DOSIMETRY STUDIES OF SMALL FIELDS IN HOMOGENEOUS AND INHOMOGENEOUS MEDIA FOR HIGH ENERGY PHOTONS

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ABSTRACT

Dose decreases rapidly for photon field sizes smaller than the range of the laterally scattered electrons. The reduction in dose leads to dose non-uniformity and the degree of dose non-uniformity depends on the shape of the tumor. The dose reduction due to lateral electronic nonequilibrium increases with increasing photon energy. Small tumors are best treated with lower energy photons.

We modified the primary and scatter dose model to include the effect of lateral electronic nonequilibrium. The dose model was verified experimentally for various geometries and is in good agreement with the measurements.

We developed a new cavity theory which includes secondary electron backscattering from the medium into the cavity. The proposed theory gives better agreement with experiments in aluminium, copper and lead for Co-60 $\gamma$-rays and 10 MV x-rays than do the Burlin and Kearsley cavity theories. A method for obtaining an ionization chamber correction factor for measuring dose in inhomogeneous media is also given.

There is a significant dose reduction in lung as compared to normal density tissue for small fields. The dose reduction in lung increases with decreasing field size and increasing photon energy. Results of the measurements suggest that a tumor in tissue surrounding lung would have better dose uniformity if the direction of the photon beam is such that the tumor resides in the proximal side of the tumor-lung interface. Tumors in lung and surrounding the lung have better dose uniformity if treated with lower energy photons.
Significant dose reduction was also observed near the air-tissue interface. The dose perturbation increases with increasing air-cavity thickness, decreasing field size and increasing photon energy. Results of the measurements again suggest that a tumor in tissue surrounding air-cavities, such as the bronchial tube, would have better dose uniformity if the direction of the photon beam is such that the tumor resides in the proximal side of the tumor-air interface. And again, lower energy photons provide better dose uniformity for tumors surrounding the air-cavity.

The presence of bone in tissue causes only modest dose perturbation for photon energies between 2 MV and 20 MV x-rays.
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Dedication

To my parents
1.1 Small fields in radiotherapy

The goal of small field irradiation is to deliver a uniform dose to a well defined small target with a minimal dose to the surrounding normal tissue. Small field irradiation was first used as stereotactic radiosurgery* in 1951 by the Swedish neurosurgeon Lars Leksell. He used a large number of small field sizes of 200-300 kVp x-ray beams at different angles circumferentially along the stereotactic frame (Leksell, 1951, Leksell et al, 1955). In 1968, Leksell started clinical studies with a gamma unit (Gamma Knife) containing 179 sources of Co-60 to produce converging narrow beams (Leksell, 1963). However, due to the high capital and operating cost of the Gamma Knife, various linac-based techniques have been developed.

There is ongoing research on extending radiosurgery to conformal therapy to treat lesions other than intracranial lesions to obtain better tumor control probability (McGinley et al, 1992). Conformal therapy is a technique where beams are conformed to the tumor volume and thus minimize the treatment volume. Conforming to the tumor volume almost always requires an irregularly shaped small field. This demands that accurate dose distribution as well as the optimum photon energy be known for any tumor shape.

* Discipline specific technical terms are given in the glossary pp 196-199.
1.2 Dosimetry of small fields

The dosimetry of small field sizes is significantly different from that for standard radiotherapy field sizes. The range of the secondary electrons, which is a function of the photon energy spectrum, plays an important role in the shape of the beam profile. One of the most difficult problems in small fields is the determination of dose in situations where lateral electronic nonequilibrium exist. The concept of lateral nonequilibrium is illustrated in figure 1.1. Interaction of photons causes electrons to be set in motion. Transfer of photon energy to the charged particles, expressed as kinetic energy per unit mass at a point, is described by the term Kerma (Kinetic Energy Released in the Medium; the "a" has been added for phonetic reasons, ICRU, 1980). The absorption of the energy (dose) by the medium does not occur at the point of interaction but at some distance downstream. For simplicity, we have shown electrons as straight tracks of equal length and examine only the volume elements P, Q, and R. Electrons that are set in motion within the volume element are labelled as A, while the electrons that are generated outside the volume element are labelled B. Only the right hand side profiles of the absorbed dose and Kerma are shown in the illustration.

Volume element P is located at the central axis of the photon beam which is further away from the beam boundary than the maximum lateral range of the electrons. Photon interactions within the volume element P generate two electrons (A). Volume element P also receives from both sides two electrons (B) that are generated outside the volume element P. This volume element is considered to be in lateral electronic equilibrium. Volume element Q is closer to the photon beam boundary than the maximum
lateral range of the electrons. Photon interactions within the volume element Q generate two electrons (A). Volume element Q receives only one electron track, from the left side. Therefore, the dose at volume element Q is less than the dose at volume element P as volume element Q receives fewer electron tracks than volume element P. Volume element Q is considered to be in lateral electronic nonequilibrium. Volume element R is outside the photon beam boundary but still receives one electron track. This results in radiation penumbra at volume element R. The effect of lateral electronic nonequilibrium is such that the dose at volume element Q is less than the dose at volume element P although the

Figure 1.1 Diagram illustrating the concept of lateral electronic equilibrium.
Kerma values at volume element \( P \) and volume element \( Q \) are the same and volume element \( R \) receives some dose although the Kerma at volume element \( R \) is zero. In small fields, even the central axis may not be in lateral electronic equilibrium. The shape of the dose profile due to lateral electronic nonequilibrium depends on the energy and the angular distribution of the secondary electrons liberated by the photons and cannot be changed by reducing the source size or improving the collimator design (Duetreix et al. 1965).

The presence of inhomogeneities such as bone, lung and air cavities also modifies the dose distribution. A photon beam may have lateral electronic equilibrium in tissue-equivalent media but may lose it upon entering an inhomogeneity and vice versa. The conventional inhomogeneity correction factors may not be applicable in lateral electronic nonequilibrium.

Although the dose in lateral electronic nonequilibrium can be accurately calculated with Monte Carlo code systems, in routine practice it is rarely done because of the excessive time required for such calculations. Treatment field sizes are frequently modified during the planning phase to evaluate various plans (see appendix A). This requires that dose calculation be fast enough to allow interactive planning.

The presence of a dosimeter in a medium perturbs the charged particle fluence in the medium unless the dosimeter and the medium are identical with respect to atomic composition and density. Therefore, the response of a dosimeter depends on the geometry, construction and the surrounding medium. A cavity theory is used to relate the dose deposited in the cavity (sensitive volume of a detector) to that in the surrounding medium.
which may be of different atomic number or composition. The parameters for the cavity theories are arbitrary at present. This makes the calculation of absorbed dose very uncertain in an inhomogeneous medium from ionization measurements in an inhomogeneous medium.

1.3 Objectives

In this work, we investigate the effect of lateral electronic nonequilibrium in small field sizes in tissue-equivalent material, lung, bone and in the proximity of air cavities, as a function photon energy and field size and we propose the use of an electron perturbation factor to correct for it. We also investigate various cavity theories and develop a general cavity theory which is applicable to cavities with different front and back wall materials. The range of photon energies of interest are Co-60 γ-rays (average energy 1.25 MeV) to 20 MV x-rays. The range of field sizes of interest are 1 cm x 1 cm to 5 cm x 5 cm.

1.4 Overview

The following block diagram illustrates schematically the coordination of the two major elements of this thesis, development of a dose model and development of an appropriate cavity theory for the experimental ionization measurements which were used to test the model.
2.1. Introduction

Small field irradiation differs from large field irradiation in that small fields often do not achieve electronic equilibrium even on the central axis. Electronic equilibrium (EE) or charged particle equilibrium (CPE) is said to exist in a small volume $v$ of a medium if each charged particle of a given type and energy leaving $v$ is replaced by an identical particle of the same energy entering the volume $v$, in terms of expectation values (Attix, 1983). Consider the small volume $v$ inside a much larger volume $V$ in the same medium. The boundaries of $v$ and $V$ are such that they are separated by at least the maximum range of the secondary charged particle emitted by the photons. For electronic equilibrium to exist in the small volume $v$, the following conditions must be satisfied throughout the volume $V$:

(a). The photon beam boundary must be further away from the surface of the small elemental volume $v$ than the maximum range of the secondary electrons.

(b). The photon beam fluence must be uniform.

(c). The atomic composition of the medium must be homogeneous.

(d). The density of the medium must be homogenous.

It is evident that condition (a) would fail even at the central axis of the beam if the
CHAPTER 2. THEORETICAL

Photon beam boundary is closer to the central axis than the maximum lateral range of the secondary electrons, condition (b) would fail if there is significant photon attenuation within the range of the secondary electrons, condition (c) would fail if there is a fluctuation in composition such as bone, and condition (d) would fail in the presence of lung or air cavities. The distribution of secondary electrons produced by the high energy photons is not isotropic but are forward-peaked along the direction of the photon beam. This causes the lateral range of the electrons to be much smaller than the longitudinal range. Therefore, in conventional dose models for large fields, electron transport is not included and so these models are not applicable to small fields. Loevinger (1981) has discussed electronic nonequilibrium and provided the conceptual framework for cases when \( \beta < 1 \), where \( \beta \) is the ratio of the absorbed dose at a given point to the collision part of Kerma (kinetic energy released in the medium) at the same point (Attix, 1979). The accurate determination of the fraction of the absorbed dose caused by the "contaminating electrons" (electrons present in the incident photon beam reaching the phantom surface) has been investigated experimentally by several authors (Attix, 1983, Biggs, 1983, Beauvais, 1993) and theoretically by Monte Carlo simulations (Petti 1983, Nilsson et al 1986).

Dose is usually calculated by (i) analytical models, (ii) Monte Carlo code systems (Kijewski et al 1986) or (iii) convolution techniques (Boyer et al 1985, Mackie et al 1986, Iwasaki et al 1989, Woo et al 1990, Ahnesjo et al 1991, Bortfield et al 1993). In this chapter, dose calculation in a homogeneous medium is discussed in section A while dose calculation in the presence of an inhomogeneity is discussed in section B.
A. DOSE IN A HOMOGENEOUS MEDIUM

2.2 Review of existing dose models

2.2.1 The Primary and scatter dose model

This model is based on the assumption that the dose at any point can be separated into a primary component and a scattered component. The primary dose \( D_p \) is defined as the energy imparted per unit mass resulting from the charged particles set in motion by photons interacting with the medium for the first time. The scatter dose \( D_s \) is defined as the energy imparted per unit mass resulting from charged particles set in motion by photons that have interacted within the medium more than once and from bremsstrahlung and annihilation photons created in the medium (Ahnesjo et al. 1991). In general, the total dose \( D \) is a function of the field size in the phantom \( (r_p) \), depth \( d \) and photon energy. One defines the primary dose \( D_p(0,d) \) at depth \( d \) as \( D(r_p=0,d) \), where \( D(r_p=0,d) \) is the dose for 0 cm x 0 cm field size at depth \( d \), \( r_p \) is the radius of the radiation field size in the phantom and \( d \) is the depth in the phantom. The total dose \( D \) is given by

\[
D(r_p,d) = D_p(0,d) + D_s(r_p,d)
\]  

(2.1)
The scatter component is a function of the field radius in the phantom \( r_p \), depth \( d \) and the photon energy. To describe the scatter dose, Cunningham (1972) introduced the concept of scatter-air-ratio (SAR) which is defined as,

\[
SAR(r_p, d) = TAR(r_p, d) - TAR(0, d)
\]  

(2.2)

where \( TAR(0, d) \) is the tissue-air-ratio at depth \( d \) for a 0x0 field size. \( TAR(0, d) \) at depth \( d \) represents the primary dose and is given by

\[
TAR(0, d) = TAR(0, d_m) \cdot e^{-\mu (d - d_m)}
\]

(2.3)

where \( \mu \) is the effective attenuation coefficient, \( d \) is the depth and \( d_m \) is the depth of maximum dose. The radiation field is divided into \( n \) equal sectors of a circle. SAR is calculated in each of the \( n \) sectors and a Clarkson summation (1941) is performed to give the average scatter-air-ratio \(<SAR>\). Mathematically, this can be represented as,

\[
<SAR(r_p, d)> = \frac{1}{n} \sum_{i=1}^{n} SAR(r_p, d)
\]

(2.4)

where \( SAR(r_p, d) \) is the SAR for the \( i^{th} \) sector in the phantom with an effective field radius \( r_p \). For high energy x-rays, TAR is difficult to measure due to the size of the build-up cap required to achieve electronic equilibrium for dose measurement. Instead, the quantities tissue-maximum-ratio \((TMR)\) and scatter-maximum-ratio \((SMR)\) are used to calculate dose. These are analogous to TAR and SAR, but they are measured in phantoms in which
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electronic equilibrium can be achieved.

\( SMR(r_p,d) \) is defined by Khan et al (1980) as:

\[
SMR(r_p,d) = TMR(r_p,d) \cdot \frac{S_p(r_p)}{S_p(0)} - TMR(0,d) \tag{2.5}
\]

where \( S_p \) is the phantom scatter factor (Khan et al 1980). Again the average \( <SMR> \) can be found by,

\[
<SMR(r_p,d)> = \frac{1}{n} \sum_{i=1}^{n} SMR(r_i,d) \tag{2.6}
\]

where \( SMR(r_p,d) \) is the \( SMR \) for the \( i^{th} \) sector in the phantom with an effective field radius \( r_i \).

The primary \( (TAR(0,d), TMR(0,d)) \) and scatter dose \( (SAR,SMR) \) models assume either (i) the condition of lateral electronic equilibrium (i.e. \( r_p \geq \lambda \), where \( \lambda \) is lateral electron mean free path in phantom material) at the point of calculation is satisfied or (ii) the range of the secondary electrons set in motion by the photons is zero. Hence, the effects of any lack of lateral electronic equilibrium in the radiation absorbed dose are therefore not accounted for (Mohan and Chui 1985). Very often the phantom scatter factor is linearly extrapolated to zero from the data measured in relatively large fields or it is assumed to be close to unity for high energy x-rays, which also implicitly assumes lateral electronic equilibrium. This model is a good approximation for \( ^{60} \text{Co} \) photons and lower energy photon beams. However, for higher energy beams, the lateral range of the
electrons produced by the high energy photons may be up to several centimeters in soft tissue. Since the lateral electronic equilibrium is not considered in dose calculations in this primary and scatter model, it can lead to dose calculation errors greater than 25% in regions where lateral charged particle equilibrium does not exist. Empirical functions with arbitrary constants are sometimes introduced to describe dose distributions near the beam boundaries defined by collimators or blocks. In some applications, the "extended source model" (Cunningham 1972, Cunningham et al 1972) is used to provide a function to model the primary intensity to account for the beam penumbra. The shape of the beam profiles in the medium at the boundaries is due to the following factors: (1) the geometry of the source, collimating system, and blocks; (2) the scattering of photons from the source housing and collimating system; (3) the scattering of photons scattered in the medium; and (4) the lateral transport of the secondary electrons. The empirical functions in the above "extended source model" cannot account for all these factors properly, especially for the lateral transport of electrons. For $^{60}$Co and lower energy photons, the lateral range of electrons is less than two millimeters and so the penumbra is entirely due to the other factors. However, extended source analytic function models fail in high energy photon beams because the beam penumbra shape is mostly due to the lateral transport of secondary electrons (Mohan and Chui 1985). The extended source analytic function shows an effect which varies monotonically as a function of distance from the source and lateral distance from the central axis. This is incorrect because the range of the electrons is almost independent of depth and field size. Consequently, for high energy photons where the range of the secondary electrons is more than a few millimeters,
empirical functions cannot predict dose in nonequilibrium regions, such as near the
boundaries formed by the collimators or blocks.

2.2.2 Modified primary and scatter dose model

Recently, a primary and scatter dose separation model has been extended to
calculate dose in lateral nonequilibrium conditions by including the electron transport
from the Monte Carlo calculations (Woo et al 1990). The results of the Monte Carlo
calculations in electronic nonequilibrium are then added to the primary and the scattered
dose model. There is good agreement between the experimental measurements and
calculated data. However, the results of Monte Carlo calculations of primary photon
beams are not easily available in the literature.
2.2.3 Monte Carlo technique

The Monte Carlo technique is the most accurate way to calculate dose under any radiation field geometry. However, it requires the incident photon spectrum for the simulation geometry. Typically, the dose is calculated by simulating a broad parallel photon beam and integrating a central axis pencil distribution over all radii. The pencil beam distribution is calculated by following the photon until it falls below some arbitrary energy (usually 10-20 keV), after which the remaining energy is considered to be deposited locally. There is also a similar cutoff for the kinetic energy of the secondary electrons (usually at 100 keV). This technique is extremely time consuming and the photon spectra for the medical linacs are not easily available. Fields are constantly adjusted during the treatment planning phase to evaluate various plans and therefore, Monte Carlo simulations are not practical for routine clinical use.
2.2.4 Convolution technique

During the last few years a new class of algorithms called the convolution or superposition methods have been proposed by several authors. The dose is calculated by convolving the released photon energy with an energy deposition kernel. The energy deposition kernels are accurately calculated by means of a Monte Carlo method. An inherent assumption of the convolution technique is that the kernels are spatially invariant, i.e., the pencil beams are assumed to be parallel to one another rather than divergent from the source, which is not valid. However, the assumption of the pencil beams being parallel produces only a small error for the range of the SSD's used clinically. The kernels of monoenergetic primary photon beams are stored in a data base as a function of photon energy. In most algorithms, the results of the calculations are not used directly but only as correction factors applied to the open beam dose distributions measured in water (Mohan et al 1991). The kernels of monoenergetic primary photon beams are stored in a data base as a function of photon energy. Kernels for the known photon spectrum are then calculated by superposition. Most convolution type calculations contain analytical expressions for pencil beam kernels describing the primary, scatter, and charged particle contamination.

Primary dose kernels contain the energy imparted per unit mass due to the secondary charged particles liberated by the photons interacting for the first time. Secondary dose kernels contain the energy deposition per unit mass due to the charged particles liberated by photons that have interacted more than once, including the
bremsstrahlung and annihilation photons. A polyenergetic photon beam is described with an analytical equation with some fitting parameters. Alternatively, primary and secondary dose kernels can be derived directly by simulating the polyenergetic photon beams in the Monte Carlo code. The beam penumbra in photon beams arises mainly from the geometric penumbra of the incident photon fluence and the diffusion of the charged particles laterally as mentioned earlier. The primary dose distribution is then convolved with the source distribution (usually modelled by a Gaussian distribution, Ahnesjo, 1992) to derive the effective primary dose pencil beam kernel. Charged particle contamination (released by the photons as they traverse the flattening filter, blocking tray, air column, etc.) kernels are also generally approximated by a gaussian pencil beam with suitable parameters obtained from fitting to the experimental buildup curve. Leakage through the treatment head, transmission through collimators and photons scattered from the irradiated part of the patient are handled by a photon contamination kernel. Finally, the divergence of the beam is accounted for by the inverse square factor.

Algorithms based on convolution provide good accuracy in a homogeneous medium. However, they are very time consuming. Even a 2D convolution calculation is slower than the conventional methods (Ahnesjo, 1993).
2.2.5 Differential pencil beam model

The differential pencil beam (DPB) model has been suggested by Mohan et al (1985). Differential pencil beam (DPB) is defined as the dose distribution relative to the position of the first collision, per unit collision density, for a monoenergetic pencil beam of photons in an infinite unit density medium. DPB dose distribution tables are generated using the Monte Carlo method. This model provides good accuracy but is quite slow as it requires evaluation of 3D integrals of differential pencil beams and is not suitable for routine treatment planning (Mohan et al 1985).

All the conventional dose models (such as the primary and scatter dose model) do not take into account lateral electronic nonequilibrium. On the other hand, Monte Carlo methods, convolution and differential pencil beam methods are time consuming or require information which is not readily available. There is a need to develop a dose model that can handle lateral electronic nonequilibrium and that is also practical for routine use, with parameters easily determined.
2.3. Proposed model - differential scatter integration in regions of electronic nonequilibrium

In this work, we present a modification of the primary and scatter dose model to include electronic nonequilibrium. All the necessary parameters are derived from experimental measurements, without the use of the Monte Carlo code systems.

2.3.1 Background

The absorbed dose, $D$, in a phantom is the mean energy imparted ($dE$) by the radiation in a differential volume of mass $dm$.

$$D = \frac{dE}{dm} \quad (2.7)$$

Let $\Psi$ be the energy fluence vector over a volume $V$ which is surrounded by a closed surface $S$. Let $n$ be the normal unit vector for the surface $S$. The net average radiant energy ($<E>$) through the surface area $S$ is given by

$$<E> = - \oint _{S} \oint _{V} \Psi \cdot n \; dS + \sum <Q> \quad (2.8)$$

---

1A publication on this model (Haider and El-Khatib, 1994) is listed in the bibliography.
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where \( <Q> \) is the release of the rest mass energy in nuclear or other elementary particle reactions. Therefore the average dose \( (<D>) \) to the volume whose mass is \( m \) is given by,

\[
<D> = \frac{<E>}{m} = - \oint \Phi \cdot n \, ds + \sum <Q>
\]

\[ \tag{2.9} \]

Since the energy fluence is continuous throughout volume \( V \), its partial derivative is also continuous in \( V \). By using the divergence theorem we can write the above equation as,

\[
<D> = \frac{\iint \nabla \Psi \, dV + \sum <Q>}{\rho \cdot V}
\]

\[ \tag{2.10} \]

If we take the volume to be sufficiently small then the \( \nabla \Psi \) will be constant and the volume integral will be approximately equal to \( (\nabla \Psi) V \). That is,

\[
\lim_{V \to 0} \frac{\iint \nabla \Psi \, dV}{V} = \nabla \Psi
\]

\[ \tag{2.11} \]

Therefore, the dose to the volume \( V \) is,

\[
<D> = - \frac{\nabla \Psi}{\rho} + \frac{\sum <Q>}{\rho \cdot V}
\]

\[ \tag{2.12} \]
This is the equation that was derived by Rossi and Roesch (1962) and Spencer (1971). $\nabla \Psi$ is the net energy fluence out of the volume. For radiation equilibrium to exist, $\nabla \Psi = 0$. This is not attainable in a finite medium except along a line or a plane. The energy fluence $\Psi$ can be separated into two components, the energy carried by the charged particles $\Psi_c$ and that by the uncharged particles $\Psi_u$. Therefore,

$$\nabla \Psi = \nabla \Psi_c + \nabla \Psi_u$$  \hspace{1cm} (2.13)

and

$$<D> = -\frac{\nabla \Psi_c + \nabla \Psi_u}{\rho} + \frac{\sum <Q>}{\rho V}$$  \hspace{1cm} (2.14)

For charged particle equilibrium, the $\nabla \Psi_c = 0$. For lateral electronic nonequilibrium, $(\partial \Psi_c/\partial x + \partial \Psi_c/\partial y)$ is not equal to zero and $\partial \Psi_c/\partial z$ may or may not be equal to zero (z is along the beam direction). In this thesis, we only consider cases where $\partial \Psi_c/\partial z$ is zero.

The dose calculation by sector integration method assumes electronic equilibrium at the point of calculation (ie. $\nabla \Psi_c = 0$). This assumption is valid for points that lie at a greater distance than the maximum range of the charged particles liberated by the photons. For points that lie within the maximum range of electrons, $\Psi_c$ must take into account the finite range of the electrons for accurate dose calculations.

The energy transport considerations give a little more insight into this problem. The imparted energy $E_d$ is the sum of the kinetic energies of all the directly and indirectly ionizing radiation (particles and photons) ($E_d$) that have entered the differential mass $dm$,
minus the sum of the kinetic energies of all those \( E_L \) that left the \( dm \) and the energy equivalent of any increase in rest mass \( E_R \) by nuclear or other interactions.

\[
E_D = \sum E_E - \sum E_L - \sum E_R \tag{2.15}
\]

We can further subdivide the energy entering and leaving the mass element \( dm \) into energy that is carried by the charged particles \( E_{EC}, E_{LC} \) and by the uncharged entities such as the photons \( E_{EU}, E_{LU} \). Therefore, the above equation can be represented as

\[
E_D = ( \sum E_{EC} - \sum E_{LC} ) + ( \sum E_{EU} - \sum E_{LU} - \sum E_R ) \tag{2.16}
\]

If the energy of the charged particle entering \( E_{EC} \) is equal to the energy of the charged particle leaving \( E_{LC} \) then this is called charged particle equilibrium \( (CPE) \).

\[
\sum E_{EC} = \sum E_{LC} \tag{2.17}
\]

It may be noted that \( CPE \) does not require the number of charged particles entering \( dm \) to be equal the number of charged particles leaving it. It only requires that the total energies carried by the charged particles entering and leaving are equal for \( CPE \).

