.

The complete range of $Ri$ for the PEQUOD data is shown in Figure 32. About 2.7% has $Ri < 0.3$. This is remarkably similar to the 2.5% that Eriksen (1978) found to be $< 0.25$ from his internal wave array. However, due to the altogether different regimes from which the data were taken, these should not be compared further. Most of the data (30.5%) lies in the bin $3 < Ri < 10$. Very high values of $Ri$ are partly artificial, since in the calculation, $Ri$ was set to 1000 when $S$ approached zero to avoid division by zero.

These $Ri$ data were then grouped according to the associated $\epsilon$ over the respective 25 meter depth intervals. In each $Ri$ bin, the number of values which had $\epsilon$ greater than a given value were counted. These were then normalized at $10^{-8} \text{ W/m}^3$ (again, note that averaged values of $\epsilon$ less than the noise level are obtained by setting $\epsilon = 0$ if $\epsilon \leq$ noise level). The amount of data used to create each curve in Figure 33 differs. Less than 3% was available for $0.1 < Ri < 0.3$ while more than 30% was in the range $3 < Ri < 10$. This lack of data is likely why the upper curve is less smoothly varying than the others. Also, the relative errors differ from curve to curve due to both the sample size and the percentage error in $Ri$.

As one might expect, relatively more values of $\epsilon$ above a
Relative frequency of occurrence of log $R_i$ estimated from the White Horse data taken in February, 1982 over the depth ranges coincident with Camel III data.
Figure 33 - Normalized frequency of occurrence of log ε per half decade interval for the ranges of Ri noted in the key.
given value exist for the lower values of Ri, as is expressed by the relative levels and rolloffs of the constant Ri bin curves of Figure 33. At larger Ri, the distinction is blurred and the lower two curves are virtually indistinguishable from each other.
A serious problem encountered by modellers of fluid dynamics is the parameterization of those motions which have temporal and/or spatial scales smaller than the time step or grid size of the model. The problems presented to modellers of oceanic flows are discussed by Garrett (1979). Essentially, the modellers require estimates of energy sinks at scales smaller than their models can resolve and a means by which these effects can be included in the equations of motion. Generally, this is done by introducing an eddy viscosity (for momentum) and eddy diffusivity (for mass), which are used in an analogous fashion to their molecular counterparts. In contrast to the molecular coefficients, $\nu$ and $\kappa$, which are peculiar to the fluid itself, the eddy coefficients are properties of the flow field. As such, the values used must be carefully considered in the context of the flow to be modelled. In a large scale ocean circulation model one must consider the effects of momentum and mass transport due to eddies (mostly horizontal), waves (momentum only) and turbulence (mostly vertical or at least across isopycnals). As the grid size is reduced the eddies may be well resolved requiring reduced eddy coefficients and a greater relative dependence on the turbulence. In models which concentrate on the surface layers of the ocean and which require a fine vertical grid such as the equatorial model of Pacanowski and Philander (1981), the vertical mixing is expected to be entirely due to the turbulence. In this chapter, a number of eddy coefficients estimated from microstructure data are
compared using the results of Crawford (1982), Osborn (1980), Gregg (1976) and the 1982 PEQUOD measurements. A heavily averaged profile of eddy diffusivity is presented and compared to that of the WESPAC data set.

7.1 Various Estimators

An acceptable balance for the turbulent kinetic energy equation (as discussed in Chapter 2) involves the turbulent production, dissipation and the work done against buoyancy, and is written as

\[
\frac{\overline{u'u'}\overline{\partial u}}{\partial x} = -\epsilon - \frac{\overline{p'w'}}{\rho}. \quad (7.1)
\]

Two of these terms may be incorporated into the flux Richardson number, defined as the ratio of the buoyancy flux to the turbulent production,

\[
R = \frac{g\overline{p'w'}}{(\overline{\rho u'u'\partial u} / \partial x)}. \quad (7.2)
\]

Since \( \overline{p'w'} = -K \frac{\partial \rho}{\partial z} = K \frac{\rho N^2}{g} \),

\[
K = \frac{g\overline{p'w'}}{(\rho N^2)}, \quad (7.3)
\]

where \( K \) represents the coefficient for the vertical diffusion of density. With (7.1), (7.2), and (7.3) the relation for \( K \) is

\[
K = \frac{R}{(1-R)}e. \quad (7.4)
\]

Osborn (1980) recommends an upper bound of 0.15 for the value of \( R \), above which the energy going into the buoyancy flux is
sufficient to suppress the turbulence. This limit on $R_f$ requires $K < 0.2\epsilon/N^2$. Oakey (1982) makes an independent estimate of $K$ from temperature microstructure measurements ($K'$, to be defined shortly) to estimate $K'/(\epsilon/N^2)$ which he estimates to be $0.26\pm0.21$. Since the independent estimates of Osborn and Oakey do not differ significantly, given the estimated factor of two error in $\epsilon$, the factor 0.2 will be carried through for this study. The estimated $K$, with $R = 0.15$ (or $R/(1-R) = 0.2$) will henceforth be denoted as $K_0$.

A smoothed estimate of $K$ is obtained by invoking $\epsilon = a_0N$, which was discussed in Chapter 4. Calling this $K$ (as it was originally suggested by Gargett (1984)),

$$K = \frac{0.2a_0}{N} \quad (7.5)$$

To estimate the eddy viscosity, $K$, (7.1) is again used with the turbulent eddy viscosity defined as

$$K = \frac{\bar{u}'\bar{w}'}{(\partial\bar{u}/\partial z)}. \quad (7.6)$$

In this case, $u$ represents the magnitude of the horizontal velocity and $K$ is then the vertical coefficient of eddy viscosity. With (7.1) and (7.2),

$$K = \frac{\epsilon}{(1-R_f)S^2} \quad (7.7)$$
Using the upper bound of $R = 0.15$, $K < 1.2 \epsilon / S^2$. 

The available measurements allow estimates to be made of $K_O$ and $K_G$ from the WESPAC data and $K_O$, $K_G$, and $K_V$ from PEQUOD. 

With measurements of temperature microstructure, an estimate can be made of the coefficient of vertical diffusion of heat, $K_T$. No temperature microstructure measurements are reported here but the coefficients discussed above are compared to estimates of $K_T$ made by Gregg (1976) in the equatorial Pacific and Osborn and Bilodeau (1980) in the equatorial Atlantic. 

Originally derived by Osborn and Cox (1972), the model is based on a form of the temperature variance equation which balances the production of temperature variance by the buoyancy flux and the destruction by molecular diffusion,

$$\overline{w'T'T}/\partial T/\partial z = -\kappa(\overline{\partial T'}/\partial z)^2.$$ 

The eddy coefficient for vertical diffusion of heat is defined by

$$K_T = -\overline{w'T'T}/(\overline{\partial T}/\partial z) \tag{7.8}$$ 

so that

$$K_T = \kappa(\overline{\partial T'}/\partial z)^2/(\overline{\partial T}/\partial z)^2 \tag{7.9}$$

Having derived the estimates $K_O$, $K_G$, $K_T$, and $K_V$ the conditions of applicability for each must be stated. The
balance equations used for the turbulent kinetic energy and temperature variance equations have excluded the effects of advection. These coefficients must only be associated with the small scale turbulence causing local cross-isopycnal diffusion. Further, the upper bound of $R$ has been inferred from measurements of turbulence in shear flows and Kelvin-Helmholtz instabilities. As Osborn (1980) notes, the estimate $K_0$ cannot account for double diffusive mixing. The same must be true for $K_V$. In estimating $K_V$, an approximation based on internal wave arguments has been made, relegating the use of $K_G$ to deep ocean environments only, where the dominant turbulent energy source is thought to be internal waves, and away from regions such as equatorial surface waters.

7.2 Comparison Of Estimates

A comparison of the vertical eddy coefficients $K_0$, $K_G$, $K_T$ and $K_V$ from the equatorial Atlantic and Pacific is shown in Table 5. For the 1982 data, the ranges shown are estimates made from averages over those equatorial drops with concurrent White Horse measurements. The thermostad estimates are from drop 14 which was the only drop with a well-developed thermostad. The estimates of $K_V$ made by Crawford (1982) used a production-dissipation balance of the turbulent kinetic energy equation and hence no factor of $1/(1-R_f)$ appears as it does in (7.7).
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<tr>
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<td>0.05-.5</td>
<td>0.02-100</td>
<td>1.03</td>
<td>1.01</td>
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Table 5 - Comparison of vertical eddy coefficients (in units of cm²/sec) for five different equatorial data sets. The molecular value for the thermal diffusivity of water is = 0.0015 cm²/sec.
Crawford's estimates would be 20% larger with the buoyancy term included.

Osborn (1980) finds good agreement between $K_0$ and $K_T$ in and above the EUC core from the Atlantic. Furthermore, these also agree with the estimates of $K_T$ made by Gregg (1976) in the Pacific in and above the EUC core. This leads one to agree with Munk (1966) that the eddy coefficients for different scalar properties are the same. It also gives confidence in the assumptions made to derive (7.4) and (7.9). The estimates given for the Pacific, 1982 data and from Crawford (1982) represent ranges of $K_0$ and $K_V$. Reflecting the lower values of $\epsilon$ found in the Pacific in 1982, $K_0$ is significantly lower than any of the other estimates of $K_0$ and $K_T$. In fact, in and below the EUC core, the lower bound is at or smaller than the molecular value for the diffusion of heat, implying that there may be occasions when molecular effects may rival the turbulent fluxes.

The derivation of $K_G$ relies on an assumption of internal wave dependence which cannot be expected to hold true in the upper equatorial waters and, as expected, $K_G$ is much smaller than $K_T$ except at depths below 300 meters. The values of $K_V$ below the core from the Pacific, 1982 are considerably smaller than those from the Atlantic, 1974 and, as has been discussed, considerably smaller values of $\epsilon$ were found below the core in
the Pacific in both 1979 and 1982 than in the Atlantic in 1974. In and above the EUC core, the agreement between the three data sets is quite good.

Figure 36 shows the 25 meter averaged estimates of $K_0$ for the eight equatorial profiles of PEQUOD (open circles). The two upper values (20-70 meters) are between 1 and 2 cm$^2$/sec. The minimum values in the core of the EUC are two factors of ten smaller or about 10$K_0$. The distinctive minimum at 400 meters is associated with the maximum in $N$ below the thermostad and the local minimum in $\epsilon$ which occurs just above the local maximum at 500 meters (see Figures 25 and 26). Below 500 meters, $K_0$ ranges from 0.06 to 0.1 cm$^2$/sec.

In Figure 34 the values of $\epsilon$ and $N$ plotted in Figure 16 were used to generate more heavily averaged estimates of $K_0$ and $K_0$ from the PEQUOD data. The lack of agreement above 400 meters is, of course, due to the differing assumptions involved in the two estimates. Below 400 meters the agreement is quite good. In this averaged profile the surface currents are poorly resolved. The deep values of $K_0$ and $K_0$ are about 0.1 cm$^2$/sec at 900 meters. Figure 35 includes similarly derived profiles from WESPAC calculated from the $\epsilon$ and $N$ values of Figures 14a,b. At 900 meters, $K_0$ and $K_0$ are slightly greater than 0.1 cm$^2$/sec. The greater values of $K_0$ and $K_0$ are expected in light of the greater turbulence levels from the deeper waters of WESPAC and
Figure 34 - Vertical profiles of $K$ (equation 7.4) and $K_O$ (equation 7.5) for the PEQUOD data. $\epsilon$ and $N$ were averaged vertically over 100 meters and then over all of the profiles.
Figure 35 - Vertical profiles of $K_o$ and $K_g$ for the WESPAC data.
incorporated into the coefficient $a_0$ estimated from Figure 16. Below 900 meters, $K$ steadily increases and is about 0.5 cm$^2$/sec below 2000 meters, assuming that $K$ is truly a smoothed representation of $K$.

### 7.3 Deep Ocean Estimates

The deep estimates of $K$ approach that of Munk(1966) who estimated a constant value of $K$ to be about 1 cm$^2$/sec in the deep ocean from a balance of the density equation between the upward advection of dense water and the turbulent diffusion downwards of lighter water. The balance can be expressed as

$$wp - (K \rho z) = 0,$$

where the subscript $z$ represents differentiation with respect to the vertical coordinate $z$. This may be rewritten (and Gargett(1984) does this) as

$$\rho (w-(K \rho z) - K \rho z z) = 0.$$

Munk(1966) assumed $K$ to be a constant value in the deep ocean between 1 and 4 kilometers in order to simplify the analysis. With the estimates of Figure 35 this assumption may be checked. By estimating the rate of formation of Antarctic Bottom Water and assuming a uniform spreading (rising) rate over the remainder of the world's oceans, Munk(1966) estimated a globally averaged upwelling speed, $w$, of $1 \times 10^{-5}$ cm/sec. From Figure 35, $(K \rho z)$ is estimated from the values at 500 and 2000 meters to be
0.3 cm²/sec in 1500 meters or 2x10⁻⁶ cm/sec. This is 20% of Munk's value for w and does not affect the order of magnitude estimate made for K. A point to note is that a constant (K) = 2x10⁻⁶ cm/sec gives a value of K = 1 cm²/sec at 5000 meters.

The indication is that K increases with depth. A quick calculation, though, shows that this trend does not necessarily mean that the mass flux increases with depth. The mass flux is

\[ \rho \bar{w} = -K_p \rho \]

With the estimate K for K,

\[ \bar{G} = \rho \]

\[ \rho \bar{w} = -0.2a_o \rho /N \]

but \( N^2 = -g \rho /\rho \) so that

\[ \rho \bar{w} = 0.2a_o \rho N/g \]

which indicates decreased mass flux with depth.

7.4 Comparison With Equatorial Model Values

In a recent paper by Pacanowski and Philander (1981) the authors express the need for a proper parameterization of vertical mixing, especially as it applies to their numerical model of the tropical ocean. The forms used for the eddy viscosity and diffusivity are

\[ K_{vPP} = 1 + 50/(1+5R_i)^2 \] (7.10)

\[ K_{rPP} = 0.1 + K_{vPP} / (1+5R_i) \] (7.11)
In Figures 36 and 37 the values of $K$ and $K$ are compared to $r_{PP}$ and $v_{PP}$ $K$ and $K$ estimated from (7.4) and (7.7). The value of $Ri$ used to evaluate $K$ and $K$ is calculated from the N and S $r_{PP}$ $v_{PP}$ profiles of Figures 24 and 25.

