INTERMEDIATE ENERGY PION PRODUCTION

by

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INTERMEDIATE ENERGY PION PRODUCTION

ABSTRACT

An experimental study of intermediate energy pion production has been conducted at TRIUMF with polarized protons. Angular distributions of differential cross sections and polarization analysing powers are presented for (p,π) reactions on ¹H with 305 to 425 MeV protons, on ²H with 305 to 400 MeV protons, and on 9Be and 12C with 200 MeV protons. In addition the inclusive ${}^{12}C(p,\pi^+)X$ reaction has been studied for protons with 330 to 425 MeV energy. Theoretical model calculations of the ¹H(p, π^+)²H reaction involve relatively few transition amplitudes in the 'threshold' kinematic region (where only sand p-wave pions contribute). Prior to this experiment the threshold region was thought to extend to 140 MeV above threshold (289 MeV). The new data show that the threshold description is only valid up to 320 MeV. A second important observation Cf this experiment is the similarity of the analysing power for ${}^{1}H(p,\pi^{+})pn$ to that for ${}^{1}H(p,\pi^{+}){}^{2}H$ for pions with center of mass energies 10 to 20 MeV below the two body similarity is also observed in the $^{2}H(p,\pi^{+})X$ reaction. The reaction, where the inclusive reaction analysing powers were slightly more positive than for $^{2}H(p,\pi^{+})^{3}H_{\bullet}$ In the 200 MeV nuclear (p,π) reactions studied, the shape of the angular distributions of cross section agree well with earlier 185 MeV results. The angular distributions of analysing power in the

exclusive nuclear (p, π) reactions, never previously measured, show a remarkably large magnitude in the forward direction and enable a thorough testing of current theoretical models. These results are an indication of the explicit role of the two nucleon mechanism in nuclear pion production. Finally the results for the inclusive reaction, ${}^{12}C(p,\pi)X$ have been compared with a semiclassical model. The model results are reasonable, however at these low energies uncertainties in the input NN \rightarrow NN π distributions lead to an underestimate of the cross section for low energy pions.

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PREFACE

Nuclear physicists' understanding of nature has frequently evolved through accurate, complete experimental descriptions of nuclear systems and by making analogies between the interactions of nuclear particles with those of easier to observe classical systems. More subtle aspects of nuclear physics have been observed and understood by relating new complex results with either better understood or more completely observed, simpler reactions.

Frequently analogies based upon unrealistic assumptions have had little value, and sometimes have served to retard our understanding. In this dissertation special care has been taken to evaluate the assumptions which are built into several models of pion production reactions in addition to the description of experimental techniques and results.

The pion production experiments described in the following chapters were initiated by members of The University of British Columbia Nuclear Physics Group before July, 1976 when the author joined in the studies. In particular the majority of the pion detection system was completed in 1976 through the efforts of G.Jones, E.G. Auld, R.R.Johnson, P.Walden, T.Masterson, D.Ottewell and several UBC summer students. It is a pleasure to thank these people for their patience throughout the pion production experiments and particularly G.Jones for his physical insight which has lead the way in interpreting the experimental results. E.G.Auld's special interest in nuclear pion production

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and his "teutonic thoroughness" have been greatly appreciated. P.Walden deserves a special thanks for enduring the author's stubbornness while jointly working on numerous experimental The technicians of The University of British Columbia details. Nuclear Physics Group, notably A.Bishop, A.Morgan, A.Stephenson and have often contributed to the experiment, C.Stevens particularly in the design and construction of a complex liquid hydrogen target system which operated flawlessly throughout the experiment. Throughout the majority of the data analysis extensive benefits were derived from the computer programming efforts of A.Haynes, D.Sample and the TRIUMF computing group. During the summer of 1979, S.Mann performed much of the Legendre polynomial fitting of the analysing powers.

The two beam nature of the TRIUMF machine makes operations difficult at times; thus it is with pleasure that the friendly cooperation of the operations staff and other TRIUMF experimenters is acknowledged.

Two of the experiments mentioned in the following chapters, the large angle ${}^{2}\text{H}(p,\pi^+){}^{3}\text{H}$ and the small angle ${}^{9}\text{Be}(p,\pi^+){}^{10}\text{Be}$, involved collaborations with a University of Alberta group under W.C.Olsen and several TRIUMF Medium Range Spectrometer users under J.Rogers respectively. The author has benefited from these efforts and gratefully acknowledges the joint work and discussions with members of those groups. The author is also grateful to D.Beder for making available the inclusive nuclear pion production program developed by P.Bendix and D.Beder. The author would like to acknowledge the financial support from The University of British Columbia and, through research grant IEP 18, from the Natural Sciences And Engineering Research Council Of Canada.

Finally the author would like to thank his wife, Linda, for her patience and encouragement throughout these studies. This thesis is dedicated to R. Mathie, who enthusiastically followed this work but was denied the satisfaction which we enjoy through its completion.

CHAPTER 1

INTRODUCTION

The understanding of nuclear interactions and particles has been largely governed by the tools used for the experimental Rutherford (1911) used low energy alpha particles from studies. natural radicactive decays to establish that an atom was composed of a dense heavy nucleus and cloud of electrons. Without the alpha particle probe the existence of the nucleus would not have been ຣ໐ graphically demonstrated. As experimental capabilities improved and low energy particle accelerators became more common a more thorough understanding of nuclei arose. A wealth of information became available, and several models of the nucleus were proposed to describe the gross characteristics of nuclei (Mayer 1955, Serber 1947. Tobocoman proton, 1954) • Electron, deuteron and alpha scattering reactions were used to investigate the mass and charge distributions of the nucleus, as well as the momentum distribution of nucleons in the nucleus, (Hendrie 1968, Feshbach 1954).

The advent of medium energy accelerators has allowed major extensions of this work with higher energy elastic scattering reactions, one and two particle transfer reactions, and studies of mesonic and muonic atoms. Medium energy electrons have been used to accurately determine the momentum distribution of nuclear particles which may be compared to the Fermi nuclear gas model (Findlay, 1978). Mesonic atom studies have given charge distributions of various nuclei (Ford, 1969). Proton, deuteron and alpha scattering results may be compared with calculations based on nuclear models (Jackson 1974). In particular the shell structure of nuclei can be clearly demonstrated with experiment (Sheline 1964, Sweet 1964). One and two particle transfer reactions might be used to infer more information about the momentum distribution of nucleons with momenta above 200 MeV/c, if the reaction mechanisms are understood, (Hoistad 1977, James 1979). These reactions may also be used to study nuclei not normally observed with elastic scattering reactions. Examples of these reactions are (p,d), (p,2p) and (p, π). Several (p, π) reactions have recently been studied and are discussed in this dissertation.

is difficult to summarize the history of (p,π) studies, It however major reviews are given by Hoistad(1976, 1977), Aslanides(1976), Measday(1978), and Spuller(1975). The most Leautiful nuclear (p,π) studies have been made at Uppsala, where Dahlgren et al (1967, 1971, 1973a,b,c, 1974a,b) have explored the low proton energy kinematic range, clearly distinguishing many discrete states of the residual nucleus. Other low proton energy nuclear pion production experiments have been conducted at Indiana (Bent, 1978) and at Orsay (LeBornec 1974, 1976). Numerous theoretical calculations have since been made for the nuclear (p,π) reaction, notably those by Miller(1974). Noble(1975) and Dillig(1977).

Fign production on systems involving few nucleons has been studied experimentally by Albrow(1971), Dolnick(1970) and Crawford(1955). Reviews of these studies and the inverse

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reactions have been presented by Richard-Serre(1970), Spuller(1975), and more recently by Jones(1977). The most successful theoretical studies have been made by Niskanen(1978).

Inclusive production of relatively low energy pions from nuclear targets was of early concern for engineering purposes as providing another testing ground for nuclear theories. well as The practical engineering applications arose with the desire to optimally construct high intensity meson lines. Experimental studies by Heer (1969), Cochran (1972), James (1975), Mathie (1976) and Crawford (1979) have been made for these purposes. Beder (1971) and Silbar (1972) have promoted reasonably successful theoretical calculations of nuclear inclusive (p,π) with their semi-classical models.

the experimental study of nuclear (p,π) reactions at In TRIUMF , explicit attempts to determine the reaction mechanism have been made to enable the extraction of nuclear structure information from this and cther experiments. The experimental techniques employed are described in Chapter 2. This experiment has greatly improved the existing data for the reactions ¹H (p, π^+)²H and ¹H (p, π^+) pn in the threshold region, which has for the first time been experimentally defined to be below proton energies cf 320 MeV. The use of a polarized proton beam and measurement of the resulting left-right pion asymmetries has demonstrated that significant d-wave pion production is observed for proton energies as low as 330 MeV. The ${}^{1}H(p,\pi^{+}){}^{2}H$, ¹H (p, π^+) pn, ²H (p, π^+) ³H, and ²H (p, π^+) X results are discussed in

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Chapter 3, where the phenomenological description of the ${}^{4}\text{H}(p,\pi^{+}){}^{2}\text{H}$ reaction is given both in terms of a power series of $\cos \theta^{*}$ and of Legendre pclynomials.

Angular distributions of single differential cross section for picn production from light nuclei have been added to the Uppsala, Orsay and Indiana data to extend these studies through the whole "threshold" region. Resolution of peaks is not as clear the lower energy results, however the as angular distributions of asymmetry, never previously so extensively have been found to be strongly reaction mechanism measured, dependent. The results are discussed in Chapter 4.

A large quantity of data for the inclusive production of low energy pions from proton bombardment of carbon has been added to the meagre supply of data available at intermediate energies. The angular distributions of double differential cross section are presented in Chapter 5.

Information from each of these experiments have been interpreted as manifestations of the (p, π) reaction mechanism in Chapter 6. In the final chapter conclusions from these experiments are drawn and recommendations for continued experimental (p, π) studies are presented.

Throughout the thesis a * designates a center of mass quantity.

CHAPTER 2

THE EXPERIMENT

The general location of the experimental area for the pion production experiments is shown in Figure 1. Work described in this thesis was performed in the meson hall where the Pembrooke spectrograph system was located in beam line 1a. The continuing UBC pion production studies are being conducted at experimental stations on beam lines 1b and 4b.



Figure 1 TRIUMF Experimental halls showing the location of the pion production experiments using the Pembrooke spectrometer.

In this chapter the use of a broad range momentum spectrograph and scintillation counters to detect and identify pions is described.

SECTION 2.1 THE PEMEROOKE SPECTROGRAPH SYSTEM

The core of the Pembrooke system is a 50 cm Browne-Buechner magnetic spectrograph (Browne 1956), which was originally intended for low energy alpha scattering experiments at The University of British Columbia. The magnetic optics of the system are described by Lee (1975) from which the important characteristics are reproduced in Table I (p.17) . The particle detection was accomplished with scintillation counters mounted at the magnet aperture (Ca), along the focal plane (Hj), and in three positions above the focal plane (C0,C1,C3) as shown in Figure 2.

The aperture counter was required to provide clear timing signals, which enabled a background reduction of 90% in a typical nuclear (p,π) run. High count rates in the aperture counter forced reduction of the incident proton beam current to less than 1.5 nA when the spectrograph was at forward angles. The 0.075 cm thick aperture counter has obviously contributed to an energy broadening of two body reaction peaks, however this contribution for a central ray, shown as a function of magnetic field in Figure 3 in comparison with the total peak width for a typical two body reaction, is very small. The typical total resolution obtained with the Pembrooke was $\Delta P/P=0.020$. The full angular acceptance of the Pembrooke was 2.8.

Part of the hodoscope counter array along the focal plane is shown in Figure 4. The 3 cm wide counters had a nominal 1cm overlap with their neighbouring counters giving 47 momentum bins

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Figure 2 Schematic diagram of the Pembrooke pion spectrograph as seen looking down beamline 1a.

from only 24 counters. The momentum bin labels are j,k,l for the jth, kth and lth single counters firing and j.5,k.5, etc... for the jth and kth, kth and lth, etc... counters firing simultaneously. All of the hodoscope counter photomultiplier tubes were magnetically shielded against stray magnetic fields.

The large "C-counters" , located above the focal plane, provided both timing and pulse height information for the

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Figure 3 The components of the Pembrooke resolution, $\Delta P/P$, due aberration, Ra; the detector size, Rd ; and the target to: vertical spot size, Rt have been taken from Lee (1975). Along Rk, the breadening due to the proton beam energy width and with reaction kinematics, these factors added in quadrature agree with the width from the Monte Carlo "geometrical" distributions for the reaction ${}^{1}\text{H}(p,\pi^{+}){}^{2}\text{H}_{\bullet}$ The effective increase in the vertical spot size due to multiple scattering in the Ca counter and pole face scattering lead to more broadening, clearly quite significant at lower magnetic fields. The pole face scattering Rpfs, was determined from the Carlo contribution, Monte simulation.



Figure 4 Schematic diagram of part of the hodoscope array showing the labelling, orientation and overlapping of the counters.

particle identification and background elimination. All Ccounters had RCA8575 phototubes on each end, thus two signals could be observed from each scintillator.

A simplified block diagram of the detection system electronics is given in Figure 5 . An event was typically defined as

$$E = CA \bullet \sum HJ \bullet CO \bullet C1$$

and a random event as

2 Erand =
$$\sum HJ \bullet CO \bullet C1 \bullet CA (delayed)$$

For each event a bit pattern which indicated every counter which fired, eight pulse heights and eight timings (including the time of an event with respect to the cyclotron rf pulses) were recorded in a computer buffer. After every 500 events, the data in the buffer and a bank of 24 CAMAC scalers were generally



Figure 5 The Pembrooke electronics, showing how any of the hodoscope counters in coincidence with the C-counters may trigger an event.

transferred to magnetic tape. For diagnostic purposes the data did not have to be transferred to tape, and any part of the bit pattern, pulse height or timing spectra could be observed on line.

A simple estimate of the solid angle of the system mav be The counters made. were all sufficiently wide such that the magnet apertures essentially define the solid angle. In the vertical dimension of the entrance aperture at a particular path length of 40.311 cm is 10.8 cm and the width of the exit aperture at a nominal path length of 127.13 cm is 1.906 cm, from which a geometrical solid angle is given by equation 3.

3
$$\Delta \mathcal{N} = 4 \arctan\left\{\frac{10.8}{2 \times 40.31}\right\} \arctan\left\{\frac{1.906}{2 \times 127.13}\right\} = 3.99 \text{ msr}$$

Using this value of the solid angle is somewhat naive because of several effects. In realistic situations the beam spot on target was larger than a point source. Frequently targets were not surrounded by vacuum, giving rise to a scattering of pions in air which could mean either a net gain or loss of particles through the magnet entrance. A very important effect is that of pcle face scattering, which is small angle Coulomb multiple scattering of the pions from the pole face steel. Pole face scattering leads to a larger solid angle than the naive calculation above. To accurately determine the system solid angle a Monte Carlo simulation of the system was made.

The Monte Carlo program REVMOC (Kitching 1973, and 1971. Kost 1977) was improved with the addition of two major subroutines and numerous modifications. These changes and REVMOC calculations are discussed in Appendix A. Typical variation in the calculated "geometrical" solid angle VS hodoscope counter for two magnetic fields is shown in Figure 6 . The solid angle including scattering effects depends upon the momentum distribution of pions and a general correction technique is discussed below.

In addition to the above solid angle considerations a simulation of the Pembrooke system was desirable to accurately determine correction factors for pion decays and to resolve the



Figure 6 The Pembrooke solid angle as a function of hodoscope counter from the Monte Carlo simulation when no multiple or pole face scattering has been included. The error bars reflect the statistical uncertainty in the calculation. The lines are least squares quadratic fits to the data shown for B=8.6 kG and (data not shown) for B=6.5 kG. Note the ordinate scale is far from zero.

ambiguity in the fraction of thicker targets which could be "seen" by the spectrograph.

TWO factors make the pion decay problem difficult. The decaying fraction of pions with a particular energy travelling along a certain path length may be readily determined, however the pions focussed at a particular hodoscope counter will have different path lengths through the magnet, depending had many upon their incident direction. The second problem, demonstrated schematically in Figure 7, is to determine the fraction of the decay muons which travel through the remainder of the system and can not be separated from pions. Typical pion decay corrections for the case of incident pion beams with gaussian momentum distributions are shown in Figure 8 .



Figure 7 Two possible cases of a decay muon being detected as a pion in which (a) the correct hodoscope bin was incremented, and (b) an incorrect counter was triggered by the decay muon. Timing and pulse heights cuts do not generally discriminate against mucns.

To correct the observed distributions for both pion decay, Coulomb scattering and the variations in solid angle with a unique factor is not possible because of the effects mentioned above. For any particular magnetic field the observed distribution can be corrected with themultiplication of matrices of coefficients, which have been derived from a large number of Monte Carlo simulations. The off-diagonal elements of the matrices reflect the shifts and broadening introduced into hodoscope distribution by pole face scattering and the muon the tails from pion decays below the focal plane.



Figure 8 The fraction of pions which survive as a function of hodoscope counter.

