AN INVESTIGATION OF THE FREQUENCY SHIFT MECHANISMS OF IPDP TYPE PULSATIONS

by

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B.Sc. (Hons), University of British Columbia, 1978

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THE FACULTY OF GRADUATE STUDIES
(Department of Geophysics and Astronomy)

We accept this thesis as conforming to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA
October, 1980
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Date  **Oct. 24/80**
ABSTRACT

Possible frequency shift mechanisms for IPDP micropulsation events are examined here. Micropulsation data collected from a north-south chain of stations in British Columbia is used for this study, along with normal-run magnetograms obtained from observatories near this chain, from equatorial observatories, and from observatories to the east of the chain.

Four theories proposing to explain the IPDP frequency shift have been advanced; the inward motion theory, the azimuthal drift theory, the increasing field theory, and the decreasing plasma density theory. It is found that two of these mechanisms, as described in the inward motion and azimuthal drift theories, can, acting together, account for the observed frequency shift in the events detected on the B.C. chain. The greater part of the frequency rise in these events is produced by the inward motion mechanism.

It is also noted that the ionospheric duct strongly affects the propagation of the IPDP hydromagnetic waves through the upper atmosphere.
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Many other people have also contributed to the success of this project. Dr. R.E. Horita of the Physics department, University of Victoria, and fellow graduate student David Boteler helped with the setting up and operation of the B.C. north-south chain and provided many useful discussions on the topic in question. I am also indebted to Dr. S. Watanabe and Brian Chapel for their help with the field operations, and to Dr. K. Hayashi, Dr. R.D. Russell, and the technical staff of the department for their help with the instrumentation used in these operations. I must also offer my thanks to my mother, Mrs. S. Koleszar, and to Miss Sandy Patience for continually correcting the English of this manuscript.

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1. INTRODUCTION

Geomagnetic micropulsations are small, transitory fluctuations in Earth's magnetic field which propagate through the magnetosphere in the form of hydromagnetic waves. The periods of these pulsations, typically between 0.1 seconds and 10 minutes, are short when compared to other magnetospheric phenomena, such as storm-time and diurnal variations. The amplitudes displayed range from less than one gamma ($10^{-9}$ tesla) to, on rare occasions, as high as a few hundreds of gammas, thus seldom exceeding one part in $10^3$ of the strength of Earth's main field.

Like magnetospheric substorms, micropulsations have an external, or solar, origin, as opposed to the internal origin of main field and secular variations. They are generated either directly or indirectly as a result of solar wind - magnetosphere interactions, and the energy required for their generation is supplied by the solar wind.

Geomagnetic micropulsations leave no lasting effects in the magnetosphere, but they can be very useful as natural probes into magnetospheric processes.

Saito (1976) divided micropulsations into two broad categories; continuous pulsations (denoted Pc), and irregular pulsations (denoted Pi) (see Table 1). Following an earlier classification scheme by Jacobs et al. (1964), the Pc section is further divided into six subgroups, and the Pi section into three subgroups. These subdivisions are based on major morphological properties of the micropulsations, such as period, amplitude, and time of occurrence. Saito has offered an even
### Table 1. Classification of Geomagnetic Micropulsations

#### Continuous Pulsations (Pc)

<table>
<thead>
<tr>
<th>Period (sec)</th>
<th>Type</th>
<th>Sub-type</th>
<th>Name</th>
</tr>
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<tr>
<td>0.2-5</td>
<td>Pc1</td>
<td>PP</td>
<td>Pearl pulsation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>HMC</td>
<td>Hydromagnetic chorus</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IPDP</td>
<td>Interval of pulsation of diminishing period</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CE</td>
<td>Continuous emission</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Others</td>
</tr>
<tr>
<td>5-10</td>
<td>Pc2</td>
<td>AIP</td>
<td>Auroral irregular pulsation</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Others</td>
</tr>
<tr>
<td>10-45</td>
<td>Pc3</td>
<td>Pc3</td>
<td>Pc3</td>
</tr>
<tr>
<td></td>
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<td>Pc4</td>
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<td></td>
<td></td>
<td>Pg</td>
<td>Giant pulsation</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Others</td>
</tr>
<tr>
<td>150-600</td>
<td>Pc5</td>
<td>Pc5</td>
<td>Pc5</td>
</tr>
<tr>
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<td>TF</td>
<td>Tail fluttering</td>
</tr>
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#### Irregular Pulsations (Pi)

<table>
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<tr>
<td>1-40</td>
<td>Pi1</td>
<td>Spt</td>
<td>Short-period Pt</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PiB</td>
<td>Pi burst</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PiC</td>
<td>Pi (continuous)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PiD</td>
<td>Daytime Pi</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Psc1,2,3</td>
<td>Sc(Si)-associated Pc1,2,3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Psi1,2,3</td>
<td>Others</td>
</tr>
<tr>
<td>40-150</td>
<td>Pi2</td>
<td>Pi2</td>
<td>Pi2 (formerly Pt)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Psfe</td>
<td>Sfe-associated pulsation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Psc4</td>
<td>Sc(Si)-associated Pc4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Psi4</td>
<td>Others</td>
</tr>
<tr>
<td>150-</td>
<td>Pi3</td>
<td>Psc5,6</td>
<td>Sc(Si)-associated Pc5,6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Psi5,6</td>
<td>Others</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Pip</td>
<td>Polar irregular pulsation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ps6</td>
<td>Substorm-associated long-period pulsation</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Others</td>
</tr>
</tbody>
</table>
more detailed classification within these subgroups which is based largely on the dynamic spectra of the pulsations concerned.

Micropulsations of the $\text{Pc1}$ subgroup have been separated into four categories: pearl pulsations, hydromagnetic chorus, continuous emission, and intervals of pulsations of diminishing period. While all these pulsations exhibit similar morphological characteristics, the dynamic spectra of the intervals of pulsations of diminishing period (IPDPs) are easily distinguished by the continuous rise in frequency of the pulsation throughout the event (see Fig. 1). It is this type of micropulsation that will be the subject of this thesis.

Troitskaya (1961) was the first to study the IPDP phenomenon in depth. Since then the subject has attracted much attention, leaving us with a well founded knowledge of the morphological properties of IPDPs, and a number of theories concerning their origins.

IPDPs consist of a more or less narrow noise band within which occasional elements of higher intensity are interspersed (see Fig. 2). Though the IPDP waveform generally resembles more strongly the $\text{Pc1}$ type of waveform, Roxburgh (1970) has suggested that it also shows many of the characteristics of the more irregular $\text{Pi1}$ waveforms.

The mid-frequency of both the noise band and the higher intensity structural elements of an IPDP increases over the course of each event, though the rate of this increase varies widely between events. A typical event may last anywhere from 20 minutes to as long as two hours. IPDPs usually occur in the
Fig. 1. Dynamic spectrum of IPDP event recorded at Fort St. John, B.C. (L=4.6), on August 9, 1979.
sub-auroral zone, 55° to 65° North or South, geomagnetic coordinates, between 1700 hours and 0100 hours local time, at the rate of a few per month.

Theories of the generation mechanism of IPDPs have been proposed by; Gendrin et al. (1967) and Heacock (1967) (Inward motion theory), Fukunishi (1969, 1973) (Azimuthal drift theory), Roxburgh (1970) (Increasing field theory), and Lin and Parks (1976) (Decreasing plasma density theory). Since an understanding of the IPDP generation mechanism could be very important in understanding magnetospheric substorm processes, more work directed at determining the relevance and relative importance of these theories would be quite useful.

This thesis will test these theories against data collected from a longitudinal line (~291° East, magnetic coordinates) of magnetic stations. A review of the current theories will be given in chapter four, and the experimental results will be presented in chapter five. Chapter six will offer a discussion of these results as they affect the theories in question, and offer suggestions for further work.
Fig. 2. Waveform of a section of the IPDF event recorded at Fort St. John, B.C., on August 8, 1979.
2. DATA COLLECTION AND ANALYSIS

This chapter will discuss the collection and processing of the micropulsation data, as well as the sources of the magnetic field data and Kp indices.

2.1 Micropulsation and Magnetic Field Data Sources

For the purposes of the research presented in this thesis, it was necessary to collect micropulsation data from a number of stations located along a line of geomagnetic longitude. Data was obtained from three such stations in north-central British Columbia, ranging from Fort St. John to the north, to Prince George, and Williams Lake to the south. See Table 2 for the geographic and geomagnetic coordinates of these stations. Data from a more southerly station, located at Pemberton, B.C., proved to be unusable as a result of a tape recorder malfunction. These stations were operated continuously throughout most of the month of August, 1979.

