STIMULATED SCATTERING IN A PLASMA
FILLING AN OPTICAL CAVITY

by

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ABSTRACT

Stimulated scattering processes in a homogeneous plasma inside an optical cavity are studied theoretically. In particular, attention is focussed on the Raman and Brillouin instabilities. The coupled equations for the wave amplitudes are solved subject to optical cavity boundary conditions and it is shown that for a wide range of plasma lengths and cavity mirror reflectivities, the threshold values for instability are approximately those for the temporal problem (waves uniform in space, growth or decay in time). Sample results are calculated numerically for the Raman instability in a typical laboratory plasma. The threshold values of incident laser intensity are easily exceeded with presently available, high powered (200 MW) CO_2 lasers. Finally, the physical significance of these results is discussed, and a generalization to the case of an inhomogeneous plasma is suggested.

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TABLE OF SELECTED NOTATION

t	time coordinate
x	spatial coordinate
A, A_0, A_-	general vector potential, pump wave, backscattered wave
N_j	density of j th species j = electrons or ions
γ	ratio of specific heats
$\underline{v}_j$	velocity of j th species
q_j	charge of j th species $q_i = e; q_e = -e$
c	velocity of light
ϕ	scalar potential
N_0	equilibrium density of electrons and ions
J	current density
γ_A, γ_-	phenomenological damping rates of light wave (/sec)
γ_p	phenomenological damping rates of ion acoustic waves or electron plasma waves (/sec)
ω_p	plasma frequency
ω_{pe}	electron plasma frequency
ω_{pi}	ion plasma frequency
m_e, m_i	electron, ion mass
θ_j	temperature of j th species in energy units $\theta_j = k_{\text{Bolt}} T_j$
n_j	density perturbation of j th species

v_j	thermal velocity $v_j^2 = \theta_j/m_j$
v_0	quiver velocity of electrons in pump electric field
ω_0, k_0	pump frequency, wavenumber
ω_-, k_-	backscattered frequency, wavenumber
ω, k	electrostatic frequency, wavenumber
$\tilde{n}_i, \tilde{n}_e, \tilde{A}_0, \tilde{A}_-$	amplitudes of sinusoidally varying quantities (superscript denotes direction of propagation)
v_p, v_-	group velocities of plasma and light waves
γ_0	temporal growth rate, far above threshold (specifies pump intensity)
c_j	dimensionless form of $\tilde{A}_-^j$
f_j^*	dimensionless form of $\tilde{n}_e^j$
ϵ	ratio of group velocities
β_c	dimensionless form of γ_-
β_f	dimensionless form of γ_p
γ_i	ratio of specific heats for ions
τ_R	rise time of laser pulse
τ_s	characteristic time for purely growing Brillouin instability
ω_s	ion acoustic frequency; $\omega_s = kc_s$
c_s	ion sound speed
δ	dimensionless growth rate
λ_0	plasma length
w	$= (\beta_f + \delta) + \epsilon (\beta_c + \delta)$

z	$= (\beta_f + \delta) - \varepsilon (\beta_c + \delta)$
Δ	$= (w^2 - 4)^{\frac{1}{2}}$
α	$= (4 - w^2)^{\frac{1}{2}}$
ζ	$= L\alpha/2$
E	electric field
B	magnetic field
r_1, r_2	voltage reflection coefficients
R	system reflectivity coefficient $= (r_1 \ r_2)^{\frac{1}{2}}$
Γ	damping coefficient describing mirror losses
$\frac{D}{Dt}$	$= \frac{\partial}{\partial t} + v_{c,f} \frac{\partial}{\partial x}$
ζ_-, ζ_p	coupling coefficients
S_-, S_p	energy fluxes of light, plasma waves
$\mathcal{E}_-, \mathcal{E}_p$	energy densities in light, plasma waves

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Chapter 1

INTRODUCTION

1.1 Nonlinear Plasma Theory

One major area in modern plasma physics is that of nonlinear phenomena. In particular, plasma waves and instabilities have been the subject of much research, both theoretical and experimental. While a general theory of large amplitude plasma waves and turbulence has yet to be developed, there has been much progress in the area of weakly nonlinear processes (Krall, 1973). These processes can be studied by extending the linear perturbation theory where one assumes small departures from equilibrium.

Even though such an approach has obvious inherent limitations, the theory is still quite complicated, and to gain useful information one must make further restrictions. In the case of plasma waves, one can single out a few waves and consider the nonlinear interactions occurring among them. Alternatively, one may wish to study changes in the bulk plasma properties owing to the presence of a large number of waves randomly superimposed on each other. These theories are known respectively as the "theory of weak coherent waves", and the "theory of weak turbulence". The work which follows is concerned with the former.

1.2 Stimulated Scattering

There are a number of nonlinear plasma phenomena which fall in the category of 3-wave interactions. In general, one considers the effects of coupling between waves which would exist independently in the plasma in a linear theory. Several 3-wave interactions are discussed by C.N. Lashmore-Davies (Lashmore-Davies, 1975).

In many situations of interest, one considers an initial wave of large amplitude, and two other small amplitude waves which perhaps grow up out of the background radiation or thermal noise of the plasma. In this case, it is appropriate to study the initial stages of the process by treating the large amplitude, or pump wave, as a given quantity, and solving the plasma equations for the space-time development of the excited waves.

Stimulated Brillouin scattering and stimulated Raman scattering are examples of 3-wave interactions. The pump wave is a transverse electromagnetic (light) wave and the excited waves are, respectively, a light wave and an ion wave; and, a light wave and an electron plasma wave. Stimulated scattering may be viewed as the product of two simpler physical processes, namely, wave mixing and normal scattering.

In wave mixing, one considers two light waves as given, and solves the plasma equations for the induced density wave. In normal scattering, one light wave and a spectrum of density waves (arising from thermal correlations) are given, and one solves for the scattered signal. Both of these processes can

occur simultaneously and in fact feed on each other. A scattered light wave can wave mix with the incident beam to induce (or enhance) a density wave. The density wave can, in turn, increase the amount of scattering. It is this type of feedback which gives rise to the backscattering instabilities.

From such a picture, one might expect stimulated scattering to dominate many physical systems. In actuality, the inclusion of the effects of wave damping and other energy losses shows that there exist threshold values in the intensity of the incident beam, below which the feedback mechanism is insufficient to cause instability. Part of the aim of this study is to derive the threshold conditions for stimulated scattering for a plasma within an optical cavity.

1.3 Optical Cavities

The interactions occurring in plasmas partially filling microwave or optical cavities have been the subject of investigation in the past (Heckenberg, 1973; Tait, 1973; Lochte-Holtgreven, 1968). Diagnostic techniques employing interferometry and the mode structure in microwave cavities (Shohet, 1970) have been useful tools in plasma physics. The theory of such systems is well understood.

In the following we consider the stimulated scattering process for a plasma within an optical cavity. The large amplitude pump wave is resonant within the cavity. This situation could be achieved in practice by using partially reflecting mirrors, or by actually inserting the plasma into the resonant cavity of a laser (Figure 1.1).

