INVERSION OF COSMOGENIC NUCLIDE DATA FROM IRON METEORITES

by

Steven James Pearce

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Department of GEOPHYSICS & ASTRONOMY

The University of British Columbia
1956 Main Mall
Vancouver, Canada
V6T 1Y3

Date 01 Aug. '84

ABSTRACT

The long accepted conclusion that the galactic cosmic ray flux has been "fairly" constant over the past billion years or so is based upon both weak inferences and self-inconsistent interpretations of the data. For example, the well known exposure age discrepancy between the analyses based on ^{40}K and those of the shorter-lived radionuclides (*i.e.*, ^{10}Be , ^{26}Al , and ^{36}Cl) has yet to be properly reconciled. Recent work by Schaeffer *et al.* (1981) on space erosion rates for irons suggests that only a variation in the galactic cosmic ray flux yields the satisfactory explanation.

Reformulating this problem within the framework of linear inverse theory (Backus and Gilbert, 1967, 1968, 1970) allows for an unprecedented perspective on this problem as compared with any previous analyses.

In theory, it is found that smoothed estimates of the long term galactic cosmic ray prehistory can be formed with arbitrarily good resolution. The degree of smoothing is determined primarily by the relative exposure age differences among adjacent meteorite samples as well as their associated experimental uncertainties. Moreover, this is the only unique information available to this entire study.

The near absense of *a priori* constraints promotes a simple objective philosophy of interpretation. Further, the lack of any independent exposure age determination presents an inherent non-linearity. But, it can be shown that a single model for both the cosmic ray flux and the exposure ages, mutually consistent with the observations and associated errors, can be derived under several fundamental criteria.

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INTRODUCTION

Over four decades ago a geochronological study of several iron meteorites employing the He-U method produced a perplexing scatter of ages (Arrol *et al.*, 1942), several greatly exceeding the current canonical age of the solar system. In an attempt to reconcile with the oldest ages Bauer (1947) and Huntley (1948) independently suggested that ^4He is efficiently produced by cosmic ray interactions with meteorites while in interplanetary space. If this were so then ^3He was also expected to be anomalously abundant in the samples. Subsequent analysis revealed the ratio $^3\text{He}/^4\text{He}$ to be of order unity in meteorites (Paneth *et al.*, 1952) in contrast to the primordial helium ratio¹ of roughly 10^{-4} (Tolstikhin, 1978). Similar anomalies were observed for the isotopes of neon (Reasbeck and Mayne, 1955) and argon (Gentner and Zähringer, 1957). Moreover, short-lived radioisotopes are observed in freshly fallen meteorites, lunar samples, and retrieved spacecraft. All such observations give overwhelming support to the Bauer-Huntley hypothesis which has long since been accepted as fact. Isotopes produced in matter interacting with cosmic radiation are generically known as *cosmogenic nuclides*.

A simple description concerning the physics of this problem is identical to that of a particle beam accelerator experiment in which a target of known composition is exposed to a high energy incident particle flux for an arbitrary time T . Some high energy nuclear (spallation) reactions will occur between incident species and *target* isotopes resulting in the accumulation of *product* nuclides (interaction debris) within the body. The nature of the spallation process allows for the production of any isotope of atomic mass below that of the target nuclides. A key goal for accelerator experiments is the empirical determination of product yields, information crucial to the interpretation of cosmogenic nuclide data. Unlike a

¹This refers to the helium abundances trapped during the condensation of the solar nebula. It is the largest helium ratio observed in the solar system apart from cosmic ray interaction products and cosmic rays themselves.

controlled laboratory experiment however the history of both target and irradiation conditions are unknown in the cosmogenic nuclide problem. It is assumed that there exists some collisional and/or erosional process responsible for producing interplanetary debris with dimensions corresponding to the mean free path of cosmic ray particles within the body.¹ The ideal description involves an initially shielded fragment suddenly exposed to the cosmic radiation. Certain "*cosmogenic clocks*" of interest are subsequently switched on and the meteoroid continues uneventfully in its orbit. After a clearly defined exposure time the body encounters the earth and becomes reshielded by the atmosphere.

Figure 1.1: The ideal description of the cosmogenic nuclide problem invokes a collisional process for the production of meteoroids since space erosion promotes an ambiguous interpretation of *exposure age*. Space erosion will be discussed in section (2.3).

¹A few meters in stone or iron.

The traditional objectives of cosmogenic nuclide research have been twofold. One approach addresses the galactic cosmic ray flux prehistory and is consequently of greatest interest to high energy astrophysicists. The other viewpoint concerns the chronology of meteoroids as small bodies in the solar system and hence is of principal interest to researchers of meteoritics and the planetary sciences. In simple terms, the mutual goals are to derive information on *both the unknown time dependence of a particle flux and the unknown exposure time of a target* using measured abundances of spallation products in tandem with experimentally determined yields. Clearly, the two desired quantities depend upon one another in the interpretation of the data. Moreover, independent *a priori* knowledge which can be used to constrain either the flux or meteoroid prehistory is grossly inadequate. Consequently, the natural dichotomy of the problem arises when researchers primarily interested in exposure ages are forced to make strict assumptions on the flux prehistory and vice versa. Evidently, the current status of this problem allows for many traps. Although research has been rewarding where valid inferences can be drawn, unfortunately, questionable practices exist in the literature.

It is currently held that the galactic cosmic ray flux has been constant to within experimental precision. In fact, these conclusions were originally based upon the analysis of radionuclides with half-lives on the order of 10^6 years and less (Arnold *et al.*, 1961). The validity of the constant flux model over this time frame is easily demonstrated (see Appendix A). However, there is strong evidence to suggest that this model is incorrect over greater time scales. The analysis of ^{40}K , the next radionuclide with respect to increasing half-life that is produced in appreciable yield within meteorites, indicates that there has been a general increase of the galactic flux over the past 10^9 years. The only competing interpretation of the ^{40}K observations, space erosion, is apparently inadequate (Schaeffer *et al.*, 1981). Nevertheless, the constant flux model is widely assumed over a gigayear throughout the literature resulting in self-inconsistent interpretations of the data.

Another misconception in this problem is the tendency to assume that continually improved observations inevitably lead to increasingly accurate understandings. All physical problems possess inherent limits to the interpretation of real observables regardless of their quantity and quality. For a given problem one could spend great time and effort to amass an arsenal of data only to find marginal improvements over the explicative powers of a few crude observations.

Let us now take a broader perspective on the problem of interpreting abundances of cosmogenic nuclides. Clearly, using these data to say something meaningful about prehistoric irradiation conditions is an exercise in inference based upon indirect observations. Over the past few years this familiar class of problem has undergone a significant advance and the philosophy of such has come to be known as *inverse theory*¹.

There are two general kinds of inverse problems depending upon whether or not the working equations are linear. There is presently little known about the solutions of non-linear inverse problems and so it is usual to elicit assumptions which reduce them to linear ones. If a problem can be posed linearly then inverse theory becomes a robust framework from which to address key questions concerning the innate information capacity of the data, effects of observational uncertainties, experimental design, and the construction of models.

The goal of this paper is to reformulate the cosmogenic nuclide problem through inverse theory. It is claimed that this analysis affords a substantial advantage over traditional methods. In particular, attention will be focused on the *long term problem* ($\sim 10^7$ years) based upon the analysis of existing data from iron meteorites. An attempt is made to present a broad overview of the possible interpretations and the limitations therein. In addition, it is clearly shown where effective improvements to the problem can be made. Moreover, it is

¹Much of the earlier developmental work dealt with problems concerning geo- and planetary physics. Consequently, the field is also known as geophysical inverse theory.

demonstrated how objective models of the galactic cosmic ray flux prehistory can be constructed which are mutually consistent with all current observations. The goal of this paper hereafter is to set upon these problems.

It is assumed that the reader has some familiarity with inverse theory.¹ However, this is not essential as the work presented here is largely self-contained.

In what follows the description of all physical processes, and assumptions therein, crucial to understanding the origin of observables is henceforth referred to as the *forward problem*. Conversely, the procedure by which the data are implemented for the purpose of inferring the prehistory of one or more physical quantities akin to the forward problem is henceforth referred to as the *inverse problem*. As such, the inverse problem is intangible until the forward problem is adequately understood.

The intent of the next two chapters is to adequately review and codify the forward problem so that proper attention can be given to inversion. The reader may wish to refer to one or more reviews on the cosmogenic nuclide problem for further details (*i.e.*, Schaeffer, 1962; Geiss *et al.*, 1962; Honda and Arnold, 1967; Kohman and Bender, 1967; Kirsten and Schaeffer, 1971; Lal, 1972). Later chapters will be concerned with inversion itself.

¹An excellent review of inverse theory was given by Parker (1977). In addition, Twomey (1977) provides a comprehensive overview of the subject. However, the actual analysis presented in this work is largely based upon the series of rigorous papers by Backus and Gilbert (1967, 1968, 1970).

2

ESSENTIALS OF THE COSMOGENIC NUCLIDE PROBLEM

Cosmogenic nuclide production may be viewed as consisting of three primary phenomena, respectively, concerned with the nature of

- i) the incident particle flux (*i.e.*, *cosmic radiation*),
- ii) the matter interaction mechanisms (*i.e.*, *pertinent nuclear chemistry*), and
- iii) the target (*i.e.*, *meteoroids, cosmic dust, lunar surface, spacecraft, etc.*).

The forward problem entails a thorough enough physical understanding of i), ii), and iii), as well as adequate knowledge concerning the histories of i) and iii), to allow for predictions of cosmogenic nuclide abundances. Of course, the inverse problem also invokes physical insight into all three components. However, independent information on the history of either the cosmic ray flux or the target is sparse and highly conjectural. Consequently, it becomes necessary to define a quantity which describes the progression of *both* the cosmic radiation and the target in time; namely, the *irradiation history*. Thus, the actual goal of the inverse problem is to use observed cosmogenic nuclide abundances to recover information on the *irradiation prehistory*, related to the target(s) under study, based upon current physical understandings of i), ii), and iii).¹

The purpose of this chapter is to present the required understanding of these three components for the purpose of describing the forward problem. Effort is made to provide extended references wherever appropriate as only those aspects of immediate concern to this work are addressed.

¹We elicit this important concept in order to emphasize that the current status of the cosmogenic nuclide problem does not properly allow one to address either the flux or target prehistory separately over long time scales.

2.1 Cosmic Radiation:

The term *cosmic ray* applies to high energy¹ particles or quanta of extraterrestrial origin. Although the existence of cosmic radiation has been known for over seventy years fundamental questions concerning their origins and large scale distributions are still inadequately understood. Nearly all that is known about such problems comes from indirect observations of one form or another. Such knowledge is crucial to understanding possible long term variations of the galactic flux which should be observed from the inner solar system. One would assume that these issues should be brought forth in order to provide necessary *a priori* information to assist in restricting the possible interpretations. Indeed, even a cursory overview of the arguments that support the current concensus of interpretations among cosmic ray researchers would be an overwhelming undertaking (*i.e.*, Brecher and Burbidge, 1972; Cesarsky, 1980; Longair, 1981). Moreover, we shall see that the results of inversion presented in this work do not possess adequate detail to warrent the effort. Rather, it is natural to view the results contained herein as an additional indirect observation of the galactic cosmic ray flux to further assist in the understanding of the origin and distribution problems.

In this work interest lies primarily in quantitatively understanding those aspects of the incident flux that are essential to the development of the forward problem (*i.e.*, composition, flux, energy spectra, arrival directions *etc.*). Let us begin with a general description of energetic particles observed near the earth.

In 1942, it was found that particles with energies in excess of 1 MeV originating at the sun can encounter the earth (Forbush, 1946). The steady emission of lower energy particles, the so-called *solar wind*, was later confirmed by experimental results from unmanned spacecraft. Unlike the solar wind however the high energy component is emitted

¹Loosely defined to be above 1 MeV.

discontinuously, resulting from solar flare events in the chromosphere. They are generally referred to as *solar cosmic rays*.

Superimposed on the solar contribution is a relatively tenuous omnidirectional flux of higher energy, averaging roughly 2 GeV, wherein some cases individual particle energies have been reported as large as 10^{20} eV! Since the flux originates from outside of the solar system the term *galactic cosmic ray* has been adopted. In addition, there are compelling arguments favouring an extragalactic origin for individuals possessing energies greater than 10^{19} eV (Sreekantan, 1979; Kiraly *et al.*, 1979; Watson, 1980). They are referred to as *extragalactic cosmic rays*.

The usual definition of cosmic ray composition is in terms of charge and energy spectra. The elemental abundances in cosmic radiation has been extensively reviewed by Cameron (1973) and by Shapiro and Silberberg (1975). Galactic cosmic ray nuclei are primarily protons and alpha particles. In addition to atomic nuclei, galactic cosmic radiation also contains electrons, positrons, and neutral species such as gamma-rays and X-rays. Approximate abundances are given in Table (2.1).

Table 2.1: Composition of Galactic Cosmic Radiation

FAMILY	PARTICLE	ABUNDANCE
Baryons	protons helium nuclei heavier nuclides anti-baryons	~85% ~15% ≤2% predicted
Leptons	electrons positrons neutrinos	≤1%† detected predicted
Photons	X-rays gamma-rays	

†Relative to the proton abundance for $E > 1$ GeV.

In the next section the interaction mechanisms responsible for the production of cosmogenic nuclides are reviewed. We shall see that these *high energy reactions* exclude all processes slower than those determined by the *strong force*. Consequently, only the baryon component is considered in the cosmogenic nuclide problem.¹ This is mentioned ahead of time in order to avoid a lengthy, and eventually superfluous, discussion of incident leptons and photons.

Unlike the galactic flux, the chemical composition of solar cosmic rays is found to vary considerably from flare to flare. The average composition is roughly 70% protons, 30% alpha particles, and >1% heavier nuclei. Evidently, the variable nature of both the flux and the composition of solar cosmic radiation makes it difficult to describe them quantitatively. Moreover, because of the lesser energies involved the effects of solar cosmic rays are restricted to the near surface regions of targets which suffer from the effects of erosion in space (section 2.3). The situation is more severe for meteorites as they have undergone atmospheric ablation. Consequently, we will be concerned with data taken deep enough in targets (say $100 \text{ g}\cdot\text{cm}^{-2}$) so that the solar flux can be ignored. Other techniques for filtering out the effects of lower energy particles in the analysis of cosmogenic nuclide data will be discussed in later sections.

Similarly, the contribution of possible extragalactic cosmic rays to the production of cosmogenic nuclides is negligible. Although the energy spectrum of the extra-solar flux extends to enormous energies the total energy is insignificant there. This conclusion is further qualified in section 3.3 in a discussion concerned with the energy band limitations of this problem (see Figure 3.6).

Thus we have considerably simplified our discussion of cosmic radiation to the baryon component of the galactic flux.

¹In section 3.3 we will see that only protons need be considered!