<table>
<thead>
<tr>
<th>Station</th>
<th>Geographic</th>
<th>Geomagnetic</th>
<th>L Shell</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lat. (N)</td>
<td>Long. (E)</td>
<td>Lat. (N)</td>
</tr>
<tr>
<td>Fort St. John</td>
<td>56°14'</td>
<td>239°05'</td>
<td>62.3°</td>
</tr>
<tr>
<td>Prince George</td>
<td>53°55'</td>
<td>237°11'</td>
<td>59.5°</td>
</tr>
<tr>
<td>Williams Lake</td>
<td>52°08'</td>
<td>237°51'</td>
<td>57.9°</td>
</tr>
</tbody>
</table>
Each micropulsation station consisted of three induction magnetometers (measuring dB/dt), their associated amplifiers, and a slow speed tape recorder. Clock and WWVB time code signals were recorded as well as the magnetic signals. The induction magnetometers were high-μ metal cored solenoids, aligned in the H (magnetic north), D (magnetic east), and Z (vertical) directions. Since the response of each sensor-amplifier system was somewhat different, all were calibrated in place with artificial magnetic signals.

Magnetic field data from points near the line of micropulsation stations, as well as from points to its north and east, and from equatorial stations, was also required for this research project. This data, in the form of normal-run magnetograms, was purchased from World Data Centre A for Solid Earth Geophysics in Boulder, Colorado. Data coverage was obtained for all of August, 1979. The stations from which data was acquired are listed in Table 3.

The Kp indices, which provide an estimate of high latitude geomagnetic activity, were also needed for this report. This quasi-logarithmic scale is calculated from 13 high latitude stations every three hours (Rostoker, 1972). The Kp indices are published monthly in the Journal of Geophysical Research by J. Virginia Lincoln, Editor. Those for August, 1979 are listed in Appendix 1.
**Table 3. Location of Magnetic Observatories**

<table>
<thead>
<tr>
<th>Station</th>
<th>Geographic</th>
<th></th>
<th>Geomagnetic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lat. (N)</td>
<td>Long. (E)</td>
<td>Lat. (N)</td>
</tr>
<tr>
<td>Baker Lake</td>
<td>64°10'</td>
<td>264°30'</td>
<td>73.9°</td>
</tr>
<tr>
<td>College</td>
<td>64°52'</td>
<td>212°10'</td>
<td>64.6°</td>
</tr>
<tr>
<td>Fort Churchill</td>
<td>58°45'</td>
<td>266°00'</td>
<td>68.8°</td>
</tr>
<tr>
<td>Fredricksburg</td>
<td>38°12'</td>
<td>282°38'</td>
<td>49.6°</td>
</tr>
<tr>
<td>Great Whale River</td>
<td>55°20'</td>
<td>282°10'</td>
<td>66.8°</td>
</tr>
<tr>
<td>Guam</td>
<td>13°35'</td>
<td>144°52'</td>
<td>4.0°</td>
</tr>
<tr>
<td>Honolulu</td>
<td>21°19'</td>
<td>202°00'</td>
<td>21.1°</td>
</tr>
<tr>
<td>Meanook</td>
<td>54°36'</td>
<td>246°42'</td>
<td>61.9°</td>
</tr>
<tr>
<td>Newport</td>
<td>48°16'</td>
<td>242°53'</td>
<td>55.1°</td>
</tr>
<tr>
<td>Ottawa</td>
<td>45°25'</td>
<td>284°17'</td>
<td>57.0°</td>
</tr>
<tr>
<td>San Juan</td>
<td>18°07'</td>
<td>293°51'</td>
<td>29.6°</td>
</tr>
<tr>
<td>Sitka</td>
<td>57°04'</td>
<td>224°40'</td>
<td>60.0°</td>
</tr>
<tr>
<td>St. John's</td>
<td>47°34'</td>
<td>307°19'</td>
<td>58.7°</td>
</tr>
<tr>
<td>Tucson</td>
<td>32°15'</td>
<td>249°10'</td>
<td>40.0°</td>
</tr>
<tr>
<td>Victoria</td>
<td>48°26'</td>
<td>236°40'</td>
<td>54.3°</td>
</tr>
<tr>
<td>Yellowknife</td>
<td>62°30'</td>
<td>245°31'</td>
<td>69.1°</td>
</tr>
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</table>
2.2 IPDP Event Selection

The rising frequency structure is the dominant feature of the IPDP class of micropulsation, separating it from the other Pc1-type pulsations (see Fig. 3). Therefore, it was this feature, in conjunction with the known diurnal variation of occurrence of IPDPs, which was used to isolate them from the wealth of micropulsation data collected.

Since virtually all IPDP events occur between 1500 hours and 0100 hours local time, dynamic spectra of the Fort St. John data were taken covering the period between 1200 hours and 0200 hours local time for each day data was recorded. Fort St. John was chosen due to this station's working clock and more sensitive instruments. It was also expected that any IPDP events would appear more strongly there. Any rising frequency structures found in these spectra were then checked for total elapsed time, amount and rate of frequency rise, and repetition of the rising structures. Pearl pulsations (also in the Pc1 group) also exhibit rising frequency structures, but they can be distinguished from IPDPs on several points. They are shorter in duration (5 minutes as opposed to 20 minutes for IPDPs), they display a smaller change in frequency, and they repeat at regular intervals (see Fig. 3).

The IPDP events isolated above were compared to slow speed charts (1 day = 1.08 metres) and their waveforms identified (see Fig. 4). The slow speed charts from the other stations were then checked for similar waveforms. No new events were found.

Since the research to be done required that an IPDP event be clearly evident in the data from at least two stations,
Fig. 3. Dynamic spectra of a) IPDP, b) PP, and c) CE. Note the differences in the rising frequency structures of IPDP and PP (parts b) and c) from Heacock, 1970).
Fig. 4. Dynamic spectrum and slow speed chart recording of the IPDP event recorded at Fort St. John on August 9, 1979.
dynamic spectra were taken at all three stations at the event times found from the Fort St. John data. Figure 5 shows three such spectra. A number of events detected at Fort St. John were completely absent or only weakly visible at the other stations, and therefore were not usable. Only three events proved clearly visible at two or more stations. These events, which were detected at all three stations, are listed in Table 4.

<table>
<thead>
<tr>
<th>Date</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. August 6, 1979</td>
<td>2100 - 2120 LT</td>
</tr>
<tr>
<td>2. August 8, 1979</td>
<td>2115 - 2145 LT</td>
</tr>
<tr>
<td>3. August 9, 1979</td>
<td>2135 - 2215 LT</td>
</tr>
</tbody>
</table>

2.3 Micropulsation Data Analysis Methods

Digital time series analysis was performed on the IPDP events selected above to determine the power levels and frequencies present at various points throughout each event. To do this two different techniques were employed, the periodogram approach and the maximum entropy method. The periodograms were used primarily to check the results of the maximum entropy spectra. Before analysis with the fast fourier transform, the data was tapered with a cosine bell function, and the mean was removed. The resulting periodogram was smoothed with a Hanning window. The results of the periodogram method were checked by comparison with a similar analysis of synthetic signals.

The main problem incurred by the use of the maximum entropy
Fig. 5. Dynamic spectra of the August 9, 1979 IPDP event from a) Fort St. John, b) Prince George, and c) Williams Lake.
method is in determining the reliability of the resulting spectra. This necessitates careful consideration in choosing the order of the prediction error filter, since an attempt to increase the resolution too much by using a higher filter order will result in split and spurious peaks in the spectra. The filter length that gives the minimum final prediction error (FPE) provides the best compromise between resolution and error. This minimum FPE filter was used in this study, after having searched for the minimum FPE with filter orders of up to half the sample length. This resulted in short filters, ~10% of the sample length, which is in accordance with the Akaike criterion for filter orders (Akaike, 1969a,b, 1970).

Many spectra were computed with filter orders both above and below the order which produced the minimum FPE, but if the order was more than a few points lower than this value the resolution would be noticeably decreased, or, if the order was a few points higher, the existing peaks would begin to split and spurious peaks would appear. Extensive comparisons with the maximum entropy spectra of synthetic signals and with the previously mentioned periodograms were also made. These tests tended to confirm the choice of filter lengths indicated above, and thus confirmed the reliability of the resulting maximum entropy spectra. Figure 6 shows a periodogram spectrum and a maximum entropy spectrum superimposed, indicating the good agreement between these two methods. The lower power levels in the periodogram spectrum are due to the smoothing process.

