ANALYSIS OF THE DECAY $\tau \to \rho \nu$

by

Stephen E. Bougerolle
B. Sc., The University of Lethbridge, 1987
M. Sc., The University of British Columbia, 1989

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Department of Physics

We accept this thesis as conforming
to the required standard

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JULY 14, 1992
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Department of Physics
The University of British Columbia
Vancouver, Canada

Date July 14, 1992
Abstract

An analysis of the decay $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ has been undertaken in the OPAL experiment at CERN, in Geneva, Switzerland. From the 1990 experimental run of the LEP particle accelerator, a sample of 3310 $e^+e^- \rightarrow \tau^+\tau^-$ events was selected, with an estimated contamination of 1.9%. Requirements were applied to select a subsample of $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ decays, resulting in 650 decays being found. From studies with simulated data, the non-$\rho$ contamination in this sample was estimated to be approximately 22%, and the $\rho$ selection efficiency to be approximately 33%.

The Branching fraction for the decay is measured to be $B(\tau^\pm \rightarrow \rho^\pm \nu_\tau) = 0.234 \pm 0.009{\text{(stat.)}} \pm 0.010{\text{(syst.)}}$. The mean $\tau$ polarisation at the peak of the $Z^0$ resonance is measured to be $P_\tau = -0.17 \pm 0.10{\text{(stat.)}} \pm 0.08{\text{(syst.)}}$. From the $\tau$ polarisation we extract a measurement of the electroweak mixing $\sin^2 \theta_W = 0.225 \pm 0.015$. The $\tau$ polarisation asymmetry at the peak of the $Z^0$ resonance is also measured, $A_{P_\tau}^{Z^0} = -0.09 \pm 0.13 \pm 0.05$. From the polarisation measurement, $v_\tau/a_\tau = 0.09 \pm 0.06$, and from the polarisation asymmetry $v_e/a_e = 0.06 \pm 0.10$. Combined with previous LEP measurements of the $\tau$ polarisation, the electroweak mixing is found to be $\sin^2 \theta_W = 0.2308 \pm 0.0042$. Combined with measurements from other experiments, one obtains $\sin^2 \theta_W = 0.2303 \pm 0.0013$. 
# Contents

Abstract ............................................. ii

List of Tables ....................................... v

List of Figures ....................................... vii

Acknowledgements ..................................... viii

1 Introduction ......................................... 1

2 Theory .................................................. 7
  2.1 The Standard Model ................................ 7
     2.1.1 The Neutral Weak Current ................... 10
  2.2 Electron-Positron Collisions ..................... 11
     2.2.1 Forward-Backward Asymmetry ................. 12
  2.3 Polarisation ....................................... 14
     2.3.1 Measurement .................................. 16
     2.3.2 Tau Leptonic Decays ........................ 19
     2.3.3 Tau Semi-Hadronic Decays .................. 19
  2.4 Radiative Corrections ............................. 24

3 Experimental Apparatus ............................. 25
  3.1 LEP ............................................... 25
     3.1.1 Injector ..................................... 25
     3.1.2 Storage Ring ................................ 27
  3.2 The OPAL Detector ................................ 27
     3.2.1 Inner Detector ................................ 29
     3.2.2 Time-of-Flight System ....................... 31
     3.2.3 Electromagnetic Calorimetry ................. 31
     3.2.4 Hadronic Calorimetry ....................... 32
     3.2.5 Muon Detector ................................ 32
     3.2.6 Trigger ....................................... 33
     3.2.7 Data Readout ................................ 34
# Data

4.1 Data ................................................. 36
4.2 Simulated Data ................................. 36
  4.2.1 Simulated $\tau$-pairs ...................... 37
  4.2.2 Other Simulated Data ...................... 38
4.3 Reconstruction ................................. 39

# Preselection of $e^+e^- \rightarrow \tau^+\tau^-$ Events

5.1 Data Quality and Status Requirements ........ 41
5.2 Elimination of Cosmic Ray Background ........ 43
5.3 Cone Analysis and Fiducial Acceptance .......... 43
5.4 Elimination of $e^+e^- \rightarrow q\bar{q}$ Background .......... 47
5.5 Elimination of $e^+e^- \rightarrow e^+e^-$ Background ........ 47
5.6 Elimination of $e^+e^- \rightarrow e^+e^-X$ Background .......... 47
5.7 Elimination of $e^+e^- \rightarrow \mu^+\mu^-$ Background .......... 49
5.8 Summary of Results ............................ 49

# Selection of $\tau^\pm \rightarrow \rho^\pm\nu_\tau$ decays

6.1 Selection Criteria ............................... 53
6.2 Summary of Results ............................ 59

# Analysis

7.1 Branching Fraction ............................. 61
  7.1.1 Branching Fraction Uncertainty ............. 62
7.2 Tau Polarisation .................................. 68
7.3 Forward-Backward Polarisation Asymmetry ....... 83

# Summary of Results

8.1 Branching Fraction ............................. 89
8.2 Polarisation ...................................... 89
  8.2.1 Polarisation Asymmetry ..................... 91
8.3 Lepton Universality ............................ 91
8.4 The Weinberg Angle .............................. 94

Abstract ................................. 100

A Derivation of Polarisation Angle Equations 100
List of Tables

1.1 Elementary particles described by the Standard Model .......................... 4
2.1 Standard Model fermions and quantum numbers ................................. 8
2.2 Neutral current couplings ......................................................... 11
4.1 $e^+e^- \rightarrow \tau^+\tau^-$ data samples ........................................... 37
4.2 Branching fractions used in simulation ............................................. 38
5.1 Contamination in $\tau$-pair sample .................................................. 52
6.1 $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ selection requirements ............................... 53
6.2 Sources of background in selected data ........................................... 59
7.1 Alternative branching fraction weighting schemes in Monte Carlo data .... 66
7.2 Uncertainties arising from decay-mode weighting ................................. 66
7.3 Uncertainties arising from $a_1$ weighting ......................................... 67
7.4 Summary of uncertainties in the branching fraction measurement ............ 67
7.5 Results of simulated analysis with MC data ....................................... 83
7.6 Summary of uncertainties in the polarisation measurement ...................... 84
8.1 Past $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ branching fraction measurements .................. 91
8.2 Tau polarisation measurements ...................................................... 92
8.3 OPAL polarisation asymmetry measurements ..................................... 92
8.4 Measurements of $\sin^2 \theta_W$ (for $M_{top} = M_{Higgs} = 100$ GeV) ........ 95
List of Figures

1.1 Diagram for beta decay $n \rightarrow p e \bar{\nu}_e$ ........................................... 3
1.2 Troublesome weak processes (with couplings) ......................................... 4
1.3 Higgs boson production ................................................................. 4

2.1 Feynman diagrams for $e^+e^- \rightarrow f \bar{f}$ ($f \neq e$) ......................... 11
2.2 Born-level cross-section for $\tau^+\tau^-$ production .................................. 13
2.3 Extra Feynman diagram for $e^+e^- \rightarrow e^+e^-$ ................................... 14
2.4 Forward-backward asymmetry, $\tau$ polarisation and polarisation asymmetry........ 15
2.5 $X$ distribution of $\tau$ leptonic decays ................................................... 17
2.6 Possible spin configurations in $\tau^\pm \rightarrow \pi^\pm \nu_\tau$ decay .................... 18
2.7 Possible spin configurations in $\tau^\pm \rightarrow \rho^\mp \nu_\tau$ decay .................... 18
2.8 Decay angles in $\tau^\pm \rightarrow \rho^\mp \nu_\tau$ .............................................. 20
2.9 Momentum distribution of $\tau^\pm \rightarrow \rho^\mp \nu_\tau$ .................................. 23

3.1 Schematic diagram of the LEP injector ....................................................... 26
3.2 Geographic location of LEP ...................................................................... 28
3.3 The OPAL detector .................................................................................. 30
3.4 1990 ECAL energy resolution ................................................................... 35
4.1 Feynman diagram for $e^+e^- \rightarrow e^+e^-X$ ............................................ 40

5.1 1990 distributions of track parameters ....................................................... 42
5.2 Useful distributions for cosmic ray elimination .......................................... 44
5.3 Normalised distributions of $\tau$-pair selection quantities (simulated data) .... 45
5.4 Bhabha-scattering and two-photon event characteristics (simulated data) .... 46
5.5 Two-photon event characteristics (simulated data) ..................................... 48
5.6 Di-muon event characteristics (simulated data) ......................................... 50
5.7 Di-muon event characteristics (simulated data) ......................................... 51

6.1 Cone size and neutral cluster requirements ................................................. 55
6.2 Values used in neutral cluster requirements ............................................. 56
6.3 Cone mass .............................................................................................. 57
6.4 $E_{ass}/P_{ctrk}$ ...................................................................................... 58
6.5 Agreement between data and Monte Carlo data for significant variables ...... 60

7.1 Variation of branching fraction with parameter changes ........................... 64
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.2</td>
<td>Variation of branching fraction with parameter changes</td>
<td>65</td>
</tr>
<tr>
<td>7.3</td>
<td>Correction terms for one-dimensional polarisation measurement</td>
<td>70</td>
</tr>
<tr>
<td>7.4</td>
<td>Data at varying stages of correction</td>
<td>71</td>
</tr>
<tr>
<td>7.5</td>
<td>One- and two-dimensional $E_{cm}$ corrections (2-D, slices in $\cos \psi$)</td>
<td>72</td>
</tr>
<tr>
<td>7.6</td>
<td>Two-dimensional $E_{cm}$ corrections, slices in $\cos \theta^*$</td>
<td>73</td>
</tr>
<tr>
<td>7.7</td>
<td>Two-dimensional polarisation corrections, slices in $\cos \psi$</td>
<td>74</td>
</tr>
<tr>
<td>7.8</td>
<td>Two-dimensional polarisation corrections, slices in $\cos \psi$</td>
<td>75</td>
</tr>
<tr>
<td>7.9</td>
<td>Two-dimensional polarisation corrections, slices in $\cos \theta^*$</td>
<td>76</td>
</tr>
<tr>
<td>7.10</td>
<td>Two-dimensional polarisation corrections, slices in $\cos \theta^*$</td>
<td>77</td>
</tr>
<tr>
<td>7.11</td>
<td>Data at varying stages of correction, slices in $\cos \psi$</td>
<td>78</td>
</tr>
<tr>
<td>7.12</td>
<td>Data at varying stages of correction, slices in $\cos \theta^*$</td>
<td>79</td>
</tr>
<tr>
<td>7.13</td>
<td>Data at varying stages of correction, slices in $\cos \theta^*$</td>
<td>80</td>
</tr>
<tr>
<td>7.14</td>
<td>Data at varying stages of correction, slices in $\cos \theta^*$</td>
<td>81</td>
</tr>
<tr>
<td>7.15</td>
<td>Variation of $\tau$ polarisation with parameter changes (1-D fit)</td>
<td>85</td>
</tr>
<tr>
<td>7.16</td>
<td>Variation of $\tau$ polarisation with parameter changes (1-D fit)</td>
<td>86</td>
</tr>
<tr>
<td>7.17</td>
<td>Variation of $\tau$ polarisation with parameter changes (2-D fit)</td>
<td>87</td>
</tr>
<tr>
<td>7.18</td>
<td>Variation of $\tau$ polarisation with parameter changes (2-D fit)</td>
<td>88</td>
</tr>
<tr>
<td>8.1</td>
<td>$\tau^\pm \rightarrow \rho^\pm \nu_\tau$ branching fraction measurements</td>
<td>90</td>
</tr>
<tr>
<td>8.2</td>
<td>Tau polarisation measurements</td>
<td>93</td>
</tr>
<tr>
<td>8.3</td>
<td>$\sin^2 \theta_W$ measurements</td>
<td>96</td>
</tr>
</tbody>
</table>
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Chapter 1

Introduction

As with all work in particle physics, the purpose of this analysis is to further our understanding of fundamental physical interactions. Conventional theory now divides these into three categories; gravity, the strong nuclear interaction, and the electroweak interaction. Two older theories describing the weak and electromagnetic interactions have been united by Glashow, Salam and Weinberg [1] in the Standard Model of Electroweak Interactions. Among the results of this work are tests of aspects of the Standard Model and a measurement of one of its fundamental parameters.

The first particle interaction studied was that between electrons and protons, now described by the theory of electromagnetism. Although electromagnetism provided a qualitatively correct picture of atomic structure, a quantitative explanation required the development of quantum mechanics. Similarly, electromagnetism correctly explained the wave nature of light but had philosophical problems which were finally resolved with the theory of special relativity.

In 1928, Dirac united the two fields that had developed from electromagnetism, quantum mechanics and special relativity, creating the theory of relativistic quantum mechanics [2]. On the basis of this new theory, he predicted the existence of “antiparticles”, with the same mass as particles but opposite-sign electric charge. The antiparticle of an electron is a positron (discovered in 1932 [3]) and the antiparticle of a proton is an antiproton (discovered in 1955 [4]). Relativistic quantum mechanics was further expanded through the use of field-theory techniques and developed into quantum electrodynamics (QED) [5], which describes the interactions of electrons and positrons with “field quanta” termed photons. QED reactions can be thought of as occurring by the exchange of photons between charged particles, with the strength of these interactions given by the coupling constant \( \alpha = e^2/(4\pi\hbar c) \).

Concurrent with the development of QED were the discoveries of several new particles. The neutron [6] was the first observed after the electron and proton. Then the muon [7], pion [8], \( \pi \), and kaon [9], \( K \) were discovered in observations of cosmic-ray particles. Experimenters made these observations while trying to test the Yukawa theory of strong interactions, which had developed as an attempt to understand the formation and structure of atomic nuclei. Their goal was to discover an exchanged particle similar to the photon, responsible for mediating the strong interaction. Instead they discovered
several new particles. This prompted the development of experiments using particle accelerators; the rate of particle production with these new facilities was much more intense than the natural cosmic-ray collision rate, so new particle searches could be carried out more efficiently.

The zoo of new particles naturally encouraged particle taxonomy. The muon did not itself undergo strong interactions, and was classed together with the electron under the category of "leptons." The other particles, all subject to the strong interaction, were classed together as "hadrons." Hadrons were further classed as "mesons," lighter particles with integral spin, and "baryons," heavier particles with half-integral spin[10].

Hadronic structure can be explained by the quark model, in which "quarks" are fermions with fractional charge. Mesons are combinations of a quark and anti-quark (and therefore have integral spin), while baryons are combinations of three quarks or three anti-quarks (and therefore have half-integral spin). Quark-quark interactions are thought to be well-modelled by a theory called Quantum Chromodynamics or QCD (modelled on Quantum Electrodynamics). Whereas QED describes interactions between particles with electric charge by the exchange of an uncharged photon, QCD describes interactions between particles with "colour" by the exchange of a particle called a "gluon" which (in contrast to the photon) is itself coloured.

The strong and electromagnetic interactions could not account for all phenomena, however. Early on, it was observed that certain nuclei also underwent a process called Beta decay (for example $^3H \rightarrow ^3He + e^- + \text{Energy}$), the characteristics of which were a change in electric charge of the original particle accompanied by production of an electron or positron plus a certain amount of "missing energy." The explanation for this was given by Pauli, who postulated the existence of a new particle carrying the missing energy (the neutrino $\nu$), and by Fermi, who postulated a new "weak" force to which neutrinos were subject.

Weak processes were modelled as a four-fermion, "point-like" interaction in which doublets of particles were coupled together with a constant strength $G_F$ (analogous to $\alpha$), known as the "Fermi constant." The possible doublets in question were

$$\begin{pmatrix} \nu \\ e \end{pmatrix}, \begin{pmatrix} \nu \\ \mu \end{pmatrix}, \begin{pmatrix} H \\ H' \end{pmatrix}$$

where $H$ and $H'$ were two hadrons. Beta decay was then a reaction between a doublet ($p \ n$) and a doublet ($\nu \ e$). It was observed that neutrinos are associated with a particular species of charged lepton [11]. That is to say, the electron neutrino $\nu_e$ and muon neutrino $\nu_\mu$ are different. Weak interactions were noted to have another feature; they were not invariant under parity transformations (inversion of the spatial axes)[12].

Fermi's original point-like coupling presented theoretical difficulties, however. While accurate for low particle energies, the predicted reaction rates became unphysical as the energy increased. To remedy this problem, the existence of an exchanged particle similar to the photon was postulated, a "weak vector boson" with charge $\pm 1$ and spin 1 (see figure 1.1). Because of the charge of this $W^\pm$ boson, W-exchange reactions are also called "charged current" reactions.
Although the introduction of the W boson reconciled weak theory with experimental observation, it also created new problems; several other processes should be permitted by the new theory, in particular $e^+e^- \rightarrow W^+W^-$ (see figure 1.2a). However, the reaction rates calculated for this grow with energy, giving unphysical results. A theoretical solution to this problem was found by postulating the existence of a new class of “neutral current” interactions (see figure 1.2b). If the couplings for the two processes are similar ($e \simeq g_W$) the divergences in each process almost cancel. Neutral current reactions (specifically $e^- + \bar{\nu}_\mu \rightarrow e^- + \bar{\nu}_\mu$) were first observed at CERN in 1973[13].

The form of the neutral current is a key feature of the Standard Model. In it, the weak and electromagnetic forces are modelled as combinations of more fundamental interactions, “weak isospin” and “weak hypercharge.” The old coupling constants $\alpha$ and $G_F$ are related to more fundamental constants $g$ and $g'$ through a new parameter $\Theta_W$, called the Weinberg angle. The Weinberg angle can be thought of as giving the relative proportion of $\gamma$ and $Z^0$ exchange contributions to electroweak reactions. The electroweak mixing is $\sin^2 \theta_W$ rather than $\theta_W$ itself. The structure of the Standard Model is described more fully in Chapter 2.

If the Standard Model is correct, it should be possible to produce the quanta associated with the charged and neutral currents, $W^\pm$ and $Z^0$ bosons. An observation of the $Z^0$ is particularly convincing as its existence is a unique feature of the Standard Model. The $Z^0$ was in fact observed at CERN by the UA1[14] experiment in 1983, as were the $W^\pm$ [15]. This was a major success for the new theory.

The introduction of the $Z^0$ did not quite cure the problems noted above. In the Standard Model, these are completely eliminated by the introduction of another exchanged quantum called the Higgs boson $H^0$ (see figure 1.3). Unlike the $W^\pm$, the Higgs boson is uncharged and unlike all bosons we have described so far, it is spinless. It couples in proportion to the mass of the interacting particle. Although the $W^\pm$ and $Z^0$ bosons have been seen, the existence of the Higgs boson has yet to be verified.

At the current time, the Standard Model accurately describes electroweak interactions among all the known particles and their antiparticles, listed in table 1.1 (the t quark and $\nu_t$, as well as the $H^0$, have yet to be observed). Although the Standard Model has been notably successful, it has many parameters which must be determined by experiment. These include all the coupling constants ($\alpha$, $G_F$ and $\sin^2 \theta_W$) as well as the individual particle masses.
Figure 1.2: Troublesome weak processes (with couplings)

Figure 1.3: Higgs boson production

<table>
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<th>Bosons</th>
<th>$\gamma, W^\pm, Z^0, H^0$</th>
</tr>
</thead>
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<tr>
<td>Charged leptons</td>
<td>$e^-, \mu^-, \tau^-$</td>
</tr>
<tr>
<td>Neutrinos</td>
<td>$\nu_e, \nu_\mu, \nu_\tau$</td>
</tr>
<tr>
<td>Quarks</td>
<td>d, u, s, c, b, t</td>
</tr>
</tbody>
</table>

Table 1.1: Elementary particles described by the Standard Model
Accurate measurement of the Weinberg angle is a high priority; whereas the other fundamental constants are known to better than one part per million, the mixing $\sin^2 \theta_W$ is only known to an accuracy of less than one part per thousand. A more accurate measurement is needed for reliable calculations using the Standard Model. When combined with other data (such as a measurement of the $Z^0$ boson mass) and higher-order calculations, an accurate measurement can be used to place limits on the $t$ quark and Higgs boson masses. The discovery (or non-discovery) of these particles in the correct mass range will provide the best proof to date of the Standard Model's validity. Measurements of $\theta_W$ have been performed by examining neutrino-scattering reactions. Complementary measurements are being made using the LEP facility at CERN in Geneva, Switzerland. LEP is built to study electron-positron annihilation into a $Z^0$ boson, which decays into any fermion-antifermion pair, $e^+e^- \rightarrow Z^0 \rightarrow ff$ with $m_f < m_Z/2$. The angular distribution and spin polarisation of the outgoing fermions are studied and the Weinberg angle extracted.