Let \( E_{cal} \) be the imparted energy in the mass element \( dm \) for a reference field size i.e. calibrated imparted energy. Therefore,

\[
E_{cal} = ( \sum E_{EC} - \sum E_{LC} )_{cal} + ( \sum E_{EU} - \sum E_{LU} - \sum E_R )_{cal} \tag{2.18}
\]
The energy deposited to the same mass element with the same collimator opening but a different field size \( r \) in the phantom, such as may be produced by blocking, is given by

\[
E_r = \left( \sum E_{EC} - \sum E_{LC} \right)_r + \left( \sum E_{EU} - \sum E_{LU} - \sum E_R \right)_r 
\]  
(2.19)

\[
= \left( \sum E_{EC} - \sum E_{LC} \right)_{cal} + \left( \sum E_{EU} - \sum E_{LU} - \sum E_R \right)_{cal} + \left( \sum E_{EC} - \sum E_{LC} \right)_{r-cal} + \left( \sum E_{EU} - \sum E_{LU} - \sum E_R \right)_{r-cal} 
\]  
(2.20)

\[
= E_{cal} + \left( \sum E_{EC} - \sum E_{LC} \right)_{r-cal} + \left( \sum E_{EU} - \sum E_{LU} - \sum E_R \right)_{r-cal} 
\]  
(2.21)

If \( E_{cal} \) is calibrated under CPE then

\[
E_r = E_{cal} + \left( \sum E_{EC} - \sum E_{LC} \right)_r + \left( \sum E_{EU} - \sum E_{LU} - \sum E_R \right)_{r-cal} 
\]  
(2.22)

If the collimator opening is changed, then the energy deposited to the mass element is given by

\[
E_r = E_{cal} + \left( \sum E_{EC} - \sum E_{LC} \right)_r + \left( \sum E_{EU} - \sum E_{LU} - \sum E_R \right)_{r-cal} + \Delta E_{col} 
\]  
(2.23)

where \( \Delta E_{col} \) is the change in energy deposition due to the different collimator opening.

From equation 2.23, we can see that there are 3 parameters needed to determine the relative energy deposition in any radiation field size. One parameter is needed for the
electron transport, one parameter is needed for the photon transport and another parameter to describe the effects of the collimator. To a first approximation, all the parameters can be considered independent of each other and therefore can be expressed in a factored form. This is the basis of our proposed dose model.
2.3.2 Dose model

Holt et al (1972) derived the equation for the relative dose factor (RDF) normalized to a reference field size and depth of maximum dose, by separating the effect of phantom scatter from the collimator scatter as

\[ \text{RDF}(r_p, r_c) = S_{pt}(r_p) \cdot S_{ct}(r_c) \]  

(2.24)

where \( S_{pt} \) is the phantom scatter factor for a field size of radius \( r_p \) in the phantom, \( S_{ct} \) is the collimator scatter factor for the collimator opening of radius \( r_c \). Each of these factors is normalized to a reference field size, usually 10 cm x 10 cm. In addition, RDF and \( S_{pt} \) are normalized at a reference depth, usually at the depth of maximum dose for the reference field size. The phantom scatter factor (\( S_{pt} \)) contains the effects of (i) photon scatter in the phantom under electronic equilibrium and (ii) electron transport when electronic equilibrium is not attained. In order to study these two components separately, we define \( S_{pt}(r_p) \) as,

\[ S_{pt}(r_p) = S_{pe}(r_p) \cdot S_{pl}(r_p) \]  

(2.25)

where \( S_{pe}(r_p) \) is the electron nonequilibrium factor which describes the effect when dose is less than Kerma in regions of lateral electronic nonequilibrium (\( E_{EC} < E_{LC} \), i.e. \( \beta<1 \)); \( S_{pl}(r_p) \) is the total dose factor at the point of measurement under electronic equilibrium.
(i.e. $\beta=1$). Both factors are normalized to those for a reference field size and depth. Now we have the three parameters ($S_{PL}$, $S_{PE}$, $S_{CT}$) necessary to describe the relative dose. For field sizes smaller than that required for lateral electronic equilibrium, $S_{PL}$ must be calculated by Monte Carlo methods or by extrapolation from large field data. The $S_{PE}(r_p)$ for $r_p \geq$ lateral equilibrium thickness is unity by definition.

The dose at any point can be separated into the dose due to the primary beam and that due to the scattered beam. Let $D_0$ be the primary dose in a phantom at $SSD_0+d_0$ in a reference field size $r_0$ at a depth $d_0$ where $SSD_0$ is the reference source to surface distance($SSD$). Then the dose, $D_R$ due to the primary beam at the depth of the maximum dose ($d_m$) under electronic equilibrium is given by:

$$D_R(r_0,d_m) = \frac{D_0(r_0,SSD_0+d_0) \cdot (SSD_0+d_0)^2 \cdot S_{CT}(r_0) \cdot S_{PL}(0)}{TMR(r_0,d_0) \cdot (SSD+d_0)^2} \quad (2.26)$$

where $D_0(r_0,SSD_0+d_0)$ is the calibrated dose in the phantom in a reference field size $r_0$ as shown in figure 2.1 and $TMR$ is the tissue maximum ratio. The calibration can be done either with an $SSD$ set up or an isocentric set up. It may be noted that the primary dose is affected by the collimator opening ($r_c$) and is independent of field size in the phantom.

The primary dose at any depth $d$, is given by:

$$D(r_c,d) = D_R(r_c,d_m) \cdot TMR(0,d) \quad (2.27)$$
The scatter dose can be calculated by a Clarkson type sector integration to give:

$$D_s(r_p,d) = D_R(r_c,d_m) \cdot \frac{1}{n} \sum_{i=1}^{n} SMR(r_i,d)$$  \hspace{1cm} (2.28)

where $r_p$ is the field size in the phantom (which may be different from the collimator opening $r_c$, due to field shaping by blocks), $r_i$ is the radius of the $i^{th}$ sector of the field size in the phantom at depth $d$ and $n$ is the total number of sectors. SMR is the scatter-maximum-ratio (Khan et al 1980).

The total dose at any depth is then given by:

$$D_d(r_c,r_p,d) = D_R(r_c,d_m) \cdot \left( TMR(0,d) + \frac{1}{n} \sum_{i=1}^{n} SMR(r_i,d) \right)$$  \hspace{1cm} (2.29)

If the point of calculation is not under lateral electronic equilibrium then we apply an electron nonequilibrium perturbation factor (EPF). The EPF is a nonlinear function of field size, depth, energy, and the angular distribution of the electrons liberated by the photons. The effective perturbation for any given field can also be approximated by a sector integration. So the above equation can be written as:

$$D_d(r_c,r_p,d) = D_R(r_c,d_m) \cdot \left( TMR(0,d) + \frac{1}{n} \left( \sum_{i=1}^{n} SMR(r_i,d) - \sum_{i=1}^{n} EPF(r_i,d) \right) \right)$$  \hspace{1cm} (2.30)
Figure 2.1. The beam geometry and the relationship between distances and radiation field sizes used in the dose calculations are shown for both an isocentrically and an SSD calibrated linac.
where the negative sign for $EPF$ indicates a reduction of dose due to lateral electronic nonequilibrium. The electron perturbation factor ($EPF$) is calculated from the equation below.

$$EPF(r_p,d) = TMR(r_p,d) \cdot (1 - S_{PE}(r_p))$$ \hspace{1cm} (2.31)

where $S_{PE}(r_p)$ is the electron nonequilibrium factor measured at the depth of maximum dose. The $EPF$ is zero for field sizes greater than that required for lateral electronic equilibrium. The above equations apply to a uniform incident beam. In order to calculate the dose at off-axis points for nonuniform beams, equation (2.29) is multiplied by an off-centre ratio ($OCR$). To calculate dose under a block the equation is modified as below:

$$D(r,c,r_p,d) = D(r,c,d_m) \cdot OCR \cdot \left( K_p \cdot TMR(0,d) + \frac{1}{n} \sum_{i=1}^{n} (K_i \cdot SMR(r_i,d) - K_q \cdot EPF(r_i,d)) \right)$$ \hspace{1cm} (2.32)

where $K_p$ is the attenuation factor of the primary beam over the point of calculation, $r_i$ is the radius of the $i^{th}$ sector from the point of calculation to the radiation field edge, $K_s$ is the modifying factor of the incident beam over the $i^{th}$ sector and $K_q$ is the modifying factor over the $i^{th}$ sector for the net electrons going in or out of the point of calculation.
2.3.3 Relationship between scatter-to-primary ratio (SPR) and $S_{PL}(0)$

The empirical parameter $S_{PL}(0)$ defined in our model is closely related to the scatter-to-primary ratio (SPR). $S_{PL}(0)$ is the dose from a pencil beam at a reference depth under electronic equilibrium normalized to a reference field size and depth. Therefore, it includes the photon scatter (including backscatter) from the pencil beam of photons as it travels from the surface to the reference depth. Let the photon scatter along the pencil beam at the reference depth be $S_{long}$. Then from the definition of the $S_{PL}$, we get

\[
S_{PL}(0) = \frac{P(0,d_{ref}) + S_{long}(pencil\ beam)}{P(0,d_{ref}) + S(r_{ref}d_{ref})}
\]  

(2.33)

where $P(0,d_{ref})$ is the dose due to the primary beam at the reference depth $d_{ref}$, and $S(r_{ref}d_{ref})$ is the scatter dose at the reference depth and field size. We define $\eta$ for a pencil beam as the following

\[
\eta = \frac{P(0,d_{ref})}{P(0,d_{ref}) + S_{long}(pencil\ beam)}
\]  

(2.34)

Therefore,

\[
\eta . S_{PL}(0) = \frac{1}{1 + SPR(r_{ref}d_{ref})}
\]  

(2.35)
where $SPR (S(r_{ref},d_{ref},P(0,d_{ref}))$ is the scatter-to-primary ratio at the reference depth. It can be seen from the above equation that $\eta S_{PL}(0)$ is a normalization factor that establishes the magnitude of the primary dose relative to the total dose under charged particle equilibrium.
2.3.4 Computer calculations

Many commercial treatment planning computer programs extensively use tissue-air-ratios (TAR\((r,d)\)) for separating dose contributions from a photon beam into the primary and scatter components. In practice, neither TMR or TAR is measured for use in treatment planning computers. Instead, TMR and TAR are generally calculated from the percentage depth dose and the conversion formulae are given in the appendix B. TMR is normalized at the depth of maximum dose. This depth varies widely from the smallest field size to the largest field size due to differences in the contributions of scatter photons and contamination electrons from the collimator jaws, flattening filter, air etc. Therefore, to determine the true scatter at any depth under CPE, the TMR must be multiplied by the relative photon scatter factor (\(S_{PL}\)). TAR preserves this normalization. There has been a lot of discussion about the definition of TAR. The definition of TAR requires that the dose in air be measured in a phantom that is just large enough to provide electronic equilibrium. This is accomplished by fitting the ionization chamber with a "build-up cap" of sufficient thickness to ensure electronic equilibrium. However, the build-up cap required for high energy photons can be quite large (e.g 3 cm radius for 18 MV x-rays). Because the field size in nonequilibrium is smaller than the thickness of the build-up cap, the TAR definition was extended by Woo and Cunningham (1990). The in-air dose was replaced by the collision Kerma. This allows dose calculation to be performed in either format. However, TAR is more convenient and its computation much faster.

An equation for calculating dose for a nonuniform beam based on the concept of
separating the tissue-air-ratio into primary and scatter components was proposed by Cunningham (1972) and can be written as

\[
\frac{D(d,x,y)}{D_a(d)} = f(x,y) \cdot \text{TAR}(d,0) + \sum_{i=0}^{N} \frac{\Delta \theta_i}{2\pi} \sum_{j=1}^{J} f(r_j, \theta_j) \cdot \frac{\Delta \text{SAR}}{\Delta r} [d(r_j, \theta_j), r_j] \Delta r_j \tag{2.36}
\]

where \(D(d,x,y)\) is the dose to the point \(Q\) at depth \(d\) with lateral cartesian coordinates \((x,y)\), in a cross section of a beam. \(D_a(d)\) is the reference dose in air along the central axis of the beam at point \(Q\) as shown in figure 2.2. \(f(x,y)\) is a function that models the primary beam intensity and penumbra according to the "extended source model" (Johns and Cunningham, 1983) and \(f(r_j, \theta_j)\) is a function that adjusts the intensity of the beam incident in the scattering volume. The first term on the right is the dose due to the effective primary photons under CPE and the second term is the dose due to the scattered photons reaching point \(Q\) from the whole irradiated volume. The volume is represented by a series of volume elements which are increments along the radii of sectors of circular cones centered on point \(Q\) (figure 2.2). The first summation with index \(i\) is over \(N\) sectors with angular width \(2\pi/N\) and the second summation with index \(j\) is carried out radially to the field edge. \(\Delta \text{SAR}/\Delta r\) is the differential scatter contribution at point \(Q\) from the pencil beam of scattered photons between \(r_j\) and \(r_j + \Delta r_j\) and between \(\theta_i\) and \(\theta_i + \Delta \theta_i\). \(J_i\) is such that \(r_j\) is the radius to the field edge. For dose calculation in nonequilibrium, \(EPF_a\) is added to \(SAR\), where \(EPF_a\) is the quotient of \(EPF\) and \(S_{pl}\). Then the above equation can be written as
Figure 2.2 Diagram for the calculation of dose at point $Q$ at depth $d$. 
\[
\frac{D(d,x,y)}{D_a(d)} = f(x,y) \cdot TAR(d,0) + \sum_{i=1}^{N} \frac{\Delta \theta_i}{2\pi} \sum_{j=1}^{J_i} f(r_j,\theta_j).
\]

\[
\left( \frac{\Delta SMR}{\Delta r} [d(r_j,\theta_j),r_j] \Delta r_j \right) - \sum_{i=1}^{N} \frac{\Delta \theta_i}{2\pi} \sum_{i=1}^{J_i} f(r_j,\theta_j) \left( \frac{\Delta EPF_a}{\Delta r} [d(r_j,\theta_j),r_j] \Delta r_j \right)
\]

(2.37)

\( J_e \) is such that \( r_j \) is the radius of the maximum range of the lateral electrons or the field edge whichever is reached first.

The integral form of the above equation is

\[
\frac{D(d,x,y)}{D_a(d)} = f(x,y) \cdot TAR(d,0) + \frac{1}{2\pi} \int_{\theta=0}^{2\pi} \int_{r=0}^{R(\theta)} f(r,\theta) \frac{dSAR}{dr} [d(r,\theta),r] \ drd\theta
\]

\[
- \frac{1}{2\pi} \int_{\theta=0}^{2\pi} \int_{r=0}^{R(\theta)} f(r,\theta) \frac{dEPF_a}{dr} [d(r,\theta),r] \ drd\theta
\]

(2.38)

where \( dSAR/dr \) is the differential scatter-air-ratio and \( dEPF_a/dr \) is the differential electron perturbation factor \( EPF_a \). The limits of the inner integral of the first term on the right side of the equation is from the point of calculation to the field edge at angle \( \theta \) and the limits of the inner integral of the second term on the right side of the equation is from the point of calculation to the maximum lateral range of the electrons or the field edge at angle \( \theta \), whichever is reached first.
2.3.5 Relative dose factor (RDF) in lateral electronic nonequilibrium

Methods for calculating the relative dose factor (RDF) involve finding an equivalent square or a circle of the same area as the irregular field. Sterling et al (1964) have proposed that square fields and rectangular fields are equivalent if the ratio of area to perimeter for the two field sizes are the same. This method works very well for large fields where lateral electronic equilibrium exists. Due to the nonlinearity of lateral electronic equilibrium with field size, the relative dose factor (RDF) also varies nonlinearly in small field sizes, and calculation of equivalent field sizes using area to perimeter ratios fails to predict the RDF.

The same equivalent square but different phantom geometry can have RDF values varying by more than several percent. We describe a simple method to calculate RDF that is valid in lateral electronic nonequilibrium. This method is intended only for points that are at least 0.4 cm away from the photon beam boundary.

The relative dose factor (RDF) defined in section 2.3.2 for circular field sizes can be modified to accommodate any rectangular field sizes as following:

\[
RDF(x_1, x_2, r_p) = S_{CT}(x_1, x_2) \cdot <S_{pf}(r_p)>
\]  

(2.39)

where \(S_{CT}\) is the total collimator scatter factor, \(x_1, x_2\) are the positions of the upper and the lower collimators, \(r_p\) is the field size in the phantom (which may be different from the collimator opening due to external blocking) and \(<S_{pf}\>\) is the mean phantom scatter for
the radiation field. It can be calculated by a sector integration as in the Clarkson method (Clarkson, 1941). The irregular field is projected onto a plane containing the point perpendicular to the central axis at the isocenter. The projected field size is then divided into \( n \) sectors with radii emanating from the point of calculation to the field boundary.

\[
<S_{PT}(r_p)> = \frac{1}{n} \sum_{i=1}^{n} S_{PT}(r_{pi})
\]  

(2.40)

where \( r_{pi} \) is the radius of the \( i^{th} \) sector at isocenter, \( n \) is the total number of sectors given by \( 2\pi/\theta \), where \( \theta \) is the sector angle width. The phantom scatter factor, \( S_{PT} \), contains the electron transport as well as the photon transport and therefore, it is valid in electronic nonequilibrium. If the point of calculation is off-axis, the dose is multiplied by an off center ratio OCR, which can be derived from the lateral beam profile measured in the phantom at the reference depth where RDF is defined. In general, the variation in OCR is negligible for small fields (Luxton, 1991) and therefore will not be considered further.

The total collimator scatter factor (\( S_{CT} \)) contains (i) the scattered radiation from the flattening filter, (i) radiation backscattered to the monitor chamber and (iii) the collimator forward scatter. The components of the head scatter have been studied by Luxton (1988), Kubo (1989) and Fontenia et al (1992).
2.3.6. Monitor unit calculation

The energy fluence in a medical Linac is monitored by an ionization chamber which is located in the treatment head between the upper jaws and the flattening filter. RDF for an arbitrary field size is calibrated as a dose per monitor unit normalized to a reference field size and depth. The effects of field size dependent output factors have been studied by many authors (Khan et al 1986, Kase 1986, Watts et al 1987). The change in RDF at a point in the phantom is due to the head scatter and the phantom scatter. The dose per monitor unit to a point from a single stationary beam treated isocentrically is given by

$$D(r_p,d) = M \cdot RDF(x_1, x_2, r_p) \cdot TMR(r_p,d) \quad (2.41)$$

where $d$ is the depth in tissue on the central axis to the point of calculation, $r_p$ is the radius of the effective field size and $M$ is the calibration factor for the machine monitor units to cGy. It may be noted that TMR calculated from percent depth doses (PDD) will be in error for small field sizes at large depths due to the divergence of the photon beam irradiating a larger phantom area, unless, they are corrected for lateral electronic nonequilibrium. The following relationship can be used to convert PDD to TMR:

$$TMR(r_p,d) = \frac{1}{100} \cdot \frac{PDD(r_p,F_r,d)}{S_{FT}(r_p,d)} \cdot \frac{S_{FT}(r_p)}{S_{FT}(r_p)} \cdot \left( \frac{SSD+d}{SSD+d_m} \right)^2 \quad (2.42)$$
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where SSD is the source to surface distance, \( r_p \) is the field size at the isocenter and \( r_{pd} \) is the field size at depth in the phantom. \( r_p \) and \( r_{pd} \) are related through a simple geometric relationship \( r_p/ r_{pd} = (SSD+d_m)/(SSD+d) \). The above equation is similar to the equation described by Khan et al (1983) except that the peak scatter factor (PSF) in their formulation is replaced by \( S_{PT} \) here, which includes the effects lateral electronic nonequilibrium in addition to the photon scatter.
2.3.7 Penumbra modelling

The beam penumbra which is handled by the function $f(x,y)$, can be described by the "extended source model" (Johns and Cunningham, 1983). The radiation beam is radially symmetric and therefore can be described by just $f(r)$. Johns and Cunningham suggested that

\[ f(r) = 1 - 0.5 e \left( \frac{\alpha_1}{p} \frac{W_d}{2} - |r| \right) \text{ for } |r| \leq \frac{W_d}{2} \]  \hspace{1cm} (2.43)

\[ f(r) = t + (0.5 - t)e \left( \frac{\alpha_2}{p} \frac{W_d}{2} - |r| \right) \text{ for } |r| > \frac{W_d}{2} \]  \hspace{1cm} (2.44)

where $|r|$ is the absolute value of the distance from the central axis; $W_d/2$ is the half width of the beam at depth $d$ and $p$ is the geometric penumbra. $\alpha_1$ and $\alpha_2$ are empirical coefficients that describe the way in which the actual penumbra region differs from the geometric penumbra region, and $t$ is the photon transmission coefficient through the collimator. The geometric penumbra ($p$) can be calculated from the following equation.

\[ p = \frac{S.(SSD-SCD)}{SCD} \]  \hspace{1cm} (2.45)
where $S$ is the source size and $SCD$ is the source to collimator distance as shown in figure 2.3. The effects of the empirical coefficients $\alpha_1$ and $\alpha_2$ are shown in figure 2.4.
Figure 2.3 Geometric penumbra $p$ due to the finite source size.

Figure 2.4 The effects of the empirical coefficients $\alpha_1$ and $\alpha_2$. 
2.4 Comparison between proposed model and existing models

Our model has a very close relationship to the extended primary and scatter dose model of Woo et al (1990), in which the electronic nonequilibrium data are derived from Monte Carlo calculational methods. The terms \( \{ \text{TAR}(0,d) + \text{dEPF}_d \text{dr}(r_p,d) \} \) in equation 2.38 can be combined together as one term \( \text{dP}/\text{drr}(r_p,d) \) to represent the dose due to the effective primary beam \( P \) of radius \( r_p \) in the phantom at depth \( d \). Woo et al calculated the effective primary beam \( P \) from Monte Carlo methods as a function of depth and field size.

Equation 2.38 can be rewritten by including \( \text{EPF} \) into the primary beam under CPE.

\[
\frac{D_p(d,x,y)}{D_d(d)} = \frac{1}{2\pi} \int_{\theta=0}^{2\pi} \int_{r=0}^{R^2(r)} f(r,\theta) \frac{dP}{dr}[d(r,\theta),r] \, dr \, d\theta \tag{2.46}
\]

where \( D_p \) is the dose due to the primary beam which may or may not be under CPE and \( dP/dr \) is the differential effective primary beam. The limits of the integrals in the above equation are the same as the third term in equation 2.38 containing \( \text{dEPF}_d / \text{dr} \). The primary and scatter model also have close analogy to the convolution methods of dose calculation suggested by Mackie et al (1985), Boyer and Mok (1985) and Mohan et al (1986). Woo et al also pointed out that the result of the convolution method is reduced to the differential primary beam in nonequilibrium when the integration is precalculated along the depth, although no derivation was given. Treuer (1987) showed that the energy deposition kernel for scattered photons integrated in all directions is equal to the scatter-
Consider a photon fluence $\Phi_0$ (photons/cm$^2$) of energy $E_0$ moving along the $z$ direction and incident perpendicularly upon the phantom. Let the source of the photons be at a distance $SSD$ from the surface of the phantom. The primary fluence distribution can be calculated by tracing the photons from the radiation source and applying inverse square law and exponential attenuation. The phantom can be divided into discrete volume elements (voxels). The point of calculation is located at the center of the voxels and its coordinate is given by the vectors $\xi$ relative to the source as the origin. The primary fluence ($\Phi_p(\xi)$) at $\xi$ can be calculated by

$$\Phi_p(\xi) = \Phi_0 \cdot \left( \frac{SSD}{SSD+d} \right)^2 \cdot \prod_i e^{-\mu \Delta d_i} = \Phi_0 \cdot w(r,\theta,z)$$

where $\mu$ is the total linear attenuation coefficient (cm$^{-1}$) for energy $E_0$ at the $i^{th}$ voxel($\Delta$) along the ray trace path from the surface of the phantom, $d$ is the depth in the phantom and $w(r,\theta,z)$ describes the attenuation and the inverse square fall-off of the primary photon fluence $\Phi_0$ in cylindrical coordinates. $\prod$ denotes the product series of the $\exp(-\mu \Delta d_i)$ term.

The primary fluence Kerma ($K_p(\xi)$) at the center of the voxel is given by

$$K_p(\xi) = \Phi_p(\xi) \cdot \frac{\mu_{en}(E_0)}{\rho} \cdot E_o$$
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The Kerma does not result in absorbed dose at point $s$ but at some distance $s'$ as the energy is transported by the electrons (Boyer and Mok, 1985).

The electron transport can be calculated by Monte Carlo techniques and can be represented by an array $k(s, s')$ which describes the dose deposited at $s'$ by the electrons set in motion at $s$. Since the total energy loss by the electrons set in motion at $s'$ must be equal to Kerma, $k(s')$ is normalized such that when integrated over all space yields unity.

$$\int \int k(s') \, dv' = 1 \quad (2.49)$$

Therefore, the dose due to primary Kerma is given by

$$D_p(s) = \int \int \Phi_p(s) \cdot \frac{\mu_{en}(E_0)}{\rho} \cdot E_0 \cdot k(s-s') \, dv' \quad (2.50)$$

The above integral is a convolution of the primary fluence with a function called the "pencil beam kernel" or the "dose spread kernel". $k(s-s')$ can be viewed as a Green's function for radiation transport (Boyer and Mok, 1985). $k(s-s')$ is calculated for the entire radiation transport processes and can be interpreted as the three dimensional impulse response of the dose transport process.

