STUDIES OF THE MICROWAVE SURFACE RESISTANCE OF PURE, ZINC, AND NICKEL DOPED YBCO CRYSTALS

by

KUAN ZIANG

B.Sc., Fudan University, 1984
M.Sc., Institute of Technical Physics, Chinese Academy of Science, 1987

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Department of Physics

The University of British Columbia
Vancouver, Canada

Date August 29, 1995
Abstract

An apparatus utilizing a superconducting cavity resonator has been developed for the systematic study of the microwave surface resistance of high purity (twinned and untwinned), zinc and nickel doped YBa$_2$Cu$_3$O$_{7-\delta}$ single crystals at 35 GHz. The objective was to obtain the intrinsic microwave properties of high temperature superconductors. By using our highly sensitive apparatus to study the effects of twinning and doping, we have been able to confirm that what we have obtained is indeed the intrinsic properties of YBa$_2$Cu$_3$O$_{7-\delta}$.

The results show that the intrinsic microwave properties of YBa$_2$Cu$_3$O$_{7-\delta}$ are very different from that of conventional s-wave BCS superconductors. Firstly, the quasiparticle scattering rate is strongly temperature dependent, and drops rapidly below the transition temperature, which implies a quasiparticle-quasiparticle scattering mechanism. Secondly, both the surface resistance and the real part of the conductivity vary linearly with $T$ below 35K (except for the zinc doped crystals), suggesting an unconventional superconducting pairing state with line nodes in the order parameter. Thirdly, the real part of the conductivity when extrapolated to the 0K is very small, in contrast to previously existing data, and approaches the limit predicted for a d-wave superconductor.

A practical result of our studies is that we have shown that the microwave loss of high temperature superconductors can be reduced by doping with certain impurities. This is an important result in the area of microwave applications, where one generally wants as low a surface resistance as possible.
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Notation

\[ A \] potential vector of electromagnetic field
\[ B \] magnetic induction
\[ E \] electric field
\[ H \] magnetic field
\[ J_s \] surface current density
\[ j \] current density
\[ k \text{ or } q \] wave vector
\[ k_F \] Fermi wave vector
\[ c^*, c \] electron creation, annihilation operator
\[ e \] electron charge
\[ f \] Fermi Dirac function
\[ f \] microwave frequency
\[ f_0, \omega_0 \] resonant frequency
\[ \hbar \] Plank’s constant
\[ k_B \] Boltzmann’s constant
\[ l \] mean free path
\[ m \] mass of the charge carriers
\[ N \] density of states
\[ N \] number of charge carriers
\[ n \] charge carrier density
\[ Q \] quality factor
\[ R_s \] surface resistance
$T_c$  superconducting transition temperature

$W$  energy stored in a cavity resonator

$X_r$  surface reactance

$x_s$  superfluid fraction

$Z$  surface impedance

$Z$  impedance

$\beta$  coupling constant between a resonant circuit and external circuit

$\Delta$  superconducting order parameter

$\delta_{sd}$  skin depth

$\Phi$  magnetic flux

$\gamma^*$ or $\gamma$  quasiparticle creation or annihilation operator

$\lambda$  magnetic penetration depth

$\mu$  permeability

$\omega$ or $\Omega$  microwave angular frequency

$\omega$  quasiparticle frequency

$\rho$  resistivity

$\sigma$  conductivity

$\sigma_1$  real part of $\sigma$

$\sigma_2$  imaginary part of $\sigma$

$\tau$  quasiparticle scattering time

$\xi$  coherence length
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Chapter 1

Introduction

The history of superconductors goes back to 1911, when Kamerlingh Onnes discovered that mercury became superconductive below 4.2 K \(^1\). In 1934, a two-fluid model was proposed by Gorter and Casimir \(^2\) to explain the phenomena. In this model it is assumed that there are two kinds of electrons in a superconductor. One kind, called normal electrons, acts as those of a normal metal, while the other kind, called superconductive electrons, forms a frictionless superfluid, which is responsible for the screening of magnetic fields from the superconductor at low frequencies. The penetration depth of the field into the superconductor is determined by the superfluid density through the London equation, first proposed by F. and H. London in 1935 \(^3\) and which can be expressed as

\[
\mu_0 \Lambda \mathbf{j} = -\mathbf{A}, \tag{1.1}
\]

where \(\Lambda\) is the vector potential, \(\mathbf{j}\) is the current density. \(\Lambda\) is a function of the superfluid density in the two fluid model: \(\Lambda = m / (\mu_0 N_s e^2)\). As long as \(\Lambda\) is positive, the field decays exponentially within the superconductor, with the characteristic penetration length \(\lambda = \sqrt{\Lambda}\). Therefore the London equation implies the Meissner effect, i.e., the ability of a superconductor to expel the magnetic flux when it is cooled below the transition temperature, provided that the field is not too strong. The two fluid model provides a reasonable qualitative macroscopic description of some of the thermodynamic and
electrodynamic properties of superconductors. However, the first satisfactory microscopic theory was given by Bardeen, Cooper and Schrieffer (BCS) in 1957. The theory is based on electron-phonon interactions and provides explanations for most of the properties of the superconductors known at that time. In 1958 Mattis and Bardeen derived an expression relating current density and an applied field for superconductors using this theory. From their result a relation similar to that of the London equation can be derived, where a quantity analogous to the superfluid density in the two-fluid model can be defined. The qualitative behavior of the temperature dependence of the penetration depth in conventional superconductors can be explained very well by this relation. A key feature of the theory is an energy gap at the Fermi surface, with the density of states sharply enhanced right below and above the gap. This enhancement of the density of states gives rise to a peak (coherence peak) right below the transition temperature for the electromagnetic absorption. This peak was in fact experimentally observed in the real part of the conductivity and in the NMR relaxation rate. The coherence peak is a feature not found in the usual two fluid model.

The discovery of high temperature superconductors in 1986 has inspired a wave in studying the fundamental mechanism of the new system. A number of basic experimental results for these new superconductors are found to be not described by the BCS theory, at least not by the conventional s-wave BCS theory. Difficulties in making samples of sufficient quality further complicates the matter. Recently, several experiments suggest that the order parameter of the high temperature superconductors have d-wave symmetry. For such a symmetry the energy gap is not isotropic, and instead it has nodes on the Fermi surface.
These experiments include NMR\textsuperscript{9,10}, penetration depth\textsuperscript{11,12,13}, photoemission\textsuperscript{14}, Josephson tunneling\textsuperscript{15}, etc. The possibility of a d-wave pairing state was explored by a number of theorists. Scalapino and co-workers have successfully analyzed the temperature dependence of the Cu(2) and O(2,3) nuclear relaxation rates of YBa\textsubscript{2}Cu\textsubscript{3}O\textsubscript{7-\delta} in terms of a weak-coupling antiferromagnetic Fermi-liquid theory\textsuperscript{16,17,18}. They have also analyzed the results from microwave impedance measured mainly in our group\textsuperscript{19}, and found that the data fit better to the theoretical result for a superconductor with a $d_{x^2-y^2}$ symmetry. Based on the experimental result on the NMR relaxation rate of YBa\textsubscript{2}Cu\textsubscript{3}O\textsubscript{7-\delta}, Millis, Monien and Pines\textsuperscript{20} proposed a phenomenological model which assumes that the spin-spin correlation function strongly peaks at ($\pm \pi/a$, $\pm \pi/a$) positions in the Brillouin zone. This assumption is qualitatively consistent with the result from the neutron scattering experiment\textsuperscript{21,22,23}. Using these results Monthoux, Balatsky and Pines proposed a phenomenological theory\textsuperscript{24}, where it is argued that the antiferromagnetic copper spin fluctuations in the system are responsible for the superconductivity. An interaction between the quasiparticles via spin fluctuations provides pairing of the quasiparticles and favors an energy gap with a $d_{x^2-y^2}$ symmetry.

Measurements of the microwave surface impedance of classical superconductors were carried out by H. London, A. B. Pippard and other researchers in the forties and fifties and played an important role in the development of BCS theory\textsuperscript{25,26,27}. The imaginary part of the surface impedance, the surface reactance, is directly related to the magnetic penetration depth, $\lambda$, which provides a measure of the superfluid density. Its temperature dependence
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reflects the quasiparticle density of states near the Fermi surface. The real part of the surface impedance, the surface resistance, \(R_s\), provides information about the real part of the conductivity \(\sigma_1\), which is proportional to the electromagnetic absorption inside the superconductor in a given electric field. The real part of the conductivity involves both the density of states and the quasiparticle scattering rate. For the high temperature superconductors, the surface impedance has also received extensive scrutiny, both in crystals and films, reflecting its importance for both the fundamental and application aspects of these materials\(^{28,29,30,31}\). Techniques used to study the surface impedance include that using a parallel plate resonator (Stanford\(^{29,32}\), a solenoid coil(U. of Tokyo\(^{33}\), and cavity perturbation (Northeastern\(^{34}\), UBC\(^{11,35}\), UCLA\(^{31}\), Maryland\(^{36}\)). Early results on the penetration depth of \(\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}\) thin films using the parallel resonator technique have shown an exponential temperature dependence at low temperatures, taken as an evidence for a finite minimum gap in the material. The measurements made at UBC on high quality \(\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}\) single crystals have shown a linear temperature dependence at low \(T\), giving strong evidence for a gap with line nodes. This linear temperature dependence was also confirmed by Mao et al.\(^{36}\), but with a much larger slope. Ma et al.\(^{32}\) reported their data on \(\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8\) single crystals and \(\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}\) thin films, also using the parallel plate resonator technique. They obtained a quadratic temperature dependence at low temperatures. This \(T^2\) behavior can be explained by a calculation done by Hirschfeld and Goldenfeld for a d-wave superconductor with strong impurity scattering\(^{37}\). Data obtained for the surface resistance on those materials varies enormously, depending on the quality of
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the material. Taking YBa$_2$Cu$_3$O$_{7-x}$ as an example, some of the data shows that the surface resistance decreases monotonically with temperature, whereas other data show a small plateau or even a peak in the region near 40 K. In addition to the different qualitative features, the magnitudes vary drastically.$^{32,38,39}$ For poor quality samples the surface resistance can be dominated by extrinsic factors. In samples with grain boundaries, for example, weak links at the boundaries which are partially normal can dominate the surface resistance. For samples of good quality, the surface resistance reflects the intrinsic properties of the material and is determined by the real part of the conductivity $\sigma_1$. Bonn et al., with a generalized two fluid model in mind, have taken $\sigma_1$ to be proportional to the product of the normal fluid density and quasiparticle scattering time.$^{40}$ Recently, Scalapino et al. have shown that for layered materials, this ansatz is in fact appropriate, with the use of an appropriately averaged scattering time.$^{41}$

In this thesis the microwave surface resistance of YBa$_2$Cu$_3$O$_{7-x}$ has been systematically investigated at 34.8 GHz using cavity perturbation methods, in an apparatus designed and built as part of the Ph.D. project. The central part of the apparatus is a superconducting cavity resonator operating in the $TE_{011}$ mode. The samples in this study were all very high quality crystals grown in our group by Liang et al.$^{42}$ We have measured the surface resistance of pure crystals, and as well as those with zinc and nickel doping. Using untwinned crystals we were also able to measure the anisotropy of the surface resistance in the $ab$ plane. Finally, the real part of the conductivity was derived from our $R_s$ data by combining it with measurements of the penetration depth, obtained on the same crystals by
our group and the McMaster group\textsuperscript{43}. 

Before this Ph.D. project began, a broad peak in the surface resistance had been observed by Bonn \textit{et al} in a 3.8 GHz apparatus\textsuperscript{40}. This broad peak was claimed to be an intrinsic property of the material, resulting from a peak in the real part of the conductivity. It was proposed that the peak in the conductivity was due to a rapid drop of the quasiparticle scattering rate below $T_c$. A good method to confirm these claims would be to obtain the frequency dependence of the surface resistance and hence the frequency dependence of the real part of the conductivity for the material: a low scattering rate combined with a higher frequency would eventually cause the product $\omega\tau$ to be close to or even greater than one, and accordingly this would alter the overall shape of the electromagnetic absorption in the material. This was one of the motivations for developing the 35 GHz apparatus. The existence of the broad peaks in $R_s$ and $\sigma_1$ were confirmed at this frequency with a shift of the peaks toward higher temperatures. The direction of this shift is exactly that expected for a superconductor with a rapid drop in the quasiparticle scattering rate\textsuperscript{35,44}. The broad peak in the surface resistance was also confirmed by Achkir \textit{et al}.\textsuperscript{45} and Mao \textit{et al}.\textsuperscript{36}

The surface resistance data of some state-of-the-art thin films have raised questions on whether the broad peak in $R_s$ is intrinsic or extrinsic. The surface resistance of some films not only lacks the broad peak, but is also found to be lower than that of the crystals at low temperatures\textsuperscript{32,38,39}. Needless to say, it was very important to decide which data represented the intrinsic properties of the material. In order to confirm that what we were measuring was
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indeed intrinsic, and also to further explore the influence that impurities have on the surface resistance, we decided to work with crystals with zinc and nickel impurities deliberately introduced. The measurement of the surface resistance of these doped samples has led to very convincing results and provides important information on the quasiparticle scattering mechanism \(^{46,47}\). It also suggests a method to obtain lower surface resistance in thin films, something that is important for many applications.

It is well known that the amount of oxygen in YBa\(_2\)Cu\(_3\)O\(_{\gamma-\delta}\) strongly affects the properties of the material. We were interested in whether oxygen vacancies on the Cu-O chains would also act as scattering centers and thus change the scattering rate of the quasiparticles in the system. Crystals with a range of oxygen concentrations have been produced and their surface resistance studied \(^{44}\).

In common with all of the copper oxide superconductors, YBa\(_2\)Cu\(_3\)O\(_{\gamma-\delta}\) is a layered material with atomic layers stacked along the c-axis. This structure results in the c axis transport being very different from that in the ab plane. Apart from this anisotropy, there are Cu-O chains along the b direction, causing the structure to be orthorhombic and leaving the transport properties also different in the a and b directions. Friedmann et al.\(^{48}\) and R. Gagnon et al.\(^{49}\) reported that the normal state resistivity in the a direction is more than twice as large as that in the b direction. Cohn et al.\(^{50}\) and Yu et al.\(^{51}\) found a quite remarkable anisotropy in the thermal conductivity in both the normal and superconducting state. However, microwave measurements performed on YBCO crystals before this study all obtained only
averaged values in the $a$ and $b$ directions. We were particularly interested in the role, if any, that the Cu-O chains play in the unusual temperature dependence of the surface resistance.

The anisotropic microwave surface resistance of YBCO system in the $ab$ plane has been investigated and found to be quite large. However, the qualitative temperature dependence of the surface impedance is the same in the $a$ and $b$ directions\textsuperscript{52}.

The mechanism giving rise to high temperature superconductivity is not yet known. We believe that our measurements of the microwave surface resistance, and thus the real part of the conductivity and the quasiparticle scattering rate of pure and impurity doped crystals, provide important information for understanding the YBCO system. The study also reveals the anisotropic electromagnetic response of the YBCO material in the superconducting state. We hope that in the very near future, with the continuous efforts of the community, the nature of high temperature superconductivity will eventually be revealed to us.
Chapter 2

Properties of YBa$_2$Cu$_3$O$_{7-\delta}$

The high temperature superconductors generally refer to a family of superconductors containing copper oxides in the form of CuO$_2$ planes. The most exciting feature of these materials is the high superconducting transition temperatures, for some of them well above 77 K, the boiling temperature of liquid nitrogen under atmospheric conditions. Because of the relatively easy access to liquid nitrogen temperatures, the discovery of these superconductors opens tremendous opportunities in the area of applications. Consequently, the materials have been intensively studied by a large number of groups world wide. The first copper oxide superconductor was found by Bednorz and Müller in 1986; it had the formula La$_{2-x}$Ba$_x$CuO$_4$ and its $T_c$ was greater than 30 K$^8$. Fairly soon after, YBa$_2$Cu$_3$O$_{7-\delta}$$^{53}$, Bi$_2$Sr$_2$CaCu$_2$O$_{8}$$^{54}$, Tl$_2$Ba$_2$Ca$_{n-1}$Cu$_n$O$_{2n+4}$$^{55}$, etc. became members of the family. Among them YBa$_2$Cu$_3$O$_{7-\delta}$ is the most studied, probably because high quality samples are more easily obtained. For microwave experiments, results are very sensitive to the sample quality, in part due to the fact that the microwave field only penetrates a thin layer of the material so that defects in the surface layer can qualitatively change the features of the surface impedance. Consequently, YBa$_2$Cu$_3$O$_{7-\delta}$ crystals, with their high crystalline perfection and smooth surface, became the best choice for studying the intrinsic properties of the copper oxide superconductors.
Chapter 2. Properties of $YBa_2Cu_3O_{7-z}$

In the following two sections we present the structure and the electromagnetic properties of $YBa_2Cu_3O_{7-z}$. We will only discuss properties which are directly related to our work, since the purpose of this short chapter is to provide a necessary introduction to later chapters.
Chapter 2. Properties of YBa₂Cu₃O₇-

2.1 Structure of YBa₂Cu₃O₇-

The crystal structure of YBa₂Cu₃O₇-δ is orthorhombic for values of δ less than 0.6. The conventional unit cell of the crystal can be thought of as consisting of several layers stacked along the c-axis. Sequentially, they are layers of: CuO, BaO, CuO₂, Y, CuO₂, BaO, CuO. In figure 2.1 we show such a unit cell for a value of δ equal to zero. On the CuO basal plane, CuO forms Cu-O chains along the crystal’s b axis, and causes this material to have an orthorhombic symmetry. Ideally, if δ = 0, all the oxygen sites are occupied, leaving perfect Cu-O chains. An increase in δ produces vacancies in the chain oxygen sites (labeled O(1)). It is well known that the properties of the system can be strongly affected by the oxygen content. A reduction in hole density in the CuO plane, for example, is observed with increasing δ. It is also found that the transition temperature is quite sensitive to the value of δ. In figure 2.2 we show a plot of $T_c$ versus x (x=1-δ) taken from Schleger’s Ph.D. thesis. The data were from various groups. The maximum $T_c$ can be seen to occur when x is very close to one. For our samples we have found that the transition temperature changed from 93.5K to 89K in a small range of δ (0.02–0.15). When δ is roughly 0.05, $T_c$ is highest around 93.5K. Crystals with a δ of this value are often said to be optimally oxygen doped. Most of the crystals studied in this project have optimal oxygen doping.

The chains cause the length of the unit cell along the b direction to be slightly longer than that in the a direction. From x-ray and the electron diffraction they are 3.89Å and
Figure 2.1 The crystal structure of YBa$_2$Cu$_3$O$_y$. Note that Cu(1) and O(1) form CuO chains in the $b$ direction.
3.82 Å respectively. In the c direction, the unit length is 11.68 Å, with the distance between the two CuO₂ planes being 3.36 Å, and that between the CuO and CuO₂ planes being 4.16 Å. During the oxygenation of the crystals, the CuO chains are formed as the sample passes through a tetragonal to orthorhombic transition, and the chains can be oriented in either of two perpendicular directions in the crystal. As a consequence, in certain regions the CuO chains are along one direction and in other regions the chains are along a perpendicular direction. A region where all the CuO chains are in one direction is called a single domain, and the boundary between these two regions is called a twin boundary. The twin boundary is usually along a direction having an angle of 45° with respect to the crystal’s a or b axis, and across the boundary the orientation of the CuO chains changes by 90°. An idealized picture of the region at the twin boundary is shown in figure 2.3. Because of the mismatch of the lattice constants in the a and b directions, properties may slightly change in the boundary region comparing with that in a single domain. For the twinned crystals we have studied, the domains have widths varying from a few μm to 100 μm.