Fig. 1.1. Schematic of the Physical System

Similar problems have already been considered for the case of SRS (Stimulated Raman Scattering) in substances other than plasmas (Lugovoi, 1969). In these cases, the medium couples pump and backscattered radiation through molecular transitions at the Raman frequency, whereas in a plasma this is accomplished by a longitudinal density wave. It will be seen that the nonzero group velocity of the density wave gives rise to effects which distinguish the behaviours of the two systems. For many values of the system parameters, the effect of convection of the density wave is negligible. The consequence of this for plasma experiments will be discussed.

1.4 Motivation

Experimental work has been done on wave mixing in plasmas and unionized gases in optical cavities (Meyer, 1976). The behaviour of gases can resemble that of plasmas since atoms and molecules can be given induced dipole moments. The polarizability of a gas is thus related to the charge density in a plasma.

There is experimental evidence to support the existence of an instability in gases analagous to the Brillouin instability. For this case, and for the experiments with plasmas, the threshold incident laser intensity seems to be considerably lower than expected. These experimental results provide motivation for obtaining a better understanding of the effects of optical cavity boundary conditions on stimulated scattering.

Chapter 2

BASIC EQUATIONS AND RELATION TO PREVIOUS WORK

2.1 The Plasma Equations

We consider a finite, homogeneous plasma in the fluid limit, completely filling an optical cavity of length ℓ . The standard fluid and Maxwell equations are:

$$\frac{\partial \underline{v}_j}{\partial t} + \underline{v}_j \cdot \nabla \underline{v}_j = \frac{q_j}{m} (-\nabla \phi - \frac{1}{c} \frac{\partial \underline{A}}{\partial t}) + \frac{q_j}{m_j c} \underline{v}_j \times (\nabla \times \underline{A}) - \frac{\gamma \theta_j}{m_j N_j} \nabla N_j \quad (2.1a)$$

$$\frac{\partial N_j}{\partial t} + \nabla \cdot (N_j \underline{v}_j) = 0 \quad (2.1b)$$

$$\nabla^2 \underline{A} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \underline{A} = -\frac{4\pi}{c} \underline{j} + \frac{1}{c} \frac{\partial}{\partial t} \nabla \phi \quad (2.1c)$$

$$\nabla^2 \phi = -4\pi \sum_j q_j N_j \quad (2.1d)$$

Here, $\underline{A}$ is the vector potential in the Coulomb gauge ($\nabla \cdot \underline{A} = 0$), N_j is the density of the j th species ($j = e$ or i), $\underline{v}_j$ the fluid velocity, ϕ the electric potential, $\underline{j}$ the particle current density, and θ_j the temperature in energy units ($\theta = k_{\text{Bolt}} T$). Thus, a Maxwellian plasma is assumed with each species being characterized by its own temperature. γ is

defined from the equation of state relating pressure and density. As usual, we take $p = N^\gamma \times \text{constant}$. For the usual case of waves of sufficiently high frequency, the adiabatic assumption is valid and γ is the ratio of specific heats. An exception to this case will be seen to occur in section 2.3 where the isothermal approximation is more appropriate and $\gamma = 1$.

Following Forslund, Kindel, and Lindman (Forslund, 1975) we may obtain coupled wave equations for the density waves and electromagnetic waves in the plasma for the one dimensional problem (slab geometry).

$$\left(\frac{\partial^2}{\partial t^2} + 2\gamma_A \frac{\partial}{\partial t} + \omega_p^2 - c^2 \nabla^2 \right) \underline{A} = -\omega_p^2 \frac{n_e}{N_0} \underline{A} \quad (2.2a)$$

$$\frac{\partial^2 n_e}{\partial t^2} + 2\gamma_p \frac{\partial n_e}{\partial t} - \gamma v_e^2 \frac{\partial^2 n_e}{\partial x^2} + \omega_{pe}^2 (n_e - n_i) =$$

$$\frac{e^2 N_0}{2m_e^2 c^2} \nabla^2 (\underline{A} \cdot \underline{A}) \quad (2.2b)$$

$$\frac{\partial^2 n_i}{\partial t^2} - 3v_i^2 \frac{\partial^2 n_i}{\partial x^2} + \omega_{pi}^2 (n_i - n_e) = 0 \quad (2.2c)$$

Here γ_A and γ_p are phenomenological damping rates, $v_j^2 = \theta_j/m_j$, $\omega_{pj}^2 = 4\pi N_0 e^2/m_j$, N_0 is the equilibrium density of electrons and ions, and n_j is the density perturbation. In this one dimensional situation, $\underline{A}$ is perpendicular to $\hat{x}$, the direction of propagation of the waves. Thus the total electric field

$$\underline{E} = -\underline{\nabla}\phi - \frac{1}{c} \frac{\partial}{\partial t} \underline{A}$$

is conveniently split up into the electrostatic field $-\nabla\phi$ in the x direction, and the electromagnetic field $-\frac{1}{c} \frac{\partial}{\partial t} \underline{A}$ in the y - z plane. In these equations, $\underline{A}$ describes the total electromagnetic signal, namely, both the pump and excited light waves.

The left hand sides of equations (2.2) are the usual linearized wave equations while the right hand sides of (2.2a) and (2.2b) contain nonlinear terms which can couple the waves. We shall see that the right hand side of (2.2a) describes the scattering of a light wave off of a macroscopic density wave in the plasma. The right hand side of (2.2b) arises from inclusion of the $\underline{v} \times \underline{B}$ force on the electrons and describes the mixing of light waves in the plasma (Recall $\underline{v} \propto \underline{A}$, $\underline{B} = \nabla \times \underline{A}$, and $\nabla \cdot \{ \underline{A} \times (\nabla \times \underline{A}) \} \propto \nabla^2 (\underline{A} \cdot \underline{A})$).

These equations are to be supplemented with boundary conditions which describe total absorption of the outgoing electrostatic waves at plasma boundaries and reflection of the outgoing electromagnetic waves at mirrors near the plasma boundaries.

The problem at hand involves six waves since one has pump, backscattered, and electrostatic waves travelling in both directions.

We focus our attention on three processes which can be characterized by the nature of the excited electrostatic wave. These are the Raman instability corresponding to an electron plasma wave, the Brillouin instability corresponding to an ion acoustic wave, and the purely growing ($\text{Re}\omega = 0$, $\text{Im}\omega < 0$) instability resulting from the mixing of the pump at $(\omega_0, \underline{k}_0)$ with the pump at $(\omega_0, -\underline{k}_0)$ corresponding to a standing

density modulation. In the theory which follows, where we neglect pump depletion, these processes are uncoupled and can be treated independently.

One could proceed by solving equations (2.2) directly. The solution would indicate the existence of modes within the cavity whose characteristic frequencies and wavenumbers would be determined by the cavity length and plasma parameters. For the case we consider, where the cavity is long compared to the wavelengths involved, the intermode spacing becomes small. Thus we may consider ω and k as continuous variables, with the understanding that, what is really meant, is the nearest cavity mode (ω_q, k_q) where $\omega \approx \omega_q$, $k \approx k_q$. In what follows, we shall only be interested in the amplitudes of the waves as a function of position and time. This allows a great simplification of the equations at the expense of losing phase information.

