ASPECTS OF ENERGY TRANSPORT
IN A VORTEX STABILIZED ARC

by

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Abstract

Vortex stabilized argon arcs are of interest as sources of high intensity light. Previous workers in the field have found that the heat transported to the wall of the arc vessel greatly exceeded that predicted by theory. Two modifications to the theory have been proposed to account for the observed values of heat transport.

This thesis describes a specialized arc vessel which has been constructed to allow the measurement of the axial profile of heat transported to the wall. The axial profile of radiation produced by the arc has also been measured. These experiments elucidate the mechanisms responsible for the large values of heat transported to the wall. In this work a 225 A d.c. arc was used. It was stabilized by an argon vortex at a pressure of 5.5 atm.

It is found that the electrode regions of the arc contribute significantly to the total heat transported to the wall. Midway between the electrodes however, the profile is found to be quite flat. The experiments also indicate that some heat is transported upstream from the arc, and it has been shown that this is due to a reverse axial flow core in the gas vortex. The measured profile of radiation produced by the arc is found to be very uniform in the arc column. In the region midway between the electrodes the dependence of the radiation and heat transported to the wall on the gas flow rate in the vortex were examined. The measured radiation is found to be 30-35% less than predicted by theory. It is shown that this may be due to the axial transport of energy in the arc. The scaling of the amount of heat transported to the wall is found to be in good agreement with the predictions of a model for the arc which includes turbulent heat transport by using a mixing length model. The d.c. power supply used in this work produces a waveform with considerable ripple. Time dependent measurements were therefore made of the radiation produced by the arc and the electrical power input to the arc column. These results are compared with a time dependent theory of the arc column.
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CHAPTER 1

Introduction

The electric arc has been known since 1808 when Davey and Ritter first demonstrated a free burning arc discharge. It was not until the 1930's however, that physicists began to use the arc extensively as an experimental source for the study of plasma physics. Since then, in various forms, it has served as a source of plasma for experiments in spectroscopy and for the measurement of transport properties of high temperature gases. More recently, interest has grown in a number of applications of high current arcs including gas heaters, light sources and materials processing. While a complete theoretical description of the arc is available in certain specific instances, this is not generally the case. In the field of applications for instance, the operating conditions of the arc are dictated by practical considerations and in this field our understanding of the arc is incomplete. In particular, there continues to be interest in the study of the interaction of the arc with gas flow fields and with magnetic fields. This thesis concerns itself with the interaction between the arc and the gas flow in a high current arc lamp.

A strict definition of an arc is difficult since the term has been applied to a broad range of electrical discharges. Nevertheless it is possible to describe a number of criteria, on which most authors agree, that may be used to classify a particular discharge as an arc.

The feature which is most characteristic of an arc is the high current density in the cathode spot. Unlike a glow discharge, which has a broad cathode attachment region, the cathode attachment in an arc is constricted to a small spot. In this spot the current densities may be as high as $10^4 \text{A/cm}^2$ (Edels (1973)). While
the current densities throughout the bulk of the arc column are lower, typically \( \sim 100 \text{ A/cm}^2 \), they are still much higher than the current densities of \( 1-10 \text{ mA/cm}^2 \) which are characteristic of glow discharges.

Another feature of the cathode region which is characteristic of an arc is demonstrated in Figure 1.1, which shows the distribution of potential in an arc column. In the arc, the cathode fall voltage, from the cathode to the positive column of the arc, is of the order of the ionization potential of the gas, rather than something greater than 100 V as in the case of the glow discharge. This is a consequence of the more efficient mechanism of electron production which proceeds by a combination of thermionic and field emission in the cathode spot of the arc.

![Figure 1.1 Potential distribution in the arc column.](image)

Because of its high temperature the arc is convectively unstable; a free burning arc tends to bow upwards in an "arc" shaped discharge. Stabilization of the discharge has been effected through a number of means.
Short arcs may be stabilized by the electrodes. For many practical applications however, a longer length of uniform arc column is often desired and the arc must be stabilized by some other means.

The wall stabilized arc operates by confining the arc in a narrow tube whose walls are strongly cooled. An excursion of the arc towards the wall results in a cooling of the fringe of the arc and reduced electrical conductivity there. This forces the arc back to the centre of the tube. In order to dissipate the large heat flux into the wall, the arc vessel is usually constructed of water cooled copper rings, separated by insulators (Maecker (1956), Maecker (1960)). The wall stabilized arc has been used extensively for spectroscopic studies (Olsen (1963), Krey and Morris (1970), Bacri et al. (1972)) and measurements of the transport properties of gases in the temperature range 7000 K to 20000 K (Emmons and Land (1962), Emmons (1967), Bues et al. (1967), Bauder and Maecker (1971), Kopainsky (1971)). In the 1960's it was also studied extensively with a view to its application as a gas heater to produce high enthalpy gas flows for aerospace research (Stine and Watson (1964), Watson and Pegot (1967)). For a review of the field of gas heater design and technology the interested reader is referred to the article by Pfender (1978).

More recently, interest has developed in the use the arc as a high intensity light source (Anderson et al. (1965), Malliaris et al. (1970), Tam and Gibbs (1972), Nodwell and Camm (1975), and Jaax and Mentel (1983)). Since the opaque wall of the wall stabilized arc is incompatible with this application, arc lamps are vortex stabilized. In this scheme a strong gas vortex is generated in the arc vessel as is shown in Fig. 1.2. The hot conducting core of the arc, being less dense than the cooler surrounding gas, is centralized on the vessel axis by the centripetal field. In the arc used in this thesis, the vortex typically provides a centripetal acceleration of approximately $1 \times 10^4 \text{ ms}^{-2}$ at a radius half way between the centreline and the wall of the arc vessel. Since the gas flow takes care of the stabilization, the arc
vessel may now have a larger radius, and thus the heat flux into the wall is reduced. A water cooled quartz wall is then appropriate for moderate power arc lamps. For high power arc lamps however, the radiation output is in excess of 45 kW and the cooling must be supplied by a water film on the inside of the quartz tube. This film is injected with a large spin so that it sticks centrifugally to the inside of the quartz tube. This is the principle behind the high power arc lamps developed by Vortek Industries Ltd. (Nodwell and Camm (1975)).

Figure 1.2 The vortex stabilized arc. The gas vortex serves to stabilize the arc on the vessel axis.

The principle of vortex stabilization of arcs was first reported by Schoenherr (1908), although it was not until the 1960's that the physics of this arc began to be investigated in connection with gas heaters (Andrada and Erfurth (1963), Bez (1963), Marlotte et al. (1968)and Cann (1973)). In its subsequent applications as a light source however, the vortex stabilized arc was operated under conditions
that were significantly different from those used in gas heater arcs. In order to increase the radiation output of the arc over that of gas heaters, the pressure was raised to several atmospheres. In contrast to gas heater design, where radiation transport in the arc may be neglected, the radiation transport in an arc used as a light source may be as high as 30-40% of the input electrical power. Furthermore, since heating large amounts of gas is no longer required, the vortex stabilized arc lamp is characterized by much lower gas flow rates than those used in arc heaters.

Recent studies in this lab have concentrated on high pressure argon arcs of moderate power (15-45 kW). The work has focused on the interaction between the arc and the gas flow and on ways of improving the radiative efficiency and electrode lifetime (Gettel (1980), Neilson (1981), Gettel and Curzon (1982)). The gas flow pattern used in these arcs differs from that used in most of the other arc lamp work which has been reported in the literature. In most arc lamps, the gas exits the arc chamber through a hole in the centre of the downstream electrode (usually the anode). In such a configuration there is inherently a strong radial gas flow which affects the radial transport of energy in the arc. In fact, in a case such as that of Jaax and Mentel (1983), the radial transport of heat by convection becomes so important that heat conduction is not considered in the arc model. In contrast, the arcs investigated in this lab operate in a gas vortex which exits the arc chamber through holes located outside of, and downstream from, the the downstream electrode. The use of this configuration has been motivated by the fact that the Vortek arc lamp has proven it to be one of considerable practical interest. In this case there is no inherent radial convection in the arc chamber, and heat conduction becomes much more important in the physics of the arc. In addition, these arcs have been run at much lower gas flow rates than most of those previously reported. This means that convection in the axial direction and the loss of heat to the incoming cold gas will not be as important in these arcs.
Gettel performed a comparative study of a.c. and d.c. vortex stabilized arcs, using calorimetry to study the properties of the arc itself. Neilson's work on a d.c. vortex stabilized arc included calorimetric measurements, but also used spectroscopy to measure the temperature profile of the arc. In both cases the experiments found that a larger amount of heat was transported from the arc to the vessel wall than was predicted using equilibrium arc theory. Two different mechanisms were put forward to explain the disagreement. Gettel suggested that turbulence in the arc and the surrounding cold gas would lead to enhanced radial transport of heat. Neilson proposed a disc model of axial convection which could account for increased overall heat transport to the wall, without invoking turbulent heat transport.

If one wishes to predict the effects of the gas flow on the characteristics of the arc, especially the radiative efficiency, it is important to be able to distinguish whether one or the other, (or both) of these mechanisms is responsible for the observed thermal wall loading. In this work, the thermal wall loading is defined to be the heat absorbed by the wall of the arc vessel due to conduction and convection, but excluding radiation. A good deal of knowledge about the arc could be obtained if we could measure the distribution of thermal wall loading and radiation as a function of position along the length of the arc vessel. Measurements made near the upstream end of the arc would allow a comparison with the disc model of axial convection, which predicts a delayed onset of wall loading since the gas is cold when it enters the arc. Further downstream the flowing gas should come into equilibrium with the arc and an axially invariant region should exist. Measurements of the local properties of the arc in this region would allow a comparison with equilibrium arc theory. Since the theories provide an indication of how the profile should depend on the gas flow rate, it would be useful to examine the dependence of the arc parameters in the various regions of the arc on the gas flow conditions.
The work described in this thesis had therefore, as its objective, the measurement of the axial profile of thermal wall loading and radiation in a high pressure, argon vortex stabilized, d.c. arc. As well, it was desired to examine the dependence of these profiles on the gas flow conditions in the arc. These results should enable one to determine which are the important mechanisms contributing to energy transport in the arc. Ultimately, the objective is to try to improve the performance of high power arc lamps through increased radiative efficiency, and increased component lifetime.

In Chapter 2 the equilibrium theory of the arc column is outlined, and the numerical solution of the resulting equation is described. Modifications to the model, to include the effects of the disc model of axial convection and the centrifugal field, are also described. Chapter 3 contains a description of the vortex stabilized arc used in this work, and discusses the gas, water cooling and electrical systems. In Chapter 4 a description is given of the specialized apparatus used in this work, including the partitioned tube arc vessel, used to measure the wall loading profile, and the apparatus for measuring the radiation profile. The calibration of the partitioned tube and several checks of the experimental results are also outlined. The results of the axial profile experiments are presented in Chapter 5. It is shown that there is a rapid onset of wall loading adjacent to the upstream electrode and that the arc contains an axially invariant region. In this axially invariant region, the thermal wall loading is found to scale with gas flow rate as would be expected if turbulent heat transport were important. The measured radiation, on the other hand, is lower than predicted by theory. Since there was considerable ripple in the output voltage of the power supply, it was suspected that time dependent effects might be responsible for this discrepancy. Experiments were therefore undertaken to measure the time dependent behaviour of the arc in the axially invariant region. These experiments, which are outlined in Chapter 6, involved the measurement of the radiation output and the electrical power input for the arc, resolved in time
over a cycle of the power supply waveform. In order to make a valid comparison with the experiments, it was necessary to solve the time dependent equation for the arc, subject to a driving function corresponding to the ripple of the power supply. Chapter 7 discusses this time dependent model of the arc and compares the results with the experimental measurements. In Chapter 8 the results of the work are summarized, and the conclusions drawn from the work are outlined. This chapter also contains a statement of the original contributions of this work to the field and suggestion for further work.

Appendix A contains a description of measurements of the gas flow direction and velocity in a cold gas vortex. An attempt to make this measurement using the technique of double exposure speckle photography is also described. Since the aerosol used in the speckle photography work constitutes a potential safety hazard, the safety precautions taken these experiments are also outlined in this appendix.
CHAPTER 2

Equilibrium Theory of the Arc Column

2.1 Introduction

We begin by treating the theory for an arc which is unaffected by the flow of gas in the stabilizing vortex. The effect of axial convection on the arc is then considered, using the disc model suggested by Neilson (1981). Finally an estimation is made of the influence of the azimuthal velocity of the vortex on the arc. A discussion of enhanced radial transport due to turbulence is left to Chapter 5.

If the arc is sufficiently long there will be a significant portion of the column which is unaffected by the presence of the electrodes. This part is termed the positive column (for historical reasons only). In high pressure arcs the effect of the electrodes is confined to a small area near the electrodes, and the behaviour of the arc is dominated by the positive column. In the theory of the arc we therefore consider only the positive column, and neglect any effect of the electrodes.

For arcs such as the one used in this work the effect of the self magnetic field may be neglected in the positive column. The effect of the magnetic field is to induce a radial pressure gradient in the arc due to the pinch effect. The magnitude of this effect may be estimated by calculating the pressure difference ($\Delta p$) which would exist across an isothermal arc column (see for example Lowke (1979))

$$\Delta p = \frac{\mu I^2}{4\pi A}$$

(2.1)
where:

\[ \begin{align*}
    I &= \text{current in the arc} \\
    A &= \text{cross sectional area of the arc} \\
    \mu &= \text{the magnetic permeability} = \mu_0
\end{align*} \]

For parameters typical of the arcs used in this work (I=225 A, A=1 \times 10^{-4} \text{ m}^2) the pressure difference across the arc column would be \( \sim 5 \times 10^{-4} \text{ atm} \). Since this pressure gradient is small compared to the gas dynamic pressure gradients in the arc its effect is neglected in this work. It is further assumed that the positive column is in local thermodynamic equilibrium (LTE). This means that the plasma may be treated as a single fluid with equal temperatures for the electron and heavy particle kinetic temperatures, excitation temperature and ionization temperature (all equal to temperature \( T \)). It is generally accepted on the basis of experiments (Bauder (1968), Bober and Tankin (1970), Farmer and Haddad (1984)) that in argon arcs, at pressures above 3 atm and temperatures below 12000 K, the assumption of LTE is valid.

2.2 The Elenbaas-Heller Equation

When the positive column of the arc is unaffected by the stabilizing gas flow or the presence of the electrodes it will be axially invariant. Since the arc is stabilized by a strong vortex it is also reasonable to assume that the arc is radially symmetric. The arc may then be described in cylindrical coordinates as a function of radius \( r \) only.

For steady state equilibrium the governing equation is the conservation of energy equation which may be expressed in power balance form as

\[ \sigma(T)E^2 = Q_R(T) - \frac{1}{r} \frac{d}{dr} \left( r\kappa(T) \frac{dT}{dr} \right). \quad (2.2) \]
where:

\[ r = \text{radius} \]
\[ T = T(r) = \text{temperature} \]
\[ \kappa(T) = \text{thermal conductivity} \]
\[ Q_R(T) = \text{radiative exitance (optically thin radiation leaving the arc per unit volume)} \]
\[ \sigma(T) = \text{electrical conductivity} \]
\[ E = \text{electric field in the column} \]

The term on the left hand side represents the ohmic heating, while the two terms on the right hand side are respectively the radiation loss and the thermal conduction loss. This equation is a modified form of the Elenbaas-Heller equation. Strictly speaking, the Elenbaas-Heller equation expresses the power balance condition for an arc where radiation transport may be neglected. For convenience throughout the thesis however, the modified form of the Elenbaas-Heller equation will be referred to simply as the Elenbaas-Heller equation. Integrating equation (2.2) once with respect to \( r \) gives

\[
\frac{dT}{dr} = \frac{1}{r\kappa(T)} \int_0^r \left( Q_R(T) - \sigma(T)E^2 \right) dr'.
\] (2.3)

This equation may be integrated to give \( T(r) \) once the material functions \( \sigma(T), \kappa(T) \) and \( Q_R(T) \) are known. Unfortunately the highly nonlinear form of these functions precludes, in general, the analytic evaluation of the integral. One possible recourse lies in the use of a channel model of the arc. In this case it is assumed that the arc runs in a hot, isothermal arc channel, surrounded by cold gas. The electrical conductivity and the radiation produced by the arc are assumed to be zero outside the arc channel. For a full review of such models the interested reader is referred to the text by Hoyaux (1968). Alternatively the solution may be obtained by use of numerical methods. In the case of this work, the solution of the Elenbaas-Heller equation has been obtained by numerically integrating equation (2.3), subject to appropriate boundary conditions.
2.3 The Transport Properties

The transport properties needed to integrate the Elenbaas-Heller equation were obtained from the experimentally measured properties of argon found in the literature. In most cases the data were obtained from wall stabilized arc experiments.

The transport properties are functions of both temperature and pressure. Both $\kappa$ and $\sigma$ however, depend only very weakly on the pressure. The radiative exitance ($Q_R$), on the other hand, is proportional to the pressure. Since all the experiments in this work were carried out at an arc chamber pressure of 5.5 atm the transport coefficients need only be determined for this one pressure.

The electrical conductivity was assumed to be zero below 5000 K. Above this temperature it has been modelled by means of an empirical scaling law which is commonly found in the literature (Devoto and Mukherjee (1971), Poisel (1983))

$$\sigma = AT^{-B} \exp\left(\frac{-C}{T}\right).$$  \hfill (2.4)

Since the electrical conductivity is a weak function of pressure (Bauder et al (1973), Kopainsky (1971), Poisel (1983)) experimental values for the conductivity at 5.0 atm were used to predict the operating characteristics of the arc at 5.5 atm. In this case we find:

$$A = 1.90 \times 10^{14} \, \Omega^{-1} \, \text{m}^{-1} \, \text{K}^{2.03}$$
$$B = 2.03$$
$$C = 6.38 \times 10^4 \, \text{K}.$$  

The fit of this model to several experimental measurements of electrical conductivity is shown in Fig. (2.1).

In the case of the thermal conductivity consideration must be given to the effects of energy transport by trapped radiation. The radiation produced in the
Figure 2.1 Model of electrical conductivity. The model is compared to several sources of experimental data which were obtained at a pressure of 5 atm.
arc column includes argon resonance radiation which falls in the far u.v.. At 5 atm pressure this resonance radiation is strongly trapped in the arc with a short mean free path, so that it escapes by ordinary diffusion of radiation. The energy transported in the u.v. thus behaves like energy transported by thermal conduction. In this work therefore, experimental values of thermal conductivity have been used which include the effects of thermal conduction and the transport of heat by optically thick u.v. radiation. It has been assumed that the radiation produced in the arc is either optically thick or optically thin, and therefore there is no radiation for which long range diffusion must be considered. Kopainsky (1971) has studied the problem of radiation transport in argon in a wall stabilized arc and finds that at 5 atm long range diffusion of radiation is not significant below 19000 K. Evans and Tankin (1967) measured the radiation output of a 10 mm diameter argon arc at 1 atm pressure and found that the correction for self absorption was less than 5% at temperatures below 12000 K. Since the centreline temperature in vortex stabilized arcs is typically less than 12000 K (Neilson (1981)) this assumption should be valid.

The dependence of thermal conductivity on temperature may be divided into two separate temperature regimes. Below 6200 K the shock tube data of Collins and Menard (1966) were used. They found that their data obeyed a power law of the form

\[ \kappa = \kappa_w \left( \frac{T}{T_w} \right)^{0.703}, \]

where:

\[ T_w = 300 \text{ K} \]
\[ \kappa_w = 0.0178 \text{ W m}^{-1} \text{ K}^{-1}. \]

It was assumed that \( \kappa \) is independent of pressure.

At high temperatures the wall stabilized arc data of Kopainsky (1971) at 5 atm have been used. They have been fitted with an exponential over the range
6200 K ≤ T ≤ 12000 K.

\[ \kappa = A \exp\left( -\frac{T}{T_0} \right) \]  

(2.6)

where:

- \( A = 0.0155 \text{ W m}^{-1} \text{K}^{-1} \)
- \( T_0 = 2740 \text{ K} \)

The models for thermal conductivity are shown along with the experimental data in Fig. 2.2.

In accordance with the preceding discussion on optically thick radiation the radiative exitance \((Q_R)\) is comprised of the optically thin radiation which escapes the arc column. The radiative exitance at 5.5 atm was calculated from Kopainsky’s experimental data at 5 atm. In order to outline how this was done, the scaling of the radiative exitance with pressure is discussed briefly below.