The nearest surface values of $K$ and $K$ are each too $r_{PP}$ $v_{PP}$ low by about a factor of 4-5, likely due to the inability of the $Ri$-dependence to account for the wind mixed layer. The value of $K$ in the large shear region above the EUC core is more nearly $r_{PP}$ equal to $K$ but, beneath this, and to 300 meters, $K$ is smaller $O$ than $K$ by at least a factor of two and up to a factor of ten. $r_{PP}$

In the region of the EUC core itself, the difference is about a factor of six. The asymptotic value of 0.1 cm$^2$/sec appears to be in good agreement with $K$ but as discussed above, we expect $O$ $K$ to increase with depth. It is not known whether the strong $O$ minimum at 400 meters is anomalous. In general, the shapes of $K$ and $K$ agree in the upper water column indicating that the $O$ $r_{PP}$ Richardson number dependence gives the right sense for $K$. $r_{PP}$

The consistently greater values of $K$ compared to $K$ should $r_{PP}$ $O$ not be of great concern due to the lower estimates of $\varepsilon$ from 1982 which may represent an anomalously low data set. Certainly, the values of $K$ are in better agreement with the $r_{PP}$
Figure 36 - Vertical profiles of 25 metre averages of $K$ and $K_0$ (equation 7.11) from PEQUOD.
Figure 37 - Vertical profiles of 25 metre averages of $K_v$ and $K_{vPP}$ (equation 7.10) from PEQUOD.
other estimates of Table 5. In the large shear region of the upper equatorial ocean, then, the Ri dependence appears to agree reasonably well. However, we expect $K$ to increase with depth.

In Figure 37, the curvature of $K'$ actually mirrors that of $V$. But, besides the very low value of $K$ (factor of ten less than $K'$) at 75 meters, the estimates are within a factor of two agreement (this is better than the estimated error for the estimate of $K$ when one considers that $\delta \varepsilon / \varepsilon \approx 1$). In the EUC core itself, the estimates of 1-2 cm$^2$/sec agree well with all of the data. Below the core, $K$ is much greater than $K'$ but is closer to the Atlantic estimate of Crawford (1982) in Table 5. Perhaps, different oceans require different parameterizations of eddy coefficients. The asymptotic value of $K$ at depth appears to be too low.
VIII. COMPARISON OF DATA SETS AND PATCH SIZE STATISTICS

Some statistics of $\varepsilon$ and the distribution of the turbulence are presented in this chapter. These provide a common ground for comparison of the two data sets involved in this study as well as to other data sets already in existence and to those yet to be compiled.

Table 6 lists heavily averaged values of $\varepsilon$ over the depth ranges indicated for PEQUOD, WESPAC, and also the Vancouver Island slope data of Lueck, Crawford and Osborn (1983). Because of the relatively small amount of data from depths greater than 1000 meters from PEQUOD, no results are given for that range.

For the upper range of data from PEQUOD (20-300 meters), $\bar{\varepsilon}$ is $85 \times 10^{-7}$ W/m$^3$, which is about eight times larger than $\bar{\varepsilon}$ from the comparable range from WESPAC and also eight times $\bar{\varepsilon}$ from 25-500 meters from the Vancouver Island slope data. This large value is, of course, due to the enormous influence of the surface current structure near the equator. While almost 50% (%turbulent or PCT) of the independent estimates of $\varepsilon$ are $>10^{-6}$ W/m$^3$ from this range from PEQUOD, only 28% are turbulent from WESPAC. The relative ratios $\bar{\varepsilon}$/PCT indicate that, as well as a greater portion of the water column being turbulent, the dissipation averaged over the turbulent portion is more than four times greater (compare $\bar{\varepsilon}$/PCT = 1.7 for PEQUOD 20-300, .47 for PEQUOD 300-1000, .39 for WESPAC 20-300, .42 for WESPAC 300-
Table 6 - Average values of $\epsilon$ from PEQUOD and WESPAC data sets compared to Vancouver Island slope values from Lueck, Crawford and Osborn (1983). These values may slightly underestimate true values of $\epsilon$ since noise levels were set = 0. The quantity of data refers to the total amount of data taken in each depth range.
1000, and .32 for WESPAC >1000 meters).

More interesting is the range 300-1000 meters, for which $\bar{e}$ from WESPAC is twice that from PEQUOD and also of the Vancouver Island slope data for the range designated by Lueck, Crawford and Osborn (1983) as > 500 meters (these include three drops to 1100 meters). The reason for the large value of $\bar{e}$ here is because twice as much of the water column is turbulent (since the relative values of $\bar{e}/PCT$ are about equal from PEQUOD and WESPAC, although PCT was not computed for the Vancouver Island slope data set). This fact is particularly interesting in light of the $\epsilon = a_0 N$ relation posed in Chapter 4. From the information in Table 6 and to follow, it appears that the values of $\bar{e}$ in the range 300-1000 meters from WESPAC (in the range of the second maximum in $N$) are higher, not because individual estimates are substantially higher but because more of the water column is turbulent.

Below 1000 meters, thw WESPAC data show the smallest $\bar{e}$ estimates yet made (3.8x10^{-6}W/m^3, which is only marginally above the instrumental noise level). However, a considerable amount of turbulence still exists at depth ($\approx 12\%$ of almost 6000 meters of vertical profiling), indicating that the deep ocean is not quiet. In fact, a slightly greater fraction from WESPAC > 1000 meters is turbulent than in the shallower depth range 300-1000 meters from the equatorial data.
8.1 Lognormal Properties Of $\varepsilon$

It has been proposed by Gurvich and Yaglom (1967) as well as others that small scale turbulent properties such as $\varepsilon$ are random variables which follow a lognormal distribution. Stewart, Wilson and Burling (1970) and Gibson, Stegen and Williams (1970) studied the turbulent boundary layer of the atmosphere over the ocean. Their work indicates that both the spatial derivatives of turbulent velocities and their squares (as are used to calculate $\varepsilon$) have lognormal properties over a range of their values which is sufficiently above the instrumental noise level but below the largest expected values. Lueck and Osborn (1982) briefly presented the lognormal properties of $\varepsilon$ in and below the wind-mixed layer and their data indicate that there are ranges over which $\varepsilon$ behaves lognormally.

The dissipation values from each subgroup in Table 6 were grouped in quarter decade intervals. The cumulative frequency of occurrence of each quarter decade was computed and plotted on normal probability paper. Each point in Figures 38-42 represents the cumulative percentage of observations with values of $\varepsilon$ less than or equal to that indicated on the horizontal axis. A straight line fit of the data on normal probability paper indicates that the random variable follows a normal probability distribution. Similarly, a straight line fit of the logarithm of the random variable indicates that the random variable follows a lognormal distribution. The form of the probability distribution function is (from Stewart, Wilson and Burling (1970))

$$P(y) = \exp[-(\ln(y) - \mu)^2 / 2\sigma^2] / (2\pi)\frac{1}{2}\sigma y$$
where \( \mu = \ln(y) \) and \( \sigma^2 = \ln(y)^2 - \ln(y)^2 \). The mean value of \( y \) is given by \( \overline{y} = \exp[\mu + \sigma^2/2] \). Since the values of \( \epsilon \) in Figures 38-42 are plotted as base 10 logarithms, it is necessary to change the parameters \( \mu \) and \( \sigma \) to base 10 logarithms from natural logarithms \( \log(y) = 2.303 \ln(y) \) to calculate \( \overline{y} \). In Figures 38-42, \( \mu \) and \( \sigma \) are estimated from the plotted straight lines and the mean value, \( \epsilon_0 \) is estimated from these. For comparison, the mean value \( \bar{\epsilon} \) estimated from the data and listed in Table 6 is included. The number of estimates of \( \epsilon \) used to construct each plot is \( n \).

At small values of \( \epsilon \) (<3x10^-7 W/m^3 which is the noise level) there is a distinct deviation of the behaviour of the data from that of the data above the noise level. The frequency of occurrence of small values of \( \epsilon \) predicted from the lognormal plot is too small. In fact, the distribution is truncated at the noise level of the instrumentation and small values are not measured although there is no reason to believe that they do not exist. A somewhat different problem exists at high values of \( \epsilon \) which was discussed by Stewart, Wilson and Burling (1970) and Gibson, Stegen and Williams (1970) and Lueck and Osborn (1982) and which is especially apparent in Figures 38, 40 and 41. It was found that the highest values of \( \epsilon \) generally deviated from the straight line fit and this deviation was attributed to undersampling of the very few expected large values.

Each of the Figures 38-42 exhibit ranges of \( \epsilon \) over which the data follows a straight line reasonably well. One estimate
Figure 38 - Cumulative distribution of the base 10 logarithm of dissipation values from PEQUOD 20-300m plotted against normal probability co-ordinates. The parameters $\mu$ and $\sigma$ are the mean and standard deviation of the natural logarithm of $\varepsilon$, estimated from the straight line. $\varepsilon_0$ is estimated from $\mu$ and $\sigma$. $\bar{\varepsilon}$ is the observed mean from Table 6. The number of independent 2 meter estimates of $\varepsilon$ is $n$. 

$\mu = -6.05$

$\sigma = 0.90$

$\varepsilon = 85 \times 10^{-7} \text{W/m}^3$

$\varepsilon_0 = 76 \times 10^{-7} \text{W/m}^3$

$n = 1855$
Figure 39 - Cumulative distribution of the base 10 logarithm of dissipation values from PEQUOD 300-1000m.

- $\mu = -7.02$
- $\sigma = 0.60$
- $\bar{\varepsilon} = 4.7 \times 10^{-7} \text{W/m}^3$
- $\varepsilon_0 = 2.5 \times 10^{-7} \text{W/m}^3$
- $n = 4657$
Figure 40 - Cumulative distribution of the base 10 logarithm of dissipation values from WESPAC 20-300m.

Mathematical data:
- $\mu = -6.37$
- $\sigma = 0.60$
- $\bar{\epsilon} = 1.1 \times 10^{-7} \text{ W/m}^3$
- $\epsilon_0 = 1.1 \times 10^{-7} \text{ W/m}^3$
- $n = 1079$
Figure 41 - Cumulative distribution of the base 10 logarithm of dissipation values from WESPAC 300-1000m.

\[
\begin{align*}
\mu &= -6.54 \\
\sigma &= 0.50 \\
\bar{\varepsilon} &= 9.2 \times 10^{-7} \text{W/m}^3 \\
\varepsilon_0 &= 5.6 \times 10^{-7} \text{W/m}^3 \\
n &= 2634
\end{align*}
\]
Figure 42 - Cumulative distribution of the base 10 logarithm of dissipation values from WESPAC >1000m.

\[ \mu = -6.74 \]
\[ \sigma = .53 \]
\[ \bar{\epsilon} = 3.8 \times 10^{-7} \text{W/m}^3 \]
\[ \epsilon_0 = 3.8 \times 10^{-7} \text{W/m}^3 \]
\[ n = 3269 \]
of the degree to which the straight line fitted to the points is representative of the data is the agreement of the estimate \( \epsilon_0 \) made from the line with \( \bar{\epsilon} \). The estimates of the mean values of the 20-300m data sets are the best, likely due to the extended dynamic range, especially in the case of the PEQUOD data. All of the estimates, however, are within a factor of two of the observed mean values.

8.2 Patch Size Statistics

To get an idea of the distribution of both turbulent patch sizes and their relative contribution to the spatially averaged dissipation values discussed above, a turbulent patch was defined. For the purposes of this study, a turbulent patch must have at least one independent estimate of \( \epsilon > 10^{-6} \text{ W/m}^3 \) (thereby limiting the smallest patch size to \( \approx 2 \) meters). Inside of the patch itself, no two successive independent estimates of \( \epsilon \) may be \( < 10^{-6} \text{ W/m}^3 \). This second criterion does not actually change the statistics considerably but does allow for single 2 meter thick quiet stretches to occur within a much larger patch. The motivation for this came from looking at lab studies of Kelvin-Helmholtz billows (Turner(1973), Van Dyke(1982)), which appear to be quite stable in the centre during the initial stages of instability while the large shear at the edges suggests turbulence. While this definition is certainly not meant to be rigorous it does provide an objective criterion which can be used to derive some interesting comparisons.

Prior to discussing the contents of Tables 7-11, the means
of deriving these are shown. From the respective data sets, turbulent patches (as defined above) were sorted according to their vertical scale \((t = \text{thickness or patch size})\). The number of patches which fell in each Range \(<3, 3-10, 10-30, >30\text{ meters}\) were counted and listed under #Patches. These ranges represent half decade and therefore order of magnitude ranges. The patch sizes in each range were summed and listed under Total Thickness. Their relative importance was estimated by dividing the total thickness of each range by the total data \((H)\) in the data subset, and the average patch size, \(\bar{t}\), was computed. Within each patch, the patch-averaged dissipation, \(\bar{e}\), was calculated, summed over all of the patches in the range, multiplied by \(\bar{t}\) and divided by \(H\bar{e}\) where \(\bar{e}\) is the average dissipation over the entire data subset (and is included in Table 5). This statistic, \(\frac{100 \Sigma \bar{e}}{(H\bar{e})}\), indicates the relative contribution of each range to the vertically integrated dissipation. The dissipation averaged over all of the patches in a single range is \(\bar{e}\) and this isolates the actual magnitude of \(e\) values within the range from the relative contribution (which is also a function of the fraction of the water column which is turbulent).

As expected, in the upper 300 meters of the PEQUOD data (Table 7), large patches due to the large mean shear dominate the water column; 34% of the entire upper 300 meters has
<table>
<thead>
<tr>
<th>Range</th>
<th>#Patches</th>
<th>Total Thickness</th>
<th>%Total</th>
<th>$\bar{e}$</th>
<th>$\Sigma e$</th>
<th>$100\Sigma e/(H\bar{e})$</th>
<th>$\bar{e} = \Sigma e /\Sigma t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;3</td>
<td>33</td>
<td>76</td>
<td>2.0%</td>
<td>2.3</td>
<td>5.4x10^{-5}</td>
<td>12x10^{-5}</td>
<td>0.4%</td>
</tr>
<tr>
<td>3-10</td>
<td>32</td>
<td>211</td>
<td>5.6</td>
<td>6.6</td>
<td>7.6</td>
<td>50</td>
<td>1.6</td>
</tr>
<tr>
<td>10-30</td>
<td>16</td>
<td>266</td>
<td>7.0</td>
<td>17.</td>
<td>6.4</td>
<td>106</td>
<td>3.3</td>
</tr>
<tr>
<td>&gt;30</td>
<td>20</td>
<td>1290</td>
<td>34%</td>
<td>65%</td>
<td>40%</td>
<td>2570</td>
<td>80%</td>
</tr>
</tbody>
</table>

Total data = 3780 meters

$\bar{e} = 85.10^{-7}$ W/m$^2$

$H\bar{e} = 32.10^{-3}$ W/m$^2$

PEQUOD 20-300 meters

Table 7 - Patch size statistics for the PEQUOD data set over the depth range 20-300 meters. Details of construction of this table are in the text.
### Table 8 - Patch size statistics for the PEQUOD data set over the depth range 300-1000 meters.