In this spirit we introduce the slow space-time amplitude and phase modulations appropriate for weak coupling (i.e., growth rate $\ll$ frequency)

$$\tilde{n}_e^j(x,t), \quad \tilde{n}_i^j(x,t), \quad \tilde{A}_-^j(x,t), \quad \tilde{A}_0^j$$

where the index $j = 1$ or 2 to describe propagation in both directions. Assuming coherent, monochromatic, plane waves, let

$$n_i = \tilde{n}_i^1(x,t) \cos(\omega t - kx) + \tilde{n}_i^2(x,t) \cos(\omega t + kx) \quad (2.3a)$$

$$n_e = \tilde{n}_e^1(x,t) \cos(\omega t - kx) + \tilde{n}_e^2(x,t) \cos(\omega t + kx) \quad (2.3b)$$

$$A = \tilde{A}_0^1 \cos(\omega_0 t - k_0 x) + \tilde{A}_0^2 \cos(\omega_0 t + k_0 x) +$$

$$\tilde{A}_-^1(x, t) \cos(\omega_- t - k_- x) + \tilde{A}_-^2(x, t) \cos(\omega_- t + k_- x) \quad (2.3c)$$

where $\omega_- = \omega - \omega_0$, $k_- = k - k_0$ and we consider ω , k real.

The equations for the slowly varying amplitudes are obtained by plugging (2.3) into (2.2) and looking at the Fourier components of the resulting equations for a specific value of ω and k . This is equivalent to the Van der Pol method of space-time averaging over the fast oscillations (Tsytovich, 1970). For maximum growth, one chooses (ω, k) such that the wavenumber and frequency matching conditions as well as the uncoupled dispersion relations for light and plasma waves are simultaneously satisfied.

2.2 The Van der Pol Method

The Raman Instability

For this case, the ions may be taken to be stationary (i.e., $n_i \approx 0$), $\gamma = 3$, and we consider $\omega \gtrsim \omega_{pe}$. Neglecting forcing terms far off resonance, equations (2.2) yield in complex notation

$$\left[\frac{\partial^2}{\partial t^2} + 2\gamma \frac{\partial}{\partial t} - c^2 \frac{\partial^2}{\partial x^2} + \omega_p^2 \right] \tilde{A}_-^j e^{i(\omega_- t \pm k_- x)} =$$

$$- \frac{\omega_p^2}{2N_0} \tilde{n}_e^j \tilde{A}_0^{j*} e^{i(\omega_- t \pm k_- x)} \quad (2.4a)$$

$$\left[\frac{\partial^2}{\partial t^2} + 2\gamma_p \frac{\partial}{\partial t} - 3v_e^2 \frac{\partial^2}{\partial x^2} + \omega_p^2 \right] \tilde{n}_e^j e^{i(\omega t \pm kx)} =$$

$$\frac{e^2 N_0}{2m^2 c^2} \frac{\partial^2}{\partial x^2} \left[\tilde{A}_0^j \tilde{A}_-^j e^{i(\omega t \pm kx)} \right] \quad (2.4b)$$

Throwing away the nonresonant terms uncouples the $j = 1$ and $j = 2$ modes inside the plasma. (They are of course coupled through the boundary conditions). The electron waves excited by the standing pump are thus merely a superposition of those excited independently by two travelling pumps propagating in opposite directions.

Carrying out the differentiation in equations (2.4) and taking into account the slowly varying nature of the amplitudes, i.e.,

$$\frac{\partial^2 \tilde{n}_e}{\partial x^2} \ll k \frac{\partial \tilde{n}_e}{\partial x} \ll k^2 \quad (2.5a)$$

$$\frac{\partial^2 \tilde{n}_e}{\partial t^2} \ll \omega \frac{\partial \tilde{n}_e}{\partial t} \ll \omega^2 \quad (2.5b)$$

(with similar inequalities for $\tilde{A}_-$), one obtains to order ω^2 , ω_-^2 the uncoupled dispersion relations

$$\omega^2 = \omega_p^2 + 3v_e^2 k^2 \quad (2.6a)$$

$$\omega_-^2 = \omega_p^2 + c^2 k^2 \quad (2.6b)$$

and to order $\omega \frac{\partial \tilde{n}_e}{\partial t}$, $\omega_- \frac{\partial \tilde{A}_-}{\partial t}$ the desired equations for the amplitudes

$$\left[\frac{\partial}{\partial t} + \gamma_p \pm v_p \frac{\partial}{\partial x} \right] \tilde{n}_e^j = - \frac{k^2 e^2 N_0}{4i\omega m^2 c^2} \tilde{A}_0^j \tilde{A}_-^j \quad (2.7a)$$

$$\left[\frac{\partial}{\partial t} + \gamma_- \pm v_- \frac{\partial}{\partial x} \right] \tilde{A}_-^j = - \frac{\omega_p^2}{4i\omega_- N_0} \tilde{A}_0^{j*} \tilde{n}_e^j \quad (2.7b)$$

where the group velocities v_p and v_- are given by

$$\left| \frac{\partial \omega}{\partial k} \right| = v_p = \left| \frac{3v_e^2 k}{\omega} \right| \quad \frac{\partial \omega_-}{\partial k_-} = v_- = \left| \frac{c^2 k_-}{\omega_-} \right| \quad (2.8)$$

and the $\pm$ sign is determined according as the wave is traveling to the right or left. In terms of the dimensionless variables

$$X \equiv \frac{x\gamma_0}{(v_p v_-)^{1/2}} \quad T = t\gamma_0 \left(\frac{v_p}{v_-} \right)^{1/2} \quad (2.9)$$

where

$$\gamma_0 = \frac{kv_0}{2(2)^{1/2}} \frac{\omega_p}{[\omega(\omega_0 - \omega)]^{1/2}}$$

$$v_0^2 = \frac{|\tilde{A}_0|^2 e^2}{2m^2 c^2}$$

and the renormalized amplitudes

$$c_j = \left(\frac{k_-}{k_0} \right)^{1/2} \frac{\tilde{A}_-^j}{\tilde{A}_0^{j*}} \quad (2.10a)$$

$$f_j^* = \left(\frac{3}{2} \right)^{1/2} \frac{v_e}{v_0} \frac{\omega_p}{c(k_0 k)^{1/2} N_0} \tilde{n}_e^j \quad (2.10b)$$

equations (2.7) become

$$\epsilon(\beta_c c_j + \dot{c}_j) \pm c_j' = f_j^* \quad (2.11a)$$

$$\beta_f f_j^* + \dot{f}_j^* \pm f_j^{*'} = c_j \quad (2.11b)$$

where

$$\epsilon = \frac{v_p}{v_-} \quad \beta_f = \frac{\gamma_p}{\gamma_0} \left(\frac{v_-}{v_p} \right)^{\frac{1}{2}} \quad \beta_c = \frac{\gamma_-}{\gamma_0} \left(\frac{v_-}{v_p} \right)^{\frac{1}{2}}$$

$$\cdot = \frac{\partial}{\partial T} \quad , \quad = \frac{\partial}{\partial X}$$

A few comments may be made concerning equations (2.11). Instead of describing growth/decay of the waves by the slowly varying amplitudes $\tilde{n}_e$ and $\tilde{A}_-$, one could have used the formalism of complex ω and k . Equations (2.2) would then have yielded a complex dispersion relation. In this formalism, the assumption of weak coupling (formerly expressed by equation (2.5)) takes the form

$$\text{Im}(\omega) \ll \text{Re}(\omega)$$

$$\text{Im}(k) \ll \text{Re}(k)$$

The imaginary part of the dispersion relations would then yield an equation equivalent to the pair (2.11).