The arc radiation is made up of line radiation, free-bound radiation and free-free radiation. Together, the free-bound and free-free radiation make up the continuum radiation produced by the arc. The total radiation source strength is the sum of the radiation from all three processes,

\[ u_t = u_l + u_c, \]  

(2.7)

where:

- \( u_t = \text{total radiation source strength (power per unit volume)} \)
- \( u_l = \text{line radiation source strength (power per unit volume)} \)
- \( u_c = \text{continuum source strength (power per unit volume)} \)

If one defines a line factor

\[ L = \frac{u_l}{u_c}, \]  

(2.8)
Figure 2.2 Thermal conductivity model. The model is compared with Kopainsky's experimental data which were obtained at 5 atm and with other low temperature data which were obtained at 1 atm.
then the total radiation source strength may be expressed as

\[ u_t = L \times u_c. \]  

(2.9)

In order to determine the pressure dependence of \( u_t \) at constant temperature, the pressure dependence of \( u_c \) must be determined. The continuum source strength is proportional to the product of the electron number density and the ion number density, \( (n_e n_i) \), since both the free-bound and free-free radiation source strengths are proportional to \( n_e n_i \). It then remains to relate \( n_e n_i \) to the pressure \( (p) \). This may be done by using the Saha equation and the ideal gas equation of state. The Saha equation relates the number density of electrons \( (n_e) \), ions \( (n_i) \) and atoms \( (n_a) \) in a plasma which is in local thermodynamic equilibrium. If only singly ionized atoms are present (as is the case for argon over the temperature range of interest in this work) the Saha equation may be written as

\[ n_e n_i = n_a \frac{2Z_i (2\pi m_e kT)^{3/2}}{h^3} \exp(-E_i/kT), \]  

(2.10)

where:

- \( T \) = the temperature,
- \( E_i \) = the ionization potential,
- \( Z_i \) = the partition function for the ion,
- \( Z_a \) = the partition function for the atom,
- \( m_e \) = the mass of the electron,
- \( k \) = Boltzmann's constant,
- \( h \) = Planck's constant.

Thus the continuum source strength may be expressed in terms of the atom number density as

\[ u_c = n_a \times \text{function}(T). \]  

(2.11)
Since the arc has only a small degree of ionization, the ideal gas equation of state,

\[ p = (n_e + n_i + n_a)kT, \]  

may be re-expressed as

\[ p = n_a kT. \]  

Finally, using equations (2.9), (2.11) and (2.13) the total radiation source strength may be expressed as

\[ u_t = L \times p \times \text{function}(T). \]  

Bauder (1968) has shown that in argon, for pressures less than 100 atm, the line factor \( L \) is independent of pressure. Thus, the dependence of radiation source strength on temperature may be determined at any pressure if the temperature dependence at one pressure is known. The linear dependence of radiation on pressure, shown by equation (2.14), was used to calculate the radiative exitance at 5.5 atm from Kopainsky's experimental values at 5 atm.

At temperatures below 5000 K the radiative exitance was assumed to be zero. Above this temperature the experimental data of Kopainsky at 5 atm have been fitted with an exponential over the range 5000 K < \( T < 12000 \) K.

\[ Q_R = A \exp\left(\frac{T}{T_r}\right) \]  

where:

\[ A = 2.402 \times 10^4 \text{ W m}^{-3} \]
\[ T_r = 967.5 \text{ K} \]

This model of the radiative exitance is plotted along with Kopainsky's data in Fig. 2.3. In the model of radiative exitance used to integrate the Elenbaas-Heller
equation numerically, these data were multiplied by a factor of 5.5/5.0 in order to model the radiation at 5.5 atm.

2.4 Numerical Solution of the Elenbaas-Heller Equation

Since the pressure variation in the arc is small it is assumed that the arc is isobaric, with pressure \( p(r) = p = 5.5 \text{ atm} \), so that a knowledge of the transport properties is only required at 5.5 atm. A treatment of the arc with radial pressure gradients appears in section 2.6.

Radial symmetry imposes the condition that \( \frac{dT}{dr} = 0 \) at \( r = 0 \). The second boundary condition is that imposed by the cooled quartz wall. In this case the temperature at \( r = R_{\text{wall}} = 13.5 \text{ mm} \) has been forced to be 600 K.

It turns out that the overall properties of the arc column are relatively insensitive to the wall temperatures so that an approximate calculation of what the wall temperature is in an operating arc will suffice. Under typical operating conditions the thermal power deposited into the quartz wall is of the order of 23 kW/(m arc length). For the 27 mm inside diameter arc vessel this corresponds to a heat flux of 270 kW m\(^{-2}\). The temperature difference required to drive this heat through the 1.5 mm thick quartz wall \( (\kappa = 1.38 \text{ W m}^{-1} \text{ K}^{-1}) \) is \( \Delta T = \left( \frac{Q}{\kappa} \right) \Delta x \approx 300 \text{ K} \). Since the cooling water holds the temperature of the outer surface of the quartz at about 300 K the inner surface will be at about 600 K. This 600 K wall temperature was the boundary condition that was imposed in all the computer calculations.

The solution of the Elenbaas-Heller equation is accomplished by assuming a centreline temperature for the arc and numerically integrating equation (2.3) outwards to determine the temperature profile. The temperature calculated at the wall is compared with the 600 K boundary condition, and an appropriate adjustment
Figure 2.3 Model of radiative exitance. The model is compared with Kopainsky's experimental data which were obtained at 5 atm. At lower temperatures, in the range 6000 K to 10000 K, the model parallels the experimental data at a pressure of 1 atm (where there are considerably more data available in the literature).
is made to the guess of the centreline temperature. The iteration is continued until the boundary condition is adequately satisfied.

In practice, the process of convergence to a solution which satisfies the 600 K wall temperature is complicated by the fact that for a given value of electric field imposed on the arc column there are, in general, three such solutions. These are the high current, low current and zero current solutions, and their existence may be illustrated as follows. Suspend for the moment the boundary condition \( T_{\text{wall}} = 600 \text{ K} \) and consider the effect of integrating equation (2.3) outwards to the wall starting from a centreline temperature \( T_{\text{cl}} \). For a given value of electric field, one may calculate the wall temperature for a wide variety of initial centreline temperatures, \( T_{\text{cl}} \), where \( 600 \text{ K} \leq T_{\text{cl}} \leq 10000 \text{ K} \). Fig. 2.4 shows a plot of \( T_{\text{wall}} \) versus \( T_{\text{cl}} \) over this range of temperatures. In order to prevent the appearance of negative temperatures for certain guesses of \( T_{\text{cl}} \), the integration routine interrupts the integration if the temperature falls below a preset value, and sets the temperature for the rest of the profile (out to and including the wall) to zero. Consider the effect of starting with a centreline temperature \( T_{\text{cl}} = 600 \text{ K} \) and gradually increasing it. The solution on the left hand side of the figure is the trivial solution with \( T_{\text{cl}} = 600 \text{ K} \) and the arc current equal to zero. The electric field is imposed across an insulating cold gas, and the temperature is everywhere equal to the centreline temperature. This is the case for an arc which has been extinguished and will not spontaneously restart.

Consider now what happens when the centreline temperature is raised. Up to 5000 K there is no ohmic heating since \( \sigma = 0 \). Since there is no energy input, the temperature profile remains flat and the wall temperature rises, remaining equal to the centreline temperature (Fig. 2.4, regime I). Above 5000 K an arc proper begins to run. In this temperature range the radiation contribution is quite small and the temperature profile is determined by the balance between ohmic heating and
Figure 2.4 Wall temperature vs. centreline temperature. The horizontal line at $T_{wall} = 600 \text{ K}$ links the three possible operating points which are indicated with open circles. Calculated with $R_{wall} = 13.5 \text{ mm}$ and $E = 490 \text{ V/m}$.

thermal conduction. In this regime (Fig. 2.4, regime II) the electrical conductivity rises more quickly with temperature than the thermal conductivity so that as the centreline temperature is increased the temperature gradient must increase to allow the $-\kappa \nabla T$ term to balance the $\sigma E^2$ term. The required increase in temperature gradient is dramatic enough to drive the wall temperature down even though the centreline temperature is increasing. Increasing the core temperature eventually forces the wall temperature to drop to 600 K, and the second operating point for the arc is reached (Fig. 2.4, regime II). Beyond this point the temperature gradient required to balance the ohmic heating continues to increase, until the wall temperature would be required to be negative. It is over this range that the integration routine holds the temperature at zero wherever it tries to go negative.

As the centreline temperature continues to increase however, the radiation begins to become a factor and eventually the arc becomes radiation dominated
(Fig. 2.4, regime III). In this temperature range the radiative exitance \( Q_R \) is increasing with temperature more quickly than \( \sigma \). Thus, as the temperature is increased, \(-\kappa \nabla T\) must increase slowly (or even decrease) in order to satisfy the power balance equation. However, since \( \kappa \) is still increasing significantly with temperature, \( \nabla T \) must be reduced. The wall temperature therefore begins to rise once again and eventually the third operating point is reached (Fig. 2.4, regime III).

The existence of multiple operating points for a given value of \( E \) field may also be seen on a plot of the \( EI \) characteristic of the arc. Such a characteristic is shown in Fig. 2.5, where the centreline temperature is marked as a parameter along the curve. Here, the three points may be seen to correspond to the zero current, low current and high current solutions. The low current solution corresponds to the low temperature case, where radiation does not play a major role, while the high current solution corresponds to the radiation dominated case.

The existence of two non-zero current solutions for one value of \( E \) field complicates the convergence of the numerical procedure. Care must be taken when adjusting the centreline temperature to try to match the 600 K boundary condition, that one does not jump from one branch of the solution to the other. The numerical routine locates the two separate roots by starting with a centreline temperature of 10000 K and approaching the high temperature root strictly from above, and then by restarting with a centreline temperature of 6000 K and approaching the low temperature root strictly from below. The full \( EI \) characteristic as shown in Fig. 2.5 may be generated by varying the \( E \) field and solving the Elenbaas-Heller equation for each value.

The details of the arc power supply determine, in practice, at which of the two non-zero current operating points the arc will run. If the arc were run from a current supply it would operate at a unique operating point determined by that current. However if the arc were run from a voltage supply it would appear that
Figure 2.5 The $EI$ characteristic of the arc. The centerline temperature of the arc is shown as a parameter along the curve. The three possible operating points for an arc with an electric field of $E = 490$ V/m are also indicated by the open circles. ($R_{wall} = 13.5$ mm)

there would be two possible operating currents for the arc. Stability dictates however, that unless ballast resistance is added in series with the arc, the high current solution will be the operating point. The low current solution is unstable for a voltage supply as can be seen in the following argument. If the current increases slightly the required voltage falls and the power supply simply supplies the extra current. This can continue, with the current increasing, until the operating point slides around onto the positive dynamic impedance half of the $EI$ curve. Once
the high current operating point is reached a further increase in current is prohibited since this would require a voltage in excess of the output voltage of the power supply.

In practice, the arcs used in this work were not run at currents below 50 A, so they were always running on the positive dynamic characteristic and current stabilization was not necessary.

Once the temperature profile has been calculated for a given set of operating conditions, the rest of the arc parameters may be determined from the profile. The total ohmic heat input and total radiation output are obtained simply by integrating $\sigma E^2$ and $Q_R$ over the profile. The thermal wall loading is given by

$$-2\pi R_{wall}\kappa(T_{wall})(d\frac{T}{dr}|_{R_{wall}})$$

(\text{where } R_{wall} = \text{radius of the wall} = 13.5 \text{ mm, and } T_{wall} = 600 K).$$

Table 2.1 and Figs. 2.6 and 2.7 show typical arc parameters, calculated using the equilibrium theory for an arc 100 mm long, running at a current $I = 225$ A.

Table 2.1

Arc Parameters Predicted from Equilibrium Elenbaas-Heller Equation

\begin{table}
\centering
\begin{tabular}{|c|c|}
\hline
\text{arc length} = 100 \text{ mm, } I = 225 \text{ A} & \\
\hline
\end{tabular}
\end{table}

\begin{align*}
P_{\text{input}} &= 11.06 \text{ kW} \\
Q_{\text{wall}} &= 1.88 \text{ kW} \\
Q_{\text{rad}} &= 9.18 \text{ kW} \\
E &= 490. \text{ V/m}
\end{align*}
Figure 2.6 Temperature profile of the arc. The profile results from the solution of the equilibrium Elenbaas-Heller equation at a pressure of 5.5 atm and with \( R_{wall} = 13.5 \) mm.
Figure 2.7 Arc parameters as a function of radial position. The ohmic heating ($P_{ohmic}$) and radiation ($P_{rad}$) in an annulus of radius $r$, thickness $dr$ and unit length are shown. The total heat flow (heat conduction) through the surface of the annulus ($H$) is also shown. The data shown were obtained by solving the equilibrium Elenbaas-Heller equation for the case of $I = 225$ A, $P = 5.5$ atm and $R_{wall} = 13.5$ mm. The annulus is of thickness ($dr$) = 13.5 μm.

2.5 Axial Convection

To explain some of the calorimetry results obtained during the course of his Ph.D. thesis work, Neilson (1981) proposed that axial convection plays an important role in the energy transport in the arc. He modelled the convection by using a disc model for the axial flow of gas. At the upstream end of the arc (Fig. 2.8) cold gas is fed into the chamber by the jets. As this gas flows down the tube it encounters the arc column and begins to be heated. Subsequently the heat in the gas is conducted
radially until it reaches the quartz wall. Due to the axial motion of the gas, the onset of this wall loading is expected to occur beyond the location where the gas is first heated by the arc column (Fig. 2.8). Neilson calculated the location of the onset of the wall loading by means of a model which neglects radial convection. In this model the arc is represented by a heat reservoir at 10000 K occupying a cylinder of radius 6.6 mm. Cold gas flows over the cylinder and is heated as it moves along. It is assumed that the axial velocity of the gas is independent of radius, so that one may move with a disc of gas. The problem then reduces to solving the heat diffusion equation in the gas disc,

\[
\frac{dT}{dt} = \frac{1}{\rho c_p r} \frac{d}{dr} \left( r \kappa \frac{dT}{dr} \right)
\]  

(2.16)

where:

\[
\rho = \text{the gas density}
\]

\[
c_p = \text{the specific heat at constant pressure}.
\]

The boundary conditions taken for the problem are \( T = 10000 \text{ K at } r = 6.6 \text{ mm} \) and \( T = 600 \text{ K at } r = 13.5 \text{ mm} \) while the initial condition is the cold gas condition \( T(0, r) = 600 \text{ K for } 6.6 \text{ mm} < r < 13.5 \text{ mm} \).

Once again, because of the nonlinear dependence of \( \kappa \) on temperature, the equation is solved numerically. This determines the time evolution of the temperature profile and the heat flux into the wall. The results may then be re-expressed in terms of position along the arc by calculating the position of the centre of mass of the gas disc as a function of time. The velocity of the gas \( (v) \) may be calculated from the mass flow rate \( (\dot{m}) \).

\[
v = \frac{\dot{m}}{\int_0^{R_{\text{wall}}} 2\pi r \rho(r) dr}
\]  

(2.17)
The disc model of axial convection. Cold gas entering at the upstream end of the arc delays the onset of thermal wall loading so that the equilibrium value of wall loading is not reached until further downstream in the arc. The isotherms corresponding to $T = 600 \text{ K}$ and $T = 5000 \text{ K}$ are shown.

Here the isobaric assumption is used to allow the density $(\rho(r))$ to be calculated from $T(r)$ and the ideal gas law. This equation may then be integrated to give the position of the disc as a function of time.

Neilson calculated that for the parameters of his arc the wall loading should have its onset 50 mm downstream of the upstream electrode. The experiments in this work were performed at higher mass flow rates than Neilson's work, so the calculations have been repeated for conditions appropriate to this work. Fig. 2.9 shows the calculated wall loading as a function of position along the arc for a mass flow rate of 2 gm/s, which is typical of the lowest flow rates used in these experiments. As can be seen, the onset of wall loading is located 80-100 mm from the upstream electrode in this case.
Figure 2.9 The onset of wall loading calculated from the disc model of axial convection. The distance $z$ is measured from the upstream end of the 10000 K reservoir. The mass flow rate in this case is $\dot{m} = 2.0$ g/s.

A further understanding of the effect of convection on the arc, in a model such as this, may be had by looking at the properties of the arc upstream of the point of onset of wall loading. The completely rigorous theory of the arc in this region would involve a two dimensional model, since the arc is no longer axially invariant. Such calculations have been carried out for wall stabilized arcs (Watson and Pegot (1967), Clark and Incropera (1972)) and recently for the vortex stabilized arc (Korneyev et al. (1983)). Since it is desired here simply to predict features of the arc which would be indicative of a convection dominated regime, a simpler approximate model has been used. This model nevertheless predicts qualitatively the same effects as those predicted by the rigorous treatment. In Fig. 2.10 the upstream half of the arc column is again shown, but this time we look at a radial slice of the arc fixed in axial position ($z$), as shown in the figure. The radial positions of the isotherms in the slice of gas will of course depend on how far the
chosen slice is from the upstream electrode. As we move away from the electrode the 600 K isotherm moves radially outward until it reaches the wall.

Figure 2.10 The upstream portion of the arc. The disc model of axial convection has been used to calculate the approximate properties of the arc in this region.

Consider the case, shown in Fig. 2.10, where the 600 K isotherm is located somewhere midway between the core and the wall. The approximation is then made that the heat flow is predominantly in the radial direction, and that we may therefore neglect the axial transport of energy. Since the temperature profile at any value of $z$ is independent of time, one can imagine the arc column at this value of $z$ as being described by the Elenbaas-Heller equation where the wall has been moved in to the position of the 600 K isotherm. The heat which would normally flow into the wall is now used to heat cold gas outside the 600 K isotherm and shift the isotherm radially outwards.
This model is obviously only an approximation, but it gives us an idea of what qualitative differences to expect in the upstream portion of the arc column. Fig. 2.11 shows the effect of varying the radius of the wall ($T = 600$ K boundary) on the arc parameters, when the arc current is held fixed. It can be seen that when the radius of the wall is reduced the heat conducted away from the arc column is in fact increased, but also that the radiation output is increased. This is in agreement with the well known result of arc constriction. When an arc is constricted (either by moving the wall in, or strongly cooling its fringes) the current is forced to flow in a smaller central core. If the same current is to be maintained, the temperature must rise in order that the conductivity be increased. This rise in temperature however, also causes a rise in $Q_R$. Since $Q_R$ is such a dramatic function of temperature, the increase in $Q_R$ more than makes up for the loss of radiating volume caused by the constriction, and the overall radiation output increases. Such an effect can be seen in the numerical calculations of Korneyev et al. (1983) if one computes the total radiation from their published temperature profiles.

The significant predictions of the disc model of axial convection may be summarized briefly. The model predicts the onset of wall loading should occur significantly downstream of the upstream electrode. The distance downstream should scale as the mass flow rate of the gas ($\dot{m}$). Finally, the arc core upstream of the point of onset for the wall loading should show enhanced radiation output when compared with the downstream part of the arc which is in equilibrium with the gas flow.

### 2.6 Model of Arc in a Centripetal Field

The arc is stabilized in the vessel by means of a strong vortex flow and it is important to estimate the effect of the vortex on the arc properties. The main effect of the vortex is to produce a radial pressure gradient, and a simplified model has been used to investigate the operation of the arc in such a gradient. It has
Figure 2.11 Arc parameters as a function of $R_{wall}$. The calculations were performed for an arc running at a current $I = 225$ A at a pressure of 5.5 atm. It has been assumed that the vortex rotates as a solid body and that the annulus next to the wall, where the velocity falls to zero, is sufficiently thin to be neglected. Since the pressure does not appear explicitly in the Elenbaas-Heller equation, the pressure gradient only exerts its influence through the pressure dependence of the transport properties ($\kappa, Q_R, \sigma$). The pressure variations expected are small, so that the dependence of $\kappa$ and $\sigma$ on pressure has been neglected. The radiative exitance however, depends directly on the pressure

$$Q_R(T, p) = \frac{p}{p_0} Q_R(T, p_0)$$  \hspace{1cm} (2.18)
where \( Q_R(T, p_0) \) is the known dependence of radiative exitance on temperature at the reference pressure \( p_0 \) (in this case \( p_0 = 5.5 \) atm).

For a vortex in solid body rotation (angular velocity = \( \omega \)) the radial pressure gradient may be expressed as

\[
\frac{dp}{dr} = \omega^2 \rho r. \tag{2.19}
\]

If we use the ideal gas equation of state and assume a small degree of ionization the density (\( \rho \)) may be eliminated from the right hand side and the equation then integrated with respect to \( r \) to give

\[
p(r) = p_c \exp \left( \frac{\omega^2}{\mathcal{R}} \int_0^r \frac{r'}{T(r')} dr' \right), \tag{2.20}
\]

where:

- \( p_c \) = pressure at centre (\( r = 0 \))
- \( \mathcal{R} \) = gas constant for argon = \( 2.08 \times 10^2 \) J(kg)\(^{-1}\)K\(^{-1}\).

When equations (2.19) and (2.20) are combined and inserted into the Elenbaas-Heller equation (eqtn. (2.3)) it becomes

\[
\frac{dT}{dr} = \frac{1}{rk} \int_0^r \left( \frac{p_c}{p_0} \exp \left( \frac{\omega^2}{\mathcal{R}} \int_0^{r'} \frac{r''}{T(r'')} dr'' \right) Q_R(T, p_0) - \sigma(T) E^2 \right) r' dr' \tag{2.21}
\]

where \( p_c \), the centreline pressure, is taken to be 5.5 atm. This equation may be integrated numerically in a manner similar to that described for the vortex free Elenbaas-Heller equation.