Let us now consider the galactic cosmic ray energy spectra for the remaining species of interest. Direct flux measurements are influenced by solar modulations (Webber, 1961). It was found that the average flux of galactic particles is roughly $5 \text{ nucleons}\cdot\text{cm}^{-2}\cdot\text{sec}^{-1}\cdot\text{sr}^{-1}$ during solar minimum and about $1 \text{ nucleon}\cdot\text{cm}^{-2}\cdot\text{sec}^{-1}\cdot\text{sr}^{-1}$ during solar maximum. This modulation becomes progressively less important towards higher energies. For example, the flux of galactic particles with energies greater than 3 GeV is about $1 \text{ nucleon}\cdot\text{cm}^{-2}\cdot\text{sec}^{-1}\cdot\text{sr}^{-1}$ during solar minimum dropping to roughly half of this value during solar maximum. The attenuation of galactic cosmic radiation throughout the solar system can be of principal concern when studying targets which have fairly eccentric orbits. However, research into the prehistory of galactic cosmic radiation using the appropriate isotopes cannot generally provide the resolution necessary to detect such orbital effects over the long term (see chapter 4).

Clearly, solar modulation introduces a certain degree of uncertainty to the lower end of the galactic cosmic ray energy spectrum (below 1 GeV). Again however, low energy filtering greatly reduces the effect of this problem. There is still the possibility of significant long term solar modulation effects (*i.e.*, changes in solar luminosity). We will address this question again in the final chapter.

The energy spectra of galactic cosmic ray protons and helium nuclei are displayed in the right center plot of Figure (2.1). Between 10^9 and 10^{15} eV/n^1 both spectra are effectively described by a differential power law of the form

$$S(E) dE = S_0 E^{-2.6} dE. \quad (2.1)$$

The spectra of the heavier elements are similar (see Meyer, 1969).

¹Electron volts per nucleon.

Figure 2.1: Some classification and nomenclature commonly used in the description of energetic particles of extraterrestrial origin appear in the left hand figure. A major classification can be established according to whether or not the radiation is of solar origin. The solar flux consists of the solar wind (SW) and the more energetic particles accelerated during solar flare events (F). Some of the latter particles attain cosmic ray energies (SCR). However, negligible amounts are observed with energies greater than about 1 GeV. The differential energy spectra of solar protons and helium nuclei are given in the top right figure (from Fan *et al.*, 1970). The extra-solar component consists of galactic cosmic rays (GCR) together with a possible contribution from extragalactic sources (EGCR). The differential energy spectrum of the galactic flux is given in the right center figure (Gloeckler and Jokipii, 1967). Note that solar modulation begins to play an important role below 1 GeV. The galactic contribution below 10 MeV in the inner solar system is poorly known. For completeness the integral energy spectrum of the extra-solar flux above 10 GeV is given in the lower right figure (Meyer, 1980).

Equation (2.1) describes the energy spectrum of the galactic cosmic ray flux in space; that is, the spectrum of the *primary* flux. However, it is the resultant flux within a target, the so-called *secondary* flux, which is of concern to the description of cosmogenic nuclide production. Therefore, we will elicit the necessary empirical observations of galactic cosmic radiation within targets in later sections concerned more with target interactions.

Next, let us proceed on to briefly discuss the pertinent facts concerning cosmic ray arrival directions. These observations have been instrumental in providing insight into theories of cosmic ray sources and large scale distributions. However, we are only interested in the solid angle dependence of the galactic radiation so as to provide a complete understanding of "*flux*" in the forward problem.

We have previously noted that the galactic flux is omnidirectional. One would expect to detect anisotropies, measured from the inner solar system, on the basis of physical arguments such as

- i) the spatial distribution of cosmic ray sources,
- ii) the Compton-Getting effect¹,
- iii) anisotropies in the interstellar confinement regions, and
- iv) the effects of the interplanetary magnetic field (for lower energies).

However, arrival directions are isotropic to within present experimental precision (Cachon, 1962; Elliot *et al.*, 1970). These results apply to particle energies between 10^{11} eV and 2×10^{13} eV. We must note that these measurements are barely precise enough to detect the net streaming motion of the solar system.

The only evidence of extra-solar cosmic ray anisotropies comes from very high energy radiation. Particles with energies in excess of 10^{19} eV tend to arrive preferentially from high

¹The effect produced from the relative motion of the solar system with respect to the interstellar medium.

galactic latitudes (Krasilnikov, 1979). But this is of little concern to the cosmogenic nuclide problem.

Thus, let us conclude that the galactic cosmic ray flux is *presently* omnidirectional and independent of solid angle. However, this may not always have been the case if, for example, the solar system passed very close to large and/or dense interstellar clouds.

This completes the discussion concerning those aspects of the primary cosmic ray flux which are essential to the forward problem. In summary, the analysis of cosmogenic nuclide production undertaken in this work elicits a model of the primary incident flux which is characterized as an isotropic omnidirectional flux of galactic cosmic ray baryons possessing a power law energy spectrum with an exponent of -2.6 throughout the applicable range.

This simple picture becomes inadequate when interest turns to the description of incident high energy radiation within relatively dense matter. The complicated interplay between cosmic ray species and target nuclei at such high energies requires our attention to shift now to the nuclear sciences.

2.2 Pertinent Nuclear Chemistry:

The process by which a nucleus interacts with another nucleus, an elementary particle, or a photon, within a time frame of 10^{-12} seconds or less is termed a nuclear reaction. In such a case the kinetic energies involved are sufficient for the incident particle to enter the nuclear potential well of the target nuclide. Subsequently, it will collide with a target nucleon releasing all kinetic energy into a small volume whose dimensions are prescribed by the *strong force* (a simple but edifying model developed by Enrico Fermi in 1950). Although the process is ephemeral the strong interaction is assumed to be sufficient to allow the establishment of statistical equilibrium in the volume. Hence, it appears that one can exclude all processes

slower than those determined by the strong interaction as noted in section (2.1).

Such a process can yield one or more other nuclei, and possibly other particles depending upon the relative energy of the interaction. There are essentially three nuclear reaction mechanisms, classified somewhat arbitrarily according to energy, which have come to be known as

- i) compound nuclear reactions ($E \leq 10$ MeV),
- ii) direct nuclear reactions ($10 \leq E \leq 100$ MeV), and
- iii) high energy nuclear reactions ($E \geq 100$ MeV).

The incident particle of a compound nuclear interaction is completely absorbed by the target nucleus implying that this is inherently a low energy process. The lifetime of the resulting excited or *compound* nucleus is on the order of 10^{-16} seconds. However, the transit time of the incident particle across the target nucleus is roughly 10^{-22} seconds and thus the total incident kinetic energy is randomly shared among all nucleons within the system. Moreover, a compound nucleus has no "*memory*" of how it formed and a succession of particles is emitted from the excited nucleus having relatively low kinetic energies; a process known as *evaporation*.

Unlike the compound nuclear model, the kinetic energy of the incident particle involved in a direct nuclear reaction is not assumed to be randomly distributed among all nucleons within the target nucleus. The time scales are now close to the nuclear transit time and consequently there is "*memory*" between the incident and final channels. Moreover, there are three types of direct reactions which are generally difficult to distinguish.¹

¹Referred to as *pickup, stripping, and knockout* reactions.

Let us now consider the de Broglie wavelength of the incident particle (*i.e.*, the ratio of Planck's constant to the momentum of the particle). At energies of 10 MeV and 100 MeV the particle size (in a quantum mechanical sense) is about 9 fm and 2.8 fm respectively.¹ Thus, the incident particle effectively *sees* the entire nucleus in the compound and direct reactions. But for higher energies, say 1 GeV and 500 GeV, the nucleus becomes more and more *transparent* as the de Broglie wavelength of the particle decreases; *i.e.*, 0.7 fm and 0.0025 fm respectively. In fact, at energies above 1 GeV the incident particle interacts with individual nucleons within the target nucleus resulting in a very complicated intranuclear collision process. The resulting outcome of particles has been coined *nuclear minestrone*. This final class, the high energy reaction, is of principal interest both to the study of cosmic ray nuclei and to the production of cosmogenic nuclides and must be studied greater detail.

Consider the interaction between an incident high energy proton and some heavier target nucleus. At high enough energies, the number of particles with which the proton will interact is simply the number of nucleons along the line of sight through the target nucleus. In fact, a reasonable model of high energy nuclear interactions considers the incident proton as executing a *random walk* inside the nucleus. Hence, Monte Carlo techniques are often applied to calculate the yields of arbitrary nuclides resulting from the interaction. Moreover, any product nuclide can result as long as its atomic mass is below that of the target isotope. In effect, the incident proton chips or spalls nuclear fragments out of the target nucleus, and hence the term *spallation*. The resulting de-excited nuclide is termed a *spallation product*. What we have described is collectively known as the *cascade-evaporation model*.

An elucidative display of information concerning high energy reactions is the so-called *mass yield curve* (Figure 2.2). The most striking feature is the broadening of the spectrum at higher energies. In fact, the 3 GeV curve extends over all accessible mass numbers, varying

¹The *fermi* (1 fm = 10^{-13} cm.) is a unit used frequently for nuclear and elementary particle phenomena.

by no greater than an order of magnitude. Evidently, the incident proton has sufficient energy to literally blow the target nucleus apart. The 480 MeV curve exhibits two prominent peaks. The first peak, closest to the target, corresponds to the spallation products whereas the next peak, centered at roughly half the atomic mass of the target (lead and bismuth), results from *high energy fission*. However, the fission mechanism becomes important only for rather high mass targets. Also, notice the formation, in high yield, of very light products during high energy interactions (3 GeV curve). This effect is referred to as *fragmentation* and again is principally associated with relatively heavy targets. Since the targets of interest to this work do not contain significant amounts of fissionable heavy elements and because the energy of the primary radiation is in the GeV range, the principal reaction mechanism of concern to the forward problem is the spallation process.

Figure 2.2: Mass yield curves are displayed for proton irradiation of heavy target nuclides (lead isotopes and ^{209}Bi) at three different energy regimes. (From Miller and Hudis, 1959).

The construction of mass yield curves is naturally based upon knowledge of product yields. The most general quantitative concept describing the probability of a nuclear reaction is that of the *cross section* σ . The probability for a reaction between an impinging particle and a target nucleus can be classically thought of as being proportional to the cross sectional area of the latter and hence is expressed in units of area.¹

For a beam of strongly interacting particles incident upon a "*thin target*" the cross section for a given process, or *partial cross section*, can be defined from the equation

$$R_i = N_t I \sigma_i \quad (2.2)$$

where R_i is the number of processes in question occurring per unit mass of target per unit time, N_t is the number of target nuclei per unit mass, I is the number of incident particles per unit area and unit time, and σ_i is the cross section for the particular process (unit area). The term *thin target* formally applies to a body which attenuates the beam only infinitesimally.² On the other hand, one can employ "*thick targets*" for so-called beam attenuation experiments in which the goal is to determine the total cross section σ_t from the removal of the incident particles from the beam (*i.e.*, attenuation). Such experiments cannot be used to determine partial cross sections that are only a part of the entire interaction.

Thin target experiments provide insight into the actual interaction between incident and target species. The results are often referred to as *thin target partial cross sections*. However, several complications are introduced when considering thick target interactions.

¹Note that this description fails for charged particle reactions that must overcome the Coulomb barrier. Nonetheless, the concept of cross section is a useful measure of expressing reaction probabilities.

²Nevertheless, the definition is often massaged to suit the particular information sought.

Consider a single particle incident upon a target of density ρ . Clearly, the probability of interaction is directly related to the number of target nuclides in the line of sight which in turn can be related to the target density. For example, a sub-atomic particle passing through 1 centimeter of water "sees" more entities with which it can interact than it would if it were traversing the same distance through air at STP. But if the air path is increased to 8 meters then the total quantity of absorber would be the same as for 1 centimeter of water since the density ratio of water to air at STP is 800. Evidently, one needs to specify both the *density* and the *path length*. Thus, the product of these two quantities yields a valid measure of the total mass of absorber traversed, and is referred to as the *surface density* having the dimensions in *cgs* units of $\text{g}\cdot\text{cm}^{-2}$.

Another important concept involves the product of the number of absorbing centers per unit volume and the effective area (or geometric cross section) of each. Clearly, this is the number of centers encountered by a single incident particle in traversing a unit distance through the target. Its reciprocal is defined as the *interaction mean free path* " l_i ". Moreover, if incident particles still continue onward after a collision with sufficient kinetic energy to strongly interact with more target nuclides then the concept of *absorption mean free path*, or *absorption length* " l_a ", comes into play. Plainly, $l_a > l_i$ by definition.

As more of the target is encountered more collisions will occur and consequently the relatively low energy flux of high energy interaction debris will increase. Hence, one now must consider a primary and a secondary flux of strongly interacting particles, both of which are depth dependent within the target (see Figure 3.2). It is sometimes convenient to define the empirical concept of *thick target cross section* which take all of these phenomena into consideration. Otherwise, a thorough quantitative understanding of the radiation throughout the thick target is required when applying thin target yields to estimate the production rate of given nuclides.

It would be ideal to possess the partial cross section for every product nuclide resulting for each combination of target isotope, incident particle, and energy of concern to the forward problem being developed in this work. Unfortunately, present experimental results are largely inadequate. Many product species of interest to the study of cosmogenic nuclide production are exceedingly difficult to directly measure (*i.e.*, ^{10}Be) and further, the energies studied are determined for the most part by the limitations of current particle accelerators. More data exists on the partial cross sections of short lived radionuclides since long lived species, which are of key concern to this work, require long and intense bombardments. Absolute abundances of these species are commonly determined by chemical separation techniques or by coincidence counting methods. The lack of precision of the data involved here cannot be overstated. Chemical separation and mass spectrometric analyses are used to determine absolute and relative abundances respectively of stable products. Data on inert isotope production is relatively good. However, the multitude of undetermined partial cross sections must be estimated in part by extrapolation into unknown energy regions which elicits an understanding of a further concept.

The relation between the variation of a given reaction cross section and the kinetic energy of the interaction is called an *excitation function*. It has been experimentally found that compound nuclear excitation functions (*i.e.*, for low energy interactions) always drop off at higher energies (see Figure 2.3). This is because there is less chance to concentrate more energy on a single particle and hence the probability of the reaction decreases. However, excitation functions for products formed only in high energy interactions are quite different. In particular, high energy excitation functions rise very slowly above the threshold, with an increase of many hundreds of MeV of bombarding energy before the maximum is reached (see Figure 2.4). Compare this with the abrupt rise of the low energy excitation functions.

Another significant observation of high energy excitation functions is that, after typically levelling off above 1 GeV, they are either virtually constant or exhibit a slight decrease with

Figure 2.3: Excitations functions for low energy (compound nuclear) reactions involving proton irradiations of ^{63}Cu . From Meadows (1953).

Figure 2.4: Excitation functions for the production of ^{24}Na , ^{41}Ar , and ^{52}Mn based on data from high energy proton irradiation of copper. From Hudis, 1968.

increasing energy.

It is also important to note that excitation functions for neighboring products from neighboring targets are nearly identical in shape. This observation has been crucial to previous analyses concerned with exposure age determinations based upon cosmogenic nuclide data (see Appendix B).

Hence, unknown partial cross sections of a given species require one to extrapolate excitation functions beyond the measurements made at attainable energies. In addition, particularly well determined excitation functions can be used to estimate those of poorly sampled isotopes by analogies and systematics, and by pertinent calculations based upon nuclear theory. The results of such are known as *semi-empirical excitation functions*.