One advantage of using (2.11) instead of the complex dispersion relation is that one eliminates the 'variables' $\text{Re}(\omega)$, $\text{Re}(k)$, which for weak coupling are assumed to be fixed anyhow by solving the wavenumber and frequency matching conditions together with the uncoupled dispersion relations. Also, having separate equations for each wave instead of a single dispersion relation for the system permits easy generalization to include effects which may act on one wave and not the other (e.g., reflection at a boundary).

Equations (2.11) in various more general forms have

been used by many authors (Forslund, 1975; Kroll, 1965; Fuchs; Dubois, 1973) to study the behaviour of parametric processes. A more complete derivation of them is given by Tsytovich (Tsytovich, 1970). These equations will be taken as the starting point for the present study.

The Brillouin Instability

In this case one considers a low frequency ion acoustic wave at (ω, k) . Take $\gamma = 1$ and specialize to the region of the ion wave where $3v_i^2 k^2 \ll \omega^2 \ll k^2 v_e^2 \ll \omega_{pe}^2$. Then proceeding as before by neglecting the offresonant forcing terms in (2.2a) and (2.2b), the $j = 1$ and $j = 2$ modes again become uncoupled. (Neglecting offresonant forcing terms for the Brillouin instability where $\omega \ll \omega_{oi}$ is a much more serious approximation than in the Raman case, since offresonant terms are now only slightly mismatched. The approximations introduced here should be valid as long as we assume perfect matching of the resonant wave, since then the stronger resonant process will dominate. Lashmore-Davies gives equations for imperfect matching, when apparently our $j = 1$ and $j = 2$ modes can be coupled. In a moment, we will consider a special case of the matched wave being nonresonant, namely the case of $\omega = 0$. Otherwise, the thresholds for the imperfectly matched case are larger than those for perfect matching.)

With the $j = 1$ and $j = 2$ modes uncoupled, the analysis of the previous section goes through with slight modifications, since now one must additionally consider the ions.

It can be shown that the desired equations again take the form of (2.11) with the same renormalized amplitudes as in

equations (2.10), except that now one must use the γ_0 , v_p , v_- appropriate to the ion acoustic wave (Forslund, 1975)

$$v_p = \left(\frac{\theta_e}{m_i} \right)^{1/2} \quad \gamma_0 = \frac{kv_0}{2(2)^{1/2}} \frac{\omega_{pi}}{(\omega_0\omega)^{1/2}} \quad (2.12)$$

In neglecting the offresonant forcing terms in (2.2b) we threw away a term proportional to $\tilde{A}_0^1 \tilde{A}_0^2 \cos(2k_0 x)$ in both the Raman and Brillouin cases. Since the dispersion relation for the electron plasma wave exhibits a cutoff at ω_{pe} and this forcing term is at $\omega = 0$, the effects on the electron plasma wave are small. This is not the case for the low frequency ion wave however. Furthermore, while the resonant forcing term is proportional to $\tilde{A}_0$, this term is much larger, being proportional to $|\tilde{A}_0|^2$. The next section deals with the process resulting from this term, which can be viewed as a special case of the Brillouin instability where $\omega = 0$ hence the $j = 1$ and $j = 2$ terms in equations (2) coalesce as do the $\tilde{A}_0$ and $\tilde{A}_-$ waves.

2.3 The Purely Growing Brillouin Instability

Let

$$n_i = \tilde{n}_i(x,t) \cos kx \quad (2.13a)$$

$$n_e = \tilde{n}_e(x,t) \cos kx \quad (2.13b)$$

$$A = \tilde{A}_0 \cos(\omega_0 t - k_0 x) + \cos(\omega_0 t + k_0 x) \quad (2.13c)$$

In this section, we relax the condition that $\tilde{n}_e(x,t)$, $\tilde{A}_-(x,t)$ be slowly varying. Equation (2.2a) describes the process

whereby an incoming wave $\tilde{A}_0 \cos(\omega_0 t - k_0 x)$ can interact with the density wave $\tilde{n}_e \cos kx$ to produce a backscattered wave $A_0 \cos(\omega_0 t + k_0 x)$ for $k = 2k_0$. In linear theory, where one considers a nondepleted pump, this equation is of no further interest. Equations (2.2b), (2.2c) give

$$\left[\frac{\partial^2}{\partial t^2} + 2\gamma_p \frac{\partial}{\partial t} - v_e^2 \frac{\partial^2}{\partial x^2} \right] \tilde{n}_e e^{ikx} + \omega_{pe}^2 (\tilde{n}_e - \tilde{n}_i) e^{ikx} = \frac{e^2 N_0}{2m^2 c^2} \frac{\partial^2}{\partial x^2} \left[\tilde{A}_0^2 e^{2ik_0 x} \right] \quad (2.14a)$$

$$\left[\frac{\partial^2}{\partial t^2} - \gamma_i v_i^2 \frac{\partial^2}{\partial x^2} + \omega_{pi}^2 \right] \tilde{n}_i e^{ikx} - \omega_{pi}^2 \tilde{n}_e e^{ikx} = 0 \quad (2.14b)$$

Since the forcing terms are independent of position, and the density waves have zero group velocity, hence do not propagate out of the interaction region, it is reasonable to look for solutions independent of position (i.e., $\frac{\partial \tilde{n}}{\partial x} = 0$).

$$\left[\frac{\partial^2}{\partial t^2} + 2\gamma_p \frac{\partial}{\partial t} + v_e^2 k^2 \right] \tilde{n}_e + \omega_{pe}^2 (\tilde{n}_e - \tilde{n}_i) = -N_0 k^2 v_0^2 \quad (2.15a)$$

$$\left[\frac{\partial^2}{\partial t^2} + \gamma_i v_i^2 k^2 \right] \tilde{n}_i + \omega_{pi}^2 (\tilde{n}_i - \tilde{n}_e) = 0 \quad (2.15b)$$

The above system possesses a steady state solution $\tilde{n}_e^{ss}$, $\tilde{n}_i^{ss}$ given by

$$\frac{\tilde{n}_e^{ss}}{N_0} = - \frac{v_0^2 (3v_i^2 k^2 + \omega_{pi}^2)}{\omega_{pi}^2 v_e^2 + \gamma_i \omega_{pe}^2 v_i^2} \quad (2.16a)$$

$$\frac{\tilde{n}_i^{ss}}{N_0} = - \frac{v_0^2 \omega_{pi}^2}{\omega_{pi}^2 v_e^2 + \gamma_i \omega_{pe}^2 v_i^2} \quad (2.16b)$$

Typically $\gamma_i v_i^2 k^2 \ll \omega_{pi}^2$ in which case the ion and electron perturbations are equal and

$$\frac{\tilde{n}^{ss}}{N_0} = - \frac{v_0^2}{v_e^2 + \gamma_i \frac{m_i}{m_e} v_i^2} \quad (2.17a)$$

which can also be written as

$$\frac{\tilde{n}^{ss}}{N_0} = - \frac{e^2 E_0^2}{2 \omega_0^2 m_e} \frac{1}{\theta_e + \gamma_i \theta_i} \quad (2.17b)$$

where E_0 is the maximum value of the electric pump field. For such a steady state solution, it is appropriate to take the ions to be isothermal (i.e., $\gamma_i = 1$). One thus recovers a special case of Meyer and Stansfield's result for enhanced density fluctuations due to optical mixing (Meyer, 1971).