In the table of particles above, it will be noticed that there is a third charged lepton, the $\tau^-$. The tau, first discovered in 1975[16], differs from other leptons in its mass and decay properties. The electron is stable. The muon has a relatively long lifetime, and decays by $\mu \rightarrow e\bar{\nu}_e \nu_\mu$ into an electron, the only charged particle lighter than itself. But the $\tau$, with a mass of 1784 MeV, has a very short lifetime and can decay into many particles which are lighter than itself (likely possibilities are $\tau^\pm \rightarrow \rho^\pm \nu_\tau$, $\tau^\pm \rightarrow \mu^\pm \bar{\nu}_\mu \nu_\tau$, $\tau^\pm \rightarrow e^\pm \bar{\nu}_e \nu_\tau$ and $\tau^\pm \rightarrow \nu_\tau$).

The spin polarisation of the $\tau$ can be studied by measuring the energy distribution of its decay products. From this quantity the electroweak couplings of the $Z^0$ to the $\tau$ and electron can be found. A problem with the measured $\tau$ lifetime gives reason to doubt that the couplings of the $W^\pm$ to the electron and $\tau$ are identical, and therefore that the assumption of "lepton universality" (that charged leptons behave identically in weak interactions) is valid. A check on the equivalent $Z^0$ couplings is a complementary test of this hypothesis. If lepton universality is assumed valid, the polarisation measurement can be used to calculate the Weinberg angle. The theory behind the polarisation measurement and calculation of related quantities is given in Chapter 2.

In this work, I have chosen to study a sub-sample of all $\tau$ lepton decays, those where the $\tau^\pm$ decays into a $\rho^\pm$ meson and a neutrino ($\tau^\pm \rightarrow \rho^\pm \nu_\tau$). This in theory provides the best measurement of the $\tau$ polarisation possible from any $\tau$ sub-sample. The experimental facilities used are described in Chapter 3. The $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ selection process and its results are detailed in Chapters 4-6. Other members of the OPAL collaboration have also studied the $\tau^\pm \rightarrow \pi^\pm \nu_\tau$, $\tau^\pm \rightarrow e^\pm \bar{\nu}_e \nu_\tau$, and $\tau^\pm \rightarrow \mu^\pm \bar{\nu}_\mu \nu_\tau$ sub-samples of $\tau$-decays, obtaining independent polarisation measurements[17].

The $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ decay is also important in the field of $\tau$ physics, distinct from electroweak theory. The fraction of $\tau$ leptons which decay by this process (its "branching fraction") has been measured before, as have the branching fractions of other decays. But when the sums of past measurements are taken together, they do not add up to one within experimental uncertainties. This "branching fraction problem" has become controversial with time and more data are desired to help resolve it. Chapter 7 contains an analysis of the $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ sample, with a measurement of its branching fraction as
well as the details of the \( \tau \) polarisation measurement. A summary of the results is given in Chapter 8.
Chapter 2

Theory

The basic features of the Standard Model were given in Chapter 1. In this chapter, I present the mathematical structure of the theory. The interested reader will find a more thorough discussion in reference [5]. An analysis is given of the reaction $e^+e^- \rightarrow f\bar{f}$, detailing the origins of polarisation and its significance. Following this is a brief overview of the structure of $\tau$ decays, with emphasis on their angular distributions and the relevance of these to the $\tau$ polarisation measurement.

2.1 The Standard Model

It is convenient to describe the Standard Model using the helicity formalism, where helicity is defined as the component of spin of a particle along its line of motion. High energy spin-$\frac{1}{2}$ fermions such as those in the Standard Model are helicity eigenstates, with eigenvalues $\pm \frac{1}{2}$. Positive helicity states are termed "right-handed," and negative helicity states are "left-handed." It is experimentally observed that the electromagnetic interaction couples equally to right- and left-handed states, the weak charged current couples only to left-handed states and the weak neutral current couples differently to right- and left-handed states.

Standard Model fermions are classified to reflect experimentally observed symmetries. In much the same way that protons and neutrons are observed to behave similarly in strong interactions, left-handed particles in a weak interaction doublet are observed to behave similarly in charged-current weak interactions. For the strong interaction, this invariance was reflected in theory by an "isospin" symmetry, modelled on SU(2) spin calculations. The proton and neutron were grouped together in a doublet with isospin $I = \frac{1}{2}$ in which each particle was a distinct eigenstate with component eigenvalues $I_3 = \pm \frac{1}{2}$. For charged-current weak interactions, this idea is taken over as "weak isospin." Analogously, left-handed particles (for example, the $e_L$ and $\nu_e$) are grouped together in a doublet with weak isospin $T = \frac{1}{2}$ in which each particle is a distinct weak eigenstate with component eigenvalues $T_3 = \pm \frac{1}{2}$. Where the strong interaction was approximately invariant under SU(2) isospin rotations, the charged-current weak interaction is invariant under SU(2)$_L$, weak isospin rotations.
Fermion states

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<td>0</td>
<td>1/2</td>
<td>1/2</td>
<td>-1</td>
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<tr>
<td>$\nu_\mu$</td>
<td>-1</td>
<td>1/2</td>
<td>-1/2</td>
<td>-1</td>
</tr>
<tr>
<td>$\nu_\tau$</td>
<td>-1/3</td>
<td>1/2</td>
<td>-1/2</td>
<td>1/3</td>
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Table 2.1: Standard Model fermions and quantum numbers

To incorporate the neutral weak current and electromagnetic interaction, these states are also labelled according to “weak hypercharge” (another term taken from earlier attempts to model hadronic structure), which is related to the electric charge and weak isospin by

$$Q = T_3 + \frac{Y}{2}$$  \hspace{1cm} (2.1)

where $Q$ is the electric charge and $Y$ is the weak hypercharge. These quantum numbers are collected for all the Standard Model fermions in table 2.1. States are labelled with a subscript $L$ or $R$ according to their handedness.

While weak interactions between leptons are seen to observe the doublet structure above, quarks behave differently. Reactions between $s$ and $u$ quarks are observed, for example, as well as reactions between $d$ and $u$ quarks (which would otherwise form a natural doublet). In other words, quark mass eigenstates are not weak interaction eigenstates. This difficulty is incorporated in the Standard Model by defining weak interaction eigenstates $d'$, $s'$ and $b'$ which are linear combinations of quark mass states $d$, $s$ and $b$:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} K_{d'd} & K_{d's} & K_{d'b} \\ K_{s'd} & K_{s's} & K_{s'b} \\ K_{b'd} & K_{b's} & K_{b'b} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$  \hspace{1cm} (2.2)

The unitary matrix formed by the coefficients $K_{ij}$ is called the Cabibbo-Kobayashi-Maskawa matrix[18]. The primed states are incorporated into the Standard Model, generalising the “generation” structure that was evident for leptons. The Standard Model places no limit on the number of generations, but assuming that neutrinos are massless, recent results from LEP have shown there to be three generations[19].

These elements are all tied together in the Standard Model Lagrangian. This depicts the weak and electromagnetic interactions as resulting from more fundamental interactions involving weak isospin and hypercharge:
where $g$ and $g'$ are new coupling constants relating the strength of the weak hypercharge and weak isospin interactions, $J_{\mu}^i$ and $J_{\mu}^Y$ are the weak isospin and weak hypercharge currents. $W^i$ and $B$ represent four “gauge fields” interacting with the currents.

The weak isospin and hypercharge fields are connected with the observed electromagnetic and weak boson fields by substituting

$$W_{\mu}^\pm = \left( \frac{1}{2} W_{\mu}^1 \mp i W_{\mu}^2 \right)$$

$$A_{\mu} = B_{\mu} \cos \theta_W + W_{\mu}^3 \sin \theta_W$$

$$Z_{\mu} = -B_{\mu} \sin \theta_W + W_{\mu}^3 \cos \theta_W$$

where $W_{\mu}^\pm$ are the weak charged boson fields, $A_{\mu}$ is the photon field, and $Z_{\mu}$ represents the weak neutral boson field. The parameter $\theta_W$ is the Weinberg angle, mentioned in Chapter 1. This demonstrates its role; the Weinberg angle gives the degree of mixing between weak hypercharge and weak isospin. The coupling constants $g$ (equivalent to $G_F$) and $g'$ are related to the electromagnetic coupling $\alpha$ by

$$e = g \sin \theta_W = g' \cos \theta_W$$

The Standard Model Lagrangian as shown above has a problem; the observed weak bosons have masses, whereas the fields $W_{\mu}^i$ and $B_{\mu}$ given are massless. This difficulty is solved by adding in another element known as the Higgs mechanism. A full discussion of this is outside the realm of this work[5]. We note only its effect; to fix the relation between the gauge boson masses while leaving the weak isospin and hypercharge invariance intact.

The Standard Model is a field theory and like all field theories has another potential problem: is it possible to make perturbation calculations from it? The answer is yes, but to do so requires renormalisation of the couplings in the theory. As with the Higgs mechanism, a full discussion of this does not belong here. However, one of the major results must be mentioned.

The weak mixing can be defined in two different ways; in terms of the boson masses $\sin^2 \theta_W = 1 - M_{\nu}^2/M_Z^2$ and in terms of the weak couplings $\sin^2 \theta_W = g'^2/(g^2 + g'^2)$. At the lowest level these are equivalent but when higher order corrections are included and the results renormalised the two definitions are no longer equivalent. The former is taken as the standard definition of $\sin^2 \theta_W$ and the latter is taken to define $\sin^2 \theta_W$, the effective weak mixing angle at the energy of the experiment. At LEP energies, the two are related by $\sin^2 \theta_W = 1.013 \sin^2 \theta_W[32]$ (assuming the top quark and Higgs boson masses to be each 100 GeV).
2.1.1 The Neutral Weak Current

As mentioned in Chapter 1, the form of the neutral current is one of the key features of the Standard Model. It is related to the electromagnetic and weak charged currents by

\[ J_{\mu}^{NC} = J_{\mu}^{3} - \sin^{2} \theta_{W} J_{\mu}^{em} \]  \hspace{1cm} (2.8)

The electromagnetic and weak charged currents, in turn, are

\[ J_{\mu}^{em} = \bar{\psi} \gamma^{\mu} Q \psi \]  \hspace{1cm} (2.9)

\[ J_{\mu}^{3} = \bar{\chi} \gamma^{\mu} \frac{1}{2} \tau_{3} \chi \]  \hspace{1cm} (2.10)

where \( \psi \) is a particle spinor, \( \chi \) a left-handed particle doublet, \( \gamma^{\mu} \) are the Dirac matrices, and \( \tau_{3} \) is the SU(2)\(_{L}\) "Pauli matrix."

The neutral current is conventionally written in a more useful parametrised form:

\[ J_{\mu}^{NC} = \bar{\psi} \gamma_{\mu} \frac{1}{2} (v - a \gamma^{5}) \psi \]  \hspace{1cm} (2.11)

The coefficients \( v \) and \( a \) represent the couplings of the \( Z^{0} \) boson to the vector and axial-vector parts of the current, respectively, and \( \gamma^{5} = i \gamma^{0} \gamma^{1} \gamma^{2} \gamma^{3} \). The values of \( v \) and \( a \) are obtained by comparison with our original interaction Lagrangian:

\[ a = T_{3} \]  \hspace{1cm} (2.12)

\[ v = T_{3} - 2 \sin^{2} \theta_{W} Q \]  \hspace{1cm} (2.13)

Because \( \sin^{2} \theta_{W} \) "runs" (varies with energy), these couplings also run. In the same way we replace \( \sin^{2} \theta_{W} \) with the effective mixing \( \sin^{2} \theta_{W} \), we replace the couplings \( v \) and \( a \) with \( \bar{v} \) and \( \bar{a} \). These are the couplings actually measured in this analysis.

The neutral current can be written in a different form, by separating out the couplings to left- and right-handed particles:

\[ J_{\mu}^{NC} = \bar{\psi} \gamma_{\mu} \frac{1}{2} (c_{R}(1 + \gamma^{5}) + c_{L}(1 - \gamma^{5})) \psi \]  \hspace{1cm} (2.14)

The left- and right-handed couplings are related to \( v \) and \( a \) through

\[ v = \frac{1}{2} (c_{L} + c_{R}) \]  \hspace{1cm} (2.15)

\[ a = \frac{1}{2} (c_{L} - c_{R}) \]  \hspace{1cm} (2.16)

These coupling constants are listed in table 2.2. The asymmetry between couplings to left- and right-handed particles is clear. As we will see later, this causes measurable effects in the reactions studied at LEP. Working backwards, measurement of these asymmetries allows an estimation of the couplings and from that, a calculation of \( \sin^{2} \theta_{W} \).
From the couplings of leptons to the photon and \( Z^0 \), it is straightforward to analyse the reaction \( e^+e^- \rightarrow f\bar{f} \). Here, we discuss only first-order (“Born level”) calculations. Higher-order corrections are discussed later.

From the Feynman diagrams in figure 2.1, one obtains the Born-level differential cross section for the case where the fermions are not an \( e^+e^- \) pair:

\[
\frac{d\sigma_{\text{Born}}}{d\cos\theta}(s,\cos\theta,p) = \frac{\pi\alpha^2}{2s}[(F_0 - pF_2)(1 + \cos^2\theta) + 2(F_1 - pF_3)\cos\theta] \tag{2.17}
\]

Here \( \sqrt{s} = E_{e^-} + E_{e^+} \), \( \cos\theta \) is the scattering angle of the fermion \( f \) with respect to the \( e^- \), \( p \) is the helicity polarisation of the fermion \( f \), and \( F_0, F_1, F_2, F_3 \) are given by:

\[
F_0 = q_e^2 q_f^2 + 2\Re(\chi)q_e q_f v_e v_f + |\chi|^2(v_e^2 + a_e^2)(v_f^2 + a_f^2) \tag{2.18}
\]

\[
F_1 = 2\Re(\chi)q_e q_f a_e a_f + |\chi|^2 4v_e a_e v_f a_f \tag{2.19}
\]

\[
F_2 = 2\Re(\chi)q_e q_f v_e a_f + |\chi|^2(v_e^2 + a_e^2)2v_f a_f \tag{2.20}
\]

\[
F_3 = 2\Re(\chi)q_e q_f a_e v_f + |\chi|^2 2v_e a_e(v_f^2 + a_f^2) \tag{2.21}
\]

with
In the equations, the vector and axial coupling constants \( v_v, v_f, a_v, a_f \) are labelled according to the particles to which they couple. The charge of the fermion produced is given by \( q_f \). The factor of \( i \Gamma_Z/M_Z \) in \( \chi \) is inserted to account for the \( Z^0 \) resonance width. The effect of this resonance term \( \chi \) is that particle production by \( Z^0 \) exchange increases greatly when the centre of mass energy \( \sqrt{s} \approx M_Z \). This can be seen in figure 2.2.

The reaction \( e^+e^- \rightarrow e^+e^- \) ("Bhabha scattering") is complicated by the fact that the incoming and outgoing particles are identical. Because of this it is necessary to include an extra diagram when calculating the cross section (see figure 2.3), and the above equation is incorrect. The actual Bhabha-scattering cross section increases strongly at small angles.

### 2.2.1 Forward-Backward Asymmetry

For \( f \neq e \), the terms proportional to \( p \cos \theta \) above induce an asymmetry between the integrated cross-sections in the forward \((\cos \theta > 0)\) and backward \((\cos \theta < 0)\) hemispheres. This is given by

\[
A_{FB} = \frac{1}{\sigma_{Born}} [\sigma(\cos \theta > 0) - \sigma(\cos \theta < 0)] = \frac{3F_1}{4F_0} \tag{2.23}
\]

where \( \sigma \) represents the integrated cross section, and \( \sigma_{Born} \), in particular is the integrated total cross section. When \( \sqrt{s} = M_Z \), terms proportional to \( \Re(\chi) \) drop out, the \( \gamma \)-exchange term in \( F_0 \) becomes negligible, and the forward-backward asymmetry is given by

\[
A_{FB} \approx \frac{3}{4} \left( \frac{2v_v a_v}{v_v^2 + a_v^2} \right) \left( \frac{2v_f a_f}{v_f^2 + a_f^2} \right) = \frac{3}{4} \lambda_v \lambda_f \tag{2.24}
\]

In the equation above, we have introduced a term \( \lambda_x \), defined by

\[
\lambda_x \equiv \frac{2v_x a_x}{v_x^2 + a_x^2} = \frac{2(v_x/a_x)}{1 + (v_x/a_x)^2} \tag{2.25}
\]

where the subscript \( x \) identifies a particular particle and its coupling constants. This is done for convenience; the functional form of \( \lambda \) also occurs later when we discuss the \( \tau \) polarisation and \( \tau \) polarisation asymmetry.

The forward-backward asymmetry is plotted as a function of \( \sqrt{s} \) in figure 2.4. In the same diagram are shown the \( \tau \) polarisation and \( \tau \) polarisation asymmetry. It can be seen that they are inherently more sensitive to changes in \( \sin^2 \theta_W \). Also, \( A_{FB} \) varies...
Figure 2.2: Born-level cross-section for $\tau^+\tau^-$ production
Plotted on the vertical scale is the expected Born-level cross section for the interaction $e^+e^- \rightarrow \tau^+\tau^-$, as a function of centre-of-mass energy. The resonance structure of the $Z^0$ is seen in the spike near 90 GeV.
Figure 2.3: Extra Feynman diagram for $e^+e^- \rightarrow e^+e^-$

more with the energy of the $\tau$-pair and as a result is more sensitive to higher-order radiative corrections. Furthermore, in the expected case where $\lambda_e = \lambda_\tau$, the latter two measurements are sensitive to the sign of $\lambda$, but the forward-backward asymmetry is not.

2.3 Polarisation

Because of the difference between the coupling constants $c_L$ and $c_R$ mentioned above, the outgoing fermions in $e^+e^- \rightarrow f\bar{f}$ will be preferentially left-handed, and the outgoing antifermions will be preferentially right-handed. This helicity polarisation is directly dependent on $\sin^2\theta_W$; measurement of the polarisation provides a measurement of the Weinberg angle.

If the produced particles are quarks, the initial helicity state is usually destroyed as the quarks bind into hadrons. If, however, the particles are $\mu$ or $\tau$ pairs, their polarisation can in theory be measured. For this reason, I limit discussion to the reaction $e^+e^- \rightarrow \ell^+\ell^-$ hereafter, where $\ell^-$ is a lepton.

By convention, the "polarisation" refers to the mean helicity polarisation of the $\ell^-$ produced in the collision, taking into account the entire angular range. It is a feature of electron-positron collisions that helicity is almost completely conserved. As a result of this (and the fact that the $Z^0$ has spin 1), the produced $\ell^+$ has opposite helicity to the $\ell^-$ and therefore opposite polarisation. Measurement of one is equivalent to measurement of the other.

The cross sections for left- and right-handed charged lepton pair production as a function of $s$ are given by

$$P_\ell = \frac{\sigma_R - \sigma_L}{\sigma_{Born}} = -\frac{F_2}{F_0}$$

(2.27)

where $\sigma_R$ and $\sigma_L$ are the cross sections for the values $p = +1, -1$, integrated over $\cos \theta$. At the peak of the $Z^0$ resonance, we have $\sqrt{s} = M_Z$. Therefore the real part of $\chi$ vanishes, $|\chi| \gg 1$ and the polarisation equation becomes
Figure 2.4: Forward-backward asymmetry, \( \tau \) polarisation and polarisation asymmetry. The change in analysis variables is plotted as a function of energy. The different lines represent different values of \( w = \sin^2 \theta_W \) (legend at top). The polarisation is seen to be slightly more sensitive than the polarisation asymmetry, but both are inherently more sensitive than the forward-backward asymmetry. As well, the latter suffers more from radiative corrections due to its stronger energy dependence.
\[ P_\ell = -\frac{2\nu_\ell a_\ell}{v_\ell^2 + a_\ell^2} = -\lambda_\ell \quad (2.28) \]

We can also calculate the lepton-polarisation asymmetry, defined by

\[ A_{pol}^{FB} = \frac{1}{\sigma_{Born}} \left[ \sigma(\cos \theta > 0, p = +1) - \sigma(\cos \theta > 0, p = -1) 
- \sigma(\cos \theta < 0, p = +1) + \sigma(\cos \theta < 0, p = -1) \right] 
= \frac{3}{4} \frac{F_3}{F_0} \quad (2.29) \]

At the \(Z^0\) resonance, this equation simplifies in much the same way as that for the polarisation, giving:

\[ A_{pol}^{FB} = -\frac{3}{4} \frac{2\nu_e a_e}{v_e^2 + a_e^2} = -\frac{3}{4} \lambda_e \quad (2.30) \]

From this, we see that a measurement of the lepton polarisation actually gives the neutral-current couplings to the outgoing lepton, whereas the polarisation asymmetry gives the couplings to the incoming electron. The polarisation, polarisation asymmetry and forward-backward asymmetry are complementary measurements. The first two are more sensitive and allow a test of lepton universality (in their equality or non-equality), whereas the latter can be measured with more statistical precision.