The charge state of the copper ion on the plane is Cu²⁺(3d⁹), with one 3d orbit being half filled. This configuration leads to a spin of 1/2 for each Cu²⁺. It is now generally believed that zinc and nickel prefer to substitute for the copper on the CuO₂ plane⁵⁹,⁶⁰,⁶¹. It is interesting to notice that the Zn and Ni both carry a charge 2⁺ in the system. For the copper sites on the basal plane, the substitutions usually have a valence 3⁺, as for Co, Al, or Fe. For small concentrations of Zn²⁺ or Ni²⁺ in the CuO₂ plane, the oxygen content, the
Figure 2.2. The transition temperature of YBCO versus oxygen content. Data were collected from Jorgensen et al., Cava et al., Brewer et al., and Poulsen et al. (from Schleger).
Figure 2.3 The twin boundary caused by the change of orientation of CuO chains in the CuO basal layer.
Chapter 2. Properties of YBa$_2$Cu$_3$O$_{7-\delta}$

chain structure and the orthorhombic phase remain unchanged; whereas a small amount of Co, Al and Fe quickly causes an orthorhombic to tetragonal phase change$^{67}$, as shown in figure 2.4.
Figure 2.4. The variation of lattice constants $a$ and $b$ versus doping concentration after Tarascon et al.\textsuperscript{57}. For crystal structure with the tetragonal symmetry, $a$ equals to $b$. One can see that a small amount of Co, Al and Fe quickly causes an orthorhombic to tetragonal phase change.
Chapter 2. Properties of YBa$_2$Cu$_3$O$_{7-\delta}$

2.2 Some Basic Features of YBa$_2$Cu$_3$O$_{7-\delta}$

As expected from its structure, the system is very anisotropic. In the normal state, the $c$-axis resistivity is about 60 times as large as that in the $ab$ plane. From magnetization experiments, the $c$-axis coherence length in the superconducting state is found to be about 3Å and the $ab$ plane coherence length to be roughly 16Å\cite{68}. The magnetic field penetration depth $\lambda$ is also very anisotropic. The $c$ direction penetration depth is found to be about 11,000Å\cite{69}, and in the $ab$ plane, Basov et al.\cite{70} report values of 1000Å in the $b$ direction and 1600Å in the $a$ direction from far infrared spectroscopy measurements. Thus YBCO is a so-called type II superconductor, with the penetration depth much larger than the coherence length $\xi$. The coherence length $\xi$ is related to the upper critical fields $H_{c2}$ by

$$H_{c2} = \frac{\Phi_0}{2\pi\xi^2},$$

where $\Phi_0$ is the flux quanta equaling to $2.07 \times 10^{-15}$ Wb. The short coherence length means a very high upper critical field for the material. From Hall effect experiments\cite{71,72}, $1/R_H$ was found to decrease linearly in $T$ in the normal state, which is rather anomalous. The $ab$ plane mean free path is estimated to be of order $10^4$Å at low temperature in the superconducting state. When the current is in the $ab$ plane, the penetration depth into the $c$ direction (It is called the $ab$ plane penetration depth because it is determined by the conductivity in the $ab$ plane.) is believed to be much larger than the $c$-axis mean free path. This ensures that the scattering process occurs in a range of almost constant field. As a result, the electrodynamic
Chapter 2. Properties of $\text{YBa}_2\text{Cu}_4\text{O}_{7-x}$

properties can be described in the local limit, to be discussed in the following chapter.

Although there are no substantial structural changes accompanying zinc or nickel doping, the transport properties change drastically, reflecting the crucial importance of the CuO$_2$ plane to the superconductivity in the system. In figure 2.5 a plot of $T_c$ versus the doping concentrations for $\text{YBa}_2(\text{Cu}_1-x\text{M}_x)_3\text{O}_{6.95}$ from Markert et al.\textsuperscript{73} is displayed. The curves may vary somewhat from group to group, but it is evident that zinc doping suppresses $T_c$ more than nickel does. In particular, 5% zinc (the percentage is compared with the total copper content) results in a drop of the transition temperature from 93.4K to about 45K. The NMR experiment on zinc and nickel doped YBCO show that 1% zinc doping into the CuO$_2$ plane causes a strong suppression of the normal state Cu nuclear relaxation rate near zinc impurities, whereas 5% nickel doping does not have such an effect\textsuperscript{74}. It was argued that spinless zinc impurity acts as magnetic impurity and leads a local collapse of antiferromagnetic spin correlation in the system, and results in a suppression in $T_c$.

In figure 2.6 we compare the resistivity curves of pure crystals with that of zinc and nickel doped ones. Data in figure 2.6a were obtained by Baar et al.\textsuperscript{75} on the crystals grown within the group, and figure 2.6b was copied from a paper published by T. R. Chien et al.\textsuperscript{76} For a pure crystal, we see that the resistivity is remarkably linear in $T$ in the normal state, and extrapolates close to zero at zero temperature. This feature is another unusual property of the high temperature superconductors and is not yet understood. Some attempts have been made to explain the phenomenon. For examples, Moriya et al.\textsuperscript{77} and Monien et al.\textsuperscript{78} have
Figure 2.5. $T_c$ versus doping concentration after Markert et al.\textsuperscript{73} Open diamonds are for zinc doped crystals and filled diamonds for nickel doped crystals.
obtained a quite linear resistivity for these materials using an antiferromagnetic-Fermi-liquid theory combined with the NMR experimental data; nonetheless, the calculated curves inevitably have a negative intercept at zero temperature. The value of the resistivity in the \( ab \) plane at 120K is about 75 \( \mu \Omega \cdot \text{cm} \). The surface resistance corresponding to this value is 0.32 \( \Omega \) at 35 GHz. For the doped crystals, the resistivity is found to be much larger in the normal state, as indicated in figure 2.6.

In figure 2.7 we show the normal state resistivity curves in the \( a \) and \( b \) directions by T. A. Friedmann et al.\textsuperscript{48}. From their experiment they concluded that the \( \rho_a/\rho_b \) anisotropy is 2.2±0.2 between 150K to 275K. This \( ab \) plane anisotropy has been also found in the thermal conductivity. We show in the next figure the thermal conductivity in the two directions in a report by R. C. Yu et al.\textsuperscript{51}. The anisotropy persists all the way from the normal state into the superconducting state, with the thermal conductivity in the \( b \) direction being larger than that in the \( a \) direction. In addition, they have also observed in both directions a broad peak in the thermal conductivity versus \( T \) in the superconducting state, which they interpreted as due to the contribution of the electrons, rather than the phonons in the system.

This concludes our basic introduction to the structure and relevant electromagnetic behavior of the YBCO material.
Figure 2.6a. Resistivity versus temperature for the pure and 0.75\% nickel doped samples in the \textit{ab} plane, taken by Baar et al.\textsuperscript{75}
Figure 2.6b Resistivity versus temperature for pure and zinc doped samples in the ab plane, after reference (76).
Figure 2.7. The resistivity versus temperature in $a$, $b$, and $c$ directions for a YBCO single crystal, after Friedmann et al.\textsuperscript{46} The resistivity in both $a$ and $b$ directions is quite linear.
Figure 2.8. Temperature dependence of the thermal conductivity in the $a$ and $b$ directions for a YBCO single crystal. Plot is from reference (51).
Chapter 3

Electromagnetic Properties of Superconductors at Microwave Frequencies

The aim of this chapter is to provide an overview of the general electromagnetic properties of conventional superconductors at microwave frequencies. The chapter is divided into three sections. In the first section we will introduce the surface impedance, which can be measured by microwave techniques, and the relationship between the surface impedance, the complex conductivity, and the penetration depth. In the second section the phenomenological two fluid model will be discussed. The complex conductivity has a simple form and interpretation within this model. In the final section we will review the electromagnetic properties derived from weak coupling BCS theory.
3.1. Microwave Surface Impedance

An important measurable quantity describing the properties of a normal metal or a superconductor in the presence of microwave radiation is the surface impedance $Z$, which is defined as:

$$Z = \left( \frac{E_z}{H_y} \right)_{z=0} \quad (3.1.1)$$

where $E_z$ and $H_y$ are the components of the electric and magnetic fields in the plane parallel to the surface, with $z=0$ at the surface of the material to be considered, as shown in figure 3.1. To take a perfect metal as an example, where the electric field can only be perpendicular to the surface, the surface impedance is obviously zero. We have assumed in (3.1.1) that the material is isotropic in the $X-Y$ plane, as in this case the electric and magnetic fields in the plane are perpendicular to one another. If the material is anisotropic in the plane, one has to decompose the fields along the two principal axes and define the anisotropic surface impedance for each direction.

For the purpose of studying high $T_c$ superconductors, we need only discuss the electrodynamics in the local limit. In this limit the field varies slowly in space on the scale of the mean free path $l$ of a normal metal or the coherence length $\xi$ of a superconductor. In this limit the current density at a point in the material is determined locally by the electric field at that point, i.e., $j = \sigma E$, where $\sigma$ is the conductivity. At a microwave frequency $\omega$, the electric field inside the material may be written in the exponential form, $E = E_{x0} e^{-kz+i\omega t}$, with
Figure 3.1 The definition of the surface impedance.
\( E_{x0} \) being the amplitude of the electric field at the surface and \( z \) the depth into the material.

Using Maxwell’s equations and Ohm’s law, one obtains \( k = \sqrt{i\mu_0 \omega \sigma} \), and the surface impedance is (see appendix A):

\[
Z = \sqrt{\frac{i\mu_0 \omega \sigma}{\sigma}} = R_s + iX_s. \tag{3.1.2}
\]

The real part of the surface impedance \( R_s \) (called the surface resistance), is essentially the sheet resistance of the surface layer into which the fields penetrate, and has the units of Ohms (Ohms per square). The imaginary part \( X_s \), the surface reactance, is the reactance of such a layer. In the case when the material is a normal metal, the thickness of the surface layer is the classical skin depth \( \delta_{cl} \); and in the case when the material is superconductive, it is the penetration depth \( \lambda \).

The surface resistance is proportional to the total electromagnetic loss inside the material. Let us consider the real part of the normal component of the complex Poynting vector at the surface, which is equal to the power absorbed per unit square by the metal or the superconductor:

\[
\text{Re} \, S_z = \text{Re} \left( \frac{1}{2} E_z \times H_y^* \right) = \text{Re} \left( \frac{1}{2} ZH_{y0}^2 \right)
\]

\[= \text{Re} \left( \frac{1}{2} ZJ_{x0}^2 \right) = \frac{1}{2} R_s J_{x0}^2 \tag{3.1.3}
\]
where $H_{yo}$ is the amplitude of the magnetic field along $y$ axis and $J_o$ is the surface current density. Due to the strong reflection from the metal, the total magnetic field at the surface $H_{yo}$ is about twice as large as the amplitude of the incident field. The square of $H_{yo}$ is then proportional to the incident power. Therefore from equation (3.1.3) we have the ratio of the power absorbed to the incident power to be proportional to $R_s$.

In general, the magnetic penetration depth is defined as:

$$\lambda = \int_0^\infty \frac{B_z(z)}{B_y(0)} dz .$$

3.1.4

Using Maxwell’s equations and the definition of the surface impedance (3.1.1), one obtains the relationship between $\lambda$ and $Z$ (also see Appendix A):

$$\lambda = \frac{Z}{\mu_0 \omega i} .$$

3.1.5

Thus the magnetic penetration depth is a complex quantity. For a normal metal unless the frequency is very high, the conductivity is real, and the surface resistance is then equal to the surface reactance. The magnetic penetration depth can be calculated:

$$\lambda = \delta_i - i \delta_r = \frac{X_i}{\mu_0 \omega} - i \frac{R_i}{\mu_0 \omega} = (1 - i) \sqrt{\frac{1}{2 \mu_0 \omega \sigma}} ,$$

3.1.6

where $\delta_i$ and $\delta_r$ are the reactive skin depth and resistive skin depth respectively. The classical skin depth $\delta_{ci}$ is defined as the depth in which the amplitude of the field drops
to $1/e$ of its original value. It can be easily found that $\delta_d$ is actually $2\delta_i$ or $2\delta_r$.

If the metal is superconductive, one usually has the relation $X_\lambda \gg R_s$, and the magnetic penetration depth $\lambda$ becomes the reactive skin depth $\delta_i$. In this local limit the reactive skin depth $\delta_i$ is called the London penetration depth. It is independent of the frequency and is thus also the d.c. magnetic penetration depth, $\lambda = \delta_i(\omega = 0)$.

In general, the conductivity $\sigma$ is complex, $\sigma = \sigma_1 - i\sigma_2$. The surface resistance can be calculated from equation (3.1.2) in terms of the real part $\sigma_1$ and the imaginary part $\sigma_2$ of it:

$$R_s = \frac{\mu_0 \omega \sqrt{\sigma_1^2 + \sigma_2^2} - \sigma_2}{2(\sigma_1^2 + \sigma_2^2)}.$$  \hspace{1cm} 3.1.7

If $\sigma_2 \gg \sigma_1$, which is true for a superconductor at relatively low frequencies and at temperature not too close to $T_c$, one can simplify equation (3.1.7) and obtain:

$$R_s = \frac{\mu_0 \omega \sigma_1}{2(\sqrt{\sigma_2})^3} = \frac{\mu_0^2 \omega^2 \lambda^3 \sigma_1}{2},$$  \hspace{1cm} 3.1.8

where we have used equation (3.1.6) to write $\sigma_2$ in terms of $\lambda$. The surface resistance is proportional to the real part of the conductivity for a superconductor.
3.2. The Two Fluid Model

The two fluid model was proposed by Gorter and Casimir\(^7\) as a tool to describe some of the properties of superconductors. When combined with a London equation\(^3\), it was used to explain the electrodynamic properties, before the more sophisticated microscopic BCS theory was developed. It is still very useful now, since one can borrow some simple concepts from the model to help in interpreting the BCS theory, particularly when working in the area of applied science. In the two fluid model picture, it is considered that there are two types of electrons in the superconductor, the normal electrons (the normal fluid), and the superconducting electrons (the superfluid). In a given field, the normal electrons are assumed to behave like electrons in the normal state and therefore dissipate energy by scattering. Their motion is described by the usual transport equation:

\[
m \frac{dj_n}{dt} + \frac{mj_n}{\tau} = N_n e^2 E \tag{3.2.1}
\]

where \(j_n = N_n e v_n\), with \(j_n\) being the normal current density, \(N_n\) the density, \(v_n\) the drift velocity of the normal electrons, and \(\tau\) the relaxation time. The superconducting electrons do not dissipate energy as they move; however, they do show inertial effects due to their mass. The equation describing their motion is:

\[
m \frac{dj_s}{dt} = N_s e^2 E \tag{3.2.2}
\]
where $j_s$ is the supercurrent density and $N_s$ is the superconducting electron density. The total electron density is then $N_n + N_s$, and the total current density is the sum of $j_n + j_s$.

A schematic circuit may be used to represent the idea of this model as shown in figure 3.2. When there is only d.c. current, the superconducting electrons carry all the current and there is no loss. When there is an ac current, a voltage builds up across the inductance $L_s$ because of the inertia of the superconducting electrons. This voltage causes normal electrons to flow through the resistance $R_n$ and causes loss.

Assuming an applied electric field varies with time at a frequency $\omega$ as: $E = E_0 e^{i\omega t}$, both real and imaginary parts of the conductivity can be derived from equation 3.2.1 and 3.2.2,

$$\sigma_1 = \frac{N_n e^2 \tau}{m(1 + \omega^2 \tau^2)}$$  \hspace{1cm} 3.2.3

and

$$\sigma_2 = \frac{N_s e^2}{m\omega} + \frac{N_n e^2 \omega \tau^2}{m(1 + \omega^2 \tau^2)}.$$  \hspace{1cm} 3.2.4

The normal electrons are responsible for the real part of the conductivity. If the frequency is not too high, such that $\omega \tau << 1$, one has $\sigma_2 >> \sigma_1$. The surface reactance can be calculated in terms of the conductivities from equation (3.1.2). Using equation (3.1.6) and (3.2.4), the magnetic penetration depth becomes $\lambda = X_s / \mu_0 \omega = (m/\mu_0 N_s e^2)^{1/2}$, which is just the London
Figure 3.2 A circuit representing the two-fluid model.
penetration depth $\lambda_z$, originally derived from London equation. At absolute zero, it is considered that all the carriers are superconducting electrons. We therefore have the superfluid fraction of the total carriers as

$$x_s = \frac{N_s(T)}{N_0} = \frac{\lambda_z^2(0)}{\lambda_z^2(T)} \quad \text{3.2.5}$$

The penetration depth is an important quantity because it is a measurable quantity that is related to the density of the superconducting electrons. Microwave techniques provide a direct measure of the penetration depth, and thus the superfluid density.

In the model, the electric field at the surface and inside the superconductor is mainly determined by the density of superconducting electrons. Because $X_s >> R_s$, the electric field at the surface is $E_0 = ZH_0 \equiv i X_s H_0$, with $|H_0|^2$ proportional to the incident power. Inside the material, the amplitude of the field varies with depth as $|E| = |E_0| e^{-x/\lambda}$. The loss per unit square is:

$$P = \int_0^1 \sigma_1 |E_0|^2 e^{-2x/\lambda} dx = \frac{1}{4} \lambda \sigma_1 X_s^2 |H_0|^2$$

$$= \frac{\mu_0 \sigma_1 J_z^2}{2} \quad \text{3.2.6}$$

$$= \frac{\sqrt{\mu_0 \sigma_1} J_z^2}{2(\sqrt{\sigma_z})^3}.$$
Here we have expressed $H_0$ in terms of $J_e$ and $X_2$ in terms of $\lambda$. Comparing this result with equation (3.1.3), we found the same expressions for the surface resistance as equation (3.1.8).

The surface resistance is also an important quantity, since combined with the penetration depth, one is able to get the real part of conductivity $\sigma_1$, which is proportional to the electromagnetic absorption inside the superconductor in a given field. The absorption can be due to the relaxation process such as inelastic or elastic scattering, or due to the direct excitations from superconducting electrons to normal electrons. Therefore the real part of the conductivity provides information on the scattering process.
3.3 The Electromagnetic Properties of a BCS Superconductor.

Following most of the literature, we will use c.g.s. units in this section. We will start with the perturbation Hamiltonian describing the applied field, and then describe the derivation of the penetration depth and microwave absorption in the transition region.