More generally, one can consider the nonsteady solutions to equations (2.15) which may arise due to the pump, v_0^2 , rising from an initial value, zero, to its maximum value. If the rise time is τ_R and we let

$$\tau_S = \frac{1}{kc_S} = \frac{1}{k} \left(\frac{m_i}{\theta_e + \gamma_i \theta_i} \right)^{1/2} \quad (2.18)$$

then the appropriate solution is determined by the ratio τ_R/τ_S . If $\tau_S \ll \tau_R$ the quasistatic approximation is valid and equation (2.17) holds with time dependent v_0^2 . On the other hand, for $\tau_R \gg \tau_S$, the impulse approximation is valid. For this case, the general (nonsteady) solution to equation (2.15) for $\gamma_i v_i^2 k^2 \ll \omega_{pi}^2$ with initial conditions

$$\tilde{n}(0) = \frac{\partial \tilde{n}(0)}{\partial t} = 0 \quad (2.19a)$$

and with

$$v_0^2 = \begin{cases} 0 & t < 0 \\ \text{constant} & t > 0 \end{cases} \quad (2.19b)$$

is

$$\frac{\tilde{n}}{N_0} = - \frac{v_0^2}{v_e^2 + \gamma_i \frac{m_i}{m_e} v_i^2} \left[1 - e^{-\gamma_p t} \cos \omega_s t \right] \quad (2.20a)$$

where

$$\omega_s^2 = \frac{k^2}{m_i} (\theta_e + \gamma_i \theta_i) \quad (2.20b)$$

Since we have $n_i \approx n_e = \tilde{n}_e e^{ikx}$, equation (2.20) shows that the transient introduced by turning the pump on suddenly at $t = 0$ consists of the term

$$\cos \omega_s t \cos kx = \frac{1}{2} \left[\cos(\omega_s t + kx) + \cos(\omega_s t - kx) \right]$$

which is to say that as well as setting up the offresonant $\omega = 0$, $k = 2k_0$ density perturbation, the pump launches two resonant, large amplitude ion acoustic waves. These waves may seed the normal Brillouin instability or produce observable density fluctuations even when one is below the power threshold for instability.

For a typical z-pinch plasma ($\theta_e \sim 10$ ev.) and CO_2 laser, $\tau_s \sim 30$ ps. so that one is in the quasistatic regime.

2.4 The Spatial Problem

Before proceeding with the Brillouin and Raman

problems for a plasma within an optical cavity, it may be best to review the solution for a travelling pump incident on a finite plasma (the spatial problem) in the present notation. This work was done by Kroll (Kroll, 1965) for the case of a solid, and is quoted and further discussed by Forslund et al. for the case of a plasma.

For the pump incident from the left, the equations are

$$\epsilon(\beta_c c + \dot{c}) - c' = f^* \quad (2.21a)$$

$$\beta_f f^* + \dot{f}^* - f^{*'} = c \quad (2.21b)$$

Assuming solutions of the form $e^{q(x - \delta t)}$

$$[\epsilon(\beta_c + \delta) - q]c = f^* \quad (2.22a)$$

$$[\beta_f + \delta + q] f^* = c \quad (2.22b)$$

The dispersion relation is

$$[\epsilon(\beta_c + \delta) - q][\beta_f + \delta + q] = 1 \quad (2.23)$$

Approximate solutions may be obtained by neglecting ϵ which is always small. We shall write β for β_f in the remainder of this section.

$$-q(\beta + \delta - q) = 1 \quad (2.24)$$

Define q_1 and q_2 by

$$q_1 = -\frac{1}{2} \left\{ \beta + \delta \pm \left[(\beta + \delta)^2 - 4 \right]^{\frac{1}{2}} \right\} \quad (2.25)$$

The solution satisfying the boundary conditions

$$\begin{aligned} f(0,t) &= 0 \\ c(L,t) &= c_0 e^{\delta t} \end{aligned} \quad (2.26)$$

is

$$c = c_0 e^{\delta t} \frac{e^{q_1 x} - (q_1/q_2) e^{q_2 x}}{e^{q_1 L} - (q_1/q_2) e^{q_2 L}} \quad (2.27a)$$

$$f^* = -q_1 c_0 e^{\delta t} \frac{e^{q_1 x} - e^{q_2 x}}{e^{q_1 L} - (q_1/q_2) e^{q_2 L}} \quad (2.27b)$$

Normally, the optical input into the plasma is due to noise, thus $\delta = 0$ and c_0 is a small constant. The amplification of the noise on propagating through the plasma may be calculated from (2.27). This behaviour is termed convective instability and is predominant for $\beta > 2$.

For $\beta < 2$, the differential equations can also admit a nontrivial solution satisfying boundary conditions corresponding to zero optical input, $f^*(0,t) = c(L,t) = 0$. This solution is

$$c = A e^{\delta t} \left[e^{q_1 x} - (q_1/q_2) e^{q_2 x} \right] \quad (2.28a)$$

$$f^* = -q_1 A e^{\delta t} \left[e^{q_1 x} - e^{q_2 x} \right] \quad (2.28b)$$

where δ , q_1 , q_2 are chosen to satisfy

$$e^{q_1 L} - \frac{q_1}{q_2} e^{q_2 L} = 0 \quad (2.29)$$

and the dispersion relation (2.24). These can be combined into the single equation

$$\left[1 - \frac{1}{4}(\beta + \delta)^2\right]^{\frac{1}{2}} = -\frac{1}{2}(\beta + \delta) \tan \left\{ \left[1 - \frac{1}{4}(\beta + \delta)^2\right]^{\frac{1}{2}} L \right\} \quad (2.30)$$

When (2.30) possesses a solution with $\delta > 0$, the plasma wave is absolutely unstable. It can be shown that such a solution with positive δ can exist only when

$$(i) \quad \beta < 2$$

$$(ii) \quad L > L_c(\beta) \geq \pi/2$$

The first condition is one on the power density of the pump and the damping rates, the second states that the plasma must be longer than some critical length which is at least $\pi/2$.

(Here L is in the dimensionless units of equation (2.9)).

Similar phenomena will be seen to occur in the next section where we consider a plasma within an optical cavity.