The matrices are applied in a matrix equation:

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$$\mathcal{E}_{JT}^{PFS} \left[\mathcal{E}_{TK}^{\pi\mu} N_{\kappa}^{\text{geometrical}} \right] = N_{J}^{\text{observed}}$$

where Nobserved is the hcdoscope distribution observed along the N geometrical Pembrooke focal plane. is the hodoscope distribution which would have been observed without the distorting effects of pole face scattering and pion decays. The two matrices were by column, by observing defined cclumn the final simulated distribution for a very narrow momentum impulse to particular ccunters across the hodoscope array. The two matrices were separately defined by not allowing pole face scattering in the

pion decay simulation and vice versa. To reduce costs. simulations were made for between four and twelve of the counters receiving impulses at a particular magnetic field, and then the remaining matrix elements defined by quadratic interpolations along the diagonals. The matrix corrections are



Figure 9 The fraction of the 3.46cm diameter liquid hydrogen target from which pions can be observed.

accurate to within 10 %, and are best used by making an estimate of the $N_{\kappa}^{geometrical}$, performing the matrix multiplications and comparing the result with the $N_{\pi}^{observed}$. This iterative technique has been frequently used in the calculations discussed in chapters 3 and 4.

Both matrices were expected to vary with pion energy and were derived from a series of Monte Carlo simulations over a range of magnetic fields. Each matrix element was then fit with a quadratic in magnetic field and all subsequent calculations referred to the 47x47x3 matrix of coefficients.

A significant fraction of the ¹H and all of the ²H studies discussed in charter 3 involved the use of a 3.16 cm thick liquid hydrogen target which is larger than the 1.91 cm wide entrance aperture of the magnet hence, for example, pions from the front face of the target could not directly enter the system without scattering. Both the pion decay corrections and an effective target thickness for the liquid hydrogen target were determined from the REVMOC Monte Carlo calculations described in Appendix A. The LH2 effective target thickness has been shown as a function of pion lab angle in Figure 9.

The unequal overlaps of the hodoscope counters lead to disjointed distributions, as is often the case with this type of arrangement. For graphical purposes a weighting function was derived from inclusive ${}^{12}C(p,\pi^+)X$ spectra which vary smoothly with momentum. The weighted distributions were easier to recognise but the unweighted distributions were used for cross section calculations.
TABLE I (Lee, 1975)TABLE OF PEMBROOKE CHARACTERISTICSPHYSICAL DETAILS :GAP WIDTH _______ 1.905 cm

MAXIMUM MAGNETIC FIELD 12.0 kG POLE FACE RADIUS 50.0 cm EFFECTIVE FIELD RADIUS 51.39 cm

PROPERTIES AT IOKG FIELD: CENTRAL PION KINETIC ENERGY 65.2 MeV CENTRAL PION MOMENTUM___ 149.9 MeV/c MOMENTUM RANGE (AP/B) ____ 0.42 FOR CENTRAL RAY. MAGNIFICATION _____ 2.38 DISPERSION _____222.9 $(cm/(\Delta P/P))$ ABERATION _____ ____1.436 cm **RESOLUTION DUE TO =** TARGET SIZE = I cm ___ 0.0107 TARGET SIZE = Imm_0.0011 COUNTER SIZE _____0.0096 ABERRATION____0.0065 TOTAL RESOLUTION = TARGET SIZE=1cm_0.0157

TARGET SIZE = Imm_0.0116

SECTION 2.2

BEAM MONITORING

Mcnitoring of both beam intensity and polarization for the experiments on beamline 1 was accomplished with a proton polarimeter. The two arm polarimeter assembly was similar to one designed for use in the TRIUMF - BASQUE experiments on beamline 4 (Ludgate, 1976), however different sized counters were used. From the polarimeter and its electronic logic diagram in Figure 10 it is clear how the monitor scalers were inhibited during incident beam spin flipping and when the computer was busy processing an event.

In principle the counters are arranged to detect elastically scattered protons from the hydrogen atoms of a thin CH2 target, however a significant background from quasi-elastic scattering of protons with the carbon also leads to monitor events. A monitor count corresponded to the coincident detection of a proton scattered at 26° to the right (left) with respect to the beam direction and a recoil proton detected at 60° to the left (right).

The analysing power of this monitor has been determined from a recent phase shift analysis of p,p scattering (Bugg ,1978) and the effect of a carbon background has been determined by Ludgate (1976). A graph of the analysing power as a function of incident proton energy is given in Figure 11. Typical beam polarizations were of the order of 65 %.

The number of monitor counts is related to the number of incident protons by the calibration parameter, CA, which is



Figure 10 A schematic of the polarimeter and its electronics. symbol indicates a random delayed signal. The \sim During the Pembrooke runs the polarization was either off or up/down so that only two sets of L, Lrand, R, and Rrand scalers were required. An off run would have two identical sets of the scalers recorded. All scalers and event interrupts are inhibited during spin flips.

defined below:

5
$$C_{A} = (L+R) rate / (2 pt I)$$

where (L+R) is the total number of polarimeter counts/second; (ρ t) is the polarimeter target areal density in mg/cm²; and I is the beam current in (nA).

The absolute intensity calibration of the polarimeter has been accomplished in two ways. A series of carbon activations



Figure 11 The polarimeter polarization analysing power as a function of proton energy (Bugg 1978). The line is an empirical fit.

have been used directly measure CA(Tp) to for the CH2 polarimeter the target, and contribution from quasifree CA(Tp) carbon from carbon. scattering using a thin carbon polarimeter target. This technique is described in Appendix B. The uncertainty of Leam normalization is 15%, largely due to the 11C production cross section uncertainties and the of measurement the NaI detector solid angle , important in the determination of the number of nuclei produced 11C in the activation target.

The free p-p scattering cross sections (Bugg et al, 1978) can also be used to absolutely calibrate the beam intensity monitor provided the solid angle of the polarimeter and the quasifree scattering contributions from carbon are known. In a similar manner as in the Pembrooke solid angle calculation, a Monte Carlc approach enables inclusion of beam spot and coulomb



Figure 12 The polarimeter intensity calibration parameter, CA based upon carbon activation measurements (the linear fit to the data) and the polarimeter Monte Carlo simulation for both protons and CH2 targets.

multiple scattering effects. In addition a model to calculate the quasifree scattering contribution can be explored with a Monte Carlo approach. To do these calculations a Monte Carlo program, POL6, was written to simulate the operation of the polarimeter. This computer program is described in Appendix C. A graph of the calibration parameter, CA as a function of incident proton energy is given in Figure 12.

SECTION 2.3 SINGLE DIFFERENTIAL CROSS SECTION AND

POLARIZATION ANALYSING POWER CALCULATIONS

In all cases where two body reactions have been observed a single differential cross section has been calculated with the fcllcwing equation.

$$\frac{d\sigma}{d\Omega} = \frac{(1-B_P) N_{T}}{N_{tgt} N_P E \Delta \Omega}$$

6

the evaluation of the efficiency factor, \mathcal{E} and solid angle, $\Delta \mathcal{R}$ were discussed in section 2.1. Bp is the fraction of background particles counted under the two-body peak. This correction was determined by fitting the cbserved distributions with simulated distributions for each two body peak and for the general pion background from inclusive (p,π) reactions. The number of pions, N_{n} is determined by integration of the appropriate two body peak after background effects have been removed in the data analysis. The constraints determined in the preliminary analysis utilizing the pulse height and timing dimensions of data stored on tape varied significantly for the various reactions studied anđ are separately discussed in the appropriate sections. The multi-dimensional analysis program, KIOWA (Stetz, 1975) was used for these calculations. The number of protons to strike the target during the run, Np, has been determined in section 2.2 to be:

$$N_{p} = \frac{(L+R)}{(pt) C_{A}} \times 3.125 \times 10^{9} \text{ protons}$$

Ntgt is the areal density of the target in use, including an effective increase due to target angle with respect to the beam direction. Ntgt is expressed in units of target atoms/cm².

The polarization analysing power is given by:

8

$$A_{\pi} = \frac{\frac{d\sigma}{d\Omega}(+)}{\frac{d\sigma}{d\Omega}(+)} - \frac{d\sigma}{d\Omega}(-)}{\frac{d\sigma}{d\Omega}(+)} + \frac{\rho(+)}{d\sigma} \frac{d\sigma}{d\sigma}(-)}{\frac{d\sigma}{d\Omega}(+)}$$

where the \pm signs indicate proton spin orientation and P is the measured beam polarization. The sign convention for A_{TT} is the Madison convention (Barschall, 1971). That is $d\sigma^{(4)}/d\Omega$ is the differential cross section measured when the proton spin is in the direction $\vec{k_p} \times \vec{k_m}$.

SECTION 2.4 DOUBLE DIFFERENTIAL CROSS SECTION

AND POLARIZATION ANALYSING FOWER CALCULATIONS

An inclusive reaction is one in which the final state includes more bodies than were detected. For all of the inclusive reactions which have been observed with the Pembrooke system a double differential cross section has been calculated with the following equation.

9
$$\frac{d^2 \sigma}{dT_{\pi} dN} = \frac{N_{\pi}}{N_{tot} N_{p}} \frac{P_{b} E_{\pi} (H+22.6)(H+72.47)}{\Delta \Omega 49.87 P^{2}}$$

The number of picns, N_{γ} , is the number counted by a particular hodoscope counter. The factors Ntgt and Np are the same as previously discussed. The remaining factors result from calculating the momentum bite from an empirical relation of the hodoscope counter, Hj; magnetic field, B; and the momentum, P; and converting to an energy bite from a momentum bite. The empirical relation of P,B and Hj is

10
$$B = \frac{P}{38.48} \begin{bmatrix} 1 + \frac{49.87}{H+22.6} \end{bmatrix}$$

The Pembrooke data has been reduced with KIOWA calculations as described in the discussions of each reaction studied. The resulting hodoscope distributions were too bulky to be further reduced by hand, so the computer programs CROSS7 and CROSS8 were written to calculate the factors of equation 9. Descriptions of CROSS7 and CROSS8 are given in Appendix D. There is an

important difference between the programs in the technique of applying the pole face scattering and pion decay corrections. N is presumed to be distribution In CROSS8 the a "geometrical" distribution, corrected for pion decays and pole face scattering elsewhere. In CROSS7 N_{rr} is presumed to be an "observed" distribution and is explicitly corrected for pion decays using the picn decay matrix and a correction for pole face scattering which assumed a near uniform incident momentum distribution. The latter correction is believed reasonable for momentum distributions without significant peaks.

The analysing power for inclusive reactions is given in analogy with section 2.3 to be:

11

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$$A_{\pi} = \frac{\frac{d^{2}\sigma}{dT_{\pi}d\mathcal{N}}}{p^{(-)}\frac{d^{2}\sigma}{dT_{\pi}d\mathcal{N}}} - \frac{\frac{d^{2}\sigma}{dT_{\pi}d\mathcal{N}}}{\frac{dT_{\pi}d\mathcal{N}}{dT_{\pi}d\mathcal{N}}}$$

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CHAPTER 3 PION PRODUCTION REACTIONS IN FEW NUCLEON

SYSTEMS NEAR THRESHOLD

Pion production in few nucleon systems is very important. Reasonable wave functions for the deuteron enable explicit theoretical calculations which may be compared with the wealth of ${}^{1}H(p,\pi^{+}){}^{2}H$ data. Calculations based on models of this simplest of (p, m) reactions give a measure of our understanding of the production and, through the inverse reaction, absorption of pions. Models of the two nucleon reaction have been found to be very important factors in the nuclear (p,π) calculations (Fearing 1975). Experimental studies of few nucleon (p, f)justified in the interest of testing our reactions are also theoretical understanding of reactions involving pions and the deuteron. In this experiment the $^{2}H(p,\pi^{+})X$ reactions have been studied.

In addition the 'H(p,π^+)²H reaction has provided a ready source of picns for calibration purposes of other pion reactions. The prolific 'H(p,π^+)²H reaction was very useful for setting up the spectrometer at the beginning of any particular running period. The reaction was also useful as a test of the Monte Carlo simulation because of the very well defined two body peak in the pion energy spectrum.

SECTION 3.1 PHENCMENOLOGICAL DESCRIPTIONS OF

1H (p,π+) 2H

The phenomenological description of this reaction is normally expressed in terms of a partial-wave expansion (Marshak 1954, Gellman 1954, Mandl 1955, Weddigen 1978, Hsieh 1978). A summary of the transition amplitudes is given in Table II. The seven transition amplitudes are essentially the matrix elements, μ_{e} , in the scattering matrix for the transitions from initial

to final states with up to d-wave pions being included (Mandl 1955). For a proton beam with polarization \overrightarrow{P} incident on an unpolarized target, the single differential cross section can be expressed as

$$\begin{array}{rcl}
12 & 32\pi \, d\sigma &= \vartheta_0 + \vartheta_2 \cos^2 \theta^* + \vartheta_4 \cos^2 \theta^* \\
& d\Omega^* \\
& + \vec{P} \cdot \hat{n} & \sin \theta^* \left\{ \lambda_0 + \lambda_1 \cos \theta^* + \lambda_2 \cos^2 \theta^* + \lambda_3 \cos^3 \theta^* \right\}
\end{array}$$

where $\hat{\pi}$ is the unit vector in the direction $k_{p} \times k_{\pi}$, and where δ_{i} and λ_{i} are related to the amplitudes discussed above. Historically the cosine expansion has been used. Due to the non-orthogonality of powers of cosine, it is desirable to express the single differential cross section in an expansion of normalized Legendre polynomials. With sufficient data the Legendre expansion coefficients are not as sensitive to truncation of the series.

$$32\pi d\sigma = G_0 P_0(\cos \theta^*) + G_2 P_2(\cos \theta^*) + A_3 P_3(\cos \theta^*) + P_{2n} \left\{ A_1 P_1'(\cos \theta^*) + A_2 P_2'(\cos \theta^*) + A_3 P_3'(\cos \theta$$

13

where for the normalized Legendre Polynomials : $\int P_{\mu}(x)P_{\mu}(x)dx = \delta_{\mu}n$ and for the associated Legendre Functions : $\int P_{\mu}(x)P_{\mu}(x)dx = \delta_{\mu}n$

In this case the Gi and Ai coefficients are related to the transition amplitudes. The Gi and Ai coefficients may be determined by utilizing the orthonormality of the Legendre polynomials (Hsieh, 1978), however the data must span wide a angular range to enable integration of d($\cos \theta^{\star}$).

More stringent tests of the theoretical predictions are made with both polarized team and polarized target (see for example, Aprile 1979).

Using a deuteron radius of 4fm, one would naively expect d-wave pions to become significant at a center of mass pion momentum of:

 $P^{*} = \sqrt{2(2+1)} \hbar/4fm = \frac{1}{20.6} MeV/c$

This center of mass pion momentum corresponds to a proton energy of 395 MeV in the reaction $^{-1}$ H(p, π^{+})²H.

TABLE I (JONES 1977)

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(a) ANGULAR MOMENTUM DECOMPOSITION OF THE 1 H(p, π^{+})²H reaction

ia

1

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i.

Initial pp state (deuteron state $ \pi\rangle$)	Amplitude
¹ S ₀ (³ S ₁ ρ) ₀	٩٥
³ P ₁ (³ S ₁ s) ₁	a,
¹ D ₂ (³ S ₁ p) ₂	a,
³ p(³ S_d)	2 0 ج
³ P ₂ (³ S ₁ d) ₂	Q A
³ F ₂ (³ S ₁ d) ₂	G _R
³ F_ (³ S, d)	۵ م
5 1 5	0
(b) EXPANSION OF γ_i in terms of \mathfrak{a}_i	
Term Type γ ₀ γ ₂	Υ4
la ₀ 1 ² p1 ² 2 0	0
$ a_1 _1^2$ $ s ^2$ 2 0	0
la ₂ 1 ² 1p1 ² 1 3	0
la ₃ 1 ² Id1 ² 5/2 -3/2	0
la ₄ l ² Idl ² 5/2 5/2	0
la ₅ l ² idl ² 5/7 30/7	-25/7
la ₆ l ² Idl ² 5/4 3/2	5/4
Re a*a ₂ p-p 2√2 -6√2	0
Rea <mark>*</mark> a ₃ s−d − √2 3√2	0
(c) EXPANSION OF λ_{1} in terms of a	
Term Type λ_0 λ_1	λα
$\lim_{n \to \infty} a^{\dagger}_{n}a$, $s-p$ $2\sqrt{2}$ O	0
$\lim_{n \to \infty} a_n^2 = a_n - a_n $	0
lm a [*] a _n s−p −2 0	0
$\lim_{n \to \infty} \frac{1}{2} = \frac{1}{2}$ $\lim_{n \to \infty} \frac{1}{2} = \frac{1}{2}$	0
$\lim_{n \to \infty} a_{n+1}^* = p - d = -2\sqrt{2} = 0$	9√2
$\lim_{n \to \infty} a_n^2 = a_n - a_n = 0$	6√5/2
$\lim_{a \to a} a = \frac{1}{2}$	0
$\lim_{n \to \infty} a_n^{\pm} a_n = p - d = 3 = 0$	15
$\lim_{n \to \infty} a_{\pi}^* a_{\pi} = s - d = 0 - 6\sqrt{5/7}$	0
lm a [#] a_ p−a O O	6√5/7

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<u>SUMMARY AND H(p.m+)²H</u> <u>1H(p.m+)pn_RESULTS</u>

¹H (p, π^+)²H reaction was extensively studied using both The CH2 and liquid hydrogen targets. For either target appropriately scaled background spectra (carbon or empty target were subtracted. In addition runs) random events were subtracted in the preliminary KIOWA analysis. The peak was very and no additional cuts in pulse heights or timing were clear required to eliminate backgrounds. A significant fraction of the ${}^{1}H(p,\pi^{+}){}^{2}H$ and ${}^{1}H(p,\pi^{+})pn$ data runs were simulated and the Monte Carlo simulated distributions were then scaled to fit the experimental distributions. Several of these fits to data are shown in Figure 13.