### 2.3.1 Measurement

For the polarisation measurement, the \(\tau\) is the most convenient of the leptons to study as the polarisation of a \(\tau\) sample can be determined from the energy distribution of its decay products. The angular distribution of decay particles in the \(\tau\) centre-of-mass frame is dependent on the \(\tau\) polarisation. When these particles are boosted forward by the momentum of the \(\tau\) in the lab frame, this distribution in angle becomes a distribution in energy. Thus, knowing the energies of the particles measured in the lab enables us to calculate their decay angle distribution in the \(\tau\) centre-of-mass frame, from which the polarisation is extracted. While the same is true of a muon, the long lifetime of the \(\mu\) at LEP energies means we do not observe many decays.

Also, a measurement of the \(\tau\) polarisation is unique in giving the neutral-current \(\tau\) couplings. While it is well established that the electron and \(\mu\) have the same coupling constants, the same can not be said of the \(\tau\). The \(\tau\) polarisation measurement provides a needed check on the Standard Model.

Although the examples given here always refer to the \(\tau^-\) decay, the distributions for \(\tau^+\) decay are related by a simple charge-parity conjugation (CP) transformation, in such a way that the final results differ only by the sign attached to the particle helicities. But as mentioned above, the helicity \(H_\tau^+ = -H_\tau^-\), with the happy result that these two effects cancel and we can ignore the charge of the \(\tau\) in the polarisation analysis.
Figure 2.5: X distribution of $\tau$ leptonic decays

The variable $x = E_\ell/E_\tau$ is plotted on the horizontal axis. The vertical axis gives the differential decay distribution, for different values of the $\tau$ polarisation.
Figure 2.6: Possible spin configurations in \( \tau^\pm \rightarrow \pi^\pm \nu_\tau \) decay

\[
\begin{align*}
H_\tau &= \pm \frac{1}{2} \\
(\tau \text{ Centre-of-mass frame})
\end{align*}
\]

Figure 2.7: Possible spin configurations in \( \tau^\pm \rightarrow \rho^\pm \nu_\tau \) decay

\[
\begin{align*}
H_\rho &= 0 \\
(\tau \text{ Centre-of-mass frame})
\end{align*}
\]
2.3.2 Tau Leptonic Decays

The decays $\tau^{\pm} \rightarrow e^{\pm} \nu_e \nu_\tau$ and $\tau^{\pm} \rightarrow \mu^{\pm} \nu_\mu \nu_\tau$ are identical in form to the well-studied decay $\mu^{\pm} \rightarrow e^{\pm} \nu_e \bar{\nu}_\mu$. Rather than repeat details, I quote only the final result here.

The distribution of number of decays in terms of the final-state lepton energy is given by

$$\frac{1}{N} \frac{dN}{dx} = \frac{1}{3} [(5 - 9x^2 + 4x^3) + P(1 - 9x^2 + 8x^3)]$$

(2.31)

where $x = E_\ell/E_{\text{max}}$, the normalised energy of the decay lepton in the laboratory frame, and $P$ is the $\tau$ polarisation. In arriving at this expression, the mass of the lepton has been ignored, as have radiative corrections to the decay matrix element. This distribution is plotted in figure 2.5.

2.3.3 Tau Semi-Hadronic Decays

Semi-hadronic modes such as $\tau^{\pm} \rightarrow \pi^{\pm} \nu_\tau$ and $\tau^{\pm} \rightarrow p^{\pm} \nu_\tau$ are valuable for polarisation studies because of their two-body final state. This means that the angular distribution of semi-hadronic decays is more sensitive to the $\tau$ polarisation than is the angular distribution for leptonic decays.

The transition amplitude (or “matrix element”) for a two-body decay can be written in general form[20]

$$M(\theta^*, \phi) = \sqrt{\frac{2j + 1}{4\pi}} (D_{m_m'}^{j}(\alpha, \beta, \gamma))^* (\phi, \theta^*, -\phi^*) f_{m_1 m_2}$$

(2.32)

where $m$ is the component of $\tau$ spin along its direction of motion (the helicity), $m_1$ and $m_2$ are the helicities of the decay particles, $m' = m_1 - m_2$, and $\theta^*, \phi^*$ are the polar and azimuthal decay angles of the hadron with respect to the $\tau$ direction of motion. $D_{m_m'}^{j}(\alpha, \beta, \gamma)$ are the rotation matrix elements for a spin-$j$ system through the Euler angles $\alpha, \beta, \gamma$, and $f_{m_1 m_2}$ is a kinematic term dependent on the final state spin components.

The simplest semi-hadronic $\tau$ decay is $\tau^{\pm} \rightarrow \pi^{\pm} \nu_\tau$. Here there is only one possible spin configuration for the final state, with $m_1 = 0$, $m_2 = -1/2$ and $m' = 1/2$ (see figure 2.6). There are then two matrix elements, one for each possible value of $m$:

$$M(m = +1/2, \theta^*, \phi) = f_{0, -\frac{1}{2}} \cos \frac{\theta^*}{2}$$

$$M(m = -1/2, \theta^*, \phi) = f_{0, -\frac{1}{2}} (-\sin \frac{\theta^*}{2}) e^{i\phi}$$

(2.33)

The kinematic parts are equal in this case so we can factor them out and write the angular distributions (after integrating out $\phi$):

$$\frac{1}{N} \frac{dN_+}{d\cos \theta^*} = |M(m = +1/2, \theta^*)|^2 = \frac{1}{2} (1 + \cos \theta^*)$$
The complete distribution is the mean of the number of decays in each $\tau$ helicity state:

$$\frac{1}{N} \frac{dN}{d \cos \theta^*} = \frac{1}{2} \left( \frac{N_+}{N_+ + N_-} \frac{dN_+}{d \cos \theta^*} + \frac{N_-}{N_+ + N_-} \frac{dN_-}{d \cos \theta^*} \right)$$

$$= \frac{1}{2} \left( 1 + \frac{N_+ - N_-}{N_+ + N_-} \cos \theta^* \right)$$

$$= \frac{1}{2} \left( 1 + P \cos \theta^* \right)$$

(2.35)

where $\theta^*$ is the decay angle of the $\pi^\pm$ in the $\tau^\pm$ rest frame (as in leptonic decays), and $N_{\pm}$ are the numbers of decays with positive and negative $\tau$ helicities.

The case for decays such as $\tau^\pm \to \rho^\pm \nu_\tau$ is complicated by the $\rho^\pm$ having spin 1. In the $\tau$ rest frame, there are then two allowed final spin states, corresponding to $m_{\rho} = -1$ and $m_{\rho} = 0$ (see figure 2.7). Proceeding as for $\tau^\pm \to \pi^\pm \nu_\tau$, we have four matrix elements to consider:

$$\mathcal{M}(m = +1/2, m_{\rho} = -1, \theta^*, \phi^*) = \frac{i}{2} e^{-i\phi^*} \sin \frac{\theta^*}{2}$$

$$\mathcal{M}(m = +1/2, m_{\rho} = 0, \theta^*, \phi^*) = f_{0,-\frac{1}{2}} \cos \frac{\theta^*}{2}$$

$$\mathcal{M}(m = -1/2, m_{\rho} = -1, \theta^*, \phi^*) = f_{-1,-\frac{1}{2}} e^{-i\phi^*} \sin \frac{\theta^*}{2}$$

$$\mathcal{M}(m = -1/2, m_{\rho} = 0, \theta^*, \phi^*) = f_{0,-\frac{1}{2}} e^{i\phi^*} (-\sin \frac{\theta^*}{2})$$

(2.36)
Unlike the $\tau^\pm \to \pi^\pm \nu_\tau$ decay, here the kinematic parts are not equal. However, from the components of the $\rho$ polarisation vector (given by standard relativistic quantum mechanics) we can deduce their relative amplitudes and assign

$$f_{0,-\frac{1}{2}} = \frac{A m_\tau}{\sqrt{2} m_\rho} \quad (2.38)$$

$$f_{-1,-\frac{1}{2}} = A \quad (2.39)$$

where $A$ represents some unknown scale.

Squaring the matrix elements, we factor out the constant term $A$. The azimuthal angle $\phi$ integrates out as with $\tau^\pm \to \pi^\pm \nu_\tau$, and we obtain the decay distributions

$$\frac{1}{N} \frac{dN}{d \cos \theta^*} = \frac{1}{2} \left(1 + \cos \theta^* \right)$$

We can then sum over helicity states, equivalent to the treatment of $\tau^\pm \to \pi^\pm \nu_\tau$, and obtain

$$\frac{1}{N} \frac{dN}{d \cos \theta^*} = \frac{1}{2} \left(1 + \alpha P \cos \theta^* \right) \quad (2.41)$$

where $P$ is the $\tau$ polarisation and $\alpha$ relates the contributions from $\rho$ helicity states:

$$\alpha = \frac{M_\tau^2 - 2M_\rho^2}{M_\tau^2 + 2M_\rho^2} \quad (2.42)$$

As expected, this is of the same form as the distribution for the decay $\tau^\pm \to \pi^\pm \nu_\tau$ (see equation 2.36), except for the factor of $\alpha$, which represents the cancelling effect of different $\rho$ helicities. Although the $\rho$ is a resonance, one can insert a mean mass of 768.3 MeV to obtain $\alpha = 0.46$.

The presence of $\alpha$ implies a diminished sensitivity to the $\tau$ polarisation, and for this reason it has conventionally been thought that these decays were inferior to the $\tau^\pm \to \pi^\pm \nu_\tau$ decay as a method of measuring the $\tau$ polarisation. However, we can make use of the fact that the $\rho$ decays via $\rho^\pm \to \pi^\pm \pi^0$ and determine the decay distribution as a function of $\cos \theta^*$ and $\cos \psi$, where $\psi$ is the decay angle of the $\pi^\pm$ in the centre-of-mass frame of the $\rho$, with respect to the $\rho$ helicity axis (see figure 2.8)[21].

First, we note that the above matrix elements are calculated in the $\tau$ centre-of-mass frame, where we can not observe the $\rho$ helicity. To analyse the $\rho^\pm \to \pi^\pm \pi^0$ decay we need
to know the matrix elements in the laboratory frame. This involves a “Wigner rotation” of the original matrix elements

\[ M_{m'_{\text{lab}}, m_{\text{lab}}}^{\text{lab}} = \sum_{m'} d_{m_m'}^{m_{\text{lab}}}(\eta) M_{m_{\text{lab}}, m_{\text{lab}}}^{\text{lab}} \] (2.43)

where \( \eta \) is given by

\[ \cos \eta = \frac{m_{\beta}^2 - m_{\rho}^2 + (m_{\tau}^2 + m_{\rho}^2) \cos \theta^*}{m_{\tau}^2 + m_{\rho}^2 + (m_{\tau}^2 - m_{\rho}^2) \cos \theta^*} \] (2.44)

(for \( E_\tau \gg m_{\tau} \))

Re-applying the general decay equation 2.32 for the case of \( \rho^\pm \to \pi^\pm \pi^0 \) gives the spatial components of the \( \rho \) decay matrix element:

\[ \mathcal{M}^\rho(m_\rho = 0, \psi, \phi') = \cos \psi \]
\[ \mathcal{M}^\rho(m_\rho = -1, \psi, \phi') = \frac{1}{\sqrt{2}} e^{i\phi'} \sin \psi \] (2.45)

These distributions are incorporated by re-calculating the matrix element to represent the decay to final-state \( \pi \)s. Integrating out the redundant angle \( \phi' \) as we did earlier for \( \phi^* \), we obtain the final result:

\[ \frac{1}{N d \cos \theta^* d \cos \psi} = \frac{3}{8(m_\pi^2 + 2m_\rho^2)} [(1 + P_\tau) W_+ + (1 - P_\tau) W_-] \] (2.46)

where

\[ W_+ = \sin^2 \psi \left((m_\tau \sin \eta \cos \frac{\theta^*}{2} - m_\rho \cos \eta \sin \frac{\theta^*}{2})^2 + m_\rho^2 \sin^2 \frac{\theta^*}{2} + 2 \cos^2 \psi(m_\tau \cos \eta \cos \frac{\theta^*}{2} + m_\rho \sin \eta \sin \frac{\theta^*}{2})^2 \right) \] (2.47)

and

\[ W_- = \sin^2 \psi \left((m_\tau \sin \eta \sin \frac{\theta^*}{2} + m_\rho \cos \eta \cos \frac{\theta^*}{2})^2 + m_\rho^2 \cos^2 \frac{\theta^*}{2} + 2 \cos^2 \psi(m_\tau \cos \eta \sin \frac{\theta^*}{2} - m_\rho \sin \eta \cos \frac{\theta^*}{2})^2 \right) \] (2.48)

This distribution, along with that for \( \cos \theta^* \) alone, is plotted in figure 2.9, for \( P_\tau = -1, P_\tau = +1 \) and \( P_\tau = -0.15 \) (the Standard Model expectation for \( \sin^2 \theta_W = 0.231 \)).
Figure 2.9: Momentum distribution of $\tau^\pm \rightarrow \rho^\pm \nu_\tau$

Plotted on the vertical scale is the differential decay distribution, as a function of the decay angles. The value $P = -0.15$ is the default in our simulated data set, and is approximately equal to the polarisation measured later.
2.4 Radiative Corrections

In addition to the Born-level calculations we have been doing so far, there are higher-order corrections which must be taken into account. Some of these are “internal” and result in renormalisation of the coupling constants, as described above. More serious are the cases where extra photons are produced along with the final-state $\tau$-pair.

The terms arising from calculations with these extra photons are called radiative corrections. There are three types; “initial-state” radiation where the photons are radiated from the colliding $e^+$ and $e^-$, “final-state” radiation where the photons are produced along with the $\tau^+\tau^-$, and “decay-state” radiation where photons appear with the $\tau$ decay products (for example, $\tau^\pm \rightarrow \rho^\pm \gamma \nu_\tau$).

For each of these corrections, there is a direct and indirect effect on the $\tau$ polarisation. Direct effect means a spin flip; the helicity of the $\tau^\pm$ is changed by the addition of a photon, in effect changing the polarisation. The indirect effect is to distort the amount of energy measured for particles in the experiment, causing mis-calculation of the scattering angles.

Estimates have been made of the degree to which the $\tau$ polarisation is affected by these corrections[22]. It was found that the direct effects are very small ($\delta P \simeq 0.2\%$), but the indirect effects are larger ($\delta P \simeq 4\%$). However, it is possible to simulate and correct for the indirect effects of radiation very easily, and this is done in my analysis. After these corrections, the remaining uncertainty due to direct radiative effects is so small compared to that from the limited sample size in our measurement ($\simeq 9\%$) that I feel safe in ignoring the direct effect.
Chapter 3

Experimental Apparatus

The parts of the experimental apparatus used fall into two large divisions; one consists of the LEP accelerator facilities which annihilate electrons and positrons, the other consists of the OPAL detector and its subsystems, used to observe the products of this annihilation. A description of LEP is given below, followed by a description of the parts of the OPAL detector relevant to this analysis. Finally, the OPAL trigger system is described; this is the process by which individual particle annihilation “events” are selected for analysis.

3.1 LEP

The first proposals for a high-energy electron-positron collider were made at CERN in 1976 and after a period of design work, the construction of LEP was authorised in 1981. The original design of LEP was optimised to collide particles at a centre-of-mass (CMS) energy between 80-100 GeV[23]. With the later discovery of the $Z^0$ it was decided to operate LEP at a CMS energy close to the mass of this particle (91.18 GeV), turning LEP into a “$Z^0$ Factory.”

A schematic view of LEP is shown in Figure 3.1. The facility consists of two main sections; an Injector which produces electrons and positrons in bunches, and a Storage Ring, which circulates and collides the particle bunches. It is the latter which is usually referred to as “LEP.”

3.1.1 Injector

The injector has several components. To produce positrons, a beam of electrons is accelerated in a linear accelerator (linac) to an energy of 200 MeV and then dumped into a converter target. In the converter target, the impacting electrons radiate bremsstrahlung photons which “convert” into electrons and positrons by pair-production. The positrons are separated out and, along with electrons from another source, enter a second linac where they attain an energy of 600 MeV.
Figure 3.1: Schematic diagram of the LEP injector
The LEP injector encompasses all the accelerators operating at CERN. Four experiments are situated at different points around the storage ring.
Both electrons and positrons are accumulated in separate bunches in a small racetrack-shaped storage ring called the Electron-Positron Accumulator (EPA). From the EPA these bunches are dumped to the CERN Proton Synchrotron (PS), an accelerator ring, in which they are further accelerated to an energy of approximately 3.5 GeV. Then they are shunted on to the Super Proton Synchrotron (SPS), another accelerator ring, where they reach an energy of 20 GeV.

From the SPS, the particle bunches are injected finally into the LEP storage ring.

3.1.2 Storage Ring

The LEP storage ring (actually a storage/accelerator ring in that it both accelerates and holds particles) was constructed in a tunnel running underneath the Pays de Gex region of France and the Swiss Canton of Geneva. It is roughly circular, approximately 27 km in circumference, situated in such a way as to avoid geological problem areas and population centres (see Figure 3.2).

Although approximately circular, LEP is more properly described as a rounded octagon. It consists of eight straight sections, connected by eight rounded corners. Particles collide in four of the eight straight sections and it is here that the four LEP experiments (OPAL, ALEPH, DELPHI and L3) are located. The other four straight sections are unused by experiments.

The particle bunches in LEP rotate approximately every 90 µs, which implies a beam-crossing rate of \( \approx 45 \) KHz. Each ideally contains \( \approx 4.16 \times 10^{11} \) particles, with expected dimensions \( \sigma_x = 155 - 190\mu m, \sigma_y = 6 - 12\mu m, \sigma_z = 1.17cm \).

The performance of an accelerator is usually measured by giving its luminosity \( \mathcal{L} \), a quantity defined in such a way that \( N = \mathcal{L} \cdot \sigma \) where \( \sigma \) is the cross-section for collision of the accelerated particles and \( N \) is the frequency of collisions observed. LEP is designed to operate at a luminosity of \( \mathcal{L} = 1.6 \times 10^{31}\text{cm}^{-2}\text{s}^{-1} \). During the collection of the data here, the luminosity was typically \( \mathcal{L} = 0.4 \times 10^{31}\text{cm}^{-2}\text{s}^{-1} \).

3.2 The OPAL Detector

OPAL has been designed with a cylindrical geometry typical of such experiments. It has a barrel section and two endcaps. The barrel contains a solenoid coil of radius 2.18 m, which generates a magnetic field of 0.435 T parallel to the z axis. Inside the coil is the Inner Detector, which contains subdetectors used to track particles as they emerge from the beam interaction point. Outside the coil is the Outer Detector, which contains subdetectors used to identify particles and measure their energies.

Each Endcap contains subdetectors which perform the same function as the Outer Detector, and in addition contains a Forward Detector which measures positions and energies of tracks which emerge at very small angles with respect to the beam line.

A complete description of the detector is given elsewhere[24]. For this analysis, the Inner Detector and parts of the Outer Detector are relied upon almost exclusively, corresponding to a region of uniformly good performance. My description here focuses on
LEP is situated in the Pays de Gex region of France and the Swiss Canton of Geneva. Although the tunnel crosses the international boundary, all access points to LEP are in France, as are the four LEP experiments. The city of Geneva is located to the south of LEP.
these sections of the detector.