Chapter 3

THE OPTICAL CAVITY PROBLEM

3.1 Equations and Solution

In accordance with the preceding section, we consider the following equations for light waves described by c_1 and c_2 and plasma waves described by f_1^* and f_2^*

$$\epsilon(\beta_c c_1 + \dot{c}_1) - c_1' = f_1^* \quad (3.1a)$$

$$\beta_f f_1^* + \dot{f}_1^* + f_1^{*'} = c_1 \quad (3.1b)$$

$$\epsilon(\beta_c c_2 + \dot{c}_2) + c_2' = f_2^* \quad (3.1c)$$

$$\beta_f f_2^* + \dot{f}_2^* - f_2^{*'} = c_2 \quad (3.1d)$$

subject to the boundary conditions

$$r_1 c_1(0, t) = c_2(0, t) \quad (3.2a)$$

$$c_1(L, t) = r_2 c_2(L, t) \quad (3.2b)$$

$$f_1^*(0, t) = 0 \quad (3.2c)$$

$$f_2^*(L, t) = 0 \quad (3.2d)$$

In the above, L is the length of the plasma (measured in

dimensionless units) which is also taken to be the length of the optical cavity. r_1 and r_2 are the reflection coefficients of the cavity mirrors. The density gradients near the ends of the plasma are assumed to be gentle, i.e.,

$$\frac{|\nabla n|}{N_0} \ll k_0$$

so that we neglect reflections of both light and plasma waves at the plasma boundaries. It can be shown that the threshold conditions for the case where the plasma does not completely fill the cavity and damping of the light wave outside the plasma is negligible are identical to those derived below.

The pair (3.1a), (3.1b) possess exponential solutions $e^{qx + \delta t}$. The resulting dispersion relation is

$$\left[\epsilon(\beta_c + \delta) - q \right] \left[\beta_f + \delta + q \right] = 1 \quad (3.3)$$

Let the solutions of this quadratic be q_1 and q_2 .

$$q_{1,2} = -\frac{1}{2} \left[\beta_f + \delta - \epsilon(\beta_c + \delta) \pm \left([\beta_f + \delta + \epsilon(\beta_c + \delta)]^2 - 4 \right)^{\frac{1}{2}} \right] \quad (3.4)$$

Then the solutions satisfying the boundary conditions are of the form

$$c_1 = (Ae^{q_1 x} + Be^{q_2 x}) e^{\delta t} \quad (3.5a)$$

$$f_1 = (De^{q_1 x} + Ee^{q_2 x}) e^{\delta t} \quad (3.5b)$$

$$c_2 = (Fe^{-q_1 x} + Ge^{-q_2 x}) e^{\delta t} \quad (3.5c)$$

$$f_{\frac{1}{2}} = (He^{-q_1 x} + Je^{-q_2 x}) e^{\delta t} \quad (3.5d)$$

Making use of the set (3.1), (3.2) to eliminate the amplitudes A, B...J, one obtains the condition determining δ , the growth rate of the system.

$$\frac{\epsilon(\beta_c + \delta) - q_1}{\epsilon(\beta_c + \delta) - q_2} = \frac{(r_1 r_2)^{\frac{1}{2}} \pm e^{q_1 L}}{(r_1 r_2)^{\frac{1}{2}} \pm e^{q_2 L}} \quad (3.6)$$

As usual (Forslund, 1975; Kroll, 1965; Fuchs) the solution exists in two distinct regimes according to whether $(\beta_f + \epsilon\beta_c)$ is greater than or less than 2. A clue to this is given by equation (3.4) since in the latter case, there exists the possibility of the roots q_1 and q_2 coalescing or going complex for non-negative δ . It is convenient to make the substitutions

$$w = (\beta_f + \delta) + \epsilon(\beta_c + \delta) \quad (3.7a)$$

$$z = (\beta_f + \delta) - \epsilon(\beta_c + \delta) \quad (3.7b)$$

$$R = (r_1 r_2)^{\frac{1}{2}} \quad (3.7c)$$

After some rearrangement, equation (3.6) may be expressed in the form

$$z = \frac{2}{L} \ln \left| \left(\cosh \frac{L\Delta}{2} + \frac{w}{\Delta} \sinh \frac{L\Delta}{2} \right) \frac{1}{R} \right| \quad (3.8a)$$

where $\Delta = (w^2 - 4)^{\frac{1}{2}}$. This result is still valid for $w < 2$.

Equivalently, for this case one obtains

$$z = \frac{2}{L} \ln \left| \left(\cos \frac{L\alpha}{2} + \frac{w}{\alpha} \sin \frac{L\alpha}{2} \right) \frac{1}{R} \right| \quad (3.8b)$$

where $\alpha = (4 - w^2)^{\frac{1}{2}}$. These forms are convenient for numerical computations and also for deducing some analytical properties. The curves of marginal stability ($\delta = 0$) are shown in Figure 3.1 for the case $R = 1$.

For a given plasma (γ_- , γ_p , ℓ , v_- , v_p) and given light wave beam parameters (ω_0 , v_0) one can calculate the quantities L , $\epsilon\beta_c$, β_f . The pair ($\epsilon\beta_c$, β_f) determine a point on the graph and L determines a particular marginal stability curve. Instability is indicated when the point lies below the curve.

There is a regime in which the character of the marginal stability curve changes dramatically. A clue to interpret this behaviour is found in equation (3.8b), in the singularity of the logarithm function. Thus implicitly we consider $w < 2$. The condition for the argument to vanish can be expressed as

$$\tan \zeta = - \frac{\zeta/L}{(1 - \zeta^2/L^2)^{\frac{1}{2}}} \quad (3.9)$$

where $\zeta = \frac{L\alpha}{2}$ and $0 < \zeta < L$. Equation (3.9) has a solution when $L > \pi/2$. For a long plasma, there are many solutions, which correspond to values of w denoted by w_m .

The similarity between the present situation and that discussed in section 2.4 should be noted. In the latter case we had a similar critical length appearing, below which there could be no absolute instability. While the optical cavity plasma considered here has some new features and provides different conditions for instability, some of the old features

Fig. 3.1. Marginal Stability Curves
 Parameters are $R=1.0$ and $L=10, 1, .1, .01$ as indicated.
 The $L=10$ and $L=\infty$ curves are coincident to drawing accuracy.

still remain. Physically, this could be expected since the limit of small mirror reflectivities must return us to the solutions of the spatial problem.

Returning to (3.8b), we see that in the neighbourhood of a singular point, w_m , the marginal stability curve is well described by the equations

$$w = \text{constant} = w_m$$

$$z \rightarrow \infty$$

or equivalently

$$\beta_f + \epsilon \beta_c = \text{constant} = w_m \quad (3.10)$$

Thus for a long plasma, the marginal stability curve can be multivalued indicating the presence of more than one unstable mode. The modes spoken of here are not to be confused with the cavity modes corresponding to different frequencies.

As the reflectivity approaches zero, the marginal stability solutions approach the singularity and one recovers the spatial thresholds as quoted by Forslund et al. Figure 3.2 displays the curve for $R = 0.1$. As R is decreased further, the pattern moves up and to the left, until finally the straight line solutions occupy the region of usual physical interest.