A picn decay and solid angle correction have been determined from the Monte Carlo simulations for the momentum cuts applied to each peak. The product of these factors is shown in Figure 14 . The fraction of pions arising from the ¹H (p,π^+) pn reaction was also determined from the fitting of simulations to the observed data. Pions from the breakup reaction make up 0.06 cf the pions in the peak. From these parameters a single differential cross section was calculated using equation 2. For some of the simulations a small shift of the peak in the momentum distribution is observed with respect to the data. These shifts are possibly due to the effective edge model of the magnetic field used in the Monte Carlo program, or to actual vertical shifts of the beam on target



Figure 13 Four typical Monte Carlo simulations fitted to data for ${}^{1}\text{H}(p,\pi)$ at a proton energy of 400 MeV. In each case the inset shows the contributions from ${}^{1}\text{H}(p,\pi^{+}){}^{2}\text{H}$ and from ${}^{1}\text{H}(p,\pi^{+})$ pn. The Monte Carlo program is discussed in Appendix A.



product of pion decay efficiency and solid angle Figure 14 The as a function of magnetic field for the¹H(p,π+)²H reaction. included in 6.5 hodoscope counters have been the peak integration.

which are reflected as shifts along the focal plane. In all of the simulations, the $pn\pi^+$ and π^+d distributions were shifted so that the two body peaks overlapped.

it has been common to assume that only s-Near threshold and p-wave pions play a role in the ${}^{1}H(p,\pi^{+}){}^{2}H$ reaction. It is from studying the amplitudes in Table II that in the clear threshold region only the δ_{o}, δ_{2} and λ_{o} coefficients are to this experimental work the maximum proton non-zero. Prior energy for which the threshold description is valid was believed to be 425 MeV (Albrow et al, 1971). Representative samples of single differential cross section and analysing powers for

¹H (p, π^+)²H for several proton energies are shown in Figure 15 and Figure 16 . Comparisons with earlier work (Dolnick 1970) have been shown in Figure 17 . Clearly the data are in agreement in the limited kinematical region of the earlier data. A11 of the new ¹H(p,π^+)²H results are tabulated in Tables III and IV. The analysing power results are shown in Tables V:, VI and VII. The relative uncertainties quoted are discussed below. Significant shifts were observed in the two cases where a general comparison of the cross sections based upon data from the CH2 and LH2 targets at the same proton energy. These have been attributed to variations in the Pembrooke acceptance as a function of beam position on target which varied between running periods with different beam tunes. These variations lead to an absolute uncertainty of \pm 10 % in cross sections which, coupled with the pclarimeter intensity calibration uncertainty, gives a total absolute uncertainty of \pm 17% for all data. The typically 6% relative uncertainty in the cross sections arises from a sum in quadrature of the 2% statistical uncertainty, the 2% pion background fraction (Bp) uncertainty, and the 5% uncertainty in the product of efficiency and solid angle. For the analysing powers an absolute uncertainty of ±5% accounts for the uncertainty in the pp polarization and the contribution of quasi-free scattering from the carbon in the CH2 target.

The asymmetry of analysing powers about 90° in figure 17 is a clear indication cf non-zero coefficients of odd powers of $\cos \theta^*$ in equation 2. A complete evaluation of the χ_i coefficients of equation 2, requires a knowledge of the



Figure 15 The angular distributions of single differential cross section for the reaction ${}^{1}\text{H}(p,\pi^{+}){}^{2}\text{H}$ at several proton energies compared with CERN2 (Richard-Serre 1978). The open and closed circles for the 425 MeV data correspond to data using CH2 and LH2 targets.

unpolarized single differential cross sections over a wider range of θ^{*} than could be achieved with the limited angular range of the Pembrcoke spectrometer. Thus to estimate the ratio of δ_0/δ_2 , the new data were fitted along with other recent data (Dolnick 1970, Axen 1976, Aebischer 1976, Preedom 1978) to $\gamma \gamma$, the center of mass pion momentum. Two functions cf of 80/82 functional forms have been used giving values within 10% (Walden 1979). A convention begun by consistent to



Figure 16 Polarization analysing powers for the reaction ${}^{1}\text{H}(p,\pi^+){}^{2}\text{H}$ as a function of center of mass pion angle for several proton energies. The lines are Legendre polynomial fits to the data.



Figure 17 Polarization analysing powers for the reaction ${}^{1}\text{H}(p,\pi^+){}^{2}\text{H}$ at 425 MeV proton energy compared with the earlier results of Dolnick (1970). The lines are Legendre polynomial fits to the data.

Akimov (1958) of presenting the λ_i coefficients normalized by $\forall_0 + 1/3 \forall_2$, a quantity approximately proportional to the total cross section in this energy range, has been followed. The results are shown in Figure 18 and are clearly consistent with the trends from the higher kinematical realm (Albrow 1971). Detailed results are given in Table VIII.

Clearly the non-zero values of λ_1 , and λ_2 indicate that d-wave pion production is playing a significant role for pion



Figure 18 Normalized λ_i parameters from the $\cos heta^{st}$ expansion of A_{π} for the reaction ${}^{1}H(p,\pi^{+}){}^{2}H$. The open circles are from the data of Albrow (1971) and the triangle is from a combination of this work with Dolnick's (1970) . The x symbols are from this experiment. The line is an interpolation of Niskanen's theoretical calculation (1978). The non-zero λ_i coefficient as low as η =0.45 is an indication of d-wave pions. Thus the threshold region, in which only s- and p-wave pions are observed, is up to proton energy of 320 MeV.

cms momenta as low as γ =0.5.

The non-zero d-wave contributions have been predicted by Niskanen (1978) and Lazard (1970) .

A11 analyses to date have fitted series which were truncated after $\cos^2 \Theta^{*}$. It is not correct to assume for example $\lambda_3 = 0.0$ because of that thecorrelations between the coefficients in a non-orthogonal series. However the limited angular range of this experiment leads to the problem that four parameters cannot be fit with certainty. The correlation of coefficients is in theory avoided when the single differential cross section is expressed as in equation 13 which uses an expansion of crthogonal Legendre polynomials. The results for the Ai coefficients are shown in Figure 19 and tabulated in Table IX. The statistical uncertainty of the coefficients was unfortunately not improved in this case because of the limited number of data points. The uncertainties due to truncating either series were explored at 425 MeV, where Dolnick's data (1970) was combined with the new data. For this more complete data set the cosine and Legendre polynomial expansions were fit with up to nine parameters. The uncertainty of the cosine expansion rarameters was larger and the values varied significantly between orders of the expansion. The Legendre expansion ccefficients however showed little variation from order four upwards.

All of the ¹H(p,π^+)pn data were recorded simultaneously with the two body ¹H(p,π^+)²H data, and the preliminary data





Figure 19 Normalized Ai parameters from the Legendre polynomial expansion of A_{π} for the ¹H(p, π +)²H reaction as a function of the center of mass pion momentum. The symbols are the same as in fig.18



Figure 20 CMS double differential cross section for $^{1}H(p,\pi^{+})pn$ as a function of Θ^{*} for pions with CMS energies ~ 20 MeV below the two body peak energy.

reduction was made in the same KIOWA analysis as the two body data. Direct simulations of these two reactions were made for several data runs. In the ${}^{1}H(p,\pi^{+}){}^{2}H$ simulation the momentum distribution of the pn π^+ reaction was presumed to be uniform. Although these simulated distributions might not reflect any energy dependence of the three body data, the fits were adequate in the determination of backgrounds to the two body reaction. In order to allow a more general correction of the $pn\pi^+$ data with less restrictive assumptions, four distributions were prepared with simple assumptions for the incident momentum distributions. The incident distribution for the two body part of the spectrum was a gaussian based upon fits to the Monte Carlo simulations to the two body reaction. The other three distributions were non-zero up to the $pn\pi^+$ threshold and were chosen as a constant, linear in momentum and quadratic in momentum. Each of these incident or geometrical distributions was multiplied by the pion decay and pole face scattering give four observed distributions. A matrices to linear ccmbination cf these four distributions were then fitted to the observed data.

$$Data_{i}(P_{i}) = aF_{i}(G(P_{i})) + b_{i}F_{i}(i) + b_{2}F_{i}(P_{i}) + b_{3}F_{i}(P_{i}^{2})$$

Here G(Pi) is the gaussian part, and

$$F_i(x_i) = \mathcal{E}_{i\varrho}^{PFS} \mathcal{E}_{ek}^{\pi\mu} x_k$$

The pnm^+ geometrical distribution was then taken as the sum of the product of the three relevant fit parameters and their respective geometrical distributions.

 $Geo_{i}(P_{i}) = a G(P_{i}) + b_{i}(I) + b_{2}(P_{i}) + b_{3}(P_{i}^{2})$

From this distribution cross sections were calculated with equation 9 using the program, CROSS8.

The cross sections for $H(p,\pi^+)$ pn reactions are important factors in theoretical nuclear pion production calculations of Beder (1971) and Hsieh (1978). These results give a measure of the importance of the nucleon-nucleon isospin 1-1 transition, whereas the ${}^{1}H(p,\pi^{+}){}^{2}H$ is a pure isospin 1-0 transition. Representative samples of the calculated double differential section and analysing power are shown in Figure 20 and cross Figure 21. The analysing powers for a particular proton energy do not tend to vary much with pion energy in this kinematic and are similar to those of the ${}^{1}H(p,\pi^{+}){}^{2}H$ reaction at realm that angle. This is an indication that the spin dependence of the reaction is essentially independent of the degree of np binding. Caution must be exercised in this conclusion for pions very near the two body peak because of the dramatic experimental influence of the pole face scattering tail of that peak. The double differential cross section have been tabulated in Table X and the analysing powers in Table XI for only one or two points



Figure 21 Polarization analysing powers for ¹H(p, π^+)pn with pion energies ~ 20 MeV below the two body peak. The line is the ¹H(p, π^+)²H analysing power for 400 MeV. The open circles are for proton energy 425 MeV and the closed circles are for 400 MeV.

at each angle, because of the very large tail correction for pions near the peak. The cross section quoted is for pions well below the two body peak. The relative uncertainty is dominated by the 10% accuracy associated with using the pion decay and pole face scattering matrices. The absolute uncertainty is as discussed above. The analysing powers have only the statistical uncertainty quoted.

			L <u>d</u> σ (<u>μb</u> dΩ* (sr) [`] H(p,π [*]) [−] F	1	
	Tp=400 MeV	- 1997 - 1 99		Tp= 42	25 MeV	
θ*	$\frac{d\sigma}{d\Omega^*}$ (CH ₂)	θ*	<u>d</u> σ(CH ₂) dΩ*	θ*	$\frac{d\sigma}{d\Omega^*}$ (LH ₂)	
61.7	48.0 (2.9)	84.0	40.9(2.5)	75.7	42.3 (2.5)	
71.0	36.8 (2.2)	90.0	401 (24)	82.7	38.9 (2.3)	
81.0	32.8 (2.0)	96.7	41.6 (2.5)	90.0	34.7 (2.1)	
90.2	33.8 (2.0)	108.0	62.4 (3.7)	97,3	39.1 (2.3)	
91. 3	28.5 (1.7)	4.	74.3 (4.5)	104.0	42.8 (2.6)	
98.8	30.9 (1.9)	120.3	81.9(4.9)	111.1	55.1 (3.3)	
109.0	39.2 (2.4)	135.1	151.5 (9.1)	118.0	73.6 (4.4)	
118.9	54.8 (3.3)	144.1	127.2(7.6)	132.3	124.2 (7.4)	
12 7.1	69.7 (4.2)			139.0	143.8(86)	
135 1	877 (53)					

4,

+2 1....

 $^{1}H(p,\pi^{+})^{2}H$ TABLE IV

Tp=350 MeV -Tp=375MeVθ^{*} θ^{*} $(CH_2) \frac{d\sigma}{d\Omega^*} \frac{\mu b}{sr}$ θ^* (LH) (CH2) <u>dσ μb</u> dσ μb dΩ^{*′}sr $d\Omega^*$ sr 62.0 36.6 (2.2) 65.2 27.5 (1.7) 65.8 19.5 (1.2) 89.7 20.0(1.2)70.4 24.1 (1.5) 73.1 17.1 (1.0) 102.3 23.2 (1.4) 76.1 21.0 (1.3) 78.0 14.0 (0.8) 118.0 41.3 (2.5) 90.0 19.4 (1.2) 84.0 12.9 (0.8) 118.2 41.9 (2.5) 104.3 23.4 (1.4) 90.3 12.0 (0.7) 110.6 26.6 (1.6) 96.0 14.2(0.8) 116.3 30.8 (18) 15.0 (0.9) 102.0 107.0 17.5 (1.1) 114.8 26.6(1.6) - Tp= 320 MeV-- Tp = 330 MeV θ* θ^{*} (LH₂) $\begin{array}{c} (CH_2) \quad \frac{d\sigma}{d\Omega^{k}} \quad \frac{\mu b}{sr} \end{array}$ dσ dor μb dΩ* sr 67.4 18.3 (1.1) 74.2 9.1 (0.5) 70.0 13.7 (0.8) 80.2 8.3 (0.5) 74.7 16.4 (1.0) 86.1 7.7 (0.5) 14.4 (0.9) 90.7

				r) H	
Tp	≖400 MeV		Tp= 425	MeV	
<i>θ</i> * (c	$(H_2) \land \pi$	<i>θ</i> * (c	Ή,) А π	<i>₿</i> *(∟н₂)	Απ
61.7	-0.062(0.015)	78.0	-0.247(0.018)	75.7	-0.184(0.011)
7 1.0	-0.212 (0.017)	8 9.9	-0.386(0.016)	82.7	-0.312(0.011)
81.0	-0.363 (0.022)	90.0	-0.369(0.017)	90.3	-0.342(0.010)
90.2	-0.458(0.022)	96.7	-0.369(0.017)	97.3	-0.315(0.011)
91.3	-0.481 (0.022)	102.0	-0.318(0.016)	104.0	-0.258(0.011)
9 8.8	-0.408(0.019)	108.0	-0.267 (0.015)	111.1	-0.174(0.011)
109.0	- 0.268 (0.022)	4.	-0.091 (0.012)	118.0	-0.071(0.010)
118.9	-0.145(0.011)	120.3	-0.103(0.012)	124.4	-0.032(0.010)
127.1	-0.066(0.016)	12 7.1	-0.030(0.012)	132.3	0.010(0.010)
135.1	-0.027(0.012)	135.1	0.028(0.011)	139.0	0.039(0.009)
145.0	-0.008(0016)	144.1	0.022(0.013)	145.4	0.051(0.009)
		155.0	0.037(0012)	153.2	0.035(0.009)
			TABLE TT H	ӊ <i>π</i>) ² Н	
	—Tp = 375 MeV—.				
∂ * (0	CH_2) $A\pi$	θ* .	(CH ₂) A π	<i>ө</i> * (lн	ε) Απ
62.0	-0.153 (0.016)	64.0	-0.194 (0.018)	65.8	-0.253 (0.008)
77.8	-0.327 (0.021)	6 5.2	-0232(0.015)	73.1	-0.358 (0.008)
89.7	-0.513 (0.014)	66.9	-0.206(0.075)	78.0	-0.471 (0.007)
102.5	-0.462 (0.0 7)	70.4	-0.302(0.014)	84.0	-0.502 (0.008)
118.1	-0.2 45 (0.014)	76.1	-0.422(0.015)	90.3	-0.532 (0.008)
154.9	-0.082 (0.017)	89.8	-0.554 (0.018)	96.0	-0.546 (0.008)
		90.0	-0.581 (0.018)	102.0	-0.478 (0.007)
		104.3	-0.475 (0.016)	107.0	-0.429 (0.008)
		110.6	-0.431 (0.015)	114.8	-0.339 (0.0 0)
		116.3	-0.392 (0.015)		
		123.6	-0.237(0.012)		
		127.0	-0.195 (0.012)		
		1303	-0.162 (0.014)		

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		TABLE	VII	^ι Η(p,π ⁺) ² Η	1
T	p= 330 MeV			Tp = 320 Me	/
θ*	(CH ₂) Δ _π		θ *	(LH ₂) A	π
67.4	-0.305(0.0	14)	74.2	-0.38	5(0.006)
70.0	-0.332 0.0	15	80.2	-0.44	B0.016
74.7	-0.358 0.0	13	86. I	-0.45	11008
90.7	-0.537 0.0	17	90.6	-0.47	0 0.008
107.0	-0.447 0.0	8	100.8	-0.46	3 0.006
111.0	-0.408 0.0	21	105.8	-0.42	40.008
7	p = 310 MeV			-To= 305 MeV	
<i>θ</i> *	(LH_2) A _{π}		θ*	(LH ₂) A	· · ·
83.3	-0.39 (0.0	1)	9 0.0	-0.34	"(0.0!)
90.5	-0.42 0.0	1	94 .0	-0.35	0.01
97.8	-0.38 0.0)1	95.0	-0.37	10.0
97.8	-0.40 0.0)t			
105.0	-0.39 0.0	2			