3.3.1 The Perturbation Hamiltonian

In the BCS theory the external microwave field can be treated as a perturbation, which derives from the change in the canonical momentum in the presence of a field, and the BCS potential term of the electron-phonon interaction remains unchanged. The field can be expressed by a vector potential $A(r)$, which is supposed to be the total real field seen by the electrons (i.e., $A(r)$ is not merely the external field). The kinetic energy term of the Hamiltonian is then $(p - eA/c)^2/2m$. If the field is weak, the non linear term containing $A^2$ can be ignored. If the system is clean, the impurity scattering is weak, and its contribution to the perturbation can be ignored. The perturbation term for the total system can then be written: 

$$
H' = -\frac{e}{2mc} \int dr \psi^* (r) \left( p \cdot A + A \cdot p \right) \psi(r) \\
= -\frac{\hbar c}{2mc} \int dr \psi^* (r) \psi(r) d\mathbf{q} \ a(q) e^{i\mathbf{q} \cdot \mathbf{q}} \cdot (2k + q) \\
= -\frac{\hbar c}{2mc} \sum_{k,\sigma} a(q) \cdot (2k + q) c_{k+q,\sigma}^* c_{k,\sigma}
$$

3.3.1
where we have used the Fourier transform of the vector potential

\[
a(q) = \frac{1}{V} \int dr A(r) e^{-iqr},
\]

and the free electron field operators

\[
\psi(r) = \sum_{k,\sigma} c_{k,\sigma} e^{i k r}.
\]

In the BCS scenario, the excitations are described by quasiparticles rather than electrons. The relation between the quasiparticle and electron operators are as follows:\(^{80}\)

\[
\begin{align*}
  c_{k\uparrow} &= u_k \gamma_{k0} + \nu_k \gamma_{k1}^* \\
  c_{-k\downarrow}^* &= -\nu_k \gamma_{k0}^* + u_k \gamma_{k1}.
\end{align*}
\]

When one substitutes (3.3.2) into (3.3.1), one gets terms of the form \(\gamma^* \gamma\) or \(\gamma \gamma\), corresponding to creating or annihilating a pair of quasiparticles, and terms of the form \(\gamma^* \gamma\), corresponding to quasiparticle scattering from one state to another. For a finite gap, as long as \(\hbar \omega\) of the microwave field is less than \(2\Delta\), the processes of creation and annihilation may be ignored, and we will only be interested in the scattering term (The issue is more complicated in the case of a d-wave ground state where the gap goes to zero at certain lines on the Fermi surface).

In terms of the quasiparticle operators, the perturbation Hamiltonian (3.3.1) can then be written as (omitting pair annihilation and creation terms): \(^{81}\)
\[ H' = -\frac{e \hbar}{2mc} \sum_{k,q,\sigma} a(q) \cdot (2k + q)(c^\dagger_{k+q,\sigma}c_{k,\sigma} - c^\dagger_{k,\sigma}c_{k+q,\sigma}) \]

\[ = -\frac{e \hbar}{2mc} \sum_{k,q,\sigma} a(q) \cdot (2k + q)(u_k u_{k+q} + \nu_k \nu_{k+q})(\gamma^\dagger_{k+q,\sigma} \gamma_{k,\sigma} - \gamma_{k,\sigma}^\dagger \gamma_{k+q,\sigma}) \]

The factor in front of the quasiparticle operators is called the coherence factor, and here it is \(u_k u_{k+q} + \nu_k \nu_{k+q}\) for the particular Hamiltonian under consideration. In general, for an interaction Hamiltonian of the form \(\mathcal{H} = B_{k',k}(c_{k',\sigma}^\dagger c_{k,\sigma} + c_{k,\sigma}^\dagger c_{k',\sigma})\), the positive sign corresponds to what BCS called case I, and a coherence factor of the form \(u_k u_{k+q} + \nu_k \nu_{k+q}\) appears once it is written in terms of the quasiparticle operators. This type of interaction is appropriate to describe processes such as ultrasonic attenuation. The negative sign (case II) applies for interaction of the electron with the electromagnetic field, as for the microwave absorption and for NMR relaxation, and a coherence factor of the form \(u_k u_{k+q} - \nu_k \nu_{k+q}\) appears. The different coherence factors result in very different behaviors near \(T_c\) for ultrasonic attenuation, as opposed to that expected for microwave absorption and nuclear relaxation. A correct description of the different coherence effects was one of the biggest successes of the BCS theory.

At microwave frequencies, the wavelength is much larger than both the coherence length and mean free path, and the vector potential \(\mathbf{A}\) may be considered uniform in space, i.e., we make the approximation \(q=0\). The Hamiltonian (3.3.3) can then be simplified. The usual quantum mechanical formula for the current density operator can be calculated in terms of electron operators.
Chapter 3. Electromagnetic Properties of Superconductors at Microwave Frequency

\[ j(r) = \frac{e}{2m} \{ \psi^*(p - \frac{eA}{c})\psi - [(p + \frac{eA}{c})\psi^*\psi] \} \]

\[ = \frac{e}{2m} \{ \psi^*p\psi - (p\psi^*)\psi \} - \frac{e^2}{2mc} 2A \psi^*\psi \quad \text{(3.3.4)} \]

\[ = \frac{\hbar}{2mV} \sum_k c_{k\sigma}^* c_{k\sigma}(2k) - \frac{e^2}{mc} A \psi^*\psi . \]

Because \( A \) varies slowly in space, the second term of the equation can be treated as the product of a constant and the number density operator. Therefore the expectation value of the second term is \(-ne^2A/mc\), where \( n \) is the density of electrons. This term is sometimes called the diamagnetic term since it tries to cancel the applied magnetic field. The expectation value of the first term can also be calculated\(^\text{79}\), and is called the paramagnetic term since it opposes the diamagnetic current.(If we ignore this term, equation 3.3.4 is in analog with equation 3.2.1 for a normal metal, with the scattering rate equaling zero.). The diamagnetic term is dominant in the superconducting state. However, the paramagnetic term is associated with the fraction of excited quasiparticles, and its temperature dependence provides important information on the low energy density of states, and thus the pairing state.

### 3.3.2 The penetration Depth

As long as the relation between the current density \( j \) and the potential vector \( A \) is linear: \( j = -A/cA \), the magnetic field would vary exponentially with distance inside the
superconductor. Because \[\mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} = -\frac{i\omega}{c} \mathbf{A} , \quad \sigma = \frac{4\pi \omega c^2 i}{Z^2}\] (equation 3.1.2), and \[Z^2 = -(4\pi \omega \lambda)^2\] (equation 3.1.5), one can obtain:

\[
j = \sigma \mathbf{E} \\
= -\frac{i\omega \sigma}{c} \mathbf{A} \\
= -\frac{c}{4\pi \lambda^2} \mathbf{A} .
\]

This relation can be considered as a definition of the penetration depth in a special case, valid in the local limit, of the more general definition of (3.1.4), where the variation of the field does not have to be of the exponential form.

Assuming we work with a single microwave frequency and that the wave vector \(\mathbf{q}\) is taken to be 0, the vector potential \(\mathbf{A}\) becomes its \(q=0\) Fourier component, \(\mathbf{a}(0)\). The paramagnetic term in the current response involves the electron operators which should be written in terms of quasiparticle operators. Its expectation value was calculated from equation (3.3.4). The total current density is

\[
j(q=0) = \frac{2e^2 \hbar^2}{m^2 c} \sum_k [\mathbf{a} \cdot \mathbf{k}] k (-\frac{\partial f}{\partial E_k}) - \frac{ne^2}{mc} a ,
\]

where \(f\) is the Fermi function and \(E_k\) is the \(k\) dependent quasiparticle energy, \(E_k=(\epsilon^2+A^2)^{1/2}\), with \(\epsilon=(\hbar k)^2/2m-(\hbar k_F)^2/2m\) measured from the Fermi surface. The sum is over all the quasiparticle states above the Fermi surface. One can express the penetration depth using
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(3.3.5) and (3.3.6) as

\[
\frac{\lambda^2(0)}{\lambda^2(T)} = 1 - \lambda^2(0) \frac{2e^2\hbar^2}{mc^2} \sum_k k^2 \cos^2 \alpha \left(-\frac{\partial f}{\partial E_k}\right),
\]

3.3.7

where \(\lambda^2(0) = mc^2/4\pi ne^2\) is the penetration depth at \(T=0\), and \(\alpha\) is the angle between \(k\) and the external field. The value \(\lambda^2(0)/\lambda^2(T)\) is defined as the superfluid fraction, first defined in the two fluid model (equation 3.2.5).

Equation (3.3.7) can be evaluated for different gap structures. If the gap function is isotropic, the sum becomes

\[
\int dk k^2 \cos^2 \alpha \left(-\frac{\partial f}{\partial E_k}\right)\alpha = \frac{k_F^2}{3} \alpha \int d\varepsilon N(\varepsilon) \left(-\frac{\partial f}{\partial \varepsilon}\right)
\]

3.3.8

\[
= \frac{k_F^2}{3} \alpha \int dE N(E) \left(-\frac{\partial f}{\partial E}\right)
\]

where one assumes that \(k\) is not far from \(k_F\). \(N(E)\) is the density of states of the quasiparticles, and \(N(\varepsilon)\) is the density of states of electrons at the Fermi surface in the normal state. If the gap function is not isotropic, the quasiparticle energy \(E_k\) will be dependent on the angle, and the transformation of the integrating parameter from \(\int dk \cos^2 \alpha (-\partial f/\partial E_k)\) to \(\int dE N(E) (-\partial f/\partial E)\) involves angular integration. In principle, the superfluid fraction can be written in both cases as

\[
\frac{\lambda^2(0)}{\lambda^2(T)} = 1 - c \int dE N(E) \left(-\frac{\partial f}{\partial E}\right).
\]

3.3.9
Therefore the temperature dependence of the penetration depth is dependent only on the quasiparticle density of states. A relevant case for high $T_c$ superconductors is a gap function with line nodes on the Fermi surface, where it is well known that the density of states is linear with energy for the low excitations. At low temperatures, the largest contribution to the integral comes from the low energy excitations because of the factor $\frac{-df}{dE}$, and the density of states in the integral may be approximated by a linear term $cE$. The fractional normal fluid density is then proportional to $T \cdot \int_0^x \frac{df}{dx} \, dx$, where $x=E/T$. Therefore the normal fluid fraction is linear in $T$ at low temperature. In figure 3.3 we display the penetration depth data on YBCO crystals obtained by our group\(^{11}\). The graph was plotted as $\lambda(0)^2/\lambda(T)^2$ versus $T$, i.e., the superfluid fraction $x_s$ versus temperature. From the graph it can be found that the normal fluid fraction ($x_n = 1-x_s$) is linear in $T$, suggesting that there are line nodes in the gap.

3.3.3 The Absorption

One of the great successes of the BCS theory was an explanation of the coherence peak, which is predicted by the perturbation Hamiltonian of case II as in (3.3.3). The Hamiltonian is

\[
H' = -\frac{e\hbar}{2mc} \sum_{k,q,\sigma} \epsilon(q) \cdot (2k + q) \left( \mu_{k+q}u_k + \nu_{k+q}v_k \right) \left( \gamma^*_{k+q,\sigma} \gamma_{k,\sigma} - \gamma^*_{-k,-\sigma} \gamma_{-k+q,-\sigma} \right)
\]

\[
= \sum_{k,q,\sigma} M_{k,q} \left( \mu_{k+q}u_k + \nu_{k+q}v_k \right) \left( \gamma^*_{k+q,\sigma} \gamma_{k,\sigma} - \gamma^*_{-k,-\sigma} \gamma_{-k+q,-\sigma} \right). \tag{3.3.10}
\]
Figure 3.3 The ratio $\lambda(0)^2/\lambda(T)^2$, i.e., the superfluid fraction, $x_5$ versus $T$ for YBCO crystals (open squares). At low temperatures the curve is approximately linear, suggesting that there are line nodes in the gap function. The solid line is from a calculation using the s-wave BCS theory.
Chapter 3. Electromagnetic Properties of Superconductors at Microwave Frequency

At microwave frequencies, $q$ is taken to be zero. The coherence factor becomes $u^2 + v^2 = 1$. For the scattering from states with energy $E$ to states with energy $E + \hbar \omega$, one can use the Golden rule to estimate the net transition rate, $W$, which is proportional to

$$W = \int |M|^2 N_s(E)N_s(E + \hbar \omega)[f(E) - f(E + \hbar \omega)]dE.$$  \hspace{1cm} 3.3.11

In the normal state, the gap is zero, and the integral reduces to $\hbar \omega |M|^2 N_s(0)$. Therefore around the transition region the ratio of absorption just below $T_c$ to that above $T_c$ is:

$$\sigma_i/\sigma_n \equiv \int \frac{N_s^2(E)\partial f}{N_s^2(0)\partial E}dE$$  \hspace{1cm} 3.3.12

where $\sigma_n$ is the conductivity in the normal state. For an isotropic s-wave gap, the density of states is highly peaked at the gap edge. Since $\int N_s(E)dE = \int N_n(\varepsilon)d\varepsilon$, one should have $\int N_s^2(E)dE >> \int N_n^2(\varepsilon)d\varepsilon$. This causes the real part of the conductivity in the superconducting state to be larger than that in the normal state. Thus an absorption peak (coherence peak) is predicted just below $T_c$. In real superconductors, the coherence peak in s-wave superconductors has been observed experimentally just below $T_c$, and at lower temperatures $\sigma_i$ decreases exponentially. However, the observed peak is usually much smaller than that predicted, which has been argued to be due to anisotropy in the gap function and other mechanisms. An example of an experimentally observed peak for a classical superconductor Nb is presented in figure 3.4. For a d-wave superconductor, due to the
highly anisotropic gap, the density of states is not sharply peaked, and in addition there are
alternations in the sign of the gap function. As a result a coherence peak is not expected.

A more complete treatment of the absorption process in the presence of impurities was
first given by Mattis and Bardeen,\(^5\) as well as by Abrikosov et al.\(^8^3\) In the work of Mattis
and Bardeen the scattering was treated in zero order of the Hamiltonian, and the external
field as a perturbation. By a plausible argument, the existence of the scattering centers was
seen as to merely bring into the current expression a range factor which exponentially
decreases with the mean free path. The complex conductivity was then evaluated for type I
superconductors, for which the penetration depth was shorter than the coherence length.
Recently, Hirschfeld et al. studied the electrodynamic absorption for unconventional
superconductors\(^8^4\), defined as those for which the symmetry of the superconducting order
parameter is lower than that of the crystal lattice, and have derived a remarkably simple
expression for \(\sigma_1\) in the London limit \((q \to 0)^8^5\)

\[
\sigma_{xx} \equiv \left( \frac{n e^2}{m^*} \right) \int_{-\infty}^{\infty} d\omega \left( \frac{-\partial f}{\partial \omega} \right) N(\omega) \text{Im} \left( \frac{1}{\Omega - i/\tau(\omega)} \right), \tag{3.3.13}
\]

where \(\Omega\) is the microwave frequency, \(\omega\) is the quasiparticle frequency, and \(N(\omega)\) is the
density of states normalized to that at the Fermi surface in the normal state. This expression
predicts a power law behavior of the absorption at low temperatures rather than an
exponential behavior for an isotropic s-wave superconductor.
Figure 3.4. The microwave absorption of a Nb sample in the superconducting state. The absorption peak right below $T_c$ is called the coherence peak, after reference (82).
Chapter 4

The Microwave Circuit

The following chapter describes the basic features of resonant circuits, and the cavity perturbation techniques used in surface impedance measurements. In section 4.1, the basic $RLC$ equivalent circuit used to represent the cavity resonator is introduced, and the quality factor of the resonator and the coupling of an external circuit to such a resonant circuit is discussed. In section 4.2 small perturbations to a cylindrical cavity resonator operated in the $TE_{011}$ mode, which forms the basis of our microwave measurement techniques, will be discussed.
4.1 The Resonant Circuit

In our experiments the cavity is designed such that only a single mode is excited by the external microwave source. For this situation, the microwave circuit can be described by an equivalent low frequency $RLC$ resonant circuit, either in the form of a series or parallel $RLC$ circuit. For convenience, we will only discuss the series case, shown in figure 4.1. The properties of this circuit are completely described by three circuit elements, $R$, $L$ and $C$.

The usual definition for the quality factor of a resonant circuit is

$$Q = 2\pi \frac{W}{W_L}$$  \hspace{1cm} 4.1.1

where $W$ is the energy stored in the circuit, $W_L$ is the energy loss per cycle.

For a cavity resonator, the quality factor becomes

$$Q = \omega_0 \int_H \frac{H^2(r)dr}{R_s H^2(r)ds},$$  \hspace{1cm} 4.1.2

where $\omega_0$ is the resonant frequency, $H$ is the magnetic field, $R_s$ is the surface resistance of the cavity wall, and $dV$ and $dS$ are volume and area elements, respectively.

In the equivalent low frequency circuit shown in figure 4.1, the energy stored is the sum of stored magnetic $W_H$ and electric energy $W_E$. At resonance,

$$\langle W_E \rangle = \langle W_H \rangle = \frac{1}{4} LI_0^2 = \frac{I_0^2}{4\omega_0^2 C},$$  \hspace{1cm} 4.1.3
Figure 4.1 The equivalent series $RLC$ circuit.
where $I_0$ is the maximum amplitude of the current, and the brackets refer to the value averaged over time. The quality factor $Q$ of the cavity is expressed by the parameters of its equivalent low frequency circuit as

\[
Q_0 = \frac{\omega_0}{\frac{1}{2} \frac{L}{R}} = \frac{\omega_0 L}{R} = \frac{X_L}{R},
\]

where $X_L$ is the reactance of inductor in circuit. Equation 4.1.4 can be regarded as another definition of the quality factor.

In order to perform measurements, the resonant circuit has to be coupled to an external circuit. A typical circuit representing the transmission configuration is shown in figure 4.2. Here the couplings between the resonant circuit and the input and output circuits are represented by the mutual inductances $M_1$ and $M_2$, respectively. In general, the coupling itself would contain both reactive and resistive parts, shown as $L_1$, $L_2$, $R_1$ and $R_2$ in the figure. However, it is usually a good approximation to neglect the resistive contributions, which simplifies the analysis of the circuit. The input and output circuits are seen by the resonant circuit through the couplings. The effective series impedances added to the resonant circuit by coupling to the input and output circuits are $\omega_0^2 M_1^2 / (Z_1 + i \omega_0 L_1)$ and $\omega_0^2 M_2^2 / (Z_2 + i \omega_0 L_2)$ respectively, with $Z_1$ and $Z_2$ being the loads seen at position $aa$ for the input circuit and position $bb$ for the output circuit. If one has matched loads, $Z_1$ and $Z_2$ would just be the characteristic impedance of the corresponding transmission lines.
Figure 4.2 The coupled circuit. The internal resonant circuit sees the impedance of external circuits through the mutual inductances, $M_1$ and $M_2$. 
According to (4.1.4), the quality factor of this coupled circuit is the total $X_L$ divided by the total resistive part:

$$Q_T = \frac{\omega_0 L - \beta_1 R X_1 / Z_1 - \beta_2 R X_2 / Z_2}{R (1 + \beta_1 + \beta_2)}$$

4.1.5

$$\equiv \frac{Q_0}{1 + \beta}.$$

Here $\beta = \beta_1 + \beta_2$, and $\beta_1$ and $\beta_2$ are the coupling constant between the resonant circuit and the input and output circuits, respectively, and are defined as:

$$\beta_1 = \frac{\omega_0^2 M_1^2}{Z_1 R (1 + X_1^2 / Z_1^2)}$$

4.1.6

$$\beta_2 = \frac{\omega_0^2 M_2^2}{Z_2 R (1 + X_2^2 / Z_2^2)}.$$

For the second step in 4.1.5 we have made the approximation that the second and the third terms in the numerator could be ignored, which is usually true since $\omega_0 L$ is much larger than $R$ and $\beta$ is smaller than 1. Equation (4.1.5) can also be written as

$$\frac{1}{Q_T} = \frac{1}{Q_0} + \frac{\beta}{Q_0} = \frac{1}{Q_0} + \frac{1}{Q_{EXT}}.$$ 4.1.5'

As shown in the definition of the quality factor, the total loss of the internal resonant circuit is proportional to $1/Q_0$. Therefore $1/Q_{EXT}$ represents the loss due to the coupling with the external circuits, and is found to be independent of the resistance of the internal resonant circuit, $R$. 
Returning to the microwave resonator, if a small sample is added to the cavity, the perturbation to the field pattern is small enough that the total energy stored in the cavity stays almost the same; however, there will be a change in the internal reactance and resistance of the equivalent circuit due to the presence of the sample. As a consequence, there will be a change in the resonant frequency and quality factor of the internal resonant circuit. The change of the quality factor is directly related to the loss due to the sample. Because $Q_{\text{ext}}$ is independent of $R$, and provided that the mutual inductance is kept unchanged, $Q_{\text{ext}}$ would remain unchanged after the introduction of the sample. Thus the change in the loss of the whole circuit is the same as that of the internal resonant circuit, $\Delta (1/Q_T) = \Delta (1/Q_0)$.