It is interesting to look at some special cases analytically. In the limit of large L , we can look for a solution with $w \gg 2$. Then (3.8) gives immediately

Fig. 3.2. Marginal Stability Curve
 $L=10$ $R=0.10$

$$z = \Delta$$

$$(\beta_f - \epsilon\beta_c)^2 = (\beta_f + \epsilon\beta_c)^2 - 4 \quad (3.11)$$

$$\epsilon\beta_f\beta_c = 1$$

The last equation is the usual threshold condition for the temporal problem. Reverting to dimensional variables this takes the form

$$\gamma_0^2 = \gamma_- \gamma_p \quad (3.12)$$

The solution is valid for $w \gg 2$ which together with (3.11) means $\beta_f \gg \epsilon\beta_c$. This condition is almost always satisfied owing to the small values of ϵ , the ratio of the group velocities of the plasma and light waves.

To have recovered the threshold for the temporal problem is a bit surprising, since the temporal problem corresponds to the case where both the light and plasma waves are reflected back into the interaction region. Furthermore, it is known that the threshold and growth rate for the finite plasma in the spatial problem (where no waves are reflected) do not approach the temporal values in the large L limit.

Experimentally, this may be significant since the temporal thresholds are easily exceeded, and the condition $L \gg 1$ is considerably less stringent than the requirement for significant growth in the spatial problem, $L \gg \beta_f$; and, of course, also considerably less stringent than the condition for absolute instability in the spatial problem, $\beta_f + \epsilon\beta_c < 2$.

In the limit of small L , the threshold condition becomes independent of the damping of the plasma wave. For arbitrary w , one obtains on expanding (3.8a) or (3.8b) to order L^2 ,

$$\epsilon\beta_c = \frac{1}{2}L. \quad (3.13)$$

Physically, the damping of the plasma wave is now unimportant because the main source of dissipation of plasma waves is the convection of energy into the plasma boundaries, governed by the parameters v_p and L . In dimensional variables, the condition is just

$$\gamma_0^2 = \gamma_- \frac{2v_p}{\ell} \quad (3.14)$$

which is one of the same form as (3.12).

3.2 Sample Results

Solutions in terms of the dimensionless variables are difficult to apply directly to a given experimental situation since the time and length scales are themselves functions of the incident laser power. In order to obtain the growth rate, δ , as an explicit function of incident laser power, it is necessary to resort to numerical techniques. The computations have been done for the Raman Instability in a typical plasma:

$$N_0 = 10^{17} \text{ cm}^{-3}, \theta_e = 10 \text{ eV}.$$

The values of the various parameters used in this sample calculation are listed in Table 3.1. Some comments are in order regarding the damping coefficients of the electro-

magnetic and electrostatic waves. While it is possible to obtain a good theoretical estimate of the Landau damping of the electron plasma wave, the same cannot be said for collisional damping. Lacking any better estimate, the collisional damping coefficients for the plasma wave and light wave have been obtained from the equations

$$\gamma_- = \frac{1}{2} \nu_{ei} \frac{\omega_p^2}{\omega^2} \quad \gamma_p = \frac{1}{2} \nu_{ei}$$

where ν_{ei} is the effective electron-ion collision frequency given by

$$\nu_{ei} = \frac{\pi N_0 e^4 \ln \Lambda}{m_e^{1/2} \theta_e^{3/2}} \quad ; \quad \Lambda = \frac{3}{2} \frac{\theta_e^{3/2}}{(\pi N_0)^{1/2}} \frac{1}{Ze^3}$$

The validity of the concept of an electron-ion collision frequency for a fully ionized plasma may well be questioned, and the results quoted should be viewed with some skepticism. However, it is not our aim to predict accurate numerical results, but merely to compare the features of the unstable plasma in an optical cavity with situations previously considered. This point shall be returned to in the conclusions of Chapter 4.

For the sample plasma investigated here, it will be noted that Landau damping is completely negligible at the wavelength of the excited electron plasma waves (since $k\lambda_D \ll 1$). For many situations of interest this will not be the case and the contribution of Landau damping to the total damping coefficient must be included.

Table 3.1 Summary of Parameters for Sample Calculation
Raman Instability

Electron Density	10^{17} cm^{-3}
Electron Temperature	10 eV
Laser Wavelength (in vacuum)	$1.06 \times 10^{-3} \text{ cm}$
Laser Frequency	$1.78 \times 10^{14} \text{ rad/sec}$
Laser Wavenumber (in plasma)	$5.90 \times 10^3 \text{ cm}^{-1}$
Backscattered Frequency	$1.60 \times 10^{14} \text{ rad/sec}$
Backscattered Wavenumber (in plasma)	$5.29 \times 10^3 \text{ cm}^{-1}$
Electrostatic Frequency	$1.80 \times 10^{13} \text{ rad/sec}$
Electrostatic Wavenumber	$1.12 \times 10^4 \text{ cm}^{-1}$
Electron Plasma Frequency	$1.78 \times 10^{13} \text{ rad/sec}$
Electron Debye Length	$7.44 \times 10^{-6} \text{ cm}$
Landau Damping Coefficient	$2.02 \times 10^{-14} \text{ sec}^{-1}$
Collisional Damping Coefficient	$3.02 \times 10^{10} \text{ sec}^{-1}$
Total Damping Coefficient for Electrostatic Wave	$3.02 \times 10^{10} \text{ sec}^{-1}$
Total Damping Coefficient for Electromagnetic Wave	$3.76 \times 10^8 \text{ sec}^{-1}$

Fig. 3.3. Raman Instability Growth Rates in a Typical Plasma within an Optical Cavity. $R=0.8$ and plasma lengths, l , in cm are indicated for each curve. See table 3.1. for plasma parameters.

Fig. 3.4. Raman Instability Thresholds in a Typical Plasma within an Optical Cavity. Plasma lengths, l , in cm are indicated. See Table 3.1. for plasma parameters.

In Figure 3.3 the growth rate of the Raman Instability is shown as a function of incident laser power. The mirror reflectivity coefficient for this calculation is $R = 0.80$. Plasma lengths are $\ell = 1000, 10, 1$ and 0.1 cm. For the case of a perfect optical cavity ($R = 1.00$), the curves for the four ℓ values all coincide with the $\ell = \infty$ curve in this plot, to within pen accuracy. The rightmost curve is for the spatial problem ($R = 0$). Again, the curves for the four ℓ values are nearly coincident. The dependence of the threshold power levels on the mirror reflectivity coefficient, R , is given in Figure 3.4. One of the surprising features here is the dramatic drop off of the thresholds as one moves away from $R = 0$. For a long plasma, the transition from the spatial to the temporal value is particularly sudden. It is possible to explain and reproduce these curves from a simple analytical model.

First consider an infinite plasma inside an optical cavity. As long as the incident laser power is slightly above the temporal threshold, the waves will grow in their own reference frame as they propagate. Thus for an infinite plasma, the wave amplitudes will become arbitrarily large at infinity. No matter how small the reflectivity coefficient is, so long as it is nonzero, there is always an infinite amount of energy reflected back into the central region of the plasma, hence one has absolute instability.