<table>
<thead>
<tr>
<th>Range</th>
<th>#Patches</th>
<th>Total Thickness</th>
<th>$%_{\text{Total}}$</th>
<th>$\overline{\varepsilon}$</th>
<th>$\overline{\varepsilon_{\text{t}}}$</th>
<th>$100\overline{\varepsilon_{\text{t}}}/(\overline{H\varepsilon})$</th>
<th>$\overline{\varepsilon_{\text{t}}}/\overline{\varepsilon}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;3</td>
<td>64</td>
<td>139</td>
<td>1.7</td>
<td>2.2</td>
<td>$11.4 \times 10^{-5}$</td>
<td>$23.4 \times 10^{-5}$</td>
<td>6.1%</td>
</tr>
<tr>
<td>3-10</td>
<td>43</td>
<td>218</td>
<td>2.7</td>
<td>5.1</td>
<td>10.</td>
<td>52</td>
<td>14</td>
</tr>
<tr>
<td>10-30</td>
<td>20</td>
<td>293</td>
<td>3.7</td>
<td>15.</td>
<td>5.1</td>
<td>75</td>
<td>20</td>
</tr>
<tr>
<td>&gt;30</td>
<td>3</td>
<td>134</td>
<td>1.7</td>
<td>45.</td>
<td>1.3</td>
<td>58</td>
<td>15</td>
</tr>
</tbody>
</table>

Total data = 8015 meters

$\overline{\varepsilon} = 4.7 \times 10^{-7}$ W/m$^3$

$H\overline{\varepsilon} = 3.8 \times 10^{-3}$ W/m$^3$

PEQUOD 300-1000 meters

$\overline{\varepsilon} = $ drop averaged dissipation

$\overline{\varepsilon_{\text{t}}} = $ individual patch average

$\overline{H\varepsilon_{\text{t}}} = $ bin averaged dissipation

$\overline{t} = $ patch thickness

$\overline{\varepsilon_{\text{t}}} = $ bin averaged patch thickness
<table>
<thead>
<tr>
<th>Range</th>
<th>#Patches</th>
<th>Total Thickness</th>
<th>%Total</th>
<th>$\bar{e}$</th>
<th>$\bar{\Sigma e}$</th>
<th>$100\bar{\Sigma e}/(H_{\bar{e}})$</th>
<th>$\bar{e} = \Sigma e/\Sigma t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;3</td>
<td>29</td>
<td>64</td>
<td>2.9%</td>
<td>2.2</td>
<td>$4.4 \times 10^{-5}$</td>
<td>$10 \times 10^{-6}$</td>
<td>$4.1%$</td>
</tr>
<tr>
<td>3-10</td>
<td>20</td>
<td>109</td>
<td>5.0</td>
<td>5.5</td>
<td>4.2</td>
<td>23</td>
<td>9.5</td>
</tr>
<tr>
<td>10-30</td>
<td>12</td>
<td>254</td>
<td>12.1</td>
<td>21.1</td>
<td>4.8</td>
<td>100</td>
<td>42.0</td>
</tr>
<tr>
<td>&gt;30</td>
<td>4</td>
<td>191</td>
<td>8.8</td>
<td>48.6</td>
<td>1.3</td>
<td>64</td>
<td>27.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total data = 2180 meters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{e} = 1.1 \times 10^{-7}$ W/m$^3$</td>
</tr>
<tr>
<td>$H_{\bar{e}} = 2.4 \times 10^{-3}$ W/m$^2$</td>
</tr>
</tbody>
</table>

WESPAC 20-300 meters

Table 9 - Patch size statistics for the WESPAC data set over the depth range 20-300 meters.
<table>
<thead>
<tr>
<th>Range</th>
<th>#Patches</th>
<th>Total Thickness</th>
<th>#Total %</th>
<th>$\bar{t}$</th>
<th>$\sum t$</th>
<th>$\sum t = \sum t / \bar{t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;3</td>
<td>73</td>
<td>159</td>
<td>3.1%</td>
<td>2.2</td>
<td>12.8x10^{-8}</td>
<td>5.6% 1.7x10^{-4}</td>
</tr>
<tr>
<td>3-10</td>
<td>62</td>
<td>387</td>
<td>7.6%</td>
<td>6.2</td>
<td>14.86</td>
<td>18.2 2.2</td>
</tr>
<tr>
<td>10-30</td>
<td>27</td>
<td>401</td>
<td>7.9%</td>
<td>15.</td>
<td>8.7 130</td>
<td>21.6 3.2</td>
</tr>
<tr>
<td>&gt;30</td>
<td>5</td>
<td>181</td>
<td>3.6%</td>
<td>36.</td>
<td>2.7 98</td>
<td>73% 5.4</td>
</tr>
</tbody>
</table>

Total data = 5085 meters

$$\bar{\epsilon} = 9.2x10^{-7} \text{ W/m}^3$$

$$\overline{Ht} = 4.7x10^{-3} \text{ W/m}^3$$

WESPAC 300-1000 meters

Table 10 - Patch size statistics for the WESPAC data set over the depth range 300-1000 meters.
<table>
<thead>
<tr>
<th>Range</th>
<th>#Patches</th>
<th>Total Thickness</th>
<th>%Total</th>
<th>$t$</th>
<th>$\Sigma e$</th>
<th>$\Sigma e / t$</th>
<th>$100 \Sigma e / (H \overline{e})$</th>
<th>$\overline{e} = \Sigma e / Et$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;3</td>
<td>51</td>
<td>107</td>
<td>1.8%</td>
<td>2.1</td>
<td>$8.8 \times 10^{-5}$</td>
<td>$19 \times 10^{-5}$</td>
<td>8.4%</td>
<td>1.7x10^{-6}</td>
</tr>
<tr>
<td>3-10</td>
<td>47</td>
<td>280</td>
<td>4.8</td>
<td>6.0</td>
<td>8.7</td>
<td>52</td>
<td>24.0</td>
<td>1.9</td>
</tr>
<tr>
<td>10-30</td>
<td>17</td>
<td>257</td>
<td>4.4</td>
<td>15.</td>
<td>3.8</td>
<td>58</td>
<td>26.0</td>
<td>2.2</td>
</tr>
<tr>
<td>&gt;30</td>
<td>1</td>
<td>33</td>
<td>0.6</td>
<td>33.</td>
<td>0.33</td>
<td>11</td>
<td>4.9</td>
<td>3.3</td>
</tr>
</tbody>
</table>

Total data = 5805 meters

$\overline{e} = 3.8 \times 10^{-7}$ W/m$^3$

$H \overline{e} = 2.2 \times 10^{-3}$ W/m$^3$

WESPAC > 1000 meters

Table 11 - Patch size statistics for the WESPAC data set for depths > 1000 meters.

$H$ = total data.
$\overline{e}$ = drop averaged dissipation
$e$ = individual patch average
$t$ = patch thickness
$\overline{e}$ = bin averaged dissipation
$t$ = patch thickness
$\overline{e}$ = bin averaged patch thickness
turbulent patches > 30 meters thick ($\bar{t} = 65$ meters). 80% of the vertically integrated dissipation ($\overline{H\epsilon}$) is concentrated in this 34% of the data.

Below 300 meters (Table 8), only 3 patches with $t > 30$ meters were found, representing < 2% of the total data. Indicative of the relatively quiescent deep waters, only 10% is turbulent. Further, only 55% of the vertically integrated dissipation is contained in patches with $\epsilon > 10^{-6}$ W/m$^3$, indicating that a relatively large contribution must be from regions where $\epsilon$ is just above the noise level but $< 10^{-6}$ W/m$^3$ (since, in averaging, values of $\epsilon <$ noise were set = 0). The notable trend is for the greater patch-averaged dissipations, $\bar{\epsilon}$, to be concentrated in the larger patches. In fact, this trend is evident in all of the other data subsets with a minor contradiction in Table 9. Successively larger patch sizes have successively larger averaged dissipations. But the relative contribution to the vertically integrated dissipation is in all cases (other than the upper equatorial waters) concentrated in the patches of range 10-30 meters simply because a greater proportion of the turbulence in the water column exists in this range of patch sizes.

The $\bar{\epsilon}$-$t$ dependence is shown in Figures 43 and 44. These are plotted on log-log scale so that half decade ranges (or bins) are equally spaced (although successive estimates of $\bar{\epsilon}$ are
Figure 43 - Log-log plot of average patch-averaged dissipations vs average patch thickness for the PEQUOD data below 300 meters.
Figure 44 - Log-log plot of average patch-averaged dissipations vs average patch thickness for the WESPAC data below 300 meters.
not necessarily equally spaced). The PEQUOD data below 300 meters is shown in Figure 43 (the numbers are directly from Table 8) while Figure 44 combines the data of Tables 10 and 11. The plots show quite clearly that successively larger patch sizes have successively larger values of $\bar{\varepsilon}$. Similar plots were made using smaller, linearly spaced bins (0-3, 3-6, 6-9, ...) and the trend was equally evident with, however, somewhat more noise due to the small sample sizes in some bins. A comparison of Figures 43 and 44 shows that neither the estimates of $\bar{e}$ nor the estimates of $\bar{t}$ differ significantly between the data sets. However, half of the data from WESPAC are below 1000 meters compared to none in this range from PEQUOD.

The appropriate length scale to which these patch sizes may be compared and which can be estimated from the available data is the buoyancy length scale, defined by $L_b = (\varepsilon/N^3)^{1/2}$ (Turner, 1973, p143), which represents the scale of motion where buoyancy forces become of the same order as the inertial forces. $L_b$, then, is an appropriate scale for the largest eddies which are able to overturn, given the background turbulence and stratification. Gargett et al. (1981) discuss the significance of $L_b$ and describes the successful scaling of the buoyancy subrange of high wavenumber oceanic energy spectra using buoyancy parameters, $\varepsilon$ and $N$. 
L was estimated by determining patch averages over all of the data below 300 meters for which there were also CTD data to calculate N. For each patch, $\epsilon$ and N were estimated, grouped into half decade bins and further averaged to get $\overline{L}_b$ for each $t_b$. These variables are plotted in Figures 45 and 46.

From PEQUOD, $\overline{L}_b$ ranges from 25 to 40 cm and is between 30 and 40 cm from WESPAC. The estimate for the upper bin from PEQUOD (>30 meters) is not plotted because there were too few patches for a comparable average. Although the 10-30 meter value of $\overline{L}_b$ from PEQUOD is higher than the other two there is no evidence of a trend from these plots. In fact, $\overline{L}_b$ is remarkably constant. Individual estimates actually range from 9 to 113 cm but most fall in the range 20-40 cm, as do the averages. (The value of 113 cm is from 2030 meters of WESPAC drop 11 where $\epsilon = 3 \times 10^{-6}$ W/m$^3$ and N was estimated from the nearest CTD station at drop 12 to be $\approx 0.0013$ rad/sec. This is by far the largest value of $L_b$). Unfortunately, these scales are smaller than the vertical spatial resolution of the $\epsilon$ measurements and are concealed in the lower bin (<3 meters). 114 of 311 patches counted from both data sets below 300 meters were <3 meters, representing 46% of all of the patches. However, these only
Figure 45 - Log-log plot of average buoyancy length scale vs average patch thickness for the PEQUOD data below 300 meters.
Figure 46 - Log-log plot of average buoyancy length scale vs average patch thickness for the WESPAC data below 300 meters.
represent 310 of 2079 meters of data with $\epsilon > 10^{-6}$ W/m$^3$, or about 15%, and contribute only 6% to the vertically averaged dissipation. The dominant patches, then, are many times $L_b$ in thickness.
IX. DISCUSSION AND CONCLUSIONS

The main contribution of this thesis is the addition to the global data set of ocean turbulence measurements. In comparing the measurements of this thesis to those of other workers, the $\overline{\varepsilon - N}$ plot of Figure 16 provided a convenient standard. In order to make better use of the various data sets which now exist, further meaningful formats must be established. Certainly, averaging is one. But, also, the statistical comparison of data made here in Chapter 8 provides a number of standards for comparing large data sets. Not only are averages made over discrete vertical intervals, but estimates are made of the fraction of the water column which is turbulent; patch thicknesses; contribution of ranges of patch sizes to the vertically integrated dissipation; and patch-averaged dissipations.

The data from depths > 300 meters indicate a strong relationship between heavily averaged values of $\varepsilon$ and $N$. Further, it is seen that this trend is due to more frequently occurring turbulence rather than to much higher individual estimates of $\varepsilon$. Scaling arguments for energy and rate of loss of energy in the internal wave field suggest a relation $\varepsilon \propto N^1$. The constant of proportionality, $a_0$, is estimated for four data sets and ranges from $(1.4 \text{ to } 4) \times 10^{-7}$ m$^2$/sec$^3$·sec. An estimate of the time scale for the decay of the internal wave field is $\tau = TE/\varepsilon$, and depends on the surface-extrapolated buoyancy frequency, $N_0$, and $a_0$. The best range of estimates from the
data sets are 10-100 days, which is in the range of estimates made independently by Olbers (1983) and Garrett and Munk (1979). A major impediment to the reliability of the estimate is the factor of two uncertainty in the GM spectral estimates. A better estimate of the time scale must await joint measurements of turbulent kinetic energy dissipation and internal wave spectra.

A very simple model is presented which accounts for the described dependence of $\epsilon$ and $N$ in at least a qualitative manner. The prediction is based on the assumption that the local turbulent dissipation is due to the breaking of internal waves and that the occurrence of breaking events is determined by the probability that the Richardson number is locally reduced to a value less than 1/4 due to a random superposition of internal wave shears. A reasonable fit is made to the distribution of the turbulence in the water column through the $-1/N \epsilon$ dependence of $Pr(Ri)$, although it is not clear how to relate the magnitudes of the data parameter PCT (percent turbulence) and the model parameter $Pr(Ri<1/4)$.

Nearly synoptic Camel III and White Horse measurements from the equatorial Pacific permit a comparison of the turbulent dissipation estimates to the local shear and difference Richardson numbers. Independent 25 meter estimates indicate a poor correlation between both $\epsilon-S$ and $\epsilon-Ri$. On a heavily averaged basis, however, a strong dependence exists between $\epsilon$ and $S$ and $\epsilon$ and $Ri$ (for $Ri < 10$). The poor correlations for individual estimates should not be too surprising if one
considers that large shears (and low $\text{Ri}$) which cause the turbulence may, in fact, locally decrease (increase) due to the overturning event. In this case, the turbulence measured may be in a slightly decayed state from the original event. Conversely, regions of large shear (or low $\text{Ri}$) which have not yet overturned may have quite low values of $\epsilon$. Perhaps a simpler reason is the fact that the Camel measurements were not synoptic with the White Horse measurements in either time or space.