Each IPDP event to be studied was first divided into a number of short segments, though these segments were chosen to
Fig. 6. The periodogram and maximum entropy spectra of a segment of the August 9 event at Fort St. John. Note the good agreement between these two methods.
be of sufficient length to avoid the problems inherent in computing the spectra of short records. The segments were then analyzed to yield a profile of the changes that took place during each event.

The noise levels in the data did create occasional problems in computing the spectra. One very weak event was rendered unusable by noise problems, since, using both of the spectral analysis methods available, it proved impossible to obtain reliable spectra for enough of the segments to give an adequate picture of the changes taking place during the event.
3. PROPERTIES OF IPDPS

In this chapter, the general morphology of IPDPs will be examined. This will include a discussion of their physical characteristics, occurrence, and relationship to other geomagnetic phenomena.

3.1 Physical Characteristics

Though IPDPs are generally classed as Pc1 pulsations (period range: 5 sec. - 0.2 sec.), their initial periods may be as long as 20 seconds (Heacock, 1967), which is in the Pc3 range. However, in the case of a more typical event the initial period of pulsation would be between 10 and five seconds, and may be as short as three seconds. The shortest periods reached at the end of an IPDP event usually fall between three seconds and one second, but can occasionally be as long as five seconds, or as short as 0.3 seconds (Tepley and Amundsen, 1964). As is evident above, there are no sharp bounds limiting the range of either the minimum or maximum frequencies present in IPDP events. Those events identified in Chapter 2 had initial periods ranging from 5.6 seconds to 4.2 seconds, and final periods of between 2.8 and 1.9 seconds.

The total amount of increase in frequency can vary quite widely between individual IPDP events. This is also true of the rate of this increase, which can range from a very low value up to five hertz/hour (Roxburgh, 1970), but will more probably be between 0.2 hertz/hour and two hertz/hour. Rates of increase
between about 0.35 and 0.65 hertz/hour were found for the events mentioned above.

The duration of an IPDP event is typically between 20 minutes and two hours, though Roxburgh (1970) has suggested that some may be as short as 10 minutes. The events identified in Chapter 2 were all between 20 and 40 minutes in length. Occasionally two or more IPDP events will occur in sequence (see Fig. 7). There is no regular period of repetition in such cases, and each event is considered to be a separate entity.

Gendrin (1970) reported the mean amplitude of IPDPs to be ~0.1 gamma. However, the structural elements, which appear at irregular intervals in many IPDP events, are of a significantly higher intensity. The frequency increase continues steadily throughout both the noise band and the structural elements.

It has also been noted that IPDPs have identical spectra at conjugate points, and that there is essentially no phase shift observed between these points (Saito, 1969).

3.2 Occurrence of IPDPs

The occurrence of IPDP events is almost entirely concentrated in the evening sector of the magnetosphere. The majority of events take place between 1700 hours and 2400 hours local time, with a strong peak of occurrence at about 2000 hours local time (see Fig. 8). Fukunishi (1969) and Heacock (1971) reported that IPDPs occurring earlier in the day (further from midnight) had lower rates of frequency increase, though Roxburgh (1970) could not support this conclusion.
Fig. 7. Two consecutive IPDP events at Fort St. John on August 6, 1979. Only the second of these two events was evident at stations further south, indicating that they are probably separate occurrences.
Fig. 8. Diurnal variation of occurrence of all IPDPs observed at Seattle, Wash., over an 11 month period (Knaflich and Kenney, 1967).
Generally, IPDPs appear at the rate of a few per month, with a somewhat enhanced rate of occurrence during the summer months (Heacock, 1967). They also tend to take place on active days \((20 < \Sigma Kp < 35;\) Jacobs, 1970). The events selected in Chapter 2 occurred on days with total Kp indices (the sum of the three-hourly indices; Kp) of 14, 19+, and 26-.

Most IPDPs occur in the high sub-auroral zone, between 55° and 65° geomagnetic latitude, with the greater concentration of events in the upper part of this range. Events will occasionally occur outside of this range, in middle or lower latitudes, or they may be propagated to lower latitudes within the ionospheric duct. The longitudinal extent of IPDPs is usually quite limited, though at times one event can be recorded at two stations which are as much as 30° apart.

Although, as mentioned, most IPDPs appear at higher latitudes, those which do occur further towards the equator tend to do so on very active days with high \(\Sigma Kp\) indices (Roxburgh, 1970). Events taking place at these lower latitudes are also inclined to exhibit higher frequencies than their higher latitude counterparts. The relatively low \(\Sigma Kp\) indices for the days of the events mentioned above seems to indicate that these were higher latitude events.

3.3 Relation to other Geomagnetic Phenomena

When IPDPs were first studied (Troitskaya, 1961) it was noted that they occurred on active days, indicating a possible relationship with polar magnetic substorms. Subsequently,
Fukunishi (1969), Roxburgh (1970), and Heacock (1971) all found that IPDP events occurred shortly after, within one hour, of the initiation of the expansion phase of a substorm. Figure 9 illustrates the relation between IPDPs and substorms.

The beginning of the expansion phase is marked by the onset of a sharp negative bay in the auroral zone near local midnight. The expansion phase is that part of a polar magnetic substorm in which energy previously built up in the magnetosphere is supplied explosively to the ionosphere through the acceleration and injection of energetic particles. The dominant current feature in the ionosphere at this time is the auroral zone westward electrojet, which extends from near local midnight through the early morning hours. This electrojet is responsible for the night side negative bays recorded at high latitude ground stations. The repetition of IPDP events is a result of substorms recurring at short intervals. Not all magnetic substorms result in IPDP activity.

Pi2 micropulsations appear to be generated in conjunction with the onset of the expansion phase of a substorm, and are therefore also often observed in association with IPDP events. Heacock (1971) noted that Pi bursts (Pi1 + Pi2), occurring in association with substorms, were centred near local midnight at approximately 70° latitude. This shows that these micropulsations, as well as the substorm produced negative bays, take place to the east and north of the subsequently occurring IPDP events.

Other micropulsations have also been reported in connection with IPDPs. Roxburgh (1970) showed that occasionally IPDPs were
Fig. 9. Magnetic field $H$ component from Great Whale River and IPDP dynamic spectrum from Fort St. John. Note the sharp onset of a negative bay shortly before the IPDP event begins (August 8, 1979).
immediately followed by PP events, and Heacock (1967, 1971) found that unstructured Pc1 - Pc2 activity often preceded the IPDPs recorded at College, Alaska (see Fig. 10). This activity included CE pulsations (Pc1) with periods typically near four or five seconds (also known as 4-second band pulsations).

In addition, other geomagnetic phenomena, such as changes in auroral luminosity, X-ray bursts, and intensity changes in the radiation belts, have been reported in association with IPDP events. Fukunishi (1973) found proton aurorae occurring with IPDPs, and increased cosmic noise absorption (CNA) events have also been detected in connection with IPDPs (Fukunishi, 1973; Lukkari et al., 1977).
Fig. 10. IPDP event from conjugate stations at Macquarie Is. and Kotzebue, Alaska. Note the Pc1 - Pc2 activity leading up to the event (Heacock et al., 1976).
4. IPDP GENERATION MECHANISMS

It is now generally accepted that IPDPs are produced by the proton cyclotron instability process in a region of the equatorial plane of the afternoon/evening sector of the magnetosphere. It is also apparent that their generation is related to the polar magnetic substorm process. This chapter will describe the general mechanism for the generation of IPDPs as well as a number of specific proposals for the frequency shift mechanism.

4.1 General Generation Process

Hydromagnetic emissions in the Pc1 range (PP, CE), which have been extensively studied, are believed to be generated by the proton cyclotron instability process. Roxburgh (1970) and Heacock (1971) both noted that the occurrence of IPDPs is sometimes closely related to these Pc1 pulsations (see sec. 3.3 and Fig. 10). The period ranges and amplitude structures of these pulsations are also quite similar to those of the IPDP events when they occur together. Therefore, the conclusion was drawn that IPDPs may also be generated by the proton cyclotron instability mechanism. Other evidence, including the observation of proton aurorae occurring in connection with IPDPs (Fukunishi, 1973), and the satellite observation of the protons involved in the generation (Horita et al., 1979), has supported this conclusion.