The axis of the OPAL barrel is nearly that of the LEP beam line. All measurements are done in a co-ordinate system where the $x$ axis is horizontal and points roughly towards the centre of LEP, the $y$ axis is vertical and the $z$ axis points in the direction of motion of the circulating electrons. A spherical polar co-ordinate system is also defined in which the polar angle $\theta$ is measured from the $z$ axis, the azimuthal angle $\phi$ is measured from the $x$ axis about the $z$ axis, and the radius $r = \sqrt{x^2 + y^2 + z^2}$ (see figure 3.3).

### 3.2.1 Inner Detector

The inner detector is an array of three concentric cylindrical subdetectors sitting inside the OPAL coil. Electrons and positrons collide at the Interaction Point, inside the LEP beam pipe. Any charged particles resulting from these collisions fly outwards, tracing out spiral orbits due to the presence of the magnetic field generated by the coil.

Innermost of the three subdetectors is the vertex detector, which gives precision locations of charged particle tracks close to the interaction point. Next is the jet chamber, which gives momenta and further position data of tracks. Outermost are the Z-chambers, which measure track positions entering the coil. All three of these are wire chambers of varying designs, operating in a common pressure vessel at 4 bar gas pressure. They complement each other, providing complete information about a track when their output is combined.

The vertex detector is a cylindrical drift chamber 1 m long and 47 cm in diameter, surrounding the vacuum pipe containing the electron and positron beams as they circulate through LEP. It is divided into two layers consisting of 36 cells each, an inner layer with “axial wires” parallel to the $z$ axis and an outer layer with “stereo wires” oriented at an angle of four degrees to the $z$ axis. The axial-wire layout allows a measurement of track positions in $r$ and $\phi$ to an accuracy of 50 $\mu$m. Resolution in $z$ is approximately 1.5 mm.

The jet chamber is 4 m long, with an inner diameter of 0.5 m and an outer diameter of 3.7 m. It is divided into 24 sectors, each of which contains one wire plane with wires parallel to the $z$ axis. The wire position, drift time, and charge division give coordinates in $r$, $\phi$ and $z$. The average resolution in $r$ and $\phi$ is approximately 135 $\mu$m, and in $z$ is approximately 6 cm. Since the jet chamber provides measurement of track positions at many points in space, the information it provides can be used to calculate the curvature of the track. From this, knowing the strength of the OPAL magnetic field, the track momentum can be calculated with an average resolution of $\delta p/p^2 = 2.2 \times 10^{-3}$ GeV$^{-1}$.

There are 24 Z-chambers in the form of a cylinder occupying the space between the coil and jet chamber. Each is 4 m long, 50 cm wide and 59 mm thick, split into eight cells 50 x 50 cm in size with six wires running in the $\phi$ direction. $z$ positions are given by the wire hits, and the $\phi$ co-ordinate is read out by charge division. With this arrangement, the Z-chambers are able to measure the $z$ co-ordinate of tracks to an accuracy of approximately 300 $\mu$m and the $r$-$\phi$ position to approximately 1.5 cm.
Figure 3.3: The OPAL detector

The different components of the OPAL detector are visible in this cutaway view. In the bottom left, the co-ordinate axes used for analyses are indicated.
3.2.2 Time-of-Flight System

The first of the outer detectors is the Time-of-Flight system (TOF), which is just outside the OPAL coil. This consists of 160 scintillating counters, each 6.8 m long, 45 mm thick and approximately 90 mm wide. The counters are placed parallel to each other to form a barrel of mean radius 2.36 m.

When a particle passes through the TOF, a pulse of light results and is read out at each end of the scintillating bar by photomultipliers. This gives a measurement of the time at which the particle passed, which is useful for elimination of cosmic ray events. As well, the difference in time between the light pulses at each end of a bar can be used to estimate the z position of the particle. The accuracy of signal timing is approximately 220 ps and the accuracy in z measurement is approximately 5.5 cm.

3.2.3 Electromagnetic Calorimetry

As charged particles pass through the lead glass they undergo electromagnetic interactions. Photons are radiated from the particles by bremsstrahlung, and electrons are produced both by ionisation of the atoms in the material and pair-production from the bremsstrahlung photons. This results in a chain of electrons and photons, called an electromagnetic shower.

The electromagnetic calorimeter (ECAL) is optimised for measurement of electromagnetic showers. It is split into a barrel (EB) and two endcaps (EE), each made up of lead glass blocks. The depth of glass (approximately 25-27 radiation lengths) and block size are chosen to correspond to the expected dimensions of an electromagnetic shower. Shower electrons emit Čerenkov radiation in the lead glass. This radiated light is then read out by photomultipliers, giving a measurement of the energy deposited by the shower particles. The accuracy of the energy measurement is limited by fluctuations in the number of particles per shower.

There are two differences between EE and EB. The first is their geometry; EB is segmented in φ into 160 blocks and in z into 59 blocks. Each half of EE consists of a dome-shaped array of 1132 blocks. In EB the blocks are oriented such that their axes point roughly to the interaction point. In EE the block axes are parallel to the beam line.

The other difference between EB and EE is the way in which the Čerenkov light is read out. EB is outside the OPAL coil and can therefore use photomultiplier tubes, which can operate in the slight residual magnetic field. EE operates in the full magnetic field and therefore must use a new type of photomultiplier called a vacuum photo triode, which is not affected by the field.

The lead glass has an intrinsic energy resolution of $\sigma_E/E = 0.2% + 6.3%/\sqrt{E}$ (E in GeV), among the best possible of calorimeter designs. However, this intrinsic resolution is degraded by the calorimeter's position behind the coil. The effect of this is to degrade resolution at very low energies (see figure 3.4)

$^{1}$75% PbO by weight, refractive index $n = 1.8467$ at $\lambda = 586$ nm
3.2.4 Hadronic Calorimetry

The hadron calorimeter (HCAL) is designed to provide position and energy measurements of hadronic showers. These are showers of hadrons resulting from collisions between a hadronic particle and nuclei in the detector material. Typically they take place at much greater depths in material than do electromagnetic showers.

HCAL is divided into a barrel, two endcaps, and two “pole tips.” The barrel calorimeter is made up of 9 layers of wire chambers alternating with 8 iron slabs, and is positioned from radii 3.39 m to 4.39 m. Each slab is 100 mm thick and each chamber 25 mm. Hadronic showers are initiated in the iron (which also serves as the flux return for the coil) and their position and energy are read out in the wire chambers. The electromagnetic and hadronic calorimeters combined are approximately 6 nuclear absorption lengths deep, corresponding to the expected depth of a hadronic shower.

The wire chambers vary in length and width, but each consists of a series of channels at least 5 m long, running parallel to the z axis, made of PVC plastic with an anode wire running down the centre of the channel. The PVC is coated with graphite which serves as a cathode, and the channels are operated in limited streamer mode.

The charge deposited by a shower is read out in an arrangement of pads and strips. Pads are large conducting surfaces, roughly the expected size of a hadronic shower. They sit against and outside the plastic of the chambers. Charge is induced on them from that deposited in the chambers and the pads are taken to define towers, the total charge in which corresponds to that deposited by a hadronic shower.

Strips sit inside the detector plastic and run along one wall of each channel. The charge deposited on a strip gives a rough measurement of energy and, by charge-division, gives a more precise measurement of the location of the hadronic shower.

The endcap calorimeter is of much the same design, but here there are 8 layers of chambers and 7 layers of 100 mm thick iron, with 35 mm gaps between the iron. The chambers are oriented with their wires pointing along the x axis.

The pole tip calorimeter is similar to the endcap, but covers the extreme range of $\theta$, $0.91 \leq \cos \theta \leq 0.99$. It has 10 chambers and 9 layers of iron, each some 70 mm thick.

The hadronic calorimeter has an energy resolution varying from $\sigma_E/E = 100%/\sqrt{E}$ to $\sigma_E/E = 140%/\sqrt{E}$, depending on the energy of the particle initiating the shower. It must be noted, though, that some particles initiate electromagnetic showers before reaching the hadronic calorimeter, so an accurate energy measurement must sum the energies in both the electromagnetic and hadronic calorimeters.

3.2.5 Muon Detector

The muon detector consists of large-area drift chambers placed outside all other detector elements, next to the hadron calorimeter. It has a barrel and two endcaps. Muon identification is accomplished by accurate measurement of the outgoing track direction and comparison to tracks in the central chamber to measure the angle of multiple scattering. The main purpose of the muon detector is therefore to accurately measure the position of outgoing tracks.
The barrel part of the detector consists of 110 wire chambers, each 1.2 m wide and 90 mm deep. Forty-four are mounted on each side of the barrel, plus 10 in a separate module on top and 12 in another on the bottom.

The \( \phi \) coordinate is measured by the barrel detector to an accuracy of approximately 1.5 mm, and the \( z \) coordinate to an accuracy of 2 mm. It has an efficiency for detecting muons above 3 GeV of approximately 100\%, and a probability of mis-identifying an isolated 5 GeV pion as a muon of \( \leq 1\% \).

Each endcap detector consists of 8 chambers 6 \( \times \) 6 m in size and 4 patch chambers 3 \( \times \) 2.5 m in size. The chambers contain two layers of streamer tubes, similar in design to those used in the hadron calorimeter. One layer is oriented horizontally and another vertically. Instead of a pad-and-strip arrangement, each layer has two sets of strips, one set parallel to the wires and one perpendicular.

The endcap measures positions to an accuracy of approximately 3 mm. Muon misidentification was measured to be 0.2\% for 10-50 GeV hadrons passing through the chamber at an angle of 45\°.

### 3.2.6 Trigger

To properly identify electron-positron collisions, signals from the different OPAL subdetectors are combined in various ways to produce a logical trigger signal telling whether the data in the subdetectors should be recorded or not. The full details of the trigger system are written elsewhere\[25\] but I will describe its basic features here.

Where possible, signals from the different subdetectors are divided into bins in \( \theta \) and \( \phi \). In each bin, a trigger signal is generated if certain conditions are met, and the results from the bins are combined by a trigger processor which decides whether to keep the event or not. As well, some subdetectors generate independent "direct" trigger signals not subject to this binning. Times of all signals are referenced to another signal generated by LEP at the moment the beams cross.

The vertex detector and jet chamber produce a signal called the "track trigger." Observed wire chamber hits are divided into bins by their values for \( r/z \), which is equivalent to \( \theta \). A peak in one \( \theta \) bin indicates the presence of a track.

The time-of-flight detector produces a trigger by looking for any hit in a block of 24 scintillator bars within \( \pm 50 \) ns of receiving a beam-crossing signal from LEP. If more than six such blocks show hits, a trigger is generated.

ECAL signals are counted by binning in \( \theta \) and \( \phi \) the energy read out from lead-glass blocks. If this energy passes different threshold values, trigger signals are produced. HCAL triggers are produced in much the same way, summing the energy in pads.

The muon detector barrel and endcap sections generate separate triggers. In the barrel, a trigger is generated if hits are observed in at least three of the four layers of the detector, in one \( \phi \) bin of approximately 15\° size. Triggers are generated in the muon endcap by summing the charge collected in 64-128 adjacent strips. If at least two planes of the detector show that this charge is above a threshold value, and the measured position is consistent with a track from the interaction point, a trigger signal is generated.
The Forward Detector trigger compares the energy deposited in the detectors against a threshold value, and generates a trigger signal if the total energy is above threshold and is deposited in a configuration consistent with a physical event. This signal is used mainly to detect-Bhabha scattering events.

In 1990, these signals were combined and an overall trigger was generated if any of a list of conditions were met. The most relevant of these for this analysis were:

- More than two tracks are observed in the barrel region
- More than three tracks are observed anywhere
- A track is observed with an associated TOF trigger
- Associated TOF and EM triggers are observed
- At least one EB bin has \( \geq 2.6 \text{ GeV} \) deposited
- \( \geq 4 \text{ GeV} \) energy is deposited in the EM barrel, and there is either an associated track or an associated TOF trigger

These triggers are independent of each other and redundant to a high degree. As a result, we have a high trigger efficiency. For detection of \( e^+e^- \rightarrow \tau^+\tau^- \) events, this was estimated to be \( 99.9 \pm 0.1\% \).

### 3.2.7 Data Readout

For each subdetector, there is a specialised processor which accumulates raw signals from the subdetectors and processes them, producing more readily usable output. These are called the Local System Crates (LSCs).

The processed signals from the LSCs are collected in a parallel-processing system called the Event Builder. This is capable of handling events at a rate of 10 Hz, each of up to 100 Kbyte in size.

From the Event Builder, a special processor called a filter performs a quick analysis of the event and labels it as coming either from an electron-positron collision or from one of several categories of “background” events (ex. cosmic rays, interactions of beam particles with material in LEP). In 1990, the filter ran in a VME-bus microprocessor system, as did the Event Builder and LSCs.

Events which make it through the event builder were, in 1990, written out to cartridge tape by a VAX 8700, regardless of whether they passed filter requirements or not. As well, events which were labelled acceptable by the filter were passed on to another computer system which performed “event reconstruction.” The raw data from the detector was converted into physically meaningful quantities such as the number of observed particles, their energies and momenta.
Figure 3.4: 1990 ECAL energy resolution
The inherent energy resolution of the electromagnetic calorimeter is seen to decrease because of the material situated in front of it.
Chapter 4

Data

In addition to the experimental data, simulated "Monte Carlo" data representing all significant processes at LEP are required to estimate efficiency and contamination. Production of these data sets begins with simulation of particle production, including some higher-order field theoretical corrections. The output of this simulation is then processed by a detector simulation which produces "raw" data in the same form as actual OPAL events. Finally, both data and Monte Carlo data are passed through the same process of reconstruction, whereby physical quantities such as particle momenta and directions are obtained from raw data.

4.1 Data

The events used for this analysis were collected during the OPAL running period from May-August of 1990. They correspond to an integrated luminosity of $\mathcal{L} = 5.8\text{pb}^{-1}$, or 3310 $\tau$-pair events. This sample was taken at different energy values from 88.22 GeV to 94.22 GeV, with about two-thirds near the $Z^0$ peak at 91.175 GeV (see table 4.1).

4.2 Simulated Data

Simulation of OPAL data proceeds through two steps. First, the fundamental particle data are created; particle types, their four-momenta, charge, and spin. A number of "generator" computer programs are used for these, according to the specific process being simulated.

After generation, the interaction of these particles is simulated in the GOPAL[26] computer program. This is built around GEANT[27], a standard modelling system for particle physics experiments. GEANT tracks the location and momentum of each particle at a certain time, calculates changes after a small increment in time, then repeats this procedure, "swimming" the particle through space until it leaves the region of the detector. Interactions between particles are taken into account, as are decay lifetimes and particle-detector interactions.
4.2.1 Simulated $\tau$-pairs

$\tau$-pairs are generated using the KORALZ\[28] 3.7 computer program. This is a standard for LEP experiments, designed to provide accurate simulation of these effects:

- "Initial-state" radiation of photons from the colliding $e^+$ and $e^-$ particles
- "Final-state" radiation of photons from the produced $\tau^+$ and $\tau^-$ particles
- Higher-order weak corrections
- Final state spin-angle correlations
- Simulation of all significant $\tau$ decay modes
- Effects on decay distributions resulting from $\tau$ polarisation
- Effects on $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ decay distributions due to $\rho$ helicity

A main sample of fifty thousand $e^+e^- \rightarrow \tau^+\tau^-$ events was made and then processed by GOPAL to provide simulated raw data. The parameters generated for each decay (its type, momenta, and decay angles) are stored with each simulated event. By comparing the reconstructed event quantities with those generated, it is possible to calculate efficiencies, final-sample contamination, and correction factors needed for analysis.

Three types of weights are applied to each $\tau$ decay in this sample. Decays are weighted by mode to correct the "missing mode problem" of $\tau$ physics. World average branching
fractions are used for one-prong modes and OPAL measurements are used for the three-prong and five-prong modes[29]. As the sum of these modes is not equal to one, each fraction is increased in proportion to its error (roughly 0.8 standard deviations) until the sum of branching fractions for all modes is one. The branching fractions used are listed in table 4.2.

Decays proceeding by $\tau^\pm \rightarrow a_1^{\mp} \nu_\tau \rightarrow \pi^\pm(2\pi^0)\nu_\tau$ are also weighted according to the actual mass of the $a_1$ meson, given by KORALZ, in such a way that the weighted mass spectrum has the current best-measured mass and width (the original KORALZ sample contained less accurate values for these parameters than those that are now known).

Finally, each $\tau$-pair is weighted according to the helicity of the $\tau^-$. These weights may be varied, allowing simulation of any degree of polarisation.

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>B.F. (measured[29], %)</th>
<th>B.F. (weighted, %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^\pm \rightarrow \rho^\pm \nu_\tau$</td>
<td>$23.1 \pm 0.6$</td>
<td>$23.6$</td>
</tr>
<tr>
<td>$\tau^\pm \rightarrow e^\pm \bar{\nu}<em>e \nu</em>\tau$</td>
<td>$17.9 \pm 0.2$</td>
<td>$18.1$</td>
</tr>
<tr>
<td>$\tau^\pm \rightarrow \mu^\pm \bar{\nu}<em>\mu \nu</em>\tau$</td>
<td>$17.5 \pm 0.2$</td>
<td>$17.7$</td>
</tr>
<tr>
<td>$\tau^\pm \rightarrow \pi^\pm \pi^\mp \pi^- (\geq 0\pi^0) \nu_\tau$</td>
<td>$15.2 \pm 0.3$</td>
<td>$15.4$</td>
</tr>
<tr>
<td>$\tau^\pm \rightarrow (\pi, K)^\pm \nu_\tau$</td>
<td>$12.0 \pm 0.3$</td>
<td>$12.3$</td>
</tr>
<tr>
<td>$\tau^\pm \rightarrow \pi^\pm (2\pi^0) \nu_\tau$</td>
<td>$8.3 \pm 0.7$</td>
<td>$8.9$</td>
</tr>
<tr>
<td>$\tau^\pm \rightarrow \pi^\pm (\geq 3\pi^0) \nu_\tau$</td>
<td>$1.6 \pm 0.6$</td>
<td>$2.1$</td>
</tr>
<tr>
<td>$\tau^\pm \rightarrow (K^*)^\pm \nu_\tau$</td>
<td>$1.4 \pm 0.2$</td>
<td>$1.6$</td>
</tr>
<tr>
<td>$\tau^\pm \rightarrow \pi^\pm \pi^\mp \pi^- \pi^+ (\geq 0\pi^0) \nu_\tau$</td>
<td>$0.25 \pm 0.08$</td>
<td>$0.3$</td>
</tr>
<tr>
<td>Total</td>
<td>$97.3 \pm 1.2$</td>
<td>$100.0$</td>
</tr>
</tbody>
</table>

Table 4.2: Branching fractions used in simulation

In addition to the main sample of $\tau$-pairs, Monte Carlo subsamples are also generated consisting of events where each $\tau^\pm$ decays by $\tau^\pm \rightarrow \rho^\pm \nu_\tau$, at a range of $\tau$ energies (see table 4.1). These samples are required to calculate correction factors in the polarisation analysis, where the particle decay angles are needed. As these decays are not used anywhere else, they are not passed through the entire simulation chain, only produced by KORALZ.

### 4.2.2 Other Simulated Data

A sample of 167,000 multi-hadron events was generated using two computer programs, HERWIG[30] and JETSET[31]. While these differ in their treatment of QCD processes, the difference is of little importance to $\tau$ studies, where we are interested only in the gross qualities of these events; the number of charged tracks and ECAL clusters produced. Hence, both samples are combined and treated as one.

Bhabha scattering is studied using a sample of 24,000 events, limited in range to $12.5^\circ \leq \theta \leq 167.5^\circ$ so the sample will be useful for background studies in the barrel.
region. Di-muon background is estimated from a sample of 20,000 events, generated using KORALZ as were the \( \tau \)-pair samples.

Last, simulated “two-photon” data sets \( (e^+e^- \rightarrow e^+e^- X) \) were generated. The name “two-photon” refers to the photon-fusion process in which extra particles \( X \) are created (see figure 4.1). Approximately 19,000 of these events were generated in total, distributed among several possibilities for the extra particles \( X (=e^+e^-, \mu^+\mu^-, \tau^+\tau^-, q\bar{q}) \).

All these background samples have been processed by GOPAL, following the same chain as the \( \tau \)-pair sample.

