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DOCTOR OF PHILOSOPHY

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M. A., University of Toronto

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Abstract

Traditional methods of interpretation of the results of magnetic surveys neglect effects due to permanent magnetization. Recent geomagnetic research on the remanent magnetization of rocks has shown this to be unjustified. Moreover techniques now being employed provide better measurements of magnetic field variations than have ordinarily been available in the past. In order to take advantage of these new developments, equations for the magnetic field over a point dipole, a horizontal line of dipoles, a thin dipping sheet, a thick dipping sheet and a sloping step are derived in the cases when both the directions of measurement and polarization are arbitrary.

It is found that these directions combine with other properties of the bodies to form parameters, which determine various features of the magnetic anomalies over the bodies. In terms of these combined parameters, it is possible to give expressions for the higher derivatives of the fields over these bodies, and to develop methods of determining the unknown parameters of the
bodies when magnetic profiles over them are given. Further, it is shown that the field over four of these bodies treated can be obtained by successive differentiation of a single function. This fact is used in drawing charts for computing values of the fields and their derivatives at points along profiles over any of these bodies. Tables of the higher derivatives are given, as well as graphs showing the position of special points such as peaks and inflection points on the profiles for any direction of polarization and measurement.

It is shown how these more general methods may be applied to the interpretation of aeromagnetic surveys, and examples are given of their use in the analysis of magnetic survey data over the La-Plonge area, Saskatchewan, and Texada Island, British Columbia. In the latter area, the general question of what geological information may be obtained from magnetic data is considered and a comparison is made of aeromagnetic anomalies with structural data obtained from aerial photographs.
PUBLICATIONS


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Advanced Electronics  R. E. Burgess
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Other Studies:

Differential Equations  C. A. Swanson
Probability  S. W. Nash
Computational Methods  F. M. C. Goodspeed
THE GEOPHYSICAL ANALYSIS OF MAGNETIC ANOMALIES

by

DONALD HERBERT HALL

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Department of Physics

The University of British Columbia, Vancouver 8, Canada.

Date April 6th, 1959
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1. **INTRODUCTION**

Research on the physics of rocks and minerals, and on the interpretation of geophysical surveys, has established that the distribution of values of physical properties within any geological unit may, under favourable conditions, be interpreted to give information on the distribution of rock type and structure, on the distribution of elements, and on the geological history of the unit. From laboratory measurements of the magnetic susceptibility of core samples, for example, the position of various units in the geological section may be determined \textsuperscript{[Brodin, 1952]}\textsuperscript{\textsuperscript{1}}. Measurements of the magnetic susceptibility of rock samples may be used to delineate zones of varying concentration of magnetite in igneous rocks, \textsuperscript{[Hawes, 1952]}\textsuperscript{\textsuperscript{2}} and measures of the remanent magnetization have been used to deduce the presence of stress, as for example, in the past history of a geological unit \textsuperscript{[Kalashnikov, 1952]}\textsuperscript{\textsuperscript{3}}. Alternatively, values of physical properties may be sampled by geophysical surveys made over the formations in question. The latter methods have the advantage of widespread and rapid coverage, particularly when air surveys are used, and the advantage of better sampling which comes from measurements on the formation in place. It is this second method, using geophysical surveys to provide the geological information, which is the subject of study in the present thesis.

Magnetic surveys share with the other members of the so-called "force-field" group of methods of geophysics \textsuperscript{[Brant, 1948, p.561]}\textsuperscript{\textsuperscript{4}} the advantage of speed of operation, and lend themselves well to airborne operations. Furthermore, the magnetic properties of rocks have been subject to considerable study in
present-day geophysics, with a view to clarifying their relationship to geological conditions. The magnetic properties of rocks have long been known to be sensitive indicators of the particular geological history of the rocks [Davis, 1935]. For these reasons, the geological interpretation of the data of magnetic surveys has been chosen as the particular subject of this investigation, which consists of two parts. The first consists of the development of the equations for the magnetic force over a number of particular types of geological body, and of methods of interpreting the observed field, so that a more complete specification of the polarization of bodies is possible from the survey data than is commonly obtained at the present time. The second is an application of the methods of interpretation of magnetic survey data to an actual case to illustrate the types of geological information that should be looked for in magnetic survey data, with a view to increasing the usefulness of magnetic surveys in solving geological problems.

Consider first the interpretation of magnetic surveys. In this work we are concerned only with the "anomaly field" of the earth, [Nettleton, 1940, p. 168] extracted from values of the total field observed along a well-controlled network of lines over a plane at or above the earth's surface. From this it is generally possible to extract quantitative information about particular bodies of magnetized rock. This may be done by fitting equations giving the theoretical effects of various idealized bodies, taken to represent geological conditions, to the observed effects and attempting to discover that body giving the best representation.
Finally it may be possible to evaluate parameters giving the position, size, and magnetic properties of the actual body in the subsurface.

Equations for the magnetic field set up by bodies of various shapes and characteristics have received considerable attention. Expressions for the components of the field in the vertical and horizontal directions are quite simple when the polarization also is along one of these directions, and these have been developed for a wide range of bodies [Nettleton, 1942]. For polarization in certain other directions, but with measurements vertical or horizontal, the equations can still be developed without difficulty, but are considerably more complicated [Heiland, 1946, pp. 389-400]. Practical difficulties in handling the equations increase still more, when the directions of polarization and of measurement are both required to be in some direction other than the vertical or horizontal as is the case for aeromagnetic data. This accounts in part for the very small number of explicit equations for the interpretation of aeromagnetic data that have appeared in the literature to date, a fact also remarked upon by Smellie [1956, p. 1021]. This has led to a good many alternative methods, such as reducing to vertical and horizontal components, [Hughes and Pondrom, 1947], or constructing model fields, either experimentally or theoretically from bodies with various values of the parameters [Vacquier, 1951; Zietz and Henderson, 1956]. These methods suffer the limitation that they cannot be easily extended to cases where the particular parameters incorporated in the models do not hold.
For a more general treatment, a flexible set of equations relating field to body is required.

This has become more urgent by the recognition, largely as the result of experimental work on the magnetic properties of rocks, that a large variety of directions of polarization are found in formations. At the same time the equations of interpretation as given in the literature either do not allow for this at all, or are not framed in such a way as to allow for variations of this additional parameter.

Most attempts to incorporate arbitrary directions of measurement and polarization begin by using explicitly the azimuths and inclinations of the directions involved, as for example, in the developments of Sutton and Mumme [1957]. As we shall see later, the shape of the anomalies does not depend explicitly upon these quantities, but upon certain parameters combining these directions. Thus equations involving the directions themselves are unnecessarily complex. Once the combined parameters are formulated on the other hand, it is possible to obtain quite simple expressions for the force-field over bodies from the simplest to more complex ones. In fact, the extra generality makes it possible to include the effects of a larger number of parameters, with greater over-all simplicity, than is possible when these more general quantities involving direction are not employed. It is one of the purposes of this research to formulate these parameters, and apply them to the problems of magnetic interpretation. It is possible by this method not only to include the direction of polarization among
the parameters to be found, but also to derive expressions for the aeromagnetic anomalies over bodies which have not been treated previously in the literature.

In the second part, in the application of the methods of magnetic interpretation to an actual aeromagnetic survey, certain general propositions about the variation of the values of the physical properties of rocks and formations are applied to a particular area (Texada Island). It is shown how to extract the most significant geological information contained in magnetic measurements. The distribution of magnetization deduced in this way is correlated with existing knowledge on the migration of iron during the formation of ore deposits on the island, and this in turn is related to the structural history of the area as revealed in the pattern of linears observed on aerial photographs, and in the general physiography of the island.

In the first period of the development of methods of interpreting magnetic surveys see, for example, Smythe [1896], Heiland [1946], and Nettleton [1942], an extensive body of literature arose for use in interpreting surveys making measurements of the vertical and horizontal components of the magnetic force at the surface of the earth. With these limitations on the direction of measurement, it was possible to derive a set of equations for the magnetic field and its derivatives over bodies of a wide variety of form.

However, newer instruments for measuring the field of the earth are not, like the older ones, confined to measurements of the vertical and horizontal components, and for this reason
bring a new problem to magnetic interpretation. The fluxgate magnetometer, for example, although capable of measuring any desired component of the field, is much more accurate for measurements in the direction of the total field [Jakosky, 1949; p.237], [Logachev, 1955; p.179]. Instruments based on nuclear free precession will record measurements only along this direction [Waters, 1956]. The problem of adapting the equations of interpretation to be of use when measurements are made with these instruments may be approached in a number of ways.

The component, measured along the total field, of the force over a magnetized body may be obtained from the components along the vertical and horizontal directions, if this force is small with respect to the field due to the earth as a whole, by the following equation [Vacquier, 1951; p.46]

\[ F_T = F_H \cos i + F_Z \sin i \]  

(1.1)

where \( i \) is the inclination of the earth's field (see Figure 1.1).

For any bodies where expressions for \( F_H \) and \( F_Z \) have already been obtained, the transformation of the existing set of equations into those for the results of measurements with the fluxgate or nuclear free precession magnetometer presents no difficulty in theory. In practice, little difficulty is encountered in the case of bodies such as single poles or lines and sheets of poles, where no direction of polarization need be
assumed. For bodies which must be considered as aggregates of dipoles, however, where a direction of polarization must be specified, the application of (1.1) becomes difficult except in the simplest cases where polarization can be taken to be vertical or along the earth's field. A number of useful methods, based on these assumptions, have appeared such as for obtaining depths of polarized bodies [Henderson and Zietz, 1948, 1958; Smellie, 1956], or for obtaining depth, intensity of magnetization and size of the body [Vacquier, 1951; Miller, 1956; Paterson, 1957].

The assumption of these special directions encounters a fundamental objection arising from studies of rock magnetism. For the basis of the long-held belief that polarization generally lies along the earth's field is the contention that the majority of rocks exhibiting magnetic properties owe their polarization largely to induction in the earth's field. Factors such as remanent magnetization or demagnetization effects in various directions, which would tend to cause the total vector of the intensity of magnetization to deviate from the total field direction, are commonly thought to be negligible [SLICHTER, 1929, p.249].

However, evidence to the contrary has been accumulating for a considerable period of time, and such cases have appeared since the earliest days of geophysics (see for example Heiland [1930]). Cases of remanent magnetization several times as great as the induced component [Kruglyakova, 1956] and in directions other than along the earth's field [Balsley and
Buddington, 1957] have been found in laboratory testing of specimens, and the possibility that similar cases might constitute a sizeable proportion of the rocks of the crust has been recognized. The results of field surveys, in which directions of polarization other than along the earth's field, also confirm the laboratory work and add strength to the belief that such cases are relatively widespread [Heiland, 1929; Garland, 1951; Mikov, 1952; Lundbak, 1956].

The work of Balsley and Buddington[1957], furthermore, in correlating direction of magnetization with metamorphic structures, is a recent demonstration of the fact that magnetic properties are closely connected with geological conditions and a sensitive indicator of the latter. This has been known for some time, and confirms earlier work such as that by Davis [1935] and his co-workers, or by Kalashnikov [1952].

This gives additional importance to direction of polarization as a parameter to be solved for in the interpretation of magnetic surveys, indicating as it does that the magnitude of this quantity is dependent on geological conditions. The comparison of values of direction of polarization across an area may be a valuable indicator of geological change. Any simplified assumptions, then, as to the direction of polarization may render equations for the fields over various bodies inadequate to meet the requirements of actual conditions, and necessitate generalization of the equations to include any desired direction.
Among the first attempts at such a generalization were those of Mikov [1953] and Voskoboynikov [1955] who gave a theoretical treatment of the effect of arbitrary direction of polarization on the vertical and horizontal components of field over a horizontal cylinder. The equations for the bodies presented in this thesis were already developed when a paper appeared by Sutton and Mumme [1957], confirming the possibility of including the direction of polarization in the equations of aeromagnetic interpretation, where the measurements have been taken with the fluxgate magnetometer. They considered the case of a single dipole and a line of dipoles and developed equations for the magnetic force over these bodies when the direction of polarization is arbitrary. They also showed the effect on profiles over these bodies of various directions of polarization and angles of inclination of the direction of measurement. A method of determining depth to the line of dipoles, using half-maximum width of the anomaly, is given. The present thesis involves a different aspect of the effect of the direction of polarization, viz. the combination of directions into shape factors, its effect on the peak and inflection points of the profiles, and methods of obtaining equations for more complex bodies and for higher derivatives of the field. The results of these studies are presented here.

The method used in the interpretation of the observed field determines those features of the theoretical equations to be emphasized. The comparison of one curve with another (the observed profile with the theoretical type-curve) is basically
a process of curve-fitting, and the methods of curve-fitting, well-known in mathematical analysis are applied in one form or another. The choice of these methods governs the scheme of interpretation adopted. The method of elementary decompositions leads to the "direct" methods of interpretation [Bullard and Cooper, 1948; Peters, 1948]; consideration of the position of special points such as peaks, half maxima, inflection points, zeros, minima is typical of the most frequently employed "indirect method" (examples are [Nettleton, 1942; Smellie, 1956]). Derived quantities involving areas under the curve etc. lead to schemes of interpretation such as proposed by Kogbetlianz [1945] to determine the centres of gravity of attracting bodies. Least squares fitting of theoretical profiles to observed ones has been developed by Hall [1958].

Special points on type curves and profiles, especially the maxima and the inflection points flanking them, are considered in this thesis. This selection of points to be considered was made because in the interpretation of observed magnetic fields over the earth, these are in general the only special points that can be located with certainty. Furthermore, to a greater or lesser accuracy, these points can be located for most anomalies; the position of the peaks can always be found on the intensity maps, while for most areas, second vertical derivative maps can be constructed [Vacquier, 1951]. This is done as routine analysis in modern practice, and the zeros give the position of the inflection points for elongated anomalies. Consequently, the inflection points may be located by numerical methods, on almost any profile.
Thus the means are at hand of using theoretical results concerning derivatives, and inflection points, of the magnetic intensity over disturbing bodies, to obtain information about the earth's crust. This gives additional justification for such developments: apart from their theoretical importance, they may be easily applied to the study of the earth. From this study, it will become evident that in addition to the parameters that are normally found to-day by geophysical interpretation viz. the depth, size, and intensity of magnetization, of the disturbing body, another parameter, related to the fundamental magnetic properties of the body viz. the direction of polarization, may also be found, knowing the position of the peak and inflection points. Since these points are commonly presented in maps covering wide areas, as part of a routine modern analysis of magnetic data, it is important to emphasize again the possible application of the present study to the mapping of the variations in the values of physical properties of the earth's crust over wide areas.

2. **THE MAGNETIC FIELD OVER SOME ELEMENTARY BODIES**

   a. **The point dipole** - see Figure 2.2

   The factors that must be allowed for are dictated by the method of surveying, and by the particular conditions of magnetization to be expected. The advent of airborne surveys makes it necessary to allow for the point of observation to be anywhere in space, and considering what has been learned about the magnetization of rocks, dipoles with axes in any direction
Heiland [1946, pp.391-3] summarizes the application of dipoles to magnetic interpretation, the equations being for straight-line, meridional traverses, with vertical or horizontal components of the magnetic intensity over dipoles polarized in the direction of the earth's field. Further generalization, to include meridional traverses over dipoles with axes along the earth's field, and measurements made of the component of intensity along this direction came with the advent of the airborne magnetometer, [Henderson and Zietz, 1948; Smellie, 1956]. With increasing knowledge about the directions of polarization in rocks, came a further step of generalization, to allow for an arbitrary direction of polarization. Examples of such a development are the paper of Sutton and Mumme [1957] and the present investigation. The latter develops the equations to a slightly more general form, in which they are suitable for further generalization to represent the fields over more complex bodies. These equations will now be developed.

