TWO-LOOP CONTRIBUTIONS TO THE $B - \bar{B}$ MIXING AMPLITUDE

By

Simon Fortin

B. Sc. (Physics) Université Laval

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

in

THE FACULTY OF GRADUATE STUDIES

PHYSICS

We accept this thesis as conforming

to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

September 1991

Table of Contents

In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the head of my department or by his or her representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of PHYSICS

The University of British Columbia
Vancouver, Canada

Date 11-12-91

DE-6 (2/88)

Abstract

It is known that the Standard Model, being a spontaneously broken gauge theory, violates the decoupling theorem. In practice, this means that amplitudes for low-energy processes grow without limit as the mass of fermions or scalars is made large. As a result of the recent determination of a lower bound of 90 GeV on the mass of the top quark and the general expectation of a large mass for the Higgs boson, this effect could lead to large higher order corrections to the observables of the theory. In this spirit, we calculate the two-loop corrections to the $B_d - \bar{B}_d$ mixing amplitude in the two limits of a very large top quark mass or a very large Higgs mass. Analytical expressions are obtained for the leading terms. The results are found to be much smaller than what one would naively expect.

List of Figures

Acknowledgement

I would like to thank Prof. D. Beder for suggesting the problem that became the subject of this thesis. I would also like to thank my supervisor John Ng for his constant help and his great patience, my collaborators and fellow students Gilles Couture, Q.C. Geng and Dan Peters for their stimulating conversations, and especially Pankaj Agrawal whose help throughout this work was invaluable. Finally, a big hug goes to those who made my years at UBC very pleasant ones: Kodi McPhee, Susan Kauffmann, Craig Morgan and the Habs of '86, Sergei Prokofiev, Karen Heiderich, Arvo Pärt, Duncan Rogers, David Fayegh and others.

Cette thèse est dédiée à Milo.

Chapter 1

Introduction

The Standard Model of particle physics, which is extremly successful in accounting for all elementary interactions up to about 100 GeV, predicts the existence of two as yet unseen particles. These particles, the top quark and the Higgs boson, have thus far eluded our most energetic searches. Their discovery would represent a spectacular confirmation of the model and would undoubtedly shed light on the mechanism responsible for mass-generation in nature.

The top quark and the Higgs are, however, extremly difficult to produce and identify directly. To produce a particle, we must create a collision with at least as much energy as the mass of the particle. This is a challenge in the case of the Higgs and the top since they are very heavy (the present limits are $m_t > 90\,GeV$, $M_H > 54\,GeV$). Once produced, these particles can be difficult to identify because high-energy collisions tend to produce a lot of "debris ", especially in the case of hadron colliders.

These circumstances make it worthwhile to look for indirect ways of detecting and measuring the properties of the top and the Higgs. In this approach, low-energy observables are measured and their values are compared with a calculation which includes quantum corrections. The corrections will in general depend on all of the parameters of the Standard Model including the top quark mass and the Higgs mass. Since the other parameters of the model are quite precisely known, this allows us, in principle, to extract the value of these masses from the data. Of course, or this to be at all feasible, the uncertainty on the value of the observables must be smaller than the size of the corrections.

A peculiar feature of the Standard Model makes this task easier: the "weak" interactions of the top quark and the Higgs are not at all weak. These interactions are mass-dependent and, for suitable values of m_t and M_H, can in fact be stronger than the "strong" interactions. The corrections due to the top quark and the Higgs are therefore expected to be quite large and this is indeed what is found for the lowest order correction. But this immediately raises a question: since the first-order correction is large, shouldn't we consider the contributions of higher-order corrections as well?

In this thesis, we address this question by calculating the leading parts of the second order corrections to a specific observable, the $B_d - \overline{B}_d$ mixing amplitude. This observable is particularly sensitive to the existence of the top quark since it would be essentially zero in its absence. The fact that it has been measured [1] and is quite large represents possibly the strongest evidence for the existence of the top. Before explaining what the $B_d - \overline{B}_d$ mixing amplitude is, I would like to back-track a bit and discuss the role of the Higgs boson and the top quark in the Standard Model.

1.1 The role of the top quark and the Higgs boson

1.1.1 The Standard Model

The forces of nature are usually considered to be of 4 different kinds:

1. The *strong* force, which is responsible, among other things, for the existence and stability of nucleons, mesons and nuclei.

2. The *weak* force, involved in the slow decay of nuclei.

3. The *electromagnetic* force which holds atoms and molecules together.

4. The familiar *gravitational* force, to which all matter is subjected.

The Standard Model of particle physics deals with the first three of those forces in a single framework, that of *gauge-theories.*

In this section, I will introduce the participants of the Standard Model, namely the elementary particles, as well as some of the necessary terminology connected with them. I will give only a short overview here; more details can be found in Chapter 2.

Elementary particles can be broadly divided into two categories according to the quantized value of their angular momentum or *spin*: with angular momentum measured in units of $\hbar \equiv h/2\pi$, h being Planck's constant, the spin of *fermions* is a half-integer $(1/2,\ 3/2,\ldots)$ while that of *bosons* is an integer or zero.

All fermions present in the Standard Model have spin 1/2. They come in two varieties: the *quarks*, which participate in the strong interactions, and the *leptons*, which do not. As a result of their strong interactions, quarks form bound states (or atoms). (The B_d particle is one such bound state, consisting, as we shall see later, of a bottom and an anti-down quark.) The forces that bind these atoms together are so strong that one would need an infinite amount of energy to separate their constituent quarks. Quarks are therefore permanently confined inside bound states and can never be found isolated. Quarks and leptons interact with one another through the exchange of bosons. There are two kinds of bosons: the *gauge-bosons*, of spin 1, and the lonely Higgs boson, the only particle in the theory with spin zero.

There are, in all, 12 gauge-bosons: the $W^{\pm}$ and the Z, which mediate the weak interactions, the familiar photon (A), responsible for the electromagnetic interactions as well as 8 *gluons* (G) which are at the origin of the strong interactions. The *electro-weak* bosons (A, Z and W) interact with one another and also with the fermions.

There are 48 fermions generally grouped into three generations: the electron (e), the electron-neutrino (ν_e) and the up (u) and down (d) quarks in the first, the muon (μ), its associated neutrino (ν_μ) and the charm (c) and strange (s) quarks in the second and the tau (τ), its neutrino (ν_τ) and the bottom (b) and top (t) quarks in the third. Also, for every one of these particles there is an anti-particle. The different varieties of quarks (i.e. u, d, ...) are called *flavors*. There are three quarks of each flavor. Quarks of the same flavor are identical in every respect but one, called *color*, which is the degree of freedom of the strong interactions.

The fermions in each generation are conveniently grouped in doublets which reflect the symmetries of the theory:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$
$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

The two members of a doublet always have a different electric charge: the neutrinos,as their name indicate, are neutral, the electron, muon and tau have charge -1, the upper components of the quark doublets (u,c,t) have charge 2/3 and the lower components (d,s,b) have charge $-1/3$. Here, all the charges are given in units of the proton charge.

We can now turn to a discussion of the role of the Higgs in the model.

1.1.2 The Higgs boson

The Higgs boson plays a very special role in the Standard Model. In its absence, all particles would be massless and their interactions would be highly symmetrical. The Higgs is introduced in order to destroy this symmetry. The way in which this is done will be described in Chapter 2. We will only mention here that this mass generation mechanism is related to the interactions of the particles with the Higgs boson: if a particle doesn't interact with the Higgs, it has to be massless. Note that this accounts for the fact that the neutrinos, the gluons and the photon remain massless.

This mechanism of symmetry breaking has several important consequences. One of these has to do with the strength of the Higgs-top interactions. Since the particle masses are generated through interactions with the Higgs, the masses will be proportional to the strength of these interactions. Turning the argument around, we see that the interaction strength between the Higgs and a given particle is proportional to the mass of the particle. More specifically, if we characterize the strength of the electromagnetic interactions by $\alpha \approx 1/137$, then the strength λ_X of the interaction of the Higgs with particle X is approximately

$$\lambda_X \approx \alpha \frac{M_X^2}{M_W^2} \tag{1.1}$$

where M_W is the mass of the $W^\pm$ boson($\approx$ 80 GeV) and M_X is the mass of X. In particular, ordinary matter couples very weakly to the Higgs (a factor of at least 10^4 smaller than electromagnetism). However, the W and Z bosons couple with electromagnetic strength. More interestingly, the top-Higgs interactions could be about 10 times stronger than electromagnetism (for $m_t \approx 3M_W$).These interactions could then be the strongest interactions around and could perhaps be strong enough to invalidate the use of perturbation theory. The situation is even more drastic in the case of the Higgs self interactions. Formula (1.1) is not valid in this case. Instead, a quartic dependence on the mass of the Higgs is found. The main consequence of all this is that the corrections to observables due to Higgs-Higgs and Higgs-top interactions are probably going to be very large and must therefore be taken into account.

Another interesting consequence of symmetry breaking via the Higgs is the existence of *flavor-changing neutral currents* (FCNC). In the absence of the Higgs, all fermions are massless. Any two fermions of the same charge are then physically undistinguishable. Now, in general, the gauge bosons interact with a pair of fermions as in fig. 1.1 . Given the undistinguishability of, say, u and c, an interaction where $f1$ is an up quark and $f2$

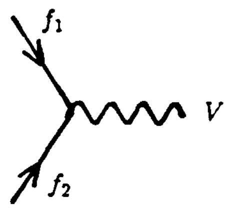


Figure 1.1: Interaction of a gauge boson (V) and two fermions (f_1 and f_2)

a charm quark would appear physically meaningless. In fact, it is possible in this case to redefine what is meant by the "up quark" and the "charm quark" so that gauge bosons can only interact with fermions which are part of the same doublet. In particular, this implies that the only interactions between quarks of a different flavors are mediated by the charged W bosons, a situation referred to as the absence of flavor-changing neutral-currents.

This is indeed what happens with leptons: here the neutrinos remain massless even in the presence of the Higgs since they do not interact with it. This leads to *lepton-number conservation.* However, all quarks acquire a different mass through their interactions with the Higgs. As a result, FCNC occur in the Standard Model albeit in a rather indirect way. The only fundamental interactions (of the type shown in fig. 1.1) between quarks in different doublets are mediated by the *charged* W boson. This, by itself, is not a FCNC. However, by using 2 such interactions as in fig. 1.2, we can generate flavor-changing neutral-currents. The existence of FCNC is crucial for the mixing of B_d and $\overline{B}_d$ since this process involves *two* changes of flavor, i.e. $b \rightarrow t \rightarrow s$.

In view of the central role it plays in the Standard Model, it may come as a surprise that the Higgs is not, strictly speaking, necessary in the theory. There are many ways in which we can induce symmetry breaking in the model. For instance, one can postulate the existence of "techniquarks" interacting through a new force ("technicolor"). The

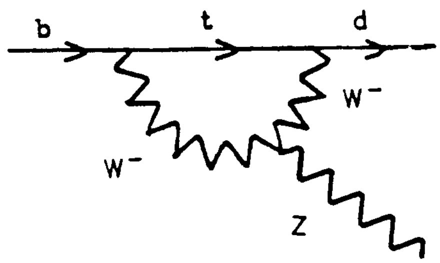


Figure 1.2: Example of flavor-changing neutral-current

role of the Higgs is then played, loosely speaking, by a bound state of techniquark-anti-techniquark. Another approach has the top quark itself as the cause of symmetry breaking, again thanks to one of its bound states. At any rate, all of the alternatives invented to date contain an object similar to a Higgs, namely a spinless, neutral particle. The "standard" Higgs mechanism of inducing symmetry breaking can then be regarded either as a fundamental, irreducible part of the theory or as a limiting case of a more general dynamics.

1.1.3 The top quark

As can be seen from the doublet structure of the theory, the top quark occupies a rather undistinguished position in the Standard Model. However, its existence is necessary, both from a theoretical and an experimental point of view.

There is a quite convincing experimental proof of the existence of the top, although it is somewhat indirect. It comes from a study of the bottom quark, the top quark's partner in the doublet. The interactions of the b quark (or any quark for that matter) with the gauge-bosons are dictated by the symmetry properties of the quark. In practice, this means that the structure of the b quark's interactions is influenced by the number of partners it has. In the Standard Model, it has only one partner: the top quark. It is possible to imagine theories in which the b quark has an arbitrary number of partners (including zero). By comparing the predictions of these theories with the measured properties of the b such as its decay rate and its neutral current couplings it has been determined that the most likely number of partners of the b quark is one, consistent with the prediction of the Standard Model.

The top quark's existence is also needed for the self-consistency of the theory. In general, calculations in quantum field theories involve infinite quantities. There exists a class of theories, the renormalizable theories, for which the infinities cancel, leaving finite results for the physical observables. The Standard Model is one such theory. At the same time, quantum field theories are susceptible of anomalies. These arise when a symmetry present in the classical theory cannot be self-consistently realised in its quantum version. Anomalies usually generate extra infinities which can cancel only if certain constraints are satisfied. Such is the case in the Standard Model. The corresponding constraint is that the sum of the charges of all the fermions in any generation should be zero. This can be the case only if the top quark exists <u>and</u> if there are three colors of quarks.

1.2 The B_d-$\overline{B}_d$ mixing

We must now answer the question: what is B_d-$\overline{B}_d$ mixing? Of all the possible results of collisions that one can imagine in particle physics, only some are actually realised. Those that are realised are found to satisfy certain conservation laws. The best known of these are undoubtedly the conservation of energy, the conservation of total linear and angular momentum and the conservation of electric charge. On the other hand, whenever a reaction is not expressly forbidden by the conservation laws, we can safely assume that it *will* happen, no matter how unlikely it may seem in the first place. Such is the case with B_d-$\overline{B}_d$ mixing 1.3. Here, a particle becomes its own anti-particle in the course

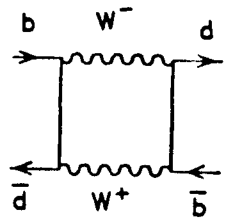


Figure 1.3: $B_d - \bar{B}_d$ mixing. The b and $\bar{d}$ quarks to the left of the figure are held together by the strong interactions (not shown) and form a B_d meson.

of its evolution or, more precisely, it oscillates between particle and anti-particle states. None of the above mentioned conservation laws are violated since B_d is a neutral, spinless particle and the particle and the anti-particle have the same mass.These oscillations have been observed not only for B_d but also for kaons (a bound state of a strange and an anti-down quarks). In the case of kaons, the oscillations have been observed over a length of a few meters!

It is not enough that the phenomenon can occur for it to be observable though. There must be some means of distinguishing between the particle and the anti-particle. If the particle is identical in every respect to its anti-particle then the mixing makes the particle turn into itself, hardly a spectacular event. In the case of B_d, flavor is what allows us to tell B_d and $\overline{B}_d$ apart. Thanks to the Higgs, flavor is not conserved in the Standard Model, which makes the transition possible. On the other hand, particles with different flavors have very different decay properties, which makes it possible to identify them. This will be discussed in depth in Chapter 4.

1.3 Outlook

We will now briefly discuss what is to come in the remainder of this thesis.

Chapter 2 contains a description of the Standard Model and introduces some field theoretical machinery that will be needed in the discussion of renormalization. Also, a few power counting rules pertaining to the behavior of Feynman integrals when one of the particles has a very large mass will be presented.

In Chapter 3, the relationship between the mixing amplitude and the observables is discussed. A full calculation of the one-loop amplitude is also shown. We will also present a brief survey of previous two-loop calculations in electroweak theory. This Chapter contains most of the physics of this work.

Chapter 4 describes the on-shell renormalization scheme used in our calculations. The discussion is kept quite general with the Appendices adapting the results for the case under study.

Finally, Chapter 5 presents the 2-loop calculation of the leading contributions. For completeness, the entire set of diagrams is shown. Most of the technical material is left to the Appendices.

Chapter 2

Field Theory and the Standard Model

In this chapter, we will establish the necessary machinery for the calculation of observables in the Standard Model. We will first show how to reduce the calculation of observables to that of the connected Green functions of the theory. We will then relate these to a knowledge of the Lagrangian via the path-integral formalism. The Lagrangian of the Standard Model will then be derived with special attention being paid to the symmetry breaking mechanism of the theory. This material is covered in several textbooks and will be presented here only for ease of reference. We will therefore be brief and omit most of the proofs. Finally, we will close this chapter with an exposition of an important theorem pertaining to the behavior of Feynman graphs when one particle in the theory has a very large mass.

2.1 Field-theoretical preliminaries

The field-theoretical object of interest to particle physicists is the scattering matrix (S-matrix). The matrix element S_{fi} is related to the probability of a transition between the initial state i and the final state f. More specifically, if we write $S = 1 + iT$, the probability per unit volume per unit time P_{fi} of a transition from i to f is

$$P_{fi} = (2\pi)^4 \delta^4(P_f - P_i) |R_{fi}|^2$$

where P_f and P_i are the momentum of the final and initial state respectively and R_{fi} is defined by

$$T_{fi} = (2\pi)^4 \delta^4(P_f - P_i)\, R_{fi}.$$

In the remainder of this section, we will show how to calculate the S-matrix given a knowledge of the Lagrangian of the theory. A more detailed treatment of this material can be found in several textbooks, an excellent one being Itzykson and Zuber [2]. A careful discussion can also be found in Aoki et al. [4].

2.1.1 The S-matrix and the Green functions

In this and the next section, we will often use a theory consisting of a single self-interacting scalar field ϕ of mass m. This will lead to more compact expressions since there is no spin structure to worry about. The appropriate changes when fermions and vector (spin-1) particles are considered will be indicated.

In general, the initial and final states are characterized by the number (and type) of particles present as well as some of their properties (linear momentum, energy, angular momentum ...). By initial and final, we mean here long before ($t = -\infty$) and long after ($t = +\infty$) the interactions occur. For theories of a single scalar field, the states are completely characterized by the number of particles present and their momenta. The S-matrix element connecting a final state of n particles of momenta p_i ($i = 1,\ldots,n$) to an initial state of m particles of momenta q_j ($j = 1,\ldots,m$) is given by:

$$\begin{aligned}\langle p_1,\ldots,p_n|S|q_1,\ldots,q_m\rangle &= (i\tilde{Z}^{-1/2})^{n+m} \int d^4y_1 \ldots d^4x_m \exp\left(i\sum_{k=1}^{n} p_k \cdot y_k - i\sum_{r=1}^{m} q_r \cdot x_r\right) \times \\ &\quad (\Box_{y_1} + m^2)\ldots(\Box_{x_m} + m^2) G_{n+m}(y_1,\ldots,x_m). \qquad (2.2)\end{aligned}$$

The notations and conventions are spelled out in Appendix A. Note that formula 2.2 applies only for transitions where all the particles are affected, that is none of the particles are "spectators" of the collision.

The functions $G_n(x_1, \ldots, x_n)$ are the connected Green functions of the theory. We will have more to say about them in the next section. The S-matrix element is then the Fourier transform of a function obtained from the Green functions by repeated application of the operator $\tilde{Z}^{-1/2}(\Box + m^2)$ (once for each external particle). Here, $\tilde{Z}$ is a calculable number between 0 and 1. A free (non-interacting) *classical* scalar field $\phi_{\rm cl}$ satisfies a wave equation involving this same operator:

$$(\Box + m^2)\phi_{\rm cl} = 0$$

This ensures that the momentum and the mass of $\phi_{\rm cl}$ satisfy the usual mass-shell relation $p^2 = m^2$. (The correspondence between the operators appearing in 2.2 and the wave function operator for the classical field will form the basis of the generalization to fermionic external states.)

The factors $(\Box_{x_i} + m^2)$ in equation 2.2 become $-p_i^2 + m^2$ after a Fourier transform. If the external states satisfy the mass-shell relation (as they should, for physical particles), it appears that the S-matrix vanishes identically! Fortunately, this is not the case because the Green functions have poles that precisely cancel the $-p_i^2 + m^2$ factors. This, as well as the meaning of the factor $\tilde{Z}$, will be discussed in Chapter 4.

If fermions are present as external particles, we must replace the "wave-function" operator $i\tilde{Z}^{-1/2}(\Box + m^2)$ by the corresponding operator for fermions. There are four cases:

a fermion in the initial state : $i\tilde{Z}_2^{-1/2}(i\not\partial + m)u(k, \epsilon)$
an anti-fermion in the initial state : $i\tilde{Z}_2^{-1/2}(i\not\partial - m)\bar{v}(k, \epsilon)$
a fermion in the final state : $-i\tilde{Z}_2^{-1/2}(i\not\partial - m)\bar{u}(k, \epsilon)$
an anti-fermion in the final state : $-i\tilde{Z}_2^{-1/2}(i\not\partial + m)v(k, \epsilon)$

where k and ϵ are the momentum and the polarisation of the fermion. The spinors u and

v satisfy the equations:

$$\begin{aligned}(\not{k} - m)u(k,\epsilon) &= 0 \\ (\not{k} + m)v(k,\epsilon) &= 0\end{aligned}$$

2.1.2 Green functions and the generating functional

We are left with the task of calculating the connected Green functions. These can be defined via a path integral:

$$G_n(x_1, \ldots, x_n) = \frac{1}{N} \int [d\phi]\, \phi(x_1) \ldots \phi(x_n) \exp\left(iS[\phi]\right) \tag{2.3}$$

where we assumed again a theory with only one type of scalar particles. The normalization factor N is given by:

$$N = \int [d\phi] \exp\left(iS[\phi]\right)$$

The symbol $[d\phi]$ stands for the measure of the path integral. Roughly speaking, $[d\phi]$ is the product $d\phi(x_1)\, d\phi(x_2) \ldots d\phi(x_k) \ldots$ extended over *all* space-time points x_i. Since there is an uncountable infinity of such points, it is understandably quite tricky to define $[d\phi]$ precisely. We will not attempt this here but simply note that this measure has properties that are similar to finite-dimensional integrals (e.g. translation invariance $[d\phi] = [d(\phi + f)]$ for f independent of ϕ, etc.). In what follows, we will only use those properties and never calculate a path integral explicitly.

The functional $S[\phi]$ that appears in 2.3 is the *action* of the theory. It is obtained by integrating the *Lagrangian density* $\mathcal{L}(\phi, \partial^\mu \phi)$:

$$S[\phi] = \int d^4x\, \mathcal{L}(\phi, \partial^\mu \phi)$$

The Lagrangian density (or Lagrangian for short) is in general a function of the field ϕ, its derivatives and possibly of the space-time point x.

In order to make progress in the calculation of Green functions, we must add a source term to the action, i.e. we replace $S[\phi]$ by $\bar{S}[\phi] = S[\phi] + S_{\text{source}}$ where

$$S_{\text{source}} = \int d^4x\, j(x)\phi(x).$$

Here, $j(x)$ is called a *source*. This will not change anything as long as we take the source to zero at the end of the calculation. But now, using the functional differentiation rule

$$\frac{\delta}{\delta j(x)} j(y) = \delta^4(x-y) \tag{2.4}$$

we find:

$$-i\frac{\delta}{\delta j(x)} e^{iS_{\text{source}}} = \phi(x)e^{iS_{\text{source}}}. \tag{2.5}$$

Using this, equation 2.3 reduces to

$$G_n(x_1,\ldots,x_n) = \frac{1}{Z[j]}\left(-i\frac{\delta}{\delta j(x_1)}\right)\cdots\left(-i\frac{\delta}{\delta j(x_1)}\right)Z[j]\bigg|_{j=0} \tag{2.6}$$

where

$$Z[j] = \int [d\phi] \exp\left(S[\phi] + \int d^4x\, j(x)\phi(x)\right) \tag{2.7}$$

is the *generating functional*. These steps are familiar from statistical mechanics.

To calculate the generating functional, we must specify the Lagrangian. To keep the writing simple, we will consider the Lagrangian

$$\mathcal{L} = \frac{1}{2}\left[(\partial^\mu\phi)(\partial_\mu\phi) - m^2\phi^2\right] - \frac{\lambda}{4!}\phi^4$$

We now write $\mathcal{L}$ as the sum of a *free* part $\mathcal{L}_0$ which contains only terms quadratic in ϕ and $\mathcal{L}_I$, the *interaction Lagrangian*, which is everything else in $\mathcal{L}$. Note that $\mathcal{L}_I$ may contain some quadratic terms, a possibility that will be exploited in Chapter 4. For now, however, we will take $\mathcal{L}_0 = \frac{1}{2}[(\partial_\mu\phi)(\partial^\mu\phi) - m^2\phi^2]$ and $\mathcal{L}_I = -\frac{\lambda}{4!}\phi^4$. We now substitute $\mathcal{L}$ in 2.7 and use 2.5 to find

$$Z[j] = \exp\left(i\int d^4x\, \mathcal{L}_I\left(-i\frac{\delta}{\delta j(x)}\right)\right) Z_0[j] \tag{2.8}$$

where

$$Z_0[j] = \int [d\phi] \exp\Big(i \int d^4x \left\{ \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{m^2}{2}\phi^2 + j\phi \right\}\Big) \tag{2.9}$$

The exponential of differential operators in 2.8 is defined by its power series

$$\exp\left\{ -\frac{i\lambda}{4!} \int d^4x \left(\frac{\delta}{\delta j(x)} \right)^4 \right\} = 1 - \frac{i\lambda}{4!} \int d^4x \left(\frac{\delta}{\delta j(x)} \right)^4 + \cdots$$

All that remains to do is to calculate $Z_0(j)$. To this end, we focus on the argument of the exponential in 2.9. Using $(\partial_\mu \phi)(\partial^\mu \phi) = \partial_\mu(\phi \partial^\mu \phi) - \phi \Box \phi$ and partial integration, we can write it as:

$$\int d^4x \left\{ -\frac{1}{2}\phi(x)(\Box + m^2)\phi(x) + j(x)\phi(x) \right\}$$

where we have assumed that the fields go to zero sufficiently fast at infinity for the surface term to vanish. We now seek a function $G(x, y)$ such that:

$$(\Box_x + m^2) G(x, y) = \delta^4(x - y) \tag{2.10}$$

After a Fourier transformation, one finds

$$G(x, y) = -\int \frac{d^4p}{(2\pi)^4} \frac{e^{ip\cdot(x-y)}}{p^2 - m^2} \tag{2.11}$$

We are now in position to make the change of variable $\phi(x) \to \phi(x) + \int d^4y \, G(x, y) j(y)$ in 2.9. Note that the measure $[d\phi]$ is invariant under this translation since $\int d^4y \, G(x, y) j(y)$ is independent of ϕ. We get:

$$Z_0[j] = \exp\left(\frac{i}{2} \int d^4x \, d^4y \, j(x) G(x, y) j(y) \right) \int [d\phi] \exp\left\{ i \int d^4x \, \mathcal{L}_0(\phi) \right\}$$

The path integral is now independent of $j(x)$ and is seen to cancel in the expression for the Green functions 2.6. We can therefore drop it and obtain:

$$Z_0[j] = \exp\left(\frac{i}{2} \int d^4x \, d^4y \, j(x) G(x, y) j(y) \right) \tag{2.12}$$

with $G(x,y)$ given by 2.11. It was in order to effect this separation that we kept only the terms quadratic in ϕ in $\mathcal{L}_0$.

All we have to do now is to use 2.6, 2.8 and 2.12, perform the indicated differentiations and collect those terms that survive in the limit $j=0$. This can be rather tedious. The results of this exercise are usually summarized by means of *Feynman rules.* (The Feynman rules for the Standard Model are given in Appendix B.)

Some important changes must be made to generalize this formalism to fermions. First of all, there are two independent sources (η and $\bar{\eta}$) that couple to the fermion. The source term reads:

$$\mathcal{L}_{\rm source} = \int d^4x \, \left\{ \bar{\eta}(x)\psi(x) + \bar{\psi}(x)\eta(x) \right\}$$

where the fermion is denoted by ψ. Fermions carry a Dirac index and the notation $\bar{\eta}(x)\psi(x)$ really stands for $\bar{\eta}_\alpha(x)\psi^\alpha(x)$. The analog of 2.4 is:

$$\frac{\delta}{\delta\eta_\alpha(x)}\eta_\beta(y) = \frac{\delta}{\delta\bar{\eta}_\alpha(x)}\bar{\eta}_\beta(y) = \delta^\alpha{}_\beta \delta^4(x-y)$$

An important point here is that any two fermionic quantities (wether they are sources, differential operators or fermions proper) anti-commute with one another. For instance, we have:

$$\frac{\delta}{\delta\eta_\beta(x)}\psi^\alpha(y)\eta_\alpha(y) = -\psi^\beta(y)\delta^4(x-y)$$

As a result, the analog of 2.5 is

$$\begin{aligned}
-i\frac{\delta}{\delta\bar{\eta}(x)}e^{iS_{\rm source}} &= \psi(x)e^{iS_{\rm source}} \\
i\frac{\delta}{\delta\eta(x)}e^{iS_{\rm source}} &= \bar{\psi}(x)e^{iS_{\rm source}}
\end{aligned}$$

and the corresponding changes must be made in 2.6 and 2.8. Finally, the free part of the fermionic Lagrangian is $\bar{\psi}(x)(i\partial\!\!\!/ - m)\psi(x)$ which translates into

$$G_F(x,y) = -\int \frac{d^4p}{(2\pi)^4} \frac{e^{ip\cdot(x-y)}}{p\!\!\!/ - m}$$

The case of vector bosons will be considered when we construct the Lagrangian of the Standard Model, a task to which we now turn.

2.2 The Standard Model

The Standard Model of electroweak interactions is, in the jargon of field-theory, a spontaneously broken $SU_2 \otimes U_1$ gauge theory. It shares a crucial property with other gauge theories: it is *renormalizable.* This essentially means that we can do calculations in gauge theories without ever encountering a logical inconsistency. This is not true of all field theories. We will not attempt to justify the choice of $SU_2 \otimes U_1$ as the symmetry of the model. It is sufficient to remark that its consequences are amply verified experimentally. This section is based largely on the excellent text of Cheng and Li [3].

2.2.1 Gauge theories

Gauge theories can be thought of as an attempt to generalize the known symmetries of the free field theories. Recall that the free part of a fermionic Lagrangian is:

$$\mathcal{L}_F = \bar{\psi}(x)\,(i\partial\!\!\!/ - m)\,\psi(x) \tag{2.13}$$

for a fermion ψ of mass m. This Lagrangian is invariant under the transformation $\psi(x) \rightarrow e^{-i\theta}\psi(x)$ (which implies $\bar{\psi}(x) \rightarrow \bar{\psi}(x)e^{i\theta}$). We can now try and generalize 2.13 so that it becomes invariant under a local version of the symmetry:

$$\psi(x) \rightarrow e^{-i\theta(x)}\psi(x)$$

Under this transformation, the derivative in 2.13 generates an extra term.

$$\mathcal{L}_F \rightarrow \mathcal{L}_F + \bar{\psi}(x)\,(\partial\!\!\!/\theta)\,\psi(x)$$

However, if we generalize this derivative to $D^\mu = \partial^\mu + ieB^\mu$ and insist that the field B^μ transform as $B^\mu \rightarrow B^\mu(1/e)\partial^\mu\theta(x)$, we find that the Lagrangian

$$\mathcal{L}_{QED} = \bar{\psi}(i\not{D} - m)\psi \tag{2.14}$$

is invariant. This is the Lagrangian of quantum electrodynamics (QED). The arbitrary parameter "e" that enter these expressions is the electric charge of ψ. It is not fixed by the theory and could take different values for different particles. (The fact that the elementary particles found so far have charge $0, \pm 1/3, \pm 2/3, \pm 1$ is unexplained.)

The transformations

$$\psi(x) \rightarrow e^{-i\theta(x)}\psi(x) \tag{2.15}$$

$$B^\mu(x) \rightarrow B^\mu(x) + (1/e)\partial^\mu\theta(x) \tag{2.16}$$

form a U_1 gauge transformation. To give a physical meaning to the field B^μ, we must find a free Lagrangian for it that is invariant under 2.16. The Lagrangian

$$\mathcal{L}_B = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{2.17}$$

where

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{2.18}$$

does the job. Notice that this Lagrangian contains no self-interactions of the field B^μ, i.e. $\mathcal{L}_B$ is purely quadratic in B^μ. This is a peculiar feature of the U_1 symmetry.

We can generalize this further by considering two degenerate fermions ψ_1 and ψ_2 of common mass m. The free Lagrangian is:

$$\mathcal{L}_F^{(2)} = \bar{\psi}_1(i\not{\partial} - m)\psi_1 + \bar{\psi}_2(i\not{\partial} - m)\psi_2$$

which can be regrouped as

$$\mathcal{L}_F^{(2)} = \bar{\psi}(i\not{\partial} - m)\psi \tag{2.19}$$

with

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}.$$

In this form, it is apparent that the Lagrangian is invariant under $\psi \to U\psi$ where U is a 2×2 unitary matrix. Actually, we can restrict ourselves to matrices U of determinant 1 (SU_2 matrices) since any unitary matrix can be written as the product of a SU_2 matrix and a U_1 phase which we have already considered. The SU_2 matrices can be parametrized as

$$U(\theta) = \exp\left\{-\frac{i\boldsymbol{\tau}\cdot\boldsymbol{\theta}}{2}\right\}$$

where $\boldsymbol{\tau} = (\tau_1, \tau_2, \tau_3)$ are the Pauli matrices (discussed in Appendix A). Again, we attempt to generalize this by taking $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3)$ to be spacetime dependent. This can be done at the expense of changing the derivative in 2.19

$$\partial^\mu \to D^\mu = \partial^\mu - ig\frac{\boldsymbol{\tau}\cdot\boldsymbol{W}^\mu}{2}$$

The fields $\mathbf{W}^\mu = (W_1^\mu, W_2^\mu, W_3^\mu)$ obey the rather complex transformation law

$$\frac{\boldsymbol{\tau}\cdot\mathbf{W}^\mu}{2} \to U(\theta)\left(\frac{\boldsymbol{\tau}\cdot\mathbf{W}^\mu}{2}\right)U^{-1}(\theta) - \frac{i}{g}\left(\partial^\mu U(\theta)\right)U^{-1}(\theta) \tag{2.20}$$

Finally, the Lagrangian for the fields $\mathbf{W}^\mu$ that is invariant under these transformations is:

$$\mathcal{L}_W = -\frac{1}{4}F^i_{\mu\nu}F_i^{\mu\nu} \tag{2.21}$$

where

$$F^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\epsilon^{ijk}W^j_\mu W^k_\nu$$

and ϵ^{ijk} is the totally antisymmetric Levi-Civita tensor ($\epsilon^{123} = +1$). This Lagrangian *does* contain interaction terms between the gauge bosons. These interactions depend on the coupling g. Therefore, every doublet interacting with the $\mathbf{W}$ will do so with the same coupling g. As in QED, the strength of this coupling is arbitrary.

The combined set of transformations

$$\psi \rightarrow U(\theta)\psi \tag{2.22}$$

$$\frac{\boldsymbol{\tau}\cdot\mathbf{W}^{\mu}}{2} \rightarrow U(\theta)\left(\frac{\boldsymbol{\tau}\cdot\mathbf{W}^{\mu}}{2}\right)U^{-1}(\theta) - \frac{i}{g}\left(\partial^{\mu}U(\theta)\right)U^{-1}(\theta) \tag{2.23}$$

constitute a SU_2 transformation. This transformation applies only to doublets. Fermions can also tranform as *singlets* under SU_2. Singlets do not interact at all with the $\mathbf{W}$ fields. There also exist SU_2 triplets, etc... but these higher representations are not utilised in the Standard Model nor are they necessary.

The Standard Model has a $SU_2 \otimes U_1$ gauge symmetry which is a combination of the two symmetries discussed above. The gauge self-interactions can be used without modifications in the model:

$$\mathcal{L}_{\text{GAUGE}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F^{i}_{\mu\nu}F^{\mu\nu}_{i} \tag{2.24}$$

with

$$F_{\mu\nu} = \partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu}$$

and

$$F^{i}_{\mu\nu} = \partial_{\mu}W^{i}_{\nu} - \partial_{\nu}W^{i}_{\mu} + g\epsilon^{ijk}W^{j}_{\mu}W^{k}_{\nu}$$

Note that this Lagrangian does not contain any mass terms i.e. terms of the form $M_B^2 B^{\mu}B_{\mu}$ or $M_W^2 W^{i\mu}W^{i}_{\mu}$. These terms are forbidden by the gauge symmetry. Therefore, the fields B^{μ} and $W^{i\mu}$ are massless at this stage.

We will now apply the formalism of this section to the interactions of fermions and gauge bosons in the Standard Model.

2.2.2 Helicity basis, parity violation and the fermion interactions

The *helicity* σ_p of a fermion is defined as the component of its spin along the direction of its momentum:

$$\sigma_p = \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{|\mathbf{p}|}$$

In the limiting case that $\sigma_p = \hbar/2$, we say that the fermion is *right-handed* and, conversely, it is *left-handed* if $\sigma_p = -\hbar/2$. The parity transformation $\mathbf{r} \rightarrow -\mathbf{r}$ changes the sign of the momentum but leaves σ invariant since it is an angular momentum ($\sim \mathbf{r} \times \mathbf{p}$). Therefore, the left-handed and right-handed states are interchanged under a parity transformation.

The corresponding entities in field theory are the right-handed and left-handed fields ψ_R and ψ_L (see Appendix A of [21]) where $\psi_R = R\psi$, $\psi_L = L\psi$ and $L = (1 - \gamma_5)/2$, $R = (1 + \gamma_5)/2$. (This correspondence holds only in the massless limit but we will see later that the fermions have to be massless.) Experimentally, the weak interactions are *not* invariant under parity. Therefore, the left-handed and right-handed fields must have dissimilar interactions. In the Standard Model, this is accomplished by taking the left-handed fields to transform as doublets under SU_2 and the right-handed fields as singlets. Before we do this, we must establish some notation.

We define:

$$\begin{aligned} e'_a &= (e', \mu', \tau') \\ \nu'_a &= (\nu'_e, \nu'_\mu, \nu'_\tau) \\ u'_a &= (u', c', t') \\ d'_a &= (d', s', b') \end{aligned}$$

The reason behind the primes will become clear when we discuss symmetry breaking.

Define further:

$$l_{aL} = \begin{pmatrix} \nu'_a \\ e'_a \end{pmatrix} \;,\; q_{aL} = \begin{pmatrix} u'_a \\ d'_a \end{pmatrix}$$

The fundamental fields are then l_{aL}, q_{aL} as well as e'_{aR}, u'_{aR}, d'_{aR} i.e. the left-handed fields are doublets and the right-handed ones singlet. Note in particular the absence of right-handed neutrinos. Note also that it is impossible to form a mass term (a term of the form $m\bar{\psi}_L\psi_R$) which is invariant under SU_2. Therefore, the fermions have to be massless.

The Lagrangian will then be:

$$\mathcal{L}_{\text{FERMIONS}} = \sum_j \bar{\psi}_j i \not{D}_j \psi_j$$

where the sum runs over l, q, e', p' and n'. The covariant derivative D^μ takes the general form:

$$D^\mu_j = \partial^\mu - ig\mathbf{T}_j \cdot \mathbf{W}^\mu - i\frac{g'}{2} Y_j B^\mu$$

where $\mathbf{T}_j = \boldsymbol{\tau}/2$ if ψ_j is a doublet and $\mathbf{T}_j = 0$ if ψ_j is a singlet. The *hypercharge* Y is related to the electric charge Q by $Q = T_3 + Y/2$ where $T_3 = 1/2$ for the upper component of a doublet, $-1/2$ for the lower component and 0 for a singlet. Explicitly,

$$\begin{aligned} &Y(q_L) = 1/3 \;, \quad Y(u'_R) = 4/3 \;, \quad Y(d'_R) = -2/3 \\ &Y(l_L) = -1 \;, \quad Y(e'_R) = -2 \end{aligned}$$

These values are not fixed in the model; they are chosen so as to reproduce the correct value of the electric charge of the various particles. This is considered a major flaw of the theory. Finally, g and g' are two arbitrary couplings. Explicitly, the fermionic Lagrangian reads:

$$\begin{aligned} \mathcal{L}_{\text{FERMIONS}} &= \bar{l}_{aL}(i\not{\partial} + \frac{g}{2}\boldsymbol{\tau} \cdot \not{\mathbf{W}} - \frac{g'}{2}\not{B})l_{aL} + \bar{e}'_{aR}(i\not{\partial} - g'\not{B})e'_{aR} \\ &+\bar{q}_{aL}(i\not{\partial} + \frac{g}{2}\boldsymbol{\tau} \cdot \not{\mathbf{W}} + \frac{g'}{2}\not{B})q_{aL} + \bar{u}'_{aR}(i\not{\partial} + \frac{2g'}{3}\not{B})u'_{aR} \end{aligned}$$

$$+\bar{d}'_{aR}(i\partial\!\!\!/ - \frac{g'}{3}B\!\!\!/)d'_{aR}. \tag{2.25}$$

2.2.3 The Higgs boson and symmetry breaking

So far, all the particles in the theory are massless, including the gauge bosons. In analogy with QED, we would then expect the presence of long range forces associated with these massless bosons. Since only the familiar electromagnetic force is observed to be long range, it is necessary to generate masses for three of the four bosons in order for the theory to describe short range weak interactions. This can be done in an ingenious way with the Higgs mechanism.

To this end, we introduce a complex scalar doublet Φ (the Higgs doublet) of hypercharge 1. Its interactions with the gauge bosons are then automatically fixed and we can write the Higgs Lagrangian:

$$\mathcal{L}_{\mathrm{HIGGS}} = (D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi) \tag{2.26}$$

where

$$D_\mu\Phi = (\partial_\mu - i\frac{g}{2}\tau\cdot\mathbf{W}_\mu - i\frac{g'}{2}B_\mu)\Phi.$$

The *Higgs potential* $V(\Phi)$ contains the self-interactions of the Higgs. Its general form is dictated by gauge invariance and considerations of renormalizability:

$$V(\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 \tag{2.27}$$

Note the peculiar sign of the quadratic term. If μ^2 is negative, this is an ordinary mass term. However, a positive μ^2 gives an imaginary mass, an unstable situation. In this case, the potential looks as in fig. 2.4. The minimum of the potential is then realized for a non-zero value of Φ. It is convenient to write

$$\Phi = \phi_0 + \phi' \tag{2.28}$$

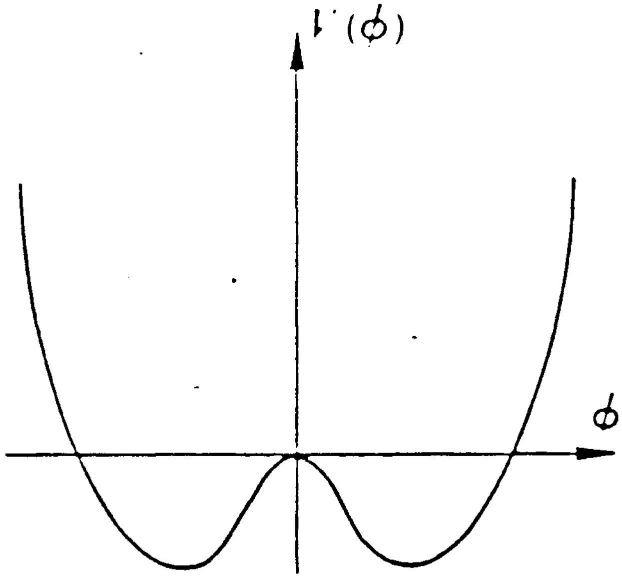


Figure 2.3: The Higgs potential for $\mu^2 > 0$.

and choose ϕ_0 so that $\phi' = 0$ at the minimum of V. The *vacuum expectation value* (VEV) ϕ_0 can always be chosen as $\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$ with v real. This is because the most general form $\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ can always be brought into the previous form by a suitable $SU_2 \otimes U_1$ transformation. In terms of the parameters of the Higgs potential,

$$v = \sqrt{\mu^2/\lambda} \tag{2.29}$$

We can now substitute 2.28 in 2.27 and parametrize ϕ' by

$$\phi' = \begin{pmatrix} \phi^+ \\ (H + i\chi)/\sqrt{2} \end{pmatrix}$$

The result is quite complex. Focus first on the mass terms. Writing:

$$W_\mu^\pm \;=\; \frac{1}{\sqrt{2}}(W_\mu^1 \mp W_\mu^2)$$

$$\begin{aligned} Z_\mu &= \cos\theta_W W^3_\mu - \sin\theta_W B_\mu \\ A_\mu &= \sin\theta_W W^3_\mu + \cos\theta_W B_\mu \end{aligned} \tag{2.30}$$

with

$$\tan\theta_W = \frac{g'}{g} \tag{2.31}$$

we find:

$$\mathcal{L}_{\mathrm{MASS}} = \frac{g^2v^2}{4} W^+_\mu W^{-\mu} + \frac{g^2v^2}{8\cos^2\theta_W} Z_\mu Z^\mu + v^2\lambda H^2 \tag{2.32}$$

There is therefore a charged W boson of mass $M_W = gv/2$, a neutral Z boson of mass $M_Z = gv/2\cos\theta_W$ and a neutral Higgs boson of mass $M_H = \sqrt{2\lambda v^2}$. The last gauge boson is the photon, which remains massless. There are also three other scalars , the $\phi^\pm$ and the χ, known as the *would-be Goldstone bosons.* These "particles" are unphysical: they cannot appear in external states. Their role in the model is to provide the massive $W^\pm$ and Z bosons with a third polarization state (which they would not have as massless particles).