### 4.3 Reconstruction

Raw information from the Inner Detector subdetectors is combined into a series of quantities corresponding to “charged tracks.” Each of the Outer Detector subdetectors produces a “cluster” by grouping together adjacent hits. A charged track should correspond to the trail left by a particle, and a cluster should correspond to the energy left behind by a shower.

Tracks and clusters are associated with each other based on their nearness in \( \theta \) and \( \phi \). One cluster may have several associated tracks, but each track is associated with at most one cluster per subdetector.

Unless stated otherwise, the only clusters we make use of here are ECAL clusters. For these, the total energy deposited in a contiguous group of lead-glass blocks (the “raw energy”) is corrected to account for losses in the coil and to neighbouring blocks. The result is the “cluster energy” (or “corrected energy”). Cluster locations in \( \theta \) and \( \phi \) are taken to be the cluster centroid.

After reconstruction, it is observed that the energy and momentum resolution in the Monte Carlo data are significantly better than in the data. To make the Monte Carlo data more faithfully represent reality, Monte Carlo ECAL energies and charged track transverse momenta are “smeared” — random numbers are added to each track momentum and cluster energy in such a way that the distributions of each become broader. The exact change is

\[
\delta E_{\text{ecal}} = 0.03E_{\text{ecal}} \quad (4.1)
\]

\[
\delta P_{\text{ctrk}}^\perp = 1.52 \times 10^{-3} \times (P_{\text{ctrk}}^\perp)^2 \quad (4.2)
\]

where \( E_{\text{ecal}} \) is the corrected energy of an ECAL cluster and \( P_{\text{ctrk}}^\perp \) is the transverse momentum of a charged track.
Figure 4.1: Feynman diagram for $e^+e^- \rightarrow e^+e^-X$
Chapter 5

Preselection of $e^+e^- \rightarrow \tau^+\tau^-$ Events

A sample of $\tau$-pair events is collected using requirements which have been standardised for all analyses in the OPAL $\tau$ polarisation working group. These ensure minimal contamination from other processes. $\tau$-pair events are selected with approximately 55% efficiency, with little effect on the $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ branching fraction or polarisation calculations. Although requirements are aimed at eliminating certain types of background, many are effective against more than one type, and an estimate of the background is given only when all requirements are combined.

5.1 Data Quality and Status Requirements

For all events, the Jet Chamber and Electromagnetic Calorimetry must have been functional, producing both data and triggers. Also, the Forward Detector, Vertex Chamber, Time-of-Flight counter, Hadron Calorimeter Barrel and Muon Detector Barrel all had to be operational and producing data.

In all preselection requirements except those which eliminate cosmic rays, we refer to "good" charged tracks and clusters, which are defined as follows:

- Tracks: $|d_0| \leq 2$ cm, $|z_0| \leq 50$ cm, $N_{\text{Jet hits}} \geq 20$, $P_\perp \geq 0.1$ GeV, and $R_{\text{wire min}} \leq 75$ cm
- ECAL barrel clusters: $E_{\text{raw}} \geq 100$ MeV
- ECAL endcap clusters: $E_{\text{raw}} \geq 200$ MeV, $N_{\text{blks}} \geq 2$, $F_{\text{max}} \leq 0.99$

Here $d_0$ is the distance of closest approach between the track and the z axis, $z_0$ is the z value of the track at the point of closest approach, and $P_\perp$ is the transverse momentum of the track. Distributions of track parameters in 1990 are shown in figure 5.1.

$R_{\text{wire min}}$ is the radius of the first wire hit in the jet chamber, and $F_{\text{max}}$ is the fraction of energy contributed by the highest-energy block in an electromagnetic calorimeter cluster. $E_{\text{raw}}$ is the total cluster energy before the above-mentioned corrections are applied.

These requirements are very loose, intended to be the minimum necessary to ensure that tracks and clusters are due to physical particles from the interaction point.
Figure 5.1: 1990 distributions of track parameters

The parameters $d_0$ and $z_0$ are, respectively, the impact parameter and $z$ coordinate at the point of closest approach to the $z$ axis of the charged track. $P_T$ is the track's transverse momentum. Arrows mark positions where requirements are applied.
5.2 Elimination of Cosmic Ray Background

Cosmic rays are eliminated with these requirements:

At least one TOF counter must satisfy $|t_{\text{meas}} - t_{\text{exp}}| \leq 10$ ns, where $t_{\text{exp}}$ is the time given by the LEP beam crossing signal and $t_{\text{meas}}$ is the time of the TOF signal. This "gate" should be sufficient to include most events arising from $e^+e^-$ annihilation (see figure 5.2a). If all TOF signals from counter pairs separated by more than $165^\circ$ have time differences greater than 10 ns, the event is rejected. For $e^+e^-$ annihilation, the time difference for these "back-to-back" counters is approximately $\delta t = 0 \pm 6$ ns, but for cosmic ray events the time difference is seen to vary between roughly $15 \leq \delta t \leq 25$ ns. This can be seen in figure 5.2b, where two peaks are evident, one for cosmic ray particles and one for annihilation products.

At least one track must satisfy $d_0 \leq 0.5$ cm and $|z_0| \leq 20$ cm, and the average $z_0$ for all tracks must satisfy $|<z_0>| \leq 20$ cm. These two requirements ensure that the event is consistent with production of particles from the interaction point. Distributions of $|d_0|_{\text{min}}$ vs. $|z_0|_{\text{min}}$ and of $<z_0>$ are shown in figure 5.2c,d for events which are rejected by the TOF requirements (ie cosmic ray events). Cosmic ray events are seen to spread uniformly, whereas annihilation events are narrowly distributed around the origin.

5.3 Cone Analysis and Fiducial Acceptance

All backgrounds other than that from cosmic rays are eliminated using requirements based on a cone analysis. The procedure for determining a cone is as follows:

All charged tracks in the event are listed in decreasing order of their momenta. The highest-momentum track is taken to define a cone direction, and then the list is searched for the next-highest momentum track within $35^\circ$ of the cone. When it is found, the momenta are added together to define a new cone direction. Then the procedure is repeated until no new tracks can be added to the cone. New cones are further defined, until all tracks in the event have been accounted for.

In a $\tau$-pair event, all the $\tau$ decay products should be collimated due to the high momentum of the $\tau$, producing only two cones. Therefore events are required to have exactly two cones. Each cone must have at least one charged track and at least 1% of the LEP beam energy. The amount of multi-hadron background deleted by this requirement can be seen in figure 5.3a.

The acolinearity of the two cones must be less than $15^\circ$ ($0.13$ rad), to eliminate events with highly energetic radiated photons. The expected distribution of the acolinearity of two-cone $\tau$-pair events is shown in figure 5.3b. It is estimated that this requirement rejects about 2% of $\tau$ pairs.

The average $|\cos \theta|$ of the two cones must satisfy $|\cos \theta| < 0.68$. This requirement eliminates 41% of all $\tau$-pair data. This acceptance limit is necessary so that events occur within a region of the detector where response is uniform and well-understood. By increasing the $\cos \theta$ limit, one encounters problems in the "overlap region" of the detector, where the endcap and barrel meet.
Figure 5.2: Useful distributions for cosmic ray elimination

Plot (a) shows the spread in impact time for particles from annihilation. Plot (b) indicates the time difference between hits in opposing TOF sectors. The two peaks indicate data and cosmic ray times. Plot (c) shows the random locations of cosmic ray tracks, and plot (d) in contrast shows the tight constraints on tracks from annihilation.
Figure 5.3: Normalised distributions of $\tau$-pair selection quantities (simulated data).
Plot (a) shows the number of charged cones in $\tau$-pair and multi-hadron events. Plot (b) shows the acolinearity of two-jet $\tau$-pairs. Plots (c) and (d) show, respectively, the number of good charged tracks and number of good clusters in $\tau$-pairs and multi-hadron events.
Figure 5.4: Bhabha-scattering and two-photon event characteristics (simulated data)
The two upper plots show the dramatic contrast between the characteristic energy de-
posits of $\tau$-pairs (a) and Bhabha-scattering events (b). The two lower plots contrast the
energy seen in the detector for two-photon events (c) and $\tau$-pairs (d).
5.4 Elimination of $e^+e^- \rightarrow q\bar{q}$ Background

Multi-hadron events are characterised by large numbers of particles produced in "jets." Since one jet corresponds roughly to a cone, some multi-hadron background is rejected by the two-cone requirement. To further reduce this background, we make two more requirements:

- The total number of good charged tracks must satisfy $2 \leq N_{\text{ctrk}} \leq 6$.
- The total number of good ECAL clusters must satisfy $N_{\text{ecal}} \leq 10$.

The distribution of the number of good charged tracks for $\tau$-pairs and multi-hadron events is shown in figure 5.3c, and the distribution of the number of good ECAL clusters in figure 5.3d. There is clear separation between the two types of events.

5.5 Elimination of $e^+e^- \rightarrow e^+e^-$ Background

Since electrons almost always deposit their entire energy in the electromagnetic calorimeter, an $e^+e^- \rightarrow e^+e^-$ event will be characterised by total ECAL energy near that of the colliding particles.

Therefore, events must satisfy

$$\Sigma E_{\text{ecal}} < 0.8E_{\text{cm}},$$

where $E_{\text{cm}}$ is the centre-of-mass energy of the colliding electron and positron

or

$$\Sigma E_{\text{ecal}} + 0.3\Sigma P_{\text{ctrk}} < E_{\text{cm}}$$

where the sum is over all good charged tracks and clusters in the cone.

The distribution of $\tau$-pair events in $\Sigma E_{\text{ecal}}$ and $\Sigma P_{\text{ctrk}}$ is shown in figure 5.4a and the same distribution for Bhabha-scattering events in figure 5.4b.

5.6 Elimination of $e^+e^- \rightarrow e^+e^-X$ Background

Two-photon events $e^+e^- \rightarrow e^+e^-X$ are characterised by two cones of secondary particles $X$ having very low energy and transverse momenta. To identify them, we define a quantity called the visible energy $E_{\text{vis}}$, as the sum over all tracks and clusters of the energy for each (the energy is calculated assuming the charged track to be a $\pi^\pm$). When the charged track and cluster are associated the maximum of the two energies is taken. The aim of this definition is to accurately estimate the total energy of the particles in the event.

We require $E_{\text{vis}} > 0.03E_{\text{cm}}$

If $E_{\text{vis}} < 0.2E_{\text{cm}}$ then the track and cluster total transverse momenta must satisfy

$$P_{1\perp} > 2 \text{ GeV} \quad \text{or} \quad P_{\text{ecal}} > 2 \text{ GeV}.$$

Figures 5.4c,d show distributions of $E_{\text{vis}}$ for two-photon and $\tau$-pair events. Figure 5.5 shows $P_{\text{ecal}}$ vs. $P_{1\perp}$, also for two-photon and $\tau$-pair events. In both cases there is clear separation, as desired.
Figure 5.5: Two-photon event characteristics (simulated data)
The momentum profiles for two-photon events (a) and $\tau$-pairs (b) show a sharp contrast.
5.7 Elimination of $e^+e^- \rightarrow \mu^+\mu^-$ Background

To be rejected, events must have two charged tracks which can be identified as muons ("muon candidates"). Muon candidates must satisfy these requirements:

- $p_t > 2$ GeV
- $p > 6$ GeV
- $|d_0| < 1$ cm
- $|z_0| < 50$ cm
- $N_{\text{hits}}^{CV+CJ+CZ} \geq 30$

and at least one of these requirements:

- At least two associated hits in MB or ME
- At least 4 associated HCAL hits, with $\geq 1$ in the last 3 layers of HB if $|\cos \theta| < 0.65$
- $P > 15$ GeV and associated ECAL energy less than 3 GeV

We require $E_{\text{tot}} > 0.6E_{\text{cm}}$ for the event to be rejected, where $E_{\text{tot}}$ is the scalar sum of track momenta and highest single cluster energy. Also, the azimuthal angles of the two muon candidates must satisfy $\delta \phi > 0.32$ rad for the event to be rejected.

Distributions of these quantities for simulated muons and $\tau$-pairs are shown in figures 5.6 and 5.7.

5.8 Summary of Results

As estimated from our simulated data, the preselection requirements identify $\tau^+\tau^-$ pairs with $54.7 \pm 0.7\%$ efficiency ($59\%$ from the acceptance requirement alone, as stated above). The final sample contains a contamination of $1.9\%$ non-$\tau$ pair events, broken down in table 5.1. The fraction of $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ events in the preselected sample is estimated to be enhanced by a factor of $1.02 \pm 0.005$. 

49
Figure 5.6: Di-muon event characteristics (simulated data)

The upper two plots compare the number of hits left in the muon barrel detector for (a) muons and (b) $\tau$-decay products. The bottom two plots compare the total energy seen for (c) di-muons and (d) $\tau$-pairs. The missing energy arising from $\tau$ decay is clearly visible.
Figure 5.7: Di-muon event characteristics (simulated data)
The upper two plots compare the hadron calorimeter interaction profiles for (a) a muon and (b) particles from τ decay. The penetration (outermost HCAL layer hit) is plotted against the number of layers hit in total. These two plots demonstrate the penetrating power of muons. The lower two plots show the electromagnetic calorimeter interaction profiles for (c) muons and (d) τ decay products. It is clearly seen that the muons deposit less energy in ECAL.
<table>
<thead>
<tr>
<th>Background</th>
<th>Contamination (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^- \rightarrow e^+e^-$</td>
<td>0.2 ± 0.2</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow \mu^+\mu^-$</td>
<td>0.8 ± 0.9</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow q\bar{q}$</td>
<td>0.7 ± 0.3</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow e^+e^-X$</td>
<td>0.2 ± 0.2</td>
</tr>
<tr>
<td>Total</td>
<td>1.9 ± 1.0</td>
</tr>
</tbody>
</table>

Table 5.1: Contamination in $\tau$-pair sample
Chapter 6

Selection of $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ decays

The complete chain for a $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ decay is

$$\tau^\pm \rightarrow \rho^\pm \nu_\tau \rightarrow \pi^+\pi^0\nu_\tau \rightarrow \pi^+\gamma\nu_\tau$$

The data are subjected to certain requirements, designed to select events where the final-state $\pi^+$ and $\pi^0$ are well separated spatially in the detector. In addition to reducing contamination these requirements allow accurate calculation of the decay angles $\cos\theta^*$ and $\cos\psi$, necessary for polarisation studies.

6.1 Selection Criteria

My requirements, summarised in table 6.1, are designed to select events where the final-state $\pi^+$ and $\pi^0$ are well separated spatially in the detector. Tracks and clusters are examined within a decay cone of half-angle 0.3 rad around the direction of the cone as calculated in preselection. The theoretical cone size for $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ decays is plotted in 6.1a. This is defined by the angular spread from the direction of the $\tau$ decay cone (as given by preselection) of the final-state $\pi^+$ and photons, where the photons are required

<table>
<thead>
<tr>
<th>Variable</th>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cone size</td>
<td>0.3 rad</td>
</tr>
<tr>
<td>Neutral cluster energy threshold</td>
<td>2.0 GeV</td>
</tr>
<tr>
<td>Number of charged tracks in cone</td>
<td>1</td>
</tr>
<tr>
<td>Minimum # of neutral ECAL clusters</td>
<td>1</td>
</tr>
<tr>
<td>Maximum # of neutral ECAL clusters</td>
<td>2</td>
</tr>
<tr>
<td>Minimum cone mass</td>
<td>550 MeV</td>
</tr>
<tr>
<td>Maximum cone mass</td>
<td>950 MeV</td>
</tr>
<tr>
<td>Maximum E/p</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table 6.1: $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ selection requirements
to have at least 2.0 GeV of energy to be consistent with a "neutral cluster threshold" defined below. As can be seen, the cone size used is very conservative.

Tracks are defined to be in the decay cone if their initial momentum vector points within the cone. I require there to be exactly one such good charged track. This greatly reduces background from $\tau$ decay modes with 3 or 5 charged particles (see figure 6.1b).

Events are required to have either one or two neutral ECAL clusters inside the cone, where a neutral cluster is unassociated with the good charged track and must have at least 2.0 GeV energy before corrections are applied. The particular value of the neutral cluster energy threshold was chosen considering several factors. ECAL energy corrections are made assuming that the energy deposited is due to an incident electron. For low energy photons, the corrections are of little use and consequently the measured energy is in doubt. As well, there is evidence of more general problems with the Monte Carlo modelling of low-energy clusters. This can be seen by comparing the distribution of neutral-cluster energy for events with one neutral cluster, at varying threshold values. The $\chi^2$ for this distribution is plotted in figure 6.1c. It is seen to reach a fairly steady low point at approximately 1.6 GeV. Aside from these measurement difficulties, higher threshold values also reduce the background from $\tau^\pm \rightarrow \pi^\pm \nu_\tau$ decays. These very often produce more than one cluster, with secondary clusters typically low in energy (see figures 6.2a,b).

The distribution of the number of neutral clusters for $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ and the main background sources is shown in figure 6.2c.

The four-momenta of the charged track and each neutral cluster are calculated:

$$P_{\text{ctrk}} = ((p_{\text{ctrk}}^2 + m_{\pi^+}^2)^{1/2}, E_{\text{ctrk}})$$

$$E_{\text{ecal}} = (E_{\text{ecal}}, E_{\text{ecal}} n_{\text{ecal}})$$

where $m_{\pi^+}$ is the mass of the $\pi^+$, $p_{\text{ctrk}}$ is the momentum of the charged track, $E_{\text{ecal}}$ the energy of the neutral cluster, and $n_{\text{ecal}}$ the direction vector of the neutral cluster.

The cone mass $M_{\text{cone}}$ is calculated as the magnitude of the sum of all four-momenta in the event

$$M_{\text{cone}} = \sqrt{(\Sigma P_0)^2 - (\Sigma P_1)^2 - (\Sigma P_2)^2 - (\Sigma P_3)^2}$$

I require that this mass be consistent with that of a $\rho$ meson: $0.55 < M_{\text{cone}} < 0.95$ GeV/c². This reduces the background from $\tau^\pm \rightarrow \pi^\pm 2\pi^0 \nu_\tau$ decays and other hadronic modes. The cone mass distribution is shown after all other cuts for data and simulated data in figure 6.3a. The distributions for the major background sources are shown in figure 6.3b-d.

To further reduce the background from $\tau^\pm \rightarrow \pi^\pm (\geq 2)\pi^0 \nu_\tau$ modes, I also require that the energy in the cluster associated with the charged track satisfy $E_{\text{ass}}/P_{\text{ctrk}} < 0.9c$. The aim of this requirement is to remove events where clusters from a $\pi^0$ overlap with that associated with the charged track. The $E/p$ distribution is plotted after all other requirements in figure 6.4a, for data and simulated data. The distributions for the major background sources are shown in figures 6.4b-d.
Figure 6.1: Cone size and neutral cluster requirements
The cone size (a) is from simulated $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ decays, counting the angular separation of the $\pi^\pm$ and photons with more than 2.0 GeV of energy from the cone direction. The distribution of number of good charged tracks (b) is shown after preselection but before $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ selection. Plot (c) shows the $\chi^2$ of the neutral cluster energies for events passing $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ selection, as a function of the neutral cluster threshold energy. The $\chi^2$ is defined as the sum over 20 energy bins of the difference between the number of events given by the data and the number of events predicted from the Monte Carlo data: $\chi^2 = \sum_i (N_i - N_i^{MC})^2 / N_i$. 