The voltage reflection and transmission coefficient $\Gamma$ and $T$, which are defined as the ratio of the reflected voltage at $aa$ and $bb$ to the input voltage respectively, can be found for such a circuit. Supposing the characteristic impedance of the input and output transmission line is $Z_0$, the impedance seen from the input side into the network is:

$$Z = \frac{\omega^2 \beta_1 M_1^2}{R + j(\omega L - \frac{1}{\omega C}) + \frac{\omega^2 \beta_2 M_2^2}{Z_0}}$$

$$= \frac{\beta_1^2 Z_0 R}{R + j(\omega L - \frac{1}{\omega C}) + \beta_2^2 R}.$$  

4.1.7

Here $L_1$, $L_2$, $R_1$ and $R_2$ have been ignored. The voltage reflection coefficient at $aa$ by the network is:
Chapter 4. The Microwave Circuit

\[ \Gamma = \frac{Z - Z_0}{Z + Z_0} , \]  

4.1.8

with the relations between the incoming voltage \( V_{in} \), reflected voltage \( V_{re} \) and the total voltage \( V_a \) at position \( aa \) being

\[ V_{in} = V_a \frac{Z + Z_0}{2Z} , \quad V_{re} = V_a \frac{Z - Z_0}{2Z} . \]

If \( \beta_2 \) is zero and \( \beta_1 \) is one, \( Z = Z_0 \) at the resonant frequency and \( \Gamma \) is zero at \( aa \), or no voltage is reflected from the network. The resonant circuit is then said to be critically coupled.

The voltage transmission coefficient is,

\[ T = \frac{V_{out}}{V_{in}} = \left( \frac{2\omega^2 M_1 M_2}{(Z + Z_0) \left[ R + j\omega L + 1/j\omega C + \omega^2 (M_1^2/Z_0 + M_2^2/Z_0) \right]} \right) , \]  

4.1.9

where \( V_{out} \) is the voltage traveling out of the network at position \( bb \). It can be seen again that the effect of the coupling is to add resistive terms in the form \( \omega^2 M^2/Z \) to the resonant circuit. The total resistive part of the circuit is then \( R_T \), the sum of the internal resistance and the additional terms caused by the external circuits.

In our measurement situation, the coupling between the input wave guide and the cavity mode is quite small compared to one. \( Z \) is much less than \( Z_0 \) and the network resembles a short circuit termination for the input wave guide. The factor \( 1/(Z + Z_0) \) in 4.1.9 can be written as \( 1/Z_0 \). Using equation 4.1.3 and 4.1.9, one has the maximum transmission

55
coefficient at the resonant frequency $\omega_0$:

$$T(\omega_0) = \frac{2\sqrt{\beta_1 \beta_2}}{1 + \beta_1 + \beta_2}$$

$$= 2\sqrt{\beta_1 \beta_2} \frac{Q_T}{Q_0}. \quad (4.1.10)$$

Therefore the voltage transmission coefficient at the resonant frequency is dependent on the coupling coefficients as $\sqrt{\beta_1 \beta_2}$.

From an engineering point of view, the circuit between $aa$ and $bb$ is a two-port network, and can be completely described by a scattering matrix $S$. In appendix B the elements of $S$ are defined. It was found that $S_{11} = S_{22} = 1$, and $S_{12} = S_{21} = T$. Any other electrical quantities associated with the network can be derived through $S$.

The power transmission $|T|^2$ as a function of the frequency is given by:

$$|T|^2 = \frac{4\beta_1 \beta_2}{(1 + \beta_1 + \beta_2)^2 + Q_0^2 \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^2}$$

$$= \frac{T^2(\omega_0)}{1 + Q_T^2 \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^2}. \quad (4.1.11)$$

In figure 4.3 we plot the power transmission versus frequency $f$ for such a circuit. The width of the peak at the half maximum power, $\Delta f$, can be seen through equation 4.1.11 to be
\[ \Delta f = \frac{\omega_0}{2\pi Q_T}. \] Therefore the quality factor

\[ Q_T = \frac{\omega_0}{2\pi\Delta f} = \frac{\omega_0}{\Delta \omega}. \] \hspace{1cm} 4.1.12

In practice, the perturbation due to the sample can sometimes be large enough to alter the mutual inductance, \( M_1 \) and \( M_2 \). For example, the field pattern close to a coupling loop in the cavity may be slightly distorted because of the introduction of the sample. Then the external loss \( 1/Q_{\text{EXT}} \) is no longer constant and the relation \( \Delta(1/Q_T) = \Delta(1/Q_0) \) would not be strictly valid. Under these circumstances one can still get accurate results by simply making the coupling constants sufficiently small. As one can see from equation 4.1.5', for small coupling constants \( 1/Q_{\text{EXT}} \) approaches zero and \( Q_T \) approaches \( Q_0 \). Another method one can use is to extrapolate the quality factor of the internal resonant circuit by varying the coupling strength. The voltage transmission coefficient at resonance is

\[ T = \frac{2\omega_0^2 M_1 M_2}{\left( RZ_0 + \omega_0^2 (M_1^2 + M_2^2) \right)}. \] \hspace{1cm} 4.1.13

Introducing a parameter \( c = (M_1^2 + M_2^2)/2M_1M_2 \), one has the following relation between the quality factors and the voltage transmission coefficient:

\[ \frac{Q_T}{Q_0} = \frac{1}{1 + \beta} \]

\[ = \frac{RZ_0}{RZ_0 + \omega_0^2 (M_1^2 + M_2^2)} \]

\[ = 1 - cT. \]
Figure 4.3 The power transmission through a resonant circuit. The full width at half peak power is equal to $\omega_0/2\pi Q$. 


If the coupling is made symmetric, the parameter $c$ becomes 1. The quality factor $Q_T$ of the coupled circuit is linear with the transmission coefficient, $T$. In the experiments one can measure the output voltage $V_{out}$ at the resonant frequency and $Q_T$. A plot of $Q_T$ versus $V_{out}$ should be a straight line, and the extrapolated $V_{out} = 0$ intercept should be the quality factor of the internal resonant circuit, $Q_0$. Practically, $M_1$ and $M_2$ will not be exactly equal; however, this is not a requirement to obtain $Q_0$ from extrapolation. As long as the ratio of $M_1$ to $M_2$ is kept reasonably constant, $c$ would be constant, and $Q_0$ can still be found from this method.
4.2 Cavity Perturbation Method

In chapter 3 when the electromagnetic properties of a metal were discussed, the relative permeability \( \mu_r \) was set to be 1, i.e., the material is assumed to have free carriers, but intrinsic magnetism is negligible. In such a treatment, one has:

\[
B = \mu_0(\mathbf{H} + \mathbf{M}) = \mu_r \mu_0 \mathbf{H} \\
\mathbf{n} \times (\mathbf{H}_{\text{out}} - \mathbf{H}_{\text{in}}) = \mathbf{j}_s
\]

where \( \mathbf{j}_s \) is the surface current, the subscripts 'out' and 'in' stand for quantities outside or inside the metal, and the \( \mathbf{n} \) is the normal unit vector of the metal surface. The magnetization \( \mathbf{M} = (\mu_r - 1) \mathbf{H} = 0 \), and the magnetic field \( \mathbf{H} \) inside the metal is the sum of the external field (here it means the field when the sample is not present) and the field induced by the conducting currents at the surface. As a result, inside the metal \( \mathbf{H} = 0 \) and the magnetic induction \( B = \mu_0 \mu_r \mathbf{H} = 0 \). The surface current is determined by \( \mathbf{H}_{\text{out}} \) right at the surface. In discussing the cavity perturbation method, one may take a different approach toward the problem. For a perfect conductor (skin depth=0), the relative permeability \( \mu_r \) can be set to zero (perfect diamagnet). In this case the set of equations 4.2.1 becomes

\[
B = \mu_0(\mathbf{H} + \mathbf{M}) = \mu_r \mu_0 \mathbf{H} = 0 \\
\mathbf{n} \times (\mathbf{M}_{\text{out}} - \mathbf{M}_{\text{in}}) = \mathbf{j}_s
\]

The magnetization inside (caused by the conducting current) \( \mathbf{M} = (\mu_r - 1) \mathbf{H} = -\mathbf{H} \), and \( \mathbf{H} \) is the magnetic field inside the conductor, which is proportional to the external field (again it is the
field before the sample is present) by some demagnetizing factor. However, the magnetic induction is still zero, $B = \mu_0 (H + M) = \mu_0 H = 0$. The surface current is determined by $M_{\text{in}}$, which is equal to $-\mathbf{H}_{\text{out}}$ at the surface. In this approach one can apply the general results of the cavity perturbation techniques to a metal by putting $\mu_r = 0$.

In our experiment a cylindrical resonant cavity operated at 34.8 GHz is used to measure the sample loss. We use the $TE_{011}$ mode, where the electric field everywhere in the cavity is perpendicular to the axis of the cylinder. The fields for this mode, in polar coordinates, are:

\[
\begin{align*}
E_r &= 0 \\
E_\theta &= \frac{i \omega_0 \mu_0}{k} AJ_0' (kr) \sin \frac{\pi z}{h} \\
E_z &= 0 \\
H_r &= \frac{\pi}{kh} AJ_0' (kr) \cos \frac{\pi z}{h} \\
H_\theta &= 0 \\
H_z &= AJ_0 (kr) \sin \frac{\pi z}{h}
\end{align*}
\]

and,

\[
k^2 = \omega_0^2 \varepsilon_0 \mu_0 - \frac{\pi^2}{h^2} = \frac{\chi^2}{b^2},
\]

where $A$ (amperes per meter) is a normalizing factor, which is imaginary, $b$ is the radius, $h$ is the height of the cavity, $J_m$ is the $m$th Bessel function, and $\chi$ is a zero for $J_0' (kr)$ and is equal
to 3.832 for this mode. Thus for the $TE_{011}$ mode, the field is independent of \( \phi \), \( J_0'(kr) \) (i.e., \( H_r \)) reaches its first zero when \( kr = \chi \), and \( H_z \) has one maximum.

In figure 4.4 the field in the cavity and the current patterns in the cavity walls are displayed. The total energy stored in the cavity is calculated from equations (4.2.3)\(^87\):

\[
W = -\frac{0.403^2 \mu_0 A^2 V_0}{2 \cdot (1 - \frac{\lambda_0^2}{4h^2})},
\]

where \( V_0 \) is the volume of the cavity and \( \lambda_0 \) is the wavelength in free space corresponding to the resonant frequency.

If the surface resistance (defined in chapter 3) of the cavity walls, \( R_s \), is known, the quality factor of the cavity can be easily calculated through equations 4.2.3 and 4.2.5 (Appendix C.):

\[
Q_0 = \frac{V_0 \omega_0^2 \varepsilon_0 \mu_0^2}{\pi \left( \frac{\chi^2}{b} + \frac{2\pi^2 b^2}{h^2} \right) R_s}.
\]

As an example, for \( R_s = 30 \mu\Omega \), the quality factor of the mode is about $5 \times 10^7$.

One approximation we are making for the cavity perturbation is that the fields are only slightly modified after introduction of the sample. Far from the sample, the fields are the same as those in the unperturbed case. Since the sample is very small compared to the volume of the cavity, the total energy stored is approximately the same as that of the
Figure 4.4 Field and current patterns inside the cavity(a, b), and on the cavity walls(c).
unperturbed case. The quality factor of the cavity with sample, $Q_0'$, is then $2\pi W/(W_L + W_S)$, where $W_S$ is the loss per cycle due to the sample. By comparing with equation (4.1.1), we can write:

$$\frac{W_S}{2\pi W} = \frac{1}{Q_0'} - \frac{1}{Q_0} = \Delta \frac{1}{Q_0} .$$  \hspace{1cm} (4.2.7)

If the sample to be measured is put in a fairly uniform region of field in the cavity, the calculation of the loss per cycle $W_S$ is dramatically simplified. (It would be a reasonable approximation if the sample is small compared to the volume of the cavity and is not near a node in the field of interest.) In such an approximation $W$ and $W_S$ can be calculated and compared to $\Delta(1/Q_0)$ measured from an experiment.

To find the loss due to the sample, the magnetic field at the sample surface has to be known. Because of the strong reflection, the magnetic fields at the surface of a sample are very close to that of a perfect conductor, although the parallel electric fields will be different from zero. Therefore, we can treat the sample as a perfect conductor as far as the magnetic field at the sample surface is concerned. If a sample is placed in a uniform external magnetic field $H_0$, the final magnetic field $H$ at the surface is

$$H = \frac{H_0}{1 + (\mu - 1)N} ,$$  \hspace{1cm} (4.2.8)

where $N$ is the demagnetizing factor, and $\mu$ is taken to be zero in our case. The
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demagnetizing factor is a pure number, which is dependent on the shape of the sample, not the nature of the material. Values of $N$ for samples of various shapes can be found in the literature\textsuperscript{88}.

The conducting current at the surface is calculated from the boundary condition given in equation (4.2.2). One can get the amplitude of the surface current density, $J_{s0} = H_p$, where $H_p$ is the parallel component of $H$ obtained from equation (4.2.8) at the sample surface. The loss per cycle $W_t$ due to the sample with a surface resistance $R_{s0}$ is obviously

\[ W_t = \frac{1}{f_0} \frac{1}{2} J_{s0}^2 R_{s0} dS = \frac{1}{f_0} \frac{1}{2} H_p^2 R_{s0} dS . \tag{4.2.9} \]

where $f_0$ is the resonant frequency.

In the following we are going to consider two relevant cases. In the first case, a sample in the shape of a thin plate is put at the center of the cavity such that the principal axis of the cylindrical cavity is parallel to the major surface of this sample, as shown in figure 4.5a. In the second case a sample in the shape of an ellipsoid is considered, with a dimension $c \ll a$ and $c \ll b$, and this sample is put at the center of the cavity such that the principal axis of the cylindrical cavity is parallel to $c$ axis of the ellipsoid, as shown in figure 4.5b.

\textit{a) Sample is a thin plate}

In this situation, the demagnetizing factor is 0, and the magnetic field $H_p$ at the surface is just the external field $H_0$. We may take $H_0$ to be the field at the center of the cavity, which
is \( A \) from equation (4.2.3). The loss per cycle is then

\[
W_s = \frac{SA^2}{f_0} R_s,
\]

where \( S \) is the area of one major surface of the sample. The relation between the surface resistance and the \( \Delta(1/Q_0) \) can be written (see equation 4.2.5 and 4.2.7):

\[
\Delta \frac{1}{Q_0} = \frac{W_s}{2\pi W} = \frac{S(\frac{\lambda_0^2}{4\pi^2} - 1)}{0.403^2\pi f_0 \mu_0 V_0} R_s.
\]

For our 34.8 GHz cavity, we obtain

\[
\Delta \frac{1}{Q_0} = 0.32 \cdot 10^{-4} S R_s,
\]

where \( S \) is in \( \text{mm}^2 \) and \( R_s \) is in \( \Omega \).

\( b) \) Sample is an ellipsoid

In this situation the surface of the sample satisfies the equation

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.
\]

One may make the further approximation that \( a=b \). Using equation 4.2.9, the loss per cycle can then be written

\[
W_s = \frac{1}{2f_0} \int_0^S dx 4\pi H_x^2 R_s \sqrt{1 + \left( \frac{dz}{dx} \right)^2}
\]

\[
= \frac{2\pi H^2 R_s}{f_0} \int_0^S dx \frac{x}{1 + \left( \frac{dz}{dx} \right)^2} \sqrt{1 + \left( \frac{dz}{dx} \right)^2}.
\]
Figure 4.5 Samples in the form of a plate and an ellipsoid in magnetic fields.
where \( \frac{dz}{dx} = cx \left[ a^2 (1 - x^2/a^2)^{1/2} \right] \), and \( H \) is the field in the center of the cavity determined by equation (4.2.3) and (4.2.8). The demagnetizing factor of such an ellipsoid placed this way in the field is \( 1 - \frac{c \pi}{2a} \), with the condition \( a \gg c \). Thus one has \( H = 2aA/c \pi \). The integration (4.2.12) can be calculated numerically for various sizes of samples. For example, for an ellipsoid having dimensions \( a = 0.5 \) mm and \( c = 0.02 \) mm, we obtain \( W_s = 4.8 \times 10^{-17} (m^2 s) R \). Therefore we can obtain the relation between the surface resistance and \( \Delta (1/Q) \) for this particular sample

\[
\Delta \frac{1}{Q} = \frac{W_s}{2\pi W} = 0.56 \cdot 10^{-4} R_s. \tag{4.2.13}
\]

From the above calculations we find that if \( S \) is 1 mm\(^2\) and \( R_s \) is around 100 \( \mu \Omega \), \( \Delta (1/Q_0) \) is around \( \sim 0.5 \times 10^{-8} \). For a quality factor \( Q_0 \) about \( 4 \times 10^6 \), introduction of such a sample into the cavity corresponds to a change of 2\% in \( Q \). As we can see, the sensitivity of the apparatus depends on having a high, and also stable, quality factor.

In both cases of a thin plate and an ellipsoid, one has the relation \( \Delta (1/Q_0) = \gamma R_s \), where \( \gamma \) is called the geometrical factor. It is dependent on the shape and the size of the sample and also the position of the sample in the cavity, since the field strength depends on position. Once \( \gamma \) is determined, the surface resistance can be calculated straightforwardly.