An analytic expression which correlates the yield data from several light targets bombarded by protons of energy up to 25 GeV was presented by Rudstam *et al.* (1962), Rudstam (1966), and by Rudstam and Sorensen (1965). Here, the basic assumptions are that

- i) spallation yields decrease exponentially with decreasing A,
- ii) the relative isobaric yields have a Gaussian distribution centered about the most likely charge and is a smoothly varying functions of A, and
- iii) the production cross section becomes constant at higher energies.

Unfortunately, this relation is inapplicable for products which are forty atomic mass units below that of the target (*i.e.*, $A_t - A_p \equiv \Delta A > 40$).

An improved version of this was introduced by Silberberg and Tsao (1973) in an extension of Rudstam's work. They presented a working relation which extended the calculation of thin target partial cross sections into new regimes which are useful to the study of cosmogenic nuclides (*i.e.*, ^{10}Be). In addition, these estimates are more reliable than previous ones. The relation is applicable for calculating cross sections for targets in the mass range

$9 \leq A_t \leq 209$ and products in the range $6 \leq A_p \leq 200$ (except for very large or very small values of ΔA) and has the form

$$\sigma = \sigma_0 e^{-f[\Delta A]} e^{-g[A_p]} \Omega \eta \quad (2.3)$$

where σ_0 is the normalizing factor (in millibarns) and the remaining parameters are chosen according to the given regimes of interest. In particular, the first exponential term describes the decrease of cross section as ΔA increases, the second exponential term describes the distribution of cross sections for the production of various isotopes of an element of atomic number Z , and the parameters Ω and η relate to the nuclear structure and the nucleon pairing respectively. The interested reader is requested to refer to Silberberg and Tsao (1973) for explicit detail concerning this relation as a complete treatment would cover the next few pages of this section.

Relative standard deviations could be estimated where actual measured yields were available. They reported values between 20 and 30 percent for $6 \leq Z_t \leq 20$ at energies above 150 MeV and as high as 40 percent for $21 \leq Z_t \leq 26$ above 2 GeV (note that the assumption of constant cross section above threshold is retained here). These estimates are claimed to be more accurate than those based upon both Monte Carlo and previous semi-empirical methods.

Hence, one now has an analytic expression describing thin target partial cross sections. We will see that (2.3) satisfies the yield requirement of the forward problem.

Evidently, we still require a sufficient quantitative understanding concerning the evolution of particle fluxes when considering thick targets. Naturally, a review of the target, those aspects of which are of immediate concern to the forward problem, must first be addressed. We will soon see that when interest is focused on the long term irradiation prehistory then iron meteoroids become the only suitable target.

2.3 Meteoritic Data and the Fatal Gap:

Of the various classes of targets currently available for the study of cosmogenic nuclides only meteorites are of concern to this work.¹ Furthermore, meteorites fall under four broad classifications; namely,

- i) chondrites,
- ii) achondrites,
- iii) stoney-irons, and
- iv) irons.

In addition, both chondrites and achondrites are usually referred to as stones. It is thought that these distinctions have a genetic significance since it is probable that members of a single class share similar histories. Thus, it is important to establish a set of criteria, in light of the goals of this work, from which to choose from one meteorite class or another.

Surprisingly, one important such criterion is based upon meteoroid chronologies inferred from the analysis of cosmogenic nuclides measured in meteorites. As noted in the first chapter the current consensus on interpretations based upon cosmogenic nuclide abundances hold that the galactic cosmic ray flux has been constant to within experimental precision.² Thus, one can easily establish apparent exposure ages of respective meteorite samples. An analytic treatment of such exposure age calculations is presented in Appendix B. The important results of this research are referred to in this work as *exposure age histograms* (Figure 2.5), the salient features of which are summarized as follows:

- i) Cosmic ray exposure ages are very much shorter than the accepted age of the solar system (*i.e.*, ages inferred from the Rb-Sr and Pb-U methods).

¹A discussion of targets for the analysis of cosmogenic nuclides is given in Appendix C.

²Recall that this conclusion is justified, for the past 10^6 years, in Appendix A.

Figure 2.5: *Exposure age histograms* of nonordinary chondrites (a) and of iron meteorites (b) based upon constant irradiation conditions. The stone exposure ages are based upon ³He content (Zähringer, 1968) whereas the exposure ages of irons are deduced from ²¹Ne content (Wänke, 1966) and from the ratio ⁴⁰K/⁴¹K (Voshage, 1967).

- ii) The mean exposure age deduced from iron meteorites (≈ 0.7 Gyr) is an order of magnitude larger than that of stoney meteorites.
- iii) All meteorite classes and sub-classes exhibit different exposure age distributions.
- iv) ^{41}K - ^{40}K exposure ages are systematically about 50 percent higher than those based upon the ^{36}Ar - ^{36}Cl , ^{36}Ar - ^{10}Be , and ^{39}Ar - ^{39}Ar methods (Voshage and Hintenberger, 1963; Schaeffer *et al.*, 1981).

The latter observation is unequivocal evidence that the respective production rates have not been constant.

Researchers generally agree upon a multi-collisional model involving more massive *parent bodies* as the principal account of the various peaked exposure age distributions. In this interpretation the largest peaks signify catastrophic events.

Another process that can be used to interpret the exposure age distributions is *space erosion*. The slow removal of meteoroid surfaces is executed by the action of ion sputtering with the solar wind and by microcratering via cosmic dust impact at planetary velocities. Consequently, a sample from an arbitrary depth below the surface of a given meteorite actually resided on the average at a greater depth. This effect conspires to significantly lower the calculated exposure ages (Whipple and Fireman, 1959; Fireman and DeFelice, 1960; Fisher, 1966).

Schaeffer *et al.* (1981) calculated the rate of space erosion on stone and iron meteoroids based upon observations of cosmic dust, meteoroid orbitals, and the cratering record throughout the solar system and upon the results of microcratering experiments. They arrived at erosion rates of $65 \text{ cm}\cdot\text{Gyr}^{-1}$ and $2.2 \text{ cm}\cdot\text{Gyr}^{-1}$ for stoney and iron meteoroids respectively. Evidently, space erosion can be neglected in the study of iron meteorites based upon these conclusions.

It is noteworthy that the more competent meteorites are the oldest ones. Plainly, this is also reflected in the conclusions drawn on space erosion rates above.

As interest lies with the reinterpretation of the irradiation prehistory, as deciphered from meteoritic data, the exposure ages presented heretofore (*i.e.*, Figure 2.5) cannot be considered for integration into the forward problem developed in this work. However, two important *a priori* considerations can be retained. Firstly, these results enable one to know the relative order of the exposure ages corresponding to a set of meteorite samples. More important however is the realization that meteorite exposure ages group into well defined clusters. This will have a significant bearing on material of the forthcoming chapter dealing with resolution studies.

It is clear from the preceeding discussions that iron meteorites are best suited to provide information on the long term irradiation prehistory. However, there are several other equally strong arguments one can put forth to further bolster this position.

Foremost is the fact that iron meteorites possess the simplest target chemistry of all natural thick targets. They are essentially iron-nickel alloys and only contain minute amounts of cobalt, phosphorus, carbon, and sulphur. Moreover, the excitation functions of all pertinent iron and nickel isotopes are nearly identical (see Figure 2.6) so that it suffices to model the bulk composition of most iron meteorites simply by the most abundant such nuclide; namely ^{56}Fe .¹ This common assumption of simple target chemistry is adopted in this work.

In addition, iron and nickel isotopes are the heaviest common nuclides present in all meteorite classes and consequently are best suited as target nuclides. This is because the range of accessible product nuclides is maximized and because more high energy products (*i.e.*, large

¹The discrepancy between assuming an ^{56}Fe model to a more accurate compositional model amounts to less than 5% for all samples studied in this work. This is well within the precision associated with experimentally determined yields.

Figure 2.6: Excitation functions for the production of ${}^{40}\text{K}$ from the four most abundant isotopes found in the iron meteorite Yardymly (Aroos). M refers to "minestrone" (section 2.2).

ΔA) are available (recall the discussions on low energy filtering in section 2.1).

Furthermore, there is an additional point dealing again with the competent nature of iron meteorites. A meteorite has undergone adverse environmental conditions before reaching the laboratory (*i.e.*, atmospheric ablation and impact). One would expect the destructive effects of such to be less evident for the iron class. Consequently, one has a better idea of the original target geometry during cosmic ray bombardment (see section 3.2).

A final point favouring irons is that rare gas losses are least expected from this meteorite class (Imamura *et al.*, 1975). This is an important consideration since their interpretation is crucial to the long term irradiation problem. Stable isotopes act as integrators of the flux and consequently are capable of containing information on prehistoric irradiation conditions spanning enormous time scales. This is in contrast to radionuclides, which are a differential effect of cosmic ray interactions, whose temporal limits on possible interpretations are on the order of their respective mean lives.

Having said this, it is appropriate to present a survey of cosmogenic species produced in iron meteoroids which are useful to the development of the forward problem.

An account of all radionuclides with half-lives greater than one year found in meteorites is given in Figure (2.7). Evidently, ^{40}K is the only radioisotope that lends itself to the long term irradiation problem. The analysis of ^{10}Be , ^{26}Al , and ^{36}Cl can only span a few million years within current experimental precision. Hence, the gap in half-life between ^{40}K and ^{10}Be provides a significant natural barrier to the resolution possible on the long term irradiation prehistory. This will be fully qualified in chapter 4. We aptly refer to this feature as the "*fatal gap*"¹ in the cosmogenic nuclide problem.

¹Adopted in part from comments made by Masatake Honda in a report on contributions to "Session OG 12 A and B" held in Tokyo, Japan in 1979.

Figure 2.7: Cosmic ray produced radionuclides with half-lives greater than one year that can be observed in iron meteorites. Note the three order-of-magnitude gap between ^{53}Mn and ^{40}K . This is referred to as the "fatal gap" in this work. Perhaps, useful observations of ^{129}I (in triolite inclusions, say) may reduce this gap by an order-of-magnitude.

We will restrict the discussion of stable nuclides to the inert gases as they have been most heavily scrutinized upon both in meteorite research and in the determination of spallation yields. In turn, only inert isotopes of low "natural" abundance are considered in this work. We will in fact rely specifically on ^{21}Ne when attention focuses on modelling the long term irradiation prehistory.

Finally, let us conclude this discussion on current theories regarding the source(s) of meteorites. We have already noted that knowledge of meteoroid orbits can be of concern to the cosmogenic nuclide problem. Theoretical models are based upon both physical and chemical arguments put forth regarding the condensation of the protostellar accretion disk. In addition, dynamical studies have been used to discount the hypothesis of the planetary origins of meteoroids (*i.e.*, from highly energetic explosions). However, few direct observations are available for corroboration.

The orbits of three chondrites have been reliably established based upon data obtained from camera networks. They are found to be roughly between 0.7 AU and 5 AU (see Figure 2.8). Interestingly, both short term comets and the Apollo and Amor asteroids possess similar orbits.

The current consensus among researchers of meteoritics hold the most likely origins of meteoroids to be short-period comets and/or the main-belt asteroids. Moreover, Monte Carlo simulations of 34 asteroids in Mars-crossing orbits suggest that they could be the dominant source of iron meteoroids encountering the earth (Anders, 1964).

Observations of the radial modulation of galactic cosmic rays via the interplanetary magnetic field have been made by spacecraft. In particular, results from Pioneer 10 (Figure 2.9) indicate that this radial modulation is of little consequence to the cosmogenic nuclide problem as developed in this work if one accepts the inferred orbits of iron meteoroids given above.

Figure 2.8: Inferred orbits of the three chondrites Pribram (P), Lost City (LC), and Innisfree (I) based upon data from camera networks. In addition, the orbit of an Apollo asteroid and the main-belt asteroids (dotted) are shown. From Sears, 1978.

Figure 2.9: The ratio of galactic cosmic radiation measured at increasing distances from the sun by Pioneer 10 to that measured by IMP 5 (near the earth). From McKibben *et al.*, 1973.

In summary, iron meteoroids are the only target suitable to the long term irradiation problem. Moreover, the principal assumptions made are that

- i) space erosion is negligible,
- ii) the exposure ages are unknown,
- iii) the target chemistry is simply ^{56}Fe , and
- iv) the radial modulation of galactic cosmic radiation in the solar system can be ignored for the orbits of iron meteoroids.

Hence, we have presented an adequate survey of the essentials concerning the forward problem. We must now translate this into a quantitative understanding of the cosmogenic nuclide budget.

3

ANALYTIC STATEMENTS OF THE FORWARD PROBLEM

One is now prepared for a quantitative description of the cosmogenic nuclide budget within a target meteoroid orbiting through interplanetary space. For generality, the derivations will pertain to radioactive nuclides since stable isotopes constitute the special case where the decay constants vanish in the equations.

Consider the production of radioactive nuclides within a target exposed to an arbitrary high energy flux of strongly interacting particles. Let $n_i(t)$ be the number of the i^{th} species present at time t per unit mass of target. Clearly, the first order rate equation is given as

$$\frac{d}{dt} [n_i(t)] = P_i(t) - \lambda_i n_i(t) \quad (3.1)$$

where $P_i(t)$ is the i^{th} production rate per unit mass of target at time t and λ_i is the associated decay constant.

The production rate is highly uncertain since it depends upon many complex and poorly understood relationships. In theory, one must account for different incident particle species, including secondaries, and possess an accurate description of the isotopic composition of the target. Possible time dependence of both the particle flux and energy spectrum must also be considered. In addition, excitation functions and the depth dependence of both particle flux and energy spectrum within thick targets are based upon empirical methods which in turn are largely founded on inadequate data. Finally, assumptions must be made on both space erosion and meteoroid geometry.

Understandably, the thick target phenomena significantly contribute to the complexity of this problem. We shall thus begin with an analysis of thin targets.

3.1 Thin Target Production Rates:

The simplest form of the production term is obtained when the target is isotopically pure and when the incident flux is time-invariant, mono-energetic, and consists of a single strongly interacting particle species. Clearly, the thin target assumption must be invoked in order to satisfy the latter point and to neglect depth dependence. In this case, the production term is merely expressed as

$$P_i = N_t \Phi \sigma_i \quad (3.2)$$

where N_t is the number of target atoms per kilogram, Φ is the omnidirectional flux (incident particles·m⁻²·sec⁻¹) and σ_i is the partial cross section of the i^{th} species at the energy in question (see Section 2.2).

The goal of this paper is to extract information on the cosmic ray flux prehistory. Therefore, one must allow for a time dependent particle flux, $\Phi(t)$.