55
Figure 6.2: Values used in neutral cluster requirements

Plot (a) shows the number of ECAL clusters not associated with the one good charged track required, and plot (b) shows the total energy in these clusters. From this it is apparent that a high threshold energy for neutral clusters will eliminate most of these pion secondary clusters. In plot (c), the distribution of number of neutral clusters is shown after preselection but before $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ selection. The distributions shown are normalised to the branching fractions in table 4.2.
Figure 6.3: Cone mass

Figure 2. Cone mass, (a) data and Monte Carlo data, and (b-d) most important sources of background. The arrows in plot (a) represent limits required. The Monte Carlo data have been normalised to the same number of preselected $\tau$-pairs as the data. All plots are after all selection requirements other than those on cone mass. In plot (d), distributions from $\tau^\pm \rightarrow e^\pm \nu_e \nu_\tau$, $\tau^\pm \rightarrow \mu^\pm \nu_\mu \nu_\tau$ and $\tau^\pm \rightarrow \pi^\pm \nu_\tau$ are combined. The data are shown as points, Monte Carlo data by the empty histogram, and estimated background by the hatched histogram.
Figure 6.4: $E_{ass}/P_{ctrk}$

Figure 3. $E_{ass}/P_{ctrk}$, (a) data and Monte Carlo data, and (b-d) most important sources of background. The arrow in plot (a) represents the required limit. The Monte Carlo data have been normalised to the same number of preselected $\tau$-pairs as the data. All plots are after all requirements other than that on $E/p$. In plot (d), distributions from $\tau^\pm \to e^\pm \nu_e \nu_\tau$, $\tau^\pm \to \mu^\pm \nu_\mu \nu_\tau$ and $\tau^\pm \to \pi^\pm \nu_\tau$ are combined. The data are shown as points, Monte Carlo data by the empty histogram, and estimated background by the hatched histogram.
<table>
<thead>
<tr>
<th>Background Decay Mode</th>
<th>Fraction of candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau^\pm \rightarrow \pi^\pm 2\pi^0 \nu_\tau )</td>
<td>16.06 ± 1.35</td>
</tr>
<tr>
<td>( \tau^\pm \rightarrow (K^*)^\pm \nu_\tau )</td>
<td>1.54 ± 0.21</td>
</tr>
<tr>
<td>( \tau^\pm \rightarrow (K^\pm , \pi^\pm ) \nu_\tau )</td>
<td>1.32 ± 0.03</td>
</tr>
<tr>
<td>( \tau^\pm \rightarrow \pi^\pm (\geq 3\pi^0) )</td>
<td>1.90 ± 0.71</td>
</tr>
<tr>
<td>( \tau^\pm \rightarrow \mu^\pm \bar{\nu}<em>\mu \nu</em>\tau )</td>
<td>0.46 ± 0.05</td>
</tr>
<tr>
<td>( \tau^\pm \rightarrow e^\pm \nu_e \nu_\tau )</td>
<td>0.50 ± 0.06</td>
</tr>
<tr>
<td>Total</td>
<td>21.78 ± 1.54</td>
</tr>
</tbody>
</table>

Table 6.2: Sources of background in selected data

### 6.2 Summary of Results

After making all these requirements, the selection efficiency for \( \tau^\pm \rightarrow \rho^\pm \nu_\tau \) decays is estimated to be 32.7 ± 0.8%, with contamination 21.8 ± 1.5%. The main sources of background are listed in table 6.2.

As this analysis depends strongly on the quality of the Monte Carlo data, several checks are made on its reliability. There is good agreement on the most basic quantities; the numbers of charged tracks and neutral clusters. This is evident from figures 6.5a and b, where the distributions of the number of charged tracks and the number of neutral clusters are shown respectively, after all other requirements have been applied. To check the cone mass, I study the effect of requiring that the good charged track has been measured in the Z-chambers. If there are systematic problems with the measurement of \( \theta \) in charged tracks, they may be eliminated by this requirement (at the cost of a lower \( \tau^\pm \rightarrow \rho^\pm \nu_\tau \) selection efficiency). No significant difference was seen in the branching fraction or \( \tau \) polarisation (\( \delta B = 2.0 \times 10^{-4}, \delta P_\tau = 0.014 \)).

A variable which provides a useful check on the modelling of ECAL is the angle between the ECAL cluster associated to the charged track and the nearest neutral cluster. One sees reasonable agreement between data and simulated data here, as well (see figure 6.5c).
Figure 6.5: Agreement between data and Monte Carlo data for significant variables

The plots show comparisons of data and Monte Carlo data in (a) number of charged tracks, (b) number of neutral clusters, and (c) the associated-nearest neutral cluster angle. The Monte Carlo data have been normalised to the same number of preselected $\tau$-pairs as the data. Plots (a) and (b) are shown after all requirements other than those on the variables shown, plot (c) after all requirements. The data are shown as points, Monte Carlo data by the empty histogram, and estimated background by the hatched histogram.
Chapter 7
Analysis

From the sample of \( \tau^\pm \rightarrow \rho^\pm \nu_\tau \) decays selected, several measurements are made. The branching fraction is most straightforward and is calculated first. Then the \( \tau \) polarisation analysis is detailed, with results given both for it and the \( \tau \) polarisation asymmetry \( A_{FB}^{Pol} \).

### 7.1 Branching Fraction

The branching fraction is calculated by:

\[
B(\tau^\pm \rightarrow \rho^\pm \nu_\tau) = \frac{N_{\text{Cand}}^\rho (1 - f_{\text{bkg}}^{\text{non}-\tau})}{N_{\text{Cand}}^\tau (1 - f_{\text{bkg}}^{\text{non}-\tau})} \cdot \frac{1}{c^\rho} \cdot F_{\text{bias}}^\rho
\]

(7.1)

where \( N_{\text{Cand}}^\rho \) is the number of \( \tau \) candidates which pass the \( \tau^\pm \rightarrow \rho^\pm \nu_\tau \) selection requirements and \( N_{\text{Cand}}^\tau \) is the number of \( \tau \) candidates observed (twice the number of \( \tau \)-pairs which passed preselection requirements). These two terms refer to the data. The other terms are all estimated from Monte Carlo data.

The two factors \( f_{\text{bkg}}^{\text{non}-\tau} \) and \( f_{\text{bkg}}^{\text{non}-\rho} \) are, respectively, the estimated fraction of events in the preselected data sample arising from processes other than \( \tau \)-pair production, and the estimated fraction of decays in the final data sample which arise from processes other than \( \tau^\pm \rightarrow \rho^\pm \nu_\tau \).

The term \( c^\rho \) represents the efficiency for detecting \( \tau^\pm \rightarrow \rho^\pm \nu_\tau \) decays after preselection. This is calculated as the ratio of the number of decays which pass all requirements over the number which pass all preselection requirements.

The term \( F_{\text{bias}}^\rho \) represents the artificial enhancement of the \( \tau^\pm \rightarrow \rho^\pm \nu_\tau \) signal as a result of applying the preselection requirements. This is the ratio of the relative fraction of \( \tau^\pm \rightarrow \rho^\pm \nu_\tau \) decays after preselection over the relative fraction before preselection.

For this analysis,

- \( N_{\text{Cand}}^\rho = 650 \)
- \( N_{\text{Cand}}^\tau = 6620 \)
- \( f_{\text{bkg}}^{\text{non}-\rho} = 0.218 \pm 0.015(MC\text{stat.}) \pm 0.002(meas.) \)
• $f_{bkg}^{non-\tau} = 0.019 \pm 0.01$
• $\epsilon^p = 0.327 \pm 0.007(MC\,stat.) \pm 0.004(\,conv.)$
• $F_{bias}^p = 1.021 \pm 0.005$

with the result

$$B(\tau \to p \nu_\tau) = 0.234 \pm 0.009(stat.) \pm 0.010(syst.)$$

### 7.1.1 Branching Fraction Uncertainty

Uncertainties in the branching fraction measurement can be divided into three classes. The statistical uncertainty is that arising from the sample size of the data. It is calculated as

$$\delta B = \left| \frac{\partial B}{\partial B_\tau} \right| \delta B_\tau = F_c(B_\tau, N_{\text{Cand}}) \left( \frac{1 - f_{bkg}^{non-\tau}} {1 - f_{bkg}^{non-\tau}} \right) \frac{1}{\epsilon^p} F_{bias}^p$$

(7.2)

where

$$B_\tau = N_{\text{Cand}}^\rho / N_{\text{Cand}}^\tau$$

(7.3)

$$\delta B_\tau = F_c(B_\tau, N_{\text{Cand}}^\tau)$$

(7.4)

and

$$F_c(B_\tau, N) = \sqrt{\frac{R(1-R)}{N}}$$

(7.5)

The function $F_c$ gives the “binomial error,” the statistical uncertainty for any fraction $R$ of $N$ events taken from a binomial distribution. Here, the original distribution is that of all preselected $\tau$-pair events, and the final distribution is that of all selected $\tau$ decays. Inserting the appropriate values (given above), I estimate the statistical uncertainty at $\delta B_{\text{stat}} = 0.87\%$.

Some uncertainties can be easily estimated from the parameters in the branching fraction calculation. The uncertainties $\delta f_{bkg}^{non-\rho}$ and $\delta F_{bias}^p$ are taken from $\tau$ preselection. The former is based on comparisons between data and Monte Carlo data, the latter propagated through from Monte Carlo statistics.

The non-$\rho$ background uncertainty $\delta f_{bkg}^{non-\rho}$ contains a binomial term (labelled “MC stat.” above) and an additional uncertainty arising from the uncertainty in the measured branching fractions for each background decay mode (labelled “meas.” above):

$$(\delta f_{bkg}^{\text{meas}})^2 = \sum_{\text{mode}\neq\rho} (f_{\text{mode}}(\delta B_{\text{mode}})^{\text{meas}})^2$$

(7.6)
where the sum is taken over all modes present in the selected data sample (other than $\tau^\pm \rightarrow \rho^\pm \nu_\tau$), $f_{mode}$ is the fraction of each in the final sample, and $\delta B_{\text{mode}}^{\text{meas}}$ is the measured branching fraction uncertainty for each mode.

Similarly, the efficiency uncertainty $\delta e'$ is given by a binomial term "MC stat." plus an extra relative uncertainty of 2% to allow for inaccuracy due to poor modelling of the rate of conversion of photons in the detector (labelled "conv."). It is estimated that some 10% of $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ decays have such a conversion, with a 20% mismatch in this rate between estimates from data and Monte Carlo data.

These uncertainties are easily propagated through to obtain a total $\delta B$. There are other uncertainties which do not lend themselves to this easy estimation from parameters, though. To estimate these I vary analysis parameters and measure the resulting change in the branching fraction. The spread upwards and downwards from the nominal branching fraction is taken to give the uncertainty in each direction. The final uncertainty quoted is thus asymmetric.

The parameters varied include the neutral cluster threshold, cone size, degree of polarisation in the Monte Carlo data, and all the specific values chosen for $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ selection requirements. The cone size is varied by roughly twice its resolution, $\pm 0.05$ rad (see figure 7.1a). The neutral cluster threshold is varied by $\pm 200$ MeV, also twice its expected resolution. As can be seen in figure 7.1b, the variation range chosen for the neutral cluster threshold is high enough to avoid a divergence in the branching fraction due to the low-energy region of poor neutral cluster modelling mentioned earlier. The actual range chosen and the variation of branching fraction within this range are plotted in figure 7.1c.

The degree of Monte Carlo polarisation is varied by $\pm 0.14$, an estimate of the final uncertainty of the polarisation from this analysis. This is shown in figure 7.1d.

To vary the values used in $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ selection requirements, an estimate is needed of how well these quantities are simulated in the Monte Carlo data. The peak position and width of the mass distribution are seen to differ in the data and Monte Carlo data by no more than $\pm 10$ MeV. From this, I conservatively set the limit on variation of the mass cuts at $\pm 50$ MeV. The results of this variation are shown in figures 7.2a and 7.2b. The $E/p$ requirement is varied by $\pm 0.3$, a factor of roughly three times its resolution, to allow for systematic effects (see figure 7.2c). Although I see no evidence for problems with the $E/p$ distribution here, other OPAL analyses have shown some evidence of modelling problems at low $E/p$ values.

Another parameter that is varied is a requirement on the angle between the associated cluster and nearest neutral cluster. This is ordinarily set to zero but is varied up to roughly one ECAL block width, 0.05 rad, to obtain an uncertainty from any problems with the ECAL modelling of electromagnetic shower shapes. It is expected that such problems would be most evident in the merging of the associated and neutral clusters, and that this parameter is a direct way of testing how well this merging is modelled. The results of variation are plotted in figure 7.2d.

An additional uncertainty is estimated by varying the degree of weighting attached to different decay modes in the Monte Carlo. Due to the "missing mode problem," these weights are somewhat arbitrary. To attach an uncertainty to the weights chosen, I use two
Figure 7.1: Variation of branching fraction with parameter changes
Plot (a) shows the change in the branching fraction as the cone size is varied. Plot (b) shows the divergence of the branching fraction as the neutral cluster threshold is lowered, and plot (c) shows the variance of the branching fraction over the range used to calculate the uncertainty from the neutral cluster threshold. Plot (d) shows the dependence of the branching fraction on the degree of polarisation assumed in the simulated data. The error bars shown represent the possible variation from point to point accounting for correlations.
Figure 7.2: Variation of branching fraction with parameter changes
Plots (a) and (b) show the amount by which the branching fraction varies as the cone mass requirements are changed. Plot (c) shows the dependence of the branching fraction on the $E/p$ requirement. Plot (d) shows the branching fraction dependence on the angle between the associated and nearest neutral clusters, a variable not used to select events but which gives an indication of the reliability of our simulation of electromagnetic shower shapes.
different weighting schemes to maximise and then minimise (within reasonable bounds) the signal-to-noise ratio in the final sample. The first set of weights is put together by varying the $\rho$ branching fraction up by its best-measured uncertainty, 0.006, then scaling up all other modes in proportion to their uncertainties until the sum of all branching fractions is one. The second set of weights is put together by varying the $\rho$ branching fraction down by 0.006, then scaling all other modes up till the sum of all branching fractions is one. The weighted branching fractions used are listed in table 7.1. It is seen that the results using the nominal weighting scheme lie between those using the two extremes (see table 7.2).

As mentioned in Chapter 4, weights are also applied to correct the shape of the $a_1$ resonance in the Monte Carlo sample. To account for uncertainty in the new parameters of the $a_1$, the mass and width are varied separately and the resulting variations summed quadratically to give a total uncertainty estimate (see table 7.3).

All uncertainty estimates for the branching fraction measurement are summarised in table 7.4. The most significant potential sources of error are seen to be those associated with the background, branching fraction weights, and efficiency. The cone mass requirements also contribute significant uncertainty.
<table>
<thead>
<tr>
<th>$M_{a_1}$ (GeV)</th>
<th>$\Gamma_{a_1}$ (GeV)</th>
<th>$B$ (%)</th>
<th>$P_1$</th>
<th>$P_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.251</td>
<td>0.599</td>
<td>23.44</td>
<td>-0.042</td>
<td>-0.169</td>
</tr>
<tr>
<td>1.251</td>
<td>0.500</td>
<td>23.56</td>
<td>-0.054</td>
<td>-0.170</td>
</tr>
<tr>
<td>1.251</td>
<td>0.700</td>
<td>23.33</td>
<td>-0.034</td>
<td>-0.171</td>
</tr>
<tr>
<td>$\delta \Gamma^-$</td>
<td></td>
<td>0.11</td>
<td>0.012</td>
<td>0.002</td>
</tr>
<tr>
<td>$\delta \Gamma^+$</td>
<td></td>
<td>0.12</td>
<td>0.008</td>
<td>0.000</td>
</tr>
<tr>
<td>1.275</td>
<td>0.599</td>
<td>23.54</td>
<td>-0.050</td>
<td>-0.171</td>
</tr>
<tr>
<td>1.226</td>
<td>0.599</td>
<td>23.32</td>
<td>-0.035</td>
<td>-0.171</td>
</tr>
<tr>
<td>$\delta M^-$</td>
<td></td>
<td>0.12</td>
<td>0.008</td>
<td>0.002</td>
</tr>
<tr>
<td>$\delta M^+$</td>
<td></td>
<td>0.10</td>
<td>0.007</td>
<td>0.000</td>
</tr>
<tr>
<td>$\delta^-$</td>
<td></td>
<td>0.16</td>
<td>0.014</td>
<td>0.002</td>
</tr>
<tr>
<td>$\delta^+$</td>
<td></td>
<td>0.16</td>
<td>0.011</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 7.3: Uncertainties arising from $a_1$ weighting

<table>
<thead>
<tr>
<th>Source of Uncertainty</th>
<th>Variation range</th>
<th>$\delta B^-$ (%)</th>
<th>$\delta B^+$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistics</td>
<td>—</td>
<td>0.87</td>
<td>0.87</td>
</tr>
<tr>
<td>Efficiency uncertainty (conversions)</td>
<td>—</td>
<td>0.47</td>
<td>0.47</td>
</tr>
<tr>
<td>Non-rho bkg. uncertainty (measurement)</td>
<td>—</td>
<td>0.46</td>
<td>0.46</td>
</tr>
<tr>
<td>Variation of $M_{cone}$ lower limit</td>
<td>±50 MeV</td>
<td>0.11</td>
<td>0.45</td>
</tr>
<tr>
<td>Variation of $M_{cone}$ upper limit</td>
<td>±50 MeV</td>
<td>0.34</td>
<td>0.02</td>
</tr>
<tr>
<td>Efficiency uncertainty (MC stats)</td>
<td>—</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td>Non-tau background uncertainty</td>
<td>—</td>
<td>0.24</td>
<td>0.24</td>
</tr>
<tr>
<td>Branching fraction weights</td>
<td>see above</td>
<td>0.21</td>
<td>0.18</td>
</tr>
<tr>
<td>E/p variation</td>
<td>±0.30</td>
<td>0.17</td>
<td>0.19</td>
</tr>
<tr>
<td>Monte Carlo polarisation</td>
<td>±0.14</td>
<td>0.16</td>
<td>0.17</td>
</tr>
<tr>
<td>$a_1$ parameter weighting</td>
<td>see above</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>Bias uncertainty</td>
<td>—</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>Variation of neutral cluster threshold</td>
<td>±200 MeV</td>
<td>0.11</td>
<td>0.10</td>
</tr>
<tr>
<td>Ass.-nearest neutral cluster angle</td>
<td>0-0.05 rad</td>
<td>0.04</td>
<td>0.11</td>
</tr>
<tr>
<td>Non-rho bkg. uncertainty (MC stats)</td>
<td>—</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Cone size variation</td>
<td>±0.05 rad</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td>Total systematic</td>
<td></td>
<td>0.92</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Table 7.4: Summary of uncertainties in the branching fraction measurement
7.2 Tau Polarisation

As we saw in Chapter 1, the \( \tau \)-polarisation determines the momentum distribution of the \( \tau \) decay products. To measure the polarisation, the decay angles are calculated and the resulting distributions fit to the theoretical functions above. The angles \( \cos \theta^* \) and \( \cos \psi \) are given by

\[
\cos \theta^* = \frac{2M_{\tau}^2}{M_{\tau}^2 - M_\rho^2} \left( \frac{E_\rho}{E_{beam}} \right) - \frac{M_{\tau}^2 + M_\rho^2}{M_{\tau}^2 - M_\rho^2}
\]

and

\[
\cos \psi = \frac{M_\rho}{(M_\rho^2 - (M_{\pi^+} + M_{\pi^0})^2)^{1/2}} \frac{2E_{\pi^+} - E_\rho}{P_\rho}
\]

where \( E_{beam} \) is the energy of the colliding e\(^-\), with \( E_\tau \approx E_{beam} \).

Data are binned two ways; a two-dimensional distribution in \( \cos \theta^* \) and \( \cos \psi \) (5 \( \times \) 4 bins), and a one-dimensional distribution in \( \cos \theta^* \) alone (10 bins). Corrections are then made to the data on a bin-by-bin basis to recover the actual distributions from those measured. I discuss only the two-dimensional case here. The one-dimensional case is the same in method.

\[
N_{ij}^{corr} = C_{ij}^{Bias} C_{ij}^{ECM} C_{ij}^{Det} \frac{1}{\epsilon_{ij}} (1 - f^{bg}_{ij}) N_{ij}^{meas}
\]

Most of these terms are analogous to those used for the branching fraction calculation, taken here to apply to the numbers of events per bin instead of the sample as a whole. All corrections are calculated from the Monte Carlo sample, using either the actual angles \( \cos \theta^*_a \) and \( \cos \psi_a \) as given by KORALZ or the reconstructed angles \( \cos \theta^* \) and \( \cos \psi_r \), given by the equations above.

The term \( C_{ij}^{ECM} \) represents the change in shape of the distribution resulting from the fact that the Monte Carlo data are produced with only one value for the centre-of-mass energy, whereas the data have been collected at a range of values. To calculate it, KORALZ is run, producing \( \cos \theta^*_a \times \cos \psi_a \) distributions at centre-of-mass energies corresponding to those scanned in 1990. These are normalised and then their sum is taken, weighting each according to the number of events at each centre-of-mass energy in the selected data. KORALZ is run again at 91.16 GeV (the \( Z^0 \) mass used in the Monte Carlo sample), and the new distribution is divided by the weighted sum distribution to obtain a correction factor.