In practice, the geometrical factor is not usually calculated because of the irregular shape of the samples. Instead, it is determined by a calibration method to be discussed in the following chapter.
Chapter 5

Experimental Preparation

This chapter deals with experimental details. In the first section the samples we have investigated and their characterization are briefly reviewed. In the second section the designs of the cryostat, the resonant cavity, the sample probe and some technical details are described. In the following section we discuss our methods for determining the quality factor of the resonant circuit, and the calibration constant for different measuring geometries. In the final section, we discuss the signal generation and detection, and give a measurement circuit diagram. We also record basic features of the instruments we have used in this section.
5.1 Samples

The samples that have been investigated are all very high quality YBa$_2$Cu$_3$O$_x$ single crystals, with and without zinc and nickel doping. Most of the crystals had $x$ set at 6.95, but we have also studied some of the nominally pure crystals with various oxygen contents. All such samples fall under the general label of YBCO single crystals. These crystals have been grown by a CuO-BaO flux method using yttria stabilized zirconia (YSZ) crucibles. It is believed from chemical analysis that YSZ is one of the best crucible materials for YBCO crystal growth. Pure YBCO samples have been characterized by magnetization, dc resistivity, surface impedance and specific heat measurements. The transition temperature obtained from magnetization measurement for the crystals with oxygen content 6.95 is around 93.2K and from specific heat measurement is around 93.4K. The bulk transition width is narrower than 0.25K from specific heat measurement, as shown in figure 5.1. The d.c. resistivity above the transition temperature is about 70 $\mu\Omega\cdot cm$ in the $ab$ plane and a factor of sixty larger along the $c$-axis. For zinc and nickel doped crystals, the transition temperatures are 89.5K for 0.31% zinc, 91.5 for 0.15% zinc and 90.8 for 0.75% nickel doped, respectively. Here the percentages are relative to the total amount of copper in the crystals. The transition temperatures of pure crystals with oxygen concentrations from 6.80 to 6.98 are all around 92K. The typical size of samples in this study is about 1×1 mm$^2$ in the $a$ and $b$ directions of the crystal axes and 10 $\mu m$ to 50 $\mu m$ in thickness ($c$ direction). The crystals are black with a mirror like surface. In figure 5.2 we show a picture of one of the twinned crystals taken with a polarizing microscope. It can be seen that the twinned
Figure 5.1 Specific heat at transition temperature for one of the pure YBCO crystals, after R. Liang et al.\textsuperscript{42}
Figure 5.2 A picture of one of the pure YBCO crystals taken with a polarizing microscope. One can easily see the twin boundaries at $45^\circ$ to the $a$ and $b$ axes.
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boundaries are clearly exposed in the polarized light.
5.2 The Experimental Apparatus

A cavity perturbation system for measuring surface resistance around 35 GHz was designed and built as part of the thesis project. The central part of the apparatus is a cylindrical cavity resonator operating on the $TE_{011}$ mode, shown in figure 5.3. The cavity assembly is made of deoxygenated (OFHC) copper. The inside of the cavity is 1.27 cm in diameter and 0.76 cm in height. According to the mode chart given in standard microwave engineering handbooks\(^9\), the $TE_{011}$ and $TM_{111}$ modes are degenerate and are at 34.7 GHz for the above dimensions. To separate them, a blind hole in the center of the bottom plate is added as a perturbation (see figure 5.3). Such a perturbation has little influence on the $TE_{011}$ mode, since the fields are quite weak at the center of the bottom plate. For the $TM_{111}$ mode, the hole is at an antinode in the magnetic field and consequently has a larger influence on the frequency. Experimentally, the $TE_{011}$ mode is at 34.8 GHz and the $TM_{111}$ mode is at 33.5 GHz. This separation is much larger than the width of the resonances and we expect no interference between the two modes.

After machining and cleaning, the cavity is electroplated with Pb-Sn alloy. We use an alloy of 95% Pb and 5% Sn as the anode in a Pb-Sn fluoborate solution, and control the plating current density at $3 \times 10^{-2} \text{Acm}^{-2}$. This alloy is superconducting at about 7K, and at 4.2K the quality factor is already quite high, about $2 \times 10^6$.

The coupling between the cavity and the input waveguide (and also the output waveguide) is realized by making a through hole connecting the two, and putting a piece of
Figure 5.3 The 34.8 GHz cavity. The cavity is made of oxygen free copper and then plated with Pb-Sn alloy.
conducting wire partially encased by a Teflon tube inside the hole (see both figures 5.3 and 5.4). The conducting wire and the wall of the hole form a short piece of coaxial line. One end of it is coupled to the waveguide mode, and the other end is bent and couples to the magnetic field of the cavity mode. The Teflon tube is connected to a piece of phosphor bronze, which can be moved along the longitudinal direction of the Teflon tube. The motion of this piece of bronze via a coupling adjuster changes the length of the conducting wire inside the cavity, and thus adjusts the coupling strength, i.e., $\beta_1$ and $\beta_2$.

The assembly including the waveguides is situated inside a hollow stainless steel cylinder, with the top and bottom plates made of brass. The cylinder is pumped to a high vacuum before helium is transferred into the dewar. To keep the cavity at a temperature close to that of the helium bath, we use a copper braid to connect the bottom of the cylinder to the bottom plate of a block housing the cavity assembly. This block holds the cavity assembly tightly, and its bottom is made of copper to ensure a good thermal link. A thermal calculation based on the structure, but ignoring the waveguides, indicates that the temperature of the cavity assembly should be close to that of the bath. However, we have found that the heat leak down the waveguides contributes significantly to the heating of the cavity assembly, and we have installed a refrigerator consisting of a long thin stainless tube passing through the top and the bottom plate of the cylinder. During its operation, the upper end is pumped while the lower end is kept in the helium bath. The flow of cold gas in the tube cools whatever is heat-sunk to it. With the refrigerator operating, the temperature of
Figure 5.4 The lower end of the apparatus. The copper braid is attached to the copper part of the bottom plate of the cylinder, which is not shown.
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the cavity is fairly close to the bath temperature. The quality factor can be as high as 2x10^7 once the bath temperature is at 1.3K.

A 0.5 in OD tube is directly connected to the block housing the copper cavity assembly, via a section of thermal insulating material (The imperial unit system is used here since these materials are manufactured with dimensions given in inches.). The sample holder (figure 5.5) consisting of a long piece of one-eighth inch stainless steel tubing is secured and centered by a set of brass fingers inside the above mentioned 0.5 in tube (figure 5.4.). The upper section of the 0.5 in tube is also made of stainless steel for the purpose of thermal insulation. The lower section is made of brass and is heat-sunk to the refrigerator. The brass fingers also act as thermal link between the sample holder and this section of the 0.5 in tubing. The lower end of the sample holder is a copper chamber, where the thermometer and heater are located. The leads for the thermometer and heater run up the inside of the 0.125 in stainless steel tube and connect to an electrical feedthrough at the upper end of the holder. The sample is attached to a sapphire rod or strip, depending on the desired orientation of the sample. This piece of sapphire is in turn glued to a copper piece which can be screwed into the copper chamber assembly.

On the top plate of the cavity there is a through hole for sample access. To prevent microwave radiation loss from the cavity, the diameter of this hole is made small (about 2.3mm), which restricts the size of sample that can be measured. During a measurement the sample can be inserted into, and removed from, the cavity.
Figure 5.5 The lower end of the sample holder. The sapphire rod can be replaced by a sapphire plate so that one can orient the sample with the ab plane parallel to the r.f. H field.
In order to position the sample in the cavity with its c-axis parallel to the r.f. $H$ field, a sapphire rod is used. Sapphire is chosen for its excellent thermal conductivity (~10 W/cmK at 10K, close to OFHC Copper) and low dielectric loss at low temperatures\(^9^0\). The sample is attached to the end of the rod using silicone grease. Since the sapphire rod itself substantially perturbs the cavity, it is extremely important to know its loss and what it does to other cavity modes. In fact, we found that a 1mm diameter rod can bring down the frequency of some of the high index modes far enough to interfere with the working ($TE_{011}$) mode. The thinner sapphire rods are quite expensive, so we have ground down the 1mm rod ourselves, using diamond paste. After grinding and polishing, one end of the rod had a square cross section about 0.6 mm\(^2\). The processed rod was then ultrasonically cleaned in an acetone bath and boiled in a solution of sulfuric and nitric acids for 30 minutes. The frequency shift of the high frequency modes was then sufficiently reduced that they did not interfere with the working mode. It was determined experimentally that the loss due to the sapphire was essentially negligible compared to the loss produced by the YBCO samples. When a sample needed to be placed in the cavity with its c-axis perpendicular to the r.f. $H$ field, a sapphire strip cut from a 100\(\mu m\) thick plate was used. The sample was attached to the lower end of one side of the strip by silicone grease. Because of its small volume, the frequency shift due to the sapphire strip was very small.

With no exchange gas present, the lowest sample temperature reached was about 11K. To obtain data below this temperature, a tiny amount of helium exchange gas had to be admitted to the apparatus. By carefully controlling the amount of exchange gas in the
apparatus, any desired temperature could be reached down to a temperature close to that of
the bath.
5.3 Determination of the Surface Resistance

As we have mentioned in chapter 4, the quality factor of a resonant circuit can be determined by measuring the voltage transmission $V_{out}$ through the resonance. From equation (4.1.7), we have:

$$T^2 = C \cdot \frac{1}{(R_T / 2L)^2 + (\omega - \omega_0)^2},$$

where $C$ is a constant, and $R_T$ is the total resistive part of the coupled circuit. For a constant input, $V_{out}$ is proportional to the voltage transmission coefficient. It is therefore peaked at the resonant frequency and the shape of $V_{out}^2$ versus frequency is a Lorentzian. In our experiment a microwave synthesizer steps through the resonance at discrete frequencies and constant input power. The curve $V_{out}^2$ versus $\omega$ can then be fitted to a Lorentzian distribution by using $\chi$-square fitting, and the quality factor and the resonant frequency can be obtained.

The measurement position determines the pattern of the currents in a sample. In the center of the cavity, the magnitude of the $H$ field of the $TE_{011}$ mode is maximum and the direction of the field is parallel to the central axis of the cavity. If a sample is placed with $c$ axis parallel to the $H$ field of the mode (See Figure 4.5a, and this position will be called the parallel position later.), the screening current would be only in the $ab$ plane. For this measuring position, the loss is proportional to a mixture of the surface resistance in the $a$ and
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*b* directions. Therefore we can measure an averaged surface resistance in the *a*-*b* plane, provided that the constant of proportionality, called the geometrical factor \( \gamma \), can be obtained. For twinned crystals, this is the measurement position we have used. If the sample is oriented with the *a* axis parallel to the r.f. \( H \) field (In this case *c* axis is perpendicular to \( H \) field, referred to later as the perpendicular position; and the same holds when *b* is parallel to the r.f. \( H \) field.), the screening current will be along the *b* direction in the *ab* plane and along the *c* direction in the *ac* plane. The total loss is then proportional to \( J_s^2A(BR_{ab}+CR_{ac}) \), where \( J_s \) is the surface current density and \( A, B \) and \( C \) are dimensions of the sample in the *a*, *b* and *c* directions respectively. To obtain \( R_{ab} \) accurately, the condition \( BR_{ab}>>CR_{ac} \) is required. Samples usually have \( A, B>>C \), and we can find untwinned samples with \( A, B=120C \). In such a case the total loss would be proportional to \( R_{ab} \), provided that \( R_{ac} \) is not much larger than \( R_{ab} \). After obtaining the geometrical factor, \( R_{ab} \) can be calculated. Similarly, \( R_{ac} \) can be measured by orienting the *a* axis parallel to the r.f. \( H \) field.

To ensure that contamination by \( R_{ac} \) is not a concern in the perpendicular position, we have estimated \( R_{ac} \) for several twinned samples of different thickness. We first measured \( R_{tab} \) in the parallel position, then measured \( BR_{tab}+CR_{ac} \) in the perpendicular position. From the normal state resistivity data and the size of the sample, \( R_{tab}+(C/B)R_{ac} \) can be found. (The calibration formula using the normal state resistivity will be discussed in the following paragraphs.) The *c* axis surface resistance \( R_{ac} \) can then be obtained by subtracting \( R_{tab} \) from \( R_{tab}+(C/B)R_{ac} \). We found that \( R_{ac} \) obtained by this method is of the same order as that in the *a* and *b* directions in the superconducting state. Therefore it is safe to measure \( R_{ac} \) and \( R_{ab} \) in
the superconducting state using thin samples with the perpendicular position, where the loss contributed by $R_{sc}$ can be neglected. In the normal state, however, the dc resistivity in the $c$ direction, $\rho_c$, is much higher than that in the $a$ or $b$ directions, resulting in a higher $R_{sc}$ than $R_{sa}$ and $R_{sb}$. The loss due to $R_{sc}$ usually contributes a few percent to the total loss.

The geometrical factor $\gamma$ is obtained by measuring the total loss of samples in the normal state and comparing it to the surface resistance calculated from the measured dc resistivity. In the parallel position for twinned samples, the currents are assumed to be equal in the $a$ and $b$ directions, and the total loss is then given by:

$$\Delta \left( \frac{1}{Q} \right) = \gamma \frac{R_{sa} + R_{sb}}{2} = \gamma \frac{\rho_a}{2\mu_0\omega} + \frac{\rho_b}{2\mu_0\omega},$$

where $\rho_a$ and $\rho_b$ are resistivities in the $a$ and $b$ directions. A measurement of $\Delta (1/Q)$, $\rho_a$ and $\rho_b$ therefore gives $\gamma$.

For the perpendicular position (See figure 4.5b.), the surface resistance can be found in a similar way. Taking $a \parallel H$ as an example, the total loss of a sample in the normal state can be written as:

$$\Delta \left( \frac{1}{Q} \right) = \gamma \cdot \left( R_{sb} + \frac{C}{B} R_{sc} \right) = \gamma \cdot \left( \frac{\rho_b}{2\mu_0\omega} + \frac{C}{B} \frac{\rho_c}{2\mu_0\omega} \right).$$

From dimensions $C$, $B$, the measured normal state resistivities $\rho_a$, $\rho_b$ and the measured value of $\Delta(1/Q)$, $\gamma$ can be found. In the superconducting state, for thin samples, the term involving $R_{sc}$ is neglected, and $R_{sb}$ is obtained from the measured $\Delta (1/Q)$, and $\gamma$.
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5.4 Circuit Diagram and Instrumentation

In figure 5.6 we present the measurement circuit diagram. The microwave source is an HP83620A 0.01-20 GHz Synthesized Sweeper, which scans a series of discrete frequencies under its own sweep mode or via computer control. An HP8349B 2-20 GHz Microwave Amplifier amplifies the signal from the synthesizer and drives a HP83554A Source Module, which doubles the input frequency. There is an interface cable connecting the sweeper and the Source Module, allowing the sweeper to monitor the output signal from the module and adjust the power to a designated value. The microwave signal is then brought to the input waveguide of our apparatus. When the source scans through the resonant frequency of the cavity, the power versus frequency transmitted to the output waveguide of the apparatus is in the shape of a Lorentzian (See equation 5.3.1.). Since in our experiment the output signal was quite low due to the weak couplings, i.e., small β's, and was at a frequency where we had no amplifiers, the signal was brought down to a lower frequency using a microwave oscillator and a mixer, and then amplified. This oscillator was set close to 35 GHz, which produced a signal at around 0.5 GHz. This signal was amplified by a broad band amplifier and then detected by a crystal detector operated in the square law region. The output of the detector, which is a d.c. signal proportional to the transmitted power, is digitized by an A/D converter and sent to a computer. The computer controls the scanning and collects the signal amplitude at each frequency. A data set consisting of transmitted power versus frequency, typically covering a region about ten times wider than the resonance width at the half maximum power Δf, is stored and analyzed by the computer, from which the quality factor
Figure 5.6 Measurement circuit diagram.
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of the circuit is obtained.

In the following, some of the basic features of these instruments will be recorded.

a) *HP83620A Synthesized Sweeper* \(^9^1\)

The HP83620A Synthesizer Sweeper (shown in figure 5.7a) is a high performance, computer programmable and broadband frequency synthesizer. The frequency range is from 10 MHz to 20 GHz. It can be used in continuous wavelength (CW) and start/stop frequency sweeping operations. In CW operation, a single frequency, low noise, synthesized signal is produced. The leveled power range for this operation is from -20dBm and +25dBm. In start/stop frequency sweeping operation, the synthesizer sweeps a frequency span which can be as wide as the frequency range of the instrument, or as narrow as 0 Hz (equivalent to sweeping CW). A sweep from the selected start frequency to the selected stop frequency is produced in this mode. The frequency resolution is one hertz. The total sweep time can be changed from a few milliseconds to more than a minute. The leveled power range is also from -20dBm to +25dBm. The Power Slope Operation allows the output power to increase linearly as the frequency increases to compensate for frequency dependent attenuation in a system when a wide sweep is being used. In the CW operation, the power can also be swept starting with a lower level and stopping at a higher level.

b) *HP8349B Microwave Amplifier* \(^9^2\)

The HP8349B Microwave Amplifier is used as a driver for the millimeter wave source
module (Figure 5.7b). It amplifies signals in the 11 to 20 GHz microwave region to levels greater than +16 dBm. A built-in source module interface controls signals required by the source module (which is the HP83554A source module in our case). As a general power amplifier, its frequency range is from 2.0 GHz to 20.0 GHz. The minimum leveled power output for a 5 dBm input is 19 dBm for the frequency from 2 GHz to 18.6 GHz and 17 dBm from 18.6 GHz to 20.0 GHz. For small signals, the gain is 15 dB and 12 dB for the above corresponding bands.

c) HP83554A Source Module

The HP83544A Source Module (Figure 5.7c) is designed to be used with the HP synthesized sweeper and HP8349B amplifier. The power required to drive the source module is about 20 dBm, with an input frequency range of 13.25 to 20.0 GHz. The output microwave signal is doubled to a range from 26.5 GHz to 40.0 GHz. A source module interface cable is also required to complete the interfacing. The specification of the maximum leveled power output when combined with the HP83620A synthesized sweeper and the HP8349B amplifier is not given. In actual operation, we usually get a maximum leveled power output of around 12 dBm, somewhat larger than the specification value of +7 to +8 dBm when it is combined with an HP8350 series sweeper and the HP8349B amplifier.

d) Other Items

The microwave oscillator mentioned in the beginning of this section is a sweepable Backward Wave Oscillator (BWO) with a frequency range from 26.5 to 40 GHz. It is
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Figure 5.7 (a) HP83620A Synthesized Sweeper
Figure 5.7 (b) HP8349B Microwave Amplifier, and (c) HP83554A Source Module
operated in the CW mode with the frequency about 0.5 GHz away from the output frequency of the source module. The mixer is an HP11517A harmonic mixer with sensitivity of -60 dBm and burnout power level of 1mW at 35 GHz. The low frequency amplifier is a Watkins-Johnson 6201-312 amplifier with frequency range from 5 to 1000 MHz. The small signal gain is > 22dB with noise figure < 6.5dB. The detector is an HP8473B crystal detector with frequency range from 0.01 to 18 GHz, and the sensitivity is > 0.5 mV/μW. The sample thermometer used is a Lakeshore Carbon Glass Resistor (CGR-1-1000 series). The sample heater is home-made using Evanohm wire with a resistance 81.4 Ω/ft and a total resistance of 33 Ω. A UBC-made 85003 Temperature Controller and a 83-047 Automatic Resistance Bridge are used to control the temperature of the samples during measurements.
Chapter 6

Data Analysis and Discussion

In this chapter we present the surface resistance data obtained using our 34.8 GHz apparatus. The chapter is separated into four sections. In the first section, background information such as the losses due to the bare sapphire rod and strip, the silicone grease used to attach samples onto the tip of the sapphire rod or strip, and the Pb:Sn reference samples are given. We also show how the resonant frequency shifts because of the presence of these items. In the second and third sections, the microwave surface resistance and the real part of the conductivity of YBCO crystals including those with a range of oxygen content, and that of zinc and nickel doped samples will be displayed and discussed. The data in these two sections were all obtained on twinned crystals and thus give averaged values in the $ab$ plane of the crystals. In the final section, we present and analyze the anisotropy in the microwave surface resistance and the real part of the conductivity within the $ab$ plane for pure, untwinned $\text{YBa}_2\text{Cu}_3\text{O}_{6.95}$ single crystals.
6.1 Background Runs

In this section the major experimental error sources are discussed. The error associated with our measurements on the surface resistance of the samples has been determined to be less than 100 $\mu\Omega$ in the region below the transition temperature and about 5 $m\Omega$ in the normal state.