To test the effect of the vortex on the calculated properties of the arc, equation (2.21) has been solved for a value of \( \omega = 1.48 \times 10^4 \) rad/s. This corresponds to a circumferential velocity of 200 m/s at the wall. Such a value is typical of the flow velocity in the inlet gas jets, as calculated from the known mass flow rate of the gas and the jet diameter. On the other hand, it is typically 5 to 10 times
greater than the highest azimuthal velocities measured in the cold gas vortex, across a cross section 10 mm away from the jets. These measurements are described in Appendix A. The calculation of the arc properties with \( \omega = 1.48 \times 10^4 \) rad/s should therefore define an upper limit on the effect the vortex has on the arc.

The consequence of considering the vortex in the calculation of the arc parameters is that the ohmic heating and the radiation output of the arc are increased slightly, while the wall loading is somewhat decreased. For a 225 A arc with \( \omega = 1.48 \times 10^4 \) the ohmic heating is increased by 0.09%, the radiation is increased by 0.152% and the wall loading is decreased by 0.2%. The pressure on axis of the vortex is 0.1 atm lower (1.8% lower) than the pressure at the wall. Since the influence of the vortex on the arc is so small, its effect has been neglected in the rest of this work.
CHAPTER 3

The Vortex Stabilized Arc

3.1 Introduction

This chapter contains a description of the arc vessel and the various subsystems which allow an arc to be run in the vessel. The arc vessel described here is the standard double walled vessel which is similar to those used previously to study arcs in this lab (Gettel (1980), Neilson (1981), Gettel and Curzon (1982)). A description of the partitioned tube vessel, which is unique to this work, follows in Chapter 4. The partitioned tube however, uses support systems which are the same as those described in this chapter.

The apparatus can be broken into several subsystems. The arc vessel itself and the electrodes; the electrical system, which supplies d.c. power for the arc; the water cooling system, which cools all the arc components; and the gas system, which generates the gas vortex in which the arc runs.

3.2 Arc Vessel and Electrodes

The arc is run in a double walled arc vessel, as shown in Fig. 3.1. The 27 mm inside diameter inner tube is made from quartz to enable it to withstand the high radiation flux and heat produced by the arc. The 50 mm inside diameter outer tube is made from pyrex so as to absorb any u.v. radiation produced by the arc. Both tubes have walls whose thickness is nominally 1.5 mm. A vigorous water flow is maintained between the two tubes to cool the quartz inner wall.
Figure 3.1 The vortex stabilized arc vessel.

The arc is stabilized inside the quartz tube (as described in Chapter 1) by means of a gas vortex. This vortex is generated by a pair of inlet nozzles in the gas jet assembly which admit the gas to the arc chamber with a large azimuthal velocity component. After swirling through the arc vessel the gas passes immediately into a water cooled heat exchanger, which cools the gas to room temperature.

The arc itself runs between two water cooled tungsten tipped electrodes which are \( \frac{1}{2} \) inch in diameter. The positions of the electrodes in the arc vessel are controlled by rack and pinion drive systems which clamp to each electrode. Sliding O-ring seals in the inlet gas jet assembly and the exit gas heat exchanger accommodate the movement of the electrodes. For most of this work the electrode separation was 100 mm and the arc was run with the upstream electrode positioned 35 to 80 mm
from the inlet gas jets. The electrode tips (manufactured by Vortek Industries Ltd.) consist of a tungsten tip approximately 10 mm long, which is bonded to a copper substrate using a proprietary process. The copper was then machined to the shape shown in Fig. 3.2, and silver soldered to the brass tubing. The machined shape assures a smooth flow of cooling water from the central brass tube over the copper surface and out the outside brass tube with no flow stagnation points.

![Diagram of electrode design](image)

**Figure 3.2** The electrode design (cathode).

The limiting step in the cooling of the electrodes is the rate of heat transfer from the metal to the cooling water. It is because of this that copper, instead of tungsten, is used at the metal/water interface. The high thermal conductivity of the copper makes a larger surface available for heat transfer to the water. Water flow rates for both electrodes are maintained at 200 ml/s which gives adequate cooling at the arc currents used in these experiments.
The finished shape of the tungsten tip depended on whether it was intended for use as an anode or cathode. The anode was first ground flat and then the sharp corner around the circumference was broken by grinding it to a 1 mm radius. This radius discouraged the arc from attaching to the edge of the electrode. The cathode was ground to a 45 degree truncated cone as shown in Fig. 3.2. Once again the corners were rounded to eliminate sharp edges. This cathode shape has proved most successful in stabilizing the cathode jet. The conical shape encourages the cathode spot to attach to the electrode in the centre of the vortex, while the flat region at the tip serves to direct the cathode flame along the arc axis.

3.3 Electrical System

The arc is run from a d.c. power supply capable of supplying up to 500 A. Figure 3.3 shows a schematic diagram of the power supply. A water cooled bridge rectifier is used to provide a full wave rectified d.c. voltage from the 208 V a.c. line. The choke, placed in series with the arc, serves to reduce the current ripple. Nevertheless, the current ripple is of the order of 75% for a 100 mm arc running at 225 A. The effect of this ripple on the arc is discussed later in this thesis. The arc current is controlled by means of a saturable reactor in series with the bridge rectifier. This allows the output current to be varied from 150 A to 500 A.

Starting of the arc is accomplished with a high voltage r.f. starting pulse. The ground electrode is first moved to within 5 mm of the live electrode. A 60 kV 4 MHz r.f. starting pulse is then applied to break the gap down. This pulse is generated using a commercial r.f. starter (L.P. Associates Inc. Model 750) which operates by series injection of the pulse. In this scheme the r.f. voltage is generated across the secondary of a transformer which is in series with the arc (Fig. 3.3). A return path, to complete the circuit for r.f. frequencies only, is provided by capacitor $C_2$. As soon as the gap is broken down, the main d.c. power supply provides current to sustain the arc. In order to prevent excessive current
flow through the short arc, the current is limited by maintaining the ballast resistors in the circuit (shorting switch open). Once the arc is burning, the ground electrode is withdrawn to a position 100 mm from the live electrode and the ballast resistors are then shorted out. This starting technique eliminates the electrode damage caused by touching the electrodes together to start the arc and yet allows the arc to be started with reasonable voltages at the operating chamber pressure.

The electrical system also has provision for measuring the voltage and current in the arc. The voltage is measured directly across the electrodes with a 10:1 voltage divider. The current is measured by means of a water cooled current shunt which has a calibrated resistance of $1.2 \times 10^{-4} \, \Omega$. Both voltage and current signals are displayed on an oscilloscope. The current signal may also be averaged using an RC circuit, the output of which is displayed on a digital voltmeter. This reading is used to maintain the arc current at a constant value during an experiment.
The total electrical input power to the arc may be computed from the arc voltage and current signals. A polaroid photograph is taken of the oscilloscope traces of the signals and a digitizer is used to input this data into the computer. The power is then calculated by averaging the instantaneous product of $V$ and $I$ over two full cycles of the waveform. The power calculated in this way balances with the total output power measured with the calorimetry system to within ±5%.

3.4 Water Cooling System

Because of the high thermal loading all the components of the arc vessel must be water cooled. A turbine pump draws water from a reservoir tank and supplies it to the cooling systems at a pressure of 150 psig. The pump is capable of supplying $1\text{ l/s}$ at this pressure. In addition a second centrifugal pump supplies up to $\frac{1}{3}\text{ l/s}$ of water to the cooling jacket of the arc vessel. There are five separate loops in the cooling system, one for each of the electrodes, the arc vessel, the water jacket, the exit gas heat exchanger and the radiation absorber. After cooling the arc components, the water is discharged to the drain. A schematic diagram of the cooling system is shown in Figure 3.4.

A calorimetry system allows the heat flowing into each of these components to be measured separately. In each cooling loop the water flow rate is measured with a rotameter type flowmeter calibrated in this lab. The temperature rise for each component is measured with a solid state temperature sensor (National Semiconductor LM335). These are clamped to the outside of a short copper tube placed in each polyflow cooling line and good thermal contact is maintained by the use of thermal jointing compound. One sensor measures a common inlet temperature for the cooling water, while the others monitor the output temperature for each arc component (see Fig. 3.4). Each output temperature has the common inlet temperature subtracted from it and then the result is digitized and displayed on a readout panel. The power absorbed by each component may then be calculated.
from $P = \Delta T \times \text{(water flow rate)} \times \text{(heat capacity of water)}$. In order to ensure that the calorimetry system has come to equilibrium the arc is allowed to run for 5 minutes after changing the operating conditions before any calorimetry data are taken.

Figure 3.4 The water cooling and calorimetry system. The water flow rates are measured with the flow meters (F) and the temperatures are measured with temperature sensors (T).

The electrodes are cooled by a coaxial arrangement of two tubes as described in section 3.2. For these experiments the water flow is maintained at a minimum of 200 ml/s for each electrode.

The arc vessel itself is cooled by water which is fed through a set of tangent inlet jets, so that the flow is introduced with high swirl. This ensures efficient and uniform cooling of the inner quartz tube by preventing the formation of stagnation regions in the flow. At the downstream end of the cooling jacket, the water is removed by a tangential exit port. Flow in the water jacket is maintained at
250 ml/s. Since the gas which leaves the arc is very hot, an exit gas heat exchanger is required before the gas can be recirculated. This heat exchanger is mounted so that it projects into the downstream end of the arc vessel. The gas exits the arc chamber through five tubes \( \frac{1}{8} \) inch in diameter which run axially through the length of the heat exchanger. Cooling water which flows through the heat exchanger surrounds the tubes and serves to cool both the gas flowing through the tubes and that part of the heat exchanger which protrudes into the arc chamber. The cooling water flow to the heat exchanger is maintained at a value of 200 ml/s. A sixth hole in the face of the heat exchanger is used as a static pressure tap. From this hole a small bore quartz tube leads to an external fitting where a connection may be made to a pressure gauge. The pressure measured at this static pressure tap is referred to as the arc chamber pressure throughout this thesis.

The vortex stabilized arc used in these experiments produces radiation equivalent to 30-40% of its input power. In order to protect the experimenter, as well as to measure this radiation, the arc is surrounded by a water cooled radiation absorber. The top, bottom and front of the arc are covered by a three sided radiation absorber which slides over the arc vessel. The fourth side (back) is covered with an absorber which has a slit running along its length to allow viewing of the arc. Cooling water is supplied to the radiation absorbers at a total flow rate of 225 ml/s. The absorbers were built from modules each consisting of two brass plates. Channels for water flow were cut in one plate and then the plates were soldered together. The inner surface of the absorber was blackened with a commercial compound called Brass Black, which gives it a 90% absorbancy in the visible (Gettel (1980)). When computing the arc radiation output, based on the power absorbed by the radiation absorber, account is taken of radiation which escapes through the slit in the absorber and through the ends.
3.5 The Gas System and Gas Inlet Jets

The argon gas used in these experiments is recirculated. The main reason for this is to increase the purity of the laboratory grade argon used by recycling it through a dessicant. The removal of residual water vapour from the argon proved crucial to these experiments since this markedly reduced the build-up of deposits on the inner quartz tube. These deposits absorb radiation and it is therefore important for the accuracy of the thermal wall loading and radiation measurements that the inner quartz tube be kept clean.

Figure 3.5 shows a schematic diagram of the gas system. A gas recirculator (Vortek Industries Ltd. Model 050) is used to recompress the argon. The recirculator uses a two stage compressor to recompress the gas on its return from the arc. It is then cooled and supplied to the high pressure reservoir at 120 psig. From the high pressure tank the gas passes through a dessicator containing approximately 2 l of silica gel. Prior to an experiment, the gas is allowed to recirculate through the dessicant overnight at a flow rate of 2 gm/s. Overnight dessication ensured upwards of 16 hours of arc operation before significant staining occurred on the quartz tube.

After passing through the dessicant, a line pressure regulator is used to reduce the gas pressure from 120 psig to the value needed to maintain the arc chamber pressure at 66 psig (5.5 atm). The small pressure drop across this regulator (typically 40 psi) is another advantage of using a gas recirculation system. If the arc were run from a gas bottle the pressure drop across the regulator would be as much as 2000 psi when the bottle is full. This high pressure drop would lead to excessive cooling and the danger of freezing of the regulator when high gas flow rates are used. By recirculating the gas through a 120 psig high pressure tank, higher gas flow rates can be used without appreciable cooling of the regulator. This feature
proved useful when the scaling of the arc parameters with gas flow rate was being measured.

The mass flow rate of the gas is measured with both a rotameter type gas flow meter, and a venturi mounted in the inlet flow line (Fig. 3.5). The differential pressure across the venturi is measured with a methanol manometer. While the rotameter is calibrated for air at room temperature, the argon flow rate may be determined by correcting the flowmeter reading for gas density.

The gas is admitted to the arc chamber through a pair of gas jets which give the flow a high degree of swirl. In these experiments three different types of jets were used. They differ both in the size of jet holes and in the angle at which the flow is initially launched (see Fig. 3.6). The first set of jets is identical to a set which had been previously been used in this lab (Gettel (1980), Gettel and Curzon (1982)). Since much of the motivation for this work came from those experiments,
a good deal of the initial work was carried out with these jets. They consist of two large inlet jet holes ($\phi = 1.60\,\text{mm}$) located on an angled ramp (Fig. 3.6), so that the gas leaves the jets with significant axial velocity. These jets were therefore termed the large angled jets.

Figure 3.6 The gas jet geometries. The hole through the centre of the jet assemblies allows the electrode to pass through.

Because of interest in the interaction between the arc and the gas flow, experiments were also done with two other geometries of jets. These jets allow the creation of different vortex flow conditions in the arc vessel. The second set of jets has the same outlet area as the first set, but the flow is admitted in a purely azimuthal direction, with no component along the axis of the tube (Fig. 3.6). These jets were termed the large azimuthal jets. It was hoped that by removing any imposed axial components of the inlet flow, the vortex flow structure would be simplified.
The third set of jets was designed with the same purely azimuthal inlet as the second set, but with one half the outlet area so as to produce higher jet velocities. These jets were termed the small azimuthal jets. The axial flow velocity of the gas in the arc vessel is determined mainly by the mass flow rate of the gas, while the amount of swirl is determined by the azimuthal velocity of the gas. Therefore, it should be possible, by using the large and small azimuthal jets, running at the same mass flow rate, to create vortex flows with similar axial velocities, but different amounts of swirl. Thus, the effect of the swirl velocity on the arc may be studied independent of the effects of axial convection.

After travelling through the arc chamber the gas exits axially through the holes of the exit gas heat exchanger. The back pressure regulator downstream of the heat exchanger (Fig. 3.5) is used in conjunction with the line pressure regulator to control the gas flow rate and the chamber pressure. By adjusting the two regulators, the flow rate and chamber pressure may be adjusted independently. This system is used to maintain the chamber pressure at 66 psig (5.5 atm) for all the experiments, while the mass flow rate may be varied from 2.0 to 4.8 gm/s. From the back pressure regulator the cooled gas is returned to the low pressure tank of the gas recirculator, which is held at 30 psig.

3.6 Summary of Standard Diagnostics

The diagnostics described in this chapter are those used on the basic arc vessel. They are summarized below:

- calorimetry system to measure thermal power deposited in all arc components
- measurement of arc voltage and current
- measurement of average electrical power input to the arc
- arc chamber pressure measurement
- gas mass flow rate measurement.
In this work a variety of other specialized diagnostics have been developed. Descriptions of these appear in Chapters 4 and 6, but in all experiments the standard diagnostics listed above were used to supplement the new apparatus.
CHAPTER 4

Apparatus to Measure Axial Profiles

4.1 Introduction

During the course of this work several specialized diagnostics were developed to measure the axial dependence of the arc properties. These include the partitioned tube, designed to measure the axial profile of thermal wall loading; and the radiometer experiment, designed to measure the axial profile of radiation. In addition photographs of the arc were used as an aid in interpreting the experimental results.

4.2 The Axial Profile of Thermal Wall Loading

4.2.1 The Partitioned Tube

The partitioned tube experiments utilize a specially designed arc vessel to measure the thermal wall loading on the inner quartz wall of the tube as a function of axial position along the arc. These measurements aid in understanding the transport of energy in the arc in two ways. In the neighbourhood of the upstream electrode, the measured profile of wall loading may be compared with the prediction of the onset of wall loading based on the disc model of axial convection. The partitioned tube allows an experimental determination of the location of the onset of wall loading for an operating arc. The location of this point may be determined for a variety of mass flow rates of the gas and compared to theory. The profile
CHAPTER 4: Measurement also provides a means of testing whether or not there is a region of the arc which exhibits axial invariance. In the upstream portion of the arc the gas undergoes a transient heating phase. In the downstream portion the gas flow should come into thermal equilibrium with the arc and the properties of the arc should become axially invariant. A profile of the wall loading allows measurements to be made locally in this region of the arc and a comparison made with the wall loading calculated from the equilibrium theory for the axially invariant arc column. Previously non-local measurements have been made by measuring the average value of wall loading over the whole length of the arc. These could be in error because of the transient heating of the cold gas when it enters the arc, the cooling of the hot gas when it leaves the arc or the possible influence of the electrodes on the wall loading in their immediate vicinity.

Vortex stabilized arcs have important applications as light sources and so for these experiments it is important that, in addition to being able to measuring the wall loading, the radiation produced by the arc should be allowed to escape the vessel and be measured externally. Since transparent plastics are damaged by the high radiation flux from the arc, this dictates, practically speaking, that the arc vessel be constructed from glass.

The first attempt to measure the axial profile of wall loading used an arc vessel which had a single isolated cooling section. This section was located in the middle of the tube and had a length of approximately 30 mm. The profile was measured by shifting both electrodes axially, keeping their separation fixed, so as to move the arc past the separately cooled chamber. The shortcoming of this technique was that the arc had to be shifted through 150-200 mm in order to measure the full arc profile. When this was done it was found that the overall properties of the arc changed significantly, presumably due to variations in the structure of the gas vortex.
It was this problem which motivated the construction of the multiple partitioned tube. This new tube allows a profile to be measured with little or no shifting of the arc. The partitioned tube vessel is designed so that short sections of the inner quartz tube (15 mm long each) can be cooled with a separate water cooling supply. By measuring the flow rate and temperature rise of this water a calorimetric determination of the thermal wall loading for that short section of the arc can be made. In practice, this was accomplished by isolating short sections of the water cooling jacket with partitions and supplying each of them with their own cooling water through sidearms (see Fig. 4.1). An important feature of the design of the partitioned tube is that the inner tube is removeable. It therefore could be replaced when it was cracked by the arc, or when it started to become discoloured.

Figure 4.1 The partitioned tube. Side view (left) and end view (right) showing the partitions which divide the water cooling jacket into separate chambers. Cooling water for each chamber is supplied and removed via a pair of pyrex sidearms.
The tube was built with twelve partitions, dividing the tube into thirteen chambers, each 15 mm long, as shown in Fig. 4.1. In practice, a profile was measured by shifting the arc through two steps of 5 mm each in order to increase the resolution of the measurement. The result is that the profile represents a measurement taken every 5 mm along the arc, of the average wall loading for a 15 mm long section of the wall.

The partitions were made by drilling a hole in the centre of a circular pyrex flat. A jig was then used to hold the twelve partitions in place while they were welded to the inside of the outer pyrex tube. The inner quartz tube was fitted simply by sliding it through the holes in the partitions. This quartz tube was selected from a large stock of 30 mm o.d. tubing to be as round and straight as possible. By choosing the largest diameter pieces of tube which would fit through the partitions a snug fit of the inner tube was assured. This helps to minimize the mixing of water between chambers.

In order to mount the partitioned tube and make seals at each end some special modifications were made to the regular arc mounting. Since the inner and outer tubes of this design are only free to slide axially with respect to one another, the mounting design must allow for the possibility that the two tubes do not have coincident axes. If this is not done it may not be possible to make leak tight O-ring seals to both tubes. As well, allowance was made for the outer tube (which had been extensively worked by the glass blower) to be out of round. A diagram of one of the mounting end caps is shown in Fig. 4.2. When the tube is mounted in the endcap a seal is first made to the inner tube by tightening the inner O-ring compressor ring. A seal is then made to the outer tube by tightening the segmented outer O-ring compressor ring. This ring is divided into eight segments which may be tightened independently, so that equal pressures may be kept on each one. In this way a seal may be made to an out of round tube since each sector of
the O-ring may be pushed up against the tube regardless of the shape of the tube. Only after a seal has been made to both inner and outer tubes are the two halves of the endcap tightened together. The bolt holes used for this purpose were drilled oversize and the seal between the two halves made with an O-ring so that the axes of the inner and outer tubes may lie up to 3 mm apart.

Figure 4.2 The endcap mounting for the partitioned tube.