The preselection bias \( C_{ij}^{Bias} \) represents the change in shape of the distribution resulting from preselection; it is the ratio of the \( \cos \theta^*_a \times \cos \psi_a \) distribution before preselection over the distribution after preselection. This calculation is limited to events with \( |\cos \theta_{cone}| < 0.68 \).

The efficiency is obtained analogously to the preselection bias, by taking the ratio of the \( \cos \theta^*_a \times \cos \psi_a \) distribution before \( \tau^+ \rightarrow \rho^\pm \nu_\tau \) selection over the distribution after selection. A notable bias is seen towards low values of \( \cos \theta^*_a \). This is equivalent to a bias
towards low $\rho^\pm$ energies and is an understandable consequence of requiring neutral clusters; the cone size of an event increases as the $\rho$ energy drops, increasing the probability of separate clusters forming.

The factor $G_{ij}^{Det}$ corrects systematic mis-measurements of $\cos \theta^*_\tau$ and $\cos \phi_\tau$ (due to radiative corrections, mass smearing, and possibly other things). The correction is the ratio of the $\cos \theta^*_\tau \times \cos \phi_\tau$ distribution over the $\cos \theta^*_\tau \times \cos \psi_\tau$ distribution.

The background is calculated by taking the ratio of the $\cos \theta^*_\tau \times \cos \phi_\tau$ distribution of non-$\tau^\pm \rightarrow \rho^\pm \nu_\tau$ decays in the final sample over the distribution of all events in the final sample.

The correction factors and data for the one-dimensional fit are shown in figures 7.3-7.5. The correction factors and data for the two-dimensional fit are shown in figures 7.5-7.14.

After all the corrections are made, a $\chi^2$ fitting procedure is used to extract the polarisation. As above, I detail only the two-dimensional case here; the one-dimensional fit is the same in method. The $\chi^2$ is calculated by

$$\chi^2 = \sum_{i,j} \left( \frac{N_{ij}^{Corr} - N_{ij}^{Theory}}{\sigma_{ij}} \right)^2$$

where $\sigma_{ij}$ represents one of two possible uncertainty estimates (detailed below) and

$$N_{ij}^{Theory} = N_{Tot}^{Corr} \int_{(\theta^*_\tau)_{low}}^{(\theta^*_\tau)_{high}} \int_{(\psi_\tau)_{low}}^{(\psi_\tau)_{high}} W(\theta^*_\tau, \psi_\tau) d\theta^*_\tau d\psi_\tau$$

where $W(\theta^*_\tau, \psi_\tau)$ is the theoretical decay distribution given above, the high and low superscripts refer to the limits of each bin in $\theta^*_\tau$ and $\psi_\tau$, and $N_{Tot}^{Corr}$ is the total number of selected decays.

The $\chi^2$ is minimised as a function of the polarisation $P_\tau$, with the statistical uncertainty in the polarisation given by the change in $P_\tau$ (as returned from the fit) for a change of $\pm 1$ in the $\chi^2$. The theoretical distribution, all corrections to the real distribution and $\sigma_{ij}$ are functions of $P_\tau$ and change as it is varied.

For the two-dimensional fit, events in a bin of range $0.6 \leq \cos \theta^*_\tau < 1.0$ are combined with those in the bin of range $0.2 \leq \cos \theta^*_\tau < 0.6$ and the same $\psi$ if the number of events in the former (before corrections) is less than 10. This results in the elimination of two bins, so the fit uses 18 points instead of 20.

Two fits are done, one to obtain the statistical uncertainty and another to obtain both the nominal polarisation and the uncertainty from limited Monte Carlo statistics. For the first fit, I use $\sigma_{ij} = \sigma_{ij}^0$ where $\sigma_{ij}^0$ is defined by

$$\sigma_{ij}^0 = \left( \frac{N_{ij}^{MC} N_{Tot}^{Corr}}{N_{Tot}^{MC}} \right)^{1/2} \chi^2_{ij} \epsilon_{ij}^{Det} C_{ij}^{Bias} C_{ij}^{ECM} C_{ij}^{Det} (1 - f_{ij}^{Bkg})$$

$N_{ij}^{MC}$ is the number of Monte Carlo events in bin $(i,j)$: $N_{Tot}^{MC}$ and $N_{Tot}^{Corr}$ are the numbers of Monte Carlo and real events passing all requirements, respectively. This is an estimate of the statistical uncertainty in each bin.
Figure 7.3: Correction terms for one-dimensional polarisation measurement

The horizontal scale shows the cosine of the decay angle of the $\rho^\pm$ in the $\tau^\pm$ rest frame, $\cos \theta^*$. The preselection bias correction is the inverse of the $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ efficiency loss in $\tau$-pair preselection. The detector effects correction removes the effects of $\cos \theta^*$ smearing due to radiative corrections and systematic problems. The efficiency plotted here is the $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ selection efficiency, and the background is the fraction of non-$\tau^\pm \rightarrow \rho^\pm \nu_\tau$ decays in the final sample. The corrections are applied by first subtracting the background, then dividing by the efficiency. The data are then multiplied by the detector effects and preselection bias corrections.
The vertical scale shows the number of events per bin, as correction factors are applied in sequence to the data. The horizontal scale shows the cosine of the decay angle of the $\rho^\pm$ in the $\tau^\pm$ rest frame, $\cos \theta^\ast$. The raw data plot is extended in range to show the slight degree of smearing beyond $\cos \theta^\ast = \pm 1$. The hatched histogram represents the estimated background, normalised to the number of selected events. The fit shown in the final results plot is the integrated number of events per bin.
Figure 7.5: One- and two-dimensional $E_{cm}$ corrections (2-D, slices in $\cos \psi$)

The horizontal scale shows the cosine of the decay angle of the $\rho^\pm$ in the $\tau^\pm$ rest frame, $\cos \theta^*$. This reflects the difference in polarisation distributions on the $Z^0$ peak and scattered over several values of $E_{cm}$ (as in our data). Error bars are shown but too small to be easily visible. These corrections are the result of a high-statistics KORALZ analysis.
Figure 7.6: Two-dimensional $E_{cm}$ corrections, slices in $\cos \theta^*$

The horizontal scale shows the cosine of the decay angle of the $\pi^\pm$ in the $p^\pm$ rest frame, $\cos \psi$. This reflects the difference in polarisation distributions on the $Z^0$ peak and scattered over several values of $E_{cm}$ (as in our data). Error bars are shown but too small to be easily visible. These corrections are the result of a high-statistics KORALZ analysis.
Figure 7.7: Two-dimensional polarisation corrections, slices in $\cos \psi$

The horizontal scale shows the cosine of the decay angle of the $\rho^\pm$ in the $\tau^\pm$ rest frame, $\cos \theta^*$. The efficiency plotted here is the $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ selection efficiency, and the background is the fraction of non-$\tau^\pm \rightarrow \rho^\pm \nu_\tau$ decays in the final sample. The corrections are applied by first subtracting the background, then dividing by the efficiency. Then the data are multiplied by the detector effects and preselection bias corrections.
Figure 7.8: Two-dimensional polarisation corrections, slices in $\cos \psi$

The horizontal scale shows the cosine of the decay angle of the $\rho^\pm$ in the $\tau^\pm$ rest frame, $\cos \theta^*$. The preselection bias correction is the inverse of the $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ efficiency loss in $\tau$-pair preselection. The detector effects correction removes the effects of $\cos \theta^*$ smearing due to radiative corrections and systematic problems. The corrections are applied by first subtracting the background, then dividing by the efficiency. Then the data are multiplied by the detector effects and preselection bias corrections.
The horizontal scale shows the cosine of the decay angle of the $\pi^\pm$ in the $p^\pm$ rest frame, $\cos \psi$. The efficiency plotted here is the $\tau^\pm \rightarrow p^\pm \nu_e$ selection efficiency, and the background is the fraction of non-$\tau^\pm \rightarrow p^\pm \nu_e$ decays in the final sample. The corrections are applied by first subtracting the background, then dividing by the efficiency. Then the data are multiplied by the detector effects and preselection bias corrections.
Figure 7.10: Two-dimensional polarisation corrections, slices in cos $\theta^*$

The horizontal scale shows the cosine of the decay angle of the $\pi^\pm$ in the $p^\pm$ rest frame, cos $\psi$. The efficiency plotted here is the $\tau^\pm \to p^\pm \nu$ selection efficiency, and the background is the fraction of non-$\tau^\pm \to p^\pm \nu$ decays in the final sample. The corrections are applied by first subtracting the background, then dividing by the efficiency. Then the data are multiplied by the detector effects and preselection bias corrections.
Figure 7.11: Data at varying stages of correction, slices in $\cos \psi$

The vertical scale shows the number of events per bin, as correction factors are applied in sequence to the data. The horizontal scale shows the cosine of the decay angle of the $\rho^\pm$ in the $\tau^\pm$ rest frame, $\cos \theta^*$. The hatched histograms represent the estimated background, normalised to the number of selected events. The raw data plot is extended in range to show the slight degree of smearing beyond $\cos \theta^* = \pm 1$. 

78
Figure 7.12: Data at varying stages of correction, slices in $\cos \psi$

The vertical scale shows the number of events per bin, as correction factors are applied in sequence to the data. The horizontal scale shows the cosine of the decay angle of the $\rho^\pm$ in the $\tau^\pm$ rest frame, $\cos \theta^*$. The fit shown in the final results plot is the integrated number of events per bin.
Figure 7.13: Data at varying stages of correction, slices in $\cos \theta^*$

The vertical scale shows the number of events per bin, as correction factors are applied in sequence to the data. The horizontal scale shows the cosine of the decay angle of the $\pi^\pm$ in the $\rho^\pm$ rest frame, $\cos \psi$. The hatched histograms represent the estimated background, normalised to the number of selected events. The raw data plot is extended in range to show the slight degree of smearing beyond $\cos \theta^* = \pm 1$. 
Figure 7.14: Data at varying stages of correction, slices in $\cos \theta^*$

The vertical scale shows the number of events per bin, as correction factors are applied in sequence to the data. The horizontal scale shows the cosine of the decay angle of the $\pi^\pm$ in the $\rho^\pm$ rest frame, $\cos \psi$. The fit shown in the final results plot is the integrated number of events per bin.
For the second fit I define $\sigma_{ij} = \sigma_{ij}^{Tot}$, where

$$(\sigma_{ij}^{Tot})^2 = (\sigma_{ij}^0)^2 + (\sigma_{ij}^{MC-stat})^2$$  \hspace{1cm} (7.13)$$

The term $\sigma_{ij}^{MC-stat}$ is the binomial error on the Monte Carlo correction factors. This is an estimate of the total uncertainty in each bin from statistics and Monte Carlo statistics.

The uncertainty of $P_\tau$ from limited Monte Carlo statistics is taken as the quadratic difference between the fit uncertainty in the first pass $\delta P_\tau^0$ and that in the second pass $\delta P_\tau$:

$$\delta (P_\tau)_{MC-stat}^2 = (\delta P_\tau)^2 - (\delta P_\tau^0)^2$$  \hspace{1cm} (7.14)$$

There are further uncertainties not easy to estimate on a bin-by-bin basis. The uncertainty from non-$\rho$ background is obtained by varying the amount of each type of background in the final spectrum up and down by the measured error on that background, taking the uncertainty of $P_\tau$ itself to be the resulting change in $P_\tau$.

It is necessary to attach an uncertainty to any systematic problems with calculation of $\cos \theta^\ast$ and $\cos \psi$. Since by far the biggest potential source of such a problem is mis-measurement of the $\rho$ mass, I look for differences between the real and Monte Carlo mass measurements to estimate this uncertainty. Smearing due to inherently limited mass resolution is not at issue here, as this is removed by the detector effects correction $C_{ij}^{Dest}$. I am only interested in differences between data and Monte Carlo data.

It is seen that there could be an additional smearing of up to 1% as well as a possible peak shift of up to $\pm 10$ MeV. To estimate the $\cos \theta^\ast$ and $\cos \psi$ uncertainty I re-run the analysis, adding in separately these systematic effects in the Monte Carlo data. The resulting change in $P_\tau$ taken as an uncertainty estimate.

Other parameters can be varied in the same way as was done for the branching fraction calculation. As can be seen in figures 19a and 19b, the long-range neutral cluster threshold variations that were troublesome for the branching fraction are not evident in the polarisation measurement.

The uncertainties from all these procedures are summarised in table 7.6. The final results from the polarisation measurements are:

- Two-dimensional fit: $P_2 = -0.17 \pm 0.10 \pm 0.08$ ($\chi^2/D = 16.8/17$)
- One-dimensional fit: $P_1 = -0.04 \pm 0.14 \pm 0.13$ ($\chi^2/D = 7.9/9$)

Here $D$ represents the number of degrees of freedom in each procedure. In Chapter 8 these results are analysed in the framework of the Standard Model. The difference in the values returned by each fit is well within the limits of statistical fluctuation. This has been checked by processing samples of Monte Carlo data of roughly the same size as the data set. The results are displayed in table 7.5. The $\chi^2$ in this table is the sum of differences between $(P_{fit})_i$ and $(P_{MC})_i$ over the uncertainty $(\delta P_{fit})_i$. 

82
Table 7.5: Results of simulated analysis with MC data

<table>
<thead>
<tr>
<th>MC events</th>
<th>MC events, sel.</th>
<th>$P_1 \pm \delta P_{fit}$</th>
<th>$P_2 \pm \delta P_{fit}$</th>
<th>$P_1 - P_2$</th>
<th>$P_{MC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12500</td>
<td>667</td>
<td>-0.100 ± 0.143</td>
<td>-0.098 ± 0.114</td>
<td>0.002</td>
<td>-0.166</td>
</tr>
<tr>
<td>12500</td>
<td>675</td>
<td>-0.109 ± 0.137</td>
<td>-0.091 ± 0.110</td>
<td>0.108</td>
<td>-0.189</td>
</tr>
<tr>
<td>12500</td>
<td>673</td>
<td>-0.011 ± 0.142</td>
<td>-0.013 ± 0.114</td>
<td>0.002</td>
<td>-0.155</td>
</tr>
<tr>
<td>12500</td>
<td>674</td>
<td>-0.121 ± 0.137</td>
<td>-0.231 ± 0.107</td>
<td>0.110</td>
<td>-0.119</td>
</tr>
<tr>
<td>12500</td>
<td>666</td>
<td>-0.183 ± 0.132</td>
<td>-0.300 ± 0.105</td>
<td>0.117</td>
<td>-0.163</td>
</tr>
<tr>
<td>12500</td>
<td>640</td>
<td>+0.052 ± 0.133</td>
<td>-0.107 ± 0.109</td>
<td>0.059</td>
<td>-0.100</td>
</tr>
<tr>
<td>12500</td>
<td>650</td>
<td>-0.006 ± 0.138</td>
<td>-0.122 ± 0.109</td>
<td>0.116</td>
<td>-0.135</td>
</tr>
<tr>
<td>12500</td>
<td>695</td>
<td>-0.372 ± 0.138</td>
<td>-0.391 ± 0.100</td>
<td>0.019</td>
<td>-0.194</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>-0.106</td>
<td>-0.169</td>
<td>0.066</td>
<td>-0.153</td>
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<td>Standard deviation</td>
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<td>0.131</td>
<td>0.126</td>
<td>0.052</td>
<td>0.033</td>
</tr>
<tr>
<td>$\chi^2$/degree of freedom</td>
<td></td>
<td>5.45/8</td>
<td>9.40/8</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>100000</td>
<td>5340</td>
<td>-0.116 ± 0.049</td>
<td>-0.160 ± 0.039</td>
<td>0.044</td>
<td>-0.153</td>
</tr>
</tbody>
</table>

7.3 Forward-Backward Polarisation Asymmetry

The forward-backward polarisation asymmetry is obtained by calculating separately the $\tau$ polarisation in the forward ($\cos \theta_+ > 0$) and backward ($\cos \theta_- < 0$) hemispheres, then taking

$$A_{FB}^{pol} = \frac{1}{2}(P_F - P_B) \left( \frac{3 + c^2}{4c} F_{eff} \right)$$

(7.15)

where $c = |\cos \theta|_{\text{max}}$ is the acceptance ($c=0.68$ here), $P_F$ and $P_B$ are the polarisation results from using data only in these hemispheres. The factor $F_{eff}$ is a small correction term arising from integrating the Born cross-section for $\tau$-pair production over $\theta$, taking into account efficiency variations. I calculate $F_{eff} = 0.98 \pm 0.02$.

The measured values of these parameters are

$$P_F = -0.24 \pm 0.15 \pm 0.08$$

$$P_B = -0.09 \pm 0.15 \pm 0.08$$

where the systematic uncertainty of each measurement is taken to be that from the mean polarisation measurement above. The polarisation asymmetry is then

$$A_{FB}^{pol} = -0.09 \pm 0.13 \pm 0.05$$

The statistical uncertainty of the polarisation asymmetry is obtained by propagating through the statistical uncertainties from the individual polarisation measurements. The
### Table 7.6: Summary of uncertainties in the polarisation measurement

<table>
<thead>
<tr>
<th>Source of Uncertainty</th>
<th>(\delta P_{1}^{-})</th>
<th>(\delta P_{1}^{+})</th>
<th>(\delta P_{2}^{-})</th>
<th>(\delta P_{2}^{+})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistics</td>
<td>0.136</td>
<td>0.136</td>
<td>0.104</td>
<td>0.104</td>
</tr>
<tr>
<td>(M_{\text{cone}}) upper limit</td>
<td>0.112</td>
<td>0.002</td>
<td>0.056</td>
<td>0.011</td>
</tr>
<tr>
<td>(M_{\text{cone}}) lower limit</td>
<td>0.014</td>
<td>0.051</td>
<td>0.016</td>
<td>0.048</td>
</tr>
<tr>
<td>MC statistics</td>
<td>0.036</td>
<td>0.036</td>
<td>0.032</td>
<td>0.032</td>
</tr>
<tr>
<td>Ass.-neutral cluster angle</td>
<td>0.001</td>
<td>0.046</td>
<td>0.009</td>
<td>0.029</td>
</tr>
<tr>
<td>(\theta^{*}/\psi) mis-measurement</td>
<td>0.034</td>
<td>0.079</td>
<td>0.024</td>
<td>0.022</td>
</tr>
<tr>
<td>Neutral cluster threshold</td>
<td>0.013</td>
<td>0.032</td>
<td>0.004</td>
<td>0.020</td>
</tr>
<tr>
<td>(E/p) variation</td>
<td>0.022</td>
<td>0.038</td>
<td>0.017</td>
<td>0.015</td>
</tr>
<tr>
<td>Non-(\rho) background measurement</td>
<td>0.042</td>
<td>0.042</td>
<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td>B.F. weights</td>
<td>0.010</td>
<td>0.011</td>
<td>0.007</td>
<td>0.008</td>
</tr>
<tr>
<td>(a_{1}) weights</td>
<td>0.014</td>
<td>0.011</td>
<td>0.002</td>
<td>0.000</td>
</tr>
<tr>
<td>Cone size</td>
<td>0.004</td>
<td>0.000</td>
<td>0.004</td>
<td>0.000</td>
</tr>
<tr>
<td>Total systematic</td>
<td>0.134</td>
<td>0.129</td>
<td>0.075</td>
<td>0.075</td>
</tr>
</tbody>
</table>

Systematic uncertainty is obtained assuming that the uncertainty of the polarisation in each hemisphere is the uncertainty of their difference, plus a contribution from \(\delta F_{\text{eff}}\).
Figure 7.15: Variation of $\tau$ polarisation with parameter changes (1-D fit)

Plot (a) shows the change in the polarisation as the cone size is varied. Plot (b) shows the change in polarisation as the neutral cluster threshold is lowered. The divergence seen in the branching fraction is absent here. Plot (c) shows the variance over the range used to calculate the uncertainty from the neutral cluster threshold.
Figure 7.16: Variation of $\tau$ polarisation with parameter changes (1-D fit)
Plots (a) and (b) show the amount by which the polarisation varies as the cone mass requirements are changed. Plot (c) shows the dependence of the polarisation on the $E/p$ requirement. Plot (d) shows the dependence on the angle between the associated and nearest neutral clusters, a variable not used to select events but which gives an indication of the reliability of Monte Carlo simulation of electromagnetic shower shapes.
Figure 7.17: Variation of $\tau$ polarisation with parameter changes (2-D fit)
Plot (a) shows the change in the polarisation as the cone size is varied. Plot (b) shows the change in polarisation as the neutral cluster threshold is lowered. The divergence seen in the branching fraction is absent here. Plot (c) shows the variance over the range used to calculate the uncertainty from the neutral cluster threshold.
Figure 7.18: Variation of \( \tau \) polarisation with parameter changes (2-D fit)

Plots (a) and (b) show the amount by which the polarisation varies as the cone mass requirements are changed. Plot (c) shows the dependence of the polarisation on the \( E/p \) requirement. Plot (d) shows the dependence on the angle between the associated and nearest neutral clusters, a variable not used to select events but which gives an indication of the reliability of Monte Carlo simulation of electromagnetic shower shapes.
Chapter 8

Summary of Results

8.1 Branching Fraction

The measured $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ branching fraction is

$$B(\tau^\pm \rightarrow \rho^\pm \nu_\tau) = 0.234 \pm 0.009 \text{(stat.)} \pm 0.010 \text{(syst.)}$$

Past measurements are listed in table 8.1. These are combined by treating the systematic uncertainties as if they were statistical in origin, and the total uncertainty for a measurement as if it is associated with normally distributed data. The combined branching fraction is then given by

$$B_{\text{mean}} = \left( \frac{\delta B_{\text{mean}}}{\delta B_i} \right)^2 \sum_i \frac{B_i}{(\delta B_i)^2}$$

and the combined uncertainty

$$\delta B_{\text{mean}} = \left( \sum_i \frac{1}{(\delta B_i)^2} \right)^{-1/2}$$

From the data in table 8.1 we obtain $B_{\text{mean}} = 23.2 \pm 0.5\%$. This is to be compared to $23.1 \pm 0.6\%$, the average without the result presented here. One can also look at the value of $22.7 \pm 0.8\%$ published in 1990 by the Particle Data Group[32]. The individual measurements, new average and old average are plotted in figure 8.1. All are seen to agree well within uncertainty limits.