By equating equation 2.46 and equation 2.50, we get

$$\frac{D_p(d)}{2\pi} \int_{0}^{2\pi} \int_{-\infty}^{\infty} f(r, \theta) \, dr \, d\theta = \frac{\mu_{en}(E_0)}{\rho} \int \int \Phi_p(s) \cdot k(s-s') \, dv' \quad (2.51)$$
The right hand side of the above equation is in vector form and therefore we must convert it to cylindrical coordinates. If we take $R_e$ equal to the width of the dose spread kernel then the right side of the equation becomes

$$\frac{1}{2\pi} \mu_{en}(E_0) \cdot E_0 \cdot \frac{2\pi R_e}{\rho} \int_0^{\frac{2\pi}{\rho}} \int_0^\infty \Phi_p(r,\theta,z) \cdot k(r,d-z) \cdot r \cdot d\theta \cdot dr \cdot dz \quad (2.52)$$

The dose to the air ($D_a$) is given by

$$D_a(d) = \frac{\mu_{en}(E_0)}{\rho} \cdot E_0 \cdot \Phi_0 \cdot w(r,\theta,z) \quad (2.53)$$

By substituting the above equation to the left hand side of equation 2.51 and substituting equation 2.53 in the right hand side of equation 2.44 and simplifying, one gets

$$f(r,\theta) \cdot \frac{dP}{dr}[d(r,\theta),r] = r\int_z^\infty w(r,\theta,z) \cdot k(r,d-z) \cdot dz \quad (2.54)$$

Therefore, the convolution method reduces to the modified primary and scatter dose model when the integration is carried out along depth in the $z$ direction. The smallest radius over which our model can be calculated corresponds to the voxel size in the convolution methods. The relationship between scatter kernel and SAR or SMR have been discussed by Treuer et al (1987).
2.5 Lateral range of electrons based on Compton interaction only

From the calculation using the Continuous Slowing Down Approximation ($R_{CSDA}$), the range of Compton electrons increases with increasing photon energy. Electrons with longer ranges broaden the penumbra as they travel further. On the other hand, as the energy of the photons increases, the average angle of the Compton electrons decreases. This latter process would decrease the penumbra. We have investigated the combined effect of these two competing processes.

The Compton electron scattering angle ($\theta_1$) can be calculated from the kinematics of Compton interactions and is given by

\[
\cot \theta_1 = \left(1 + \frac{h\nu}{m_0c^2}\right)\tan\left(\frac{\theta_2}{2}\right) \tag{2.55}
\]

where $h\nu$ is the energy of the incident photon, $m_0c^2$ is the rest mass energy of electron and $\theta_2$ is angle of the scattered photon. $\theta_2$ can be calculated from the following equation

\[
E = h\nu \cdot \frac{\alpha(1-\cos \theta_2)}{1 + \alpha(1-\cos \theta_2)} \tag{2.56}
\]

where $E$ is the energy of the recoil electron and $\alpha$ is the ratio of the energy of the incoming photon ($h\nu$) and the rest mass energy of the electron ($m_0c^2$). The Compton interaction coefficient of a photon with a free electron is calculated from the Klein-Nishina coefficient
\[
\frac{d\sigma}{d\Omega} - \frac{d\sigma_0}{d\Omega} \cdot F_{KN} - \frac{r_0^2}{2} (1 + \cos^2\theta_2) \cdot F_{KN} \tag{2.57}
\]

where \(d\sigma/d\Omega\) is the differential Compton cross section per unit solid angle \((d\Omega)\), \(d\sigma_0/d\Omega\) is the Rutherford scattering coefficient per unit solid angle \((d\Omega)\) and \(F_{KN}\) is the Klein-Nishina coefficient. \(F_{KN}\) is given by

\[
F_{KN} = \left( \frac{1}{1 + \alpha(1 - \cos\theta_2)} \right)^2 \left( 1 + \frac{\alpha^2(1 - \cos\theta_2)^2}{(1 + \alpha(1 - \cos\theta_2))(1 + \cos^2\theta_2)} \right) \tag{2.58}
\]

The total Compton coefficient is given by integrating over all angles for equation 2.57 using \(F_{KN}\) in equation 2.58 giving

\[
\sigma = \frac{3}{4} \sigma_0 \left[ \frac{1 + \alpha}{1 + 2\alpha} \left( \frac{2(1 + 2\alpha)}{1 + 2\alpha} \frac{\ln(1 + 2\alpha)}{\alpha} \right), \frac{\ln(1 + 2\alpha)}{2\alpha} - \frac{1 + 3\alpha}{(1 + 2\alpha)^2} \right] \tag{2.59}
\]

The average fraction of the photon energy given to the electrons \((<E>/\hbar\nu)\) is calculated by multiplying \(d\sigma/d\Omega\) with the fraction of the energy given to the recoil electrons to give

\[
\frac{d\sigma_{\text{ir}}}{d\Omega} = \frac{d\sigma_0}{d\Omega} \cdot F_{KN} \cdot \left( \frac{\alpha(1 - \cos\theta_2)}{1 + \alpha(1 - \cos\theta_2)} \right) \tag{2.60}
\]
where \( d\sigma_{\gamma}/d\Omega \) is the differential energy transfer coefficient per unit solid angle. Integrating the above equation over all angles yields

\[
\sigma_{\gamma} = 0.75\sigma_0 \left( \frac{2(1+\alpha)^2}{\alpha^2(1+2\alpha)} - \frac{(1+3\alpha)}{(1+2\alpha)^2} + \frac{(1+\alpha)(1+2\alpha-2\alpha^2)}{\alpha^2(1+2\alpha)^2} \right)
\]

\[
-0.75\sigma_0 \left( \frac{4\alpha^2}{3(1+2\alpha)^3} + \frac{1+\alpha}{\alpha^3} - \frac{1}{2\alpha} + \frac{1}{2\alpha^3} \ln(1+2\alpha) \right)
\]

(2.61)

The effect of the binding energy of the electron is not included in the calculation as it has minimal effect in low Z media. The average fraction of the incident photon’s energy given to the electrons is also given by Attix (1986)

\[
\frac{\langle E \rangle}{h\nu} = \frac{\sigma_{\gamma}}{\sigma_0}
\]

(2.62)

Substituting the equation 2.62 into equation 2.56 gives

\[
\sigma_{\gamma} = \sigma_0 \cdot \frac{\alpha(1-\cos \langle \theta_\gamma \rangle)}{1+\alpha(1-\cos \langle \theta_\gamma \rangle)}
\]

(2.63)

where \( \langle \theta_\gamma \rangle \) is the mean angle of the scattered photons produced by monoenergetic photons. Solving for \( \langle \theta_\gamma \rangle \) in the above equation and substituting in equation 2.55 yields the mean Compton electron emission angle \( \langle \theta_1 \rangle \). \( \langle \theta_1 \rangle \) is plotted as a function of photon energy in figure 2.5 (mass attenuation coefficients and mass energy transfer coefficients data are due to Hubbell, as given by Evans (1968)). The Compton electrons suffer many small angle collisions and can be ignored to a first approximation. Therefore, the mean
lateral range $<R_L>$ from a monoenergetic photon can be calculated by

$$R_L = \frac{<E>}{S_{tot}(E)} \cdot \sin <\theta_1>$$

(2.64)

where $S_{tot}$ is the total stopping power (including bremsstrahlung) of an electron at energy $E$ for water. Lateral range in water is calculated from equation 2.64 is shown in figure 2.6.

The minimum field diameter required for CPE is twice the lateral range for an initially parallel photon beam. It may be noted that the range of the electrons may be much greater than the lateral range. For example, Co-60 produces on average 0.6 MeV electrons. The CSDA range for 0.6 MeV electron is 2.2 mm while the lateral range is only 1.1 mm. The ratio of lateral range and CSDA range deceases with increasing photon energy due to the smaller average Compton emission angle, although the lateral range increases with increasing energy for the range of energies investigated.
Figure 2.5 Mean Compton electron emission angle as a function of photon energy.

Figure 2.6 Mean lateral range $R_{CSDA}$ calculated from equation 2.64 as a function of the Compton electron energy.
2.6 Inhomogeneity correction factors

The presence of inhomogeneities such as bone, lung or air cavities perturbs the unit density dose calculations. The dose perturbation varies depending on the location of the point of calculation with respect to the inhomogeneity, the lateral dimensions of the inhomogeneity and the photon field size. In this research, we investigate only slab type inhomogeneities whose lateral dimensions are much larger than the photon field size. Some of the conventional methods of dose correction are discussed below.

2.6.1. Effective attenuation coefficient method

This is a one dimensional dose correction model. The assumption in this model is that the perturbation to dose in a water phantom is affected only by the material lying in the path of the ray joining the computation point and the source of radiation but is independent of the material outside the path of the source-point ray. The effective pathlength or the radiological pathlength \( d_e \) is given by

\[
    d_e = \rho_e \cdot d \tag{2.65}
\]
where $\rho_e$ is the electron density of the inhomogeneity and $d$ is the physical pathlength in the inhomogeneity. The correction factor (C.F.) for the dose at a point is given by

$$C.F. = e^{\mu(d-d_e)}$$  \hspace{1cm} (2.66)$$

where $\mu$ is the effective linear attenuation coefficient for the photon beam which is determined empirically. By definition, it cannot handle electronic nonequilibrium.

### 2.6.2 Tissue-air-ratio (TAR) method

This is a more sophisticated correction factor which accounts for the depth and the field size through the use of the tissue-air-ratio (TAR). The correction factor is given by

$$C.F. = \frac{TAR(d_e, r_d)}{TAR(d, r_d)}$$  \hspace{1cm} (2.67)$$

where $r_d$ is the radius of the field size at depth. This also does not correct for the electronic nonequilibrium.

### 2.6.3 Power law method

This method was proposed by Batho (1964) and Young and Gaylord (1970) for points lying within the inhomogeneity and was extended to points below an inhomogeneity by Sontag and Cunningham (1977). The general expression for the correction factor is given by
where $\rho_1$ is the density of the material in which the point of calculation lies, $d_1$ is the distance, along a ray, from the point to the anterior surface of that material. $\rho_2$ is the density of the overlying material; and $d_2$ is the distance to its anterior surface from the point of calculation as shown in figure 2.7 (Johns and Cunningham, 1983). It has been shown that the power law method underestimates dose in low density media and overestimates it in high density media (Wong and Henkelman, 1982). This correction factor also cannot account for electronic nonequilibrium.

**Figure 2.7 Example of an inhomogeneous phantom with a point of calculation.**

### 2.6.4 Differential scatter-air-ratio (DSAR) method

This method was proposed by Beaudoin (1968) and Cunningham (1972). A 3D
algorithm based on this method was implemented by Larson and Prasad (1978). This model also assumed local energy deposition and first-order ray tracing. The calculation with DSAR was reported to be not much better than 1D Batho calculations (Wong and Purdy (1990). Wong and Purdy also pointed out that DSAR in commercial treatment planning computers are not based on 3D volumetric DSAR defined by Cunningham (1972) but on SAR values differentiated with respect to depth and radius. In particular, the commercial DSAR method is an extension of the Batho 1-D power law.

2.6.5 Delta-Volume method

The delta-volume method (Beaudoin, 1968, Wong and Henkelman, 1983) of dose calculations is a variation of the three dimensional differential scatter-air-ratio (DSAR). The DSAR values are deduced by differentiating SAR values with respect to depth and radius. This method incorporates the transport of scattered photons but ignores the transport of electrons. This model gives highly accurate results for Co-60 beams. For practical reasons, only first-order scatter ray tracing is performed. The assumption of local energy deposition in the earlier model was unsuitable for high energy photons. In recent years, Monte Carlo simulation data have been used as the input data and therefore the method is able to handle electronic nonequilibrium. This method is extremely time consuming. Calculation of a crude 24 cubed element matrix with a 30 megaflop array processor requires three hours and on a Vax-11/780 it takes 100 hours to calculate the dose distribution for one beam.
2.6.6 Effective tissue-air-ratio (ETAR) method

Sontag and Cunningham (1978) suggested a modification of the tissue-air-ratio method. The essence of ETAR is the idea that an effective tissue-air-ratio may be determined for phantoms containing non-water equivalent materials by scaling the depth and the field size. The correction factor becomes

\[
C.F. = \frac{TAR(d_s, r_s)}{TAR(d, r_d)}
\]  

(2.69)

where \(d_s\) and \(r_s\) are the scaled depth and field radius. Scaling of the depth is the same as the radiological pathlength and the field size is scaled as follows

\[
r_s = r \cdot \rho_{\text{eff}}
\]

(2.70)

where \(\rho_{\text{eff}}\) is the electron density relative to that of water of the volume element at index \(i,j,k\) and \(W_{ijk}\) are weighting factors, one for each of the density elements, \(\rho_{ijk}\). The ETAR model uses the input of computed tomography (CT) to determine the equivalent density for field size scaling. The effective density can be considered as the convolution of the weighting factors. The weighting factors express the relative importance of each of the volume elements in contributing to the dose due to the scattered radiation at the point of
calculation. Weighting factors are generally derived empirically for a homogeneous water medium. This is the only method that explicitly corrects for scatter dose. The detailed description of determining the weighting factors is given by Sontag and Cunningham (1977). A 3-D ETAR using fast fourier transform (FFT) has been implemented by Yu and Wong (1990).

The modelling deficiency in the ETAR method currently implemented on treatment planning systems is its assumption that electronic equilibrium is always preserved (Yu and Wong, 1993). The ETAR method has been shown to be inadequate for low density inhomogeneities irradiated with small fields (Mackie et al 1985). Woo and Cunningham (1990) have derived a new set of weighting factors from theoretically generated dosimetric data (Monte Carlo methods). Dose calculated from these new weighting factors gives good agreement with experimental measurements in electronic nonequilibrium. This is the only method currently available that takes into account the three dimensional nature of the irradiated volume which can be used for interactive treatment planning.
2.6.7 Inhomogeneity correction factor applied to the proposed model

Although any inhomogeneity correction factors can be applied to our model, only the ETAR is 3-D in nature and is able to handle electronic equilibrium with proper weighting factors. Masterson et al (1991) observed more than 40% discrepancies in dose in low density media and as much as 12% discrepancy in dose in the presence of high density media with the conventional 1-D inhomogeneity correction factors. Almost all the errors can be attributed to the inability of conventional dose calculation models to properly incorporate the transport of scattered photons and electrons. A set of weighting factors for the voxels can be derived from the $S_{PT}$ function described in section 2.3.2. Methods for measurement of $S_{PT}$ in electronic nonequilibrium are described in section 3.1.

We describe a 2-D correction factor that is conceptually similar to the ETAR method that is better than all 1-D conventional correction factors and is fast enough to permit interactive treatment planning in electronic nonequilibrium.

We used a scatter integration method by Clarkson to derive inhomogeneity correction factors. We made an assumption that each scatter sector is homogeneous and the electron and photon scattering contribution are independent. There is only one ray tracing done for each point calculation. Although more tracings can be done, computation time will increase proportionately. For complex geometries, three or four ray tracings may be required to get good accuracy. We perform ray tracings by connecting the point of calculation to the source and finding the effective radiological path length. The correction factor ($C.F.$) is given by
where the first term in the numerator corrects for the photon fluence (same as the radiological depth), the first term inside the parenthesis corrects for the scattered photons (SMR) and the second term inside the parenthesis corrects for electron scattering (EPF).

Both terms inside the parenthesis are in a plane perpendicular to the beam axis. Both SMR and EPF contain the effect of the collimator scatter and the ratio of the collimator scatter factors (SCF) corrects for the collimator effects. The denominator is the dose in unit density medium.

To calculate dose inside the inhomogeneity, the C.F. is multiplied by the ratio of the mass energy absorption coefficients ($\mu_\text{en}/\rho$) of the inhomogeneity and water. The following expression is suggested to account for the location of the inhomogeneity boundary with respect to the point of calculation:

\[
(C.F.)_T = C.F. \exp(-\rho_e d_1 \alpha_p) + (1 - \exp(-\rho_e d_1 \alpha_p))
\]

(2.72)

where $d_1$ is the distance from the inhomogeneity boundary to the point of calculation and $\alpha_p$ is the dose build-up coefficient derived experimentally in a water equivalent phantom. $\alpha_p$ is a function of field size and photon energy and decreases with increasing field size and photon energy.

The correction factor proposed above may be extended to 3-D by integrating in...
a 3-D volume around the point of calculation. The method of 3-D differential scatter integration by Clarkson under charged particle equilibrium has been investigated by Ayyanger et al (1993). The correction factor at the point of calculation is given by

\[ C.F. = \frac{TMR(O, \rho_e d)}{TMR(O, d) + \frac{1}{n} \sum_{i=1}^{n} (SMR(r_i d) - EPF(r_i d))} \]

\[ \frac{S_{CT}(r_e)}{S_{CT}(\rho_e r_e)} \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{J} \left( \frac{\Delta SMR}{\Delta r} (\rho_{ij} r_{ij} \rho_{e d}) - \frac{\Delta EPF}{\Delta r} (\rho_{e r_{ij}} \rho_{e d}) \right) \]

\[ TMR(O, d) + \frac{1}{n} \sum_{i=1}^{n} (SMR(r_i d) - EPF(r_i d)) \]  

(2.73)

where \( \Delta SMR/\Delta r \) is the differential scatter-maximum-ratio, \( \Delta EPF/\Delta r \) is the differential electron perturbation factor. The first summation with index \( i \) is over \( n \) sectors with angular width \( 2\pi/n \) and the second summation with index \( j \) is carried out radially to the field edge. \( \Delta SMR/\Delta r \) is the differential scatter contribution at the point of calculation (\( Q \)) from the pencil beam of scattered photons between \( r_j \) and \( r_j + \Delta r_j \) and between \( \theta_i \) and \( \theta_i + \Delta \theta_i \). \( J_i \) is such that \( r_{ij} \) is the radius to the field edge. \( J_e \) is such that \( r_{je} \) is the radius of the maximum range of the lateral electrons or the field edge, whichever is reached first.

We do not use equation 2.73 as it is extremely time consuming. The dose calculation error from using equations 2.71 and 2.72 is expected to be less than 5% except for points located close to the interface, a point that is supported by the relatively good agreement between our model calculation and the measured dose in, for example, figures 4.16a
through 4.16d. A dose calculation using equations 2.71 and 2.72 is expected to be several times faster than a dose calculation based on equation 2.73 depending on the number of ray tracings done for each point.

2.7 Dosimetry in inhomogeneous media

The presence of a dosimeter in a medium perturbs the charged particle fluence in the medium unless the dosimeter and the medium are identical with respect to atomic composition and density. The response of a dosimeter, therefore, depends on the geometry, construction and the surrounding medium. A cavity theory is used to relate the dose deposited in the cavity (sensitive volume of the detector) to that in the surrounding medium which may be of different atomic number or composition. The various cavity theories are briefly reviewed in the next section.
2.8 Cavity theories

2.8.1 Bragg-Gray cavity theory

The simplest cavity theory is the Bragg-Gray cavity theory (Bragg, 1910, Gray, 1929, 1936). Consider a fluence of identical charged particles $\Phi_e$ of kinetic energy $T$ passing through an interface between two different media $g$ and $w$ as shown in figure 2.8. The absorbed dose in medium $g$ is given by

$$D_g = \Phi_e \cdot \left( \frac{dT}{\rho dx} \right)_{c,g}$$

(2.74)

where $(dT/\rho dx)_{c,g}$ is the mass collision stopping power of the medium $g$, evaluated at $T$, and $\rho t$ (g/cm$^2$) is the particle pathlength through the medium. The absorbed dose in $w$ is given by

$$D_w = \Phi_e \cdot \left( \frac{dT}{\rho dx} \right)_{c,w}$$

(2.75)

where $(dT/\rho dx)_{c,w}$ is the mass collision stopping power of the medium $w$, evaluated at energy $T$. If $\Phi_e$ is continuous across the interface and electron backscattering is absent at the interface then the ratio of the absorbed doses between the two media adjacent to the boundary is given by
If a thin layer of medium $g$ is sandwiched between two regions containing medium $w$, and if $\Phi_e$ is continuous across both interfaces, the ratio of doses is also given by equation 2.76 and is called the Bragg-Gray relation. The above equation applies to each charged particle of kinetic energy $T$. For a differential energy distribution of $\Phi_{e,T}$, the average mass collision stopping power over the entire spectrum in any medium is given by

$$<S|\rho> = \frac{\int \Phi_{e,T} \cdot \left( \frac{dT}{\rho dx} \right) dT}{\int \Phi_{e,T} dT}$$  \hspace{1cm} (2.77)$$

Therefore, the Bragg-Gray relation for a spectrum of charged particles is given by

$$\frac{D_g}{D_w} = \frac{<S|\rho>_g}{<S|\rho>_w}$$  \hspace{1cm} (2.78)$$

The following conditions are assumed in the derivation of the Bragg-Gray relation.

a. The thickness of the $g$ layer (cavity) is so small compared to the range of the charged particles that the presence of the cavity does not perturb the charged particle field. This requires that the scattering properties of medium
Figure 2.8 A fluence $\Phi_e$ of charged particles crossing a thin layer of medium $g$.

$w$ and medium $g$ are sufficiently similar. In particular, backscattering is the same at $w$-$g$, $g$-$w$, and $w$-$w$ interfaces.

b. The energy deposited in the cavity is entirely by the charged particles crossing the cavity, ie. no charged particle is produced in the cavity.

However, the Bragg-Gray cavity theory (B-G theory) is not in agreement with experimental measurements, especially with cavity walls of high atomic number. The stopping power in the B-G theory is evaluated under the assumption of the continuous slowing down approximation (CSDA). However, the delta rays produced by the knock-on
electron-electron collisions enhance the equilibrium spectrum of lower energy electrons crossing the cavity. The resulting spectrum is further enhanced at low energies as the energy of the electrons is decreased because the Møller (1931) differential cross section for delta ray production is inversely proportional to the square of the energy. Since the stopping power increases with decreasing electron energy, the presence of additional low energy electrons in the equilibrium spectrum as a result of delta ray production, would give a larger stopping power ratio for low and high Z media than that evaluated using the CSDA assumption.
2.9.2 Spencer-Attix cavity theory

Spencer and Attix (1955) rederived the B-G theory to include the effect of the production of delta rays. The Spencer-Attix theory assumes CPE and the absence of bremsstahlung generation in addition to the two B-G conditions. The equilibrium electron spectrum is divided into two groups in the Spencer-Attix theory.

a. Electrons that have energy \( T > \Delta \) and are able to cross the cavity. These electrons can therefore transport energy into the cavity.

b. Electrons that have energy \( T < \Delta \) and are not able to cross the cavity as they deposit their energy locally. These electrons are therefore not able to transport energy into the cavity.

According to the Spencer-Attix theory, the ratio of dose in medium \( g \) and medium \( w \) is given by

\[
\frac{D_g}{D_w} = \frac{\int \frac{R(T_o';T)}{(dT/\rho dx)_g} \cdot (S/\rho)_g(T,\Delta) \, dT}{\int \frac{R(T_o';T)}{(dT/\rho dx)_w} \cdot (S/\rho)_w(T,\Delta) \, dT}
\]  

(2.79)
where \( R(T_0, T) \) is the ratio of the differential electron fluence including delta rays to that of primary alone. The difference between the Bragg-Gray theory and Spencer-Attix theory becomes negligible, as the cavity size is increased, due to the diminishing influence of delta rays. The stopping power ratios given by Burlin (1968) show that for Co-60 \( \gamma \)-rays, and a cavity size equal to the range of a 0.08 MeV electron, the theories agree to within 1\%. 
2.8.3 Burlin cavity theory

Burlin (1966) proposed a general cavity theory to include all cavity sizes. The size of the cavity is defined relative to the range of the electrons. A cavity is considered small when the range of the electrons entering the cavity is very much greater than the cavity dimensions. The electron spectrum within the cavity is solely determined by the medium surrounding the cavity. The ratio of absorbed dose in the cavity to that in the surrounding medium is given by the Spencer-Attix theory. When the cavity dimensions are many times larger than the range of the most energetic electrons, the electron spectrum within the cavity is determined by the cavity material itself. The ratio of the absorbed dose in the cavity to the absorbed dose in the medium is given by

$$D \left( \frac{E}{J(\rho)} \right) = \frac{D_g}{D_w} = \frac{\langle \mu_{en}/\rho \rangle_g}{\langle \mu_{en}/\rho \rangle_w}$$

(2.80)

where $\langle \mu_{en}/\rho \rangle$ is the mean mass energy absorption coefficient. Cavities whose dimensions are comparable to the range of electrons entering the cavity have a spectrum within the cavity that is partially determined by the medium and partially determined by the cavity material. The Burlin cavity theory in its simplest form can be written as follows:

$$f(T, \Delta) = d \cdot \langle S/\rho \rangle_w^{g} + (1-d) \cdot \langle \mu_{en}/\rho \rangle_w^{g}$$

(2.81)
where $d$ is a parameter that is related to the cavity dimensions and approaches unity for small cavity sizes and zero for large ones. The above equation is not valid for doses in the medium adjacent to the cavity as the dose is perturbed by the presence of the cavity. The theory requires that $D_{med} = K_{med}$ (although not explicitly stated by Burlin, but implied in the equation 2.81) and is therefore only valid for points further from the cavity than the maximum range of the electrons. The presence of the cavity modifies the electron spectrum in two ways (Burlin, 1965). First, it alters the relative distribution of the electrons in the energy spectrum. This results from two simultaneous processes.

a. The electron spectrum generated by the photons in the medium and which is partially or completely absorbed as it crosses the cavity.

b. The photons interacting in the cavity will generate an electron spectrum that will build up toward equilibrium at increasing distances from the cavity/medium interface.