The substantially smaller averaged dissipations from the equatorial Pacific in 1982 (as opposed to the 1979 results of Crawford (1982)) must be considered to be indicative of the spatial and temporal variability of the turbulence in the region. Apparently the variation in the mean shear is not as great as the variation in $\epsilon$, either from a comparison of individual drops or from a comparison of 1979 and 1982 data sets. Nor is a direct relationship (on the basis of individual drops) found between shear, $\text{Ri}$ or wind speed. The findings may not be surprising in light of the production-dissipation model of the EUC proposed by Crawford and Osborn (1979b, 1981b). In this balance, the dominant terms are $\epsilon$ and the production of turbulent kinetic energy by the Reynolds stresses working against the mean shear and hence to relate the variability of $\epsilon$ requires a measure of the variability of the Reynolds stresses as well as of the mean shear. In terms of the balance of terms of the mean kinetic energy equation, the much smaller dissipations measured in 1982 are not large enough to balance
the work done by the zonal pressure gradient between the level of zero zonal velocity and the undercurrent core. Hence, other terms in the mean kinetic energy equation must be important.

Estimates of eddy coefficients from a number of sources compare favourably with those made here ranging from 1-40 cm$^2$/sec in the large mean shear above the EUC core and from .003-.5 cm$^2$/sec in the region of the core itself. At depths greater than 300 meters, the estimates made from both the equatorial and the western Pacific data agree very closely. The eddy coefficient for mass, $K$, increases with depth, extrapolating to Munk's (1966) value of 1 cm$^2$/sec at 5000 meters. The increase of $K$ with depth does not imply greater turbulent fluxes with depth.

An analysis of patch statistics indicates that i) average values of dissipation averaged over a single patch, $\overline{\epsilon}$, are larger for thicker patches than for thinner patches and ii) the buoyancy length scale $L_b$ is virtually constant, regardless of patch size, $\overline{t}_b$, implying that patches are many times $L_b$ in thickness. The parameter $L_b$ indicates the largest overturning scales, and hence, a single turbulent patch would be comprised of a number of adjacent overturning events.
**BIBLIOGRAPHY**


APPENDIX A - HYDRODYNAMICS

The instrument, Camel III, is designed to be positively buoyant in water. Ballast is provided by lead weights fastened to the body by AWG20 wire which is in series with a pressure sensitive shear wire cylinder release system.

The buoyant force is chiefly due to the main pressure tube, which measures 30.5 cm in diameter, 152 cm long and 1.9 cm thick. The material is an aluminum alloy (6061-T6) with a specific gravity of 2.77. The lift given by this geometry and material is then simply calculated to be 42.0 kg over the tube length (using a nominal seawater density of 1028 kg/m$^3$).

The end caps to the pressure tube are a cast aluminum alloy (A536-T6). The shape is hemispherical but with an irregular surface making it difficult to calculate the net effect on the buoyancy of the instrument. It can, however, be measured. The remaining components of the instrument, other than the preamplifier case, act solely as ballast.

The instrument was weighed in Monterey harbour from the R/V Acania, operated by the United States Naval Postgraduate School. It was found to be 1.2 kg heavy after 12.2 kg of expendable lead weights plus 3.1 kg of lead attached to the nosepiece were added. Another 2.0 kg were added to the nosepiece to ensure that the instrument would continue to sink in a region where very large density change with depth occurs. The added weight acts further to stabilize the instrument (by adding mass below the centre of gravity) and increase the fall rate, thereby changing the shear sensitivity. An undesirable side effect is
the increased energy available for vibration, which is the limiting factor in the resolution of the dissipation calculation (see Appendix G).

Two other safety criteria are met by the system. First of all, the instrument is positively buoyant should only one weight fall off the instrument, allowing for some margin of safety in case of a malfunction in the pressure release mechanism. As well, the weak link in the system is thought to be the pressure transducer link to the ambient pressure, which is a threaded connection to the preamplifier case. The instrument will be positively buoyant should the preamplifier case flood, and only one weight fall off.

The fall speed of the instrument is governed by the negative buoyancy of the instrument and the drag force on the instrument in motion. The drag force is empiricized in the form of a squared drag law \( D = kW^2 \) where \( k = \rho AC /2g \) (A is the frontal cross-section and C is the drag coefficient) is a quantity which can be directly calculated knowing the mass and the fall rate of the instrument.

The drag parameter, \( k \), was determined from drops made in Howe Sound in April, 1981 aboard the CNAV Endeavour and these measurements were confirmed on other cruises. With 8.5 kg of added lead weight (7.7 kg in water) the measured fall rate was 1.1 m/s, resulting in a value for \( k \) of 27.5 kg/m.

It is interesting to calculate the drag coefficient, \( C \), for the instrument under these circumstances. The area
projected to the flow is \( A = \pi (0.305/2)^2 = 0.07 \text{m}^2 \). Using a nominal value of 1028 kg/m\(^3\) for seawater, \( C_d = 0.8 \). To compare, Hoerner (1965) cites \( C_d = 0.35 \) as a measured drag coefficient for a torpedo and \( C_d = 0.8 \) for a blunt-nosed cylinder aligned longitudinally to the flow. The shape of Camel III is certainly not blunt but added projections on the surface and especially aft of the main body likely contribute substantially to the drag.
The pressure is sensed by a Viatran model 104 strain gauge type pressure transducer. The pressure signal is preamplified by the circuit shown in Figure B.1. The second stage amplifier is also shown. Switches on the second stage amplifier allow different full scale gains to be selected. The single pole \( R_{16}C_2 \) is a low pass filter with a measured -3db point at 1.4 Hz.

Calibration of the pressure signal is conducted in the lab using an Amthor type 452 dead weight tester. The calibration curve is linear over the full transducer range. A typical calibration data table is shown in Table B.1.

The accuracy of the transducer quoted by the manufacturer is 0.2 percent of full scale. For the PEQUOD and WESPAC cruises a transducer with 0-5000 psi range was used, resulting in a worst case accuracy of 10 psi or 6.8 dbar.

The resolution of the pressure measurement is governed by the electronic noise added to the signal by the FM/tape recording system. This noise can be measured in the lab. A tape is recorded on the Camel III tape cassettes with DC inputs to each VCO. The recorded signal is played back on a JVC KDA11 tape deck and demodulated using a Sonex FM discriminator system. The spectral density of the noise on a particular FM channel is determined with an HP3582A spectrum analyser. Integration of the spectrum over the frequency range of interest yields the noise expected due to the signal processing. Peaks in this spectrum are attributed to incomplete tape speed compensation. On the 400 Hz FM channel (which is that used to carry the
pressure signal) there is an rms noise level of 1 mv over 0-0.5 Hz which is equivalent to 1.0 psi or 0.7 dbar. If the peak to peak value of the noise is taken to be five times the rms level the resolution of the pressure measurement is about 3.5 dbar.

Figure B.1 - Camel III preamplifier and lowpass filter-amplifier.
### Table B.1 - Camel III pressure calibration data.

<table>
<thead>
<tr>
<th>Pressure</th>
<th>( V )</th>
<th>( P_{fit} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>-2.440</td>
<td>24</td>
</tr>
<tr>
<td>50</td>
<td>-2.414</td>
<td>49</td>
</tr>
<tr>
<td>100</td>
<td>-2.363</td>
<td>99</td>
</tr>
<tr>
<td>200</td>
<td>-2.260</td>
<td>200</td>
</tr>
<tr>
<td>300</td>
<td>-2.157</td>
<td>300</td>
</tr>
<tr>
<td>400</td>
<td>-2.054</td>
<td>401</td>
</tr>
<tr>
<td>500</td>
<td>-1.952</td>
<td>501</td>
</tr>
<tr>
<td>600</td>
<td>-1.850</td>
<td>600</td>
</tr>
<tr>
<td>800</td>
<td>-1.645</td>
<td>801</td>
</tr>
<tr>
<td>1000</td>
<td>-1.440</td>
<td>1001</td>
</tr>
<tr>
<td>1200</td>
<td>-1.236</td>
<td>1201</td>
</tr>
<tr>
<td>1400</td>
<td>-1.031</td>
<td>1401</td>
</tr>
<tr>
<td>1600</td>
<td>-0.827</td>
<td>1600</td>
</tr>
<tr>
<td>1800</td>
<td>-0.622</td>
<td>1801</td>
</tr>
<tr>
<td>2000</td>
<td>-0.418</td>
<td>2000</td>
</tr>
<tr>
<td>2200</td>
<td>-0.214</td>
<td>2199</td>
</tr>
<tr>
<td>2400</td>
<td>-0.009</td>
<td>2400</td>
</tr>
<tr>
<td>2600</td>
<td>0.195</td>
<td>2599</td>
</tr>
<tr>
<td>2800</td>
<td>0.400</td>
<td>2800</td>
</tr>
<tr>
<td>3000</td>
<td>0.605</td>
<td>3000</td>
</tr>
<tr>
<td>3200</td>
<td>0.809</td>
<td>3199</td>
</tr>
<tr>
<td>3400</td>
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<td>3400</td>
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<tr>
<td>3600</td>
<td>1.219</td>
<td>3600</td>
</tr>
<tr>
<td>3800</td>
<td>1.423</td>
<td>3799</td>
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<tr>
<td>4000</td>
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<td>4000</td>
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<tr>
<td>4200</td>
<td>1.833</td>
<td>4200</td>
</tr>
<tr>
<td>4400</td>
<td>2.038</td>
<td>4401</td>
</tr>
</tbody>
</table>

HP33E linear regression routine gives \( P = 977.4V + 2408.6 \).
APPENDIX C - FALL RATE

The fall rate of the instrument is calculated by electronic differentiation of the signal from the pressure preamplifier. The circuit diagram is shown in Figure C.1. A calibration similar to that described in Appendix B yields a linear fit to the output voltage, $V_p$, of the pressure preamplifier of the form

$$V_p = a + bP_p$$

where $P$ is the measured calibration voltage. Then,

$$\frac{\partial V_p}{\partial t} = b \frac{\partial P_p}{\partial t}.$$

The differentiator output is

$$V_f = K \frac{\partial V_p}{\partial t} = K b \frac{\partial P_p}{\partial t}.$$ 

$\frac{\partial P_p}{\partial t}$ is easily converted to $\frac{\partial z}{\partial t} = W$ which represents the instrument's fall rate. The differentiator gain is plotted in Figure C.2. The gain is $412 \pm 5$ seconds with the $-3$db point measured at 0.8 Hz. A similar measurement to that described in Appendix B shows the resolution to be about 1.5 cm/s.
Figure C.1 - Camel III fall rate circuit (pressure differentiator).
Figure C.2 - Camel III Pressure derivative transfer function.

-5 db at 0.01 Hz

i/p HP3582A random noise source to PP-11
o/p PP-12
uniform window used for transfer function
APPENDIX D - TEMPERATURE

Temperature is measured using a Thermometrics Fastip Thermoprobe model FP07.

The thermistor preamplifier and low pass filter are shown in Figure D.1. \( R \) represents the thermistor in the bridge. The low pass filter has a measured -3db point at 7 Hz.

Calibration of individual thermistors is carried out in a temperature bath in the lab. Ice water is allowed to warm to room temperature very slowly while the bath is thoroughly mixed using a jet stirrer.

The flow past the thermistor is of the order of 1 m/s. The temperature is measured using a Dymec model 2801A quartz thermometer which is positioned as close as possible to the thermistor. The ice point is recorded, as are the voltages \( V_{tst4} \) and \( V_{tst5} \).

\( V_{tst4} \) is the input to the temperature VCO, while \( V_{tst5} \) is differentiated in order to calculate the temperature gradient. A cubic polynomial fit is made to the temperature data, which gives agreement to about 0.01°C. A typical calibration sheet and the resulting fit are shown in Table D.1.

The accuracy is limited by the electronic noise added by the FM/tape recording system. Noise measurements show that the temperature resolution of the system is about 0.1°C.
Figure D.1 - Thermistor preamplifier and temperature circuit.
<table>
<thead>
<tr>
<th>T (°C)</th>
<th>V (TST5)</th>
<th>T fit (°C)</th>
<th>V (TST4)</th>
<th>3V/3T (TST4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.500</td>
<td>0.061</td>
<td>6.497</td>
<td>-0.044</td>
<td>0.162</td>
</tr>
<tr>
<td>7.510</td>
<td>0.856</td>
<td>7.510</td>
<td>-0.681</td>
<td>0.161</td>
</tr>
<tr>
<td>8.500</td>
<td>0.656</td>
<td>8.500</td>
<td>-0.523</td>
<td>0.161</td>
</tr>
<tr>
<td>9.550</td>
<td>0.443</td>
<td>9.558</td>
<td>-0.353</td>
<td>0.160</td>
</tr>
<tr>
<td>10.500</td>
<td>0.254</td>
<td>10.500</td>
<td>-0.202</td>
<td>0.159</td>
</tr>
<tr>
<td>11.540</td>
<td>0.047</td>
<td>11.537</td>
<td>-0.036</td>
<td>0.158</td>
</tr>
<tr>
<td>12.600</td>
<td>0.164</td>
<td>12.601</td>
<td>0.130</td>
<td>0.157</td>
</tr>
<tr>
<td>13.500</td>
<td>0.341</td>
<td>13.499</td>
<td>0.272</td>
<td>0.156</td>
</tr>
<tr>
<td>14.500</td>
<td>0.537</td>
<td>14.500</td>
<td>0.427</td>
<td>0.155</td>
</tr>
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<td>15.500</td>
<td>0.731</td>
<td>15.499</td>
<td>0.582</td>
<td>0.154</td>
</tr>
<tr>
<td>16.510</td>
<td>0.926</td>
<td>16.511</td>
<td>0.736</td>
<td>0.153</td>
</tr>
<tr>
<td>17.500</td>
<td>1.114</td>
<td>17.497</td>
<td>0.887</td>
<td>0.151</td>
</tr>
<tr>
<td>18.510</td>
<td>1.305</td>
<td>18.508</td>
<td>1.039</td>
<td>0.150</td>
</tr>
<tr>
<td>19.500</td>
<td>1.490</td>
<td>19.498</td>
<td>1.185</td>
<td>0.148</td>
</tr>
<tr>
<td>20.500</td>
<td>1.675</td>
<td>20.499</td>
<td>1.333</td>
<td>0.146</td>
</tr>
<tr>
<td>21.520</td>
<td>1.862</td>
<td>21.522</td>
<td>1.482</td>
<td>0.145</td>
</tr>
<tr>
<td>22.500</td>
<td>2.041</td>
<td>22.513</td>
<td>1.623</td>
<td>0.143</td>
</tr>
<tr>
<td>23.500</td>
<td>2.217</td>
<td>23.499</td>
<td>1.764</td>
<td>0.141</td>
</tr>
<tr>
<td>24.510</td>
<td>2.394</td>
<td>24.503</td>
<td>1.905</td>
<td>0.139</td>
</tr>
</tbody>
</table>

\[ T = 11.77 + 6.32V + 0.12V^2 + 0.04V^3 \quad \text{fit TST4 TST4 TST4} \]

\[ T = 11.77 + 5.03V + 0.07V^2 + 0.02V^3 \quad \text{fit TST5 TST5 TST5} \]

Table D.1 - Camel III temperature calibration data and result of cubic polynomial fit.
APPENDIX E - VELOCITY SHEAR

The calibration and the behaviour of the airfoil probes used at the Department of Oceanography, University of British Columbia have been described by Osborn and Crawford (1980). The same technique for probe calibration was used for this study. The electronic processing of the signal, however, is somewhat different.