4.0

TABLE XIII i H(p, π^{*})²H NORMALIZED λ_{i} COEFFICIENTS OF COS θ^{*} POWER SERIES OF A π

Tp(Me∨)	7	Target	$\frac{\gamma_0}{\gamma_2}$	$\frac{\lambda_0}{\gamma_0 + \frac{1}{3}\gamma_2}$	$\frac{\lambda_1}{\gamma_0 + \frac{1}{3}\gamma_2}$	$\frac{\lambda_2}{\gamma_0 + \frac{1}{3}\gamma_2}$
305	0.323	LH ₂	1.20 (0.20)	-0.27 (0.020)	· <u> </u>	
310	0.367	LH ₂	0.93 (0.15)	-0.296(0.010)	0.011(0.073)	-0.18 (0.47)
320	0.444	LH ₂	0.67 (0.10)	-0.318(0.011)	0.056(0.028)	0.06(0.20)
330	0.513	CH2	0.52 (0.10)	- 0.306(0.020)	0.108(0.027)	007(0.19)
350	0.641	CH2	0.38(0.04)	-0.314 (0.008)	0.144(0.013)	0.36 (0.04)
350	0.641	LH ₂	0.38 (0.04)	-0.291 (0.006)	0.086(0.012)	0.24 (0.06)
375	0.774	CH ₂	0.30 (0.0 3)	-0.239 (0.009)	0.102(0.019)	0.28 (0.05)
400	0.881	CH2	0.27 (0.0 2)	-0.202(0.006)	0.079(0.014)	0.43(0.04)
425	0.9 85	CH2	0.24 (0.02)	-0161 (0.004)	0.074 (0.025)	0.48(0.05)
425	0.985	LH2	024 (0.02)	-0.149 (0.003)	0.069(0.015)	0.48(0.03)

TABLE IX I H(P, π)²H NORMALIZED A COEFFICIENTS OF LEGENDRE SERIES FOR A π

Tp(MeV) η		TARGET	$\frac{\gamma_0}{\gamma_2}$	$\frac{A_1}{\chi_{0+\frac{1}{3}\chi_2}}$	$\frac{A_2}{\chi_0^2 + \frac{1}{3}\chi_2}$	$\frac{A_3}{\chi_{0+\frac{1}{2}}\gamma_2}$
710	0 2 6 7		-	-0777 (0000)		- 9 -
510	0.367	LH2	0.95(015)	-0.535(0.089)	0.004(0.024)	-0.025(0.063)
320	0.444	LH2	0 67(0 ,10)	-0306(0.031)	0.019(0.009)	0.008(0.026)
330	0.513	CH2	0.52(0.10)	-0.292(0.026)	0.036 (0.009)	0.009(0.025)
350	0.641	CH ₂	0.38(004)	-0.24i (0.005)	0.048 (0.004)	0.048(0.006)
350	0.641	LH ₂	0.38(0.04)	-0244(0007)	0.029 (0.004)	0.0 32 (0.00 8)
375	0.774	CH2	0.30(0.03)	-0183 (0.008)	0.034 (0.006)	0.037(0.007)
400	0.881	CH ₂	0.26 (00.2.)	-0116 (0.005)	0.026 (0.005)	0 057(0 005)
425	0.985	CH2	0.24 (<u>0</u> .02)	-0.05 2(0.006)	0.023 (0.005)	0.064(0.004)
425	0.985	LH2	0.24 (0.02)	-0.065(0.010)	0.025 (0.008)	0.064(0.006)

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θ* (17) 80.1 86.6 11.0	<i>θ</i> * (1 96.9 97.0 103.5 109.0 120.7 127.0 137.5 137.5	$ heta_{\pi}^{*}$ 82.4 88.0 96.9 97.0 103.5 120.7 127.0 137.5 80.1 138.4 111.0	
Tp=350) T# (MeV) 13.8 14.0 14.3 19.5	Тр=425 #4=58 ме 37.7 . 33.6 . 34.6 . 34.6 . 37.3 . 37.5 . 37.7 . 37.5 .	Т# 13.8 14.3 15.3 14.3 14.3 15.3 14.3 15.3 14.3 15.3 14.3 15.3 14.3 15.3	
Mev Α ₇ -0.41(028) -0.28(0.28) -0.51(023) -0.42(0.23)	MeV 0.30 (0.16) 0.29 (0.15) 0.41 (0.19) 0.40 (0.15) 0.40 (0.14) 0.38 (0.15) 0.38 (0.15) 0.38 (0.15) 0.38 (0.15)	25 MeV 1 V) d ² a (16) 1 B (0.17 1.18 (0.17 1.18 (0.17 1.10 (0.17 1.59 (0.17 1.58 (0.17 1.58 (0.17 1.59 (0.17 1.58 (0.17 1.58 (0.17 1.59 (0.17 1.59 (0.17 0.57 (0.0 0.57 (0.0	
θ* 74.5 80.6 87.7 99.2	TABLE <i>θ</i> * 66.3 98.1 104.8 104.8 105.7 116.7 126.1	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	
Tp=33 T#(Me) #d=18 M 11.3 8.6 9.3 12.3	Tp=40 元=48m 29.2 29.2 29.5 31.8 30.1 30.1		
0 MeV 1) A ₇₇ eV) -0.31(0,31) -0.42(0.22) -0.30(025) -0.58(027)	H(p,π)pn) Aπ -008 (0.14) -0.36 (0.14) -0.40 (020) -0.40 (020) -0.47 (0.17) -0.35 (0.14) -0.23 (0.14)	¹ H(p,π ¹)pn 400 MeV IeV) <u>d²σ</u> (<u>μb</u> <u>dT6Ω7</u> ⁴ (<u>μb</u> 0.62 0.07 0.90 (0.11) 0.73 (0.09) 0.81 (0.10) 0.81 (0.10) 0.98 (0.12) 0.96 (0.12) 0.96 (0.12) 1.05 (0.13) 1.05 (0.13) 1.52 (0.18) 1.52 (0.18) <u>dT6Ω7</u> ⁴ srMeV 8MeV) 0.45 (0.06) 0.53 (0.06) 0.53 (0.06)	
	69.2 94.8 123.0		
	Tp=375 MeV T# (MeV) Aπ fmd=37 MeV) 19.6 -0.23 (0.03) 27.5 -0.29 (0.03) 28.8 -0.16 (0.18)	Tp = 375 MeV Tm (MeV) d ² σ μb Tm (MeV) d ² σ (μb dT bΩ2 19.6 0.48 (0.06) 8 27.5 0.46 (0.06) 9 15.8 0.42 (0.05) 0 23.1 0.42 (0.05) 0 28.8 0.91 (0.11)	
			46

SECTION 3.3

SUMMARY OF ${}^{2}H(p,\pi^{+}){}^{3}H$ AND ${}^{2}H(p,\pi^{+})X$ RESULTS

Data for the reactions ${}^{2}H(p,\pi^{+}){}^{3}H$ and ${}^{2}H(p,\pi^{+})X$ were obtained by utilizing the Pembrooke to identify pions produced at different angles from a liquid ${}^{2}H$ target. As in the case of the studies using a liquid ${}^{1}H$ target a properly normalized empty target background had to be subtracted.

KIOWA calculations precisely the same as discussed in section 3.2 were used to reduce the event by event data.

Monte Carlo simulations of the 400 MeV, 120° , ${}^{2}H(p,\pi^{+})^{3}H$ reaction have an efficiency and solid angle correction in agreement with the ${}^{1}H(p,\pi^{+})^{2}H$ simulations, so the same corrections were applied to these data. Unlike the ${}^{1}H$ data however the rapid relative variation of the two body peak height and the breakup distribution height leads to a wide variation in Bp, the fraction of pions within the two body cuts which arise from the breakup reaction. Bp was determined for each run by fitting the data with the same four parameter algorithm as used for the ${}^{1}H(p,\pi^{+})pn$ calculations. The results are shown in Figure 22 and Figure 23 for the ${}^{2}H(p,\pi^{+})^{3}H$ single differential cross section and analysing power, compared with the previous data of Carrol¹, Kallne², Frank (1954), Dollhopf (1973), Auld

 The data of J. Carrol et al. Nucl. Phys. A305 (1978) 502 have been included after multiplying their ²H(p,π[•])³He cross sections by two assuming charge independence.
 The data of J. Kallne et al. Phys. Rev. Lett. 40 (1978) 378 have been included after assuming detailed balance and charge independence.



Figure 22 Semilog graphs of CMS single differential cross sections for the reaction ${}^{2}\mathrm{H}(p, \pi^{+}){}^{3}\mathrm{H}$. Data of comparable proton energies are shown in each of the four parts. The Carrol ${}^{2}\mathrm{H}(p, \pi^{\circ}){}^{3}\mathrm{H}e$ data have been multiplied by two for comparison with the ${}^{2}\mathrm{H}(p, \pi^{+}){}^{3}\mathrm{H}$ data by charge independence. The Kallne ${}^{3}\mathrm{He}(\pi^{\bullet}, n){}^{2}\mathrm{H}$ data have been converted by both detailed balance and charge independence.



Figure 23 Polarization analysing powers for ${}^{2}H(p, \pi^{+}){}^{3}H$ for several proton energies compared with the 400 MeV results of Auld (1979) (open boxes).

(1979), Crewe³, Harting⁴, and Gabathuler⁵, and with the theoretical calculations of Bludman⁶, Fearing (1975) and Green⁷. A more complete tabulation of the results is given in Tablex XII and XIII. The relative uncertainties and the additional absolute uncertainties are as discussed in section 3.2.

Clearly this reaction is strongly forward peaked. The new 400 MeV data is in agreement with the Auld (1980) data in the small region of angular overlap. The 470 MeV Auld data is in disagreement with the pronounced large angle peak of Dollhopf

^{3.} A. V. Crewe et al. Phys. Rev. 118 (1960) 1091. D. Harting et al. Phys. Rev. 119 (1960) 1716. 4. K. Gabathuler et al. Nucl. Phys. B40 (1972) 32. 5. S. A. Bludman Phys. Rev. 94 (1954) 1722. 6. 7.



Figure 24 CMS double differential cross section for the reaction ${}^{2}\text{H}(p,\pi^+)X$ for pions with CMS energy ~ 20 MeV less than the two body peak. The backward peaking is similar to that observed in ${}^{1}\text{H}(p,\pi^+){}^{2}\text{H}$ and ${}^{1}\text{H}(p,\pi^+)\text{pn}$.

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Figure 25 Polarization analysing powers for ${}^{2}H(p,\eta^{+})X$. The lines serve to guide the eye.

(1973). The Carrol data and theoretical calculations at 377 MeV show a large angle increase in cross sections which differ with the new data. These data have not been scaled to remove any possible kinematic effects. The Fearing calculation is otherwise in agreement with data. The Green calculation is typically a factor of four above the data.

The CROSS8 program has been used to calculate double differential cross sections from the ${}^{2}H(p, \pi^{*})X$ data. These cross sections are presented in Table XIV and in Figure 24. The inclusive reaction analysing powers were calculated in the initial KIOWA data handling and are presented in Figure 25 and Table XV. These analysing powers are slightly different from the analysing powers for the two body reaction, showing a possible dependence on the degree of binding on the residual nucleons.

			ТΔ		² H(n	ر ال ³ ام			
		Tp=400 M	eV	To=33	SO MeV	<i>y, 1</i> 11	To=3	05 MeV	
	θ^*	dσ, µ	⊳, θ *	do	<u>, ш</u> в	θ*	.,	σ. <u>μ</u> b.	
		$d\Omega^{r}$ si	,)	Ωb	;*() ;* sr		đ	$\overline{\Omega}$ sr	
	125.5	0.71 (0	.04) 75	- 3 1.50	(009)	68.0) 2	.04(0.12)	•
	138.0	0.68 0	04) 81	0 1.0	5 (0.06)	89.0	0	93(0.06)
	145.5	0.870	1.05/ 90	0.9	2 (0.06)	113.0	0	57(0.03)
			107	.0 0.7	6(0.05)	144.0	o o	57(0.03)
			116	5 0.8	35(0.05)				
			120	.0 0.	5(0.04)				
			TAR	1 F 3 700	2 _{Hín 1}	a ³			
			TAD	To: 33	Πψ.// Ο Μογ	<i>()</i> [1	To - 3	OFMAN	
	θ *	Δ_	6 *	۱۳-33 - ۵	-	A*	10-2		
	125.5	-0.27 (0.0	3) 75.3	3 -0.47	r 7(0.02)	68.0	-0.3	5π 6 (0.0l)	
	138.0	-0.25 (0.0	4) BIC	-0.53	8(0.02)	89.0	-0.6	0(0.01)	
	145.5	-0.13 (0.0	3) 904	0 -0.5	3(0.02)	113.0	-0.4	0 (0.02)	
			107.	0 -0.43	3(0.02)	144.0	-0.1	4 (0.03)	
			116.	5 -0.2	6(0.03)				
			126.	0 -0.2	1(0.02)				
			145.	0 -0.0	9(0.06)				
			TAD	C W (1)	2, " •				
	To a	} 400 MaV	IAB		Н(р,π' ЗО Ма∨)X		T 705	
م	τĽ(Me	V d ² σ_{ub}	A	כ-קו ע₀עT ¹ (M₀V			₽	1p= 503	
0	· #	dTdΩsrMeV	0	T T T T T T		sr Me∨	0	iπ (Me	$\frac{d\sigma}{dTd\Omega} \left(\frac{\mu D}{srMeV}\right)$
ר)	# t =112	Me∨)	(T#1=73 Me	V)		(Ti	# = 58 Me	V)
		070/004							
126.0	87.2	0.36 (0.04)	76.2	54.3	0.23(0	0.03)	68.9	42.3	020(0.02)
136.0	84.9 84.9	105 (0.05)	62.U 01.P	50.4 54 C		202)	93.3	37.2	0.19(0.02)
140.0	040	1.03 (0.137	1080	52 B	0.19 (0	1021	117.8	54.1 40.5	0.29(0.03)
			119.0	49.0	0.33 (0	0.037 0.04)	147.0	40.0	0.24(0.05)
			128.3	49.0	0.64 (0	0.08)			
			147.0	47.6	0. 38 (0	0.05)			
		TA	BLE XX		² Η(p,π)Χ				
	Tp= 400	MeV		Tp=330	Me∨			Tp= 305	Me∨
<i>•</i> - (मुँ	Tir (MeV = 112Me)Α _π ∨)	θ* (Τ	T# (MeV) ≭=73MeV	Α π		θ * (τ	T∰(MeV) ⊒=58 Me	Α π
126.0	87.2 (0.06(0.04)	76.2	54.3	-0.23(0	0.04)	68.9	42.3	-0.20(0.05)
138.0 . 6	84.9 (0.11(0.02)	82.0	55.4	-0.27(0	0.05)	93.3	37.1	-0.46(0.03)
146.0	84.9 0).13(0.03)	91.8	54.6	-0.37(0	0.04)	117.8	34.7	-0.28(0.02)
			108.0	, 52.8	-0.37((0.03)	147.0	40.6	-0.10 (0.04)
			119.0	49.1	-0.18 (004)			
			128.3	49.0	-0.11 (0.02)			
			147.5	47.6	-0.01 (0.03)			
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CHAPTER 4 THE (p.m) REACTION ON LIGHT NUCLEI

NEAR THRESHOLD

Single differential cross section and analysing powers for the reactions ${}^{12}C(p,\pi^+){}^{13}C$ and ${}^{9}Be(p,\pi^+){}^{10}Be$ leading to discrete final states in the residual nuclei have been measured. The most significant aspect of these data with proton energies higher than the initial studies at Uppsala (Dahlgren 1973b, 1973c), is the angular distribution of polarization analysing power, A_{π} .

SECTION 4.1 ANALYSIS OF LOW CROSS SECTION DATA

The single differential cross section for the few nucleon reactions discussed in chapter 3 were sufficiently large (p.1) that backgrounds were dealt with simply by subtraction of suitably normalized empty target runs and random flagged events. reactions with light nuclear targets the runs were For the longer and cross sections smaller so that backgrounds had to be more carefully eliminated. A net time of flight has been derived from the separately recorded time of flights of particles between the CA counter and any hodoscope counter, and between that hodoscope counter and the C-counters. In runs where the picn energy was clearly high enough for the pion to travel through the entire array of C-counters a pulse in C3 was required which helped eliminate low energy background. A typical timing distribution is shown in Figure 26 . In addition one side of one of the C-counters was the pulse height frcm commonly used to identify pions by their rate of energy loss, A typical pulse height distribution is shown dE/dx. in Figure 27 . In some runs the time measured between any hcdoscope counter firing and the cyclotron rf pulses clearly showed evidence for more than one primary proton beam burst per cyclotron rf period. This has been attributed to H- ions within the cyclotron which pass the stripping foil, slip out of phase with the accelerator and de-accelerate back to the stripper. These de-accelerated ions then are stripped giving rise to a small beam burst on target shifted in time from the main burst. A typical time distribution with respect to the rf is shown in



Figure 26 Net time of flight of pions as a function of hodoscope counter. The pion band is obvious and the impact of a C31.C3r cut clearly reduces some of the background. These data are for pions from $^{12}C(p,\pi^+)^{13}C$ at 50°.



Figure 27 Pulse height versus hodoscope counter histogram which shows the pion band.

Figure 28 and the more uncommon multiple beams are clearly shown in Figure 29.