This proton cyclotron instability process which produces
IPDP events involves an interaction between energetic protons and left-handed circularly polarized hydromagnetic waves. This resonance condition will occur when the wave frequency, doppler shifted to the protons' velocity parallel to the magnetic field lines, equals the gyrofrequency of the protons about these field lines. The result of this cyclotron instability process is a transfer of some of the kinetic energy of the protons to the growing hydromagnetic waves. The protons involved in this process are subsequently precipitated into the ionosphere, where they can result in proton aurorae. A further discussion of the proton cyclotron instability process will be presented in Appendix 2.

The protons involved in the generation of IPDPs are accelerated in the magnetotail and injected in towards the midnight sector of the inner magnetosphere at the beginning of the expansion phase of a polar magnetic substorm. They then become trapped on closed field lines, and move westward towards the IPDP generation region under the influence of the curvature and gradient drift mechanisms. The pitch angle anisotropy needed for the proton cyclotron instability process to occur is believed to be created by the loss of low pitch angle protons during this westward drift. Some of the particles accelerated in the tail are not trapped on closed field lines, but are injected directly into the ionosphere at high latitudes near midnight, creating the westward electrojet and negative bays characteristic of substorms. This picture is supported by the observed diurnal variation of occurrence, and association with substorms exhibited by IPDPs, and Frank (1970) has detected a
Fig. 11. A model current system for a typical polar magnetic substorm, showing the partial ring current on the evening side (Kamide and Fukushima, 1972).
partial ring current carried by westward drifting protons in the evening sector of the magnetosphere. Figure 11 shows a model current system for polar magnetic substorms, illustrating the occurrence of this partial ring current. A schematic diagram outlining the particle acceleration in the tail and the westward drift of the trapped protons, followed by the cyclotron turbulence and precipitation of these protons, is presented in Figure 12.

The ionospheric section completing the partial ring current circuit is believed to be the eastward electrojet. This feature, which appears on the ground as a high latitude positive bay in the evening sector, does not occur with all substorms (Boteler, 1980), implying that the entire partial ring current system may not appear with every substorm. This could account for the observed fact that IPDP events also do not appear with every substorm. Figure 13 shows an IPDP event and the associated high latitude positive bay.

As the protons drift westward from the injection region, the cyclotron turbulence is believed to occur in or near the equatorial plane. Such a generation region location is required to produce the identical spectra observed for IPDP events at conjugate points. The hydromagnetic waves thus generated near the equator then propagate down the geomagnetic field lines to the sub-auroral zones of Earth.

Ground based estimates of the energy of the protons involved in the cyclotron instability process range from 10 to 100 KeV (Gendrin et al., 1967; Heacock, 1973; Kangas et al., 1974). Satellite measurements of proton energies in the IPDP
Fig. 12. Schematic illustration of particle acceleration, drift, and precipitation during a substorm (Boteler, 1980).
Fig. 13. Normal-run magnetogram from College, Alaska, and IPDP dynamic spectrum from Fort St. John. Note the positive bay occurring at College. There is a slight time difference between the IPDP and the peak of this bay.
generation region yielded a range of one to 100 KeV (Horita et al., 1979), which is in good agreement with the earlier estimates.

The radial position in the equatorial plane of the generation region is uncertain, though most estimates put it between L shells five and eight (Gendrin et al., 1967; Troitskaya et al., 1968; Fukunishi, 1969; Heacock et al., 1976). Horita et al. (1979) mentioned L values of between 4.7 and 5.5 for the onset of IPDP generation.

The steady rise in frequency over the course of each event is the most prominent characteristic of IPDP pulsations. What physical mechanism is behind this feature is not clear, though many have been proposed. Gendrin et al. (1967) put forward the idea that a continuously increasing background magnetic field in the generation region, due to an earthward motion of this region, resulted in a steadily increasing ion gyrofrequency. This rising gyrofrequency would then produce a constant increase in the frequency of the hydromagnetic waves generated by the proton cyclotron instability mechanism. Fukunishi (1969, 1973) proposed that the frequency shift was due to the greater azimuthal drift velocity of the higher energy protons moving around from the injection region. Since the frequency of the cyclotron instability produced waves varies with the proton energy as $1/E^{1/2}$, the higher energy protons, which would arrive in the generation region first, would result in lower frequency waves. As slower protons of progressively lower energies arrived, the wave frequencies would rise. Roxburgh (1970) also attempted to explain the frequency shift with an increasing
background magnetic field. Unlike previous work, however, the generation region was stationary and the increase in the field was attributed to the decay of the partial ring current. It has also been pointed out (Lin and Parks, 1976) that a decreasing cold plasma density in the generation region could produce a rising frequency structure. All of the these mechanisms will be discussed in greater detail in the next section.

4.2 Frequency Shift Mechanisms

It can be shown (see Appendix 2) that the frequency of the waves generated by the proton cyclotron instability process is given by:

$$w \propto \frac{B^2}{(N_p \omega)^{1/2}} \quad (1)$$

where $B$ is the Earth's dipole field, $N_p$ is the background plasma density, and $\omega$ is the particle (proton) kinetic energy. Each of the frequency shift theories discussed below postulates a change in one of the parameters on the righthand side of (1) in order to produce the rising tone of IPDPs.

Inward Motion Theory. In this theory, the rising IPDP frequency is attributed to the inward diffusion of energetic protons across geomagnetic field lines. (Gendrin et al., 1967, Heacock, 1967). As the protons move inward toward regions of increasingly higher background magnetic field, their interaction with hydromagnetic waves produces steadily higher frequencies.
It is evident from (1) that, assuming steady state magnetospheric conditions, under which \( N_0 \propto B \), and no energy gain by the diffusing particles, the wave frequency would be:

\[
\omega \propto B^{3/4} \tag{2}
\]

If we represent the Earth's dipole field as:

\[
B = \frac{B_0}{R'} \tag{3}
\]

where \( B_0 \) is the field strength on Earth's equator and \( R \) is the distance from the centre of the Earth measured in Earth radii, then (2) becomes:

\[
\omega \propto \frac{1}{R^{7/4}} \tag{4}
\]

It is obvious from this relation that inward motion of the generation region (decreasing \( R \)) will produce a rising tone structure.

The total change in field strength seen by a particle moving through a magnetic field is given by:

\[
\frac{d\vec{B}}{dt} = \frac{\partial \vec{B}}{\partial t} + (\vec{v} \cdot \nabla) \vec{B} \tag{5}
\]

However, for the inward diffusion theory, the assumption that \( \partial B/\partial t = 0 \) (steady state conditions) is made. Therefore, the total field change seen by the diffusing protons is:

\[
\frac{d\vec{B}}{dt} = (\vec{v}_p \cdot \nabla) \vec{B} \tag{6}
\]

Where \( v_p \) is the proton diffusion velocity (directed inward). If
the steady state condition is to be maintained, then it must be assumed that the diffusion process does not affect the background plasma. It is evident from the equation:

$$\frac{\delta B}{\delta t} = \nabla \times (v_x \vec{B}) \quad (7)$$

where $v_x$ is the velocity of the bulk plasma, that any such effect would produce a non-zero $\delta B/\delta t$ term.

Substituting equation (3) into (6), we get:

$$\frac{DB}{Dt} = \frac{3v_x B}{R} \quad (8)$$

From (2) and (8) it can then be shown (Roxburgh, 1970) that the inward motion required to produce the rising tone of IPDPs is:

$$v_x = \frac{2R Dw}{9w Dt} \quad (9)$$

For a generation region distance of $R=6$, the diffusion velocity necessary would be $\sim 6$ km/sec. However, the uncertainty in this distance makes it difficult to obtain reliable velocity estimates.

It has been suggested (Lacourly, 1969) that the inward drift is due to a westward electric field in the equatorial plane. The diffusion velocity would then be given by:

$$\vec{v}_x = \frac{\vec{E} \times \vec{B}^2}{B^3} \quad (10)$$

The electric fields necessary to produce the drift velocities given by (9) can now be estimated, again subject to the uncertainty in the distance to the generation region. Lacourly
Fig. 14. Diagram showing the inward motion of the generation region due to the inward motion of the plasmapause. The plasmapause is represented by the solid and (later) dashed curved lines. The ground station is represented by G, and L and T indicate L-shell and time, respectively (Horita et al., 1979).
gave a value of $-5 \times 10^{-4}$ volts/m for this field, and, for a distance of $R=6$, Roxburgh found an average value of $E=8.8 \times 10^{-4}$ volts/m.