The cross stream force per unit length of the probe is

\[ f = \frac{1}{2} \rho U^2 (dA/dx) \sin(2\alpha) \]

where \( \rho \) is the density of the fluid, \( U \) is the flow speed, \( \alpha \) is the angle of attack and \( dA/dx \) is the rate of change of cross-section perpendicular to the flow with distance along the body. This is illustrated in Figure E.1.

The net force is obtained by integrating over \( x \) from the probe tip to the point at which \( dA/dx = 0 \) and is

\[ F = \frac{1}{2} \rho U^2 A \sin(2\alpha) \]

but, since the downstream and cross stream components of the flow are, respectively, \( W = U \cos \alpha \) and \( u = U \sin \alpha \), and using the trigonometric identity \( \sin(2\alpha) = 2 \sin \alpha \cos \alpha \), \( F \) is given by

\[ F = \rho A W u \quad (E.1) \]

The piezoceramic beam mounted in the tip of the probe generates a voltage which is proportional to the force applied to it. Since \( A \) is constant while \( \rho \) and \( W \) are both slowly varying and independently measured, the probe force/voltage has a linear dependence on the cross stream velocity component of
Figure E.1 — The airfoil probe showing flow components.
the flow, $u$, which is expressed in (E.1).

For calibration purposes, the probe is mounted in a jet of water (the calibrator is described in detail by Crawford (1976)). The angle of attack of the flow against the shear probe, $a$, is varied from $-22^\circ$ to $+22^\circ$, and the probe is rotated about its axis. The sinusoidal voltage generated by the rotating motion in the jet is transmitted to a preamplifier, bandpass filter and rms meter. The instantaneous voltage,

$$E = \rho U^2 \sin 2a \sin \omega t,$$

(where $\omega$ is the rotational frequency in radians/second), is directly proportional to the cross stream force on the probe. It is convenient to consider the output of the rms meter, $E_{\text{rms}}$,

$$E_{\text{rms}} \approx \rho U^2 \sin(2a).$$

The constant of proportionality, termed the probe sensitivity, $S$, is determined by plotting $E_{\text{rms}} / (\rho U^2)$ vs $\sin(2a)$. The sensitivity is the slope of this curve,

$$S = \frac{d(E_{\text{rms}} / (\rho U^2))}{d(\sin(2a))}$$

A typical calibration curve is shown in Figure E.2.

$S$ has units of volts/(dyne/cm$^2$), and is approximately constant over angles of attack of less than $10^\circ$ (see Osborn and Crawford (1980)). At greater angles of attack the sensitivity increases, illustrating the importance of maintaining vehicle stability. Any large tilting of the instrument will cause the probe to 'see' a large angle of attack thereby changing the sensitivity of the probe. Accelerometers mounted in the
Figure E.2 - Typical airfoil probe calibration curve.
instrument body are monitored. Experience indicates that occasional tilting of 2° in regions of large mean shear is both a maximum value and rare. Thus, a constant value of $S$ may be used.

Expressing the probe voltage in terms of the sensitivity,

$$E = S\rho U^2 \sin(2\alpha)$$

$$= 2S\rho Wu$$

and the peak voltage, $E$, for a sinusoid is $\sqrt{2E}$, so that

$$E = 2\sqrt{2S\rho Wu}$$

(E.2)

This signal is transmitted to a preamplifier, a 100 Hz low pass filter, and to a second stage amplifier prior to being sent to a voltage controlled-oscillator (VCO) and added to the FM signal. This processed signal is routed to a further amplification stage and this is also added to the FM signal. The purpose of this second amplified signal is to improve the signal to noise ratio at low values of velocity shear. However, a voltage limiting circuit is required to prevent saturation of the VCO at high values of velocity shear and possible contamination of adjacent FM bands. A block diagram of the processing (Figure E.3) is included.

The preamplifier differentiates the input signal from the probe (Figure E.4). Its transfer function is given by

$$\frac{V}{V} = 1 + \frac{(j\omega R_1 C_2)}{(1+j\omega R_1 C_1)(1+j\omega R_2 C_2)}$$

where the circuit parameters are given in Figure E.4. The poles are
Figure E.3 - Camel III velocity shear signal processing.
Figure E.4 - Velocity shear preamplifiers.
The measured preamplifier gain is shown in Figure E.5. It is plotted as a constant differentiator gain in dbv (the actual gain is divided by $2\pi f$ and converted to $\text{dbv} = 20\log(V/V_0)$). The gain is approximately constant at 0.85 seconds, and the -3db point is at 150 Hz.

The 100 Hz Cauer elliptic filter is manufactured by Frequency Devices (#7438). The second stage of amplification is provided by the bandpass amplifier shown in Figure E.6. This was used prior to the inclusion of the 7438 filter into the circuit and has been retained as an amplifier although no longer required as a low pass filter. The transfer function of this amplifier is

$$V/V_0 = \frac{(-j\omega R_1 C_1)/(1+j\omega R_5 C_1+j\omega R_6 C_1+j\omega R_6 C_1)}{(1+j\omega R_7 C_1+j\omega R_6 C_1+j\omega R_6 C_1)(1+j\omega R_6 C_1)}$$

The poles are

$$(2\pi R_7 C_1)^{-1} = 0.11 \text{ Hz}$$
$$(2\pi R_6 C_1)^{-1} = 0.5 \text{ Hz}$$
$$(2\pi R_7 C_4)^{-1} = 706 \text{ Hz}$$
$$(2\pi R_6 C_2)^{-1} = 234 \text{ Hz}$$
$$(2\pi R_6 C_1)^{-1} = 1.6 \text{ Hz}$$

The total measured transfer function of the complete circuit which processes the probe signal and sends it to the VCO is plotted as a differentiator gain in Figure E.7. It is seen that the gain is approximately 2.7 seconds with -3db points at f.
Figure E.5 - Velocity shear preamplifier transfer function.
Figure E.6 - Velocity shear amplifiers.
Figure E.7 - Complete shear circuit transfer function.
< 1 Hz and f > 100 Hz.

Further amplification for low values of velocity shear is provided by the amplifier and voltage limiting circuit shown in Figure E.8. The gain is given by

\[ \frac{V}{V_o} = \frac{R_1 + R_2}{R_2} \]

which is theoretically equal to about 30.5 given the nominal circuit parameters. Actual values are slightly different and the measured transfer function is plotted in Figure E.9.

Processing of the velocity shear signal can be represented by a differentiator with a gain of GK, as below,

\[ E(t) \rightarrow \frac{\partial E}{\partial t} \rightarrow E_{vco} \]

\[ E_{vco} = GKE \quad k = \frac{G}{2\pi f} \]

where \( E \) is the probe voltage and \( E_{vco} \) is the VCO input voltage. \( E_{vco} \)

Then,

\[ E_{vco} = GKE = Kk\partial E/\partial t \]

where \( k \) is the differentiator gain. If \( E \) is represented by a functional form \( \exp(i\omega t) \),

\[ E_{vco} = \omega KkE = 2\pi f KkE. \]

Using equation (E.2),

\[ E_{vco} = Kk\partial E/\partial t = Kk2\nu/2\rho w\omega/\partial t \]
Figure E.8 - Velocity shear high gain amplifier.
Figure E.9 - Velocity shear high gain amplifier transfer function.
provided that \( u \) is the only time dependent parameter. Then, invoking Taylor's frozen flow hypothesis, which can be stated as

\[
\frac{\partial u}{\partial t} = W \frac{\partial u}{\partial z},
\]

one obtains

\[
\frac{\partial u}{\partial z} = \frac{E}{(2v/2KkSpW^2)}.
\]

It is necessary to include another factor to account for the inherent gain of the VCO-FM system (which is termed \( a \)). Then,

\[
\frac{\partial u}{\partial z} = \frac{aE}{(2v/2KkSpW^2)},
\]

where \( E \) is the voltage of the demultiplexed signal. The gain factor, \( K \), is equal to 1 for the standard velocity shear signal and equal to the gain of the final stage amplifier (10 for PEQUOD and 30 for WESPAC) when the amplified shear signal is used for low signal values.
APPENDIX F - CALCULATION OF SPECTRA

The method of calculating spectra is described here. The spectral values are calculated in units of bits$^2$/Hz and converted, using the suitable calibration constants (see Appendix E) to units of velocity shear spectral density (sec$^{-2}$/Hz).

Due to the importance of this calculation to the thesis, the method of calculating the spectra and the calibration of the routine for doing this calculation is outlined here.

The data were originally recorded onto analogue tape using the Phideck tape recording units mounted inside of Camel III. Copies of each tape were made on board ship and the two identical sets of tapes were transported to UBC by different means. At UBC the data were digitized at a rate of 400 Hz using an LSI-11 computer. The mode of digitization was as 12-bit two's complement numbers with full scale corresponding to ±5 volts. The conversion from bits to volts, then, is

$$
\begin{align*}
-2048 \text{ bits} &= -5 \text{ volts} \\
0 \text{ bits} &= 0 \text{ volts} \\
2047 \text{ bits} &= 4.997 \text{ volts} \\
1 \text{ bit} &= (5/2048) \text{ volts}
\end{align*}
$$

The spectral routine was originally written at UBC by P. Leszko for the Dolphin data of T.R. Osborn, and subsequently tailored to the needs of this project. The calibration scheme for the routine used blocks of 4096 data points and operated as follows:
a) first differencing to detrend the data,
b) removal of the mean

c) a cosine taper is applied to the first and last 200 points of the block, to force the time series to zero at the end points. The form of the taper is $1 - \cos\left(1 - \frac{k-1}{200}\right)$,
d) an FFT is done on the tailored time series,
e) the calibration data are used to convert the frequency domain data to the appropriate velocity shear units,
f) the spectral values are squared and,
g) averaged over 8 adjacent frequency bands.

The output data are printed together with the frequency and the cumulative variance. The lowest frequency band, 0.98 Hz, is not included in the cumulative value. A sample program and output listing are shown in Table F.1. The average fall rate over the 4096 points is used to calculate the velocity shear and is printed on the bottom of the output listing.

Final spectral calculations were made using 1024 points for improved spatial resolution and the appropriate changes were made to the routine.

The cosine taper reduces the variance of the signal. For a signal having a white spectrum, the variance with a full cosine taper is $3/8$'s of the total variance. In this case only 400 of the points are tapered. The variance, then, is

$$1 - \frac{400}{4096}(1 - 3/8) = .939$$

of the total variance. To recover the total variance the output is divided by the factor .939.
The spectral routine used to estimate the vertical shear spectrum and sample output.
121
122
123
124
129
128
127

158

129
130
131
132
133
134
135
13S
137
138
139
140
141
142
143
144
145
146
147
148
149
160

C

C

C
C

C

C
C

C

169

170
171
172
173
174
175
176
177
178
179
180

DO

C
C

26

128-1,312

AVP(I28)-0.
26 CONTINUE -

DO «B 188-1,IFLA1
NUP-NL0.15
USUB-O.
00 124 H24-NL0.NUP
124
USUB«USUB+US(I124)«100.
USUB-USUB/16.
NLO-NLO+16
USUB - VELOCITY OF SUBMARINE (M/SEC)
IFfI77.E0.1.0R.I77.E0.2)FACT-(B.»PI«A/<2O4B.»S0RT(2.)-R0
»»S»USUB««2«0F))««2/.a390
IF(I77.EQ.3)FACT-4.3E-6/USUB»«3
IFU77.EQ.4.0R. I77.E0.B)FACT-< 1 .7/FL0AT(3048)>«-2
IF(I77.EQ.6)FACT-(I./FLOAT(3048))««3
CALL PAWEL2(C)
1

161

152
153
154
155
156
157
158
159
160
161
162
163
164
169
166
167 '
168

FREO<O-I./DT/N
IN1-IN-1
DO 2T 127*1.INI
FRE0(2-I27)-0F*FL0AT(I2T)
2T
FRE0(2-I27*1)«FRE0<2M27)
FREQ(BI3)-DF«2S6.

13

1 H-I.N
0ATA{I1)-C(I77.I1*1)-C(IT7,I1)
CALL AVER(DATA.N)

00

00

13 1 1 3 - 1 , N

0ATA(I13)-SCB(I13)«REAL(0ATA(113))

CALL F0UR2I0ATA.N,1.1,1)
0 0 2 I2-2.NDIM
A1-REAL(0ATA(I2>)
A2-AIMAG(DATA(12) )
2 CONTINUE
H-0.
00 3 13-2.8
H-H»P(I3)
3 CONTINUE
AVP(3)-»VP(2)*H/7./FL0AT(IFLA1)
AVP(1)-AVP(2)
00 4 14-2,IN
H-0.
00 6 IB-1.8
H-H*P(16*(I4-1)»B>
9
CONTINUE
AVP(2'I4)-AVP(2M4)*H/8./FL0AT(IFlA1)
AVP(2-I4-1)-AVP(2M4)
4 CONTINUE
88 CONTINUE
00 84 194-1,N8
84 AINT(I94)-0.