In order to treat this quantitatively, Burlin used the experimental results of the beta-ray absorption properties. The beta rays are absorbed nearly exponentially with an absorption coefficient $\beta$. Therefore, the electron spectrum in the cavity at a distance $x$ from the medium will be reduced by the factor, $\exp(-\beta x)$, while the electrons generated in the cavity build-up to $1 - \exp(-\beta x)$. Burlin defined $d$ as the mean value of the electron transmission factor, $\exp(-\beta x)$, in the cavity.
where \( l \, (\text{g cm}^{-2}) \) is the mean chord length along the path \( x \) from the medium to any point in the cavity. The corresponding relation for \( 1-d \) is the mean value of the electron build-up in the cavity and is given by

\[
1 - d = \frac{\langle \Phi_{e,g} \rangle}{\Phi_{e,g,c}} \cdot \frac{\int_0^l \Phi_{e,m,c} e^{-\beta x} \, dx}{\int_0^l \Phi_{e,m,c} \, dx} = \frac{1 - e^{-\beta l}}{\beta l} \quad (2.83)
\]

For an inhomogeneous medium the value of the absorption coefficient is not the same for the cavity- and medium-generated electrons and therefore,

\[
\frac{\langle \Phi_{e,g} \rangle}{\Phi_{e,g,c}} = d' \neq (1-d) \quad (2.84)
\]

and

\[
d + d' \neq 1 \quad (2.85)
\]

The Burlin theory ignores this possibility.
2.8.4 Horowitz modified Burlin cavity theory

Horowitz et al (1983) have proposed a modification of Burlin's cavity theory based on the proof (Horowitz and Dubi, 1982) that, in general, the average path length for electrons crossing the cavity is not equal to the average path length for electrons created within the cavity. Burlin's expression was modified to the following:

\[
f(T,A) = (Z/A)^{F} \left[ 1 + d \left( (Z/A)^{w} <S_{g}^{w}> - 1 \right) + d' \left( (Z/A)^{w} <\mu_{en}^{w}> - 1 \right) \right]
\]  

(2.86)

where \(d\) is the same as in Burlin model and \(d'\) is given by

\[
d' = \frac{\int_{0}^{g'} (1 - e^{-\beta x}) dx}{\int_{0}^{g'} dx} = \frac{\beta g' + e^{-\beta g'} - 1}{\beta g'}
\]

(2.87)

where \(g'\) is the average pathlength for electrons created by photon fluence interaction within the cavity. Although conceptually interesting, the Burlin-Horowitz model did not make any significant improvement on the Burlin model (Kearsley, 1984).
2.8.5 Kearsley cavity theory

Kearsley proposed a general cavity theory (1984) that takes into account secondary electron scattering at the cavity boundary. An outstanding feature of this model is the ability to calculate dose distributions inside plane parallel cavities surrounded by medium.

The Kearsley cavity theory gives the following expression:

\[ f_K = d_1 <S/\rho>^k_w + d_2 <\mu_{en}/\rho>^k_w \]  

(2.88)

where

\[ d_1 = (1-b_w) \cdot <\lambda_1(w,g)> \]  

(2.89)

\[ d_2 = 1 + <\lambda_1(w,g)> b_w (1-b_g) (1-e^{-\gamma}) - <\lambda_1(g,g)> (1-b_g) [1 + b_g (1-e^{-\gamma})] \]  

(2.90)

\[ <\lambda_1(w,g)> = b_w e^{-2\beta_w} \frac{(e^{-\beta_w l} - 1)}{\beta l} + (1-b_w b_g l) \frac{(1-e^{-\beta_l})}{\beta l} \]  

(2.91)

\[ <\lambda_1(g,g)> = b_g e^{-2\beta_g} \frac{(e^{-\beta_g l} - 1)}{\beta l} + (1-b_g^2) \frac{(1-e^{-\beta_g l})}{\beta l} \]  

(2.92)

\[ b_g(l) = b_g \cdot (1 - e^{-\alpha l}) \]  

(2.93)
where \( \lambda_i(w,g) \) is the multiple backscattering coefficient between medium \( w \) and medium \( g \), \( \lambda_i(g,g) \) is the multiple backscattering coefficient between medium \( g \) and medium \( g \), \( l \) is the cavity thickness, \( \beta \) is the effective mass absorption coefficient of electrons, \( b \) is the electron backscatter factor, \( \alpha \) and \( \gamma \) are cavity electron backscatter and electron production coefficients, respectively. Kearsley suggests that \( l \) to be taken as the average path length of electrons crossing the cavity to the medium. In the absence of secondary electron backscattering, the Kearsley cavity theory reduces to Burlin's cavity theory.

We do not discuss the cavity theory by Burch (1955). The cavity theory is so detailed as to preclude a full numerical solution (Burlin, 1966).
2.9 A new cavity theory

2.9.1 Introduction

The Burlin cavity theory ignores the electron backscattered into the cavity from the rear interface. The Kearsley theory includes electron backscatter from the wall material. However, the Kearsley theory has poor correlation with experimental results in high Z media. The magnitude of the input parameters is arbitrary at present (Ogunleye, 1986) and therefore, the dose to the cavity depends on the choice of the parameters. Here, we have developed a new cavity theory that can relate the dose in the cavity to the dose in the medium (front wall or back wall) when front wall medium, cavity medium and the back wall medium are all of different atomic composition. The parameters of the model are extracted systematically.
2.9.2 Theory

The total electron fluence in the cavity \( (\Phi_{e,g}(z)) \) of thickness \( t \) is divided into three groups.

a. The electron fluence spectrum that originated in the cavity, including its backscatter \( (\Phi_{e,g}(z)) \).

b. The electron fluence spectrum that originated in the front wall medium and its backscatter as it crosses the cavity \( (\Phi_{e,w}(z)) \).

c. The electron fluence spectrum in the cavity resulting from the differences in the backscattering coefficient of the cavity and the back wall medium \( (\Phi_{e,m,b}(z)) \).

From here on, we refer to \( w \) as the front wall medium, \( g \) as the cavity medium, \( m \) as the back wall medium and \( b \) is the backscattering coefficient.

We assume that Compton interaction is the dominant radiation interaction and thus the applicable energy range of the theory is from 500 kV to 20 MV x-rays. The energy loss cross-section of the electron stopping power is determined by the excitation energy of the molecules and the Compton scatter cross-section. When the Compton process is dominant, the difference between the mean excitation energies per electron of different materials is small and so the stopping power ratio is equal to the ratio of the electron densities, making the ratio independent of cavity dimensions.

The electron fluence in the cavity is given by
Therefore, the dose to the cavity is given by

\[ D_g = \Phi_{e,g}(z) \cdot <S/p>_g = (\Phi_{e,g}(z) + \Phi_{e,w}(z) + \Phi_{e,m,b}(z)) \cdot <S/p>_g \] (2.95)

In the absence of the cavity, the dose to the wall is given by

\[ D_w = \Phi_{e,w,c} \cdot <S/p>_w \] (2.96)

where the subscript \( c \) denotes the equilibrium fluence in the wall medium (i.e. under charged particle equilibrium). Therefore, the ratio \( f_H \) of the average dose to the cavity \( (g) \) and to the front wall medium \( (w) \) is given by

\[ f_H = \frac{\Phi_{e,g}(z) + \Phi_{e,w}(z) + \Phi_{e,m,b}(z)}{\Phi_{e,w,c}} \cdot <S/p>_w \] (2.97)

\( \Phi_{e,g}, \Phi_{e,w} \) and \( \Phi_{e,m,b} \) are functions of the distance \( z \) from the front cavity interface (figure 2.9). Each term can be represented by a product of a distance-independent electron fluence term at the interface and a distance-dependent weighting factor. Therefore,

\[ f_H = \frac{C_1(z) \cdot \Phi_{e,g,c} + C_2(z) \cdot \Phi_{e,w-g,i} + C_3(z) \cdot \Phi_{e,g-m,b}}{\Phi_{e,w,c}} \cdot <S/p>_w \] (2.98)
where $\Phi_{eg,c}$ is the equilibrium cavity fluence, $\Phi_{e,w-g,i}$ is the electron fluence at the front wall-cavity ($w$-$g$) interface, $\Phi_{e,g-m,b}$ is the electron fluence at the back wall-cavity ($g$-$m$) interface and $C_1$, $C_2$ and $C_3$ are $z$ dependent weighting factors. $C_1$ is the ratio of the electron fluence generated in the cavity medium at a distance $z$ from the front wall-cavity interface and the equilibrium fluence in the cavity medium, $C_2$ is the ratio of the electron fluence generated in the front wall medium at a distance $z$ from the wall-cavity ($w$-$g$) interface (figure 2.9) and the electron fluence in the front wall-cavity ($w$-$g$) interface, and $C_3$ is the ratio of the backscattered electron fluence at a distance $(t-z)$ from the front wall-cavity ($w$-$g$) interface to the backscattered electron fluence at the cavity-back wall ($g$-$m$) interface.

The ratio of the mean dose to the cavity and to the front wall $<f_H>$ can be found by using the mean values of $C_1$, $C_2$ and $C_3$ in the cavity. Therefore, $<f_H>$ is given by

$$<f_H> = \frac{d_1 \Phi_{eg,c} + d_2 \Phi_{e,w-g,i} + d_3 \Phi_{e,g-m,b}}{\Phi_{e,w,c}} \cdot <\rho^g_w>$$

(2.99)

where $d_1$, $d_2$ and $d_3$ are the mean weighting factors in the cavity and are discussed in section 2.8.4.b. Now we will relate each term in the numerator to $\Phi_{e,w,c}$.

If the cavity and the wall materials are irradiated by the same photon fluence separately, then the dose ratio between the cavity and the wall under CPE is related by the ratio of the average energy mass absorption coefficients. Therefore,

$$\frac{D_g}{D_w} = \langle \mu_{eff} / \rho \rangle^g_w$$

(2.100)
Figure 2.9 Illustration of cavity parameters on a linear scale. $\Phi_{e,g}(z)$ is the electron fluence spectrum that originated in the cavity (g), including its backscatter. $\Phi_{e,w}(z)$ is the electron fluence spectrum that originated in the front wall medium (w) and its backscatter as it crosses the cavity. $\Phi_{e,m,b}(z)$ is the electron fluence spectrum in the cavity resulting from the differences in the backscattering coefficient of the cavity (g) and the back wall medium (m).
The dose ratio between the cavity and the wall is also related by

$$\frac{D_g}{D_w} = \frac{\Phi_{e,g,c}}{\Phi_{e,w,c}} \cdot \langle S/\rho \rangle_w^g$$ \hspace{1cm} (2.101)

Equating equation 2.100 and equation 2.101, we get

$$\Phi_{e,g,c} = \Phi_{e,w,c} \cdot \langle \mu_{ed}/\rho \rangle_w^g \cdot \langle S/\rho \rangle_w^w$$ \hspace{1cm} (2.102)

Let $F$ be the fraction of the total number of electrons per unit area which crosses a reference plane $r$ from left to right in a medium under CPE, and let $b$ be the probability of backscattering per electron crossing the reference plane $r$ from left to right. Therefore, in a homogeneous medium, the equilibrium fluence ($\Phi_{e,c}$) is simply given by

$$F \cdot \Phi_{e,c} \cdot (1+b) = \Phi_{e,c}$$ \hspace{1cm} (2.103)

If the medium to the left of the reference plane $r$ is $w$ and to the right of the reference plane is $g$ then the electron fluence at the interface of $w$-$g$ ($\Phi_{e,w-g,i}$) is given by

$$\Phi_{e,w-g,i} = \Phi_{e,w,c} \cdot F_w \cdot (1 + b_g)$$ \hspace{1cm} (2.104)

where $F_w$ is the fractional forward electron fluence originating in the $w$ medium. If medium $g$ of thickness $t$ is sandwiched between medium $w$ and medium $m$, then the relative backscattered electron fluence between medium $g$ and medium $m$ at the interface
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\( g-m (\Phi_{e,g,m,b}) \) is given by

\[
\Phi_{e,g-m,b} = F_{e,g-m,i} \cdot \Phi_{e,g-m,i} \cdot (b_m - b_g)
\]  

(2.105)

where \( F_{e,g-m,i} \) is the fractional forward electron fluence of the total electron fluence, \( \Phi_{e,g-m,i} \), arriving at the interface \( g-m \). Electron backscattering within the medium \( g \) is already accounted for in \( \Phi_{e,g} \) and \( \Phi_{e,w} \). \( \Phi_{e,g-m,i} \) consists of \( \Phi_{e,g} \) and \( \Phi_{e,w} \) evaluated at cavity thickness \( t \) and is given by

\[
F_{e,g-m,i} \cdot \Phi_{e,g-m,i} = F_g \cdot \Phi_{e,g}(t) + F_w \cdot \Phi_{e,w}(t)
\]  

(2.106)

where \( F_g \) is the fractional forward electron fluence originating in the cavity medium and \( F_w \) is the fractional forward electron fluence originating in the front wall medium. \( \Phi_{e,g}(t) \) and \( \Phi_{e,w}(t) \) can be expressed by a product of cavity size-independent electron fluence terms and cavity size-dependent weighting factors. Therefore,

\[
F_{e,g-m,i} \cdot \Phi_{e,g-m,i} = d_4 \cdot F_g \cdot \Phi_{e,g,c} + d_5 \cdot F_w \cdot \Phi_{e,w-g,i}
\]  

(2.107)

where \( d_4 \) and \( d_5 \) are the fractions of the electron fluences \( \Phi_{e,g,c} \) and \( \Phi_{e,w-g,i} \), respectively, that arrive at the interface \( g-m \) of cavity thickness \( t \) and is discussed in section 2.8.4.b.

Substituting for \( \Phi_{e,g,c} \) in equation 2.102, \( \Phi_{e,w-g,i} \) in equation 2.104 and \( \Phi_{e,g-m,i} \) in equation 2.105 into equation 2.99, we get
$$<f_H> = \frac{d_1 \langle \mu_{en}/\rho \rangle_w^g \langle S/\rho \rangle_w^w \Phi_{e,w,c} + d_2 F_w (1 + b_g) \Phi_{e,w,c}}{\Phi_{e,w,c}} \cdot \langle S/\rho \rangle_w^g +$$

$$d_3 \frac{d_4 F_g \langle \mu_{en}/\rho \rangle_w^g \langle S/\rho \rangle_w^w \Phi_{e,w,c} + d_5 F_w (1 + b_g) \Phi_{e,w,c}}{\Phi_{e,w,c}} \cdot (b_m - b_g) \cdot \langle S/\rho \rangle_w^g \quad (2.108)$$

Factoring the above equation we get

$$<f_H> = d_1 \cdot \langle \mu_{en}/\rho \rangle_w^g + d_2 \cdot F_w \cdot (1 + b_g) \cdot \langle S/\rho \rangle_w^g +$$

$$d_3 \left( d_4 F_g \cdot \langle \mu_{en}/\rho \rangle_w^g + d_5 F_w \cdot \langle S/\rho \rangle_w^w \right) \cdot (b_m - b_g) \quad (2.109)$$

When the cavity size is very large compared to the range of the electrons, the above equation reduces to the ratio of the mass energy absorption coefficients of cavity and medium as in the Burlin and the Kearsley cavity theories. When the cavity size is very small compared to the range of the electrons and the front wall medium and back-wall medium are identical, $<f_H>$ reduces to the Bragg-Gray theory. However, when the front and the back wall are not identical, $<f_H>$ does not reduce to Bragg-Gray theory. Since the contribution from the front wall medium, cavity medium and the backscattering from the back wall medium is calculated separately, all three media could be of different atomic composition.
2.9.3 Determination of parameters

2.9.3.a Backscattering coefficient

Theoretical treatment of electron scattering based on diffusion (Bethe et al, 1938) and single elastic collision (Everhart, 1960), suggests that the backscatter is almost independent of the electron energy. This was experimentally shown to be correct (Bethe, 1949, Everhart, 1960) for low energy electrons. For high energy electrons (above 2 MeV), the energy losses due to continuous slowing down of the particle traversing the scatterer are more dominant than those due to scattering. So at high energy, electron backscattering is inversely proportional to energy (Wright and Trump, 1962). Relative electron backscattering (EBF) values for high energy electrons have been measured (Klevenhagen et al, 1982, Gagnon and Cundiff, 1980). Dutreix and Bernard (1966) have derived $b$ for several materials from experimental measurements of dose near interfaces between dissimilar media for various photon energies; values are given in the appendix C.
2.9.4.b Weighting factors

We also assume as in Burlin’s theory that the secondary electrons generated in the wall are exponentially attenuated in the cavity with an attenuation coefficient $\beta_a$ and the electrons generated in the cavity build-up exponentially with a build-up coefficient $\beta_b$ and therefore, their sum is unity at all depths. This assumption is based on the empirical observation of the dose build-up curve for high energy x-rays. The average weighting factor $d_1$ is given by,

$$d_1 = \frac{\int_0^t e^{-\beta_a z} dz}{\int_0^t dz} = \frac{1 - e^{-\beta_a t}}{\beta_a t} \quad (2.110)$$

where $\beta_a$ is the effective electron absorption coefficient (g/cm²) in the cavity, and $t$ is the cavity thickness. The weighting factor $d_2$ is given by

$$d_2 = 1 - d_1 \quad (2.111)$$

The weighting factor $d_3$ is given by

$$d_3 = \frac{\int_0^t e^{-\beta_b (t-z)} dz}{\int_0^t dz} = \frac{1 - e^{-\beta_b t}}{\beta_b t} \quad (2.112)$$
where $\beta_a$ is the effective electron absorption coefficient (cm$^2$/g) in the cavity for electrons that are backscattered from the back wall medium.

The weighting factors $d_4$ and $d_5$ are given by

$$d_4 = (1-d_5) = e^{-\beta_a t}$$  \hspace{1cm} (2.113)

There are only two parameters $\beta_a$ and $\beta_b$ that need to be known in order to evaluate $d_1$, $d_2$, $d_3$, $d_4$ and $d_5$.

Burlin's assumption of the exponential attenuation of electron spectra is based on spectra of $\beta$-rays emitted by radioactive atoms. The $\beta$-ray spectrum is known to be absorbed exponentially (Schmidt, 1906, Hahn and Meitner, 1908) and is usually characterized by the maximum energy of the electrons to determine the effective attenuation coefficient. Compton electron spectra, which are generally different from the $\beta$-ray spectra, cannot be characterized by the maximum energy of the Compton electron to determine its effective attenuation coefficient. In the following section, we describe a way to determine $\beta_a$.

Experimental measurements of $\beta$-rays show that $\beta_a$ is approximately inversely proportional to the maximum energy of the electrons. $\beta_b$ for the backscattered electron can be obtained from the product of $\beta_a$ and fractional energy backscattered per electron. Backscattered electrons have much lower energy than the equilibrium spectra and therefore, they are attenuated much more rapidly. If the fractional energy backscatter data are not available, we recommend $\beta_b = 3.5\beta_a$ for photon beams less than 4 MV and $\beta_b = 3\beta_a$ for photon beams 4-20 MV. The recommendation is based on the fact that the
backscattered electrons have 1/3 to 1/4 the energy of the incident electrons and decrease slowly with increasing energy (estimated from Tabata, 1967).
2.9.3.c Determination of electron absorption coefficient ($\beta_a$)

Let $\Phi_e$ be the number of electrons originating in a medium through the interaction of photons at any depth $z$. Let the electron production coefficient in the cavity be $\beta_p$. The electrons originating in the cavity will be absorbed as they lose their energy. Let the absorption coefficient be $\beta_a$. Photons in the medium will be absorbed exponentially with an absorption coefficient $\mu$. This process is analogous to the radioactive parent-daughter decay scheme. The analogies are as follows:

| Cavity theory | photon | electron | depth ($z$) | $\mu$ | $\beta_a$ | $\Phi_e(0) = 0$
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Parent-daughter relation</td>
<td>parent</td>
<td>daughter</td>
<td>time ($t$)</td>
<td>decay constant of parent</td>
<td>decay constant of daughter</td>
<td>concentration of daughter at $t=0$ is zero</td>
</tr>
</tbody>
</table>

At any depth $z$, the rate of change of $\Phi_e$ is given by

$$\frac{d\Phi_e(z)}{dz} = \beta_p(\Phi_p) - \beta_a\Phi_e(z)$$

(2.114)

or

$$\frac{d\Phi_e(z)}{dz} = \beta_p(\Phi_p)_0 e^{(-\mu z)} - \beta_a\Phi_e(z)$$

(2.115)

The general solution of the above differential equation is given by

$$\Phi_e(z) = (\Phi_p)_0 (h.e^{-\mu z} + k.e^{-\beta_a z})$$

(2.116)
where \((\Phi_p)_0\) is the incident photon fluence in the medium, \(h\) and \(k\) are coefficients to be determined. \(h\) and \(k\) are related as shown later.

Differentiating the equation 2.116 with respect to depth \((z)\) gives

\[
\frac{d\Phi_e(z)}{dz} = (\Phi_p)_0 \left( h(-\mu)e^{-\mu z} + k(-\beta_a)e^{-\beta_a z} \right)
\]  

Substituting the equation 2.116 and equation 2.117 into equation 2.114, we get

\[
- (\Phi_p)_0 \left( h \cdot e^{-\mu z} + k \cdot \beta_a \cdot e^{-\beta_a z} \right) = \beta_p \cdot (\Phi_p)_0 \cdot e^{-\mu z} - \\
\beta_a \cdot (\Phi_p)_0 \cdot (h \cdot e^{-\mu z} + k \cdot e^{-\beta_a z})
\]

collecting the like terms and simplifying, we get

\[
h(\Phi_p)_0 e^{-\mu z} (\beta_a - \mu) + k(0) - (\Phi_p)_0 (\beta_p) e^{-\mu z} = 0
\]  

or

\[
e^{-\mu z} \cdot (h \cdot (\Phi_p)_0 \cdot (\beta_a - \mu) - \beta_p \cdot (\Phi_p)_0) = 0
\]

Since \(e^{-\mu z}\) is not zero, therefore

\[
h \cdot (\Phi_p)_0 \cdot (\beta_a - \mu) = \beta_p \cdot (\Phi_p)_0
\]

\[
\therefore h = \frac{\beta_p}{\beta_a - \mu}
\]
At the surface of the phantom \((z=0)\), the electron fluence is zero (i.e. the photon beam is electron "contamination free" and there are no electrons backscattered from the surface). Therefore, from equation 2.116, we get

\[
(\Phi_p)_0 \cdot (h \cdot e^{-\mu z} + k \cdot e^{-\beta_a z}) = 0 \tag{2.123}
\]

or

\[
(\Phi_p)_0 \cdot (h + k) = 0 \tag{2.124}
\]

\[
\therefore k = -h = \left(\frac{-\beta_p}{\beta_a - \mu}\right) \tag{2.125}
\]

Substituting the above equation into the general solution (equation 2.116) yields

\[
\Phi_e(z) = \left(\frac{(\Phi_p)_0 \cdot \beta_p}{\beta_a - \mu}\right) \cdot (e^{-\mu z} - e^{-\beta_a z}) \tag{2.126}
\]

If we normalize the electron fluence at the depth of maximum ionization \(d_{\text{max}}\) in the phantom, the above equation becomes

\[
\frac{\Phi_e(z)}{\Phi_e(d_{\text{max}})} = \frac{(e^{-\mu z} - e^{-\beta_a z})}{(e^{-\mu d_{\text{max}}} - e^{-\beta_a d_{\text{max}}})} \tag{2.127}
\]
\( \Phi_e(z) / \Phi_e(d_{\text{max}}) \) is also equal to the TMR. Therefore,

\[
TMR = \frac{\Phi_e(z)}{\Phi_e(d_{\text{max}})} = A \cdot \left( e^{-\mu z} - e^{-\beta_e z} \right) \tag{2.128}
\]

\( \beta_e \) and \( \mu \) can be determined by fitting the above equation to the measured TMR. Alternatively, one may take the derivative of the above equation and get the depth of maximum dose as

\[
d_{\text{max}} = \frac{\ln(\mu / \beta_e)}{(\mu - \beta_e)} \tag{2.129}
\]

\( \mu \) can be determined by from the measured TMR data beyond the depth of maximum dose \( (d_{\text{max}}) \). Since this procedure uses only one data point, we recommend the use of equation 2.128.

The first data point for a dose build-up curve measurement should be taken at 0.03-0.04 cm depth in a phantom to avoid the contaminating electrons from the collimator assembly. The build-up curve is extrapolated to zero depth by forcing the dose to go to zero at depth zero. If the medium \( g \) is tissue equivalent then the TMR measured in water may be used to determine the coefficients \( \beta_e \) and \( \mu \). It may be noted that there are no path length corrections (for oblique electron trajectories) in the evaluation of the weighting factors in our model as in the Burlin and Kearsley cavity theories. The effect of path lengthening is included in the determination of \( \beta_e \) and \( \beta_b \).
2.10 Conversion of ionization to dose in inhomogeneous media

It is evident from the analysis of the cavity ionization that the cavity wall medium and the cavity should be of the same composition as the phantom material to minimize dose perturbations. When the dosimeter wall and cavity differ significantly from the phantom material in atomic composition, then the dose to the cavity depends on the relative number of the charged particles originating in the wall material. For simple dosimeters such as LiF TLD, it is possible to find a correction factor to give the true dose in the phantom material. It is very difficult to estimate a correction factor for an ionization chamber when the wall of the ionization chamber does not match the medium. The response of the ionization chamber depends on the differences of the atomic composition of the wall and the phantom, the size and shape of the wall material and the charged particle energy spectrum. We recommend the following procedure to correct the ionization reading for any ionization chamber.