For every run a preview of these distributions was first made, then in subsequent passes with the analysis program KIOWA, applied to the time of flight vs hodoscope counter, cuts were pulse height vs hodoscope counter and to the rf timing spectrum. Data runs with significant multiple peaks in the rf timing spectrum were rejected because of the uncertainty in the beam polarization and the potential small energy shifts in the proton beam and changes in beam optics. The number of polarimeter monitor counts was adjusted for those particles rejected by the rf cut applied for runs where the second peak was small.



Figure 28 Histograms for time of flight with respect to the cyclotron rf pulses for both real and random events.

Figure 29 rf timing histograms for the abnormal de-accelerated beam conditions occasionally encountered.





Figure 30 Hodoscope distributions for real and random events. A larger number of randoms are expected for the higher hodoscope counters because of their proximity to the C-counters.

The hodoscope distribution of pions from the $1^{2}C(p,\pi^{+})$ at several lab angles with a variety of cuts is shown reaction in Figure 30 , Figure 31 and Figure 32 . The threshold of the first three body reaction with pion energies below the ground state is indicated in each figure showing the region where a real pion background is expected to begin. Clearly the ground state could be resolved, however the first three excited states of 13C could not be separately resolved. A similar situation existed for the 'Be reaction, where excited states beyond thefirst excited state of 10Be could not be resolved.

Monte Carlo simulations of the reactions to the ground state, the first excited state of the residual nucleus and of



Figure 31 Hodoscope distributions with pulse height, aperture counter and rf timing cuts.

the inclusive reaction were made at several angles. In the same manner as for the ¹H simulations the product of \mathcal{EAA} was evaluated for the appropriate momentum cuts. This correction factor is shown in Figure 33 . A fit of the Monte Carlo simulations for two cases is shown in Figure 34 .

The fraction of pions within a particular peak due to the pole face scattering tail of pions from other peaks, Bp, was determined for the Be reactions by five parameter fitting of "observed" distributions for the first three states and the inclusive reaction. These "observed" distributions were determined from the multiplication of the pole face scattering decay matrices with three appropriate gaussian and pion



Figure 32 Hodoscope distribution of real events with all cuts applied and finally the distribution of real-random events in which the distribution above the ^{13}C ground state is essentially zero.

distributions and with distributions constant and linear in momentum up to the 10Be breakup threshold. Similarly a four parameter fit to the carbon data (based on the first two peaks and the inclusive distribution) was used to determine Bp for the number of pions associated with each state. N_m, а particular residual nuclear state, is the sum in the peak. Single differential cross sections were calculated with equation 6. The quality of the fits obtained with the matrix technique were generally as good as from the Monte Carlo simulations. In cases where the inclusive part of the spectrum had an energy dependence the matrix fits were in fact better. The costs of using the matrix technique were insignificant



Figure 33 Graph of the pion decay correction x solid angle for two body peaks as a function of magnetic field with 3.0 counters included in the hodoscope cut.

compared with the expensive direct simulations.



Figure 34 Fits of Monte Carlo distributions to ${}^{9}\text{Be}(p,\pi^{+}){}^{10}\text{Be}$ data at small angles were quite successful, however fits to the large angle data were not as good for the ground state of the very low cross section data, and the correspondingly poor statistics.

SECTION 4.2 SUMMARY OF RESULTS FOR 12C (p. T+) 13C AND

9Ee (p, π+) 10Be

The angular distributions of both single differential cross section and analysing power for reactions leaving the residual ¹³C in its ground state and first three excited states are shown in Figure 35 and Figure 36. Similar results for ¹⁰Be left in its ground state and first excited state are given in Figure 37 and Figure 38. Wherever possible the earlier cross section



Figure 35 Angular distributions of single differential cross section for ${}^{12}C(p,\pi^+){}^{13}C$. The data are for the ${}^{13}C$ ground state (circles) and for the sum of the ${}^{13}C$ first two excited states (boxes). The open symbols are from Dahlgren (1973c) at a proton energy of 185 MeV. The circles with x's are recent data from Bent (1978) and the present data (closed symbols) are both for 200 MeV protons. The lines are Legendre polynomial fits.

data of Dahlgren (1973) and Bent (1978) for these target nuclei are shown in the appropriate figures. The only analysing powers



Figure 36 Angular distributions of $A_{\tau\tau}$ for ${}^{12}C(p,\tau){}^{13}C$ ground state (circles) and first excited states (boxes). The lone circle with an x is the only previous data (Heer 1958). The lines are Legendre polynomial fits.

pricr to this data (Heer et al, 1958) are shown in the measured first figures. Poor resolution data was available for aluminum at two angles in the Heer data. The new results are listed in Table XVI and Table XVII. The relative uncertainties in single differential cross section include 5% for the product of efficiency and sclid angle, typically 10% in the pion background from other reactions (Br) and the statistical uncertainty which varied significantly. In addition further absolute a uncertainty of 17% should be used for comparisons with other experiments, as discussed in section 3.2. The uncertainty quoted in the analysing power is primarily statistical.

It is clear that the trends in single differential cross section reviewed by Hoistadt (1976) are also true at the higher



Figure 37 Angular distributions of single differential cross section for ${}^{9}\text{Be}(p,\pi^{+}){}^{10}\text{Be}$ ground state (circles) and first excited state (boxes). The open symbols are the 185 MeV data of Dahlgren (1973) included for comparison with the current 200 MeV data of this work (closed symbols).

energy. In particular the ${}^{12}C(p,\pi^+) {}^{13}Cgs$, ${}^{9}Be(p,\pi^+) {}^{10}Begs$ and ${}^{9}Be(p,\pi^+) {}^{10}Be3.3$ single differential cross section distributions have a relatively structureless slope typical of all cases where the neutron is captured into a p-shell. In cases where the neutron is captured into an s- or d-shell, such as for the excited states of ${}^{13}C$, the angular distribution of the cross section has a dramatic upswing at large angles.

The most striking feature of these 200 MeV results is the shape of the analysing power which is very large and negative near 60° lab angle for all cases observed in this work. This feature is in fact also true for a 2 MeV wide bite of pions at 10 MeV excitation above the 10Be ground state. The statistical



Figure 38 Angular distributions of $A_{\pi\tau}$ for ${}^{9}Be(p,\pi^{+}){}^{10}Be$ ground state (circles) and first excited state (boxes).

uncertainty of the analysing power data is relatively large making interpretation of the subtle differences in distributions for different nuclear states doubtful.

As in the few nucleon data a least squares polynomial fit to both cross section analysing power and is а dood representation the data. of The nuclear (p.π) single differential cross section may be represented in the following expansion:

$$\frac{d\sigma}{d\Omega^*} = \sum_{i=0}^{4} G_i \widetilde{P_i}(\cos \theta^*) + \widetilde{P} \cdot \widehat{m} \sum_{i=1}^{4} A_i \widetilde{P_i}(\cos \theta^*)$$

14

The expansion coefficients are given in Table XVIII. The data

could not be fit to reasonable accuracy with fewer than five parameters. The limited amount of data prevented investigations with higher order expansions. Most of the cross section distributions have large even-order contributions except for ${}^{12}C(p,\pi^+){}^{13}C3.3,3.7$ in which the large angle peaking brings up the odd-order contribution. There is little variation in the coefficients for the analysing power expansion for the ${}^{10}Be$ reactions and the reaction to the ${}^{13}C$ excited states.

		TABLE X	VI	
	⁹ Be(p,π) ⁶ Be _{0.0}	Tp=200 M	eV ⁹ B	$e(p,\pi)^{10}Be_{3,4}$
θ	dσ μb	Απ΄	dσ μb	Απ
	dΩ sr		$\overline{\mathbf{d}\Omega}$ sr	
36. I	0.124 (0.022)	-0.388(0.084)	0.198 (0.028)	-0.052(0.056)
50.2	0.068 (0.009)	-0.707(0.075)	0.146 (0.018)	-0.554(0.047)
60.0	0.059 (0.012)	-0.580(0.088)	0.158 (0.022)	-0.633 (0.054)
70.0	0.042 (0.008)	-0.672(0106)	0.140(0.028)	-0.705(0.059)
90.0	0.025 (0.005)	-0.539 (0.168)	0.144(0.023)	-0.603(0.074)
105.0	0.022 (0.007)	-0.303 (0.096)	0.147 (0.020)	-0442(0036)
129.9	0.109 (0.004)	-0.145(0.153)	0.086(0.012)	-0.134 (0.060)

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TABLE XVII

	12(C(p, π)^{!3}Co	0.0	Tp=200 M	leV ¹²	C(p,π) ¹³ C;	3.1,3.7	
θ	<u>dσ</u> dΩ	<u>µb</u> sr	Απ		đσ dΩ	<u>μb</u> sr	Απ	
35.3	0.402	(0.056)	-0.345	5 (0.053)	0.605	(0.069)	-0.521(0.034)
43.2	0.277	(0.034)	-0.46	1 (0.062)	0.548	(0.063)	-0.728	(0.034)
50.8	0.196	(0.024)	-0.72	6 (0.072)	0.350	(0.039)	-0.764	(0.040)
600	0.096	(0.016)	-0.74	0(0.094)	0.192	(0.025)	-0.888	(0.056)
67.8	0.073	(0.014)	-0.74	4 (0.096)	0.132	(0.016)	-0.851	(0.060)
75.1	0.030	(0.005)	-0.61	2 (0. 130)	0.052	(0.007)	-0.730	(0.095)
85.2	0.015	(0.002)	-0.47	5(0.110)	0.045	(0.006)	-0.444	(0.059)
90.6	0.016	(0.003)	-0.50	9 (0.120)	0.103	(0.012)	-0.399	(0.060)
100.0	0.015	(0.004) -0.58	5(0.100)	0.148	(0.018)	-0.355	(0.073)
120.0	0.009	(0.002) -0.29	7(0.119)	0.283	(0.030)	-0.355	(0.029)
122.8	0.015	(0.005	1-0.44	0(0.141)	0.349	10.060	J-0.200	10.0017

TABLE	XVIII	LEGEN	IDRE	POLYNO	MIAL	COEFF	ICIENT	S		
REACTION	Go	G _I	G2	G3	G ₄	Α,	A ₂	Α3	Δ4	
¹² C(p, <i>π</i>) ¹³ C _{0.0}	117.0	187.8	155.9	69.6	29.3	0.14	0.14	0.15	0.19	
	(2.0)	(32)	(5.0)	(2.8)	(3.0)	(0.04)	(0.05)	(0.05)	(0.06)	~
¹² C(p; r) ¹³ C	366.7	282	438.0	48.6	50.3	-0.69	-0.21	-0.07	0.18	
	(4.0)	(5.6)	(10.7)	(4.9)	(6.9)	(0.0 9)	(800)	(0.09)	(0,14)	
9 Ве(р,π)Ве _{о.о}	45.9	48.4	26.5	12.6	6.7	-0.60	-0.20	0.06	0.06	
	(2.3)	(2.9)	(5.8)) (3.1)	(4.5)	(0. 8)	(0.19)	(0.22)	(0.21)	
9 _{Be} (n 71 ⁰ Be	1324	473	- 19 6	40.8	-25	-061	-0.07	0 22	0 12	
DC (P,#/ DC3.3	(37)	(4.2)	(85)	(51)	(74)	(012)	(013)	0.22	(0.15)	
		∖ , , <i>∠</i> /		(0.17	(1)	(UIE)	10:107			

THE INCLUSIVE (p.π) REACTION ON

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In inclusive (p, m) reactions the outgoing reaction channels not limited to two body final states. These reactions have are been studied in detail for three main reasons. The relatively high cross sections enabled early experiments to accumulate data which has been used to evaluate several theoretical models (Beder 1971, Silbar 1972). Explicit testing of model calculations is a way to test our understanding of the process and has implications in other kinematic realms. The second reason for studying these reactions evolved from the desire to build "meson factories". It became necessary to understand the dependance of meson production cross sections with energy and production angles so that secondary meson channels could be optimally engineered. Finally the single nucleon induced pion production inclusive cross sections have recently become important data in developing an empirical representation of pion production in heavy ion collisions.

The data in presented this chapter often appears incomplete. These new inclusive ${}^{12}C(p,\pi^+)X$ cross sections arise from a thorough utilization of data which were recorded as а background in the ¹H(p,π^+)²H studies in which a CH2 target was used. The kinematic variables for each run were thus generally set to observe the two body reaction giving a useful but less than complete set of carbon observations.

SECTION 5.1 ANALYSIS OF INCLUSIVE (p. m) DATA

The preliminary analysis of these data was made at the same time as that for the two body reactions discussed in section 3.2 using the KIOWA program. After using the KIOWA prepared C-data files for background subtraction from the CH2 files, the inclusive cross sections were calculated with the CROSS7 program (see Appendix D) using equation 9.

As in the case of the ¹H and ²H inclusive (p,π) data the pion analysing powers given by equation 11, were actually calculated directly in the preliminary analysis.

SECTION 5.2 SUMMARY OF $12C(p,\pi^+)X$ RESULTS

For all practical purposes it is most convenient and logical to present the inclusive reaction cross sections as results averaged in T_{π} =5 MeV steps, although this generally was rather larger binning than was calculated in the CROSS7 program where a calculation is made for each hodoscope counter.

Investigations of backgrounds eliminated by time of flight and dE/dx cuts indicate the calculated cross sections could at most be shifted downwards by 10%. These possible background cuts have not been applied to the results presented here. The cross sections for five incident proton energies from 330 to 425 MeV have been presented in Tables XIX to XXIII. The statistical uncertainties are typically $\pm 2\%$, which are small compared to the 10% uncertainty inherent in the pion decay and pole face scattering corrections. The absolute uncertainty of 17%, has been discussed in section 3.4.

representative sample of the 425 MeV double differential A cross sections have been plotted in Figure 39 . Several qualitative features are readily observed. As the pion production angle increases there is a clear peaking of the distribution at progressively lower pion energies. For pion energies greater than 50 MeV, the cross sections are steeply climbing at small forward angles. These cross sections are 1.26 1.60 times larger than the earlier UVIC-TRIUMF experiment to (Mathie 1976) in the limited kinematical region of overlap. The earlier data of Lillethun (1962) has a normalization error



Figure 39 Superimposed distributions of ${}^{12}C(p,\pi^+)X$ double differential cross section versus pion kinetic energy for a variety of angles and proton energy of 425 MeV. The lines are simply to help guide the eye.

of 0.35, tending to overestimate the cross section. This is true when the proton energy dependence based on Mathie (1976) has been removed. The new 425 MeV data; the 425 MeV (400 and 450 MeV average), renormalized, 60° data of Mathie (1976) ; and renormalized data of Lillethun have been replotted in Figure 40 as a function of angle with fixed pion energy parameters.

Several features of the pion production cross sections as a function of incident proton energy are clearly shown in the new data. At small pion angles the cross section for low energy pions doesn't appreciably change with proton energy. This is demonstrated in Figure 41 where small angle cross sections have



Figure 4Q Distribution of ${}^{12}C(p,\pi^+)X$ double differential cross section versus pion kinetic energy for 60° and 425 MeV. Both the renormalized Lillethun (1962) data at 450 MeV and the renormalized Mathie (1976) average of 400 and 450 MeV data help to show the more complete energy spectrum.

against pion energy been plotted six incident for proton energies ranging from 330 to 730 MeV. At larger pion angles the increase with proton energy is more dramatic. In Figure 42, ູ90**ັ** cross sections for 40 MeV pions are shown from several for proton energies experiments 350 MeV to 730 MeV. An engineering application of these results is to note that low



Figure 41 Distributions of ${}^{12}C(p,\pi^+)X$ double differential cross section at small angles for various proton energies demonstrate that there is little variation for low energy pions.

energy pion fluxes in forward angle meson channels will not appreciably change over a large span in proton energies, however typical large angle channels will have their largest flux at the highest available energy.

The analysing powers for the inclusive ${}^{12}C(p,\pi^+)X$ reaction at 400 MeV proton energy have been summarized in Table XXIV. No earlier experiments have measured this quantity. A contour plot of the 400 MeV results is given in Figure 43.



Figure 42 The double differential cross section for $T_{\rm T}$ =40 MeV pions produced at 90° for several proton energies.

Unfortunately $\gamma_{T} + /\gamma_{T}$ production ratios were not commonly measured in this experiment, however negative pion cross sections were measured for 15 to 100 MeV pions produced at 34.5° by 500 MeV protons incident on carbon. The double differential cross section presented in Figure 44 , are significantly different from the preliminary analysis of these data by Poon(1977). As in the case for low energy positive pions



Figure 43 Contour plct of polarization analysing powers as a function of pion energy and angle with 400 MeV protons in the reaction $^{12}C(p,\pi^+)$ X.

produced at forward angles, there is no significant increase in a cross section at proton energy 730 MeV.



Figure 44 The double differential cross section for negative pions produced at 34.5° by 500 MeV protons in the reaction $^{12}C(p,\pi^-)X$.