Table

F.I

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216
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21B
2 19
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221
222
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2 28
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336
237
338
239
24Q

0 0 85 193-2.IN
AINT(2*I9S)-AtNT(2*!93-2)+0F*AVP(2*I9S)
AINr(2-I95-1)-.S»(AINT(3*I85)-AINT(2<I8B-3))
00 86 I86-1.N3
6010 FORMAT(IX, 3(0PI3, 3X, F6.2, JX,1PE9.3,2X,E8.3,6X))
WRITE(6.6123)USUB
6123 FORMAT{// 5IHAVG VALUE OF FALL RATE USED TO CALCULATE SHEAR
4.F10.4.9H
CM/SEC)
C
CALL PLOUREO.AVP.S^BO.WT)
93

c
c

C

c

RETURN
END
FUNCTION SCB(K)
DATA PI/3.1415927/
1F(K.OT.200)GO TO 1
SCB-.S«(1.-COS(P1«K/2CO.))
RETURN
1 IF(K.GT.3B96)G0 TO 2
SCB-1.
RETURN
2 5CB-.5-(l.-C0S(PI«(4O96-K)/2OO.))
RETURN
END
SUBROUTINE AVER(A.N)
COMPLEX'S A(ai82)
SUM-O.
00 1 I-1.N
1 SUM-SUM*REAL(A(I))
SUM-SUMZFLOAT(N)
DO 2 K-1.N
2 A(K)-REAL(A(K))-SUH
RETURN
END
SUBROUTINE PAWEL2CC)
INTEGER'S
C(10.4097),A(3B60).LEN
INTEGER
LMJM

c

c

7
2
1
20

00 1 11-1. 17
CALL READ(A,LEN.0.LNUM,B,630)
DO 3 13-1.256
K-2S6-(I1-1)+12
IFfK.QT.4087)00 TO 30
DO 7 17-1. 10
C(I7.K)-A(IO-(I2-l)*I7)
CONTINUE
CONTINUE
CONTINUE
RETURN

30 STOP 30
END
SUBROUTINE AXL0G(XO.YO.IFLAG.NMIN.DN. I)
DIMENSION TNUM(IS)
DATA TNUM/2H-1.2H-2.3H-3.2H-4,2H-5.2H-6.2H-7.2H-8.2H-8,
»
3H-10.3H-11,3H-12,3H-13,3H-14.3H-18/

cont'd


CALL AXCTRL('YORI', YORI+YSIZE)}
302 CALL AXCTRL('YORI', YORI+YSIZE)}
303 CALL AXCTRL('YORI', YORI+YSIZE)}
304 CALL AXCTRL('YORI', YORI+YSIZE)}
305 CALL AXCTRL('YORI', YORI+YSIZE)}
306 IF(USUB.GE.1.)GO TO 3
307 ALD=ALOG10(1./USUB+3)
308 MM=NMIN-1+INT(ALD)
309 YORI=YORI+MM-INT(ALD)}
310 GO TO 4
311 3 AL=ALOG10(USUB**2)
312 MM=NMIN+INT(AL)
313 XORI=YORI+MM-INT(AL)}
314 4 CALL PLOT(XORI+XSIZE, YORI, 2)
315 CALL AXCTRL('YORI', YORI)
316 CALL AXCTRL('YORI', YORI)
317 CALL AXCTRL('YORI', YORI)
318 CALL AXCTRL('YORI', YORI)
319 CALL AXCTRL('YORI', YORI)
320 CALL AXCTRL(XORI+XSIZE, YORI, 2)
321 CALL AXCTRL(XORI+XSIZE, YORI, 2)
322 CALL AXCTRL(XORI+XSIZE, YORI, 2)
323 CALL AXCTRL(XORI+XSIZE, YORI, 2)
324 CALL AXCTRL(XORI+XSIZE, YORI, 2)
325 CALL AXCTRL(XORI+XSIZE, YORI, 2)
326 CALL AXCTRL(XORI+XSIZE, YORI, 2)
327 CALL AXCTRL(XORI+XSIZE, YORI, 2)
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333 CALL AXCTRL(XORI+XSIZE, YORI, 2)
334 CALL AXCTRL(XORI+XSIZE, YORI, 2)
335 CALL AXCTRL(XORI+XSIZE, YORI, 2)
336 CALL AXCTRL(XORI+XSIZE, YORI, 2)
337 1 IF(N.I.EQ.1)G0 1
338 CALL LINE(X0, Y0, XSIZE, YSIZE)
339 3 CALL PLOT(X0, Y0, XSIZE, YSIZE)
340 CALL PLOT(X0, Y0, XSIZE, YSIZE)
341 CALL PLOT(X0, Y0, XSIZE, YSIZE)
342 CALL PLOT(X0, Y0, XSIZE, YSIZE)
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344 CALL PLOT(X0, Y0, XSIZE, YSIZE)
345 CALL PLOT(X0, Y0, XSIZE, YSIZE)
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368 CALL PLOT(X0, Y0, XSIZE, YSIZE)
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370 CALL PLOT(X0, Y0, XSIZE, YSIZE)
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372 CALL PLOT(X0, Y0, XSIZE, YSIZE)
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374 CALL PLOT(X0, Y0, XSIZE, YSIZE)
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378 CALL PLOT(X0, Y0, XSIZE, YSIZE)
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381 CALL PLOT(X0, Y0, XSIZE, YSIZE)
382 CALL PLOT(X0, Y0, XSIZE, YSIZE)
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394 CALL PLOT(X0, Y0, XSIZE, YSIZE)
395 CALL PLOT(X0, Y0, XSIZE, YSIZE)
396 CALL PLOT(X0, Y0, XSIZE, YSIZE)
397 CALL PLOT(X0, Y0, XSIZE, YSIZE)
398 CALL PLOT(X0, Y0, XSIZE, YSIZE)
399 CALL PLOT(X0, Y0, XSIZE, YSIZE)
400 CALL PLOT(X0, Y0, XSIZE, YSIZE)

Table F.1 - cont'd
IF(N.LT.1) RETURN
CALL SYMBOL(X+10./T.*HTVL+2./T.*HTYS,Y+HTVL-HTYS,HTYS,
& TNUM(N),0,J)
CONTINUE
RETURN
END

END BLOCK DATA
COMMON/NUM/TNUM
COMMON/TAPE/MOUN
COMMON/FILE/FSR
COMMON/RECORD/FSR
LOGICAL*1 POSN(8)/'P','O','S','N,'.'0', '.'/.
LOGICAL*1 TNUM(10)/'1','2', '3', '4', '5', '6', '7', '8', '9', '0'/
INTEGER FSR(2)/'FSR '/
INTEGER*2 MOUN(30)/30'/

Table F.1 - cont'd
Table F.1 - cont'd
As a means of checking the calculation performed by this routine, a time series was generated and input to the routine. The form of this time series was \( g(t) = \sum_{i=1}^{4} a_i \sin(\omega_i t) \), where \( a_i \) represent the amplitudes of the specific sinusoid, \( \omega_i = \frac{2\pi n_i}{N \Delta t} \) is the frequency, \( N \) is the total number of data points, and \( \Delta t \) is the inverse of the sampling rate.

For the test function, a sampling rate of 500 Hz was used, so that \( \Delta t = 0.002 \) seconds. The parameters used were

- \( a_1 = 25 \) bits, \( n_1 = 100 \)
- \( a_2 = 50 \) \( n_2 = 150 \)
- \( a_3 = 75 \) \( n_3 = 200 \)
- \( a_4 = 100 \) \( n_4 = 300 \)

These values of \( n \) give \( f_i = \frac{n_i}{N \Delta t} \) values of

- \( f_1 = 12.2 \) Hz
- \( f_2 = 18.3 \) Hz
- \( f_3 = 24.4 \) Hz
- \( f_4 = 36.6 \) Hz

The variance is \( \sigma^2 = \frac{1}{T} \int_0^T g^2(t) \, dt \). Using integer values for \( n \) reduces the form to \( \sigma^2 = \sum_{i} \frac{a_i^2}{2} \), which is \( 9375 \) bits\(^2\) for the parameters given above.
<table>
<thead>
<tr>
<th>f(Hz)</th>
<th>numerical cumulative variance, $\sigma^2_N$</th>
<th></th>
<th></th>
<th>f(Hz)</th>
<th>analytical cumulative variance, $\sigma^2_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.72</td>
<td>3.4</td>
<td></td>
<td></td>
<td>12.2</td>
<td>312.5</td>
</tr>
<tr>
<td>12.70</td>
<td>312.0</td>
<td></td>
<td></td>
<td>13.67</td>
<td>314.8</td>
</tr>
<tr>
<td>17.58</td>
<td>322.9</td>
<td></td>
<td></td>
<td>18.55</td>
<td>1547.</td>
</tr>
<tr>
<td>19.53</td>
<td>1563.</td>
<td></td>
<td></td>
<td>23.44</td>
<td>1575.</td>
</tr>
<tr>
<td>24.41</td>
<td>1619.</td>
<td></td>
<td></td>
<td>25.39</td>
<td>4367.</td>
</tr>
<tr>
<td>26.37</td>
<td>4377.</td>
<td></td>
<td></td>
<td>36.13</td>
<td>4424.</td>
</tr>
<tr>
<td>37.11</td>
<td>9310.</td>
<td></td>
<td></td>
<td>38.09</td>
<td>9358.</td>
</tr>
<tr>
<td>39.06</td>
<td>9362.</td>
<td></td>
<td></td>
<td>12.2</td>
<td>0.7%</td>
</tr>
<tr>
<td>18.3</td>
<td>0.03%</td>
<td></td>
<td></td>
<td>24.4</td>
<td>0.05%</td>
</tr>
<tr>
<td>36.6</td>
<td>0.14%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table F.2 - Comparison of cumulative variances calculated using the spectral routine with the expected values (units are bits$^2$).
The directly calculated output is listed in Table F.2 along with the variance calculated numerically by the spectral routine, which is 9363 bits². The discrepancy is due to the fact that the generated test function is not random but is equal to zero at the beginning of the block, and only a small number of cycles occur in the tapered region. In this respect the test signal is not representative of the actual shear signal to which the routine is applied.

The reliability of the program in converting the data in bits to appropriate shear units was confirmed by applying the appropriate calibration factors to

\[ \frac{\partial u}{\partial z} = \frac{(2.5/4)(5/2048)X}{(2\sqrt{2}KkS\rho W^2)} \]

where \( K, k, S, \rho \) and \( W \) have been defined in Appendix E. \( X \) is the magnitude in bits of the spectral value, \((5/2048)\) converts from bits to volts and \((2.5/4)\) is the FM gain. For the following values of the calibration constants:

\begin{align*}
k & = 2.7 \text{ seconds} \\
S & = 4 \times 10^{-5} \text{ volts/(dyne/cm}^2) \\
K & = 1 \\
\rho & = 1.028 \text{ gm/cm}^3 \\
W & = 48 \text{ cm/sec}
\end{align*}

it follows that

\[ \frac{\partial u}{\partial z} = 2.11 \times 10^{-3} X \]

The analytical and numerical calculations are shown in Table F.3.
<table>
<thead>
<tr>
<th>$f(\text{Hz})$</th>
<th>Numerical cumulative variance, $\sigma^2$</th>
<th>$f(\text{Hz})$</th>
<th>Analytical cumulative variance, $\sigma^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.72</td>
<td>1.493x10^{-8}</td>
<td>12.2</td>
<td>1.379x10^{-3}</td>
</tr>
<tr>
<td>12.70</td>
<td>1.382x10^{-3}</td>
<td>13.67</td>
<td>1.394x10^{-3}</td>
</tr>
<tr>
<td>17.58</td>
<td>1.430x10^{-3}</td>
<td>18.55</td>
<td>6.853x10^{-2}</td>
</tr>
<tr>
<td>19.53</td>
<td>6.923x10^{-2}</td>
<td>23.44</td>
<td>6.976x10^{-3}</td>
</tr>
<tr>
<td>24.41</td>
<td>7.173x10^{-2}</td>
<td>25.39</td>
<td>1.935x10^{-2}</td>
</tr>
<tr>
<td>26.37</td>
<td>1.939x10^{-2}</td>
<td>36.13</td>
<td>1.959x10^{-2}</td>
</tr>
<tr>
<td>37.11</td>
<td>4.125x10^{-2}</td>
<td>38.09</td>
<td>4.146x10^{-2}</td>
</tr>
<tr>
<td>39.06</td>
<td>4.147x10^{-2}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$f(\text{Hz})$</th>
<th>$((\sigma^2 - \sigma_o^2)/\sigma^2) \times 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.2</td>
<td>0.80%</td>
</tr>
<tr>
<td>18.3</td>
<td>0.38%</td>
</tr>
<tr>
<td>24.4</td>
<td>0.40%</td>
</tr>
<tr>
<td>36.6</td>
<td>0.20%</td>
</tr>
</tbody>
</table>

Table F.3 - Comparison of cumulative variances calculated using the spectral routine with the expected values (units are sec^{-2}).
APPENDIX G - RESOLUTION OF THE DISSIPATION MEASUREMENT

The resolution of a measuring technique is determined by the contribution to the noise of the various components of the system. The fundamental measurement made in this case is of small scale shear using the airfoil probe. The probe is mounted on an instrument housing and the signal is processed electronically. Sources of noise associated with the measurement are:

1) inherent noise in the airfoil probe itself
2) electronic noise
3) instrument vibration picked up by the probe.

A number of tests were made to determine both the predominant source of noise to the measurement and the actual noise level or resolution of the dissipation calculation. The various noise sources will be discussed in turn.

Initial tests made to determine the noise level of the probe itself were conducted in the anechoic chamber of the Mechanical Engineering Department at the University of British Columbia. The measurement was affected, however, by an unfortunate mislaying of an electrical conduit which served to couple the anechoic chamber to the building, and it appeared that the signal picked up by the probe was chiefly building noise.

An alternate test was conducted in the Oceanography huts. A probe was potted up in epoxy resin to restrain its motion and suspended by a rubber band (which performed the role of a rather
crude vibration isolation system) in a metal pail shielded from 60 Hz using aluminum foil. The signal was preamplified and differentiated and the output spectrum (measured with the HP3582A spectrum analyser) is shown in Figure G.1. The data and the calculation made to determine the noise are shown in Table G.1. and will be considered to be representative of the other calculations in this section. The noise spectral density integrated to 40 Hz is 62 μvolts. Converted to units of dissipation using the nominal circuit values, the probe sensitivity of $3 \times 10^{-5}$ volts/(dyne/cm$^2$) and a fall rate of 75 cm/sec, this is $2 \times 10^{-10}$ W/m$^3$.

The relative contribution to the noise due to FM transmission of the signal and the tape recording were measured in the lab. To measure the FM contribution, the airfoil probe connections were shorted, and the VCO outputs fed directly into the Sonex FM demodulation system. The outputs of the discriminators used for the shear signals (3.9 and 5.4 kHz) as well as for the amplified shear signals (1.3 and 1.7 kHz) were input to the HP3582A and the noise spectral density function measured and integrated to 40 Hz.

To determine the effect of the tape recording system the mixed FM signal was recorded on one channel of the instrument's Phideck recorder while a tape speed reference signal was recorded on a second channel to provide a means of accounting for wow and flutter of the tape drive. This tape was played back, the signal demodulated and the noise spectral density calculated.
The equivalent dissipation noise is in Table 6.1 discussed in the text. Calculation of the probe was discussed in the text. Noise spectral density measured with a ported shear.
date March 26, 1981 time 2000 PST

bandwidth (BW) = 0.6 Hz

<table>
<thead>
<tr>
<th>f(Hz)</th>
<th>V(µvolts)</th>
<th>V/VBW</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-5</td>
<td>3.</td>
<td>4.</td>
</tr>
<tr>
<td>5-10</td>
<td>7.</td>
<td>9.</td>
</tr>
<tr>
<td>15-20</td>
<td>8.</td>
<td>10.</td>
</tr>
<tr>
<td>20-25</td>
<td>7.</td>
<td>9.</td>
</tr>
<tr>
<td>30-35</td>
<td>8.</td>
<td>10.</td>
</tr>
</tbody>
</table>

\[ V = \sqrt{\sum (V/VBW)^2} \Delta f = 62\mu V \]

Preamplifier gain = .85 seconds

\[ S = 3 \times 10^{-5} \text{ volts/(dyne/cm}^2) \quad W = 75 \text{ cm/sec} \]

\[ (\partial u/\partial z) = V / (5/V2SW^2/\pi) = 1.6 \times 10^{-4} \text{ sec}^{-1} \]

\[ \epsilon = 7.5\nu(\partial u/\partial z)^2 = 2 \times 10^{-10} \text{ W/m}^3 \]

**Table G.1** - Integration of the noise spectral density function and calculation of equivalent dissipation due to the inherent noise of the shear probe.
The results of these tests are summarized in Table G.2, in terms of equivalent dissipation units. Use of the amplified shear signals reduces the noise level by a factor of 10 and the addition of the tape recording system substantially increases the noise. The very best we can do, using the tape recording system and both amplified shears is $2.2 \times 10^{-10}$ W/m$^3$, which rivals the inherent noise level of the probe.