Now, consider an energy spectrum $S(E)$ over the incident particle flux. If this distribution is also time-variable then the production term becomes

$$P_i(t) = N_t \Phi(t) \bar{\sigma}_i(t) \quad (3.3)$$

where $\bar{\sigma}_i(t)$ is the average partial cross section of the i^{th} species, over the entire energy range at time t , weighted according to the energy spectrum. That is,

$$\bar{\sigma}_i(t) = \frac{\int_0^{\infty} \sigma_i(E) S(E,t) dE}{\int_0^{\infty} S(E,t) dE} \quad (3.4)$$

Further, if the incident flux is composed of K particle species and if the target is impure, composed of M target isotopes, then one defines σ_{ikm} as the partial cross section of the i^{th} nuclide associated with the inelastic interaction between the k^{th} incident species, with respective time dependent energy spectrum $S_k(E,t)$, and the m^{th} target isotope. Then the *mean-target average partial cross section* is defined as

$$\Sigma_{ik}(t) = \sum_{m=1}^M W_m \bar{\sigma}_{ikm}(t) = \frac{\sum_{m=1}^M W_m \int_0^{\infty} \sigma_{ikm}(E) S_k(E,t) dE}{\int_0^{\infty} S_k(E,t) dE} \quad (3.5)$$

where W_m is the fraction of the m^{th} target isotope averaged over the analysis region of the sample; that is, defining N_m as the number of isotopes "m" per kilogram of target then

$$\frac{N_m}{N_t} = W_m \quad (3.6)$$

Hence, an expression for the production rate of the i^{th} cosmogenic nuclide in a general thin target exposed to an omnidirectional particle flux is given as

$$P_i(t) = N_t \sum_{k=1}^K \Phi_k(t) \Sigma_{ik}(t) \quad (3.7)$$

where $\Phi_k(t)$ is flux of the k^{th} strongly interacting incident species.

3.2 Thick Target Considerations:

Recall the interaction mean free path l_i from section (2.2). For a high energy proton in iron,

$$l_i = \frac{1}{\rho_{\text{Fe}} \sigma_{p, \text{Fe}}} \approx 10 \text{ cm.}$$

Consequently, a hand specimen of iron meteorite must exhibit the inter-nuclear cascade phenomena on a macroscopic scale.

In a discussion of thick target experiments, it is appropriate to consider the production rate as depth dependent since one is dealing with a slab of material exposed to a collimated beam of particles. However, if the incident particle flux is omnidirectional then the concept of depth dependence is applicable only to smooth spherically symmetrical targets (this includes the semi-infinite slab as a limiting case). Indeed, if the target is arbitrary in geometry then, if its shape is well known, it is fitting to regard the production rate as positionally dependent within the body since the meaning of depth is ambiguous here. If the geometry is poorly known then it is convenient to consider surfaces of constant shielding conditions within the target. Clearly, a given shielding condition in such a body can be related to similar shielding effects at an appropriate depth in a slab exposed to a particle beam (as above). This gives rise to the concept of *effective depth*, henceforth denoted by "d".

More specifically, it is both the energy spectrum and primary particle flux which vary within a body. In addition, the cascade particle contribution to the flux is also positionally dependent. Hence, a general expression for the production rate within an arbitrary target of known geometry is

$$P_i(t, r) = N_i \sum_{k=1}^{K+K'(r)} \Phi_k(t, r) \Sigma_{ik}(t, r) \quad (3.8)$$

where r is the vector relating a given position to the origin of some convenient coordinate system and $K'(r)$ is the number of strongly interacting secondary species, not present in the primary radiation, as a function of position.

In practice, even the basic geometry of an iron meteoroid may not be well known. The present sample delivery system is violent and does not guarantee recovery of the complete specimen. It is possible to estimate the original surface profile before atmospheric ablation by mapping concentration contours of a suitable cosmogenic species (see Figure 3.1).

Figure 3.1: Contours of ^3He in a center cross section from the Grant iron meteorite indicate surfaces of constant shielding. In this case the contours indicate that the present shape is similar to the original one. The concentration map is based on mass spectrometric measurements (crosses) in bars cut from the slab. The values indicate content in millionths of a cubic centimeter per gram. (From Reynolds, 1960).

This is equivalent to mapping surfaces of constant shielding conditions as mentioned earlier. However, the sample must be largely complete and, moreover, such descriptions are nearly absent in the literature.

Spherical geometry is widely adopted as a first order approximation for iron meteoroids. For example, Kohman and Bender (1967) calculated the specific production rate of several nuclides within iron spheres of various radii (see Figure 3.2). They concluded that a comparison of their results with most observations from medium-sized and larger meteorites yields good order-of-magnitude agreement. They also admit, in a comprehensive review of their suppositions, that the spherical assumption "*is probably the weakest*" since the majority of recovered samples are quite irregular.

Hence, it is concluded for this work that current geometrical schemes are inadequate for assigning accurate estimates of shielding effects throughout a given meteorite. One must seek a more indirect method of addressing this problem.

Recall that the partial cross sections of various nuclides in a given target differ in their energy dependence. Hence, a comparison of certain nuclear abundances should provide a sensitive indicator of the effective irradiation hardness¹ which, in turn, can be related to the position within the target.

The most successful analyses are based upon inert isotope data alone (Signer and Nier, 1960; Voshage and Feldmann, 1978). The depth dependence of particularly useful noble gas species produced from iron spallation is displayed in Figure (3.3). In particular, note the abrupt initial increase in ⁴He production. Clearly, this nuclide is made in appreciable yield by the lower energy secondary particles. On the other hand, ²¹Ne exhibits a sharp monotonic

¹Irradiation hardness relates to the penetrability of ionizing radiation and is customarily reserved for short wavelength X-rays. In cosmic ray physics the effective irradiation hardness relates to the degree of attenuation in the *hard* primary component coupled with the development of secondary particles and the associated *soft* component.

Figure 3.2: Calculated specific production rates of a) ^{10}Be , b) ^{26}Al , and c) ^{36}Cl as a function of depth within iron spheres exposed to the primary cosmic ray flux. Note the initial increase in the first few centimeters. This is where the secondary flux is maximum. (From Kohman and Bender, 1967).

decrease indicating that it is a high energy product. Hence, the concentration ratio of two such nuclides can serve as an *effective irradiation hardness indicator*.

TABLE 3.1: Effective Irradiation Hardness Indicators.

	EFFECTIVE IRRADIATION HARDNESS†	
Ratio	Very Hard Radiation (shallow targets)	Very Soft Radiation (deep targets)
${}^3\text{He}/{}^4\text{He}$	0.315	0.235
${}^4\text{He}/{}^{21}\text{Ne}$	230	460
${}^{38}\text{Ar}/{}^{21}\text{Ne}$	4.05	6.4

†Voshage (1968).

Figure 3.3: Predicted radial variations of $[{}^{21}\text{Ne}]$, $[{}^{38}\text{Ar}]$, $[{}^3\text{He}]$, and $[{}^4\text{He}]$ in a spherical iron target exposed to the cosmic ray flux. The ordinate is defined so that an infinitesimally small target would yield unit concentration. (From Signer and Nier, 1960). Concentrations are in kilograms.

Three such isotopic ratios are given, for both hard and soft radiation, in Table (3.1). Evidently, the $^4\text{He}/^{21}\text{Ne}$ ratio is most sensitive to irradiation conditions. This ratio is an integral part of the current ^{40}K dating technique for iron meteorites as outlined in Appendix B (Voshage, 1967; 1978). In addition, it is important to note that almost all helium and neon in iron meteoroids is produced by cosmic ray interactions (Buchwald, 1975).

Finally, let us examine the factors in equation (3.8). From the previous discussion the general positional dependence should be revised to describe the effective depth dependence so that the factors now read as $\Phi_k(t,d)$ and $\Sigma_{ik}(t,d)$. Recall from section 2.2 that there are two main approaches in modelling production rates within thick targets. In one method, the evolution of the flux and energy spectra are empirically tied into thick target cross sections which are derived from accelerator experiments. When considering cosmic ray primaries natural conditions have to be simulated closely in order to obtain production rates directly (Honda, 1962; Shedlovsky and Rayudu, 1964; Van Ginneken and Turkevich, 1970). Nuclear systematics and multiple-group diffusion theory are used as more indirect techniques of estimation (Kohman and Bender, 1967; Trivedi and Goel, 1969). Note that the estimates in Figure (3.2) are based upon the latter analysis.

The alternative approach is more allied to the physics of the problem and shall be adopted in this work. In this method, excitation functions (*i.e.*, thin target cross sections) are employed in conjunction with empirical relations describing the evolution of particle fluxes and spectra within thick targets. The spectral relations were determined by comparing the star size distribution (*i.e.*, nuclear disintegrations) in nuclear emulsions flown at various altitudes in helium balloon experiments (Birnbaum *et al.*, 1952; Powell *et al.*, 1959) and are applicable from 100 to 3000 MeV.

For greater energies the spectrum is essentially the same as the primary energy spectrum. In this work a low energy cut-off E_l is forced at 100 MeV since the adopted excitation function (equation 2.3) is inapplicable for lower energies. The consequences will be

discussed in the next section. The interested reader should refer to Reedy and Arnold (1972) for an extension of the spectrum to lower energies.

Arnold *et al.* (1961) claimed that the nuclear emulsion results were transferable to iron targets. They calculated the flux and energy spectrum at various depths in iron. Their results are displayed in Figure (3.4). Secondary particles in this energy range are mostly intra-nuclear cascade nucleons together with copious amounts of charged pions. The neutron is the dominant strongly interacting particle below 500 MeV since most charged particles at lower energies are strongly impaired by ionization energy losses before interacting with target nuclei.

Figure 3.4: The differential energy spectrum at 100 g·cm^{-2} in an iron meteoroid as calculated by Arnold *et al.* (1961) from data provided by Powell *et al.* (1959). The contributions of protons, neutrons, and charged pions are shown separately along with the total. (From Arnold *et al.*, 1961).

Empirical relations for the differential energy spectrum of all strongly interacting particles were devised for 10 and 100 g·cm⁻² depths (see Table 3.2). Note that no significant variation in the energy spectrum is expected beyond the latter depth albeit the total flux diminishes rapidly.

TABLE 3.2: Energy Spectrum at Two Different Depths in Iron Resulting from Incident Cosmic Radiation.†

ENERGY RANGE (GeV)	SPECTRUM‡	
	10 g · cm ⁻² depth	100 g · cm ⁻² depth
0.1 - 3.0	$7.2(0.4+E)^{-2.5}$	$3.3(0.2+E)^{-2.5}$
>3.0	$11(1+E)^{-2.5}$	$5.8(1+E)^{-2.5}$

†Arnold et al. (1961).

‡Units are particles · cm⁻² · sec⁻¹ · GeV⁻¹.

An extension of this model to other depths was presented by Reedy and Arnold (1972). They gave the general form of the differential spectrum at some effective depth d as

$$S(E,d)dE = S_0(d)[a(d)+E]^{-2.5}dE \quad (3.9)$$

where a determines the shape of the distribution, and S_0 normalizes the expression (as in Table 3.2). In their model, a decreases with depth as the ratio of soft to hard radiation

increases. The normalizing constant is evaluated from the integral flux of particles ($E > 1$ GeV/nucleon) at each depth. The estimated effective depth dependence in these parameters is shown in Figure (3.5).

Figure 3.5: The parameters for the energy spectrum as a function of depth (equation 3.9). The top curve represents the shape parameter $a(d)$, whereas the bottom curve is the normalizing constant $S_0(d)$. (From Reedy and Arnold, 1972).

As a final note, the spectral index must be modified in light of more recent findings regarding the primary energy spectrum (see Meyer, 1969). More explicitly, the exponent in equation (3.9) should be modified from 2.5 to 2.6 (see equation 2.1).

3.3 Problems and Assumptions:

Let us recapitulate as follows. The production rate of the i^{th} cosmogenic nuclide at time t and effective depth d in a thick target, of number density N_t , composed of M different target isotopes, bombarded by $K+K'(d)$ strongly interacting particle species with respective energy spectra $S_k(E,t,d)$ is written as

$$P_i(t,d) = N_t \sum_{k=1}^{K+K'(d)} \left[\frac{\Phi_k(t,d) \sum_{m=1}^M W_m \int_{E_L}^{\infty} \sigma_{ikm}(E) S_k(E,t,d) dE}{\int_{E_L}^{\infty} S_k(E,t,d) dE} \right] \quad (3.10)$$

The immediate problems are that

- i) only semi-empirical excitation functions for high energy proton-nucleus reactions are available,
- ii) the cosmic-ray-produced spectra of the individual secondary species are not available,
- iii) the limits of integration are not rigorously established, and
- iv) the time dependence of energy spectra has not been addressed.

The obvious consequence of i) and ii) is that all nuclear reactions are treated as high energy proton-nucleus interactions, henceforth referred to as the *proton-nucleus assumption*. This means that $K=1$ and $K'(d)=0$ in equation (3.10). Let us analyse the validity of this forced simplification before addressing the latter items.

To this writer's knowledge, all cosmic ray induced production rate problems ignore $Z > 2$ incident primaries since they constitute less than 2% of the total flux. This is essentially a statement of ignorance concerning these reactions. In fact, very little is known even about high energy alpha-nucleus reactions (Silberberg and Tsao, 1973). Those few authors

that attempt to include the effects of alpha-nucleus reactions (*i.e.*, Reedy and Arnold, 1972) are forced to use a phenomenological approach which has little to do with the actual nuclear interaction mechanisms involved.

The important differences between proton-nucleus and alpha-nucleus reactions were reviewed by Silberberg and Tsao (1973) and are summarized as follows:

- a) Reaction mechanisms which contribute to the high peak of (p,pn) reactions at low energies (see Figure 2.3) are inoperative for (α ,pn) reactions (Crandall *et al.*, 1956; Lindner and Osborne, 1953; Radin, 1970).
- b) The partial cross sections for lithium and beryllium from targets like carbon and nitrogen are enhanced for alpha-nucleus reactions as compared with proton-nucleus reactions (Fonts *et al.*, 1971; Jung *et al.*, 1969).
- c) If one considers the kinetic energy per nucleus rather than per nucleon then the energy dependence of the proton- and helium-induced partial cross sections appear similar (Crespo *et al.*, 1964; Davis *et al.*, 1963; Korteling, 1962; Korteling and Hyde, 1964) albeit the latter are larger by a factor of two. This is because the energy deposition from proton and helium ion bombardments is similar at the same kinetic energy per nucleus.
- d) The partial cross sections for $\Delta A \geq 3$ products in alpha-nucleus reactions at high energies are only 1.6X, rather than 2X larger than in proton-nucleus reactions (Radin, 1970).

The first two items are not pertinent to a discussion of iron spallation. However, c) strongly supports the proton-nucleus assumption, at least for slightly more massive primaries. If one accepts this then the latter point, d), should be considered in the analysis of uncertainties (section 5.2).

Similar conclusions can be drawn for pion-nucleus interactions (Orth *et al.*, 1978; Friedlander *et al.*, 1981) and neutron-nucleus interactions (Korteling, 1962; Davis *et al.*, 1963). Hence, the proton-nucleus assumption appears to be justifiable.

Let us now examine problem iii). High energy cosmogenic nuclide production is primarily sensitive to a narrow range of the spectrum. This inherent band-limitation is completely specified by the average cross section. The integrand in the numerator of equation

(3.5) is the product of two monotonic functions; an increasing excitation function and a decreasing spectral function. It is displayed in Figure (3.6) for several high energy products at a specific depth. ^{26}Al clearly exhibits the expected band limitation for high energy products above $E_l=100$ MeV. The lower limit need only be reduced a few tens of MeV in order to obtain similar distributions for the heavier nuclides. The picture is certainly more complicated for ^{10}Be , however, since nuclear evaporation supersedes spallation as the principal production mechanism for such light nuclei at lower energies. Nevertheless, the major contribution to the average cross section appears to be well accounted for above 100 MeV.