In order to measure the surface resistance of samples using our apparatus, the tip of the sample holder, as well as the sample have to be introduced into the cavity. Therefore it is very important to know the loss and frequency shift due to the tip of the sample holder alone, before a meaningful measurement can be made. As mentioned earlier, sapphire was chosen as the tip material because of its excellent thermal properties and low dielectric loss. The silicone grease used to attach the sample onto the tip has mediocre but adequate thermal properties; its dielectric loss are also acceptable. In figure 6.1 the change of the resonant frequency due to the sapphire rod versus its position in the cavity is shown. In this experiment the temperature of the cavity was held at 4.2 K, and the quality factor of the resonator was about $2 \times 10^6$. The $x$ coordinate stands for the depth of the sapphire rod inserted into the cavity, but with arbitrary origin. Because of the different thermal expansions for the sample holder and the 0.5 in stainless steel tube, it is difficult to determine precisely the value $x=x_0$ corresponding to the position where the rod started to enter the cavity. Roughly, we estimate $x_0=1.9$ cm, although the resonant frequency starts to change at $x=1.8$ cm due to the penetration of the field into the insertion hole. From the height of
Figure 6.1 The frequency shift of the resonator due to perturbation by the bare sapphire rod.
the cavity, we know the center of the cavity should correspond to $x=2.3$ cm. We can see from the graph that the frequency shift due to the sapphire rod is negative, as expected from cavity perturbation theory. (Physically, the presence of the sapphire as a piece of dielectric in the cavity makes the space it occupies effectively larger for the electric field. This results in an apparent volume of the cavity that is larger, which shifts the resonant frequency down.) We can see a length of 0.5 cm of this sapphire rod caused a shift about 20 MHz. Compared to the resonant frequency of 35 GHz, this shift is relatively small. The upwards shift at about $x=2.5$ cm is probably caused by the uneven thickness of the rod, resulting from an imperfect grinding job.

In figure 6.2 we show the extra loss of the resonator due to the perturbation of the sapphire rod and the sapphire strip, as a function of temperature. Here the loss is presented by the value of $\Delta(1/Q)=1/Q_l-1/Q_0$ because they are proportional to each other, where $Q_l$ is the quality factor in the presence of the sample, sapphire, etc., and $Q_0$ is the quality factor without any perturbation. The two loss curves for the same sapphire rod are from different runs. Between the two runs the resonator was disassembled, re-plated with Pb:Sn, and then reassembled. It can be seen that the loss for both the rod and the strip is in the range $1 \times 10^{-8}$, which is quite low compared with that due to a typical YBCO crystal in the superconducting state. This low loss is partly a result of the sapphire being at a node of the electric field (antinode of the magnetic field). The increase in the loss is also in the range $1 \times 10^{-8}$ from low temperatures to 100 K, and the curve is approximately linear versus temperature.
Figure 6.2 The loss due to bare sapphire. The filled and unfilled circles are data points for the sapphire rod from two different runs. The stars are data points for the sapphire strip.
Furthermore, the perturbation losses extrapolated to zero temperature are not equal between each plating and reassembling. We believe that this zero temperature perturbation is related to the re-arrangement of the field pattern and currents in some weak links in the joint between the top plate and the body of the cavity, caused by the perturbing object. A small extra loss (it can be either a positive or negative systematic error) is then observed. For each run, this systematic error is different due to slightly different conditions for plating and reassembling, and is carefully determined and corrected using pieces of Pb:Sn as reference samples. The correction procedure will be discussed later in this section.

Before making measurements on a sample, we also have to know the loss caused by the silicone grease used to attach a sample to the tip of the sapphire rod. In figure 6.3 the loss curve versus temperature is displayed. On the tip we have put a tiny amount of high vacuum silicone grease (Dow Corning high vacuum grease), which is just enough to hold a sample. Again the loss is found to be in the range $1 \times 10^{-8}$, with a little larger slope than that of the bare sapphire. The amount of the grease used is hard to control accurately, and may vary slightly from run to run. In the parallel configuration a superconducting sample tends to shield grease from the electric fields and this loss is not a concern. In the vertical configuration, the loss due to the grease may introduce a small systematic error in the measurements.

In figure 6.4a we show the frequency shift versus sample position along the central axis of the cavity for an aluminum contaminated YBCO crystal (batch RL1087) in the parallel
Figure 6.3 The loss due to the silicone grease and the sapphire rod.
measuring position (see chapter 4, section 2 for the geometry of this position). The sample size is 1.2 mm in length and 1.1 mm in width, with thickness unmeasured. We can see that the frequency shift is strongly dependent on the sample position. The maximum frequency shift is about 100 MHz for this sample and occurs when the sample is at 2.31 cm. This position would correspond to the center of the cavity if the perturbation by the sapphire rod were negligible. Close to the maximum shift region, a change of position by 0.25 mm caused a shift of around one MHz. It is not unusual for the thermal motion of the sample holder to cause a frequency shift of a comparable amount during a wide temperature ramp (from a few Kelvin to more than a hundred Kelvin). The resonant frequency is also dependent on the microwave penetration depth for the sample. For our experiment, a change in microwave skin depth of 2 µm causes a change of about 1.0 MHz in the resonant frequency. It is thus evident that the observed frequency shift is completely dominated by motion of the sample due to thermal effects, and the apparatus in its current configuration is not suitable for precision measurements of changes in penetration depth of YBCO samples, where a resolution of several angstroms is desired.

In figure 6.4b the loss versus position for this sample at a temperature of 12.5 K is displayed. A strong dependence of loss on the sample position is observed. The maximum loss position is at 2.34 cm, quite close to the maximum frequency shift position. The maximum loss occurs when the sample is at the center of the cavity, where the magnetic field is maximum and thus the shielding current is the largest. We also notice that in a range about 0.05 cm around the center, the variation of the loss is only 2%. Therefore, as
Figure 6.4 The frequency shift (a) and the loss (b) due to a sample versus its position in the cavity. The data were taken when the sample temperature was 12.5K.
long as the sample is placed at the center of the cavity, the error caused by the thermal expansion of the sample holder is negligible. The apparatus in its current configuration is thus suitable for good measurements of the surface resistance. We also notice that the loss of this particular sample at 12.5K is quite high, with $\Delta(1/Q)=5\times10^{-7}$. This high loss was later found to be the result of aluminum contamination of the crystal during fabrication. High quality YBCO, and zinc or nickel doped samples have losses that are considerably lower at low temperatures, with $\Delta(1/Q)$ around $5\times10^{-8}$ at a few Kelvin. Since this is comparable to the $1\times10^{-8}$ systematic errors, it becomes important to know the systematic error for each run when measuring very low losses at low temperature.

In order to accurately correct for the systematic error, a Pb:Sn reference sample is measured either before or after a YBCO sample is measured. The Pb:Sn sample is cut to a size close to that of the YBCO sample such that when it is placed in the cavity, the field pattern resembles that when the YBCO sample is present. To determine the surface resistance of the YBCO sample, we first obtain the difference in the loss of the YBCO sample and the 1.3 K loss of the Pb:Sn sample of the same size, then use the calibration method discussed in section 5.3 to calculate the difference in $R_s$, and finally add the $R_s$ of the Pb:Sn alloy at 1.3K to the data set. The surface resistance of the Pb:Sn alloy was estimated by determining the surface resistance of Pb:Sn plated on the cavity walls from the quality factor of a fresh plated cavity, using the equation relating the quality factor and the surface resistance (equation 4.2.6). In this way we obtained a value around $40 \mu\Omega$ for Pb:Sn.
at 1.3K. We may also estimate it from the fact that the quality factor of the cavity resonator increases by a factor of ten from 4.2 K to 1.3 K; or in other words, from the fact that the $R_s$ of Pb:Sn at 1.3 K is at least ten times less than that at 4.2 K. The $R_s$ of our calibration Pb:Sn sample at 1.3 K can then be calculated from its value at 4.2 K, since the $R_s$ at 4.2 K is substantially larger than the systematic error and can thus be determined quite accurately. The surface resistance of Pb:Sn at 4.2 K obtained from the experiment is about 300 $\mu\Omega$, which is very close to the value of 340 $\mu\Omega$ from the BCS calculation for Pb\textsuperscript{94}. Using this method, a slightly smaller $R_s$, about 30 $\mu\Omega$ at 1.3 K, is obtained. We emphasize that these estimated corrections are less than the usual scatter of our data. In figure 6.5 we show the loss curves for several Pb:Sn reference samples of various sizes from various runs. We can see from the figure that the systematic errors are all in the range of $1.5 \times 10^8$, which roughly correspond to a value of $R_s$ around 100 $\mu\Omega$.

Other error sources come from temperature variations of the cavity and sample and the possible mechanical vibration of sample holder. These errors are responsible for the scatter of the data and are about 30 $\mu\Omega$ and 50 $\mu\Omega$ for measurements in the parallel and vertical configuration respectively in the superconducting state, and are about 5 $m\Omega$ in the normal state. As pointed out earlier, the possible error caused by the thermal expansion of the sample holder should be less than 2% of the $R_s$ values. We have also checked whether $R_s$ depends on the input microwave power level. It was found that $R_s$ of our samples was essentially power independent within a fairly wide range; however, we have always kept the
power level at the lower end of such a range during measurements.

To summarize, we apply a systematic correction of order 100 $\mu\Omega$ to the raw data, which itself is not much larger than the scatter in the data. We believe our results to have an error of order 50 $\mu\Omega$ at temperatures below 30 K, at most 100 $\mu\Omega$ for the entire region in the superconducting state, and about 5 $m\Omega$ in the normal state, for crystals of nominal dimensions $1 \text{ mm} \times 1 \text{ mm}$.
Figure 6.5 The loss of a series of Pb:Sn samples. The value at 1.3 K typically corresponds to a value of $100 \mu \Omega$, which consists of two parts, the $R_s$ of the Pb:Sn and the systematic error ($T_c$ of Pb:Sn is about 7.2 K).
6.2 Microwave Surface Resistance and Conductivity of Pure YBa$_2$Cu$_3$O$_{7-\delta}$ Crystals

In figure 6.6 we show the surface resistance versus temperature for a pure YBCO ($\delta=0.05$) crystal from sample batch RL1095. The dc resistivity in the $ab$ plane for this batch of samples is about $75 \ \mu\Omega\cdot cm$ at 100 K. This corresponds to a value of $0.316 \ \Omega$ for the surface resistance at 34.8 GHz. Below the transition temperature, the surface resistance drops rapidly, has a minimum near 72 K, then rises to a broad peak around 50 K, and finally decreases at low temperatures. The broad peak in $R_s$ is typical for almost all pure crystals that we have measured, and was first observed in our lab at around 40 K$^{40}$ in a low frequency ($f=3.8$ GHz) apparatus. This peak was attributed to a rapid drop in the scattering rate of the quasiparticles in this material.

To clearly show the features of $R_s$ in the superconducting state, in figure 6.7 we display the data on a linear scale. below the broad peak, the surface resistance at 34.8 GHz is quite linear at low temperatures, which is typical for all pure YBCO crystals that we have studied. The slope of the linear term for these crystals is typically about 50 to 55 $\mu\Omega\cdot K^{-1}$. The residual $R_s$ extrapolated to 0 K is roughly 1 $m\Omega$, which is much larger than our systematic error. We note that the residual $R_s$ values in samples of earlier batches tended to be higher than that of later batches, presumably due to slightly different growth conditions.

In the same figure we show the data of Bonn et al.$^{40}$ taken at 3.8 GHz. For comparison, the low frequency data have been scaled to the higher frequency by a factor $r=(34.8\text{GHz}/3.8\text{GHz})^2$, on the assumption that $R_s$ is proportional to the square of the
Figure 6.6 The surface resistance of a sample from batch RL1095. We found the residual $R_s$ is relatively high for this sample, which is typical for samples from earlier batches.
Figure 6.7 The comparison of the surface resistance at two frequencies, 3.8 GHz (Bonn et al.\textsuperscript{49}) and 34.8 GHz (this work). The error is essentially the scatter in the data.
frequency. It is found that the scaled data match the higher frequency data quite well from $T_c$ down to 50 K, but start to deviate below 50 K. In chapter 3 the relation between the surface resistance and the real part of the conductivity, $\sigma_1$ has been derived. As a reminder, the simplified expression is given again:

$$R_s = \frac{\mu_0^2 \omega^2 \lambda^3 \sigma_1}{2},$$

3.1.8

valid for $\sigma_2 \gg \sigma_1$, which for our frequencies occurs within a fraction of a degree below $T_c$. Provided that there is no much difference in the penetration depth at the two frequencies, we are actually comparing the difference in the real part of the conductivity $\sigma_1$ at the two frequencies. The real part of the conductivity was therefore calculated and is displayed in figure 6.8. To derive $\sigma_1$, we have used penetration depth data from other experiments on the same crystals. The absolute penetration depth at 0 K, $\lambda(0)$, was measured using far infrared spectroscopy $^{70}$, and its temperature dependence was obtained by our group at 1.0 GHz $^{11}$. Here we have assumed that the penetration depth at 3.8 and 34.8 GHz is the same as that at 1.0 GHz. This is equivalent to assuming that the screening of the microwave fields is only caused by the superfluid. In the two fluid model, this assumption is true if the product of the microwave angular frequency and the scattering time, $\omega \tau$, is much less than one. If $\omega \tau$ approaches one, then the normal fluid contributes to the screening and an error is introduced if one uses the penetration depth at 1.0 GHz instead of its effective value at higher frequencies. Within this model, we will find later from the calculation of the scattering rate, that for pure crystals $\omega \tau$ does approach unity at 34.8 GHz at temperatures
Figure 6.8 The real part of the conductivity of a sample at two frequencies. An error of up to 15% may be introduced into $\sigma_1$ in the region below 30 K for the higher frequency data because of the use of the low frequency penetration depth data in the calculation (see text).
below 30 K. However, since we do not have penetration depth data at 34.8 GHz, and since its frequency dependence is not necessarily well described by the two fluid model, the low frequency penetration depth data were used to derive the real part of the conductivity. For such a treatment, we estimate that an error of up to 15% may be introduced into $\sigma_1$ for the pure sample in the region below 30 K at the higher frequency. It is seen in the figure that there are strong broad peaks in $\sigma_1$. It is worth noting that the peaks are not associated with the coherence peak expected for s-wave superconductors, which should have risen sharply, immediately below $T_c$. At low temperatures, a strikingly linear behavior of the conductivity is observed.

The broad peak in the surface resistance can be traced to the peak in the real part of the conductivity. In the two fluid model where the real part of conductivity is expressed in equation (3.2.3), the peak can be seen to result from a competition between the rapid rise of the scattering time and the relatively slow drop of the normal fluid density below the transition temperature. If the frequency is high enough such that $\omega \tau$ starts to approach one at low temperatures, a factor $1/(1+\omega^2 \tau^2)$ would play a role in the real part of the conductivity, and would result in a frequency dependent $\sigma_1$. The observed difference for the two data sets at the low temperatures is consistent with the claim that the scattering rate drops rapidly below $T_c$, such that $1/\tau$ approaches the higher frequency, $2\pi \times 34.8$ GHz $\sim 2.2 \times 10^{11}$ s$^{-1}$.

The sudden rise in $\sigma_1$ close to the transition temperature is due in part to fluctuation
effects, where the dc $\sigma_1$ in the normal state diverges at $T_c$. It is also a region where $R_e$ and $\lambda$, which are measured in the different apparatuses, are both varying rapidly, and a small difference in the two temperature calibrations can produce spurious effects.

Within the two fluid model, the scattering time of all the quasiparticles is considered the same, independent of frequency, and it can actually be calculated from equation (3.2.3) and (3.2.4), using the conductivity and penetration depth data. The calculated scattering rate for 34.8 GHz data is displayed on a log-linear scale in figure 6.9. We can see that the scattering rate drops by a factor of 10 from the transition temperature to 50 K, and starts to saturate below 35 K. The scattering rate below 40K is approximately $3.5 \times 10^{11}$ s$^{-1}$, which causes $\omega \tau$ to be around 0.6 at 34.8 GHz.

In a more sophisticated BCS-like microscopic theory however, the density of states and the dynamic properties of the quasiparticles such as their velocity, vary rapidly with respect to energy, because of the presence of an energy gap at the Fermi surface. In particular, the scattering rate is strongly dependent on the energy for the quasiparticles. The expression derived by Hirschfeld et al. for the real part of the conductivity for d-wave pairing,

$$\sigma_{xx} \equiv \left( \frac{ne^2}{m^*} \right) \int_{-\infty}^{\infty} d\omega \left( \frac{-\partial f}{\partial \omega} \right) N(\omega) \text{Im} \left( \frac{1}{\Omega - i / \tau(\omega)} \right),$$

was used to estimate the low temperature behavior. They derived expressions for the scattering rate for a $d_{x^2-y^2}$ state due to different types of impurities and obtained a quadratic
Figure 6.9 The scattering rate in the superconducting state derived from the two fluid model for a pure YBCO sample.
Figure 6.10 The calculated real part of the conductivity, after Hirschfeld et al\textsuperscript{19}. The upper curve corresponds to a frequency at 3.89 GHz and the lower curve corresponds to 38.9 GHz.
or constant behavior for the low temperature real part of the conductivity at the low temperatures in the presence of magnetic or non-magnetic impurities. By setting the scattering rate equal to the sum of the unitary impurity scattering (strong scattering with phase shift $= \pi/2$) rate and the inelastic rate estimated using a spin-fluctuation model by Quinlan et al., they calculated the real part of the conductivity and fitted it to our data. The results are displayed in figure 6.10. It was found that the overall shape of the curves agrees quite well with the data, however, the linearity of our data at low temperatures clearly deviates from the calculated curves. This linearity could not be explained by either a simple s-wave or a simple d-wave model.

The quantity calculated from the two fluid model can be treated as an energy averaged quasiparticle scattering rate, which can be defined as (in the limit $\omega \tau << 1$)

$$
\langle \tau \rangle = \frac{\int dE \left( -\frac{\partial \tau}{\partial E} \right) N(E) \tau(E)}{\int dE \left( -\frac{\partial \tau}{\partial E} \right) N(E)}.
$$

6.2.2

For a pure d-wave superconductor, this averaged rate corresponds to that of a typical quasiparticle with an energy of order $k_B T$ above the Fermi surface. Most of the heavily occupied states with such an energy are those in the region of the nodes of the energy gap. The inelastic scattering of such quasiparticles by spin-fluctuations was predicted to drop rapidly below the transition temperature. In figure 6.11 we display the calculation for the scattering rate using the spin-fluctuation model by Quinlan et al., which was fitted to our
Figure 6.11. Calculated scattering rate versus temperature compared to our data, after Quinlan et al.\textsuperscript{96}  a) $\log_{10}(\tau(T)/\tau(T_c))$ calculated from an s-wave model with $2\Delta_0/k_B T_c = 4$ (dotted), 6 (dashed), and 8 (solid). b) $\log_{10}(\tau(T)/\tau(T_c))$ calculated from a d-wave model with $2\Delta_0/k_B T_c \sim 6$ (dashed) and 8 (solid). (The sharp features near $T_c$ are due to a combination of fluctuation effects and differences in temperature calibration for the penetration depth and surface resistance apparatuses. See discussion following figure 6.8.)
data. From the figure, the d-wave gap results with $2\Delta_0 \sim 6$ to $8 \ k_BT_c$ appear to follow the experimental data fairly well in the inelastic region, which is above $T=0.5T_c$.