For a plasma of finite length, the processes of convective e-foldings, and mirror losses compete. For uncoupled light waves in an optical cavity (i.e., no interaction with density waves, etc.) the effective damping coefficient describing amplitude loss due to the mirrors is

$$\Gamma = \frac{v_c}{\ell} \ln \frac{1}{R} \quad (3.15)$$

The equation describing the gain of the light wave is

$$\frac{D}{Dt} \tilde{A}_- = -\gamma_- \tilde{A}_- + \zeta_- \tilde{n}_e \quad (3.16)$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_c \frac{\partial}{\partial x}$$

and ζ_- is the coupling coefficient (see equation 2.7b). At threshold, neglecting convection of the density wave due to the smallness of v_p , we take $\tilde{n}_e$ to be given from (see equation 2.7a).

$$0 = -\gamma_p \tilde{n}_e + \zeta_p \tilde{A}_- \quad (3.17)$$

The gain coefficient for the light wave is thus approximately

$$-\gamma_- + \frac{\zeta_- \zeta_p}{\gamma_p} = \frac{\gamma_0^2}{\gamma_p} - \gamma_- \quad (3.18)$$

An estimate of the threshold is then given by

$$\begin{aligned} \frac{v_c}{\ell} \ln \frac{1}{R} &= \left(\frac{\gamma_0^2}{\gamma_p} - \gamma_- \right) \\ \gamma_0^2 &= \gamma_p \frac{v_c}{\ell} \ln \frac{1}{R} + \gamma_- \gamma_p \end{aligned} \quad (3.19)$$

This simple formula describes the results excellently for $R \neq 0$. In the limit of small R , the simple description is invalid due to the breakdown of equation (3.15) giving Γ . It is clear physically, however, that the thresholds for this problem can never exceed those of the spatial problem, and rise monotonically to them as R tends to zero.

3.3 Discussion

Previously, various authors have derived the temporal and spatial thresholds for stimulated scattering processes. For most experimental situations, the spatial thresholds, which are several orders of magnitude larger, have been taken to be the relevant ones. In contrast, Figures 3.1, 3.3, and 3.4 illustrate the relevance of temporal thresholds to the onset of absolute instability in a wide variety of experimental situations where a modest fraction of backscattered radiation is reflected back into the interaction region of the plasma.

From a theoretical point of view, the optical cavity problem provides a smooth link between the spatial and temporal problems.

It is not surprising that reflection of the excited waves at the plasma boundaries could convert convective instability into absolute instability; however, since stimulated scattering generates two types of excited waves, namely, electrostatic and electromagnetic, it is not obvious what the effect of reflection of the light waves alone would be. The results indicate that the temporal description is a good approximation for a moderately long plasma, and reasonable mirror reflectivities. The mirrors effectively constrain the light waves to be practically uniform in space, hence any instability must be temporal.

It is useful to consider the energy transport by the excited waves. Again, we take the Raman instability for illustration. Using conservation of wave action, we obtain for the ratio of energy fluxes

$$\frac{S_-}{S_p} = \frac{\mathcal{E}_- v_-}{\mathcal{E}_p v_p} = \frac{\omega_- v_c}{\omega v_p} = \frac{c^2 k_-}{3 v_e^2 k} \sim \frac{c^2}{6 v_e^2} \quad (3.20)$$

where $\mathcal{E}_j$ is the energy density. Thus in a typical situation, the energy transported from the interaction region by the density wave is negligible. Containment of the excited light waves is practically equivalent to periodic boundary conditions for the system.

It may be possible to generalize these results for a homogeneous plasma, to the case of a plasma with density

gradients. Normally, stimulated scattering is inhibited by the presence of a density gradient due to the fact that the excited waves of a particular wavelength propagate out of the region where they are resonant for further interaction. Looked at in another way, the excited frequency and wave-number is a function of position in the plasma, thus the effect of propagation prevents significant growth of any one wave (Chen, 1974).

In the optical cavity problem, we have seen that the light waves can dominate the behaviour of the system. In a plasma with density gradients, a light wave with a particular frequency and wavenumber gets to cross the region where it is resonant for parametric interaction many times. Thus the density gradient scale length loses its significance. Under these circumstances, one again expects absolute instability to set in near the homogeneous, temporal thresholds.

The threshold conditions in various regimes of interest all have simple physical explanations in terms of energy balance. The pump provides the input, dissipative losses due to the damping of the light wave and plasma wave, and convection of energy out of the plasma due to propagation of the light and plasma waves provide the losses which must be counter-balanced. These effects are characterized respectively by γ_0 , γ_- , γ_p , and various combinations of the parameters v_- , v_p , ℓ , and R . In the temporal problem, the loss mechanisms are dissipative, thus

$$\gamma_0^2 = \gamma_- \gamma_p \quad (3.21)$$

In the spatial problem, the main factors to be accounted for are the damping of the plasma wave and the fact that the excited plasma and light waves propagate away from each other as they grow.

$$\gamma_0 = \frac{1}{2}(v_-/v_p)^{\frac{1}{2}} \gamma_p \quad (3.22)$$

For an extremely short plasma within an optical cavity, the dominant effects are convection of the plasma wave and damping of the light wave.

$$\gamma_0^2 = \gamma_- \frac{2v_p}{\ell} \quad (3.23)$$

Finally, for a plasma of more reasonable length ($L \gg 1$) inside an optical cavity, the loss mechanisms are damping of the plasma wave, damping of the light wave, and mirror losses; hence, the threshold is

$$\gamma_0^2 = \gamma_- \gamma_p + \gamma_p \frac{v_c}{\ell} \ln \frac{1}{R} \quad (3.24)$$

Chapter 4

SUMMARY AND CONCLUSIONS

It has been shown that a plasma is considerably more unstable to stimulated scattering processes when it is placed inside an optical cavity. Solutions for the slowly varying wave amplitudes indicate that the temporal thresholds are relevant to many such experimental situations, especially where the plasma is long or the cavity losses are small. Since the temporal thresholds are generally several orders of magnitude below the spatial thresholds for absolute instability, this result may be quite significant.

It would be rash to suggest that the numerical values calculated for the thresholds and growth rates are accurate in an absolute sense. Estimates of collisional and Landau damping are necessarily crude. In addition, there are many assumptions and restrictions specific to this model, which may be difficult or impossible to achieve in practice. Perhaps the most serious of these are the assumptions of isotropy in the y - z plane (perpendicular to the direction of propagation of the waves) and homogeneity of the plasma. Such a one dimensional model cannot strictly apply to schemes where the laser beam is converging, as is quite common in practice, to achieve high power densities in the focal spot. In addition, stimulated side scattering at arbitrary angles to the pump wave is not accounted for.

Frequently, experiments involve lasers with an extremely short pulse. It might be noted that, for the above cavity analysis to be valid, one must have the round trip transit time for a photon very much smaller than the time for which the laser pulse exists. It is also obvious that for significant growth, one must have the pulse existing for many e-folding times of the system.

The threshold intensity levels for many other nonlinear processes in a plasma (e.g., self focussing) lie in roughly the same range as those for stimulated scattering, though generally are slightly higher. If beam filamentation exists, so that local flux densities exceed the average values, then these other nonlinear processes may modify the plasma, or couple directly to the effect being studied. This creates further difficulties which are awkward or impossible to handle theoretically and may dominate the experimental results.

While the exact numbers which come out of such a theory may thus be quite unreliable, the trends and basic conclusions concerning the effect of the optical cavity should still stand.

The experimental observation and study of nonlinear plasma phenomena is obviously crucial to our understanding of the field. The optical cavity may well facilitate such work.

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