Clearly, one need not be too concerned with energies greater than 10 GeV, particularly in light of the uncertainties accrued thus far. Consequently, an upper limit to the energy E_u can be established. Moreover, only two or so decades of the energy spectrum are of concern to the cosmogenic nuclide budget.

Finally, let us examine the latter item iv). One could invoke various hypothetical arguments regarding the cause of temporal variations in the cosmic ray flux. Clearly, all such arguments are equally valid in a discussion concerning the possible time dependence of the primary energy spectrum. For it is difficult to imagine a process which can perturb the observed particle flux by a common factor at all energies, thus leaving the shape of the entire spectrum unaltered. Indeed, this is practically impossible for cosmic rays since

- a) the spectrum spans at least twenty-one decades of energy, and
- b) there are likely to be a host of diverse and uncorrelated sources involved.

However, the previous discussion on the inherent energy filtering of the problem weakens the former point. This will prove important after first discussing some pertinent observations.

Several independent approaches have been put forth to delineate the spectral prehistory of cosmic rays. For instance, Geiss *et al.* (1962) obtained a closed solution to their production rate equations which, they claimed, enabled one to deduce the effective energy spectrum over

Figure 3.6: The energy dependence in the production of several important high energy spallation products at $100 \text{ g}\cdot\text{cm}^{-2}$ in ^{56}Fe is displayed in the bottom figure. Each of these curves is the product of the energy spectrum at the respective depth (top figure) and corresponding excitation function (center figure). Note that the excitation functions for ^{26}Al , ^{36}Cl , and ^{40}K have respectively increasing cross sections for lower energies. This reflects the fact that larger ΔA products are preferentially produced at higher energies. However, this rule of thumb breaks down for lighter nuclides where nuclear evaporation becomes the dominant production channel from targets like iron. This is exemplified by ^{10}Be which is intermediate between ^{26}Al and ^{36}Cl . In addition, note that this analysis applied to ^{21}Ne , a nuclide crucial to delineating information on the long term irradiation prehistory, yields qualitatively similar results to that of ^{26}Al as expected. All curves in the lower figure are expected to decline rapidly below 100 MeV.

the irradiation history of a given target¹. This analysis conducted on the Grant iron meteorite suggests that the average energy spectrum above 1 GeV has been similar to that presently observed.

A fundamentally different approach is based on the analysis of fossil tracks due to cosmic rays in meteorites and lunar samples (Amin *et al.*, 1969; Bhandari *et al.*, 1971a). These features can be rendered visible by chemical etching only if the primary ionization of an incident charged particle exceeds a critical value (Fleischer *et al.*, 1967a, b; Lal 1969; Bhandari *et al.*, 1971b, c, d, e; Price *et al.*, 1968; Katz and Kobetich, 1968). This condition is satisfied when the incident particle possesses an appropriate combination of atomic number and velocity. Moreover, the minimum atomic number required for producing etchable tracks in silicates is $Z \geq 20$ (Lal, 1972). But iron nuclei are the most abundant species above this threshold. Thus, most etchable cosmic ray tracks in stone targets are produced when the kinetic energy of incident iron nuclei drops below a critical value². Clearly, the depth distribution of etchable tracks is a measure of the effective energy spectrum received by a target assuming that

- a) the cosmic ray iron flux has always been relatively high,
- b) the iron spectrum is representative of the proton spectrum, and
- c) space erosion can be accounted for.

The conclusion, as one may have surmised, is that the iron energy spectrum averaged over the past few million years is similar to the present spectrum.

¹Stauffer and Honda (1962) independently arrived at this result as an empirical correlation relation at about the same time. Refer here for additional details concerning sample analysis.

²The greatest recordable track length in pyroxenes and olivines is roughly 15μ corresponding to a critical kinetic energy of 2.5 MeV/n for iron (Lal, 1969).

Unfortunately, the conclusions put forth from the preceeding discussions provide a false sense of security. From the onset, the reader must have been wary of the potential for cyclic arguments arising from the intimate tie between flux and spectrum. In order to say something quantitative about the spectrum these authors must make suppositions regarding the flux¹. And, as is typical in cosmogenic research, it is assumed to be constant. This is clearly incompatible with the current work.

On an intuitive level, however, it appears reasonable that changes in the flux should be more dramatic than variations in the shape of the spectrum over the two decades of energy in question. Of course, our poor understanding of cosmic ray origin, acceleration mechanisms, attenuation enroute, etc., make this claim difficult to rigorously defend at present. Nonetheless, it is certainly no weaker an assertion than the purely arbitrary claim of constant flux used in long term spectral prehistory investigations. Hence, under this qualitative line of reasoning and without information to the contrary the *weak* assumption of spectral time invariance is adopted².

In summary, a tractable version of equation (3.10) has been obtained by considering

- i) only proton-nucleus reactions and
- ii) a time invariant energy spectrum.

Hence, equation (3.1) can now be used as the differential equation of the cosmogenic nuclide budget within an iron meteoroid of known isotopic composition.

Finally, let us extend this analysis to include the spallation product budget of several meteoroids simultaneously. This requires a common origin in time for all samples. Therefore,

¹Obviously, the opposite is being done in this work!

²This conclusion automatically assumes that space erosion is negligible for iron meteoroids (section 2.3) and that spectral variations due to orbital histories within the solar system can be ignored.

it is necessary to define a time scale with its origin fixed at the present¹ and positive backwards with respect to real time. In addition, let us restrict ourselves to some arbitrarily fixed effective depth d_0 . Thus, the revised and final form of equations (3.1) and (3.10) describing the budget of the i^{th} cosmogenic nuclide in the j^{th} meteorite is given as

$$\boxed{\frac{d}{dt} [j_{n_i}(t, d_0)] = \lambda_i j_{n_i}(t, d_0) - j_{N_i} \Phi(t, d_0) j_{\Sigma_i}(d_0)} \quad (3.11)$$

where equation (3.5) has been modified as

$$\boxed{j_{\Sigma_i}(d_0) = \frac{\sum_{m=1}^M j_{W_m} \int_{E_L}^{E_U} \sigma_{ik}(E) S(E, d_0) dE}{\int_{E_L}^{E_U} S(E, d_0) dE}} \quad (3.12)$$

¹This assumes that the terrestrial residence times of meteorites are negligible (*i.e.*, for the study of shorter lived isotopes).

3.4 Functional Description of Cosmogenic Nuclide Abundances:

Let us now engage in a formal discussion concerned with inversion. The linear inverse problem is defined as one in which a set of "N" observables e_i is connected to some unknown model m by a set of *linear functional¹ relationships* G_i . That is,

$$e_i = G_i(m) \quad i=1, \dots, N. \quad (3.13)$$

Furthermore, the linear problem is assumed to have a special form in which there exists a square integrable function $g_i(x)$ for each G_i such that

$$e_i = \int_I g_i(x) m(x) dx \quad i=1, \dots, N. \quad (3.14)$$

where I is a real interval.² The unknown model $m(x)$ in (3.14) is additionally square integrable and in principal should adequately represent the physical system under scrutiny.

Moreover, it is clear that the functions $g_i(x)$ contain all of the physics and assumptions therein, apart from the unknown function $m(x)$, and are henceforth referred to as *data kernels* (or kernel functions). Clearly, equation (3.14) is an analytic statement of the forward problem.

The goal of the inverse problem is centered upon the following question:

Given (3.14) what can be said about $m(x)$?

¹A functional maps a set of functions into the set of real (or complex) numbers.

²Note that observables in almost all physical problems are related to the model by an integral transform. In particular, (3.14) is a Fredholm equation of the first kind.

An important concept which greatly facilitates one's understanding of this question is rooted to the accordance between equations like (3.14) and inner product spaces. Moreover, the analysis of the inverse problem in the Hilbert space framework (*i.e.*, a complete inner product space) yields simple geometric analogues to most of our operations. Within this guise the inverse problem amounts to estimating an unknown vector m from only a few of its components.

Most of what follows is based on a series of papers presented by Backus and Gilbert (1967, 1968, 1970). They used this perspective to demonstrate what unique information can be obtained from the finite set of inaccurate data e_i and they showed how one can construct models which reproduce such observations and further, how to contend with the problem of stability therein.

Let us now reformulate this problem to comply with the inversion scheme just outlined. We seek a linear functional relationship between the measured isotopic abundances in a given set of meteorites and the cosmic ray flux prehistory. Hence, the immediate objective is to transform equation (3.11) into integral form.

The procedure begins by introducing an integrating factor $\Theta_i(t)$ to equation (3.11). Clearly, for the *single sample* case

$$\Theta_i(t) = e^{-\lambda t} \quad (3.15)$$

and hence

$$\frac{d}{dt} [n_i(t, d_0)] = \lambda_i n_i(t, d_0) - N_i \Phi(t, d_0) \Sigma_i(d_0)$$

$$\frac{d}{dt} [e^{-\lambda t} n_i(t, d_0)] = -e^{-\lambda t} N_i \Phi(t, d_0) \Sigma_i(d_0) .$$

Integration over the target exposure time T yields

$$n_i(0, d_0) = N_i \Sigma_i(d_0) \int_0^T e^{-\lambda t} \Phi(t, d_0) dt \quad (3.16)$$

where it has been assumed that

- i) the initial concentrations (at time T) of the given cosmogenic species are negligible, and
- ii) T is well known.

Plainly, the more general *multi-sample* case can take the convenient form

$$j_{n_i}(0, d_0) = j_{N_i} j_{\Sigma_i(d_0)} \int_0^\infty e^{-\lambda t} H(T_j - t) \Phi(t, d_0) dt \quad (3.17)$$

where $H(T_j - t)$ is the Heaviside step function corresponding to the j^{th} sample.

Returning to the nomenclature set out in the beginning of this section one can readily identify (3.17) as the data functional (3.14) through which the data $j_{n_i}(0, d_0)$ are related to an unknown model $\Phi(t, d_0)$. Clearly, the kernel function

$$j_g(t, d_0) = j_{N_i} j_{\Sigma_i(d_0)} e^{-\lambda t} H(T_j - t) \quad (3.18)$$

embodies all the physics, apart from the flux function, and suppositions made thus far.

Equation (3.17) is of the precise form sought in the beginning of this section. However, there are two computational problems that must be addressed initially. First of all,

both sides of (3.17) are immense, when using any standard convention of units, thus abetting numerical instabilities. Secondly, it shall be seen that the success of inversion rests heavily upon the precision of the data kernels.

Assuming, for the moment, that the exposure ages are well known then the errors in (3.18) lie almost entirely in the first two factors j_{N_i} and $j_{\Sigma_i}(d_0)$. Normalizing (3.17) by these factors avoids the latter problem for all intents and purposes albeit the numbers are still unmanageably large.

However, the former difficulty can be adequately resolved by an appropriate choice of working units.

Henceforth, the form of the data functional and units employed for computational purposes will be as follows:

$$j_{e_i} = \int_0^{\infty} j_{g_i}(t) m(t) dt \quad (3.19)$$

where

$$j_{e_i} = \frac{j_{n_i}(0, d_0)}{j_{N_i} j_{\Sigma_i}(d_0)} \left[\frac{\text{product nuclides } kg^{-1}}{\text{target nuclides } kg^{-1} \cdot nm^2} \right],$$

$$j_{g_i}(t) = e^{-\lambda_i H(T_j - t)} \quad [Gyr], \text{ and}$$

$$m(t) = \Phi(l, d_0) \quad [\text{incident particles} \cdot Gyr^{-1} \cdot nm^{-2}].$$

Note that the temporal unit is the *gigayear*. Of course, all results will be presented in conventional units.

4

RESOLUTION ON THE GALACTIC COSMIC RAY PREHISTORY

What kind of structures can one hope to resolve on the prehistory of galactic cosmic radiation given observed abundances of cosmogenic nuclides? This question is intimately tied to the problem of nonuniqueness in inverse theory and was cleverly addressed by Backus and Gilbert (1968).

Let us refer back to the functional description of the general forward problem; namely, equation (3.14). Suppose one is interested in very localized information concerning the model, say at an arbitrary point $x=x_0$. In theory, if the kernel function g_i in equation (3.14) is the Dirac function $\delta(x-x_0)$ then one could recover the model $m(x)$ uniquely at x_0 given perfectly accurate data. Realistically however, the best one can achieve is to mimic the Dirac function as closely as possible with an appropriate combination of the data kernels. That is, we form a "pseudo-delta function" $\tilde{\delta}(x, x_0)$ by taking linear combinations of the data; *i.e.*,

$$\sum_i a_i(x_0) e_i = \int_I \tilde{\delta}(x, x_0) m(x) dx \quad (4.1)$$

where

$$\tilde{\delta}(x, x_0) = \sum_i a_i(x_0) g_i(x). \quad (4.2)$$

and where the coefficients $a_i(x_0)$ are determined from the methods of constrained optimization.

As such, $\tilde{\delta}(x, x_0)$ provides a blurred estimate of the model at x_0 . The degree of blurring is clearly related to the "closeness" of $\tilde{\delta}(x, x_0)$ to the Dirac function. Equation (4.1) provides the only unique information on the model obtainable from a finite set of inaccurate observations since it only depends upon both the coefficients and the data. Clearly, this

information is simply weighted averages of $m(x)$ and consequently $\tilde{\delta}(x, x_0)$ is often referred to as an *averaging function*.¹

Evidently, the width and general form of the averaging functions provide crucial information regarding the resolving power of the data. Hence, they are also referred to as *resolving kernels*. Unfortunately, the interpretation of averaging functions can be somewhat ambiguous for many problems. This dilemma has been effectively addressed by Oldenburg (1983). However, these problems will prove to be of little consequence to the long term irradiation problem in its present status.

4.1 Derivation of the Resolving Kernels:

Let us now derive the coefficients which satisfy (4.2). The method adopted here is frequently referred to as the *first Dirichlet criterion* (Oldenburg, 1976). When concerned with accurate data the goal is to minimize the function

$$R(x_0) \equiv \int_I |\tilde{\delta}(x, x_0) - \delta(x - x_0)|^2 dx \quad (4.3)$$

subject to the unimodular constraint

$$N(x_0) \equiv \int_I \tilde{\delta}(x, x_0) dx = 1 \quad (4.4)$$

the latter being introduced by Lagrange's method of undetermined multipliers. With this one will attain a set of coefficients which yield the maximum resolution theoretically possible from a given set of data.

¹Actually, averaging functions are defined as any unimodular combination of the data kernels.

However, all observations are inaccurate. Hence, the averages given by (4.1) must also be inaccurate. Consequently, it becomes necessary to introduce the variance of the average

$$\text{VAR}[\sum_i a_i(x_0)e_i]$$

into our evolving objective function. In addition, we will adopt the usual statistical assumption of uncorrelated Gaussian errors in order to make the solution tractable.

In addition, normalizing both the data and the kernel functions by the respective standard deviations s_i greatly assists in the derivation of resolving kernels. Henceforth in this section, this normalization is so assumed in the notation e_i and $g_i(x)$. Thus, both the data and the kernel functions are now dimensionless and possess unit variance.

Backus and Gilbert (1970) proved that the cost of improved resolution is diminished statistical reliability and vice versa. Therefore, one should introduce the term describing measurement accuracy A above into the objective function in accordance with this fact. In particular, the coefficients a_i must now be considered as a function of some parameter θ which governs the trade-off between R and A .¹ We write the objective function as

$$\psi(x_0, \theta) = \cos \theta R(x_0) + \sin \theta A(x_0) + 2LN(x_0) \quad (4.5)$$

where L is the Lagrange multiplier.