Another version of the inward motion theory is discussed by Horita et al. (1979). In this case, only the area in which the conditions are suitable for ion cyclotron turbulence moves inward. No inward diffusion of energetic protons is required; the protons drifting westward from the midnight injection region continually resupply the generation region as it moves inward (see Fig. 14). The inward motion is believed to be caused by an inward motion of the plasmapause. The proton cyclotron instability is thought to be excited when the drifting protons meet the evening side plasmasphere bulge. Equation (2) still controls the frequency of the generated waves.

**Azimuthal Drift Theory.** Fukunishi (1969) attributed the rising tone structure of IFDPs to a gradually softening beam of protons. This beam is produced by the westward drift of the protons injected near local midnight during magnetic substorms. The gradient and curvature of the geomagnetic field produce a combined azimuthal drift velocity of:

$$v_{az} = \frac{w(1+\cos^3 \alpha)}{eBR_e}$$  \hspace{1cm} (11)

where $\alpha$ and $R_e$ are, respectively, the pitch angle of the protons, and the radius of curvature of the field lines in the region where the drift is occurring. The magnetosphere is assumed to be in a steady state condition.

It is obvious from (11) that the azimuthal drift velocity
is greater for higher energy protons and they will therefore arrive in the generation region first. Protons of lower energies will arrive later, producing the softening energy spectrum of the beam required for IPDP generation. Assuming dB/dt=0, and a constant background plasma density in the generation region, the generated wave frequency from (1) would become:

$$w \propto \frac{1}{W^{1/n}} \quad (12)$$

Therefore the earlier arriving higher energy protons would produce a lower frequency. As progressively lower energy protons arrived, the generated frequency would go up, resulting in the steadily rising tone of IPDP pulsations.

From equations (11) and (12) it can be shown that the wave frequency resulting from the differential azimuthal drift velocities can be related to the elapsed time $t$ since the substorm expansion phase onset (particle injection) by:

$$w \propto t^{1/n} \quad (13)$$

Therefore, the rate of increase of $w$ becomes:

$$\frac{dw}{dt} \propto \frac{1}{t^{1/n}} \quad (14)$$

It is now evident from (14) that longer delays between the expansion phase onset and the generation of an IPDP event will produce lower rates of frequency rise. This effect has been noted by some authors (see sec. 3.2).

Fukunishi indicated that the cyclotron turbulence generating IPDPs would occur when the pitch angle anisotropy
caused by the loss of small pitch angle particles during the westward drift became pronounced enough. It has also been suggested the cyclotron instability occurs near the plasmapause as the westward drifting protons meet the plasmasphere bulge (Heacock, 1973; Horita et al., 1979).

**Increasing Field Theory.** Roxburgh (1970) suggested that the rising tone of IPDPs was the result of an increasing magnetic field in the generation region. The inward motion theory also accounts for the rising tone with an increasing magnetic field, but attributes this increase to the motion of the generation region inward toward areas of higher field strength. In the increasing field theory, the generation region does not move, but the field strength changes with time.

Since the generation region is stationary, it is the second term on the right-hand side of (5) which is set to zero ([\(\vec{V} \cdot \vec{V})B=0\)]. Therefore, the total change in the magnetic field as seen by the protons in the generation region is given by:

\[
\frac{DB}{Dt} = \frac{d}{dt}
\]

In actual fact, the \((\vec{V} \cdot \vec{V})B\) term may be non-zero. Equation (7) shows that, if \(\lambda B/\lambda t \neq 0\), then \(\vec{V} \neq 0\). This effect would create a non-zero \((\vec{V} \cdot \vec{V})B\) term in (5). However, it is believed that this term will remain very small \((\vec{V} \cdot \vec{V})B < \lambda B/\lambda t\). Roxburgh (1970) showed that, at its extreme maximum, it is only of the same order of magnitude as the \(\lambda B/\lambda t\) term.

If it is assumed that both \(N_e\) and \(W\) are constant, then (1)
shows that, for the increasing field theory, $w$ will be given by:

$$w \propto B^3$$  \hspace{1cm} (16)

Roxburgh outlined the conditions necessary to produce the increasing magnetic field and IPDP pulsations as follows. Prior to the expansion phase of a magnetic substorm, the field in the IPDP generation area is slowly depressed by the formation of the partial ring current outside this area. This current, which occurs only in the evening quadrant, is believed to be composed of westward drifting protons. At the start of the expansion phase, the source of these protons, which is near local midnight, is cut off. The ring current then rapidly decays, resulting in the recovery of the magnetic field in the generation region to its normal strength. A source of energetic protons with an anisotropic pitch angle distribution is needed for the proton cyclotron instability to occur. It is thought that such protons are injected into the inner magnetosphere shortly before the onset of the substorm expansion phase. These protons would then drift westward, and be in the IPDP generation region when the magnetic field was recovering. Any hydromagnetic waves then generated by these protons, via the proton cyclotron instability mechanism, in the presence of the increasing magnetic field would have the characteristic rising tone of IPDPs.

Though some aspects of this mechanism, such as the time of the proton injection and the role of the partial ring current, are not in perfect agreement with the general mechanism
discussed in section 4.1, Roxburgh's work must still be considered.

**Decreasing Plasma Density Theory.** Lin and Parks (1976) investigated in detail the role of the proton cyclotron instability process in clouds of particles allowed to drift in the magnetosphere. The particles considered were injected near midnight onto closed field lines at L=7, then allowed to drift westward around to the IPDP generation region. Calculations of the growth rate of hydromagnetic waves generated by the proton cyclotron instability process in these drifting particles indicated that both the azimuthal drift effects discussed earlier, and the effect of a changing cold plasma density must be accounted for in order to understand the frequency characteristics of these waves. It was shown that either the energy dependent azimuthal drift or a decreasing cold plasma density could each, independently, result in a rising tone frequency structure.

In a constant magnetic field, the frequencies generated by drifting particles with a constant background cold plasma density is described by (12). However, if $N$ is allowed to vary, with the drift effects not included, the frequency of the generated waves is given by:

$$w \propto \frac{1}{N^{\alpha}}$$

which shows that it is also possible to have rising tones when $N_{\rho}$ is decreasing. The effects of each of these two conditions
Fig. 15. Growth rate contours for static cold plasma density with drift effects included (top), and decreasing cold plasma density with no drift (bottom) (Lin and Parks, 1976).
Fig. 16. Growth rate contours for various cold plasma density profiles, all with drift effects included. Note the rising tones for the constant and decreasing cold plasma density plots (Lin and Parks, 1976).
are illustrated separately in Figure 15.

It is then clear from the work of Lin and Parks that azimuthal drift or decreasing density effects can produce IPDP-type spectra. It is also pointed out that it is quite possible that both these effects may operate together to produce rising tone structures. This situation is illustrated in Figure 16.

4.3 Discussion of Frequency Shift Mechanisms

At present, the relative importance of each of the above mechanisms proposed to account for the frequency shift of IPDPs is not clear. It is quite probable that more than one of these processes may be involved in the generation of an IPDP event, though what combinations are possible, and the relative contributions to the frequency rise of each mechanism in a combination, remains to be determined. Many of the assumptions made for each of the theories, such as the time constant magnetic fields required by the inward motion and azimuthal theories and the static particle density of the increasing field theory, can be broken without invalidating the theory. These assumptions are made primarily to facilitate the evaluation and comparison of the various theories. If they are found to be invalid it would, in most cases, merely mean that two or more mechanisms were operating simultaneously. An example of this type of situation is the superposition of the azimuthal drift and decreasing plasma density mechanisms proposed by Lin and Parks (1976).

For the version of the inward motion theory proposed by
Gendrin et al. (1967) the assumption was made that the diffusion process affected only the protons involved in the cyclotron turbulence, and did not disturb the background plasma. It is difficult to see how this background plasma could remain completely unaffected, especially if the inward diffusion is caused by an $\mathbf{E} \times \mathbf{B}$ drift. However, if a bulk motion of the plasma does create a non-zero $\mathbf{B}/\mathbf{t}$ term (from (7)), the frequency rise due to the inward motion will be affected (from (5)). The extent of this effect is not known, since it is very difficult to calculate the $\mathbf{B}/\mathbf{t}$ term created by the background plasma motion.