It is now possible to express the parameters v, μ and λ in terms of the masses of the W and Higgs bosons:

$$\begin{aligned} v &= \frac{2M_W}{g} \\ \lambda &= \frac{g^2}{8}\frac{M_H{}^2}{M_W^2} \\ \mu^2 &= \frac{M_H^2}{2} \end{aligned} \tag{2.33}$$

Note also the relation $M_Z = M_W/\cos\theta_W$. In terms of these, the Higgs Lagrangian reads:

$$\begin{aligned} \mathcal{L}_{\mathrm{HIGGS}} = \; & (\partial_\mu\phi^-)(\partial^\mu\phi^+) + \frac{1}{2}(\partial_\mu H)(\partial^\mu H) + \frac{1}{2}(\partial_\mu\chi)(\partial^\mu\chi) + M_W^2 W^+_\mu W^{-\mu} + M_Z^2 Z^\mu Z_\mu/2 \\ & + \left(e^2 A_\mu A^\mu + \frac{eg}{c}(c^2 - s^2)A_\mu Z^\mu + \frac{g^2}{4c^2}(c^2-s^2)^2 Z_\mu Z^\mu\right)\phi^+\phi^- \\ & + gW^{+\mu}W^-_\mu(M_W H + \frac{g}{4}H^2 + \frac{g}{4}\chi^2 + \frac{g}{2}\phi^+\phi^-) \end{aligned}$$

$$
\begin{aligned}
&+\frac{g}{c}Z^\mu Z_\mu(M_Z H+\frac{g}{8c}H^2+\frac{g}{8c}\chi^2)\\
&+e\left[\phi^+W^{-\mu}(M_W A_\mu+\frac{g}{2}HA_\mu-i\frac{g}{2}\chi A_\mu-sM_Z Z_\mu-\frac{e}{2c}HZ_\mu+i\frac{e}{2c}\chi Z_\mu)+\text{h.c.}\right]\\
&+\left[iM_W W^{-\mu}\partial_\mu\phi^+ + i\frac{g}{2}\left\{\left(2sA^\mu+\frac{(c^2-s^2)}{c}Z^\mu\right)\phi^-+(H-i\chi)W^{-\mu}\right\}+\text{h.c.}\right]\\
&+\left[M_Z Z^\mu\partial_\mu\chi+i\frac{g}{2}\left\{W_\mu^+\phi^- -\frac{1}{2c}(H-i\chi)Z_\mu\right\}(\partial^\mu H+i\partial^\mu\chi)+\text{h.c.}\right]\\
&-V(\Phi)
\end{aligned}
\tag{2.34}
$$

Here, h.c. stands for "hermitian conjugate", $e = g\sin\theta_W$ and the abbreviations $c = \cos\theta_W$ and $s = \sin\theta_W$ have been used. The potential V is given by:

$$
\begin{aligned}
V(\Phi) &= \frac{1}{2}M_H^2H^2+\frac{g}{4}\frac{M_H^2}{M_W^2}(H^3+\chi^2H+2\phi^+\phi^-H)\\
&+\frac{g^2}{32}\left(\frac{M_H^2}{M_W^2}\right)\left(H^4+2H^2\chi^2+\chi^4+4(\phi^+\phi^-)^2+4\phi^+\phi^-H^2+4\phi^+\phi^-\chi^2\right)
\end{aligned}
$$

Note the presence of an M_H^2 dependence in the couplings.

The Lagrangian 2.34 contains some problematic terms of the form $W^{+\mu}\partial_\mu\phi^-$. These terms are very difficult to interpret. Fortunately, they cancel against the corresponding pieces of the gauge-fixing Lagrangian which will be presented in section 2.2.5.

The gauge Lagrangian 2.24 must also be written in terms of the physical fields A, Z and $W^\pm$. This gives another lengthy expression.

$$
\begin{aligned}
\mathcal{L}_{\text{GAUGE}} &= (\partial^\mu W^{+\nu})(\partial_\nu W_\mu^-)-(\partial^\mu W^{+\nu})(\partial_\mu W_\nu^-)+\frac{1}{2}\left[(\partial^\mu A^\nu)(\partial_\nu A_\mu)-(\partial^\mu A^\nu)(\partial_\mu A_\nu)\right]\\
&+\frac{1}{2}\left[(\partial^\mu Z^\nu)(\partial_\nu Z_\mu)-(\partial^\mu Z^\nu)(\partial_\mu Z_\nu)\right]-e^2\left[(A^\mu A_\mu)(W^{+\nu}W_\nu^-)-(A^\mu W_\mu^+)(A^\nu W_\nu^-)\right]\\
&-g^2c^2\left[(Z^\mu Z_\mu)(W^{+\nu}W_\nu^-)-(Z^\mu W_\mu^+)(Z^\nu W_\nu^-)\right]-\frac{g^2}{2}\left[(W^{+\mu}W_\mu^-)(W^{+\nu}W_\nu^-)\right.\\
&\left.-(W^{+\mu}W_\mu^+)(W^{-\nu}W_\nu^-)\right]-\left\{egc\left[(A^\mu Z_\mu)(W^{+\nu}W_\nu^-)-(W^{+\mu}Z_\mu)(W^{-\nu}A_\nu)\right]\right.\\
&\left.+ie\left[(W_\mu^+A_\nu-W_\nu^+A_\mu)(\partial^\mu W^{-\nu})-(W^{+\mu}W^{-\nu})(\partial_\mu A_\nu)\right]+\text{h.c.}\right\}\\
&+igc\left\{\left[(W_\mu^+Z_\nu-W_\nu^+Z_\mu)(\partial^\mu W^{-\nu})-(W^{+\mu}W^{-\nu})(\partial_\mu Z_\nu)\right]+\text{h.c.}\right\}
\end{aligned}
\tag{2.35}
$$

We have seen that the Higgs has given a mass to the W and Z bosons and kept the photon massless. In the next section, we will see how it can be used to generate masses for the fermions.

2.2.4 The Yukawa interactions

The Higgs doublet has been introduced to generate the vector bosons masses.. However, there is no reason why it should not interact with the fermions in the model. All that is required is that the interaction terms obey the $SU_2 \otimes U_1$ gauge-symmetry and that they be renormalizable. The last condition means that the couplings involved cannot have a dimension of $(\text{mass})^{-x}$ where x is a positive number. The gauge invariance is easily satisfied by coupling the Higgs doublet to a fermion doublet and a fermion singlet and demanding that the hypercharges sum to zero.

It is convenient to introduce the conjugate of Φ by

$$\tilde{\Phi} = i\tau_2 \Phi^*.$$

Explicitly:

$$\tilde{\Phi} = \begin{pmatrix} (v + H - i\chi)/\sqrt{2} \\ -\phi^- \end{pmatrix}$$

This field has the same transformation properties under SU_2 than Φ but it has the opposite hypercharge, $Y(\tilde{\Phi}) = -1$. The most general form of interactions between the Higgs and the fermions is:

$$\mathcal{L}_Y = f_{ab}^{(e)} \bar{l}_{aL} \Phi e'_{bR} + f_{ab}^{(u)} \bar{q}_{aL} \tilde{\Phi} u'_{bR} + f_{ab}^{(d)} \bar{q}_{aL} \Phi d'_{bR} + \text{h.c.} \tag{2.36}$$

This is called the *Yukawa Lagrangian.* The matrices f in $\mathcal{L}_Y$ are 3×3 complex matrices of dimensionless coefficients. This gives a total of 54 arbitrary parameters. However, not all of these are meaningful. We will see that only 13 of them are free parameters.

We now introduce the explicit form of the doublets l, q, Φ and $\tilde{\Phi}$ in 2.36 and simultaneously rotate the fermion fields.

$$\begin{aligned} e'_{bR} &= S^{(e)}_{bc} e_{cR} \quad e'_{bL} = T^{(e)}_{bc} e_{cL} \quad \nu'_{bL} = T^{(e)}_{bc} \nu_{bL} \\ u'_{bR} &= S^{(u)}_{bc} u_{cR} \quad u'_{bL} = T^{(u)}_{bc} u_{cL} \\ d'_{bR} &= S^{(d)}_{bc} d_{cR} \quad d'_{bL} = T^{(d)}_{bc} d_{cL} \end{aligned} \tag{2.37}$$

The S and T matrices are unitary. They are chosen to satisfy

$$\frac{v}{\sqrt{2}} T^{(e)*}_{ad} f^{(e)}_{ab} S^{(e)}_{bc} = m^{(e)}_{d} \delta_{cd} \tag{2.38}$$

with similar equations for u and d. It is always possible to find S and T that satisfy 2.38 [3]. The parameters m are the masses of the fermions. We see then directly that the masses are proportionnal to the interaction strength f between the Higgs and the fermions.

With all these transformations, the Yukawa Lagrangian becomes:

$$\begin{aligned} \mathcal{L}_Y &= -\bar{e}_a m^{(e)}_a e_a - \bar{u}_a m^{(u)}_a u_a - \bar{d}_a m^{(d)}_a d_a \\ & -\frac{g}{2M_W}\left\{H\bar{e}_a m^{(e)}_a e_a + H\bar{u}_a m^{(u)}_a u_a + H\bar{d}_a m^{(d)}_a d_a\right\} \\ & -\frac{g}{2M_W}\left\{i\chi\bar{e}_a m^{(e)}_a \gamma_5 e_a - i\chi\bar{p}_a m^{(p)}_a \gamma_5 p_a + i\chi\bar{n}_a m^{(n)}_a n_a\right\} \\ & -\frac{g}{\sqrt{2}M_W}\left\{\bar{\nu}_a m^{(e)}_a R e_a \phi^+ - \bar{u}_a (m^{(u)}_a L - m^{(d)}_a R) d_b V_{ab} \phi^+ + \text{h.c.}\right\} \end{aligned} \tag{2.39}$$

where the relations $M_W = gv/2$, $\psi_R = R\psi$, etc. have been used. Also, the Kobayashi-Maskawa (KM) matrix V_{ab} has been defined by $V_{ab} = S^{(p)*}_{ca} S^{(n)}_{cb}$. Therefore, V is unitary as it is a product of two unitary matrices.

There are then 9 mass parameters as well as the 3×3 unitary matrix V_{ab} as free parameters in $\mathcal{L}_Y$. A general 3×3 unitary matrix can be parametrized by 9 real numbers. However, V_{ab} appears only in the charged interactions of quarks (interactions between quarks of different charge). We are always free to redefine the fields by a phase: $\psi \rightarrow e^{i\theta}\psi$. When we do this, the combination $\bar{u}_a d_b$ that enters the expression of the charged interactions changes by a phase $e^{i\theta_{ab}}$. There are 5 independent such phases and they can be used to remove 5 parameters from V_{ab}. Therefore, $\mathcal{L}_Y$ contains 13 free parameters; 4 in V_{ab} and 9 masses.

Note also that, as promised, the interactions of the Higgs and the fermions are proportional to the masses of the fermions. Moreover, the would-be Goldstone bosons ϕ and χ also couple to the mass of the fermions. Since these can be thought of as the longitudinal components of the W and Z bosons, we can expect that they too will behave as though they couple to the fermions with a strength proportional to their masses.

We can perform the field redefinitions 2.37 and 2.30 in the fermionic Lagrangian 2.25 as well. The result is:

$$\begin{aligned}
\mathcal{L}_{\text{FERMIONS}} &= \bar{\nu}_a i\partial\!\!\!/ \nu_a + \bar{e}_a i\partial\!\!\!/ e_a + \bar{u}_a i\partial\!\!\!/ u_a + \bar{d}_a i\partial\!\!\!/ d_a \\
&+\frac{g}{2c}\bar{\nu}_a Z\!\!\!/ L\nu_a + \frac{g}{2c}\bar{e}_a Z\!\!\!/ \left((s^2-c^2)L + s^2 R\right) e_a - e\,\bar{e}_a A\!\!\!/ e_a \\
&+\frac{g}{6c}\bar{u}_a Z\!\!\!/ \left((3c^2-s^2)L - 4s^2 R\right) u_a + \frac{2e}{3}\bar{u}_a A\!\!\!/ u_a \\
&+\frac{g}{6c}\bar{d}_a Z\!\!\!/ \left(-(3c^2+s^2)L + 2s^2 R\right) d_a - \frac{e}{3}\bar{d}_a A\!\!\!/ d_a \\
&+\frac{g}{\sqrt{2}}\left(\bar{\nu}_a W\!\!\!\!/^{+} L e_a + \bar{u}_a V_{ab} W\!\!\!\!/^{+} L d_a + \text{h.c.}\right) \qquad (2.40)
\end{aligned}$$

2.2.5 Gauge-fixing

The parts of the Lagrangian involving the gauge fields (eq. 2.35 and 2.34) are plagued with problems. One of them, the existence of cross-terms $W_\mu^+\partial^\mu\phi^-$, has been mentionned before. Another becomes apparent when we study the expression 2.3 for the Green functions. Since the action S is invariant under a gauge transformation, all the field configurations related by a gauge transformation will contribute the same value to the Green functions. This produces infinities in the path integral. We will now show how to factor out these infinities, using a method originally due to Fadeev and Popov. This will, at the same time, cure the cross-term problem.

We consider the expression of the generating functional

$$Z[j] = \int [d\phi] \, \exp\{iS[\phi] + iS_{\text{source}}\} \tag{2.41}$$

where ϕ now stands for all the fields in the theory. We insert the identity

$$\int [d\theta] \, \Delta_f[B] \delta\left[f_a(B_i^\theta) - C^a(x)\right] = 1 \tag{2.42}$$

in 2.41. Here, B_i^θ stands for the "gauge transformed" gauge fields ($\mathbf{W}$ and B) as in 2.16 and 2.20 and f_a and C_a ($a = 1, 2, 3, 4$) are arbitrary functions (C_a is independent of θ). The delta function appearing in 2.42 is a functional delta function i.e. a product of ordinary delta functions at every space-time point. Its purpose is to "fix" the gauge by ensuring that only those field configurations satisfying the constraints $f_a(B_i^\theta) - C_a(x) = 0$ contribute to the integral. Finally, Δ_f is a functional determinant

$$\Delta_f[B] = \det\left[\frac{\delta f_a}{\delta \theta_b}\right] \tag{2.43}$$

It is simply the Jacobian of the transformation when we pass from B^θ to f_a. The generating functional reads:

$$Z[j] = \int [d\theta] \int [d\phi] \, \Delta_f[B] \delta\left[f_a(B_i^\theta) - C_a(x)\right] \exp\{iS[\phi] + iS_{\text{source}}\} \tag{2.44}$$

A remarkable fact, which we shall not prove, is that, inasmuch as we are interested only in gauge-invariant Green functions, the integrand of 2.44 is independent of θ. (A proof is given in [5].) We can then replace B_i^θ by B_i and drop the $[d\theta]$ integral (it cancels in the Green functions). The result is:

$$Z[j] = \int [d\phi] \, \Delta_f[B] \delta(f_a(B_i) - C_a(x)) \exp\{iS[\phi] + iS_{\text{source}}\} \tag{2.45}$$

We can now write the δ function in 2.45 in exponential form by multiplying Z by a constant

$$\int [dC] \exp\left\{-\frac{i}{2\xi}\int d^4x\, C_a(x) C^a(x)\right\}$$

where ξ is an arbitrary parameter known as the gauge parameter. Performing the C integration, we find:

$$Z[j] = \int [d\phi]\, \Delta_f[B] \exp\left\{iS[\phi] + iS_{\text{source}} + iS_{GF}\right\} \tag{2.46}$$

where $S_{GF} = \int d^4x\, \mathcal{L}_{GF}$ and the *gauge-fixing Lagrangian* $\mathcal{L}_{GF}$ is

$$\mathcal{L}_{GF} = -\frac{1}{2\xi} f^a(B_i) f_a(B_i)$$

With the choice of function

$$\begin{aligned} f_i &= \partial_\mu W_i^\mu + ig\xi\left(\phi'^\dagger \frac{\tau_i}{2}\phi_0 - \phi_0^\dagger \frac{\tau_i}{2}\phi'\right) \ (i = 1,2,3) \\ f_4 &= \partial_\mu B^\mu + ig'\frac{\xi}{2}\left(\phi'^\dagger \phi_0 - \phi_o^\dagger \phi'\right) \end{aligned} \tag{2.47}$$

the gauge-fixing Lagrangian becomes:

$$\begin{aligned} \mathcal{L}_{GF} = & -\frac{1}{2\xi}\left[2(\partial^\nu W_\nu^+)(\partial^\mu W_\mu^-) + (\partial^\nu A_\nu)(\partial^\mu A_\mu) + (\partial^\nu Z_\nu)(\partial^\mu Z_\mu)\right] \\ & -iM_W(\phi^- \partial^\nu W_\nu^+ - \phi^+ \partial^\nu W_\nu^-) \\ & -M_Z \chi \partial^\nu Z_\nu - \xi M_W^2 \phi^+ \phi^- - \xi M_Z^2 \chi^2/2 \end{aligned} \tag{2.48}$$

Three points are noteworthy about $\mathcal{L}_{GF}$

- The cross-terms in $\mathcal{L}_{GF}$ cancel precisely those in $\mathcal{L}_{\text{HIGGS}}$ (eq. 2.34). The functions F_i were chosen to accomplish this.
- The would-be Goldstone bosons ϕ and χ acquire the mass $\sqrt{\xi} M_W$ and $\sqrt{\xi} M_Z$ respectively.

- There are quadratic terms in the gauge-fields which complement those in $\mathcal{L}_{\text{GAUGE}}$ (eq. 2.35).

We can now construct the free part of the gauge boson Lagrangian. Consider the Z boson for example. Collecting all quadratic terms in Z, we find:

$$S_{\text{FREE}} = \frac{1}{2}\int d^4x\, Z_\mu \left\{ \Box g^{\mu\nu} - (1-\frac{1}{\xi})\partial^\mu\partial^\nu + M_Z^2 g^{\mu\nu} \right\} Z_\nu$$

which gives the propagator

$$G_Z(x,y) = \int \frac{d^4p}{(2\pi)^4}\frac{e^{ip\cdot(x-y)}}{p^2-M_Z^2}\left\{g^{\mu\nu} + (\xi-1)p^\mu p^\nu/(p^2-\xi M_Z^2)\right\}$$

There is one remaining factor in the generating functional 2.46, the Jacobian $\Delta_f(B)$. It too exponentiates thanks to the following remarkable property of integration over anti-commuting variables

$$\int [dc][dc^\dagger]\, e^{c^\dagger A c} = \det A.$$

We will skip the details here; they can be found in Appendix B of [3]. Suffice to say that 4 complex anti-commuting fields $\omega^\pm$, ω_Z, ω_A have to be introduced. (Note that$(\omega^+)^\dagger \neq \omega^-$ since the ω are complex.)

These fields are called ghosts and, like the would-be Goldstone bosons, they cannot be found in the initial and final states. The Lagrangian that one constructs with this exponentiation is called the *Fadeev-Popov Lagrangian* $\mathcal{L}_{FP}$. It depends on the gauge parameter ξ. However, this parameter is totally arbitrary and, therefore, unphysical. No observables can depend on ξ. Two commonly used values of this parameter are: $\xi = 0$, known as the Landau gauge and the Feynman gauge $\xi = 1$. We will use the latter in this work.

The total Lagrangian of the Standard Model is given by the sum of 2.34, 2.35, 2.39, 2.40, 2.48 and $\mathcal{L}_{FP}$. We can then follow the procedure of section 1 to extract the Feynman rules of the theory. These are given in Appendix B.

2.3 A word about the strong interactions

The strong interactions are introduced in the model in a way that continues the pattern of section 2.2.1. Every flavor of quark is assumed to come in 3 colors. They are grouped in triplets, say

$$\psi = \begin{pmatrix} \psi_a \\ \psi_b \\ \psi_c \end{pmatrix}$$

where a, b, c are the color labels. The free Lagrangian $\bar{\psi}(i\partial\!\!\!/ - m)\psi$ is invariant under a SU_3 transformation (3×3 unitary matrices of determinant 1). Once more, we make this symmetry local by the introduction of 8 gauge-fields: the *gluons*. The gluons do not couple to the Higgs and remain massless. This theory is called Quantum Chromo-Dynamics (QCD). We will not go in the details here but simply mention some of the model's most important properties.

First, QCD is assumed to lead to *confinement*. This means that the external states admissible by the theory must be invariant under SU_3. No free particle can be found that carries a net color. This implies in particular that the quarks themselves cannot occur as free particles: they are permanently confined inside bound states. The bound states must also have a special structure. For instance, the B_d meson has the quark content $\bar{d}^a b_a$ where a is the color index and a sum is implied over a. Only this combination is invariant under SU_3.

Confinement is , so far, only a conjectured property of QCD; it hasn't been proved. *Asymptotic freedom* is an interesting property of QCD that <u>has</u> been proved. In an asymptotically free theory, the coupling can be considered to decrease with energy. Now, a coupling is a fixed number and cannot depend on energy so this statement is at best ambiguous. To define it more precisely, we have to consider the gluon-quark-antiquark

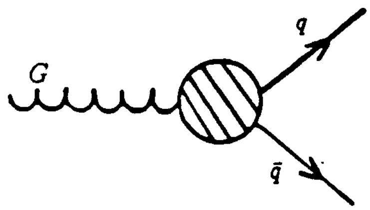


Figure 2.5: The gluon-quark-antiquark connected Green function.The blob represent all possible insertions of quarks and gluon lines

connected Green function (fig. 2.5). In lowest order of perturbation theory, this is $ig_s\gamma^\mu\lambda^A$ where g_s is the strong coupling and λ^A is a Gell-Mann matrix. If we calculate this function to all orders in perturbation theory and keep only the largest terms in the limit that the energy of the gluon becomes large, the perturbation series sums to $ig(E)\gamma^\mu\lambda^A$ which is of the same form as before but with an effective coupling that depends on energy. It is this effective coupling that decreases with energy.In practice, asymptotic freedom means that the strong interactions of quarks and gluons will not be so strong at high energy, and that perturbation theory can be used reliably in this case.

2.4 An important theorem

In chapter 5, we will be concerned with the calculation of Feynman diagrams when the mass of one of the particles involved becomes much larger than all the others. In those circumstances, we expect to find:

$$D \approx M^{\lambda(D)} \left(1 + \mathcal{O}(\frac{m}{M})\right)$$

where D is any diagram, M is the large mass and m any small mass. This should hold, up to logarithmic corrections in M. In this section, we will show how to obtain an upper bound on $\lambda(D)$ for any given graph D with at least one loop. We will not prove the result here. (See Collins [6] and references therein for a more detailed discussion)

We begin with some terminology. A line in the graph is said to be *massive* (*massless*) if it stands for the large mass particle (a small mass particle). We will restrict our considerations to graphs with massless external lines. The graph's internal lines are further divided into heavy lines and light lines. This classification is motivated by the fact that, for the purpose of power counting, the internal lines can be considered to carry either a large ($\approx M$) momentum or a small one ($\ll M$).*Heavy* (*light*) lines are the lines carrying large (small) momentum. Since we consider processes at small external momentum, conservation of momentum implies that heavy lines must form closed loops. Finally, the lines are either fermionic or bosonic according to whether they represent a fermion or a boson.

The contributions to $\lambda(D)$ are then as follows:

- $+4n$ where n is the number of independent loops formed by the heavy lines.
- -2 (-1) for a heavy bosonic (fermionic) line, irrespective of wether it is massive or massless.
- -2 (-1) for a light, massive bosonic (fermionic) line.
- 0 for a light, massless bosonic or fermionic line.

This is essentially power counting. Note that these contributions depend on which lines we choose to call heavy. There are, in general, several ways of doing this (subject only to the condition that the heavy lines must close on themselves). If we call $\lambda^c(D)$ the value obtained for one particular choice of heavy lines then $\lambda(D)$ is the maximum of $\lambda^c(D)$ over all possible such choices. We proceed with a pair of examples.

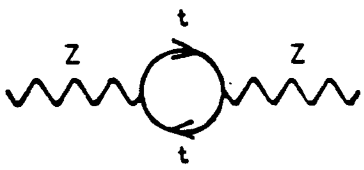


Figure 2.6: A 1-loop illustration of the power counting theorem: the top-quark contribution to the self-energy of the Z boson

The first example is the top-quark contribution to the self-energy of the Z boson at 1-loop (fig. 2.6). We want to obtain the leading behavior as $m_t \gg M_Z$. The two top-quark lines in the graph are heavy, massive fermionic lines. There is only one choice of heavy lines and the power counting gives:$+4-1-1=+2$. We then expect the graph to behave as m_t^2 for large m_t. The behavior found is $m_t^2 \ln m_t$ which is compatible with the theorem.

A more complex example can be taken from the $B_d - \bar{B}_d$ mixing at two loops. The basic graph is shown in fig. 2.7 where the lines b, c, d, and e are W bosons, line a is a Higgs boson and f and g are top-quarks. We want to extract the leading behavior in M_H as M_H gets large. Therefore, the only massive line is a. There are 4 possible choices of heavy lines (see fig 2.8). We give below the value of $\lambda^c(D)$ for each choice, listing in detail the contribution of every line.

Graph (a): lines a, b and d are heavy bosonic $\rightarrow -6$

line f is heavy fermionic $\rightarrow -1$

lines c, e, g are light, massless $\rightarrow 0$

Since there is one loop, $\lambda^{(a)} = -3$.

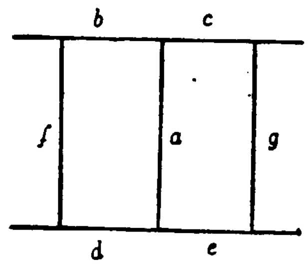


Figure 2.7: A 2-loop illustration of the power counting theorem: contribution to $B_d - \bar{B}_d$ mixing. The meaning of the labels is explained in the text.

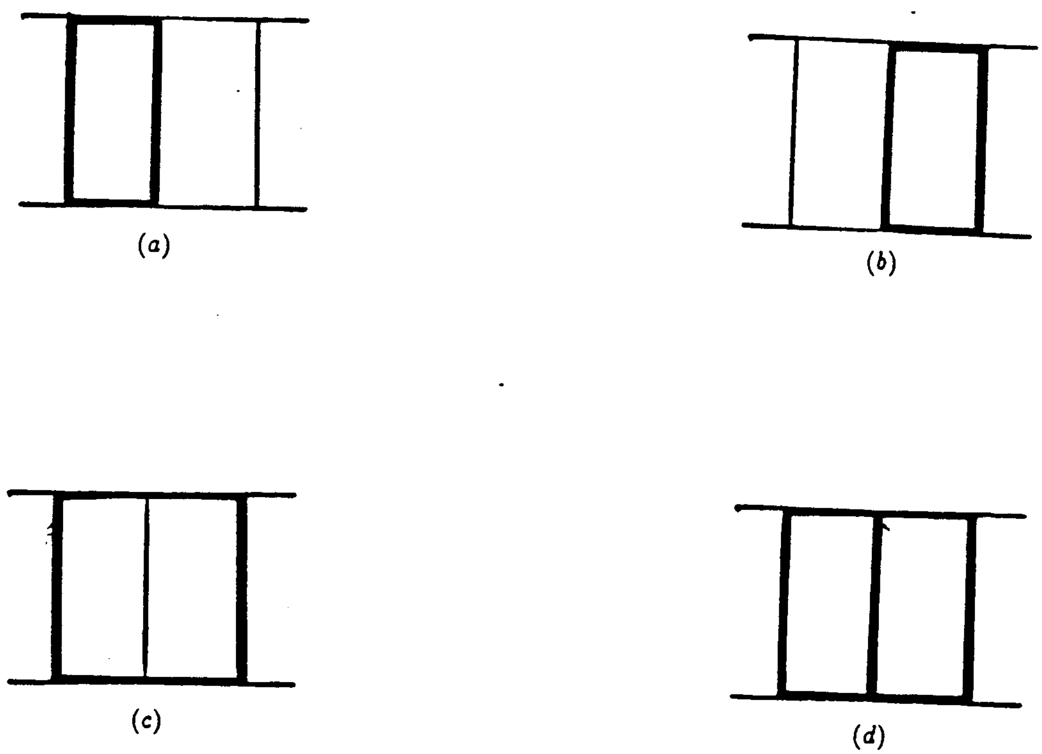


Figure 2.8: The 4 choices of heavy lines (indicated by thick lines) for the previous graph

Graph (b) is the same as graph (a).

Graph (c): lines b, c, d, e are heavy, bosonic $\rightarrow -8$

lines f, g are heavy fermionic $\rightarrow -2$

line a is light, massive, bosonic $\rightarrow -2$

With one loop, we get $\lambda^{(c)} = -8$.

Graph (d): lines a, b, c, d, e are heavy bosonic $\rightarrow -10$

lines f, g are heavy fermionic $\rightarrow -2$

There are two loops, so $\lambda^{(d)} = -4$

The maximum value is therefore -3. The actual behavior is $M_H^{-4}\ln^2(M_H)$. This illustrates an important point: the procedure described above yields only an upper bound; the actual behavior may be "softer". Generally, however, the exponent found with this method is the correct leading exponent.

This theorem will be used in Chapter 5 to determine which graphs are leading graphs.

Chapter 3

$B_d - \bar{B}_d$ mixing and radiative corrections

In this chapter, we will examine the physics of $B_d - \bar{B}_d$ mixing. We will begin in section 1 by the study of mixing of two stable states in quantum mechanics. In section 2, we will generalize this result to unstable states and move on to a discussion of $B_d - \bar{B}_d$ oscillations. The expected signature of the phenomenon will also be touched upon. We will then proceed with the one-loop calculation of the mixing parameters in section 3. We will restrict ourselves to calculations in the Standard Model. We conclude in section 4 with a brief summary of the status of two-loop calculations in electroweak theory.

3.1 A quantum mechanical example

It is well known that two-state quantum mechanical systems can exhibit oscillations in time between the two states. We will review this phenomenon here as it represents a simplified form of $B_d - \bar{B}_d$ mixing.

The most general Hamiltonian for a two-state system can be written, after a suitable rescaling

$$H = \begin{pmatrix} 1 & \epsilon/2 \\ \epsilon^*/2 & 1+\beta \end{pmatrix} \tag{3.49}$$

This Hamiltonian is expressed in the basis $|1\rangle$, $|2\rangle$ where

$$|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad , \quad |2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The transitions between $|1\rangle$ and $|2\rangle$ are induced by ϵ, the non-diagonal term of H. In practice, ϵ is a small perturbation and 1 and $1+\beta$ are the unperturbed energies. The parameter β is therefore a measure of the energy difference between the unperturbed states.

Suppose we know that the system is in the state $|1\rangle$ at $t=0$. We want to find the probability that the system be found in state $|2\rangle$ at an arbitrary time $t>0$. This is given by the square of the modulus of the transition amplitude T_{12}

$$T_{12} = \langle 2|e^{-iHt}|1\rangle$$

To calculate T_{12}, we introduce the eigenstates $|+\rangle$ and $|-\rangle$ of H with the corresponding eigenvalues E_+ and E_- and use completeness

$$T_{12} = e^{-iE_+t}\langle 2|+\rangle\langle +|1\rangle + e^{-iE_-t}\langle 2|-\rangle\langle -|1\rangle$$

The eigenvalues and eigenvectors can be found easily from 3.49. The result is remarkably simple

$$T_{12} = -ie^{-i(1+\beta/2)t}\left(\frac{\epsilon^*}{\sqrt{\beta^2+|\epsilon|^2}}\right)\sin\left(\sqrt{\beta^2+|\epsilon|^2}t\right)$$

The transition probability is

$$P_{12} = \frac{|\epsilon|^2}{\beta^2+|\epsilon|^2}\sin^2\left(\sqrt{\beta^2+|\epsilon|^2}t\right) \tag{3.50}$$

The time-averaged value takes a very simple form

$$\langle P_{12}\rangle = \frac{1}{2}\frac{|\epsilon|^2}{\beta^2+|\epsilon|^2}$$

From these expressions, it is apparent that the oscillations can be sizeable only if the energy difference β isn't much larger than the perturbation. This explains why the phenomenon isn't very common. In the case of two perfectly degenerate states ($\beta=0$), the time averaged transition probability reduces to 1/2, independently of the size of ϵ. The eigenvectors also become independent of ϵ in that limit

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|1\rangle \pm |2\rangle)$$

This particular case is important for the $B_d - \bar{B}_d$ mixing since B_d and $\bar{B}_d$ have exactly the same mass.

3.2 The $B_d - \bar{B}_d$ mixing

The $B_d - \bar{B}_d$ oscillations involve one feature that hasn't been discussed so far: the B mesons can decay. If the oscillations are to be observable, they must occur before the mesons decay. A look at formula 3.50 for the transition probability shows that this will be possible if the perturbation ϵ is larger than the width (or inverse life-time) Γ_B of the meson.

These notions can be made more precise by the introduction of an effective Hamiltonian [7] for the time evolution of the $B - \bar{B}$ system (we will drop the subscript d from now on). In the rest frame of the B

$$\boldsymbol{H}\begin{pmatrix} B \\ \bar{B} \end{pmatrix} = \left(\boldsymbol{M} - i\frac{\boldsymbol{\Gamma}}{2}\right)\begin{pmatrix} B \\ \bar{B} \end{pmatrix} = \begin{pmatrix} M - i\Gamma/2 & M_{12} - i\Gamma_{12}/2 \\ M_{12}^* - i\Gamma_{12}^*/2 & M - i\Gamma/2 \end{pmatrix}\begin{pmatrix} B \\ \bar{B} \end{pmatrix} \tag{3.51}$$

This "Hamiltonian" is not hermitian since probability isn't conserved in the $B - \bar{B}$ system as a result of the decays of B and $\bar{B}$. However, both $\boldsymbol{M}$ and $\boldsymbol{\Gamma}$ are hermitian but their eigenvectors need not be the same. Note also that the diagonal elements are equal. This is a consequence of the CPT symmetry. This symmetry, which is obeyed in all quantum field theories, interchanges final and initial states and replaces particles with anti-particles and all constants by their complex conjugate.

The eigenvectors of $\boldsymbol{H}$ can be written down in the form:

$$|B_{1,2}\rangle = \frac{(1+\epsilon)|B\rangle \pm (1-\epsilon)|\bar{B}\rangle}{\sqrt{2(1+|\epsilon|^2)}} \tag{3.52}$$

Here, ϵ (which should not be confused with the ϵ of the previous section) is given implicitly by

$$\eta \equiv \frac{1-\epsilon}{1+\epsilon} = \sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}} \tag{3.53}$$

It is a measure of CP-violation in the system. We will see later that it can be neglected in the $B_d - \bar{B}_d$ system.The corresponding eigenvalues are $M_{1,2} - i\Gamma_{1,2}/2$ where:

$$\begin{aligned} M_{1,2} &= M \pm \mathrm{Re}\, Q \\ \Gamma_{1,2} &= \Gamma \mp 2\mathrm{Im}\, Q \end{aligned} \tag{3.54}$$

with

$$Q = \sqrt{(M_{12} - i\Gamma_{12}/2)(M_{12}^* - i\Gamma_{12}^*/2)} \tag{3.55}$$

Suppose we start at $t = 0$ with a pure $|B\rangle$ state

$$|\psi(t=0)\rangle = |B\rangle = \frac{\sqrt{1+|\epsilon|^2}}{\sqrt{2}(1+\epsilon)}\left(|B_1\rangle + |B_2\rangle\right) \tag{3.56}$$

The time evolution of $|B_{1,2}\rangle$ is very simple since they are eigenstates of H. We get, at time t:

$$|\psi(t)\rangle = \frac{\sqrt{1+|\epsilon|^2}}{\sqrt{2}(1+\epsilon)}\left[e^{-i(M_1 - i\Gamma_1/2)t}|B_1\rangle + e^{-i(M_2 - i\Gamma_2/2)t}|B_2\rangle\right] \tag{3.57}$$

We can use eq. 3.52 to eliminate $|B_1\rangle$ and $|B_2\rangle$ and express $|\psi(t)\rangle$ as a superposition of $|B\rangle$ and $|\bar{B}\rangle$. The coefficients of this superposition are the transition amplitudes $A(B \to B)$ and $A(B \to \bar{B})$

$$\begin{aligned} A(B \to B) &= \frac{1}{2}\left[e^{-iM_1t}e^{-\Gamma_1 t/2} + e^{-iM_2t}e^{-\Gamma_2 t/2}\right] \\ A(B \to \bar{B}) &= \frac{1}{2}\left(\frac{1-\epsilon}{1+\epsilon}\right)\left[e^{-iM_1t}e^{-\Gamma_1 t/2} - e^{-iM_2t}e^{-\Gamma_2 t/2}\right] \end{aligned}$$

A useful quantity is the ratio r of time averaged probabilities:

$$r = \frac{\int_o^\infty dt\,|A(B \to \bar{B})|^2}{\int_0^\infty dt\,|A(B \to B)|^2} \tag{3.58}$$

A direct calculation gives

$$r = \left|\frac{1-\epsilon}{1+\epsilon}\right|^2 \frac{x^2+y^2}{2+x^2-y^2} \tag{3.59}$$

where $x = \Delta M/\Gamma$ and $y = \Delta\Gamma/2\Gamma$ with

$$\begin{aligned} \Delta M &= M_1 - M_2 \\ \Delta\Gamma &= \Gamma_1 - \Gamma_2 \end{aligned}$$

Note that

$$\begin{aligned} 0 &\leq x^2 \leq \infty \\ 0 &\leq y^2 = \left(\frac{\Gamma_1 - \Gamma_2}{\Gamma_1 + \Gamma_2}\right)^2 \leq 1 \end{aligned}$$

Then, neglecting CP-violation ($\epsilon = 0$), we have

$$0 \leq r \leq 1$$

Values of r close to 1 obtain in two cases:

1. $x \to \infty$. This can happen when ΔM is much greater than the width either because the particle is stable ($\Gamma \to 0$) or the perturbation ($M_{12} - i\Gamma_{12}/2$) is large.

2. $y \to 1$. This happens if one particle has a much greater lifetime than the other. This is the case for $K - \bar{K}$ mixing.

Similarly, we could obtain

$$\bar{r} = \left|\frac{1+\epsilon}{1-\epsilon}\right|^2 \frac{x^2+y^2}{2+x^2-y^2} \tag{3.60}$$

where

$$\bar{r} = \frac{\int_0^\infty dt\, |A(\bar{B} \to B)|^2}{\int_0^\infty dt\, |A(\bar{B} \to \bar{B})|^2}$$

Note that $r = \bar{r}$ in the absence of CP-violation. Therefore, the asymmetry

$$a = \frac{r - \bar{r}}{r + \bar{r}}$$

is a measure of CP-violation.

To make connection with the real world, we must specify how this phenomenon is to be observed. The standard way of doing this is to first produce a $b\bar{b}$ bound state, the $\Upsilon(4s)$ resonnance. This bound state then decays almost immediately to a $B_d - \bar{B}_d$ pair via the strong interactions. The B mesons then evolve in time according to the previously described formalism. Because of the oscillations, there are three different types of final states that can be observed: $B\bar{B}$, BB and $\bar{B}\bar{B}$. The $B\bar{B}$ final state includes the possibility of a "double flip" $B\bar{B} \to \bar{B}B$. As will be seen later, these final states are identified by their decay properties.

Typically, r is measured through the ratio

$$R = \frac{N(BB) + N(\bar{B}\bar{B})}{N(B\bar{B}) + N(\bar{B}B)}$$

where $N(BB)$ is the number of BB final states observed etc. The final states $B\bar{B}$ and $\bar{B}B$ are undistinguishable and are kept separate only as a reminder of the possibility of a double flip.

A naive evaluation of R in terms of r and $\bar{r}$ gives

$$R = \frac{r + \bar{r}}{1 + r\bar{r}}$$

This result is correct when the B and $\bar{B}$ evolve independently. It must be modified when they form a coherent pair, as is the case at the $\Upsilon(4s)$ resonnance [8]. This bound state has $J = 1$ and $C = -1$. The $B_d - \bar{B}_d$ pair it decays into must have the same quantum numbers. This means that the produced state is

$$|B(k_1,t)\rangle|\bar{B}(k_2,t)\rangle - |\bar{B}(k_1,t)\rangle|B(k_2,t)\rangle$$

where, neglecting CP-violation:

$$\begin{aligned} |B(t)\rangle &= R(t)|B\rangle + C(t)|\bar{B}\rangle \\ |\bar{B}(t)\rangle &= C(t)|B\rangle + R(t)|\bar{B}\rangle \end{aligned}$$

The functions $R(t)$ and $C(t)$ are the amplitudes to "remain" or to "change", namely

$$\begin{aligned} R &= A(B \to B) = A(\bar{B} \to \bar{B}) \\ C &= A(B \to \bar{B}) = A(\bar{B} \to B) \end{aligned}$$

If the two decays take place at $t = t_1$ and $t = t_2$ respectively, the superposition becomes

$$(R_1C_2 - C_1R_2)\left(|BB\rangle - |\bar{B}\bar{B}\rangle\right) + (R_1R_2 - C_1C_2)\left(|B\bar{B}\rangle - |\bar{B}B\rangle\right)$$

where $R_1C_2 = R(t_1)C(t_2)$ etc. Therefore, we get:

$$R = \frac{\int dt_1\, dt_2\, |R_1C_2 - C_1R_2|^2}{\int dt_1\, dt_2\, |R_1R_2 - C_1C_2|^2}$$

A lengthy calculation gives $R = r$.

Finally, we will say a word about the identification of the final states. The B_d meson lifetime is very short ($\approx 10^{-12}$ seconds) [20]. Furthermore, the $B_d - \bar{B}_d$ pair is produced almost at rest. Therefore, the mesons will not travel at all and will decay at the collision point. They must be identified by their decay products.

The B_d meson is the lightest meson carrying the b flavor. Its decay must therefore involve a change of flavor and, consequently, the emission of a W boson. In the spectator

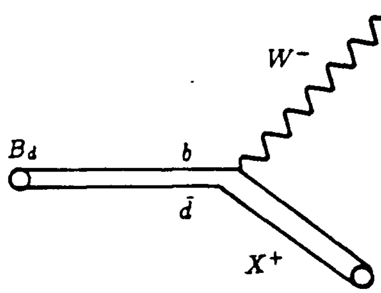


Figure 3.9: B_d meson decay in the spectator approximation. X^+ is usually the D^+ meson.

approach, the W is assumed to be emitted by the b quark with the $\bar{d}$ quark a "spectator" of the process (see fig. 3.9). This approach, which is reasonable since the b quark is much heavier than the d quark, is known to work very well. A consequence of this is that B_d will always decay through the emission of a W^- while $\bar{B}_d$ decays to a W^+. This is what allows us to differentiate between the two mesons.

The $W^{\pm}$ in these decays is necessarily virtual: there is not enough energy available to produce a real W. It is identified by its decay. The $W^-(W^+)$ decays roughly one third of the time into a charged lepton $l^-(l^+)$ and a neutrino $\bar{\nu}(\nu)$, the neutrino escaping detection. This mode is very specific to the W and allows for a clean identification. In short, the final states sought experimentally consist of two charged leptons and some hadrons, the leptons being both negatively charged for a BB state, both positively charged for a $\bar{B}\bar{B}$ state and carrying different charges for a $B\bar{B}$ state. The complete process can be represented schematically as in fig. 3.10 when no mixing occurs and as in fig. 3.11 for a case with mixing. In both cases, the charged leptons (either muon or electron) have to be observed. A great deal of care must be taken to distinguish these leptons from those coming from other sources (X and Y decays, photo-production, etc.). The reader is referred to [1] for the details.

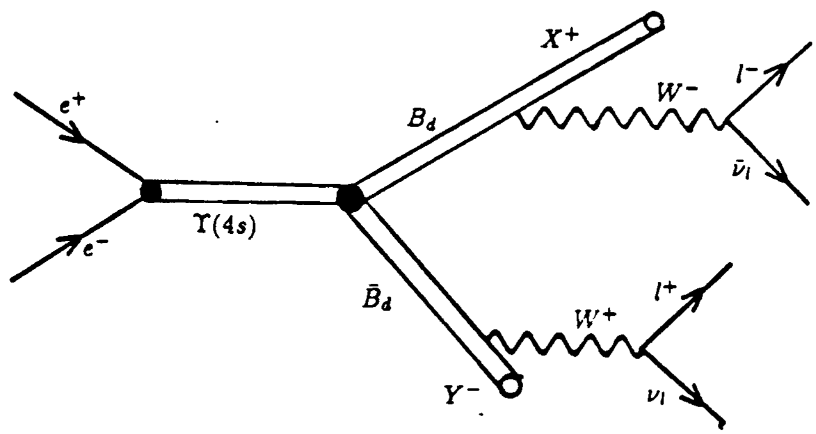


Figure 3.10: Production and decay of a $B_d - \bar{B}_d$ pair (without mixing). X and Y are charged mesons (usually $D^{\pm}$).

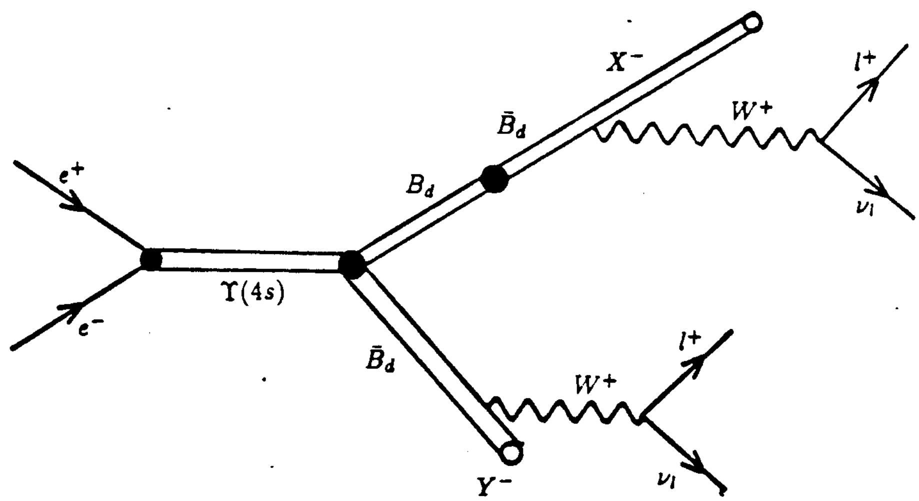


Figure 3.11: Production and decay of a $B_d - \bar{B}_d$ pair. The blob on the B_d line represents the mixing transition $B_d \to \bar{B}_d$. X and Y are as above.