Let a magnetometer Q at the point \((x, y, z)\) measure \(F\), the component in the direction \(\{l, m, n\}\) of the field due to a dipole \(P\) of moment \(\mu\), polarized in the direction \(\{L, M, N\}\) and located at the point \((a, b, c)\). Then \(F\) is given by [Jeans, 1948, p. 372]
Figure 2.1 - Traverse over dipole

\[ \mathcal{F} = \mu \frac{\partial^2}{\partial t^2} \left( \frac{1}{r^2} \right) \]  

(2.1)

where \( \frac{\partial}{\partial t} \) is differentiation in the direction \( \{l, m, n\} \), that is in the direction of the component to be measured, and \( \frac{\partial}{\partial S} \) is differentiation in the direction \( \{L, M, N\} \), that is in the direction of the north pole of the dipole, and

\[ r^2 = (x-a)^2 + (y-b)^2 + (z-c)^2 \]  

(2.2)

If \( f \) and \( g \) are any two functions of \( x, y, \) and \( z \):

\[ \frac{\partial f}{\partial s} = L \frac{\partial f}{\partial x} + M \frac{\partial f}{\partial y} + N \frac{\partial f}{\partial z} \]  

(2.3)

and

\[ \frac{\partial g}{\partial t} = l \frac{\partial g}{\partial x} + m \frac{\partial g}{\partial y} + n \frac{\partial g}{\partial z} \]  

(2.4)

These expressions can be used to expand (2.1) in the form:

\[ \mathcal{F} = \frac{\mu}{r^5} \left\{ \alpha_{11} (x-a)^2 + \alpha_{22} (y-b)^2 + \alpha_{33} (z-c)^2 + \alpha_{12} (x-a)(y-b) ight. 
\]

\[ + \left. \alpha_{13} (x-a)(z-c) + \alpha_{23} (y-b)(z-c) \right\} \]  

(2.5)

where

\[ \alpha_{11} = 2Ll - Mm - Nn \quad \alpha_{12} = 3(Ml + Lm) \]

\[ \alpha_{22} = 2Mm - Nn - lL \quad \alpha_{13} = 3(Nl + Lm) \]

\[ \alpha_{33} = 2Nn - lL - Mm \quad \alpha_{23} = 3(Nm + Mn) \]
Observation in a horizontal plane above a point dipole

Taking the z-axis downward, and the dipole at (0, 0, h) we have the case of observation in the x-y plane, which is a distance h above the dipole.

Thus in (2.5), \( z = a = b = 0 \), and \( c = h \),

and hence

\[
\mathcal{F} = \frac{\mu}{(x^2 + y^2 + h^2)^{5/2}} \left\{ \alpha_{11}x^2 + \alpha_{22}y^2 + \alpha_{33}h^2 + \alpha_{12}xy - \alpha_{13}xh - \alpha_{23}yh \right\}
\]

Some simplification results particularly in performing actual computation, from writing distances in units of \( h \) and going over to what we will call the "reduced form" of the expression for \( \mathcal{F} \).

Letting

\[
\xi = \frac{x}{h}, \quad \eta = \frac{y}{h}
\]

we have

\[
\mathcal{F} = \frac{\mu}{h^3} \left\{ \frac{\alpha_{11} \xi^2 + \alpha_{22} \eta^2 + \alpha_{33} + \alpha_{12} \xi \eta - \alpha_{13} \xi - \alpha_{23} \eta}{(\xi^2 + \eta^2 + 1)^{5/2}} \right\}
\]

Thus the direction of polarization and of measurement, as well as depth, are among the unknown parameters of a body which determine the shape of the corresponding anomaly. The various charts given by Vacquier [1951] illustrate this with prismatic bodies for the special case \( \{L, M, N\} = \{l, m, n\} \).

The peak of the anomaly, and the line of minima are given by the condition:

\[
\frac{\partial \mathcal{F}}{\partial \xi} = \frac{\partial \mathcal{F}}{\partial \eta} = 0
\]
From (2.9) we obtain:
\[
\frac{\partial F}{\partial \xi} = \frac{\mu}{h^3(\xi^2+\eta^2+1)^{3/2}} \left\{ (\alpha_{12} \eta - \alpha_{13}) + \frac{2 \alpha_{12} - 5(\alpha_{22} \xi^2 - \alpha_{23} \xi + \alpha_{33}) \eta}{4 (\alpha_{12} \xi - \alpha_{13}) \xi^2 - 3 \alpha_{11} \xi^3} \right\}
\]
and
\[
\frac{\partial F}{\partial \eta} = \frac{\mu}{h^3(\xi^2+\eta^2+1)^{3/2}} \left\{ (\alpha_{12} \xi - \alpha_{13}) + \frac{2 \alpha_{12} - 5(\alpha_{22} \xi^2 - \alpha_{23} \xi + \alpha_{33}) \eta}{4 (\alpha_{12} \xi - \alpha_{13}) \eta^2 - 3 \alpha_{22} \eta^3} \right\}
\]
(2.11) (2.12)

When substituted into (2.10) these give a pair of simultaneous third order equations in $\xi$ and $\eta$ which could, if required, be solved numerically to find the positions of the peak and minima corresponding to given directions of polarization and of measurement.

If the axes are taken so that the $y$-axis is along the magnetic meridian, (2.7) above is equivalent to equation (6) of Sutton and Mumme [1957]. In this form it is suitable for calculating the actual contours over a point dipole, or as is done by the above-mentioned authors, for studying the profiles in special directions such as along or perpendicular to the magnetic meridian.

However, if the equations for the dipole are to be integrated to obtain the profiles over more complex bodies, the more general equation (2.7) is preferable.
(ii) Observations along a traverse.

If Q (Figure 2.1) is confined to the x-axis, then $\eta = 0$ and the equations are further simplified. Substituting $\eta = 0$ into (2.9) and (2.11) we obtain:

$$\mathcal{F} = \frac{\mu_0 \kappa}{h^2 (\kappa^2 + 1)^{3/2}} \left\{ \kappa^2 - C_1 \kappa + C_2 \right\}$$

with

$$C_1 = \frac{\alpha_{13}}{\alpha_{11}}, \quad C_2 = \frac{\alpha_{33}}{\alpha_{11}}$$  \hspace{1cm} (2.13)

and

$$\frac{\partial \mathcal{F}}{\partial \kappa} = -\frac{\mu_0 \kappa}{h^3 (\kappa^2 + 1)^{3/2}} \left\{ 3 \kappa^3 - 4 C_1 \kappa^2 + (5 C_2 - 2) \kappa + C_1 \right\}$$  \hspace{1cm} (2.14)

from (2.14) we obtain further:

$$\frac{\partial^2 \mathcal{F}}{\partial \kappa^2} = \frac{\mu_0 \kappa}{h^3 (\kappa^2 + 1)^{3/2}} \left\{ 12 \kappa^4 - 20 C_1 \kappa^3 + 3(10 C_2 - 7) \kappa^2 + 15 C_1 \kappa - (5 C_2 - 2) \right\}$$  \hspace{1cm} (2.15)

The shape of the profile, then is determined by three quantities: $h$, $C_1$, and $C_2$.

If the position of the dipole in the horizontal plane can be determined from other methods, for instance from a gravity survey, or from geological indications, then the origin of the co-ordinate system in (2.13) to (2.15) may be fixed. Using the positions of peaks and inflection points along one, or a number of profiles through the origin, values of $L$, $M$, $N$, and $h$, can be determined without difficulty. If the origin cannot be located by independent means, a contour map of the magnetic values in the vicinity may be used to locate this origin. An iterative
Table I - Derivatives of field values along traverse over a single dipole

to follow page 16.

Referring to Figure 2.2, and writing $D^n F = \frac{d^n F}{dx^n}$, $D^n F = \frac{d^n F}{dz^n}$,
we have:

\[
F = \mu \left\{ \alpha_{11} x^2 - \alpha_{13} hx + \alpha_{33} h^2 \right\} \quad \frac{1}{(x^2 + y^2 + h^2)^{3/2}}
\]

\[
D^2 F = \mu \left\{ \frac{3\alpha_{11} x^3 - 4\alpha_{13} hx^2 + (5\alpha_{33} - 2\alpha_{11}) h^2 x + \alpha_{13} h^3 x}{(x^2 + y^2 + h^2)^{3/2}} \right\}
\]

\[
D_z F = \mu \left\{ \frac{3\alpha_{33} z^3 + 4\alpha_{13} xz^2 + (2\alpha_{33} - 5\alpha_{11}) x^2 z - \alpha_{13} x^3}{(x^2 + y^2 + h^2)^{3/2}} \right\}
\]

\[
D^2 F = \mu \left\{ \frac{12\alpha_{11} x^4 - 20\alpha_{13} hx^3 + 3(10\alpha_{33} - 7\alpha_{11}) h^2 x^2 + 15\alpha_{13} h^2 x}{(x^2 + y^2 + h^2)^{9/2}} \right\}
\]

\[
D_z^2 F = \mu \left\{ \frac{(2\alpha_{33} - 5\alpha_{11}) x^4 + 15\alpha_{13} x^3 z + 3 (10\alpha_{11} - 7\alpha_{33}) x^2 z^2 - 20\alpha_{13} xz^3 + 12\alpha_{33} z^4}{(x^2 + y^2 + h^2)^{9/2}} \right\}
\]
process is possible, starting from an arbitrary origin, for which the corresponding values of L, M, N and h are computed. These values, when substituted into (2.11) and (2.12) give a new, more correct position of the origin. The process may be continued for as long as is necessary. The solution would be laborious, however, since new profiles would have to be drawn for each re-determination of the origin.

A single dipole approximates a compact, uniformly magnetized body at depth, with dimensions roughly the same in all directions. Such bodies are encountered especially in mining exploration (see, for example, Yüngül [1956]), and for this reason the equations developed for the dipole may on occasion be required for the solution of practical problems.

They are intended here, however, as the starting point for developing equations for the profiles over more complex bodies.

(b) The horizontal line of dipoles (see Figure 2.3) -

Let the magnetic moment have a constant line density \( \mu_L \) per unit length, and consider a straight, horizontal traverse perpendicular to the line and passing over it at a height h. If we take axes with z vertically downward, and the traverse along Ox, then any element, length dt, of the line at \( b = t \) can be considered as a dipole with magnetic moment \( \mu_L \text{dt} \), producing a field \( d\mathbf{F} \) at the magnetometer, given by (2.5) in which \( y = z = a = 0, b = t, c = h, \) and \( r = x^2 + h^2 + t^2 \).
we have:
\[ d\mathcal{F} = \mu_L \frac{1}{r^5} \left\{ \alpha_{11} x^2 + \alpha_{22} t^2 + \alpha_{33} h^2 - \alpha_{12} x t - \alpha_{13} x h + \alpha_{23} t h \right\} \, dt \]

writing distances in units of \( h \), with
\[ \xi = x/h, \quad \tau = t/h, \]
we have:
\[ d\mathcal{F} = \frac{\mu_L}{h^2 (\xi^2 + \tau^2 + 1)} \left\{ \alpha_{11} \xi^2 + \alpha_{22} \tau^2 + \alpha_{33} - \alpha_{12} \xi \tau - \alpha_{13} \xi \tau + \alpha_{23} \tau \right\} \, d\tau \]

If \( \mathcal{F}_{T_1} \), is the force due to a line terminating at \( y = \pm T_1 \)
\[ \mathcal{F}_{T_1} = \int_{-T_1}^{T_1} d\mathcal{F} \] (2.19)

Substituting from (2.18) into (2.19) and carrying out the integration we find:
\[ \mathcal{F}_h = \frac{2 T_1^3 \mu_L}{h^2 (\xi^2 + 1)(\xi^2 + \tau^2 + 1)^{3/2}} \left\{ \frac{\alpha_{11} \xi^4 + 3 \frac{C_5}{T_1^2} \xi^3 + \left[ C_4 + \frac{1}{T_1^2} (\alpha_{11} + \alpha_{33}) \right] \xi^2}{\left( 1 + \frac{3}{T_1^2} \right) C_5 \xi + \left[ C_6 + \frac{\alpha_{33}}{T_1^2} \right]} \right\} \] (2.20)

where \( C_4 = -C_6 = (1L - nN) \)

and \( C_5 = -2(1N + Ln) \). (2.21)
If \( T_1 \to \infty \), we have the case of an infinite line of dipoles, and \( \mathcal{F}_T \to \mathcal{F} \), given by:

\[
\mathcal{F} = \frac{2\mu_L}{h^2} \left\{ \frac{\xi^2 + \Lambda_1 \xi - 1}{(\xi^2 + 1)^2} \right\}
\]

(2.22)

where \( \Lambda_1 = \frac{C_5}{C_4} = \frac{2(1N + nL)}{(nN - 1L)} \). This may be called the "polarization function". (2.23)

(c) The infinite line of dipoles

This body, with profile given in (2.22), is of considerable use in interpretation, and has been the object of considerable attention. The effect of an arbitrary direction of polarization on the areas under the profiles of the vertical and horizontal intensity of anomalies over this body has been studied by Mikov [1953] and by Voskoboynikov [1955], and a method for calculating the unknown parameters of the body is given by these authors. Profiles of the total field of the anomaly for a number of directions of polarization have been drawn by Sutton and Mumme [1957].

Additional quantities are necessary for the more complete analysis of profiles, and these will be developed in later sections.

3. Extension and generalization of methods.

The equations for the magnetic field and derivatives over two elementary bodies - the single dipole and the line of dipoles, have been generalized to include cases where both the directions of polarization and measurement are
arbitrary. These developments are of value in direct applications to geological conditions which can be approximated by these elementary forms. Within reasonable limits of accuracy, such elementary forms do have a wide application in practical interpretation [Nettleton, 1942, p.293], but if the methods are to be used to study the direction of polarization over a wide variety of geological situations, or if generalizations are to be made, a wider range of bodies must be treated. These include cylinders of arbitrary cross section, thick sheets, sloping steps and bodies derivable from the latter (synclines, anticlines, and other forms as shown for example by Heiland, [1946, p.396]). At a further stage of generalization, various parameters have been found which may be evaluated even though the particular form of the body is not known [Kogbetliantz, 1945; 1948; Strakov, 1956].

Various aspects of the direction of polarization for elementary forms have been studied in the past and these have already been mentioned. It is believed that more complex bodies have not been treated at all in this way, and methods of performing such a generalization are now examined.

Equations for a number of the more complex bodies may be derived from those for the single dipole, line of dipoles and the thin sheet, by using these bodies as elements, and integrating between the required limits. For others where the actual integration is difficult, charts are used as an aid in the summation [Logachev, 1955, pp.77-80];
These charts are based on elementary bodies, and any equations for these which take into account the direction of polarization may be used to modify the charts to account for this parameter.

a. Bodies obtainable by integration.

In an earlier section, the expression was derived for the magnetic field observed on a traverse perpendicular to the strike, over an infinite line of dipoles (eq. 2.22). Expressing distance along the traverse in terms of $x$ and $h$ rather than $x/h$ and $h$ as in (2.22) we may write:

$$\mathcal{F} = 2\mu \left\{ \frac{C_4 x^2 + C_5 h x - C_4 h^2}{(x^2 + h^2)^2} \right\}$$

(3.1)

with $C_4$ and $C_5$ as defined in (2.21).

This equation is the basis of further generalization to long, cylindrical bodies with various cross-section shapes.

The magnetic force due to any body may be obtained by integration over elementary volumes, each taken as a dipole [Jeans, 1948, p.375]. Thus for a cylindrical body, a line of dipoles is an admissible element of integration. All forces derived will be for profiles along traverses perpendicular to the strike of the body.

(i) The thin, dipping semi-infinite sheet of dipoles

(see Figure 3.4)
The magnetic force at Q is determined by the intensity of magnetization, I of the sheet, by the distances x and h, and by the directions of polarization and measurement, expressed as direction cosines with respect to the axes \(0 \mathbin{x} z\). The derivation of the expression for the force in terms of these quantities may be carried out most easily by first deriving it in terms of distances and directions relative to the axes \(0^1 \mathbin{x} 1^1 z^1\). This procedure avoids having to perform the laborious integrations which otherwise would be necessary.