Dose to the medium $w$ is given by

$$D_w = \Phi_{e,w} \cdot \langle S/p \rangle_w$$  \hspace{1cm} (2.130)

Dose to the medium $w$ is also related by

$$D_w = \langle Q_{g,w}/m \rangle \cdot (W/e) \cdot \frac{\langle S/p \rangle_w}{\langle S/p \rangle_g}$$ \hspace{1cm} (2.131)
where $Q_{g,w}$ is the charge of either sign produced in the air cavity in an ionization chamber which does not perturb the medium, $(W/e)$ is the mean energy expended per unit charge produced in the air and $m$ is the mass of air (medium $g$) in the ionization chamber. Therefore,

$$Q_w = \frac{\Phi_w \cdot m \cdot \langle S/p \rangle_g}{(W/e)}$$

(2.132)

If another ionization chamber is placed in medium $y$ which also does not perturb that medium $y$ and which has the same mass of air ($m$), then the charge collected in medium $y$ is given by

$$Q_y = \frac{\Phi_y \cdot m \cdot \langle S/p \rangle_g}{(W/e)}$$

(2.133)

If the charged particle spectrum crossing the air cavity is the same in both media then the stopping power in air is also the same. Therefore, the ratio of the ionizations in the two media is given by

$$\frac{Q_w}{Q_y} = \frac{\Phi_w}{\Phi_y}$$

(2.134)

From equation 2.102, the ratio of the electron fluence at electronic equilibrium under the same photon fluence in medium $w$ and in medium $y$ is given by
Substituting equation 2.135 into 2.134 yields

\[ \frac{Q_w}{Q_y} = \left< \mu \frac{e}{\rho} \right>_y \cdot \left< \frac{S}{\rho} \right>_w \]  \hspace{1cm} (2.136)

Let the ionization chamber to be calibrated give the true ionization in medium \( w \) (\( Q_w \)) (i.e. charge collection corrected for chamber perturbation) and an ionization \( (Q_{y,\text{un}}) \) in medium \( y \), which requires an ionization correction factor \( f \). Therefore the above equation can be written as,

\[ \frac{Q_w}{Q_{y,\text{un}}} \cdot f = \frac{Q_w}{Q_y} = \left< \mu \frac{e}{\rho} \right>_y \cdot \left< \frac{S}{\rho} \right>_w \]  \hspace{1cm} (2.137)

The ionization correction factor \( f \) is then given by

\[ f = \frac{Q_w}{Q_{y,\text{un}}} \cdot \left< \mu \frac{e}{\rho} \right>_y \cdot \left< \frac{S}{\rho} \right>_w \]  \hspace{1cm} (2.138)

It is important to note that the ionization chamber must be exposed to the same photon fluence in both media. The ionization correction factor \( f \) in aluminium for a PTW 0.05 cm\(^3\) Markus chamber is in the order of 1.20. The large mechanical support (for the parallel plate chamber) accounts for the relatively large correction factor. It may be
noted that the TG-21 protocol (AAPM 1983) cannot give the proper correction factor to be used in an inhomogeneous medium because the protocol does not consider secondary electron backscattering from the chamber back wall and the protocol is only valid in tissue-equivalent media. Ignoring secondary electron backscattering results ionization correction factor to be unity for Markus parallel plate ionization chamber regardless of the medium.
CHAPTER 3 - MATERIALS AND METHODS

3.1 Materials

3.1.1 Accelerators

The photon beams for the experimental measurements were obtained either from a Varian Clinac 2100C (Varian Associates, Palo Alto, Ca) which produces 6 MV and 10 MV x-rays or from a Mitsubishi EXL-13 (Mitsubishi International Corporation, Philadelphia, PA) which produces 6 MV x-rays. A review of the theory and the operation of medical linear accelerators has been described by Karzmark (1983).

3.1.2 Dosimetry phantoms

Water is the recommended material for dosimetry phantoms (AAPM, 1983). However, for many experiments, polystyrene has been used instead of water for practical reasons. The electron density and the physical density of polystyrene relative to water is 1.008 and 1.04 respectively. Balsa wood and cedar wood were used for measurements in low density medium. Lung was simulated with balsa wood and bone was simulated with
3.1.3 Radiation detectors

Dosimetry measurements in large fields were carried out with a cylindrical Farmer type ionization chamber (PTW 0.6 cm³ or PTW 0.3 cm³, PTW-Freiburg, Gottingen, Germany). Measurements in small fields require the size of the radiation detector to be as small as possible. Typically, a small volume ionization chamber or a diode is used for such measurements. A parallel plate ionization chamber (PTW Markus 0.05 cm³) was used for measurements in polystyrene and in inhomogeneous media for central axis measurements. The Markus chamber has an active diameter of 4.6 mm and is operated with a negative bias voltage of 300 volts. The smallest field size used for measurements with the Markus chamber was 1.5 cm x 1.5 cm.

A p-type semiconductor diode (Therados, Uppsala, Sweden) was used for field sizes up to 0.7 cm diameter. It has superior spatial resolution (active diameter ~2.5 mm) and a better signal to noise ratio because of lower ionization potential and higher physical density. The theory and operation of diodes has been described in detail by Dearnaley et al (1966), Brown et al (1969) and Bertolini et al (1968). The silicon diode was operated without bias because the DC leakage current decreases with the bias voltage more rapidly than the radiation induced current (Attix, 1986). The Markus ionization chamber or the diode was connected to a digital electrometer (Victoreen Model 500, Nuclear Associates,
Cleveland, OH or Keithley 614, Keithley Instruments, Cleveland, OH) for charge measurements.

Absorbed dose measurements with a diode in the penumbra region can exhibit significant perturbation (Bjarngard et al 1990). The response of a radiation detector is due to (1) secondary electrons that are produced outside the sensitive volume and the encapsulation of the detector and (2) secondary electrons that are produced within the sensitive volume and its encapsulation (Dawson et al 1984). For a thin wall ionization chamber, nearly all of the measured dose is due to secondary electrons originating outside the sensitive volume ((1), above) since the contribution from electrons originating in the gas of the collection volume is negligible. A diode detector in an x-ray beam measures a convolution of (1) and (2) above. Profile measurements with the same diode but different orientation result in slightly different measured penumbra. The diode’s sensitive volume has been determined to be 0.35 mm inside the encapsulation for the Scandatronix photon diode detector, from comparison of depth dose measurements at a depth of 10 cm in water with a PTW parallel plate Markus chamber (0.05 cc). The thickness of the sensitive volume is 60 μm (as given by Scandatronix, Uppsala). The p-type silicon substrate is stated by the manufacturer to be approximately 0.5 mm thick with a density of 2.3 g/cm$^3$ (Beddar et al, 1994). The backing of the diode is made of tungsten/epoxy (2.0 g/cm$^3$) and epoxy resin (1.2 g/cm$^3$); it is used to absorb the low energy backscattered photons. When the diode is oriented with its axis perpendicular to the beam axis and
parallel to the scan axis (figure 3.1 A) the detector size from the beam’s eye view is 0.5mm. The diameter of the encapsulation is approximately 7mm with an equal amount of resin around the diode. However, the epoxy resin and the diode are denser than water and therefore, the range of the lateral electrons is slightly reduced compared to water, which tends to give a sharper penumbra. On the other hand, the finite size of the detector broadens the penumbra. Therefore, an ideal design of a detector should be such that these two competing processes compensate each other. Obviously, this would make diodes photon energy sensitive. Perhaps a diode could be designed for Co-60 with provisions to add on sleeves of different thickness over the diode to obtain the correct beam profile.

It is tempting to orient the diode axis parallel to the beam axis and perpendicular
to the scanning direction due to the very small physical thickness (figure 3.1 B and C). This orientation can be achieved in two ways, one with the stem of the diode pointing to the central axis (figure 3.1 B) and the other pointing away from the central axis (Figure 3.1 C). Profiles obtained from these two configurations not only differ from each other (0.8 mm) but also are asymmetric. This is due to the different density and atomic composition (different from water) of the diode and its encapsulation (Beddar, 1994). Therefore, this orientation was not used. Rikner (1983) points out that this orientation causes problems when the diode is energy compensated. Recently Beddar (1994) investigated this same problem in more detail and concluded our choice of orientation to be the optimum orientation for an energy compensated diode.

A Wellhoffer 3D automated water phantom scanner and a diode were used for all percentage dose (PDD) measurements.

3.1.4 External collimators

An external collimator was made to produce various small field sizes (figure 3.2). An external collimator reduces the geometric penumbra of the radiation beam. The collimator frame is made of aluminium and the collimator inserts were made of 10 cm thick cerrobend (low melting temperature lead, bismuth and tin alloy). The collimator assembly has four adjustment screws at the base which allows precision movement of the collimator to better than 0.05 cm. This allows the collimator to be centered along the beam axis. The design of the collimator is a variation of that of Lutz et al (1988).
Figure 3.2 External collimator built for the experiments. The design of the collimator is a variation of that of Lutz et al (1988).
3.2 METHODS

3.2.1. Determination of zero area TMR

The TMR's were measured with a Baldwin Farmer type thimble ionization chamber (PTW N23333, 0.6 cm$^3$) in a polystyrene phantom. Since there is no experimental method to measure either primary or scatter dose, a number of approximate methods based on the extrapolation of TMR data to zero field size have been developed (Van Dyk 1977, Day 1983, Bjarngard et al 1988, Nizin et al 1993). The primary dose can also be calculated from the attenuation coefficients for the known photon energy spectrum (Storm and Israel 1970, Hubbell et al 1975, Seltzer 1993). The first extrapolation procedure relies on the exponential nature of the TMR's beyond a certain depth, usually 10 cm. The effective attenuation coefficients are plotted for each field size and extrapolated to zero area field size. Although this procedure may work for certain energies it is unlikely that an attenuation coefficient extrapolated from TMR measurements in electronic equilibrium can describe the zero area TMR for the whole range of depths of interest because the scattered photon contribution is a nonlinear function of field size and depth. It may be pointed out that if there is no detectable difference between the TMR's of two field sizes which are larger than the maximum range of the electrons liberated by the photons, it does not guarantee narrow beam geometry. It simply indicates that the ratio of incremental scatter between $d_{max}$ and depth $d$, is the same for both of the field sizes. The second procedure is to plot TMR at a depth against field size and linearly extrapolate to zero area. However, the scatter dose is a nonlinear function of field size.
Ideally, one should derive zero area $TMR$ from narrow beam conditions under electronic equilibrium but this is not possible with high energy photons. All approximation methods must satisfy the field size dependence of scatter-doses by requiring that

$$\lim_{x \to 0} D_s(x,d) = 0 \quad (3.1)$$

where $D_s$ is the scatter dose and $x$ is the length of a side of the square field. We used a nonlinear extrapolation method to derive zero area $TMR$. The following equation was used to fit $TMR$ vs. $x$ for various depths, and was extrapolated to zero area field size.

$$TMR(x,d) = D(d) - A(d) \cdot \exp(b \cdot x) \quad (3.2)$$

where $D,A$ and $b$ are parameters that are determined from the nonlinear least squares fit of $TMR$ for field sizes larger than that required for lateral electronic equilibrium. It is obvious from the above equation that $D$ is the maximum $TMR$ obtainable from the largest field size at a particular depth and $(D-A)$ is the $TMR$ for zero area under charged particle equilibrium. The zero area $TMR$ represents the dose due to the effective primary component of the beam under charged particle equilibrium. The dose contribution due to the scattered photons is small and increases with depth and field size. The parameters for the fitted functions are varied at each depth to account for the spectral changes of photons with depth. The zero area $TMR$ data from several depths is again fitted to the following function, which gives the value of $TMR$ at any depth including the buildup region.
where \(d\) is the depth in water in centimetres and \(L, c, M, e, N\) and \(f\) are parameters determined from nonlinear least squares minimization fitting of \(TMR\). The above equation can be used to describe \(TMR\) with 0.5\% accuracy for all depths and field sizes.
3.2.2. Collimator scatter factor ($S_{ct}$)

The collimator scatter factor ($S_{ct}$) varies with field size due to the backscattered radiation arising from the secondary collimator jaws of the linear accelerator. The contribution of this backscattered radiation to the beam monitor current will change as the collimator opening changes (Figure 3.3). Backscattered radiation from the secondary collimator jaws to a beam monitor chamber was measured on a Clinac 2100C for 6 MV and 10 MV x-rays in a similar way as described by Kubo (1989) and is shown in figure 3.4. A Farmer chamber (PTW 0.6 cm$^3$) was positioned at 100 cm source-to-axis distance SAD in air with appropriate build-up cap (1.5 g/cm$^2$ for 6 MV x-rays, 2.5 g/cm$^2$ for 10 MV x-rays). An external collimator with 2 cm diameter insert was placed at the blocking tray. We used a single slit instead of a double slit as used by Kubo. In order to study the collimator backscatter from the upper jaw (UJ) independently, the lower jaw (LJ) was fixed at 40 cm and the upper collimator was varied. The scatter from the lower jaws is a function of both the upper jaws and the lower jaws because the upper jaws partly block the lower jaws from the monitor chamber point of view. Therefore, for the same lower jaw setting, the value of LJ may vary depending on the position of the upper collimator jaw.

The treatment head as a whole is a source of scattered radiation. Experimental measurements by Kase and Svensson (1986) have shown that the forward photon scatter originates mainly from the flattening filter for most linear accelerators. The forward
Figure 3.3 Configuration of beam defining components for a Varian Clinac 2100C.

Figure 3.4 Measurement set-up for the study of backscattered radiation reaching the beam monitor chamber from the collimator jaws.
photon scatter can be derived from the ratio of collimator scatter with and without the external collimator in place for each position of the secondary collimators of the linac (Luxton and Astrahan, 1988).

The effect of external collimators or blocks placed at the shadow blocking tray has been discussed by several authors (Khan et al 1986). The external collimator effect was measured at approximately 330 cm source-to-surface distance (SSD) in order to irradiate the build-up cap fully for the smallest field size. All data are normalized to a 10 cm x 10 cm field size at 100 cm source to axis distance (SAD).
3.2.3. Relative dose factor (RDF) and photon scatter factor ($S_{PL}$)

Relative dose factors (RDF) were measured in polystyrene. They are normalized to a 10 cm x 10 cm field size at 100 cm source to axis distance (SAD) at a fixed depth (1.5 g/cm² for 6 MV x-rays, 2.5 g/cm² for 10 MV x-rays). The photon scatter factor ($S_{PL}(x)$) for large fields ($S_{PL}=1$) is calculated from equation 2.23 and equation 2.24. The calculated $S_{PL}(x)$ values are then fitted to the equation below and extrapolated to zero area field size.

$$S_{PL} = P - Q \exp(-\omega x)$$

(3.4)

where $x$ is the side of the equivalent square and $P, Q, \omega$ (cm⁻¹) are fitting parameters.
3.2.3. Electron nonequilibrium factor ($S_{PE}$) and phantom scatter factor ($S_{PT}$)$^2$

We describe a method for measuring the phantom scatter factor ($S_{PT}$) which does not require measurements in very small fields or at extended SSD. Our method is based on the density scaling theorem as described by O'Connor (1957).

The density scaling theorem states that the ratio of the primary to the scattered photons remains unchanged when the density of the irradiated phantom is changed such that all the linear dimensions of the phantom are altered in inverse proportion to the change in density. This is the basis of dose calculations based on effective radiological pathlength. This theorem requires that the probability of an interaction of any specific kind occurring in a small volume element is directly proportional to the number of interaction sites in the volume and that there are no cooperative phenomena that depend on the relative spatial positions of the interaction sites or any change in interaction probability. Compton scattering satisfies all of the above conditions and pair production satisfies these if the effective atomic number, $Z$, for pair production is not changed in the materials used. We also assume that the differences in polarization effect between the media are negligible.

The scaling theorem is expected to apply if the ratio of the effective attenuation coefficient for each type of interaction remains unchanged from one medium to the other. Kornelson (1983) has demonstrated experimentally the applicability of the density scaling theorem to a 10 MV x-ray beam.

From the density scaling theorem we get the following relationships:

\[ A \text{ publication on this method (Haider and El-Khatib, 1993) is listed in the bibliography.} \]
\[ S_{PT}(p_1r_1) = S_{PT}(p_2r_2) \text{ if } p_1r_1 = p_2r_2 \]  \hspace{1cm} (3.5)

where \( p_1 \) and \( p_2 \) are the electron densities relative to water in medium 1 and medium 2 respectively, and \( r_1 \) and \( r_2 \) are the corresponding field sizes in the media. Therefore, \( S_{PT}(r) \) for a small field could be calculated by measuring \( S_{PT}(r) \) in a large field of a low density medium. To avoid measuring \( S_{PT}(r) \) directly in a low density medium, we use the following relationships. Let the dose be \( D_1 \) in medium 1 with density \( p_1 \), and \( D_2 \) in medium 2 with density \( p_2 \) at the depth of maximum dose, for field size \( r \) at a 100 cm source to detector distance. Medium 1 and medium 2 are similar biological material, hence have the same atomic number \( Z \). Then for ionization dosimetry the following Bragg-Gray (Bragg, 1910, Gray, 1929, 1936) relations apply:

\[ D_1 = \frac{Q_1}{m} \cdot \frac{<W/e>}{<S/\rho>_{g1}} \]  \hspace{1cm} (3.6)

\[ D_2 = \frac{Q_2}{m} \cdot \frac{<W/e>}{<S/\rho>_{g2}} \]  \hspace{1cm} (3.7)

where \( Q_1 \) and \( Q_2 \) are the ionization charges collected in the gas, \( m \) is the mass of the gas (g) in the ionization chamber, \( <W/e> \) is the mean energy spent per ion pair produced and \( <S/\rho> \) is the average mass collision stopping power. The above equations do not require charged particle equilibrium. If the average mass collision stopping power for both the media, averaged over the whole spectrum of electrons generated is the same, then the
doses to medium 1 and medium 2 are related by the ratio of the ionization measurements. This assumes that in the region of electronic nonequilibrium the average energy of the electron spectrum is not changed. The error introduced by this is of the order of 1% for the field sizes used in our experiment (Andreo and Brahme, 1985, Wu et al 1993).

Therefore,
\[
\frac{D_1}{D_2} = \frac{Q_1}{Q_2}
\]  

(3.8)

If the $S_{PT}$ in both media are normalized to the same reference field size in a reference medium then the dose to medium 1 and medium 2 is related by,
\[
\frac{D_1}{D_2} = \frac{(S_{CT}(r_c), (S_{PT}(r_p))_1}{(S_{CT}(r_c), (S_{PT}(r_p))_2}
\]  

(3.9)

where $S_{CT}(r)$ is the collimator scatter factor for each medium. The dose due to the effective primary beam and the collimator scatter are the same in both the media. Therefore,
\[
\frac{D_1}{D_2} = \frac{(S_{PT}(r_p))_1}{(S_{PT}(r_p))_2} = \gamma = \frac{Q_1}{Q_2}
\]  

(3.10)

and
\[
(S_{PT}(r_p))_1 = (S_{PT}(r_p))_2 \cdot \gamma
\]  

(3.11)
or in terms of equivalent radii in the two media

\[ S_{PT}(r, \rho_1) = S_{PT}(r, \rho_2) \cdot \gamma \quad (3.12) \]

If medium 2 is water then

\[ S_{PT}(r, \rho_1) = S_{PT}(r) \cdot \gamma \quad (3.13) \]

Ionization charge measurements were made with a PTW N2332 (0.3 cc) chamber connected to a Victoreen Model 500 electrometer. \( \gamma \) was calculated as the ratio of ionization measured in low density material to that measured in water for the same field size at the same source to detector distance as shown in figure 3.5. The low density materials used were balsa wood which had an average electron density of 0.16±0.01 relative to water and cedar wood which had an average electron density of 0.32±0.01 relative to water, according to computed tomography (CT) measurements. The depths of maximum ionization were determined at 100 cm source-to-detector distance in balsa, cedar and water for a 10 cm x 10 cm radiation field size and were found to be 15.6 cm, 8.0 cm and 2.5 cm respectively for the 10 MV beam. The collimator scatter factor was measured with the ionization chamber fitted with the 2.5 g/cm² plexiglas buildup cap and is normalized to a 10 cm x 10 cm field. The relative dose factor was measured in water with the ionization chamber with the buildup cap removed and normalized to a 10 cm x 10 cm field size. The \( S_{PT}(r) \) for field sizes larger than 3 cm x 3 cm was calculated from equation 2.24. \( \gamma \) is measured for field sizes 18 cm x 18 cm to 3 cm x 3 cm in balsa and...
cedar. The $S_{pr}$ for field sizes less than 4.5 cm x 4.5 cm was calculated from equation 3.13 using the large field $S_{pr}$ and $\gamma$.

Figure 3.5 Experimental setup used for measuring $\gamma$. (a) Ionization chamber placed at source to axis distance (SAD) of 100 cm at the depth of maximum ionization in low density material for a given field size; (b) Ionization chamber placed at the depth of maximum ionization in water for the same field size at the SAD. $\gamma$ is the ratio of the two ionizations.

For example, $S_{pr}$ for 4.5 cm x 4.5 cm is found by multiplying the $S_{pr}(14$ cm x 14 cm)
cm) in water by $\gamma$, which is the ratio of the ionization in cedar ($\rho_c = 0.32$) and in water
for a 4.5 cm x 4.5 cm collimator setting. $S_{pr}(14 \times 0.32) = S_{pr}(4.5) = S_{pr}(14) \times \gamma(14)$. The
measured data can be fitted to $S_{pr}(r) = 1 - \Gamma e^{-\Gamma r}$ for $r \leq 4$ cm within 1% accuracy. We
used approximately 2.6 cm thickness of balsa and cedar as a back scattering material
under the ionization chamber for the measurements of $\gamma$. Additional 10 cm of polystyrene
phantom material was used under the 2.5 cm effective depth, which may increase the
value of $\gamma$ up to 1% compared to the case with all low density material behind. This
result is expected due to the increased scattering in polystyrene compared to the low
density material used.
3.3. Experimental verifications of the proposed dose model

Two sets of experiments with 10 MV x-rays were performed to check the validity of the dose calculation model. The experiments were designed to represent examples typical of lateral electronic nonequilibrium in a radiation beam. The first situation is that of small, narrow beams and the second situation is for larger beams blocked by narrow blocks. A 0.7 cm x 5 cm and a 1 cm x 5.5 cm radiation field at 100 cm SAD was perpendicularly incident upon a water tank as shown in figure 3.6B (the beam was directed vertically downward). The fields were produced by setting the linac collimator at 5.5 cm x 5.5 cm at isocenter. An external collimator was used to reduce the geometric penumbra of the radiation beam. The collimator inserts were made of 10 cm thick cerrobend and additional collimation was obtained by placing 7 cm thick cerrobend collimators on top of the external collimator holder (see figure 3.6 A). The external collimator was centered along the central axis of the beam. The relative dose normalized to 10 cm x 10 cm at 2.5 g/cm² depth was measured at the isocenter at depths of 7.8 cm and 12.8 cm. The doses are then calculated for this geometry using equation 2.30. In the second set of experiments, a 10 cm x 10 cm radiation field at 100 cm SAD was perpendicularly incident upon a water tank. A 10 cm x 1 cm (as projected at 100 cm) block was placed in the tray holder of the gantry and centered along the central axis of the beam as shown in figure 3.5D. The relative doses normalized to the open field at depths of 7.8 cm and 12.5 cm were measured at 100 cm SAD under the block with a p-type semiconductor diode (Therados, Uppsala, Sweden). Then a circular larynx block
Figure 3.6 (A) Schematic diagram of the experimental setup. B, C, and D are beam's eye views of the fields for which measurements and calculations are being compared. The shaded areas represent blocked areas. Dose is measured at the central axis of the beam. (B) A 0.7 cm x 5.5 cm or 1 cm x 5.5 cm field size. (C) A 10 cm x 10 cm field is blocked with a 10 cm x 1 cm block at the central axis. (D) A 10 cm x 10 cm field is blocked with a circular block of 1.8 cm diameter at the central axis.
of 1.8 cm diameter (at SAD, figure 3.6D) was placed at the centre of the field and the
dose was measured under the block at the isocenter, at depths of 2.5 and 15 cm. The
block transmission factor was also measured under a large block (8 cm x 8 cm x 7 cm)
of the same thickness (7 cm) as the other blocks.
3.4. Depth dose and penumbra measurements

The depth dose profiles were measured with a p-type semiconductor diode (Therados, Uppsala, Sweden) connected to a Wellhofer automated water phantom scanner for 6 MV and 10 MV x-rays for various field sizes. Beam profiles were scanned along a line perpendicular to the central beam axis at various depths. The penumbra (arbitrarily defined as the distance for dose to fall from 90% to 10% of the central axis value) is calculated from the profile by the software. The water phantom and the gantry of the linear accelerator were aligned such that the center of the diode remained within ±1 mm of the central beam axis from the surface to 30 cm depth in water.
3.5 Measurements in inhomogeneous media

3.5.1. Measurements in balsa to simulate lung

Balsa wood was used to simulate lung because it is comparable to tissue in composition and has an effective atomic number equal to the tissue-equivalent polystyrene. Therefore, the ratio of ionization measurements in balsa and polystyrene is assumed to be equal to the ratio of absorbed dose for balsa and polystyrene. This was shown to be valid by Rice et al (1988). The measurements were carried out for 6 MV x-rays using various field sizes from 1.5 cm x 1.5 cm to 20 cm x 20 cm. All measurements were taken at a fixed source to chamber distance of 100 cm SAD in order to remove the inverse square dependence of source to measurement point. All data are normalized to the depth of maximum dose (1.5 cm) for 10 cm x 10 cm. We refer this normalized data as the tissue-output-ratio \((TOR)\).
3.5.2. Measurements in the proximity of air cavities

Measurements in the proximity of air cavities in a polystyrene phantom were made using a PTW Markus chamber, at the same source to axis distance (SAD=100 cm) and normalized to a 10 cm x 10 cm field in polystyrene at 1.5 cm depth for various thicknesses of air cavities.
3.5.3 Measurements in aluminium to simulate bone

There are two kinds of absorbed dose in bone reported in the literature, (i) absorbed dose to the soft tissue within bone (bone marrow) and (ii) absorbed dose to the bone itself. In this research, we report absorbed dose to the bone itself. Aluminium was used to simulate bone. We used a PTW Markus ionization chamber for measurements in aluminium. The Markus chamber has a large mechanical support made of lucite (perspex). When a Markus chamber is used with a phantom of similar atomic number, a homogeneous dosimetry system is achieved and the chamber supporting material is assumed to cause no perturbation. However, when the measurements are carried out in a phantom of significantly different atomic number, chamber response varies according to differences in the backscattered and laterally scattered electron fluence between the medium and the chamber supporting material. Therefore, ionization measurements must be corrected for ionization chamber perturbations. A method for determining the ionization correction factor \( f \) to be applied to the raw ionization reading has been discussed in section 2.10. The determination of this ionization correction factor \( f \) requires that ionization measurements be made in tissue-equivalent medium and in aluminium under the same photon fluence. The following is a description of the required measurements.