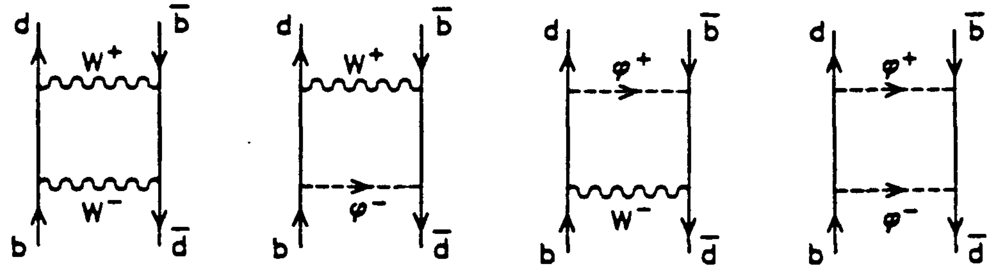


Figure 3.12: Lowest order contributions to M_{12}. The final state is at the top of the diagram and the initial state at the bottom.

3.3 M_{12} and the box-diagrams

The formalism of the previous section relates the mixing ratio r to a knowledge of the effective Hamiltonian of eq. 3.51. In this section, we will calculate the parameters M_{12} and Γ_{12} of this Hamiltonian to lowest order in perturbation theory. (The other parameters, M and Γ, are known experimentally). Chapters 4 and 5 are devoted to higher order calculations of the same parameters.

3.3.1 The box-diagrams

M_{12} and Γ_{12} cause transitions between the B_d and the $\bar{B}_d$ states. In the Standard Model, the lowest order diagrams thet contribute to M_{12} are shown in fig. 3.12. The quarks in the loop can be t, c or u but we will see later that the contribution of the top quark dominates. There is also the corresponding s-channel diagrams (fig. 3.13) but these can be found from the previous graphs via a generalized Fierz transformation.

Γ_{12} is the absorptive part of these diagrams. The contributions come from the region of loop momentum where one or more of the particles in the loop are on-shell, corresponding to a real decay of the B_d (or $\bar{B}_d$). However, as we will discuss in greater depth in Chapter 5, we will use the approximation that $m_b = 0$ in the loop integrals. This means immediately that $\Gamma_{12} = 0$ since a massless particle cannot decay. This result allows for a

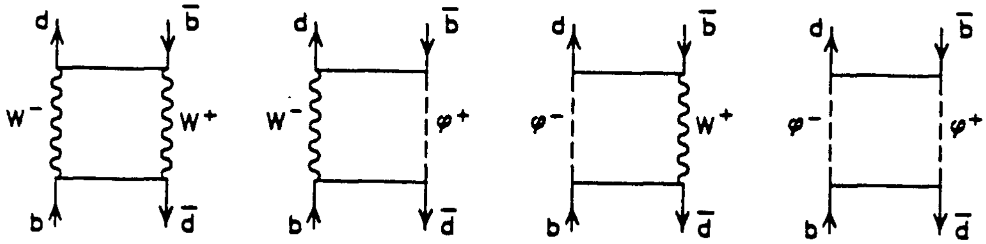


Figure 3.13: The s-channel diagrams

considerable simplification of the formulas for the mixing ratios r and $\bar{r}$ namely:

$$r = \bar{r} = \frac{x^2}{2 + x^2}$$

where $x = 2|M_{12}|/\Gamma$. Also, the CP-violation parameter ϵ becomes purely imaginary in this limit and can be neglected since it can be removed by a phase redefinition of the $|B_d\rangle$.

There are two quantities which influence the relative size of the contributions of the different internal quarks (u, c, t) to M_{12}: the mass of the quark and the KM matrix elements.

The exchange of an up-type quark between the b and $\bar{d}$ quarks involves the combination of KM matrix elements $V_{qd}^* V_{qb}$ where $q = u$, c or t. Using Wolfenstein's parametrization of the KM matrix (see eq. 4.89), we get:

$$\begin{aligned} V_{ud}^* V_{ub} &= A\lambda^3 \rho e^{i\phi} \\ V_{cd}^* V_{cb} &= -A\lambda^3 \\ V_{td}^* V_{tb} &= A\lambda^3 (1 - \rho e^{-i\phi}) \end{aligned}$$

Since the size of these expressions is mainly governed by λ, we see that all three of them are of the same order of magnitude.

The dominant factor determining the relative size of the contribution of the various quarks is their mass. The sum of the graphs of fig. 3.12 behaves as m_q^2/M_W^2 for a quark mass $m_q \gg M_W$ and as a constant if $m_q \ll M_W$. We can factor out the dependence on the KM matrix elements to get

$$\text{Box} = \sum_{i,j} \alpha_i \alpha_j E(m_i, m_j) \tag{3.61}$$

where $\alpha_i = V_{id}^* V_{ib}$ and the sum extends over u, c and t. We can take $m_u = m_c = 0$ in 3.61. Using the unitarity of the KM matrix ($\sum_i \alpha_i = 0$), we find:

$$\text{Box} = \alpha_t^2 \left(E(m_t, m_t) - 2E(m_t, 0) + E(0,0) \right)$$

where $E(0, m_t) = E(m_t, 0)$ has been used. Only the first term contributes to the leading m_t^2 behavior. Using the methods of Appendix F, we find:

$$\text{Box} = -\frac{ig^4}{32\pi^2} \left(\frac{m_t^2}{M_W^4} \right) f(x) \eta_1 \tag{3.62}$$

where

$$\begin{aligned} \eta_1 &= \bar{v}_d \gamma_\mu L v_b \bar{u}_d \gamma^\mu L u_b \\ f(x) &= \frac{1}{4(1-x)^3}(1 - 12x + 27x^2 - 4x^3 - 6x \ln x) \end{aligned}$$

with $x = M_W^2/m_t^2$. The function $f(x)$ increases monotonically from $1/4$ at $x = 0$ to 1 as $x \to \infty$. It has therefore a very weak dependence on m_t.

The diagrams of fig. 3.13 sum to the value 3.62 with the replacement

$$\eta_1 \to \eta'_1 = \bar{v}_d \gamma_\mu L u_b \bar{u}_d \gamma^\mu L v_b$$

As mentioned earlier, η_1 and η'_1 can be related.

Referring to figs 3.12 and 3.13, we attach a subscript i (f) to spinors in the initial (final) state. We get:

$$\begin{aligned} \eta_1 &= \bar{v}_{di} \gamma_\mu L v_{bf} \bar{u}_{df} \gamma^\mu L u_{bi} \\ \eta'_1 &= \bar{v}_{di} \gamma_\mu L u_{bi} \bar{u}_{df} \gamma^\mu L v_{bf} \end{aligned}$$

We see that η'_1 is a product of two quantities involving only the initial or the final state while η_1 involves "mixed" terms. It is convenient to disentangle η_1 by a Fierz transformation.

If we write the color indices and the spin structure explicitly in η_1, we find:

$$\eta_1 = \delta_{ae}\delta_{cf}(\gamma^\mu L)_{\alpha\beta}(\gamma_\mu L)_{\rho\tau}\left(\bar{v}_d^{a\alpha} v_b^{e\beta} \bar{u}_d^{c\rho} u_b^{f\tau}\right)$$

A well known Fierz transformation gives

$$(\gamma^\mu L)_{\alpha\beta}(\gamma_\mu L)_{\rho\tau} = -(\gamma^\mu L)_{\alpha\tau}(\gamma_\mu L)_{\rho\beta} \tag{3.63}$$

We can also transform the color term. Writing

$$\delta_{ae}\delta_{cf} = A\delta_{af}\delta_{ec} + h_{AB}\lambda^A_{af}\lambda^B_{ec}$$

where the λ's are the Gell-Mann matrices, we find $A = 1/3$ and $h_{AB} = 2\delta_{AB}$. The identities

$$\begin{aligned} \mathrm{Tr}\lambda^A &= 0 \\ \mathrm{Tr}\lambda^A\lambda^B &= \frac{1}{2}\delta^{AB} \end{aligned}$$

have been used to derive this result. Putting it all together:

$$\eta_1 = \frac{1}{3}\eta'_1 + 2\bar{v}_d\lambda^A\gamma^\mu L u_b \bar{u}_d \lambda_A \gamma_\mu L v_b$$

where the $-$ sign of 3.63 is cancelled by the permutation of the anticommuting spinors.

3.3.2 The strong interaction corrections

There are two types of strong interaction corrections to this result. First, there are the short range corrections due to the exchange of gluons between the quarks in the box-diagrams. These can be computed in perturbative QCD. These calculations have been done only in the unrealistic limit $m_t \ll M_W$. We will not consider them further here.

Secondly, it is necessary to correct for the fact that the results of the previous section apply to free quarks while in reality they are confined inside the mesons. To this effect, we must first construct an *effective Hamiltonian* H_{eff} that is, a Hamiltonian (or equivalently an interaction Lagrangian $\mathcal{L}_{\text{I}} = -H_{\text{eff}}$) which reproduces 3.62 for the transition $b\bar{d} \to \bar{b}d$ when evaluated in first order in perturbation theory. This is done by replacing the spinors in 3.62 by the corresponding quark operators and multiplying the result by $i/2$ where the 1/2 is a symmetry factor. We get:

$$H_{\text{eff}} = \frac{g^4}{32\pi^2}\left(\frac{m_t^2}{M_W^4}\right) f(x) \left(\frac{1}{2}\right) \left\{\frac{4}{3}(\bar{d}\gamma_\mu Lb)^2 + 2(\bar{d}\lambda_A\gamma_\mu Lb)^2\right\}$$

To account for the strong interactions, we must take the matrix element of H_{eff} between a B_d and a $\bar{B}_d$ state instead of between free quarks:

$$M_{12} = \langle B_d|H_{\text{eff}}|\bar{B}_d\rangle \tag{3.64}$$

The matrix element M_{12} must be evaluated in a non-perturbative model of the strong interactions, for example a QCD lattice calculation. An approach which is often discussed in the litterature is the vacuum saturation approximation. The matrix element of an axial current between a B_d and the vacuum is defined by

$$\langle 0|\bar{d}\gamma_\mu\gamma_5 b|\bar{B}_d\rangle = \frac{ip_\mu}{\sqrt{2E_p}} f_B \tag{3.65}$$

where p_μ is the 4-momentum of the meson and E_p the corresponding energy. The constant f_B could, in principle, be determined from a study of the leptonic decay modes of B_d. The matrix element of the vector current can be neglected since B_d is (mainly) a pseudoscalar.

The calculation then proceeds with the insertion of a complete set of states in H_{eff} i.e.

$$\langle B_d|(\bar{d}\gamma_\mu Lb)^2|\bar{B}_d\rangle = \sum_n \langle B_d|\bar{d}\gamma^\mu Lb|n\rangle\langle n|\bar{d}\gamma_\mu Lb|\bar{B}_d\rangle$$

The approximation consists in neglecting all states but the vacuum in the sum. Under those conditions, we get, using eq. 3.65

$$\langle B_d|(\bar{d}\gamma_\mu Lb)^2|\bar{B}_d\rangle = \frac{1}{4}f_B^2 m_B$$

in the rest frame of the meson. Note that in this approximation, the octet term $(\bar{d}\lambda^a\gamma^\mu Lb)^2$ doesn't contribute at all since both the vacuum and B_d are color singlets.

Putting it all together, we get:

$$M_{12} = \frac{g^4}{192\pi^2}\left(\frac{m_t^2}{M_W^4}\right) f(x) f_B^2 B_B m_B$$

where the factor B_B is inserted to take possible deviations from the vacuum saturation approximation into account. The value of $f_B^2 B_B$ is subject to rather large uncertainties and must be calculated in an other model (typically lattice QCD) or taken from experiments involving B meson decays. This is done in the case of kaons.

3.4 Status of two-loop calculations

The physics of the gauge-bosons has received a lot of attention lately as a result of the LEP I experiments [17]. It is therefore no surprise that the only other calculation at the two-loop level in electroweak theory is that of the ρ-parameter which is defined by:

$$\rho = \left(\frac{M_Z \cos\theta_W}{M_W}\right)_{\text{exp}}$$

Unfortunately, many different definitions of the experimental quantities $M_{Z\text{exp}}$, $M_{W\text{exp}}$ and $(\cos\theta_W)_{\text{exp}}$ have been used in the literature [9], yielding different ρ-parameters. Furthermore, even when two authors agree on their definition of ρ, their result may differ because of the use of different renormalization schemes. We will not attempt to give a precise definition of ρ here. We will instead give some of its generic properties that have been found to be independent of the specific definition.

First, every reasonable definition of the experimental quantities is such that they reduce to the corresponding renormalized parameters at tree level (e.g. $M_{Z\text{exp}} = M_Z$). Since the relation $M_Z \cos\theta_W = M_W$ holds between the renormalized parameters (in any renormalization scheme), we obtain $\rho = 1$ at tree level.

When we consider loop corrections, the calculated value of ρ can be expressed in the form

$$\rho = 1 + \alpha\delta\rho_1 + \alpha^2\delta\rho_2 + \mathcal{O}(\alpha^3)$$

We are interested in the dependence of $\delta\rho_1$ and $\delta\rho_2$ on M_H and m_t. The 1-loop coefficient $\delta\rho_1$ depends quadratically on m_t (for large m_t) [10]. This is what is expected by power counting. (It is in contrast, however, to the situation in unbroken gauge-theories where the dependence of every low-energy quantity on m_t has to vanish as $m_t \to \infty$ [12]. However, it depends only logarithmically on M_H [11]. The M_H^2 terms cancel when summing over all graphs. This effect, which generalizes to all other low-energy observables in the Standard Model, has been called screening by Veltman: the effects of the Higgs are effectively "screened" from low-energy physics. These properties of $\delta\rho_1$ are independent of the renormalization scheme used to calculate it.

The two-loop corrections $\delta\rho_2$ have been calculated by Van der Bij and Hoogeven [13] for large m_t and by Van der Bij and Veltman [14] for large M_H. In the case of large m_t, a quartic dependence was found, which is again what is expected by power counting. In the large M_H case, the behaviour found was M_H^2. Again, the behavior is softer than what would be expected by power counting. The coefficients of the M_H^2 and m_t^4 terms depend strongly on the renormalization scheme. Note however that the authors of ref. [14] neglected the $t\bar{t}H$ coupling which cannot be neglected anymore. To our knowledge, this thesis is the first calculation that studies the limit when both m_t and M_H are large. This will add to our understanding of the "decoupling theorem" [12] in field theory.

As will be seen in Chapter 5, the leading behavior found for the $B_d - \bar{B}_d$ mixing amplitude at two loops is m_t^4 for large m_t and $a \ln^2 M_H + b \ln M_H$ for large M_H. This explicit dependence has not been obtained before.

Chapter 4

Renormalization

4.1 Introduction

In Chapter 2, we have described how to obtain the S-matrix from the Lagrangian by a combination of functional differentiation and integration. However, when we try to carry out this program, we encounter some difficulties: some of the integrals that occur are divergent. The task of renormalization in field theory is to make sense of these divergent expressions.

The type of divergences encountered depends on the method of approximation used in the calculations. In pertubation theory, divergences occur in Feynman graphs with one or more closed loops. An example is a bosonic self energy (fig. 4.1). The result is proportional to

$$\int d^4p \, \frac{1}{(p^2 - M_1^2)\,((p-k)^2 - M_2^2)} \tag{4.66}$$

which behaves as $\int \frac{d^4p}{p^4}$ for large p. This is therefore logarithmically divergent. In order

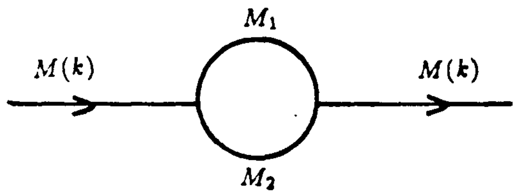


Figure 4.14: Self-energy of a boson ϕ of momentum k and mass M. The bosons in the loop have mass M_1 and M_2.

to ascribe a value to such integrals, we have to find a systematic procedure that replaces them by finite integrals. This is called *regularization.* One possibility is to introduce a cutoff in 4.66: after the angular integrations are performed, the radial integral is taken to extend to Λ instead of infinity. The integral is then a well defined function of k, M_1, M_2 and Λ that becomes infinite when $\Lambda \to \infty$. Another possibility follows from the observation that the integral 4.66 would be well defined in, say, 3 dimensions. This is generalized by the replacement of every 4-dimensional integral by a n-dimensional one. The integrals then become functions of n that go to infinity as $n \to 4$. This is the method used in this work. It is called dimensional regularization [15] and will be discussed in more details in the next section.

It appears, therefore, that we have to introduce extra parameters in the theory in order to regularize the integrals. These parameters (Λ, $n \ldots$) are called *regulators.* The various integrals that appear in the calculation of observables become infinite when the regulators take their *physical value* ($\Lambda = \infty$, $n = 4 \ldots$). In the remainder of this section, we will consider that there is only one regulator, ϵ, of physical value 0. The expressions "is finite" and "is independent of ϵ" will be taken to mean "remains finite as $\epsilon \to 0$" and conversely for "infinite" and "depends on ϵ".

It could be hoped at this stage that the S-matrix elements (calculated via 2.2) could be independent of ϵ since they are, after all, complicated combinations of integrals and the ϵ dependence could cancel among these. This is not the case. However, for a certain class of theories called renormalizable, it is possible to circumvent this problem.

The starting point is the observation that the original parameters of the Lagrangian (called *bare parameters*) have no a priori numerical values. We are therefore free to redefine them in any way we please. It is convenient to define the bare parameters (say

x_0^i) as functions of a set of *renormalized parameters* x^i and of ϵ.

$$x_0^i = f^i(x^j, \epsilon) \tag{4.67}$$

In a renormalizable theory such as the Standard Model, it is possible to choose the ϵ dependence so as to cancel the divergences in the S-matrix.

The above procedure is not sufficient to make the Green functions finite. To this end, one must carry out wave-function renormalizations. This uses the fact that the normalization of fields is physically irrelevant (much like the normalization of states in quantum mechanics). The fields of the original Lagrangian (now called *bare fields* ψ_0) can then be rescaled as follows

$$\psi_0 \to Z_\psi^{1/2}\psi \tag{4.68}$$

Here, ψ is *the renormalized field* and Z_ψ, called the wave-function renormalization constant, depends on ϵ in such a way that it cancels the infinities of the Green functions.

The substitution of 4.67 and 4.68 in the original Lagrangian (now rechristened *bare Lagrangian*) $\mathcal{L}_0(\psi_0, x_0^i)$ gives the renormalized Lagrangian $\mathcal{L}(\psi, x)$

$$\mathcal{L}(\psi, x) = \mathcal{L}_0(Z_\psi^{1/2}\psi, f^i(x^j, \epsilon))$$

It is also convenient to define the counterterm Lagrangian $\mathcal{L}_c$ by:

$$\mathcal{L}_c = \mathcal{L}(\psi, x) - \mathcal{L}_0(\psi, x)$$

Section 3 will be devoted to the construction of $\mathcal{L}_c$.

4.2 Dimensional regularization

Our first task is to make sense of the various divergent integrals we encounter in the calculations. We will do so through the use of dimensional regularization i.e. the integrals will be considered to be n-dimensional rather than 4-dimensional. Divergent integrals will then be replaced by functions of n that are singular as $n \to 4$.

4.2.1 Field and coupling dimension

When we change the dimension of integrals, we must also change the dimension of the fields and the couplings in order to keep the action S dimensionless. Consider for instance the action for a free fermion. In n-dimension, it reads

$$S_{\mathrm{FREE}} = \int d^n x \, \left\{ \bar{\psi}(x)(i\partial\!\!\!/ - m)\psi(x) \right\}$$

In order for S_{FREE} to be dimensionless, $\psi(x)$ must have mass dimension $\left(\frac{n-1}{2}\right)$. This holds for any fermion. Similar considerations yield the mass dimension $\left(\frac{n-2}{2}\right)$ for bosons.

A typical interaction term between two fermions and a boson ϕ is of the form $S_{\mathrm{INT}} = g \int d^n x \bar{\psi}(x)\psi(x)\phi(x)$. In 4 dimensions, g is dimensionless. In n dimensions, we obtain:

$$\begin{aligned} \dim[S_{\mathrm{INT}}] = 0 &= \dim[g] - n + 2\left(\frac{n-1}{2}\right) + \left(\frac{n-2}{2}\right) \\ &= \dim[g] + \left(\frac{n-4}{2}\right) \end{aligned}$$

Therefore, the coupling has dimension $(4-n)/2$. It is convenient to replace this dimensionful coupling by a dimensionless one via:

$$g \rightarrow g\mu^{(4-n)/2} \tag{4.69}$$

where μ has dimension of mass. With this substitution, the couplings in the theory are all dimensionless.

The parameter μ appearing in 4.69 is totally arbitrary: it cancels out of all physical quantities. Throughout most of this work, we will take it equal to 1.

4.2.2 Integration in n-dimension

Integration in n-dimension poses no problem of definition if n is an integer. Here, however, we want to be able to take the limit of our result as $n \rightarrow 4$. Hence, our results must be defined for all complex values of n. We will not attempt to give a precise definition here; the interested reader is referred to the excellent text of Collins [6] for a thorough exposition. We will only give the properties of n-dimensional integration that are relevant to our calculations.

1. Linearity: for any complex numbers a and b

$$\int d^n p \, [a\, f(p) + b\, g(p)] = a \int d^n p\, f(p) + b \int d^n p\, g(p)$$

2. Scaling: for any number s

$$\int d^n p\, f(sp) = s^{-n} \int d^n p\, f(p)$$

3. Translation invariance: for any vector k (independent of p)

$$\int d^n p\, f(p+k) = \int d^n p\, f(p)$$

4. Differentiation:

$$\frac{\partial}{\partial k^\mu} \int d^n p\, f(p,k,\ldots) = \int d^n p\, \frac{\partial}{\partial k^\mu} f(p,k,\ldots)$$

5. Partial integration:

$$\int d^n p\, \partial f(p)/\partial p^\mu = 0$$

The first 4 properties are straightforward generalizations of ordinary integration. The last is unusual in that the integral of a derivative is always zero, irrespective of the surface term. Finally,

6. Multiple integrals are independent of the order of integration:

$$\int d^n p \int d^n q\, f(p,q) = \int d^n q \int d^n p\, f(p,q)$$

This is the same as for ordinary integrals. However, multiple integrals with different dimensions n and n' can be interchanged only if their integrand is independent of $p \cdot q$.

Appendix C contains the explicit values of the integrals needed in this work.

4.2.3 Metric and Dirac matrices

The metric $g^{\mu\nu}$ is defined so that

$$g^{\mu\nu} g_{\mu\nu} = n$$

(an explicit definition of $g^{\mu\nu}$ is given in Collins [6]). The usual relations hold:

$$a^\mu b_\mu = a_\mu b^\mu = g^{\mu\nu} a_\mu b_\nu = g_{\mu\nu} a^\mu b^\nu = a \cdot b$$

Also,

$$\int d^n p\, p^\mu p^\nu g(p^2) = \frac{1}{n} \int d^n p\, p^2 g(p^2)$$

The Dirac matrices satisfy the usual equation

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \tag{4.70}$$

This implies in particular

$$\begin{aligned}
\gamma^\mu \gamma_\mu &= n \\
\gamma^\mu \gamma^\alpha \gamma_\mu &= (2-n)\gamma^\alpha \\
\gamma^\mu \gamma^\alpha \gamma^\beta \gamma_\nu &= 4g^{\alpha\beta} + (n-4)\gamma^\alpha \gamma^\beta
\end{aligned}$$

Traces can be evaluated by using $Tr(1) = 4$ and the fact that the trace of an odd number of γ matrices is zero.

The definition of the matrix γ_5 poses serious difficulties. In 4-dimensions, γ_5 is defined by (see Appendix A)

$$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3 = -\frac{i}{4}\epsilon^{\alpha\beta\lambda\sigma}\gamma_\alpha\gamma_\beta\gamma_\lambda\gamma_\sigma$$

The Levi-Civita tensor $\epsilon^{\alpha\beta\lambda\sigma}$ and γ_5 are intrinsically 4-dimensional objects. The problems that occur when we try to generalize them to n dimensions become apparent when we consider the quantity

$$T = Tr(\gamma^\alpha \gamma^\beta \gamma^\lambda \gamma^\sigma \gamma_5) \tag{4.71}$$

In 4-dimensions, $T = i\epsilon^{\alpha\beta\lambda\sigma}$. In general, if we insert the identity $\gamma^\mu\gamma_\mu = n$ in 4.71 and use the cyclicity of the trace, we get $(n-4)T = 0$. If this is to hold for every value of n, we must have $T = 0$.

In practice, the situation is not as bad as it may seem. Fermion lines either form closed loops or are ultimately attached to external states (this follows from conservation of angular momentum). For loops, it is sufficient to take

$$\begin{aligned} \{\gamma^\mu, \gamma_5\} &= 0 \\ Tr\gamma_5 &= 0 \\ Tr\gamma^\mu\gamma^\nu\gamma_5 &= 0 \end{aligned} \tag{4.72}$$

along with 4.70. This is known to work for up to two-loops [16]. In other words, in this case the trace 4.71 is never needed. For the external lines, we can use the 4-dimensional Dirac algebra of Appendix A; external lines therefore pose no problem. However, this introduces an unappealing asymmetry between the loops and the external lines.

In this work, we will use the n-dimensional algebra (4.70 and 4.72) for both the loops and the external lines. This introduces a slight complication. A glance at Appendix G shows that the two-loop diagrams can all be expressed as $A_1\eta_1 + A_2\eta_2$ where $\eta_1 = \bar{v}_d\gamma^\mu L u_b \bar{u}_d \gamma_\mu L v_b$ and $\eta_2 = \bar{v}_d\gamma^\mu\gamma^\nu\gamma^\rho L u_b \bar{u}_d \gamma_\rho\gamma_\nu\gamma_\mu L v_b$. In 4-dimensions, use of the reduction formula A.95 of Appendix A allows us to show that $\eta_2 = 4\eta_1$. In n-dimensions, this doesn't necessarily hold. We can only assume that $\eta_2 = 4\eta_1 + \epsilon\Delta$ where $\epsilon = (n-4)$ and Δ is an arbitrary number. This can create an ambiguity in the result since, in general, A_2 has "pole terms" of the form $1/\epsilon$. However, we know we could use 4-dimensionnal Dirac algebra for the external lines if we so desired, so there cannot be any ambiguities in the final result. This means that the pole terms in A_2 must cancel when we sum over all diagrams. Since they must also cancel out of the full result, we get that when A_1 and A_2 are summed over all diagrams, their $1/\epsilon$ terms independently cancel. This provides a

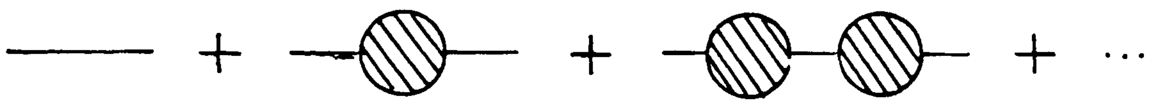


Figure 4.15: Expansion of propagators

check on the calculation.

4.3 Counterterm Lagrangian and renormalization conditions in the on-shell scheme

In this section, we will construct the counterterm Lagrangian and present the renormalization conditions that define the on-shell renormalization scheme. We will not prove that the choice of renormalization constants implied by this procedure leads to finite Green functions and hence finite S-matrix. (A proof can be found in [4].) We will be satisfied with a description of the equations and methods which prove useful in practical calculations.

4.3.1 Propagators and the on-shell conditions.

The full propagator Δ_0 of the bare field ϕ_0 is defined as the two point connected Green function of ϕ_0

$$\Delta_0(x-y) = \langle \phi_0(x)\phi_0(y) \rangle \equiv \int [d\phi_0]\, \phi_0(x)\phi_0(y) e^{iS} / \int [d\phi_0]\, e^{iS} \tag{4.73}$$

(we use again a theory of a single scalar field as an example). Note that everything in 4.73 is expressed in terms of the bare field ϕ_0. Its expansion in terms of Feynman diagrams is shown in fig. 4.15. The blob in fig. 4.15 is the proper self-energy of ϕ_0. It consists of the sum of all one particle irreducible graphs. These are the graphs that do not become disconnected upon cutting a single line. If we denote the proper self-energy by $i\Sigma(p^2)$ (where p is the momentum of ϕ_0), then the series in fig 4.15 sums to (in Fourier space)

$$\Delta_0 = \frac{i}{p^2 - m^2} + \left(\frac{i}{p^2 - m^2}\right) i\Sigma_0(p^2) \left(\frac{i}{p^2 - m^2}\right) + \cdots = \frac{i}{p^2 - m^2 + \Sigma_0(p^2)}$$

Here, m is the renormalized mass:

$$m_0^2 = m^2 + \delta m^2$$

In the on-shell renormalization scheme, the renormalization constant δm^2 is determined by requiring that the real part of the pole of Δ_0 be at $p^2 = m^2$. This is equivalent to

$$\mathrm{Re}\left\{\Sigma_0(m^2)\right\} = 0 \tag{4.74}$$

Assuming that 4.74 holds, we can expand Δ_0 in the form:

$$\Delta_0 = \frac{i\tilde{Z}}{(p^2 - m^2) + \mathcal{O}\left((p^2 - m^2)^2\right)} \tag{4.75}$$

The quantity $\tilde{Z}$ that appears in 4.75 is precisely the same as the one in the expression for the S-matrix 2.2. We will use this fact shortly.

In practice, it is convenient to replace the bare field ϕ_0 by the renormalized field ϕ via $\phi_0 \rightarrow Z^{1/2}\phi$. The most natural quantity to study is therefore the renormalized propagator $\Delta \equiv \langle\phi(x)\phi(y)\rangle$, not Δ_0. A look at 4.73 reveals that$\Delta = \frac{1}{Z}\Delta_0$. Inserting 4.75 in this relation, we find that

$$\Delta = \frac{\tilde{Z}}{Z} \frac{i}{(p^2 - m^2) + \mathcal{O}\left((p^2 - m^2)^2\right)}$$

We can now choose Z so that $Z = \tilde{Z}$. This can be implemented directly in terms of Δ by choosing the value of Z so that

$$\frac{\partial}{\partial p^2}(\Delta^{-1}) \bigg|_{p^2=m^2} = -i \tag{4.76}$$

is satisfied.

The conditions 4.74 and 4.76 take a much simpler form when expressed in terms of the self-energy $\Sigma(p^2)$ defined through:

$$\Delta = \frac{i}{p^2 - m^2 + \Sigma(p^2)}$$

The on-shell conditions are then:

$$\Sigma(m^2) = 0 \qquad (4.77)$$

$$\Sigma'(m^2) = 0 \qquad (4.78)$$

An advantage of this renormalization scheme becomes apparent when we look at formula 2.2 for the S-matrix: with the conditions 4.77 and 4.78 in place, the factors $i\tilde{Z}^{-1/2}(\Box + m^2)$ precisely cancel the propagators on the external legs of the Green functions. In this case, the calculation of the S-matrix reduces to a calculation of the amputated Green functions. These are the Green functions with the external propagators removed.

The formalism developed so far is adequate for a single scalar particle; as such, it can be used for the mass and wave-function renormalization of the Higgs boson. We will now consider the case of vector particles and fermions.

Vector particle propagators have a non-trivial Lorentz structure: for any vector boson V^μ, we can write the propagator $\Delta^{\mu\nu}$ as

$$\Delta^{\mu\nu} = \mathcal{F}\langle V^\mu(x)V^\nu(y)\rangle = A(p^2)g^{\mu\nu} + B(p^2)\frac{p^\mu p^\nu}{p^2}$$

where $\mathcal{F}$ stands for Fourier transform. The coefficient of the metric can be parametrized as

$$A(p^2) = \frac{-i}{p^2 - m^2 + \Sigma(p^2)}$$

The renormalization conditions on Σ are the the same as for a scalar particle (eqns 4.77 and 4.78). (The coefficient B is then automatically made finite.) To satisfy them, we can adjust the mass renormalization constant of the vector boson as well as the rescaling factor Z that enters the expression $V_0^\mu \rightarrow ZV^\mu$.

The case of fermions is more complicated. First of all, the left-handed and right-handed fields (section 2.2.2) renormalize independently

$$\psi_{L0} \rightarrow Z_L^{1/2}\psi_L \tag{4.79}$$

$$\psi_{R0} \rightarrow Z_R^{1/2}\psi_R \tag{4.80}$$

Due to the lack of invariance of the weak interactions under parity, the coefficients Z_L and Z_R are in general different. We also have the mass renormalization $m_0 \rightarrow m + \delta m$. The parameters Z_L, Z_R and δm are then adjusted to satisfy equations analogous to 4.77 and 4.78.

To find the explicit form of these equations, we parametrize the fermion propagator in the form

$$\Delta_F \equiv \mathcal{F}\langle\psi(x)\bar{\psi}(y)\rangle = \frac{i}{\not{p} - m + \Sigma(p)}$$

Note that Σ is a 4×4 matrix in Dirac space. We then introduce the on-shell spinor $u(p)$ which satisfies:

$$\begin{aligned} (\not{p} - m)u(p) &= 0 \\ \bar{u}(p)(\not{p} - m) &= 0 \end{aligned}$$

The renormalization conditions are then

$$\begin{aligned} \Sigma(p)u(p) &= 0 \\ \bar{u}(p)\Sigma(p) &= 0 \\ \frac{1}{\not{p} - m}\Sigma(p)u(p) &= 0 \end{aligned} \tag{4.81}$$

$$\bar{u}(p)\Sigma(p)\frac{1}{\not{p}-m} = 0 \tag{4.82}$$

We will see more explicitly later what this implies.

A further complication is caused by the existence of different particles with the same spin and charge (e.g. the u-type quarks u, c, t and the d-type d, s, b). In this case, the propagator $\Delta_{ij} \equiv \mathcal{F}\langle\psi_i(x)\bar{\psi}_j(y)\rangle$ is a matrix (in flavor space) which, in general, is not diagonal. We can then generalize 4.79 and 4.80 to read (in an unescapably cluttered notation)

$$\begin{aligned} \psi^i_{L0} &\rightarrow (Z^{ij}_L)^{1/2}\psi^j_L \\ \psi^i_{R0} &\rightarrow (Z^{ij}_R)^{1/2}\psi^j_R \end{aligned}$$

The matrices Z_L and Z_R have no particular symmetry. Especially, they are not necessarily unitary. We also have at our disposal the mass renormalization constants δm^i (from $m^i_0 \rightarrow m^i + \delta m^i$). These parameters are determined from the following renormalization conditions:

$$\begin{aligned} \Sigma^{ij}(p)u^j(p) &= 0 \\ \bar{u}^i(p)\Sigma^{ij}(p) &= 0 \\ \frac{1}{\not{p}-m}\Sigma^{ii}(p)u^i(p) &= 0 \\ \bar{u}^i(p)\Sigma^{ii}(p)\frac{1}{\not{p}-m} &= 0 \quad \text{(no sum over i)} \end{aligned} \tag{4.83}$$

where the propagator Δ_{ij} is written as:

$$\Delta_{ij} = \frac{i}{(\not{p}-m)\delta^{ij} + \Sigma^{ij}(p)}$$

Equations 4.83 say that the corrections to the propagator vanish when taken on shell. In practice, this means that graphs with loops on the external legs (see fig.4.16 for an example) identically vanish. A more detailed form of 4.83 can be obtained by writing

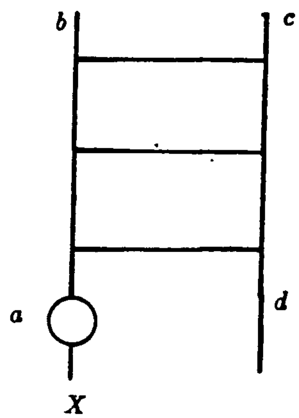


Figure 4.16: Corrections to external legs. The external legs are a, b, c, and d. This graph is identically zero if particle X is on-shell, no matter what the particles in the graph are.

the Dirac structure explicitly.

$$\Sigma^{ij}(p) = \Sigma_1^{ij} + \Sigma_5^{ij}\gamma_5 + \Sigma_\gamma^{ij}\not{p} + \Sigma_{5\gamma}^{ij}\not{p}\gamma_5$$

We get:

$$\begin{aligned} \Sigma_1^{ii}(m_i^2) + m_i\Sigma_\gamma^{ii}(m_i^2) &= 0 \\ 2m_i\Sigma_1^{ii\prime}(m_i^2) + \Sigma_\gamma^{ii}(m_i^2) + 2m_i^2\Sigma_\gamma^{ii\prime}(m_i^2) &= 0 \\ \Sigma_5^{ii}(m_i^2) &= 0 \\ \Sigma_{5\gamma}^{ii}(m_i^2) &= 0 \end{aligned} \tag{4.84}$$

for $i = j$ (no sum over i), and

$$\begin{aligned} \Sigma_1^{ij}(m_j^2) + m_j\Sigma_\gamma^{ij}(m_j^2) &= 0 \\ \Sigma_1^{ij}(m_i^2) + m_i\Sigma_\gamma^{ij}(m_i^2) &= 0 \\ \Sigma_5^{ij}(m_j^2) - m_j\Sigma_{5\gamma}^{ij}(m_j^2) &= 0 \\ \Sigma_5^{ij}(m_i^2) + m_i\Sigma_{5\gamma}^{ij}(m_i^2) &= 0 \end{aligned} \tag{4.85}$$

for $i \neq j$.

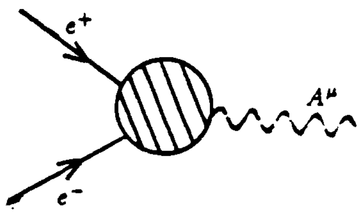


Figure 4.17: The electron-photon 3-point function

Finally, it should be noted that the Z-boson and the photon also mix in this way. We must introduce the wave-function renormalization constants as follows:

$$\begin{aligned} Z_0^\mu &\to Z_{ZZ}^{1/2} Z^\mu + Z_{ZA}^{1/2} A^\mu \\ A_0^\mu &\to Z_{AZ}^{1/2} Z^\mu + Z_{AA}^{1/2} A^\mu \end{aligned}$$

Here, $Z_{ZA} = Z_{AZ}$ is determined by $\Sigma_{AZ}(0) = 0$.

4.3.2 The other parameters

We have so far given the renormalization conditions for the masses of all physical particles. We will take as independent parameters M_W, M_Z, M_H and the masses of all the quarks and leptons. This doesn't exhaust the list of independent parameters of the Standard Model. We also need to define a coupling and to consider the Kobayashi-Maskawa matrix.

There are several couplings in the theory. Only one of them can be taken as an independent parameter. It is customary to take the charge of the electron as the independent parameter. It is defined via the 3-point Green function (see fig. 4.17)

$$\mathcal{F}\langle \bar{e}(x) A^\mu(y) e(z) \rangle \equiv \Gamma^\mu$$

The renormalization condition imposed is

$$\bar{u}_e \Gamma^\mu u_e \Big|_{p=0} = 0 \tag{4.86}$$

where p is the photon momentum. This definition has the advantage that the usual definition of the fine structure constant α in terms of Thomson scattering gives $\alpha = e^2/4\pi$ exactly.

To satisfy 4.86, we introduce the charge renormalization constant Y through $e_0 \to Ye$ and adjust Y.

To complete the description of the parameters, we need to consider the Kobayashi-Maskawa matrix V. This matrix is relevant to the interactions of quarks with the charged W-boson (as well as with the corresponding would-be Goldstone bosons $\phi^{\pm}$). A typical interaction term is of the form

$$\mathcal{L}_I = \bar{u}_0^i \gamma^\mu L d_0^j V_{ij} W_0^{+\mu} \tag{4.87}$$

where everything is expressed in terms of the bare fields. If we substitute the expressions for the renormalized fields in 4.87, we get (only the left-handed components of quarks contribute to this interaction)

$$\mathcal{L}_I = \bar{u}^k \gamma^\mu L d^l (Z_{(u)L}^{ki\dagger})^{1/2} (Z_{(d)L}^{lj})^{1/2} V_{ij} Z_W^{1/2} W^{+\mu}$$

The main point is that any divergences present in V_{ij} can be absorbed by a redefinition of $Z_{(u)L}$ and $Z_{(d)L}$, these being then determined as before. There is, therefore, no need to impose any renormalization conditions on V_{ij}. Note, however, that we could impose some renormalization conditions on V_{ij} in order to bring it closer to the experimental quantities (see [18], for instance). We will not do this here since no consensus has emerged as to the definition of V_{ij} in terms of experimental quantities. Hence, we will not introduce any counterterms for V_{ij}.

4.3.3 The tadpoles, gauge-fixing and the counterterm Lagrangian

We can make use of a further renormalization condition to get rid of a nuisance in the model: the occurence of *tadpole* graphs (fig. 4.18). These are graphs with only one Higgs

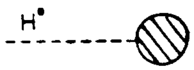


Figure 4.18: A tadpole graph

line sticking out. We can eliminate them by the introduction of a counterterm in the Higgs potential.

Until further notice, all quantities appearing in the equations are bare quantities even though they do not carry the subscript 0. We start from the Higgs potential 2.27

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2$$

Again, we insert $\Phi = \phi_0 + \phi'$ with

$$\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} , \ \phi' = \begin{pmatrix} \phi^+ \\ (H + i\chi)/\sqrt{2} \end{pmatrix}$$

Unlike section 2.3, we will not choose v so that $\phi' = 0$ at the minimum of V. Instead, we define

$$T = v(\mu^2 - \lambda v^2)$$

This, along with the usual relations

$$\begin{aligned} M_W &= \frac{gv}{2} \\ M_H^2 &= 2\mu^2 \end{aligned}$$

gives the following modification of eqn 2.33

$$\begin{aligned} v &= \frac{2M_W}{g} \\ \mu^2 &= \frac{M_H^2}{2} \\ \lambda &= \frac{g^2}{8M_W^2}\left(M_H^2 - \frac{gT}{M_W}\right) \end{aligned} \tag{4.88}$$

With these modifications, the potential $V(\Phi)$ becomes

$$\begin{aligned}
V(\Phi) &= -TH + \left(\frac{M_H^2}{2} - \frac{3}{4}\frac{gT}{M_W}\right)H^2 - \frac{1}{4}\left(\frac{gT}{M_W}\right)\chi^2 - \frac{1}{2}\left(\frac{gT}{M_W}\right)\phi^+\phi^- \\
&+ \frac{g}{4M_W}\left(M_H^2 - \frac{gT}{M_W}\right)(H^3 + H\chi^2 + 2H\phi^+\phi^-) \\
&\frac{g^2}{8M_W^2}\left(M_H^2 - \frac{gT}{M_W}\right)\left(\frac{H^4}{4} + \frac{\chi^4}{4} + \frac{H^2\chi^2}{2} + H^2\phi^+\phi^- + \chi^2\phi^+\phi^- + \phi^+\phi^-\phi^+\phi^-\right)
\end{aligned}$$

The tadpole counterterm T is then determined by requiring that the sum of all tadpoles vanish.

We are now ready to express the Lagrangian in terms of the renormalized quantities. We use the full Lagrangian of Chapter 2 with two exceptions:

- we use $V(\Phi)$ given above instead of the one of Chapter 2
- the gauge-fixing Lagrangian will be treated separately

We affix a subscript 0 to all fields and parameters to remind ourselves that they are bare quantities. We then make the substitutions:

$$\begin{aligned}
Z_0^\mu &\rightarrow Z_{ZZ}^{1/2} Z^\mu + Z_{ZA}^{1/2} A^\mu \\
A_0^\mu &\rightarrow Z_{AZ}^{1/2} Z^\mu + Z_{AA}^{1/2} A^\mu \\
W_0^{+\mu} &\rightarrow Z_W^{1/2} W^{+\mu} \\
H_0 &\rightarrow Z_H^{1/2} H \\
\phi_0^+ &\rightarrow Z_\phi^{1/2}\phi^+ \\
\chi_0 &\rightarrow Z_\chi^{1/2}\chi
\end{aligned}$$

We do the same for the fermions

$$\begin{aligned}
u_{L0}^i &\rightarrow \left(Z_{(u)L}^{ij}\right)^{1/2} u_L^j \\
u_{R0}^i &\rightarrow \left(Z_{(u)R}^{ij}\right)^{1/2} u_R^j
\end{aligned}$$

$$\begin{aligned} d^i_{L0} &\rightarrow \left(Z^{ij}_{(d)L}\right)^{1/2} d^j_L \\ d^i_{R0} &\rightarrow \left(Z^{ij}_{(d)R}\right)^{1/2} d^j_R \end{aligned}$$

and,

$$\begin{aligned} e^i_{L0} &\rightarrow \left(Z^{ii}_{(e)L}\right)^{1/2} e^i_L \\ e^i_{R0} &\rightarrow \left(Z^{ii}_{(e)R}\right)^{1/2} e^i_R \\ \nu^i_{L0} &\rightarrow \left(Z^{ii}_{(\nu)L}\right)^{1/2} \nu^i_L \end{aligned}$$

(There is no mixing of leptons.) The masses of the bosons (Z, W^+ and H) are renormalized quadratically according to

$$M_0^2 \rightarrow M^2 + \delta M^2$$

while those of fermions are renormalized linearly

$$m_0 \rightarrow m + \delta m$$

Also, the electron charge is renormalized by

$$e_0 \rightarrow Ye$$

From these relations, we can deduce the renormalization of the other parameters of the model. For instance,

$$(\cos\theta_W)_0 = \frac{M_{W0}}{M_{Z0}} \rightarrow \frac{M_W\sqrt{1+\frac{\delta M_W^2}{M_W^2}}}{M_Z\sqrt{1+\frac{\delta M_Z^2}{M_Z^2}}} = \cos\theta_W\sqrt{\frac{1+\delta M_W^2/M_W^2}{1+\delta M_Z^2/M_Z^2}}$$

where $\cos\theta_W = \frac{M_W}{M_Z}$,

$$(\sin\theta_W)_0 = \sqrt{1-(\cos^2\theta_W)_0} \rightarrow \sin\theta_W\sqrt{1-\frac{\cos^2\theta_W}{\sin^2\theta_W}\frac{(\delta M_W^2/M_W^2 - \delta M_Z^2/M_Z^2)}{(1+\delta M_Z^2/M_Z^2)}}$$

where $\sin\theta_W = \sqrt{1-\cos^2\theta_W}$ and

$$g_0 = \frac{e_0}{s_0} \to \frac{gY}{\sqrt{1 - \frac{\cos^2\theta_W}{\sin^2\theta_W}\frac{(\delta M_W^2 - \delta M_Z^2)}{(1+\delta M_Z^2/M_Z^2)}}}$$

where $g = e/s$.