The transformations are as follows, referring to Figures 3.1 and 3.2.
Consider an elementary horizontal cylinder with axis passing through the point \((u^1, h^1)\), (Figure 3.1) cross sectional area \(t\, du\), and a magnetic moment per unit length of \(I\, t\, du\). By (3.1) the contribution of the cylinder to the force at Q in the direction \(\{l^1, m^1, n^1\}\) is given by

\[
\left. d\mathcal{F} = 2\, I\, t\, \frac{C_4^1(u^1 + x^1)^2 + C_5^1 h^1(u^1 + x^1) - C_4^1 h^{12}}{(u^1 + x^1)^2 + h^{12}}\right|_0^\infty \; du^1
\]

(3.5)

Thus the force at Q due to the sheet is given by

\[
\mathcal{F} = 2\, I\, t\int_{u=-\infty}^0 d\mathcal{F} = \text{It}\left\{ \frac{2C_4^1 x^1 + C_5^1 h^1}{x^1 + h^1} \right\}
\]

(3.6)

where \(C_4^1\), and \(C_5^1\) are referred to axes \(O^1x^1z^1\). The integration is simply done by \((18)\) of appendix I. Substituting from (3.2) into (3.6)

\[
\mathcal{F} = \text{It}\left\{ \frac{2C_4^1(x\cos d + h\sin d) + C_5^1(-x\sin d + h\cos d)}{x^2 + h^2} \right\}
\]

(3.7)
\[ \mathcal{F} = \frac{1}{t} \left( 2C_4 \cos d - C_5 \sin d \right) x + \left( 2C_4 \sin d + C_5 \cos d \right) h \]

\[ \frac{x^2 + h^2}{\mathcal{F}} \]  \hspace{1cm} (3.8)

It can be shown from (3.3) and (3.4) that

\[ C_4 = C_4 \cos 2d + \frac{1}{2} C_5 \sin 2d \]

and \[ C_5 = -2C_4 \sin 2d + C_5 \cos 2d \]  \hspace{1cm} (3.9)

Substituting these expressions into (3.8) collecting terms in \( C_4 \) and \( C_5 \), and using the identities:

\[ \sin d = \sin (2d-d) = \sin 2d \cos d - \cos 2d \sin d \]
\[ \cos d = \cos (2d-d) = \cos 2d \cos d + \sin 2d \sin d, \]

we finally obtain:

\[ \mathcal{F} = \frac{1}{t} \left( C_7 x + C_8 \right) h \]
\[ \frac{x^2 + h^2}{\mathcal{F}} \]  \hspace{1cm} (3.10)

where \[ C_7 = 2C_4 \cos d + C_5 \sin d \]
\[ C_8 = -2C_4 \sin d + C_5 \cos d \]  \hspace{1cm} (3.11)

Similar to the expression for the profile over a point dipole (2.9) or over an infinite line of dipoles (2.22), the corresponding expression for the thin, dipping sheet, (3.10) may be written in reduced form.

Dividing numerator and denominator by \( h^2 \), and factoring out \( C_8 \), we have:

\[ \mathcal{F} = \frac{1}{t} \frac{C_8}{h} \left\{ \frac{\Lambda_2 x^2 + 1}{x^2 + 1} \right\} \]  \hspace{1cm} (3.12)

where \( x = x/h \), and

\[ \Lambda_2 = \frac{C_7}{C_8} = \frac{\Lambda_1 \tan d + 2}{\Lambda_1 - 2 \tan d} \]  \hspace{1cm} (3.13)
The reduced form is important, for it separates the parameters into two groups: the "scale parameters", which are effective in determining the size of the anomaly (here \( I, t, C_8 \) and \( h \)); and the "shape parameters", which are effective in determining "type curves", representing shape of the anomaly. Here \( \Lambda_2 \) is the parameter of a family of type curves obtained by plotting the function \( \frac{\Lambda_2 \xi + 1}{\xi^2 + 1} \) against \( \xi \).

(ii) The thick, dipping polarized sheet: (see Figure 3.3)

Take as an element a thin sheet whose top is at \((u, h)\) and width \(du \sin d\), then from (3.10) the force at \(Q\) is given by:

\[
dF = I \sin d \frac{C_7(x-u) + C_8h}{(x-u)^2 + h^2} \, du
\]

(3.14)

The force due to the whole sheet is

\[
F = \int_{u=-b}^{b} dF = I \sin d \int_{u=-b}^{b} \frac{C_7(x-u) + C_8h}{(x-u)^2 + h^2} \, du
\]

\[
= -I \sin d \left[ \frac{C_7 \log_e((x^2 + h^2)^{1/2}) + C_8 \tan^{-1} \frac{x-u}{h}}{2} \right]_{-b}^{+b}
\]

\[
= +I \sin d \left[ \frac{C_7 \log_e(x^2 + h^2) + C_8 \tan^{-1} \frac{x}{h}}{2} \right]_{1}^{2}
\]

where 2 and 1 represent the distances from \((-b, h)\) and \((b, h)\) to \(Q\).
(iii) The sloping step (see Figure 3.4)

In this case the force $\mathbf{F}$ observed at $Q$ is that due to a sloping step of mean depth $h$ and with the origin at a distance $x$ from $Q$. $\mathbf{F}$ may be expressed in terms of these distances and the direction of polarization of the step and the direction of measurement. If the distances and directions were measured with respect to axes $0^1 x^1 z^1$, however, $\mathbf{F}$ would be given by the expression for a thick polarized sheet of dip $\delta$. Furthermore distances and directions relative to $0^1 x^1 z^1$ may be transformed into those relative to $0 x z$ by equations (3.2), (3.3) and (3.4) with $d = \beta$. The force at $Q$ is thus given by:

$$\mathbf{F} = I \sin \delta \left[ \frac{c_1}{2} \log_e \left( x^{12} + h^{12} \right) + c_8 \tan^{-1} \frac{x^1}{h^1} \right]^2$$

(3.16)

where $c_7 = 2c_4 \cos \delta + c_5 \sin \delta$

and $c_8 = -2c_4 \sin \delta + c_5 \cos \delta$

since equation (3.15) for the dipping sheet may be used with distances and directions referred to $0^1 x^1 z^1$. 
There are some advantages in expressing the slope of the face of the step in terms of the angle $\beta = 180^\circ - \xi$ rather than $\xi$.

The quantities $x^1, h^1, C_4^1, C_5^1$ referred to $O^1 x^1 z^1$ must be transformed into the corresponding quantities $x, h, C_4$ and $C_5$ referred to $O x z$. This is facilitated by noticing that $-C_7^1$ and $-C_8^1$ (expressed in terms of $\beta$) are of the same form as the coefficients of $x$ and $h$ respectively in equation (3.8). These were transformed into $C_7$ and $C_8$ respectively, as given in equation (3.11) by the same transformation (with $\beta = \beta$) as those we are now applying. Thus in the present case $C_7^1$ becomes $-C_7$ and $C_8^1$ becomes $-C_8$ on transforming to axes $O x z$, where
\[
C_7 = 2C_4 \cos \beta + C_5 \sin \beta \quad (3.17)
\]
and
\[
C_8 = -2C_4 \sin \beta + C_5 \cos \beta
\]

Finally using equation (3.2) (with $d = \beta$), equation (3.16) becomes:
\[
\mathcal{J} = I \sin \beta \left[ -\frac{C_7}{2} \log_e (x^2 + h^2) - C_8 \tan^{-1} \left( \frac{x \cos \beta + h \sin \beta}{-x \sin \beta + h \cos \beta} \right) \right]^2
\]

Since
\[
\tan^{-1} \left( \frac{x \cos \beta + h \sin \beta}{-x \sin \beta + h \cos \beta} \right) = \tan^{-1} \left( \frac{x + \tan \beta}{1 - x \tan \beta} \right)
\]

we finally have, writing $d$ for $\beta$ for the sake of uniformity,
\[
\mathcal{J} = I \sin d \left[ \frac{C_7}{2} \log_e (x^2 + h^2) + C_8 \tan^{-1} \frac{x}{h} \right]^2
\]

(3.19)
(iv) **Special cases.**

None of the equations for the force field over dipping sheets, or the sloping step appear in the literature for arbitrary directions of polarization and measurement. However, various special cases do exist all of which can be obtained from the more general equations given above, one of which follows as an example.

1. **The thin, dipping sheet**

Equations for Z and H, the vertical and horizontal components of $\mathbf{F}$, for a vertically-polarized sheet are given by Logachev [1955, (eq. 18.7) p. 72] viz:

$$Z = 4ib \sin \alpha \frac{h \sin \alpha + x \cos \alpha}{h^2 + x^2}, \quad \text{and}$$

$$H = 4ib \sin \alpha \frac{h \cos \alpha - x \sin \alpha}{h^2 + x^2},$$

where $2b \sin \alpha$ is the thickness of the sheet, and $\alpha$ is the dip (the complement of $d$ in equation (3.10)). I, $x$ and $h$ have the same meaning. Thus in the present notation:

$$Z = 2It \frac{h \sin d - x \cos d}{h^2 + x^2}, \quad \text{and}$$

$$H = 2It \frac{-h \cos d - x \sin d}{h^2 + x^2}, \quad (3.20)$$

$Z$ is the value of $\mathbf{F}$ when $l = 0, n = 1$. For vertical polarization, $L = 0, N = 1$, making $C_4 = -1$ and $C_5 = 0$. Then $C_7 = -2 \cos d$ and $C_8 = 2 \sin d$, so that from (3.10),

$$Z = 2It \frac{h \sin d - x \cos d}{h^2 + x^2}, \quad \text{as obtained by Logachev.}$$

The expression for $H$ may be similarly obtained.
b. **A system of equations of interpretation.**

Reviewing the equations for the field over an infinite line of dipoles (3.1), a thin, dipping, polarized sheet (3.10), and a thick, polarized sheet (whether traversed as a dipping sheet (3.13) or as a sloping step (3.17)), we may note that each equation is derived from the preceding one, apart from transformations of axes which change only the coefficients, by integration with respect to x.

The form of the equation for the field over a thick sheet is \( \mathcal{F} = cF \), where \( c \) is a constant, and

\[
F = \alpha \tan^{-1} \frac{x}{h} + \beta \log_e (x^2 + h^2)
\]  

(3.21)

It follows from this and the relations between equations 3.1, 3.10 and 3.17 that the form of the equations for any of these bodies or any order of their derivatives with respect to x may be obtained from (3.21) by differentiation.

Expressions for the higher derivatives of \( F \) are given in Appendix I, and the first eight are given in Table II. Comparing these with the equations for the various bodies, we see certain similarities, which are summarized in the following table.
b. A system of equations of interpretation.

Reviewing the equations for the field over an infinite line of dipoles (3.1), a thin, dipping, polarized sheet (3.10), and a thick, polarized sheet (whether traversed as a dipping sheet (3.13) or as a sloping step (3.17)), we may note that each equation is derived from the preceding one, apart from transformations of axes which change only the coefficients, by integration with respect to $x$.

The form of the equation for the field over a thick sheet is $\mathcal{F} = c\mathcal{F}$, where $c$ is a constant, and

$$F = \alpha \tan^{-1} \frac{x}{h} + \beta \log_e (x^2 + h^2) \quad (3.21)$$

It follows from this and the relations between equations 3.1, 3.10 and 3.17 that the form of the equations for any of these bodies or any order of their derivatives with respect to $x$ may be obtained from (3.21) by differentiation.

Expressions for the higher derivatives of $F$ are given in Appendix I, and the first eight are given in Table II. Comparing these with the equations for the various bodies, we see certain similarities, which are summarized in the following table.
TABLE II - Derivatives of \( F = \alpha \tan^{-1} \frac{x}{h} + \beta \log_e (x^2 + h^2) \)

- to follow page 29.

Referring to Appendix I, eq. 10, writing \( \frac{D^n F}{dx^n} = d^n F \), and \( r^2 = x^2 + h^2 \), we have:

\[
\begin{align*}
\text{n} = 1 & \quad D^1 F = \frac{1}{r^2} \left\{ 2 \beta x + \alpha h \right\} \\
\text{n} = 2 & \quad D^2 F = - \frac{1}{r^4} \left\{ 2 \beta x^2 + 2 \alpha hx - 2 \beta h^2 \right\} \\
\text{n} = 3 & \quad D^3 F = \frac{2}{r^6} \left\{ 2 \beta x^3 + 3 \alpha hx^2 - 6 \beta h^2 x - \alpha h^3 \right\} \\
\text{n} = 4 & \quad D^4 F = - \frac{6}{r^8} \left\{ 2 \beta x^4 + 4 \alpha hx^3 - 12 \beta h^2 x^2 - 4 \alpha h^3 x + 2 \beta h^4 \right\} \\
\text{n} = 5 & \quad D^5 F = \frac{24}{r^{10}} \left\{ 2 \beta x^5 + 5 \alpha hx^4 - 20 \beta h^2 x^3 - 10 \alpha h^3 x^2 + 10 \beta h^4 x + \alpha h^5 \right\} \\
\text{n} = 6 & \quad D^6 F = - \frac{120}{r^{12}} \left\{ 2 \beta x^6 + 6 \alpha hx^5 - 30 \beta h^2 x^4 - 20 \alpha h^3 x^3 + 30 \beta h^4 x^2 + 6 \alpha h^5 x - 2 \beta h^6 \right\} \\
\text{n} = 7 & \quad D^7 F = \frac{720}{r^{14}} \left\{ 2 \beta x^7 + 7 \alpha hx^6 - 42 \beta h^2 x^5 - 35 \alpha h^3 x^4 + 70 \beta h^4 x^3 + 21 \alpha h^5 x^2 - 14 \beta h^6 x - \alpha h^7 \right\} \\
\text{n} = 8 & \quad D^8 F = - \frac{5040}{r^{16}} \left\{ 2 \beta x^8 + 8 \alpha hx^7 - 56 \beta h^2 x^6 - 56 \alpha h^3 x^5 + 140 \beta h^4 x^4 + 56 \alpha h^5 x^3 - 56 \beta h^6 x^2 - 8 \alpha h^7 x + 2 \beta h^8 \right\}
\end{align*}
\]
### TABLE III

<table>
<thead>
<tr>
<th>$D^n F$</th>
<th>$\mathcal{F} = k D^n F$</th>
<th>Body</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n = 2$</td>
<td>$k = \mathcal{U}_L$</td>
<td>line of dipoles</td>
<td>$C_5$</td>
<td>$C_4$</td>
</tr>
<tr>
<td>1</td>
<td>It</td>
<td>thin, dipping sheet</td>
<td>$C_8$</td>
<td>$C_{7/2}$</td>
</tr>
<tr>
<td>0</td>
<td>$I \sin d$</td>
<td>thick, dipping sheet</td>
<td>$C_8$</td>
<td>$C_{7/2}$</td>
</tr>
<tr>
<td>0</td>
<td>$I \sin d$</td>
<td>sloping step</td>
<td>$C_8$</td>
<td>$C_{7/2}$</td>
</tr>
</tbody>
</table>

We may also summarize the quantities represented by the different orders of $D^n F$ for the bodies, as in Table IV.

Using the relations between $\mathcal{F}$ over the various bodies treated in this section and the function $F$ as given in Table III, it is quite easy to utilize Table II to construct tables of $\mathcal{F}$ and its derivatives (up to the third) for the various bodies. These are done in Tables V to IX, both explicitly in terms of $x$ and $h$ and in reduced form. The first and second derivatives with respect to $z$, obtained by (15), Appendix I, are also included.
<table>
<thead>
<tr>
<th>n</th>
<th>line of dipoles</th>
<th>thin sheet</th>
<th>thick sheet or step. (with boundary conditions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>---</td>
<td>area under profile</td>
<td>value of field</td>
</tr>
<tr>
<td>1</td>
<td>area under profile</td>
<td>value of field</td>
<td>slope of profile</td>
</tr>
<tr>
<td>2</td>
<td>value of field</td>
<td>slope of profile</td>
<td>second derivative. (with respect to x or z)</td>
</tr>
<tr>
<td>3</td>
<td>slope of profile</td>
<td>second derivative (with respect to x or z)</td>
<td>third derivative (with respect to x)</td>
</tr>
<tr>
<td>4</td>
<td>second derivative (with respect to x or z)</td>
<td>third derivative (with respect to x)</td>
<td>fourth derivative (with respect to x or z)</td>
</tr>
<tr>
<td>5</td>
<td>third derivative (with respect to x)</td>
<td>fourth derivative (with respect to x or z)</td>
<td>fifth derivative (with respect to x)</td>
</tr>
</tbody>
</table>
Table V - Derivatives of field values along traverse over infinite line of dipoles

- to follow page 31

Referring to Tables II and III, and Figure 2.3, writing

\[ D^n\mathcal{F} = \frac{d^n\mathcal{F}}{dx^n}, \quad D^m\mathcal{F} = \frac{d^n\mathcal{F}}{dz^n} \]

we have:

\[ \mathcal{F} = 2\mu_L \left\{ \frac{c_4 x^2 + c_5 hx - c_4 h^2}{(x^2 + h^2)^2} \right\} \]

\[ D\mathcal{F} = -2\mu_L \left\{ \frac{2c_4 x^3 + 3c_5 hx^2 - 6c_4 h^2 x - c_5 h^3}{(x^2 + h^2)^3} \right\} \]

\[ D_z\mathcal{F} = -2\mu_L \left\{ \frac{2c_4 h^3 - 3c_5 x h^2 - 6c_4 x^2 h + c_5 x^3}{(x^2 + h^2)^3} \right\} \]

\[ D^2\mathcal{F} = D^2\mathcal{F} = 12\mu_L \left\{ \frac{c_4 x^4 + 2c_5 hx^3 - 6c_4 h^2 x^2 - 2c_5 h^3 x + c_4 h^4}{(x^2 + h^2)^4} \right\} \]

\[ D^3\mathcal{F} = -24\mu_L \left\{ \frac{2c_4 x^5 + 5c_5 hx^4 - 20c_4 h^2 x^3 - 10c_5 h^3 x^2 + 10c_4 h^4 x + c_5 h^5}{(x^2 + h^2)^5} \right\} \]
Table VI - Derivatives of field values along traverse over infinite line of dipoles - reduced form:

- to follow page 31.