The broad peak in the surface resistance of YBCO single crystals was later confirmed by the Sherbrooke\textsuperscript{45} and Maryland\textsuperscript{36} groups, both using a cavity perturbation technique. Shibauchi \textit{et al.}\textsuperscript{97} reported their $\sigma_1$ data obtained on YBCO single crystals, also using the cavity perturbation method. However, the measured $\sigma_1$ appears to increase with decreasing temperature. They argued that the large residual surface resistance obtained in their samples, which is likely extrinsic, could strongly affect the extracted values of $\sigma_1$ at low temperatures, and the intrinsic behavior was in fact expected to have a broad peak.

The sample used for the data presented above was also used to study the possible role that the oxygen vacancy sites on the chain would play in the scattering rate. The sample had originally been annealed in oxygen gas to have an oxygen content $x=6.95$. After the measurement we ultrasonically cleaned the sample in a heptane bath to remove the silicone grease, and annealed it again to have a different oxygen content. In this way a sample with the same geometry, but different oxygen content was obtained. In figure 6.12, the surface resistance of this sample with several different oxygen contents is displayed. Surprisingly, the surface resistance is found to be quite insensitive to oxygen content for $\delta$ from 0.02 to 0.15, although in this range $T_c$ varies by 4 degrees. This range of $\delta$ corresponds to a factor of 7 change in the number of oxygen vacancies on the chains. The result indicates that the chain vacancies contribute little to the qualitative behavior of the surface resistance, and in
particular, the quasiparticle scattering process is not strongly affected by vacancy sites in the 
CuO chain layers.
Figure 6.12. The surface resistance of YBCO crystals with various oxygen contents at 34.8 GHz. Filled circles correspond to $\delta = 0.05$, open circles correspond to $\delta = 0.10$, filled squares correspond to $\delta = 0.15$, and open squares correspond to $\delta = 0.02$. 
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6.3 Microwave Surface Resistance and Conductivity of Zinc and Nickel Doped YBCO Crystals

As pointed out in chapter 1, the microwave surface resistance plays an important role in understanding the fundamental property of high temperature superconductors; and it is also one of the key parameters to be understood in terms of various applications of this material. Consequently, it has been extensively studied. However, the results are very sample dependent. In figure 6.13 we show the surface resistance of YBCO samples measured by three different groups (Klein et al. 39, Ma et al. 32, and Mogro-Campero et al. 98). Most of the data were obtained on thin films. The magnitudes vary significantly from sample to sample, and most of them decrease monotonically with decreasing temperature in the superconducting state, while the surface resistance of a few samples shows a plateau around 50 K to 60 K, and then falls at lower temperatures. These data are qualitatively different from what we have observed in our pure crystals. Because the surface resistance is proportional to the square of the frequency in the two fluid model (which neglecting coherence effects, we expect to be accurate at medium or high temperature, where the scattering rate is not comparable to the frequency), it is usually scaled to some common frequency, say 10 GHz for comparison purposes. When all of the data is scaled to 10 GHz, the surface resistance of some state-of-the-art thin films is actually lower than that observed in the crystals at low temperatures.

As it is known that the surface resistance is very sensitive to the quality of the samples.
In samples of poor quality, grain boundaries and structural defects may be partially normal and may dominate the surface resistance. However we do not believe these sort of defects play a role either in our single crystals or in very good films. The intrinsic surface resistance is proportional to the real part of the conductivity, as shown by equation 3.1.8. For high quality samples one expects that the introduction of a proper amount of impurities would increase the scattering rate, which would reduce the real part of the conductivity, and thereby lower the surface resistance. It is worth pointing out that this property of superconductors is very different from that of normal metals, for which the surface resistance is inversely proportional to the square root of the conductivity, and the introduction of the impurities would lead to a higher surface resistance.

In order to confirm that the peak in $R_s(T)$ that we measured was indeed the intrinsic behavior of the material, and to further explore the influence that the impurities have on the surface resistance, zinc and nickel doped crystals were grown in our lab. Both nickel and zinc dopants will predominantly substitute for coppers on the Cu-O planes in YBCO\textsuperscript{59,60,61}. These dopants would thus act as impurity scattering centers for the quasiparticles, and produce a qualitative change in the behavior of the surface resistance in the superconducting state. In figure 6.14 we present the surface resistance of three samples in the superconducting state, with nominal zinc concentrations of 0, 0.15 and 0.31%. In figure 6.15 we present the surface resistance of two samples with nominal nickel concentrations of 0, and 0.75%. (The percentage is given with respect to the total copper concentration in the crystals.) All the samples here are twinned. For the pure sample, we found a much lower
Figure 6.13 The surface resistance of various YBCO samples measured by three groups. The upper graph is from reference 39 with a frequency at 18.9 GHz, the middle one is from reference 32 with a frequency scaled to 10 GHz using $R_s \propto f^2$, and the lower is from reference 98 with a frequency at 12 GHz.
residual $R_s$ (value extrapolated to 0 K) than that of earlier batches. At 77K, the value of $R_s$ is about 2.2 $m\Omega$, or about 180 $\mu\Omega$ when scaled to 10 GHz, which is quite a bit lower than that for the best films reported. This very low loss at 77K is another indication that extrinsic factors play little or no role in our crystals.

The overall shape of the surface resistance of doped crystals was found to resemble that of the best films, which may be seen by comparing figure 6.14 and 6.15 with figure 6.13. In the case of 0.15% zinc doping, the broad peak near 50 K is reduced to a plateau at 60 K, and the overall surface resistance is lower than that of the pure crystals. For the sample with 0.31% zinc concentration, there is no longer even a plateau apparent, and the surface resistance decreases monotonically in temperature, to an even lower value. We also note that the linear behavior of the surface resistance at low temperature is modified by zinc doping. Both zinc doped curves exhibit curvature near 10 K, with an obvious departure from linearity for the larger concentration. For the 0.75% nickel doped sample, the overall surface resistance is similarly much lower than that of pure samples. However, the linearity persists in a greater range of temperatures, with a small slope of about 12 $\mu\Omega \cdot K^{-1}$. This slope is less than one quarter of that for the pure samples at low temperatures. It is also interesting to note that the dosage of 0.75% nickel is not as effective as that of 0.31% zinc in shifting down the transition temperature, which agrees with the results of other groups. Furthermore, we found that a relatively high transition temperature for nickel doped sample make $R_s$ particularly low at 77K. When scaled to 10 GHz, the surface resistance is about 130 $\mu\Omega$, which is lower than the value 160 $\mu\Omega$ for the 0.31% zinc doped sample, and
Figure 6.14 The superconducting state surface resistance of zinc doped YBCO crystals at 34.8GHz. The zinc concentrations compared to that of copper are: open circle, 0; filled circle, 0.15%; and open square, 0.31%. The error is about ±30 μΩ.
Figure 6.15 The superconducting state surface resistance of nickel doped YBCO crystals. The nickel concentrations are: open circle, 0%; filled circle, 0.75%. The error is about ±30 μΩ.
substantially lower than that of the pure crystals with twins. The residual loss at 0 K is found to be higher for the nickel doped versus the zinc doped crystals, suggesting that nickel is a different type of scattering source than zinc. However, we should keep in mind that the residual loss is a complicated quantity, since twin boundaries, defects, or surface contamination can all contribute to it. We note that the above measurements have been repeated on different samples with the same doping concentration, and with results essentially the same as displayed here.

For the doped crystals, the product $\omega \tau$ is much less than one, because of the increase of the impurity scattering. The assumption of $\lambda$ being independent of frequency is thus probably better; however, $\lambda(0)$ values for these doped samples have not been measured yet and have to be assumed in order to derive the real part of the conductivity. We know from pure samples that $\lambda(0)$ is around 1300 Å$^{70}$ Because the impurity concentrations are quite low, it is reasonable to use $\lambda(0)=1300$ Å for all samples. The impurities may increase $\lambda(0)$ slightly which leads to an over-estimate of the magnitude of $\sigma_1$, but this will not affect the overall shape of the curves.

In figure 6.16 and 6.17 we show the real part of the conductivity of the pure sample and the samples with zinc and nickel doping. When compared to the data for the pure crystal, it is seen that both zinc and nickel impurities suppress the amplitude of the peak in $\sigma_1$, and shift the maximum position to a higher temperature. The much lower $\sigma_1$ is a clear indication that the impurity scattering limits the rapid drop of the scattering rate. The shift of
the peak towards higher temperature is caused by saturation of the scattering rate at a higher temperature, i.e., the point where the impurities start to dominate the scattering process. We note that doubling the amount of zinc impurities roughly halves the real part of the conductivity at low temperatures, showing that the impurity scattering dominates at temperatures below 30 K.

Zinc and nickel doping are seen to have different effects on the low temperature behavior of $\sigma_1$. Zinc doping changes the linear behavior of pure sample to a nearly quadratic behavior at 0.31% dosage, while 0.75% nickel doping has no such effect. In the simple two fluid model picture, the scattering rate is taken to be independent of energy, and reaches a limit determined by the impurities at low temperatures. The dependence of $\sigma_1$ at low temperatures can thus be traced to the temperature dependence of the normal fluid density. The linear behavior for the pure and nickel doped samples suggests a linear temperature dependence for the normal fluid, and the quadratic behavior for zinc doped sample indicates $T^2$ behavior. Within the simple two fluid picture, these features are consistent with the temperature dependence of the penetration depth and thus also the normal fluid density measured on these crystals. As indicated earlier, the microscopic theory for superconductors with line nodes in the gap predicts a quadratic temperature dependence for $\sigma_1$ for unitary scattering and a constant $T$ dependence for Born scattering (weak scattering with small phase shift) at low temperatures, inconsistent with our experimental results on pure samples. Nevertheless, these predictions appear to agree with the experimental data for 0.31% zinc and 0.75 nickel doped samples if Zn$^{2+}$ and Ni$^{2+}$ are assumed to be magnetic and non-
Figure 6.16 The effect of zinc doping on the real part of the conductivity. In the figure the open circles stand for that of pure sample, filled circle for 0.15% zinc doped sample, and open square for 0.31% zinc doped sample.
Figure 6.17 The effect of nickel doping on the real part of the conductivity, open circles are data points for the pure sample and filled circles are data points for the doped sample.
magnetic impurities respectively. The actual fitting to our zinc doped data using the model by Hirschfeld et al.\textsuperscript{85} are shown in figure 6.18. It was found that the size and position of the prominent maximum in the real part of the conductivity are reproduced qualitatively, but it is clear that the low temperature behavior of the data does not agree in detail with the predictions of the model being proposed.

Finally, the averaged scattering rate versus temperature for each dopant and each concentration is displayed in figure 6.19. The two fluid model is again used to derive this energy independent scattering rate, which can be considered as an averaged scattering rate. As we see in the figure, the main effect of doping is an increase in the quasiparticle scattering rate at low temperatures.

The doping effect is a very important result. It not only supports the conclusion reached from the pure samples that the scattering rate drops rapidly below the transition temperature for this superconductor, but also demonstrates the possibility of reducing the surface resistance for these materials at a temperature as high as 77K. The surface resistance is one of the key parameters in terms of applications, especially in the area of microwave devices, where strong efforts have been focused on the reduction of the surface resistance. The result on crystals indicates that one might be able to produce lower loss high temperature superconducting thin films by slightly doping the material with impurities.
Figure 6.18 The calculated curves (solid line) of the real part of the conductivity at 34.8 GHz, which are compared to our data. In the figure, \( \Gamma \) is a parameter proportional to the density of impurities, and the frequency should be 34.8 GHz instead of 34.8 Hz, after Hirschfeld et al.\(^{85} \)
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Figure 6.19 The scattering rate derived from the two fluid model. Filled circles are data points for the 0.75% nickel doped sample, open square are for the 0.31% zinc doped sample, filled squares are for 0.15% zinc doped sample, and open circles are for the pure sample.
6.4 Anisotropic Properties in the ab Plane

The existence of CuO chains in the structure of YBCO inevitably causes anisotropic transport properties in the ab plane. In the normal state, transport properties in the b(chain) direction have been found to be quite different from that in the a direction\(^48\), and it is important to see how this anisotropy manifests itself in the electrodynamics below \(T_c\). In addition, the role of twin boundaries in the qualitative behavior of the surface resistance was not known. We particularly wanted to know how the twin boundaries would affect the residual surface resistance, which is of special importance in d-wave superconductors. Furthermore, there were suggestions in the literature that the qualitative feature of the surface resistance could be seriously affected by the weak links at the twin boundaries\(^99\), and that the Cu-O chain might play a key role in the qualitative behavior of the surface impedance\(^100,101\). For these reasons, we set out to measure the microwave properties of twin free single crystals.

Figure 6.20 shows the surface resistance of two untwinned samples on a log scale. Sample 1 is naturally untwinned and has a size 1.2×1.2×0.01 mm\(^3\). Sample 2 was mechanically detwinned and has a size 1.1×1.0×0.035 mm\(^3\). As discussed in section 4.3, with a ratio of the length to the thickness exceeding 100, the contamination of \(R_{sc}\) can be ignored for sample 1 for the measurement in the superconducting state. When scaled to 10 GHz, we found that the surface resistance \(R_{sb}\) for sample 1 corresponded to 125 µΩ at 77 K, and the average of \(R_{sa}\) and \(R_{sb}\) corresponded to 140 µΩ. These values are substantially lower than
Figure 6.20 The surface resistance of two untwinned samples. $R_s$ of sample 2 matches that of sample 1 at low temperatures but is higher in the normal state, due to the contribution from $R_{sc}$. The error is ±50 $\mu\Omega$ below $T_c$ and ±5 $m\Omega$ above $T_c$. 

\begin{figure}[h]  
\centering  
\includegraphics[width=\textwidth]{Figure6.20.png}  
\caption{The surface resistance of two untwinned samples. $R_s$ of sample 2 matches that of sample 1 at low temperatures but is higher in the normal state, due to the contribution from $R_{sc}$. The error is ±50 $\mu\Omega$ below $T_c$ and ±5 $m\Omega$ above $T_c$.}  
\end{figure}
all reported surface resistance values for YBCO material at 77 K. We note that in the normal state, the surface resistance of sample 2 is slightly higher due to contamination by $R_{sc}$. In the superconducting state, it also has a higher residual resistivity at 0 K and higher values at 77 K, perhaps indicating slightly lower quality. Furthermore, its thickness to length ratio was more than three times larger than that of sample 1. For these reasons, the rest of the analysis concentrates on sample 1.

In figure 6.21 we show the surface resistance of sample 1 plotted to emphasize the normal state surface resistance. Because the sample is approximately square, it is reasonable to assume the same geometrical factor for the measurement of $R_{sa}$ and $R_{sb}$. Therefore we obtained the ratio of the dc resistivity values in the two directions from the ratio of the surface resistance in the normal state in this measurement. Knowing the averaged dc resistivity in the plane from the twinned samples, the d.c. resistivity values in the $a$ and $b$ directions can be derived. The geometrical factor can thus be determined from the loss in the $a$ and $b$ directions and their respective d.c. resistivity values. It can be seen from the figure that $R_{sa}$ is about 1.5 to 1.6 times as large as $R_{sb}$ at 121 K. This corresponds to a ratio of 2.4 for the resistivities in the $a$ and $b$ directions, which agrees with the highest ratios reported so far for the d.c. resistivities.

In figure 6.22, the surface resistance is shown on a linear scale in order to highlight the low temperature behavior. The center curve is the averaged surface resistance. It was obtained using a measurement method similar to that used for the $a$ and $b$ directions, as discussed in chapter 4, except that the current ran at 45° to the $a$ or $b$ directions. This curve
Figure 6.21 The normal state surface resistance of sample 1 in the $a$ and $b$ direction. The surface resistance in the $a$ direction is roughly 1.6 times as high as that in the $b$ direction.
Figure 6.22 The surface resistance of sample 1 in the superconducting state. The error is ±50 μΩ as indicated by the scatter in the data. $R_{sa}$ is 80% larger than $R_{sb}$ at low temperatures, and has a more prominent peak.
Chapter 6. Data Analysis and Discussion

is found to be close to the average of $R_{sa}$ and $R_{sb}$. The values of $R_{sa}$ are seen to have a very prominent broad peak at about 48 K, whereas the peak for $R_{sb}$ is much smaller. The minimum, occurring at nearly 72 K in twinned samples, is shifted up to around 75 K and is now deeper. This could be explained by the twin boundaries producing extra losses, that vary only slowly with temperature. Below 35 K, both $R_{sa}$ and $R_{sb}$ are very linear, with the $R_{sa}$ curve about 1.8 times steeper than that of $R_{sb}$. If we fit the data below 30 K to a straight line, and scale the values to 10 GHz, we obtain $R_{sa}(0) = 2.5 \pm 2 \mu \Omega$ and $R_{sb}(0) = 0.8 \pm 1 \mu \Omega$. Given the fact that the error on each point is of order $\pm 5 \mu \Omega$, these extrapolated values should not be taken too seriously. However, these appear to be the lowest residual surface resistance values ever reported for YBCO material. For comparison, the typical residual surface resistance of our twinned crystals is about 25 $\mu \Omega$ when scaled to 10 GHz.

The absolute penetration depths in the two directions have been measured on sample 2 at McMaster university\textsuperscript{70}. They obtained a value of 1000 Å for $\lambda_a(0)$ and a value of 1600 Å for $\lambda_a(0)$. The temperature dependence of $\lambda_a$ and $\lambda_b$ was measured on sample 1 by our own group at 1 GHz\textsuperscript{102}. Using these data we are able to derive the real part of the conductivity in the two directions. We should keep in mind that an error up to 15% in the region 10 to 30 K might be introduced, for the same reasons discussed in section 6.2. The result is displayed in figure 6.23. It is interesting to note that the large anisotropy in $\lambda(0)$ has caused $R_{sa}$ to be larger than $R_{sb}$, in spite of the fact that $\sigma_{1a}$ is smaller than $\sigma_{1b}$. At temperatures just below $T_c$, $\sigma_{1b}$ is about 2.4 times as large as $\sigma_{1a}$, consistent with the corresponding normal state
conductivities estimated from the surface resistance data. Both $\sigma_{1a}$ and $\sigma_{1b}$ rise 6 to 7 times from their values at $T_c$ to the peak values around 42 K, showing again the rapid drop of quasiparticle scattering rates in both directions. Throughout the range down to the low temperatures, $\sigma_{1b}$ remains about a factor of two larger than $\sigma_{1a}$. At low temperatures, both curves are linear up to 15 K and somewhat sub-linear up to 30 K.