Finally, we must take care of the gauge-fixing Lagrangian. We can proceed in the same way as in Chapter 2 to get equation 2.46. We can use 2.47 for the functions f but we will use their renormalized versions i.e. the fields and the parameters that enter 2.47 are understood to be the renormalized ones. This means that we do not get any counterterm vertices from the gauge-fixing Lagrangian.

The counterterm Lagrangian is the Lagrangian obtained by the above procedure minus its value when the renormalization constants are zero. The parts of it that are relevant to our calculation are shown in Appendix D.

4.4 Numerical values of the parameters

The parameters of the model have been defined in section 3. In this section, we will give the numerical values of these parameters as determined by a variety of experiments.

First, the electric charge e is determined by the fine structure constant

$$\alpha = \frac{e^2}{4\pi} = 1/137.036$$

with negligible uncertainty. This gives $e = .303$. Note that e is not very small. Yet, the small coupling expansion works because the successsive terms in the perturbation series are usually down by a factor $\alpha/2\pi$.

The masses of the leptons are very small and can be neglected. So can most of the quark masses except of course for the top quark ($m_t \geq 90GeV$) and, to some extent, the bottom quark ($m_b = 5.0GeV$). The masses of the W and Z bosons are:

$$\begin{aligned} M_W &= 80.11 \pm .15 \\ M_Z &= 91.175 \pm .005 \end{aligned}$$

Also, the mass of the Higgs is unknown ($M_H \geq 50GeV$).

Finally, the Kobayashi-Maskawa matrix can be parametrized as

$$\begin{aligned} V &= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \\ &= \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3\rho e^{i\phi} \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho e^{-i\phi}) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) \end{aligned} \tag{4.89}$$

where $\lambda = 0.221 \pm .002$ and $A = 1.05 \pm 0.17$. As for ρ, we can take $\rho = 0.55 \pm .14$. The CP-violating phase ϕ is subject to large uncertainties. We will take it as completely arbitrary.

We can now find the values of the derived parameters $\sin\theta_W$ and g:

$$\begin{aligned} \sin^2\theta_W &= 1 - \frac{M_W^2}{M_Z^2} = .228 \pm .002 \\ g &= \frac{e}{s} = .63 \end{aligned}$$

Chapter 5

The two-loop calculations

The two-loop calculations of the $B_d - \bar{B}_d$ mixing amplitude will form the subject of this chapter. We will begin with a description of the approximations made in the calculations. We will follow this with an enumeration of the graphs that contribute to this process. We will list all those graphs that survive in the limit of zero external momentum and zero mass of the d-type quarks. This will give an idea of the complexity of the full calculation. We will also indicate which of these graphs contribute under our approximations. Sections 4 and 5 are devoted to a description of the techniques used in the calculations. Many details are left to the Appendices. Section 6 summarizes the results and we present our conclusions in section 7.

5.1 The approximations

As will be shown in the following section, the complete calculation of $B_d - \bar{B}_d$ mixing at two-loops involves more than 10,000 diagrams. This is clearly beyond our reach (at least for the moment). We will therefore limit ourselves to two limiting cases of interest:

1. We assume that the mass of the top quark is much larger than all other masses in the model. We then expand the diagrams in the asymptotic limit $m_t \to \infty$ and keep only the dominant contribution. This turns out to be of the form m_t^4. Therefore, we neglect the terms of order m_t^2 or lower as $m_t \to \infty$. (Inclusion of the m_t^2 terms would be virtually equivalent to doing the full calculation.) This we will call the large m_t limit.

2. The second limit is somewhat more complicated. We assume that the mass of the Higgs boson is much larger than all other masses. Again, we expand the diagrams in the asymptotic limit $M_H \to \infty$ but now we keep all terms that go to infinity as $M_H \to \infty$. In general, we get contributions of the form M_H^4, M_H^2, $\ln^2 \frac{M_H}{m_t}$ and $\ln \frac{M_H}{m_t}$. when we sum over all diagrams, the quartic and quadratic terms cancel. The coefficients of the logarithmic terms are functions of m_t and M_W which behave like m_t^4 when $m_t \gg M_W$. The terms neglected so far also behave in this way. It is therefore advisable to include the terms in the diagrams that are independent of M_H, expand them for large m_t and keep the m_t^4 terms. This is what we shall do. We will call this expansion the large M_H limit.

We will also neglect the masses of all quarks but the top. This means that the Higgs and the would-be Goldstone boson χ couple only to the top quark and the bosons. Also the coupling of the ϕ^+ to quarks becomes purely right-handed. (In general, it is of the form $aL + bR$ where a and b are mass dependent coefficients.) This approximation is valid up to corrections of order $m_b^2/M_W^2 \approx .4\%$.

Finally, we will also make some approximations relative to the Kobayashi-Maskawa matrix. A look at 4.89 shows that the diagonal elements of the KM matrix are larger than the off-diagonal ones. We will limit ourselves to calculations involving the least possible number of off-diagonal elements in the two-loop diagrams, namely two. This approximation is valid up to corrections of order $\lambda^4 \approx .3\%$.

5.2 The two-loop diagrams

In this section, we will systematically enumerate the 2-loop diagrams involved in the calculation of $b\bar{d} \to d\bar{b}$. We will also indicate which of these graphs contribute in the two limits considered in this work, namely the large m_t limit and the large M_H limit. Since

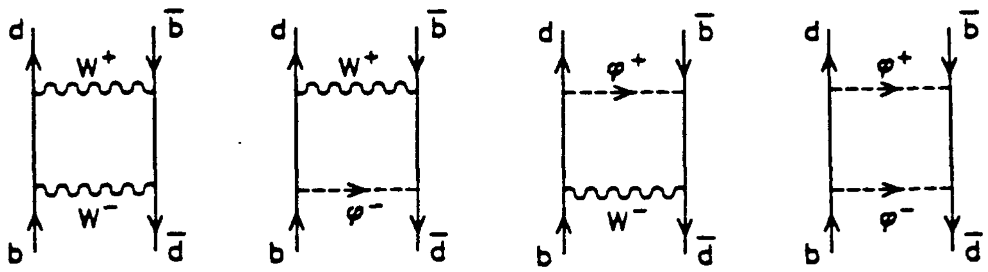


Figure 5.19: The one-loop diagrams

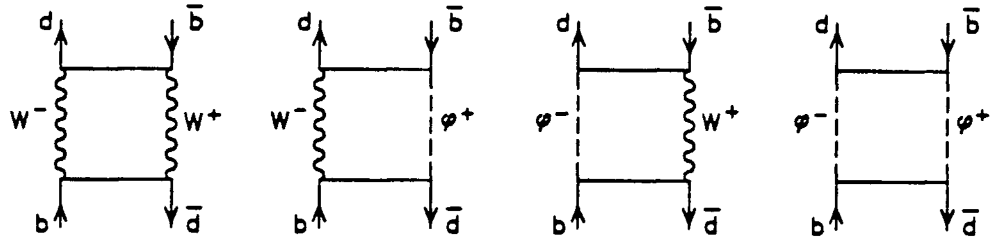


Figure 5.20: The s-channel diagrams at one-loop

the masses of the external quarks are small compared to M_W, we shall set them equal to zero.

In the Standard Model, the lowest order diagrams that lead to $B_d - \bar{B}_d$ transitions are the 1-loop diagrams shown in fig. 5.19. The intermediate state quarks can be u, c, or t. Furthermore, there are the corresponding s-channel diagrams (fig. 5.20). The intermediate state quark we are most interested in is the t-quark.

At the two-loop level, there are seven different types of diagrams to be evaluated. We shall systematically discuss them below. It should be kept in mind that every diagram shown stands in reality for 18 diagrams generated by the 9 combinations of quarks in the loop and their s-channel counterparts.

Type I: These are generated from the 1-loop graphs by exchanging a Z, a Higgs or a χ between the quark lines. Since the coupling of the Higgs (or the χ) to fermions is proportional to m_f, the graphs involving the Higgs or the χ coupling to external quarks can be neglected. This gives rise to the set of graphs depicted in fig 5.21.

There are 112 graphs all together. The leading behavior in M_H and m_t is given by the first two and the last two graphs.

Type II graphs are generated by Z, H and χ exchange between the charged bosons. The exchange of the Higgs boson between the $W^{\pm}$ and $\phi^{\pm}$ will generate the 16 graphs shown in fig. 5.22. All of these depend on M_H, hence they must all be evaluated. Since the Z-boson and photon couplings to W^+W^-, $\phi^+\phi^-$ and $W^{\pm}\phi^{\mp}$, there are 32 graphs with the H replaced by the Z or the photon. However, these will not have any M_H dependence and are also subleading in m_t.

The exchange of the Goldstone boson χ will generate fewer graphs of this type. This is because the only couplings we are allowed are $\chi W^{\pm}\phi^{\mp}$. Hence, we have the graphs of fig. 5.23 to consider. None of these will contribute to leading order.

Therefore there are 52 type II graphs of which only the 16 involving the Higgs exchange will have an M_H dependence and leading behavior in m_t.

The diagrams of type III are generated by the exchange of a Z boson or a photon between the internal bosons and an external quark. They are shown in fig. 5.24. None of these 64 diagrams contribute to leading order.

The type IV graphs are obtained by pinching the bosonic lines of the one-loop diagrams as in fig. 5.25. Again, none of these diagrams need to be calculated to leading order.

Further diagrams are generated from the basic 1-loop diagrams by the renormalization procedure. These we classify as type V and VI.

Type V graphs are vertex correction insertions into fig. 5.19. Schematically they are represented by the diagrams of fig. 5.26. The blobs represent the 1-loop vertex corrections to $td(b)W$ and $td(b)\phi$ vertices and they are to be inserted in all possible ways. Hence,

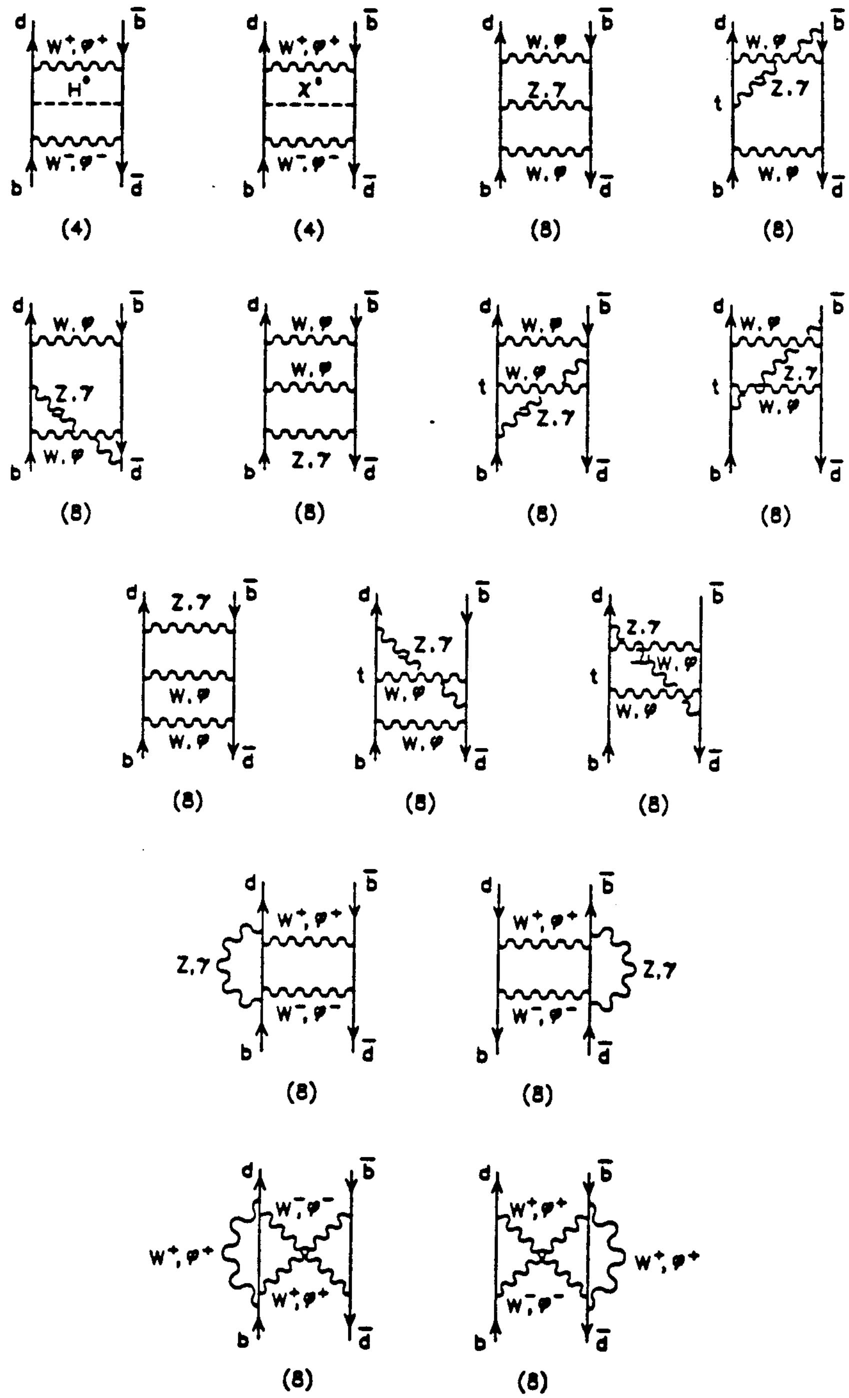


Figure 5.21: Type I diagrams

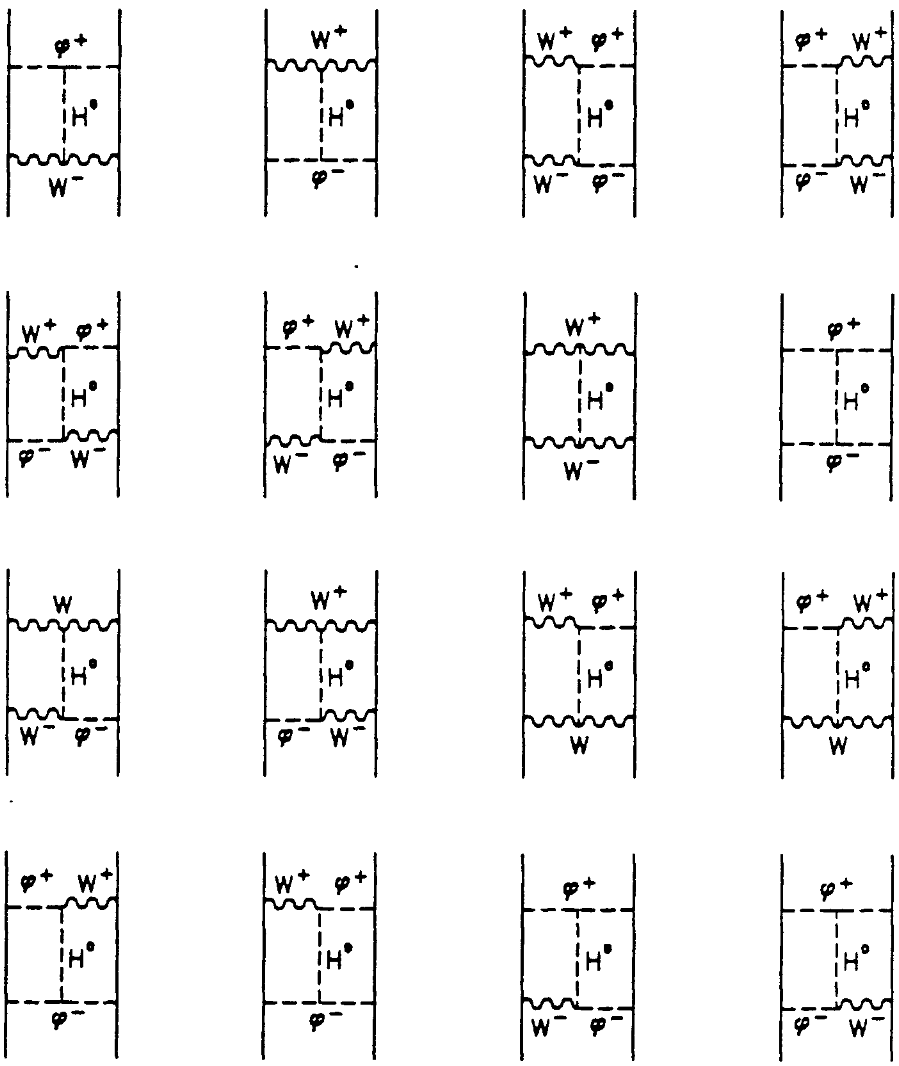


Figure 5.22: Type II diagrams involving the Higgs

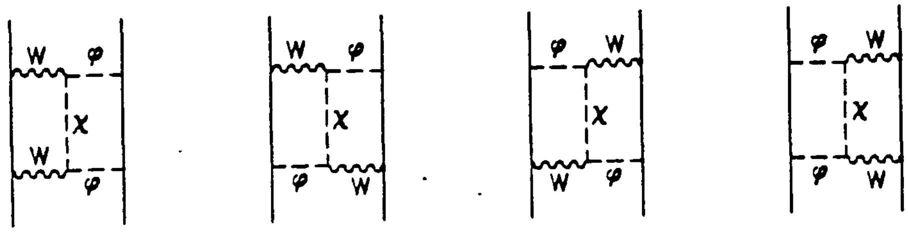


Figure 5.23: Type II diagrams involving the χ

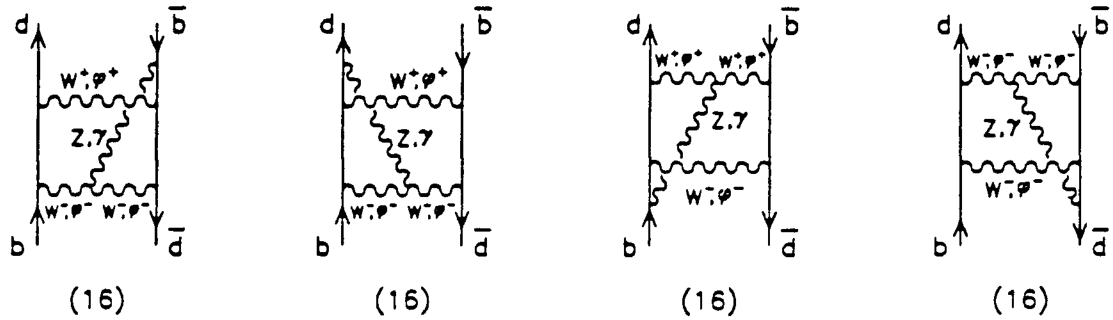


Figure 5.24: Type III diagrams

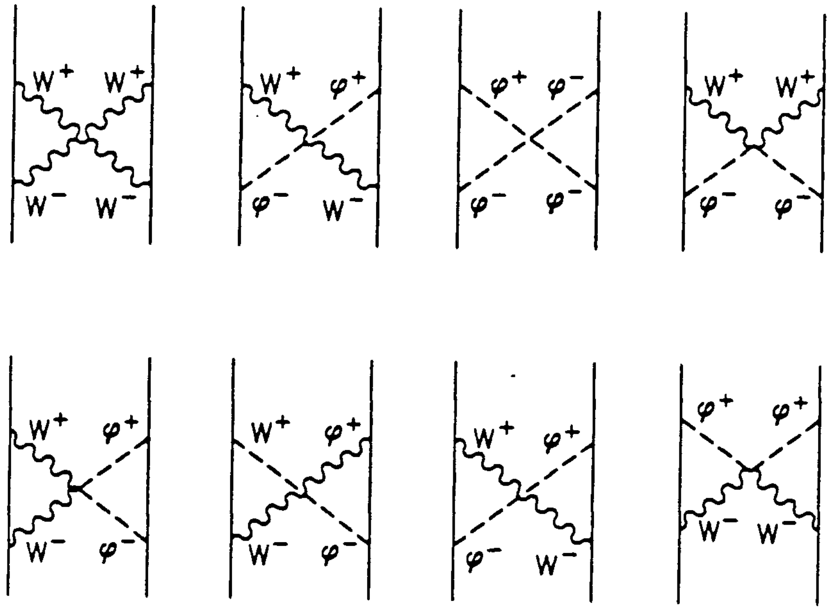


Figure 5.25: Type IV diagrams

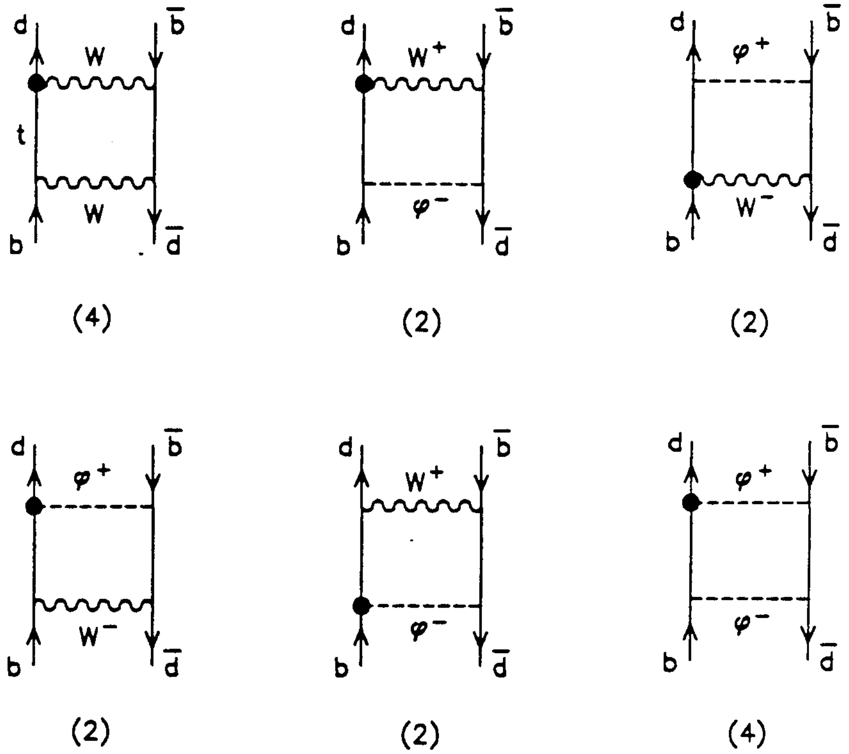


Figure 5.26: Type V graphs: vertex corrections

there are 4 insertions in the first and last graphs and two insertions each for the rest giving a total of 16 such vertex graphs.

The fermion-fermion-W vertex corrections are given by the expansion shown in fig. 5.27 (the tdW vertex corrections are given by the same expansion with the b-quark replaced by the d-quark). Of the 13 graphs, only the first two depend on M_H and none contribute to the leading m_t^4 behaviour. We note that the tbW vertex corrections consist of 104 graphs, of which 16 are needed for the leading behaviour.

Similarly, the 13 graphs contributing to the $td\phi$ (or $tb\phi$) vertex corrections are shown in fig. 5.28. Again, only the first two need to be calculated.

Furthermore, the counterterms (fig. 5.29) that remove the divergences in these vertex corrections have to be inserted. Discussion of the counterterms and their calculation can be found in Chapter 4 and in the following section. Adding these gives a total of 208

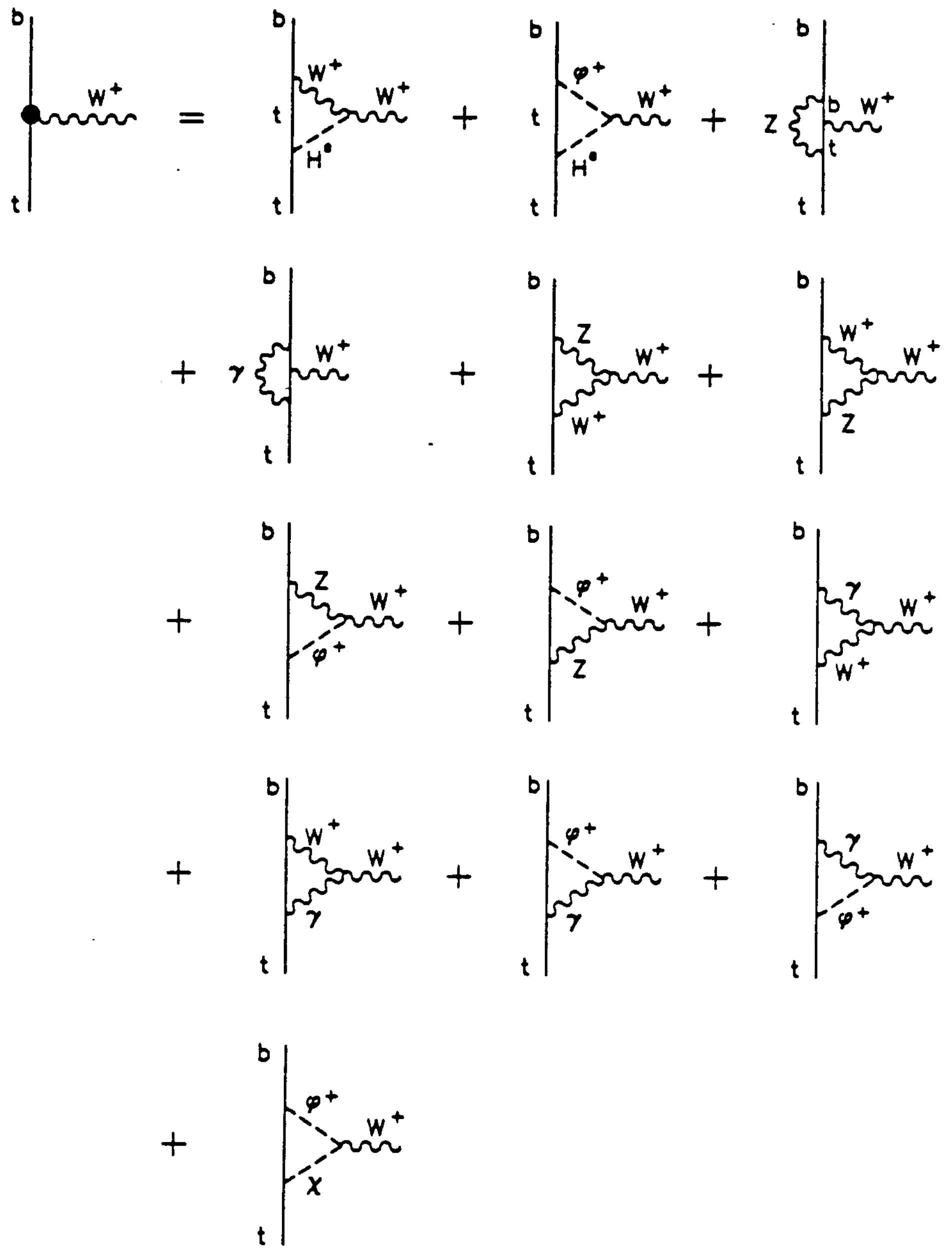


Figure 5.27: The tbW vertex

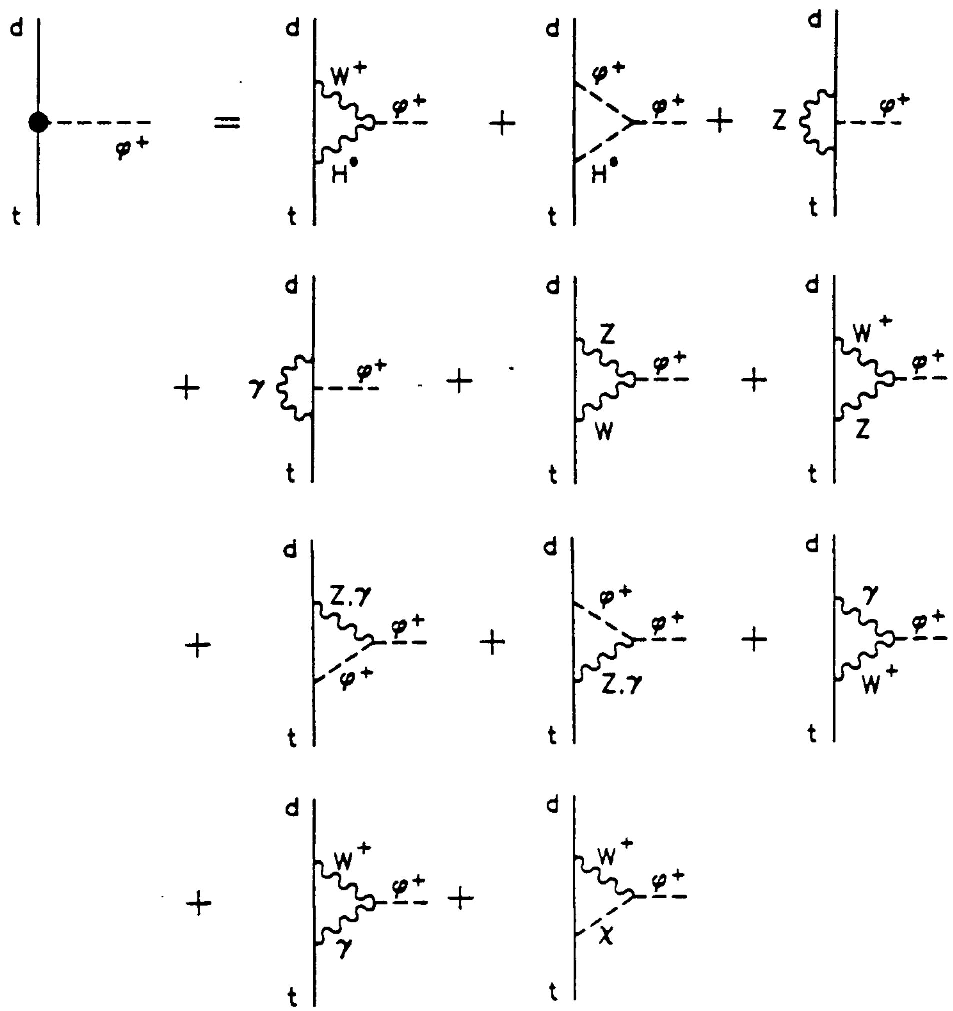


Figure 5.28: The $td\phi$ vertex

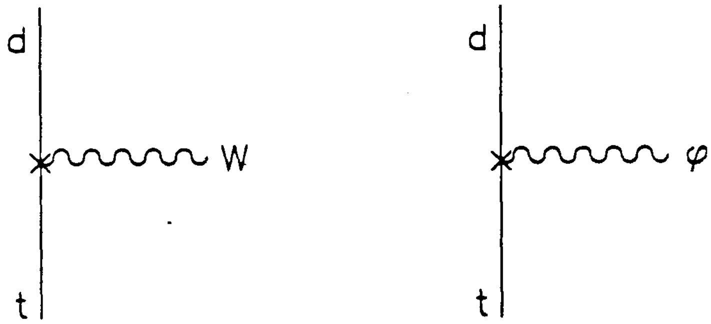


Figure 5.29: Vertex counterterms

vertex graphs, of which 48 enter our discussion.

The second kind of renormalization graphs are the propagator corrections. These are the type VI graphs shown in fig. 5.30. There are 8 top quark self-energy insertions, 4 W-boson self-energy insertion, 4 ϕ-boson self-energy insertions and 8 graphs with W-ϕ transition insertions. These self-energy terms are given in figs 5.31 to 5.34.

The diagrams with the crosses denote the counterterms. For the fermion self-energy, only the first three graphs are relevant. The first four and the first seven of the WW and $\phi\phi$ respectively are of relevance and the first three of the $W\phi$ transitions will contribute to our calculation. This gives a total of 116 graphs including 24 counterterm contributions. The full calculation would involve 324 graphs from this type. We note that in the renormalization scheme we have adopted, tadpole contributions to the self-energies are cancelled by the tadpole counterterm and hence not shown.

Finally, we list the two-loop diagrams which are not obtained from the basic 1-loop graphs (fig. 5.35). These are one-particle reducible graphs (sometimes called "dipenguins") and we label them as type VII. We show here only the dbZ vertex corrections. There are similar corrections involving the other neutral bosons but they are subleading.

This completes our enumeration of the two-loop diagrams for the flavor-changing transition $b\bar{d}$-$\bar{b}d$ in the limit of zero external quark masses and vanishing external momenta.

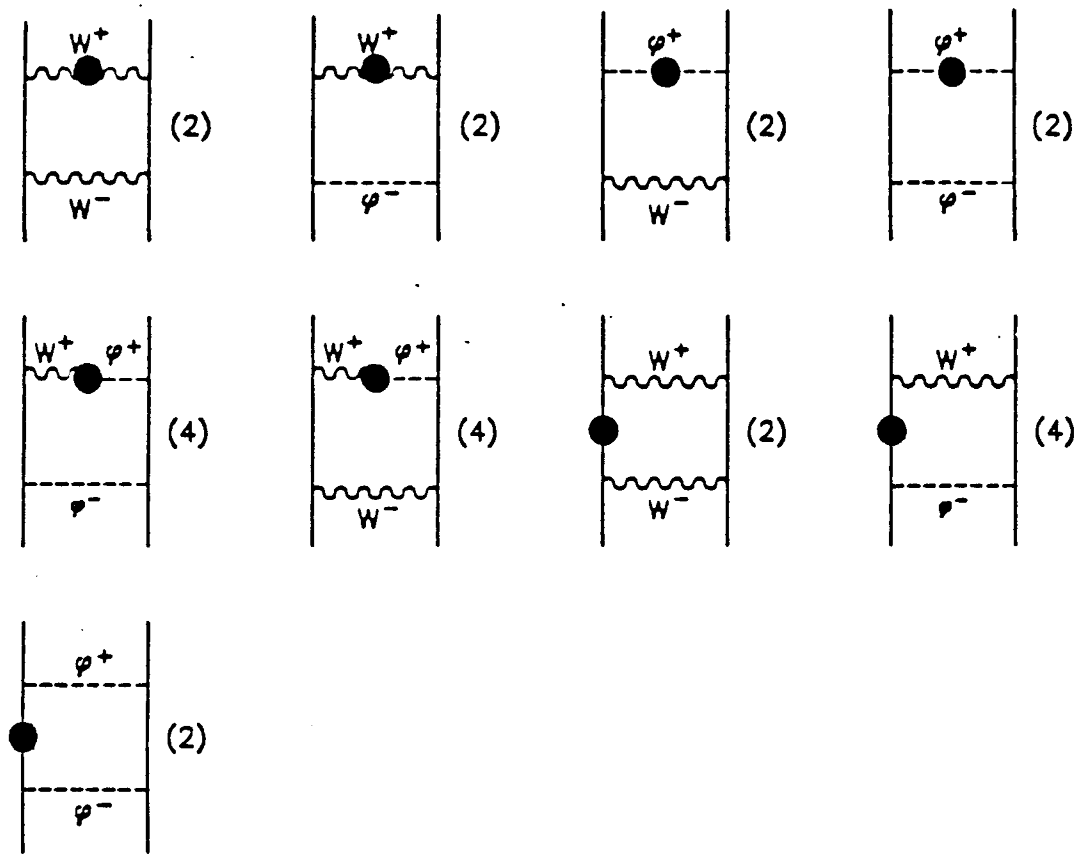


Figure 5.30: Propagator corrections

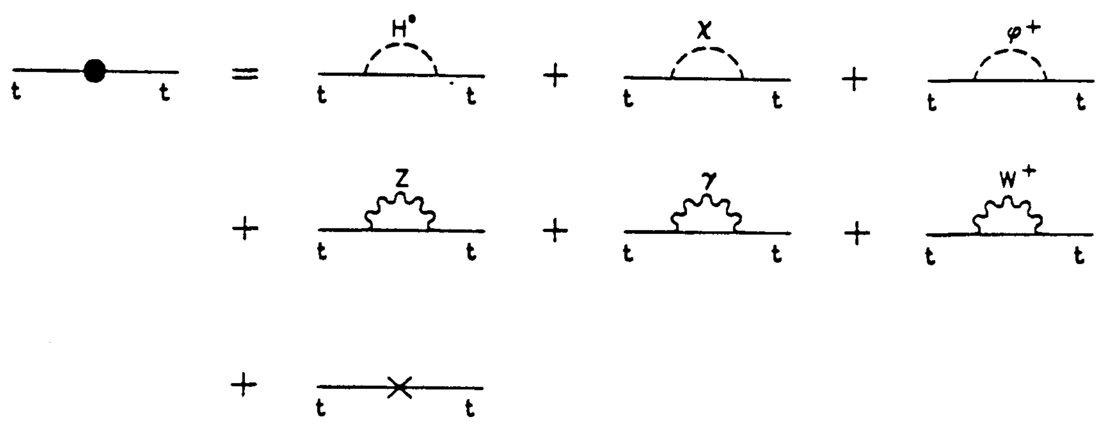


Figure 5.31: Top-quark self-energy

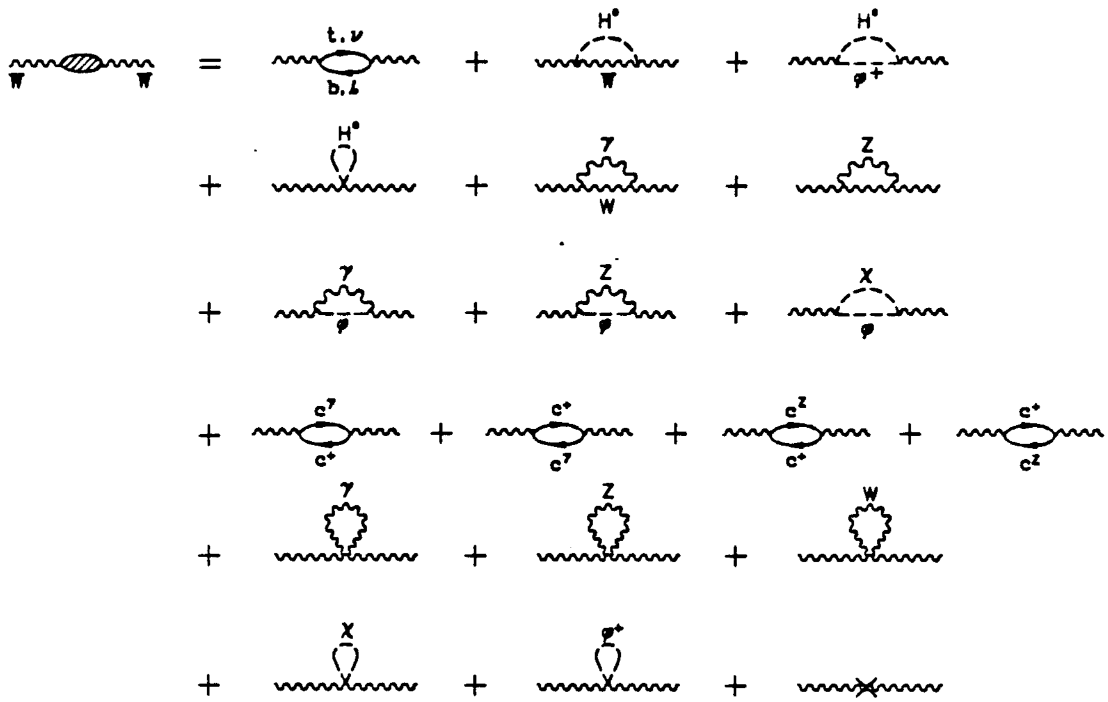


Figure 5.32: W self-energy

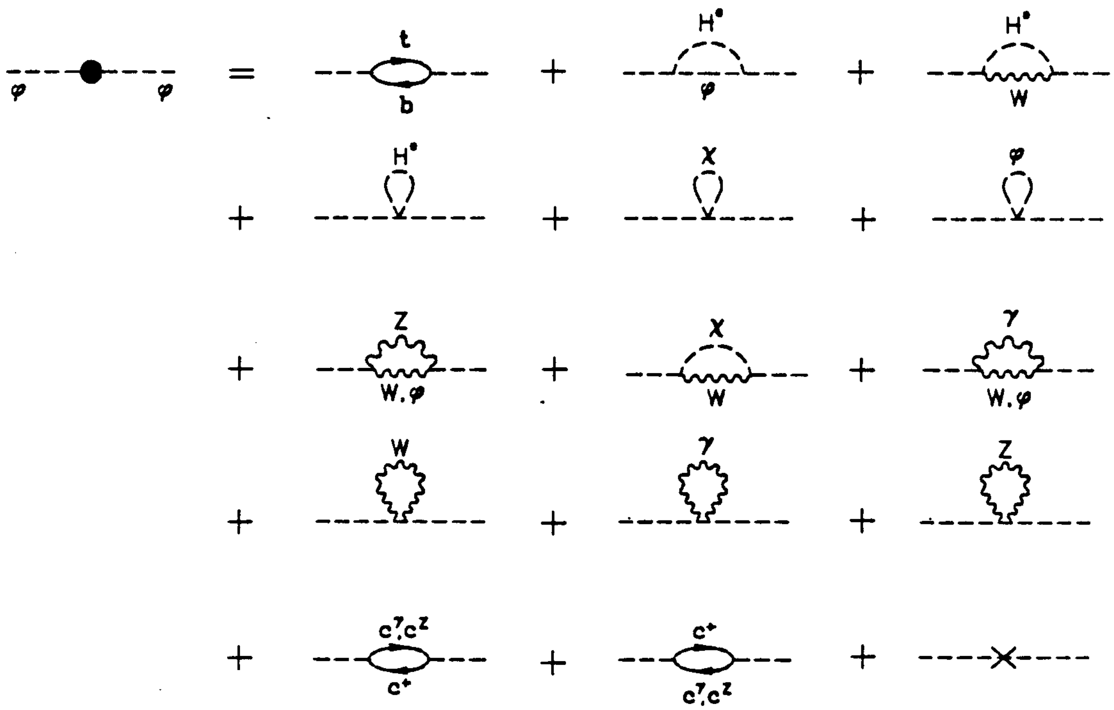


Figure 5.33: ϕ self-energy

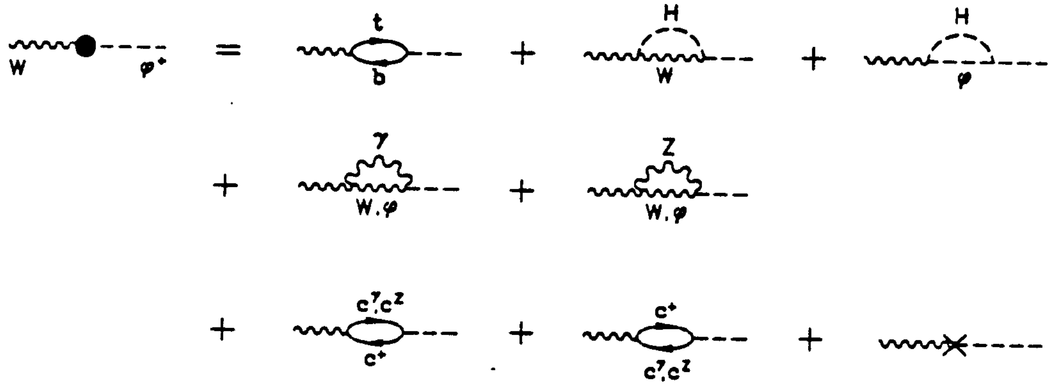
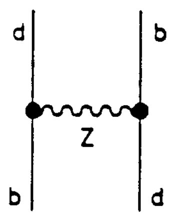


Figure 5.34: W-ϕ transition two-point function



where

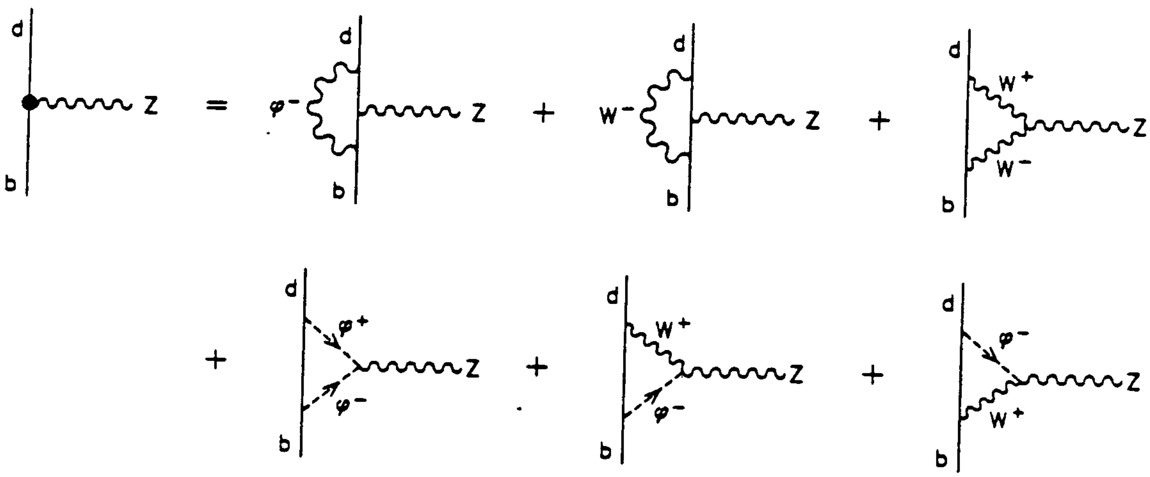


Figure 5.35: Type VII diagrams: dipenguins

5.3 Calculation of the counterterms

We will present here the diagrams needed for the calculation of the renormalization constants and the counterterms. The evaluation of the diagrams is straightforward and use of Appendix D yields the explicit value of the counterterms. These are listed in Appendix E.

We will keep all diagrams that depend on M_H as well as all those diagrams that behave as m_t^4 or m_t^2 for large m_t. The quadratic terms in m_t must be kept in the diagrams since they can generate a quartic behavior when inserted in the box diagrams.