Referring to Table V, writing $D^n F(\xi) = \frac{d^n F(\xi)}{dx^n}$, $D^n F(\xi) = \frac{d^n F(\xi)}{dz^n}$, we have:

$$F(\xi) = \frac{2\mu L C_4}{h^2} \left\{ \frac{\xi^2 + \Lambda_1 \xi - 1}{(\xi^2 + 1)^2} \right\}$$

$$D F(\xi) = -\frac{2\mu L C_4}{h^3} \left\{ \frac{2\xi^3 + 3\Lambda_1 \xi^2 - 6\xi - \Lambda_1}{(\xi^2 + 1)^3} \right\}$$

$$D_z F(\xi) = -\frac{2\mu L C_4}{h^3} \left\{ \frac{\Lambda_1 \xi^3 - 3\xi^2 - 3\Lambda_1 \xi + 2}{(\xi^2 + 1)^3} \right\}$$

$$D^2 F(\xi) = D_z^2 F(\xi) = \frac{12\mu L C_4}{h^4} \left\{ \frac{\xi^4 + 2\Lambda_1 \xi^3 - 6\xi^2 - 2\Lambda_1 \xi + 1}{(\xi^2 + 1)^4} \right\}$$

$$D^3 F(\xi) = -\frac{24\mu L C_4}{h^5} \left\{ \frac{2\xi^5 + 5\Lambda_1 \xi^4 - 20\xi^3 - 10\Lambda_1 \xi^2 + 10\xi + \Lambda_1}{(\xi^2 + 1)^5} \right\}$$
Table VII - Derivatives of field values along traverse over thin, dipping sheet

Referring to Tables II and III, and Figure 3.1, writing

\[ \frac{d^n \Phi}{dx^n}, \frac{d^n \Phi}{dz^n} \], we have:

\[ \Phi' = It \left\{ \frac{C_7 x + C_8 h}{x^2 + h^2} \right\} \]

\[ D\Phi = -2It \left\{ \frac{C_7 x^2 + 2C_8 hx - C_7 h^2}{(x^2 + h^2)^2} \right\} \]

\[ D_z \Phi = -2It \left\{ \frac{C_8 h^2 + 2C_7 hx - C_8 x^2}{(x^2 + h^2)^2} \right\} \]

\[ D^2 \Phi = D_z^2 \Phi = 2It \left\{ \frac{C_7 x^3 + 3C_8 hx^2 - 3C_7 h^2 x - C_8 h^3}{(x^2 + h^2)^3} \right\} \]

\[ D^3 \Phi = -6It \left\{ \frac{C_7 x^4 + 4C_8 hx^3 - 6C_7 h^2 x^2 - 4C_8 h^3 x + C_7 h^4}{(x^2 + h^2)^4} \right\} \]
Table VIII - Derivatives of field values along traverse over thin, dipping sheet - reduced form

- to follow page 31.

Referring to Table VII, writing $D^{n}_{x}(\xi) = \frac{d^{n}F(\xi)}{dx^{n}}$, $D^{n}_{y}(\xi) = \frac{d^{n}F(\xi)}{dy^{n}}$, we have:

$$F(\xi) = \frac{ItC_{8}}{h} \left\{ \frac{\Lambda_{2}\xi + 1}{\xi^{2} + 1} \right\}$$

$$D^{1}_{x}F(\xi) = -2ItC_{8} \left\{ \frac{\Lambda_{2}\xi^{2} + 2\xi - \Lambda_{2}}{(\xi^{2} + 1)^{2}} \right\}$$

$$D_{x}F(\xi) = -2ItC_{8} \left\{ \frac{1 + 2\Lambda_{2}\xi - \xi^{2}}{(\xi^{2} + 1)^{2}} \right\}$$

$$D^{2}_{x}F(\xi) = D^{2}_{y}F(\xi) = 2ItC_{8} \left\{ \frac{\Lambda_{2}\xi^{3} + 3\xi^{2} - 3\Lambda_{2}\xi - 1}{(\xi^{2} + 1)^{3}} \right\}$$

$$D^{3}_{x}F(\xi) = -6ItC_{8} \left\{ \frac{\Lambda_{2}\xi^{4} + 4\xi^{3} - 6\Lambda_{2}\xi^{2} - 4\xi + \Lambda_{2}}{(\xi^{2} + 1)^{4}} \right\}$$
Table IX - Derivatives of field values along traverse over thick, dipping dike or sloping step.

- to follow page 31.

Referring to Tables II and III, and Figures 3.3 and 3.5, writing $D^n f = \frac{d^n f}{dx^n}$, we have:

\[
\mathcal{F} = I \sin d \left\{ \frac{C_7 \log_e(x^2 + h^2)}{2} + C_8 \tan^{-1} \frac{x}{h} \right\}_2
\]

\[
D\mathcal{F} = I \sin d \left\{ \frac{C_7x + C_8h}{x^2 + h^2} \right\}_2
\]

\[
D^2\mathcal{F} = -I \sin d \left\{ \frac{C_7x^2 + 2C_8xh - C_7h^2}{(x^2 + h^2)^2} \right\}_2
\]

Reduced form of the equations.

\[
\mathcal{F} = I \sin d \left\{ \Lambda_2 \left( \log_e(\xi^2 + 1) \right) \frac{1}{\xi} - \frac{\log_e h^2}{\xi^2 + 1} + \tan^{-1} \xi \right\}_2
\]

\[
D\mathcal{F} = +I \sin d \frac{C_8}{h} \left\{ \frac{\Lambda_2 \xi + 1}{\left( \xi^2 + 1 \right)} \right\}_2
\]

\[
D^2\mathcal{F} = -I \sin d \frac{C_8}{h^2} \left\{ \frac{\Lambda_2 \xi^2 + 2\xi - \Lambda_2}{\left( \xi^2 + 1 \right)^2} \right\}_2
\]
Referring to Table IV, it is evident that since each derivative of \( F \) corresponds to some feature of the profile over each of the four bodies treated, calculation of a relatively small number of values of these derivatives will produce quantities which can be used in a very much larger number of situations in interpretation. Quite often when the basic parameters for a body are given, it is required to calculate the value of the field and some of its derivatives at given points. Such calculations are laborious especially when an extra parameter such as direction of polarization is to be taken into account, and the economy resulting from treating these four bodies as particular cases of a single function is of great practical value.

These derivatives are expressed in polynomial form in the preceding tables. This form is best for illustrating the relations between the fields and their derivatives over the various bodies. In a later section the significance of these equations will be discussed.

For the calculation of values of \( D^n F \), however, the trigonometric form (7) as derived in Appendix I, is more convenient. This is:

\[
D^n F = (-1)^{n-1} \left( \frac{n-1}{h} \right)! \sin^n \theta \left\{ \alpha \sin n\theta + 2\beta \cos n\theta \right\}
\]

where \( \cot \theta = x/h \)  

(7)
Substitution of the appropriate parameters and multiplication of $D^nF$ by the corresponding value of $k$ in Table III will give the value of $J_\theta$ over the body required.

A method of calculation using graphs $\sin^n\theta \sin n\theta$ and $\sin^n\theta \cos n\theta$ will be used. For lower values of $n$, these functions appear in expressions for the vertical and horizontal components of the anomalies over various bodies, and such curves have been published previously. Three of the curves for $n = 1$ and $n = 2$ appear in Nettleton's "Master Curves" [1942, p.299]. However, the system as a whole does not appear in the literature, consequently these functions have been calculated and appear in Plate I, plotted against $\xi$, position on traverse in depth units.

For each value of $n$, one of the functions is equal to zero for $\xi = 0$ and the other has an absolute value of 1 at this point.

Thus for $n = 1$, $\sin^n\theta \sin n\theta = \sin^2\theta = 1$ when $\xi = 0$; at the same time, $\sin^n\theta \cos n\theta = \sin\theta \cos\theta = 0$.

For $n = 2$, $\sin^n\theta \sin n\theta = \sin^2\theta \sin 2\theta = 0$ when $\xi = 0$; at the same time, $\sin^n\theta \cos n\theta = \sin^2\theta \cos 2\theta = -1$. 

---

Figure 5.1 - Relation of $\theta$ to $x$ and $h$. 
VALUES OF THE FUNCTION $f_0$

TABLE OF SIGNS FOR $f_0$

values on curves to be prefixed by:
    the following signs

<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+</td>
<td>++</td>
<td>--</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>2</td>
<td>+</td>
<td>++</td>
<td>--</td>
<td>++</td>
<td>---</td>
</tr>
<tr>
<td>3</td>
<td>+</td>
<td>++</td>
<td>--</td>
<td>++</td>
<td>++</td>
</tr>
<tr>
<td>4</td>
<td>+</td>
<td>++</td>
<td>--</td>
<td>++</td>
<td>++</td>
</tr>
<tr>
<td>5</td>
<td>+</td>
<td>++</td>
<td>--</td>
<td>++</td>
<td>++</td>
</tr>
</tbody>
</table>
Any of these functions which is zero for \( \xi = 0 \) will be denoted by \( f_0 \), and those whose absolute value is 1 at \( \xi = 0 \) by \( f_1 \). Graphs of \( f_0 \) for various values of \( n \) are shown on Plate I, and of \( f_1 \) on Plate II. These are plotted against \( |\xi| \), the absolute value of \( \xi \), and represent \( f_0 \) and \( f_1 \) when preceded by the appropriate sign in the "table of signs". For example, \( f_0 \) for \( n = 3 \) is the negative of the values on the \( n = 3 \) curve of Plate I for \( \xi < 0 \).

As a numerical example consider the horizontal line of dipoles for which (referring to Table III and (2.22))

\[
\mathcal{F} = \frac{2\mu L}{h^2} \left\{ C_4 f_1 + \frac{C_5}{2} f_0 \right\} \quad \text{for } n = 2 \quad (3.22)
\]

In calculating the value of \( \mathcal{F} \) for the same body at \( \xi = -0.23 \), it is seen from the "table of signs" that \( f_1 \) is equal to the negative of the value on the \( n = 2 \) curve of Plate II for \( |\xi| = 0.23 \), and \( f_0 \) is the negative of the value on the \( n = 2 \) curve of Plate I at \( |\xi| = 0.23 \). Thus:

\[
\mathcal{F} = \frac{2\mu L}{h^2} \left\{ -0.856 C_4 - 0.412 C_5 \right\} \quad (3.23)
\]

Similarly, any of the quantities listed in Table III may be calculated.

(ii) **An explicit form for the derivatives of the field over a thick, polarized sheet.**

Referring to Table VII, the expression for the derivatives of the field over a thick dipping dike or a sloping step are expressed as:
VALUES OF THE FUNCTION $f_1$

TABLE OF SIGNS FOR $f_1$
values on curves to be prefixed by the following signs

<table>
<thead>
<tr>
<th>$n$</th>
<th>$p_0$</th>
<th>$y_{c0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>2</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>
\[ D^n\mathcal{J} = I \sin d\left\{ C_8 \sin^n \theta \sin n\theta + C_7 \sin n\theta \cos n\theta \right\}^2 \]  

(3.24)

This is convenient for calculating values of the derivatives, as outlined in the preceding section. However, since the form of the equations alters considerably when the limits are inserted, this must be done before examining their significance.

(a) the thick, dipping dike

For the first derivative

\[ D^1\mathcal{J} = I \sin d \left[ \frac{C_7x + C_8h}{x^2 + h^2} \right]^2 \]  

(3.25)

Inserting the limits

\[ D^1\mathcal{J} = I \sin d \left[ \frac{C_7(x+b)+C_8h}{(x+b)^2 + h^2} - \frac{C_7(x-b)+C_8h}{(x-b)^2 + h^2} \right] \]

\[ = -I \frac{2b \sin d}{h^2} \left[ \frac{C_7x^2 + 2C_8xh - C_7(h^2 + b^2)}{x^4 + 2(h^2 - b^2)x^2 + (h^2 + b^2)^2} \right] \]

(3.26)

\[ = -2bC_8I \sin d \left[ \frac{\Lambda_2 - \Lambda_1}{\xi^4 + 2(1-\beta^2)\xi^2 + (1+\beta^2)^2} \right] \]

(3.27)

where \( \xi = x/h, \beta = b/h \) and \( \Lambda_2 = \Lambda_1 \tan d + 2, \Lambda_1 = 2(1N+nL) \)

(3.27)

Applying (16) of Appendix I, we obtain further that:

\[ D^2\mathcal{J} = I 2b \sin d \left[ \frac{2C_7x^5 + 6C_8hx^4 - 4C_7(h^2 + b^2)x^3 + 4C_8h(h^2 - b^2)x^2}{(x^4 + 2(h^2 - b^2)x^2 + (h^2 + b^2)^2)^2} \right] \]

(3.28)
(b) the sloping step

Applying the corresponding limits to (3.25) for this body,

\[ DJ = I \sin d \left[ \frac{C_7(x+b) + C_8(h+b \cot d) - C_7(x-b)+C_8(h-b \cot d)}{(x+b)^2 + (h+b \cot d)^2} \right] - \left[ \frac{C_7(x-b)+C_8(h-b \cot d)}{(x-b)^2+(h-b \cot d)^2} \right] \]

\[ = 2bI \sin d \left[ (C_8-C_7 \cot d) x^2 - 2(C_8 \cot d-C_7)xh + (C_7 \cot d-C_8)h^2 \right] - \frac{+b^2 \csc^2 d (C_7 \cot d + C_8)}{x^4 + 2(h^2+b^2 \cot^2 d)x^2 - 8b^2 \cot d x+2b^2(b^2 \cot^2 d-h^2)} \]

\[ = 2(C_8-C_7 \cot d)bI \sin d \left[ \frac{\xi^2 + \Lambda_1 \xi + (\beta^2 \Lambda_3 - 1)}{\xi^4 + 2(1+\alpha)\xi^2 - 8\beta^2 \cot d - 2\beta^2} \right] \]

where \( C = 1 - \beta^2 \cot^2 d \)

\[ \Lambda_3 = - (\Lambda_1 \cot d + \cot^2 d - 1) \]

4. METHODS OF SOLUTION

A system of equations for the magnetic force over several bodies has been derived in terms of a number of new parameters, and a method of calculating the values of the field and its derivatives over a specified body has been given.

It is now necessary to discuss methods of applying these to magnetic maps, when the field is given but the body is unknown, so that they may be used to extract geological information from magnetic data.
Consider the latter to be given in the form of a contour map of magnetic field values, corrected for all other influences and fields, except those due to magnetized geological formations and structures. From this it is possible to determine in many cases the position of various bodies relative to the plane of observation, their size, shape, average intensity of magnetization and average direction of polarization.

If the values of \( \mathcal{F} \), observed at points in the horizontal plane, were represented on a vertical axis, an undulating surface would result representing the field values. Information about the bodies connected with the various anomalies on the map may be obtained by fitting the theoretical expressions for the fields due to particular bodies, either to the three-dimensional surface for \( \mathcal{F} \), as is done for example by Vacquier [1951], or as is more common, to profiles obtained from the surface by taking vertical sections in various directions. This latter, the method of profiles will be adopted here.