8.2 Polarisation

For the nominal polarisation, I take the value returned by the two-dimensional fit, and combine the systematic and statistical uncertainties:

$$P_\tau = -0.17 \pm 0.13$$
Published $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ branching fraction measurements, except for that from DASP, which had error bars too large for convenient display. The dotted line indicates the published world average in 1990, from the Particle Data Group’s Review of Particle Properties[32]. The solid line indicates the new average.
Table 8.1: Past $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ branching fraction measurements

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Measurement (%)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARGUS</td>
<td>22.3 ± 1.0</td>
<td>[33]</td>
</tr>
<tr>
<td>ALEPH</td>
<td>24.6 ± 1.1</td>
<td>[34]</td>
</tr>
<tr>
<td>Crystal Ball</td>
<td>21.3 ± 2.1</td>
<td>[35]</td>
</tr>
<tr>
<td>Mark II</td>
<td>25.8 ± 3.0</td>
<td>[36]</td>
</tr>
<tr>
<td>Mark III</td>
<td>23.0 ± 2.1</td>
<td>[37]</td>
</tr>
<tr>
<td>CELLO</td>
<td>22.2 ± 1.7</td>
<td>[38]</td>
</tr>
<tr>
<td>DASP</td>
<td>24.0 ± 9.2</td>
<td>[39]</td>
</tr>
</tbody>
</table>

Other measurements of the $\tau$ polarisation are given in table 8.2. Combining OPAL results, the mean polarisation is found to be

$$P^{\text{OPAL}} = -0.06 \pm 0.07$$

The most accurate measurement can be obtained by combining all results from LEP, listed in table 8.2 and plotted in figure 8.2. As has been remarked earlier, the $\tau$ polarisation is energy-dependent so only measurements at the same energy can be averaged. From all LEP measurements, we obtain the polarisation at $\sqrt{s} = M_{\rho^0}$:

$$P^{\text{LEP}} = -0.12 \pm 0.04$$

8.2.1 Polarisation Asymmetry

Although the polarisation asymmetry analysis proceeds in parallel with that of the polarisation, other LEP experiments have not presented results for this measurement. OPAL measurements[17] are summarised in table 8.3. The combined polarisation asymmetry is $A_{FB}^{\text{pol}} = -0.17 \pm 0.08$.

8.3 Lepton Universality

From Chapter 2, we recall the definition of $\lambda$ and its relation to the effective vector and axial coupling constants for leptons:

$$\lambda_e \equiv \frac{2(\hat{v}_e/\hat{a}_e)}{1 + (\hat{v}_e/\hat{a}_e)^2} = -\frac{4}{3} A_{FB}^{\text{pol}}$$  \hspace{1cm} (8.3)

$$\lambda_{\tau} \equiv \frac{2(\hat{v}_{\tau}/\hat{a}_\tau)}{1 + (\hat{v}_{\tau}/\hat{a}_\tau)^2} = -P_{\tau}$$  \hspace{1cm} (8.4)

These equations can be solved to obtain $\hat{v}_e/\hat{a}_e$ and $\hat{v}_{\tau}/\hat{a}_\tau$. From the $\tau$ polarisation we have $\lambda_{\tau} = 0.17 \pm 0.13$, yielding
<table>
<thead>
<tr>
<th>Experiment</th>
<th>Decay modes</th>
<th>Measurement (%)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>OPAL</td>
<td>$\tau^\pm \rightarrow \rho^\pm \nu_\tau$</td>
<td>$-0.17 \pm 0.13$</td>
<td>[17]</td>
</tr>
<tr>
<td>OPAL</td>
<td>$\tau^\pm \rightarrow e^\pm \bar{\nu}<em>e \nu</em>\tau$</td>
<td>$+0.20 \pm 0.15$</td>
<td></td>
</tr>
<tr>
<td>OPAL</td>
<td>$\tau^\pm \rightarrow \mu^\pm \bar{\nu}<em>\mu \nu</em>\tau$</td>
<td>$-0.17 \pm 0.19$</td>
<td>[17]</td>
</tr>
<tr>
<td>OPAL</td>
<td>$\tau^\pm \rightarrow \pi^\pm \nu_\tau$</td>
<td>$-0.08 \pm 0.12$</td>
<td>[17]</td>
</tr>
<tr>
<td>OPAL</td>
<td>Combined</td>
<td>$-0.06 \pm 0.07$</td>
<td></td>
</tr>
<tr>
<td>ALEPH</td>
<td>$\tau^\pm \rightarrow e^\pm \bar{\nu}<em>e \nu</em>\tau$</td>
<td>$-0.36 \pm 0.18$</td>
<td>[40]</td>
</tr>
<tr>
<td>ALEPH</td>
<td>$\tau^\pm \rightarrow \mu^\pm \bar{\nu}<em>\mu \nu</em>\tau$</td>
<td>$-0.19 \pm 0.14$</td>
<td>[40]</td>
</tr>
<tr>
<td>ALEPH</td>
<td>$\tau^\pm \rightarrow \pi^\pm \nu_\tau$</td>
<td>$-0.13 \pm 0.08$</td>
<td>[40]</td>
</tr>
<tr>
<td>ALEPH</td>
<td>$\tau^\pm \rightarrow \rho^\pm \nu_\tau$</td>
<td>$-0.12 \pm 0.07$</td>
<td>[40]</td>
</tr>
<tr>
<td>ALEPH</td>
<td>$\tau^\pm \rightarrow \pi^\pm 2\pi^0 \nu_\tau$</td>
<td>$-0.15 \pm 0.17$</td>
<td>[40]</td>
</tr>
<tr>
<td>ALEPH</td>
<td>Combined</td>
<td>$-0.15 \pm 0.05$</td>
<td>[40]</td>
</tr>
<tr>
<td>DELPHI</td>
<td>$\tau^\pm \rightarrow e^\pm \bar{\nu}<em>e \nu</em>\tau$</td>
<td>$-0.10 \pm 0.22$</td>
<td>[41]</td>
</tr>
<tr>
<td>DELPHI</td>
<td>$\tau^\pm \rightarrow \mu^\pm \bar{\nu}<em>\mu \nu</em>\tau$</td>
<td>$-0.09 \pm 0.21$</td>
<td>[41]</td>
</tr>
<tr>
<td>DELPHI</td>
<td>$\tau^\pm \rightarrow \pi^\pm \nu_\tau$</td>
<td>$-0.28 \pm 0.13$</td>
<td>[41]</td>
</tr>
<tr>
<td>DELPHI</td>
<td>$\tau^\pm \rightarrow \rho^\pm \nu_\tau$</td>
<td>$-0.17 \pm 0.12$</td>
<td>[41]</td>
</tr>
<tr>
<td>DELPHI</td>
<td>Combined</td>
<td>$-0.18 \pm 0.08$</td>
<td>[41]</td>
</tr>
<tr>
<td>LEP</td>
<td>Combined</td>
<td>$-0.130 \pm 0.034$</td>
<td></td>
</tr>
</tbody>
</table>

Table 8.2: Tau polarisation measurements

<table>
<thead>
<tr>
<th>Mode</th>
<th>$A_{FB}^{P_{\tau}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^\pm \rightarrow e^\pm \bar{\nu}<em>e \nu</em>\tau$</td>
<td>$-0.16 \pm 0.19$</td>
</tr>
<tr>
<td>$\tau^\pm \rightarrow \mu^\pm \bar{\nu}<em>\mu \nu</em>\tau$</td>
<td>$-0.08 \pm 0.22$</td>
</tr>
<tr>
<td>$\tau^\pm \rightarrow \pi^\pm \nu_\tau$</td>
<td>$-0.34 \pm 0.16$</td>
</tr>
<tr>
<td>$\tau^\pm \rightarrow \rho^\pm \nu_\tau$</td>
<td>$-0.09 \pm 0.14$</td>
</tr>
<tr>
<td>Combined</td>
<td>$-0.17 \pm 0.08$</td>
</tr>
</tbody>
</table>

Table 8.3: OPAL polarisation asymmetry measurements
Figure 8.2: Tau polarisation measurements
The solid line indicates the average polarisation.
\[ \frac{\hat{\alpha}_\tau}{\hat{\sigma}_\tau} = 0.09 \pm 0.06 \]

From the polarisation asymmetry we have \( \lambda_e = 0.12 \pm 0.19 \), yielding

\[ \frac{\hat{\alpha}_e}{\hat{\sigma}_e} = 0.06 \pm 0.10 \]

The LEP average polarisation yields \( \lambda_e = 0.130 \pm 0.034 \) and \( \frac{\hat{\alpha}_e}{\hat{\sigma}_e} = 0.065 \pm 0.017 \).

From the OPAL average polarisation asymmetry, we obtain \( \lambda_e = 0.23 \pm 0.11 \), and \( \frac{\hat{\alpha}_e}{\hat{\sigma}_e} = 0.12 \pm 0.06 \). Comparing this with the polarisation results, we see reasonable agreement and conclude there is no evidence here for violation of lepton universality.

Further data would make it possible to refine this comparison more. However, independent numbers have not been made available by the other LEP experiments.

### 8.4 The Weinberg Angle

Given a value of \( \hat{\alpha}/\hat{\sigma} \), \( \sin^2 \theta_W \) is obtained using

\[ \frac{\hat{\alpha}}{\hat{\sigma}} = 1 - 4 \sin^2 \theta_W \quad (8.5) \]

One then calculates \( \sin^2 \theta_W \) using the relation \( \sin^2 \theta_W = 1.013 \sin^2 \theta_W \) (for assumed Higgs boson and top quark masses of 100 GeV each). Using the value of \( \hat{\alpha}/\hat{\sigma} \) derived from the \( \tau \) polarisation, we find

\[ \sin^2 \theta_W = 0.225 \pm 0.015 \]

Using \( \hat{\alpha}/\hat{\sigma} \) from the polarisation asymmetry yields

\[ \sin^2 \theta_W = 0.232 \pm 0.025 \]

These results should be compared to those from other \( \tau \) polarisation measurements, the relative uncertainties of which we have already seen in table 8.2. It is seen that this result compares quite favourably with those from other experiments. Combining all LEP \( \tau \) polarisation measurements yields \( P_e = -0.130 \pm 0.034 \), \( \hat{\alpha}/\hat{\sigma} = 0.065 \pm 0.017 \) and \( \sin^2 \theta_W = 0.2308 \pm 0.0042 \). In comparison, the combined LEP forward-backward asymmetry measurements give \( \sin^2 \theta_W = 0.2307 \pm 0.0014 \).

Also of interest are measurements of \( \sin^2 \theta_W \) obtained by other methods, summarised in table 8.4 and shown in figure 8.3. From the LEP experiments, measurements are also obtained from the forward-backward asymmetries for different fermion types and from a measurement of the mass of the \( Z^0 \).

The most accurate measurements of the weak mixing before LEP were made by examining neutral current "deep inelastic" scattering of \( \mu \) neutrinos off quarks. In this type of analysis the energy distribution of outgoing neutrinos is fit to a theoretical prediction to obtain \( \sin^2 \theta_W \). Another, similar, experiment can be done where elastic scattering of \( \mu \) neutrinos off electrons is studied. In this case, the ratio of reaction rates for incident neutrinos and antineutrinos gives \( \sin^2 \theta_W \).
Table 8.4: Measurements of $\sin^2 \theta_W$ (for $M_{top} = M_{Higgs} = 100$ GeV)

<table>
<thead>
<tr>
<th>Method</th>
<th>Measured $\sin^2 \theta_W$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEP $\tau$ polarisation</td>
<td>$0.2308 \pm 0.0042$</td>
<td></td>
</tr>
<tr>
<td>LEP fwd-bwd asymmetry</td>
<td>$0.2307 \pm 0.0014$</td>
<td>[42]</td>
</tr>
<tr>
<td>$M_W/M_Z$</td>
<td>$0.219 \pm 0.009$</td>
<td>[32]</td>
</tr>
<tr>
<td>Deep inelastic $\nu$ scattering</td>
<td>$0.233 \pm 0.006$</td>
<td>[32]</td>
</tr>
<tr>
<td>Elastic $\nu e$ scattering</td>
<td>$0.222 \pm 0.011$</td>
<td>[32]</td>
</tr>
<tr>
<td>Elastic $\nu p$ scattering</td>
<td>$0.207 \pm 0.032$</td>
<td>[32]</td>
</tr>
<tr>
<td>Inelastic $eN$ scattering</td>
<td>$0.217 \pm 0.020$</td>
<td>[32]</td>
</tr>
<tr>
<td>Atomic parity violation</td>
<td>$0.215 \pm 0.018$</td>
<td>[32]</td>
</tr>
<tr>
<td>Combined</td>
<td>$0.2303 \pm 0.0013$</td>
<td></td>
</tr>
</tbody>
</table>

Other variations on the scattering method include elastic scattering of neutrinos off protons rather than electrons, and inelastic scattering of electrons off nuclear targets. In electron scattering, the weak mixing is retrieved by studying the asymmetry in the reaction cross sections for left- and right- handed electrons, a method inherently similar to the analysis of $\tau$ polarisation.

The weak mixing can also be extracted by studying parity violation in atomic systems. Here the Coulomb potential is modified by $Z^0$ exchange, causing “forbidden” atomic transitions which can be measured.

Combining all results, we obtain the best measurement of the electroweak mixing, $\sin^2 \theta_W = 0.2303 \pm 0.0013$. This is to be compared with the electromagnetic coupling $\alpha = 7.2974 \times 10^{-3}$ (known to 0.045 parts per million[32]) and the Fermi constant $G_F = 1.1664 \times 10^{-5}$ GeV$^{-2}$ (known to 17 parts per million[32]). As these are the fundamental parameters of the Standard Model, it is clear that the precision of Standard Model predictions is still limited by the accuracy of $\sin^2 \theta_W$.

Combining $\sin^2 \theta_W$ with the mass of the $Z^0$ ($91.175 \pm 0.021$ GeV, also from LEP[42]) the mass of the $W^\pm$ boson is estimated to be $80.01 \pm 0.32$ GeV. This is consistent with direct measurement. The mass of the top quark is also loosely constrained; including other results, one finds $m_{top} = 132 \pm 35$ GeV[42], but no tight limit can be placed on the mass of the Higgs boson yet. With further work the accuracy of $\sin^2 \theta_W$ can be improved, giving better mass limits. The consistency of the Standard Model can then be checked by direct measurement of these masses.
Figure 8.3: $\sin^2 \theta_W$ measurements
The solid line indicates the average $\sin^2 \theta_W$. 
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Appendix A

Derivation of Polarisation Angle Equations

The energy of the $\rho$ in the $\tau$ decay frame is given by

$$E_{\rho}^c = \frac{m_{\tau}^2 + m_{\rho}^2}{2m_{\tau}}$$  \hspace{1cm} (A.1)

and its momentum by

$$P_{\rho}^c = \frac{m_{\tau}^2 - m_{\rho}^2}{2m_{\tau}}$$  \hspace{1cm} (A.2)

If $\theta^*$ is the decay angle of the $\rho$ in the $\tau$ rest frame, then the component of the $\rho$ momentum parallel to the $\tau$ line of motion is $P_{\parallel} = P_{\rho}^c \cos \theta^*$.

Boosting to the lab frame, we have

$$E_{\rho} = \gamma_{\tau}(E_{\rho}^c + \beta_{\tau}P_{\rho}^c \cos \theta^*)$$  \hspace{1cm} (A.3)

with

$$\gamma_{\tau} = \frac{E_{\tau}}{m_{\tau}}$$  \hspace{1cm} (A.4)

Since $E_{\tau} \simeq M_{\tau}/2$, we may safely approximate $\beta_{\tau} = 1$ and substitute the expressions for $E_{\rho}^c$ and $P_{\rho}^c$, yielding

$$E_{\rho} = \frac{E_{\tau}}{2m_{\tau}^2}(m_{\tau}^2 + m_{\rho}^2 + (m_{\tau}^2 - m_{\rho}^2) \cos \theta^*)$$  \hspace{1cm} (A.5)

Defining $X = E_{\rho}/E_{\tau}$ we re-arrange to give $\cos \theta^*$

$$\cos \theta^* = \frac{2m_{\tau}^2}{m_{\tau}^2 - m_{\rho}^2} X - \frac{m_{\tau}^2 + m_{\rho}^2}{m_{\tau}^2 - m_{\rho}^2}$$  \hspace{1cm} (A.6)

Derivation of the second decay angle $\cos \psi$ proceeds similarly. In the $\rho$ decay frame the energy of the charged pion is given by
\[ E_{\pi^+}^c = \frac{m_\rho^2 + m_{\pi^+}^2 - m_{\pi^0}^2}{2m_\rho} \]  

and its momentum by

\[ P_{\pi^+}^c = \left[ \frac{\left( m_\rho^2 - (m_{\pi^+}^2 + m_{\pi^0}^2)(m_{\pi^+}^2 - (m_{\pi^+}^2 - m_{\pi^0}^2) \right)}{4m_\rho^2} \right]^{\frac{1}{2}} \]  

Since the masses of the pions are small compared to that of the \( \rho \) we can approximate \( m_{\pi^+} = m_{\pi^0} \equiv m_\pi \). Then these equations reduce to

\[ E_{\pi^+}^c \approx \frac{m_\rho}{2} \]  

\[ P_{\pi^+}^c = \frac{1}{2} \sqrt{m_\rho^2 - 4m_\pi^2} \]  

We then boost to the lab frame again

\[ E_\pi = \gamma_\rho (E_{\pi^+}^c + \beta_\rho P_{\pi^+}^c \cos \psi) \]  

Here we can not safely assume \( \beta_\rho \approx 1 \), unlike the case of the \( \tau \) decay.

Inserting values for \( E_{\pi^+}^c, P_{\pi^+}^c \) and \( \gamma_\rho = E_\rho/m_\rho \), we find

\[ E_\pi = \frac{E_\rho}{2m_\rho} (m_\rho + \beta_\rho \sqrt{m_\rho^2 - 4m_\pi^2} \cos \psi) \]  

Re-arranging yields

\[ \cos \psi = \frac{m_\rho}{\beta_\rho \sqrt{m_\rho^2 - 4m_\pi^2}} \left( \frac{2E_\pi - E_\rho}{E_\rho} \right) \]  

or

\[ \cos \psi = \frac{m_\rho}{\sqrt{m_\rho^2 - 4m_\pi^2}} \left( \frac{2E_\pi - E_\rho}{P_\rho} \right) \]