Cooling water is supplied to, and removed, from each partition by a pair of pyrex sidearms as shown in Fig. 4.1. At the end of the sidearms a length of flexible reinforced tygon tubing is connected with a hose clamp. The 90 degree bend in the sidearm allows the tygon tubing to be shielded behind the radiation absorber. On the inlet side the tygon tubing connects the arc vessel to the inlet
manifold, while on the outlet side connections are made to thirteen fully immersed thermometers, and thence to the outlet manifold (see Fig. 4.3). A common inlet temperature is measured by a fully immersed thermometer in the line supplying the inlet manifold. The total flow rate of the cooling water is measured by a calibrated rotameter type flowmeter downstream of the outlet manifold. For the initial experiments it was assumed that the flow divides equally amongst all the partitions so that the heat deposited in each chamber by the arc is given by 

\[ P = \left( \frac{\text{total flow}}{13} \right) \times (T_{\text{out}} - T_{\text{inlet}}) \times (\text{heat capacity of water}). \]

The purpose of the manifolds is to ensure that each chamber is fed by, and discharges into, a uniform pressure. Furthermore, each sidearm has a precise restriction near its joint to the arc vessel. This restriction, created by drawing a piece of accurate diameter carbon through the hot glass tubing, ensures that each channel presents an equal impedance to the flow. Taken together, these features
serve to maintain the pressures in all thirteen chambers nearly equal, and in this way minimize mixing.

In order to test the amount of error introduced by the mixing of water between the chambers, a model of the arc vessel was tested before the actual partitioned tube was built. This consisted of four chambers, with pyrex partitions separating them and a quartz inner tube. The outer tube and the sidearms were however made from lucite. The model was instrumented with four inlet and four outlet thermometers and connected to inlet and outlet manifolds. Between the inlet manifold and the vessel however, the feed tubes were made sufficiently long that a 10 foot length of one of them could be immersed in a hot water bath. In this way, any one of the chambers could be fed with water that was as much as 2 K hotter than the water feeding the other chambers. Using the calorimetry system the heat loss from this chamber and the gain by the other chambers was measured. Experiments showed that under these conditions the heat loss from a chamber is in the range of 5-10% depending on how tightly the inner quartz tube fits.

4.2.2 Calibration of the Partitioned Tube

Early measurements taken with the partitioned tube yielded a profile which showed discontinuities from channel to channel. It was suspected that this might be due to mixing of heated water between chambers, so measurements were made of the water flow rate into, and out of, each chamber. The measurements were made by means of a venturi, which could be inserted into the tygon tubing between the partitioned tube and either the inlet or outlet manifold. Care was taken that the throat of the venturi was significantly larger than the bore of the sidearms, so that the flow was not appreciably altered by inserting the venturi. The pressure difference in the venturi was measured with a manometer and the venturi was calibrated by timing the discharge into a container of known volume with a stopwatch.
Measurements were made of the inlet and outlet flow rate of each chamber without the arc running, but with the water cooling system operating as it normally would otherwise. It was found that between inlet and outlet the flow rate of some of the chambers increased while others decreased. The maximum change in flow for any chamber was 8%, while the average change was about ±3%. Unfortunately, the mixing between chambers happens right next to the inner quartz tube, where the water is the hottest (when the arc is running). Therefore, for instance, a loss of 3% of the average flow from one chamber may result in the loss of more than 3% of the average heat being deposited into that chamber by the arc. In order to overcome this difficulty the partitioned tube was calibrated by using an axially uniform heater of known power in place of the arc.

If the calibration is to be meaningful the power density supplied by the heater must be of the same order as the thermal wall loading supplied by the arc. Since the heater is almost twice as long as the arc this implies that it must be capable of supplying approximately 3\(\frac{1}{2}\) kW. In order to avoid convective effects and maintain uniform heating, an electrical radiant heater was chosen.

The heater coil was made from four twisted strands of 1 mm tungsten wire, heated red hot and wound around a mandrel. Electrical connections were made by clamping the coil to the electrodes, and an alumina ceramic rod running through the coil provided mechanical support when the coil was heated. The inside of the quartz tube was blackened so as to be totally opaque and absorbing by coating it with soot from a methane/air flame. To eliminate convection and force the heater to operate radiantly, the arc chamber was evacuated to 10 \(\mu\)mHg before the calibration experiment was done. During a calibration run at full power the coil drew 115 A rms at a voltage of 30 V rms.

The total power absorbed by the partitioned tube was calculated by summing the powers absorbed by each chamber, based on the outlet flow rate of each chamber.
To calibrate the tube it was assumed that this heat should be uniformly absorbed by all the chambers and that the measured non-uniformity was due to mixing of water between chambers. A calibration factor, which corrects the individual measured powers to agree with the average, was then calculated for each chamber.

When these calibration factors were applied to the measurements of the axial profile of wall loading, taken with the arc running, a great improvement was found. The profile was then found to vary smoothly, with no large discontinuities between adjacent chambers.

This type of calibration can be expected to be quite good in the regions where the profile of arc wall loading is relatively flat. In regions with steep gradients however, it will not be entirely accurate in accounting for mixing. In order to investigate any residual errors in the experiment after calibration a number of checks were made on the accuracy of the measured arc profiles.

4.2.3 Checks on the Accuracy of the Partitioned Tube Results

The simplest check that can be made on the partitioned tube results is to compare the overall values of the arc properties with those measured in a standard double wall arc chamber. Under identical operating conditions a comparison can be made for the total wall loading over the length of the tube, the radiation, electrode loading and heat in the exit gas heat exchanger. When this is done it is found that the results agree to within ±5%.

The profile of wall loading, as measured by the partitioned tube, was then checked for self-consistency. The partitioned tube was demounted and turned end for end, so that chambers that were previously measuring the wall loading upstream of the arc were now measuring the wall loading downstream. If there are no systematic errors in the individual chambers of the partitioned tube, the profile should be identical irrespective of the relative orientation of the arc and the tube. In
practice, it was found that profiles measured in the two orientations gave results which agreed to within ±10%.

A second check was made by shifting the axial position of the arc electrodes and the vortex in the partitioned tube. This was done by inserting the gas jet assembly and upstream electrode further into the tube, while the gas exit heat exchanger and the downstream electrode were withdrawn by a corresponding amount. The amount of shift was arranged to be 15 mm so that the wall loading profile should remain the same, but be shifted one chamber downstream. When the experiment was performed it was found that the profiles agreed to within an accuracy of ±10%.

A final piece of evidence that the calibration had reduced the errors, due to mixing, to the limits described above, is the agreement between the general shape of the profiles measured with this multiple partitioned tube, and those measured with the original single chamber partitioned tube where mixing could not occur.

In summary, this part of the work is important in establishing the accuracy of the data measured with the partitioned tube and in checking for systematic errors. The experiments show that the wall loading values have a systematic error of about ±10%. Since the errors in the calorimetry system are typically less than this, this value is taken as representative of the error for the partitioned tube wall loading results.

4.2.4 Radiation Absorption Correction and Data Reduction

The optically thin spectrum of the arc contains contributions in the i.r. and the u.v. The i.r. radiation may be absorbed by the water in the partitioned tube, while the u.v. may be absorbed by the pyrex of the tube itself. This absorption reduces the radiation measured at the radiation absorber and artificially increases the measured wall loading.
CHAPTER 4

In order to correct for the absorption the spectrum of the arc must be known. Estimates for the spectrum were made by using spectra from the work done by Anderson, Eschenbach and Troue (1965) on vortex stabilized arcs and from spectra of the Vortek arc lamp (Camm (1984)). These spectra show no significant amount of optically thin arc radiation outside the wavelengths 0.2 \(\mu m\) to 2.0 \(\mu m\). The absorption spectra for water between 0.2 \(\mu m\) and 2.0 \(\mu m\) was taken from the International Critical Tables and for pyrex from the Corning Quickfit Catalogue.

The absorption of radiation in the partitioned tube was calculated numerically from this data. Figure 4.4 shows the geometry used for the calculation. The arc was assumed to be a uniformly radiating column of diameter 16 mm and length 100 mm. At each wavelength (\(\lambda\)) the contribution to the radiation absorption at point \(P\), due to radiation emitted at point \(r\), was calculated based on the absorption spectrum of water at wavelength \(\lambda\) and the path lengths \(p_A\) and \(p_W\). The total absorption at point \(P\) was then obtained by integrating numerically over the volume of the source and over the spectrum of wavelengths produced by the arc. Finally, the total radiation absorbed in a slice of the water jacket of thickness \(dz\) was obtained by integrating over the thickness of the water jacket 15 mm < \(r_P\) < 25 mm.

Since the absorption of radiation by the pyrex makes a smaller contribution to the correction, this absorption was modelled in a more approximate way. The pyrex was considered as if it were all located at the radius of the outer wall of the vessel. Using the total volume of pyrex in the partitions and the outer tube, an equivalent thickness of outer tube was calculated, supposing the pyrex were all located at this radius. By convolving the spectrum of radiation produced by the arc with the absorption spectrum of such a thickness of pyrex it was found that 3% of the radiation is absorbed. In any slice through the arc vessel (such as shown in Fig. 4.4) the total absorption by water plus pyrex was therefore calculated by
adding to the absorption due to water alone, an absorption corresponding to 3% of the remaining radiation escaping the outer boundary of the water jacket.

By repeating this calculation for a number of axial positions \( z \) of the cross section through the arc vessel, the amount of radiation absorption per unit length of arc vessel was determined as a function of position along the arc column. Integration of this function over the length of the arc vessel also provided the total absorption for the vessel. For the thickness of water jacket and pyrex used in the partitioned tube, approximately 20% of the arc radiation was absorbed by the arc vessel. This can be broken down into 17% absorption by the water and 3% by the pyrex.

To test these calculations a series of runs with unpartitioned double walled arc tubes was done. In each run the arc conditions were the same, but the arc vessel was altered by fitting outer pyrex tubes of different diameters. The effect
of this was to vary the thickness of the water jacket from a minimum of 3.5 mm to a maximum of 14 mm. For each geometry the radiation and thermal wall loading was measured at three values of arc current (corresponding to three values of total arc power). Figure 4.5 shows the thermal wall loading data, and Figure 4.6 shows the measured total arc radiation. It can be seen that as the water jacket thickness is increased for any given value of current (total arc power), the radiation goes down and the wall loading goes up. Thus radiation absorption by the water jacket is significant in this apparatus. The figures also show the result of correcting the data for radiation absorption using the calculation described above with the appropriate water jacket thickness. These results verify that the absorption calculation correctly accounts for the absorption of radiation in the water jacket.

In calculating the absorption it was assumed that none of the optically thin radiation produced by the arc has a short absorption length in quartz. This radiation would be strongly absorbed before entering the water. Since most of the u.v. radiation produced by the arc is strongly trapped radiation, this assumption seems reasonable. The radiation absorption in the body of the partitioned tube arc vessel seems therefore to be well described by this absorption correction calculation.

A further correction was, however, included for the effect of the pyrex sidearms of the partitioned tube which carry the cooling water. To measure the amount of radiation absorbed in the sidearms a model of a sidearm was built. This consisted of a loop of pyrex tubing bent into the shape of two sidearms and then joined together (rather than each joining to the arc vessel). Cooling water could be run through the "sidearm", and the loop placed almost touching the arc vessel. With the arc running, the power absorbed by the "sidearm" was measured for a number of axial positions of the sidearm along the arc. In this way, a correction was developed for radiation absorption by the sidearms of each chamber in the partitioned tube, as a function of the position of that chamber along the arc.
Figure 4.5 Measured wall loading vs. water jacket thickness. The open data points represent the data uncorrected for radiation absorption, while the solid data points include the correction for radiation absorption in the water jacket.
Figure 4.6 Measured radiation vs. water jacket thickness. The open data points represent the data uncorrected for radiation absorption, while the solid data points include the correction for radiation absorption in the water jacket.
In order to apply the calibration and absorption corrections to the data for each partitioned tube run a data reduction computer routine was written. For each chamber in the tube the routine subtracts the outlet temperature from the inlet temperature and uses the outlet flow rate for that chamber to compute the power absorbed. This power is then multiplied by the calibration factor for that channel, to account for mixing. Finally, using the position of the chamber with respect to the arc, the appropriate correction is made for radiation absorption by the partitioned tube and the pyrex sidearms. The results are then plotted as a function of chamber position to yield the axial profile of wall loading. The routine also corrects the value of radiation measured by the radiation absorber, to account for the loss of the radiation which is absorbed by the partitioned tube.

4.3 The Axial Profile of Radiation

The radiometer experiments were used to measure the axial profile of radiation produced by the arc. The purpose of these experiments was twofold. First, it was desired to measure the radiation locally, in the downstream part of the arc, to search for a region where the arc was in equilibrium with the gas flow, and to compare the results, in this region, with the predictions of equilibrium theory. Second, it was desired to measure the radiation in the upstream part of the arc, to look for enhanced radiation output, such as is predicted by the disc model of axial convection.

In order to measure the axial profile of radiation produced by the arc, a system of slits and a radiometer is used. A vertical slit 25 mm high by 3 mm wide is used to select a cross section of the arc of limited axial extent as shown in Fig. 4.7. The brass plate, in which the slit was cut, is water cooled so as to maintain its temperature near room temperature. A radiometer views the arc through the slit from a distance of 0.45 m. In addition, a second water cooled circular aperture placed 20 mm in front of the radiometer prevents the radiometer from viewing
anything but the immediate vicinity of the first slit. This shields the radiometer from scattered light and thermal radiation from hot pieces of apparatus near the arc. The radiometer used was an Eppley circular 16-junction model, which is sensitive to radiation of wavelengths between 0.2 \( \mu m \) and 2.0 \( \mu m \). It had been calibrated in terms of absolute intensity by N.R.C., so that intensities at the sensitive surface may be measured directly in \( W/cm^2 \). This means that by taking account of the geometry of the experiment, and by assuming that the arc radiates uniformly in all directions, the radiation output of a short localized length of the arc column can be determined.

![Diagram of apparatus to measure the axial profile of radiation. The translation stage moves the slit, the aperture and the thermopile along the axis of the arc.](image)

**Figure 4.7** Apparatus to measure the axial profile of radiation. The translation stage moves the slit, the aperture and the thermopile along the axis of the arc.
The radiometer and the slits are all mounted on a stage which can be translated parallel to the arc axis. In this way, the radiation produced by the arc was measured as a function of position along the arc. Thus the radiometer experiments complement the partitioned tube experiments. With the two experiments information is available about the local value of wall loading and radiation produced at each position along the arc.

4.4 Photography

As a general qualitative diagnostic, photographs were taken of the arc. These were obtained by enclosing the area around the arc with opaque curtains and running with the radiation absorber removed. A welder's filter was arranged so that photographs could be taken through the filter from a distance of approximately 2 meters. Lenses of 50 mm and 75 mm focal length were used.
CHAPTER 5

Results of Axial Profile Experiments and Discussion

5.1 Results

5.1.1 Introduction

The axial profiles of thermal wall loading and radiation were measured under a variety of vortex flow conditions. The first experiments were carried out with a set of angled inlet jets identical to those used by Gettel. Later two sets of azimuthal inlet jets were used to investigate more thoroughly the dependence of the arc parameters on the vortex flow conditions. For each set of jets used, the mass flow rate of gas could be varied from 2.0 g/s to 4.8 g/s. In all cases however, the arc chamber pressure was maintained at 66 psig (5.5 atm) independent of the flow rate. The arc current was maintained at an average current of 225 A for all experiments by adjusting the arc power supply.

The general features of the axial profiles of the arc may be seen by looking at the measurements made using the angled inlet jets at a mass flow rate of 4.8 g/s. Figure 5.1 shows a typical axial profile of thermal wall loading obtained with the partitioned tube. The most obvious feature of the profile is the effect that the electrodes have on the local value of wall loading. The extra voltage drop associated with the cathode and anode fall potentials is responsible for a high energy input
Figure 5.1 Typical axial profile of wall loading taken using the partitioned tube. The thermal wall loading per unit length of the wall ($P_{wall}$) is shown as a function of the distance from the jets ($D$). The arc was run using the large angled jets at a mass flow rate of 4.8 g/s.

to the plasma in the immediate vicinity of the electrodes. The wall loading measurements show that some of this energy is transported to the wall as heat which appears immediately adjacent to the electrode.

Between the electrodes there is, however, a plateau region where the wall loading is relatively flat. This region will be discussed in further detail in the next section. An important point to note however, is that the heat flow from the electrode regions significantly increases the total value of wall loading above that due to the positive column of the arc alone.
A second noticeable feature of the wall loading profile is the existence of wall loading upstream of the upstream electrode, as is illustrated in Figure 5.2. In this case the arc has been shifted downstream in the tube by 20 mm to allow investigation of the wall loading further upstream of the electrode. When the arc is shifted in this manner the wall loading in the plateau region remains substantially constant. There is, however, a slight reduction in the wall loading adjacent to the upstream electrode (in this case the anode). This is accompanied by an increase in the heating of the electrode itself. Thus shifting the arc slightly downstream results in a redirection of the heat flow in the electrode sheath plasma. The heat flow to the downstream electrode (in this case the cathode) remains substantially constant under the shift.

This example also illustrates another property common to all the wall loading profiles measured, namely the rapid onset of wall loading immediately adjacent to the upstream electrode. It is also found that the wall loading falls rapidly to a small residual value downstream of the downstream electrode (see Fig. 5.1).

Figure 5.3 shows the axial profile of arc radiation obtained from scanning the arc with the radiometer. The arc was operated under the same conditions as in the wall loading experiment shown in Figure 5.1. In the case of the radiation the axial profile is very flat with the exception of a dramatic peak in front of the cathode. This peak is produced by the radiation emitted from the hot cathode flame and accounts for approximately 15% of the total radiation emitted by the arc. Beyond the electrodes the radiation falls rapidly to zero.
Figure 5.2 The wall loading profile showing upstream wall loading. The arc was run using the large angled jets at a mass flow rate of 4.8 g/s.
Figure 5.3 The axial profile of radiation. The arc was run using the large angled jets at a mass flow rate of 4.8 g/s.

5.1.2 The Plateau Region

The existence of a plateau in the axial profiles suggests that the gas flow past the arc in this region has reached a steady state temperature profile and is in equilibrium with the arc column. If this is the case then lengthening the arc column should simply lengthen the plateau region. Figure 5.4 shows the wall loading profile for an arc which is 135 mm long. Apart from a 35 mm increase in arc length
the experimental conditions are identical to those for the arc of Figure 5.1. In particular, the saturable reactor control is re-adjusted in order to hold the current constant as the arc is lengthened. In the plateau region the wall loading is found to agree with that measured for the short arc to within experimental error. The existence of a lengthened plateau, in the case of the longer arc, is good evidence that the wall loading in this region is representative of the equilibrium value for the positive column. In the case of the radiation profile a similar result is found. Lengthening the arc results in the addition of more arc column which radiates at the same radiative power per unit length.

**Figure 5.4** The wall loading profile for an extended length arc. The arc was run at a length of 135 mm instead of the usual length of 100 mm. (Large angled jets, $\dot{m}=4.8$ g/s).
In order to test further whether the measured value of wall loading in the plateau region is representative of the actual value of the wall loading in the positive column of the arc, one additional experiment was performed. The positions of the cathode and anode in the vortex were reversed and the axial profiles were remeasured. When this is done it is found that there are changes in the wall loading immediately adjacent to the electrodes (see Fig. 5.5), however a plateau region remains in the centre of the arc. The measured value of wall loading in this plateau region changes by approximately 12% under the reversal of the electrode positions. The wall loading is found to be slightly higher when the anode is located downstream in the vortex. The measured value of wall loading upstream of the arc is not found to be affected (to within experimental error) by the reversal of the electrode positions.

In the case of the axial profile of radiation, a plateau region is once again found when the positions of the electrodes in the vortex are reversed. The measured value of radiation per unit length of arc in this region is found to be independent of electrode position to within ±10%. There is however, a noticeable change in the radiation associated with the electrode regions. The behaviour of the plasma associated with the electrodes is discussed in further detail in Section 5.1.4.

Since it was found that the reversal of the positions of the electrodes in the vortex made only a small difference in the measured characteristics of the arc, the balance of the experiments were carried out with the anode located upstream in the vortex. The choice of which electrode was to be the upstream electrode was somewhat arbitrary, although the slightly reduced wall loading associated with locating the anode upstream was felt to reduce the chance of cracking the inner quartz tube of the arc vessel. Furthermore, other experiments being conducted in this lab at the same time suggested that locating the anode upstream in the
Figure 5.5 The wall loading profile with the positions of the anode and cathode in the vortex reversed. (Large angled jets, $\dot{m}=4.8$ g/s).

vortex might afford a reduction in the erosion rate of the anode and therefore this configuration might be of more practical importance.