The most distinctive points on a profile are the maxima and minima. These mark the location of anomalies, and separate one from the other. The minima however are not usually reliable points, being subject to the disturbing influences of neighboring bodies. The maxima, on the other hand, are among the most reliable points on the profile. Their position are given by the condition that:

\[
\frac{d \mathcal{F}}{dx} = 0. \tag{4.1}
\]
The maxima and minima are flanked by inflection points, and those on each side of the peak are often easy to determine. They lie on the zero line on second derivative maps [Elkins 1951], and such maps are commonly constructed, especially in connection with aeromagnetic surveying. Being near the central portion of the anomaly, these points, like the maxima, are relatively free - (especially in the aeromagnetic anomalies) - from the disturbing influence of neighboring bodies. Their location is found by solving the equation:

\[ \frac{d^2 \tau}{dx^2} = 0. \] (4.2)

The peaks of the second vertical derivative are easily located on maps of this quantity, and the condition for these points is:

\[ \frac{d^3 \tau}{dxdz^2} = 0. \] (4.3)

For infinitely long bodies of uniform magnetization, \( \frac{d^2 \tau}{dx^2} = \frac{d^2 \tau}{dz^2} \) by Laplace's equation, (4.4)

so that equation (4.3) becomes:

\[ \frac{d^3 \tau}{dx^3} = 0. \] (4.5)

Since expressions for these derivatives have been obtained, conditions such as (4.2), (4.3) and (4.5) result in considerable simplification, treatment of these special points is a suitable beginning in applying the new parameters to magnetic interpretation.

a. Special points for the bodies of Table III
The expressions for \( \mathcal{F} \) are obtained from one of the derivatives of \( F = \alpha \tan^{-1} \frac{x}{h} + \beta \log_e (x^2 + h^2) \), and the special points from

\[ D^n_F = 0. \quad (4.6) \]

From the trigonometric form of \( D^n_F \),

\[ \theta_n = \frac{1}{n} \tan^{-1} (-2/\lambda), \quad \text{where} \quad (4.7) \]

\( \theta_n \) is the value of \( \lambda \) corresponding to \( x_n \), the position of the special point in question. \( \lambda = \frac{\alpha}{\beta} \) and has the value \( C_5/C_4 \), or \( 2C_8/C_7 \) depending on whether the body is a horizontal line of dipoles, or whether it is one of the three sheet-like bodies.

In Plate III, a graph of the principal values of \( \tan^{-1} (-2/\lambda) \) is plotted against \( \lambda \), from which the roots of \( D^n_F = 0 \) may be obtained for any value of \( n \). For example, when \( \lambda = 1 \), the principal value of \( \tan^{-1} (-2/\lambda) \) may be read from this curve as \(-1.107\). Other values are \(-1.107 - \pi \), \(-1.107 - 2\pi \), \(-1.107 - 3\pi \) etc. If \( n = 3 \), for example, 3 values of \( \theta \) satisfying (4.7) lie in the range 0 to \( \pi \). These are \( \theta_3 = -0.369, -1.416, \) and \(-2.463 \), hence \( \zeta_3 = x_3/h = \cot \theta_3 = -2.58, -0.16 \) and \( 1.24 \). For the line of dipoles these would represent the position of a minimum, the peak and the other minimum respectively.

(i) Infinite line of dipoles and thin dipping dike

Referring to Table IV, for these two bodies, putting \( n = 1 \) (4.7) gives for the dipping dike

\[ \frac{1}{\Lambda_2} = \zeta_1 \quad (4.8) \]

which is the value of \( \zeta = x/h \) at which the profile intersects base level (\( \zeta_0 \)).
PRINCIPAL VALUES OF $n\theta$ WHEN DERIVATIVES OF THE FUNCTION $F$ EQUAL ZERO
For \( n = 2 \),
\[ \theta_2 = \frac{1}{2} \tan^{-1} \left( \frac{-2}{\gamma} \right) \]  
(4.9)

For the thin dipping dike, this is the condition for obtaining \( \xi \) at the maxima or minima (\( \xi_M \) or \( \xi_m \)), while for the line of dipoles, it gives the points where the profile cuts base level (\( \xi_0 \), primes marking points for \( \xi < 0 \)).

(ii) the thick dipping dike and the sloping step

The conditions for special points on profiles over these bodies are most easily obtained from the explicit forms given in equations (3.26), (3.30) and (3.31).

Considering only conditions for maxima and minima, for the thick dipping dike:
\[ \Lambda_2 \xi^2 + 2 \xi - \Lambda_2 (1 + \beta^2) = 0 \]  
(4.10)

and for the sloping step:
\[ \xi^2 + \Lambda_1 \xi + (\beta^2 \Lambda_3 - 1) = 0 \]  
(4.11)

Solving the latter equation
\[ \xi = -\frac{\Lambda_1}{2} \pm \sqrt{\frac{\Lambda_1^2}{4} + (1 - \beta^2 \Lambda_3)} \]  
(4.12)

the positive sign before the root giving \( \xi_M \) and the negative sign \( \xi_m \).

Thus:
\[ \xi_M - \xi_m = \sqrt{\frac{\Lambda_1^2}{4} + 4 \beta^2 (\Lambda_1 \cot d + \cot^2 d - 1)} \]  
(4.13)

and from this,
\[ \Lambda_1 = -2 \beta^2 \cot d \pm \sqrt{4 \beta^4 \cot^2 d + (\xi_M - \xi_m)^2 - 4 - 4 \beta^2 (\cot^2 d - 1)} \]  
(4.14)
This last formula may be used to calculate $\Lambda_1$ and hence the direction of polarization of a step of known dimensions and position, when the positions of the maximum and minimum are also known.

Special cases:
When $d = 90^\circ$, cot $d = 0$ and (4.12) becomes
\[ \xi = -\Lambda_1 \pm \sqrt{\frac{\Lambda_1^2}{4} + (1 - \beta^2)}, \text{ from which} \]
\[ \Lambda_1 = \pm \sqrt{(\xi_M - \xi_m)^2 - 4(1 - \beta^2)} \] (4.15)

When $d = 45^\circ$, cot $d = 1$ and (4.12) becomes
\[ \xi = -\Lambda_1 \pm \sqrt{\frac{\Lambda_1^2}{4} + (1 + \Lambda_1 \beta^2)}, \text{ from which} \]
\[ \Lambda_1 = -2\beta^2 \pm \sqrt{4\beta^4 + (\xi_M - \xi_m)^2 - 4} \] (4.16)

5. **CALCULATION OF PARAMETERS OF BODIES**

The equations developed in the preceding sections may be used to calculate various unknown parameters of certain bodies from their magnetic profiles. Methods of doing this will be illustrated in the case of the infinite line of dipoles, the thin dipping sheet, and the sloping step.

For each of these bodies, the polarization function $\Lambda_1$, the dip $d$ of any surfaces bounding the body, their depths and magnetic moment per unit length along the strike combine in a way typical of the particular body to determine the size and shape of the anomaly.
a. The polarization function

This function depends upon the direction of polarization, the direction of measurement and the angle at which the magnetic meridian cuts the strike of the body. It is most simply expressed in terms of direction cosines, and as defined in section (2),

\[ \Lambda_1 = \frac{2(1N + L_n)}{nN - L_L} \]  

(2.23)

l and n are always known and \( \Lambda_1 \) is determined from the anomaly, leaving L and N as the unknown quantities. The relation of these to the direction of polarization is shown in figure (5.1)

\[ L = \cos \ i_p \sin \ a_p \]
\[ M = \cos \ i_p \cos a_p \]
\[ N = \sin i_p \]

For infinitely long, uniform bodies, \( M = 0 \). This is the case for all the bodies treated in this section.
L an N are then not independent, for

\[ L^2 + N^2 = 1 \]  \hspace{1cm} (5.2)

The calculation of \( L \) an \( N \) from \( A \) is facilitated by defining a quantity

\[ K = \frac{\Lambda_1 n - 2l}{\Lambda_1 l + 2n} \]  \hspace{1cm} (5.3)

Substitution into (2.23) gives

\[ N = \frac{1}{\sqrt{1 + K^2}}, \quad L = \frac{K}{\sqrt{1 + K^2}} \]  \hspace{1cm} (5.4)

A graph of \( N \) and \( L \) against \( K \) (Plate IV) is given for use in calculating these quantities.

b. Calculation of the parameters

(i) The infinite line of dipoles (refer to figure 2.2 and equation (2.22))

The direction of polarization, which can be represented by \( \Lambda_1 \), the depth \( h \) and the magnetic moment per unit length \( \mu_L \), are the unknown parameters.

The following three quantities, observable on the profile, are sufficient to determine the unknowns:

\[ x_{M} - x_{I} = w', \quad x_{M} - x_{I} = w, \quad F_{M} - F_{I} \]

(or \( F_{M} - F_{I}' \)). These are shown in figure 5.2.

![Figure 5.2 - Quantities observed on profile.](image)
Referring to equations (4.10) and (4.11) \( x_M \) is given by one of the solutions of
\[
\tan 3\Theta = -\frac{2}{\Lambda_1}, \text{ where } \cot \Theta = \frac{x}{h} \tag{5.5}
\]
and \( x_I \) and \( x_J \) are two of the solutions of
\[
\tan 4\Theta = -\frac{2}{\Lambda_1} \tag{5.6}
\]
Consequently, \( \frac{w}{h} \) and \( \frac{w'}{h} \) are functions of \( \Lambda_1 \) alone, as is \( \frac{w}{w'} \). All these quantities may be calculated from the equations above, and are shown in Plate V plotted against \( \Lambda_1 \). It should be noted that \( w \) and \( w' \) are so related that the value of one for \( -\Lambda_1 \) is equal to that of the other for \( +\Lambda_1 \). Thus \( \frac{w}{w'} \) for \( -\Lambda_1 \) is equal to the inverse of the value of \( \frac{w}{w'} \) at \( \Lambda_1 \).

\( \frac{w}{w'} \) can be calculated from the profile, and the corresponding value of \( \Lambda_1 \) obtained from the graph. Knowing \( \Lambda_1 \), the graphs supply values of \( \frac{w}{h} \) or \( \frac{w'}{h} \); since \( w \) and \( w' \) are known from the profile, \( h \) can thus be calculated.

From equation (3.22) for an infinite line of dipoles it follows that
\[
\mathcal{F}_M - \mathcal{F}_I = \frac{2\mu_L C_4}{h^2} \left\{ (f_1)_M - (f_1)_I + \frac{\Lambda_1}{2} \left[ (f_0)_M - (f_0)_I \right] \right\} \tag{5.7}
\]
Since \( \mu_L \) is the only unknown, it can be calculated from the above equation.
DIRECTION COSINES OF POLARIZATION VECTOR FOR BODIES WHERE $M = 0$

$$N = \frac{1}{\Delta + K^2} \quad L = \frac{K}{\Delta + K^2} \quad \frac{N}{L} = \tan \varphi \quad \text{where} \varphi \text{ is the dip of the magnetization vector below the} \ x-axis$$

$$K = \frac{\Delta n - 21}{2n + \Delta}$$
(ii) The thin, dipping, polarized sheet (refer to figure 2.4. and equation (3.10)).

The direction of polarization, represented by $\lambda_1$, the depth $h$, dip $d$ and the magnetic moment per unit surface area of the sheet, $\mu$, are the unknown parameters.

Exactly the same quantities as for the line of dipoles may be measured on the profile (Figure 5.2). For this body, referring to equations (4.9) and (4.10) $x_M$ is one of the solutions of

$$\tan 2\theta = -\frac{1}{\lambda_2} \quad \text{cot} \theta = \frac{x}{h} \quad (5.8)$$

and $x_1$ and $x_1'$ are two of the solutions of

$$\tan 3\theta = -\frac{1}{\lambda_2} \quad (5.9)$$

These may be used to calculate values of $\frac{w}{h}$, $\frac{w'}{h}$ and $w/w'$, which are functions of $\lambda_2$ alone. These quantities are all shown on Plate VI plotted against $\lambda_2$. These can be used in the same manner as the corresponding graphs for the line of dipoles, to calculate $h$ and $\lambda_2$ for the dipping sheet, given a profile over that body.

Referring to equation (3.12) it follows that for this body,

$$F_M - F_I = \frac{Ic_{g8}}{h} \left\{ (f_{1})_M - (f_{1})_I + \lambda_2 \left[ (f_{0})_M - (f_{0})_I \right] \right\} \quad (5.10)$$

This may be used to calculate $I\mu$.

Since $\lambda_2 = \frac{\lambda_1 \tan d + 2}{\lambda_1 - 2 \tan d} \quad (3.13)$
SHIFT OF PEAK AND INFLECTION POINTS

\[ \Lambda = +\Lambda_2 \text{ for sheet, } \Lambda = -\Lambda_1 \text{ for line} \]

\[ +\xi_I \text{ for } \Lambda > 0 \]
\[ -\xi_I \text{ for } \Lambda < 0 \]

line of dipoles

thin sheet

\[ +\xi_M \text{ for } \Lambda > 0 \]
\[ -\xi_M \text{ for } \Lambda < 0 \]

line of dipoles

thin sheet
SEPARATION OF PEAK AND INFLECTION POINTS

PLATE VI to follow p. 45

\[
\Lambda = \frac{\Lambda_2}{\Lambda_1} \text{ for sheet, } \Lambda = \frac{\Lambda_1}{\Lambda_1} \text{ for line}
\]
both the polarization function and the dip for this body combine jointly to determine the shape of the profile.
Thus unless \( d \) is known from some other source of information \( \Lambda_1 \) and hence the direction of polarization cannot be obtained.

(iii) **Tests for type of body**

Profiles over an infinite line of dipoles and a dipping sheet are very similar (compare figures n. Plate VII and tests to distinguish one from the other are necessary. There are a number of ways in which this may be done.

1. **Height of inflection points and peak**

Referring to equation (3.22) it follows that

\[
\frac{\mathcal{F}_M - \mathcal{F}_I}{\mathcal{F}_M} = (f_1)_M - (f_1)_I + \Lambda_n \left[ (f_0)_M - (f_0)_I \right]
\]

with \( n = 1 \) in the case of a thin, dipping sheet, and \( n = 2 \) for a line of dipoles.

This ratio depends then, on \( \Lambda_1 \) or \( \Lambda_2 \) alone, and may be calculated for various values of these parameters, and plotted as in Plates V and VI. These graphs may be used to determine how closely a profile corresponds to a line of dipoles or a thin dipping sheet when one or the other is suspected. Conversely, if the type of body is known or assumed, the corresponding value of \( \frac{\mathcal{F}_M - \mathcal{F}_I}{\mathcal{F}_M} \) when \( \Lambda_n \) is known, may be used to find the base level of profiles on which only \( \mathcal{F}_M - \mathcal{F}_I \) is known.
2. Other special points and slopes

As further tests to determine whether the type of body assumed corresponds to the profile, other special points and slopes may be calculated from the parameters, and compared with the observed profile.

(i) Zero points and width of anomaly at base level

Putting $\xi$ equal to zero in equation 2.22 gives

$$\xi_0 = \pm \sqrt{\frac{\Lambda^2}{4} + 4} + \frac{1}{2} \Lambda_1$$

and

$$\xi_0 = -\frac{1}{2} \sqrt{\Lambda^2 + 4} + \frac{1}{2} \Lambda_1$$

so that

$$\xi^1 - \xi_0 = \sqrt{\frac{\Lambda^2}{4} + 4}$$

in the case of the line of dipoles.

For the dipping dike, the profile cuts the base level at one point,

$$\xi_0 = -\frac{1}{\Lambda_2}$$

(ii) Slopes at any point on the profile

For the line of dipoles, $(n=2)$, referring to section 3,

$$f_1 = -4 \mu L C_4 \left\{ \frac{f_0 + \Lambda_1 f_1}{2} \right\}$$

and for the thin lipping sheet, $(n=1)$

$$f_1 = -\frac{1}{2} \frac{f_0 + \Lambda_2 f_1}{h^2}$$

These equations may be used to obtain the slope at any point, using the calculated parameters. Comparison with
observed slopes is a test of the accuracy of the interpretation.