Another example of the possible overlap of frequency shift mechanisms would be the combination of the inward motion and azimuthal drift effects. If the generation region is moving inward due to the Earthward displacement of the plasmapause, and the protons involved in the ion cyclotron turbulence in this region are being continuously resupplied by protons drifting azimuthally on progressively lower L shells, then both these effects would contribute to the rising tone of hydromagnetic waves generated. However, in this case, the effects on the azimuthal drift process of the changing drift velocity due to the higher magnetic field strength at lower L shells and decreasing drift path length on lower L shells must be taken into account. Under these conditions, and assuming that the Earth's magnetic field is the dipole field represented by (3), equation (13) describing the azimuthal drift effects becomes:

$$w \propto (\mathbf{Rt})^{1/2} \quad (18)$$
Note that this results in a decreased frequency shift effect from the azimuthal drift mechanism.

On the other hand, if Gendrin's picture of the inward motion theory is correct (protons diffusing inward across field lines), then the two mechanisms could be superimposed without affecting each other. Here, the protons would drift westward on one L shell, then diffuse inward in the generation region under the influence of an electric field.

An increasing magnetic field in a stationary generation region would also influence the azimuthal drift mechanism. From (11) it is apparent that the increasing field would cause a reduction in the drift velocity. If the increasing field occurs, as Roxburgh postulates, throughout the evening sector, the result would be to slow the softening of the proton beam, and therefore slow the frequency rise due to the azimuthal drift.

It appears, however, that most of the potential combinations involve only the simple superposition of the mechanisms on one another without affecting the processes involved in each mechanism.

There is evidence to support each of the theories examined in section 4.2 and their possible superposition. The satellite (Explorer 45) observations of the resonant protons (Horita et al., 1979) seem to support the azimuthal drift theory, but the possibility of inward motion of the generation region in conjunction with this drift is left open. Other authors, notably Heacock (1973) and Kangas et al. (1974), have found that the azimuthal drift effects alone are insufficient to produce the observed frequency shift. They suggest that inward motion, and
possibly cold plasma density decreases, must also be involved.

Heacock et al. (1976) indicated that the inward drift mechanism could be quite important, especially for IPDPs generated closer to midnight. Gul'elmi (1974) points out evidence for the superposition of the inward motion and azimuthal drift theories in the generation of IPDPs, and also suggests that the effect of an increasing background magnetic field should be taken into account. The results from the Finnish north-south chain presented by Lukkari et al. (1977) indicate an inward motion of the generation region. It is also suggested that changing cold plasma densities could have a strong effect. Roxburgh (1970), using magnetic field data from satellite ATS-1, presented strong evidence for the existence of an increasing magnetic field in the generation region during IPDP events.
5. EXPERIMENTAL RESULTS

The experimental work reported on in this chapter deals with the IPDP frequency shift mechanisms discussed in sections 4.2 and 4.3 of the preceding chapter. In Chapter 2 three IPDP events were identified and the analysis of their power spectra was reviewed. Using the information obtained from this analysis, and the normal-run magnetograms also discussed in Chapter 2, an attempt is made here to determine which mechanism or mechanisms are responsible for the frequency rise observed in the IPDP events under consideration.

5.1 Inward Motion of Generation Region

A north-south line of micropulsation stations, such as the B.C. chain, is ideal for the detection and analysis of a possible inward movement of the IPDP generation region. If the generation region moved inward in the equatorial plane, the hydromagnetic waves produced would travel along successively lower field lines to lower latitudes on Earth. This would create a shift towards the equator of the IPDP event as observed on a north-south line of stations.

However, after propagating down to the ionosphere, the hydromagnetic waves of IPDPs may travel horizontally in the ionospheric duct. They could therefore appear simultaneously at all the stations of a short chain, such as the one in B.C. providing data for this study. Even in such cases, though, an equatorial displacement of the peak amplitude of the IPDP event
should be discernible on a north-south chain if an inward motion of the generation region is responsible for the rising frequency of the event.

If ionospheric ducting is taking place during the IPDP events recorded on the B.C. chain, then these events should appear at all three stations simultaneously with each event exhibiting very similar frequency and power profiles along the whole chain. Figure 17 shows the peak frequency and peak power profiles of one IPDP event. It is evident that the frequency evolution of this IPDP is almost identical at all three stations. The shape of the power profiles obtained from each of the stations are also very similar, though the absolute power levels are quite different, and the relative power levels between the stations vary somewhat as the event progresses. It is then consistent with this evidence that the hydromagnetic waves were propagating horizontally in the ionospheric duct during this IPDP event. The characteristics described above which lead to this conclusion were also observed in each of the other two events selected from the B.C. chain data.

The analysis producing the frequency and power information discussed above was conducted only on the H component of the micropulsation records. The power present in the Z component was very much less than that in the H component, and therefore it need not be considered. However, the power in the D component was of the same order of magnitude as that in the H component, hence it would have a strong effect on the shape of the total power profiles of the IPDP events. On a high speed chart recording, the amplitude profiles of the H and D components
Fig. 17. The frequency (top) and power (bottom) profiles of an IPDP event recorded on the B.C. north-south chain.
appeared to be quite similar at all three stations. To verify this, a combined H - D component power profile was compared to the corresponding H component profile for one event. The results showed good agreement between the two (see Fig. 18), demonstrating the validity of the use of the H component alone.

The ionospheric duct is a natural waveguide formed by the sharply varying ion concentrations in the upper atmosphere. The high level of ionization at the peak of the ionospheric F2 layer produces a minimum in the Alfvén wave velocity there, creating the possibility of hydromagnetic wave energy becoming trapped in this layer and travelling horizontally as in a waveguide. The local time, season, and level of magnetic activity all effect the ability of this waveguide to propagate hydromagnetic waves, since these factors all affect the level and location of the ionization peaks in the ionosphere. During the day, for instance, the reflectivity of the waveguide's lower wall (the E region) decreases, resulting in a greater attenuation for waves propagating in the duct.

The lower cut-off frequency for propagation in the ionospheric duct is believed to be ~0.5 hertz (Nishida, 1978). This means that micropulsations below this frequency, including the majority of IPDPs, will not be effectively contained within this waveguide. The wave energy will leak across both the upper and lower walls of the waveguide as it travels horizontally, spreading upward throughout the magnetosphere and penetrating down to the Earth's surface where the waves can be detected as micropulsations. This rapid attenuation may account for the relatively small area over which most IPDPs are observed.
Fig. 18. Fort St. John power profiles for the $H$ component (solid line) and the total horizontal component ($H+D$) (dashed line). Since this relative relationship between $H$ and $H+D$ appeared to be the same at all three stations, it is feasible to use the $H$ component alone.
Since the ionospheric duct is obviously affecting the IPDP wave propagation during the events studied, an equatorial displacement of the peak IPDP amplitude must, as mentioned, be looked for in order to detect an inward motion of the generation region. Such a shift could be detected by searching for any continuous changes in the relative amplitudes between station pairs. In particular, if the IPDP generation region were on an L shell such that the waves travelling down the field lines arrived in the ionospheric duct above Fort St. John, then the amplitude seen on the ground at Fort St. John should be much greater than that seen at Prince George or Williams Lake. If the generation region then began moving inward to lower L shells, the waves would arrive in the duct at lower latitudes closer to Prince George. Therefore, the amplitude at Prince George, relative to Fort St. John, would increase. If the generation region were stationary, then the relative amplitudes would not change.

For each event, plots were made of the peak power (at the IPDP midfrequency) recorded at each ground station relative to that recorded at each of the other stations. In every case, consistent regular changes in the relative power levels were found which indicated an inward motion of the IPDP generation region (see Fig. 19). By examining the form of these power ratio plots it is possible to estimate roughly where the generation region is, and to what extent it is moving. However, if rough modelling calculations can be carried out to determine the strength on the ground of the IPDP signal after it has travelled along the ionospheric duct, then more quantitative results can
Fig. 19. The power ratio profiles for each possible station pairing for an IPDP observed on the B.C. chain. Note the regular changes indicating a non-stationary generation region.
be obtained for the location and movement of the generation region.

Such calculations can be performed if the attenuation in and the height of the ionospheric duct are known. The positions of the ground stations and the power ratios found from these stations can then be used to determine the geomagnetic latitude at which the IPDP signal entered the ionospheric duct. Each power ratio would yield two possible latitudes; however, as described above, the shape of the power ratio curves would be sufficient to determine which was the correct result.