Since the smallest measurements made have been no smaller than $5 \times 10^{-8}$ W/m$^3$ and generally are more like $1-3 \times 10^{-7}$ W/m$^3$ in quiet stretches of the data record, the probe noise and electronic noise are not limiting factors. Arguments have been made for the existence of a background turbulence level required to dissipate the tidal energy (Lambeck(1977)) and to maintain the energy balance in the internal wave field (Olbers(1983)). The estimates given by Lambeck and Olbers are in the range of the smallest dissipation measurements made to date. This prompted an investigation of hydrodynamically induced noise sources which is reported in Moum and Lueck(1984). From a series of profiles made in a local British Columbia inlet noted for its turbulent quiescence and for which accelerometer measurements were compared to shear probe measurements, it was determined that vehicle accelerations which must be hydrodynamically induced by flow over the instrument body result in an upper limit to the noise of $3 \times 10^{-7}$ W/m$^3$. It is significant that this noise level can be reduced further by at least a factor of 4 by removal of the rear recovery ring, due to the replacement of broadband vibrational chatter (1-10Hz) which
contaminates the desired signal by low frequency tilting of the instrument which can be filtered out. This implies that any background turbulence levels must be less than the noise level of the Camel. It also provides guidelines for the ongoing redesign of Camels.
date August 10, 1981

noise spectral density function integrated to 40 Hz

\[ S = 3 \times 10^{-5} \text{ volts/(dyne/cm}^2\text{)} \]

\[ W = 75 \text{ cm/sec} \]

<table>
<thead>
<tr>
<th></th>
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<th>( e)</th>
<th>( e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FM</td>
<td>1.3 \times 10^{-9}</td>
<td>1.2 \times 10^{-9}</td>
<td>1.1 \times 10^{-11}</td>
<td>1.4 \times 10^{-11}</td>
</tr>
<tr>
<td>FM/TR</td>
<td>2.3 \times 10^{-9}</td>
<td>2.7 \times 10^{-9}</td>
<td>2.2 \times 10^{-10}</td>
<td>7.4 \times 10^{-11}</td>
</tr>
</tbody>
</table>

**Table G.2** - Comparison of noise levels due to FM and tape recording systems on all of the shear channels.
APPENDIX H - ERRORS IN THE DISSIPATION CALCULATION

A discussion of the errors in the calculation of $\epsilon$ is necessary so that we may be well aware of individual contributions and may, at some future time, be able to correct the faults that exist in the system and improve the accuracy of our estimates. In no particular order of importance the recognized errors are due to:

1) the inaccuracy in the measurement of the gain of the electronics used to process the signal
2) the estimate of the fall rate
3) the estimate of the shear probe sensitivity
4) the lack of correction for the frequency response of the electronics
5) the assumption of isotropic turbulence
6) the limited spatial response of the probe
7) the spectral variance missed since the integration scheme does not track the viscous cutoff wavenumber exactly.

The first four sources of error on the list are the most obvious and are more easily discussed. Measurement of electronics gain using the HP3582A spectrum analyser is no likely worse than 1%. As a squared term in the estimate for $\epsilon$ (Equation 2.6) this results in a 2% error in $\epsilon$. From Appendix C, the fall rate is known within $\pm 1.5$ cm/sec. Since the instrument slows considerably as the density increases, the smallest fall rates of about 50 cm/sec occur at depth, giving a
3% error in the fall rate, and hence a 12% error in \( \varepsilon \), since \( \varepsilon \propto W^{-1/4} \). Lueck, Crawford and Osborn (1983) estimate that neglecting the frequency response of the electronics causes a direct 5% error, or 10% in \( \varepsilon \). The calibration of the shear probes is accurate to 10% (R. Ninnis, personal communication). This results in an error of 20% in \( \varepsilon \).

The problem of how far to integrate the spectrum to recover the appropriate variance for computing \( \varepsilon \) is more subtle and hence warrants more attention. A closely related problem is the choice of high or low gain shear channel used in the computation. These decisions were made automatically by my routine 'DISSIPATION'. The criteria upon which the decisions were based are discussed here.

Figure H.1 shows four shear spectra which range over approximately four decades in \( \varepsilon \) and are from a preliminary test drop made in Quatsino Sound on Vancouver Island in April, 1981. These have been computed using the routine discussed in Appendix F and are labelled in the figure in descending magnitude. Also computed is the Kolmogoroff microscale, \( \eta = (\nu^3/\varepsilon)^{1/4} \), which is converted to a cyclic frequency using the fall rate of 50 cm/sec, \( f = \omega/2\pi \eta \). The expected downward and leftward shift in frequency of the peak of the shear spectra as the energy dissipation decreases is seen. The constancy (in dimensionless wavenumber \( k/k_s \), where \( k_s = 1/\eta \)) of the peak in the dissipation spectrum has been generally accepted due to investigations which have found that: i) \( k/k_s = 0.1 \) in a tidal channel (Grant et
Four shear spectra from Quatsino Sound on Vancouver Island taken in April, 1981. The dissipations calculated from each spectrum are: 1) $6.2 \times 10^{-4}$; 2) $1.0 \times 10^{-4}$; 3) $1.3 \times 10^{-5}$; and 4) $1.3 \times 10^{-6}$ W/m$^3$. The fall rate was 48 cm/sec. The arrows refer to $k/k = 0.15$ for each spectrum.
al.(1962)) and ii) \( k/k \approx 0.15 \) in a grid produced turbulence

(Stewart and Townsend(1951)). For comparison, an arrow is drawn
over the spectrum at \( k/k = 0.15 \) and the agreement with the peak
is quite good - excepting spectrum 4, which is quite likely
overestimated using the low gain shear which was used for this
calculation, thereby shifting the estimated peak to the right.

Spectrum 4 in Figure H.1 illustrates the necessity of high
gain amplification of the shear signal prior to recording. The
peaks at 6 and 12 Hz alone of spectrum 4 account for 30 percent
of the variance integrated to 20 Hz and are directly
attributable to unwanted signal added by the tape recording
system in the form of incomplete compensation for tape speed wow
and flutter by the FM discriminator system used. The effect of
incomplete tape speed compensation is easily demonstrated by
examining the spectrum of the output of a tape recorded signal
with grounded input. The effectiveness of the high gain channel
in overcoming this problem will be discussed further.

The ambient high frequency noise is constant among the four
spectra in Figure H.1 and becomes of increasing importance for
smaller dissipations. Spectrum 1 may be integrated to 60 Hz
(120 cyc/m) before noise is encountered. However, integration
of spectrum 3 past 15 Hz leads to the inclusion of noise in the
estimate, and spectrum 4 must be replaced by the high gain shear
since it is dominated by noise.

The spectral integration cutoff frequencies used are shown
in Figure H.2, plotted with the integrated universal dissipation
Figure H.2 - Percentage of the variance resolved by the shear probe vs normalized wave number. The curve is the integrated universal dissipation wavenumber spectrum. Symbols at the very bottom represent the probe cutoff (70 cyc/m) while smaller symbols above represent integration cutoffs.
curve. The original energy spectrum is from Nasmyth(1970), converted to the transverse spectrum and dissipation spectrum by Oakey(1982) and integrated to show the cumulative variance by Lueck, Crawford and Osborn(1983). It shows the variance resolved by integrating to dimensionless wavenumber \( k/k \). The upper limit to the integration must account for the spatial rolloff of the probe, estimated to be approximately 70 cyc/m by Ninnis(1984). The equivalent dimensionless wavenumber \( k/k \) for 70 cyc/m is shown adjacent to the \( k/k \) axis for values of \( \epsilon \) denoted by the symbols in the key and a kinematic viscosity, \( \nu = 0.01 \text{ cm}^2/\text{sec} \). At quite high dissipation rates of \( 10^{-2} \text{ W/m}^3 \), barely 50% of the variance is resolved by integrating to the probe cutoff, while at \( 10^{-3} \text{ W/m}^3 \), 80% is resolved. Since no values were measured either from PEQUOD or WESPAC which were greater than \( 10^{-2} \text{ W/m}^3 \), and only a dozen (of more than ten thousand) were greater than \( 10^{-3} \text{ W/m}^3 \) (and the largest of these was \( 3 \times 10^{-3} \)) which were integrated to 56 Hz (70 cyc/m at 80 cm/sec), the spatial resolution problem was not considered to be a serious one. Below \( 10^{-3} \text{ W/m}^3 \), integration to the probe cutoff leads to the inclusion of noise in the variance computation, and hence the integration is terminated at 30 Hz (38 cyc/m at 80 cm/sec). Below about \( 5 \times 10^{-5} \text{ W/m}^3 \), however the probe cutoff wavenumber is greater than the viscous cutoff wavenumber, and does not affect the measurement. For values of \( \epsilon < 10^{-5} \text{W/m}^3 \), the integration was terminated at 15 Hz (19 cyc/m at 80 cm/sec).

The other choice to be made, besides the frequency at which
to terminate integration of the spectrum is the use of low or high gain shear. Figures H.3 and H.4 show a continuous time series with respective spectra from four adjacent data blocks computed from a drop made in Monterey Bay in November, 1981. Shown are the two low gain time series with one of the high gain shear time series and all of the spectra from the four signals for each section (numbered 57-60). The dissipation computed from each low or high gain pair is shown in H.3. The time series have not been calibrated and are only included for qualitative comparison.

Noise in the low gain shear channel is attributed to incomplete tape speed compensation of the signal recorded on the internal cassette recorder of Camel III. At dissipations less than about $10^{-5} \text{ W/m}^3$ (as computed from the high gain shear channel) this noise dominates the signal and the dissipation estimated from the low gain shear channels is considerably higher (a factor of 20 at $4\times10^{-7} \text{ W/m}^3$). High gain shear spectra indicate the reduction of the high frequency noise by about 2 decades in shear units. Spectra for block 57, where the computed dissipation is $1.5\times10^{-5} \text{ W/m}^3$ agree much better due to the higher $\frac{\text{turbulent velocity signal}}{\text{tape recorder noise}}$ level. On the other hand, block 60 shows the predominance of the noise above the turbulence spectra in the low gain channels, while the high gain channels exhibit attenuated noise peaks. These spectra also indicate the agreement between the two orthogonally mounted shear probes. At higher dissipations, not shown here, the amplified signal is clipped by the VCO to which
Figure H.3 - Example of shear probe outputs from drop made in Monterey Bay in November, 1981. The signal on the right is the high gain amplified signal of that on the left while the middle signal is the high gain amplified shear signal from a probe which is mounted perpendicular to the first. These were divided into blocks of 1024 points for which the dissipation was calculated from both regular and high gain signals (subscript A).
Figure H.4 - Spectra from the four adjacent blocks (57-60) of the time series shown in Figure H.3. The horizontal axis is log frequency (Hz) and the vertical axis is shear spectral density (sec$^{-2}$/Hz). These are scaled in lower left hand corner. The reference dot on each spectrum is located at (1Hz, $10^{-5}$ sec$^{-2}$/Hz).
it is input and hence underestimates the dissipation and the low gain channel must be used. A plot of the ratio of the dissipation computed by the low and high gain shears plotted against that computed by the low gain shear indicates that a threshold level of $10^{-4}$ W/m$^3$ should be used when the amplifier gain is 10 (as for PEQUOD), above which the low gain shear is used. When the gain is 30 (WESPAC), a threshold level of $10^{-5}$ W/m$^3$ was used. It is very difficult to estimate the error due to a poor choice of high or low gain shear. Obviously, it is critical away from the threshold level but since the range over which the threshold level can be chosen appears to be relatively broad, we can be reasonably well assured that the best choice has been made and accept the computed estimate from the chosen signal.

The remaining contribution to the error is the initial assumption of isotropic turbulence which is made to enable the use of Equation (2.6) for computing $\epsilon$. We have no information on the vertical shear component, $\partial w/\partial z$, and can only compare the two horizontal components, $\partial u/\partial z$ and $\partial v/\partial z$. The value of the two horizontal components can vary by up to a factor of 2, although not frequently, and a variation of about 40% is more usual. As discussed in Chapter 2, however, a recent paper by Gargett et al. (1984) indicates that the assumption of isotropy at dissipation scales may be a good assumption.

Errors 1)-4) contribute 44% when the absolute values are summed, representing the worst case. Dissipations less than $10^{-4}$ W/m$^3$ are resolved to 80% or better, although larger but
infrequent values are more poorly resolved due to spatial rolloff of the probe. \(10^{-4} \text{ W/m}^3\) represents the worst integration error since the spectrum becomes limited by noise at high frequency and integration cannot be continued to the probe cutoff wavenumber. The high gain shear improves the signal to noise ratio here but occasional large turbulent bursts may be electronically clipped, resulting in possibly worse errors. These errors represent, then, a worst case of 64%. Considering these and the assumption of isotropy made, I believe that the estimate of \(e\) is good to within a factor of 2.
APPENDIX I - UNITS OF $\epsilon$

There are a number of commonly quoted units used for $\epsilon$. The units used in this thesis are W/m$^3$, which represent rate of change of energy per unit volume, and the assumption of constant density of seawater has been made (1028 kg/m$^3$). This assumption will cause a worst case error of 4% in the estimate for $\epsilon$, which is small compared to the factor of 2 which I believe is the accuracy. This unit is useful in comparing energy transfer rates, which are commonly in units of J/m$^3$.

Units of dissipation rate per unit mass are, however, more fundamentally correct if the correct density is not included in the calculation. These are cm$^2$/sec$^3$ and m$^2$/sec$^3$. If one chooses to put the density into the calculation it is easily shown that m$^2$/sec$^3$ is approximately equal to W/kg.