6. **EXAMPLES OF CALCULATION OF THE PARAMETERS IN THE CASE OF AN INFINITE LINE OF DIPLODES.**

Plate VII shows a theoretically-calculated profile over an infinite line of dipoles. It is instructive to calculate the parameters of the body from such a "perfect" profile. Assume that the profile is over a body in high magnetic latitudes, where to a good approximation, \( l = 0 \) and \( n = 1 \).

a. **Calculation of depth and polarization function**

Assume that the inflection points have been located as shown. Then:

\[
\begin{align*}
\omega &= 0.68 \text{ miles, } \omega' = 0.60 \text{ miles}, \\
\mathcal{F}_M - \mathcal{F}_I &= 335\sigma, \text{ and } \mathcal{F}_M - \mathcal{F}_{I'} &= 190\sigma.
\end{align*}
\]

Adopting the procedure of section 5,

\[\frac{\omega}{\omega'} = 1.13, \text{ and hence } \Lambda_1 = 1.50. \text{ From this value of } \Lambda_1, \frac{w}{h} = 0.45 \text{ and } \frac{w'}{h} = 0.40.\]

Knowing both \( w \), and \( w' \), two estimates may be made of \( h \). Both of these give \( h = 1.50 \) miles. Also from Plate V

\[
\begin{align*}
\mathcal{F}_M &= -0.22, \mathcal{F}_I = +0.23, \mathcal{F}_{I'} = -0.62, \text{ and thus}
\end{align*}
\]

\[
\begin{align*}
x_M &= -0.33 \text{ miles, } x_I = +0.35 \text{ miles and } x_{I'} = -0.93 \text{ miles.}
\end{align*}
\]

Referring to section (3) and Plates I and II:

At \( \mathcal{F}_M, \mathcal{F} = \frac{2\mu L^4}{h^2} \left\{ -0.866 - 0.75 \times 0.396 \right\}, \frac{2\mu L^4}{h^2} (1.163)\]
Anomaly field (gamma)

PROFILE OVER HORIZONTAL LINE OF DIPOLES

PROFILE OVER THIN, DIPPING, POLARIZED SHEET

PLATE VII to follow p. 48
\[ F_I = \frac{2\mu L C_4}{h^2} \left\{ -0.856 + 0.75(0.412) \right\} = \frac{2\mu L C_4}{h^2} (0.542) \]

and at \( F_I = \frac{2\mu L C_4}{h^2} \left\{ -0.324 - 0.75(0.649) \right\} = \frac{2\mu L C_4}{h^2} (0.810) \]

Thus \[ F_M - F_I = - \frac{2\mu L C_4}{h^2} \left\{ (0.621) \right\} = -335 \gamma \]

and in both cases \[ \frac{2\mu L C_4}{h^2} = -539 \gamma \]

b. Calculation of the polarization

\( \Lambda_1 \) was found to be 1.50, and it was assumed that \( l = 0, n = 1 \). Hence for equation (5.3),

\[ K = \frac{\Lambda_1}{2} = 0.75. \]

From plate IV, \( L = 0.60 \) \( N = 0.80 \)

\[ C_4 = 1L - nN = -0.80. \]

The intensity of magnetization can now be computed from the value of \( \frac{2\mu L C_4}{h^2} \), giving

\[ \mu_L = 5.39 \times \left( \frac{1.5}{2} \right)^2 \times (1.609)^2 \times 10^7 = 1.98 \times 10^8 \]

c.g.s. units.

c. Determination of base level

From Plate V \[ \frac{F_M - F_I}{F_M} = 0.55, \frac{F_M - F_I'}{F_M} = 0.31 \]

for \( \Lambda_1 = 1.5 \). Substituting the observed values of \( F_M - F_I \) and \( F_M - F_I' \), the base level is found to be 610 \( \gamma \) below the peak.
From equation (5.12), $Q$ may be calculated to be $-0.50$, giving $x_0 = -0.75$ miles as observed.

To summarize: from the distances between peak and inflection points on the observed profile, values of a number of parameters have been obtained as follows -

- $\Lambda_1$, polarization function: 1.50,
- $h$, depth to line: 1.50 miles,
- $\{L, O, N\}$ direction cosines of polarization vector $\{0.60, 0, 0.80\}$,
- $\mu L$, magnetic moment per unit length of line: $1.98 \times 10^8$ c.g.s. units.
These are the parameters that were used to calculate the original profile.

d. Calculation of slopes at the inflection points

It is a useful check to compute these slopes from the calculated parameters and compare with the observed values. The inflection points occur at $f_I = +0.23$ and $f_I' = -0.62$.

Referring to equation 5.13 and Plates I and II,

- $Df = 719 \{ -0.75 (-0.725) - 0.580 \} \frac{\gamma}{\text{mile}} = -810 \frac{\gamma}{\text{mile}}$ at $x_I$,
- $Df = 719 \{ -0.75 (-0.054) + 0.616 \} \frac{\gamma}{\text{mile}} = +470 \frac{\gamma}{\text{mile}}$ at $x_I'$.

7. **Example in the case of a thin, dipping, polarized sheet**

a. Calculation of $h$ and $\Lambda_2$

Plate VII shows a theoretically-calculated profile over a thin, dipping sheet with $\Lambda_2 = -0.85$ and $h = 0.50$ miles.
These constants were chosen to produce a curve of similar size and shape to that of Figure 6.1, illustrating the similarity that can exist between profiles over these two bodies.

Assuming that the inflection points have been located as shown,

\[ w' = 0.32 \text{ miles}, \quad w'' = 0.30 \text{ miles} \]

\[ F_M - F_I = 240\gamma, \quad F_M' - F_I' = 105\gamma. \]

As outlined in section 5, the solution proceeds as follows:

\[ \frac{w'}{w''} = 1.07 \quad \text{; consequently} \ A_2 = -0.85 \quad \text{, and} \]

\[ w' / h = 0.65, \quad w'' / h = 0.60. \quad \text{Knowing both} \ w \ \text{and} \ w'', \ \text{two estimates may be made of} \ h. \ \text{Both of these give} \ h = 0.50 \text{ miles. Also Table VI gives} \]

\[ F_M = -0.35, \quad F_I = +0.30, \quad F_I' = -0.95, \ \text{leading to} \ x_M = -0.18 \text{ miles,} \ x_I = 0.15 \text{ miles, and} \]

\[ x_I' = -0.48 \text{ miles.} \]

Referring to section 3 and Plates I and II, at \( F_M \)

\[ F = \frac{ItC_8}{h} \left\{ A_2 f_0 + f_1 \right\} \]

\[ = \frac{ItC_8}{h} \left\{ + 0.85(0.312) + 0.890 \right\} = 1.155 \frac{ItC_8}{h} ; \]

at \( F_I \),

\[ F = \frac{ItC_8}{h} \left\{ -0.85 (0.274) + 0.918 \right\} = 0.686 \frac{ItC_8}{h} ; \]

and at \( F_I' \)

\[ F = \frac{ItC_8}{h} \left\{ + 0.85 (0.499) + 0.527 \right\} = 0.950 \frac{ItC_8}{h} . \]
Thus \( \mathcal{F}_M - \mathcal{F}_I = 0.469 \frac{\text{ItC}_8}{h} \),
\[ \mathcal{F}_M - \mathcal{F}_I' = 0.205 \frac{\text{ItC}_8}{h} \]
and in both cases \( \text{ItC}_8 = 512 \).

b. Calculation of the dip and polarization

Referring to equations 3.12 and 3.13, the shape of the profile depends only on \( h \) and \( \Lambda_2 \). Furthermore, \( \Lambda_2 \) is a function of both the polarization function and the dip. Thus neither of these last two quantities can be determined from the magnetic profile unless the other is known from some independent source of information.

As an example, suppose the dip of the sheet was suspected to be 65° S.

The polarization function in terms of \( \Lambda_2 \) and \( d \) is given by:
\[
\Lambda_1 = 2 \left( \frac{\Lambda_2 \tan d + 1}{\Lambda_2 - \tan d} \right)
\]
thus
\[
\Lambda_1 = 2 \left( \frac{-0.85(2.14) + 1}{-0.85 - 2.14} \right) = 0.55.
\]
If the magnetic latitude and strike of the body are such that \( l = 0.31 \) and \( n = 0.95 \), then \( k = -0.05 \), corresponding approximately to vertical polarization, with \( N = 1 \) and \( L = 0 \). In this case \( C_4 = -0.95 \), \( C_5 = -0.62 \), and \( C_8 = 1.46 \).
Thus \( \text{It}_8 \), the magnetic moment per unit area of face of the sheet is:
It = \frac{512 \times 0.50 \times 1.61}{1.46} = 282 \text{ c.g.s units}

C. Determination of the base level

From Plate VI, \( \frac{F_M - F_I'}{F_M} = 0.41 \) and 
\( \frac{F_M - F_I}{F_M} = 0.18 \) for \( \Lambda_2 = -0.85 \). Substituting the observed values of \( F_M - F_I \) and \( F_M - F_I', F_0 \), the base level is found to be 585\( \gamma \) below the peak. From equation (4.8)
\[ \frac{F_0}{\Lambda_2} = 1.18, \text{ as observed on the profile.} \]

To summarize

From the distances measured on the profile a number of parameters of the body have been found as follows:
\[ \Lambda_2 = -0.85 \]

\( h \), depth to upper edge = 0.50 miles.
Assuming a dip of 65° S, and knowing \( l \) and \( n \), the direction of polarization is given by \( L = 0, N = 1 \), corresponding to a polarization function of 0.55.
The quantity \( F_0 \) is equal to 282 c.g.s units. These are the parameters from which the original profile was calculated.
d. Calculation of slopes at the inflection points.

Referring to section 3,
\[ \mathcal{F}' = - \frac{I t C g}{h^2} \left\{ \sum f_1 + f_0 \right\} \quad \text{for } n = 2 \quad (7.3) \]

For \( \xi = \xi_1 = +0.30, \)
\[ \mathcal{F}' = -1024 \left\{ -0.85(-0.766) + 0.504 \right\} \text{ } \gamma/\text{mile} \]
\[ = -1185 \text{ } \gamma/\text{mile} \]

For \( \xi = \xi_1' = -0.95, \)
\[ \mathcal{F}' = -1024 \left\{ -0.85(-0.026) - 0.525 \right\} \text{ } \gamma/\text{mile} \]
\[ = 515 \text{ } \gamma/\text{mile}. \]

These slopes are observed on the profile.

8. **APPLICATIONS TO THE INTERPRETATION OF AEROMAGNETIC MAPS.**

a. **Anomalies approximating profiles over infinite lines of dipoles or thin, dipping sheets.**

Plate VIII shows the interpreted magnetic trends in a portion of an aeromagnetic map of the La-Plonge area of Saskatchewan, Canada [Geological Survey of Canada, 1952]. Two trends are shown, marking the peaks of anomalies of a type which frequently occurs over one or other of the bodies treated in this section.

The profile along a traverse ABC, chosen so that it crosses the bodies perpendicular to the strike is also shown in Plate VIII. The two peaks are numbered 1 and 2 respectively, and the inflection points flanking these, as determined by numerical analysis, are marked.
(1) **Analysis of anomaly 1 as being due to a line of dipoles**

If this is due to a line of dipoles, then since \( w = w^1 = 0.38 \) miles, \( \Lambda_1 = 0 \), \( w/h = w^1/h = 0.42 \) (Plate VI). Consequently \( h = 0.91 \) miles. For \( \Lambda_1 = 0 \), \( \frac{F_M - F_I}{F_M} = \frac{F_I - F_{I'}}{F_M} = 0.40 \). Since the inflection points are \( 82^\circ \) below the peak, \( F_M = 205^\circ \).

Writing \( \frac{\mu L C_4}{h^2} = k \) in (5.7), we have when \( \Lambda_1 = 0 \):

\[
\frac{F_M - F_I}{F_M} = k \left[ (f^1)_M - (f^1)_I \right].
\]

From the curve for \( n = 2 \), Plate II, \( (F_M - F_I) = k (1 - 0.60) \) since the peak is at \( \xi = 0 \) and \( \xi_I = w/h = 0.42 \).

Thus \( k = \frac{82}{0.40} = 205^\circ \), equal to \( F_M \) in this case.

As a test of the accuracy of the parameters obtained, the values \( F \) at various points on the traverse may be computed. Substituting the value of \( k \) obtained above into (5.7) for \( \Lambda_1 = 0 \),

\[
F = 205 f_1(\xi) \quad \text{for} \quad n = 2, \quad \text{where} \quad \xi = x/h = x/0.91. \quad (8.1)
\]

Values of \( F \) calculated from (8.1) are plotted on the profile in Plate VIII. \( \Lambda_1 = 0 \) means that \( L = 0, N = 1 \), and that polarization is vertical.

(2) **Analysis of anomaly 2 as due to a line of dipoles**

From a numerical analysis of the profile, there are three points which might possibly be inflection points. That to the south, numbered 1 is definite, while 2 and 3 mark the possible limits of the one to the north, obscured perhaps by a minor trend paralleling the main one. Assuming that there actually exists a single inflection point on the north, obscured by a minor
trend, it is possible to obtain first an approximate solution, and then to follow this with an adjustment of the parameters obtained to give more exact values.

Inflection point 1 is situated so that the ratios between \( w \) and \( w_1 \), and \( F_M - F_I \) and \( F_M - F_I' \) match those expected if the profile were due to a line of dipoles or a thin, dipping sheet, if the other inflection were at point 4 as marked.

Assuming that the body is a line of dipoles, \( w = 0.60 \) miles, \( w_1 = 0.52 \) miles, \( w/w_1 = 1.15 \) and from Plate VI, \( A_1 = 1.75 \), \( w/h = 0.46 \), \( w_1/h = 0.40 \). Referring to Plate V, \( \xi_M = -0.25 \), \( \xi_I' = -0.65 \), \( \xi_I = 0.22 \). Thus \( h = 1.30 \) miles and \( x_M = 0.30 \) miles. These are first approximations and give:

\[
F = k \left\{ 0.88 f_0(\xi) + f_1(\xi) \right\} \quad \text{for} \quad n = 2 \quad (8.3)
\]

where \( \xi = \frac{x}{1.30} \)

\( F_I \) is a reliable point and for this value, \( \xi_I = 0.22 \), \( F_I = -0.53 k \). \( F_o - F_I = k (-1 + 0.53) = -0.47k \), where \( F_o \) is the value of the field at \( x = 0 \). \( F_o - F_I = 70 \xi \), thus \( k = -149 \xi \); the base level is 149 \( \xi \) below \( F_o \) at 1690 \( \xi \).

To adjust the parameters, choose two widely-separated points on the profile and proceed as follows. Consider points \( x \) and \( -x \), and the corresponding field values \( F^+ \) and \( F^- \).

\[
F^+ = k \left\{ \frac{A_1}{2} f^+_0 + f^+_1 \right\} \quad (8.4)
\]

and \( F^- = k \left\{ \frac{A_1}{2} f^-_0 + f^-_1 \right\} \)

writing \( \frac{F^+}{F^-} = x \), it follows that

\[
\frac{A_1}{2} = \frac{f^+_1 - x f^-_1}{f^+ - x f^-} \quad (8.5)
\]
Two successive applications of (8.5) gives the corrected value of $\Lambda$ to be 1.10. Corresponding to this, $w/h = 0.44$ and $w^1/h = 0.40$, giving from $w/h$ that $h = 1.36$ miles. Thus the corresponding value of $w^1$ is 0.54 miles, $\mathcal{F}_M = -0.18$, $\mathcal{F}_I = 0.27$, $\mathcal{F}_I' = -0.58$, and $x_M = -0.25$ miles. The adjusted value for $\mathcal{F}_I$ is $-0.55 k$ and $\mathcal{F}_o - \mathcal{F}_I = -0.45 k = 65 \gamma$, hence $k = -144 \gamma$, giving base level at 1720 $\gamma$.

Thus $\mathcal{F} = -144 (0.55 f_o (\mathcal{F}) + f_1 (\mathcal{F}))$ for $n = 2$  

where $\mathcal{F} = \frac{x}{1.36}$.

Values of $\mathcal{F}$ calculated from (8.6) are shown on the profile in Plate VIII, as well as the composite curve for both bodies.

3. Analysis of the anomalies as due to thin, dipping sheets

A preliminary trial shows that each anomaly can also be fitted by the theoretical curve over a thin, dipping sheet. As in the case of assuming lines of dipoles, these predict about equal base levels for both anomalies, about 100 $\gamma$ below those obtained when assuming lines of dipoles. It is reasonable then, to assume that the anomalies are due to either two lines of dipoles or two dipping sheets, rather than to one of each.

In the analysis assuming lines of dipoles it was possible to treat each anomaly separately, since the field due to such bodies drops off sufficiently rapidly with distance, that neighboring bodies do not materially distort the anomalies. For sheets, however, as can be seen from Plates I and II, the field drops off much more slowly with distance, and for anomalies as close together as those treated in the present section, correction must be made for the effect of the neighboring body.
As a start of the solution, let us find the thin, dipping sheet which is the best-fit to anomaly 2, ignoring the effect of the other sheet. For \( w = 0.60 \) miles, \( w' = 0.54 \) miles, we obtain \( \Lambda_2 = -0.55 \). For the dipping sheet, the equation for adjustment of the parameters, corresponding to (8.5) is:

\[-\Lambda_2 = \frac{f_1^+ - Xf_1}{f_0^+ - Xf_0} \quad \text{for} \ n = 1 \] (8.7)

Making adjustment, allowing for the effect of the sheet giving best-fit to anomaly 1, and then adjusting the latter so as to allow for the sheet connected with anomaly 2, and so on may be continued as an iteration, which converges to the following values of the parameters for the sheets: for anomaly 2, \( \Lambda_2 = -0.45 \), \( h = 0.98 \) miles, and \( x_M = -0.23 \) miles; for anomaly 1, \( \Lambda_2 = 0 \), \( h = 0.53 \) miles. The profile over the bodies, and the predicted base levels are shown on Plate VIII.