5.1.3 Dependence of the Axial Profile Results on $\dot{m}$ and $v_{\text{azimuthal}}$

The dependence of the axial profiles on the mass flow rate of the gas ($\dot{m}$) and on the azimuthal velocity of the gas ($v_{\text{azimuthal}}$) is of interest for two reasons. The disc model of axial convection predicts that the axial location of the onset of wall loading depends on the axial flow rate of the gas. According to the model, a doubling of the mass flow rate of the gas should lead to a doubling of the distance from the upstream electrode to the point of onset. Measuring the mass flow rate
dependence of the position of onset of thermal wall loading, would therefore allow one to examine the convective behaviour of the arc. In the plateau region of the arc, on the other hand, one is interested in the properties of the axially invariant positive column of the arc. For an arc in laminar flow the thermal wall loading and radiation of the arc in this region should be independent of gas flow rate. However, if turbulence were important for the transport of heat from the arc column to the wall one would expect a dependence of the wall loading (and also radiation) on the flow rate of the gas. Examining the scaling of the local value of thermal wall loading and radiation with flow rate, in the plateau region, therefore sheds light on the role of turbulent transport in the arc column.

Finally, it would be useful to determine the cause of the wall loading which has been measured upstream of the upstream electrode. Measurements of any flow rate dependence of this phenomenon would be useful in pinpointing its origin.

In order to make the results of these experiments as easy to interpret as possible, a pair of special jets was constructed with the aim of simplifying the flow pattern in the vortex. Details of the jet construction are discussed in Chapter 2. The essential feature of these new jets however, is that the gas is introduced purely azimuthally, without any axial velocity. Two sets of these jets were built, one with large inlet jets, equal in cross sectional area to the previous angled jets and one with smaller inlet jets having an area equal to half that of the large jets. By operating the two sets of azimuthal jets at the same mass flow rate it was hoped that vortices with the same axial flow velocity but with different azimuthal velocities could be obtained. Thus the scaling of arc parameters with vortex swirl at fixed mass flow rate (axial velocity) may be determined. It is of particular interest to look at the local value of wall loading in the plateau region and determine whether it scales with azimuthal velocity, while keeping the axial velocity (which governs axial convective effects in the disc model of axial convection) fixed. Alternatively both sets of jets
may be used individually to determine the scaling of arc parameters with gas mass flow rate for fixed jet geometry.

Before the scaling experiments were performed, a series of checks was done on the arc, to compare the arc performance with the new jets, to that which had been measured with the old angled jets. These experiments are described briefly below. The arcs run with the azimuthal jets are found to have profiles of wall loading and radiation which show the same general features as those of the angled jets. Shown in Figures 5.6 and 5.7 are the wall loading and radiation profiles taken using the large azimuthal jets at a mass flow rate of 4.8 g/s. In comparing this arc with one run using the angled jets (equal jet inlet area) at the same mass flow rate, it is seen that the wall loading in the plateau region is approximately 40% higher for the azimuthal jets, while the radiation is 5% higher. In general it is found that the large azimuthal jets produce arcs with wall loadings between 10% and 40% higher than the angled jets and radiation values between 5% lower and 5% higher depending somewhat on the flow rate and the position of the arc in the vortex. Once again it is found that there is measurable wall loading upstream of the upstream electrode. A behaviour similar to that observed using the angled jets is also found for the balance between the heating of the upstream electrode and amount of heat conducted to the wall in the neighbourhood of the electrode. When the arc is moved 30 mm downstream (in order to measure the upstream wall loading) the heat conducted to the wall falls, while the heating of the electrode rises. The heat balance for the downstream electrode shows little change under this shift. The shift does, however, have an influence on the measured value of wall loading and radiation in the plateau region. In this case instead of being independent of the shift, as was the case for the angled jets, the wall loading in the plateau region decreases as the arc is shifted 30 mm downstream. The decrease is 5% at low gas flow rates (2.0 g/s) and 15% at the high flow rates (4.8 g/s). The plateau value for
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Figure 5.6 The wall loading profile of the arc run using the large azimuthal jets. (mass flow rate $\dot{m}=4.5$ g/s.)

the radiation stays essentially constant under such a shift in the case of the large azimuthal jets, but increases by about 15% in the case of the small azimuthal jets.

As with the angled jets several checks were performed on the plateau region. When the arc is lengthened it is found that both the wall loading and the radiation plateaus are lengthened, while the measured values in the plateau region remain substantially the same. Reversing the positions of the anode and cathode in the vortex causes the wall loading in the plateau region to drop by 12% at low gas flow rates. At high gas flow rates the wall loading is found to be independent of electrode position. Thus for these jets, the measurement of a unique value of the wall loading in the positive column of the arc has about the same accuracy as for
the previous set of angled jets. Having established the general properties of an arc run with the azimuthal jets, the flow rate scaling experiments themselves could now be undertaken.

The flow rate scaling experiments were mostly conducted with the anode as the upstream electrode in the vortex. A test of the scaling was made with the anode downstream, using the large azimuthal jets, and similar scaling results were found.

The dependence of the arc parameters on mass flow rate and velocity was investigated by measuring the axial profiles of wall loading and radiation at various mass flow rates, using both the large azimuthal and small azimuthal jets. All
experiments were carried out at an average arc current of 225 A, although the total electrical power input varied considerably depending on the vortex conditions.

Comparisons may be made between the results from a given set of jets operating at different mass flow rates. Figure 5.8 shows for example the wall loading profiles obtained using the large azimuthal jets at mass flow rates of 2.1, 3.4 and 4.5 g/s.

Alternatively, by looking at results from the large and small sets of jets operating at the same mass flow rate, one may make comparisons between arcs running at different inlet flow velocities, but at the same overall mass flow rate.

The results of these experiments are summarized in Table 5.1 which shows the measured values of thermal wall loading and radiation in the plateau region of the arc as a function of vortex conditions. The level of wall loading measured upstream of the upstream electrode is also included since this has proved useful in understanding the interaction between the arc and the gas flow. Figure 5.9 shows the scaling of the wall loading values in the plateau region with mass flow rate, for both sets of jets.

For both sets of jets it is seen that the wall loading increases with mass flow rate, while the radiation remains the same or rises slightly. At the same time it is seen that the upstream wall loading increases with mass flow rate. Switching to the smaller jets while maintaining the same mass flow rate results in increased wall loading and slightly decreased radiation. The upstream wall loading is also seen to increase when switching to the smaller jets.

5.1.4 The Electrode Regions

The work in this thesis is not primarily concerned with the electrode regions of the arc plasma. During the course of the work however, some electrode
Figure 5.8 The profiles of wall loading at mass flow rates of 2.1, 3.4 and 4.5 g/s. (Large azimuthal jets.)
Table 5.1

The Scaling of Arc Parameters with Flow Rate

<table>
<thead>
<tr>
<th>JETS</th>
<th>MASS FLOW RATE (g/s)</th>
<th>PLATEAU VALUE $P_{wall}$ (kW/m)</th>
<th>PLATEAU VALUE $P_{rad}$ (kW/m)</th>
<th>UPPER WALL LOADING (kW/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Azimuthal</td>
<td>2.1</td>
<td>25.0</td>
<td>60.3</td>
<td>4.0</td>
</tr>
<tr>
<td>Large Azimuthal</td>
<td>3.4</td>
<td>28.5</td>
<td>**</td>
<td>7.5</td>
</tr>
<tr>
<td>Large Azimuthal</td>
<td>4.5</td>
<td>29.5</td>
<td>60.3</td>
<td>10.0</td>
</tr>
<tr>
<td>Small Azimuthal</td>
<td>2.1</td>
<td>28.5</td>
<td>50.6</td>
<td>6.0</td>
</tr>
<tr>
<td>Small Azimuthal</td>
<td>3.4</td>
<td>34.0</td>
<td>**</td>
<td>10.5</td>
</tr>
<tr>
<td>Small Azimuthal</td>
<td>4.5</td>
<td>36.0</td>
<td>57.8</td>
<td>12.0</td>
</tr>
</tbody>
</table>

related effects were found which have a bearing on the interaction between the arc and the gas flow. While some qualitative conclusions may be drawn from these results, caution must be exercised, since the electrode regions of the arc plasma are exceedingly complex (see for instance Pfender (1978) or Guile (1971)).

The particular results concern the behaviour of the plasma associated with the anode when the location of the anode in the vortex is reversed. When the anode is upstream in the vortex an axial profile of radiation such as was previously shown in Figure 5.3 is obtained. When the anode is moved to the downstream position however a second peak appears in the radiation profile immediately in front of the anode. Figure 5.10 shows the profile for an arc with the anode located downstream, but with all operating conditions otherwise identical to those of the arc in Figure 5.3. The appearance of a peak in the radiation profile at the anode is accompanied by a 15-20% decrease in the radiation associated with the cathode, while the radiation level in the plateau region changes by less than 10%.
Figure 5.9 The scaling of wall loading with mass flow rate. Data are shown for arcs run with both the large and small azimuthal inlet gas jets.
Figure 5.10 The radiation profile for the arc run with the anode located downstream. The arc conditions are otherwise the same as those of the arc in Figure 5.6.

Photographs taken of the arc running in these two configurations show that the increased radiation from the anode region comes from a concentrated clump of hot luminous plasma, which forms in front of the anode when it is downstream. The different behaviour of the anode plasma for the cases of anode upstream and anode downstream is shown by the two photographs in Figures 5.11 and 5.12.
Figure 5.11 Photograph of the anode plasma (anode upstream).

Figure 5.12 Photograph of the anode plasma (anode downstream).
5.2 Discussion

5.2.1 The Plateau Region

Both the radiation measurements and the partitioned tube measurements indicate that there is an axially invariant region in the arc. The fact that the level of this plateau remains constant, and the plateau lengthens as the arc is lengthened, confirms that these are reliable measurements of the local properties of the arc in the positive column.

Outside the plateau region, near the ends of the arc, there is a marked influence on the wall loading caused by the plasma associated with the electrodes. Of the two electrodes the cathode contributes the most to the wall loading. This contribution increases the total thermal wall loading for the whole water cooling jacket above what it would be due to the positive column alone. Measurements of the wall loading made by averaging over the whole cooling jacket would therefore overestimate the wall loading in the positive column. This explains in part, but only in part, why previous measurements have found a wall loading which exceeded that predicted by the equilibrium theory. Further comparisons between the local wall loading in the plateau and theory are made in Section 5.2.4.

An interesting result concerns the contribution of the upstream electrode to the wall loading. When the arc is shifted downstream in the vortex, so that the upstream electrode is moved from 50 mm downstream of the jets to 75 mm downstream of the jets, the wall loading adjacent to the electrode falls, while the heat absorbed by the electrode increases. The variation in the vortex as one moves downstream, apparently results in decreased heat transport to the wall, and therefore increased heat transport to the electrode. Because of the difficulty of making measurements of the flow velocity with the arc running the exact mechanics of this
tradeoff between electrode heating and wall loading are not understood, but this result may be of practical significance in the design of arc lamps. In this case, the ability to reduce electrode heating by moving the placement of the electrode in the vortex, may result in reduced electrode erosion rates.

5.2.2 Convection

The partitioned tube measurements show that there is a rapid rise in wall loading immediately adjacent to the upstream electrode. This is a general feature, which has been seen in all the wall loading profiles measured in the course of this work. An increase in the mass flow rate of the gas by a factor of two has not been found to drive the point of onset downstream measurably. This phenomenon is beneficial, in that it has allowed the measurement of the thermal wall loading in an axially invariant region of the positive column. On the other hand, the rapid onset of wall loading immediately adjacent to the upstream electrode disagrees with the predictions of the disc model of axial convection, which was discussed in Chapter 2. In this model, the onset of wall loading is predicted to occur 80-100 mm downstream of the upstream electrode for low mass flow rates. Apparently such a convective model, with an axial velocity which is independent of radius is inappropriate.

Further evidence for a dependence of axial velocity on radius may be obtained by looking at the power balance of the arc. The existence of relatively flat plateau regions in the wall loading and radiation profiles suggest that the gas flow in this region is in reasonable equilibrium with the arc column. In that case, the radial temperature profile of the gas there should be that predicted by the equilibrium theory for an arc operating at 225 A. If we assume that the gas flows axially with uniform velocity (as a gas disc) the power to heat the incoming cold gas to the equilibrium temperature profile may be calculated based on the mass flow rate of the gas. At a mass flow rate of 4.8 g/s it would take 11.5 kW to heat the incoming
cold gas to the equilibrium temperature profile. Under these flow conditions, measurements show that the arc runs at a total input power of 17.0 kW. The measured total radiation output is 7.0 kW and the heat absorbed by both electrodes sums to 5.0 kW. Even disregarding entirely the heat conducted to the wall of the arc it is clear that there is insufficient power to heat the gas to the equilibrium profile if all the gas flows with a uniform axial velocity. The existence of a plateau in the arc properties therefore implies that most of the mass flow must occur in a cooler layer near the wall of the arc, leaving the arc running in a comparatively stagnant region where the axial flow velocities are much reduced. Such a flow pattern is not unlikely, as it is known from studies on arc gas heaters (Watson and Pegot (1967)) that in many cases much of the gas tends to flow around the periphery of the arc and is not heated.

5.2.3 Reverse Axial Flow

In the course of this work there have been a number of indications of the existence of a core of reverse axial flow in the arc vortex. The strongest of these is the presence of significant thermal wall loading upstream of the upstream electrode. It is important to be able to show that the wall loading in this region is due to heat which is convected from the arc column and not due simply to radiation which is absorbed in the arc vessel and not properly accounted for. If the upstream wall loading were due to radiation one would expect that it would scale with the total radiation output of the arc. Looking at Table 5.1 however we see that this is not the case. For either set of jets a doubling of the mass flow rate leads to approximately a doubling of the value of upstream wall loading. The radiation output of the arc meanwhile remains roughly constant or increases by 15%. An even more dramatic demonstration is to compare the results from the two different types of jets operating at the same flow rate. In this case when one switches from the large azimuthal jets to the small azimuthal jets the wall loading increases by
20-50\% while the radiation decreases by 5-20\%. Thus the upstream wall loading is predominantly due to heat convection from the arc.

The effect of the reverse flow core in the vortex is to carry heat upstream from the arc as shown in Figure 5.13. (b) This leads to the observed upstream wall loading as well as a preheating of the incoming cold gas, an effect which contributes to the rapid onset of wall loading. This matter is discussed more thoroughly later in this chapter.

**Figure 5.13** The effects of reverse axial flow on the arc. Figure 5.13 (a) shows an arc with no reverse axial flow, while Figure 5.13 (b) shows the effects of reverse axial flow including upstream wall loading and preheating of the incoming cold gas flow.

Vorticities with reverse axial flow cores have been reported previously in arc chambers. Bez (1961) observed reverse flow in a nitrogen vortex stabilized arc by injecting Boron Carbide powder into the gas flow. In Bez's work however, the gas
exited the arc chamber through holes in the centre of both electrodes, so that the vortex flow pattern was quite different than the flow pattern in this work.

The dependence of the upstream wall loading on the inlet gas mass flow rate and jet size may be understood by looking at the mechanism which is responsible for the reverse flow (Nissan and Bresan (1961), King and Syred (1980)). The vortex is created by the vigorous swirl imparted to the flow by the inlet jets (Fig 5.14). This swirl leads to a radial pressure gradient given by

\[
\frac{dp}{dr} = \rho \frac{v^2}{r},
\]

(5.1)

where \( v \) = the azimuthal velocity of the gas.

This in turn leads to a low pressure on the axis of the vortex. At the downstream end of the tube the vortex strength is reduced due to the effects of viscosity and the drag of the stationary quartz tube on the vortex. Thus at the downstream end of the tube the pressure difference between the wall and the axis is less than at the upstream end. Figure 5.14 illustrates this for a simplified case where most of the vortex rotates as a rigid body. If the vortex is a strong one, with an azimuthal velocity that is much greater than the axial velocity (as is the case for the arc vortex), the radial pressure gradients will dominate over the axial gradients. There will therefore be only a relatively small pressure gradient along the wall of the arc vessel in the axial direction, namely that required to cause the swirling flow to be driven down the tube at a relatively low axial flow rate. This means that the pressure gradient on the axis of the vortex is strongly influenced by the difference in the radial pressure gradients at the upstream and downstream ends of the chamber. Since the radial pressure drop is less in the downstream part of the vortex than in
CHAPTER 5: The vortex and reverse axial flow. On the right the pressures at the four points A,B,C,D are shown schematically. It is assumed that the vortex rotates as a solid body, as shown on the left. The resulting pressure difference $P_D - P_A$ is responsible for driving the reverse axial flow.

In the upstream part, there may exist a reverse pressure gradient on the vortex axis which drives a reverse axial flow core in the vortex (see Fig. 5.14).

The scaling of the upstream wall loading with flow may now be understood in terms of the relative radial pressure gradients, at the ends of the chamber, for the different vortex flows. As the azimuthal velocity of the vortex is increased the pressure difference in the upstream end of the vessel, where the vortex is initially generated, will be increased. At the downstream end of the chamber the azimuthal velocity may also increase somewhat, but owing to the influence of viscosity, and possibly turbulence, the increase will not be as much as that at the upstream end of the chamber. Thus when the azimuthal velocity of the vortex is increased, the pressure on axis at the downstream end will not drop as much as that at the upstream end. The effect of this is to increase the pressure gradient in the reverse
direction on the axis of the vortex, and consequently drive more reverse flow in the core of the vortex. Thus, at each of the three mass flow rates used in these experiments, the replacement of the large inlet azimuthal jets by the small inlet azimuthal jets results in a 20-50% increase in the upstream wall loading. A similar situation would seem to occur when the mass flow rate is increased for a given jet geometry, although in this case the situation is complicated by the increase of net axial mass flow as well as the increase of azimuthal velocity.

The existence of reverse axial flow is further supported by photographs of the anode plasma (see Fig 5.11 and 5.12). Two photographs were taken, one when the anode was downstream in the vortex, and the other when it was upstream in the vortex (all conditions remain otherwise the same). The photograph taken with the anode upstream shows a diffuse plasma in front of the electrode. The plasma spills over to the sides of the electrode, much as might be expected if the electrode were actually downstream of the arc in the local axial flow. The photograph taken with the anode downstream complements this observation. In this case, the anode plasma appears to be concentrated in a clump in front of the electrode, which does not completely cover the whole face of the electrode. This would be consistent with a local axial flow in which the anode was actually upstream of the arc. A similar influence may operate at the cathode, but the existence of a pronounced cathode flame obscures the flow effects.

A related experiment, conducted in this lab by S. Dindo, allows us to infer the existence of reverse axial flow in yet another way. In this case, the flow pattern was determined from the trajectories of particles in the arc. The experiments were designed primarily to compare the erosion of the arc electrodes when the arc was run from d.c. and a.c. power supplies. However the arc chamber used was very similar in geometry to the one used in this work and similar mass flow rates of gas were used. In the course of these experiments it was found that at high erosion
rates (produced when water vapour was added to the argon) small tungsten droplets melted on the electrode surface and were carried away by the flow. When these droplets hit the inside wall of the quartz tube they left short tracks on its surface which could be examined after the arc was shut off. The droplet tracks revealed an interesting fact. Some of the tracks had a left handed spiral, whereas the inlet jets imparted a right handed spiral to the gas flow. Since it seems unlikely that the azimuthal velocity of the vortex is reversed anywhere, the droplets must have left the electrode with an axial velocity directed towards the jets.

A final check on the hypothesis of reverse axial flow in the vortex was made by measuring the flow direction in the cold gas vortex, without the arc running. Because of the high heat flux and intense radiation it has not been practical to make such a flow measurement with the arc running. These experiments, which are described in Appendix A, show that for the cold gas vortex there is reverse axial flow in the region of the vessel upstream of the upstream electrode. Furthermore, it is found that the reverse flow becomes more pronounced when the large azimuthal jets are replaced with the small azimuthal jets. This is in keeping with the observed upstream wall loading results.

The rapid onset of wall loading adjacent to the upstream electrode may now be understood in terms of two features of the flow pattern in the arc. In general, most of the mass flow of the gas through the arc is concentrated in a thin layer near the wall, leaving the arc in a relatively stagnant region (as far as axial flow is concerned). The heat from the arc therefore only has to diffuse across a thin layer of cold gas before it reaches the wall. Furthermore, the existence of a reverse flow core in the vortex draws hot gas from the arc and carries it upstream, towards the gas inlet jets. This has the additional effect of preheating the incoming cold gas. Thus, the onset of wall loading occurs very close to the upstream electrode and its position in not dramatically affected by the gas flow rate.
5.2.4 Comparison to Equilibrium Theory

The existence of a plateau region in the arc suggests that it would be meaningful to compare the local values of wall loading and radiation here against the predictions of the equilibrium Elenbaas-Heller equation. In all cases the arc was run at a chamber pressure of 66 psig (5.5 atm) and an average current of 225 A. Under these conditions the Elenbaas-Heller equation predicts the arc parameters shown in Table 5.2.