$ frac{d^2\sigma}{dTd\Omega}$ ($ frac{\mu b}{sr MeV}$) TAB		TABLE Tp=3	ABLE XIX Tp= 330 MeV		¹² C(p, <i>π</i>)X			d ² σ_(μb) d T dΩ sr MeV				TABLE : Tp=350	^{۱2} C(p,π ⁺)X				
$\theta^{T_{\pi}}$	17.5 (MeV	22.5)	27.5	32.5	37.5	42.5	47.5	θ ^{, T} π	7.5 (Me\	22.5 /)	27.5	32.5	37.5	42.5	475	52.5	57.5
36.0			4.8 (0.5)	5. l (0.5)	5.8 (0.6)	6. I (0.6)		39.0					7.6 (0.8)	7.9 (0.8)	8.5 (0.9)	9.3 (0.9)	9.5 (1.0)
39.0			5.1 (0.5)	5.8 (0.6)	6.4 (0.6)	6.8 (0.7)		43.0				5.7 (0.6)	5.9 (0.6)	0.0 (a0)	6.7 (0.7)	7.0 (0.7)	
48.0		4.6 (0.6)	5.6 (0.6)	6.0 (0.6)				51.0	÷		4.5 (0.5)	5.6 (0.6)	6. I (0.6)	6.2 (0.6)			
58.0	4.4 (0.4)	52 (05)						61.0		5.0 (0.5)	5.6 (0.6)	6.1 (0.6)	6.2 (0.6)				
61.0	4.3 (0.4)	45 (05)						66.0	7.0 (0.7)	7.6 (0.8)	8.0 (0.8)						
63.0	5.0 (0.5)	in .						71.0	4.5 (0.5)	5.7 (0.6)							-
								77.0	5.7 (0.6)								

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							T A BL	E XXI							
			<u>ь</u> ть	$\frac{2\sigma}{d\Omega} \frac{\mu b}{\text{srMe}}$	- V		Tp=375	ŏMeV			¹² C(ρ, π) Χ			
θΤπ	17.5 (MeV	22.5)	27.5	32.5	37.5	42.5	47.5	52.5 _.	57.5	62.5	67.5	72. 5	77.5	82.5	87.5
35.0								7.4 (0.7)	7.7 (0.8)	8.5 (Q9)	8.9 (0.9)	9.9 (1.0)	10.1 (1.0)	10.6 (1.1)	
46.0						6.1 (0.6)	6.1 (0.6)	63 (06)	6.9 (07)	7.1 (0.7)	7.4 (0.7)				
54.0					5.9 (0.6)	6.0 (0.6)	6.2 (0.6)	6.5 (0.7)	6.7 (0.7)						
63.0			5.5 (0.6)	5.6 (0.6)	6.1 (0.6)	6.2 (0.6)	6.4 (0,6)								
76 .0		5.5 (0.6)	6.6 (0.7)	6.7 (<u>0.</u> 7)											
94.0	6.6 (0.7)														

							٦	TABLE	XXII									
			_	$\frac{d^2\sigma}{d T d\Omega}$ s	μ <u>b</u> r MeV		Tp=400 MeV						"ζC(ρ, π)Χ					
	$\theta^{T_{\boldsymbol{\pi}}}$	I 7.5 (Me∨	22.5	27.5	32.5	37.5	42.5	47.5	52.5	575	62.5	67.5	72.5	77.5	82.5	87.5	92.5	
1	36.0										7.6 (0.8)	8.3 (0.8)	9.4 (0.9)	10.1 (1.0)	11.3 (-1.1)	10. 8 (1.1)	11.5 (1.2)	
	42.0									7.0	7.4	8.2	8.4	9.5	9.5	9.6		
	49.0								6.1	(0.7) 6.2	(0.7) 6.7	(0.8) 6.9	(0.8) 7.3	(1.0) 7.1	(1.0) 7.6	(1.0)		
									(0.6)	(0.6)	(0.7)	(0.7)	(0.7)	(0.7)	(0.8)			
	55.0							7.1	7.2	7.4	7.9	7.6	8.1					
								(0.7)	(0.7)	(0.7)	(0.8)	(0.8)	(0.8)					
	56.0						·	6.8 (0.7)	6.6 (0.7)	6.7 (0.7)	7.3 (0.7)	7. I (0. 7)	7.4 (0.7)					
	60.0				6.7 (0.7)	6.7 (0.7)	6.7 (0.7)	6.8 (0.7)	6.8 (0.7)	6.4 (0.7)	6.8 (0.7)	6.7 (0.7)	6.7 (0.7)	6.2 (0.6)	6.2 (0.6)	5.9 (0.6)		
!	6 2 .0					6.7 (0.7)	6.6 (0,7)	6.7 (0.7)	7.1 (0.7)	6.9 (0.7)	7.4 (0.7)	-						
	70.0				6 · 5 (0. 7)	6.6 (0.7)	7.2 (0.7)	7. (0.7)										
•	80.0			5.4 (0.5)	7.2 (0.7)	7.5 (0.8)	•											
•	88.0		5.9 (0.6)	7.6 (0.8)	8.0 (0.8)													
	97.0	6.2 (0.6)	7.9 (0.8)	8.1 (08)														
	11 0.0	10.6 (1.1)											•					

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		POL	ARISA	TION	ANALYS	ING P	OWER	Tp=	400 M	eV	¹² C(p,7	·)X				
Ťπ	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	575	62.5	67.5	72.5	77.5	82.5	87.5	92.5
θ	(MeV)															
36.0										-0.20	-0.19	-0.19	-0.17	-0.13	-0.12	-0.07
										(0.03)	(0.04)	(0.03)	(0.04)	(0.03)	(0.03)	(0.03)
42.0									-0.26	-0.24	-0.24	-0.22	-0.19	-0.19	-0.16	
									(0.18)	(0.17)	(0.16)	(0.13)	(0.09)	(80.0)	(0.08)	
49.0								-0.25	-0.28	-0.28	-0.25	-0.27	-0.22	-0.19		
								(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)		
55.0							-0.32	-0.31	-0.31	-0.32	-0.32	-0.31				
						·	(0.05)	(0.05)	(0.05)	(0.04)	(0.05)	(0.05)				
56.0							-0.35	-0.28	-0.31	-0.31	-0.33	-0.28				
							(0.05)	(0.05)	(0.05)	(0.04)	(0.05)	(0.05)				
62.0					-0.30	-0.31	-0.31	-0.29	-0.26							
					(0.05)	(0.05)	(0.04)	(0.04)	(0.05)							
70.0				-0.25	-0.27	-0.28	-0.27									
				(0.04)	(0.04)	(0.03)	(0.04)									
80.0			-0.2	-0.22	-0.23	-0.26										
			(0.05)	(0.04)	(0.04)	(0.04)										
88.0		-0.19	-0.25	-0.21	•••••	• •										
		(0.09)	(0.08)	(0.07))											
97.0	-0.15	-0.14														
	(0.06)	(0.05)														

TABLE XXIV

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				d² dTd	$\frac{\sigma}{\Omega} \frac{\mu b}{\text{srMe}}$	Ī		Tp=42	5 MeV		l	²⊄p, #)X					
. •	θ Τπ	7.5 (MeV)	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5	72.5	77.5	82.5	87.5	92.5
	47.0										8,1 (0.8)	8.5 (0.9)	9.4 (0.9)	9.7 (1.0)	10.6 (1. 1)	10.1 (1.0)	10.0 (1.0)
	51.0									7. 7	7.9	8.3	8.6	9.5	9.2	8.9	8.8
										(0.8)	(0.8)	(0.8)	(0.9)	(1.0)	(0.9)	(0.9)	(0.9)
	56.0								7.5 (0.8)	7.6 (0.8)	7.7 (0.8)	8.0 (0.8)	8.4 (0.8)	8 .0 (0.8)	7.9 (0.8)		
	61.0							7.9	7.6	7.8	7.9	8. İ	7.9	7.8		·	
								(0.8)	(0.8)	(0.8)	(0.8)	(0.8)	(0.8)	(0.8)			
	66.0						8.2 (0.8)	8.0 (0.8)	8.0 (0.8)	8.5 (0.9)	8.3 (0.8)	8.4 (0.8)					
	71.0					8.4	8.4	8. I	8.6	8.5	8.5						
						(0.8)	(0.8)	(0.8)	(0.9)	(0.9)	(0.9)						
. '	76 .0					8.0 (0.8)	8.0 (0.8)	8.5 (0.9)	8.2 (0.8)								
	90.0			8.6 (0. 9)	9.1 (0.9)	9.6 (1.0)	9.5 (1.0)										
	99 .0		8.7 (0.9)	10.1 (1.0)	10.4 (1.0)												
	111.0	7.0 (0.7)	8.8 (0.9.)														

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<u>CHAPTER 6</u> <u>THEORETICAL CONSIDERATIONS</u>

The discussions of the theory of pion production have been arbitrarily divided into two broad classes, the single or one nucleon model (SNM) and the two nucleon model (TNM) . The definitions cf these models were clarified by Fearing (1979), who suggested that if the interaction Hamiltonian for the only the incident nucleon reaction explicitly included coordinates then the calculation be called an SNM calculation.



Figure 45 Schematic diagram of (a) the two nucleon mechanism and (b) the single nucleon mechanism. Miller (1974) considered (c) pion pre-emission, (d) pion rescattering and (e) inelastic proton scattering and pion emission in his SNM calculations.

A two nucleon calculation correspondingly has the coordinates of incident and one target nucleon explicitly included in the the interaction Hamiltonian. descriptions The two emphasize different aspects of the reaction. The averaged effect of all the target and residual nucleons in a SNM calculation are included as corrections on the expectation that they play a

secondary role, whereas TNM calculations emphasize these effects. Clearly the SNM calculation with corrections appears similar to the TNM calculation when the individual parts of the latter are shown diagrammatically in Figure 45. It may be argued that the effects shown separately are all implicit in the two nucleon mcdel. An advantage of the two nucleon model is that uncertainties in these microscopic details become less important when the measured nucleon-nucleon results are used. Phenomenological TNM calculations rely on an empirical representation of the two nucleon interaction, and shifts the microscopic calculational details from the whole reaction to the two nucleon reaction. An important object of the (p, π) experiments is to evaluate these calculations. More proper comparisons of theory and experiment require simultaneous calculation of both cross sections and analysing powers. Clearly any free parameters used in the calculation must be constrained by both the cross section and analysing power .

SECTION 6.1 SINGLE NUCLEON MODEL

The initial impetus for the single nucleon model came from the similarity of the observed (p, π) reactions to the older and more thoroughly studied (d,p) stripping reactions. In (d, p)calculations the incident d is frequently thought to dissociate into its constituent p and n as the d interacts with the nucleus (Pearson, 1966). Subsequently the p scatters off the target nucleus and the n is captured into the target nucleus forming the residual nucleus. The analogy is appealing however one must remember the d is a relatively weakly bound nucleus and that most (d,p) studies were made at very low energies. The relative momentum transfer to the nucleon is much higher in the (p,π) case as well. The momentum transfers for several cases of (d,p) and (p,π) reactions are shown in Figure 46 .

The neutron pickup model (the inverse reaction to stripping) has been used in calculations of the (p,d) reaction, it not been as successful in calculating however has polarization analysing powers for recent measurements at hiqh momentum transfer as those at low momentum transfer (Cameron This may imply similar problems for the inverse reaction 1979). For the stripping reaction at high momentum transfer. the unpolarized single differential cross section is given by:

15 dor = [phase space] $\sum_{fi} |\langle f|H|i \rangle|^2$ dor = [factors] $\sum_{fi} |\langle f|H|i \rangle|^2$



Figure 46 The center of mass momentum transfer versus angle for several reactions on ${}^{12}C_{\bullet}$. The q for picn elastic scattering with pions of the same kinetic energy as those from a typical (p,π) reaction energy is much lower than the q from the latter reaction. This is not true in the corresponding (p,p) and (d,p) reactions.

The interaction hamiltonian is typically written in the form given below.

16
$$H_{\pi N} \sim \frac{f_{\pi N}}{M_{\pi}} \left[\left(1 + \frac{\lambda E_{\pi}}{2M} \right) \sigma \cdot \nabla_{\pi} - \frac{\lambda E_{\pi}}{2M} \sigma \cdot \nabla_{N} \right] \uparrow \cdot \phi_{\pi}$$

where ∇_N acts only on the incident nucleon and σ, τ are the Pauli spin, isospin matrices



Figure 47 Miller's (1974) calculated angular distribution of single differential cross section compared to Dahlgren's (1973) Note the huge variations due to changing the pion 185 MeV data. optical potential (dash x dash versus solid line) and also variations in the captured neutron binding potential (long versus short dashed curves).

The "galilean invariant" form is given by λ =1 in equation 16, however many theorists use the "static form" given by λ =0 in equation 16.

To calculate an asymmetry the spin dependent cross section

must be calculated from a revised equation.

17
$$\frac{d\sigma}{d\Omega}(\pm) = \begin{bmatrix} phase \ space \\ factors \end{bmatrix} \int_{f} M_{fi} \left[\frac{1 \pm \sigma_{y} P}{2} \right] M_{fi}$$

where Mfi is the matrix element identified in equation 16 and ∇_y is the spin projection operator. The ± signs indicate the spin of the incident prcton.

From equation 17 an asymmetry can be directly calculated as in equation 8.

Early calculations with equation 15 were surprisingly successful in producing the general shape of the angular distribution of cross section however the normalization was grossly in error (Keating 1973, Miller 1974). In the more refined calculations of Miller (1974b), the impact of the final neutron wave function and the final state interaction of the cutgoing pion with the residual nucleus, "pion or distortions", were explored. For a given pion-nucleus optical potential, the sensitivity to the final neutron wave function used to describe the neutron in the residual nucleus gives new information about the high momentum components of the neutron function by virtue of the large momentum transfer to that wave neutron. Unfortunately independent evaluations of the pion optical potential parameters based upon fitting the pion scattering data failed to distinguish between several pion scattering cases. In his most recent and most successful attempts to calculate the (p, η) angular distributions , Miller (1974b) has used a different pion optical potential . Miller's
results are shown in Figure 47, in which the sensitivity to the choice of pion optical potential and neutron binding potential is shown.



Figure 48 Noble's (1975) calculation of polarization analysing power for the reaction to the ¹³C ground state and first two excited states compared to the ground state data of this work.

In Noble's (1975) SNN calculation special attention is paid to proton distortions; that is, the net interaction of the incident proton with the target nucleons. He calculates analysing powers for ${}^{12}C(p,\pi^+){}^{13}C$ reactions leaving the nucleus



Figure 49 Eisenberg's (1978) calculations of the polarization analysing power for the reaction to the ¹³C ground state compared to this work showing the dramatic effect in the calculation due to different neutron binding potentials.

in the $1p_{1/2}$ ground state, the $1d_{5/2}$ third excited state, and zero analysing power for the $2s_{1/2}$ state for 200 MeV incident protons. In the new Pembrooke data the first two excited states of 13C were not resolved, however the Dahlgren (1973c) cross sections at 185 MeV were used to weigh the observed analysing power at 200 MeV. If the differential cross section for state k is written:

18
$$\sigma_{\nu} = \sigma_{\sigma_{K}} + \sigma_{PK} \equiv \sigma_{\sigma_{K}} + H_{K} \sigma_{\sigma_{K}}$$



Figure 50 Eisenberg's (1978) calculations for the polarization analysing power for the reaction to the first two excited states of ${}^{13}C_{\bullet}$

then clearly the weighted analysing power for a composite of two states is given by:

19
$$A_{1+2} = \frac{\sigma_{p_1} + \sigma_{p_2}}{\sigma_{o_1} + \sigma_{o_2}} = \frac{A_1 \sigma_{o_1} + A_2 \sigma_{o_2}}{\sigma_{o_1} + \sigma_{o_2}}$$

The appropriately weighted A $_{\pi}$ from Noble's calculation is shown in Figure 48 .

Eisenberg and Weber (1978) have more recently calculated A_{π} for 200 MeV protons and have also paid special attention to the effects of the proton distortions. The results for the



Figure 51 Gibbs' calculation (1978) of polarization analysing power for the (p,π) reaction to the ¹³C ground state showing the effect of choosing the galilean invariant operator (lower curve) and the static operator (upper curve) for the interaction Hamiltonian.

ground state are shown in Figure 49 , and once again the Dahlgren cross sections were used to weigh the $A_{\tau\tau}$ calculated for the first two excited states for comparison to the Pembrooke data. This comparison is made in Figure 50 . The sensitivity neutron binding potentials is also indicated in Figure 49. to Eisenberg and Weber indicate the cross sections they have calculated have normalizations four times too big for the 13C ground state, and 100 times too large for the first excited



Figure 52 Gibbs[•] calculation of polarization analysing power for the first two excited states of the ¹³C. The sign of these calculations is clearly wrong compared with the data of this experiment.

states.

In their stripping calculations, Gibbs and Young (1978) were primarily concerned with demonstrating the importance of pion rescattering or pion distortions with respect to the analysing power. The sign of their calculations for the ¹³C is correct, however the ¹³C excited state calculation clearly fails. In their figures, reproduced in Figure 51 and Figure 52 , large effects are observed when the form of the bound neutron wave function or the interaction Hamiltonian were varied.

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SECTION 6.2 TWO NUCLEON MODELS AT LOW PROTON ENERGY

Within the framework of the TNM the typically large (p,π) reaction momentum transfer is absorbed by two nucleons in the



Figure 53 Dillig's (1977) field theoretic two nucleon calculation of ${}^{12}C(p,\pi^+){}^{13}C$ compared with Dahlgren's (1973) 185 MeV data. The dashed curve is for a plane wave Born approximation and the solid line is for a distorted wave Born approximation.

residual nucleus.