4. Comparison of the interpretations

In the area under consideration, the inclination of the earth's magnetic field is 79°N, and the declination is N21°E. This is the direction of the component of the field measured by the airborne magnetometer. Thus in Figure 2.2, \( i = 79^\circ \) for both anomalies; for anomaly 1, \( a = 10^\circ \) and for anomaly 2, \( a = 0^\circ \).

a. Anomaly 1 as due to a line of dipoles

\( \Lambda_1 = 0 \), \( h = 0.91 \) miles. The peak is vertically above the line. \( l = \cos 79^\circ \sin 10^\circ = 0.033 \), \( n = \sin 79^\circ = 0.982 \). Thus \( K = 0.07 = 0.036 \), and referring to Plate IV, \( L = 0 \), \( N = 1 \), and 1.96 polarization is roughly vertical. Base level is calculated to
be at 1705°.

b. Anomaly 2 as due to a line of dipoles -

\[ \Lambda_1 = 1.10, \ h = 1.36 \text{ miles}. \]  The peak is shifted 0.25 miles to the southeast of the line. \( l = \cos 79^\circ \sin 0^\circ = 0, \ n = \sin 79^\circ = 0.982. \) Thus \( K = 1.10 \left( 0.982 \right) = 0.55. \) From Plate IV, \( L = 0.48, \ N = 0.87, \) and \( \varphi = 61.1^\circ. \) Base level is calculated to be at 1720°.

c. Anomaly 1 as due to a thin, dipping sheet

\[ \Lambda_2 = 0, \ h = 0.53 \text{ miles}, \]  and the peak is directly above the top of the sheet. Base level is calculated to be at 1607°.

\[ \Lambda_2 = 0, \]  referring to (3.13), means that \(-2/\Lambda_1 = \tan d.\) Measurement along the earth's field (as with the airborne magnetometer) means that \( l \approx 0 \) and \( n \approx 1, \) making \( \Lambda_1 \approx \frac{2L}{N}. \) Thus \(-\tan d = \tan \varphi, \) (where \( \varphi \) is as defined in Plate IV), and the polarization is directed downwards along the dip of the sheet.

d. Anomaly 2 as due to a thin, dipping sheet

\[ \Lambda_2 = -0.45, \ h = 0.98 \text{ miles}, \]  and the peak is shifted 0.23 miles to the southeast of the top of the body. Base level is calculated to be at 1607°. For vertical polarization, as an example, \( L = 0, \ N = 1. \) Since for this body, \( l = 0 \) this would mean that \( \Lambda_1 = 0. \) By rearranging (3.13) we see that

\[ d = \tan^{-1} \left\{ \frac{\Lambda_1 \Lambda_2}{2\Lambda_2 + \Lambda_1} \right\}, \quad (8.8) \]

and we would have

\[ d = \tan^{-1} \left( \frac{-2}{-2 \times 0.45} \right) = 65.8^\circ. \]
Thus if the body were polarized entirely by induction in the earth's field, it would be dipping 65.8°SE to give $\Lambda_2 = -0.45$.

e. Summary and most likely interpretation

Near the peaks of the profiles, both bodies give an equally close fit. The predicted profiles differ at the outer parts of the anomaly, however, and in the values of the base levels that would result from each assumption. This affords a possibility of distinguishing between these two proposed interpretations.

The profiles corresponding to the dipping sheets combine to fit the observed profile very closely, and the predicted base level of about 1600$\gamma$ is close to that observed nearby in areas relatively free of anomalies. It appears that this interpretation is the more likely.

9. THE GEOLOGICAL SIGNIFICANCE OF MAGNETIC DATA - AN INTERPRETATION OF AN AIRBORNE SURVEY OVER TEXADA ISLAND, B.C.

The mathematical models of the preceding sections are based on the assumption that the fields observed over magnetized formations reflect geological conditions. Areas where good magnetic surveys exist, and where significant comparisons can be made with geological data afford a test of this hypothesis, and a guide in applying quantitative methods of interpretation.

In February, 1957, the British Columbia Department of Mines released Map A.M.57-3, an aeromagnetic survey of a portion of Texada Island, B.C. As conditions are favourable in this area
for the observation of the geology, and since the original magnetometer traces and flight data were available at the Department of Mines, it was decided to use this survey as a test both of the degree to which geological information can be reflected in magnetic data and of the applicability of some of the methods of interpretation developed in previous sections.

The contours of equal magnetic intensity shown on the map may be taken to represent the attractions of magnetized rocks at and below the surface of the ground. In detail, these form closures - localized "anomalies" or area of high or low field, reflecting local concentration or other variations in the magnetization of the rocks. On a broader scale, they divide into zones, each having a characteristic average level of field. The entire area is divided into magnetic zones on this basis, and the divisions between zones are considered to represent lateral discontinuities in magnetic properties. Some of these postulated discontinuities are tested using magnetic profiles across them, and analyzed by the methods developed in the previous sections. Finally, comparison is made with the known geology of the island, and with the results of a structural study of the area carried out with the aid of aerial photographs.

When such zoning of magnetic properties is observed to occur, it is likely to have considerable geological significance. Quite apart from the general geological environment, many factors such as the intrinsic magnetic moments of the various ions composing the minerals, the crystal structure, the past states of temperature, pressure and stress all influence the amount, kind,
and stability of the magnetization. It would thus be surprising if the properties peculiar to each region were not reflected also in its magnetic constitution. Experience with magnetic surveys shows that this is true in very many cases.

Texada Island is a locality where the geological structure is well expressed in topographic features: differing rock types occupy areas of differing elevation; faults and other fractures are clearly visible, there are many exposed areas where the trends of the formations may be plotted from the air. Under such conditions it has become widespread practice to obtain much of the geological information from a study of aerial photographs. Such studies have been found to be particularly valuable in correlating aeromagnetic trends with geological formations, and correlations of this type were found in the present study.

Some of the main magnetic zones are bounded by major linear features determined from the aerial photographs. Smaller linear features are often followed by minor trends in the magnetic values. Thus an airphoto study can possibly supply a geological explanation of some of the magnetic trends.

a. **Analysis of the magnetic trends**

Most of this analysis was carried out on data obtained from the aeromagnetic map, which presents magnetic contours on a scale of \( \frac{1}{2} \) mile = 1 inch, and with a contour interval of 25\%. The map was made from continuous magnetic profiles with a line spacing of \( \frac{1}{4} \) mile and with 500 feet ground clearance, recorded with a fluxgate magnetometer towed behind a fixed wing aircraft.
Anomalies of particular interest were studied directly on the original magnetometer and radio altimeter records, made available by the kind co-operation of Dr. H. Sargent, Chief Mineralogist, British Columbia Department of Mines.

It is possible to divide magnetic trends and anomalies into four main groups as follows: large, distinct anomalies with height exceeding 500$\gamma$; smaller anomalies in the range 200-500 $\gamma$; smaller anomalies still, with height below 200$\gamma$; and finally, straight, continuous lows, often apparently isolated, but usually associated with the larger anomalies. Trends were divided according to this criterion, those over 500$\gamma$ height being called "first order anomalies", those with height in the range 200-500$\gamma$, "second order anomalies", and those below 200$\gamma$ - "third order anomalies".

(i) The first order anomalies

These fall into long, even, narrow trends, interrupted only by occasional cross displacements. This may be taken as an indication of a system originally uniform, but later complicated by faulting. The first-order trends lie in two belts, one near the northeast coast of the island, and averaging a 4000$\gamma$ field at the peaks with a standard deviation of 450$\gamma$ among the highs along its length. This is shown as Zone I on maps 3 and 4. The second belt of first-order anomalies lies near the southwest coast of the island and averages a 3500$\gamma$ field at the peaks, with a standard deviation of 400$\gamma$. This is shown as Zone III. These are separated by a belt, marked as Zone II on the maps, which is free of major anomalies.
The boundaries between the zones are paralleled by remarkably continuous and regular belts of lows averaging a 3000\(\gamma\) field, with standard deviation of 400\(\gamma\). An examination of the northern belt indicated that two systems of sharp, straight boundaries running east-west and north-south respectively cut through the belts and further subdivided them. These subdivisions are shows as Ia, Ib, Ic, Id, and Ie.

(ii) Second and third order anomalies

These add detail to the zones and from the basis of final divisions into regions of differing magnetization, as shown on Map 4.

(b) Analysis of structural trends

Vertical aerial photograph coverage on a scale of ¼ mile = 1 inch was obtained, and the whole area studied under a stereoscope. Many fractures, straight scarps and other features suggestive of a fracturing pattern were visible, and these were plotted, using the standard criterea and techniques of analysis of fracturing patterns from airphoto linears, such as those employed previously by the author (Hall, 1950) (see also Wilson, 1948), and used in recent years as preliminary data in planning Geological Survey mapping. This pattern is shown on Map 2. Comparison with Map 1 shows a great similarity between this pattern and the divisions made on the basis of the magnetic map, and suggests certain ideas regarding the structural history of the region, which are outlined in the following section.
(c) **Comparison of magnetic and structural data**

The most striking feature of the two sets of data is the direction of trends. The major boundaries between the different magnetic regions are paralleled by structural trends, and the directions of minor magnetic trends within the regions are frequently paralleled by the smaller linears. There is no doubt that the divisions appearing on the magnetic map reflect geological boundaries, and that magnetic contrasts, representing segregation of magnetization in varying degrees from one geological division to the next, are reflected in the magnetic field values. This correlation with rock type is remarked on in a publication describing ground checks of some of the anomalies, that were made by geologists from the *British Columbia Department of Mines* [1958]. This gives support to the idea that significant geological information may be obtained by calculating the magnetization of various bodies and zones from the anomalies over them.

The major trends suggest the following structural history. The three major belts shown in Map 2 were originally continuous. Subject to thrusting from the southwest, they were displaced along a set of transcurrent faults in a north-south and east-west direction, suffering a displacement of west side north especially along a line between Vanada and Welcome Bay and again on a line between Davie and Mouat Bays. This would mean that the strong magnetic trend running on the south side of the island is displaced first at Welcome Bay and then again at Mouat Bay, until it runs into the straits of Georgia in the vicinity of Davie Bay.
Similarly the magnetic trend from Limekiln bay to Priest Lake is displaced so as to run on the south side of Comet mountain, to the south side of Mount Davies and ultimately into Sabine channel north of Jedediah island.

The above analysis was based on theories of fracturing and displacement as put forward by Wilson [1948], which appears to explain the observed pattern satisfactorily. An alternative theory, put forward by Moody and Hills [1956] was also tried as a possible explanation but without success.

The possibility of such a structural dislocation is confirmed by the topography of the island, as show in Map 5. Variations in topography reflect the divisions made on the basis of the magnetic and structural trends. For example, the southern boundary of Zone I from Comet mountain south is followed closely by the 1000-foot contour; the highland on the north west tip of the island falls in magnetic division IIIa; and Zone II is followed by a strip of low land.

(d) Quantitative calculations for magnetic profiles

The foregoing qualitative comparisons leave no doubt that the magnetic contours contain significant geological information. Certain interesting points are raised, which can be tested by quantitative analysis of the magnetic values. This will help settle some points raised in the interpretation, and will serve to indicate the value of the methods of calculation developed in the earlier sections, when applied to actual cases.
(i) **Magnetic high over Comet mountain**

The two magnetic highs over this mountain are the highest on the sheet, and magnetic Zone Ib (Map 3) comprising them is the most magnetic of all the divisions. The boundary between Ib and Ic constitutes a major magnetic division, apparently connected with anomaly east of Pocohontas Bay. This rises to 4924\(\gamma\) and has the appearance of an anomaly originating from a pair of sloping steps, as shown in Plate IX. This figure shows also the topographic profile along flight line 21, which was flown with a constant ground clearance of 500 feet as checked with the radio altimeter record. Consequently the airborne traverse is as shown, and in relation to the topographic profile, is the equivalent of a horizontal traverse over a pair of sloping steps, the south-east one coinciding with the major boundary mentioned above. This boundary is represented on the aerial photographs by a prominent linear. The step to the north-west is sufficiently far removed not to affect values over the one at the boundary, which we are interested in, as was found by trial calculation. Consequently, the latter step may be treated independently.

In previous sections the following equation was obtained for the force field over a step:

\[
\mathbf{F} = I \sin d \left[ \frac{C_7}{2} \log_e(x^2+h^2) + C_8 \tan^{-1} \frac{x}{h} \right] \right]_1^2
\]

and from equation (3.31) the condition for maxima and minima over the profile may be obtained, viz:
PLATE IX

TOPOGRAPHIC PROFILE OVER BOUNDARY BETWEEN
I-b AND I-c

CROSS SECTION OF POSTULATED BODY,
CALCULATED PROFILE, AND OBSERVED VALUES
(EAST OF COMET MOUNTAIN, TEXADA ISLAND, B.C.)
(section taken along flight line 21, northwest of fiducial 409)
\[ \xi^2 + \Lambda_1 \xi + (\beta^2 \Lambda_3 - 1) = 0 \]  
(9.1)

where \( \Lambda_3 = -(\Lambda_1 \cot d + \cot^2 d - 1) \), \( \beta = b/h \), and \( \xi = x/h \).

A step with the observed thickness of the topographic feature will not produce an anomaly with the observed positions of maximum and minimum for any value of \( \Lambda_1 \). This may be seen from Plate IX: no value of \( \Lambda_1 \) will result in both \( \xi_M \) and \( \xi_m \) satisfying this equation.

If the boundary were the surface trace of a magnetic boundary going to depth, the anomaly would be the result, approximately, of the magnetic attraction of a thicker step. A few trials for the thickness show that for \( b = 550' \), values of \( \xi_M \) and \( \xi_m \) result which both satisfy (9.1) for \( \Lambda_1 = 0.68 \), the latter value corresponding to \( L = 0.19 \), \( N = 0.98 \), a direction of polarization approximately that of induction in the earth's field. On this assumption, the field values at various positions of the body can be calculated.

In equation (3.29), let:

\[ X = \frac{x - b \cot d}{h - b}, \quad Y = \frac{x + b \cot d}{h + b} \]

then, inserting the limits,

\[
\mathcal{F} = I \sin d \left\{ \frac{C_7}{2} \log_e \frac{X^2 + 1}{Y^2 + 1} + C_7 \log_e \frac{h - b}{h + b} + C_8 (\tan^{-1} X - \tan^{-1} Y) \right\}
= I \sin d \left\{ \frac{C_7}{2} \log_e \frac{X^2 + 1}{Y^2 + 1} + C_7 \log_e \frac{h - b}{h + b} + C_8 \tan^{-1} \frac{X - Y}{1 + XY} \right\}
\]

(9.2)

Since \( d = 45^\circ \), and the dip, declination and strike of body are
such as to give $l = 0.13$ and $n = 0.95$, $C_4 = -0.89$, $C_5 = -0.50$ for polarization along the earth's field, and $C_7 = -1.83$, and $C_8 = 1.05$. The value of $I$ may be calculated from the height between maximum and minimum, and is found to be $4.97 \times 10^{-3}$ c.g.s.u.

Substitution into equations (9.2) gives values of $f$ at various points along the profile, and these are shown in Plate IX together with the observed aeromagnetic profile taken from the original magnetometer records. The closeness of fit shows that the hypothesis of a marked contrast in magnetic properties at boundaries like that tested can be supported by exact calculations.

The method of calculating anomalies for arbitrary directions of polarization and measurement proves to be quite applicable in such tests.

(ii) Magnetic high over Pochontas Mountain

This high, rising to $4000\gamma$ lies on the boundary between Zones I and II. The drop-off from the high to the adjacent low ($3000\gamma$) appears to be connected with the boundary. Topography is steep here, however, and there is the possibility that the anomaly is due to topography without any particular magnetic contrast. Equation (9.1) may be used to test this hypothesis. The topographic profile shown on Plate X indicates that the feature may be treated as a step. The positions of measurement of the maximum and minimum are shown.
In a similar analysis to that in the previous section it can be shown that equation (9.1) cannot be satisfied for any value of $\Lambda$, and that therefore the anomaly is not due to a uniformly magnetized topographic feature. The only alternative is a true lateral contrast of magnetic properties between the two zones.