Minimizing with respect to each coefficient a_i yields

$$\frac{\partial \psi}{\partial a_i} = \cos \theta \frac{\partial R}{\partial a_i} + \sin \theta \frac{\partial}{\partial a_i} \left\{ \text{VAR} \left[\sum_i a_i e_i \right] \right\} - 2L \int_1 \frac{\partial}{\partial a_i} \tilde{\delta}(x, x_0) dx = 0$$

¹This leads to the well known "trade-off curves" of Backus and Gilbert (1970).

$$\Leftrightarrow \cos \theta \left[\sum_i \mathbf{a}_i \int_1 g_i(x) g_f(x) dx - g_i(x_0) \right] + \sin \theta \mathbf{a}_i - L \int_1 g_i(x) dx = 0.$$

Let us define the matrix Γ_{ij} to account for the elements generated by the integral in the first term of this equation.¹ In addition, let the vector U_i represent the elements generated by the integral of the last term. Thus, in the more convenient vector notation one can write

$$\cos \theta [\mathbf{a} \cdot \Gamma - \mathbf{g}_0] + \sin \theta \mathbf{a} - L\mathbf{U} = 0 \quad (4.6)$$

with the unimodular constraint

$$\mathbf{a} \cdot \mathbf{U} = 1. \quad (4.7)$$

Now, it becomes convenient to decompose the matrix Γ according to its singular values. Since Γ is symmetric and positive definite we can always write

$$\Gamma = \mathbf{R} \cdot \Lambda \cdot \mathbf{R}^T \quad (4.8)$$

where $\mathbf{R}$ is an orthogonal matrix² (i.e., $\mathbf{R}^{-1} = \mathbf{R}^T$) and where Λ is a diagonal matrix containing the respective singular values. Substituting (4.8) into (4.6) and multiplying by $\mathbf{R}$ yields

¹This will later be referred to as the *inner product matrix* when considering model construction.

²Sometimes referred to as the *rotation matrix*. See Strang (1980).

$$\cos\theta \left[\underbrace{\mathbf{a} \cdot \mathbf{R} \cdot \boldsymbol{\Lambda}}_{\hat{\mathbf{a}}} - \underbrace{\mathbf{g}_0 \cdot \mathbf{R}}_{\hat{\mathbf{g}}} \right] + \sin\theta \underbrace{\mathbf{a} \cdot \mathbf{R}}_{\hat{\mathbf{a}}} - \underbrace{L\mathbf{U} \cdot \mathbf{R}}_{L\hat{\mathbf{U}}} = 0$$

 $\Leftrightarrow$

$$\cos\theta [\hat{\mathbf{a}} \cdot \boldsymbol{\Lambda} - \hat{\mathbf{g}}_0] + \sin\theta \hat{\mathbf{a}} - L\hat{\mathbf{U}} = 0.$$

Defining the diagonal matrix

$$\mathbf{D} = \cos\theta \boldsymbol{\Lambda} + \sin\theta \mathbf{I},$$

where $\mathbf{I}$ is the identity matrix, then the *rotated coefficients* can be written as

$$\hat{\mathbf{a}} = [L\hat{\mathbf{U}} + \cos\theta \hat{\mathbf{g}}_0] \cdot \mathbf{D}^{-1}. \quad (4.9)$$

Clearly, the unimodular condition (4.7) is invariant under rotation (i.e., $\hat{\mathbf{a}} \cdot \hat{\mathbf{U}} = \mathbf{a} \cdot \mathbf{U} = 1$).

Thus, the Lagrange multiplier can be evaluated as

$$L = [1 - \cos\theta \hat{\mathbf{g}}_0 \cdot \mathbf{D}^{-1} \cdot \hat{\mathbf{U}}] / [\hat{\mathbf{U}} \cdot \mathbf{D}^{-1} \cdot \hat{\mathbf{U}}].$$

Multiplying (4.9) by $\mathbf{R}^T$ places the analysis back into the original unrotated coordinate system.

Hence, the coefficients sought are given as

$$\boxed{\mathbf{a}(x_0, \theta) = \hat{\mathbf{a}} \cdot \mathbf{R}^T = [L\hat{\mathbf{U}} + \cos\theta \hat{\mathbf{g}}_0] \cdot \mathbf{D}^{-1} \cdot \mathbf{R}^T.} \quad (4.10)$$

Finally, the resolving kernels for the cosmogenic nuclide problem, based on the budget of "I" nuclide species analysed in "J" meteorite samples, takes the form

$$\tilde{\delta}(t, t_0, \theta) = \sum_{j=1}^J \sum_{i=1}^I j a_{i,j}(t_0, \theta) j_{g_i}(t) \quad (4.11)$$

where $j a_{i,j}(t_0, \theta)$ and $j_{g_i}(t)$ are given by equations (4.10) and (3.19) respectively. The desired trade-off between accuracy and resolution is governed by θ . It is usual at this point to construct so called trade-off curves (Backus and Gilbert, 1970) so that a reasonable value of θ can be assigned. However, let us first proceed to discuss the theoretical resolution limit (i.e., perfectly accurate data) by setting $\theta = 0$. We will see that the nature of this problem does not warrant such an account of data precision.

4.2 Single Sample Analysis:

Let us consider the maximum resolution theoretically possible on the galactic cosmic ray prehistory from cosmogenic nuclide abundances observed in an individual meteorite sample of known exposure age. Only the radionuclides ^{40}K ($t_{1/2} = 1.25 \times 10^9$ years), ^{10}Be ($t_{1/2} = 1.6 \times 10^6$ years), ^{26}Al ($t_{1/2} = 7.12 \times 10^5$ years), and ^{36}Cl ($t_{1/2} = 3.1 \times 10^5$ years) are considered here since shorter lived radioisotopes produced in meteoroids (below ^{36}Cl) tend to be lower energy products (see Figure 2.7). Furthermore, we will see that adding more than one stable isotope into the analysis is redundant in the absence of uncertainties. In addition, we need not be concerned with the identity of the latter throughout this chapter.

The averaging functions based on these four radioisotopes located at three evenly spaced positions t_0 are displayed in Figure (4.1) corresponding to exposure ages of 10^6 , 10^7 , 10^8 , and 10^9 years respectively. The most obvious observation is the dramatic loss of

resolution between 10^7 and 10^8 years. Clearly, only ^{40}K has a response time greater than roughly 2×10^7 years. This is the source of distinction between the short term problem and the long term problem.

The bulk of the short term averaging functions are well centered over their respective positions albeit significant "side lobes" exist with some negative contributions. Nevertheless, they all indicate that the typical resolution is never better than a third of the exposure age intervals given.

On the other hand, the long term averaging functions are all essentially the ^{40}K data kernel as a consequence of the *fatal gap*. Thus, we are now dealing with a one data problem. Evidently, there is no hope of recovering any features of the cosmic ray model other than general monotonic trends using this datam alone.

Figure 4.1: Data kernels and corresponding averaging functions (resolving kernels) based on ^{40}K , ^{10}Be , ^{26}Al , and ^{36}Cl . They are generated at three evenly spaced locations t_0 over four different decades of exposure age (arbitrary height).

Further, the addition of a stable isotope (*i.e.*, constant kernel function) cannot ameliorate this situation.

A more edifying display of the results generated by equation (4.11) is presented in Figure (4.2). Here we have adjacently stacked the resolving kernels with respectively increasing values of t_0 in a perspective view throughout the corresponding exposure age interval. The resulting surface is henceforth referred to as a *resolving surface*. The relative resolving strength for positions of interest over the exposure age interval becomes immediately apparent.

Figure 4.2: Resolving surfaces for exposure ages of 10^6 and 10^7 years respectively based upon ^{40}K , ^{10}Be , ^{26}Al , and ^{36}Cl .

The resolving surface corresponding to an exposure age of 10^6 years (left figure) displays two prominent features. Firstly, the diagonal ridge clearly indicates that the resolving kernels are fairly well centered about their respective positions t_0 . Let us refer to this feature as the *ridge of resolution*. Secondly, the optimal resolving power is apparently associated with

the boundaries of the exposure age interval (*i.e.*, the spikes near grid locations (0,0) and (T,T) respectively). It is difficult to put forth physical arguments which can reconcile with improved resolution near the initial exposure time. Rather, this probably reflects the advantage of having precise knowledge concerning the exposure age. We suggest that any detailed interpretation of the resolving kernels constructed at the most distant epoch of the irradiation prehistory is dangerous practice. Moreover, the overwhelming problems of determining both precise and accurate exposure age estimates immediately suggests that these features are of little practical consequence. Nevertheless, we leave this as a curiosity of theoretical interest.

On the other hand, the resolving surface corresponding to an exposure age of 10^7 years (right figure) is nearly void of both features. The height of the resolving kernels near the origin relative to the rest of the interval is striking. Beyond 2 Myrs the functions become broad and moreover the meaning of their respective locations is ambiguous.

There is clearly no need to continue on to the long term problem at this point.

For completeness, let us introduce a stable isotope into this analysis. In Figure 4.3 this is done for an exposure age of 10^7 years. One immediately notices marked changes when comparing to Figure 4.2 (right) albeit the differences are not profound. Clearly, the resolving power has improved near the initial exposure time. Once more, the resolution appears to be optimal around the exposure age boundaries. Consequently, the same argument employed above can be used to mute the apparent interpretive significance of this feature (especially in light of uncertainties). Nevertheless, one is safe in concluding that the addition of another data kernel, a "box car" in this case, can only improve upon the construction of resolving kernels as indicated by Figure (4.3). Unfortunately, the improvement is clearly inadequate here.

Figure 4.3: Resolving surface for an exposure age of 10^7 based on ^{40}K , ^{10}Be , ^{26}Al , ^{36}Cl , and a stable isotope.

The addition of a stable isotope into the previous example ($T=10^6$ years) yields even poorer results. This is due to the low concentration of ^{40}K over this relatively small frame of time. Consequently, the decay rate is small and hence the ^{40}K data kernel mimics that of the stable isotope. Thus, little is added to the overall resolution.

We have already noted that ^{40}K and stable isotopes are the only nuclides available to the long term irradiation problem. Another way of stating this is that only their respective kernel functions span the sufficient time scale. But one cannot combine both a constant function and a broad monotonic function (exponential) in a way that successfully mimics a Dirac function! We have just demonstrated this simple fact.

Hence, we conclude that observations of cosmogenic nuclides from a single meteorite sample cannot provide localized unique averages with respect to the long term cosmic ray flux prehistory.

4.3 Multi-Sample Analysis:

In any analysis, the introduction of additional independent observations is equivalent to augmenting the amount of information that can become available for interpretation. For example, we demonstrated in the last section that adding a stable isotope into that analysis has a positive albeit minute effect on resolution. Therefore, then one would intuitively expect a marked increase in information from the simultaneous analysis of the said isotopes from a set of meteorite samples of differing exposure ages. In doing so, we have increased the number of data by the factor J which is equal to the number of meteorite samples in the analysis. In particular, the long term problem is now a $2J$ data problem.

Let us arbitrarily begin by reconsidering the analysis represented by Figure 4.2 (left) but now with twice the amount of data introduced via a second meteorite sample (Figure 4.4). In addition, we will assume the exposure age of the new sample to be half that of the former one.

Clearly the resolution has significantly improved. For instance, a third spike has been introduced through the additional boundary (center of the plot). Again, the interpretation of this feature strongly depends upon the precision of the shorter exposure age. Furthermore, the ridge of resolution has become more localized by roughly a factor of two. More important however, the resolving kernels are less ambiguous, with respect to the entire interval, than in the previous case as exhibited by the two vanished quadrants.

Figure 4.4: Resolving surface based upon the same radionuclides as per Figure 4.2 (left) but with the addition of data from a second meteorite sample with an exposure age of 5×10^5 years.

Let us discuss the origin of the two nil quadrants of Figure (4.4). Each resolving kernel is formed by taking linear combinations of the eight data kernels. However, the data kernels are paired according to isotope, the pairing resulting from separate observations in individual meteorite specimens. For example, the only difference between the two ^{40}K data kernels, derived from the two respective meteorite analyses, is their respective spans of temporal distance. That is, one spans the interval $[0, T_1]$ whereas the other spans the interval $[0, T_2]$, where $T_2 = 2T_1$ in this example. Therefore, they are identical within the interval $[0, T_1]$.

Hence, when linearly combining these two functions either one can be completely subtracted out in order to localize the resolving function. More explicitly, from equation (4.2),

$$\delta(t, t_0) = a_1 g_1(t) + a_2 g_2(t)$$

where

$$g_i = e^{-\lambda t} H(T_j - t), \quad i=1, 2.$$

Clearly, a resolving kernel centered at $t_0 \in (T_1, T_2]$ would require that $a_1 = -a_2$, so that the difference of $g_1(t)$ and $g_2(t)$ vanishes over the region $[0, T_1]$. The resulting resolving function would appear as the shaded portion of Figure (4.5) if only considering the ^{40}K observations. One can plainly form resolving kernels in the complementary region $[0, T_1]$ in a similar fashion. Hence, one has a complete understanding of the nil regions in Figure (4.4).

Figure 4.5: Schematic of two kernel functions based upon the measurements of a given isotopic species (a radionuclide in this example) from two different meteorite samples of respective ages T_1 and T_2 . Clearly, they can be linearly combined so that the result vanishes over certain temporal locations.

What we have just described has profound consequences. For in theory, one can obtain the Dirac function simply as

$$\lim_{T_1 \rightarrow T_2} \tilde{\delta}(t, t_0) = \delta(t - t_0).^1$$

A practical example of such enhanced resolution is displayed in Figure (4.6). Compare this to any previous resolving surface!

Figure 4.6: Averaging surface based upon ^{40}K and a stable isotope measured in two hypothetical meteorite samples possessing exposure ages of 5×10^7 and 5.5×10^7 years respectively.

¹This statement is formally correct if the two data kernels pertain to a stable isotope. Recall that the Dirac function is defined from a unit "box car" whose width vanishes.

Evidently, it is more important to know the relative differences of exposure ages than it is to know individual ones accurately. Moreover, parameter errors (*i.e.*, in exposure age) outweigh the importance of data errors. This fact lessens the importance of constructing trade-off curves (Backus and Gilbert, 1970) at the present time.

Recall from section (2.3) that meteorite exposure ages fall into well defined clusters. Thus, our problem is naturally suited to this analysis of resolution. Moreover, this immediately suggests that good resolution on the galactic cosmic ray flux prehistory is generally associated with exposure age clusters. The situation is promising in view of Figure (2.5).

Indeed, it is surprising that this analysis only requires measuring a single isotopic species from two separate meteorite samples. Clearly, the choice of cosmogenic species depends upon the most reliable isotopic measurements whereas the choice of meteorite samples is restricted to both the precision (and accuracy to a lesser degree) of respective exposure ages and to the temporal location of interest (which again depends upon exposure age accuracy). Thus, we have established several criteria for limiting the kind of improvements in the data base that are required for a given investigation.