We will first show that the corrections to Y and Z_{ZA} can be neglected in our approximations. The constant Z_{ZA} is determined from the mixing self-energy $\Sigma_{ZA}^{\mu\nu}$ of the photon and the Z shown in fig. 5.36. Since the Higgs doesn't couple to the photon, $\Sigma_{ZA}^{\mu\nu}$ is independent of M_H. The dependence on m_t comes from the top loop. Power counting reveals that $\Sigma_{ZA}^{\mu\nu}$ can, in principle, behave as m_t^2. However, because of the U_1 gauge-invariance of electromagnetism, $\Sigma_{ZA}^{\mu\nu}$ can only be of the form

$$\Sigma_{ZA}^{\mu\nu} = (k^2 g^{\mu\nu} - k^\mu k^\nu)\Pi_{ZA}(k^2, m_t^2)$$

The function Π is dimensionless and is well defined for $k = 0$. This means that its expansion for large m_t is:

$$\Pi_{ZA} = a + b\frac{k^2}{m_t^2} + \cdots$$

where the coefficients a and b are at most logarithmically dependent on m_t. Therefore, there is no m_t^2 behavior (or dependence on M_H) in $\Sigma_{ZA}^{\mu\nu}$ and we can take $Z_{ZA} = 0$.

The case of the charge renormalization Y is a bit more complex. It is determined from the e^+e^--photon vertex (fig. 5.37) in the limit that the photon momentum is zero (i.e. the photon is on-shell). In addition to Y, we have the wave-function renormalizations $Z_L^{(e)}$, $Z_R^{(e)}$ and Z_{AA} at our disposal to carry out the renormalization. We consider them

Figure 5.36: The Z-photon mixing self-energy.

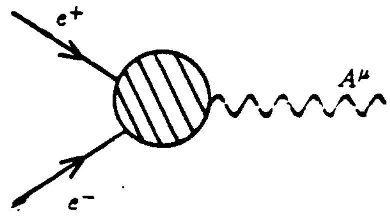


Figure 5.37: The e^+e^--photon vertex.

in turn.

Z_{AA} is determined from the photon self-energy; the argument used for Z_{ZA} applies here as well and hence $\delta Z_{AA} = Z_{AA} - 1 = 0$. Also, the wave-function renormalization constants of the electron are determined from the electron self-energy $\Sigma^{(e)}$. Since the electron doesn't couple to either the top quark or the Higgs, we can set $\delta Z^{(e)}_{L(R)} = 0$.

The renormalization of Γ^μ is therefore completely determined by Y. Again, the Higgs doesn't contribute to Γ^μ since it doesn't couple to the photon nor the electron in the massless limit. The only potential dependence on m_t comes from the diagrams of fig. 5.38. However, these are exactly zero in the on-shell scheme. Therefore, we can take $\delta Y = Y - 1 = 0$.

The vertex counterterms that are needed for this calculation are the following: WW, $W\phi$, $\phi\phi$, $t\bar{t}$, tbW, $tb\phi$, tdW, $td\phi$ and dbZ. Appendix D lists the general form of these counterterms. From this Appendix, we deduce that we need the following renormalization constants: T, δM_W^2, δM_Z^2, δZ_W, δZ_{ZZ}, δZ_ϕ, δm_t,$\delta Z^{tt}_{L(R)}$, δZ^{bb}_L and δZ^{bd}_L. The last of these

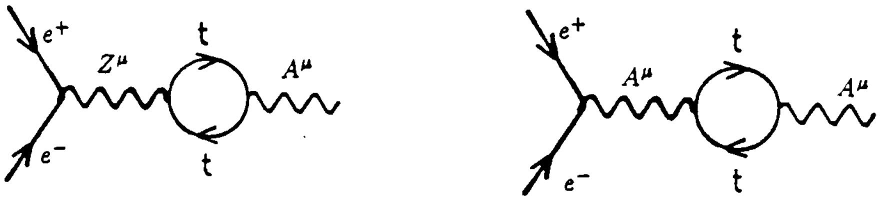


Figure 5.38: Top-quark contribution to the e^+e^--photon vertex function

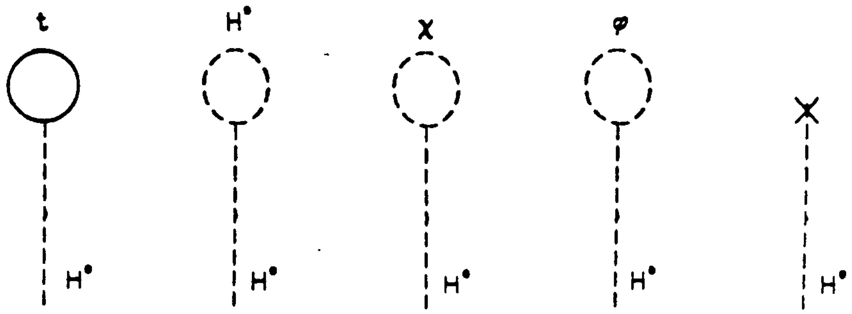


Figure 5.39: The tadpole graphs

is needed because the leading contribution to the renormalization of the $td\phi$ vertex is $\delta Z_L^{bd}[tb\phi]$.

The diagrams needed for the tadpole T are shown in fig. 5.39. The last two are needed since the $H\phi^+\phi^-$ and $H\chi^2$ couplings are proportionnal to M_H^2. The renormalization constants δM_W^2 and δZ_W are determined from the W self-energy (fig. 5.40). The Z self-energy (fig. 5.41) yields δM_Z^2 and δZ_{ZZ}. Note that we need to calculate the renormalization constants of the Z propagator even though the Z boson is not part of any of the "leading" two-loop graphs. This is a peculiar feature of the on-shell renormalization scheme.

We get only one renormalization constant, δZ_ϕ, from the ϕ self-energy of fig. 5.42 since the mass renormalization is already taken care of by the tadpole. The renormalization

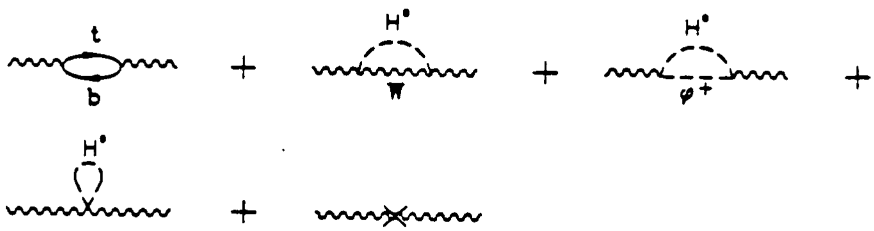


Figure 5.40: The W self-energy

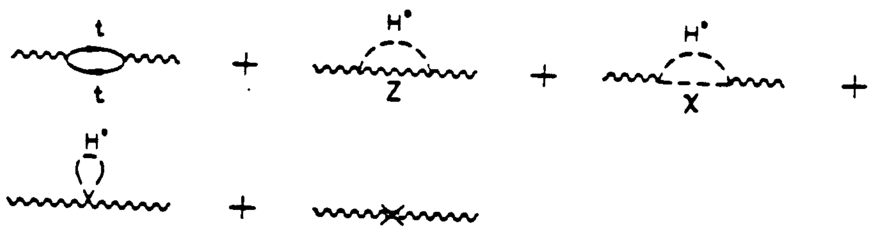


Figure 5.41: The Z self-energy

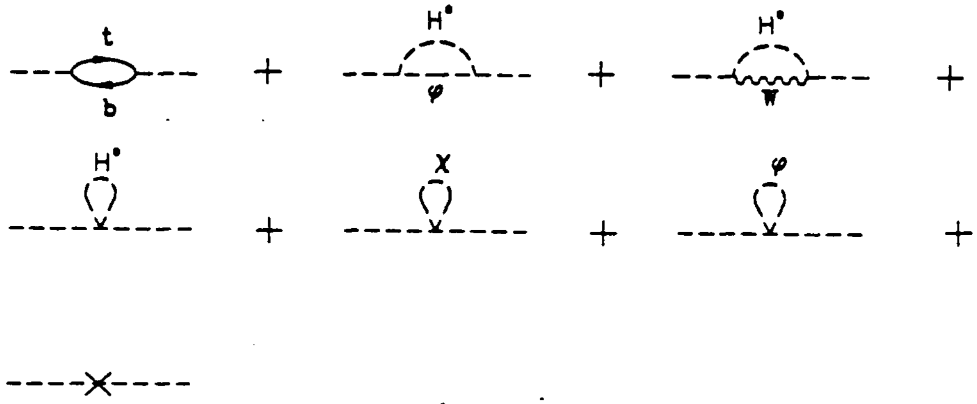


Figure 5.42: ϕ self-energy

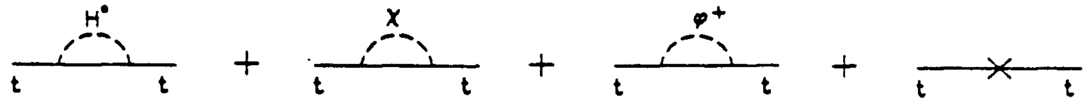


Figure 5.43: Self-energy of the top quark

constants δm_t, δZ_L^{tt} and δZ_R^{tt} are obtained from the top quark self-energy (fig. 5.43). Finally, only one graph contributes to the bb and bd self-energies (fig. 5.44) and the determination of δZ_L^{bb} and δZ_L^{bd} (the mass renormalizations and δZ_R are both zero).

The complete results for the counterterms are shown in Appendix E.

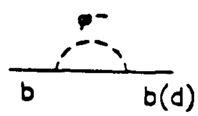


Figure 5.44: Self-energy of the b-quark

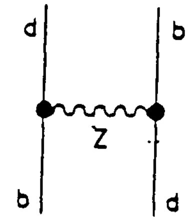


Figure 5.45: The di-penguin diagrams

5.4 The di-penguins

We are now in position to tackle the two-loop calculation. We begin with the Z "di-penguin" diagrams of fig. 5.45. These can be extracted readily from the work of Inami and Lim [19]. These authors calculated the $Z\bar{b}d$ vertex at one-loop. Their assumptions are less restrictive than ours and therefore their results could be used directly. We will not do this here but instead present a calculation of the vertex in the on-shell scheme in the large m_t limit. Since the result is independent of the renormalization scheme used, this will give a check on Inami and Lim's calculation.

The dipenguin SP of fig. 5.45 is related to the $Z\bar{b}d$ vertex function $\Gamma^\mu_Z(0)$ (fig.5.46) by

$$SP = \frac{i}{M_Z^2}\Gamma^\mu_Z(0)\Gamma_{\mu Z}(0) \tag{5.90}$$

In the large m_t limit, only three graphs contribute to Γ^μ_Z (see fig. 5.47). Graph (c) is trivial to evaluate given the counterterm of Appendix E. Graphs (a) and (b) are called penguin diagrams, hence the name dipenguin for fig. 5.45. They can be expressed in terms of the integral

$$H^1(m) = \int \frac{d^np}{(2\pi)^n}\frac{1}{(p^2-m^2)}$$

This is so because the external momenta are taken to be zero. The most general form of

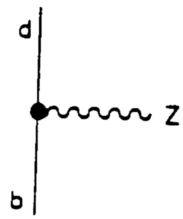


Figure 5.46: The flavor-changing three-point function Zbd

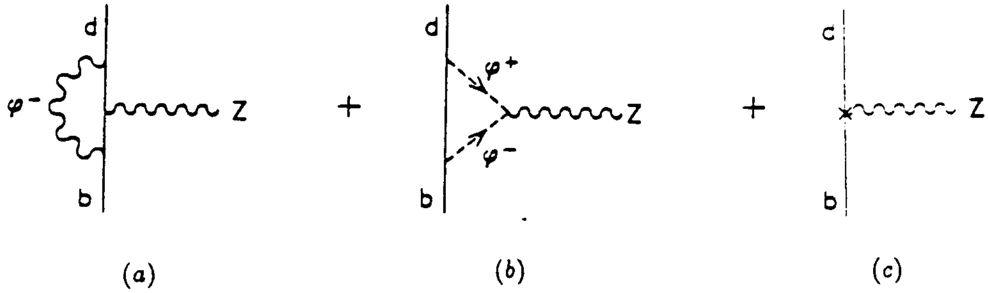


Figure 5.47: Contributions to the Zbd vertex

a 1-loop integral under those circumstances is

$$J_1 = \int \frac{d^n p}{(2\pi)^n} \frac{(p^2)^\alpha}{D} \tag{5.91}$$

with

$$D = \prod_{i=1}^{k} (p^2 - m_i^2)^{\beta_i} \tag{5.92}$$

where α, k and β_i are integers. We can use partial fractions to write

$$\frac{1}{D} = \sum_j \frac{C_j}{(p^2 - m_j^2)^{\gamma_j}}$$

where the coefficients C_j and the integers γ_j can be found from the explicit form of D (eq.5.92). Substituting this in 5.91, we see that we can write J_1 as a sum of integrals of the form

$$\int \frac{d^n p}{(2\pi)^n} \frac{(p^2)^\alpha}{(p^2 - m^2)^\beta}.$$

This in turn can be written as a sum of integrals of the form

$$H^k(m) = \int \frac{d^n p}{(2\pi)^n} \frac{1}{(p^2 - m^2)^k} \tag{5.93}$$

via the replacement $p^2 \to (p^2 - m^2) + m^2$ and an expansion of the numerator. Finally, $H^k(m)$ can be recursively related to $H^1(M)$. Note that

$$H^k(M) = \frac{1}{(k-1)} \frac{\partial}{\partial m^2} H^{k-1}(m).$$

Also, the scaling $p \to mp'$ in 5.93 shows that $H^k(m) = (m^2)^{(n/2-k)} A_k$ where A_k is independent of m. Therefore,

$$m^2 \frac{\partial}{\partial m^2} H^k(m) = \left(\frac{n}{2} - k\right) H^k(m)$$

and finally

$$H^k(m) = \frac{(n/2 - k + 1)}{(k-1)} \frac{H^{k-1}(m)}{m^2}$$

All we need to calculate explicitly is $H^1(m)$. This can be done easily using the results of Appendix C.

$$H^1(m) = \frac{im^2}{16\pi^2} \left(\frac{2}{\epsilon} - \gamma_E + 1 - \ln \frac{m^2}{4\pi}\right)$$

Using this and the counterterm formula of Appendix E for the $\bar{b}dZ^\mu$ vertex, we find explicitly:

$$SP = -\frac{ig^6}{4096\pi^4} \left(\frac{m_t^4}{M_W^6}\right) \tag{5.94}$$

This result holds in both limits since, as we will see, the dipenguins are independent of M_H.

There are other possible dipenguin diagrams, namely those where the Z is replaced by any other neutral boson. However, these are all suppressed. If the particle exchanged is a photon or a gluon, gauge-invariance ensures that the amplitude behaves at most as m_t^2. The reasoning that leads to this conclusion is similar to that of section 5.3. The

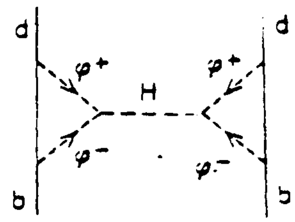


Figure 5.48: A penguin diagram involving the Higgs

three point function $\bar{b}dA^\mu = \Gamma_A^\mu$ vanishes at zero external momentum by gauge-invariance (see [19]). When we construct the dipenguin,we take $\Gamma_A^\mu(0)\Gamma_A^\nu(0)P_{\mu\nu}(0)$ where $P_{\mu\nu}$ is the photon propagator $P_{\mu\nu} = -ig_{\mu\nu}/k^2$. Owing to the pole in the photon propagator at zero momentum, this expression is finite but dimensional analysis shows that it can behave at most as m_t^2 since it lacks the $1/M_Z^2$ factor of eq. 5.90.

The case of the Higgs (or χ) exchange is interesting. The three point function Γ_H has a contribution of the form shown in fig. 5.48. Since the coupling $\phi^+\phi^- H$ is proportionnal to M_H^2, we would expect this to be very large. However, in the limit $m_b = m_d = 0$ amd zero external momentum, the loop integral is actually zero. (The integrand is an odd function of p.) This means that, had we kept the mass of the b quark non-zero, we would have found a result proportionnal to $m_b^2 M_H^2$ for the loop and the dipenguin would behave as $(m_b^2 M_H^2)^2/M_H^2 = m_b^4 M_H^2$ where the $1/M_H^2$ comes from the Higgs propagator. This could be comparable to the leading m_t^4 term if

$$\frac{m_b^4 M_H^2}{M_W^4} \geq \frac{m_t^4}{M_Z^2}$$

or $M_H \geq 30TeV$ for $m_t = 90GeV$. We will ignore this possibility here. The same conclusion applies to the χ exchange.

This completes our study of the dipenguin diagrams. We will now turn to the box diagrams.

5.5 The box diagrams

The evaluation of the box diagrams is much more involved. We will give an overview here and leave the details of the calculations to Appendix F. The results are shown in Appendix G.

After first taking care of the Dirac and Lorentz structure we are left with a scalar integral of the form $\int f(p,q)/D$ where

$$D = \prod_{i=1}^{k_1}(p^2 - m_i^2)^{\alpha_i} \prod_{j=1}^{k_2}(q^2 - m_j^2)^{\beta_j} \left((p-q)^2 - M^2\right)$$

and $f(p,q)$ is a quartic polynomial in p and q. (In this and the following, the integration measure $\dfrac{d^n p}{(2\pi)^n}\dfrac{d^n q}{(2\pi)^n}$ is not written explicitly. Partial fractions reduce this integral to a sum of terms $\int f(p,q)/D'$ where

$$D' = (p^2 - m_1^2)^i (q^2 - m_2^2)^j ((p-q)^2 - m_3^2)$$

The polynomial $f(p,q)$ can be eliminated by using $p^2 = (p^2 - m_1^2) + m_1^2$ and other such relations. We are left with two types of terms

- Products of one-loop integrals $H^i(M_1)H^j(M_2)$
- Terms of the form $I^{i,j}(m_1, m_2, m_3)$ where $I^{i,j} = \int 1/D'$

The first entry in Appendix G gives the value of the diagrams in terms of $I^{i,j}$ and H^k. Partial differentiation allows us to express all of the $I^{i,j}$ in term of $I^{2,1}$ and $I^{1,1}$. $I^{2,1}$ can also be related to $I^{1,1}$ by partial differentiation but we will not do so. (In Appendix F, we derive an algebraic relationship between $I^{1,1}$ and $I^{2,1}$.) All we have to do now is to evaluate $I^{2,1}(m_1, m_2, m_3)$. This can be done analytically in terms of dilogarithms. We can then expand $I^{2,1}$ for large m_t (or large M_H) and collect the leading terms. The second entry in Appendix G corresponds to the large M_H limit and the third to the large m_t limit.

Only the first step (reduction of digrams to scalar integrals) is done "by hand", the rest is done by computer using Mathematica programs. To minimize the possibility of error, two different codes were written independently (with co-worker Pankaj Agrawal) [22] and the results compared.

5.6 The results

The results of Appendix G exhibit the behavior previously mentioned, namely the large M_H results consist of terms of the form M_H^4, M_H^2, $\ln^2 M_H$, $\ln M_H$, and m_t^4 while the large m_t results show only the m_t^4 behavior. Several features of these results deserve further examination.

In diagrams of type V and VI, there is a tendency for the leading behavior to cancel when the basic graph and its counterterm are added. This is most visible in the self-energy insertions (type VI) and can be seen to follow from our choice of renormalization scheme.

The general expression of an unrenormalized self-energy for a particle of mass M can be expanded in, say, the large m_t limit:

$$\Sigma(k^2) = m_t^2 \left(a + b\frac{k^2}{m_t^2} + ck^4 m_t^4 + \cdots \right)$$

The leading behavior is m_t^2. In the on-shell scheme, the renormalized self-energy is given by:

$$\Sigma_R(k^2) = \Sigma(k^2) - \Sigma(M^2)$$

Clearly, Σ_R does not have an m_t^2 term. This would not be true with minimal subtraction (for instance) as the renormalization scheme since in MS, only the divergent part of a is removed by renormalization.

This argument applies only to the physical particles. The unphysical particles $\phi^\pm$ and χ are not renormalized on-shell but rather through the tadpole counterterm. However,

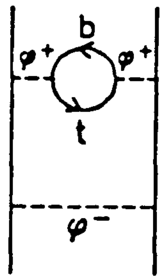


Figure 5.49: ϕ self-energy correction

an explicit calculation shows that the leading behavior cancels in them as well. This means that the leading behavior cancels in all one-loop self-energy graphs. We can't conclude from this that every two-loop diagram with a self-energy insertion will share this property: the argument merely suggests that it is a possibility. The actual calculation gives a cancellation of the leading term for all but one of the type VI graphs. The exception, shown in fig. 5.49 is the quark contribution to the self-energy of the $\phi^{\pm}$ which exhibits an m_t^4 behavior. The reason why this occurs is still unclear.

In the large M_H limit, the first two leading orders (M_H^4 and M_H^2) cancel in both the type V and type VI graphs. This is another manifestation of the screening theorem discussed in section 3.4. The behavior left-over is only logarithmic in M_H but is usually quite complicated in m_t. There is also a $\log^2 M_H$ term in graph $D5$ (fig. 5.50). The dependence on m_t of this term is remarkably simple, namely m_t^4 without any corrections. Again, the reason for this is unknown.

The sum of all graphs of Appendix G and the dipenguin term (eq. 5.94) is given by:

- In the large m_t limit:
$$-\frac{ig^6\alpha_t^2\eta_1}{16384\pi^4}\left(\frac{m_t^4}{M_W^6}\right)\left(17-6\frac{c^2}{s^2}\right)$$
- In the large M_H limit:
$$-\frac{ig^6\alpha_t^2\eta_1}{1179648\pi^4}\left(\frac{m_t^4}{M_W^6}\right)\left(491+60\pi^2-432\frac{c^2}{s^2}\right)$$

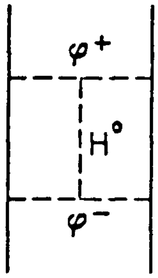


Figure 5.50: Diagram $D5$. This diagram is the only one to have a $\log^2 M_H$ dependence.

$$
\begin{aligned}
&-\frac{ig^6\alpha_t^2\eta_1}{32768\pi^4}\left(\frac{m_t^4}{M_W^6}\right)\ln^2(M_H^2/m_t^2)\\
&+\frac{ig^6\alpha_t^2\eta_1}{49152\pi^4}\left(\frac{m_t^4}{M_W^6}\right)\ln(M_H^2/m_t^2)\Big\{5-60x+340x^2-420x^3-141x^4\\
&+452x^5-176x^6+x^2\ln x(93+104x-373x^2+176x^3)\Big\}
\end{aligned}
$$

where $c=\cos\theta_W$, $s=\sin\theta_W$ and $x=M_W^2/m_t^2$. An important comment about this result is in order. Recall from eq. 3.61 that the result for the box diagrams can be written as:

$$\text{Box}=\alpha_t^2\left(E(m_t,m_t)-2E(m_t,0)+E(0,0)\right)$$

The above results include only $E(m_t,m_t)$ and $E(0,0)$. The case with different quark masses requires a separate calculation. this is inconsequential for the pure m_t^4 terms and for the $\ln^2\frac{M_H^2}{m_t^2}$ term since in these cases, the neglected term is subleading. However, $E(m_t,0)$ is needed for the $\ln(M_H^2/m_t^2)$ term. This is reserved for future work. In order to facilitate comparison with the 1-loop result, we can recast these formulas as corrections to $f(x)$ of eq. 3.62, that is

$$
\begin{aligned}
M_{12} &= \langle B_d|H_{\text{eff}}|\bar{B}_d\rangle\\
H_{\text{eff}} &= \frac{g^4}{32\pi^2}\left(\frac{m_t^2}{M_W^4}\right)\frac{1}{2}\Big\{\frac{4}{3}(\bar{d}\gamma_\mu Lb)^2+2(\bar{d}\lambda_A\gamma_\mu Lb)^2\Big\}(f(x)+\Delta)
\end{aligned}
$$

where

$$\Delta=\frac{g^2}{512\pi^2}\left(\frac{m_t^2}{M_W^2}\right)\kappa$$

with

$$
\begin{aligned}
\kappa &= \left(17 - 6\frac{c^2}{s^2}\right) \quad \text{large } m_t \\
&= \frac{1}{72}\left(491 + 60\pi^2 - 432\frac{c^2}{s^2}\right) + \frac{1}{2}\ln^2\frac{M_H^2}{m_t^2} \quad \text{large } M_H
\end{aligned}
$$

where the $\ln M_H^2$ terms have not been included since their calculation is incomplete. Numerically, the correction is negative and very small.

$$
\begin{aligned}
\Delta &= -1 \times 10^{-4}\left(\frac{m_t^2}{M_W^2}\right) \quad \text{large } m_t \\
&= -1 \times 10^{-4}\left(\frac{m_t^2}{M_W^2}\right)\left(3 - \frac{1}{2}\ln^2\frac{M_H^2}{m_t^2}\right) \quad \text{large } M_H
\end{aligned}
$$

Chapter 6

Discussion and conclusion.

We have shown in the previous chapter that the 2-loop correction to the $B_d - \bar{B}_d$ mixing is much smaller than the 1-loop result for reasonable values of m_t ($m_t \leq 250\ GeV$). Hence, predictions based on the 1-loop result are unaffected by our calculation. In practice, this means that if the top quark is found with a mass larger than that allowed by the present experimental value of $B_d - \bar{B}_d$ mixing (which yields the bound $m_t \leq 200\ GeV$), the source of the discrepancy will not be higher order corrections in electroweak theory in the Standard Model. In that case, one would have to reevaluate the hadronic matrix elements and/or include higher order strong interaction corrections. More interestingly, the existence of new particles such as those predicted by the supersymmetric models may then be required.

It is interesting to compare this result to the similar calculation of the ρ-parameter at two-loops by Van der Bij and Hoogeven [13]. In both cases, the ratio of the 2-loop to the 1-loop result is small and negative. It may be a little premature to generalize from these observations. Nevertheless, if we assume that the pattern holds for all low-energy processes, the two following conclusions can be drawn.

The first conclusion, which has already been alluded to, is that the perurbation series for low-energy processes converges a lot faster than what one would naively expect. In practice, this means that the lowest order calculations will often be accurate enough to describe a phenomenon completely.

The second conclusion has to do with the point at which perturbation theory fails.

We have already mentioned several times that the 2-loop result is small if $m_t \leq 250\ GeV$. If we do not impose this restriction on m_t, the 2-loop correction can be quite large. In fact, for $m_t \approx 10\ TeV$, the 2-loop and 1-loop amplitudes are of the same order. This is also what is found in ref. [13]. Given that the 1 and 2-loop results have opposite signs, they tend to cancel for $m_t \approx 10\ TeV$. Therefore, for such a large value of m_t, decoupling appears to hold: the low-energy observables do not depend on m_t. There is a problem with this argument, however: if the two-loop corrections are large, the three-loop corrections might very well be large too! A careful discussion of possible decoupling would require calculations at three-loops or higher. Given the difficulty involved in the two-loop calculation, it appears unlikely that the three-loop calculation can be done, unless some new calculation technique or approximate theories are developped. This is indeed a very active field of research (see, for instance, ref. [23] where an effective field theory for very massive particles is constructed). Besides establishing the accuracy of perturbation theory for $m_t < 250\ GeV$, our result can also be used to check the validity and consistency of these recently developed effective field theories by comparing their result at two-loop with ours.

It is also perfectly possible that the similarities between our calculation and that of Hoogeven and Van der Bij are totally accidental. In this case, there could well exist some low-energy observables for which the two-loop corrections are large, even for modest m_t. A possibility is that of rare kaon decays such as $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ which are known to be sensitive to the value of the mass of the top quark. Besides the box diagram, this kaon decay also proceeds through the three-point function $Z^\mu ds$ (fig. 6.51). This function is expected to have a dependence on m_t and M_H of the same form as the one found here but perhaps with larger coefficients. This is reserved for future work.

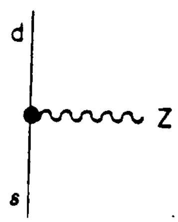


Figure 6.51: The $Z^\mu ds$ three-point function

Bibliography

[1] Argus collab. Phys. Lett. B192 (1987) 245.

[2] C. Itzykson, J.B. Zuber. Quantum Field Theory. M^{c}Graw-Hill (1980).

[3] T8.P. Cheng, L.F. Li. Gauge Theory of Elementary Particle Physics. Oxford University Press (1984).

[4] K. Aoki et al. Supp. Prog. Theor. Phys. 73 (1982) 1.

[5] J.C. Taylor. Gauge Theories of Weak Interactions. Cambridge University Press (1976).

[6] J. Collins. Renormalization. Cambridge University Press (1984).

[7] E.D. Cummins, P.H. Bucksbaum. Weak Interactions of Leptons and Quarks. Cambridge University Press (1983).

[8] R.L. Kingsley. Phys. Lett. B63 (1976) 329.

8I. Bigi, A.I. Sanda. Phys. Lett. B171 (1986) 320.

[9] See for instance:

A. Sirlin. Phys. Rev. D22 (1980) 971.

W.J. Marciano, A. Sirlin. Phys. Rev. D22 (1980) 2695.

D.J. Peters. Masters Thesis. U.B.C. (1991).

[10] M. Veltman. Nucl. Phys. B123 (1977) 89.

[11] M.B. Einhorn et al. Nucl. Phys. B191 (1981) 146.

[12] T. Applequist, J. Carrazone. Phys. Rev. D11 (1974) 2856.

[13] J.J. Van der Bij, F. Hoogeven. Nucl. Phys. B283 (1987) 477.

[14] J. Van der Bij, M. Veltman. Nucl. Phys. B231 (1983).

[15] G. 't Hooft, M. Veltman. Nucl. Phys. B44 (1972) 189.

[16] D. Espriu, R. Tarrach. Z. Phys. C16 (1982) 77.

[17] See, for instance:

Z Physics at LEP I G. Altarelli, R. Kleiss, C. Verzegnassi eds. CERN 89-08 Vol. 1 (1989)

for a theorist's account.

[18] See, for instance:

A.J. Buras et al. Nucl. Phys. B238 (1983) 529.

[19] T. Inami, C.S. Lim. Prog. Theor. Phys. C5 (1981) 297.

[20] Aleph collab., D. Decamp et al. Phys. Lett. B257 (1991) 492.

[21] F. Mandl, G. Shaw. Quantum Field Theory. Wiley Interscience (1984)

[22] P. Agrawal, S. Fortin, J.N. Ng. In preparation.

[23] G.L. Lin, H. Steger, Y.P. Yao. CERN Preprint TH.6006/91 (1991)

Appendix A

Notations and conventions

A.1 Metric

The metric used is $(\mu, \nu = 0, 1, 2, 3)$

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Summation over repeated indices is understood unless explicitly stated.

$$a \cdot b = a_\mu b^\mu = a^\mu b_\mu = g_{\mu\nu} a^\mu b^\nu = g^{\mu\nu} a_\mu b_\nu$$

The abbreviations

$$\partial^\mu \equiv \frac{\partial}{\partial x_\mu} \, , \; \partial_\mu \equiv \frac{\partial}{\partial x^\mu} \, , \; \Box = \partial^\mu \partial_\mu$$

are often used.

Three-dimensional vectors are denoted by boldface letters.

A.2 Dirac matrices

The Dirac matrices γ^μ satisfy

$$\{\gamma^\mu, \gamma^\nu\} \equiv \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}$$

with γ^0 hermitian and γ^i antihermitian $(i = 1, 2, 3)$.

The matrix γ_5 is defined by

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = -\frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma = \gamma_5^\dagger$$

with

$$\gamma_5^2 = 1 \;,\; \{\gamma_5, \gamma^\mu\} = 0$$

The totally antisymmetric Levi-Civita tensor is given by:

$$\epsilon^{\mu\nu\rho\sigma} = \begin{cases} +1 \text{ if } \{\mu,\nu,\rho,\sigma\} \text{ is an even permutation of } \{0,1,2,3\} \\ \text{-1 if it is an odd permutation} \\ \text{0 otherwise} \end{cases}$$

<u>Identities</u>:

$$\epsilon^{\mu\nu\rho\sigma} = -\epsilon_{\mu\nu\rho\sigma}$$
$$\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu\nu}{}^{\rho'\sigma'} = -2(g^{\rho\rho'}g^{\sigma\sigma'} - g^{\rho\sigma'}g^{\rho'\sigma})$$
$$\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu\nu\rho}{}^{\sigma'} = -6g^{\sigma\sigma'}$$
$$\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu\nu\rho\sigma} = -24$$

<u>Reduction formula</u>

$$\gamma_\alpha\gamma_\beta\gamma_\lambda = g_{\alpha\beta}\gamma_\lambda + g_{\beta\lambda}\gamma_\alpha - g_{\alpha\lambda}\gamma_\beta + i\epsilon_{\mu\alpha\beta\lambda}\gamma^\mu\gamma_5 \tag{A.95}$$

<u>Traces</u>:

Trace of an odd product of γ^μ matrices vanishes

$$Tr(\gamma_5\gamma^\mu) = 0$$
$$Tr(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu}$$
$$Tr(\gamma^\mu\gamma^\nu\gamma_5) = 0$$
$$Tr(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho})$$
$$Tr(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5) = -4i\epsilon^{\mu\nu\rho\sigma}$$

Completeness.

Let $\Gamma_S = 1$, $\Gamma_V = \gamma_\mu$, $\Gamma_T = \frac{i}{2}[\gamma^\mu, \gamma^\nu] \equiv \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$, $\Gamma_A = \gamma_\mu\gamma_5$, $\Gamma_P = \gamma_5$.

An arbitrary 4×4 matrix can be expressed as a combination of the Γ with the (generally complex) coefficients found by taking traces.

A.3 The Pauli matrices

The Pauli matrices $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

They satisfy

$$[\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma^k$$

$$\{\sigma_i, \sigma_j\} = 2\delta_{ij}$$

$$Tr(\sigma_i\sigma_j) = 2\delta_{ij}$$

with ϵ_{ijk} totally antisymmetric and $\epsilon_{123} = 1$. Also:

$$\sigma_{iab}\sigma^i{}_{cd} = 2(\delta_{bc}\delta_{ad} - \tfrac{1}{2}\delta_{ab}\delta_{cd})$$

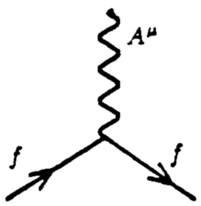
Appendix B

Feynman Rules

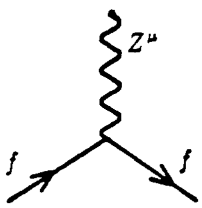
The Feynman rules for the vertices in the Standard Model of electroweak theory are given below. The list does not include the vertices involving the ghosts. These can be found elsewhere (see for instance [2]). The vertices can be derived using the methods of Chapter 2. We also give the expression of the propagators for fermions and spin 0 and 1 bosons.

B.1 Vertices involving fermions

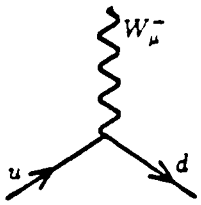
In each diagram, u can be replaced by c or t and d can be replaced by s or b, with the corresponding change in the Feynman rule. Similarly, e^- can be replaced by μ^- or τ^-, provided that ν_e is correspondingly changed to ν_μ or ν_τ. The symbol f refers to any of the quarks or leptons.



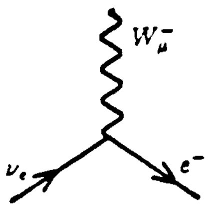
$$= ieQ_f\gamma_\mu$$



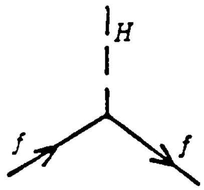
$$= \frac{ig}{2c}\gamma_\mu(c_L^f L + c_R^f R)$$



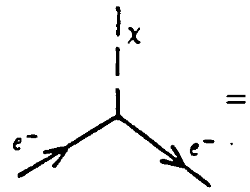
$$= \frac{ig}{\sqrt{2}}V_{ud}\gamma_\mu L$$



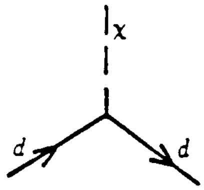
$$= \frac{ig}{\sqrt{2}}\gamma_\mu L$$



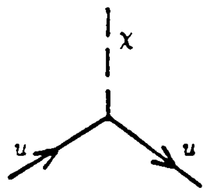
$$= -\frac{igm_f}{2M_W}$$



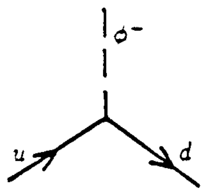
$$= \frac{gm_e}{2M_W}\gamma_5$$



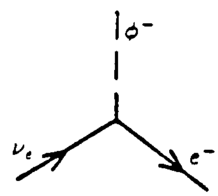
$$= \frac{gm_d}{2M_W}\gamma_5$$



$$= -\frac{gm_u}{2M_W}\gamma_5$$



$$= -\frac{ig}{\sqrt{2}M_W}V_{ud}(m_d L - m_u R)$$



$$= -\frac{igm_e}{\sqrt{2}M_W}L$$

Here, Q_f, c_L^f, and c_R^f are given in the following table:

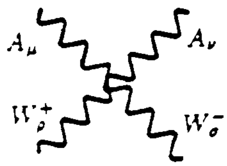
f	Q_f	c_L^f	c_R^f
u,c,t	$\frac{2}{3}$	$\frac{1}{2} - \frac{2}{3}s^2$	$-\frac{2}{3}s^2$
d,s,b	$-\frac{1}{3}$	$-\frac{1}{2} + \frac{1}{3}s^2$	$\frac{1}{3}s^2$
ν_e,ν_μ,ν_τ	0	$\frac{1}{2}$	0
e^-,μ^-,τ^-	-1	$-\frac{1}{2} + s^2$	s^2

and $s = \sin\theta_W$, $c = \cos\theta_W$. V is the Kobayashi-Maskawa mixing matrix.

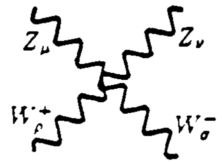
B.2 Vertices involving the gauge-bosons

In these diagrams, k_1, k_2, k_3 refer to the momenta of the particles. All particles are considered to be entering the vertex for the purpose of determining the sign of the momentum or charge. The following definitions are used:

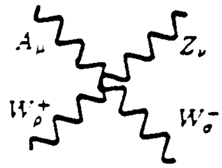
$$\begin{aligned} C_{\mu\nu\rho}(k_1, k_2, k_3) &= (k_1 - k_2)_\rho g_{\mu\nu} + (k_2 - k_3)_\mu g_{\nu\rho} + (k_3 - k_1)_\nu g_{\mu\rho} \\ S_{\mu\nu,\rho\sigma} &= 2g_{\mu\nu} g_{\rho\sigma} - g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho} \end{aligned}$$



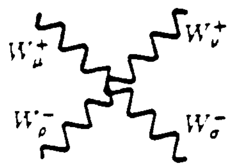
$= -ie^2 S_{\mu\nu,\rho\sigma}$



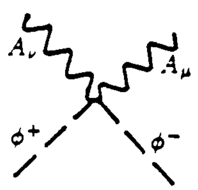
$= -ig^2 c^2 S_{\mu\nu,\rho\sigma}$



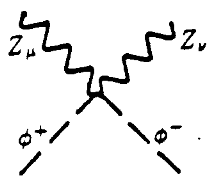
$= -iegc S_{\mu\nu,\rho\sigma}$



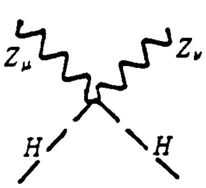
$= ig^2 S_{\mu\nu,\rho\sigma}$



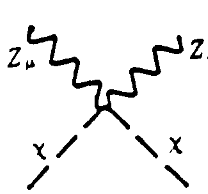
$= 2ie^2 g_{\mu\nu}$



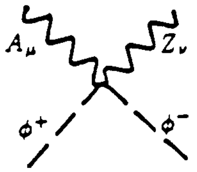
$= \dfrac{ig^2}{2c^2}(1 - 2s^2)^2 g_{\mu\nu}$



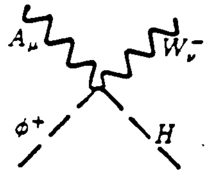
$= \dfrac{ig^2}{2c^2} g_{\mu\nu}$



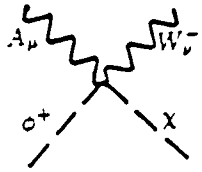
$= \dfrac{ig^2}{2c^2} g_{\mu\nu}$



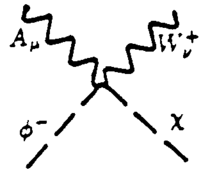
$$= -\frac{ieg}{c}(1-2s^2)g_{\mu\nu}$$



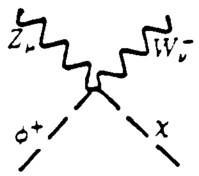
$$= \frac{ieg}{2}g_{\mu\nu}$$



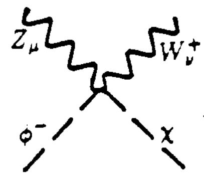
$$= \frac{eg}{2}g_{\mu\nu}$$



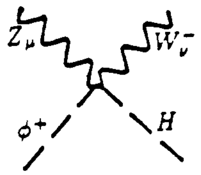
$$= -\frac{eg}{2}g_{\mu\nu}$$



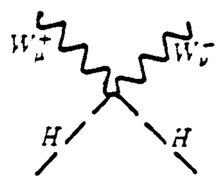
$$= -\frac{e^2}{2c}g_{\mu\nu}$$



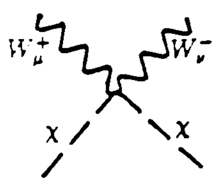
$$= \frac{e^2}{2c}g_{\mu\nu}$$



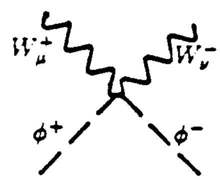
$$= -\frac{ie^2}{2c}g_{\mu\nu}$$



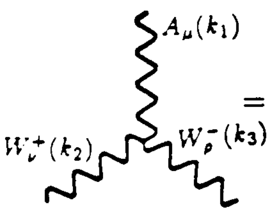
$$= \frac{ig^2}{2}g_{\mu}\nu$$



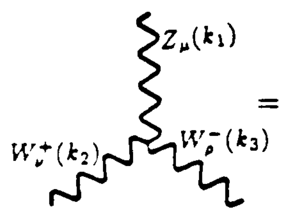
$$= \frac{ig^2}{2}g_{\mu\nu}$$



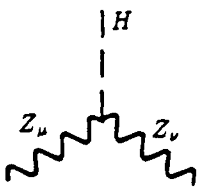
$$= \frac{ig^2}{2}g_{\mu\nu}$$



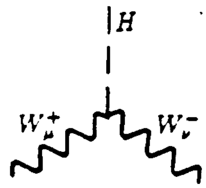
$$= -ieC_{\mu\nu\rho}(k_1,k_2,k_3)$$



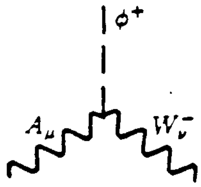
$$= -igcC_{\mu\nu\rho(k_1,k_2,k_3)}$$



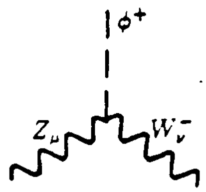
$$= \frac{ig}{c} M_Z g_{\mu\nu}$$



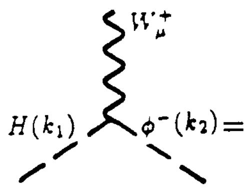
$$= igM_W g_{\mu\nu}$$



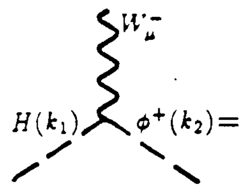
$$= ieM_W g_{\mu\nu}$$



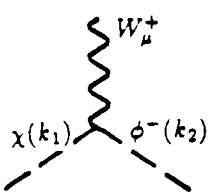
$$= -iesM_Z g_{\mu\nu}$$



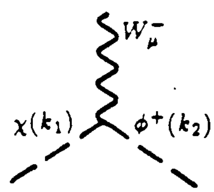
$$= \frac{ig}{2}(k_1 - k_2)_\mu$$



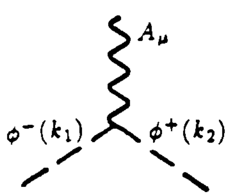
$$= \frac{ig}{2}(k_2 - k_1)_\mu$$



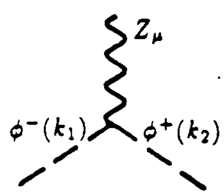
$$= \frac{g}{2}(k_2 - k_1)_\mu$$



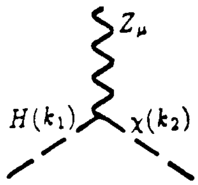
$$= \frac{g}{2}(k_2 - k_1)_\mu$$



$$= ie(k_2 - k_1)_\mu$$



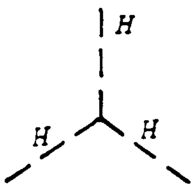
$$= \frac{ig}{2c}(1 - 2s^2)(k_2 - k_1)_\mu$$



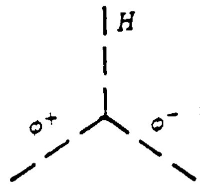
$$= \frac{g}{2c}(k_2 - k_1)_\mu$$

B.3 Higgs vertices

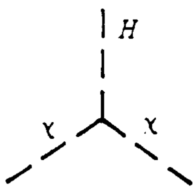
The following diagrams come from the Higgs potential.



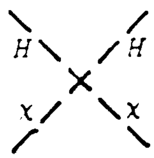
$$= -\frac{3ig}{2}\frac{M_H^2}{M_W}$$



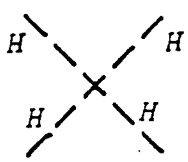
$$= -\frac{ig}{2}\frac{M_H^2}{M_W}$$



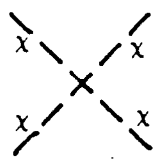
$$= -\frac{ig}{2}\frac{M_H^2}{M_W}$$



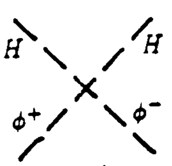
$$= -\frac{ig^2}{4}\frac{M_H^2}{M_W^2}$$



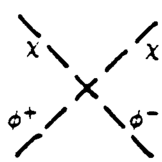
$$= -\frac{3ig^2}{4}\frac{M_H^2}{M_W^2}$$



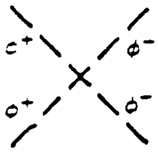
$$= -\frac{3ig^2}{4}\frac{M_H^2}{M_W^2}$$



$$= -\frac{ig^2}{4}\frac{M_H^2}{M_W^2}$$



$$= -\frac{ig^2}{4}\frac{M_H^2}{M_W^2}$$



$$= -\frac{ig^2}{2}\frac{M_H^2}{M_W^2}$$

B.4 Propagators

Vector-bosons

$$= \frac{-i}{k^2 - M^2}[g_{\mu\nu} + (\xi - 1)k_\mu k_\nu/(k^2 - \xi M^2)]$$

where M is the mass of the boson and the gauge parameter ξ will be set equal to 1.