A measurement is made by placing 1.5 cm of aluminium directly over the Markus chamber and 5 cm of polystyrene over the aluminium as shown in figure 3.7. In the
Figure 3.7 Experimental set-up for deriving the ionization correction factor \( f \).

In the second experiment, 5 cm of polystyrene is placed directly on top of the ionization chamber and 1.5 cm of aluminium was placed over the polystyrene. Since the aluminium is much further from the ionization chamber than the maximum range of the secondary electrons, it only affects the photon transport and not the electron transport. Therefore,
the electrons generated by photon interaction in aluminium are not able to reach the ionization chamber. The only difference in geometry between the first experiment and the second experiment is the order of stacking of aluminium and polystyrene. This guarantees that both measurements are done under the same photon fluence. The ratio of these two ionization measurements is substituted into equation 2.138 to yield the ionization correction factor \((f)\) for the Markus chamber.

Measurements in aluminium were made in 1 mm steps for various field sizes. All ionization readings within aluminium were corrected using the ionization correction factor \((f)\). All measurements were done at the same source to axis distance (SAD) and normalized to a 10 cm x 10 cm field in polystyrene at 1.5 cm depth.
4.1 Results of the dose model

4.1.1 Zero area \( TMR \)

The tissue-maximum-ratio \( TMR \) curves as a function of field size for various depths are shown in figure 4.1 for 10 MV x-rays. Recently, Bjarngard and Petti (1988) have shown that the scatter dose is a linear function of a geometrical parameter \( z \), where \( z \) is defined as \( z = r_p d / (r_p + d) \). To test the validity of our extrapolation method, the scatter dose calculated from equation 2.27, for 10 MV x-rays at 10 cm, 15 cm and 20 cm depth is plotted as a function of \( z \) and shown in figure 4.2. The scatter-dose calculated from this equation is a linear function of \( z \) and satisfies the equation, approaching zero as \( z \rightarrow 0 \). \( TMR \) data for 10 cm x 10 cm and 0 cm x 0 cm are shown in figure 4.3.

4.1.2 Collimator scatter factor \( (S_{CT}) \)

The effects of the collimator backscattering into the beam monitor chamber (see figure 3.3) for 6 MV and 10 MV x-rays are shown in figure 4.4.a and figure 4.4.b., normalized to a 10 cm x 10 cm field size. The effects of the upper jaws (UJ) were
Figure 4.1 \(TMR\) versus field size for a number of depths for 10 MV x-rays. The crosses (+) indicate measured data. Measurement uncertainty is ±2%, somewhat smaller than the symbols for each data point.

Figure 4.2 The scatter dose as a function of the parameter \(z\) defined by \(r_p d/(r_p + d)\) for 10 MV x-rays.
Figure 4.3 TMR for 0 cm x 0 cm and 10 cm x 10 cm field size for 10 MV x-rays.

Figure 4.4.a The effect of upper jaw (UJ) and lower jaw (LJ) openings for 6 MV x-rays. The effect of upper jaws (UJ) and lower jaws (LJ) together is given by square fields. Measurement uncertainty is ±2%.
Figure 4.4.b The effect of upper jaw (UJ) and lower jaw (LJ) openings for 10 MV x-rays. The effect of upper jaws (UJ) and lower jaws (LJ) together is given by square fields. Measurement uncertainty is ±2%.
measured by varying the upper jaw openings while the lower jaw was fixed at an opening of 40 cm. Similarly, the effects of the lower jaws (LJ) were measured by varying the lower jaw openings while the upper jaw (UJ) was fixed at an opening of 40 cm (see figure 3.3). The effect of upper jaws (UJ) and lower jaws (LJ) together is given by square fields. The results indicate important differences between the upper collimator jaws (UJ) and the lower collimator jaws (LJ). The collimator scatter factor for the upper collimator jaws closely follows that for square fields of different opening sizes. This shows that the effect of the lower jaws is much smaller than that of the upper jaws, due to the fact that the upper jaws are positioned very close to the beam monitor chamber. This finding is consistent with Kase and Svensson (1986), Khan (1986) and Duzenli et al (1993).

The effect of the external collimator scatter for 6 MV and 10 MV x-rays is shown in figure 4.5. The external collimation or blocking does not significantly alter the collimator scatter factor (S_e). This is consistent with the findings of Khan et al (1969) and Khan et al (1986).

4.1.4 Photon Scatter factor (S_pl)

The S_pl for 10 MV x-rays calculated from equation 2.24 is shown in figure 4.6. The value of S_pl(0) was found to be 0.972.
Figure 4.5 The effect of the external collimator for various field sizes for 6 MV and 10 MV x-rays. Measurement uncertainty is ±2%.

Figure 4.6 The photon scatter factor ($S_{pl}$) as a function of field size for 10 MV x-rays.
CHAPTER 4. DATA ANALYSIS AND RESULTS

4.1.5 Phantom scatter factor ($S_{pr}$) and Electron nonequilibrium factor ($S_{pe}$)

The depths of maximum ionization ($d$) for a 10 cm x 10 cm radiation field size were determined at 100 cm source to detector distance in balsa ($\rho_c=0.16\pm0.01$), in cedar ($\rho_c=0.32\pm0.01$) and in water, and were found to be 15.6 cm, 8.0 cm and 2.5 cm, respectively, for the 10 MV beam. It may be noted that the depth of maximum ionization is at $\rho_c d$, independent of the medium. The relative dose factor ($RDF$) was measured in water and normalized to a 10 cm x 10 cm field size, as given in table 4.1. The $S_{pr}(x)$ for field sizes 3 cm x 3 cm and larger, was calculated from equation 2.23 and is given in table 4.1. Results of $\gamma$ measured for field sizes 18 cm x 18 cm to 3 cm x 3 cm in balsa and cedar are given in table 4.2 and table 4.3. The $S_{pr}$ for field sizes less than 4.5 cm x 4.5 cm was calculated from equation 3.13 using the large field $S_{pr}$ and $\gamma$. For example, $S_{pr}$ for 4.5 x 4.5 cm is found by multiplying the $S_{pr}(14 \text{ cm x 14 cm})$ in water by $\gamma$, which is the ratio of the ionization for cedar ($\rho_c=0.32$) and for water for a 4.5 cm x 4.5 cm collimator setting. $S_{pr}(14\times0.32)=S_{pr}(4.5)=S_{pr}(14) \times \gamma(14)=1.012 \times 0.984=0.992$.

The Phantom scatter factors ($S_{pr}$) calculated from equation 3.13 in balsa wood and cedar wood are plotted in figure 4.7 and figure 4.8. The phantom scatter factors measured in balsa and cedar agree within the experimental errors, for field sizes larger than 0.96 cm x 0.96 cm (water equivalent). For the equivalent field size of 0.96 cm x 0.96 cm in water (the smallest field size used with cedar), the phantom scatter factor in balsa is about 6% higher than that in cedar. This discrepancy remains unexplained. The zero area $S_{pr}$ was
extrapolated from the measured data in balsa and cedar and was found to be 0.49 and 0.41 respectively. The measured data can be fitted to $S_{PT}(r) = 1 - \Gamma e^{\xi r}$ for $r \leq 4$ cm within 1% accuracy. The value of $\Gamma$ was found to be 0.59 and $\xi = 1.112$ cm$^{-1}$ for cedar and $\Gamma=0.51$ and $\xi=1.165$ cm$^{-1}$ for balsa. The average value of $S_{PT}$ from balsa and cedar provides the best estimate for $S_{PT}$ values in water and is plotted in figure 4.9 and fitted to the same equation as above. For this new curve the $\Gamma$ is 0.51 and the $\xi$ is 1.085 cm$^{-1}$.

$S_{PT}$ values for small and large field sizes are plotted in figure 4.10. Values of $S_{PE}$, as calculated from equation 2.24, are shown in figure 4.11. The extrapolated zero area $S_{PT}$ from the average data is 0.45. We used 2.7 cm of balsa and cedar as a backscattering material under the ionization chamber for the measurements of $\gamma$. An additional 10 cm of polystyrene phantom material was used under the 2.7 cm balsa or cedar, which may increase the value of $\gamma$ up to 1% compared to all low density material behind. This is expected due to the increased scattering in polystyrene compared to the low density material used.

We also calculated the phantom scatter factor from the data published by Podgorsak et al.(1987) and Sixel et al.(1993). In our calculations, we used the relative dose factors as given by Podgorsak et al. and the collimator scatter factors ($S_{CT}$) as given by Sixel et al. which are reported to be independent of the stereotactic collimator size for diameters in the range 15-30mm for 10 MV photons. $S_{PT}$ values calculated by using equation 2.23 and the data of Podgorsak et al.(1987) and Sixel et al. (1993) are within 1%
Table 4.1. Measured relative dose factors (RDF), collimator scatter factors ($S_{ct}$) and the derived phantom scatter factors ($S_{pt}$) are shown for field sizes of 3 cm x 3 cm to 18 cm x 18 cm. Measurement uncertainty is ±2%.

<table>
<thead>
<tr>
<th>Field size in water (cm)$^2$</th>
<th>Measured Relative dose factor $RDF$</th>
<th>Measured collimator scatter factor $S_{ct}$</th>
<th>Phantom scatter factor $S_{pt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 x 18</td>
<td>1.050</td>
<td>1.027</td>
<td>1.022</td>
</tr>
<tr>
<td>16 x 16</td>
<td>1.041</td>
<td>1.024</td>
<td>1.017</td>
</tr>
<tr>
<td>14 x 14</td>
<td>1.030</td>
<td>1.018</td>
<td>1.012</td>
</tr>
<tr>
<td>12 x 12</td>
<td>1.017</td>
<td>1.009</td>
<td>1.008</td>
</tr>
<tr>
<td>10 x 10</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>9 x 9</td>
<td>0.992</td>
<td>0.993</td>
<td>0.999</td>
</tr>
<tr>
<td>8 x 8</td>
<td>0.981</td>
<td>0.983</td>
<td>0.998</td>
</tr>
<tr>
<td>7 x 7</td>
<td>0.969</td>
<td>0.974</td>
<td>0.995</td>
</tr>
<tr>
<td>6 x 6</td>
<td>0.956</td>
<td>0.966</td>
<td>0.990</td>
</tr>
<tr>
<td>5 x 5</td>
<td>0.940</td>
<td>0.957</td>
<td>0.982</td>
</tr>
<tr>
<td>4 x 4</td>
<td>0.920</td>
<td>0.947</td>
<td>0.972</td>
</tr>
<tr>
<td>3 x 3</td>
<td>0.897</td>
<td>0.932</td>
<td>0.962</td>
</tr>
</tbody>
</table>
Table 4.2. The measured $\gamma$ and phantom scatter factors ($S_{pr}$) are given for the radiation field sizes in balsa and the corresponding equivalent field sizes in water. Measurement uncertainty is ±2%.

<table>
<thead>
<tr>
<th>Field size in balsa (cm$^2$)</th>
<th>Eq. side of a square in water (cm)</th>
<th>Measured $\gamma$</th>
<th>Phantom scatter factor $S_{pr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 x 18</td>
<td>2.88</td>
<td>0.954</td>
<td>0.975</td>
</tr>
<tr>
<td>16 x 16</td>
<td>2.56</td>
<td>0.950</td>
<td>0.966</td>
</tr>
<tr>
<td>14 x 14</td>
<td>2.24</td>
<td>0.948</td>
<td>0.959</td>
</tr>
<tr>
<td>12 x 12</td>
<td>1.92</td>
<td>0.938</td>
<td>0.945</td>
</tr>
<tr>
<td>10 x 10</td>
<td>1.60</td>
<td>0.921</td>
<td>0.921</td>
</tr>
<tr>
<td>9 x 9</td>
<td>1.44</td>
<td>0.902</td>
<td>0.902</td>
</tr>
<tr>
<td>8 x 8</td>
<td>1.28</td>
<td>0.885</td>
<td>0.883</td>
</tr>
<tr>
<td>7 x 7</td>
<td>1.12</td>
<td>0.875</td>
<td>0.870</td>
</tr>
<tr>
<td>6 x 6</td>
<td>0.96</td>
<td>0.849</td>
<td>0.840</td>
</tr>
<tr>
<td>5 x 5</td>
<td>0.80</td>
<td>0.830</td>
<td>0.815</td>
</tr>
<tr>
<td>4 x 4</td>
<td>0.64</td>
<td>0.767</td>
<td>0.749</td>
</tr>
<tr>
<td>3 x 3</td>
<td>0.48</td>
<td>0.720</td>
<td>0.693</td>
</tr>
</tbody>
</table>
Table 4.3. The measured $\gamma$ and phantom scatter factors ($S_{PT}$) are given for the radiation field sizes in cedar and the corresponding equivalent field sizes in water. Measurement uncertainty is $\pm 2\%$.

<table>
<thead>
<tr>
<th>Field size in cedar (cm²)</th>
<th>Eq. side of a square in water (cm)</th>
<th>Measured $\gamma$</th>
<th>Phantom scatter factor $S_{PT}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>14 x 14</td>
<td>4.50</td>
<td>0.984</td>
<td>0.987</td>
</tr>
<tr>
<td>12 x 12</td>
<td>3.84</td>
<td>0.980</td>
<td>0.986</td>
</tr>
<tr>
<td>10 x 10</td>
<td>3.20</td>
<td>0.979</td>
<td>0.973</td>
</tr>
<tr>
<td>9 x 9</td>
<td>2.88</td>
<td>0.972</td>
<td>0.972</td>
</tr>
<tr>
<td>8 x 8</td>
<td>2.56</td>
<td>0.968</td>
<td>0.966</td>
</tr>
<tr>
<td>7 x 7</td>
<td>2.24</td>
<td>0.955</td>
<td>0.950</td>
</tr>
<tr>
<td>6 x 6</td>
<td>1.92</td>
<td>0.938</td>
<td>0.929</td>
</tr>
<tr>
<td>5 x 5</td>
<td>1.60</td>
<td>0.928</td>
<td>0.911</td>
</tr>
<tr>
<td>4 x 4</td>
<td>1.28</td>
<td>0.880</td>
<td>0.860</td>
</tr>
<tr>
<td>3 x 3</td>
<td>0.96</td>
<td>0.820</td>
<td>0.789</td>
</tr>
</tbody>
</table>
Figure 4.7 The phantom scatter factor ($S_p$) as a function of water equivalent field size measured in balsa wood for 10 MV x-rays.

Figure 4.8 The phantom scatter factor ($S_p$) as a function of water equivalent field size measured in cedar wood for 10 MV x-rays.
Figure 4.9 Average $S_{pt}$ measured in balsa and cedar wood as a function of water equivalent field size for 10 MV x-rays.

Figure 4.10 The $S_{pt}$ as a function of water equivalent field size for 10 MV x-rays.
Figure 4.11 $S_{PE}$ as a function of water equivalent field size for 10 MV x-rays.
of our measured data. This is expected as the phantom scatter factor is only a function of the energy spectrum of the photons and the composition of the phantom material and is independent of the collimator system by definition.

We have described a method of measuring phantom scatter factors for very small radiation fields (see section 3.2.3). Our measurements of \( S_{pr} \) are within 3% of the values published for field sizes larger than 2 cm x 2 cm. Our measurement of \( S_{pr} \) for 1 cm x 1 cm is 10% - 12% lower than the data for the LMR-13 linear accelerator manufactured by Toshiba (Houdek et al 1983). There are no phantom scatter factor data available in the literature for field sizes smaller than 1 cm x 1 cm.

4.1.6 Electron perturbation factor (EPF)

The \( EPF \) was calculated from equation 2.30 and is given in table 4.4. for 10 MV x-rays. The \( EPF \) for field sizes whose radius is greater than 1.9 cm is essentially zero as, for this energy, lateral electronic equilibrium is established once a radius of this size is reached.
Table 4.4. Electron Perturbation factor (EPF) for various radii of field sizes for 10 MV x-rays.

<table>
<thead>
<tr>
<th>Depth (cm)</th>
<th>0.14 cm</th>
<th>0.28 cm</th>
<th>0.42 cm</th>
<th>0.56 cm</th>
<th>0.70 cm</th>
<th>0.84 cm</th>
<th>0.98 cm</th>
<th>1.13 cm</th>
<th>1.27 cm</th>
<th>1.41 cm</th>
<th>1.55 cm</th>
<th>1.69 cm</th>
<th>1.83 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>0.391</td>
<td>0.284</td>
<td>0.207</td>
<td>0.151</td>
<td>0.110</td>
<td>0.080</td>
<td>0.058</td>
<td>0.042</td>
<td>0.031</td>
<td>0.023</td>
<td>0.016</td>
<td>0.011</td>
<td>0.007</td>
</tr>
<tr>
<td>5.0</td>
<td>0.365</td>
<td>0.265</td>
<td>0.193</td>
<td>0.146</td>
<td>0.103</td>
<td>0.075</td>
<td>0.054</td>
<td>0.039</td>
<td>0.029</td>
<td>0.022</td>
<td>0.015</td>
<td>0.010</td>
<td>0.007</td>
</tr>
<tr>
<td>6.0</td>
<td>0.349</td>
<td>0.254</td>
<td>0.186</td>
<td>0.141</td>
<td>0.099</td>
<td>0.072</td>
<td>0.052</td>
<td>0.038</td>
<td>0.028</td>
<td>0.021</td>
<td>0.014</td>
<td>0.010</td>
<td>0.007</td>
</tr>
<tr>
<td>8.0</td>
<td>0.321</td>
<td>0.234</td>
<td>0.171</td>
<td>0.136</td>
<td>0.096</td>
<td>0.067</td>
<td>0.049</td>
<td>0.035</td>
<td>0.025</td>
<td>0.019</td>
<td>0.013</td>
<td>0.009</td>
<td>0.006</td>
</tr>
<tr>
<td>10.0</td>
<td>0.298</td>
<td>0.217</td>
<td>0.159</td>
<td>0.117</td>
<td>0.086</td>
<td>0.062</td>
<td>0.045</td>
<td>0.033</td>
<td>0.024</td>
<td>0.018</td>
<td>0.013</td>
<td>0.009</td>
<td>0.006</td>
</tr>
<tr>
<td>12.0</td>
<td>0.277</td>
<td>0.202</td>
<td>0.147</td>
<td>0.109</td>
<td>0.080</td>
<td>0.058</td>
<td>0.042</td>
<td>0.031</td>
<td>0.023</td>
<td>0.017</td>
<td>0.012</td>
<td>0.008</td>
<td>0.005</td>
</tr>
<tr>
<td>14.0</td>
<td>0.258</td>
<td>0.188</td>
<td>0.138</td>
<td>0.102</td>
<td>0.074</td>
<td>0.054</td>
<td>0.039</td>
<td>0.029</td>
<td>0.021</td>
<td>0.016</td>
<td>0.011</td>
<td>0.008</td>
<td>0.005</td>
</tr>
<tr>
<td>16.0</td>
<td>0.241</td>
<td>0.175</td>
<td>0.129</td>
<td>0.095</td>
<td>0.069</td>
<td>0.050</td>
<td>0.037</td>
<td>0.027</td>
<td>0.020</td>
<td>0.015</td>
<td>0.010</td>
<td>0.007</td>
<td>0.005</td>
</tr>
<tr>
<td>18.0</td>
<td>0.224</td>
<td>0.164</td>
<td>0.120</td>
<td>0.088</td>
<td>0.065</td>
<td>0.047</td>
<td>0.034</td>
<td>0.025</td>
<td>0.019</td>
<td>0.014</td>
<td>0.009</td>
<td>0.007</td>
<td>0.004</td>
</tr>
<tr>
<td>20.0</td>
<td>0.209</td>
<td>0.153</td>
<td>0.112</td>
<td>0.082</td>
<td>0.060</td>
<td>0.044</td>
<td>0.032</td>
<td>0.023</td>
<td>0.017</td>
<td>0.013</td>
<td>0.009</td>
<td>0.006</td>
<td>0.004</td>
</tr>
<tr>
<td>22.0</td>
<td>0.195</td>
<td>0.143</td>
<td>0.105</td>
<td>0.077</td>
<td>0.056</td>
<td>0.041</td>
<td>0.030</td>
<td>0.022</td>
<td>0.016</td>
<td>0.012</td>
<td>0.008</td>
<td>0.006</td>
<td>0.004</td>
</tr>
<tr>
<td>24.0</td>
<td>0.181</td>
<td>0.134</td>
<td>0.098</td>
<td>0.072</td>
<td>0.059</td>
<td>0.038</td>
<td>0.028</td>
<td>0.020</td>
<td>0.015</td>
<td>0.011</td>
<td>0.008</td>
<td>0.005</td>
<td>0.003</td>
</tr>
<tr>
<td>26.0</td>
<td>0.169</td>
<td>0.125</td>
<td>0.092</td>
<td>0.067</td>
<td>0.049</td>
<td>0.036</td>
<td>0.026</td>
<td>0.019</td>
<td>0.014</td>
<td>0.010</td>
<td>0.007</td>
<td>0.005</td>
<td>0.003</td>
</tr>
<tr>
<td>28.0</td>
<td>0.156</td>
<td>0.116</td>
<td>0.086</td>
<td>0.063</td>
<td>0.046</td>
<td>0.034</td>
<td>0.024</td>
<td>0.018</td>
<td>0.013</td>
<td>0.010</td>
<td>0.007</td>
<td>0.004</td>
<td>0.003</td>
</tr>
<tr>
<td>30.0</td>
<td>0.147</td>
<td>0.110</td>
<td>0.080</td>
<td>0.058</td>
<td>0.043</td>
<td>0.031</td>
<td>0.023</td>
<td>0.016</td>
<td>0.012</td>
<td>0.009</td>
<td>0.006</td>
<td>0.004</td>
<td>0.003</td>
</tr>
</tbody>
</table>
4.1.7 Scatter maximum ratio (SMR)

The scatter maximum ratio \((SMR)\) was calculated from \(S_{PL}\) and \(TMR\) as follows:

\[
SMR(r_p,d) = \frac{S_{PL}(r_p) \cdot TMR(r_p,d)}{S_{PL}(0)} - TMR(0,d)
\]  

(3.27)

The \(SMR\) values for up to 50 cm radius are calculated as a function of depth for 10 MV x-rays and are given in table 4.5.
### Table 4.5. Scatter maximum ratio (SMR) for square field sizes for 10 MV x-rays.