It has been pointed out that the conductivity of a $d_{x^2-y^2}$ state superconductor approaches a low temperature limit $\sigma_{00}=ne^2/m^*\pi\Delta(0)$ that is independent of the scattering rate$^{85,103}$, with $n$ being the total carrier density, $m^*$ the effective mass, and $\Delta(0)$ the maximum gap at $T=0$. If one takes $h/\pi(T_c) \sim 2k_BT_c$ and $\Delta(0) \sim 2k_BT_c$, then $\sigma_{00} \sim 0.3 \sigma_{dc}(T_c)$. If we fit our low temperature conductivities to a straight line we obtain residual conductivities of $\sigma_{1a}(T\rightarrow0) \sim (0.45\pm0.15) \times10^6 \ \Omega^{-1}m^{-1} \sim 0.45\pm0.15 \ \sigma_{1a, \ dc}(T_c)$ and $\sigma_{1b}(T\rightarrow0) \sim (0.7\pm0.2) \times10^6 \ \Omega^{-1}m^{-1} \sim 0.35\pm0.10 \ \sigma_{1b, \ dc}(T_c)$. The residual conductivity is near our resolution limit and close to the predicted $\sigma_{00}$ for a d-wave superconductor.

The anisotropy in the penetration depth implies a value of 2.4 for the ratio $(n/m^*)_b$ to $(n/m^*)_a$. This is the simplest explanation for the anisotropy in the conductivity. Other contributions may come from the difference in the quasiparticle scattering rate in the two directions or a favored position of the nodes in the gap toward one of the $a$ and $b$ directions in $k$ space, where the momentum a typical quasiparticle carries would be different in the two directions. At the present stage, the error transferred to the real part of the conductivity from the measurement of $\lambda(0)$ and also the use of low frequency (1 GHz) $\Delta\lambda(T)$ at 35 GHz
Figure 6.23 The real part of the conductivity in the two directions at 34.8 GHz. Note that it is the larger anisotropy in λ that causes $R_{ba} > R_{ab}$, in spite of the fact that $\sigma_{1b}$ is larger than $\sigma_{1a}$. 
overturned any chance to distinguish the difference in the scattering rate in the two directions. Therefore the role that $\tau$ plays in the anisotropy is not yet clear. The positions of the nodes in the gap are not easily probed by microwave techniques. Direct or indirect measurements involving large momentum (order of $k_F$) but small energy may help determine those positions.
Chapter 7

Conclusions

We have systematically studied the surface resistance of YBCO based materials. The measurements were performed using a 34.8 GHz cavity perturbation apparatus constructed as part of the Ph.D. project. For a sample of typical size (1.0 mm in length and 1.0 mm in width), the apparatus is capable of measuring the surface resistance from 1.3 K to 130 K, and has a sensitivity of about 30 $\mu\Omega$ for measurements of the average $R$, in the plane and 50 $\mu\Omega$ for the $R$, along a particular direction. An important part of the design is the ability to move the sample, which is mounted on a slender sapphire rod or strip, in and out the cavity at the operating temperatures. This allows an in-situ measurement of the unperturbed cavity quality factor $Q$.

The samples investigated were both twinned (with and without impurity doping) and untwinned YBCO single crystals. In the next three sections, we summarize the measurements on the various kinds of samples, and at the end of the chapter we present conclusions for the whole thesis project.

1) Undoped, twinned samples

The surface resistance of undoped YBCO crystals at 34.8 GHz just below $T_c$ and the broad peak around 50K indicate a rapid drop of the quasiparticle scattering rate and a
relatively slow decrease in the number of quasiparticle excitations in the superconducting state. This slow decrease of the excitation density at low temperatures indicates the existence of low lying states, which suggests the presence of nodes in the gap function. The linear behavior of the surface resistance at low temperatures has no satisfactory explanation at the current stage.

The lack of the coherence peak in the real part of the conductivity in the transition region suggests a non BCS s-wave behavior, a picture consistent with a d-wave gap, where the density of states has only a logarithmic singularity at the maximum gap position and also the coherence factor vanishes for the quasiparticle scattering across a line node on the Fermi surface\textsuperscript{18,104}. The source of the broad peak around 50 K in the real part of the conductivity is the same as that for the surface resistance. Just as $R_s$ does, $\sigma_1$ decreases linearly in $T$ at low temperatures, a behavior which is not consistent with the exponential behavior of an s-wave superconductor nor with the quadratic law behavior expected for a simple d-wave superconductor.

Our data have been fitted by Hirschfeld et al.\textsuperscript{85} They have obtained results for $\sigma_1$ by using the numerical results of Quinlan et al.\textsuperscript{96} for an inelastic scattering rate based on a spin fluctuation model with a $d_{x^2-y^2}$ state, and an impurity scattering rate in the unitary limit. The overall shape of the fitting and the data agree reasonably well, but there are obvious difficulties for the fit with regard to the linear behavior in the low temperature region. The averaged quasiparticle scattering rate was extracted from our data through the two fluid
Chapter 7. Conclusions

model. The data were compared to the quasiparticle scattering rates calculated for an s-wave and a d-wave superconductor associated with spin-fluctuation scattering by Quinlan et al.\textsuperscript{96} The results from the d-wave model appear to follow the experimental data in the region $0.5 T_c < T < T_a$, where the dominant scattering process is expected to be inelastic.

The measurement on YBCO with various oxygen contents indicates that both the oxygen vacancies in the CuO chains and the changes in doping of the holes in the CuO\textsubscript{2} plane have no large effect on the surface resistance of the material, although the effect on the normal state resistivity and the transition temperature is more obvious. Therefore the quasiparticle scattering process remains mainly within the plane and is not very sensitive to the small amount of oxygen vacancies existing on the chains.

2) Zinc and nickel doped samples

The experiments performed on zinc and nickel doped samples show that both dopants generally suppress $R_s$ below $T_c$ and in particular suppress the broad peak in the temperature dependence of the surface resistance, a result indicating that the inelastic scattering drops rapidly below $T_c$ and thus the impurity scattering plays an important role in the magnitude of $R_s$. The extreme sensitivity of the surface resistance to impurities explains the large variation of the $R_s$ values in the literature, and the similarity of both magnitude and the shape of the temperature dependence of $R_s$ of doped crystals to that of the best films suggests that extrinsic factors no longer dominate $R_s$ for the highest quality films. A surface resistance value of 130 $\mu$\Omega at 77K when scaled to 10 GHz was achieved for the 0.75%
nickel doped sample, the lowest \( R_s \) value for YBCO materials of which we are aware. This result suggests that the microwave loss of high temperature superconducting thin films might be improved by slightly doping the material with impurities.

The doping effect on the real part of the conductivity, given that the doping levels do not appreciably change the penetration depth \( \lambda \), directly reflects the changes in \( R_s \). As expected, the huge peak in the conductivity is suppressed by doping, with a stronger effect with the larger doping concentration. The qualitative features at low temperatures remain the same as that for the surface resistance. The data for zinc doped samples presented in this work have also been fitted to the d-wave model by Hirschfeld et al.\(^{85} \) Although overall qualitative agreement is obtained, there are also obvious difficulties remaining with regard to the details at low temperatures, as for pure crystals.

3) Untwinned crystals

A large anisotropy in the surface resistance within the \( ab \) plane of the untwinned crystals was revealed for the first time by our study. The qualitative features in the two directions are similar. However, the surface resistance in the \( a \) direction is found to be remarkably larger than that in the \( b \) direction, and at low temperatures, the surface resistance in both directions is very linear, with the \( R_{sa} \) curve about 1.8 times steeper than that of \( R_{sb} \). The data on untwinned YBCO crystals also show a very small residual surface resistance, about 100 \( \mu \Omega \) at 1.3K at 35 GHz. This result indicates that the twin boundaries might contribute to the \( R_s \) of twinned crystals, where a typical residual \( R_s \) is about 300 \( \mu \Omega \).
Chapter 7. Conclusions

The real part of the conductivity was extracted using recently obtained values of \( \lambda_a(T) \) and \( \lambda_b(T) \). We found \( \sigma_{ib} \) to be about twice as large as \( \sigma_{ia} \), similar to the normal state conductivity anisotropy. It is interesting to note that the large anisotropy in the penetration depth causes \( R_{aa} > R_{ab} \), in spite of the fact that \( \sigma_{ia} < \sigma_{ib} \). At low temperatures the \( \sigma_i \)'s also exhibit linear behavior in both directions. If we fit our averaged low temperature conductivity to a straight line, we obtain residual conductivities of \( \sigma_1(T \rightarrow 0) \equiv 0.6 \pm 0.2 \times 10^6 \Omega^{-1} m^{-1} \). The residual conductivity is near our resolution limit and close to the predicted \( \sigma_1(0) \equiv 0.3 \sigma_{1, DC} (T_c) \) for a d-wave superconductor by Lee and Hirschfeld.\(^{45,103}\) The experiments on twin free crystals not only allowed a determination of the anisotropic microwave properties of YBa$_2$Cu$_3$O$_{6.95}$ in the \( ab \) plane for the first time, but they have also clearly demonstrated that the unusual features of the surface resistance in our twinned samples were not caused by twin boundaries. However, we concluded that much of the residual conductivity previously observed was in fact caused by twin boundaries.

The broad peak in the surface resistance was later confirmed by other groups on YBCO single crystals, also using the cavity perturbation technique(Mao et al.\(^{36}\), Achkîr et al.\(^{45}\)). The plateau in the surface resistance observed in some thin film samples (Ma et al.\(^{32}\)) and also in 0.15% zinc doped crystals can be well explained as due to a somewhat suppressed peak in \( \sigma_1 \), caused by impurity scattering. Even stronger impurity scattering can completely suppress the broad peak, as observed in some thin film samples(Ma et al.\(^{32}\), Klein et al.\(^{39}\)) and also in our 0.31% zinc and 0.75% nickel doped crystals.
Finally, let us conclude the project by the following:

1. For undoped crystals there is a broad peak in both the surface resistance and the real part of the conductivity. The peak is attributed to the rapid drop of the quasiparticle scattering rate in the superconducting state.

2. The position of the peak in $\sigma_1$ at 34 GHz is at higher temperature in comparison with that at 3.8 GHz, which further confirms the rapid drop of the scattering rate.

3. As a result of the low scattering rate in the superconducting state, the $R_s$ and $\sigma_1$ are extremely sensitive to a small amount of impurities, which has also been confirmed by our measurements on doped crystals.

4. The temperature dependence of $\sigma_1$ clearly does not show an isotropic s-wave behavior, for which the absorption would exponentially decrease at low temperatures, and would also have a coherence peak right below $T_c$. The overall shape of the real part of the conductivity agrees with the calculation from a d-wave model; however, there are obvious difficulties for the theory with regard to the details for the low temperature behavior. The averaged quasiparticle scattering rate derived from the two fluid model was explained by a d-wave model of spin fluctuations in the region $0.5 T_c < T < T_c$.

5. The residual real part of the conductivity extrapolated to zero temperature is quite small, and is roughly equal to two-fifths of its value at $T_c$, close to the value predicted by Lee$^{103}$ and Hirschfeld$^{85}$ for a d-wave superconductor.

6. Our study also provides information on how one might produce low loss, high
quality thin films for commercial applications. In particular, the microwave loss of high temperature superconducting thin films may be improved by slightly doping the material with impurities. Because the doping effect is most effective at a temperature relatively far away from the transition temperature, materials with a $T_c$ higher than that of YBCO such as the TI based compounds may be better candidates for reduction in $R_s$ by doping.

7. A large anisotropy in electrodynamic responses exists within the $ab$ plane. The differences in magnitude can largely be subsumed under an anisotropy in $n/m^*$, with $m^*$ being the effective mass. Such an anisotropy explains most of the difference in the magnitudes of $\sigma_{1a}$ and $\sigma_{1b}$ below $T_c$ and is also consistent with the overall size of the dc resistivity anisotropy above $T_c$.

8. The linearity in both the surface resistance and the conductivity at low temperatures in both $a$ and $b$ directions indicates that the CuO chains are not likely to be the sole source of the low-lying states responsible for the linear $T$ dependencies. All of the qualitative features of the electrodynamics are the same in the $a$ and $b$ directions, it is the magnitudes that differ. The presence of the CuO chains may lead to a large anisotropy in $n/m^*$ without affecting any of the qualitative features of the electrodynamics below $T_c$.

Our study of the microwave surface resistance of YBCO based materials reveals important clues to the pairing mechanism of the new superconducting system. The results
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generally favor a d-wave pairing state. However, there is disagreement with regard to the
detailed low temperature behavior of $\sigma_1$ between the d-wave calculation and the
experimental data, indicating that something may be missing in the model used in the
calculation.
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Appendix A

In this appendix some basic relations between the surface impedance $Z$ and the conductivity, $\sigma$, the penetration depth, $\lambda$, are derived.

Starting with Maxwell's equations (omitting displacement current which is negligible comparing to $j$):

$$\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \times \mathbf{H} = j,$$

plus Ohm's law, $j = \sigma \mathbf{E}$, we obtain

$$\nabla^2 \mathbf{E} = \mu \sigma \frac{\partial \mathbf{E}}{\partial t}.$$  

The solutions are therefore of the form

$$E_x = E_{x0} e^{-kz+i\omega t},$$

$$H_{y0} = \frac{k}{\mu \omega i} E_{x0},$$

where $k = \sqrt{\mu \omega \sigma i}$ and $E_{x0}$ and $H_{y0}$ are the amplitudes of the fields at the surface of the material.

The surface impedance according to the definition 3.1.1 is then:
\[ Z = \frac{E_{x0}}{H_{y0}} = \frac{i \mu_0 \omega}{k} = \sqrt{\frac{i \mu_0 \omega}{\sigma}}, \]

where we let \( \mu = \mu_0 \) for non-magnetic metal. A3 is just equation 3.1.2.

The magnetic penetration depth is:

\[ \lambda = \int_0^\infty \frac{B_x}{B_{y0}} \, dz = \frac{\int_0^\infty \frac{1}{i \omega} \frac{\partial E_x}{\partial z} \, dz}{\mu_0 H_{y0}} = \frac{Z}{i \omega \mu_0}. \]

Above we have used A1, A2 and \( \frac{\partial B_y}{\partial t} = i \omega B_y \). A4 is just equation 3.1.5.
Appendix B

We would like to relate the elements of the scattering matrix of the network discussed in chapter 4 to the voltage transmission coefficient $T$ and the reflection voltage ratio $\Gamma$.

For the resonant network between $aa$ and $bb$ in Figure 4.2, we denote the voltages and the currents flowing to the network at $aa$ and $bb$ to be $V_a$, $V_b$, $I_a$, and $I_b$. The scattering matrix is defined by the equation\(^{105}\)

\[
\begin{pmatrix}
  c_a \\
  d_b
\end{pmatrix} = \begin{pmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{pmatrix} \begin{pmatrix}
  e_a \\
  f_b
\end{pmatrix},
\]

where $c_a$ and $d_b$ are the reflection parameters and $e_a$ and $f_b$ are the incident parameters at ports $aa$ and $bb$ correspondingly, and are defined by

\[
c_a = \frac{1}{2} \left( \frac{V_a}{\sqrt{Z_0}} - \sqrt{Z_0} I_a \right),
\]

\[
d_b = \frac{1}{2} \left( \frac{V_b}{\sqrt{Z_0}} - \sqrt{Z_0} I_b \right),
\]

\[
e_a = \frac{1}{2} \left( \frac{V_a}{\sqrt{Z_0}} + \sqrt{Z_0} I_a \right),
\]

\[
f_b = \frac{1}{2} \left( \frac{V_b}{\sqrt{Z_0}} + \sqrt{Z_0} I_b \right),
\]

where $Z_0$ is the characteristic impedance of the input and output transmission line. If the reflection parameter at a certain port is zero, then there is no reflection from this port. Similarly, a zero incident parameter at certain port indicates that there is no incident voltage.
at the port. For example, if the incident parameter \( f_b = 0 \), one has \( V_b = -Z_0 I_b \). Noting that \( I_b \) is defined *flowing* into the network at \( bb \), this condition corresponds to the situation where the transmission line connected to port \( bb \) is terminated by its characteristic impedance, i.e., there is only an outgoing wave at position \( bb \). \( S_{11} \) can then be expressed as

\[
S_{11} = \left. \frac{c_a}{e_a} \right|_{f_a=0} = \frac{V_a}{\sqrt{Z_0}} - \frac{\sqrt{Z_0} I_a}{V} = \frac{V_a}{\sqrt{Z_0}} + \frac{\sqrt{Z_0} I_a}{V} = \frac{Z - Z_0}{Z + Z_0} = \Gamma,
\]

where \( Z = V_a / I_a \) is the input impedance at \( aa \). \( S_{21} \) can be found in a similar way:

\[
S_{21} = \left. \frac{d_b}{e_a} \right|_{f_a=0} = \frac{V_b}{\sqrt{Z_0}} - \frac{\sqrt{Z_0} I_b}{V} = \frac{V_b}{\sqrt{Z_0}} + \frac{\sqrt{Z_0} I_a}{V} = \frac{2V_b}{V_a(1 + Z_0/Z)} = \frac{V_{\text{out}}}{V_{\text{in}}} = T.
\]

Above we have used the relation between \( V_a \) and the incoming voltage \( V_{\text{in}} \) at port \( aa \), \( V_{\text{in}} = V_a \frac{Z + Z_0}{2Z} \), and the relation between \( V_b \) and output voltage \( V_{\text{out}} \) at port \( bb \), \( V_b = V_{\text{out}} \).

From the symmetry of the network, one has \( S_{22} = S_{11} = \Gamma \), and \( S_{12} = S_{21} = T \).
Appendix C

Here we would like to find the relation between the quality factor and the surface resistance of the walls of a cylindrical cavity operated at $TE_{011}$ mode.

The loss per unit time of the cavity is

$$L = \oint_{S_z} \frac{1}{2} |H|^2 R_s ds,$$

where $R_s$ is the surface resistance and $H$ is the amplitude of the magnetic field given by equation 4.2.3. First let us consider one of the bottom plates with $z=0$. In this case the amplitude of the magnetic field is

$$H_r = \frac{\pi}{kh} A J_0'(kr),$$
$$H_\theta = H_z = 0.$$

Using the formula for Bessel functions:

$$\int_0^\infty J_0^2 (kr)rdr = \frac{b^2}{2} J_0^2(\chi),$$

the loss due to the bottom plate per unit time $L_{bp}$ is

$$L_{bp} = \frac{\pi^2 b^2 R_s}{2 k^2 h^2} |A|^2 J_0^2(\chi).$$
The amplitude of the magnetic field on the cylindrical wall is

\[ H_r = H_\theta = 0, \]
\[ H_z = A_2 J_0(\chi) \sin \frac{\pi z}{h}. \]

The loss due to the cylindrical wall is then

\[ L_{cw} = \frac{1}{2} R_s \pi b h |A|^2 J_0^2(\chi). \]  

The total loss is \(2L_{dy} + L_{cw}\). The quality factor of the resonant cavity can be derived from the energy stored and the loss per unit time from equation 4.2.5, C2, and C3\textsuperscript{106}:

\[
Q = \omega_0 \frac{\mu_0 V_0}{2(1 - \frac{\chi^2}{\lambda_0^2})(\frac{1}{4} \pi b h + \frac{\pi^2 b^2}{k^2 h^2})R_s} = \frac{V_0 \omega_0^2 \varepsilon_0 \mu_0^2}{\pi(\frac{\chi^2 h}{b} + \frac{2\pi^2 b^2}{h^2})R_s}.
\]

Above we have used the relation:

\[ k^2 = \omega_0^2 \varepsilon_0 \mu_0 - \frac{\pi^2}{h^2} = \omega_0^2 \varepsilon_0 \mu_0 (1 - \frac{\lambda_0^2}{4h^2}). \]

Equation C4 is just equation 4.2.6.