In summary, deriving unique averages where sample exposure ages allow provides various smoothed estimates of flux intensities at their respective temporal locations. Hence, one has an indication of flux variations throughout the prehistory. One could employ current exposure age determinations (Voshage, 1967, 1978) to choose from respective iron meteorite samples according to their relative age differences. However, the temporal location of these estimates would be uncertain since long term exposure ages are based upon a flux model which is inconsistent with several crucial observations.

One can address the long term flux variation problem from a different perspective and simultaneously contend with the uncertainty of the iron exposure ages by now turning to model construction.

5

MODELLING THE LONG TERM IRRADIATION PREHISTORY

Let us finally consider the problem of both modelling the long term galactic cosmic ray flux prehistory and of simultaneously recalibrating the iron meteorite exposure age histogram. Recall the general data functional (3.14). Clearly, there are an infinite number of models $m(x)$ which reproduce a finite set of inaccurate observations e_i (recall that m is an element of a Hilbert space). Further, it is clearly understood that *a priori* information is largely absent for limiting the possibilities. Hence, we must adopt an *objective* approach by minimizing key structure(s) associated with the model. In particular, since we are refuting the long term constant flux interpretation then it becomes natural to minimize the *gradient*. Such a model is colloquially referred to as the *flattest model* and is denoted here by m_f .

Furthermore, the problem is clearly non-linear since the exposure ages are assumed to be unknown. However, the flattest model provides an additional constraint which will prove instrumental in contending with this problem; namely, an initial value $m(0)$. We will expound upon this after first formulating the flattest model for the linear problem.

5.1 Derivation of the Flattest Model:

It is usual to apply the methods of variational calculus on the Hilbert space of models in order to seek those which *optimize* arbitrary criteria. Let us henceforth refer to this space as "*model space*" and denote it as M . In addition, it is customary to minimize in a least squares sense which implies that M is in fact the real space $L_2[a,b]$ where the norm of the model is defined as

$$||m|| = \left(\int_a^b m^2(x) dx \right)^{1/2}. \quad (5.1)$$

Moreover, the data equation (3.14) is simply the inner product of the data kernel and the model.

For convenience, let us adopt Dirac's "*bra*" and "*ket*" notation whereupon (3.14) and (5.1) become $\langle g|m \rangle$ and $\langle m|m \rangle$ respectively. Similarly, a linear vector equation defined by

$$x = Ay$$

is written as $|x\rangle = A|y\rangle$ in this convention.

Consider that model which has the smallest norm in M and simultaneously satisfies (4.13). Let us denote this model by m_f . This model can be found by minimizing (5.1) subject to the data constraint (equation 3.14) introduced through the Lagrange multipliers L_i . That is, we must minimize the objective function

$$\psi = \langle m|m \rangle + 2 \sum_i L_i [e_i - \langle g_i|m \rangle]. \quad (5.2)$$

Clearly, m_f is that model possessing the smallest gradient of all models in M . Hence, we must reformulate the data functional (3.14) so that $m'(x)$ takes the place of $m(x)$ in (5.2).

This is done by integrating (3.14) by parts to yield

$$f_i \equiv m(b)h_i(b) - e_i = \langle h_i|m' \rangle \quad (5.3)$$

where h_i is the indefinite integral of the i^{th} data kernel. Thus, the flattest model can be found by integrating the function which results from minimizing

$$\psi = \langle m'|m' \rangle + 2 \sum_i L_i [f_i - \langle h_i|m' \rangle]. \quad (5.4)$$

Let us proceed to do this by the method of stationary values.

Consider an arbitrary infinitesimal perturbation of the model $\delta m'$. Clearly, m' is a minimum if

$$\psi_{(m' + \delta m')} - \psi_{(m')} = \delta \psi_{(m')} = 0.$$

More explicitly,

$$\begin{aligned} \delta \psi_{(m')} &= 2 \langle m' | \delta m' \rangle + 2 \sum_i L_i (-1) \langle h_i | \delta m' \rangle = 0 \\ \langle m' - \sum_i L_i h_i | \delta m' \rangle &= 0. \end{aligned} \quad (5.5)$$

Since $\delta m'$ is arbitrary then (5.5) is satisfied for all $\delta m'$. Thus,

$$m' = \sum_i \beta_i h_i \quad (5.6)$$

where $\beta_i \equiv L_i$ are the coefficients which must be determined.

The coefficients are evaluated by substituting (5.6) into (5.3). That is,

$$\begin{aligned} f_i &= \langle h_i | \sum_j \beta_j h_j \rangle \\ \Leftrightarrow |f\rangle &= \Gamma | \beta \rangle \end{aligned}$$

where the inner product matrix Γ has been already defined in section (4.1). Γ is invertable since it is symmetric and positive definite. Thus, the coefficients are evaluated simply from the relation

$$| \beta \rangle = \Gamma^{-1} | f \rangle. \quad (5.7)$$

Hence, the flattest model can be evaluated as

$$m_f(t) = \sum_i \beta_i \int h_i(t) dt + C \quad (5.8)$$

where C is the constant of integration.

Let us now be explicit in terms of the cosmogenic nuclide problem. The indefinite integral of (5.3), pertaining to the i^{th} isotopic species measured in the j^{th} meteorite sample, is evaluated as

$$j h_i(t) = \int_{T_j}^t e^{-\lambda \tau} d\tau = \begin{cases} \frac{1}{\lambda_i} (e^{-\lambda_i T_j} - e^{-\lambda_i t}), & \lambda_i \neq 0 \\ t - T_j, & \lambda_i = 0. \end{cases} \quad (5.9)$$

Hence, the new data $j f_i$ is given from (5.3) as

$$j f_i = \begin{cases} \frac{\Phi(0)}{\lambda_i} (1 - e^{-\lambda_i T_j}) - j e_i, & \lambda_i \neq 0 \\ \Phi(0) T_j - j e_i, & \lambda_i = 0 \end{cases} \quad (5.10)$$

and consequently we have introduced the present value of the flux $\Phi(0)$ into the analysis.

The inner product matrix Γ_{ik} can be written as

$$\langle h_i | h_k \rangle = \begin{cases} \frac{e^{-(\lambda_i - \lambda_j)T}}{\lambda_i \lambda_j} \left[T_j - \frac{1}{\lambda_i \lambda_j} + \frac{1}{\lambda_i} + \frac{1}{\lambda_j} \right] - \frac{e^{-\lambda_i T}}{\lambda_j^2 \lambda_i} - \frac{e^{-\lambda_j T}}{\lambda_i^2 \lambda_j} + \frac{1}{\lambda \lambda (\lambda_i + \lambda_j)}, & \lambda_{i,k} \neq 0 \\ \frac{1}{\lambda_j^2} (T_j - \frac{1}{\lambda_j}) + \frac{1}{\lambda_j} e^{-\lambda_j T} (\frac{1}{\lambda_j^2} - \frac{T_j^2}{2}), & \lambda_k = 0 \\ T_j^3/3, & \lambda_{i,k} = 0. \end{cases} \quad (5.11)$$

for $T_j > T_i$. Clearly, replacing the index "j" for "i" in equation (5.11) holds for the case where $T_j > T_i$. Notice that the situation where $T_j \neq T_i$ has not been specified. This is because the success of inversion heavily depends upon the *condition* of Γ . Thus, inversion fails if the two exposure ages are identical since two vectors in Γ would therefore be linearly dependent.¹

The *condition number* of Γ relates the relative change in the data to the relative change in the model. This naturally leads into the question of model stability. The usual method of dealing with data errors is to expand Γ according to its singular values. Then, one proceeds to construct a set of orthonormal basis functions from the data kernels incorporating this expansion. It is found that the smaller singular values are associated with the finer model structure, most of which being statistically unreliable. Thus, the model can be smoothed by winnowing out the smaller singular values based upon some suitable "*misfit*" criterion (i.e., the Chi-squared test). Obviously, the model so described is identical to (5.8) if all singular values are retained. What we have just described is collectively referred to as the *spectral decomposition* method (Parker, 1977).

However, it is clear from (5.8) that model structure is largely dependent upon the collective structure of the data kernels. But only ^{40}K and an arbitrary stable isotope are

¹Similarly, only one stable isotope can be explicitly introduced into this model. Another way of understanding this is that two identical kernel functions cannot add any more structure to the model than can either one alone.

available to the long term irradiation problem. Consequently, one is only capable of generating monotonic long term models given one meteorite sample and its true exposure age. In this case, equation (5.8) will suffice in generating all single sample models of interest to this problem.¹ That is,

$$\Phi_f(t) = \sum_i \beta_i \left[\begin{array}{ll} \frac{1}{\lambda} [te^{-\lambda_i T} + \frac{1}{\lambda}(e^{-\lambda_i T} - 1)] + \Phi(0) & \lambda_i \neq 0 \\ t^2/2 - Tt + \Phi(0) & \lambda_i = 0. \end{array} \right] \quad (5.12)$$

Furthermore, models based upon the multi-sample analysis can attain structures whose "wavelengths" are on the order of the respective exposure age differences (section 4.3). These models have been investigated using the spectral decomposition method discussed in section (5.1). In all cases, fine structures are lost when the models are misfit² based upon conservative estimates of data errors (section 5.2). Hence, the multi-sample problem will not be pursued in this work.

5.2 Data and Estimates of Uncertainties:

We have emphasized throughout this work that existing cosmogenic nuclide data and the respective partial cross sections are largely inadequate. This is reflected in the fact that extensive tabulations of absolute cosmogenic radionuclide abundances do not exist in the literature. Voshage (1967, 1978) has presented exhaustive results of both ⁴⁰K and of stable

¹However, all model construction in this work is in fact based upon spectral expansion. Hence, this claim has been verified using reasonable estimates of uncertainties (section 5.2).

²Employing the Chi-squared criterion.

nuclide analyses in iron meteorites. Unfortunately, the ^{40}K data are only relative concentrations since the analysis is based upon the mass spectrometer. One could calibrate these values using accurate absolute abundance measurements of some well known stable species. Then the data uncertainties would be largely due to the precision of both this latter measurement and the two pertinent yields. However, we will not pursue this method.

Data on absolute abundances of cosmogenic nuclides from specific meteorite samples varies greatly throughout the literature. In particular, such data on ^{40}K is nearly absent as it is a very difficult species to measure precisely.

On the other hand, data on the lighter inert isotopes are relatively good as stated in section (2.2). In particular, ^{21}Ne abundances (and corresponding yields from iron) are well known. Moreover, it is well suited since its "natural" abundance is very low and because it is a high energy product with respect to an iron target. Thus, we will adopt ^{21}Ne as the stable nuclide used in the long term irradiation problem.

In addition, the choice of meteorite samples should be biased against both large terrestrial ages and multiple exposure ages¹ as interpretations should be as simple as possible. Furthermore, samples must have experienced similar cosmic ray shielding for use in the multi-sample analysis.

The data adopted in this work is from the celebrated paper by Honda and Arnold (1964). The samples which come closest to satisfying our requirements are listed in Table (5.1). Only half of the original samples were adopted here. Although improved measurements can be found scattered throughout the literature it is important to restrict our analysis to data from a single source.

¹Some meteorites exhibit more than one exposure age for differing sample locations indicating a more complicated irradiation history.

Table 5.1: Data for Use in the Long Term Irradiation Problem.

METEORITE	DATE OF FALL	WEIGHT (kg)	[⁴⁰ K], 10 ⁻⁹ g/g	[²¹ Ne], 10 ⁻⁸ cm ³ /g†
Mt. Ayliff	(find)	013.6	0.60 ± 0.02	8.9
Yardymly‡	1959	150	0.49 ± 0.03	8.15
Williamstown	(find)*	031	0.37 ± 0.02	5.3
Carbo	(find)	454	0.22 ± 0.01	3.2

†At STP. Uncertainties are not available.

‡Also known as Aroos.

*²⁶Al activities indicate short terrestrial age (Honda *et al.*, 1961).

This is clearly not an ideal data set. As such, it illustrates the severe problems facing current cosmogenic nuclide research. One should be concerned about the large scatter in sample weights as well as the fact that only one meteorite is an observed fall. The latter point can be remedied by observing large activities of shorter-lived radionuclides indicating small terrestrial residence times. However, this information was not obtained for two of the three samples in question. This is not to say that these measurements are nonexistent; one must be well versed in the this literature in order to make such a claim. Again, we stress the need for a compendium of cosmogenic nuclide data.

Furthermore, ⁴He data was not found and consequently irradiation hardness correlations among respective samples cannot be made using the methods outlined in section (3.2). Instead, the unorthodox correlation based on ³He/²¹Ne has been utilized to establish Table (5.1). This writer is unaware of committing any "crimes" in doing so although one worries when referring back to Figure (3.3)!

For completeness, let us conclude this section with a summary of all the pertinent data and further, present crude estimates of the associated experimental errors. The points just given all indicate that large systematic errors exist in the data. Unfortunately, we have not

provided enough information from which to quantify them as such. Quoted precision in Table (5.1) is under 5 percent although a more realistic value may be substantially higher.

Recall equation (3.19). According to our formulation both the yield and the target number density values are defined as part of the data. Quoted errors in the latter were not found. On the other hand, we discussed the relative standard deviations associated with individual yield estimates in section (2.2). However, one must look to Figure 3.6 (bottom) in order to translate yield errors into errors in average partial cross section. Silberberg and Tsao (1973) found the largest errors in equation (2.3) to be associated with energies above 2 GeV for the nuclides of interest. Fortunately, the average cross sections all fall dramatically in this energy range in accordance with equation (3.9). Once again however, one can argue about implicit systematic errors as above.

Both equation (3.12) and Table (3.2) were employed to evaluate the pertinent average partial cross sections. The respective computing codes were rigorously tested against the data provided in Silberberg and Tsao (1973) and by Arnold *et al.* (1961). The integration was accomplished by implementing both spline and numerical integration algorithms (courtesy of Prof. D. Oldenburg). The necessary steps were taken to assure that accuracy was maintained. The results based upon the spallation of ^{56}Fe are as follows:

$$^{21}\text{Ne} - 5.0 \times 10^{-28} \text{ cm}^2,$$

$$^{40}\text{K} - 3.1 \times 10^{-27} \text{ cm}^2,$$

$$^{10}\text{Be} - 7.9 \times 10^{-28} \text{ cm}^2,$$

$$^{26}\text{Al} - 4.3 \times 10^{-28} \text{ cm}^2, \text{ and}$$

$$^{36}\text{Cl} - 2.6 \times 10^{-27} \text{ cm}^2.$$

Also, the target number density based on ^{56}Fe for the accuracy required is

$$N_i = 1.1 \times 10^{25} \text{ kg}^{-1}.$$

Finally, one must possess a value of the present omnidirectional flux at an effective depth of $d_0 = 100 \text{ g}\cdot\text{cm}^{-1}$. There are two methods of achieving the value investigated in this work. Firstly, a freshly fallen meteorite can be analysed for short-lived radionuclides at the prescribed depth. Assuming secular equilibrium (*i.e.*, $\lambda_i T \gg 1$ and $\Phi = \text{constant}$) over the short term then equation (3.19) becomes

$$A_i(0, d_0)/N_i \Sigma_i(d_0) = \Phi(0, d_0) \quad (5.13)$$

where $A_i = n_i / \lambda_i$ is the i^{th} activity in question and $\Phi(0, d_0)$ is the present flux.