<table>
<thead>
<tr>
<th>Depth (cm)</th>
<th>1x1 cm²</th>
<th>4x4 cm²</th>
<th>6x6 cm²</th>
<th>8x8 cm²</th>
<th>10x10 cm²</th>
<th>12x12 cm²</th>
<th>15x15 cm²</th>
<th>20x20 cm²</th>
<th>25x25 cm²</th>
<th>30x30 cm²</th>
<th>35x35 cm²</th>
<th>40x40 cm²</th>
<th>50x50 cm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>0.005</td>
<td>0.012</td>
<td>0.019</td>
<td>0.024</td>
<td>0.029</td>
<td>0.034</td>
<td>0.042</td>
<td>0.051</td>
<td>0.060</td>
<td>0.0680</td>
<td>0.075</td>
<td>0.081</td>
<td>0.092</td>
</tr>
<tr>
<td>5.0</td>
<td>0.006</td>
<td>0.026</td>
<td>0.036</td>
<td>0.043</td>
<td>0.050</td>
<td>0.055</td>
<td>0.062</td>
<td>0.070</td>
<td>0.079</td>
<td>0.087</td>
<td>0.095</td>
<td>0.101</td>
<td>0.111</td>
</tr>
<tr>
<td>6.0</td>
<td>0.011</td>
<td>0.031</td>
<td>0.038</td>
<td>0.054</td>
<td>0.064</td>
<td>0.070</td>
<td>0.079</td>
<td>0.088</td>
<td>0.097</td>
<td>0.106</td>
<td>0.115</td>
<td>0.120</td>
<td>0.131</td>
</tr>
<tr>
<td>8.0</td>
<td>0.017</td>
<td>0.041</td>
<td>0.058</td>
<td>0.070</td>
<td>0.085</td>
<td>0.090</td>
<td>0.103</td>
<td>0.114</td>
<td>0.126</td>
<td>0.136</td>
<td>0.165</td>
<td>0.152</td>
<td>0.164</td>
</tr>
<tr>
<td>10</td>
<td>0.018</td>
<td>0.044</td>
<td>0.063</td>
<td>0.079</td>
<td>0.092</td>
<td>0.103</td>
<td>0.118</td>
<td>0.131</td>
<td>0.144</td>
<td>0.156</td>
<td>0.166</td>
<td>0.174</td>
<td>0.187</td>
</tr>
<tr>
<td>12</td>
<td>0.017</td>
<td>0.047</td>
<td>0.065</td>
<td>0.083</td>
<td>0.098</td>
<td>0.109</td>
<td>0.126</td>
<td>0.142</td>
<td>0.156</td>
<td>0.169</td>
<td>0.180</td>
<td>0.187</td>
<td>0.199</td>
</tr>
<tr>
<td>14</td>
<td>0.015</td>
<td>0.047</td>
<td>0.066</td>
<td>0.085</td>
<td>0.102</td>
<td>0.113</td>
<td>0.131</td>
<td>0.149</td>
<td>0.164</td>
<td>0.177</td>
<td>0.190</td>
<td>0.199</td>
<td>0.212</td>
</tr>
<tr>
<td>16</td>
<td>0.015</td>
<td>0.047</td>
<td>0.066</td>
<td>0.085</td>
<td>0.102</td>
<td>0.115</td>
<td>0.134</td>
<td>0.152</td>
<td>0.169</td>
<td>0.184</td>
<td>0.195</td>
<td>0.204</td>
<td>0.216</td>
</tr>
<tr>
<td>18</td>
<td>0.015</td>
<td>0.046</td>
<td>0.064</td>
<td>0.084</td>
<td>0.101</td>
<td>0.115</td>
<td>0.134</td>
<td>0.155</td>
<td>0.171</td>
<td>0.186</td>
<td>0.200</td>
<td>0.207</td>
<td>0.220</td>
</tr>
<tr>
<td>20</td>
<td>0.014</td>
<td>0.045</td>
<td>0.062</td>
<td>0.082</td>
<td>0.100</td>
<td>0.112</td>
<td>0.133</td>
<td>0.154</td>
<td>0.172</td>
<td>0.187</td>
<td>0.201</td>
<td>0.210</td>
<td>0.221</td>
</tr>
<tr>
<td>22</td>
<td>0.014</td>
<td>0.044</td>
<td>0.060</td>
<td>0.079</td>
<td>0.097</td>
<td>0.110</td>
<td>0.131</td>
<td>0.153</td>
<td>0.171</td>
<td>0.186</td>
<td>0.200</td>
<td>0.209</td>
<td>0.220</td>
</tr>
<tr>
<td>24</td>
<td>0.013</td>
<td>0.041</td>
<td>0.057</td>
<td>0.075</td>
<td>0.094</td>
<td>0.107</td>
<td>0.128</td>
<td>0.149</td>
<td>0.168</td>
<td>0.183</td>
<td>0.198</td>
<td>0.206</td>
<td>0.219</td>
</tr>
<tr>
<td>26</td>
<td>0.013</td>
<td>0.040</td>
<td>0.055</td>
<td>0.072</td>
<td>0.091</td>
<td>0.102</td>
<td>0.124</td>
<td>0.146</td>
<td>0.165</td>
<td>0.181</td>
<td>0.194</td>
<td>0.203</td>
<td>0.216</td>
</tr>
<tr>
<td>28</td>
<td>0.013</td>
<td>0.038</td>
<td>0.051</td>
<td>0.068</td>
<td>0.086</td>
<td>0.098</td>
<td>0.119</td>
<td>0.142</td>
<td>0.160</td>
<td>0.176</td>
<td>0.190</td>
<td>0.199</td>
<td>0.212</td>
</tr>
<tr>
<td>30</td>
<td>0.011</td>
<td>0.037</td>
<td>0.048</td>
<td>0.065</td>
<td>0.081</td>
<td>0.094</td>
<td>0.114</td>
<td>0.138</td>
<td>0.156</td>
<td>0.171</td>
<td>0.187</td>
<td>0.194</td>
<td>0.196</td>
</tr>
</tbody>
</table>
4.2 Experimental verification of the proposed dose model

The results of the measured dose and calculated dose (equation 2.31) are shown in table 4.6 and table 4.7. The calculated dose including the electronic nonequilibrium perturbation factor (EPF) are in good agreement with the experimental dose while the standard dose model (equation 2.28) overestimates the dose by as much as 25%. The standard model dose was calculated without the EPF (equation 2.28) and with $S_{PL}$ set to unity as is sometimes assumed for high energy x-rays. For positions under field blocks, the calculated dose without the EPF is significantly lower than the measured dose, with the worst case being when $S_{PL}$ is assumed as unity. It may be noted that the convolution technique and the extended data derived from Monte Carlo methods for the primary and scatter dose model have similar accuracy, which is only limited by the quality of the kernels used.
TABLE 4.6. Comparison of measured and calculated dose for 10 MV x-rays. All doses expressed as a fraction of the dose for a 10 cm x 10 cm field size at 2.5 cm depth. Measurement uncertainty is ±3%.

<table>
<thead>
<tr>
<th>Field size cm²</th>
<th>Measured Dose</th>
<th>Calculated dose with EPF</th>
<th>Calculated dose without EPF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7 x 5.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d = 7.8 cm</td>
<td>0.637</td>
<td>0.620</td>
<td>0.809</td>
</tr>
<tr>
<td>d = 12.5 cm</td>
<td>0.537</td>
<td>0.540</td>
<td>0.682</td>
</tr>
<tr>
<td>1.0 x 5.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d = 7.8 cm</td>
<td>0.678</td>
<td>0.682</td>
<td>0.820</td>
</tr>
<tr>
<td>d = 12.5 cm</td>
<td>0.589</td>
<td>0.597</td>
<td>0.696</td>
</tr>
</tbody>
</table>

Table 4.7. Comparison of measured and calculated dose under small blocks for 10 MV x-rays. All doses expressed as a fraction of the dose for a 10 cm x 10 cm field size at 2.5 cm depth. Measurement uncertainty is ±3%.

<table>
<thead>
<tr>
<th></th>
<th>Measured dose</th>
<th>Calculated dose with EPF</th>
<th>Calculated dose without EPF</th>
<th>Calculated dose without EPF and S₁₀=0.972</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectang. blk</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depth 2.5 cm</td>
<td>0.095</td>
<td>0.104</td>
<td>0.055</td>
<td>0.041</td>
</tr>
<tr>
<td>Depth 15 cm</td>
<td>0.174</td>
<td>0.164</td>
<td>0.125</td>
<td>0.113</td>
</tr>
<tr>
<td>circular blk</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depth 2.5 cm</td>
<td>0.108</td>
<td>0.109</td>
<td>0.06</td>
<td>0.041</td>
</tr>
<tr>
<td>Depth 15 cm</td>
<td>0.195</td>
<td>0.187</td>
<td>0.141</td>
<td>0.134</td>
</tr>
</tbody>
</table>
4.3 Results of penumbra and depth dose measurements

The penumbra measured (arbitrarily defined as the distance for the dose to fall off from 90% of the central axis value to 10% of the central axis value) from the beam profiles as a function of depth and field size for 6 MV and 10 MV x-rays are shown in figure 4.12.a through figure 4.12.e. As expected, the penumbra increases with increasing photon energy. Penumbra also increases with depth in water and with field size, due to the multiply scattered photons and beam divergence. But at any given depth and field size, 6 MV always produces smaller penumbra than 10 MV x-rays.

Percentage depth doses were measured for various field sizes for 6 MV x-rays and 10 MV x-rays; samples of these are shown in figure 4.13.a and figure 4.13.b. The reproducibility of the measurements was approximately ±2%.
Figure 4.12.a. Penumbra as a function of depth for 6 MV and 10 MV x-rays for a 1.3 cm collimator diameter. The curves serve only as a guide to the eye.

Figure 4.12.b. Penumbra as a function of depth for 6 MV and 10 MV x-rays for a 1.9 cm collimator diameter. The curves serve only as a guide to the eye.
Figure 4.12.c. Penumbra as a function of depth for 6 MV and 10 MV x-rays for a 2.6 cm collimator diameter. The curves serve only as a guide to the eye.

Figure 4.12.d. Penumbra as a function of depth for 6 MV and 10 MV x-rays for a 3.3 cm collimator diameter. The curves serve only as a guide to the eye.
Figure 4.2.e. Penumbra as a function of depth for 6 MV and 10 MV x-rays for a 4.0 cm collimator diameter. The curves serve only as a guide to the eye.
Figure 4.13.a Percentage depth dose for 6 MV x-rays for a 2.0 cm diameter field size. Measurement uncertainty is ±2%.
Figure 4.13.b Percentage depth dose for 10 MV x-rays for a 2.0 cm diameter field size. Measurement uncertainty is ±2%.
4.4 Results of the proposed cavity theory

Ogunleye et al (1980) and Ogunleye (1987) performed a careful experimental test of the Burlin cavity theory for $^{60}$Co γ-rays and 10 MV x-rays. The experiment was conducted with lithium fluoride thermoluminescence dosimeters (0.4 mm x 3 mm x 3 mm, hot-pressed LiF TLD-100). Stacks of dosimeters were surrounded by homogeneous lithium fluoride, aluminium, copper or lead. The absorbed dose in the LiF was derived from the integrated thermoluminescent light. The experimental results are the mean doses of four coplanar dosimeters, each with a standard deviation of ±1.5%. We have used these experimental results to compare with our results, predicted by the proposed cavity theory. The ratios of the cavity dose to the equilibrium wall dose ($f$) for various materials were calculated from the proposed cavity theory. Comparisons of the cavity theory and experiments are plotted in figures 4.14.a, 4.14.b, 4.14.c for $^{60}$Co γ-rays and in figures 4.15.a, 4.15.b and 4.15.c for 10 MV x-rays. The results of the Burlin and Kearsley cavity theories (Ogunleye et al 1980, Ogunleye 1987, Kearsley 1984) are also included for comparison. The chi-squared ($\chi^2/n$) values are calculated to indicate the goodness of fit between the various theories and the experiments. They are calculated by

$$\chi^2/n = \frac{1}{n} \sum_{i}^{n} \left( \frac{(f_{i,exp})_m - f_{i,theory}}{(\sigma_{i,exp})_m} \right)^2$$

where $(f_{i,exp})_m$ is the mean dose to the $i^{th}$ thickness of TLD and $(\sigma_{i,exp})_m$ is the standard error of the mean dose to the $i^{th}$ thickness of TLD (Press et al, 1989). The standard error $(\sigma_{i,exp})_m$ of the mean dose is given by (Eichholz and Poston, 1985)
\[(\sigma_{i,exp})_m = \frac{\sigma_{i,exp}}{\sqrt{M}}\]  

(4.3)

where \(\sigma_{i,exp}\) is the standard deviation of \(M\) dose measurements for the \(i^{th}\) thickness of TLD.

Our proposed theory has significantly lower \(\chi^2/n\) values in all media, both for \(^{60}\)Co \(\gamma\)-rays and for 10 MV x-rays.

Table 4.7 \(\chi^2/n\) values for various cavity theories for \(^{60}\)Co \(\gamma\)-rays.

<table>
<thead>
<tr>
<th>(^{60})Co (\gamma)-rays (\chi^2/n)</th>
<th>Burlin theory</th>
<th>Kearsley theory</th>
<th>Proposed theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium</td>
<td>4.9</td>
<td>2.5</td>
<td>0.6</td>
</tr>
<tr>
<td>Copper</td>
<td>17.2</td>
<td>3.9</td>
<td>1.2</td>
</tr>
<tr>
<td>Lead</td>
<td>57.2</td>
<td>67.6</td>
<td>12'</td>
</tr>
</tbody>
</table>

Table 4.8 \(\chi^2/n\) values for various cavity theories for 10 MV x-rays.

<table>
<thead>
<tr>
<th>10 MV x-rays (\chi^2/n)</th>
<th>Burlin theory</th>
<th>Kearsley theory</th>
<th>Proposed theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium</td>
<td>1.3</td>
<td>1.1</td>
<td>0.5</td>
</tr>
<tr>
<td>Copper</td>
<td>2.72</td>
<td>4</td>
<td>0.5</td>
</tr>
<tr>
<td>Lead</td>
<td>24</td>
<td>125</td>
<td>17.6'</td>
</tr>
</tbody>
</table>

The proposed cavity theory takes into account the secondary electrons backscattering at the cavity boundaries. The theory can calculate the mean dose to the cavity and the equilibrium dose in the front wall (or back wall), even if the front and back walls are of different atomic composition. The model is very easy to use. It has only a few parameters and these are well defined, and it provides better agreement with experimental data than the other theories.

* See appendix D for comments on model comparison using \((\chi^2/n)\) values.
Figure 4.14.a and 4.14.b Average dose to the cavity (LiF dosimeters) surrounded by aluminium and copper, divided by the equilibrium LiF dose as calculated by various cavity theories and as measured by Ogunleye et al (1980) for $^{60}$Co $\gamma$-rays.
Figure 4.14.c Average dose to the cavity (LiF dosimeters) surrounded by lead, divided by the equilibrium LiF dose as calculated by various cavity theories and as measured by Ogunleye et al (1980) for $^{60}$Co $\gamma$-rays.
Figure 4.15.a and 4.15.b  Average dose to the cavity (LiF dosimeters) surrounded by aluminium and copper, divided by the equilibrium LiF dose as calculated by various cavity theories and as measured by Ogunleye (1987) for 10 MV x-rays.
Figure 4.15.c  Average dose to the cavity (LiF dosimeters) surrounded by lead, divided by the equilibrium LiF dose as calculated by various cavity theories and as measured by Ogunleye (1987) for 10 MV x-rays.
4.5 Results of measurements in inhomogeneous media

4.5.1 Measurements in balsa wood to simulate lung

The tissue-output-ratio (TOR defined as a dose normalized to 10 cm x 10 cm at 1.5 cm depth) measured in balsa wood (used to simulate lung) is plotted in figure 4.16.a through figure 4.16.d as a function of depth. The TOR in polystyrene is also plotted for comparison. The results of the measurements show that there is a significant dose reduction in balsa as compared to polystyrene. The dose reduction in balsa increases with decreasing field sizes as expected, due to the loss of lateral electronic equilibrium. The dose reduction at 3 cm depth in balsa wood for 6 MV x-rays is approximately 12%, 22% and 30% for 5 cm x 5 cm, 2 cm x 2 cm and 1.5 cm x 1.5 cm field sizes, respectively. There is also a significant dose build-up in the distal balsa-polystyrene interface which extends to 3 mm depth in polystyrene for 5 cm x 5 cm field size and gradually increases to about 1 cm depth for 1.5 cm x 1.5 cm field size. The magnitude of the dose build-up is approximately 10%, 18% and 30% for 5 cm x 5 cm, 2 cm x 2 cm and 1.5 cm x 1.5 cm field sizes, respectively. The reproducibility of each of these measurements was approximately ±2%. There is high dose gradient within balsa, close to the proximal interface and this increases with decreasing field size. This initial dose reduction in balsa is due to lateral electronic nonequilibrium. But at greater depths in balsa, a higher
Figure 4.16.a and 4.16.b Measured and calculated tissue-output-ratios (TOR) for 1.5 cm x 1.5 cm and 2.0 cm x 2.0 cm field sizes in balsa wood and polystyrene for 6 MV x-rays.
Figure 4.16.c and 4.16.d Measured and calculated tissue-output-ratios (TOR) for 5 cm x 5 cm and 20 cm x 20 cm field size in balsa wood and polystyrene for 6 MV x-rays.
CHAPTER 4. DATA ANALYSIS AND RESULTS

photon fluence in balsa as compared to polystyrene (due to higher photon transmission in balsa than in polystyrene) tends to compensate for this dose reduction. It is interesting to note that for a 20 cm x 20 cm field size (figure 4.16.d), the dose in balsa is always greater than that in polystyrene as lateral electronic equilibrium is established in balsa for that field size. Also, there is no significant dose perturbation in polystyrene at the proximal polystyrene-balsa interface. This suggests that a tumor in tissue surrounding the lung would have better dose uniformity if the direction of the photon beam is such that the tumor resides on the proximal side of the tumor-lung interface.

Conventional 1D inhomogeneity correction factors are not able to model lateral electronic nonequilibrium and consequently may overestimate dose in lung by 25% or more, depending on the photon energy, beam geometry and the electron density of lung. We developed a 2D inhomogeneity correction factor that is valid in lateral electronic nonequilibrium. There is good agreement between the proposed model and experiment for 6 MV x-rays. Doses calculated from the proposed model and conventional 1D effective TMR are also shown in figure 4.16.a through 4.16.c for comparison. The correction factors are applied to individually measured points in polystyrene and therefore contain the inherent uncertainty associated with each measurement. TMR acquired by an automated water phantom scanner and smoothed by appropriate smoothing functions would yield smooth curves. There is good agreement between the dose calculated from the proposed model and experimental measurements. The disagreement close to the interface between dose calculated with the proposed model and measured experimentally is expected as the longitudinal electronic nonequilibrium is not included in our model.
Algorithms incorporating 3D equivalent tissue-air-ratios (ETAR) (not available commercially at the time of writing) with extended data derived from Monte Carlo methods (Woo et al, 1990) would be expected to give accurate results.
4.5.2 Measurements in the proximity of an air cavity

Measurements in the proximity of an air-cavity of different dimensions for various field sizes are shown in figure 4.17.a through figure 4.19.c. The measurements show that there is a significant dose reduction\(^3\) of up to 7\% at the proximal polystyrene-air interface for a 1.5 cm x 1.5 cm field size and it extends 2-3 mm backward into the polystyrene (fig 4.17.a). The dose reduction decreases to about 2\% as the field size increases to 5 cm x 5 cm (fig. 4.17.c). The protective cap used for the ionization chamber limits the proximity of measurements to 0.5 mm from the interface. The actual interface dose is likely to be much lower than the observed dose as significant backscattering of electrons takes place within the first few tenths of a millimeter of polystyrene beyond the measuring point (Tabata, 1967) and that backscattering (from air) will be greatly reduced at the proximal interface. The dose reduction at the distal air-polystyrene interface depends on the thickness of the air cavity and inversely on the field size. Compared to the dose profile that would exist if the air gap were replaced by polystyrene (the "in polystyrene" curve), the dose reduction at the distal air-polystyrene interface increases from 8\% for 2 cm x 2 cm field size (fig. 4.17.b) to 10\% for 1.5 cm x 1.5 cm field size for a 0.5 cm air cavity (fig. 4.17.a); from 16\% for 2 cm x 2 cm field size (fig. 4.18.b) to 22\% for 1.5 cm x 1.5 cm field size (fig. 4.18.a) for a 1 cm air cavity and from 24\% for 5 cm x 5 cm field size (fig. 4.19.b) to 32\% for 1.5 cm x 1.5 cm field size (fig. 4.19.a) for a 1.5 cm air cavity. There was no significant dose reduction for 5 cm x 5 cm fields for any of 0.5 cm, 1.0 cm,

\(^3\)Compared to the dose that would be observed if the air cavity were replaced by polystyrene.
or 1.5 cm air cavities (fig. 4.17.c, fig. 4.18.c, and fig. 4.19.c). The thickness required for longitudinal electronic equilibrium to be established also increases with air cavity thickness from 3 mm for a 0.5 cm air cavity to 1 cm for a 1.5 cm air cavity for 2 cm x 2 cm field size and from 7 mm for a 0.5 cm air cavity to 1 cm for a 1.5 cm air cavity, for 1.5 cm x 1.5 cm field size. Dose gradients are also high in the build-up region for these field sizes. There was no significant dose build-up for 5 cm x 5 cm field size for any of the air cavities investigated. The reproducibility of measurements was approximately ±2.5%. Since the dose reduction in the proximal interface is minimal, tumor surrounding air cavities (such as the bronchial tubes) would have better dose uniformity if the direction of the photon beam is such that the tumor resides in the proximal side of the tissue-air interface. Since we do not model longitudinal electronic equilibrium, our calculations do not include this contribution to dose reduction and dose build-up at the interface.
Figure 4.17.a

Figure 4.17.b

Figure 4.17.a and 4.17.b Measured and calculated tissue-output-ratio (TOR) in polystyrene with 0.5 cm air cavity thickness for 1.5 cm x 1.5 cm and 2 cm x 2 cm field sizes of 6 MV x-rays. The dashed line represents measurements in polystyrene only, with no air gap.
Figure 4.17.c Measured and calculated tissue-output-ratio (TOR) in polystyrene with 0.5 cm air cavity thickness for 5 cm x 5 cm field size of 6 MV x-rays. The dashed line represents measurements in polystyrene only, with no air gap.
Figure 4.18.a and 4.18.b Measured and calculated tissue-output-ratio (TOR) in polystyrene with 1.0 cm air cavity thickness for 1.5 cm x 1.5 cm and 2 cm x 2 cm field sizes of 6 MV x-rays. The dashed line represents measurements in polystyrene only, with no air gap.
Figure 4.18.c Measured and calculated tissue-output-ratio (TOR) in polystyrene with 1.0 cm air cavity thickness for 5 cm x 5 cm field size of 6 MV x-rays. The dashed line represents measurements in polystyrene only, with no air gap.
Figure 4.19.a and 4.19.b Measured and calculated tissue-output-ratio (TOR) in polystyrene with 1.5 cm air cavity thickness for 1.5 cm x 1.5 cm and 2 cm x 2 cm field sizes of 6 MV x-rays. The dashed line represents measurements in polystyrene only, with no air gap.
Figure 4.19.c Measured and calculated tissue-output-ratio (TOR) in polystyrene with 1.5 cm air cavity thickness for 5 cm x 5 cm field size of 6 MV x-rays. The dashed line represents measurements in polystyrene only, with no air gap.
4.5.3 Measurements in aluminium to simulate bone

Ionization measurements in aluminium must be corrected for the perturbation caused by the presence of the ionization chamber itself. The ionization correction factor \( f \) for the PTW Markus 0.05 cm\(^3\) ionization chamber for 6 MV x-rays was found to be 1.20. Raw ionization data, corrected ionization data and dose to aluminium are shown in figure 4.20.a.

The large mechanical support of the Markus chamber accounts for 13% of the dose perturbation according to the proposed general cavity theory (section 2.8 of chapter 2). The differences in lateral scattering in aluminium and in the Markus ionization chamber and the complicated composition of the Markus ionization chamber itself are assumed to cause the remaining 7% dose perturbation.

The absorbed dose to the aluminium is plotted as function of equivalent depth in polystyrene for various field sizes and is shown in figure 4.20.b and figure 4.20.c. The ratio of the absorbed dose in aluminium (representing bone) relative to that in polystyrene (representing soft tissue) is approximately 0.9 for 6 MV x-rays. There is a slight dose build-up in polystyrene at the distal aluminium-polystyrene interface; it is approximately 5% for a 5 cm x 5 cm field size and 9% for a 1.6 cm x 1.6 cm field size. The thickness required for dose build-up is approximately 5 mm for all field sizes investigated. There is a slight dose build-up expected at the proximal polystyrene-aluminium interface due to the electrons backscattering into polystyrene from aluminium. This was not investigated. Reproducibility of measurements was approximately ±2%.
Determinations of absorbed dose to the aluminium calculated from the proposed model and from the conventional model are similar for all field sizes investigated and therefore only the results of our model are plotted.

The dose reduction in bone would actually be much less than in aluminium, as bone has a much lower effective $Z$ ($Z=10$) than aluminium ($Z=13$). The ratio of mass-energy absorption for compact bone and soft tissue is approximately 0.956 for 6 MV x-rays and remains close to unity for up to 20 MV x-rays. Therefore, the presence of bone does not significantly perturb the dose.

![Graph](image)

Figure 4.20.a Raw ionization, corrected ionization and relative dose to the aluminium and polystyrene for a 5 cm x 5 cm field size for 6 MV x-rays.
Figure 4.20.b and 4.20.c Relative dose to the aluminium and polystyrene for 1.6 cm x 1.6 cm and 2 cm x 2 cm field sizes for 6 MV x-rays.
CHAPTER 5 - GENERAL DISCUSSIONS

5.1 Homogeneous Media

The proposed modified primary and scatter dose model offers significant improvement over the conventional dose model which is not able to account for electronic nonequilibrium. All the parameters of the model are derived experimentally without the use of Monte Carlo code systems. The electron transport data for the dose model were derived experimentally in low density media. We have shown the relationship between our model and the Monte Carlo-based extended primary and scatter dose model. We have also shown that the convolution method reduces to our model if the kernel integration is carried out along the z-axis. There is good agreement between the dose calculated by the proposed model and experimental measurements of dose. The model is also very easy to implement in most commercial treatment planning systems which use the primary and scatter dose model.

The phantom scatter factors as a function of field size, normalized to 10 cm x 10 cm for various energies are shown in figure 5.1. The data for $^{60}$Co are derived from the British Journal of Radiology, Supplement 17 (1982). The data for 15 MV x-rays and 24
MV x-rays are calculated from the data of Bortfeld et al (1993) and Rice et al (1988), using equation 2.24. We make use of the fact that the external collimator does not significantly alter the collimator scatter factor and the collimator scatter factor remains constant as the field size is reduced by blocking. We found this to be true for 0.7 cm to 4 cm diameter collimators for both 6 MV x-rays and 10 MV x-rays. It can be seen from figure 5.1 that the effect of electronic nonequilibrium increases nearly exponentially with decreasing field size. It also increases with increasing photon energy. Consider the two geometries as shown in figure 5.2 containing point A and point B. For simplicity, we define dose uniformity as the ratio of dose at point A and point B. We then calculate
Figure 5.2 Geometries used in dose calculations given in table 5.1.