Table 5.2

<table>
<thead>
<tr>
<th>Arc Parameters Predicted from Elenbaas-Heller Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{input} = 110.6 \text{ kW/m}$</td>
</tr>
<tr>
<td>$P_{wall} = 18.8 \text{ kW/m}$</td>
</tr>
<tr>
<td>$P_{rad} = 91.8 \text{ kW/m}$</td>
</tr>
</tbody>
</table>

The experiments show that the local wall loading in the plateau region does depend on the vortex conditions. However, in all cases the measured wall loading is higher than that predicted by theory. The radiation output of the arc is also found to depend on vortex conditions, but does not vary as dramatically as the wall loading. In all cases however, it is found that the radiation values measured are lower than those predicted by theory. Table 5.3 summarizes the experimental measurements.

The enhanced thermal wall loading in the plateau region could be as the result of turbulence, as suggested by Gettel. In fact, the scaling of the wall loading with flow rate is consistent with what one would expect from turbulent heat transfer. From Table 5.1 it can be seen that the wall loading increases as the mass flow rate of
Table 5.3

Experimental Measurements of Arc Parameters

\[ P_{\text{wall}} = 21 - 36 \text{ kW/m} \]
\[ P_{\text{rad}} = 50 - 65 \text{ kW/m} \]

the gas is increased. Furthermore, the wall loading is found to increase when the mass flow rate is held constant, but the azimuthal injection velocity is increased by using the small inlet jets. In the next section, the results are therefore compared with a simple equilibrium model which includes turbulent heat transport.

5.2.5 Equilibrium Model with Turbulent Heat Conduction

It does not seem justified, at this time, to attempt to model the turbulence in other than a fairly simple manner. The gas flow pattern in the arc chamber is quite complex, and it has not been feasible to measure the flow velocities experimentally. The effect of turbulence in the arc was therefore accounted for by using a mixing length model for turbulent heat transport. In this case the contribution of turbulent heat transport is represented by the presence of a turbulent thermal conductivity \( \kappa_t \) in the heat conduction term of the Elenbaas-Heller equation. The equation then becomes, (see equation (2.2) for reference)

\[
\sigma(T)E^2 = Q_R(T) - \frac{1}{r} \frac{d}{dr} \left( r(\kappa(T) + \kappa_t) \frac{dT}{dr} \right). \tag{5.2}
\]

The effect of turbulence is thus to enhance the transport of heat from the arc to the wall. Mixing length theory (see for example Tennekes and Lumley (1972), p50) estimates the turbulent thermal conductivity to be

\[
\kappa_t = C \rho c_p u l \tag{5.3}
\]
where

\[ C = \text{a dimensionless constant} \]
\[ \rho = \text{the gas density} \]
\[ c_p = \text{the specific heat capacity at constant pressure} \]
\[ u = \text{a characteristic velocity scale for the flow} \]
\[ l = \text{a characteristic length scale for the flow}. \]

Since the purpose of this model is to allow one to compare the measured scaling of the wall loading with the predictions of a model which includes turbulence, this relationship may be further simplified. The characteristic length scale is taken to be the radius of the vessel minus the radius of the arc and the characteristic velocity is taken to be that of the gas flow between the arc and the wall. In order to make use of the mixing length concept, one has to choose values for \( \rho \) and \( c_p \) which are representative of the values of these quantities in the region between the arc and the wall. It is then assumed that when the gas flow conditions are changed, by changing \( u \), the temperature profile is modified sufficiently little that one is justified in neglecting the changes in \( \rho, c_p \) and \( l \) compared to the change in \( u \). In this case it can be seen that the turbulent thermal conductivity is proportional to the characteristic velocity in the region between the arc and the wall.

\[ \kappa_t = C_t u, \quad (5.4) \]

where:

\[ C_t = \text{a constant} = C \rho c_p l. \]

The results of the equilibrium model with turbulent heat transport were arrived at by solving equation (5.2) numerically using the same techniques which were outlined in Chapter 2 for the non-turbulent Elenbaas-Heller equation. The equation was solved for a value of \( E \) which corresponds to a total arc current of 225 A (the experimentally measured value). Calculations were made for a range of turbulent thermal conductivities, from 0.0 to 0.3 W/mK. In order to give some feel
for these values, the value of $\kappa_t$ is also expressed in terms of the average molecular thermal conductivity in the arc vessel. The molecular thermal conductivity averaged over the equilibrium arc temperature profile ($\bar{\kappa}$) has a value of 0.283 W/mK. The normalized values of turbulent thermal conductivity used in these calculations were $0 \leq \frac{\kappa_t}{\bar{\kappa}} \leq 1.06$. In Figure 5.15 the values of wall loading calculated from this equilibrium turbulent model are shown as a function of $\kappa_t$. The results show an almost linear scaling of the wall loading with $\kappa_t$, with a no turbulence ($\kappa_t \rightarrow 0$) limit of 18.8 kW/m.

In order to compare the experimentally determined scaling of the wall loading with this model, equation (5.4) is used. A comparison would be possible if the velocity in the region between the arc and the wall were known. Since it was impractical to measure this velocity, because of the intense heat and radiation from the arc, it has been assumed that the velocity ($u$) is proportional to the gas velocity in the inlet jet nozzle ($v_{jet}$). A jet velocity was therefore calculated for each set of experimental conditions used in the scaling experiments. Figure 5.16 shows the scaling of the wall loading in the plateau region (for both sets of jets) plotted against jet velocity (instead of mass flow rate as in Figure 5.14). It is seen that by using the jet velocity the data from both sets of jets are reduced to a common scaling, which is consistent with the scaling expected for turbulent heat transport. Furthermore, if one extrapolates the data linearly back to the no-flow condition, a wall loading value of $21.0 \pm 2$ kW/m is found which compares favourably with the equilibrium theory value of 18.8 kW/m. The anomalously high values of wall loading in the plateau region therefore appear consistent with an enhanced thermal transport in the arc by means of turbulent heat transport.

The model also predicts the effect of turbulent heat transport on the radiation produced by the arc. Figure 5.17 shows that the radiation is almost independent of the turbulent thermal conductivity ($\kappa_t$), rising by about 5% at the highest values
Figure 5.15 The dependence of the calculated wall loading on the turbulent thermal conductivity ($\kappa_t$). For reference the turbulent thermal conductivity has also been shown normalized to the average molecular thermal conductivity in the arc column $\bar{\kappa}$. 
Figure 5.16 The scaling of the experimentally measured values of wall loading with jet velocity ($v_{jet}$). The theoretical scaling of wall loading with $v_{jet}$ is shown by the solid curve. This curve was obtained by assuming that $\kappa_t$ is proportional to $v_{jet}$ and then fitting the theory to the data with a constant of proportionality which gave the best fit.
of \( \kappa_4 \) used (\( \kappa_\lambda = 1.06\kappa \)). Thus the experimentally measured values of radiation still lie below the predictions of equilibrium theory.

There is, however, another complicating factor, namely that the power supply used to run the arc does not produce ripple free d.c. current. Since the supply is simply a single phase full wave rectified supply with a series choke, the resulting voltage and current waveforms for the arc have as much as 75% ripple at an average arc current of 225 A. It may not be possible therefore, to describe the time averaged properties of the arc by means of an equilibrium theory which uses the average current in the arc (225 A). It was decided therefore to make time resolved measurements of some of the arc properties over the cycle of the arc. One could then determine whether the arc could be described by equilibrium theory over at least some part of the cycle.
Figure 5.17 The theoretical scaling of arc radiation with turbulent thermal conductivity ($\kappa_t$). The turbulent thermal conductivity is also shown in dimensionless form by normalizing it to the average molecular thermal in the arc $\bar{\kappa}$. 
CHAPTER 6

The Time Resolved Arc Experiments

6.1 Introduction

While the equilibrium Elenbaas-Heller equation, with the addition of turbulent heat transport, adequately accounts for the measured values of thermal wall loading, the measured values of radiation fall below that predicted by theory. Because of the presence of ripple in the voltage and current waveforms however, it may not be possible to describe the arc by equilibrium theory over the full cycle. Time resolved measurements were therefore undertaken to determine the characteristics of the arc over a cycle of the waveform and determine if the equilibrium theory was applicable over a part of the cycle. The measurements were made in the plateau region of the arc.

Because of the long time constants of the water cooling jacket and the inner quartz tube it is not possible to make time resolved measurements of the thermal wall loading using the partitioned tube. However, it is possible to make measurements of both the radiation output and the electrical power input to the arc column, time resolved over the cycle of the arc.

6.2 Stroboscopic Measurement of Radiation

The response time of the Eppley thermopile detector used in the radiation profile experiments is of the order of 2 seconds, so that it cannot, by itself, be used to time resolve the radiation over the 8.33 ms cycle of the arc. However, since the current waveform of the arc is periodic, a detector with a slow response time
such as this may be employed if a stroboscopic chopper is placed in front of the detector. The detector then only receives radiation for a short time interval at the same phase in every cycle. By shifting the phase of the chopper with respect to the arc current waveform, the radiation output may be measured at a number of points over the cycle.

The experimental setup is similar to that of the radiation profile experiment shown in Figure 4.7 except that a stroboscopic chopper is added immediately in front of the water cooled aperture (see Figure 6.1). Since it is the plateau region of the arc that is of interest, the translation stage is moved to an axial position corresponding to the centre of the plateau in the wall loading profile and fixed in position there.

The stroboscopic chopper consists of an 1800 rpm synchronous motor, to which is attached a chopper disc having four small radial slots (Figure 6.1). This gives the stroboscope a frequency of 120 Hz, which, of course, matches that of the full wave rectified d.c. power supply. The phase of the chopped signal relative to the current waveform of the arc is adjusted by rotating the housing of the synchronous motor. To this end, the motor is mounted on an “L” shaped baseplate as shown in Figure 6.1. A handle attached to the baseplate can be used to rotate the motor and baseplate about an axis aligned with the motor shaft. In order to determine the phase of the radiation signal relative to the current waveform, a second radiation sampling hole was drilled in the water cooled plate which is situated immediately in front of the radiometer. This hole is located directly above the hole for the radiometer and a piece of fiber optic cable, mounted in this hole, leads the light to a photodiode (see Fig. 6.1). By displaying both the photodiode signal and the arc current waveform simultaneously on an oscilloscope the chopper can be accurately adjusted to measure the radiation at any phase of the current waveform.
Figure 6.1 The stroboscopic chopper apparatus used to make time resolved radiation measurements. By rotating the handle on the motor mount the radiation may be measured at any desired phase angle in the current waveform. The photodiode provides a reference signal to monitor accurately the phase of the chopper wheel with respect to the current waveform. Note: the distance from the motor to the chopper wheel has been exaggerated in this diagram for the sake of clarity.

The experiment was performed on an arc running with the angled inlet jets at a mass flow rate of 4.8 g/s. This arc had a radiation output in the plateau region which was representative of most of the arcs examined in the course of this work. Figure 6.2 shows both the arc current and the radiation per unit length plotted against time for one cycle.
Figure 6.2 Time resolved measurement of radiation and current. The arc was run with the large angled jets at a mass flow rate of 4.8 g/s.
6.3 Variable Length Arc Measurement of $E(t), I(t)$ and $P(t)$.

The electrical power input in the positive column of the arc may be determined if the electric field in the positive column and the arc current are known. Since the same total current flows through the arc and the external circuit, the current in the positive column is easily determined. Unfortunately, the determination of the local electric field in the positive column is complicated by the effect of the electrodes. At each of the electrodes there is a significant voltage drop (of the order of 10-15 V) in the direction of current flow. These drops occur in a small axial distance, within a fraction of a millimeter, from each electrode and result in high electric field strengths in these regions. In the positive column, between the electrode fall regions, the electric field will be relatively uniform. If it is assumed that the electrode fall voltages are independent of the position of the electrode in the vortex, and that the electric field in the positive column is uniform, then the electric field may be determined by varying the arc length while holding the current constant. Lengthening the arc by an amount $\Delta l$ will then simply add an extra length $\Delta l$ of uniform positive column and increase the arc voltage by an amount $\Delta V = E \Delta l$.

The arc was therefore run at five different lengths from 60 mm to 140 mm. In each case the average arc current was held at 225 A by adjusting the arc power supply. At each length the arc voltage and current over two cycles of the waveform were displayed simultaneously on an oscilloscope. Polaroid photographs were taken of the waveforms and these photographs were digitized and entered into the computer for processing.

Each waveform was time resolved by looking at 50 points equally spaced in time over one cycle. At each point in time the arc voltage was plotted as a function of arc length. A least squares fit to a straight line allowed the slope, which corresponds to the electric field in the column, to be found. By determining
the y intercept, the sum of the electrode voltage drops could also be measured. Using this method the electric field was found at 50 points over the waveform. It is then a simple matter to compute the electrical power input to the arc, since at each time $P_{input}(t) = E(t) \times I(t)$. Figure 6.3 shows the resultant electric field and input power waveforms measured under the same arc conditions as those used in the time resolved radiation measurements of Figure 6.2.

6.4 Discussion

Before comparing the experimental results with equilibrium theory, let us first examine some conclusions which may be drawn from the results themselves. Since the electrical power input for the positive column of the arc has now been measured, we may look at the power balance for the positive column. The variable length arc experiments have given us a measure of the input power which is independent of the high power input in the regions near the electrodes. This should be a good measure of the power input in the plateau region, where the local values of thermal wall loading and radiation have been measured. In this way the power balance of a radial slice of the arc in the plateau region may be determined.

Since the wall loading is, by necessity, a time averaged measurement, the power balance must be examined on a time averaged basis. This may be done by simply averaging the measured input power over a cycle. For the case of an arc run using the angled inlet jets at a flow rate of 4.8 g/s the measured power balance in a radial slice of the arc (in the positive column) is as follows

\[
\begin{align*}
P_{input} &= 99 \text{ kW/m} \\
P_{wall} &= 22 \text{ kW/m} \\
P_{rad} &= 58 \text{ kW/m}.
\end{align*}
\]
Figure 6.3 Time resolved measurement of $P_{\text{input}}(t)$ and $E(t)$. The arc was run with the large angled jets at a mass flow rate of 4.8 g/s.
Thus there is a net energy flow leaving the slice in an axial direction. This energy flow accounts for about 20% of the total input power. It should be pointed out that despite this radial imbalance, the arc does exhibit overall power balance. That is, the average total electrical input power derived from the total arc voltage and current measurements does balance with the sum of all the power outputs measured by the calorimetry system, to within 5% or better.

When the radial power balance is measured for arcs with different vortex structures it is found that, in general, in the plateau region the local input electrical power exceeds the sum of the local wall loading and radiation. For the vortices used in this work between 10% and 20% of the input power is carried away axially. This limits somewhat the applicability of a radial model for the arc, such as that described by the Elenbaas-Heller equation. Nevertheless some useful insight may be gained by using such a model.

A comparison between the measurements and equilibrium theory may be made by solving the Elenbaas-Heller equation for values of arc current which correspond to values of current taken from the measured arc current waveform. It is assumed that at each point in time the ohmic heating, wall loading and radiation are those of an equilibrium arc with a current equal to the measured current flowing in the arc at that instant of time. Since the Elenbaas-Heller equation uses the electric field in the arc column, rather than the total current in the column, an iterative process is required to arrive at the desired value of current when the computation is done. The computation has been performed with a turbulent thermal conductivity ($\kappa_t$) equal to 0.045 W/mK or $\xi_h = 0.16$. This value was found to produce reasonable agreement with the average value of wall loading observed in the time resolved arc experiments. Figures 6.4 and 6.5 show a comparison between the equilibrium theory and the experimental results time resolved over one arc cycle.
Figure 6.4 Comparison between equilibrium theory and experimental measurements of $P_{\text{input}}(t)$, $P_{\text{rad}}(t)$. The theory includes turbulent heat transport with $\kappa_t/\bar{\kappa} = 0.16$. 
Figure 6.5 Comparison between equilibrium theory and experimental measurements of $E(t)$ and $I(t)$. The theory includes turbulent heat transport with $\kappa_t/\kappa = 0.16$. 
Over parts of the cycle the ohmic heating seems to be reasonably well predicted by the equilibrium model. Since the wall loading measurement is not time resolved, a comparison in this case is possible only by means of averaging the theoretical value over the cycle. When this is done, it is found that the equilibrium theory predicts an average wall loading of 22.1 kW/m, while the experimentally measured value is 22 kW/m. When the radiation is considered however, it is found that over the whole cycle the measured radiation falls significantly below theory. In the case of the electric field, a comparison shows that the theoretical value remains too high during the low current part of the cycle, resulting in an electric field waveform which is much flatter than the measured waveform.

In summary then, the time resolved experiments have shown that the measured ohmic heating and thermal wall wall loading are in reasonably good agreement with the theory over much of the cycle. The measured arc radiation, on the other hand, falls below the prediction of the equilibrium model. There is a considerable amount of error in the measurement of the electric field in the arc however, and this introduces a rather large uncertainty into the measurement of $P_{input}$. It may be possible therefore to make a modification to the model which would produce better agreement with the measured radiation, while still giving acceptable agreement (within the range of experimental error) with $P_{input}$. One possible explanation for the observed discrepancy in the radiation, is that time dependent effects may be important throughout the arc cycle. If the time constants associated with the arc column are significant when compared to the 8.33 ms period of the arc cycle, the arc would not be adequately described by the equilibrium model. The development of a time dependent model for the arc is therefore addressed in the next chapter.
The Time Dependent Elenbaas-Heller Equation

Since there is considerable ripple in the voltage supplied to the arc, time dependent effects in the arc column could significantly influence the characteristics of the arc. In order to judge the importance of such effects, the time dependent Elenbaas-Heller equation has been solved using the experimentally measured current waveform as the driving function.

7.1 Time Dependent Model

When the time dependent terms are added to the Elenbaas-Heller equation the equation becomes:

\[
\frac{dT}{dt} = \frac{1}{c_p} \left( \sigma E^2 - Q_R + \frac{1}{r} \frac{d}{dr} \left( r \kappa \frac{dT}{dr} \right) \right).
\]  

(7.1)

where:

\( c_p = \) heat capacity at constant pressure
\( \rho = \) gas density

In deriving this expression radial convection has been neglected, so that the time dependent Elenbaas-Heller equation has the form of a time dependent heat flow equation. It is further assumed that the pressure distribution in the arc column is isobaric. The density (\( \rho \)) may then be re-expressed as a function of pressure (\( p \)) and temperature (\( T \)), using the ideal gas equation of state

\[
\frac{1}{\rho} = \frac{(1 + \alpha) RT}{p},
\]  

(7.2)
where:

\[ \alpha = \text{degree of ionization} \]

\[ R = \text{specific gas constant for argon} = 208 \, \text{J/(kg\textsuperscript{-1}K\textsuperscript{-1})}. \]

For the small degree of ionization found in these arcs, one may let \( \alpha \to 0 \) and equation 7.1 then becomes

\[
\frac{dT}{dt} = \frac{RT}{pc_p} \left( \sigma E^2 - Q_R + \frac{1}{r} \frac{d}{dr} \left( r\kappa \frac{dT}{dr} \right) \right).
\] (7.3)

Once again, the temperature dependence of the transport properties \( \sigma, \kappa \) and \( Q_R \) causes the equation to be highly nonlinear and it is only practical to obtain a solution by numerical means. The values of the electrical conductivity and the radiative exitance are modelled in the same way as described in Chapter 2 for the equilibrium equation. The thermal conductivity, on the other hand, is modified to include the effects of turbulent heat transport, by using the mixing length theory outlined in Chapter 5. In order to do this, the thermal conductivity \( (\kappa) \) in equation 7.3 is replaced by the total thermal conductivity \( (\kappa + \kappa_t) \). For these calculations, the value of the turbulent thermal conductivity used was 0.045 W/mK (or \( \kappa_t/\kappa = 0.16 \)).

In addition however, a model is required for the heat capacity at constant pressure \( (c_p) \). At 5 atm the value of \( c_p \) remains constant, at a value equal to its room temperature value \( (c_p = 0.519 \, \text{kJ/kgK}) \) up to 8200 K. (for example see Drellishak et al. (1962)). Beyond that temperature it begins to increase as the result of the onset of ionization. In this high temperature range it is modelled with an exponential, which has been fitted to Drellishak's calculated values at 5 atm. Figure 7.1 shows Drellishak's data and the model used in this work.

The desired solution is, of course, the periodic response of the arc to the periodic driving function. In this case the driving function is the experimentally
Figure 7.1 Model used for $c_p(T)$. The model is compared to Drellishak's values in the range $8000 \, K \leq T \leq 11000 \, K$.

measured arc current $I(t)$. The steady state periodic solution is approached by starting from a d.c. current, which corresponds to the peak current of the current waveform. The initial temperature profile can therefore be set to the equilibrium profile for the peak current. The current is then made to follow the driving waveform and the arc parameters are monitored for several cycles of current until the transients have died away and a true steady state periodic response is achieved.