Microscopic model calculations have been made by Dillig others in a field theoretic calculation of (1977)and 12C(p, T+) 13C for 185 MeV incident protons. In these calculations the microscopic details explicitly include pion and rho meson exchanges in the interaction Hamiltonian. The results for DWBA and PWEA calculations have been presented in Figure 53

with quite reasonable agreement with the Dahlgren data.

Phenomenological calculations use empirical an representation of free two nucleon reaction data. This approach has been used with some success by Ingram (1971) to calculate small angle ²H(p, π^+) ³H and a more general calculation by Fearing (1975) agrees favourably with the data for the reaction at all angles. Unfortunately neither of these models have been extended to light nuclei and can't be used to calculate an analysing rower.

SECTION 6.3 TWO NUCLEON MODELS AT MEDIUM ENERGIES

FCR INCLUSIVE (p,m)

proton energies 400 - 600 MeV above threshold several At phenomenological models which involve empirical representations the two nucleon reactions ${}^{1}H(p,\pi)$ and $n(p,\pi)$ have been of somewhat successful (Beder 1971, Silbar 1972). Some of these calculations have been recently extended to lower energies. The Beder model results for 425 MeV proton energy are compared with the new inclusive (p, m) data in Figure 54 and Figure 55 . The Beder mcdel gives the inclusive nuclear production rate as a momentum averaged free nucleon-nucleon pion production rate corrected for absorption of the pions. There are essentially no free parameters, and the model has proved reasonable at higher proton energies. The input data to the program are experimental NN- NNT CIOSS sections. At present the program uses extrapolations from high energy proton data due to the lack of full distributions of cross sections at low proton energy, particularly at large pion angles.



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Figure 54 Beder's model results (solid circles) compared to the experimental double differential cross sections for ${}^{12}C(p,\pi^+)X$ at 60° for 425 MeV incident protons. The renormalized Lillethun (1962) results (triangles) and renormalized Mathie (1976) results (boxes) and present data (open circles) tend to peak at lower pion energies than the calculation. The curves simply help to guide the eye.



Figure 55 Beder's model results (solid symbols) compared to the new experimental double differential cross section for $^{12}C(p,\pi^+)X$ at various angles for 425 MeV incident protons.

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SECTION 6.4 KINEMATICAL NATURE OF A IN THM

In the phenomenological model, calculations of cross the reactions $A(p,\pi^{T}) + 1$ are reasonably easy to sections for visualize. The calculation involves the interaction of the incident proton with any of the bound target protons giving rise for π^+ ,²H final state. In example to a the reaction ¹²C (p, π^+)¹³C, the incident proton interacts with a bound proton from the 12C nucleus in the presence of a 11B spectator nucleus,



Figure 56 Kinematical model calculation of polarization analysing power compared to the experimental results for ${}^{12}C(p,\pi^+){}^{13}C$.

where the target proton has a typical momentum from the 12 C single particle momentum distribution. If the ²H and ¹¹B do not recombine an inclusive (p, π) reaction is observed, or if they

combine to form 13C the pion for the net two body reaction is In the ${}^{1}H(p,\pi^{+}){}^{2}H$ cross section calculation, observed. the kinematics must be evaluated for the case of a moving target proton, and an angle such that the pion comes out as observed in the net reaction , and finally so that the 11B and the **2**H can combine to form 13C. The empirical representation of the ¹H (p,m⁺) ²H cross section, a function of pion center of mass angle (Richard-Serre momentum and 1970) is used in the calculation. Thus the calculation is largely dependent upon



Figure 57 Kinematical model calculations of polarization analysing power compared to the experimental results for ${}^{9}\text{Be}(p,\pi^+){}^{1}{}^{0}\text{Be}$.

determining the kinematical variables to evaluate the cross section.

In a similar manner it seems reasonable to evaluate an empirical function representing the ${}^{1}\text{H}(p,\pi^{+}){}^{2}\text{H}$ analysing power with the same kinematical variables. Clearly the kinematics used above to evaluate the cross sections can be used to evaluate an A_{π} function. The naivety of this assumption is that for the unpolarized nuclear cross section, a sum over spins is implicit within a squared matrix element found in the free two nucleon cross sections. In the case of the analysing power the sum should not be made, and the combination of matrix elements does not simply lead to the two body analysing power.

A computer code in which the kinematic variables are calculated dynamically for random choices of bound proton momentum from a reasonable momentum distribution, and an A_{π} is evaluated as described in Appendix E. Calculated angular distributions of A_{π} for the ¹²C and ⁹Be reactions are shown in Figure 56 and Figure 57 . These curves clearly reflect the minimum in A_{π} at forward lab angles near 60° which is observed in all data sets. The A_{π} data did not indicate much variation for different final states and the model predicts none.

CHAPTER 7

CONCLUSIONS

A program of experiments to study proton induced pion production has been undertaken at TRIUMF . The first studies, which utilized the Pembrooke spectrograph have been discussed in this dissertation. Analysis of the Pembrooke data was complicated by the effect of pole face scattering, the multiple scattering cf richs in the pole face steel. Pole face scattering was the largest factor in the Pembrooke resolution (36% of the typical total $\Delta P/P$ of 0.02), and generally was the cause of the largest relative uncertainty in the cross sections (5-10%). The pole face scattering, pion decays, and multiple scattering in air and counters were studied in a Monte Carlo simulation of the entire system. From the simulations it Was possible to determine the solid angle of the spectrograph and make corrections for pole face scattering, pion decays and other less important effects.

Single differential cross sections and polarization analysing powers, A_{π} for the reaction ¹H(p, π^+)²H have been determined. The cross sections are in good agreement with the empirical fits of data from previous experiments. The analysing in powers have largely filled a void of such data in the threshold region. For proton energies as low as 320 MeV d-wave pion production is significant, contrary to the understanding from previous experiments. The coupled channels calculation of Niskanen (1978) has proved to be the most successful when compared to these data.

Dcuble differential cross sections and Arr have been determined for pions produced from the reaction $^{1}H(p,\pi^{+})$ pn with center of mass kinetic energies 10 to 20 MeV below that of pions from ¹H(p, π^+)²H. Energy distributions of these cross sections function of pion energy may be determined only when the as а three body final state is experimentally separated. Unfortunately this was not possible with the Pembrooke system because the pole face scattering effect led to a large. uncertain tail on the two body peak. Future experiments would benefit from a coincidence configuration in which both the pion and deuteron are detected, thus unambiguously defining the final state. The A_{π} for pions produced in ${}^{1}H(p,\pi^{+})$ pn is very similar to that observed for the two body reaction, indicating very little dependence upon the degree of binding of the residual n and p.

Pion production from deuterium for both the inclusive and two body reactions, has been observed for several incident proton energies. The 2 H(p, π^{+})³H cross sections of Frank (1954) fall 20% below the present data, however in the small angular overlap with Auld (1979) the two newer experiments are in agreement. Fearing's calculation of the $^{2}H(p,\pi^{+})^{3}H$ angular distribution is in reasonable agreement with the data. In the new data the angular distributions of analysing powers look very similar to those of the ${}^{1}H(p, \pi^{+}){}^{2}H$ reaction. The magnitude of the A_{π} in the 400 MeV experiment of Auld (1979) do not agree with the present data, leading to the conclusion that more extensive $^{2}H(p,\pi^{+})^{3}H$ measurements are required. The analysing powers for the inclusive reaction giving pions with center of mass energy 20 to 30 MeV below the two body peak are slightly more positive than for the two body reaction. The dependence of A_{π} on excitation energy may be important when compared to the corresponding ¹H(p, π) reactions, where the degree of binding of the residual nucleons was not important.

Angular distributions of cross section and analysing power for the reactions ${}^{12}C(p,n^+){}^{13}C$ and ${}^{9}Be(p,n^+){}^{10}Be$ with 200 MeV protons have been measured. These data agree well in most respects with the preliminary cross sections of Bent (1978) however at the largest angles these data differ by a factor of two. The shape of the angular distribution of cross sections agree with those determined at 185 MeV by Dahlgren (1973) and are in agreement with Hcistad's review of the cases where the residual nuclecn is captured into a p-shell as in the ¹³Cgs, ¹⁰Begs, and ¹⁰Be3.3 final states or into an s- or d-shell in the cases of 13C3.1 and 13C3.7 states. Possibly the most as exciting aspect of these studies was the angular distribution of analysing powers for these nuclear (p,π) reactions, which had a about 60° for all of the nuclear reactions deep minimum at observed. These data were preceeded by only one experiment which had very poor resolution addid not measure an angular distribution. Numerous attempts to calculate the analysing power using the stripping model were prompted by the new experiment. Huge variations in theoretically calculated cross sections and analysing powers have been calculated primarily due a variety of techniques for including the effects of proton to

distortions in the initial state. None of the authors have treated the cross sections and analysing powers in the same calculation, simultaneously varying the degrees of freedom within their models. This must be done if a model is to be credible and be used in a predictive sense which is the final test and usefulness of any model.

The similarity of the analysing powers for the nuclear pion production reactions to the ${}^{1}\text{H}(p,\pi^{+}) {}^{2}\text{H}$ reactions leads one to question if the single nucleon model with distortions is best. The dominance of two nucleon effects as indicated by the above similarity may be better represented in two nucleon models, in which the model Hamiltonian explicitly includes the two nucleon aspects rather than adding them as a correction. Two nucleon calculations have not yet been seriously applied to calculating analysing powers, however they have been at least as successful as single nucleon model calculations of the cross sections.

Experimentally the nuclear (p,π) reactions have to be studied much more. To properly evaluate the reaction mechanism uncertainties discussed above, transitions to a wider variety of final states must be studied to further test the single nucleon calculations. Possible targets are: ¹⁰B (two easily resolved states in ¹¹B); ¹⁶O (resolvable 1d5/2 and 2s1/2 states in ¹⁷O); ⁶⁰Ca (1f7/2 ground state in ⁶¹Ca); and ²⁰⁸Pb (g9/2, i11/2, and j15/2 states of ²⁰⁹Pb). The nuclear (p,π) reactions must also be studied as a function of incident proton energy. If the similarity to the ¹H (p,π^+) ²H analysing powers is to be tested, somewhat higher proton energies must be studied. The ¹H(p, π^+)²H A_{π} varies from the largely negative distributions observed near threshold to all positive distributions at energies above 500 MeV.

The final (p,π) reactions discussed in this dissertation were inclusive reactions leading to relatively low energy pions from carbon targets. These data show the peaking of double differential cross section at higher pion energies for forward angle pion production. An important characteristic of these data is how small the variations of forward produced low energy pions is for large differences in proton energy. Theoretical calculations of these crcss sections with Beder's model (1971)indicate that for these low proton energies the contribution from $^{1}H(p,\pi^{+})pn$ is being underestimated. This empirical improved with improvements in the existing calculation may be $NN \rightarrow NN\pi$ data. Clearly this model stresses the importance of two nucleon effects in the medium energy inclusive reactions.

The amcunt of data for the inclusive (p,π) reactions is very poor. These reactions have relatively high cross sections and an experiment with several targets (possibly ⁹Be, ¹²C, ⁶³Cu and ²⁰⁸Pb) for proton energies from 300 to 500 MeV would be useful for engineering purposes, further model testing, and as input data for new heavy ion calculations.

The use of multiwire chambers to determine the focal plane distribution is suggested in further experiments because this would enable elimination of the pole face scattering problem through ray-tracing back to the pole face.

BIBLIOGRAPHY

Aebischer D., Favier B., Greeniaus L. G., Hess R., Junod A., Lechanoine C., Nikles J-C., Rapin D., and Werren D.W. 1976 Nucl.Phys <u>B108</u> 214-238

Akimov Yu.K, Savchenko O.V., Soroko L.M. 1958 Nucl. Phys <u>8</u> 637-649

Albrcw-M.G. 1971 Phys.Lett. <u>34B</u> 337-342

Aprile-E. et al 1979 High Energy Physics and Nuclear Structure Vanccuver (unpublished contributed paper 3e24)

Aslanides E. 1976 International Topical Conference on Meson Nuclear Physics Pittsburgh (New York: AIP conf. Ser. 33) 204-220

Auld E.G., Haynes A., Johnson R.R., Jones G., Masterson T., Mathie E.L., Ottewell D., Walden P., and Tatischeff B. 1978 Phys.Rev.Lett. <u>41</u> 462-465

Auld E.G., Johnson R.R., Jones G., Mathie E.L., Walden P., Hutcheon D., Kitching P., Olsen W.C., Perdrisat C.F., and Tatischeff B. 1979 (to be published)

Axen D., Duesdieker G., Felawka L., Ingram Q., Johnson R.R., Jones G., Lepatourel D., Salomon M., and Westlund W. 1976 Nucl.Phys. <u>A256</u> 387+413

Barschall H.H and Haeberli W. (editors) 1971 Polarization Phenomena in Nuclear Reactions (Madison: University of Wisconsin Press) xxv

Beder L. and Bendix P. 1971 Nucl. Phys. <u>B26</u> 597-610

Bent R.D., Debevec P.T., Pile P.H., Pollock R.E., Marrs R.E., and Green M.C. 1978 Phys.Rev.Lett. <u>40</u> 495-498

Bent R.D. et al 1978 IUCF annual report

Block M.M. et al 1952 Phys.Rev. <u>88</u> 1239-1247

Browne C.P. And Buechner W.W. (1956) Rev.Sci.Instr. 27 899

Bugg D.V. et al 1978 J.Phys. G4 1025

Cameron J.M. et al 1979 High Energy Physics and Nuclear Structure Vancouver (unpublished contributed paper 4A15)

Cochran D.R.F., Dean P.N., Gram P.A.M., Knapp E.A., Martin E.R., Nagle D.E., Perkins R.B., Schlaer W.J., Thiessen H.A., and Theriot E.D. 1972 Phys Rev <u>D6</u> 3085 3116 Crawford F.S. and Stevenson M.L. 1955 Phys.Rev. <u>97</u> 1305-1313 Crawford J. et al 1979 High Energy Physics and Nuclear Structure Vancouver (unpublished contributed paper 4A1 Cumming J.B. 1963 Ann. Rev. of Nucl. Sci. <u>13</u> 261-286 Hasselgren D., Hoistad B., Dahlgren S., Ingemarsson A., Jchansson A., Renberg F.U., Sundberg O., and Tibell G. 1967 Nucl. Phys. <u>A90</u> 673-695 Dahlren S., Hcistad B., and Grafstrom P. 1971 Phys.Lett. 35B 219-221 Dahlgren S., Grafstrom P., Hoistad B., and Asberg A. 1973a Phys.Lett. 47B 439-441 Dahlgren S., Grafstrcm P., Hoistad B., and Asberg A. 1973b Nucl.Phys. A204 53-64 Dahlgren S., Grafstrom P., Hoistad B., and Asberg A. 1973c Nucl. Phys <u>A211</u> 243-253 Dahlgren S., Grafstrom P., Hoistad B., and Asberg A. 1974 Nucl. Phys. <u>A227</u> 245-256 Dillig. N. and Huber N. 1977 Phys.Lett. 69B 429-432 Dollhopf W., Roberts W.K., Lunke C., Perdrisat C.F., Kitching P., Olsen W.C., and Priest J.R. 1973 Nucl.Phys. A217 381-399 Dolnick C.L. 1970 Nucl. Phys. <u>B22</u> 461-477 Eisenberg J.M. and Weber H.J. 1979 Meson-Nuclear Physics Houstin (New York, AIP conf. ser 54) 190 191 Evans R.D. 1955 The Atcmic Nucleus (New York, McGraw-Hill) 474-484 Fearing H.W. 1975 Phys.Rev. <u>C11</u> 1210-1226 Fearing H.W. 1979 Western Regional Nuclear Physics Conference Banff (unpublished invited paper) Fernbach S., Serber R., and Taylor T.B. 1949 Phys.Rev. 75 1352-1355 Feshbach H., Porter C.E., and Weisskopf V.F. 1954 Phys.Rev. 96 448-464 Findlay D.J.S. and Owens R.O. 1978 Phys.Lett. 74B 305 308 Ford K.W. and Wills J.G. 1969 Phys.Rev. <u>185</u> 1429-1438

Frank W.J., Bandtel K.C., Masley R., and Mayer B.J. 1954 Phys.Rev. <u>94</u> 1716-1721 Gellman M. and Watson K.M. 1954 Ann.Rev.Nucl.Sci. <u>4</u> 219-270 Gibbs W.R. and Young S. 1978 Phys.Rev. C17 837-841 Michaelis E.R., Heer E., Hirt W., Martin M., Serre C., Skarek P. and Wright B.T. 1969 CERN report 69-24 Heer E., Roberts A., and Tinlot J. 1958 Phys.Rev. 111 640-644 Hendrie D.L., Glendenning N.K., Harvey B.G., Jarvis O.N., Duhm H.H., Sandinos J., Mahoney J. 1968 Phys.Lett. B26 127-130 Hirt W. 1969 Nucl. Phys. <u>B9</u> 447-450 Hodgson P.E. 1971 Nuclear Reactions and Nuclear Structure (Oxford: Clarendon Press) 28 Hoistad B., Dahlgren S., Grafstrom P., and Asberg A. 1974 Physica Scripta <u>9</u> 201-207 Hoistad B. 1976 Topical Meeting on Intermediate Energy Physics Zuoz (Villigen: SIN Bibliothek) 537-563 Hoistad B. 1977 High Energy Physics and Nuclear Structure Zurich (Basel: Birkhauser) 215-223 Hsieh W. 1978 A New Hamiltonian for Systems of Nucleons and Pions (Vancouver: University of British Columbia MSc Thesis) Ingram C.H.Q., Tanner N.W., Domingo J.J. and Rohlin J. Nucl.Phys. <u>B31</u> 331-348 Jackson D.F. 1974 Rep.Prog.Phys. 37 55-146 James A.N., McDonald W.J., Cameron J.M., Miller C.A., Hutcheon D.A., Kitching P., Neilson G.C., and Stinson G.M. 1979 Nucl.Phys. <u>A324</u> 253-265 James F.W. 1975 Low Energy, Large Angle Pion Production by 580 MeV Froton Bcmbardment of Various Nuclei (Victoria: University of Victoria PhD Thesis) Jones G. 1977 Nucleon-Nucleon Interactions Vancouver (New York: AIP conf. Ser. 41) 292-304 Jones G. 1978 Few Body Problems and Nuclear Forces Graz, Austria (Berlin, Springer-Verlag) 142-151 Keating M.P. and Wills J.G. 1973 Phys.Rev. C7 1336-1340 Kitching P. 1971 TRIUMF internal report TRI-71-2