To achieve the results described above rough assumptions must be made concerning the degree of attenuation suffered in the ionospheric duct and the height of the lower boundary of the duct. These assumptions are necessary to determine where the IPDP signal must have entered the duct to produce the observed signal strengths at the ground stations. Due to the fact that the IPDPs observed were below the duct cut-off frequency, the signal strength was assumed to fall off in proportion to the square of the distance as the waves travelled through the duct and then through the lower atmosphere to the ground. This assumption is considered to be very rough, and may well over estimate the attenuation undergone, though it is supported by the latitudinal amplitude profile of Pi2 micropulsations given by Jacobs (1970). Using this assumption, calculations to determine the latitude of the incoming waves were attempted using a number of heights for the lower duct boundary. An approximate idea of the lower boundary height can be gained from a study of the ionospheric layers, although, as mentioned, the
positions and strengths of these layers are not constant. For the events studied, an altitude of 150 kilometres seemed to provide the best results.

Once the geomagnetic latitude of the area in which the IPDP hydromagnetic waves are arriving in the ionosphere is known, the L value can be easily calculated. This value gives the radial position of the generation region in the equatorial plane, in units of Earth radii. For each event, L values were calculated from all three possible power ratios. The results obtained from the Prince George/Fort St. John and Williams Lake/Fort St. John ratios were quite consistent (<5% apart). However, the Williams Lake/Prince George L values were not close to those of the other two station pairs. The much weaker signals recorded at the Prince George and Williams Lake stations may have increased the errors noticeably. Also, these two stations have essentially the same separation in longitude as they do in latitude. Since all changes in the power ratios are assumed to be due to latitudinal effects, any unknown longitudinal effects could strongly influence the Williams Lake/Prince George results. Since the latitudinal separations between Fort St. John and the other two stations are much greater than their corresponding longitudinal separations (see Table 2), such longitudinal effects would be much less evident in the results from these station pairs.

Accordingly, the results of only the Prince George/Fort St. John and Williams Lake/Fort St. John calculations were considered to produce the L value profiles for each event. These results, which demonstrate the inward motion of the IPDP generation region, are shown in Figure 20 for all three events.
Using equation (4) from Chapter 4, and substituting in the $L$ values found above for $R$, the relative frequency rise due to the inward motion can be calculated. From (4) we have:

$$\frac{w}{w_0} = \left( \frac{L_0}{L} \right)^{3/2} \quad (19)$$

where $w_0$ and $L_0$ are the initial wave frequency and initial $L$ value of the generation region, respectively.

The relative frequency increases found from (19) are also included in Figure 20, along with the actual relative frequency increases for these events as obtained from the experimental data. On comparing these measured and calculated frequency shifts, it must be noted that the calculated shifts are generally below the measured ones when every point throughout all three of the events is considered. Although, in the case of one of these IPDPs, the final total measured and calculated frequency increases are very close, it is thought that the generally higher frequencies measured from the experimental data during this event are more significant.

On the basis of the above conclusion, it is believed that, for the three events studied here, the frequency rise due to the inward motion of the IPDP generation region is insufficient to account for the total increase observed. The remainder of this increase must then be generated by one (or more) of the other mechanisms discussed in the last chapter. This point will be further examined in the following sections.
Fig. 20. The inward motion of the IPDP generation region in terms of L values, and the frequency shift predicted from these values. The actual frequency shift is included for comparison. Part (a), August 6 event; part (b), August 8 event; and part (c), August 9 event (following page).
Fig. 20. Part (c).
5.2 Frequency Shift from Azimuthal Drift Effects

There has been a great deal of evidence published in support of the azimuthal drift frequency shift mechanism, and it would seem to be a likely candidate to produce the remaining part of the frequency rise not accounted for by the inward motion mechanism.

If the inward motion of the generation region is due to the inward diffusion of the resonant protons, then the azimuthal drift and inward motion mechanisms can be simply superimposed (see sec. 4.3). In this case, the frequency shift from the azimuthal drift will be given by (13), and with the inward motion effect from (4), the total relative frequency shift becomes:

\[
\frac{w}{w_o} = \left( \frac{L_o}{L} \right)^{3/2} \left( \frac{t}{t_o} \right)^{1/2}
\]

(20)

where \( t_o \) is the start time of the IPDP event. The delay time \( t \) is set to zero at the time of the sharp onset of the negative bay recorded near local midnight at Great Whale River shortly before each IPDP event.

In general, the frequency increase given by (20) is slightly closer to the actual increase observed than that predicted by the inward motion alone (from (19)). However, the combined increase from these two mechanisms in simple superposition consistently overestimates the actual increase, as opposed to the underestimation of the inward motion mechanism. In an effort to achieve an even closer match between the real and predicted frequency shifts, the azimuthal drift mechanism
will now be examined in association with the inward motion mechanism where this motion is now caused by the Earthward displacement of the plasmapause, with the resonant protons continuously drifting into the generation region from the east.

As pointed out in section 4.3, the drift mechanism is now affected by the inward motion, and its wave emission frequency is now controlled by \( (18) \). In this case, the total relative frequency rise due to the azimuthal drift and inward motion mechanisms is now not given by \( (20) \), but by:

\[
\frac{w}{w_0} = \left( \frac{L}{L_e} \right) \frac{\sqrt{L}}{L_e} \frac{\sqrt{L}}{L_e} \frac{t}{t_e} \sqrt{L_e}
\]

which produces a slightly slower rate of increase. Figure 21 shows the frequency increase due to the westward drift effects (from \( (18) \)) as well as the total increase due to the combination of mechanisms under consideration as given by \( (21) \). It can be seen that the total predicted frequency shift matches the real shift reasonably well throughout all three events. It appears then that this combined process of plasmapause associated inward motion of the IPDP generation region, together with the energy dependent azimuthal drift of the resonant protons into this region during its movement, can account for the observed spectra of these IPDPs.

Further evidence for the involvement of the azimuthal drift mechanism can be found by studying the relation between the rate of the frequency rise and the delay time between the substorm onset and the IPDP event. For the IPDPs considered here, it was clear that the longer delay times were associated with lower rates of frequency increase as produced by the azimuthal drift
Fig. 21. The frequency shift due to the azimuthal drift mechanism as corrected for the inward motion of the generation region, and the total shift due to the combination of these two mechanisms (as given by (21)). The real frequency shift is included for comparison. Part (a), August 6 event; part (b), August 8 event; and part (c), August 9 event (following page).
Fig. 21. Part (c).
mechanism alone (from equation (18)). This effect has been observed by other authors (Chapter 3) and, from (14), is a predicted consequence of the azimuthal drift mechanism.

5.3 Increasing Field and Decreasing Plasma Density Processes

Since it is evident that the frequency shift of the IPDP events under study can be attributed entirely to the inward motion - azimuthal drift mechanism as described above, no contribution is needed from the other two mechanisms discussed in Chapter 4; the increasing field theory and the decreasing plasma density theory.

Though the increasing field theory cannot be evaluated by a chain of micropulsation stations, normal-run magnetograms obtained from observatories near the chain and from equatorial regions at approximately the same longitude as the chain can be used to determine whether or not this process is contributing to the frequency rise of an IPDP. According to the increasing field theory, the IPDP pulsations are generated as the partial ring current decays, and thereby produces a rising background magnetic field in the IPDP generation region. The decay of the partial ring current, and therefore of the entire associated current system including the eastward electrojet, can be seen on the ground with the aid of the normal-run magnetograms.

The decay of the partial ring current can be seen directly as the recovery of the substorm associated equatorial region $H$ component negative bay in the afternoon/evening sector. If an IPDP event is observed at the same time and at the same
longitude as this field recovery, then the increasing magnetic field resulting from the partial ring current decay may be contributing to the frequency rise of the IPDP. The decay of the partial ring current system can also be observed as the recovery of an H component positive bay at high latitudes as the eastward electrojet abates. Again, if an IPDP appears during this positive bay recovery, then the increasing field mechanism could be at least in part responsible for the frequency increase. If an IPDP event does not occur at the same time as these bay recoveries, then the increasing field theory cannot be involved in the generation of the event since the partial ring current decay necessary to produce the increasing field will not be occurring.