Figure 7.2 shows the approach of the system to steady state. Strictly speaking, the solution to the equation is the temperature profile of the arc ($T(r,t)$), but at each point in time, the total electrical input power, the total radiation and the
Figure 7.2 The approach of the time dependent solution to the periodic steady state. The total input power ($P_{\text{input}}$), radiation ($P_{\text{rad}}$) and thermal wall loading ($P_{\text{wall}}$) are shown over the time interval during which the transients associated with the d.c. starting conditions die away. The calculation includes the effect of turbulent heat transport ($\kappa_e/\kappa = 0.16$).

thermal wall loading may be found from the temperature profile at that point in time.

The numerical solution starts from the initial temperature profile and steps forward in time. At each point in time a new temperature profile is calculated from the profile at the last time step using equation 7.3. Unfortunately, this equation uses the electric field ($E(t)$) as the driving term, while it is easier to measure the current ($I(t)$) in an experiment. To relate the two the conductance of the arc column ($G$) is used.

$$E(t) = \frac{I(t)}{G(t)} = \frac{I(t)}{2\pi \int_0^r r'\sigma(T)dr'}$$  \hspace{1cm} (7.4)
This relationship is used to update the value of $E(t)$ (which is used as a driving function in equation 7.3) based on the measured current waveform $I(t)$. This is done every 0.1 ms or approximately 80 times over each arc cycle.

### 7.2 Comparison with Experimental Results

Figures 7.3 and 7.4 show the time dependent properties of the arc calculated from the time dependent Elenbaas-Heller equation. The properties are shown over one cycle of the current waveform, after the transients have died away and the response has become periodic. Also shown are the experimentally measured values of $P_{\text{input}}$, $P_{\text{rad}}$ and $E(t)$. These are the results of the experiments described in the last chapter using the large angled jets operating at a mass flow rate of 4.8 g/s.

When the results of the time dependent model are compared with the results of the equilibrium model, it can be seen that the addition of time dependent terms produces several differences. The electric field now falls more during the low current phase of the waveform, although it still remains higher than the measured field. Both the total electrical input power and the radiation are slightly higher than in the equilibrium case, while the thermal wall loading, although showing much less variation with time, retains about the same average value. The result is that the input power and the thermal wall loading still agree reasonably well with the measured values. On the other hand, the predicted radiation remains higher than the experimentally measured value. The value of radiation predicted is still about 35% above the measured value of radiation, when one averages over a full cycle. It therefore seems that time dependent effects do not account for the disagreement between the theoretical and experimental values of radiation.

It must be remembered however, that the experimental measurements have shown that up to 20% of the input electrical power is lost from a radial slice of the arc. A 35% discrepancy in radiation would correspond to approximately a 20%
Figure 7.3 The time dependent model with turbulent heat transport ($\kappa_t/\bar{\kappa} = 0.16$). The results of this model are compared with the experimentally measured values of $P_{\text{input}}(t)$, $P_{\text{rad}}(t)$.
Figure 7.4 The time dependent model with turbulent heat transport ($\kappa_t/\kappa = 0.16$). The results of this model are compared with the experimentally measured values of $E(t)$ and $I(t)$. 
discrepancy in the total input power. Since a radial model such as this does not account for any axial transport it cannot be expected to agree with the experimental results to within better than 20% of the total arc power. In the next section an argument is put forward which suggests that axial losses may in fact be responsible for reducing the radiation output of the arc.

7.3 Discussion

The results of modelling the arc, using the time dependent Elenbaas-Heller equation, have shown that this approach gives good agreement with the experimentally measured values of wall loading, if the radial heat transport is enhanced by means of an enhanced thermal conductivity. This has been done by using a mixing length model for the turbulent heat transport. A thermal conductivity of 1.06 times the average molecular thermal conductivity in the arc (or a total conductivity \( \kappa + \kappa_t = 2.06\kappa \)) is sufficient to account for the values of wall loading measured in these experiments. In addition, it is found that there is good agreement between the predictions of the time dependent model for the total heat input due to ohmic heating, and the values measured in the variable length arc experiments.

The experiments indicate however, that even in the plateau region a radial slice of the arc loses up to 20% of the electrical input power in the axial direction. Thus a radial model, such as the time dependent model developed in this chapter, cannot be expected to be accurate to better than 20%. In fact discrepancies do exist, as evidenced by the fact that the measured radiation is lower than that predicted from the model. A plausible argument can be made however, for this being due to the transport, in the axial direction, of 20% of the energy in the slice.

Since the axial velocities in the arc are not known quantitatively it is not possible, at this time, to model the axial flow of energy in detail. It is possible however, to modify the arc model in an approximate way so as to indicate what
effect such axial transport might have on the arc. In order to reduce the computer
time required to do this, the calculations were made using the equilibrium theory
model of the arc, rather than the time dependent model.

In the modified model it is assumed that only 80% of the ohmic heating in-
put \((\sigma E^2)\) to the arc column is available to supply the radiation and wall loading.
This was initially done simply by multiplying the \(\sigma E^2\) term in the Elenbaas-Heller
equation by 0.8. The equation was then solved by using the same numerical tech-
nique that was used for the conventional equilibrium model. It was found that the
effect of this modification is to increase the total ohmic heating by 10% while the
radiation is reduced by 13%.

It is also feasible however, that the removal of heat from a radial slice of the
arc does not take place uniformly over the slice. If 25% of the ohmic heat input
to the core of the slice, where the radius is less than 9 mm, is removed, (and for
\(r \geq 9\) mm no heat is removed) then the overall loss from the slice remains at 20%.
The model shows that the effect of this is to increase the total ohmic heating by
8%, while the radiation decreases by 17% (when compared to the zero axial loss
case). The wall loading is found to increase by 6%. Further increases in the axial
loss from the arc core are found to accentuate this effect.

If it is assumed that a similar effect would also occur in the case of the time
dependent model, then a considerable improvement is realized in the agreement
between theory and experiment. The axial transport of energy has the effect of
reducing considerably the radiation output of the arc without increasing the wall
loading and ohmic heating to unacceptably high levels. By including axial trans-
port it is therefore possible to model the ohmic heating, wall loading and radiation
in such a way as to obtain agreement with the experimentally measured values
to within ±18%. While this is obviously only a crude model of the rather compli-
cated phenomena involved in the axial transport of energy in the arc, it nevertheless
demonstrates that the presence of axial transport in the arc is a possible explanation for the low values of radiation which have been measured.
CHAPTER 8

Conclusions

8.1 Introduction

The work presented in this thesis was directed towards gaining further understanding of the physical mechanisms responsible for energy transport in the vortex stabilized arc. The ultimate goal of such research is to improve the performance of vortex stabilized arc lamps, particularly in the areas of radiative efficiency and arc component lifetime.

Of particular interest was an understanding of the mechanisms which are responsible for the thermal wall loading. Previous workers in the field have suggested that either turbulence or axial convection could be responsible for the unexpectedly high values of wall loading. The experiments in this work were therefore designed to measure the contribution of each of these mechanisms to the flow of energy in the arc. In this chapter the results of this investigation are summarized, the original contributions are noted, and suggestions are made for future work.

8.2 Conclusions

The axial profiles of both thermal wall loading and radiation were measured for a variety of vortex flow conditions. All the experiments were carried out in argon at a pressure of 5.5 atm and an arc current of 225 A. The experiments have shown that the central section of the arc exhibits a plateau region both in wall loading and radiation output. In this region the properties of the arc are approximately axially invariant.
At the ends of the arc however, the electrodes exhibit a profound influence on the wall loading and the radiation. The wall loading is increased in the immediate neighbourhood of the electrodes, particularly near the cathode, where it may be twice as large as the value in the plateau region. The radiation shows a similar peak associated with the cathode flame, as well as one associated with the anode, if the anode is located downstream in the vortex.

The effect of the electrode regions of the arc is to increase the overall wall loading of the arc vessel, above that due to the positive column alone. It is, in part, because of this that previous measurements which averaged over the whole arc vessel found wall loading values that were unexpectedly large. If one wishes to measure the characteristics of the positive column, one must therefore use a technique such as the partitioned tube, which allows one to measure the local value of wall loading adjacent to the positive column (see Figure 8.1).

In practical terms, the existence of peaks in the distribution of heat flux is important for the design of quartz walls or water walls for arc vessels. It is particularly interesting that by moving the upstream electrode in the vortex, a tradeoff may be made between the heat which flows from the electrode plasma to the wall, and the heat which flows to the electrode. This could have important consequences for electrode erosion and electrode lifetime in arc lamps.

The measurements of the axial profile of the wall loading have also shown a rapid onset of the wall loading immediately adjacent to the upstream electrode and a marked fall in loading adjacent to the downstream electrode. It was found that the position of onset could not be driven measureably downstream even when the mass flow rate of the gas was doubled. Furthermore the axial profile of radiation showed that there was no significant enhancement of radiation in the upstream portion of the arc column. These two aspects of the arc behaviour disagree with the predictions of the disc model of axial convection where the axial velocity is
Figure 8.1 Wall loading profile showing the plateau region and the high heat flux near the electrodes.

assumed to be uniform. Furthermore, measurements in the plateau region have shown that there is power balance in a radial slice of the arc, to within 20% or better. Thus axial convection does not play a dominant role in the heat transport from the column.

Another feature of the wall loading profile is the existence of wall loading upstream of the upstream electrode. This, together with evidence from photographs of the electrode plasmas and the trajectories of tungsten droplets eroded from the electrodes, indicates that the arc vortex contains a reverse axial flow core. The behaviour of this core has been shown to depend on the vortex flow conditions. An increase in the azimuthal velocity of the vortex results in an increase in the
reverse flow in the core, and consequently increased thermal wall loading upstream of the upstream electrode.

A consideration of the overall energy balance of the arc has shown that there is insufficient power to heat the gas to an equilibrium temperature profile if the axial velocity is uniform across the arc. Since a plateau region with approximately invariant arc properties exists, most of the gas must flow through a layer near the wall, where the temperature is relatively low.

The axial flow pattern is therefore seen to be quite different from that assumed in the disc model of axial convection. Since most of the incoming gas passes through a relatively thin cool layer near the wall heat may be conducted across this layer from the arc to the wall relatively quickly. The enhancement of thermal conductivity by turbulent heat transport also ensures the rapid transport of heat across this layer. Furthermore, the reverse flow core carries heat upstream from the arc, and this has the effect of preheating the incoming gas. These features of the gas flow in the arc vessel, when taken together, allow us to explain the rapid onset of wall loading adjacent to the upstream electrode (see Figure 8.2).

The immediate onset of wall loading, and the achievement of a plateau region closer to the upstream electrode than predicted from the disc model of axial convective have implications for arc design. The wall loading and radiation in the upstream end of the arc column have much different values than they would have if the axial velocity were uniformly distributed across the radius of the arc. Existence of a reverse flow core is also important to the understanding of electrode erosion studies. Studies on arcs of the type used in this work have shown that the erosion rate for the anode is higher when the anode is located downstream in the vortex. This observation may be interpreted in terms of the flow pattern in the vortex. The wall loading measurements and the photographs of the electrode plasmas show that the reverse flow core extends to a radius greater than that of the electrode.
Figure 8.2 Overall pattern of heat transport and gas flow, showing reverse flow, high mass flow near wall, and increased heat transport due to turbulence.

Consequently the flow direction at the electrodes will be opposite to that expected naively from the location of the inlet jets and the exit ports in the chamber. In this work it has been shown that the anode plasma is much hotter and more concentrated when the anode is located "downstream". In fact the anode is actually upstream in the local flow, and the plasma is confined to the face of the electrode by the flow of gas from behind the electrode. Thus an anode located "downstream" with respect to the inlet jets is exposed to a hotter, more concentrated clump of plasma, and might be expected to exhibit higher erosion rates.

In an arc such as the Vortek arc lamp, on the other hand, the situation may be different. In this case, the gas vortex is in contact with a rotating water wall instead of a stationary quartz tube. Because of this, the vortex will be more vigorous at the gas exit end of the chamber, and a reverse flow core may not exist. Caution must therefore be exercised in predicting electrode erosion rates for a Vortek type
lamp, based on experiments conducted in an arc vessel such as the one used in this work.

In the plateau region of the arc, comparisons may be made between the measured local values of the arc properties and the theoretically predicted values. The wall loading in the plateau region has been found to scale almost linearly with inlet jet velocity. This has been shown to be consistent with the scaling expected from a turbulent heat transport model which uses mixing length theory. Furthermore, it has been found that when the data is extrapolated back to the no-flow condition, the value of wall loading is in good agreement with the prediction of the Elenbaas-Heller equation with no turbulent heat transport.

This enhanced radial transport influences the performance of practical arc lamps in two ways. First, since the radiation is not as strongly affected by the flow conditions as the wall loading (neither in the predictions of theory nor in the experimental measurements) the enhanced radial transport reduces the radiative efficiency of the arc. Second, the increased value of wall loading, in itself, affects the performance of the quartz or water wall surrounding the arc.

A time dependent model for the arc has been developed to account for the effect of the ripple in the power supply waveform. The measured time dependent values of ohmic heating and thermal wall loading in the arc agree reasonably well with the model, if mixing length theory is used to include turbulence in the model.

The radiation measurements however, show that the arc radiates about 30% less power than expected from the time dependent model, even when the enhanced radial transport due to turbulence is taken into account. While the mechanism responsible for this is not understood in detail, an argument has been put forward which shows that the cause may lie with the axial transport of energy in the arc. The experiments have shown however, that the overall effect of this axial heat flow
is limited to transporting 20% or less of the electrical energy input in the plateau region of the arc.

In summary, it is found that because of the axial flow patterns which exist in the arc, the arc is not strongly affected by axial convection. The onset of wall loading therefore occurs immediately adjacent to the upstream electrode, and the arc exhibits a plateau region where the wall loading and radiation are approximately axially invariant. In this region the thermal wall loading is found to increase with the flow rate of the gas in the arc in a way which agrees with the predictions of turbulent heat transport theory. The zero flow wall loading obtained by extrapolating these data agrees well with the non-turbulent Elenbaas-Heller equation prediction. The measured values of total input power and thermal wall loading are found to be reasonably well described by the time dependent turbulent Elenbaas-Heller equation. The measured radiation values are about 30-35% less than that expected from theory.

8.3 Original Contributions

These measurements of the wall loading profile are the first axially resolved measurements of the thermal wall loading in a vortex stabilized arc. It is important that in these experiments, it is also possible to measure the total radiation output of the arc at the same time. The measurements have been made possible by the development of a partitioned tube arc vessel, with a replaceable inner tube. The ability to replace the inner tube was crucial to these experiments since it allowed a new tube to be fitted when the old tube became discoloured or was cracked by the arc. The experiments have demonstrated that the electrode regions of the arc make an important contribution to the wall loading. The measured profiles have also allowed us to establish the existence of a reverse axial flow core by measuring its influence on the heat transfer in the arc. Using the upstream wall loading as a measure of the reverse flow, it has been shown that increasing the azimuthal flow
velocity in the vortex increases the reverse flow. In addition, it has been pointed out that the existence of reverse flow in the vortex is important in interpreting other experiments, which show that the anode erosion rate depends on the position of the anode in the vortex.

Measuring the wall loading locally in the region adjacent to the positive column of the arc has allowed a comparison between theory and the actual wall loading in the positive column of a vortex stabilized arc. Using these results it has been shown that it is necessary to invoke turbulent heat transport to account for the measured values of wall loading. A model for the arc has been developed which uses mixing length theory to account for turbulent heat transport in the arc. Furthermore it was found that the effect of axial convection is limited to transporting 20% or less of the input power in the plateau region.

Finally, as described in Appendix A, the technique of double exposure speckle photography has been used to measure the flow velocity of an aerosol seeded gas in a pipe.

8.4 Suggestions for Future Work

It would be useful if experiments could be done with a low ripple d.c. power supply. This would simplify the arc system, by removing time dependent effects, and facilitate the comparison with theory. If a three phase power supply with full wave rectification were used the ripple could be reduced to less than 5%. It would then be useful to repeat the partitioned tube experiments and the radiation profile measurements.

Further progress could also be made if the flow velocities and directions in the arc chamber could be measured while the arc is running. At this time this has not proved practical because of the difficulties of making these measurements in the intense heat and radiation fluxes produced by the arc.
Another approach to this problem would be to try to produce a simpler flow pattern in the vortex, by eliminating the reverse flow core. It would appear possible to do this by spinning the arc vessel like a centrifuge. The gas would enter and exit the chamber through porous plugs at each end, and should maintain reasonable solid body rotation, free from reverse axial flow. An arc vessel such as this is shown schematically in Fig. 8.3. While it would not be practical to measure the axial profile of thermal wall loading in such an apparatus, one could measure the radiation profile, and determine the electric field in the arc column using the same techniques described in this thesis. Such experiments would allow a direct comparison between the predictions of the non-turbulent Elenbaas-Heller equation, and the results from a vortex stabilized arc. In practical terms the gas flow rate could be reduced to zero, since the vortex is driven by the rotating vessel itself. This should result in an increase in the radiative efficiency of the arc.
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Figure 8.3 The rotating arc vessel. Electrical connections to the rotating electrodes would be made using slip rings.


APPENDIX A

Velocity Measurements

A.1 Introduction

It would be beneficial to our understanding of the arc if the flow pattern in the arc vortex were known. However, because of the high temperatures and large radiation fluxes associated with the arc, it is very difficult to make direct flow measurements with the arc running. In order to obtain an approximate idea of the properties of the vortex, a series of measurements was therefore made in the gas vortex without the arc running. While there will obviously be differences between the flow pattern with and without the arc, the cold gas measurements might be expected to show some of the overall features of the vortex.

In particular, one would like to look for the presence of a core of reverse axial flow in the cold vortex. Certain results from the partitioned tube experiments are best explained in terms of such a reverse flow pattern, and its existence in the cold gas vortex would lend support to this hypothesis. Furthermore, in order to calculate the effect of the centripetal field on the Elenbaas-Heller equation one needs to know the angular velocity of the vortex. By measuring the azimuthal gas velocity near the jets in the cold vortex, an approximate idea is obtained of the angular velocity of the vortex which impinges on the arc.

Some idea of the reliability of these measurements may be had by looking at the overall characteristics of the gas flow, with and without the arc running. When the arc is switched on, the pressure measured in the arc vessel increases by about
$1\frac{1}{2}\%$, while the mass flow rate decreases by less than $10\%$. Thus the flow through the gas inlet jets is disturbed very little by the presence of the arc.

### A.2 Velocity Direction Measurements

In order to test for the existence of a reverse axial flow core in the vortex, measurements were made of the flow direction. It was assumed that the radial flow velocity is negligible throughout most of the vortex and so only the contributions of the axial and azimuthal flow components were considered.

The flow direction was measured with a two port differential pressure probe as shown in Figure A.1. This probe was similar to the one used by Andrada and Erfurth (1963). The two ports were drilled 90 degrees apart and each port is connected to one end of a differential manometer. If the tube is rotated until the differential pressure is zero, the bisector of the two ports is then known to be aligned against the direction of the flow (Fig. A.1). A protractor clamped to the tube allows the flow angle to be measured.

The direction measuring system was mounted along a diameter in a lucite tube, of the same bore as the inner quartz tube of the arc vessel, as shown in Figure A.2. A seal was made between the lucite tube and the direction probe by means of an O-ring. This allowed the probe to be rotated, and to be traversed radially across the arc vessel, so that the flow direction could be measured as a function of radius. Provision was made to mount gas inlet jets at one end of the lucite tube and an assembly of gas exit holes, identical to those in the heat exchanger, at the opposite end, so that the flow pattern in the arc vessel could be reproduced.

Direction measurements were made with each of the three inlet jet assemblies used in the arc experiments, and for each jet a representative range of gas mass flow rates was covered. The measurements were made along a diameter of the arc
Fig. A.1 Flow direction measurement apparatus. Two pressure measurement holes are drilled 90 degrees apart in a rod as is show in cross section above. Each hole is connected to one end of a differential manometer so that the pressure difference between the two may be measured. To measure the flow direction the probe is rotated until the pressure difference is zero. The bisector of the angle between the two holes then points in the direction of the incoming flow, and the flow angle (β) may be determined.

A vessel at a position 6 mm upstream of the end of the upstream electrode. A hole was drilled through the electrode to allow the probe to pass through (see Fig. A.2). In these experiments the flow direction is quoted in terms of the angle measured from pure downstream flow. Flow whose axial component is directed from the jets to the exit is designated by angles less than 90 degrees, while reverse flow angles are greater than 90 degrees. Thus pure circumferential flow would have a flow angle of +90 degrees.

For all the vortices investigated in these experiments a reverse flow core was found in the region adjacent to the electrode. Typical flow direction results are shown for two different inlet jet assemblies in Figure A.3. For a given set of jets, it
Figure A.2 The apparatus used to measure the flow direction in the cold gas vortex.

was found that the flow direction pattern was almost independent of the gas flow rate. The flow angle at any point changed only ±2 degrees when the mass flow rate was increased from 2 g/s to 4 g/s. Nevertheless, at the higher mass flow rates the flow velocities would be higher, even though the flow angles remained the same.