Since all of these units have been used, and are continuing to be used by various researchers, the following table is presented for the convenience of the reader.

\[
\begin{align*}
1 \text{ m}^2/\text{sec}^3 &= 10^{-4} \text{ cm}^2/\text{sec}^3 \\
1 \text{ W/m}^3 &= 10^{-1} \text{ cm}^2/\text{sec}^3 \\
1 \text{ W/kg} &= 1 \text{ m}^2/\text{sec}^3 \\
1 \text{ erg. sec}^{-1}/\text{cm}^3 &= 1 \text{ cm}^2/\text{sec}^3 \\
1 \text{ erg. sec}^{-1}/\text{gm} &= 1 \text{ cm}^2/\text{sec}^3.
\end{align*}
\]
White Horse velocity data were received from J. Luyten and G. Needell of WHOI. A magnetic tape was prepared of the upper 1000 m of each White Horse profile which corresponded to a Camel III profile. The velocities were calculated at 25 m intervals. White Horse profiles shown in this thesis are of these data. In order to calculate shears, however, the profiles were smoothed using a 3 point running mean and the shear calculated from first differences of the smoothed data, \( S = \Delta U/\Delta z \), where \( U^2 = (u^2 + v^2) \), and \( u, v \) are the E-W and and N-S velocity components.

Some of the plots in this thesis require a discussion of errors in the estimate of the parameters in order to understand the significance of trends. A discussion of the limitations of the White Horse measurements is in Luyten, Needell and Thompson (1982). They infer that the velocity is resolved to 4 cm/sec using a 25 dbar spacing. The amount of smoothing, however, using the 3 point running mean is substantial and, for the purpose of calculating differences in velocity (as opposed to the absolute values of the velocity), I have used an error in the estimate of the velocity of \( \delta u = 1 \) cm/sec. The error in the magnitude of the velocity \( U \), then is

\[
\delta U = |\partial U/\partial u|\delta u + |\partial U/\partial v|\delta v
\]

\[
\delta U = |u/U|\delta u + |v/U|\delta v.
\]

But \( \delta u = \delta v \) so \( \delta U = (|u| + |v|)/U \delta u \) which is largest for the smallest values of \( u, v \). Since the limiting values are 1 cm/sec, for which \( U = \sqrt{2} \) cm/sec, I will take a worst case estimate of \( \delta U \)
\[ \sqrt{2}\delta u = \sqrt{2}\text{ cm/sec}. \]

The accuracy of the pressure measurement is quoted as 0.1 percent of full scale (0.1/100\times6500 \text{ dbar} = 6.5 \text{ dbar}). However, the resolution is the critical parameter in determining differences and must be considerably smaller. The 12-bit system of the White Horse results in an error of \( \delta z = 6500/2^{12} \approx 1.6 \text{ meters}. \)

The worst case error in differencing two adjacent points is twice the error of each individual value, \( \delta(\Delta U) = 2\delta U, \quad \delta(\Delta z) = 2\delta z, \) where \( \delta \) refers to the error and \( \Delta \) to the difference in adjacent values. The error in the shear estimate, \( S = \Delta U/\Delta z, \) is

\[
\delta S = |\frac{\partial S}{\partial (\Delta U)}| \delta(\Delta U) + |\frac{\partial S}{\partial (\Delta z)}| \delta(\Delta z)
\]

giving

\[
\frac{\delta S}{S} = 2|\delta U/\Delta U| + 2|\delta z/\Delta z|.
\]

\( \Delta z \) is fixed at 25 dbar so the error in the shear estimate depends critically on the velocity difference between the two adjacent points. In the equatorial surface current - undercurrent interface region where velocity differences over 25 dbar may reach 50 cm/sec, the resulting error is \( \frac{\delta S}{S} = 2\sqrt{2}/50 + 2(1.6)/25 = 0.2. \) However, where velocity differences are smaller, say 1 cm/sec, the signal to noise ratio is considerably worse, \( \frac{\delta S}{S} = 3. \)

White Horse CTD data from the PEQUOD trip was also provided by J.Luyten and G.Needell along with the velocity data. P.Niiler of Scripps Institute of Oceanography provided the CTD data from the WESPAC trip. In both cases the data was fully calibrated and given at 2 meter intervals.
The in situ density is calculated using the 1980 UNESCO international equation of state (as given by Pond and Pickard(1983)). The speed of sound used to calculate the Brunt-Vaisala frequency, N, is calculated from the form given by Del Grosso(1974), and N is calculated from

\[ N^2 = -g \Delta \rho / \rho \Delta z - g^2 / c^2 , \]

where \( g \) is the acceleration due to gravity, \( c \) is the speed of sound in seawater and \( \rho = \rho(S,T,p) \) is the in situ density.

Since estimating the error in \( N \) calculated from CTD data is a long, tedious process, since I do not have all of the specifics of the instrumental behaviour and since it has already been done in a quite general manner by Gregg(1979), I rely on his interpretation of the error in \( N \) for this work.

The CTD mounted on the White Horse is a 12-bit system and although it is not at all certain that the limitation is quantization noise, I shall use Figure 11 of Gregg(1979) as a lower bound on the noise in the calculation of \( N \) from PEQUOD and as an upper bound to the error in \( N \) from the 16-bit system used for WESPAC. \( N \) ranges from \( 10^{-3} \) rad/sec to \( 10^{-2} \) rad/sec in most of the ocean with spikes, for example, in the equatorial undercurrent core at greater than \( 3 \times 10^{-2} \) rad/sec occurring only rarely. For \( \Delta z = 20 \text{m} \) which results in a slightly greater error than for the \( \Delta z = 25 \text{m} \) used in this study, these values of \( N \) are associated with the following errors.

<table>
<thead>
<tr>
<th>( N(\text{rad/sec}) )</th>
<th>( \delta N/N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^{-3} )</td>
<td>1.0</td>
</tr>
<tr>
<td>( 10^{-2} )</td>
<td>0.03</td>
</tr>
</tbody>
</table>
A bulk Richardson number is calculated from these parameters as $R_i = N^2/S^2$, and the error in the estimate of $R_i$ can be estimated from $\delta S$ and $\delta N$, 

$$\delta R_i = |\partial R_i/\partial S|\delta S + |\partial R_i/\partial N|\delta N,$$

or,

$$\delta R_i/R_i = 2|\delta S/S| + 2|\delta N/N|.$$

Since $R_i$ depends on independent values of $N$ and $S$, I have estimated the ranges of each for a number of values of $R_i$ to give a feel for the error in $R_i$.

<table>
<thead>
<tr>
<th>$R_i$</th>
<th>$N$ (rad/s)</th>
<th>$S$ (s$^{-1}$)</th>
<th>$\delta N/N$</th>
<th>$\delta S/S$</th>
<th>$\delta R_i/R_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.15</td>
<td>(.002-.02)</td>
<td>(.005-.03)</td>
<td>.05</td>
<td>-.15</td>
<td>.40</td>
</tr>
<tr>
<td>.50</td>
<td>(.0005-.01)</td>
<td>(.0005-.01)</td>
<td>.25</td>
<td>.30</td>
<td>1.1</td>
</tr>
<tr>
<td>2.0</td>
<td>(.0004-.01)</td>
<td>(.0002-.005)</td>
<td>.30</td>
<td>1.0</td>
<td>2.6</td>
</tr>
<tr>
<td>10.</td>
<td>(.0003-.003)</td>
<td>(.0001-.001)</td>
<td>.50</td>
<td>2.0</td>
<td>5.0</td>
</tr>
</tbody>
</table>

It is fortunate that smaller and more interesting values of $R_i$ have a better signal to noise ratio.
APPENDIX K - PEQUOD DROPS

This appendix contains individual plots of the logarithm to base ten of turbulent kinetic energy dissipation for each of the drops of the PEQUOD cruise. These are plotted as histograms with the right end of each bar representing the dissipation in units of W/m^3 calculated over approximately 2 meter intervals. The log \( \epsilon \) axis spans 5 decades from \( 10^{-7} \) - \( 10^{-2} \) W/m^3. The vertical axis has a tic every 200 dbar. Preceding the plots is a drop log which lists relevant data pertaining to each drop. The White Horse net code is listed under WH NET. For those drops which were accompanied by White Horse drops, velocity, temperature, salinity and Brunt-Vaisala frequency profiles are plotted. Temperature is a solid line while salinity is dashed. The ranges on the horizontal scales are 0 - 30°C, 34.5 - 35.7 ppt, and 0. - 0.024 rad/sec. The velocity profiles show zonal velocities as solid lines (with eastward flow > 0) and meridional velocities dashed (northward flow >0). Full scale on the plots is ± 100 cm/sec.

Drops made prior to drop 13 were started approximately four hours after the White Horse had been dropped. This procedure gave the crew the opportunity to recover the instrument before launching the Camel. However, drops 13 and following were made nearly simultaneously with White Horse drops, as the crew gained confidence in our operational procedures. Due to the much shorter drop time of the Camel (the White Horse goes all the way to bottom), we were able to launch and recover the Camel within one White Horse cycle.
<table>
<thead>
<tr>
<th>DROP</th>
<th>DATE</th>
<th>TIME</th>
<th>POSITION</th>
<th>GOOD DATA(m)</th>
<th>WH NET</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>02/07/82</td>
<td>1900</td>
<td>2N, 138W</td>
<td>20-1540</td>
<td>B</td>
</tr>
<tr>
<td>3</td>
<td>02/08/82</td>
<td>1502</td>
<td>50N, 138W</td>
<td>20-1245</td>
<td>D</td>
</tr>
<tr>
<td>4</td>
<td>02/09/82</td>
<td>1210</td>
<td>0N, 138W</td>
<td>20-900</td>
<td>E</td>
</tr>
<tr>
<td>5</td>
<td>02/09/82</td>
<td>2014</td>
<td>50S, 138W</td>
<td>20-365</td>
<td>F</td>
</tr>
<tr>
<td>6</td>
<td>02/10/82</td>
<td>1247</td>
<td>1.25S, 138W</td>
<td>20-920</td>
<td>G</td>
</tr>
<tr>
<td>7</td>
<td>02/13/82</td>
<td>1236</td>
<td>0N, 144.7W</td>
<td>20-835</td>
<td>none</td>
</tr>
<tr>
<td>8</td>
<td>02/13/82</td>
<td>1449</td>
<td>0N, 144.7W</td>
<td>20-130, 270-820</td>
<td>none</td>
</tr>
<tr>
<td>10</td>
<td>02/15/82</td>
<td>0835</td>
<td>0N, 145W</td>
<td>200-565, 590-810</td>
<td>K</td>
</tr>
<tr>
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<td>1108</td>
<td>25N, 144.7W</td>
<td>20-900</td>
<td>none</td>
</tr>
<tr>
<td>13</td>
<td>02/20/82</td>
<td>0417</td>
<td>50S, 145W</td>
<td>20-920</td>
<td>L</td>
</tr>
<tr>
<td>14</td>
<td>02/20/82</td>
<td>1054</td>
<td>0N, 145W</td>
<td>20-920</td>
<td>K</td>
</tr>
<tr>
<td>15</td>
<td>02/20/82</td>
<td>1947</td>
<td>50N, 145W</td>
<td>20-900</td>
<td>none</td>
</tr>
<tr>
<td>16</td>
<td>02/21/82</td>
<td>0604</td>
<td>1.25N, 145W</td>
<td>20-930</td>
<td>O</td>
</tr>
<tr>
<td>17</td>
<td>02/21/82</td>
<td>1618</td>
<td>0N, 145W</td>
<td>20-930</td>
<td>K</td>
</tr>
<tr>
<td>18</td>
<td>02/22/82</td>
<td>1355</td>
<td>0N, 148W</td>
<td>200-280, 390-815</td>
<td>none</td>
</tr>
<tr>
<td>19</td>
<td>02/24/82</td>
<td>0845</td>
<td>0N, 153W</td>
<td>20-220, 550-925</td>
<td>Q</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TOTAL DATA</th>
<th>12335m</th>
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<tbody>
<tr>
<td>20-300m</td>
<td>3780</td>
</tr>
<tr>
<td>300-1000m</td>
<td>8015</td>
</tr>
<tr>
<td>&gt;1000m</td>
<td>540</td>
</tr>
</tbody>
</table>

Table K.1 - PEQUOD drop log.
LOG ε (W/m²)

PRESSURE (dbar)

HORIZONTAL VELOCITY (cm/sec)

PEQUOD DROP 3

PEQUOD NET D 08/02/82
PEQUOD DROP 7
LOG ε (W/m²)

PRESURE (dbar)

PEQUOD DROP 15
LOG e (W/m³)

HORIZONTAL VELOCITY (cm/sec)

PRESSURE (dbar)

PEQUOD DROP 17

PEQUOD NET K 21/02/82
APPENDIX L - WESPAC DROPS

Vertical profiles of $\epsilon$ and associated temperature, salinity and Brunt-Vaisala frequency profiles from the WESPAC cruise along with a drop log are included here. The scales are the same as for Appendix K, except for the salinity which ranges from 33.8 - 35.0 ppt. Note that the deepest drops 11-13 have been photographically reduced in order to accommodate them on standard page size. Drops were timed so that the Camel broke surface shortly after the CTD had been brought back on deck.
<table>
<thead>
<tr>
<th>DROP</th>
<th>DATE</th>
<th>TIME</th>
<th>POSITION</th>
<th>GOOD DATA(m)</th>
<th>CTD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>05/24/82</td>
<td>1010</td>
<td>22.7N, 149E</td>
<td>20-840</td>
<td>none</td>
</tr>
<tr>
<td>2</td>
<td>05/26/82</td>
<td>0154</td>
<td>27.7N, 152E</td>
<td>20-1470</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>05/27/82</td>
<td>0833</td>
<td>28.5N, 152E</td>
<td>20-1400</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>05/28/82</td>
<td>0847</td>
<td>30N, 152E</td>
<td>20-670</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>05/28/82</td>
<td>1830</td>
<td>30.7N, 152E</td>
<td>20-975</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>05/30/82</td>
<td>1447</td>
<td>32.5N, 152E</td>
<td>20-135, 195-1400</td>
<td>11</td>
</tr>
<tr>
<td>9</td>
<td>05/31/82</td>
<td>1240</td>
<td>34N, 152E</td>
<td>20-1655</td>
<td>13, 14</td>
</tr>
<tr>
<td>10</td>
<td>06/01/82</td>
<td>1239</td>
<td>35N, 152E</td>
<td>420-1510</td>
<td>16</td>
</tr>
<tr>
<td>11</td>
<td>06/03/82</td>
<td>0121</td>
<td>37.5S, 152E</td>
<td>930-2240</td>
<td>none</td>
</tr>
<tr>
<td>12</td>
<td>06/03/82</td>
<td>0837</td>
<td>38.25N, 152E</td>
<td>1380-2270</td>
<td>21</td>
</tr>
<tr>
<td>13</td>
<td>06/05/82</td>
<td>0213</td>
<td>41N, 152E</td>
<td>20-340, 990-2240</td>
<td>24</td>
</tr>
</tbody>
</table>

TOTAL DATA 13070m

20-300m 2180
300-1000m 5085
>1000m 5805

Table L.1 - WESPAC drop log.
LOG α (W/m²)

PRESURE (dbar)

WESPAC DROP 6
TEMPERATURE (°C)

BUOYANCY FREQUENCY (rad/sec)

SALINITY (ppt)

PRESSURE (dbar)