Scalars

$$- - - - - - - - \quad = \frac{i}{k^2 - M^2}$$

where $M^2 = \xi M_W^2$ for $\phi^\pm$, $M^2 = \xi M_Z^2$ for χ, and $M^2 = M_H^2$ for the Higgs.

Fermions

$$\overline{\qquad\qquad} \quad = \frac{i}{\not{k} - m}$$

Appendix C

Integration in n-dimensions

We present here a few identities that are useful for the calculation of dimensionaly regularized Feynman integrals.

The Feynman identity

$$\frac{1}{A_1^{\alpha_1}\cdots A_n^{\alpha_n}} = \frac{\Gamma(\alpha_1+\cdots+\alpha_n)}{\Gamma(\alpha_1)\cdots\Gamma(\alpha_n)}\int dx_1\cdots dx_n\,\delta(1-x_1-\cdots-x_n)\times$$
$$\times\frac{x_1^{\alpha_1-1}\cdots x_n^{\alpha_n-1}}{(A_1x_1+\cdots+A_nx_n)^{\alpha_1+\cdots+\alpha_n}}$$

allows to combine the various propagators. Some explicit formulas for n-dimensional integrals are:

$$\begin{aligned}
J_0 &\equiv \int d^np/(-p^2+2p\cdot k+M^2)^\alpha \\
&= i\pi^{n/2}(k^2+M^2)^{n/2-\alpha}\Gamma(\alpha-n/2)/\Gamma(\alpha) \\
J_1^\mu &\equiv \int d^np\,p^\mu/(-p^2+2p\cdot k+M^2)^\alpha \\
&= i\pi^{n/2}(k^2+M^2)^{n/2-\alpha}k^\mu\Gamma(\alpha-n/2)/\Gamma(\alpha) \\
J_2^{\mu\nu} &\equiv \int d^np\,p^\mu p^\nu/(-p^2+2p\cdot k+M^2)^\alpha = i\pi^{n/2}(k^2+M^2)^{n/2-\alpha}\times \\
&\quad \left[\Gamma(\alpha-n/2)k^\mu k^\nu-\Gamma(\alpha-1-n/2)g^{\mu\nu}(k^2+M^2)/2\right]/\Gamma(\alpha)
\end{aligned}$$

Here, the Gamma-function is defined by:

$$\Gamma(x)=\int_0^\infty dt\,t^{x-1}e^{-t}$$

In particular, for small x:

$$\Gamma(x)\approx\frac{1}{x}-\gamma_E \qquad (x\approx 0)$$

where γ_E is Euler's constant ($\gamma_E = 0.534$).

An interesting identity that does not, strictly speaking, follow from J_0 is

$$\int d^n p\, p^\alpha = 0$$

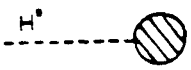
for all α and n.

Appendix D

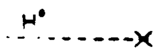
Renormalization conditions and counterterms.

In this Appendix, we tabulate the counterterms that are needed for our calculation and the renormalization conditions associated with them. We give both a parametrization of the Green functions and the structure of the counterterms. In the counterterm formulas, it should be understood that the part that survives when all the counterterms are taken to zero (the tree level value) should be subtracted.

D.1 Tadpole



=tadpole



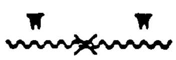
$=iT$

Renormalization condition : tadpole=0

Renormalization constant : T

D.2 Two-point functions(self-energies).

$$= \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right) A_W(k^2) + \frac{k^\mu k^\nu}{k^2} B_W(k^2)$$

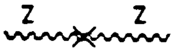


$$= i Z_W g^{\mu\nu} \left[\delta M_W^2 + (M_W^2 - k^2) \right] + i Z_W k^\mu k^\nu$$

$$\text{Renormalization conditions}: \quad \begin{aligned} A_W(M_W^2) &= 0 \\ A'_W(M_W^2) &= 0 \end{aligned}$$

Renormalization constants : Z_W, δM_W^2

$$=\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{k^2}\right)A_Z(k^2)+\frac{k^\mu k^\nu}{k^2}B_Z(k^2)$$



$$=ig^{\mu\nu}\left(Z_{ZZ}(M_Z^2+\delta M_Z^2)-k^2(Z_{ZZ}+Z_{AZ})\right)+ik^\mu k^\nu(Z_{ZZ}+Z_{AZ})$$

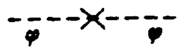
Renormalization conditions : $A_Z(M_Z^2) = 0$

$A'_Z(M_Z^2) = 0$

Renormalization constants : Z_{ZZ}, δM_Z^2

(For our purpose, Z_{AZ} can be set to zero.)

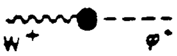
$$=F_\phi(k^2)$$



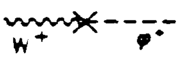
$$=-ik^2Z_\phi+i\frac{gTY}{2M_W}G_2H^{-1}Z_\phi$$

Renormalization condition : $F'_\phi(M_W^2)=0$

Renormalization constant : Z_ϕ



$$=ik^\mu C^+(k^2)$$



$$=-ik^\mu M_W G_W Z_W^{1/2}Z_\phi^{1/2}$$

There is no new renormalization constants needed for this function.

$$=\Sigma_1^{ij}(k^2)+\Sigma_5^{ij}(k^2)\gamma_5+\Sigma_\gamma^{ij}(k^2)\not{k}+\Sigma_{5\gamma}^{ij}(k^2)\not{k}\gamma_5$$

$$\begin{aligned}
= \; & i\left[-\frac{1}{2}(Z^{il\dagger}_{(u)L})^{1/2}(m_l^{(u)}+\delta m_l^{(u)})(Z^{lj}_{(u)R})^{1/2}-\frac{1}{2}(Z^{il\dagger}_{(u)R})^{1/2}(m_l^{(u)}+\delta m_l^{(u)})(Z^{lj}_{(u)L})^{1/2}\right]\\
&+i\gamma_5\left[-\frac{1}{2}(Z^{il\dagger}_{(u)L})^{1/2}(m_l^{(u)}+\delta m_l^{(u)})(Z^{lj}_{(u)R})^{1/2}+\frac{1}{2}(Z^{il\dagger}_{(u)R})^{1/2}(m_l^{(u)}+\delta m_l^{(u)})(Z^{lj}_{(u)L})^{1/2}\right]\\
&+i\not{k}\left[\frac{1}{2}(Z^{il\dagger}_{(u)L})^{1/2}(Z^{lj}_{(u)L})^{1/2}+\frac{1}{2}(Z^{il\dagger}_{(u)R})^{1/2}(Z^{lj}_{(u)R})^{1/2}\right]\\
&+i\not{k}\gamma_5\left[-\frac{1}{2}(Z^{il\dagger}_{(u)L})^{1/2}(Z^{lj}_{(u)L})^{1/2}+\frac{1}{2}(Z^{il\dagger}_{(u)R})^{1/2}(Z^{lj}_{(u)R})^{1/2}\right]
\end{aligned}$$

Renormalization conditions :

$$\Sigma_1^{ii}(m_i^2) + m_i \Sigma_\gamma^{ii}(m_i^2) = 0$$

$$\Sigma_5^{ii}(m_i^2) = 0$$

$$\Sigma_{5\gamma}(m_i^2) = 0$$

$$2m_i \Sigma_1^{ii\prime}(m_i^2) + \Sigma_\gamma^{ii}(m_i^2) + 2m_i^2 \Sigma_\gamma^{ii\prime}(m_i^2) = 0$$

for $i = j$ and

$$\Sigma_1^{ij}(m_j^2) + m_j \Sigma_\gamma^{ij}(m_j^2) = 0$$

$$\Sigma_5^{ij} - m_j \Sigma_{5\gamma}^{ij}(m_j^2) = 0$$

$$\Sigma_1^{ij}(m_i^2) + m_i \Sigma_\gamma^{ij}(m_i^2) = 0$$

$$\Sigma_5^{ij}(m_i^2) + m_i \Sigma_{5\gamma}^{ij}(m_i^2) = 0$$

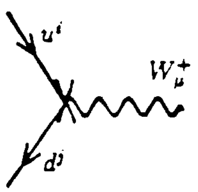
for $i \neq j$.

Renormalization constants : $Z_{(u)L}$, $Z_{(u)R}$, $m^{(u)}$.

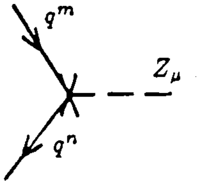
There is also the two-point function for the down-type quarks, which is the same as for the up-type quarks with $u \to d$.

D.3 The three-point functions

No new renormalization constants are needed for the three-point functions (it is argued in Chapter 5 that the charge renormalization is zero in our approximations, which means that we can take $Y = 1$ in the following formulas). We will use the notation $[\phi_1\phi_2\phi_3]$ for the three-point vertex $\phi_1 - \phi_2 - \phi_3$ of Appendix B and the subscripts R and L will refer to the right-handed and left-handed parts of the vertex respectively. We will only give the counterterms here.



$$= Y G_1 (Z_{(u)L}^{ik\dagger})^{1/2} (Z_{(d)L}^{lj})^{1/2} [W_\mu^+ d_l \bar{u}_k]$$



$$
\begin{aligned}
= \; & YG_1G_2Z_{ZZ}^{1/2}(Z_{(q)L}^{ml\dagger})^{1/2}[Z_\mu\bar{q}_l q_l]_L(Z_{(q)L}^{ln})^{1/2} \\
& +YG_1^{-1}G_2Z_{ZZ}^{1/2}(Z_{(q)\bar{\psi}}^{ml})^{1/2}[Z_\mu\bar{q}_l q_l]_Q(Z_{(q)\psi}^{ln})^{1/2}
\end{aligned}
$$

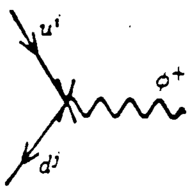
Here, $q = u$ or d and

$$
\begin{aligned}
Z_{(q)\psi}^{1/2} &= Z_{(q)R}^{1/2}R + Z_{(q)L}^{1/2}L \\
Z_{(q)\bar{\psi}}^{1/2} &= Z_{(q)L}^{1/2\dagger}R + Z_{(q)R}^{1/2\dagger}L
\end{aligned}
$$

Also, the vertex $[Z^\mu\bar{q}_l q_l]$ is defined as $\frac{ig}{2c}\gamma^\mu(A_qL - 2\sin^2\theta_W Q_q)$ where Q_q is the charge of the quark and

$$
\begin{aligned}
A_u &= 1/2 \\
A_d &= -1/2
\end{aligned}
$$

Finally, we also show the counterterm to the interactions between the charged would-be-Goldstone bosons and the quarks.



$$
\begin{aligned}
= \; & YG_W^{-1}G_1Z_\phi^{1/2}(Z_{(u)R}^{jk\dagger})^{1/2}G_{m_k}^{(d)}[\phi^+\bar{u}_k d_l]_L(Z_{(d)L}^{li})^{1/2} \\
& +YG_W^{-1}G_1Z_\phi^{1/2}(Z_{(u)L}^{jk\dagger})^{1/2}G_{m_k}^{(u)}[\phi^+\bar{u}_k d_l]_R(Z_{(d)R}^{li})^{1/2}
\end{aligned}
$$

The following abbreviations have been used throughout:

$$
\begin{aligned}
G_W &= \sqrt{1 + \frac{\delta M_W^2}{M_W^2}} \\
G_Z &= \sqrt{1 + \frac{\delta M_Z^2}{M_Z^2}} \\
G_{m_k}^{(q)} &= 1 + \frac{\delta m_k^{(q)}}{m_k^{(q)}}
\end{aligned}
$$

$$
\begin{aligned}
H &= \sqrt{1 + \frac{\delta M_Z^2 - \delta M_W^2}{M_Z^2 - M_W^2}} \\
G_1 &= G_Z / H \\
G_2 &= G_Z / G_W
\end{aligned}
$$

Appendix E

Renormalization constants and counterterms.

In this Appendix, we present the renormalization constants and the counterterms needed for our calculations.

The renormalization constants, as determined with the formulas of Appendix D, are as follows:

$$
\begin{aligned}
\delta(Z_L^{tt})^{1/2} = & -\frac{g^2}{128\pi^2}\left(\frac{m_t}{M_W}\right)^2 \left\{\frac{2}{\epsilon} - \gamma_E - \int_0^1 dx\,(1-x)\Big[\ln\Big(\frac{M_H^2(1-x)+m_t^2x^2}{4\pi}\Big)\right. \\
& + \ln\Big(\frac{M_Z^2(1-x)+m_t^2x^2}{4\pi}\Big)\Big] \\
& \left. +2m_t^2\int_0^1 dx\left[\frac{x(1-x)(2-x)}{M_H^2(1-x)+m_t^2x^2} - \frac{x^2(1-x)}{M_Z^2(1-x)+m_t^2x^2} + \frac{x(1-x)}{M_W^2-m_t^2x}\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
\delta(Z_R^{tt})^{1/2} = & -\frac{g^2}{128\pi^2}\left(\frac{m_t}{M_W}\right)^2 \left\{2\left(\frac{2}{\epsilon} - \gamma_E\right) - \int_0^1 dx\,(1-x)\Big[2\ln\Big(\frac{M_W^2(1-x)-m_t^2x(1-x)}{4\pi}\Big)\right. \\
& + \ln\Big(\frac{M_H^2(1-x)+m_t^2x^2}{4\pi}\Big) + \ln\Big(\frac{M_H^2(1-x)+m_t^2x^2}{4\pi}\Big)\Big] \\
& \left. +2m_t^2\int_0^1 dx\left[\frac{x(1-x)(2-x)}{M_H^2(1-x)+m_t^2x^2} - \frac{x^2(1-x)}{M_Z^2(1-x)+m_t^2x^2} + \frac{x(1-x)}{M_W^2-m_t^2x}\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\delta m_t}{m_t} = & \frac{g^2}{64\pi^2}\left\{\frac{3}{2}\left(\frac{2}{\epsilon} - \gamma_E\right) + \int_0^1 dx\,\ln\Big(\frac{M_Z^2(1-x)+m_t^2x^2}{M_H^2(1-x)+m_t^2x^2}\Big)\right. \\
& - \int_0^1 dx\,(1-x)\Big[\ln\Big(\frac{M_W^2(1-x)-m_t^2x(1-x)}{4\pi}\Big) + \ln\Big(\frac{M_H^2(1-x)+m_t^2x^2}{4\pi}\Big) \\
& \left. + \ln\Big(\frac{M_Z^2(1-x)+m_t^2x^2}{4\pi}\Big)\Big]\right\}
\end{aligned}
$$

$$T = \frac{g}{64\pi^2 M_W}\left\{\left(\frac{2}{\epsilon}-\gamma_E+1\right)(24m_t^4-3M_H^4-2M_H^2M_W^2-M_H^2M_Z^2)-24m_t^4\ln\frac{m_t^2}{4\pi}\right.$$
$$\left.+3M_H^4\ln\frac{M_H^2}{4\pi}+2M_H^2M_W^2\ln\frac{M_W^2}{4\pi}+M_H^2M_Z^2\ln\frac{M_Z^2}{4\pi}\right\}$$

$$\delta Z_\phi = -\frac{g^2}{64\pi^2 M_W^2}\left\{(M_H^4-2M_H^2M_W^2)\int_0^1 dx\,\frac{x(1-x)}{M_W^2x^2+M_H^2(1-x)}-2M_W^2\left[\frac{2}{\epsilon}-\gamma_E\right.\right.$$
$$\left.-\int_0^1 dx\,\ln\left(\frac{M_W^2x^2+M_H^2(1-x)}{4\pi}\right)\right]+6m_t^2\int_0^1 dx\left[\frac{2}{\epsilon}-\gamma_E+1-\ln\left(\frac{m_t^2-M_W^2x}{4\pi}\right)\right]$$
$$\left.-6m_t^2\int_0^1 dx\,\frac{x}{1-M_W^2x/m_t^2}\right\}$$

$$\delta M_W^2 = -\frac{g^2}{64\pi^2}\left\{+2\int_0^1 dx\,\left(M_W^2x^2+M_H^2(1-x)\right)\left[\frac{2}{\epsilon}-\gamma_E+1-\ln\left(\frac{M_W^2x^2+M_H^2(1-x)}{4\pi}\right)\right]\right.$$
$$-M_H^2\left[\frac{2}{\epsilon}-\gamma_E+1-\ln\frac{M_H^2}{4\pi}\right]-4M_W^2\int_0^1 dx\left[\frac{2}{\epsilon}-\gamma_E-\ln\left(\frac{M_W^2x^2+M_H^2(1-x)}{4\pi}\right)\right]$$
$$-12\int_0^1 dx\,\left(m_t^2x-M_W^2x(1-x)\right)\left[\frac{2}{\epsilon}-\gamma_E+1-\ln\left(\frac{m_t^2x-M_W^2x(1-x)}{4\pi}\right)\right]$$
$$\left.+12m_t^2\left[\frac{2}{\epsilon}-\gamma_E+1-\ln\frac{m_t^2}{4\pi}\right]\right\}$$

$$\delta Z_W = \frac{g^2}{32\pi^2}\left\{6\int_0^1 dx\,x(1-x)\left[\frac{2}{\epsilon}-\gamma_E-\ln\left(\frac{m_t^2x-M_W^2x(1-x)}{4\pi}\right)\right]-\int_0^1 dx\,x(1-x)\times\right.$$
$$\left.\times\left[\frac{2}{\epsilon}-\gamma_E-\ln\left(\frac{M_W^2x^2+M_H^2(1-x)}{4\pi}\right)\right]-4M_W^2\int_0^1 dx\,\frac{x(1-x)}{M_W^2x^2+M_H^2(1-x)}\right\}$$

$$\delta M_Z^2 = -\frac{g^2}{64\pi^2c^2}\left\{6m_t^2\int_0^1 dx\left[\frac{2}{\epsilon}-\gamma_E-\ln\left(\frac{m_t^2-M_Z^2x(1-x)}{4\pi}\right)\right]\right.$$
$$+24(C_L^{t2}+C_R^{t2})m_t^2\left[\frac{2}{\epsilon}-\gamma_E+1-\ln\frac{m_t^2}{4\pi}\right]$$
$$-24(C_L^{t2}+C_R^{t2})\int_0^1 dx\,\left(m_t^2-M_Z^2x(1-x)\right)\left[\frac{2}{\epsilon}-\gamma_E+1-\ln\left(\frac{m_t^2-M_Z^2x(1-x)}{4\pi}\right)\right]$$
$$+2\int_0^1 dx\,\left(M_Z^2x^2+M_H^2(1-x)\right)\left[\frac{2}{\epsilon}-\gamma_E+1-\ln\left(\frac{M_Z^2x^2+M_H^2(1-x)}{4\pi}\right)\right]$$
$$\left.-4M_Z^2\int_0^1 dx\left[\frac{2}{\epsilon}-\gamma_E-\ln\left(\frac{M_Z^2x^2+M_H^2(1-x)}{4\pi}\right)\right]-M_H^2\left[\frac{2}{\epsilon}-\gamma_E+1-\ln\frac{M_H^2}{4\pi}\right]\right\}$$

$$\delta Z_Z = \frac{g^2}{32\pi^2 c^2}\Bigg\{ 3m_t^2 \int_0^1 dx \, \frac{x(1-x)}{m_t^2 - M_Z^2 x(1-x)} + 12(C_L^{t2} + C_R^{t2}) \int_0^1 dx \, x(1-x) \times$$
$$\times \left[\frac{2}{\epsilon} - \gamma_E - \ln\Big(\frac{m_t^2 - M_Z^2 x(1-x)}{4\pi} \Big) \right] - 2M_Z^2 \int_0^1 dx \, \frac{x(1-x)}{M_Z^2 x^2 + M_H^2(1-x)}$$
$$- \int_0^1 dx \, x(1-x) \left[\frac{2}{\epsilon} - \gamma_E - \ln\Big(\frac{M_Z^2 x^2 + M_H^2(1-x)}{4\pi} \Big) \right] \Bigg\}$$

$$\delta(Z_L^{bb})^{1/2} = \frac{\delta(Z_L^{bs})^{1/2}}{V_{ts}} = -\frac{g^2}{64\pi^2} \int_0^1 dx \, x \left[\frac{2}{\epsilon} - \gamma_E - \ln\Big(\frac{M_W^2 x + m_t^2(1-x)}{4\pi} \Big) \right]$$

These formulas take a simplified form when we consider them in the two limits mentionned in Chapter 5. In the following, we will give the form of the renormalization constants first in the large M_H limit and second in the large m_t limit (denoted by a ♠). (If the two limits happen to coincide, we give only one result.)

$$\delta(Z_L^{tt})^{1/2} = -\frac{g^2}{128\pi^2} \left(\frac{m_t}{M_W} \right)^2 \left\{ \frac{2}{\epsilon} - \gamma_E - \frac{1}{4} - \frac{1}{2} \ln \frac{m_t^2}{4\pi} - \frac{1}{2} \ln \frac{M_H^2}{4\pi} \right\}$$
$$♠ \qquad = -\frac{g^2}{128\pi^2} \left(\frac{m_t}{M_W} \right)^2 \left\{ \frac{2}{\epsilon} - \gamma_E - 4 - 2\ln \frac{M_H^2}{4\pi} + \ln \frac{m_t^2}{4\pi} \right\}$$

$$\delta(Z_R^{tt})^{1/2} = -\frac{g^2}{128\pi^2} \left(\frac{m_t}{M_W} \right)^2 \left\{ 2\left(\frac{2}{\epsilon} - \gamma_E \right) + \frac{7}{4} - \frac{3}{2} \ln \frac{m_t^2}{4\pi} - \frac{1}{2} \ln \frac{M_H^2}{4\pi} \right\}$$
$$♠ \qquad = -\frac{g^2}{64\pi^2} \left(\frac{m_t}{M_W} \right)^2 \left\{ \frac{2}{\epsilon} - \gamma_E - 1 - \ln \frac{M_H^2}{4\pi} \right\}$$

$$\frac{\delta m_t}{m_t} = \frac{g^2}{64\pi^2} \left(\frac{m_t}{M_W} \right)^2 \left\{ \frac{3}{2} \left(\frac{2}{\epsilon} - \gamma_E \right) + \frac{7}{4} - \frac{3}{2} \ln \frac{M_H^2}{4\pi} \right\}$$
$$♠ \quad = \frac{g^2}{64\pi^2} \left(\frac{m_t}{M_W} \right)^2 \left\{ \frac{3}{2} \left(\frac{2}{\epsilon} - \gamma_E \right) + 4 - \frac{3}{2} \ln \frac{m_t^2}{4\pi} \right\}$$

$$T = \frac{g}{64\pi^2 M_W} \left\{ \left(\frac{2}{\epsilon} - \gamma_E + 1 \right) (24m_t^4 - 3M_H^4 - 2M_H^2 M_W^2 - M_H^2 M_Z^2) - 24m_t^4 \ln \frac{m_t^2}{4\pi} \right.$$
$$\left. + 3M_H^4 \ln \frac{M_H^2}{4\pi} + 2M_H^2 M_W^2 \ln \frac{M_W^2}{4\pi} + M_H^2 M_Z^2 \ln \frac{M_Z^2}{4\pi} \right\}$$
$$♠ = \frac{3gm_t^4}{8\pi^2 M_W} \left\{ \frac{2}{\epsilon} - \gamma_E + 1 - \ln \frac{m_t^2}{4\pi} \right\}$$

$$\delta Z_\phi = -\frac{g^2}{64\pi^2 M_W^2}\left\{\frac{M_H^2}{2} + M_W^2 \ln\frac{M_W^2}{M_H^2} + 2M_W^2 \ln\frac{M_H^2}{4\pi} + 6m_t^2\left(\frac{2}{\epsilon} - \gamma_E + \frac{1}{2} - \ln\frac{m_t^2}{4\pi}\right)\right\}$$

$$\spadesuit \quad = -\frac{3g^2 m_t^2}{32\pi^2 M_W^2}\left\{\frac{2}{\epsilon} - \gamma_E + \frac{1}{2} - \ln\frac{m_t^2}{4\pi}\right\}$$

$$\delta M_W^2 = -\frac{g^2}{32\pi^2}\left\{\frac{5M_W^2}{3}\ln\frac{M_H^2}{4\pi} + \frac{M_H^2}{4} + 3m_t^2\left(\frac{2}{\epsilon} - \gamma_E + \frac{1}{2} - \ln\frac{m_t^2}{4\pi}\right)\right\}$$

$$\spadesuit \quad = -\frac{3g^2 m_t^2}{32\pi^2}\left\{\frac{2}{\epsilon} - \gamma_E + \frac{1}{2} - \ln\frac{m_t^2}{4\pi}\right\}$$

$$\delta Z_W = \frac{g^2}{192\pi^2}\ln\frac{M_H^2}{4\pi}$$

$$\spadesuit \quad = 0$$

$$\delta M_Z^2 = -\frac{g^2}{32\pi^2 c^2}\left\{\frac{5}{3}M_Z^2 \ln\frac{M_H^2}{4\pi} + \frac{M_H^2}{4} + 3m_t^2\left(\frac{2}{\epsilon} - \gamma_E - \ln\frac{m_t^2}{4\pi}\right)\right\}$$

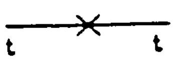
$$\spadesuit \quad = -\frac{3g^2 m_t^2}{32\pi^2 c^2}\left\{\frac{2}{\epsilon} - \gamma_E - \ln\frac{m_t^2}{4\pi}\right\}$$

$$\delta Z_{ZZ} = \frac{g^2}{192\pi^2}\ln\frac{M_H^2}{4\pi}$$

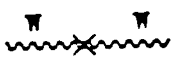
$$\spadesuit \quad = 0$$

$$\delta(Z_L^{bb})^{1/2} = \frac{\delta(Z_L^{bs})^{1/2}}{V_{ts}} = -\frac{g^2}{128\pi^2}\left(\frac{m_t}{M_W}\right)^2\left\{\frac{2}{\epsilon} - \gamma_E - \ln\frac{m_t^2}{4\pi} + \frac{3}{2}\right\}$$

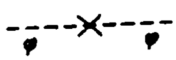
The counterterms are given by:



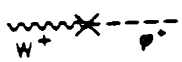
$= i(m_t A_t + R_t \not{k} R + L_t \not{k} L)$



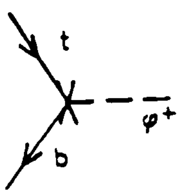
$= i\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\left(A_W + (M_W^2 - k^2)B_W\right) + i\frac{k^\mu k^\nu}{k^2}C_W$



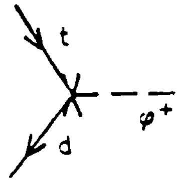
$= i(A_\phi + B_\phi k^2)$



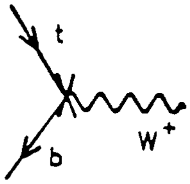
$= -iM_W k^\mu A_{W\phi}$



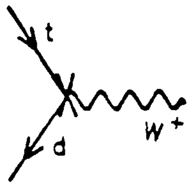
$= A_{\phi tb}[\phi^- \bar{b} t]$



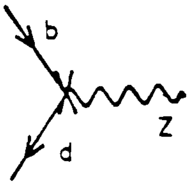
$$=A_{\phi tb}[\phi^- \bar{d} t]$$



$$=A_{Wtb}[W^{-\mu}\bar{b}t]$$



$$=A_{Wtb}[W^{-\mu}\bar{d}t]$$



$$=-\frac{g^2}{64\pi^2}\left(\frac{m_t^2}{M_W^2}\right)\alpha_t\left(\frac{2}{\epsilon}-\gamma_E-\ln\frac{m_t^2}{4\pi}\right)[Z^\mu t\bar{t}]$$

where,

$$R_T = 2\delta(Z_R^{tt})^{1/2}$$

$$L_T = 2\delta(Z_L^{tt})^{1/2}$$

$$A_T = -\left(\frac{R_T}{2}+\frac{L_T}{2}+\frac{\delta m_t}{m_t}\right)$$

$$A_W = \delta M_W^2$$

$$B_W = \delta Z_W$$

$$C_W = A_W + M_W^2 B_W$$

$$A_\phi = \frac{g}{2M_W}T$$

$$B_\phi = \delta Z_\phi$$

$$A_{W\phi} = \frac{1}{2}\left(\frac{A_W}{M_W^2}+B_W+B_\phi\right)$$

$$A_{\phi tb} = -\frac{A_W}{2M_W^2}-\frac{1}{2s^2M_Z^2}(c^2\delta M_Z^2-\delta M_W^2)+\frac{1}{2}B_\phi+\delta(Z_L^{bb})^{1/2}+\frac{\delta m_t}{m_t}+2R_T$$

$$A_{Wtb} = \frac{1}{2}\delta Z_W+\frac{L_T}{2}+\delta(Z_L^{bb})^{1/2}-\frac{1}{2s^2M_Z^2}(c^2\delta M_Z^2-\delta M_W^2)$$

Appendix F

Calculation of two-loop box diagrams

We will present here the details of the calulation of two-loop box diagrams. First, an application of the Feynman rules of Appendix B gives the following general form of the diagram:

$$\int \frac{d^n p}{(2\pi)^n} \frac{d^n q}{(2\pi)^n} f(p,q)/D$$

where $D = \prod_{i=1}^{k}(p^2 - m_1^2)^{\alpha_i} \prod_{j=1}^{l}(q^2 - m_j^2)^{\beta_j} ((p-q)^2 - M^2)$ and $f(p,q)$ is a quartic polynomial in p and q. To simplify this integral, we can use the identities (we omit the measure $\frac{d^n p}{(2\pi)^n} \frac{d^n q}{(2\pi)^n}$)

$$\begin{aligned}
\int p^\mu p^\nu g(p,q) &= \frac{1}{n} \int p^2 g(p,q) g^{\mu\nu} \\
\int p^\mu q^\nu g(p,q) &= \frac{1}{n} \int p \cdot q\, g(p,q) g^{\mu\nu} \\
\int p^\alpha p^\beta p^\mu q^\nu g(p,q) &= \frac{1}{n(n+2)} \int p^2 p \cdot q\, g(p,q) \{ g^{\mu\nu} g^{\alpha\beta} + g^{\mu\alpha} g^{\nu\beta} + g^{\mu\beta} g^{\nu\alpha} \} \\
\int p^\alpha p^\beta q^\mu q^\nu g(p,q) &= \frac{1}{n(n-1)(n+2)} \{ A g^{\mu\nu} g^{\alpha\beta} + B(g^{\mu\alpha} g^{\nu\beta} + g^{\mu\beta} g^{\nu\alpha}) \}
\end{aligned}$$

where $A = \int \{(n+1)p^2 q^2 - 2(p\cdot q)^2\} g(p,q)$, $B = \int \{n(p\cdot q)^2 - p^2 q^2\} g(p,q)$ and g is an arbitrary function of p and q. These identities are easy to derive when we realize that the Lorentz structure of the integrals follows immediately from the symmetries of the integrand and the fact that we can only use the metric in their construction (there is no external momentum). The coefficients can then be found by contraction of the indices.

As a second step, we can reduce to a minimum the number of γ matrices present by the use of the identities of Appendix C. An arbitrary diagram can now be written as

$A_1\eta_1 + A_2\eta_2$ where $\eta_1 = \bar{u}_b\gamma^\alpha L u_d \bar{v}_d \gamma_\alpha L v_b$ and $\eta_2 = \bar{u}_b\gamma^\alpha\gamma^\beta\gamma^\delta L u_d \bar{v}_d \gamma_\delta\gamma_\beta\gamma_\alpha L v_b$ and the A_i are integrals of the form $\int h(p,q)/D$ where h is a scalar function quartic in p and q.

Using partial fractions, we can write this as a sum of terms of the form $\int h(p,q)K^{i,j}(m_1,m_2,m_3)$ where

$$K^{i,j}(m_1,m_2,m_3) = \frac{1}{(p^2-m_1^2)^i(q^2-m_2^2)^j((p-q)^2-m_3^2)}$$

The following identities prove useful:

$$\begin{aligned}
\int (p\cdot q)^2 K^{i,j}(m_1,m_2,m_3) &= \int (p^2+q^2-m_3^2)p\cdot q K^{i,j}(m_1,m_2,m_3)/2 \\
\int p^2 p\cdot q K^{i,j}(m_1,m_2,m_3) &= \int [(p^2+q^2-m_3^2)p^2 K^{i,j}(m_1,m_2,m_3)/2] \\
&\quad -H^{i-1}(m_1)H^j(m_2)/2 - m_1^2 H^i(m_1)H^j(m_2) \\
\int p^4 K^{i,j}(m_1,m_2,m_3) &= m_1^2\int [p^2 K^{i,j}(m_1,m_2,m_3)] + H^{j-1}(m_2)H^1(m_3) \\
&\quad +(m_2^2+m_3^2)H^j(m_2)H^i(m_3) \qquad i=1 \\
&= \int p^2[K^{i-1,j}(m_1,m_2,m_3) + m_1^2 K^{i,j}(m_1,m_2,m_3)] \quad i>1 \\
\int p^2q^2 K^{i,j}(m_1,m_2,m_3) &= \int K^{i-1,j-1}(m_1,m_2,m_3) \\
&\quad \int +(m_1^2q^2 + m_2^2p^2 - m_1^2m_2^2)K^{i,j}(m_1,m_2,m_3) \\
\int p\cdot q K^{i,j}(m_1,m_2,m_3) &= \int [(p^2+q^2-m_3^2)K^{i,j}(m_1,m_2,m_3)/2] - H^i(m_1)H^j(m_2)/2 \\
\int p^2 K^{i,j}(m_1,m_2,m_3) &= \int [K^{i-1,j}(m_1,m_2,m_3) + m_1^2 K^{i,j}(m_1,m_2,m_3)]
\end{aligned}$$

as well as the identities obtained from those above vie the interchange

$$q \leftrightarrow q \ , \ m_1 \leftrightarrow m_2 \ , \ i \leftrightarrow j$$

Here, $H^i(m)$ is the 1-loop integral

$$H^i(m) = \int \frac{d^n p}{(2\pi)^n}\frac{1}{(p^2-m^2)^i} \tag{F.96}$$

Its value will be given later.

These identities can be derived easily by the substitutions $p^2 \to (p^2 - m_1^2) + m_1^2$, $2p\cdot q = -[((p-q)^2 - m_3^2) - p^2 - q^2 + m_3^2]$ and so on. Their repeated application allows us to express the integrals as a sum of terms of the form

$$I^{i,j}(m_1,m_2,m_3) = \int K^{i,j}(m_1,m_2,m_3)$$

We can reduce the values of i and j with the help of partial differentiation

$$\begin{aligned} I^{i,j}(m_1,m_2,m_3) &= \frac{1}{(i-1)}\frac{\partial}{\partial m_1^2} I^{i-1,j}(m_1,m_2,m_3) \\ &= \frac{1}{(j-1)}\frac{\partial}{\partial m_2^2} I^{i,j-1}(m_1,m_2,m_3) \end{aligned}$$

We could therefore write all of them in terms of $I^{1,1}$. However, it is preferable to express them in terms of $I^{2,1}$, which is easier to calculate. Note that we cannot obtain $I^{1,1}$ in terms of $I^{2,1}$ by partial differentiation. We can, however, use a different technique. Notice that $I^{1,1}$ is a homogeneous function of degree $2(n-3)$, that is

$$I^{1,1}(\lambda m_1, \lambda m_2, \lambda m_3) = \lambda^{2(n-3)} I^{1,1}(m_1,m_2,m_3)$$

By Euler's theorem, we get

$$\begin{aligned} I^{1,1}(m_1,m_2,m_3) &= \frac{1}{2(n-3)}(m_1\frac{\partial}{\partial m_1} + m_2\frac{\partial}{\partial m_2} + m_3\frac{\partial}{\partial m_3}) I^{1,1}(m_1,m_2,m_3) \\ &= \frac{1}{(n-3)}(m_1^2\frac{\partial}{\partial m_1^2} + m_2^2\frac{\partial}{\partial m_2^2} + m_3^2\frac{\partial}{\partial m_3^2}) I^{1,1}(m_1,m_2,m_3) \end{aligned}$$

Now,

$$\begin{aligned} \frac{\partial}{\partial m_2^2} I^{1,1}(m_1,m_2,m_3) &= \int \frac{1}{(p^2-m_1^2)(q^2-m_2^2)^2((p-q)^2-m_3^2)} \\ &= \int \frac{1}{(p^2-m_2^2)^2(q^2-m_1^2)((p-q)^2-m_3^2)} \\ &= I^{2,1}(m_2,m_1,m_3) = I^{2,1}(m_2,m_3,m_1) \end{aligned}$$

Similarly,

$$\begin{aligned} \partial/\partial m_1^2 I^{1,1}(m_1,m_2,m_3) &= I^{2,1}(m_1,m_2,m_3) \\ \partial/\partial m_3^2 I^{1,1}(m_1,m_2,m_3) &= I^{2,1}(m_3,m_1,m_2) \end{aligned}$$

and we get:

$$I^{1,1}(m_1,m_2,m_3) = \frac{1}{(n-3)}\Big\{m_1^2 I^{2,1}(m_1,m_2,m_3) + m_2^2 I^{2,1}(m_2,m_3,m_1) + m_3^2 I^{2,1}(m_3,m_1,m_2)\Big\}$$

The function $I^{2,1}(m_1,m_2,m_3)$ can be calculated after a Feynman parametrization. The result is:

$$\begin{aligned} I^{2,1}(m_1,m_2,m_3) &= \frac{-1}{256\pi^4}\left\{\frac{2}{\epsilon^2}+\frac{1}{\epsilon}\left(1-2\gamma_E-2\ln\frac{m_1^2}{4\pi}\right)+\frac{1}{4}\left(1-2\gamma_E-2\ln\frac{m_1^2}{4\pi}\right)^2\right. \\ &\left. +\frac{5}{4}+\frac{\pi^2}{12}-J^{2,1}\left(\frac{m_2^2}{m_1^2},\frac{m_3^2}{m_1^2}\right)\right\} \end{aligned} \tag{F.97}$$

where

$$J^{2,1}(x,y) \equiv \int_0^1 dz_1 \int_0^1 dz_2\, \frac{1-z_2}{z_2}\ln[1+(\mu^2-1)z_2] \tag{F.98}$$

with

$$\mu^2 \equiv \frac{x}{1-z_1}+\frac{y}{z_1}$$

Explicitly:

$$\begin{aligned} J^{2,1}(x,y) &= 1+\frac{1}{2}\ln x\ln y+\frac{1-x-y}{\sqrt{\lambda}}\left\{\frac{\pi^2}{6}+\frac{1}{4}\ln^2\frac{2x}{1-x-y-\sqrt{\lambda}}\right. \\ &+ \frac{1}{4}\ln^2\frac{2y}{1-x-y-\sqrt{\lambda}}+\frac{1}{4}\ln\frac{y}{x}\ln\left[\left(\frac{1+x-y+\sqrt{\lambda}}{1+x-y-\sqrt{\lambda}}\right)\left(\frac{1-x+y-\sqrt{\lambda}}{1-x+y+\sqrt{\lambda}}\right)\right] \\ &+ \left.\mathrm{Li}_2\left(\frac{-2x}{1-x-y-\sqrt{\lambda}}\right)+\mathrm{Li}_2\left(\frac{-2y}{1-x-y-\sqrt{\lambda}}\right)\right\} \\ \lambda &= 1-2x-2y-2xy+x^2+y^2 \end{aligned}$$

Our last task is to obtain the expansions of the $I^{i,j}$ when one or more of the masses are made large. In practice, we will encounter only $I^{1,1}$, $I^{2,1}$, $I^{2,2}$ and $I^{3,1}$. Furthermore, $I^{1,1}$ is algebraically related to $I^{2,1}$. Therefore, we only need to expand $I^{2,1}$, $I^{3,1}$ and $I^{2,2}$. As we have seen, $I^{3,1}$ and $I^{2,2}$ are related to $I^{2,1}$ by partial differentiation. However, we cannot obtain their expansions by differentiation of the expansion of $I^{2,1}$ since, in general, these operations do not commute. We must first obtain a general expression for $I^{2,2}$ and $I^{3,1}$ and then expand them. It is convenient to define $J^{2,2}$ and $J^{3,1}$ by

$$\begin{aligned} I^{2,2}(m_1, m_2, m_3) &= \frac{1}{256\pi^4 m_1^2} J^{2,2}(\frac{m_2^2}{m_1^2}, \frac{m_3^2}{m_1^2}) \\ I^{3,1}(m_1, m_2, m_3) &= \frac{1}{512\pi^4 m_1^2} \left\{ \frac{2}{\epsilon} - 2\gamma_E - 2\ln\frac{m_1^2}{4\pi} + 1 + J^{3,1}(\frac{m_2^2}{m_1^2}, \frac{m_3^2}{m_1^2}) \right\} \end{aligned}$$

The expansions for $J^{2,1}$, $J^{2,2}$ and $J^{3,1}$ will be given below. In those expressions, $x \ll 1$, $y \ll 1$ and a is not too large ($ax < 1$). In practice, a is almost less than 1. The expansion parameters x and y are $(\frac{M_W}{m_t})^2$, $(\frac{M_W}{M_H})^2$ or $(\frac{m_t}{M_H})^2$. We assume all of these ratios to be small. The coefficients of the J's can be as high as m_t^{12} and M_H^8. Therefore, we need to expand these functions to at least the fourth order in the small quantities. The expansions we give are valid to the fifth order. This is done by computer algebra using two independent codes. Furthermore, the first three of these functions have been checked against the results of ref. [13].

$$\begin{aligned} J^{2,1}(x,y) =& 1/2 - \pi^2/12 + x + x^2/4 + x^3/9 + x^4/16 + x^5/25 + (1-\pi^2/3)xy/2 \\ &+(3 - 2\pi^2/3)x^2y + (11/2 - \pi^2)x^3y + (74/9 - 4\pi^2/3)x^4y \\ &+(49/8 - 3\pi^2/2)x^2y^2 + (35 - 8\pi^2)x^3y^2 - \ln x(x + x^2/2 + x^3/3 + x^4/4 + x^5/5) \\ &-y\ln x(x + 4x^2 + 8x^3 + 38x^4/3) - y^2\ln x(x + 21x^2/2 + 40x^3) \\ &-y^3\ln x(x + 20x^2) - y^4 x\ln x - y\ln x\ln y(x/2 + 2x^2 + 3x^3 + 4x^4) \\ &-y^2\ln x\ln y(9x^2/2 + 24x^3) \\ &+x \leftrightarrow y \end{aligned}$$

$$\begin{aligned} J^{2,1}(a, 1/x) =& J^{2,1}(1/x, a) \\ =& 1 + \pi^2/6 + (\ln^2 x)/2 - x\{1 + a - \pi^2 a/3 - a\ln ax - a\ln a\ln x - \ln^2 x\} \\ &-x^2\Big\{1/4 + 3a - 2\pi^2 a/3 + a^2(9/4 - \pi^2/3) - a\ln a(1 + 5a/2) \\ &-\ln x(1/2 + 4a + 5a^2/2) - a(2+a)\ln ax\ln x\Big\} \end{aligned}$$

$$
\begin{aligned}
&-x^3\Big\{1/9 + (11/2 - \pi^2)a + (9 - 2\pi^2)a^2 + (47/18 - \pi^2/3)a^3 \\
&-a\ln a(1 + 8a + 10a^2/3) - \ln x(1/3 + 8a + 13a^2 + 10a^3/3) \\
&-a\ln ax\ln x(3 + 6a + a^2)\Big\} \\
&-x^4\Big\{1/16 + 74a/9 - 4\pi^2 a/3 + 107a^2/4 - 6\pi^2 a^2 + (20 - 8\pi^2)a^3 \\
&+(401/144 - \pi^2/3)a^4 - a\ln a(1 + 33a/2 + 23a^2 + 47a^3/12) \\
&-\ln x(1/4 + 38a/3 + 42a^2 + 30a^3 + 47a^4/12) \\
&-a\ln ax\ln x(4 + 18a + 12a^2 + a^3)\Big\} \\
&-x^5\Big\{1/25 + 797a/12 - 5\pi^2 a/3 + 569a^2/9 - 40\pi^2 a^2/3 + 177a^3/2 - 20\pi^2 a^3 \\
&+328a^4/9 - 20\pi^2 a^4/3 + 5197a^5/1800 - \pi^2 a^5/3 \\
&-a\ln a(1 + 28a + 82a^2 + 142a^3/3 + 131a^4/30) \\
&-\ln x(1/5 + 107a/6 + 308a^2/3 + 144a^3 + 169a^4/3 + 131a^5/30) \\
&-a\ln ax\ln x(5 + 40a + 60a^2 + 20a^3 + a^4)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
J^{2,1}(1,x) = J^{2,1}&(x,1) \\
&1 + x + 5x^2/36 + 47x^3/1800 + 319x^4/58800 + 1879x^5/1587600 \\
&-x\ln x(2 + x/3 + x^2/15 + x^3/70 + x^4/315)
\end{aligned}
$$

$$
\begin{aligned}
J^{2,2}(x,y) = &-y\Big\{\pi^2/3 - x(2 - 4\pi^2/3) - x^2(17/2 - 3\pi^2) - x^3(182/9 - 16\pi^2/3) \\
&-x^4(2701/72 - 25\pi^2/3)\Big\} + y^2\Big\{2 - 2\pi^2/3 + x(14 - 6\pi^2) + x^2(65 - 24\pi^2) \\
&+x^3(1921/9 - 200\pi^2/3)\Big\} + y^3\Big\{9/2 - \pi^2 + x(50 - 16\pi^2) + x^2(597/2 - 100\pi^2)\Big\} \\
&+y^4\Big\{65/9 - 4\pi^2/3 + x(1133/9 - 100\pi^2/3)\Big\} + y^5\Big\{725/72 - 5\pi^2/3\Big\} \\
&-\ln x\Big\{1 + x + x^2 + x^3 + x^4 + x^5 + y(1 + 8x + 24x^2 + 152x^3/3 + 535x^4/6) \\
&+y^2(1 + 21x + 120x^2 + 1240x^3/3) + y^3(1 + 40x + 370x^2) + y^4(1 + 65x) + y^5\Big\} \\
&-y\ln y\Big\{2 + 4x + 6x^2 + 8x^3 + 10x^4 + y(6 + 30x + 84x^2 + 180x^3) \\
&+y^2(11 + 104x + 470x^2) + y^3(50/3 + 770x/3) + 137y^4/6\Big\} \\
&-y\ln y\ln x\Big\{1 + 4x + 9x^2 + 16x^3 + 25x^4 + y(2 + 18x + 72x^2 + 200x^3) \\
&+y^2(3 + 48x + 300x^2) + y^3(4 + 100x) + 5y^4\Big\}
\end{aligned}
$$

$$
\begin{aligned}
J^{2,2}(a,1/x)=&x\{\pi^2/3+\ln a+2\ln x+\ln ax\ln x\}\\
&-x^2\Big\{2-2\pi^2/3+2a-2\pi^2a/3-\ln a-5a\ln a-6(1+a)\ln x\\
&-2(1+a)\ln ax\ln x\Big\}-x^3\Big\{9/2-\pi^2+(10-4\pi^2)a+(9/2-\pi^2)a^2\\
&-2a\ln a(5+8a)-\ln x(11+32a+11a^2)-3\ln ax\ln x(1+4a+a^2)\Big\}\\
&-x^4\Big\{65/9-4\pi^2/3+(37-12\pi^2)a(1+a)+(65/9-4\pi^2/3)a^3\\
&-\ln a(1+33a+69a^2+47a^3/3)-\ln x(50/3+102a(1+a)+50a^3/3)\\
&-4\ln ax\ln x(1+9a(1+a)+a^3)\Big\}\\
&-x^5\Big\{725/72-5\pi^2/3+(886/9-80\pi^2/3)a+(367/2-60\pi^2)a^2\\
&+(886/9-80\pi^2/3)a^3+(725/72-5\pi^2/3)a^4-\ln a(1+56a+246a^2\\
&+568a^3/3+131a^4/6)-\ln x(137/6+736a/3+492a^2+736a^3/3+137a^4/6)\\
&-5\ln ax\ln x(1+16a+36a^2+16a^3+a^4)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
J^{2,2}(1,x)=&1-x/9-16x^2/225-249x^3/9800-782x^4/99225-17645x^5/7683984\\
&+x\ln x(1/6+x/15+3x^2/140+2x^3/315+5x^4/2772)
\end{aligned}
$$

$$
\begin{aligned}
J^{2,2}(x,1)=&1/2+7x/36+37x^2/600+533x^3/29400+1627x^4/317520+18107x^5/12806640\\
&-\ln x(1/2+x/6+x^2/20+x^3/70+x^4/252+x^5/924)
\end{aligned}
$$

$$
\begin{aligned}
J^{3,1}(x,y)=&xy\Big\{\pi^2/3-(4-2\pi^2)x-(13-4\pi^2)x^2-(247/9-20\pi^2/3)x^3\Big\}\\
&-x^2y^2\Big\{14-6\pi^2+(115-40\pi^2)x\Big\}+\ln x\Big\{x+x^2+x^3+x^4+x^5+3xy+14x^2y\\
&+35x^3y+202x^4y/3+5xy^2+51x^2y^2+224x^3y^2+7xy^3+124x^2y^3+9xy^4\Big\}\\
&+xy\ln x\ln y\{1+6x+12x^2+20x^3+18xy+120x^2y\}\\
&+x\leftrightarrow y
\end{aligned}
$$

$$
\begin{aligned}
J^{3,1}(a,1/x)=&J^{3,1}(1/x,a)=\ln x+x\{a\ln ax+\ln x\}\\
&+x^2\{2\pi^2a/3+3a\ln a+a^2\ln ax+\ln x+6a\ln x+2a\ln ax\ln x\}\\
&+x^3\Big\{-4a(1+a)(1-\pi^2/2)+a\ln a(5+14a+a^2)+\ln x(1+19a+19a^2+a^3)
\end{aligned}
$$

$$
\begin{aligned}
&+6a(1+a)\ln ax\ln x\Big\}\\
&+x^4\Big\{-13a+4\pi^2a-28a^2+12\pi^2a^2-13a^3+4\pi^2a^3+\ln a(7a+51a^2\\
&+35a^3+a^4)+\ln x(1+42a+102a^2+42a^3+a^4)+12a\ln ax\ln x(1+3a+a^2)\Big\}\\
&+x^5\Big\{-(247/9-20\pi^2/3)a-(115-40\pi^2)a^2(1+a)-247a^4/9+20\pi^2a^4/3\\
&+a\ln a(9+124a+224a^2+202a^3/3+a^4)+\ln x(1+229a/3+348a^2(1+a)\\
&+229a^4/3+a^5)+20a\ln ax\ln x(1+6a+6a^2+a^3)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
J^{3,1}(1,x)=&J^{3,1}(x,1)\\
&-1-7x/18-37x^2/300-533x^3/1470-1627x^4/158760-18107x^5/6403320\\
&+x\ln x(5/6-2x/5+x^2/35+x^3/126+x^4/462)
\end{aligned}
$$

Finally, we must give the value of $H^i(m)$. It could be calculted directly from F.96 but, as shown in section ?, it can also be found from a recursion relation

$$
H^k(m)=\frac{n/2-k+1}{k-1}\frac{H^{k-1}(m)}{m^2}
$$

All we need is the value of $H^1(m)$, which is

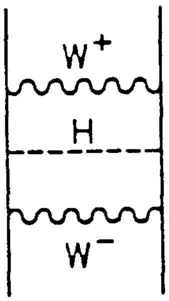
$$
H^1(m)=\frac{i}{16\pi^2}\left(\frac{2}{\epsilon}-\left(\gamma_E-1+\ln\frac{m^2}{4\pi}\right)+\epsilon\left(\frac{\pi^2}{24}+\frac{1}{4}+\frac{1}{4}\left(\gamma_E-1+\ln\frac{m^2}{4\pi}\right)^2\right)\right)
$$

The $\mathcal{O}(\epsilon)$ term must be calculated because products of two H^1 functions often arise in the decomposition of two-loop integrals.