A noticeable change in flow pattern was apparent however, when changes were made in the type of inlet jets being used. The reverse axial flow became more and more pronounced as the jets were changed from the large inlet angled jets, to the large inlet azimuthal jets and finally to the small inlet azimuthal jets. In the case of each change the boundary radius of the reverse flow core increased and the angle of the reverse flow in the core also increased. This effect can be seen...
Figure A.3 Flow direction vs. radius in the cold gas vortex. Measurements are shown for both sets of azimuthal jets. Since it was found that the flow direction is quite insensitive to mass flow rate, the data for each set of jets represents an average over the range of mass flow rates used in the arc experiments.
in the flow direction pattern measured with the large inlet angled jets and the small inlet azimuthal jets as shown in Figure A.3.

A.3 Flow Velocity Measurements

In order to estimate the azimuthal velocity in the gas vortex, a series of measurements were made of the static pressure in a lucite model of the arc chamber. This model was once again constructed with the same bore as the inner quartz tube, and had an identical set of exit holes in the downstream end (see Fig. A.4). In order to accommodate a number of static pressure taps at the upstream end, the inlet jet assembly was modified slightly. Inlet holes were drilled through the walls of the lucite so as to emerge tangent to the inside diameter (Fig. A.4). Two different jet hole sizes were used corresponding to the small and large inlet azimuthal jet assemblies used in the arc experiments. In these experiments only pure azimuthal jet injection could be obtained; there was no provision to make measurements on the angled jet assembly.

In the end wall, at the upstream end of the chamber, seven static pressure tap holes were drilled, spaced evenly along a radius from the chamber centre-line to the outside wall. Tubing leading from each of the pressure tap holes allowed the differential pressure between them to be measured by means of a methanol manometer. In order to extract velocity information from the pressure difference measurements, some assumptions must be made about the flow. It was assumed that the flow in the chamber is predominantly azimuthal in direction, so that the effects of variations in axial and radial flow velocities on the static pressure at the pressure tap holes may be neglected. The radial pressure gradient is then just that needed to provide the centripetal acceleration of the flow:

\[
\frac{dp}{dr} = \frac{\rho v^2}{r} \tag{A.1}
\]
Figure A.4 The apparatus used to measure the flow velocity in the cold gas vortex. The pressure taps in the upstream endcap wall of the vessel could be connected to a differential manometer, so that the pressure difference between any two ports could be measured.

where: \( v = \text{azimuthal velocity} \).

Since the pressure difference between the centre and outside of the vortex is small, the density \( (\rho) \) may be assumed to be constant, independent of radius. If the \( i^{th} \) pressure tap is at a radius \( r_i \) (refer to Figure A.5 for nomenclature) and the pressure tap immediately inside it, (the \( (i - 1)^{th} \) tap) is at a radius \( r_{i-1} \), then the pressure difference between adjacent taps is:

\[
p_i - p_{i-1} = \Delta p = \rho \int_{r_{i-1}}^{r_i} \frac{(v(r))^2}{r} \, dr. \tag{A.2}
\]

If we further assume that between pressure taps the vortex velocity varies linearly with radius, then equation A.2 may be integrated. This assumption corresponds to linearizing the velocity profile in the neighbourhood of the midpoint.
Figure A.5 Nomenclature for the vortex velocity measurement calculation. It is assumed that between pressure measuring taps the azimuthal velocity of the gas \((v(r))\) varies linearly with radius \((r)\).

between the pressure taps, so that the velocity there may be written as:

\[
v(r) = v(r_{i-1}) + (r - r_{i-1}) \frac{\Delta v}{\Delta r},
\]

(A.3)

where: (see Figure A.5)

\[
\Delta v = v(r_i) - v(r_{i-1})
\]

\[
\Delta r = r_i - r_{i-1}.
\]

Substituting equation (A.3) into equation (A.2) and performing the integration, one
obtains, after some manipulation:

\[
\frac{\Delta p}{\rho} = \left(\frac{\Delta v}{\Delta r}\right)^2 \left(\frac{r_i^2}{2} - \frac{r_{i-1}^2}{2} - 2\Delta r r_{i-1} + r_{i-1}^2 \ln\left(\frac{r_i}{r_{i-1}}\right)\right) + \left(\frac{\Delta v}{\Delta r}\right) \left(2\Delta r v(r_{i-1}) - 2r_{i-1} v(r_{i-1}) \ln\left(\frac{r_i}{r_{i-1}}\right)\right) + v^2(r_{i-1}) \ln\left(\frac{r_i}{r_{i-1}}\right)
\]  

(A.4)

Using the measured pressure difference (\(\Delta p\)) and the known radii of the pressure ports (\(r_i, r_{i-1}\) and \(\Delta r\)) this equation may be solved to determine the velocity difference between any two ports (\(\Delta v\)). It was assumed that the velocity is zero on the axis of the vessel (\(v(r_i) = 0\)), so that the velocity at all the ports may be calculated by starting from the centre and working outward (\(v(r_i) = v(r_{i-1}) + \Delta v\)).

Figure A.6 shows the results of measurements taken with the small inlet (azimuthal) jets on the lucite model, at three different mass flow rates. The vortex seems to show reasonable solid body rotation near the centre, but there is clearly evidence of the effect of the inlet jets in the high velocities near the wall. Repeating the experiments with the large aperture jets and the same mass flow rates produced results that were qualitatively similar but with lower flow velocities.

The most important result of these experiments was the estimation of an upper bound on the vortex angular velocity for use in the calculations on an arc running in a centripetal field. The experiments have shown that none of the jets used in the arc experiment produced maximum azimuthal velocities in excess of 200 m/s, while throughout the core of the vortex the velocities were typically 20 m/s or less. The angular velocity for solid body rotation based on 200 m/s at \(r = 1.2\) cm is \(\omega = 1.67 \times 10^4\) rad/s. Since the solid body rotation in the centre of the vortex is much slower than this, such a figure will certainly provide an upper bound for the calculation of the effect of the vortex on the arc.
Figure A.6 Velocity profile of the vortex at three mass flow rates. The arc was run using the small azimuthal jets.
A.4 Double Exposure Speckle Photography

The static pressure measurements which have been described in the previous section are limited in that they measure only the azimuthal component of the vortex velocity, and only in the plane of the back wall of the arc chamber. In order to overcome these limitations an attempt was made to measure flow velocities in the cold gas vortex using double exposure speckle photography. This technique has been used previously to measure displacements in transparent solids (Barker and Fourney (1976)) and velocities in a liquid (Barker and Fourney (1977)).

This technique was chosen primarily because it allows the measurement of velocities throughout the vortex, without the disturbance produced by inserting flow measuring probes into the vortex. A further advantage is that the velocity everywhere in a cross section through the flow is obtained in a single "snapshot" of the flow.

As an introduction to the technique, consider the setup shown in Figure A.7. The diffuser is illuminated with a pulsed laser beam and the lens images the diffuser onto the film plane. Because the diffuser is illuminated with coherent light its image on the film will be covered with a speckle pattern. Consider then the effect of shifting the diffuser in a direction perpendicular to the line of imaging (as shown in Fig. A.7) and making a second exposure on the plate with a second laser pulse. Since the relative phase at all the scattering centres on the diffuser remains unchanged by this shift, the second exposure will simply be a shifted version of the original speckle pattern. The amount of movement of the diffuser between exposures may be determined by measuring the separation of the correlated speckles on the doubly exposed film and accounting for the magnification of the imaging system.

To measure the spacing of the speckle pairs on the developed, film a HeNe laser is used to illuminate a portion of the film as shown in Figure A.8. The screen
Figure A.7 Illustration of the principle of the double exposure speckle photography measurement.

is placed so that the far field diffraction pattern may be observed. The zero order stop serves to remove the undiffracted HeNe beam. Since the speckle pattern on the film is a random pattern, it acts as a diffuser. Thus, the far field diffraction pattern will be covered with speckle. However, since the doubly exposed film is actually covered with pairs of speckles the diffraction pattern will be modulated with Young's interference fringes. A typical diffraction pattern obtained from a shifted diffuser is shown in Figure A.9.

By measuring the angular separation of the Young's fringes (angle $\alpha$), the separation of the speckle pairs on the film ($\xi_i$) may be determined

$$
\sin \alpha = \frac{\lambda}{\xi_i}
$$

(A.5)

where $\lambda$ is the wavelength of the probe light $\lambda_{HeNe} = 632.8$ nm.
Figure A.8 The measurement of the speckle separation. The pairs of speckles in the doubly exposed film modulate the diffraction pattern with Young's fringes. By measuring the angular separation of the fringes, the separation of the speckles on the film may be determined.

One limitation of the double exposure speckle photography technique is in the amount of displacement of the diffuser which it can measure. There is a lower bound, given by the requirement that the displacement be sufficient to move the speckle pattern at least one speckle diameter. If this requirement is not met, there will not be distinct speckle pairs with which to form Young's fringes. On the other hand, the displacement in the film plane should not be much greater than \( \sim 20 \) speckle diameters, since this would crowd the fringes together in the diffraction pattern and make the fringe spacing difficult to measure.

The speckle size in the film plane is determined by the \( f/# \) of the imaging lens and the wavelength of the light used in making the exposure (\( \lambda \)). The smallest speckles will have a diameter comparable to the diffraction spot size produced by
Figure A.9 A typical diffraction pattern obtained when measuring the speckle separation.

the lens

\[ s_i = 1.22\lambda f/\#(m + 1) \]  \hspace{1cm} (A.6)

where \( m = d_i/d_o \) = magnification. In these experiments the imaging was done with \( f/4 \) optics at a magnification of 2 and a pulsed ruby laser (\( \lambda = 694.3 \text{ nm} \)) was used. This gave a speckle size \( s_i \approx 10 \mu\text{m} \) on the film which corresponds to a speckle size of 5 \( \mu\text{m} \) in the object plane. Therefore movements of 7 to 100 \( \mu\text{m} \) could be observed.

A.4.1 The Laser System

To implement such a double exposure speckle photography system one needs a laser which will produce two pulses of roughly equal energy. Ideally the temporal separation between the pulses should be easily adjustable. In this experiment this was achieved by double Q-switching a ruby laser oscillator. The laser cavity is
set up much as it would be for normal Q-switch operation (see Fig. A.10). Some modifications are made, however, to the Pockels cell circuitry, in order to extract two pulses. The technique used is similar to that of Wetzels and Alfs (1969), except that in this case cable discharges are used to control the Pockels cell voltage.

![Diagram of laser oscillator](image)

**Figure A.10** The laser oscillator used to produce a double Q switched laser pulse.

While the rod is being pumped, the Pockels cell voltage is held at its $\frac{1}{4}$ wave value, so that the Q of the cavity remains low and a substantial inversion is built up. When the first pulse is desired, the voltage on the Pockels cell is lowered but not dropped to zero. The value of the voltage is chosen such that the Q of the cavity allows approximately one half of the inversion to escape in a Q switched pulse. After sufficient time has elapsed to ensure the generation of a Q switched pulse the Pockels cell voltage is returned to its $\frac{1}{4}$ wave value. When the second pulse is desired, the voltage on the Pockels cell is dropped to zero, so that the remainder
of the inversion is Q switched out. The time between pulses may varied from 1 to 50 μs. Since the characteristic time for pumping the rod with the flashlamps is of the order of 1000 μs there is a negligible amount of pumping between the pulses.

In practice, the voltage applied to the Pockels cell is controlled using several cable discharges, as shown in Figure A.11. While the rod is being pumped cable 2 is held charged at $V_{\frac{1}{4}}$, the $\frac{1}{4}$ wave voltage of the Pockels cell. Cable 1 is uncharged, so that the full $\frac{1}{4}$ wave voltage appears across the Pockels cell. Cable 0 is, however, charged to an adjustable voltage $V$. When the first pulse is desired cable 0 is shorted into cable 1 to produce a square pulse of voltage $\frac{1}{2}V$ and of duration equal to the round trip time of cable 0. This pulse is applied to the "ground" electrode of the Pockels cell to reduce the voltage across the cell and allow the first Q switch pulse to be generated. The value of $V$ may be adjusted so that roughly half the inversion is dumped in this pulse. When the second pulse is desired cable 2 is shorted so that the voltage across the Pockels cell is dropped to zero. Figure A.12 shows the timing of the pulses.

In order to monitor the energy in the two pulses light leaking through the back mirror of the cavity was directed onto a photodiode. The value of voltage $V$ was then adjusted until two equal energy pulses were produced.

Following their generation in the oscillator the two pulses were passed through a single ruby rod amplifier so that the energy in each pulse could be boosted to as much as 250 mJ.

A.4.2 The Aerosol System

To make measurements of fluid velocities using this system, the fluid must act as a diffuse scatterer, so that its displacement may be measured as discussed in the preceding section. In order to arrange for this in a gas flow, an aerosol was injected into the inlet gas stream. If the droplets are to follow the flow to a useful
Figure A.11 The cable discharge circuit used to drive the Pockels cell. The switching was achieved using Krytron gas filled switching tubes (EG & G model KN-22).

degree of approximation they must be less than 1 $\mu$m in diameter. Aerosols of this size range may be produced in an aerosol generator similar to those developed at N.R.L. (Echols and Young (1963)). The generator produces an aerosol of oil droplets by forcing the carrier gas through a small slit orifice under the surface of an oil bath (see Figure A.13). In the high shear regions at the jet/liquid boundary small droplets of oil are torn from the liquid and entrained within the gas flow. The gas bubbles to the surface and then is carried to a second chamber where the larger droplets are stripped from the flow. This is done by accelerating the flow through a nozzle and letting it impinge on a flat plate. This has the effect of introducing a strong curvature into the flow. Droplets larger than a certain size are unable
Figure A.12 The timing of the Pockels cell voltage pulses. The voltages refer to the labelled voltage reference points in Figure A.11.

To follow the curvature of the flow and impact on the flat plate. By adjusting the distance from the nozzle to the plate, the upper limit on the drop size may be varied (Echols and Young (1963)).

For these experiments the geometry was set to produce droplets < 1 \( \mu m \) in diameter. The average droplet diameter was 0.7 \( \mu m \) and the concentration of droplets in the carrier gas was \( 7 \times 10^5 \text{ cm}^{-3} \). This implies an average interdroplet spacing of \( \sim 110 \mu m \) and has the effect of increasing the average density of the fluid by 0.1% .

During this experiment a problem arose in that the oil (Di(2-ethylhexyl)-phthalate or Di-octyl-phthalate) chosen by N.R.L., partly because it was non-toxic,
Figure A.13 The aerosol generator.

was reclassified as a suspected carcinogen. Therefore some time was spent re-searching the literature on D.E.H.P. before proceeding with the experiment. The study which resulted in the reclassification of D.E.H.P. was a feed study using rats and mice, done for the National Toxicology Program (National Toxicology Program T.R.S. No. 217). Based on the levels of D.E.H.P. per unit body weight used in this study, and on values for maximum permissible concentrations of D.E.H.P. in air (from the Handbook of Dangerous Materials) it was decided to proceed with the experiment subject to certain precautions. The exhaust from the system was filtered with a two stage coalescing filter to remove 99.9% of all droplets in the size range of the aerosol. After filtering the gas was vented out of doors with a fan. Furthermore, all components used in the experiment were stored in sealed plastic
bags. The plastic parts were disposed of by high temperature incineration, while the metal ones were rinsed in varsol under a fume hood and then heated with a flame to burn off any residue. The varsol was disposed of by high temperature incineration. In conclusion, it should be noted that for an oil to be useful in generating small aerosol droplets it should have a low vapour pressure. This fact is beneficial when handling the liquid, once the aerosol has been allowed to settle out.

A.4.3 The Velocimetry System

In order to use the speckle principle to measure flow velocities in a cold gas vortex a lucite model of the arc chamber was used. Figure A.14 shows a diagram of the apparatus.

![Diagram](image)

Figure A.14 The vortex velocity measurement system.
The gas flow was first passed through a calibrated venturi, so that the mass flow rate could be measured, and then through the aerosol generator. Finally, it was introduced into the vortex chamber through two tangential jets. The gas exited from the chamber through five axial exit holes, just as in the arc chamber.

The vortex was illuminated along a diameter with a 2 mm diameter beam from the doubly pulsed ruby laser. The scattered light from the laser was imaged by the camera lens onto the film. A clear lucite end window in the vortex chamber made this feasible (see Fig. A.14). For these experiments, the magnification of the camera system was 2 and the imaging was done with f/4 optics, so that the speckle size in the film plane was $10\,\mu m$. This small speckle size means that high resolution film must be used. In these experiments two types of holographic plates were used, Kodak Type 120-02 and Agfa 10E75, both of which have resolutions of 1000 lines/mm or better. The temporal separation used between the two pulses was $6\,\mu s$, which for the anticipated circumferential velocities in the vortex, would give a speckle pattern displacement of approximately 6 speckle diameters.

After exposure and development the plate may be scanned with a HeNe laser and the velocity determined at a number of points across the diameter.

As a check on the velocity measuring technique, a free jet was also set up so that measurements could be made on a system whose velocity profile could be determined from simple theory. This system is shown in Figure A.15. Using the same technique just described for the vortex, one can measure the radial profile of the axial velocity in the jet.

### A.4.4 Results and Conclusions

When the experiment was run on the vortex chamber the speckle patterns showed no fringes at any radius in the flow. In order to ensure that the velocity had not simply been mis-estimated, shots were done with pulse separations ranging from
Figure A.15 Apparatus used to measure the velocity profile in a gas jet. The measurement was made immediately above the end of the inner tube, so that the velocity profile could be compared to the theoretical profile for pipe flow. The flow of gas in the outer tube was adjusted so as to stabilize the gas jet.

$6 \mu s$ to $15 \mu s$. These were chosen to bracket the expected speed as measured from the pressure probe experiments. Furthermore, since the circumferential velocity exhibits a radial profile, a scan across the radius would give velocities that ranged over at least an order of magnitude. Despite the use of several pulse separations and scanning over the vortex vessel radius, nowhere were any fringes found in the flow field.

In order to check the validity of the speckle technique for measuring velocities in gas flows and to try to understand the absence of fringes in the vortex experiments, the free jet experiments described in the last section were undertaken. Experiments were begun with slow flows in the jet ($Re=1000$) and it was found that clear sets of fringes could be obtained from the developed doubly exposed
plates. When these plates were scanned with a HeNe laser a radial velocity distribution could be obtained. The velocity distribution measured for the jet is shown in Fig. A.16. The peak velocity which occurs on axis was measured to be $1.34 \text{ m/s}$. This agrees well with a value of $1.38 \text{ m/s}$ calculated from the mass flow rate through the pipe measured by the venturi and the velocity profile for laminar flow (Massey (1970) p.143)

$$v(r) = 2v_{avg} \left(1 - \frac{r^2}{R^2}\right) \quad (A.7)$$

where:

$v_{avg}$ = average velocity of fluid flow
$R$ = radius of the pipe

The mass flow rate of the jet was then increased and the experiment was repeated. When the Reynolds number of the jet reached $\sim 2000$ the fringes evident in the speckle pattern began to degrade. At some points in the flow field the contrast of the fringes was reduced, while at other points the fringes disappeared altogether.

It seems that what is happening is that between exposures the scattering centres move with respect to each other in the flow. This has the effect of changing the relative phase of the scattered wave from each droplet so that the two speckle patterns created in the film plane are different. Thus instead of having two speckle patterns which are shifted versions of one another, one has two uncorrelated speckle patterns. The lack of correlation destroys the fringes and as well any hope of extracting information on the amount of shift the object has undergone between exposures. If the relative shift of the scatterers is small, or if only a small fraction of them are shifted, a fringe pattern may still be seen, but with reduced fringe contrast.
Figure A.16 Velocity profile measured in the gas jet. The data are compared with the theoretical velocity profile for pipe flow (solid line) based on the measured mass flow rate of the gas and the pipe diameter.
This experiment does, however, allow one to determine a lower bound for the turbulence velocities in the flow. In order to change the speckle pattern between exposures, a typical scattering centre would have to move of the order of half a wavelength with respect to its neighbours. In this experiment this implies a movement of 0.35 $\mu$m in a time between exposures of 6 $\mu$s. This corresponds to a velocity of 0.06 m/s. One might think that the problem caused by relative motion of the droplets could be overcome by simply shortening the time between exposures, but it must be remembered that the speckles must be displaced by at least one speckle diameter in order to make their separation measurable. Alternatively, it would help to make the f/# of the camera larger, but this reduces the amount of scattered light being collected. Since high resolution film must be used the film speed is quite slow. It was therefore not feasible to increase the f/# substantially and still collect enough light to obtain an adequate exposure.

For laminar or mildly turbulent flows therefore, the double exposure speckle photography technique may be successful, but for highly turbulent flows it seems inappropriate. In the case of the vortex flow used here it gives a lower limit on the fluctuating part of the instantaneous velocity of 0.06 m/s. Taking typical values of mean flow velocity from the pressure tap measurements of 5 to 10 m/s, this gives a turbulence intensity ($v_{fluctuating}/v_{average}$) of 1% as a lower bound. It seems therefore that the cold gas vortex is turbulent.