The H component magnetograms from Honolulu show no significant field recoveries near the times at which the IPDPs were recorded. Even though the Honolulu observatory is somewhat north of the equator and west of the B.C. micropulsation chain, this must be considered to be viable evidence against the possibility of the increasing field process being involved in the generation of these events. As well, the correlation of the IPDPs studied here with high latitude H component positive bays generally did not support the involvement of the increasing field mechanism. The IPDP events recorded on August 8 and August 9 did not occur during a positive bay recovery (see Fig. 22). However, the situation on August 6 was somewhat more confused. Higher latitude stations, such as College and Meanook, did record a positive bay recovery during the IPDP event (see Fig. 13, Chapter 4), although magnetograms from lower latitude
Fig. 22. A positive bay recorded at College simultaneously with the observation of an IPDP event on the B.C. chain. The time of occurrence of the IPDP is marked on the magnetogram by the vertical lines. Note that the IPDP does not occur on the recovery side of the bay.
stations (Victoria, Sitka, Newport) could not confirm this observation. The possibility remains, though, that for at least this event, the increasing field mechanism could have contributed to the IPDP frequency rise, although this is not considered likely, since the entire shift has already been accounted for (see sec. 5.2). In general, then, the magnetograms from both the equatorial and northern regions can be seen as confirming that the increasing field frequency shift process does not contribute to the frequency rise of the IPDP events studied here.

No data was obtained for the evaluation of the decreasing plasma density theory. However, since the inward motion and azimuthal drift theories can account for the entire frequency rise, there is no evidence to support the inclusion of this effect in the generation process of the IPDPs under consideration.

5.4 Discussion

It has been shown that, using rough calculations to determine the L shell of the IPDP generation region and using the onset of the midnight sector negative bay as the start time of the westward drift, the combination of the inward motion and azimuthal drift frequency shift mechanisms described by (21) can account for the observed frequency rise of the IPDPs analyzed. Equation (21) can be regarded as describing the frequency rise of the IPDPs as produced by the inward motion process corrected for the effects of the energy dependent azimuthal drift of the
protons to the generation region along different paths through different background magnetic field strengths.

As pointed out in section 5.3, the conditions may have been right for Roxburgh's increasing field mechanism to contribute to the frequency shift of one of the IPDP events studied, even though the total frequency increase for this event can be accounted for in other ways. It is quite possible, however, that errors induced by the approximate nature of the calculations (say from the assumptions concerning the pure dipole nature of the background magnetic field or the conditions of the ionospheric duct) used to predict this frequency shift could leave room for the possible contribution to the increase by the increasing field process. The same argument also holds true for the decreasing plasma density theory, though it is believed that the contributions from these two processes would be small if they are present at all.

It was mentioned in Chapter 2 that a number of IPDP events were recorded only at the Fort St. John station. Since the relative amplitudes found from at least one pair of stations are required to find the L value of the generation region, any possible motion of the generation region for these events could not be analyzed. These recordings could represent the southernmost extent of IPDPs moving southward from higher latitudes, or they could be events with stationary generation regions located near the L shell of Fort St. John. Poorer propagation in the ionospheric duct could account for these IPDPs not being detectable further to the south, at Prince George or Williams Lake.
6. Conclusions and Future Experiments

With the aid of the micropulsation data from the B.C. north-south chain and the normal-run magnetograms from Great Whale River it has been established that the frequency shift of all three of the IPDP events studied could be accounted for by an inward motion of the generation region combined with the energy dependent westward drift effects on the resonant protons. The frequency of the IPDP hydromagnetic waves produced by the proton cyclotron instability process rises due to the increasing magnetic field which results from the inward motion to areas of higher field strength of the plasmapause, near which the instability process occurs. The steadily softening nature of the beam of protons arriving in the generation region during its Earthward movement also contributes to the frequency shift.

It must be pointed out, however, that there are limitations on the conclusions which can be drawn from the study of only three events. While it is now apparent that IPDPs can be generated by the mechanism described above, it is not necessary that all events be produced in this manner. IPDPs which appear at different latitudes and/or different longitudes, or under different substorm conditions, may involve different frequency shift mechanisms, either singly or in combination. The fact that all the events studied here occurred at approximately the same latitude and local time may explain why the same generation mechanism was observed in each case.

The detailed study of many more IPDP events will be necessary before their generation is well understood. An extended longitudinal (north-south) chain would be necessary
to determine the location and motion of the generation region of a wide range of events. In addition, a chain of micropulsation stations along a line of constant latitude could provide useful information on the longitudinal extent and variation of individual IPDPs. Such data could be instrumental in confirming the role of the azimuthal drift mechanism, as well as aiding in the removal of the effects of any longitudinal variations in an IPDP event from the results produced by a north-south chain with stations at slightly different longitudes.

Both ground-based and satellite-based observations, carried out in conjunction with micropulsation observations, on the polar magnetic substorm current system in general, and on the partial ring current system in particular, could be very important to the eventual understanding of IPDPs and their relation to the magnetospheric substorm process. Such data would be useful especially for the evaluation of the increasing field theory. Satellite particle observations would also be required for the direct evaluation of the decreasing plasma density theory.

A specific experiment aimed at confirming the occurrence of the generation mechanism presented in this thesis to account for the frequency shift of the three events studied could be carried out with the aid of an east-west (latitudinal) chain of micropulsation stations. Due to the shape of the evening side plasmasphere bulge (see Fig. 14, Chapter 4) IPDPs occurring closer to midnight should tend to appear at lower latitudes since the westward drift would have to occur on lower L shells in order to intersect the plasmapause, which approaches closer
to Earth towards the midnight sector (away from the bulge).

Though the assumptions concerning the attenuation in and the height of the ionospheric duct seemed to produce reasonable results for the inward motion of the IPDP generation region, the degree of confidence in the overall mechanism proposed for the frequency shift of the events studied in this thesis could also be increased significantly by a much more detailed and in depth study of the propagation of IPDP-type waves in the duct. Such a study, however, would become quite involved, and must therefore be left for future work.
REFERENCES


APPENDIX 1

KP INDICES FOR AUGUST 1979

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APPENDIX 2
PROTON CYCLOTRON INSTABILITY FREQUENCY

The dispersion relation for a left hand polarized ion cyclotron wave propagating parallel to the background magnetic field in a plasma consisting of protons and electrons is (Jacobs, 1970):

\[ w^2 - c^2 k^2 - \frac{n_t^2 w}{w + w_e} - \frac{n_e^2 w}{w - w_p} = 0 \]  \hspace{1cm} (i)

where \( n_t \) (plasma frequency) is:

\[ n_t = \frac{4\pi N_l e^2}{m_t}, \quad l = p, e \] \hspace{1cm} (ii)

and \( \omega_t \) (cyclotron frequency) is:

\[ \omega_t = \frac{|eB_0|}{m_t c}, \quad l = p, e \] \hspace{1cm} (iii)

Since, for resonance to occur, the wave frequency must be doppler shifted to the proton velocity parallel to the background magnetic field, the resonance condition can be expressed as:

\[ w - kV_z - w_p = 0 \] \hspace{1cm} (iv)

where \( V_z \) is the proton streaming velocity.

Using the approximations \( m_e << m_p \) and \( w << w_e \), and assuming that \( N_e = N_p \), (i) can be written as:

\[ w^2 - c^2 k^2 - \frac{4\pi N_p ecw^2}{B_o (w - w_p)} = 0 \] \hspace{1cm} (v)
Now, using (iv) to substitute into (v) for \( k \), and the approximation \( V_s \ll c \), we have:

\[
\frac{4\pi N_p e c w^2}{B_0 (w - w_p)} \approx -c^2 (w - w_p)^2 \quad (vi)
\]

which, on algebraic manipulation, becomes:

\[
w^2 = \frac{B_0}{N_p W_{\parallel}} \cdot \frac{e^2}{8\pi m_p c^2} \cdot \left(1 - \frac{w}{w_p}\right)^3 \quad (vii)
\]

where \( W_{\parallel} \) is the parallel energy of the protons. It is then obvious that, using the assumption that \( w \ll w_p \), (vii) yields:

\[
w \propto \frac{B_0^2}{(N_p W_{\parallel})^{3/2}} \quad (viii)
\]

Since \( W_{\parallel} \) is related to the total energy \( W \) by the pitch angle \( (W = W \cos^2 \alpha) \) (viii) becomes:

\[
w \propto \frac{B_0^2}{(N_p W)^{3/2}} \quad (ix)
\]

which is equation (1) of Chapter 4.