Appendix G

The box diagrams at two-loops

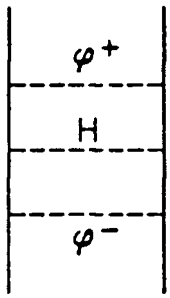
In this Appendix, we list the values of the leading diagrams as determined by the methods of Appendix F. For each diagram, there are two or three values given. The first one is the exact value expressed in terms of $I^{i,j}$ and H^k. The second one is the expansion of the first in the large M_H limit. It is labelled ♠. Finally, if the expansion for large m_t gives a different result than for large M_H, it is given in a third entry labelled ♣. If not, only two values are given. Each result should be multiplied by $ig^6\alpha_t^2$.



(1)

$$D1 = -\frac{m_t^4}{16M_W^2}\eta_1 \Big\{ \left(H^{(1)}(m_t) - H^{(1)}(M_W)\right)^2 + (m_t^2 - M_W^2)^2 \left(H^{(2)}(m_t)\right)^2 - 2(m_t^2 - M_W^2)\times$$
$$\times \left(H^{(1)}(m_t) - H^{(1)}(M_W)\right) H^{(2)}(m_t) + (M_H^2 - 4m_t^2)(m_t^2 - M_W^2)^2 I^{2,2}(m_t, m_t, M_H)$$
$$+(M_H^2 - M_W^2)\left(I^{1,1}(m_t, m_t, M_H) - 2I^{1,1}(m_t, M_W, M_H) + I^{1,1}(M_W, M_W, M_H)\right)$$
$$-2(m_t^2 - M_W^2)(M_H^2 - 2m_t^2 - 2M_W^2)\left(I^{2,1}(m_t, m_t, M_H) - I^{2,1}(m_t, M_W, M_H)\right)\Big\}$$

♠ =0

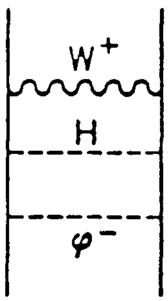


(1)

$$D2 = \frac{m_t^4}{4M_W^4} D1$$

$$\spadesuit = 0$$

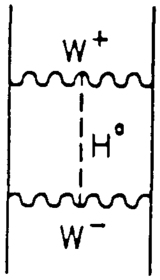
$$\clubsuit = \frac{\eta_1}{49152\pi^4} \frac{m_t^4}{M_W^6} (15 - 2\pi^2)$$



(2)

$$\begin{aligned}
D3 = -\frac{m_t^4}{64M_W^4}\eta_1 \Big\{ & 2(M_H^2 - 2M_W^2 - 4m_t^2)\left(H^1(m_t) - H^1(M_W)\right)^2 \\
& + 2(m_t^2 - M_W^2)^2 \left(H^2(m_t)\right)^2 (M_H^2 - 6m_t^2) \\
& + (m_t^2 - M_W^2)(M_H^2 - 5m_t^2 - M_W^2)\left(H^1(m_t) - H^1(M_W)\right) H^2(m_t) \\
& + \left((M_H^2 - 4m_t^2)^2 + (M_H^2 - 4M_W^2)^2 - 12(m_t^2 - M_W^2)^2\right) I^{1,1}(m_t, m_t, M_H) \\
& - 2\left((M_H^2 - 4m_t^2)^2 + (M_H^2 - 4M_W^2)^2 - 10(m_t^2 - M_W^2)^2\right) I^{1,1}(m_t, M_W, M_H) \\
& + \left(3(M_H^2 - 4m_t^2)^2 + (M_H^2 - 4M_W^2)^2 - 8(m_t^2 - M_W^2)^2\right) I^{1,1}(M_W, M_W, M_H) \\
& - (m_t^2 - M_W^2)\left(3(M_H^2 - 4m_t^2)^2 + (M_H^2 - 4M_W^2)^2 - 16(m_t^2 - M_W^2)^2\right) I^{2,1}(m_t, m_t, M_H) \\
& + (m_t^2 - M_W^2)\left(3(M_H^2 - 4m_t^2)^2 + (M_H^2 - 4M_W^2)^2 - 12(m_t^2 - M_W^2)^2\right) I^{2,1}(m_t, M_W, M_H) \\
& + 2(m_t^2 - M_W^2)^2 (M_H^2 - 4m_t^2)^2 I^{2,2}(m_t, m_t, M_H) \Big\}
\end{aligned}$$

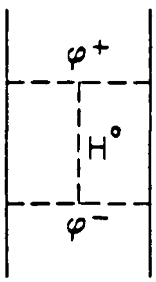
♠ =0



(1)

$$D4 = -\frac{M_W^2}{8}\eta_1\Big\{\Big(H^1(m_t) - H^1(M_W)\Big)^2 - 2(m_t^2 - M_W^2)\Big(H^1(m_t) - H^1(M_W)\Big)H^2(M_W)$$
$$+(m_t^2 - M_W^2)^2\Big[\Big(H^2(M_W)\Big)^2 + (M_H^2 - 2M_W^2)I^{2,2}(M_W, M_W, M_H)\Big]$$
$$+(M_H^2 - 2m_t^2)\Big(I^{1,1}(m_t, m_t, M_H) - 2I^{1,1}(m_t, M_W, M_H) + I^{1,1}(M_W, M_W, M_H)\Big)$$
$$-2(m_t^2 - M_W^2)(M_H^2 - m_t^2 - M_W^2)\Big(I^{2,1}(M_W, m_t, M_H) - I^{2,1}(M_W, M_W, M_H)\Big)\Big\}$$

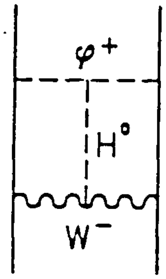
♠ =0



(1)

$$D5 = \frac{M_H^4 m_t^4}{16 M_W^8} D4$$
$$= \frac{\eta_1}{393216\pi^4}\frac{m_t^4}{M_W^6}\Big\{15 - 4\pi^2 - 12\ln^2\frac{M_H^2}{m_t^2} + 24\frac{\ln(M_H^2/m_t^2)}{(1-x)^2}\Big((1-x)(1-2x)$$
$$+x(2-x)\ln x\Big)\Big\}$$

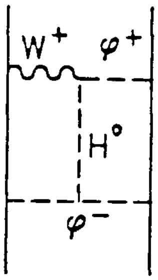
♠ =0



(2)

$$D6=-\frac{M_H^2m_t^4}{4M_W^2}\eta_1\Big\{I^{1,1}(m_t,m_t,M_H)-2I^{1,1}(m_t,M_W,M_H)+I^{1,1}(M_W,M_W,M_H)$$
$$-2(m_t^2-M_W^2)\left(I^{2,1}(M_W,m_t,M_H)-I^{2,1}(M_W,M_W,M_H)\right)$$
$$+(m_t^2-M_W^2)^2I^{2,2}(M_W,M_W,M_H)\Big\}$$

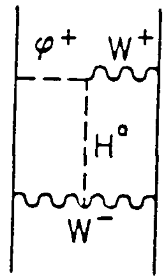
♠ $=0$



(4)

$$D7=\frac{M_H^2m_t^4}{32M_W^4}\eta_1\Big\{\left(H^1(m_t)-H^1(M_W)-(m_t^2-M_W^2)H^2(M_W)\right)^2$$
$$(M_H^2+2m_t^2)\left(I^{1,1}(m_t,m_t,M_H)-2I^{1,1}(m_t,M_W,M_H)+I^{1,1}(M_W,M_W,M_H)\right)$$
$$-2(m_t^2-M_W^2)(M_H^2+m_t^2+M_W^2)\left(I^{2,1}(M_W,m_t,M_H)-I^{2,1}(M_W,M_W,M_H)\right)$$
$$+(m_t^2-M_W^2)^2(M_H^2+2m_t^2)I^{2,2}(M_W,M_W,M_H)\Big\}$$
$$=\frac{\eta_1}{2048\pi^4}\frac{m_t^2}{M_W^4}\frac{\ln(M_H^2/m_t^2)}{(1-x)^2}(1-x+x\ln x)$$

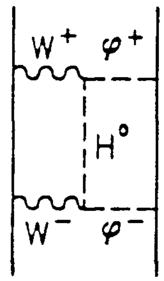
♠ $=0$



(4)

$$D8 = \frac{m_t^2}{2}\eta_1\Big\{\left(H^1(m_t) - H^1(M_W) - (m_t^2 - M_W^2)H^2(M_W)\right)^2$$
$$+(M_H^2 - m_t^2)\left(I^{1,1}(m_t,m_t,M_H) - 2I^{1,1}(m_t,M_W,M_H) + I^{1,1}(M_W,M_W,M_H)\right)$$
$$-(m_t^2 - M_W^2)(2M_H^2 - m_t^2 - M_W^2)\left(I^{2,1}(M_W,m_t,M_H) - I^{2,1}(M_W,M_W,M_H)\right)$$
$$+(m_t^2 - M_W^2)^2(M_H^2 - M_W^2)I^{2,2}(M_W,M_W,M_H)\Big\}$$

♠ =0

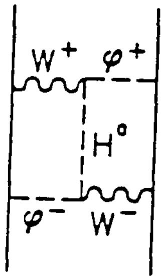


(2)

$$D9 = -\frac{m_t^2}{64M_W^2}\eta_1\Big\{5\left(H^1(m_t) - H^1(M_W)\right)^2 m_t^2 - 5(m_t^4 - M_W^4)H^2(M_W)\left(H^1(m_t) - H^1(M_W)\right)$$
$$+5(m_t^2 - M_W^2)^2\left(H^2(M_W)\right)^2 + m_t^2(5M_H^2 - 2m_t^2)I^{1,1}(m_t,m_t,M_H)$$
$$-\left(2(5M_H^2 - 2m_t^2)m_t^2 + 5(m_t^2 - M_W^2)^2\right)I^{1,1}(m_t,M_W,M_H)$$
$$+\left((5M_H^2 - 2m_t^2)m_t^2 + 5(m_t^2 - M_W^2)^2\right)I^{1,1}(M_W,M_W,M_H)$$
$$-(m_t^2 - M_W^2)\left((5M_H^2 - 2m_t^2)(m_t^2 + M_W^2) - (m_t^2 - M_W^2)(3m_t^2 - 5M_W^2)\right)\times$$
$$\times I^{2,1}(M_W,m_t,M_H) + (m_t^2 - M_W^2)\Big(m_t^2(5M_H^2 - 2m_t^2) + M_W^2(5M_H^2 - 2M_W^2)$$
$$+2(m_t^2 - M_W^2)^2\Big)I^{2,1}(M_W,M_W,M_H) + (m_t^2 - M_W^2)^2(5M_H^2 - 2M_W^2)\times$$

$I^{2,2}(M_W, M_W, M_H)\Big\}$

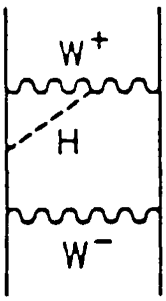
♠ =0



(2)

$$D10 = \frac{m_t^4}{64 M_W^2}\eta_1 \Big\{ 5\left(H^1(m_t) - H^1(M_W) - (m_t^2 - M_W^2)H^2(M_W)\right)^2$$
$$+(5M_H^2 - 2m_t^2)\left(I^{1,1}(m_t, m_t, M_H) - 2I^{1,1}(m_t, M_W, M_H) + I^{1,1}(M_W, M_W, M_H)\right)$$
$$-2(m_t^2 - M_W^2)(5M_H^2 - m_t^2 - M_W^2)\left(I^{2,1}(M_W, m_t, M_H) - I^{2,1}(M_W, M_W, M_H)\right)$$
$$+(m_t^2 - M_W^2)^2(5M_H^2 - 2M_W^2)I^{2,2}(M_W, M_W, M_H)\Big\}$$

♠ =0

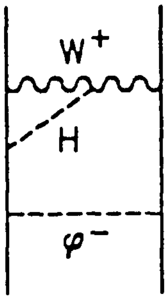


(4)

$$D11 = \frac{m_t^2}{4}\eta_1 \Big\{ 2\left(H^1(m_t) - H^1(M_W)\right)^2 - (m_t^2 - M_W^2)\left(H^1(m_t) - H^1(M_W)\right) \times$$
$$\times \left(H^2(m_t) + H^2(M_W)\right) + (2M_H^2 - 5m_t^2 - 3M_W^2)I^{1,1}(m_t, m_t, M_H)$$
$$+(2M_H^2 - 3m_t^2 - 5M_W^2)I^{1,1}(M_W, M_W, M_H) - 4(M_H^2 - 2m_t^2 - 2M_W^2)\times$$
$$\times I^{1,1}(m_t, M_W, M_H) - (M_H^2 - 4m_t^2)\left(I^{2,1}(m_t, m_t, M_H) - I^{2,1}(M_W, M_W, M_H)\right) \times$$
$$\times (m_t^2 - M_W^2) + (m_t^2 - M_W^2)(M_H^2 - 3m_t^2 - M_W^2)I^{2,1}(m_t, M_W, M_H)$$

$$-(m_t^2 - M_W^2)(M_H^2 - m_t^2 - 3M_W^2)I^{2,1}(M_W, M_W, M_H)\Big\}$$

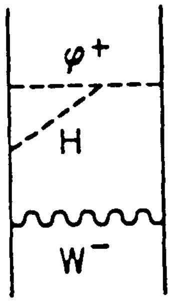
♠ $=0$



(4)

$$D12 = -\frac{m_t^4}{4M_W^2}\eta_1\Big\{2\left(H^1(m_t) - H^1(M_W)\right)^2$$
$$-(m_t^2 - M_W^2)\left(H^1(m_t) - H^1(M_W)\right)\left(H^2(m_t) + H^2(M_W)\right)$$
$$+(2M_H^2 - 7m_t^2 - M_W^2)I^{1,1}(m_t, m_t, M_H) + (2M_H^2 - 5m_t^2 - 3M_W^2)I^{1,1}(M_W, M_W, M_H)$$
$$-4(M_H^2 - 3m_t^2 - M_W^2)I^{1,1}(m_t, M_W, M_H) - (m_t^2 - M_W^2)(M_H^2 - 4m_t^2)I^{2,1}(m_t, m_t, M_H)$$
$$+(m_t^2 - M_W^2)(M_H^2 - 3m_t^2 - M_W^2)\left(I^{2,1}(m_t, M_W, M_H) - I^{2,1}(M_W, m_t, M_H)\right)$$
$$+(m_t^2 - M_W^2)(M_H^2 - 2m_t^2 - 2M_W^2)I^{2,1}(M_W, M_W, M_H)\Big\}$$

♠ $=0$

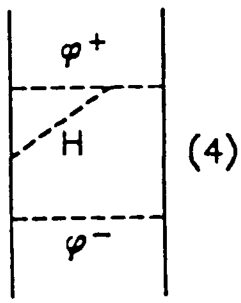


(4)

$$D13 = \frac{M_H^2}{2M_W^2}D12$$

♠ $= \frac{\eta_1}{1024\pi^4}\frac{m_t^2}{M_W^4}\frac{\ln(M_H^2/m_t^2)}{(1-x)^3}\Big(2(1-x) + (1+x)\ln x\Big)$

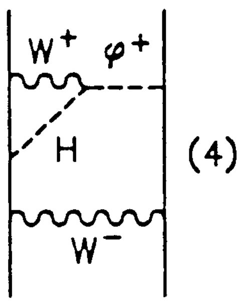
♣ $=0$



$$D14 = \frac{M_H^2 m_t^4}{8 M_W^6} D11$$

$$\spadesuit \quad = -\frac{\eta_1}{8192\pi^4}\frac{m_t^4}{M_W^6}\left[1 + \frac{2\ln(M_H^2/m_t^2)}{(1-x)^3}(1 - x^2 + 2x\ln x)\right]$$

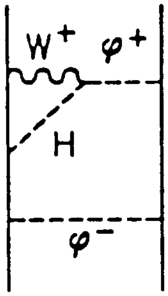
$$\clubsuit \quad = 0$$



$$\begin{aligned} D15 = &-\frac{m_t^4}{8M_W^2}\eta_1\Big\{2\left(H^1(m_t) - H^1(M_W)\right)^2 \\ &-(m_t^2 - M_W^2)\left(H^1(m_t) - H^1(M_W)\right)\left(H^2(m_t) + H^2(M_W)\right) \\ &+(2M_H^2 - 3m_t^2 - 5M_W^2)I^{1,1}(m_t, m_t, M_H) + (2M_H^2 - 5m_t^2 - 3M_W^2)I^{1,1}(M_W, M_W, M_H) \\ &-4(M_H^2 - 2m_t^2 - 2M_W^2)I^{1,1}(m_t, M_W, M_H) \\ &-(m_t^2 - M_W^2)(M_H^2 - 4m_t^2)\left(I^{2,1}(m_t, m_t, M_H) - I^{2,1}(M_W, M_W, M_H)\right) \\ &+(m_t^2 - M_W^2)(M_H^2 - 5m_t^2 + M_W^2)I^{2,1}(m_t, M_W, M_H) \\ &-(M_H^2 - m_t^2 + 5M_W^2)I^{2,1}(M_W, m_t, M_H)\Big\} \end{aligned}$$

$$\spadesuit \quad = -\frac{\eta_1}{1024\pi^4}\frac{\ln(M_H^2/m_t^2)}{M_W^2(1-x)^3}\Big(2(1-x) + (1+x)\ln x\Big)$$

$$\clubsuit \quad = 0$$



(4)

$$D16 = \frac{m_t^4 \eta_1}{8n M_W^4} \Big\{ 2M_W^2 \left(H^1(m_t) - H^1(M_W) \right)^2$$

$$-(m_t^2 - M_W^2) \left(H^1(m_t) - H^1(M_W) \right) \left(m_t^2 H^2(m_t) + (m_t^2 - 2M_W^2) H^2(M_W) \right)$$

$$+ \left(2M_W^2 (M_H^2 - 4m_t^2) - m_t^2 (m_t^2 - M_W^2) \right) I^{1,1}(m_t, m_t, M_H)$$

$$+ \left(2M_W^2 (M_H^2 - 4m_t^2) - 3m_t^2 (m_t^2 - M_W^2) \right) I^{1,1}(M_W, M_W, M_H)$$

$$-4 \left(M_W^2 (M_H^2 - 4m_t^2) - m_t^2 (m_t^2 - M_W^2) \right) I^{1,1}(m_t, M_W, M_H)$$

$$-m_t^2 \left((M_H^2 - 4m_t^2) I^{2,1}(m_t, m_t, M_H) - (M_H^2 - 5m_t^2 + M_W^2) I^{2,1}(m_t, M_W, M_H) \right) \times$$

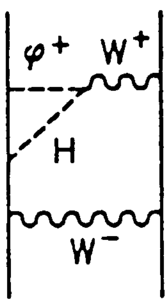
$$\times (m_t^2 - M_W^2) + (m_t^2 - M_W^2) \left((M_H^2 - 5m_t^2 + 4M_W^2)(m_t^2 - 2M_W^2) + 4(m_t^2 - M_W^2)^2 \right) \times$$

$$\times I^{2,1}(M_W, m_t, M_H) - (m_t^2 - M_W^2) \left(M_H^2 (m_t^2 - 2M_W^2) + 2M_W^2 (m_t^2 + M_W^2) \right) \times$$

$$\times I^{2,1}(M_W, M_W, M_H) \Big\}$$

$$\spadesuit \quad = \frac{\eta_1}{4096\pi^4} \frac{m_t^2}{M_W^4} \frac{\ln(M_H^2/m_t^2)}{(1-x)^3} (1 - x^2 + 2x \ln x)$$

$$\clubsuit \quad = 0$$



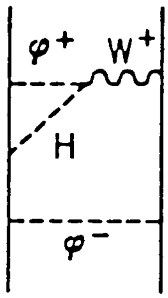
(4)

$$D17 = \frac{m_t^2 \eta_2}{8M_W^2 n(n-1)} \Big\{ 2(M_H^2 - m_t^2 - M_W^2) \left(H^1(m_t) - H^1(M_W) \right)^2$$

$$-(m_t^2 - M_W^2)\left(H^1(m_t) - H^1(M_W)\right)\left(H^2(m_t) + H^2(M_W)\right)(M_H^2 - m_t^2 - M_W^2)$$
$$-(m_t^2 - M_W^2)\left(H^1(m_t) - H^1(M_W)\right)\left(2H^1(M_H) - H^1(m_t) - H^1(M_W)\right)$$
$$-(m_t^2 - M_W^2)^2\left[\left(H^1(M_H) - H^1(m_t)\right)H^2(m_t) + \left(H^1(M_H) - H^1(M_W)\right)H^2(M_W)\right]$$
$$+2M_H^2\Big((M_H^2 - 3m_t^2 - M_W^2)I^{1,1}(m_t, m_t, M_H) - 2(M_H^2 - 2m_t^2 - 2M_W^2)\times$$
$$\times I^{1,1}(m_t, M_W, M_H)\Big) + 2M_H^2(M_H^2 - m_t^2 - 3M_W^2)I^{1,1}(M_W, M_W, M_H)$$
$$-M_H^2(m_t^2 - M_W^2)\left[(M_H^2 - 4m_t^2)I^{2,1}(m_t, m_t, M_H) - (M_H^2 - 4M_W^2)I^{2,1}(M_W, M_W, M_H)\right]$$
$$+\left(M_H^2(M_H^2 - 2M_W^2 - 2m_t^2) + (m_t^2 - M_W^2)^2\right)(m_t^2 - M_W^2)\times$$
$$\times\left[I^{2,1}(m_t, M_W, M_H) - I^{2,1}(M_W, m_t, M_H)\right]\Big\}$$
$$-\frac{m_t^2\eta_1}{8M_W^2(n-1)}\Big\{\left(3(1-n)m_t^2 + 2nM_H^2 - (1+3n)M_W^2\right)\left(H^1(m_t) - H^1(M_W)\right)^2$$
$$+nm_t^2(m_t^2 - M_W^2)\left(H^1(m_t) - H^1(M_W)\right)\left(2H^1(M_H) - 3H^1(m_t) + H^1(M_W)\right)$$
$$+\left(3(1-3n)M_H^2m_t^2 - 7(1-n)m_t^4 + 2nM_H^4 - (1+n)M_H^2M_W^2 - (1-n)m_t^2M_W^2\right)\times$$
$$\times I^{1,1}(m_t, m_t, M_H) + \Big(-4nM_H^4 + (1-n)(7m_t^4 + 6m_t^2M_W^2 + 3M_W^4)$$
$$-2(3-7n)M_H^2m_t^2 + 2(1+3n)M_H^2M_W^2\Big)I^{1,1}(m_t, M_W, M_H) + \Big(2nM_H^4 + (3-5n)\times$$
$$\times M_H^2m_t^2 - (1+5n)M_H^2M_W^2 - (1-n)M_W^2(5m_t^2 - 3M_W^2)\Big)I^{1,1}(M_W, M_W, M_H)$$
$$+(m_t^2 - M_W^2)\left(n(M_H^2 - M_W^2)^2 + m_t^4 + (1-3n)M_H^2m_t^2 - (5-3n)m_t^2M_W^2\right)\times$$
$$\times I^{2,1}(m_t, M_W, M_H) + (m_t^2 - M_W^2)\Big((2-3n)m_t^4 - nM_H^4 - 2(1-2n)M_H^2m_t^2$$
$$+(1+n)M_H^2M_W^2 + (3-n)m_t^2M_W^2 - M_W^4\Big)I^{2,1}(M_W, m_t, M_H)$$
$$+(m_t^2 - M_W^2)\left(nM_H^4 + 2(1-n)(M_H^2m_t^2 - m_t^2M_W^2 - M_W^4) - (1+3n)M_H^2M_W^2\right)\Big\}$$

$$\spadesuit \quad = \frac{\eta_2}{8192\pi^4}\frac{\ln(M_H^2/m_t^2)}{M_W^2(1-x)^3}(1 - x^2 + 2x\ln x)$$

$$\clubsuit \quad = 0$$

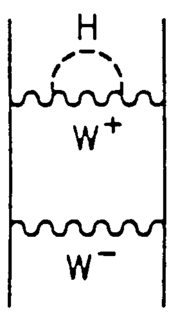


(4)

$$D18 = \frac{m_t^2\eta_1}{8nM_W^4}\Big\{\Big(2\left(H^1(m_t) - H^1(M_W)\right) - (m_t^2 - M_W^2)\left(H^2(m_t) + H^2(M_W)\right)\Big) \times$$
$$\times\left(H^1(m_t) - H^1(M_W)\right) + \left(I^{1,1}(m_t, m_t, M_H) + I^{1,1}(M_W, M_W, M_H)\right) \times$$
$$\times(2M_H^2 - 9m_t^2 + M_W^2) - 4(M_H^2 - 2m_t^2 - 2M_W^2)I^{1,1}(m_t, M_W, M_H)$$
$$-(m_t^2 - M_W^2)(M_H^2 + m_t^2 - 5M_W^2)I^{2,1}(m_t, M_W, M_H)$$
$$-(m_t^2 - M_W^2)(M_H^2 - 4m_t^2)\left(I^{2,1}(m_t, m_t, M_H) - I^{2,1}(M_W, M_W, M_H)\right)$$
$$-(m_t^2 - M_W^2)(M_H^2 - 5m_t^2 + M_W^2)I^{2,1}(M_W, m_t, M_H)\Big\}$$

$$\spadesuit \quad = -\frac{\eta_1}{2048\pi^4}\left(\frac{m_t^2}{M_W^4}\right)\frac{\ln(M_H^2/m_t^2)}{(1-x)^3}\Big(2(1-x) + (1+x)\ln x\Big)$$

$$\clubsuit \quad = 0$$

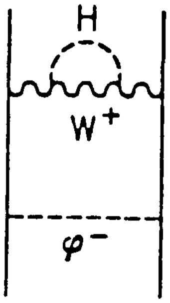


(2)

$$D19 = \frac{M_W^2}{2n}\eta_2\Big\{-(2m_t^2 + M_W^2)\left(I^{1,1}(m_t, M_W, M_H) - I^{1,1}(M_W, M_W, M_H)\right)$$
$$+(m_t^2 - M_W^2)\left(m_t^2 I^{2,1}(m_t, M_W, M_H) + (m_t^2 + M_W^2)I^{2,1}(M_W, M_W, M_H)\right)$$
$$+M_W^2(m_t^2 - M_W^2)^2 I^{3,1}(M_W, M_W, M_H)\Big\}$$

$$\spadesuit \quad = -\frac{\eta_2}{4096\pi^4}\left(\frac{M_W^2}{m_t^4}\right)\frac{\ln(M_H^2/m_t^2)}{(1-x)^4}\Big((1-x)(5+x)+2(1+2x)\ln x\Big)$$

$$\clubsuit \quad =0$$

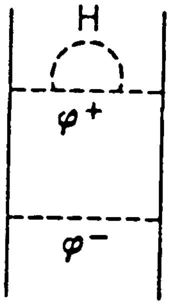


(2)

$$D20=\frac{m_t^4}{2}\eta_1\Big\{3\left(I^{1,1}(m_t,M_W,M_H)-I^{1,1}(M_W,M_W,M_H)\right)-(m_t^2-M_W^2)\times$$
$$\times\left(2I^{2,1}(m_t,M_W,M_H)+2I^{2,1}(M_W,M_W,M_H)-(m_t^2+M_W^2)I^{3,1}(M_W,M_W,M_H)\right)\Big\}$$

$$\spadesuit \quad =\frac{\eta_1}{1024\pi^4}\frac{\ln(M_H^2/m_t^2)}{M_W^2(1-x)^4}\Big((1-x)(1+5x)+2x(2+x)\ln x\Big)$$

$$\clubsuit \quad =0$$



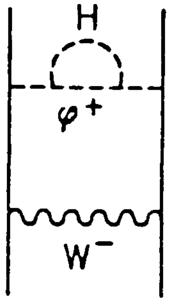
(2)

$$D21=\frac{M_H^4 m_t^4}{4M_W^8}\frac{\eta_1}{\eta_2}D19$$

$$\spadesuit \quad =\frac{\eta_1}{32768\pi^4}\left(\frac{M_H^2}{M_W^6}\right)\frac{1}{(1-x)^4}\Big\{M_H^2\Big[2\left(\frac{2}{\epsilon}-2\gamma_E-\ln\frac{m_t^2}{4\pi}-\ln\frac{M_H^2}{4\pi}\right)\Big((1-x)(5+x)$$
$$+2(1+2x)\ln x\Big)\Big)+(1-x)(23+7x)+(2+12x+x^2)\ln x-2(1+2x)\ln^2 x\Big]$$
$$+m_t^2\Big[(1-x)(2+5x-x^2)+2x(8-4x-x^2)\ln x+4x(1+2x)\ln^2 x$$

$$-2x\ln(M_H^2/m_t^2)\Big((5+x)(1-x)+2(1+2x)\ln x\Big)\Big]\Bigg\}$$
$$+\frac{\eta_1}{49152\pi^4}\left(\frac{m_t^4}{M_W^6}\right)\left[2-\frac{\ln(M_H^2/m_t^2)}{(1-x)^4}\Big((1-x)(1+3x+33x^2-x^3)+12x^2(2+x)\ln x\Big)\right]$$

♣ $=0$

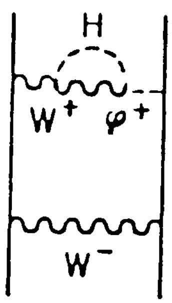


(2)

$$D22=\frac{M_H^4}{4M_W^4}D20$$

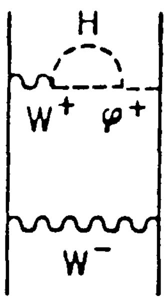
♠ $$=-\frac{\eta_1}{8192\pi^4}\left(\frac{M_H^2}{M_W^6}\right)\frac{1}{(1-x)^4}\Bigg\{2M_H^2\Bigg[\left(\frac{2}{\epsilon}-2\gamma_E-\ln\frac{m_t^2}{4\pi}-\ln\frac{M_H^2}{4\pi}\right)\Big((1-x)(1+5x)$$
$$+2x(2+x)\ln x\Big)+(1-x)(1+11x)-(1-6x-7x^2)\ln x-x(2+x)\ln^2 x\Bigg]$$
$$+M_W^2\Big[(1-x)(5+x)+2(2+6x-5x^2)\ln x+4x(2+x)\ln^2 x$$
$$-2\ln(M_H^2/m_t^2)\Big((1-x)(1+5x)+2x(2+x)\ln x\Big)\Big]\Bigg\}$$
$$+\frac{\eta_1}{2048\pi^4}\frac{1}{M_W^2(1-x)^4}\Big(3(1-x^2)+(1+4x+x^2)\ln x\Big)$$

♣ $=0$



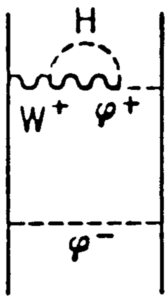
(4)

$$
\begin{aligned}
D23 =& \frac{m_t^2}{4}\eta_1 \Big\{ \Big[-3\left(H^1(m_t) - H^1(M_W)\right) + (m_t^2 - M_W^2)\left(H^2(m_t) + 2H^2(M_W)\right) \\
& + (m_t^2 - M_W^2)^2 H^3(M_W)\Big]\left(H^1(M_H) - H^1(M_W)\right) \\
& + 3(M_H^2 + 2m_t^2)\left(I^{1,1}(m_t, M_W, M_H) - I^{1,1}(M_W, M_W, M_H)\right) \\
& - (m_t^2 - M_W^2)\Big[(M_H^2 + 3m_t^2 - M_W^2)I^{2,1}(m_t, M_W, M_H) \\
& + (2M_H^2 + 3m_t^2 + M_W^2)I^{2,1}(M_W, M_W, M_H)\Big] \\
& - (m_t^2 - M_W^2)^2(M_H^2 + M_W^2)I^{3,1}(M_W, M_W, M_H)\Big\} \\
\spadesuit \quad =& \frac{3\eta_1}{2048\pi^4}\frac{\ln(M_H^2/m_t^2)}{m_t^2(1-x)^4}\Big((1-x)(5+x) + 2(1+2x)\ln x\Big) \\
\clubsuit \quad =& 0
\end{aligned}
$$



(4)

$$
\begin{aligned}
D24 =& \frac{M_H^2 m_t^2}{4M_W^2}\eta_1 \Big\{ \Big[3\left(H^1(m_t) - H^1(M_W)\right) - (m_t^2 - M_W^2)\left(H^2(m_t) + 2H^2(M_W)\right) \\
& - (m_t^2 - M_W^2)^2 H^3(M_W)\Big]\left(H^1(M_H) - H^1(M_W)\right) \\
& - 3(M_H^2 - M_W^2)\left(I^{1,1}(m_t, M_W, M_H) - I^{1,1}(M_W, M_W, M_H)\right) \\
& + (M_H^2 - M_W^2)\left(I^{2,1}(m_t, M_W, M_H) + 2I^{2,1}(M_W, M_W, M_H) + I^{3,1}(M_W, M_W, M_H)\right) \times \\
& \times (m_t^2 - M_W^2)\Big\} \\
\spadesuit \quad =& \frac{\eta_1}{4096\pi^4}\frac{1}{m_t^2(1-x)^4}\left(\frac{M_H^2}{M_W^2} - 2\ln(M_H^2/m_t^2)\right)\Big((1-x)(5+x) + 2(1+2x)\ln x\Big) \\
\clubsuit \quad =& 0
\end{aligned}
$$

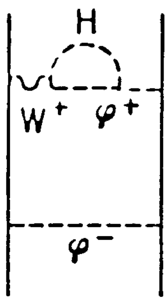


(4)

$$D25 = -\frac{m_t^2}{n M_W^2} D23$$

$$\spadesuit \quad = -\frac{3\eta_1}{8192\pi^4} \frac{\ln(M_H^2/m_t^2)}{M_W^2(1-x)^4}\Big((1-x)(5+x) + 2(1+2x)\ln x\Big)$$

$$\clubsuit \quad = 0$$

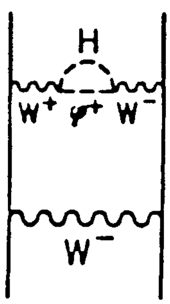


(4)

$$D26 = -\frac{m_t^2}{n M_W^2} D24$$

$$\spadesuit \quad = -\frac{\eta_1}{16384\pi^4} \frac{1}{M_W^2(1-x)^4}\left(\frac{M_H^2}{M_W^2} - 2\ln(M_H^2/m_t^2)\right)\Big((1-x)(5+x) + 2(1+2x)\ln x\Big)$$

$$\clubsuit \quad = 0$$

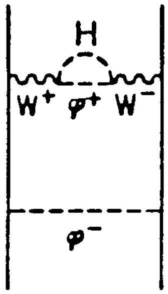


(2)

$$D27 = -\frac{\eta_2}{8n(n-1)}\Big\{\Big(H^1(m_t) - H^1(M_W)\Big) H^1(M_H)(3M_H^2 + 2m_t^2 - 2M_W^2)$$
$$-(m_t^2 - M_W^2)H^1(M_W)(3M_H^2 - 2m_t^2 - 4M_W^2)$$
$$-(m_t^2 - M_W^2)H^1(M_H)\Big(H^2(m_t)(M_H^2 + m_t^2 - M_W^2) + H^2(M_W)(2M_H^2 + m_t^2 - M_W^2)\Big)$$
$$+(m_t^2 - M_W^2)H^1(M_W)\Big(H^2(m_t)(M_H^2 - m_t^2 - M_W^2) + H^2(M_W)(2M_H^2 - m_t^2 - 3M_W^2)\Big)$$
$$+(m_t^2 - M_W^2)^2 H^3(M_W)\Big(H^1(M_W)(M_H^2 - 2M_W^2) - M_H^2 H^1(M_H)\Big)$$
$$-(3M_H^4 - 4M_H^2 m_t^2 + m_t^4 - 8M_H^2 m_t^2 - 2m_t^2 M_W^2 + M_W^4)\times$$
$$\times\Big(I^{1,1}(m_t, M_W, M_H) - I^{1,1}(M_W, M_W, M_H)\Big)$$
$$+(m_t^2 - M_W^2)\Big(M_H^2(M_H^2 - 2m_t^2 - 2M_W^2) + (m_t^2 - M_W^2)^2\Big) I^{2,1}(m_t, M_W, M_H)$$
$$+2M_H^2(m_t^2 - M_W^2)(M_H^2 - m_t^2 - 3M_W^2)I^{2,1}(M_W, M_W, M_H)$$
$$+M_H^2(m_t^2 - M_W^2)^2(M_H^2 - 4M_W^2)I^{3,1}(M_W, M_W, M_H)\Big\}$$
$$-\frac{\eta_1}{8(n-1)}\Big\{-\Big(H^1(m_t) - H^1(M_W)\Big) H^1(M_H)\Big(2(2-n)m_t^2 + 3nM_H^2 + 2(1-2n)M_W^2\Big)$$
$$+\Big(H^1(m_t) - H^1(M_W)\Big) H^1(M_W)\Big(3nM_H^2 - 2(2-n)m_t^2 - 2(1+n)M_W^2\Big)$$
$$+H^2(m_t)\Big[\Big((2-n)m_t^2 + n(M_H^2 - M_W^2)\Big) H^1(M_H) + \Big((2-n)m_t^2 - n(M_H^2 - M_W^2)\Big)\Big]\times$$
$$\times(m_t^2 - M_W^2) + (m_t^2 - M_W^2)H^2(M_W)\Big[\Big((2-n)m_t^2 + 2nM_H^2 + (2-3n)M_W^2\Big) H^1(M_H)$$
$$+\Big((2-n)m_t^2 - 2nM_H^2 + (2+n)M_W^2\Big)\Big]$$
$$+(m_t^2 - M_W^2)^2 H^3(M_W)\Big[\Big(nM_H^2 + 2(1-n)M_W^2\Big) H^1(M_H) - (nM_H^2 - 2M_W^2)H^1(M_W)\Big]$$
$$+\Big(3nM_H^4 + m_t^4 - 4M_H^2 m_t^2 - 2m_t^2 M_W^2 - 2(1+3n)M_H^2 M_W^2 - (2-3n)M_W^4\Big)\times$$
$$\times\Big(I^{1,1}(m_t, M_W, M_H) - I^{1,1}(M_W, M_W, M_H)\Big)$$
$$-(m_t^2 - M_W^2)\Big(n(M_H^2 - M_W^2)^2 + m_t^2(m_t^2 - 2M_H^2 - 2M_W^2)\Big) I^{2,1}(m_t, M_W, M_H)$$
$$-2(m_t^2 - M_W^2)\Big(n(M_H^2 - M_W^2)^2 - M_W^4 - M_H^2 M_W^2 - M_H^2 m_t^2\Big) I^{2,1}(M_W, M_W, M_H)$$
$$-(m_t^2 - M_W^2)^2\Big(n(M_H^2 - M_W^2)^2 - M_W^2(M_W^2 + 2M_H^2)\Big) I^{3,1}(M_W, M_W, M_H)\Big\}$$
$$\spadesuit \quad = -\frac{\eta_2}{16384\pi^4}\left(\frac{M_H^2}{m_t^4}\right)\frac{1}{(1-x)^4}\Bigg[\left(\frac{2}{\epsilon} - 2\gamma_E - \ln\frac{m_t^2}{4\pi} - \ln\frac{M_H^2}{4\pi}\right)\Big((1-x)(5+x)$$
$$+2(1+2x)\ln x\Big) + 2(1-x)(7+2x) + (3+14x+x^2)\ln x + (1+2x)\ln^2 x\Bigg]$$
$$+\frac{\ln(M_H^2/m_t^2)}{24576\pi^4}\frac{1}{m_t^2(1-x)^4}\Big\{\eta_1[-(1-x)(1-5x-2x^2) + 6x^2\ln x]$$

$$+2\eta_2[(1-x)(2+5x-x^2)+6x\ln x]\Big\}$$

♣ $=0$

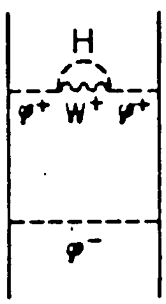


(2)

$$D28=\frac{m_t^4\eta_1}{8nM_W^2}\Big\{2\Big[-3\left(H^1(m_t)-H^1(M_W)\right)+(m_t^2-M_W^2)\left(H^2(m_t)+2H^2(M_W)\right.$$
$$\left.+(m_t^2-M_W^2)H^3(M_W)\right)\Big]\left(H^1(M_H)-H^1(M_W)\right)$$
$$-(6M_H^2+2m_t^2-5M_W^2)\left(I^{1,1}(m_t,M_W,M_H)-I^{1,1}(M_W,M_W,M_H)\right)$$
$$+(m_t^2-M_W^2)\left((2M_H^2-m_t^2+2M_W^2)I^{2,1}(m_t,M_W,M_H)\right.$$
$$\left.+(4M_H^2-m_t^2+3M_W^2)I^{2,1}(M_W,M_W,M_H)\right)$$
$$+(m_t^2-M_W^2)^2(2M_H^2+M_W^2)I^{3,1}(M_W,M_W,M_H)\Big\}$$