(iii) Postulated fault line west of Ponchontas Mountain

The boundary between I -5 and 9 and I-6 and 8, Map 4 shows prominently on the magnetic contours and on the aerial photographs, and is one of the main set of postulated transcurrent fractures in section c. This was also postulated as a fault by geologists of the British Columbia Department of Mines during ground checks on the aeromagnetic map [B.C. Department of Mines, 1958, p.22]. Plate X shows an example of a profile across this zone, traced from the original magnetic records. There are a number of similar anomalies in the vicinity of the zone, all with shape suggesting their connection with a sloping step. When fitted by steps with $i = 90^\circ$, these indicate a contrast going to a depth of about 500', with a difference in the intensity of magnetization of $5 \times 10^{-3}$ c.g.s. units or more, if polarization is taken to be parallel to the earth's field. These results are consistent with the idea that a fault is connected with the zone.

(e) General summary

An interpretation of certain features of the aeromagnetic map of Texada Island has been made. This shows that the magnetic anomalies reflect geological conditions. Some of the equations of interpretation developed in earlier sections are applied to
AEROMAGNETIC PROFILE ACROSS POSTULATED FAULT ZONE
EAST OF POCOHONTAS MOUNTAIN, TEXADA ISLAND, B.C.

Projected in a direction perpendicular to the strike of the anomalies
— taken from original magnetometer record, line 14

SOUTHWEST SIDE OF POCOHONTAS MOUNTAIN

projection of direction of measurement

strike of zone, N 20° W

postulated fault zone — on basis of aeromagnetic map and aerial photographs

direction of induced magnetization

magnetic contrast as interpreted from the magnetic anomalies

To follow p. 70
test hypotheses arising from the comparison of the magnetic results with the geology. This illustrates how quantitative methods of analysis of magnetic fields when sufficiently flexible, can be used to provide geological information, and check geological theories.

10. **SUMMARY AND CONCLUSIONS**

Expressions for the magnetic force and its derivatives over a number of uniformly magnetized bodies have been derived for the case when both the directions of polarization and measurement are arbitrary. This provides a greater generality in the equations for interpreting magnetic anomalies than in the forms previously published in the literature. In the present treatment the equations may be applied equally well to vertical or horizontal directions of measurement or to the total field direction as measured by nuclear free precession magnetometer or by the type of fluxgate magnetometer used in present day aeromagnetic surveying. In addition, any arbitrary direction of polarization may be incorporated into the calculation of the field, or as an unknown parameter.

For each of the bodies treated: the single dipole, the infinite line of dipoles, the thin dipping sheet, the thick, dipping sheet, and the sloping step, expressions for the field and its derivatives have been expressed in a reduced form, which separates those parameters of the body which determine the shape of the anomaly, from those which determine its size. Since inclusion of the direction of polarization gives a more complete
set of parameters than those given in previous treatments, it is possible to make a more thorough assessment of the geological information obtainable from an analysis of the shape and size of magnetic anomalies.

It is concluded that from magnetic anomalies, the depth, pole strength, and direction of polarization of a single dipole or a horizontal line of dipoles can be determined. For dipping sheets or a sloping step, depth, pole strength per unit surface area, and a polarization function combining the direction of polarization and the dip of the inclined faces, can be determined. The polarization function, and the pole strength (which is determined jointly by the size and the intensity of magnetization of the body) can yield values of their component quantities only when values of one or other of them are obtained from some other source. Methods are presented for the solution of these parameters using data obtained from magnetic anomalies. Examples are given, and applications to magnetic surveys in the vicinity of La la Plonge, Saskatchewan, and Texada Island, British Columbia illustrate their use in the interpretation of aeromagnetic surveys.

The equations for fields and their derivatives over infinite lines of dipoles, dipping sheets and sloping steps are expressed as derivatives of a single function, and a general expression for derivatives of any order is given. Thus a single set of equations can be used to compute values of the fields and their derivatives over these bodies. Graphs are presented as an aid to their computation.
Points at which fields or their derivatives are zero are given special consideration; equations and a graph are given from which any of these points can be easily computed. These values give for changing directions of measurement and polarization the shift of base levels, maxima and minima and inflection points. The base level is shown to be a useful indicator of the type of body causing the anomaly.

Finally as an application of magnetic interpretation to geological problems an interpretation of an aeromagnetic survey of Texada Island, British Columbia, is given. Here it is shown that certain zones may be outlined, each with a characteristic level of magnetization. The boundaries between these zones are found to co-incident with structural trends observed on aerial photographs of the area. That the boundaries represent true lateral contrasts of magnetic properties, and not topographic effects is proved in a number of cases by the application of the quantitative methods developed in earlier section. It is finally concluded that evidence of faulting can be deduced from the aeromagnetic map and the aerial photographs and a structural history of the region is suggested on the basis of these observed patterns of faulting.
Appendix I

Derivatives and integrals of special functions

Techniques of analyzing geophysical force-field data require the numerical computation of the derivatives often to quite high order, of the observed distribution of field values, and the matching of these to the theoretically-calculated derivatives of various trial distributions of matter. [Evjen, 1936; Henderson and Zietz, 1949; Elkins, 1951].

The progress of theoretical work in geophysical interpretation depends to some extent upon the availability of general methods for obtaining derivatives of higher orders, as the labour involved in successive differentiations is usually prohibitive. There is little reference in the geophysical literature to methods of obtaining higher derivatives of the functions commonly required in interpretation. Related functions have been dealt with elsewhere, and references are given by Schwatt [1924; p3] and by Hobson [1931; p 124], but none of these are directly applicable to the functions appearing in equations for magnetic interpretation.

Tables I to IX show that functions of the type $\frac{1}{r^m}, \frac{1}{r^m} (b_0 + b_1 x + \ldots + b_n x^n), \log_e(x^2+h^2)$ and $\tan^{-1} x/h$ frequently occur. Methods of obtaining higher derivatives of these will now be developed.
1. Higher derivatives of \( \frac{1}{r^m} \):

Consider the function \( y = \frac{1}{r^m} \), where \( r^2 = x^2 + h^2 \), and \( m \) and \( h \) are constants. For \( y = f(u) \) where \( u = x^2 \), we may write

\[
\frac{d^n y}{dx^n} = \sum_{r=0}^{\infty} \frac{n(n-1)...(n-2r+1)}{r!} (2x)^{n-2r} f^{(n-r)}(u) \tag{1}
\]

where \( q = \frac{1}{2}n \) if \( n \) is even,

\[= \frac{1}{2}(n-1) \text{ if } n \text{ is odd.}\]

Since \( f(u) = \frac{1}{(x^2 + h^2)^{m/2}} \), it can be shown that \( f^{(n-r)}(u) = (-1)^{n-r} \frac{m(m+2)...(m+2(n-r-1))}{2^{n-r}} \left(\frac{1}{x^2 + h^2}\right)^{m+2(n-r)} \)

if we let \( \theta = \cot^{-1} \frac{x}{h} \)

and substitute from (2) into (1) we finally obtain:

\[
\frac{d^n y}{dx^n} = (-1)^n \frac{\sin m+n}{h^{m+n}} \left[ \sum_{r=0}^{\infty} \alpha_r \cos^{n-2r} \right] \tag{3}
\]

where \( \alpha_r = \frac{n(n-1)(n-2)...(n-2r+1)m(m+2)...(m+2(n-r-1))}{2^r r!} (-1)^r \)

In the special case \( m = 2 \), (3) may be shown to reduce to:

\[
\left[ \text{Gibson, 1931; p. 85} \right]
\]

\[
D^n \left( \frac{1}{x^2 + h^2} \right) = (-1)^n \frac{n! \sin n+1 \theta \sin (n+1) \theta}{h^{n+2}} \tag{4}
\]

writing \( \frac{d^n y}{dx^n} = D^n y \). The related function \( \frac{x}{x^2 + h^2} \) may be shown to have derivatives given by \( \left[ \text{Gibson, 1931; p. 85} \right] \):

\[
D^n \left( \frac{x}{x^2 + h^2} \right) = (-1)^n \frac{n! \sin n+1 \theta \cos (n+1) \theta}{h^{n+1}} \tag{5}
\]

These will be used in the following section.
2. Higher derivatives of $F = \alpha \tan^{-1} \frac{x}{h} + \beta \log_e (x^2 + h^2)$

$$D^n F = D^{n-1} \left( \frac{\alpha h}{x^2 + h^2} + \frac{2 \beta x}{x^2 + h^2} \right)$$

$$= (-1)^{n-1}(n-1)! \sin^n \Theta \left\{ \sin n\Theta + 2\beta \cos n\Theta \right\}$$

(substituting into (6) from (4) and (5))

Thus $D^n F = 0$ if $\frac{1}{n} \tan^{-1} (-\frac{2}{\lambda})$

where $\lambda = \frac{\alpha}{\beta}$

The above expression for $D^n F$ may be written in terms of $x$ and $h$ by expanding $\sin n\Theta$ and $\cos n\Theta$ in terms of $\sin \Theta$ and $\cos \Theta$ and substituting $\frac{h}{(x^2 + h^2)^{1/2}}$ and $\frac{x}{(x^2 + h^2)^{1/2}}$ respectively for the last quantities.

Thus $\alpha \sin n\Theta + 2\beta \cos n\Theta = \alpha \left\{ \left( \frac{x}{x^2 + h^2} \right)^{n/2} - \frac{x}{(x^2 + h^2)^{n/2}} \right\}$

leading to:

$$D^n F = (-1)^{n-1}(n-1)! \left\{ 2\beta \frac{x^n}{x^n + h(x^{n-1} - 2\beta h^2 x^{n-2})} - \alpha \frac{x^n}{h^3 x^{n-3} + 2\beta h^4 x^{n-4}} + \alpha \frac{x^n}{h^5 x^{n-5}} \right\}$$

These derivatives for $n = 1$ to 8 are given in Table III.

3. Derivatives of $\mathcal{F} = b_0 + \ldots + b_n x^n$

Writing $\mathcal{F}$ as

$$\mathcal{F} = \frac{f(x)}{r^m}$$

$$d \mathcal{F} = - \left\{ \frac{mxf(x) - f'(x)r^2}{r^{m+2}} \right\}$$
When \( f(x) \) is a polynomial, \( b_0 + \ldots + b_n x^n \), this leads to the expression:

\[
\frac{d f}{dx} = \frac{d}{dx} \left\{ \frac{b_0 + \ldots + b_n x^n}{r^m} \right\} = \left\{ \frac{a_0 + \ldots + a_{n+1} x^{n+1}}{r^{m+2}} \right\}
\]

\[
= \frac{1}{r^{m+2}} \sum_{k=0}^{n+1} \left\{ \sum_{j=0}^{n} \left( \begin{array}{c}
F_{jk} \end{array} \right) \right\} X^k
\]

(13)

(14)

where \( F_{jk} \) is defined by the array:

<table>
<thead>
<tr>
<th>( k )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( j )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-b_0 m</td>
<td>0</td>
<td>2b_2 h^2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>(-m)b_1</td>
<td>0</td>
<td>3b_3 h^2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>(2-m)b_2</td>
<td>0</td>
<td>4b_4 h^2</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(3-m)b_3</td>
<td>0</td>
<td>5b_5 h^2</td>
</tr>
</tbody>
</table>

where \( h^2 = r^2 - x^2 \).

As an example of the method, suppose it is required to obtain the expression for \( D \mathcal{F} \) in Table I from the expression for \( \mathcal{F} \). Referring to the Table, we have from the integrand.

\[
b_0 = \alpha_{33} h^2
\]

\[
b_1 = \alpha_{13} h
\]

\[
b_2 = \alpha_{11}
\]

\[
m = 5
\]

applying (15) we obtain:
\[
a_0 = b_1 h^2 = -\alpha_{13} h^3
\]
\[
a_1 = -b_0 m + 2b_2 h^2 = -(5\alpha_{33} - 2\alpha_{11}) h^2
\]
\[
a_2 = (1-m)b_1 + 4\alpha_{13} h
\]
\[
a_3 = (2-m)b_2 = -3\alpha_{11}
\]
and \[
D\mathcal{F} = -\mu \left\{ \frac{3\alpha_{11} x^3 - 4\alpha_{13} h x^2 + 5(\alpha_{33} - 2\alpha_{11}) h^2 x + \alpha_{13} h^3}{(x^2 + y^2 + h^2)^{7/2}} \right\}
\]
as given.

4. Derivatives of \[
\frac{\partial}{\partial x} \mathcal{F} = \frac{b_0 + \cdots + b_n x^n}{(c_0 + \cdots + c_m x^m)^\nu}
\]
The results of the preceding section may be generalized as follows:

\[
\frac{d\mathcal{F}}{dx} = \frac{d}{dx} \left\{ \frac{b_0 + \cdots + b_n x^n}{(c_0 + \cdots + c_m x^m)^\nu} \right\} = \left\{ \frac{a_0 + \cdots + a_{n+m-1} x^{n+m-1}}{(c_0 + \cdots + c_m x^m)^{\nu+1}} \right\}
\]

\[
= \sum_{k} \sum_{j} F_{jk} X^k
\]

It can be shown that \( F_{jk} \) is defined by the array:

<table>
<thead>
<tr>
<th>K=</th>
<th>j= 0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-\nu c_0 b_0</td>
<td>b_1 c_0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-2\nu c_0 b_0</td>
<td>(1-\nu) c_0 b_1</td>
<td>2b_2 c_0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-3\nu c_0 b_0</td>
<td>(1-2\nu) c_2 b_1</td>
<td>(2-\nu) c_0 b_2</td>
<td>3b_3 c_0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>3</td>
<td>-4\nu c_0 b_0</td>
<td>(1-3\nu) c_3 b_1</td>
<td>(2-2\nu) c_0 b_2</td>
<td>(3-\nu) c_1 b_3</td>
<td>4b_4 c_0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>-5\nu c_0 b_0</td>
<td>(1-4\nu) c_4 b_1</td>
<td>(2-3\nu) c_0 b_2</td>
<td>(3-2\nu) c_1 b_3</td>
<td>(4-\nu) c_1 b_4</td>
<td>5b_5 c_0</td>
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</tbody>
</table>
5. Integration

If the function \( f = a_0 + \ldots + a_n x^n \) is known to be the derivative of a function of similar form, i.e. if

\[
\int f \, dx = b_0 + \ldots + b_{n-1} x^{n-1}
\]

then the coefficients \( a_k \) may be obtained from the \( b_k \) by means of (15). A set of equations may be obtained expressing \( a_k \) in terms of the \( b_k \) and \( b_{k+1} \). These equations may be solved to obtain expression for the \( b_k \) in terms of the coefficients \( a_k \), as follows:

\[
\begin{align*}
b_{n-1} & = \frac{a_n}{n-m-1} \\
b_{n-2} & = \frac{a_{n-1}}{n-m} \\
b_{n-3} & = \frac{(n-1)n^2b_{n-1} - a_{n-2}}{(n-m-1)} \\
b_{n-4} & = \frac{(n-2)n^2b_{n-2} - a_{n-3}}{(n-m-2)}
\end{align*}
\]

Example:

From Table II we see that:

\[
\int -\frac{2\beta x^2 - 2\alpha h x + 2\beta h^2}{r^4} \, dx = \frac{2\beta x + \alpha h}{r^2}
\]

thus in (17) \( a_0 = 2\beta h^2 \), \( a_1 = -2\alpha h \), \( a_2 = -2\beta \), \( m = 4 \), \( n = 2 \)

Hence from (18) \( b_1 = \frac{-2\beta}{-1} = 2\beta ; \) \( b_0 = \frac{2\alpha h}{2} = \alpha h \)

as in (19).
REFERENCES


HEILAND, C.A., Possible causes of abnormal polarization of magnetic formations, Zeits für Geophysik, v.6, pp. 228-232 (1930).


KOGBETLIANZ, E.G., Quantitative interpretation of magnetic and gravitational anomalies, Geophysics, v. 9, p. 463, 1948.


YUNGUL, S., Prospecting for Chromite with the gravimeter and magnetometer over rugged topography in east Turkey, Geophysics, v. 21, pp. 433-454, 1956.