Equation (5.13) was evaluated for radioisotopes observed in the iron meteorite Yardymly (Arnold *et al.*, 1961); namely, for ^{10}Be , ^{26}Al , ^{36}Cl , ^{44}Ti , ^{14}C , and ^{39}Ar . The average value obtained is $\Phi(0, d_0) = 10 \pm 2 \text{ cm}^{-2}\cdot\text{sec}^{-1}$.

The second such approach is based upon integration of an empirical depth dependent cosmic ray spectrum (see Table 3.2 or equation 3.9). The result of $14 \pm 2 \text{ cm}^{-2}\cdot\text{sec}^{-1}$ was established over the energy interval $E_l = 100 \text{ MeV}$ and $E_u = 10 \text{ GeV}$. However, many of lower energy particles involved here do not enter into the budget of high energy products. Hence this latter figure is likely to be high.

We will adopt the former figure.

Evidently, the data errors are fairly uncertain although they are clearly significant. Thus, a conservative estimate of data error based on the preceeding discussions is on the order of 50 percent; a value typically quoted for uncertainties of results derived from research into cosmogenic nuclides.

5.3 Contending with Non-Linearity:

We now possess both the data and the machinery from which to address the linear inverse problem. However, equation (3.19) is non-linear since T_j is unknown. Our remedy to this apparent impasse is simply an extension of the objectivist philosophy adopted in this work. In particular, we will now consider the flattest model to be a function of the variable T_j . Thus, the goal is to obtain that flattest model m_F which possesses the smallest norm with respect to variable exposure age. That is,

$$m_F = \min ||m_F(T_j)||$$

$$\Leftrightarrow \min < \beta_i(T_j) | \Gamma(T_j) | \beta_i(T_j) >, \quad \forall T_j.$$

In so doing, we establish a criterion for objectively choosing both an element of model space and a set of corresponding exposure ages. Furthermore, one can impose various degrees of constraint to such choices depending on the number of meteorite samples used in the analysis.

Examples of single sample analyses using synthetic data are given in Figures (5.1) and (5.2). They are respectively based upon a constant flux model and an exponentially increasing flux¹ model, both spanning 1 Gyr. Both respective L_2 -norm plots exhibit subtle differences in character although in general they are similar. For example, the most dramatic increase in the norm occurs while approaching the origin after having passed the global minimum. Intuitively, this is because we are trying to squeeze the "energy" of the true model into an integration time which is less than the exposure age. This is a milestone observation when contending with the multi-sample problem as more than one minimum can occur.

¹With respect to real time.

Figure 5.1: Flattest models generated for five different exposure ages based upon synthetic data (left). The data is generated from a constant flux model and consequently the central curve is the true model. The right figure is a plot of the L_2 -norm spanning the exposure ages in question.

Figure 5.2: Flattest models generated for five different exposure ages based upon synthetic data (left). The data is generated from an exponential flux model. The right figure is a plot of the L_2 -norm spanning the exposure ages in question.

Consider the two-sample case. Further, let us generate models using constant flux synthetic data and observe the behavior of the norm (Figure 5.3). Again, the global minimum occurs for the true respective ages since the constant flux model is the flattest possible model. However, notice the strong diagonal ridge. Recall that inversion fails when $T_1 = T_2$. Hence, this feature is aptly referred to in this work as the *ridge of ill-conditionality* (with respect to the inner product matrix).

Evidently, model construction based upon multi-sample data is best suited to meteorites with largely dissimilar ages. This is in contrast to our results of the resolution studies conducted in section (4.3).

The introduction of more meteorite samples clearly becomes more cumbersome. The Monte Carlo approach could be employed to sample sufficiently the corresponding J -dimensional space of model norms in order to establish the global minimum. However, in a cursory investigation of this we found several nearly identical minima to occur while using only four hypothetical samples. Clearly, the global minimum norm criterion is ambiguous under certain conditions. It is clear that adequate theory is required before rigorously pursuing the multi-sample data problem.

Further, it was stated in section (5.1) why it is currently impractical to construct models based upon multi-sample data. Thus we shall refrain from any further discussion concerning the multi-sample problem.

Finally, using only a single datum, an observation of a single isotopic species from an individual meteorite sample, is too unconstrained for this analysis. One can always construct a constant flux model which matches the current flux value by finding the appropriate exposure age (see equation 3.19). Hence, both ^{40}K and ^{21}Ne must be used together in this analysis.

Figure 5.3: Contour plot of the L_2 -norms of models corresponding to synthetic constant flux data. Here, ^{40}K and ^{21}Ne are analysed from two hypothetical meteorite samples (the true ages marked with the cross).

5.4 Results and Conclusions:

Let us apply the data from Table (5.1) to equation (5.12) with regards to individual meteorite analyses. The model norms for the Mt. Ayliff meteorite are displayed in Figure (5.4). This is representative of the other three samples under study. The minimum norm ages of Carbo and Yardmly are systematically about 20 percent lower than those reported by Voshage (1967). Ages for the remaining two meteorites were determined but no such comparison could be made. The *minimum norm ages* recovered are reported here as follows:

Carbo	0.59 Gyr
Williamstown	0.67 Gyr
Yardmly	0.76 Gyr
Mt. Ayliff	0.87 Gyr.

The corresponding models are sampled in Figure (5.5). Moreover, the global flattest models for all four specimens are presented in Figure (5.6). Evidently, all models are in qualitative agreement as to the general increase of the flux with respect to real time. The systematic flattening of the respective models is merely an artifact of having performed independent inversions of the respective meteoritic data, all under the the criterion of global flatness.

Various tests for robustness have been administered (*i.e.*, varying the adopted current flux value about conservative limits) which support this result. Hence, we have constructed models of the galactic cosmic ray flux prehistory, involving recalibrated exposure ages, which are mutually consistent with the respective observations of cosmogenic nuclide abundances.

Moreover, the flux appears to have been increasing over the past billion years in accordance with the claim put forth by Schaeffer *et al.* (1981). Recall that space erosion has

Figure 5.4: Model norms based upon ^{40}K and ^{21}Ne observations from the iron meteorite Mt. Ayliff (Arnold and Honda, 1964). The minimum occurs for an exposure age of 870 million years. Figure courtesy of B. Narod.

Figure 5.5: Flattest models generated for five different exposure ages based upon ^{40}K and ^{21}Ne data from the iron meteorite Mt. Ayliff (Arnold and Honda, 1964). The center curve is the flattest possible model which can reproduce these data in the context of this work. Figure courtesy of B. Narod.

Figure 5.6: Global flattest models based upon ^{40}K and ^{21}Ne data from the iron meteorite Mt. Ayliff (Arnold and Honda, 1964). All exhibit a monotonic increase with respect to real time. Figure courtesy of B. Narod.

been rejected as a cause for apparent changes in production rates for iron meteorites in section (2.3). In addition, long term changes in solar luminosity are predicted. Consequently, long term solar modulation effects could be used to explain the variation in production rates. However, theoretical studies of stellar evolution suggest that the solar luminosity L has been steadily increasing at an average rate

$$\frac{d}{dt} \ln L = 0.08 \times 10^9 \text{yr}^{-1}$$

(Newkirk, 1983). Thus, one would expect the galactic flux to be decreasing if there exists a direct relation between solar luminosity and solar modulation (as one may expect). This is in contrast to interpretations of cosmogenic nuclide observations. Unfortunately, such a relationship is not presently realized.

Thus, it is difficult to elicit local effects which can conspire to produce time dependent production rates. Indeed, if the variation is in fact due to changes in the galactic cosmic ray flux then this model provides a vital clue to researchers trying to address the problem of cosmic ray origins.

On the other hand, we have demonstrated that drastic changes in exposure age interpretations are not required in order to build a mutually consistent model of the irradiation prehistory. The conclusions drawn from exposure age histograms are unaffected since the results presented herein only suggest that a calibration error exists regarding the long term ages.

Also, it is clear that improvements in experimental precision are essential. However, we have shown how one can determine what data and associated precision are necessary in order to address certain problems of interest.

Finally, we have clearly outlined where the most serious problems currently exist in the interpretation of cosmogenic nuclide observations. The so-called fatal gap is perhaps the most severe since it is a natural barrier. However, the multi-sample approach may greatly facilitate this situation when the required data precision exists. But the lack of an independent method for exposure age determinations will always hinder the reliability of all such interpretations.

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APPENDIX A:

Previous Inferences Drawn on the Cosmic Ray Flux Prehistory

Let us describe the reasoning which has led to the current model of the short term galactic cosmic ray flux (over the past 10^6 years). The theme has been centered upon comparing predicted production rates to ones calculated from cosmic ray produced radionuclides spanning 10^6 years in half-life (Figure 2.7) from the same sample of meteorite. The production rate of the i^{th} radioisotope in pure iron can be written as

$$P_i(t,d) = N_i \int_E \sigma_i(E) S(E,t,d) dE \quad (A-1)$$

where all symbols are described in chapter 3. On the other hand, the respective decay rate, or activity A_i , measured at the end of irradiation period T (i.e., at $t=0$) is

$$A_i(0,d) = \lambda_i n_i(0,d) = \lambda_i \int_0^T e^{-\lambda_i t} P_i(t,d) dt \quad (A-2)$$

(see equation 3.16).

In particle accelerator research the irradiation conditions are usually held constant. Consequently, after a given time the condition $T \gg 1/\lambda_i$ applies. This is always the case for all isotopes except for ^{40}K found in iron meteorites (compare Figures 2.5 and 2.7). Thus, equation (A-2) now reads as

$A_i(d) = P_i(d)$

(A-3)

a condition referred to as *secular equilibrium*.

Logically, the only inference that can be drawn from observed activities then is to say that the irradiation conditions have not been constant if condition (A-3) fails.¹ However, this analysis conducted on freshly fallen meteorites supports condition (A-3) (*i.e.*, Arnold *et al.*, 1961) and it has been customary to assume that the galactic cosmic ray flux has been constant to within experimental precision (by about a factor of two). Of course, the problem is that a constant flux model is only one of an infinite set of models which reproduce this finite set of real observations.

Nonetheless, this model is acceptable when one acknowledges the lack of contradictory information. The minimum structure model must be given favour above all others for the sake of objective interpretation. This is the same reasoning which has led to the flattest model formulation contained in this work.

Since ^{10}Be is the longest lived radionuclide to which this analysis can be applied then the inference of constant irradiation conditions holds for a temporal span on the order of its half-life; namely, 10^6 years.

¹Assuming that (A-1) is evaluated correctly.

APPENDIX B:

Exposure Age Calculations Based Upon Constant Irradiation Conditions

All cosmic ray exposure age calculations are based upon constant irradiation conditions. Further, the analysis requires two cosmogenic species, a radionuclide which provides a measure of the flux (*i.e.*, assuming secular equilibrium) and a stable species which serves as an integrator of the flux. From equation (3.16) the abundance of a stable species n_s produced under constant irradiation conditions after a time T is

$$n_s(0,d) = N_t \sigma_s(E) \Phi(d,E) T \quad (B-1)$$

where all symbols and the time scale are defined in chapter 3. Similarly, the abundance of radionuclides n_r is

$$n_r(0,d) = N_t \sigma_r(E) \Phi(d,E) [1 - e^{-\lambda_r T}] / \lambda_r$$

If $T \gg 1/\lambda_r$ then the activity may be written as

$$A_r(0,d) = N_t \sigma_r(E) \Phi(d,E) \quad (B-2)$$

The ratio of (B-1) and (B-2) yields the basic expression for the exposure age as

$$T = R n_s(0,d) / A_r(0,d) \quad (B-3)$$

where R is the ratio of radioisotope yield to stable product yield. Clearly, the energy dependence of the respective reaction cross sections is assumed to be identical. Hence, the

nuclides are chosen to be either isobaric or isotopes of the same element so that this simplification holds.

Many variations of equation (B-3) exist throughout the literature. Most notable is that of Voshage (1967, 1978) for the analysis of iron meteorites. It is based upon the radionuclide ^{40}K and the stable species ^{41}K . Note that ^{40}K is not in secular equilibrium since $T \sim 1/\lambda$. In addition, an additional stable isotope ^{39}K is introduced empirically to establish the effective depth d . However, systematic discrepancies exist among ages determined from Voshage and those based upon shorter-lived isotopes. Thus, the assumption of constant irradiation conditions must be incorrect and consequently one expects the exposure age estimates of meteorites to become progressively more inaccurate with increasing exposure age.

APPENDIX C:

A Survey of Targets for the Study of Cosmogenic Nuclides

The study of cosmogenic nuclides is confined to those targets which can be analysed in the laboratory. Presently, we have the following kinds of targets available for radiochemical and mass spectrometric analyses:

- a) the upper atmosphere,
- b) meteorites,
- c) cosmic dust,
- d) the lunar surface,
- e) high altitude balloon payloads, and
- f) retrieved spacecraft.

Perhaps the best known example of cosmic ray interaction with matter is the production of ^{14}C from the reaction $^{14}\text{N}(n,p)^{14}\text{C}$ in the stratosphere and upper troposphere. However, long lived or stable nuclides produced in the earth's atmosphere will be transported away from their point of origin and perhaps undergo various geochemical reactions, all of which are not quantitatively well understood. Consequently, the terrestrial cosmic ray fossil record is extremely difficult to decipher.

Cosmic dust could be useful for the analysis of thin target interactions by cosmic radiation. However, the chemical composition of cosmic dust is still not clearly understood, and is usually difficult to distinguish from terrestrial dust. Moreover, sufficient sample weights are currently unobtainable for the detection of radionuclides except for reservoirs on the earth which unfortunately are plagued with terrestrial contamination.

On the other hand, targets carried aloft in high altitude (helium) balloon payloads and targets in the form of retrievable spacecraft have both a well known chemical composition and a well known history of irradiation. The former are particularly useful for direct short term flux measurements and for the determination of empirical relationships describing the cosmic ray thick target energy spectrum (see section 3.2). The latter has been particularly useful for studying the effects caused by the geomagnetic field but has little to do with the galactic cosmic ray flux. One exception is the Surveyor 3 spacecraft which resided on the lunar surface for two and a half years prior to retrieval, in part, by the Apollo 12 astronauts.

In order to examine the effects of galactic cosmic radiation over longer time scales one must turn to lunar surface samples and to meteorites. Lunar samples afford the advantage of having a well known location within the solar system in contrast to meteoroids whose orbits are poorly understood.¹ The analysis of cosmogenic nuclides from deep lunar core samples yields information on the lunar irradiation prehistory involving galactic cosmic radiation which in turn involves both the galactic cosmic ray flux prehistory and the stratigraphic evolution of the sampled selenographic region. Clearly, the latter constitutes a significantly complicated target prehistory.² Moreover, lunar material has a relatively complicated target chemistry. The fact that data concerning product yields from a wide variety of target nuclides is both sparse and imprecise (section 2.2) clearly makes the simplest targets most attractive.

The discussion of meteoroids is of principal interest to this paper and the discussion of such is given in section (2.3).

¹However, this is probably not too important since the orbits are known not to be eccentric enough to sample significant variations in the galactic flux as a consequence of solar modulation within current experimental precision. Besides, such effects would average when considering the long term problem.

²But one can argue that good independent observations are available for constraining the possible interpretations (*i.e.*, impact craters, estimated influx of cosmic dust, *etc.*).