Table 5.1 Relative dose at point A and point B calculated from equation 2.32.

<table>
<thead>
<tr>
<th>Photon energy</th>
<th>A circular (a)</th>
<th>B circular (a)</th>
<th>B/A circular (a)</th>
<th>A irregular (b)</th>
<th>B irregular (b)</th>
<th>B/A irregular (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Co-60</td>
<td>0.99</td>
<td>0.99</td>
<td>1</td>
<td>0.99</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>6 MV</td>
<td>0.98</td>
<td>0.93</td>
<td>0.95</td>
<td>0.99</td>
<td>0.84</td>
<td>0.85</td>
</tr>
<tr>
<td>10 MV</td>
<td>0.95</td>
<td>0.87</td>
<td>0.92</td>
<td>0.93</td>
<td>0.73</td>
<td>0.79</td>
</tr>
<tr>
<td>15 MV</td>
<td>0.86</td>
<td>0.78</td>
<td>0.90</td>
<td>0.92</td>
<td>0.68</td>
<td>0.74</td>
</tr>
</tbody>
</table>

the dose to point A and point B with our model using equation 2.32, at the nominal depth of maximum dose for each energy (0.5 cm for Co-60 γ-rays, 1.5 cm for 6 MV, 2.5 cm for 10 MV x-rays and 3 cm for 15 MV x-rays). The results of the calculations,
to a 10 cm x 10 cm field size, are summarized in table 5.1.

We see from table 5.1 that dose uniformity varies greatly with the irradiation geometry and the shape of the irradiation geometry is very important in selecting the optimum photon energy for the best dose uniformity. For circular fields, dose uniformity decreases with increasing photon energy. For this example, the difference in dose uniformity between low energy and high energy photons is less than 12%, with lower energy photons providing better dose uniformity. This finding is consistent with Chierego et al (1988) and Serago et al (1992). Chierego et al came to the conclusion that the optimum energy was about 4 MV, based on film dosimetry in selected planes. Serago et al compared dose volume histograms for a beam diameter of 2 cm (almost too large to see the differences between low and high energy, figure 5.1). Serago et al found almost no differences in the dose volume histograms above the 70% isodose line. For volumes encompassed by the 50% and 60% isodose lines, however, significant differences were observed. Thomas (1994) also noted that the field diameters required to give a 90% isodose surrounding a 2.5 cm tumor were 2.85 cm at 5 MV, 3.0 cm at 8 MV and 3.1 cm at 16 MV. Our model predicts that such compensations in field size, in order to maintain a particular level of dose uniformity for different energy photon beams, would need to increase as nominal field size decreases. For irregularly shaped geometries, as in figure 5.2b, the difference in dose uniformity between higher and lower energy photons becomes much larger (>25%), with lower energy photons providing better dose uniformity. It may be noted that lower energy photons also have a sharper radiation penumbra (figure 4.12.a to figure 4.12.d). Therefore, we would expect $^{60}$Co to provide the optimum dose.
homogeneity. However, the geometric penumbra of $^{60}$Co is very large due to its large source size. A linac with a nominal accelerating potential of about 3 MV (almost equivalent to $^{60}$Co \( \gamma \)-rays but with very small geometric penumbra due to the small focal spot size) has the best potential for better dose uniformity for irregular small fields. We do not discuss integral dose as it is generally much less important than dose uniformity for treatment with small field sizes in the megavoltage range.

Therefore, for any shape tumor, lower energy megavoltage photons provide better dose uniformity. The difference in dose uniformity between low and high energy photons is least for a spherical tumor.
5.2 Inhomogeneous media

The presence of a dosimeter in a medium perturbs the charged particle fluence in the medium unless the dosimeter and the medium are identical with respect to atomic composition and density. We developed a new cavity theory that gives better agreement with experimental measurements in LiF thermoluminescent dosimeters surrounded by aluminium, copper and lead for $^{60}$Co $\gamma$-rays and 10 MV x-rays. The parameters for the model are derived systematically. The theory is difficult to apply for dosimeters having complex geometry such as cylindrical ionization chambers or parallel plate chambers in inhomogeneous media. We developed a procedure for correcting the ionization reading for the perturbation caused by an ionization chamber in an inhomogeneous medium. The ionization correction factor for the Markus chamber inside aluminium was found to be 1.20.

There is good agreement between the proposed inhomogeneity correction factor and experimental measurement for 6 MV x-rays. The dose reduction in balsa as compared to polystyrene at 5 cm downstream from 2.5 cm of overlying polystyrene, at the same radiological depth\(^4\), for various photon energies and for a 5 cm x 5 cm field size is shown in figure 5.3. It can be seen from figure 5.3 that the dose in balsa decreases with increasing photon energy. Measurements and theory also show that there is a dose build-up in polystyrene in the distal balsa-polystyrene interface, which is similar in magnitude

\(^4\)Radiological depth ($d_r$) in a phantom is defined as $d_r = \rho_e d$, where $\rho_e$ is electron density of the phantom relative to water and $d$ is the depth in the phantom.
to the dose reduction in balsa. There was no dose reduction observed on the proximal polystyrene-balsa interface. This suggests that a tumor in tissue surrounding lung would have better dose uniformity if the direction of the photon beam were such that the tumor resides on the proximal side of the tumor-lung interface. And lower energy photons provide the least dose perturbation in lung and in tissue posterior to the lung.

Figure 5.3 Lung dose correction factors as a function of accelerating potential (MV) for 5 cm x 5 cm field size.

Measurements in the proximity of air-cavities show that there is a significant dose reduction in the proximal polystyrene-air interface and dose build-up in the distal air-
polystyrene interface. The magnitude of dose reduction and dose build-up increases with decreasing field size and increasing air-cavity thickness. While the range of the dose reduction is only 2-3 mm from the interface, the dose build-up extends to about a centimeter from the interface for a 1.5 cm air cavity, for all field sizes investigated (1.5 cm x 1.5 cm to 5 cm x 5 cm). Since the extent of dose reduction in the proximal interface is minimal, tumor surrounding an air-cavity (such as bronchial tubes) would have better dose uniformity if the direction of the photon beam is such that the tumor resides in the proximal side of the tissue-air interface. The range and the magnitude of the dose build-up is expected to increase with increasing photon energy as both the longitudinal range and the lateral range of the secondary electrons increases with increasing photon energy (figure 2.6). Lower energy photons require the least thickness (2mm for 3 MV x-rays) to establish longitudinal electronic equilibrium.

The dose perturbation in aluminium (simulating bone) and in the proximity of aluminium is in the order of a few percent for 6 MV x-rays for all field sizes investigated. The dose perturbation in compact bone is much less than that in aluminium as (i) bone has a much lower effective Z (Z=10) than aluminium (Z=13) and (ii) bone has a much lower electron density relative to water, \( \rho_e = 1.67 \) for compact bone) than aluminium \( \rho_e = 2.33 \). The ratio of mass-energy absorption for compact bone and tissue is approximately 0.956 for 6 MV x-rays and remains close to unity for up to 20 MV x-rays. Therefore, the presence of bone does not significantly perturb the dose at any photon energy.

Lateral charged particle equilibrium plays an important role in the shape of the beam.
profile and the sharpness of the penumbra. The ideal beam should have (i) minimum scatter of the primary beam and (ii) minimum energy transfer to the secondary electrons. Heavy charged particles are much better than photons in this respect. For photons, the maximum energy $E_{\text{max}}(e)$ transferred to the secondary electron via Compton scattering (which is the dominant interaction for the clinically useful range under consideration in this thesis) is given by

$$E_{\text{max}}(e) = \frac{2\alpha}{1 + 2\alpha} \cdot E(h\nu)$$

(5.1)

where $E(h\nu)$ is the energy of the incident photon and $\alpha$ is the ratio of energy of the photon to the rest mass energy the electron.

On the other hand, for heavy charged particles, the maximum energy transfer is given by

$$E_{\text{max}}(e) = \frac{4A}{(1 + A)^2} \cdot E(p)$$

(5.2)

where $E(p)$ is the energy of the heavy charged particle and $A$ is the ratio of the rest mass of the charged particles to that of the electron. From the above, it can be seen that heavy charged particles transfer much less energy to the electrons than do photons. This leads to much less scattering for the primary beam and also much smaller lateral range of the electrons.

Furthermore, heavy charged particle beams have a very sharp Bragg peak which
can be associated with a high relative biological effectiveness (RBE), giving a higher effective dose. This also makes the distal penumbra sharper. Hence, when radiation beams with very sharp penumbra are needed, such as that required in radiosurgery, heavy charged particles can be advantageous over photons.
CHAPTER 6 - CONCLUSIONS

The following conclusions are made from this research:

1. Absorbed dose decreases very rapidly for field sizes smaller than that required by lateral charged particle equilibrium. The reduction in dose leads to dose non-uniformity and the degree of dose non-uniformity depends on the shape of the tumor. The dose reduction due to lateral electronic nonequilibrium increases with increasing photon energy. Higher photon energies also have a slightly larger radiation penumbra. For spherical tumors, the difference in dose uniformity between low energy photons and high energy photons is more modest. However, for irregularly shaped tumors, the dose uniformity is much better with lower energy photons. Therefore, small tumors are best treated with approximately 3 MV x-rays.

2. Dose models that are not based on Monte Carlo systems or whose data are not derived from Monte Carlo methods are not able to calculate dose in situations of lateral electronic nonequilibrium. Failure to account for lateral electronic nonequilibrium may lead to dose calculation errors of more than 25%, depending on photon beam energy and tumor geometry. We modified the primary and scatter dose model to include the effect of lateral electronic nonequilibrium. The electron transport data for the model were derived experimentally. The dose model was verified experimentally for various
geometries and was in good agreement with measurements.

3. We developed a new cavity theory (used to relate the dose to the cavity and to the surrounding medium) which includes secondary electron backscattering from the medium into the cavity. The proposed model contains few parameters and a systematic way of extracting the parameters. The cavity theory was found to be in good agreement with the experimental data of Ogunleye et al (1980) and Ogunleye (1987) for \(^{60}\text{Co}\) \(\gamma\)-rays and 10 MV \(x\)-rays. We also give a method for obtaining an ionization chamber correction factor for measuring dose in inhomogeneous media. The ionization correction factor in aluminium for a PTW 0.05 cm\(^3\) Markus chamber is 1.20.

4. There is a significant dose reduction in lung as compared to normal density tissue for small fields due to lateral electronic nonequilibrium. The dose reduction in lung (simulated by balsa wood with \(\rho_e=0.16\)) at the same radiological depth as in tissue for 6 MV \(x\)-rays is approximately 20% and 30% for 5 cm x 5 cm and 1.5 cm x 1.5 cm fields, respectively. There is also a dose build-up of similar magnitude in the distal lung-tissue interface which extends to about a centimeter from the interface. The dose perturbation in tissue due to the inhomogeneity vanishes at a point that is located further from the inhomogeneity than the maximum range of the electrons. Dose gradients within lung close to the proximal interface are high and increase with decreasing field size. There are no significant dose perturbations in tissue upstream from the proximal tissue-lung interface. This suggests that a tumor in tissue surrounding a lung would have better dose uniformity if the direction of the photon beam is such that the tumor resides on the proximal side of the tumor-lung interface.
5. Conventional 1-D inhomogeneity correction factors are not able to model lateral electronic nonequilibrium and consequently may overestimate dose in lung by 25% or more, depending on photon energy, beam geometry and the electron density of lung. 3-D equivalent tissue-air-ratios, whose weighting factors are derived from Monte Carlo calculations, are expected to provide the most accurate dose calculations (not available commercially at the time of writing). We developed a 2-D inhomogeneity correction factor that is valid in conditions of lateral electronic nonequilibrium. There is good agreement between the proposed model and experiment for 6 MV x-rays. Dose reduction calculated from our model in lung ($\rho_e=0.16$) at the same radiological depth as water for a 5 cm x 5 cm field is 5% for 2 MV x-rays, 20% for 6 MV x-rays, 30% for 10 MV x-rays, 38% for 15 MV x-rays and over 50% for 20 MV x-rays. The region of dose build-up in tissue at the distal lung-tissue interface also increases with increasing photon energy. Therefore, tumors in lung and surrounding lung would have better dose uniformity if treated with lower energy photons.

6. Significant dose reduction was observed in the proximal and the distal interface of the air-tissue interface. The dose perturbation increases with increasing air-cavity thickness. The dose reduction for 6 MV x-rays at the distal interface increases from 8% for 2 cm x 2 cm field size to 10% for 1.5 cm x 1.5 cm field size for a 0.5 cm air cavity (infinite lateral dimensions); from 16% for 2 cm x 2 cm field size to 22% for 1.5 cm x 1.5 cm field size for a 1 cm air cavity and from 24% for 2 cm x 2 cm field size to 32% for 1.5 cm x 1.5 cm field size for a 1.5 cm air cavity. There was no significant dose reduction observed for 5 cm x 5 cm fields for any of 0.5 cm, 1.0 cm or 1.5 cm air
cavities. The dose reduction at the proximal interface extends only 2-3 mm upstream from the interface, while the dose build-up at the distal interface extends for up to 1 cm for field sizes smaller than 2 cm x 2 cm. Lower energy photons requires the least thickness for dose build-up because the range of the secondary electrons is smaller. Therefore, lower energy photons, once again, are more suitable to treat tumors that surround an air-cavity (such as the bronchial tubes). A tumor in tissue surrounding an air-cavity would have better dose uniformity if the direction of the photon beam is such that the tumor resides on the proximal side of the tumor-air interface.

7. The presence of bone (simulated by aluminium) in tissue also perturbs the dose. The dose in aluminium for 6 MV x-rays is approximately 90% of the dose to the polystyrene (tissue equivalent). There is considerable dose build-up (7%) at the distal aluminium-polystyrene interface for a 1.6 cm x 1.6 cm field; this diminishes (to 3%) gradually with increasing field size to 5 cm x 5 cm. The effective Z and the electron density of compact bone are much less than for aluminium and therefore the magnitude of the correction factor will be much less in bone. The ratio of the mass-energy absorption coefficients, bone to tissue, is just slightly less than unity. This tends to be off-set by increased lateral scattering in bone, due to its higher physical density relative to tissue. Therefore, the presence of bone is expected to cause only a small dose perturbation for x-ray beams less than 20 MV.

Lateral charged particle equilibrium plays an important role in the shape of the dose profile and penumbra. The ideal beam should have minimum scattering of the primary beam and minimum energy transfer to the secondary electrons. This is not
possible with photons, but heavy charged particles have this characteristic. Lateral scattering in heavy charged particle beams is very small, and the Bragg peak produces a high tumor to normal tissue dose ratio. Heavy charged particles such as protons and alpha particles are superior to any photon beam even if one does not take advantage of the Bragg peak. They can also have higher radiobiological effectiveness (RBE) than photons, making them even more attractive.
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**Glossary**

Absorbed dose - Defined as the energy deposited by ionizing radiation per unit mass of material. The SI unit is gray (J/kg).

Charged particle equilibrium or electronic equilibrium - Charged particle equilibrium or electronic equilibrium exists in a volume $v$ if each charged particle of a given type and energy leaving $v$ is replaced by an identical particle of the same energy, in terms of expectation value.

Collimator scatter factor ($S_{CT}$) - Defined as the ratio of dose in air for a given field size to the dose in air for a reference field size.

Electronic nonequilibrium factor ($S_{PE}$) - An empirical factor that describes the effect when dose is less than Kerma in regions of lateral electronic nonequilibrium.

Electron perturbation factor ($EPF$) - An empirical factor that corrects dose in conditions of lateral electronic nonequilibrium.

Field size - Defined as the projected collimator opening at the source-to-axis distance.

Gamma Knife - Specialized irradiation device designed by Leksell for stereotactic radiosurgery. It contains a large number of cobalt sources focused to a point.

Homogeneous medium - A medium that is comparable to unit density soft tissue in terms of electron density, physical density and atomic composition.

Inhomogeneous medium - A medium that is different from unit density soft tissue in terms of electron density, physical density and atomic composition.

Intracranial - Inside skull.
Isocenter - Point of intersection of the central beam axis and the gantry rotation axis.

Kerma - This quantity is relevant only for indirectly ionizing radiations (photons or neutrons). Kerma is defined as the quotient of $dE_\nu/dm$, where $dE_\nu$ is the expectation value of the energy transferred to charged particles in a small mass $dm$ including radiative-loss energy but excluding energy passed from a primary to a secondary charged particle (Attix, 1986).

Collision Kerma - Defined as the expectation value of the energy transferred to charged particles per unit mass at the point of interest, excluding both the radiative-loss energy and energy passed from a primary to a secondary charged particle.

Radiative Kerma - Defined as the part of the Kerma where some energy of the charged particles is carried away by photons (e.g., positron in-flight annihilation and bremsstrahlung).

kVp - Applied peak kilovoltage potential between anode and cathode.

Lateral electronic equilibrium - Secondary charged particles are produced uniformly throughout the volume $V$ by an external source. Lateral charged particle equilibrium is said to exist in a smaller internal volume $v$ if the minimum distance separating the boundaries $V$ and $v$ is greater than the lateral range of the electrons (see figure 1.1).

Mass Stopping power ($dT/pdx$) - Defined as the expectation value of the rate of energy loss in a medium, per unit path length, per unit density, by a charged particle.

Mass collision stopping power ($S/p$) - Defined as the expectation value of the rate of energy loss per unit path length by a charged particle through soft and hard collisions only. Radiative loss by the charged particle is not included.

Restricted mass stopping power ($L/p$) - Defined as the fraction of the mass collision stopping power that includes all the soft collisions plus the hard collisions resulting in delta rays with energies less than a cutoff value.
Penumbra (p) - Defined as the region at the edge of a radiation beam over which the dose rate changes rapidly as function of distance from the beam central axis. The geometric penumbra is the portion of the penumbra caused by the finite radiation source size and the position of the collimators. Penumbra caused by the lateral scattering of the charged particles is called the radiation penumbra.

Percentage depth dose (PDD) - Defined as the quotient, expressed as a percentage, of the absorbed dose at any depth to the absorbed dose at a fixed reference depth along the central axis of the beam (Khan, 1984).

Phantom scatter factor (S_p) - Defined as the ratio of the dose rate for a given field size at reference depth to the dose rate at the same depth for the reference field size with the same collimator opening.

Photon scatter factor (S_p) - Defined as the ratio of the total dose at a given point in a phantom under electronic equilibrium to the dose at a reference depth and field size.

Polarization effect - The nucleus and the orbital electrons of a medium are attracted in different directions by the electric field of a passing charged particle which results in the creation of electric dipoles with their own electric field. The atoms and the molecules become polarized developing a screening field against the influence of the passing particle which minimizes interaction probability reducing energy losses.

Radiological depth or pathlength (d_e) - Defined as \( d_e = \rho_e d \), where \( \rho_e \) is the electron density of a phantom relative to the water and \( d \) is the depth in the phantom.

Relative dose factor (RDF) - Defined as ratio of the dose rate for given field size at a reference depth to the dose rate at the same depth for the reference field size.

Scatter-air-ratio (SAR) - Defined as a ratio of the scattered dose at a given point in a phantom to the dose in free space at the same point.

Scatter-maximum-ratio (SMR) - Defined as a ratio of the scattered dose at a given point in a phantom to the primary dose at the same point at the reference depth of maximum
dose.

Simulator - A machine which emulates the geometry of a treatment machine, but which uses diagnostic energy x-rays to take images of the patient in the treatment position.

Source-to-axis distance (SAD) - Defined as the distance from the source to the axis of gantry rotation.

Source-to-surface distance (SSD) - Defined as the distance from the source to the surface of a phantom.

Stereotactic radiosurgery - Precision radiation treatment of a particular (usually small) intracranial structure or lesion. For these treatments the patient’s head must be firmly constrained to a precise position in relation to the radiation beam. The head fixation is usually done using a head fixation device bolted to the skull.

Tissue-air-ratio (TAR) - Defined as the ratio of the absorbed dose at a given point in tissue or a tissue equivalent phantom to the absorbed dose at the same spatial point within a volume of the phantom material just large enough to provide the maximum electronic build-up at the point of reference. Both measurements are taken at the same source-to-axis distance and field size. (Johns, Whitmore, Watson and Umberg, 1953; ICRU, 1963).

Tissue-maximum-ratio (TMR) - Defined as the ratio of the absorbed dose at a given depth and field size to the absorbed dose at the same field size and source-to-axis distance but at the depth of maximum dose (Khan, 1983).

Tissue-output-ratio (TOR) - Defined as the ratio of the dose rate for given field size at a depth to the dose rate at a reference field size and depth.
The following is an outline of the procedures involved for a typical cancer patient undergoing radiotherapy treatment:

1. Computed tomography, Magnetic resonance images etc.
2. Simulation of treatment fields on a simulator
3. Treatment planning
4. Fabrication of field defining blocks
5. Treatment

Treatment planning:
- Field size
- Beam energy
- Beam direction
- Custom blocking

Isodose distributions
- Modifications of field size, beam directions and energy and blocking
- Optimum treatment plan
Conversion of tissue air ratio (TAR) to tissue maximum ratio (TMR)

\[ \text{TAR}(r,d) = \frac{\text{TMR}(r,d)}{\text{PSF}(r)} \]

where \( r \) is the radius of the field size at depth \( d \) and \( \text{PSF} \) is the peak scatter factor. Peak scatter factor is a special case of tissue air ratio (TAR) when the depth \( d \) is equal to the depth of maximum dose \( d_{\text{max}} \).
APPENDIX C

Ratios of mass-energy absorption coefficients \((\mu_{en}/\rho)_{LiF}/(\mu_{en}/\rho)_w\) and of mean mass-collision stopping powers \(<S/p>_{LiF}/<S/p>_w\) for \(^{60}\)Co \(\gamma\)-rays for LiF/(wall material) (Ogunleye et al 1980). Backscatter factors \((b)\) from Dutreix and Bernard (1965).

<table>
<thead>
<tr>
<th>Wall material (w)</th>
<th>((\mu_{en}/\rho)<em>{LiF}/(\mu</em>{en}/\rho)_w)</th>
<th>(&lt;S/p&gt;_{LiF}/&lt;S/p&gt;_w)</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lithium Fluoride</td>
<td>1.000</td>
<td>1.000</td>
<td>0.15</td>
</tr>
<tr>
<td>Aluminium</td>
<td>0.961</td>
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</tr>
<tr>
<td>Copper</td>
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<tr>
<td>Lead</td>
<td>0.802</td>
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<td>0.58</td>
</tr>
</tbody>
</table>

Ratios of mass-energy absorption coefficients \((\mu_{en}/\rho)_{LiF}/(\mu_{en}/\rho)_w\) and of mass-collision stopping powers \(<S/p>_{LiF}/<S/p>_w\) for 10 MV x-rays for LiF/(wall material) (Ogunleye 1987). Backscatter factors \((b)\) from Dutreix and Bernard (1965).

<table>
<thead>
<tr>
<th>Wall material (w)</th>
<th>((\mu_{en}/\rho)<em>{LiF}/(\mu</em>{en}/\rho)_w)</th>
<th>(&lt;S/p&gt;_{LiF}/&lt;S/p&gt;_w)</th>
<th>(b)</th>
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</thead>
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<tr>
<td>Lithium Fluoride</td>
<td>1.000</td>
<td>1.000</td>
<td>0.14</td>
</tr>
<tr>
<td>Aluminium</td>
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<tr>
<td>Copper</td>
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</tr>
<tr>
<td>Lead</td>
<td>0.686</td>
<td>1.502</td>
<td>0.54</td>
</tr>
</tbody>
</table>

The electron absorption coefficient \((\beta_e)\) from equation 2.128 was found to be 9.8 cm\(^2\)/g for \(^{60}\)Co and 1.85 cm\(^2\)/g for 10 MV x-rays.
APPENDIX D

Comments on model comparison using chi-squared ($\chi^2/n$) values

Figure AD.1 A hypothetical experimental dose distribution and dose values obtained from arbitrary models 1 and 2.

Suppose an experiment yields dose values ($f_{exp}$) that are normally distributed for a particular dosimeter as illustrated in figure AD.1. The standard deviation of the measurements ($\sigma_{exp}$) is very small. The predicted dose is then calculated with model 1 and model 2 as shown in the figure. The chi-squared ($\chi^2/n$) values can be used to indicate the relative goodness of fit between the models and the experiment. Chi-squared ($\chi^2/n$) values are given by
A large value of \( \chi^2/n \) indicates a poor fit between the model and the experiment. It is evident from the illustration shown in figure AD.1 that the dose values obtained from model 1 and model 2 fit poorly with the experimental dose values. However, the calculated dose value obtained from model 1 has a greater probability of being from the experimental dose distribution than the dose value obtained from model 2. Therefore, it is useful to compare \( \chi^2/n \) values of various models to determine the relative success of the models. In such a case, \( \chi^2/n \) values are not used to predict whether the experimental and theoretical dose values are from the same distribution or not. It may be noted that one may obtain a very large \( \chi^2/n \) value if the experimental standard deviation is very small, even though, the dose value from the model may only differ by a few percent from the mean dose value from the experiments.