♠ $$=\frac{\eta_1}{8192\pi^4}\left(\frac{M_H^2}{M_W^4}\right)\frac{1}{(1-x)^4}\left[2\left(\frac{2}{\epsilon}-2\gamma_E-\ln\frac{m_t^2}{4\pi}-\ln\frac{M_H^2}{4\pi}\right)\left((1-x)(1+5x)\right.\right.$$
$$\left.\left.+2x(2+x)\ln x\right)+3(1-x)(1+9x)-2(1-8x-8x^2)\ln x-2x(2+x)\ln^2 x\right]$$
$$+\frac{\eta_1}{16384\pi^4}\frac{\ln(M_H^2/m_t^2)}{M_W^2(1-x)^4}\left((1-x)(1-19x)+2(1-6x-4x^2)\right)$$

♣ $=0$

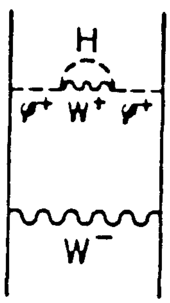


(2)

$$
\begin{aligned}
D29 = & \frac{m_t^4\eta_1}{8nM_W^4}\Big\{-\Big(H^1(M_H)-2H^1(M_W)\Big)\left(2m_t^2+M_W^2\right)\Big(H^1(m_t)-H^1(M_W)\Big) \\
& +\Big(H^1(M_H)+H^1(M_W)\Big)m_t^2(m_t^2-M_W^2)H^2(m_t) \\
& +\Big(H^1(M_H)-2H^1(M_W)\Big)(m_t^2-M_W^2)\times \\
& \times\Big[(m_t^2+M_W^2)H^2(M_W)+M_W^2(m_t^2-M_W^2)H^3(M_W)\Big] \\
& +(4M_H^2m_t^2+2m_t^4+2M_H^2M_W^2+2m_t^2M_W^2-M_W^4)\Big(I^{1,1}(m_t,M_W,M_H) \\
& -I^{1,1}(M_W,M_W,M_H)\Big)-(m_t^2-M_W^2)m_t^2(2M_H^2+2m_t^2-M_W^2)I^{2,1}(m_t,M_W,M_H) \\
& -(m_t^2-M_W^2)(2M_H^2m_t^2+2M_H^2M_W^2+3m_t^2M_W^2-M_W^4)I^{2,1}(M_W,M_W,M_H) \\
& -M_W^2(m_t^2-M_W^2)^2(2M_H^2+M_W^2)I^{3,1}(M_W,M_W,M_H)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\spadesuit \quad = & -\frac{\eta_1}{32768\pi^4}\left(\frac{M_H^2}{M_W^4}\right)\frac{1}{(1-x)^4}\Big[2\left(\frac{2}{\epsilon}-2\gamma_E-\ln\frac{m_t^2}{4\pi}-\ln\frac{M_H^2}{4\pi}\right)\Big((1-x)(5+x) \\
& +2(1+2x)\ln x\Big)+(1-x)(23+7x)+2(2+12x+x^2)\ln x-2(1+2x)\ln^2 x\Big] \\
& +\frac{\eta_1}{16384\pi^4}\left(\frac{m_t^2}{M_W^4}\right)\frac{\ln(M_H^2/m_t^2)}{(1-x)^4}\Big((1-x)(4+15x-x^2)+2x(2x+7)\ln x\Big)
\end{aligned}
$$

$$\clubsuit \quad =0$$

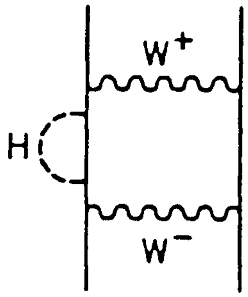


(2)

$$
\begin{aligned}
D30 = & \frac{m_t^4}{8M_W^2}\eta_1\Big\{\Big[3\Big(H^1(m_t)-H^1(M_W)\Big)+(m_t^2-M_W^2)\Big(H^2(m_t)-2H^2(M_W) \\
& -(m_t^2-M_W^2)H^3(M_W)\Big)\Big]\Big(H^1(M_H)-H^1(M_W)\Big) \\
& -(6M_H^2+4m_t^2-M_W^2)\Big(I^{1,1}(m_t,M_W,M_H)-I^{1,1}(M_W,M_W,M_H)\Big) \\
& +\Big((2M_H^2+2m_t^2-M_W^2)I^{2,1}(m_t,M_W,M_H)+2(2M_H^2+m_t^2)I^{2,1}(M_W,M_W,M_H)\Big)\times \\
& \times(m_t^2-M_W^2)+(m_t^2-M_W^2)^2(2M_H^2+M_W^2)I^{3,1}(M_W,M_W,M_H)\Big\}
\end{aligned}
$$

$$\spadesuit \quad = \frac{\eta_1}{4096\pi^4}\left(\frac{M_H^2}{M_W^4}\right)\frac{1}{(1-x)^4}\Bigg[\left(\frac{2}{\epsilon}-2\gamma_E-\ln\frac{m_t^2}{4\pi}-\ln\frac{M_H^2}{4\pi}\right)\Big((1-x)(1+5x)$$
$$+2x(2x+1)\ln x\Big)+(1-x)(1+11x)-(1-6x-7x^2)\ln x - x(x+2)\ln^2 x\Bigg]$$
$$-\frac{\eta_1}{4096\pi^4}\frac{\ln(M_H^2/m_t^2)}{M_W^2(1-x)^4}\Big((1-x)(11+7x)+2(2+6x+x^2)\ln x\Big)$$

$$\clubsuit \quad =0$$

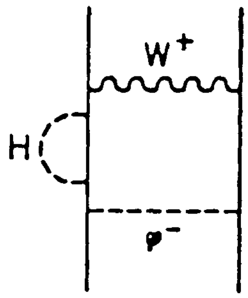


(2)

$$D31 = \frac{m_t^2\eta_2}{16nM_W^2}\Bigg\{2\left[(2m_t^2+M_W^2)\left(H^1(m_t)-H^1(M_W)\right)-(m_t^2-M_W^2)^2H^3(m_t)\right]\times$$
$$\times(H^1(M_H)-H^1(m_t))-\left((3m_t^2+M_W^2)H^2(m_t)+(m_t^2+M_W^2)H^2(M_W)\right)\times$$
$$\times(m_t^2-M_W^2)(H^1(M_H)-H^1(m_t))-\left(I^{1,1}(m_t,m_t,M_H)-I^{1,1}(M_W,m_t,M_H)\right)\times$$
$$\times(4M_H^2m_t^2-9m_t^4+2M_H^2M_W^2-14m_t^2M_W^2-M_W^4)$$
$$+(m_t^2-M_W^2)(3M_H^2m_t^2-8m_t^4+M_H^2M_W^2-8m_t^2M_W^2)I^{2,1}(m_t,m_t,M_H)$$
$$+(m_t^2-M_W^2)(M_H^2m_t^2-m_t^4+M_H^2M_W^2-6m_t^2M_W^2-M_W^4)I^{2,1}(M_W,M_W,M_H)$$
$$+2m_t^2(m_t^2-M_W^2)^2(4m_t^2-M_H^2)I^{3,1}(m_t,m_t,M_H)\Bigg\}$$

$$\spadesuit \quad = \frac{\eta_2}{16384\pi^4}\frac{\ln(M_H^2/m_t^2)}{M_W^2(1-x)^4}\Big((1-x)(2+15x+x^2)+2x(5+4x)\ln x\Big)$$

$$\clubsuit \quad =0$$

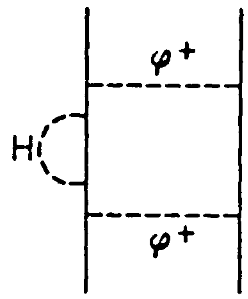


(4)

$$D32=\frac{m_t^6}{4M_W^4}\eta_1\Big\{-\Big[3\left(H^1(m_t)-H^1(M_W)\right)+(m_t^2-M_W^2)\left(2H^2(m_t)+H^2(M_W)\right.$$
$$\left.+(m_t^2-M_W^2)H^3(m_t)\right)\Big]\left(H^1(M_H)-H^1(M_W)\right)$$
$$+(3M_H^2-8m_t^2-4M_W^2)\left(I^{1,1}(m_t,m_t,M_H)-I^{1,1}(m_t,M_W,M_H)\right)$$
$$-\left(2(M_H^2-3m_t^2-M_W^2)I^{2,1}(m_t,m_t,M_H)+(M_H^2-2m_t^2-2M_W^2)I^{2,1}(M_W,m_t,M_H)\right)\times$$
$$\times(m_t^2-M_W^2)+(m_t^2-M_W^2)^2(M_H^2-4m_t^2)I^{3,1}(m_t,m_t,M_H)\Big\}$$

♠ $$=-\frac{\eta_1}{2048\pi^4}\left(\frac{m_t^2}{M_W^4}\right)\frac{\ln(M_H^2/m_t^2)}{(1-x)^4}\left((1-x)(7+11x)+2(1+6x+2x^2)\ln x\right)$$

♣ $$=0$$

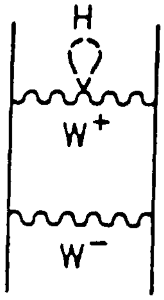


(2)

$$D33=\frac{m_t^4}{M_W^4}\frac{\eta_1}{\eta_2}D31$$

♠ $$=-\frac{\eta_1}{16384\pi^4}\left(\frac{m_t^4}{M_W^6}\right)\Big[\frac{4}{\epsilon}-4\gamma_E+16-4\ln\frac{m_t^2}{4\pi}-\frac{\ln(M_H^2/m_t^2)}{(1-x)^4}$$
$$\left((1-x)(2+15x+x^2)+2x(5+4x)\ln x\right)\Big]$$

♣ $$=-\frac{\eta_1}{4096\pi^4}\left(\frac{m_t^4}{M_W^6}\right)\left[\frac{1}{\epsilon}-\gamma_E+\frac{21}{4}-\frac{\pi^2}{3}-\ln\frac{m_t^2}{4\pi}\right]$$

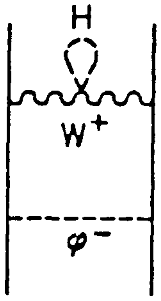


(2)

$$D34 = \frac{\eta_2}{8n} H^1(M_H)\Big\{-(2m_t^2 + M_W^2)\left(H^1(m_t) - H^1(M_W)\right)$$
$$+(m_t^2 - M_W^2)\left(m_t^2 H^2(m_t) + (m_t^2 + M_W^2)H^2(M_W) + M_W^2(m_t^2 - M_W^2)H^3(M_W)\right)\Big\}$$

$\spadesuit$ $$= \frac{\eta_2}{32768\pi^4}\left(\frac{M_H^2}{m_t^4}\right)\frac{1}{(1-x)^4}\Bigg[2\left(\frac{2}{\epsilon} - 2\gamma_E - \ln\frac{m_t^2}{4\pi} - \ln\frac{M_H^2}{4\pi}\right)\Big((1-x)(5+x)$$
$$+2(1+2x)\ln x\Big) + (1-x)(23+7x) + 2(2+12x+x^2)\ln x - 2(1+2x)\ln^2 x\Bigg]$$

$\clubsuit$ $=0$

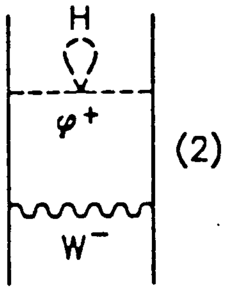


(2)

$$D35 = \frac{m_t^4}{8M_W^2}\eta_1 H^1(m_t)\Big\{3\left(H^1(m_t) - H^1(M_W)\right)$$
$$-(m_t^2 - M_W^2)\left(H^2(m_t) + 2H^2(M_W) + (m_t^2 - M_W^2)H^3(M_W)\right)\Big\}$$

$\spadesuit$ $$= -\frac{\eta_1}{4096\pi^4}\left(\frac{M_H^2}{M_W^4}\right)\frac{1}{(1-x)^4}\Bigg[\left(\frac{2}{\epsilon} - 2\gamma_E - \ln\frac{m_t^2}{4\pi} - \ln\frac{M_H^2}{4\pi}\right)\Big((1-x)(1+5x)$$
$$+2x(2+x)\ln x\Big) + (1-x)(1+11x) - (1-6x-7x^2)\ln x - x(2+x)\ln^2 x\Bigg]$$

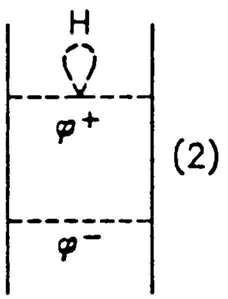
$\clubsuit$ $=0$



$$D36=\frac{M_H^2}{2M_W^2}D35$$

$$\spadesuit \quad =-\frac{\eta_1}{8192\pi^4}\left(\frac{M_H^4}{M_W^6}\right)\frac{1}{(1-x)^4}\left[\left(\frac{2}{\epsilon}-2\gamma_E-\ln\frac{m_t^2}{4\pi}-\ln\frac{M_H^2}{4\pi}\right)\Big((1-x)(1+5x)\right.$$
$$\left.+2x(2+x)\ln x\Big)+(1-x)(1+11x)-(1-6x-7x^2)\ln x-x(x+2)\ln^2 x\right]$$

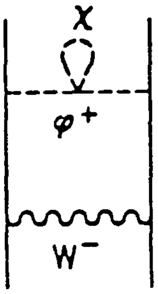
$$\clubsuit \quad =0$$



$$D37=\frac{M_H^2}{2M_W^2}\frac{\eta_1}{\eta_2}D36$$

$$\spadesuit \quad =\frac{\eta_1}{65536\pi^4}\left(\frac{M_H^4}{M_W^6}\right)\frac{1}{(1-x)^4}\left[2\left(\frac{2}{\epsilon}-2\gamma_E-\ln\frac{m_t^2}{4\pi}-\ln\frac{M_H^2}{4\pi}\right)\Big((1-x)(5+x)\right.$$
$$\left.+2(1+2x)\ln x\Big)+(1-x)(23+7x)+2(2+12x+x^2)\ln x-2(1+2x)\ln^2 x\right]$$

$$\clubsuit \quad =0$$

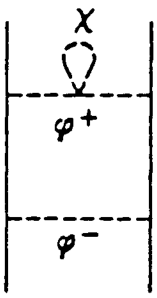


(2)

$D38{=}D36$ with $H^1(M_H) \to H^1(M_Z)$

$$\spadesuit \quad = -\frac{\eta_1}{8192\pi^4}\left(\frac{M_H^2 M_Z^2}{M_W^6}\right)\frac{1}{(1-x)^4}\left[\left(\frac{2}{\epsilon} - 2\gamma_E - \ln\frac{m_t^2}{4\pi} - \ln\frac{M_Z^2}{4\pi}\right)\Big((1-x)(1+5x) + 2x(2+x)\ln x\Big) + (1-x)(1+11x) - (1-6x-7x^2)\ln x - x(2+x)\ln^2 x\right]$$

$$\clubsuit \quad =0$$

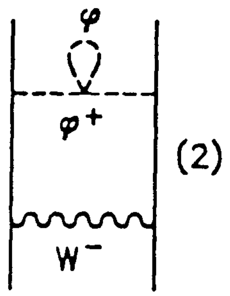


(2)

$D39{=}D37$ with $H^1(M_H) \to H^1(M_Z)$

$$\spadesuit \quad = \frac{\eta_1}{65536\pi^4}\left(\frac{M_H^2 M_Z^2}{M_W^6}\right)\frac{1}{(1-x)^4}\left[2\left(\frac{2}{\epsilon} - 2\gamma_E - \ln\frac{m_t^2}{4\pi} - \ln\frac{M_Z^2}{4\pi}\right)\Big((1-x)(5+x) + 2(1+2x)\ln x\Big) + (1-x)(23+7x) + 2(2+12x+x^2)\ln x - 2(1+2x)\ln^2 x\right]$$

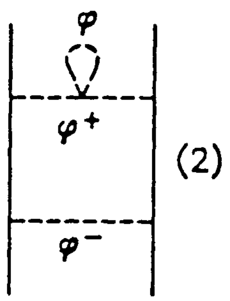
$$\clubsuit \quad =0$$



$D40{=}4 \times D36$ with $H^1(M_H) \rightarrow H^1(M_W)$

$$\spadesuit \quad =-\frac{\eta_1}{2048\pi^4}\left(\frac{M_H^2}{M_W^4}\right)\frac{1}{(1-x)^4}\left[\left(\frac{2}{\epsilon}-2\gamma_E-\ln\frac{m_t^2}{4\pi}-\ln\frac{M_W^2}{4\pi}\right)\Big((1-x)(1+5x)\right.$$
$$\left.+2x(2+x)\ln x\Big)+(1-x)(1+11x)-(1-6x-7x^2)\ln x-x(2+x)\ln^2 x\right]$$

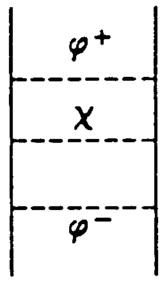
$$\clubsuit \quad =0$$



$D41{=}4 \times D37$ with $H^1(M_H) \rightarrow H^1(M_W)$

$$\spadesuit \quad =\frac{\eta_1}{16384\pi^4}\left(\frac{M_H^2}{M_W^4}\right)\frac{1}{(1-x)^4}\left[2\left(\frac{2}{\epsilon}-2\gamma_E-\ln\frac{m_t^2}{4\pi}-\ln\frac{M_W^2}{4\pi}\right)\Big((1-x)(5+x)\right.$$
$$\left.+2(1+2x)\ln x\Big)+(1-x)(23+7x)+2(2+12x+x^2)-2(1+2x)\ln^2 x\right]$$

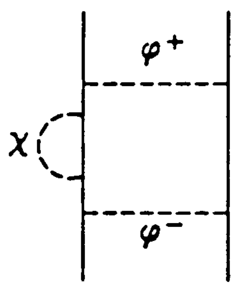
$$\clubsuit \quad =0$$



(1)

$$D42=\frac{m_t^8}{64M_W^6}\eta_1\Big\{\Big(H^1(m_t)-H^1(M_W)-(m_t^2-M_W^2)H^2(m_t)\Big)^2$$
$$-I^{1,1}(m_t,m_t,M_Z)+2I^{1,1}(m_t,M_W,M_Z)-I^{1,1}(M_W,M_W,M_Z)$$
$$+\Big(2I^{2,1}(m_t,M_W,M_Z)-2I^{2,1}(m_t,m_t,M_Z)+(m_t^2-M_W^2)I^{2,2}(m_t,m_t,M_Z)\Big)\times$$
$$\times(m_t^2-M_W^2)\Big\}$$

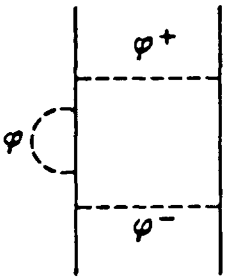
$$\spadesuit \quad =-\frac{\eta_1}{16384\pi^4}\left(\frac{m_t^4}{M_W^6}\right)$$



(2)

$$D43=\frac{m_t^6\eta_1}{16nM_W^6}\Big\{-2\left(H^1(m_t)-H^1(M_Z)\right)(2m_t^2+M_W^2)\left(H^1(m_t)-H^1(M_W)\right)$$
$$+\left(H^1(m_t)-H^1(M_Z)\right)m_t^2(m_t^2-M_W^2)^2H^3(m_t)$$
$$+(m_t^2-M_W^2)\left(H^1(m_t)-H^1(M_Z)\right)\left((3m_t^2+M_W^2)H^2(m_t)+(m_t^2+M_W^2)H^2(M_W)\right)$$
$$+\left((m_t^2-M_W^2)^2-2M_Z^2(2m_t^2+M_W^2)\right)\left(I^{1,1}(m_t,m_t,M_Z)-I^{1,1}(m_t,M_W,M_Z)\right)$$
$$-(m_t^2-M_W^2)\left((m_t^2-M_W^2)^2-M_Z^2(m_t^2+M_W^2)\right)I^{2,1}(M_W,m_t,M_Z)$$
$$+M_Z^2(m_t^2-M_W^2)\left((3m_t^2+M_W^2)I^{2,1}(m_t,m_t,M_Z)-2m_t^2(m_t^2-M_W^2)I^{3,1}(m_t,m_t,M_Z)\right)\Big\}$$

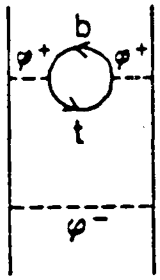
$$♠ \quad = \frac{\eta_1}{16384\pi^4}\left(\frac{m_t^4}{M_W^6}\right)\left[\frac{4}{\epsilon} - 4\gamma_E + 5 - 4\ln\frac{m_t^2}{4\pi}\right]$$



(2)

$$\begin{aligned} D44 = \frac{m_t^6\eta_1}{8nM_W^6}\Big\{ & (m_t^2 + 2M_W^2)H^1(M_W)\left(H^1(m_t) - H^1(M_W)\right) \\ & -(m_t^2 - M_W^2)H^1(M_W)\left((m_t^2 + M_W^2)H^2(m_t) + M_W^2 H^2(M_W) - m_t^2(m_t^2 - M_W^2)H^3(m_t)\right) \\ & +M_W^2(m_t^2 - M_W^2)\left(I^{1,1}(m_t,0,M_W) - I^{1,1}(m_t,0,M_W) - (m_t^2 - M_W^2)I^{2,1}(m_t,0,M_W)\right) \\ & +m_t^2(m_t^2 - M_W^2)^3 I^{3,1}(m_t,0,M_W)\Big\} \end{aligned}$$

$$♠ \quad = \frac{\eta_1}{32768\pi^4}\left(\frac{m_t^4}{M_W^6}\right)\left[\frac{4}{\epsilon} - 4\gamma_E + 3 - 4\ln\frac{m_t^2}{4\pi}\right]$$

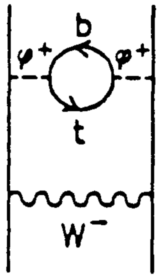


(2)

$$\begin{aligned} D45 = \frac{3m_t^6}{4nM_W^6}\eta_1\Big\{ & (2m_t^2 + M_W^2)H^1(m_t)\left(H^1(m_t) - H^1(M_W)\right) \\ & -(m_t^2 - M_W^2)H^1(m_t)\left(m_t^2 H^2(m_t) + (m_t^2 + M_W^2)H^2(M_W) + M_W^2(m_t^2 - M_W^2)H^3(M_W)\right) \\ & +m_t^2(m_t^2 - M_W^2)\left(I^{1,1}(m_t,m_t,0) - I^{1,1}(m_t,M_W,0)\right) \\ & -(m_t^2 - M_W^2)^2\left(m_t^2 I^{2,1}(M_W,m_t,0) - M_W^2(m_t^2 - M_W^2)I^{3,1}(M_W,m_t,0)\right)\Big\} \end{aligned}$$

$$♠ \quad = -\frac{3\eta_1}{4096\pi^4}\left(\frac{m_t^4}{M_W^6}\right)\left[4\left(\frac{1}{\epsilon} - \gamma_E\right)(2 - \ln x) - \ln^2 x - 4\ln x \ln\frac{m_t^2}{4\pi} + 2\ln x\right.$$

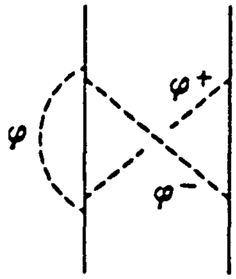
$$-8\ln\frac{m_t^2}{4\pi}+10\Big]$$



(2)

$$D46=\frac{3m_t^6}{4M_W^4}\eta_1\Big\{-3\left(H^1(m_t)-H^1(M_W)\right)+(m_t^2-M_W^2)^2H^1(m_t)H^3(M_W)$$
$$+(m_t^2-M_W^2)\left[H^1(m_t)\left(H^2(m_t)+2H^2(M_W)\right)+I^{1,1}(m_t,M_W,0)-I^{1,1}(m_t,m_t,0)\right]$$
$$+(m_t^2-M_W^2)^2\left(I^{2,1}(M_W,m_t,0)+(m_t^2-M_W^2)I^{3,1}(M_W,m_t,0)\right)\Big\}$$

♠ $$=\frac{3\eta_1}{1024\pi^4}\left(\frac{m_t^4}{M_W^6}\right)\left[\frac{2}{\epsilon}-2\gamma_E+1-\ln\frac{m_t^2}{4\pi}-\ln\frac{M_W^2}{4\pi}\right]$$



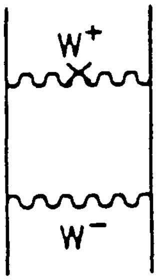
(2)

$$D47=\frac{m_t^6}{32M_W^6}\eta_1\Big\{\left(H^1(m_t)-H^1(M_W)\right)^2+(m_t^2-M_W^2)^2H^2(m_t)H^2(M_W)-2(m_t^2-M_W^2)\times$$
$$\times\left(m_t^2I^{1,1}(m_t,m_t,0)-(m_t^2+M_W^2)I^{1,1}(m_t,M_W,0)+M_W^2I^{1,1}(M_W,M_W,0)\right)$$
$$+(m_t^2-M_W^2)^3\left(I^{2,1}(m_t,M_W,0)+I^{2,1}(M_W,m_t,0)+(m_t^2+M_W^2)I^{2,2}(m_t,M_W,0)\right)\Big\}$$

♠ $$=\frac{\eta_1}{24576\pi^4}\left(\frac{m_t^4}{M_W^6}\right)(9-\pi^2)$$

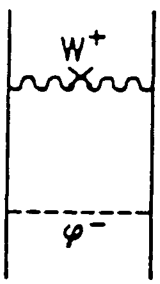
The following diagrams are generated by the insertion of the counterterm vertices of Appendix E in the 1-loop diagrams. The notation for the counterterms is the same as in

Appendix E.



(2)

$$
\begin{aligned}
C1=&\frac{i}{2n}\Big\{\Big[\Big(A_W(2m_t^2+M_W^2)-B_W(m_t^4-M_W^4)\Big)(\eta_2-n\eta_1)+nC_W(2m_t^2+M_W^2)\eta_1\Big]\times\\
&\Big(H^1(m_t)-H^1(M_W)\Big)-\Big[m_t^2\Big(A_W-B_W(m_t^2-M_W^2)\Big)(\eta_2-n\eta_1)+nC_W m_t^2\eta_1\Big]\times\\
&\times(m_t^2-M_W^2)H^2(m_t)-\Big[\Big((m_t^2+M_W^2)A_W-M_W^2(m_t^2-M_W^2)B_W\Big)(\eta_2-n\eta_1)\\
&+nC_W(m_t^2+M_W^2)\eta_1\Big](m_t^2-M_W^2)H^2(M_W)-M_W^2\left[A_W(\eta_2-n\eta_1)+nC_W\eta_1\right]\times\\
&\times(m_t^2-M_W^2)^2H^3(M_W)\Big\}\\
\spadesuit =&\frac{\eta_2}{32768\pi^4}\left(\frac{M_H^2}{m_t^4}\right)\Big((1-x)(5+x)+2(1+2x)\ln x\Big)\\
&-\frac{\ln(M_H^2/m_t^2)}{24576\pi^4}\frac{1}{m_t^2(1-x)^4}\Big[2\Big((1-x)(2+5x-x^2)+6x\ln x\Big)\eta_1\\
&-\Big((1-x)(1+25x+4x^2)+6x(2+3x)\ln x\Big)\eta_2\Big]\\
\clubsuit =&0
\end{aligned}
$$



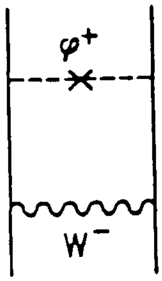
(2)

$$
\begin{aligned}
C2=&\frac{im_t^4\eta_1}{2nM_W^2}\Big\{\Big(H^1(m_t)-H^1(M_W)\Big)\Big[(1-n)\Big(3A_W-2(m_t^2-M_W^2)B_W\Big)-3C_W\Big]\\
&-(m_t^2-M_W^2)H^2(m_t)\Big[(1-n)\Big(A_W-(m_t^2-M_W^2)B_W\Big)-C_W\Big]
\end{aligned}
$$

$$
\begin{aligned}
&-(m_t^2 - M_W^2)H^2(M_W)\left[(1-n)\left(2A_W - (m_t^2 - M_W^2)B_W\right) - 2C_W\right] \\
&-(m_t^2 - M_W^2)^2 H^3(M_W)\left[(1-n)A_W - C_W\right]\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\spadesuit \ =& -\frac{\eta_1}{8192\pi^4}\left(\frac{M_H^2}{M_W^4}\right)\left((1-x)(1+5x) + 2x(2+x)\ln x\right) \\
&-\frac{\eta_1}{16384\pi^4}\frac{\ln(M_H^2/m_t^2)}{M_W^2(1-x)^4}\left((1-x)(17+61x) + 2(1+26x+12x^2)\ln x\right)
\end{aligned}
$$

$$\clubsuit \ = 0$$

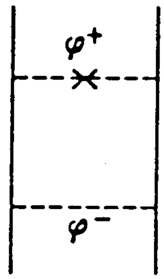


(2)

$$
\begin{aligned}
C3 =& \frac{i m_t^4 \eta_1}{2M_W^2}\Big\{\left(H^1(m_t) - H^1(M_W)\right)\left(3A_\phi + (2m_t^2 + M_W^2)B_\phi\right) \\
&-(m_t^2 - M_W^2)\left[H^2(m)(A_\phi + m_t^2 B_\phi) + H^2(M_W)\left(2A_\phi + (m_t^2 + M_W^2)B_\phi\right)\right] \\
&-(m_t^2 - M_W^2)^2 H^3(M_W)(A_\phi + M_W^2 B_\phi)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\spadesuit \ =& \frac{3\eta_1}{8192\pi^4}\left(\frac{M_H^4}{M_W^6}\right)\frac{1}{(1-x)^4}\left[\left(\frac{2}{\epsilon} - 2\gamma_E - \ln\frac{m_t^2}{4\pi} - \ln\frac{M_H^2}{4\pi}\right)\left((1-x)(1+5x)\right.\right. \\
&\left.\left.+2x(2+x)\ln x\right) + (1-x)(1+11x) - (1-6x-7x^2)\ln x - x(2+x)\ln^2 x\right] \\
&+\frac{\eta_1}{8192\pi^4}\left(\frac{M_H^2}{M_W^4}\right)\frac{1}{(1-x)^4}\left[4\left(\frac{1}{\epsilon} - \gamma_E - \ln\frac{m_t^2}{4\pi}\right)\left((1-x)(1+5x)\right.\right. \\
&\left.\left.+2x(2+x)\ln x\right) + (1-x)(7+23x) - 2(1-4x-12x^2)\ln x - 6x(2+x)\ln^2 x\right] \\
&+\frac{\eta_1}{8192\pi^4}\left(\frac{M_H^2 M_Z^2}{M_W^6}\right)\frac{1}{(1-x)^4}\left[\left(\frac{2}{\epsilon} - 2\gamma_E - \ln\frac{m_t^2}{4\pi} - \ln\frac{M_Z^2}{4\pi}\right)\left((1-x)(1+5x)\right.\right. \\
&\left.\left.+2x(2+x)\ln x\right) + (1-x)(1+11x) - (1-6x-7x^2)\ln x - x(2+x)\ln^2 x\right] \\
&-\frac{3\eta_1}{1024\pi^4}\left(\frac{m_t^4}{M_W^6}\right)\left(\frac{2}{\epsilon} - 2\gamma_E - \ln\frac{m_t^2}{4\pi} - \ln\frac{M_W^2}{4\pi} - 1\right) \\
&+\frac{\eta_1}{4096\pi^4}\frac{\ln(M_H^2/m_t^2)}{M_W^2(1-x)^4}\left((1-x)(5+x) + 2(2x+1)\ln x\right)
\end{aligned}
$$

$$♣ = -\frac{3\eta_1}{1024\pi^4}\left(\frac{m_t^4}{M_W^6}\right)\left(\frac{2}{\epsilon} - 2\gamma_E - \ln\frac{m_t^2}{4\pi} - \ln\frac{M_W^2}{4\pi} + 1\right)$$

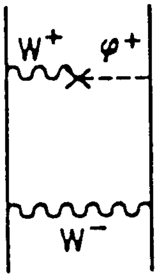


(2)

$$C4 = \frac{im_t^4\eta_1}{2nM_W^4}\Big\{ - \left(2(2m_t^2 + M_W^2)A_\phi + (m_t^2 + 2M_W^2)B_\phi\right)\left(H^1(m_t) - H^1(M_W)\right)$$
$$+(m_t^2 - M_W^2)H^2(M_W)\left((m_t^2 + M_W^2)A_\phi + 2m_t^2 M_W^2 B_\phi\right)$$
$$+m_t^2(m_t^2 - M_W^2)H^2(m_t)(A_\phi + m_t^2 B_\phi) + M_W^2(m_t^2 - M_W^2)^2 H^3(M_W)(A_\phi + M_W^2 B_\phi)\Big\}$$

$$♠ = -\frac{3\eta_1}{65536\pi^4}\left(\frac{M_H^4}{M_W^6}\right)\frac{1}{(1-x)^4}\Big[2\left(\frac{2}{\epsilon} - 2\gamma_E - \ln\frac{m_t^2}{4\pi} - \ln\frac{M_H^2}{4\pi}\right)\Big((1-x)(5+x)$$
$$+2(1+2x)\ln x\Big) + (1-x)(23+7x) + 2(2+12x+x^2)\ln x - 2(1+2x)\ln^2 x\Big]$$
$$-\frac{\eta_1}{16384\pi^4}\left(\frac{M_H^2 m_t^2}{M_W^6}\right)\frac{1}{(1-x)^4}\Big[2\left(\frac{1}{\epsilon} - \gamma_E - \ln\frac{m_t^2}{4\pi}\right)x\Big((1-x)(5+x) + 2(1+2x)\ln x\Big)$$
$$+(1-x)(1+14x+3x^2) + 2x^2(8+x)\ln x - 3x(1+2x)\ln^2 x\Big]$$
$$-\frac{\eta_1}{65536\pi^4}\left(\frac{M_H^2 M_Z^2}{M_W^6}\right)\frac{1}{(1-x)^4}\Big[2\left(\frac{2}{\epsilon} - 2\gamma_E - \ln\frac{m_t^2}{4\pi} - \ln\frac{M_Z^2}{4\pi}\right)\Big((1-x)(5+x)$$
$$+2(1+2x)\ln x\Big) + (1-x)(23+7x) + 2(2+12x+x^2)\ln x - 2(1+2x)\ln^2 x\Big]$$
$$+\frac{3\eta_1}{8192\pi^4}\left(\frac{m_t^4}{M_W^6}\right)\left[19 + 4\ln x - 2\ln^2 x + 8(2+\ln x)\left(\frac{1}{\epsilon} - \gamma_E - \ln\frac{m_t^2}{4\pi}\right)\right]$$
$$-\frac{\eta_1}{16384\pi^4}\left(\frac{m_t^2}{M_W^4}\right)\frac{\ln(M_H^2/m_t^2)}{(1-x)^4}\Big((1-x)(2+5x-x^2) + 6\ln x\Big)$$

$$♣ = \frac{3\eta_1}{8192\pi^4}\left(\frac{m_t^4}{M_W^6}\right)\left[\left(\frac{2}{\epsilon} - 2\gamma_E - \ln\frac{m_t^2}{4\pi} - \ln\frac{M_W^2}{4\pi}\right)(8 + 4\ln x) + 19 + 12\ln x + 2\ln^2 x\right]$$

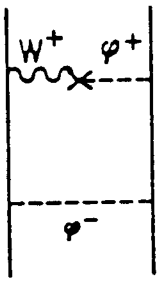


(4)

$$C5 = i m_t^2 A_{W\phi} \eta_1 \Big\{ -(2m_t^2 + M_W^2)\left(H^1(m_t) - H^1(M_W)\right)$$
$$+(m_t^2 - M_W^2)\left(m_t^2 H^2(m_t) + (m_t^2 + M_W^2)H^2(M_W) + M_W^2(m_t^2 - M_W^2)H^3(M_W)\right)\Big\}$$

$$\spadesuit \; = -\frac{\eta_1}{4096\pi^4}\frac{1}{m_t^2(1-x)^4}\left(\frac{M_H^2}{M_W^2} + 4\ln(M_H^2/m_t^2)\right)\left((1-x)(5+x) + 2(1+2x)\ln x\right)$$

$$\clubsuit \; = 0$$

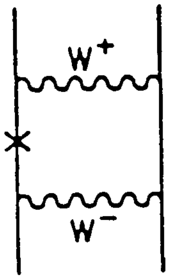


(4)

$$C6 = -\frac{m_t^2}{M_W^2} C5$$

$$\spadesuit \; = \frac{\eta_1}{16384\pi^4}\frac{1}{M_W^2(1-x)^4}\left(\frac{M_H^2}{M_W^2} + 4\ln(M_H^2/m_t^2)\right)\left((1-x)(5+x) + 2(1+2x)\ln x\right)$$

$$\clubsuit \; = 0$$

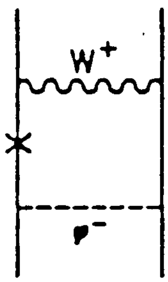


(2)

$$
\begin{aligned}
C7 = &\frac{i\eta_2}{2}\Big\{(2A_t + R_t)\Big[m_t^2(2M_W^2 + m_t^2)\Big(H^1(m_t) - H^1(M_W)\Big)\\
&-(m_t^2 - M_W^2)\Big(m_t^2(m_t^2 + M_W^2)H^2(m_t) + M_W^2 m_t^2 H^2(M_W) - m_t^4(m_t^2 - M_W^2)H^3(M_W)\Big)\Big]\\
&-L_t(m_t^2 - M_W^2)\Big(2M_W^2 m_t^2 H^2(m_t) + M_W^4 H^2(M_W) - m_t^4(m_t^2 - M_W^2)H^3(M_W)\Big)\\
&L_t M_W^4\Big(H^1(m_t) - H^1(M_W)\Big)\Big\}
\end{aligned}
$$

$$\spadesuit = -\frac{\eta_2}{16384\pi^4}\frac{\ln(M_H^2/m_t^2)}{M_W^2(1-x)^4}\Big((1-x)(2+15x+x^2) + 2x(5+4x)\ln x\Big)$$

$$\clubsuit = 0$$

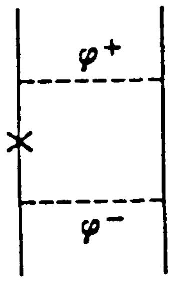


(4)

$$
\begin{aligned}
C8 = &-\frac{i m_t^4 \eta_1}{M_W^2}\Big\{A_t\Big[2(2m_t^2 + M_W^2)\Big(H^1(m_t) - H^1(M_W)\Big) - (m_t^2 - M_W^2)\times\\
&\times\Big((3m_t^2 + M_W^2)H^2(m_t) + (m_t^2 + M_W^2)H^2(M_W) - 2m_t^2(m_t^2 - M_W^2)H^3(m_t)\Big)\Big]\\
&+(L_t + R_t)\Big[(m_t^2 + 2M_W^2)\Big(H^1(m_t) - H^1(M_W)\Big)\\
&-(m_t^2 - M_W^2)\Big((m_t^2 + M_W^2)H^2(m_t) + M_W^2 H^2(M_W) - m_t^2(m_t^2 - M_W^2)H^3(m_t)\Big)\Big]\Big\}
\end{aligned}
$$

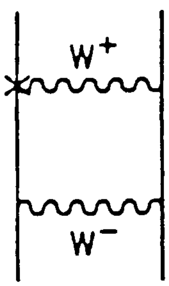
$$\spadesuit = \frac{\eta_1}{2048\pi^4}\left(\frac{m_t^2}{M_W^4}\right)\frac{\ln(M_H^2/m_t^2)}{(1-x)^4}\Big((1-x)(7+11x) + 2(1+6x+2x^2)\ln x\Big)$$

$$\clubsuit = 0$$



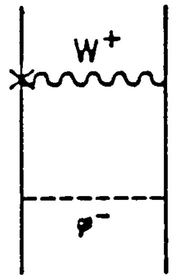
(2)

$$C9=\frac{im_t^4\eta_1}{2nM_W^4}\Big\{m_t^2(2A_t+L_t)\Big[(m_t^2+2M_W^2)\left(H^1(m_t)-H^1(M_W)\right)$$
$$-(m_t^2-M_W^2)\left((m_t^2+M_W^2)H^2(m_t)+M_W^2H^2(M_W)-m_t^2(m_t^2-M_W^2)H^3(m_t)\right)\Big]$$
$$+R_t\Big[M_W^2(2m_t^2+M_W^2)\left(H^1(m_t)-H^1(M_W)\right)$$
$$-(m_t^2-M_W^2)\left(2m_t^2M_W^2H^2(m_t)+M_W^4H^2(M_W)+m_t^4(m_t^2-M_W^2)H^3(m_t)\right)\Big]\Big\}$$
$$\spadesuit\ =-\frac{\eta_1}{32768\pi^4}\left(\frac{m_t^4}{M_W^6}\right)\left[4\left(\frac{1}{\epsilon}-\gamma_E-\ln\frac{m_t^2}{4\pi}\right)-3\right]$$
$$-\frac{\eta_1}{16384\pi^4}\left(\frac{m_t^4}{M_W^6}\right)\frac{\ln(M_H^2/m_t^2)}{(1-x)^4}\left((1-x)(2+15x+x^2)+2x(5+4x)\ln x\right)$$
$$\clubsuit\ =\frac{\eta_1}{32768\pi^4}\left(\frac{m_t^4}{M_W^6}\right)\left[-4\left(\frac{1}{\epsilon}-\gamma_E-\ln\frac{m_t^2}{4\pi}\right)+27+8\ln(M_H^2/m_t^2)\right]$$



(4)

$$C10=\frac{i(m_t^2-M_W^2)\eta_2}{n}A_{Wtb}\Big\{(m_t^2+M_W^2)\left(H^1(m_t)-H^1(M_W)\right)$$
$$-(m_t^2-M_W^2)\left(m_t^2H^2(m_t)+M_W^2H^2(M_W)\right)\Big\}$$
$$\spadesuit\ =-\frac{\eta_1}{49152\pi^4}\frac{\ln(M_H^2/m_t^2)(3+22x)}{M_W^2(1-x)^3}(1-x^2+2x\ln x)$$
$$\clubsuit\ =0$$

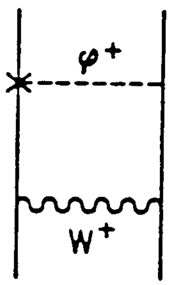


(4)

$$C11 = -i\frac{m_t^4}{M_W^2}(m_t^2 - M_W^2)A_{Wtb}\Big\{2\left(H^1(m_t) - H^1(M_W)\right)$$
$$-(m_t^2 - M_W^2)\left(H^2(m_t) + H^2(M_W)\right)\Big\}$$

♠ $$= \frac{\eta_1}{12288\pi^4}\left(\frac{m_t^2}{M_W^4}\right)\frac{\ln(M_H^2/m_t^2)(3+22x)}{(1-x)^3}\Big(2(1-x) + (1+x)\ln x\Big)$$

♣ $=0$

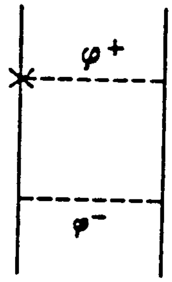


(4)

$$C12 = \frac{A_{\phi tb}}{A_{Wtb}}C11$$

♠ $$= -\frac{\eta_1}{12288\pi^4}\left(\frac{m_t^2}{M_W^4}\right)\frac{(15-34x)\ln(M_H^2/m_t^2)}{(1-x)^3}\Big(2(1-x) + (1+x)\ln x\Big)$$

♣ $=0$



(4)

$$
\begin{aligned}
C13 =& \, i\frac{m_t^4}{nM_W^4}(m_t^2 - M_W^2)A_{\phi tb}\Big\{(m_t^2 + M_W^2)\left(H^1(m_t) - H^1(M_W)\right) \\
& -(m_t^2 - M_W^2)\left(m_t^2 H^2(m_t) + M_W^2 H^2(M_W)\right)\Big\} \\
\spadesuit \quad =& -\frac{\eta_1}{98304\pi^4}\left(\frac{m_t^4}{M_W^6}\right)\left[3 - 36c^2/s^2 - 2\frac{(15-34x)\ln(M_H^2/m_t^2)}{(1-x)^3}(1 - x^2 + 2x\ln x)\right] \\
\clubsuit \quad =& \frac{\eta_1}{16384\pi^4}\left(\frac{m_t^4}{M_W^6}\right)\left[-17 + 6c^2/s^2 - 4\ln(M_H^2/m_t^2)\right]
\end{aligned}
$$