Kitching P. and Stinson G. 1973 TRIUMF internal report TRI-DNA-73-4 Kost C., Hsieh W., and Reeve P. 1977 TRIUMF internal report TRI-DN-77-6 Lazard C., Ballot J.L. and Becker F. 1970 Nuovo Cimento 65B 117-146 LeBornec Y., Tatischeff B., Bimbot L., Brissaud I., Garron J.P., Holmgren H.D., Reide F., and Willis N. 1974 Phys.Lett. 49B 434 Tatischeff B., Bimbot L., LeBornec Y., LeBornec Y., Tatischeff B., Bim Holmgren H.D., Kallne J., Reide F., Brissaud I., and Willis N. 1974 Phys.Lett. <u>61B</u> 47 Lee C. 1975 Offical Properties of 50 cm Browne-Buechner Spectrograph (Vanccuver: University of British Columbia MSC Thesis) Lillethun E. 1962 Phys.Rev. <u>125</u> 665-676 Ludgate G. 1976 Rutherford Lab Report HEP/T/59 Mandl F. and Regge T. 1955 Phys.Rev. 99 1478-1483 Marshak R.E. And Messiah A.M.L. 1954 Nuovo Cimento 11 337 Mathie E.L. 1976 Iow Energy Fion Production by Protons With Incident Energies Between 400 and 500 MeV (Victoria: University of Victoria MSc Thesis) Mayer M.G. and Jenson J.H. 1955 Elementary Theory of Nuclear Shell Structure (New York: Wiley, Chapman and Hall) Measday D.F. and Miller G.A. 1979 Ann.Rev.Nucl.Sci. 29 121-160 Miller G.A. 1974 Nucl. Phys. <u>A224</u> 269-300 Miller G.A. and Phatak S.C. 1974 Phys.Lett 51B 129-132 Niskanen J.A. 1978 Phys.Lett <u>B79</u> 190 Niskanen J.A. 1978 Nucl. Phys. <u>A298</u> 417-451 Noble J.V. 1975 Nucl.Phys. A244 526-532 Pearson C.A. and Coz M. 1966 Ann. Of Phys. <u>39</u> 199-215 Pile P.H., Bent R.D., Pollock R.E., Debevec P.T., Marrs R.E., Green M.C., Sjoreen T.P., and Soga F. 1979 Phys.Rev.Lett. 42 1461 Poon M. 1977 Optimization Studies of the TRIUMF Biomedical Pion

Beam (Vancouver: University of British Columbia MSc thesis) Preedom B.m. et al 1978 Phys.Rev. C17 1402-1407 Richard-Serre C., Hirt W., Measday D.F., Michaelis E.G., Saltmarsh M.J.M., and Skarek P. 1970 Nucl. Phys B20 413 440 Rutherford E. 1911 Phil.Mag. 21 669 Serber R. 1947 Phys.Rev. 72 1008-1016 Sheline R.K., Watson C. and Hamburger E.W. 1964 Phys.Lett. 8 121-124 Silbar R. and Sternheim M. 1972 Phys. Rev. D6 3117-3126 Spuller J. and Measday D.F. 1975 Phys.Rev. D12 3550-3555 Stetz A.W. 1975 University of Alberta internal report 81 Sweet R.F., Bhatt K.H. and Ball J.B. 1964 Phys.Lett. 8 131-133 Tobocman W. 1954 Phys.Rev. <u>94</u> 1655-1663 Tobocman W. 1959 Phys.Rev. <u>115</u> 98-107 Trippe T.G. et al 1976 Rev. Mod. Phys. 48 S45 Walden P, Ottewell D., Mathie E.L., Masterson T., Jones G., Johnson R.R., Haynes A., and Auld E.G. 1979 Phys.Lett. <u>81B</u> 156-160 Weddigen Ch. 1978 Nucl. Phys. A312 330

APPENDIX A REVMOCE, A MONTE CARLO SIMULATION

OF THE PEMBROOKE SPECTROMETER

A recent version of the general TRIUMF Monte Carlo program (Kost 1978, Kitching 1973 and Kitching 1971), REVMOC has been modified to be used to simulate the Pembrooke spectrometer. The modifications and additions were made to four principle areas of the existing program. A flowchart for REVMOC6 is given in Figure 58.



Figure 58 Flowchart of the REVMOC6 Monte Carlo program.

The target region simulation has been modified so that any two body reaction may be included. In the two body case a random direction for the farticle to be traced is chosen within the user defined angular ranges. The two body kinematics are used to define each ray's momentum rather than the usual random choice of momentum. In addition the target geometry may be an upright cylinder rather than a cube. The final modification to the target region simulation is the explicit inclusion of energy losses of the primary and secondary particle in the target.

The most significant modification to the REVMOC program was the total change of the dipole magnet simulation. The new routine simulates a dipole with circular pole faces. The dipole center may be displaced from the optic axis of the system, as often happens when large elements are slightly misaligned. The particle direction is altered as it crosses the effective edges of the magnet field to account for the force imposed as it crosses the field gradient. first call to the BEND In the subroutine no detailed interactions are allowed to occur. Particles which strike the magnet pole faces are considered lost and pion decays are not calculated. The resulting distribution of this pass through the system is called the "geometrical" distribution. In the second call these interactions may occur. If the particle is to decay, the decay product is traced through the remainder of the system from the point of decay. If the particle (primary or decay product) strikes the pole face it is allowed to multiply scatter through up to eight arbitrary slices of the pole face steel, after which the particle is presumed lost. At the end of each slice, a test is made to determine if the particle has scattered back into the magnet gap in which case it would be traced through the remainder of the system. In

the Pembrooke simulation 1cm slices were chosen, consistent with the range of pions in steel.

subroutines were written to enable simulation of the TWO complicated hodoscope box with its overlapping array of 24 counters. Ihe counting of rays which were successful in striking any of the 47 bins was made in the same manner as the data acquisition system, except every count was flagged as being related to the first pass of the system where only geometrical limits were imposed, or related to the final pass as an undecayed pion , a muon from pion decays before the hodoscope array or as a pion which subsequently decayed above the focal plane and whose decay muon was counted in the C-counters.

These four distributions were separately printed out to both the paper record of the run and to computer memory storage for latter use.

APFENDIX B

CARBON ACTIVATIONS

The principle of the activation technique is that an unknown number of beam particles will create ,via a well understood reaction, radioactive nuclei which can be detected at a later time out of the primary beam. The determination of proton fluxes with carbon activations employs the well understood (Cumming 1963) reaction ¹²C(p,pn)¹¹C and general gamma spectroscopy techniques for determining the number of ¹¹C nuclei produced.

It can be shown (Evans 1955) that the number of ¹¹C nuclei at the end of a proton blast of duration Tb is given by:

20
$$N_o = I\left(\frac{A\rho \times}{GMW}\right) \sigma T_b \left(1 - e^{\lambda_b T_b}\right) \equiv IK$$

where I is the proton beam current, \mathcal{S} is the ¹¹C production cross section and $\left(\frac{A\rho \times}{GMW}\right)$ is the areal density of the carbon activation target.

After the proton blast the ¹¹C nuclei decay according to the radiative decay law decay law:

$$N(t) = N_o e^{-\lambda_b t}$$

where T=0 at the end of the blast. A measurement of ¹¹C decays between times t1 and t2 will yield:

22
$$D = IK(E \Delta R) \frac{LT}{RT} \left(e^{-\lambda_b T_1} - e^{-\lambda_b T_2} \right)$$

where LT and RT are the livetime and real time of the measurement respectively and $\mathcal{E}\Delta\mathcal{R}$ is the product of detector

efficiency and solid angle determined from measurements with a calibrated ²2Na source.

The variation of the ${}^{11}C$ production cross section is given in Figure 59 .



Figure 59 The production cross sections for ¹¹C as a function of incident proton energy, based on a review by Cumming (1963).

To calibrate the proton polarimeter or any other beam intensity monitor it is simply necessary to record the number of monitor counts during the activation blast and relate these to the beam current determined in equation 23 . The calibration factor ,CA defined in equation 5 was based upon a series of 22 activation measurements including four with a carbon polarimeter target.

APPENDIX C POLG, A MONTE CARLO SIMULATION

OF A PROTON POLARIMETER

Throughout the course of the carbon activation measurements considerable variations in the calibration parameter, Ca which are reflected in its quoted uncertainties were observed. In addition the cross sections for the reaction seemed to vary with running conditions! To help determine if these variations could be attributed to changes in the beam characteristics at the polarimeter and to provide essentially independent an calibration of the polarimeter a Monte Carlo simulation was written.

The program flow logic is shown in Figure 60 . There are two major routes through the program, controlled by the pointer 'J' , in which protons from proton elastic scattering, or from the reaction ¹²C(p,2p)¹¹B are traced through the counter telescopes. In either case, the effect of including multiple scattering may be separately evaluated. The directions of the proton momentum are randomly chosen in the pp center of mass frame, so that the protons ccme out "back to back" as in elastic scattering. An appropriate Lorentz transform is used to find momenta of the protons in the lab system. Rotations are the made so that directions are described with respect to the beam line direction, rather than the direction of the incident proton which need not be on axis, or the direction of the pp cms motion in the pC case.

Multiple scattering is included (J=1 or 3) simply by



Figure 60 Flowchart of the polarimeter Monte Carlo program.

rotating the proton through an angle Θ chosen from a gaussian distribution with standard deviation $\sqrt{2} \Theta_{plone}^{rms}$, and ϕ , a random azimuthal angle between 0 and 2π . The angle Θ_{plone}^{rms} is given below for a particle with charge Zproj and momentum P to be (Trippe, 1976):

23
$$\Theta_{\text{plane}}^{\text{rms}} = (1+\epsilon) \frac{15 \text{ MeV/c}}{P\beta} \sqrt{\frac{5}{Lr}}$$

where δ is the path length through a material with radiation length Lr.

The ¹²C(p,2p)¹¹B contribution has been calculated by

assuming a quasi-elastic model which allows proton elastic scattering with a ¹¹B spectator. The incident proton scatters from a nuclear proton which has a momentum randomly chosen from a reasonable nuclear single particle momentum distribution.

The number of polarimeter counts is given by:

24

$$N(L+R) = \frac{I'(pt)}{1000} \frac{2A_0}{GmW} \frac{d\sigma}{dx^*} \Delta x^* (2arms)(6.2415 \times 10^9)$$

where I' is the number of protons in nanocoulombs; (f) is the polarimeter areal density in mg/cm²; GMW is the gram molecular weight of the target and Ao is Avogadro's number.

For a CH2 target equation 24 may be evaluated to give:

$$C_{A} = 5.36 \times 10^{29} d\sigma \Delta \Omega_{PP}^{*}$$

and

26
$$C_{A}^{carbon} = 2.68 \times 10^{29} \frac{1}{dR^*} \Delta \mathcal{N}_{pc}^{*}$$

In the Monte Carlo simulation the protons are uniformly distributed into a fraction of the total p,p center of mass solid angle, $f \mathcal{N}^*$. Each ray is tracked through the polarimeter and counted with the simulated counters. The center of mass solid angle of the polarimeter is then given by:

27
$$\Delta \mathcal{R}^{*} = \frac{N \text{ successes } (f \mathcal{R}^{*})}{N \text{ triols}}$$

The pp scattering cross sections of Bugg(1978) have been used together with the Monte Carlo calculations of $\Delta \mathcal{A}^{\#}$ to estimate Ca as a function of proton energy. The results were included in figure 12. Variations in Ca due to reasonable changes in beam size and angular dispersion were found to be within the 5% statistical uncertainty of the Monte Carlo calculations. In addition the simulated left-right asymmetry was consistent with zerc for these beam variations.

APPENDIX D CROSS SECTION CALCULATIONAL PROGRAMS

Calculation of single differential cross sections for two body reactions entailed integrating the counts from a resolvable peak and applying corrections to efficiency and solid angle appropriate to that peak. To calculate double differential cross sections for the cases where a slowly varying distribution of rions were observed was more complicated and involved correcting much more data. The programs CROSS7 and CROSS8 were written to perform this task which was largely of a bookkeeping nature. A general flowchart for both programs is shown in Figure 61. The purpose of both programs is to evaluate number of counts in each full hodoscope equation 9 for the counter and the solid angle appropriate to that counter when no multiple or pole face scatterings had caused a change in the number of picns observed. Of course these effects do change the effective solid angle for each counter as explained in chapter 2 so corrections the data. matrix must be made to The of correction factors for pion decays described in chapter 2 is applied in CROSS7, however an average correction is made for pole face scattering rather than employing the pole face scattering correction matrix. This average correction is based dependent ratio of the solid angle derived from on the energy Monte Carlo runs which included multiple scattering and pole face scattering throughout the system to the geometrical solid angle for cases where a uniform momentum distribution was incident. Neither of these corrections are applied in CROSS8 which presumes the user has previously corrected the data for



Figure 61 Flowchart for the double differential cross section programs.

decays and scattering by the methods discussed for example in chapter 3, where cross sections for the reaction $^{1}H(p,\pi^{+})pn$ are determined.
AFPENDIX E PA4, A KINEMATICAL PROGRAM FOR MODEL

CALCULATIONS OF NUCLEAR (p, m) REACTIONS

The similarities of the angular distributions of the analysing power for various nuclear $A(p,\pi)A+1$ reactions may be an indication of a strong reaction mechanism dependence. If the mechanism should be dominated by reaction a two nucleon interaction then the nuclear (p,π) analysing power may be related to the A for the two nucleon reaction $^{1}H(p,\pi^{+})^{2}H$ which is the most important nucleon-nucleon interaction in the intermediate energy realm.

A simple kinematical model for the nuclear (p,π) reaction has been incorporated into the computer code PA4 to estimate analysing powers. In this model the incident proton interacts with a bound nuclear proton via a ${}^{1}\text{H}(p,\pi^{+}){}^{2}\text{H}$ reaction in the presence of an (A-1) spectator nucleus.

The momentum of the bound proton is chosen randomly from a momentum distribution which is the fourier sine transform of a Woods-Saxon density distribution with radius, $Rc=(1.106 + 1.05 \times 10^{4} \text{A}) \text{ A}^{1/3}$ fm, and surface-diffuseness, ac=0.502 fm (Hodgson 1971). The direction of the bound proton is random. The struck proton and resultant deuteron are off-shell, that is for the proton:

$$E_p^2 \neq M_p^2 + P_p^2$$

The reaction is shown schematically in Figure 62 , from





Figure 62 The kinematics for the nuclear reaction model showing; (a) the overall reaction, (b) the incident channel in the center of mass (cms), (c) the final state in cms, (d) the reaction with spectator nucleus 2, (e) the reaction with the spectator nucleus in the cms, and finally (f) the recombination of the spectator and the deuteron to give the residual nucleus. P_{NA} and P_{NN} are momenta for the appropriate centers of mass.

which the overall reaction kinematics, the two nucleon kinematics, and finally the deuteron and spectator nucleus recombination kinematics may be derived.

A flowchart for PA4 is given in Figure 63. Upon reading in the run details such as proton energy, target mass and spectrometer angle a call is made to the two body kinematics routine to calculate the overall reaction kinematics(*1 in figure 63). From figure 62 it is clear that:

$$E_{A_1}^2 = P_2^2 + M_{A_1}^2$$



Figure 63 Flowchart of the kinematical model calculations.

and

$$E_2 = M_A - E_{AT}$$

giving the bound proton mass:

31
$$M_2^2 = E_2^2 - P_2^2$$

similarly from the deuteron and spectator nucleus recombination it can be seen that:

32
$$P_d^2 = P_{4}^2 + P_{2}^2 + 2P_{4}P_{2}\cos(\theta_{4} + \theta_{2})$$

and

$$E_d = E_4 - E_A$$

giving the deuteron mass (*2 in figure 63):

$$M_d^2 = E_d^2 - P_d^2$$

a second call to the two body kinematics routine is made with the moving target proton and deuteron which have the masses calculated above (*3 in figure 63). From this calculation the Θ_{π}^{*} (with respect to the incident particle direction) and γ , the pion cms momentum in units of the pion mass are determined. The ¹H(p, π^+)²H cross sections and analysing powers have been parametrized in terms of Θ^{*} and γ (*4 in figure 63). These calculations are made for a specified number of target proton momenta and directions with the final A_{π} taken as the mean of the A_{π} from each calculation weighted by the total cross sections.

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